Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/300-6.2.2

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**111**]. This is test number [300].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (111)	0.00 (0)
Mathematica	100.00 (111)	0.00 (0)
Maple	100.00 (111)	0.00 (0)
Fricas	100.00 (111)	0.00 (0)
Giac	63.96 (71)	36.04 (40)
Maxima	57.66 (64)	42.34 (47)
Reduce	36.94 (41)	63.06 (70)
Sympy	23.42 (26)	76.58 (85)
Mupad	18.02 (20)	81.98 (91)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

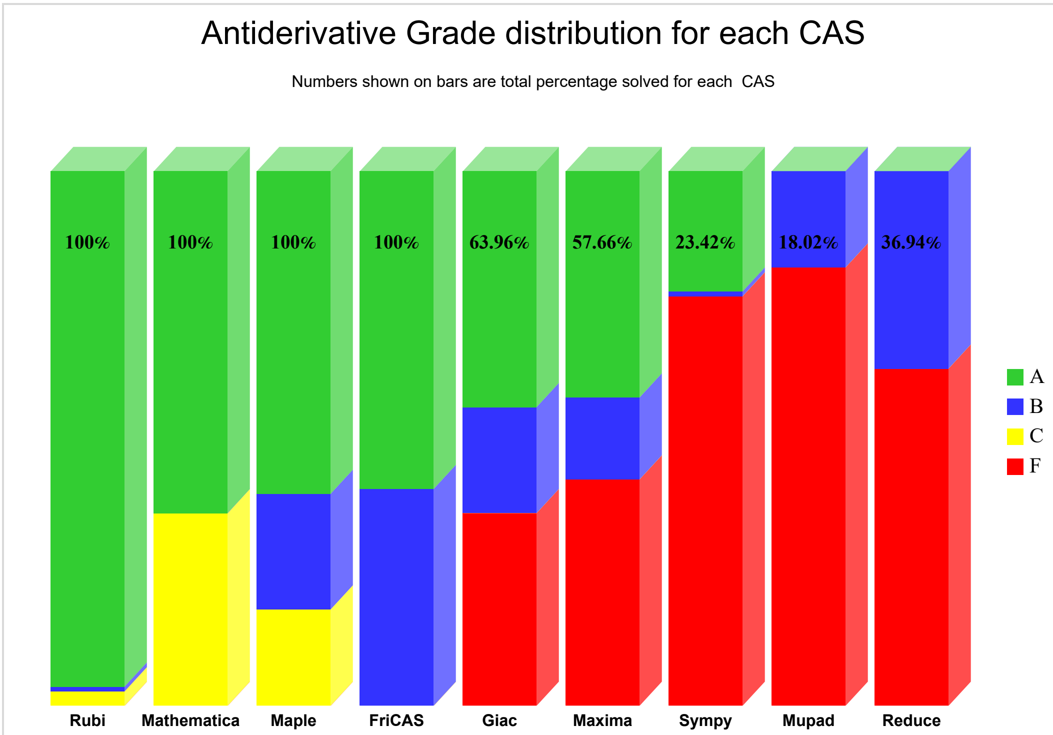
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

