Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/299-6.2.1

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**183**]. This is test number [299].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (183)	0.00 (0)
Mathematica	99.45 (182)	0.55 (1)
Fricas	81.97 (150)	18.03 (33)
Maxima	78.14 (143)	21.86 (40)
Maple	60.66 (111)	39.34 (72)
Giac	56.28 (103)	43.72 (80)
Mupad	38.25 (70)	61.75 (113)
Reduce	33.33 (61)	66.67 (122)
Sympy	33.33 (61)	66.67 (122)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

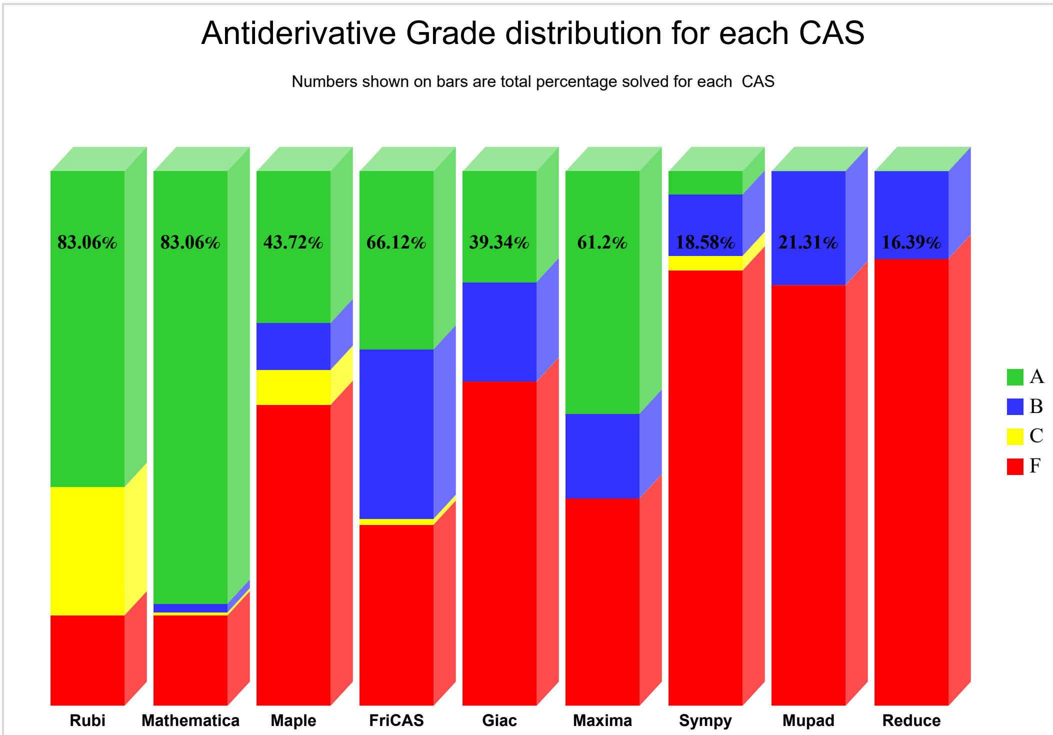
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

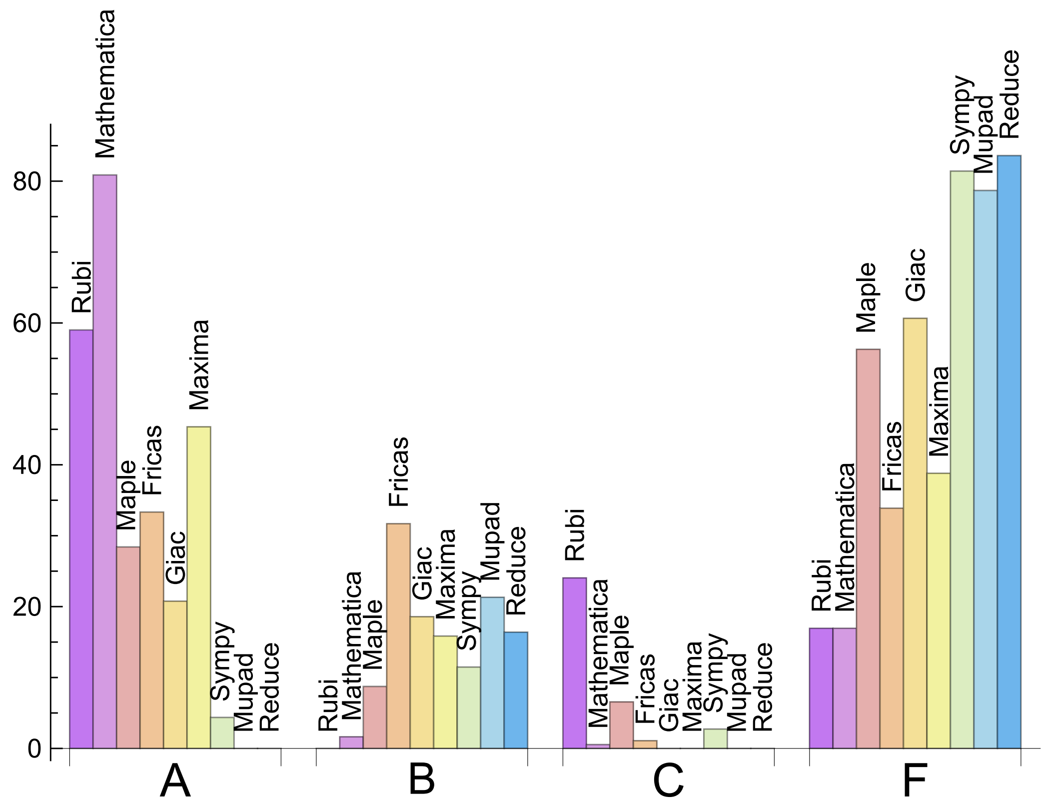
System	% A grade	% B grade	% C grade	% F grade
Mathematica	80.874	1.639	0.546	16.940
Rubi	59.016	0.000	24.044	16.940
Maxima	45.355	15.847	0.000	38.798
Fricas	33.333	31.694	1.093	33.880
Maple	28.415	8.743	6.557	56.284
Giac	20.765	18.579	0.000	60.656
Sympy	4.372	11.475	2.732	81.421
Mupad	0.000	21.311	0.000	78.689
Reduce	0.000	16.393	0.000	83.607

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	1	0.00	100.00	0.00
Fricas	33	18.18	0.00	81.82
Maxima	40	80.00	0.00	20.00
Maple	72	100.00	0.00	0.00
Giac	80	100.00	0.00	0.00
Mupad	113	0.00	100.00	0.00
Sympy	122	75.41	13.11	11.48
Reduce	122	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Maxima	0.15
Reduce	0.19
Giac	0.45
Rubi	0.56
Maple	0.71
Mupad	1.59
Mathematica	2.48
Sympy	3.31

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	93.54	1.32	45.50	1.12
Rubi	127.96	1.03	96.00	1.00
Maple	130.12	1.35	81.00	1.11
Sympy	158.89	1.80	76.00	1.47
Maxima	160.73	2.52	116.00	1.10
Mathematica	187.51	1.07	78.50	0.92
Giac	192.99	1.90	107.00	1.15
Fricas	472.70	2.77	168.00	1.87
Reduce	1270.52	57.57	108.00	1.96

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

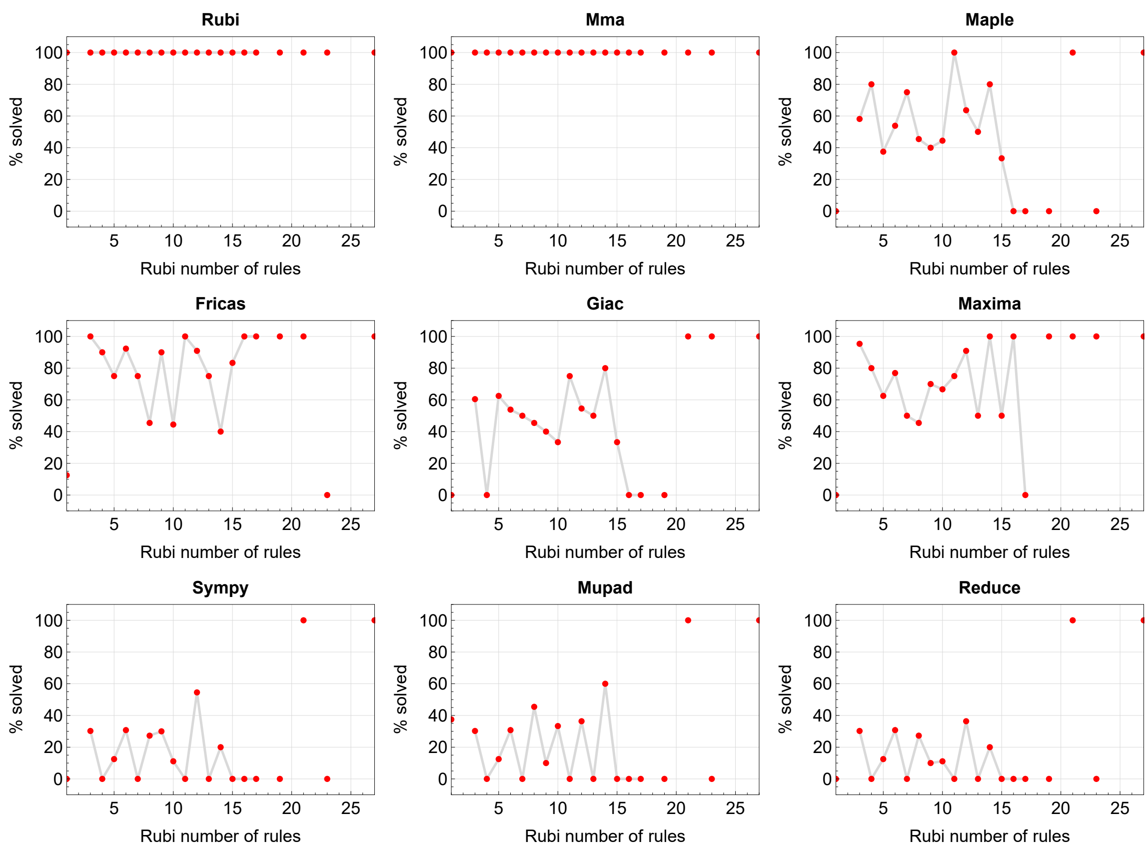


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

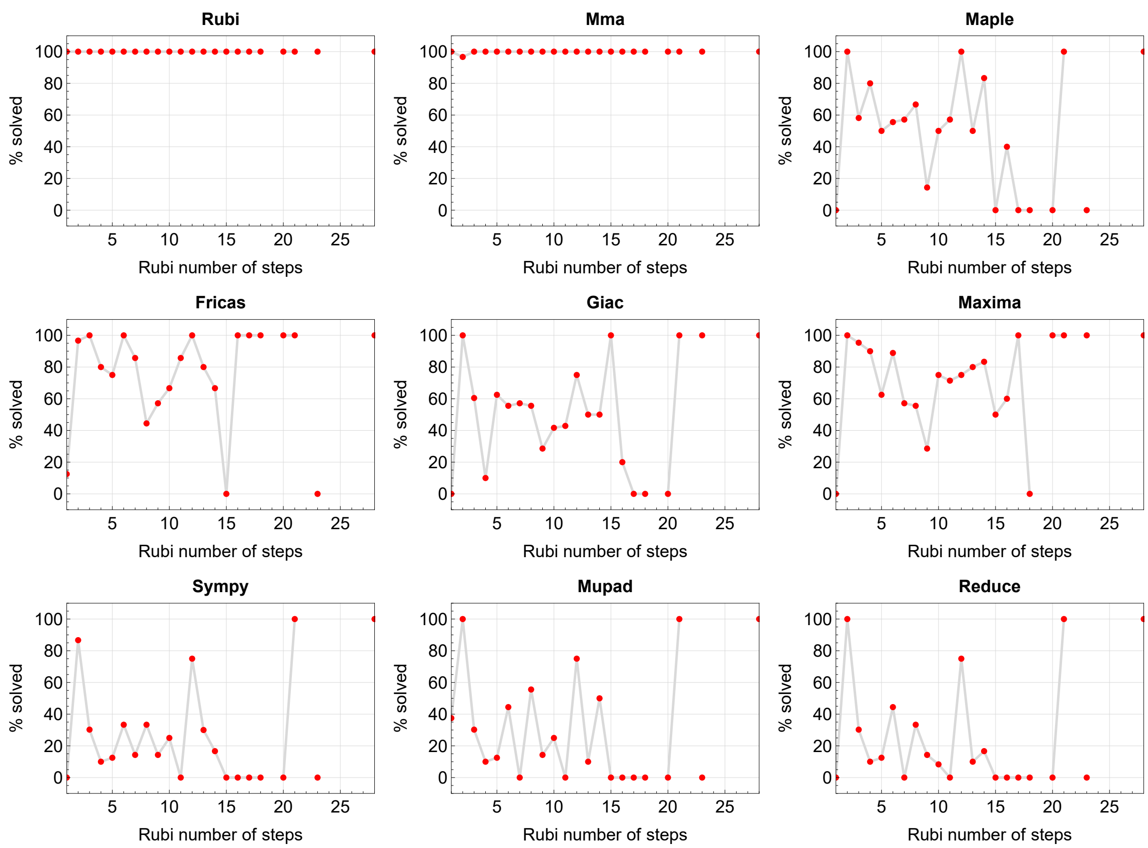


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

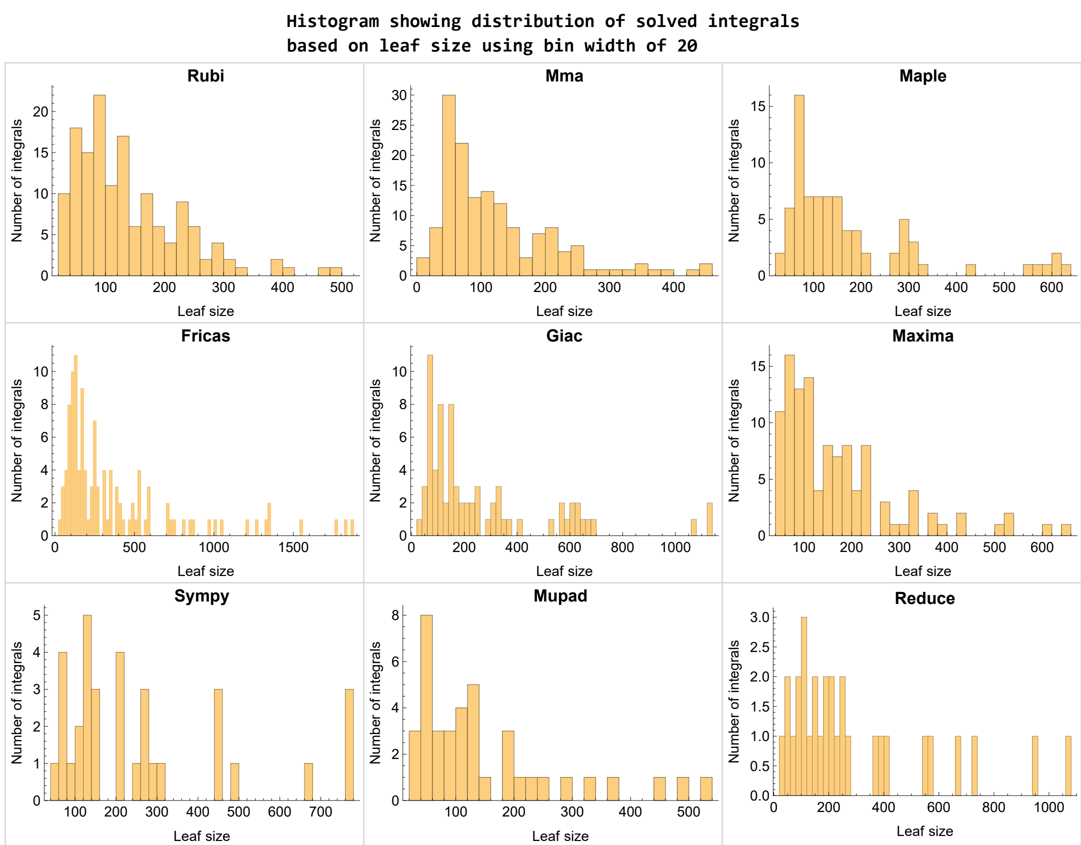


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

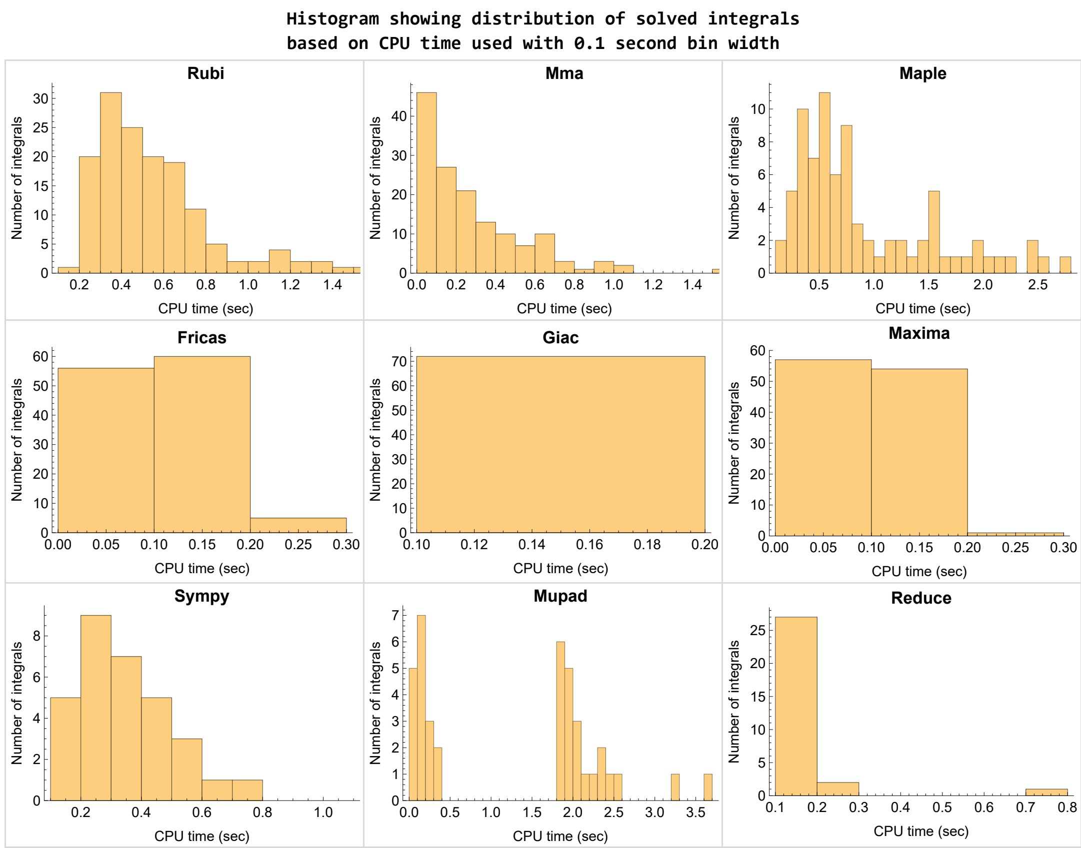


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

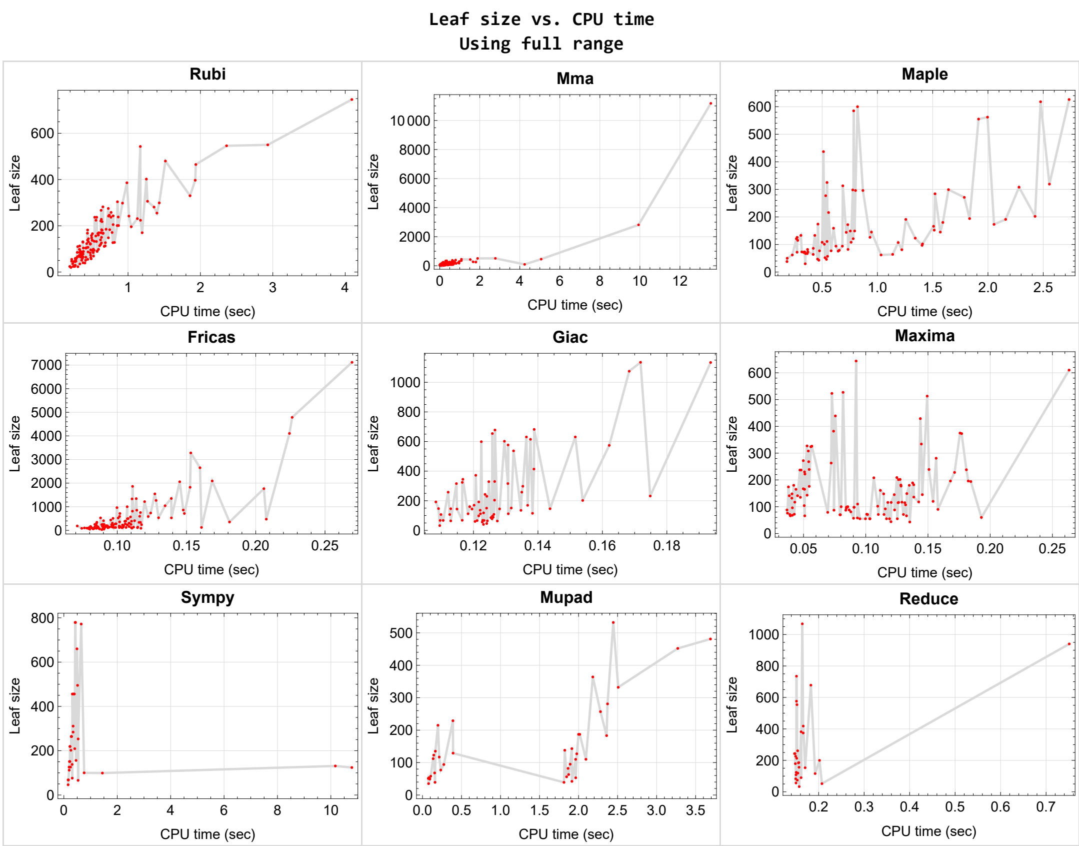


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{29, 30, 34, 35, 39, 40, 68, 69, 70, 75, 79, 80, 114, 115, 119, 120, 142, 143, 147, 148, 149, 150, 154, 155, 171, 172, 176, 177, 178, 182, 183\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{\}$

Mathematica $\{71, 73, 151, 152, 174, 175\}$

Maple $\{100, 101\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

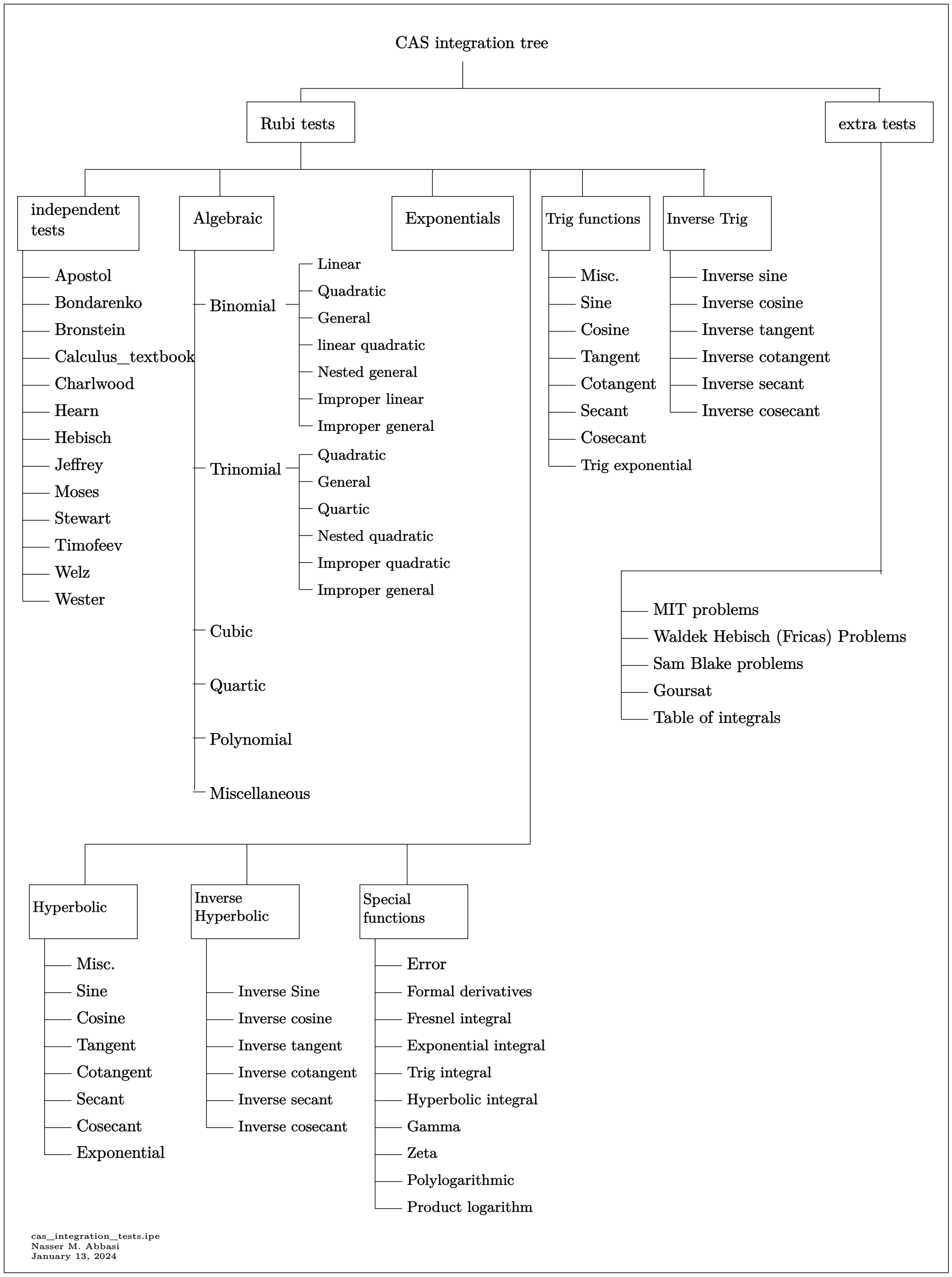
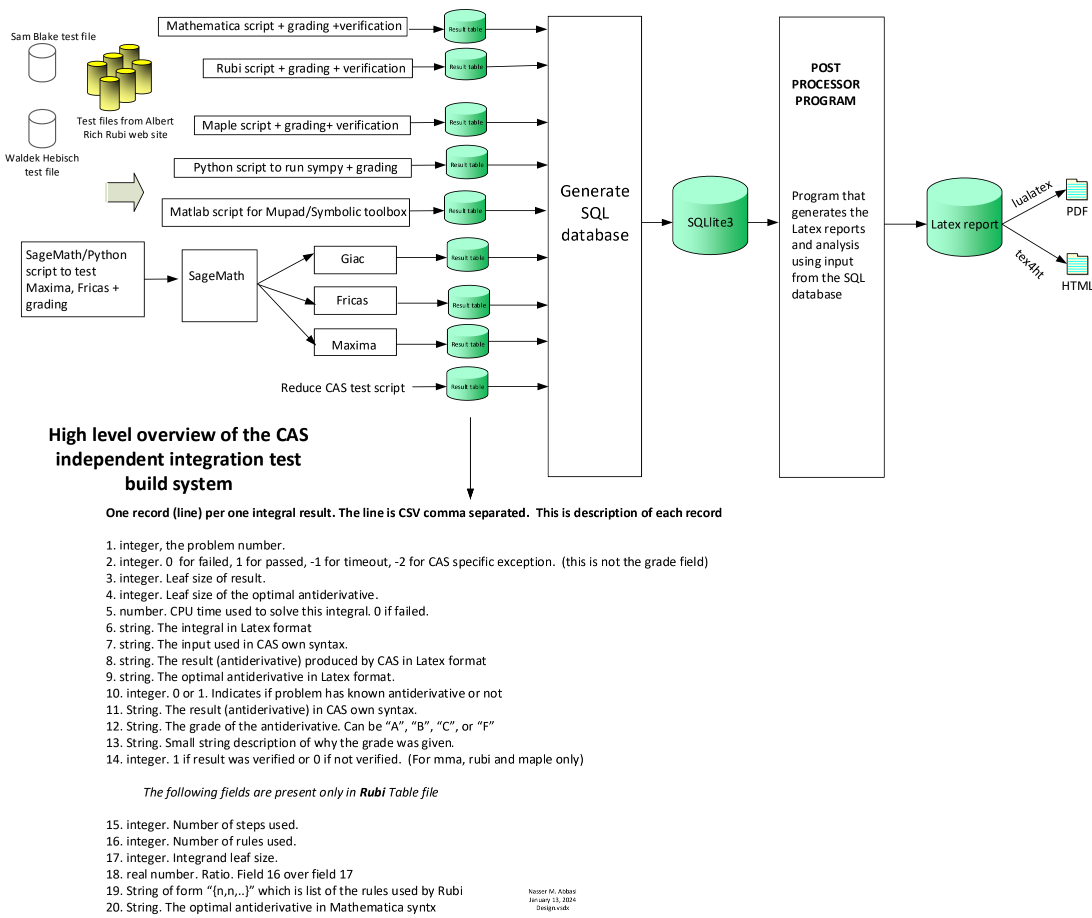


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 4, 5, 8, 9, 10, 11, 12, 14, 19, 20, 22, 23, 24, 25, 26, 27, 28, 33, 36, 37, 38, 44, 48, 49, 50, 51, 53, 55, 58, 59, 61, 65, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 113, 118, 123, 124, 129, 130, 131, 135, 136, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

B grade { }

C grade { 1, 2, 3, 6, 7, 13, 15, 16, 17, 18, 21, 31, 32, 41, 42, 43, 45, 46, 47, 52, 54, 56, 57, 60, 62, 63, 64, 66, 67, 109, 111, 112, 116, 117, 121, 122, 125, 126, 127, 128, 132, 133, 134, 137 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 31, 32, 33, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 72, 73, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 175, 179, 180, 181 }

B grade { 71, 173, 174 }

C grade { 74 }

F normal fail { }

F(-1) timedout fail { 40 }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 16, 17, 18, 19, 20, 21, 23, 24, 25, 28, 33, 99, 100, 101, 102, 103, 105, 106, 107, 108, 109, 113, 117, 118, 121, 122, 123, 127, 128, 129, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166 }

B grade { 7, 14, 15, 22, 31, 32, 38, 104, 110, 111, 112, 116, 161, 167, 170, 175 }

C grade { 63, 64, 65, 66, 67, 81, 82, 83, 84, 85, 86, 87 }

F normal fail { 26, 27, 36, 37, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 71, 72, 73, 74, 76, 77, 78, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 173, 174, 179, 180, 181 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 9, 10, 11, 12, 18, 19, 20, 23, 24, 25, 44, 51, 59, 65, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 99, 100, 101, 102, 103, 105, 106, 107, 108, 151, 152, 153, 156, 157, 158, 159, 160, 162, 163, 164, 165, 166, 179, 180, 181 }

B grade { 6, 7, 8, 13, 14, 15, 16, 17, 21, 22, 26, 27, 28, 33, 36, 37, 38, 41, 42, 43, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 60, 61, 62, 63, 64, 66, 67, 72, 104, 109, 110, 111, 112, 113, 116, 117, 118, 161, 167, 168, 169, 170, 173, 174, 175 }

C grade { 31, 32 }

F normal fail { 139, 140, 141, 144, 145, 146 }

F(-1) timedout fail { }

F(-2) exception fail { 70, 71, 73, 74, 95, 96, 97, 98, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 149 }

Maxima

A grade { 5, 6, 7, 10, 11, 12, 13, 14, 15, 20, 21, 22, 23, 24, 25, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 107, 108, 109, 110, 113, 121, 122, 123, 127, 128, 129, 133, 134, 135, 151, 152, 153, 158, 159, 160, 161, 163, 164, 165, 166, 167, 179, 180, 181 }

B grade { 1, 2, 3, 4, 8, 9, 16, 17, 18, 19, 31, 33, 41, 42, 43, 44, 63, 64, 65, 99, 100, 105, 106, 111, 116, 118, 156, 157, 162 }

C grade { }

F normal fail { 26, 27, 28, 32, 36, 37, 38, 71, 72, 73, 74, 95, 96, 97, 98, 112, 117, 124, 125, 126, 130, 131, 132, 136, 137, 138, 139, 140, 141, 144, 145, 146 }

F(-1) timedout fail { }

F(-2) exception fail { 93, 94, 168, 169, 170, 173, 174, 175 }

Giac

A grade { 4, 5, 10, 11, 12, 19, 20, 23, 24, 25, 41, 42, 43, 44, 51, 63, 64, 65, 101, 102, 107, 108, 113, 121, 122, 123, 124, 125, 126, 133, 134, 135, 136, 137, 158, 159, 164, 165 }

B grade { 1, 2, 3, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 21, 22, 33, 99, 100, 103, 104, 105, 106, 109, 110, 118, 138, 156, 157, 160, 161, 162, 163, 166, 167 }

C grade { }

F normal fail { 26, 27, 28, 31, 32, 36, 37, 38, 45, 46, 47, 48, 49, 50, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 66, 67, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 111, 112, 116, 117, 127, 128, 129, 130, 131, 132, 139, 140, 141, 144, 145, 146, 151, 152, 153, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

F(-1) timedout fail { }

F(-2) exception fail { }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 23, 24, 25, 33, 71, 72, 73, 99, 100, 101, 105, 106, 107, 113, 118, 121, 122, 123, 127, 128, 129, 156, 157, 158, 162, 163, 164 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 111, 112, 116, 117, 124, 125, 126, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 159, 160, 161, 165, 166, 167, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

F(-2) exception fail { }

Sympy

A grade { 4, 19, 23, 24, 25, 101, 118, 158 }

B grade { 1, 2, 3, 8, 9, 10, 11, 16, 17, 18, 99, 100, 105, 106, 107, 113, 156, 157, 162, 163, 164 }

C grade { 63, 64, 65, 66, 67 }

F normal fail { 5, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 33, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 57, 58, 59, 60, 61, 62, 71, 72, 74, 76, 77, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 108, 109, 110, 111, 112, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 159, 165, 166, 167, 168, 169, 170 }

F(-1) timedout fail { 6, 7, 55, 56, 73, 103, 104, 150, 160, 161, 173, 174, 175, 176, 177, 178 }

F(-2) exception fail { 78, 81, 82, 83, 84, 85, 86, 87, 151, 152, 153, 179, 180, 181 }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 8, 9, 10, 11, 16, 17, 18, 19, 23, 24, 25, 33, 99, 100, 101, 105, 106, 107, 113, 118, 156, 157, 158, 162, 163, 164 }

C grade { }

F normal fail { 5, 6, 7, 12, 13, 14, 15, 20, 21, 22, 26, 27, 28, 31, 32, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 76, 77, 78, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 102, 103, 104, 108, 109, 110, 111, 112, 116, 117, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 151, 152, 153, 159, 160, 161, 165, 166, 167, 168, 169, 170, 173, 174, 175, 179, 180, 181 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	114	76	144	326	171	311	324	243	215
N.S.	1	1.25	0.84	1.58	3.58	1.88	3.42	3.56	2.67	2.36
time (sec)	N/A	0.608	0.177	0.718	0.057	0.092	0.346	0.117	0.147	0.198

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	83	61	111	222	111	202	204	153	143
N.S.	1	1.19	0.87	1.59	3.17	1.59	2.89	2.91	2.19	2.04
time (sec)	N/A	0.477	0.115	0.544	0.051	0.086	0.264	0.127	0.170	1.914

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	44	77	135	64	112	112	83	82
N.S.	1	1.20	0.90	1.57	2.76	1.31	2.29	2.29	1.69	1.67
time (sec)	N/A	0.369	0.087	0.476	0.047	0.083	0.200	0.118	0.150	1.865

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	30	68	30	46	46	33	35
N.S.	1	1.00	0.96	1.07	2.43	1.07	1.64	1.64	1.18	1.25
time (sec)	N/A	0.264	0.068	0.347	0.041	0.089	0.159	0.124	0.157	0.078

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	82	57	94	0	56	16	0
N.S.	1	1.00	0.96	1.61	1.12	1.84	0.00	1.10	0.31	0.00
time (sec)	N/A	0.415	0.057	0.368	0.090	0.078	0.000	0.123	0.154	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	84	65	133	81	150	0	615	76	0
N.S.	1	1.18	0.92	1.87	1.14	2.11	0.00	8.66	1.07	0.00
time (sec)	N/A	0.519	0.162	0.434	0.088	0.081	0.000	0.138	0.153	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	88	277	95	254	0	298	103	0
N.S.	1	1.07	0.85	2.66	0.91	2.44	0.00	2.87	0.99	0.00
time (sec)	N/A	0.671	0.325	0.532	0.084	0.090	0.000	0.135	0.178	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	167	132	145	382	312	660	372	576	332
N.S.	1	1.03	0.81	0.90	2.36	1.93	4.07	2.30	3.56	2.05
time (sec)	N/A	0.583	0.364	0.946	0.074	0.085	0.491	0.121	0.151	2.508

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	129	104	121	263	209	456	243	382	229
N.S.	1	1.04	0.84	0.98	2.12	1.69	3.68	1.96	3.08	1.85
time (sec)	N/A	0.378	0.264	0.781	0.072	0.085	0.393	0.124	0.161	0.390

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	97	75	79	165	123	264	136	226	127
N.S.	1	1.02	0.79	0.83	1.74	1.29	2.78	1.43	2.38	1.34
time (sec)	N/A	0.333	0.184	0.655	0.044	0.098	0.275	0.122	0.150	1.977

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	51	52	88	66	126	63	108	58
N.S.	1	1.00	0.93	0.95	1.60	1.20	2.29	1.15	1.96	1.05
time (sec)	N/A	0.222	0.166	0.526	0.037	0.082	0.196	0.127	0.150	0.105

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	64	97	72	104	0	68	18	0
N.S.	1	1.00	0.82	1.24	0.92	1.33	0.00	0.87	0.23	0.00
time (sec)	N/A	0.379	0.157	1.404	0.101	0.076	0.000	0.123	0.158	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	94	75	152	88	164	0	574	198	0
N.S.	1	1.16	0.93	1.88	1.09	2.02	0.00	7.09	2.44	0.00
time (sec)	N/A	0.532	0.270	1.514	0.087	0.081	0.000	0.162	0.149	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	148	102	299	99	278	0	330	408	0
N.S.	1	1.32	0.91	2.67	0.88	2.48	0.00	2.95	3.64	0.00
time (sec)	N/A	0.497	0.606	1.643	0.085	0.099	0.000	0.127	0.163	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	171	121	555	110	409	0	537	688	0
N.S.	1	1.06	0.75	3.43	0.68	2.52	0.00	3.31	4.25	0.00
time (sec)	N/A	0.780	0.554	1.915	0.093	0.090	0.000	0.132	0.166	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	330	158	173	644	528	772	654	1068	532
N.S.	1	1.47	0.70	0.77	2.86	2.35	3.43	2.91	4.75	2.36
time (sec)	N/A	1.856	0.730	2.055	0.092	0.088	0.650	0.126	0.164	2.445

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	224	122	284	439	343	495	414	678	364
N.S.	1	1.28	0.70	1.62	2.51	1.96	2.83	2.37	3.87	2.08
time (sec)	N/A	1.172	0.635	1.521	0.076	0.085	0.513	0.139	0.183	2.183

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	150	93	191	272	199	284	230	374	183
N.S.	1	1.22	0.76	1.55	2.21	1.62	2.31	1.87	3.04	1.49
time (sec)	N/A	0.633	0.360	1.257	0.050	0.092	0.345	0.124	0.166	2.359

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	52	62	143	95	126	98	156	77
N.S.	1	1.00	0.69	0.83	1.91	1.27	1.68	1.31	2.08	1.03
time (sec)	N/A	0.366	0.183	1.033	0.053	0.082	0.249	0.122	0.157	0.234

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	166	117	186	0	112	18	0
N.S.	1	1.00	0.84	1.37	0.97	1.54	0.00	0.93	0.15	0.00
time (sec)	N/A	0.477	0.257	1.508	0.129	0.071	0.000	0.131	0.157	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	158	196	271	145	305	0	1075	167	0
N.S.	1	1.09	1.35	1.87	1.00	2.10	0.00	7.41	1.15	0.00
time (sec)	N/A	0.495	0.442	1.786	0.145	0.109	0.000	0.168	0.165	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	242	218	562	145	527	0	602	226	0
N.S.	1	1.32	1.18	3.05	0.79	2.86	0.00	3.27	1.23	0.00
time (sec)	N/A	1.012	0.590	1.996	0.123	0.139	0.000	0.130	0.165	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	237	100	102	176	195	253	150	243	129
N.S.	1	1.38	0.58	0.59	1.02	1.13	1.47	0.87	1.41	0.75
time (sec)	N/A	0.764	0.278	1.408	0.055	0.105	0.535	0.124	0.148	0.392

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	181	90	81	132	147	209	118	179	94
N.S.	1	1.35	0.67	0.60	0.99	1.10	1.56	0.88	1.34	0.70
time (sec)	N/A	0.642	0.109	1.221	0.041	0.086	0.407	0.123	0.149	0.273

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	85	53	64	96	114	138	86	119	68
N.S.	1	1.06	0.66	0.80	1.20	1.42	1.72	1.08	1.49	0.85
time (sec)	N/A	0.299	0.123	1.138	0.041	0.110	0.309	0.125	0.154	0.155

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	179	185	343	0	0	497	0	0	71	0
N.S.	1	1.03	1.92	0.00	0.00	2.78	0.00	0.00	0.40	0.00
time (sec)	N/A	0.697	0.346	0.000	0.000	0.108	0.000	0.000	0.183	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	121	199	0	0	305	0	0	50	0
N.S.	1	1.02	1.67	0.00	0.00	2.56	0.00	0.00	0.42	0.00
time (sec)	N/A	0.481	0.202	0.000	0.000	0.096	0.000	0.000	0.169	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	85	101	0	157	0	0	29	0
N.S.	1	1.00	1.39	1.66	0.00	2.57	0.00	0.00	0.48	0.00
time (sec)	N/A	0.288	0.057	0.520	0.000	0.092	0.000	0.000	0.149	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	12	16	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	0.86	1.14	1.14	1.29
time (sec)	N/A	0.202	2.692	0.137	0.320	0.086	0.463	0.282	0.163	1.871

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	27	14	16	27	18
N.S.	1	1.00	1.14	1.00	1.14	1.93	1.00	1.14	1.93	1.29
time (sec)	N/A	0.202	5.910	0.142	0.351	0.087	0.666	1.439	0.157	1.905

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	127	145	298	238	1332	0	0	543	0
N.S.	1	1.23	1.41	2.89	2.31	12.93	0.00	0.00	5.27	0.00
time (sec)	N/A	0.687	0.609	0.777	0.181	0.111	0.000	0.000	0.163	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	93	91	159	0	715	0	0	261	0
N.S.	1	1.27	1.25	2.18	0.00	9.79	0.00	0.00	3.58	0.00
time (sec)	N/A	0.504	0.469	0.603	0.000	0.102	0.000	0.000	0.160	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	51	57	72	161	0	78	90	50
N.S.	1	1.00	1.76	1.97	2.48	5.55	0.00	2.69	3.10	1.72
time (sec)	N/A	0.272	0.086	0.549	0.043	0.109	0.000	0.126	0.161	0.094

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	102	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	6.38	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.225	25.298	0.119	0.143	0.088	0.485	0.114	0.173	1.926

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	156	29	15	18	29	18
N.S.	1	1.00	1.12	1.00	9.75	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.230	21.722	0.134	0.172	0.086	0.677	0.175	0.146	1.964

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	296	306	455	0	0	4785	0	0	0	0
N.S.	1	1.03	1.54	0.00	0.00	16.17	0.00	0.00	0.00	0.00
time (sec)	N/A	1.269	5.068	0.000	0.000	0.226	0.000	0.000	0.160	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	175	183	270	0	0	2651	0	0	919	0
N.S.	1	1.05	1.54	0.00	0.00	15.15	0.00	0.00	5.25	0.00
time (sec)	N/A	0.747	1.667	0.000	0.000	0.160	0.000	0.000	0.159	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	104	156	216	0	1267	0	0	383	0
N.S.	1	1.02	1.53	2.12	0.00	12.42	0.00	0.00	3.75	0.00
time (sec)	N/A	0.418	0.465	0.558	0.000	0.128	0.000	0.000	0.157	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	321	18	14	18	18	18
N.S.	1	1.00	1.12	1.00	20.06	1.12	0.88	1.12	1.12	1.12
time (sec)	N/A	0.223	141.939	0.127	0.360	0.079	0.496	2.012	0.165	1.938

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	0	16	405	29	15	18	29	18
N.S.	1	1.00	0.00	1.00	25.31	1.81	0.94	1.12	1.81	1.12
time (sec)	N/A	0.224	0.000	0.131	0.418	0.093	0.695	26.676	0.164	1.998

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	171	201	107	0	308	523	0	232	62	0
N.S.	1	1.18	0.63	0.00	1.80	3.06	0.00	1.36	0.36	0.00
time (sec)	N/A	0.851	0.041	0.000	0.054	0.101	0.000	0.175	0.163	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	146	167	107	0	268	387	0	202	36	0
N.S.	1	1.14	0.73	0.00	1.84	2.65	0.00	1.38	0.25	0.00
time (sec)	N/A	0.652	0.066	0.000	0.054	0.110	0.000	0.154	0.159	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	123	138	105	0	230	302	0	169	15	0
N.S.	1	1.12	0.85	0.00	1.87	2.46	0.00	1.37	0.12	0.00
time (sec)	N/A	0.505	0.051	0.000	0.051	0.108	0.000	0.137	0.156	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	105	0	180	123	0	89	17	0
N.S.	1	1.00	1.01	0.00	1.73	1.18	0.00	0.86	0.16	0.00
time (sec)	N/A	0.383	0.027	0.000	0.042	0.106	0.000	0.126	0.166	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	119	137	118	0	104	338	0	0	29	0
N.S.	1	1.15	0.99	0.00	0.87	2.84	0.00	0.00	0.24	0.00
time (sec)	N/A	0.507	0.241	0.000	0.051	0.116	0.000	0.000	0.156	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	149	168	150	0	115	534	0	0	46	0
N.S.	1	1.13	1.01	0.00	0.77	3.58	0.00	0.00	0.31	0.00
time (sec)	N/A	0.658	0.429	0.000	0.126	0.130	0.000	0.000	0.186	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	205	191	0	115	853	0	0	63	0
N.S.	1	1.18	1.10	0.00	0.66	4.90	0.00	0.00	0.36	0.00
time (sec)	N/A	0.794	0.270	0.000	0.131	0.114	0.000	0.000	0.160	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	246	137	0	281	1001	0	0	68	0
N.S.	1	1.03	0.57	0.00	1.18	4.19	0.00	0.00	0.28	0.00
time (sec)	N/A	0.741	0.310	0.000	0.157	0.107	0.000	0.000	0.180	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	211	218	137	0	239	755	0	0	40	0
N.S.	1	1.03	0.65	0.00	1.13	3.58	0.00	0.00	0.19	0.00
time (sec)	N/A	0.641	0.150	0.000	0.151	0.117	0.000	0.000	0.169	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	129	0	189	590	0	0	17	0
N.S.	1	1.00	0.78	0.00	1.14	3.55	0.00	0.00	0.10	0.00
time (sec)	N/A	0.508	0.287	0.000	0.138	0.102	0.000	0.000	0.173	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	141	0	107	155	0	115	19	0
N.S.	1	1.00	1.02	0.00	0.78	1.12	0.00	0.83	0.14	0.00
time (sec)	N/A	0.441	0.095	0.000	0.132	0.102	0.000	0.138	0.157	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	142	160	152	0	116	569	0	0	31	0
N.S.	1	1.13	1.07	0.00	0.82	4.01	0.00	0.00	0.22	0.00
time (sec)	N/A	0.591	0.253	0.000	0.126	0.110	0.000	0.000	0.160	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	174	221	156	0	118	861	0	0	48	0
N.S.	1	1.27	0.90	0.00	0.68	4.95	0.00	0.00	0.28	0.00
time (sec)	N/A	0.624	0.819	0.000	0.143	0.148	0.000	0.000	0.150	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	220	243	204	0	116	1350	0	0	65	0
N.S.	1	1.10	0.93	0.00	0.53	6.14	0.00	0.00	0.30	0.00
time (sec)	N/A	0.795	0.282	0.000	0.120	0.139	0.000	0.000	0.166	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	251	304	222	0	116	1825	0	0	82	0
N.S.	1	1.21	0.88	0.00	0.46	7.27	0.00	0.00	0.33	0.00
time (sec)	N/A	0.854	0.516	0.000	0.118	0.153	0.000	0.000	0.153	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	381	546	195	0	513	2092	0	0	68	0
N.S.	1	1.43	0.51	0.00	1.35	5.49	0.00	0.00	0.18	0.00
time (sec)	N/A	2.362	0.352	0.000	0.149	0.169	0.000	0.000	0.191	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	326	465	209	0	429	1545	0	0	40	0
N.S.	1	1.43	0.64	0.00	1.32	4.74	0.00	0.00	0.12	0.00
time (sec)	N/A	1.937	0.321	0.000	0.144	0.127	0.000	0.000	0.178	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	210	0	334	1217	0	0	17	0
N.S.	1	1.00	0.76	0.00	1.21	4.43	0.00	0.00	0.06	0.00
time (sec)	N/A	0.728	0.211	0.000	0.145	0.120	0.000	0.000	0.158	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	192	0	177	253	0	0	19	0
N.S.	1	1.00	0.84	0.00	0.78	1.11	0.00	0.00	0.08	0.00
time (sec)	N/A	0.631	0.168	0.000	0.128	0.110	0.000	0.000	0.171	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	246	267	246	0	196	1344	0	0	31	0
N.S.	1	1.09	1.00	0.00	0.80	5.46	0.00	0.00	0.13	0.00
time (sec)	N/A	0.628	0.575	0.000	0.182	0.114	0.000	0.000	0.154	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	277	402	253	0	194	2058	0	0	48	0
N.S.	1	1.45	0.91	0.00	0.70	7.43	0.00	0.00	0.17	0.00
time (sec)	N/A	1.253	1.805	0.000	0.184	0.145	0.000	0.000	0.158	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	331	480	376	0	196	3280	0	0	65	0
N.S.	1	1.45	1.14	0.00	0.59	9.91	0.00	0.00	0.20	0.00
time (sec)	N/A	1.516	0.630	0.000	0.168	0.153	0.000	0.000	0.158	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	B	B	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	132	51	133	174	191	131	145	40	0
N.S.	1	1.19	0.46	1.20	1.57	1.72	1.18	1.31	0.36	0.00
time (sec)	N/A	0.563	0.012	0.308	0.038	0.093	10.168	0.128	0.154	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	B	B	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	107	48	121	148	138	100	108	76	0
N.S.	1	1.16	0.52	1.32	1.61	1.50	1.09	1.17	0.83	0.00
time (sec)	N/A	0.447	0.010	0.267	0.041	0.084	0.759	0.126	0.156	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	B	A	C	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	48	72	117	59	66	60	16	0
N.S.	1	1.00	0.62	0.94	1.52	0.77	0.86	0.78	0.21	0.00
time (sec)	N/A	0.349	0.008	0.273	0.043	0.087	0.533	0.120	0.161	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	106	67	115	76	136	99	0	22	0
N.S.	1	1.20	0.76	1.31	0.86	1.55	1.12	0.00	0.25	0.00
time (sec)	N/A	0.434	0.027	0.279	0.037	0.115	1.443	0.000	0.157	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	C	A	B	C	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	133	78	126	58	179	124	0	22	0
N.S.	1	1.17	0.68	1.11	0.51	1.57	1.09	0.00	0.19	0.00
time (sec)	N/A	0.534	0.069	0.274	0.093	0.101	10.786	0.000	0.155	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	15	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	0.94	1.12
time (sec)	N/A	0.238	3.063	0.212	0.849	0.088	1.204	0.152	0.158	1.901

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	14	16	16	15	16	17	18
N.S.	1	1.00	1.12	0.88	1.00	1.00	0.94	1.00	1.06	1.12
time (sec)	N/A	0.230	4.461	0.227	0.928	0.107	0.884	0.129	0.169	1.907

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	8	10	0	10	10	11	10
N.S.	1	1.00	1.20	0.80	1.00	0.00	1.00	1.00	1.10	1.00
time (sec)	N/A	0.348	2.441	0.069	0.168	0.000	40.170	0.127	0.160	1.820

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0	19	39
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.95	1.95
time (sec)	N/A	0.211	0.293	0.000	0.000	0.000	0.000	0.000	0.171	0.161

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	109	0	0	25	42
N.S.	1	1.00	0.67	0.00	0.00	4.54	0.00	0.00	1.04	1.75
time (sec)	N/A	0.195	0.073	0.000	0.000	0.079	0.000	0.000	0.161	1.915

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0	21	110
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.45	2.34
time (sec)	N/A	0.221	0.457	0.000	0.000	0.000	0.000	0.000	0.159	2.094

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0	23	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.236	0.656	0.000	0.000	0.000	0.000	0.000	0.181	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	20	20	17	20	22	20
N.S.	1	1.00	1.11	1.00	1.11	1.11	0.94	1.11	1.22	1.11
time (sec)	N/A	0.237	2.091	0.131	0.206	0.093	8.984	0.149	0.166	1.986

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	205	0	161	340	0	0	249	0
N.S.	1	1.00	0.86	0.00	0.68	1.43	0.00	0.00	1.05	0.00
time (sec)	N/A	0.567	0.135	0.000	0.112	0.117	0.000	0.000	0.158	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	132	0	102	241	0	0	317	0
N.S.	1	1.00	0.92	0.00	0.71	1.67	0.00	0.00	2.20	0.00
time (sec)	N/A	0.453	0.141	0.000	0.110	0.118	0.000	0.000	0.159	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	0	79	168	0	0	113	0
N.S.	1	1.00	0.93	0.00	0.72	1.53	0.00	0.00	1.03	0.00
time (sec)	N/A	0.305	0.041	0.000	0.069	0.089	0.000	0.000	0.169	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	16	16	14	16	16	18
N.S.	1	1.00	1.14	1.00	1.14	1.14	1.00	1.14	1.14	1.29
time (sec)	N/A	0.206	4.239	0.093	0.139	0.110	1.949	0.116	0.153	1.899

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	18	18	15	18	153	18
N.S.	1	1.00	1.12	1.00	1.12	1.12	0.94	1.12	9.56	1.12
time (sec)	N/A	0.215	2.739	0.107	0.171	0.100	4.279	0.114	0.184	1.945

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	226	0
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	3.83	0.00
time (sec)	N/A	0.280	0.028	0.342	0.095	0.117	0.000	0.000	0.162	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	320	0
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	5.42	0.00
time (sec)	N/A	0.285	0.015	0.371	0.103	0.084	0.000	0.000	0.161	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	86	0	0	76	0
N.S.	1	1.00	0.92	1.24	0.93	1.46	0.00	0.00	1.29	0.00
time (sec)	N/A	0.285	0.023	0.337	0.100	0.085	0.000	0.000	0.177	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	54	73	55	78	0	0	83	0
N.S.	1	1.00	0.92	1.24	0.93	1.32	0.00	0.00	1.41	0.00
time (sec)	N/A	0.271	0.012	0.316	0.103	0.079	0.000	0.000	0.208	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	49	67	43	78	0	0	15	0
N.S.	1	1.00	1.00	1.37	0.88	1.59	0.00	0.00	0.31	0.00
time (sec)	N/A	0.272	0.016	0.350	0.135	0.090	0.000	0.000	0.193	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	52	67	55	86	0	0	15	0
N.S.	1	1.00	0.95	1.22	1.00	1.56	0.00	0.00	0.27	0.00
time (sec)	N/A	0.276	0.015	0.365	0.119	0.087	0.000	0.000	0.179	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	71	55	86	0	0	15	0
N.S.	1	1.00	0.93	1.20	0.93	1.46	0.00	0.00	0.25	0.00
time (sec)	N/A	0.283	0.019	0.334	0.125	0.074	0.000	0.000	0.214	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	728	0
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	8.47	0.00
time (sec)	N/A	0.367	0.075	0.000	0.115	0.096	0.000	0.000	0.191	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	78	0	71	136	0	0	510	0
N.S.	1	1.00	0.92	0.00	0.84	1.60	0.00	0.00	6.00	0.00
time (sec)	N/A	0.351	0.069	0.000	0.101	0.104	0.000	0.000	0.204	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	79	0	71	136	0	0	330	0
N.S.	1	1.00	0.92	0.00	0.83	1.58	0.00	0.00	3.84	0.00
time (sec)	N/A	0.345	0.073	0.000	0.110	0.094	0.000	0.000	0.209	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	76	0	71	122	0	0	190	0
N.S.	1	1.00	0.89	0.00	0.84	1.44	0.00	0.00	2.24	0.00
time (sec)	N/A	0.329	0.059	0.000	0.130	0.099	0.000	0.000	0.186	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	64	0	55	117	0	0	17	0
N.S.	1	1.00	0.89	0.00	0.76	1.62	0.00	0.00	0.24	0.00
time (sec)	N/A	0.327	0.043	0.000	0.117	0.101	0.000	0.000	0.186	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	73	0	0	136	0	0	17	0
N.S.	1	1.00	0.88	0.00	0.00	1.64	0.00	0.00	0.20	0.00
time (sec)	N/A	0.348	0.068	0.000	0.000	0.091	0.000	0.000	0.183	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	84	0	0	136	0	0	17	0
N.S.	1	1.00	1.00	0.00	0.00	1.62	0.00	0.00	0.20	0.00
time (sec)	N/A	0.373	0.073	0.000	0.000	0.112	0.000	0.000	0.212	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	21	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	0.88	0.00
time (sec)	N/A	0.263	0.056	0.000	0.000	0.000	0.000	0.000	0.224	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	17	0	0	0	0	0	25	0
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	1.04	0.00
time (sec)	N/A	0.243	0.194	0.000	0.000	0.000	0.000	0.000	0.193	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	0	0	0	0	0	21	0
N.S.	1	1.00	0.96	0.00	0.00	0.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.281	0.083	0.000	0.000	0.000	0.000	0.000	0.182	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	0	0	0	0	0	25	0
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	0.38	0.00
time (sec)	N/A	0.355	0.095	0.000	0.000	0.000	0.000	0.000	0.191	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	122	172	237	168	264	258	200	187
N.S.	1	1.00	1.37	1.93	2.66	1.89	2.97	2.90	2.25	2.10
time (sec)	N/A	0.384	0.333	0.732	0.048	0.093	0.287	0.135	0.202	2.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	80	77	141	102	151	146	116	112
N.S.	1	1.00	1.19	1.15	2.10	1.52	2.25	2.18	1.73	1.67
time (sec)	N/A	0.324	0.214	0.581	0.045	0.091	0.209	0.144	0.192	0.137

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	52	43	66	51	68	64	52	53
N.S.	1	1.00	1.16	0.96	1.47	1.13	1.51	1.42	1.16	1.18
time (sec)	N/A	0.271	0.400	0.470	0.039	0.090	0.158	0.124	0.207	0.081

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	54	94	70	111	0	67	30	0
N.S.	1	1.00	0.84	1.47	1.09	1.73	0.00	1.05	0.47	0.00
time (sec)	N/A	0.417	0.122	0.683	0.079	0.103	0.000	0.123	0.210	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	68	149	87	162	0	631	175	0
N.S.	1	1.00	0.78	1.71	1.00	1.86	0.00	7.25	2.01	0.00
time (sec)	N/A	0.448	0.259	0.794	0.074	0.090	0.000	0.152	0.156	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	90	296	98	274	0	316	355	0
N.S.	1	1.00	0.73	2.41	0.80	2.23	0.00	2.57	2.89	0.00
time (sec)	N/A	0.479	0.355	0.869	0.091	0.106	0.000	0.131	0.160	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	217	180	527	395	779	577	735	452
N.S.	1	1.00	0.97	0.80	2.35	1.76	3.48	2.58	3.28	2.02
time (sec)	N/A	0.557	0.928	1.592	0.082	0.106	0.423	0.131	0.151	3.271

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	192	123	327	227	456	329	418	257
N.S.	1	1.00	1.14	0.73	1.95	1.35	2.71	1.96	2.49	1.53
time (sec)	N/A	0.439	0.407	1.342	0.053	0.097	0.361	0.125	0.166	2.281

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	81	126	167	113	219	151	186	123
N.S.	1	1.00	0.83	1.29	1.70	1.15	2.23	1.54	1.90	1.26
time (sec)	N/A	0.328	0.586	0.933	0.050	0.115	0.229	0.132	0.156	0.145

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	139	113	191	149	228	0	135	53	0
N.S.	1	0.96	0.78	1.32	1.03	1.57	0.00	0.93	0.37	0.00
time (sec)	N/A	0.620	0.498	2.160	0.133	0.104	0.000	0.135	0.158	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	170	207	308	182	359	0	1134	369	0
N.S.	1	1.08	1.32	1.96	1.16	2.29	0.00	7.22	2.35	0.00
time (sec)	N/A	0.596	0.606	2.281	0.139	0.102	0.000	0.194	0.170	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	298	353	618	202	596	0	682	753	0
N.S.	1	1.44	1.71	2.99	0.98	2.88	0.00	3.29	3.64	0.00
time (sec)	N/A	0.924	1.079	2.476	0.128	0.116	0.000	0.139	0.175	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	128	178	325	228	438	0	0	476	0
N.S.	1	1.09	1.52	2.78	1.95	3.74	0.00	0.00	4.07	0.00
time (sec)	N/A	0.812	0.964	0.544	0.171	0.095	0.000	0.000	0.164	0.000

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	98	133	174	0	243	0	0	225	0
N.S.	1	1.11	1.51	1.98	0.00	2.76	0.00	0.00	2.56	0.00
time (sec)	N/A	0.592	0.616	0.462	0.000	0.089	0.000	0.000	0.159	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	51	70	47	71	92	76	66	74	53
N.S.	1	1.04	1.43	0.96	1.45	1.88	1.55	1.35	1.51	1.08
time (sec)	N/A	0.359	0.558	0.459	0.039	0.111	0.313	0.121	0.153	1.963

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	98	27	27	22	30	22
N.S.	1	1.00	1.10	1.00	4.90	1.35	1.35	1.10	1.50	1.10
time (sec)	N/A	0.245	7.586	0.235	0.171	0.087	1.065	0.115	0.162	1.898

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	154	51	58	22	119	22
N.S.	1	1.00	1.10	1.00	7.70	2.55	2.90	1.10	5.95	1.10
time (sec)	N/A	0.240	5.968	0.234	0.214	0.084	1.834	0.168	0.169	1.909

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	506	600	610	1863	0	0	1485	0
N.S.	1	1.00	1.98	2.35	2.39	7.31	0.00	0.00	5.82	0.00
time (sec)	N/A	1.399	1.892	0.820	0.263	0.111	0.000	0.000	0.173	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	196	295	313	0	963	0	0	669	0
N.S.	1	0.98	1.48	1.56	0.00	4.82	0.00	0.00	3.34	0.00
time (sec)	N/A	1.045	0.977	0.687	0.000	0.100	0.000	0.000	0.166	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	119	114	82	239	385	156	192	184	138
N.S.	1	0.97	0.93	0.67	1.94	3.13	1.27	1.56	1.50	1.12
time (sec)	N/A	0.621	0.663	0.734	0.055	0.105	0.456	0.108	0.156	1.823

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	593	57	54	22	53	22
N.S.	1	1.00	1.10	1.00	29.65	2.85	2.70	1.10	2.65	1.10
time (sec)	N/A	0.277	20.004	0.420	0.392	0.088	1.754	0.120	0.163	1.911

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	710	99	105	22	100	22
N.S.	1	1.00	1.10	1.00	35.50	4.95	5.25	1.10	5.00	1.10
time (sec)	N/A	0.282	21.124	0.404	0.589	0.090	4.323	0.227	0.176	1.999

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	126	53	108	120	0	0	147	18	117
N.S.	1	1.15	0.48	0.98	1.09	0.00	0.00	1.34	0.16	1.06
time (sec)	N/A	0.632	0.209	0.500	0.154	0.000	0.000	0.109	0.156	0.214

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	99	44	86	90	0	0	107	18	95
N.S.	1	1.12	0.50	0.98	1.02	0.00	0.00	1.22	0.20	1.08
time (sec)	N/A	0.520	0.155	0.420	0.158	0.000	0.000	0.110	0.156	1.893

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	66	34	64	60	0	0	67	16	56
N.S.	1	1.25	0.64	1.21	1.13	0.00	0.00	1.26	0.30	1.06
time (sec)	N/A	0.378	0.251	0.419	0.193	0.000	0.000	0.111	0.155	1.847

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	57	54	0	0	0	0	32	18	0
N.S.	1	0.69	0.65	0.00	0.00	0.00	0.00	0.39	0.22	0.00
time (sec)	N/A	0.491	0.192	0.000	0.000	0.000	0.000	0.110	0.156	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	110	89	75	0	0	0	0	68	52	0
N.S.	1	0.81	0.68	0.00	0.00	0.00	0.00	0.62	0.47	0.00
time (sec)	N/A	0.573	0.138	0.000	0.000	0.000	0.000	0.117	0.162	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	151	113	97	0	0	0	0	107	18	0
N.S.	1	0.75	0.64	0.00	0.00	0.00	0.00	0.71	0.12	0.00
time (sec)	N/A	0.685	0.180	0.000	0.000	0.000	0.000	0.113	0.162	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	69	33	62	88	0	0	0	14	63
N.S.	1	1.01	0.49	0.91	1.29	0.00	0.00	0.00	0.21	0.93
time (sec)	N/A	0.524	0.083	0.230	0.123	0.000	0.000	0.000	0.159	1.873

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	55	31	50	66	0	0	0	14	51
N.S.	1	1.04	0.58	0.94	1.25	0.00	0.00	0.00	0.26	0.96
time (sec)	N/A	0.444	0.065	0.184	0.133	0.000	0.000	0.000	0.157	0.074

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	F(-2)	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	35	22	38	44	0	0	0	12	39
N.S.	1	1.09	0.69	1.19	1.38	0.00	0.00	0.00	0.38	1.22
time (sec)	N/A	0.335	0.048	0.182	0.120	0.000	0.000	0.000	0.158	1.813

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	23	0	0	0	0	0	14	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.61	0.00
time (sec)	N/A	0.308	0.007	0.000	0.000	0.000	0.000	0.000	0.153	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	39	33	0	0	0	0	0	35	0
N.S.	1	0.93	0.79	0.00	0.00	0.00	0.00	0.00	0.83	0.00
time (sec)	N/A	0.404	0.074	0.000	0.000	0.000	0.000	0.000	0.157	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	67	63	44	0	0	0	0	0	14	0
N.S.	1	0.94	0.66	0.00	0.00	0.00	0.00	0.00	0.21	0.00
time (sec)	N/A	0.486	0.083	0.000	0.000	0.000	0.000	0.000	0.166	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	185	170	70	0	180	0	0	192	29	0
N.S.	1	0.92	0.38	0.00	0.97	0.00	0.00	1.04	0.16	0.00
time (sec)	N/A	1.195	0.240	0.000	0.135	0.000	0.000	0.122	0.167	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	145	127	54	0	136	0	0	144	29	0
N.S.	1	0.88	0.37	0.00	0.94	0.00	0.00	0.99	0.20	0.00
time (sec)	N/A	0.791	0.178	0.000	0.139	0.000	0.000	0.115	0.169	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	73	56	0	92	0	0	96	25	0
N.S.	1	0.82	0.63	0.00	1.03	0.00	0.00	1.08	0.28	0.00
time (sec)	N/A	0.482	0.085	0.000	0.116	0.000	0.000	0.126	0.169	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	55	40	36	0	0	0	0	40	29	0
N.S.	1	0.73	0.65	0.00	0.00	0.00	0.00	0.73	0.53	0.00
time (sec)	N/A	0.379	0.016	0.000	0.000	0.000	0.000	0.123	0.158	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	F(-2)	F	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	79	64	53	0	0	0	0	112	52	0
N.S.	1	0.81	0.67	0.00	0.00	0.00	0.00	1.42	0.66	0.00
time (sec)	N/A	0.375	0.092	0.000	0.000	0.000	0.000	0.122	0.163	0.000

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	109	91	69	0	0	0	0	170	29	0
N.S.	1	0.83	0.63	0.00	0.00	0.00	0.00	1.56	0.27	0.00
time (sec)	N/A	0.574	0.047	0.000	0.000	0.000	0.000	0.120	0.160	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	383	238	213	0	0	0	0	0	31	0
N.S.	1	0.62	0.56	0.00	0.00	0.00	0.00	0.00	0.08	0.00
time (sec)	N/A	0.864	0.231	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	269	170	163	0	0	0	0	0	31	0
N.S.	1	0.63	0.61	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.626	0.172	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	157	104	139	0	0	0	0	0	29	0
N.S.	1	0.66	0.89	0.00	0.00	0.00	0.00	0.00	0.18	0.00
time (sec)	N/A	0.418	0.325	0.000	0.000	0.000	0.000	0.000	0.161	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	30	17	18	30	18
N.S.	1	1.00	1.11	0.89	1.00	1.67	0.94	1.00	1.67	1.00
time (sec)	N/A	0.252	2.549	0.184	0.196	0.090	1.875	0.473	0.159	1.862

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	34	19	18	34	18
N.S.	1	1.00	1.11	0.89	1.00	1.89	1.06	1.00	1.89	1.00
time (sec)	N/A	0.254	1.222	0.187	0.198	0.092	3.552	1.258	0.158	1.866

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	402	231	234	0	0	0	0	0	29	0
N.S.	1	0.57	0.58	0.00	0.00	0.00	0.00	0.00	0.07	0.00
time (sec)	N/A	1.129	0.646	0.000	0.000	0.000	0.000	0.000	0.160	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	248	148	161	0	0	0	0	0	29	0
N.S.	1	0.60	0.65	0.00	0.00	0.00	0.00	0.00	0.12	0.00
time (sec)	N/A	0.728	0.354	0.000	0.000	0.000	0.000	0.000	0.156	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	140	96	110	0	0	0	0	0	27	0
N.S.	1	0.69	0.79	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.484	0.141	0.000	0.000	0.000	0.000	0.000	0.159	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	36	14	14	29	14
N.S.	1	1.00	1.14	0.86	1.00	2.57	1.00	1.00	2.07	1.00
time (sec)	N/A	0.284	7.771	0.132	0.178	0.130	8.243	0.112	0.159	2.018

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	12	14	42	15	14	35	14
N.S.	1	1.00	1.14	0.86	1.00	3.00	1.07	1.00	2.50	1.00
time (sec)	N/A	0.292	8.714	0.131	0.202	0.106	14.150	0.115	0.169	2.008

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	16	18	0	15	18	20	18
N.S.	1	1.00	1.11	0.89	1.00	0.00	0.83	1.00	1.11	1.00
time (sec)	N/A	0.278	2.327	0.192	0.234	0.000	1.618	0.166	0.165	1.908

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	0	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10	1.10
time (sec)	N/A	0.263	4.587	0.175	0.200	0.099	0.000	0.184	0.159	2.017

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	402	386	429	0	373	713	0	0	816	0
N.S.	1	0.96	1.07	0.00	0.93	1.77	0.00	0.00	2.03	0.00
time (sec)	N/A	0.985	1.523	0.000	0.177	0.148	0.000	0.000	0.177	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	263	258	302	0	209	493	0	0	560	0
N.S.	1	0.98	1.15	0.00	0.79	1.87	0.00	0.00	2.13	0.00
time (sec)	N/A	0.766	0.682	0.000	0.125	0.107	0.000	0.000	0.166	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	189	0	100	249	0	0	275	0
N.S.	1	1.00	1.44	0.00	0.76	1.90	0.00	0.00	2.10	0.00
time (sec)	N/A	0.453	0.219	0.000	0.080	0.090	0.000	0.000	0.160	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	133	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	6.65	1.10
time (sec)	N/A	0.280	3.376	0.158	0.137	0.091	1.497	0.110	0.175	1.978

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	39	29	22	4857	22
N.S.	1	1.00	1.10	1.00	1.10	1.95	1.45	1.10	242.85	1.10
time (sec)	N/A	0.265	6.600	0.333	0.201	0.088	11.246	0.120	0.264	2.020

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	123	108	237	168	264	258	213	187
N.S.	1	1.00	1.38	1.21	2.66	1.89	2.97	2.90	2.39	2.10
time (sec)	N/A	0.344	0.287	0.762	0.047	0.092	0.288	0.112	0.152	2.015

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	83	76	141	102	151	146	124	110
N.S.	1	1.00	1.24	1.13	2.10	1.52	2.25	2.18	1.85	1.64
time (sec)	N/A	0.302	0.181	0.645	0.039	0.090	0.227	0.113	0.151	1.963

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	46	46	66	51	68	64	56	49
N.S.	1	1.00	1.02	1.02	1.47	1.13	1.51	1.42	1.24	1.09
time (sec)	N/A	0.248	0.102	0.539	0.051	0.080	0.173	0.113	0.150	0.093

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	57	94	70	111	0	67	32	0
N.S.	1	1.00	0.89	1.47	1.09	1.73	0.00	1.05	0.50	0.00
time (sec)	N/A	0.352	0.098	0.625	0.080	0.093	0.000	0.110	0.152	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	71	149	87	162	0	631	179	0
N.S.	1	1.00	0.82	1.71	1.00	1.86	0.00	7.25	2.06	0.00
time (sec)	N/A	0.369	0.257	0.755	0.084	0.096	0.000	0.136	0.154	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	95	296	98	274	0	316	361	0
N.S.	1	1.00	0.77	2.41	0.80	2.23	0.00	2.57	2.93	0.00
time (sec)	N/A	0.429	0.386	0.802	0.091	0.099	0.000	0.115	0.155	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	232	194	523	409	779	599	940	481
N.S.	1	1.00	0.98	0.82	2.21	1.73	3.29	2.53	3.97	2.03
time (sec)	N/A	0.540	0.753	1.834	0.073	0.113	0.435	0.122	0.751	3.690

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	252	145	324	240	456	345	554	281
N.S.	1	1.00	1.38	0.80	1.78	1.32	2.51	1.90	3.04	1.54
time (sec)	N/A	0.442	0.584	1.569	0.056	0.096	0.318	0.117	0.152	2.371

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	96	107	165	122	219	160	261	135
N.S.	1	1.00	0.85	0.95	1.46	1.08	1.94	1.42	2.31	1.19
time (sec)	N/A	0.332	4.242	1.188	0.051	0.161	0.229	0.119	0.154	0.165

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	133	202	148	230	0	144	59	0
N.S.	1	1.00	0.85	1.29	0.95	1.47	0.00	0.92	0.38	0.00
time (sec)	N/A	0.555	0.200	2.426	0.114	0.107	0.000	0.120	0.156	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	233	319	181	355	0	1135	398	0
N.S.	1	1.00	1.27	1.74	0.99	1.94	0.00	6.20	2.17	0.00
time (sec)	N/A	0.635	0.406	2.556	0.128	0.181	0.000	0.172	0.156	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	394	626	201	586	0	678	791	0
N.S.	1	1.00	1.63	2.59	0.83	2.42	0.00	2.80	3.27	0.00
time (sec)	N/A	0.730	0.793	2.734	0.126	0.122	0.000	0.127	0.159	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	436	397	336	0	0	1042	0	0	350	0
N.S.	1	0.91	0.77	0.00	0.00	2.39	0.00	0.00	0.80	0.00
time (sec)	N/A	1.928	0.300	0.000	0.000	0.135	0.000	0.000	0.173	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	320	299	247	0	0	736	0	0	246	0
N.S.	1	0.93	0.77	0.00	0.00	2.30	0.00	0.00	0.77	0.00
time (sec)	N/A	1.431	0.172	0.000	0.000	0.124	0.000	0.000	0.159	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	201	152	437	0	473	0	0	144	0
N.S.	1	0.99	0.75	2.15	0.00	2.33	0.00	0.00	0.71	0.00
time (sec)	N/A	0.869	0.086	0.512	0.000	0.208	0.000	0.000	0.152	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	27	17	22	31	22
N.S.	1	1.00	1.10	1.00	1.10	1.35	0.85	1.10	1.55	1.10
time (sec)	N/A	0.266	0.811	0.186	0.193	0.154	6.041	0.127	0.152	1.943

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	51	19	22	126	22
N.S.	1	1.00	1.10	1.00	1.10	2.55	0.95	1.10	6.30	1.10
time (sec)	N/A	0.244	0.755	0.191	0.254	0.093	35.764	0.291	0.169	1.975

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	823	746	11178	0	0	7116	0	0	0	0
N.S.	1	0.91	13.58	0.00	0.00	8.65	0.00	0.00	0.00	0.00
time (sec)	N/A	4.092	13.543	0.000	0.000	0.270	0.000	0.000	0.233	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	593	550	2814	0	0	4105	0	0	0	0
N.S.	1	0.93	4.75	0.00	0.00	6.92	0.00	0.00	0.00	0.00
time (sec)	N/A	2.930	9.932	0.000	0.000	0.224	0.000	0.000	0.190	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	281	509	585	0	1765	0	0	0	0
N.S.	1	1.03	1.86	2.14	0.00	6.44	0.00	0.00	0.00	0.00
time (sec)	N/A	1.359	2.779	0.785	0.000	0.206	0.000	0.000	0.165	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	416	55	0	22	66	22
N.S.	1	1.00	1.10	1.00	20.80	2.75	0.00	1.10	3.30	1.10
time (sec)	N/A	0.274	25.012	0.347	0.597	0.199	0.000	0.398	0.168	2.078

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	622	96	0	22	121	22
N.S.	1	1.00	1.10	1.00	31.10	4.80	0.00	1.10	6.05	1.10
time (sec)	N/A	0.263	25.304	0.358	0.906	0.092	0.000	1.301	0.178	2.170

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	F(-1)	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	0	22	22	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.00	1.10	1.10	1.10
time (sec)	N/A	0.256	2.930	0.136	0.204	0.092	0.000	0.224	0.160	2.026

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	543	543	447	0	375	813	0	0	1232	0
N.S.	1	1.00	0.82	0.00	0.69	1.50	0.00	0.00	2.27	0.00
time (sec)	N/A	1.170	1.084	0.000	0.176	0.111	0.000	0.000	0.170	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	282	282	254	0	208	509	0	0	652	0
N.S.	1	1.00	0.90	0.00	0.74	1.80	0.00	0.00	2.31	0.00
time (sec)	N/A	0.653	0.482	0.000	0.106	0.097	0.000	0.000	0.158	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	A	F(-2)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	119	0	100	249	0	0	284	0
N.S.	1	1.00	0.91	0.00	0.76	1.90	0.00	0.00	2.17	0.00
time (sec)	N/A	0.376	0.120	0.000	0.086	0.084	0.000	0.000	0.152	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	22	17	22	46	22
N.S.	1	1.00	1.10	1.00	1.10	1.10	0.85	1.10	2.30	1.10
time (sec)	N/A	0.236	1.095	0.135	0.159	0.077	1.591	0.115	0.232	1.839

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	22	38	19	22	62433	22
N.S.	1	1.00	1.10	1.00	1.10	1.90	0.95	1.10	3121.65	1.10
time (sec)	N/A	0.256	4.833	0.329	0.175	0.071	13.242	0.124	1.125	1.998

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [16] had the largest ratio of [1.68750000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	14	14	1.25	14	1.000
2	C	12	12	1.19	14	0.857
3	C	8	8	1.20	14	0.571
4	A	6	6	1.00	12	0.500
5	A	7	7	1.00	14	0.500
6	C	11	11	1.18	14	0.786
7	C	13	13	1.07	14	0.929
8	A	9	9	1.03	16	0.562
9	A	6	6	1.04	16	0.375
10	A	6	6	1.02	16	0.375
11	A	3	3	1.00	14	0.214
12	A	3	3	1.00	16	0.188
13	C	11	11	1.16	16	0.688
14	A	6	6	1.32	16	0.375
15	C	14	14	1.06	16	0.875
16	C	28	27	1.47	16	1.688
17	C	21	21	1.28	16	1.312
18	C	13	12	1.22	16	0.750
19	A	8	8	1.00	14	0.571
20	A	3	3	1.00	16	0.188
21	C	3	3	1.09	16	0.188

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	11	11	1.32	16	0.688
23	A	12	12	1.38	12	1.000
24	A	12	12	1.35	12	1.000
25	A	5	5	1.06	10	0.500
26	A	7	6	1.03	14	0.429
27	A	6	5	1.02	14	0.357
28	A	5	4	1.00	12	0.333
29	N/A	2	0	1.00	14	0.000
30	N/A	2	0	1.00	14	0.000
31	C	11	10	1.23	16	0.625
32	C	10	9	1.27	16	0.562
33	A	6	6	1.00	14	0.429
34	N/A	2	0	1.00	16	0.000
35	N/A	2	0	1.00	16	0.000
36	A	11	10	1.03	16	0.625
37	A	9	8	1.05	16	0.500
38	A	7	6	1.02	14	0.429
39	N/A	2	0	1.00	16	0.000
40	N/A	2	0	1.00	16	0.000
41	C	16	15	1.18	16	0.938
42	C	13	12	1.14	16	0.750
43	C	10	9	1.12	16	0.562
44	A	7	6	1.00	16	0.375
45	C	10	9	1.15	16	0.562
46	C	13	12	1.13	16	0.750
47	C	16	15	1.18	16	0.938
48	A	6	6	1.03	18	0.333
49	A	6	6	1.03	18	0.333
50	A	3	3	1.00	18	0.167
51	A	3	3	1.00	18	0.167
52	C	10	9	1.13	18	0.500
53	A	6	6	1.27	18	0.333

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	C	13	12	1.10	18	0.667
55	A	9	9	1.21	18	0.500
56	C	20	19	1.43	18	1.056
57	C	17	16	1.43	18	0.889
58	A	3	3	1.00	18	0.167
59	A	3	3	1.00	18	0.167
60	C	3	3	1.09	18	0.167
61	A	11	10	1.45	18	0.556
62	C	14	13	1.45	18	0.722
63	C	13	12	1.19	12	1.000
64	C	10	9	1.16	12	0.750
65	A	7	6	1.00	12	0.500
66	C	10	9	1.20	12	0.750
67	C	13	12	1.17	12	1.000
68	N/A	2	0	1.00	16	0.000
69	N/A	2	0	1.00	16	0.000
70	N/A	4	0	1.00	10	0.000
71	A	1	1	1.00	17	0.059
72	A	1	1	1.00	20	0.050
73	A	1	1	1.00	20	0.050
74	A	1	1	1.00	21	0.048
75	N/A	2	0	1.00	18	0.000
76	A	3	3	1.00	16	0.188
77	A	3	3	1.00	16	0.188
78	A	4	4	1.00	14	0.286
79	N/A	2	0	1.00	14	0.000
80	N/A	2	0	1.00	16	0.000
81	A	4	4	1.00	12	0.333
82	A	4	4	1.00	12	0.333
83	A	4	4	1.00	12	0.333
84	A	4	4	1.00	10	0.400
85	A	4	4	1.00	12	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	4	1.00	12	0.333
87	A	4	4	1.00	12	0.333
88	A	3	3	1.00	14	0.214
89	A	3	3	1.00	14	0.214
90	A	3	3	1.00	14	0.214
91	A	3	3	1.00	12	0.250
92	A	3	3	1.00	14	0.214
93	A	3	3	1.00	14	0.214
94	A	3	3	1.00	14	0.214
95	A	1	1	1.00	20	0.050
96	A	1	1	1.00	20	0.050
97	A	1	1	1.00	20	0.050
98	A	1	1	1.00	24	0.042
99	A	3	3	1.00	18	0.167
100	A	3	3	1.00	18	0.167
101	A	3	3	1.00	16	0.188
102	A	3	3	1.00	18	0.167
103	A	3	3	1.00	18	0.167
104	A	3	3	1.00	18	0.167
105	A	3	3	1.00	20	0.150
106	A	3	3	1.00	20	0.150
107	A	3	3	1.00	18	0.167
108	A	5	5	0.96	20	0.250
109	C	5	5	1.08	20	0.250
110	A	7	7	1.44	20	0.350
111	C	13	12	1.09	20	0.600
112	C	12	11	1.11	20	0.550
113	A	8	8	1.04	18	0.444
114	N/A	2	0	1.00	20	0.000
115	N/A	2	0	1.00	20	0.000
116	C	16	15	1.00	20	0.750
117	C	16	15	0.98	20	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	10	10	0.97	18	0.556
119	N/A	2	0	1.00	20	0.000
120	N/A	2	0	1.00	20	0.000
121	C	14	14	1.15	18	0.778
122	C	10	10	1.12	18	0.556
123	A	8	8	1.25	16	0.500
124	A	9	9	0.69	18	0.500
125	C	13	13	0.81	18	0.722
126	C	15	15	0.75	18	0.833
127	C	14	14	1.01	14	1.000
128	C	10	10	1.04	14	0.714
129	A	8	8	1.09	12	0.667
130	A	4	4	1.00	14	0.286
131	A	8	8	0.93	14	0.571
132	C	10	10	0.94	14	0.714
133	C	23	23	0.92	14	1.643
134	C	15	14	0.88	14	1.000
135	A	10	10	0.82	12	0.833
136	A	5	5	0.73	14	0.357
137	C	5	5	0.81	14	0.357
138	A	8	8	0.83	14	0.571
139	A	9	8	0.62	18	0.444
140	A	8	7	0.63	18	0.389
141	A	7	6	0.66	16	0.375
142	N/A	2	0	1.00	18	0.000
143	N/A	2	0	1.00	18	0.000
144	A	13	12	0.57	14	0.857
145	A	11	10	0.60	14	0.714
146	A	9	8	0.69	12	0.667
147	N/A	2	0	1.00	14	0.000
148	N/A	2	0	1.00	14	0.000

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
149	N/A	2	0	1.00	18	0.000
150	N/A	2	0	1.00	20	0.000
151	A	5	5	0.96	20	0.250
152	A	5	5	0.98	20	0.250
153	A	3	3	1.00	18	0.167
154	N/A	2	0	1.00	20	0.000
155	N/A	2	0	1.00	20	0.000
156	A	3	3	1.00	18	0.167
157	A	3	3	1.00	18	0.167
158	A	3	3	1.00	16	0.188
159	A	3	3	1.00	18	0.167
160	A	3	3	1.00	18	0.167
161	A	3	3	1.00	18	0.167
162	A	3	3	1.00	20	0.150
163	A	3	3	1.00	20	0.150
164	A	3	3	1.00	18	0.167
165	A	3	3	1.00	20	0.150
166	A	3	3	1.00	20	0.150
167	A	3	3	1.00	20	0.150
168	A	10	9	0.91	20	0.450
169	A	9	8	0.93	20	0.400
170	A	8	7	0.99	18	0.389
171	N/A	2	0	1.00	20	0.000
172	N/A	2	0	1.00	20	0.000
173	A	18	17	0.91	20	0.850
174	A	16	15	0.93	20	0.750
175	A	14	13	1.03	18	0.722
176	N/A	2	0	1.00	20	0.000
177	N/A	2	0	1.00	20	0.000
178	N/A	2	0	1.00	20	0.000
179	A	3	3	1.00	20	0.150

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
180	A	3	3	1.00	20	0.150
181	A	3	3	1.00	18	0.167
182	N/A	2	0	1.00	20	0.000
183	N/A	2	0	1.00	20	0.000

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int (c+dx)^4 \cosh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 91

$$\int (c+dx)^4 \cosh(a+bx)\,dx = -\frac{24d^3(c+dx)\cosh(a+bx)}{b^4} - \frac{4d(c+dx)^3\cosh(a+bx)}{b^2} + \frac{24d^4\sinh(a+bx)}{b^5} + \frac{12d^2(c+dx)^2\sinh(a+bx)}{b^3} + \frac{(c+dx)^4\sinh(a+bx)}{b}$$

output
```
-24*d^3*(d*x+c)*cosh(b*x+a)/b^4-4*d*(d*x+c)^3*cosh(b*x+a)/b^2+24*d^4*sinh(
b*x+a)/b^5+12*d^2*(d*x+c)^2*sinh(b*x+a)/b^3+(d*x+c)^4*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.84

$$\int (c+dx)^4 \cosh(a+bx)\,dx = \frac{-4bd(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)+\left(24d^4+12b^2d^2(c+dx)^2+b^4(c+dx)^4\right)\sinh(a+bx)}{b^5}$$

input
```
Integrate[(c + d*x)^4*Cosh[a + b*x],x]
```

output
```
(-4*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + (24*d^4 + 12*b
^2*d^2*(c + d*x)^2 + b^4*(c + d*x)^4)*Sinh[a + b*x])/b^5
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.61 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.25, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^4\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}-\frac{4id\int -i(c+dx)^3\sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}-\frac{4d\int (c+dx)^3\sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}-\frac{4d\int -i(c+dx)^3\sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\int (c+dx)^3\sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int (c+dx)^2\cosh(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int-i(c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int(c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int-i(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)}{b}\right)}{b}$$

$\Big\downarrow$ 3117

$$\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^4*Cosh[a + b*x],x]
```

output
```
((c + d*x)^4*Sinh[a + b*x])/b + ((4*I)*d*((I*(c + d*x)^3*Cosh[a + b*x])/b
- ((3*I)*d*(((c + d*x)^2*Sinh[a + b*x])/b + ((2*I)*d*((I*(c + d*x)*Cosh[a
+ b*x])/b - (I*d*Sinh[a + b*x])/b^2))/b))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.58

method	result
parallelrisch	$\frac{12\left(\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2+2d^2\right)b\,d^2x\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+2\left(-(dx+c)^4b^4-12d^2(dx+c)^2b^2-24d^4\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+8b\left(\left(x^2d^2\right.\right.}{b^5\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)}$
orering	$-\frac{8d\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2+4b^4c^3dx+b^4c^4+9b^2d^4x^2+18b^2c\,d^3x+9b^2c^2d^2+12d^4\right)\cosh(bx+a)}{b^6(dx+c)}+\frac{\left(d^4x^4b^4\right.}{}$
risch	$\frac{\left(d^4x^4b^4+4b^4c\,d^3x^3+6b^4c^2d^2x^2-4b^3d^4x^3+4b^4c^3dx-12b^3c\,d^3x^2+b^4c^4-12b^3c^2d^2x+12b^2d^4x^2-4b^3c^3d+24b^2c\,d^3x+1\right.}{2b^5}$
parts	$\frac{\sinh(bx+a)d^4x^4}{b}+\frac{4\sinh(bx+a)c\,d^3x^3}{b}+\frac{6\sinh(bx+a)c^2d^2x^2}{b}+\frac{4\sinh(bx+a)c^3dx}{b}+\frac{\sinh(bx+a)c^4}{b}-\frac{4d\left(\frac{d^3}{}\right.}{}$
meijerg	$-\frac{16id^4\cosh(a)\sqrt{\pi}\left(-\frac{ixb\left(\frac{5x^2b^2}{2}+15\right)\cosh(bx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}x^4b^4+\frac{15}{2}x^2b^2+15\right)\sinh(bx)}{10\sqrt{\pi}}\right)}{b^5}-\frac{16d^4\sinh(a)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}x^4b\right.}{}\right.}{}$
derivativedivides	$\frac{c^4\sinh(bx+a)+\frac{4d\,c^3((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{4d^3a^3c\sinh(bx+a)}{b^3}+\frac{6d^2a^2c^2\sinh(bx+a)}{b^2}-\frac{4da\,c^3\sinh(bx+a)}{b}-\frac{4d^4a\left(\right.}{}}{}$
default	$\frac{c^4\sinh(bx+a)+\frac{4d\,c^3((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{4d^3a^3c\sinh(bx+a)}{b^3}+\frac{6d^2a^2c^2\sinh(bx+a)}{b^2}-\frac{4da\,c^3\sinh(bx+a)}{b}-\frac{4d^4a\left(\right.}{}}{}$

input
```
int((d*x+c)^4*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(6*((1/3*x^2*d^2+c*d*x+c^2)*b^2+2*d^2)*b*d^2*x*tanh(1/2*b*x+1/2*a)^2+(-(
d*x+c)^4*b^4-12*d^2*(d*x+c)^2*b^2-24*d^4)*tanh(1/2*b*x+1/2*a)+4*b*((d^2*x^
2+c*d*x+c^2)*b^2+6*d^2)*d*(1/2*d*x+c))/b^5/(tanh(1/2*b*x+1/2*a)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.88

$$\int (c+dx)^4 \cosh(a+bx)\,dx =$$
$$-\frac{4\left(b^3d^4x^3+3\,b^3cd^3x^2+b^3c^3d+6\,bcd^3+3\left(b^3c^2d^2+2\,bd^4\right)x\right)\cosh\left(bx+a\right)-\left(b^4d^4x^4+4\,b^4cd^3x^3+b^4c\ldots\right)}{b^5}$$

input
```
integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-(4*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2 + b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^
2 + 2*b*d^4)*x)*cosh(b*x + a) - (b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + b^4*c^4 +
 12*b^2*c^2*d^2 + 24*d^4 + 6*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 4*(b^4*c^3*d
+ 6*b^2*c*d^3)*x)*sinh(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. 2(92) = 184.

Time = 0.35 (sec) , antiderivative size = 311, normalized size of antiderivative = 3.42

$$\int (c+dx)^4 \cosh(a+bx)\,dx$$
$$= \begin{cases} \frac{c^4\sinh\left(a+bx\right)}{b}+\frac{4c^3dx\sinh\left(a+bx\right)}{b}+\frac{6c^2d^2x^2\sinh\left(a+bx\right)}{b}+\frac{4cd^3x^3\sinh\left(a+bx\right)}{b}+\frac{d^4x^4\sinh\left(a+bx\right)}{b}-\frac{4c^3d\cosh\left(a+bx\right)}{b^2}-\frac{12c^2d\ldots}{} \\ \left(c^4x+2c^3dx^2+2c^2d^2x^3+cd^3x^4+\frac{d^4x^5}{5}\right)\cosh\left(a\right) \end{cases}$$

input
```
integrate((d*x+c)**4*cosh(b*x+a),x)
```

output
```
Piecewise((c**4*sinh(a + b*x)/b + 4*c**3*d*x*sinh(a + b*x)/b + 6*c**2*d**2
*x**2*sinh(a + b*x)/b + 4*c*d**3*x**3*sinh(a + b*x)/b + d**4*x**4*sinh(a +
 b*x)/b - 4*c**3*d*cosh(a + b*x)/b**2 - 12*c**2*d**2*x*cosh(a + b*x)/b**2
- 12*c*d**3*x**2*cosh(a + b*x)/b**2 - 4*d**4*x**3*cosh(a + b*x)/b**2 + 12*
c**2*d**2*sinh(a + b*x)/b**3 + 24*c*d**3*x*sinh(a + b*x)/b**3 + 12*d**4*x**
*2*sinh(a + b*x)/b**3 - 24*c*d**3*cosh(a + b*x)/b**4 - 24*d**4*x*cosh(a +
b*x)/b**4 + 24*d**4*sinh(a + b*x)/b**5, Ne(b, 0)), ((c**4*x + 2*c**3*d*x**
2 + 2*c**2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cosh(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(91) = 182$.

Time = 0.06 (sec) , antiderivative size = 326, normalized size of antiderivative = 3.58

$$\int (c+dx)^4\cosh(a+bx)\,dx = \frac{c^4e^{(bx+a)}}{2\,b} + \frac{2\,(bxe^a - e^a)c^3de^{(bx)}}{b^2} - \frac{c^4e^{(-bx-a)}}{2\,b} - \frac{2\,(bx+1)c^3de^{(-bx-a)}}{b^2} + \frac{3\,(b^2x^2e^a - 2\,bxe^a + 2\,e^a)c^2d^2e^{(bx)}}{b^3} - \frac{3\,(b^2x^2 + 2\,bx + 2)c^2d^2e^{(-bx-a)}}{b^3} + \frac{2\,(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)cd^3e^{(bx)}}{b^4} - \frac{2\,(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)cd^3e^{(-bx-a)}}{b^4} + \frac{(b^4x^4e^a - 4\,b^3x^3e^a + 12\,b^2x^2e^a - 24\,bxe^a + 24\,e^a)d^4e^{(bx)}}{2\,b^5} - \frac{(b^4x^4 + 4\,b^3x^3 + 12\,b^2x^2 + 24\,bx + 24)d^4e^{(-bx-a)}}{2\,b^5}$$

input
```
integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^4*e^(b*x + a)/b + 2*(b*x*e^a - e^a)*c^3*d*e^(b*x)/b^2 - 1/2*c^4*e^(-
b*x - a)/b - 2*(b*x + 1)*c^3*d*e^(-b*x - a)/b^2 + 3*(b^2*x^2*e^a - 2*b*x*e
^a + 2*e^a)*c^2*d^2*e^(b*x)/b^3 - 3*(b^2*x^2 + 2*b*x + 2)*c^2*d^2*e^(-b*x
- a)/b^3 + 2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*c*d^3*e^(b*
x)/b^4 - 2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*c*d^3*e^(-b*x - a)/b^4 + 1/2*
(b^4*x^4*e^a - 4*b^3*x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*d^4*e
^(b*x)/b^5 - 1/2*(b^4*x^4 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*d^4*e^(-
b*x - a)/b^5
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 324, normalized size of antiderivative = 3.56

$$\int (c+dx)^4\cosh(a+bx)\,dx$$
$$=\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2-4\,b^3d^4x^3+4\,b^4c^3dx-12\,b^3cd^3x^2+b^4c^4-12\,b^3c^2d^2x+12\,b^2d^4x^2-4}{2\,b^5}$$
$$-\frac{(b^4d^4x^4+4\,b^4cd^3x^3+6\,b^4c^2d^2x^2+4\,b^3d^4x^3+4\,b^4c^3dx+12\,b^3cd^3x^2+b^4c^4+12\,b^3c^2d^2x+12\,b^2d^4x^2+}{2\,b^5}$$

input
```
integrate((d*x+c)^4*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4
*b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*
x^2 - 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*
d^3 + 24*d^4)*e^(b*x + a)/b^5 - 1/2*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4
*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4
+ 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^
2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.36

$$\int (c+dx)^4\cosh(a+bx)\,dx=\frac{\sinh(a+b\,x)\,\left(b^4\,c^4+12\,b^2\,c^2\,d^2+24\,d^4\right)}{b^5}-\frac{4\cosh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^4}-\frac{4\,d^4\,x^3\cosh(a+b\,x)}{b^2}-\frac{12\,x\cosh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^4}+\frac{d^4\,x^4\sinh(a+b\,x)}{b}+\frac{4\,x\sinh(a+b\,x)\,\left(b^2\,c^3\,d+6\,c\,d^3\right)}{b^3}+\frac{6\,x^2\sinh(a+b\,x)\,\left(b^2\,c^2\,d^2+2\,d^4\right)}{b^3}-\frac{12\,c\,d^3\,x^2\cosh(a+b\,x)}{b^2}+\frac{4\,c\,d^3\,x^3\sinh(a+b\,x)}{b}$$

input
```
int(cosh(a + b*x)*(c + d*x)^4,x)
```

output
```
(sinh(a + b*x)*(24*d^4 + b^4*c^4 + 12*b^2*c^2*d^2))/b^5 - (4*cosh(a + b*x)
*(6*c*d^3 + b^2*c^3*d))/b^4 - (4*d^4*x^3*cosh(a + b*x))/b^2 - (12*x*cosh(a
 + b*x)*(2*d^4 + b^2*c^2*d^2))/b^4 + (d^4*x^4*sinh(a + b*x))/b + (4*x*sinh
(a + b*x)*(6*c*d^3 + b^2*c^3*d))/b^3 + (6*x^2*sinh(a + b*x)*(2*d^4 + b^2*c
^2*d^2))/b^3 - (12*c*d^3*x^2*cosh(a + b*x))/b^2 + (4*c*d^3*x^3*sinh(a + b*
x))/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.67

$$\int (c+dx)^4 \cosh(a+bx)\,dx$$
$$= \frac{-4\cosh(bx+a)\,b^3c^3d - 12\cosh(bx+a)\,b^3c^2d^2x - 12\cosh(bx+a)\,b^3c\,d^3x^2 - 4\cosh(bx+a)\,b^3d^4x^3 -}{}$$

input
```
int((d*x+c)^4*cosh(b*x+a),x)
```

output
```
( - 4*cosh(a + b*x)*b**3*c**3*d - 12*cosh(a + b*x)*b**3*c**2*d**2*x - 12*c
osh(a + b*x)*b**3*c*d**3*x**2 - 4*cosh(a + b*x)*b**3*d**4*x**3 - 24*cosh(a
 + b*x)*b*c*d**3 - 24*cosh(a + b*x)*b*d**4*x + sinh(a + b*x)*b**4*c**4 + 4
*sinh(a + b*x)*b**4*c**3*d*x + 6*sinh(a + b*x)*b**4*c**2*d**2*x**2 + 4*sin
h(a + b*x)*b**4*c*d**3*x**3 + sinh(a + b*x)*b**4*d**4*x**4 + 12*sinh(a + b
*x)*b**2*c**2*d**2 + 24*sinh(a + b*x)*b**2*c*d**3*x + 12*sinh(a + b*x)*b**
2*d**4*x**2 + 24*sinh(a + b*x)*d**4)/b**5
```

3.2 $\int (c+dx)^3 \cosh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 70

$$\int (c+dx)^3 \cosh(a+bx)\,dx = -\frac{6d^3\cosh(a+bx)}{b^4} - \frac{3d(c+dx)^2\cosh(a+bx)}{b^2} + \frac{6d^2(c+dx)\sinh(a+bx)}{b^3} + \frac{(c+dx)^3\sinh(a+bx)}{b}$$

output
```
-6*d^3*cosh(b*x+a)/b^4-3*d*(d*x+c)^2*cosh(b*x+a)/b^2+6*d^2*(d*x+c)*sinh(b*
x+a)/b^3+(d*x+c)^3*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.87

$$\int (c+dx)^3 \cosh(a+bx)\,dx = \frac{-3d(2d^2+b^2(c+dx)^2)\cosh(a+bx)+b(c+dx)\left(6d^2+b^2(c+dx)^2\right)\sinh(a+bx)}{b^4}$$

input
```
Integrate[(c + d*x)^3*Cosh[a + b*x],x]
```

output
```
(-3*d*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] + b*(c + d*x)*(6*d^2 + b^2*(
c + d*x)^2)*Sinh[a + b*x])/b^4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.48 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.857, Rules used = {3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3id\int -i(c+dx)^2\sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int (c+dx)^2\sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int -i(c+dx)^2\sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\int (c+dx)^2\sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\cosh(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \; 3777$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 26$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int \sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \; 3118$$

$$\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^3*Cosh[a + b*x],x]
```

output
```
((c + d*x)^3*Sinh[a + b*x])/b + ((3*I)*d*((I*(c + d*x)^2*Cosh[a + b*x])/b
- ((2*I)*d*(-((d*Cosh[a + b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.59

method	result
parallelrisch	$\frac{6b^2d^2x(\frac{dx}{2}+c)\tanh(\frac{bx}{2}+\frac{a}{2})^2-2(dx+c)b((dx+c)^2b^2+6d^2)\tanh(\frac{bx}{2}+\frac{a}{2})+6((\frac{1}{2}x^2d^2+cdx+c^2)b^2+2d^2)d}{b^4(\tanh(\frac{bx}{2}+\frac{a}{2})^2-1)}$
orering	$-\frac{6d(d^2x^2b^2+2b^2cdx+b^2c^2+4d^2)\cosh(bx+a)}{b^4}+\frac{(d^2x^2b^2+2b^2cdx+b^2c^2+6d^2)(3(dx+c)^2\cosh(bx+a)d+(dx+c)^3b\,\mathrm{s}\cdots}{(dx+c)^2b^4}$
risch	$\frac{(d^3x^3b^3+3b^3c\,d^2x^2+3b^3c^2dx-3b^2d^3x^2+b^3c^3-6b^2c\,d^2x-3b^2c^2d+6b\,d^3x+6bc\,d^2-6d^3)\mathrm{e}^{bx+a}}{2b^4}-\frac{(d^3x^3b^3+3b^3c\,d^2x^2\cdots}{}$
parts	$\frac{\sinh(bx+a)d^3x^3}{b}+\frac{3\sinh(bx+a)c\,d^2x^2}{b}+\frac{3\sinh(bx+a)c^2dx}{b}+\frac{\sinh(bx+a)c^3}{b}-\frac{3d(\frac{d^2((bx+a)^2\cosh(bx+a)-2(bx\cdots}{}}{}$
derivativedivides	$\frac{\frac{d^3((bx+a)^3\sinh(bx+a)-3(bx+a)^2\cosh(bx+a)+6(bx+a)\sinh(bx+a)-6\cosh(bx+a))}{b^3}-\frac{3d^3a((bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(b\cdots}{b^3}}{}$
default	$\frac{\frac{d^3((bx+a)^3\sinh(bx+a)-3(bx+a)^2\cosh(bx+a)+6(bx+a)\sinh(bx+a)-6\cosh(bx+a))}{b^3}-\frac{3d^3a((bx+a)^2\sinh(bx+a)-2(bx+a)\cosh(b\cdots}{b^3}}{}$
meijerg	$\frac{8d^3\cosh(a)\sqrt{\pi}(\frac{3}{4\sqrt{\pi}}-\frac{(\frac{3x^2b^2}{2}+3)\cosh(bx)}{4\sqrt{\pi}}+\frac{xb(\frac{x^2b^2}{2}+3)\sinh(bx)}{4\sqrt{\pi}})}{b^4}-\frac{8id^3\sinh(a)\sqrt{\pi}(\frac{ixb(\frac{5x^2b^2}{2}+15)\cosh(bx)}{20\sqrt{\pi}}-\cdots}{b^4}$

input
```
int((d*x+c)^3*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(3*b^2*d^2*x*(1/2*d*x+c)*tanh(1/2*b*x+1/2*a)^2-(d*x+c)*b*((d*x+c)^2*b^2+
6*d^2)*tanh(1/2*b*x+1/2*a)+3*((1/2*x^2*d^2+c*d*x+c^2)*b^2+2*d^2)*d)/b^4/(t
anh(1/2*b*x+1/2*a)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.59

$$\int (c+dx)^3\cosh(a+bx)\,dx = -\frac{3\left(b^2d^3x^2+2\,b^2cd^2x+b^2c^2d+2\,d^3\right)\cosh\left(bx+a\right)-\left(b^3d^3x^3+3\,b^3cd^2x^2+b^3c^3+6\,bcd^2+3\left(b^3c^2d+2\right.\right.}{b^4}$$

input
```
integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-(3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + b^2*c^2*d + 2*d^3)*cosh(b*x + a) - (b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + b^3*c^3 + 6*b*c*d^2 + 3*(b^3*c^2*d + 2*b*d^3)
*x)*sinh(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. 2(70) = 140.

Time = 0.26 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.89

$$\int (c+dx)^3\cosh(a+bx)\,dx = \begin{cases} \frac{c^3\sinh\left(a+bx\right)}{b}+\frac{3c^2dx\sinh\left(a+bx\right)}{b}+\frac{3cd^2x^2\sinh\left(a+bx\right)}{b}+\frac{d^3x^3\sinh\left(a+bx\right)}{b}-\frac{3c^2d\cosh\left(a+bx\right)}{b^2}-\frac{6cd^2x\cosh\left(a+bx\right)}{b^2}-\frac{3d^3x^2\,c}{} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cosh\left(a\right) \end{cases}$$

input
```
integrate((d*x+c)**3*cosh(b*x+a),x)
```

output
```
Piecewise((c**3*sinh(a + b*x)/b + 3*c**2*d*x*sinh(a + b*x)/b + 3*c*d**2*x*
*2*sinh(a + b*x)/b + d**3*x**3*sinh(a + b*x)/b - 3*c**2*d*cosh(a + b*x)/b*
*2 - 6*c*d**2*x*cosh(a + b*x)/b**2 - 3*d**3*x**2*cosh(a + b*x)/b**2 + 6*c*
d**2*sinh(a + b*x)/b**3 + 6*d**3*x*sinh(a + b*x)/b**3 - 6*d**3*cosh(a + b*
x)/b**4, Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4
)*cosh(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(70) = 140$.

Time = 0.05 (sec) , antiderivative size = 222, normalized size of antiderivative = 3.17

$$\begin{aligned}\int (c+dx)^3 \cosh(a+bx)\,dx &= \frac{c^3 e^{(bx+a)}}{2\,b} + \frac{3\,(bxe^a - e^a)c^2 de^{(bx)}}{2\,b^2} \\ &- \frac{c^3 e^{(-bx-a)}}{2\,b} - \frac{3\,(bx+1)c^2 de^{(-bx-a)}}{2\,b^2} \\ &+ \frac{3\,(b^2x^2e^a - 2\,bxe^a + 2\,e^a)cd^2 e^{(bx)}}{2\,b^3} \\ &- \frac{3\,(b^2x^2 + 2\,bx + 2)cd^2 e^{(-bx-a)}}{2\,b^3} \\ &+ \frac{(b^3x^3e^a - 3\,b^2x^2e^a + 6\,bxe^a - 6\,e^a)d^3 e^{(bx)}}{2\,b^4} \\ &- \frac{(b^3x^3 + 3\,b^2x^2 + 6\,bx + 6)d^3 e^{(-bx-a)}}{2\,b^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^3*e^(b*x + a)/b + 3/2*(b*x*e^a - e^a)*c^2*d*e^(b*x)/b^2 - 1/2*c^3*e^
(-b*x - a)/b - 3/2*(b*x + 1)*c^2*d*e^(-b*x - a)/b^2 + 3/2*(b^2*x^2*e^a - 2
*b*x*e^a + 2*e^a)*c*d^2*e^(b*x)/b^3 - 3/2*(b^2*x^2 + 2*b*x + 2)*c*d^2*e^(-
b*x - a)/b^3 + 1/2*(b^3*x^3*e^a - 3*b^2*x^2*e^a + 6*b*x*e^a - 6*e^a)*d^3*e
^(b*x)/b^4 - 1/2*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*d^3*e^(-b*x - a)/b^4
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(70) = 140$.

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.91

$$\int (c+dx)^3 \cosh(a+bx)\,dx = \frac{\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx - 3\,b^2d^3x^2 + b^3c^3 - 6\,b^2cd^2x - 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 6\,d^3\right)e^{(bx+a)}}{2\,b^4} - \frac{\left(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + 3\,b^2d^3x^2 + b^3c^3 + 6\,b^2cd^2x + 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 6\,d^3\right)e^{(-bx-a)}}{2\,b^4}$$

input
```
integrate((d*x+c)^3*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x - 3*b^2*d^3*x^2 + b^3*c
^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 6*d^3)*e^(b*x +
 a)/b^4 - 1/2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*b^2*d^3*x
^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 6*d^3
)*e^(-b*x - a)/b^4
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.04

$$\int (c+dx)^3 \cosh(a+bx)\,dx = \frac{\sinh(a+b\,x)\,\left(b^2\,c^3+6\,c\,d^2\right)}{b^3} - \frac{3\cosh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^4} - \frac{3\,d^3\,x^2\cosh(a+b\,x)}{b^2} + \frac{d^3\,x^3\sinh(a+b\,x)}{b} + \frac{3\,x\sinh(a+b\,x)\,\left(b^2\,c^2\,d+2\,d^3\right)}{b^3} - \frac{6\,c\,d^2\,x\cosh(a+b\,x)}{b^2} + \frac{3\,c\,d^2\,x^2\sinh(a+b\,x)}{b}$$

input
```
int(cosh(a + b*x)*(c + d*x)^3,x)
```

output
```
(sinh(a + b*x)*(6*c*d^2 + b^2*c^3))/b^3 - (3*cosh(a + b*x)*(2*d^3 + b^2*c^
2*d))/b^4 - (3*d^3*x^2*cosh(a + b*x))/b^2 + (d^3*x^3*sinh(a + b*x))/b + (3
*x*sinh(a + b*x)*(2*d^3 + b^2*c^2*d))/b^3 - (6*c*d^2*x*cosh(a + b*x))/b^2
+ (3*c*d^2*x^2*sinh(a + b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.19

$$\int (c+dx)^3 \cosh(a+bx)\, dx = \frac{-3\cosh(bx+a)\, b^2c^2d - 6\cosh(bx+a)\, b^2c\, d^2x - 3\cosh(bx+a)\, b^2d^3x^2 - 6\cosh(bx+a)\, d^3 + \sinh(bx$$

input
```
int((d*x+c)^3*cosh(b*x+a),x)
```

output
```
( - 3*cosh(a + b*x)*b**2*c**2*d - 6*cosh(a + b*x)*b**2*c*d**2*x - 3*cosh(a
 + b*x)*b**2*d**3*x**2 - 6*cosh(a + b*x)*d**3 + sinh(a + b*x)*b**3*c**3 +
3*sinh(a + b*x)*b**3*c**2*d*x + 3*sinh(a + b*x)*b**3*c*d**2*x**2 + sinh(a
+ b*x)*b**3*d**3*x**3 + 6*sinh(a + b*x)*b*c*d**2 + 6*sinh(a + b*x)*b*d**3*
x)/b**4
```

3.3 $\int (c+dx)^2 \cosh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (c+dx)^2 \cosh(a+bx)\,dx = -\frac{2d(c+dx)\cosh(a+bx)}{b^2} + \frac{2d^2\sinh(a+bx)}{b^3} + \frac{(c+dx)^2\sinh(a+bx)}{b}$$

output
```
-2*d*(d*x+c)*cosh(b*x+a)/b^2+2*d^2*sinh(b*x+a)/b^3+(d*x+c)^2*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

$$\int (c+dx)^2 \cosh(a+bx)\,dx = \frac{-2bd(c+dx)\cosh(a+bx) + (2d^2 + b^2(c+dx)^2)\sinh(a+bx)}{b^3}$$

input
```
Integrate[(c + d*x)^2*Cosh[a + b*x],x]
```

output
```
(-2*b*d*(c + d*x)*Cosh[a + b*x] + (2*d^2 + b^2*(c + d*x)^2)*Sinh[a + b*x])
/b^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int -i(c+dx)\sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int (c+dx)\sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int -i(c+dx)\sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int (c+dx)\sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}$$

input
```
Int[(c + d*x)^2*Cosh[a + b*x],x]
```

output
```
((c + d*x)^2*Sinh[a + b*x])/b + ((2*I)*d*((I*(c + d*x)*Cosh[a + b*x])/b -
(I*d*Sinh[a + b*x])/b^2))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.57

method	result
parallelrisch	$\frac{2x\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}b\,d^{2}+2\left(-(dx+c)^{2}b^{2}-2d^{2}\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+4bd\left(\frac{dx}{2}+c\right)}{b^{3}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)}$
parts	$\frac{\sinh(bx+a)x^{2}d^{2}}{b}+\frac{2\sinh(bx+a)cdx}{b}+\frac{\sinh(bx+a)c^{2}}{b}-\frac{2d\left(\frac{d((bx+a)\cosh(bx+a)-\sinh(bx+a))}{b}-\frac{da\cosh(bx+a)}{b}+c\cosh\right.}{b^{2}}$
risch	$\frac{(d^{2}x^{2}b^{2}+2b^{2}cdx+b^{2}c^{2}-2b\,d^{2}x-2bcd+2d^{2})e^{bx+a}}{2b^{3}}-\frac{(d^{2}x^{2}b^{2}+2b^{2}cdx+b^{2}c^{2}+2b\,d^{2}x+2bcd+2d^{2})e^{-bx-a}}{2b^{3}}$
orering	$-\frac{4d(d^{2}x^{2}b^{2}+2b^{2}cdx+b^{2}c^{2}+d^{2})\cosh(bx+a)}{b^{4}(dx+c)}+\frac{(d^{2}x^{2}b^{2}+2b^{2}cdx+b^{2}c^{2}+2d^{2})\left(2(dx+c)\cosh(bx+a)d+(dx+c)^{2}b\sin\right.}{b^{4}(dx+c)^{2}}$
derivativedivides	$\frac{\frac{d^{2}\left((bx+a)^{2}\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh(bx+a)\right)}{b^{2}}-\frac{2d^{2}a((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b^{2}}+\frac{2dc((bx+a)\sinh(bx+a)-}{b}}{b}$
default	$\frac{\frac{d^{2}\left((bx+a)^{2}\sinh(bx+a)-2(bx+a)\cosh(bx+a)+2\sinh(bx+a)\right)}{b^{2}}-\frac{2d^{2}a((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b^{2}}+\frac{2dc((bx+a)\sinh(bx+a)-}{b}}{b}$
meijerg	$\frac{4id^{2}\cosh(a)\sqrt{\pi}\left(\frac{ixb\cosh(bx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^{2}b^{2}}{2}+3\right)\sinh(bx)}{6\sqrt{\pi}}\right)}{b^{3}}+\frac{4d^{2}\sinh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^{2}b^{2}}{2}+1\right)\cosh(bx)}{2\sqrt{\pi}}-\frac{xb\sinh(bx)}{2\sqrt{\pi}}\right)}{b^{3}}$

input
```
int((d*x+c)^2*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
2*(x*tanh(1/2*b*x+1/2*a)^2*b*d^2+(-(d*x+c)^2*b^2-2*d^2)*tanh(1/2*b*x+1/2*a
)+2*b*d*(1/2*d*x+c))/b^3/(tanh(1/2*b*x+1/2*a)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int (c+dx)^2\cosh(a+bx)\,dx = -\frac{2\left(bd^{2}x+bcd\right)\cosh\left(bx+a\right)-\left(b^{2}d^{2}x^{2}+2\,b^{2}cdx+b^{2}c^{2}+2\,d^{2}\right)\sinh\left(bx+a\right)}{b^{3}}$$

input
```
integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-(2*(b*d^2*x + b*c*d)*cosh(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2
 + 2*d^2)*sinh(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2\cosh(a+bx)\,dx = \begin{cases} \frac{c^2\sinh(a+bx)}{b} + \frac{2cdx\sinh(a+bx)}{b} + \frac{d^2x^2\sinh(a+bx)}{b} - \frac{2cd\cosh(a+bx)}{b^2} - \frac{2d^2x\cosh(a+bx)}{b^2} + \frac{2d^2\sinh(a+bx)}{b^3} & \text{for } b\neq 0 \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)**2*cosh(b*x+a),x)
```

output
```
Piecewise((c**2*sinh(a + b*x)/b + 2*c*d*x*sinh(a + b*x)/b + d**2*x**2*sinh
(a + b*x)/b - 2*c*d*cosh(a + b*x)/b**2 - 2*d**2*x*cosh(a + b*x)/b**2 + 2*d
**2*sinh(a + b*x)/b**3, Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cosh
(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(49) = 98$.

Time = 0.05 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.76

$$\int (c+dx)^2\cosh(a+bx)\,dx = \frac{c^2e^{(bx+a)}}{2\,b} + \frac{(bxe^a - e^a)cde^{(bx)}}{b^2} - \frac{c^2e^{(-bx-a)}}{2\,b} - \frac{(bx+1)cde^{(-bx-a)}}{b^2} + \frac{(b^2x^2e^a - 2\,bxe^a + 2\,e^a)d^2e^{(bx)}}{2\,b^3} - \frac{(b^2x^2 + 2\,bx + 2)d^2e^{(-bx-a)}}{2\,b^3}$$

input
```
integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c^2*e^(b*x + a)/b + (b*x*e^a - e^a)*c*d*e^(b*x)/b^2 - 1/2*c^2*e^(-b*x
- a)/b - (b*x + 1)*c*d*e^(-b*x - a)/b^2 + 1/2*(b^2*x^2*e^a - 2*b*x*e^a + 2
*e^a)*d^2*e^(b*x)/b^3 - 1/2*(b^2*x^2 + 2*b*x + 2)*d^2*e^(-b*x - a)/b^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (c+dx)^2\cosh(a+bx)\,dx = \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2-2\,bd^2x-2\,bcd+2\,d^2)e^{(bx+a)}}{2\,b^3} - \frac{(b^2d^2x^2+2\,b^2cdx+b^2c^2+2\,bd^2x+2\,bcd+2\,d^2)e^{(-bx-a)}}{2\,b^3}$$

input
```
integrate((d*x+c)^2*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*d^2*x - 2*b*c*d + 2*d^2)*e^
(b*x + a)/b^3 - 1/2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + 2*b*d^2*x + 2*b
*c*d + 2*d^2)*e^(-b*x - a)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int (c+dx)^2\cosh(a+bx)\,dx = \frac{\sinh(a+b\,x)\,\left(b^2\,c^2+2\,d^2\right)}{b^3} + \frac{d^2\,x^2\,\sinh(a+b\,x)}{b} - \frac{2\,c\,d\cosh(a+b\,x)}{b^2} - \frac{2\,d^2\,x\cosh(a+b\,x)}{b^2} + \frac{2\,c\,d\,x\sinh(a+b\,x)}{b}$$

input
```
int(cosh(a + b*x)*(c + d*x)^2,x)
```

output
```
(sinh(a + b*x)*(2*d^2 + b^2*c^2))/b^3 + (d^2*x^2*sinh(a + b*x))/b - (2*c*d
*cosh(a + b*x))/b^2 - (2*d^2*x*cosh(a + b*x))/b^2 + (2*c*d*x*sinh(a + b*x)
)/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int (c+dx)^2 \cosh(a+bx)\,dx$$
$$= \frac{-2\cosh(bx+a)\,bcd - 2\cosh(bx+a)\,b\,d^2x + \sinh(bx+a)\,b^2c^2 + 2\sinh(bx+a)\,b^2cdx + \sinh(bx+a)\,b}{b^3}$$

input
```
int((d*x+c)^2*cosh(b*x+a),x)
```

output
```
( - 2*cosh(a + b*x)*b*c*d - 2*cosh(a + b*x)*b*d**2*x + sinh(a + b*x)*b**2*
c**2 + 2*sinh(a + b*x)*b**2*c*d*x + sinh(a + b*x)*b**2*d**2*x**2 + 2*sinh(
a + b*x)*d**2)/b**3
```

3.4 $\int (c + dx) \cosh(a + bx) \, dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (c + dx) \cosh(a + bx) \, dx = -\frac{d \cosh(a + bx)}{b^2} + \frac{(c + dx) \sinh(a + bx)}{b}$$

output
```
-d*cosh(b*x+a)/b^2+(d*x+c)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (c + dx) \cosh(a + bx) \, dx = \frac{-d \cosh(a + bx) + b(c + dx) \sinh(a + bx)}{b^2}$$

input
```
Integrate[(c + d*x)*Cosh[a + b*x],x]
```

output
```
(-(d*Cosh[a + b*x]) + b*(c + d*x)*Sinh[a + b*x])/b^2
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int \sin(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}$$

input
```
Int[(c + d*x)*Cosh[a + b*x],x]
```

output
```
-((d*Cosh[a + b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{(dx+c)b\sinh(bx+a)-d(\cosh(bx+a)+1)}{b^2}$
parts	$\frac{\sinh(bx+a)dx}{b}+\frac{\sinh(bx+a)c}{b}-\frac{d\cosh(bx+a)}{b^2}$
orering	$-\frac{2d\cosh(bx+a)}{b^2}+\frac{\cosh(bx+a)d+(dx+c)b\sinh(bx+a)}{b^2}$
risch	$\frac{(dxb+cb-d)\mathrm{e}^{bx+a}}{2b^2}-\frac{(dxb+cb+d)\mathrm{e}^{-bx-a}}{2b^2}$
derivativedivides	$\frac{\frac{d((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{da\sinh(bx+a)}{b}+c\sinh(bx+a)}{b}$
default	$\frac{\frac{d((bx+a)\sinh(bx+a)-\cosh(bx+a))}{b}-\frac{da\sinh(bx+a)}{b}+c\sinh(bx+a)}{b}$
meijerg	$-\frac{2d\cosh(a)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(bx)}{2\sqrt{\pi}}-\frac{xb\sinh(bx)}{2\sqrt{\pi}}\right)}{b^2}+\frac{d\sinh(a)(\cosh(bx)xb-\sinh(bx))}{b^2}+\frac{c\cosh(a)\sinh(bx)}{b}-\frac{c\sinh$

input
```
int((d*x+c)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*b*sinh(b*x+a)-d*(cosh(b*x+a)+1))/b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (c + dx) \cosh(a + bx) \, dx = -\frac{d \cosh(bx + a) - (bdx + bc) \sinh(bx + a)}{b^2}$$

input
```
integrate((d*x+c)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-(d*cosh(b*x + a) - (b*d*x + b*c)*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (c + dx) \cosh(a + bx) \, dx = \begin{cases} \frac{c \sinh(a+bx)}{b} + \frac{dx \sinh(a+bx)}{b} - \frac{d \cosh(a+bx)}{b^2} & \text{for } b \neq 0 \\ \left(cx + \frac{dx^2}{2}\right) \cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*cosh(b*x+a),x)
```

output
```
Piecewise((c*sinh(a + b*x)/b + d*x*sinh(a + b*x)/b - d*cosh(a + b*x)/b**2,
 Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int (c+dx)\cosh(a+bx)\,dx = \frac{ce^{(bx+a)}}{2\,b} + \frac{(bxe^a - e^a)de^{(bx)}}{2\,b^2} - \frac{ce^{(-bx-a)}}{2\,b} - \frac{(bx+1)de^{(-bx-a)}}{2\,b^2}$$

input
```
integrate((d*x+c)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*c*e^(b*x + a)/b + 1/2*(b*x*e^a - e^a)*d*e^(b*x)/b^2 - 1/2*c*e^(-b*x -
a)/b - 1/2*(b*x + 1)*d*e^(-b*x - a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (c+dx)\cosh(a+bx)\,dx = \frac{(bdx + bc - d)e^{(bx+a)}}{2\,b^2} - \frac{(bdx + bc + d)e^{(-bx-a)}}{2\,b^2}$$

input
```
integrate((d*x+c)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b*d*x + b*c - d)*e^(b*x + a)/b^2 - 1/2*(b*d*x + b*c + d)*e^(-b*x - a)
/b^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (c+dx)\cosh(a+bx)\,dx = \frac{c\sinh(a+b\,x) + d\,x\sinh(a+b\,x)}{b} - \frac{d\cosh(a+b\,x)}{b^2}$$

input
```
int(cosh(a + b*x)*(c + d*x),x)
```

output
```
(c*sinh(a + b*x) + d*x*sinh(a + b*x))/b - (d*cosh(a + b*x))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int (c + dx) \cosh(a + bx) \, dx = \frac{-\cosh(bx + a) d + \sinh(bx + a) bc + \sinh(bx + a) bdx}{b^2}$$

input
```
int((d*x+c)*cosh(b*x+a),x)
```

output
```
( - cosh(a + b*x)*d + sinh(a + b*x)*b*c + sinh(a + b*x)*b*d*x)/b**2
```

3.5 $\int \frac{\cosh(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx = \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d} + \frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

output
```
cosh(a-b*c/d)*Chi(b*c/d+b*x)/d+sinh(a-b*c/d)*Shi(b*c/d+b*x)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx = \frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)+\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x),x]
```

output
```
(Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x] + Sinh[a - (b*c)/d]*SinhInt
egral[(b*c)/d + b*x])/d
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{3784}$$

$$\cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{26}$$

$$\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3042}$$

$$\sinh\left(a-\frac{bc}{d}\right)\int -\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{26}$$

$$\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx$$

$$\Big\downarrow \text{3779}$$

$$\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{3782}$$

$$\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}$$

input
```
Int[Cosh[a + b*x]/(c + d*x),x]
```

output
```
(Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d + (Sinh[a - (b*c)/d]*Sin
hIntegral[(b*c)/d + b*x])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.61

method	result	size
risch	$-\frac{\mathrm{e}^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_{1}\left(bx+a-\frac{ad-cb}{d}\right)}{2d}-\frac{\mathrm{e}^{\frac{ad-cb}{d}} \operatorname{expIntegral}_{1}\left(-bx-a-\frac{-ad+cb}{d}\right)}{2d}$	82

input
```
int(cosh(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-1/2/d*exp((a*d-b*c)/d)*Ei
(1,-b*x-a-(-a*d+b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.84

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx = \frac{\left(\operatorname{Ei}\left(\frac{bdx+bc}{d}\right)+\operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\operatorname{Ei}\left(\frac{bdx+bc}{d}\right)-\operatorname{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\sinh\left(-\frac{bc-ad}{d}\right)}{2\,d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) + (
Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/d
```

Sympy [F]

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx = \int \frac{\cosh\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx = -\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{2\,d} - \frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 1/2*e^(a - b*c/d)
*exp_integral_e(1, -(d*x + c)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx = \frac{\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + \mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}}{2\,d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/
d))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx=\int \frac{\cosh(a+b\,x)}{c+d\,x}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x),x)
```

output
```
int(cosh(a + b*x)/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{c+dx}\,dx=\int \frac{\cosh(bx+a)}{dx+c}dx$$

input
```
int(cosh(b*x+a)/(d*x+c),x)
```

output
```
int(cosh(a + b*x)/(c + d*x),x)
```

3.6 $\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = -\frac{\cosh(a+bx)}{d(c+dx)}+\frac{b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{d^2}+\frac{b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d^2}$$

output
```
-cosh(b*x+a)/d/(d*x+c)+b*Chi(b*c/d+b*x)*sinh(a-b*c/d)/d^2+b*cosh(a-b*c/d)*
Shi(b*c/d+b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = \frac{-\frac{d\cosh(a+bx)}{c+dx}+b\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)\sinh\left(a-\frac{bc}{d}\right)+b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x)^2,x]
```

output
```
(-((d*Cosh[a + b*x])/(c + d*x)) + b*CoshIntegral[b*(c/d + x)]*Sinh[a - (b*
c)/d] + b*Cosh[a - (b*c)/d]*SinhIntegral[b*(c/d + x)])/d^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.786, Rules used = {3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}+\frac{ib\int -\frac{i\sinh(a+bx)}{c+dx}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{b\int \frac{\sinh(a+bx)}{c+dx}dx}{d}-\frac{\cosh(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}+\frac{b\int -\frac{i\sin(ia+ibx)}{c+dx}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\int \frac{\sin(ia+ibx)}{c+dx}dx}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(a-\frac{bc}{d}\right)\int-\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\cosh(a+bx)}{d(c+dx)}-\frac{ib\left(\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}$$

input
```
Int[Cosh[a + b*x]/(c + d*x)^2,x]
```

output
```
-(Cosh[a + b*x]/(d*(c + d*x))) - (I*b*((I*CoshIntegral[(b*c)/d + b*x]*Sinh
[a - (b*c)/d])/d + (I*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.87

method	result	siz
risch	$-\frac{b\,e^{-bx-a}}{2d(dxb+cb)}+\frac{b\,e^{-\frac{ad-cb}{d}}\,\mathrm{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{2d^2}-\frac{b\,e^{bx+a}}{2d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,e^{\frac{ad-cb}{d}}\,\mathrm{expIntegral}_1\left(-bx-a-\frac{-ad+cb}{d}\right)}{2d^2}$	13

input
```
int(cosh(b*x+a)/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*b*exp(-b*x-a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a
*d-b*c)/d)-1/2*b/d^2*exp(b*x+a)/(b*c/d+b*x)-1/2*b/d^2*exp((a*d-b*c)/d)*Ei(
1,-b*x-a-(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(71) = 142$.

Time = 0.08 (sec) , antiderivative size = 150, normalized size of antiderivative = 2.11

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = -\frac{2\,d\cosh\left(bx+a\right)-\left((bdx+bc)\text{Ei}\left(\frac{bdx+bc}{d}\right)-(bdx+bc)\text{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)-\left((bdx+bc)\text{Ei}\left(\frac{b}{}\right.\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*d*cosh(b*x + a) - ((b*d*x + b*c)*Ei((b*d*x + b*c)/d) - (b*d*x + b*
c)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - ((b*d*x + b*c)*Ei((b*d*x +
 b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^3*
x + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = \frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{d} - \frac{e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{d}\right)}{2\,d} - \frac{\cosh\left(bx+a\right)}{(dx+c)d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
1/2*b*(e^(-a + b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - e^(a - b*c/d)*e
xp_integral_e(1, -(d*x + c)*b/d)/d)/d - cosh(b*x + a)/((d*x + c)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(71) = 142.

Time = 0.14 (sec) , antiderivative size = 615, normalized size of antiderivative = 8.66

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx =$$

$$-\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(-\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)}{d}\right.\right.}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{}{}\right.\right.}$$

$$+\frac{\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}+b^3c\mathrm{Ei}\left(\frac{(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+b}{d}\right.\right.}{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+}\right.\right.}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b
- b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + b^3*c
*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*
c - a*d)/d) - a*b^2*d*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) +
 b*c - a*d)/d)*e^((b*c - a*d)/d) + b^2*d*e^(-(d*x + c)*(b - b*c/(d*x + c)
+ a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d
^4 + b*c*d^4 - a*d^5)*b) + 1/2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
c))*b^2*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*
e^(-(b*c - a*d)/d) + b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c
)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*
x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - b^2*d*e^((d*x
 + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d))*d^2/(((d*x + c)*(b - b*c/(d*
x + c) + a*d/(d*x + c))*d^4 + b*c*d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cosh(a+b\,x)}{{(c+d\,x)}^2}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x)^2,x)
```

output
```
int(cosh(a + b*x)/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^2}\,dx = \frac{e^{2a}\left(\int \frac{e^{bx}}{d^2x^2+2cdx+c^2}dx\right)+\int \frac{1}{e^{bx}c^2+2e^{bx}cdx+e^{bx}d^2x^2}dx}{2e^a}$$

input
```
int(cosh(b*x+a)/(d*x+c)^2,x)
```

output
```
(e**(2*a)*int(e**(b*x)/(c**2 + 2*c*d*x + d**2*x**2),x) + int(1/(e**(b*x)*c
**2 + 2*e**(b*x)*c*d*x + e**(b*x)*d**2*x**2),x))/(2*e**a)
```

3.7 $\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx = -\frac{\cosh(a+bx)}{2d(c+dx)^2} + \frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{2d^3} - \frac{b\sinh(a+bx)}{2d^2(c+dx)} + \frac{b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{2d^3}$$

output
```
-1/2*cosh(b*x+a)/d/(d*x+c)^2+1/2*b^2*cosh(a-b*c/d)*Chi(b*c/d+b*x)/d^3-1/2*
b*sinh(b*x+a)/d^2/(d*x+c)+1/2*b^2*sinh(a-b*c/d)*Shi(b*c/d+b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx = \frac{b^2\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right) - \frac{d(d\cosh(a+bx)+b(c+dx)\sinh(a+bx))}{(c+dx)^2} + b^2\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x)^3,x]
```

output
```
(b^2*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] - (d*(d*Cosh[a + b*x] + b
*(c + d*x)*Sinh[a + b*x]))/(c + d*x)^2 + b^2*Sinh[a - (b*c)/d]*SinhIntegra
l[b*(c/d + x)])/(2*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.67 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 3778, 26, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^3}dx$$

$$\Big\downarrow \ 3778$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}+\frac{ib\int -\frac{i\sinh(a+bx)}{(c+dx)^2}dx}{2d}$$

$$\Big\downarrow \ 26$$

$$\frac{b\int \frac{\sinh(a+bx)}{(c+dx)^2}dx}{2d}-\frac{\cosh(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \ 3042$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}+\frac{b\int -\frac{i\sin(ia+ibx)}{(c+dx)^2}dx}{2d}$$

$$\Big\downarrow \ 26$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\int \frac{\sin(ia+ibx)}{(c+dx)^2}dx}{2d}$$

$$\Big\downarrow \ 3778$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\int\frac{\cosh(a+bx)}{c+dx}dx}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{c+dx}dx}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\left(\cosh\left(a-\frac{bc}{d}\right)\int\frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int\frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\left(\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\left(\sinh\left(a-\frac{bc}{d}\right)\int-\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\left(\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\left(\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\cosh(a+bx)}{2d(c+dx)^2}-\frac{ib\left(\frac{ib\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d}-\frac{i\sinh(a+bx)}{d(c+dx)}\right)}{2d}$$

input
```
Int[Cosh[a + b*x]/(c + d*x)^3,x]
```

output
```
-1/2*Cosh[a + b*x]/(d*(c + d*x)^2) - ((I/2)*b*(((-I)*Sinh[a + b*x])/(d*(c
+ d*x)) + (I*b*((Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/d + (Sinh[
a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d))/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(96) = 192$.

Time = 0.53 (sec) , antiderivative size = 277, normalized size of antiderivative = 2.66

method	result
risch	$\frac{b^3 e^{-bx-a} x}{4d(d^2x^2b^2+2b^2cdx+b^2c^2)} + \frac{b^3 e^{-bx-a} c}{4d^2(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{b^2 e^{-bx-a}}{4d(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{b^2 e^{-\frac{ad-cb}{d}} \text{expIntegral}_1\left(bx+a-\frac{ad-}{}\right.}{4d^3}$

input
```
int(cosh(b*x+a)/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x+1/4*b^3*exp(-b*x
-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/4*b^2*exp(-b*x-a)/d/(b^2*d^2
*x^2+2*b^2*c*d*x+b^2*c^2)-1/4*b^2/d^3*exp(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*
c)/d)-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)^2-1/4*b^2/d^3*exp(b*x+a)/(b*c/d+b
*x)-1/4*b^2/d^3*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(96) = 192$.

Time = 0.09 (sec) , antiderivative size = 254, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,d^2\cosh\left(bx+a\right) - \left(\left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\text{Ei}\left(\frac{bdx+bc}{d}\right) + \left(b^2d^2x^2+2\,b^2cdx+b^2c^2\right)\text{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cos}{}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*d^2*cosh(b*x + a) - ((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei((b*d
*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-(b*d*x + b*c)/d))
*cosh(-(b*c - a*d)/d) + 2*(b*d^2*x + b*c*d)*sinh(b*x + a) - ((b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*
x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx = \frac{b\left(\frac{e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{(dx+c)d} - \frac{e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{(dx+c)d}\right)}{4\,d} - \frac{\cosh\left(bx+a\right)}{2\,(dx+c)^2d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="maxima")
```

output
```
1/4*b*(e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - e^(
a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d))/d - 1/2*cosh(b
*x + a)/((d*x + c)^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(96) = 192$.

Time = 0.14 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.87

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{b^2d^2x^2\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + b^2d^2x^2\text{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + 2\,b^2cdx\text{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + 2\,b^2cdx\text{Ei}\left(-\frac{bdx+bc}{d}\right)}{}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(b^2*d^2*x^2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + b^2*d^2*x^2*Ei(-(b*d*
x + b*c)/d)*e^(-a + b*c/d) + 2*b^2*c*d*x*Ei((b*d*x + b*c)/d)*e^(a - b*c/d)
 + 2*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + b^2*c^2*Ei((b*d*x + b
*c)/d)*e^(a - b*c/d) + b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) - b*d^2
*x*e^(b*x + a) + b*d^2*x*e^(-b*x - a) - b*c*d*e^(b*x + a) + b*c*d*e^(-b*x
- a) - d^2*e^(b*x + a) - d^2*e^(-b*x - a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cosh(a+b\,x)}{{(c+d\,x)}^3}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x)^3,x)
```

output
```
int(cosh(a + b*x)/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^3}\,dx = \frac{e^{2a}\left(\int \frac{e^{bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)+\int \frac{1}{e^{bx}c^3+3e^{bx}c^2dx+3e^{bx}c\,d^2x^2+e^{bx}d^3x^3}dx}{2e^a}$$

input
```
int(cosh(b*x+a)/(d*x+c)^3,x)
```

output
```
(e**(2*a)*int(e**(b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)
+ int(1/(e**(b*x)*c**3 + 3*e**(b*x)*c**2*d*x + 3*e**(b*x)*c*d**2*x**2 + e*
*(b*x)*d**3*x**3),x))/(2*e**a)
```

3.8 $\int (c+dx)^4 \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 162

$$\int (c+dx)^4 \cosh^2(a+bx)\,dx = \frac{3d^4x}{4b^4}+\frac{d(c+dx)^3}{2b^2}+\frac{(c+dx)^5}{10d}-\frac{3d^3(c+dx)\cosh^2(a+bx)}{2b^4}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{3d^4\cosh(a+bx)\sinh(a+bx)}{4b^5}+\frac{3d^2(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b^3}+\frac{(c+dx)^4\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
3/4*d^4*x/b^4+1/2*d*(d*x+c)^3/b^2+1/10*(d*x+c)^5/d-3/2*d^3*(d*x+c)*cosh(b*
x+a)^2/b^4-d*(d*x+c)^3*cosh(b*x+a)^2/b^2+3/4*d^4*cosh(b*x+a)*sinh(b*x+a)/b
^5+3/2*d^2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^4*cosh(b*x+a)
*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.81

$$\int (c+dx)^4\cosh^2(a+bx)\,dx$$
$$=\frac{8b^5x(5c^4+10c^3dx+10c^2d^2x^2+5cd^3x^3+d^4x^4)-20bd(c+dx)\left(3d^2+2b^2(c+dx)^2\right)\cosh(2(a+bx))+}{80b^5}$$

input
```
Integrate[(c + d*x)^4*Cosh[a + b*x]^2,x]
```

output
```
(8*b^5*x*(5*c^4 + 10*c^3*d*x + 10*c^2*d^2*x^2 + 5*c*d^3*x^3 + d^4*x^4) - 2
0*b*d*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + 10*(3*d^4
+ 6*b^2*d^2*(c + d*x)^2 + 2*b^4*(c + d*x)^4)*Sinh[2*(a + b*x)])/(80*b^5)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.562, Rules used = {3042, 3792, 17, 3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\cosh^2(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (c+dx)^4\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$
$$\Big\downarrow \text{ 3792}$$
$$\frac{3d^2\int (c+dx)^2\cosh^2(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^4dx-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}$$
$$\Big\downarrow \text{ 17}$$

$$\frac{3d^2\int(c+dx)^2\cosh^2(a+bx)dx}{b^2}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{b^2}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3792}$$

$$\frac{3d^2\left(\frac{d^2\int\cosh^2(a+bx)dx}{2b^2}+\frac{1}{2}\int(c+dx)^2dx-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{b^2}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{17}$$

$$\frac{3d^2\left(\frac{d^2\int\cosh^2(a+bx)dx}{2b^2}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\left(\frac{d^2\int\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b^2}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3d^2\left(\frac{d^2\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

$$\Big\downarrow \text{24}$$

$$\frac{3d^2\left(-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{d^2\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}\right)}{b^2}-\frac{d(c+dx)^3\cosh^2(a+bx)}{b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^5}{10d}$$

input
```
Int[(c + d*x)^4*Cosh[a + b*x]^2,x]
```

output
```
(c + d*x)^5/(10*d) - (d*(c + d*x)^3*Cosh[a + b*x]^2)/b^2 + ((c + d*x)^4*Co
sh[a + b*x]*Sinh[a + b*x])/(2*b) + (3*d^2*((c + d*x)^3/(6*d) - (d*(c + d*x
)*Cosh[a + b*x]^2)/(2*b^2) + ((c + d*x)^2*Cosh[a + b*x]*Sinh[a + b*x])/(2*
b) + (d^2*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/(2*b^2)))/b^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.90

method	result
parallelrisch	$\frac{\left(2(dx+c)^4b^4+6d^2(dx+c)^2b^2+3d^4\right)\sinh(2bx+2a)+4\left(-\left((dx+c)^2b^2+\frac{3d^2}{2}\right)(dx+c)d\cosh(2bx+2a)+x\left(\frac{1}{5}d^4x^4+c\,d^3x^3\right.\right.}{8b^5}$
risch	$\frac{d^4x^5}{10}+\frac{d^3c\,x^4}{2}+d^2c^2x^3+d\,c^3x^2+\frac{c^4x}{2}+\frac{c^5}{10d}+\frac{(2d^4x^4b^4+8b^4c\,d^3x^3+12b^4c^2d^2x^2-4b^3d^4x^3+8b^4c^3dx-}{}$
orering	$\frac{(2b^6d^6x^7+14b^6c\,d^5x^6+42b^6c^2d^4x^5+70b^6c^3d^3x^4+70b^6c^4d^2x^3-20b^4d^6x^5+40b^6c^5d\,x^2-100b^4c\,d^5x^4+10b^6c^6x-200b^4}{10}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^4*b^4+6*d^2*(d*x+c)^2*b^2+3*d^4)*sinh(2*b*x+2*a)+4*(-((d*x
+c)^2*b^2+3/2*d^2)*(d*x+c)*d*cosh(2*b*x+2*a)+x*(1/5*d^4*x^4+c*d^3*x^3+2*c^
2*d^2*x^2+2*c^3*d*x+c^4)*b^4+b^2*c^3*d+3/2*d^3*c)*b)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 312 vs. 2(148) = 296.

Time = 0.09 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.93

$$\int (c+dx)^4\cosh^2(a+bx)\,dx$$
$$=\frac{2\,b^5d^4x^5+10\,b^5cd^3x^4+20\,b^5c^2d^2x^3+20\,b^5c^3dx^2+10\,b^5c^4x-5\,(2\,b^3d^4x^3+6\,b^3cd^3x^2+2\,b^3c^3d+3\,bcd^3}{}$$

input
```
integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/20*(2*b^5*d^4*x^5 + 10*b^5*c*d^3*x^4 + 20*b^5*c^2*d^2*x^3 + 20*b^5*c^3*d
*x^2 + 10*b^5*c^4*x - 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3
*b*c*d^3 + 3*(2*b^3*c^2*d^2 + b*d^4)*x)*cosh(b*x + a)^2 + 5*(2*b^4*d^4*x^4
 + 8*b^4*c*d^3*x^3 + 2*b^4*c^4 + 6*b^2*c^2*d^2 + 3*d^4 + 6*(2*b^4*c^2*d^2
+ b^2*d^4)*x^2 + 4*(2*b^4*c^3*d + 3*b^2*c*d^3)*x)*cosh(b*x + a)*sinh(b*x +
 a) - 5*(2*b^3*d^4*x^3 + 6*b^3*c*d^3*x^2 + 2*b^3*c^3*d + 3*b*c*d^3 + 3*(2*
b^3*c^2*d^2 + b*d^4)*x)*sinh(b*x + a)^2)/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 660 vs. $2(156) = 312$.

Time = 0.49 (sec) , antiderivative size = 660, normalized size of antiderivative = 4.07

$$\int (c+dx)^4 \cosh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cosh(b*x+a)**2,x)
```

output
```
Piecewise((-c**4*x*sinh(a + b*x)**2/2 + c**4*x*cosh(a + b*x)**2/2 - c**3*d
*x**2*sinh(a + b*x)**2 + c**3*d*x**2*cosh(a + b*x)**2 - c**2*d**2*x**3*sin
h(a + b*x)**2 + c**2*d**2*x**3*cosh(a + b*x)**2 - c*d**3*x**4*sinh(a + b*x
)**2/2 + c*d**3*x**4*cosh(a + b*x)**2/2 - d**4*x**5*sinh(a + b*x)**2/10 +
d**4*x**5*cosh(a + b*x)**2/10 + c**4*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 2
*c**3*d*x*sinh(a + b*x)*cosh(a + b*x)/b + 3*c**2*d**2*x**2*sinh(a + b*x)*c
osh(a + b*x)/b + 2*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/b + d**4*x**4*s
inh(a + b*x)*cosh(a + b*x)/(2*b) - c**3*d*sinh(a + b*x)**2/b**2 - 3*c**2*d
**2*x*sinh(a + b*x)**2/(2*b**2) - 3*c**2*d**2*x*cosh(a + b*x)**2/(2*b**2)
- 3*c*d**3*x**2*sinh(a + b*x)**2/(2*b**2) - 3*c*d**3*x**2*cosh(a + b*x)**2
/(2*b**2) - d**4*x**3*sinh(a + b*x)**2/(2*b**2) - d**4*x**3*cosh(a + b*x)*
*2/(2*b**2) + 3*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)/(2*b**3) + 3*c*d**3*
x*sinh(a + b*x)*cosh(a + b*x)/b**3 + 3*d**4*x**2*sinh(a + b*x)*cosh(a + b*
x)/(2*b**3) - 3*c*d**3*sinh(a + b*x)**2/(2*b**4) - 3*d**4*x*sinh(a + b*x)*
*2/(4*b**4) - 3*d**4*x*cosh(a + b*x)**2/(4*b**4) + 3*d**4*sinh(a + b*x)*co
sh(a + b*x)/(4*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**2*d**2*x*
*3 + c*d**3*x**4 + d**4*x**5/5)*cosh(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(148) = 296$.

Time = 0.07 (sec) , antiderivative size = 382, normalized size of antiderivative = 2.36

$$\int (c+dx)^4 \cosh^2(a+bx)\,dx$$
$$= \frac{1}{4}\left(4\,x^2 + \frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{b^2} - \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)c^3d$$
$$+\frac{1}{8}\left(8\,x^3 + \frac{3\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{b^3} - \frac{3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{b^3}\right)c^2d^2$$
$$+\frac{1}{8}\left(4\,x^4 + \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^4} - \frac{\left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx-2}}{b^4}\right.$$
$$+\frac{1}{80}\left(8\,x^5 + \frac{5\left(2\,b^4x^4e^{(2\,a)} - 4\,b^3x^3e^{(2\,a)} + 6\,b^2x^2e^{(2\,a)} - 6\,bxe^{(2\,a)} + 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^5} - \frac{5\left(2\,b^4x^4 + 4\,b^3x^3 + \right.}{}\right.$$
$$+\frac{1}{8}\,c^4\left(4\,x + \frac{e^{(2\,bx+2\,a)}}{b} - \frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/4*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c^3*d + 1/8*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
 e^(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*
c^2*d^2 + 1/8*(4*x^4 + (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(2
*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2
*b*x - 2*a)/b^4)*c*d^3 + 1/80*(8*x^5 + 5*(2*b^4*x^4*e^(2*a) - 4*b^3*x^3*e^
(2*a) + 6*b^2*x^2*e^(2*a) - 6*b*x*e^(2*a) + 3*e^(2*a))*e^(2*b*x)/b^5 - 5*(
2*b^4*x^4 + 4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^5)*d^4 +
 1/8*c^4*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(148) = 296$.

Time = 0.12 (sec) , antiderivative size = 372, normalized size of antiderivative = 2.30

$$\int (c+dx)^4 \cosh^2(a+bx)\,dx = \frac{1}{10}d^4x^5 + \frac{1}{2}cd^3x^4 + c^2d^2x^3 + c^3dx^2 + \frac{1}{2}c^4x$$
$$+ \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 8\,b^4c^3dx - 12\,b^3cd^3x^2 + 2\,b^4c^4 - 12\,b^3c^2d^2x + 6\,b^2d^4x}{16\,b^5}$$
$$- \frac{(2\,b^4d^4x^4 + 8\,b^4cd^3x^3 + 12\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 8\,b^4c^3dx + 12\,b^3cd^3x^2 + 2\,b^4c^4 + 12\,b^3c^2d^2x + 6\,b^2d^4x}{16\,b^5}$$

input

```
integrate((d*x+c)^4*cosh(b*x+a)^2,x, algorithm="giac")
```

output

```
1/10*d^4*x^5 + 1/2*c*d^3*x^4 + c^2*d^2*x^3 + c^3*d*x^2 + 1/2*c^4*x + 1/16*
(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 8*
b^4*c^3*d*x - 12*b^3*c*d^3*x^2 + 2*b^4*c^4 - 12*b^3*c^2*d^2*x + 6*b^2*d^4*
x^2 - 4*b^3*c^3*d + 12*b^2*c*d^3*x + 6*b^2*c^2*d^2 - 6*b*d^4*x - 6*b*c*d^3
 + 3*d^4)*e^(2*b*x + 2*a)/b^5 - 1/16*(2*b^4*d^4*x^4 + 8*b^4*c*d^3*x^3 + 12
*b^4*c^2*d^2*x^2 + 4*b^3*d^4*x^3 + 8*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + 2*b^
4*c^4 + 12*b^3*c^2*d^2*x + 6*b^2*d^4*x^2 + 4*b^3*c^3*d + 12*b^2*c*d^3*x +
6*b^2*c^2*d^2 + 6*b*d^4*x + 6*b*c*d^3 + 3*d^4)*e^(-2*b*x - 2*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.05

$$\int (c+dx)^4\cosh^2(a+bx)\,dx = \frac{c^4\,x}{2}+\frac{d^4\,x^5}{10}+c^3\,d\,x^2+\frac{c\,d^3\,x^4}{2}+\frac{c^4\sinh(2\,a+2\,b\,x)}{4\,b}+\frac{3\,d^4\sinh(2\,a+2\,b\,x)}{8\,b^5}+c^2\,d^2\,x^3-\frac{c^3\,d\cosh(2\,a+2\,b\,x)}{2\,b^2}-\frac{3\,c\,d^3\cosh(2\,a+2\,b\,x)}{4\,b^4}-\frac{3\,d^4\,x\cosh(2\,a+2\,b\,x)}{4\,b^4}+\frac{3\,c^2\,d^2\sinh(2\,a+2\,b\,x)}{4\,b^3}-\frac{d^4\,x^3\cosh(2\,a+2\,b\,x)}{2\,b^2}+\frac{d^4\,x^4\sinh(2\,a+2\,b\,x)}{4\,b}+\frac{3\,d^4\,x^2\sinh(2\,a+2\,b\,x)}{4\,b^3}+\frac{3\,c^2\,d^2\,x^2\sinh(2\,a+2\,b\,x)}{2\,b}+\frac{c^3\,d\,x\sinh(2\,a+2\,b\,x)}{b}+\frac{3\,c\,d^3\,x\sinh(2\,a+2\,b\,x)}{2\,b^3}-\frac{3\,c^2\,d^2\,x\cosh(2\,a+2\,b\,x)}{2\,b^2}-\frac{3\,c\,d^3\,x^2\cosh(2\,a+2\,b\,x)}{2\,b^2}+\frac{c\,d^3\,x^3\sinh(2\,a+2\,b\,x)}{b}$$

input
```
int(cosh(a + b*x)^2*(c + d*x)^4,x)
```

output
```
(c^4*x)/2 + (d^4*x^5)/10 + c^3*d*x^2 + (c*d^3*x^4)/2 + (c^4*sinh(2*a + 2*b
*x))/(4*b) + (3*d^4*sinh(2*a + 2*b*x))/(8*b^5) + c^2*d^2*x^3 - (c^3*d*cosh
(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*cosh(2*a + 2*b*x))/(4*b^4) - (3*d^4*x*co
sh(2*a + 2*b*x))/(4*b^4) + (3*c^2*d^2*sinh(2*a + 2*b*x))/(4*b^3) - (d^4*x^
3*cosh(2*a + 2*b*x))/(2*b^2) + (d^4*x^4*sinh(2*a + 2*b*x))/(4*b) + (3*d^4*
x^2*sinh(2*a + 2*b*x))/(4*b^3) + (3*c^2*d^2*x^2*sinh(2*a + 2*b*x))/(2*b) +
 (c^3*d*x*sinh(2*a + 2*b*x))/b + (3*c*d^3*x*sinh(2*a + 2*b*x))/(2*b^3) - (
3*c^2*d^2*x*cosh(2*a + 2*b*x))/(2*b^2) - (3*c*d^3*x^2*cosh(2*a + 2*b*x))/(
2*b^2) + (c*d^3*x^3*sinh(2*a + 2*b*x))/b
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 576, normalized size of antiderivative = 3.56

$$\int (c+dx)^4 \cosh^2(a+bx)\,dx$$
$$= \frac{10e^{4bx+4a}b^4d^4x^4 - 20e^{4bx+4a}b^3c^3d - 20e^{4bx+4a}b^3d^4x^3 + 30e^{4bx+4a}b^2c^2d^2 + 15e^{4bx+4a}d^4 - 10b^4c^4 + 40e^{4bx+}}{}$$

input
```
int((d*x+c)^4*cosh(b*x+a)^2,x)
```

output
```
(10*e**(4*a + 4*b*x)*b**4*c**4 + 40*e**(4*a + 4*b*x)*b**4*c**3*d*x + 60*e*
*(4*a + 4*b*x)*b**4*c**2*d**2*x**2 + 40*e**(4*a + 4*b*x)*b**4*c*d**3*x**3
+ 10*e**(4*a + 4*b*x)*b**4*d**4*x**4 - 20*e**(4*a + 4*b*x)*b**3*c**3*d - 6
0*e**(4*a + 4*b*x)*b**3*c**2*d**2*x - 60*e**(4*a + 4*b*x)*b**3*c*d**3*x**2
 - 20*e**(4*a + 4*b*x)*b**3*d**4*x**3 + 30*e**(4*a + 4*b*x)*b**2*c**2*d**2
 + 60*e**(4*a + 4*b*x)*b**2*c*d**3*x + 30*e**(4*a + 4*b*x)*b**2*d**4*x**2
- 30*e**(4*a + 4*b*x)*b*c*d**3 - 30*e**(4*a + 4*b*x)*b*d**4*x + 15*e**(4*a
 + 4*b*x)*d**4 + 40*e**(2*a + 2*b*x)*b**5*c**4*x + 80*e**(2*a + 2*b*x)*b**
5*c**3*d*x**2 + 80*e**(2*a + 2*b*x)*b**5*c**2*d**2*x**3 + 40*e**(2*a + 2*b
*x)*b**5*c*d**3*x**4 + 8*e**(2*a + 2*b*x)*b**5*d**4*x**5 - 10*b**4*c**4 -
40*b**4*c**3*d*x - 60*b**4*c**2*d**2*x**2 - 40*b**4*c*d**3*x**3 - 10*b**4*
d**4*x**4 - 20*b**3*c**3*d - 60*b**3*c**2*d**2*x - 60*b**3*c*d**3*x**2 - 2
0*b**3*d**4*x**3 - 30*b**2*c**2*d**2 - 60*b**2*c*d**3*x - 30*b**2*d**4*x**
2 - 30*b*c*d**3 - 30*b*d**4*x - 15*d**4)/(80*e**(2*a + 2*b*x)*b**5)
```

3.9 $\int (c+dx)^3 \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 124

$$\begin{aligned}\int (c+dx)^3 \cosh^2(a+bx)\,dx = {} & \frac{3d(c+dx)^2}{8b^2}+\frac{(c+dx)^4}{8d}-\frac{3d^3\cosh^2(a+bx)}{8b^4} \\ & -\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2} \\ & +\frac{3d^2(c+dx)\cosh(a+bx)\sinh(a+bx)}{4b^3} \\ & +\frac{(c+dx)^3\cosh(a+bx)\sinh(a+bx)}{2b}\end{aligned}$$

output
```
3/8*d*(d*x+c)^2/b^2+1/8*(d*x+c)^4/d-3/8*d^3*cosh(b*x+a)^2/b^4-3/4*d*(d*x+c
)^2*cosh(b*x+a)^2/b^2+3/4*d^2*(d*x+c)*cosh(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x
+c)^3*cosh(b*x+a)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.84

$$\int (c+dx)^3 \cosh^2(a+bx)\,dx$$
$$= \frac{2b^4x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)-3d(d^2+2b^2(c+dx)^2)\cosh(2(a+bx))+2b(c+dx)\left(3d^2+2b^2(c+\right.}{16b^4}$$

input
```
Integrate[(c + d*x)^3*Cosh[a + b*x]^2,x]
```

output
```
(2*b^4*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) - 3*d*(d^2 + 2*b^2*(c
 + d*x)^2)*Cosh[2*(a + b*x)] + 2*b*(c + d*x)*(3*d^2 + 2*b^2*(c + d*x)^2)*S
inh[2*(a + b*x)])/(16*b^4)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3792, 17, 3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \cosh^2(a+bx)\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int (c+dx)^3 \sin\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$
$$\Big\downarrow \text{ 3792}$$
$$\frac{3d^2\int (c+dx)\cosh^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^3dx-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}$$
$$\Big\downarrow \text{ 17}$$

$$\frac{3d^2\int(c+dx)\cosh^2(a+bx)dx}{2b^2}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b^2}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{3791}$$

$$\frac{3d^2\left(\frac{1}{2}\int(c+dx)dx-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

$$\Big\downarrow \text{17}$$

$$\frac{3d^2\left(-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^2}{4d}\right)}{2b^2}-\frac{3d(c+dx)^2\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^4}{8d}$$

input
```
Int[(c + d*x)^3*Cosh[a + b*x]^2,x]
```

output
```
(c + d*x)^4/(8*d) - (3*d*(c + d*x)^2*Cosh[a + b*x]^2)/(4*b^2) + ((c + d*x)
^3*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (3*d^2*((c + d*x)^2/(4*d) - (d*Cos
h[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/(2
*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

method	result
parallelrisch	$\frac{4(dx+c)b\left(\left(dx+c\right)^{2}b^{2}+\frac{3d^{2}}{2}\right)\sinh\left(2bx+2a\right)-6d\left(\left(dx+c\right)^{2}b^{2}+\frac{d^{2}}{2}\right)\cosh\left(2bx+2a\right)+2\left(d^{3}x^{4}+4d^{2}c\,x^{3}+6d\,c^{2}x^{2}+4c^{3}x\right)b^{4}}{16b^{4}}$
risch	$\frac{d^{3}x^{4}}{8}+\frac{d^{2}c\,x^{3}}{2}+\frac{3d\,c^{2}x^{2}}{4}+\frac{c^{3}x}{2}+\frac{c^{4}}{8d}+\frac{\left(4d^{3}x^{3}b^{3}+12b^{3}c\,d^{2}x^{2}+12b^{3}c^{2}dx-6b^{2}d^{3}x^{2}+4b^{3}c^{3}-12b^{2}c\,d^{2}x-6b^{2}c^{2}\right.}{32b^{4}}$
orering	$\frac{\left(b^{4}d^{5}x^{6}+6b^{4}c\,d^{4}x^{5}+15b^{4}c^{2}d^{3}x^{4}+20b^{4}c^{3}d^{2}x^{3}+14b^{4}c^{4}d\,x^{2}-6b^{2}d^{5}x^{4}+4b^{4}c^{5}x-24b^{2}c\,d^{4}x^{3}-39b^{2}c^{2}d^{3}x^{2}-30b^{2}c^{3}d^{2}x-\right.}{4b^{4}\left(dx+c\right)^{2}}$
derivativedivides	$\frac{d^{3}\left(\frac{\left(bx+a\right)^{3}\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2}+\frac{\left(bx+a\right)^{4}}{8}-\frac{3\left(bx+a\right)^{2}\cosh\left(bx+a\right)^{2}}{4}+\frac{3\left(bx+a\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{4}+\frac{3\left(bx+a\right)^{2}}{8}-\frac{3\cosh\left(bx\right.}{8}\right.}{b^{3}}$
default	$\frac{d^{3}\left(\frac{\left(bx+a\right)^{3}\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{2}+\frac{\left(bx+a\right)^{4}}{8}-\frac{3\left(bx+a\right)^{2}\cosh\left(bx+a\right)^{2}}{4}+\frac{3\left(bx+a\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)}{4}+\frac{3\left(bx+a\right)^{2}}{8}-\frac{3\cosh\left(bx\right.}{8}\right.}{b^{3}}$

input
```
int((d*x+c)^3*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*(d*x+c)*b*((d*x+c)^2*b^2+3/2*d^2)*sinh(2*b*x+2*a)-6*d*((d*x+c)^2*b
^2+1/2*d^2)*cosh(2*b*x+2*a)+2*(d^3*x^4+4*c*d^2*x^3+6*c^2*d*x^2+4*c^3*x)*b^
4+6*b^2*c^2*d+3*d^3)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.69

$$\int (c+dx)^3\cosh^2(a+bx)\,dx$$
$$=\frac{2\,b^4d^3x^4+8\,b^4cd^2x^3+12\,b^4c^2dx^2+8\,b^4c^3x-3\,(2\,b^2d^3x^2+4\,b^2cd^2x+2\,b^2c^2d+d^3)\cosh\,(bx+a)^2+4\,(2}{}$$

input
```
integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/16*(2*b^4*d^3*x^4 + 8*b^4*c*d^2*x^3 + 12*b^4*c^2*d*x^2 + 8*b^4*c^3*x - 3
*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x + a)^2 + 4*(
2*b^3*d^3*x^3 + 6*b^3*c*d^2*x^2 + 2*b^3*c^3 + 3*b*c*d^2 + 3*(2*b^3*c^2*d +
 b*d^3)*x)*cosh(b*x + a)*sinh(b*x + a) - 3*(2*b^2*d^3*x^2 + 4*b^2*c*d^2*x
+ 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2)/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(119) = 238.

Time = 0.39 (sec) , antiderivative size = 456, normalized size of antiderivative = 3.68

$$\int (c+dx)^3\cosh^2(a+bx)\,dx$$
$$=\begin{cases}-\frac{c^3x\sinh^2(a+bx)}{2}+\frac{c^3x\cosh^2(a+bx)}{2}-\frac{3c^2dx^2\sinh^2(a+bx)}{4}+\frac{3c^2dx^2\cosh^2(a+bx)}{4}-\frac{cd^2x^3\sinh^2(a+bx)}{2}+\frac{cd^2x^3\cosh^2(a+bx)}{2} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cosh^2(a)\end{cases}$$

input
```
integrate((d*x+c)**3*cosh(b*x+a)**2,x)
```

output
```
Piecewise((-c**3*x*sinh(a + b*x)**2/2 + c**3*x*cosh(a + b*x)**2/2 - 3*c**2
*d*x**2*sinh(a + b*x)**2/4 + 3*c**2*d*x**2*cosh(a + b*x)**2/4 - c*d**2*x**
3*sinh(a + b*x)**2/2 + c*d**2*x**3*cosh(a + b*x)**2/2 - d**3*x**4*sinh(a +
 b*x)**2/8 + d**3*x**4*cosh(a + b*x)**2/8 + c**3*sinh(a + b*x)*cosh(a + b*
x)/(2*b) + 3*c**2*d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) + 3*c*d**2*x**2*si
nh(a + b*x)*cosh(a + b*x)/(2*b) + d**3*x**3*sinh(a + b*x)*cosh(a + b*x)/(2
*b) - 3*c**2*d*sinh(a + b*x)**2/(4*b**2) - 3*c*d**2*x*sinh(a + b*x)**2/(4*
b**2) - 3*c*d**2*x*cosh(a + b*x)**2/(4*b**2) - 3*d**3*x**2*sinh(a + b*x)**
2/(8*b**2) - 3*d**3*x**2*cosh(a + b*x)**2/(8*b**2) + 3*c*d**2*sinh(a + b*x
)*cosh(a + b*x)/(4*b**3) + 3*d**3*x*sinh(a + b*x)*cosh(a + b*x)/(4*b**3) -
 3*d**3*sinh(a + b*x)**2/(8*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d*x**2/2 +
 c*d**2*x**3 + d**3*x**4/4)*cosh(a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(112) = 224$.

Time = 0.07 (sec) , antiderivative size = 263, normalized size of antiderivative = 2.12

$$\int (c+dx)^3\cosh^2(a+bx)\,dx$$
$$= \frac{3}{16}\left(4\,x^2 + \frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{b^2} - \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)c^2d$$
$$+ \frac{1}{16}\left(8\,x^3 + \frac{3\left(2\,b^2x^2e^{(2\,a)} - 2\,bxe^{(2\,a)} + e^{(2\,a)}\right)e^{(2\,bx)}}{b^3} - \frac{3\left(2\,b^2x^2 + 2\,bx + 1\right)e^{(-2\,bx-2\,a)}}{b^3}\right)cd^2$$
$$+ \frac{1}{32}\left(4\,x^4 + \frac{\left(4\,b^3x^3e^{(2\,a)} - 6\,b^2x^2e^{(2\,a)} + 6\,bxe^{(2\,a)} - 3\,e^{(2\,a)}\right)e^{(2\,bx)}}{b^4} - \frac{\left(4\,b^3x^3 + 6\,b^2x^2 + 6\,bx + 3\right)e^{(-2\,bx-}}{b^4}\right.$$
$$+ \frac{1}{8}\,c^3\left(4\,x + \frac{e^{(2\,bx+2\,a)}}{b} - \frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
3/16*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*c^2*d + 1/16*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a)
 + e^(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3
)*c*d^2 + 1/32*(4*x^4 + (4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a) + 6*b*x*e^(
2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - (4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-
2*b*x - 2*a)/b^4)*d^3 + 1/8*c^3*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a
)/b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(112) = 224.

Time = 0.12 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.96

$$\int (c+dx)^3\cosh^2(a+bx)\,dx = \frac{1}{8}d^3x^4+\frac{1}{2}cd^2x^3+\frac{3}{4}c^2dx^2+\frac{1}{2}c^3x + \frac{(4b^3d^3x^3+12b^3cd^2x^2+12b^3c^2dx-6b^2d^3x^2+4b^3c^3-12b^2cd^2x-6b^2c^2d+6bd^3x+6bcd^2-3d^3)e}{32b^4} - \frac{(4b^3d^3x^3+12b^3cd^2x^2+12b^3c^2dx+6b^2d^3x^2+4b^3c^3+12b^2cd^2x+6b^2c^2d+6bd^3x+6bcd^2+3d^3)e}{32b^4}$$

input
```
integrate((d*x+c)^3*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/8*d^3*x^4 + 1/2*c*d^2*x^3 + 3/4*c^2*d*x^2 + 1/2*c^3*x + 1/32*(4*b^3*d^3*
x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x - 6*b^2*d^3*x^2 + 4*b^3*c^3 - 12*b
^2*c*d^2*x - 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 3*d^3)*e^(2*b*x + 2*a)/
b^4 - 1/32*(4*b^3*d^3*x^3 + 12*b^3*c*d^2*x^2 + 12*b^3*c^2*d*x + 6*b^2*d^3*
x^2 + 4*b^3*c^3 + 12*b^2*c*d^2*x + 6*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 3
*d^3)*e^(-2*b*x - 2*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.85

$$\int (c+dx)^3 \cosh^2(a+bx)\,dx$$
$$= \frac{4\,b^4\,c^3\,x-\frac{3\,d^3\cosh(2\,a+2\,b\,x)}{2}+2\,b^3\,c^3\sinh(2\,a+2\,b\,x)+b^4\,d^3\,x^4-3\,b^2\,c^2\,d\cosh(2\,a+2\,b\,x)+6\,b^4\,c^2\,d\,x^2+}{}$$

input
```
int(cosh(a + b*x)^2*(c + d*x)^3,x)
```

output
```
(4*b^4*c^3*x - (3*d^3*cosh(2*a + 2*b*x))/2 + 2*b^3*c^3*sinh(2*a + 2*b*x) +
 b^4*d^3*x^4 - 3*b^2*c^2*d*cosh(2*a + 2*b*x) + 6*b^4*c^2*d*x^2 + 4*b^4*c*d
^2*x^3 - 3*b^2*d^3*x^2*cosh(2*a + 2*b*x) + 2*b^3*d^3*x^3*sinh(2*a + 2*b*x)
 + 3*b*c*d^2*sinh(2*a + 2*b*x) + 3*b*d^3*x*sinh(2*a + 2*b*x) - 6*b^2*c*d^2
*x*cosh(2*a + 2*b*x) + 6*b^3*c^2*d*x*sinh(2*a + 2*b*x) + 6*b^3*c*d^2*x^2*s
inh(2*a + 2*b*x))/(8*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 382, normalized size of antiderivative = 3.08

$$\int (c+dx)^3 \cosh^2(a+bx)\,dx$$
$$= \frac{4e^{4bx+4a}b^3c^3+12e^{4bx+4a}b^3c^2dx+12e^{4bx+4a}b^3c\,d^2x^2+4e^{4bx+4a}b^3d^3x^3-6e^{4bx+4a}b^2c^2d-12e^{4bx+4a}b^2c\,d^2x}{}$$

input
```
int((d*x+c)^3*cosh(b*x+a)^2,x)
```

output
```
(4*e**(4*a + 4*b*x)*b**3*c**3 + 12*e**(4*a + 4*b*x)*b**3*c**2*d*x + 12*e**
(4*a + 4*b*x)*b**3*c*d**2*x**2 + 4*e**(4*a + 4*b*x)*b**3*d**3*x**3 - 6*e**
(4*a + 4*b*x)*b**2*c**2*d - 12*e**(4*a + 4*b*x)*b**2*c*d**2*x - 6*e**(4*a
+ 4*b*x)*b**2*d**3*x**2 + 6*e**(4*a + 4*b*x)*b*c*d**2 + 6*e**(4*a + 4*b*x)
*b*d**3*x - 3*e**(4*a + 4*b*x)*d**3 + 16*e**(2*a + 2*b*x)*b**4*c**3*x + 24
*e**(2*a + 2*b*x)*b**4*c**2*d*x**2 + 16*e**(2*a + 2*b*x)*b**4*c*d**2*x**3
+ 4*e**(2*a + 2*b*x)*b**4*d**3*x**4 - 4*b**3*c**3 - 12*b**3*c**2*d*x - 12*
b**3*c*d**2*x**2 - 4*b**3*d**3*x**3 - 6*b**2*c**2*d - 12*b**2*c*d**2*x - 6
*b**2*d**3*x**2 - 6*b*c*d**2 - 6*b*d**3*x - 3*d**3)/(32*e**(2*a + 2*b*x)*b
**4)
```

3.10 $\int (c+dx)^2 \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int (c+dx)^2 \cosh^2(a+bx)\,dx = \frac{d^2x}{4b^2} + \frac{(c+dx)^3}{6d} - \frac{d(c+dx)\cosh^2(a+bx)}{2b^2} + \frac{d^2\cosh(a+bx)\sinh(a+bx)}{4b^3} + \frac{(c+dx)^2\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
1/4*d^2*x/b^2+1/6*(d*x+c)^3/d-1/2*d*(d*x+c)*cosh(b*x+a)^2/b^2+1/4*d^2*cosh
(b*x+a)*sinh(b*x+a)/b^3+1/2*(d*x+c)^2*cosh(b*x+a)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.79

$$\int (c+dx)^2 \cosh^2(a+bx)\,dx = \frac{4b^3x(3c^2+3cdx+d^2x^2) - 6bd(c+dx)\cosh(2(a+bx)) + 3(d^2+2b^2(c+dx)^2)\sinh(2(a+bx))}{24b^3}$$

input
```
Integrate[(c + d*x)^2*Cosh[a + b*x]^2,x]
```

output
```
(4*b^3*x*(3*c^2 + 3*c*d*x + d^2*x^2) - 6*b*d*(c + d*x)*Cosh[2*(a + b*x)] +
 3*(d^2 + 2*b^2*(c + d*x)^2)*Sinh[2*(a + b*x)])/(24*b^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3792, 17, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \cosh^2(a+bx) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2 \sin\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{d^2 \int \cosh^2(a+bx)dx}{2b^2}+\frac{1}{2}\int (c+dx)^2dx-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$\frac{d^2 \int \cosh^2(a+bx)dx}{2b^2}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2 \int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2 dx}{2b^2}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \text{3115}$$

$$\frac{d^2\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

$$\Big\downarrow \; 24$$

$$-\frac{d(c+dx)\cosh^2(a+bx)}{2b^2}+\frac{d^2\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{2b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^3}{6d}$$

input
```
Int[(c + d*x)^2*Cosh[a + b*x]^2,x]
```

output
```
(c + d*x)^3/(6*d) - (d*(c + d*x)*Cosh[a + b*x]^2)/(2*b^2) + ((c + d*x)^2*C
osh[a + b*x]*Sinh[a + b*x])/(2*b) + (d^2*(x/2 + (Cosh[a + b*x]*Sinh[a + b*
x])/(2*b)))/(2*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{\left(2(dx+c)^2b^2+d^2\right)\sinh(2bx+2a)+4b\left(-\frac{d(dx+c)\cosh(2bx+2a)}{2}+x\left(\frac{1}{3}x^2d^2+cdx+c^2\right)b^2+\frac{cd}{2}\right)}{8b^3}$
risch	$\frac{d^2x^3}{6}+\frac{dc\,x^2}{2}+\frac{c^2x}{2}+\frac{c^3}{6d}+\frac{(2d^2x^2b^2+4b^2cdx+2b^2c^2-2b\,d^2x-2bcd+d^2)\,\mathrm{e}^{2bx+2a}}{16b^3}-\frac{(2d^2x^2b^2+4b^2cdx+2b^2c^2}{}$
derivativedivides	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^3}{6}-\frac{(bx+a)\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx}{}\right.}{}$
default	$\frac{d^2\left(\frac{(bx+a)^2\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^3}{6}-\frac{(bx+a)\cosh(bx+a)^2}{2}+\frac{\cosh(bx+a)\sinh(bx+a)}{4}+\frac{bx}{4}+\frac{a}{4}\right)}{b^2}-\frac{2d^2a\left(\frac{(bx+a)\cosh(bx}{}\right.}{}$
orering	$\frac{(4b^4d^4x^5+20b^4c\,d^3x^4+40b^4c^2d^2x^3+36b^4c^3d\,x^2+12b^4c^4x-12b^2d^4x^3-42b^2c\,d^3x^2-48b^2c^2d^2x-12b^2c^3d-12d^4x-3d^3}{12(dx+c)^2b^4}$

input
```
int((d*x+c)^2*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*((2*(d*x+c)^2*b^2+d^2)*sinh(2*b*x+2*a)+4*b*(-1/2*d*(d*x+c)*cosh(2*b*x+
2*a)+x*(1/3*x^2*d^2+c*d*x+c^2)*b^2+1/2*c*d))/b^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.29

$$\int (c+dx)^2\cosh^2(a+bx)\,dx$$
$$= \frac{2\,b^3d^2x^3+6\,b^3cdx^2+6\,b^3c^2x-3\,(bd^2x+bcd)\cosh\left(bx+a\right)^2+3\,(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+d^2)\cosh}{12\,b^3}$$

input
```
integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/12*(2*b^3*d^2*x^3 + 6*b^3*c*d*x^2 + 6*b^3*c^2*x - 3*(b*d^2*x + b*c*d)*co
sh(b*x + a)^2 + 3*(2*b^2*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + d^2)*cosh(b*x
 + a)*sinh(b*x + a) - 3*(b*d^2*x + b*c*d)*sinh(b*x + a)^2)/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. 2(85) = 170.

Time = 0.28 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.78

$$\int (c+dx)^2\cosh^2(a+bx)\,dx$$
$$= \begin{cases} -\frac{c^2x\sinh^2(a+bx)}{2}+\frac{c^2x\cosh^2(a+bx)}{2}-\frac{cdx^2\sinh^2(a+bx)}{2}+\frac{cdx^2\cosh^2(a+bx)}{2}-\frac{d^2x^3\sinh^2(a+bx)}{6}+\frac{d^2x^3\cosh^2(a+bx)}{6}+\frac{c^2}{} \\ \left(c^2x+cdx^2+\frac{d^2x^3}{3}\right)\cosh^2(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cosh(b*x+a)**2,x)
```

output
```
Piecewise((-c**2*x*sinh(a + b*x)**2/2 + c**2*x*cosh(a + b*x)**2/2 - c*d*x*
*2*sinh(a + b*x)**2/2 + c*d*x**2*cosh(a + b*x)**2/2 - d**2*x**3*sinh(a + b
*x)**2/6 + d**2*x**3*cosh(a + b*x)**2/6 + c**2*sinh(a + b*x)*cosh(a + b*x)
/(2*b) + c*d*x*sinh(a + b*x)*cosh(a + b*x)/b + d**2*x**2*sinh(a + b*x)*cos
h(a + b*x)/(2*b) - c*d*sinh(a + b*x)**2/(2*b**2) - d**2*x*sinh(a + b*x)**2
/(4*b**2) - d**2*x*cosh(a + b*x)**2/(4*b**2) + d**2*sinh(a + b*x)*cosh(a +
 b*x)/(4*b**3), Ne(b, 0)), ((c**2*x + c*d*x**2 + d**2*x**3/3)*cosh(a)**2,
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.74

$$\int (c+dx)^2\cosh^2(a+bx)\,dx$$
$$=\frac{1}{8}\left(4\,x^2+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)cd$$
$$+\frac{1}{48}\left(8\,x^3+\frac{3\left(2\,b^2x^2e^{(2\,a)}-2\,bxe^{(2\,a)}+e^{(2\,a)}\right)e^{(2\,bx)}}{b^3}-\frac{3\left(2\,b^2x^2+2\,bx+1\right)e^{(-2\,bx-2\,a)}}{b^3}\right)d^2$$
$$+\frac{1}{8}\,c^2\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/8*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*b
*x - 2*a)/b^2)*c*d + 1/48*(8*x^3 + 3*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) +
e^(2*a))*e^(2*b*x)/b^3 - 3*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3)*d
^2 + 1/8*c^2*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.43

$$\int (c+dx)^2\cosh^2(a+bx)\,dx$$
$$=\frac{1}{6}\,d^2x^3+\frac{1}{2}\,cdx^2+\frac{1}{2}\,c^2x+\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2-2\,bd^2x-2\,bcd+d^2\right)e^{(2\,bx+2\,a)}}{16\,b^3}$$
$$-\frac{\left(2\,b^2d^2x^2+4\,b^2cdx+2\,b^2c^2+2\,bd^2x+2\,bcd+d^2\right)e^{(-2\,bx-2\,a)}}{16\,b^3}$$

input
```
integrate((d*x+c)^2*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/6*d^2*x^3 + 1/2*c*d*x^2 + 1/2*c^2*x + 1/16*(2*b^2*d^2*x^2 + 4*b^2*c*d*x
+ 2*b^2*c^2 - 2*b*d^2*x - 2*b*c*d + d^2)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2
*d^2*x^2 + 4*b^2*c*d*x + 2*b^2*c^2 + 2*b*d^2*x + 2*b*c*d + d^2)*e^(-2*b*x
- 2*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.34

$$\int (c+dx)^2\cosh^2(a+bx)\,dx = \frac{c^2\,x}{2}+\frac{d^2\,x^3}{6}+\frac{c^2\sinh(2\,a+2\,b\,x)}{4\,b}+\frac{d^2\sinh(2\,a+2\,b\,x)}{8\,b^3}+\frac{c\,d\,x^2}{2}-\frac{d^2\,x\cosh(2\,a+2\,b\,x)}{4\,b^2}+\frac{d^2\,x^2\sinh(2\,a+2\,b\,x)}{4\,b}-\frac{c\,d\cosh(2\,a+2\,b\,x)}{4\,b^2}+\frac{c\,d\,x\sinh(2\,a+2\,b\,x)}{2\,b}$$

input
```
int(cosh(a + b*x)^2*(c + d*x)^2,x)
```

output
```
(c^2*x)/2 + (d^2*x^3)/6 + (c^2*sinh(2*a + 2*b*x))/(4*b) + (d^2*sinh(2*a +
2*b*x))/(8*b^3) + (c*d*x^2)/2 - (d^2*x*cosh(2*a + 2*b*x))/(4*b^2) + (d^2*x
^2*sinh(2*a + 2*b*x))/(4*b) - (c*d*cosh(2*a + 2*b*x))/(4*b^2) + (c*d*x*sin
h(2*a + 2*b*x))/(2*b)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.38

$$\int (c+dx)^2\cosh^2(a+bx)\,dx = \frac{6e^{4bx+4a}b^2c^2+12e^{4bx+4a}b^2cdx+6e^{4bx+4a}b^2d^2x^2-6e^{4bx+4a}bcd-6e^{4bx+4a}b\,d^2x+3e^{4bx+4a}d^2+24e^{2bx+2a}b^3}{48e^{2bx+2a}b^3}$$

input
```
int((d*x+c)^2*cosh(b*x+a)^2,x)
```

output
```
(6*e**(4*a + 4*b*x)*b**2*c**2 + 12*e**(4*a + 4*b*x)*b**2*c*d*x + 6*e**(4*a
 + 4*b*x)*b**2*d**2*x**2 - 6*e**(4*a + 4*b*x)*b*c*d - 6*e**(4*a + 4*b*x)*b
*d**2*x + 3*e**(4*a + 4*b*x)*d**2 + 24*e**(2*a + 2*b*x)*b**3*c**2*x + 24*e
**(2*a + 2*b*x)*b**3*c*d*x**2 + 8*e**(2*a + 2*b*x)*b**3*d**2*x**3 - 6*b**2
*c**2 - 12*b**2*c*d*x - 6*b**2*d**2*x**2 - 6*b*c*d - 6*b*d**2*x - 3*d**2)/
(48*e**(2*a + 2*b*x)*b**3)
```

3.11 $\int (c+dx)\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int (c+dx)\cosh^2(a+bx)\,dx=\frac{(c+dx)^2}{4d}-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
1/4*(d*x+c)^2/d-1/4*d*cosh(b*x+a)^2/b^2+1/2*(d*x+c)*cosh(b*x+a)*sinh(b*x+a
)/b
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int (c+dx)\cosh^2(a+bx)\,dx=\frac{-d\cosh(2(a+bx))+2b(2ac+bx(2c+dx)+(c+dx)\sinh(2(a+bx)))}{8b^2}$$

input
```
Integrate[(c + d*x)*Cosh[a + b*x]^2,x]
```

output
```
(-(d*Cosh[2*(a + b*x)]) + 2*b*(2*a*c + b*x*(2*c + d*x) + (c + d*x)*Sinh[2*
(a + b*x)]))/(8*b^2)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3791, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{1}{2}\int (c+dx)dx-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$-\frac{d\cosh^2(a+bx)}{4b^2}+\frac{(c+dx)\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^2}{4d}$$

input
```
Int[(c + d*x)*Cosh[a + b*x]^2,x]
```

output
```
(c + d*x)^2/(4*d) - (d*Cosh[a + b*x]^2)/(4*b^2) + ((c + d*x)*Cosh[a + b*x]
*Sinh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{2b(dx+c)\sinh(2bx+2a)-d\cosh(2bx+2a)+(2d\,x^2+4cx)b^2+d}{8b^2}$
risch	$\frac{d\,x^2}{4}+\frac{cx}{2}+\frac{(2dxb+2cb-d)\mathrm{e}^{2bx+2a}}{16b^2}-\frac{(2dxb+2cb+d)\mathrm{e}^{-2bx-2a}}{16b^2}$
derivativedivides	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^2}{4}-\frac{\cosh(bx+a)^2}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a}{2}\right.}{b}$
default	$\frac{\frac{d\left(\frac{(bx+a)\cosh(bx+a)\sinh(bx+a)}{2}+\frac{(bx+a)^2}{4}-\frac{\cosh(bx+a)^2}{4}\right)}{b}-\frac{da\left(\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}\right)}{b}+c\left(\frac{\cosh(bx+a)\sinh(bx+a}{2}\right.}{b}$
orering	$\frac{(2b^2d^3x^4+8b^2c\,d^2x^3+10b^2c^2d\,x^2+4b^2c^3x-3d^3x^2-6c\,d^2x-2d\,c^2)\cosh(bx+a)^2}{4b^2(dx+c)^2}+\frac{(2x^2d^2+4cdx+c^2)\left(d\cosh(bx+a)\right.}{4(d}$

input
```
int((d*x+c)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b*(d*x+c)*sinh(2*b*x+2*a)-d*cosh(2*b*x+2*a)+(2*d*x^2+4*c*x)*b^2+d)/
b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.20

$$\int (c+dx)\cosh^2(a+bx)\,dx$$
$$=\frac{2\,b^2dx^2+4\,b^2cx-d\cosh\left(bx+a\right)^2+4\left(bdx+bc\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)-d\sinh\left(bx+a\right)^2}{8\,b^2}$$

input
```
integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(2*b^2*d*x^2 + 4*b^2*c*x - d*cosh(b*x + a)^2 + 4*(b*d*x + b*c)*cosh(b*
x + a)*sinh(b*x + a) - d*sinh(b*x + a)^2)/b^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. 2(48) = 96.

Time = 0.20 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.29

$$\int (c+dx)\cosh^2(a+bx)\,dx$$
$$=\begin{cases}-\frac{cx\sinh^2\left(a+bx\right)}{2}+\frac{cx\cosh^2\left(a+bx\right)}{2}-\frac{dx^2\sinh^2\left(a+bx\right)}{4}+\frac{dx^2\cosh^2\left(a+bx\right)}{4}+\frac{c\sinh\left(a+bx\right)\cosh\left(a+bx\right)}{2b}+\frac{dx\sinh\left(a+bx\right)\cosh\left(a\right.}{2b}\\\left(cx+\frac{dx^2}{2}\right)\cosh^2\left(a\right)\end{cases}$$

input
```
integrate((d*x+c)*cosh(b*x+a)**2,x)
```

output
```
Piecewise((-c*x*sinh(a + b*x)**2/2 + c*x*cosh(a + b*x)**2/2 - d*x**2*sinh(
a + b*x)**2/4 + d*x**2*cosh(a + b*x)**2/4 + c*sinh(a + b*x)*cosh(a + b*x)/
(2*b) + d*x*sinh(a + b*x)*cosh(a + b*x)/(2*b) - d*sinh(a + b*x)**2/(4*b**2
), Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.60

$$\int (c+dx)\cosh^2(a+bx)\,dx$$
$$=\frac{1}{16}\left(4x^2+\frac{\left(2\,bxe^{(2\,a)}-e^{(2\,a)}\right)e^{(2\,bx)}}{b^2}-\frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{b^2}\right)d$$
$$+\frac{1}{8}\,c\left(4\,x+\frac{e^{(2\,bx+2\,a)}}{b}-\frac{e^{(-2\,bx-2\,a)}}{b}\right)$$

input
```
integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/16*(4*x^2 + (2*b*x*e^(2*a) - e^(2*a))*e^(2*b*x)/b^2 - (2*b*x + 1)*e^(-2*
b*x - 2*a)/b^2)*d + 1/8*c*(4*x + e^(2*b*x + 2*a)/b - e^(-2*b*x - 2*a)/b)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.15

$$\int (c+dx)\cosh^2(a+bx)\,dx=\frac{1}{4}\,dx^2+\frac{1}{2}\,cx+\frac{(2\,bdx+2\,bc-d)e^{(2\,bx+2\,a)}}{16\,b^2}-\frac{(2\,bdx+2\,bc+d)e^{(-2\,bx-2\,a)}}{16\,b^2}$$

input
```
integrate((d*x+c)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/4*d*x^2 + 1/2*c*x + 1/16*(2*b*d*x + 2*b*c - d)*e^(2*b*x + 2*a)/b^2 - 1/1
6*(2*b*d*x + 2*b*c + d)*e^(-2*b*x - 2*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.05

$$\int (c+dx)\cosh^2(a+bx)\,dx$$
$$=\frac{b^2\,d\,x^2-\frac{d\cosh(2\,a+2\,b\,x)}{2}+b\,c\,\sinh(2\,a+2\,b\,x)+2\,b^2\,c\,x+b\,d\,x\,\sinh(2\,a+2\,b\,x)}{4\,b^2}$$

input
```
int(cosh(a + b*x)^2*(c + d*x),x)
```

output
```
(b^2*d*x^2 - (d*cosh(2*a + 2*b*x))/2 + b*c*sinh(2*a + 2*b*x) + 2*b^2*c*x +
 b*d*x*sinh(2*a + 2*b*x))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.96

$$\int (c+dx)\cosh^2(a+bx)\,dx$$
$$=\frac{2e^{4bx+4a}bc+2e^{4bx+4a}bdx-e^{4bx+4a}d+8e^{2bx+2a}b^2cx+4e^{2bx+2a}b^2d\,x^2-2bc-2bdx-d}{16e^{2bx+2a}b^2}$$

input
```
int((d*x+c)*cosh(b*x+a)^2,x)
```

output
```
(2*e**(4*a + 4*b*x)*b*c + 2*e**(4*a + 4*b*x)*b*d*x - e**(4*a + 4*b*x)*d +
8*e**(2*a + 2*b*x)*b**2*c*x + 4*e**(2*a + 2*b*x)*b**2*d*x**2 - 2*b*c - 2*b
*d*x - d)/(16*e**(2*a + 2*b*x)*b**2)
```

3.12 $\int \frac{\cosh^2(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 78

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx = \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}$$

output
```
1/2*cosh(2*a-2*b*c/d)*Chi(2*b*c/d+2*b*x)/d+1/2*ln(d*x+c)/d+1/2*sinh(2*a-2*
b*c/d)*Shi(2*b*c/d+2*b*x)/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx = \frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right)+\log(c+dx)+\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x),x]
```

output
```
(Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] + Log[c + d*x] + Si
nh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{c+dx}dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{\cosh(2a+2bx)}{2(c+dx)}+\frac{1}{2(c+dx)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x),x]
```

output
```
(Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*
x]/(2*d) + (Sinh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.40 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24

method	result	size
risch	$\frac{\ln(dx+c)}{2d}-\frac{e^{-\frac{2(ad-cb)}{d}}\operatorname{expIntegral}_1\left(2bx+2a-\frac{2(ad-cb)}{d}\right)}{4d}-\frac{e^{\frac{2ad-2cb}{d}}\operatorname{expIntegral}_1\left(-2bx-2a-\frac{2(-ad+cb)}{d}\right)}{4d}$	97

input
```
int(cosh(b*x+a)^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d*x+c)/d-1/4/d*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/
4/d*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.33

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx$$
$$=\frac{\left(\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)+\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{2\,(bc-ad)}{d}\right)+\left(\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\sinh\left(-\frac{2\,(bc-a}{d}\right.}{4\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*((Ei(2*(b*d*x + b*c)/d) + Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/
d) + (Ei(2*(b*d*x + b*c)/d) - Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/
d) + 2*log(d*x + c))/d
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx = \int \frac{\cosh^2{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(cosh(a + b*x)**2/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx = -\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_1\left(\frac{2\,(dx+c)b}{d}\right)}{4\,d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_1\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,d} + \frac{\log\left(dx+c\right)}{2\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(1, 2*(d*x + c)*b/d)/d - 1/4*e^(2*a
- 2*b*c/d)*exp_integral_e(1, -2*(d*x + c)*b/d)/d + 1/2*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx = \frac{\operatorname{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+\operatorname{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+2\,\log\left(dx+c\right)}{4\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + Ei(-2*(b*d*x + b*c)/d)*e^(-
2*a + 2*b*c/d) + 2*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx = \int \frac{\cosh(a+b\,x)^2}{c+d\,x}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x),x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{c+dx}\,dx = \int \frac{\cosh\left(bx+a\right)^2}{dx+c}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c),x)
```

output
```
int(cosh(a + b*x)**2/(c + d*x),x)
```

3.13 $\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 81

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\cosh^2(a+bx)}{d(c+dx)} + \frac{b\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{d^2} + \frac{b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^2}$$

output
```
-cosh(b*x+a)^2/d/(d*x+c)+b*Chi(2*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^2+b*cosh
(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = \frac{-\frac{d\cosh^2(a+bx)}{c+dx} + b\text{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right) + b\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{d^2}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x)^2,x]
```

output
```
(-((d*Cosh[a + b*x]^2)/(c + d*x)) + b*CoshIntegral[(2*b*(c + d*x))/d]*Sinh
[2*a - (2*b*c)/d] + b*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d
])/d^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3794}$$

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}+\frac{2ib\int -\frac{i\sinh(2a+2bx)}{2(c+dx)}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\int \frac{\sinh(2a+2bx)}{c+dx}dx}{d}-\frac{\cosh^2(a+bx)}{d(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}+\frac{b\int -\frac{i\sin(2ia+2ibx)}{c+dx}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\int \frac{\sin(2ia+2ibx)}{c+dx}dx}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{i\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 26

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 3042

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int-\frac{i\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 26

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}$$

$\Big\downarrow$ 3779

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}$$

$\Big\downarrow$ 3782

$$-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(\frac{i\sinh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x)^2,x]
```

output
```
-(Cosh[a + b*x]^2/(d*(c + d*x))) - (I*b*((I*CoshIntegral[(2*b*c)/d + 2*b*x
]*Sinh[2*a - (2*b*c)/d])/d + (I*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)
/d + 2*b*x])/d))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.88

method	result
risch	$-\frac{1}{2(dx+c)d}-\frac{b\,\mathrm{e}^{-2bx-2a}}{4d(dxb+cb)}+\frac{b\,\mathrm{e}^{-\frac{2(ad-cb)}{d}}\,\mathrm{expIntegral}_1\left(2bx+2a-\frac{2(ad-cb)}{d}\right)}{2d^2}-\frac{b\,\mathrm{e}^{2bx+2a}}{4d^2\left(\frac{bc}{d}+bx\right)}-\frac{b\,\mathrm{e}^{\frac{2ad-2cb}{d}}\,\mathrm{expIntegral}_1\left(-\right.}{2d^2}$

input
```
int(cosh(b*x+a)^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(d*x+c)/d-1/4*b*exp(-2*b*x-2*a)/d/(b*d*x+b*c)+1/2*b/d^2*exp(-2*(a*d-b
*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/4*b/d^2*exp(2*b*x+2*a)/(b*c/d+b*x)-
1/2*b/d^2*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. 2(81) = 162.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.02

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{d\cosh\left(bx+a\right)^2+d\sinh\left(bx+a\right)^2-\left((bdx+bc)\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)-(bdx+bc)\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)\right)\cosh\left(-\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(d*cosh(b*x + a)^2 + d*sinh(b*x + a)^2 - ((b*d*x + b*c)*Ei(2*(b*d*x +
 b*c)/d) - (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d))*cosh(-2*(b*c - a*d)/d) -
((b*d*x + b*c)*Ei(2*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-2*(b*d*x + b*c)/d
))*sinh(-2*(b*c - a*d)/d) + d)/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(cosh(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.09

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = -\frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_2\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_2\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)d} - \frac{1}{2\,(d^2x+cd)}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*e^(-2*a + 2*b*c/d)*exp_integral_e(2, 2*(d*x + c)*b/d)/((d*x + c)*d) -
 1/4*e^(2*a - 2*b*c/d)*exp_integral_e(2, -2*(d*x + c)*b/d)/((d*x + c)*d) -
 1/2/(d^2*x + c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 574 vs. $2(81) = 162$.

Time = 0.16 (sec) , antiderivative size = 574, normalized size of antiderivative = 7.09

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = -\frac{\left(2\,(dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)b^2\text{Ei}\left(-\frac{2\left((dx+c)\left(b-\frac{bc}{dx+c}+\frac{ad}{dx+c}\right)+bc-ad\right)}{d}\right)e^{\left(\frac{2\,(bc-ad)}{d}\right)}+2\,b^3c\text{Ei}\left(-\frac{2\left((dx+c)\left(b-\right.\right.}{}\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-2*((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) +
 2*b^3*c*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d)*e^(2*(b*c - a*d)/d) - 2*a*b^2*d*Ei(-2*((d*x + c)*(b - b*c/(d*x + c) +
a*d/(d*x + c)) + b*c - a*d)/d)*e^(2*(b*c - a*d)/d) - 2*(d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c))*b^2*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) - 2*b^3*c*Ei(2*((d*x + c)*(b
 - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-2*(b*c - a*d)/d) + 2
*a*b^2*d*Ei(2*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/
d)*e^(-2*(b*c - a*d)/d) + b^2*d*e^(2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d
*x + c))/d) + b^2*d*e^(-2*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d)
 + 2*b^2*d)*d^2/(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*d^4 + b*c*
d^4 - a*d^5)*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{{\cosh(a+b\,x)}^2}{{(c+d\,x)}^2}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^2,x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^2}\,dx = \frac{e^{2a}\left(\int \frac{e^{2bx}}{d^2x^2+2cdx+c^2}dx\right)c^2+e^{2a}\left(\int \frac{e^{2bx}}{d^2x^2+2cdx+c^2}dx\right)cdx+\left(\int \frac{1}{e^{2bx+2a}c^2+2e^{2bx+2a}cdx+e^{2bx+2a}d^2x^2}dx\right)c^2+\left(\int \frac{}{e^{2}}\right.}{4c\,(dx+c)}$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^2,x)
```

output
```
(e**(2*a)*int(e**(2*b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 + e**(2*a)*i
nt(e**(2*b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + int(1/(e**(2*a + 2*b
*x)*c**2 + 2*e**(2*a + 2*b*x)*c*d*x + e**(2*a + 2*b*x)*d**2*x**2),x)*c**2
+ int(1/(e**(2*a + 2*b*x)*c**2 + 2*e**(2*a + 2*b*x)*c*d*x + e**(2*a + 2*b*
x)*d**2*x**2),x)*c*d*x + 2*x)/(4*c*(c + d*x))
```

3.14 $\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 112

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx = -\frac{\cosh^2(a+bx)}{2d(c+dx)^2} + \frac{b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{d^3} - \frac{b\cosh(a+bx)\sinh(a+bx)}{d^2(c+dx)} + \frac{b^2\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d^3}$$

output
```
-1/2*cosh(b*x+a)^2/d/(d*x+c)^2+b^2*cosh(2*a-2*b*c/d)*Chi(2*b*c/d+2*b*x)/d^
3-b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)+b^2*sinh(2*a-2*b*c/d)*Shi(2*b*c/d+
2*b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx = \frac{2b^2\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2b(c+dx)}{d}\right) - \frac{d\left(d\cosh^2(a+bx)+b(c+dx)\sinh(2(a+bx))\right)}{(c+dx)^2} + 2b^2\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2b(c+dx)}{d}\right)}{2d^3}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x)^3,x]
```

output
```
(2*b^2*Cosh[2*a - (2*b*c)/d]*CoshIntegral[(2*b*(c + d*x))/d] - (d*(d*Cosh[
a + b*x]^2 + b*(c + d*x)*Sinh[2*(a + b*x)]))/(c + d*x)^2 + 2*b^2*Sinh[2*a
- (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x))/d])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.32, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3042, 3795, 16, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{2b^2\int \frac{\cosh^2(a+bx)}{c+dx}dx}{d^2}-\frac{b^2\int \frac{1}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\cosh^2(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{2b^2\int \frac{\cosh^2(a+bx)}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{c+dx}dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{2b^2\int \left(\frac{\cosh(2a+2bx)}{2(c+dx)}+\frac{1}{2(c+dx)}\right)dx}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b^2\log(c+dx)}{d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2b^2\left(\frac{\cosh\left(2a-\frac{2bc}{d}\right)\text{Chi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\sinh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{2d}+\frac{\log(c+dx)}{2d}\right)}{d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{d^2(c+dx)}-\frac{\cosh^2(a+bx)}{2d(c+dx)^2}-\frac{b^2\log(c+dx)}{d^3}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x)^3,x]
```

output
```
-1/2*Cosh[a + b*x]^2/(d*(c + d*x)^2) - (b^2*Log[c + d*x])/d^3 - (b*Cosh[a
+ b*x]*Sinh[a + b*x])/(d^2*(c + d*x)) + (2*b^2*((Cosh[2*a - (2*b*c)/d]*Cos
hIntegral[(2*b*c)/d + 2*b*x])/(2*d) + Log[c + d*x]/(2*d) + (Sinh[2*a - (2*
b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/(2*d)))/d^2
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(110) = 220$.

Time = 1.64 (sec) , antiderivative size = 299, normalized size of antiderivative = 2.67

method	result
risch	$-\frac{1}{4(dx+c)^2 d}+\frac{b^3 \mathrm{e}^{-2bx-2a} x}{4d(d^2x^2b^2+2b^2cdx+b^2c^2)}+\frac{b^3 \mathrm{e}^{-2bx-2a} c}{4d^2(d^2x^2b^2+2b^2cdx+b^2c^2)}-\frac{b^2 \mathrm{e}^{-2bx-2a}}{8d(d^2x^2b^2+2b^2cdx+b^2c^2)}-\frac{b^2 \mathrm{e}^{-\frac{2(ad-cb)}{d}} \exp$

input
```
int(cosh(b*x+a)^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/(d*x+c)^2/d+1/4*b^3*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^
2)*x+1/4*b^3*exp(-2*b*x-2*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/8*b
^2*exp(-2*b*x-2*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-1/2*b^2/d^3*exp(-2*
(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/8*b^2/d^3*exp(2*b*x+2*a)/(b*c
/d+b*x)^2-1/4*b^2/d^3*exp(2*b*x+2*a)/(b*c/d+b*x)-1/2*b^2/d^3*exp(2*(a*d-b*
c)/d)*Ei(1,-2*b*x-2*a-2*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(110) = 220$.

Time = 0.10 (sec) , antiderivative size = 278, normalized size of antiderivative = 2.48

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx =$$

$$-\frac{d^2\cosh(bx+a)^2+d^2\sinh(bx+a)^2+4\left(bd^2x+bcd\right)\cosh(bx+a)\sinh(bx+a)+d^2-2\left(\left(b^2d^2x^2+\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(d^2*cosh(b*x + a)^2 + d^2*sinh(b*x + a)^2 + 4*(b*d^2*x + b*c*d)*cosh
(b*x + a)*sinh(b*x + a) + d^2 - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*E
i(2*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*d*x
+ b*c)/d))*cosh(-2*(b*c - a*d)/d) - 2*((b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^
2)*Ei(2*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*Ei(-2*(b*
d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cosh^2{(a+bx)}}{(c+dx)^3}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**3,x)
```

output
```
Integral(cosh(a + b*x)**2/(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx = -\frac{1}{4\,(d^3x^2+2\,cd^2x+c^2d)} - \frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_3\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_3\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^2d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4/(d^3*x^2 + 2*c*d^2*x + c^2*d) - 1/4*e^(-2*a + 2*b*c/d)*exp_integral_e
(3, 2*(d*x + c)*b/d)/((d*x + c)^2*d) - 1/4*e^(2*a - 2*b*c/d)*exp_integral_
e(3, -2*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(110) = 220$.

Time = 0.13 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.95

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx = \frac{4\,b^2d^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+4\,b^2d^2x^2\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}+8\,b^2cdx\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}+\cdots}{\cdots}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*x^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*d^2*x^2
*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 8*b^2*c*d*x*Ei(2*(b*d*x + b*c
)/d)*e^(2*a - 2*b*c/d) + 8*b^2*c*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*
c/d) + 4*b^2*c^2*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) + 4*b^2*c^2*Ei(-2
*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b*d^2*x*e^(2*b*x + 2*a) + 2*b*d^2
*x*e^(-2*b*x - 2*a) - 2*b*c*d*e^(2*b*x + 2*a) + 2*b*c*d*e^(-2*b*x - 2*a) -
 d^2*e^(2*b*x + 2*a) - d^2*e^(-2*b*x - 2*a) - 2*d^2)/(d^5*x^2 + 2*c*d^4*x
+ c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx = \int \frac{{\cosh\left(a+b\,x\right)}^2}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^3,x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^3}\,dx$$
$$= \frac{e^{2a}\left(\int \frac{e^{2bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c^2d+2e^{2a}\left(\int \frac{e^{2bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c\,d^2x+e^{2a}\left(\int \frac{e^{2bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right.}{}$$

input

```
int(cosh(b*x+a)^2/(d*x+c)^3,x)
```

output

```
(e**(2*a)*int(e**(2*b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x
)*c**2*d + 2*e**(2*a)*int(e**(2*b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3),x)*c*d**2*x + e**(2*a)*int(e**(2*b*x)/(c**3 + 3*c**2*d*x + 3*c*
d**2*x**2 + d**3*x**3),x)*d**3*x**2 + int(1/(e**(2*a + 2*b*x)*c**3 + 3*e**
(2*a + 2*b*x)*c**2*d*x + 3*e**(2*a + 2*b*x)*c*d**2*x**2 + e**(2*a + 2*b*x)
*d**3*x**3),x)*c**2*d + 2*int(1/(e**(2*a + 2*b*x)*c**3 + 3*e**(2*a + 2*b*x
)*c**2*d*x + 3*e**(2*a + 2*b*x)*c*d**2*x**2 + e**(2*a + 2*b*x)*d**3*x**3),
x)*c*d**2*x + int(1/(e**(2*a + 2*b*x)*c**3 + 3*e**(2*a + 2*b*x)*c**2*d*x +
 3*e**(2*a + 2*b*x)*c*d**2*x**2 + e**(2*a + 2*b*x)*d**3*x**3),x)*d**3*x**2
 - 1)/(4*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.15 $\int \frac{\cosh^2(a+bx)}{(c+dx)^4} dx$

Optimal result

Integrand size = 16, antiderivative size = 162

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4} dx = \frac{b^2}{3d^3(c+dx)} - \frac{\cosh^2(a+bx)}{3d(c+dx)^3} - \frac{2b^2\cosh^2(a+bx)}{3d^3(c+dx)} + \frac{2b^3\text{Chi}\left(\frac{2bc}{d}+2bx\right)\sinh\left(2a-\frac{2bc}{d}\right)}{3d^4} - \frac{b\cosh(a+bx)\sinh(a+bx)}{3d^2(c+dx)^2} + \frac{2b^3\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{3d^4}$$

output
```
1/3*b^2/d^3/(d*x+c)-1/3*cosh(b*x+a)^2/d/(d*x+c)^3-2/3*b^2*cosh(b*x+a)^2/d^
3/(d*x+c)+2/3*b^3*Chi(2*b*c/d+2*b*x)*sinh(2*a-2*b*c/d)/d^4-1/3*b*cosh(b*x+
a)*sinh(b*x+a)/d^2/(d*x+c)^2+2/3*b^3*cosh(2*a-2*b*c/d)*Shi(2*b*c/d+2*b*x)/
d^4
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx$$
$$= \frac{4b^3\text{Chi}\left(\frac{2b(c+dx)}{d}\right)\sinh\left(2a-\frac{2bc}{d}\right)-\frac{d((d^2+2b^2(c+dx)^2)\cosh(2(a+bx))+d(d+b(c+dx)\sinh(2(a+bx))))}{(c+dx)^3}+4b^3\cosh\left(2a-\right.}{6d^4}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x)^4,x]
```

output
```
(4*b^3*CoshIntegral[(2*b*(c + d*x))/d]*Sinh[2*a - (2*b*c)/d] - (d*((d^2 +
2*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)] + d*(d + b*(c + d*x)*Sinh[2*(a + b*x)
])))/(c + d*x)^3 + 4*b^3*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*(c + d*x)
)/d])/(6*d^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.78 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.06, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3795, 17, 3042, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^4}dx$$
$$\Big\downarrow \text{3795}$$
$$\frac{2b^2\int \frac{\cosh^2(a+bx)}{(c+dx)^2}dx}{3d^2}-\frac{b^2\int \frac{1}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}$$

$$\Big\downarrow \text{17}$$

$$\frac{2b^2\int \frac{\cosh^2(a+bx)}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^2}dx}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3794}$$

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}+\frac{2ib\int -\frac{i\sinh(2a+2bx)}{2(c+dx)}dx}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b^2\left(\frac{b\int \frac{\sinh(2a+2bx)}{c+dx}dx}{d}-\frac{\cosh^2(a+bx)}{d(c+dx)}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}+\frac{b\int -\frac{i\sin(2ia+2ibx)}{c+dx}dx}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{26}$$

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\int \frac{\sin(2ia+2ibx)}{c+dx}dx}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$$\Big\downarrow \text{3784}$$

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{i\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$\big\downarrow$ 26

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\cosh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sinh\left(\frac{2bc}{d}+2bx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$\big\downarrow$ 3042

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2a-\frac{2bc}{d}\right)\int-\frac{i\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$\big\downarrow$ 26

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx\right)}{c+dx}dx\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$\big\downarrow$ 3779

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(i\sinh\left(2a-\frac{2bc}{d}\right)\int\frac{\sin\left(\frac{2ibc}{d}+2ibx+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\text{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

$\big\downarrow$ 3782

$$\frac{2b^2\left(-\frac{\cosh^2(a+bx)}{d(c+dx)}-\frac{ib\left(\frac{i\sinh\left(2a-\frac{2bc}{d}\right)\mathrm{Chi}\left(\frac{2bc}{d}+2bx\right)}{d}+\frac{i\cosh\left(2a-\frac{2bc}{d}\right)\mathrm{Shi}\left(\frac{2bc}{d}+2bx\right)}{d}\right)}{d}\right)}{3d^2}-\frac{b\sinh(a+bx)\cosh(a+bx)}{3d^2(c+dx)^2}-\frac{\cosh^2(a+bx)}{3d(c+dx)^3}+\frac{b^2}{3d^3(c+dx)}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x)^4,x]
```

output
```
b^2/(3*d^3*(c + d*x)) - Cosh[a + b*x]^2/(3*d*(c + d*x)^3) - (b*Cosh[a + b*
x]*Sinh[a + b*x])/(3*d^2*(c + d*x)^2) + (2*b^2*(-(Cosh[a + b*x]^2/(d*(c +
d*x))) - (I*b*((I*CoshIntegral[(2*b*c)/d + 2*b*x]*Sinh[2*a - (2*b*c)/d])/d
 + (I*Cosh[2*a - (2*b*c)/d]*SinhIntegral[(2*b*c)/d + 2*b*x])/d))/d))/(3*d^
2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(150) = 300$.

Time = 1.92 (sec) , antiderivative size = 555, normalized size of antiderivative = 3.43

method	result
risch	$-\frac{1}{6(dx+c)^3 d}-\frac{b^5 e^{-2bx-2a} x^2}{6d(b^3d^3x^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)}-\frac{b^5 e^{-2bx-2a} cx}{3d^2(b^3d^3x^3+3b^3c\,d^2x^2+3b^3c^2dx+b^3c^3)}-\frac{b^5 e^{-2bx-2}}{6d^3(b^3d^3x^3+3b^3c\,d^2x^2+}$

input
```
int(cosh(b*x+a)^2/(d*x+c)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/6/(d*x+c)^3/d-1/6*b^5*exp(-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*
b^3*c^2*d*x+b^3*c^3)*x^2-1/3*b^5*exp(-2*b*x-2*a)/d^2/(b^3*d^3*x^3+3*b^3*c*
d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c*x-1/6*b^5*exp(-2*b*x-2*a)/d^3/(b^3*d^3*x^
3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c^2+1/12*b^4*exp(-2*b*x-2*a)/d/(b
^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*x+1/12*b^4*exp(-2*b*x-2*
a)/d^2/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)*c-1/12*b^3*exp(
-2*b*x-2*a)/d/(b^3*d^3*x^3+3*b^3*c*d^2*x^2+3*b^3*c^2*d*x+b^3*c^3)+1/3*b^3/
d^4*exp(-2*(a*d-b*c)/d)*Ei(1,2*b*x+2*a-2*(a*d-b*c)/d)-1/12*b^3/d^4*exp(2*b
*x+2*a)/(b*c/d+b*x)^3-1/12*b^3/d^4*exp(2*b*x+2*a)/(b*c/d+b*x)^2-1/6*b^3/d^
4*exp(2*b*x+2*a)/(b*c/d+b*x)-1/3*b^3/d^4*exp(2*(a*d-b*c)/d)*Ei(1,-2*b*x-2*
a-2*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(150) = 300$.

Time = 0.09 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.52

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx =$$

$$-\frac{d^3 + (2\,b^2d^3x^2 + 4\,b^2cd^2x + 2\,b^2c^2d + d^3)\cosh{(bx+a)}^2 + 2\,(bd^3x + bcd^2)\cosh{(bx+a)}\sinh{(bx+a)} +}{}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="fricas")
```

output
```
-1/6*(d^3 + (2*b^2*d^3*x^2 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*cosh(b*x +
 a)^2 + 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) + (2*b^2*d^3*x^2
 + 4*b^2*c*d^2*x + 2*b^2*c^2*d + d^3)*sinh(b*x + a)^2 - 2*((b^3*d^3*x^3 +
3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) - (b^3*d^
3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))
*cosh(-2*(b*c - a*d)/d) - 2*((b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*
x + b^3*c^3)*Ei(2*(b*d*x + b*c)/d) + (b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + b^3*c^3)*Ei(-2*(b*d*x + b*c)/d))*sinh(-2*(b*c - a*d)/d))/(d^7*
x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**4,x)
```

output
```
Integral(cosh(a + b*x)**2/(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.68

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx = -\frac{1}{6\,(d^4x^3+3\,cd^3x^2+3\,c^2d^2x+c^3d)} - \frac{e^{\left(-2\,a+\frac{2\,bc}{d}\right)}E_4\left(\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d} - \frac{e^{\left(2\,a-\frac{2\,bc}{d}\right)}E_4\left(-\frac{2\,(dx+c)b}{d}\right)}{4\,(dx+c)^3d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="maxima")
```

output
```
-1/6/(d^4*x^3 + 3*c*d^3*x^2 + 3*c^2*d^2*x + c^3*d) - 1/4*e^(-2*a + 2*b*c/d
)*exp_integral_e(4, 2*(d*x + c)*b/d)/((d*x + c)^3*d) - 1/4*e^(2*a - 2*b*c/
d)*exp_integral_e(4, -2*(d*x + c)*b/d)/((d*x + c)^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 537 vs. 2(150) = 300.

Time = 0.13 (sec) , antiderivative size = 537, normalized size of antiderivative = 3.31

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx = \frac{4\,b^3d^3x^3\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)} - 4\,b^3d^3x^3\mathrm{Ei}\left(-\frac{2\,(bdx+bc)}{d}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)} + 12\,b^3cd^2x^2\mathrm{Ei}\left(\frac{2\,(bdx+bc)}{d}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^4,x, algorithm="giac")
```

output
```
1/12*(4*b^3*d^3*x^3*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*d^3*x^
3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) + 12*b^3*c*d^2*x^2*Ei(2*(b*d*x
 + b*c)/d)*e^(2*a - 2*b*c/d) - 12*b^3*c*d^2*x^2*Ei(-2*(b*d*x + b*c)/d)*e^(
-2*a + 2*b*c/d) + 12*b^3*c^2*d*x*Ei(2*(b*d*x + b*c)/d)*e^(2*a - 2*b*c/d) -
 12*b^3*c^2*d*x*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*b*c/d) - 2*b^2*d^3*x^2*
e^(2*b*x + 2*a) - 2*b^2*d^3*x^2*e^(-2*b*x - 2*a) + 4*b^3*c^3*Ei(2*(b*d*x +
 b*c)/d)*e^(2*a - 2*b*c/d) - 4*b^3*c^3*Ei(-2*(b*d*x + b*c)/d)*e^(-2*a + 2*
b*c/d) - 4*b^2*c*d^2*x*e^(2*b*x + 2*a) - 4*b^2*c*d^2*x*e^(-2*b*x - 2*a) -
2*b^2*c^2*d*e^(2*b*x + 2*a) - b*d^3*x*e^(2*b*x + 2*a) - 2*b^2*c^2*d*e^(-2*
b*x - 2*a) + b*d^3*x*e^(-2*b*x - 2*a) - b*c*d^2*e^(2*b*x + 2*a) + b*c*d^2*
e^(-2*b*x - 2*a) - d^3*e^(2*b*x + 2*a) - d^3*e^(-2*b*x - 2*a) - 2*d^3)/(d^
7*x^3 + 3*c*d^6*x^2 + 3*c^2*d^5*x + c^3*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx = \int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^4}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^4,x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^4, x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^4}\,dx = \frac{3e^{2a}\left(\int \frac{e^{2bx}}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)c^3d+9e^{2a}\left(\int \frac{e^{2bx}}{d^4x^4+4c\,d^3x^3+6c^2d^2x^2+4c^3dx+c^4}dx\right)c^2d^2x+9e^{2a}\left(\int \frac{}{d^4x^4+}\right.}{}$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^4,x)
```

output
```
(3*e**(2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**
3*x**3 + d**4*x**4),x)*c**3*d + 9*e**(2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d
*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*c**2*d**2*x + 9*e**(
2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d*x + 6*c**2*d**2*x**2 + 4*c*d**3*x**3
+ d**4*x**4),x)*c*d**3*x**2 + 3*e**(2*a)*int(e**(2*b*x)/(c**4 + 4*c**3*d*x
 + 6*c**2*d**2*x**2 + 4*c*d**3*x**3 + d**4*x**4),x)*d**4*x**3 + 3*int(1/(e
**(2*a + 2*b*x)*c**4 + 4*e**(2*a + 2*b*x)*c**3*d*x + 6*e**(2*a + 2*b*x)*c*
*2*d**2*x**2 + 4*e**(2*a + 2*b*x)*c*d**3*x**3 + e**(2*a + 2*b*x)*d**4*x**4
),x)*c**3*d + 9*int(1/(e**(2*a + 2*b*x)*c**4 + 4*e**(2*a + 2*b*x)*c**3*d*x
 + 6*e**(2*a + 2*b*x)*c**2*d**2*x**2 + 4*e**(2*a + 2*b*x)*c*d**3*x**3 + e*
*(2*a + 2*b*x)*d**4*x**4),x)*c**2*d**2*x + 9*int(1/(e**(2*a + 2*b*x)*c**4
+ 4*e**(2*a + 2*b*x)*c**3*d*x + 6*e**(2*a + 2*b*x)*c**2*d**2*x**2 + 4*e**(
2*a + 2*b*x)*c*d**3*x**3 + e**(2*a + 2*b*x)*d**4*x**4),x)*c*d**3*x**2 + 3*
int(1/(e**(2*a + 2*b*x)*c**4 + 4*e**(2*a + 2*b*x)*c**3*d*x + 6*e**(2*a + 2
*b*x)*c**2*d**2*x**2 + 4*e**(2*a + 2*b*x)*c*d**3*x**3 + e**(2*a + 2*b*x)*d
**4*x**4),x)*d**4*x**3 - 2)/(12*d*(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3))
```

3.16 $\int (c+dx)^4 \cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 225

$$\begin{aligned}\int (c+dx)^4 \cosh^3(a+bx)\,dx = &-\frac{160d^3(c+dx)\cosh(a+bx)}{9b^4} \\ &-\frac{8d(c+dx)^3\cosh(a+bx)}{3b^2}-\frac{8d^3(c+dx)\cosh^3(a+bx)}{27b^4} \\ &-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{488d^4\sinh(a+bx)}{27b^5} \\ &+\frac{80d^2(c+dx)^2\sinh(a+bx)}{9b^3}+\frac{2(c+dx)^4\sinh(a+bx)}{3b} \\ &+\frac{4d^2(c+dx)^2\cosh^2(a+bx)\sinh(a+bx)}{9b^3} \\ &+\frac{(c+dx)^4\cosh^2(a+bx)\sinh(a+bx)}{3b} \\ &+\frac{8d^4\sinh^3(a+bx)}{81b^5}\end{aligned}$$

output
```
-160/9*d^3*(d*x+c)*cosh(b*x+a)/b^4-8/3*d*(d*x+c)^3*cosh(b*x+a)/b^2-8/27*d^
3*(d*x+c)*cosh(b*x+a)^3/b^4-4/9*d*(d*x+c)^3*cosh(b*x+a)^3/b^2+488/27*d^4*s
inh(b*x+a)/b^5+80/9*d^2*(d*x+c)^2*sinh(b*x+a)/b^3+2/3*(d*x+c)^4*sinh(b*x+a
)/b+4/9*d^2*(d*x+c)^2*cosh(b*x+a)^2*sinh(b*x+a)/b^3+1/3*(d*x+c)^4*cosh(b*x
+a)^2*sinh(b*x+a)/b+8/81*d^4*sinh(b*x+a)^3/b^5
```

Mathematica [A] (verified)

Time = 0.73 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.70

$$\int (c+dx)^4\cosh^3(a+bx)\,dx$$
$$=\frac{-972bd(c+dx)\left(6d^2+b^2(c+dx)^2\right)\cosh(a+bx)-12bd(c+dx)\left(2d^2+3b^2(c+dx)^2\right)\cosh(3(a+bx))+}{}$$

input
```
Integrate[(c + d*x)^4*Cosh[a + b*x]^3,x]
```

output
```
(-972*b*d*(c + d*x)*(6*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 12*b*d*(c +
d*x)*(2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[3*(a + b*x)] + 2*(2920*d^4 + 1476*b^
2*d^2*(c + d*x)^2 + 135*b^4*(c + d*x)^4 + (8*d^4 + 36*b^2*d^2*(c + d*x)^2
+ 27*b^4*(c + d*x)^4)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(324*b^5)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.86 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.47, number of steps used = 28, number of rules used = 27, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.688, Rules used = {3042, 3792, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^4\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^4\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 3792$$

$$\frac{4d^2\int(c+dx)^2\cosh^3(a+bx)dx}{3b^2}+\frac{2}{3}\int(c+dx)^4\cosh(a+bx)dx-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^4\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3777

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}-\frac{4id\int-i(c+dx)^3\sinh(a+bx)dx}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 26

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}-\frac{4d\int(c+dx)^3\sinh(a+bx)dx}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}-\frac{4d\int-i(c+dx)^3\sin(ia+ibx)dx}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 26

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\int(c+dx)^3\sin(ia+ibx)dx}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3777

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int(c+dx)^2\cosh(a+bx)dx}{b}\right)}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3777

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int-i(c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 26

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int(c+dx)\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int-i(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int(c+dx)\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{4d^2\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)}{b}\right)-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{4d^2\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right.}{b}}{b}\right.$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3117

$$\frac{4d^2\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3792

$$\frac{4d^2\left(\frac{2d^2\int\cosh^3(a+bx)dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\cosh(a+bx)dx-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{4d^2\left(\frac{2d^2\int\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{9b^2}+\frac{2}{3}\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3113

$$\frac{4d^2\left(\frac{2id^2\int(\sinh^2(a+bx)+1)d(-i\sinh(a+bx))}{9b^3}+\frac{2}{3}\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+\ldots}{3\ldots}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-\ldots$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 2009

$$\frac{4d^2\left(\frac{2}{3}\int(c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\ldots}{3b}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-\ldots$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int -i(c+dx)\sinh(a+bx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+\cdots}{\cdots}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-\cdots$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 26

$$\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int (c+dx)\sinh(a+bx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\cdots}{\cdots}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-\cdots$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int -i(c+dx)\sin(ia+ibx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+\cdots}{}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-\cdots$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 26

$$\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int (c+dx)\sin(ia+ibx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx\cdots}{}\right.}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-\cdots$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3}{9b^2}}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{4d^2\left(\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)}{}}{3b^2}$$

$$\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)$$

$$\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3117

$$
-\frac{4d(c+dx)^3\cosh^3(a+bx)}{9b^2}+
$$

$$
\frac{2}{3}\left(\frac{(c+dx)^4\sinh(a+bx)}{b}+\frac{4id\left(\frac{i(c+dx)^3\cosh(a+bx)}{b}-\frac{3id\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)}{b}\right)-
$$

$$
\frac{4d^2\left(\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right.\right.}{3b^2}
$$

$$
\frac{(c+dx)^4\sinh(a+bx)\cosh^2(a+bx)}{3b}
$$

input
```
Int[(c + d*x)^4*Cosh[a + b*x]^3,x]
```

output
```
(-4*d*(c + d*x)^3*Cosh[a + b*x]^3)/(9*b^2) + ((c + d*x)^4*Cosh[a + b*x]^2*
Sinh[a + b*x])/(3*b) + (4*d^2*((-2*d*(c + d*x)*Cosh[a + b*x]^3)/(9*b^2) +
((c + d*x)^2*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b) + (((2*I)/9)*d^2*((-I)*S
inh[a + b*x] - (I/3)*Sinh[a + b*x]^3))/b^3 + (2*(((c + d*x)^2*Sinh[a + b*x
])/b + ((2*I)*d*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2))
/b))/3))/(3*b^2) + (2*(((c + d*x)^4*Sinh[a + b*x])/b + ((4*I)*d*((I*(c + d
*x)^3*Cosh[a + b*x])/b - ((3*I)*d*(((c + d*x)^2*Sinh[a + b*x])/b + ((2*I)*
d*((I*(c + d*x)*Cosh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2))/b))/b))/b))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 2.06 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.77

method	result
parallelrisch	$\frac{\left(27(dx+c)^4b^4+36d^2(dx+c)^2b^2+8d^4\right)\sinh(3bx+3a)-36(dx+c)b\left((dx+c)^2b^2+\frac{2d^2}{3}\right)d\cosh(3bx+3a)+243\left((dx+c)^4b^4}{324b^5}$
risch	$\frac{(27d^4x^4b^4+108b^4c\,d^3x^3+162b^4c^2d^2x^2-36b^3d^4x^3+108b^4c^3dx-108b^3c\,d^3x^2+27b^4c^4-108b^3c^2d^2x+36b^2d^4x^2-36b^3c^3}{648b^5}$
orering	$-\frac{16d(135b^6d^6x^6+810b^6c\,d^5x^5+2025b^6c^2d^4x^4+2700b^6c^3d^3x^3+2025b^6c^4d^2x^2+891b^4d^6x^4+810b^6c^5dx+3564b^4c\,d^5x}{243b^8}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int((d*x+c)^4*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/324*((27*(d*x+c)^4*b^4+36*d^2*(d*x+c)^2*b^2+8*d^4)*sinh(3*b*x+3*a)-36*(d
*x+c)*b*((d*x+c)^2*b^2+2/3*d^2)*d*cosh(3*b*x+3*a)+243*((d*x+c)^4*b^4+12*d^
2*(d*x+c)^2*b^2+24*d^4)*sinh(b*x+a)-972*(((d*x+c)^2*b^2+6*d^2)*(d*x+c)*cos
h(b*x+a)+28/27*b^2*c^3+488/81*d^2*c)*b*d)/b^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(205) = 410$.

Time = 0.09 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.35

$$\int (c+dx)^4\cosh^3(a+bx)\,dx =$$
$$-\frac{12\,(3\,b^3d^4x^3+9\,b^3cd^3x^2+3\,b^3c^3d+2\,bcd^3+(9\,b^3c^2d^2+2\,bd^4)x)\cosh{(bx+a)}^3+36\,(3\,b^3d^4x^3+9\,b^3c}{}$$

input
```
integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/324*(12*(3*b^3*d^4*x^3 + 9*b^3*c*d^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9
*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a)^3 + 36*(3*b^3*d^4*x^3 + 9*b^3*c*d
^3*x^2 + 3*b^3*c^3*d + 2*b*c*d^3 + (9*b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x +
 a)*sinh(b*x + a)^2 - (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 27*b^4*c^4 + 3
6*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + 36*(3*b^4*c^3
*d + 2*b^2*c*d^3)*x)*sinh(b*x + a)^3 + 972*(b^3*d^4*x^3 + 3*b^3*c*d^3*x^2
+ b^3*c^3*d + 6*b*c*d^3 + 3*(b^3*c^2*d^2 + 2*b*d^4)*x)*cosh(b*x + a) - 3*(
81*b^4*d^4*x^4 + 324*b^4*c*d^3*x^3 + 81*b^4*c^4 + 972*b^2*c^2*d^2 + 1944*d
^4 + 486*(b^4*c^2*d^2 + 2*b^2*d^4)*x^2 + (27*b^4*d^4*x^4 + 108*b^4*c*d^3*x
^3 + 27*b^4*c^4 + 36*b^2*c^2*d^2 + 8*d^4 + 18*(9*b^4*c^2*d^2 + 2*b^2*d^4)*
x^2 + 36*(3*b^4*c^3*d + 2*b^2*c*d^3)*x)*cosh(b*x + a)^2 + 324*(b^4*c^3*d +
 6*b^2*c*d^3)*x)*sinh(b*x + a))/b^5
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 772 vs. $2(226) = 452$.

Time = 0.65 (sec) , antiderivative size = 772, normalized size of antiderivative = 3.43

$$\int (c+dx)^4 \cosh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**4*cosh(b*x+a)**3,x)
```

output
```
Piecewise((-2*c**4*sinh(a + b*x)**3/(3*b) + c**4*sinh(a + b*x)*cosh(a + b*
x)**2/b - 8*c**3*d*x*sinh(a + b*x)**3/(3*b) + 4*c**3*d*x*sinh(a + b*x)*cos
h(a + b*x)**2/b - 4*c**2*d**2*x**2*sinh(a + b*x)**3/b + 6*c**2*d**2*x**2*s
inh(a + b*x)*cosh(a + b*x)**2/b - 8*c*d**3*x**3*sinh(a + b*x)**3/(3*b) + 4
*c*d**3*x**3*sinh(a + b*x)*cosh(a + b*x)**2/b - 2*d**4*x**4*sinh(a + b*x)*
*3/(3*b) + d**4*x**4*sinh(a + b*x)*cosh(a + b*x)**2/b + 8*c**3*d*sinh(a +
b*x)**2*cosh(a + b*x)/(3*b**2) - 28*c**3*d*cosh(a + b*x)**3/(9*b**2) + 8*c
**2*d**2*x*sinh(a + b*x)**2*cosh(a + b*x)/b**2 - 28*c**2*d**2*x*cosh(a + b
*x)**3/(3*b**2) + 8*c*d**3*x**2*sinh(a + b*x)**2*cosh(a + b*x)/b**2 - 28*c
*d**3*x**2*cosh(a + b*x)**3/(3*b**2) + 8*d**4*x**3*sinh(a + b*x)**2*cosh(a
 + b*x)/(3*b**2) - 28*d**4*x**3*cosh(a + b*x)**3/(9*b**2) - 80*c**2*d**2*s
inh(a + b*x)**3/(9*b**3) + 28*c**2*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(3*
b**3) - 160*c*d**3*x*sinh(a + b*x)**3/(9*b**3) + 56*c*d**3*x*sinh(a + b*x)
*cosh(a + b*x)**2/(3*b**3) - 80*d**4*x**2*sinh(a + b*x)**3/(9*b**3) + 28*d
**4*x**2*sinh(a + b*x)*cosh(a + b*x)**2/(3*b**3) + 160*c*d**3*sinh(a + b*x
)**2*cosh(a + b*x)/(9*b**4) - 488*c*d**3*cosh(a + b*x)**3/(27*b**4) + 160*
d**4*x*sinh(a + b*x)**2*cosh(a + b*x)/(9*b**4) - 488*d**4*x*cosh(a + b*x)*
*3/(27*b**4) - 1456*d**4*sinh(a + b*x)**3/(81*b**5) + 488*d**4*sinh(a + b*
x)*cosh(a + b*x)**2/(27*b**5), Ne(b, 0)), ((c**4*x + 2*c**3*d*x**2 + 2*c**
2*d**2*x**3 + c*d**3*x**4 + d**4*x**5/5)*cosh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(205) = 410$.

Time = 0.09 (sec) , antiderivative size = 644, normalized size of antiderivative = 2.86

$$\int (c+dx)^4 \cosh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/18*c^3*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e
^(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2) + 1/24*c^4*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b -
e^(-3*b*x - 3*a)/b) + 1/36*c^2*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2
*e^(3*a))*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3
 - 81*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(
-3*b*x - 3*a)/b^3) + 1/54*c*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) +
6*b*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 + 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a
 + 6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e
^(-b*x - a)/b^4 - (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4
) + 1/648*d^4*((27*b^4*x^4*e^(3*a) - 36*b^3*x^3*e^(3*a) + 36*b^2*x^2*e^(3*
a) - 24*b*x*e^(3*a) + 8*e^(3*a))*e^(3*b*x)/b^5 + 243*(b^4*x^4*e^a - 4*b^3*
x^3*e^a + 12*b^2*x^2*e^a - 24*b*x*e^a + 24*e^a)*e^(b*x)/b^5 - 243*(b^4*x^4
 + 4*b^3*x^3 + 12*b^2*x^2 + 24*b*x + 24)*e^(-b*x - a)/b^5 - (27*b^4*x^4 +
36*b^3*x^3 + 36*b^2*x^2 + 24*b*x + 8)*e^(-3*b*x - 3*a)/b^5)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(205) = 410$.

Time = 0.13 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.91

$$\int (c+dx)^4 \cosh^3(a+bx)\,dx$$
$$= \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 - 36\,b^3d^4x^3 + 108\,b^4c^3dx - 108\,b^3cd^3x^2 + 27\,b^4c^4 - 108\,b^3c^2d^2x}{648\,b^5}$$
$$+ \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 - 4\,b^3d^4x^3 + 4\,b^4c^3dx - 12\,b^3cd^3x^2 + b^4c^4 - 12\,b^3c^2d^2x + 12\,b^2d^4x^2}{8\,b^5}$$
$$- \frac{3\,(b^4d^4x^4 + 4\,b^4cd^3x^3 + 6\,b^4c^2d^2x^2 + 4\,b^3d^4x^3 + 4\,b^4c^3dx + 12\,b^3cd^3x^2 + b^4c^4 + 12\,b^3c^2d^2x + 12\,b^2d^4x^2}{8\,b^5}$$
$$- \frac{(27\,b^4d^4x^4 + 108\,b^4cd^3x^3 + 162\,b^4c^2d^2x^2 + 36\,b^3d^4x^3 + 108\,b^4c^3dx + 108\,b^3cd^3x^2 + 27\,b^4c^4 + 108\,b^3c^2d}{648\,b^5}$$

input
```
integrate((d*x+c)^4*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/648*(27*b^4*d^4*x^4 + 108*b^4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 - 36*b^3*d
^4*x^3 + 108*b^4*c^3*d*x - 108*b^3*c*d^3*x^2 + 27*b^4*c^4 - 108*b^3*c^2*d^
2*x + 36*b^2*d^4*x^2 - 36*b^3*c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 - 24
*b*d^4*x - 24*b*c*d^3 + 8*d^4)*e^(3*b*x + 3*a)/b^5 + 3/8*(b^4*d^4*x^4 + 4*
b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 - 4*b^3*d^4*x^3 + 4*b^4*c^3*d*x - 12*b^3
*c*d^3*x^2 + b^4*c^4 - 12*b^3*c^2*d^2*x + 12*b^2*d^4*x^2 - 4*b^3*c^3*d + 2
4*b^2*c*d^3*x + 12*b^2*c^2*d^2 - 24*b*d^4*x - 24*b*c*d^3 + 24*d^4)*e^(b*x
+ a)/b^5 - 3/8*(b^4*d^4*x^4 + 4*b^4*c*d^3*x^3 + 6*b^4*c^2*d^2*x^2 + 4*b^3*
d^4*x^3 + 4*b^4*c^3*d*x + 12*b^3*c*d^3*x^2 + b^4*c^4 + 12*b^3*c^2*d^2*x +
12*b^2*d^4*x^2 + 4*b^3*c^3*d + 24*b^2*c*d^3*x + 12*b^2*c^2*d^2 + 24*b*d^4*
x + 24*b*c*d^3 + 24*d^4)*e^(-b*x - a)/b^5 - 1/648*(27*b^4*d^4*x^4 + 108*b^
4*c*d^3*x^3 + 162*b^4*c^2*d^2*x^2 + 36*b^3*d^4*x^3 + 108*b^4*c^3*d*x + 108
*b^3*c*d^3*x^2 + 27*b^4*c^4 + 108*b^3*c^2*d^2*x + 36*b^2*d^4*x^2 + 36*b^3*
c^3*d + 72*b^2*c*d^3*x + 36*b^2*c^2*d^2 + 24*b*d^4*x + 24*b*c*d^3 + 8*d^4)
*e^(-3*b*x - 3*a)/b^5
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.36

$$\int (c+dx)^4\cosh^3(a+bx)\,dx$$
$$= \frac{\cosh(a+b\,x)^2\sinh(a+b\,x)\,\left(27\,b^4\,c^4+252\,b^2\,c^2\,d^2+488\,d^4\right)}{27\,b^5} - \frac{2\sinh(a+b\,x)^3\,\left(27\,b^4\,c^4+360\,b^2\,c^2\,d^2+728\,d^4\right)}{81\,b^5} - \frac{4\cosh(a+b\,x)^3\,\left(21\,b^2\,c^3\,d+122\,c\,d^3\right)}{27\,b^4} + \frac{8\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(3\,b^2\,c^3\,d+20\,c\,d^3\right)}{9\,b^4} - \frac{28\,d^4\,x^3\cosh(a+b\,x)^3}{9\,b^2} - \frac{4\,x\cosh(a+b\,x)^3\,\left(63\,b^2\,c^2\,d^2+122\,d^4\right)}{27\,b^4} - \frac{2\,d^4\,x^4\sinh(a+b\,x)^3}{3\,b} - \frac{8\,x\sinh(a+b\,x)^3\,\left(3\,b^2\,c^3\,d+20\,c\,d^3\right)}{9\,b^3} - \frac{4\,x^2\sinh(a+b\,x)^3\,\left(9\,b^2\,c^2\,d^2+20\,d^4\right)}{9\,b^3} + \frac{2\,x^2\cosh(a+b\,x)^2\sinh(a+b\,x)\,\left(9\,b^2\,c^2\,d^2+14\,d^4\right)}{3\,b^3} - \frac{28\,c\,d^3\,x^2\cosh(a+b\,x)^3}{3\,b^2} + \frac{d^4\,x^4\cosh(a+b\,x)^2\sinh(a+b\,x)}{b} + \frac{8\,d^4\,x^3\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{3\,b^2} - \frac{8\,c\,d^3\,x^3\sinh(a+b\,x)^3}{3\,b} + \frac{8\,x\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(9\,b^2\,c^2\,d^2+20\,d^4\right)}{9\,b^4} + \frac{4\,x\cosh(a+b\,x)^2\sinh(a+b\,x)\,\left(3\,b^2\,c^3\,d+14\,c\,d^3\right)}{3\,b^3} + \frac{4\,c\,d^3\,x^3\cosh(a+b\,x)^2\sinh(a+b\,x)}{b} + \frac{8\,c\,d^3\,x^2\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2}$$

input
```
int(cosh(a + b*x)^3*(c + d*x)^4,x)
```

output
```
(cosh(a + b*x)^2*sinh(a + b*x)*(488*d^4 + 27*b^4*c^4 + 252*b^2*c^2*d^2))/(
27*b^5) - (2*sinh(a + b*x)^3*(728*d^4 + 27*b^4*c^4 + 360*b^2*c^2*d^2))/(81
*b^5) - (4*cosh(a + b*x)^3*(122*c*d^3 + 21*b^2*c^3*d))/(27*b^4) + (8*cosh(
a + b*x)*sinh(a + b*x)^2*(20*c*d^3 + 3*b^2*c^3*d))/(9*b^4) - (28*d^4*x^3*c
osh(a + b*x)^3)/(9*b^2) - (4*x*cosh(a + b*x)^3*(122*d^4 + 63*b^2*c^2*d^2))
/(27*b^4) - (2*d^4*x^4*sinh(a + b*x)^3)/(3*b) - (8*x*sinh(a + b*x)^3*(20*c
*d^3 + 3*b^2*c^3*d))/(9*b^3) - (4*x^2*sinh(a + b*x)^3*(20*d^4 + 9*b^2*c^2*
d^2))/(9*b^3) + (2*x^2*cosh(a + b*x)^2*sinh(a + b*x)*(14*d^4 + 9*b^2*c^2*d
^2))/(3*b^3) - (28*c*d^3*x^2*cosh(a + b*x)^3)/(3*b^2) + (d^4*x^4*cosh(a +
b*x)^2*sinh(a + b*x))/b + (8*d^4*x^3*cosh(a + b*x)*sinh(a + b*x)^2)/(3*b^2
) - (8*c*d^3*x^3*sinh(a + b*x)^3)/(3*b) + (8*x*cosh(a + b*x)*sinh(a + b*x)
^2*(20*d^4 + 9*b^2*c^2*d^2))/(9*b^4) + (4*x*cosh(a + b*x)^2*sinh(a + b*x)*
(14*c*d^3 + 3*b^2*c^3*d))/(3*b^3) + (4*c*d^3*x^3*cosh(a + b*x)^2*sinh(a +
b*x))/b + (8*c*d^3*x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 1068, normalized size of antiderivative = 4.75

$$\int (c+dx)^4 \cosh^3(a+bx)\,dx$$

$$= \frac{243e^{4bx+4a}b^4d^4x^4 - 972e^{4bx+4a}b^3c^3d - 972e^{4bx+4a}b^3d^4x^3 + 2916e^{4bx+4a}b^2c^2d^2 + 5832e^{4bx+4a}d^4 - 27b^4c^4 +}{}$$

input
```
int((d*x+c)^4*cosh(b*x+a)^3,x)
```

output
```
(27*e**(6*a + 6*b*x)*b**4*c**4 + 108*e**(6*a + 6*b*x)*b**4*c**3*d*x + 162*
e**(6*a + 6*b*x)*b**4*c**2*d**2*x**2 + 108*e**(6*a + 6*b*x)*b**4*c*d**3*x*
*3 + 27*e**(6*a + 6*b*x)*b**4*d**4*x**4 - 36*e**(6*a + 6*b*x)*b**3*c**3*d
- 108*e**(6*a + 6*b*x)*b**3*c**2*d**2*x - 108*e**(6*a + 6*b*x)*b**3*c*d**3
*x**2 - 36*e**(6*a + 6*b*x)*b**3*d**4*x**3 + 36*e**(6*a + 6*b*x)*b**2*c**2
*d**2 + 72*e**(6*a + 6*b*x)*b**2*c*d**3*x + 36*e**(6*a + 6*b*x)*b**2*d**4*
x**2 - 24*e**(6*a + 6*b*x)*b*c*d**3 - 24*e**(6*a + 6*b*x)*b*d**4*x + 8*e**
(6*a + 6*b*x)*d**4 + 243*e**(4*a + 4*b*x)*b**4*c**4 + 972*e**(4*a + 4*b*x)
*b**4*c**3*d*x + 1458*e**(4*a + 4*b*x)*b**4*c**2*d**2*x**2 + 972*e**(4*a +
 4*b*x)*b**4*c*d**3*x**3 + 243*e**(4*a + 4*b*x)*b**4*d**4*x**4 - 972*e**(4
*a + 4*b*x)*b**3*c**3*d - 2916*e**(4*a + 4*b*x)*b**3*c**2*d**2*x - 2916*e*
*(4*a + 4*b*x)*b**3*c*d**3*x**2 - 972*e**(4*a + 4*b*x)*b**3*d**4*x**3 + 29
16*e**(4*a + 4*b*x)*b**2*c**2*d**2 + 5832*e**(4*a + 4*b*x)*b**2*c*d**3*x +
 2916*e**(4*a + 4*b*x)*b**2*d**4*x**2 - 5832*e**(4*a + 4*b*x)*b*c*d**3 - 5
832*e**(4*a + 4*b*x)*b*d**4*x + 5832*e**(4*a + 4*b*x)*d**4 - 243*e**(2*a +
 2*b*x)*b**4*c**4 - 972*e**(2*a + 2*b*x)*b**4*c**3*d*x - 1458*e**(2*a + 2*
b*x)*b**4*c**2*d**2*x**2 - 972*e**(2*a + 2*b*x)*b**4*c*d**3*x**3 - 243*e**
(2*a + 2*b*x)*b**4*d**4*x**4 - 972*e**(2*a + 2*b*x)*b**3*c**3*d - 2916*e**
(2*a + 2*b*x)*b**3*c**2*d**2*x - 2916*e**(2*a + 2*b*x)*b**3*c*d**3*x**2 -
972*e**(2*a + 2*b*x)*b**3*d**4*x**3 - 2916*e**(2*a + 2*b*x)*b**2*c**2*d...
```

3.17 $\int (c+dx)^3 \cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 175

$$\int (c+dx)^3 \cosh^3(a+bx)\,dx = -\frac{40d^3\cosh(a+bx)}{9b^4} - \frac{2d(c+dx)^2\cosh(a+bx)}{b^2} - \frac{2d^3\cosh^3(a+bx)}{27b^4} - \frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2} + \frac{40d^2(c+dx)\sinh(a+bx)}{9b^3} + \frac{2(c+dx)^3\sinh(a+bx)}{3b} + \frac{2d^2(c+dx)\cosh^2(a+bx)\sinh(a+bx)}{9b^3} + \frac{(c+dx)^3\cosh^2(a+bx)\sinh(a+bx)}{3b}$$

output
```
-40/9*d^3*cosh(b*x+a)/b^4-2*d*(d*x+c)^2*cosh(b*x+a)/b^2-2/27*d^3*cosh(b*x+
a)^3/b^4-1/3*d*(d*x+c)^2*cosh(b*x+a)^3/b^2+40/9*d^2*(d*x+c)*sinh(b*x+a)/b^
3+2/3*(d*x+c)^3*sinh(b*x+a)/b+2/9*d^2*(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)/b^
3+1/3*(d*x+c)^3*cosh(b*x+a)^2*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.70

$$\int (c+dx)^3\cosh^3(a+bx)\,dx$$
$$=\frac{-486d(2d^2+b^2(c+dx)^2)\cosh(a+bx)-2d(2d^2+9b^2(c+dx)^2)\cosh(3(a+bx))+12b(c+dx)\,(82d^2+}{216b^4}$$

input
```
Integrate[(c + d*x)^3*Cosh[a + b*x]^3,x]
```

output
```
(-486*d*(2*d^2 + b^2*(c + d*x)^2)*Cosh[a + b*x] - 2*d*(2*d^2 + 9*b^2*(c +
d*x)^2)*Cosh[3*(a + b*x)] + 12*b*(c + d*x)*(82*d^2 + 15*b^2*(c + d*x)^2 +
(2*d^2 + 3*b^2*(c + d*x)^2)*Cosh[2*(a + b*x)])*Sinh[a + b*x])/(216*b^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.17 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.28, number of steps used = 21, number of rules used = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.312, Rules used = {3042, 3792, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\cosh^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^3\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{3792}$$
$$\frac{2d^2\int(c+dx)\cosh^3(a+bx)dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\cosh(a+bx)dx-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\int(c+dx)^3\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3id\int-i(c+dx)^2\sinh(a+bx)dx}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int(c+dx)^2\sinh(a+bx)dx}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}-\frac{3d\int-i(c+dx)^2\sin(ia+ibx)dx}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\int(c+dx)^2\sin(ia+ibx)dx}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\cosh(a+bx)dx}{b}\right)}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int-i\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 26

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int\sinh(a+bx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int-i\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 26

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int\sin(ia+ibx)dx}{b}\right)}{b}\right)}{b}\right)-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3118

$$\frac{2d^2\int(c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3791

$$\frac{2d^2\left(\frac{2}{3}\int(c+dx)\cosh(a+bx)dx-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{2d^2\left(\frac{2}{3}\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \textcolor{blue}{3777}$$

$$\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \textcolor{blue}{26}$$

$$\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \textcolor{blue}{3042}$$

$$\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 26$$

$$\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int \sin(ia+ibx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3118$$

$$\frac{2d^2\left(\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}\right)}{3b^2}-\frac{d(c+dx)^2\cosh^3(a+bx)}{3b^2}+\frac{2}{3}\left(\frac{(c+dx)^3\sinh(a+bx)}{b}+\frac{3id\left(\frac{i(c+dx)^2\cosh(a+bx)}{b}-\frac{2id\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)}{b}\right)}{b}\right)+\frac{(c+dx)^3\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^3*Cosh[a + b*x]^3,x]
```

output
```
-1/3*(d*(c + d*x)^2*Cosh[a + b*x]^3)/b^2 + ((c + d*x)^3*Cosh[a + b*x]^2*Si
nh[a + b*x])/(3*b) + (2*d^2*(-1/9*(d*Cosh[a + b*x]^3)/b^2 + ((c + d*x)*Cos
h[a + b*x]^2*Sinh[a + b*x])/(3*b) + (2*(-((d*Cosh[a + b*x])/b^2) + ((c + d
*x)*Sinh[a + b*x])/b))/3))/(3*b^2) + (2*(((c + d*x)^3*Sinh[a + b*x])/b + (
(3*I)*d*((I*(c + d*x)^2*Cosh[a + b*x])/b - ((2*I)*d*(-((d*Cosh[a + b*x])/b
^2) + ((c + d*x)*Sinh[a + b*x])/b))/b))/b))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.62

method	result
parallelrisch	$\frac{126b^2d^2x\left(\frac{dx}{2}+c\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^6-54(dx+c)\left((dx+c)^2b^2+\frac{14d^2}{3}\right)b\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5+\left(\left(-27d^3x^2-54c\,d^2x+162d\,c^2\right)b^2+\right.}{}$
risch	$\frac{\left(9b^3d^3x^3+27b^3c\,d^2x^2+27b^3c^2dx-9b^2d^3x^2+9b^3c^3-18b^2c\,d^2x-9b^2c^2d+6b\,d^3x+6bc\,d^2-2d^3\right)e^{3bx+3a}}{216b^4}+\frac{3\left(b^3d^3x^3+3\right.}{}$
orering	$-\frac{20d\left(9d^4x^4b^4+36b^4c\,d^3x^3+54b^4c^2d^2x^2+36b^4c^3dx+9b^4c^4+22b^2d^4x^2+44b^2c\,d^3x+22b^2c^2d^2-72d^4\right)\cosh(bx+a)^3}{27b^6(dx+c)^2}+$
derivativedivides	$\frac{\frac{d^3\left(\frac{2(bx+a)^3\sinh(bx+a)}{3}+\frac{(bx+a)^3\sinh(bx+a)\cosh(bx+a)^2}{3}-2(bx+a)^2\cosh(bx+a)+\frac{40(bx+a)\sinh(bx+a)}{9}-\frac{40\cosh(bx+a)}{9}-\frac{(bx\right.}{}}{b^3}}{}$
default	$\frac{\frac{d^3\left(\frac{2(bx+a)^3\sinh(bx+a)}{3}+\frac{(bx+a)^3\sinh(bx+a)\cosh(bx+a)^2}{3}-2(bx+a)^2\cosh(bx+a)+\frac{40(bx+a)\sinh(bx+a)}{9}-\frac{40\cosh(bx+a)}{9}-\frac{(bx\right.}{}}{b^3}}{}$

input
```
int((d*x+c)^3*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/27*(126*b^2*d^2*x*(1/2*d*x+c)*tanh(1/2*b*x+1/2*a)^6-54*(d*x+c)*((d*x+c)^
2*b^2+14/3*d^2)*b*tanh(1/2*b*x+1/2*a)^5+((-27*d^3*x^2-54*c*d^2*x+162*c^2*d
)*b^2+252*d^3)*tanh(1/2*b*x+1/2*a)^4+36*((d*x+c)^2*b^2+38/3*d^2)*(d*x+c)*b
*tanh(1/2*b*x+1/2*a)^3+((-27*d^3*x^2-54*c*d^2*x-216*c^2*d)*b^2-480*d^3)*ta
nh(1/2*b*x+1/2*a)^2-54*(d*x+c)*((d*x+c)^2*b^2+14/3*d^2)*b*tanh(1/2*b*x+1/2
*a)+(63*d^3*x^2+126*c*d^2*x+126*c^2*d)*b^2+244*d^3)/b^4/(tanh(1/2*b*x+1/2*
a)-1)^3/(tanh(1/2*b*x+1/2*a)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(161) = 322$.

Time = 0.09 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.96

$$\int (c+dx)^3\cosh^3(a+bx)\,dx = -\frac{(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d+2\,d^3)\cosh{(bx+a)}^3+3\,(9\,b^2d^3x^2+18\,b^2cd^2x+9\,b^2c^2d+2\,d^3)\cosh}{}$$

input
```
integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/108*((9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*cosh(b*x +
a)^3 + 3*(9*b^2*d^3*x^2 + 18*b^2*c*d^2*x + 9*b^2*c^2*d + 2*d^3)*cosh(b*x +
 a)*sinh(b*x + a)^2 - 3*(3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 + 3*b^3*c^3 + 2*b
*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*sinh(b*x + a)^3 + 243*(b^2*d^3*x^2 + 2
*b^2*c*d^2*x + b^2*c^2*d + 2*d^3)*cosh(b*x + a) - 9*(9*b^3*d^3*x^3 + 27*b^
3*c*d^2*x^2 + 9*b^3*c^3 + 54*b*c*d^2 + (3*b^3*d^3*x^3 + 9*b^3*c*d^2*x^2 +
3*b^3*c^3 + 2*b*c*d^2 + (9*b^3*c^2*d + 2*b*d^3)*x)*cosh(b*x + a)^2 + 27*(b
^3*c^2*d + 2*b*d^3)*x)*sinh(b*x + a))/b^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 495 vs. $2(173) = 346$.

Time = 0.51 (sec) , antiderivative size = 495, normalized size of antiderivative = 2.83

$$\int (c+dx)^3\cosh^3(a+bx)\,dx = \begin{cases} -\frac{2c^3\sinh^3{(a+bx)}}{3b}+\frac{c^3\sinh{(a+bx)}\cosh^2{(a+bx)}}{b}-\frac{2c^2dx\sinh^3{(a+bx)}}{b}+\frac{3c^2dx\sinh{(a+bx)}\cosh^2{(a+bx)}}{b}-\frac{2cd^2x^2\sinh^3{(a+bx)}}{b} \\ \left(c^3x+\frac{3c^2dx^2}{2}+cd^2x^3+\frac{d^3x^4}{4}\right)\cosh^3{(a)} \end{cases}$$

input
```
integrate((d*x+c)**3*cosh(b*x+a)**3,x)
```

output
```
Piecewise((-2*c**3*sinh(a + b*x)**3/(3*b) + c**3*sinh(a + b*x)*cosh(a + b*
x)**2/b - 2*c**2*d*x*sinh(a + b*x)**3/b + 3*c**2*d*x*sinh(a + b*x)*cosh(a
+ b*x)**2/b - 2*c*d**2*x**2*sinh(a + b*x)**3/b + 3*c*d**2*x**2*sinh(a + b*
x)*cosh(a + b*x)**2/b - 2*d**3*x**3*sinh(a + b*x)**3/(3*b) + d**3*x**3*sin
h(a + b*x)*cosh(a + b*x)**2/b + 2*c**2*d*sinh(a + b*x)**2*cosh(a + b*x)/b*
*2 - 7*c**2*d*cosh(a + b*x)**3/(3*b**2) + 4*c*d**2*x*sinh(a + b*x)**2*cosh
(a + b*x)/b**2 - 14*c*d**2*x*cosh(a + b*x)**3/(3*b**2) + 2*d**3*x**2*sinh(
a + b*x)**2*cosh(a + b*x)/b**2 - 7*d**3*x**2*cosh(a + b*x)**3/(3*b**2) - 4
0*c*d**2*sinh(a + b*x)**3/(9*b**3) + 14*c*d**2*sinh(a + b*x)*cosh(a + b*x)
**2/(3*b**3) - 40*d**3*x*sinh(a + b*x)**3/(9*b**3) + 14*d**3*x*sinh(a + b*
x)*cosh(a + b*x)**2/(3*b**3) + 40*d**3*sinh(a + b*x)**2*cosh(a + b*x)/(9*b
**4) - 122*d**3*cosh(a + b*x)**3/(27*b**4), Ne(b, 0)), ((c**3*x + 3*c**2*d
*x**2/2 + c*d**2*x**3 + d**3*x**4/4)*cosh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(161) = 322$.

Time = 0.08 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.51

$$\int (c+dx)^3\cosh^3(a+bx)\,dx$$
$$= \frac{1}{24}c^2d\left(\frac{(3\,bxe^{(3\,a)}-e^{(3\,a)})e^{(3\,bx)}}{b^2}+\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)$$
$$+\frac{1}{24}c^3\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)$$
$$+\frac{1}{72}cd^2\left(\frac{(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)})e^{(3\,bx)}}{b^3}+\frac{81\,(b^2x^2e^a-2\,bxe^a+2\,e^a)e^{(bx)}}{b^3}-\frac{81\,(b^2x^2+2\,bx+}{b^3}\right.$$
$$+\frac{1}{216}d^3\left(\frac{(9\,b^3x^3e^{(3\,a)}-9\,b^2x^2e^{(3\,a)}+6\,bxe^{(3\,a)}-2\,e^{(3\,a)})e^{(3\,bx)}}{b^4}+\frac{81\,(b^3x^3e^a-3\,b^2x^2e^a+6\,bxe^a-6\,e^a)}{b^4}\right.$$

input
```
integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*c^2*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e
^(b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/
b^2) + 1/24*c^3*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b -
e^(-3*b*x - 3*a)/b) + 1/72*c*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e
^(3*a))*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 -
 81*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3
*b*x - 3*a)/b^3) + 1/216*d^3*((9*b^3*x^3*e^(3*a) - 9*b^2*x^2*e^(3*a) + 6*b
*x*e^(3*a) - 2*e^(3*a))*e^(3*b*x)/b^4 + 81*(b^3*x^3*e^a - 3*b^2*x^2*e^a +
6*b*x*e^a - 6*e^a)*e^(b*x)/b^4 - 81*(b^3*x^3 + 3*b^2*x^2 + 6*b*x + 6)*e^(-
b*x - a)/b^4 - (9*b^3*x^3 + 9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*x - 3*a)/b^4)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(161) = 322$.

Time = 0.14 (sec) , antiderivative size = 414, normalized size of antiderivative = 2.37

$$\int (c+dx)^3 \cosh^3(a+bx)\,dx$$

$$= \frac{(9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 27\,b^3c^2dx - 9\,b^2d^3x^2 + 9\,b^3c^3 - 18\,b^2cd^2x - 9\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 2\,d^3)e^{(3}}{216\,b^4}$$
$$+ \frac{3\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx - 3\,b^2d^3x^2 + b^3c^3 - 6\,b^2cd^2x - 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 - 6\,d^3)e^{(bx+a}}{8\,b^4}$$
$$- \frac{3\,(b^3d^3x^3 + 3\,b^3cd^2x^2 + 3\,b^3c^2dx + 3\,b^2d^3x^2 + b^3c^3 + 6\,b^2cd^2x + 3\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 6\,d^3)e^{(-bx-}}{8\,b^4}$$
$$- \frac{(9\,b^3d^3x^3 + 27\,b^3cd^2x^2 + 27\,b^3c^2dx + 9\,b^2d^3x^2 + 9\,b^3c^3 + 18\,b^2cd^2x + 9\,b^2c^2d + 6\,bd^3x + 6\,bcd^2 + 2\,d^3)e}{216\,b^4}$$

input
```
integrate((d*x+c)^3*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/216*(9*b^3*d^3*x^3 + 27*b^3*c*d^2*x^2 + 27*b^3*c^2*d*x - 9*b^2*d^3*x^2 +
 9*b^3*c^3 - 18*b^2*c*d^2*x - 9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 - 2*d^3)
*e^(3*b*x + 3*a)/b^4 + 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x
- 3*b^2*d^3*x^2 + b^3*c^3 - 6*b^2*c*d^2*x - 3*b^2*c^2*d + 6*b*d^3*x + 6*b*
c*d^2 - 6*d^3)*e^(b*x + a)/b^4 - 3/8*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^
3*c^2*d*x + 3*b^2*d^3*x^2 + b^3*c^3 + 6*b^2*c*d^2*x + 3*b^2*c^2*d + 6*b*d^
3*x + 6*b*c*d^2 + 6*d^3)*e^(-b*x - a)/b^4 - 1/216*(9*b^3*d^3*x^3 + 27*b^3*
c*d^2*x^2 + 27*b^3*c^2*d*x + 9*b^2*d^3*x^2 + 9*b^3*c^3 + 18*b^2*c*d^2*x +
9*b^2*c^2*d + 6*b*d^3*x + 6*b*c*d^2 + 2*d^3)*e^(-3*b*x - 3*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 364, normalized size of antiderivative = 2.08

$$\int (c+dx)^3\cosh^3(a+bx)\,dx = \frac{\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(3\,b^2\,c^3+14\,c\,d^2\right)}{3\,b^3} - \frac{2\,\sinh(a+b\,x)^3\,\left(3\,b^2\,c^3+20\,c\,d^2\right)}{9\,b^3} - \frac{\cosh(a+b\,x)^3\,\left(63\,b^2\,c^2\,d+122\,d^3\right)}{27\,b^4} + \frac{2\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2\,\left(9\,b^2\,c^2\,d+20\,d^3\right)}{9\,b^4} - \frac{2\,x\,\sinh(a+b\,x)^3\,\left(9\,b^2\,c^2\,d+20\,d^3\right)}{9\,b^3} - \frac{7\,d^3\,x^2\,\cosh(a+b\,x)^3}{3\,b^2} - \frac{2\,d^3\,x^3\,\sinh(a+b\,x)^3}{3\,b} - \frac{14\,c\,d^2\,x\,\cosh(a+b\,x)^3}{3\,b^2} + \frac{x\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)\,\left(9\,b^2\,c^2\,d+14\,d^3\right)}{3\,b^3} + \frac{d^3\,x^3\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{b} + \frac{2\,d^3\,x^2\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2} - \frac{2\,c\,d^2\,x^2\,\sinh(a+b\,x)^3}{b} + \frac{3\,c\,d^2\,x^2\,\cosh(a+b\,x)^2\,\sinh(a+b\,x)}{b} + \frac{4\,c\,d^2\,x\,\cosh(a+b\,x)\,\sinh(a+b\,x)^2}{b^2}$$

input
```
int(cosh(a + b*x)^3*(c + d*x)^3,x)
```

output
```
(cosh(a + b*x)^2*sinh(a + b*x)*(14*c*d^2 + 3*b^2*c^3))/(3*b^3) - (2*sinh(a
 + b*x)^3*(20*c*d^2 + 3*b^2*c^3))/(9*b^3) - (cosh(a + b*x)^3*(122*d^3 + 63
*b^2*c^2*d))/(27*b^4) + (2*cosh(a + b*x)*sinh(a + b*x)^2*(20*d^3 + 9*b^2*c
^2*d))/(9*b^4) - (2*x*sinh(a + b*x)^3*(20*d^3 + 9*b^2*c^2*d))/(9*b^3) - (7
*d^3*x^2*cosh(a + b*x)^3)/(3*b^2) - (2*d^3*x^3*sinh(a + b*x)^3)/(3*b) - (1
4*c*d^2*x*cosh(a + b*x)^3)/(3*b^2) + (x*cosh(a + b*x)^2*sinh(a + b*x)*(14*
d^3 + 9*b^2*c^2*d))/(3*b^3) + (d^3*x^3*cosh(a + b*x)^2*sinh(a + b*x))/b +
(2*d^3*x^2*cosh(a + b*x)*sinh(a + b*x)^2)/b^2 - (2*c*d^2*x^2*sinh(a + b*x)
^3)/b + (3*c*d^2*x^2*cosh(a + b*x)^2*sinh(a + b*x))/b + (4*c*d^2*x*cosh(a
+ b*x)*sinh(a + b*x)^2)/b^2
```

Reduce [B] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 678, normalized size of antiderivative = 3.87

$$\int (c+dx)^3 \cosh^3(a+bx)\,dx$$
$$= \frac{81e^{4bx+4a}b^3d^3x^3 - 243e^{4bx+4a}b^2c^2d - 243e^{4bx+4a}b^2d^3x^2 + 486e^{4bx+4a}bc\,d^2 + 486e^{4bx+4a}b\,d^3x - 27b^3c^2dx - \cdots}{}$$

input
```
int((d*x+c)^3*cosh(b*x+a)^3,x)
```

output

```
(9*e**(6*a + 6*b*x)*b**3*c**3 + 27*e**(6*a + 6*b*x)*b**3*c**2*d*x + 27*e**
(6*a + 6*b*x)*b**3*c*d**2*x**2 + 9*e**(6*a + 6*b*x)*b**3*d**3*x**3 - 9*e**
(6*a + 6*b*x)*b**2*c**2*d - 18*e**(6*a + 6*b*x)*b**2*c*d**2*x - 9*e**(6*a
+ 6*b*x)*b**2*d**3*x**2 + 6*e**(6*a + 6*b*x)*b*c*d**2 + 6*e**(6*a + 6*b*x)
*b*d**3*x - 2*e**(6*a + 6*b*x)*d**3 + 81*e**(4*a + 4*b*x)*b**3*c**3 + 243*
e**(4*a + 4*b*x)*b**3*c**2*d*x + 243*e**(4*a + 4*b*x)*b**3*c*d**2*x**2 + 8
1*e**(4*a + 4*b*x)*b**3*d**3*x**3 - 243*e**(4*a + 4*b*x)*b**2*c**2*d - 486
*e**(4*a + 4*b*x)*b**2*c*d**2*x - 243*e**(4*a + 4*b*x)*b**2*d**3*x**2 + 48
6*e**(4*a + 4*b*x)*b*c*d**2 + 486*e**(4*a + 4*b*x)*b*d**3*x - 486*e**(4*a
+ 4*b*x)*d**3 - 81*e**(2*a + 2*b*x)*b**3*c**3 - 243*e**(2*a + 2*b*x)*b**3*
c**2*d*x - 243*e**(2*a + 2*b*x)*b**3*c*d**2*x**2 - 81*e**(2*a + 2*b*x)*b**
3*d**3*x**3 - 243*e**(2*a + 2*b*x)*b**2*c**2*d - 486*e**(2*a + 2*b*x)*b**2
*c*d**2*x - 243*e**(2*a + 2*b*x)*b**2*d**3*x**2 - 486*e**(2*a + 2*b*x)*b*c
*d**2 - 486*e**(2*a + 2*b*x)*b*d**3*x - 486*e**(2*a + 2*b*x)*d**3 - 9*b**3
*c**3 - 27*b**3*c**2*d*x - 27*b**3*c*d**2*x**2 - 9*b**3*d**3*x**3 - 9*b**2
*c**2*d - 18*b**2*c*d**2*x - 9*b**2*d**3*x**2 - 6*b*c*d**2 - 6*b*d**3*x -
2*d**3)/(216*e**(3*a + 3*b*x)*b**4)
```

3.18 $\int (c+dx)^2 \cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int (c+dx)^2 \cosh^3(a+bx)\,dx = -\frac{4d(c+dx)\cosh(a+bx)}{3b^2} - \frac{2d(c+dx)\cosh^3(a+bx)}{9b^2} + \frac{14d^2\sinh(a+bx)}{9b^3} + \frac{2(c+dx)^2\sinh(a+bx)}{3b} + \frac{(c+dx)^2\cosh^2(a+bx)\sinh(a+bx)}{3b} + \frac{2d^2\sinh^3(a+bx)}{27b^3}$$

output

```
-4/3*d*(d*x+c)*cosh(b*x+a)/b^2-2/9*d*(d*x+c)*cosh(b*x+a)^3/b^2+14/9*d^2*si
nh(b*x+a)/b^3+2/3*(d*x+c)^2*sinh(b*x+a)/b+1/3*(d*x+c)^2*cosh(b*x+a)^2*sinh
(b*x+a)/b+2/27*d^2*sinh(b*x+a)^3/b^3
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.76

$$\int (c+dx)^2\cosh^3(a+bx)\,dx$$
$$=\frac{-162bd(c+dx)\cosh(a+bx)-6bd(c+dx)\cosh(3(a+bx))+2(82d^2+45b^2(c+dx)^2+(2d^2+9b^2(c+}{108b^3}$$

input
```
Integrate[(c + d*x)^2*Cosh[a + b*x]^3,x]
```

output
```
(-162*b*d*(c + d*x)*Cosh[a + b*x] - 6*b*d*(c + d*x)*Cosh[3*(a + b*x)] + 2*
(82*d^2 + 45*b^2*(c + d*x)^2 + (2*d^2 + 9*b^2*(c + d*x)^2)*Cosh[2*(a + b*x
)])*Sinh[a + b*x])/(108*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.22, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\cosh^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{3792}$$
$$\frac{2d^2\int\cosh^3(a+bx)dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\cosh(a+bx)dx-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$
$$\Big\downarrow \text{3042}$$

$$\frac{2d^2\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^3 dx}{9b^2}+\frac{2}{3}\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3113}$$

$$\frac{2id^2\int \left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{9b^3}+\frac{2}{3}\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{3}\int (c+dx)^2\sin\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2id\int -i(c+dx)\sinh(a+bx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int (c+dx)\sinh(a+bx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}-\frac{2d\int -i(c+dx)\sin(ia+ibx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\int(c+dx)\sin(ia+ibx)dx}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\cosh(a+bx)dx}{b}\right)}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\int\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{b}\right)}{b}\right)+\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3117}$$

$$\frac{2id^2\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{9b^3}-\frac{2d(c+dx)\cosh^3(a+bx)}{9b^2}+\frac{2}{3}\left(\frac{(c+dx)^2\sinh(a+bx)}{b}+\frac{2id\left(\frac{i(c+dx)\cosh(a+bx)}{b}-\frac{id\sinh(a+bx)}{b^2}\right)}{b}\right)+\frac{(c+dx)^2\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^2*Cosh[a + b*x]^3,x]
```

output
```
(-2*d*(c + d*x)*Cosh[a + b*x]^3)/(9*b^2) + ((c + d*x)^2*Cosh[a + b*x]^2*Si
nh[a + b*x])/(3*b) + (((2*I)/9)*d^2*((-I)*Sinh[a + b*x] - (I/3)*Sinh[a + b
*x]^3))/b^3 + (2*(((c + d*x)^2*Sinh[a + b*x])/b + ((2*I)*d*((I*(c + d*x)*C
osh[a + b*x])/b - (I*d*Sinh[a + b*x])/b^2))/b))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.55

method	result
parallelrisch	$\frac{42x\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}b\,d^{2}+\left(-54(dx+c)^{2}b^{2}-84d^{2}\right)\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+108b\left(-\frac{dx}{6}+c\right)d\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\left(36(dx+c)^{2}b^{2}+1\right.}{27b^{3}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^{3}\left(t\right.}$
risch	$\frac{(9d^{2}x^{2}b^{2}+18b^{2}cdx+9b^{2}c^{2}-6b\,d^{2}x-6bcd+2d^{2})\,\mathrm{e}^{3bx+3a}}{216b^{3}}+\frac{3(d^{2}x^{2}b^{2}+2b^{2}cdx+b^{2}c^{2}-2b\,d^{2}x-2bcd+2d^{2})\,\mathrm{e}^{bx+a}}{8b^{3}}-3$
derivativedivides	$\frac{d^{2}\left(\frac{2(bx+a)^{2}\sinh(bx+a)}{3}+\frac{(bx+a)^{2}\sinh(bx+a)\cosh(bx+a)^{2}}{3}-\frac{4(bx+a)\cosh(bx+a)}{3}+\frac{40\sinh(bx+a)}{27}-\frac{2(bx+a)\cosh(bx+a)^{3}}{9}+\frac{2\sin}{}\right.}{b^{2}}$
default	$\frac{d^{2}\left(\frac{2(bx+a)^{2}\sinh(bx+a)}{3}+\frac{(bx+a)^{2}\sinh(bx+a)\cosh(bx+a)^{2}}{3}-\frac{4(bx+a)\cosh(bx+a)}{3}+\frac{40\sinh(bx+a)}{27}-\frac{2(bx+a)\cosh(bx+a)^{3}}{9}+\frac{2\sin}{}\right.}{b^{2}}$
orering	$-\frac{40d(9d^{4}x^{4}b^{4}+36b^{4}c\,d^{3}x^{3}+54b^{4}c^{2}d^{2}x^{2}+36b^{4}c^{3}dx+9b^{4}c^{4}+b^{2}d^{4}x^{2}+2b^{2}c\,d^{3}x+b^{2}c^{2}d^{2}-12d^{4})\cosh(bx+a)^{3}}{81b^{6}(dx+c)^{3}}+\frac{2(45}{}$

input
```
int((d*x+c)^2*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/27*(42*x*tanh(1/2*b*x+1/2*a)^6*b*d^2+(-54*(d*x+c)^2*b^2-84*d^2)*tanh(1/2
*b*x+1/2*a)^5+108*b*(-1/6*d*x+c)*d*tanh(1/2*b*x+1/2*a)^4+(36*(d*x+c)^2*b^2
+152*d^2)*tanh(1/2*b*x+1/2*a)^3-144*(1/8*d*x+c)*b*d*tanh(1/2*b*x+1/2*a)^2+
(-54*(d*x+c)^2*b^2-84*d^2)*tanh(1/2*b*x+1/2*a)+84*b*d*(1/2*d*x+c))/b^3/(ta
nh(1/2*b*x+1/2*a)-1)^3/(tanh(1/2*b*x+1/2*a)+1)^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.62

$$\int (c+dx)^2\cosh^3(a+bx)\,dx =$$

$$-\frac{6\left(bd^{2}x+bcd\right)\cosh\left(bx+a\right)^{3}+18\left(bd^{2}x+bcd\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{2}-\left(9\,b^{2}d^{2}x^{2}+18\,b^{2}cdx+9\right.}{}$$

input
```
integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/108*(6*(b*d^2*x + b*c*d)*cosh(b*x + a)^3 + 18*(b*d^2*x + b*c*d)*cosh(b*
x + a)*sinh(b*x + a)^2 - (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 2*d^2
)*sinh(b*x + a)^3 + 162*(b*d^2*x + b*c*d)*cosh(b*x + a) - 3*(27*b^2*d^2*x^
2 + 54*b^2*c*d*x + 27*b^2*c^2 + (9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2
+ 2*d^2)*cosh(b*x + a)^2 + 54*d^2)*sinh(b*x + a))/b^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(121) = 242$.

Time = 0.35 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.31

$$\int (c+dx)^2 \cosh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{2c^2\sinh^3(a+bx)}{3b} + \frac{c^2\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{4cdx\sinh^3(a+bx)}{3b} + \frac{2cdx\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2d^2x^2\sinh^3(a+bx)}{3b} + \\ \left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right)\cosh^3(a) \end{cases}$$

input
```
integrate((d*x+c)**2*cosh(b*x+a)**3,x)
```

output
```
Piecewise((-2*c**2*sinh(a + b*x)**3/(3*b) + c**2*sinh(a + b*x)*cosh(a + b*
x)**2/b - 4*c*d*x*sinh(a + b*x)**3/(3*b) + 2*c*d*x*sinh(a + b*x)*cosh(a +
b*x)**2/b - 2*d**2*x**2*sinh(a + b*x)**3/(3*b) + d**2*x**2*sinh(a + b*x)*c
osh(a + b*x)**2/b + 4*c*d*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) - 14*c*d
*cosh(a + b*x)**3/(9*b**2) + 4*d**2*x*sinh(a + b*x)**2*cosh(a + b*x)/(3*b*
*2) - 14*d**2*x*cosh(a + b*x)**3/(9*b**2) - 40*d**2*sinh(a + b*x)**3/(27*b
**3) + 14*d**2*sinh(a + b*x)*cosh(a + b*x)**2/(9*b**3), Ne(b, 0)), ((c**2*
x + c*d*x**2 + d**2*x**3/3)*cosh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(111) = 222$.

Time = 0.05 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.21

$$\int (c+dx)^2\cosh^3(a+bx)\,dx$$
$$=\frac{1}{36}\,cd\left(\frac{(3\,bxe^{(3\,a)}-e^{(3\,a)})e^{(3\,bx)}}{b^2}+\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)+\frac{1}{24}\,c^2\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)+\frac{1}{216}\,d^2\left(\frac{(9\,b^2x^2e^{(3\,a)}-6\,bxe^{(3\,a)}+2\,e^{(3\,a)})e^{(3\,bx)}}{b^3}+\frac{81\,(b^2x^2e^a-2\,bxe^a+2\,e^a)e^{(bx)}}{b^3}-\frac{81\,(b^2x^2+2\,bx+}{b^3}\right.$$

input
```
integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/36*c*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^(
b*x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^
2) + 1/24*c^2*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^
(-3*b*x - 3*a)/b) + 1/216*d^2*((9*b^2*x^2*e^(3*a) - 6*b*x*e^(3*a) + 2*e^(3
*a))*e^(3*b*x)/b^3 + 81*(b^2*x^2*e^a - 2*b*x*e^a + 2*e^a)*e^(b*x)/b^3 - 81
*(b^2*x^2 + 2*b*x + 2)*e^(-b*x - a)/b^3 - (9*b^2*x^2 + 6*b*x + 2)*e^(-3*b*
x - 3*a)/b^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(111) = 222$.

Time = 0.12 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

$$\int (c+dx)^2 \cosh^3(a+bx)\,dx = \frac{(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 - 6\,bd^2x - 6\,bcd + 2\,d^2)e^{(3\,bx+3\,a)}}{216\,b^3} + \frac{3\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 - 2\,bd^2x - 2\,bcd + 2\,d^2)e^{(bx+a)}}{8\,b^3} - \frac{3\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2 + 2\,bd^2x + 2\,bcd + 2\,d^2)e^{(-bx-a)}}{8\,b^3} - \frac{(9\,b^2d^2x^2 + 18\,b^2cdx + 9\,b^2c^2 + 6\,bd^2x + 6\,bcd + 2\,d^2)e^{(-3\,bx-3\,a)}}{216\,b^3}$$

input
```
integrate((d*x+c)^2*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/216*(9*b^2*d^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 - 6*b*d^2*x - 6*b*c*d + 2*
d^2)*e^(3*b*x + 3*a)/b^3 + 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 - 2*b*
d^2*x - 2*b*c*d + 2*d^2)*e^(b*x + a)/b^3 - 3/8*(b^2*d^2*x^2 + 2*b^2*c*d*x
+ b^2*c^2 + 2*b*d^2*x + 2*b*c*d + 2*d^2)*e^(-b*x - a)/b^3 - 1/216*(9*b^2*d
^2*x^2 + 18*b^2*c*d*x + 9*b^2*c^2 + 6*b*d^2*x + 6*b*c*d + 2*d^2)*e^(-3*b*x
 - 3*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.49

$$\int (c+dx)^2 \cosh^3(a+bx)\,dx = \frac{\frac{3\,d^2\sinh(a+b\,x)}{2} + \frac{d^2\sinh(3\,a+3\,b\,x)}{54} + \frac{3\,b^2\,c^2\sinh(a+b\,x)}{4} + \frac{b^2\,c^2\sinh(3\,a+3\,b\,x)}{12} + \frac{3\,b^2\,d^2\,x^2\sinh(a+b\,x)}{4} - \frac{b\,c\,d\cosh(3\,a+3\,b\,x)}{18} - \cdots}{}$$

input
```
int(cosh(a + b*x)^3*(c + d*x)^2,x)
```

output
```
((3*d^2*sinh(a + b*x))/2 + (d^2*sinh(3*a + 3*b*x))/54 + (3*b^2*c^2*sinh(a
+ b*x))/4 + (b^2*c^2*sinh(3*a + 3*b*x))/12 + (3*b^2*d^2*x^2*sinh(a + b*x))
/4 - (b*c*d*cosh(3*a + 3*b*x))/18 - (3*b*d^2*x*cosh(a + b*x))/2 + (b^2*d^2
*x^2*sinh(3*a + 3*b*x))/12 - (b*d^2*x*cosh(3*a + 3*b*x))/18 - (3*b*c*d*cos
h(a + b*x))/2 + (b^2*c*d*x*sinh(3*a + 3*b*x))/6 + (3*b^2*c*d*x*sinh(a + b*
x))/2)/b^3
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 374, normalized size of antiderivative = 3.04

$$\int (c+dx)^2 \cosh^3(a+bx)\,dx$$
$$= \frac{9e^{6bx+6a}b^2c^2 + 18e^{6bx+6a}b^2cdx + 9e^{6bx+6a}b^2d^2x^2 - 6e^{6bx+6a}bcd - 6e^{6bx+6a}b\,d^2x + 2e^{6bx+6a}d^2 + 81e^{4bx+4a}b^2}{}$$

input
```
int((d*x+c)^2*cosh(b*x+a)^3,x)
```

output
```
(9*e**(6*a + 6*b*x)*b**2*c**2 + 18*e**(6*a + 6*b*x)*b**2*c*d*x + 9*e**(6*a
 + 6*b*x)*b**2*d**2*x**2 - 6*e**(6*a + 6*b*x)*b*c*d - 6*e**(6*a + 6*b*x)*b
*d**2*x + 2*e**(6*a + 6*b*x)*d**2 + 81*e**(4*a + 4*b*x)*b**2*c**2 + 162*e*
*(4*a + 4*b*x)*b**2*c*d*x + 81*e**(4*a + 4*b*x)*b**2*d**2*x**2 - 162*e**(4
*a + 4*b*x)*b*c*d - 162*e**(4*a + 4*b*x)*b*d**2*x + 162*e**(4*a + 4*b*x)*d
**2 - 81*e**(2*a + 2*b*x)*b**2*c**2 - 162*e**(2*a + 2*b*x)*b**2*c*d*x - 81
*e**(2*a + 2*b*x)*b**2*d**2*x**2 - 162*e**(2*a + 2*b*x)*b*c*d - 162*e**(2*
a + 2*b*x)*b*d**2*x - 162*e**(2*a + 2*b*x)*d**2 - 9*b**2*c**2 - 18*b**2*c*
d*x - 9*b**2*d**2*x**2 - 6*b*c*d - 6*b*d**2*x - 2*d**2)/(216*e**(3*a + 3*b
*x)*b**3)
```

3.19 $\int (c+dx)\cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 75

$$\int (c+dx)\cosh^3(a+bx)\,dx = -\frac{2d\cosh(a+bx)}{3b^2} - \frac{d\cosh^3(a+bx)}{9b^2} + \frac{2(c+dx)\sinh(a+bx)}{3b} + \frac{(c+dx)\cosh^2(a+bx)\sinh(a+bx)}{3b}$$

output
```
-2/3*d*cosh(b*x+a)/b^2-1/9*d*cosh(b*x+a)^3/b^2+2/3*(d*x+c)*sinh(b*x+a)/b+1
/3*(d*x+c)*cosh(b*x+a)^2*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

$$\int (c+dx)\cosh^3(a+bx)\,dx = -\frac{27d\cosh(a+bx) + d\cosh(3(a+bx)) - 3b(c+dx)(9\sinh(a+bx) + \sinh(3(a+bx)))}{36b^2}$$

input
```
Integrate[(c + d*x)*Cosh[a + b*x]^3,x]
```

output
```
-1/36*(27*d*Cosh[a + b*x] + d*Cosh[3*(a + b*x)] - 3*b*(c + d*x)*(9*Sinh[a
+ b*x] + Sinh[3*(a + b*x)]))/b^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{2}{3}\int (c+dx)\cosh(a+bx)dx-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\int (c+dx)\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{id\int -i\sinh(a+bx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int \sinh(a+bx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\int -i\sin(ia+ibx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}+\frac{id\int \sin(ia+ibx)dx}{b}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{ 3118}$$

$$\frac{2}{3}\left(\frac{(c+dx)\sinh(a+bx)}{b}-\frac{d\cosh(a+bx)}{b^2}\right)-\frac{d\cosh^3(a+bx)}{9b^2}+\frac{(c+dx)\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)*Cosh[a + b*x]^3,x]
```

output
```
-1/9*(d*Cosh[a + b*x]^3)/b^2 + ((c + d*x)*Cosh[a + b*x]^2*Sinh[a + b*x])/(
3*b) + (2*(-((d*Cosh[a + b*x])/b^2) + ((c + d*x)*Sinh[a + b*x])/b))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.83

method	result
parallelrisch	$\frac{3b(dx+c)\sinh(3bx+3a)-d\cosh(3bx+3a)+27(dx+c)b\sinh(bx+a)-27\cosh(bx+a)d-28d}{36b^2}$
risch	$\frac{(3dxb+3cb-d)\mathrm{e}^{3bx+3a}}{72b^2}+\frac{3(dxb+cb-d)\mathrm{e}^{bx+a}}{8b^2}-\frac{3(dxb+cb+d)\mathrm{e}^{-bx-a}}{8b^2}-\frac{(3dxb+3cb+d)\mathrm{e}^{-3bx-3a}}{72b^2}$
derivativedivides	$\frac{\frac{d\left(\frac{2(bx+a)\sinh(bx+a)}{3}+\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{3}-\frac{2\cosh(bx+a)}{3}-\frac{\cosh(bx+a)^3}{9}\right)}{b}-\frac{da\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}+c}{b}$
default	$\frac{\frac{d\left(\frac{2(bx+a)\sinh(bx+a)}{3}+\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^2}{3}-\frac{2\cosh(bx+a)}{3}-\frac{\cosh(bx+a)^3}{9}\right)}{b}-\frac{da\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}+c}{b}$
orering	$-\frac{4d(5d^2x^2b^2+10b^2cdx+5b^2c^2-2d^2)\cosh(bx+a)^3}{9b^4(dx+c)^2}+\frac{2(5d^2x^2b^2+10b^2cdx+5b^2c^2-4d^2)\left(d\cosh(bx+a)^3+3(dx+c)\right.}{9b^4(dx+c)^2}$

input
```
int((d*x+c)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/36*(3*b*(d*x+c)*sinh(3*b*x+3*a)-d*cosh(3*b*x+3*a)+27*(d*x+c)*b*sinh(b*x+
a)-27*cosh(b*x+a)*d-28*d)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.27

$$\int (c+dx)\cosh^3(a+bx)\,dx = -\frac{d\cosh(bx+a)^3 + 3\,d\cosh(bx+a)\sinh(bx+a)^2 - 3\,(bdx+bc)\sinh(bx+a)^3 + 27\,d\cosh(bx+a) - \cdots}{36\,b^2}$$

input
```
integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/36*(d*cosh(b*x + a)^3 + 3*d*cosh(b*x + a)*sinh(b*x + a)^2 - 3*(b*d*x +
b*c)*sinh(b*x + a)^3 + 27*d*cosh(b*x + a) - 9*(3*b*d*x + (b*d*x + b*c)*cos
h(b*x + a)^2 + 3*b*c)*sinh(b*x + a))/b^2
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.68

$$\int (c+dx)\cosh^3(a+bx)\,dx$$
$$= \begin{cases} -\frac{2c\sinh^3(a+bx)}{3b} + \frac{c\sinh(a+bx)\cosh^2(a+bx)}{b} - \frac{2dx\sinh^3(a+bx)}{3b} + \frac{dx\sinh(a+bx)\cosh^2(a+bx)}{b} + \frac{2d\sinh^2(a+bx)\cosh(a+bx)}{3b^2} - \cdots \\ \left(cx+\frac{dx^2}{2}\right)\cosh^3(a) \end{cases}$$

input
```
integrate((d*x+c)*cosh(b*x+a)**3,x)
```

output
```
Piecewise((-2*c*sinh(a + b*x)**3/(3*b) + c*sinh(a + b*x)*cosh(a + b*x)**2/
b - 2*d*x*sinh(a + b*x)**3/(3*b) + d*x*sinh(a + b*x)*cosh(a + b*x)**2/b +
2*d*sinh(a + b*x)**2*cosh(a + b*x)/(3*b**2) - 7*d*cosh(a + b*x)**3/(9*b**2
), Ne(b, 0)), ((c*x + d*x**2/2)*cosh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(67) = 134$.

Time = 0.05 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.91

$$\int (c+dx)\cosh^3(a+bx)\,dx$$
$$=\frac{1}{72}d\left(\frac{\left(3\,bxe^{(3\,a)}-e^{(3\,a)}\right)e^{(3\,bx)}}{b^2}+\frac{27\,(bxe^a-e^a)e^{(bx)}}{b^2}-\frac{27\,(bx+1)e^{(-bx-a)}}{b^2}-\frac{(3\,bx+1)e^{(-3\,bx-3\,a)}}{b^2}\right)$$
$$+\frac{1}{24}c\left(\frac{e^{(3\,bx+3\,a)}}{b}+\frac{9\,e^{(bx+a)}}{b}-\frac{9\,e^{(-bx-a)}}{b}-\frac{e^{(-3\,bx-3\,a)}}{b}\right)$$

input
```
integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/72*d*((3*b*x*e^(3*a) - e^(3*a))*e^(3*b*x)/b^2 + 27*(b*x*e^a - e^a)*e^(b*
x)/b^2 - 27*(b*x + 1)*e^(-b*x - a)/b^2 - (3*b*x + 1)*e^(-3*b*x - 3*a)/b^2)
 + 1/24*c*(e^(3*b*x + 3*a)/b + 9*e^(b*x + a)/b - 9*e^(-b*x - a)/b - e^(-3*
b*x - 3*a)/b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.31

$$\int (c+dx)\cosh^3(a+bx)\,dx=\frac{(3\,bdx+3\,bc-d)e^{(3\,bx+3\,a)}}{72\,b^2}+\frac{3\,(bdx+bc-d)e^{(bx+a)}}{8\,b^2}-\frac{3\,(bdx+bc+d)e^{(-bx-a)}}{8\,b^2}-\frac{(3\,bdx+3\,bc+d)e^{(-3\,bx-3\,a)}}{72\,b^2}$$

input
```
integrate((d*x+c)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/72*(3*b*d*x + 3*b*c - d)*e^(3*b*x + 3*a)/b^2 + 3/8*(b*d*x + b*c - d)*e^(
b*x + a)/b^2 - 3/8*(b*d*x + b*c + d)*e^(-b*x - a)/b^2 - 1/72*(3*b*d*x + 3*
b*c + d)*e^(-3*b*x - 3*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.03

$$\int (c+dx)\cosh^3(a+bx)\,dx$$
$$=\frac{\frac{3\,c\sinh(a+b\,x)}{4}+\frac{c\sinh(3\,a+3\,b\,x)}{12}+\frac{d\,x\sinh(3\,a+3\,b\,x)}{12}+\frac{3\,d\,x\sinh(a+b\,x)}{4}}{b}-\frac{d\cosh(3\,a+3\,b\,x)}{36\,b^2}-\frac{3\,d\cosh(a+b\,x)}{4\,b^2}$$

input

```
int(cosh(a + b*x)^3*(c + d*x),x)
```

output

```
((3*c*sinh(a + b*x))/4 + (c*sinh(3*a + 3*b*x))/12 + (d*x*sinh(3*a + 3*b*x)
)/12 + (3*d*x*sinh(a + b*x))/4)/b - (d*cosh(3*a + 3*b*x))/(36*b^2) - (3*d*
cosh(a + b*x))/(4*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.08

$$\int (c+dx)\cosh^3(a+bx)\,dx$$
$$=\frac{3e^{6bx+6a}bc+3e^{6bx+6a}bdx-e^{6bx+6a}d+27e^{4bx+4a}bc+27e^{4bx+4a}bdx-27e^{4bx+4a}d-27e^{2bx+2a}bc-27e^{2bx+2}}{72e^{3bx+3a}b^2}$$

input

```
int((d*x+c)*cosh(b*x+a)^3,x)
```

output

```
(3*e**(6*a + 6*b*x)*b*c + 3*e**(6*a + 6*b*x)*b*d*x - e**(6*a + 6*b*x)*d +
27*e**(4*a + 4*b*x)*b*c + 27*e**(4*a + 4*b*x)*b*d*x - 27*e**(4*a + 4*b*x)*
d - 27*e**(2*a + 2*b*x)*b*c - 27*e**(2*a + 2*b*x)*b*d*x - 27*e**(2*a + 2*b
*x)*d - 3*b*c - 3*b*d*x - d)/(72*e**(3*a + 3*b*x)*b**2)
```

3.20 $\int \frac{\cosh^3(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 121

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = \frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d} + \frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d} + \frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

output
```
3/4*cosh(a-b*c/d)*Chi(b*c/d+b*x)/d+1/4*cosh(3*a-3*b*c/d)*Chi(3*b*c/d+3*b*x
)/d+3/4*sinh(a-b*c/d)*Shi(b*c/d+b*x)/d+1/4*sinh(3*a-3*b*c/d)*Shi(3*b*c/d+3
*b*x)/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = \frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(b\left(\frac{c}{d}+x\right)\right)+\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3b(c+dx)}{d}\right)+3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(b\left(\frac{c}{d}+x\right)\right)+\sinh\left(3\right.}{4d}$$

input
```
Integrate[Cosh[a + b*x]^3/(c + d*x),x]
```

output
```
(3*Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] + Cosh[3*a - (3*b*c)/d]*Cos
hIntegral[(3*b*(c + d*x))/d] + 3*Sinh[a - (b*c)/d]*SinhIntegral[b*(c/d + x
)] + Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))/d])/(4*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = $\{3042, 3793, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx$$
$$\Big\downarrow \text{ 3793}$$
$$\int \left(\frac{3\cosh(a+bx)}{4(c+dx)}+\frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}$$

input
```
Int[Cosh[a + b*x]^3/(c + d*x),x]
```

output
```
(3*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b
*c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d) + (3*Sinh[a - (b*c)/d]*SinhI
ntegral[(b*c)/d + b*x])/(4*d) + (Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c
)/d + 3*b*x])/(4*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{\mathrm{e}^{-\frac{3(ad-cb)}{d}} \operatorname{expIntegral}_1\left(3bx+3a-\frac{3(ad-cb)}{d}\right)}{8d}-\frac{3\,\mathrm{e}^{-\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{8d}-\frac{3\,\mathrm{e}^{\frac{ad-cb}{d}} \operatorname{expIntegral}_1\left(-bx-a-\right.}{8d}$

input
```
int(cosh(b*x+a)^3/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*exp(-3*(a*d-b*c)/d)*Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)-3/8/d*exp(-(a*d-b
*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/8/d*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*
c)/d)-1/8/d*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = \frac{3\left(\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)+\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)\right)\cosh\left(-\frac{bc-ad}{d}\right)+\left(\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)+\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)\right)\cosh\left(-\frac{3\,(bc-ad)}{d}\right)+3}{8\,d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
1/8*(3*(Ei((b*d*x + b*c)/d) + Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) +
 (Ei(3*(b*d*x + b*c)/d) + Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) +
 3*(Ei((b*d*x + b*c)/d) - Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) + (Ei
(3*(b*d*x + b*c)/d) - Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/d
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = \int \frac{\cosh^3{(a+bx)}}{c+dx}\,dx$$

input
```
integrate(cosh(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(cosh(a + b*x)**3/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.97

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = -\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_1\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_1\left(\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_1\left(-\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_1\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
-1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(1, 3*(d*x + c)*b/d)/d - 3/8*e^(-a +
 b*c/d)*exp_integral_e(1, (d*x + c)*b/d)/d - 3/8*e^(a - b*c/d)*exp_integra
l_e(1, -(d*x + c)*b/d)/d - 1/8*e^(3*a - 3*b*c/d)*exp_integral_e(1, -3*(d*x
 + c)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx$$
$$= \frac{\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}+3\,\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)}+3\,\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)}+\mathrm{Ei}\left(-\frac{3\,(bdx+bc)}{d}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{8\,d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
1/8*(Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*Ei((b*d*x + b*c)/d)*e^(a
- b*c/d) + 3*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + Ei(-3*(b*d*x + b*c)/d)*
e^(-3*a + 3*b*c/d))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = \int \frac{\cosh(a+b\,x)^3}{c+d\,x}\,dx$$

input
```
int(cosh(a + b*x)^3/(c + d*x),x)
```

output
```
int(cosh(a + b*x)^3/(c + d*x), x)
```

Reduce [F]

$$\int \frac{\cosh^3(a+bx)}{c+dx}\,dx = \int \frac{\cosh(bx+a)^3}{dx+c}\,dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c),x)
```

output
```
int(cosh(a + b*x)**3/(c + d*x),x)
```

3.21 $\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 145

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx = -\frac{\cosh^3(a+bx)}{d(c+dx)} + \frac{3b\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right)}{4d^2} + \frac{3b\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right)}{4d^2} + \frac{3b\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d^2} + \frac{3b\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d^2}$$

output
```
-cosh(b*x+a)^3/d/(d*x+c)+3/4*b*Chi(3*b*c/d+3*b*x)*sinh(3*a-3*b*c/d)/d^2+3/
4*b*Chi(b*c/d+b*x)*sinh(a-b*c/d)/d^2+3/4*b*cosh(a-b*c/d)*Shi(b*c/d+b*x)/d^
2+3/4*b*cosh(3*a-3*b*c/d)*Shi(3*b*c/d+3*b*x)/d^2
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.35

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx = -\frac{3\cosh(a)\cosh(bx)}{4d(c+dx)} - \frac{\cosh(3a)\cosh(3bx)}{4d(c+dx)} - \frac{3\sinh(a)\sinh(bx)}{4d(c+dx)} - \frac{\sinh(3a)\sinh(3bx)}{4d(c+dx)} - \frac{3b\left(-2\text{Chi}\left(\frac{3bc}{d}+3bx\right)\sinh\left(3a-\frac{3bc}{d}\right) - 2\text{Chi}\left(\frac{bc}{d}+bx\right)\sinh\left(a-\frac{bc}{d}\right) - 2\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right) - \right.}{8d^2}$$

input
```
Integrate[Cosh[a + b*x]^3/(c + d*x)^2,x]
```

output
```
(-3*Cosh[a]*Cosh[b*x])/(4*d*(c + d*x)) - (Cosh[3*a]*Cosh[3*b*x])/(4*d*(c +
 d*x)) - (3*Sinh[a]*Sinh[b*x])/(4*d*(c + d*x)) - (Sinh[3*a]*Sinh[3*b*x])/(
4*d*(c + d*x)) - (3*b*(-2*CoshIntegral[(3*b*c)/d + 3*b*x]*Sinh[3*a - (3*b*
c)/d] - 2*CoshIntegral[(b*c)/d + b*x]*Sinh[a - (b*c)/d] - 2*Cosh[a - (b*c)
/d]*SinhIntegral[(b*c)/d + b*x] - 2*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*
b*c)/d + 3*b*x]))/(8*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 3794}$$

$$-\frac{\cosh^3(a+bx)}{d(c+dx)}+\frac{3ib\int\left(-\frac{i\sinh(a+bx)}{4(c+dx)}-\frac{i\sinh(3a+3bx)}{4(c+dx)}\right)dx}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\cosh^3(a+bx)}{d(c+dx)}+\frac{3ib\left(-\frac{i\sinh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}-\frac{i\sinh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{i\cosh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}-\frac{i\cosh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}\right)}{d}$$

input
```
Int[Cosh[a + b*x]^3/(c + d*x)^2,x]
```

output
```
-(Cosh[a + b*x]^3/(d*(c + d*x))) + ((3*I)*b*(((-1/4*I)*CoshIntegral[(3*b*c
)/d + 3*b*x]*Sinh[3*a - (3*b*c)/d])/d - ((I/4)*CoshIntegral[(b*c)/d + b*x]
*Sinh[a - (b*c)/d])/d - ((I/4)*Cosh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*
x])/d - ((I/4)*Cosh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)/d + 3*b*x])/d))/
d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
 - 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.87

method	result
risch	$-\frac{b\,\mathrm{e}^{-3bx-3a}}{8d(dxb+cb)}+\frac{3b\,\mathrm{e}^{-\frac{3(ad-cb)}{d}}\operatorname{expIntegral}_1\left(3bx+3a-\frac{3(ad-cb)}{d}\right)}{8d^2}-\frac{3b\,\mathrm{e}^{-bx-a}}{8d(dxb+cb)}+\frac{3b\,\mathrm{e}^{-\frac{ad-cb}{d}}\operatorname{expIntegral}_1\left(bx+a-\frac{ad-cb}{d}\right)}{8d^2}$

input
```
int(cosh(b*x+a)^3/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*b*exp(-3*b*x-3*a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-3*(a*d-b*c)/d)*Ei(1,3*
b*x+3*a-3*(a*d-b*c)/d)-3/8*b*exp(-b*x-a)/d/(b*d*x+b*c)+3/8*b/d^2*exp(-(a*d
-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/8*b/d^2*exp(b*x+a)/(b*c/d+b*x)-3/8*b/d^
2*exp((a*d-b*c)/d)*Ei(1,-b*x-a-(-a*d+b*c)/d)-1/8*b/d^2*exp(3*b*x+3*a)/(b*c
/d+b*x)-3/8*b/d^2*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-3*a-3*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(137) = 274.

Time = 0.11 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.10

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx =$$
$$-\frac{2\,d\cosh\left(bx+a\right)^3+6\,d\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+6\,d\cosh\left(bx+a\right)-3\left((bdx+bc)\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)-(b$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*d*cosh(b*x + a)^3 + 6*d*cosh(b*x + a)*sinh(b*x + a)^2 + 6*d*cosh(b
*x + a) - 3*((b*d*x + b*c)*Ei((b*d*x + b*c)/d) - (b*d*x + b*c)*Ei(-(b*d*x
+ b*c)/d))*cosh(-(b*c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) -
 (b*d*x + b*c)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) - 3*((b*d*x
+ b*c)*Ei((b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*
c - a*d)/d) - 3*((b*d*x + b*c)*Ei(3*(b*d*x + b*c)/d) + (b*d*x + b*c)*Ei(-3
*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cosh^3{(a+bx)}}{(c+dx)^2}\,dx$$

input
```
integrate(cosh(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(cosh(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx = -\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_2\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_2\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_2\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_2\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(2, 3*(d*x + c)*b/d)/((d*x + c)*d) -
 3/8*e^(-a + b*c/d)*exp_integral_e(2, (d*x + c)*b/d)/((d*x + c)*d) - 3/8*e
^(a - b*c/d)*exp_integral_e(2, -(d*x + c)*b/d)/((d*x + c)*d) - 1/8*e^(3*a
- 3*b*c/d)*exp_integral_e(2, -3*(d*x + c)*b/d)/((d*x + c)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. 2(137) = 274.

Time = 0.17 (sec) , antiderivative size = 1075, normalized size of antiderivative = 7.41

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
-1/8*(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(-3*((d*x + c)
*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) +
 3*b^3*c*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)
/d)*e^(3*(b*c - a*d)/d) - 3*a*b^2*d*Ei(-3*((d*x + c)*(b - b*c/(d*x + c) +
a*d/(d*x + c)) + b*c - a*d)/d)*e^(3*(b*c - a*d)/d) + 3*(d*x + c)*(b - b*c/
(d*x + c) + a*d/(d*x + c))*b^2*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*
x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) + 3*b^3*c*Ei(-((d*x + c)*(b - b*
c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*c - a*d)/d) - 3*a*b^2*d
*Ei(-((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^((b*
c - a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))*b^2*Ei(((d*x
 + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/
d) - 3*b^3*c*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d
)/d)*e^(-(b*c - a*d)/d) + 3*a*b^2*d*Ei(((d*x + c)*(b - b*c/(d*x + c) + a*d
/(d*x + c)) + b*c - a*d)/d)*e^(-(b*c - a*d)/d) - 3*(d*x + c)*(b - b*c/(d*x
 + c) + a*d/(d*x + c))*b^2*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
 c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) - 3*b^3*c*Ei(3*((d*x + c)*(b - b
*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e^(-3*(b*c - a*d)/d) + 3*a*b
^2*d*Ei(3*((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c)) + b*c - a*d)/d)*e
^(-3*(b*c - a*d)/d) + b^2*d*e^(3*(d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x +
 c))/d) + 3*b^2*d*e^((d*x + c)*(b - b*c/(d*x + c) + a*d/(d*x + c))/d) +...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int(cosh(a + b*x)^3/(c + d*x)^2,x)
```

output
```
int(cosh(a + b*x)^3/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^2}\,dx$$
$$= \frac{e^{4a}\left(\int \frac{e^{3bx}}{d^2x^2+2cdx+c^2}dx\right)+3e^{2a}\left(\int \frac{e^{bx}}{d^2x^2+2cdx+c^2}dx\right)+e^{a}\left(\int \frac{1}{e^{3bx+3a}c^2+2e^{3bx+3a}cdx+e^{3bx+3a}d^2x^2}dx\right)+3\left(\int \frac{1}{e^{bx}c^2+2 \cdots}\right.}{8e^{a}}$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^2,x)
```

output
```
(e**(4*a)*int(e**(3*b*x)/(c**2 + 2*c*d*x + d**2*x**2),x) + 3*e**(2*a)*int(
e**(b*x)/(c**2 + 2*c*d*x + d**2*x**2),x) + e**a*int(1/(e**(3*a + 3*b*x)*c*
*2 + 2*e**(3*a + 3*b*x)*c*d*x + e**(3*a + 3*b*x)*d**2*x**2),x) + 3*int(1/(
e**(b*x)*c**2 + 2*e**(b*x)*c*d*x + e**(b*x)*d**2*x**2),x))/(8*e**a)
```

3.22 $\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 184

$$\begin{aligned}\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx = &-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}+\frac{3b^2\cosh\left(a-\frac{bc}{d}\right)\mathrm{Chi}\left(\frac{bc}{d}+bx\right)}{8d^3}\\&+\frac{9b^2\cosh\left(3a-\frac{3bc}{d}\right)\mathrm{Chi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\\&-\frac{3b\cosh^2(a+bx)\sinh(a+bx)}{2d^2(c+dx)}+\frac{3b^2\sinh\left(a-\frac{bc}{d}\right)\mathrm{Shi}\left(\frac{bc}{d}+bx\right)}{8d^3}\\&+\frac{9b^2\sinh\left(3a-\frac{3bc}{d}\right)\mathrm{Shi}\left(\frac{3bc}{d}+3bx\right)}{8d^3}\end{aligned}$$

output
```
-1/2*cosh(b*x+a)^3/d/(d*x+c)^2+3/8*b^2*cosh(a-b*c/d)*Chi(b*c/d+b*x)/d^3+9/
8*b^2*cosh(3*a-3*b*c/d)*Chi(3*b*c/d+3*b*x)/d^3-3/2*b*cosh(b*x+a)^2*sinh(b*
x+a)/d^2/(d*x+c)+3/8*b^2*sinh(a-b*c/d)*Shi(b*c/d+b*x)/d^3+9/8*b^2*sinh(3*a
-3*b*c/d)*Shi(3*b*c/d+3*b*x)/d^3
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.18

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{6d\cosh(bx)(d\cosh(a)+b(c+dx)\sinh(a))+2d\cosh(3bx)(d\cosh(3a)+3b(c+dx)\sinh(3a))+6d(b(c$$

input
```
Integrate[Cosh[a + b*x]^3/(c + d*x)^3,x]
```

output
```
-1/16*(6*d*Cosh[b*x]*(d*Cosh[a] + b*(c + d*x)*Sinh[a]) + 2*d*Cosh[3*b*x]*(
d*Cosh[3*a] + 3*b*(c + d*x)*Sinh[3*a]) + 6*d*(b*(c + d*x)*Cosh[a] + d*Sinh
[a])*Sinh[b*x] + 2*d*(3*b*(c + d*x)*Cosh[3*a] + d*Sinh[3*a])*Sinh[3*b*x] -
 6*b^2*(c + d*x)^2*(Cosh[a - (b*c)/d]*CoshIntegral[b*(c/d + x)] + 3*Cosh[3
*a - (3*b*c)/d]*CoshIntegral[(3*b*(c + d*x))/d] + Sinh[a - (b*c)/d]*SinhIn
tegral[b*(c/d + x)] + 3*Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*(c + d*x))
/d]))/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.32, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 3795, 3042, 3784, 26, 3042, 26, 3779, 3782, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^3}dx$$
$$\Big\downarrow \text{ 3795}$$

$$\frac{9b^2\int \frac{\cosh^3(a+bx)}{c+dx}dx}{2d^2}-\frac{3b^2\int \frac{\cosh(a+bx)}{c+dx}dx}{d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{3b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{c+dx}dx}{d^2}+\frac{9b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 3784}$$

$$\frac{9b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(\cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int \frac{i\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{9b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(\sinh\left(a-\frac{bc}{d}\right)\int \frac{\sinh\left(\frac{bc}{d}+bx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\cosh\left(\frac{bc}{d}+bx\right)}{c+dx}dx\right)}{d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{9b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(\sinh\left(a-\frac{bc}{d}\right)\int -\frac{i\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx+\cosh\left(a-\frac{bc}{d}\right)\int \frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{9b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx\right)}{c+dx}dx\right)}{d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 3779

$$-\frac{3b^2\left(\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}+\cosh\left(a-\frac{bc}{d}\right)\int\frac{\sin\left(\frac{ibc}{d}+ibx+\frac{\pi}{2}\right)}{c+dx}dx\right)}{d^2}+\frac{9b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 3782

$$\frac{9b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx}{2d^2}-\frac{3b^2\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 3793

$$\frac{9b^2\int\left(\frac{3\cosh(a+bx)}{4(c+dx)}+\frac{\cosh(3a+3bx)}{4(c+dx)}\right)dx}{2d^2}-\frac{3b^2\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

$\Big\downarrow$ 2009

$$-\frac{3b^2\left(\frac{\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{d}+\frac{\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{d}\right)}{d^2}+\frac{9b^2\left(\frac{3\cosh\left(a-\frac{bc}{d}\right)\text{Chi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\cosh\left(3a-\frac{3bc}{d}\right)\text{Chi}\left(\frac{3bc}{d}+3bx\right)}{4d}+\frac{3\sinh\left(a-\frac{bc}{d}\right)\text{Shi}\left(\frac{bc}{d}+bx\right)}{4d}+\frac{\sinh\left(3a-\frac{3bc}{d}\right)\text{Shi}\left(\frac{3bc}{d}+3bx\right)}{4d}\right)}{2d^2}-\frac{3b\sinh(a+bx)\cosh^2(a+bx)}{2d^2(c+dx)}-\frac{\cosh^3(a+bx)}{2d(c+dx)^2}$$

input
```
Int[Cosh[a + b*x]^3/(c + d*x)^3,x]
```

output
```
-1/2*Cosh[a + b*x]^3/(d*(c + d*x)^2) - (3*b*Cosh[a + b*x]^2*Sinh[a + b*x])
/(2*d^2*(c + d*x)) - (3*b^2*((Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x
])/d + (Sinh[a - (b*c)/d]*SinhIntegral[(b*c)/d + b*x])/d))/d^2 + (9*b^2*((
3*Cosh[a - (b*c)/d]*CoshIntegral[(b*c)/d + b*x])/(4*d) + (Cosh[3*a - (3*b*
c)/d]*CoshIntegral[(3*b*c)/d + 3*b*x])/(4*d) + (3*Sinh[a - (b*c)/d]*SinhIn
tegral[(b*c)/d + b*x])/(4*d) + (Sinh[3*a - (3*b*c)/d]*SinhIntegral[(3*b*c)
/d + 3*b*x])/(4*d)))/(2*d^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 561 vs. $2(172) = 344$.

Time = 2.00 (sec) , antiderivative size = 562, normalized size of antiderivative = 3.05

method	result
risch	$\frac{3b^3 e^{-3bx-3a} x}{16d(d^2x^2b^2+2b^2cdx+b^2c^2)} + \frac{3b^3 e^{-3bx-3a} c}{16d^2(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{b^2 e^{-3bx-3a}}{16d(d^2x^2b^2+2b^2cdx+b^2c^2)} - \frac{9b^2 e^{-\frac{3(ad-cb)}{d}} \operatorname{expIntegral}_1\left(3bx\right.}{16d^3}$

input
```
int(cosh(b*x+a)^3/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
3/16*b^3*exp(-3*b*x-3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*x+3/16*b^3*ex
p(-3*b*x-3*a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*c-1/16*b^2*exp(-3*b*x-
3*a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-9/16*b^2/d^3*exp(-3*(a*d-b*c)/d)*
Ei(1,3*b*x+3*a-3*(a*d-b*c)/d)+3/16*b^3*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*
d*x+b^2*c^2)*x+3/16*b^3*exp(-b*x-a)/d^2/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)*
c-3/16*b^2*exp(-b*x-a)/d/(b^2*d^2*x^2+2*b^2*c*d*x+b^2*c^2)-3/16*b^2/d^3*ex
p(-(a*d-b*c)/d)*Ei(1,b*x+a-(a*d-b*c)/d)-3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x
)^2-3/16*b^2/d^3*exp(b*x+a)/(b*c/d+b*x)-3/16*b^2/d^3*exp((a*d-b*c)/d)*Ei(1
,-b*x-a-(-a*d+b*c)/d)-1/16*b^2/d^3*exp(3*b*x+3*a)/(b*c/d+b*x)^2-3/16*b^2/d
^3*exp(3*b*x+3*a)/(b*c/d+b*x)-9/16*b^2/d^3*exp(3*(a*d-b*c)/d)*Ei(1,-3*b*x-
3*a-3*(-a*d+b*c)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(172) = 344$.