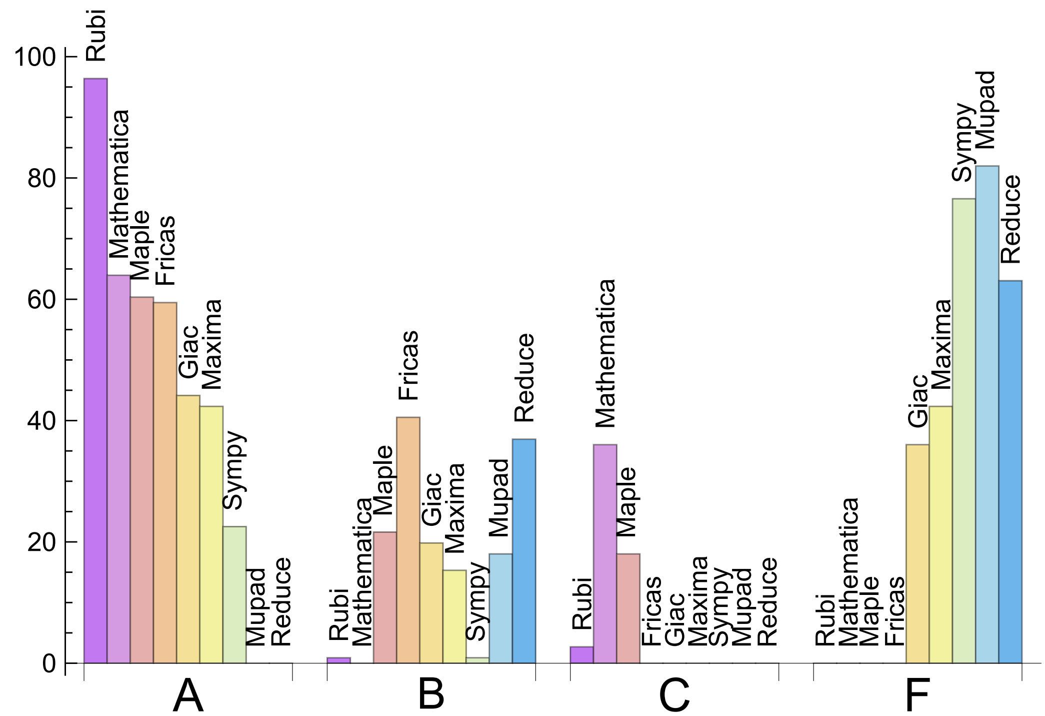
System	% A grade	% B grade	% C grade	% F grade
Rubi	96.396	0.901	2.703	0.000
Mathematica	63.964	0.000	36.036	0.000
Maple	60.360	21.622	18.018	0.000
Fricas	59.459	40.541	0.000	0.000
Giac	44.144	19.820	0.000	36.036
Maxima	42.342	15.315	0.000	42.342
Sympy	22.523	0.901	0.000	76.577
Mupad	0.000	18.018	0.000	81.982
Reduce	0.000	36.937	0.000	63.063

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Maple	0	0.00	0.00	0.00
Giac	40	92.50	0.00	7.50
Maxima	47	87.23	12.77	0.00
Reduce	70	100.00	0.00	0.00
Sympy	85	67.06	32.94	0.00
Mupad	91	0.00	100.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.10
Fricas	0.11
Giac	0.12
Reduce	0.15
Mathematica	0.60
Maple	0.70
Sympy	0.80
Rubi	0.95
Mupad	1.44

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Mupad	118.50	0.99	115.00	0.97
Sympy	139.15	1.28	127.50	1.23
Reduce	167.66	1.41	153.00	1.54
Maxima	183.64	1.67	170.00	1.72
Mathematica	192.48	0.81	140.00	0.78
Rubi	291.83	1.03	178.00	1.00
Giac	398.31	2.64	199.00	1.66
Maple	513.52	1.71	259.00	1.60
Fricas	652.41	1.82	200.00	1.71