Time = 0.14 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.86

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx =$$
$$-\frac{2\,d^2\cosh\left(bx+a\right)^3+6\,d^2\cosh\left(bx+a\right)\sinh\left(bx+a\right)^2+6\left(bd^2x+bcd\right)\sinh\left(bx+a\right)^3+6\,d^2\cosh\left(bx+\right.}{}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/16*(2*d^2*cosh(b*x + a)^3 + 6*d^2*cosh(b*x + a)*sinh(b*x + a)^2 + 6*(b*
d^2*x + b*c*d)*sinh(b*x + a)^3 + 6*d^2*cosh(b*x + a) - 3*((b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*Ei(-(b*d*x + b*c)/d))*cosh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) + (b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*cosh(-3*(b*c - a*d)/d) + 6*(b*d^2*x + b*c*
d + 3*(b*d^2*x + b*c*d)*cosh(b*x + a)^2)*sinh(b*x + a) - 3*((b^2*d^2*x^2 +
 2*b^2*c*d*x + b^2*c^2)*Ei((b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*Ei(-(b*d*x + b*c)/d))*sinh(-(b*c - a*d)/d) - 9*((b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*Ei(3*(b*d*x + b*c)/d) - (b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*Ei(-3*(b*d*x + b*c)/d))*sinh(-3*(b*c - a*d)/d))/(d^5*x^2 + 2*c*d
^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx = \int \frac{\cosh^3\left(a+bx\right)}{\left(c+dx\right)^3}\,dx$$

input
```
integrate(cosh(b*x+a)**3/(d*x+c)**3,x)
```

output
```
Integral(cosh(a + b*x)**3/(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx = -\frac{e^{\left(-3\,a+\frac{3\,bc}{d}\right)}E_3\left(\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)^2d} - \frac{3\,e^{\left(-a+\frac{bc}{d}\right)}E_3\left(\frac{(dx+c)b}{d}\right)}{8\,(dx+c)^2d} - \frac{3\,e^{\left(a-\frac{bc}{d}\right)}E_3\left(-\frac{(dx+c)b}{d}\right)}{8\,(dx+c)^2d} - \frac{e^{\left(3\,a-\frac{3\,bc}{d}\right)}E_3\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,(dx+c)^2d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/8*e^(-3*a + 3*b*c/d)*exp_integral_e(3, 3*(d*x + c)*b/d)/((d*x + c)^2*d)
 - 3/8*e^(-a + b*c/d)*exp_integral_e(3, (d*x + c)*b/d)/((d*x + c)^2*d) - 3
/8*e^(a - b*c/d)*exp_integral_e(3, -(d*x + c)*b/d)/((d*x + c)^2*d) - 1/8*e
^(3*a - 3*b*c/d)*exp_integral_e(3, -3*(d*x + c)*b/d)/((d*x + c)^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(172) = 344.

Time = 0.13 (sec) , antiderivative size = 602, normalized size of antiderivative = 3.27

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx = \frac{9\,b^2d^2x^2\mathrm{Ei}\left(\frac{3\,(bdx+bc)}{d}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)} + 3\,b^2d^2x^2\mathrm{Ei}\left(\frac{bdx+bc}{d}\right)e^{\left(a-\frac{bc}{d}\right)} + 3\,b^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+bc}{d}\right)e^{\left(-a+\frac{bc}{d}\right)} + 9\,b^2d^2x^2}{}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/16*(9*b^2*d^2*x^2*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*b^2*d^2*x^
2*Ei((b*d*x + b*c)/d)*e^(a - b*c/d) + 3*b^2*d^2*x^2*Ei(-(b*d*x + b*c)/d)*e
^(-a + b*c/d) + 9*b^2*d^2*x^2*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) +
18*b^2*c*d*x*Ei(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 6*b^2*c*d*x*Ei((b*d
*x + b*c)/d)*e^(a - b*c/d) + 6*b^2*c*d*x*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/
d) + 18*b^2*c*d*x*Ei(-3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) + 9*b^2*c^2*Ei
(3*(b*d*x + b*c)/d)*e^(3*a - 3*b*c/d) + 3*b^2*c^2*Ei((b*d*x + b*c)/d)*e^(a
 - b*c/d) + 3*b^2*c^2*Ei(-(b*d*x + b*c)/d)*e^(-a + b*c/d) + 9*b^2*c^2*Ei(-
3*(b*d*x + b*c)/d)*e^(-3*a + 3*b*c/d) - 3*b*d^2*x*e^(3*b*x + 3*a) - 3*b*d^
2*x*e^(b*x + a) + 3*b*d^2*x*e^(-b*x - a) + 3*b*d^2*x*e^(-3*b*x - 3*a) - 3*
b*c*d*e^(3*b*x + 3*a) - 3*b*c*d*e^(b*x + a) + 3*b*c*d*e^(-b*x - a) + 3*b*c
*d*e^(-3*b*x - 3*a) - d^2*e^(3*b*x + 3*a) - 3*d^2*e^(b*x + a) - 3*d^2*e^(-
b*x - a) - d^2*e^(-3*b*x - 3*a))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx=\int \frac{{\cosh\left(a+b\,x\right)}^3}{{\left(c+d\,x\right)}^3}\,dx$$

input
```
int(cosh(a + b*x)^3/(c + d*x)^3,x)
```

output
```
int(cosh(a + b*x)^3/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^3}\,dx = \frac{e^{4a}\left(\int \frac{e^{3bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)+3e^{2a}\left(\int \frac{e^{bx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)+e^{a}\left(\int \frac{1}{e^{3bx+3a}c^3+3e^{3bx+3a}c^2dx+3e^{3bx+3a}c\,d^2x^2}\right.}{8e^{a}}$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^3,x)
```

output

```
(e**(4*a)*int(e**(3*b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x
) + 3*e**(2*a)*int(e**(b*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
),x) + e**a*int(1/(e**(3*a + 3*b*x)*c**3 + 3*e**(3*a + 3*b*x)*c**2*d*x + 3
*e**(3*a + 3*b*x)*c*d**2*x**2 + e**(3*a + 3*b*x)*d**3*x**3),x) + 3*int(1/(
e**(b*x)*c**3 + 3*e**(b*x)*c**2*d*x + 3*e**(b*x)*c*d**2*x**2 + e**(b*x)*d*
*3*x**3),x))/(8*e**a)
```

3.23 $\int x^3 \cosh^4(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 172

$$\begin{aligned}\int x^3 \cosh^4(a+bx)\,dx = {} & \frac{45x^2}{128b^2}+\frac{3x^4}{32}-\frac{45\cosh^2(a+bx)}{128b^4} \\ & -\frac{9x^2\cosh^2(a+bx)}{16b^2}-\frac{3\cosh^4(a+bx)}{128b^4} \\ & -\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{45x\cosh(a+bx)\sinh(a+bx)}{64b^3} \\ & +\frac{3x^3\cosh(a+bx)\sinh(a+bx)}{8b} \\ & +\frac{3x\cosh^3(a+bx)\sinh(a+bx)}{32b^3} \\ & +\frac{x^3\cosh^3(a+bx)\sinh(a+bx)}{4b}\end{aligned}$$

output
```
45/128*x^2/b^2+3/32*x^4-45/128*cosh(b*x+a)^2/b^4-9/16*x^2*cosh(b*x+a)^2/b^
2-3/128*cosh(b*x+a)^4/b^4-3/16*x^2*cosh(b*x+a)^4/b^2+45/64*x*cosh(b*x+a)*s
inh(b*x+a)/b^3+3/8*x^3*cosh(b*x+a)*sinh(b*x+a)/b+3/32*x*cosh(b*x+a)^3*sinh
(b*x+a)/b^3+1/4*x^3*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.58

$$\int x^3\cosh^4(a+bx)\,dx$$
$$=\frac{-192(1+2b^2x^2)\cosh(2(a+bx))-3(1+8b^2x^2)\cosh(4(a+bx))+4bx(24b^3x^3+32(3+2b^2x^2)\sinh(2(a}{1024b^4}$$

input
```
Integrate[x^3*Cosh[a + b*x]^4,x]
```

output
```
(-192*(1 + 2*b^2*x^2)*Cosh[2*(a + b*x)] - 3*(1 + 8*b^2*x^2)*Cosh[4*(a + b*
x)] + 4*b*x*(24*b^3*x^3 + 32*(3 + 2*b^2*x^2)*Sinh[2*(a + b*x)] + (3 + 8*b^
2*x^2)*Sinh[4*(a + b*x)]))/(1024*b^4)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.38, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3792, 3042, 3791, 3042, 3791, 15, 3792, 15, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{3\int x\cosh^4(a+bx)dx}{8b^2}+\frac{3}{4}\int x^3\cosh^2(a+bx)dx-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx}{8b^2}+\frac{3}{4}\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\downarrow$ 3791

$$\frac{3\left(\frac{3}{4}\int x\cosh^2(a+bx)dx-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\downarrow$ 3042

$$\frac{3\left(\frac{3}{4}\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\downarrow$ 3791

$$\frac{3\left(\frac{3}{4}\left(\frac{\int xdx}{2}-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\downarrow$ 15

$$\frac{3}{4}\int x^3\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3\left(\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\downarrow$ 3792

$$\frac{3}{4}\left(\frac{3\int x\cosh^2(a+bx)dx}{2b^2}+\frac{\int x^3dx}{2}-\frac{3x^2\cosh^2(a+bx)}{4b^2}+\frac{x^3\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3\left(\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\downarrow$ 15

$$\frac{3}{4}\left(\frac{3\int x\cosh^2(a+bx)dx}{2b^2}-\frac{3x^2\cosh^2(a+bx)}{4b^2}+\frac{x^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^4}{8}\right)-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3\left(\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\big\downarrow$ 3042

$$\frac{3}{4}\left(\frac{3\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b^2}-\frac{3x^2\cosh^2(a+bx)}{4b^2}+\frac{x^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^4}{8}\right)-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3\left(\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\big\downarrow$ 3791

$$\frac{3}{4}\left(\frac{3\left(\frac{\int xdx}{2}-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}\right)}{2b^2}-\frac{3x^2\cosh^2(a+bx)}{4b^2}+\frac{x^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^4}{8}\right)-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3\left(\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$\big\downarrow$ 15

$$-\frac{3x^2\cosh^4(a+bx)}{16b^2}+\frac{3\left(\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)}{8b^2}+\frac{3}{4}\left(-\frac{3x^2\cosh^2(a+bx)}{4b^2}+\frac{3\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)}{2b^2}+\frac{x^3\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^4}{8}\right)+\frac{x^3\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

input
```
Int[x^3*Cosh[a + b*x]^4,x]
```

output
```
(-3*x^2*Cosh[a + b*x]^4)/(16*b^2) + (x^3*Cosh[a + b*x]^3*Sinh[a + b*x])/(4
*b) + (3*(-1/16*Cosh[a + b*x]^4/b^2 + (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(4
*b) + (3*(x^2/4 - Cosh[a + b*x]^2/(4*b^2) + (x*Cosh[a + b*x]*Sinh[a + b*x]
)/(2*b)))/4))/(8*b^2) + (3*(x^4/8 - (3*x^2*Cosh[a + b*x]^2)/(4*b^2) + (x^3
*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (3*(x^2/4 - Cosh[a + b*x]^2/(4*b^2)
+ (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/(2*b^2)))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.41 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.59

method	result
parallelrisch	$\frac{(-384x^2b^2-192)\cosh(2bx+2a)+(-24x^2b^2-3)\cosh(4bx+4a)+(256x^3b^3+384bx)\sinh(2bx+2a)+(32x^3b^3+12bx)\sin}{1024b^4}$
risch	$\frac{3x^4}{32}+\frac{(32x^3b^3-24x^2b^2+12bx-3)e^{4bx+4a}}{2048b^4}+\frac{(4x^3b^3-6x^2b^2+6bx-3)e^{2bx+2a}}{32b^4}-\frac{(4x^3b^3+6x^2b^2+6bx+3)e^{-2bx-}}{32b^4}$
derivativedivides	$\frac{-a^3\left(\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}+\frac{3(bx+a)\cosh(bx+a)s}{8}\right.}{}$
default	$\frac{-a^3\left(\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}\right)+3a^2\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}+\frac{3(bx+a)\cosh(bx+a)s}{8}\right.}{}$
orering	$\frac{(32x^6b^6-240x^4b^4-330x^2b^2+945)\cosh(bx+a)^4}{128x^2b^6}+\frac{5(40x^4b^4+24x^2b^2-243)\left(3x^2\cosh(bx+a)^4+4x^3\cosh(bx+a)^3b\sin\right.}{256x^4b^6}$

input
```
int(x^3*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/1024*((-384*b^2*x^2-192)*cosh(2*b*x+2*a)+(-24*b^2*x^2-3)*cosh(4*b*x+4*a)
+(256*b^3*x^3+384*b*x)*sinh(2*b*x+2*a)+(32*b^3*x^3+12*b*x)*sinh(4*b*x+4*a)
+96*x^4*b^4+195)/b^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.13

$$\int x^3\cosh^4(a+bx)\,dx$$
$$=\frac{96\,b^4x^4-3\,(8\,b^2x^2+1)\cosh\left(bx+a\right)^4+16\,(8\,b^3x^3+3\,bx)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-3\,(8\,b^2x^2+1)\,\mathrm{s}}{}$$

input
```
integrate(x^3*cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/1024*(96*b^4*x^4 - 3*(8*b^2*x^2 + 1)*cosh(b*x + a)^4 + 16*(8*b^3*x^3 + 3
*b*x)*cosh(b*x + a)*sinh(b*x + a)^3 - 3*(8*b^2*x^2 + 1)*sinh(b*x + a)^4 -
192*(2*b^2*x^2 + 1)*cosh(b*x + a)^2 - 6*(64*b^2*x^2 + 3*(8*b^2*x^2 + 1)*co
sh(b*x + a)^2 + 32)*sinh(b*x + a)^2 + 16*((8*b^3*x^3 + 3*b*x)*cosh(b*x + a
)^3 + 16*(2*b^3*x^3 + 3*b*x)*cosh(b*x + a))*sinh(b*x + a))/b^4
```

Sympy [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.47

$$\int x^3\cosh^4(a+bx)\,dx$$
$$=\begin{cases}\frac{3x^4\sinh^4(a+bx)}{32}-\frac{3x^4\sinh^2(a+bx)\cosh^2(a+bx)}{16}+\frac{3x^4\cosh^4(a+bx)}{32}-\frac{3x^3\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{5x^3\sinh(a+bx)\cosh^3(a+\cdots}{8b}\\ \frac{x^4\cosh^4(a)}{4}\end{cases}$$

input
```
integrate(x**3*cosh(b*x+a)**4,x)
```

output
```
Piecewise((3*x**4*sinh(a + b*x)**4/32 - 3*x**4*sinh(a + b*x)**2*cosh(a + b
*x)**2/16 + 3*x**4*cosh(a + b*x)**4/32 - 3*x**3*sinh(a + b*x)**3*cosh(a +
b*x)/(8*b) + 5*x**3*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 45*x**2*sinh(a
+ b*x)**4/(128*b**2) - 9*x**2*sinh(a + b*x)**2*cosh(a + b*x)**2/(64*b**2)
- 51*x**2*cosh(a + b*x)**4/(128*b**2) - 45*x*sinh(a + b*x)**3*cosh(a + b*x
)/(64*b**3) + 51*x*sinh(a + b*x)*cosh(a + b*x)**3/(64*b**3) + 45*sinh(a +
b*x)**4/(256*b**4) - 51*cosh(a + b*x)**4/(256*b**4), Ne(b, 0)), (x**4*cosh
(a)**4/4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 176, normalized size of antiderivative = 1.02

$$\begin{aligned}\int x^3\cosh^4(a+bx)\,dx=&\ \frac{3}{32}x^4+\frac{\left(32\,b^3x^3e^{(4\,a)}-24\,b^2x^2e^{(4\,a)}+12\,bxe^{(4\,a)}-3\,e^{(4\,a)}\right)e^{(4\,bx)}}{2048\,b^4}\\&+\frac{\left(4\,b^3x^3e^{(2\,a)}-6\,b^2x^2e^{(2\,a)}+6\,bxe^{(2\,a)}-3\,e^{(2\,a)}\right)e^{(2\,bx)}}{32\,b^4}\\&-\frac{(4\,b^3x^3+6\,b^2x^2+6\,bx+3)e^{(-2\,bx-2\,a)}}{32\,b^4}\\&-\frac{(32\,b^3x^3+24\,b^2x^2+12\,bx+3)e^{(-4\,bx-4\,a)}}{2048\,b^4}\end{aligned}$$

input
```
integrate(x^3*cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
3/32*x^4 + 1/2048*(32*b^3*x^3*e^(4*a) - 24*b^2*x^2*e^(4*a) + 12*b*x*e^(4*a
) - 3*e^(4*a))*e^(4*b*x)/b^4 + 1/32*(4*b^3*x^3*e^(2*a) - 6*b^2*x^2*e^(2*a)
 + 6*b*x*e^(2*a) - 3*e^(2*a))*e^(2*b*x)/b^4 - 1/32*(4*b^3*x^3 + 6*b^2*x^2
+ 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 - 1/2048*(32*b^3*x^3 + 24*b^2*x^2 + 12*b
*x + 3)*e^(-4*b*x - 4*a)/b^4
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.87

$$\int x^3\cosh^4(a+bx)\,dx = \frac{3}{32}x^4 + \frac{(32b^3x^3 - 24b^2x^2 + 12bx - 3)e^{(4bx+4a)}}{2048b^4} + \frac{(4b^3x^3 - 6b^2x^2 + 6bx - 3)e^{(2bx+2a)}}{32b^4} - \frac{(4b^3x^3 + 6b^2x^2 + 6bx + 3)e^{(-2bx-2a)}}{32b^4} - \frac{(32b^3x^3 + 24b^2x^2 + 12bx + 3)e^{(-4bx-4a)}}{2048b^4}$$

input
```
integrate(x^3*cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/32*x^4 + 1/2048*(32*b^3*x^3 - 24*b^2*x^2 + 12*b*x - 3)*e^(4*b*x + 4*a)/b
^4 + 1/32*(4*b^3*x^3 - 6*b^2*x^2 + 6*b*x - 3)*e^(2*b*x + 2*a)/b^4 - 1/32*(
4*b^3*x^3 + 6*b^2*x^2 + 6*b*x + 3)*e^(-2*b*x - 2*a)/b^4 - 1/2048*(32*b^3*x
^3 + 24*b^2*x^2 + 12*b*x + 3)*e^(-4*b*x - 4*a)/b^4
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.75

$$\int x^3\cosh^4(a+bx)\,dx = \frac{3x^4}{32} - \frac{\frac{3\cosh(2a+2bx)}{16} + \frac{3\cosh(4a+4bx)}{1024} + b^2\left(\frac{3x^2\cosh(2a+2bx)}{8} + \frac{3x^2\cosh(4a+4bx)}{128}\right) - b\left(\frac{3x\sinh(2a+2bx)}{8} + \frac{3x\sinh(4a+\cdots}{256}\right.}{b^4}$$

input
```
int(x^3*cosh(a + b*x)^4,x)
```

output
```
(3*x^4)/32 - ((3*cosh(2*a + 2*b*x))/16 + (3*cosh(4*a + 4*b*x))/1024 + b^2*
((3*x^2*cosh(2*a + 2*b*x))/8 + (3*x^2*cosh(4*a + 4*b*x))/128) - b*((3*x*si
nh(2*a + 2*b*x))/8 + (3*x*sinh(4*a + 4*b*x))/256) - b^3*((x^3*sinh(2*a + 2
*b*x))/4 + (x^3*sinh(4*a + 4*b*x))/32))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.41

$$\int x^3 \cosh^4(a+bx)\,dx$$
$$= \frac{32e^{8bx+8a}b^3x^3 - 24e^{8bx+8a}b^2x^2 + 12e^{8bx+8a}bx - 3e^{8bx+8a} + 256e^{6bx+6a}b^3x^3 - 384e^{6bx+6a}b^2x^2 + 384e^{6bx+6a}}{}$$

input
```
int(x^3*cosh(b*x+a)^4,x)
```

output
```
(32*e**(8*a + 8*b*x)*b**3*x**3 - 24*e**(8*a + 8*b*x)*b**2*x**2 + 12*e**(8*
a + 8*b*x)*b*x - 3*e**(8*a + 8*b*x) + 256*e**(6*a + 6*b*x)*b**3*x**3 - 384
*e**(6*a + 6*b*x)*b**2*x**2 + 384*e**(6*a + 6*b*x)*b*x - 192*e**(6*a + 6*b
*x) + 192*e**(4*a + 4*b*x)*b**4*x**4 - 256*e**(2*a + 2*b*x)*b**3*x**3 - 38
4*e**(2*a + 2*b*x)*b**2*x**2 - 384*e**(2*a + 2*b*x)*b*x - 192*e**(2*a + 2*
b*x) - 32*b**3*x**3 - 24*b**2*x**2 - 12*b*x - 3)/(2048*e**(4*a + 4*b*x)*b*
*4)
```

3.24 $\int x^2 \cosh^4(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 134

$$\begin{aligned}\int x^2\cosh^4(a+bx)\,dx &= \frac{15x}{64b^2}+\frac{x^3}{8}-\frac{3x\cosh^2(a+bx)}{8b^2}-\frac{x\cosh^4(a+bx)}{8b^2}\\ &\quad+\frac{15\cosh(a+bx)\sinh(a+bx)}{64b^3}\\ &\quad+\frac{3x^2\cosh(a+bx)\sinh(a+bx)}{8b}\\ &\quad+\frac{\cosh^3(a+bx)\sinh(a+bx)}{32b^3}+\frac{x^2\cosh^3(a+bx)\sinh(a+bx)}{4b}\end{aligned}$$

output
```
15/64*x/b^2+1/8*x^3-3/8*x*cosh(b*x+a)^2/b^2-1/8*x*cosh(b*x+a)^4/b^2+15/64*
cosh(b*x+a)*sinh(b*x+a)/b^3+3/8*x^2*cosh(b*x+a)*sinh(b*x+a)/b+1/32*cosh(b*
x+a)^3*sinh(b*x+a)/b^3+1/4*x^2*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.67

$$\int x^2\cosh^4(a+bx)\,dx$$
$$=\frac{32b^3x^3-64bx\cosh(2(a+bx))-4bx\cosh(4(a+bx))+32\sinh(2(a+bx))+64b^2x^2\sinh(2(a+bx))+\text{s}}{256b^3}$$

input
```
Integrate[x^2*Cosh[a + b*x]^4,x]
```

output
```
(32*b^3*x^3 - 64*b*x*Cosh[2*(a + b*x)] - 4*b*x*Cosh[4*(a + b*x)] + 32*Sinh
[2*(a + b*x)] + 64*b^2*x^2*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)] + 8*b^2*x
^2*Sinh[4*(a + b*x)])/(256*b^3)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.35, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3792, 3042, 3115, 3042, 3115, 24, 3792, 15, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{\int\cosh^4(a+bx)dx}{8b^2}+\frac{3}{4}\int x^2\cosh^2(a+bx)dx-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin \left(ia+ibx+\frac{\pi}{2}\right)^4 dx}{8b^2}+\frac{3}{4}\int x^2 \sin \left(ia+ibx+\frac{\pi}{2}\right)^2 dx-\frac{x \cosh^4(a+bx)}{8b^2}+\frac{x^2 \sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2 \sin \left(ia+ibx+\frac{\pi}{2}\right)^2 dx-\frac{x \cosh^4(a+bx)}{8b^2}+\frac{x^2 \sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \textbf{3042}$$

$$\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin \left(ia+ibx+\frac{\pi}{2}\right)^2 dx}{8b^2}+\frac{3}{4}\int x^2 \sin \left(ia+ibx+\frac{\pi}{2}\right)^2 dx-\frac{x \cosh^4(a+bx)}{8b^2}+\frac{x^2 \sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \textbf{3115}$$

$$\frac{\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}}{8b^2}+\frac{3}{4}\int x^2 \sin \left(ia+ibx+\frac{\pi}{2}\right)^2 dx-\frac{x \cosh^4(a+bx)}{8b^2}+\frac{x^2 \sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \textbf{24}$$

$$\frac{3}{4}\int x^2 \sin \left(ia+ibx+\frac{\pi}{2}\right)^2 dx-\frac{x \cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2 \sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \textbf{3792}$$

$$\frac{3}{4}\left(\frac{\int \cosh^2(a+bx)dx}{2b^2}+\frac{\int x^2dx}{2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2 \sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{x \cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2 \sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \textbf{15}$$

$$\frac{3}{4}\left(\frac{\int \cosh^2(a+bx)dx}{2b^2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\left(\frac{\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{2b^2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\left(\frac{\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}}{2b^2}-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{3}{4}\left(-\frac{x\cosh^2(a+bx)}{2b^2}+\frac{\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}}{2b^2}+\frac{x^2\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^3}{6}\right)-\frac{x\cosh^4(a+bx)}{8b^2}+\frac{\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)}{8b^2}+\frac{x^2\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

input
```
Int[x^2*Cosh[a + b*x]^4,x]
```

output
```
-1/8*(x*Cosh[a + b*x]^4)/b^2 + (x^2*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) +
 ((Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a
+ b*x])/(2*b)))/4)/(8*b^2) + (3*(x^3/6 - (x*Cosh[a + b*x]^2)/(2*b^2) + (x^
2*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (x/2 + (Cosh[a + b*x]*Sinh[a + b*x]
)/(2*b))/(2*b^2)))/4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.60

method	result
parallelrisch	$\frac{(64x^2b^2+32)\sinh(2bx+2a)+(8x^2b^2+1)\sinh(4bx+4a)+32\left(x^2b^2-2\cosh(2bx+2a)-\frac{\cosh(4bx+4a)}{8}\right)xb}{256b^3}$
risch	$\frac{x^3}{8}+\frac{(8x^2b^2-4bx+1)e^{4bx+4a}}{512b^3}+\frac{(2x^2b^2-2bx+1)e^{2bx+2a}}{16b^3}-\frac{(2x^2b^2+2bx+1)e^{-2bx-2a}}{16b^3}-\frac{(8x^2b^2+4bx+1)e^{-4b}}{512b^3}$
derivativedivides	$a^2\left(\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(}{8}\right.$
default	$a^2\left(\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}\right)-2a\left(\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(}{8}\right.$
orering	$\frac{(32x^6b^6-120x^4b^4-30x^2b^2+135)\cosh(bx+a)^4}{96x^3b^6}+\frac{5(112x^4b^4+6x^2b^2-171)\left(2x\cosh(bx+a)^4+4x^2\cosh(bx+a)^3b\sinh\right.}{768x^4b^6}$

input
```
int(x^2*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/256*((64*b^2*x^2+32)*sinh(2*b*x+2*a)+(8*b^2*x^2+1)*sinh(4*b*x+4*a)+32*(x
^2*b^2-2*cosh(2*b*x+2*a)-1/8*cosh(4*b*x+4*a))*x*b)/b^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.10

$$\int x^2\cosh^4(a+bx)\,dx$$
$$=\frac{8\,b^3x^3-bx\cosh\left(bx+a\right)^4-bx\sinh\left(bx+a\right)^4+\left(8\,b^2x^2+1\right)\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3-16\,bx\cosh\left(b\right.}{}$$

input
```
integrate(x^2*cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/64*(8*b^3*x^3 - b*x*cosh(b*x + a)^4 - b*x*sinh(b*x + a)^4 + (8*b^2*x^2 +
 1)*cosh(b*x + a)*sinh(b*x + a)^3 - 16*b*x*cosh(b*x + a)^2 - 2*(3*b*x*cosh
(b*x + a)^2 + 8*b*x)*sinh(b*x + a)^2 + ((8*b^2*x^2 + 1)*cosh(b*x + a)^3 +
16*(2*b^2*x^2 + 1)*cosh(b*x + a))*sinh(b*x + a))/b^3
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.56

$$\int x^2\cosh^4(a+bx)\,dx$$
$$=\begin{cases} \frac{x^3\sinh^4(a+bx)}{8} - \frac{x^3\sinh^2(a+bx)\cosh^2(a+bx)}{4} + \frac{x^3\cosh^4(a+bx)}{8} - \frac{3x^2\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{5x^2\sinh(a+bx)\cosh^3(a+bx)}{8b} \\ \frac{x^3\cosh^4(a)}{3} \end{cases}$$

input
```
integrate(x**2*cosh(b*x+a)**4,x)
```

output
```
Piecewise((x**3*sinh(a + b*x)**4/8 - x**3*sinh(a + b*x)**2*cosh(a + b*x)**
2/4 + x**3*cosh(a + b*x)**4/8 - 3*x**2*sinh(a + b*x)**3*cosh(a + b*x)/(8*b
) + 5*x**2*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 15*x*sinh(a + b*x)**4/(6
4*b**2) - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/(32*b**2) - 17*x*cosh(a +
b*x)**4/(64*b**2) - 15*sinh(a + b*x)**3*cosh(a + b*x)/(64*b**3) + 17*sinh(
a + b*x)*cosh(a + b*x)**3/(64*b**3), Ne(b, 0)), (x**3*cosh(a)**4/3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99

$$\int x^2\cosh^4(a+bx)\,dx = \frac{1}{8}x^3 + \frac{\left(8b^2x^2e^{(4a)} - 4bxe^{(4a)} + e^{(4a)}\right)e^{(4bx)}}{512b^3} + \frac{\left(2b^2x^2e^{(2a)} - 2bxe^{(2a)} + e^{(2a)}\right)e^{(2bx)}}{16b^3} - \frac{(2b^2x^2 + 2bx + 1)e^{(-2bx-2a)}}{16b^3} - \frac{(8b^2x^2 + 4bx + 1)e^{(-4bx-4a)}}{512b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/8*x^3 + 1/512*(8*b^2*x^2*e^(4*a) - 4*b*x*e^(4*a) + e^(4*a))*e^(4*b*x)/b^
3 + 1/16*(2*b^2*x^2*e^(2*a) - 2*b*x*e^(2*a) + e^(2*a))*e^(2*b*x)/b^3 - 1/1
6*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*b*x - 2*a)/b^3 - 1/512*(8*b^2*x^2 + 4*b*x
+ 1)*e^(-4*b*x - 4*a)/b^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.88

$$\int x^2 \cosh^4(a+bx)\,dx = \frac{1}{8}x^3 + \frac{(8b^2x^2-4bx+1)e^{(4bx+4a)}}{512b^3} + \frac{(2b^2x^2-2bx+1)e^{(2bx+2a)}}{16b^3} - \frac{(2b^2x^2+2bx+1)e^{(-2bx-2a)}}{16b^3} - \frac{(8b^2x^2+4bx+1)e^{(-4bx-4a)}}{512b^3}$$

input
```
integrate(x^2*cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
1/8*x^3 + 1/512*(8*b^2*x^2 - 4*b*x + 1)*e^(4*b*x + 4*a)/b^3 + 1/16*(2*b^2*
x^2 - 2*b*x + 1)*e^(2*b*x + 2*a)/b^3 - 1/16*(2*b^2*x^2 + 2*b*x + 1)*e^(-2*
b*x - 2*a)/b^3 - 1/512*(8*b^2*x^2 + 4*b*x + 1)*e^(-4*b*x - 4*a)/b^3
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.70

$$\int x^2 \cosh^4(a+bx)\,dx = \frac{\frac{\sinh(2a+2bx)}{8} + \frac{\sinh(4a+4bx)}{256} - b\left(\frac{x\cosh(2a+2bx)}{4} + \frac{x\cosh(4a+4bx)}{64}\right) + b^2\left(\frac{x^2\sinh(2a+2bx)}{4} + \frac{x^2\sinh(4a+4bx)}{32}\right)}{b^3} + \frac{x^3}{8}$$

input
```
int(x^2*cosh(a + b*x)^4,x)
```

output
```
(sinh(2*a + 2*b*x)/8 + sinh(4*a + 4*b*x)/256 - b*((x*cosh(2*a + 2*b*x))/4
+ (x*cosh(4*a + 4*b*x))/64) + b^2*((x^2*sinh(2*a + 2*b*x))/4 + (x^2*sinh(4
*a + 4*b*x))/32))/b^3 + x^3/8
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.34

$$\int x^2\cosh^4(a+bx)\,dx$$
$$=\frac{8e^{8bx+8a}b^2x^2-4e^{8bx+8a}bx+e^{8bx+8a}+64e^{6bx+6a}b^2x^2-64e^{6bx+6a}bx+32e^{6bx+6a}+64e^{4bx+4a}b^3x^3-64e^{2bx}}{512e^{4bx+4a}b^3}$$

input
```
int(x^2*cosh(b*x+a)^4,x)
```

output
```
(8*e**(8*a + 8*b*x)*b**2*x**2 - 4*e**(8*a + 8*b*x)*b*x + e**(8*a + 8*b*x)
+ 64*e**(6*a + 6*b*x)*b**2*x**2 - 64*e**(6*a + 6*b*x)*b*x + 32*e**(6*a + 6
*b*x) + 64*e**(4*a + 4*b*x)*b**3*x**3 - 64*e**(2*a + 2*b*x)*b**2*x**2 - 64
*e**(2*a + 2*b*x)*b*x - 32*e**(2*a + 2*b*x) - 8*b**2*x**2 - 4*b*x - 1)/(51
2*e**(4*a + 4*b*x)*b**3)
```

3.25 $\int x\cosh^4(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 80

$$\int x\cosh^4(a+bx)\,dx = \frac{3x^2}{16}-\frac{3\cosh^2(a+bx)}{16b^2}-\frac{\cosh^4(a+bx)}{16b^2}+\frac{3x\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{x\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

output
```
3/16*x^2-3/16*cosh(b*x+a)^2/b^2-1/16*cosh(b*x+a)^4/b^2+3/8*x*cosh(b*x+a)*s
inh(b*x+a)/b+1/4*x*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.66

$$\int x\cosh^4(a+bx)\,dx = -\frac{16\cosh(2(a+bx))+\cosh(4(a+bx))-4bx(6bx+8\sinh(2(a+bx))+\sinh(4(a+bx)))}{128b^2}$$

input
```
Integrate[x*Cosh[a + b*x]^4,x]
```

output
```
-1/128*(16*Cosh[2*(a + b*x)] + Cosh[4*(a + b*x)] - 4*b*x*(6*b*x + 8*Sinh[2
*(a + b*x)] + Sinh[4*(a + b*x)]))/b^2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3791, 3042, 3791, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4}\int x\cosh^2(a+bx)dx-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3}{4}\int x\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3791}$$

$$\frac{3}{4}\left(\frac{\int xdx}{2}-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{15}$$

$$\frac{3}{4}\left(-\frac{\cosh^2(a+bx)}{4b^2}+\frac{x\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x^2}{4}\right)-\frac{\cosh^4(a+bx)}{16b^2}+\frac{x\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

input
```
Int[x*Cosh[a + b*x]^4,x]
```

output
```
-1/16*Cosh[a + b*x]^4/b^2 + (x*Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(
x^2/4 - Cosh[a + b*x]^2/(4*b^2) + (x*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/
4
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{24x^2b^2+4bx\sinh(4bx+4a)+32bx\sinh(2bx+2a)-\cosh(4bx+4a)-16\cosh(2bx+2a)+17}{128b^2}$
risch	$\frac{3x^2}{16}+\frac{(4bx-1)\mathrm{e}^{4bx+4a}}{256b^2}+\frac{(2bx-1)\mathrm{e}^{2bx+2a}}{16b^2}-\frac{(2bx+1)\mathrm{e}^{-2bx-2a}}{16b^2}-\frac{(4bx+1)\mathrm{e}^{-4bx-4a}}{256b^2}$
derivativedivides	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{8}+\frac{3(bx+a)^2}{16}-\frac{\cosh(bx+a)^4}{16}-\frac{3\cosh(bx+a)^2}{16}-a\Bigg(\Bigg(\frac{\cosh(bx+a)}{4}}{b^2}$
default	$\frac{\frac{(bx+a)\sinh(bx+a)\cosh(bx+a)^3}{4}+\frac{3(bx+a)\cosh(bx+a)\sinh(bx+a)}{8}+\frac{3(bx+a)^2}{16}-\frac{\cosh(bx+a)^4}{16}-\frac{3\cosh(bx+a)^2}{16}-a\Bigg(\Bigg(\frac{\cosh(bx+a)}{4}}{b^2}$
orering	$\frac{(8x^4b^4-15x^2b^2+5)\cosh(bx+a)^4}{16b^4x^2}+\frac{5(2x^2b^2-1)\left(\cosh(bx+a)^4+4x\cosh(bx+a)^3b\sinh(bx+a)\right)}{16b^4x^2}-\frac{5(x^2b^2-1)\Big(8\,\mathrm{co}}{}$

input
```
int(x*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/128*(24*x^2*b^2+4*b*x*sinh(4*b*x+4*a)+32*b*x*sinh(2*b*x+2*a)-cosh(4*b*x+
4*a)-16*cosh(2*b*x+2*a)+17)/b^2
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.42

$$\int x\cosh^4(a+bx)\,dx$$
$$=\frac{16\,bx\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+24\,b^2x^2-\cosh\left(bx+a\right)^4-\sinh\left(bx+a\right)^4-2\left(3\,\cosh\left(bx+a\right)^2+8\right)}{128\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/128*(16*b*x*cosh(b*x + a)*sinh(b*x + a)^3 + 24*b^2*x^2 - cosh(b*x + a)^4
 - sinh(b*x + a)^4 - 2*(3*cosh(b*x + a)^2 + 8)*sinh(b*x + a)^2 - 16*cosh(b
*x + a)^2 + 16*(b*x*cosh(b*x + a)^3 + 4*b*x*cosh(b*x + a))*sinh(b*x + a))/
b^2
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.72

$$\int x\cosh^4(a+bx)\,dx$$
$$=\begin{cases}\frac{3x^2\sinh^4(a+bx)}{16}-\frac{3x^2\sinh^2(a+bx)\cosh^2(a+bx)}{8}+\frac{3x^2\cosh^4(a+bx)}{16}-\frac{3x\sinh^3(a+bx)\cosh(a+bx)}{8b}+\frac{5x\sinh(a+bx)\cosh^3(a+bx}{8b}\\ \frac{x^2\cosh^4(a)}{2}\end{cases}$$

input
```
integrate(x*cosh(b*x+a)**4,x)
```

output
```
Piecewise((3*x**2*sinh(a + b*x)**4/16 - 3*x**2*sinh(a + b*x)**2*cosh(a + b
*x)**2/8 + 3*x**2*cosh(a + b*x)**4/16 - 3*x*sinh(a + b*x)**3*cosh(a + b*x)
/(8*b) + 5*x*sinh(a + b*x)*cosh(a + b*x)**3/(8*b) + 3*sinh(a + b*x)**4/(32
*b**2) - 5*cosh(a + b*x)**4/(32*b**2), Ne(b, 0)), (x**2*cosh(a)**4/2, True
))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.20

$$\int x\cosh^4(a+bx)\,dx = \frac{3}{16}\,x^2 + \frac{\left(4\,bxe^{(4\,a)} - e^{(4\,a)}\right)e^{(4\,bx)}}{256\,b^2} + \frac{\left(2\,bxe^{(2\,a)} - e^{(2\,a)}\right)e^{(2\,bx)}}{16\,b^2} - \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
3/16*x^2 + 1/256*(4*b*x*e^(4*a) - e^(4*a))*e^(4*b*x)/b^2 + 1/16*(2*b*x*e^(
2*a) - e^(2*a))*e^(2*b*x)/b^2 - 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 - 1/
256*(4*b*x + 1)*e^(-4*b*x - 4*a)/b^2
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.08

$$\int x\cosh^4(a+bx)\,dx = \frac{3}{16}\,x^2 + \frac{(4\,bx-1)e^{(4\,bx+4\,a)}}{256\,b^2} + \frac{(2\,bx-1)e^{(2\,bx+2\,a)}}{16\,b^2} - \frac{(2\,bx+1)e^{(-2\,bx-2\,a)}}{16\,b^2} - \frac{(4\,bx+1)e^{(-4\,bx-4\,a)}}{256\,b^2}$$

input
```
integrate(x*cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/16*x^2 + 1/256*(4*b*x - 1)*e^(4*b*x + 4*a)/b^2 + 1/16*(2*b*x - 1)*e^(2*b
*x + 2*a)/b^2 - 1/16*(2*b*x + 1)*e^(-2*b*x - 2*a)/b^2 - 1/256*(4*b*x + 1)*
e^(-4*b*x - 4*a)/b^2
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.85

$$\int x\cosh^4(a+bx)\,dx$$
$$=\frac{3\,x^2}{16}-\frac{\frac{3\cosh(a+b\,x)^2}{16}+\frac{\cosh(a+b\,x)^4}{16}-b\left(\frac{x\sinh(a+b\,x)\cosh(a+b\,x)^3}{4}+\frac{3\,x\sinh(a+b\,x)\cosh(a+b\,x)}{8}\right)}{b^2}$$

input
```
int(x*cosh(a + b*x)^4,x)
```

output
```
(3*x^2)/16 - ((3*cosh(a + b*x)^2)/16 + cosh(a + b*x)^4/16 - b*((x*cosh(a +
 b*x)^3*sinh(a + b*x))/4 + (3*x*cosh(a + b*x)*sinh(a + b*x))/8))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.49

$$\int x\cosh^4(a+bx)\,dx$$
$$=\frac{4e^{8bx+8a}bx-e^{8bx+8a}+32e^{6bx+6a}bx-16e^{6bx+6a}+48e^{4bx+4a}b^2x^2-32e^{2bx+2a}bx-16e^{2bx+2a}-4bx-1}{256e^{4bx+4a}b^2}$$

input
```
int(x*cosh(b*x+a)^4,x)
```

output
```
(4*e**(8*a + 8*b*x)*b*x - e**(8*a + 8*b*x) + 32*e**(6*a + 6*b*x)*b*x - 16*
e**(6*a + 6*b*x) + 48*e**(4*a + 4*b*x)*b**2*x**2 - 32*e**(2*a + 2*b*x)*b*x
 - 16*e**(2*a + 2*b*x) - 4*b*x - 1)/(256*e**(4*a + 4*b*x)*b**2)
```

3.26 $\int (c+dx)^3 \operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 179

$$\int (c+dx)^3 \operatorname{sech}(a+bx)\,dx = \frac{2(c+dx)^3 \arctan\left(e^{a+bx}\right)}{b} - \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} + \frac{3id(c+dx)^2 \operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{6id^2(c+dx) \operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{6id^2(c+dx) \operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{6id^3 \operatorname{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} + \frac{6id^3 \operatorname{PolyLog}\left(4,ie^{a+bx}\right)}{b^4}$$

output
```
2*(d*x+c)^3*arctan(exp(b*x+a))/b-3*I*d*(d*x+c)^2*polylog(2,-I*exp(b*x+a))/
b^2+3*I*d*(d*x+c)^2*polylog(2,I*exp(b*x+a))/b^2+6*I*d^2*(d*x+c)*polylog(3,
-I*exp(b*x+a))/b^3-6*I*d^2*(d*x+c)*polylog(3,I*exp(b*x+a))/b^3-6*I*d^3*pol
ylog(4,-I*exp(b*x+a))/b^4+6*I*d^3*polylog(4,I*exp(b*x+a))/b^4
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 343, normalized size of antiderivative = 1.92

$$\int (c+dx)^3 \text{sech}(a+bx)\,dx$$
$$= \frac{i\left(-2ib^3c^3\arctan\left(e^{a+bx}\right)+3b^3c^2dx\log\left(1-ie^{a+bx}\right)+3b^3cd^2x^2\log\left(1-ie^{a+bx}\right)+b^3d^3x^3\log\left(1-ie^{a+bx}\right)\right.}{}$$

input
```
Integrate[(c + d*x)^3*Sech[a + b*x],x]
```

output
```
(I*((-2*I)*b^3*c^3*ArcTan[E^(a + b*x)] + 3*b^3*c^2*d*x*Log[1 - I*E^(a + b*
x)] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(a + b*x)] + b^3*d^3*x^3*Log[1 - I*E^(a
+ b*x)] - 3*b^3*c^2*d*x*Log[1 + I*E^(a + b*x)] - 3*b^3*c*d^2*x^2*Log[1 + I
*E^(a + b*x)] - b^3*d^3*x^3*Log[1 + I*E^(a + b*x)] - 3*b^2*d*(c + d*x)^2*P
olyLog[2, (-I)*E^(a + b*x)] + 3*b^2*d*(c + d*x)^2*PolyLog[2, I*E^(a + b*x)
] + 6*b*c*d^2*PolyLog[3, (-I)*E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, (-I)*E^(
a + b*x)] - 6*b*c*d^2*PolyLog[3, I*E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, I*E
^(a + b*x)] - 6*d^3*PolyLog[4, (-I)*E^(a + b*x)] + 6*d^3*PolyLog[4, I*E^(a
 + b*x)]))/b^4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \text{sech}(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int (c+dx)^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 4668$$

$$-\frac{3id\int (c+dx)^2 \log \left(1-ie^{a+bx}\right)dx}{b}+\frac{3id\int (c+dx)^2 \log \left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)^3 \arctan \left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{3id\left(\frac{2d\int (c+dx) \text{PolyLog}\left(2,-ie^{a+bx}\right)dx}{b}-\frac{(c+dx)^2 \text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3id\left(\frac{2d\int (c+dx) \text{PolyLog}\left(2,ie^{a+bx}\right)dx}{b}-\frac{(c+dx)^2 \text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{2(c+dx)^3 \arctan \left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{7163}$$

$$\frac{3id\left(\frac{2d\left(\frac{(c+dx) \text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{d\int \text{PolyLog}\left(3,-ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{(c+dx)^2 \text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3id\left(\frac{2d\left(\frac{(c+dx) \text{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{d\int \text{PolyLog}\left(3,ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{(c+dx)^2 \text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{2(c+dx)^3 \arctan \left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{3id\left(\frac{2d\left(\frac{(c+dx) \text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{d\int e^{-a-bx} \text{PolyLog}\left(3,-ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{(c+dx)^2 \text{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3id\left(\frac{2d\left(\frac{(c+dx) \text{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{d\int e^{-a-bx} \text{PolyLog}\left(3,ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{(c+dx)^2 \text{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}+\frac{2(c+dx)^3 \arctan \left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2(c+dx)^3\arctan\left(e^{a+bx}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{d\operatorname{PolyLog}\left(4,-ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b}-\frac{d\operatorname{PolyLog}\left(4,ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b}\right)}{b}$$

input
```
Int[(c + d*x)^3*Sech[a + b*x],x]
```

output
```
(2*(c + d*x)^3*ArcTan[E^(a + b*x)])/b + ((3*I)*d*(-(((c + d*x)^2*PolyLog[2
, (-I)*E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b
 - (d*PolyLog[4, (-I)*E^(a + b*x)])/b^2))/b))/b - ((3*I)*d*(-(((c + d*x)^2
*PolyLog[2, I*E^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, I*E^(a + b*x)
])/b - (d*PolyLog[4, I*E^(a + b*x)])/b^2))/b))/b
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int (dx + c)^3 \operatorname{sech}(bx + a)\,dx$$

input
```
int((d*x+c)^3*sech(b*x+a),x)
```

output
```
int((d*x+c)^3*sech(b*x+a),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 497 vs. 2(146) = 292.

Time = 0.11 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.78

$$\int (c+dx)^3 \operatorname{sech}(a+bx)\,dx$$
$$= \frac{6i\,d^3 \operatorname{polylog}(4, i\,\cosh(bx+a) + i\,\sinh(bx+a)) - 6i\,d^3 \operatorname{polylog}(4, -i\,\cosh(bx+a) - i\,\sinh(bx+a)) - \cdots}{\cdots}$$

input
```
integrate((d*x+c)^3*sech(b*x+a),x, algorithm="fricas")
```

output
```
(6*I*d^3*polylog(4, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*I*d^3*polylog(4
, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 3*(-I*b^2*d^3*x^2 - 2*I*b^2*c*d^2*
x - I*b^2*c^2*d)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - 3*(I*b^2*d^3*x
^2 + 2*I*b^2*c*d^2*x + I*b^2*c^2*d)*dilog(-I*cosh(b*x + a) - I*sinh(b*x +
a)) + (I*b^3*c^3 - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d^2 - I*a^3*d^3)*log(cosh
(b*x + a) + sinh(b*x + a) + I) + (-I*b^3*c^3 + 3*I*a*b^2*c^2*d - 3*I*a^2*b
*c*d^2 + I*a^3*d^3)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^3*d^3*x
^3 - 3*I*b^3*c*d^2*x^2 - 3*I*b^3*c^2*d*x - 3*I*a*b^2*c^2*d + 3*I*a^2*b*c*d
^2 - I*a^3*d^3)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b^3*d^3*x^
3 + 3*I*b^3*c*d^2*x^2 + 3*I*b^3*c^2*d*x + 3*I*a*b^2*c^2*d - 3*I*a^2*b*c*d^
2 + I*a^3*d^3)*log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1) - 6*(I*b*d^3*x
+ I*b*c*d^2)*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) - 6*(-I*b*d^3*x
 - I*b*c*d^2)*polylog(3, -I*cosh(b*x + a) - I*sinh(b*x + a)))/b^4
```

Sympy [F]

$$\int (c+dx)^3\text{sech}(a+bx)\,dx=\int (c+dx)^3\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sech(b*x+a),x)
```

output
```
Integral((c + d*x)**3*sech(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^3\text{sech}(a+bx)\,dx=\int (dx+c)^3\,\text{sech}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)^3*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*c^3*arctan(e^(-b*x - a))/b + 2*integrate((d^3*x^3*e^a + 3*c*d^2*x^2*e^a
 + 3*c^2*d*x*e^a)*e^(b*x)/(e^(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int (c+dx)^3\operatorname{sech}(a+bx)\,dx=\int (dx+c)^3\operatorname{sech}(bx+a)\,dx$$

input
```
integrate((d*x+c)^3*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\operatorname{sech}(a+bx)\,dx=\int \frac{(c+d\,x)^3}{\cosh(a+b\,x)}\,dx$$

input
```
int((c + d*x)^3/cosh(a + b*x),x)
```

output
```
int((c + d*x)^3/cosh(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^3\operatorname{sech}(a+bx)\,dx = \frac{2atan\left(e^{bx+a}\right)c^3+\left(\int \operatorname{sech}(bx+a)\,x^3dx\right)b\,d^3+3\left(\int \operatorname{sech}(bx+a)\,x^2dx\right)bc\,d^2+3\left(\int \operatorname{sech}(bx+a)\,xdx\right)b\,c}{b}$$

input
```
int((d*x+c)^3*sech(b*x+a),x)
```

output
```
(2*atan(e**(a + b*x))*c**3 + int(sech(a + b*x)*x**3,x)*b*d**3 + 3*int(sech
(a + b*x)*x**2,x)*b*c*d**2 + 3*int(sech(a + b*x)*x,x)*b*c**2*d)/b
```

3.27 $\int (c+dx)^2 \text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 119

$$\int (c+dx)^2 \text{sech}(a+bx)\,dx = \frac{2(c+dx)^2 \arctan\left(e^{a+bx}\right)}{b} - \frac{2id(c+dx)\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} + \frac{2id(c+dx)\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{2id^2\,\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{2id^2\,\text{PolyLog}\left(3,ie^{a+bx}\right)}{b^3}$$

output
```
2*(d*x+c)^2*arctan(exp(b*x+a))/b-2*I*d*(d*x+c)*polylog(2,-I*exp(b*x+a))/b^
2+2*I*d*(d*x+c)*polylog(2,I*exp(b*x+a))/b^2+2*I*d^2*polylog(3,-I*exp(b*x+a
))/b^3-2*I*d^2*polylog(3,I*exp(b*x+a))/b^3
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.67

$$\int (c+dx)^2 \operatorname{sech}(a+bx)\,dx$$
$$= \frac{i\left(-2ib^2c^2\arctan\left(e^{a+bx}\right)+2b^2cdx\log\left(1-ie^{a+bx}\right)+b^2d^2x^2\log\left(1-ie^{a+bx}\right)-2b^2cdx\log\left(1+ie^{a+bx}\right)-\right.}{}$$

input
```
Integrate[(c + d*x)^2*Sech[a + b*x],x]
```

output
```
(I*((-2*I)*b^2*c^2*ArcTan[E^(a + b*x)] + 2*b^2*c*d*x*Log[1 - I*E^(a + b*x)
] + b^2*d^2*x^2*Log[1 - I*E^(a + b*x)] - 2*b^2*c*d*x*Log[1 + I*E^(a + b*x)
] - b^2*d^2*x^2*Log[1 + I*E^(a + b*x)] - 2*b*d*(c + d*x)*PolyLog[2, (-I)*E
^(a + b*x)] + 2*b*d*(c + d*x)*PolyLog[2, I*E^(a + b*x)] + 2*d^2*PolyLog[3,
 (-I)*E^(a + b*x)] - 2*d^2*PolyLog[3, I*E^(a + b*x)]))/b^3
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \operatorname{sech}(a+bx)\,dx$$
$$\Big\downarrow \; 3042$$
$$\int (c+dx)^2 \csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$
$$\Big\downarrow \; 4668$$
$$-\frac{2id\int (c+dx)\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{2id\int (c+dx)\log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)^2\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2id\left(\frac{d\int \text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{(c+dx)\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2id\left(\frac{d\int \text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{(c+dx)\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{2(c+dx)^2\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{2id\left(\frac{d\int e^{-a-bx}\text{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2id\left(\frac{d\int e^{-a-bx}\text{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}+\frac{2(c+dx)^2\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{2(c+dx)^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2id\left(\frac{d\,\text{PolyLog}(3,-ie^{a+bx})}{b^2}-\frac{(c+dx)\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2id\left(\frac{d\,\text{PolyLog}(3,ie^{a+bx})}{b^2}-\frac{(c+dx)\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}$$

input
```
Int[(c + d*x)^2*Sech[a + b*x],x]
```

output
```
(2*(c + d*x)^2*ArcTan[E^(a + b*x)])/b + ((2*I)*d*(-(((c + d*x)*PolyLog[2,
(-I)*E^(a + b*x)])/b) + (d*PolyLog[3, (-I)*E^(a + b*x)])/b^2))/b - ((2*I)*
d*(-(((c + d*x)*PolyLog[2, I*E^(a + b*x)])/b) + (d*PolyLog[3, I*E^(a + b*x
)])/b^2))/b
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int (dx + c)^2 \operatorname{sech}(bx + a) \, dx$$

input
```
int((d*x+c)^2*sech(b*x+a),x)
```

output
```
int((d*x+c)^2*sech(b*x+a),x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(96) = 192$.

Time = 0.10 (sec) , antiderivative size = 305, normalized size of antiderivative = 2.56

$$\int (c+dx)^2 \operatorname{sech}(a+bx)\,dx = \frac{-2i\,d^2\operatorname{polylog}(3, i\cosh(bx+a)+i\sinh(bx+a)) + 2i\,d^2\operatorname{polylog}(3, -i\cosh(bx+a)-i\sinh(bx+a))}{}$$

input
```
integrate((d*x+c)^2*sech(b*x+a),x, algorithm="fricas")
```

output
```
(-2*I*d^2*polylog(3, I*cosh(b*x + a) + I*sinh(b*x + a)) + 2*I*d^2*polylog(
3, -I*cosh(b*x + a) - I*sinh(b*x + a)) - 2*(-I*b*d^2*x - I*b*c*d)*dilog(I*
cosh(b*x + a) + I*sinh(b*x + a)) - 2*(I*b*d^2*x + I*b*c*d)*dilog(-I*cosh(b
*x + a) - I*sinh(b*x + a)) + (I*b^2*c^2 - 2*I*a*b*c*d + I*a^2*d^2)*log(cos
h(b*x + a) + sinh(b*x + a) + I) + (-I*b^2*c^2 + 2*I*a*b*c*d - I*a^2*d^2)*l
og(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b^2*d^2*x^2 - 2*I*b^2*c*d*x -
2*I*a*b*c*d + I*a^2*d^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b
^2*d^2*x^2 + 2*I*b^2*c*d*x + 2*I*a*b*c*d - I*a^2*d^2)*log(-I*cosh(b*x + a)
 - I*sinh(b*x + a) + 1))/b^3
```

Sympy [F]

$$\int (c+dx)^2 \operatorname{sech}(a+bx)\,dx = \int (c+dx)^2 \operatorname{sech}(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sech(b*x+a),x)
```

output
```
Integral((c + d*x)**2*sech(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)^2\operatorname{sech}(a+bx)\,dx=\int (dx+c)^2\operatorname{sech}(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*sech(b*x+a),x, algorithm="maxima")
```

output
```
-2*c^2*arctan(e^(-b*x - a))/b + 2*integrate((d^2*x^2*e^a + 2*c*d*x*e^a)*e^
(b*x)/(e^(2*b*x + 2*a) + 1), x)
```

Giac [F]

$$\int (c+dx)^2\operatorname{sech}(a+bx)\,dx=\int (dx+c)^2\operatorname{sech}(bx+a)\,dx$$

input
```
integrate((d*x+c)^2*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\operatorname{sech}(a+bx)\,dx=\int \frac{(c+d\,x)^2}{\cosh(a+b\,x)}\,dx$$

input
```
int((c + d*x)^2/cosh(a + b*x),x)
```

output
```
int((c + d*x)^2/cosh(a + b*x), x)
```

Reduce [F]

$$\int (c+dx)^2\text{sech}(a+bx)\,dx$$
$$= \frac{2atan\left(e^{bx+a}\right)c^2+\left(\int \text{sech}(bx+a)\,x^2dx\right)b\,d^2+2\left(\int \text{sech}(bx+a)\,xdx\right)bcd}{b}$$

input
```
int((d*x+c)^2*sech(b*x+a),x)
```

output
```
(2*atan(e**(a + b*x))*c**2 + int(sech(a + b*x)*x**2,x)*b*d**2 + 2*int(sech
(a + b*x)*x,x)*b*c*d)/b
```

3.28 $\int (c+dx)\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 61

$$\int (c+dx)\text{sech}(a+bx)\,dx = \frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b} - \frac{id\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} + \frac{id\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}$$

output
```
2*(d*x+c)*arctan(exp(b*x+a))/b-I*d*polylog(2,-I*exp(b*x+a))/b^2+I*d*polylo
g(2,I*exp(b*x+a))/b^2
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.39

$$\int (c+dx)\text{sech}(a+bx)\,dx = -\frac{c\cot^{-1}(\sinh(a+bx))}{b} + \frac{id\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{b^2}$$

input
```
Integrate[(c + d*x)*Sech[a + b*x],x]
```

output
```
-((c*ArcCot[Sinh[a + b*x]])/b) + (I*d*(b*x*(Log[1 - I*E^(a + b*x)] - Log[1
 + I*E^(a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a + b*
x)]))/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4668}$$

$$-\frac{id\int \log\left(1-ie^{a+bx}\right)dx}{b}+\frac{id\int \log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{id\int e^{-a-bx}\log\left(1-ie^{a+bx}\right)de^{a+bx}}{b^2}+\frac{id\int e^{-a-bx}\log\left(1+ie^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}$$

input
```
Int[(c + d*x)*Sech[a + b*x],x]
```

output
```
(2*(c + d*x)*ArcTan[E^(a + b*x)])/b - (I*d*PolyLog[2, (-I)*E^(a + b*x)])/b
^2 + (I*d*PolyLog[2, I*E^(a + b*x)])/b^2
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.66

method	result
derivativedivides	$\frac{\frac{d\left(i(bx+a)\left(\ln\left(1-i e^{bx+a}\right)-\ln\left(1+i e^{bx+a}\right)\right)-i\,\mathrm{dilog}\left(1+i e^{bx+a}\right)+i\,\mathrm{dilog}\left(1-i e^{bx+a}\right)\right)}{b}-\frac{2da\arctan\left(e^{bx+a}\right)}{b}+2c\arctan\left(e^{bx+a}\right)}{b}$
default	$\frac{\frac{d\left(i(bx+a)\left(\ln\left(1-i e^{bx+a}\right)-\ln\left(1+i e^{bx+a}\right)\right)-i\,\mathrm{dilog}\left(1+i e^{bx+a}\right)+i\,\mathrm{dilog}\left(1-i e^{bx+a}\right)\right)}{b}-\frac{2da\arctan\left(e^{bx+a}\right)}{b}+2c\arctan\left(e^{bx+a}\right)}{b}$
parts	$\frac{\arctan(\sinh(bx+a))dx}{b}+\frac{\arctan(\sinh(bx+a))c}{b}-\frac{d\left(x\arctan(\sinh(bx+a))-\frac{i(bx+a)\left(\ln\left(1-i e^{bx+a}\right)-\ln\left(1+i e^{bx+a}\right)\right)}{}\right.}{b}$
risch	$\frac{2c\arctan\left(e^{bx+a}\right)}{b}-\frac{id\ln\left(1+i e^{bx+a}\right)x}{b}-\frac{id\ln\left(1+i e^{bx+a}\right)a}{b^2}+\frac{id\ln\left(1-i e^{bx+a}\right)x}{b}+\frac{id\ln\left(1-i e^{bx+a}\right)a}{b^2}-\frac{id\,\mathrm{dilo}}{}$

input
```
int((d*x+c)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(d/b*(I*(b*x+a)*(ln(1-I*exp(b*x+a))-ln(1+I*exp(b*x+a)))-I*dilog(1+I*ex
p(b*x+a))+I*dilog(1-I*exp(b*x+a)))-2*d/b*a*arctan(exp(b*x+a))+2*c*arctan(e
xp(b*x+a)))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(48) = 96$.

Time = 0.09 (sec) , antiderivative size = 157, normalized size of antiderivative = 2.57

$$\int (c+dx)\text{sech}(a+bx)\,dx = \frac{i\,d\text{Li}_2(i\,\cosh{(bx+a)}+i\,\sinh{(bx+a)})-i\,d\text{Li}_2(-i\,\cosh{(bx+a)}-i\,\sinh{(bx+a)})+(i\,bc-i\,ad)\log{(}$$

input
```
integrate((d*x+c)*sech(b*x+a),x, algorithm="fricas")
```

output
```
(I*d*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) - I*d*dilog(-I*cosh(b*x + a)
 - I*sinh(b*x + a)) + (I*b*c - I*a*d)*log(cosh(b*x + a) + sinh(b*x + a) +
I) + (-I*b*c + I*a*d)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (-I*b*d*x -
 I*a*d)*log(I*cosh(b*x + a) + I*sinh(b*x + a) + 1) + (I*b*d*x + I*a*d)*log
(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/b^2
```

Sympy [F]

$$\int (c+dx)\text{sech}(a+bx)\,dx = \int (c+dx)\,\text{sech}\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*sech(b*x+a),x)
```

output
```
Integral((c + d*x)*sech(a + b*x), x)
```

Maxima [F]

$$\int (c+dx)\text{sech}(a+bx)\,dx = \int (dx+c)\,\text{sech}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*sech(b*x+a),x, algorithm="maxima")
```

output
```
2*d*integrate(x*e^(b*x + a)/(e^(2*b*x + 2*a) + 1), x) - 2*c*arctan(e^(-b*x
 - a))/b
```

Giac [F]

$$\int (c+dx)\text{sech}(a+bx)\,dx = \int (dx+c)\,\text{sech}\,(bx+a)\,\,dx$$

input
```
integrate((d*x+c)*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)*sech(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\text{sech}(a+bx)\,dx = \int \frac{c+d\,x}{\cosh\,(a+b\,x)}\,dx$$

input
```
int((c + d*x)/cosh(a + b*x),x)
```

output
```
int((c + d*x)/cosh(a + b*x), x)
```

Reduce [F]

$$\int (c + dx)\text{sech}(a + bx)\,dx = \frac{2atan\left(e^{bx+a}\right)c + \left(\int \text{sech}(bx + a)\,xdx\right)bd}{b}$$

input
```
int((d*x+c)*sech(b*x+a),x)
```

output
```
(2*atan(e**(a + b*x))*c + int(sech(a + b*x)*x,x)*b*d)/b
```

3.29 $\int \frac{\text{sech}(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{sech}(a+bx)}{c+dx}\,dx = \text{Int}\left(\frac{\text{sech}(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(sech(b*x+a)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.69 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{sech}(a+bx)}{c+dx}\,dx = \int \frac{\text{sech}(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Sech[a + b*x]/(c + d*x),x]
```

output
```
Integrate[Sech[a + b*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}dx$$

input
```
Int[Sech[a + b*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)}{dx+c}dx$$

input
```
int(sech(b*x+a)/(d*x+c),x)
```

output
```
int(sech(b*x+a)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{sech}(bx+a)}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c),x)
```

output
```
Integral(sech(a + b*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{sech}(bx+a)}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\mathrm{sech}(a+bx)}{c+dx}\,dx=\int \frac{\mathrm{sech}\left(bx+a\right)}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\mathrm{sech}(a+bx)}{c+dx}\,dx=\int \frac{1}{\cosh\left(a+b\,x\right)\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cosh(a + b*x)*(c + d*x)),x)
```

output
```
int(1/(cosh(a + b*x)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}(bx+a)}{dx+c}dx$$

input

```
int(sech(b*x+a)/(d*x+c),x)
```

output

```
int(sech(a + b*x)/(c + d*x),x)
```

3.30 $\int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\text{sech}(a+bx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(sech(b*x+a)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.91 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\text{sech}(a+bx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Sech[a + b*x]/(c + d*x)^2,x]
```

output
```
Integrate[Sech[a + b*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Sech[a + b*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)}{(dx+c)^2}dx$$

input
```
int(sech(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sech(b*x+a)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.67 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)**2,x)
```

output
```
Integral(sech(a + b*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}(bx+a)}{(dx+c)^2}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)/(d*x + c)^2, x)
```

Giac [N/A]

Not integrable

Time = 1.44 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx=\int \frac{1}{\cosh\left(a+b\,x\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cosh(a + b*x)*(c + d*x)^2),x)
```

output
```
int(1/(cosh(a + b*x)*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\operatorname{sech}(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}(bx+a)}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sech(b*x+a)/(d*x+c)^2,x)
```

output
```
int(sech(a + b*x)/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.31 $\int (c+dx)^3 \mathbf{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 103

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx = \frac{(c+dx)^3}{b} - \frac{3d(c+dx)^2 \log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{3d^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3} + \frac{3d^3\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{2b^4} + \frac{(c+dx)^3\tanh(a+bx)}{b}$$

output
```
(d*x+c)^3/b-3*d*(d*x+c)^2*ln(1+exp(2*b*x+2*a))/b^2-3*d^2*(d*x+c)*polylog(2
,-exp(2*b*x+2*a))/b^3+3/2*d^3*polylog(3,-exp(2*b*x+2*a))/b^4+(d*x+c)^3*tan
h(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.41

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx$$

$$= \frac{-\frac{de^{2a}\left(\frac{4e^{-2a}(c+dx)^3}{d}+\frac{6\left(1+e^{-2a}\right)(c+dx)^2\log\left(1+e^{-2(a+bx)}\right)}{b}-\frac{3d\left(1+e^{-2a}\right)\left(2b(c+dx)\operatorname{PolyLog}\left(2,-e^{-2(a+bx)}\right)+d\operatorname{PolyLog}\left(3,-e^{-2(a+bx)}\right)\right)}{b^3}\right)}{1+e^{2a}} + 2\cdots}{2b}$$

input
```
Integrate[(c + d*x)^3*Sech[a + b*x]^2,x]
```

output
```
(-((d*E^(2*a)*((4*(c + d*x)^3)/(d*E^(2*a)) + (6*(1 + E^(-2*a))*(c + d*x)^2
*Log[1 + E^(-2*(a + b*x))])/b - (3*d*(1 + E^(-2*a))*(2*b*(c + d*x)*PolyLog
[2, -E^(-2*(a + b*x))] + d*PolyLog[3, -E^(-2*(a + b*x))]))/b^3))/(1 + E^(2
*a))) + 2*(c + d*x)^3*Sech[a]*Sech[a + b*x]*Sinh[b*x])/(2*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.23, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \csc\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{(c+dx)^3 \tanh(a+bx)}{b} - \frac{3id\int -i(c+dx)^2 \tanh(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^3 \tanh(a+bx)}{b} - \frac{3d\int (c+dx)^2 \tanh(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^3 \tanh(a+bx)}{b} - \frac{3d\int -i(c+dx)^2 \tan(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^3 \tanh(a+bx)}{b} + \frac{3id\int (c+dx)^2 \tan(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{(c+dx)^3\tanh(a+bx)}{b}+\frac{3id\left(2i\int\frac{e^{2(a+bx)}(c+dx)^2}{1+e^{2(a+bx)}}dx-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{(c+dx)^3\tanh(a+bx)}{b}+\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{d\int(c+dx)\log\left(1+e^{2(a+bx)}\right)dx}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{(c+dx)^3\tanh(a+bx)}{b}+\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{d\left(\frac{d\int\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)dx}{2b}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{(c+dx)^3\tanh(a+bx)}{b}+\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{d\left(\frac{d\int e^{-2(a+bx)}\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{(c+dx)^3\tanh(a+bx)}{b}+\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{d\left(\frac{d\operatorname{PolyLog}\left(3,-e^{2(a+bx)}\right)}{4b^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(a+bx)}\right)}{2b}\right)}{b}\right)-\frac{i(c+dx)^3}{3d}\right)}{b}$$

input
```
Int[(c + d*x)^3*Sech[a + b*x]^2,x]
```

output
```
((3*I)*d*(((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((c + d*x)^2*Log[1 + E^(2*(a +
 b*x))])/(2*b) - (d*(-1/2*((c + d*x)*PolyLog[2, -E^(2*(a + b*x))])/b + (d*
PolyLog[3, -E^(2*(a + b*x))])/(4*b^2)))/b)))/b + ((c + d*x)^3*Tanh[a + b*x
])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(101) = 202$.

Time = 0.78 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.89

method	result
risch	$-\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{(1+e^{2bx+2a})b}+\frac{6d^3a^2\ln(e^{bx+a})}{b^4}-\frac{6d^3a^2x}{b^3}+\frac{2d^3x^3}{b}-\frac{4d^3a^3}{b^4}-\frac{3d^3\ln(1+e^{2bx+2a})x^2}{b^2}+\frac{3d^3\operatorname{polylog}(\ldots}{2b\ldots}$

input
```
int((d*x+c)^3*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/(1+exp(2*b*x+2*a))/b+6/b^4*d^3*a^2*
ln(exp(b*x+a))-6/b^3*d^3*a^2*x+2/b*d^3*x^3-4/b^4*d^3*a^3-3/b^2*d^3*ln(1+ex
p(2*b*x+2*a))*x^2+3/2*d^3*polylog(3,-exp(2*b*x+2*a))/b^4-3/b^3*d^3*polylog
(2,-exp(2*b*x+2*a))*x+6/b*d^2*c*x^2+6/b^3*d^2*c*a^2-6/b^2*d^2*c*ln(1+exp(2
*b*x+2*a))*x-3/b^3*d^2*c*polylog(2,-exp(2*b*x+2*a))-12/b^3*d^2*c*a*ln(exp(
b*x+a))+12/b^2*d^2*c*a*x+6/b^2*d*c^2*ln(exp(b*x+a))-3/b^2*d*c^2*ln(1+exp(2
*b*x+2*a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1332, normalized size of antiderivative = 12.93

$$\int (c+dx)^3\operatorname{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="fricas")
```

output

```
-(2*b^3*c^3 - 6*a*b^2*c^2*d + 6*a^2*b*c*d^2 - 2*a^3*d^3 - 2*(b^3*d^3*x^3 +
 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3
)*cosh(b*x + a)^2 - 4*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2 + a^3*d^3)*cosh(b*x + a)*sinh(b*x + a) - 2*(b^3
*d^3*x^3 + 3*b^3*c*d^2*x^2 + 3*b^3*c^2*d*x + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
 + a^3*d^3)*sinh(b*x + a)^2 + 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*c
osh(b*x + a)^2 + 2*(b*d^3*x + b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*d^
3*x + b*c*d^2)*sinh(b*x + a)^2)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) +
 6*(b*d^3*x + b*c*d^2 + (b*d^3*x + b*c*d^2)*cosh(b*x + a)^2 + 2*(b*d^3*x +
 b*c*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*d^3*x + b*c*d^2)*sinh(b*x + a)^
2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + 3*(b^2*c^2*d - 2*a*b*c*d^2
+ a^2*d^3 + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)^2 + 2*(b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x + a)*sinh(b*x + a) + (b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*sinh(b*x + a)^2)*log(cosh(b*x + a) + sinh(b*x + a) +
 I) + 3*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3 + (b^2*c^2*d - 2*a*b*c*d^2 + a^
2*d^3)*cosh(b*x + a)^2 + 2*(b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*cosh(b*x +
a)*sinh(b*x + a) + (b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*sinh(b*x + a)^2)*lo
g(cosh(b*x + a) + sinh(b*x + a) - I) + 3*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*
a*b*c*d^2 - a^2*d^3 + (b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*d^3
)*cosh(b*x + a)^2 + 2*(b^2*d^3*x^2 + 2*b^2*c*d^2*x + 2*a*b*c*d^2 - a^2*...
```

Sympy [F]

$$\int (c+dx)^3\operatorname{sech}^2(a+bx)\,dx=\int (c+dx)^3\operatorname{sech}^2(a+bx)\,dx$$

input

```
integrate((d*x+c)**3*sech(b*x+a)**2,x)
```

output

```
Integral((c + d*x)**3*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(100) = 200$.

Time = 0.18 (sec) , antiderivative size = 238, normalized size of antiderivative = 2.31

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx = 3\,c^2 d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b} - \frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\right) - \frac{3\,\left(2\,bx\log\left(e^{(2\,bx+2\,a)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)\right)cd^2}{b^3} + \frac{2\,c^3}{b\left(e^{(-2\,bx-2\,a)}+1\right)} - \frac{2\,\left(d^3x^3+3\,cd^2x^2\right)}{be^{(2\,bx+2\,a)}+b} - \frac{3\,\left(2\,b^2x^2\log\left(e^{(2\,bx+2\,a)}+1\right)+2\,bx\mathrm{Li}_2\left(-e^{(2\,bx+2\,a)}\right)-\mathrm{Li}_3\left(-e^{(2\,bx+2\,a)}\right)\right)d^3}{2\,b^4} + \frac{2\,\left(b^3d^3x^3+3\,b^3cd^2x^2\right)}{b^4}$$

input
```
integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
3*c^2*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x + 2*a
) + 1)*e^(-2*a))/b^2) - 3*(2*b*x*log(e^(2*b*x + 2*a) + 1) + dilog(-e^(2*b*
x + 2*a)))*c*d^2/b^3 + 2*c^3/(b*(e^(-2*b*x - 2*a) + 1)) - 2*(d^3*x^3 + 3*c
*d^2*x^2)/(b*e^(2*b*x + 2*a) + b) - 3/2*(2*b^2*x^2*log(e^(2*b*x + 2*a) + 1
) + 2*b*x*dilog(-e^(2*b*x + 2*a)) - polylog(3, -e^(2*b*x + 2*a)))*d^3/b^4
+ 2*(b^3*d^3*x^3 + 3*b^3*c*d^2*x^2)/b^4
```

Giac [F]

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx = \int (dx+c)^3 \operatorname{sech}(bx+a)^2\, dx$$

input
```
integrate((d*x+c)^3*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sech(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx = \int \frac{(c+d\,x)^3}{\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int((c + d*x)^3/cosh(a + b*x)^2,x)
```

output
```
int((c + d*x)^3/cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^3 \operatorname{sech}^2(a+bx)\,dx = \frac{12e^{2bx+2a}\left(\int \frac{x^2}{e^{4bx+4a}+2e^{2bx+2a}+1}dx\right)b^3d^3 + 24e^{2bx+2a}\left(\int \frac{x}{e^{4bx+4a}+2e^{2bx+2a}+1}dx\right)b^3c\,d^2 + 12e^{2bx+2a}\left(\int \frac{x}{e^{4bx+4a}+2e}\right.}{}$$

input
```
int((d*x+c)^3*sech(b*x+a)^2,x)
```

output
```
(12*e**(2*a + 2*b*x)*int(x**2/(e**(4*a + 4*b*x) + 2*e**(2*a + 2*b*x) + 1),
x)*b**3*d**3 + 24*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x) + 2*e**(2*a + 2
*b*x) + 1),x)*b**3*c*d**2 + 12*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x) +
2*e**(2*a + 2*b*x) + 1),x)*b**2*d**3 - 6*e**(2*a + 2*b*x)*log(e**(2*a + 2*
b*x) + 1)*b**2*c**2*d - 6*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1)*b*c*d
**2 - 3*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1)*d**3 + 4*e**(2*a + 2*b*
x)*b**3*c**3 + 12*e**(2*a + 2*b*x)*b**3*c**2*d*x + 12*e**(2*a + 2*b*x)*b**
2*c*d**2*x + 6*e**(2*a + 2*b*x)*b*d**3*x + 12*int(x**2/(e**(4*a + 4*b*x) +
 2*e**(2*a + 2*b*x) + 1),x)*b**3*d**3 + 24*int(x/(e**(4*a + 4*b*x) + 2*e**
(2*a + 2*b*x) + 1),x)*b**3*c*d**2 + 12*int(x/(e**(4*a + 4*b*x) + 2*e**(2*a
 + 2*b*x) + 1),x)*b**2*d**3 - 6*log(e**(2*a + 2*b*x) + 1)*b**2*c**2*d - 6*
log(e**(2*a + 2*b*x) + 1)*b*c*d**2 - 3*log(e**(2*a + 2*b*x) + 1)*d**3 - 12
*b**3*c*d**2*x**2 - 4*b**3*d**3*x**3 - 6*b**2*d**3*x**2)/(2*b**4*(e**(2*a
+ 2*b*x) + 1))
```

3.32 $\int (c+dx)^2 \text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 73

$$\int (c+dx)^2 \text{sech}^2(a+bx)\,dx = \frac{(c+dx)^2}{b} - \frac{2d(c+dx)\log\left(1+e^{2(a+bx)}\right)}{b^2} - \frac{d^2\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{b^3} + \frac{(c+dx)^2\tanh(a+bx)}{b}$$

output
```
(d*x+c)^2/b-2*d*(d*x+c)*ln(1+exp(2*b*x+2*a))/b^2-d^2*polylog(2,-exp(2*b*x+
2*a))/b^3+(d*x+c)^2*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.25

$$\int (c+dx)^2 \text{sech}^2(a+bx)\,dx$$
$$= \frac{-\frac{2b(c+dx)\left(b(c+dx)+d\left(1+e^{2a}\right)\log\left(1+e^{-2(a+bx)}\right)\right)}{1+e^{2a}} + d^2\,\text{PolyLog}\left(2,-e^{-2(a+bx)}\right) + b^2(c+dx)^2\text{sech}(a)\text{sech}(a+bx)\,\text{s}}{b^3}$$

input
```
Integrate[(c + d*x)^2*Sech[a + b*x]^2,x]
```

output
```
((-2*b*(c + d*x)*(b*(c + d*x) + d*(1 + E^(2*a))*Log[1 + E^(-2*(a + b*x))])
)/(1 + E^(2*a)) + d^2*PolyLog[2, -E^(-2*(a + b*x))] + b^2*(c + d*x)^2*Sech
[a]*Sech[a + b*x]*Sinh[b*x])/b^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\csc\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}-\frac{2id\int -i(c+dx)\tanh(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}-\frac{2d\int (c+dx)\tanh(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}-\frac{2d\int -i(c+dx)\tan(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{2id\int (c+dx)\tan(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{4201}$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{2id\left(2i\int \frac{e^{2(a+bx)}(c+dx)}{1+e^{2(a+bx)}}dx-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \; 2620$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{d\int\log\left(1+e^{2(a+bx)}\right)dx}{2b}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \; 2715$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(a+bx)}+1\right)}{2b}-\frac{d\int e^{-2(a+bx)}\log\left(1+e^{2(a+bx)}\right)de^{2(a+bx)}}{4b^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

$$\Big\downarrow \; 2838$$

$$\frac{(c+dx)^2\tanh(a+bx)}{b}+\frac{2id\left(2i\left(\frac{d\,\text{PolyLog}\left(2,-e^{2(a+bx)}\right)}{4b^2}+\frac{(c+dx)\log\left(e^{2(a+bx)}+1\right)}{2b}\right)-\frac{i(c+dx)^2}{2d}\right)}{b}$$

input
```
Int[(c + d*x)^2*Sech[a + b*x]^2,x]
```

output
```
((2*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c + d*x)*Log[1 + E^(2*(a + b
*x))])/(2*b) + (d*PolyLog[2, -E^(2*(a + b*x))])/(4*b^2))))/b + ((c + d*x)^
2*Tanh[a + b*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(73) = 146$.

Time = 0.60 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.18

method	result
risch	$-\frac{2(x^2d^2+2cdx+c^2)}{b(1+e^{2bx+2a})}+\frac{4dc\ln(e^{bx+a})}{b^2}-\frac{2dc\ln(1+e^{2bx+2a})}{b^2}+\frac{2d^2x^2}{b}+\frac{4d^2xa}{b^2}+\frac{2d^2a^2}{b^3}-\frac{2d^2\ln(1+e^{2bx+2a})x}{b^2}-$ d^2 po

input
```
int((d*x+c)^2*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*(d^2*x^2+2*c*d*x+c^2)/b/(1+exp(2*b*x+2*a))+4/b^2*d*c*ln(exp(b*x+a))-2/b
^2*d*c*ln(1+exp(2*b*x+2*a))+2/b*d^2*x^2+4/b^2*d^2*x*a+2/b^3*d^2*a^2-2/b^2*
d^2*ln(1+exp(2*b*x+2*a))*x-d^2*polylog(2,-exp(2*b*x+2*a))/b^3-4/b^3*d^2*a*
ln(exp(b*x+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 715, normalized size of antiderivative = 9.79

$$\int (c+dx)^2 \text{sech}^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
-2*(b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d
 - a^2*d^2)*cosh(b*x + a)^2 - 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c*d - a
^2*d^2)*cosh(b*x + a)*sinh(b*x + a) - (b^2*d^2*x^2 + 2*b^2*c*d*x + 2*a*b*c
*d - a^2*d^2)*sinh(b*x + a)^2 + (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)
*sinh(b*x + a) + d^2*sinh(b*x + a)^2 + d^2)*dilog(I*cosh(b*x + a) + I*sinh
(b*x + a)) + (d^2*cosh(b*x + a)^2 + 2*d^2*cosh(b*x + a)*sinh(b*x + a) + d^
2*sinh(b*x + a)^2 + d^2)*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)) + (b*c*
d - a*d^2 + (b*c*d - a*d^2)*cosh(b*x + a)^2 + 2*(b*c*d - a*d^2)*cosh(b*x +
 a)*sinh(b*x + a) + (b*c*d - a*d^2)*sinh(b*x + a)^2)*log(cosh(b*x + a) + s
inh(b*x + a) + I) + (b*c*d - a*d^2 + (b*c*d - a*d^2)*cosh(b*x + a)^2 + 2*(
b*c*d - a*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*c*d - a*d^2)*sinh(b*x + a)
^2)*log(cosh(b*x + a) + sinh(b*x + a) - I) + (b*d^2*x + a*d^2 + (b*d^2*x +
 a*d^2)*cosh(b*x + a)^2 + 2*(b*d^2*x + a*d^2)*cosh(b*x + a)*sinh(b*x + a)
+ (b*d^2*x + a*d^2)*sinh(b*x + a)^2)*log(I*cosh(b*x + a) + I*sinh(b*x + a)
 + 1) + (b*d^2*x + a*d^2 + (b*d^2*x + a*d^2)*cosh(b*x + a)^2 + 2*(b*d^2*x
+ a*d^2)*cosh(b*x + a)*sinh(b*x + a) + (b*d^2*x + a*d^2)*sinh(b*x + a)^2)*
log(-I*cosh(b*x + a) - I*sinh(b*x + a) + 1))/(b^3*cosh(b*x + a)^2 + 2*b^3*
cosh(b*x + a)*sinh(b*x + a) + b^3*sinh(b*x + a)^2 + b^3)
```

Sympy [F]

$$\int (c+dx)^2 \text{sech}^2(a+bx)\,dx = \int (c+dx)^2\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sech(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**2*sech(a + b*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2\text{sech}^2(a+bx)\,dx = \int (dx+c)^2\,\text{sech}\,(bx+a)^2\;dx$$

input
```
integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
-2*d^2*(x^2/(b*e^(2*b*x + 2*a) + b) - 2*integrate(x/(b*e^(2*b*x + 2*a) + b
), x)) + 2*c*d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*
x + 2*a) + 1)*e^(-2*a))/b^2) + 2*c^2/(b*(e^(-2*b*x - 2*a) + 1))
```

Giac [F]

$$\int (c+dx)^2\text{sech}^2(a+bx)\,dx = \int (dx+c)^2\,\text{sech}\,(bx+a)^2\;dx$$

input
```
integrate((d*x+c)^2*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sech(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\text{sech}^2(a+bx)\,dx = \int \frac{(c+d\,x)^2}{\cosh\left(a+b\,x\right)^2}\,dx$$

input
```
int((c + d*x)^2/cosh(a + b*x)^2,x)
```

output
```
int((c + d*x)^2/cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \operatorname{sech}^2(a+bx)\,dx$$
$$= \frac{4e^{2bx+2a}\left(\int \frac{x}{e^{4bx+4a}+2e^{2bx+2a}+1}dx\right)b^2d^2 - 2e^{2bx+2a}\log\left(e^{2bx+2a}+1\right)bcd - e^{2bx+2a}\log\left(e^{2bx+2a}+1\right)d^2 + 2e^{2bx}}{}$$

input
```
int((d*x+c)^2*sech(b*x+a)^2,x)
```

output
```
(4*e**(2*a + 2*b*x)*int(x/(e**(4*a + 4*b*x) + 2*e**(2*a + 2*b*x) + 1),x)*b
**2*d**2 - 2*e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1)*b*c*d - e**(2*a +
2*b*x)*log(e**(2*a + 2*b*x) + 1)*d**2 + 2*e**(2*a + 2*b*x)*b**2*c**2 + 4*e
**(2*a + 2*b*x)*b**2*c*d*x + 2*e**(2*a + 2*b*x)*b*d**2*x + 4*int(x/(e**(4*
a + 4*b*x) + 2*e**(2*a + 2*b*x) + 1),x)*b**2*d**2 - 2*log(e**(2*a + 2*b*x)
 + 1)*b*c*d - log(e**(2*a + 2*b*x) + 1)*d**2 - 2*b**2*d**2*x**2)/(b**3*(e*
*(2*a + 2*b*x) + 1))
```

3.33 $\int (c+dx)\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 29

$$\int (c+dx)\text{sech}^2(a+bx)\,dx = -\frac{d\log(\cosh(a+bx))}{b^2} + \frac{(c+dx)\tanh(a+bx)}{b}$$

output
```
-d*ln(cosh(b*x+a))/b^2+(d*x+c)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.76

$$\int (c+dx)\text{sech}^2(a+bx)\,dx = -\frac{d\log(\cosh(a+bx))}{b^2} + \frac{dx\text{sech}(a)\text{sech}(a+bx)\sinh(bx)}{b} + \frac{dx\tanh(a)}{b} + \frac{c\tanh(a+bx)}{b}$$

input
```
Integrate[(c + d*x)*Sech[a + b*x]^2,x]
```

output
```
-((d*Log[Cosh[a + b*x]])/b^2) + (d*x*Sech[a]*Sech[a + b*x]*Sinh[b*x])/b +
(d*x*Tanh[a])/b + (c*Tanh[a + b*x])/b
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{4672}$$

$$\frac{(c+dx)\tanh(a+bx)}{b}-\frac{id\int -i\tanh(a+bx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\int \tanh(a+bx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\int -i\tan(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)\tanh(a+bx)}{b}+\frac{id\int \tan(ia+ibx)dx}{b}$$

$$\Big\downarrow \text{3956}$$

$$\frac{(c+dx)\tanh(a+bx)}{b}-\frac{d\log(\cosh(a+bx))}{b^2}$$

input
```
Int[(c + d*x)*Sech[a + b*x]^2,x]
```

output
```
-((d*Log[Cosh[a + b*x]])/b^2) + ((c + d*x)*Tanh[a + b*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.97

method	result	siz
risch	$\frac{2dx}{b}+\frac{2da}{b^{2}}-\frac{2(dx+c)}{\left(1+{\mathrm e}^{2bx+2a}\right)b}-\frac{d\ln\left(1+{\mathrm e}^{2bx+2a}\right)}{b^{2}}$	57
parallelrisch	$\frac{-\cosh(bx+a)d\ln\left(2-\operatorname{sech}\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\right)+2\cosh(bx+a)d\ln\left(1-\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)\right)+(dx\cosh(bx+a)+\sinh(bx+a)(dx+c))b}{b^{2}\cosh(bx+a)}$	86

input
```
int((d*x+c)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
2*d/b*x+2*d/b^2*a-2*(d*x+c)/(1+exp(2*b*x+2*a))/b-d/b^2*ln(1+exp(2*b*x+2*a)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(29) = 58$.

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 5.55

$$\int (c+dx)\text{sech}^2(a+bx)\,dx$$
$$= \frac{2\,bdx\cosh{(bx+a)}^2 + 4\,bdx\cosh{(bx+a)}\sinh{(bx+a)} + 2\,bdx\sinh{(bx+a)}^2 - 2\,bc - \left(d\cosh{(bx+a)}^2\right.}{b^2\cosh{(bx+a)}^2 + 2\,b^2\cosh{(bx+a)}\sinh{(bx+}}$$

input
```
integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
(2*b*d*x*cosh(b*x + a)^2 + 4*b*d*x*cosh(b*x + a)*sinh(b*x + a) + 2*b*d*x*s
inh(b*x + a)^2 - 2*b*c - (d*cosh(b*x + a)^2 + 2*d*cosh(b*x + a)*sinh(b*x +
 a) + d*sinh(b*x + a)^2 + d)*log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x
 + a))))/(b^2*cosh(b*x + a)^2 + 2*b^2*cosh(b*x + a)*sinh(b*x + a) + b^2*si
nh(b*x + a)^2 + b^2)
```

Sympy [F]

$$\int (c+dx)\text{sech}^2(a+bx)\,dx = \int (c+dx)\,\text{sech}^2\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*sech(b*x+a)**2,x)
```

output
```
Integral((c + d*x)*sech(a + b*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 72 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 72, normalized size of antiderivative = 2.48

$$\int (c+dx)\operatorname{sech}^2(a+bx)\,dx = d\left(\frac{2\,xe^{(2\,bx+2\,a)}}{be^{(2\,bx+2\,a)}+b} - \frac{\log\left(\left(e^{(2\,bx+2\,a)}+1\right)e^{(-2\,a)}\right)}{b^2}\right) + \frac{2\,c}{b(e^{(-2\,bx-2\,a)}+1)}$$

input
```
integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
d*(2*x*e^(2*b*x + 2*a)/(b*e^(2*b*x + 2*a) + b) - log((e^(2*b*x + 2*a) + 1)
*e^(-2*a))/b^2) + 2*c/(b*(e^(-2*b*x - 2*a) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(29) = 58$.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.69

$$\int (c+dx)\operatorname{sech}^2(a+bx)\,dx = \frac{2\,bdxe^{(2\,bx+2\,a)} - de^{(2\,bx+2\,a)}\log\left(e^{(2\,bx+2\,a)}+1\right) - 2\,bc - d\log\left(e^{(2\,bx+2\,a)}+1\right)}{b^2e^{(2\,bx+2\,a)}+b^2}$$

input
```
integrate((d*x+c)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
(2*b*d*x*e^(2*b*x + 2*a) - d*e^(2*b*x + 2*a)*log(e^(2*b*x + 2*a) + 1) - 2*
b*c - d*log(e^(2*b*x + 2*a) + 1))/(b^2*e^(2*b*x + 2*a) + b^2)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.72

$$\int (c+dx)\operatorname{sech}^2(a+bx)\,dx=\frac{2\,d\,x}{b}-\frac{2\,(c+d\,x)}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}-\frac{d\,\ln\left(\mathrm{e}^{2\,a}\,\mathrm{e}^{2\,b\,x}+1\right)}{b^2}$$

input
```
int((c + d*x)/cosh(a + b*x)^2,x)
```

output
```
(2*d*x)/b - (2*(c + d*x))/(b*(exp(2*a + 2*b*x) + 1)) - (d*log(exp(2*a)*exp
(2*b*x) + 1))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 3.10

$$\int (c+dx)\operatorname{sech}^2(a+bx)\,dx = \frac{-e^{2bx+2a}\log\left(e^{2bx+2a}+1\right)d+2e^{2bx+2a}bc+2e^{2bx+2a}bdx-\log\left(e^{2bx+2a}+1\right)d}{b^2\left(e^{2bx+2a}+1\right)}$$

input
```
int((d*x+c)*sech(b*x+a)^2,x)
```

output
```
( - e**(2*a + 2*b*x)*log(e**(2*a + 2*b*x) + 1)*d + 2*e**(2*a + 2*b*x)*b*c
+ 2*e**(2*a + 2*b*x)*b*d*x - log(e**(2*a + 2*b*x) + 1)*d)/(b**2*(e**(2*a +
 2*b*x) + 1))
```

3.34 $\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)}{c+dx},x\right)$$

output
```
Defer(Int)(sech(b*x+a)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 25.30 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx$$

input
```
Integrate[Sech[a + b*x]^2/(c + d*x),x]
```

output
```
Integrate[Sech[a + b*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc \left(ia+ibx+\frac{\pi}{2}\right)^2}{c+dx}dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}dx$$

input
```
Int[Sech[a + b*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)^2}{dx+c}dx$$

input
```
int(sech(b*x+a)^2/(d*x+c),x)
```

output
```
int(sech(b*x+a)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}^2\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sech(b*x+a)**2/(d*x+c),x)
```

output
```
Integral(sech(a + b*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 102, normalized size of antiderivative = 6.38

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^2}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="maxima")
```

output
```
-4*d*integrate(1/2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^(2*a) + 2
*b*c*d*x*e^(2*a) + b*c^2*e^(2*a))*e^(2*b*x)), x) - 2/(b*d*x + b*c + (b*d*x
*e^(2*a) + b*c*e^(2*a))*e^(2*b*x))
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx = \int \frac{\operatorname{sech}(bx+a)^2}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx = \int \frac{1}{\cosh(a+b\,x)^2\,(c+d\,x)}\,dx$$

input
```
int(1/(cosh(a + b*x)^2*(c + d*x)),x)
```

output
```
int(1/(cosh(a + b*x)^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}(bx+a)^2}{dx+c}dx$$

input
```
int(sech(b*x+a)^2/(d*x+c),x)
```

output
```
int(sech(a + b*x)**2/(c + d*x),x)
```

3.35 $\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(sech(b*x+a)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 21.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx = \int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2} dx$$

input
```
Integrate[Sech[a + b*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Sech[a + b*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^2}dx$$

$$\Big\downarrow \text{4680}$$

$$\int \frac{\text{sech}^2(a+bx)}{(c+dx)^2}dx$$

input
```
Int[Sech[a + b*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}\left(bx+a\right)^2}{\left(dx+c\right)^2}dx$$

input
```
int(sech(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sech(b*x+a)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.68 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{sech}^2\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)**2/(d*x+c)**2,x)
```

output
```
Integral(sech(a + b*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 156, normalized size of antiderivative = 9.75

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-4*d*integrate(1/(b*d^3*x^3 + 3*b*c*d^2*x^2 + 3*b*c^2*d*x + b*c^3 + (b*d^3
*x^3*e^(2*a) + 3*b*c*d^2*x^2*e^(2*a) + 3*b*c^2*d*x*e^(2*a) + b*c^3*e^(2*a)
)*e^(2*b*x)), x) - 2/(b*d^2*x^2 + 2*b*c*d*x + b*c^2 + (b*d^2*x^2*e^(2*a) +
 2*b*c*d*x*e^(2*a) + b*c^2*e^(2*a))*e^(2*b*x))
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cosh\left(a+b\,x\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cosh(a + b*x)^2*(c + d*x)^2),x)
```

output
```
int(1/(cosh(a + b*x)^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{sech}^2(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^2}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sech(b*x+a)^2/(d*x+c)^2,x)
```

output
```
int(sech(a + b*x)**2/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.36 $\int (c+dx)^3 \mathrm{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 296

$$\int (c+dx)^3 \mathrm{sech}^3(a+bx)\,dx = -\frac{6d^2(c+dx)\arctan\left(e^{a+bx}\right)}{b^3} + \frac{(c+dx)^3\arctan\left(e^{a+bx}\right)}{b} + \frac{3id^3\,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{b^4} - \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} - \frac{3id^3\,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{b^4} + \frac{3id(c+dx)^2\,\mathrm{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} + \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{3id^2(c+dx)\,\mathrm{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} - \frac{3id^3\,\mathrm{PolyLog}\left(4,-ie^{a+bx}\right)}{b^4} + \frac{3id^3\,\mathrm{PolyLog}\left(4,ie^{a+bx}\right)}{b^4} + \frac{3d(c+dx)^2\mathrm{sech}(a+bx)}{2b^2} + \frac{(c+dx)^3\mathrm{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
-6*d^2*(d*x+c)*arctan(exp(b*x+a))/b^3+(d*x+c)^3*arctan(exp(b*x+a))/b+3*I*d
^3*polylog(2,-I*exp(b*x+a))/b^4-3/2*I*d*(d*x+c)^2*polylog(2,-I*exp(b*x+a))
/b^2-3*I*d^3*polylog(2,I*exp(b*x+a))/b^4+3/2*I*d*(d*x+c)^2*polylog(2,I*exp
(b*x+a))/b^2+3*I*d^2*(d*x+c)*polylog(3,-I*exp(b*x+a))/b^3-3*I*d^2*(d*x+c)*
polylog(3,I*exp(b*x+a))/b^3-3*I*d^3*polylog(4,-I*exp(b*x+a))/b^4+3*I*d^3*p
olylog(4,I*exp(b*x+a))/b^4+3/2*d*(d*x+c)^2*sech(b*x+a)/b^2+1/2*(d*x+c)^3*s
ech(b*x+a)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 5.07 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.54

$$\int (c+dx)^3 \operatorname{sech}^3(a+bx)\,dx = \frac{i\left(-2ib^3c^3\arctan\left(e^{a+bx}\right)+12ibcd^2\arctan\left(e^{a+bx}\right)+3b^3c^2dx\log\left(1-ie^{a+bx}\right)-6bd^3x\log\left(1-ie^{a+bx}\right)+\cdots\right.}{}$$

input
```
Integrate[(c + d*x)^3*Sech[a + b*x]^3,x]
```

output
```
(I*((-2*I)*b^3*c^3*ArcTan[E^(a + b*x)] + (12*I)*b*c*d^2*ArcTan[E^(a + b*x)
] + 3*b^3*c^2*d*x*Log[1 - I*E^(a + b*x)] - 6*b*d^3*x*Log[1 - I*E^(a + b*x)
] + 3*b^3*c*d^2*x^2*Log[1 - I*E^(a + b*x)] + b^3*d^3*x^3*Log[1 - I*E^(a +
b*x)] - 3*b^3*c^2*d*x*Log[1 + I*E^(a + b*x)] + 6*b*d^3*x*Log[1 + I*E^(a +
b*x)] - 3*b^3*c*d^2*x^2*Log[1 + I*E^(a + b*x)] - b^3*d^3*x^3*Log[1 + I*E^(
a + b*x)] - 3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, (-I)*E^(a + b*x)] +
3*d*(-2*d^2 + b^2*(c + d*x)^2)*PolyLog[2, I*E^(a + b*x)] + 6*b*c*d^2*PolyL
og[3, (-I)*E^(a + b*x)] + 6*b*d^3*x*PolyLog[3, (-I)*E^(a + b*x)] - 6*b*c*d
^2*PolyLog[3, I*E^(a + b*x)] - 6*b*d^3*x*PolyLog[3, I*E^(a + b*x)] - 6*d^3
*PolyLog[4, (-I)*E^(a + b*x)] + 6*d^3*PolyLog[4, I*E^(a + b*x)]) + b^2*(c
+ d*x)^2*Sech[a + b*x]*(3*d + b*(c + d*x)*Tanh[a + b*x]))/(2*b^4)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 306, normalized size of antiderivative = 1.03, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3042, 4674, 3042, 4668, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4674}$$

$$-\frac{3d^2\int (c+dx)\text{sech}(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\text{sech}(a+bx)dx+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3d^2\int (c+dx)\csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b^2}+\frac{1}{2}\int (c+dx)^3\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{4668}$$

$$-\frac{3d^2\left(-\frac{id\int \log(1-ie^{a+bx})dx}{b}+\frac{id\int \log(1+ie^{a+bx})dx}{b}+\frac{2(c+dx)\arctan(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3id\int (c+dx)^2\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{3id\int (c+dx)^2\log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)^3\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \text{2715}$$

$$-\frac{3d^2\left(-\frac{id\int e^{-a-bx}\log(1-ie^{a+bx})de^{a+bx}}{b^2}+\frac{id\int e^{-a-bx}\log(1+ie^{a+bx})de^{a+bx}}{b^2}+\frac{2(c+dx)\arctan(e^{a+bx})}{b}\right)}{b^2}+\frac{1}{2}\left(-\frac{3id\int(c+dx)^2\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{3id\int(c+dx)^2\log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)^3\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\big\downarrow$ 2838

$$\frac{1}{2}\left(-\frac{3id\int(c+dx)^2\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{3id\int(c+dx)^2\log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)^3\arctan\left(e^{a+bx}\right)}{b}\right)-\frac{3d^2\left(\frac{2(c+dx)\arctan(e^{a+bx})}{b}-\frac{id\text{PolyLog}(2,-ie^{a+bx})}{b^2}+\frac{id\text{PolyLog}(2,ie^{a+bx})}{b^2}\right)}{b^2}+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\big\downarrow$ 3011

$$\frac{1}{2}\left(\frac{3id\left(\frac{2d\int(c+dx)\text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{(c+dx)^2\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{3id\left(\frac{2d\int(c+dx)\text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{(c+dx)^2\text{Pol}}{}\right.}{b}\right.$$

$$\frac{3d^2\left(\frac{2(c+dx)\arctan(e^{a+bx})}{b}-\frac{id\text{PolyLog}(2,-ie^{a+bx})}{b^2}+\frac{id\text{PolyLog}(2,ie^{a+bx})}{b^2}\right)}{b^2}+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\big\downarrow$ 7163

$$\frac{1}{2}\left(\frac{3id\left(\frac{2d\left(\frac{(c+dx)\text{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{d\int\text{PolyLog}\left(3,-ie^{a+bx}\right)dx}{b}\right)}{b}-\frac{(c+dx)^2\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{3id\left(\frac{2d\left(\frac{(c+dx)\text{PolyLog}\left(3,ie^{\cdots}\right.}{b}\right.}{}\right.}{}\right.$$

$$\frac{3d^2\left(\frac{2(c+dx)\arctan(e^{a+bx})}{b}-\frac{id\text{PolyLog}(2,-ie^{a+bx})}{b^2}+\frac{id\text{PolyLog}(2,ie^{a+bx})}{b^2}\right)}{b^2}+\frac{3d(c+dx)^2\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\big\downarrow$ 2720

$$\frac{1}{2}\left(\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{d\int e^{-a-bx}\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)de^{a+bx}}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\frac{3id\left(\frac{2d\left(\frac{(c+dx)}{}\right.}{}\right.}{}\right.$$

$$\frac{3d^2\left(\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}-\frac{id\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)}{b^2}+$$

$$\frac{3d(c+dx)^2\operatorname{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

$\big\downarrow$ 7143

$$-\frac{3d^2\left(\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}-\frac{id\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)}{b^2}+$$

$$\frac{1}{2}\left(\frac{2(c+dx)^3\arctan\left(e^{a+bx}\right)}{b}+\frac{3id\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b}-\frac{d\operatorname{PolyLog}\left(4,-ie^{a+bx}\right)}{b^2}\right)}{b}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b}\right)}{b}-\right.$$

$$\frac{3d(c+dx)^2\operatorname{sech}(a+bx)}{2b^2}+\frac{(c+dx)^3\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

input
```
Int[(c + d*x)^3*Sech[a + b*x]^3,x]
```

output
```
(-3*d^2*((2*(c + d*x)*ArcTan[E^(a + b*x)])/b - (I*d*PolyLog[2, (-I)*E^(a +
 b*x)])/b^2 + (I*d*PolyLog[2, I*E^(a + b*x)])/b^2))/b^2 + ((2*(c + d*x)^3*
ArcTan[E^(a + b*x)])/b + ((3*I)*d*(-(((c + d*x)^2*PolyLog[2, (-I)*E^(a + b
*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, (-I)*E^(a + b*x)])/b - (d*PolyLog[4
, (-I)*E^(a + b*x)])/b^2))/b))/b - ((3*I)*d*(-(((c + d*x)^2*PolyLog[2, I*E
^(a + b*x)])/b) + (2*d*(((c + d*x)*PolyLog[3, I*E^(a + b*x)])/b - (d*PolyL
og[4, I*E^(a + b*x)])/b^2))/b))/b)/2 + (3*d*(c + d*x)^2*Sech[a + b*x])/(2*
b^2) + ((c + d*x)^3*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int (dx+c)^3 \operatorname{sech}(bx+a)^3 \, dx$$

input
```
int((d*x+c)^3*sech(b*x+a)^3,x)
```

output
```
int((d*x+c)^3*sech(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 4785 vs. $2(242) = 484$.

Time = 0.23 (sec) , antiderivative size = 4785, normalized size of antiderivative = 16.17

$$\int (c+dx)^3 \operatorname{sech}^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^3\text{sech}^3(a+bx)\,dx=\int (c+dx)^3\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**3*sech(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**3*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^3\text{sech}^3(a+bx)\,dx=\int (dx+c)^3\,\text{sech}\,(bx+a)^3\;dx$$

input
```
integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
b^2*d^3*integrate(x^3*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 3*b^2*
c*d^2*integrate(x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 3*b^2*c^
2*d*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - c^3*(arctan(
e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a)
 + e^(-4*b*x - 4*a) + 1))) - 6*d^3*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x +
 2*a) + b^2), x) - 6*c*d^2*arctan(e^(b*x + a))/b^3 + ((b*d^3*x^3*e^(3*a) +
 3*c^2*d*e^(3*a) + 3*(b*c*d^2 + d^3)*x^2*e^(3*a) + 3*(b*c^2*d + 2*c*d^2)*x
*e^(3*a))*e^(3*b*x) - (b*d^3*x^3*e^a - 3*c^2*d*e^a + 3*(b*c*d^2 - d^3)*x^2
*e^a + 3*(b*c^2*d - 2*c*d^2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*
e^(2*b*x + 2*a) + b^2)
```

Giac [F]

$$\int (c+dx)^3\text{sech}^3(a+bx)\,dx=\int (dx+c)^3\,\text{sech}\,(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^3*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3\text{sech}^3(a+bx)\,dx=\int \frac{(c+d\,x)^3}{\cosh\,(a+b\,x)^3}\,dx$$

input
```
int((c + d*x)^3/cosh(a + b*x)^3,x)
```

output
```
int((c + d*x)^3/cosh(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3\text{sech}^3(a+bx)\,dx=\text{too large to display}$$

input
```
int((d*x+c)^3*sech(b*x+a)^3,x)
```

output

```
(9*e**(4*a + 4*b*x)*atan(e**(a + b*x))*b**3*c**3 + 36*e**(4*a + 4*b*x)*ata
n(e**(a + b*x))*b**2*c**2*d + 24*e**(4*a + 4*b*x)*atan(e**(a + b*x))*b*c*d
**2 + 8*e**(4*a + 4*b*x)*atan(e**(a + b*x))*d**3 + 18*e**(2*a + 2*b*x)*ata
n(e**(a + b*x))*b**3*c**3 + 72*e**(2*a + 2*b*x)*atan(e**(a + b*x))*b**2*c*
*2*d + 48*e**(2*a + 2*b*x)*atan(e**(a + b*x))*b*c*d**2 + 16*e**(2*a + 2*b*
x)*atan(e**(a + b*x))*d**3 + 9*atan(e**(a + b*x))*b**3*c**3 + 36*atan(e**(
a + b*x))*b**2*c**2*d + 24*atan(e**(a + b*x))*b*c*d**2 + 8*atan(e**(a + b*
x))*d**3 + 24*e**(5*a + 4*b*x)*int((e**(b*x)*x**3)/(e**(6*a + 6*b*x) + 3*e
**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**4*d**3 + 72*e**(5*a + 4*b*
x)*int((e**(b*x)*x**2)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a
+ 2*b*x) + 1),x)*b**4*c*d**2 + 96*e**(5*a + 4*b*x)*int((e**(b*x)*x**2)/(e*
*(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**3*d**3
 + 72*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*
b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**4*c**2*d + 192*e**(5*a + 4*b*x)*int((
e**(b*x)*x)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) +
1),x)*b**3*c*d**2 + 64*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x)
 + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**2*d**3 + 9*e**(3*a +
 3*b*x)*b**3*c**3 + 36*e**(3*a + 3*b*x)*b**2*c**2*d + 24*e**(3*a + 3*b*x)*
b*c*d**2 + 8*e**(3*a + 3*b*x)*d**3 + 48*e**(3*a + 2*b*x)*int((e**(b*x)*x**
3)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*...
```

3.37 $\int (c+dx)^2 \operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 175

$$\int (c+dx)^2 \operatorname{sech}^3(a+bx)\,dx = \frac{(c+dx)^2 \arctan\left(e^{a+bx}\right)}{b} - \frac{d^2 \arctan(\sinh(a+bx))}{b^3} - \frac{id(c+dx)\operatorname{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2} + \frac{id(c+dx)\operatorname{PolyLog}\left(2,ie^{a+bx}\right)}{b^2} + \frac{id^2\operatorname{PolyLog}\left(3,-ie^{a+bx}\right)}{b^3} - \frac{id^2\operatorname{PolyLog}\left(3,ie^{a+bx}\right)}{b^3} + \frac{d(c+dx)\operatorname{sech}(a+bx)}{b^2} + \frac{(c+dx)^2\operatorname{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
(d*x+c)^2*arctan(exp(b*x+a))/b-d^2*arctan(sinh(b*x+a))/b^3-I*d*(d*x+c)*pol
ylog(2,-I*exp(b*x+a))/b^2+I*d*(d*x+c)*polylog(2,I*exp(b*x+a))/b^2+I*d^2*po
lylog(3,-I*exp(b*x+a))/b^3-I*d^2*polylog(3,I*exp(b*x+a))/b^3+d*(d*x+c)*sec
h(b*x+a)/b^2+1/2*(d*x+c)^2*sech(b*x+a)*tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 1.67 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.54

$$\int (c+dx)^2\text{sech}^3(a+bx)\,dx$$
$$= \frac{i\left(-2ib^2c^2\arctan\left(e^{a+bx}\right)+4id^2\arctan\left(e^{a+bx}\right)+2b^2cdx\log\left(1-ie^{a+bx}\right)+b^2d^2x^2\log\left(1-ie^{a+bx}\right)-2b^2\right.}{}$$

input
```
Integrate[(c + d*x)^2*Sech[a + b*x]^3,x]
```

output
```
(I*((-2*I)*b^2*c^2*ArcTan[E^(a + b*x)] + (4*I)*d^2*ArcTan[E^(a + b*x)] + 2
*b^2*c*d*x*Log[1 - I*E^(a + b*x)] + b^2*d^2*x^2*Log[1 - I*E^(a + b*x)] - 2
*b^2*c*d*x*Log[1 + I*E^(a + b*x)] - b^2*d^2*x^2*Log[1 + I*E^(a + b*x)] - 2
*b*d*(c + d*x)*PolyLog[2, (-I)*E^(a + b*x)] + 2*b*d*(c + d*x)*PolyLog[2, I
*E^(a + b*x)] + 2*d^2*PolyLog[3, (-I)*E^(a + b*x)] - 2*d^2*PolyLog[3, I*E^
(a + b*x)]) + b^2*(c + d*x)^2*Sech[a]*Sech[a + b*x]^2*Sinh[b*x] + b*(c + d
*x)*Sech[a + b*x]*(2*d + b*(c + d*x)*Tanh[a]))/(2*b^3)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2\text{sech}^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^2\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{4674}$$

$$-\frac{d^2\int \text{sech}(a+bx)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\text{sech}(a+bx)dx+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \ 3042$$

$$-\frac{d^2\int \csc\left(ia+ibx+\frac{\pi}{2}\right)dx}{b^2}+\frac{1}{2}\int (c+dx)^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \ 4257$$

$$\frac{1}{2}\int (c+dx)^2\csc\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{d^2\arctan(\sinh(a+bx))}{b^3}+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \ 4668$$

$$\frac{1}{2}\left(-\frac{2id\int (c+dx)\log\left(1-ie^{a+bx}\right)dx}{b}+\frac{2id\int (c+dx)\log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)^2\arctan\left(e^{a+bx}\right)}{b}\right)-\frac{d^2\arctan(\sinh(a+bx))}{b^3}+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \ 3011$$

$$\frac{1}{2}\left(\frac{2id\left(\frac{d\int \text{PolyLog}(2,-ie^{a+bx})dx}{b}-\frac{(c+dx)\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2id\left(\frac{d\int \text{PolyLog}(2,ie^{a+bx})dx}{b}-\frac{(c+dx)\text{PolyLog}(2,ie^{a+bx})}{b}\right)}{b}\right.$$
$$\frac{d^2\arctan(\sinh(a+bx))}{b^3}+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \ 2720$$

$$\frac{1}{2}\left(\frac{2id\left(\frac{d\int e^{-a-bx}\text{PolyLog}(2,-ie^{a+bx})de^{a+bx}}{b^2}-\frac{(c+dx)\text{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2id\left(\frac{d\int e^{-a-bx}\text{PolyLog}(2,ie^{a+bx})de^{a+bx}}{b^2}-(c+d\right.}{b}\right.$$
$$\frac{d^2\arctan(\sinh(a+bx))}{b^3}+\frac{d(c+dx)\text{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$$\Big\downarrow \ 7143$$

$$-\frac{d^2\arctan(\sinh(a+bx))}{b^3}+$$
$$\frac{1}{2}\left(\frac{2(c+dx)^2\arctan\left(e^{a+bx}\right)}{b}+\frac{2id\left(\frac{d\operatorname{PolyLog}(3,-ie^{a+bx})}{b^2}-\frac{(c+dx)\operatorname{PolyLog}(2,-ie^{a+bx})}{b}\right)}{b}-\frac{2id\left(\frac{d\operatorname{PolyLog}(3,ie^{a+bx})}{b^2}-\frac{(c}{}\right.}{b}\right.$$
$$\frac{d(c+dx)\operatorname{sech}(a+bx)}{b^2}+\frac{(c+dx)^2\tanh(a+bx)\operatorname{sech}(a+bx)}{2b}$$

input
```
Int[(c + d*x)^2*Sech[a + b*x]^3,x]
```

output
```
-((d^2*ArcTan[Sinh[a + b*x]])/b^3) + ((2*(c + d*x)^2*ArcTan[E^(a + b*x)])/
b + ((2*I)*d*(-(((c + d*x)*PolyLog[2, (-I)*E^(a + b*x)])/b) + (d*PolyLog[3
, (-I)*E^(a + b*x)])/b^2))/b - ((2*I)*d*(-(((c + d*x)*PolyLog[2, I*E^(a +
b*x)])/b) + (d*PolyLog[3, I*E^(a + b*x)])/b^2))/b)/2 + (d*(c + d*x)*Sech[a
 + b*x])/b^2 + ((c + d*x)^2*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int (dx + c)^2 \operatorname{sech}(bx + a)^3 \, dx$$

input
```
int((d*x+c)^2*sech(b*x+a)^3,x)
```

output
```
int((d*x+c)^2*sech(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 2651 vs. $2(150) = 300$.

Time = 0.16 (sec) , antiderivative size = 2651, normalized size of antiderivative = 15.15

$$\int (c + dx)^2 \text{sech}^3(a + bx)\, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(b*x
+ a)^3 + 6*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d + b*d^2)*x)*cosh(
b*x + a)*sinh(b*x + a)^2 + 2*(b^2*d^2*x^2 + b^2*c^2 + 2*b*c*d + 2*(b^2*c*d
 + b*d^2)*x)*sinh(b*x + a)^3 - 2*(b^2*d^2*x^2 + b^2*c^2 - 2*b*c*d + 2*(b^2
*c*d - b*d^2)*x)*cosh(b*x + a) - 2*((-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)^4
 + 4*(-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*d^2*x -
I*b*c*d)*sinh(b*x + a)^4 - I*b*d^2*x - I*b*c*d + 2*(-I*b*d^2*x - I*b*c*d)*
cosh(b*x + a)^2 + 2*(-I*b*d^2*x - I*b*c*d + 3*(-I*b*d^2*x - I*b*c*d)*cosh(
b*x + a)^2)*sinh(b*x + a)^2 + 4*((-I*b*d^2*x - I*b*c*d)*cosh(b*x + a)^3 +
(-I*b*d^2*x - I*b*c*d)*cosh(b*x + a))*sinh(b*x + a))*dilog(I*cosh(b*x + a)
 + I*sinh(b*x + a)) - 2*((I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^4 + 4*(I*b*d^
2*x + I*b*c*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*d^2*x + I*b*c*d)*sinh(
b*x + a)^4 + I*b*d^2*x + I*b*c*d + 2*(I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^2
 + 2*(I*b*d^2*x + I*b*c*d + 3*(I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^2)*sinh(
b*x + a)^2 + 4*((I*b*d^2*x + I*b*c*d)*cosh(b*x + a)^3 + (I*b*d^2*x + I*b*c
*d)*cosh(b*x + a))*sinh(b*x + a))*dilog(-I*cosh(b*x + a) - I*sinh(b*x + a)
) + ((I*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*cosh(b*x + a)^4 - 4*(-I*b
^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)*d^2)*cosh(b*x + a)*sinh(b*x + a)^3 + (I
*b^2*c^2 - 2*I*a*b*c*d + I*(a^2 - 2)*d^2)*sinh(b*x + a)^4 + I*b^2*c^2 - 2*
I*a*b*c*d + I*(a^2 - 2)*d^2 - 2*(-I*b^2*c^2 + 2*I*a*b*c*d - I*(a^2 - 2)...
```

Sympy [F]

$$\int (c+dx)^2\text{sech}^3(a+bx)\,dx=\int (c+dx)^2\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)**2*sech(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**2*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)^2\text{sech}^3(a+bx)\,dx=\int (dx+c)^2\,\text{sech}\,(bx+a)^3\ dx$$

input
```
integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
b^2*d^2*integrate(x^2*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) + 2*b^2*
c*d*integrate(x*e^(b*x + a)/(b^2*e^(2*b*x + 2*a) + b^2), x) - c^2*(arctan(
e^(-b*x - a))/b - (e^(-b*x - a) - e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a)
 + e^(-4*b*x - 4*a) + 1))) + ((b*d^2*x^2*e^(3*a) + 2*c*d*e^(3*a) + 2*(b*c*
d + d^2)*x*e^(3*a))*e^(3*b*x) - (b*d^2*x^2*e^a - 2*c*d*e^a + 2*(b*c*d - d^
2)*x*e^a)*e^(b*x))/(b^2*e^(4*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) - 2
*d^2*arctan(e^(b*x + a))/b^3
```

Giac [F]

$$\int (c+dx)^2\text{sech}^3(a+bx)\,dx=\int (dx+c)^2\,\text{sech}\,(bx+a)^3\ dx$$

input
```
integrate((d*x+c)^2*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2\operatorname{sech}^3(a+bx)\,dx=\int \frac{(c+d\,x)^2}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((c + d*x)^2/cosh(a + b*x)^3,x)
```

output
```
int((c + d*x)^2/cosh(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^2\operatorname{sech}^3(a+bx)\,dx = \frac{48e^{4bx+5a}\left(\int \frac{e^{bx}x}{e^{6bx+6a}+3e^{4bx+4a}+3e^{2bx+2a}+1}dx\right)b^3cd+96e^{2bx+3a}\left(\int \frac{e^{bx}x}{e^{6bx+6a}+3e^{4bx+4a}+3e^{2bx+2a}+1}dx\right)b^3cd+48e^{a}\left(\right.}{}$$

input
```
int((d*x+c)^2*sech(b*x+a)^3,x)
```

output

```
(9*e**(4*a + 4*b*x)*atan(e**(a + b*x))*b**2*c**2 + 24*e**(4*a + 4*b*x)*ata
n(e**(a + b*x))*b*c*d + 8*e**(4*a + 4*b*x)*atan(e**(a + b*x))*d**2 + 18*e*
*(2*a + 2*b*x)*atan(e**(a + b*x))*b**2*c**2 + 48*e**(2*a + 2*b*x)*atan(e**
(a + b*x))*b*c*d + 16*e**(2*a + 2*b*x)*atan(e**(a + b*x))*d**2 + 9*atan(e*
*(a + b*x))*b**2*c**2 + 24*atan(e**(a + b*x))*b*c*d + 8*atan(e**(a + b*x))
*d**2 + 24*e**(5*a + 4*b*x)*int((e**(b*x)*x**2)/(e**(6*a + 6*b*x) + 3*e**(
4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**3*d**2 + 48*e**(5*a + 4*b*x)*
int((e**(b*x)*x)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*
x) + 1),x)*b**3*c*d + 64*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*
x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**2*d**2 + 9*e**(3*a
 + 3*b*x)*b**2*c**2 + 24*e**(3*a + 3*b*x)*b*c*d + 8*e**(3*a + 3*b*x)*d**2
+ 48*e**(3*a + 2*b*x)*int((e**(b*x)*x**2)/(e**(6*a + 6*b*x) + 3*e**(4*a +
4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**3*d**2 + 96*e**(3*a + 2*b*x)*int((e
**(b*x)*x)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1
),x)*b**3*c*d + 128*e**(3*a + 2*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x) +
3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**2*d**2 - 9*e**(a + b*x)
*b**2*c**2 - 48*e**(a + b*x)*b**2*c*d*x - 24*e**(a + b*x)*b**2*d**2*x**2 +
 24*e**(a + b*x)*b*c*d - 16*e**(a + b*x)*b*d**2*x + 8*e**(a + b*x)*d**2 +
24*e**a*int((e**(b*x)*x**2)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**
(2*a + 2*b*x) + 1),x)*b**3*d**2 + 48*e**a*int((e**(b*x)*x)/(e**(6*a + 6...
```

3.38 $\int (c+dx)\text{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 102

$$\int (c+dx)\text{sech}^3(a+bx)\,dx = \frac{(c+dx)\arctan\left(e^{a+bx}\right)}{b} - \frac{id\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{2b^2} + \frac{id\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{2b^2} + \frac{d\text{sech}(a+bx)}{2b^2} + \frac{(c+dx)\text{sech}(a+bx)\tanh(a+bx)}{2b}$$

output
```
(d*x+c)*arctan(exp(b*x+a))/b-1/2*I*d*polylog(2,-I*exp(b*x+a))/b^2+1/2*I*d*
polylog(2,I*exp(b*x+a))/b^2+1/2*d*sech(b*x+a)/b^2+1/2*(d*x+c)*sech(b*x+a)*
tanh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.53

$$\int (c+dx)\text{sech}^3(a+bx)\,dx = \frac{c\arctan(\sinh(a+bx))}{2b} + \frac{id\left(bx\left(\log\left(1-ie^{a+bx}\right)-\log\left(1+ie^{a+bx}\right)\right)-\text{PolyLog}\left(2,-ie^{a+bx}\right)+\text{PolyLog}\left(2,ie^{a+bx}\right)\right)}{2b^2} + \frac{d\text{sech}(a)\text{sech}(a+bx)(\cosh(a)+bx\sinh(a))}{2b^2} + \frac{dx\text{sech}(a)\text{sech}^2(a+bx)\sinh(bx)}{2b} + \frac{c\text{sech}(a+bx)\tanh(a+bx)}{2b}$$

input
```
Integrate[(c + d*x)*Sech[a + b*x]^3,x]
```

output
```
(c*ArcTan[Sinh[a + b*x]])/(2*b) + ((I/2)*d*(b*x*(Log[1 - I*E^(a + b*x)] -
Log[1 + I*E^(a + b*x)]) - PolyLog[2, (-I)*E^(a + b*x)] + PolyLog[2, I*E^(a
 + b*x)]))/b^2 + (d*Sech[a]*Sech[a + b*x]*(Cosh[a] + b*x*Sinh[a]))/(2*b^2)
 + (d*x*Sech[a]*Sech[a + b*x]^2*Sinh[b*x])/(2*b) + (c*Sech[a + b*x]*Tanh[a
 + b*x])/(2*b)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\csc\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{4673}$$

$$\frac{1}{2}\int (c+dx)\text{sech}(a+bx)dx+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 3042

$$\frac{1}{2}\int (c+dx)\csc\left(ia+ibx+\frac{\pi}{2}\right)dx+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 4668

$$\frac{1}{2}\left(-\frac{id\int \log\left(1-ie^{a+bx}\right)dx}{b}+\frac{id\int \log\left(1+ie^{a+bx}\right)dx}{b}+\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 2715

$$\frac{1}{2}\left(-\frac{id\int e^{-a-bx}\log\left(1-ie^{a+bx}\right)de^{a+bx}}{b^2}+\frac{id\int e^{-a-bx}\log\left(1+ie^{a+bx}\right)de^{a+bx}}{b^2}+\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}\right)+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

$\Big\downarrow$ 2838

$$\frac{1}{2}\left(\frac{2(c+dx)\arctan\left(e^{a+bx}\right)}{b}-\frac{id\,\text{PolyLog}\left(2,-ie^{a+bx}\right)}{b^2}+\frac{id\,\text{PolyLog}\left(2,ie^{a+bx}\right)}{b^2}\right)+\frac{d\text{sech}(a+bx)}{2b^2}+\frac{(c+dx)\tanh(a+bx)\text{sech}(a+bx)}{2b}$$

input
```
Int[(c + d*x)*Sech[a + b*x]^3,x]
```

output
```
((2*(c + d*x)*ArcTan[E^(a + b*x)])/b - (I*d*PolyLog[2, (-I)*E^(a + b*x)])/
b^2 + (I*d*PolyLog[2, I*E^(a + b*x)])/b^2)/2 + (d*Sech[a + b*x])/(2*b^2) +
 ((c + d*x)*Sech[a + b*x]*Tanh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [B] (verified)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(87) = 174$.

Time = 0.56 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.12

method	result
risch	$\frac{e^{bx+a}\left(bdx\, e^{2bx+2a}+bc\, e^{2bx+2a}-dxb+e^{2bx+2a}d-cb+d\right)}{b^2\left(1+e^{2bx+2a}\right)^2}+\frac{c\arctan\left(e^{bx+a}\right)}{b}-\frac{id\ln\left(1+ie^{bx+a}\right)x}{2b}-\frac{id\ln\left(1+ie^{bx+a}\right)a}{2b^2}+\frac{id\ln}{}$

input
```
int((d*x+c)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
exp(b*x+a)*(b*d*x*exp(2*b*x+2*a)+b*c*exp(2*b*x+2*a)-d*x*b+exp(2*b*x+2*a)*d
-c*b+d)/b^2/(1+exp(2*b*x+2*a))^2+1/b*c*arctan(exp(b*x+a))-1/2*I/b*d*ln(1+I
*exp(b*x+a))*x-1/2*I/b^2*d*ln(1+I*exp(b*x+a))*a+1/2*I/b*d*ln(1-I*exp(b*x+a
))*x+1/2*I/b^2*d*ln(1-I*exp(b*x+a))*a-1/2*I/b^2*d*dilog(1+I*exp(b*x+a))+1/
2*I/b^2*d*dilog(1-I*exp(b*x+a))-1/b^2*d*a*arctan(exp(b*x+a))
```

Fricas [B] (verification not implemented)

Both result and optimal contain complex but leaf count of result is larger than twice the leaf count of optimal. 1267 vs. $2(81) = 162$.

Time = 0.13 (sec) , antiderivative size = 1267, normalized size of antiderivative = 12.42

$$\int (c + dx)\text{sech}^3(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/2*(2*(b*d*x + b*c + d)*cosh(b*x + a)^3 + 6*(b*d*x + b*c + d)*cosh(b*x +
a)*sinh(b*x + a)^2 + 2*(b*d*x + b*c + d)*sinh(b*x + a)^3 - 2*(b*d*x + b*c
- d)*cosh(b*x + a) + (I*d*cosh(b*x + a)^4 + 4*I*d*cosh(b*x + a)*sinh(b*x +
 a)^3 + I*d*sinh(b*x + a)^4 + 2*I*d*cosh(b*x + a)^2 - 2*(-3*I*d*cosh(b*x +
 a)^2 - I*d)*sinh(b*x + a)^2 - 4*(-I*d*cosh(b*x + a)^3 - I*d*cosh(b*x + a)
)*sinh(b*x + a) + I*d)*dilog(I*cosh(b*x + a) + I*sinh(b*x + a)) + (-I*d*co
sh(b*x + a)^4 - 4*I*d*cosh(b*x + a)*sinh(b*x + a)^3 - I*d*sinh(b*x + a)^4
- 2*I*d*cosh(b*x + a)^2 - 2*(3*I*d*cosh(b*x + a)^2 + I*d)*sinh(b*x + a)^2
- 4*(I*d*cosh(b*x + a)^3 + I*d*cosh(b*x + a))*sinh(b*x + a) - I*d)*dilog(-
I*cosh(b*x + a) - I*sinh(b*x + a)) + ((I*b*c - I*a*d)*cosh(b*x + a)^4 - 4*
(-I*b*c + I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (I*b*c - I*a*d)*sinh(b*x
+ a)^4 - 2*(-I*b*c + I*a*d)*cosh(b*x + a)^2 - 2*(3*(-I*b*c + I*a*d)*cosh(b
*x + a)^2 - I*b*c + I*a*d)*sinh(b*x + a)^2 + I*b*c - I*a*d - 4*((-I*b*c +
I*a*d)*cosh(b*x + a)^3 + (-I*b*c + I*a*d)*cosh(b*x + a))*sinh(b*x + a))*lo
g(cosh(b*x + a) + sinh(b*x + a) + I) + ((-I*b*c + I*a*d)*cosh(b*x + a)^4 -
 4*(I*b*c - I*a*d)*cosh(b*x + a)*sinh(b*x + a)^3 + (-I*b*c + I*a*d)*sinh(b
*x + a)^4 - 2*(I*b*c - I*a*d)*cosh(b*x + a)^2 - 2*(3*(I*b*c - I*a*d)*cosh(
b*x + a)^2 + I*b*c - I*a*d)*sinh(b*x + a)^2 - I*b*c + I*a*d - 4*((I*b*c -
I*a*d)*cosh(b*x + a)^3 + (I*b*c - I*a*d)*cosh(b*x + a))*sinh(b*x + a))*log
(cosh(b*x + a) + sinh(b*x + a) - I) + ((-I*b*d*x - I*a*d)*cosh(b*x + a)...
```

Sympy [F]

$$\int (c+dx)\text{sech}^3(a+bx)\,dx=\int (c+dx)\,\text{sech}^3\,(a+bx)\,dx$$

input
```
integrate((d*x+c)*sech(b*x+a)**3,x)
```

output
```
Integral((c + d*x)*sech(a + b*x)**3, x)
```

Maxima [F]

$$\int (c+dx)\text{sech}^3(a+bx)\,dx=\int (dx+c)\,\text{sech}\,(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
d*(((b*x*e^(3*a) + e^(3*a))*e^(3*b*x) - (b*x*e^a - e^a)*e^(b*x))/(b^2*e^(4
*b*x + 4*a) + 2*b^2*e^(2*b*x + 2*a) + b^2) + 8*integrate(1/8*x*e^(b*x + a)
/(e^(2*b*x + 2*a) + 1), x)) - c*(arctan(e^(-b*x - a))/b - (e^(-b*x - a) -
e^(-3*b*x - 3*a))/(b*(2*e^(-2*b*x - 2*a) + e^(-4*b*x - 4*a) + 1)))
```

Giac [F]

$$\int (c+dx)\text{sech}^3(a+bx)\,dx=\int (dx+c)\,\text{sech}\,(bx+a)^3\,dx$$

input
```
integrate((d*x+c)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*sech(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\text{sech}^3(a+bx)\,dx=\int \frac{c+d\,x}{\cosh\left(a+b\,x\right)^3}\,dx$$

input
```
int((c + d*x)/cosh(a + b*x)^3,x)
```

output
```
int((c + d*x)/cosh(a + b*x)^3, x)
```

Reduce [F]

$$\int (c+dx)\text{sech}^3(a+bx)\,dx$$
$$= \frac{3e^{4bx+4a}atan\left(e^{bx+a}\right)bc+4e^{4bx+4a}atan\left(e^{bx+a}\right)d+6e^{2bx+2a}atan\left(e^{bx+a}\right)bc+8e^{2bx+2a}atan\left(e^{bx+a}\right)d+3ata}{}$$

input
```
int((d*x+c)*sech(b*x+a)^3,x)
```

output
```
(3*e**(4*a + 4*b*x)*atan(e**(a + b*x))*b*c + 4*e**(4*a + 4*b*x)*atan(e**(a
 + b*x))*d + 6*e**(2*a + 2*b*x)*atan(e**(a + b*x))*b*c + 8*e**(2*a + 2*b*x
)*atan(e**(a + b*x))*d + 3*atan(e**(a + b*x))*b*c + 4*atan(e**(a + b*x))*d
 + 8*e**(5*a + 4*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b
*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**2*d + 3*e**(3*a + 3*b*x)*b*c + 4*e**(3
*a + 3*b*x)*d + 16*e**(3*a + 2*b*x)*int((e**(b*x)*x)/(e**(6*a + 6*b*x) + 3
*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**2*d - 3*e**(a + b*x)*b*c
 - 8*e**(a + b*x)*b*d*x + 4*e**(a + b*x)*d + 8*e**a*int((e**(b*x)*x)/(e**(
6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b**2*d)/(3*
b**2*(e**(4*a + 4*b*x) + 2*e**(2*a + 2*b*x) + 1))
```

3.39 $\int \frac{\operatorname{sech}^3(a+bx)}{c+dx} \, dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx} \, dx = \operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{c+dx}, x\right)$$

output
```
Defer(Int)(sech(b*x+a)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 141.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx} \, dx = \int \frac{\operatorname{sech}^3(a+bx)}{c+dx} \, dx$$

input
```
Integrate[Sech[a + b*x]^3/(c + d*x),x]
```

output
```
Integrate[Sech[a + b*x]^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)^3}{c+dx}dx$$
$$\Big\downarrow \text{4680}$$
$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}dx$$

input
```
Int[Sech[a + b*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(bx+a)^3}{dx+c}dx$$

input
```
int(sech(b*x+a)^3/(d*x+c),x)
```

output
```
int(sech(b*x+a)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.50 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}^3\left(a+bx\right)}{c+dx}\,dx$$

input
```
integrate(sech(b*x+a)**3/(d*x+c),x)
```

output
```
Integral(sech(a + b*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 321, normalized size of antiderivative = 20.06

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="maxima")
```

output
```
((b*d*x*e^(3*a) + (b*c - d)*e^(3*a))*e^(3*b*x) - (b*d*x*e^a + (b*c + d)*e^
a)*e^(b*x))/(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2 + (b^2*d^2*x^2*e^(4*a) +
2*b^2*c*d*x*e^(4*a) + b^2*c^2*e^(4*a))*e^(4*b*x) + 2*(b^2*d^2*x^2*e^(2*a)
+ 2*b^2*c*d*x*e^(2*a) + b^2*c^2*e^(2*a))*e^(2*b*x)) + 8*integrate(1/8*(b^2
*d^2*x^2*e^a + 2*b^2*c*d*x*e^a + (b^2*c^2 - 2*d^2)*e^a)*e^(b*x)/(b^2*d^3*x
^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 + (b^2*d^3*x^3*e^(2*a) + 3*
b^2*c*d^2*x^2*e^(2*a) + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)
), x)
```

Giac [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^3}{dx+c}\,dx$$

input
```
integrate(sech(b*x+a)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{1}{{\cosh\left(a+b\,x\right)}^3\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/(cosh(a + b*x)^3*(c + d*x)),x)
```

output
```
int(1/(cosh(a + b*x)^3*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(a+bx)}{c+dx}\,dx=\int \frac{\operatorname{sech}(bx+a)^3}{dx+c}dx$$

input
```
int(sech(b*x+a)^3/(d*x+c),x)
```

output
```
int(sech(a + b*x)**3/(c + d*x),x)
```

3.40 $\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx=\operatorname{Int}\left(\frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2},x\right)$$

output
```
Defer(Int)(sech(b*x+a)^3/(d*x+c)^2,x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx=\$\text{Aborted}$$

input
```
Integrate[Sech[a + b*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2} \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^2}dx$$

$$\Big\downarrow \text{ 4680}$$

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}dx$$

input

```
Int[Sech[a + b*x]^3/(c + d*x)^2,x]
```

output

```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}\left(bx+a\right)^{3}}{\left(dx+c\right)^{2}}dx$$

input

```
int(sech(b*x+a)^3/(d*x+c)^2,x)
```

output

```
int(sech(b*x+a)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.69 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{sech}^3\left(a+bx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)**3/(d*x+c)**2,x)
```

output
```
Integral(sech(a + b*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 405, normalized size of antiderivative = 25.31

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
((b*d*x*e^(3*a) + (b*c - 2*d)*e^(3*a))*e^(3*b*x) - (b*d*x*e^a + (b*c + 2*d
)*e^a)*e^(b*x))/(b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3 +
 (b^2*d^3*x^3*e^(4*a) + 3*b^2*c*d^2*x^2*e^(4*a) + 3*b^2*c^2*d*x*e^(4*a) +
b^2*c^3*e^(4*a))*e^(4*b*x) + 2*(b^2*d^3*x^3*e^(2*a) + 3*b^2*c*d^2*x^2*e^(2
*a) + 3*b^2*c^2*d*x*e^(2*a) + b^2*c^3*e^(2*a))*e^(2*b*x)) + 8*integrate(1/
8*(b^2*d^2*x^2*e^a + 2*b^2*c*d*x*e^a + (b^2*c^2 - 6*d^2)*e^a)*e^(b*x)/(b^2
*d^4*x^4 + 4*b^2*c*d^3*x^3 + 6*b^2*c^2*d^2*x^2 + 4*b^2*c^3*d*x + b^2*c^4 +
 (b^2*d^4*x^4*e^(2*a) + 4*b^2*c*d^3*x^3*e^(2*a) + 6*b^2*c^2*d^2*x^2*e^(2*a
) + 4*b^2*c^3*d*x*e^(2*a) + b^2*c^4*e^(2*a))*e^(2*b*x)), x)
```

Giac [N/A]

Not integrable

Time = 26.68 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}\left(bx+a\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(sech(b*x+a)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)^3/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{1}{\cosh\left(a+b\,x\right)^3\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(cosh(a + b*x)^3*(c + d*x)^2),x)
```

output
```
int(1/(cosh(a + b*x)^3*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{sech}^3(a+bx)}{(c+dx)^2}\,dx = \int \frac{\operatorname{sech}(bx+a)^3}{d^2x^2+2cdx+c^2}dx$$

input
```
int(sech(b*x+a)^3/(d*x+c)^2,x)
```

output
```
int(sech(a + b*x)**3/(c**2 + 2*c*d*x + d**2*x**2),x)
```

3.41 $\int (c+dx)^{5/2}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 171

$$\begin{aligned}\int (c+dx)^{5/2}\cosh(a+bx)\,dx &= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{2b^2} \\ &+ \frac{15d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} - \frac{15d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{7/2}} \\ &+ \frac{15d^2\sqrt{c+dx}\sinh(a+bx)}{4b^3} + \frac{(c+dx)^{5/2}\sinh(a+bx)}{b}\end{aligned}$$

output
```
-5/2*d*(d*x+c)^(3/2)*cosh(b*x+a)/b^2+15/16*d^(5/2)*exp(-a+b*c/d)*Pi^(1/2)*
erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/16*d^(5/2)*exp(a-b*c/d)*Pi^(
1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)+15/4*d^2*(d*x+c)^(1/2)*si
nh(b*x+a)/b^3+(d*x+c)^(5/2)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.63

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx = -\frac{d^3e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{b(c+dx)}{d}\right)\right)}{2b^4\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cosh[a + b*x],x]
```

output
```
-1/2*(d^3*E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[7/2, -(
(b*(c + d*x))/d)] + E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (b*(c +
 d*x))/d]))/(b^4*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.85 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{5/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}-\frac{5id\int -i(c+dx)^{3/2}\sinh(a+bx)dx}{2b}$$

$$\Big\downarrow \; 26$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}-\frac{5d\int (c+dx)^{3/2}\sinh(a+bx)dx}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}-\frac{5d\int -i(c+dx)^{3/2}\sin(ia+ibx)dx}{2b}$$

$$\Big\downarrow \; 26$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\int (c+dx)^{3/2}\sin(ia+ibx)dx}{2b}$$

$$\Big\downarrow \; 3777$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\cosh(a+bx)dx}{2b}\right)}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)}{2b}$$

$$\Big\downarrow \; 3777$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{id\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \; 26$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \; 3042$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \ 26$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\int\frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \ 3789$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{1}{2}i\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \ 2611$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \ 2633$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right)}{2b}$$

$$\downarrow \text{2634}$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}\right)}{2b}$$

input
```
Int[(c + d*x)^(5/2)*Cosh[a + b*x],x]
```

output
```
((c + d*x)^(5/2)*Sinh[a + b*x])/b + (((5*I)/2)*d*((I*(c + d*x)^(3/2)*Cosh[
a + b*x])/b - (((3*I)/2)*d*(((I/2)*d*(((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*
Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b
*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))
/b + (Sqrt[c + d*x]*Sinh[a + b*x])/b))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \cosh (bx+a)\, dx$$

input
```
int((d*x+c)^(5/2)*cosh(b*x+a),x)
```

output
```
int((d*x+c)^(5/2)*cosh(b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. 2(131) = 262.

Time = 0.10 (sec) , antiderivative size = 523, normalized size of antiderivative = 3.06

$$\int (c+dx)^{5/2} \cosh(a+bx)\, dx = \frac{15\sqrt{\pi}\left(d^3\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - d^3\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^3\cosh\left(-\frac{bc-ad}{d}\right) - d\right.}{}$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/16*(15*sqrt(pi)*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x +
 a)*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d^3*sinh(-(b*c - a*
d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 15*sqrt(pi)
*(d^3*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-(b*c -
a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d))*sinh(b*x +
 a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(4*b^3*d^2*x^2 + 4*b^3*c
^2 + 10*b^2*c*d + 15*b*d^2 - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*
b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x + a)^2 - 2*(4*b^3*d^2*x^2 +
4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 + 2*(4*b^3*c*d - 5*b^2*d^2)*x)*cosh(b*x
+ a)*sinh(b*x + a) - (4*b^3*d^2*x^2 + 4*b^3*c^2 - 10*b^2*c*d + 15*b*d^2 +
2*(4*b^3*c*d - 5*b^2*d^2)*x)*sinh(b*x + a)^2 + 2*(4*b^3*c*d + 5*b^2*d^2)*x
)*sqrt(d*x + c))/(b^4*cosh(b*x + a) + b^4*sinh(b*x + a))
```

Sympy [F]

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx=\int (c+dx)^{\frac{5}{2}}\cosh(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*cosh(b*x+a),x)
```

output
```
Integral((c + d*x)**(5/2)*cosh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(131) = 262.

Time = 0.05 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.80

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx=\frac{32\,(dx+c)^{\frac{7}{2}}\cosh(bx+a)-\frac{\left(\frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^4\sqrt{-\frac{b}{d}}}-\frac{105\,\sqrt{\pi}d^4\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^4\sqrt{\frac{b}{d}}}+\frac{2\left(8\,(dx+c)^{\frac{7}{2}}b^3d\cdots\right.}{\cdots}\right.}{\cdots}}{\cdots}$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/112*(32*(d*x + c)^(7/2)*cosh(b*x + a) - (105*sqrt(pi)*d^4*erf(sqrt(d*x +
 c)*sqrt(-b/d))*e^(a - b*c/d)/(b^4*sqrt(-b/d)) - 105*sqrt(pi)*d^4*erf(sqrt
(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^4*sqrt(b/d)) + 2*(8*(d*x + c)^(7/2)
*b^3*d*e^(b*c/d) + 28*(d*x + c)^(5/2)*b^2*d^2*e^(b*c/d) + 70*(d*x + c)^(3/
2)*b*d^3*e^(b*c/d) + 105*sqrt(d*x + c)*d^4*e^(b*c/d))*e^(-a - (d*x + c)*b/
d)/b^4 + 2*(8*(d*x + c)^(7/2)*b^3*d*e^a - 28*(d*x + c)^(5/2)*b^2*d^2*e^a +
 70*(d*x + c)^(3/2)*b*d^3*e^a - 105*sqrt(d*x + c)*d^4*e^a)*e^((d*x + c)*b/
d - b*c/d)/b^4)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.36

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx =$$

$$-\frac{\frac{15\,\sqrt{\pi}d^4\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^3}-\frac{15\,\sqrt{\pi}d^4\,\mathrm{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^3}-\frac{2\left(4\,(dx+c)^{\frac{5}{2}}\,b^2d-10\,(dx+c)^{\frac{3}{2}}\,bd^2+15\,\sqrt{dx+c}d^3\right)e^{\left(\frac{(dx+\cdots}{}\right.}}{b^3}\cdots}{16\,d}$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a),x, algorithm="giac")
```

output
```
-1/16*(15*sqrt(pi)*d^4*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(
sqrt(b*d)*b^3) - 15*sqrt(pi)*d^4*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c
 - a*d)/d)/(sqrt(-b*d)*b^3) - 2*(4*(d*x + c)^(5/2)*b^2*d - 10*(d*x + c)^(3
/2)*b*d^2 + 15*sqrt(d*x + c)*d^3)*e^(((d*x + c)*b - b*c + a*d)/d)/b^3 + 2*
(4*(d*x + c)^(5/2)*b^2*d + 10*(d*x + c)^(3/2)*b*d^2 + 15*sqrt(d*x + c)*d^3
)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^3)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx = \int \cosh(a+b\,x)\,\left(c+d\,x\right)^{5/2}\,dx$$

input
```
int(cosh(a + b*x)*(c + d*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cosh(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cosh(bx+a)\,x^2dx\right)d^2 + 2\left(\int \sqrt{dx+c}\,\cosh(bx+a)\,xdx\right)cd + \left(\int \sqrt{dx+c}\,\cosh(bx+a)\,dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cosh(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*cosh(a
+ b*x)*x,x)*c*d + int(sqrt(c + d*x)*cosh(a + b*x),x)*c**2
```

3.42 $\int (c+dx)^{3/2}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 146

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx = -\frac{3d\sqrt{c+dx}\cosh(a+bx)}{2b^2} + \frac{3d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{3d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8b^{5/2}} + \frac{(c+dx)^{3/2}\sinh(a+bx)}{b}$$

output
```
-3/2*d*(d*x+c)^(1/2)*cosh(b*x+a)/b^2+3/8*d^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*er
f(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/8*d^(3/2)*exp(a-b*c/d)*Pi^(1/2)
*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+(d*x+c)^(3/2)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.73

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx = \frac{de^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(-\frac{e^{2a}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} - \frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b^2}$$

input
```
Integrate[(c + d*x)^(3/2)*Cosh[a + b*x],x]
```

output
```
(d*E^(-a - (b*c)/d)*Sqrt[c + d*x]*(-((E^(2*a)*Gamma[5/2, -((b*(c + d*x))/d
)])/Sqrt[-((b*(c + d*x))/d)]) - (E^((2*b*c)/d)*Gamma[5/2, (b*(c + d*x))/d]
)/Sqrt[(b*(c + d*x))/d]))/(2*b^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3777, 26, 3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3id\int -i\sqrt{c+dx}\sinh(a+bx)dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\sinh(a+bx)dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int -i\sqrt{c+dx}\sin(ia+ibx)dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\int \sqrt{c+dx}\sin(ia+ibx)dx}{2b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}$$

$$\Big\downarrow \text{3788}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \text{2611}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+}{}$$

$$\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}$$

input
```
Int[(c + d*x)^(3/2)*Cosh[a + b*x],x]
```

output
```
(((3*I)/2)*d*((I*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a + (b*c)
/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (
E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]
*Sqrt[d])))/b))/b + ((c + d*x)^(3/2)*Sinh[a + b*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \cosh (bx+a)\, dx$$

input
```
int((d*x+c)^(3/2)*cosh(b*x+a),x)
```

output
```
int((d*x+c)^(3/2)*cosh(b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(110) = 220.

Time = 0.11 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.65

$$\int (c+dx)^{3/2}\cosh(a+bx)\, dx = \frac{3\sqrt{\pi}\left(d^2\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right) - d^2\cosh(bx+a)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d^2\cosh\left(-\frac{bc-ad}{d}\right) - d^2\right.\right.}{}$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(pi)*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a
)*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh(-(b*c - a*d)
/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 3*sqrt(pi)*(d
^2*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-(b*c - a*d
)/d) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a)
)*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) - 2*(2*b^2*d*x + 2*b^2*c - (2*b
^2*d*x + 2*b^2*c - 3*b*d)*cosh(b*x + a)^2 - 2*(2*b^2*d*x + 2*b^2*c - 3*b*d
)*cosh(b*x + a)*sinh(b*x + a) - (2*b^2*d*x + 2*b^2*c - 3*b*d)*sinh(b*x + a
)^2 + 3*b*d)*sqrt(d*x + c))/(b^3*cosh(b*x + a) + b^3*sinh(b*x + a))
```

Sympy [F]

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\cosh{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)*cosh(b*x+a),x)
```

output
```
Integral((c + d*x)**(3/2)*cosh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(110) = 220$.

Time = 0.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.84

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx=\frac{16\,(dx+c)^{\frac{5}{2}}\cosh{(bx+a)}+\frac{\left(\frac{15\,\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}+\frac{15\,\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}-\frac{2\left(4\,(dx+c)^{\frac{5}{2}}b^2de^{\left(\cdots\right.}}{\cdots}\right.}{\cdots}}{4\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/40*(16*(d*x + c)^(5/2)*cosh(b*x + a) + (15*sqrt(pi)*d^3*erf(sqrt(d*x + c
)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) + 15*sqrt(pi)*d^3*erf(sqrt(d*
x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) - 2*(4*(d*x + c)^(5/2)*b^
2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 15*sqrt(d*x + c)*d^3*
e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 - 2*(4*(d*x + c)^(5/2)*b^2*d*e^a - 1
0*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x + c)*b/d -
 b*c/d)/b^3)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.38

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx = -\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b^2}+\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b^2}-\frac{2\left(2(dx+c)^{\frac{3}{2}}bd-3\sqrt{dx+c}d^2\right)e^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b^2}+\frac{2\left(2\right.}{}}{8d}$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a),x, algorithm="giac")
```

output
```
-1/8*(3*sqrt(pi)*d^3*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sq
rt(b*d)*b^2) + 3*sqrt(pi)*d^3*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c -
a*d)/d)/(sqrt(-b*d)*b^2) - 2*(2*(d*x + c)^(3/2)*b*d - 3*sqrt(d*x + c)*d^2)
*e^(((d*x + c)*b - b*c + a*d)/d)/b^2 + 2*(2*(d*x + c)^(3/2)*b*d + 3*sqrt(d
*x + c)*d^2)*e^(-((d*x + c)*b - b*c + a*d)/d)/b^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx = \int \cosh(a+b\,x)\,\left(c+d\,x\right)^{3/2}\,dx$$

input
```
int(cosh(a + b*x)*(c + d*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\cosh(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cosh(bx+a)\,x dx\right)d + \left(\int \sqrt{dx+c}\,\cosh(bx+a)\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*cosh(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)*x,x)*d + int(sqrt(c + d*x)*cosh(a + b*x),x
)*c
```

3.43 $\int \sqrt{c+dx}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 123

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx = \frac{\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} - \frac{\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4b^{3/2}} + \frac{\sqrt{c+dx}\sinh(a+bx)}{b}$$

output
```
1/4*d^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3
/2)-1/4*d^(1/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/
b^(3/2)+(d*x+c)^(1/2)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.85

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx = \frac{e^{-a-\frac{bc}{d}}\sqrt{c+dx}\left(\frac{e^{2a}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)}{\sqrt{-\frac{b(c+dx)}{d}}} - \frac{e^{\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{b(c+dx)}{d}\right)}{\sqrt{\frac{b(c+dx)}{d}}}\right)}{2b}$$

input
```
Integrate[Sqrt[c + d*x]*Cosh[a + b*x],x]
```

output
```
(E^(-a - (b*c)/d)*Sqrt[c + d*x]*((E^(2*a)*Gamma[3/2, -((b*(c + d*x))/d)])/
Sqrt[-((b*(c + d*x))/d)] - (E^((2*b*c)/d)*Gamma[3/2, (b*(c + d*x))/d])/Sqr
t[(b*(c + d*x))/d]))/(2*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{id\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}$$

$$\Big\downarrow \text{3789}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{1}{2}i\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}$$

input
```
Int[Sqrt[c + d*x]*Cosh[a + b*x],x]
```

output
```
((I/2)*d*(((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/
Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sinh[a +
 b*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int \sqrt{dx + c}\, \cosh (bx + a)\, dx$$

input
```
int((d*x+c)^(1/2)*cosh(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*cosh(b*x+a),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(91) = 182$.

Time = 0.11 (sec) , antiderivative size = 302, normalized size of antiderivative = 2.46

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx$$
$$= \frac{\sqrt{\pi}\left(d\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right) - d\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right) + \left(d\cosh\left(-\frac{bc-ad}{d}\right) - d\sinh\left(-\frac{bc-ad}{d}\right)\right.}{}$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh
(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(
b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + sqrt(pi)*(d*cosh(b*x +
a)*cosh(-(b*c - a*d)/d) + d*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + (d*cosh(-
(b*c - a*d)/d) + d*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqr
t(d*x + c)*sqrt(-b/d)) + 2*(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x
 + a) + b*sinh(b*x + a)^2 - b)*sqrt(d*x + c))/(b^2*cosh(b*x + a) + b^2*sin
h(b*x + a))
```

Sympy [F]

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx = \int \sqrt{c+dx}\cosh\left(a+bx\right)\,dx$$

input
```
integrate((d*x+c)**(1/2)*cosh(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*cosh(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(91) = 182$.

Time = 0.05 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.87

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx = \frac{8\,(dx+c)^{\frac{3}{2}}\cosh\left(bx+a\right) - \frac{\left(\frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{2\left(2\,(dx+c)^{\frac{3}{2}}bde^{\left(\frac{bc}{d}\right)}+3\sqrt{dx+c}d^2\right.}{b^2}}{d}}{12\,d}$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/12*(8*(d*x + c)^(3/2)*cosh(b*x + a) - (3*sqrt(pi)*d^2*erf(sqrt(d*x + c)*
sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) - 3*sqrt(pi)*d^2*erf(sqrt(d*x +
 c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) + 2*(2*(d*x + c)^(3/2)*b*d*e
^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^2 + 2*(
2*(d*x + c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*
c/d)/b^2)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.37

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx = -\frac{\frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc-ad}{d}\right)}}{\sqrt{bd}b} - \frac{\sqrt{\pi}d^2\operatorname{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-ad}{d}\right)}}{\sqrt{-bd}b} - \frac{2\sqrt{dx+c}de^{\left(\frac{(dx+c)b-bc+ad}{d}\right)}}{b} + \frac{2\sqrt{dx+c}de^{\left(-\frac{(dx+c)b-b}{d}\right.}}{b}}{4\,d}$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a),x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*d^2*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^((b*c - a*d)/d)/(sqrt
(b*d)*b) - sqrt(pi)*d^2*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - a*d)/d
)/(sqrt(-b*d)*b) - 2*sqrt(d*x + c)*d*e^(((d*x + c)*b - b*c + a*d)/d)/b + 2
*sqrt(d*x + c)*d*e^(-((d*x + c)*b - b*c + a*d)/d)/b)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx = \int \cosh(a+b\,x)\,\sqrt{c+d\,x}\,dx$$

input
```
int(cosh(a + b*x)*(c + d*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cosh(a+bx)\,dx = \int \sqrt{dx+c}\,\cosh(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*cosh(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x),x)
```

3.44 $\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 104

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} + \frac{e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

output
```
1/2*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1
/2)+1/2*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/
d^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.01

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{e^{-a-\frac{bc}{d}}\left(e^{2a}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right) - e^{\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{b(c+dx)}{d}\right)\right)}{2b\sqrt{c+dx}}$$

input
```
Integrate[Cosh[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(E^(-a - (b*c)/d)*(E^(2*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d
*x))/d)] - E^((2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (b*(c + d*x))/d]
))/(2*b*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int \frac{ie^{-a-bx}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int \frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int \frac{e^{a+bx}}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{2611}$$

$$\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}$$

input
```
Int[Cosh[a + b*x]/Sqrt[c + d*x],x]
```

output
```
(E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b
]*Sqrt[d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/(2*Sqrt[b]*Sqrt[d])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int \frac{\cosh(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(cosh(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(cosh(b*x+a)/(d*x+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.18

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)-\sinh\left(-\frac{bc-ad}{d}\right)\right)\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{bc-ad}{d}\right)+\sinh\left(-\frac{bc-ad}{d}\right)\right)}{2b}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c - a*d)/d))*erf(
sqrt(d*x + c)*sqrt(b/d)) - sqrt(pi)*sqrt(-b/d)*(cosh(-(b*c - a*d)/d) + sin
h(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(cosh(a + b*x)/sqrt(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. $2(74) = 148$.

Time = 0.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.73

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \frac{4\sqrt{dx+c}\cosh(bx+a) + \frac{\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} + \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - \frac{2\sqrt{dx+c}de^{\left(a+\frac{(dx+c)b}{d}-\frac{bc}{d}\right)}}{b} - 2\sqrt{dx+c}d \cdots}{d}}{2d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*sqrt(d*x + c)*cosh(b*x + a) + (sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(-b
/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) + sqrt(pi)*d*erf(sqrt(d*x + c)*sqrt(b/d)
)*e^(-a + b*c/d)/(b*sqrt(b/d)) - 2*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d -
b*c/d)/b - 2*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b)*b/d)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\left(\frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{bc}{d}\right)}}{\sqrt{bd}} + \frac{\sqrt{\pi}d\,\mathrm{erf}\left(-\frac{\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{bc-2\,ad}{d}\right)}}{\sqrt{-bd}}\right)e^{(-a)}}{2\,d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(pi)*d*erf(-sqrt(b*d)*sqrt(d*x + c)/d)*e^(b*c/d)/sqrt(b*d) + sqr
t(pi)*d*erf(-sqrt(-b*d)*sqrt(d*x + c)/d)*e^(-(b*c - 2*a*d)/d)/sqrt(-b*d))*
e^(-a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh(a+b\,x)}{\sqrt{c+d\,x}}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(cosh(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(cosh(a + b*x)/sqrt(c + d*x),x)
```

3.45 $\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cosh(a+bx)}{d\sqrt{c+dx}} - \frac{\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

output
```
-2*cosh(b*x+a)/d/(d*x+c)^(1/2)-b^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*
(d*x+c)^(1/2)/d^(1/2))/d^(3/2)+b^(1/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*
(d*x+c)^(1/2)/d^(1/2))/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.99

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-a}\left(-e^{-bx}\left(1+e^{2(a+bx)}\right)+e^{\frac{bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},b\left(\frac{c}{d}+x\right)\right)+e^{2a-\frac{bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{b(c}{}\right.\right.}{d\sqrt{c+dx}}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(-((1 + E^(2*(a + b*x)))/E^(b*x)) + E^((b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamm
a[1/2, b*(c/d + x)] + E^(2*a - (b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2
, -((b*(c + d*x))/d)])/(d*E^a*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.51 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.562$, Rules used = {3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{2b\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}$$

input
```
Int[Cosh[a + b*x]/(c + d*x)^(3/2),x]
```

output
```
(-2*Cosh[a + b*x])/(d*Sqrt[c + d*x]) - ((2*I)*b*(((-1/2*I)*E^(-a + (b*c)/d
)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2
)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]
*Sqrt[d])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int \frac{\cosh(bx+a)}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int(cosh(b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(cosh(b*x+a)/(d*x+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(91) = 182$.

Time = 0.12 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.84

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{\sqrt{\pi}\left((dx+c)\cosh\left(bx+a\right)\cosh\left(-\frac{bc-ad}{d}\right)-(dx+c)\cosh\left(bx+a\right)\sinh\left(-\frac{bc-ad}{d}\right)+\left((dx+c)\cosh\left(-\frac{bc-}{d}\right.\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(
b*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) - (d*x + c
)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/
d)) + sqrt(pi)*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + (d*x + c)*c
osh(b*x + a)*sinh(-(b*c - a*d)/d) + ((d*x + c)*cosh(-(b*c - a*d)/d) + (d*x
 + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sq
rt(-b/d)) + sqrt(d*x + c)*(cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a)
 + sinh(b*x + a)^2 + 1))/((d^2*x + c*d)*cosh(b*x + a) + (d^2*x + c*d)*sinh
(b*x + a))
```

Sympy [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh\left(a+bx\right)}{\left(c+dx\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.87

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{\frac{\left(\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}-\frac{\sqrt{\pi}\operatorname{erf}\left(\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-a+\frac{bc}{d}\right)}}{\sqrt{\frac{b}{d}}}\right)b}{d}-\frac{2\cosh(bx+a)}{\sqrt{dx+c}}}{d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
((sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/sqrt(-b/d) - sqrt(p
i)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/sqrt(b/d))*b/d - 2*cosh(b*x
 + a)/sqrt(d*x + c))/d
```

Giac [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh(bx+a)}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh(a+b\,x)}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh\left(bx+a\right)}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(cosh(b*x+a)/(d*x+c)^(3/2),x)
```

output
```
int(cosh(a + b*x)/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.46 $\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 149

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}} + \frac{2b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4b\sinh(a+bx)}{3d^2\sqrt{c+dx}}$$

output
```
-2/3*cosh(b*x+a)/d/(d*x+c)^(3/2)+2/3*b^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^
(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+2/3*b^(3/2)*exp(a-b*c/d)*Pi^(1/2)*erf
i(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)-4/3*b*sinh(b*x+a)/d^2/(d*x+c)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.01

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{e^{-a}\left(-e^{-bx}\left(d\left(1+e^{2(a+bx)}\right)+2b\left(-1+e^{2(a+bx)}\right)(c+dx)+2de^{b\left(\frac{c}{d}+x\right)}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma(\right.\right.}{3d^2(c+dx)^{3/2}}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(-((d*(1 + E^(2*(a + b*x))) + 2*b*(-1 + E^(2*(a + b*x)))*(c + d*x) + 2*d*E
^(b*(c/d + x))*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, b*(c/d + x)])/E^(b*x)) -
 2*d*E^(2*a - (b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x
))/d)])/(3*d^2*E^a*(c + d*x)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3778, 26, 3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{2ib\int -\frac{i\sinh(a+bx)}{(c+dx)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{2b\int \frac{\sinh(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}+\frac{2b\int -\frac{i\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\int \frac{\sin(ia+ibx)}{(c+dx)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\cosh(a+bx)}{3d(c+dx)^{3/2}}-\frac{2ib\left(\frac{2ib\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}-\frac{2i\sinh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}$$

input
```
Int[Cosh[a + b*x]/(c + d*x)^(5/2),x]
```

output
```
(-2*Cosh[a + b*x])/(3*d*(c + d*x)^(3/2)) - (((2*I)/3)*b*(((2*I)*b*((E^(-a
+ (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[
d]) + (E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*
Sqrt[b]*Sqrt[d])))/d - ((2*I)*Sinh[a + b*x])/(d*Sqrt[c + d*x])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int \frac{\cosh(bx+a)}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int(cosh(b*x+a)/(d*x+c)^(5/2),x)
```

output
```
int(cosh(b*x+a)/(d*x+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. 2(111) = 222.

Time = 0.13 (sec) , antiderivative size = 534, normalized size of antiderivative = 3.58

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{2\sqrt{\pi}\left((bd^2x^2+2\,bcdx+bc^2)\cosh(bx+a)\cosh\left(-\frac{bc-ad}{d}\right)-(bd^2x^2+2\,bcdx+bc^2)\,\mathrm{co}\right.}{}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-(b*c
- a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-(b*c - a*d
)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a*d)/d) - (b*d^2*x^2
+ 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sq
rt(d*x + c)*sqrt(b/d)) - 2*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(
b*x + a)*cosh(-(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)*sinh(-(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-(b*c - a
*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-(b*c - a*d)/d))*sinh(b*x +
a))*sqrt(-b/d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (2*b*d*x - (2*b*d*x + 2*b*c
 + d)*cosh(b*x + a)^2 - 2*(2*b*d*x + 2*b*c + d)*cosh(b*x + a)*sinh(b*x + a
) - (2*b*d*x + 2*b*c + d)*sinh(b*x + a)^2 + 2*b*c - d)*sqrt(d*x + c))/((d^
4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a) + (d^4*x^2 + 2*c*d^3*x + c^2*d^
2)*sinh(b*x + a))
```

Sympy [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh\left(a+bx\right)}{\left(c+dx\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c)**(5/2),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{\frac{\left(\frac{\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}-\frac{\sqrt{-\frac{(dx+c)b}{d}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}\right)b}{d}-\frac{2\cosh(bx+a)}{(dx+c)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
1/3*((sqrt((d*x + c)*b/d)*e^(-a + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d
*x + c) - sqrt(-(d*x + c)*b/d)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/s
qrt(d*x + c))*b/d - 2*cosh(b*x + a)/(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh(bx+a)}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh(a+b\,x)}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh(bx+a)}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(cosh(b*x+a)/(d*x+c)^(5/2),x)
```

output
```
int(cosh(a + b*x)/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c + d
*x)*d**2*x**2),x)
```

3.47 $\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 174

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}} - \frac{8b^2\cosh(a+bx)}{15d^3\sqrt{c+dx}} - \frac{4b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{4b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{4b\sinh(a+bx)}{15d^2(c+dx)^{3/2}}$$

output
```
-2/5*cosh(b*x+a)/d/(d*x+c)^(5/2)-8/15*b^2*cosh(b*x+a)/d^3/(d*x+c)^(1/2)-4/
15*b^(5/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/
2)+4/15*b^(5/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/
d^(7/2)-4/15*b*sinh(b*x+a)/d^2/(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-a}\left(2e^{2a}\left(-3d^2e^{bx} - 2be^{-\frac{bc}{d}}(c+dx)\left(e^{b\left(\frac{c}{d}+x\right)}(d+2b(c+dx)) + 2d\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\right.\right.\right.}{}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(2*E^(2*a)*(-3*d^2*E^(b*x) - (2*b*(c + d*x)*(E^(b*(c/d + x))*(d + 2*b*(c +
 d*x)) + 2*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b*(c + d*x))/d)]))/E
^((b*c)/d)) + (-6*d^2 + 4*b*d*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*d^2*E^(b*(
c/d + x))*((b*(c + d*x))/d)^(5/2)*Gamma[1/2, (b*(c + d*x))/d])/E^(b*x))/(3
0*d^3*E^a*(c + d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.938$, Rules used = {3042, 3778, 26, 3042, 26, 3778, 3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}+\frac{2ib\int -\frac{i\sinh(a+bx)}{(c+dx)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{2b\int \frac{\sinh(a+bx)}{(c+dx)^{5/2}}dx}{5d}-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}+\frac{2b\int -\frac{i\sin(ia+ibx)}{(c+dx)^{5/2}}dx}{5d}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\int\frac{\sin(ia+ibx)}{(c+dx)^{5/2}}dx}{5d}$$

$\big\downarrow$ 3778

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\int\frac{\cosh(a+bx)}{(c+dx)^{3/2}}dx}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 3042

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 3778

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\int-\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 26

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(\frac{2b\int\frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 3042

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int-\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 26

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int\frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 3789

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 2611

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 2633

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

$\big\downarrow$ 2634

$$-\frac{2\cosh(a+bx)}{5d(c+dx)^{5/2}}-\frac{2ib\left(\frac{2ib\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{3d}-\frac{2i\sinh(a+bx)}{3d(c+dx)^{3/2}}\right)}{5d}$$

input
```
Int[Cosh[a + b*x]/(c + d*x)^(7/2),x]
```

output
```
(-2*Cosh[a + b*x])/(5*d*(c + d*x)^(5/2)) - (((2*I)/5)*b*((((2*I)/3)*b*((-2
*Cosh[a + b*x])/(d*Sqrt[c + d*x]) - ((2*I)*b*(((-1/2*I)*E^(-a + (b*c)/d)*S
qrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E
^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sq
rt[d])))/d))/d - (((2*I)/3)*Sinh[a + b*x])/(d*(c + d*x)^(3/2))))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [F]

$$\int \frac{\cosh (bx + a)}{(dx + c)^{\frac{7}{2}}}dx$$

input
```
int(cosh(b*x+a)/(d*x+c)^(7/2),x)
```

output
```
int(cosh(b*x+a)/(d*x+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 853 vs. 2(132) = 264.

Time = 0.11 (sec) , antiderivative size = 853, normalized size of antiderivative = 4.90

$$\int \frac{\cosh(a + bx)}{(c + dx)^{7/2}}\, dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/15*(4*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^
3)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3
 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) - (b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))
*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 4*sqrt(pi)*((b^2*
d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-(
b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*
cosh(b*x + a)*sinh(-(b*c - a*d)/d) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b
^2*c^2*d*x + b^2*c^3)*cosh(-(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^
2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/
d)*erf(sqrt(d*x + c)*sqrt(-b/d)) + (4*b^2*d^2*x^2 + 4*b^2*c^2 - 2*b*c*d +
(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*co
sh(b*x + a)^2 + 2*(4*b^2*d^2*x^2 + 4*b^2*c^2 + 2*b*c*d + 3*d^2 + 2*(4*b^2*
c*d + b*d^2)*x)*cosh(b*x + a)*sinh(b*x + a) + (4*b^2*d^2*x^2 + 4*b^2*c^2 +
 2*b*c*d + 3*d^2 + 2*(4*b^2*c*d + b*d^2)*x)*sinh(b*x + a)^2 + 3*d^2 + 2*(4
*b^2*c*d - b*d^2)*x)*sqrt(d*x + c))/((d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x
+ c^3*d^3)*cosh(b*x + a) + (d^6*x^3 + 3*c*d^5*x^2 + 3*c^2*d^4*x + c^3*d^3)
*sinh(b*x + a))
```

Sympy [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c)**(7/2),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.66

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{\frac{\left(\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}-\frac{\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(a-\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}\right)b}{d}-\frac{2\cosh(bx+a)}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
1/5*((((d*x + c)*b/d)^(3/2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x
 + c)^(3/2) - (-(d*x + c)*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*
b/d)/(d*x + c)^(3/2))*b/d - 2*cosh(b*x + a)/(d*x + c)^(5/2))/d
```

Giac [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh(bx+a)}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh(a+b\,x)}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x)^(7/2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh(bx+a)}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input

```
int(cosh(b*x+a)/(d*x+c)^(7/2),x)
```

output

```
int(cosh(a + b*x)/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sqrt(
c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.48 $\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = \frac{5d(c+dx)^{3/2}}{16b^2} + \frac{(c+dx)^{7/2}}{7d} - \frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2} + \frac{15d^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} - \frac{15d^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{256b^{7/2}} + \frac{(c+dx)^{5/2}\cosh(a+bx)\sinh(a+bx)}{2b} + \frac{15d^2\sqrt{c+dx}\sinh(2a+2bx)}{64b^3}$$

output
```
5/16*d*(d*x+c)^(3/2)/b^2+1/7*(d*x+c)^(7/2)/d-5/8*d*(d*x+c)^(3/2)*cosh(b*x+
a)^2/b^2+15/512*d^(5/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-15/512*d^(5/2)*exp(2*a-2*b*c/d)*2^(1/2
)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)+1/2*(d*x+c)
^(5/2)*cosh(b*x+a)*sinh(b*x+a)/b+15/64*d^2*(d*x+c)^(1/2)*sinh(2*b*x+2*a)/b
^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.57

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx=\frac{\frac{64(c+dx)^4}{d}-\frac{7\sqrt{2}d^3e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},-\frac{2b(c+dx)}{d}\right)}{b^4}-\frac{7\sqrt{2}d^3e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{7}{2},\frac{2b(c+dx)}{d}\right)}{b^4}}{448\sqrt{c+dx}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cosh[a + b*x]^2,x]
```

output
```
((64*(c + d*x)^4)/d - (7*Sqrt[2]*d^3*E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*
x))/d)]*Gamma[7/2, (-2*b*(c + d*x))/d])/b^4 - (7*Sqrt[2]*d^3*E^(-2*a + (2*
b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[7/2, (2*b*(c + d*x))/d])/b^4)/(448*Sqr
t[c + d*x])
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3792, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int (c+dx)^{5/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \ 3792$$

$$\frac{15d^2\int\sqrt{c+dx}\cosh^2(a+bx)dx}{16b^2}+\frac{1}{2}\int (c+dx)^{5/2}dx-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$\frac{15d^2\int\sqrt{c+dx}\cosh^2(a+bx)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{15d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx}{16b^2}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{3793}$$

$$\frac{15d^2\int\left(\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)+\frac{1}{2}\sqrt{c+dx}\right)dx}{16b^2}-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{5d(c+dx)^{3/2}\cosh^2(a+bx)}{8b^2}+\frac{15d^2\left(\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}\right)}{16b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{7/2}}{7d}$$

input

```
Int[(c + d*x)^(5/2)*Cosh[a + b*x]^2,x]
```

output

```
(c + d*x)^(7/2)/(7*d) - (5*d*(c + d*x)^(3/2)*Cosh[a + b*x]^2)/(8*b^2) + ((
c + d*x)^(5/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b) + (15*d^2*((c + d*x)^(3/
2)/(3*d) + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*S
qrt[c + d*x]])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b*c)/d)*Sqrt[P
i/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x]])/Sqrt[d]])/(16*b^(3/2)) + (Sqrt[c
 + d*x]*Sinh[2*a + 2*b*x])/(4*b)))/(16*b^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \cosh (bx+a)^2 \, dx$$

input
```
int((d*x+c)^(5/2)*cosh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(5/2)*cosh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1001 vs. $2(183) = 366$.

Time = 0.11 (sec) , antiderivative size = 1001, normalized size of antiderivative = 4.19

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/3584*(105*sqrt(2)*sqrt(pi)*(d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
 d^4*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d)
- d^4*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(
-2*(b*c - a*d)/d) - d^4*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a
))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 105*sqrt(2)*sqrt(pi)*(
d^4*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b*x + a)^2*sinh(-2*(
b*c - a*d)/d) + (d^4*cosh(-2*(b*c - a*d)/d) + d^4*sinh(-2*(b*c - a*d)/d))*
sinh(b*x + a)^2 + 2*(d^4*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^4*cosh(b
*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt
(d*x + c)*sqrt(-b/d)) - 4*(112*b^3*d^3*x^2 + 112*b^3*c^2*d + 140*b^2*c*d^2
 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c
*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)^4 - 28*(16*b^3*d^3*x^2 + 16*b^3*c^2*d -
 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*cosh(b*x + a)*si
nh(b*x + a)^3 - 7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3
 + 4*(8*b^3*c*d^2 - 5*b^2*d^3)*x)*sinh(b*x + a)^4 + 105*b*d^3 - 128*(b^4*d
^3*x^3 + 3*b^4*c*d^2*x^2 + 3*b^4*c^2*d*x + b^4*c^3)*cosh(b*x + a)^2 - 2*(6
4*b^4*d^3*x^3 + 192*b^4*c*d^2*x^2 + 192*b^4*c^2*d*x + 64*b^4*c^3 + 21*(16*
b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4*(8*b^3*c*d^2 - 5*
b^2*d^3)*x)*cosh(b*x + a)^2)*sinh(b*x + a)^2 + 28*(8*b^3*c*d^2 + 5*b^2*d^3
)*x - 4*(7*(16*b^3*d^3*x^2 + 16*b^3*c^2*d - 20*b^2*c*d^2 + 15*b*d^3 + 4...
```

Sympy [F]

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = \int (c+dx)^{\frac{5}{2}}\cosh^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**(5/2)*cosh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(5/2)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.18

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = \frac{512\,(dx+c)^{\frac{7}{2}} - \frac{105\,\sqrt{2}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}} + \frac{105\,\sqrt{2}\sqrt{\pi}d^3\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}} - \frac{28\,\cdots}{\cdots}}{\cdots}$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/3584*(512*(d*x + c)^(7/2) - 105*sqrt(2)*sqrt(pi)*d^3*erf(sqrt(2)*sqrt(d*
x + c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^3*sqrt(-b/d)) + 105*sqrt(2)*sqrt(p
i)*d^3*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^3*sqrt(b
/d)) - 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*b*c/d) + 20*(d*x + c)^(3/2)*b*d^2
*e^(2*b*c/d) + 15*sqrt(d*x + c)*d^3*e^(2*b*c/d))*e^(-2*a - 2*(d*x + c)*b/d
)/b^3 + 28*(16*(d*x + c)^(5/2)*b^2*d*e^(2*a) - 20*(d*x + c)^(3/2)*b*d^2*e^
(2*a) + 15*sqrt(d*x + c)*d^3*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^3)/d
```

Giac [F]

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = \int (dx+c)^{\frac{5}{2}}\cosh\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = \int \cosh(a+b\,x)^2\,(c+d\,x)^{5/2}\,dx$$

input
```
int(cosh(a + b*x)^2*(c + d*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^2*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cosh^2(a+bx)\,dx = \left(\int \sqrt{dx+c}\,\cosh\left(bx+a\right)^2 x^2dx\right)d^2 + 2\left(\int \sqrt{dx+c}\,\cosh\left(bx+a\right)^2 xdx\right)cd + \left(\int \sqrt{dx+c}\,\cosh\left(bx+a\right)^2 dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cosh(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)**2*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*cosh
(a + b*x)**2*x,x)*c*d + int(sqrt(c + d*x)*cosh(a + b*x)**2,x)*c**2
```

3.49 $\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 211

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx = \frac{3d\sqrt{c+dx}}{16b^2}+\frac{(c+dx)^{5/2}}{5d}-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{3d^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{3d^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{5/2}}+\frac{(c+dx)^{3/2}\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
3/16*d*(d*x+c)^(1/2)/b^2+1/5*(d*x+c)^(5/2)/d-3/8*d*(d*x+c)^(1/2)*cosh(b*x+
a)^2/b^2+3/128*d^(3/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(1
/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+3/128*d^(3/2)*exp(2*a-2*b*c/d)*2^(1/2)*
Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+1/2*(d*x+c)^(
3/2)*cosh(b*x+a)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.65

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx=\frac{\sqrt{c+dx}\left(32(c+dx)^2-\frac{5\sqrt{2}d^2e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{5}{2},-\frac{2b(c+dx)}{d}\right)}{b^2\sqrt{-\frac{b(c+dx)}{d}}}-\frac{5\sqrt{2}d^2e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{5}{2},\frac{2b(c+dx)}{d}\right)}{b^2\sqrt{\frac{b(c+dx)}{d}}}\right)}{160d}$$

input
```
Integrate[(c + d*x)^(3/2)*Cosh[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*(32*(c + d*x)^2 - (5*Sqrt[2]*d^2*E^(2*a - (2*b*c)/d)*Gamma[
5/2, (-2*b*(c + d*x))/d])/(b^2*Sqrt[-((b*(c + d*x))/d)]) - (5*Sqrt[2]*d^2*
E^(-2*a + (2*b*c)/d)*Gamma[5/2, (2*b*(c + d*x))/d])/(b^2*Sqrt[(b*(c + d*x)
)/d])))/(160*d)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}=0.333$, Rules used = {3042, 3792, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{3d^2\int\frac{\cosh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}+\frac{1}{2}\int (c+dx)^{3/2}dx-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{17}$$

$$\frac{3d^2\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{3d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx}{16b^2}-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{3793}$$

$$\frac{3d^2\int \left(\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx}{16b^2}-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{3d^2\left(\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{16b^2}-\frac{3d\sqrt{c+dx}\cosh^2(a+bx)}{8b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{(c+dx)^{5/2}}{5d}$$

input
```
Int[(c + d*x)^(3/2)*Cosh[a + b*x]^2,x]
```

output
```
(c + d*x)^(5/2)/(5*d) - (3*d*Sqrt[c + d*x]*Cosh[a + b*x]^2)/(8*b^2) + (3*d
^2*(Sqrt[c + d*x]/d + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b
]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt
[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]))
)/(16*b^2) + ((c + d*x)^(3/2)*Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \cosh (bx+a)^2 \, dx$$

input
```
int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(159) = 318$.

Time = 0.12 (sec) , antiderivative size = 755, normalized size of antiderivative = 3.58

$$\int (c + dx)^{3/2} \cosh^2(a + bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/640*(15*sqrt(2)*sqrt(pi)*(d^3*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d
^3*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) -
d^3*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2
*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 15*sqrt(2)*sqrt(pi)*(d^3
*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x + a)^2*sinh(-2*(b*c
 - a*d)/d) + (d^3*cosh(-2*(b*c - a*d)/d) + d^3*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a)^2 + 2*(d^3*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^3*cosh(b*x
+ a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*
x + c)*sqrt(-b/d)) - 4*(20*b^2*d^2*x - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^
2)*cosh(b*x + a)^4 - 20*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)*
sinh(b*x + a)^3 - 5*(4*b^2*d^2*x + 4*b^2*c*d - 3*b*d^2)*sinh(b*x + a)^4 +
20*b^2*c*d + 15*b*d^2 - 32*(b^3*d^2*x^2 + 2*b^3*c*d*x + b^3*c^2)*cosh(b*x
+ a)^2 - 2*(16*b^3*d^2*x^2 + 32*b^3*c*d*x + 16*b^3*c^2 + 15*(4*b^2*d^2*x +
 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^2)*sinh(b*x + a)^2 - 4*(5*(4*b^2*d^2*x
 + 4*b^2*c*d - 3*b*d^2)*cosh(b*x + a)^3 + 16*(b^3*d^2*x^2 + 2*b^3*c*d*x +
b^3*c^2)*cosh(b*x + a))*sinh(b*x + a))*sqrt(d*x + c))/(b^3*d*cosh(b*x + a)
^2 + 2*b^3*d*cosh(b*x + a)*sinh(b*x + a) + b^3*d*sinh(b*x + a)^2)
```

Sympy [F]

$$\int (c + dx)^{3/2} \cosh^2(a + bx) \, dx = \int (c + dx)^{\frac{3}{2}} \cosh^2(a + bx) \, dx$$

input
```
integrate((d*x+c)**(3/2)*cosh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**(3/2)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.13

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx = \frac{128\,(dx+c)^{\frac{5}{2}} + \frac{15\,\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{15\,\sqrt{2}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} - \frac{20\left(4\left(\cdots\right.\right.}{\cdots}}{640\,d\cdots}$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/640*(128*(d*x + c)^(5/2) + 15*sqrt(2)*sqrt(pi)*d^2*erf(sqrt(2)*sqrt(d*x
+ c)*sqrt(-b/d))*e^(2*a - 2*b*c/d)/(b^2*sqrt(-b/d)) + 15*sqrt(2)*sqrt(pi)*
d^2*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b^2*sqrt(b/d)
) - 20*(4*(d*x + c)^(3/2)*b*d*e^(2*b*c/d) + 3*sqrt(d*x + c)*d^2*e^(2*b*c/d
))*e^(-2*a - 2*(d*x + c)*b/d)/b^2 + 20*(4*(d*x + c)^(3/2)*b*d*e^(2*a) - 3*
sqrt(d*x + c)*d^2*e^(2*a))*e^(2*(d*x + c)*b/d - 2*b*c/d)/b^2)/d
```

Giac [F]

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\cosh\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx=\int \cosh(a+b\,x)^2\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cosh(a + b*x)^2*(c + d*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^2*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\cosh^2(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\cosh(bx+a)^2\,xdx\right)d+\left(\int \sqrt{dx+c}\,\cosh(bx+a)^2\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*cosh(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)**2*x,x)*d + int(sqrt(c + d*x)*cosh(a + b*x
)**2,x)*c
```

3.50 $\int \sqrt{c+dx}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 166

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx=\frac{(c+dx)^{3/2}}{3d}+\frac{\sqrt{d}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{d}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}$$

output

```
1/3*(d*x+c)^(3/2)/d+1/32*d^(1/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2^
(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/32*d^(1/2)*exp(2*a-2*b*c/d)
*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+1/4*
(d*x+c)^(1/2)*sinh(2*b*x+2*a)/b
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.78

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx = \frac{1}{48}\sqrt{c+dx}\left(\frac{16(c+dx)}{d}+\frac{3\sqrt{2}e^{2a-\frac{2bc}{d}}\Gamma\left(\frac{3}{2},-\frac{2b(c+dx)}{d}\right)}{b\sqrt{-\frac{b(c+dx)}{d}}}-\frac{3\sqrt{2}e^{-2a+\frac{2bc}{d}}\Gamma\left(\frac{3}{2},\frac{2b(c+dx)}{d}\right)}{b\sqrt{\frac{b(c+dx)}{d}}}\right)$$

input
```
Integrate[Sqrt[c + d*x]*Cosh[a + b*x]^2,x]
```

output
```
(Sqrt[c + d*x]*((16*(c + d*x))/d + (3*Sqrt[2]*E^(2*a - (2*b*c)/d)*Gamma[3/
2, (-2*b*(c + d*x))/d])/(b*Sqrt[-((b*(c + d*x))/d)]) - (3*Sqrt[2]*E^(-2*a
+ (2*b*c)/d)*Gamma[3/2, (2*b*(c + d*x))/d])/(b*Sqrt[(b*(c + d*x))/d])))/48
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3793}$$

$$\int \left(\frac{1}{2}\sqrt{c+dx}\cosh(2a+2bx)+\frac{1}{2}\sqrt{c+dx}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{\frac{2bc}{d}-2a}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{2}}\sqrt{d}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{c+dx}\sinh(2a+2bx)}{4b}+\frac{(c+dx)^{3/2}}{3d}$$

input
```
Int[Sqrt[c + d*x]*Cosh[a + b*x]^2,x]
```

output
```
(c + d*x)^(3/2)/(3*d) + (Sqrt[d]*E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt
[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(2*a - (2*b
*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/
2)) + (Sqrt[c + d*x]*Sinh[2*a + 2*b*x])/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \sqrt{dx+c}\,\cosh(bx+a)^2\,dx$$

input
```
int((d*x+c)^(1/2)*cosh(b*x+a)^2,x)
```

output
```
int((d*x+c)^(1/2)*cosh(b*x+a)^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 590 vs. $2(122) = 244$.

Time = 0.10 (sec) , antiderivative size = 590, normalized size of antiderivative = 3.55

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/96*(3*sqrt(2)*sqrt(pi)*(d^2*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - d^2
*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) - d^
2*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(
b*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*s
qrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(2)*sqrt(pi)*(d^2*co
sh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sinh(-2*(b*c -
a*d)/d) + (d^2*cosh(-2*(b*c - a*d)/d) + d^2*sinh(-2*(b*c - a*d)/d))*sinh(b
*x + a)^2 + 2*(d^2*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + d^2*cosh(b*x + a
)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x +
 c)*sqrt(-b/d)) + 4*(3*b*d*cosh(b*x + a)^4 + 12*b*d*cosh(b*x + a)*sinh(b*x
 + a)^3 + 3*b*d*sinh(b*x + a)^4 + 8*(b^2*d*x + b^2*c)*cosh(b*x + a)^2 + 2*
(4*b^2*d*x + 9*b*d*cosh(b*x + a)^2 + 4*b^2*c)*sinh(b*x + a)^2 - 3*b*d + 4*
(3*b*d*cosh(b*x + a)^3 + 4*(b^2*d*x + b^2*c)*cosh(b*x + a))*sinh(b*x + a))
*sqrt(d*x + c))/(b^2*d*cosh(b*x + a)^2 + 2*b^2*d*cosh(b*x + a)*sinh(b*x +
a) + b^2*d*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx = \int \sqrt{c+dx}\cosh^2{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*cosh(b*x+a)**2,x)
```

output
```
Integral(sqrt(c + d*x)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.14

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx = -\frac{\frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2a-\frac{2bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{3\sqrt{2}\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2a+\frac{2bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} - 32\,(dx+c)^{\frac{3}{2}} - \frac{12\sqrt{dx+c}de^{\left(2a+\frac{2(c\cdots}{}\right.}}{b}\cdots}{96\,d}$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/96*(3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a -
 2*b*c/d)/(b*sqrt(-b/d)) - 3*sqrt(2)*sqrt(pi)*d*erf(sqrt(2)*sqrt(d*x + c)*
sqrt(b/d))*e^(-2*a + 2*b*c/d)/(b*sqrt(b/d)) - 32*(d*x + c)^(3/2) - 12*sqrt
(d*x + c)*d*e^(2*a + 2*(d*x + c)*b/d - 2*b*c/d)/b + 12*sqrt(d*x + c)*d*e^(
-2*a - 2*(d*x + c)*b/d + 2*b*c/d)/b)/d
```

Giac [F]

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx = \int \sqrt{dx+c}\cosh\left(bx+a\right)^2\,dx$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx=\int \cosh(a+b\,x)^2\,\sqrt{c+d\,x}\,dx$$

input
```
int(cosh(a + b*x)^2*(c + d*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^2*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cosh^2(a+bx)\,dx=\int \sqrt{dx+c}\,\cosh(bx+a)^2\,dx$$

input
```
int((d*x+c)^(1/2)*cosh(b*x+a)^2,x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)**2,x)
```

3.51 $\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 138

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{c+dx}}{d} + \frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}} + \frac{e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}$$

output
```
(d*x+c)^(1/2)/d+1/8*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1/2)*erf(2^(1/2)*b^(1/2)
*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)+1/8*exp(2*a-2*b*c/d)*2^(1/2)*Pi^(1
/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.02

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{c+dx}}{d} + \frac{e^{2a-\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}} - \frac{e^{-2a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\,\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)}{4\sqrt{2}b\sqrt{c+dx}}$$

input
```
Integrate[Cosh[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
Sqrt[c + d*x]/d + (E^(2*a - (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
 (-2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c + d*x]) - (E^(-2*a + (2*b*c)/d)*
Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d])/(4*Sqrt[2]*b*Sqrt[c +
 d*x])
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx$$

$$\Big\downarrow \text{ 3793}$$

$$\int \left(\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}$$

input
```
Int[Cosh[a + b*x]^2/Sqrt[c + d*x],x]
```

output
```
Sqrt[c + d*x]/d + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sq
rt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c)/d)*Sqrt[Pi/
2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \frac{\cosh(bx+a)^2}{\sqrt{dx+c}}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)
```

output
```
int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.12

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2(bc-ad)}{d}\right)-d\sinh\left(-\frac{2(bc-ad)}{d}\right)\right)\sqrt{\frac{b}{d}}\operatorname{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{2}\sqrt{\pi}\left(d\cosh\left(-\frac{2(bc-ad)}{d}\right.\right.}{8\,bd}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*(d*cosh(-2*(b*c - a*d)/d) - d*sinh(-2*(b*c - a*d)/d)
)*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(2)*sqrt(pi)*(d*cos
h(-2*(b*c - a*d)/d) + d*sinh(-2*(b*c - a*d)/d))*sqrt(-b/d)*erf(sqrt(2)*sqr
t(d*x + c)*sqrt(-b/d)) + 8*sqrt(d*x + c)*b)/(b*d)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh^2{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**(1/2),x)
```

output
```
Integral(cosh(a + b*x)**2/sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.78

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(2\,a-\frac{2\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}} + \frac{\sqrt{2}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{2}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-2\,a+\frac{2\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}} + 8\sqrt{dx+c}}{8\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/8*(sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))*e^(2*a - 2*b*c
/d)/sqrt(-b/d) + sqrt(2)*sqrt(pi)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d))*e^(
-2*a + 2*b*c/d)/sqrt(b/d) + 8*sqrt(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = -\frac{\left(\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{bd}\sqrt{dx+c}}{d}\right)e^{\left(\frac{2bc}{d}\right)}}{\sqrt{bd}}+\frac{\sqrt{2}\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{2}\sqrt{-bd}\sqrt{dx+c}}{d}\right)e^{\left(-\frac{2(bc-2ad)}{d}\right)}}{\sqrt{-bd}}-8\sqrt{dx+c}e^{(2a)}\right)e^{(-2a)}}{8d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
-1/8*(sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(b*d)*sqrt(d*x + c)/d)*e^(2*b*c/
d)/sqrt(b*d) + sqrt(2)*sqrt(pi)*d*erf(-sqrt(2)*sqrt(-b*d)*sqrt(d*x + c)/d)
*e^(-2*(b*c - 2*a*d)/d)/sqrt(-b*d) - 8*sqrt(d*x + c)*e^(2*a))*e^(-2*a)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh(a+b\,x)^2}{\sqrt{c+d\,x}}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh(bx+a)^2}{\sqrt{dx+c}}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(1/2),x)
```

output
```
int(cosh(a + b*x)**2/sqrt(c + d*x),x)
```

3.52 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 142

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}} - \frac{\sqrt{b}e^{-2a+\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{b}e^{2a-\frac{2bc}{d}}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

output
```
-2*cosh(b*x+a)^2/d/(d*x+c)^(1/2)-1/2*b^(1/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^
(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)+1/2*b^(1/2)*exp(2
*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d
^(3/2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.07

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-2\left(a+b\left(\frac{c}{d}+x\right)\right)}\left(\sqrt{2}e^{4a+2bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+e^{\frac{2bc}{d}}\left(-\left(1+e^{2(a+bx)}\right)^2+\sqrt{2}e\right.\right.}{2d\sqrt{c+dx}}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[2]*E^(4*a + 2*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*
x))/d] + E^((2*b*c)/d)*(-(1 + E^(2*(a + b*x)))^2 + Sqrt[2]*E^((2*b*(c + d*
x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d]))/(2*d*E^(2*(a
+ b*(c/d + x)))*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.13, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3794, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3794}$$

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{4ib\int -\frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{2b\int \frac{\sinh(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int -\frac{i\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int \frac{\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}$$

$$\Big\downarrow \text{3789}$$

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int\frac{e^{2(a+bx)}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-2(a+bx)}}{\sqrt{c+dx}}dx\right)}{d}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{2\left(a-\frac{bc}{d}\right)+\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}$$

$$\Big\downarrow \text{2634}$$

$$-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x)^(3/2),x]
```

output
```
(-2*Cosh[a + b*x]^2)/(d*Sqrt[c + d*x]) - ((2*I)*b*(((-1/2*I)*E^(-2*a + (2*
b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*
Sqrt[d]) + ((I/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cosh (bx+a)^2}{(dx+c)^{\frac{3}{2}}}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(3/2),x)
```

output
```
int(cosh(b*x+a)^2/(d*x+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. 2(109) = 218.

Time = 0.11 (sec) , antiderivative size = 569, normalized size of antiderivative = 4.01

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/2*(sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) -
 (d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(b*
c - a*d)/d) - (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*x
+ c)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-
2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(
b/d)) + sqrt(2)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d)
 + (d*x + c)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((d*x + c)*cosh(-2*(
b*c - a*d)/d) + (d*x + c)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((d*
x + c)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh
(-2*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sq
rt(-b/d)) + (cosh(b*x + a)^4 + 4*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x
+ a)^4 + 2*(3*cosh(b*x + a)^2 + 1)*sinh(b*x + a)^2 + 2*cosh(b*x + a)^2 + 4
*(cosh(b*x + a)^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*sqrt(d*x + c))/((d^2
*x + c*d)*cosh(b*x + a)^2 + 2*(d^2*x + c*d)*cosh(b*x + a)*sinh(b*x + a) +
(d^2*x + c*d)*sinh(b*x + a)^2)
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh^2{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**2/(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{\frac{\sqrt{2}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{2\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{2}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{2\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{4}{\sqrt{dx+c}}}{4\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/4*(sqrt(2)*sqrt((d*x + c)*b/d)*e^(2*(b*c - a*d)/d)*gamma(-1/2, 2*(d*x +
 c)*b/d)/sqrt(d*x + c) + sqrt(2)*sqrt(-(d*x + c)*b/d)*e^(-2*(b*c - a*d)/d)
*gamma(-1/2, -2*(d*x + c)*b/d)/sqrt(d*x + c) + 4/sqrt(d*x + c))/d
```

Giac [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh(bx+a)^2}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh(bx+a)^2}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(3/2),x)
```

output
```
int(cosh(a + b*x)**2/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.53 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 174

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}} + \frac{2b^{3/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2b^{3/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{3d^2\sqrt{c+dx}}$$

output
```
-2/3*cosh(b*x+a)^2/d/(d*x+c)^(3/2)+2/3*b^(3/2)*exp(-2*a+2*b*c/d)*2^(1/2)*P
i^(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+2/3*b^(3/2)*exp
(2*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))
/d^(5/2)-8/3*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.82 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.90

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2e^{-2\left(a+\frac{bc}{d}\right)}\left(\sqrt{2}de^{4a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{2b(c+dx)}{d}\right)+\sqrt{2}de^{\frac{4bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\frac{2b(c+dx)}{d}\right)+e^{2\left(a+\frac{bc}{d}\right)}(d\cos}{3d^2(c+dx)^{3/2}}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(-2*(Sqrt[2]*d*E^(4*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-2*b*(c + d*
x))/d] + Sqrt[2]*d*E^((4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/2, (2*b*(
c + d*x))/d] + E^(2*(a + (b*c)/d))*(d*Cosh[a + b*x]^2 + 2*b*(c + d*x)*Sinh
[2*(a + b*x)])))/(3*d^2*E^(2*(a + (b*c)/d))*(c + d*x)^(3/2))
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.27, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3795, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{16b^2\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{16b^2\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{3793}$$

$$\frac{16b^2 \int \left(\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16b^2\left(\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\text{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\text{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x)^(5/2),x]
```

output
```
(-16*b^2*Sqrt[c + d*x])/(3*d^3) - (2*Cosh[a + b*x]^2)/(3*d*(c + d*x)^(3/2)
) + (16*b^2*(Sqrt[c + d*x]/d + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[
2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*b*c
)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*
Sqrt[d])))/(3*d^2) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*Sqrt[c + d*x
])
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\cosh(bx+a)^2}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

output
```
int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(134) = 268$.

Time = 0.15 (sec) , antiderivative size = 861, normalized size of antiderivative = 4.95

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/6*(4*sqrt(2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*c
osh(-2*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*si
nh(-2*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d
)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x +
a)^2 + 2*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d
)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d
))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 4*sqrt(
2)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*cosh(-2*(b*c
- a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*sinh(-2*(b*c -
 a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-2*(b*c - a*d)/d) + (b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b
*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b*d^
2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x
+ a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d)) - ((4*b*d*x + 4*b*c
 + d)*cosh(b*x + a)^4 + 4*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)*sinh(b*x + a
)^3 + (4*b*d*x + 4*b*c + d)*sinh(b*x + a)^4 - 4*b*d*x + 2*d*cosh(b*x + a)^
2 + 2*(3*(4*b*d*x + 4*b*c + d)*cosh(b*x + a)^2 + d)*sinh(b*x + a)^2 - 4*b*
c + 4*((4*b*d*x + 4*b*c + d)*cosh(b*x + a)^3 + d*cosh(b*x + a))*sinh(b*x +
 a) + d)*sqrt(d*x + c))/((d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)^2 +
 2*(d^4*x^2 + 2*c*d^3*x + c^2*d^2)*cosh(b*x + a)*sinh(b*x + a) + (d^4*x...
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh^2{(a+bx)}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**(5/2),x)
```

output
```
Integral(cosh(a + b*x)**2/(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.68

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{\frac{3\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{3\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{2}{(dx+c)^{\frac{3}{2}}}}{6\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-1/6*(3*sqrt(2)*((d*x + c)*b/d)^(3/2)*e^(2*(b*c - a*d)/d)*gamma(-3/2, 2*(d
*x + c)*b/d)/(d*x + c)^(3/2) + 3*sqrt(2)*(-(d*x + c)*b/d)^(3/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-3/2, -2*(d*x + c)*b/d)/(d*x + c)^(3/2) + 2/(d*x + c)^(3/
2))/d
```

Giac [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh(bx+a)^2}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh(bx+a)^2}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(5/2),x)
```

output
```
int(cosh(a + b*x)**2/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c
+ d*x)*d**2*x**2),x)
```

3.54 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 220

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{16b^2}{15d^3\sqrt{c+dx}} - \frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}} - \frac{32b^2\cosh^2(a+bx)}{15d^3\sqrt{c+dx}} - \frac{8b^{5/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} + \frac{8b^{5/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{15d^{7/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{15d^2(c+dx)^{3/2}}$$

output

```
16/15*b^2/d^3/(d*x+c)^(1/2)-2/5*cosh(b*x+a)^2/d/(d*x+c)^(5/2)-32/15*b^2*co
sh(b*x+a)^2/d^3/(d*x+c)^(1/2)-8/15*b^(5/2)*exp(-2*a+2*b*c/d)*2^(1/2)*Pi^(1
/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+8/15*b^(5/2)*exp(2*
a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^
(7/2)-8/15*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{-6d^2+e^{2a}\left(-3d^2e^{2bx}-4be^{-\frac{2bc}{d}}(c+dx)\left(e^{\frac{2b(c+dx)}{d}}(d+4b(c+dx))+4\sqrt{2}d\left(-\frac{b(c+d}{d}\right.\right.\right.}{}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(-6*d^2 + E^(2*a)*(-3*d^2*E^(2*b*x) - (4*b*(c + d*x)*(E^((2*b*(c + d*x))/d
)*(d + 4*b*(c + d*x)) + 4*Sqrt[2]*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2,
(-2*b*(c + d*x))/d]))/E^((2*b*c)/d)) + (-3*d^2 + 4*b*(c + d*x)*(d - 4*b*(c
 + d*x) + 4*Sqrt[2]*d*E^((2*b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[
1/2, (2*b*(c + d*x))/d]))/E^(2*(a + b*x)))/(30*d^3*(c + d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3795, 17, 3042, 3794, 27, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{ 3795}$$

$$\frac{16b^2\int \frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b^2\int \frac{1}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{ 17}$$

$$\frac{16b^2\int\frac{\cosh^2(a+bx)}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{16b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^{3/2}}dx}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3794}$$

$$\frac{16b^2\left(-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{4ib\int-\frac{i\sinh(2a+2bx)}{2\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{16b^2\left(\frac{2b\int\frac{\sinh(2a+2bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{16b^2\left(-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int-\frac{i\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{16b^2\left(-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int\frac{\sin(2ia+2ibx)}{\sqrt{c+dx}}dx}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$$\Big\downarrow \text{ 3789}$$

$$\frac{16b^2\left(-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int\frac{e^{2(a+bx)}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-2(a+bx)}}{\sqrt{c+dx}}dx\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 2611

$$\frac{16b^2\left(-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{2\left(a-\frac{bc}{d}\right)+\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 2633

$$\frac{16b^2\left(-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-2\left(a-\frac{bc}{d}\right)-\frac{2b(c+dx)}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

$\Big\downarrow$ 2634

$$\frac{16b^2\left(-\frac{2\cosh^2(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{15d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{15d^2(c+dx)^{3/2}}-\frac{2\cosh^2(a+bx)}{5d(c+dx)^{5/2}}+\frac{16b^2}{15d^3\sqrt{c+dx}}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x)^(7/2),x]
```

output
```
(16*b^2)/(15*d^3*Sqrt[c + d*x]) - (2*Cosh[a + b*x]^2)/(5*d*(c + d*x)^(5/2)
) + (16*b^2*((-2*Cosh[a + b*x]^2)/(d*Sqrt[c + d*x]) - ((2*I)*b*(((-1/2*I)*
E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d
]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(2*a - (2*b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2
]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d))/(15*d^2) - (8*b
*Cosh[a + b*x]*Sinh[a + b*x])/(15*d^2*(c + d*x)^(3/2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\cosh (bx+a)^2}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(7/2),x)
```

output
```
int(cosh(b*x+a)^2/(d*x+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1350 vs. $2(172) = 344$.

Time = 0.14 (sec) , antiderivative size = 1350, normalized size of antiderivative = 6.14

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
-1/30*(16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x
 + b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*
c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d
) + ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*
c - a*d)/d) - (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*si
nh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2
+ 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) - (b^2*d^3
*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b
*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)
) + 16*sqrt(2)*sqrt(pi)*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x +
b^2*c^3)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d
^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) +
 ((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(-2*(b*c -
 a*d)/d) + (b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*sinh(
-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^2*d^3*x^3 + 3*b^2*c*d^2*x^2 + 3
*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^2*d^3*x^
3 + 3*b^2*c*d^2*x^2 + 3*b^2*c^2*d*x + b^2*c^3)*cosh(b*x + a)*sinh(-2*(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(-b/d))
 + (16*b^2*d^2*x^2 + (16*b^2*d^2*x^2 + 16*b^2*c^2 + 4*b*c*d + 3*d^2 + 4*(8
*b^2*c*d + b*d^2)*x)*cosh(b*x + a)^4 + 4*(16*b^2*d^2*x^2 + 16*b^2*c^2 +...
```

Sympy [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh^2{(a+bx)}}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**(7/2),x)
```

output
```
Integral(cosh(a + b*x)**2/(c + d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.53

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{\frac{5\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{5\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{2(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{2(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{1}{(dx+c)^{\frac{5}{2}}}}{5\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-1/5*(5*sqrt(2)*((d*x + c)*b/d)^(5/2)*e^(2*(b*c - a*d)/d)*gamma(-5/2, 2*(d
*x + c)*b/d)/(d*x + c)^(5/2) + 5*sqrt(2)*(-(d*x + c)*b/d)^(5/2)*e^(-2*(b*c
 - a*d)/d)*gamma(-5/2, -2*(d*x + c)*b/d)/(d*x + c)^(5/2) + 1/(d*x + c)^(5/
2))/d
```

Giac [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh\left(bx+a\right)^2}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^(7/2),x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh\left(bx+a\right)^2}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(7/2),x)
```

output
```
int(cosh(a + b*x)**2/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sq
rt(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.55 $\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 251

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{16b^2}{105d^3(c+dx)^{3/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}} - \frac{32b^2\cosh^2(a+bx)}{105d^3(c+dx)^{3/2}} + \frac{32b^{7/2}e^{-2a+\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} + \frac{32b^{7/2}e^{2a-\frac{2bc}{d}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{105d^{9/2}} - \frac{8b\cosh(a+bx)\sinh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{128b^3\cosh(a+bx)\sinh(a+bx)}{105d^4\sqrt{c+dx}}$$

output

```
16/105*b^2/d^3/(d*x+c)^(3/2)-2/7*cosh(b*x+a)^2/d/(d*x+c)^(7/2)-32/105*b^2*
cosh(b*x+a)^2/d^3/(d*x+c)^(3/2)+32/105*b^(7/2)*exp(-2*a+2*b*c/d)*2^(1/2)*P
i^(1/2)*erf(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(9/2)+32/105*b^(7/2)*
exp(2*a-2*b*c/d)*2^(1/2)*Pi^(1/2)*erfi(2^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/
2))/d^(9/2)-8/35*b*cosh(b*x+a)*sinh(b*x+a)/d^2/(d*x+c)^(5/2)-128/105*b^3*c
osh(b*x+a)*sinh(b*x+a)/d^4/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 222, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \frac{2\Big(8b^2d(c+dx)^2 - 15d^3\cosh^2(a+bx) - 16b^2d(c+dx)^2\cosh^2(a+bx) + 16\sqrt{2}b^3e^{2a}$$

input
```
Integrate[Cosh[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(2*(8*b^2*d*(c + d*x)^2 - 15*d^3*Cosh[a + b*x]^2 - 16*b^2*d*(c + d*x)^2*Co
sh[a + b*x]^2 + 16*Sqrt[2]*b^3*E^(2*a - (2*b*c)/d)*(c + d*x)^3*Sqrt[-((b*(
c + d*x))/d)]*Gamma[1/2, (-2*b*(c + d*x))/d] - 16*Sqrt[2]*b^3*E^(-2*a + (2
*b*c)/d)*(c + d*x)^3*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (2*b*(c + d*x))/d] -
 6*b*d^2*(c + d*x)*Sinh[2*(a + b*x)] - 32*b^3*(c + d*x)^3*Sinh[2*(a + b*x)
]))/(105*d^4*(c + d*x)^(7/2))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3795, 17, 3042, 3795, 17, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^{9/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{16b^2\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2} - \frac{8b^2\int \frac{1}{(c+dx)^{5/2}}dx}{35d^2} - \frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}} - \frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{16b^2\int \frac{\cosh^2(a+bx)}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{(c+dx)^{5/2}}dx}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3795}$$

$$\frac{16b^2\left(\frac{16b^2\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b^2\int \frac{1}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{17}$$

$$\frac{16b^2\left(\frac{16b^2\int \frac{\cosh^2(a+bx)}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{16b^2\left(\frac{16b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^2}{\sqrt{c+dx}}dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{16b^2\left(\frac{16b^2\int \left(\frac{\cosh(2a+2bx)}{2\sqrt{c+dx}}+\frac{1}{2\sqrt{c+dx}}\right)dx}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\frac{16b^2\sqrt{c+dx}}{3d^3}\right)}{35d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16b^2\left(\frac{16b^2\left(\frac{\sqrt{\frac{\pi}{2}}e^{\frac{2bc}{d}-2a}\operatorname{erf}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{2}}e^{2a-\frac{2bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{2}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4\sqrt{b}\sqrt{d}}+\frac{\sqrt{c+dx}}{d}\right)}{3d^2}-\frac{8b\sinh(a+bx)\cosh(a+bx)}{3d^2\sqrt{c+dx}}-\frac{2\cosh^2(a+bx)}{3d(c+dx)^{3/2}}-\right.}{35d^2}$$

$$\frac{8b\sinh(a+bx)\cosh(a+bx)}{35d^2(c+dx)^{5/2}}-\frac{2\cosh^2(a+bx)}{7d(c+dx)^{7/2}}+\frac{16b^2}{105d^3(c+dx)^{3/2}}$$

input
```
Int[Cosh[a + b*x]^2/(c + d*x)^(9/2),x]
```

output
```
(16*b^2)/(105*d^3*(c + d*x)^(3/2)) - (2*Cosh[a + b*x]^2)/(7*d*(c + d*x)^(7
/2)) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(35*d^2*(c + d*x)^(5/2)) + (16*b^
2*((-16*b^2*Sqrt[c + d*x])/(3*d^3) - (2*Cosh[a + b*x]^2)/(3*d*(c + d*x)^(3
/2)) + (16*b^2*(Sqrt[c + d*x]/d + (E^(-2*a + (2*b*c)/d)*Sqrt[Pi/2]*Erf[(Sq
rt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[b]*Sqrt[d]) + (E^(2*a - (2*
b*c)/d)*Sqrt[Pi/2]*Erfi[(Sqrt[2]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(4*Sqrt[
b]*Sqrt[d])))/(3*d^2) - (8*b*Cosh[a + b*x]*Sinh[a + b*x])/(3*d^2*Sqrt[c +
d*x])))/(35*d^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\cosh\left(bx+a\right)^{2}}{\left(dx+c\right)^{\frac{9}{2}}}dx$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(9/2),x)
```

output
```
int(cosh(b*x+a)^2/(d*x+c)^(9/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1825 vs. $2(199) = 398$.

Time = 0.15 (sec) , antiderivative size = 1825, normalized size of antiderivative = 7.27

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="fricas")
```

output
```
1/210*(64*sqrt(2)*sqrt(pi)*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2
*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) - (
b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^
4)*cosh(b*x + a)^2*sinh(-2*(b*c - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^
3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) -
(b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c
^4)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^
3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2
*(b*c - a*d)/d) - (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b
^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*sinh(-2*(b*c - a*d)/d))*sinh(b*x + a))
*sqrt(b/d)*erf(sqrt(2)*sqrt(d*x + c)*sqrt(b/d)) - 64*sqrt(2)*sqrt(pi)*((b^
3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)
*cosh(b*x + a)^2*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 +
 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)^2*sinh(-2*(b*c
 - a*d)/d) + ((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c
^3*d*x + b^3*c^4)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4 + 4*b^3*c*d^3*x^3
+ 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*sinh(-2*(b*c - a*d)/d))*sin
h(b*x + a)^2 + 2*((b^3*d^4*x^4 + 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b
^3*c^3*d*x + b^3*c^4)*cosh(b*x + a)*cosh(-2*(b*c - a*d)/d) + (b^3*d^4*x^4
+ 4*b^3*c*d^3*x^3 + 6*b^3*c^2*d^2*x^2 + 4*b^3*c^3*d*x + b^3*c^4)*cosh(b...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**2/(d*x+c)**(9/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.46

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = -\frac{\frac{14\sqrt{2}\left(\frac{(dx+c)b}{d}\right)^{\frac{7}{2}} e^{\left(\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},\frac{2\,(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{14\sqrt{2}\left(-\frac{(dx+c)b}{d}\right)^{\frac{7}{2}} e^{\left(-\frac{2\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{7}{2},-\frac{2\,(dx+c)b}{d}\right)}{(dx+c)^{\frac{7}{2}}} + \frac{1}{(dx+c)^{\frac{7}{2}}}}{7\,d}$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="maxima")
```

output
```
-1/7*(14*sqrt(2)*((d*x + c)*b/d)^(7/2)*e^(2*(b*c - a*d)/d)*gamma(-7/2, 2*(
d*x + c)*b/d)/(d*x + c)^(7/2) + 14*sqrt(2)*(-(d*x + c)*b/d)^(7/2)*e^(-2*(b
*c - a*d)/d)*gamma(-7/2, -2*(d*x + c)*b/d)/(d*x + c)^(7/2) + 1/(d*x + c)^(
7/2))/d
```

Giac [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\cosh(bx+a)^2}{(dx+c)^{\frac{9}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^2/(d*x+c)^(9/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^2/(d*x + c)^(9/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\cosh(a+b\,x)^2}{(c+d\,x)^{9/2}}\,dx$$

input
```
int(cosh(a + b*x)^2/(c + d*x)^(9/2),x)
```

output
```
int(cosh(a + b*x)^2/(c + d*x)^(9/2), x)
```

Reduce [F]

$$\int \frac{\cosh^2(a+bx)}{(c+dx)^{9/2}}\,dx = \int \frac{\cosh(bx+a)^2}{\sqrt{dx+c}\,c^4+4\sqrt{dx+c}\,c^3dx+6\sqrt{dx+c}\,c^2d^2x^2+4\sqrt{dx+c}\,c\,d^3x^3+\sqrt{dx+c}\,d^4$$

input
```
int(cosh(b*x+a)^2/(d*x+c)^(9/2),x)
```

output
```
int(cosh(a + b*x)**2/(sqrt(c + d*x)*c**4 + 4*sqrt(c + d*x)*c**3*d*x + 6*sq
rt(c + d*x)*c**2*d**2*x**2 + 4*sqrt(c + d*x)*c*d**3*x**3 + sqrt(c + d*x)*d
**4*x**4),x)
```

3.56 $\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 381

$$\begin{aligned}\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx &= -\frac{5d(c+dx)^{3/2}\cosh(a+bx)}{3b^2} \\ &- \frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} + \frac{45d^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} \\ &+ \frac{5d^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} \\ &- \frac{45d^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{64b^{7/2}} - \frac{5d^{5/2}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{576b^{7/2}} \\ &+ \frac{45d^2\sqrt{c+dx}\sinh(a+bx)}{16b^3} + \frac{2(c+dx)^{5/2}\sinh(a+bx)}{3b} \\ &+ \frac{(c+dx)^{5/2}\cosh^2(a+bx)\sinh(a+bx)}{3b} + \frac{5d^2\sqrt{c+dx}\sinh(3a+3bx)}{144b^3}\end{aligned}$$

output
```
-5/3*d*(d*x+c)^(3/2)*cosh(b*x+a)/b^2-5/18*d*(d*x+c)^(3/2)*cosh(b*x+a)^3/b^
2+45/64*d^(5/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/
b^(7/2)+5/1728*d^(5/2)*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1
/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-45/64*d^(5/2)*exp(a-b*c/d)*Pi^(1/2)*erf
i(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)-5/1728*d^(5/2)*exp(3*a-3*b*c/d)*3
^(1/2)*Pi^(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(7/2)+45/16*
d^2*(d*x+c)^(1/2)*sinh(b*x+a)/b^3+2/3*(d*x+c)^(5/2)*sinh(b*x+a)/b+1/3*(d*x
+c)^(5/2)*cosh(b*x+a)^2*sinh(b*x+a)/b+5/144*d^2*(d*x+c)^(1/2)*sinh(3*b*x+3
*a)/b^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 195, normalized size of antiderivative = 0.51

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^{3/2}\left(\sqrt{3}be^{6a}(c+dx)\Gamma\left(\frac{7}{2},-\frac{3b(c+dx)}{d}\right)+243be^{4a+\frac{2bc}{d}}(c+dx)\Gamma\left(\frac{7}{2},-\frac{b(c+dx)}{d}\right)-d \cdots\right)}{648b^2\left(-\frac{b(c+dx)}{d}\right)^{5/2}}$$

input
```
Integrate[(c + d*x)^(5/2)*Cosh[a + b*x]^3,x]
```

output
```
((c + d*x)^(3/2)*(Sqrt[3]*b*E^(6*a)*(c + d*x)*Gamma[7/2, (-3*b*(c + d*x))/
d] + 243*b*E^(4*a + (2*b*c)/d)*(c + d*x)*Gamma[7/2, -((b*(c + d*x))/d)] -
d*E^((4*b*c)/d)*Sqrt[-((b^2*(c + d*x)^2)/d^2)]*(243*E^(2*a)*Gamma[7/2, (b*
(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[7/2, (3*b*(c + d*x))/d])))/(64
8*b^2*E^(3*(a + (b*c)/d))*(-((b*(c + d*x))/d))^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.36 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.43, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.056, Rules used = {3042, 3792, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{5/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{5d^2\int\sqrt{c+dx}\cosh^3(a+bx)dx}{12b^2}+\frac{2}{3}\int(c+dx)^{5/2}\cosh(a+bx)dx-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\int(c+dx)^{5/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}-\frac{5id\int-i(c+dx)^{3/2}\sinh(a+bx)dx}{2b}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}-\frac{5d\int(c+dx)^{3/2}\sinh(a+bx)dx}{2b}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}-\frac{5d\int-i(c+dx)^{3/2}\sin(ia+ibx)dx}{2b}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 26

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\int(c+dx)^{3/2}\sin(ia+ibx)dx}{2b}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int\sqrt{c+dx}\cosh(a+bx)dx}{2b}\right)}{2b}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3042

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx}{2b}\right)}{2b}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3777

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{id\int-\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}}{}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 26$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int\frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}}{}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}-\frac{d\int-\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}}{}\right)-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 26$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\int\frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)}{2b}\right)-\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3789

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{1}{2}i\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)}{2b}\right)-\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 2611

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+\cdots}{d}}\cdots}{\cdots}\right)}{2b}\cdots\right)}{2b}\cdots\right)}{2b}\cdots\right)$$

$$\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 2633$$

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c\cdots}{\cdots}}\cdots}{\cdots}\right)}{2b}\cdots\right)}{2b}\cdots\right)}{2b}\cdots\right)$$

$$\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 2634

$$\frac{5d^2\int\sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx}{12b^2}-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{e}\cdots}{2\cdots}\cdots\right.}{2b}\cdots\right.}{2b}\cdots\right.}{2b}\cdots\right.$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3793

$$\frac{5d^2 \int \left(\frac{3}{4}\sqrt{c+dx}\cosh(a+bx) + \frac{1}{4}\sqrt{c+dx}\cosh(3a+3bx)\right) dx}{12b^2} - \frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2} +$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b} + \frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b} - \frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b} + \frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}} - \frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{e}\cdots}{2\sqrt{\cdots}} \cdots\right)}{2b} \cdots\right)}{2b} \cdots\right)}{2b} \cdots\right)$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\downarrow \text{2009}$$

$$-\frac{5d(c+dx)^{3/2}\cosh^3(a+bx)}{18b^2}+\frac{5d^2\left(\frac{3\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}\right.}{12b^2}$$

$$\frac{2}{3}\left(\frac{(c+dx)^{5/2}\sinh(a+bx)}{b}+\frac{5id\left(\frac{i(c+dx)^{3/2}\cosh(a+bx)}{b}-\frac{3id\left(\frac{\sqrt{c+dx}\sinh(a+bx)}{b}+\frac{id\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{er}\cdots}{2\sqrt{\cdots}}\right.}{2b}\right.}{2b}\right.}{2b}\right.$$

$$\frac{(c+dx)^{5/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

input
```
Int[(c + d*x)^(5/2)*Cosh[a + b*x]^3,x]
```

output
```
(-5*d*(c + d*x)^(3/2)*Cosh[a + b*x]^3)/(18*b^2) + ((c + d*x)^(5/2)*Cosh[a
+ b*x]^2*Sinh[a + b*x])/(3*b) + (2*(((c + d*x)^(5/2)*Sinh[a + b*x])/b + ((
(5*I)/2)*d*((I*(c + d*x)^(3/2)*Cosh[a + b*x])/b - (((3*I)/2)*d*(((I/2)*d*(
((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/
(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c +
 d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/b + (Sqrt[c + d*x]*Sinh[a + b*x])/b))
/b))/b))/3 + (5*d^2*((3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(16*b^(3/2)) + (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi
/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (3*Sqrt[d
]*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3
/2)) - (Sqrt[d]*E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]])/(48*b^(3/2)) + (3*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b) +
 (Sqrt[c + d*x]*Sinh[3*a + 3*b*x])/(12*b)))/(12*b^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^{\frac{5}{2}} \cosh (bx+a)^3 \, dx$$

input
```
int((d*x+c)^(5/2)*cosh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(5/2)*cosh(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2092 vs. $2(291) = 582$.

Time = 0.17 (sec) , antiderivative size = 2092, normalized size of antiderivative = 5.49

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/1728*(5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d
^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d) -
d^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-3
*(b*c - a*d)/d) - d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^
2 + 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*si
nh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*s
qrt(b/d)) + 5*sqrt(3)*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d)
 + d^3*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^3*cosh(-3*(b*c - a*d)/d
) + d^3*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cos
h(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x +
 a)^2 + 3*(d^3*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^3*cosh(b*x + a)^
2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x +
 c)*sqrt(-b/d)) + 1215*sqrt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d)
- d^3*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) - d
^3*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^3*cosh(b*x + a)*cosh(-(b*c
 - a*d)/d) - d^3*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(
d^3*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d^3*cosh(b*x + a)^2*sinh(-(b*c
- a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 1215*sq
rt(pi)*(d^3*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d^3*cosh(b*x + a)^3*sin
h(-(b*c - a*d)/d) + (d^3*cosh(-(b*c - a*d)/d) + d^3*sinh(-(b*c - a*d)/d...
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**(5/2)*cosh(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 513, normalized size of antiderivative = 1.35

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx =$$

$$-\frac{\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-\frac{5\sqrt{3}\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^3\sqrt{\frac{b}{d}}}+\frac{1215\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^3\sqrt{-\frac{b}{d}}}-}{}$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/1728*(5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3
*a - 3*b*c/d)/(b^3*sqrt(-b/d)) - 5*sqrt(3)*sqrt(pi)*d^3*erf(sqrt(3)*sqrt(d
*x + c)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^3*sqrt(b/d)) + 1215*sqrt(pi)*d^3*
erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^3*sqrt(-b/d)) - 1215*sqrt(p
i)*d^3*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^3*sqrt(b/d)) + 162*(
4*(d*x + c)^(5/2)*b^2*d*e^(b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(b*c/d) + 1
5*sqrt(d*x + c)*d^3*e^(b*c/d))*e^(-a - (d*x + c)*b/d)/b^3 + 6*(12*(d*x + c
)^(5/2)*b^2*d*e^(3*b*c/d) + 10*(d*x + c)^(3/2)*b*d^2*e^(3*b*c/d) + 5*sqrt(
d*x + c)*d^3*e^(3*b*c/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^3 - 6*(12*(d*x + c)
^(5/2)*b^2*d*e^(3*a) - 10*(d*x + c)^(3/2)*b*d^2*e^(3*a) + 5*sqrt(d*x + c)*
d^3*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^3 - 162*(4*(d*x + c)^(5/2)*b^
2*d*e^a - 10*(d*x + c)^(3/2)*b*d^2*e^a + 15*sqrt(d*x + c)*d^3*e^a)*e^((d*x
 + c)*b/d - b*c/d)/b^3)/d
```

Giac [F]

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx=\int (dx+c)^{\frac{5}{2}}\cosh\left(bx+a\right)^3\,dx$$

input
```
integrate((d*x+c)^(5/2)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(5/2)*cosh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx=\int \cosh(a+b\,x)^3\,{(c+d\,x)}^{5/2}\,dx$$

input
```
int(cosh(a + b*x)^3*(c + d*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^3*(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int (c+dx)^{5/2}\cosh^3(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\cosh\left(bx+a\right)^3x^2dx\right)d^2+2\left(\int \sqrt{dx+c}\,\cosh\left(bx+a\right)^3xdx\right)cd+\left(\int \sqrt{dx+c}\,\cosh\left(bx+a\right)^3dx\right)c^2$$

input
```
int((d*x+c)^(5/2)*cosh(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)**3*x**2,x)*d**2 + 2*int(sqrt(c + d*x)*cosh
(a + b*x)**3*x,x)*c*d + int(sqrt(c + d*x)*cosh(a + b*x)**3,x)*c**2
```

3.57 $\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 326

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx = -\frac{d\sqrt{c+dx}\cosh(a+bx)}{b^2} - \frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2} + \frac{9d^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{d^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{9d^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{32b^{5/2}} + \frac{d^{3/2}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{96b^{5/2}} + \frac{2(c+dx)^{3/2}\sinh(a+bx)}{3b} + \frac{(c+dx)^{3/2}\cosh^2(a+bx)\sinh(a+bx)}{3b}$$

output

```
-d*(d*x+c)^(1/2)*cosh(b*x+a)/b^2-1/6*d*(d*x+c)^(1/2)*cosh(b*x+a)^3/b^2+9/3
2*d^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2
)+1/288*d^(3/2)*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*
x+c)^(1/2)/d^(1/2))/b^(5/2)+9/32*d^(3/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2
)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+1/288*d^(3/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi
^(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(5/2)+2/3*(d*x+c)^(3/
2)*sinh(b*x+a)/b+1/3*(d*x+c)^(3/2)*cosh(b*x+a)^2*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.64

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^{5/2}\left(\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{3b(c+dx)}{d}\right)+81e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{5}{2},-\frac{b(c+dx)}{d}\right)+e^{\frac{4bc}{d}}\cdots\right)}{216d\left(-\frac{b^2(c+dx)^2}{d^2}\right)^{3/2}}$$

input
```
Integrate[(c + d*x)^(3/2)*Cosh[a + b*x]^3,x]
```

output
```
((c + d*x)^(5/2)*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, (-3*b*(
c + d*x))/d] + 81*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[5/2, -((
b*(c + d*x))/d)] + E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(81*E^(2*a)*Gamm
a[5/2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[5/2, (3*b*(c + d*x))
/d])))/(216*d*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.94 (sec) , antiderivative size = 465, normalized size of antiderivative = 1.43, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.889$, Rules used = {3042, 3792, 3042, 3777, 26, 3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \text{3792}$$

$$\frac{d^2\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\cosh(a+bx)dx-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\int (c+dx)^{3/2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3777

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3id\int -i\sqrt{c+dx}\sinh(a+bx)dx}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 26

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int \sqrt{c+dx}\sinh(a+bx)dx}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3042

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}-\frac{3d\int -i\sqrt{c+dx}\sin(ia+ibx)dx}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 26

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\int \sqrt{c+dx}\sin(ia+ibx)dx}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\big\downarrow$ 3777

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{2b}\right)}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{3788}$$

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \text{26}$$

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{2b}\right)}{2b}\right)-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 2611$$

$$\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{2b}\right)}{2b}\right)-\frac{d^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 2633$$

$$\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}\right)-\frac{d^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}+\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$$\Big\downarrow \; 2634$$

$$\frac{d^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{12b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}\right)+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 3793

$$\frac{d^2\int\left(\frac{3\cosh(a+bx)}{4\sqrt{c+dx}}+\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{12b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}\right)+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

$\downarrow$ 2009

$$\frac{d^2\left(\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right)}{12b^2}-\frac{d\sqrt{c+dx}\cosh^3(a+bx)}{6b^2}+\frac{2}{3}\left(\frac{(c+dx)^{3/2}\sinh(a+bx)}{b}+\frac{3id\left(\frac{i\sqrt{c+dx}\cosh(a+bx)}{b}-\frac{id\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{2b}\right)}{2b}\right)+\frac{(c+dx)^{3/2}\sinh(a+bx)\cosh^2(a+bx)}{3b}$$

input

```
Int[(c + d*x)^(3/2)*Cosh[a + b*x]^3,x]
```

output

```
-1/6*(d*Sqrt[c + d*x]*Cosh[a + b*x]^3)/b^2 + (d^2*((3*E^(-a + (b*c)/d)*Sqr
t[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(-3*a
 + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*
Sqrt[b]*Sqrt[d]) + (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x]
)/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sq
rt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d])))/(12*b^2) + ((
c + d*x)^(3/2)*Cosh[a + b*x]^2*Sinh[a + b*x])/(3*b) + (2*((((3*I)/2)*d*((I
*Sqrt[c + d*x]*Cosh[a + b*x])/b - ((I/2)*d*((E^(-a + (b*c)/d)*Sqrt[Pi]*Erf
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a - (b*c)/d)*
Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d])))/b))/
b + ((c + d*x)^(3/2)*Sinh[a + b*x])/b))/3
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^{\frac{3}{2}} \cosh (bx+a)^3 \, dx$$

input
```
int((d*x+c)^(3/2)*cosh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(3/2)*cosh(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(246) = 492.

Time = 0.13 (sec) , antiderivative size = 1545, normalized size of antiderivative = 4.74

$$\int (c+dx)^{3/2} \cosh^3(a+bx) \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/288*(sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d^2*
cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) - d^2
*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*(b
*c - a*d)/d) - d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 +
 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(
-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt
(b/d)) - sqrt(3)*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d^
2*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d^2*cosh(-3*(b*c - a*d)/d) + d
^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-3*
(b*c - a*d)/d) + d^2*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2
 + 3*(d^2*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + d^2*cosh(b*x + a)^2*sin
h(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*s
qrt(-b/d)) + 81*sqrt(pi)*(d^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) - d^2*c
osh(b*x + a)^3*sinh(-(b*c - a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) - d^2*sinh
(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d^2*cosh(b*x + a)*cosh(-(b*c - a*d)
/d) - d^2*cosh(b*x + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d^2*cos
h(b*x + a)^2*cosh(-(b*c - a*d)/d) - d^2*cosh(b*x + a)^2*sinh(-(b*c - a*d)/
d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) - 81*sqrt(pi)*(d
^2*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d^2*cosh(b*x + a)^3*sinh(-(b*c -
 a*d)/d) + (d^2*cosh(-(b*c - a*d)/d) + d^2*sinh(-(b*c - a*d)/d))*sinh(b...
```

Sympy [F]

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx=\int (c+dx)^{\frac{3}{2}}\cosh^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(3/2)*cosh(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**(3/2)*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.32

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx = \frac{\frac{\sqrt{3}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}} + \frac{\sqrt{3}\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b^2\sqrt{\frac{b}{d}}} + \frac{81\sqrt{\pi}d^2\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b^2\sqrt{-\frac{b}{d}}}}{}$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/288*(sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
 3*b*c/d)/(b^2*sqrt(-b/d)) + sqrt(3)*sqrt(pi)*d^2*erf(sqrt(3)*sqrt(d*x + c
)*sqrt(b/d))*e^(-3*a + 3*b*c/d)/(b^2*sqrt(b/d)) + 81*sqrt(pi)*d^2*erf(sqrt
(d*x + c)*sqrt(-b/d))*e^(a - b*c/d)/(b^2*sqrt(-b/d)) + 81*sqrt(pi)*d^2*erf
(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c/d)/(b^2*sqrt(b/d)) - 54*(2*(d*x + c)
^(3/2)*b*d*e^(b*c/d) + 3*sqrt(d*x + c)*d^2*e^(b*c/d))*e^(-a - (d*x + c)*b/
d)/b^2 - 6*(2*(d*x + c)^(3/2)*b*d*e^(3*b*c/d) + sqrt(d*x + c)*d^2*e^(3*b*c
/d))*e^(-3*a - 3*(d*x + c)*b/d)/b^2 + 6*(2*(d*x + c)^(3/2)*b*d*e^(3*a) - s
qrt(d*x + c)*d^2*e^(3*a))*e^(3*(d*x + c)*b/d - 3*b*c/d)/b^2 + 54*(2*(d*x +
 c)^(3/2)*b*d*e^a - 3*sqrt(d*x + c)*d^2*e^a)*e^((d*x + c)*b/d - b*c/d)/b^2
)/d
```

Giac [F]

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx = \int (dx+c)^{\frac{3}{2}}\cosh(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^(3/2)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^(3/2)*cosh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx=\int \cosh(a+b\,x)^3\,(c+d\,x)^{3/2}\,dx$$

input
```
int(cosh(a + b*x)^3*(c + d*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^3*(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int (c+dx)^{3/2}\cosh^3(a+bx)\,dx=\left(\int \sqrt{dx+c}\,\cosh(bx+a)^3\,xdx\right)d+\left(\int \sqrt{dx+c}\,\cosh(bx+a)^3\,dx\right)c$$

input
```
int((d*x+c)^(3/2)*cosh(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)**3*x,x)*d + int(sqrt(c + d*x)*cosh(a + b*x
)**3,x)*c
```

3.58 $\int \sqrt{c+dx}\cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 275

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx = \frac{3\sqrt{d}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} + \frac{\sqrt{d}e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} - \frac{3\sqrt{d}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}} - \frac{\sqrt{d}e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}} + \frac{3\sqrt{c+dx}\sinh(a+bx)}{4b} + \frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}$$

output
```
3/16*d^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(
3/2)+1/144*d^(1/2)*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*
(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-3/16*d^(1/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(
1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)-1/144*d^(1/2)*exp(3*a-3*b*c/d)*3^(1/2)
*Pi^(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(3/2)+3/4*(d*x+c)^
(1/2)*sinh(b*x+a)/b+1/12*(d*x+c)^(1/2)*sinh(3*b*x+3*a)/b
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.76

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx$$
$$= \frac{e^{-3\left(a+\frac{bc}{d}\right)}\sqrt{c+dx}\left(\sqrt{3}e^{6a}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{3b(c+dx)}{d}\right)+27e^{4a+\frac{2bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\Gamma\left(\frac{3}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{4bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\right.}{72b\sqrt{-\frac{b^2(c+dx)^2}{d^2}}}$$

input
```
Integrate[Sqrt[c + d*x]*Cosh[a + b*x]^3,x]
```

output
```
(Sqrt[c + d*x]*(Sqrt[3]*E^(6*a)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, (-3*b*(c
+ d*x))/d] + 27*E^(4*a + (2*b*c)/d)*Sqrt[(b*(c + d*x))/d]*Gamma[3/2, -((b*
(c + d*x))/d)] - E^((4*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*(27*E^(2*a)*Gamma[
3/2, (b*(c + d*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[3/2, (3*b*(c + d*x))/d
])))/(72*b*E^(3*(a + (b*c)/d))*Sqrt[-((b^2*(c + d*x)^2)/d^2)])
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{c+dx}\sin\left(ia+ibx+\frac{\pi}{2}\right)^3dx$$
$$\Big\downarrow \text{3793}$$
$$\int \left(\frac{3}{4}\sqrt{c+dx}\cosh(a+bx)+\frac{1}{4}\sqrt{c+dx}\cosh(3a+3bx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}\sqrt{d}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}+\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{\frac{3bc}{d}-3a}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}-\frac{3\sqrt{\pi}\sqrt{d}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{16b^{3/2}}-\frac{\sqrt{\frac{\pi}{3}}\sqrt{d}e^{3a-\frac{3bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{48b^{3/2}}+\frac{3\sqrt{c+dx}\sinh(a+bx)}{4b}+\frac{\sqrt{c+dx}\sinh(3a+3bx)}{12b}$$

input
```
Int[Sqrt[c + d*x]*Cosh[a + b*x]^3,x]
```

output
```
(3*Sqrt[d]*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])
/(16*b^(3/2)) + (Sqrt[d]*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt
[b]*Sqrt[c + d*x])/Sqrt[d]])/(48*b^(3/2)) - (3*Sqrt[d]*E^(a - (b*c)/d)*Sqr
t[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(16*b^(3/2)) - (Sqrt[d]*E^(3*
a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(
48*b^(3/2)) + (3*Sqrt[c + d*x]*Sinh[a + b*x])/(4*b) + (Sqrt[c + d*x]*Sinh[
3*a + 3*b*x])/(12*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \sqrt{dx+c}\,\cosh\left(bx+a\right)^{3}dx$$

input
```
int((d*x+c)^(1/2)*cosh(b*x+a)^3,x)
```

output
```
int((d*x+c)^(1/2)*cosh(b*x+a)^3,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1217 vs. $2(201) = 402$.

Time = 0.12 (sec) , antiderivative size = 1217, normalized size of antiderivative = 4.43

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/144*(sqrt(3)*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) - d*cosh
(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) - d*sinh(-3
*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/
d) - d*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b
*x + a)^2*cosh(-3*(b*c - a*d)/d) - d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d
))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)
*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^3*si
nh(-3*(b*c - a*d)/d) + (d*cosh(-3*(b*c - a*d)/d) + d*sinh(-3*(b*c - a*d)/d
))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) + d*cosh(b*
x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh
(-3*(b*c - a*d)/d) + d*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x +
a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d)) + 27*sqrt(pi)*(d*cosh
(b*x + a)^3*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d)
+ (d*cosh(-(b*c - a*d)/d) - d*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*(d
*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - d*cosh(b*x + a)*sinh(-(b*c - a*d)/d)
)*sinh(b*x + a)^2 + 3*(d*cosh(b*x + a)^2*cosh(-(b*c - a*d)/d) - d*cosh(b*x
 + a)^2*sinh(-(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*s
qrt(b/d)) + 27*sqrt(pi)*(d*cosh(b*x + a)^3*cosh(-(b*c - a*d)/d) + d*cosh(b
*x + a)^3*sinh(-(b*c - a*d)/d) + (d*cosh(-(b*c - a*d)/d) + d*sinh(-(b*c -
a*d)/d))*sinh(b*x + a)^3 + 3*(d*cosh(b*x + a)*cosh(-(b*c - a*d)/d) + d*...
```

Sympy [F]

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx = \int \sqrt{c+dx}\cosh^3{(a+bx)}\,dx$$

input
```
integrate((d*x+c)**(1/2)*cosh(b*x+a)**3,x)
```

output
```
Integral(sqrt(c + d*x)*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.21

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx = -\frac{\frac{\sqrt{3}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3a-\frac{3bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{\sqrt{3}\sqrt{\pi}d\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3a+\frac{3bc}{d}\right)}}{b\sqrt{\frac{b}{d}}} + \frac{27\sqrt{\pi}d\operatorname{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{b\sqrt{-\frac{b}{d}}} - \frac{27\sqrt{}}{}}{}$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/144*(sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a -
3*b*c/d)/(b*sqrt(-b/d)) - sqrt(3)*sqrt(pi)*d*erf(sqrt(3)*sqrt(d*x + c)*sqr
t(b/d))*e^(-3*a + 3*b*c/d)/(b*sqrt(b/d)) + 27*sqrt(pi)*d*erf(sqrt(d*x + c)
*sqrt(-b/d))*e^(a - b*c/d)/(b*sqrt(-b/d)) - 27*sqrt(pi)*d*erf(sqrt(d*x + c
)*sqrt(b/d))*e^(-a + b*c/d)/(b*sqrt(b/d)) - 6*sqrt(d*x + c)*d*e^(3*a + 3*(
d*x + c)*b/d - 3*b*c/d)/b - 54*sqrt(d*x + c)*d*e^(a + (d*x + c)*b/d - b*c/
d)/b + 54*sqrt(d*x + c)*d*e^(-a - (d*x + c)*b/d + b*c/d)/b + 6*sqrt(d*x +
c)*d*e^(-3*a - 3*(d*x + c)*b/d + 3*b*c/d)/b)/d
```

Giac [F]

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx = \int \sqrt{dx+c}\cosh(bx+a)^3\,dx$$

input
```
integrate((d*x+c)^(1/2)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*cosh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx = \int \cosh(a+b\,x)^3\,\sqrt{c+d\,x}\,dx$$

input
```
int(cosh(a + b*x)^3*(c + d*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^3*(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{c+dx}\cosh^3(a+bx)\,dx = \int \sqrt{dx+c}\,\cosh\left(bx+a\right)^3 dx$$

input
```
int((d*x+c)^(1/2)*cosh(b*x+a)^3,x)
```

output
```
int(sqrt(c + d*x)*cosh(a + b*x)**3,x)
```

3.59 $\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 228

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{3e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{e^{-3a+\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{3e^{a-\frac{bc}{d}}\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}} + \frac{e^{3a-\frac{3bc}{d}}\sqrt{\frac{\pi}{3}}\,\text{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

output
```
3/8*exp(-a+b*c/d)*Pi^(1/2)*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1
/2)+1/24*exp(-3*a+3*b*c/d)*3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1
/2)/d^(1/2))/b^(1/2)/d^(1/2)+3/8*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c
)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)+1/24*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1/2)*er
fi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/b^(1/2)/d^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.84

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx$$
$$= \frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(\sqrt{3}e^{6a}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+9e^{4a+\frac{2bc}{d}}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{b(c+dx)}{d}\right)-e^{\frac{4bc}{d}}\sqrt{\frac{b(c+dx)}{d}}\left(9e^{2a}\Gamma\right.\right.}{24b\sqrt{c+dx}}$$

input
```
Integrate[Cosh[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
(Sqrt[3]*E^(6*a)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*x))/d] +
 9*E^(4*a + (2*b*c)/d)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, -((b*(c + d*x))
/d)] - E^((4*b*c)/d)*Sqrt[(b*(c + d*x))/d]*(9*E^(2*a)*Gamma[1/2, (b*(c + d
*x))/d] + Sqrt[3]*E^((2*b*c)/d)*Gamma[1/2, (3*b*(c + d*x))/d]))/(24*b*E^(3
*(a + (b*c)/d))*Sqrt[c + d*x])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{3793}$$
$$\int \left(\frac{3\cosh(a+bx)}{4\sqrt{c+dx}}+\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}$$

input
```
Int[Cosh[a + b*x]^3/Sqrt[c + d*x],x]
```

output
```
(3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt
[b]*Sqrt[d]) + (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[
c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (3*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi
[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(3*a - (3*b*c)
/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*S
qrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int \frac{\cosh (bx+a)^3}{\sqrt{dx+c}}dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^(1/2),x)
```

output
```
int(cosh(b*x+a)^3/(d*x+c)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\sqrt{3}\sqrt{\pi}\sqrt{\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad)}{d}\right)-\sinh\left(-\frac{3(bc-ad)}{d}\right)\right)\operatorname{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)-\sqrt{3}\sqrt{\pi}\sqrt{-\frac{b}{d}}\left(\cosh\left(-\frac{3(bc-ad}{d}\right.\right.\cdots}{\cdots}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
1/24*(sqrt(3)*sqrt(pi)*sqrt(b/d)*(cosh(-3*(b*c - a*d)/d) - sinh(-3*(b*c -
a*d)/d))*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) - sqrt(3)*sqrt(pi)*sqrt(-b/d
)*(cosh(-3*(b*c - a*d)/d) + sinh(-3*(b*c - a*d)/d))*erf(sqrt(3)*sqrt(d*x +
 c)*sqrt(-b/d)) + 9*sqrt(pi)*sqrt(b/d)*(cosh(-(b*c - a*d)/d) - sinh(-(b*c
- a*d)/d))*erf(sqrt(d*x + c)*sqrt(b/d)) - 9*sqrt(pi)*sqrt(-b/d)*(cosh(-(b*
c - a*d)/d) + sinh(-(b*c - a*d)/d))*erf(sqrt(d*x + c)*sqrt(-b/d)))/b
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh^3{(a+bx)}}{\sqrt{c+dx}}\,dx$$

input
```
integrate(cosh(b*x+a)**3/(d*x+c)**(1/2),x)
```

output
```
Integral(cosh(a + b*x)**3/sqrt(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.78

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx = \frac{\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(3\,a-\frac{3\,bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{3}\sqrt{dx+c}\sqrt{\frac{b}{d}}\right)e^{\left(-3\,a+\frac{3\,bc}{d}\right)}}{\sqrt{\frac{b}{d}}}+\frac{9\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx+c}\sqrt{-\frac{b}{d}}\right)e^{\left(a-\frac{bc}{d}\right)}}{\sqrt{-\frac{b}{d}}}+\frac{9\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c\cdots}\right.}{}}{24\,d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
1/24*(sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(-b/d))*e^(3*a - 3*b*
c/d)/sqrt(-b/d) + sqrt(3)*sqrt(pi)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d))*e^
(-3*a + 3*b*c/d)/sqrt(b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(-b/d))*e^(a
 - b*c/d)/sqrt(-b/d) + 9*sqrt(pi)*erf(sqrt(d*x + c)*sqrt(b/d))*e^(-a + b*c
/d)/sqrt(b/d))/d
```

Giac [F]

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh\left(bx+a\right)^3}{\sqrt{dx+c}}\,dx$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3/sqrt(d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh(a+b\,x)^3}{\sqrt{c+d\,x}}\,dx$$

input
```
int(cosh(a + b*x)^3/(c + d*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^3/(c + d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\cosh\left(bx+a\right)^3}{\sqrt{dx+c}}dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^(1/2),x)
```

output
```
int(cosh(a + b*x)**3/sqrt(c + d*x),x)
```

3.60 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 246

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = -\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}} - \frac{3\sqrt{b}e^{-a+\frac{bc}{d}}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} - \frac{\sqrt{b}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{3\sqrt{b}e^{a-\frac{bc}{d}}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}} + \frac{\sqrt{b}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{4d^{3/2}}$$

output

```
-2*cosh(b*x+a)^3/d/(d*x+c)^(1/2)-3/4*b^(1/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(b^
(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)-1/4*b^(1/2)*exp(-3*a+3*b*c/d)*3^(1/2)
*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)+3/4*b^(1/2)*e
xp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(3/2)+1/4*b^(1/
2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^
(1/2))/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \frac{e^{-3\left(a+b\left(\frac{c}{d}+x\right)\right)}\left(\sqrt{3}e^{6a+3bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)+3e^{4a+\frac{2bc}{d}+3bx}\sqrt{-\frac{b(c+dx)}{d}}\Gamma\left(\frac{1}{2},-\right.\right.}{}$$

input
```
Integrate[Cosh[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
(Sqrt[3]*E^(6*a + 3*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2, (-3*b*(c + d*
x))/d] + 3*E^(4*a + (2*b*c)/d + 3*b*x)*Sqrt[-((b*(c + d*x))/d)]*Gamma[1/2,
 -((b*(c + d*x))/d)] + E^((3*b*c)/d)*(-(1 + E^(2*(a + b*x)))^3 + 3*E^(2*a
+ (b*c)/d + 3*b*x)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, b*(c/d + x)] + Sqrt[3]
*E^((3*b*(c + d*x))/d)*Sqrt[(b*(c + d*x))/d]*Gamma[1/2, (3*b*(c + d*x))/d]
))/(4*d*E^(3*(a + b*(c/d + x)))*Sqrt[c + d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.09, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx$$

$$\Big\downarrow \text{3794}$$

$$-\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}}+\frac{6ib\int\left(-\frac{i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}}+\frac{6ib\left(\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\operatorname{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\operatorname{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\operatorname{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right)}{d}$$

input
```
Int[Cosh[a + b*x]^3/(c + d*x)^(3/2),x]
```

output
```
(-2*Cosh[a + b*x]^3)/(d*Sqrt[c + d*x]) + ((6*I)*b*(((I/8)*E^(-a + (b*c)/d)
*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/8)
*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[
d]])/(Sqrt[b]*Sqrt[d]) - ((I/8)*E^(a - (b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqr
t[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - ((I/8)*E^(3*a - (3*b*c)/d)*Sqrt[
Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

Maple [F]

$$\int \frac{\cosh\left(bx+a\right)^{3}}{\left(dx+c\right)^{\frac{3}{2}}}dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^(3/2),x)
```

output
```
int(cosh(b*x+a)^3/(d*x+c)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1344 vs. $2(182) = 364$.

Time = 0.11 (sec) , antiderivative size = 1344, normalized size of antiderivative = 5.46

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(3)*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) -
 (d*x + c)*cosh(b*x + a)^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*
c - a*d)/d) - (d*x + c)*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x
+ c)*cosh(b*x + a)*cosh(-3*(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)*sinh(-
3*(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(
b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x
 + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*sqrt(b/d)) + sqrt(3)*sqrt(pi)*(
(d*x + c)*cosh(b*x + a)^3*cosh(-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)
^3*sinh(-3*(b*c - a*d)/d) + ((d*x + c)*cosh(-3*(b*c - a*d)/d) + (d*x + c)*
sinh(-3*(b*c - a*d)/d))*sinh(b*x + a)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(
-3*(b*c - a*d)/d) + (d*x + c)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/d))*sinh(b
*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2*cosh(-3*(b*c - a*d)/d) + (d*x + c
)*cosh(b*x + a)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sq
rt(3)*sqrt(d*x + c)*sqrt(-b/d)) + 3*sqrt(pi)*((d*x + c)*cosh(b*x + a)^3*co
sh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^3*sinh(-(b*c - a*d)/d) + ((d*
x + c)*cosh(-(b*c - a*d)/d) - (d*x + c)*sinh(-(b*c - a*d)/d))*sinh(b*x + a
)^3 + 3*((d*x + c)*cosh(b*x + a)*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x
 + a)*sinh(-(b*c - a*d)/d))*sinh(b*x + a)^2 + 3*((d*x + c)*cosh(b*x + a)^2
*cosh(-(b*c - a*d)/d) - (d*x + c)*cosh(b*x + a)^2*sinh(-(b*c - a*d)/d))*si
nh(b*x + a))*sqrt(b/d)*erf(sqrt(d*x + c)*sqrt(b/d)) + 3*sqrt(pi)*((d*x ...
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh^3{(a+bx)}}{(c+dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)**3/(d*x+c)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**3/(c + d*x)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.80

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx =$$

$$-\frac{\frac{\sqrt{3}\sqrt{\frac{(dx+c)b}{d}}e^{\left(\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{3\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{\sqrt{3}\sqrt{-\frac{(dx+c)b}{d}}e^{\left(-\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{1}{2},-\frac{3\,(dx+c)b}{d}\right)}{\sqrt{dx+c}}+\frac{3\sqrt{\frac{(dx+c)b}{d}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{1}{2},\frac{(dx+c)b}{d}\right)}{\sqrt{dx+c}}}{8\,d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(3)*sqrt((d*x + c)*b/d)*e^(3*(b*c - a*d)/d)*gamma(-1/2, 3*(d*x +
 c)*b/d)/sqrt(d*x + c) + sqrt(3)*sqrt(-(d*x + c)*b/d)*e^(-3*(b*c - a*d)/d)
*gamma(-1/2, -3*(d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt((d*x + c)*b/d)*e^(-a
 + b*c/d)*gamma(-1/2, (d*x + c)*b/d)/sqrt(d*x + c) + 3*sqrt(-(d*x + c)*b/d
)*e^(a - b*c/d)*gamma(-1/2, -(d*x + c)*b/d)/sqrt(d*x + c))/d
```

Giac [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh\left(bx+a\right)^3}{(dx+c)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3/(d*x + c)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^{3/2}}\,dx$$

input
```
int(cosh(a + b*x)^3/(c + d*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^3/(c + d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}\,dx = \int \frac{\cosh(bx+a)^3}{\sqrt{dx+c}\,c+\sqrt{dx+c}\,dx}dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^(3/2),x)
```

output
```
int(cosh(a + b*x)**3/(sqrt(c + d*x)*c + sqrt(c + d*x)*d*x),x)
```

3.61 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 277

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx = -\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}} + \frac{b^{3/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{b^{3/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{b^{3/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} + \frac{b^{3/2}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2d^{5/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{d^2\sqrt{c+dx}}$$

output
```
-2/3*cosh(b*x+a)^3/d/(d*x+c)^(3/2)+1/2*b^(3/2)*exp(-a+b*c/d)*Pi^(1/2)*erf(
b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+1/2*b^(3/2)*exp(-3*a+3*b*c/d)*3^(1/
2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+1/2*b^(3/2)
*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(5/2)+1/2*b^(
3/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/
d^(1/2))/d^(5/2)-4*b*cosh(b*x+a)^2*sinh(b*x+a)/d^2/(d*x+c)^(1/2)
```

Mathematica [A] (verified)

Time = 1.81 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.91

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \frac{e^{-3\left(a+\frac{bc}{d}\right)}\left(-3\sqrt{3}de^{6a}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},-\frac{3b(c+dx)}{d}\right)-3de^{4a+\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{3/2}\Gamma\left(\frac{1}{2},\right.\right.}{}$$

input
```
Integrate[Cosh[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
(-3*Sqrt[3]*d*E^(6*a)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(c + d*x
))/d] - 3*d*E^(4*a + (2*b*c)/d)*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, -((b
*(c + d*x))/d)] - 3*d*E^(2*a + (4*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (b*(c + d*x))/d] - 3*Sqrt[3]*d*E^((6*b*c)/d)*((b*(c + d*x))/d)^(3/2)*Ga
mma[1/2, (3*b*(c + d*x))/d] - 4*E^(3*(a + (b*c)/d))*Cosh[a + b*x]^2*(d*Cos
h[a + b*x] + 6*b*(c + d*x)*Sinh[a + b*x]))/(6*d^2*E^(3*(a + (b*c)/d))*(c +
 d*x)^(3/2))
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 402, normalized size of antiderivative = 1.45, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3795, 3042, 3788, 26, 2611, 2633, 2634, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{5/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{12b^2\int \frac{\cosh^3(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{8b^2\int \frac{\cosh(a+bx)}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx}{d^2}+\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3788}$$

$$-\frac{8b^2\left(\frac{1}{2}i\int-\frac{ie^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{ie^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d^2}+\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{26}$$

$$-\frac{8b^2\left(\frac{1}{2}\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx+\frac{1}{2}\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx\right)}{d^2}+\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{2611}$$

$$-\frac{8b^2\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d^2}+\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{8b^2\left(\frac{\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{\sqrt{c+dx}}dx}{d^2}-\frac{8b^2\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$$\Big\downarrow \text{3793}$$

$$\frac{12b^2\int\left(\frac{3\cosh(a+bx)}{4\sqrt{c+dx}}+\frac{\cosh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d^2}-\frac{8b^2\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

$\downarrow$ 2009

$$-\frac{8b^2\left(\frac{\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}+\frac{\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d^2}+\frac{12b^2\left(\frac{3\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{3\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}\right)}{d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{d^2\sqrt{c+dx}}-\frac{2\cosh^3(a+bx)}{3d(c+dx)^{3/2}}$$

input
```
Int[Cosh[a + b*x]^3/(c + d*x)^(5/2),x]
```

output
```
(-2*Cosh[a + b*x]^3)/(3*d*(c + d*x)^(3/2)) - (8*b^2*((E^(-a + (b*c)/d)*Sqr
t[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d]) + (E^(a -
(b*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(2*Sqrt[b]*Sqrt[d
])))/d^2 + (12*b^2*((3*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x
])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(S
qrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d]) + (3*E^(a - (b
*c)/d)*Sqrt[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(8*Sqrt[b]*Sqrt[d])
 + (E^(3*a - (3*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sq
rt[d]])/(8*Sqrt[b]*Sqrt[d])))/d^2 - (4*b*Cosh[a + b*x]^2*Sinh[a + b*x])/(d
^2*Sqrt[c + d*x])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\cosh(bx+a)^3}{(dx+c)^{\frac{5}{2}}}dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^(5/2),x)
```

output
```
int(cosh(b*x+a)^3/(d*x+c)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2058 vs. $2(209) = 418$.

Time = 0.15 (sec) , antiderivative size = 2058, normalized size of antiderivative = 7.43

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="fricas")
```

output
```
1/12*(6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*
cosh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^3*s
inh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*c - a*
d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh(b*x +
 a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*c - a*
d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c - a*d)/
d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*c
osh(-3*(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)^2*si
nh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(b/d)*erf(sqrt(3)*sqrt(d*x + c)*s
qrt(b/d)) - 6*sqrt(3)*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)^3*cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
a)^3*sinh(-3*(b*c - a*d)/d) + ((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(-3*(b*
c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*sinh(-3*(b*c - a*d)/d))*sinh
(b*x + a)^3 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*cosh(-3*(b*
c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a)*sinh(-3*(b*c -
 a*d)/d))*sinh(b*x + a)^2 + 3*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
a)^2*cosh(-3*(b*c - a*d)/d) + (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x + a
)^2*sinh(-3*(b*c - a*d)/d))*sinh(b*x + a))*sqrt(-b/d)*erf(sqrt(3)*sqrt(d*x
 + c)*sqrt(-b/d)) + 6*sqrt(pi)*((b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +
 a)^3*cosh(-(b*c - a*d)/d) - (b*d^2*x^2 + 2*b*c*d*x + b*c^2)*cosh(b*x +...
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx=\int \frac{\cosh^3{(a+bx)}}{(c+dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)**3/(d*x+c)**(5/2),x)
```

output
```
Integral(cosh(a + b*x)**3/(c + d*x)**(5/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx =$$

$$-\frac{3\left(\frac{\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},\frac{3\,(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-\frac{3\,(bc-ad)}{d}\right)}\Gamma\left(-\frac{3}{2},-\frac{3\,(dx+c)b}{d}\right)}{(dx+c)^{\frac{3}{2}}}+\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{3}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\frac{3}{2},\right.}{(dx+c)^{\frac{3}{2}}}\right.}{8\,d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="maxima")
```

output
```
-3/8*(sqrt(3)*((d*x + c)*b/d)^(3/2)*e^(3*(b*c - a*d)/d)*gamma(-3/2, 3*(d*x
 + c)*b/d)/(d*x + c)^(3/2) + sqrt(3)*(-(d*x + c)*b/d)^(3/2)*e^(-3*(b*c - a
*d)/d)*gamma(-3/2, -3*(d*x + c)*b/d)/(d*x + c)^(3/2) + ((d*x + c)*b/d)^(3/
2)*e^(-a + b*c/d)*gamma(-3/2, (d*x + c)*b/d)/(d*x + c)^(3/2) + (-(d*x + c)
*b/d)^(3/2)*e^(a - b*c/d)*gamma(-3/2, -(d*x + c)*b/d)/(d*x + c)^(3/2))/d
```

Giac [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh\left(bx+a\right)^3}{(dx+c)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3/(d*x + c)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^{5/2}}\,dx$$

input
```
int(cosh(a + b*x)^3/(c + d*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^3/(c + d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{5/2}}\,dx = \int \frac{\cosh\left(bx+a\right)^3}{\sqrt{dx+c}\,c^2+2\sqrt{dx+c}\,cdx+\sqrt{dx+c}\,d^2x^2}dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^(5/2),x)
```

output
```
int(cosh(a + b*x)**3/(sqrt(c + d*x)*c**2 + 2*sqrt(c + d*x)*c*d*x + sqrt(c
+ d*x)*d**2*x**2),x)
```

3.62 $\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 331

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{16b^2\cosh(a+bx)}{5d^3\sqrt{c+dx}} - \frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}} - \frac{24b^2\cosh^3(a+bx)}{5d^3\sqrt{c+dx}} - \frac{b^{5/2}e^{-a+\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{3b^{5/2}e^{-3a+\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{b^{5/2}e^{a-\frac{bc}{d}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} + \frac{3b^{5/2}e^{3a-\frac{3bc}{d}}\sqrt{3\pi}\,\mathrm{erfi}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{5d^{7/2}} - \frac{4b\cosh^2(a+bx)\sinh(a+bx)}{5d^2(c+dx)^{3/2}}$$

output
```
16/5*b^2*cosh(b*x+a)/d^3/(d*x+c)^(1/2)-2/5*cosh(b*x+a)^3/d/(d*x+c)^(5/2)-2
4/5*b^2*cosh(b*x+a)^3/d^3/(d*x+c)^(1/2)-1/5*b^(5/2)*exp(-a+b*c/d)*Pi^(1/2)
*erf(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)-3/5*b^(5/2)*exp(-3*a+3*b*c/d)*
3^(1/2)*Pi^(1/2)*erf(3^(1/2)*b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+1/5*b^
(5/2)*exp(a-b*c/d)*Pi^(1/2)*erfi(b^(1/2)*(d*x+c)^(1/2)/d^(1/2))/d^(7/2)+3/
5*b^(5/2)*exp(3*a-3*b*c/d)*3^(1/2)*Pi^(1/2)*erfi(3^(1/2)*b^(1/2)*(d*x+c)^(
1/2)/d^(1/2))/d^(7/2)-4/5*b*cosh(b*x+a)^2*sinh(b*x+a)/d^2/(d*x+c)^(3/2)
```

Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.14

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \frac{e^{-3a}\left(2e^{6a}\left(-d^2e^{3bx}-2be^{-\frac{3bc}{d}}(c+dx)\left(e^{\frac{3b(c+dx)}{d}}(d+6b(c+dx))+6\sqrt{3}d\left(-\frac{b(c+dx)}{d}\right.\right.\right.\right.}{}$$

input
```
Integrate[Cosh[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
(2*E^(6*a)*(-(d^2*E^(3*b*x)) - (2*b*(c + d*x)*(E^((3*b*(c + d*x))/d)*(d +
6*b*(c + d*x)) + 6*Sqrt[3]*d*(-((b*(c + d*x))/d))^(3/2)*Gamma[1/2, (-3*b*(
c + d*x))/d]))/E^((3*b*c)/d)) + 2*E^(4*a)*(-3*d^2*E^(b*x) - (2*b*(c + d*x)
*(E^(b*(c/d + x))*(d + 2*b*(c + d*x)) + 2*d*(-((b*(c + d*x))/d))^(3/2)*Gam
ma[1/2, -((b*(c + d*x))/d)]))/E^((b*c)/d)) + E^(2*a - b*x)*(-6*d^2 + 4*b*d
*(c + d*x) - 8*b^2*(c + d*x)^2 + 8*d^2*E^(b*(c/d + x))*((b*(c + d*x))/d)^(
5/2)*Gamma[1/2, (b*(c + d*x))/d]) + (2*(-d^2 + 2*b*(c + d*x)*(d - 6*b*(c +
 d*x) + 6*Sqrt[3]*d*E^((3*b*(c + d*x))/d)*((b*(c + d*x))/d)^(3/2)*Gamma[1/
2, (3*b*(c + d*x))/d])))/E^(3*b*x))/(40*d^3*E^(3*a)*(c + d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.52 (sec) , antiderivative size = 480, normalized size of antiderivative = 1.45, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {3042, 3795, 3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{7/2}}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{12b^2\int \frac{\cosh^3(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\int \frac{\cosh(a+bx)}{(c+dx)^{3/2}}dx}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{8b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)}{(c+dx)^{3/2}}dx}{5d^2}+\frac{12b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3778}$$

$$\frac{12b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2ib\int -\frac{i\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{12b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(\frac{2b\int \frac{\sinh(a+bx)}{\sqrt{c+dx}}dx}{d}-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{12b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}+\frac{2b\int -\frac{i\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{26}$$

$$\frac{12b^2\int \frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\int \frac{\sin(ia+ibx)}{\sqrt{c+dx}}dx}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3789}$$

$$\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{1}{2}i\int\frac{e^{a+bx}}{\sqrt{c+dx}}dx-\frac{1}{2}i\int\frac{e^{-a-bx}}{\sqrt{c+dx}}dx\right)}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{2611}$$

$$\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\int e^{a+\frac{b(c+dx)}{d}-\frac{bc}{d}}d\sqrt{c+dx}}{d}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{2633}$$

$$-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\int e^{-a-\frac{b(c+dx)}{d}+\frac{bc}{d}}d\sqrt{c+dx}}{d}\right)}{d}\right)}{5d^2}+\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{12b^2\int\frac{\sin\left(ia+ibx+\frac{\pi}{2}\right)^3}{(c+dx)^{3/2}}dx}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{3794}$$

$$\frac{12b^2\left(-\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}}+\frac{6ib\int\left(-\frac{i\sinh(a+bx)}{4\sqrt{c+dx}}-\frac{i\sinh(3a+3bx)}{4\sqrt{c+dx}}\right)dx}{d}\right)}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{12b^2\left(-\frac{2\cosh^3(a+bx)}{d\sqrt{c+dx}}+\frac{6ib\left(\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}+\frac{i\sqrt{\frac{\pi}{3}}e^{\frac{3bc}{d}-3a}\text{erf}\left(\frac{\sqrt{3}\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{8\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\frac{\pi}{3}}e^{3a-\frac{3bc}{d}}\text{erfi}\cdots}{8\sqrt{b}\cdots}\right)}{d}\right)}{5d^2}-\frac{8b^2\left(-\frac{2\cosh(a+bx)}{d\sqrt{c+dx}}-\frac{2ib\left(\frac{i\sqrt{\pi}e^{a-\frac{bc}{d}}\text{erfi}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}-\frac{i\sqrt{\pi}e^{\frac{bc}{d}-a}\text{erf}\left(\frac{\sqrt{b}\sqrt{c+dx}}{\sqrt{d}}\right)}{2\sqrt{b}\sqrt{d}}\right)}{d}\right)}{5d^2}-\frac{4b\sinh(a+bx)\cosh^2(a+bx)}{5d^2(c+dx)^{3/2}}-\frac{2\cosh^3(a+bx)}{5d(c+dx)^{5/2}}$$

input
```
Int[Cosh[a + b*x]^3/(c + d*x)^(7/2),x]
```

output
```
(-2*Cosh[a + b*x]^3)/(5*d*(c + d*x)^(5/2)) - (8*b^2*((-2*Cosh[a + b*x])/(d
*Sqrt[c + d*x]) - ((2*I)*b*(((-1/2*I)*E^(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[
b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) + ((I/2)*E^(a - (b*c)/d)*Sqr
t[Pi]*Erfi[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d])))/d))/(5*d^
2) + (12*b^2*((-2*Cosh[a + b*x]^3)/(d*Sqrt[c + d*x]) + ((6*I)*b*(((I/8)*E^
(-a + (b*c)/d)*Sqrt[Pi]*Erf[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqr
t[d]) + ((I/8)*E^(-3*a + (3*b*c)/d)*Sqrt[Pi/3]*Erf[(Sqrt[3]*Sqrt[b]*Sqrt[c
 + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - ((I/8)*E^(a - (b*c)/d)*Sqrt[Pi]*Erf
i[(Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b]*Sqrt[d]) - ((I/8)*E^(3*a - (3
*b*c)/d)*Sqrt[Pi/3]*Erfi[(Sqrt[3]*Sqrt[b]*Sqrt[c + d*x])/Sqrt[d]])/(Sqrt[b
]*Sqrt[d])))/d))/(5*d^2) - (4*b*Cosh[a + b*x]^2*Sinh[a + b*x])/(5*d^2*(c +
 d*x)^(3/2))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

Maple [F]

$$\int \frac{\cosh (bx+a)^3}{(dx+c)^{\frac{7}{2}}}dx$$

input
```
int(cosh(b*x+a)^3/(d*x+c)^(7/2),x)
```

output
```
int(cosh(b*x+a)^3/(d*x+c)^(7/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3280 vs. 2(253) = 506.

Time = 0.15 (sec) , antiderivative size = 3280, normalized size of antiderivative = 9.91

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh^3(a+bx)}{(c+dx)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)**3/(d*x+c)**(7/2),x)
```

output
```
Integral(cosh(a + b*x)**3/(c + d*x)**(7/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.59

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = -\frac{3\left(\frac{3\sqrt{3}\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{3\sqrt{3}\left(-\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-\frac{3(bc-ad)}{d}\right)}\Gamma\left(-\frac{5}{2},-\frac{3(dx+c)b}{d}\right)}{(dx+c)^{\frac{5}{2}}}+\frac{\left(\frac{(dx+c)b}{d}\right)^{\frac{5}{2}}e^{\left(-a+\frac{bc}{d}\right)}\Gamma\left(-\right.}{(dx+c)^{\frac{5}{2}}}}{8d}$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="maxima")
```

output
```
-3/8*(3*sqrt(3)*((d*x + c)*b/d)^(5/2)*e^(3*(b*c - a*d)/d)*gamma(-5/2, 3*(d
*x + c)*b/d)/(d*x + c)^(5/2) + 3*sqrt(3)*(-(d*x + c)*b/d)^(5/2)*e^(-3*(b*c
 - a*d)/d)*gamma(-5/2, -3*(d*x + c)*b/d)/(d*x + c)^(5/2) + ((d*x + c)*b/d)
^(5/2)*e^(-a + b*c/d)*gamma(-5/2, (d*x + c)*b/d)/(d*x + c)^(5/2) + (-(d*x
+ c)*b/d)^(5/2)*e^(a - b*c/d)*gamma(-5/2, -(d*x + c)*b/d)/(d*x + c)^(5/2))
/d
```

Giac [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh\left(bx+a\right)^3}{(dx+c)^{\frac{7}{2}}}\,dx$$

input
```
integrate(cosh(b*x+a)^3/(d*x+c)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^3/(d*x + c)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh(a+b\,x)^3}{(c+d\,x)^{7/2}}\,dx$$

input
```
int(cosh(a + b*x)^3/(c + d*x)^(7/2),x)
```

output
```
int(cosh(a + b*x)^3/(c + d*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{\cosh^3(a+bx)}{(c+dx)^{7/2}}\,dx = \int \frac{\cosh(bx+a)^3}{\sqrt{dx+c}\,c^3+3\sqrt{dx+c}\,c^2dx+3\sqrt{dx+c}\,c\,d^2x^2+\sqrt{dx+c}\,d^3x^3}dx$$

input

```
int(cosh(b*x+a)^3/(d*x+c)^(7/2),x)
```

output

```
int(cosh(a + b*x)**3/(sqrt(c + d*x)*c**3 + 3*sqrt(c + d*x)*c**2*d*x + 3*sq
rt(c + d*x)*c*d**2*x**2 + sqrt(c + d*x)*d**3*x**3),x)
```

3.63 $\int (dx)^{3/2}\cosh(fx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 111

$$\int (dx)^{3/2}\cosh(fx)\,dx = -\frac{3d\sqrt{dx}\cosh(fx)}{2f^2} + \frac{3d^{3/2}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{3d^{3/2}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{8f^{5/2}} + \frac{(dx)^{3/2}\sinh(fx)}{f}$$

output
```
-3/2*d*(d*x)^(1/2)*cosh(f*x)/f^2+3/8*d^(3/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1
/2)/d^(1/2))/f^(5/2)+3/8*d^(3/2)*Pi^(1/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2)
)/f^(5/2)+(d*x)^(3/2)*sinh(f*x)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.46

$$\int (dx)^{3/2}\cosh(fx)\,dx = \frac{d^2\left(\sqrt{-fx}\,\Gamma\left(\frac{5}{2},-fx\right) - \sqrt{fx}\,\Gamma\left(\frac{5}{2},fx\right)\right)}{2f^3\sqrt{dx}}$$

input
```
Integrate[(d*x)^(3/2)*Cosh[f*x],x]
```

output
```
(d^2*(Sqrt[-(f*x)]*Gamma[5/2, -(f*x)] - Sqrt[f*x]*Gamma[5/2, f*x]))/(2*f^3
*Sqrt[d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3777, 26, 3042, 26, 3777, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (dx)^{3/2}\cosh(fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (dx)^{3/2}\sin\left(\frac{\pi}{2}+ifx\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(dx)^{3/2}\sinh(fx)}{f}-\frac{3id\int -i\sqrt{dx}\sinh(fx)dx}{2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(dx)^{3/2}\sinh(fx)}{f}-\frac{3d\int \sqrt{dx}\sinh(fx)dx}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(dx)^{3/2}\sinh(fx)}{f}-\frac{3d\int -i\sqrt{dx}\sin(ifx)dx}{2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\int \sqrt{dx}\sin(ifx)dx}{2f}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\int\frac{\cosh(fx)}{\sqrt{dx}}dx}{2f}\right)}{2f}$$

$\big\downarrow$ 3042

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\int\frac{\sin\left(ifx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{2f}\right)}{2f}$$

$\big\downarrow$ 3788

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{1}{2}i\int-\frac{ie^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int\frac{ie^{-fx}}{\sqrt{dx}}dx\right)}{2f}\right)}{2f}$$

$\big\downarrow$ 26

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{1}{2}\int\frac{e^{-fx}}{\sqrt{dx}}dx+\frac{1}{2}\int\frac{e^{fx}}{\sqrt{dx}}dx\right)}{2f}\right)}{2f}$$

$\big\downarrow$ 2611

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\int e^{fx}d\sqrt{dx}}{d}\right)}{2f}\right)}{2f}$$

$\big\downarrow$ 2633

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)}{2f}$$

$\big\downarrow$ 2634

$$\frac{(dx)^{3/2}\sinh(fx)}{f}+\frac{3id\left(\frac{i\sqrt{dx}\cosh(fx)}{f}-\frac{id\left(\frac{\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}\right)}{2f}$$

input
```
Int[(d*x)^(3/2)*Cosh[f*x],x]
```

output
```
(((3*I)/2)*d*((I*Sqrt[d*x]*Cosh[f*x])/f - ((I/2)*d*((Sqrt[Pi]*Erf[(Sqrt[f]
*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d
*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])))/f))/f + ((d*x)^(3/2)*Sinh[f*x])/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.20

method	result	si
meijerg	$-\frac{2i(dx)^{\frac{3}{2}}\sqrt{2}\,\sqrt{\pi}\left(-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}(10fx+15)\,\mathrm{e}^{-fx}}{80\sqrt{\pi}\,f^{2}}-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{5}{2}}(-10fx+15)\,\mathrm{e}^{fx}}{80\sqrt{\pi}\,f^{2}}+\frac{3(if)^{\frac{5}{2}}\sqrt{2}\,\operatorname{erf}(\sqrt{x}\,\sqrt{f})}{32f^{\frac{5}{2}}}+\frac{3(if)^{\frac{5}{2}}\sqrt{2}\,\operatorname{erfi}(\sqrt{x}\,\sqrt{f})}{32f^{\frac{5}{2}}}\right)}{x^{\frac{3}{2}}(if)^{\frac{3}{2}}f}$	1

input
```
int((d*x)^(3/2)*cosh(f*x),x,method=_RETURNVERBOSE)
```

output
```
-2*I*(d*x)^(3/2)/x^(3/2)*2^(1/2)/(I*f)^(3/2)*Pi^(1/2)/f*(-1/80/Pi^(1/2)*x^
(1/2)*2^(1/2)*(I*f)^(5/2)*(10*f*x+15)/f^2*exp(-f*x)-1/80/Pi^(1/2)*x^(1/2)*
2^(1/2)*(I*f)^(5/2)*(-10*f*x+15)/f^2*exp(f*x)+3/32*(I*f)^(5/2)*2^(1/2)/f^(
5/2)*erf(x^(1/2)*f^(1/2))+3/32*(I*f)^(5/2)*2^(1/2)/f^(5/2)*erfi(x^(1/2)*f^
(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. 2(77) = 154.

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.72

$$\int (dx)^{3/2}\cosh(fx)\,dx = \frac{3\sqrt{\pi}(d^2\cosh(fx)+d^2\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-3\sqrt{\pi}(d^2\cosh(fx)+d^2\mathrm{si}}{}$$

input
```
integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="fricas")
```

output
```
1/8*(3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sq
rt(f/d)) - 3*sqrt(pi)*(d^2*cosh(f*x) + d^2*sinh(f*x))*sqrt(-f/d)*erf(sqrt(
d*x)*sqrt(-f/d)) - 2*(2*d*f^2*x - (2*d*f^2*x - 3*d*f)*cosh(f*x)^2 - 2*(2*d
*f^2*x - 3*d*f)*cosh(f*x)*sinh(f*x) - (2*d*f^2*x - 3*d*f)*sinh(f*x)^2 + 3*
d*f)*sqrt(d*x))/(f^3*cosh(f*x) + f^3*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.17 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.18

$$\int (dx)^{3/2}\cosh(fx)\,dx = \frac{5d^{\frac{3}{2}}x^{\frac{3}{2}}\sinh(fx)\Gamma\left(\frac{5}{4}\right)}{4f\Gamma\left(\frac{9}{4}\right)} - \frac{15d^{\frac{3}{2}}\sqrt{x}\cosh(fx)\Gamma\left(\frac{5}{4}\right)}{8f^2\Gamma\left(\frac{9}{4}\right)} + \frac{15\sqrt{2}\sqrt{\pi}d^{\frac{3}{2}}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{5}{4}\right)}{16f^{\frac{5}{2}}\Gamma\left(\frac{9}{4}\right)}$$

input
```
integrate((d*x)**(3/2)*cosh(f*x),x)
```

output
```
5*d**(3/2)*x**(3/2)*sinh(f*x)*gamma(5/4)/(4*f*gamma(9/4)) - 15*d**(3/2)*sq
rt(x)*cosh(f*x)*gamma(5/4)/(8*f**2*gamma(9/4)) + 15*sqrt(2)*sqrt(pi)*d**(3
/2)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqrt(pi))*ga
mma(5/4)/(16*f**(5/2)*gamma(9/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. 2(77) = 154.

Time = 0.04 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.57

$$\int (dx)^{3/2}\cosh(fx)\,dx = \frac{16\,(dx)^{\frac{5}{2}}\cosh(fx) + \frac{f\left(\frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^3\sqrt{\frac{f}{d}}} + \frac{15\sqrt{\pi}d^3\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^3\sqrt{-\frac{f}{d}}} - \frac{2\left(4\,(dx)^{\frac{5}{2}}df^2 - 10\,(dx)^{\frac{3}{2}}d^2f + 15}{f^3}}{d}}{40\,d}$$

input
```
integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="maxima")
```

output
```
1/40*(16*(d*x)^(5/2)*cosh(f*x) + f*(15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(f/d
))/(f^3*sqrt(f/d)) + 15*sqrt(pi)*d^3*erf(sqrt(d*x)*sqrt(-f/d))/(f^3*sqrt(-
f/d)) - 2*(4*(d*x)^(5/2)*d*f^2 - 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)*d^3)*
e^(f*x)/f^3 - 2*(4*(d*x)^(5/2)*d*f^2 + 10*(d*x)^(3/2)*d^2*f + 15*sqrt(d*x)
*d^3)*e^(-f*x)/f^3)/d)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.31

$$\int (dx)^{3/2} \cosh(fx)\,dx =$$

$$-\frac{1}{8}d\left(\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f^2}+\frac{2\left(2\sqrt{dx}d^2fx+3\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}}{d^2}+\frac{\frac{3\sqrt{\pi}d^3\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f^2}-\frac{2\left(2\sqrt{dx}d^2fx-3\sqrt{dx}d^2\right)e^{(fx)}}{f^2}}{d^2}\right)$$

input
```
integrate((d*x)^(3/2)*cosh(f*x),x, algorithm="giac")
```

output
```
-1/8*d*((3*sqrt(pi)*d^3*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f^2) + 2*(2
*sqrt(d*x)*d^2*f*x + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d^2 + (3*sqrt(pi)*d^3*
erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d*f)*f^2) - 2*(2*sqrt(d*x)*d^2*f*x - 3
*sqrt(d*x)*d^2)*e^(f*x)/f^2)/d^2)
```

Mupad [F(-1)]

Timed out.

$$\int (dx)^{3/2} \cosh(fx)\,dx = \int \cosh(f\,x)\,(d\,x)^{3/2}\,dx$$

input
```
int(cosh(f*x)*(d*x)^(3/2),x)
```

output
```
int(cosh(f*x)*(d*x)^(3/2), x)
```

Reduce [F]

$$\int (dx)^{3/2}\cosh(fx)\,dx = \frac{\sqrt{d}\,d\left(-6\sqrt{x}\,\cosh\left(fx\right)+4\sqrt{x}\,\sinh\left(fx\right)fx+3\left(\int\frac{\cosh(fx)}{\sqrt{x}}dx\right)\right)}{4f^2}$$

input
```
int((d*x)^(3/2)*cosh(f*x),x)
```

output
```
(sqrt(d)*d*( - 6*sqrt(x)*cosh(f*x) + 4*sqrt(x)*sinh(f*x)*f*x + 3*int(cosh(
f*x)/sqrt(x),x)))/(4*f**2)
```

3.64 $\int \sqrt{dx}\cosh(fx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 92

$$\int \sqrt{dx}\cosh(fx)\,dx=\frac{\sqrt{d}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}-\frac{\sqrt{d}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{4f^{3/2}}+\frac{\sqrt{dx}\sinh(fx)}{f}$$

output
```
1/4*d^(1/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))/f^(3/2)-1/4*d^(1/2)*
Pi^(1/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/f^(3/2)+(d*x)^(1/2)*sinh(f*x)/f
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

$$\int \sqrt{dx}\cosh(fx)\,dx=-\frac{d\left(\sqrt{-fx}\,\Gamma\left(\frac{3}{2},-fx\right)+\sqrt{fx}\,\Gamma\left(\frac{3}{2},fx\right)\right)}{2f^2\sqrt{dx}}$$

input
```
Integrate[Sqrt[d*x]*Cosh[f*x],x]
```

output
```
-1/2*(d*(Sqrt[-(f*x)]*Gamma[3/2, -(f*x)] + Sqrt[f*x]*Gamma[3/2, f*x]))/(f^
2*Sqrt[d*x])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.45 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3777, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{dx}\cosh(fx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{dx}\sin\left(\frac{\pi}{2}+ifx\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{id\int -\frac{i\sinh(fx)}{\sqrt{dx}}dx}{2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{d\int \frac{\sinh(fx)}{\sqrt{dx}}dx}{2f}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}-\frac{d\int -\frac{i\sin(ifx)}{\sqrt{dx}}dx}{2f}$$

$$\Big\downarrow \text{26}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\int \frac{\sin(ifx)}{\sqrt{dx}}dx}{2f}$$

$$\Big\downarrow \text{3789}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{1}{2}i\int \frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{e^{-fx}}{\sqrt{dx}}dx\right)}{2f}$$

$$\Big\downarrow \text{2611}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{2f}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{2f}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt{dx}\sinh(fx)}{f}+\frac{id\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{2f}$$

input
```
Int[Sqrt[d*x]*Cosh[f*x],x]
```

output
```
((I/2)*d*(((-1/2*I)*Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sq
rt[f]) + ((I/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[
f])))/f + (Sqrt[d*x]*Sinh[f*x])/f
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.32

method	result	size
meijerg	$-\frac{i\sqrt{dx}\,\sqrt{2}\,\sqrt{\pi}\left(\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{3}{2}}\,e^{fx}}{4\sqrt{\pi}\,f}-\frac{\sqrt{x}\,\sqrt{2}\,(if)^{\frac{3}{2}}\,e^{-fx}}{4\sqrt{\pi}\,f}+\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erf}(\sqrt{x}\,\sqrt{f})}{8f^{\frac{3}{2}}}-\frac{(if)^{\frac{3}{2}}\sqrt{2}\,\mathrm{erfi}(\sqrt{x}\,\sqrt{f})}{8f^{\frac{3}{2}}}\right)}{\sqrt{x}\,\sqrt{if}\,f}$	121

input
```
int((d*x)^(1/2)*cosh(f*x),x,method=_RETURNVERBOSE)
```

output
```
-I*(d*x)^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)*Pi^(1/2)/f*(1/4/Pi^(1/2)*x^(1/2
)*2^(1/2)*(I*f)^(3/2)/f*exp(f*x)-1/4/Pi^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(3/2)/
f*exp(-f*x)+1/8*(I*f)^(3/2)*2^(1/2)/f^(3/2)*erf(x^(1/2)*f^(1/2))-1/8*(I*f)
^(3/2)*2^(1/2)/f^(3/2)*erfi(x^(1/2)*f^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.50

$$\int \sqrt{dx}\cosh(fx)\,dx$$
$$= \frac{\sqrt{\pi}(d\cosh(fx)+d\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)+\sqrt{\pi}(d\cosh(fx)+d\sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right. \cdots}{4\left(f^2\cosh(fx)+f^2\sinh(fx)\right)}$$

input
```
integrate((d*x)^(1/2)*cosh(f*x),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d
)) + sqrt(pi)*(d*cosh(f*x) + d*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sqrt(-f
/d)) + 2*(f*cosh(f*x)^2 + 2*f*cosh(f*x)*sinh(f*x) + f*sinh(f*x)^2 - f)*sqr
t(d*x))/(f^2*cosh(f*x) + f^2*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.09

$$\int \sqrt{dx}\cosh(fx)\,dx = \frac{3\sqrt{d}\sqrt{x}\sinh(fx)\Gamma\left(\frac{3}{4}\right)}{4f\Gamma\left(\frac{7}{4}\right)} - \frac{3\sqrt{2}\sqrt{\pi}\sqrt{d}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{3}{4}\right)}{8f^{\frac{3}{2}}\Gamma\left(\frac{7}{4}\right)}$$

input
```
integrate((d*x)**(1/2)*cosh(f*x),x)
```

output
```
3*sqrt(d)*sqrt(x)*sinh(f*x)*gamma(3/4)/(4*f*gamma(7/4)) - 3*sqrt(2)*sqrt(p
i)*sqrt(d)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)/sqr
t(pi))*gamma(3/4)/(8*f**(3/2)*gamma(7/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(62) = 124$.

Time = 0.04 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.61

$$\int \sqrt{dx}\cosh(fx)\,dx = \frac{8\,(dx)^{\frac{3}{2}}\cosh(fx) + \frac{f\left(\frac{3\,\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f^2\sqrt{\frac{f}{d}}} - \frac{3\,\sqrt{\pi}d^2\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f^2\sqrt{-\frac{f}{d}}} - \frac{2\left(2\,(dx)^{\frac{3}{2}}\,df - 3\,\sqrt{dx}d^2\right)e^{(fx)}}{f^2} - \frac{2\left(2\,(dx)^{\frac{3}{2}}\,df + 3\,\sqrt{dx}d^2\right)e^{(-fx)}}{f^2}\right)}{d}}{12\,d}$$

input
```
integrate((d*x)^(1/2)*cosh(f*x),x, algorithm="maxima")
```

output
```
1/12*(8*(d*x)^(3/2)*cosh(f*x) + f*(3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(f/d))
/(f^2*sqrt(f/d)) - 3*sqrt(pi)*d^2*erf(sqrt(d*x)*sqrt(-f/d))/(f^2*sqrt(-f/d
)) - 2*(2*(d*x)^(3/2)*d*f - 3*sqrt(d*x)*d^2)*e^(f*x)/f^2 - 2*(2*(d*x)^(3/2
)*d*f + 3*sqrt(d*x)*d^2)*e^(-f*x)/f^2)/d)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.17

$$\int \sqrt{dx}\cosh(fx)\,dx = -\frac{\frac{\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}f} + \frac{2\,\sqrt{dx}de^{(-fx)}}{f}}{4\,d} + \frac{\frac{\sqrt{\pi}d^2\,\mathrm{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}f} + \frac{2\,\sqrt{dx}de^{(fx)}}{f}}{4\,d}$$

input
```
integrate((d*x)^(1/2)*cosh(f*x),x, algorithm="giac")
```

output
```
-1/4*(sqrt(pi)*d^2*erf(-sqrt(d*f)*sqrt(d*x)/d)/(sqrt(d*f)*f) + 2*sqrt(d*x)
*d*e^(-f*x)/f)/d + 1/4*(sqrt(pi)*d^2*erf(-sqrt(-d*f)*sqrt(d*x)/d)/(sqrt(-d
*f)*f) + 2*sqrt(d*x)*d*e^(f*x)/f)/d
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{dx}\cosh(fx)\,dx = \int \cosh(f\,x)\,\sqrt{d\,x}\,dx$$

input
```
int(cosh(f*x)*(d*x)^(1/2),x)
```

output
```
int(cosh(f*x)*(d*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{dx}\cosh(fx)\,dx$$
$$= \frac{\sqrt{d}\left(\sqrt{\pi}\,e^{fx}\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\,i\right)i + 2\sqrt{x}\,\sqrt{f}\,e^{2fx} + \sqrt{f}\,e^{fx}\left(\int \frac{\sqrt{x}}{e^{fx}x}dx\right) - 2\sqrt{x}\,\sqrt{f}\right)}{4\sqrt{f}\,e^{fx}f}$$

input
```
int((d*x)^(1/2)*cosh(f*x),x)
```

output
```
(sqrt(d)*(sqrt(pi)*e**(f*x)*erf(sqrt(x)*sqrt(f)*i)*i + 2*sqrt(x)*sqrt(f)*e
**(2*f*x) + sqrt(f)*e**(f*x)*int(sqrt(x)/(e**(f*x)*x),x) - 2*sqrt(x)*sqrt(
f)))/(4*sqrt(f)*e**(f*x)*f)
```

3.65 $\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 77

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

output
```
1/2*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(1/2)/f^(1/2)+1/2*Pi^(1/2)
*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(1/2)/f^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx=\frac{\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right)-\sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)}{2f\sqrt{dx}}$$

input
```
Integrate[Cosh[f*x]/Sqrt[d*x],x]
```

output
```
(Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] - Sqrt[f*x]*Gamma[1/2, f*x])/(2*f*Sqrt[d*
x])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(fx)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ifx\right)}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -\frac{ie^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{ie^{-fx}}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int \frac{e^{-fx}}{\sqrt{dx}}dx+\frac{1}{2}\int \frac{e^{fx}}{\sqrt{dx}}dx$$

$$\Big\downarrow \text{2611}$$

$$\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\int e^{fx}d\sqrt{dx}}{d}$$

$$\Big\downarrow \text{2633}$$

$$\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

$$\Big\downarrow \text{2634}$$

$$\frac{\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}$$

input
```
Int[Cosh[f*x]/Sqrt[d*x],x]
```

output
```
(Sqrt[Pi]*Erf[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi
]*Erfi[(Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.94

method	result	size
meijerg	$-\frac{i\sqrt{\pi}\,\sqrt{x}\,\sqrt{2}\,\sqrt{if}\left(\frac{\sqrt{if}\,\sqrt{2}\,\operatorname{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{2\sqrt{f}}+\frac{\sqrt{if}\,\sqrt{2}\,\operatorname{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{2\sqrt{f}}\right)}{2\sqrt{dx}\,f}$	72

input
```
int(cosh(f*x)/(d*x)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*I*Pi^(1/2)/(d*x)^(1/2)*x^(1/2)*2^(1/2)*(I*f)^(1/2)/f*(1/2*(I*f)^(1/2)
*2^(1/2)/f^(1/2)*erf(x^(1/2)*f^(1/2))+1/2*(I*f)^(1/2)*2^(1/2)/f^(1/2)*erfi
(x^(1/2)*f^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.77

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx=\frac{\sqrt{\pi}\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)-\sqrt{\pi}\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{2\,f}$$

input
```
integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(pi)*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/d)) - sqrt(pi)*sqrt(-f/d)*erf
(sqrt(d*x)*sqrt(-f/d)))/f
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx = \frac{\sqrt{2}\sqrt{\pi}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(\frac{1}{4}\right)}{4\sqrt{d}\sqrt{f}\Gamma\left(\frac{5}{4}\right)}$$

input
```
integrate(cosh(f*x)/(d*x)**(1/2),x)
```

output
```
sqrt(2)*sqrt(pi)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*exp(I*pi/4)
/sqrt(pi))*gamma(1/4)/(4*sqrt(d)*sqrt(f)*gamma(5/4))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. 2(49) = 98.

Time = 0.04 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.52

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx = \frac{4\sqrt{dx}\cosh\left(fx\right) - \frac{\left(\frac{2\sqrt{dx}de^{(fx)}}{f} + \frac{2\sqrt{dx}de^{(-fx)}}{f} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{f\sqrt{\frac{f}{d}}} - \frac{\sqrt{\pi}d\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{f\sqrt{-\frac{f}{d}}}\right)f}{d}}{2\,d}$$

input
```
integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="maxima")
```

output
```
1/2*(4*sqrt(d*x)*cosh(f*x) - (2*sqrt(d*x)*d*e^(f*x)/f + 2*sqrt(d*x)*d*e^(-
f*x)/f - sqrt(pi)*d*erf(sqrt(d*x)*sqrt(f/d))/(f*sqrt(f/d)) - sqrt(pi)*d*er
f(sqrt(d*x)*sqrt(-f/d))/(f*sqrt(-f/d)))*f/d)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.78

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx = -\frac{\frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{df}\sqrt{dx}}{d}\right)}{\sqrt{df}} + \frac{\sqrt{\pi}d\operatorname{erf}\left(-\frac{\sqrt{-df}\sqrt{dx}}{d}\right)}{\sqrt{-df}}}{2\,d}$$

input
```
integrate(cosh(f*x)/(d*x)^(1/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(pi)*d*erf(-sqrt(d*f)*sqrt(d*x)/d)/sqrt(d*f) + sqrt(pi)*d*erf(-s
qrt(-d*f)*sqrt(d*x)/d)/sqrt(-d*f))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx = \int \frac{\cosh(f\,x)}{\sqrt{d\,x}}\,dx$$

input
```
int(cosh(f*x)/(d*x)^(1/2),x)
```

output
```
int(cosh(f*x)/(d*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh(fx)}{\sqrt{dx}}\,dx = \frac{\int \frac{\cosh(fx)}{\sqrt{x}}dx}{\sqrt{d}}$$

input
```
int(cosh(f*x)/(d*x)^(1/2),x)
```

output
```
int(cosh(f*x)/sqrt(x),x)/sqrt(d)
```

3.66 $\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = -\frac{2\cosh(fx)}{d\sqrt{dx}} - \frac{\sqrt{f}\sqrt{\pi}\operatorname{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}} + \frac{\sqrt{f}\sqrt{\pi}\operatorname{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{d^{3/2}}$$

output
```
-2*cosh(f*x)/d/(d*x)^(1/2)-f^(1/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/d^(1/2
))/d^(3/2)+f^(1/2)*Pi^(1/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = \frac{e^{-fx}x\left(-1-e^{2fx}+e^{fx}\sqrt{-fx}\,\Gamma\left(\frac{1}{2},-fx\right)+e^{fx}\sqrt{fx}\,\Gamma\left(\frac{1}{2},fx\right)\right)}{(dx)^{3/2}}$$

input
```
Integrate[Cosh[f*x]/(d*x)^(3/2),x]
```

output
```
(x*(-1 - E^(2*f*x) + E^(f*x)*Sqrt[-(f*x)]*Gamma[1/2, -(f*x)] + E^(f*x)*Sqr
t[f*x]*Gamma[1/2, f*x]))/(E^(f*x)*(d*x)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3778, 26, 3042, 26, 3789, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ifx\right)}{(dx)^{3/2}}dx$$

$$\Big\downarrow \; 3778$$

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}+\frac{2if\int -\frac{i\sinh(fx)}{\sqrt{dx}}dx}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{2f\int \frac{\sinh(fx)}{\sqrt{dx}}dx}{d}-\frac{2\cosh(fx)}{d\sqrt{dx}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}+\frac{2f\int -\frac{i\sin(ifx)}{\sqrt{dx}}dx}{d}$$

$$\Big\downarrow \; 26$$

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\int \frac{\sin(ifx)}{\sqrt{dx}}dx}{d}$$

$$\Big\downarrow \; 3789$$

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{1}{2}i\int \frac{e^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int \frac{e^{-fx}}{\sqrt{dx}}dx\right)}{d}$$

$$\Big\downarrow \; 2611$$

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\int e^{fx}d\sqrt{dx}}{d}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{d}$$

$$\Big\downarrow \ 2633$$

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\int e^{-fx}d\sqrt{dx}}{d}\right)}{d}$$

$$\Big\downarrow \ 2634$$

$$-\frac{2\cosh(fx)}{d\sqrt{dx}}-\frac{2if\left(\frac{i\sqrt{\pi}\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}-\frac{i\sqrt{\pi}\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}$$

input
```
Int[Cosh[f*x]/(d*x)^(3/2),x]
```

output
```
(-2*Cosh[f*x])/(d*Sqrt[d*x]) - ((2*I)*f*(((-1/2*I)*Sqrt[Pi]*Erf[(Sqrt[f]*S
qrt[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f]) + ((I/2)*Sqrt[Pi]*Erfi[(Sqrt[f]*Sqrt
[d*x])/Sqrt[d]])/(Sqrt[d]*Sqrt[f])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3789
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[I
/2   Int[(c + d*x)^m/E^(I*(e + f*x)), x], x] - Simp[I/2   Int[(c + d*x)^m*E
^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e, f, m}, x]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

method	result	size
meijerg	$-\frac{i\sqrt{\pi}\, x^{\frac{3}{2}} \sqrt{2}\, (if)^{\frac{3}{2}} \left(-\frac{2\sqrt{2}\, e^{fx}}{\sqrt{\pi}\, \sqrt{x}\, \sqrt{if}}-\frac{2\sqrt{2}\, e^{-fx}}{\sqrt{\pi}\, \sqrt{x}\, \sqrt{if}}-\frac{2\sqrt{2}\, \sqrt{f}\, \operatorname{erf}\left(\sqrt{x}\, \sqrt{f}\right)}{\sqrt{if}}+\frac{2\sqrt{2}\, \sqrt{f}\, \operatorname{erfi}\left(\sqrt{x}\, \sqrt{f}\right)}{\sqrt{if}}\right)}{4(dx)^{\frac{3}{2}} f}$	115

input
```
int(cosh(f*x)/(d*x)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*I*Pi^(1/2)/(d*x)^(3/2)*x^(3/2)*2^(1/2)*(I*f)^(3/2)/f*(-2/Pi^(1/2)/x^(
1/2)*2^(1/2)/(I*f)^(1/2)*exp(f*x)-2/Pi^(1/2)/x^(1/2)*2^(1/2)/(I*f)^(1/2)*e
xp(-f*x)-2/(I*f)^(1/2)*2^(1/2)*f^(1/2)*erf(x^(1/2)*f^(1/2))+2/(I*f)^(1/2)*
2^(1/2)*f^(1/2)*erfi(x^(1/2)*f^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(62) = 124$.

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.55

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = -\frac{\sqrt{\pi}(dx\cosh(fx) + dx\sinh(fx))\sqrt{\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) + \sqrt{\pi}(dx\cosh(fx) + dx\sinh(fx))\sqrt{-\frac{f}{d}}\operatorname{erf}\left(\sqrt{dx}\right.}{d^2x\cosh(fx) + d^2x\sinh(fx)}$$

input
```
integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="fricas")
```

output
```
-(sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(f/d)*erf(sqrt(d*x)*sqrt(f/
d)) + sqrt(pi)*(d*x*cosh(f*x) + d*x*sinh(f*x))*sqrt(-f/d)*erf(sqrt(d*x)*sq
rt(-f/d)) + sqrt(d*x)*(cosh(f*x)^2 + 2*cosh(f*x)*sinh(f*x) + sinh(f*x)^2 +
 1))/(d^2*x*cosh(f*x) + d^2*x*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = -\frac{\sqrt{2}\sqrt{\pi}\sqrt{f}e^{-\frac{3i\pi}{4}}S\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\Gamma\left(\frac{3}{4}\right)} + \frac{\cosh(fx)\Gamma\left(-\frac{1}{4}\right)}{2d^{\frac{3}{2}}\sqrt{x}\Gamma\left(\frac{3}{4}\right)}$$

input
```
integrate(cosh(f*x)/(d*x)**(3/2),x)
```

output
```
-sqrt(2)*sqrt(pi)*sqrt(f)*exp(-3*I*pi/4)*fresnels(sqrt(2)*sqrt(f)*sqrt(x)*
exp(I*pi/4)/sqrt(pi))*gamma(-1/4)/(2*d**(3/2)*gamma(3/4)) + cosh(f*x)*gamm
a(-1/4)/(2*d**(3/2)*sqrt(x)*gamma(3/4))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = -\frac{\frac{f\left(\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right)}{\sqrt{\frac{f}{d}}}-\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{-\frac{f}{d}}\right)}{\sqrt{-\frac{f}{d}}}\right)}{d}+\frac{2\cosh(fx)}{\sqrt{dx}}}{d}$$

input
```
integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="maxima")
```

output
```
-(f*(sqrt(pi)*erf(sqrt(d*x)*sqrt(f/d))/sqrt(f/d) - sqrt(pi)*erf(sqrt(d*x)*
sqrt(-f/d))/sqrt(-f/d))/d + 2*cosh(f*x)/sqrt(d*x))/d
```

Giac [F]

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = \int \frac{\cosh(fx)}{(dx)^{\frac{3}{2}}}\,dx$$

input
```
integrate(cosh(f*x)/(d*x)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x)/(d*x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = \int \frac{\cosh(f\,x)}{(d\,x)^{3/2}}\,dx$$

input
```
int(cosh(f*x)/(d*x)^(3/2),x)
```

output
```
int(cosh(f*x)/(d*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{\cosh(fx)}{(dx)^{3/2}}\,dx = \frac{\int \frac{\cosh(fx)}{\sqrt{x}\,x}dx}{\sqrt{d}\,d}$$

input
```
int(cosh(f*x)/(d*x)^(3/2),x)
```

output
```
int(cosh(f*x)/(sqrt(x)*x),x)/(sqrt(d)*d)
```

3.67 $\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 114

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = -\frac{2\cosh(fx)}{3d(dx)^{3/2}} + \frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} + \frac{2f^{3/2}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{3d^{5/2}} - \frac{4f\sinh(fx)}{3d^2\sqrt{dx}}$$

output
```
-2/3*cosh(f*x)/d/(d*x)^(3/2)+2/3*f^(3/2)*Pi^(1/2)*erf(f^(1/2)*(d*x)^(1/2)/
d^(1/2))/d^(5/2)+2/3*f^(3/2)*Pi^(1/2)*erfi(f^(1/2)*(d*x)^(1/2)/d^(1/2))/d^
(5/2)-4/3*f*sinh(f*x)/d^2/(d*x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.68

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = \frac{x\left(-2e^{fx}(1+2fx) - 4(-fx)^{3/2}\Gamma\left(\frac{1}{2},-fx\right) + e^{-fx}\left(-2+4fx-4e^{fx}(fx)^{3/2}\Gamma\left(\frac{1}{2},fx\right)\right)\right)}{6(dx)^{5/2}}$$

input
```
Integrate[Cosh[f*x]/(d*x)^(5/2),x]
```

output
```
(x*(-2*E^(f*x)*(1 + 2*f*x) - 4*(-(f*x))^(3/2)*Gamma[1/2, -(f*x)] + (-2 + 4
*f*x - 4*E^(f*x)*(f*x)^(3/2)*Gamma[1/2, f*x])/E^(f*x)))/(6*(d*x)^(5/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.53 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.17, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3778, 26, 3042, 26, 3778, 3042, 3788, 26, 2611, 2633, 2634}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ifx\right)}{(dx)^{5/2}}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}+\frac{2if\int -\frac{i\sinh(fx)}{(dx)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{26}$$

$$\frac{2f\int \frac{\sinh(fx)}{(dx)^{3/2}}dx}{3d}-\frac{2\cosh(fx)}{3d(dx)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}+\frac{2f\int -\frac{i\sin(ifx)}{(dx)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\int \frac{\sin(ifx)}{(dx)^{3/2}}dx}{3d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\left(\frac{2if\int\frac{\cosh(fx)}{\sqrt{dx}}dx}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)}{3d}$$

$\downarrow$ 3042

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\left(\frac{2if\int\frac{\sin\left(ifx+\frac{\pi}{2}\right)}{\sqrt{dx}}dx}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)}{3d}$$

$\downarrow$ 3788

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\left(\frac{2if\left(\frac{1}{2}i\int-\frac{ie^{fx}}{\sqrt{dx}}dx-\frac{1}{2}i\int\frac{ie^{-fx}}{\sqrt{dx}}dx\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)}{3d}$$

$\downarrow$ 26

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\left(\frac{2if\left(\frac{1}{2}\int\frac{e^{-fx}}{\sqrt{dx}}dx+\frac{1}{2}\int\frac{e^{fx}}{\sqrt{dx}}dx\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)}{3d}$$

$\downarrow$ 2611

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\left(\frac{2if\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\int e^{fx}d\sqrt{dx}}{d}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)}{3d}$$

$\downarrow$ 2633

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\left(\frac{2if\left(\frac{\int e^{-fx}d\sqrt{dx}}{d}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)}{3d}$$

$\downarrow$ 2634

$$-\frac{2\cosh(fx)}{3d(dx)^{3/2}}-\frac{2if\left(\frac{2if\left(\frac{\sqrt{\pi}\,\text{erf}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}+\frac{\sqrt{\pi}\,\text{erfi}\left(\frac{\sqrt{f}\sqrt{dx}}{\sqrt{d}}\right)}{2\sqrt{d}\sqrt{f}}\right)}{d}-\frac{2i\sinh(fx)}{d\sqrt{dx}}\right)}{3d}$$

input
```
Int[Cosh[f*x]/(d*x)^(5/2),x]
```

output
```
(-2*Cosh[f*x])/(3*d*(d*x)^(3/2)) - (((2*I)/3)*f*(((2*I)*f*((Sqrt[Pi]*Erf[(
Sqrt[f]*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f]) + (Sqrt[Pi]*Erfi[(Sqrt[f]
*Sqrt[d*x])/Sqrt[d]])/(2*Sqrt[d]*Sqrt[f])))/d - ((2*I)*Sinh[f*x])/(d*Sqrt[
d*x])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2611
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/Sqrt[(c_.) + (d_.)*(x_)], x_Symbol] :
> Simp[2/d   Subst[Int[F^(g*(e - c*(f/d)) + f*g*(x^2/d)), x], x, Sqrt[c + d
*x]], x] /; FreeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2633
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erfi[(c + d*x)*Rt[b*Log[F], 2]]/(2*d*Rt[b*Log[F], 2])), x] /; FreeQ[{
F, a, b, c, d}, x] && PosQ[b]
```

rule 2634
```
Int[(F_)^((a_.) + (b_.)*((c_.) + (d_.)*(x_))^2), x_Symbol] :> Simp[F^a*Sqrt
[Pi]*(Erf[(c + d*x)*Rt[(-b)*Log[F], 2]]/(2*d*Rt[(-b)*Log[F], 2])), x] /; Fr
eeQ[{F, a, b, c, d}, x] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.11

method	result	size
meijerg	$-\frac{i\sqrt{\pi}\,x^{\frac{5}{2}}\sqrt{2}\,(if)^{\frac{5}{2}}\left(-\frac{8\sqrt{2}\left(-fx+\frac{1}{2}\right){\mathrm e}^{-fx}}{3\sqrt{\pi}\,x^{\frac{3}{2}}(if)^{\frac{3}{2}}}-\frac{8\sqrt{2}\left(fx+\frac{1}{2}\right){\mathrm e}^{fx}}{3\sqrt{\pi}\,x^{\frac{3}{2}}(if)^{\frac{3}{2}}}+\frac{8\sqrt{2}\,f^{\frac{3}{2}}\,\mathrm{erf}\left(\sqrt{x}\,\sqrt{f}\right)}{3(if)^{\frac{3}{2}}}+\frac{8\sqrt{2}\,f^{\frac{3}{2}}\,\mathrm{erfi}\left(\sqrt{x}\,\sqrt{f}\right)}{3(if)^{\frac{3}{2}}}\right)}{8(dx)^{\frac{5}{2}}f}$	126

input
```
int(cosh(f*x)/(d*x)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/8*I*Pi^(1/2)/(d*x)^(5/2)*x^(5/2)*2^(1/2)*(I*f)^(5/2)/f*(-8/3/Pi^(1/2)/x
^(3/2)*2^(1/2)/(I*f)^(3/2)*(-f*x+1/2)*exp(-f*x)-8/3/Pi^(1/2)/x^(3/2)*2^(1/
2)/(I*f)^(3/2)*(f*x+1/2)*exp(f*x)+8/3/(I*f)^(3/2)*2^(1/2)*f^(3/2)*erf(x^(1
/2)*f^(1/2))+8/3/(I*f)^(3/2)*2^(1/2)*f^(3/2)*erfi(x^(1/2)*f^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(78) = 156.

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.57

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = \frac{2\sqrt{\pi}(dfx^2\cosh(fx) + dfx^2\sinh(fx))\sqrt{\frac{f}{d}}\,\mathrm{erf}\left(\sqrt{dx}\sqrt{\frac{f}{d}}\right) - 2\sqrt{\pi}(dfx^2\cosh(fx) + dfx$$

input
```
integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="fricas")
```

output
```
1/3*(2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt(f/d)*erf(sqrt
(d*x)*sqrt(f/d)) - 2*sqrt(pi)*(d*f*x^2*cosh(f*x) + d*f*x^2*sinh(f*x))*sqrt
(-f/d)*erf(sqrt(d*x)*sqrt(-f/d)) - ((2*f*x + 1)*cosh(f*x)^2 + 2*(2*f*x + 1
)*cosh(f*x)*sinh(f*x) + (2*f*x + 1)*sinh(f*x)^2 - 2*f*x + 1)*sqrt(d*x))/(d
^3*x^2*cosh(f*x) + d^3*x^2*sinh(f*x))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 10.79 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.09

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = -\frac{\sqrt{2}\sqrt{\pi}f^{\frac{3}{2}}e^{-\frac{i\pi}{4}}C\left(\frac{\sqrt{2}\sqrt{f}\sqrt{x}e^{\frac{i\pi}{4}}}{\sqrt{\pi}}\right)\Gamma\left(-\frac{3}{4}\right)}{d^{\frac{5}{2}}\Gamma\left(\frac{1}{4}\right)} + \frac{f\sinh\left(fx\right)\Gamma\left(-\frac{3}{4}\right)}{d^{\frac{5}{2}}\sqrt{x}\Gamma\left(\frac{1}{4}\right)} + \frac{\cosh\left(fx\right)\Gamma\left(-\frac{3}{4}\right)}{2d^{\frac{5}{2}}x^{\frac{3}{2}}\Gamma\left(\frac{1}{4}\right)}$$

input
```
integrate(cosh(f*x)/(d*x)**(5/2),x)
```

output
```
-sqrt(2)*sqrt(pi)*f**(3/2)*exp(-I*pi/4)*fresnelc(sqrt(2)*sqrt(f)*sqrt(x)*e
xp(I*pi/4)/sqrt(pi))*gamma(-3/4)/(d**(5/2)*gamma(1/4)) + f*sinh(f*x)*gamma
(-3/4)/(d**(5/2)*sqrt(x)*gamma(1/4)) + cosh(f*x)*gamma(-3/4)/(2*d**(5/2)*x
**(3/2)*gamma(1/4))
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.51

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = \frac{\frac{f\left(\frac{\sqrt{fx}\Gamma\left(-\frac{1}{2},fx\right)}{\sqrt{dx}} - \frac{\sqrt{-fx}\Gamma\left(-\frac{1}{2},-fx\right)}{\sqrt{dx}}\right)}{d} - \frac{2\cosh(fx)}{(dx)^{\frac{3}{2}}}}{3\,d}$$

input
```
integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="maxima")
```

output
```
1/3*(f*(sqrt(f*x)*gamma(-1/2, f*x)/sqrt(d*x) - sqrt(-f*x)*gamma(-1/2, -f*x
)/sqrt(d*x))/d - 2*cosh(f*x)/(d*x)^(3/2))/d
```

Giac [F]

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = \int \frac{\cosh(fx)}{(dx)^{\frac{5}{2}}}\,dx$$

input
```
integrate(cosh(f*x)/(d*x)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(f*x)/(d*x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = \int \frac{\cosh(f\,x)}{(d\,x)^{5/2}}\,dx$$

input
```
int(cosh(f*x)/(d*x)^(5/2),x)
```

output
```
int(cosh(f*x)/(d*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{\cosh(fx)}{(dx)^{5/2}}\,dx = \frac{\int \frac{\cosh(fx)}{\sqrt{x}\,x^2}dx}{\sqrt{d}\,d^2}$$

input
```
int(cosh(f*x)/(d*x)^(5/2),x)
```

output
```
int(cosh(f*x)/(sqrt(x)*x**2),x)/(sqrt(d)*d**2)
```

3.68 $\int \sqrt{c+dx}\,\mathbf{sech}(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx = \mathrm{Int}\Big(\sqrt{c+dx}\,\mathrm{sech}(a+bx),x\Big)$$

output
```
Defer(Int)((d*x+c)^(1/2)*sech(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 3.06 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx = \int \sqrt{c+dx}\,\mathrm{sech}(a+bx)\,dx$$

input
```
Integrate[Sqrt[c + d*x]*Sech[a + b*x],x]
```

output
```
Integrate[Sqrt[c + d*x]*Sech[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{c+dx}\text{sech}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{c+dx}\csc\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{4680}$$

$$\int \sqrt{c+dx}\text{sech}(a+bx)dx$$

input
```
Int[Sqrt[c + d*x]*Sech[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{dx+c}\,\text{sech}\,(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*sech(b*x+a),x)
```

output
```
int((d*x+c)^(1/2)*sech(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\text{sech}(a+bx)\,dx=\int \sqrt{dx+c}\,\text{sech}\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^(1/2)*sech(b*x+a),x, algorithm="fricas")
```

output
```
integral(sqrt(d*x + c)*sech(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\text{sech}(a+bx)\,dx=\int \sqrt{c+dx}\,\text{sech}\left(a+bx\right)dx$$

input
```
integrate((d*x+c)**(1/2)*sech(b*x+a),x)
```

output
```
Integral(sqrt(c + d*x)*sech(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\text{sech}(a+bx)\,dx=\int \sqrt{dx+c}\,\text{sech}\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^(1/2)*sech(b*x+a),x, algorithm="maxima")
```

output
```
integrate(sqrt(d*x + c)*sech(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \sqrt{c+dx}\text{sech}(a+bx)\,dx = \int \sqrt{dx+c}\,\text{sech}\left(bx+a\right)\,dx$$

input
```
integrate((d*x+c)^(1/2)*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate(sqrt(d*x + c)*sech(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \sqrt{c+dx}\text{sech}(a+bx)\,dx = \int \frac{\sqrt{c+d\,x}}{\cosh\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)^(1/2)/cosh(a + b*x),x)
```

output
```
int((c + d*x)^(1/2)/cosh(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \sqrt{c+dx}\operatorname{sech}(a+bx)\,dx = \int \sqrt{dx+c}\,\operatorname{sech}(bx+a)\,dx$$

input
```
int((d*x+c)^(1/2)*sech(b*x+a),x)
```

output
```
int(sqrt(c + d*x)*sech(a + b*x),x)
```

3.69 $\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}},x\right)$$

output
```
Defer(Int)(sech(b*x+a)/(d*x+c)^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 4.46 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx = \int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$

input
```
Integrate[Sech[a + b*x]/Sqrt[c + d*x],x]
```

output
```
Integrate[Sech[a + b*x]/Sqrt[c + d*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\csc\left(ia+ibx+\frac{\pi}{2}\right)}{\sqrt{c+dx}}dx$$
$$\Big\downarrow \text{ 4680}$$
$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}dx$$

input
```
Int[Sech[a + b*x]/Sqrt[c + d*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{sech}(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(sech(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(sech(b*x+a)/(d*x+c)^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="fricas")
```

output
```
integral(sech(b*x + a)/sqrt(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.88 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{sech}\left(a+bx\right)}{\sqrt{c+dx}}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)**(1/2),x)
```

output
```
Integral(sech(a + b*x)/sqrt(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.93 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sech(b*x + a)/sqrt(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{sech}\left(bx+a\right)}{\sqrt{dx+c}}\,dx$$

input
```
integrate(sech(b*x+a)/(d*x+c)^(1/2),x, algorithm="giac")
```

output
```
integrate(sech(b*x + a)/sqrt(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{1}{\cosh\left(a+b\,x\right)\,\sqrt{c+d\,x}}\,dx$$

input
```
int(1/(cosh(a + b*x)*(c + d*x)^(1/2)),x)
```

output
```
int(1/(cosh(a + b*x)*(c + d*x)^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}(a+bx)}{\sqrt{c+dx}}\,dx=\int \frac{\operatorname{sech}(bx+a)}{\sqrt{dx+c}}dx$$

input
```
int(sech(b*x+a)/(d*x+c)^(1/2),x)
```

output
```
int(sech(a + b*x)/sqrt(c + d*x),x)
```

3.70 $\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx = -\frac{\cosh^{\frac{3}{2}}(x)}{2x^2} - \frac{3\sqrt{\cosh(x)}\sinh(x)}{4x} - \frac{3}{8}\text{Int}\left(\frac{1}{x\sqrt{\cosh(x)}},x\right) + \frac{9}{8}\text{Int}\left(\frac{\cosh^{\frac{3}{2}}(x)}{x},x\right)$$

output
```
-1/2*cosh(x)^(3/2)/x^2-3/4*cosh(x)^(1/2)*sinh(x)/x-3/8*Defer(Int)(1/x/cosh
(x)^(1/2),x)+9/8*Defer(Int)(cosh(x)^(3/2)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.44 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx$$

input
```
Integrate[Cosh[x]^(3/2)/x^3,x]
```

output
```
Integrate[Cosh[x]^(3/2)/x^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$\frac{9}{8}\int \frac{\cosh^{\frac{3}{2}}(x)}{x}dx-\frac{3}{8}\int \frac{1}{x\sqrt{\cosh(x)}}dx-\frac{\cosh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sinh(x)\sqrt{\cosh(x)}}{4x}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3}{8}\int \frac{1}{x\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx+\frac{9}{8}\int \frac{\sin\left(ix+\frac{\pi}{2}\right)^{3/2}}{x}dx-\frac{\cosh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sinh(x)\sqrt{\cosh(x)}}{4x}$$

$$\Big\downarrow \text{3807}$$

$$\frac{9}{8}\int \frac{\cosh^{\frac{3}{2}}(x)}{x}dx-\frac{3}{8}\int \frac{1}{x\sqrt{\cosh(x)}}dx-\frac{\cosh^{\frac{3}{2}}(x)}{2x^2}-\frac{3\sinh(x)\sqrt{\cosh(x)}}{4x}$$

input
```
Int[Cosh[x]^(3/2)/x^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\cosh\left(x\right)^{\frac{3}{2}}}{x^{3}}dx$$

input
```
int(cosh(x)^(3/2)/x^3,x)
```

output
```
int(cosh(x)^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(x)^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 40.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cosh^{\frac{3}{2}}\left(x\right)}{x^{3}}\,dx$$

input
```
integrate(cosh(x)**(3/2)/x**3,x)
```

output
```
Integral(cosh(x)**(3/2)/x**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cosh{(x)}^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(cosh(x)^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(cosh(x)^(3/2)/x^3, x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3}\,dx = \int \frac{\cosh{(x)}^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate(cosh(x)^(3/2)/x^3,x, algorithm="giac")
```

output
```
integrate(cosh(x)^(3/2)/x^3, x)
```

Mupad [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} \, dx = \int \frac{\cosh(x)^{3/2}}{x^3} \, dx$$

input
```
int(cosh(x)^(3/2)/x^3,x)
```

output
```
int(cosh(x)^(3/2)/x^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

$$\int \frac{\cosh^{\frac{3}{2}}(x)}{x^3} \, dx = \int \frac{\sqrt{\cosh(x)}\,\cosh(x)}{x^3} dx$$

input
```
int(cosh(x)^(3/2)/x^3,x)
```

output
```
int((sqrt(cosh(x))*cosh(x))/x**3,x)
```

3.71 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=-4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}$$

output
```
-4*cosh(x)^(1/2)+2*x*sinh(x)/cosh(x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

Time = 0.29 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=\frac{2\sinh(x)\left(x-\frac{2\cosh(x)\sinh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(-1+\cosh(x))^{3/2}\sqrt{1+\cosh(x)}}\right)}{\sqrt{\cosh(x)}}$$

input
```
Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

output
```
(2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2
)*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

input
```
Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

output
```
-4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\cosh(x)^{\frac{3}{2}}} + x\sqrt{\cosh(x)}\right) dx$$

input
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = \int \frac{x(\cosh^2(x) + 1)}{\cosh^{\frac{3}{2}}(x)}\, dx$$

input
```
integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
```

output
```
Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=\int x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=\int x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=-\frac{2\sqrt{\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}}\,\left(x+2\,\mathrm{e}^{2\,x}-x\,\mathrm{e}^{2\,x}+2\right)}{\mathrm{e}^{2\,x}+1}$$

input
```
int(x*cosh(x)^(1/2) + x/cosh(x)^(3/2),x)
```

output
```
-(2*(exp(-x)/2 + exp(x)/2)^(1/2)*(x + 2*exp(2*x) - x*exp(2*x) + 2))/(exp(2
*x) + 1)
```

Reduce [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right) dx = \int \frac{\sqrt{\cosh (x)}\, x}{\cosh (x)^{2}}dx+\int \sqrt{\cosh (x)}\, xdx$$

input
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

output
```
int((sqrt(cosh(x))*x)/cosh(x)**2,x) + int(sqrt(cosh(x))*x,x)
```

3.72 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \frac{4}{3\sqrt{\cosh(x)}} + \frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

output
```
4/3/cosh(x)^(1/2)+2/3*x*sinh(x)/cosh(x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \frac{2(2 + x\tanh(x))}{3\sqrt{\cosh(x)}}$$

input
```
Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

output
```
(2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

output
```
4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\cosh\left(x\right)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\cosh\left(x\right)}}\right)dx$$

input
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(16) = 32$.

Time = 0.08 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.54

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \frac{4\left((x+2)\cosh(x)^3 + 3(x+2)\cosh(x)\sinh(x)^2 + (x+2)\sinh(x)^3 - (x-2)\cosh(x) + (3(x+2)\cosh\right.}{3\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 2\left(3\cosh(x)^2+1\right)\sinh(x)^2 + 2\cosh(x)^2 + 4(\cosh\right.}$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
 (x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2
 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = -\frac{\int \left(-\frac{3x}{\cosh^{\frac{5}{2}}(x)}\right) dx + \int \frac{x}{\sqrt{\cosh(x)}}\, dx}{3}$$

input
```
integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

output
```
-(Integral(-3*x/cosh(x)**(5/2), x) + Integral(x/sqrt(cosh(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \frac{4\,e^{x}\,\sqrt{\frac{e^{-x}}{2} + \frac{e^{x}}{2}}\,\left(2\,e^{2\,x} - x + x\,e^{2\,x} + 2\right)}{3\,\left(e^{2\,x} + 1\right)^2}$$

input
```
int(x/cosh(x)^(5/2) - x/(3*cosh(x)^(1/2)),x)
```

output
```
(4*exp(x)*(exp(-x)/2 + exp(x)/2)^(1/2)*(2*exp(2*x) - x + x*exp(2*x) + 2))/
(3*(exp(2*x) + 1)^2)
```

Reduce [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = -\frac{\left(\int \frac{\sqrt{\cosh(x)}\,x}{\cosh(x)} dx\right)}{3} + \int \frac{\sqrt{\cosh(x)}\,x}{\cosh(x)^3} dx$$

input
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

output
```
( - int((sqrt(cosh(x))*x)/cosh(x),x) + 3*int((sqrt(cosh(x))*x)/cosh(x)**3,
x))/3
```

3.73 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

output
```
4/15/cosh(x)^(3/2)-12/5*cosh(x)^(1/2)+2/5*x*sinh(x)/cosh(x)^(5/2)+6/5*x*si
nh(x)/cosh(x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cosh(x)}\right) dx = \frac{1}{5}\sqrt{\cosh(x)}\left(-\frac{12\sinh^2(x)}{\sqrt{-1+\cosh(x)}(1+\cosh(x))^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}} + 6x\tanh(x) + \operatorname{sech}^2(x)\left(\frac{4}{3} + 2x\tanh(x)\right)\right)$$

input
```
Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

output
```
(Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sq
rt[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)} + \frac{3}{5}x\sqrt{\cosh(x)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)} - \frac{12\sqrt{\cosh(x)}}{5} + \frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)} + \frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

input
```
Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

output
```
4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/
2)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}+\frac{3x\sqrt{\cosh\left(x\right)}}{5}\right)dx$$

input
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\text{Timed out}$$

input
```
integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.34

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right) dx = \frac{e^{2x}\left(\frac{8x}{5}+\frac{16}{15}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{\left(e^{2x}+1\right)^2} - \left(\frac{6x}{5}+\frac{12}{5}\right)\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}} + \frac{12x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\left(e^{2x}+1\right)} - \frac{16x\,e^{2x}\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\left(e^{2x}+1\right)^3}$$

input
```
int((3*x*cosh(x)^(1/2))/5 + x/cosh(x)^(7/2),x)
```

output
```
(exp(2*x)*((8*x)/5 + 16/15)*(exp(-x)/2 + exp(x)/2)^(1/2))/(exp(2*x) + 1)^2
 - ((6*x)/5 + 12/5)*(exp(-x)/2 + exp(x)/2)^(1/2) + (12*x*exp(2*x)*(exp(-x)
/2 + exp(x)/2)^(1/2))/(5*(exp(2*x) + 1)) - (16*x*exp(2*x)*(exp(-x)/2 + exp
(x)/2)^(1/2))/(5*(exp(2*x) + 1)^3)
```

Reduce [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right) dx = \int \frac{\sqrt{\cosh(x)}\,x}{\cosh(x)^4}dx + \frac{3\left(\int \sqrt{\cosh(x)}\,x dx\right)}{5}$$

input
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

output
```
(5*int((sqrt(cosh(x))*x)/cosh(x)**4,x) + 3*int(sqrt(cosh(x))*x,x))/5
```

3.74 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

output
```
-8*x*cosh(x)^(1/2)-16*I*EllipticE(I*sinh(1/2*x),2^(1/2))+2*x^2*sinh(x)/cos
h(x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.66 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx$$
$$=\frac{4\sqrt{\cosh(x)}(\cosh(x)+\sinh(x))\left(-4(-2+x)\cosh(x)+x^2\sinh(x)+8\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\right.\right.}{1+e^{2x}}$$

input
```
Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

output
```
(4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] +
8*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1
+ Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)$$

input
```
Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

output
```
-8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[C
osh[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x^2}{\cosh (x)^{\frac{3}{2}}}+x^2\sqrt{\cosh (x)}\right)dx$$

input
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

output
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx = \int \frac{x^2(\cosh^2(x)+1)}{\cosh^{\frac{3}{2}}(x)}\,dx$$

input
```
integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
```

output
```
Integral(x**2*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=\int x^2\sqrt{\cosh{(x)}}+\frac{x^2}{\cosh{(x)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=\int x^2\sqrt{\cosh{(x)}}+\frac{x^2}{\cosh{(x)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=\int x^2\,\sqrt{\cosh{(x)}}+\frac{x^2}{{\cosh{(x)}}^{3/2}}\,dx$$

input
```
int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2),x)
```

output
```
int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)}\right) dx = \int \frac{\sqrt{\cosh(x)}\, x^2}{\cosh(x)^2} dx + \int \sqrt{\cosh(x)}\, x^2 dx$$

input
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

output
```
int((sqrt(cosh(x))*x**2)/cosh(x)**2,x) + int(sqrt(cosh(x))*x**2,x)
```

3.75 $\int (c+dx)^m (b\cosh(e+fx))^n\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m (b\cosh(e+fx))^n\,dx = \mathrm{Int}\left((c+dx)^m (b\cosh(e+fx))^n, x\right)$$

output
```
Defer(Int)((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 2.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cosh(e+fx))^n\,dx = \int (c+dx)^m (b\cosh(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(b*Cosh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(b*Cosh[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(b\cosh(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\left(b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3807}$$

$$\int (c+dx)^m(b\cosh(e+fx))^n dx$$

input
```
Int[(c + d*x)^m*(b*Cosh[e + f*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m\,(b\cosh(fx+e))^n\,dx$$

input
```
int((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\cosh(e+fx))^n\,dx = \int (dx+c)^m(b\cosh(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Sympy [N/A]

Not integrable

Time = 8.98 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int (c+dx)^m(b\cosh(e+fx))^n\,dx = \int (b\cosh(e+fx))^n\,(c+dx)^m\,dx$$

input
```
integrate((d*x+c)**m*(b*cosh(f*x+e))**n,x)
```

output
```
Integral((b*cosh(e + f*x))**n*(c + d*x)**m, x)
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(b\cosh(e+fx))^n\,dx = \int (dx+c)^m(b\cosh(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cosh(e+fx))^n\,dx = \int (dx+c)^m (b\cosh(fx+e))^n\,dx$$

input
```
integrate((d*x+c)^m*(b*cosh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*cosh(f*x + e))^n, x)
```

Mupad [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m (b\cosh(e+fx))^n\,dx = \int (b\cosh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((b*cosh(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((b*cosh(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int (c+dx)^m (b\cosh(e+fx))^n\,dx = b^n \left(\int (dx+c)^m \cosh\left(fx+e\right)^n dx\right)$$

input
```
int((d*x+c)^m*(b*cosh(f*x+e))^n,x)
```

output
```
b**n*int((c + d*x)**m*cosh(e + f*x)**n,x)
```

3.76 $\int (c+dx)^m \cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^m \cosh^3(a+bx)\,dx &= \frac{3^{-1-m}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)}{8b}\\ &\quad+\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{8b}\\ &\quad-\frac{3e^{-a+\frac{bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{8b}\\ &\quad-\frac{3^{-1-m}e^{-3a+\frac{3bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3b(c+dx)}{d}\right)}{8b}\end{aligned}$$

output
```
1/8*3^(-1-m)*exp(3*a-3*b*c/d)*(d*x+c)^m*GAMMA(1+m,-3*b*(d*x+c)/d)/b/((-b*(
d*x+c)/d)^m)+3/8*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*
x+c)/d)^m)-3/8*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c
)/d)^m)-1/8*3^(-1-m)*exp(-3*a+3*b*c/d)*(d*x+c)^m*GAMMA(1+m,3*b*(d*x+c)/d)/
b/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.86

$$\int (c+dx)^m \cosh^3(a+bx)\,dx$$
$$= \frac{3^{-1-m}e^{-3\left(a+\frac{bc}{d}\right)}(c+dx)^m\left(-\frac{b^2(c+dx)^2}{d^2}\right)^{-m}\left(e^{6a}\left(b\left(\frac{c}{d}+x\right)\right)^m\Gamma\left(1+m,-\frac{3b(c+dx)}{d}\right)+3^{2+m}e^{4a+\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right.\right.\right.}{8b}$$

input
```
Integrate[(c + d*x)^m*Cosh[a + b*x]^3,x]
```

output
```
(3^(-1 - m)*(c + d*x)^m*(E^(6*a)*(b*(c/d + x))^m*Gamma[1 + m, (-3*b*(c + d
*x))/d] + 3^(2 + m)*E^(4*a + (2*b*c)/d)*(b*(c/d + x))^m*Gamma[1 + m, -((b*
(c + d*x))/d)] - E^((4*b*c)/d)*(-((b*(c + d*x))/d))^m*(3^(2 + m)*E^(2*a)*G
amma[1 + m, (b*(c + d*x))/d] + E^((2*b*c)/d)*Gamma[1 + m, (3*b*(c + d*x))/
d])))/(8*b*E^(3*(a + (b*c)/d))*(-((b^2*(c + d*x)^2)/d^2))^m)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)(c+dx)^m\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^3(c+dx)^m dx$$
$$\Big\downarrow \ 3793$$
$$\int \left(\frac{3}{4}\cosh(a+bx)(c+dx)^m+\frac{1}{4}\cosh(3a+3bx)(c+dx)^m\right)dx$$
$$\Big\downarrow \ 2009$$

$$\frac{3^{-m-1}e^{3a-\frac{3bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3b(c+dx)}{d}\right)}{8b}+\frac{3e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{b(c+dx)}{d}\right)}{8b}-\frac{3e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{b(c+dx)}{d}\right)}{8b}-\frac{3^{-m-1}e^{\frac{3bc}{d}-3a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3b(c+dx)}{d}\right)}{8b}$$

input
```
Int[(c + d*x)^m*Cosh[a + b*x]^3,x]
```

output
```
(3^(-1 - m)*E^(3*a - (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (-3*b*(c + d*x))/
d])/(8*b*(-((b*(c + d*x))/d))^m) + (3*E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1
+ m, -((b*(c + d*x))/d)])/(8*b*(-((b*(c + d*x))/d))^m) - (3*E^(-a + (b*c)/
d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x))/d])/(8*b*((b*(c + d*x))/d)^m) -
(3^(-1 - m)*E^(-3*a + (3*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (3*b*(c + d*x))/
d])/(8*b*((b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^m \cosh(bx+a)^3 \, dx$$

input
```
int((d*x+c)^m*cosh(b*x+a)^3,x)
```

output
```
int((d*x+c)^m*cosh(b*x+a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 340, normalized size of antiderivative = 1.43

$$\int (c+dx)^m \cosh^3(a+bx) \, dx =$$

$$-\frac{\cosh\left(\frac{dm\log\left(\frac{3b}{d}\right)-3bc+3ad}{d}\right)\Gamma\left(m+1,\frac{3(bdx+bc)}{d}\right)+9\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-9\cosh}{}$$

input
```
integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/24*(cosh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d)*gamma(m + 1, 3*(b*d*x + b*
c)/d) + 9*cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d)
 - 9*cosh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) -
cosh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d)*gamma(m + 1, -3*(b*d*x + b*c)/d)
 - gamma(m + 1, 3*(b*d*x + b*c)/d)*sinh((d*m*log(3*b/d) - 3*b*c + 3*a*d)/d
) - 9*gamma(m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + 9
*gamma(m + 1, -(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d) + gamm
a(m + 1, -3*(b*d*x + b*c)/d)*sinh((d*m*log(-3*b/d) + 3*b*c - 3*a*d)/d))/b
```

Sympy [F]

$$\int (c+dx)^m \cosh^3(a+bx)\, dx = \int (c+dx)^m \cosh^3(a+bx)\, dx$$

input
```
integrate((d*x+c)**m*cosh(b*x+a)**3,x)
```

output
```
Integral((c + d*x)**m*cosh(a + b*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.68

$$\int (c+dx)^m \cosh^3(a+bx)\, dx = -\frac{(dx+c)^{m+1} e^{\left(-3\,a+\frac{3\,bc}{d}\right)} E_{-m}\left(\frac{3\,(dx+c)b}{d}\right)}{8\,d} - \frac{3\,(dx+c)^{m+1} e^{\left(-a+\frac{bc}{d}\right)} E_{-m}\left(\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{3\,(dx+c)^{m+1} e^{\left(a-\frac{bc}{d}\right)} E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{8\,d} - \frac{(dx+c)^{m+1} e^{\left(3\,a-\frac{3\,bc}{d}\right)} E_{-m}\left(-\frac{3\,(dx+c)b}{d}\right)}{8\,d}$$

input
```
integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
-1/8*(d*x + c)^(m + 1)*e^(-3*a + 3*b*c/d)*exp_integral_e(-m, 3*(d*x + c)*b
/d)/d - 3/8*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*
b/d)/d - 3/8*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)
*b/d)/d - 1/8*(d*x + c)^(m + 1)*e^(3*a - 3*b*c/d)*exp_integral_e(-m, -3*(d
*x + c)*b/d)/d
```

Giac [F]

$$\int (c+dx)^m \cosh^3(a+bx)\,dx = \int (dx+c)^m \cosh{(bx+a)}^3\,dx$$

input
```
integrate((d*x+c)^m*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cosh(b*x + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cosh^3(a+bx)\,dx = \int \cosh(a+b\,x)^3\,(c+d\,x)^m\,dx$$

input
```
int(cosh(a + b*x)^3*(c + d*x)^m,x)
```

output
```
int(cosh(a + b*x)^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cosh^3(a+bx)\,dx = \frac{e^{6bx+6a}(dx+c)^m+9e^{4bx+4a}(dx+c)^m-9e^{2bx+2a}(dx+c)^m-(dx+c)^m-e^{3bx+6a}\left(\int \frac{e^{3bx}(dx+c)^m}{dx+c}dx\right)dm}{24e^{3bx+}}$$

input
```
int((d*x+c)^m*cosh(b*x+a)^3,x)
```

output
```
(e**(6*a + 6*b*x)*(c + d*x)**m + 9*e**(4*a + 4*b*x)*(c + d*x)**m - 9*e**(2
*a + 2*b*x)*(c + d*x)**m - (c + d*x)**m - e**(6*a + 3*b*x)*int((e**(3*b*x)
*(c + d*x)**m)/(c + d*x),x)*d*m - 9*e**(4*a + 3*b*x)*int((e**(b*x)*(c + d*
x)**m)/(c + d*x),x)*d*m + e**(3*a + 3*b*x)*int((c + d*x)**m/(e**(3*a + 3*b
*x)*c + e**(3*a + 3*b*x)*d*x),x)*d*m + 9*e**(2*a + 3*b*x)*int((c + d*x)**m
/(e**(b*x)*c + e**(b*x)*d*x),x)*d*m)/(24*e**(3*a + 3*b*x)*b)
```

3.77 $\int (c+dx)^m \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 144

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = \frac{(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-3-m}e^{-2a+\frac{2bc}{d}}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}$$

output
```
1/2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*exp(2*a-2*b*c/d)*(d*x+c)^m*GAMMA(1+m,-2
*b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-2^(-3-m)*exp(-2*a+2*b*c/d)*(d*x+c)^m*GA
MMA(1+m,2*b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.92

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = \frac{1}{8}(c+dx)^m \left(\frac{4c+4dx}{d+dm} + \frac{2^{-m}e^{2a-\frac{2bc}{d}}\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2b(c+dx)}{d}\right)}{b} - \frac{2^{-m}e^{-2a+\frac{2bc}{d}}\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2b(c+dx)}{d}\right)}{b}\right)$$

input
```
Integrate[(c + d*x)^m*Cosh[a + b*x]^2,x]
```

output
```
((c + d*x)^m*((4*c + 4*d*x)/(d + d*m) + (E^(2*a - (2*b*c)/d)*Gamma[1 + m,
(-2*b*(c + d*x))/d])/(2^m*b*(-((b*(c + d*x))/d))^m) - (E^(-2*a + (2*b*c)/d
)*Gamma[1 + m, (2*b*(c + d*x))/d])/(2^m*b*((b*(c + d*x))/d)^m)))/8
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2(c+dx)^m dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2}\cosh(2a+2bx)(c+dx)^m+\frac{1}{2}(c+dx)^m\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2^{-m-3}e^{2a-\frac{2bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2b(c+dx)}{d}\right)}{b}-\frac{2^{-m-3}e^{\frac{2bc}{d}-2a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2b(c+dx)}{d}\right)}{b}+\frac{(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*Cosh[a + b*x]^2,x]
```

output
```
(c + d*x)^(1 + m)/(2*d*(1 + m)) + (2^(-3 - m)*E^(2*a - (2*b*c)/d)*(c + d*x
)^m*Gamma[1 + m, (-2*b*(c + d*x))/d])/(b*(-((b*(c + d*x))/d))^m) - (2^(-3
- m)*E^(-2*a + (2*b*c)/d)*(c + d*x)^m*Gamma[1 + m, (2*b*(c + d*x))/d])/(b*
((b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int (dx+c)^m \cosh(bx+a)^2\, dx$$

input
```
int((d*x+c)^m*cosh(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.67

$$\int (c+dx)^m \cosh^2(a+bx)\, dx = -\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2b}{d}\right)-2bc+2ad}{d}\right)\Gamma\left(m+1,\frac{2(bdx+bc)}{d}\right)-(dm+d)\cosh\left(\frac{dm\log\left(-\frac{2b}{d}\right)+2bc-2ad}{d}\right)\Gamma\left(m\right.}{}$$

input
```
integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/8*((d*m + d)*cosh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d)*gamma(m + 1, 2*(b
*d*x + b*c)/d) - (d*m + d)*cosh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d)*gamma
(m + 1, -2*(b*d*x + b*c)/d) - (d*m + d)*gamma(m + 1, 2*(b*d*x + b*c)/d)*si
nh((d*m*log(2*b/d) - 2*b*c + 2*a*d)/d) + (d*m + d)*gamma(m + 1, -2*(b*d*x
+ b*c)/d)*sinh((d*m*log(-2*b/d) + 2*b*c - 2*a*d)/d) - 4*(b*d*x + b*c)*cosh
(m*log(d*x + c)) - 4*(b*d*x + b*c)*sinh(m*log(d*x + c)))/(b*d*m + b*d)
```

Sympy [F]

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = \int (c+dx)^m \cosh^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*cosh(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.71

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = -\frac{(dx+c)^{m+1}e^{\left(-2a+\frac{2bc}{d}\right)}E_{-m}\left(\frac{2(dx+c)b}{d}\right)}{4d} - \frac{(dx+c)^{m+1}e^{\left(2a-\frac{2bc}{d}\right)}E_{-m}\left(-\frac{2(dx+c)b}{d}\right)}{4d} + \frac{(dx+c)^{m+1}}{2d(m+1)}$$

input
```
integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(d*x + c)^(m + 1)*e^(-2*a + 2*b*c/d)*exp_integral_e(-m, 2*(d*x + c)*b
/d)/d - 1/4*(d*x + c)^(m + 1)*e^(2*a - 2*b*c/d)*exp_integral_e(-m, -2*(d*x
 + c)*b/d)/d + 1/2*(d*x + c)^(m + 1)/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = \int (dx+c)^m \cosh(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cosh^2(a+bx)\,dx = \int \cosh(a+b\,x)^2\,(c+d\,x)^m\,dx$$

input
```
int(cosh(a + b*x)^2*(c + d*x)^m,x)
```

output
```
int(cosh(a + b*x)^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cosh^2(a+bx)\,dx$$
$$= \frac{e^{4bx+4a}(dx+c)^m dm + e^{4bx+4a}(dx+c)^m d + 4e^{2bx+2a}(dx+c)^m bc + 4e^{2bx+2a}(dx+c)^m bdx - (dx+c)^m c}{}$$

input
```
int((d*x+c)^m*cosh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x)*(c + d*x)**m*d*m + e**(4*a + 4*b*x)*(c + d*x)**m*d + 4*e
**(2*a + 2*b*x)*(c + d*x)**m*b*c + 4*e**(2*a + 2*b*x)*(c + d*x)**m*b*d*x -
 (c + d*x)**m*d*m - (c + d*x)**m*d - e**(4*a + 2*b*x)*int((e**(2*b*x)*(c +
 d*x)**m)/(c + d*x),x)*d**2*m**2 - e**(4*a + 2*b*x)*int((e**(2*b*x)*(c + d
*x)**m)/(c + d*x),x)*d**2*m + e**(2*a + 2*b*x)*int((c + d*x)**m/(e**(2*a +
 2*b*x)*c + e**(2*a + 2*b*x)*d*x),x)*d**2*m**2 + e**(2*a + 2*b*x)*int((c +
 d*x)**m/(e**(2*a + 2*b*x)*c + e**(2*a + 2*b*x)*d*x),x)*d**2*m)/(8*e**(2*a
 + 2*b*x)*b*d*(m + 1))
```

3.78 $\int (c+dx)^m \cosh(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int (c+dx)^m \cosh(a+bx)\,dx = \frac{e^{a-\frac{bc}{d}}(c+dx)^m \left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{b(c+dx)}{d}\right)}{2b} - \frac{e^{-a+\frac{bc}{d}}(c+dx)^m \left(\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{b(c+dx)}{d}\right)}{2b}$$

output
```
1/2*exp(a-b*c/d)*(d*x+c)^m*GAMMA(1+m,-b*(d*x+c)/d)/b/((-b*(d*x+c)/d)^m)-1/
2*exp(-a+b*c/d)*(d*x+c)^m*GAMMA(1+m,b*(d*x+c)/d)/b/((b*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int (c+dx)^m \cosh(a+bx)\,dx = \frac{e^{-a-\frac{bc}{d}}(c+dx)^m \left(e^{2a}\left(-\frac{b(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{b(c+dx)}{d}\right) - e^{\frac{2bc}{d}}\left(b\left(\frac{c}{d}+x\right)\right)^{-m} \Gamma\left(1+m,\frac{b(c+dx)}{d}\right)\right)}{2b}$$

input
```
Integrate[(c + d*x)^m*Cosh[a + b*x],x]
```

output
```
(E^(-a - (b*c)/d)*(c + d*x)^m*((E^(2*a)*Gamma[1 + m, -((b*(c + d*x))/d)])/
(-((b*(c + d*x))/d))^m - (E^((2*b*c)/d)*Gamma[1 + m, (b*(c + d*x))/d])/(b*
(c/d + x))^m))/(2*b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)(c+dx)^m dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}(c+dx)^m dx-\frac{1}{2}i\int ie^{-a-bx}(c+dx)^m dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}(c+dx)^m dx+\frac{1}{2}\int e^{a+bx}(c+dx)^m dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{e^{a-\frac{bc}{d}}(c+dx)^m\left(-\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{b(c+dx)}{d}\right)}{2b}-\frac{e^{\frac{bc}{d}-a}(c+dx)^m\left(\frac{b(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{b(c+dx)}{d}\right)}{2b}$$

input
```
Int[(c + d*x)^m*Cosh[a + b*x],x]
```

output
```
(E^(a - (b*c)/d)*(c + d*x)^m*Gamma[1 + m, -((b*(c + d*x))/d)])/(2*b*(-((b*
(c + d*x))/d))^m) - (E^(-a + (b*c)/d)*(c + d*x)^m*Gamma[1 + m, (b*(c + d*x
))/d])/(2*b*((b*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [F]

$$\int (dx + c)^m \cosh(bx + a) \, dx$$

input
```
int((d*x+c)^m*cosh(b*x+a),x)
```

output
```
int((d*x+c)^m*cosh(b*x+a),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.53

$$\int (c+dx)^m \cosh(a+bx)\,dx =$$

$$-\frac{\cosh\left(\frac{dm\log\left(\frac{b}{d}\right)-bc+ad}{d}\right)\Gamma\left(m+1,\frac{bdx+bc}{d}\right)-\cosh\left(\frac{dm\log\left(-\frac{b}{d}\right)+bc-ad}{d}\right)\Gamma\left(m+1,-\frac{bdx+bc}{d}\right)-\Gamma\left(m+1,\frac{bd\cdots}{}\right.}{2b}$$

input
```
integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh((d*m*log(b/d) - b*c + a*d)/d)*gamma(m + 1, (b*d*x + b*c)/d) - c
osh((d*m*log(-b/d) + b*c - a*d)/d)*gamma(m + 1, -(b*d*x + b*c)/d) - gamma(
m + 1, (b*d*x + b*c)/d)*sinh((d*m*log(b/d) - b*c + a*d)/d) + gamma(m + 1,
-(b*d*x + b*c)/d)*sinh((d*m*log(-b/d) + b*c - a*d)/d))/b
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m \cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.72

$$\int (c+dx)^m \cosh(a+bx)\,dx = -\frac{(dx+c)^{m+1}e^{\left(-a+\frac{bc}{d}\right)}E_{-m}\left(\frac{(dx+c)b}{d}\right)}{2\,d} - \frac{(dx+c)^{m+1}e^{\left(a-\frac{bc}{d}\right)}E_{-m}\left(-\frac{(dx+c)b}{d}\right)}{2\,d}$$

input
```
integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(d*x + c)^(m + 1)*e^(-a + b*c/d)*exp_integral_e(-m, (d*x + c)*b/d)/d
- 1/2*(d*x + c)^(m + 1)*e^(a - b*c/d)*exp_integral_e(-m, -(d*x + c)*b/d)/d
```

Giac [F]

$$\int (c+dx)^m \cosh(a+bx)\,dx = \int (dx+c)^m \cosh(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m \cosh(a+bx)\,dx = \int \cosh(a+b\,x)\,(c+d\,x)^m\,dx$$

input
```
int(cosh(a + b*x)*(c + d*x)^m,x)
```

output
```
int(cosh(a + b*x)*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m \cosh(a+bx)\,dx$$
$$= \frac{e^{2bx+2a}(dx+c)^m - (dx+c)^m - e^{bx+2a}\left(\int \frac{e^{bx}(dx+c)^m}{dx+c}dx\right)dm + e^{bx}\left(\int \frac{(dx+c)^m}{e^{bx}c+e^{bx}dx}dx\right)dm}{2e^{bx+a}b}$$

input
```
int((d*x+c)^m*cosh(b*x+a),x)
```

output
```
(e**(2*a + 2*b*x)*(c + d*x)**m - (c + d*x)**m - e**(2*a + b*x)*int((e**(b*
x)*(c + d*x)**m)/(c + d*x),x)*d*m + e**(b*x)*int((c + d*x)**m/(e**(b*x)*c
+ e**(b*x)*d*x),x)*d*m)/(2*e**(a + b*x)*b)
```

3.79 $\int (c+dx)^m \operatorname{sech}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int (c+dx)^m \operatorname{sech}(a+bx)\,dx = \operatorname{Int}((c+dx)^m \operatorname{sech}(a+bx),x)$$

output
```
Defer(Int)((d*x+c)^m*sech(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 4.24 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{sech}(a+bx)\,dx = \int (c+dx)^m \operatorname{sech}(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sech[a + b*x],x]
```

output
```
Integrate[(c + d*x)^m*Sech[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)(c+dx)^m dx$$

$$\Big\downarrow \text{4680}$$

$$\int \operatorname{sech}(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sech[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \operatorname{sech}(bx+a)\,dx$$

input
```
int((d*x+c)^m*sech(b*x+a),x)
```

output
```
int((d*x+c)^m*sech(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{sech}(a+bx)\,dx = \int (dx+c)^m \operatorname{sech}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sech(b*x+a),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sech(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int (c+dx)^m \operatorname{sech}(a+bx)\,dx = \int (c+dx)^m \operatorname{sech}(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sech(b*x+a),x)
```

output
```
Integral((c + d*x)**m*sech(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \operatorname{sech}(a+bx)\,dx = \int (dx+c)^m \operatorname{sech}(bx+a)\,dx$$

input
```
integrate((d*x+c)^m*sech(b*x+a),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sech(b*x + a), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \text{sech}(a+bx)\,dx=\int (dx+c)^m\,\text{sech}\,(bx+a)\;dx$$

input
```
integrate((d*x+c)^m*sech(b*x+a),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sech(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int (c+dx)^m \text{sech}(a+bx)\,dx=\int \frac{(c+d\,x)^m}{\cosh\left(a+b\,x\right)}\,dx$$

input
```
int((c + d*x)^m/cosh(a + b*x),x)
```

output
```
int((c + d*x)^m/cosh(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int (c+dx)^m \text{sech}(a+bx)\,dx = \int (dx+c)^m\,\text{sech}(bx+a)\,dx$$

input
```
int((d*x+c)^m*sech(b*x+a),x)
```

output
```
int((c + d*x)**m*sech(a + b*x),x)
```

3.80 $\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx = \operatorname{Int}\left((c+dx)^m \operatorname{sech}^2(a+bx),x\right)$$

output
```
Defer(Int)((d*x+c)^m*sech(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 2.74 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx = \int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx$$

input
```
Integrate[(c + d*x)^m*Sech[a + b*x]^2,x]
```

output
```
Integrate[(c + d*x)^m*Sech[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(a+bx)(c+dx)^m\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \csc\left(ia+ibx+\frac{\pi}{2}\right)^2(c+dx)^m dx$$

$$\Big\downarrow \ 4680$$

$$\int \operatorname{sech}^2(a+bx)(c+dx)^m dx$$

input
```
Int[(c + d*x)^m*Sech[a + b*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int (dx+c)^m \operatorname{sech}(bx+a)^2\,dx$$

input
```
int((d*x+c)^m*sech(b*x+a)^2,x)
```

output
```
int((d*x+c)^m*sech(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*sech(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 4.28 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx = \int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate((d*x+c)**m*sech(b*x+a)**2,x)
```

output
```
Integral((c + d*x)**m*sech(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \operatorname{sech}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*sech(b*x + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \text{sech}^2(a+bx)\,dx = \int (dx+c)^m \operatorname{sech}(bx+a)^2\,dx$$

input
```
integrate((d*x+c)^m*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*sech(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int (c+dx)^m \text{sech}^2(a+bx)\,dx = \int \frac{(c+d\,x)^m}{\cosh(a+b\,x)^2}\,dx$$

input
```
int((c + d*x)^m/cosh(a + b*x)^2,x)
```

output
```
int((c + d*x)^m/cosh(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 153, normalized size of antiderivative = 9.56

$$\int (c+dx)^m \text{sech}^2(a+bx)\,dx$$
$$= \frac{2e^{2a}\left(e^{2bx}(dx+c)^m - e^{2bx+2a}\left(\int \frac{e^{2bx}(dx+c)^m}{e^{2bx+2a}c+e^{2bx+2a}dx+c+dx}dx\right)dm - \left(\int \frac{e^{2bx}(dx+c)^m}{e^{2bx+2a}c+e^{2bx+2a}dx+c+dx}dx\right)dm\right)}{b\left(e^{2bx+2a}+1\right)}$$

input
```
int((d*x+c)^m*sech(b*x+a)^2,x)
```

output
```
(2*e**(2*a)*(e**(2*b*x)*(c + d*x)**m - e**(2*a + 2*b*x)*int((e**(2*b*x)*(c
 + d*x)**m)/(e**(2*a + 2*b*x)*c + e**(2*a + 2*b*x)*d*x + c + d*x),x)*d*m -
 int((e**(2*b*x)*(c + d*x)**m)/(e**(2*a + 2*b*x)*c + e**(2*a + 2*b*x)*d*x
+ c + d*x),x)*d*m))/(b*(e**(2*a + 2*b*x) + 1))
```