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

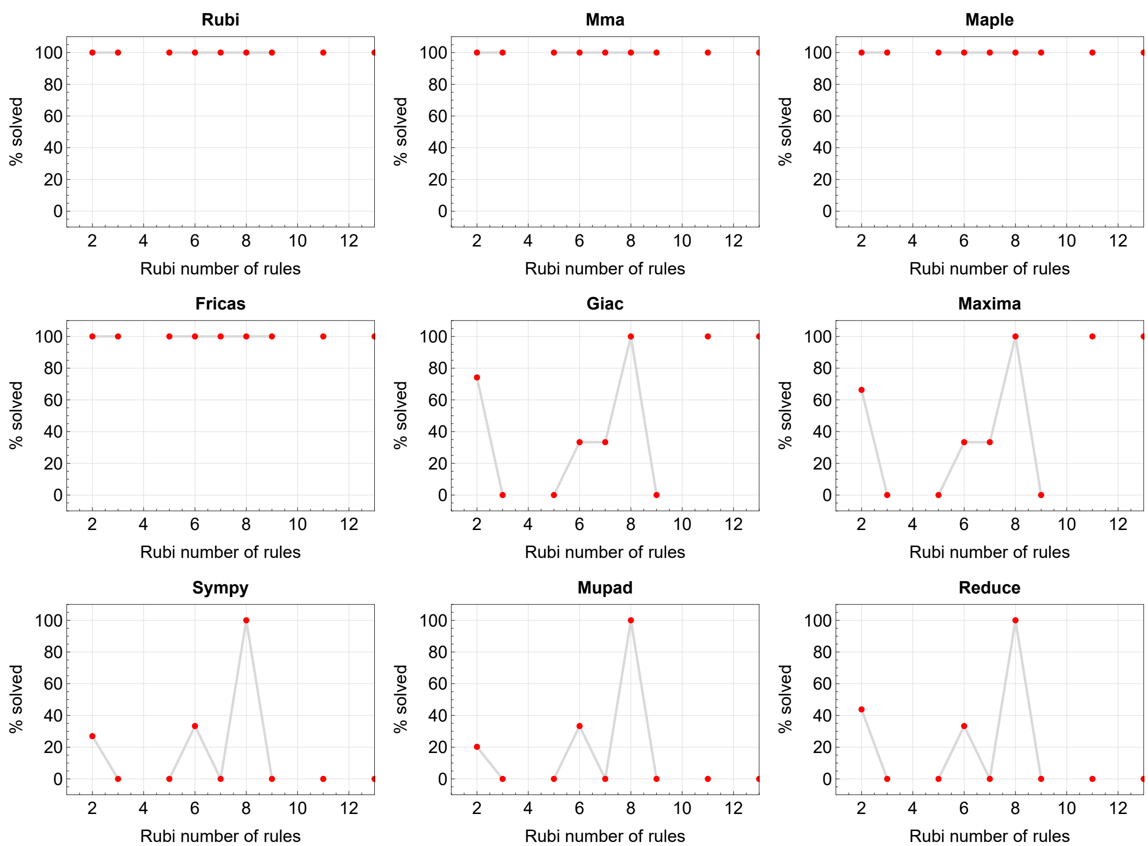


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

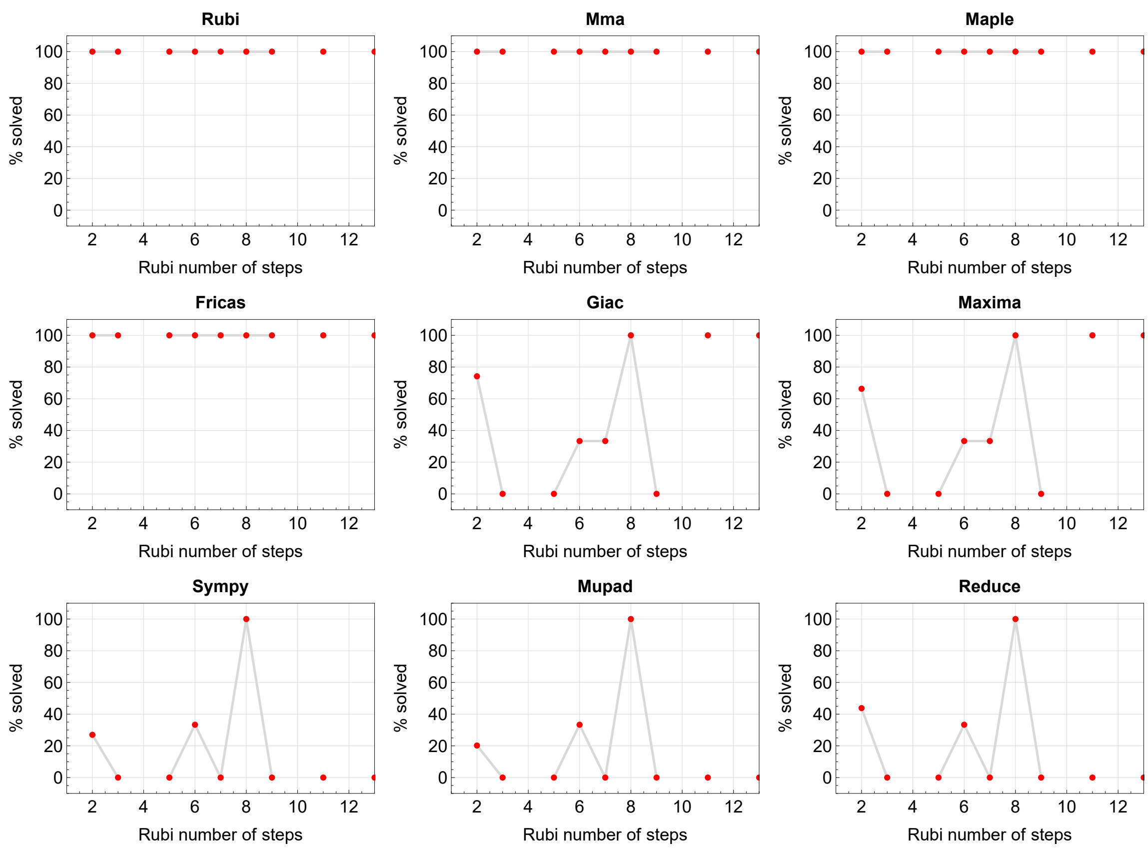


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

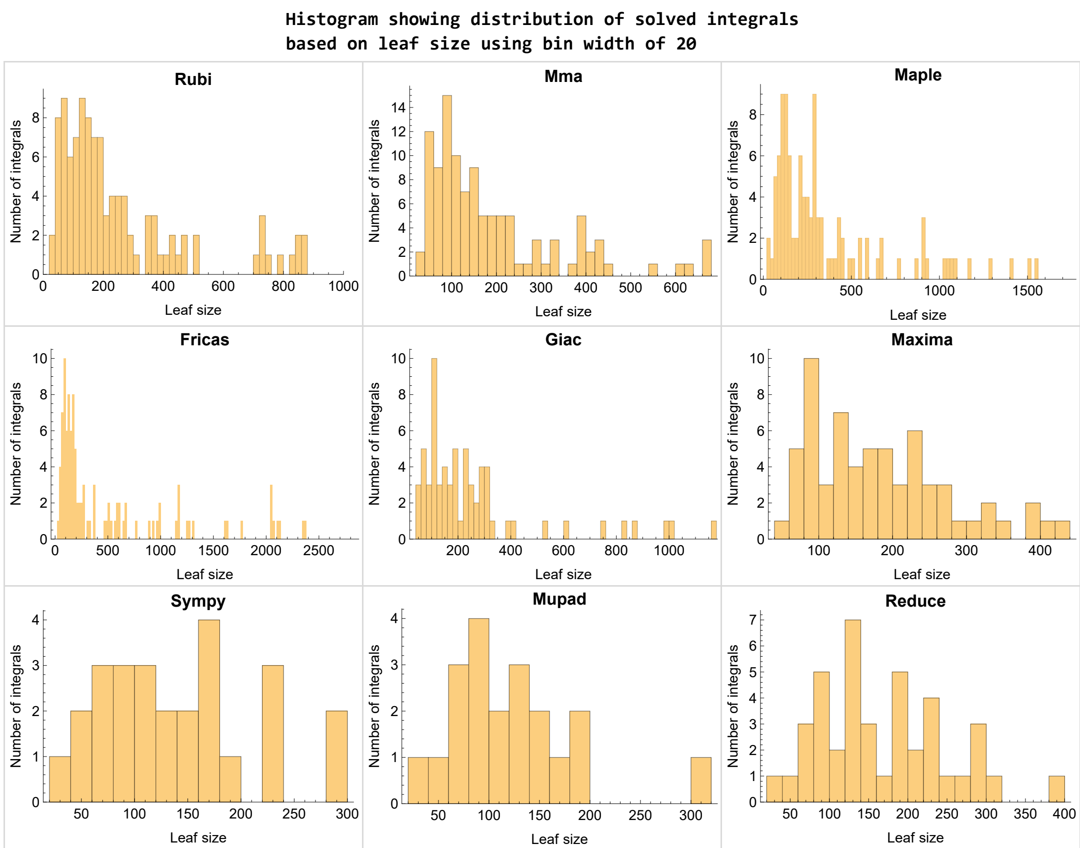