3.81 $\int x^{3+m}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{3+m}\cosh(a+bx)\,dx=-\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)}{2b^4}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}$$

output
```
-1/2*exp(a)*x^m*GAMMA(4+m,-b*x)/b^4/((-b*x)^m)-1/2*x^m*GAMMA(4+m,b*x)/b^4/
exp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{3+m}\cosh(a+bx)\,dx=-\frac{e^a x^m(-bx)^{-m}\Gamma(4+m,-bx)+e^{-a}x^m(bx)^{-m}\Gamma(4+m,bx)}{2b^4}$$

input
```
Integrate[x^(3 + m)*Cosh[a + b*x],x]
```

output
```
-1/2*((E^a*x^m*Gamma[4 + m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[4 + m, b*x])/
(E^a*(b*x)^m))/b^4
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+3}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}x^{m+3}dx-\frac{1}{2}i\int ie^{-a-bx}x^{m+3}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^{m+3}dx+\frac{1}{2}\int e^{a+bx}x^{m+3}dx$$

$$\Big\downarrow \text{2612}$$

$$-\frac{e^a x^m(-bx)^{-m}\Gamma(m+4,-bx)}{2b^4}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+4,bx)}{2b^4}$$

input
```
Int[x^(3 + m)*Cosh[a + b*x],x]
```

output
```
-1/2*(E^a*x^m*Gamma[4 + m, -(b*x)])/(b^4*(-(b*x))^m) - (x^m*Gamma[4 + m, b
*x])/(2*b^4*E^a*(b*x)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{4+m} \operatorname{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{1}{2},3+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \cosh\left(a\right)}{4+m}+\frac{b\,x^{5+m} \operatorname{hypergeom}\left(\left[\frac{5}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{7}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \sinh\left(a\right)}{5+m}$	73

input
```
int(x^(3+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[1/2,3+1/2*m],1/4*x^2*b^2)*cosh(a)+b/(
5+m)*x^(5+m)*hypergeom([5/2+1/2*m],[3/2,7/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{3+m}\cosh(a+bx)\,dx = -\frac{\cosh\left((m+3)\log\left(b\right)+a\right)\Gamma(m+4,bx) - \cosh\left((m+3)\log\left(-b\right)-a\right)\Gamma(m+4,-bx) + \Gamma(m+4,-bx}{2\,b}$$

input
```
integrate(x^(3+m)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh((m + 3)*log(b) + a)*gamma(m + 4, b*x) - cosh((m + 3)*log(-b) -
a)*gamma(m + 4, -b*x) + gamma(m + 4, -b*x)*sinh((m + 3)*log(-b) - a) - gam
ma(m + 4, b*x)*sinh((m + 3)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{3+m}\cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(3+m)*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{3+m}\cosh(a+bx)\,dx = -\frac{1}{2}\,(bx)^{-m-4}\,x^{m+4}e^{(-a)}\Gamma(m+4,bx) - \frac{1}{2}\,(-bx)^{-m-4}\,x^{m+4}e^{a}\Gamma(m+4,-bx)$$

input
```
integrate(x^(3+m)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(b*x)^(-m - 4)*x^(m + 4)*e^(-a)*gamma(m + 4, b*x) - 1/2*(-b*x)^(-m -
4)*x^(m + 4)*e^a*gamma(m + 4, -b*x)
```

Giac [F]

$$\int x^{3+m}\cosh(a+bx)\,dx=\int x^{m+3}\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x^(3+m)*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\cosh(a+bx)\,dx=\int x^{m+3}\cosh(a+b\,x)\,dx$$

input
```
int(x^(m + 3)*cosh(a + b*x),x)
```

output
```
int(x^(m + 3)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^{3+m}\cosh(a+bx)\,dx$$
$$=\frac{-x^m\cosh\left(bx+a\right)b^2m\,x^2-3x^m\cosh\left(bx+a\right)b^2x^2-x^m\cosh\left(bx+a\right)m^3-6x^m\cosh\left(bx+a\right)m^2-11}{}$$

input
```
int(x^(3+m)*cosh(b*x+a),x)
```

output
```
( - x**m*cosh(a + b*x)*b**2*m*x**2 - 3*x**m*cosh(a + b*x)*b**2*x**2 - x**m
*cosh(a + b*x)*m**3 - 6*x**m*cosh(a + b*x)*m**2 - 11*x**m*cosh(a + b*x)*m
- 6*x**m*cosh(a + b*x) + x**m*sinh(a + b*x)*b**3*x**3 + x**m*sinh(a + b*x)
*b*m**2*x + 5*x**m*sinh(a + b*x)*b*m*x + 6*x**m*sinh(a + b*x)*b*x + int((x
**m*cosh(a + b*x))/x,x)*m**4 + 6*int((x**m*cosh(a + b*x))/x,x)*m**3 + 11*i
nt((x**m*cosh(a + b*x))/x,x)*m**2 + 6*int((x**m*cosh(a + b*x))/x,x)*m)/b**
4
```

3.82 $\int x^{2+m}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{2+m}\cosh(a+bx)\,dx=\frac{e^a x^m(-bx)^{-m}\Gamma(3+m,-bx)}{2b^3}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(3+m,bx)}{2b^3}$$

output
```
1/2*exp(a)*x^m*GAMMA(3+m,-b*x)/b^3/((-b*x)^m)-1/2*x^m*GAMMA(3+m,b*x)/b^3/e
xp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{2+m}\cosh(a+bx)\,dx=\frac{e^{-a}x^m\left(e^{2a}(-bx)^{-m}\Gamma(3+m,-bx)-(bx)^{-m}\Gamma(3+m,bx)\right)}{2b^3}$$

input
```
Integrate[x^(2 + m)*Cosh[a + b*x],x]
```

output
```
(x^m*((E^(2*a)*Gamma[3 + m, -(b*x)])/(-(b*x))^m - Gamma[3 + m, b*x]/(b*x)^
m))/(2*b^3*E^a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}x^{m+2}dx-\frac{1}{2}i\int ie^{-a-bx}x^{m+2}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^{m+2}dx+\frac{1}{2}\int e^{a+bx}x^{m+2}dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{e^a x^m(-bx)^{-m}\Gamma(m+3,-bx)}{2b^3}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+3,bx)}{2b^3}$$

input
```
Int[x^(2 + m)*Cosh[a + b*x],x]
```

output
```
(E^a*x^m*Gamma[3 + m, -(b*x)])/(2*b^3*(-(b*x))^m) - (x^m*Gamma[3 + m, b*x]
)/(2*b^3*E^a*(b*x)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.37 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{3+m} \operatorname{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{5}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \cosh \left(a\right)}{3+m}+\frac{b\,x^{4+m} \operatorname{hypergeom}\left(\left[2+\frac{m}{2}\right],\left[\frac{3}{2},3+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \sinh \left(a\right)}{4+m}$	73

input
```
int(x^(2+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[1/2,5/2+1/2*m],1/4*x^2*b^2)*cosh(a)
+b/(4+m)*x^(4+m)*hypergeom([2+1/2*m],[3/2,3+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{2+m}\cosh(a+bx)\,dx = -\frac{\cosh\left((m+2)\log\left(b\right)+a\right)\Gamma(m+3,bx)-\cosh\left((m+2)\log\left(-b\right)-a\right)\Gamma(m+3,-bx)+\Gamma(m+3,-bx}{2\,b}$$

input
```
integrate(x^(2+m)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh((m + 2)*log(b) + a)*gamma(m + 3, b*x) - cosh((m + 2)*log(-b) -
a)*gamma(m + 3, -b*x) + gamma(m + 3, -b*x)*sinh((m + 2)*log(-b) - a) - gam
ma(m + 3, b*x)*sinh((m + 2)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{2+m}\cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(2+m)*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{2+m}\cosh(a+bx)\,dx = -\frac{1}{2}\,(bx)^{-m-3}\,x^{m+3}e^{(-a)}\Gamma(m+3,bx) - \frac{1}{2}\,(-bx)^{-m-3}\,x^{m+3}e^{a}\Gamma(m+3,-bx)$$

input
```
integrate(x^(2+m)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(b*x)^(-m - 3)*x^(m + 3)*e^(-a)*gamma(m + 3, b*x) - 1/2*(-b*x)^(-m -
3)*x^(m + 3)*e^a*gamma(m + 3, -b*x)
```

Giac [F]

$$\int x^{2+m}\cosh(a+bx)\,dx=\int x^{m+2}\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x^(2+m)*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\cosh(a+bx)\,dx=\int x^{m+2}\cosh(a+b\,x)\,dx$$

input
```
int(x^(m + 2)*cosh(a + b*x),x)
```

output
```
int(x^(m + 2)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^{2+m}\cosh(a+bx)\,dx$$
$$=\frac{x^me^{2bx+2a}b^2x^2-x^me^{2bx+2a}bmx-2x^me^{2bx+2a}bx+x^me^{2bx+2a}m^2+3x^me^{2bx+2a}m+2x^me^{2bx+2a}-e^{bx+2a}\Big(}{}$$

input
```
int(x^(2+m)*cosh(b*x+a),x)
```

output
```
(x**m*e**(2*a + 2*b*x)*b**2*x**2 - x**m*e**(2*a + 2*b*x)*b*m*x - 2*x**m*e*
*(2*a + 2*b*x)*b*x + x**m*e**(2*a + 2*b*x)*m**2 + 3*x**m*e**(2*a + 2*b*x)*
m + 2*x**m*e**(2*a + 2*b*x) - e**(2*a + b*x)*int((x**m*e**(b*x))/x,x)*m**3
 - 3*e**(2*a + b*x)*int((x**m*e**(b*x))/x,x)*m**2 - 2*e**(2*a + b*x)*int((
x**m*e**(b*x))/x,x)*m + e**(b*x)*int(x**m/(e**(b*x)*x),x)*m**3 + 3*e**(b*x
)*int(x**m/(e**(b*x)*x),x)*m**2 + 2*e**(b*x)*int(x**m/(e**(b*x)*x),x)*m -
x**m*b**2*x**2 - x**m*b*m*x - 2*x**m*b*x - x**m*m**2 - 3*x**m*m - 2*x**m)/
(2*e**(a + b*x)*b**3)
```

3.83 $\int x^{1+m}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{1+m}\cosh(a+bx)\,dx = -\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx)}{2b^2} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}$$

output
```
-1/2*exp(a)*x^m*GAMMA(2+m,-b*x)/b^2/((-b*x)^m)-1/2*x^m*GAMMA(2+m,b*x)/b^2/
exp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^{1+m}\cosh(a+bx)\,dx = -\frac{e^a x^m(-bx)^{-m}\Gamma(2+m,-bx) + e^{-a}x^m(bx)^{-m}\Gamma(2+m,bx)}{2b^2}$$

input
```
Integrate[x^(1 + m)*Cosh[a + b*x],x]
```

output
```
-1/2*((E^a*x^m*Gamma[2 + m, -(b*x)])/(-(b*x))^m + (x^m*Gamma[2 + m, b*x])/
(E^a*(b*x)^m))/b^2
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+1}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}x^{m+1}dx-\frac{1}{2}i\int ie^{-a-bx}x^{m+1}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^{m+1}dx+\frac{1}{2}\int e^{a+bx}x^{m+1}dx$$

$$\Big\downarrow \text{2612}$$

$$-\frac{e^a x^m(-bx)^{-m}\Gamma(m+2,-bx)}{2b^2}-\frac{e^{-a}x^m(bx)^{-m}\Gamma(m+2,bx)}{2b^2}$$

input
```
Int[x^(1 + m)*Cosh[a + b*x],x]
```

output
```
-1/2*(E^a*x^m*Gamma[2 + m, -(b*x)])/(b^2*(-(b*x))^m) - (x^m*Gamma[2 + m, b
*x])/(2*b^2*E^a*(b*x)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.34 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{2+m} \operatorname{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{1}{2},2+\frac{m}{2}\right],\frac{x^2 b^2}{4}\right) \cosh(a)}{2+m}+\frac{b\,x^{3+m} \operatorname{hypergeom}\left(\left[\frac{3}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{5}{2}+\frac{m}{2}\right],\frac{x^2 b^2}{4}\right) \sinh(a)}{3+m}$	73

input
```
int(x^(1+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[1/2,2+1/2*m],1/4*x^2*b^2)*cosh(a)+b/(
3+m)*x^(3+m)*hypergeom([3/2+1/2*m],[3/2,5/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{1+m}\cosh(a+bx)\,dx = -\frac{\cosh\left((m+1)\log\left(b\right)+a\right)\Gamma(m+2,bx)-\cosh\left((m+1)\log\left(-b\right)-a\right)\Gamma(m+2,-bx)+\Gamma(m+2,-bx}{2\,b}$$

input
```
integrate(x^(1+m)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh((m + 1)*log(b) + a)*gamma(m + 2, b*x) - cosh((m + 1)*log(-b) -
a)*gamma(m + 2, -b*x) + gamma(m + 2, -b*x)*sinh((m + 1)*log(-b) - a) - gam
ma(m + 2, b*x)*sinh((m + 1)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{1+m}\cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(1+m)*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{1+m}\cosh(a+bx)\,dx = -\frac{1}{2}\,(bx)^{-m-2}\,x^{m+2}e^{(-a)}\Gamma(m+2,bx) - \frac{1}{2}\,(-bx)^{-m-2}\,x^{m+2}e^{a}\Gamma(m+2,-bx)$$

input
```
integrate(x^(1+m)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(b*x)^(-m - 2)*x^(m + 2)*e^(-a)*gamma(m + 2, b*x) - 1/2*(-b*x)^(-m -
2)*x^(m + 2)*e^a*gamma(m + 2, -b*x)
```

Giac [F]

$$\int x^{1+m}\cosh(a+bx)\,dx=\int x^{m+1}\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x^(1+m)*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m}\cosh(a+bx)\,dx=\int x^{m+1}\cosh(a+b\,x)\,dx$$

input
```
int(x^(m + 1)*cosh(a + b*x),x)
```

output
```
int(x^(m + 1)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^{1+m}\cosh(a+bx)\,dx$$
$$=\frac{-x^{m}\cosh\left(bx+a\right)m-x^{m}\cosh\left(bx+a\right)+x^{m}\sinh\left(bx+a\right)bx+\left(\int\frac{x^{m}\cosh(bx+a)}{x}dx\right)m^{2}+\left(\int\frac{x^{m}\cosh(bx+}{x}\right.}{b^{2}}$$

input
```
int(x^(1+m)*cosh(b*x+a),x)
```

output
```
( - x**m*cosh(a + b*x)*m - x**m*cosh(a + b*x) + x**m*sinh(a + b*x)*b*x + i
nt((x**m*cosh(a + b*x))/x,x)*m**2 + int((x**m*cosh(a + b*x))/x,x)*m)/b**2
```

3.84 $\int x^m \cosh(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 59

$$\int x^m \cosh(a+bx)\,dx = \frac{e^a x^m (-bx)^{-m}\Gamma(1+m,-bx)}{2b} - \frac{e^{-a} x^m (bx)^{-m}\Gamma(1+m,bx)}{2b}$$

output
```
1/2*exp(a)*x^m*GAMMA(1+m,-b*x)/b/((-b*x)^m)-1/2*x^m*GAMMA(1+m,b*x)/b/exp(a
)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int x^m \cosh(a+bx)\,dx = \frac{e^{-a} x^m \left(e^{2a}(-bx)^{-m}\Gamma(1+m,-bx) - (bx)^{-m}\Gamma(1+m,bx)\right)}{2b}$$

input
```
Integrate[x^m*Cosh[a + b*x],x]
```

output
```
(x^m*((E^(2*a)*Gamma[1 + m, -(b*x)])/(-(b*x))^m - Gamma[1 + m, b*x]/(b*x)^
m))/(2*b*E^a)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}x^m dx - \frac{1}{2}i\int ie^{-a-bx}x^m dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^m dx + \frac{1}{2}\int e^{a+bx}x^m dx$$

$$\Big\downarrow \text{2612}$$

$$\frac{e^a x^m(-bx)^{-m}\Gamma(m+1,-bx)}{2b} - \frac{e^{-a}x^m(bx)^{-m}\Gamma(m+1,bx)}{2b}$$

input
```
Int[x^m*Cosh[a + b*x],x]
```

output
```
(E^a*x^m*Gamma[1 + m, -(b*x)])/(2*b*(-(b*x))^m) - (x^m*Gamma[1 + m, b*x])/
(2*b*E^a*(b*x)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.32 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24

method	result	size
meijerg	$\frac{x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{x^2 b^2}{4}\right) \cosh(a)}{1+m}+\frac{b\, x^{2+m} \operatorname{hypergeom}\left(\left[1+\frac{m}{2}\right],\left[\frac{3}{2},2+\frac{m}{2}\right],\frac{x^2 b^2}{4}\right) \sinh(a)}{2+m}$	73

input
```
int(x^m*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(1+m)*x^(1+m)*hypergeom([1/2+1/2*m],[1/2,3/2+1/2*m],1/4*x^2*b^2)*cosh(a)
+b/(2+m)*x^(2+m)*hypergeom([1+1/2*m],[3/2,2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.32

$$\int x^m \cosh(a+bx)\,dx =$$
$$-\frac{\cosh\left(m\log\left(b\right)+a\right)\Gamma(m+1,bx)-\cosh\left(m\log\left(-b\right)-a\right)\Gamma(m+1,-bx)+\Gamma(m+1,-bx)\sinh\left(m\log\right.}{2\,b}$$

input
```
integrate(x^m*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh(m*log(b) + a)*gamma(m + 1, b*x) - cosh(m*log(-b) - a)*gamma(m +
 1, -b*x) + gamma(m + 1, -b*x)*sinh(m*log(-b) - a) - gamma(m + 1, b*x)*sin
h(m*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^m \cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**m*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^m \cosh(a+bx)\,dx = -\frac{1}{2}\left(bx\right)^{-m-1}x^{m+1}e^{(-a)}\Gamma(m+1,bx) - \frac{1}{2}\left(-bx\right)^{-m-1}x^{m+1}e^{a}\Gamma(m+1,-bx)$$

input
```
integrate(x^m*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(b*x)^(-m - 1)*x^(m + 1)*e^(-a)*gamma(m + 1, b*x) - 1/2*(-b*x)^(-m -
1)*x^(m + 1)*e^a*gamma(m + 1, -b*x)
```

Giac [F]

$$\int x^m \cosh(a+bx)\,dx = \int x^m \cosh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh(a+bx)\,dx = \int x^m \cosh(a+b\,x)\,dx$$

input
```
int(x^m*cosh(a + b*x),x)
```

output
```
int(x^m*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^m \cosh(a+bx)\,dx = \frac{x^m e^{2bx+2a} - e^{bx+2a}\left(\int \frac{x^m e^{bx}}{x}dx\right)m + e^{bx}\left(\int \frac{x^m}{e^{bx}x}dx\right)m - x^m}{2e^{bx+a}b}$$

input
```
int(x^m*cosh(b*x+a),x)
```

output
```
(x**m*e**(2*a + 2*b*x) - e**(2*a + b*x)*int((x**m*e**(b*x))/x,x)*m + e**(b
*x)*int(x**m/(e**(b*x)*x),x)*m - x**m)/(2*e**(a + b*x)*b)
```

3.85 $\int x^{-1+m}\cosh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 49

$$\int x^{-1+m}\cosh(a+bx)\,dx=-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)$$

output
```
-1/2*exp(a)*x^m*GAMMA(m,-b*x)/((-b*x)^m)-1/2*x^m*GAMMA(m,b*x)/exp(a)/((b*x
)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int x^{-1+m}\cosh(a+bx)\,dx=-\frac{1}{2}e^a x^m(-bx)^{-m}\Gamma(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)$$

input
```
Integrate[x^(-1 + m)*Cosh[a + b*x],x]
```

output
```
-1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m - (x^m*Gamma[m, b*x])/(2*E^a*(b
*x)^m)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-1}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}x^{m-1}dx-\frac{1}{2}i\int ie^{-a-bx}x^{m-1}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^{m-1}dx+\frac{1}{2}\int e^{a+bx}x^{m-1}dx$$

$$\Big\downarrow \text{2612}$$

$$-\frac{1}{2}e^ax^m(-bx)^{-m}\Gamma(m,-bx)-\frac{1}{2}e^{-a}x^m(bx)^{-m}\Gamma(m,bx)$$

input
```
Int[x^(-1 + m)*Cosh[a + b*x],x]
```

output
```
-1/2*(E^a*x^m*Gamma[m, -(b*x)])/(-(b*x))^m - (x^m*Gamma[m, b*x])/(2*E^a*(b
*x)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.35 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.37

method	result	size
meijerg	$\frac{x^{m} \operatorname{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{1}{2},1+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \cosh \left(a \right)}{m}+\frac{b \,x^{1+m} \operatorname{hypergeom}\left(\left[\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{3}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \sinh \left(a \right)}{1+m}$	67

input
```
int(x^(-1+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/m*x^m*hypergeom([1/2*m],[1/2,1+1/2*m],1/4*x^2*b^2)*cosh(a)+b/(1+m)*x^(1+
m)*hypergeom([1/2+1/2*m],[3/2,3/2+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int x^{-1+m}\cosh(a+bx)\,dx = -\frac{\cosh\left((m-1)\log\left(b\right)+a\right)\Gamma(m,bx)-\cosh\left((m-1)\log\left(-b\right)-a\right)\Gamma(m,-bx)+\Gamma(m,-bx)\sinh\left((m-\right.}{2\,b}$$

input
```
integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh((m - 1)*log(b) + a)*gamma(m, b*x) - cosh((m - 1)*log(-b) - a)*g
amma(m, -b*x) + gamma(m, -b*x)*sinh((m - 1)*log(-b) - a) - gamma(m, b*x)*s
inh((m - 1)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-1+m}\cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(-1+m)*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88

$$\int x^{-1+m}\cosh(a+bx)\,dx = -\frac{x^m e^{(-a)}\Gamma(m,bx)}{2\,(bx)^m}-\frac{x^m e^a\Gamma(m,-bx)}{2\,(-bx)^m}$$

input
```
integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*x^m*e^(-a)*gamma(m, b*x)/(b*x)^m - 1/2*x^m*e^a*gamma(m, -b*x)/(-b*x)^
m
```

Giac [F]

$$\int x^{-1+m}\cosh(a+bx)\,dx=\int x^{m-1}\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-1+m)*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\cosh(a+bx)\,dx=\int x^{m-1}\cosh(a+b\,x)\,dx$$

input
```
int(x^(m - 1)*cosh(a + b*x),x)
```

output
```
int(x^(m - 1)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^{-1+m}\cosh(a+bx)\,dx=\int\frac{x^{m}\cosh\left(bx+a\right)}{x}dx$$

input
```
int(x^(-1+m)*cosh(b*x+a),x)
```

output
```
int((x**m*cosh(a + b*x))/x,x)
```

3.86 $\int x^{-2+m} \cosh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 55

$$\int x^{-2+m}\cosh(a+bx)\,dx = \frac{1}{2}be^a x^m(-bx)^{-m}\Gamma(-1+m,-bx) - \frac{1}{2}be^{-a}x^m(bx)^{-m}\Gamma(-1+m,bx)$$

output
```
1/2*b*exp(a)*x^m*GAMMA(-1+m,-b*x)/((-b*x)^m)-1/2*b*x^m*GAMMA(-1+m,b*x)/exp
(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.95

$$\int x^{-2+m}\cosh(a+bx)\,dx = \frac{1}{2}be^{-a}x^m\left(e^{2a}(-bx)^{-m}\Gamma(-1+m,-bx) - (bx)^{-m}\Gamma(-1+m,bx)\right)$$

input
```
Integrate[x^(-2 + m)*Cosh[a + b*x],x]
```

output
```
(b*x^m*((E^(2*a)*Gamma[-1 + m, -(b*x)])/(-(b*x))^m - Gamma[-1 + m, b*x]/(b
*x)^m))/(2*E^a)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^{m-2}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{ 3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}x^{m-2}dx-\frac{1}{2}i\int ie^{-a-bx}x^{m-2}dx$$

$$\Big\downarrow \text{ 26}$$

$$\frac{1}{2}\int e^{-a-bx}x^{m-2}dx+\frac{1}{2}\int e^{a+bx}x^{m-2}dx$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{1}{2}e^{a}bx^{m}(-bx)^{-m}\Gamma(m-1,-bx)-\frac{1}{2}e^{-a}bx^{m}(bx)^{-m}\Gamma(m-1,bx)$$

input
```
Int[x^(-2 + m)*Cosh[a + b*x],x]
```

output
```
(b*E^a*x^m*Gamma[-1 + m, -(b*x)])/(2*(-(b*x))^m) - (b*x^m*Gamma[-1 + m, b*
x])/(2*E^a*(b*x)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.36 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

method	result	size
meijerg	$\frac{x^{-1+m} \operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{1}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \cosh \left(a \right)}{-1+m}+\frac{b \,x^{m} \operatorname{hypergeom}\left(\left[\frac{m}{2}\right],\left[\frac{3}{2},1+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \sinh \left(a \right)}{m}$	67

input
```
int(x^(-2+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[1/2,1/2+1/2*m],1/4*x^2*b^2)*cosh
(a)+b/m*x^m*hypergeom([1/2*m],[3/2,1+1/2*m],1/4*x^2*b^2)*sinh(a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.56

$$\int x^{-2+m} \cosh(a+bx)\,dx = -\frac{\cosh\left((m-2)\log\left(b\right)+a\right)\Gamma(m-1,bx)-\cosh\left((m-2)\log\left(-b\right)-a\right)\Gamma(m-1,-bx)+\Gamma(m-1,-bx}{2\,b}$$

input
```
integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh((m - 2)*log(b) + a)*gamma(m - 1, b*x) - cosh((m - 2)*log(-b) -
a)*gamma(m - 1, -b*x) + gamma(m - 1, -b*x)*sinh((m - 2)*log(-b) - a) - gam
ma(m - 1, b*x)*sinh((m - 2)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-2+m} \cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(-2+m)*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int x^{-2+m} \cosh(a+bx)\,dx = -\frac{1}{2}\left(bx\right)^{-m+1}x^{m-1}e^{(-a)}\Gamma(m-1,bx) - \frac{1}{2}\left(-bx\right)^{-m+1}x^{m-1}e^{a}\Gamma(m-1,-bx)$$

input
```
integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(b*x)^(-m + 1)*x^(m - 1)*e^(-a)*gamma(m - 1, b*x) - 1/2*(-b*x)^(-m +
1)*x^(m - 1)*e^a*gamma(m - 1, -b*x)
```

Giac [F]

$$\int x^{-2+m}\cosh(a+bx)\,dx=\int x^{m-2}\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-2+m)*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\cosh(a+bx)\,dx=\int x^{m-2}\cosh(a+b\,x)\,dx$$

input
```
int(x^(m - 2)*cosh(a + b*x),x)
```

output
```
int(x^(m - 2)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^{-2+m}\cosh(a+bx)\,dx=\int\frac{x^m\cosh\left(bx+a\right)}{x^2}dx$$

input
```
int(x^(-2+m)*cosh(b*x+a),x)
```

output
```
int((x**m*cosh(a + b*x))/x**2,x)
```

3.87 $\int x^{-3+m} \cosh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 59

$$\int x^{-3+m}\cosh(a+bx)\,dx = -\frac{1}{2}b^2e^ax^m(-bx)^{-m}\Gamma(-2+m,-bx) - \frac{1}{2}b^2e^{-a}x^m(bx)^{-m}\Gamma(-2+m,bx)$$

output
```
-1/2*b^2*exp(a)*x^m*GAMMA(-2+m,-b*x)/((-b*x)^m)-1/2*b^2*x^m*GAMMA(-2+m,b*x
)/exp(a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int x^{-3+m}\cosh(a+bx)\,dx = \frac{1}{2}b^2e^{-a}x^m\left(-e^{2a}(-bx)^{-m}\Gamma(-2+m,-bx) - (bx)^{-m}\Gamma(-2+m,bx)\right)$$

input
```
Integrate[x^(-3 + m)*Cosh[a + b*x],x]
```

output
```
(b^2*x^m*(-((E^(2*a)*Gamma[-2 + m, -(b*x)])/(-(b*x))^m) - Gamma[-2 + m, b*
x]/(b*x)^m))/(2*E^a)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3788, 26, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-3}\sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3788}$$

$$\frac{1}{2}i\int -ie^{a+bx}x^{m-3}dx-\frac{1}{2}i\int ie^{-a-bx}x^{m-3}dx$$

$$\Big\downarrow \text{26}$$

$$\frac{1}{2}\int e^{-a-bx}x^{m-3}dx+\frac{1}{2}\int e^{a+bx}x^{m-3}dx$$

$$\Big\downarrow \text{2612}$$

$$-\frac{1}{2}e^ab^2x^m(-bx)^{-m}\Gamma(m-2,-bx)-\frac{1}{2}e^{-a}b^2x^m(bx)^{-m}\Gamma(m-2,bx)$$

input
```
Int[x^(-3 + m)*Cosh[a + b*x],x]
```

output
```
-1/2*(b^2*E^a*x^m*Gamma[-2 + m, -(b*x)])/(-(b*x))^m - (b^2*x^m*Gamma[-2 +
m, b*x])/(2*E^a*(b*x)^m)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3788
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + Pi*(k_.) + (f_.)*(x_)], x_Symbol
] :> Simp[I/2   Int[(c + d*x)^m/(E^(I*k*Pi)*E^(I*(e + f*x))), x], x] - Simp
[I/2   Int[(c + d*x)^m*E^(I*k*Pi)*E^(I*(e + f*x)), x], x] /; FreeQ[{c, d, e
, f, m}, x] && IntegerQ[2*k]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4.

Time = 0.33 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.20

method	result	size
meijerg	$\frac{x^{-2+m} \operatorname{hypergeom}\left(\left[-1+\frac{m}{2}\right],\left[\frac{1}{2},\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \cosh \left(a \right)}{-2+m}+\frac{b \,x^{-1+m} \operatorname{hypergeom}\left(\left[-\frac{1}{2}+\frac{m}{2}\right],\left[\frac{3}{2},\frac{1}{2}+\frac{m}{2}\right],\frac{x^{2} b^{2}}{4}\right) \sinh \left(a \right)}{-1+m}$	71

input
```
int(x^(-3+m)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/(-2+m)*x^(-2+m)*hypergeom([-1+1/2*m],[1/2,1/2*m],1/4*x^2*b^2)*cosh(a)+b/
(-1+m)*x^(-1+m)*hypergeom([-1/2+1/2*m],[3/2,1/2+1/2*m],1/4*x^2*b^2)*sinh(a
)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.46

$$\int x^{-3+m}\cosh(a+bx)\,dx = -\frac{\cosh\left((m-3)\log\left(b\right)+a\right)\Gamma(m-2,bx)-\cosh\left((m-3)\log\left(-b\right)-a\right)\Gamma(m-2,-bx)+\Gamma(m-2,-bx}{2\,b}$$

input
```
integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(cosh((m - 3)*log(b) + a)*gamma(m - 2, b*x) - cosh((m - 3)*log(-b) -
a)*gamma(m - 2, -b*x) + gamma(m - 2, -b*x)*sinh((m - 3)*log(-b) - a) - gam
ma(m - 2, b*x)*sinh((m - 3)*log(b) + a))/b
```

Sympy [F(-2)]

Exception generated.

$$\int x^{-3+m}\cosh(a+bx)\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x**(-3+m)*cosh(b*x+a),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\begin{aligned}\int x^{-3+m}\cosh(a+bx)\,dx = &-\frac{1}{2}\left(bx\right)^{-m+2}x^{m-2}e^{(-a)}\Gamma(m-2,bx)\\ &-\frac{1}{2}\left(-bx\right)^{-m+2}x^{m-2}e^{a}\Gamma(m-2,-bx)\end{aligned}$$

input
```
integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*(b*x)^(-m + 2)*x^(m - 2)*e^(-a)*gamma(m - 2, b*x) - 1/2*(-b*x)^(-m +
2)*x^(m - 2)*e^a*gamma(m - 2, -b*x)
```

Giac [F]

$$\int x^{-3+m}\cosh(a+bx)\,dx = \int x^{m-3}\cosh\left(bx+a\right)\,dx$$

input
```
integrate(x^(-3+m)*cosh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*cosh(b*x + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\cosh(a+bx)\,dx = \int x^{m-3}\cosh(a+b\,x)\,dx$$

input
```
int(x^(m - 3)*cosh(a + b*x),x)
```

output
```
int(x^(m - 3)*cosh(a + b*x), x)
```

Reduce [F]

$$\int x^{-3+m}\cosh(a+bx)\,dx = \int \frac{x^m\cosh\left(bx+a\right)}{x^3}dx$$

input
```
int(x^(-3+m)*cosh(b*x+a),x)
```

output
```
int((x**m*cosh(a + b*x))/x**3,x)
```

3.88 $\int x^{3+m} \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x^{3+m} \cosh^2(a+bx)\,dx = \frac{x^{4+m}}{2(4+m)} - \frac{2^{-6-m}e^{2a}x^m(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4} - \frac{2^{-6-m}e^{-2a}x^m(bx)^{-m}\Gamma(4+m,2bx)}{b^4}$$

output
```
x^(4+m)/(8+2*m)-2^(-6-m)*exp(2*a)*x^m*GAMMA(4+m,-2*b*x)/b^4/((-b*x)^m)-2^(
-6-m)*x^m*GAMMA(4+m,2*b*x)/b^4/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int x^{3+m} \cosh^2(a+bx)\,dx = \frac{1}{64}x^m\left(\frac{32x^4}{4+m} - \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(4+m,-2bx)}{b^4} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(4+m,2bx)}{b^4}\right)$$

input
```
Integrate[x^(3 + m)*Cosh[a + b*x]^2,x]
```

output
```
(x^m*((32*x^4)/(4 + m) - (E^(2*a)*Gamma[4 + m, -2*b*x])/(2^m*b^4*(-(b*x))^
m) - Gamma[4 + m, 2*b*x]/(2^m*b^4*E^(2*a)*(b*x)^m)))/64
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+3}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m+3}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int\left(\frac{1}{2}x^{m+3}\cosh(2a+2bx)+\frac{x^{m+3}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{e^{2a}2^{-m-6}x^m(-bx)^{-m}\Gamma(m+4,-2bx)}{b^4}-\frac{e^{-2a}2^{-m-6}x^m(bx)^{-m}\Gamma(m+4,2bx)}{b^4}+\frac{x^{m+4}}{2(m+4)}$$

input
```
Int[x^(3 + m)*Cosh[a + b*x]^2,x]
```

output
```
x^(4 + m)/(2*(4 + m)) - (2^(-6 - m)*E^(2*a)*x^m*Gamma[4 + m, -2*b*x])/(b^4
*(-(b*x))^m) - (2^(-6 - m)*x^m*Gamma[4 + m, 2*b*x])/(b^4*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{3+m}\cosh\left(bx+a\right)^{2}dx$$

input
```
int(x^(3+m)*cosh(b*x+a)^2,x)
```

output
```
int(x^(3+m)*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

$$\int x^{3+m}\cosh^2(a+bx)\,dx$$
$$= \frac{4\,bx\cosh\left((m+3)\log\left(x\right)\right)-(m+4)\cosh\left((m+3)\log\left(2\,b\right)+2\,a\right)\Gamma(m+4,2\,bx)+(m+4)\cosh\left((m+\right.}{}$$

input
```
integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cosh((m + 3)*log(x)) - (m + 4)*cosh((m + 3)*log(2*b) + 2*a)*gam
ma(m + 4, 2*b*x) + (m + 4)*cosh((m + 3)*log(-2*b) - 2*a)*gamma(m + 4, -2*b
*x) + (m + 4)*gamma(m + 4, 2*b*x)*sinh((m + 3)*log(2*b) + 2*a) - (m + 4)*g
amma(m + 4, -2*b*x)*sinh((m + 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 3)*log
(x)))/(b*m + 4*b)
```

Sympy [F]

$$\int x^{3+m}\cosh^2(a+bx)\,dx = \int x^{m+3}\cosh^2{(a+bx)}\,dx$$

input
```
integrate(x**(3+m)*cosh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 3)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int x^{3+m}\cosh^2(a+bx)\,dx = -\frac{1}{4}\,(2\,bx)^{-m-4}\,x^{m+4}e^{(-2\,a)}\Gamma(m+4,2\,bx) - \frac{1}{4}\,(-2\,bx)^{-m-4}\,x^{m+4}e^{(2\,a)}\Gamma(m+4,-2\,bx) + \frac{x^{m+4}}{2\,(m+4)}$$

input
```
integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 4)*x^(m + 4)*e^(-2*a)*gamma(m + 4, 2*b*x) - 1/4*(-2*b*x
)^(-m - 4)*x^(m + 4)*e^(2*a)*gamma(m + 4, -2*b*x) + 1/2*x^(m + 4)/(m + 4)
```

Giac [F]

$$\int x^{3+m}\cosh^2(a+bx)\,dx=\int x^{m+3}\cosh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(3+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 3)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{3+m}\cosh^2(a+bx)\,dx=\int x^{m+3}\cosh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 3)*cosh(a + b*x)^2,x)
```

output
```
int(x^(m + 3)*cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{3+m}\cosh^2(a+bx)\,dx$$
$$=\frac{10e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}dx\right)m^4+35e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}dx\right)m^3+50e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}dx\right)m^2+24e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}\cdots\right.}{\cdots}$$

input
```
int(x^(3+m)*cosh(b*x+a)^2,x)
```

output

```
(8*x**m*e**(4*a + 4*b*x)*b**3*m*x**3 + 32*x**m*e**(4*a + 4*b*x)*b**3*x**3
- 4*x**m*e**(4*a + 4*b*x)*b**2*m**2*x**2 - 28*x**m*e**(4*a + 4*b*x)*b**2*m
*x**2 - 48*x**m*e**(4*a + 4*b*x)*b**2*x**2 + 2*x**m*e**(4*a + 4*b*x)*b*m**
3*x + 18*x**m*e**(4*a + 4*b*x)*b*m**2*x + 52*x**m*e**(4*a + 4*b*x)*b*m*x +
 48*x**m*e**(4*a + 4*b*x)*b*x - x**m*e**(4*a + 4*b*x)*m**4 - 10*x**m*e**(4
*a + 4*b*x)*m**3 - 35*x**m*e**(4*a + 4*b*x)*m**2 - 50*x**m*e**(4*a + 4*b*x
)*m - 24*x**m*e**(4*a + 4*b*x) + e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,
x)*m**5 + 10*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**4 + 35*e**(4*a
 + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**3 + 50*e**(4*a + 2*b*x)*int((x**m*
e**(2*b*x))/x,x)*m**2 + 24*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m +
 32*x**m*e**(2*a + 2*b*x)*b**4*x**4 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x
)*m**5 + 10*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**4 + 35*e**(2*b*x)*int
(x**m/(e**(2*b*x)*x),x)*m**3 + 50*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m*
*2 + 24*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m - 8*x**m*b**3*m*x**3 - 32*
x**m*b**3*x**3 - 4*x**m*b**2*m**2*x**2 - 28*x**m*b**2*m*x**2 - 48*x**m*b**
2*x**2 - 2*x**m*b*m**3*x - 18*x**m*b*m**2*x - 52*x**m*b*m*x - 48*x**m*b*x
- x**m*m**4 - 10*x**m*m**3 - 35*x**m*m**2 - 50*x**m*m - 24*x**m)/(64*e**(2
*a + 2*b*x)*b**4*(m + 4))
```

3.89 $\int x^{2+m}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 85

$$\int x^{2+m}\cosh^2(a+bx)\,dx=\frac{x^{3+m}}{2(3+m)}+\frac{2^{-5-m}e^{2a}x^m(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-5-m}e^{-2a}x^m(bx)^{-m}\Gamma(3+m,2bx)}{b^3}$$

output
```
x^(3+m)/(6+2*m)+2^(-5-m)*exp(2*a)*x^m*GAMMA(3+m,-2*b*x)/b^3/((-b*x)^m)-2^(
-5-m)*x^m*GAMMA(3+m,2*b*x)/b^3/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int x^{2+m}\cosh^2(a+bx)\,dx=\frac{1}{32}x^m\left(\frac{16x^3}{3+m}+\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(3+m,-2bx)}{b^3}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(3+m,2bx)}{b^3}\right)$$

input
```
Integrate[x^(2 + m)*Cosh[a + b*x]^2,x]
```

output
```
(x^m*((16*x^3)/(3 + m) + (E^(2*a)*Gamma[3 + m, -2*b*x])/(2^m*b^3*(-(b*x))^
m) - Gamma[3 + m, 2*b*x]/(2^m*b^3*E^(2*a)*(b*x)^m)))/32
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+2}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int x^{m+2}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{ 3793}$$

$$\int\left(\frac{1}{2}x^{m+2}\cosh(2a+2bx)+\frac{x^{m+2}}{2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{e^{2a}2^{-m-5}x^m(-bx)^{-m}\Gamma(m+3,-2bx)}{b^3}-\frac{e^{-2a}2^{-m-5}x^m(bx)^{-m}\Gamma(m+3,2bx)}{b^3}+\frac{x^{m+3}}{2(m+3)}$$

input
```
Int[x^(2 + m)*Cosh[a + b*x]^2,x]
```

output
```
x^(3 + m)/(2*(3 + m)) + (2^(-5 - m)*E^(2*a)*x^m*Gamma[3 + m, -2*b*x])/(b^3
*(-(b*x))^m) - (2^(-5 - m)*x^m*Gamma[3 + m, 2*b*x])/(b^3*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{2+m} \cosh\left(bx+a\right)^{2} dx$$

input
```
int(x^(2+m)*cosh(b*x+a)^2,x)
```

output
```
int(x^(2+m)*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.60

$$\int x^{2+m} \cosh^2(a+bx)\,dx = \frac{4\,bx\cosh\left((m+2)\log\left(x\right)\right) - (m+3)\cosh\left((m+2)\log\left(2\,b\right)+2\,a\right)\Gamma(m+3,2\,bx) + (m+3)\cosh\left((m+\right.}{}$$

input
```
integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cosh((m + 2)*log(x)) - (m + 3)*cosh((m + 2)*log(2*b) + 2*a)*gam
ma(m + 3, 2*b*x) + (m + 3)*cosh((m + 2)*log(-2*b) - 2*a)*gamma(m + 3, -2*b
*x) + (m + 3)*gamma(m + 3, 2*b*x)*sinh((m + 2)*log(2*b) + 2*a) - (m + 3)*g
amma(m + 3, -2*b*x)*sinh((m + 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 2)*log
(x)))/(b*m + 3*b)
```

Sympy [F]

$$\int x^{2+m}\cosh^2(a+bx)\,dx = \int x^{m+2}\cosh^2{(a+bx)}\,dx$$

input
```
integrate(x**(2+m)*cosh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 2)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^{2+m}\cosh^2(a+bx)\,dx = -\frac{1}{4}\,(2\,bx)^{-m-3}\,x^{m+3}e^{(-2\,a)}\Gamma(m+3,2\,bx) - \frac{1}{4}\,(-2\,bx)^{-m-3}\,x^{m+3}e^{(2\,a)}\Gamma(m+3,-2\,bx) + \frac{x^{m+3}}{2\,(m+3)}$$

input
```
integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 3)*x^(m + 3)*e^(-2*a)*gamma(m + 3, 2*b*x) - 1/4*(-2*b*x
)^(-m - 3)*x^(m + 3)*e^(2*a)*gamma(m + 3, -2*b*x) + 1/2*x^(m + 3)/(m + 3)
```

Giac [F]

$$\int x^{2+m}\cosh^2(a+bx)\,dx=\int x^{m+2}\cosh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(2+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 2)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{2+m}\cosh^2(a+bx)\,dx=\int x^{m+2}\cosh(a+b\,x)^2\,dx$$

input
```
int(x^(m + 2)*cosh(a + b*x)^2,x)
```

output
```
int(x^(m + 2)*cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{2+m}\cosh^2(a+bx)\,dx$$
$$=\frac{-e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}dx\right)m^4-6e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}dx\right)m^3-11e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}dx\right)m^2-6e^{2bx+4a}\left(\int\frac{x^m e^{2bx}}{x}d\right.}{}$$

input
```
int(x^(2+m)*cosh(b*x+a)^2,x)
```

output

```
(4*x**m*e**(4*a + 4*b*x)*b**2*m*x**2 + 12*x**m*e**(4*a + 4*b*x)*b**2*x**2
- 2*x**m*e**(4*a + 4*b*x)*b*m**2*x - 10*x**m*e**(4*a + 4*b*x)*b*m*x - 12*x
**m*e**(4*a + 4*b*x)*b*x + x**m*e**(4*a + 4*b*x)*m**3 + 6*x**m*e**(4*a + 4
*b*x)*m**2 + 11*x**m*e**(4*a + 4*b*x)*m + 6*x**m*e**(4*a + 4*b*x) - e**(4*
a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**4 - 6*e**(4*a + 2*b*x)*int((x**m*
e**(2*b*x))/x,x)*m**3 - 11*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**
2 - 6*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m + 16*x**m*e**(2*a + 2*
b*x)*b**3*x**3 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**4 + 6*e**(2*b*x)
*int(x**m/(e**(2*b*x)*x),x)*m**3 + 11*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x
)*m**2 + 6*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m - 4*x**m*b**2*m*x**2 -
12*x**m*b**2*x**2 - 2*x**m*b*m**2*x - 10*x**m*b*m*x - 12*x**m*b*x - x**m*m
**3 - 6*x**m*m**2 - 11*x**m*m - 6*x**m)/(32*e**(2*a + 2*b*x)*b**3*(m + 3))
```

3.90 $\int x^{1+m}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 86

$$\int x^{1+m}\cosh^2(a+bx)\,dx=\frac{x^{2+m}}{2(2+m)}-\frac{2^{-4-m}e^{2a}x^m(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-4-m}e^{-2a}x^m(bx)^{-m}\Gamma(2+m,2bx)}{b^2}$$

output
```
x^(2+m)/(4+2*m)-2^(-4-m)*exp(2*a)*x^m*GAMMA(2+m,-2*b*x)/b^2/((-b*x)^m)-2^(
-4-m)*x^m*GAMMA(2+m,2*b*x)/b^2/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int x^{1+m}\cosh^2(a+bx)\,dx=\frac{1}{16}x^m\left(\frac{8x^2}{2+m}-\frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(2+m,-2bx)}{b^2}-\frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(2+m,2bx)}{b^2}\right)$$

input
```
Integrate[x^(1 + m)*Cosh[a + b*x]^2,x]
```

output
```
(x^m*((8*x^2)/(2 + m) - (E^(2*a)*Gamma[2 + m, -2*b*x])/(2^m*b^2*(-(b*x))^m
) - Gamma[2 + m, 2*b*x]/(2^m*b^2*E^(2*a)*(b*x)^m)))/16
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m+1}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int x^{m+1}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \; 3793$$

$$\int\left(\frac{1}{2}x^{m+1}\cosh(2a+2bx)+\frac{x^{m+1}}{2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{e^{2a}2^{-m-4}x^m(-bx)^{-m}\Gamma(m+2,-2bx)}{b^2}-\frac{e^{-2a}2^{-m-4}x^m(bx)^{-m}\Gamma(m+2,2bx)}{b^2}+\frac{x^{m+2}}{2(m+2)}$$

input
```
Int[x^(1 + m)*Cosh[a + b*x]^2,x]
```

output
```
x^(2 + m)/(2*(2 + m)) - (2^(-4 - m)*E^(2*a)*x^m*Gamma[2 + m, -2*b*x])/(b^2
*(-(b*x))^m) - (2^(-4 - m)*x^m*Gamma[2 + m, 2*b*x])/(b^2*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{1+m}\cosh{(bx+a)}^{2}\,dx$$

input
```
int(x^(1+m)*cosh(b*x+a)^2,x)
```

output
```
int(x^(1+m)*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.58

$$\int x^{1+m}\cosh^{2}(a+bx)\,dx = \frac{4\,bx\cosh\left((m+1)\log\left(x\right)\right)-(m+2)\cosh\left((m+1)\log\left(2\,b\right)+2\,a\right)\Gamma(m+2,2\,bx)+(m+2)\cosh\left((m+\right.}{}$$

input
```
integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cosh((m + 1)*log(x)) - (m + 2)*cosh((m + 1)*log(2*b) + 2*a)*gam
ma(m + 2, 2*b*x) + (m + 2)*cosh((m + 1)*log(-2*b) - 2*a)*gamma(m + 2, -2*b
*x) + (m + 2)*gamma(m + 2, 2*b*x)*sinh((m + 1)*log(2*b) + 2*a) - (m + 2)*g
amma(m + 2, -2*b*x)*sinh((m + 1)*log(-2*b) - 2*a) + 4*b*x*sinh((m + 1)*log
(x)))/(b*m + 2*b)
```

Sympy [F]

$$\int x^{1+m}\cosh^2(a+bx)\,dx=\int x^{m+1}\cosh^2{(a+bx)}\,dx$$

input
```
integrate(x**(1+m)*cosh(b*x+a)**2,x)
```

output
```
Integral(x**(m + 1)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.83

$$\int x^{1+m}\cosh^2(a+bx)\,dx=-\frac{1}{4}\,(2\,bx)^{-m-2}\,x^{m+2}e^{(-2\,a)}\Gamma(m+2,2\,bx)-\frac{1}{4}\,(-2\,bx)^{-m-2}\,x^{m+2}e^{(2\,a)}\Gamma(m+2,-2\,bx)+\frac{x^{m+2}}{2\,(m+2)}$$

input
```
integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 2)*x^(m + 2)*e^(-2*a)*gamma(m + 2, 2*b*x) - 1/4*(-2*b*x
)^(-m - 2)*x^(m + 2)*e^(2*a)*gamma(m + 2, -2*b*x) + 1/2*x^(m + 2)/(m + 2)
```

Giac [F]

$$\int x^{1+m} \cosh^2(a+bx)\,dx = \int x^{m+1} \cosh\left(bx+a\right)^2 \,dx$$

input
```
integrate(x^(1+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m + 1)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{1+m} \cosh^2(a+bx)\,dx = \int x^{m+1} \cosh(a+b\,x)^2 \,dx$$

input
```
int(x^(m + 1)*cosh(a + b*x)^2,x)
```

output
```
int(x^(m + 1)*cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{1+m} \cosh^2(a+bx)\,dx$$
$$= \frac{2x^m e^{4bx+4a} bmx + 4x^m e^{4bx+4a} bx - x^m e^{4bx+4a} m^2 - 3x^m e^{4bx+4a} m - 2x^m e^{4bx+4a} + e^{2bx+4a} \left(\int \frac{x^m e^{2bx}}{x} dx\right) m^3}{}$$

input
```
int(x^(1+m)*cosh(b*x+a)^2,x)
```

output
```
(2*x**m*e**(4*a + 4*b*x)*b*m*x + 4*x**m*e**(4*a + 4*b*x)*b*x - x**m*e**(4*
a + 4*b*x)*m**2 - 3*x**m*e**(4*a + 4*b*x)*m - 2*x**m*e**(4*a + 4*b*x) + e*
*(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m**3 + 3*e**(4*a + 2*b*x)*int((x
**m*e**(2*b*x))/x,x)*m**2 + 2*e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*
m + 8*x**m*e**(2*a + 2*b*x)*b**2*x**2 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x)
,x)*m**3 + 3*e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**2 + 2*e**(2*b*x)*int
(x**m/(e**(2*b*x)*x),x)*m - 2*x**m*b*m*x - 4*x**m*b*x - x**m*m**2 - 3*x**m
*m - 2*x**m)/(16*e**(2*a + 2*b*x)*b**2*(m + 2))
```

3.91 $\int x^m \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int x^m \cosh^2(a+bx)\,dx = \frac{x^{1+m}}{2(1+m)} + \frac{2^{-3-m}e^{2a}x^m(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-3-m}e^{-2a}x^m(bx)^{-m}\Gamma(1+m,2bx)}{b}$$

output
```
x^(1+m)/(2+2*m)+2^(-3-m)*exp(2*a)*x^m*GAMMA(1+m,-2*b*x)/b/((-b*x)^m)-2^(-3
-m)*x^m*GAMMA(1+m,2*b*x)/b/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.89

$$\int x^m \cosh^2(a+bx)\,dx = \frac{1}{8}x^m\left(\frac{4x}{1+m} + \frac{2^{-m}e^{2a}(-bx)^{-m}\Gamma(1+m,-2bx)}{b} - \frac{2^{-m}e^{-2a}(bx)^{-m}\Gamma(1+m,2bx)}{b}\right)$$

input
```
Integrate[x^m*Cosh[a + b*x]^2,x]
```

output
```
(x^m*((4*x)/(1 + m) + (E^(2*a)*Gamma[1 + m, -2*b*x])/(2^m*b*(-(b*x))^m) -
Gamma[1 + m, 2*b*x]/(2^m*b*E^(2*a)*(b*x)^m)))/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^m \sin\left(ia+ibx+\frac{\pi}{2}\right)^2 dx$$

$$\Big\downarrow \text{3793}$$

$$\int \left(\frac{1}{2}x^m\cosh(2a+2bx)+\frac{x^m}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{e^{2a}2^{-m-3}x^m(-bx)^{-m}\Gamma(m+1,-2bx)}{b}-\frac{e^{-2a}2^{-m-3}x^m(bx)^{-m}\Gamma(m+1,2bx)}{b}+\frac{x^{m+1}}{2(m+1)}$$

input
```
Int[x^m*Cosh[a + b*x]^2,x]
```

output
```
x^(1 + m)/(2*(1 + m)) + (2^(-3 - m)*E^(2*a)*x^m*Gamma[1 + m, -2*b*x])/(b*(
-(b*x))^m) - (2^(-3 - m)*x^m*Gamma[1 + m, 2*b*x])/(b*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^m \cosh{(bx+a)}^2\,dx$$

input
```
int(x^m*cosh(b*x+a)^2,x)
```

output
```
int(x^m*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.44

$$\int x^m \cosh^2(a+bx)\,dx = \frac{4\,bx\cosh{(m\log{(x)})} - (m+1)\cosh{(m\log{(2\,b)}+2\,a)}\,\Gamma(m+1,2\,bx) + (m+1)\cosh{(m\log{(-2\,b)}-2\,a}$$

input
```
integrate(x^m*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cosh(m*log(x)) - (m + 1)*cosh(m*log(2*b) + 2*a)*gamma(m + 1, 2*
b*x) + (m + 1)*cosh(m*log(-2*b) - 2*a)*gamma(m + 1, -2*b*x) + (m + 1)*gamm
a(m + 1, 2*b*x)*sinh(m*log(2*b) + 2*a) - (m + 1)*gamma(m + 1, -2*b*x)*sinh
(m*log(-2*b) - 2*a) + 4*b*x*sinh(m*log(x)))/(b*m + b)
```

Sympy [F]

$$\int x^m \cosh^2(a+bx)\,dx = \int x^m \cosh^2{(a+bx)}\,dx$$

input
```
integrate(x**m*cosh(b*x+a)**2,x)
```

output
```
Integral(x**m*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.84

$$\int x^m \cosh^2(a+bx)\,dx = -\frac{1}{4}\,(2\,bx)^{-m-1}\,x^{m+1}e^{(-2\,a)}\Gamma(m+1,2\,bx) - \frac{1}{4}\,(-2\,bx)^{-m-1}\,x^{m+1}e^{(2\,a)}\Gamma(m+1,-2\,bx) + \frac{x^{m+1}}{2\,(m+1)}$$

input
```
integrate(x^m*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*b*x)^(-m - 1)*x^(m + 1)*e^(-2*a)*gamma(m + 1, 2*b*x) - 1/4*(-2*b*x
)^(-m - 1)*x^(m + 1)*e^(2*a)*gamma(m + 1, -2*b*x) + 1/2*x^(m + 1)/(m + 1)
```

Giac [F]

$$\int x^m \cosh^2(a+bx)\,dx = \int x^m \cosh{(bx+a)}^2\,dx$$

input
```
integrate(x^m*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^m \cosh^2(a+bx)\,dx = \int x^m\,{\cosh\left(a+b\,x\right)}^2\,dx$$

input
```
int(x^m*cosh(a + b*x)^2,x)
```

output
```
int(x^m*cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^m \cosh^2(a+bx)\,dx = \frac{x^m e^{4bx+4a}m + x^m e^{4bx+4a} - e^{2bx+4a}\left(\int \frac{x^m e^{2bx}}{x}dx\right)m^2 - e^{2bx+4a}\left(\int \frac{x^m e^{2bx}}{x}dx\right)m + 4x^m e^{2bx+2a}bx + e^{2bx}\left(\int \frac{}{e}\right.}{8e^{2bx+2a}b\,(m+1)}$$

input
```
int(x^m*cosh(b*x+a)^2,x)
```

output
```
(x**m*e**(4*a + 4*b*x)*m + x**m*e**(4*a + 4*b*x) - e**(4*a + 2*b*x)*int((x
**m*e**(2*b*x))/x,x)*m**2 - e**(4*a + 2*b*x)*int((x**m*e**(2*b*x))/x,x)*m
+ 4*x**m*e**(2*a + 2*b*x)*b*x + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m**2
 + e**(2*b*x)*int(x**m/(e**(2*b*x)*x),x)*m - x**m*m - x**m)/(8*e**(2*a + 2
*b*x)*b*(m + 1))
```

3.92 $\int x^{-1+m} \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 72

$$\int x^{-1+m} \cosh^2(a+bx)\,dx = \frac{x^m}{2m} - 2^{-2-m}e^{2a}x^m(-bx)^{-m}\Gamma(m,-2bx) - 2^{-2-m}e^{-2a}x^m(bx)^{-m}\Gamma(m,2bx)$$

output
```
1/2*x^m/m-2^(-2-m)*exp(2*a)*x^m*GAMMA(m,-2*b*x)/((-b*x)^m)-2^(-2-m)*x^m*GA
MMA(m,2*b*x)/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89

$$\int x^{-1+m} \cosh^2(a+bx)\,dx = \frac{1}{4}x^m\left(\frac{2}{m} - 2^{-m}e^{2a}(-bx)^{-m}\Gamma(m,-2bx) - 2^{-m}e^{-2a}(bx)^{-m}\Gamma(m,2bx)\right)$$

input
```
Integrate[x^(-1 + m)*Cosh[a + b*x]^2,x]
```

output
```
(x^m*(2/m - (E^(2*a)*Gamma[m, -2*b*x])/(2^m*(-(b*x))^m) - Gamma[m, 2*b*x]/
(2^m*E^(2*a)*(b*x)^m)))/4
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-1}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-1}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int\left(\frac{1}{2}x^{m-1}\cosh(2a+2bx)+\frac{x^{m-1}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2a}\left(-2^{-m-2}\right)x^m(-bx)^{-m}\Gamma(m,-2bx)-e^{-2a}2^{-m-2}x^m(bx)^{-m}\Gamma(m,2bx)+\frac{x^m}{2m}$$

input
```
Int[x^(-1 + m)*Cosh[a + b*x]^2,x]
```

output
```
x^m/(2*m) - (2^(-2 - m)*E^(2*a)*x^m*Gamma[m, -2*b*x])/(-(b*x))^m - (2^(-2
- m)*x^m*Gamma[m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-1+m} \cosh \left(bx + a\right)^{2} dx$$

input
```
int(x^(-1+m)*cosh(b*x+a)^2,x)
```

output
```
int(x^(-1+m)*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.62

$$\int x^{-1+m} \cosh^2(a + bx)\, dx$$
$$= \frac{4\, bx \cosh\left((m-1)\log\left(x\right)\right) - m \cosh\left((m-1)\log\left(2\, b\right) + 2\, a\right) \Gamma(m, 2\, bx) + m \cosh\left((m-1)\log\left(-2\, b\right) - \right.}{}$$

input
```
integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cosh((m - 1)*log(x)) - m*cosh((m - 1)*log(2*b) + 2*a)*gamma(m,
2*b*x) + m*cosh((m - 1)*log(-2*b) - 2*a)*gamma(m, -2*b*x) + m*gamma(m, 2*b
*x)*sinh((m - 1)*log(2*b) + 2*a) - m*gamma(m, -2*b*x)*sinh((m - 1)*log(-2*
b) - 2*a) + 4*b*x*sinh((m - 1)*log(x)))/(b*m)
```

Sympy [F]

$$\int x^{-1+m}\cosh^2(a+bx)\,dx=\int x^{m-1}\cosh^2(a+bx)\,dx$$

input
```
integrate(x**(-1+m)*cosh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 1)*cosh(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.76

$$\int x^{-1+m}\cosh^2(a+bx)\,dx=-\frac{x^m e^{(-2a)}\Gamma(m,2bx)}{4\,(2bx)^m}-\frac{x^m e^{(2a)}\Gamma(m,-2bx)}{4\,(-2bx)^m}+\frac{x^m}{2m}$$

input
```
integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*x^m*e^(-2*a)*gamma(m, 2*b*x)/(2*b*x)^m - 1/4*x^m*e^(2*a)*gamma(m, -2*
b*x)/(-2*b*x)^m + 1/2*x^m/m
```

Giac [F]

$$\int x^{-1+m}\cosh^2(a+bx)\,dx=\int x^{m-1}\cosh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(-1+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 1)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-1+m}\cosh^2(a+bx)\,dx=\int x^{m-1}\cosh(a+b\,x)^2\,dx$$

input
```
int(x^(m - 1)*cosh(a + b*x)^2,x)
```

output
```
int(x^(m - 1)*cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-1+m}\cosh^2(a+bx)\,dx=\int\frac{x^m\cosh\left(bx+a\right)^2}{x}dx$$

input
```
int(x^(-1+m)*cosh(b*x+a)^2,x)
```

output
```
int((x**m*cosh(a + b*x)**2)/x,x)
```

3.93 $\int x^{-2+m}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 83

$$\int x^{-2+m}\cosh^2(a+bx)\,dx=-\frac{x^{-1+m}}{2(1-m)}+2^{-1-m}be^{2a}x^m(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-1-m}be^{-2a}x^m(bx)^{-m}\Gamma(-1+m,2bx)$$

output
```
-1/2*x^(-1+m)/(1-m)+2^(-1-m)*b*exp(2*a)*x^m*GAMMA(-1+m,-2*b*x)/((-b*x)^m)-
2^(-1-m)*b*x^m*GAMMA(-1+m,2*b*x)/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.88

$$\int x^{-2+m}\cosh^2(a+bx)\,dx=\frac{1}{2}x^m\left(\frac{1}{(-1+m)x}+2^{-m}be^{2a}(-bx)^{-m}\Gamma(-1+m,-2bx)-2^{-m}be^{-2a}(bx)^{-m}\Gamma(-1+m,2bx)\right)$$

input
```
Integrate[x^(-2 + m)*Cosh[a + b*x]^2,x]
```

output
```
(x^m*(1/((-1 + m)*x) + (b*E^(2*a)*Gamma[-1 + m, -2*b*x])/(2^m*(-(b*x))^m)
- (b*Gamma[-1 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)))/2
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-2}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-2}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int\left(\frac{1}{2}x^{m-2}\cosh(2a+2bx)+\frac{x^{m-2}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$e^{2a}b2^{-m-1}x^m(-bx)^{-m}\Gamma(m-1,-2bx)-e^{-2a}b2^{-m-1}x^m(bx)^{-m}\Gamma(m-1,2bx)-\frac{x^{m-1}}{2(1-m)}$$

input
```
Int[x^(-2 + m)*Cosh[a + b*x]^2,x]
```

output
```
-1/2*x^(-1 + m)/(1 - m) + (2^(-1 - m)*b*E^(2*a)*x^m*Gamma[-1 + m, -2*b*x])
/(-(b*x))^m - (2^(-1 - m)*b*x^m*Gamma[-1 + m, 2*b*x])/(E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-2+m} \cosh (bx+a)^2 \, dx$$

input
```
int(x^(-2+m)*cosh(b*x+a)^2,x)
```

output
```
int(x^(-2+m)*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.64

$$\int x^{-2+m} \cosh^2(a+bx) \, dx = \frac{4\,bx \cosh\left((m-2)\log\left(x\right)\right) - (m-1)\cosh\left((m-2)\log\left(2\,b\right)+2\,a\right)\Gamma(m-1,2\,bx) + (m-1)\cosh\left((m-\right.}{}$$

input
```
integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cosh((m - 2)*log(x)) - (m - 1)*cosh((m - 2)*log(2*b) + 2*a)*gam
ma(m - 1, 2*b*x) + (m - 1)*cosh((m - 2)*log(-2*b) - 2*a)*gamma(m - 1, -2*b
*x) + (m - 1)*gamma(m - 1, 2*b*x)*sinh((m - 2)*log(2*b) + 2*a) - (m - 1)*g
amma(m - 1, -2*b*x)*sinh((m - 2)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 2)*log
(x)))/(b*m - b)
```

Sympy [F]

$$\int x^{-2+m}\cosh^2(a+bx)\,dx = \int x^{m-2}\cosh^2{(a+bx)}\,dx$$

input
```
integrate(x**(-2+m)*cosh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 2)*cosh(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{-2+m}\cosh^2(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-2>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int x^{-2+m}\cosh^2(a+bx)\,dx = \int x^{m-2}\cosh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^(-2+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 2)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-2+m}\cosh^2(a+bx)\,dx = \int x^{m-2}\cosh(a+b\,x)^2\,dx$$

input
```
int(x^(m - 2)*cosh(a + b*x)^2,x)
```

output
```
int(x^(m - 2)*cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-2+m}\cosh^2(a+bx)\,dx = \int \frac{x^m\cosh\left(bx+a\right)^2}{x^2}dx$$

input
```
int(x^(-2+m)*cosh(b*x+a)^2,x)
```

output
```
int((x**m*cosh(a + b*x)**2)/x**2,x)
```

3.94 $\int x^{-3+m} \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 84

$$\int x^{-3+m}\cosh^2(a+bx)\,dx = -\frac{x^{-2+m}}{2(2-m)} - 2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx) - 2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)$$

output
```
-1/2*x^(-2+m)/(2-m)-b^2*exp(2*a)*x^m*GAMMA(-2+m,-2*b*x)/(2^m)/((-b*x)^m)-b
^2*x^m*GAMMA(-2+m,2*b*x)/(2^m)/exp(2*a)/((b*x)^m)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00

$$\int x^{-3+m}\cosh^2(a+bx)\,dx = -\frac{x^{-2+m}}{2(2-m)} - 2^{-m}b^2e^{2a}x^m(-bx)^{-m}\Gamma(-2+m,-2bx) - 2^{-m}b^2e^{-2a}x^m(bx)^{-m}\Gamma(-2+m,2bx)$$

input
```
Integrate[x^(-3 + m)*Cosh[a + b*x]^2,x]
```

output
```
-1/2*x^(-2 + m)/(2 - m) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(
b*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^{m-3}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^{m-3}\sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3793}$$

$$\int\left(\frac{1}{2}x^{m-3}\cosh(2a+2bx)+\frac{x^{m-3}}{2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-e^{2a}b^2 2^{-m}x^m(-bx)^{-m}\Gamma(m-2,-2bx)-e^{-2a}b^2 2^{-m}x^m(bx)^{-m}\Gamma(m-2,2bx)-\frac{x^{m-2}}{2(2-m)}$$

input
```
Int[x^(-3 + m)*Cosh[a + b*x]^2,x]
```

output
```
-1/2*x^(-2 + m)/(2 - m) - (b^2*E^(2*a)*x^m*Gamma[-2 + m, -2*b*x])/(2^m*(-(
b*x))^m) - (b^2*x^m*Gamma[-2 + m, 2*b*x])/(2^m*E^(2*a)*(b*x)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

Maple [F]

$$\int x^{-3+m} \cosh (bx + a)^2 \, dx$$

input
```
int(x^(-3+m)*cosh(b*x+a)^2,x)
```

output
```
int(x^(-3+m)*cosh(b*x+a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.62

$$\int x^{-3+m} \cosh^2(a + bx) \, dx = \frac{4\,bx\cosh\left((m-3)\log\left(x\right)\right) - (m-2)\cosh\left((m-3)\log\left(2\,b\right) + 2\,a\right)\Gamma(m-2, 2\,bx) + (m-2)\cosh\left((m-\right.}{}$$

input
```
integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*b*x*cosh((m - 3)*log(x)) - (m - 2)*cosh((m - 3)*log(2*b) + 2*a)*gam
ma(m - 2, 2*b*x) + (m - 2)*cosh((m - 3)*log(-2*b) - 2*a)*gamma(m - 2, -2*b
*x) + (m - 2)*gamma(m - 2, 2*b*x)*sinh((m - 3)*log(2*b) + 2*a) - (m - 2)*g
amma(m - 2, -2*b*x)*sinh((m - 3)*log(-2*b) - 2*a) + 4*b*x*sinh((m - 3)*log
(x)))/(b*m - 2*b)
```

Sympy [F]

$$\int x^{-3+m}\cosh^2(a+bx)\,dx=\int x^{m-3}\cosh^2{(a+bx)}\,dx$$

input
```
integrate(x**(-3+m)*cosh(b*x+a)**2,x)
```

output
```
Integral(x**(m - 3)*cosh(a + b*x)**2, x)
```

Maxima [F(-2)]

Exception generated.

$$\int x^{-3+m}\cosh^2(a+bx)\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(m-3>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int x^{-3+m}\cosh^2(a+bx)\,dx=\int x^{m-3}\cosh{(bx+a)}^2\,dx$$

input
```
integrate(x^(-3+m)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^(m - 3)*cosh(b*x + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int x^{-3+m}\cosh^2(a+bx)\,dx=\int x^{m-3}\cosh(a+b\,x)^2\,dx$$

input
```
int(x^(m - 3)*cosh(a + b*x)^2,x)
```

output
```
int(x^(m - 3)*cosh(a + b*x)^2, x)
```

Reduce [F]

$$\int x^{-3+m}\cosh^2(a+bx)\,dx=\int\frac{x^m\cosh(bx+a)^2}{x^3}dx$$

input
```
int(x^(-3+m)*cosh(b*x+a)^2,x)
```

output
```
int((x**m*cosh(a + b*x)**2)/x**3,x)
```

3.95 $\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)\,dx=-\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}$$

output
```
-4/9/sech(x)^(3/2)+2/3*x*sinh(x)/sech(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)\,dx=\frac{2(-2+3x\tanh(x))}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

input
```
Integrate[x/Sech[x]^(3/2) - (x*Sqrt[Sech[x]])/3,x]
```

output
```
(2*(-2 + 3*x*Tanh[x]))/(9*Sech[x]^(3/2))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right) \, dx$$

$$\Big\downarrow \; 2009$$

$$\frac{2x\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{4}{9\operatorname{sech}^{\frac{3}{2}}(x)}$$

input
```
Int[x/Sech[x]^(3/2) - (x*Sqrt[Sech[x]])/3,x]
```

output
```
-4/(9*Sech[x]^(3/2)) + (2*x*Sinh[x])/(3*Sqrt[Sech[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\operatorname{sech}\left(x\right)^{\frac{3}{2}}}-\frac{x\sqrt{\operatorname{sech}\left(x\right)}}{3}\right)dx$$

input
```
int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)
```

output
```
int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right) dx = -\frac{\int \left(-\frac{3x}{\operatorname{sech}^{\frac{3}{2}}(x)}\right) dx + \int x\sqrt{\operatorname{sech}(x)}\,dx}{3}$$

input
```
integrate(x/sech(x)**(3/2)-1/3*x*sech(x)**(1/2),x)
```

output
```
-(Integral(-3*x/sech(x)**(3/2), x) + Integral(x*sqrt(sech(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right) dx = \int -\frac{1}{3}\,x\sqrt{\operatorname{sech}(x)} + \frac{x}{\operatorname{sech}(x)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x*sqrt(sech(x)) + x/sech(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)\,dx=\int -\frac{1}{3}\,x\sqrt{\operatorname{sech}\left(x\right)}+\frac{x}{\operatorname{sech}\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x*sqrt(sech(x)) + x/sech(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)\,dx=-\int \frac{x\,\sqrt{\frac{1}{\cosh(x)}}}{3}-\frac{x}{{\left(\frac{1}{\cosh(x)}\right)}^{3/2}}\,dx$$

input
```
int(x/(1/cosh(x))^(3/2) - (x*(1/cosh(x))^(1/2))/3,x)
```

output
```
-int((x*(1/cosh(x))^(1/2))/3 - x/(1/cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x\sqrt{\operatorname{sech}(x)}\right)\,dx=\int \frac{\sqrt{\operatorname{sech}\left(x\right)}\,x}{\operatorname{sech}\left(x\right)^{2}}dx-\frac{\left(\int \sqrt{\operatorname{sech}\left(x\right)}\,x dx\right)}{3}$$

input
```
int(x/sech(x)^(3/2)-1/3*x*sech(x)^(1/2),x)
```

output
```
(3*int((sqrt(sech(x))*x)/sech(x)**2,x) - int(sqrt(sech(x))*x,x))/3
```

3.96 $\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=-\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}$$

output
```
-4/25/sech(x)^(5/2)+2/5*x*sinh(x)/sech(x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.71

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=\frac{2(-2+5x\tanh(x))}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

input
```
Integrate[x/Sech[x]^(5/2) - (3*x)/(5*Sqrt[Sech[x]]),x]
```

output
```
(2*(-2 + 5*x*Tanh[x]))/(25*Sech[x]^(5/2))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x\sinh(x)}{5\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{4}{25\operatorname{sech}^{\frac{5}{2}}(x)}$$

input
```
Int[x/Sech[x]^(5/2) - (3*x)/(5*Sqrt[Sech[x]]),x]
```

output
```
-4/(25*Sech[x]^(5/2)) + (2*x*Sinh[x])/(5*Sech[x]^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\operatorname{sech}(x)^{\frac{5}{2}}}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)dx$$

input
```
int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)
```

output
```
int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=-\frac{\int \left(-\frac{5x}{\operatorname{sech}^{\frac{5}{2}}(x)}\right)\,dx+\int \frac{3x}{\sqrt{\operatorname{sech}(x)}}\,dx}{5}$$

input
```
integrate(x/sech(x)**(5/2)-3/5*x/sech(x)**(1/2),x)
```

output
```
-(Integral(-5*x/sech(x)**(5/2), x) + Integral(3*x/sqrt(sech(x)), x))/5
```

Maxima [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=\int -\frac{3\,x}{5\,\sqrt{\operatorname{sech}\left(x\right)}}+\frac{x}{\operatorname{sech}\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-3/5*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=\int -\frac{3\,x}{5\,\sqrt{\operatorname{sech}\left(x\right)}}+\frac{x}{\operatorname{sech}\left(x\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-3/5*x/sqrt(sech(x)) + x/sech(x)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=-\int \frac{3\,x}{5\,\sqrt{\frac{1}{\cosh(x)}}}-\frac{x}{{\left(\frac{1}{\cosh(x)}\right)}^{5/2}}\,dx$$

input
```
int(x/(1/cosh(x))^(5/2) - (3*x)/(5*(1/cosh(x))^(1/2)),x)
```

output
```
-int((3*x)/(5*(1/cosh(x))^(1/2)) - x/(1/cosh(x))^(5/2), x)
```

Reduce [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{5}{2}}(x)}-\frac{3x}{5\sqrt{\operatorname{sech}(x)}}\right)\,dx=\int \frac{\sqrt{\operatorname{sech}\left(x\right)}\,x}{\operatorname{sech}\left(x\right)^{3}}dx-\frac{3\left(\int \frac{\sqrt{\operatorname{sech}(x)}\,x}{\operatorname{sech}(x)}dx\right)}{5}$$

input
```
int(x/sech(x)^(5/2)-3/5*x/sech(x)^(1/2),x)
```

output
```
(5*int((sqrt(sech(x))*x)/sech(x)**3,x) - 3*int((sqrt(sech(x))*x)/sech(x),x
))/5
```

3.97 $\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) \, dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) \, dx=-\frac{4}{49\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{20}{63\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{2x\sinh(x)}{7\operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10x\sinh(x)}{21\sqrt{\operatorname{sech}(x)}}$$

output
```
-4/49/sech(x)^(7/2)-20/63/sech(x)^(3/2)+2/7*x*sinh(x)/sech(x)^(5/2)+10/21*
x*sinh(x)/sech(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) \, dx=\sqrt{\operatorname{sech}(x)}\left(-\frac{167}{882}-\frac{88}{441}\cosh(2x)-\frac{1}{98}\cosh(4x)+\frac{13}{42}x\sinh(2x)+\frac{1}{28}x\sinh(4x)\right)$$

input
```
Integrate[x/Sech[x]^(7/2) - (5*x*Sqrt[Sech[x]])/21,x]
```

output
```
Sqrt[Sech[x]]*(-167/882 - (88*Cosh[2*x])/441 - Cosh[4*x]/98 + (13*x*Sinh[2
*x])/42 + (x*Sinh[4*x])/28)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{20}{63 \operatorname{sech}^{\frac{3}{2}}(x)}-\frac{4}{49 \operatorname{sech}^{\frac{7}{2}}(x)}+\frac{2 x \sinh(x)}{7 \operatorname{sech}^{\frac{5}{2}}(x)}+\frac{10 x \sinh(x)}{21 \sqrt{\operatorname{sech}(x)}}$$

input
```
Int[x/Sech[x]^(7/2) - (5*x*Sqrt[Sech[x]])/21,x]
```

output
```
-4/(49*Sech[x]^(7/2)) - 20/(63*Sech[x]^(3/2)) + (2*x*Sinh[x])/(7*Sech[x]^(
5/2)) + (10*x*Sinh[x])/(21*Sqrt[Sech[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\operatorname{sech}(x)^{\frac{7}{2}}} - \frac{5x\sqrt{\operatorname{sech}(x)}}{21} \right) dx$$

input
```
int(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x)
```

output
```
int(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)} - \frac{5}{21} x\sqrt{\operatorname{sech}(x)} \right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)} - \frac{5}{21} x\sqrt{\operatorname{sech}(x)} \right) dx = -\frac{\int \left(-\frac{21x}{\operatorname{sech}^{\frac{7}{2}}(x)} \right) dx + \int 5x\sqrt{\operatorname{sech}(x)}\, dx}{21}$$

input
```
integrate(x/sech(x)**(7/2)-5/21*x*sech(x)**(1/2),x)
```

output
```
-(Integral(-21*x/sech(x)**(7/2), x) + Integral(5*x*sqrt(sech(x)), x))/21
```

Maxima [F]

$$\int\left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{sech}(x)}\right)dx=\int-\frac{5}{21}\,x\sqrt{\operatorname{sech}\left(x\right)}+\frac{x}{\operatorname{sech}\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-5/21*x*sqrt(sech(x)) + x/sech(x)^(7/2), x)
```

Giac [F]

$$\int\left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{sech}(x)}\right)dx=\int-\frac{5}{21}\,x\sqrt{\operatorname{sech}\left(x\right)}+\frac{x}{\operatorname{sech}\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-5/21*x*sqrt(sech(x)) + x/sech(x)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int\left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)}-\frac{5}{21}x\sqrt{\operatorname{sech}(x)}\right)dx=-\int\frac{5\,x\,\sqrt{\frac{1}{\cosh(x)}}}{21}-\frac{x}{\left(\frac{1}{\cosh(x)}\right)^{7/2}}\,dx$$

input
```
int(x/(1/cosh(x))^(7/2) - (5*x*(1/cosh(x))^(1/2))/21,x)
```

output
```
-int((5*x*(1/cosh(x))^(1/2))/21 - x/(1/cosh(x))^(7/2), x)
```

Reduce [F]

$$\int \left(\frac{x}{\operatorname{sech}^{\frac{7}{2}}(x)} - \frac{5}{21} x \sqrt{\operatorname{sech}(x)}\right) dx = \int \frac{\sqrt{\operatorname{sech}(x)}\, x}{\operatorname{sech}(x)^4} dx - \frac{5\left(\int \sqrt{\operatorname{sech}(x)}\, x dx\right)}{21}$$

input
```
int(x/sech(x)^(7/2)-5/21*x*sech(x)^(1/2),x)
```

output
```
(21*int((sqrt(sech(x))*x)/sech(x)**4,x) - 5*int(sqrt(sech(x))*x,x))/21
```

3.98 $\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 66

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)dx=-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{16}{27}i\sqrt{\cosh(x)}\operatorname{EllipticF}\left(\frac{ix}{2},2\right)\sqrt{\operatorname{sech}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}+\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}$$

output
```
-8/9*x/sech(x)^(3/2)-16/27*I*cosh(x)^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)
)*sech(x)^(1/2)+16/27*sinh(x)/sech(x)^(1/2)+2/3*x^2*sinh(x)/sech(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)\,dx=\frac{2}{27}\sqrt{\operatorname{sech}(x)}\left(-8i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)+\cosh(x)\left(-12x\cosh(x)+\left(8+9x^2\right)\sinh(x)\right)\right)$$

input
```
Integrate[x^2/Sech[x]^(3/2) - (x^2*Sqrt[Sech[x]])/3,x]
```

output
```
(2*Sqrt[Sech[x]]*((-8*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2] + Cosh[x]*(-1
2*x*Cosh[x] + (8 + 9*x^2)*Sinh[x])))/27
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.042, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2x^2\sinh(x)}{3\sqrt{\operatorname{sech}(x)}}-\frac{8x}{9\operatorname{sech}^{\frac{3}{2}}(x)}+\frac{16\sinh(x)}{27\sqrt{\operatorname{sech}(x)}}-\frac{16}{27}i\sqrt{\cosh(x)}\sqrt{\operatorname{sech}(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)$$

input
```
Int[x^2/Sech[x]^(3/2) - (x^2*Sqrt[Sech[x]])/3,x]
```

output
```
(-8*x)/(9*Sech[x]^(3/2)) - ((16*I)/27)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2]
*Sqrt[Sech[x]] + (16*Sinh[x])/(27*Sqrt[Sech[x]]) + (2*x^2*Sinh[x])/(3*Sqrt
[Sech[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x^2}{\operatorname{sech}\left(x\right)^{\frac{3}{2}}}-\frac{x^2\sqrt{\operatorname{sech}\left(x\right)}}{3}\right)dx$$

input
```
int(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x)
```

output
```
int(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)dx = -\frac{\int \left(-\frac{3x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}\right)dx+\int x^2\sqrt{\operatorname{sech}\left(x\right)}\,dx}{3}$$

input
```
integrate(x**2/sech(x)**(3/2)-1/3*x**2*sech(x)**(1/2),x)
```

output
```
-(Integral(-3*x**2/sech(x)**(3/2), x) + Integral(x**2*sqrt(sech(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right) dx = \int -\frac{1}{3}\,x^2\sqrt{\operatorname{sech}\left(x\right)} + \frac{x^2}{\operatorname{sech}\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x^2*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right) dx = \int -\frac{1}{3}\,x^2\sqrt{\operatorname{sech}\left(x\right)} + \frac{x^2}{\operatorname{sech}\left(x\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x^2*sqrt(sech(x)) + x^2/sech(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)} - \frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right) dx = -\int \frac{x^2\sqrt{\frac{1}{\cosh(x)}}}{3} - \frac{x^2}{\left(\frac{1}{\cosh(x)}\right)^{3/2}}\,dx$$

input
```
int(x^2/(1/cosh(x))^(3/2) - (x^2*(1/cosh(x))^(1/2))/3,x)
```

output
```
-int((x^2*(1/cosh(x))^(1/2))/3 - x^2/(1/cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x^2}{\operatorname{sech}^{\frac{3}{2}}(x)}-\frac{1}{3}x^2\sqrt{\operatorname{sech}(x)}\right)dx=\int \frac{\sqrt{\operatorname{sech}(x)}\,x^2}{\operatorname{sech}(x)^2}dx-\frac{\left(\int \sqrt{\operatorname{sech}(x)}\,x^2dx\right)}{3}$$

input
```
int(x^2/sech(x)^(3/2)-1/3*x^2*sech(x)^(1/2),x)
```

output
```
(3*int((sqrt(sech(x))*x**2)/sech(x)**2,x) - int(sqrt(sech(x))*x**2,x))/3
```

3.99 $\int (c+dx)^3(a+a\cosh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx = \frac{a(c+dx)^4}{4d}-\frac{6ad^3\cosh(e+fx)}{f^4} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6ad^2(c+dx)\sinh(e+fx)}{f^3} + \frac{a(c+dx)^3\sinh(e+fx)}{f}$$

output
```
1/4*a*(d*x+c)^4/d-6*a*d^3*cosh(f*x+e)/f^4-3*a*d*(d*x+c)^2*cosh(f*x+e)/f^2+
6*a*d^2*(d*x+c)*sinh(f*x+e)/f^3+a*(d*x+c)^3*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.37

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx = a\left(\frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) - \frac{3d(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)}{f^4} + \frac{(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\sinh(e+fx)}{f^3}\right)$$

input
```
Integrate[(c + d*x)^3*(a + a*Cosh[e + f*x]),x]
```

output
```
a*((x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*d*(c^2*f^2 + 2*c
*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x])/f^4 + ((c + d*x)*(c^2*f^2 + 2
*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x])/f^3)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a\cosh(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^3\cosh(e+fx)+a(c+dx)^3\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{6ad^2(c+dx)\sinh(e+fx)}{f^3} - \frac{3ad(c+dx)^2\cosh(e+fx)}{f^2} + \frac{a(c+dx)^3\sinh(e+fx)}{f} + \frac{a(c+dx)^4}{4d} - \frac{6ad^3\cosh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + a*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) - (6*a*d^3*Cosh[e + f*x])/f^4 - (3*a*d*(c + d*x)^2*C
osh[e + f*x])/f^2 + (6*a*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (a*(c + d*x)^3
*Sinh[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.93

method	result
parallelrisch	$\frac{a\left(\left(\left(\frac{1}{2}x^2d^2+cdx+c^2\right)f^2+6d^2\right)x f^2\left(\frac{dx}{2}+c\right)\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)^2-2(dx+c)\left((dx+c)^2f^2+6d^2\right)f\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)+\left(-\frac{1}{4}d^3x^4-\right.\right.}{f^4\left(\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)^2-1\right)}$
risch	$\frac{a\,d^3x^4}{4}+a\,d^2c\,x^3+\frac{3ad\,c^2x^2}{2}+a\,c^3x+\frac{a\,c^4}{4d}+\frac{a\left(d^3x^3f^3+3c\,d^2f^3x^2+3c^2d\,f^3x-3d^3f^2x^2+c^3f^3-6c\,d^2f^2\right.}{2f^4}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{a\left(\frac{d^3\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{3d^3e\left((fx+e)^2\sinh(fx\right.}{}\right.}{}$
orering	$\frac{\left(d^5f^4x^6+6c\,d^4f^4x^5+15c^2d^3f^4x^4+20c^3d^2f^4x^3+14c^4d\,f^4x^2-24d^5f^2x^4+4c^5f^4x-96c\,d^4f^2x^3-156c^2d^3f^2x^2-120c^3\right.}{4f^4(dx+c)^2}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3a\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3ea\left((f\right.}{}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3a\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3ea\left((f\right.}{}}{}$

input
```
int((d*x+c)^3*(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
a*(((1/2*x^2*d^2+c*d*x+c^2)*f^2+6*d^2)*x*f^2*(1/2*d*x+c)*tanh(1/2*f*x+1/2*
e)^2-2*(d*x+c)*((d*x+c)^2*f^2+6*d^2)*f*tanh(1/2*f*x+1/2*e)+(-1/4*d^3*x^4-d
^2*c*x^3-3/2*x^2*c^2*d-x*c^3)*f^4+3*(d^3*x^2+2*c*d^2*x+2*c^2*d)*f^2+12*d^3
)/f^4/(tanh(1/2*f*x+1/2*e)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int(c+dx)^3(a+a\cosh(e+fx))\,dx$$
$$=\frac{ad^3f^4x^4+4\,acd^2f^4x^3+6\,ac^2df^4x^2+4\,ac^3f^4x-12\left(ad^3f^2x^2+2\,acd^2f^2x+ac^2df^2+2\,ad^3\right)\cosh\left(fx+\right.}{4\,f^4}$$

input
```
integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 - 12*(a*d^3*f^2*x^2 + 2*a*c*d^2*f^2*x + a*c^2*d*f^2 + 2*a*d^3)*cosh(f*x +
 e) + 4*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + a*c^3*f^3 + 6*a*c*d^2*f + 3*(
a*c^2*d*f^3 + 2*a*d^3*f)*x)*sinh(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(88) = 176$.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.97

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx$$
$$= \begin{cases} ac^3x + \frac{ac^3\sinh(e+fx)}{f} + \frac{3ac^2dx^2}{2} + \frac{3ac^2dx\sinh(e+fx)}{f} - \frac{3ac^2d\cosh(e+fx)}{f^2} + acd^2x^3 + \frac{3acd^2x^2\sinh(e+fx)}{f} - \frac{6acd^2x\operatorname{co}}{f} \\ (a\cosh(e)+a)\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+a*cosh(f*x+e)),x)
```

output
```
Piecewise((a*c**3*x + a*c**3*sinh(e + f*x)/f + 3*a*c**2*d*x**2/2 + 3*a*c**
2*d*x*sinh(e + f*x)/f - 3*a*c**2*d*cosh(e + f*x)/f**2 + a*c*d**2*x**3 + 3*
a*c*d**2*x**2*sinh(e + f*x)/f - 6*a*c*d**2*x*cosh(e + f*x)/f**2 + 6*a*c*d*
*2*sinh(e + f*x)/f**3 + a*d**3*x**4/4 + a*d**3*x**3*sinh(e + f*x)/f - 3*a*
d**3*x**2*cosh(e + f*x)/f**2 + 6*a*d**3*x*sinh(e + f*x)/f**3 - 6*a*d**3*co
sh(e + f*x)/f**4, Ne(f, 0)), ((a*cosh(e) + a)*(c**3*x + 3*c**2*d*x**2/2 +
c*d**2*x**3 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(87) = 174$.

Time = 0.05 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx$$
$$= \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x + \frac{3}{2}\,ac^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+ \frac{3}{2}\,acd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right)$$
$$+ \frac{1}{2}\,ad^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} - \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right)$$
$$+ \frac{ac^3\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*a*c^2*d*((f*
x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*a*c*d^2*((f^2
*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x
- e)/f^3) + 1/2*a*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e
^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + a*c^3*s
inh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(87) = 174$.

Time = 0.13 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx = \frac{1}{4}\,ad^3x^4 + acd^2x^3 + \frac{3}{2}\,ac^2dx^2 + ac^3x$$
$$+ \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x - 3\,ad^3f^2x^2 + ac^3f^3 - 6\,acd^2f^2x - 3\,ac^2df^2 + 6\,ad^3fx + 6\,acd^2f}{2\,f^4}$$
$$- \frac{(ad^3f^3x^3 + 3\,acd^2f^3x^2 + 3\,ac^2df^3x + 3\,ad^3f^2x^2 + ac^3f^3 + 6\,acd^2f^2x + 3\,ac^2df^2 + 6\,ad^3fx + 6\,acd^2f}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(a*d^3*f^3*x
^3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x - 3*a*d^3*f^2*x^2 + a*c^3*f^3 - 6
*a*c*d^2*f^2*x - 3*a*c^2*d*f^2 + 6*a*d^3*f*x + 6*a*c*d^2*f - 6*a*d^3)*e^(f
*x + e)/f^4 - 1/2*(a*d^3*f^3*x^3 + 3*a*c*d^2*f^3*x^2 + 3*a*c^2*d*f^3*x + 3
*a*d^3*f^2*x^2 + a*c^3*f^3 + 6*a*c*d^2*f^2*x + 3*a*c^2*d*f^2 + 6*a*d^3*f*x
 + 6*a*c*d^2*f + 6*a*d^3)*e^(-f*x - e)/f^4
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.10

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx = \frac{\sinh(e+fx)\,\left(a\,c^3\,f^2+6\,a\,c\,d^2\right)}{f^3} - \frac{3\cosh(e+fx)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{3\,x\sinh(e+fx)\,\left(a\,c^2\,d\,f^2+2\,a\,d^3\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 - \frac{3\,a\,d^3\,x^2\cosh(e+fx)}{f^2} + \frac{a\,d^3\,x^3\sinh(e+fx)}{f} - \frac{6\,a\,c\,d^2\,x\cosh(e+fx)}{f^2} + \frac{3\,a\,c\,d^2\,x^2\sinh(e+fx)}{f}$$

input
```
int((a + a*cosh(e + f*x))*(c + d*x)^3,x)
```

output
```
(sinh(e + f*x)*(a*c^3*f^2 + 6*a*c*d^2))/f^3 - (3*cosh(e + f*x)*(2*a*d^3 +
a*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*sinh(e + f*x)*(2*a*d^3
+ a*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (3*a*d^3*x^2*cosh(
e + f*x))/f^2 + (a*d^3*x^3*sinh(e + f*x))/f - (6*a*c*d^2*x*cosh(e + f*x))/
f^2 + (3*a*c*d^2*x^2*sinh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.25

$$\int (c+dx)^3(a+a\cosh(e+fx))\,dx$$
$$= \frac{a(-12\cosh(fx+e)\,c^2d\,f^2 - 24\cosh(fx+e)\,c\,d^2f^2x - 12\cosh(fx+e)\,d^3f^2x^2 - 24\cosh(fx+e)\,d^3 +}{}$$

input
```
int((d*x+c)^3*(a+a*cosh(f*x+e)),x)
```

output
```
(a*( - 12*cosh(e + f*x)*c**2*d*f**2 - 24*cosh(e + f*x)*c*d**2*f**2*x - 12*
cosh(e + f*x)*d**3*f**2*x**2 - 24*cosh(e + f*x)*d**3 + 4*sinh(e + f*x)*c**
3*f**3 + 12*sinh(e + f*x)*c**2*d*f**3*x + 12*sinh(e + f*x)*c*d**2*f**3*x**
2 + 24*sinh(e + f*x)*c*d**2*f + 4*sinh(e + f*x)*d**3*f**3*x**3 + 24*sinh(e
 + f*x)*d**3*f*x + 4*c**3*f**4*x + 6*c**2*d*f**4*x**2 + 4*c*d**2*f**4*x**3
 + d**3*f**4*x**4))/(4*f**4)
```

3.100 $\int (c+dx)^2(a+a\cosh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx = \frac{a(c+dx)^3}{3d} - \frac{2ad(c+dx)\cosh(e+fx)}{f^2} + \frac{2ad^2\sinh(e+fx)}{f^3} + \frac{a(c+dx)^2\sinh(e+fx)}{f}$$

output
```
1/3*a*(d*x+c)^3/d-2*a*d*(d*x+c)*cosh(f*x+e)/f^2+2*a*d^2*sinh(f*x+e)/f^3+a*
(d*x+c)^2*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.19

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx = a\left(c^2x + cdx^2 + \frac{d^2x^3}{3} - \frac{2d(c+dx)\cosh(e+fx)}{f^2} + \frac{(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx)}{f^3}\right)$$

input
```
Integrate[(c + d*x)^2*(a + a*Cosh[e + f*x]),x]
```

output
```
a*(c^2*x + c*d*x^2 + (d^2*x^3)/3 - (2*d*(c + d*x)*Cosh[e + f*x])/f^2 + ((c
^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^3)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a\cosh(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2\cosh(e+fx)+a(c+dx)^2\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2ad(c+dx)\cosh(e+fx)}{f^2}+\frac{a(c+dx)^2\sinh(e+fx)}{f}+\frac{a(c+dx)^3}{3d}+\frac{2ad^2\sinh(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + a*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) - (2*a*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*a*d^2*Sin
h[e + f*x])/f^3 + (a*(c + d*x)^2*Sinh[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.58 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

method	result
parallelrisch	$\frac{\left(\left(\left(dx+c\right)^{2} f^{2}+2d^{2}\right) \sinh \left(fx+e\right)+\left(\left(-2d^{2}x-2cd\right) \cosh \left(fx+e\right)+x\left(\frac{1}{3}x^{2}d^{2}+cdx+c^{2}\right) f^{2}-2cd\right) f\right) a}{f^{3}}$
risch	$\frac{a\,d^{2}x^{3}}{3}+adc\,x^{2}+a\,c^{2}x+\frac{a\,c^{3}}{3d}+\frac{a\left(d^{2}x^{2}f^{2}+2cd\,f^{2}x+c^{2}f^{2}-2d^{2}fx-2cdf+2d^{2}\right) {\mathrm e}^{fx+e}}{2f^{3}}-\frac{a\left(d^{2}x^{2}f^{2}+2cd\,f\right.}{}$
parts	$\frac{a\left(dx+c\right)^{3}}{3d}+\frac{a\left(\frac{d^{2}\left(\left(fx+e\right)^{2} \sinh \left(fx+e\right)-2\left(fx+e\right) \cosh \left(fx+e\right)+2\sinh \left(fx+e\right)\right)}{f^{2}}-\frac{2d^{2}e\left(\left(fx+e\right) \sinh \left(fx+e\right)-\cosh \left(fx+e\right)\right)}{f^{2}}+\frac{2dc(}{}\right.}{f}$
derivativedivides	$\frac{\frac{d^{2}a\left(fx+e\right)^{3}}{3f^{2}}+\frac{d^{2}a\left(\left(fx+e\right)^{2} \sinh \left(fx+e\right)-2\left(fx+e\right) \cosh \left(fx+e\right)+2\sinh \left(fx+e\right)\right)}{f^{2}}-\frac{d^{2}ea\left(fx+e\right)^{2}}{f^{2}}-\frac{2d^{2}ea\left(\left(fx+e\right) \sinh \left(fx+e\right)-\cosh \left(\right.\right.}{f^{2}}}{}$
default	$\frac{\frac{d^{2}a\left(fx+e\right)^{3}}{3f^{2}}+\frac{d^{2}a\left(\left(fx+e\right)^{2} \sinh \left(fx+e\right)-2\left(fx+e\right) \cosh \left(fx+e\right)+2\sinh \left(fx+e\right)\right)}{f^{2}}-\frac{d^{2}ea\left(fx+e\right)^{2}}{f^{2}}-\frac{2d^{2}ea\left(\left(fx+e\right) \sinh \left(fx+e\right)-\cosh \left(\right.\right.}{f^{2}}}{}$
orering	$\frac{\left(d^{4}f^{4}x^{5}+5c\,d^{3}f^{4}x^{4}+10c^{2}d^{2}f^{4}x^{3}+9c^{3}d\,f^{4}x^{2}+3c^{4}f^{4}x-12d^{4}f^{2}x^{3}-42c\,d^{3}f^{2}x^{2}-48c^{2}d^{2}f^{2}x-12c^{3}d\,f^{2}-48d^{4}x-12d^{3}\right.}{3f^{4}\left(dx+c\right)^{2}}$

input
```
int((d*x+c)^2*(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(((d*x+c)^2*f^2+2*d^2)*sinh(f*x+e)+((-2*d^2*x-2*c*d)*cosh(f*x+e)+x*(1/3*x^
2*d^2+c*d*x+c^2)*f^2-2*c*d)*f)*a/f^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx$$
$$=\frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x-6\,(ad^2fx+acdf)\cosh{(fx+e)}+3\,(ad^2f^2x^2+2\,acdf^2x+ac^2f^2+2\,}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 6*(a*d^2*f*x + a*c*
d*f)*cosh(f*x + e) + 3*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 + 2*a*d^
2)*sinh(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx$$
$$=\begin{cases} ac^2x+\frac{ac^2\sinh{(e+fx)}}{f}+acdx^2+\frac{2acdx\sinh{(e+fx)}}{f}-\frac{2acd\cosh{(e+fx)}}{f^2}+\frac{ad^2x^3}{3}+\frac{ad^2x^2\sinh{(e+fx)}}{f}-\frac{2ad^2x\cosh{(e+fx}}{f^2} \\ (a\cosh{(e)}+a)\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+a*cosh(f*x+e)),x)
```

output
```
Piecewise((a*c**2*x + a*c**2*sinh(e + f*x)/f + a*c*d*x**2 + 2*a*c*d*x*sinh
(e + f*x)/f - 2*a*c*d*cosh(e + f*x)/f**2 + a*d**2*x**3/3 + a*d**2*x**2*sin
h(e + f*x)/f - 2*a*d**2*x*cosh(e + f*x)/f**2 + 2*a*d**2*sinh(e + f*x)/f**3
, Ne(f, 0)), ((a*cosh(e) + a)*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.05 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + acd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}\,ad^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{ac^2\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + a*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 -
 (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*a*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e
)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*c^2*sinh(f*x +
 e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.

Time = 0.14 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 - 2\,ad^2fx - 2\,acdf + 2\,ad^2)e^{(fx+e)}}{2\,f^3} - \frac{(ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2 + 2\,ad^2fx + 2\,acdf + 2\,ad^2)e^{(-fx-e)}}{2\,f^3}$$

input
```
integrate((d*x+c)^2*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(a*d^2*f^2*x^2 + 2*a*c*d*f^2*x +
 a*c^2*f^2 - 2*a*d^2*f*x - 2*a*c*d*f + 2*a*d^2)*e^(f*x + e)/f^3 - 1/2*(a*d
^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2 + 2*a*d^2*f*x + 2*a*c*d*f + 2*a*d^2
)*e^(-f*x - e)/f^3
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.67

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx = \frac{2\,a\,d^2\sinh(e+f\,x)-\frac{a\,f\,\left(6\,x\cosh(e+f\,x)\,d^2+6\,c\cosh(e+f\,x)\,d\right)}{3}+\frac{a\,f^2\,\left(3\,c^2\sinh(e+f\,x)+3\,d^2\,x^2\sinh(e+f\,x)+6\,c\,d\,x\sinh(e+f\,x)\right)}{3}}{f^3} + \frac{a\,\left(3\,c^2\,x+3\,c\,d\,x^2+d^2\,x^3\right)}{3}$$

input
```
int((a + a*cosh(e + f*x))*(c + d*x)^2,x)
```

output
```
(2*a*d^2*sinh(e + f*x) - (a*f*(6*d^2*x*cosh(e + f*x) + 6*c*d*cosh(e + f*x)
))/3 + (a*f^2*(3*c^2*sinh(e + f*x) + 3*d^2*x^2*sinh(e + f*x) + 6*c*d*x*sin
h(e + f*x)))/3)/f^3 + (a*(3*c^2*x + d^2*x^3 + 3*c*d*x^2))/3
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.73

$$\int (c+dx)^2(a+a\cosh(e+fx))\,dx = \frac{a(-6\cosh(fx+e)\,cdf-6\cosh(fx+e)\,d^2fx+3\sinh(fx+e)\,c^2f^2+6\sinh(fx+e)\,cd\,f^2x+3\sinh(\ldots}{3f^3}$$

input
```
int((d*x+c)^2*(a+a*cosh(f*x+e)),x)
```

output
```
(a*( - 6*cosh(e + f*x)*c*d*f - 6*cosh(e + f*x)*d**2*f*x + 3*sinh(e + f*x)*
c**2*f**2 + 6*sinh(e + f*x)*c*d*f**2*x + 3*sinh(e + f*x)*d**2*f**2*x**2 +
6*sinh(e + f*x)*d**2 + 3*c**2*f**3*x + 3*c*d*f**3*x**2 + d**2*f**3*x**3))/
(3*f**3)
```

3.101 $\int (c+dx)(a+a\cosh(e+fx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int (c+dx)(a+a\cosh(e+fx))\,dx=\frac{a(c+dx)^2}{2d}-\frac{ad\cosh(e+fx)}{f^2}+\frac{a(c+dx)\sinh(e+fx)}{f}$$

output
```
1/2*a*(d*x+c)^2/d-a*d*cosh(f*x+e)/f^2+a*(d*x+c)*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int (c+dx)(a+a\cosh(e+fx))\,dx=\frac{a(-2(e+fx)(de-2cf-dfx)-4d\cosh(e+fx)+4f(c+dx)\sinh(e+fx))}{4f^2}$$

input
```
Integrate[(c + d*x)*(a + a*Cosh[e + f*x]),x]
```

output
```
(a*(-2*(e + f*x)*(d*e - 2*c*f - d*f*x) - 4*d*Cosh[e + f*x] + 4*f*(c + d*x)
*Sinh[e + f*x]))/(4*f^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a\cosh(e+fx)+a)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)\cosh(e+fx)+a(c+dx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)\sinh(e+fx)}{f}+\frac{a(c+dx)^2}{2d}-\frac{ad\cosh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + a*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (a*d*Cosh[e + f*x])/f^2 + (a*(c + d*x)*Sinh[e + f*
x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{\left(f(dx+c)\sinh(fx+e)-d\cosh(fx+e)+x\left(\frac{dx}{2}+c\right)f^2-d\right)a}{f^2}$
risch	$\frac{ad\,x^2}{2}+acx+\frac{a(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^2}-\frac{a(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$
parts	$a\left(\frac{1}{2}d\,x^2+cx\right)+\frac{a\left(\frac{d((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{de\sinh(fx+e)}{f}+c\sinh(fx+e)\right)}{f}$
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{da((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{dea\sinh(fx+e)}{f}+ca(fx+e)+ca\sinh(fx+e)}{f}$
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{da((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{dea\sinh(fx+e)}{f}+ca(fx+e)+ca\sinh(fx+e)}{f}$
orering	$\frac{\left(d^3f^2x^4+4c\,d^2f^2x^3+5c^2d\,f^2x^2+2c^3f^2x-6d^3x^2-12c\,d^2x-4d\,c^2\right)(a+a\cosh(fx+e))}{2f^2(dx+c)^2}+\frac{(2x^2d^2+4cdx+c^2)(d(a+a\,c\cdots}{(\cdots}$

input
```
int((d*x+c)*(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(f*(d*x+c)*sinh(f*x+e)-d*cosh(f*x+e)+x*(1/2*d*x+c)*f^2-d)*a/f^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c+dx)(a+a\cosh(e+fx))\,dx = \frac{adf^2x^2+2\,acf^2x-2\,ad\cosh(fx+e)+2\,(adfx+acf)\sinh(fx+e)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*a*d*cosh(f*x + e) + 2*(a*d*f*x + a*c*f)
*sinh(f*x + e))/f^2
```

Sympy [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int (c+dx)(a+a\cosh(e+fx))\,dx = \begin{cases} acx + \frac{ac\sinh(e+fx)}{f} + \frac{adx^2}{2} + \frac{adx\sinh(e+fx)}{f} - \frac{ad\cosh(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a\cosh(e)+a)\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e)),x)
```

output
```
Piecewise((a*c*x + a*c*sinh(e + f*x)/f + a*d*x**2/2 + a*d*x*sinh(e + f*x)/
f - a*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a*cosh(e) + a)*(c*x + d*x**2/2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int (c+dx)(a+a\cosh(e+fx))\,dx = \frac{1}{2}adx^2 + acx + \frac{1}{2}ad\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{ac\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*a*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(
-f*x - e)/f^2) + a*c*sinh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int (c+dx)(a+a\cosh(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx + \frac{(adfx + acf - ad)e^{(fx+e)}}{2\,f^2} - \frac{(adfx + acf + ad)e^{(-fx-e)}}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*(a*d*f*x + a*c*f - a*d)*e^(f*x + e)/f^2 - 1/2*(a
*d*f*x + a*c*f + a*d)*e^(-f*x - e)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int (c+dx)(a+a\cosh(e+fx))\,dx = \frac{\frac{a\,f\,(2\,c\sinh(e+f\,x)+2\,d\,x\sinh(e+f\,x))}{2} - a\,d\cosh(e+f\,x)}{f^2} + \frac{a\,(d\,x^2+2\,c\,x)}{2}$$

input
```
int((a + a*cosh(e + f*x))*(c + d*x),x)
```

output
```
((a*f*(2*c*sinh(e + f*x) + 2*d*x*sinh(e + f*x)))/2 - a*d*cosh(e + f*x))/f^
2 + (a*(2*c*x + d*x^2))/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.16

$$\int (c+dx)(a+a\cosh(e+fx))\,dx = \frac{a(-2\cosh(fx+e)\,d+2\sinh(fx+e)\,cf+2\sinh(fx+e)\,dfx+2cf^2x+df^2x^2)}{2f^2}$$

input
```
int((d*x+c)*(a+a*cosh(f*x+e)),x)
```

output
```
(a*( - 2*cosh(e + f*x)*d + 2*sinh(e + f*x)*c*f + 2*sinh(e + f*x)*d*f*x + 2
*c*f**2*x + d*f**2*x**2))/(2*f**2)
```

3.102 $\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx=\frac{a\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}+\frac{a\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
a*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d+a*ln(d*x+c)/d-a*sinh(-e+c*f/d)*Shi(c*f/d
+f*x)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.84

$$\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx=\frac{a\left(\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+\log(c+dx)+\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d}$$

input
```
Integrate[(a + a*Cosh[e + f*x])/(c + d*x),x]
```

output
```
(a*(Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + Log[c + d*x] + Sinh[e -
(c*f)/d]*SinhIntegral[f*(c/d + x)]))/d
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cosh(e+fx)+a}{c+dx}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)}{c+dx}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a\cosh(e+fx)}{c+dx}+\frac{a}{c+dx}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{a\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{a\log(c+dx)}{d}$$

input
```
Int[(a + a*Cosh[e + f*x])/(c + d*x),x]
```

output
```
(a*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
 (a*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{a\ln(dx+c)}{d}-\frac{a\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d}-\frac{a\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d}$	94

input
```
int((a+a*cosh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d-1/2*a/d*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)-1/2*a/d*e
xp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.73

$$\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx$$
$$=\frac{\left(a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\right.}{2\,d}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((a*Ei((d*f*x + c*f)/d) + a*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d)
 + 2*a*log(d*x + c) - (a*Ei((d*f*x + c*f)/d) - a*Ei(-(d*f*x + c*f)/d))*sin
h(-(d*e - c*f)/d))/d
```

Sympy [F]

$$\int \frac{a + a\cosh(e + fx)}{c + dx}\, dx = a\left(\int \frac{\cosh(e + fx)}{c + dx}\, dx + \int \frac{1}{c + dx}\, dx\right)$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c),x)
```

output
```
a*(Integral(cosh(e + f*x)/(c + d*x), x) + Integral(1/(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{a + a\cosh(e + fx)}{c + dx}\, dx = -\frac{1}{2}a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a\log(dx + c)}{d}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*a*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*
exp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx$$
$$=\frac{a\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+a\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(a*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + a*Ei(-(d*f*x + c*f)/d)*e^(-e +
c*f/d) + 2*a*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx=\int \frac{a+a\cosh(e+f\,x)}{c+d\,x}\,dx$$

input
```
int((a + a*cosh(e + f*x))/(c + d*x),x)
```

output
```
int((a + a*cosh(e + f*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{a+a\cosh(e+fx)}{c+dx}\,dx=\frac{a\left(\left(\int \frac{\cosh(fx+e)}{dx+c}dx\right)d+\log(dx+c)\right)}{d}$$

input
```
int((a+a*cosh(f*x+e))/(d*x+c),x)
```

output
```
(a*(int(cosh(e + f*x)/(c + d*x),x)*d + log(c + d*x)))/d
```

3.103 $\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)} - \frac{a\cosh(e+fx)}{d(c+dx)} + \frac{af\,\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2} + \frac{af\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)-a*cosh(f*x+e)/d/(d*x+c)-a*f*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2
+a*f*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.78

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx = \frac{a\left(-\frac{d(1+\cosh(e+fx))}{c+dx} + f\,\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right) + f\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{d^2}$$

input
```
Integrate[(a + a*Cosh[e + f*x])/(c + d*x)^2,x]
```

output
```
(a*(-((d*(1 + Cosh[e + f*x]))/(c + d*x)) + f*CoshIntegral[f*(c/d + x)]*Sin
h[e - (c*f)/d] + f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)]))/d^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cosh(e+fx)+a}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a\cosh(e+fx)}{(c+dx)^2}+\frac{a}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{af\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{af\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{a\cosh(e+fx)}{d(c+dx)}-\frac{a}{d(c+dx)}$$

input
```
Int[(a + a*Cosh[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) - (a*Cosh[e + f*x])/(d*(c + d*x)) + (a*f*CoshIntegral[(
c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (a*f*Cosh[e - (c*f)/d]*SinhIntegral
[(c*f)/d + f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{a}{d(dx+c)}-\frac{fa\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}+\frac{fa\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{fa\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{fa\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-\right.}{2d^2}$

input
```
int((a+a*cosh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)-1/2*f*a*exp(-f*x-e)/d/(d*f*x+c*f)+1/2*f*a/d^2*exp((c*f-d*e)/d
)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*f*a/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*f*a/d^2*e
xp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.86

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx = -\frac{2\,ad\cosh\left(fx+e\right)+2\,ad-\left((adfx+acf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(adfx+acf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*cosh(f*x + e) + 2*a*d - ((a*d*f*x + a*c*f)*Ei((d*f*x + c*f)/d)
 - (a*d*f*x + a*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((a*d*f*
x + a*c*f)*Ei((d*f*x + c*f)/d) + (a*d*f*x + a*c*f)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + a\cosh(e + fx)}{(c + dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{a + a\cosh(e + fx)}{(c + dx)^2}\,dx = -\frac{1}{2}\,a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a}{d^2x + cd}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*a*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) + e^
(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x +
c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(90) = 180$.

Time = 0.15 (sec) , antiderivative size = 631, normalized size of antiderivative = 7.25

$$\int \frac{a + a\cosh(e + fx)}{(c + dx)^2}\,dx$$

$$= \frac{1}{2}a\left(\frac{\left((dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right)f^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)} - def^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}\right.}{d}\right.\right.}{\left((dx+c)d^4\left(\frac{de}{dx+c} - \frac{cf}{dx+c}\right.\right.}\right.$$

$$- \frac{a}{(dx+c)d}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/2*a*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d
*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*
f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((
d*e - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
d*e + c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d^5*e + c*d^4*f)*f) - ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^
2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(
d*e - c*f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x
+ c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+a\cosh(e+f\,x)}{(c+d\,x)^2}\,dx$$

input
```
int((a + a*cosh(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + a*cosh(e + f*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^2}\,dx = \frac{a\left(e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)c^2+e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)cdx+2e^ex+\left(\int \frac{1}{e^{fx}c^2+2e^{fx}cdx+e^{fx}d^2x^2}dx\right)c^2+\left(\int \frac{1}{e^{f}\right.\right.}{2e^ec\,(dx+c)}$$

input
```
int((a+a*cosh(f*x+e))/(d*x+c)^2,x)
```

output
```
(a*(e**(2*e)*int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 + e**(2*e)*
int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + 2*e**e*x + int(1/(e**
(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*c**2 + int(1/(e**(f
*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*c*d*x))/(2*e**e*c*(c
+ d*x))
```

3.104 $\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2} - \frac{a\cosh(e+fx)}{2d(c+dx)^2} + \frac{af^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3} - \frac{af\sinh(e+fx)}{2d^2(c+dx)} + \frac{af^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*a*cosh(f*x+e)/d/(d*x+c)^2+1/2*a*f^2*cosh(-e+c*f/d)*
Chi(c*f/d+f*x)/d^3-1/2*a*f*sinh(f*x+e)/d^2/(d*x+c)-1/2*a*f^2*sinh(-e+c*f/d
)*Shi(c*f/d+f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.73

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx = \frac{a\left(f^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(d+d\cosh(e+fx)+f(c+dx)\sinh(e+fx))}{(c+dx)^2}+f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)\right)}{2d^3}$$

input
```
Integrate[(a + a*Cosh[e + f*x])/(c + d*x)^3,x]
```

output
```
(a*(f^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(d + d*Cosh[e + f
*x] + f*(c + d*x)*Sinh[e + f*x]))/(c + d*x)^2 + f^2*Sinh[e - (c*f)/d]*Sinh
Integral[f*(c/d + x)]))/(2*d^3)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a\cosh(e+fx)+a}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a\cosh(e+fx)}{(c+dx)^3}+\frac{a}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{af^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{af^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{af\sinh(e+fx)}{2d^2(c+dx)}-\frac{a\cosh(e+fx)}{2d(c+dx)^2}-\frac{a}{2d(c+dx)^2}$$

input
```
Int[(a + a*Cosh[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - (a*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (a*f^2*Cosh
[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (a*f*Sinh[e + f*x])/(
2*d^2*(c + d*x)) + (a*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(
2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(115) = 230$.

Time = 0.87 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.41

method	result
risch	$-\frac{a}{2d(dx+c)^2}+\frac{f^3 a\,\mathrm{e}^{-fx-e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^3 a\,\mathrm{e}^{-fx-e}c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 a\,\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 a\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{ex}}{}$

input
```
int((a+a*cosh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2+1/4*f^3*a*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^
2)*x+1/4*f^3*a*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*f^2
*a*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*f^2*a/d^3*exp((c*f-
d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*a/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4
*f^2*a/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*a/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(115) = 230$.

Time = 0.11 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.23

$$\int \frac{a + a\cosh(e + fx)}{(c + dx)^3}\,dx = -\frac{2\,ad^2\cosh\left(fx+e\right) + 2\,ad^2 - \left((ad^2f^2x^2 + 2\,acdf^2x + ac^2f^2)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right) + (ad^2f^2x^2 + 2\,acdf^2x + ac^2 f\right.}{}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*d^2*cosh(f*x + e) + 2*a*d^2 - ((a*d^2*f^2*x^2 + 2*a*c*d*f^2*x +
a*c^2*f^2)*Ei((d*f*x + c*f)/d) + (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^
2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + 2*(a*d^2*f*x + a*c*d*f)*si
nh(f*x + e) + ((a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*Ei((d*f*x + c*f
)/d) - (a*d^2*f^2*x^2 + 2*a*c*d*f^2*x + a*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + a\cosh(e + fx)}{(c + dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx = -\frac{1}{2}\,a\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*a*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) +
e^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(
d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(115) = 230.

Time = 0.13 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.57

$$\int \frac{a+a\cosh(e+fx)}{(c+dx)^3}\,dx = \frac{ad^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+ad^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,acdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+2\,acdf^2x\mathrm{Ei}\left(\right.}{}$$

input
```
integrate((a+a*cosh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(a*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + a*d^2*f^2*x^2*Ei(-(
d*f*x + c*f)/d)*e^(-e + c*f/d) + 2*a*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e -
c*f/d) + 2*a*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + a*c^2*f^2*Ei(
(d*f*x + c*f)/d)*e^(e - c*f/d) + a*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*
f/d) - a*d^2*f*x*e^(f*x + e) + a*d^2*f*x*e^(-f*x - e) - a*c*d*f*e^(f*x + e
) + a*c*d*f*e^(-f*x - e) - a*d^2*e^(f*x + e) - a*d^2*e^(-f*x - e) - 2*a*d^
2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a + a\cosh(e + fx)}{(c + dx)^3}\,dx = \int \frac{a + a\cosh(e + f\,x)}{(c + d\,x)^3}\,dx$$

input
```
int((a + a*cosh(e + f*x))/(c + d*x)^3,x)
```

output
```
int((a + a*cosh(e + f*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{a + a\cosh(e + fx)}{(c + dx)^3}\,dx = \frac{a\left(e^{2e}\left(\int \frac{e^{fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c^2d + 2e^{2e}\left(\int \frac{e^{fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c\,d^2x + e^{2e}\left(\int \frac{e^{fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\right.\right.}{}$$

input
```
int((a+a*cosh(f*x+e))/(d*x+c)^3,x)
```

output
```
(a*(e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x)*c**2*d + 2*e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d
**3*x**3),x)*c*d**2*x + e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**
2*x**2 + d**3*x**3),x)*d**3*x**2 - e**e + int(1/(e**(f*x)*c**3 + 3*e**(f*x
)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*c**2*d + 2*in
t(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*
x)*d**3*x**3),x)*c*d**2*x + int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3
*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*d**3*x**2))/(2*e**e*d*(c**2
 + 2*c*d*x + d**2*x**2))
```

3.105 $\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx &= \frac{3a^2d(c+dx)^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{12a^2d^3\cosh(e+fx)}{f^4}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}\\&\quad-\frac{3a^2d^3\cosh^2(e+fx)}{8f^4}-\frac{3a^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}\\&\quad+\frac{12a^2d^2(c+dx)\sinh(e+fx)}{f^3}+\frac{2a^2(c+dx)^3\sinh(e+fx)}{f}\\&\quad+\frac{3a^2d^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{4f^3}+\frac{a^2(c+dx)^3\cosh(e+fx)\sinh(e+fx)}{2f}\end{aligned}$$

output
```
3/8*a^2*d*(d*x+c)^2/f^2+3/8*a^2*(d*x+c)^4/d-12*a^2*d^3*cosh(f*x+e)/f^4-6*a
^2*d*(d*x+c)^2*cosh(f*x+e)/f^2-3/8*a^2*d^3*cosh(f*x+e)^2/f^4-3/4*a^2*d*(d*
x+c)^2*cosh(f*x+e)^2/f^2+12*a^2*d^2*(d*x+c)*sinh(f*x+e)/f^3+2*a^2*(d*x+c)^
3*sinh(f*x+e)/f+3/4*a^2*d^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*a^2*(d
*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.93 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.97

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx$$
$$= \frac{a^2(-96d(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-3d(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2))\cosh(2(e+}{}$$

input
```
Integrate[(c + d*x)^3*(a + a*Cosh[e + f*x])^2,x]
```

output
```
(a^2*(-96*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 3*
d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] + 2*f*
(3*f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*(c + d*x)*(c^2*f
^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x] + (c + d*x)*(2*c^2*f^2
 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f*x)])))/(16*f^4)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a\cosh(e+fx)+a)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int (c+dx)^3\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(a^2(c+dx)^3\cosh^2(e+fx)+2a^2(c+dx)^3\cosh(e+fx)+a^2(c+dx)^3\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{12a^2d^2(c+dx)\sinh(e+fx)}{f^3}+\frac{3a^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}-\frac{3a^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}-\frac{6a^2d(c+dx)^2\cosh(e+fx)}{f^2}+\frac{2a^2(c+dx)^3\sinh(e+fx)}{f}+\frac{a^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{3a^2d(c+dx)^2}{8f^2}+\frac{3a^2(c+dx)^4}{8d}-\frac{3a^2d^3\cosh^2(e+fx)}{8f^4}-\frac{12a^2d^3\cosh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + a*Cosh[e + f*x])^2,x]
```

output
```
(3*a^2*d*(c + d*x)^2)/(8*f^2) + (3*a^2*(c + d*x)^4)/(8*d) - (12*a^2*d^3*Co
sh[e + f*x])/f^4 - (6*a^2*d*(c + d*x)^2*Cosh[e + f*x])/f^2 - (3*a^2*d^3*Co
sh[e + f*x]^2)/(8*f^4) - (3*a^2*d*(c + d*x)^2*Cosh[e + f*x]^2)/(4*f^2) + (
12*a^2*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (2*a^2*(c + d*x)^3*Sinh[e + f*x]
)/f + (3*a^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) + (a^2*(c
+ d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{\left((dx+c)\left((dx+c)^2 f^2+\frac{3d^2}{2}\right) f\sinh(2fx+2e)-\frac{3d\left((dx+c)^2 f^2+\frac{d^2}{2}\right)\cosh(2fx+2e)}{2}+8(dx+c)\left((dx+c)^2 f^2+6d^2\right) f\sinh(f$
risch	$\frac{3a^2d^3x^4}{8}+\frac{3a^2d^2c\,x^3}{2}+\frac{9a^2d\,c^2x^2}{4}+\frac{3a^2c^3x}{2}+\frac{3a^2c^4}{8d}+\frac{a^2(4d^3x^3f^3+12c\,d^2f^3x^2+12c^2d\,f^3x-6d^3f^2x^2+4c^3}{32}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((d*x+c)*((d*x+c)^2*f^2+3/2*d^2)*f*sinh(2*f*x+2*e)-3/2*d*((d*x+c)^2*f^
2+1/2*d^2)*cosh(2*f*x+2*e)+8*(d*x+c)*((d*x+c)^2*f^2+6*d^2)*f*sinh(f*x+e)-2
4*d*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)+(6*d^2*c*x^3+9*x^2*c^2*d+3/2*d^3*x^4
+6*x*c^3)*f^4-45/2*c^2*d*f^2-189/4*d^3)*a^2/f^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.76

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx = \frac{6\,a^2d^3f^4x^4+24\,a^2cd^2f^4x^3+36\,a^2c^2df^4x^2+24\,a^2c^3f^4x-3\left(2\,a^2d^3f^2x^2+4\,a^2cd^2f^2x+2\,a^2c^2df^2+a^2d^3\right.}{}$$

input
```
integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(6*a^2*d^3*f^4*x^4 + 24*a^2*c*d^2*f^4*x^3 + 36*a^2*c^2*d*f^4*x^2 + 24
*a^2*c^3*f^4*x - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x + 2*a^2*c^2*d*f^
2 + a^2*d^3)*cosh(f*x + e)^2 - 3*(2*a^2*d^3*f^2*x^2 + 4*a^2*c*d^2*f^2*x +
2*a^2*c^2*d*f^2 + a^2*d^3)*sinh(f*x + e)^2 - 96*(a^2*d^3*f^2*x^2 + 2*a^2*c
*d^2*f^2*x + a^2*c^2*d*f^2 + 2*a^2*d^3)*cosh(f*x + e) + 4*(8*a^2*d^3*f^3*x
^3 + 24*a^2*c*d^2*f^3*x^2 + 8*a^2*c^3*f^3 + 48*a^2*c*d^2*f + 24*(a^2*c^2*d
*f^3 + 2*a^2*d^3*f)*x + (2*a^2*d^3*f^3*x^3 + 6*a^2*c*d^2*f^3*x^2 + 2*a^2*c
^3*f^3 + 3*a^2*c*d^2*f + 3*(2*a^2*c^2*d*f^3 + a^2*d^3*f)*x)*cosh(f*x + e))
*sinh(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(228) = 456$.

Time = 0.42 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.48

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(a+a*cosh(f*x+e))**2,x)
```

output
```
Piecewise((-a**2*c**3*x*sinh(e + f*x)**2/2 + a**2*c**3*x*cosh(e + f*x)**2/
2 + a**2*c**3*x + a**2*c**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c**
3*sinh(e + f*x)/f - 3*a**2*c**2*d*x**2*sinh(e + f*x)**2/4 + 3*a**2*c**2*d*
x**2*cosh(e + f*x)**2/4 + 3*a**2*c**2*d*x**2/2 + 3*a**2*c**2*d*x*sinh(e +
f*x)*cosh(e + f*x)/(2*f) + 6*a**2*c**2*d*x*sinh(e + f*x)/f - 3*a**2*c**2*d
*sinh(e + f*x)**2/(4*f**2) - 6*a**2*c**2*d*cosh(e + f*x)/f**2 - a**2*c*d**
2*x**3*sinh(e + f*x)**2/2 + a**2*c*d**2*x**3*cosh(e + f*x)**2/2 + a**2*c*d
**2*x**3 + 3*a**2*c*d**2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 6*a**2*c
*d**2*x**2*sinh(e + f*x)/f - 3*a**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2) - 3
*a**2*c*d**2*x*cosh(e + f*x)**2/(4*f**2) - 12*a**2*c*d**2*x*cosh(e + f*x)/
f**2 + 3*a**2*c*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 12*a**2*c*d**2
*sinh(e + f*x)/f**3 - a**2*d**3*x**4*sinh(e + f*x)**2/8 + a**2*d**3*x**4*c
osh(e + f*x)**2/8 + a**2*d**3*x**4/4 + a**2*d**3*x**3*sinh(e + f*x)*cosh(e
 + f*x)/(2*f) + 2*a**2*d**3*x**3*sinh(e + f*x)/f - 3*a**2*d**3*x**2*sinh(e
 + f*x)**2/(8*f**2) - 3*a**2*d**3*x**2*cosh(e + f*x)**2/(8*f**2) - 6*a**2*
d**3*x**2*cosh(e + f*x)/f**2 + 3*a**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(
4*f**3) + 12*a**2*d**3*x*sinh(e + f*x)/f**3 - 3*a**2*d**3*sinh(e + f*x)**2
/(8*f**4) - 12*a**2*d**3*cosh(e + f*x)/f**4, Ne(f, 0)), ((a*cosh(e) + a)**
2*(c**3*x + 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 527 vs. $2(212) = 424$.

Time = 0.08 (sec) , antiderivative size = 527, normalized size of antiderivative = 2.35

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx = \frac{1}{4}\,a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}\,a^2c^2dx^2$$
$$+\frac{3}{16}\left(4\,x^2 + \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} - \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2c^2d$$
$$+\frac{1}{16}\left(8\,x^3 + \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} - \frac{3\,(2\,f^2x^2+2\,fx+1)e^{(-2\,fx-2\,e)}}{f^3}\right)a^2cd^2$$
$$+\frac{1}{32}\left(4\,x^4 + \frac{\left(4\,f^3x^3e^{(2\,e)} - 6\,f^2x^2e^{(2\,e)} + 6\,fxe^{(2\,e)} - 3\,e^{(2\,e)}\right)e^{(2\,fx)}}{f^4} - \frac{(4\,f^3x^3+6\,f^2x^2+6\,fx+3)e^{(-2\cdots}}{f^4}\right.$$
$$+\frac{1}{8}\,a^2c^3\left(4\,x + \frac{e^{(2\,fx+2\,e)}}{f} - \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^3x$$
$$+3\,a^2c^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+3\,a^2cd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2+2\,fx+2)e^{(-fx-e)}}{f^3}\right)$$
$$+a^2d^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} - \frac{(f^3x^3+3\,f^2x^2+6\,fx+6)e^{(-fx-e)}}{f^4}\right)$$
$$+\frac{2\,a^2c^3\sinh\left(fx+e\right)}{f}$$

input
```
integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 + (2*f*x
*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*
c^2*d + 1/16*(8*x^3 + 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2
*f*x)/f^3 - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*c*d^2 + 1/
32*(4*x^4 + (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(
2*e))*e^(2*f*x)/f^4 - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)
/f^4)*a^2*d^3 + 1/8*a^2*c^3*(4*x + e^(2*f*x + 2*e)/f - e^(-2*f*x - 2*e)/f)
 + a^2*c^3*x + 3*a^2*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*
x - e)/f^2) + 3*a^2*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 -
 (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a^2*d^3*((f^3*x^3*e^e - 3*f^2*x
^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6
)*e^(-f*x - e)/f^4) + 2*a^2*c^3*sinh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(212) = 424$.

Time = 0.13 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.58

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx = \frac{3}{8}\,a^2d^3x^4 + \frac{3}{2}\,a^2cd^2x^3 + \frac{9}{4}\,a^2c^2dx^2 + \frac{3}{2}\,a^2c^3x$$
$$+ \frac{(4\,a^2d^3f^3x^3 + 12\,a^2cd^2f^3x^2 + 12\,a^2c^2df^3x - 6\,a^2d^3f^2x^2 + 4\,a^2c^3f^3 - 12\,a^2cd^2f^2x - 6\,a^2c^2df^2 + 6\,a^2d^3}{32\,f^4}$$
$$+ \frac{(a^2d^3f^3x^3 + 3\,a^2cd^2f^3x^2 + 3\,a^2c^2df^3x - 3\,a^2d^3f^2x^2 + a^2c^3f^3 - 6\,a^2cd^2f^2x - 3\,a^2c^2df^2 + 6\,a^2d^3fx + 6}{f^4}$$
$$- \frac{(a^2d^3f^3x^3 + 3\,a^2cd^2f^3x^2 + 3\,a^2c^2df^3x + 3\,a^2d^3f^2x^2 + a^2c^3f^3 + 6\,a^2cd^2f^2x + 3\,a^2c^2df^2 + 6\,a^2d^3fx + 6}{f^4}$$
$$- \frac{(4\,a^2d^3f^3x^3 + 12\,a^2cd^2f^3x^2 + 12\,a^2c^2df^3x + 6\,a^2d^3f^2x^2 + 4\,a^2c^3f^3 + 12\,a^2cd^2f^2x + 6\,a^2c^2df^2 + 6\,a^2d^3}{32\,f^4}$$

input
```
integrate((d*x+c)^3*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
3/8*a^2*d^3*x^4 + 3/2*a^2*c*d^2*x^3 + 9/4*a^2*c^2*d*x^2 + 3/2*a^2*c^3*x +
1/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x - 6*a^
2*d^3*f^2*x^2 + 4*a^2*c^3*f^3 - 12*a^2*c*d^2*f^2*x - 6*a^2*c^2*d*f^2 + 6*a
^2*d^3*f*x + 6*a^2*c*d^2*f - 3*a^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a^2*d^3*f^3
*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x - 3*a^2*d^3*f^2*x^2 + a^2*c
^3*f^3 - 6*a^2*c*d^2*f^2*x - 3*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2
*f - 6*a^2*d^3)*e^(f*x + e)/f^4 - (a^2*d^3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 +
 3*a^2*c^2*d*f^3*x + 3*a^2*d^3*f^2*x^2 + a^2*c^3*f^3 + 6*a^2*c*d^2*f^2*x +
 3*a^2*c^2*d*f^2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f + 6*a^2*d^3)*e^(-f*x - e)
/f^4 - 1/32*(4*a^2*d^3*f^3*x^3 + 12*a^2*c*d^2*f^3*x^2 + 12*a^2*c^2*d*f^3*x
 + 6*a^2*d^3*f^2*x^2 + 4*a^2*c^3*f^3 + 12*a^2*c*d^2*f^2*x + 6*a^2*c^2*d*f^
2 + 6*a^2*d^3*f*x + 6*a^2*c*d^2*f + 3*a^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.02

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx$$
$$= \frac{16\,a^2\,c^3\,f^3\sinh(e+f\,x) - \frac{3\,a^2\,d^3\cosh(2\,e+2\,f\,x)}{2} - 96\,a^2\,d^3\cosh(e+f\,x) + 12\,a^2\,c^3\,f^4\,x + 2\,a^2\,c^3\,f^3\sinh(2\,e}{}$$

input
```
int((a + a*cosh(e + f*x))^2*(c + d*x)^3,x)
```

output
```
(16*a^2*c^3*f^3*sinh(e + f*x) - (3*a^2*d^3*cosh(2*e + 2*f*x))/2 - 96*a^2*d
^3*cosh(e + f*x) + 12*a^2*c^3*f^4*x + 2*a^2*c^3*f^3*sinh(2*e + 2*f*x) + 3*
a^2*d^3*f^4*x^4 + 96*a^2*c*d^2*f*sinh(e + f*x) + 96*a^2*d^3*f*x*sinh(e + f
*x) - 3*a^2*d^3*f^2*x^2*cosh(2*e + 2*f*x) + 2*a^2*d^3*f^3*x^3*sinh(2*e + 2
*f*x) - 48*a^2*c^2*d*f^2*cosh(e + f*x) + 3*a^2*c*d^2*f*sinh(2*e + 2*f*x) +
 3*a^2*d^3*f*x*sinh(2*e + 2*f*x) - 3*a^2*c^2*d*f^2*cosh(2*e + 2*f*x) + 18*
a^2*c^2*d*f^4*x^2 + 12*a^2*c*d^2*f^4*x^3 - 48*a^2*d^3*f^2*x^2*cosh(e + f*x
) + 16*a^2*d^3*f^3*x^3*sinh(e + f*x) - 6*a^2*c*d^2*f^2*x*cosh(2*e + 2*f*x)
 + 6*a^2*c^2*d*f^3*x*sinh(2*e + 2*f*x) + 48*a^2*c*d^2*f^3*x^2*sinh(e + f*x
) + 6*a^2*c*d^2*f^3*x^2*sinh(2*e + 2*f*x) - 96*a^2*c*d^2*f^2*x*cosh(e + f*
x) + 48*a^2*c^2*d*f^3*x*sinh(e + f*x))/(8*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 735, normalized size of antiderivative = 3.28

$$\int (c+dx)^3(a+a\cosh(e+fx))^2\,dx$$
$$= \frac{a^2\left(-6e^{4fx+4e}c^2d\,f^2 + 6e^{4fx+4e}c\,d^2f + 4e^{4fx+4e}d^3f^3x^3 - 6e^{4fx+4e}d^3f^2x^2 + 6e^{4fx+4e}d^3fx - 96e^{3fx+3e}c^2d\right.}{}$$

input
```
int((d*x+c)^3*(a+a*cosh(f*x+e))^2,x)
```

output
```
(a**2*(4*e**(4*e + 4*f*x)*c**3*f**3 + 12*e**(4*e + 4*f*x)*c**2*d*f**3*x -
6*e**(4*e + 4*f*x)*c**2*d*f**2 + 12*e**(4*e + 4*f*x)*c*d**2*f**3*x**2 - 12
*e**(4*e + 4*f*x)*c*d**2*f**2*x + 6*e**(4*e + 4*f*x)*c*d**2*f + 4*e**(4*e
+ 4*f*x)*d**3*f**3*x**3 - 6*e**(4*e + 4*f*x)*d**3*f**2*x**2 + 6*e**(4*e +
4*f*x)*d**3*f*x - 3*e**(4*e + 4*f*x)*d**3 + 32*e**(3*e + 3*f*x)*c**3*f**3
+ 96*e**(3*e + 3*f*x)*c**2*d*f**3*x - 96*e**(3*e + 3*f*x)*c**2*d*f**2 + 96
*e**(3*e + 3*f*x)*c*d**2*f**3*x**2 - 192*e**(3*e + 3*f*x)*c*d**2*f**2*x +
192*e**(3*e + 3*f*x)*c*d**2*f + 32*e**(3*e + 3*f*x)*d**3*f**3*x**3 - 96*e*
*(3*e + 3*f*x)*d**3*f**2*x**2 + 192*e**(3*e + 3*f*x)*d**3*f*x - 192*e**(3*
e + 3*f*x)*d**3 + 48*e**(2*e + 2*f*x)*c**3*f**4*x + 72*e**(2*e + 2*f*x)*c*
*2*d*f**4*x**2 + 48*e**(2*e + 2*f*x)*c*d**2*f**4*x**3 + 12*e**(2*e + 2*f*x
)*d**3*f**4*x**4 - 32*e**(e + f*x)*c**3*f**3 - 96*e**(e + f*x)*c**2*d*f**3
*x - 96*e**(e + f*x)*c**2*d*f**2 - 96*e**(e + f*x)*c*d**2*f**3*x**2 - 192*
e**(e + f*x)*c*d**2*f**2*x - 192*e**(e + f*x)*c*d**2*f - 32*e**(e + f*x)*d
**3*f**3*x**3 - 96*e**(e + f*x)*d**3*f**2*x**2 - 192*e**(e + f*x)*d**3*f*x
 - 192*e**(e + f*x)*d**3 - 4*c**3*f**3 - 12*c**2*d*f**3*x - 6*c**2*d*f**2
- 12*c*d**2*f**3*x**2 - 12*c*d**2*f**2*x - 6*c*d**2*f - 4*d**3*f**3*x**3 -
 6*d**3*f**2*x**2 - 6*d**3*f*x - 3*d**3))/(32*e**(2*e + 2*f*x)*f**4)
```

3.106 $\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 168

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx=\frac{a^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{2d}-\frac{4a^2d(c+dx)\cosh(e+fx)}{f^2}-\frac{a^2d(c+dx)\cosh^2(e+fx)}{2f^2}+\frac{4a^2d^2\sinh(e+fx)}{f^3}+\frac{2a^2(c+dx)^2\sinh(e+fx)}{f}+\frac{a^2d^2\cosh(e+fx)\sinh(e+fx)}{4f^3}+\frac{a^2(c+dx)^2\cosh(e+fx)\sinh(e+fx)}{2f}$$

output

```
1/4*a^2*d^2*x/f^2+1/2*a^2*(d*x+c)^3/d-4*a^2*d*(d*x+c)*cosh(f*x+e)/f^2-1/2*
a^2*d*(d*x+c)*cosh(f*x+e)^2/f^2+4*a^2*d^2*sinh(f*x+e)/f^3+2*a^2*(d*x+c)^2*
sinh(f*x+e)/f+1/4*a^2*d^2*cosh(f*x+e)*sinh(f*x+e)/f^3+1/2*a^2*(d*x+c)^2*co
sh(f*x+e)*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.14

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$$
$$= \frac{a^2(12c^2f^3x+12cdf^3x^2+4d^2f^3x^3-32df(c+dx)\cosh(e+fx)-2df(c+dx)\cosh(2(e+fx))+32d^2\sinh$$

input
```
Integrate[(c + d*x)^2*(a + a*Cosh[e + f*x])^2,x]
```

output
```
(a^2*(12*c^2*f^3*x + 12*c*d*f^3*x^2 + 4*d^2*f^3*x^3 - 32*d*f*(c + d*x)*Cos
h[e + f*x] - 2*d*f*(c + d*x)*Cosh[2*(e + f*x)] + 32*d^2*Sinh[e + f*x] + 16
*c^2*f^2*Sinh[e + f*x] + 32*c*d*f^2*x*Sinh[e + f*x] + 16*d^2*f^2*x^2*Sinh[
e + f*x] + d^2*Sinh[2*(e + f*x)] + 2*c^2*f^2*Sinh[2*(e + f*x)] + 4*c*d*f^2
*x*Sinh[2*(e + f*x)] + 2*d^2*f^2*x^2*Sinh[2*(e + f*x)]))/(8*f^3)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a\cosh(e+fx)+a)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^2\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a^2(c+dx)^2\cosh^2(e+fx)+2a^2(c+dx)^2\cosh(e+fx)+a^2(c+dx)^2\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2 d(c+dx)\cosh^2(e+fx)}{2f^2}-\frac{4a^2 d(c+dx)\cosh(e+fx)}{f^2}+\frac{2a^2(c+dx)^2\sinh(e+fx)}{f}+\frac{a^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{a^2(c+dx)^3}{2d}+\frac{4a^2d^2\sinh(e+fx)}{f^3}+\frac{a^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{a^2d^2x}{4f^2}$$

input
```
Int[(c + d*x)^2*(a + a*Cosh[e + f*x])^2,x]
```

output
```
(a^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(2*d) - (4*a^2*d*(c + d*x)*Cosh[e
+ f*x])/f^2 - (a^2*d*(c + d*x)*Cosh[e + f*x]^2)/(2*f^2) + (4*a^2*d^2*Sinh[
e + f*x])/f^3 + (2*a^2*(c + d*x)^2*Sinh[e + f*x])/f + (a^2*d^2*Cosh[e + f*
x]*Sinh[e + f*x])/(4*f^3) + (a^2*(c + d*x)^2*Cosh[e + f*x]*Sinh[e + f*x])/
(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{\left(\left((dx+c)^2f^2+\frac{d^2}{2}\right)\sinh(2fx+2e)-df(dx+c)\cosh(2fx+2e)+8\left((dx+c)^2f^2+2d^2\right)\sinh(fx+e)+6\left(-\frac{8d(dx+c)\cosh(fx\cdots}{3}\right.\right.}{4f^3}$
risch	$\frac{a^2d^2x^3}{2}+\frac{3a^2dc\,x^2}{2}+\frac{3a^2c^2x}{2}+\frac{a^2c^3}{2d}+\frac{a^2(2d^2x^2f^2+4cd\,f^2x+2c^2f^2-2d^2fx-2cdf+d^2)\,\mathrm{e}^{2fx+2e}}{16f^3}+\frac{a^2(d^2x^2f\cdots}{}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{a^2\left(\frac{d^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e)^2}{2}+\frac{\cosh(fx+e)\sinh(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}\right)}{f^2}\cdots\right.}{}$
derivativedivides	$\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2a^2\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}+\frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx\cdots}{}\right.}{}$
default	$\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2a^2\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}+\frac{d^2a^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx\cdots}{}\right.}{}$
orering	Expression too large to display

input
```
int((d*x+c)^2*(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*((((d*x+c)^2*f^2+1/2*d^2)*sinh(2*f*x+2*e)-d*f*(d*x+c)*cosh(2*f*x+2*e)+8
*((d*x+c)^2*f^2+2*d^2)*sinh(f*x+e)+6*(-8/3*d*(d*x+c)*cosh(f*x+e)+x*(1/3*x^
2*d^2+c*d*x+c^2)*f^2+17/6*c*d)*f)*a^2/f^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.35

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$$
$$=\frac{2\,a^2d^2f^3x^3+6\,a^2cdf^3x^2+6\,a^2c^2f^3x-(a^2d^2fx+a^2cdf)\cosh(fx+e)^2-(a^2d^2fx+a^2cdf)\sinh(fx+\cdots}{}$$

input
```
integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*(2*a^2*d^2*f^3*x^3 + 6*a^2*c*d*f^3*x^2 + 6*a^2*c^2*f^3*x - (a^2*d^2*f*
x + a^2*c*d*f)*cosh(f*x + e)^2 - (a^2*d^2*f*x + a^2*c*d*f)*sinh(f*x + e)^2
 - 16*(a^2*d^2*f*x + a^2*c*d*f)*cosh(f*x + e) + (8*a^2*d^2*f^2*x^2 + 16*a^
2*c*d*f^2*x + 8*a^2*c^2*f^2 + 16*a^2*d^2 + (2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*
f^2*x + 2*a^2*c^2*f^2 + a^2*d^2)*cosh(f*x + e))*sinh(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(163) = 326$.

Time = 0.36 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.71

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$$
$$= \begin{cases} -\frac{a^2c^2x\sinh^2(e+fx)}{2} + \frac{a^2c^2x\cosh^2(e+fx)}{2} + a^2c^2x + \frac{a^2c^2\sinh(e+fx)\cosh(e+fx)}{2f} + \frac{2a^2c^2\sinh(e+fx)}{f} - \frac{a^2cdx^2\sinh^2(e+fx}{2} \\ (a\cosh(e)+a)^2\left(c^2x + cdx^2 + \frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+a*cosh(f*x+e))**2,x)
```

output
```
Piecewise((-a**2*c**2*x*sinh(e + f*x)**2/2 + a**2*c**2*x*cosh(e + f*x)**2/
2 + a**2*c**2*x + a**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c**
2*sinh(e + f*x)/f - a**2*c*d*x**2*sinh(e + f*x)**2/2 + a**2*c*d*x**2*cosh(
e + f*x)**2/2 + a**2*c*d*x**2 + a**2*c*d*x*sinh(e + f*x)*cosh(e + f*x)/f +
 4*a**2*c*d*x*sinh(e + f*x)/f - a**2*c*d*sinh(e + f*x)**2/(2*f**2) - 4*a**
2*c*d*cosh(e + f*x)/f**2 - a**2*d**2*x**3*sinh(e + f*x)**2/6 + a**2*d**2*x
**3*cosh(e + f*x)**2/6 + a**2*d**2*x**3/3 + a**2*d**2*x**2*sinh(e + f*x)*c
osh(e + f*x)/(2*f) + 2*a**2*d**2*x**2*sinh(e + f*x)/f - a**2*d**2*x*sinh(e
 + f*x)**2/(4*f**2) - a**2*d**2*x*cosh(e + f*x)**2/(4*f**2) - 4*a**2*d**2*
x*cosh(e + f*x)/f**2 + a**2*d**2*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) + 4*
a**2*d**2*sinh(e + f*x)/f**3, Ne(f, 0)), ((a*cosh(e) + a)**2*(c**2*x + c*d
*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(158) = 316$.

Time = 0.05 (sec) , antiderivative size = 327, normalized size of antiderivative = 1.95

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$$
$$= \frac{1}{3}a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4x^2 + \frac{\left(2fxe^{(2e)} - e^{(2e)}\right)e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)a^2cd$$
$$+ \frac{1}{48}\left(8x^3 + \frac{3\left(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)}\right)e^{(2fx)}}{f^3} - \frac{3(2f^2x^2+2fx+1)e^{(-2fx-2e)}}{f^3}\right)a^2d^2$$
$$+ \frac{1}{8}a^2c^2\left(4x + \frac{e^{(2fx+2e)}}{f} - \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^2x$$
$$+ 2a^2cd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+ a^2d^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2+2fx+2)e^{(-fx-e)}}{f^3}\right)$$
$$+ \frac{2a^2c^2\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(
2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*c*d + 1/48*(8*x^3 + 3*(
2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 - 3*(2*f^2*x^2
+ 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*a^2*d^2 + 1/8*a^2*c^2*(4*x + e^(2*f*x +
 2*e)/f - e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a^2*c*d*((f*x*e^e - e^e)*e^(
f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + a^2*d^2*((f^2*x^2*e^e - 2*f*x*e^e
 + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a^2*c^
2*sinh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(158) = 316.

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.96

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$$
$$=\frac{1}{2}a^2d^2x^3+\frac{3}{2}a^2cdx^2+\frac{3}{2}a^2c^2x+\frac{(2a^2d^2f^2x^2+4a^2cdf^2x+2a^2c^2f^2-2a^2d^2fx-2a^2cdf+a^2d^2)e^{(2fx+2e)}}{16f^3}+\frac{(a^2d^2f^2x^2+2a^2cdf^2x+a^2c^2f^2-2a^2d^2fx-2a^2cdf+2a^2d^2)e^{(fx+e)}}{f^3}-\frac{(a^2d^2f^2x^2+2a^2cdf^2x+a^2c^2f^2+2a^2d^2fx+2a^2cdf+2a^2d^2)e^{(-fx-e)}}{f^3}-\frac{(2a^2d^2f^2x^2+4a^2cdf^2x+2a^2c^2f^2+2a^2d^2fx+2a^2cdf+a^2d^2)e^{(-2fx-2e)}}{16f^3}$$

input
```
integrate((d*x+c)^2*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d^2*x^3 + 3/2*a^2*c*d*x^2 + 3/2*a^2*c^2*x + 1/16*(2*a^2*d^2*f^2*x^
2 + 4*a^2*c*d*f^2*x + 2*a^2*c^2*f^2 - 2*a^2*d^2*f*x - 2*a^2*c*d*f + a^2*d^
2)*e^(2*f*x + 2*e)/f^3 + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2
- 2*a^2*d^2*f*x - 2*a^2*c*d*f + 2*a^2*d^2)*e^(f*x + e)/f^3 - (a^2*d^2*f^2*
x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + 2*a^2*
d^2)*e^(-f*x - e)/f^3 - 1/16*(2*a^2*d^2*f^2*x^2 + 4*a^2*c*d*f^2*x + 2*a^2*
c^2*f^2 + 2*a^2*d^2*f*x + 2*a^2*c*d*f + a^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.53

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$$
$$=\frac{16\,a^2\,d^2\sinh(e+f\,x)+\frac{a^2\,d^2\sinh(2\,e+2\,f\,x)}{2}+8\,a^2\,c^2\,f^2\sinh(e+f\,x)+6\,a^2\,c^2\,f^3\,x+a^2\,c^2\,f^2\sinh(2\,e+2\,f\,}{}$$

input
```
int((a + a*cosh(e + f*x))^2*(c + d*x)^2,x)
```

output
```
(16*a^2*d^2*sinh(e + f*x) + (a^2*d^2*sinh(2*e + 2*f*x))/2 + 8*a^2*c^2*f^2*
sinh(e + f*x) + 6*a^2*c^2*f^3*x + a^2*c^2*f^2*sinh(2*e + 2*f*x) + 2*a^2*d^
2*f^3*x^3 - a^2*c*d*f*cosh(2*e + 2*f*x) - 16*a^2*d^2*f*x*cosh(e + f*x) + a
^2*d^2*f^2*x^2*sinh(2*e + 2*f*x) + 6*a^2*c*d*f^3*x^2 - a^2*d^2*f*x*cosh(2*
e + 2*f*x) - 16*a^2*c*d*f*cosh(e + f*x) + 8*a^2*d^2*f^2*x^2*sinh(e + f*x)
+ 16*a^2*c*d*f^2*x*sinh(e + f*x) + 2*a^2*c*d*f^2*x*sinh(2*e + 2*f*x))/(4*f
^3)
```

Reduce [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 418, normalized size of antiderivative = 2.49

$$\int (c+dx)^2(a+a\cosh(e+fx))^2\,dx$$
$$= \frac{a^2\left(e^{4fx+4e}d^2+32e^{3fx+3e}d^2-32e^{fx+e}d^2-2c^2f^2-d^2+4e^{4fx+4e}cd\,f^2x+32e^{3fx+3e}cd\,f^2x+24e^{2fx+2e}cd\right.}{}$$

input
```
int((d*x+c)^2*(a+a*cosh(f*x+e))^2,x)
```

output
```
(a**2*(2*e**(4*e + 4*f*x)*c**2*f**2 + 4*e**(4*e + 4*f*x)*c*d*f**2*x - 2*e*
*(4*e + 4*f*x)*c*d*f + 2*e**(4*e + 4*f*x)*d**2*f**2*x**2 - 2*e**(4*e + 4*f
*x)*d**2*f*x + e**(4*e + 4*f*x)*d**2 + 16*e**(3*e + 3*f*x)*c**2*f**2 + 32*
e**(3*e + 3*f*x)*c*d*f**2*x - 32*e**(3*e + 3*f*x)*c*d*f + 16*e**(3*e + 3*f
*x)*d**2*f**2*x**2 - 32*e**(3*e + 3*f*x)*d**2*f*x + 32*e**(3*e + 3*f*x)*d*
*2 + 24*e**(2*e + 2*f*x)*c**2*f**3*x + 24*e**(2*e + 2*f*x)*c*d*f**3*x**2 +
 8*e**(2*e + 2*f*x)*d**2*f**3*x**3 - 16*e**(e + f*x)*c**2*f**2 - 32*e**(e
+ f*x)*c*d*f**2*x - 32*e**(e + f*x)*c*d*f - 16*e**(e + f*x)*d**2*f**2*x**2
 - 32*e**(e + f*x)*d**2*f*x - 32*e**(e + f*x)*d**2 - 2*c**2*f**2 - 4*c*d*f
**2*x - 2*c*d*f - 2*d**2*f**2*x**2 - 2*d**2*f*x - d**2))/(16*e**(2*e + 2*f
*x)*f**3)
```

3.107 $\int (c+dx)(a+a\cosh (e+fx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int (c+dx)(a+a\cosh (e+fx))^2\,dx=\frac{3a^2(c+dx)^2}{4d}-\frac{2a^2d\cosh (e+fx)}{f^2}-\frac{a^2d\cosh ^2(e+fx)}{4f^2}+\frac{2a^2(c+dx)\sinh (e+fx)}{f}+\frac{a^2(c+dx)\cosh (e+fx)\sinh (e+fx)}{2f}$$

output
```
3/4*a^2*(d*x+c)^2/d-2*a^2*d*cosh(f*x+e)/f^2-1/4*a^2*d*cosh(f*x+e)^2/f^2+2*
a^2*(d*x+c)*sinh(f*x+e)/f+1/2*a^2*(d*x+c)*cosh(f*x+e)*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.83

$$\int (c+dx)(a+a\cosh (e+fx))^2\,dx =\frac{a^2(-6(e+fx)(-2cf+d(e-fx))-16d\cosh (e+fx)-d\cosh (2(e+fx))+16f(c+dx)\sinh (e+fx)}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + a*Cosh[e + f*x])^2,x]
```

output
```
(a^2*(-6*(e + f*x)*(-2*c*f + d*(e - f*x)) - 16*d*Cosh[e + f*x] - d*Cosh[2*
(e + f*x)] + 16*f*(c + d*x)*Sinh[e + f*x] + 2*f*(c + d*x)*Sinh[2*(e + f*x)
]))/(8*f^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a\cosh(e+fx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)\cosh^2(e+fx)+2a^2(c+dx)\cosh(e+fx)+a^2(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2(c+dx)\sinh(e+fx)}{f}+\frac{a^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{3a^2(c+dx)^2}{4d}-\frac{a^2d\cosh^2(e+fx)}{4f^2}-\frac{2a^2d\cosh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + a*Cosh[e + f*x])^2,x]
```

output
```
(3*a^2*(c + d*x)^2)/(4*d) - (2*a^2*d*Cosh[e + f*x])/f^2 - (a^2*d*Cosh[e +
f*x]^2)/(4*f^2) + (2*a^2*(c + d*x)*Sinh[e + f*x])/f + (a^2*(c + d*x)*Cosh[
e + f*x]*Sinh[e + f*x])/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.93 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.29

method	result
risch	$\frac{3a^2 d x^2}{4}+\frac{3a^2 cx}{2}+\frac{a^2(2dxf+2cf-d)e^{2fx+2e}}{16f^2}+\frac{a^2(dxf+cf-d)e^{fx+e}}{f^2}-\frac{a^2(dxf+cf+d)e^{-fx-e}}{f^2}-\frac{a^2(2dxf+\cdots}{}$
parts	$a^2\left(\frac{1}{2}d x^2+cx\right)+\frac{a^2\left(\frac{d\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\left(\frac{\cosh(fx+e)\sinh(fx+e)}{2}+\frac{fx\cdots}{2}\cdots\right)}{f}\cdots\right)}{f}$
derivativedivides	$\frac{\frac{d a^2(fx+e)^2}{2f}+\frac{2d a^2((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d a^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de a^2\cdots}{}}{}$
default	$\frac{\frac{d a^2(fx+e)^2}{2f}+\frac{2d a^2((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d a^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de a^2\cdots}{}}{}$
orering	$\frac{(2d^5 f^4 x^6+12c d^4 f^4 x^5+28c^2 d^3 f^4 x^4+32c^3 d^2 f^4 x^3+18c^4 d f^4 x^2-15d^5 f^2 x^4+4c^5 f^4 x-60c d^4 f^2 x^3-85c^2 d^3 f^2 x^2-50c^3\cdots}{4f^4(dx+c)^4}$

input
```
int((d*x+c)*(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
3/4*a^2*d*x^2+3/2*a^2*c*x+1/16*a^2*(2*d*f*x+2*c*f-d)/f^2*exp(2*f*x+2*e)+a^
2*(d*f*x+c*f-d)/f^2*exp(f*x+e)-a^2*(d*f*x+c*f+d)/f^2*exp(-f*x-e)-1/16*a^2*
(2*d*f*x+2*c*f+d)/f^2*exp(-2*f*x-2*e)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.15

$$\int (c+dx)(a+a\cosh(e+fx))^2\,dx$$
$$=\frac{6\,a^2df^2x^2+12\,a^2cf^2x-a^2d\cosh\left(fx+e\right)^2-a^2d\sinh\left(fx+e\right)^2-16\,a^2d\cosh\left(fx+e\right)+4\left(4\,a^2dfx+4\right.}{8\,f^2}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*(6*a^2*d*f^2*x^2 + 12*a^2*c*f^2*x - a^2*d*cosh(f*x + e)^2 - a^2*d*sinh
(f*x + e)^2 - 16*a^2*d*cosh(f*x + e) + 4*(4*a^2*d*f*x + 4*a^2*c*f + (a^2*d
*f*x + a^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(94) = 188.

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 2.23

$$\int (c+dx)(a+a\cosh(e+fx))^2\,dx$$
$$=\begin{cases}-\frac{a^2cx\sinh^2\left(e+fx\right)}{2}+\frac{a^2cx\cosh^2\left(e+fx\right)}{2}+a^2cx+\frac{a^2c\sinh\left(e+fx\right)\cosh\left(e+fx\right)}{2f}+\frac{2a^2c\sinh\left(e+fx\right)}{f}-\frac{a^2dx^2\sinh^2\left(e+fx\right)}{4}+\frac{a}{} \\ \left(a\cosh\left(e\right)+a\right)^2\left(cx+\frac{dx^2}{2}\right)\end{cases}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e))**2,x)
```

output
```
Piecewise((-a**2*c*x*sinh(e + f*x)**2/2 + a**2*c*x*cosh(e + f*x)**2/2 + a*
*2*c*x + a**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*c*sinh(e + f*x)
/f - a**2*d*x**2*sinh(e + f*x)**2/4 + a**2*d*x**2*cosh(e + f*x)**2/4 + a**
2*d*x**2/2 + a**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) + 2*a**2*d*x*sinh(
e + f*x)/f - a**2*d*sinh(e + f*x)**2/(4*f**2) - 2*a**2*d*cosh(e + f*x)/f**
2, Ne(f, 0)), ((a*cosh(e) + a)**2*(c*x + d*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.70

$$\int (c+dx)(a+a\cosh(e+fx))^2\,dx = \frac{1}{2}\,a^2dx^2 + \frac{1}{16}\left(4\,x^2 + \frac{(2\,fxe^{(2\,e)} - e^{(2\,e)})e^{(2\,fx)}}{f^2} - \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)a^2d + \frac{1}{8}\,a^2c\left(4\,x + \frac{e^{(2\,fx+2\,e)}}{f} - \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2cx + a^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{2\,a^2c\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 + 1/16*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2
*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*a^2*d + 1/8*a^2*c*(4*x + e^(2*f*x + 2*e)/f
 - e^(-2*f*x - 2*e)/f) + a^2*c*x + a^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f
*x + 1)*e^(-f*x - e)/f^2) + 2*a^2*c*sinh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.54

$$\int (c+dx)(a+a\cosh(e+fx))^2\,dx = \frac{3}{4}\,a^2dx^2 + \frac{3}{2}\,a^2cx + \frac{(2\,a^2dfx + 2\,a^2cf - a^2d)e^{(2\,fx+2\,e)}}{16\,f^2} + \frac{(a^2dfx + a^2cf - a^2d)e^{(fx+e)}}{f^2} - \frac{(a^2dfx + a^2cf + a^2d)e^{(-fx-e)}}{f^2} - \frac{(2\,a^2dfx + 2\,a^2cf + a^2d)e^{(-2\,fx-2\,e)}}{16\,f^2}$$

input
```
integrate((d*x+c)*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
3/4*a^2*d*x^2 + 3/2*a^2*c*x + 1/16*(2*a^2*d*f*x + 2*a^2*c*f - a^2*d)*e^(2*
f*x + 2*e)/f^2 + (a^2*d*f*x + a^2*c*f - a^2*d)*e^(f*x + e)/f^2 - (a^2*d*f*
x + a^2*c*f + a^2*d)*e^(-f*x - e)/f^2 - 1/16*(2*a^2*d*f*x + 2*a^2*c*f + a^
2*d)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.26

$$\int (c+dx)(a+a\cosh(e+fx))^2\,dx = \frac{3\,a^2\,d\,x^2}{4}+\frac{3\,a^2\,c\,x}{2}-\frac{a^2\,d\cosh(e+f\,x)^2}{4\,f^2} -\frac{2\,a^2\,d\cosh(e+f\,x)}{f^2}+\frac{2\,a^2\,c\sinh(e+f\,x)}{f} +\frac{a^2\,c\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f} +\frac{2\,a^2\,d\,x\sinh(e+f\,x)}{f} +\frac{a^2\,d\,x\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f}$$

input
```
int((a + a*cosh(e + f*x))^2*(c + d*x),x)
```

output
```
(3*a^2*d*x^2)/4 + (3*a^2*c*x)/2 - (a^2*d*cosh(e + f*x)^2)/(4*f^2) - (2*a^2
*d*cosh(e + f*x))/f^2 + (2*a^2*c*sinh(e + f*x))/f + (a^2*c*cosh(e + f*x)*s
inh(e + f*x))/(2*f) + (2*a^2*d*x*sinh(e + f*x))/f + (a^2*d*x*cosh(e + f*x)
*sinh(e + f*x))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.90

$$\int (c+dx)(a+a\cosh(e+fx))^2\,dx = \frac{a^2\left(2e^{4fx+4e}cf+2e^{4fx+4e}dfx-e^{4fx+4e}d+16e^{3fx+3e}cf+16e^{3fx+3e}dfx-16e^{3fx+3e}d+24e^{2fx+2e}c\,f^2x+\right.}{16e^{2fx+2e}f^2}$$

input
```
int((d*x+c)*(a+a*cosh(f*x+e))^2,x)
```

output
```
(a**2*(2*e**(4*e + 4*f*x)*c*f + 2*e**(4*e + 4*f*x)*d*f*x - e**(4*e + 4*f*x
)*d + 16*e**(3*e + 3*f*x)*c*f + 16*e**(3*e + 3*f*x)*d*f*x - 16*e**(3*e + 3
*f*x)*d + 24*e**(2*e + 2*f*x)*c*f**2*x + 12*e**(2*e + 2*f*x)*d*f**2*x**2 -
 16*e**(e + f*x)*c*f - 16*e**(e + f*x)*d*f*x - 16*e**(e + f*x)*d - 2*c*f -
 2*d*f*x - d))/(16*e**(2*e + 2*f*x)*f**2)
```

3.108 $\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 145

$$\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx = \frac{2a^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d} + \frac{3a^2\log(c+dx)}{2d} + \frac{2a^2\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{a^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output
```
2*a^2*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d+1/2*a^2*cosh(-2*e+2*c*f/d)*Chi(2*c*f
/d+2*f*x)/d+3/2*a^2*ln(d*x+c)/d-2*a^2*sinh(-e+c*f/d)*Shi(c*f/d+f*x)/d-1/2*
a^2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.78

$$\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx$$
$$= \frac{a^2\left(4\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right)+3\log(c+dx)+4\sinh\left(e-\frac{cf}{d}\right)\operatorname{Sh}\right.}{2d}$$

input
```
Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x),x]
```

output
```
(a^2*(4*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + Cosh[2*e - (2*c*f)/d
]*CoshIntegral[(2*f*(c + d*x))/d] + 3*Log[c + d*x] + 4*Sinh[e - (c*f)/d]*S
inhIntegral[f*(c/d + x)] + Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*
x))/d]))/(2*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cosh(e+fx)+a)^2}{c+dx}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}{c+dx}dx$$
$$\Big\downarrow \text{ 3799}$$
$$4a^2\int \frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+dx}dx$$
$$\Big\downarrow \text{ 3042}$$

$$4a^2 \int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4}{c+dx}dx$$

$\Big\downarrow$ 3793

$$4a^2 \int \left(\frac{\cosh(e+fx)}{2(c+dx)}+\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{3}{8(c+dx)}\right)dx$$

$\Big\downarrow$ 2009

$$4a^2\left(\frac{\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d}+\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{8d}+\frac{\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d}+\frac{\sinh\left(\right.}{}\right.$$

input
```
Int[(a + a*Cosh[e + f*x])^2/(c + d*x),x]
```

output
```
4*a^2*((Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d) + (Cosh[2*e -
 (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(8*d) + (3*Log[c + d*x])/(8*d
) + (Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(2*d) + (Sinh[2*e - (2
*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*d))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 191, normalized size of antiderivative = 1.32

method	result
risch	$-\frac{a^2 e^{\frac{cf-de}{d}} \operatorname{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{d}-\frac{a^2 e^{-\frac{cf-de}{d}} \operatorname{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{3a^2\ln(dx+c)}{2d}-\frac{a^2 e^{-\frac{2(cf-de)}{d}} \operatorname{ex}}{}$

input
```
int((a+a*cosh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-a^2/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a^2/d*exp(-(c*f-d*e)/d)*Ei
(1,-f*x-e-(c*f-d*e)/d)+3/2*a^2*ln(d*x+c)/d-1/4*a^2/d*exp(-2*(c*f-d*e)/d)*E
i(1,-2*f*x-2*e-2*(c*f-d*e)/d)-1/4*a^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+
2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.57

$$\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx = \frac{6\,a^2\log\left(dx+c\right)+4\left(a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(a^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)+a^2\mathrm{Ei}\left(-\frac{2\,(dfx-}{d}\right.\right.}{}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(6*a^2*log(d*x + c) + 4*(a^2*Ei((d*f*x + c*f)/d) + a^2*Ei(-(d*f*x + c*
f)/d))*cosh(-(d*e - c*f)/d) + (a^2*Ei(2*(d*f*x + c*f)/d) + a^2*Ei(-2*(d*f*
x + c*f)/d))*cosh(-2*(d*e - c*f)/d) - 4*(a^2*Ei((d*f*x + c*f)/d) - a^2*Ei(
-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) - (a^2*Ei(2*(d*f*x + c*f)/d) - a^2
*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/d
```

Sympy [F]

$$\int \frac{(a + a\cosh(e + fx))^2}{c + dx}\,dx = a^2\left(\int \frac{2\cosh(e + fx)}{c + dx}\,dx + \int \frac{\cosh^2(e + fx)}{c + dx}\,dx + \int \frac{1}{c + dx}\,dx\right)$$

input
```
integrate((a+a*cosh(f*x+e))**2/(d*x+c),x)
```

output
```
a**2*(Integral(2*cosh(e + f*x)/(c + d*x), x) + Integral(cosh(e + f*x)**2/(
c + d*x), x) + Integral(1/(c + d*x), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.03

$$\int \frac{(a + a\cosh(e + fx))^2}{c + dx}\,dx = -\frac{1}{4}\,a^2\left(\frac{e^{\left(-2\,e + \frac{2\,cf}{d}\right)}E_1\left(\frac{2\,(dx+c)f}{d}\right)}{d} + \frac{e^{\left(2\,e - \frac{2\,cf}{d}\right)}E_1\left(-\frac{2\,(dx+c)f}{d}\right)}{d} - \frac{2\,\log\left(dx + c\right)}{d}\right) - a^2\left(\frac{e^{\left(-e + \frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e - \frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a^2\log\left(dx + c\right)}{d}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
 - 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d - 2*log(d*x + c)/d) - a^
2*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*exp_i
ntegral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.93

$$\int \frac{(a + a\cosh(e + fx))^2}{c + dx}\,dx = \frac{a^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)} + 4\,a^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)} + 4\,a^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)} + a^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{(-}}{4\,d}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(a^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a^2*Ei((d*f*x + c*f)/
d)*e^(e - c*f/d) + 4*a^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + a^2*Ei(-2*(
d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 6*a^2*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\cosh(e + fx))^2}{c + dx}\,dx = \int \frac{{\left(a + a\cosh(e + f\,x)\right)}^2}{c + d\,x}\,dx$$

input
```
int((a + a*cosh(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + a*cosh(e + f*x))^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+a\cosh(e+fx))^2}{c+dx}\,dx$$
$$=\frac{a^2\left(2\left(\int \frac{\cosh(fx+e)}{dx+c}dx\right)d+\left(\int \frac{\cosh(fx+e)^2}{dx+c}dx\right)d+\log(dx+c)\right)}{d}$$

input
```
int((a+a*cosh(f*x+e))^2/(d*x+c),x)
```

output
```
(a**2*(2*int(cosh(e + f*x)/(c + d*x),x)*d + int(cosh(e + f*x)**2/(c + d*x)
,x)*d + log(c + d*x)))/d
```

3.109 $\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx = -\frac{4a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)} + \frac{a^2f\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2} + \frac{2a^2f\operatorname{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2} + \frac{2a^2f\cosh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{a^2f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^2}$$

output
```
-4*a^2*cosh(1/2*f*x+1/2*e)^4/d/(d*x+c)-a^2*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+
2*c*f/d)/d^2-2*a^2*f*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2+2*a^2*f*cosh(-e+c*f
/d)*Shi(c*f/d+f*x)/d^2+a^2*f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.32

$$\int \frac{(a + a\cosh(e + fx))^2}{(c + dx)^2}\,dx$$
$$= \frac{a^2\left(-3d - 4d\cosh(e + fx) - d\cosh(2(e + fx)) + 2f(c + dx)\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e - \frac{2cf}{d}\right) + 4f(c + dx\right.}{}$$

input
```
Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(a^2*(-3*d - 4*d*Cosh[e + f*x] - d*Cosh[2*(e + f*x)] + 2*f*(c + d*x)*CoshI
ntegral[(2*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 4*f*(c + d*x)*CoshInteg
ral[f*(c/d + x)]*Sinh[e - (c*f)/d] + 4*c*f*Cosh[e - (c*f)/d]*SinhIntegral[
f*(c/d + x)] + 4*d*f*x*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 2*c*f
*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] + 2*d*f*x*Cosh[2*e
- (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d]))/(2*d^2*(c + d*x))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.60 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3799, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cosh(e + fx) + a)^2}{(c + dx)^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a + a\sin\left(ie + ifx + \frac{\pi}{2}\right)\right)^2}{(c + dx)^2}dx$$

$$\Big\downarrow \; 3799$$

$$4a^2\int \frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{(c+dx)^2}dx$$

$$\Big\downarrow \ 3042$$

$$4a^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4}{(c+dx)^2}dx$$

$$\Big\downarrow \ 3794$$

$$4a^2\left(-\frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{2if\int\left(-\frac{i\sinh(e+fx)}{4(c+dx)}-\frac{i\sinh(2e+2fx)}{8(c+dx)}\right)dx}{d}\right)$$

$$\Big\downarrow \ 2009$$

$$4a^2\left(-\frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)}+\frac{2if\left(-\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8d}-\frac{i\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{4d}-\frac{i\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{4d}\right.}{d}\right.$$

input
```
Int[(a + a*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
4*a^2*(-(Cosh[e/2 + (f*x)/2]^4/(d*(c + d*x))) + ((2*I)*f*(((-1/8*I)*CoshIn
tegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d - ((I/4)*CoshIntegral[(
c*f)/d + f*x]*Sinh[e - (c*f)/d])/d - ((I/4)*Cosh[e - (c*f)/d]*SinhIntegral
[(c*f)/d + f*x])/d - ((I/8)*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d +
 2*f*x])/d))/d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.96

method	result
risch	$-\frac{f\,a^{2} {\mathrm e}^{-f x -e}}{d \left(d x f +c f \right)}+\frac{f\,a^{2} {\mathrm e}^{\frac{c f -d e}{d}} \operatorname{expIntegral}_{1}\left(f x +e +\frac{c f -d e}{d}\right)}{d^{2}}-\frac{f\,a^{2} {\mathrm e}^{f x +e}}{d^{2} \left(\frac{c f}{d}+f x \right)}-\frac{f\,a^{2} {\mathrm e}^{-\frac{c f -d e}{d}} \operatorname{expIntegral}_{1}\left(-f x -e -\frac{c f -d e}{d}\right)}{d^{2}}$ -

input
```
int((a+a*cosh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-f*a^2*exp(-f*x-e)/d/(d*f*x+c*f)+f*a^2/d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*
f-d*e)/d)-f*a^2/d^2*exp(f*x+e)/(c*f/d+f*x)-f*a^2/d^2*exp(-(c*f-d*e)/d)*Ei(
1,-f*x-e-(c*f-d*e)/d)-3/2*a^2/d/(d*x+c)-1/4*f*a^2*exp(-2*f*x-2*e)/d/(d*f*x
+c*f)+1/2*f*a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*f
*a^2/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f*a^2/d^2*exp(-2*(c*f-d*e)/d)*Ei(1
,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 359 vs. 2(156) = 312.

Time = 0.10 (sec) , antiderivative size = 359, normalized size of antiderivative = 2.29

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx = -\frac{a^2d\cosh\left(fx+e\right)^2+a^2d\sinh\left(fx+e\right)^2+4\,a^2d\cosh\left(fx+e\right)+3\,a^2d-2\left(\left(a^2dfx+a^2cf\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)\right.}{}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(a^2*d*cosh(f*x + e)^2 + a^2*d*sinh(f*x + e)^2 + 4*a^2*d*cosh(f*x + e
) + 3*a^2*d - 2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) - (a^2*d*f*x +
a^2*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((a^2*d*f*x + a^2*c*
f)*Ei(2*(d*f*x + c*f)/d) - (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x + c*f)/d))*c
osh(-2*(d*e - c*f)/d) + 2*((a^2*d*f*x + a^2*c*f)*Ei((d*f*x + c*f)/d) + (a^
2*d*f*x + a^2*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((a^2*d*f*
x + a^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (a^2*d*f*x + a^2*c*f)*Ei(-2*(d*f*x +
c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx = a^2\left(\int \frac{2\cosh\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx+\int \frac{\cosh^2\left(e+fx\right)}{c^2+2cdx+d^2x^2}\,dx + \int \frac{1}{c^2+2cdx+d^2x^2}\,dx\right)$$

input
```
integrate((a+a*cosh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
a**2*(Integral(2*cosh(e + f*x)/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral
(cosh(e + f*x)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(1/(c**2 + 2*
c*d*x + d**2*x**2), x))
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.16

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx = -\frac{1}{4}a^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d}+\frac{2}{d^2x+cd}\right) - a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a^2}{d^2x+cd}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*a^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)
*d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d)
+ 2/(d^2*x + c*d)) - a^2*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/
((d*x + c)*d) + e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)
*d)) - a^2/(d^2*x + c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1134 vs. $2(156) = 312$.

Time = 0.19 (sec) , antiderivative size = 1134, normalized size of antiderivative = 7.22

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x +
c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d)
 - 2*a^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*a^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) + 4*(d*x + c)*a
^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*a^2*d*e*f^2*Ei(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f
)/d) + 4*a^2*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^((d*e - c*f)/d) - 4*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x +
 c) + f)*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*
f)/d)*e^(-(d*e - c*f)/d) + 4*a^2*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c
*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 4*a^2*c*f^3*Ei(-((d
*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f
)/d) - 2*(d*x + c)*a^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x
 + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f
)/d) + 2*a^2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
- d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 2*a^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - a^2*d
*f^2*e^(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*a^2*d*f^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\cosh(e + fx))^2}{(c + dx)^2}\,dx = \int \frac{\left(a + a\cosh(e + f\,x)\right)^2}{\left(c + d\,x\right)^2}\,dx$$

input
```
int((a + a*cosh(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + a*cosh(e + f*x))^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a + a\cosh(e + fx))^2}{(c + dx)^2}\,dx$$
$$= \frac{a^2\left(e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)c^2 + e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)cdx + 4e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)c^2 + 4e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx}\right.\right.}{}$$

input
```
int((a+a*cosh(f*x+e))^2/(d*x+c)^2,x)
```

output
```
(a**2*(e**(3*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 + e**(
3*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + 4*e**(2*e)*int
(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*c**2 + 4*e**(2*e)*int(e**(f*x)/(
c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + e**e*int(1/(e**(2*e + 2*f*x)*c**2 +
 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2),x)*c**2 + e**e*int
(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d*
*2*x**2),x)*c*d*x + 6*e**e*x + 4*int(1/(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x +
 e**(f*x)*d**2*x**2),x)*c**2 + 4*int(1/(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x +
 e**(f*x)*d**2*x**2),x)*c*d*x))/(4*e**e*c*(c + d*x))
```

3.110 $\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 207

$$\begin{aligned}\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx &= -\frac{2a^2\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{d(c+dx)^2}+\frac{a^2f^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d^3}\\&\quad+\frac{a^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\\&\quad-\frac{4a^2f\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)}\\&\quad+\frac{a^2f^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^3}\\&\quad+\frac{a^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\end{aligned}$$

output

```
-2*a^2*cosh(1/2*f*x+1/2*e)^4/d/(d*x+c)^2+a^2*f^2*cosh(-e+c*f/d)*Chi(c*f/d+
f*x)/d^3+a^2*f^2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/d^3-4*a^2*f*cosh(1/
2*f*x+1/2*e)^3*sinh(1/2*f*x+1/2*e)/d^2/(d*x+c)-a^2*f^2*sinh(-e+c*f/d)*Shi(
c*f/d+f*x)/d^3-a^2*f^2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d^3
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.71

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx$$
$$= \frac{a^2\left(-3d^2-4d^2\cosh(e+fx)-d^2\cosh(2(e+fx))+4f^2(c+dx)^2\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+4f^2(c\right.}{}$$

input
```
Integrate[(a + a*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
(a^2*(-3*d^2 - 4*d^2*Cosh[e + f*x] - d^2*Cosh[2*(e + f*x)] + 4*f^2*(c + d*
x)^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + 4*f^2*(c + d*x)^2*Cosh[
2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))/d] - 4*c*d*f*Sinh[e + f*x] -
 4*d^2*f*x*Sinh[e + f*x] - 2*c*d*f*Sinh[2*(e + f*x)] - 2*d^2*f*x*Sinh[2*(e
 + f*x)] + 4*c^2*f^2*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 8*c*d*f
^2*x*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*d^2*f^2*x^2*Sinh[e -
(c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhI
ntegral[(2*f*(c + d*x))/d] + 8*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegra
l[(2*f*(c + d*x))/d] + 4*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2
*f*(c + d*x))/d]))/(4*d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.44, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 3799, 3042, 3795, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cosh(e+fx)+a)^2}{(c+dx)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}{(c+dx)^3}dx$$

$\Big\downarrow$ 3799

$$4a^2\int \frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{(c+dx)^3}dx$$

$\Big\downarrow$ 3042

$$4a^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4}{(c+dx)^3}dx$$

$\Big\downarrow$ 3795

$$4a^2\left(\frac{2f^2\int \frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+dx}dx}{d^2}-\frac{3f^2\int \frac{\cosh^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{c+dx}dx}{2d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)}-\frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{2d(c+dx)^2}\right)$$

$\Big\downarrow$ 3042

$$4a^2\left(-\frac{3f^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2}{c+dx}dx}{2d^2}+\frac{2f^2\int \frac{\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4}{c+dx}dx}{d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)\cosh^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{d^2(c+dx)}-\frac{\cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)}{2d(c+dx)^2}\right.$$

$\Big\downarrow$ 3793

$$4a^2\left(-\frac{3f^2\int \left(\frac{\cosh(e+fx)}{2(c+dx)}+\frac{1}{2(c+dx)}\right)dx}{2d^2}+\frac{2f^2\int \left(\frac{\cosh(e+fx)}{2(c+dx)}+\frac{\cosh(2e+2fx)}{8(c+dx)}+\frac{3}{8(c+dx)}\right)dx}{d^2}-\frac{f\sinh\left(\frac{e}{2}+\frac{fx}{2}\right)\cosh}{d^2(c+dx}\right.$$

$\Big\downarrow$ 2009

$$4a^2\left(-\frac{3f^2\left(\frac{\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d}+\frac{\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d}+\frac{\log(c+dx)}{2d}\right)}{2d^2}+\frac{2f^2\left(\frac{\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d}+\frac{\text{Chi}(}{}\right.}{}\right.$$

input
```
Int[(a + a*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
4*a^2*(-1/2*Cosh[e/2 + (f*x)/2]^4/(d*(c + d*x)^2) - (f*Cosh[e/2 + (f*x)/2]
^3*Sinh[e/2 + (f*x)/2])/(d^2*(c + d*x)) - (3*f^2*((Cosh[e - (c*f)/d]*CoshI
ntegral[(c*f)/d + f*x])/(2*d) + Log[c + d*x]/(2*d) + (Sinh[e - (c*f)/d]*Si
nhIntegral[(c*f)/d + f*x])/(2*d)))/(2*d^2) + (2*f^2*((Cosh[e - (c*f)/d]*Co
shIntegral[(c*f)/d + f*x])/(2*d) + (Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*
c*f)/d + 2*f*x])/(8*d) + (3*Log[c + d*x])/(8*d) + (Sinh[e - (c*f)/d]*SinhI
ntegral[(c*f)/d + f*x])/(2*d) + (Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f
)/d + 2*f*x])/(8*d)))/d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(199) = 398$.

Time = 2.48 (sec) , antiderivative size = 618, normalized size of antiderivative = 2.99

method	result
risch	$\frac{f^3 a^2 \mathrm{e}^{-fx-e} x}{2d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} + \frac{f^3 a^2 \mathrm{e}^{-fx-e} c}{2d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)} - \frac{f^2 a^2 \mathrm{e}^{-fx-e}}{2d(d^2x^2f^2+2cd\,f^2x+c^2f^2)} - \frac{f^2 a^2 \mathrm{e}^{\frac{cf-de}{d}} \operatorname{expIntegral}_1\left(fx+\right.}{2d^3}$

input
```
int((a+a*cosh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^3*a^2*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/2*f^3*a^2*
exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/2*f^2*a^2*exp(-f*x-e
)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*f^2*a^2/d^3*exp((c*f-d*e)/d)*Ei(
1,f*x+e+(c*f-d*e)/d)-1/2*f^2*a^2/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/2*f^2*a^2/
d^3*exp(f*x+e)/(c*f/d+f*x)-1/2*f^2*a^2/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)-3/4*a^2/d/(d*x+c)^2+1/4*f^3*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2
+2*c*d*f^2*x+c^2*f^2)*x+1/4*f^3*a^2*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d
*f^2*x+c^2*f^2)*c-1/8*f^2*a^2*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c
^2*f^2)-1/2*f^2*a^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1
/8*f^2*a^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*f^2*a^2/d^3*exp(2*f*x+2*e)
/(c*f/d+f*x)-1/2*f^2*a^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*
e)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(199) = 398$.

Time = 0.12 (sec) , antiderivative size = 596, normalized size of antiderivative = 2.88

$$\int \frac{(a + a\cosh(e + fx))^2}{(c + dx)^3}\,dx =$$
$$-\frac{a^2d^2\cosh\left(fx+e\right)^2 + a^2d^2\sinh\left(fx+e\right)^2 + 4\,a^2d^2\cosh\left(fx+e\right) + 3\,a^2d^2 - 2\left(\left(a^2d^2f^2x^2 + 2\,a^2cdf^2x\right.\right.}{}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(a^2*d^2*cosh(f*x + e)^2 + a^2*d^2*sinh(f*x + e)^2 + 4*a^2*d^2*cosh(f
*x + e) + 3*a^2*d^2 - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)
*Ei((d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*E
i(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*
d*f^2*x + a^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (a^2*d^2*f^2*x^2 + 2*a^2*c*
d*f^2*x + a^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d*e - c*f)/d) + 4*
(a^2*d^2*f*x + a^2*c*d*f + (a^2*d^2*f*x + a^2*c*d*f)*cosh(f*x + e))*sinh(f
*x + e) + 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei((d*f*x +
 c*f)/d) - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^2*c^2*f^2)*Ei(-(d*f*x +
c*f)/d))*sinh(-(d*e - c*f)/d) + 2*((a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^
2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (a^2*d^2*f^2*x^2 + 2*a^2*c*d*f^2*x + a^
2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*x^2 + 2*c*
d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx = a^2\left(\int \frac{2\cosh(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{\cosh^2(e+fx)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{1}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx\right)$$

input
```
integrate((a+a*cosh(f*x+e))**2/(d*x+c)**3,x)
```

output
```
a**2*(Integral(2*cosh(e + f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3), x) + Integral(cosh(e + f*x)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3), x) + Integral(1/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
), x))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.98

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx$$
$$= -\frac{1}{4}a^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d}+\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right)$$
$$-a^2\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*a^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) + e^(-2*e + 2*c*f/d)*exp_integra
l_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) + e^(2*e - 2*c*f/d)*exp_integral_e
(3, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) - a^2*(e^(-e + c*f/d)*exp_integral_
e(3, (d*x + c)*f/d)/((d*x + c)^2*d) + e^(e - c*f/d)*exp_integral_e(3, -(d*
x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 682 vs. 2(199) = 398.

Time = 0.14 (sec) , antiderivative size = 682, normalized size of antiderivative = 3.29

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+a*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*a^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a^2*d^2
*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a^2*d^2*f^2*x^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*a^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*
e + 2*c*f/d) + 8*a^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 8
*a^2*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 8*a^2*c*d*f^2*x*Ei(-(d*
f*x + c*f)/d)*e^(-e + c*f/d) + 8*a^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(-
2*e + 2*c*f/d) + 4*a^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4
*a^2*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a^2*c^2*f^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*a^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
2*c*f/d) - 2*a^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a^2*d^2*f*x*e^(f*x + e) + 4*a
^2*d^2*f*x*e^(-f*x - e) + 2*a^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*a^2*c*d*f*e^(
2*f*x + 2*e) - 4*a^2*c*d*f*e^(f*x + e) + 4*a^2*c*d*f*e^(-f*x - e) + 2*a^2*
c*d*f*e^(-2*f*x - 2*e) - a^2*d^2*e^(2*f*x + 2*e) - 4*a^2*d^2*e^(f*x + e) -
 4*a^2*d^2*e^(-f*x - e) - a^2*d^2*e^(-2*f*x - 2*e) - 6*a^2*d^2)/(d^5*x^2 +
 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{\left(a+a\cosh(e+f\,x)\right)^2}{\left(c+d\,x\right)^3}\,dx$$

input
```
int((a + a*cosh(e + f*x))^2/(c + d*x)^3,x)
```

output
```
int((a + a*cosh(e + f*x))^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+a\cosh(e+fx))^2}{(c+dx)^3}\,dx$$
$$= \frac{a^2\left(e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c^2d + 2e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)c\,d^2x + e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\right.\right.}{}$$

input
```
int((a+a*cosh(f*x+e))^2/(d*x+c)^3,x)
```

output

```
(a**2*(e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3),x)*c**2*d + 2*e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x
**2 + d**3*x**3),x)*c*d**2*x + e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x
+ 3*c*d**2*x**2 + d**3*x**3),x)*d**3*x**2 + 4*e**(2*e)*int(e**(f*x)/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*c**2*d + 8*e**(2*e)*int(e**(f
*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*c*d**2*x + 4*e**(2*
e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*d**3*x*
*2 + e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e
**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c**2*d + 2*e*
*e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*e
+ 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c*d**2*x + e**e*int(
1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*e + 2*f*x
)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*d**3*x**2 - 3*e**e + 4*int(
1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)
*d**3*x**3),x)*c**2*d + 8*int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e
**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*c*d**2*x + 4*int(1/(e**(f*x)*
c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),
x)*d**3*x**2))/(4*e**e*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.111 $\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 117

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = \frac{(c+dx)^3}{af} - \frac{6d(c+dx)^2\log\left(1+e^{e+fx}\right)}{af^2} - \frac{12d^2(c+dx)\operatorname{PolyLog}\left(2,-e^{e+fx}\right)}{af^3} + \frac{12d^3\operatorname{PolyLog}\left(3,-e^{e+fx}\right)}{af^4} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

output
```
(d*x+c)^3/a/f-6*d*(d*x+c)^2*ln(1+exp(f*x+e))/a/f^2-12*d^2*(d*x+c)*polylog(
2,-exp(f*x+e))/a/f^3+12*d^3*polylog(3,-exp(f*x+e))/a/f^4+(d*x+c)^3*tanh(1/
2*f*x+1/2*e)/a/f
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.52

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx$$
$$= \frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(-\frac{6de^e\cosh\left(\frac{1}{2}(e+fx)\right)\left(\frac{e^{-e}(c+dx)^3}{3d}+\frac{\left(1+e^{-e}\right)(c+dx)^2\log\left(1+e^{-e-fx}\right)}{f}-\frac{2de^{-e}(1+e^e)\left(f(c+dx)\operatorname{PolyLog}\left(2,-e^{-e-fx}\right.\right.}{f^3}}{1+e^e}\right.}{af(1+\cosh(e+fx))}$$

input
```
Integrate[(c + d*x)^3/(a + a*Cosh[e + f*x]),x]
```

output
```
(2*Cosh[(e + f*x)/2]*((-6*d*E^e*Cosh[(e + f*x)/2]*((c + d*x)^3/(3*d*E^e) +
 ((1 + E^(-e))*(c + d*x)^2*Log[1 + E^(-e - f*x)])/f - (2*d*(1 + E^e)*(f*(c
 + d*x)*PolyLog[2, -E^(-e - f*x)] + d*PolyLog[3, -E^(-e - f*x)]))/(E^e*f^3
)))/(1 + E^e) + (c + d*x)^3*Sech[e/2]*Sinh[(f*x)/2]))/(a*f*(1 + Cosh[e + f
*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.81 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.09, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3799, 3042, 4672, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a\cosh(e+fx)+a}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^3}{a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)}\,dx$$
$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6id\int -i(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int -i(c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 4201}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\int \frac{e^{e+fx}(c+dx)^2}{1+e^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\int (c+dx)\log\left(1+e^{e+fx}\right)dx}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-e^{e+fx}\right)dx}{f}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)}{f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

$\Big\downarrow$ 2720

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\mathrm{PolyLog}\left(2,-e^{e+fx}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

$\Big\downarrow$ 7143

$$\frac{\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-e^{e+fx}\right)}{f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^3/(a + a*Cosh[e + f*x]),x]
```

output
```
(((6*I)*d*(((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((c + d*x)^2*Log[1 + E^(e + f
*x)])/f - (2*d*(-(((c + d*x)*PolyLog[2, -E^(e + f*x)])/f) + (d*PolyLog[3,
-E^(e + f*x)])/f^2))/f)))/f + (2*(c + d*x)^3*Tanh[e/2 + (f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. $2(110) = 220$.

Time = 0.54 (sec) , antiderivative size = 325, normalized size of antiderivative = 2.78

method	result
risch	$-\frac{2(d^3x^3+3c\,d^2x^2+3c^2dx+c^3)}{fa(e^{fx+e}+1)}+\frac{6d\,c^2\ln(e^{fx+e})}{a\,f^2}-\frac{6d\,c^2\ln(e^{fx+e}+1)}{a\,f^2}+\frac{6d^2c\,x^2}{af}+\frac{6d^2c\,e^2}{a\,f^3}-\frac{12d^2c\ln(e^{fx+e}+1)x}{a\,f^2}-$

input
```
int((d*x+c)^3/(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(d^3*x^3+3*c*d^2*x^2+3*c^2*d*x+c^3)/a/(exp(f*x+e)+1)+6/a/f^2*d*c^2*ln
(exp(f*x+e))-6/a/f^2*d*c^2*ln(exp(f*x+e)+1)+6/a/f*d^2*c*x^2+6/a/f^3*d^2*c*
e^2-12/a/f^2*d^2*c*ln(exp(f*x+e)+1)*x-12/a/f^3*d^2*c*polylog(2,-exp(f*x+e)
)-12/a/f^3*d^2*c*e*ln(exp(f*x+e))+12/a/f^2*d^2*c*e*x+2/a/f*d^3*x^3-4/a/f^4
*d^3*e^3-6/a/f^2*d^3*ln(exp(f*x+e)+1)*x^2+12*d^3*polylog(3,-exp(f*x+e))/a/
f^4-12/a/f^3*d^3*polylog(2,-exp(f*x+e))*x+6/a/f^4*d^3*e^2*ln(exp(f*x+e))-6
/a/f^3*d^3*e^2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 438 vs. $2(109) = 218$.

Time = 0.10 (sec) , antiderivative size = 438, normalized size of antiderivative = 3.74

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx$$
$$= \frac{2\left(d^3e^3-3\,cd^2e^2f+3\,c^2def^2-c^3f^3+\left(d^3f^3x^3+3\,cd^2f^3x^2+3\,c^2df^3x+d^3e^3-3\,cd^2e^2f+3\,c^2def^2\right)\mathrm{co}\right.}{}$$

input
```
integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
2*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3 + (d^3*f^3*x^3 + 3*c*
d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*cos
h(f*x + e) - 6*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e) + (d
^3*f*x + c*d^2*f)*sinh(f*x + e))*dilog(-cosh(f*x + e) - sinh(f*x + e)) - 3
*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x +
 c^2*d*f^2)*cosh(f*x + e) + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2)*sinh
(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + 1) + 6*(d^3*cosh(f*x + e) +
 d^3*sinh(f*x + e) + d^3)*polylog(3, -cosh(f*x + e) - sinh(f*x + e)) + (d^
3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*
c^2*d*e*f^2)*sinh(f*x + e))/(a*f^4*cosh(f*x + e) + a*f^4*sinh(f*x + e) + a
*f^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = \frac{\int \frac{c^3}{\cosh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cosh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cosh(e+fx)+1}\,dx + \int \frac{3c^2dx}{\cosh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**3/(a+a*cosh(f*x+e)),x)
```

output
```
(Integral(c**3/(cosh(e + f*x) + 1), x) + Integral(d**3*x**3/(cosh(e + f*x)
 + 1), x) + Integral(3*c*d**2*x**2/(cosh(e + f*x) + 1), x) + Integral(3*c*
*2*d*x/(cosh(e + f*x) + 1), x))/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 228 vs. $2(109) = 218$.

Time = 0.17 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = 6\,c^2d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}+af} - \frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{af^2}\right) + \frac{2\,c^3}{(ae^{(-fx-e)}+a)f} - \frac{2\,(d^3x^3+3\,cd^2x^2)}{afe^{(fx+e)}+af} - \frac{12\,\left(fx\log\left(e^{(fx+e)}+1\right)+\mathrm{Li}_2\left(-e^{(fx+e)}\right)\right)cd^2}{af^3} - \frac{6\,\left(f^2x^2\log\left(e^{(fx+e)}+1\right)+2\,fx\mathrm{Li}_2\left(-e^{(fx+e)}\right)-2\,\mathrm{Li}_3(-e^{(fx+e)})\right)d^3}{af^4} + \frac{2\,(d^3f^3x^3+3\,cd^2f^3x^2)}{af^4}$$

input
```
integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
6*c^2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1)*e^(
-e))/(a*f^2)) + 2*c^3/((a*e^(-f*x - e) + a)*f) - 2*(d^3*x^3 + 3*c*d^2*x^2)
/(a*f*e^(f*x + e) + a*f) - 12*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x +
e)))*c*d^2/(a*f^3) - 6*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(-e^(f*x
 + e)) - 2*polylog(3, -e^(f*x + e)))*d^3/(a*f^4) + 2*(d^3*f^3*x^3 + 3*c*d^
2*f^3*x^2)/(a*f^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = \int \frac{(dx+c)^3}{a\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(a*cosh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+a\cosh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^3/(a + a*cosh(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + a*cosh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+a\cosh(e+fx)}\,dx = \frac{6e^{fx+e}\left(\int \frac{x^2}{e^{2fx+2e}+2e^{fx+e}+1}dx\right)d^3f^3+12e^{fx+e}\left(\int \frac{x}{e^{2fx+2e}+2e^{fx+e}+1}dx\right)c\,d^2f^3+12e^{fx+e}\left(\int \frac{x}{e^{2fx+2e}+2e^{fx+e}+1}c\right.}{}$$

input
```
int((d*x+c)^3/(a+a*cosh(f*x+e)),x)
```

output
```
(2*(3*e**(e + f*x)*int(x**2/(e**(2*e + 2*f*x) + 2*e**(e + f*x) + 1),x)*d**
3*f**3 + 6*e**(e + f*x)*int(x/(e**(2*e + 2*f*x) + 2*e**(e + f*x) + 1),x)*c
*d**2*f**3 + 6*e**(e + f*x)*int(x/(e**(2*e + 2*f*x) + 2*e**(e + f*x) + 1),
x)*d**3*f**2 - 3*e**(e + f*x)*log(e**(e + f*x) + 1)*c**2*d*f**2 - 6*e**(e
+ f*x)*log(e**(e + f*x) + 1)*c*d**2*f - 6*e**(e + f*x)*log(e**(e + f*x) +
1)*d**3 + e**(e + f*x)*c**3*f**3 + 3*e**(e + f*x)*c**2*d*f**3*x + 6*e**(e
+ f*x)*c*d**2*f**2*x + 6*e**(e + f*x)*d**3*f*x + 3*int(x**2/(e**(2*e + 2*f
*x) + 2*e**(e + f*x) + 1),x)*d**3*f**3 + 6*int(x/(e**(2*e + 2*f*x) + 2*e**
(e + f*x) + 1),x)*c*d**2*f**3 + 6*int(x/(e**(2*e + 2*f*x) + 2*e**(e + f*x)
 + 1),x)*d**3*f**2 - 3*log(e**(e + f*x) + 1)*c**2*d*f**2 - 6*log(e**(e + f
*x) + 1)*c*d**2*f - 6*log(e**(e + f*x) + 1)*d**3 - 3*c*d**2*f**3*x**2 - d*
*3*f**3*x**3 - 3*d**3*f**2*x**2))/(a*f**4*(e**(e + f*x) + 1))
```

3.112 $\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 88

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \frac{(c+dx)^2}{af} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{af^2} - \frac{4d^2\,\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{af^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

output
```
(d*x+c)^2/a/f-4*d*(d*x+c)*ln(1+exp(f*x+e))/a/f^2-4*d^2*polylog(2,-exp(f*x+
e))/a/f^3+(d*x+c)^2*tanh(1/2*f*x+1/2*e)/a/f
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(-\frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)\left(f(c+dx)+2d(1+e^e)\log\left(1+e^{-e-fx}\right)\right)-2d^2(1+e^e)\,\mathrm{PolyLog}\left(2,-e^{-e-fx}\right)\right)}{(1+e^e)f^2}+(c+\right.}{af(1+\cosh(e+fx))}$$

input
```
Integrate[(c + d*x)^2/(a + a*Cosh[e + f*x]),x]
```

output
```
(2*Cosh[(e + f*x)/2]*((-2*Cosh[(e + f*x)/2]*(f*(c + d*x)*(f*(c + d*x) + 2*
d*(1 + E^e)*Log[1 + E^(-e - f*x)]) - 2*d^2*(1 + E^e)*PolyLog[2, -E^(-e - f
*x)]))/((1 + E^e)*f^2) + (c + d*x)^2*Sech[e/2]*Sinh[(f*x)/2]))/(a*f*(1 + C
osh[e + f*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3042, 3799, 3042, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a\cosh(e+fx)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^2}{a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{\int (c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (c+dx)^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4id\int -i(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int (c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int -i(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 26$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\int (c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \; 4201$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\int \frac{e^{e+fx}(c+dx)}{1+e^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2620$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\left(\frac{(c+dx)\log\left(e^{e+fx}+1\right)}{f}-\frac{d\int \log\left(1+e^{e+fx}\right)dx}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2715$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\left(\frac{(c+dx)\log\left(e^{e+fx}+1\right)}{f}-\frac{d\int e^{-e-fx}\log\left(1+e^{e+fx}\right)de^{e+fx}}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

$$\Big\downarrow \; 2838$$

$$\frac{\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\left(\frac{(c+dx)\log\left(e^{e+fx}+1\right)}{f}+\frac{d\,\text{PolyLog}\left(2,-e^{e+fx}\right)}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}}{2a}$$

input
```
Int[(c + d*x)^2/(a + a*Cosh[e + f*x]),x]
```

output
```
(((4*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c + d*x)*Log[1 + E^(e + f*x
)])/f + (d*PolyLog[2, -E^(e + f*x)])/f^2)))/f + (2*(c + d*x)^2*Tanh[e/2 +
(f*x)/2])/f)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. 2(82) = 164.

Time = 0.46 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{2(x^2d^2+2cdx+c^2)}{fa(\mathrm{e}^{fx+e}+1)}+\frac{4dc\ln(\mathrm{e}^{fx+e})}{a\,f^2}-\frac{4dc\ln(\mathrm{e}^{fx+e}+1)}{a\,f^2}+\frac{2d^2x^2}{af}+\frac{4d^2ex}{a\,f^2}+\frac{2d^2e^2}{a\,f^3}-\frac{4d^2\ln(\mathrm{e}^{fx+e}+1)x}{a\,f^2}-\frac{4d^2\,\mathrm{poly}}{}$

input
```
int((d*x+c)^2/(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f*(d^2*x^2+2*c*d*x+c^2)/a/(exp(f*x+e)+1)+4/a/f^2*d*c*ln(exp(f*x+e))-4/a
/f^2*d*c*ln(exp(f*x+e)+1)+2/a/f*d^2*x^2+4/a/f^2*d^2*e*x+2/a/f^3*d^2*e^2-4/
a/f^2*d^2*ln(exp(f*x+e)+1)*x-4*d^2*polylog(2,-exp(f*x+e))/a/f^3-4/a/f^3*d^
2*e*ln(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. 2(81) = 162.

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 2.76

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = -\frac{2\left(d^2e^2-2\,cdef+c^2f^2-\left(d^2f^2x^2+2\,cdf^2x-d^2e^2+2\,cdef\right)\cosh\left(fx+e\right)+2\left(d^2\cosh\left(fx+e\right)+d\right.}{}$$

input
```
integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
-2*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 +
 2*c*d*e*f)*cosh(f*x + e) + 2*(d^2*cosh(f*x + e) + d^2*sinh(f*x + e) + d^2
)*dilog(-cosh(f*x + e) - sinh(f*x + e)) + 2*(d^2*f*x + c*d*f + (d^2*f*x +
c*d*f)*cosh(f*x + e) + (d^2*f*x + c*d*f)*sinh(f*x + e))*log(cosh(f*x + e)
+ sinh(f*x + e) + 1) - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*s
inh(f*x + e))/(a*f^3*cosh(f*x + e) + a*f^3*sinh(f*x + e) + a*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \frac{\int \frac{c^2}{\cosh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cosh(e+fx)+1}\,dx + \int \frac{2cdx}{\cosh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**2/(a+a*cosh(f*x+e)),x)
```

output
```
(Integral(c**2/(cosh(e + f*x) + 1), x) + Integral(d**2*x**2/(cosh(e + f*x)
 + 1), x) + Integral(2*c*d*x/(cosh(e + f*x) + 1), x))/a
```

Maxima [F]

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \int \frac{(dx+c)^2}{a\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
-2*d^2*(x^2/(a*f*e^(f*x + e) + a*f) - 2*integrate(x/(a*f*e^(f*x + e) + a*f
), x)) + 4*c*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) +
 1)*e^(-e))/(a*f^2)) + 2*c^2/((a*e^(-f*x - e) + a)*f)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \int \frac{(dx+c)^2}{a\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(a*cosh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+a\cosh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a + a*cosh(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + a*cosh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+a\cosh(e+fx)}\,dx = \frac{4e^{fx+e}\left(\int \frac{x}{e^{2fx+2e}+2e^{fx+e}+1}dx\right)d^2f^2 - 4e^{fx+e}\log\left(e^{fx+e}+1\right)cdf - 4e^{fx+e}\log\left(e^{fx+e}+1\right)d^2 + 2e^{fx+e}c^2f^2 +}{a}$$

input
```
int((d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

output

```
(2*(2*e**(e + f*x)*int(x/(e**(2*e + 2*f*x) + 2*e**(e + f*x) + 1),x)*d**2*f
**2 - 2*e**(e + f*x)*log(e**(e + f*x) + 1)*c*d*f - 2*e**(e + f*x)*log(e**(
e + f*x) + 1)*d**2 + e**(e + f*x)*c**2*f**2 + 2*e**(e + f*x)*c*d*f**2*x +
2*e**(e + f*x)*d**2*f*x + 2*int(x/(e**(2*e + 2*f*x) + 2*e**(e + f*x) + 1),
x)*d**2*f**2 - 2*log(e**(e + f*x) + 1)*c*d*f - 2*log(e**(e + f*x) + 1)*d**
2 - d**2*f**2*x**2))/(a*f**3*(e**(e + f*x) + 1))
```

3.113 $\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 49

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{af^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af}$$

output
```
-2*d*ln(cosh(1/2*f*x+1/2*e))/a/f^2+(d*x+c)*tanh(1/2*f*x+1/2*e)/a/f
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.43

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = \frac{2\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2d\cosh\left(\frac{1}{2}(e+fx)\right)\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)+f(c+dx)\sinh\left(\frac{1}{2}(e+fx)\right)\right)}{af^2(1+\cosh(e+fx))}$$

input
```
Integrate[(c + d*x)/(a + a*Cosh[e + f*x]),x]
```

output
```
(2*Cosh[(e + f*x)/2]*(-2*d*Cosh[(e + f*x)/2]*Log[Cosh[(e + f*x)/2]] + f*(c
 + d*x)*Sinh[(e + f*x)/2]))/(a*f^2*(1 + Cosh[e + f*x]))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 3799, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a\cosh(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx}{2a}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}}{2a}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}}{2a}$$

input
```
Int[(c + d*x)/(a + a*Cosh[e + f*x]),x]
```

output
```
((-4*d*Log[Cosh[e/2 + (f*x)/2]])/f^2 + (2*(c + d*x)*Tanh[e/2 + (f*x)/2])/f
)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

method	result	size
parallelrisch	$\frac{2\ln\left(1-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d+\left((dx+c)\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)+dx\right)f}{a\,f^{2}}$	47
risch	$\frac{2dx}{af}+\frac{2de}{a\,f^{2}}-\frac{2(dx+c)}{af\left(\mathrm{e}^{fx+e}+1\right)}-\frac{2d\ln\left(\mathrm{e}^{fx+e}+1\right)}{a\,f^{2}}$	63

input
```
int((d*x+c)/(a+a*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(2*ln(1-tanh(1/2*f*x+1/2*e))*d+((d*x+c)*tanh(1/2*f*x+1/2*e)+d*x)*f)/a/f^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. 2(41) = 82.

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = \frac{2\left(dfx\cosh\left(fx+e\right)+dfx\sinh\left(fx+e\right)-cf-\left(d\cosh\left(fx+e\right)+d\sinh\left(fx+e\right)+d\right)\log\left(\cosh\left(fx+\right.\right.\right.}{af^{2}\cosh\left(fx+e\right)+af^{2}\sinh\left(fx+e\right)+af^{2}}$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
2*(d*f*x*cosh(f*x + e) + d*f*x*sinh(f*x + e) - c*f - (d*cosh(f*x + e) + d*
sinh(f*x + e) + d)*log(cosh(f*x + e) + sinh(f*x + e) + 1))/(a*f^2*cosh(f*x
 + e) + a*f^2*sinh(f*x + e) + a*f^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(37) = 74$.

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.55

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = \begin{cases} \frac{c\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} + \frac{dx\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{af} - \frac{dx}{af} + \frac{2d\log\left(\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{af^2} & \text{for } f \neq 0 \\ \frac{cx+\frac{dx^2}{2}}{a\cosh(e)+a} & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e)),x)
```

output
```
Piecewise((c*tanh(e/2 + f*x/2)/(a*f) + d*x*tanh(e/2 + f*x/2)/(a*f) - d*x/(
a*f) + 2*d*log(tanh(e/2 + f*x/2) + 1)/(a*f**2), Ne(f, 0)), ((c*x + d*x**2/
2)/(a*cosh(e) + a), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = 2\,d\left(\frac{xe^{(fx+e)}}{afe^{(fx+e)}+af} - \frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{af^2}\right) + \frac{2\,c}{\left(ae^{(-fx-e)}+a\right)f}$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
2*d*(x*e^(f*x + e)/(a*f*e^(f*x + e) + a*f) - log((e^(f*x + e) + 1)*e^(-e))
/(a*f^2)) + 2*c/((a*e^(-f*x - e) + a)*f)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.35

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = \frac{2\left(dfxe^{(fx+e)} - de^{(fx+e)}\log\left(e^{(fx+e)}+1\right) - cf - d\log\left(e^{(fx+e)}+1\right)\right)}{af^2e^{(fx+e)}+af^2}$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
2*(d*f*x*e^(f*x + e) - d*e^(f*x + e)*log(e^(f*x + e) + 1) - c*f - d*log(e^
(f*x + e) + 1))/(a*f^2*e^(f*x + e) + a*f^2)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = \frac{2\,d\,x}{a\,f} - \frac{2\,(c+d\,x)}{a\,f\,\left(\mathrm{e}^{e+f\,x}+1\right)} - \frac{2\,d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}+1\right)}{a\,f^2}$$

input
```
int((c + d*x)/(a + a*cosh(e + f*x)),x)
```

output
```
(2*d*x)/(a*f) - (2*(c + d*x))/(a*f*(exp(e + f*x) + 1)) - (2*d*log(exp(f*x)
*exp(e) + 1))/(a*f^2)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \frac{c+dx}{a+a\cosh(e+fx)}\,dx = \frac{-2e^{fx+e}\log\left(e^{fx+e}+1\right)d + 2e^{fx+e}cf + 2e^{fx+e}dfx - 2\log\left(e^{fx+e}+1\right)d}{a\,f^2\left(e^{fx+e}+1\right)}$$

input
```
int((d*x+c)/(a+a*cosh(f*x+e)),x)
```

output
```
(2*( - e**(e + f*x)*log(e**(e + f*x) + 1)*d + e**(e + f*x)*c*f + e**(e + f
*x)*d*f*x - log(e**(e + f*x) + 1)*d))/(a*f**2*(e**(e + f*x) + 1))
```

3.114 $\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+a\cosh(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 7.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\cosh(e+fx)+a)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{(c+dx)(a\cosh(e+fx)+a)}dx$$

input
```
Int[1/((c + d*x)*(a + a*Cosh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+a\cosh\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (a*d*x + a*c)*cosh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.06 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \frac{\int \frac{1}{c\cosh(e+fx)+c+dx\cosh(e+fx)+dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

output
```
Integral(1/(c*cosh(e + f*x) + c + d*x*cosh(e + f*x) + d*x), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 98, normalized size of antiderivative = 4.90

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
-2*d*integrate(1/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2*e^e +
 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x)), x) - 2/(a*d*f*x + a*c*f + (a*d*f
*x*e^e + a*c*f*e^e)*e^(f*x))
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)(a\cosh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(a*cosh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.90 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(a+a\cosh{(e+f\,x)})\,(c+d\,x)}\,dx$$

input
```
int(1/((a + a*cosh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + a*cosh(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))}\,dx = \frac{\int \frac{1}{\cosh(fx+e)c+\cosh(fx+e)dx+c+dx}dx}{a}$$

input
```
int(1/(d*x+c)/(a+a*cosh(f*x+e)),x)
```

output
```
int(1/(cosh(e + f*x)*c + cosh(e + f*x)*d*x + c + d*x),x)/a
```

3.115 $\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+a\cosh(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 5.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\cosh(e+fx)+a)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)^2(a\cosh(e+fx)+a)}dx$$

input
```
Int[1/((c + d*x)^2*(a + a*Cosh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\left(a+a\cosh\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (a*d^2*x^2 + 2*a*c*d*x + a*c^2
)*cosh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.83 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.90

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \frac{\int \frac{1}{c^2\cosh(e+fx)+c^2+2cdx\cosh(e+fx)+2cdx+d^2x^2\cosh(e+fx)+d^2x^2}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e)),x)
```

output
```
Integral(1/(c**2*cosh(e + f*x) + c**2 + 2*c*d*x*cosh(e + f*x) + 2*c*d*x +
d**2*x**2*cosh(e + f*x) + d**2*x**2), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 154, normalized size of antiderivative = 7.70

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
-4*d*integrate(1/(a*d^3*f*x^3 + 3*a*c*d^2*f*x^2 + 3*a*c^2*d*f*x + a*c^3*f
+ (a*d^3*f*x^3*e^e + 3*a*c*d^2*f*x^2*e^e + 3*a*c^2*d*f*x*e^e + a*c^3*f*e^e
)*e^(f*x)), x) - 2/(a*d^2*f*x^2 + 2*a*c*d*f*x + a*c^2*f + (a*d^2*f*x^2*e^e
 + 2*a*c*d*f*x*e^e + a*c^2*f*e^e)*e^(f*x))
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(a\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(a*cosh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \int \frac{1}{(a+a\cosh(e+f\,x))\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*cosh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + a*cosh(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 119, normalized size of antiderivative = 5.95

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))}\,dx = \frac{2e^e\left(\int \frac{e^{fx}}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2+2e^{fx+e}c^2+4e^{fx+e}cdx+2e^{fx+e}d^2x^2+c^2+2cdx+d^2x^2}dx\right)}{a}$$

input
```
int(1/(d*x+c)^2/(a+a*cosh(f*x+e)),x)
```

output
```
(2*e**e*int(e**(f*x)/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e
**(2*e + 2*f*x)*d**2*x**2 + 2*e**(e + f*x)*c**2 + 4*e**(e + f*x)*c*d*x + 2
*e**(e + f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x))/a
```

3.116 $\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 255

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx &= \frac{(c+dx)^3}{3a^2f} - \frac{2d(c+dx)^2\log\left(1+e^{e+fx}\right)}{a^2f^2} \\ &\quad + \frac{4d^3\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{a^2f^4} \\ &\quad - \frac{4d^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{a^2f^3} \\ &\quad + \frac{4d^3\,\mathrm{PolyLog}\left(3,-e^{e+fx}\right)}{a^2f^4} + \frac{d(c+dx)^2\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f^2} \\ &\quad - \frac{2d^2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{a^2f^3} + \frac{(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} \\ &\quad + \frac{(c+dx)^3\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}\end{aligned}$$

output

```
1/3*(d*x+c)^3/a^2/f-2*d*(d*x+c)^2*ln(1+exp(f*x+e))/a^2/f^2+4*d^3*ln(cosh(1
/2*f*x+1/2*e))/a^2/f^4-4*d^2*(d*x+c)*polylog(2,-exp(f*x+e))/a^2/f^3+4*d^3*
polylog(3,-exp(f*x+e))/a^2/f^4+1/2*d*(d*x+c)^2*sech(1/2*f*x+1/2*e)^2/a^2/f
^2-2*d^2*(d*x+c)*tanh(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^3*tanh(1/2*f*x+1/
2*e)/a^2/f+1/6*(d*x+c)^3*sech(1/2*f*x+1/2*e)^2*tanh(1/2*f*x+1/2*e)/a^2/f
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.98

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx$$
$$= \frac{\cosh\left(\frac{1}{2}(e+fx)\right)\left(-\frac{8d\cosh^3\left(\frac{1}{2}(e+fx)\right)\left(6d^2e^efx-3c^2e^ef^3x+3cdf^3x^2+d^2f^3x^3+6cdf^2x\log\left(1+e^{-e-fx}\right)+6cde^ef^2x\log\left(1+e^{-e-fx}\right)+\right.}{}\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Cosh[e + f*x])^2,x]
```

output
```
(Cosh[(e + f*x)/2]*((-8*d*Cosh[(e + f*x)/2]^3*(6*d^2*E^e*f*x - 3*c^2*E^e*f
^3*x + 3*c*d*f^3*x^2 + d^2*f^3*x^3 + 6*c*d*f^2*x*Log[1 + E^(-e - f*x)] + 6
*c*d*E^e*f^2*x*Log[1 + E^(-e - f*x)] + 3*d^2*f^2*x^2*Log[1 + E^(-e - f*x)]
 + 3*d^2*E^e*f^2*x^2*Log[1 + E^(-e - f*x)] - 6*d^2*Log[1 + E^(e + f*x)] -
6*d^2*E^e*Log[1 + E^(e + f*x)] + 3*c^2*f^2*Log[1 + E^(e + f*x)] + 3*c^2*E^
e*f^2*Log[1 + E^(e + f*x)] - 6*d*(1 + E^e)*f*(c + d*x)*PolyLog[2, -E^(-e -
 f*x)] - 6*d^2*(1 + E^e)*PolyLog[3, -E^(-e - f*x)]))/((1 + E^e)*f) + (c +
d*x)*Sech[e/2]*(3*d*f*(c + d*x)*Cosh[(f*x)/2] + 3*d*f*(c + d*x)*Cosh[e + (
f*x)/2] - 12*d^2*Sinh[(f*x)/2] + 3*c^2*f^2*Sinh[(f*x)/2] + 6*c*d*f^2*x*Sin
h[(f*x)/2] + 3*d^2*f^2*x^2*Sinh[(f*x)/2] + 6*d^2*Sinh[e + (f*x)/2] - 6*d^2
*Sinh[e + (3*f*x)/2] + c^2*f^2*Sinh[e + (3*f*x)/2] + 2*c*d*f^2*x*Sinh[e +
(3*f*x)/2] + d^2*f^2*x^2*Sinh[e + (3*f*x)/2])))/(3*a^2*f^3*(1 + Cosh[e + f
*x])^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.40 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3799, 3042, 4674, 3042, 4672, 26, 3042, 26, 3956, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\cosh(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^3}{\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3799}$$

$$\frac{\int (c+dx)^3\text{sech}^4\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{ 4674}$$

$$\frac{-\frac{4d^2\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f^2}+\frac{2}{3}\int (c+dx)^3\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx+\frac{2d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f^2}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2(}{3f}}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{4d^2\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx}{f^2}+\frac{2}{3}\int (c+dx)^3\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx+\frac{2d(c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{f^2}+\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{f}{2}\right.}{3f}}{4a^2}$$

$$\Big\downarrow \text{ 4672}$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6id\int -i(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{2d(c+dx)^2}{}}{4a^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int (c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{2d(c+dx)^2\text{sech}^2(}{f^2}}{4a^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{6d\int -i(c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)+\frac{2d(c+dx)^2\mathrm{s}}{}}{4a^2}$$

$$\Big\downarrow \ 26$$

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)+\frac{2d(c+dx)^2\mathrm{sech}}{f^2}}{4a^2}$$

$$\Big\downarrow \ 3956$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\int (c+dx)^2\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)}{f^2}+\frac{2d(c+dx)^2\mathrm{sech}^2(}{f^2}}{4a^2}$$

$$\Big\downarrow \ 4201$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\int \frac{e^{e+fx}(c+dx)^2}{1+e^{e+fx}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)}{f^2}+\frac{2d(c+dx)^2}{}}{4a^2}$$

$$\Big\downarrow \ 2620$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\int (c+dx)\log\left(1+e^{e+fx}\right)dx}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}\right)-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4c}{}\right)}{f^2}}{4a^2}$$

$$\Big\downarrow \ 3011$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\left(\frac{d\int \mathrm{PolyLog}\left(2,-e^{e+fx}\right)dx}{f}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}\right)-}{4a^2}$$

$\downarrow$ 2720

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\left(\frac{d\int e^{-e-fx}\text{PolyLog}\left(2,-e^{e+fx}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-e^{e+fx}\right)}{f}\right)}{f}\right)-\frac{i(c}{}}{f}\right)}{4a^2}$$

$\downarrow$ 7143

$$\frac{-\frac{4d^2\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)}{f^2}+\frac{2}{3}\left(\frac{2(c+dx)^3\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{6id\left(2i\left(\frac{(c+dx)^2\log\left(e^{e+fx}+1\right)}{f}-\frac{2d\left(\frac{d\text{PolyLog}\left(3,-e^{e+}\right)}{f^2}\right.}{}\right.\right.}{f}\right.}{4a^2}$$

input
```
Int[(c + d*x)^3/(a + a*Cosh[e + f*x])^2,x]
```

output
```
((2*d*(c + d*x)^2*Sech[e/2 + (f*x)/2]^2)/f^2 + (2*(c + d*x)^3*Sech[e/2 + (
f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(3*f) - (4*d^2*((-4*d*Log[Cosh[e/2 + (f*x)/
2]])/f^2 + (2*(c + d*x)*Tanh[e/2 + (f*x)/2])/f))/f^2 + (2*(((6*I)*d*(((-1/
3*I)*(c + d*x)^3)/d + (2*I)*(((c + d*x)^2*Log[1 + E^(e + f*x)])/f - (2*d*(
-(((c + d*x)*PolyLog[2, -E^(e + f*x)])/f) + (d*PolyLog[3, -E^(e + f*x)])/f
^2))/f)))/f + (2*(c + d*x)^3*Tanh[e/2 + (f*x)/2])/f))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(220) = 440$.

Time = 0.82 (sec) , antiderivative size = 600, normalized size of antiderivative = 2.35

method	result
risch	$-\frac{2\left(3d^3f^2x^3\mathrm{e}^{fx+e}+9c\,d^2f^2x^2\mathrm{e}^{fx+e}+d^3f^2x^3-3d^3f\,x^2\mathrm{e}^{2fx+2e}+9c^2d\,f^2x\,\mathrm{e}^{fx+e}+3c\,d^2f^2x^2-6c\,d^2fx\,\mathrm{e}^{2fx+2e}-3d^3f\,x^2\mathrm{e}^{fx+e}+3\right.}{}$

input
```
int((d*x+c)^3/(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*d^3*f^2*x^3*exp(f*x+e)+9*c*d^2*f^2*x^2*exp(f*x+e)+d^3*f^2*x^3-3*d^
3*f*x^2*exp(2*f*x+2*e)+9*c^2*d*f^2*x*exp(f*x+e)+3*c*d^2*f^2*x^2-6*c*d^2*f*
x*exp(2*f*x+2*e)-3*d^3*f*x^2*exp(f*x+e)+3*c^3*f^2*exp(f*x+e)+3*c^2*d*f^2*x
-3*c^2*d*f*exp(2*f*x+2*e)-6*c*d^2*f*x*exp(f*x+e)-6*d^3*x*exp(2*f*x+2*e)+c^
3*f^2-3*c^2*d*f*exp(f*x+e)-6*c*d^2*exp(2*f*x+2*e)-12*d^3*x*exp(f*x+e)-12*c
*d^2*exp(f*x+e)-6*d^3*x-6*d^2*c)/f^3/a^2/(exp(f*x+e)+1)^3-4/a^2/f^3*d^3*po
lylog(2,-exp(f*x+e))*x+2/a^2/f^3*d^2*c*e^2-4/a^2/f^3*d^2*c*polylog(2,-exp(
f*x+e))+2/3/a^2/f*d^3*x^3+2/a^2/f^2*d*c^2*ln(exp(f*x+e))-2/a^2/f^2*d*c^2*l
n(exp(f*x+e)+1)+2/a^2/f^4*d^3*e^2*ln(exp(f*x+e))-4/3/a^2/f^4*d^3*e^3+4*d^3
*polylog(3,-exp(f*x+e))/a^2/f^4-4/a^2/f^4*d^3*ln(exp(f*x+e))+4/a^2/f^4*d^3
*ln(exp(f*x+e)+1)+2/a^2/f*d^2*c*x^2-4/a^2/f^3*d^2*c*e*ln(exp(f*x+e))+4/a^2
/f^2*d^2*c*e*x-4/a^2/f^2*d^2*c*ln(exp(f*x+e)+1)*x-2/a^2/f^3*d^3*e^2*x-2/a^
2/f^2*d^3*ln(exp(f*x+e)+1)*x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1863 vs. $2(219) = 438$.

Time = 0.11 (sec) , antiderivative size = 1863, normalized size of antiderivative = 7.31

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
2/3*(d^3*e^3 + 3*c^2*d*e*f^2 - c^3*f^3 - 6*d^3*e + (d^3*f^3*x^3 + 3*c*d^2*
f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3
 - 2*d^3*f)*x)*cosh(f*x + e)^3 + (d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + d^3*e^3
- 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3*(c^2*d*f^3 - 2*d^3*f)*x)*sin
h(f*x + e)^3 + 3*(d^3*f^3*x^3 + d^3*e^3 - 6*d^3*e + (3*c^2*d*e + c^2*d)*f^
2 + (3*c*d^2*f^3 + d^3*f^2)*x^2 - (3*c*d^2*e^2 - 2*c*d^2)*f + (3*c^2*d*f^3
 + 2*c*d^2*f^2 - 4*d^3*f)*x)*cosh(f*x + e)^2 + 3*(d^3*f^3*x^3 + d^3*e^3 -
6*d^3*e + (3*c^2*d*e + c^2*d)*f^2 + (3*c*d^2*f^3 + d^3*f^2)*x^2 - (3*c*d^2
*e^2 - 2*c*d^2)*f + (3*c^2*d*f^3 + 2*c*d^2*f^2 - 4*d^3*f)*x + (d^3*f^3*x^3
 + 3*c*d^2*f^3*x^2 + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - 6*d^3*e + 3
*(c^2*d*f^3 - 2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e)^2 - 3*(c*d^2*e^2 -
2*c*d^2)*f + 3*(d^3*f^2*x^2 + d^3*e^3 - c^3*f^3 - 6*d^3*e + (3*c^2*d*e + c
^2*d)*f^2 - (3*c*d^2*e^2 - 4*c*d^2)*f + 2*(c*d^2*f^2 - d^3*f)*x)*cosh(f*x
+ e) - 6*(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e)^3 + (d^3*f
*x + c*d^2*f)*sinh(f*x + e)^3 + 3*(d^3*f*x + c*d^2*f)*cosh(f*x + e)^2 + 3*
(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e))*sinh(f*x + e)^2 +
3*(d^3*f*x + c*d^2*f)*cosh(f*x + e) + 3*(d^3*f*x + c*d^2*f + (d^3*f*x + c*
d^2*f)*cosh(f*x + e)^2 + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + e))*sinh(f*x + e
))*dilog(-cosh(f*x + e) - sinh(f*x + e)) - 3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x
+ c^2*d*f^2 + (d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 2*d^3)*cosh(f*...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int \frac{c^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{\cdots}{\cosh^2(e\cdots}}{a^2}$$

input
```
integrate((d*x+c)**3/(a+a*cosh(f*x+e))**2,x)
```

output
```
(Integral(c**3/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(d**
3*x**3/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(3*c*d**2*x*
*2/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(cos
h(e + f*x)**2 + 2*cosh(e + f*x) + 1), x))/a**2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 610 vs. $2(219) = 438$.

Time = 0.26 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*c^2*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + e^(f
*x + e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2*f*x + 2*e) + 3*a^2*f^2*
e^(f*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c^
3*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-
3*f*x - 3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) +
 a^2*e^(-3*f*x - 3*e) + a^2)*f)) - 2/3*(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 - 6*
d^3*x - 6*c*d^2 - 3*(d^3*f*x^2*e^(2*e) + 2*c*d^2*e^(2*e) + 2*(c*d^2*f*e^(2
*e) + d^3*e^(2*e))*x)*e^(2*f*x) + 3*(d^3*f^2*x^3*e^e - 4*c*d^2*e^e + (3*c*
d^2*f^2*e^e - d^3*f*e^e)*x^2 - 2*(c*d^2*f*e^e + 2*d^3*e^e)*x)*e^(f*x))/(a^
2*f^3*e^(3*f*x + 3*e) + 3*a^2*f^3*e^(2*f*x + 2*e) + 3*a^2*f^3*e^(f*x + e)
+ a^2*f^3) - 4*(f*x*log(e^(f*x + e) + 1) + dilog(-e^(f*x + e)))*c*d^2/(a^2
*f^3) - 4*d^3*x/(a^2*f^3) - 2*(f^2*x^2*log(e^(f*x + e) + 1) + 2*f*x*dilog(
-e^(f*x + e)) - 2*polylog(3, -e^(f*x + e)))*d^3/(a^2*f^4) + 4*d^3*log(e^(f
*x + e) + 1)/(a^2*f^4) + 2/3*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/(a^2*f^4)
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(a*cosh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^3}{(a+a\cosh\left(e+f\,x\right))^2}\,dx$$

input
```
int((c + d*x)^3/(a + a*cosh(e + f*x))^2,x)
```

output
```
int((c + d*x)^3/(a + a*cosh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+a\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^3/(a+a*cosh(f*x+e))^2,x)
```

output
```
(54*e**(3*e + 3*f*x)*int(x**2/(e**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e
**(2*e + 2*f*x) + 4*e**(e + f*x) + 1),x)*d**3*f**3 + 108*e**(3*e + 3*f*x)*
int(x/(e**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(
e + f*x) + 1),x)*c*d**2*f**3 + 198*e**(3*e + 3*f*x)*int(x/(e**(4*e + 4*f*x
) + 4*e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(e + f*x) + 1),x)*d**3*
f**2 - 54*e**(3*e + 3*f*x)*log(e**(e + f*x) + 1)*c**2*d*f**2 - 198*e**(3*e
 + 3*f*x)*log(e**(e + f*x) + 1)*c*d**2*f - 147*e**(3*e + 3*f*x)*log(e**(e
+ f*x) + 1)*d**3 + 54*e**(3*e + 3*f*x)*c**2*d*f**3*x - 18*e**(3*e + 3*f*x)
*c**2*d*f**2 + 198*e**(3*e + 3*f*x)*c*d**2*f**2*x - 66*e**(3*e + 3*f*x)*c*
d**2*f + 147*e**(3*e + 3*f*x)*d**3*f*x - 49*e**(3*e + 3*f*x)*d**3 + 162*e*
*(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e**(2*e
 + 2*f*x) + 4*e**(e + f*x) + 1),x)*d**3*f**3 + 324*e**(2*e + 2*f*x)*int(x/
(e**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(e + f*
x) + 1),x)*c*d**2*f**3 + 594*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 4*
e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(e + f*x) + 1),x)*d**3*f**2 -
 162*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*c**2*d*f**2 - 594*e**(2*e + 2*
f*x)*log(e**(e + f*x) + 1)*c*d**2*f - 441*e**(2*e + 2*f*x)*log(e**(e + f*x
) + 1)*d**3 + 162*e**(2*e + 2*f*x)*c**2*d*f**3*x + 594*e**(2*e + 2*f*x)*c*
d**2*f**2*x + 441*e**(2*e + 2*f*x)*d**3*f*x + 162*e**(e + f*x)*int(x**2/(e
**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(e + f...
```

3.117 $\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 200

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \frac{(c+dx)^2}{3a^2f} - \frac{4d(c+dx)\log\left(1+e^{e+fx}\right)}{3a^2f^2} - \frac{4d^2\,\mathrm{PolyLog}\left(2,-e^{e+fx}\right)}{3a^2f^3} + \frac{d(c+dx)\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^2} - \frac{2d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f^3} + \frac{(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f} + \frac{(c+dx)^2\mathrm{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

output
```
1/3*(d*x+c)^2/a^2/f-4/3*d*(d*x+c)*ln(1+exp(f*x+e))/a^2/f^2-4/3*d^2*polylog
(2,-exp(f*x+e))/a^2/f^3+1/3*d*(d*x+c)*sech(1/2*f*x+1/2*e)^2/a^2/f^2-2/3*d^
2*tanh(1/2*f*x+1/2*e)/a^2/f^3+1/3*(d*x+c)^2*tanh(1/2*f*x+1/2*e)/a^2/f+1/6*
(d*x+c)^2*sech(1/2*f*x+1/2*e)^2*tanh(1/2*f*x+1/2*e)/a^2/f
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.48

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx$$
$$= \frac{\cosh\left(\frac{1}{2}(e+fx)\right)\left(-\frac{8\cosh^3\left(\frac{1}{2}(e+fx)\right)\left(f(c+dx)\left(f(c+dx)+2d(1+e^e)\log\left(1+e^{-e-fx}\right)\right)-2d^2(1+e^e)\operatorname{PolyLog}\left(2,-e^{-e-fx}\right)\right)}{1+e^e}+\operatorname{sech}(\right.}{}$$

input
```
Integrate[(c + d*x)^2/(a + a*Cosh[e + f*x])^2,x]
```

output
```
(Cosh[(e + f*x)/2]*((-8*Cosh[(e + f*x)/2]^3*(f*(c + d*x)*(f*(c + d*x) + 2*
d*(1 + E^e)*Log[1 + E^(-e - f*x)]) - 2*d^2*(1 + E^e)*PolyLog[2, -E^(-e - f
*x)]))/(1 + E^e) + Sech[e/2]*(2*d*f*(c + d*x)*Cosh[(f*x)/2] + 2*d*f*(c + d
*x)*Cosh[e + (f*x)/2] - 4*d^2*Sinh[(f*x)/2] + 3*c^2*f^2*Sinh[(f*x)/2] + 6*
c*d*f^2*x*Sinh[(f*x)/2] + 3*d^2*f^2*x^2*Sinh[(f*x)/2] + 2*d^2*Sinh[e + (f*
x)/2] - 2*d^2*Sinh[e + (3*f*x)/2] + c^2*f^2*Sinh[e + (3*f*x)/2] + 2*c*d*f^
2*x*Sinh[e + (3*f*x)/2] + d^2*f^2*x^2*Sinh[e + (3*f*x)/2])))/(3*a^2*f^3*(1
 + Cosh[e + f*x])^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.98, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3799, 3042, 4674, 3042, 4254, 24, 4672, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\cosh(e+fx)+a)^2}\,dx$$
$$\Big\downarrow \text{ 3042}$$

$$\int \frac{(c+dx)^2}{\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$\Big\downarrow$ 3799

$$\frac{\int (c+dx)^2\text{sech}^4\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{\int (c+dx)^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4dx}{4a^2}$$

$\Big\downarrow$ 4674

$$\frac{\frac{2}{3}\int (c+dx)^2\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx-\frac{4d^2\int \text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{3f^2}+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}}{4a^2}$$

$\Big\downarrow$ 3042

$$\frac{\frac{2}{3}\int (c+dx)^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx-\frac{4d^2\int \csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx}{3f^2}+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\right.}{3f}}{4a^2}$$

$\Big\downarrow$ 4254

$$\frac{\frac{2}{3}\int (c+dx)^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx-\frac{8id^2\int 1d\left(-i\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3f^3}+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2}{3f}}{4a^2}$$

$\Big\downarrow$ 24

$$\frac{\frac{2}{3}\int (c+dx)^2\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{8d^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^3}}{4a^2}$$

$\Big\downarrow$ 4672

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4id\int -i(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-}{4a^2}$$

$\Big\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{8d}{}}{4a^2}$$

$\downarrow$ 3042

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\int-i(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-}{4a^2}$$

$\downarrow$ 26

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\int(c+dx)\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}-\frac{8}{}}{4a^2}$$

$\downarrow$ 4201

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\int\frac{e^{e+fx}(c+dx)}{1+e^{e+fx}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{f}{}\right.}{3f}}{4a^2}$$

$\downarrow$ 2620

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\left(\frac{(c+dx)\log\left(e^{e+fx}+1\right)}{f}-\frac{d\int\log\left(1+e^{e+fx}\right)dx}{f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx)}{}}{4a^2}$$

$\downarrow$ 2715

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\left(\frac{(c+dx)\log\left(e^{e+fx}+1\right)}{f}-\frac{d\int e^{-e-fx}\log\left(1+e^{e+fx}\right)de^{e+fx}}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}\right)+\frac{4d(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$\downarrow$ 2838

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)^2\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{4id\left(2i\left(\frac{(c+dx)\log\left(e^{e+fx}+1\right)}{f}+\frac{d\operatorname{PolyLog}\left(2,-e^{e+fx}\right)}{f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}\right)+\frac{4d(c+dx)\operatorname{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}+\frac{2(c+dx}{}}{4a^2}$$

input
```
Int[(c + d*x)^2/(a + a*Cosh[e + f*x])^2,x]
```

output
```
((4*d*(c + d*x)*Sech[e/2 + (f*x)/2]^2)/(3*f^2) - (8*d^2*Tanh[e/2 + (f*x)/2
])/(3*f^3) + (2*(c + d*x)^2*Sech[e/2 + (f*x)/2]^2*Tanh[e/2 + (f*x)/2])/(3*
f) + (2*(((4*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c + d*x)*Log[1 + E^
(e + f*x)])/f + (d*PolyLog[2, -E^(e + f*x)])/f^2)))/f + (2*(c + d*x)^2*Tan
h[e/2 + (f*x)/2])/f))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 313, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{2\left(3d^2f^2x^2\mathrm{e}^{fx+e}+6cd\,f^2x\,\mathrm{e}^{fx+e}+d^2x^2f^2-2d^2fx\,\mathrm{e}^{2fx+2e}+3c^2f^2\mathrm{e}^{fx+e}+2cd\,f^2x-2cdf\,\mathrm{e}^{2fx+2e}-2d^2fx\,\mathrm{e}^{fx+e}+c^2f^2-2cdf\,\mathrm{e}^{fx}\right.}{3f^3a^2\left(\mathrm{e}^{fx+e}+1\right)^3}$

input
```
int((d*x+c)^2/(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*d^2*f^2*x^2*exp(f*x+e)+6*c*d*f^2*x*exp(f*x+e)+d^2*x^2*f^2-2*d^2*f*
x*exp(2*f*x+2*e)+3*c^2*f^2*exp(f*x+e)+2*c*d*f^2*x-2*c*d*f*exp(2*f*x+2*e)-2
*d^2*f*x*exp(f*x+e)+c^2*f^2-2*c*d*f*exp(f*x+e)-2*exp(2*f*x+2*e)*d^2-4*exp(
f*x+e)*d^2-2*d^2)/f^3/a^2/(exp(f*x+e)+1)^3+4/3/a^2*d/f^2*c*ln(exp(f*x+e))-
4/3/a^2*d/f^2*c*ln(exp(f*x+e)+1)+2/3/a^2*d^2/f*x^2+4/3/a^2*d^2/f^2*e*x+2/3
/a^2*d^2/f^3*e^2-4/3/a^2*d^2/f^2*ln(exp(f*x+e)+1)*x-4/3*d^2*polylog(2,-exp
(f*x+e))/a^2/f^3-4/3/a^2*d^2/f^3*e*ln(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 963 vs. $2(163) = 326$.

Time = 0.10 (sec) , antiderivative size = 963, normalized size of antiderivative = 4.82

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-2/3*(d^2*e^2 - 2*c*d*e*f + c^2*f^2 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2
 + 2*c*d*e*f)*cosh(f*x + e)^3 - (d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c
*d*e*f)*sinh(f*x + e)^3 - (3*d^2*f^2*x^2 - 3*d^2*e^2 + 2*d^2 + 2*(3*c*d*e
+ c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^2 - (3*d^2*f^2*x^2 - 3*d
^2*e^2 + 2*d^2 + 2*(3*c*d*e + c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x + 3*(d^2*f^
2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*cosh(f*x + e))*sinh(f*x + e)^2
- 2*d^2 + (3*d^2*e^2 + 3*c^2*f^2 - 2*d^2*f*x - 4*d^2 - 2*(3*c*d*e + c*d)*f
)*cosh(f*x + e) + 2*(d^2*cosh(f*x + e)^3 + d^2*sinh(f*x + e)^3 + 3*d^2*cos
h(f*x + e)^2 + 3*d^2*cosh(f*x + e) + 3*(d^2*cosh(f*x + e) + d^2)*sinh(f*x
+ e)^2 + d^2 + 3*(d^2*cosh(f*x + e)^2 + 2*d^2*cosh(f*x + e) + d^2)*sinh(f*
x + e))*dilog(-cosh(f*x + e) - sinh(f*x + e)) + 2*(d^2*f*x + (d^2*f*x + c*
d*f)*cosh(f*x + e)^3 + (d^2*f*x + c*d*f)*sinh(f*x + e)^3 + c*d*f + 3*(d^2*
f*x + c*d*f)*cosh(f*x + e)^2 + 3*(d^2*f*x + c*d*f + (d^2*f*x + c*d*f)*cosh
(f*x + e))*sinh(f*x + e)^2 + 3*(d^2*f*x + c*d*f)*cosh(f*x + e) + 3*(d^2*f*
x + c*d*f + (d^2*f*x + c*d*f)*cosh(f*x + e)^2 + 2*(d^2*f*x + c*d*f)*cosh(f
*x + e))*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + 1) + (3*d^2*e^
2 + 3*c^2*f^2 - 2*d^2*f*x - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d
*e*f)*cosh(f*x + e)^2 - 4*d^2 - 2*(3*c*d*e + c*d)*f - 2*(3*d^2*f^2*x^2 - 3
*d^2*e^2 + 2*d^2 + 2*(3*c*d*e + c*d)*f + 2*(3*c*d*f^2 + d^2*f)*x)*cosh(f*x
 + e))*sinh(f*x + e))/(a^2*f^3*cosh(f*x + e)^3 + a^2*f^3*sinh(f*x + e)^...
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \frac{\int \frac{c^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx + \int \frac{2cdx}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**2/(a+a*cosh(f*x+e))**2,x)
```

output
```
(Integral(c**2/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(d**
2*x**2/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x) + Integral(2*c*d*x/(co
sh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x))/a**2
```

Maxima [F]

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2/3*d^2*((f^2*x^2 - 2*(f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + (3*f^2*x^2*e^e
- 2*f*x*e^e - 4*e^e)*e^(f*x) - 2)/(a^2*f^3*e^(3*f*x + 3*e) + 3*a^2*f^3*e^(
2*f*x + 2*e) + 3*a^2*f^3*e^(f*x + e) + a^2*f^3) - 6*integrate(1/3*x/(a^2*f
*e^(f*x + e) + a^2*f), x)) + 4/3*c*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e
) + e^(2*e))*e^(2*f*x) + e^(f*x + e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2
*e^(2*f*x + 2*e) + 3*a^2*f^2*e^(f*x + e) + a^2*f^2) - log((e^(f*x + e) + 1
)*e^(-e))/(a^2*f^2)) + 2/3*c^2*(3*e^(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^
2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*x - 3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x -
e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*x - 3*e) + a^2)*f))
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(a*cosh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+a\cosh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^2/(a + a*cosh(e + f*x))^2,x)
```

output
```
int((c + d*x)^2/(a + a*cosh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+a\cosh(e+fx))^2}\,dx$$
$$= \frac{-4e^{fx+e}\log\left(e^{fx+e}+1\right)cdf + \frac{4\left(\int \frac{x}{e^{4fx+4e}+4e^{3fx+3e}+6e^{2fx+2e}+4e^{fx+e}+1}dx\right)d^2f^2}{3} - \frac{22e^{fx+e}\log\left(e^{fx+e}+1\right)d^2}{3} + \frac{22e^{2fx+2e}d^2}{3}}{}$$

input
```
int((d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

output
```
(2*(18*e**(3*e + 3*f*x)*int(x/(e**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e
**(2*e + 2*f*x) + 4*e**(e + f*x) + 1),x)*d**2*f**2 - 18*e**(3*e + 3*f*x)*l
og(e**(e + f*x) + 1)*c*d*f - 33*e**(3*e + 3*f*x)*log(e**(e + f*x) + 1)*d**
2 + 18*e**(3*e + 3*f*x)*c*d*f**2*x - 6*e**(3*e + 3*f*x)*c*d*f + 33*e**(3*e
 + 3*f*x)*d**2*f*x - 11*e**(3*e + 3*f*x)*d**2 + 54*e**(2*e + 2*f*x)*int(x/
(e**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(e + f*
x) + 1),x)*d**2*f**2 - 54*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*c*d*f - 9
9*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*d**2 + 54*e**(2*e + 2*f*x)*c*d*f*
*2*x + 99*e**(2*e + 2*f*x)*d**2*f*x + 54*e**(e + f*x)*int(x/(e**(4*e + 4*f
*x) + 4*e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(e + f*x) + 1),x)*d**
2*f**2 - 54*e**(e + f*x)*log(e**(e + f*x) + 1)*c*d*f - 99*e**(e + f*x)*log
(e**(e + f*x) + 1)*d**2 - 27*e**(e + f*x)*c**2*f**2 - 27*e**(e + f*x)*d**2
*f**2*x**2 + 72*e**(e + f*x)*d**2*f*x + 36*e**(e + f*x)*d**2 + 18*int(x/(e
**(4*e + 4*f*x) + 4*e**(3*e + 3*f*x) + 6*e**(2*e + 2*f*x) + 4*e**(e + f*x)
 + 1),x)*d**2*f**2 - 18*log(e**(e + f*x) + 1)*c*d*f - 33*log(e**(e + f*x)
+ 1)*d**2 - 9*c**2*f**2 - 6*c*d*f - 9*d**2*f**2*x**2 + 25*d**2))/(27*a**2*
f**3*(e**(3*e + 3*f*x) + 3*e**(2*e + 2*f*x) + 3*e**(e + f*x) + 1))
```

3.118 $\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx = -\frac{2d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{3a^2f^2}+\frac{d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f^2}+\frac{(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{3a^2f}+\frac{(c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f}$$

output
```
-2/3*d*ln(cosh(1/2*f*x+1/2*e))/a^2/f^2+1/6*d*sech(1/2*f*x+1/2*e)^2/a^2/f^2
+1/3*(d*x+c)*tanh(1/2*f*x+1/2*e)/a^2/f+1/6*(d*x+c)*sech(1/2*f*x+1/2*e)^2*t
anh(1/2*f*x+1/2*e)/a^2/f
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx = \frac{\cosh\left(\frac{1}{2}(e+fx)\right)\left(-2d\cosh\left(\frac{3}{2}(e+fx)\right)\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)+\cosh\left(\frac{1}{2}(e+fx)\right)\left(2d-6d\log\left(\cosh\left(\right.\right.\right.\right.}{3a^2f^2(1+\cosh(e+fx))^2}$$

input
```
Integrate[(c + d*x)/(a + a*Cosh[e + f*x])^2,x]
```

output
```
(Cosh[(e + f*x)/2]*(-2*d*Cosh[(3*(e + f*x))/2]*Log[Cosh[(e + f*x)/2]] + Co
sh[(e + f*x)/2]*(2*d - 6*d*Log[Cosh[(e + f*x)/2]]) + f*(c + d*x)*(3*Sinh[(
e + f*x)/2] + Sinh[(3*(e + f*x))/2])))/(3*a^2*f^2*(1 + Cosh[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.97, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.556, Rules used = {3042, 3799, 3042, 4673, 3042, 4672, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\cosh(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3799}$$

$$\frac{\int (c+dx)\text{sech}^4\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4dx}{4a^2}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\frac{2}{3}\int (c+dx)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)dx+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\int (c+dx)\csc\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^2dx+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{4672}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2id\int -i\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int \tanh\left(\frac{e}{2}+\frac{fx}{2}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{2d\int -i\tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}+\frac{2id\int \tan\left(\frac{ie}{2}+\frac{ifx}{2}\right)dx}{f}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

$$\Big\downarrow \text{3956}$$

$$\frac{\frac{2}{3}\left(\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{f}-\frac{4d\log\left(\cosh\left(\frac{e}{2}+\frac{fx}{2}\right)\right)}{f^2}\right)+\frac{2(c+dx)\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f}+\frac{2d\text{sech}^2\left(\frac{e}{2}+\frac{fx}{2}\right)}{3f^2}}{4a^2}$$

input
```
Int[(c + d*x)/(a + a*Cosh[e + f*x])^2,x]
```

output
```
((2*d*Sech[e/2 + (f*x)/2]^2)/(3*f^2) + (2*(c + d*x)*Sech[e/2 + (f*x)/2]^2*
Tanh[e/2 + (f*x)/2])/(3*f) + (2*((-4*d*Log[Cosh[e/2 + (f*x)/2]])/f^2 + (2*
(c + d*x)*Tanh[e/2 + (f*x)/2])/f))/3)/(4*a^2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4672
```
Int[csc[(e_.) + (f_.)*(x_)]^2*((c_.) + (d_.)*(x_))^(m_.), x_Symbol] :> Simp
[(-(c + d*x)^m)*(Cot[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)
*Cot[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{4\ln\left(1-\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)\right)d-(dx+c)f\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)^{3}-d\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)^{2}+3\tanh\left(\frac{fx}{2}+\frac{e}{2}\right)f(dx+c)+2dxf}{6f^{2}a^{2}}$	82
risch	$\frac{2dx}{3f\,a^{2}}+\frac{2de}{3f^{2}a^{2}}-\frac{2\left(3dfx\,\mathrm{e}^{fx+e}+3cf\,\mathrm{e}^{fx+e}+dxf-\mathrm{e}^{2fx+2e}d+cf-\mathrm{e}^{fx+e}d\right)}{3f^{2}a^{2}\left(\mathrm{e}^{fx+e}+1\right)^{3}}-\frac{2d\ln\left(\mathrm{e}^{fx+e}+1\right)}{3f^{2}a^{2}}$	108

input
```
int((d*x+c)/(a+a*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*(4*ln(1-tanh(1/2*f*x+1/2*e))*d-(d*x+c)*f*tanh(1/2*f*x+1/2*e)^3-d*tanh(
1/2*f*x+1/2*e)^2+3*tanh(1/2*f*x+1/2*e)*f*(d*x+c)+2*d*x*f)/f^2/a^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(95) = 190.

Time = 0.11 (sec) , antiderivative size = 385, normalized size of antiderivative = 3.13

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx$$
$$= \frac{2\left(dfx\cosh\left(fx+e\right)^3 + dfx\sinh\left(fx+e\right)^3 + \left(3\,dfx+d\right)\cosh\left(fx+e\right)^2 + \left(3\,dfx\cosh\left(fx+e\right) + 3\,dfx +\right.}{}$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
2/3*(d*f*x*cosh(f*x + e)^3 + d*f*x*sinh(f*x + e)^3 + (3*d*f*x + d)*cosh(f*
x + e)^2 + (3*d*f*x*cosh(f*x + e) + 3*d*f*x + d)*sinh(f*x + e)^2 - c*f - (
3*c*f - d)*cosh(f*x + e) - (d*cosh(f*x + e)^3 + d*sinh(f*x + e)^3 + 3*d*co
sh(f*x + e)^2 + 3*(d*cosh(f*x + e) + d)*sinh(f*x + e)^2 + 3*d*cosh(f*x + e
) + 3*(d*cosh(f*x + e)^2 + 2*d*cosh(f*x + e) + d)*sinh(f*x + e) + d)*log(c
osh(f*x + e) + sinh(f*x + e) + 1) + (3*d*f*x*cosh(f*x + e)^2 - 3*c*f + 2*(
3*d*f*x + d)*cosh(f*x + e) + d)*sinh(f*x + e))/(a^2*f^2*cosh(f*x + e)^3 +
a^2*f^2*sinh(f*x + e)^3 + 3*a^2*f^2*cosh(f*x + e)^2 + 3*a^2*f^2*cosh(f*x +
 e) + a^2*f^2 + 3*(a^2*f^2*cosh(f*x + e) + a^2*f^2)*sinh(f*x + e)^2 + 3*(a
^2*f^2*cosh(f*x + e)^2 + 2*a^2*f^2*cosh(f*x + e) + a^2*f^2)*sinh(f*x + e))
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.27

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx$$
$$= \begin{cases} -\frac{c\tanh^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{c\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f} - \frac{dx\tanh^3\left(\frac{e}{2}+\frac{fx}{2}\right)}{6a^2f} + \frac{dx\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)}{2a^2f} - \frac{dx}{3a^2f} + \frac{2d\log\left(\tanh\left(\frac{e}{2}+\frac{fx}{2}\right)+1\right)}{3a^2f^2} - \frac{d\tanh^2\left(\right.}{6a^2} \\ \frac{cx+\frac{dx^2}{2}}{(a\cosh(e)+a)^2} \end{cases}$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e))**2,x)
```

output
```
Piecewise((-c*tanh(e/2 + f*x/2)**3/(6*a**2*f) + c*tanh(e/2 + f*x/2)/(2*a**
2*f) - d*x*tanh(e/2 + f*x/2)**3/(6*a**2*f) + d*x*tanh(e/2 + f*x/2)/(2*a**2
*f) - d*x/(3*a**2*f) + 2*d*log(tanh(e/2 + f*x/2) + 1)/(3*a**2*f**2) - d*ta
nh(e/2 + f*x/2)**2/(6*a**2*f**2), Ne(f, 0)), ((c*x + d*x**2/2)/(a*cosh(e)
+ a)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(95) = 190.

Time = 0.05 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.94

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx$$
$$= \frac{2}{3}\,d\left(\frac{fxe^{(3\,fx+3\,e)} + \left(3\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)} + e^{(fx+e)}}{a^2f^2e^{(3\,fx+3\,e)} + 3\,a^2f^2e^{(2\,fx+2\,e)} + 3\,a^2f^2e^{(fx+e)} + a^2f^2} - \frac{\log\left(\left(e^{(fx+e)}+1\right)e^{(-e)}\right)}{a^2f^2}\right)$$
$$+ \frac{2}{3}\,c\left(\frac{3\,e^{(-fx-e)}}{\left(3\,a^2e^{(-fx-e)} + 3\,a^2e^{(-2\,fx-2\,e)} + a^2e^{(-3\,fx-3\,e)} + a^2\right)f} + \frac{1}{\left(3\,a^2e^{(-fx-e)} + 3\,a^2e^{(-2\,fx-2\,e)} + a^2e^{(-3\,fx}\right.}\right.$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2/3*d*((f*x*e^(3*f*x + 3*e) + (3*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + e^(f*x
 + e))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2*f^2*e^(2*f*x + 2*e) + 3*a^2*f^2*e^
(f*x + e) + a^2*f^2) - log((e^(f*x + e) + 1)*e^(-e))/(a^2*f^2)) + 2/3*c*(3
*e^(-f*x - e)/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2*e^(-3*f*
x - 3*e) + a^2)*f) + 1/((3*a^2*e^(-f*x - e) + 3*a^2*e^(-2*f*x - 2*e) + a^2
*e^(-3*f*x - 3*e) + a^2)*f))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(95) = 190$.

Time = 0.11 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.56

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx = \frac{2\left(dfxe^{(3\,fx+3\,e)}+3\,dfxe^{(2\,fx+2\,e)}-3\,cfe^{(fx+e)}-de^{(3\,fx+3\,e)}\log\left(e^{(fx+e)}+1\right)-3\,de^{(2\,fx+2\,e)}\log\left(e^{(fx+e)}+\right.}{3\left(a^2f^2e^{(3\,fx+3\,e)}+3\,a^2f^2e^{(2\,fx+2\,e)}+3\,c\right.}$$

input
```
integrate((d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
2/3*(d*f*x*e^(3*f*x + 3*e) + 3*d*f*x*e^(2*f*x + 2*e) - 3*c*f*e^(f*x + e) -
 d*e^(3*f*x + 3*e)*log(e^(f*x + e) + 1) - 3*d*e^(2*f*x + 2*e)*log(e^(f*x +
 e) + 1) - 3*d*e^(f*x + e)*log(e^(f*x + e) + 1) - c*f + d*e^(2*f*x + 2*e)
+ d*e^(f*x + e) - d*log(e^(f*x + e) + 1))/(a^2*f^2*e^(3*f*x + 3*e) + 3*a^2
*f^2*e^(2*f*x + 2*e) + 3*a^2*f^2*e^(f*x + e) + a^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 1.82 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.12

$$\begin{aligned}\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx &= \frac{2\,d}{3\,a^2\,f^2\,\left(\mathrm{e}^{e+f\,x}+1\right)}-\frac{2\,\left(d+c\,f+d\,f\,x\right)}{3\,a^2\,f^2\,\left(2\,\mathrm{e}^{e+f\,x}+\mathrm{e}^{2\,e+2\,f\,x}+1\right)}\\ &\quad+\frac{2\,d\,x}{3\,a^2\,f}-\frac{2\,d\,\ln\left(\mathrm{e}^{f\,x}\,\mathrm{e}^{e}+1\right)}{3\,a^2\,f^2}\\ &\quad-\frac{4\,\mathrm{e}^{e+f\,x}\,\left(c+d\,x\right)}{3\,a^2\,f\,\left(3\,\mathrm{e}^{e+f\,x}+3\,\mathrm{e}^{2\,e+2\,f\,x}+\mathrm{e}^{3\,e+3\,f\,x}+1\right)}\end{aligned}$$

input
```
int((c + d*x)/(a + a*cosh(e + f*x))^2,x)
```

output
```
(2*d)/(3*a^2*f^2*(exp(e + f*x) + 1)) - (2*(d + c*f + d*f*x))/(3*a^2*f^2*(2
*exp(e + f*x) + exp(2*e + 2*f*x) + 1)) + (2*d*x)/(3*a^2*f) - (2*d*log(exp(
f*x)*exp(e) + 1))/(3*a^2*f^2) - (4*exp(e + f*x)*(c + d*x))/(3*a^2*f*(3*exp
(e + f*x) + 3*exp(2*e + 2*f*x) + exp(3*e + 3*f*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.50

$$\int \frac{c+dx}{(a+a\cosh(e+fx))^2}\,dx$$
$$= \frac{-\frac{2e^{3fx+3e}\log\left(e^{fx+e}+1\right)d}{3}+\frac{2e^{3fx+3e}dfx}{3}-\frac{2e^{3fx+3e}d}{9}-2e^{2fx+2e}\log\left(e^{fx+e}+1\right)d+2e^{2fx+2e}dfx-2e^{fx+e}\log\left(e^{fx+\cdots}\right.}{a^2f^2\left(e^{3fx+3e}+3e^{2fx+2e}+3e^{fx+e}+1\right)}$$

input
```
int((d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

output
```
(2*( - 3*e**(3*e + 3*f*x)*log(e**(e + f*x) + 1)*d + 3*e**(3*e + 3*f*x)*d*f
*x - e**(3*e + 3*f*x)*d - 9*e**(2*e + 2*f*x)*log(e**(e + f*x) + 1)*d + 9*e
**(2*e + 2*f*x)*d*f*x - 9*e**(e + f*x)*log(e**(e + f*x) + 1)*d - 9*e**(e +
 f*x)*c*f - 3*log(e**(e + f*x) + 1)*d - 3*c*f - d))/(9*a**2*f**2*(e**(3*e
+ 3*f*x) + 3*e**(2*e + 2*f*x) + 3*e**(e + f*x) + 1))
```

3.119 $\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+a\cosh(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + a*Cosh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\cosh(e+fx)+a)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)(a\cosh(e+fx)+a)^2}dx$$

input
```
Int[1/((c + d*x)*(a + a*Cosh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.42 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+a\cosh\left(fx+e\right)\right)^2}dx$$

input
```
int(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 2.85

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (a^2*d*x + a^2*c)*cosh(f*x + e)^2 + 2*(a^2*d
*x + a^2*c)*cosh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.75 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.70

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \frac{\int \frac{1}{c\cosh^2(e+fx)+2c\cosh(e+fx)+c+dx\cosh^2(e+fx)+2dx\cosh(e+fx)+dx}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e))**2,x)
```

output
```
Integral(1/(c*cosh(e + f*x)**2 + 2*c*cosh(e + f*x) + c + d*x*cosh(e + f*x)
**2 + 2*d*x*cosh(e + f*x) + d*x), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 593, normalized size of antiderivative = 29.65

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2/3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 2*d^2 + (d^2*f*x*e^(2*e) + c*d
*f*e^(2*e) - 2*d^2*e^(2*e))*e^(2*f*x) + (3*d^2*f^2*x^2*e^e + 3*c^2*f^2*e^e
 + c*d*f*e^e - 4*d^2*e^e + (6*c*d*f^2*e^e + d^2*f*e^e)*x)*e^(f*x))/(a^2*d^
3*f^3*x^3 + 3*a^2*c*d^2*f^3*x^2 + 3*a^2*c^2*d*f^3*x + a^2*c^3*f^3 + (a^2*d
^3*f^3*x^3*e^(3*e) + 3*a^2*c*d^2*f^3*x^2*e^(3*e) + 3*a^2*c^2*d*f^3*x*e^(3*
e) + a^2*c^3*f^3*e^(3*e))*e^(3*f*x) + 3*(a^2*d^3*f^3*x^3*e^(2*e) + 3*a^2*c
*d^2*f^3*x^2*e^(2*e) + 3*a^2*c^2*d*f^3*x*e^(2*e) + a^2*c^3*f^3*e^(2*e))*e^
(2*f*x) + 3*(a^2*d^3*f^3*x^3*e^e + 3*a^2*c*d^2*f^3*x^2*e^e + 3*a^2*c^2*d*f
^3*x*e^e + a^2*c^3*f^3*e^e)*e^(f*x)) - integrate(2/3*(d^3*f^2*x^2 + 2*c*d^
2*f^2*x + c^2*d*f^2 - 6*d^3)/(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^
2*c^2*d^2*f^3*x^2 + 4*a^2*c^3*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4*e^e
 + 4*a^2*c*d^3*f^3*x^3*e^e + 6*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x
*e^e + a^2*c^4*f^3*e^e)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(a*cosh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{\left(a+a\cosh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((a + a*cosh(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + a*cosh(e + f*x))^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.65

$$\int \frac{1}{(c+dx)(a+a\cosh(e+fx))^2}\,dx = \frac{\int \frac{1}{\cosh(fx+e)^2c+\cosh(fx+e)^2dx+2\cosh(fx+e)c+2\cosh(fx+e)dx+c+dx}dx}{a^2}$$

input
```
int(1/(d*x+c)/(a+a*cosh(f*x+e))^2,x)
```

output
```
int(1/(cosh(e + f*x)**2*c + cosh(e + f*x)**2*d*x + 2*cosh(e + f*x)*c + 2*c
osh(e + f*x)*d*x + c + d*x),x)/a**2
```

3.120 $\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 21.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\cosh(e+fx)+a)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)^2(a\cosh(e+fx)+a)^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + a*Cosh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\left(a+a\cosh\left(fx+e\right)\right)^2}dx$$

input
```
int(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 99, normalized size of antiderivative = 4.95

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (a^2*d^2*x^2 + 2*a^2*c*d
*x + a^2*c^2)*cosh(f*x + e)^2 + 2*(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2)*co
sh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 4.32 (sec) , antiderivative size = 105, normalized size of antiderivative = 5.25

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx$$
$$= \frac{\int \frac{1}{c^2\cosh^2(e+fx)+2c^2\cosh(e+fx)+c^2+2cdx\cosh^2(e+fx)+4cdx\cosh(e+fx)+2cdx+d^2x^2\cosh^2(e+fx)+2d^2x^2\cosh(e+fx)+d^2x^2}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)**2/(a+a*cosh(f*x+e))**2,x)
```

output
```
Integral(1/(c**2*cosh(e + f*x)**2 + 2*c**2*cosh(e + f*x) + c**2 + 2*c*d*x*
cosh(e + f*x)**2 + 4*c*d*x*cosh(e + f*x) + 2*c*d*x + d**2*x**2*cosh(e + f*
x)**2 + 2*d**2*x**2*cosh(e + f*x) + d**2*x**2), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 710, normalized size of antiderivative = 35.50

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-2/3*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 - 6*d^2 + 2*(d^2*f*x*e^(2*e) + c
*d*f*e^(2*e) - 3*d^2*e^(2*e))*e^(2*f*x) + (3*d^2*f^2*x^2*e^e + 3*c^2*f^2*e
^e + 2*c*d*f*e^e - 12*d^2*e^e + 2*(3*c*d*f^2*e^e + d^2*f*e^e)*x)*e^(f*x))/
(a^2*d^4*f^3*x^4 + 4*a^2*c*d^3*f^3*x^3 + 6*a^2*c^2*d^2*f^3*x^2 + 4*a^2*c^3
*d*f^3*x + a^2*c^4*f^3 + (a^2*d^4*f^3*x^4*e^(3*e) + 4*a^2*c*d^3*f^3*x^3*e^
(3*e) + 6*a^2*c^2*d^2*f^3*x^2*e^(3*e) + 4*a^2*c^3*d*f^3*x*e^(3*e) + a^2*c^
4*f^3*e^(3*e))*e^(3*f*x) + 3*(a^2*d^4*f^3*x^4*e^(2*e) + 4*a^2*c*d^3*f^3*x^
3*e^(2*e) + 6*a^2*c^2*d^2*f^3*x^2*e^(2*e) + 4*a^2*c^3*d*f^3*x*e^(2*e) + a^
2*c^4*f^3*e^(2*e))*e^(2*f*x) + 3*(a^2*d^4*f^3*x^4*e^e + 4*a^2*c*d^3*f^3*x^
3*e^e + 6*a^2*c^2*d^2*f^3*x^2*e^e + 4*a^2*c^3*d*f^3*x*e^e + a^2*c^4*f^3*e^
e)*e^(f*x)) - integrate(4/3*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f^2 - 12*
d^3)/(a^2*d^5*f^3*x^5 + 5*a^2*c*d^4*f^3*x^4 + 10*a^2*c^2*d^3*f^3*x^3 + 10*
a^2*c^3*d^2*f^3*x^2 + 5*a^2*c^4*d*f^3*x + a^2*c^5*f^3 + (a^2*d^5*f^3*x^5*e
^e + 5*a^2*c*d^4*f^3*x^4*e^e + 10*a^2*c^2*d^3*f^3*x^3*e^e + 10*a^2*c^3*d^2
*f^3*x^2*e^e + 5*a^2*c^4*d*f^3*x*e^e + a^2*c^5*f^3*e^e)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(a*cosh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \int \frac{1}{\left(a+a\cosh\left(e+f\,x\right)\right)^2\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/((a + a*cosh(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + a*cosh(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 100, normalized size of antiderivative = 5.00

$$\int \frac{1}{(c+dx)^2(a+a\cosh(e+fx))^2}\,dx = \frac{\int \frac{1}{\cosh(fx+e)^2c^2+2\cosh(fx+e)^2cdx+\cosh(fx+e)^2d^2x^2+2\cosh(fx+e)c^2+4\cosh(fx+e)cdx+2\cosh(fx+e)d^2x^2+c^2+2cdx+d^2x^2}dx}{a^2}$$

input
```
int(1/(d*x+c)^2/(a+a*cosh(f*x+e))^2,x)
```

output
```
int(1/(cosh(e + f*x)**2*c**2 + 2*cosh(e + f*x)**2*c*d*x + cosh(e + f*x)**2
*d**2*x**2 + 2*cosh(e + f*x)*c**2 + 4*cosh(e + f*x)*c*d*x + 2*cosh(e + f*x
)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)/a**2
```

3.121 $\int x^3 \sqrt{a+a\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 110

$$\int x^3 \sqrt{a+a\cosh(c+dx)}\,dx = -\frac{96\sqrt{a+a\cosh(c+dx)}}{d^4} - \frac{12x^2\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{48x\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3} + \frac{2x^3\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

output

```
-96*(a+a*cosh(d*x+c))^(1/2)/d^4-12*x^2*(a+a*cosh(d*x+c))^(1/2)/d^2+48*x*(a
+a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1/2*c)/d^3+2*x^3*(a+a*cosh(d*x+c))^(1/2
)*tanh(1/2*d*x+1/2*c)/d
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.48

$$\int x^3 \sqrt{a+a\cosh(c+dx)}\,dx = \frac{2\sqrt{a(1+\cosh(c+dx))}\left(-6(8+d^2x^2)+dx(24+d^2x^2)\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d^4}$$

input
```
Integrate[x^3*Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cosh[c + d*x])]*(-6*(8 + d^2*x^2) + d*x*(24 + d^2*x^2)*Tanh
[(c + d*x)/2]))/d^4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.778$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a\cosh(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int x^3\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int x^3\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6i\int -ix^2\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\int x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{6\int -ix^2\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\int x^2\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3777}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)$$

$$\Big\downarrow \text{3777}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2i\int -i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}\right.}{d}\right.}{d}\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 3042

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int-i\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)}{d}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 26

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)dx}{d}\right.}{d}\right.}{d}\right.$$

$\downarrow$ 3118

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^3\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{6i\left(\frac{2ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2}\right)}{d}\right)}{d}\right.$$

input
```
Int[x^3*Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*((2*x^3*Sinh[c/2 + (d*x)/2])
/d + ((6*I)*(((2*I)*x^2*Cosh[c/2 + (d*x)/2])/d - ((4*I)*((-4*Cosh[c/2 + (d
*x)/2])/d^2 + (2*x*Sinh[c/2 + (d*x)/2])/d))/d))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{d x+c}+1\right)^{2} \mathrm{e}^{-d x-c}}\,\left(d^{3} x^{3} \mathrm{e}^{d x+c}-d^{3} x^{3}-6 d^{2} x^{2} \mathrm{e}^{d x+c}-6 x^{2} d^{2}+24 d x\,\mathrm{e}^{d x+c}-24 d x-48\,\mathrm{e}^{d x+c}-48\right)}{\left(\mathrm{e}^{d x+c}+1\right) d^{4}}$	108

input
```
int(x^3*(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d^3*x^3*exp
(d*x+c)-d^3*x^3-6*d^2*x^2*exp(d*x+c)-6*x^2*d^2+24*d*x*exp(d*x+c)-24*d*x-48
*exp(d*x+c)-48)/d^4
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx = \int x^3\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

input
```
integrate(x**3*(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.09

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx = -\frac{\left(\sqrt{2}\sqrt{a}d^3x^3+6\sqrt{2}\sqrt{a}d^2x^2+24\sqrt{2}\sqrt{a}dx-\left(\sqrt{2}\sqrt{a}d^3x^3e^c-6\sqrt{2}\sqrt{a}d^2x^2e^c+24\sqrt{2}\sqrt{a}dxe^c-48\sqrt{2}\right.\right.}{d^4}$$

input
```
integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*d^3*x^3 + 6*sqrt(2)*sqrt(a)*d^2*x^2 + 24*sqrt(2)*sqrt(a)
*d*x - (sqrt(2)*sqrt(a)*d^3*x^3*e^c - 6*sqrt(2)*sqrt(a)*d^2*x^2*e^c + 24*s
qrt(2)*sqrt(a)*d*x*e^c - 48*sqrt(2)*sqrt(a)*e^c)*e^(d*x) + 48*sqrt(2)*sqrt
(a))*e^(-1/2*d*x - 1/2*c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.34

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx = \frac{\sqrt{2}\left(\sqrt{a}d^3x^3e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-\sqrt{a}d^3x^3e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}-6\sqrt{a}d^2x^2e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)}-6\sqrt{a}d^2x^2e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}+24\sqrt{a}dxe^{\left(\frac{1}{2}\right.}\right.}{d^4}$$

input
```
integrate(x^3*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(a)*d^3*x^3*e^(1/2*d*x + 1/2*c) - sqrt(a)*d^3*x^3*e^(-1/2*d*x
 - 1/2*c) - 6*sqrt(a)*d^2*x^2*e^(1/2*d*x + 1/2*c) - 6*sqrt(a)*d^2*x^2*e^(-
1/2*d*x - 1/2*c) + 24*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - 24*sqrt(a)*d*x*e^(
-1/2*d*x - 1/2*c) - 48*sqrt(a)*e^(1/2*d*x + 1/2*c) - 48*sqrt(a)*e^(-1/2*d*
x - 1/2*c))/d^4
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.06

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx =$$
$$-\frac{\sqrt{a+a\left(\frac{e^{c+dx}}{2}+\frac{e^{-c-dx}}{2}\right)}\left(\frac{96\,e^{c+dx}}{d^4}+\frac{48\,x}{d^3}+\frac{96}{d^4}+\frac{2\,x^3}{d}+\frac{12\,x^2}{d^2}-\frac{2\,x^3\,e^{c+dx}}{d}+\frac{12\,x^2\,e^{c+dx}}{d^2}-\frac{48\,x\,e^{c+dx}}{d^3}\right)}{e^{c+dx}+1}$$

input
```
int(x^3*(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
-((a + a*(exp(c + d*x)/2 + exp(- c - d*x)/2))^(1/2)*((96*exp(c + d*x))/d^4
 + (48*x)/d^3 + 96/d^4 + (2*x^3)/d + (12*x^2)/d^2 - (2*x^3*exp(c + d*x))/d
 + (12*x^2*exp(c + d*x))/d^2 - (48*x*exp(c + d*x))/d^3))/(exp(c + d*x) + 1
)
```

Reduce [F]

$$\int x^3\sqrt{a+a\cosh(c+dx)}\,dx = \sqrt{a}\left(\int\sqrt{\cosh(dx+c)+1}\,x^3dx\right)$$

input
```
int(x^3*(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(c + d*x) + 1)*x**3,x)
```

3.122 $\int x^2\sqrt{a+a\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 88

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx=-\frac{8x\sqrt{a+a\cosh(c+dx)}}{d^2}+\frac{16\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^3}+\frac{2x^2\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

output
```
-8*x*(a+a*cosh(d*x+c))^(1/2)/d^2+16*(a+a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1
/2*c)/d^3+2*x^2*(a+a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1/2*c)/d
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.50

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx=\frac{2\sqrt{a(1+\cosh(c+dx))}\left(-4dx+(8+d^2x^2)\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d^3}$$

input
```
Integrate[x^2*Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cosh[c + d*x])]*(-4*d*x + (8 + d^2*x^2)*Tanh[(c + d*x)/2]))
/d^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.556$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2 \sqrt{a \cosh(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2 \sqrt{a+a \sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a \cosh(c+dx)+a}\int x^2 \cosh\left(\frac{c}{2}+\frac{dx}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a \cosh(c+dx)+a}\int x^2 \sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a \cosh(c+dx)+a}\left(\frac{2x^2 \sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\int -ix \sinh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a \cosh(c+dx)+a}\left(\frac{2x^2 \sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\int x \sinh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\int -ix\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \; 26$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{4i\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \; 3777$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{4i\left(\frac{2ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2i\int\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)}{d}\right)$$

$$\Big\downarrow \; 3042$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{4i\left(\frac{2ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2i\int\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)$$

$$\Big\downarrow \; 3117$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x^2\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{4i\left(\frac{2ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2}\right)}{d}\right)$$

input
```
Int[x^2*Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*((2*x^2*Sinh[c/2 + (d*x)/2])
/d + ((4*I)*(((2*I)*x*Cosh[c/2 + (d*x)/2])/d - ((4*I)*Sinh[c/2 + (d*x)/2])
/d^2))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(\mathrm{e}^{dx+c}+1\right)^{2}\mathrm{e}^{-dx-c}}\,\left(d^{2}x^{2}\mathrm{e}^{dx+c}-x^{2}d^{2}-4dx\,\mathrm{e}^{dx+c}-4dx+8\,\mathrm{e}^{dx+c}-8\right)}{\left(\mathrm{e}^{dx+c}+1\right)d^{3}}$	86

input
```
int(x^2*(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d^2*x^2*exp
(d*x+c)-x^2*d^2-4*d*x*exp(d*x+c)-4*d*x+8*exp(d*x+c)-8)/d^3
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx = \int x^2\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

input
```
integrate(x**2*(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx = -\frac{\left(\sqrt{2}\sqrt{a}d^2x^2+4\sqrt{2}\sqrt{a}dx-\left(\sqrt{2}\sqrt{a}d^2x^2e^c-4\sqrt{2}\sqrt{a}dxe^c+8\sqrt{2}\sqrt{a}e^c\right)e^{(dx)}+8\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{d^3}$$

input
```
integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*d^2*x^2 + 4*sqrt(2)*sqrt(a)*d*x - (sqrt(2)*sqrt(a)*d^2*x
^2*e^c - 4*sqrt(2)*sqrt(a)*d*x*e^c + 8*sqrt(2)*sqrt(a)*e^c)*e^(d*x) + 8*sq
rt(2)*sqrt(a))*e^(-1/2*d*x - 1/2*c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx$$
$$=\frac{\sqrt{2}\Big(\sqrt{a}d^2x^2e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-\sqrt{a}d^2x^2e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}-4\,\sqrt{a}dxe^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}-4\,\sqrt{a}dxe^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}+8\,\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d^3}$$

input
```
integrate(x^2*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(a)*d^2*x^2*e^(1/2*d*x + 1/2*c) - sqrt(a)*d^2*x^2*e^(-1/2*d*x
 - 1/2*c) - 4*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - 4*sqrt(a)*d*x*e^(-1/2*d*x
- 1/2*c) + 8*sqrt(a)*e^(1/2*d*x + 1/2*c) - 8*sqrt(a)*e^(-1/2*d*x - 1/2*c))
/d^3
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx$$
$$=-\frac{\sqrt{a+a\,\left(\frac{\mathrm{e}^{c+d\,x}}{2}+\frac{\mathrm{e}^{-c-d\,x}}{2}\right)}\,\left(\frac{8\,x}{d^2}-\frac{16\,\mathrm{e}^{c+d\,x}}{d^3}+\frac{16}{d^3}+\frac{2\,x^2}{d}-\frac{2\,x^2\,\mathrm{e}^{c+d\,x}}{d}+\frac{8\,x\,\mathrm{e}^{c+d\,x}}{d^2}\right)}{\mathrm{e}^{c+d\,x}+1}$$

input
```
int(x^2*(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
-((a + a*(exp(c + d*x)/2 + exp(- c - d*x)/2))^(1/2)*((8*x)/d^2 - (16*exp(c
 + d*x))/d^3 + 16/d^3 + (2*x^2)/d - (2*x^2*exp(c + d*x))/d + (8*x*exp(c +
d*x))/d^2))/(exp(c + d*x) + 1)
```

Reduce [F]

$$\int x^2\sqrt{a+a\cosh(c+dx)}\,dx=\sqrt{a}\left(\int\sqrt{\cosh(dx+c)+1}\,x^2dx\right)$$

input
```
int(x^2*(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(c + d*x) + 1)*x**2,x)
```

3.123 $\int x\sqrt{a+a\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 53

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx = -\frac{4\sqrt{a+a\cosh(c+dx)}}{d^2} + \frac{2x\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}$$

output
```
-4*(a+a*cosh(d*x+c))^(1/2)/d^2+2*x*(a+a*cosh(d*x+c))^(1/2)*tanh(1/2*d*x+1/
2*c)/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.64

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx = \frac{2\sqrt{a(1+\cosh(c+dx))}\left(-2+dx\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d^2}$$

input
```
Integrate[x*Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cosh[c + d*x])]*(-2 + d*x*Tanh[(c + d*x)/2]))/d^2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a\cosh(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int x\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int x\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2i\int -i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int \sinh\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{2\int -i\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}+\frac{2i\int \sin\left(\frac{ic}{2}+\frac{idx}{2}\right)dx}{d}\right)$$

$$\Big\downarrow \text{3118}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{2x\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{4\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d^2}\right)$$

input
```
Int[x*Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*((-4*Cosh[c/2 + (d*x)/2])/d^
2 + (2*x*Sinh[c/2 + (d*x)/2])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.21

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(e^{dx+c}+1\right)^{2} e^{-dx-c}}\,\left(dx\,e^{dx+c}-dx-2\,e^{dx+c}-2\right)}{\left(e^{dx+c}+1\right) d^{2}}$	64

input
```
int(x*(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(a*(exp(d*x+c)+1)^2*exp(-d*x-c))^(1/2)/(exp(d*x+c)+1)*(d*x*exp(d*x
+c)-d*x-2*exp(d*x+c)-2)/d^2
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx = \int x\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}\,dx$$

input
```
integrate(x*(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(x*sqrt(a*(cosh(c + d*x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.13

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx$$
$$= -\frac{\left(\sqrt{2}\sqrt{a}dx - \left(\sqrt{2}\sqrt{a}dxe^{c} - 2\sqrt{2}\sqrt{a}e^{c}\right)e^{(dx)} + 2\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}}{d^2}$$

input
```
integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*d*x - (sqrt(2)*sqrt(a)*d*x*e^c - 2*sqrt(2)*sqrt(a)*e^c)*
e^(d*x) + 2*sqrt(2)*sqrt(a))*e^(-1/2*d*x - 1/2*c)/d^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx$$
$$= \frac{\sqrt{2}\left(\sqrt{a}dxe^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - \sqrt{a}dxe^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)} - 2\sqrt{a}e^{\left(\frac{1}{2}dx+\frac{1}{2}c\right)} - 2\sqrt{a}e^{\left(-\frac{1}{2}dx-\frac{1}{2}c\right)}\right)}{d^2}$$

input
```
integrate(x*(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
sqrt(2)*(sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) - sqrt(a)*d*x*e^(-1/2*d*x - 1/2*c
) - 2*sqrt(a)*e^(1/2*d*x + 1/2*c) - 2*sqrt(a)*e^(-1/2*d*x - 1/2*c))/d^2
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx = \frac{2\,x\sinh\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\cosh\left(c+d\,x\right)}}{d\cosh\left(\frac{c}{2}+\frac{d\,x}{2}\right)} - \frac{4\,\sqrt{a+a\cosh\left(c+d\,x\right)}}{d^2}$$

input
```
int(x*(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
(2*x*sinh(c/2 + (d*x)/2)*(a + a*cosh(c + d*x))^(1/2))/(d*cosh(c/2 + (d*x)/
2)) - (4*(a + a*cosh(c + d*x))^(1/2))/d^2
```

Reduce [F]

$$\int x\sqrt{a+a\cosh(c+dx)}\,dx = \sqrt{a}\left(\int\sqrt{\cosh\left(dx+c\right)+1}\,xdx\right)$$

input
```
int(x*(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(c + d*x) + 1)*x,x)
```

3.124 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 83

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx = \cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\operatorname{Chi}\left(\frac{dx}{2}\right)\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right) + \sqrt{a+a\cosh(c+dx)}\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}\right)\operatorname{Shi}\left(\frac{dx}{2}\right)$$

output
```
cosh(1/2*c)*(a+a*cosh(d*x+c))^(1/2)*Chi(1/2*d*x)*sech(1/2*d*x+1/2*c)+(a+a*
cosh(d*x+c))^(1/2)*sech(1/2*d*x+1/2*c)*sinh(1/2*c)*Shi(1/2*d*x)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.65

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx = \sqrt{a(1+\cosh(c+dx))}\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(\cosh\left(\frac{c}{2}\right)\operatorname{Chi}\left(\frac{dx}{2}\right)+\sinh\left(\frac{c}{2}\right)\operatorname{Shi}\left(\frac{dx}{2}\right)\right)$$

input
```
Integrate[Sqrt[a + a*Cosh[c + d*x]]/x,x]
```

output
```
Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(Cosh[c/2]*CoshIntegral[(d*x
)/2] + Sinh[c/2]*SinhIntegral[(d*x)/2])
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.69, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3800, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cosh(c+dx)+a}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int \frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int \frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3784}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\cosh\left(\frac{c}{2}\right)\int \frac{\cosh\left(\frac{dx}{2}\right)}{x}dx-i\sinh\left(\frac{c}{2}\right)\int \frac{i\sinh\left(\frac{dx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\sinh\left(\frac{c}{2}\right)\int \frac{\sinh\left(\frac{dx}{2}\right)}{x}dx+\cosh\left(\frac{c}{2}\right)\int \frac{\cosh\left(\frac{dx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\sinh\left(\frac{c}{2}\right)\int-\frac{i\sin\left(\frac{idx}{2}\right)}{x}dx+\cosh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx\right)$$

$\big\downarrow$ 26

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\cosh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx-i\sinh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}\right)}{x}dx\right)$$

$\big\downarrow$ 3779

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\sinh\left(\frac{c}{2}\right)\operatorname{Shi}\left(\frac{dx}{2}\right)+\cosh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx\right)$$

$\big\downarrow$ 3782

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\cosh\left(\frac{c}{2}\right)\operatorname{Chi}\left(\frac{dx}{2}\right)+\sinh\left(\frac{c}{2}\right)\operatorname{Shi}\left(\frac{dx}{2}\right)\right)$$

input
```
Int[Sqrt[a + a*Cosh[c + d*x]]/x,x]
```

output
```
Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*(Cosh[c/2]*CoshIntegral[(d*x
)/2] + Sinh[c/2]*SinhIntegral[(d*x)/2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + a\cosh\left(dx + c\right)}}{x}dx$$

input
```
int((a+a*cosh(d*x+c))^(1/2)/x,x)
```

output
```
int((a+a*cosh(d*x+c))^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x}\, dx = \int \frac{\sqrt{a\,(\cosh{(c + dx)} + 1)}}{x}\, dx$$

input
```
integrate((a+a*cosh(d*x+c))**(1/2)/x,x)
```

output
```
Integral(sqrt(a*(cosh(c + d*x) + 1))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x}\, dx = \int \frac{\sqrt{a\cosh{(dx + c)} + a}}{x}\, dx$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(d*x + c) + a)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.39

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x}\, dx = \frac{1}{2}\,\sqrt{2}\left(\sqrt{a}\mathrm{Ei}\left(\frac{1}{2}\,dx\right)e^{\left(\frac{1}{2}\,c\right)} + \sqrt{a}\mathrm{Ei}\left(-\frac{1}{2}\,dx\right)e^{\left(-\frac{1}{2}\,c\right)}\right)$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x,x, algorithm="giac")
```

output
```
1/2*sqrt(2)*(sqrt(a)*Ei(1/2*d*x)*e^(1/2*c) + sqrt(a)*Ei(-1/2*d*x)*e^(-1/2*
c))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx=\int \frac{\sqrt{a+a\cosh\left(c+d\,x\right)}}{x}\,dx$$

input
```
int((a + a*cosh(c + d*x))^(1/2)/x,x)
```

output
```
int((a + a*cosh(c + d*x))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\cosh\left(dx+c\right)+1}}{x}dx\right)$$

input
```
int((a+a*cosh(d*x+c))^(1/2)/x,x)
```

output
```
sqrt(a)*int(sqrt(cosh(c + d*x) + 1)/x,x)
```

3.125 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 110

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx = -\frac{\sqrt{a+a\cosh(c+dx)}}{x} + \frac{1}{2}d\sqrt{a+a\cosh(c+dx)}\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}\right) + \frac{1}{2}d\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)$$

output
```
-(a+a*cosh(d*x+c))^(1/2)/x+1/2*d*(a+a*cosh(d*x+c))^(1/2)*Chi(1/2*d*x)*sech
(1/2*d*x+1/2*c)*sinh(1/2*c)+1/2*d*cosh(1/2*c)*(a+a*cosh(d*x+c))^(1/2)*sech
(1/2*d*x+1/2*c)*Shi(1/2*d*x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.68

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx$$
$$= \frac{\sqrt{a(1+\cosh(c+dx))}\left(-2+dx\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)\sinh\left(\frac{c}{2}\right)+dx\cosh\left(\frac{c}{2}\right)\text{sech}\left(\frac{1}{2}(c+dx)\right)\text{Shi}\left(\frac{dx}{2}\right)\right)}{2x}$$

input
```
Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^2,x]
```

output
```
(Sqrt[a*(1 + Cosh[c + d*x])]*(-2 + d*x*CoshIntegral[(d*x)/2]*Sech[(c + d*x
)/2]*Sinh[c/2] + d*x*Cosh[c/2]*Sech[(c + d*x)/2]*SinhIntegral[(d*x)/2]))/(
2*x)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.81, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.722, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cosh(c+dx)+a}}{x^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}{x^2}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int \frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \; 3778$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}+\frac{1}{2}id\int-\frac{i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \; 26$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{1}{2}d\int\frac{\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)$$

$$\Big\downarrow \; 3042$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}+\frac{1}{2}d\int-\frac{i\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \; 26$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}-\frac{1}{2}id\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \; 3784$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}-\frac{1}{2}id\left(i\sinh\left(\frac{c}{2}\right)\int\frac{\cosh\left(\frac{dx}{2}\right)}{x}dx+\cosh\left(\frac{c}{2}\right)\int\frac{i\sinh\left(\right.}{x}\right.\right.$$

$$\Big\downarrow \; 26$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}-\frac{1}{2}id\left(i\sinh\left(\frac{c}{2}\right)\int\frac{\cosh\left(\frac{dx}{2}\right)}{x}dx+i\cosh\left(\frac{c}{2}\right)\int\frac{\sinh\left(\right.}{x}\right.\right.$$

$$\Big\downarrow \; 3042$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}-\frac{1}{2}id\left(i\sinh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{c}{2}\right)\int-\frac{i}{}\right.\right.$$

$$\Big\downarrow \; 26$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}-\frac{1}{2}id\left(i\sinh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx+\cosh\left(\frac{c}{2}\right)\int\frac{\sin\left(\right.}{}\right.\right.$$

$$\Big\downarrow \; 3779$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}-\frac{1}{2}id\left(i\sinh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx+i\cosh\left(\frac{c}{2}\right)\text{Shi}\right.\right.$$

$$\Big\downarrow \color{blue}{3782}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}-\frac{1}{2}id\left(i\sinh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)+i\cosh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\right)\right)$$

input
```
Int[Sqrt[a + a*Cosh[c + d*x]]/x^2,x]
```

output
```
Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*(-(Cosh[c/2 + (d*x)/2]/x) -
(I/2)*d*(I*CoshIntegral[(d*x)/2]*Sinh[c/2] + I*Cosh[c/2]*SinhIntegral[(d*x
)/2]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + a\cosh\left(dx + c\right)}}{x^2}dx$$

input
```
int((a+a*cosh(d*x+c))^(1/2)/x^2,x)
```

output
```
int((a+a*cosh(d*x+c))^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^2}\,dx = \int \frac{\sqrt{a\left(\cosh\left(c + dx\right) + 1\right)}}{x^2}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*(cosh(c + d*x) + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^2}\,dx = \int \frac{\sqrt{a\cosh\left(dx + c\right) + a}}{x^2}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(d*x + c) + a)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.62

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^2}\,dx = \frac{\sqrt{2}\left(\sqrt{a}dx\mathrm{Ei}\left(\frac{1}{2}\,dx\right)e^{\left(\frac{1}{2}\,c\right)} - \sqrt{a}dx\mathrm{Ei}\left(-\frac{1}{2}\,dx\right)e^{\left(-\frac{1}{2}\,c\right)} - 2\,\sqrt{a}e^{\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)} - 2\,\sqrt{a}e^{\left(-\frac{1}{2}\,dx - \frac{1}{2}\,c\right)}\right)}{4\,x}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(sqrt(a)*d*x*Ei(1/2*d*x)*e^(1/2*c) - sqrt(a)*d*x*Ei(-1/2*d*x)*
e^(-1/2*c) - 2*sqrt(a)*e^(1/2*d*x + 1/2*c) - 2*sqrt(a)*e^(-1/2*d*x - 1/2*c
))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx=\int \frac{\sqrt{a+a\cosh\left(c+d\,x\right)}}{x^2}\,dx$$

input
```
int((a + a*cosh(c + d*x))^(1/2)/x^2,x)
```

output
```
int((a + a*cosh(c + d*x))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^2}\,dx$$
$$=\frac{\sqrt{a}\left(-2\sqrt{\cosh\left(dx+c\right)+1}+\left(\int \frac{\sqrt{\cosh(dx+c)+1}\,\sinh(dx+c)}{\cosh(dx+c)x+x}dx\right)dx\right)}{2x}$$

input
```
int((a+a*cosh(d*x+c))^(1/2)/x^2,x)
```

output
```
(sqrt(a)*( - 2*sqrt(cosh(c + d*x) + 1) + int((sqrt(cosh(c + d*x) + 1)*sinh
(c + d*x))/(cosh(c + d*x)*x + x),x)*d*x))/(2*x)
```

3.126 $\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^3} dx$

Optimal result

Integrand size = 18, antiderivative size = 151

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^3} dx = -\frac{\sqrt{a+a\cosh(c+dx)}}{2x^2} + \frac{1}{8}d^2\cosh\left(\frac{c}{2}\right)\sqrt{a+a\cosh(c+dx)}\text{Chi}\left(\frac{dx}{2}\right)\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right) + \frac{1}{8}d^2\sqrt{a+a\cosh(c+dx)}\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right) - \frac{d\sqrt{a+a\cosh(c+dx)}\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4x}$$

output
```
-1/2*(a+a*cosh(d*x+c))^(1/2)/x^2+1/8*d^2*cosh(1/2*c)*(a+a*cosh(d*x+c))^(1/
2)*Chi(1/2*d*x)*sech(1/2*d*x+1/2*c)+1/8*d^2*(a+a*cosh(d*x+c))^(1/2)*sech(1
/2*d*x+1/2*c)*sinh(1/2*c)*Shi(1/2*d*x)-1/4*d*(a+a*cosh(d*x+c))^(1/2)*tanh(
1/2*d*x+1/2*c)/x
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.64

$$\int \frac{\sqrt{a+a\cosh(c+dx)}}{x^3}\,dx = \frac{\sqrt{a(1+\cosh(c+dx))}\left(-4+d^2x^2\cosh\left(\frac{c}{2}\right)\operatorname{Chi}\left(\frac{dx}{2}\right)\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)+d^2x^2\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\sinh\left(\frac{c}{2}\right)\operatorname{Shi}\right.}{8x^2}$$

input
```
Integrate[Sqrt[a + a*Cosh[c + d*x]]/x^3,x]
```

output
```
(Sqrt[a*(1 + Cosh[c + d*x])]*(-4 + d^2*x^2*Cosh[c/2]*CoshIntegral[(d*x)/2]
*Sech[(c + d*x)/2] + d^2*x^2*Sech[(c + d*x)/2]*Sinh[c/2]*SinhIntegral[(d*x
)/2] - 2*d*x*Tanh[(c + d*x)/2]))/(8*x^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.68 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.75, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cosh(c+dx)+a}}{x^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}{x^3}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int \frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}+\frac{1}{4}id\int-\frac{i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(\frac{1}{4}d\int\frac{\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x^2}dx-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}+\frac{1}{4}d\int-\frac{i\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)}{x^2}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}\right)}{x^2}dx\right)$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\int\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}dx-\frac{i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\int\frac{\sin\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)}{x}\right)\right)$$

$$\Big\downarrow \text{3784}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\left(\cosh\left(\frac{c}{2}\right)\int\frac{\cosh\left(\frac{dx}{2}\right)}{x}dx-i\sinh\left(\frac{c}{2}\right)\int\frac{i\,s\ldots}{}\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\left(\sinh\left(\frac{c}{2}\right)\int\frac{\sinh\left(\frac{dx}{2}\right)}{x}dx+\cosh\left(\frac{c}{2}\right)\int\frac{\cos\ldots}{}\right.\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\left(\sinh\left(\frac{c}{2}\right)\int-\frac{i\sin\left(\frac{idx}{2}\right)}{x}dx+\cosh\left(\frac{c}{2}\right)\int\frac{\text{s}}{}\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\left(\cosh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}+\frac{\pi}{2}\right)}{x}dx-i\sinh\left(\frac{c}{2}\right)\int\right.\right.\right.$$

$$\Big\downarrow \text{3779}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\left(\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)+\cosh\left(\frac{c}{2}\right)\int\frac{\sin\left(\frac{idx}{2}\right.}{x}\right.\right.\right.$$

$$\Big\downarrow \text{3782}$$

$$\operatorname{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a\cosh(c+dx)+a}\left(-\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2x^2}-\frac{1}{4}id\left(\frac{1}{2}id\left(\cosh\left(\frac{c}{2}\right)\text{Chi}\left(\frac{dx}{2}\right)+\sinh\left(\frac{c}{2}\right)\text{Shi}\left(\frac{dx}{2}\right)\right)\right.\right.$$

input

```
Int[Sqrt[a + a*Cosh[c + d*x]]/x^3,x]
```

output

```
Sqrt[a + a*Cosh[c + d*x]]*Sech[c/2 + (d*x)/2]*(-1/2*Cosh[c/2 + (d*x)/2]/x^
2 - (I/4)*d*(((-I)*Sinh[c/2 + (d*x)/2])/x + (I/2)*d*(Cosh[c/2]*CoshIntegra
l[(d*x)/2] + Sinh[c/2]*SinhIntegral[(d*x)/2])))
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a + a\cosh(dx + c)}}{x^3}dx$$

input
```
int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

output
```
int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^3}\,dx = \int \frac{\sqrt{a\left(\cosh\left(c + dx\right) + 1\right)}}{x^3}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*(cosh(c + d*x) + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^3}\,dx = \int \frac{\sqrt{a\cosh\left(dx + c\right) + a}}{x^3}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(d*x + c) + a)/x^3, x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.71

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^3}\, dx$$
$$= \frac{\sqrt{2}\left(\sqrt{a}d^2x^2\mathrm{Ei}\left(\frac{1}{2}\,dx\right)e^{\left(\frac{1}{2}\,c\right)} + \sqrt{a}d^2x^2\mathrm{Ei}\left(-\frac{1}{2}\,dx\right)e^{\left(-\frac{1}{2}\,c\right)} - 2\,\sqrt{a}dxe^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)} + 2\,\sqrt{a}dxe^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)} - 4\,\sqrt{a}}{16\,x^2}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2)/x^3,x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(sqrt(a)*d^2*x^2*Ei(1/2*d*x)*e^(1/2*c) + sqrt(a)*d^2*x^2*Ei(-
1/2*d*x)*e^(-1/2*c) - 2*sqrt(a)*d*x*e^(1/2*d*x + 1/2*c) + 2*sqrt(a)*d*x*e^
(-1/2*d*x - 1/2*c) - 4*sqrt(a)*e^(1/2*d*x + 1/2*c) - 4*sqrt(a)*e^(-1/2*d*x
 - 1/2*c))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^3}\, dx = \int \frac{\sqrt{a + a\cosh\left(c + d\,x\right)}}{x^3}\, dx$$

input
```
int((a + a*cosh(c + d*x))^(1/2)/x^3,x)
```

output
```
int((a + a*cosh(c + d*x))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a + a\cosh(c + dx)}}{x^3}\, dx = \sqrt{a}\left(\int \frac{\sqrt{\cosh\left(dx + c\right) + 1}}{x^3} dx\right)$$

input
```
int((a+a*cosh(d*x+c))^(1/2)/x^3,x)
```

output
```
sqrt(a)*int(sqrt(cosh(c + d*x) + 1)/x**3,x)
```

3.127 $\int x^3\sqrt{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 68

$$\int x^3\sqrt{a+a\cosh(x)}\,dx = -96\sqrt{a+a\cosh(x)}-12x^2\sqrt{a+a\cosh(x)} + 48x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right) + 2x^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)$$

output
```
-96*(a+a*cosh(x))^(1/2)-12*x^2*(a+a*cosh(x))^(1/2)+48*x*(a+a*cosh(x))^(1/2
)*tanh(1/2*x)+2*x^3*(a+a*cosh(x))^(1/2)*tanh(1/2*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.49

$$\int x^3\sqrt{a+a\cosh(x)}\,dx = 2\sqrt{a(1+\cosh(x))}\left(-6\left(8+x^2\right)+x\left(24+x^2\right)\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x^3*Sqrt[a + a*Cosh[x]],x]
```

output
```
2*Sqrt[a*(1 + Cosh[x])]*(-6*(8 + x^2) + x*(24 + x^2)*Tanh[x/2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.01, number of steps used = 14, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3\sqrt{a\cosh(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^3\cosh\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^3\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)-6i\int -ix^2\sinh\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)-6\int x^2\sinh\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)-6\int -ix^2\sin\left(\frac{ix}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\int x^2\sin\left(\frac{ix}{2}\right)dx\right)$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\int x\cosh\left(\frac{x}{2}\right)dx\right)\right)$$

$\Big\downarrow$ 3042

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx\right)\right)$$

$\Big\downarrow$ 3777

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)-2i\int -i\sinh\left(\frac{x}{2}\right)dx\right)\right)\right)$$

$\Big\downarrow$ 26

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)-2\int \sinh\left(\frac{x}{2}\right)dx\right)\right)\right)$$

$\Big\downarrow$ 3042

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)-2\int -i\sin\left(\frac{ix}{2}\right)dx\right)\right)\right)$$

$\Big\downarrow$ 26

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)+2i\int \sin\left(\frac{ix}{2}\right)dx\right)\right)\right)$$

$\Big\downarrow$ 3118

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)-4\cosh\left(\frac{x}{2}\right)\right)\right)\right)$$

input
```
Int[x^3*Sqrt[a + a*Cosh[x]],x]
```

output
```
Sqrt[a + a*Cosh[x]]*Sech[x/2]*(2*x^3*Sinh[x/2] + (6*I)*((2*I)*x^2*Cosh[x/2
] - (4*I)*(-4*Cosh[x/2] + 2*x*Sinh[x/2])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a(\mathrm{e}^{x}+1)^{2}\mathrm{e}^{-x}}\,(x^{3}\mathrm{e}^{x}-x^{3}-6x^{2}\mathrm{e}^{x}-6x^{2}+24x\,\mathrm{e}^{x}-24x-48\,\mathrm{e}^{x}-48)}{\mathrm{e}^{x}+1}$	62

input
```
int(x^3*(a+cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x^3*exp(x)-x^3-6*x^2*ex
p(x)-6*x^2+24*x*exp(x)-24*x-48*exp(x)-48)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3\sqrt{a+a\cosh(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3\sqrt{a+a\cosh(x)}\,dx = \int x^3\sqrt{a\left(\cosh\left(x\right)+1\right)}\,dx$$

input
```
integrate(x**3*(a+a*cosh(x))**(1/2),x)
```

output
```
Integral(x**3*sqrt(a*(cosh(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.29

$$\int x^3\sqrt{a+a\cosh(x)}\,dx = -\left(\sqrt{2}\sqrt{a}x^3+6\sqrt{2}\sqrt{a}x^2+24\sqrt{2}\sqrt{a}x-\left(\sqrt{2}\sqrt{a}x^3-6\sqrt{2}\sqrt{a}x^2+24\sqrt{2}\sqrt{a}x-48\sqrt{2}\sqrt{a}\right)e^x+48\sqrt{2}$$

input
```
integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*x^3 + 6*sqrt(2)*sqrt(a)*x^2 + 24*sqrt(2)*sqrt(a)*x - (sq
rt(2)*sqrt(a)*x^3 - 6*sqrt(2)*sqrt(a)*x^2 + 24*sqrt(2)*sqrt(a)*x - 48*sqrt
(2)*sqrt(a))*e^x + 48*sqrt(2)*sqrt(a))*e^(-1/2*x)
```

Giac [F]

$$\int x^3\sqrt{a+a\cosh(x)}\,dx=\int\sqrt{a\cosh(x)+a}x^3\,dx$$

input
```
integrate(x^3*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x) + a)*x^3, x)
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.93

$$\int x^3\sqrt{a+a\cosh(x)}\,dx = -\frac{\sqrt{a+a\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}\,(48x+96e^x+12x^2e^x-2x^3e^x-48xe^x+12x^2+2x^3+96)}{e^x+1}$$

input
```
int(x^3*(a + a*cosh(x))^(1/2),x)
```

output
```
-((a + a*(exp(-x)/2 + exp(x)/2))^(1/2)*(48*x + 96*exp(x) + 12*x^2*exp(x) -
 2*x^3*exp(x) - 48*x*exp(x) + 12*x^2 + 2*x^3 + 96))/(exp(x) + 1)
```

Reduce [F]

$$\int x^3\sqrt{a+a\cosh(x)}\,dx=\sqrt{a}\left(\int\sqrt{\cosh(x)+1}\,x^3dx\right)$$

input
```
int(x^3*(a+a*cosh(x))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x) + 1)*x**3,x)
```

3.128 $\int x^2 \sqrt{a + a\cosh(x)}\, dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int x^2 \sqrt{a + a\cosh(x)}\, dx = -8x\sqrt{a + a\cosh(x)} + 16\sqrt{a + a\cosh(x)} \tanh\left(\frac{x}{2}\right) + 2x^2\sqrt{a + a\cosh(x)} \tanh\left(\frac{x}{2}\right)$$

output
```
-8*x*(a+a*cosh(x))^(1/2)+16*(a+a*cosh(x))^(1/2)*tanh(1/2*x)+2*x^2*(a+a*cos
h(x))^(1/2)*tanh(1/2*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.58

$$\int x^2 \sqrt{a + a\cosh(x)}\, dx = 8\sqrt{a(1 + \cosh(x))}\left(-x + \frac{1}{4}(8 + x^2) \tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x^2*Sqrt[a + a*Cosh[x]],x]
```

output
```
8*Sqrt[a*(1 + Cosh[x])]*(-x + ((8 + x^2)*Tanh[x/2])/4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.44 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\sqrt{a\cosh(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^2\cosh\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^2\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^2\sinh\left(\frac{x}{2}\right)-4i\int -ix\sinh\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^2\sinh\left(\frac{x}{2}\right)-4\int x\sinh\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^2\sinh\left(\frac{x}{2}\right)-4\int -ix\sin\left(\frac{ix}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\int x\sin\left(\frac{ix}{2}\right)dx\right)$$

$$\Big\downarrow \text{3777}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\left(2ix\cosh\left(\frac{x}{2}\right)-2i\int\cosh\left(\frac{x}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\left(2ix\cosh\left(\frac{x}{2}\right)-2i\int\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{3117}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\left(2ix\cosh\left(\frac{x}{2}\right)-4i\sinh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Int[x^2*Sqrt[a + a*Cosh[x]],x]
```

output
```
Sqrt[a + a*Cosh[x]]*Sech[x/2]*((4*I)*((2*I)*x*Cosh[x/2] - (4*I)*Sinh[x/2])
 + 2*x^2*Sinh[x/2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.94

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a\left(e^{x}+1\right)^{2} e^{-x}}\,\left(x^{2} e^{x}-x^{2}-4 x\,e^{x}-4 x+8\,e^{x}-8\right)}{e^{x}+1}$	50

input
```
int(x^2*(a+cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x^2*exp(x)-x^2-4*x*exp(
x)-4*x+8*exp(x)-8)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2\sqrt{a+a\cosh(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2\sqrt{a+a\cosh(x)}\,dx=\int x^2\sqrt{a\left(\cosh\left(x\right)+1\right)}\,dx$$

input
```
integrate(x**2*(a+a*cosh(x))**(1/2),x)
```

output
```
Integral(x**2*sqrt(a*(cosh(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int x^2\sqrt{a+a\cosh(x)}\,dx$$
$$=-\left(\sqrt{2}\sqrt{a}x^2+4\,\sqrt{2}\sqrt{a}x-\left(\sqrt{2}\sqrt{a}x^2-4\,\sqrt{2}\sqrt{a}x+8\,\sqrt{2}\sqrt{a}\right)e^x+8\,\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}\,x\right)}$$

input
```
integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*x^2 + 4*sqrt(2)*sqrt(a)*x - (sqrt(2)*sqrt(a)*x^2 - 4*sqr
t(2)*sqrt(a)*x + 8*sqrt(2)*sqrt(a))*e^x + 8*sqrt(2)*sqrt(a))*e^(-1/2*x)
```

Giac [F]

$$\int x^2\sqrt{a+a\cosh(x)}\,dx=\int\sqrt{a\cosh\left(x\right)+a}x^2\,dx$$

input
```
integrate(x^2*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x) + a)*x^2, x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int x^2\sqrt{a+a\cosh(x)}\,dx$$
$$= -\frac{\sqrt{a+a\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)}\,(8\,x-16\,e^x-2\,x^2\,e^x+8\,x\,e^x+2\,x^2+16)}{e^x+1}$$

input
```
int(x^2*(a + a*cosh(x))^(1/2),x)
```

output
```
-((a + a*(exp(-x)/2 + exp(x)/2))^(1/2)*(8*x - 16*exp(x) - 2*x^2*exp(x) + 8
*x*exp(x) + 2*x^2 + 16))/(exp(x) + 1)
```

Reduce [F]

$$\int x^2\sqrt{a+a\cosh(x)}\,dx = \sqrt{a}\left(\int\sqrt{\cosh(x)+1}\,x^2dx\right)$$

input
```
int(x^2*(a+a*cosh(x))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x) + 1)*x**2,x)
```

3.129 $\int x\sqrt{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 32

$$\int x\sqrt{a+a\cosh(x)}\,dx=-4\sqrt{a+a\cosh(x)}+2x\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)$$

output
```
-4*(a+a*cosh(x))^(1/2)+2*x*(a+a*cosh(x))^(1/2)*tanh(1/2*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.69

$$\int x\sqrt{a+a\cosh(x)}\,dx=2\sqrt{a(1+\cosh(x))}\left(-2+x\tanh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[x*Sqrt[a + a*Cosh[x]],x]
```

output
```
2*Sqrt[a*(1 + Cosh[x])]*(-2 + x*Tanh[x/2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\sqrt{a\cosh(x)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x\cosh\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x\sinh\left(\frac{x}{2}\right)-2i\int -i\sinh\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x\sinh\left(\frac{x}{2}\right)-2\int \sinh\left(\frac{x}{2}\right)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x\sinh\left(\frac{x}{2}\right)-2\int -i\sin\left(\frac{ix}{2}\right)dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x\sinh\left(\frac{x}{2}\right)+2i\int \sin\left(\frac{ix}{2}\right)dx\right)$$

$$\Big\downarrow \text{3118}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(2x\sinh\left(\frac{x}{2}\right)-4\cosh\left(\frac{x}{2}\right)\right)$$

input
```
Int[x*Sqrt[a + a*Cosh[x]],x]
```

output
```
Sqrt[a + a*Cosh[x]]*Sech[x/2]*(-4*Cosh[x/2] + 2*x*Sinh[x/2])
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.19

method	result	size
risch	$\frac{\sqrt{2}\,\sqrt{a(\mathrm{e}^{x}+1)^{2}\mathrm{e}^{-x}}\,(x\,\mathrm{e}^{x}-x-2\,\mathrm{e}^{x}-2)}{\mathrm{e}^{x}+1}$	38

input
```
int(x*(a+cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2^(1/2)*(a*(exp(x)+1)^2*exp(-x))^(1/2)/(exp(x)+1)*(x*exp(x)-x-2*exp(x)-2)
```

Fricas [F(-2)]

Exception generated.

$$\int x\sqrt{a+a\cosh(x)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x\sqrt{a+a\cosh(x)}\,dx = \int x\sqrt{a\,(\cosh{(x)}+1)}\,dx$$

input
```
integrate(x*(a+a*cosh(x))**(1/2),x)
```

output
```
Integral(x*sqrt(a*(cosh(x) + 1)), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int x\sqrt{a+a\cosh(x)}\,dx = -\left(\sqrt{2}\sqrt{a}x - \left(\sqrt{2}\sqrt{a}x - 2\sqrt{2}\sqrt{a}\right)e^{x} + 2\sqrt{2}\sqrt{a}\right)e^{\left(-\frac{1}{2}x\right)}$$

input
```
integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*x - (sqrt(2)*sqrt(a)*x - 2*sqrt(2)*sqrt(a))*e^x + 2*sqrt
(2)*sqrt(a))*e^(-1/2*x)
```

Giac [F]

$$\int x\sqrt{a+a\cosh(x)}\,dx = \int \sqrt{a\cosh(x)+a}x\,dx$$

input
```
integrate(x*(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x) + a)*x, x)
```

Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.22

$$\int x\sqrt{a+a\cosh(x)}\,dx = -\frac{\sqrt{a+a\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}\,(2x+4e^{x}-2xe^{x}+4)}{e^{x}+1}$$

input
```
int(x*(a + a*cosh(x))^(1/2),x)
```

output
```
-((a + a*(exp(-x)/2 + exp(x)/2))^(1/2)*(2*x + 4*exp(x) - 2*x*exp(x) + 4))/
(exp(x) + 1)
```

Reduce [F]

$$\int x\sqrt{a+a\cosh(x)}\,dx=\sqrt{a}\left(\int\sqrt{\cosh(x)+1}\,xdx\right)$$

input
```
int(x*(a+a*cosh(x))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x) + 1)*x,x)
```

3.130 $\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx=\sqrt{a+a\cosh(x)}\,\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)$$

output
```
(a+a*cosh(x))^(1/2)*Chi(1/2*x)*sech(1/2*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx=\sqrt{a(1+\cosh(x))}\,\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a + a*Cosh[x]]/x,x]
```

output
```
Sqrt[a*(1 + Cosh[x])]*CoshIntegral[x/2]*Sech[x/2]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3800, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cosh(x)+a}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\cosh\left(\frac{x}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3782}$$

$$\text{Chi}\left(\frac{x}{2}\right)\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

input
```
Int[Sqrt[a + a*Cosh[x]]/x,x]
```

output
```
Sqrt[a + a*Cosh[x]]*CoshIntegral[x/2]*Sech[x/2]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a+\cosh \left(x \right) a}}{x}dx$$

input
```
int((a+cosh(x)*a)^(1/2)/x,x)
```

output
```
int((a+cosh(x)*a)^(1/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a + a\cosh(x)}}{x}\,dx = \int \frac{\sqrt{a(\cosh(x) + 1)}}{x}\,dx$$

input
```
integrate((a+a*cosh(x))**(1/2)/x,x)
```

output
```
Integral(sqrt(a*(cosh(x) + 1))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{a + a\cosh(x)}}{x}\,dx = \int \frac{\sqrt{a\cosh(x) + a}}{x}\,dx$$

input
```
integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x) + a)/x, x)
```

Giac [F]

$$\int \frac{\sqrt{a + a\cosh(x)}}{x}\,dx = \int \frac{\sqrt{a\cosh(x) + a}}{x}\,dx$$

input
```
integrate((a+a*cosh(x))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x) + a)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx=\int \frac{\sqrt{a+a\cosh{(x)}}}{x}\,dx$$

input
```
int((a + a*cosh(x))^(1/2)/x,x)
```

output
```
int((a + a*cosh(x))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\cosh{(x)}+1}}{x}dx\right)$$

input
```
int((a+a*cosh(x))^(1/2)/x,x)
```

output
```
sqrt(a)*int(sqrt(cosh(x) + 1)/x,x)
```

3.131 $\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 42

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx=-\frac{\sqrt{a+a\cosh(x)}}{x}+\frac{1}{2}\sqrt{a+a\cosh(x)}\text{sech}\left(\frac{x}{2}\right)\text{Shi}\left(\frac{x}{2}\right)$$

output
```
-(a+a*cosh(x))^(1/2)/x+1/2*(a+a*cosh(x))^(1/2)*sech(1/2*x)*Shi(1/2*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.79

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx=\frac{\sqrt{a(1+\cosh(x))}\left(-2+x\text{sech}\left(\frac{x}{2}\right)\text{Shi}\left(\frac{x}{2}\right)\right)}{2x}$$

input
```
Integrate[Sqrt[a + a*Cosh[x]]/x^2,x]
```

output
```
(Sqrt[a*(1 + Cosh[x])]*(-2 + x*Sech[x/2]*SinhIntegral[x/2]))/(2*x)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3779}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cosh(x)+a}}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\cosh\left(\frac{x}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3778}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{x}+\frac{1}{2}i\int -\frac{i\sinh\left(\frac{x}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{1}{2}\int \frac{\sinh\left(\frac{x}{2}\right)}{x}dx-\frac{\cosh\left(\frac{x}{2}\right)}{x}\right)$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{x}+\frac{1}{2}\int -\frac{i\sin\left(\frac{ix}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{x}-\frac{1}{2}i\int \frac{\sin\left(\frac{ix}{2}\right)}{x}dx\right)$$

$$\Big\downarrow \text{3779}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{\operatorname{Shi}\left(\frac{x}{2}\right)}{2}-\frac{\cosh\left(\frac{x}{2}\right)}{x}\right)$$

input
```
Int[Sqrt[a + a*Cosh[x]]/x^2,x]
```

output
```
Sqrt[a + a*Cosh[x]]*Sech[x/2]*(-(Cosh[x/2]/x) + SinhIntegral[x/2]/2)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a+\cosh(x)\, a}}{x^2}dx$$

input
```
int((a+cosh(x)*a)^(1/2)/x^2,x)
```

output
```
int((a+cosh(x)*a)^(1/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\, dx = \int \frac{\sqrt{a(\cosh(x)+1)}}{x^2}\, dx$$

input
```
integrate((a+a*cosh(x))**(1/2)/x**2,x)
```

output
```
Integral(sqrt(a*(cosh(x) + 1))/x**2, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx = \int \frac{\sqrt{a\cosh(x)+a}}{x^2}\,dx$$

input
```
integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x) + a)/x^2, x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx = \int \frac{\sqrt{a\cosh(x)+a}}{x^2}\,dx$$

input
```
integrate((a+a*cosh(x))^(1/2)/x^2,x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x) + a)/x^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx = \int \frac{\sqrt{a+a\cosh(x)}}{x^2}\,dx$$

input
```
int((a + a*cosh(x))^(1/2)/x^2,x)
```

output
```
int((a + a*cosh(x))^(1/2)/x^2, x)
```

Reduce [F]

$$\int \frac{\sqrt{a + a\cosh(x)}}{x^2}\,dx = \frac{\sqrt{a}\left(-2\sqrt{\cosh(x) + 1} + \left(\int \frac{\sqrt{\cosh(x)+1}\,\sinh(x)}{\cosh(x)x+x}dx\right)x\right)}{2x}$$

input

```
int((a+a*cosh(x))^(1/2)/x^2,x)
```

output

```
(sqrt(a)*( - 2*sqrt(cosh(x) + 1) + int((sqrt(cosh(x) + 1)*sinh(x))/(cosh(x
)*x + x),x)*x))/(2*x)
```

3.132 $\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 67

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = -\frac{\sqrt{a+a\cosh(x)}}{2x^2}+\frac{1}{8}\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right) - \frac{\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)}{4x}$$

output
```
-1/2*(a+a*cosh(x))^(1/2)/x^2+1/8*(a+a*cosh(x))^(1/2)*Chi(1/2*x)*sech(1/2*x
)-1/4*(a+a*cosh(x))^(1/2)*tanh(1/2*x)/x
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.66

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = \frac{\sqrt{a(1+\cosh(x))}\left(-4+x^2\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)-2x\tanh\left(\frac{x}{2}\right)\right)}{8x^2}$$

input
```
Integrate[Sqrt[a + a*Cosh[x]]/x^3,x]
```

output
```
(Sqrt[a*(1 + Cosh[x])]*(-4 + x^2*CoshIntegral[x/2]*Sech[x/2] - 2*x*Tanh[x/
2]))/(8*x^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3800, 3042, 3778, 26, 3042, 26, 3778, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{a\cosh(x)+a}}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\cosh\left(\frac{x}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{2x^2}+\frac{1}{4}i\int -\frac{i\sinh\left(\frac{x}{2}\right)}{x^2}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{1}{4}\int \frac{\sinh\left(\frac{x}{2}\right)}{x^2}dx-\frac{\cosh\left(\frac{x}{2}\right)}{2x^2}\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{2x^2}+\frac{1}{4}\int -\frac{i\sin\left(\frac{ix}{2}\right)}{x^2}dx\right)$$

$$\Big\downarrow \text{26}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{2x^2}-\frac{1}{4}i\int\frac{\sin\left(\frac{ix}{2}\right)}{x^2}dx\right)$$

$$\Big\downarrow \text{3778}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{2x^2}-\frac{1}{4}i\left(\frac{1}{2}i\int\frac{\cosh\left(\frac{x}{2}\right)}{x}dx-\frac{i\sinh\left(\frac{x}{2}\right)}{x}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{2x^2}-\frac{1}{4}i\left(\frac{1}{2}i\int\frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)}{x}dx-\frac{i\sinh\left(\frac{x}{2}\right)}{x}\right)\right)$$

$$\Big\downarrow \text{3782}$$

$$\operatorname{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh\left(\frac{x}{2}\right)}{2x^2}-\frac{1}{4}i\left(\frac{i\text{Chi}\left(\frac{x}{2}\right)}{2}-\frac{i\sinh\left(\frac{x}{2}\right)}{x}\right)\right)$$

input
```
Int[Sqrt[a + a*Cosh[x]]/x^3,x]
```

output
```
Sqrt[a + a*Cosh[x]]*Sech[x/2]*(-1/2*Cosh[x/2]/x^2 - (I/4)*((I/2)*CoshInteg
ral[x/2] - (I*Sinh[x/2])/x))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{\sqrt{a+\cosh(x)\,a}}{x^3}dx$$

input
```
int((a+cosh(x)*a)^(1/2)/x^3,x)
```

output
```
int((a+cosh(x)*a)^(1/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = \int \frac{\sqrt{a\left(\cosh\left(x\right)+1\right)}}{x^3}\,dx$$

input
```
integrate((a+a*cosh(x))**(1/2)/x**3,x)
```

output
```
Integral(sqrt(a*(cosh(x) + 1))/x**3, x)
```

Maxima [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = \int \frac{\sqrt{a\cosh\left(x\right)+a}}{x^3}\,dx$$

input
```
integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x) + a)/x^3, x)
```

Giac [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx = \int \frac{\sqrt{a\cosh\left(x\right)+a}}{x^3}\,dx$$

input
```
integrate((a+a*cosh(x))^(1/2)/x^3,x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x) + a)/x^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx=\int \frac{\sqrt{a+a\cosh\left(x\right)}}{x^3}\,dx$$

input
```
int((a + a*cosh(x))^(1/2)/x^3,x)
```

output
```
int((a + a*cosh(x))^(1/2)/x^3, x)
```

Reduce [F]

$$\int \frac{\sqrt{a+a\cosh(x)}}{x^3}\,dx=\sqrt{a}\left(\int \frac{\sqrt{\cosh\left(x\right)+1}}{x^3}dx\right)$$

input
```
int((a+a*cosh(x))^(1/2)/x^3,x)
```

output
```
sqrt(a)*int(sqrt(cosh(x) + 1)/x**3,x)
```

3.133 $\int x^3(a+a\cosh(x))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 185

$$\int x^3(a+a\cosh(x))^{3/2}\,dx = -\frac{1280}{9}a\sqrt{a+a\cosh(x)} - 16ax^2\sqrt{a+a\cosh(x)} - \frac{64}{27}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)} - \frac{8}{3}ax^2\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)} + \frac{32}{9}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right) + \frac{4}{3}ax^3\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right) + \frac{640}{9}ax\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right) + \frac{8}{3}ax^3\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)$$

output
```
-1280/9*a*(a+a*cosh(x))^(1/2)-16*a*x^2*(a+a*cosh(x))^(1/2)-64/27*a*cosh(1/
2*x)^2*(a+a*cosh(x))^(1/2)-8/3*a*x^2*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)+32/
9*a*x*cosh(1/2*x)*(a+a*cosh(x))^(1/2)*sinh(1/2*x)+4/3*a*x^3*cosh(1/2*x)*(a
+a*cosh(x))^(1/2)*sinh(1/2*x)+640/9*a*x*(a+a*cosh(x))^(1/2)*tanh(1/2*x)+8/
3*a*x^3*(a+a*cosh(x))^(1/2)*tanh(1/2*x)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.38

$$\int x^3(a+a\cosh(x))^{3/2}\,dx=\frac{2}{27}a\sqrt{a(1+\cosh(x))}\Big(-2\left(968+117x^2\right)+3x\left(328+15x^2\right)\tanh\left(\frac{x}{2}\right)+\cosh(x)\left(-2\left(8+9x^2\right)+3x\left(8+3x^2\right)\tanh\left(\frac{x}{2}\right)\right)\Big)$$

input
```
Integrate[x^3*(a + a*Cosh[x])^(3/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cosh[x])]*(-2*(968 + 117*x^2) + 3*x*(328 + 15*x^2)*Tanh[x
/2] + Cosh[x]*(-2*(8 + 9*x^2) + 3*x*(8 + 3*x^2)*Tanh[x/2])))/27
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.92, number of steps used = 23, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.643, Rules used = {3042, 3800, 3042, 3792, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a\cosh(x)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^3\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^3\cosh^3\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^3\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx$$

$\big\downarrow$ 3792

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x^3\cosh\left(\frac{x}{2}\right)dx+\frac{8}{3}\int x\cosh^3\left(\frac{x}{2}\right)dx+\frac{2}{3}x^3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)-\frac{4}{3}x^2\cosh^3\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x^3\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx+\frac{8}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)-\right.$$

$\big\downarrow$ 3777

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)-6i\int -ix^2\sinh\left(\frac{x}{2}\right)dx\right)+\frac{8}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sinh\right.$$

$\big\downarrow$ 26

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)-6\int x^2\sinh\left(\frac{x}{2}\right)dx\right)+\frac{8}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sinh\left(\frac{x}{2}\right.\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)-6\int -ix^2\sin\left(\frac{ix}{2}\right)dx\right)+\frac{8}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sinh\right.$$

$\big\downarrow$ 26

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\int x^2\sin\left(\frac{ix}{2}\right)dx\right)+\frac{8}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sinh\left(\right.\right.$$

$\big\downarrow$ 3777

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\int x\cosh\left(\frac{x}{2}\right)dx\right)\right)+\frac{8}{3}\int x\sin\left(\frac{ix}{2}+\right.\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx\right)\right)+\frac{8}{3}\int x\sin\left(\right.\right.$$

$\downarrow$ 3777

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)-2i\int -i\sinh\left(\frac{x}{2}\right)dx\right)\right)\right.\right.$$

$\downarrow$ 26

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)-2\int \sinh\left(\frac{x}{2}\right)dx\right)\right)\right)+\right.$$

$\downarrow$ 3042

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)-2\int -i\sin\left(\frac{ix}{2}\right)dx\right)\right)\right.\right.$$

$\downarrow$ 26

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)-4i\left(2x\sinh\left(\frac{x}{2}\right)+2i\int \sin\left(\frac{ix}{2}\right)dx\right)\right)\right)\right.$$

$\downarrow$ 3118

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{8}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)-\frac{4}{3}x^2\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x^3\sin\right.\right.$$

$\downarrow$ 3791

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{8}{3}\left(\frac{2}{3}\int x\cosh\left(\frac{x}{2}\right)dx-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^3\sinh\left(\frac{x}{2}\right)\right.$$

$\downarrow$ 3042

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{8}{3}\left(\frac{2}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^3\sinh\right.$$

$\downarrow$ 3777

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{8}{3}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)-2i\int -i\sinh\left(\frac{x}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{8}{3}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)-2\int \sinh\left(\frac{x}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right.\right.$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{8}{3}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)-2\int -i\sin\left(\frac{ix}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\right.\right.\right.$$

$$\Big\downarrow \text{26}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{8}{3}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)+2i\int \sin\left(\frac{ix}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right.\right.$$

$$\Big\downarrow \text{3118}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}x^3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)-\frac{4}{3}x^2\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x^3\sinh\left(\frac{x}{2}\right)+6i\left(2ix^2\cosh\left(\frac{x}{2}\right)\right.\right.\right.$$

input
```
Int[x^3*(a + a*Cosh[x])^(3/2),x]
```

output
```
2*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*((-4*x^2*Cosh[x/2]^3)/3 + (2*x^3*Cosh[x/
2]^2*Sinh[x/2])/3 + (8*((-4*Cosh[x/2]^3)/9 + (2*x*Cosh[x/2]^2*Sinh[x/2])/3
 + (2*(-4*Cosh[x/2] + 2*x*Sinh[x/2]))/3))/3 + (2*(2*x^3*Sinh[x/2] + (6*I)*
((2*I)*x^2*Cosh[x/2] - (4*I)*(-4*Cosh[x/2] + 2*x*Sinh[x/2]))))/3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x^{3}(a+\cosh \left(x\right) a)^{\frac{3}{2}}d x$$

input
```
int(x^3*(a+cosh(x)*a)^(3/2),x)
```

output
```
int(x^3*(a+cosh(x)*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^3(a+a\cosh(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^3(a+a\cosh(x))^{3/2}\,dx = \int x^3(a(\cosh{(x)}+1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**3*(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(x**3*(a*(cosh(x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.97

$$\int x^3(a+a\cosh(x))^{3/2}\,dx = -\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^3+18\sqrt{2}a^{\frac{3}{2}}x^2+24\sqrt{2}a^{\frac{3}{2}}x+16\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^3-18\sqrt{2}a^{\frac{3}{2}}x^2+24\sqrt{2}a^{\frac{3}{2}}x-16\sqrt{2}a^{\frac{3}{2}}\right.\right.$$

input
```
integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
-1/54*(9*sqrt(2)*a^(3/2)*x^3 + 18*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)
*x + 16*sqrt(2)*a^(3/2) - (9*sqrt(2)*a^(3/2)*x^3 - 18*sqrt(2)*a^(3/2)*x^2
+ 24*sqrt(2)*a^(3/2)*x - 16*sqrt(2)*a^(3/2))*e^(3*x) - 81*(sqrt(2)*a^(3/2)
*x^3 - 6*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(3/2)*x - 48*sqrt(2)*a^(3/2))*
e^(2*x) + 81*(sqrt(2)*a^(3/2)*x^3 + 6*sqrt(2)*a^(3/2)*x^2 + 24*sqrt(2)*a^(
3/2)*x + 48*sqrt(2)*a^(3/2))*e^x)*e^(-3/2*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.04

$$\int x^3(a+a\cosh(x))^{3/2}\,dx =$$
$$-\frac{1}{54}\sqrt{2}\Big(54\,a^{\frac{3}{2}}x^3e^{\left(-\frac{1}{2}\,x\right)}+9\,a^{\frac{3}{2}}x^3e^{\left(-\frac{3}{2}\,x\right)}+324\,a^{\frac{3}{2}}x^2e^{\left(-\frac{1}{2}\,x\right)}+18\,a^{\frac{3}{2}}x^2e^{\left(-\frac{3}{2}\,x\right)}+1296\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)}+24\,a^{\frac{3}{2}}xe^{(-$$

input
```
integrate(x^3*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
-1/54*sqrt(2)*(54*a^(3/2)*x^3*e^(-1/2*x) + 9*a^(3/2)*x^3*e^(-3/2*x) + 324*
a^(3/2)*x^2*e^(-1/2*x) + 18*a^(3/2)*x^2*e^(-3/2*x) + 1296*a^(3/2)*x*e^(-1/
2*x) + 24*a^(3/2)*x*e^(-3/2*x) + 2592*a^(3/2)*e^(-1/2*x) + 16*a^(3/2)*e^(-
3/2*x) - (9*a^(3/2)*x^3 - 18*a^(3/2)*x^2 + 24*a^(3/2)*x - 16*a^(3/2))*e^(3
/2*x) - 81*(a^(3/2)*x^3 - 6*a^(3/2)*x^2 + 24*a^(3/2)*x - 48*a^(3/2))*e^(1/
2*x) + 27*(a^(3/2)*x^3 + 6*a^(3/2)*x^2 + 24*a^(3/2)*x + 48*a^(3/2))*e^(-1/
2*x))
```

Mupad [F(-1)]

Timed out.