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

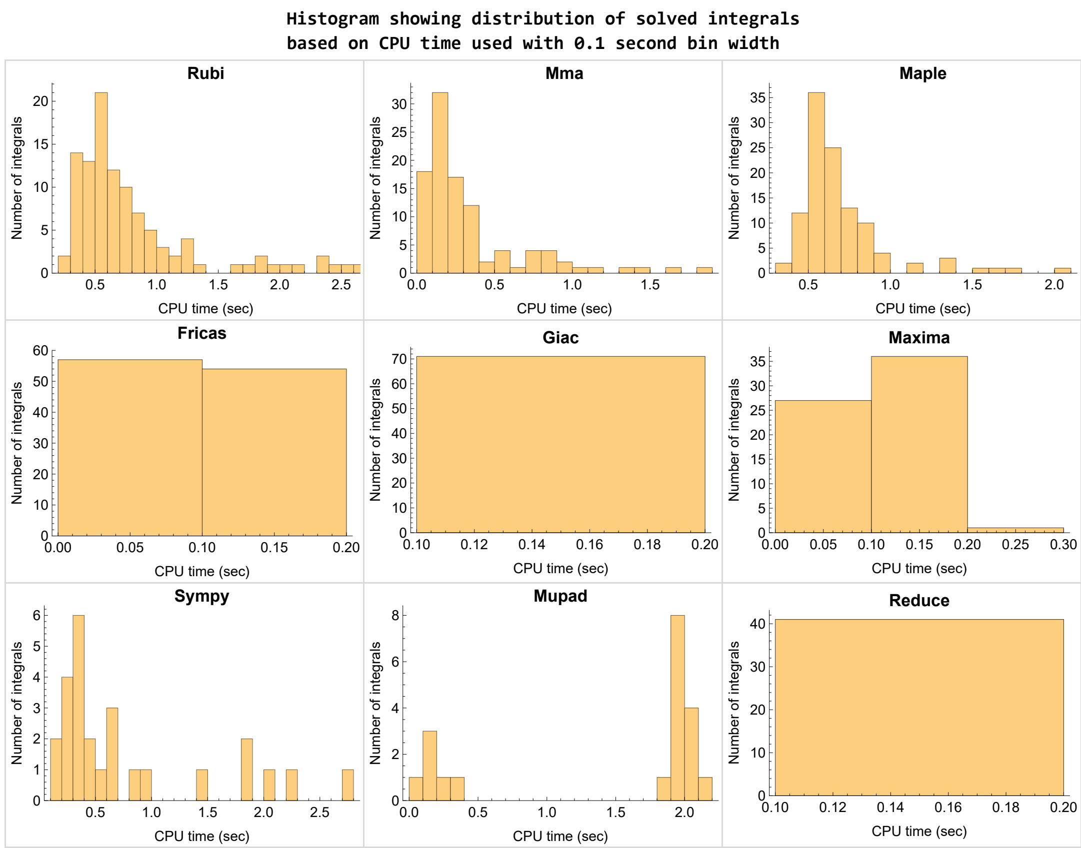


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

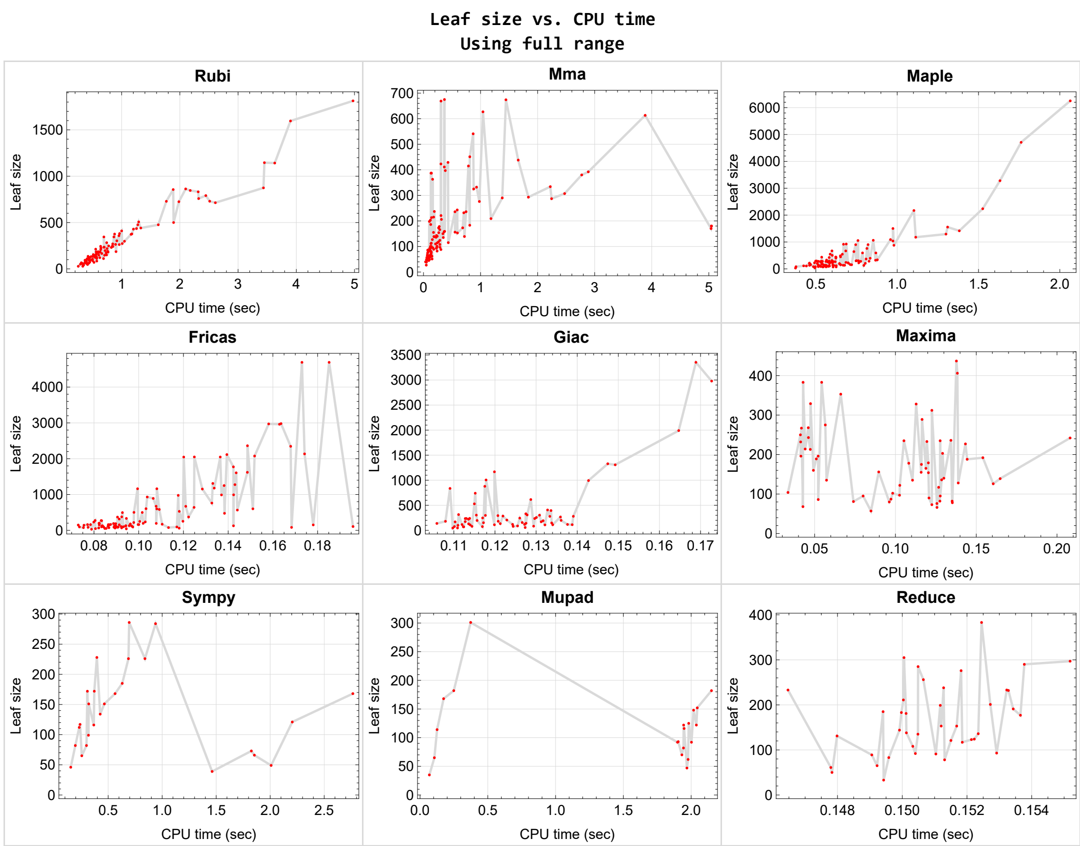


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {}

Mathematica {}

Maple {94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