$$\int x^3(a+a\cosh(x))^{3/2}\,dx = \int x^3\,{\left(a+a\,\mathrm{cosh}\left(x\right)\right)}^{3/2} \,d x$$

input
```
int(x^3*(a + a*cosh(x))^(3/2),x)
```

output
```
int(x^3*(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int x^3(a+a\cosh(x))^{3/2}\,dx=\sqrt{a}\,a\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)\,x^3dx+\int\sqrt{\cosh(x)+1}\,x^3dx\right)$$

input
```
int(x^3*(a+a*cosh(x))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(cosh(x) + 1)*cosh(x)*x**3,x) + int(sqrt(cosh(x) + 1)*x
**3,x))
```

3.134 $\int x^2(a+a\cosh(x))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 145

$$\begin{aligned}\int x^2(a+a\cosh(x))^{3/2}\,dx = &-\frac{32}{3}ax\sqrt{a+a\cosh(x)}\\ &-\frac{16}{9}ax\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax^2\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)\\ &+\frac{224}{9}a\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)+\frac{8}{3}ax^2\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)\\ &+\frac{32}{27}a\sqrt{a+a\cosh(x)}\sinh^2\left(\frac{x}{2}\right)\tanh\left(\frac{x}{2}\right)\end{aligned}$$

output
```
-32/3*a*x*(a+a*cosh(x))^(1/2)-16/9*a*x*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)+4
/3*a*x^2*cosh(1/2*x)*(a+a*cosh(x))^(1/2)*sinh(1/2*x)+224/9*a*(a+a*cosh(x))
^(1/2)*tanh(1/2*x)+8/3*a*x^2*(a+a*cosh(x))^(1/2)*tanh(1/2*x)+32/27*a*(a+a*
cosh(x))^(1/2)*sinh(1/2*x)^2*tanh(1/2*x)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.37

$$\int x^2(a+a\cosh(x))^{3/2}\,dx=\frac{2}{27}a\sqrt{a(1+\cosh(x))}\Big(-156x+\left(328+45x^2\right)\tanh\left(\frac{x}{2}\right)+\cosh(x)\left(-12x+\left(8+9x^2\right)\tanh\left(\frac{x}{2}\right)\right)\Big)$$

input
```
Integrate[x^2*(a + a*Cosh[x])^(3/2),x]
```

output
```
(2*a*Sqrt[a*(1 + Cosh[x])]*(-156*x + (328 + 45*x^2)*Tanh[x/2] + Cosh[x]*(-
12*x + (8 + 9*x^2)*Tanh[x/2])))/27
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.88, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3800, 3042, 3792, 3042, 3113, 2009, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a\cosh(x)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int x^2\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^2\cosh^3\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x^2\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 3792$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x^2\cosh\left(\frac{x}{2}\right)dx+\frac{8}{9}\int \cosh^3\left(\frac{x}{2}\right)dx+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)-\frac{8}{9}x\cosh^3\left(\right.\right.$$

$$\Big\downarrow \; 3042$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x^2\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx+\frac{8}{9}\int \sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)-\frac{8}{9}\right.$$

$$\Big\downarrow \; 3113$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x^2\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx+\frac{16}{9}i\int\left(\sinh^2\left(\frac{x}{2}\right)+1\right)d\left(-i\sinh\left(\frac{x}{2}\right)\right)+\frac{2}{3}x^2\sinh\left(\right.\right.$$

$$\Big\downarrow \; 2009$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x^2\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)+\frac{16}{9}i\left(-\frac{1}{3}i\sinh^3\left(\frac{x}{2}\right)-i\sinh\right.\right.$$

$$\Big\downarrow \; 3777$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^2\sinh\left(\frac{x}{2}\right)-4i\int -ix\sinh\left(\frac{x}{2}\right)dx\right)+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)+\frac{16}{9}i\left(-\frac{1}{3}i\right.\right.$$

$$\Big\downarrow \; 26$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^2\sinh\left(\frac{x}{2}\right)-4\int x\sinh\left(\frac{x}{2}\right)dx\right)+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)+\frac{16}{9}i\left(-\frac{1}{3}i\sin\right.\right.$$

$$\Big\downarrow \; 3042$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^2\sinh\left(\frac{x}{2}\right)-4\int -ix\sin\left(\frac{ix}{2}\right)dx\right)+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)+\frac{16}{9}i\left(-\frac{1}{3}i\right.\right.$$

$$\Big\downarrow \; 26$$

$$2a\,\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\int x\sin\left(\frac{ix}{2}\right)dx\right)+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)+\frac{16}{9}i\left(-\frac{1}{3}i\sin\right.\right.$$

$$\Big\downarrow \; 3777$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\left(2ix\cosh\left(\frac{x}{2}\right)-2i\int\cosh\left(\frac{x}{2}\right)dx\right)\right)+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\right.$$

$\big\downarrow$ 3042

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\left(2ix\cosh\left(\frac{x}{2}\right)-2i\int\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx\right)\right)+\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\text{c}\right.$$

$\big\downarrow$ 3117

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}x^2\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x^2\sinh\left(\frac{x}{2}\right)+4i\left(2ix\cosh\left(\frac{x}{2}\right)-4i\sinh\left(\frac{x}{2}\right)\right)\right)+\right.$$

input
```
Int[x^2*(a + a*Cosh[x])^(3/2),x]
```

output
```
2*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*((-8*x*Cosh[x/2]^3)/9 + (2*x^2*Cosh[x/2]
^2*Sinh[x/2])/3 + (2*((4*I)*((2*I)*x*Cosh[x/2] - (4*I)*Sinh[x/2]) + 2*x^2*
Sinh[x/2]))/3 + ((16*I)/9)*((-I)*Sinh[x/2] - (I/3)*Sinh[x/2]^3))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x^2(a+\cosh(x)\,a)^{\frac{3}{2}}\,dx$$

input
```
int(x^2*(a+cosh(x)*a)^(3/2),x)
```

output
```
int(x^2*(a+cosh(x)*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x^2(a+a\cosh(x))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x^2(a+a\cosh(x))^{3/2}\,dx = \int x^2(a(\cosh(x)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x**2*(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(x**2*(a*(cosh(x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.94

$$\int x^2(a+a\cosh(x))^{3/2}\,dx =$$
$$-\frac{1}{54}\left(9\sqrt{2}a^{\frac{3}{2}}x^2+12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}-\left(9\sqrt{2}a^{\frac{3}{2}}x^2-12\sqrt{2}a^{\frac{3}{2}}x+8\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-81\left(\sqrt{2}a^{\frac{3}{2}}x^2-4\sqrt{2}a\right.\right.$$

input
```
integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
-1/54*(9*sqrt(2)*a^(3/2)*x^2 + 12*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2) -
(9*sqrt(2)*a^(3/2)*x^2 - 12*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^(3*x)
 - 81*(sqrt(2)*a^(3/2)*x^2 - 4*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e^(2
*x) + 81*(sqrt(2)*a^(3/2)*x^2 + 4*sqrt(2)*a^(3/2)*x + 8*sqrt(2)*a^(3/2))*e
^x)*e^(-3/2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.99

$$\int x^2(a+a\cosh(x))^{3/2}\,dx =$$
$$-\frac{1}{54}\sqrt{2}\left(54\,a^{\frac{3}{2}}x^2e^{\left(-\frac{1}{2}\,x\right)}+9\,a^{\frac{3}{2}}x^2e^{\left(-\frac{3}{2}\,x\right)}+216\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)}+12\,a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}\,x\right)}+432\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)}+8\,a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,x\right)}-\right.$$

input
```
integrate(x^2*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
-1/54*sqrt(2)*(54*a^(3/2)*x^2*e^(-1/2*x) + 9*a^(3/2)*x^2*e^(-3/2*x) + 216*
a^(3/2)*x*e^(-1/2*x) + 12*a^(3/2)*x*e^(-3/2*x) + 432*a^(3/2)*e^(-1/2*x) +
8*a^(3/2)*e^(-3/2*x) - (9*a^(3/2)*x^2 - 12*a^(3/2)*x + 8*a^(3/2))*e^(3/2*x
) - 81*(a^(3/2)*x^2 - 4*a^(3/2)*x + 8*a^(3/2))*e^(1/2*x) + 27*(a^(3/2)*x^2
 + 4*a^(3/2)*x + 8*a^(3/2))*e^(-1/2*x))
```

Mupad [F(-1)]

Timed out.

$$\int x^2(a+a\cosh(x))^{3/2}\,dx = \int x^2\,(a+a\cosh(x))^{3/2}\,dx$$

input
```
int(x^2*(a + a*cosh(x))^(3/2),x)
```

output
```
int(x^2*(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int x^2(a+a\cosh(x))^{3/2}\,dx=\sqrt{a}\,a\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)\,x^2dx+\int\sqrt{\cosh(x)+1}\,x^2dx\right)$$

input
```
int(x^2*(a+a*cosh(x))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(cosh(x) + 1)*cosh(x)*x**2,x) + int(sqrt(cosh(x) + 1)*x
**2,x))
```

3.135 $\int x(a+a\cosh(x))^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 89

$$\int x(a+a\cosh(x))^{3/2}\,dx=-\frac{16}{3}a\sqrt{a+a\cosh(x)}-\frac{8}{9}a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}+\frac{4}{3}ax\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)+\frac{8}{3}ax\sqrt{a+a\cosh(x)}\tanh\left(\frac{x}{2}\right)$$

output
```
-16/3*a*(a+a*cosh(x))^(1/2)-8/9*a*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)+4/3*a*
x*cosh(1/2*x)*(a+a*cosh(x))^(1/2)*sinh(1/2*x)+8/3*a*x*(a+a*cosh(x))^(1/2)*
tanh(1/2*x)
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.63

$$\int x(a+a\cosh(x))^{3/2}\,dx=\frac{1}{9}a\sqrt{a(1+\cosh(x))}\operatorname{sech}\left(\frac{x}{2}\right)\left(-54\cosh\left(\frac{x}{2}\right)-2\cosh\left(\frac{3x}{2}\right)+3x\left(9\sinh\left(\frac{x}{2}\right)+\sinh\left(\frac{3x}{2}\right)\right)\right)$$

input
```
Integrate[x*(a + a*Cosh[x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cosh[x])]*Sech[x/2]*(-54*Cosh[x/2] - 2*Cosh[(3*x)/2] + 3*x*
(9*Sinh[x/2] + Sinh[(3*x)/2])))/9
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {3042, 3800, 3042, 3791, 3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a\cosh(x)+a)^{3/2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int x\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \; 3800$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x\cosh^3\left(\frac{x}{2}\right)dx$$

$$\Big\downarrow \; 3042$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx$$

$$\Big\downarrow \; 3791$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x\cosh\left(\frac{x}{2}\right)dx-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 3042$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\int x\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \; 3777$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)-2i\int -i\sinh\left(\frac{x}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{26}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)-2\int\sinh\left(\frac{x}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)-2\int-i\sin\left(\frac{ix}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{26}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)+2i\int\sin\left(\frac{ix}{2}\right)dx\right)-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)\right)$$

$$\Big\downarrow \text{3118}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{4}{9}\cosh^3\left(\frac{x}{2}\right)+\frac{2}{3}x\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)+\frac{2}{3}\left(2x\sinh\left(\frac{x}{2}\right)-4\cosh\left(\frac{x}{2}\right)\right)\right)$$

input
```
Int[x*(a + a*Cosh[x])^(3/2),x]
```

output
```
2*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*((-4*Cosh[x/2]^3)/9 + (2*x*Cosh[x/2]^2*S
inh[x/2])/3 + (2*(-4*Cosh[x/2] + 2*x*Sinh[x/2]))/3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int x(a+\cosh(x)\, a)^{\frac{3}{2}}\, dx$$

input
```
int(x*(a+cosh(x)*a)^(3/2),x)
```

output
```
int(x*(a+cosh(x)*a)^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int x(a+a\cosh(x))^{3/2}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >> Error detected within library code: inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int x(a+a\cosh(x))^{3/2}\,dx = \int x(a(\cosh(x)+1))^{\frac{3}{2}}\,dx$$

input
```
integrate(x*(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(x*(a*(cosh(x) + 1))**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.03

$$\int x(a+a\cosh(x))^{3/2}\,dx =$$
$$-\frac{1}{18}\left(3\sqrt{2}a^{\frac{3}{2}}x+2\sqrt{2}a^{\frac{3}{2}}-\left(3\sqrt{2}a^{\frac{3}{2}}x-2\sqrt{2}a^{\frac{3}{2}}\right)e^{(3x)}-27\left(\sqrt{2}a^{\frac{3}{2}}x-2\sqrt{2}a^{\frac{3}{2}}\right)e^{(2x)}+27\left(\sqrt{2}a^{\frac{3}{2}}x+2\sqrt{}\right.\right.$$

input
```
integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
-1/18*(3*sqrt(2)*a^(3/2)*x + 2*sqrt(2)*a^(3/2) - (3*sqrt(2)*a^(3/2)*x - 2*
sqrt(2)*a^(3/2))*e^(3*x) - 27*(sqrt(2)*a^(3/2)*x - 2*sqrt(2)*a^(3/2))*e^(2
*x) + 27*(sqrt(2)*a^(3/2)*x + 2*sqrt(2)*a^(3/2))*e^x)*e^(-3/2*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.08

$$\int x(a+a\cosh(x))^{3/2}\,dx =$$
$$-\frac{1}{18}\sqrt{2}\Big(18\,a^{\frac{3}{2}}xe^{\left(-\frac{1}{2}\,x\right)}+3\,a^{\frac{3}{2}}xe^{\left(-\frac{3}{2}\,x\right)}+36\,a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)}+2\,a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,x\right)}-\Big(3\,a^{\frac{3}{2}}x-2\,a^{\frac{3}{2}}\Big)e^{\left(\frac{3}{2}\,x\right)}-27\,\Big(a^{\frac{3}{2}}x-2\,a$$

input
```
integrate(x*(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
-1/18*sqrt(2)*(18*a^(3/2)*x*e^(-1/2*x) + 3*a^(3/2)*x*e^(-3/2*x) + 36*a^(3/
2)*e^(-1/2*x) + 2*a^(3/2)*e^(-3/2*x) - (3*a^(3/2)*x - 2*a^(3/2))*e^(3/2*x)
 - 27*(a^(3/2)*x - 2*a^(3/2))*e^(1/2*x) + 9*(a^(3/2)*x + 2*a^(3/2))*e^(-1/
2*x))
```

Mupad [F(-1)]

Timed out.

$$\int x(a+a\cosh(x))^{3/2}\,dx=\int x\,(a+a\cosh(x))^{3/2}\,dx$$

input
```
int(x*(a + a*cosh(x))^(3/2),x)
```

output
```
int(x*(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int x(a+a\cosh(x))^{3/2}\,dx=\sqrt{a}\,a\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)\,xdx+\int\sqrt{\cosh(x)+1}\,xdx\right)$$

input
```
int(x*(a+a*cosh(x))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(cosh(x) + 1)*cosh(x)*x,x) + int(sqrt(cosh(x) + 1)*x,x)
)
```

3.136 $\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 55

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \frac{3}{2}a\sqrt{a+a\cosh(x)}\,\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right) + \frac{1}{2}a\sqrt{a+a\cosh(x)}\,\text{Chi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right)$$

output
```
3/2*a*(a+a*cosh(x))^(1/2)*Chi(1/2*x)*sech(1/2*x)+1/2*a*(a+a*cosh(x))^(1/2)
*Chi(3/2*x)*sech(1/2*x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.65

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \frac{1}{2}a\sqrt{a(1+\cosh(x))}\left(3\text{Chi}\left(\frac{x}{2}\right)+\text{Chi}\left(\frac{3x}{2}\right)\right)\text{sech}\left(\frac{x}{2}\right)$$

input
```
Integrate[(a + a*Cosh[x])^(3/2)/x,x]
```

output
```
(a*Sqrt[a*(1 + Cosh[x])]*(3*CoshIntegral[x/2] + CoshIntegral[(3*x)/2])*Sec
h[x/2])/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3800, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cosh(x)+a)^{3/2}}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}{x}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x}dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3}{x}dx$$

$$\Big\downarrow \text{3793}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \left(\frac{3\cosh\left(\frac{x}{2}\right)}{4x}+\frac{\cosh\left(\frac{3x}{2}\right)}{4x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$2a\left(\frac{3\text{Chi}\left(\frac{x}{2}\right)}{4}+\frac{\text{Chi}\left(\frac{3x}{2}\right)}{4}\right)\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}$$

input
```
Int[(a + a*Cosh[x])^(3/2)/x,x]
```

output
```
2*a*Sqrt[a + a*Cosh[x]]*((3*CoshIntegral[x/2])/4 + CoshIntegral[(3*x)/2]/4
)*Sech[x/2]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a+\cosh(x)\,a)^{\frac{3}{2}}}{x}dx$$

input
```
int((a+cosh(x)*a)^(3/2)/x,x)
```

output
```
int((a+cosh(x)*a)^(3/2)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \int \frac{(a(\cosh{(x)}+1))^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+a*cosh(x))**(3/2)/x,x)
```

output
```
Integral((a*(cosh(x) + 1))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \int \frac{(a\cosh{(x)}+a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate((a*cosh(x) + a)^(3/2)/x, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.73

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \frac{1}{4}\sqrt{2}\left(a^{\frac{3}{2}}\text{Ei}\left(\frac{3}{2}\,x\right)+3\,a^{\frac{3}{2}}\text{Ei}\left(\frac{1}{2}\,x\right)+3\,a^{\frac{3}{2}}\text{Ei}\left(-\frac{1}{2}\,x\right)+a^{\frac{3}{2}}\text{Ei}\left(-\frac{3}{2}\,x\right)\right)$$

input
```
integrate((a+a*cosh(x))^(3/2)/x,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*(a^(3/2)*Ei(3/2*x) + 3*a^(3/2)*Ei(1/2*x) + 3*a^(3/2)*Ei(-1/2*x
) + a^(3/2)*Ei(-3/2*x))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx$$

input
```
int((a + a*cosh(x))^(3/2)/x,x)
```

output
```
int((a + a*cosh(x))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{(a+a\cosh(x))^{3/2}}{x}\,dx = \sqrt{a}\,a\left(\int \frac{\sqrt{\cosh(x)+1}}{x}dx+\int \frac{\sqrt{\cosh(x)+1}\,\cosh(x)}{x}dx\right)$$

input
```
int((a+a*cosh(x))^(3/2)/x,x)
```

output
```
sqrt(a)*a*(int(sqrt(cosh(x) + 1)/x,x) + int((sqrt(cosh(x) + 1)*cosh(x))/x,
x))
```

3.137 $\int \frac{(a+a\cosh(x))^{3/2}}{x^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 79

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^2}\,dx = -\frac{2a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x} + \frac{3}{4}a\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\operatorname{Shi}\left(\frac{x}{2}\right) + \frac{3}{4}a\sqrt{a+a\cosh(x)}\operatorname{sech}\left(\frac{x}{2}\right)\operatorname{Shi}\left(\frac{3x}{2}\right)$$

output
```
-2*a*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)/x+3/4*a*(a+a*cosh(x))^(1/2)*sech(1/
2*x)*Shi(1/2*x)+3/4*a*(a+a*cosh(x))^(1/2)*sech(1/2*x)*Shi(3/2*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.67

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^2}\,dx = -\frac{a\sqrt{a(1+\cosh(x))}\operatorname{sech}\left(\frac{x}{2}\right)\left(8\cosh^3\left(\frac{x}{2}\right)-3x\operatorname{Shi}\left(\frac{x}{2}\right)-3x\operatorname{Shi}\left(\frac{3x}{2}\right)\right)}{4x}$$

input
```
Integrate[(a + a*Cosh[x])^(3/2)/x^2,x]
```

output
```
-1/4*(a*Sqrt[a*(1 + Cosh[x])]*Sech[x/2]*(8*Cosh[x/2]^3 - 3*x*SinhIntegral[
x/2] - 3*x*SinhIntegral[(3*x)/2]))/x
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3042, 3800, 3042, 3794, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cosh(x)+a)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}{x^2}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^2}dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3}{x^2}dx$$

$$\Big\downarrow \text{3794}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh^3\left(\frac{x}{2}\right)}{x}+\frac{3}{2}i\int\left(-\frac{i\sinh\left(\frac{x}{2}\right)}{4x}-\frac{i\sinh\left(\frac{3x}{2}\right)}{4x}\right)dx\right)$$

$$\Big\downarrow \text{2009}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{\cosh^3\left(\frac{x}{2}\right)}{x}+\frac{3}{2}i\left(-\frac{1}{4}i\text{Shi}\left(\frac{x}{2}\right)-\frac{1}{4}i\text{Shi}\left(\frac{3x}{2}\right)\right)\right)$$

input
```
Int[(a + a*Cosh[x])^(3/2)/x^2,x]
```

output
```
2*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*(-(Cosh[x/2]^3/x) + ((3*I)/2)*((-1/4*I)*
SinhIntegral[x/2] - (I/4)*SinhIntegral[(3*x)/2]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a + \cosh(x)\, a)^{\frac{3}{2}}}{x^2} dx$$

input
```
int((a+cosh(x)*a)^(3/2)/x^2,x)
```

output
```
int((a+cosh(x)*a)^(3/2)/x^2,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^2} \, dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^2} \, dx = \int \frac{(a(\cosh(x) + 1))^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((a+a*cosh(x))**(3/2)/x**2,x)
```

output
```
Integral((a*(cosh(x) + 1))**(3/2)/x**2, x)
```

Maxima [F]

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^2} \, dx = \int \frac{(a\cosh(x) + a)^{\frac{3}{2}}}{x^2} \, dx$$

input
```
integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="maxima")
```

output
```
integrate((a*cosh(x) + a)^(3/2)/x^2, x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.42

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^2}\,dx = \frac{1}{8}\sqrt{2}\left(\frac{3\,a^{\frac{3}{2}}x\mathrm{Ei}\left(\frac{3}{2}\,x\right) + 3\,a^{\frac{3}{2}}x\mathrm{Ei}\left(\frac{1}{2}\,x\right) - a^{\frac{3}{2}}x\mathrm{Ei}\left(-\frac{1}{2}\,x\right) - 2\,a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)} - 6\,a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)} - \right.}{x}$$

input
```
integrate((a+a*cosh(x))^(3/2)/x^2,x, algorithm="giac")
```

output
```
1/8*sqrt(2)*((3*a^(3/2)*x*Ei(3/2*x) + 3*a^(3/2)*x*Ei(1/2*x) - a^(3/2)*x*Ei
(-1/2*x) - 2*a^(3/2)*e^(3/2*x) - 6*a^(3/2)*e^(1/2*x) - 2*a^(3/2)*e^(-1/2*x
))/x - (2*a^(3/2)*x*Ei(-1/2*x) + 3*a^(3/2)*x*Ei(-3/2*x) + 4*a^(3/2)*e^(-1/
2*x) + 2*a^(3/2)*e^(-3/2*x))/x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^2}\,dx = \int \frac{(a + a\cosh(x))^{3/2}}{x^2}\,dx$$

input
```
int((a + a*cosh(x))^(3/2)/x^2,x)
```

output
```
int((a + a*cosh(x))^(3/2)/x^2, x)
```

Reduce [F]

$$\int \frac{(a + a\cosh(x))^{3/2}}{x^2}\,dx = \frac{\sqrt{a}\,a\left(-2\sqrt{\cosh(x)+1} + 2\left(\int \frac{\sqrt{\cosh(x)+1}\,\cosh(x)}{x^2}dx\right)x + \left(\int \frac{\sqrt{\cosh(x)+1}\,\sinh(x)}{\cosh(x)x+x}dx\right)\right.}{2x}$$

input
```
int((a+a*cosh(x))^(3/2)/x^2,x)
```

output
```
(sqrt(a)*a*( - 2*sqrt(cosh(x) + 1) + 2*int((sqrt(cosh(x) + 1)*cosh(x))/x**
2,x)*x + int((sqrt(cosh(x) + 1)*sinh(x))/(cosh(x)*x + x),x)*x))/(2*x)
```

3.138 $\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 109

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx = -\frac{a\cosh^2\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}}{x^2} + \frac{3}{16}a\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right) + \frac{9}{16}a\sqrt{a+a\cosh(x)}\text{Chi}\left(\frac{3x}{2}\right)\text{sech}\left(\frac{x}{2}\right) - \frac{3a\cosh\left(\frac{x}{2}\right)\sqrt{a+a\cosh(x)}\sinh\left(\frac{x}{2}\right)}{2x}$$

output
```
-a*cosh(1/2*x)^2*(a+a*cosh(x))^(1/2)/x^2+3/16*a*(a+a*cosh(x))^(1/2)*Chi(1/
2*x)*sech(1/2*x)+9/16*a*(a+a*cosh(x))^(1/2)*Chi(3/2*x)*sech(1/2*x)-3/2*a*c
osh(1/2*x)*(a+a*cosh(x))^(1/2)*sinh(1/2*x)/x
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.63

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx = \frac{(a(1+\cosh(x)))^{3/2}\left(3x^2\text{Chi}\left(\frac{x}{2}\right)\text{sech}^3\left(\frac{x}{2}\right) + 9x^2\text{Chi}\left(\frac{3x}{2}\right)\text{sech}^3\left(\frac{x}{2}\right) - 8(2+3x\,\text{ta}\right.}{32x^2}$$

input
```
Integrate[(a + a*Cosh[x])^(3/2)/x^3,x]
```

output
```
((a*(1 + Cosh[x]))^(3/2)*(3*x^2*CoshIntegral[x/2]*Sech[x/2]^3 + 9*x^2*Cosh
Integral[(3*x)/2]*Sech[x/2]^3 - 8*(2 + 3*x*Tanh[x/2])))/(32*x^2)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.571, Rules used = {3042, 3800, 3042, 3795, 3042, 3782, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a\cosh(x)+a)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}{x^3}dx$$

$$\Big\downarrow \text{3800}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x^3}dx$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3}{x^3}dx$$

$$\Big\downarrow \text{3795}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{9}{8}\int \frac{\cosh^3\left(\frac{x}{2}\right)}{x}dx-\frac{3}{4}\int \frac{\cosh\left(\frac{x}{2}\right)}{x}dx-\frac{\cosh^3\left(\frac{x}{2}\right)}{2x^2}-\frac{3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\Big\downarrow \text{3042}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{3}{4}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)}{x}dx+\frac{9}{8}\int \frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3}{x}dx-\frac{\cosh^3\left(\frac{x}{2}\right)}{2x^2}-\frac{3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\Big\downarrow \text{3782}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{9}{8}\int\frac{\sin\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3}{x}dx-\frac{3\text{Chi}\left(\frac{x}{2}\right)}{4}-\frac{\cosh^3\left(\frac{x}{2}\right)}{2x^2}-\frac{3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\Big\downarrow \textbf{3793}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(\frac{9}{8}\int\left(\frac{3\cosh\left(\frac{x}{2}\right)}{4x}+\frac{\cosh\left(\frac{3x}{2}\right)}{4x}\right)dx-\frac{3\text{Chi}\left(\frac{x}{2}\right)}{4}-\frac{\cosh^3\left(\frac{x}{2}\right)}{2x^2}-\frac{3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)}{4x}\right)$$

$$\Big\downarrow \textbf{2009}$$

$$2a\text{sech}\left(\frac{x}{2}\right)\sqrt{a\cosh(x)+a}\left(-\frac{3\text{Chi}\left(\frac{x}{2}\right)}{4}+\frac{9}{8}\left(\frac{3\text{Chi}\left(\frac{x}{2}\right)}{4}+\frac{\text{Chi}\left(\frac{3x}{2}\right)}{4}\right)-\frac{\cosh^3\left(\frac{x}{2}\right)}{2x^2}-\frac{3\sinh\left(\frac{x}{2}\right)\cosh^2\left(\frac{x}{2}\right)}{4x}\right)$$

input
```
Int[(a + a*Cosh[x])^(3/2)/x^3,x]
```

output
```
2*a*Sqrt[a + a*Cosh[x]]*Sech[x/2]*(-1/2*Cosh[x/2]^3/x^2 - (3*CoshIntegral[
x/2])/4 + (9*((3*CoshIntegral[x/2])/4 + CoshIntegral[(3*x)/2]/4))/8 - (3*C
osh[x/2]^2*Sinh[x/2])/(4*x))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3795
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[(c + d*x)^(m + 1)*((b*Sin[e + f*x])^n/(d*(m + 1))), x] + (-Simp[
b*f*n*(c + d*x)^(m + 2)*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(d^2*(m + 1)
*(m + 2))), x] + Simp[b^2*f^2*n*((n - 1)/(d^2*(m + 1)*(m + 2)))   Int[(c +
d*x)^(m + 2)*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[f^2*(n^2/(d^2*(m + 1)*
(m + 2)))   Int[(c + d*x)^(m + 2)*(b*Sin[e + f*x])^n, x], x]) /; FreeQ[{b,
c, d, e, f}, x] && GtQ[n, 1] && LtQ[m, -2]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int \frac{(a+\cosh(x)\, a)^{\frac{3}{2}}}{x^3} dx$$

input
```
int((a+cosh(x)*a)^(3/2)/x^3,x)
```

output
```
int((a+cosh(x)*a)^(3/2)/x^3,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3} dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx=\int \frac{(a(\cosh{(x)}+1))^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+a*cosh(x))**(3/2)/x**3,x)
```

output
```
Integral((a*(cosh(x) + 1))**(3/2)/x**3, x)
```

Maxima [F]

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx=\int \frac{(a\cosh{(x)}+a)^{\frac{3}{2}}}{x^3}\,dx$$

input
```
integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="maxima")
```

output
```
integrate((a*cosh(x) + a)^(3/2)/x^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.56

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx=\frac{1}{32}\sqrt{2}\left(\frac{9\,a^{\frac{3}{2}}x^2\mathrm{Ei}\left(\frac{3}{2}\,x\right)+3\,a^{\frac{3}{2}}x^2\mathrm{Ei}\left(\frac{1}{2}\,x\right)+a^{\frac{3}{2}}x^2\mathrm{Ei}\left(-\frac{1}{2}\,x\right)-6\,a^{\frac{3}{2}}xe^{\left(\frac{3}{2}\,x\right)}-6\,a^{\frac{3}{2}}xe}{x^2}\right.$$

input
```
integrate((a+a*cosh(x))^(3/2)/x^3,x, algorithm="giac")
```

output
```
1/32*sqrt(2)*((9*a^(3/2)*x^2*Ei(3/2*x) + 3*a^(3/2)*x^2*Ei(1/2*x) + a^(3/2)
*x^2*Ei(-1/2*x) - 6*a^(3/2)*x*e^(3/2*x) - 6*a^(3/2)*x*e^(1/2*x) + 2*a^(3/2
)*x*e^(-1/2*x) - 4*a^(3/2)*e^(3/2*x) - 12*a^(3/2)*e^(1/2*x) - 4*a^(3/2)*e^
(-1/2*x))/x^2 + (2*a^(3/2)*x^2*Ei(-1/2*x) + 9*a^(3/2)*x^2*Ei(-3/2*x) + 4*a
^(3/2)*x*e^(-1/2*x) + 6*a^(3/2)*x*e^(-3/2*x) - 8*a^(3/2)*e^(-1/2*x) - 4*a^
(3/2)*e^(-3/2*x))/x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx=\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx$$

input
```
int((a + a*cosh(x))^(3/2)/x^3,x)
```

output
```
int((a + a*cosh(x))^(3/2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+a\cosh(x))^{3/2}}{x^3}\,dx=\sqrt{a}\,a\left(\int \frac{\sqrt{\cosh(x)+1}}{x^3}dx+\int \frac{\sqrt{\cosh(x)+1}\,\cosh(x)}{x^3}dx\right)$$

input
```
int((a+a*cosh(x))^(3/2)/x^3,x)
```

output
```
sqrt(a)*a*(int(sqrt(cosh(x) + 1)/x**3,x) + int((sqrt(cosh(x) + 1)*cosh(x))
/x**3,x))
```

3.139 $\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 383

$$\begin{aligned}\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx &= \frac{4x^3\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}}\\
&-\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}\\
&+\frac{12ix^2\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}\\
&+\frac{48ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^3\sqrt{a+a\cosh(c+dx)}}\\
&-\frac{48ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(3,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^3\sqrt{a+a\cosh(c+dx)}}\\
&-\frac{96i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^4\sqrt{a+a\cosh(c+dx)}}\\
&+\frac{96i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(4,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^4\sqrt{a+a\cosh(c+dx)}}\end{aligned}$$

output
```
4*x^3*arctan(exp(1/2*d*x+1/2*c))*cosh(1/2*d*x+1/2*c)/d/(a+a*cosh(d*x+c))^(
1/2)-12*I*x^2*cosh(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*d*x+1/2*c))/d^2/(a+
a*cosh(d*x+c))^(1/2)+12*I*x^2*cosh(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*d*x+
1/2*c))/d^2/(a+a*cosh(d*x+c))^(1/2)+48*I*x*cosh(1/2*d*x+1/2*c)*polylog(3,-
I*exp(1/2*d*x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)-48*I*x*cosh(1/2*d*x+1/2*
c)*polylog(3,I*exp(1/2*d*x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)-96*I*cosh(1
/2*d*x+1/2*c)*polylog(4,-I*exp(1/2*d*x+1/2*c))/d^4/(a+a*cosh(d*x+c))^(1/2)
+96*I*cosh(1/2*d*x+1/2*c)*polylog(4,I*exp(1/2*d*x+1/2*c))/d^4/(a+a*cosh(d*
x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.56

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx$$
$$= \frac{2i\cosh\left(\frac{1}{2}(c+dx)\right)\left(d^3x^3\log\left(1-ie^{\frac{1}{2}(c+dx)}\right)-d^3x^3\log\left(1+ie^{\frac{1}{2}(c+dx)}\right)-6d^2x^2\operatorname{PolyLog}\left(2,-ie^{\frac{1}{2}(c+dx)}\right)\right.}{}$$

input
```
Integrate[x^3/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
((2*I)*Cosh[(c + d*x)/2]*(d^3*x^3*Log[1 - I*E^((c + d*x)/2)] - d^3*x^3*Log
[1 + I*E^((c + d*x)/2)] - 6*d^2*x^2*PolyLog[2, (-I)*E^((c + d*x)/2)] + 6*d
^2*x^2*PolyLog[2, I*E^((c + d*x)/2)] + 24*d*x*PolyLog[3, (-I)*E^((c + d*x)
/2)] - 24*d*x*PolyLog[3, I*E^((c + d*x)/2)] - 48*PolyLog[4, (-I)*E^((c + d
*x)/2)] + 48*PolyLog[4, I*E^((c + d*x)/2)]))/(d^4*Sqrt[a*(1 + Cosh[c + d*x
])])
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.62, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3800, 3042, 4668, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{\sqrt{a\cosh(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x^3}{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^3\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^3\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{ 4668}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{6i\int x^2\log\left(1-ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}+\frac{6i\int x^2\log\left(1+ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}+\frac{4x^3\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{ 3011}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}-\frac{2x^2\,\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}-\frac{6i\left(\frac{4\int x\,\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}-\frac{2x^2\,\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right.}{d}\right.}{d}\right.}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{ 7163}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}-\frac{2\int\mathrm{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}\right)}{d}-\frac{2x^2\,\mathrm{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}-\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{PolyLog}\left(3,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right.}{}\right.}{}\right.}{\sqrt{a\cosh(c+dx)+a}}$$

$\Big\downarrow$ 2720

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}-\frac{4\int e^{-\frac{c}{2}-\frac{dx}{2}}\,\mathrm{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)de^{\frac{c}{2}+\frac{dx}{2}}}{d^2}\right)}{d}-\frac{2x^2\,\mathrm{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}-\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{P}}{}\right.}{}\right.}{}\right.}{\sqrt{a\cosh(c+dx)+}}$$

$\Big\downarrow$ 7143

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4x^3\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}+\frac{6i\left(\frac{4\left(\frac{2x\,\mathrm{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}-\frac{4\,\mathrm{PolyLog}\left(4,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2}\right)}{d}-\frac{2x^2\,\mathrm{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}-\frac{6i\left(\right.}{}\right.}{\sqrt{a\cosh(c+dx)+a}}$$

input
```
Int[x^3/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(Cosh[c/2 + (d*x)/2]*((4*x^3*ArcTan[E^(c/2 + (d*x)/2)])/d + ((6*I)*((-2*x^
2*PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])/d + (4*((2*x*PolyLog[3, (-I)*E^(c/2
+ (d*x)/2)])/d - (4*PolyLog[4, (-I)*E^(c/2 + (d*x)/2)])/d^2))/d))/d - ((6*
I)*((-2*x^2*PolyLog[2, I*E^(c/2 + (d*x)/2)])/d + (4*((2*x*PolyLog[3, I*E^(
c/2 + (d*x)/2)])/d - (4*PolyLog[4, I*E^(c/2 + (d*x)/2)])/d^2))/d))/d))/Sqr
t[a + a*Cosh[c + d*x]]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{\sqrt{a + a\cosh(dx + c)}}dx$$

input
```
int(x^3/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
int(x^3/(a+a*cosh(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^3}{\sqrt{a + a\cosh(c + dx)}}\,dx = \int \frac{x^3}{\sqrt{a\cosh(dx + c) + a}}\,dx$$

input
```
integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^3/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(x**3/(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(x**3/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*d^3*integrate(x^3*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^3*e^(2*d*x + 2*
c) + 2*sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3), x) + 12*sqrt(2)*d^2*integra
te(x^2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^3*e^(2*d*x + 2*c) + 2*sqrt(a)*d^3*e^
(d*x + c) + sqrt(a)*d^3), x) + 48*sqrt(2)*d*integrate(x*e^(1/2*d*x + 1/2*c
)/(sqrt(a)*d^3*e^(2*d*x + 2*c) + 2*sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3),
 x) + 96*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*
d^3)*d) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d^4)) - 2*(sqrt(2)*sqrt(a)*
d^3*x^3*e^(1/2*c) + 6*sqrt(2)*sqrt(a)*d^2*x^2*e^(1/2*c) + 24*sqrt(2)*sqrt(
a)*d*x*e^(1/2*c) + 48*sqrt(2)*sqrt(a)*e^(1/2*c))*e^(1/2*d*x)/(a*d^4*e^(d*x
 + c) + a*d^4)
```

Giac [F]

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^3/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^3/sqrt(a*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x^3}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(x^3/(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
int(x^3/(a + a*cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^3}{\sqrt{a+a\cosh(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}\,x^3}{\cosh(dx+c)+1}dx\right)}{a}$$

input
```
int(x^3/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(cosh(c + d*x) + 1)*x**3)/(cosh(c + d*x) + 1),x))/a
```

3.140 $\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 269

$$\begin{aligned}\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx &= \frac{4x^2\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}}\\ &\quad-\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}\\ &\quad+\frac{8ix\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}\\ &\quad+\frac{16i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^3\sqrt{a+a\cosh(c+dx)}}\\ &\quad-\frac{16i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\text{PolyLog}\left(3,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^3\sqrt{a+a\cosh(c+dx)}}\end{aligned}$$

output
```
4*x^2*arctan(exp(1/2*d*x+1/2*c))*cosh(1/2*d*x+1/2*c)/d/(a+a*cosh(d*x+c))^(
1/2)-8*I*x*cosh(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*d*x+1/2*c))/d^2/(a+a*c
osh(d*x+c))^(1/2)+8*I*x*cosh(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*d*x+1/2*c)
)/d^2/(a+a*cosh(d*x+c))^(1/2)+16*I*cosh(1/2*d*x+1/2*c)*polylog(3,-I*exp(1/
2*d*x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)-16*I*cosh(1/2*d*x+1/2*c)*polylog
(3,I*exp(1/2*d*x+1/2*c))/d^3/(a+a*cosh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.61

$$\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{2i\cosh\left(\frac{1}{2}(c+dx)\right)\left(d^2x^2\log\left(1-ie^{\frac{1}{2}(c+dx)}\right)-d^2x^2\log\left(1+ie^{\frac{1}{2}(c+dx)}\right)-4dx\,\text{PolyLog}\left(2,-ie^{\frac{1}{2}(c+dx)}\right)\right.}{d^3\sqrt{a(1+\cosh(c+a}}$$

input
```
Integrate[x^2/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
((2*I)*Cosh[(c + d*x)/2]*(d^2*x^2*Log[1 - I*E^((c + d*x)/2)] - d^2*x^2*Log
[1 + I*E^((c + d*x)/2)] - 4*d*x*PolyLog[2, (-I)*E^((c + d*x)/2)] + 4*d*x*P
olyLog[2, I*E^((c + d*x)/2)] + 8*PolyLog[3, (-I)*E^((c + d*x)/2)] - 8*Poly
Log[3, I*E^((c + d*x)/2)]))/(d^3*Sqrt[a*(1 + Cosh[c + d*x])])
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.63, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3800, 3042, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{\sqrt{a\cosh(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x^2}{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^2\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\int x^2\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{4i\int x\log\left(1-ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}+\frac{4i\int x\log\left(1+ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}+\frac{4x^2\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{3011}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}-\frac{2x\,\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}-\frac{4i\left(\frac{2\int \text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}-\frac{2x\,\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}\right.}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4i\left(\frac{4\int e^{-\frac{c}{2}-\frac{dx}{2}}\,\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)de^{\frac{c}{2}+\frac{dx}{2}}}{d^2}-\frac{2x\,\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}-\frac{4i\left(\frac{4\int e^{-\frac{c}{2}-\frac{dx}{2}}\,\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)de^{\frac{c}{2}+\frac{d}{}}\right.}{d}\right.}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4x^2\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}+\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2}-\frac{2x\,\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{d}-\frac{4i\left(\frac{4\,\text{PolyLog}\left(3,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2}-\frac{2x\,\text{PolyL}}{}\right.}{d}\right.}{\sqrt{a\cosh(c+dx)+a}}$$

input
```
Int[x^2/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(Cosh[c/2 + (d*x)/2]*((4*x^2*ArcTan[E^(c/2 + (d*x)/2)])/d + ((4*I)*((-2*x*
PolyLog[2, (-I)*E^(c/2 + (d*x)/2)])/d + (4*PolyLog[3, (-I)*E^(c/2 + (d*x)/
2)])/d^2))/d - ((4*I)*((-2*x*PolyLog[2, I*E^(c/2 + (d*x)/2)])/d + (4*PolyL
og[3, I*E^(c/2 + (d*x)/2)])/d^2))/d))/Sqrt[a + a*Cosh[c + d*x]]
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{\sqrt{a + a\cosh(dx + c)}}dx$$

input
```
int(x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
int(x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{x^2}{\sqrt{a + a\cosh(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cosh(dx + c) + a}}\,dx$$

input
```
integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x^2/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x^2}{\sqrt{a + a\cosh(c + dx)}}\,dx = \int \frac{x^2}{\sqrt{a(\cosh(c + dx) + 1)}}\,dx$$

input
```
integrate(x**2/(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(x**2/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*d^2*integrate(x^2*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^2*e^(2*d*x + 2*
c) + 2*sqrt(a)*d^2*e^(d*x + c) + sqrt(a)*d^2), x) + 8*sqrt(2)*d*integrate(
x*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d^2*e^(2*d*x + 2*c) + 2*sqrt(a)*d^2*e^(d*x
+ c) + sqrt(a)*d^2), x) + 16*sqrt(2)*(e^(1/2*d*x + 1/2*c)/((sqrt(a)*d^2*e^
(d*x + c) + sqrt(a)*d^2)*d) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d^3)) -
 2*(sqrt(2)*d^2*x^2*e^(1/2*c) + 4*sqrt(2)*d*x*e^(1/2*c) + 8*sqrt(2)*e^(1/2
*c))*e^(1/2*d*x)/(sqrt(a)*d^3*e^(d*x + c) + sqrt(a)*d^3)
```

Giac [F]

$$\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2/sqrt(a*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{x^2}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(x^2/(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
int(x^2/(a + a*cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x^2}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}\,x^2}{\cosh(dx+c)+1}dx\right)}{a}$$

input
```
int(x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(cosh(c + d*x) + 1)*x**2)/(cosh(c + d*x) + 1),x))/a
```

3.141 $\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 157

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{4x\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\sqrt{a+a\cosh(c+dx)}} - \frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}} + \frac{4i\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2\sqrt{a+a\cosh(c+dx)}}$$

output
```
4*x*arctan(exp(1/2*d*x+1/2*c))*cosh(1/2*d*x+1/2*c)/d/(a+a*cosh(d*x+c))^(1/
2)-4*I*cosh(1/2*d*x+1/2*c)*polylog(2,-I*exp(1/2*d*x+1/2*c))/d^2/(a+a*cosh(
d*x+c))^(1/2)+4*I*cosh(1/2*d*x+1/2*c)*polylog(2,I*exp(1/2*d*x+1/2*c))/d^2/
(a+a*cosh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.89

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{4i\cosh\left(\frac{1}{2}(c+dx)\right)\left(ic\arctan\left(e^{\frac{1}{2}(c+dx)}\right)+\frac{1}{2}(c+dx)\log\left(1-ie^{\frac{1}{2}(c+dx)}\right)-\frac{1}{2}(c+dx)\log\left(1+ie^{\frac{1}{2}(c+dx)}\right)\right.}{d^2\sqrt{a(1+\cosh(c+dx))}}$$

input
```
Integrate[x/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
((4*I)*Cosh[(c + d*x)/2]*(I*c*ArcTan[E^((c + d*x)/2)] + ((c + d*x)*Log[1 -
 I*E^((c + d*x)/2)])/2 - ((c + d*x)*Log[1 + I*E^((c + d*x)/2)])/2 - PolyLo
g[2, (-I)*E^((c + d*x)/2)] + PolyLog[2, I*E^((c + d*x)/2)]))/(d^2*Sqrt[a*(
1 + Cosh[c + d*x])])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.66, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3800, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{\sqrt{a\cosh(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{x}{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3800}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\int x\text{sech}\left(\frac{c}{2}+\frac{dx}{2}\right)dx}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\int x\csc\left(\frac{ic}{2}+\frac{idx}{2}+\frac{\pi}{2}\right)dx}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{4668}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{2i\int\log\left(1-ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}+\frac{2i\int\log\left(1+ie^{\frac{c}{2}+\frac{dx}{2}}\right)dx}{d}+\frac{4x\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{2715}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-\frac{4i\int e^{-\frac{c}{2}-\frac{dx}{2}}\log\left(1-ie^{\frac{c}{2}+\frac{dx}{2}}\right)de^{\frac{c}{2}+\frac{dx}{2}}}{d^2}+\frac{4i\int e^{-\frac{c}{2}-\frac{dx}{2}}\log\left(1+ie^{\frac{c}{2}+\frac{dx}{2}}\right)de^{\frac{c}{2}+\frac{dx}{2}}}{d^2}+\frac{4x\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}\right)}{\sqrt{a\cosh(c+dx)+a}}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{4x\arctan\left(e^{\frac{c}{2}+\frac{dx}{2}}\right)}{d}-\frac{4i\,\text{PolyLog}\left(2,-ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2}+\frac{4i\,\text{PolyLog}\left(2,ie^{\frac{c}{2}+\frac{dx}{2}}\right)}{d^2}\right)}{\sqrt{a\cosh(c+dx)+a}}$$

input
```
Int[x/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(Cosh[c/2 + (d*x)/2]*((4*x*ArcTan[E^(c/2 + (d*x)/2)])/d - ((4*I)*PolyLog[2
, (-I)*E^(c/2 + (d*x)/2)])/d^2 + ((4*I)*PolyLog[2, I*E^(c/2 + (d*x)/2)])/d
^2))/Sqrt[a + a*Cosh[c + d*x]]
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

Maple [F]

$$\int \frac{x}{\sqrt{a + a\cosh(dx + c)}}dx$$

input
```
int(x/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
int(x/(a+a*cosh(d*x+c))^(1/2),x)
```

Fricas [F]

$$\int \frac{x}{\sqrt{a + a\cosh(c + dx)}}\,dx = \int \frac{x}{\sqrt{a\cosh(dx + c) + a}}\,dx$$

input
```
integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(x/sqrt(a*cosh(d*x + c) + a), x)
```

Sympy [F]

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x}{\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(x/(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(x/sqrt(a*(cosh(c + d*x) + 1)), x)
```

Maxima [F]

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*d*integrate(x*e^(1/2*d*x + 1/2*c)/(sqrt(a)*d*e^(2*d*x + 2*c) + 2
*sqrt(a)*d*e^(d*x + c) + sqrt(a)*d), x) + 4*sqrt(2)*(e^(1/2*d*x + 1/2*c)/(
(sqrt(a)*d*e^(d*x + c) + sqrt(a)*d)*d) + arctan(e^(1/2*d*x + 1/2*c))/(sqrt
(a)*d^2)) - 2*(sqrt(2)*sqrt(a)*d*x*e^(1/2*c) + 2*sqrt(2)*sqrt(a)*e^(1/2*c)
)*e^(1/2*d*x)/(a*d^2*e^(d*x + c) + a*d^2)
```

Giac [F]

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x}{\sqrt{a\cosh\left(dx+c\right)+a}}\,dx$$

input
```
integrate(x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(x/sqrt(a*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{x}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(x/(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
int(x/(a + a*cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{x}{\sqrt{a+a\cosh(c+dx)}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}\,x}{\cosh(dx+c)+1}dx\right)}{a}$$

input
```
int(x/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(cosh(c + d*x) + 1)*x)/(cosh(c + d*x) + 1),x))/a
```

3.142 $\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx=\text{Int}\left(\frac{1}{x\sqrt{a+a\cosh(c+dx)}},x\right)$$

output
```
Defer(Int)(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx$$

input
```
Integrate[1/(x*Sqrt[a + a*Cosh[c + d*x]]),x]
```

output
```
Integrate[1/(x*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{a\cosh(c+dx)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{x\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3807}$$

$$\int \frac{1}{x\sqrt{a\cosh(c+dx)+a}}dx$$

input
```
Int[1/(x*Sqrt[a + a*Cosh[c + d*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x\sqrt{a+a\cosh\left(dx+c\right)}}dx$$

input
```
int(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
int(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a + a\cosh(c + dx)}}\,dx = \int \frac{1}{\sqrt{a\cosh(dx + c) + ax}}\,dx$$

input
```
integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cosh(d*x + c) + a)/(a*x*cosh(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{1}{x\sqrt{a + a\cosh(c + dx)}}\,dx = \int \frac{1}{x\sqrt{a(\cosh(c + dx) + 1)}}\,dx$$

input
```
integrate(1/x/(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(a*(cosh(c + d*x) + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cosh(dx+c)+a}x}\,dx$$

input
```
integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*cosh(d*x + c) + a)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.47 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cosh(dx+c)+a}x}\,dx$$

input
```
integrate(1/x/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cosh(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.86 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{x\,\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(x*(a + a*cosh(c + d*x))^(1/2)),x)
```

output
```
int(1/(x*(a + a*cosh(c + d*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{1}{x\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}}{\cosh(dx+c)x+x}dx\right)}{a}$$

input
```
int(1/x/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(c + d*x) + 1)/(cosh(c + d*x)*x + x),x))/a
```

3.143 $\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \text{Int}\left(\frac{1}{x^2\sqrt{a+a\cosh(c+dx)}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx$$

input
```
Integrate[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]),x]
```

output
```
Integrate[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2\sqrt{a\cosh(c+dx)+a}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x^2\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x^2\sqrt{a\cosh(c+dx)+a}}dx$$

input
```
Int[1/(x^2*Sqrt[a + a*Cosh[c + d*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{1}{x^2\sqrt{a+a\cosh\left(dx+c\right)}}dx$$

input
```
int(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
int(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{1}{\sqrt{a\cosh(dx+c)+ax^2}}\,dx$$

input
```
integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cosh(d*x + c) + a)/(a*x^2*cosh(d*x + c) + a*x^2), x)
```

Sympy [N/A]

Not integrable

Time = 3.55 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx=\int \frac{1}{x^2\sqrt{a\left(\cosh\left(c+dx\right)+1\right)}}\,dx$$

input
```
integrate(1/x**2/(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/(x**2*sqrt(a*(cosh(c + d*x) + 1))), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cosh(dx+c)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(sqrt(a*cosh(d*x + c) + a)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cosh(dx+c)+a}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(sqrt(a*cosh(d*x + c) + a)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 1.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{x^2\,\sqrt{a+a\cosh(c+d\,x)}}\,dx$$

input
```
int(1/(x^2*(a + a*cosh(c + d*x))^(1/2)),x)
```

output
```
int(1/(x^2*(a + a*cosh(c + d*x))^(1/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{1}{x^2\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}}{\cosh(dx+c)x^2+x^2}dx\right)}{a}$$

input
```
int(1/x^2/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(c + d*x) + 1)/(cosh(c + d*x)*x**2 + x**2),x))/a
```

3.144 $\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 402

$$\begin{aligned}\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx &= \frac{3x^2}{a\sqrt{a+a\cosh(x)}}-\frac{24x\arctan\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}\\
&+\frac{x^3\arctan\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{24i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}\\
&-\frac{3ix^2\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}-\frac{24i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}\\
&+\frac{3ix^2\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{12ix\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}\\
&-\frac{12ix\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}-\frac{24i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}\\
&+\frac{24i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(4,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}}+\frac{x^3\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}\end{aligned}$$

output
```
3*x^2/a/(a+a*cosh(x))^(1/2)-24*x*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cos
h(x))^(1/2)+x^3*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x))^(1/2)+24*I*
cosh(1/2*x)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)-3*I*x^2*cosh(1/
2*x)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)-24*I*cosh(1/2*x)*polyl
og(2,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+3*I*x^2*cosh(1/2*x)*polylog(2,I*e
xp(1/2*x))/a/(a+a*cosh(x))^(1/2)+12*I*x*cosh(1/2*x)*polylog(3,-I*exp(1/2*x
))/a/(a+a*cosh(x))^(1/2)-12*I*x*cosh(1/2*x)*polylog(3,I*exp(1/2*x))/a/(a+a
*cosh(x))^(1/2)-24*I*cosh(1/2*x)*polylog(4,-I*exp(1/2*x))/a/(a+a*cosh(x))^
(1/2)+24*I*cosh(1/2*x)*polylog(4,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+1/2*x
^3*tanh(1/2*x)/a/(a+a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.58

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\left(6x^2\cosh\left(\frac{x}{2}\right)+8i\cosh^2\left(\frac{x}{2}\right)\left(-3x\log\left(1-ie^{x/2}\right)+\frac{1}{8}x^3\log\left(1-ie^{x/2}\right)\right.\right.}{}$$

input
```
Integrate[x^3/(a + a*Cosh[x])^(3/2),x]
```

output
```
(Cosh[x/2]*(6*x^2*Cosh[x/2] + (8*I)*Cosh[x/2]^2*(-3*x*Log[1 - I*E^(x/2)] +
 (x^3*Log[1 - I*E^(x/2)])/8 + 3*x*Log[1 + I*E^(x/2)] - (x^3*Log[1 + I*E^(x
/2)])/8 - (3*(-8 + x^2)*PolyLog[2, (-I)*E^(x/2)])/4 + (3*(-8 + x^2)*PolyLo
g[2, I*E^(x/2)])/4 + 3*x*PolyLog[3, (-I)*E^(x/2)] - 3*x*PolyLog[3, I*E^(x/
2)] - 6*PolyLog[4, (-I)*E^(x/2)] + 6*PolyLog[4, I*E^(x/2)]) + x^3*Sinh[x/2
]))/(a*(1 + Cosh[x]))^(3/2)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.57, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {3042, 3800, 3042, 4674, 3042, 4668, 2715, 2838, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{(a\cosh(x)+a)^{3/2}}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{x^3}{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$\Big\downarrow$ 3800

$$\frac{\cosh\left(\frac{x}{2}\right)\int x^3\text{sech}^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a\cosh(x)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\cosh\left(\frac{x}{2}\right)\int x^3\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx}{2a\sqrt{a\cosh(x)+a}}$$

$\Big\downarrow$ 4674

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^3\text{sech}\left(\frac{x}{2}\right)dx-12\int x\text{sech}\left(\frac{x}{2}\right)dx+x^3\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)+6x^2\text{sech}\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh(x)+a}}$$

$\Big\downarrow$ 3042

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^3\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx-12\int x\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx+x^3\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)+6x^2\text{sech}\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh(x)+a}}$$

$\Big\downarrow$ 4668

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-6i\int x^2\log\left(1-ie^{x/2}\right)dx+6i\int x^2\log\left(1+ie^{x/2}\right)dx+4x^3\arctan\left(e^{x/2}\right)\right)-12\left(-2i\int\log\left(1-ie^{x/}\right.\right.\right.}{2a\sqrt{a\cosh(x)+a}}$$

$\Big\downarrow$ 2715

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-6i\int x^2\log\left(1-ie^{x/2}\right)dx+6i\int x^2\log\left(1+ie^{x/2}\right)dx+4x^3\arctan\left(e^{x/2}\right)\right)-12\left(-4i\int e^{-x/2}\log\left(1\right.\right.\right.}{2a\sqrt{a\cosh(}}$$

$\Big\downarrow$ 2838

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-6i\int x^2\log\left(1-ie^{x/2}\right)dx+6i\int x^2\log\left(1+ie^{x/2}\right)dx+4x^3\arctan\left(e^{x/2}\right)\right)-12\left(4x\arctan\left(e^{x/2}\right)-\right.\right.}{2a\sqrt{a\cosh(x)+a}}$$

$\Big\downarrow$ 3011

$$\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(6i\left(4\int x\,\text{PolyLog}\left(2,-ie^{x/2}\right)dx-2x^2\,\text{PolyLog}\left(2,-ie^{x/2}\right)\right)-6i\left(4\int x\,\text{PolyLog}\left(2,ie^{x/2}\right)dx-2x^2\,\text{P}\right.\right.\right.$$

$\big\downarrow$ 7163

$$\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(6i\left(4\left(2x\,\text{PolyLog}\left(3,-ie^{x/2}\right)-2\int\text{PolyLog}\left(3,-ie^{x/2}\right)dx\right)-2x^2\,\text{PolyLog}\left(2,-ie^{x/2}\right)\right)-6i\left(4\left(2x\,\text{Po}\right.\right.\right.\right.$$

$\big\downarrow$ 2720

$$\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(6i\left(4\left(2x\,\text{PolyLog}\left(3,-ie^{x/2}\right)-4\int e^{-x/2}\,\text{PolyLog}\left(3,-ie^{x/2}\right)de^{x/2}\right)-2x^2\,\text{PolyLog}\left(2,-ie^{x/2}\right)\right)-6\right.\right.$$

$\big\downarrow$ 7143

$$\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4x^3\arctan\left(e^{x/2}\right)+6i\left(4\left(2x\,\text{PolyLog}\left(3,-ie^{x/2}\right)-4\,\text{PolyLog}\left(4,-ie^{x/2}\right)\right)-2x^2\,\text{PolyLog}\left(2,-ie^{x/2}\right.\right.\right.\right.$$

input
```
Int[x^3/(a + a*Cosh[x])^(3/2),x]
```

output
```
(Cosh[x/2]*(-12*(4*x*ArcTan[E^(x/2)] - (4*I)*PolyLog[2, (-I)*E^(x/2)] + (4
*I)*PolyLog[2, I*E^(x/2)]) + (4*x^3*ArcTan[E^(x/2)] + (6*I)*(-2*x^2*PolyLo
g[2, (-I)*E^(x/2)] + 4*(2*x*PolyLog[3, (-I)*E^(x/2)] - 4*PolyLog[4, (-I)*E
^(x/2)])) - (6*I)*(-2*x^2*PolyLog[2, I*E^(x/2)] + 4*(2*x*PolyLog[3, I*E^(x
/2)] - 4*PolyLog[4, I*E^(x/2)])))/2 + 6*x^2*Sech[x/2] + x^3*Sech[x/2]*Tanh
[x/2]))/(2*a*Sqrt[a + a*Cosh[x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3}{(a+\cosh(x)\,a)^{\frac{3}{2}}}dx$$

input
```
int(x^3/(a+cosh(x)*a)^(3/2),x)
```

output
```
int(x^3/(a+cosh(x)*a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{x^3}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cosh(x) + a)*x^3/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{x^3}{(a(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**3/(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(x**3/(a*(cosh(x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{x^3}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
8/27*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2
)) + 36*sqrt(2)*integrate(1/9*x^3*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e
^(3*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 72*sqrt(2)*int
egrate(1/9*x^2*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*
e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 96*sqrt(2)*integrate(1/9*x*e^(3/2
*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e
^x + a^(3/2)), x) - 4/27*(9*sqrt(2)*sqrt(a)*x^3 + 18*sqrt(2)*sqrt(a)*x^2 +
 24*sqrt(2)*sqrt(a)*x + 16*sqrt(2)*sqrt(a))*e^(3/2*x)/(a^2*e^(3*x) + 3*a^2
*e^(2*x) + 3*a^2*e^x + a^2)
```

Giac [F]

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx = \int \frac{x^3}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^3/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^3/(a*cosh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx = \int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx$$

input
```
int(x^3/(a + a*cosh(x))^(3/2),x)
```

output
```
int(x^3/(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^3}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)+1}\,x^3}{\cosh(x)^2+2\cosh(x)+1}dx\right)}{a^2}$$

input
```
int(x^3/(a+a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(cosh(x) + 1)*x**3)/(cosh(x)**2 + 2*cosh(x) + 1),x))/a**
2
```

3.145 $\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 248

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx = \frac{2x}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\arctan\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4\arctan\left(\sinh\left(\frac{x}{2}\right)\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{2ix\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{2ix\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{4i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{4i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(3,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x^2\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}$$

output
```
2*x/a/(a+a*cosh(x))^(1/2)+x^2*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x
))^(1/2)-4*arctan(sinh(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x))^(1/2)-2*I*x*cos
h(1/2*x)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+2*I*x*cosh(1/2*x)*
polylog(2,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+4*I*cosh(1/2*x)*polylog(3,-I
*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)-4*I*cosh(1/2*x)*polylog(3,I*exp(1/2*x))
/a/(a+a*cosh(x))^(1/2)+1/2*x^2*tanh(1/2*x)/a/(a+a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.65

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\left(4x\cosh\left(\frac{x}{2}\right)+i\cosh^2\left(\frac{x}{2}\right)\left(16i\arctan\left(e^{x/2}\right)+x^2\log\left(1-ie^{x/2}\right)-x^2\right.\right.}{}$$

input
```
Integrate[x^2/(a + a*Cosh[x])^(3/2),x]
```

output
```
(Cosh[x/2]*(4*x*Cosh[x/2] + I*Cosh[x/2]^2*((16*I)*ArcTan[E^(x/2)] + x^2*Lo
g[1 - I*E^(x/2)] - x^2*Log[1 + I*E^(x/2)] - 4*x*PolyLog[2, (-I)*E^(x/2)] +
 4*x*PolyLog[2, I*E^(x/2)] + 8*PolyLog[3, (-I)*E^(x/2)] - 8*PolyLog[3, I*E
^(x/2)]) + x^2*Sinh[x/2]))/(a*(1 + Cosh[x]))^(3/2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.60, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 3800, 3042, 4674, 3042, 4257, 4668, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{(a\cosh(x)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x^2}{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{x}{2}\right)\int x^2\text{sech}^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{x}{2}\right)\int x^2\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx}{2a\sqrt{a\cosh(x)+a}}$$

$\big\downarrow$ 4674

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^2\text{sech}\left(\frac{x}{2}\right)dx-4\int \text{sech}\left(\frac{x}{2}\right)dx+x^2\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)+4x\text{sech}\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh(x)+a}}$$

$\big\downarrow$ 3042

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^2\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx-4\int \csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx+x^2\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)+4x\text{sech}\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh(x)+a}}$$

$\big\downarrow$ 4257

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x^2\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx-8\arctan\left(\sinh\left(\frac{x}{2}\right)\right)+x^2\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)+4x\text{sech}\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh(x)+a}}$$

$\big\downarrow$ 4668

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-4i\int x\log\left(1-ie^{x/2}\right)dx+4i\int x\log\left(1+ie^{x/2}\right)dx+4x^2\arctan\left(e^{x/2}\right)\right)-8\arctan\left(\sinh\left(\frac{x}{2}\right)\right)+x^2\right.}{2a\sqrt{a\cosh(x)+a}}$$

$\big\downarrow$ 3011

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4i\left(2\int \text{PolyLog}\left(2,-ie^{x/2}\right)dx-2x\,\text{PolyLog}\left(2,-ie^{x/2}\right)\right)-4i\left(2\int \text{PolyLog}\left(2,ie^{x/2}\right)dx-2x\,\text{PolyLo}\right.\right.}{2a\sqrt{a\cosh(x)+a}}$$

$\big\downarrow$ 2720

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4i\left(4\int e^{-x/2}\text{PolyLog}\left(2,-ie^{x/2}\right)de^{x/2}-2x\,\text{PolyLog}\left(2,-ie^{x/2}\right)\right)-4i\left(4\int e^{-x/2}\text{PolyLog}\left(2,ie^{x/2}\right)\right.\right.}{2a\sqrt{a\cosh}}$$

$\big\downarrow$ 7143

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4x^2\arctan\left(e^{x/2}\right)+4i\left(4\,\text{PolyLog}\left(3,-ie^{x/2}\right)-2x\,\text{PolyLog}\left(2,-ie^{x/2}\right)\right)-4i\left(4\,\text{PolyLog}\left(3,ie^{x/2}\right)-\right.\right.\right.}{2a\sqrt{a\cosh(x)+a}}$$

input
```
Int[x^2/(a + a*Cosh[x])^(3/2),x]
```

output
```
(Cosh[x/2]*(-8*ArcTan[Sinh[x/2]] + (4*x^2*ArcTan[E^(x/2)] + (4*I)*(-2*x*Po
lyLog[2, (-I)*E^(x/2)] + 4*PolyLog[3, (-I)*E^(x/2)]) - (4*I)*(-2*x*PolyLog
[2, I*E^(x/2)] + 4*PolyLog[3, I*E^(x/2)]))/2 + 4*x*Sech[x/2] + x^2*Sech[x/
2]*Tanh[x/2]))/(2*a*Sqrt[a + a*Cosh[x]])
```

Defintions of rubi rules used

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4674
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_))^(m_), x_Symbo
l] :> Simp[(-b^2)*(c + d*x)^m*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n
- 1))), x] + (-Simp[b^2*d*m*(c + d*x)^(m - 1)*((b*Csc[e + f*x])^(n - 2)/(f^
2*(n - 1)*(n - 2))), x] + Simp[b^2*d^2*m*((m - 1)/(f^2*(n - 1)*(n - 2)))
Int[(c + d*x)^(m - 2)*(b*Csc[e + f*x])^(n - 2), x], x] + Simp[b^2*((n - 2)/
(n - 1))   Int[(c + d*x)^m*(b*Csc[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c
, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2] && GtQ[m, 1]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2}{(a+\cosh(x)\,a)^{\frac{3}{2}}}dx$$

input
```
int(x^2/(a+cosh(x)*a)^(3/2),x)
```

output
```
int(x^2/(a+cosh(x)*a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{x^2}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cosh(x) + a)*x^2/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F]

$$\int \frac{x^2}{(a + a\cosh(x))^{3/2}}\,dx = \int \frac{x^2}{(a(\cosh(x) + 1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x**2/(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(x**2/(a*(cosh(x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x^2}{(a + a\cosh(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cosh(x) + a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
4/27*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2
)) + 36*sqrt(2)*integrate(1/9*x^2*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e
^(3*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) + 48*sqrt(2)*int
egrate(1/9*x*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3*x) + 6*a^(3/2)*e^
(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) - 4/27*(9*sqrt(2)*x^2 + 12*sqrt(2)*x
+ 8*sqrt(2))*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^
x + a^(3/2))
```

Giac [F]

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx = \int \frac{x^2}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x^2/(a*cosh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx = \int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx$$

input
```
int(x^2/(a + a*cosh(x))^(3/2),x)
```

output
```
int(x^2/(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x^2}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)+1}\,x^2}{\cosh(x)^2+2\cosh(x)+1}dx\right)}{a^2}$$

input
```
int(x^2/(a+a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(cosh(x) + 1)*x**2)/(cosh(x)**2 + 2*cosh(x) + 1),x))/a**
2
```

3.146 $\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 140

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx = \frac{1}{a\sqrt{a+a\cosh(x)}} + \frac{x\arctan\left(e^{x/2}\right)\cosh\left(\frac{x}{2}\right)}{a\sqrt{a+a\cosh(x)}} - \frac{i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,-ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{i\cosh\left(\frac{x}{2}\right)\operatorname{PolyLog}\left(2,ie^{x/2}\right)}{a\sqrt{a+a\cosh(x)}} + \frac{x\tanh\left(\frac{x}{2}\right)}{2a\sqrt{a+a\cosh(x)}}$$

output
```
1/a/(a+a*cosh(x))^(1/2)+x*arctan(exp(1/2*x))*cosh(1/2*x)/a/(a+a*cosh(x))^(
1/2)-I*cosh(1/2*x)*polylog(2,-I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+I*cosh(1
/2*x)*polylog(2,I*exp(1/2*x))/a/(a+a*cosh(x))^(1/2)+1/2*x*tanh(1/2*x)/a/(a
+a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)+i\cosh^2\left(\frac{x}{2}\right)\left(x\left(\log\left(1-ie^{x/2}\right)-\log\left(1+ie^{x/2}\right)\right)-2\,\mathrm{Pol}\right.\right.}{(a(1+\cosh(x)))^{3/2}}$$

input
```
Integrate[x/(a + a*Cosh[x])^(3/2),x]
```

output
```
(Cosh[x/2]*(2*Cosh[x/2] + I*Cosh[x/2]^2*(x*(Log[1 - I*E^(x/2)] - Log[1 + I
*E^(x/2)]) - 2*PolyLog[2, (-I)*E^(x/2)] + 2*PolyLog[2, I*E^(x/2)]) + x*Sin
h[x/2]))/(a*(1 + Cosh[x]))^(3/2)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.69, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3800, 3042, 4673, 3042, 4668, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{(a\cosh(x)+a)^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{x}{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3800}$$

$$\frac{\cosh\left(\frac{x}{2}\right)\int x\text{sech}^3\left(\frac{x}{2}\right)dx}{2a\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh\left(\frac{x}{2}\right)\int x\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)^3dx}{2a\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \text{4673}$$

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x\text{sech}\left(\frac{x}{2}\right)dx+2\text{sech}\left(\frac{x}{2}\right)+x\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\int x\csc\left(\frac{ix}{2}+\frac{\pi}{2}\right)dx+2\text{sech}\left(\frac{x}{2}\right)+x\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\right)}{2a\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \; 4668$$

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-2i\int\log\left(1-ie^{x/2}\right)dx+2i\int\log\left(1+ie^{x/2}\right)dx+4x\arctan\left(e^{x/2}\right)\right)+2\text{sech}\left(\frac{x}{2}\right)+x\tanh\left(\frac{x}{2}\right)\text{sech}(\right.}{2a\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \; 2715$$

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(-4i\int e^{-x/2}\log\left(1-ie^{x/2}\right)de^{x/2}+4i\int e^{-x/2}\log\left(1+ie^{x/2}\right)de^{x/2}+4x\arctan\left(e^{x/2}\right)\right)+2\text{sech}\left(\frac{x}{2}\right)+\right.}{2a\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \; 2838$$

$$\frac{\cosh\left(\frac{x}{2}\right)\left(\frac{1}{2}\left(4x\arctan\left(e^{x/2}\right)-4i\,\text{PolyLog}\left(2,-ie^{x/2}\right)+4i\,\text{PolyLog}\left(2,ie^{x/2}\right)\right)+2\text{sech}\left(\frac{x}{2}\right)+x\tanh\left(\frac{x}{2}\right)\text{sech}\left(\frac{x}{2}\right)\right.}{2a\sqrt{a\cosh(x)+a}}$$

input
```
Int[x/(a + a*Cosh[x])^(3/2),x]
```

output
```
(Cosh[x/2]*((4*x*ArcTan[E^(x/2)] - (4*I)*PolyLog[2, (-I)*E^(x/2)] + (4*I)*
PolyLog[2, I*E^(x/2)])/2 + 2*Sech[x/2] + x*Sech[x/2]*Tanh[x/2]))/(2*a*Sqrt
[a + a*Cosh[x]])
```

Defintions of rubi rules used

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3800
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Simp[(2*a)^IntPart[n]*((a + b*Sin[e + f*x])^FracPart[n]/Sin[e
/2 + a*(Pi/(4*b)) + f*(x/2)]^(2*FracPart[n]))   Int[(c + d*x)^m*Sin[e/2 + a
*(Pi/(4*b)) + f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &&
EqQ[a^2 - b^2, 0] && IntegerQ[n + 1/2] && (GtQ[n, 0] || IGtQ[m, 0])
```

rule 4668
```
Int[csc[(e_.) + Pi*(k_.) + (Complex[0, fz_])*(f_.)*(x_)]*((c_.) + (d_.)*(x_
))^(m_.), x_Symbol] :> Simp[-2*(c + d*x)^m*(ArcTanh[E^((-I)*e + f*fz*x)/E^(
I*k*Pi)]/(f*fz*I)), x] + (-Simp[d*(m/(f*fz*I))   Int[(c + d*x)^(m - 1)*Log[
1 - E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x] + Simp[d*(m/(f*fz*I))   Int[(c
+ d*x)^(m - 1)*Log[1 + E^((-I)*e + f*fz*x)/E^(I*k*Pi)], x], x]) /; FreeQ[{c
, d, e, f, fz}, x] && IntegerQ[2*k] && IGtQ[m, 0]
```

rule 4673
```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((c_.) + (d_.)*(x_)), x_Symbol] :>
 Simp[(-b^2)*(c + d*x)*Cot[e + f*x]*((b*Csc[e + f*x])^(n - 2)/(f*(n - 1))),
 x] + (-Simp[b^2*d*((b*Csc[e + f*x])^(n - 2)/(f^2*(n - 1)*(n - 2))), x] + S
imp[b^2*((n - 2)/(n - 1))   Int[(c + d*x)*(b*Csc[e + f*x])^(n - 2), x], x])
 /; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && NeQ[n, 2]
```

Maple [F]

$$\int \frac{x}{(a+\cosh(x)\,a)^{\frac{3}{2}}}dx$$

input
```
int(x/(a+cosh(x)*a)^(3/2),x)
```

output
```
int(x/(a+cosh(x)*a)^(3/2),x)
```

Fricas [F]

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx = \int \frac{x}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cosh(x) + a)*x/(a^2*cosh(x)^2 + 2*a^2*cosh(x) + a^2), x)
```

Sympy [F]

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx = \int \frac{x}{(a(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(x/(a*(cosh(x) + 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx = \int \frac{x}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
1/9*sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2)
) + 12*sqrt(2)*integrate(1/3*x*e^(3/2*x)/(a^(3/2)*e^(4*x) + 4*a^(3/2)*e^(3
*x) + 6*a^(3/2)*e^(2*x) + 4*a^(3/2)*e^x + a^(3/2)), x) - 4/9*(3*sqrt(2)*sq
rt(a)*x + 2*sqrt(2)*sqrt(a))*e^(3/2*x)/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^
2*e^x + a^2)
```

Giac [F]

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{x}{(a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(x/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(x/(a*cosh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx$$

input
```
int(x/(a + a*cosh(x))^(3/2),x)
```

output
```
int(x/(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{x}{(a+a\cosh(x))^{3/2}}\,dx=\frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)+1}\,x}{\cosh(x)^2+2\cosh(x)+1}dx\right)}{a^2}$$

input
```
int(x/(a+a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*int((sqrt(cosh(x) + 1)*x)/(cosh(x)**2 + 2*cosh(x) + 1),x))/a**2
```

3.147 $\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x(a+a\cosh(x))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x/(a+a*cosh(x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 7.77 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx = \int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx$$

input
```
Integrate[1/(x*(a + a*Cosh[x])^(3/2)),x]
```

output
```
Integrate[1/(x*(a + a*Cosh[x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x(a\cosh(x)+a)^{3/2}} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{x\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{x(a\cosh(x)+a)^{3/2}}dx$$

input
```
Int[1/(x*(a + a*Cosh[x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x\left(a+\cosh\left(x\right)a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x/(a+cosh(x)*a)^(3/2),x)
```

output
```
int(1/x/(a+cosh(x)*a)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.57

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cosh(x) + a)/(a^2*x*cosh(x)^2 + 2*a^2*x*cosh(x) + a^2*x),
x)
```

Sympy [N/A]

Not integrable

Time = 8.24 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{x\,(a\,(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(1/(x*(a*(cosh(x) + 1))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cosh(x) + a)^(3/2)*x), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh{(x)}+a)^{\frac{3}{2}}x}\,dx$$

input
```
integrate(1/x/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cosh(x) + a)^(3/2)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{x\,(a+a\cosh{(x)})^{3/2}}\,dx$$

input
```
int(1/(x*(a + a*cosh(x))^(3/2)),x)
```

output
```
int(1/(x*(a + a*cosh(x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 29, normalized size of antiderivative = 2.07

$$\int \frac{1}{x(a+a\cosh(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)+1}}{\cosh(x)^2 x+2\cosh(x)x+x}dx\right)}{a^2}$$

input
```
int(1/x/(a+a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x) + 1)/(cosh(x)**2*x + 2*cosh(x)*x + x),x))/a**2
```

3.148 $\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx = \text{Int}\left(\frac{1}{x^2(a+a\cosh(x))^{3/2}},x\right)$$

output
```
Defer(Int)(1/x^2/(a+a*cosh(x))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 8.71 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx = \int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx$$

input
```
Integrate[1/(x^2*(a + a*Cosh[x])^(3/2)),x]
```

output
```
Integrate[1/(x^2*(a + a*Cosh[x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x^2(a\cosh(x)+a)^{3/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{x^2\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{1}{x^2(a\cosh(x)+a)^{3/2}}dx$$

input
```
Int[1/(x^2*(a + a*Cosh[x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{1}{x^2\left(a+\cosh\left(x\right)a\right)^{\frac{3}{2}}}dx$$

input
```
int(1/x^2/(a+cosh(x)*a)^(3/2),x)
```

output
```
int(1/x^2/(a+cosh(x)*a)^(3/2),x)
```

Fricas [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
integral(sqrt(a*cosh(x) + a)/(a^2*x^2*cosh(x)^2 + 2*a^2*x^2*cosh(x) + a^2*
x^2), x)
```

Sympy [N/A]

Not integrable

Time = 14.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{x^2\,(a\,(\cosh(x)+1))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x**2/(a+a*cosh(x))**(3/2),x)
```

output
```
Integral(1/(x**2*(a*(cosh(x) + 1))**(3/2)), x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx=\int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((a*cosh(x) + a)^(3/2)*x^2), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x)+a)^{\frac{3}{2}}x^2}\,dx$$

input
```
integrate(1/x^2/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((a*cosh(x) + a)^(3/2)*x^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx = \int \frac{1}{x^2\,(a+a\cosh(x))^{3/2}}\,dx$$

input
```
int(1/(x^2*(a + a*cosh(x))^(3/2)),x)
```

output
```
int(1/(x^2*(a + a*cosh(x))^(3/2)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{1}{x^2(a+a\cosh(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)+1}}{\cosh(x)^2x^2+2\cosh(x)x^2+x^2}dx\right)}{a^2}$$

input
```
int(1/x^2/(a+a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x) + 1)/(cosh(x)**2*x**2 + 2*cosh(x)*x**2 + x**2),x
))/a**2
```

3.149 $\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \mathrm{Int}\left(\frac{\sqrt[3]{a+a\cosh(c+dx)}}{x},x\right)$$

output
```
Defer(Int)((a+a*cosh(d*x+c))^(1/3)/x,x)
```

Mathematica [N/A]

Not integrable

Time = 2.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(1/3)/x,x]
```

output
```
Integrate[(a + a*Cosh[c + d*x])^(1/3)/x, x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt[3]{a\cosh(c+dx)+a}}{x}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt[3]{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}{x}dx$$

$$\Big\downarrow \; 3807$$

$$\int \frac{\sqrt[3]{a\cosh(c+dx)+a}}{x}dx$$

input
```
Int[(a + a*Cosh[c + d*x])^(1/3)/x,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.89

$$\int \frac{(a+a\cosh(dx+c))^{\frac{1}{3}}}{x}dx$$

input
```
int((a+a*cosh(d*x+c))^(1/3)/x,x)
```

output
```
int((a+a*cosh(d*x+c))^(1/3)/x,x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [N/A]

Not integrable

Time = 1.62 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \int \frac{\sqrt[3]{a\left(\cosh\left(c+dx\right)+1\right)}}{x}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))**(1/3)/x,x)
```

output
```
Integral((a*(cosh(c + d*x) + 1))**(1/3)/x, x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \int \frac{(a\cosh(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="maxima")
```

output
```
integrate((a*cosh(d*x + c) + a)^(1/3)/x, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \int \frac{(a\cosh(dx+c)+a)^{\frac{1}{3}}}{x}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))^(1/3)/x,x, algorithm="giac")
```

output
```
integrate((a*cosh(d*x + c) + a)^(1/3)/x, x)
```

Mupad [N/A]

Not integrable

Time = 1.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt[3]{a+a\cosh(c+dx)}}{x}\,dx = \int \frac{(a+a\cosh(c+d\,x))^{1/3}}{x}\,dx$$

input
```
int((a + a*cosh(c + d*x))^(1/3)/x,x)
```

output
```
int((a + a*cosh(c + d*x))^(1/3)/x, x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{\sqrt[3]{a + a\cosh(c + dx)}}{x}\,dx = a^{\frac{1}{3}}\left(\int \frac{(\cosh(dx + c) + 1)^{\frac{1}{3}}}{x}dx\right)$$

input
```
int((a+a*cosh(d*x+c))^(1/3)/x,x)
```

output
```
a**(1/3)*int((cosh(c + d*x) + 1)**(1/3)/x,x)
```

3.150 $\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx = \text{Int}\left((c+dx)^m(a+a\cosh(e+fx))^n,x\right)$$

output
```
Defer(Int)((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 4.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx = \int (c+dx)^m(a+a\cosh(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a\cosh(e+fx)+a)^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3807}$$

$$\int (c+dx)^m(a\cosh(e+fx)+a)^n dx$$

input
```
Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m\,(a+a\cosh(fx+e))^n\,dx$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+a\cosh(e+fx))^n \, dx = \int (dx+c)^m (a\cosh(fx+e)+a)^n \, dx$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m (a+a\cosh(e+fx))^n \, dx = \text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+a*cosh(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+a\cosh(e+fx))^n \, dx = \int (dx+c)^m (a\cosh(fx+e)+a)^n \, dx$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx=\int (dx+c)^m(a\cosh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(a*cosh(f*x + e) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx=\int (a+a\cosh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((a + a*cosh(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((a + a*cosh(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+a\cosh(e+fx))^n\,dx=\int (dx+c)^m\,(\cosh(fx+e)\,a+a)^n\,dx$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^n,x)
```

output

```
int((c + d*x)**m*(cosh(e + f*x)*a + a)**n,x)
```

3.151 $\int (c+dx)^m (a+a\cosh(e+fx))^3\,dx$

Optimal result

Integrand size = 20, antiderivative size = 402

$$\begin{aligned}\int (c+dx)^m (a+a\cosh(e+fx))^3\,dx &= \frac{5a^3(c+dx)^{1+m}}{2d(1+m)}+\frac{3^{-1-m}a^3e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f}\\&+\frac{3\ 2^{-3-m}a^3e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}\\&+\frac{15a^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f}\\&-\frac{15a^3e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f}\\&-\frac{3\ 2^{-3-m}a^3e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}\\&-\frac{3^{-1-m}a^3e^{-3e+\frac{3cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}\end{aligned}$$

output
```
5/2*a^3*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*a^3*exp(3*e-3*c*f/d)*(d*x+c)^m*
GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+3*2^(-3-m)*a^3*exp(2*e-2*c*
f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+15/8*a^3*exp
(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-15/8*a^3*
exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-3*2^(-3
-m)*a^3*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)
/d)^m)-1/8*3^(-1-m)*a^3*exp(-3*e+3*c*f/d)*(d*x+c)^m*GAMMA(1+m,3*f*(d*x+c)/
d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (warning: unable to verify)

Time = 1.52 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.07

$$\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx =$$

$$-\frac{2^{-6-m}3^{-1-m}a^3e^{-3\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}(1+\cosh(e+fx))^3\left(-2^mde^{6e}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^3,x]
```

output
```
-((2^(-6 - m)*3^(-1 - m)*a^3*(c + d*x)^m*(1 + Cosh[e + f*x])^3*(-(2^m*d*E^
(6*e)*(1 + m)*((f*(c + d*x))/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d]) - 3^(2
 + m)*d*E^(5*e + (c*f)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c +
d*x))/d] - 5*2^m*3^(2 + m)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)
^m*Gamma[1 + m, -((f*(c + d*x))/d)] + 5*2^m*3^(2 + m)*d*E^(2*e + (4*c*f)/d
)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d] + 3^(2 + m)
*d*E^(e + (5*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (2*f*(c +
 d*x))/d] + 2^m*E^((3*c*f)/d)*(-20*3^(1 + m)*E^(3*e)*f*(c + d*x)*(-((f^2*(
c + d*x)^2)/d^2))^m + d*E^((3*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma
[1 + m, (3*f*(c + d*x))/d]))*Sech[(e + f*x)/2]^6)/(d*E^(3*(e + (c*f)/d))*f
*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a\cosh(e+fx)+a)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m \left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^3 dx$$

$$\Big\downarrow \text{3799}$$

$$8a^3\int (c+dx)^m\cosh^6\left(\frac{e}{2}+\frac{fx}{2}\right)dx$$

$$\Big\downarrow \text{3042}$$

$$8a^3\int (c+dx)^m\sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^6 dx$$

$$\Big\downarrow \text{3793}$$

$$8a^3\int \left(\frac{15}{32}\cosh(e+fx)(c+dx)^m+\frac{3}{16}\cosh(2e+2fx)(c+dx)^m+\frac{1}{32}\cosh(3e+3fx)(c+dx)^m+\frac{5}{16}(c+dx)^m$$

$$\Big\downarrow \text{2009}$$

$$8a^3\left(\frac{3^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3f(c+dx)}{d}\right)}{64f}+\frac{3\,2^{-m-6}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}}{f}\right.$$

input
```
Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^3,x]
```

output
```
8*a^3*((5*(c + d*x)^(1 + m))/(16*d*(1 + m)) + (3^(-1 - m)*E^(3*e - (3*c*f)
/d)*(c + d*x)^m*Gamma[1 + m, (-3*f*(c + d*x))/d])/(64*f*(-((f*(c + d*x))/d
))^m) + (3*2^(-6 - m)*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(
c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (15*E^(e - (c*f)/d)*(c + d*x)^m
*Gamma[1 + m, -((f*(c + d*x))/d)])/(64*f*(-((f*(c + d*x))/d))^m) - (15*E^(
-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(64*f*((f*(c + d*
x))/d)^m) - (3*2^(-6 - m)*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2
*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m) - (3^(-1 - m)*E^(-3*e + (3*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c + d*x))/d])/(64*f*((f*(c + d*x))/d)^m)
)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int (dx+c)^m (a+a\cosh(fx+e))^3 \, dx$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.77

$$\int (c+dx)^m(a+a\cosh(e+fx))^3 \, dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/24*((a^3*d*m + a^3*d)*cosh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(m
+ 1, 3*(d*f*x + c*f)/d) + 9*(a^3*d*m + a^3*d)*cosh((d*m*log(2*f/d) + 2*d*e
 - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 45*(a^3*d*m + a^3*d)*cosh((
d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 45*(a^3*d*m +
 a^3*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d)
 - 9*(a^3*d*m + a^3*d)*cosh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m +
 1, -2*(d*f*x + c*f)/d) - (a^3*d*m + a^3*d)*cosh((d*m*log(-3*f/d) - 3*d*e
+ 3*c*f)/d)*gamma(m + 1, -3*(d*f*x + c*f)/d) - (a^3*d*m + a^3*d)*gamma(m +
 1, 3*(d*f*x + c*f)/d)*sinh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d) - 9*(a^3*d
*m + a^3*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e -
 2*c*f)/d) - 45*(a^3*d*m + a^3*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*m*
log(f/d) + d*e - c*f)/d) + 45*(a^3*d*m + a^3*d)*gamma(m + 1, -(d*f*x + c*f
)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + 9*(a^3*d*m + a^3*d)*gamma(m + 1
, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) + (a^3*d*m
 + a^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/d)*sinh((d*m*log(-3*f/d) - 3*d*e +
 3*c*f)/d) - 60*(a^3*d*f*x + a^3*c*f)*cosh(m*log(d*x + c)) - 60*(a^3*d*f*x
 + a^3*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+a*cosh(f*x+e))**3,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 373, normalized size of antiderivative = 0.93

$$\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx = -\frac{1}{8}\left(\frac{(dx+c)^{m+1}e^{\left(-3e+\frac{3cf}{d}\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e}{}\right.$$
$$-\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2(dx+c)^{m+}}{d(m+1)}\right.$$
$$-\frac{3}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^3$$
$$+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8*((d*x + c)^(m + 1)*e^(-3*e + 3*c*f/d)*exp_integral_e(-m, 3*(d*x + c)*
f/d)/d + 3*(d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f
/d)/d + 3*(d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/
d)/d + (d*x + c)^(m + 1)*e^(3*e - 3*c*f/d)*exp_integral_e(-m, -3*(d*x + c)
*f/d)/d)*a^3 - 3/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m
, 2*(d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(
-m, -2*(d*x + c)*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^3 - 3/2*((d*x
 + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d + (d*x +
c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^3 + (d*x
+ c)^(m + 1)*a^3/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx=\int (a\cosh(fx+e)+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((a*cosh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx=\int (a+a\cosh(e+f\,x))^3\,(c+d\,x)^m\,dx$$

input
```
int((a + a*cosh(e + f*x))^3*(c + d*x)^m,x)
```

output
```
int((a + a*cosh(e + f*x))^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+a\cosh(e+fx))^3\,dx$$
$$= \frac{a^3\left(e^{6fx+6e}(dx+c)^m dm + e^{6fx+6e}(dx+c)^m d + 9e^{5fx+5e}(dx+c)^m d + 45e^{4fx+4e}(dx+c)^m d - 45e^{2fx+2}\right.}{}$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^3,x)
```

output
```
(a**3*(e**(6*e + 6*f*x)*(c + d*x)**m*d*m + e**(6*e + 6*f*x)*(c + d*x)**m*d
 + 9*e**(5*e + 5*f*x)*(c + d*x)**m*d*m + 9*e**(5*e + 5*f*x)*(c + d*x)**m*d
 + 45*e**(4*e + 4*f*x)*(c + d*x)**m*d*m + 45*e**(4*e + 4*f*x)*(c + d*x)**m
*d + 60*e**(3*e + 3*f*x)*(c + d*x)**m*c*f + 60*e**(3*e + 3*f*x)*(c + d*x)*
*m*d*f*x - 45*e**(2*e + 2*f*x)*(c + d*x)**m*d*m - 45*e**(2*e + 2*f*x)*(c +
 d*x)**m*d - 9*e**(e + f*x)*(c + d*x)**m*d*m - 9*e**(e + f*x)*(c + d*x)**m
*d - (c + d*x)**m*d*m - (c + d*x)**m*d - e**(6*e + 3*f*x)*int((e**(3*f*x)*
(c + d*x)**m)/(c + d*x),x)*d**2*m**2 - e**(6*e + 3*f*x)*int((e**(3*f*x)*(c
 + d*x)**m)/(c + d*x),x)*d**2*m - 9*e**(5*e + 3*f*x)*int((e**(2*f*x)*(c +
d*x)**m)/(c + d*x),x)*d**2*m**2 - 9*e**(5*e + 3*f*x)*int((e**(2*f*x)*(c +
d*x)**m)/(c + d*x),x)*d**2*m - 45*e**(4*e + 3*f*x)*int((e**(f*x)*(c + d*x)
**m)/(c + d*x),x)*d**2*m**2 - 45*e**(4*e + 3*f*x)*int((e**(f*x)*(c + d*x)*
*m)/(c + d*x),x)*d**2*m + e**(3*e + 3*f*x)*int((c + d*x)**m/(e**(3*e + 3*f
*x)*c + e**(3*e + 3*f*x)*d*x),x)*d**2*m**2 + e**(3*e + 3*f*x)*int((c + d*x
)**m/(e**(3*e + 3*f*x)*c + e**(3*e + 3*f*x)*d*x),x)*d**2*m + 45*e**(2*e +
3*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*d**2*m**2 + 45*e**(
2*e + 3*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*d**2*m + 9*e*
*(e + 3*f*x)*int((c + d*x)**m/(e**(2*f*x)*c + e**(2*f*x)*d*x),x)*d**2*m**2
 + 9*e**(e + 3*f*x)*int((c + d*x)**m/(e**(2*f*x)*c + e**(2*f*x)*d*x),x)*d*
*2*m))/(24*e**(3*e + 3*f*x)*d*f*(m + 1))
```

3.152 $\int (c+dx)^m (a+a\cosh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 263

$$\begin{aligned}
\int (c+dx)^m (a+a\cosh(e+fx))^2\,dx &= \frac{3a^2(c+dx)^{1+m}}{2d(1+m)} + \frac{2^{-3-m}a^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f}\\
&+ \frac{a^2e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f}\\
&- \frac{a^2e^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f}\\
&- \frac{2^{-3-m}a^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}
\end{aligned}$$

output

```
3/2*a^2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*a^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMM
A(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a^2*exp(e-c*f/d)*(d*x+c)^m*GAMM
A(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-a^2*exp(-e+c*f/d)*(d*x+c)^m*GAMMA
(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^(-3-m)*a^2*exp(-2*e+2*c*f/d)*(d*x+
c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (warning: unable to verify)

Time = 0.68 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.15

$$\int (c+dx)^m (a+a\cosh(e+fx))^2\,dx =$$

$$-\frac{2^{-5-m}a^2e^{-2\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}(1+\cosh(e+fx))^2\left(-32^{2+m}e^{2\left(e+\frac{cf}{d}\right)}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right.\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x])^2,x]
```

output
```
-((2^(-5 - m)*a^2*(c + d*x)^m*(1 + Cosh[e + f*x])^2*(-3*2^(2 + m)*E^(2*(e
+ (c*f)/d))*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m - d*E^(4*e)*(1 + m)*(
f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] - 2^(3 + m)*d*E^(3*e + (c*
f)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, -((f*(c + d*x))/d)] + 2^(3 + m)
*d*E^(e + (3*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c + d
*x))/d] + d*E^((4*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (2*f
*(c + d*x))/d])*Sech[(e + f*x)/2]^4)/(d*E^(2*(e + (c*f)/d))*f*(1 + m)*(-((
f^2*(c + d*x)^2)/d^2))^m))
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 258, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3799, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a\cosh(e+fx)+a)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m \left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\Big\downarrow \text{3799}$$

$$4a^2 \int (c+dx)^m \cosh^4\left(\frac{e}{2}+\frac{fx}{2}\right)dx$$

$\big\downarrow$ 3042

$$4a^2 \int (c+dx)^m \sin\left(\frac{ie}{2}+\frac{ifx}{2}+\frac{\pi}{2}\right)^4 dx$$

$\big\downarrow$ 3793

$$4a^2 \int \left(\frac{1}{2}\cosh(e+fx)(c+dx)^m+\frac{1}{8}\cosh(2e+2fx)(c+dx)^m+\frac{3}{8}(c+dx)^m\right)dx$$

$\big\downarrow$ 2009

$$4a^2\left(\frac{2^{-m-5}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}+\frac{e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\right.}{4f}\right.$$

input
```
Int[(c + d*x)^m*(a + a*Cosh[e + f*x])^2,x]
```

output
```
4*a^2*((3*(c + d*x)^(1 + m))/(8*d*(1 + m)) + (2^(-5 - m)*E^(2*e - (2*c*f)/
d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d*x))/d))^m
) + (E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(4*f*(-
((f*(c + d*x))/d))^m) - (E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c +
 d*x))/d])/(4*f*((f*(c + d*x))/d)^m) - (2^(-5 - m)*E^(-2*e + (2*c*f)/d)*(c
 + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 3799
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Simp[(2*a)^n   Int[(c + d*x)^m*Sin[(1/2)*(e + Pi*(a/(2*b)))
+ f*(x/2)]^(2*n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 - b^
2, 0] && IntegerQ[n] && (GtQ[n, 0] || IGtQ[m, 0])
```

Maple [F]

$$\int (dx+c)^m\,(a+a\cosh(fx+e))^2\,dx$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 493, normalized size of antiderivative = 1.87

$$\int (c+dx)^m(a+a\cosh(e+fx))^2\,dx = -\frac{(a^2dm+a^2d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2\,de-2\,cf}{d}\right)\Gamma\left(m+1,\frac{2\,(dfx+cf)}{d}\right)+8\,(a^2dm+a^2d)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right.}{}$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/8*((a^2*d*m + a^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m +
 1, 2*(d*f*x + c*f)/d) + 8*(a^2*d*m + a^2*d)*cosh((d*m*log(f/d) + d*e - c*
f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a^2*d*m + a^2*d)*cosh((d*m*log(-f
/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (a^2*d*m + a^2*d)*cos
h((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) -
(a^2*d*m + a^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2
*d*e - 2*c*f)/d) - 8*(a^2*d*m + a^2*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh(
(d*m*log(f/d) + d*e - c*f)/d) + 8*(a^2*d*m + a^2*d)*gamma(m + 1, -(d*f*x +
 c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (a^2*d*m + a^2*d)*gamma(m +
 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 12*(a^
2*d*f*x + a^2*c*f)*cosh(m*log(d*x + c)) - 12*(a^2*d*f*x + a^2*c*f)*sinh(m*
log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m (a+a\cosh(e+fx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+a*cosh(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.79

$$\int (c+dx)^m(a+a\cosh(e+fx))^2\,dx = -\frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2(dx+c)^{m+1}}{d(m+1)}\right.$$
$$-\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2$$
$$+\frac{(dx+c)^{m+1}a^2}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x + c)*
f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d*x +
c)*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a^2 - ((d*x + c)^(m + 1)*e^(-
e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d + (d*x + c)^(m + 1)*e^(e -
c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a^2 + (d*x + c)^(m + 1)*a^2/(
d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+a\cosh(e+fx))^2\,dx = \int (a\cosh(fx+e)+a)^2(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*cosh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+a\cosh(e+fx))^2\,dx=\int (a+a\cosh(e+f\,x))^2\,(c+d\,x)^m\,dx$$

input
```
int((a + a*cosh(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + a*cosh(e + f*x))^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+a\cosh(e+fx))^2\,dx = \frac{a^2\Big(e^{4fx+4e}(dx+c)^m\,dm+e^{4fx+4e}(dx+c)^m\,d+8e^{3fx+3e}(dx+c)^m\,dm+8e^{3fx+3e}(dx+c)^m\,d+12e^{2fx+}}{}$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e))^2,x)
```

output
```
(a**2*(e**(4*e + 4*f*x)*(c + d*x)**m*d*m + e**(4*e + 4*f*x)*(c + d*x)**m*d
 + 8*e**(3*e + 3*f*x)*(c + d*x)**m*d*m + 8*e**(3*e + 3*f*x)*(c + d*x)**m*d
 + 12*e**(2*e + 2*f*x)*(c + d*x)**m*c*f + 12*e**(2*e + 2*f*x)*(c + d*x)**m
*d*f*x - 8*e**(e + f*x)*(c + d*x)**m*d*m - 8*e**(e + f*x)*(c + d*x)**m*d -
 (c + d*x)**m*d*m - (c + d*x)**m*d - e**(4*e + 2*f*x)*int((e**(2*f*x)*(c +
 d*x)**m)/(c + d*x),x)*d**2*m**2 - e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d
*x)**m)/(c + d*x),x)*d**2*m - 8*e**(3*e + 2*f*x)*int((e**(f*x)*(c + d*x)**
m)/(c + d*x),x)*d**2*m**2 - 8*e**(3*e + 2*f*x)*int((e**(f*x)*(c + d*x)**m)
/(c + d*x),x)*d**2*m + e**(2*e + 2*f*x)*int((c + d*x)**m/(e**(2*e + 2*f*x)
*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m**2 + e**(2*e + 2*f*x)*int((c + d*x)**
m/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m + 8*e**(e + 2*f*x)
*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*d**2*m**2 + 8*e**(e + 2*f
*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*d**2*m))/(8*e**(2*e +
2*f*x)*d*f*(m + 1))
```

3.153 $\int (c+dx)^m(a+a\cosh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx = \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} - \frac{ae^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)+1/2*a*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/
d)/f/((-f*(d*x+c)/d)^m)-1/2*a*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/
d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.44

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx =$$
$$-\frac{ae^{-e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}(1+\cosh(e+fx))\left(-2e^{e+\frac{cf}{d}}f(c+dx)\left(-\frac{f^2(c+dx)^2}{d^2}\right)^m-de^{2e}(1+\cdots\right.}{4df(\cdots}$$

input
```
Integrate[(c + d*x)^m*(a + a*Cosh[e + f*x]),x]
```

output
```
-1/4*(a*E^(-e - (c*f)/d)*(c + d*x)^m*(1 + Cosh[e + f*x])*(-2*E^(e + (c*f)/
d)*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m - d*E^(2*e)*(1 + m)*(f*(c/d +
x))^m*Gamma[1 + m, -((f*(c + d*x))/d)] + d*E^((2*c*f)/d)*(1 + m)*(-((f*(c
+ d*x))/d))^m*Gamma[1 + m, (f*(c + d*x))/d])*Sech[(e + f*x)/2]^2)/(d*f*(1
+ m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a\cosh(e+fx)+a)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a(c+dx)^m\cosh(e+fx)+a(c+dx)^m\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{ae^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{ae^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{a(c+dx)^{m+1}}{d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + a*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) + (a*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
 + m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (a*E^(-e + (c*f)
/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m\,(a+a\cosh(fx+e))\,dx$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+a*cosh(f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.90

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx = -\frac{(adm+ad)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1,\frac{dfx+cf}{d}\right)-(adm+ad)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)\Gamma\left(m+1\right.}{}$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
-1/2*((a*d*m + a*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
 + c*f)/d) - (a*d*m + a*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1
, -(d*f*x + c*f)/d) - (a*d*m + a*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*
m*log(f/d) + d*e - c*f)/d) + (a*d*m + a*d)*gamma(m + 1, -(d*f*x + c*f)/d)*
sinh((d*m*log(-f/d) - d*e + c*f)/d) - 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x +
 c)) - 2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+a*cosh(f*x+e)),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx$$
$$= -\frac{1}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a$$
$$+\frac{(dx+c)^{m+1}a}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d
 + (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a
 + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx = \int (a\cosh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*cosh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx=\int (a+a\cosh(e+f\,x))\ (c+d\,x)^m\,dx$$

input
```
int((a + a*cosh(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + a*cosh(e + f*x))*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+a\cosh(e+fx))\,dx = \frac{a\Big(e^{2fx+2e}(dx+c)^m\,dm+e^{2fx+2e}(dx+c)^m\,d+2e^{fx+e}(dx+c)^m\,cf+2e^{fx+e}(dx+c)^m\,dfx-(dx+c)^m}{}$$

input
```
int((d*x+c)^m*(a+a*cosh(f*x+e)),x)
```

output
```
(a*(e**(2*e + 2*f*x)*(c + d*x)**m*d*m + e**(2*e + 2*f*x)*(c + d*x)**m*d +
2*e**(e + f*x)*(c + d*x)**m*c*f + 2*e**(e + f*x)*(c + d*x)**m*d*f*x - (c +
 d*x)**m*d*m - (c + d*x)**m*d - e**(2*e + f*x)*int((e**(f*x)*(c + d*x)**m)
/(c + d*x),x)*d**2*m**2 - e**(2*e + f*x)*int((e**(f*x)*(c + d*x)**m)/(c +
d*x),x)*d**2*m + e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*
d**2*m**2 + e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*d**2*
m))/(2*e**(e + f*x)*d*f*(m + 1))
```

3.154 $\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx = \text{Int}\left(\frac{(c+dx)^m}{a+a\cosh(e+fx)},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 3.38 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a\cosh(e+fx)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^m}{a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{(c+dx)^m}{a\cosh(e+fx)+a}dx$$

input
```
Int[(c + d*x)^m/(a + a*Cosh[e + f*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+a\cosh(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.50 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx = \frac{\int \frac{(c+dx)^m}{\cosh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**m/(a+a*cosh(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(cosh(e + f*x) + 1), x)/a
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx=\int \frac{(dx+c)^m}{a\cosh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*cosh(f*x + e) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx=\int \frac{(c+d\,x)^m}{a+a\cosh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(a + a*cosh(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + a*cosh(e + f*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 6.65

$$\int \frac{(c+dx)^m}{a+a\cosh(e+fx)}\,dx$$
$$=\frac{2e^e\left(e^{fx}(dx+c)^m-e^{fx+e}\left(\int \frac{e^{fx}(dx+c)^m}{e^{fx+e}c+e^{fx+e}dx+c+dx}dx\right)dm-\left(\int \frac{e^{fx}(dx+c)^m}{e^{fx+e}c+e^{fx+e}dx+c+dx}dx\right)dm\right)}{af\left(e^{fx+e}+1\right)}$$

input
```
int((d*x+c)^m/(a+a*cosh(f*x+e)),x)
```

output
```
(2*e**e*(e**(f*x)*(c + d*x)**m - e**(e + f*x)*int((e**(f*x)*(c + d*x)**m)/
(e**(e + f*x)*c + e**(e + f*x)*d*x + c + d*x),x)*d*m - int((e**(f*x)*(c +
d*x)**m)/(e**(e + f*x)*c + e**(e + f*x)*d*x + c + d*x),x)*d*m))/(a*f*(e**(
e + f*x) + 1))
```

3.155 $\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx = \operatorname{Int}\left(\frac{(c+dx)^m}{(a+a\cosh(e+fx))^2},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 6.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + a*Cosh[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\cosh(e+fx)+a)^2} \, dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{(c+dx)^m}{\left(a+a\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{ 3807}$$
$$\int \frac{(c+dx)^m}{(a\cosh(e+fx)+a)^2}dx$$

input
```
Int[(c + d*x)^m/(a + a*Cosh[e + f*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{(a+a\cosh(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(a^2*cosh(f*x + e)^2 + 2*a^2*cosh(f*x + e) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 11.25 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx=\frac{\int \frac{(c+dx)^m}{\cosh^2(e+fx)+2\cosh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**m/(a+a*cosh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(cosh(e + f*x)**2 + 2*cosh(e + f*x) + 1), x)/a**2
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx=\int \frac{(dx+c)^m}{(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*cosh(f*x + e) + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(a\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*cosh(f*x + e) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+a\cosh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^m/(a + a*cosh(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + a*cosh(e + f*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 4857, normalized size of antiderivative = 242.85

$$\int \frac{(c+dx)^m}{(a+a\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int((d*x+c)^m/(a+a*cosh(f*x+e))^2,x)
```

output

```
(4*( - 3*e**(e + f*x)*(c + d*x)**m*c*f - (c + d*x)**m*c*f - (c + d*x)**m*d
*m - 6*e**(4*e + 3*f*x)*int((e**(f*x)*(c + d*x)**m*x)/(2*e**(3*e + 3*f*x)*
c**2*f + 2*e**(3*e + 3*f*x)*c*d*f*x - e**(3*e + 3*f*x)*c*d*m - e**(3*e + 3
*f*x)*d**2*m*x + 6*e**(2*e + 2*f*x)*c**2*f + 6*e**(2*e + 2*f*x)*c*d*f*x -
3*e**(2*e + 2*f*x)*c*d*m - 3*e**(2*e + 2*f*x)*d**2*m*x + 6*e**(e + f*x)*c*
*2*f + 6*e**(e + f*x)*c*d*f*x - 3*e**(e + f*x)*c*d*m - 3*e**(e + f*x)*d**2
*m*x + 2*c**2*f + 2*c*d*f*x - c*d*m - d**2*m*x),x)*c*d**2*f**2*m + 3*e**(4
*e + 3*f*x)*int((e**(f*x)*(c + d*x)**m*x)/(2*e**(3*e + 3*f*x)*c**2*f + 2*e
**(3*e + 3*f*x)*c*d*f*x - e**(3*e + 3*f*x)*c*d*m - e**(3*e + 3*f*x)*d**2*m
*x + 6*e**(2*e + 2*f*x)*c**2*f + 6*e**(2*e + 2*f*x)*c*d*f*x - 3*e**(2*e +
2*f*x)*c*d*m - 3*e**(2*e + 2*f*x)*d**2*m*x + 6*e**(e + f*x)*c**2*f + 6*e**
(e + f*x)*c*d*f*x - 3*e**(e + f*x)*c*d*m - 3*e**(e + f*x)*d**2*m*x + 2*c**
2*f + 2*c*d*f*x - c*d*m - d**2*m*x),x)*d**3*f*m**2 + 2*e**(3*e + 3*f*x)*in
t((c + d*x)**m/(2*e**(3*e + 3*f*x)*c**2*f + 2*e**(3*e + 3*f*x)*c*d*f*x - e
**(3*e + 3*f*x)*c*d*m - e**(3*e + 3*f*x)*d**2*m*x + 6*e**(2*e + 2*f*x)*c**
2*f + 6*e**(2*e + 2*f*x)*c*d*f*x - 3*e**(2*e + 2*f*x)*c*d*m - 3*e**(2*e +
2*f*x)*d**2*m*x + 6*e**(e + f*x)*c**2*f + 6*e**(e + f*x)*c*d*f*x - 3*e**(e
 + f*x)*c*d*m - 3*e**(e + f*x)*d**2*m*x + 2*c**2*f + 2*c*d*f*x - c*d*m - d
**2*m*x),x)*c**2*d*f**2*m + e**(3*e + 3*f*x)*int((c + d*x)**m/(2*e**(3*e +
 3*f*x)*c**2*f + 2*e**(3*e + 3*f*x)*c*d*f*x - e**(3*e + 3*f*x)*c*d*m - ...
```

3.156 $\int (c+dx)^3(a+b\cosh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 89

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx = \frac{a(c+dx)^4}{4d}-\frac{6bd^3\cosh(e+fx)}{f^4} - \frac{3bd(c+dx)^2\cosh(e+fx)}{f^2} + \frac{6bd^2(c+dx)\sinh(e+fx)}{f^3} + \frac{b(c+dx)^3\sinh(e+fx)}{f}$$

output
```
1/4*a*(d*x+c)^4/d-6*b*d^3*cosh(f*x+e)/f^4-3*b*d*(d*x+c)^2*cosh(f*x+e)/f^2+
6*b*d^2*(d*x+c)*sinh(f*x+e)/f^3+b*(d*x+c)^3*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.38

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx$$
$$=\frac{1}{4}ax\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)-\frac{3bd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)}{f^4}$$
$$+\frac{b(c+dx)\left(c^2f^2+2cdf^2x+d^2(6+f^2x^2)\right)\sinh(e+fx)}{f^3}$$

input
```
Integrate[(c + d*x)^3*(a + b*Cosh[e + f*x]),x]
```

output
```
(a*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))/4 - (3*b*d*(c^2*f^2 + 2*
c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x])/f^4 + (b*(c + d*x)*(c^2*f^2
+ 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x])/f^3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^3\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a(c+dx)^3+b(c+dx)^3\cosh(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(c+dx)^4}{4d}+\frac{6bd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{3bd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{b(c+dx)^3\sinh(e+fx)}{f}-\frac{6bd^3\cosh(e+fx)}{f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) - (6*b*d^3*Cosh[e + f*x])/f^4 - (3*b*d*(c + d*x)^2*C
osh[e + f*x])/f^2 + (6*b*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (b*(c + d*x)^3
*Sinh[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.21

method	result
parallelrisch	$\frac{(dx+c)b\left(\left((dx+c)^2f^2+6d^2\right)f\sinh(fx+e)-3bd\left((dx+c)^2f^2+2d^2\right)\cosh(fx+e)+\left(\frac{dx}{2}+c\right)\left(\frac{1}{2}x^2d^2+cdx+c^2\right)ax f^4-3b c}{f^4}$
risch	$\frac{a d^3x^4}{4}+a d^2c x^3+\frac{3ad c^2x^2}{2}+a c^3x+\frac{a c^4}{4d}+\frac{b\left(d^3x^3f^3+3c d^2f^3x^2+3c^2d f^3x-3d^3f^2x^2+c^3f^3-6c d^2f^2}{2f^4}$
parts	$\frac{a(dx+c)^4}{4d}+\frac{b\left(\frac{d^3\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{3d^3e\left((fx+e)^2\sinh(fx\right.}{}\right.}{}$
orering	$\frac{(d^5f^4x^6+6c d^4f^4x^5+15c^2d^3f^4x^4+20c^3d^2f^4x^3+14c^4d f^4x^2-24d^5f^2x^4+4c^5f^4x-96c d^4f^2x^3-156c^2d^3f^2x^2-120c^3}{4f^4(dx+c)^2}$
derivativedivides	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((f\right.}{}}{}$
default	$\frac{\frac{d^3a(fx+e)^4}{4f^3}+\frac{d^3b\left((fx+e)^3\sinh(fx+e)-3(fx+e)^2\cosh(fx+e)+6(fx+e)\sinh(fx+e)-6\cosh(fx+e)\right)}{f^3}-\frac{d^3ea(fx+e)^3}{f^3}-\frac{3d^3eb\left((f\right.}{}}{}$

input
```
int((d*x+c)^3*(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
((d*x+c)*b*((d*x+c)^2*f^2+6*d^2)*f*sinh(f*x+e)-3*b*d*((d*x+c)^2*f^2+2*d^2)
*cosh(f*x+e)+(1/2*d*x+c)*(1/2*x^2*d^2+c*d*x+c^2)*a*x*f^4-3*b*c^2*d*f^2-6*d
^3*b)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.89

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx$$
$$=\frac{ad^3f^4x^4+4\,acd^2f^4x^3+6\,ac^2df^4x^2+4\,ac^3f^4x-12\left(bd^3f^2x^2+2\,bcd^2f^2x+bc^2df^2+2\,bd^3\right)\cosh\left(fx+e\right.}{4\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*(a*d^3*f^4*x^4 + 4*a*c*d^2*f^4*x^3 + 6*a*c^2*d*f^4*x^2 + 4*a*c^3*f^4*x
 - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2 + 2*b*d^3)*cosh(f*x +
 e) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + b*c^3*f^3 + 6*b*c*d^2*f + 3*(
b*c^2*d*f^3 + 2*b*d^3*f)*x)*sinh(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(88) = 176$.

Time = 0.29 (sec) , antiderivative size = 264, normalized size of antiderivative = 2.97

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx$$
$$= \begin{cases} ac^3x + \frac{3ac^2dx^2}{2} + acd^2x^3 + \frac{ad^3x^4}{4} + \frac{bc^3\sinh(e+fx)}{f} + \frac{3bc^2dx\sinh(e+fx)}{f} - \frac{3bc^2d\cosh(e+fx)}{f^2} + \frac{3bcd^2x^2\sinh(e+fx)}{f} - \\ (a+b\cosh(e))\left(c^3x + \frac{3c^2dx^2}{2} + cd^2x^3 + \frac{d^3x^4}{4}\right) \end{cases}$$

input
```
integrate((d*x+c)**3*(a+b*cosh(f*x+e)),x)
```

output
```
Piecewise((a*c**3*x + 3*a*c**2*d*x**2/2 + a*c*d**2*x**3 + a*d**3*x**4/4 +
b*c**3*sinh(e + f*x)/f + 3*b*c**2*d*x*sinh(e + f*x)/f - 3*b*c**2*d*cosh(e
+ f*x)/f**2 + 3*b*c*d**2*x**2*sinh(e + f*x)/f - 6*b*c*d**2*x*cosh(e + f*x)
/f**2 + 6*b*c*d**2*sinh(e + f*x)/f**3 + b*d**3*x**3*sinh(e + f*x)/f - 3*b*
d**3*x**2*cosh(e + f*x)/f**2 + 6*b*d**3*x*sinh(e + f*x)/f**3 - 6*b*d**3*co
sh(e + f*x)/f**4, Ne(f, 0)), ((a + b*cosh(e))*(c**3*x + 3*c**2*d*x**2/2 +
c*d**2*x**3 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(87) = 174$.

Time = 0.05 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx$$
$$=\frac{1}{4}ad^3x^4+acd^2x^3+\frac{3}{2}ac^2dx^2+ac^3x+\frac{3}{2}bc^2d\left(\frac{(fxe^e-e^e)e^{(fx)}}{f^2}-\frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+\frac{3}{2}bcd^2\left(\frac{(f^2x^2e^e-2\,fxe^e+2\,e^e)e^{(fx)}}{f^3}-\frac{(f^2x^2+2\,fx+2)e^{(-fx-e)}}{f^3}\right)$$
$$+\frac{1}{2}bd^3\left(\frac{(f^3x^3e^e-3\,f^2x^2e^e+6\,fxe^e-6\,e^e)e^{(fx)}}{f^4}-\frac{(f^3x^3+3\,f^2x^2+6\,fx+6)e^{(-fx-e)}}{f^4}\right)$$
$$+\frac{bc^3\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 3/2*b*c^2*d*((f*
x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + 3/2*b*c*d^2*((f^2
*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x
- e)/f^3) + 1/2*b*d^3*((f^3*x^3*e^e - 3*f^2*x^2*e^e + 6*f*x*e^e - 6*e^e)*e
^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6)*e^(-f*x - e)/f^4) + b*c^3*s
inh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(87) = 174$.

Time = 0.11 (sec) , antiderivative size = 258, normalized size of antiderivative = 2.90

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx=\frac{1}{4}ad^3x^4+acd^2x^3+\frac{3}{2}ac^2dx^2+ac^3x$$
$$+\frac{(bd^3f^3x^3+3\,bcd^2f^3x^2+3\,bc^2df^3x-3\,bd^3f^2x^2+bc^3f^3-6\,bcd^2f^2x-3\,bc^2df^2+6\,bd^3fx+6\,bcd^2f-}{2\,f^4}$$
$$-\frac{(bd^3f^3x^3+3\,bcd^2f^3x^2+3\,bc^2df^3x+3\,bd^3f^2x^2+bc^3f^3+6\,bcd^2f^2x+3\,bc^2df^2+6\,bd^3fx+6\,bcd^2f+}{2\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*a*d^3*x^4 + a*c*d^2*x^3 + 3/2*a*c^2*d*x^2 + a*c^3*x + 1/2*(b*d^3*f^3*x
^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x - 3*b*d^3*f^2*x^2 + b*c^3*f^3 - 6
*b*c*d^2*f^2*x - 3*b*c^2*d*f^2 + 6*b*d^3*f*x + 6*b*c*d^2*f - 6*b*d^3)*e^(f
*x + e)/f^4 - 1/2*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + 3
*b*d^3*f^2*x^2 + b*c^3*f^3 + 6*b*c*d^2*f^2*x + 3*b*c^2*d*f^2 + 6*b*d^3*f*x
 + 6*b*c*d^2*f + 6*b*d^3)*e^(-f*x - e)/f^4
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 187, normalized size of antiderivative = 2.10

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx = \frac{\sinh(e+f\,x)\,\left(b\,c^3\,f^2+6\,b\,c\,d^2\right)}{f^3} - \frac{3\cosh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^4} + \frac{a\,d^3\,x^4}{4} + a\,c^3\,x + \frac{3\,x\sinh(e+f\,x)\,\left(b\,c^2\,d\,f^2+2\,b\,d^3\right)}{f^3} + \frac{3\,a\,c^2\,d\,x^2}{2} + a\,c\,d^2\,x^3 - \frac{3\,b\,d^3\,x^2\cosh(e+f\,x)}{f^2} + \frac{b\,d^3\,x^3\sinh(e+f\,x)}{f} - \frac{6\,b\,c\,d^2\,x\cosh(e+f\,x)}{f^2} + \frac{3\,b\,c\,d^2\,x^2\sinh(e+f\,x)}{f}$$

input
```
int((a + b*cosh(e + f*x))*(c + d*x)^3,x)
```

output
```
(sinh(e + f*x)*(b*c^3*f^2 + 6*b*c*d^2))/f^3 - (3*cosh(e + f*x)*(2*b*d^3 +
b*c^2*d*f^2))/f^4 + (a*d^3*x^4)/4 + a*c^3*x + (3*x*sinh(e + f*x)*(2*b*d^3
+ b*c^2*d*f^2))/f^3 + (3*a*c^2*d*x^2)/2 + a*c*d^2*x^3 - (3*b*d^3*x^2*cosh(
e + f*x))/f^2 + (b*d^3*x^3*sinh(e + f*x))/f - (6*b*c*d^2*x*cosh(e + f*x))/
f^2 + (3*b*c*d^2*x^2*sinh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.39

$$\int (c+dx)^3(a+b\cosh(e+fx))\,dx$$
$$= \frac{-12\cosh(fx+e)\,b\,c^2d\,f^2 - 24\cosh(fx+e)\,bc\,d^2f^2x - 12\cosh(fx+e)\,b\,d^3f^2x^2 - 24\cosh(fx+e)\,b\,d}{}$$

input
```
int((d*x+c)^3*(a+b*cosh(f*x+e)),x)
```

output
```
( - 12*cosh(e + f*x)*b*c**2*d*f**2 - 24*cosh(e + f*x)*b*c*d**2*f**2*x - 12
*cosh(e + f*x)*b*d**3*f**2*x**2 - 24*cosh(e + f*x)*b*d**3 + 4*sinh(e + f*x
)*b*c**3*f**3 + 12*sinh(e + f*x)*b*c**2*d*f**3*x + 12*sinh(e + f*x)*b*c*d*
*2*f**3*x**2 + 24*sinh(e + f*x)*b*c*d**2*f + 4*sinh(e + f*x)*b*d**3*f**3*x
**3 + 24*sinh(e + f*x)*b*d**3*f*x + 4*a*c**3*f**4*x + 6*a*c**2*d*f**4*x**2
 + 4*a*c*d**2*f**4*x**3 + a*d**3*f**4*x**4)/(4*f**4)
```

3.157 $\int (c+dx)^2(a+b\cosh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 67

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx = \frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{2bd^2\sinh(e+fx)}{f^3}+\frac{b(c+dx)^2\sinh(e+fx)}{f}$$

output
```
1/3*a*(d*x+c)^3/d-2*b*d*(d*x+c)*cosh(f*x+e)/f^2+2*b*d^2*sinh(f*x+e)/f^3+b*
(d*x+c)^2*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.24

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx = \frac{1}{3}ax\left(3c^2+3cdx+d^2x^2\right)-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\sinh(e+fx)}{f^3}$$

input
```
Integrate[(c + d*x)^2*(a + b*Cosh[e + f*x]),x]
```

output
```
(a*x*(3*c^2 + 3*c*d*x + d^2*x^2))/3 - (2*b*d*(c + d*x)*Cosh[e + f*x])/f^2
+ (b*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Sinh[e + f*x])/f^3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a(c+dx)^2+b(c+dx)^2\cosh(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^3}{3d}-\frac{2bd(c+dx)\cosh(e+fx)}{f^2}+\frac{b(c+dx)^2\sinh(e+fx)}{f}+\frac{2bd^2\sinh(e+fx)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) - (2*b*d*(c + d*x)*Cosh[e + f*x])/f^2 + (2*b*d^2*Sin
h[e + f*x])/f^3 + (b*(c + d*x)^2*Sinh[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

method	result
parallelrisch	$\frac{b\left(\left(dx+c\right)^{2} f^{2}+2 d^{2}\right) \sinh \left(fx+e\right)+\left(-2 b d \left(dx+c\right) \cosh \left(fx+e\right)+a x \left(\frac{1}{3} x^{2} d^{2}+c d x+c^{2}\right) f^{2}-2 b c d\right) f}{f^{3}}$
risch	$\frac{a\,d^{2} x^{3}}{3}+a d c\,x^{2}+a\,c^{2} x+\frac{a\,c^{3}}{3 d}+\frac{b\left(d^{2} x^{2} f^{2}+2 c d\,f^{2} x+c^{2} f^{2}-2 d^{2} f x-2 c d f+2 d^{2}\right) {\mathrm e}^{f x+e}}{2 f^{3}}-\frac{b\left(d^{2} x^{2} f^{2}+2 c d\,f^{2}\right.}{}$
parts	$\frac{a \left(dx+c\right)^{3}}{3 d}+\frac{b\left(\frac{d^{2}\left(\left(fx+e\right)^{2} \sinh \left(fx+e\right)-2 \left(fx+e\right) \cosh \left(fx+e\right)+2 \sinh \left(fx+e\right)\right)}{f^{2}}-\frac{2 d^{2} e \left(\left(fx+e\right) \sinh \left(fx+e\right)-\cosh \left(fx+e\right)\right)}{f^{2}}+\frac{d^{2} e^{2}}{}\right.}{f}$
derivativedivides	$\frac{\frac{d^{2} a \left(fx+e\right)^{3}}{3 f^{2}}+\frac{d^{2} b\left(\left(fx+e\right)^{2} \sinh \left(fx+e\right)-2 \left(fx+e\right) \cosh \left(fx+e\right)+2 \sinh \left(fx+e\right)\right)}{f^{2}}-\frac{d^{2} e a \left(fx+e\right)^{2}}{f^{2}}-\frac{2 d^{2} e b \left(\left(fx+e\right) \sinh \left(fx+e\right)-\cosh \left(f\right.\right.}{f^{2}}}{}$
default	$\frac{\frac{d^{2} a \left(fx+e\right)^{3}}{3 f^{2}}+\frac{d^{2} b\left(\left(fx+e\right)^{2} \sinh \left(fx+e\right)-2 \left(fx+e\right) \cosh \left(fx+e\right)+2 \sinh \left(fx+e\right)\right)}{f^{2}}-\frac{d^{2} e a \left(fx+e\right)^{2}}{f^{2}}-\frac{2 d^{2} e b \left(\left(fx+e\right) \sinh \left(fx+e\right)-\cosh \left(f\right.\right.}{f^{2}}}{}$
orering	$\frac{\left(d^{4} f^{4} x^{5}+5 c\,d^{3} f^{4} x^{4}+10 c^{2} d^{2} f^{4} x^{3}+9 c^{3} d\,f^{4} x^{2}+3 c^{4} f^{4} x-12 d^{4} f^{2} x^{3}-42 c\,d^{3} f^{2} x^{2}-48 c^{2} d^{2} f^{2} x-12 c^{3} d\,f^{2}-48 d^{4} x-12 d^{3}\right.}{3 f^{4} \left(dx+c\right)^{2}}$

input
```
int((d*x+c)^2*(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
(b*((d*x+c)^2*f^2+2*d^2)*sinh(f*x+e)+(-2*b*d*(d*x+c)*cosh(f*x+e)+a*x*(1/3*
x^2*d^2+c*d*x+c^2)*f^2-2*b*c*d)*f)/f^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.52

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx$$
$$= \frac{ad^2f^3x^3+3\,acdf^3x^2+3\,ac^2f^3x-6\,(bd^2fx+bcdf)\cosh\,(fx+e)+3\,(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2+2\,b\cdots}{3\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*(a*d^2*f^3*x^3 + 3*a*c*d*f^3*x^2 + 3*a*c^2*f^3*x - 6*(b*d^2*f*x + b*c*
d*f)*cosh(f*x + e) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^
2)*sinh(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(65) = 130$.

Time = 0.23 (sec) , antiderivative size = 151, normalized size of antiderivative = 2.25

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx$$
$$= \begin{cases} ac^2x+acdx^2+\frac{ad^2x^3}{3}+\frac{bc^2\sinh(e+fx)}{f}+\frac{2bcdx\sinh(e+fx)}{f}-\frac{2bcd\cosh(e+fx)}{f^2}+\frac{bd^2x^2\sinh(e+fx)}{f}-\frac{2bd^2x\cosh(e+fx)}{f^2}\cdots \\ (a+b\cosh(e))\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*cosh(f*x+e)),x)
```

output
```
Piecewise((a*c**2*x + a*c*d*x**2 + a*d**2*x**3/3 + b*c**2*sinh(e + f*x)/f
+ 2*b*c*d*x*sinh(e + f*x)/f - 2*b*c*d*cosh(e + f*x)/f**2 + b*d**2*x**2*sin
h(e + f*x)/f - 2*b*d**2*x*cosh(e + f*x)/f**2 + 2*b*d**2*sinh(e + f*x)/f**3
, Ne(f, 0)), ((a + b*cosh(e))*(c**2*x + c*d*x**2 + d**2*x**3/3), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(65) = 130$.

Time = 0.04 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.10

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + bcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{1}{2}\,bd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right) + \frac{bc^2\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + b*c*d*((f*x*e^e - e^e)*e^(f*x)/f^2 -
 (f*x + 1)*e^(-f*x - e)/f^2) + 1/2*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e
)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + b*c^2*sinh(f*x +
 e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(65) = 130$.

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 2.18

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx = \frac{1}{3}\,ad^2x^3 + acdx^2 + ac^2x + \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 - 2\,bd^2fx - 2\,bcdf + 2\,bd^2)e^{(fx+e)}}{2\,f^3} - \frac{(bd^2f^2x^2 + 2\,bcdf^2x + bc^2f^2 + 2\,bd^2fx + 2\,bcdf + 2\,bd^2)e^{(-fx-e)}}{2\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
1/3*a*d^2*x^3 + a*c*d*x^2 + a*c^2*x + 1/2*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
 b*c^2*f^2 - 2*b*d^2*f*x - 2*b*c*d*f + 2*b*d^2)*e^(f*x + e)/f^3 - 1/2*(b*d
^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2 + 2*b*d^2*f*x + 2*b*c*d*f + 2*b*d^2
)*e^(-f*x - e)/f^3
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.64

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx = \frac{a\,d^2\,x^3}{3} + \frac{\sinh(e+f\,x)\,\left(b\,c^2\,f^2+2\,b\,d^2\right)}{f^3} + a\,c^2\,x + a\,c\,d\,x^2 - \frac{2\,b\,d^2\,x\cosh(e+f\,x)}{f^2} + \frac{b\,d^2\,x^2\sinh(e+f\,x)}{f} - \frac{2\,b\,c\,d\cosh(e+f\,x)}{f^2} + \frac{2\,b\,c\,d\,x\sinh(e+f\,x)}{f}$$

input
```
int((a + b*cosh(e + f*x))*(c + d*x)^2,x)
```

output
```
(a*d^2*x^3)/3 + (sinh(e + f*x)*(2*b*d^2 + b*c^2*f^2))/f^3 + a*c^2*x + a*c*
d*x^2 - (2*b*d^2*x*cosh(e + f*x))/f^2 + (b*d^2*x^2*sinh(e + f*x))/f - (2*b
*c*d*cosh(e + f*x))/f^2 + (2*b*c*d*x*sinh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.85

$$\int (c+dx)^2(a+b\cosh(e+fx))\,dx = \frac{-6\cosh(fx+e)\,bcdf - 6\cosh(fx+e)\,b\,d^2fx + 3\sinh(fx+e)\,b\,c^2f^2 + 6\sinh(fx+e)\,bcd\,f^2x + 3\sinh}{3f^3}$$

input
```
int((d*x+c)^2*(a+b*cosh(f*x+e)),x)
```

output
```
( - 6*cosh(e + f*x)*b*c*d*f - 6*cosh(e + f*x)*b*d**2*f*x + 3*sinh(e + f*x)
*b*c**2*f**2 + 6*sinh(e + f*x)*b*c*d*f**2*x + 3*sinh(e + f*x)*b*d**2*f**2*
x**2 + 6*sinh(e + f*x)*b*d**2 + 3*a*c**2*f**3*x + 3*a*c*d*f**3*x**2 + a*d*
*2*f**3*x**3)/(3*f**3)
```

3.158 $\int (c+dx)(a+b\cosh(e+fx))\,dx$

Optimal result

Integrand size = 16, antiderivative size = 45

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \frac{a(c+dx)^2}{2d} - \frac{bd\cosh(e+fx)}{f^2} + \frac{b(c+dx)\sinh(e+fx)}{f}$$

output
```
1/2*a*(d*x+c)^2/d-b*d*cosh(f*x+e)/f^2+b*(d*x+c)*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \frac{-2bd\cosh(e+fx)+f(afx(2c+dx)+2b(c+dx)\sinh(e+fx))}{2f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Cosh[e + f*x]),x]
```

output
```
(-2*b*d*Cosh[e + f*x] + f*(a*f*x*(2*c + d*x) + 2*b*(c + d*x)*Sinh[e + f*x]
))/(2*f^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\cosh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)+b(c+dx)\cosh(e+fx))dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\sinh(e+fx)}{f}-\frac{bd\cosh(e+fx)}{f^2}$$

input
```
Int[(c + d*x)*(a + b*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (b*d*Cosh[e + f*x])/f^2 + (b*(c + d*x)*Sinh[e + f*
x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.02

method	result
parallelrisch	$\frac{(dx+c)bf\sinh(fx+e)-\cosh(fx+e)bd+\left(\frac{dx}{2}+c\right)ax\,f^2-bd}{f^2}$
risch	$\frac{ad\,x^2}{2}+acx+\frac{b(dxf+cf-d)\mathrm{e}^{fx+e}}{2f^2}-\frac{b(dxf+cf+d)\mathrm{e}^{-fx-e}}{2f^2}$
parts	$a\left(\frac{1}{2}d\,x^2+cx\right)+\frac{b\left(\frac{d((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{de\sinh(fx+e)}{f}+c\sinh(fx+e)\right)}{f}$
derivativedivides	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\sinh(fx+e)}{f}+ca(fx+e)+cb\sinh(fx+e)}{f}$
default	$\frac{\frac{da(fx+e)^2}{2f}+\frac{db((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}-\frac{dea(fx+e)}{f}-\frac{deb\sinh(fx+e)}{f}+ca(fx+e)+cb\sinh(fx+e)}{f}$
orering	$\frac{(d^3f^2x^4+4c\,d^2f^2x^3+5c^2d\,f^2x^2+2c^3f^2x-6d^3x^2-12c\,d^2x-4d\,c^2)(a+b\cosh(fx+e))}{2f^2(dx+c)^2}+\frac{(2x^2d^2+4cdx+c^2)(d(a+b\cosh \ldots}{(c \ldots}$

input

```
int((d*x+c)*(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
((d*x+c)*b*f*sinh(f*x+e)-cosh(f*x+e)*b*d+(1/2*d*x+c)*a*x*f^2-b*d)/f^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.13

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \frac{adf^2x^2+2\,acf^2x-2\,bd\cosh\left(fx+e\right)+2\left(bdfx+bcf\right)\sinh\left(fx+e\right)}{2\,f^2}$$

input

```
integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(a*d*f^2*x^2 + 2*a*c*f^2*x - 2*b*d*cosh(f*x + e) + 2*(b*d*f*x + b*c*f)
*sinh(f*x + e))/f^2
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.51

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \begin{cases} acx + \frac{adx^2}{2} + \frac{bc\sinh(e+fx)}{f} + \frac{bdx\sinh(e+fx)}{f} - \frac{bd\cosh(e+fx)}{f^2} & \text{for } f \neq 0 \\ (a+b\cosh(e))\left(cx + \frac{dx^2}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*(a+b*cosh(f*x+e)),x)
```

output
```
Piecewise((a*c*x + a*d*x**2/2 + b*c*sinh(e + f*x)/f + b*d*x*sinh(e + f*x)/
f - b*d*cosh(e + f*x)/f**2, Ne(f, 0)), ((a + b*cosh(e))*(c*x + d*x**2/2),
True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.47

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \frac{1}{2}adx^2 + acx + \frac{1}{2}bd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{bc\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(
-f*x - e)/f^2) + b*c*sinh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.42

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \frac{1}{2}\,adx^2 + acx + \frac{(bdfx + bcf - bd)e^{(fx+e)}}{2\,f^2} - \frac{(bdfx + bcf + bd)e^{(-fx-e)}}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*a*d*x^2 + a*c*x + 1/2*(b*d*f*x + b*c*f - b*d)*e^(f*x + e)/f^2 - 1/2*(b
*d*f*x + b*c*f + b*d)*e^(-f*x - e)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.09

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \frac{f\,(b\,c\sinh(e+f\,x) + b\,d\,x\sinh(e+f\,x)) - b\,d\cosh(e+f\,x)}{f^2} + a\,c\,x + \frac{a\,d\,x^2}{2}$$

input
```
int((a + b*cosh(e + f*x))*(c + d*x),x)
```

output
```
(f*(b*c*sinh(e + f*x) + b*d*x*sinh(e + f*x)) - b*d*cosh(e + f*x))/f^2 + a*
c*x + (a*d*x^2)/2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.24

$$\int (c+dx)(a+b\cosh(e+fx))\,dx = \frac{-2\cosh(fx+e)\,bd+2\sinh(fx+e)\,bcf+2\sinh(fx+e)\,bdfx+2ac\,f^2x+ad\,f^2x^2}{2f^2}$$

input
```
int((d*x+c)*(a+b*cosh(f*x+e)),x)
```

output
```
( - 2*cosh(e + f*x)*b*d + 2*sinh(e + f*x)*b*c*f + 2*sinh(e + f*x)*b*d*f*x
+ 2*a*c*f**2*x + a*d*f**2*x**2)/(2*f**2)
```

3.159 $\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 64

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx=\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{d}+\frac{a\log(c+dx)}{d}+\frac{b\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d}$$

output
```
b*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d+a*ln(d*x+c)/d-b*sinh(-e+c*f/d)*Shi(c*f/d
+f*x)/d
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.89

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx=\frac{b\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+a\log(c+dx)+b\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d}$$

input
```
Integrate[(a + b*Cosh[e + f*x])/(c + d*x),x]
```

output
```
(b*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + a*Log[c + d*x] + b*Sinh[e
 - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}{c+dx}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{c+dx}+\frac{b\cosh(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\log(c+dx)}{d}+\frac{b\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{b\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d}$$

input
```
Int[(a + b*Cosh[e + f*x])/(c + d*x),x]
```

output
```
(b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (a*Log[c + d*x])/d +
 (b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.47

method	result	size
risch	$\frac{a\ln(dx+c)}{d}-\frac{b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d}-\frac{b\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{2d}$	94

input
```
int((a+b*cosh(f*x+e))/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*ln(d*x+c)/d-1/2*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*b/d*exp
(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.73

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx = \frac{\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+2\,a\log\left(dx+c\right)-\left(b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\sinh\left(-\right.}{2\,d}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
1/2*((b*Ei((d*f*x + c*f)/d) + b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d)
 + 2*a*log(d*x + c) - (b*Ei((d*f*x + c*f)/d) - b*Ei(-(d*f*x + c*f)/d))*sin
h(-(d*e - c*f)/d))/d
```

Sympy [F]

$$\int \frac{a + b\cosh(e + fx)}{c + dx}\, dx = \int \frac{a + b\cosh(e + fx)}{c + dx}\, dx$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c),x)
```

output
```
Integral((a + b*cosh(e + f*x))/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \frac{a + b\cosh(e + fx)}{c + dx}\, dx = -\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right) + \frac{a\log(dx+c)}{d}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
-1/2*b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*
exp_integral_e(1, -(d*x + c)*f/d)/d) + a*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx = \frac{b\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+b\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,a\log\left(dx+c\right)}{2\,d}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + b*Ei(-(d*f*x + c*f)/d)*e^(-e +
c*f/d) + 2*a*log(d*x + c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx = \int \frac{a+b\cosh(e+f\,x)}{c+d\,x}\,dx$$

input
```
int((a + b*cosh(e + f*x))/(c + d*x),x)
```

output
```
int((a + b*cosh(e + f*x))/(c + d*x), x)
```

Reduce [F]

$$\int \frac{a+b\cosh(e+fx)}{c+dx}\,dx = \frac{\left(\int \frac{\cosh(fx+e)}{dx+c}dx\right)bd+\log(dx+c)\,a}{d}$$

input
```
int((a+b*cosh(f*x+e))/(d*x+c),x)
```

output
```
(int(cosh(e + f*x)/(c + d*x),x)*b*d + log(c + d*x)*a)/d
```

3.160 $\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx = -\frac{a}{d(c+dx)} - \frac{b\cosh(e+fx)}{d(c+dx)} + \frac{bf\,\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2} + \frac{bf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2}$$

output
```
-a/d/(d*x+c)-b*cosh(f*x+e)/d/(d*x+c)-b*f*Chi(c*f/d+f*x)*sinh(-e+c*f/d)/d^2
+b*f*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.82

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx = \frac{-\frac{d(a+b\cosh(e+fx))}{c+dx} + bf\,\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)\sinh\left(e-\frac{cf}{d}\right) + bf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{d^2}$$

input
```
Integrate[(a + b*Cosh[e + f*x])/(c + d*x)^2,x]
```

output
```
(-((d*(a + b*Cosh[e + f*x]))/(c + d*x)) + b*f*CoshIntegral[f*(c/d + x)]*Si
nh[e - (c*f)/d] + b*f*Cosh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)])/d^2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3798}$$
$$\int \left(\frac{a}{(c+dx)^2}+\frac{b\cosh(e+fx)}{(c+dx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a}{d(c+dx)}+\frac{bf\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{bf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{b\cosh(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Cosh[e + f*x])/(c + d*x)^2,x]
```

output
```
-(a/(d*(c + d*x))) - (b*Cosh[e + f*x])/(d*(c + d*x)) + (b*f*CoshIntegral[(
c*f)/d + f*x]*Sinh[e - (c*f)/d])/d^2 + (b*f*Cosh[e - (c*f)/d]*SinhIntegral
[(c*f)/d + f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{a}{d(dx+c)}-\frac{fb\,\mathrm{e}^{-fx-e}}{2d(dxf+cf)}+\frac{fb\,\mathrm{e}^{\frac{cf-de}{d}}\operatorname{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{2d^2}-\frac{fb\,\mathrm{e}^{fx+e}}{2d^2\left(\frac{cf}{d}+fx\right)}-\frac{fb\,\mathrm{e}^{-\frac{cf-de}{d}}\operatorname{expIntegral}_1\left(-fx-e\right.}{2d^2}$

input
```
int((a+b*cosh(f*x+e))/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-a/d/(d*x+c)-1/2*f*b*exp(-f*x-e)/d/(d*f*x+c*f)+1/2*f*b/d^2*exp((c*f-d*e)/d
)*Ei(1,f*x+e+(c*f-d*e)/d)-1/2*f*b/d^2*exp(f*x+e)/(c*f/d+f*x)-1/2*f*b/d^2*e
xp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.86

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx =$$
$$-\frac{2\,bd\cosh\left(fx+e\right)+2\,ad-\left((bdfx+bcf)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)-(bdfx+bcf)\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(\left(\right.\right.}{2\left(d^3x+cd^2\right)}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*cosh(f*x + e) + 2*a*d - ((b*d*f*x + b*c*f)*Ei((d*f*x + c*f)/d)
 - (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + ((b*d*f*
x + b*c*f)*Ei((d*f*x + c*f)/d) + (b*d*f*x + b*c*f)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx = -\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a}{d^2x+cd}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/2*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/((d*x + c)*d) + e^
(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)*d)) - a/(d^2*x +
c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(90) = 180$.

Time = 0.14 (sec) , antiderivative size = 631, normalized size of antiderivative = 7.25

$$\int \frac{a + b\cosh(e + fx)}{(c + dx)^2}\,dx$$

$$= \frac{1}{2}b\left(\frac{\left((dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right)f^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c} + f\right) - de + cf}{d}\right)e^{\left(\frac{de-cf}{d}\right)} - def^2\mathrm{Ei}\left(\frac{(dx+c)\left(\frac{de}{dx+c} - \frac{cf}{dx+c}\right.}{d}\right.\right.}{\left((dx+c)d^4\left(\frac{de}{dx+c} - \frac{cf}{dx+c}\right.\right.}\right.$$

$$- \frac{a}{(dx+c)d}$$

input

```
integrate((a+b*cosh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output

```
1/2*b*(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d
*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - d*e*
f^2*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((
d*e - c*f)/d) + c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) -
d*e + c*f)/d)*e^((d*e - c*f)/d) - d*f^2*e^((d*x + c)*(d*e/(d*x + c) - c*f/
(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d^5*e + c*d^4*f)*f) - ((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^
2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(
d*e - c*f)/d) - d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
 - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + c*f^3*Ei(-((d*x + c)*(d*e/(d*x + c)
- c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) + d*f^2*e^(-(d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d))*d^2/(((d*x + c)*d^4*(d*e/(d*x
+ c) - c*f/(d*x + c) + f) - d^5*e + c*d^4*f)*f)) - a/((d*x + c)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\cosh(e+f\,x)}{{(c+d\,x)}^2}\,dx$$

input
```
int((a + b*cosh(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + b*cosh(e + f*x))/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^2}\,dx = \frac{e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)b\,c^2+e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)bcdx+2e^{e}ax+\left(\int \frac{1}{e^{fx}c^2+2e^{fx}cdx+e^{fx}d^2x^2}dx\right)b\,c^2+\left(\int\right.}{2e^{e}c\,(dx+c)}$$

input
```
int((a+b*cosh(f*x+e))/(d*x+c)^2,x)
```

output
```
(e**(2*e)*int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c**2 + e**(2*e)*i
nt(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b*c*d*x + 2*e**e*a*x + int(1/(
e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*b*c**2 + int(1/(
e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*b*c*d*x)/(2*e**e
*c*(c + d*x))
```

3.161 $\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 123

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = -\frac{a}{2d(c+dx)^2}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}+\frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(\frac{cf}{d}+fx\right)}{2d^3}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}+\frac{bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{2d^3}$$

output
```
-1/2*a/d/(d*x+c)^2-1/2*b*cosh(f*x+e)/d/(d*x+c)^2+1/2*b*f^2*cosh(-e+c*f/d)*
Chi(c*f/d+f*x)/d^3-1/2*b*f*sinh(f*x+e)/d^2/(d*x+c)-1/2*b*f^2*sinh(-e+c*f/d
)*Shi(c*f/d+f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.77

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = \frac{bf^2\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)-\frac{d(ad+bd\cosh(e+fx)+bf(c+dx)\sinh(e+fx))}{(c+dx)^2}+bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(f\left(\frac{c}{d}+x\right)\right)}{2d^3}$$

input
```
Integrate[(a + b*Cosh[e + f*x])/(c + d*x)^3,x]
```

output
```
(b*f^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] - (d*(a*d + b*d*Cosh[e
+ f*x] + b*f*(c + d*x)*Sinh[e + f*x]))/(c + d*x)^2 + b*f^2*Sinh[e - (c*f)/
d]*SinhIntegral[f*(c/d + x)])/(2*d^3)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a}{(c+dx)^3}+\frac{b\cosh(e+fx)}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a}{2d(c+dx)^2}+\frac{bf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{2d^3}+\frac{bf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{2d^3}-\frac{bf\sinh(e+fx)}{2d^2(c+dx)}-\frac{b\cosh(e+fx)}{2d(c+dx)^2}$$

input
```
Int[(a + b*Cosh[e + f*x])/(c + d*x)^3,x]
```

output
```
-1/2*a/(d*(c + d*x)^2) - (b*Cosh[e + f*x])/(2*d*(c + d*x)^2) + (b*f^2*Cosh
[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/(2*d^3) - (b*f*Sinh[e + f*x])/(
2*d^2*(c + d*x)) + (b*f^2*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/(
2*d^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(115) = 230$.

Time = 0.80 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.41

method	result
risch	$-\frac{a}{2d(dx+c)^2}+\frac{f^3 b\,\mathrm{e}^{-fx-e}x}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^3 b\,\mathrm{e}^{-fx-e}c}{4d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 b\,\mathrm{e}^{-fx-e}}{4d(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 b\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{ex}}{}$

input
```
int((a+b*cosh(f*x+e))/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a/d/(d*x+c)^2+1/4*f^3*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^
2)*x+1/4*f^3*b*exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/4*f^2
*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/4*f^2*b/d^3*exp((c*f-
d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-1/4*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)^2-1/4
*f^2*b/d^3*exp(f*x+e)/(c*f/d+f*x)-1/4*f^2*b/d^3*exp(-(c*f-d*e)/d)*Ei(1,-f*
x-e-(c*f-d*e)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(115) = 230.

Time = 0.10 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.23

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = -\frac{2\,bd^2\cosh\left(fx+e\right)+2\,ad^2-\left(\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right)\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)+\left(bd^2f^2x^2+2\,bcdf^2x+bc^2f^2\right.\right.}{}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b*d^2*cosh(f*x + e) + 2*a*d^2 - ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x +
b*c^2*f^2)*Ei((d*f*x + c*f)/d) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^
2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) + 2*(b*d^2*f*x + b*c*d*f)*si
nh(f*x + e) + ((b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei((d*f*x + c*f
)/d) - (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x + b*c^2*f^2)*Ei(-(d*f*x + c*f)/d))*s
inh(-(d*e - c*f)/d))/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.80

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = -\frac{1}{2}b\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right) - \frac{a}{2\left(d^3x^2+2\,cd^2x+c^2d\right)}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/2*b*(e^(-e + c*f/d)*exp_integral_e(3, (d*x + c)*f/d)/((d*x + c)^2*d) +
e^(e - c*f/d)*exp_integral_e(3, -(d*x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a/(
d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(115) = 230.

Time = 0.11 (sec) , antiderivative size = 316, normalized size of antiderivative = 2.57

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = \frac{bd^2f^2x^2\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+bd^2f^2x^2\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+2\,bcdf^2x\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+2\,bcdf^2x\mathrm{Ei}\left(-\right.}{}$$

input
```
integrate((a+b*cosh(f*x+e))/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/4*(b*d^2*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + b*d^2*f^2*x^2*Ei(-(
d*f*x + c*f)/d)*e^(-e + c*f/d) + 2*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e -
c*f/d) + 2*b*c*d*f^2*x*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b*c^2*f^2*Ei(
(d*f*x + c*f)/d)*e^(e - c*f/d) + b*c^2*f^2*Ei(-(d*f*x + c*f)/d)*e^(-e + c*
f/d) - b*d^2*f*x*e^(f*x + e) + b*d^2*f*x*e^(-f*x - e) - b*c*d*f*e^(f*x + e
) + b*c*d*f*e^(-f*x - e) - b*d^2*e^(f*x + e) - b*d^2*e^(-f*x - e) - 2*a*d^
2)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = \int \frac{a+b\cosh(e+f\,x)}{{(c+d\,x)}^3}\,dx$$

input
```
int((a + b*cosh(e + f*x))/(c + d*x)^3,x)
```

output
```
int((a + b*cosh(e + f*x))/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{a+b\cosh(e+fx)}{(c+dx)^3}\,dx = \frac{e^{2e}\left(\int \frac{e^{fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b\,c^2d+2e^{2e}\left(\int \frac{e^{fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)bc\,d^2x+e^{2e}\left(\int \frac{e^{fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}d\right.}{}$$

input
```
int((a+b*cosh(f*x+e))/(d*x+c)^3,x)
```

output
```
(e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*
b*c**2*d + 2*e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d*
*3*x**3),x)*b*c*d**2*x + e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d*
*2*x**2 + d**3*x**3),x)*b*d**3*x**2 - e**e*a + int(1/(e**(f*x)*c**3 + 3*e*
*(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*b*c**2*d
 + 2*int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 +
 e**(f*x)*d**3*x**3),x)*b*c*d**2*x + int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**
2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*b*d**3*x**2)/(2*e*
*e*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.162 $\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx &= \frac{3b^2d(c+dx)^2}{8f^2}+\frac{a^2(c+dx)^4}{4d}\\ &+\frac{b^2(c+dx)^4}{8d}-\frac{12abd^3\cosh(e+fx)}{f^4}\\ &-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}\\ &-\frac{3b^2d^3\cosh^2(e+fx)}{8f^4}\\ &-\frac{3b^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}\\ &+\frac{12abd^2(c+dx)\sinh(e+fx)}{f^3}\\ &+\frac{2ab(c+dx)^3\sinh(e+fx)}{f}\\ &+\frac{3b^2d^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{4f^3}\\ &+\frac{b^2(c+dx)^3\cosh(e+fx)\sinh(e+fx)}{2f}\end{aligned}$$

output
```
3/8*b^2*d*(d*x+c)^2/f^2+1/4*a^2*(d*x+c)^4/d+1/8*b^2*(d*x+c)^4/d-12*a*b*d^3
*cosh(f*x+e)/f^4-6*a*b*d*(d*x+c)^2*cosh(f*x+e)/f^2-3/8*b^2*d^3*cosh(f*x+e)
^2/f^4-3/4*b^2*d*(d*x+c)^2*cosh(f*x+e)^2/f^2+12*a*b*d^2*(d*x+c)*sinh(f*x+e
)/f^3+2*a*b*(d*x+c)^3*sinh(f*x+e)/f+3/4*b^2*d^2*(d*x+c)*cosh(f*x+e)*sinh(f
*x+e)/f^3+1/2*b^2*(d*x+c)^3*cosh(f*x+e)*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.98

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$$
$$=\frac{-96abd(c^2f^2+2cdf^2x+d^2(2+f^2x^2))\cosh(e+fx)-3b^2d(2c^2f^2+4cdf^2x+d^2(1+2f^2x^2))\cosh(2(e}{}$$

input
```
Integrate[(c + d*x)^3*(a + b*Cosh[e + f*x])^2,x]
```

output
```
(-96*a*b*d*(c^2*f^2 + 2*c*d*f^2*x + d^2*(2 + f^2*x^2))*Cosh[e + f*x] - 3*b
^2*d*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(1 + 2*f^2*x^2))*Cosh[2*(e + f*x)] + 2
*f*((2*a^2 + b^2)*f^3*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + 16*a
*b*(c + d*x)*(c^2*f^2 + 2*c*d*f^2*x + d^2*(6 + f^2*x^2))*Sinh[e + f*x] + b
^2*(c + d*x)*(2*c^2*f^2 + 4*c*d*f^2*x + d^2*(3 + 2*f^2*x^2))*Sinh[2*(e + f
*x)]))/(16*f^4)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^3 \left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a^2(c+dx)^3+2ab(c+dx)^3\cosh(e+fx)+b^2(c+dx)^3\cosh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(c+dx)^4}{4d}+\frac{12abd^2(c+dx)\sinh(e+fx)}{f^3}-\frac{6abd(c+dx)^2\cosh(e+fx)}{f^2}+\frac{2ab(c+dx)^3\sinh(e+fx)}{f}-\frac{12abd^3\cosh(e+fx)}{f^4}+\frac{3b^2d^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{4f^3}-\frac{3b^2d(c+dx)^2\cosh^2(e+fx)}{4f^2}+\frac{b^2(c+dx)^3\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{3b^2d(c+dx)^2}{8f^2}+\frac{b^2(c+dx)^4}{8d}-\frac{3b^2d^3\cosh^2(e+fx)}{8f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Cosh[e + f*x])^2,x]
```

output
```
(3*b^2*d*(c + d*x)^2)/(8*f^2) + (a^2*(c + d*x)^4)/(4*d) + (b^2*(c + d*x)^4
)/(8*d) - (12*a*b*d^3*Cosh[e + f*x])/f^4 - (6*a*b*d*(c + d*x)^2*Cosh[e + f
*x])/f^2 - (3*b^2*d^3*Cosh[e + f*x]^2)/(8*f^4) - (3*b^2*d*(c + d*x)^2*Cosh
[e + f*x]^2)/(4*f^2) + (12*a*b*d^2*(c + d*x)*Sinh[e + f*x])/f^3 + (2*a*b*(
c + d*x)^3*Sinh[e + f*x])/f + (3*b^2*d^2*(c + d*x)*Cosh[e + f*x]*Sinh[e +
f*x])/(4*f^3) + (b^2*(c + d*x)^3*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.83 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.82

method	result
parallelrisch	$\frac{4(dx+c)\left((dx+c)^2f^2+\frac{3d^2}{2}\right)b^2f\sinh(2fx+2e)-6\left((dx+c)^2f^2+\frac{d^2}{2}\right)b^2d\cosh(2fx+2e)+32(dx+c)ba\left((dx+c)^2f^2+6d^2\right.}{16}$
risch	$\frac{d^3a^2x^4}{4}+\frac{d^3b^2x^4}{8}+d^2a^2c\,x^3+\frac{d^2b^2c\,x^3}{2}+\frac{3a^2d\,c^2x^2}{2}+\frac{3d\,b^2c^2x^2}{4}+a^2c^3x+\frac{b^2c^3x}{2}+\frac{a^2c^4}{4d}+\frac{b^2c^4}{8d}$
parts	Expression too large to display
derivativedivides	Expression too large to display
default	Expression too large to display
orering	Expression too large to display

input
```
int((d*x+c)^3*(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/16*(4*(d*x+c)*((d*x+c)^2*f^2+3/2*d^2)*b^2*f*sinh(2*f*x+2*e)-6*((d*x+c)^2
*f^2+1/2*d^2)*b^2*d*cosh(2*f*x+2*e)+32*(d*x+c)*b*a*((d*x+c)^2*f^2+6*d^2)*f
*sinh(f*x+e)-96*b*a*d*((d*x+c)^2*f^2+2*d^2)*cosh(f*x+e)+16*(a^2+1/2*b^2)*(
1/2*d*x+c)*(1/2*x^2*d^2+c*d*x+c^2)*x*f^4-18*b^2*c^2*d*f^2-9*d^3*b^2)/f^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 1.73

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$$
$$= \frac{2\,(2\,a^2+b^2)d^3f^4x^4+8\,(2\,a^2+b^2)cd^2f^4x^3+12\,(2\,a^2+b^2)c^2df^4x^2+8\,(2\,a^2+b^2)c^3f^4x-3\,(2\,b^2d^3f^2x^2+}{}$$

input
```
integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/16*(2*(2*a^2 + b^2)*d^3*f^4*x^4 + 8*(2*a^2 + b^2)*c*d^2*f^4*x^3 + 12*(2*
a^2 + b^2)*c^2*d*f^4*x^2 + 8*(2*a^2 + b^2)*c^3*f^4*x - 3*(2*b^2*d^3*f^2*x^
2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*cosh(f*x + e)^2 - 3*(2*
b^2*d^3*f^2*x^2 + 4*b^2*c*d^2*f^2*x + 2*b^2*c^2*d*f^2 + b^2*d^3)*sinh(f*x
+ e)^2 - 96*(a*b*d^3*f^2*x^2 + 2*a*b*c*d^2*f^2*x + a*b*c^2*d*f^2 + 2*a*b*d
^3)*cosh(f*x + e) + 4*(8*a*b*d^3*f^3*x^3 + 24*a*b*c*d^2*f^3*x^2 + 8*a*b*c^
3*f^3 + 48*a*b*c*d^2*f + 24*(a*b*c^2*d*f^3 + 2*a*b*d^3*f)*x + (2*b^2*d^3*f
^3*x^3 + 6*b^2*c*d^2*f^3*x^2 + 2*b^2*c^3*f^3 + 3*b^2*c*d^2*f + 3*(2*b^2*c^
2*d*f^3 + b^2*d^3*f)*x)*cosh(f*x + e))*sinh(f*x + e))/f^4
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 779 vs. $2(240) = 480$.

Time = 0.43 (sec) , antiderivative size = 779, normalized size of antiderivative = 3.29

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)**3*(a+b*cosh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**3*x + 3*a**2*c**2*d*x**2/2 + a**2*c*d**2*x**3 + a**2*d*
*3*x**4/4 + 2*a*b*c**3*sinh(e + f*x)/f + 6*a*b*c**2*d*x*sinh(e + f*x)/f -
6*a*b*c**2*d*cosh(e + f*x)/f**2 + 6*a*b*c*d**2*x**2*sinh(e + f*x)/f - 12*a
*b*c*d**2*x*cosh(e + f*x)/f**2 + 12*a*b*c*d**2*sinh(e + f*x)/f**3 + 2*a*b*
d**3*x**3*sinh(e + f*x)/f - 6*a*b*d**3*x**2*cosh(e + f*x)/f**2 + 12*a*b*d*
*3*x*sinh(e + f*x)/f**3 - 12*a*b*d**3*cosh(e + f*x)/f**4 - b**2*c**3*x*sin
h(e + f*x)**2/2 + b**2*c**3*x*cosh(e + f*x)**2/2 + b**2*c**3*sinh(e + f*x)
*cosh(e + f*x)/(2*f) - 3*b**2*c**2*d*x**2*sinh(e + f*x)**2/4 + 3*b**2*c**2
*d*x**2*cosh(e + f*x)**2/4 + 3*b**2*c**2*d*x*sinh(e + f*x)*cosh(e + f*x)/(
2*f) - 3*b**2*c**2*d*sinh(e + f*x)**2/(4*f**2) - b**2*c*d**2*x**3*sinh(e +
 f*x)**2/2 + b**2*c*d**2*x**3*cosh(e + f*x)**2/2 + 3*b**2*c*d**2*x**2*sinh
(e + f*x)*cosh(e + f*x)/(2*f) - 3*b**2*c*d**2*x*sinh(e + f*x)**2/(4*f**2)
- 3*b**2*c*d**2*x*cosh(e + f*x)**2/(4*f**2) + 3*b**2*c*d**2*sinh(e + f*x)*
cosh(e + f*x)/(4*f**3) - b**2*d**3*x**4*sinh(e + f*x)**2/8 + b**2*d**3*x**
4*cosh(e + f*x)**2/8 + b**2*d**3*x**3*sinh(e + f*x)*cosh(e + f*x)/(2*f) -
3*b**2*d**3*x**2*sinh(e + f*x)**2/(8*f**2) - 3*b**2*d**3*x**2*cosh(e + f*x
)**2/(8*f**2) + 3*b**2*d**3*x*sinh(e + f*x)*cosh(e + f*x)/(4*f**3) - 3*b**
2*d**3*sinh(e + f*x)**2/(8*f**4), Ne(f, 0)), ((a + b*cosh(e))**2*(c**3*x +
 3*c**2*d*x**2/2 + c*d**2*x**3 + d**3*x**4/4), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 523 vs. $2(223) = 446$.

Time = 0.07 (sec) , antiderivative size = 523, normalized size of antiderivative = 2.21

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx = \frac{1}{4}\,a^2d^3x^4 + a^2cd^2x^3 + \frac{3}{2}\,a^2c^2dx^2$$
$$+ \frac{3}{16}\left(4\,x^2 + \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} - \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2c^2d$$
$$+ \frac{1}{16}\left(8\,x^3 + \frac{3\left(2\,f^2x^2e^{(2\,e)} - 2\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(2\,fx)}}{f^3} - \frac{3\,(2\,f^2x^2 + 2\,fx + 1)e^{(-2\,fx-2\,e)}}{f^3}\right)b^2cd^2$$
$$+ \frac{1}{32}\left(4\,x^4 + \frac{(4\,f^3x^3e^{(2\,e)} - 6\,f^2x^2e^{(2\,e)} + 6\,fxe^{(2\,e)} - 3\,e^{(2\,e)})e^{(2\,fx)}}{f^4} - \frac{(4\,f^3x^3 + 6\,f^2x^2 + 6\,fx + 3)e^{(-2}}{f^4}\right.$$
$$+ \frac{1}{8}\,b^2c^3\left(4\,x + \frac{e^{(2\,fx+2\,e)}}{f} - \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2c^3x$$
$$+ 3\,abc^2d\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right)$$
$$+ 3\,abcd^2\left(\frac{(f^2x^2e^e - 2\,fxe^e + 2\,e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2 + 2\,fx + 2)e^{(-fx-e)}}{f^3}\right)$$
$$+ abd^3\left(\frac{(f^3x^3e^e - 3\,f^2x^2e^e + 6\,fxe^e - 6\,e^e)e^{(fx)}}{f^4} - \frac{(f^3x^3 + 3\,f^2x^2 + 6\,fx + 6)e^{(-fx-e)}}{f^4}\right)$$
$$+ \frac{2\,abc^3\sinh(fx+e)}{f}$$

input
```
integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + 3/16*(4*x^2 + (2*f*x
*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*
c^2*d + 1/16*(8*x^3 + 3*(2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2
*f*x)/f^3 - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*c*d^2 + 1/
32*(4*x^4 + (4*f^3*x^3*e^(2*e) - 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) - 3*e^(
2*e))*e^(2*f*x)/f^4 - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x - 2*e)
/f^4)*b^2*d^3 + 1/8*b^2*c^3*(4*x + e^(2*f*x + 2*e)/f - e^(-2*f*x - 2*e)/f)
 + a^2*c^3*x + 3*a*b*c^2*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f*x + 1)*e^(-f*
x - e)/f^2) + 3*a*b*c*d^2*((f^2*x^2*e^e - 2*f*x*e^e + 2*e^e)*e^(f*x)/f^3 -
 (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + a*b*d^3*((f^3*x^3*e^e - 3*f^2*x
^2*e^e + 6*f*x*e^e - 6*e^e)*e^(f*x)/f^4 - (f^3*x^3 + 3*f^2*x^2 + 6*f*x + 6
)*e^(-f*x - e)/f^4) + 2*a*b*c^3*sinh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. $2(223) = 446$.

Time = 0.12 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.53

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$$
$$= \frac{1}{4}a^2d^3x^4 + \frac{1}{8}b^2d^3x^4 + a^2cd^2x^3 + \frac{1}{2}b^2cd^2x^3 + \frac{3}{2}a^2c^2dx^2 + \frac{3}{4}b^2c^2dx^2 + a^2c^3x + \frac{1}{2}b^2c^3x$$
$$+ \frac{(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + 12\,b^2c^2df^3x - 6\,b^2d^3f^2x^2 + 4\,b^2c^3f^3 - 12\,b^2cd^2f^2x - 6\,b^2c^2df^2 + 6\,b^2d^3f}{32\,f^4}$$
$$+ \frac{(abd^3f^3x^3 + 3\,abcd^2f^3x^2 + 3\,abc^2df^3x - 3\,abd^3f^2x^2 + abc^3f^3 - 6\,abcd^2f^2x - 3\,abc^2df^2 + 6\,abd^3fx +}{f^4}$$
$$- \frac{(abd^3f^3x^3 + 3\,abcd^2f^3x^2 + 3\,abc^2df^3x + 3\,abd^3f^2x^2 + abc^3f^3 + 6\,abcd^2f^2x + 3\,abc^2df^2 + 6\,abd^3fx +}{f^4}$$
$$- \frac{(4\,b^2d^3f^3x^3 + 12\,b^2cd^2f^3x^2 + 12\,b^2c^2df^3x + 6\,b^2d^3f^2x^2 + 4\,b^2c^3f^3 + 12\,b^2cd^2f^2x + 6\,b^2c^2df^2 + 6\,b^2d^3f}{32\,f^4}$$

input
```
integrate((d*x+c)^3*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/4*a^2*d^3*x^4 + 1/8*b^2*d^3*x^4 + a^2*c*d^2*x^3 + 1/2*b^2*c*d^2*x^3 + 3/
2*a^2*c^2*d*x^2 + 3/4*b^2*c^2*d*x^2 + a^2*c^3*x + 1/2*b^2*c^3*x + 1/32*(4*
b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x - 6*b^2*d^3*f^
2*x^2 + 4*b^2*c^3*f^3 - 12*b^2*c*d^2*f^2*x - 6*b^2*c^2*d*f^2 + 6*b^2*d^3*f
*x + 6*b^2*c*d^2*f - 3*b^2*d^3)*e^(2*f*x + 2*e)/f^4 + (a*b*d^3*f^3*x^3 + 3
*a*b*c*d^2*f^3*x^2 + 3*a*b*c^2*d*f^3*x - 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 -
 6*a*b*c*d^2*f^2*x - 3*a*b*c^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f - 6*a
*b*d^3)*e^(f*x + e)/f^4 - (a*b*d^3*f^3*x^3 + 3*a*b*c*d^2*f^3*x^2 + 3*a*b*c
^2*d*f^3*x + 3*a*b*d^3*f^2*x^2 + a*b*c^3*f^3 + 6*a*b*c*d^2*f^2*x + 3*a*b*c
^2*d*f^2 + 6*a*b*d^3*f*x + 6*a*b*c*d^2*f + 6*a*b*d^3)*e^(-f*x - e)/f^4 - 1
/32*(4*b^2*d^3*f^3*x^3 + 12*b^2*c*d^2*f^3*x^2 + 12*b^2*c^2*d*f^3*x + 6*b^2
*d^3*f^2*x^2 + 4*b^2*c^3*f^3 + 12*b^2*c*d^2*f^2*x + 6*b^2*c^2*d*f^2 + 6*b^
2*d^3*f*x + 6*b^2*c*d^2*f + 3*b^2*d^3)*e^(-2*f*x - 2*e)/f^4
```

Mupad [B] (verification not implemented)

Time = 3.69 (sec) , antiderivative size = 481, normalized size of antiderivative = 2.03

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx = a^2\,c^3\,x+\frac{b^2\,c^3\,x}{2}+\frac{a^2\,d^3\,x^4}{4}+\frac{b^2\,d^3\,x^4}{8}+\frac{3\,a^2\,c^2\,d\,x^2}{2}+a^2\,c\,d^2\,x^3+\frac{3\,b^2\,c^2\,d\,x^2}{4}+\frac{b^2\,c\,d^2\,x^3}{2}-\frac{3\,b^2\,d^3\cosh(2\,e+2\,f\,x)}{16\,f^4}+\frac{b^2\,c^3\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{12\,a\,b\,d^3\cosh(e+f\,x)}{f^4}+\frac{2\,a\,b\,c^3\sinh(e+f\,x)}{f}-\frac{3\,b^2\,d^3\,x^2\cosh(2\,e+2\,f\,x)}{8\,f^2}+\frac{b^2\,d^3\,x^3\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{3\,b^2\,c^2\,d\cosh(2\,e+2\,f\,x)}{8\,f^2}+\frac{3\,b^2\,c\,d^2\sinh(2\,e+2\,f\,x)}{8\,f^3}+\frac{3\,b^2\,d^3\,x\sinh(2\,e+2\,f\,x)}{8\,f^3}-\frac{3\,b^2\,c\,d^2\,x\cosh(2\,e+2\,f\,x)}{4\,f^2}+\frac{3\,b^2\,c^2\,d\,x\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{6\,a\,b\,c^2\,d\cosh(e+f\,x)}{f^2}+\frac{12\,a\,b\,c\,d^2\sinh(e+f\,x)}{f^3}+\frac{12\,a\,b\,d^3\,x\sinh(e+f\,x)}{f^3}+\frac{3\,b^2\,c\,d^2\,x^2\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{6\,a\,b\,d^3\,x^2\cosh(e+f\,x)}{f^2}+\frac{2\,a\,b\,d^3\,x^3\sinh(e+f\,x)}{f}+\frac{6\,a\,b\,c\,d^2\,x^2\sinh(e+f\,x)}{f}-\frac{12\,a\,b\,c\,d^2\,x\cosh(e+f\,x)}{f^2}+\frac{6\,a\,b\,c^2\,d\,x\sinh(e+f\,x)}{f}$$

input
```
int((a + b*cosh(e + f*x))^2*(c + d*x)^3,x)
```

output
```
a^2*c^3*x + (b^2*c^3*x)/2 + (a^2*d^3*x^4)/4 + (b^2*d^3*x^4)/8 + (3*a^2*c^2
*d*x^2)/2 + a^2*c*d^2*x^3 + (3*b^2*c^2*d*x^2)/4 + (b^2*c*d^2*x^3)/2 - (3*b
^2*d^3*cosh(2*e + 2*f*x))/(16*f^4) + (b^2*c^3*sinh(2*e + 2*f*x))/(4*f) - (
12*a*b*d^3*cosh(e + f*x))/f^4 + (2*a*b*c^3*sinh(e + f*x))/f - (3*b^2*d^3*x
^2*cosh(2*e + 2*f*x))/(8*f^2) + (b^2*d^3*x^3*sinh(2*e + 2*f*x))/(4*f) - (3
*b^2*c^2*d*cosh(2*e + 2*f*x))/(8*f^2) + (3*b^2*c*d^2*sinh(2*e + 2*f*x))/(8
*f^3) + (3*b^2*d^3*x*sinh(2*e + 2*f*x))/(8*f^3) - (3*b^2*c*d^2*x*cosh(2*e
+ 2*f*x))/(4*f^2) + (3*b^2*c^2*d*x*sinh(2*e + 2*f*x))/(4*f) - (6*a*b*c^2*d
*cosh(e + f*x))/f^2 + (12*a*b*c*d^2*sinh(e + f*x))/f^3 + (12*a*b*d^3*x*sin
h(e + f*x))/f^3 + (3*b^2*c*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) - (6*a*b*d^3*x
^2*cosh(e + f*x))/f^2 + (2*a*b*d^3*x^3*sinh(e + f*x))/f + (6*a*b*c*d^2*x^2
*sinh(e + f*x))/f - (12*a*b*c*d^2*x*cosh(e + f*x))/f^2 + (6*a*b*c^2*d*x*si
nh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.75 (sec) , antiderivative size = 940, normalized size of antiderivative = 3.97

$$\int (c+dx)^3(a+b\cosh(e+fx))^2\,dx$$
$$= \frac{4e^{4fx+4e}b^2c^3f^3 - 192e^{3fx+3e}ab\,d^3 - 192e^{fx+e}ab\,d^3 - 6b^2c^2d\,f^2 - 6b^2c\,d^2f - 4b^2d^3f^3x^3 - 6b^2d^3f^2x^2 - 6b}{}$$

input
```
int((d*x+c)^3*(a+b*cosh(f*x+e))^2,x)
```

output

```
(4*e**(4*e + 4*f*x)*b**2*c**3*f**3 + 12*e**(4*e + 4*f*x)*b**2*c**2*d*f**3*
x - 6*e**(4*e + 4*f*x)*b**2*c**2*d*f**2 + 12*e**(4*e + 4*f*x)*b**2*c*d**2*
f**3*x**2 - 12*e**(4*e + 4*f*x)*b**2*c*d**2*f**2*x + 6*e**(4*e + 4*f*x)*b*
*2*c*d**2*f + 4*e**(4*e + 4*f*x)*b**2*d**3*f**3*x**3 - 6*e**(4*e + 4*f*x)*
b**2*d**3*f**2*x**2 + 6*e**(4*e + 4*f*x)*b**2*d**3*f*x - 3*e**(4*e + 4*f*x
)*b**2*d**3 + 32*e**(3*e + 3*f*x)*a*b*c**3*f**3 + 96*e**(3*e + 3*f*x)*a*b*
c**2*d*f**3*x - 96*e**(3*e + 3*f*x)*a*b*c**2*d*f**2 + 96*e**(3*e + 3*f*x)*
a*b*c*d**2*f**3*x**2 - 192*e**(3*e + 3*f*x)*a*b*c*d**2*f**2*x + 192*e**(3*
e + 3*f*x)*a*b*c*d**2*f + 32*e**(3*e + 3*f*x)*a*b*d**3*f**3*x**3 - 96*e**(
3*e + 3*f*x)*a*b*d**3*f**2*x**2 + 192*e**(3*e + 3*f*x)*a*b*d**3*f*x - 192*
e**(3*e + 3*f*x)*a*b*d**3 + 32*e**(2*e + 2*f*x)*a**2*c**3*f**4*x + 48*e**(
2*e + 2*f*x)*a**2*c**2*d*f**4*x**2 + 32*e**(2*e + 2*f*x)*a**2*c*d**2*f**4*
x**3 + 8*e**(2*e + 2*f*x)*a**2*d**3*f**4*x**4 + 16*e**(2*e + 2*f*x)*b**2*c
**3*f**4*x + 24*e**(2*e + 2*f*x)*b**2*c**2*d*f**4*x**2 + 16*e**(2*e + 2*f*
x)*b**2*c*d**2*f**4*x**3 + 4*e**(2*e + 2*f*x)*b**2*d**3*f**4*x**4 - 32*e**
(e + f*x)*a*b*c**3*f**3 - 96*e**(e + f*x)*a*b*c**2*d*f**3*x - 96*e**(e + f
*x)*a*b*c**2*d*f**2 - 96*e**(e + f*x)*a*b*c*d**2*f**3*x**2 - 192*e**(e + f
*x)*a*b*c*d**2*f**2*x - 192*e**(e + f*x)*a*b*c*d**2*f - 32*e**(e + f*x)*a*
b*d**3*f**3*x**3 - 96*e**(e + f*x)*a*b*d**3*f**2*x**2 - 192*e**(e + f*x)*a
*b*d**3*f*x - 192*e**(e + f*x)*a*b*d**3 - 4*b**2*c**3*f**3 - 12*b**2*c*...
```

3.163 $\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx$

Optimal result

Integrand size = 20, antiderivative size = 182

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx = \frac{b^2d^2x}{4f^2}+\frac{a^2(c+dx)^3}{3d}+\frac{b^2(c+dx)^3}{6d} - \frac{4abd(c+dx)\cosh(e+fx)}{f^2} - \frac{b^2d(c+dx)\cosh^2(e+fx)}{2f^2} + \frac{4abd^2\sinh(e+fx)}{f^3} + \frac{2ab(c+dx)^2\sinh(e+fx)}{f} + \frac{b^2d^2\cosh(e+fx)\sinh(e+fx)}{4f^3} + \frac{b^2(c+dx)^2\cosh(e+fx)\sinh(e+fx)}{2f}$$

output

```
1/4*b^2*d^2*x/f^2+1/3*a^2*(d*x+c)^3/d+1/6*b^2*(d*x+c)^3/d-4*a*b*d*(d*x+c)*
cosh(f*x+e)/f^2-1/2*b^2*d*(d*x+c)*cosh(f*x+e)^2/f^2+4*a*b*d^2*sinh(f*x+e)/
f^3+2*a*b*(d*x+c)^2*sinh(f*x+e)/f+1/4*b^2*d^2*cosh(f*x+e)*sinh(f*x+e)/f^3+
1/2*b^2*(d*x+c)^2*cosh(f*x+e)*sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.38

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx = \frac{1}{24}\left(24a^2c^2x+12b^2c^2x+24a^2cdx^2+12b^2cdx^2+8a^2d^2x^3+4b^2d^2x^3-\frac{96abd(c+dx)\cosh(e+fx)}{f^2}-\frac{6b^2d(c+dx)\cosh(2(e+fx))}{f^2}+\frac{96abd^2\sinh(e+fx)}{f^3}+\frac{48abc^2\sinh(e+fx)}{f}+\frac{96abcdx\sinh(e+fx)}{f}+\frac{48abd^2x^2\sinh(e+fx)}{f}+\frac{3b^2d^2\sinh(2(e+fx))}{f^3}+\frac{6b^2c^2\sinh(2(e+fx))}{f}+\frac{12b^2cdx\sinh(2(e+fx))}{f}+\frac{6b^2d^2x^2\sinh(2(e+fx))}{f}\right)$$

input
```
Integrate[(c + d*x)^2*(a + b*Cosh[e + f*x])^2,x]
```

output
```
(24*a^2*c^2*x + 12*b^2*c^2*x + 24*a^2*c*d*x^2 + 12*b^2*c*d*x^2 + 8*a^2*d^2
*x^3 + 4*b^2*d^2*x^3 - (96*a*b*d*(c + d*x)*Cosh[e + f*x])/f^2 - (6*b^2*d*(
c + d*x)*Cosh[2*(e + f*x)])/f^2 + (96*a*b*d^2*Sinh[e + f*x])/f^3 + (48*a*b
*c^2*Sinh[e + f*x])/f + (96*a*b*c*d*x*Sinh[e + f*x])/f + (48*a*b*d^2*x^2*S
inh[e + f*x])/f + (3*b^2*d^2*Sinh[2*(e + f*x)])/f^3 + (6*b^2*c^2*Sinh[2*(e
 + f*x)])/f + (12*b^2*c*d*x*Sinh[2*(e + f*x)])/f + (6*b^2*d^2*x^2*Sinh[2*(
e + f*x)])/f)/24
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.150, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^2\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)^2+2ab(c+dx)^2\cosh(e+fx)+b^2(c+dx)^2\cosh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^3}{3d}-\frac{4abd(c+dx)\cosh(e+fx)}{f^2}+\frac{2ab(c+dx)^2\sinh(e+fx)}{f}+\frac{4abd^2\sinh(e+fx)}{f^3}-\frac{b^2d(c+dx)\cosh^2(e+fx)}{2f^2}+\frac{b^2(c+dx)^2\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{b^2(c+dx)^3}{6d}+\frac{b^2d^2\sinh(e+fx)\cosh(e+fx)}{4f^3}+\frac{b^2d^2x}{4f^2}$$

input

```
Int[(c + d*x)^2*(a + b*Cosh[e + f*x])^2,x]
```

output

```
(b^2*d^2*x)/(4*f^2) + (a^2*(c + d*x)^3)/(3*d) + (b^2*(c + d*x)^3)/(6*d) -
(4*a*b*d*(c + d*x)*Cosh[e + f*x])/f^2 - (b^2*d*(c + d*x)*Cosh[e + f*x]^2)/
(2*f^2) + (4*a*b*d^2*Sinh[e + f*x])/f^3 + (2*a*b*(c + d*x)^2*Sinh[e + f*x]
)/f + (b^2*d^2*Cosh[e + f*x]*Sinh[e + f*x])/(4*f^3) + (b^2*(c + d*x)^2*Cos
h[e + f*x]*Sinh[e + f*x])/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.80

method	result
parallelrisch	$\frac{b^2\left((dx+c)^2f^2+\frac{d^2}{2}\right)\sinh(2fx+2e)-b^2df(dx+c)\cosh(2fx+2e)+8ba\left((dx+c)^2f^2+2d^2\right)\sinh(fx+e)+4\left(-4bad(dx+c)\right.}{4f^3}$
risch	$\frac{d^2a^2x^3}{3}+\frac{d^2b^2x^3}{6}+a^2dc\,x^2+\frac{d\,b^2c\,x^2}{2}+a^2c^2x+\frac{b^2c^2x}{2}+\frac{a^2c^3}{3d}+\frac{b^2c^3}{6d}+\frac{b^2\left(2d^2x^2f^2+4cd\,f^2x+2c^2\right.}{1}$
parts	$\frac{a^2(dx+c)^3}{3d}+\frac{b^2\left(\frac{d^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^3}{6}-\frac{(fx+e)\cosh(fx+e)^2}{2}+\frac{\cosh(fx+e)\sinh(fx+e)}{4}+\frac{fx}{4}+\frac{e}{4}\right)}{f^2}\right.}{}$
derivativedivides	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx\right.}{}}{}$
default	$\frac{\frac{d^2a^2(fx+e)^3}{3f^2}+\frac{2d^2ab\left((fx+e)^2\sinh(fx+e)-2(fx+e)\cosh(fx+e)+2\sinh(fx+e)\right)}{f^2}+\frac{d^2b^2\left(\frac{(fx+e)^2\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx\right.}{}}{}$
orering	Expression too large to display

input
```
int((d*x+c)^2*(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*((d*x+c)^2*f^2+1/2*d^2)*sinh(2*f*x+2*e)-b^2*d*f*(d*x+c)*cosh(2*f*
x+2*e)+8*b*a*((d*x+c)^2*f^2+2*d^2)*sinh(f*x+e)+4*(-4*b*a*d*(d*x+c)*cosh(f*
x+e)+(a^2+1/2*b^2)*(1/3*x^2*d^2+c*d*x+c^2)*x*f^2-4*c*(a-1/16*b)*b*d)*f)/f^
3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.32

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx$$
$$= \frac{2\,(2\,a^2+b^2)d^2f^3x^3+6\,(2\,a^2+b^2)cdf^3x^2+6\,(2\,a^2+b^2)c^2f^3x-3\,(b^2d^2fx+b^2cdf)\cosh\left(fx+e\right)^2-3\,(b\cdots}{}$$

input
```
integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/12*(2*(2*a^2 + b^2)*d^2*f^3*x^3 + 6*(2*a^2 + b^2)*c*d*f^3*x^2 + 6*(2*a^2
 + b^2)*c^2*f^3*x - 3*(b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e)^2 - 3*(b^2*d
^2*f*x + b^2*c*d*f)*sinh(f*x + e)^2 - 48*(a*b*d^2*f*x + a*b*c*d*f)*cosh(f*
x + e) + 3*(8*a*b*d^2*f^2*x^2 + 16*a*b*c*d*f^2*x + 8*a*b*c^2*f^2 + 16*a*b*
d^2 + (2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + b^2*d^2)*cosh
(f*x + e))*sinh(f*x + e))/f^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 456 vs. $2(177) = 354$.

Time = 0.32 (sec) , antiderivative size = 456, normalized size of antiderivative = 2.51

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx$$
$$= \begin{cases} a^2c^2x + a^2cdx^2 + \frac{a^2d^2x^3}{3} + \frac{2abc^2\sinh\left(e+fx\right)}{f} + \frac{4abcdx\sinh\left(e+fx\right)}{f} - \frac{4abcd\cosh\left(e+fx\right)}{f^2} + \frac{2abd^2x^2\sinh\left(e+fx\right)}{f} - \frac{4abd^2x\cdots}{} \\ (a+b\cosh\left(e\right))^2\left(c^2x+cdx^2+\frac{d^2x^3}{3}\right) \end{cases}$$

input
```
integrate((d*x+c)**2*(a+b*cosh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c**2*x + a**2*c*d*x**2 + a**2*d**2*x**3/3 + 2*a*b*c**2*sin
h(e + f*x)/f + 4*a*b*c*d*x*sinh(e + f*x)/f - 4*a*b*c*d*cosh(e + f*x)/f**2
+ 2*a*b*d**2*x**2*sinh(e + f*x)/f - 4*a*b*d**2*x*cosh(e + f*x)/f**2 + 4*a*
b*d**2*sinh(e + f*x)/f**3 - b**2*c**2*x*sinh(e + f*x)**2/2 + b**2*c**2*x*c
osh(e + f*x)**2/2 + b**2*c**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*c*d
*x**2*sinh(e + f*x)**2/2 + b**2*c*d*x**2*cosh(e + f*x)**2/2 + b**2*c*d*x*s
inh(e + f*x)*cosh(e + f*x)/f - b**2*c*d*sinh(e + f*x)**2/(2*f**2) - b**2*d
**2*x**3*sinh(e + f*x)**2/6 + b**2*d**2*x**3*cosh(e + f*x)**2/6 + b**2*d**
2*x**2*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d**2*x*sinh(e + f*x)**2/(4
*f**2) - b**2*d**2*x*cosh(e + f*x)**2/(4*f**2) + b**2*d**2*sinh(e + f*x)*c
osh(e + f*x)/(4*f**3), Ne(f, 0)), ((a + b*cosh(e))**2*(c**2*x + c*d*x**2 +
 d**2*x**3/3), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.78

$$\begin{aligned}
&\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx \\
&= \frac{1}{3}a^2d^2x^3 + a^2cdx^2 + \frac{1}{8}\left(4x^2 + \frac{(2fxe^{(2e)} - e^{(2e)})e^{(2fx)}}{f^2} - \frac{(2fx+1)e^{(-2fx-2e)}}{f^2}\right)b^2cd \\
&\quad + \frac{1}{48}\left(8x^3 + \frac{3(2f^2x^2e^{(2e)} - 2fxe^{(2e)} + e^{(2e)})e^{(2fx)}}{f^3} - \frac{3(2f^2x^2+2fx+1)e^{(-2fx-2e)}}{f^3}\right)b^2d^2 \\
&\quad + \frac{1}{8}b^2c^2\left(4x + \frac{e^{(2fx+2e)}}{f} - \frac{e^{(-2fx-2e)}}{f}\right) + a^2c^2x \\
&\quad + 2abcd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) \\
&\quad + abd^2\left(\frac{(f^2x^2e^e - 2fxe^e + 2e^e)e^{(fx)}}{f^3} - \frac{(f^2x^2+2fx+2)e^{(-fx-e)}}{f^3}\right) \\
&\quad + \frac{2abc^2\sinh(fx+e)}{f}
\end{aligned}$$

input
```
integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + 1/8*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(
2*f*x)/f^2 - (2*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*c*d + 1/48*(8*x^3 + 3*(
2*f^2*x^2*e^(2*e) - 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x)/f^3 - 3*(2*f^2*x^2
+ 2*f*x + 1)*e^(-2*f*x - 2*e)/f^3)*b^2*d^2 + 1/8*b^2*c^2*(4*x + e^(2*f*x +
 2*e)/f - e^(-2*f*x - 2*e)/f) + a^2*c^2*x + 2*a*b*c*d*((f*x*e^e - e^e)*e^(
f*x)/f^2 - (f*x + 1)*e^(-f*x - e)/f^2) + a*b*d^2*((f^2*x^2*e^e - 2*f*x*e^e
 + 2*e^e)*e^(f*x)/f^3 - (f^2*x^2 + 2*f*x + 2)*e^(-f*x - e)/f^3) + 2*a*b*c^
2*sinh(f*x + e)/f
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(170) = 340$.

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.90

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx$$
$$= \frac{1}{3}a^2d^2x^3 + \frac{1}{6}b^2d^2x^3 + a^2cdx^2 + \frac{1}{2}b^2cdx^2 + a^2c^2x + \frac{1}{2}b^2c^2x + \frac{(2\,b^2d^2f^2x^2 + 4\,b^2cdf^2x + 2\,b^2c^2f^2 - 2\,b^2d^2fx - 2\,b^2cdf + b^2d^2)e^{(2\,fx+2\,e)}}{16\,f^3} + \frac{(abd^2f^2x^2 + 2\,abcdf^2x + abc^2f^2 - 2\,abd^2fx - 2\,abcdf + 2\,abd^2)e^{(fx+e)}}{f^3} - \frac{(abd^2f^2x^2 + 2\,abcdf^2x + abc^2f^2 + 2\,abd^2fx + 2\,abcdf + 2\,abd^2)e^{(-fx-e)}}{f^3} - \frac{(2\,b^2d^2f^2x^2 + 4\,b^2cdf^2x + 2\,b^2c^2f^2 + 2\,b^2d^2fx + 2\,b^2cdf + b^2d^2)e^{(-2\,fx-2\,e)}}{16\,f^3}$$

input
```
integrate((d*x+c)^2*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/3*a^2*d^2*x^3 + 1/6*b^2*d^2*x^3 + a^2*c*d*x^2 + 1/2*b^2*c*d*x^2 + a^2*c^
2*x + 1/2*b^2*c^2*x + 1/16*(2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^
2*f^2 - 2*b^2*d^2*f*x - 2*b^2*c*d*f + b^2*d^2)*e^(2*f*x + 2*e)/f^3 + (a*b*
d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2 - 2*a*b*d^2*f*x - 2*a*b*c*d*f
+ 2*a*b*d^2)*e^(f*x + e)/f^3 - (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^
2*f^2 + 2*a*b*d^2*f*x + 2*a*b*c*d*f + 2*a*b*d^2)*e^(-f*x - e)/f^3 - 1/16*(
2*b^2*d^2*f^2*x^2 + 4*b^2*c*d*f^2*x + 2*b^2*c^2*f^2 + 2*b^2*d^2*f*x + 2*b^
2*c*d*f + b^2*d^2)*e^(-2*f*x - 2*e)/f^3
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.54

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx = a^2\,c^2\,x+\frac{b^2\,c^2\,x}{2}+\frac{a^2\,d^2\,x^3}{3}+\frac{b^2\,d^2\,x^3}{6}+\frac{b^2\,c^2\,\sinh(2\,e+2\,f\,x)}{4\,f}+\frac{b^2\,d^2\,\sinh(2\,e+2\,f\,x)}{8\,f^3}+a^2\,c\,d\,x^2+\frac{b^2\,c\,d\,x^2}{2}+\frac{2\,a\,b\,c^2\,\sinh(e+f\,x)}{f}+\frac{4\,a\,b\,d^2\,\sinh(e+f\,x)}{f^3}+\frac{b^2\,d^2\,x^2\,\sinh(2\,e+2\,f\,x)}{4\,f}-\frac{b^2\,c\,d\cosh(2\,e+2\,f\,x)}{4\,f^2}-\frac{b^2\,d^2\,x\cosh(2\,e+2\,f\,x)}{4\,f^2}-\frac{4\,a\,b\,c\,d\cosh(e+f\,x)}{f^2}-\frac{4\,a\,b\,d^2\,x\cosh(e+f\,x)}{f^2}+\frac{2\,a\,b\,d^2\,x^2\,\sinh(e+f\,x)}{f}+\frac{b^2\,c\,d\,x\,\sinh(2\,e+2\,f\,x)}{2\,f}+\frac{4\,a\,b\,c\,d\,x\,\sinh(e+f\,x)}{f}$$

input
```
int((a + b*cosh(e + f*x))^2*(c + d*x)^2,x)
```

output
```
a^2*c^2*x + (b^2*c^2*x)/2 + (a^2*d^2*x^3)/3 + (b^2*d^2*x^3)/6 + (b^2*c^2*s
inh(2*e + 2*f*x))/(4*f) + (b^2*d^2*sinh(2*e + 2*f*x))/(8*f^3) + a^2*c*d*x^
2 + (b^2*c*d*x^2)/2 + (2*a*b*c^2*sinh(e + f*x))/f + (4*a*b*d^2*sinh(e + f*
x))/f^3 + (b^2*d^2*x^2*sinh(2*e + 2*f*x))/(4*f) - (b^2*c*d*cosh(2*e + 2*f*
x))/(4*f^2) - (b^2*d^2*x*cosh(2*e + 2*f*x))/(4*f^2) - (4*a*b*c*d*cosh(e +
f*x))/f^2 - (4*a*b*d^2*x*cosh(e + f*x))/f^2 + (2*a*b*d^2*x^2*sinh(e + f*x)
)/f + (b^2*c*d*x*sinh(2*e + 2*f*x))/(2*f) + (4*a*b*c*d*x*sinh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 554, normalized size of antiderivative = 3.04

$$\int (c+dx)^2(a+b\cosh(e+fx))^2\,dx$$
$$= \frac{-6e^{4fx+4e}b^2cdf + 6e^{4fx+4e}b^2d^2f^2x^2 - 6e^{4fx+4e}b^2d^2fx + 48e^{3fx+3e}ab\,c^2f^2 + 48e^{2fx+2e}a^2c^2f^3x + 16e^{2fx+2}}{}$$

input
```
int((d*x+c)^2*(a+b*cosh(f*x+e))^2,x)
```

output
```
(6*e**(4*e + 4*f*x)*b**2*c**2*f**2 + 12*e**(4*e + 4*f*x)*b**2*c*d*f**2*x -
 6*e**(4*e + 4*f*x)*b**2*c*d*f + 6*e**(4*e + 4*f*x)*b**2*d**2*f**2*x**2 -
6*e**(4*e + 4*f*x)*b**2*d**2*f*x + 3*e**(4*e + 4*f*x)*b**2*d**2 + 48*e**(3
*e + 3*f*x)*a*b*c**2*f**2 + 96*e**(3*e + 3*f*x)*a*b*c*d*f**2*x - 96*e**(3*
e + 3*f*x)*a*b*c*d*f + 48*e**(3*e + 3*f*x)*a*b*d**2*f**2*x**2 - 96*e**(3*e
 + 3*f*x)*a*b*d**2*f*x + 96*e**(3*e + 3*f*x)*a*b*d**2 + 48*e**(2*e + 2*f*x
)*a**2*c**2*f**3*x + 48*e**(2*e + 2*f*x)*a**2*c*d*f**3*x**2 + 16*e**(2*e +
 2*f*x)*a**2*d**2*f**3*x**3 + 24*e**(2*e + 2*f*x)*b**2*c**2*f**3*x + 24*e*
*(2*e + 2*f*x)*b**2*c*d*f**3*x**2 + 8*e**(2*e + 2*f*x)*b**2*d**2*f**3*x**3
 - 48*e**(e + f*x)*a*b*c**2*f**2 - 96*e**(e + f*x)*a*b*c*d*f**2*x - 96*e**
(e + f*x)*a*b*c*d*f - 48*e**(e + f*x)*a*b*d**2*f**2*x**2 - 96*e**(e + f*x)
*a*b*d**2*f*x - 96*e**(e + f*x)*a*b*d**2 - 6*b**2*c**2*f**2 - 12*b**2*c*d*
f**2*x - 6*b**2*c*d*f - 6*b**2*d**2*f**2*x**2 - 6*b**2*d**2*f*x - 3*b**2*d
**2)/(48*e**(2*e + 2*f*x)*f**3)
```

3.164 $\int (c+dx)(a+b\cosh(e+fx))^2\,dx$

Optimal result

Integrand size = 18, antiderivative size = 113

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx = \frac{a^2(c+dx)^2}{2d}+\frac{b^2(c+dx)^2}{4d}-\frac{2abd\cosh(e+fx)}{f^2}-\frac{b^2d\cosh^2(e+fx)}{4f^2}+\frac{2ab(c+dx)\sinh(e+fx)}{f}+\frac{b^2(c+dx)\cosh(e+fx)\sinh(e+fx)}{2f}$$

output
```
1/2*a^2*(d*x+c)^2/d+1/4*b^2*(d*x+c)^2/d-2*a*b*d*cosh(f*x+e)/f^2-1/4*b^2*d*
cosh(f*x+e)^2/f^2+2*a*b*(d*x+c)*sinh(f*x+e)/f+1/2*b^2*(d*x+c)*cosh(f*x+e)*
sinh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 4.24 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.85

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx = -\frac{2(2a^2+b^2)(e+fx)(-2cf+d(e-fx))+16abd\cosh(e+fx)+b^2d\cosh(2(e+fx))-16abf(c+dx}{8f^2}$$

input
```
Integrate[(c + d*x)*(a + b*Cosh[e + f*x])^2,x]
```

output
```
-1/8*(2*(2*a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x)) + 16*a*b*d*Cosh[e +
 f*x] + b^2*d*Cosh[2*(e + f*x)] - 16*a*b*f*(c + d*x)*Sinh[e + f*x] - 2*b^2
*f*(c + d*x)*Sinh[2*(e + f*x)])/f^2
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^2(c+dx)+2ab(c+dx)\cosh(e+fx)+b^2(c+dx)\cosh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\sinh(e+fx)}{f}-\frac{2abd\cosh(e+fx)}{f^2}+\frac{b^2(c+dx)\sinh(e+fx)\cosh(e+fx)}{2f}+\frac{b^2(c+dx)^2}{4d}-\frac{b^2d\cosh^2(e+fx)}{4f^2}$$

input
```
Int[(c + d*x)*(a + b*Cosh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) + (b^2*(c + d*x)^2)/(4*d) - (2*a*b*d*Cosh[e + f*x]
)/f^2 - (b^2*d*Cosh[e + f*x]^2)/(4*f^2) + (2*a*b*(c + d*x)*Sinh[e + f*x])/
f + (b^2*(c + d*x)*Cosh[e + f*x]*Sinh[e + f*x])/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{2b^2 f(dx+c)\sinh(2fx+2e)-b^2 d\cosh(2fx+2e)+16abf(dx+c)\sinh(fx+e)-16abd\cosh(fx+e)+\left(\left(2d\,x^2+4cx\right)f^2-3d\right)}{8f^2}$
risch	$\frac{a^2 d\,x^2}{2}+a^2 cx+\frac{b^2 d\,x^2}{4}+\frac{b^2 cx}{2}+\frac{b^2(2dxf+2cf-d)e^{2fx+2e}}{16f^2}+\frac{ab(dxf+cf-d)e^{fx+e}}{f^2}-\frac{ab(dxf+cf+d)e^{-f}}{f^2}$
parts	$a^2\left(\frac{1}{2}d\,x^2+cx\right)+\frac{b^2\left(\frac{d\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\left(\frac{\cosh(fx+e)\sinh(fx+e)}{2}+\frac{fx}{2}\right.}{f}\right.}{f}$
derivativedivides	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\,a^2}{}}{}$
default	$\frac{\frac{d\,a^2(fx+e)^2}{2f}+\frac{2dab((fx+e)\sinh(fx+e)-\cosh(fx+e))}{f}+\frac{d\,b^2\left(\frac{(fx+e)\cosh(fx+e)\sinh(fx+e)}{2}+\frac{(fx+e)^2}{4}-\frac{\cosh(fx+e)^2}{4}\right)}{f}-\frac{de\,a^2}{}}{}$
orering	$\frac{(2d^5 f^4 x^6+12c\,d^4 f^4 x^5+28c^2 d^3 f^4 x^4+32c^3 d^2 f^4 x^3+18c^4 d\,f^4 x^2-15d^5 f^2 x^4+4c^5 f^4 x-60c\,d^4 f^2 x^3-85c^2 d^3 f^2 x^2-50c^3}{4f^4(dx+c)^4}$

input
```
int((d*x+c)*(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(2*b^2*f*(d*x+c)*sinh(2*f*x+2*e)-b^2*d*cosh(2*f*x+2*e)+16*a*b*f*(d*x+c
)*sinh(f*x+e)-16*a*b*d*cosh(f*x+e)+((2*d*x^2+4*c*x)*f^2-3*d)*b^2+8*(1/2*d*
x+c)*a^2*x*f^2)/f^2
```

Fricas [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.08

$$\int (c + dx)(a + b\cosh(e + fx))^2\,dx$$
$$= \frac{2\,(2\,a^2 + b^2)df^2x^2 + 4\,(2\,a^2 + b^2)cf^2x - b^2d\cosh\left(fx + e\right)^2 - b^2d\sinh\left(fx + e\right)^2 - 16\,abd\cosh\left(fx + e\right) +}{8\,f^2}$$

input
```
integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/8*(2*(2*a^2 + b^2)*d*f^2*x^2 + 4*(2*a^2 + b^2)*c*f^2*x - b^2*d*cosh(f*x
+ e)^2 - b^2*d*sinh(f*x + e)^2 - 16*a*b*d*cosh(f*x + e) + 4*(4*a*b*d*f*x +
 4*a*b*c*f + (b^2*d*f*x + b^2*c*f)*cosh(f*x + e))*sinh(f*x + e))/f^2
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 219 vs. 2(105) = 210.

Time = 0.23 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.94

$$\int (c + dx)(a + b\cosh(e + fx))^2\,dx$$
$$= \begin{cases} a^2cx + \frac{a^2dx^2}{2} + \frac{2abc\sinh\left(e+fx\right)}{f} + \frac{2abdx\sinh\left(e+fx\right)}{f} - \frac{2abd\cosh\left(e+fx\right)}{f^2} - \frac{b^2cx\sinh^2\left(e+fx\right)}{2} + \frac{b^2cx\cosh^2\left(e+fx\right)}{2} + \frac{b^2c\sin}{} \\ (a + b\cosh\left(e\right))^2\left(cx + \frac{dx^2}{2}\right) \end{cases}$$

input
```
integrate((d*x+c)*(a+b*cosh(f*x+e))**2,x)
```

output
```
Piecewise((a**2*c*x + a**2*d*x**2/2 + 2*a*b*c*sinh(e + f*x)/f + 2*a*b*d*x*
sinh(e + f*x)/f - 2*a*b*d*cosh(e + f*x)/f**2 - b**2*c*x*sinh(e + f*x)**2/2
 + b**2*c*x*cosh(e + f*x)**2/2 + b**2*c*sinh(e + f*x)*cosh(e + f*x)/(2*f)
- b**2*d*x**2*sinh(e + f*x)**2/4 + b**2*d*x**2*cosh(e + f*x)**2/4 + b**2*d
*x*sinh(e + f*x)*cosh(e + f*x)/(2*f) - b**2*d*sinh(e + f*x)**2/(4*f**2), N
e(f, 0)), ((a + b*cosh(e))**2*(c*x + d*x**2/2), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.46

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx = \frac{1}{2}\,a^2dx^2 + \frac{1}{16}\left(4\,x^2 + \frac{\left(2\,fxe^{(2\,e)} - e^{(2\,e)}\right)e^{(2\,fx)}}{f^2} - \frac{(2\,fx+1)e^{(-2\,fx-2\,e)}}{f^2}\right)b^2d + \frac{1}{8}\,b^2c\left(4\,x + \frac{e^{(2\,fx+2\,e)}}{f} - \frac{e^{(-2\,fx-2\,e)}}{f}\right) + a^2cx + abd\left(\frac{(fxe^e - e^e)e^{(fx)}}{f^2} - \frac{(fx+1)e^{(-fx-e)}}{f^2}\right) + \frac{2\,abc\sinh{(fx+e)}}{f}$$

input
```
integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 + 1/16*(4*x^2 + (2*f*x*e^(2*e) - e^(2*e))*e^(2*f*x)/f^2 - (2
*f*x + 1)*e^(-2*f*x - 2*e)/f^2)*b^2*d + 1/8*b^2*c*(4*x + e^(2*f*x + 2*e)/f
 - e^(-2*f*x - 2*e)/f) + a^2*c*x + a*b*d*((f*x*e^e - e^e)*e^(f*x)/f^2 - (f
*x + 1)*e^(-f*x - e)/f^2) + 2*a*b*c*sinh(f*x + e)/f
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.42

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx = \frac{1}{2}\,a^2dx^2 + \frac{1}{4}\,b^2dx^2 + a^2cx + \frac{1}{2}\,b^2cx + \frac{(2\,b^2dfx + 2\,b^2cf - b^2d)e^{(2\,fx+2\,e)}}{16\,f^2} + \frac{(abdfx + abcf - abd)e^{(fx+e)}}{f^2} - \frac{(abdfx + abcf + abd)e^{(-fx-e)}}{f^2} - \frac{(2\,b^2dfx + 2\,b^2cf + b^2d)e^{(-2\,fx-2\,e)}}{16\,f^2}$$

input
```
integrate((d*x+c)*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/2*a^2*d*x^2 + 1/4*b^2*d*x^2 + a^2*c*x + 1/2*b^2*c*x + 1/16*(2*b^2*d*f*x
+ 2*b^2*c*f - b^2*d)*e^(2*f*x + 2*e)/f^2 + (a*b*d*f*x + a*b*c*f - a*b*d)*e
^(f*x + e)/f^2 - (a*b*d*f*x + a*b*c*f + a*b*d)*e^(-f*x - e)/f^2 - 1/16*(2*
b^2*d*f*x + 2*b^2*c*f + b^2*d)*e^(-2*f*x - 2*e)/f^2
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.19

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx = \frac{a^2\,d\,x^2}{2} + \frac{b^2\,d\,x^2}{4} + a^2\,c\,x + \frac{b^2\,c\,x}{2} - \frac{b^2\,d\,{\cosh(e+f\,x)}^2}{4\,f^2} + \frac{b^2\,c\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f} - \frac{2\,a\,b\,d\cosh(e+f\,x)}{f^2} + \frac{2\,a\,b\,c\sinh(e+f\,x)}{f} + \frac{2\,a\,b\,d\,x\sinh(e+f\,x)}{f} + \frac{b^2\,d\,x\cosh(e+f\,x)\,\sinh(e+f\,x)}{2\,f}$$

input
```
int((a + b*cosh(e + f*x))^2*(c + d*x),x)
```

output
```
(a^2*d*x^2)/2 + (b^2*d*x^2)/4 + a^2*c*x + (b^2*c*x)/2 - (b^2*d*cosh(e + f*
x)^2)/(4*f^2) + (b^2*c*cosh(e + f*x)*sinh(e + f*x))/(2*f) - (2*a*b*d*cosh(
e + f*x))/f^2 + (2*a*b*c*sinh(e + f*x))/f + (2*a*b*d*x*sinh(e + f*x))/f +
(b^2*d*x*cosh(e + f*x)*sinh(e + f*x))/(2*f)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 261, normalized size of antiderivative = 2.31

$$\int (c+dx)(a+b\cosh(e+fx))^2\,dx = \frac{2e^{4fx+4e}b^2cf+2e^{4fx+4e}b^2dfx-e^{4fx+4e}b^2d+16e^{3fx+3e}abcf+16e^{3fx+3e}abdfx-16e^{3fx+3e}abd+16e^{2fx+}}{}$$

input
```
int((d*x+c)*(a+b*cosh(f*x+e))^2,x)
```

output
```
(2*e**(4*e + 4*f*x)*b**2*c*f + 2*e**(4*e + 4*f*x)*b**2*d*f*x - e**(4*e + 4
*f*x)*b**2*d + 16*e**(3*e + 3*f*x)*a*b*c*f + 16*e**(3*e + 3*f*x)*a*b*d*f*x
 - 16*e**(3*e + 3*f*x)*a*b*d + 16*e**(2*e + 2*f*x)*a**2*c*f**2*x + 8*e**(2
*e + 2*f*x)*a**2*d*f**2*x**2 + 8*e**(2*e + 2*f*x)*b**2*c*f**2*x + 4*e**(2*
e + 2*f*x)*b**2*d*f**2*x**2 - 16*e**(e + f*x)*a*b*c*f - 16*e**(e + f*x)*a*
b*d*f*x - 16*e**(e + f*x)*a*b*d - 2*b**2*c*f - 2*b**2*d*f*x - b**2*d)/(16*
e**(2*e + 2*f*x)*f**2)
```

3.165 $\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 156

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \frac{2ab\cosh\left(e-\frac{cf}{d}\right)\mathrm{Chi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{b^2\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Chi}\left(\frac{2cf}{d}+2fx\right)}{2d} + \frac{a^2\log(c+dx)}{d} + \frac{b^2\log(c+dx)}{2d} + \frac{2ab\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(\frac{cf}{d}+fx\right)}{d} + \frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(\frac{2cf}{d}+2fx\right)}{2d}$$

output

```
2*a*b*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d+1/2*b^2*cosh(-2*e+2*c*f/d)*Chi(2*c*f
/d+2*f*x)/d+a^2*ln(d*x+c)/d+1/2*b^2*ln(d*x+c)/d-2*a*b*sinh(-e+c*f/d)*Shi(c
*f/d+f*x)/d-1/2*b^2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \frac{4ab\cosh\left(e-\frac{cf}{d}\right)\text{Chi}\left(f\left(\frac{c}{d}+x\right)\right)+b^2\cosh\left(2e-\frac{2cf}{d}\right)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)+2a^2\log(c+dx)+b^2\log(c+dx)}{2d}$$

input
```
Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x),x]
```

output
```
(4*a*b*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)] + b^2*Cosh[2*e - (2*c*f
)/d]*CoshIntegral[(2*f*(c + d*x))/d] + 2*a^2*Log[c + d*x] + b^2*Log[c + d*
x] + 4*a*b*Sinh[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + b^2*Sinh[2*e - (2
*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d])/(2*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}{c+dx}dx$$

$$\Big\downarrow \; 3798$$

$$\int \left(\frac{a^2}{c+dx}+\frac{2ab\cosh(e+fx)}{c+dx}+\frac{b^2\cosh^2(e+fx)}{c+dx}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2\log(c+dx)}{d}+\frac{2ab\mathrm{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d}+\frac{2ab\sinh\left(e-\frac{cf}{d}\right)\mathrm{Shi}\left(xf+\frac{cf}{d}\right)}{d}+\frac{b^2\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2d}+\frac{b^2\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{2d}+\frac{b^2\log(c+dx)}{2d}$$

input
```
Int[(a + b*Cosh[e + f*x])^2/(c + d*x),x]
```

output
```
(2*a*b*Cosh[e - (c*f)/d]*CoshIntegral[(c*f)/d + f*x])/d + (b^2*Cosh[2*e -
(2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*d) + (a^2*Log[c + d*x])/d +
 (b^2*Log[c + d*x])/(2*d) + (2*a*b*Sinh[e - (c*f)/d]*SinhIntegral[(c*f)/d
+ f*x])/d + (b^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2
*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 2.43 (sec) , antiderivative size = 202, normalized size of antiderivative = 1.29

method	result
risch	$-\frac{ab\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{d}-\frac{ab\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{d}+\frac{b^2\ln(dx+c)}{2d}+\frac{a^2\ln(dx+c)}{d}-$ b

input
```
int((a+b*cosh(f*x+e))^2/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-a*b/d*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*f-d*e)/d)-a*b/d*exp(-(c*f-d*e)/d)*Ei
(1,-f*x-e-(c*f-d*e)/d)+1/2*b^2*ln(d*x+c)/d+a^2*ln(d*x+c)/d-1/4*b^2/d*exp(2
*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)-1/4*b^2/d*exp(-2*(c*f-d*e)/d)*
Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.47

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \frac{4\left(ab\text{Ei}\left(\frac{dfx+cf}{d}\right)+ab\text{Ei}\left(-\frac{dfx+cf}{d}\right)\right)\cosh\left(-\frac{de-cf}{d}\right)+\left(b^2\text{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)+b^2\text{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\right)\cosh\left(-\frac{2\,(de-}{d}\right.}{}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
1/4*(4*(a*b*Ei((d*f*x + c*f)/d) + a*b*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c
*f)/d) + (b^2*Ei(2*(d*f*x + c*f)/d) + b^2*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*
(d*e - c*f)/d) + 2*(2*a^2 + b^2)*log(d*x + c) - 4*(a*b*Ei((d*f*x + c*f)/d)
 - a*b*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) - (b^2*Ei(2*(d*f*x + c*f
)/d) - b^2*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/d
```

Sympy [F]

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\cosh{(e+fx)})^2}{c+dx}\,dx$$

input
```
integrate((a+b*cosh(f*x+e))**2/(d*x+c),x)
```

output
```
Integral((a + b*cosh(e + f*x))**2/(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = -\frac{1}{4}\,b^2\left(\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_1\left(\frac{2\,(dx+c)f}{d}\right)}{d}+\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_1\left(-\frac{2\,(dx+c)f}{d}\right)}{d}-\frac{2\,\log\left(dx+c\right)}{d}\right) - ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_1\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_1\left(-\frac{(dx+c)f}{d}\right)}{d}\right)+\frac{a^2\log\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
-1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/d + e^(2*e
 - 2*c*f/d)*exp_integral_e(1, -2*(d*x + c)*f/d)/d - 2*log(d*x + c)/d) - a*
b*(e^(-e + c*f/d)*exp_integral_e(1, (d*x + c)*f/d)/d + e^(e - c*f/d)*exp_i
ntegral_e(1, -(d*x + c)*f/d)/d) + a^2*log(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.92

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \frac{b^2\mathrm{Ei}\left(\frac{2\,(dfx+cf)}{d}\right)e^{\left(2\,e-\frac{2\,cf}{d}\right)}+4\,ab\mathrm{Ei}\left(\frac{dfx+cf}{d}\right)e^{\left(e-\frac{cf}{d}\right)}+4\,ab\mathrm{Ei}\left(-\frac{dfx+cf}{d}\right)e^{\left(-e+\frac{cf}{d}\right)}+b^2\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(-2}{4\,d}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
1/4*(b^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*Ei((d*f*x + c*f)/
d)*e^(e - c*f/d) + 4*a*b*Ei(-(d*f*x + c*f)/d)*e^(-e + c*f/d) + b^2*Ei(-2*(
d*f*x + c*f)/d)*e^(-2*e + 2*c*f/d) + 4*a^2*log(d*x + c) + 2*b^2*log(d*x +
c))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\cosh(e+f\,x))^2}{c+d\,x}\,dx$$

input
```
int((a + b*cosh(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + b*cosh(e + f*x))^2/(c + d*x), x)
```

Reduce [F]

$$\int \frac{(a+b\cosh(e+fx))^2}{c+dx}\,dx = \frac{2\left(\int \frac{\cosh(fx+e)}{dx+c}dx\right)abd+\left(\int \frac{\cosh(fx+e)^2}{dx+c}dx\right)b^2d+\log(dx+c)\,a^2}{d}$$

input
```
int((a+b*cosh(f*x+e))^2/(d*x+c),x)
```

output
```
(2*int(cosh(e + f*x)/(c + d*x),x)*a*b*d + int(cosh(e + f*x)**2/(c + d*x),x
)*b**2*d + log(c + d*x)*a**2)/d
```

3.166 $\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 183

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx = -\frac{a^2}{d(c+dx)} - \frac{2ab\cosh(e+fx)}{d(c+dx)} - \frac{b^2\cosh^2(e+fx)}{d(c+dx)} + \frac{b^2 f\,\text{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2} + \frac{2abf\,\text{Chi}\left(\frac{cf}{d}+fx\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2} + \frac{2abf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(\frac{cf}{d}+fx\right)}{d^2} + \frac{b^2 f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^2}$$

output
```
-a^2/d/(d*x+c)-2*a*b*cosh(f*x+e)/d/(d*x+c)-b^2*cosh(f*x+e)^2/d/(d*x+c)-b^2
*f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/d^2-2*a*b*f*Chi(c*f/d+f*x)*sinh(-
e+c*f/d)/d^2+2*a*b*f*cosh(-e+c*f/d)*Shi(c*f/d+f*x)/d^2+b^2*f*cosh(-2*e+2*c
*f/d)*Shi(2*c*f/d+2*f*x)/d^2
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.27

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{-2a^2d-b^2d-4abd\cosh(e+fx)-b^2d\cosh(2(e+fx))+2b^2f(c+dx)\text{Chi}\left(\frac{2f(c+dx)}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)+}{}$$

input
```
Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
(-2*a^2*d - b^2*d - 4*a*b*d*Cosh[e + f*x] - b^2*d*Cosh[2*(e + f*x)] + 2*b^
2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 4*a*
b*f*(c + d*x)*CoshIntegral[f*(c/d + x)]*Sinh[e - (c*f)/d] + 4*a*b*c*f*Cosh
[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] + 4*a*b*d*f*x*Cosh[e - (c*f)/d]*Si
nhIntegral[f*(c/d + x)] + 2*b^2*c*f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*
f*(c + d*x))/d] + 2*b^2*d*f*x*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c +
 d*x))/d])/(2*d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}{(c+dx)^2}dx$$
$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a^2}{(c+dx)^2}+\frac{2ab\cosh(e+fx)}{(c+dx)^2}+\frac{b^2\cosh^2(e+fx)}{(c+dx)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2}{d(c+dx)}+\frac{2abf\text{Chi}\left(xf+\frac{cf}{d}\right)\sinh\left(e-\frac{cf}{d}\right)}{d^2}+\frac{2abf\cosh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^2}-\frac{2ab\cosh(e+fx)}{d(c+dx)}+\frac{b^2f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d^2}+\frac{b^2f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^2}-\frac{b^2\cosh^2(e+fx)}{d(c+dx)}$$

input
```
Int[(a + b*Cosh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
-(a^2/(d*(c + d*x))) - (2*a*b*Cosh[e + f*x])/(d*(c + d*x)) - (b^2*Cosh[e +
 f*x]^2)/(d*(c + d*x)) + (b^2*f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e -
 (2*c*f)/d])/d^2 + (2*a*b*f*CoshIntegral[(c*f)/d + f*x]*Sinh[e - (c*f)/d])
/d^2 + (2*a*b*f*Cosh[e - (c*f)/d]*SinhIntegral[(c*f)/d + f*x])/d^2 + (b^2*
f*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [A] (verified)

Time = 2.56 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.74

method	result
risch	$-\frac{fab\,\mathrm{e}^{-fx-e}}{d(dxf+cf)}+\frac{fab\,\mathrm{e}^{\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(fx+e+\frac{cf-de}{d}\right)}{d^2}-\frac{fab\,\mathrm{e}^{fx+e}}{d^2\left(\frac{cf}{d}+fx\right)}-\frac{fab\,\mathrm{e}^{-\frac{cf-de}{d}}\,\mathrm{expIntegral}_1\left(-fx-e-\frac{cf-de}{d}\right)}{d^2}-$

input
```
int((a+b*cosh(f*x+e))^2/(d*x+c)^2,x,method=_RETURNVERBOSE)
```

output
```
-f*a*b*exp(-f*x-e)/d/(d*f*x+c*f)+f*a*b/d^2*exp((c*f-d*e)/d)*Ei(1,f*x+e+(c*
f-d*e)/d)-1/d^2*f*a*b*exp(f*x+e)/(c*f/d+f*x)-1/d^2*f*a*b*exp(-(c*f-d*e)/d)
*Ei(1,-f*x-e-(c*f-d*e)/d)-a^2/d/(d*x+c)-1/2*b^2/(d*x+c)/d-1/4*f*b^2*exp(-2
*f*x-2*e)/d/(d*f*x+c*f)+1/2*f*b^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*
(c*f-d*e)/d)-1/4*b^2*f/d^2*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*b^2*f/d^2*exp(-2
*(c*f-d*e)/d)*Ei(1,-2*f*x-2*e-2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.94

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx =$$

$$-\frac{b^2d\cosh\left(fx+e\right)^2+b^2d\sinh\left(fx+e\right)^2+4\,abd\cosh\left(fx+e\right)+\left(2\,a^2+b^2\right)d-2\left(\left(abdfx+abcf\right)\mathrm{Ei}\left(\frac{df}{}\right.\right.}{}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
-1/2*(b^2*d*cosh(f*x + e)^2 + b^2*d*sinh(f*x + e)^2 + 4*a*b*d*cosh(f*x + e
) + (2*a^2 + b^2)*d - 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/d) - (a*b*
d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - ((b^2*d*f*x
+ b^2*c*f)*Ei(2*(d*f*x + c*f)/d) - (b^2*d*f*x + b^2*c*f)*Ei(-2*(d*f*x + c*
f)/d))*cosh(-2*(d*e - c*f)/d) + 2*((a*b*d*f*x + a*b*c*f)*Ei((d*f*x + c*f)/
d) + (a*b*d*f*x + a*b*c*f)*Ei(-(d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + ((
b^2*d*f*x + b^2*c*f)*Ei(2*(d*f*x + c*f)/d) + (b^2*d*f*x + b^2*c*f)*Ei(-2*(
d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^3*x + c*d^2)
```

Sympy [F]

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\cosh{(e+fx)})^2}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*cosh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*cosh(e + f*x))**2/(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.99

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx = -\frac{1}{4}b^2\left(\frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(2e-\frac{2cf}{d}\right)}E_2\left(-\frac{2(dx+c)f}{d}\right)}{(dx+c)d} + \frac{2}{d^2x+cd}\right) - ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_2\left(\frac{(dx+c)f}{d}\right)}{(dx+c)d} + \frac{e^{\left(e-\frac{cf}{d}\right)}E_2\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)d}\right) - \frac{a^2}{d^2x+cd}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)
*d) + e^(2*e - 2*c*f/d)*exp_integral_e(2, -2*(d*x + c)*f/d)/((d*x + c)*d)
+ 2/(d^2*x + c*d)) - a*b*(e^(-e + c*f/d)*exp_integral_e(2, (d*x + c)*f/d)/
((d*x + c)*d) + e^(e - c*f/d)*exp_integral_e(2, -(d*x + c)*f/d)/((d*x + c)
*d)) - a^2/(d^2*x + c*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(186) = 372$.

Time = 0.17 (sec) , antiderivative size = 1135, normalized size of antiderivative = 6.20

$$\int \frac{(a + b\cosh(e + fx))^2}{(c + dx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
1/4*(2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(2*((d*x +
c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d)
 - 2*b^2*d*e*f^2*Ei(2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^(2*(d*e - c*f)/d) + 2*b^2*c*f^3*Ei(2*((d*x + c)*(d*e/(d*x + c
) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(2*(d*e - c*f)/d) + 4*(d*x + c)*a
*b*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(((d*x + c)*(d*e/(d*x + c) -
c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f)/d) - 4*a*b*d*e*f^2*Ei(((
d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^((d*e - c*f
)/d) + 4*a*b*c*f^3*Ei(((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e
 + c*f)/d)*e^((d*e - c*f)/d) - 4*(d*x + c)*a*b*(d*e/(d*x + c) - c*f/(d*x +
 c) + f)*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*
f)/d)*e^(-(d*e - c*f)/d) + 4*a*b*d*e*f^2*Ei(-((d*x + c)*(d*e/(d*x + c) - c
*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f)/d) - 4*a*b*c*f^3*Ei(-((d
*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-(d*e - c*f
)/d) - 2*(d*x + c)*b^2*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x
 + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f
)/d) + 2*b^2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)
- d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - 2*b^2*c*f^3*Ei(-2*((d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) - b^2*d
*f^2*e^(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) - 4*a*b*d*f^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\cosh(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*cosh(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + b*cosh(e + f*x))^2/(c + d*x)^2, x)
```

Reduce [F]

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^2}\,dx = \frac{e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)b^2c^2 + e^{3e}\left(\int \frac{e^{2fx}}{d^2x^2+2cdx+c^2}dx\right)b^2cdx + 4e^{2e}\left(\int \frac{e^{fx}}{d^2x^2+2cdx+c^2}dx\right)ab\,c^2 + 4e^{2e}\left(\int \frac{e}{d^2x^2+}\right.}{}$$

input
```
int((a+b*cosh(f*x+e))^2/(d*x+c)^2,x)
```

output
```
(e**(3*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c**2 + e**(3
*e)*int(e**(2*f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c*d*x + 4*e**(2*e)
*int(e**(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c**2 + 4*e**(2*e)*int(e*
*(f*x)/(c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c*d*x + e**e*int(1/(e**(2*e + 2
*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2),x)*b**
2*c**2 + e**e*int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e*
*(2*e + 2*f*x)*d**2*x**2),x)*b**2*c*d*x + 4*e**e*a**2*x + 2*e**e*b**2*x +
4*int(1/(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*a*b*c**
2 + 4*int(1/(e**(f*x)*c**2 + 2*e**(f*x)*c*d*x + e**(f*x)*d**2*x**2),x)*a*b
*c*d*x)/(4*e**e*c*(c + d*x))
```

3.167 $\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 242

$$\begin{aligned}\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx = &-\frac{a^2}{2d(c+dx)^2}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}\\&+\frac{abf^2\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(\frac{cf}{d}+fx\right)}{d^3}\\&+\frac{b^2f^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\\&-\frac{abf\sinh(e+fx)}{d^2(c+dx)}-\frac{b^2f\cosh(e+fx)\sinh(e+fx)}{d^2(c+dx)}\\&+\frac{abf^2\sinh\left(e-\frac{cf}{d}\right)\operatorname{Shi}\left(\frac{cf}{d}+fx\right)}{d^3}\\&+\frac{b^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{d^3}\end{aligned}$$

output

```
-1/2*a^2/d/(d*x+c)^2-a*b*cosh(f*x+e)/d/(d*x+c)^2-1/2*b^2*cosh(f*x+e)^2/d/(
d*x+c)^2+a*b*f^2*cosh(-e+c*f/d)*Chi(c*f/d+f*x)/d^3+b^2*f^2*cosh(-2*e+2*c*f
/d)*Chi(2*c*f/d+2*f*x)/d^3-a*b*f*sinh(f*x+e)/d^2/(d*x+c)-b^2*f*cosh(f*x+e)
*sinh(f*x+e)/d^2/(d*x+c)-a*b*f^2*sinh(-e+c*f/d)*Shi(c*f/d+f*x)/d^3-b^2*f^2
*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/d^3
```

Mathematica [A] (verified)

Time = 0.79 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.63

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx =$$

$$-\frac{2a^2d^2+b^2d^2+4abd^2\cosh(e+fx)+b^2d^2\cosh(2(e+fx))-4abf^2(c+dx)^2\cosh\left(e-\frac{cf}{d}\right)\operatorname{Chi}\left(f\left(\frac{c}{d}+\right.\right.}{}$$

input
```
Integrate[(a + b*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
-1/4*(2*a^2*d^2 + b^2*d^2 + 4*a*b*d^2*Cosh[e + f*x] + b^2*d^2*Cosh[2*(e +
f*x)] - 4*a*b*f^2*(c + d*x)^2*Cosh[e - (c*f)/d]*CoshIntegral[f*(c/d + x)]
- 4*b^2*f^2*(c + d*x)^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*f*(c + d*x))
/d] + 4*a*b*c*d*f*Sinh[e + f*x] + 4*a*b*d^2*f*x*Sinh[e + f*x] + 2*b^2*c*d*
f*Sinh[2*(e + f*x)] + 2*b^2*d^2*f*x*Sinh[2*(e + f*x)] - 4*a*b*c^2*f^2*Sinh
[e - (c*f)/d]*SinhIntegral[f*(c/d + x)] - 8*a*b*c*d*f^2*x*Sinh[e - (c*f)/d
]*SinhIntegral[f*(c/d + x)] - 4*a*b*d^2*f^2*x^2*Sinh[e - (c*f)/d]*SinhInte
gral[f*(c/d + x)] - 4*b^2*c^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*
(c + d*x))/d] - 8*b^2*c*d*f^2*x*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c
 + d*x))/d] - 4*b^2*d^2*f^2*x^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c
 + d*x))/d])/(d^3*(c + d*x)^2)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}{(c+dx)^3}dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(\frac{a^2}{(c+dx)^3}+\frac{2ab\cosh(e+fx)}{(c+dx)^3}+\frac{b^2\cosh^2(e+fx)}{(c+dx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2}{2d(c+dx)^2}+\frac{abf^2\text{Chi}\left(xf+\frac{cf}{d}\right)\cosh\left(e-\frac{cf}{d}\right)}{d^3}+\frac{abf^2\sinh\left(e-\frac{cf}{d}\right)\text{Shi}\left(xf+\frac{cf}{d}\right)}{d^3}-\frac{abf\sinh(e+fx)}{d^2(c+dx)}-\frac{ab\cosh(e+fx)}{d(c+dx)^2}+\frac{b^2f^2\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d^3}+\frac{b^2f^2\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d^3}-\frac{b^2f\sinh(e+fx)\cosh(e+fx)}{d^2(c+dx)}-\frac{b^2\cosh^2(e+fx)}{2d(c+dx)^2}$$

input
```
Int[(a + b*Cosh[e + f*x])^2/(c + d*x)^3,x]
```

output
```
-1/2*a^2/(d*(c + d*x)^2) - (a*b*Cosh[e + f*x])/(d*(c + d*x)^2) - (b^2*Cosh
[e + f*x]^2)/(2*d*(c + d*x)^2) + (a*b*f^2*Cosh[e - (c*f)/d]*CoshIntegral[(
c*f)/d + f*x])/d^3 + (b^2*f^2*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d
 + 2*f*x])/d^3 - (a*b*f*Sinh[e + f*x])/(d^2*(c + d*x)) - (b^2*f*Cosh[e + f
*x]*Sinh[e + f*x])/(d^2*(c + d*x)) + (a*b*f^2*Sinh[e - (c*f)/d]*SinhIntegr
al[(c*f)/d + f*x])/d^3 + (b^2*f^2*Sinh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*
f)/d + 2*f*x])/d^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(242) = 484$.

Time = 2.73 (sec) , antiderivative size = 626, normalized size of antiderivative = 2.59

method	result
risch	$\frac{f^{3} a b \,\mathrm{e}^{-f x -e} x}{2 d \left(d^{2} x^{2} f^{2}+2 c d \,f^{2} x +c^{2} f^{2}\right)}+\frac{f^{3} a b \,\mathrm{e}^{-f x -e} c}{2 d^{2} \left(d^{2} x^{2} f^{2}+2 c d \,f^{2} x +c^{2} f^{2}\right)}-\frac{f^{2} a b \,\mathrm{e}^{-f x -e}}{2 d \left(d^{2} x^{2} f^{2}+2 c d \,f^{2} x +c^{2} f^{2}\right)}-\frac{f^{2} a b \,\mathrm{e}^{\frac{c f -d e}{d}} \operatorname{expIntegral}_{1}\left(f x +\right.}{2 d^{3}}$

input
```
int((a+b*cosh(f*x+e))^2/(d*x+c)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*f^3*a*b*exp(-f*x-e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/2*f^3*a*b*
exp(-f*x-e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/2*f^2*a*b*exp(-f*x-e
)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*f^2*a*b/d^3*exp((c*f-d*e)/d)*Ei(
1,f*x+e+(c*f-d*e)/d)-1/2/d^3*f^2*a*b*exp(f*x+e)/(c*f/d+f*x)^2-1/2/d^3*f^2*
a*b*exp(f*x+e)/(c*f/d+f*x)-1/2/d^3*f^2*a*b*exp(-(c*f-d*e)/d)*Ei(1,-f*x-e-(
c*f-d*e)/d)-1/2*a^2/d/(d*x+c)^2-1/4*b^2/(d*x+c)^2/d+1/4*f^3*b^2*exp(-2*f*x
-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*x+1/4*f^3*b^2*exp(-2*f*x-2*e)/d^
2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c-1/8*f^2*b^2*exp(-2*f*x-2*e)/d/(d^2*f
^2*x^2+2*c*d*f^2*x+c^2*f^2)-1/2*f^2*b^2/d^3*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+
2*e+2*(c*f-d*e)/d)-1/8*f^2*b^2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)^2-1/4*f^2*b^
2/d^3*exp(2*f*x+2*e)/(c*f/d+f*x)-1/2*f^2*b^2/d^3*exp(-2*(c*f-d*e)/d)*Ei(1,
-2*f*x-2*e-2*(c*f-d*e)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(242) = 484$.

Time = 0.12 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.42

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx =$$

$$-\frac{b^2d^2\cosh\left(fx+e\right)^2+b^2d^2\sinh\left(fx+e\right)^2+4\,abd^2\cosh\left(fx+e\right)+(2\,a^2+b^2)d^2-2\,\left((abd^2f^2x^2+2\,ab\right.}{}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="fricas")
```

output
```
-1/4*(b^2*d^2*cosh(f*x + e)^2 + b^2*d^2*sinh(f*x + e)^2 + 4*a*b*d^2*cosh(f
*x + e) + (2*a^2 + b^2)*d^2 - 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*
c^2*f^2)*Ei((d*f*x + c*f)/d) + (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^
2*f^2)*Ei(-(d*f*x + c*f)/d))*cosh(-(d*e - c*f)/d) - 2*((b^2*d^2*f^2*x^2 +
2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) + (b^2*d^2*f^2*x^2 +
2*b^2*c*d*f^2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*cosh(-2*(d*e - c*f)
/d) + 4*(a*b*d^2*f*x + a*b*c*d*f + (b^2*d^2*f*x + b^2*c*d*f)*cosh(f*x + e)
)*sinh(f*x + e) + 2*((a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(
(d*f*x + c*f)/d) - (a*b*d^2*f^2*x^2 + 2*a*b*c*d*f^2*x + a*b*c^2*f^2)*Ei(-(
d*f*x + c*f)/d))*sinh(-(d*e - c*f)/d) + 2*((b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^
2*x + b^2*c^2*f^2)*Ei(2*(d*f*x + c*f)/d) - (b^2*d^2*f^2*x^2 + 2*b^2*c*d*f^
2*x + b^2*c^2*f^2)*Ei(-2*(d*f*x + c*f)/d))*sinh(-2*(d*e - c*f)/d))/(d^5*x^
2 + 2*c*d^4*x + c^2*d^3)
```

Sympy [F]

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{\left(a+b\cosh\left(e+fx\right)\right)^2}{\left(c+dx\right)^3}\,dx$$

input
```
integrate((a+b*cosh(f*x+e))**2/(d*x+c)**3,x)
```

output
```
Integral((a + b*cosh(e + f*x))**2/(c + d*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.83

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx$$
$$= -\frac{1}{4}b^2\left(\frac{1}{d^3x^2+2\,cd^2x+c^2d}+\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_3\left(\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_3\left(-\frac{2\,(dx+c)f}{d}\right)}{(dx+c)^2d}\right)$$
$$-ab\left(\frac{e^{\left(-e+\frac{cf}{d}\right)}E_3\left(\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}+\frac{e^{\left(e-\frac{cf}{d}\right)}E_3\left(-\frac{(dx+c)f}{d}\right)}{(dx+c)^2d}\right)-\frac{a^2}{2\,(d^3x^2+2\,cd^2x+c^2d)}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="maxima")
```

output
```
-1/4*b^2*(1/(d^3*x^2 + 2*c*d^2*x + c^2*d) + e^(-2*e + 2*c*f/d)*exp_integra
l_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*d) + e^(2*e - 2*c*f/d)*exp_integral_e
(3, -2*(d*x + c)*f/d)/((d*x + c)^2*d)) - a*b*(e^(-e + c*f/d)*exp_integral_
e(3, (d*x + c)*f/d)/((d*x + c)^2*d) + e^(e - c*f/d)*exp_integral_e(3, -(d*
x + c)*f/d)/((d*x + c)^2*d)) - 1/2*a^2/(d^3*x^2 + 2*c*d^2*x + c^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 678 vs. $2(242) = 484$.

Time = 0.13 (sec) , antiderivative size = 678, normalized size of antiderivative = 2.80

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(f*x+e))^2/(d*x+c)^3,x, algorithm="giac")
```

output
```
1/8*(4*b^2*d^2*f^2*x^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4*a*b*d^2
*f^2*x^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a*b*d^2*f^2*x^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*b^2*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*
e + 2*c*f/d) + 8*b^2*c*d*f^2*x*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 8
*a*b*c*d*f^2*x*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 8*a*b*c*d*f^2*x*Ei(-(d*
f*x + c*f)/d)*e^(-e + c*f/d) + 8*b^2*c*d*f^2*x*Ei(-2*(d*f*x + c*f)/d)*e^(-
2*e + 2*c*f/d) + 4*b^2*c^2*f^2*Ei(2*(d*f*x + c*f)/d)*e^(2*e - 2*c*f/d) + 4
*a*b*c^2*f^2*Ei((d*f*x + c*f)/d)*e^(e - c*f/d) + 4*a*b*c^2*f^2*Ei(-(d*f*x
+ c*f)/d)*e^(-e + c*f/d) + 4*b^2*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(-2*e +
2*c*f/d) - 2*b^2*d^2*f*x*e^(2*f*x + 2*e) - 4*a*b*d^2*f*x*e^(f*x + e) + 4*a
*b*d^2*f*x*e^(-f*x - e) + 2*b^2*d^2*f*x*e^(-2*f*x - 2*e) - 2*b^2*c*d*f*e^(
2*f*x + 2*e) - 4*a*b*c*d*f*e^(f*x + e) + 4*a*b*c*d*f*e^(-f*x - e) + 2*b^2*
c*d*f*e^(-2*f*x - 2*e) - b^2*d^2*e^(2*f*x + 2*e) - 4*a*b*d^2*e^(f*x + e) -
 4*a*b*d^2*e^(-f*x - e) - b^2*d^2*e^(-2*f*x - 2*e) - 4*a^2*d^2 - 2*b^2*d^2
)/(d^5*x^2 + 2*c*d^4*x + c^2*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx = \int \frac{(a+b\cosh(e+f\,x))^2}{(c+d\,x)^3}\,dx$$

input
```
int((a + b*cosh(e + f*x))^2/(c + d*x)^3,x)
```

output
```
int((a + b*cosh(e + f*x))^2/(c + d*x)^3, x)
```

Reduce [F]

$$\int \frac{(a+b\cosh(e+fx))^2}{(c+dx)^3}\,dx = \frac{e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b^2c^2d+2e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}dx\right)b^2c\,d^2x+e^{3e}\left(\int \frac{e^{2fx}}{d^3x^3+3c\,d^2x^2+3c^2dx+c^3}\right.}{}$$

input
```
int((a+b*cosh(f*x+e))^2/(d*x+c)^3,x)
```

output
```
(e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x
)*b**2*c**2*d + 2*e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x*
*2 + d**3*x**3),x)*b**2*c*d**2*x + e**(3*e)*int(e**(2*f*x)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3),x)*b**2*d**3*x**2 + 4*e**(2*e)*int(e**(f*
x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b*c**2*d + 8*e**(2
*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),x)*a*b*c*
d**2*x + 4*e**(2*e)*int(e**(f*x)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3),x)*a*b*d**3*x**2 + e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e +
2*f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x
**3),x)*b**2*c**2*d + 2*e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*
f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**
3),x)*b**2*c*d**2*x + e**e*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*
x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3)
,x)*b**2*d**3*x**2 - 2*e**e*a**2 - e**e*b**2 + 4*int(1/(e**(f*x)*c**3 + 3*
e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*a*b*c*
*2*d + 8*int(1/(e**(f*x)*c**3 + 3*e**(f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x*
*2 + e**(f*x)*d**3*x**3),x)*a*b*c*d**2*x + 4*int(1/(e**(f*x)*c**3 + 3*e**(
f*x)*c**2*d*x + 3*e**(f*x)*c*d**2*x**2 + e**(f*x)*d**3*x**3),x)*a*b*d**3*x
**2)/(4*e**e*d*(c**2 + 2*c*d*x + d**2*x**2))
```

3.168 $\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 436

$$\begin{aligned}\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx &= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}-\frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f}\\
&\quad+\frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}\\
&\quad-\frac{3d(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}\\
&\quad-\frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3}\\
&\quad+\frac{6d^2(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3}\\
&\quad+\frac{6d^3\operatorname{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^4}\\
&\quad-\frac{6d^3\operatorname{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^4}\end{aligned}$$

output
```
(d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f-(d*x+c)
^3*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f+3*d*(d*x+c)^2*
polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2-3*d*(d*x+
c)^2*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2-6*d^
2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^3
+6*d^2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2
)/f^3+6*d^3*polylog(4,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f
^4-6*d^3*polylog(4,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^4
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx$$

$$= \frac{(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)-(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)+\frac{3d\left(f^2(c+dx)^2\operatorname{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2-b^2}}\right)-2df(c+dx)\right.}{}}{}$$

input
```
Integrate[(c + d*x)^3/(a + b*Cosh[e + f*x]),x]
```

output
```
((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - (c + d*x)^3*
Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])] + (3*d*(f^2*(c + d*x)^2*Pol
yLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 - b^2])] - 2*d*f*(c + d*x)*PolyLog[
3, (b*E^(e + f*x))/(-a + Sqrt[a^2 - b^2])] + 2*d^2*PolyLog[4, (b*E^(e + f*
x))/(-a + Sqrt[a^2 - b^2])]))/f^3 - (3*d*(f^2*(c + d*x)^2*PolyLog[2, -((b*
E^(e + f*x))/(a + Sqrt[a^2 - b^2]))] - 2*d*f*(c + d*x)*PolyLog[3, -((b*E^(
e + f*x))/(a + Sqrt[a^2 - b^2]))] + 2*d^2*PolyLog[4, -((b*E^(e + f*x))/(a
+ Sqrt[a^2 - b^2]))]))/f^3)/(Sqrt[a^2 - b^2]*f)
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.91, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.450, Rules used = {3042, 3801, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3801}$$

$$2\int \frac{e^{e+fx}(c+dx)^3}{2e^{e+fx}a+be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{2694}$$

$$2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\int(c+dx)^2\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{3d\int(c+dx)^2\log\left(\frac{}{a+}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\Big\downarrow \text{3011}$$

$$2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \; 7163$$

$$2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e-\cdots}}{a-\sqrt{a\cdots}}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\Big\downarrow \; 2720$$

$$2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyL}}{}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\downarrow 7143$$

$$2\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\operatorname{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b}}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

input
```
Int[(c + d*x)^3/(a + b*Cosh[e + f*x]),x]
```

output

```
2*((b*(((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(b*f)
- (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))
])/f) + (2*d*(((c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]
))])/f - (d*PolyLog[4, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/f^2))/f)
)/(b*f)))/(2*Sqrt[a^2 - b^2]) - (b*(((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(
a + Sqrt[a^2 - b^2])])/(b*f) - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*E^(e +
 f*x))/(a + Sqrt[a^2 - b^2]))])/f) + (2*d*(((c + d*x)*PolyLog[3, -((b*E^(e
 + f*x))/(a + Sqrt[a^2 - b^2]))])/f - (d*PolyLog[4, -((b*E^(e + f*x))/(a +
 Sqrt[a^2 - b^2]))])/f^2))/f))/(b*f)))/(2*Sqrt[a^2 - b^2]))
```

Defintions of rubi rules used

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694

```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(dx+c)^3}{a+b\cosh(fx+e)}dx$$

input
```
int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

output
```
int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1042 vs. $2(394) = 788$.

Time = 0.13 (sec) , antiderivative size = 1042, normalized size of antiderivative = 2.39

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
(6*b*d^3*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x +
 e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) - 6*b*
d^3*sqrt((a^2 - b^2)/b^2)*polylog(4, -(a*cosh(f*x + e) + a*sinh(f*x + e) -
 (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) + 3*(b*d^3*
f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*c
osh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt(
(a^2 - b^2)/b^2) + b)/b + 1) - 3*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*
d*f^2)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (
b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (b*
d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt((a^2 - b^2)/
b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 - b^2)/b^2)
 + 2*a) - (b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*sqrt
((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^
2 - b^2)/b^2) + 2*a) + (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*
x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 - b^2)/b^2)*l
og((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh(f*x + e)
)*sqrt((a^2 - b^2)/b^2) + b)/b) - (b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b
*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*sqrt((a^2 -
b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*si
nh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) - 6*(b*d^3*f*x + b*c*d^2*f)*...
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+dx)^3}{a+b\cosh\left(e+fx\right)}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*cosh(f*x+e)),x)
```

output
```
Integral((c + d*x)**3/(a + b*cosh(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\cosh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*cosh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+b\cosh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^3/(a + b*cosh(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + b*cosh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^{fx+e}b+a}{\sqrt{-a^2+b^2}}\right)c^3+2e^e\left(\int \frac{e^{fx}x^3}{e^{2fx+2e}b+2e^{fx+e}a+b}dx\right)a^2d^3f-2e^e\left(\int \frac{e^{fx}x^3}{e^{2fx+2e}b+2e^{fx+e}a+b}dx\right)b^2d^3}{}$$

input
```
int((d*x+c)^3/(a+b*cosh(f*x+e)),x)
```

output
```
(2*( - sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2)
)*c**3 + e**e*int((e**(f*x)*x**3)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a +
 b),x)*a**2*d**3*f - e**e*int((e**(f*x)*x**3)/(e**(2*e + 2*f*x)*b + 2*e**(
e + f*x)*a + b),x)*b**2*d**3*f + 3*e**e*int((e**(f*x)*x**2)/(e**(2*e + 2*f
*x)*b + 2*e**(e + f*x)*a + b),x)*a**2*c*d**2*f - 3*e**e*int((e**(f*x)*x**2
)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a + b),x)*b**2*c*d**2*f + 3*e**e*in
t((e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a + b),x)*a**2*c**2*d*
f - 3*e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a + b),x)
*b**2*c**2*d*f))/(f*(a**2 - b**2))
```

3.169 $\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 320

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{2d(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2} - \frac{2d(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2} - \frac{2d^2\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3} + \frac{2d^2\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^3}$$

output

```
(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f-(d*x+c)
^2*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f+2*d*(d*x+c)*po
lylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2-2*d*(d*x+c)
*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2-2*d^2*po
lylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^3+2*d^2*polyl
og(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^3
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 247, normalized size of antiderivative = 0.77

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx$$
$$= \frac{(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)-(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)+\frac{2d\left(f(c+dx)\operatorname{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2-b^2}}\right)-d\operatorname{PolyLog}\left(3,\right.\right.}{f^2}}{\sqrt{a^2-b^2}f}$$

input
```
Integrate[(c + d*x)^2/(a + b*Cosh[e + f*x]),x]
```

output
```
((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - (c + d*x)^2*
Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])] + (2*d*(f*(c + d*x)*PolyLog
[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 - b^2])] - d*PolyLog[3, (b*E^(e + f*x))
/(-a + Sqrt[a^2 - b^2])]))/f^2 - (2*d*(f*(c + d*x)*PolyLog[2, -((b*E^(e +
f*x))/(a + Sqrt[a^2 - b^2]))] - d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a
^2 - b^2]))]))/f^2)/(Sqrt[a^2 - b^2]*f)
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 299, normalized size of antiderivative = 0.93, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3801, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^2}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3801}$$
$$2\int \frac{e^{e+fx}(c+dx)^2}{2e^{e+fx}a+be^{2(e+fx)}+b}dx$$

$\Big\downarrow$ 2694

$$2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$\Big\downarrow$ 27

$$2\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$\Big\downarrow$ 2620

$$2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\int(c+dx)\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{2d\int(c+dx)\log\left(\frac{e^{e\cdots}}{a+\sqrt{\cdots}}\cdots\right)}{bf}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right.$$

$\Big\downarrow$ 3011

$$2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+\cdots}}{\sqrt{a^2-\cdots}}\cdots\right)}{bf}\cdots\right)}{\cdots}\right.$$

$\Big\downarrow$ 2720

$$2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)}{}\right.}{}\right.$$

$\downarrow$ 7143

$$2\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+}\right.}{bf}\right.}{}\right.$$

input
```
Int[(c + d*x)^2/(a + b*Cosh[e + f*x]),x]
```

output
```
2*((b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(b*f)
- (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])
/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/f^2))/(b*f)
))/(2*Sqrt[a^2 - b^2]) - (b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqr
t[a^2 - b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 - b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2
- b^2]))])/f^2))/(b*f)))/(2*Sqrt[a^2 - b^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dx+c)^2}{a+b\cosh(fx+e)}dx$$

input
```
int((d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

output
```
int((d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 736 vs. $2(288) = 576$.

Time = 0.12 (sec) , antiderivative size = 736, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
-(2*b*d^2*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + e) + a*sinh(f*x
+ e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) - 2*b
*d^2*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(f*x + e) + a*sinh(f*x + e)
- (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2))/b) - 2*(b*d^2
*f*x + b*c*d*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x
 + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b +
 1) + 2*(b*d^2*f*x + b*c*d*f)*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e
) + a*sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)
/b^2) + b)/b + 1) + (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 - b^2)
/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b*sqrt((a^2 - b^2)/b^2
) + 2*a) - (b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*sqrt((a^2 - b^2)/b^2)*log
(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a)
- (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 - b^
2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*cosh(f*x + e) + b*sinh
(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + (b*d^2*f^2*x^2 + 2*b*c*d*f^2*x
- b*d^2*e^2 + 2*b*c*d*e*f)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*
sinh(f*x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2)
+ b)/b))/((a^2 - b^2)*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+dx)^2}{a+b\cosh{(e+fx)}}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*cosh(f*x+e)),x)
```

output
```
Integral((c + d*x)**2/(a + b*cosh(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*cosh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+b\cosh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^2/(a + b*cosh(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + b*cosh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}\,dx$$
$$= \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^{fx+e}b+a}{\sqrt{-a^2+b^2}}\right)c^2+2e^e\left(\int \frac{e^{fx}x^2}{e^{2fx+2e}b+2e^{fx+e}a+b}dx\right)a^2d^2f-2e^e\left(\int \frac{e^{fx}x^2}{e^{2fx+2e}b+2e^{fx+e}a+b}dx\right)b^2d^2}{f\left(a^2-b^2\right)}$$

input
```
int((d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

output
```
(2*( - sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2)
)*c**2 + e**e*int((e**(f*x)*x**2)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a +
 b),x)*a**2*d**2*f - e**e*int((e**(f*x)*x**2)/(e**(2*e + 2*f*x)*b + 2*e**(
e + f*x)*a + b),x)*b**2*d**2*f + 2*e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)
*b + 2*e**(e + f*x)*a + b),x)*a**2*c*d*f - 2*e**e*int((e**(f*x)*x)/(e**(2*
e + 2*f*x)*b + 2*e**(e + f*x)*a + b),x)*b**2*c*d*f))/(f*(a**2 - b**2))
```

3.170 $\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 203

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} - \frac{(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f} + \frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2} - \frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,f^2}$$

output
```
(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f-(d*x+c)*l
n(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f+d*polylog(2,-b*exp
(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2-d*polylog(2,-b*exp(f*x+e)
/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(1/2)/f^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.75

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = \frac{f(c+dx)\left(\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)-\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)\right)+d\,\text{PolyLog}\left(2,\frac{be^{e+fx}}{-a+\sqrt{a^2-b^2}}\right)-d\,\text{PolyLog}\left(2,-\frac{}{a}\right.}{\sqrt{a^2-b^2}\,f^2}$$

input
```
Integrate[(c + d*x)/(a + b*Cosh[e + f*x]),x]
```

output
```
(f*(c + d*x)*(Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*
E^(e + f*x))/(a + Sqrt[a^2 - b^2])]) + d*PolyLog[2, (b*E^(e + f*x))/(-a +
Sqrt[a^2 - b^2])] - d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))]
)/(Sqrt[a^2 - b^2]*f^2)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3801, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{c+dx}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3801}$$

$$2\int \frac{e^{e+fx}(c+dx)}{2e^{e+fx}a+be^{2(e+fx)}+b}dx$$

$$\Big\downarrow \text{2694}$$

$$2\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{27}$$

$$2\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)$$

$$\Big\downarrow \text{2620}$$

$$2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{d\int\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{d\int\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}\right)$$

$\downarrow$ 2715

$$2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{e+fx}}{bf^2}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e}{a+}\right.}{b}\right.}{2\sqrt{a^2-b^2}}\right.$$

$\downarrow$ 2838

$$2\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}\right.$$

input
```
Int[(c + d*x)/(a + b*Cosh[e + f*x]),x]
```

output
```
2*((b*(((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(b*f) +
(d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/(b*f^2)))/(2*Sqrt
[a^2 - b^2]) - (b*(((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]
)])/(b*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(b*f^
2)))/(2*Sqrt[a^2 - b^2]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 436 vs. $2(183) = 366$.

Time = 0.51 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.15

method	result
risch	$\frac{2c\arctan\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{-a^2+b^2}}\right)}{f\sqrt{-a^2+b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{f\sqrt{a^2-b^2}}-\frac{d\ln\left(\frac{b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{f\sqrt{a^2-b^2}}+\frac{d\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)e}{f^2\sqrt{a^2-b^2}}-$

input
```
int((d*x+c)/(a+b*cosh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
2/f*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^2)^(1/2))+1
/f*d/(a^2-b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1
/2)))*x-1/f*d/(a^2-b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-
b^2)^(1/2)))*x+1/f^2*d/(a^2-b^2)^(1/2)*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a
)/(-a+(a^2-b^2)^(1/2)))*e-1/f^2*d/(a^2-b^2)^(1/2)*ln((b*exp(f*x+e)+(a^2-b^
2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*e+1/f^2*d/(a^2-b^2)^(1/2)*dilog((-b*exp(f
*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/f^2*d/(a^2-b^2)^(1/2)*dil
og((b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/f^2*d*e/(-a^2+b
^2)^(1/2)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(181) = 362$.

Time = 0.21 (sec) , antiderivative size = 473, normalized size of antiderivative = 2.33

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = \frac{bd\sqrt{\frac{a^2-b^2}{b^2}}\operatorname{Li}_2\left(-\frac{a\cosh(fx+e)+a\sinh(fx+e)+(b\cosh(fx+e)+b\sinh(fx+e))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b}+1\right)-bd\sqrt{\frac{a^2-b^2}{b^2}}\operatorname{Li}_2\left(-\frac{a\cosh(fx+e)}{}\right.}{}$$

input
```
integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
(b*d*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) + (b*
cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - b*d*s
qrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f
*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + (b*d*e - b*
c*f)*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) + 2*b
*sqrt((a^2 - b^2)/b^2) + 2*a) - (b*d*e - b*c*f)*sqrt((a^2 - b^2)/b^2)*log(
2*b*cosh(f*x + e) + 2*b*sinh(f*x + e) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) +
 (b*d*f*x + b*d*e)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x
 + e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b)
- (b*d*f*x + b*d*e)*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*
x + e) - (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b)
)/((a^2 - b^2)*f^2)
```

Sympy [F]

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx=\int \frac{c+dx}{a+b\cosh\left(e+fx\right)}\,dx$$

input
```
integrate((d*x+c)/(a+b*cosh(f*x+e)),x)
```

output
```
Integral((c + d*x)/(a + b*cosh(e + f*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = \int \frac{dx+c}{b\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*cosh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = \int \frac{c+d\,x}{a+b\cosh(e+f\,x)}\,dx$$

input
```
int((c + d*x)/(a + b*cosh(e + f*x)),x)
```

output
```
int((c + d*x)/(a + b*cosh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{c+dx}{a+b\cosh(e+fx)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^{fx+e}b+a}{\sqrt{-a^2+b^2}}\right)c+2e^e\left(\int \frac{e^{fx}x}{e^{2fx+2e}b+2e^{fx+e}a+b}dx\right)a^2df-2e^e\left(\int \frac{e^{fx}x}{e^{2fx+2e}b+2e^{fx+e}a+b}dx\right)b^2df}{f\,(a^2-b^2)}$$

input
```
int((d*x+c)/(a+b*cosh(f*x+e)),x)
```

output
```
(2*( - sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2)
)*c + e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a + b),x)
*a**2*d*f - e**e*int((e**(f*x)*x)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a +
 b),x)*b**2*d*f))/(f*(a**2 - b**2))
```

3.171 $\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx=\text{Int}\left(\frac{1}{(c+dx)(a+b\cosh(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx=\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))} \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{(c+dx)\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \ 3807$$

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + b*Cosh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\cosh\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx=\int \frac{1}{(dx+c)(b\cosh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (b*d*x + b*c)*cosh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 6.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx=\int \frac{1}{(a+b\cosh{(e+fx)})\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

output
```
Integral(1/((a + b*cosh(e + f*x))*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx=\int \frac{1}{(dx+c)(b\cosh{(fx+e)}+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.94 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(a+b\cosh(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*cosh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + b*cosh(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))}\,dx$$
$$=\int \frac{1}{\cosh\left(fx+e\right)bc+\cosh\left(fx+e\right)bdx+ac+adx}dx$$

input
```
int(1/(d*x+c)/(a+b*cosh(f*x+e)),x)
```

output
```
int(1/(cosh(e + f*x)*b*c + cosh(e + f*x)*b*d*x + a*c + a*d*x),x)
```

3.172 $\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\cosh(e+fx))},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)^2\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Cosh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\left(a+b\cosh\left(fx+e\right)\right)}dx$$

input
```
int(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*cosh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 35.76 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(a+b\cosh(e+fx))(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e)),x)
```

output
```
Integral(1/((a + b*cosh(e + f*x))*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)), x)
```

Giac [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\cosh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx = \int \frac{1}{(a+b\cosh(e+f\,x))\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*cosh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + b*cosh(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))}\,dx$$
$$= 2e^e\left(\int \frac{e^{fx}}{e^{2fx+2e}b\,c^2 + 2e^{2fx+2e}bcdx + e^{2fx+2e}b\,d^2x^2 + 2e^{fx+e}a\,c^2 + 4e^{fx+e}acdx + 2e^{fx+e}a\,d^2x^2 + b\,c^2 + 2b\right.$$

input
```
int(1/(d*x+c)^2/(a+b*cosh(f*x+e)),x)
```

output
```
2*e**e*int(e**(f*x)/(e**(2*e + 2*f*x)*b*c**2 + 2*e**(2*e + 2*f*x)*b*c*d*x
+ e**(2*e + 2*f*x)*b*d**2*x**2 + 2*e**(e + f*x)*a*c**2 + 4*e**(e + f*x)*a*
c*d*x + 2*e**(e + f*x)*a*d**2*x**2 + b*c**2 + 2*b*c*d*x + b*d**2*x**2),x)
```

3.173 $\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2} dx$

Optimal result

Integrand size = 20, antiderivative size = 823

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx = -\frac{(c+dx)^3}{(a^2-b^2)f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{3d(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} - \frac{a(c+dx)^3\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} + \frac{3ad(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} + \frac{6d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} - \frac{3ad(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} - \frac{6d^3\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^4} - \frac{6ad^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} - \frac{6d^3\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^4} + \frac{6ad^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} + \frac{6ad^3\,\mathrm{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^4} - \frac{6ad^3\,\mathrm{PolyLog}\left(4,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^4} - \frac{b(c+dx)^3\sinh(e+fx)}{(a^2-b^2)f(a+b\cosh(e+fx))}$$

output
```
-(d*x+c)^3/(a^2-b^2)/f+3*d*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2))
)/(a^2-b^2)/f^2+a*(d*x+c)^3*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^
2)^(3/2)/f+3*d*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/
f^2-a*(d*x+c)^3*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f+6
*d^2*(d*x+c)*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3+3*
a*d*(d*x+c)^2*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)
/f^2+6*d^2*(d*x+c)*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/
f^3-3*a*d*(d*x+c)^2*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)
^(3/2)/f^2-6*d^3*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^
4-6*a*d^2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(
3/2)/f^3-6*d^3*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^4+
6*a*d^2*(d*x+c)*polylog(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/
2)/f^3+6*a*d^3*polylog(4,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2
)/f^4-6*a*d^3*polylog(4,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)
/f^4-b*(d*x+c)^3*sinh(f*x+e)/(a^2-b^2)/f/(a+b*cosh(f*x+e))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 11178 vs. $2(823) = 1646$.

Time = 13.54 (sec) , antiderivative size = 11178, normalized size of antiderivative = 13.58

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^3/(a + b*Cosh[e + f*x])^2,x]
```

output
```
Result too large to show
```

Rubi [A] (verified)

Time = 4.09 (sec) , antiderivative size = 746, normalized size of antiderivative = 0.91, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.850$, Rules used = {3042, 3805, 26, 3042, 3801, 2694, 27, 2620, 3011, 6096, 2620, 3011, 2720, 7143, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^3}{\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}dx}{a^2-b^2}+\frac{3ibd\int -\frac{i(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f(a^2-b^2)}-\frac{b(c+dx)^3\sinh(e+fx)}{f(a^2-b^2)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{26}$$

$$\frac{a\int \frac{(c+dx)^3}{a+b\cosh(e+fx)}dx}{a^2-b^2}+\frac{3bd\int \frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f(a^2-b^2)}-\frac{b(c+dx)^3\sinh(e+fx)}{f(a^2-b^2)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{(c+dx)^3}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{3bd\int \frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f(a^2-b^2)}-\frac{b(c+dx)^3\sinh(e+fx)}{f(a^2-b^2)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{3801}$$

$$\frac{2a\int \frac{e^{e+fx}(c+dx)^3}{2e^{e+fx}a+be^{2(e+fx)}+b}dx}{a^2-b^2}+\frac{3bd\int \frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f(a^2-b^2)}-\frac{b(c+dx)^3\sinh(e+fx)}{f(a^2-b^2)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^3}{2\left(a+be^{e+fx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{3bd\int \frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f(a^2-b^2)}-\frac{b(c+dx)^3\sinh(e+fx)}{f(a^2-b^2)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \; 27$$

$$\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^3}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{3bd\int \frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \; 2620$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{3d\int (c+dx)^2\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bf}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{a^2-b^2}\cdots$$

$$\frac{3bd\int \frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \; 3011$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\int (c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e\cdots}}{\sqrt{a^2-\cdots}}\cdots\right)}{bf}\cdots\right)}{\cdots}\cdots\right)}{a^2-b^2}\cdots$$

$$\frac{3bd\int \frac{(c+dx)^2\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \; 6096$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e\cdots}}{\sqrt{a^2-\cdots}}\right.}{bf}\right.}{}\right.}{a^2-b^2}$$

$$\frac{3bd\left(\int\frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx+\int\frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx-\frac{(c+dx)^3}{3bd}\right)}{f\left(a^2-b^2\right)}-$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \ 2620$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e\cdots}}{\sqrt{a^2-\cdots}}\right.}{bf}\right.}{}\right.}{a^2-b^2}$$

$$\frac{3bd\left(-\frac{2d\int(c+dx)\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}-\frac{2d\int(c+dx)\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bf}+\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}+\frac{(c+dx)^2\log\left(\frac{be^{e+f\cdots}}{\sqrt{a^2-b^2}}\right.}{bf}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \ 3011$$

$$\frac{3bd\left(-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{\cdots}}\right)}{f}\cdots\right.}{bf}\cdots\right.}{f\left(a^2-b^2\right)\cdots}$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{\cdots}}{\sqrt{a^2-\cdots}}\cdots\right.}{bf}\cdots\right.}{\cdots}\cdots\right.}{a^2-b^2\cdots}$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}$$

$$\Big\downarrow \; 2720$$

$$\frac{3bd\left(-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)de^{e+f\cdots}}{f^2}\cdots\right.}{bf}\cdots\right.}{f\left(a^2-b^2\right)\cdots}$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{\cdots}}{\sqrt{a^2-\cdots}}\cdots\right.}{bf}\cdots\right.}{\cdots}\cdots\right.}{a^2-b^2\cdots}$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}$$

$$\Big\downarrow \; 7143$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\int(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+\dots}}{\sqrt{a^2-\dots}}\right)}{bf}\dots\right.}{\dots}\right.}{a^2-b^2}$$

$$\frac{3bd\left(-\frac{2d\left(\frac{d\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}-\frac{2d\left(\frac{d\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f}\dots\right.}{bf}\dots\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \; 7163$$

$$2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}\right)}{f}-\frac{(c+dx)^2\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b}}\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\frac{3bd\left(-\frac{2d\left(\frac{d\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}-\frac{2d\left(\frac{d\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f}\right.}{bf}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \ 2720$$

$$2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\int e^{-e-fx}\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(\right.}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.$$

$$\frac{3bd\left(-\frac{2d\left(\frac{d\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}-\frac{2d\left(\frac{d\operatorname{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right.}{f}\right.}{bf}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}$$

$\downarrow$ 7143

$$\frac{3bd\left(-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}-\frac{2d\left(\frac{d\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{f}\right.}{bf}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^3\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{3d\left(\frac{2d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}-\frac{d\,\mathrm{PolyLog}\left(4,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}\right)}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right.}{bf}\right.}{2\sqrt{a^2-b^2}}\right.}{}$$

$$\frac{b(c+dx)^3\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}$$

input
```
Int[(c + d*x)^3/(a + b*Cosh[e + f*x])^2,x]
```

output
```
(3*b*d*(-1/3*(c + d*x)^3/(b*d) + ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a -
 Sqrt[a^2 - b^2])])/(b*f) + ((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a + Sqrt
[a^2 - b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a
- Sqrt[a^2 - b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 -
 b^2]))])/f^2))/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x))/(a
 + Sqrt[a^2 - b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2
- b^2]))])/f^2))/(b*f)))/((a^2 - b^2)*f) + (2*a*((b*(((c + d*x)^3*Log[1 +
(b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(b*f) - (3*d*(-(((c + d*x)^2*PolyL
og[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/f) + (2*d*(((c + d*x)*Pol
yLog[3, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/f - (d*PolyLog[4, -((b*
E^(e + f*x))/(a - Sqrt[a^2 - b^2]))])/f^2))/f))/(b*f)))/(2*Sqrt[a^2 - b^2]
) - (b*(((c + d*x)^3*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(b*f)
 - (3*d*(-(((c + d*x)^2*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])
)])/f) + (2*d*(((c + d*x)*PolyLog[3, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2
]))])/f - (d*PolyLog[4, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/f^2))/f
))/(b*f)))/(2*Sqrt[a^2 - b^2])))/(a^2 - b^2) - (b*(c + d*x)^3*Sinh[e + f*x
])/((a^2 - b^2)*f*(a + b*Cosh[e + f*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{(dx+c)^3}{(a+b\cosh(fx+e))^2}dx$$

input
```
int((d*x+c)^3/(a+b*cosh(f*x+e))^2,x)
```

output
```
int((d*x+c)^3/(a+b*cosh(f*x+e))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7116 vs. $2(761) = 1522$.

Time = 0.27 (sec) , antiderivative size = 7116, normalized size of antiderivative = 8.65

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**3/(a+b*cosh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx=\int \frac{(dx+c)^3}{(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx=\int \frac{(c+d\,x)^3}{(a+b\cosh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^3/(a + b*cosh(e + f*x))^2,x)
```

output
```
int((c + d*x)^3/(a + b*cosh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+b\cosh(e+fx))^2}\,dx=\text{too large to display}$$

input
```
int((d*x+c)^3/(a+b*cosh(f*x+e))^2,x)
```

output

```
(12*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt(
- a**2 + b**2))*a**3*b*c**2*d*f**2 + 12*e**(2*e + 2*f*x)*sqrt( - a**2 + b*
*2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**3*b*c*d**2*f + 6*e*
*(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2
 + b**2))*a**3*b*d**3 - 4*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e**(
e + f*x)*b + a)/sqrt( - a**2 + b**2))*a*b**3*c**3*f**3 - 12*e**(2*e + 2*f*
x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a*
b**3*c**2*d*f**2 - 12*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e**(e +
f*x)*b + a)/sqrt( - a**2 + b**2))*a*b**3*c*d**2*f - 6*e**(2*e + 2*f*x)*sqr
t( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a*b**3*d
**3 + 24*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt(
 - a**2 + b**2))*a**4*c**2*d*f**2 + 24*e**(e + f*x)*sqrt( - a**2 + b**2)*a
tan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**4*c*d**2*f + 12*e**(e +
f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*
a**4*d**3 - 8*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/
sqrt( - a**2 + b**2))*a**2*b**2*c**3*f**3 - 24*e**(e + f*x)*sqrt( - a**2 +
 b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2*c**2*d*f*
*2 - 24*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt(
- a**2 + b**2))*a**2*b**2*c*d**2*f - 12*e**(e + f*x)*sqrt( - a**2 + b**2)*
atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2*d**3 + 12*sqr...
```

3.174 $\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 593

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = -\frac{(c+dx)^2}{(a^2-b^2)f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} + \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{2d(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^2} - \frac{a(c+dx)^2\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{2d^2\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} + \frac{2ad(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} + \frac{2d^2\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)f^3} - \frac{2ad(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} - \frac{2ad^2\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} + \frac{2ad^2\,\mathrm{PolyLog}\left(3,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^3} - \frac{b(c+dx)^2\sinh(e+fx)}{(a^2-b^2)f(a+b\cosh(e+fx))}$$

output
```
-(d*x+c)^2/(a^2-b^2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/
(a^2-b^2)/f^2+a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)
^(3/2)/f+2*d*(d*x+c)*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^2-
a*(d*x+c)^2*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f+2*d^2
*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3+2*a*d*(d*x+c)*
polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2+2*d^2*pol
ylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)/f^3-2*a*d*(d*x+c)*poly
log(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^2-2*a*d^2*polyl
og(3,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3+2*a*d^2*polylo
g(3,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f^3-b*(d*x+c)^2*sin
h(f*x+e)/(a^2-b^2)/f/(a+b*cosh(f*x+e))
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 2814 vs. $2(593) = 1186$.

Time $= 9.93$ (sec) , antiderivative size $= 2814$, normalized size of antiderivative $= 4.75$

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[(c + d*x)^2/(a + b*Cosh[e + f*x])^2,x]
```

output
```
-((4*(a^2 - b^2)^2*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f^2*x + 2*(a^2 - b^2)^2
*d^2*((a^2 - b^2)*E^(2*e))^(3/2)*f^2*x^2 + 4*a^3*Sqrt[a^2 - b^2]*Sqrt[-(a^
2 - b^2)^2]*c*d*Sqrt[(a^2 - b^2)*E^(2*e)]*f*ArcTan[(a + b*E^(e + f*x))/Sqr
t[-a^2 + b^2]] - 4*a*b^2*Sqrt[a^2 - b^2]*Sqrt[-(a^2 - b^2)^2]*c*d*Sqrt[(a^
2 - b^2)*E^(2*e)]*f*ArcTan[(a + b*E^(e + f*x))/Sqrt[-a^2 + b^2]] + (4*a*b^
2*(a^2 - b^2)^(3/2)*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f*ArcTan[(a + b*E^(e +
 f*x))/Sqrt[-a^2 + b^2]])/Sqrt[-(a^2 - b^2)^2] + (4*a^3*Sqrt[-(a^2 - b^2)^
2]*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f*ArcTan[(a + b*E^(e + f*x))/Sqrt[-a^2
+ b^2]])/Sqrt[a^2 - b^2] - 4*a*(a^2 - b^2)^(5/2)*c*d*Sqrt[(a^2 - b^2)*E^(2
*e)]*f*ArcTanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 4*a*(a^2 - b^2)^(3/2
)*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f*ArcTanh[(a + b*E^(e + f*x))/Sqrt[a^2 -
 b^2]] + 2*a*(a^2 - b^2)^(5/2)*c^2*Sqrt[(a^2 - b^2)*E^(2*e)]*f^2*ArcTanh[(
a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] + 2*a*(a^2 - b^2)^(3/2)*c^2*((a^2 - b^
2)*E^(2*e))^(3/2)*f^2*ArcTanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 2*(a^
2 - b^2)^3*c*d*Sqrt[(a^2 - b^2)*E^(2*e)]*f*Log[b + 2*a*E^(e + f*x) + b*E^(
2*(e + f*x))] - 2*(a^2 - b^2)^2*c*d*((a^2 - b^2)*E^(2*e))^(3/2)*f*Log[b +
2*a*E^(e + f*x) + b*E^(2*(e + f*x))] - 2*(a^2 - b^2)^3*d^2*Sqrt[(a^2 - b^2
)*E^(2*e)]*f*x*Log[1 + (b*E^(2*e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)
])] - 2*(a^2 - b^2)^2*d^2*((a^2 - b^2)*E^(2*e))^(3/2)*f*x*Log[1 + (b*E^(2*
e + f*x))/(a*E^e - Sqrt[(a^2 - b^2)*E^(2*e)])] - 2*a*(a^2 - b^2)^3*c*d*...
```

Rubi [A] (verified)

Time = 2.93 (sec) , antiderivative size = 550, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3805, 26, 3042, 3801, 2694, 27, 2620, 3011, 2720, 6096, 2620, 2715, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}dx}{a^2-b^2}+\frac{2ibd\int -\frac{i(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{a\int \frac{(c+dx)^2}{a+b\cosh(e+fx)}dx}{a^2-b^2}+\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{a\int \frac{(c+dx)^2}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{ 3801}$$

$$\frac{2a\int \frac{e^{e+fx}(c+dx)^2}{2e^{e+fx}a+be^{2(e+fx)}+b}dx}{a^2-b^2}+\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{ 2694}$$

$$\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{2\left(a+be^{e+fx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)^2}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}+\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{ 2620}$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{2d\int (c+dx)\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}\right.}{a^2-b^2}$$

$$\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$\big\downarrow$ 3011

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int \text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)dx}{f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right.}{bf}\right.}{\cdots}\right.}{a^2-b^2}$$

$$\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$\big\downarrow$ 2720

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{a^2-b^2}$$

$$\frac{2bd\int \frac{(c+dx)\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$\big\downarrow$ 6096

$$2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{b\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad a^2-b^2 \qquad}$$

$$\frac{2bd\left(\int\frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx+\int\frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx-\frac{(c+dx)^2}{2bd}\right)}{f\left(a^2-b^2\right)}-$$

$$\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \; 2620$$

$$2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{b\cdots}\right.}{\cdots}\right.$$

$$\overline{\qquad a^2-b^2 \qquad}$$

$$\frac{2bd\left(-\frac{d\int\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}-\frac{d\int\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{(c+dx)\cdots}{2bd}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \; 2715$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{a^2-b^2}$$

$$\frac{2bd\left(-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{e+fx}}{bf^2}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{e+fx}}{bf^2}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\cdots\right.}{\cdots}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$\Big\downarrow$ 2838

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\int e^{-e-fx}\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)de^{e+fx}}{f^2}-\frac{(c+dx)\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\cdots\right.}{\cdots}\right.}{\cdots}\right)}{a^2-b^2}$$

$$\frac{2bd\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}-\frac{(c+dx\cdots}{2bd\cdots}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$\Big\downarrow$ 7143

$$\frac{2bd\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}+\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}-\frac{(c+dx}{2bd}\right.}{f\left(a^2-b^2\right)}$$

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{2d\left(\frac{d\,\text{PolyLog}\left(3,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{f}\right)}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)^2\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\right.}{}\right.}{a^2-b^2}$$

$$\frac{b(c+dx)^2\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}$$

input
```
Int[(c + d*x)^2/(a + b*Cosh[e + f*x])^2,x]
```

output
```
(2*b*d*(-1/2*(c + d*x)^2/(b*d) + ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a - S
qrt[a^2 - b^2])])/(b*f) + ((c + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2
 - b^2])])/(b*f) + (d*PolyLog[2, -((b*E^(e + f*x))/(a - Sqrt[a^2 - b^2]))]
)/(b*f^2) + (d*PolyLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(b*f^
2)))/((a^2 - b^2)*f) + (2*a*((b*(((c + d*x)^2*Log[1 + (b*E^(e + f*x))/(a -
 Sqrt[a^2 - b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyLog[2, -((b*E^(e + f*x)
)/(a - Sqrt[a^2 - b^2]))])/f) + (d*PolyLog[3, -((b*E^(e + f*x))/(a - Sqrt[
a^2 - b^2]))])/f^2))/(b*f)))/(2*Sqrt[a^2 - b^2]) - (b*(((c + d*x)^2*Log[1
+ (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(b*f) - (2*d*(-(((c + d*x)*PolyL
og[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/f) + (d*PolyLog[3, -((b*E
^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/f^2))/(b*f))/(2*Sqrt[a^2 - b^2])))/(
a^2 - b^2) - (b*(c + d*x)^2*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[e +
f*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{(dx+c)^2}{(a+b\cosh(fx+e))^2}dx$$

input
```
int((d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

output
```
int((d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4105 vs. $2(547) = 1094$.

Time = 0.22 (sec) , antiderivative size = 4105, normalized size of antiderivative = 6.92

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2/(a+b*cosh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+b\cosh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^2/(a + b*cosh(e + f*x))^2,x)
```

output
```
int((c + d*x)^2/(a + b*cosh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+b\cosh(e+fx))^2} dx = \text{too large to display}$$

input
```
int((d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

output
```
(4*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( -
 a**2 + b**2))*a**3*b*c*d*f + 2*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan
((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**3*b*d**2 - 2*e**(2*e + 2*f*
x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a*
b**3*c**2*f**2 - 4*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x
)*b + a)/sqrt( - a**2 + b**2))*a*b**3*c*d*f - 2*e**(2*e + 2*f*x)*sqrt( - a
**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a*b**3*d**2 +
8*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2
 + b**2))*a**4*c*d*f + 4*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f
*x)*b + a)/sqrt( - a**2 + b**2))*a**4*d**2 - 4*e**(e + f*x)*sqrt( - a**2 +
 b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2*c**2*f**2
 - 8*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a
**2 + b**2))*a**2*b**2*c*d*f - 4*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e
**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2*d**2 + 4*sqrt( - a**2 +
 b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**3*b*c*d*f + 2*sq
rt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**3*b*
d**2 - 2*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**
2))*a*b**3*c**2*f**2 - 4*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sq
rt( - a**2 + b**2))*a*b**3*c*d*f - 2*sqrt( - a**2 + b**2)*atan((e**(e + f*
x)*b + a)/sqrt( - a**2 + b**2))*a*b**3*d**2 - 4*e**(3*e + 2*f*x)*int((e...
```

3.175 $\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 274

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} - \frac{a(c+dx)\log\left(1+\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f} + \frac{d\log(a+b\cosh(e+fx))}{(a^2-b^2)f^2} + \frac{ad\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} - \frac{ad\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}f^2} - \frac{b(c+dx)\sinh(e+fx)}{(a^2-b^2)f(a+b\cosh(e+fx))}$$

output
```
a*(d*x+c)*ln(1+b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f-a*(d*x+
c)*ln(1+b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2)^(3/2)/f+d*ln(a+b*cosh(
f*x+e))/(a^2-b^2)/f^2+a*d*polylog(2,-b*exp(f*x+e)/(a-(a^2-b^2)^(1/2)))/(a^
2-b^2)^(3/2)/f^2-a*d*polylog(2,-b*exp(f*x+e)/(a+(a^2-b^2)^(1/2)))/(a^2-b^2
)^(3/2)/f^2-b*(d*x+c)*sinh(f*x+e)/(a^2-b^2)/f/(a+b*cosh(f*x+e))
```

Mathematica [A] (warning: unable to verify)

Time = 2.78 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.86

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx =$$

$$-\frac{(a^2-b^2)\left(-\sqrt{-(a^2-b^2)^2}d(e+fx)+2a\sqrt{a^2-b^2}d\arctan\left(\frac{a+be^{e+fx}}{\sqrt{-a^2+b^2}}\right)+2a\sqrt{-a^2+b^2}d\operatorname{arctanh}\left(\frac{a+be^{e+fx}}{\sqrt{a^2-b^2}}\right)+2a\sqrt{-a^2+b^2}de\operatorname{arctanh}\left(\right.\right.}{}$$

input
```
Integrate[(c + d*x)/(a + b*Cosh[e + f*x])^2,x]
```

output
```
-((((a^2 - b^2)*(-(Sqrt[-(a^2 - b^2)^2]*d*(e + f*x)) + 2*a*Sqrt[a^2 - b^2]
*d*ArcTan[(a + b*E^(e + f*x))/Sqrt[-a^2 + b^2]] + 2*a*Sqrt[-a^2 + b^2]*d*A
rcTanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] + 2*a*Sqrt[-a^2 + b^2]*d*e*Arc
Tanh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] - 2*a*Sqrt[-a^2 + b^2]*c*f*ArcTa
nh[(a + b*E^(e + f*x))/Sqrt[a^2 - b^2]] + a*Sqrt[-a^2 + b^2]*d*(e + f*x)*L
og[1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])] - a*Sqrt[-a^2 + b^2]*d*(e +
f*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])] + Sqrt[-(a^2 - b^2)^2]
*d*Log[b + 2*a*E^(e + f*x) + b*E^(2*(e + f*x))] + a*Sqrt[-a^2 + b^2]*d*Pol
yLog[2, (b*E^(e + f*x))/(-a + Sqrt[a^2 - b^2])] - a*Sqrt[-a^2 + b^2]*d*Pol
yLog[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))]))/(-(a^2 - b^2)^2)^(3/2)
 + (b*f*(c + d*x)*Sinh[e + f*x])/((a - b)*(a + b)*(a + b*Cosh[e + f*x])))/
f^2)
```

Rubi [A] (verified)

Time = 1.36 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.03, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.722$, Rules used = {3042, 3805, 26, 3042, 26, 3147, 16, 3801, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{c+dx}{\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3805}$$

$$\frac{a\int \frac{c+dx}{a+b\cosh(e+fx)}dx}{a^2-b^2}+\frac{ibd\int -\frac{i\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{26}$$

$$\frac{a\int \frac{c+dx}{a+b\cosh(e+fx)}dx}{a^2-b^2}+\frac{bd\int \frac{\sinh(e+fx)}{a+b\cosh(e+fx)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\int \frac{c+dx}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{bd\int -\frac{i\cos\left(ie+ifx-\frac{\pi}{2}\right)}{a-b\sin\left(ie+ifx-\frac{\pi}{2}\right)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{26}$$

$$\frac{a\int \frac{c+dx}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{ibd\int \frac{\cos\left(\frac{1}{2}(2ie-\pi)+ifx\right)}{a-b\sin\left(\frac{1}{2}(2ie-\pi)+ifx\right)}dx}{f\left(a^2-b^2\right)}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{3147}$$

$$\frac{a\int \frac{c+dx}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{d\int \frac{1}{a+b\cosh(e+fx)}d(b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}$$

$$\Big\downarrow \text{16}$$

$$\frac{a\int \frac{c+dx}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}+\frac{d\log(a+b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3801}$$

$$\frac{2a\int \frac{e^{e+fx}(c+dx)}{2e^{e+fx}a+be^{2(e+fx)}+b}dx}{a^2-b^2}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}+\frac{d\log(a+b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{2\left(a+be^{e+fx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)(a+b\cosh(e+fx))}+\frac{d\log(a+b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 27

$$\frac{2a\left(\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{e+fx}(c+dx)}{a+be^{e+fx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}+\frac{d\log(a+b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 2620

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{d\int \log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bf}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}+\frac{d\log(a+b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 2715

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{e+fx}}{bf^2}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}-\frac{d\int e^{-e-fx}\log\left(\frac{e^{e+fx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{e+fx}}{bf^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}+\frac{d\log(a+b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}$$

$\Big\downarrow$ 2838

$$\frac{2a\left(\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a-\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{(c+dx)\log\left(\frac{be^{e+fx}}{\sqrt{a^2-b^2}+a}+1\right)}{bf}+\frac{d\,\text{PolyLog}\left(2,-\frac{be^{e+fx}}{a+\sqrt{a^2-b^2}}\right)}{bf^2}\right)}{2\sqrt{a^2-b^2}}\right)}{a^2-b^2}-\frac{b(c+dx)\sinh(e+fx)}{f\left(a^2-b^2\right)\left(a+b\cosh(e+fx)\right)}+\frac{d\log(a+b\cosh(e+fx))}{f^2\left(a^2-b^2\right)}$$

input
```
Int[(c + d*x)/(a + b*Cosh[e + f*x])^2,x]
```

output
```
(d*Log[a + b*Cosh[e + f*x]])/((a^2 - b^2)*f^2) + (2*a*((b*(((c + d*x)*Log[
1 + (b*E^(e + f*x))/(a - Sqrt[a^2 - b^2])])/(b*f) + (d*PolyLog[2, -((b*E^(
e + f*x))/(a - Sqrt[a^2 - b^2]))])/(b*f^2)))/(2*Sqrt[a^2 - b^2]) - (b*(((c
 + d*x)*Log[1 + (b*E^(e + f*x))/(a + Sqrt[a^2 - b^2])])/(b*f) + (d*PolyLog
[2, -((b*E^(e + f*x))/(a + Sqrt[a^2 - b^2]))])/(b*f^2)))/(2*Sqrt[a^2 - b^2
])))/(a^2 - b^2) - (b*(c + d*x)*Sinh[e + f*x])/((a^2 - b^2)*f*(a + b*Cosh[
e + f*x]))
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 3805
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2, x_
Symbol] :> Simp[b*(c + d*x)^m*(Cos[e + f*x]/(f*(a^2 - b^2)*(a + b*Sin[e + f
*x]))), x] + (Simp[a/(a^2 - b^2)   Int[(c + d*x)^m/(a + b*Sin[e + f*x]), x]
, x] - Simp[b*d*(m/(f*(a^2 - b^2)))   Int[(c + d*x)^(m - 1)*(Cos[e + f*x]/(
a + b*Sin[e + f*x])), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 -
b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. 2(254) = 508.

Time = 0.78 (sec) , antiderivative size = 585, normalized size of antiderivative = 2.14

method	result
risch	$\frac{2(dx+c)(a\,\mathrm{e}^{fx+e}+b)}{f(a^2-b^2)(\mathrm{e}^{2fx+2e}b+2a\,\mathrm{e}^{fx+e}+b)}+\frac{2ac\arctan\left(\frac{2b\,\mathrm{e}^{fx+e}+2a}{2\sqrt{-a^2+b^2}}\right)}{f(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{ad\ln\left(\frac{-b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{f(a^2-b^2)^{\frac{3}{2}}}-\frac{ad\ln\left(\frac{b\,\mathrm{e}^{fx+e}+\sqrt{a^2-b^2}}{a+\sqrt{a^2-b^2}}\right.}{f(a^2-b^2)^{\frac{3}{2}}}$

input

```
int((d*x+c)/(a+b*cosh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output

```
2*(d*x+c)*(a*exp(f*x+e)+b)/f/(a^2-b^2)/(exp(2*f*x+2*e)*b+2*a*exp(f*x+e)+b)
+2/f/(a^2-b^2)*a*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*b*exp(f*x+e)+2*a)/(-a^2+
b^2)^(1/2))+1/f/(a^2-b^2)^(3/2)*a*d*ln((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(
-a+(a^2-b^2)^(1/2)))*x-1/f/(a^2-b^2)^(3/2)*a*d*ln((b*exp(f*x+e)+(a^2-b^2)^
(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x+1/f^2/(a^2-b^2)^(3/2)*a*d*ln((-b*exp(f*x+e
)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*e-1/f^2/(a^2-b^2)^(3/2)*a*d*ln(
(b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*e+1/f^2/(a^2-b^2)^(3
/2)*a*d*dilog((-b*exp(f*x+e)+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/f^
2/(a^2-b^2)^(3/2)*a*d*dilog((b*exp(f*x+e)+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^
(1/2)))+1/f^2/(a^2-b^2)*d*ln(exp(2*f*x+2*e)*b+2*a*exp(f*x+e)+b)-2/f^2/(a^2
-b^2)*d*ln(exp(f*x+e))-2/f^2/(a^2-b^2)*a*d*e/(-a^2+b^2)^(1/2)*arctan(1/2*(
2*b*exp(f*x+e)+2*a)/(-a^2+b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1765 vs. 2(252) = 504.

Time = 0.21 (sec) , antiderivative size = 1765, normalized size of antiderivative = 6.44

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-(2*(a^2*b - b^3)*d*e - 2*(a^2*b - b^3)*c*f + 2*((a^2*b - b^3)*d*f*x + (a^
2*b - b^3)*d*e)*cosh(f*x + e)^2 + 2*((a^2*b - b^3)*d*f*x + (a^2*b - b^3)*d
*e)*sinh(f*x + e)^2 - (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 +
 2*a^2*b*d*cosh(f*x + e) + a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*s
inh(f*x + e))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x +
 e) + (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1
) + (a*b^2*d*cosh(f*x + e)^2 + a*b^2*d*sinh(f*x + e)^2 + 2*a^2*b*d*cosh(f*
x + e) + a*b^2*d + 2*(a*b^2*d*cosh(f*x + e) + a^2*b*d)*sinh(f*x + e))*sqrt
((a^2 - b^2)/b^2)*dilog(-(a*cosh(f*x + e) + a*sinh(f*x + e) - (b*cosh(f*x
+ e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - (a*b^2*d*f*x +
 a*b^2*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 + (a*b^2*d*f*x + a*
b^2*d*e)*sinh(f*x + e)^2 + 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e) + 2*(
a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*sinh(f*
x + e))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e) + (b*
cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + (a*b^2*d*
f*x + a*b^2*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e)^2 + (a*b^2*d*f*x
 + a*b^2*d*e)*sinh(f*x + e)^2 + 2*(a^2*b*d*f*x + a^2*b*d*e)*cosh(f*x + e)
+ 2*(a^2*b*d*f*x + a^2*b*d*e + (a*b^2*d*f*x + a*b^2*d*e)*cosh(f*x + e))*si
nh(f*x + e))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(f*x + e) + a*sinh(f*x + e)
- (b*cosh(f*x + e) + b*sinh(f*x + e))*sqrt((a^2 - b^2)/b^2) + b)/b) + 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(a+b*cosh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{dx+c}{(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{c+d\,x}{(a+b\cosh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)/(a + b*cosh(e + f*x))^2,x)
```

output
```
int((c + d*x)/(a + b*cosh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{c+dx}{(a+b\cosh(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

output
```
(2*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( -
 a**2 + b**2))*a**3*b*d - 2*e**(2*e + 2*f*x)*sqrt( - a**2 + b**2)*atan((e*
*(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a*b**3*c*f - 2*e**(2*e + 2*f*x)*sq
rt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a*b**3*
d + 4*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( -
a**2 + b**2))*a**4*d - 4*e**(e + f*x)*sqrt( - a**2 + b**2)*atan((e**(e + f
*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2*c*f - 4*e**(e + f*x)*sqrt( - a*
*2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2*d + 2
*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**2))*a**3
*b*d - 2*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( - a**2 + b**
2))*a*b**3*c*f - 2*sqrt( - a**2 + b**2)*atan((e**(e + f*x)*b + a)/sqrt( -
a**2 + b**2))*a*b**3*d - 4*e**(3*e + 2*f*x)*int((e**(f*x)*x)/(e**(4*e + 4*
f*x)*b**2 + 4*e**(3*e + 3*f*x)*a*b + 4*e**(2*e + 2*f*x)*a**2 + 2*e**(2*e +
 2*f*x)*b**2 + 4*e**(e + f*x)*a*b + b**2),x)*a**5*b**2*d*f**2 + 8*e**(3*e
+ 2*f*x)*int((e**(f*x)*x)/(e**(4*e + 4*f*x)*b**2 + 4*e**(3*e + 3*f*x)*a*b
+ 4*e**(2*e + 2*f*x)*a**2 + 2*e**(2*e + 2*f*x)*b**2 + 4*e**(e + f*x)*a*b +
 b**2),x)*a**3*b**4*d*f**2 - 4*e**(3*e + 2*f*x)*int((e**(f*x)*x)/(e**(4*e
+ 4*f*x)*b**2 + 4*e**(3*e + 3*f*x)*a*b + 4*e**(2*e + 2*f*x)*a**2 + 2*e**(2
*e + 2*f*x)*b**2 + 4*e**(e + f*x)*a*b + b**2),x)*a*b**6*d*f**2 - e**(2*e +
 2*f*x)*log(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a + b)*a**4*b*d + 2*e**...
```

3.176 $\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+b\cosh(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.01 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Cosh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{(c+dx)\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3807}$$

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)*(a + b*Cosh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\cosh\left(fx+e\right)\right)^2}dx$$

input
```
int(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\cosh{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*cosh(f*x + e)^2 + 2*(a*b*d
*x + a*b*c)*cosh(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 416, normalized size of antiderivative = 20.80

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\cosh{(fx+e)}+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*(a*e^(f*x + e) + b)/(a^2*b*c*f - b^3*c*f + (a^2*b*d*f - b^3*d*f)*x + (a^
2*b*c*f*e^(2*e) - b^3*c*f*e^(2*e) + (a^2*b*d*f*e^(2*e) - b^3*d*f*e^(2*e))*
x)*e^(2*f*x) + 2*(a^3*c*f*e^e - a*b^2*c*f*e^e + (a^3*d*f*e^e - a*b^2*d*f*e
^e)*x)*e^(f*x)) + integrate(2*(b*d + (a*d*f*x*e^e + (c*f*e^e + d*e^e)*a)*e
^(f*x))/(a^2*b*c^2*f - b^3*c^2*f + (a^2*b*d^2*f - b^3*d^2*f)*x^2 + 2*(a^2*
b*c*d*f - b^3*c*d*f)*x + (a^2*b*c^2*f*e^(2*e) - b^3*c^2*f*e^(2*e) + (a^2*b
*d^2*f*e^(2*e) - b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(2*e) - b^3*c*d
*f*e^(2*e))*x)*e^(2*f*x) + 2*(a^3*c^2*f*e^e - a*b^2*c^2*f*e^e + (a^3*d^2*f
*e^e - a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e - a*b^2*c*d*f*e^e)*x)*e^(f*
x)), x)
```

Giac [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*cosh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(a+b\cosh(e+f\,x))^2\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*cosh(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + b*cosh(e + f*x))^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.30

$$\int \frac{1}{(c+dx)(a+b\cosh(e+fx))^2}\,dx$$
$$=\int \frac{1}{\cosh(fx+e)^2 b^2c+\cosh(fx+e)^2 b^2dx+2\cosh(fx+e)abc+2\cosh(fx+e)abdx+a^2c+a^2dx}dx$$

input
```
int(1/(d*x+c)/(a+b*cosh(f*x+e))^2,x)
```

output
```
int(1/(cosh(e + f*x)**2*b**2*c + cosh(e + f*x)**2*b**2*d*x + 2*cosh(e + f*
x)*a*b*c + 2*cosh(e + f*x)*a*b*d*x + a**2*c + a**2*d*x),x)
```

3.177 $\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \mathrm{Int}\left(\frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2},x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{(c+dx)^2\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3807$$

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Cosh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\,(a+b\cosh(fx+e))^2}dx$$

input
```
int(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*cosh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*co
sh(f*x + e)), x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/(a+b*cosh(f*x+e))**2,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.91 (sec) , antiderivative size = 622, normalized size of antiderivative = 31.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*(a*e^(f*x + e) + b)/(a^2*b*c^2*f - b^3*c^2*f + (a^2*b*d^2*f - b^3*d^2*f)
*x^2 + 2*(a^2*b*c*d*f - b^3*c*d*f)*x + (a^2*b*c^2*f*e^(2*e) - b^3*c^2*f*e^
(2*e) + (a^2*b*d^2*f*e^(2*e) - b^3*d^2*f*e^(2*e))*x^2 + 2*(a^2*b*c*d*f*e^(
2*e) - b^3*c*d*f*e^(2*e))*x)*e^(2*f*x) + 2*(a^3*c^2*f*e^e - a*b^2*c^2*f*e^
e + (a^3*d^2*f*e^e - a*b^2*d^2*f*e^e)*x^2 + 2*(a^3*c*d*f*e^e - a*b^2*c*d*f
*e^e)*x)*e^(f*x)) + integrate(2*(2*b*d + (a*d*f*x*e^e + (c*f*e^e + 2*d*e^e
)*a)*e^(f*x))/(a^2*b*c^3*f - b^3*c^3*f + (a^2*b*d^3*f - b^3*d^3*f)*x^3 + 3
*(a^2*b*c*d^2*f - b^3*c*d^2*f)*x^2 + 3*(a^2*b*c^2*d*f - b^3*c^2*d*f)*x + (
a^2*b*c^3*f*e^(2*e) - b^3*c^3*f*e^(2*e) + (a^2*b*d^3*f*e^(2*e) - b^3*d^3*f
*e^(2*e))*x^3 + 3*(a^2*b*c*d^2*f*e^(2*e) - b^3*c*d^2*f*e^(2*e))*x^2 + 3*(a
^2*b*c^2*d*f*e^(2*e) - b^3*c^2*d*f*e^(2*e))*x)*e^(2*f*x) + 2*(a^3*c^3*f*e^
e - a*b^2*c^3*f*e^e + (a^3*d^3*f*e^e - a*b^2*d^3*f*e^e)*x^3 + 3*(a^3*c*d^2
*f*e^e - a*b^2*c*d^2*f*e^e)*x^2 + 3*(a^3*c^2*d*f*e^e - a*b^2*c^2*d*f*e^e)*
x)*e^(f*x)), x)
```

Giac [N/A]

Not integrable

Time = 1.30 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*cosh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx = \int \frac{1}{(a+b\cosh(e+f\,x))^2\,(c+d\,x)^2}\,dx$$

input
```
int(1/((a + b*cosh(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*cosh(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 121, normalized size of antiderivative = 6.05

$$\int \frac{1}{(c+dx)^2(a+b\cosh(e+fx))^2}\,dx$$
$$= \int \frac{1}{\cosh(fx+e)^2 b^2c^2 + 2\cosh(fx+e)^2 b^2cdx + \cosh(fx+e)^2 b^2d^2x^2 + 2\cosh(fx+e)\,ab\,c^2 + 4\cosh($$

input
```
int(1/(d*x+c)^2/(a+b*cosh(f*x+e))^2,x)
```

output
```
int(1/(cosh(e + f*x)**2*b**2*c**2 + 2*cosh(e + f*x)**2*b**2*c*d*x + cosh(e
 + f*x)**2*b**2*d**2*x**2 + 2*cosh(e + f*x)*a*b*c**2 + 4*cosh(e + f*x)*a*b
*c*d*x + 2*cosh(e + f*x)*a*b*d**2*x**2 + a**2*c**2 + 2*a**2*c*d*x + a**2*d
**2*x**2),x)
```

3.178 $\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx = \text{Int}((c+dx)^m(a+b\cosh(e+fx))^n,x)$$

output
```
Defer(Int)((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

Mathematica [N/A]

Not integrable

Time = 2.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx = \int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

input
```
Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^n,x]
```

output
```
Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^n, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3807}$$

$$\int (c+dx)^m(a+b\cosh(e+fx))^n dx$$

input
```
Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^n,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m\,(a+b\cosh(fx+e))^n\,dx$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

output
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\int (dx+c)^m(b\cosh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Sympy [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\text{Timed out}$$

input
```
integrate((d*x+c)**m*(a+b*cosh(f*x+e))**n,x)
```

output
```
Timed out
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\int (dx+c)^m(b\cosh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Giac [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\int (dx+c)^m(b\cosh(fx+e)+a)^n\,dx$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^n,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m*(b*cosh(f*x + e) + a)^n, x)
```

Mupad [N/A]

Not integrable

Time = 2.03 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\int (a+b\cosh(e+f\,x))^n\,(c+d\,x)^m\,dx$$

input
```
int((a + b*cosh(e + f*x))^n*(c + d*x)^m,x)
```

output
```
int((a + b*cosh(e + f*x))^n*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m(a+b\cosh(e+fx))^n\,dx=\int (dx+c)^m\,(\cosh(fx+e)\,b+a)^n\,dx$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^n,x)
```

output
```
int((c + d*x)**m*(cosh(e + f*x)*b + a)**n,x)
```

3.179 $\int (c+dx)^m (a+b\cosh(e+fx))^3 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 543

$$
\begin{aligned}
&\int (c+dx)^m (a+b\cosh(e+fx))^3\,dx \\
&= \frac{a^3(c+dx)^{1+m}}{d(1+m)} + \frac{3ab^2(c+dx)^{1+m}}{2d(1+m)} \\
&+ \frac{3^{-1-m}b^3 e^{3e-\frac{3cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)}{8f} \\
&+ \frac{3\ 2^{-3-m}ab^2 e^{2e-\frac{2cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\
&+ \frac{3a^2b e^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f} \\
&+ \frac{3b^3 e^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{8f} \\
&- \frac{3a^2b e^{-e+\frac{cf}{d}}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f} \\
&- \frac{3b^3 e^{-e+\frac{cf}{d}}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{8f} \\
&- \frac{3\ 2^{-3-m}ab^2 e^{-2e+\frac{2cf}{d}}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f} \\
&- \frac{3^{-1-m}b^3 e^{-3e+\frac{3cf}{d}}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(1+m,\frac{3f(c+dx)}{d}\right)}{8f}
\end{aligned}
$$

output
```
a^3*(d*x+c)^(1+m)/d/(1+m)+3/2*a*b^2*(d*x+c)^(1+m)/d/(1+m)+1/8*3^(-1-m)*b^3
*exp(3*e-3*c*f/d)*(d*x+c)^m*GAMMA(1+m,-3*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)
+3*2^(-3-m)*a*b^2*exp(2*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/(
(-f*(d*x+c)/d)^m)+3/2*a^2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)
/f/((-f*(d*x+c)/d)^m)+3/8*b^3*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/
d)/f/((-f*(d*x+c)/d)^m)-3/2*a^2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x
+c)/d)/f/((f*(d*x+c)/d)^m)-3/8*b^3*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*
x+c)/d)/f/((f*(d*x+c)/d)^m)-3*2^(-3-m)*a*b^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*G
AMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-1/8*3^(-1-m)*b^3*exp(-3*e+3*c*
f/d)*(d*x+c)^m*GAMMA(1+m,3*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 1.08 (sec) , antiderivative size = 447, normalized size of antiderivative = 0.82

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx = \frac{2^{-3-m}3^{-1-m}e^{-3\left(e+\frac{cf}{d}\right)}(c+dx)^m\left(-\frac{f^2(c+dx)^2}{d^2}\right)^{-m}\left(2^mb^3de^{6e}(1+m)\left(\frac{f(c+dx)}{d}\right)^m\Gamma\left(1+m,-\frac{3f(c+dx)}{d}\right)+3\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^3,x]
```

output
```
(2^(-3 - m)*3^(-1 - m)*(c + d*x)^m*(2^m*b^3*d*E^(6*e)*(1 + m)*((f*(c + d*x
))/d)^m*Gamma[1 + m, (-3*f*(c + d*x))/d] + 3^(2 + m)*a*b^2*d*E^(5*e + (c*f
)/d)*(1 + m)*(f*(c/d + x))^m*Gamma[1 + m, (-2*f*(c + d*x))/d] + 2^m*3^(2 +
 m)*b*(4*a^2 + b^2)*d*E^(4*e + (2*c*f)/d)*(1 + m)*((f*(c + d*x))/d)^m*Gamm
a[1 + m, -((f*(c + d*x))/d)] - E^((3*c*f)/d)*(2^m*3^(2 + m)*b*(4*a^2 + b^2
)*d*E^(2*e + (c*f)/d)*(1 + m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (f*(c +
d*x))/d] + 3^(2 + m)*a*b^2*d*E^(e + (2*c*f)/d)*(1 + m)*(-((f*(c + d*x))/d)
)^m*Gamma[1 + m, (2*f*(c + d*x))/d] + 2^m*(-4*3^(1 + m)*a*(2*a^2 + 3*b^2)*
E^(3*e)*f*(c + d*x)*(-((f^2*(c + d*x)^2)/d^2))^m + b^3*d*E^((3*c*f)/d)*(1
+ m)*(-((f*(c + d*x))/d))^m*Gamma[1 + m, (3*f*(c + d*x))/d]))))/(d*E^(3*(e
 + (c*f)/d))*f*(1 + m)*(-((f^2*(c + d*x)^2)/d^2))^m)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 543, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^3dx$$

$$\Big\downarrow \text{3798}$$

$$\int \left(a^3(c+dx)^m+3a^2b(c+dx)^m\cosh(e+fx)+3ab^2(c+dx)^m\cosh^2(e+fx)+b^3(c+dx)^m\cosh^3(e+fx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^{m+1}}{d(m+1)}+\frac{3a^2be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f}-\frac{3a^2be^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}+\frac{3ab^2 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{3ab^2 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{3ab^2(c+dx)^{m+1}}{2d(m+1)}+\frac{b^3 3^{-m-1}e^{3e-\frac{3cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{3f(c+dx)}{d}\right)}{8f}+\frac{3b^3e^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{8f}-\frac{3b^3e^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{8f}-\frac{b^3 3^{-m-1}e^{\frac{3cf}{d}-3e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{3f(c+dx)}{d}\right)}{8f}$$

input
```
Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^3,x]
```

output
```
(a^3*(c + d*x)^(1 + m))/(d*(1 + m)) + (3*a*b^2*(c + d*x)^(1 + m))/(2*d*(1
+ m)) + (3^(-1 - m)*b^3*E^(3*e - (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-3*f
*(c + d*x))/d])/(8*f*(-((f*(c + d*x))/d))^m) + (3*2^(-3 - m)*a*b^2*E^(2*e
- (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c + d*x))/d])/(f*(-((f*(c + d
*x))/d))^m) + (3*a^2*b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1 + m, -((f*(c +
d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) + (3*b^3*E^(e - (c*f)/d)*(c + d*x)
^m*Gamma[1 + m, -((f*(c + d*x))/d)])/(8*f*(-((f*(c + d*x))/d))^m) - (3*a^2
*b*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c
 + d*x))/d)^m) - (3*b^3*E^(-e + (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c +
d*x))/d])/(8*f*((f*(c + d*x))/d)^m) - (3*2^(-3 - m)*a*b^2*E^(-2*e + (2*c*f
)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
- (3^(-1 - m)*b^3*E^(-3*e + (3*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (3*f*(c +
d*x))/d])/(8*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\cosh(fx+e))^3\,dx$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^3,x)
```

output
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 813, normalized size of antiderivative = 1.50

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/24*((b^3*d*m + b^3*d)*cosh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d)*gamma(m
+ 1, 3*(d*f*x + c*f)/d) + 9*(a*b^2*d*m + a*b^2*d)*cosh((d*m*log(2*f/d) + 2
*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) + 9*((4*a^2*b + b^3)*d*m
+ (4*a^2*b + b^3)*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*
x + c*f)/d) - 9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*cosh((d*m*log(-f
/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - 9*(a*b^2*d*m + a*b^2*
d)*cosh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)
/d) - (b^3*d*m + b^3*d)*cosh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d)*gamma(m
+ 1, -3*(d*f*x + c*f)/d) - (b^3*d*m + b^3*d)*gamma(m + 1, 3*(d*f*x + c*f)/
d)*sinh((d*m*log(3*f/d) + 3*d*e - 3*c*f)/d) - 9*(a*b^2*d*m + a*b^2*d)*gamm
a(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 9*(
(4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)*d)*gamma(m + 1, (d*f*x + c*f)/d)*sin
h((d*m*log(f/d) + d*e - c*f)/d) + 9*((4*a^2*b + b^3)*d*m + (4*a^2*b + b^3)
*d)*gamma(m + 1, -(d*f*x + c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + 9
*(a*b^2*d*m + a*b^2*d)*gamma(m + 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f
/d) - 2*d*e + 2*c*f)/d) + (b^3*d*m + b^3*d)*gamma(m + 1, -3*(d*f*x + c*f)/
d)*sinh((d*m*log(-3*f/d) - 3*d*e + 3*c*f)/d) - 12*((2*a^3 + 3*a*b^2)*d*f*x
 + (2*a^3 + 3*a*b^2)*c*f)*cosh(m*log(d*x + c)) - 12*((2*a^3 + 3*a*b^2)*d*f
*x + (2*a^3 + 3*a*b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*cosh(f*x+e))**3,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.69

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx$$
$$= -\frac{3}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)a^2b$$
$$-\frac{3}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2e+\frac{2cf}{d}\right)}E_{-m}\left(\frac{2(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(2e-\frac{2cf}{d}\right)}E_{-m}\left(-\frac{2(dx+c)f}{d}\right)}{d}-\frac{2(dx+c)^{m+}}{d(m+1)}\right.$$
$$-\frac{1}{8}\left(\frac{(dx+c)^{m+1}e^{\left(-3e+\frac{3cf}{d}\right)}E_{-m}\left(\frac{3(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{3(dx+c)^{m+1}e}{}\right.$$
$$+\frac{(dx+c)^{m+1}a^3}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-3/2*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d
 + (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a
^2*b - 3/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x
 + c)*f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(
d*x + c)*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*a*b^2 - 1/8*((d*x + c)^
(m + 1)*e^(-3*e + 3*c*f/d)*exp_integral_e(-m, 3*(d*x + c)*f/d)/d + 3*(d*x
+ c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d + 3*(d*x +
 c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d + (d*x + c)
^(m + 1)*e^(3*e - 3*c*f/d)*exp_integral_e(-m, -3*(d*x + c)*f/d)/d)*b^3 + (
d*x + c)^(m + 1)*a^3/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx=\int (b\cosh{(fx+e)}+a)^3(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((b*cosh(f*x + e) + a)^3*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx=\int {\left(a+b\,\mathrm{cosh}\left(e+f\,x\right)\right)}^3\,{\left(c+d\,x\right)}^m\,dx$$

input
```
int((a + b*cosh(e + f*x))^3*(c + d*x)^m,x)
```

output
```
int((a + b*cosh(e + f*x))^3*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+b\cosh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^3,x)
```

output
```
(e**(6*e + 6*f*x)*(c + d*x)**m*b**3*d*m + e**(6*e + 6*f*x)*(c + d*x)**m*b*
*3*d + 9*e**(5*e + 5*f*x)*(c + d*x)**m*a*b**2*d*m + 9*e**(5*e + 5*f*x)*(c
+ d*x)**m*a*b**2*d + 36*e**(4*e + 4*f*x)*(c + d*x)**m*a**2*b*d*m + 36*e**(
4*e + 4*f*x)*(c + d*x)**m*a**2*b*d + 9*e**(4*e + 4*f*x)*(c + d*x)**m*b**3*
d*m + 9*e**(4*e + 4*f*x)*(c + d*x)**m*b**3*d + 24*e**(3*e + 3*f*x)*(c + d*
x)**m*a**3*c*f + 24*e**(3*e + 3*f*x)*(c + d*x)**m*a**3*d*f*x + 36*e**(3*e
+ 3*f*x)*(c + d*x)**m*a*b**2*c*f + 36*e**(3*e + 3*f*x)*(c + d*x)**m*a*b**2
*d*f*x - 36*e**(2*e + 2*f*x)*(c + d*x)**m*a**2*b*d*m - 36*e**(2*e + 2*f*x)
*(c + d*x)**m*a**2*b*d - 9*e**(2*e + 2*f*x)*(c + d*x)**m*b**3*d*m - 9*e**(
2*e + 2*f*x)*(c + d*x)**m*b**3*d - 9*e**(e + f*x)*(c + d*x)**m*a*b**2*d*m
- 9*e**(e + f*x)*(c + d*x)**m*a*b**2*d - (c + d*x)**m*b**3*d*m - (c + d*x)
**m*b**3*d - e**(6*e + 3*f*x)*int((e**(3*f*x)*(c + d*x)**m)/(c + d*x),x)*b
**3*d**2*m**2 - e**(6*e + 3*f*x)*int((e**(3*f*x)*(c + d*x)**m)/(c + d*x),x
)*b**3*d**2*m - 9*e**(5*e + 3*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x)
,x)*a*b**2*d**2*m**2 - 9*e**(5*e + 3*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c
 + d*x),x)*a*b**2*d**2*m - 36*e**(4*e + 3*f*x)*int((e**(f*x)*(c + d*x)**m)
/(c + d*x),x)*a**2*b*d**2*m**2 - 36*e**(4*e + 3*f*x)*int((e**(f*x)*(c + d*
x)**m)/(c + d*x),x)*a**2*b*d**2*m - 9*e**(4*e + 3*f*x)*int((e**(f*x)*(c +
d*x)**m)/(c + d*x),x)*b**3*d**2*m**2 - 9*e**(4*e + 3*f*x)*int((e**(f*x)*(c
 + d*x)**m)/(c + d*x),x)*b**3*d**2*m + e**(3*e + 3*f*x)*int((c + d*x)**...
```

3.180 $\int (c+dx)^m (a+b\cosh(e+fx))^2 \, dx$

Optimal result

Integrand size = 20, antiderivative size = 282

$$\begin{aligned}\int (c+dx)^m (a+b\cosh(e+fx))^2 \, dx &= \frac{a^2(c+dx)^{1+m}}{d(1+m)} + \frac{b^2(c+dx)^{1+m}}{2d(1+m)} \\ &+ \frac{2^{-3-m}b^2e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)}{f} \\ &+ \frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f} \\ &- \frac{abe^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f} \\ &- \frac{2^{-3-m}b^2e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{f}\end{aligned}$$

output
```
a^2*(d*x+c)^(1+m)/d/(1+m)+1/2*b^2*(d*x+c)^(1+m)/d/(1+m)+2^(-3-m)*b^2*exp(2
*e-2*c*f/d)*(d*x+c)^m*GAMMA(1+m,-2*f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)+a*b*e
xp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/d)/f/((-f*(d*x+c)/d)^m)-a*b*exp
(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m)-2^(-3-m)*b
^2*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m,2*f*(d*x+c)/d)/f/((f*(d*x+c)/d)^m
)
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.90

$$\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx$$
$$=\frac{(c+dx)^m\left(8a^2f(c+dx)+4b^2f(c+dx)+2^{-m}b^2de^{2e-\frac{2cf}{d}}(1+m)\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{2f(c+dx)}{d}\right)\right.}{}$$

input
```
Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x])^2,x]
```

output
```
((c + d*x)^m*(8*a^2*f*(c + d*x) + 4*b^2*f*(c + d*x) + (b^2*d*E^(2*e - (2*c
*f)/d)*(1 + m)*Gamma[1 + m, (-2*f*(c + d*x))/d])/(2^m*(-((f*(c + d*x))/d))
^m) + (8*a*b*d*E^(e - (c*f)/d)*(1 + m)*Gamma[1 + m, -((f*(c + d*x))/d)])/(
-((f*(c + d*x))/d))^m - (8*a*b*d*E^(-e + (c*f)/d)*(1 + m)*Gamma[1 + m, (f*
(c + d*x))/d])/((f*(c + d*x))/d)^m - (b^2*d*E^(-2*e + (2*c*f)/d)*(1 + m)*G
amma[1 + m, (2*f*(c + d*x))/d])/(2^m*((f*(c + d*x))/d)^m)))/(8*d*f*(1 + m)
)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a+b\cosh(e+fx))^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int (c+dx)^m \left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2 dx$$

$$\Big\downarrow \text{ 3798}$$

$$\int \left(a^2(c+dx)^m+2ab(c+dx)^m\cosh(e+fx)+b^2(c+dx)^m\cosh^2(e+fx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(c+dx)^{m+1}}{d(m+1)}+\frac{abe^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{f}-\frac{abe^{\frac{cf}{d}-e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{f}+\frac{b^2 2^{-m-3}e^{2e-\frac{2cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,-\frac{2f(c+dx)}{d}\right)}{f}-\frac{b^2 2^{-m-3}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{f}+\frac{b^2(c+dx)^{m+1}}{2d(m+1)}$$

input
```
Int[(c + d*x)^m*(a + b*Cosh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^(1 + m))/(d*(1 + m)) + (b^2*(c + d*x)^(1 + m))/(2*d*(1 + m)
) + (2^(-3 - m)*b^2*E^(2*e - (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (-2*f*(c
+ d*x))/d])/(f*(-((f*(c + d*x))/d))^m) + (a*b*E^(e - (c*f)/d)*(c + d*x)^m*
Gamma[1 + m, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) - (a*b*E^(-e
+ (c*f)/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(f*((f*(c + d*x))/d)
^m) - (2^(-3 - m)*b^2*E^(-2*e + (2*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (2*f*(
c + d*x))/d])/(f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx+c)^m (a+b\cosh (fx+e))^2\,dx$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 509, normalized size of antiderivative = 1.80

$$\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx = -\frac{(b^2dm+b^2d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)+8(abdm+abd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right.}{}$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/8*((b^2*d*m + b^2*d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m +
 1, 2*(d*f*x + c*f)/d) + 8*(a*b*d*m + a*b*d)*cosh((d*m*log(f/d) + d*e - c*
f)/d)*gamma(m + 1, (d*f*x + c*f)/d) - 8*(a*b*d*m + a*b*d)*cosh((d*m*log(-f
/d) - d*e + c*f)/d)*gamma(m + 1, -(d*f*x + c*f)/d) - (b^2*d*m + b^2*d)*cos
h((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d)*gamma(m + 1, -2*(d*f*x + c*f)/d) -
(b^2*d*m + b^2*d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2
*d*e - 2*c*f)/d) - 8*(a*b*d*m + a*b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh(
(d*m*log(f/d) + d*e - c*f)/d) + 8*(a*b*d*m + a*b*d)*gamma(m + 1, -(d*f*x +
 c*f)/d)*sinh((d*m*log(-f/d) - d*e + c*f)/d) + (b^2*d*m + b^2*d)*gamma(m +
 1, -2*(d*f*x + c*f)/d)*sinh((d*m*log(-2*f/d) - 2*d*e + 2*c*f)/d) - 4*((2*
a^2 + b^2)*d*f*x + (2*a^2 + b^2)*c*f)*cosh(m*log(d*x + c)) - 4*((2*a^2 + b
^2)*d*f*x + (2*a^2 + b^2)*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m (a+b\cosh(e+fx))^2\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*cosh(f*x+e))**2,x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.74

$$
\begin{aligned}
&\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx \\
&= -\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)ab \\
&\quad -\frac{1}{4}\left(\frac{(dx+c)^{m+1}e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_{-m}\left(\frac{2\,(dx+c)f}{d}\right)}{d} + \frac{(dx+c)^{m+1}e^{\left(2\,e-\frac{2\,cf}{d}\right)}E_{-m}\left(-\frac{2\,(dx+c)f}{d}\right)}{d} - \frac{2\,(dx+c)^{m+1}}{d(m+1)}\right)b^2 \\
&\quad + \frac{(dx+c)^{m+1}a^2}{d(m+1)}
\end{aligned}
$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d + (
d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*a*b -
 1/4*((d*x + c)^(m + 1)*e^(-2*e + 2*c*f/d)*exp_integral_e(-m, 2*(d*x + c)*
f/d)/d + (d*x + c)^(m + 1)*e^(2*e - 2*c*f/d)*exp_integral_e(-m, -2*(d*x +
c)*f/d)/d - 2*(d*x + c)^(m + 1)/(d*(m + 1)))*b^2 + (d*x + c)^(m + 1)*a^2/(
d*(m + 1))
```

Giac [F]

$$
\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx = \int (b\cosh(fx+e)+a)^2(dx+c)^m\,dx
$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((b*cosh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx=\int \left(a+b\cosh(e+f\,x)\right)^2\,{\left(c+d\,x\right)}^m\,dx$$

input
```
int((a + b*cosh(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + b*cosh(e + f*x))^2*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+b\cosh(e+fx))^2\,dx$$
$$=\frac{e^{4fx+4e}(dx+c)^m\,b^2dm+e^{4fx+4e}(dx+c)^m\,b^2d+8e^{3fx+3e}(dx+c)^m\,abdm+8e^{3fx+3e}(dx+c)^m\,abd+8e^{2}}{}$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e))^2,x)
```

output
```
(e**(4*e + 4*f*x)*(c + d*x)**m*b**2*d*m + e**(4*e + 4*f*x)*(c + d*x)**m*b*
*2*d + 8*e**(3*e + 3*f*x)*(c + d*x)**m*a*b*d*m + 8*e**(3*e + 3*f*x)*(c + d
*x)**m*a*b*d + 8*e**(2*e + 2*f*x)*(c + d*x)**m*a**2*c*f + 8*e**(2*e + 2*f*
x)*(c + d*x)**m*a**2*d*f*x + 4*e**(2*e + 2*f*x)*(c + d*x)**m*b**2*c*f + 4*
e**(2*e + 2*f*x)*(c + d*x)**m*b**2*d*f*x - 8*e**(e + f*x)*(c + d*x)**m*a*b
*d*m - 8*e**(e + f*x)*(c + d*x)**m*a*b*d - (c + d*x)**m*b**2*d*m - (c + d*
x)**m*b**2*d - e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x),x)
*b**2*d**2*m**2 - e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)**m)/(c + d*x)
,x)*b**2*d**2*m - 8*e**(3*e + 2*f*x)*int((e**(f*x)*(c + d*x)**m)/(c + d*x)
,x)*a*b*d**2*m**2 - 8*e**(3*e + 2*f*x)*int((e**(f*x)*(c + d*x)**m)/(c + d*
x),x)*a*b*d**2*m + e**(2*e + 2*f*x)*int((c + d*x)**m/(e**(2*e + 2*f*x)*c +
 e**(2*e + 2*f*x)*d*x),x)*b**2*d**2*m**2 + e**(2*e + 2*f*x)*int((c + d*x)*
*m/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*b**2*d**2*m + 8*e**(e +
2*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*a*b*d**2*m**2 + 8*e
**(e + 2*f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x)*d*x),x)*a*b*d**2*m)/
(8*e**(2*e + 2*f*x)*d*f*(m + 1))
```

3.181 $\int (c+dx)^m(a+b\cosh(e+fx))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 131

$$\begin{aligned}\int (c+dx)^m(a+b\cosh(e+fx))\,dx &= \frac{a(c+dx)^{1+m}}{d(1+m)}+\frac{be^{e-\frac{cf}{d}}(c+dx)^m\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{2f}\\&\quad-\frac{be^{-e+\frac{cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{2f}\end{aligned}$$

output
```
a*(d*x+c)^(1+m)/d/(1+m)+1/2*b*exp(e-c*f/d)*(d*x+c)^m*GAMMA(1+m,-f*(d*x+c)/
d)/f/((-f*(d*x+c)/d)^m)-1/2*b*exp(-e+c*f/d)*(d*x+c)^m*GAMMA(1+m,f*(d*x+c)/
d)/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.91

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx = \frac{1}{2}(c+dx)^m\left(\frac{2a(c+dx)}{d(1+m)} + \frac{be^{e-\frac{cf}{d}}\left(-\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,-\frac{f(c+dx)}{d}\right)}{f} - \frac{be^{-e+\frac{cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{f(c+dx)}{d}\right)}{f}\right)$$

input
```
Integrate[(c + d*x)^m*(a + b*Cosh[e + f*x]),x]
```

output
```
((c + d*x)^m*((2*a*(c + d*x))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*Gamma[1 + m
, -((f*(c + d*x))/d)])/(f*(-((f*(c + d*x))/d))^m) - (b*E^(-e + (c*f)/d)*Ga
mma[1 + m, (f*(c + d*x))/d])/(f*(f*(c/d + x))^m)))/2
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3798, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (c+dx)^m\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)dx$$

$$\Big\downarrow \text{3798}$$

$$\int (a(c+dx)^m + b(c+dx)^m \cosh(e+fx))\,dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a(c+dx)^{m+1}}{d(m+1)} + \frac{be^{e-\frac{cf}{d}}(c+dx)^m \left(-\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,-\frac{f(c+dx)}{d}\right)}{2f} - \frac{be^{\frac{cf}{d}-e}(c+dx)^m \left(\frac{f(c+dx)}{d}\right)^{-m} \Gamma\left(m+1,\frac{f(c+dx)}{d}\right)}{2f}$$

input
```
Int[(c + d*x)^m*(a + b*Cosh[e + f*x]),x]
```

output
```
(a*(c + d*x)^(1 + m))/(d*(1 + m)) + (b*E^(e - (c*f)/d)*(c + d*x)^m*Gamma[1
 + m, -((f*(c + d*x))/d)])/(2*f*(-((f*(c + d*x))/d))^m) - (b*E^(-e + (c*f)
/d)*(c + d*x)^m*Gamma[1 + m, (f*(c + d*x))/d])/(2*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3798
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Sin[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[n, 0] && (EqQ[n, 1] || IGtQ[
m, 0] || NeQ[a^2 - b^2, 0])
```

Maple [F]

$$\int (dx + c)^m (a + b\cosh(fx + e))\, dx$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e)),x)
```

output
```
int((d*x+c)^m*(a+b*cosh(f*x+e)),x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.90

$$\int (c + dx)^m (a + b\cosh(e + fx))\, dx =$$

$$-\frac{(bdm + bd)\cosh\left(\frac{dm\log\left(\frac{f}{d}\right)+de-cf}{d}\right)\Gamma\left(m+1, \frac{dfx+cf}{d}\right) - (bdm + bd)\cosh\left(\frac{dm\log\left(-\frac{f}{d}\right)-de+cf}{d}\right)\Gamma\left(m+1,\right.}{}$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
-1/2*((b*d*m + b*d)*cosh((d*m*log(f/d) + d*e - c*f)/d)*gamma(m + 1, (d*f*x
 + c*f)/d) - (b*d*m + b*d)*cosh((d*m*log(-f/d) - d*e + c*f)/d)*gamma(m + 1
, -(d*f*x + c*f)/d) - (b*d*m + b*d)*gamma(m + 1, (d*f*x + c*f)/d)*sinh((d*
m*log(f/d) + d*e - c*f)/d) + (b*d*m + b*d)*gamma(m + 1, -(d*f*x + c*f)/d)*
sinh((d*m*log(-f/d) - d*e + c*f)/d) - 2*(a*d*f*x + a*c*f)*cosh(m*log(d*x +
 c)) - 2*(a*d*f*x + a*c*f)*sinh(m*log(d*x + c)))/(d*f*m + d*f)
```

Sympy [F(-2)]

Exception generated.

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)**m*(a+b*cosh(f*x+e)),x)
```

output
```
Exception raised: TypeError >> cannot determine truth value of Relational
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx = -\frac{1}{2}\left(\frac{(dx+c)^{m+1}e^{\left(-e+\frac{cf}{d}\right)}E_{-m}\left(\frac{(dx+c)f}{d}\right)}{d}+\frac{(dx+c)^{m+1}e^{\left(e-\frac{cf}{d}\right)}E_{-m}\left(-\frac{(dx+c)f}{d}\right)}{d}\right)b + \frac{(dx+c)^{m+1}a}{d(m+1)}$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2*((d*x + c)^(m + 1)*e^(-e + c*f/d)*exp_integral_e(-m, (d*x + c)*f/d)/d
 + (d*x + c)^(m + 1)*e^(e - c*f/d)*exp_integral_e(-m, -(d*x + c)*f/d)/d)*b
 + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [F]

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx=\int (b\cosh\left(fx+e\right)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((b*cosh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx=\int (a+b\cosh(e+f\,x))\,\left(c+d\,x\right)^m\,dx$$

input
```
int((a + b*cosh(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + b*cosh(e + f*x))*(c + d*x)^m, x)
```

Reduce [F]

$$\int (c+dx)^m(a+b\cosh(e+fx))\,dx$$
$$=\frac{e^{2fx+2e}(dx+c)^m\,bdm+e^{2fx+2e}(dx+c)^m\,bd+2e^{fx+e}(dx+c)^m\,acf+2e^{fx+e}(dx+c)^m\,adfx-(dx+c)^{}}{}$$

input
```
int((d*x+c)^m*(a+b*cosh(f*x+e)),x)
```

output
```
(e**(2*e + 2*f*x)*(c + d*x)**m*b*d*m + e**(2*e + 2*f*x)*(c + d*x)**m*b*d +
 2*e**(e + f*x)*(c + d*x)**m*a*c*f + 2*e**(e + f*x)*(c + d*x)**m*a*d*f*x -
 (c + d*x)**m*b*d*m - (c + d*x)**m*b*d - e**(2*e + f*x)*int((e**(f*x)*(c +
 d*x)**m)/(c + d*x),x)*b*d**2*m**2 - e**(2*e + f*x)*int((e**(f*x)*(c + d*x
)**m)/(c + d*x),x)*b*d**2*m + e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(
f*x)*d*x),x)*b*d**2*m**2 + e**(f*x)*int((c + d*x)**m/(e**(f*x)*c + e**(f*x
)*d*x),x)*b*d**2*m)/(2*e**(e + f*x)*d*f*(m + 1))
```

3.182 $\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \text{Int}\left(\frac{(c+dx)^m}{a+b\cosh(e+fx)},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 1.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}dx$$

input
```
Int[(c + d*x)^m/(a + b*Cosh[e + f*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{a+b\cosh(fx+e)}dx$$

input
```
int((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

output
```
int((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(dx+c)^m}{b\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.59 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*cosh(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(a + b*cosh(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(dx+c)^m}{b\cosh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(dx+c)^m}{b\cosh\left(fx+e\right)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*cosh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*cosh(f*x + e) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = \int \frac{(c+d\,x)^m}{a+b\cosh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^m/(a + b*cosh(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + b*cosh(e + f*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \frac{(c+dx)^m}{a+b\cosh(e+fx)}\,dx = 2e^e\left(\int \frac{e^{fx}(dx+c)^m}{e^{2fx+2e}b+2e^{fx+e}a+b}dx\right)$$

input
```
int((d*x+c)^m/(a+b*cosh(f*x+e)),x)
```

output
```
2*e**e*int((e**(f*x)*(c + d*x)**m)/(e**(2*e + 2*f*x)*b + 2*e**(e + f*x)*a
+ b),x)
```

3.183 $\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \text{Int}\left(\frac{(c+dx)^m}{(a+b\cosh(e+fx))^2},x\right)$$

output
```
Defer(Int)((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 4.83 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$$

input
```
Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m/(a + b*Cosh[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{(c+dx)^m}{\left(a+b\sin\left(ie+ifx+\frac{\pi}{2}\right)\right)^2}dx$$
$$\Big\downarrow \text{3807}$$
$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}dx$$

input
```
Int[(c + d*x)^m/(a + b*Cosh[e + f*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(dx+c)^m}{(a+b\cosh(fx+e))^2}dx$$

input
```
int((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

output
```
int((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.90

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((d*x + c)^m/(b^2*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e) + a^2), x)
```

Sympy [N/A]

Not integrable

Time = 13.24 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx$$

input
```
integrate((d*x+c)**m/(a+b*cosh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(a + b*cosh(e + f*x))**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(b*cosh(f*x + e) + a)^2, x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(b\cosh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+b*cosh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(b*cosh(f*x + e) + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.00 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+b\cosh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^m/(a + b*cosh(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + b*cosh(e + f*x))^2, x)
```

Reduce [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 62433, normalized size of antiderivative = 3121.65

$$\int \frac{(c+dx)^m}{(a+b\cosh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m/(a+b*cosh(f*x+e))^2,x)
```

output
```
(4*( - 4*e**(e + f*x)*(c + d*x)**m*a*c**2*f - (c + d*x)**m*b*c**2*f - (c +
 d*x)**m*b*c*d*m - 4*e**(5*e + 2*f*x)*int((e**(3*f*x)*(c + d*x)**m*x)/(2*e
**(4*e + 4*f*x)*b**2*c**3*f**2 + 2*e**(4*e + 4*f*x)*b**2*c**2*d*f**2*x + e
**(4*e + 4*f*x)*b**2*c**2*d*f*m + e**(4*e + 4*f*x)*b**2*c*d**2*f*m*x - e**
(4*e + 4*f*x)*b**2*c*d**2*m**2 - e**(4*e + 4*f*x)*b**2*d**3*m**2*x + 8*e**
(3*e + 3*f*x)*a*b*c**3*f**2 + 8*e**(3*e + 3*f*x)*a*b*c**2*d*f**2*x + 4*e**
(3*e + 3*f*x)*a*b*c**2*d*f*m + 4*e**(3*e + 3*f*x)*a*b*c*d**2*f*m*x - 4*e**
(3*e + 3*f*x)*a*b*c*d**2*m**2 - 4*e**(3*e + 3*f*x)*a*b*d**3*m**2*x + 8*e**
(2*e + 2*f*x)*a**2*c**3*f**2 + 8*e**(2*e + 2*f*x)*a**2*c**2*d*f**2*x + 4*e
**(2*e + 2*f*x)*a**2*c**2*d*f*m + 4*e**(2*e + 2*f*x)*a**2*c*d**2*f*m*x - 4
*e**(2*e + 2*f*x)*a**2*c*d**2*m**2 - 4*e**(2*e + 2*f*x)*a**2*d**3*m**2*x +
 4*e**(2*e + 2*f*x)*b**2*c**3*f**2 + 4*e**(2*e + 2*f*x)*b**2*c**2*d*f**2*x
 + 2*e**(2*e + 2*f*x)*b**2*c**2*d*f*m + 2*e**(2*e + 2*f*x)*b**2*c*d**2*f*m
*x - 2*e**(2*e + 2*f*x)*b**2*c*d**2*m**2 - 2*e**(2*e + 2*f*x)*b**2*d**3*m*
*2*x + 8*e**(e + f*x)*a*b*c**3*f**2 + 8*e**(e + f*x)*a*b*c**2*d*f**2*x + 4
*e**(e + f*x)*a*b*c**2*d*f*m + 4*e**(e + f*x)*a*b*c*d**2*f*m*x - 4*e**(e +
 f*x)*a*b*c*d**2*m**2 - 4*e**(e + f*x)*a*b*d**3*m**2*x + 2*b**2*c**3*f**2
+ 2*b**2*c**2*d*f**2*x + b**2*c**2*d*f*m + b**2*c*d**2*f*m*x - b**2*c*d**2
*m**2 - b**2*d**3*m**2*x),x)*a*b**2*c**4*d*f**4 + 2*e**(5*e + 2*f*x)*int((
e**(3*f*x)*(c + d*x)**m*x)/(2*e**(4*e + 4*f*x)*b**2*c**3*f**2 + 2*e**(4...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
         fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
         gamma,loggamma,digamma,zeta,polylog,LambertW,
         elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):  #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])  #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#          Albert Rich to use with Sagemath. This is used to
#          grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#          'arctan2','floor','abs','log_integral'
#June 4, 2022 Made default grade_annotation "none" instead of "" due
#          issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral',
          'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
          'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file