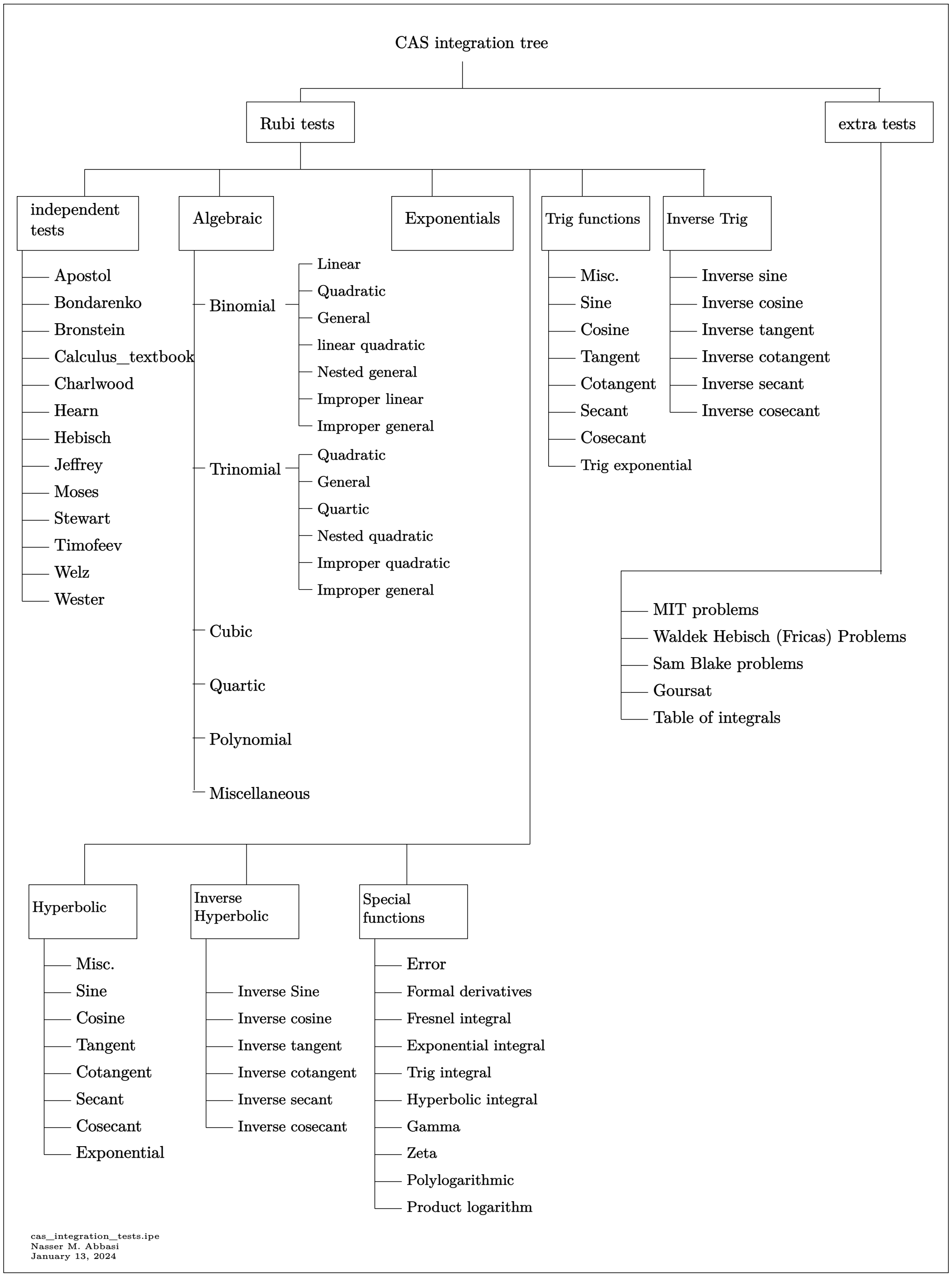
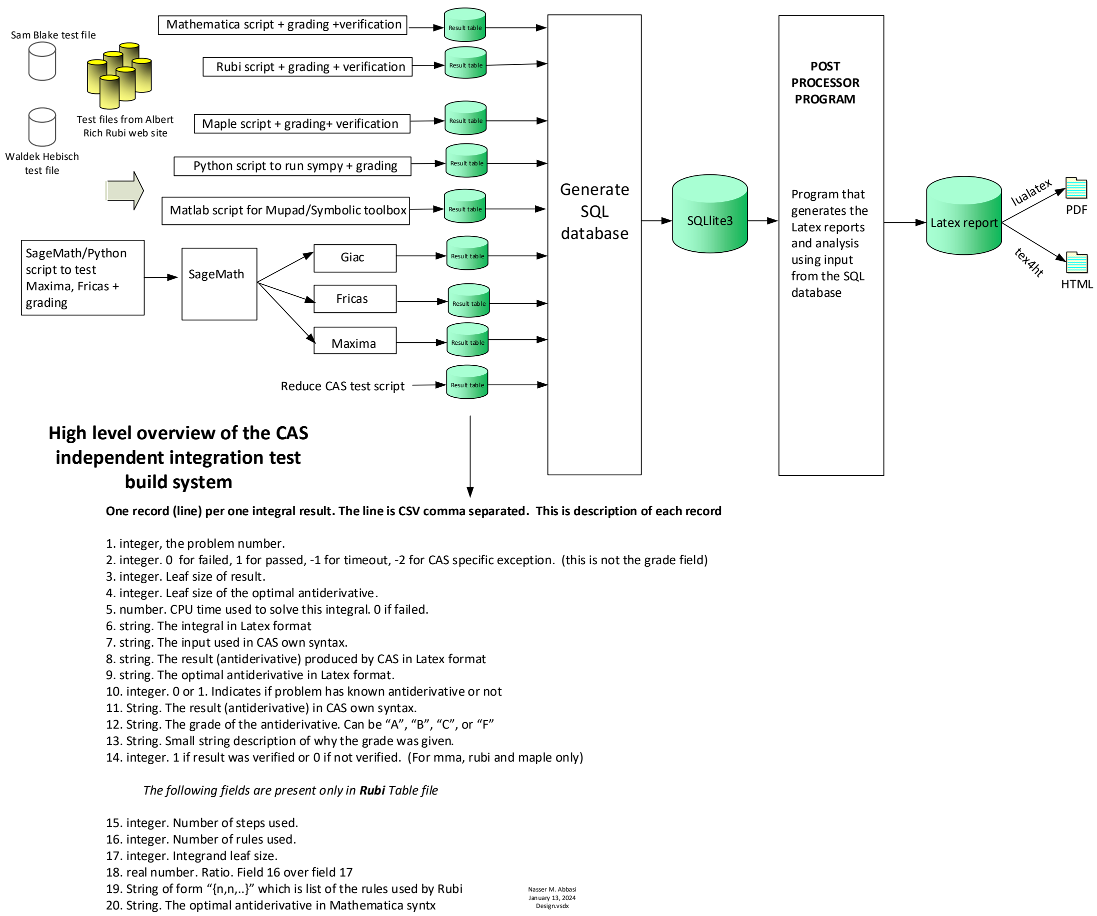


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 110, 111 }

B grade { 109 }

C grade { 12, 30, 36 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { }

C grade { 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 30, 31, 32, 37, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 69, 70, 71, 75, 76, 77, 79, 80, 81, 82, 83, 85, 86, 87, 88 }

B grade { 26, 27, 28, 29, 33, 34, 35, 36, 38, 39, 52, 53, 65, 66, 67, 68, 72, 73, 74, 78, 84, 91, 92, 93 }

C grade { 89, 90, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 34, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 60, 62, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { 30, 33, 35, 36, 37, 38, 39, 57, 58, 59, 61, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

C grade { }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 1, 6, 7, 8, 9, 10, 14, 15, 16, 17, 18, 22, 24, 25, 26, 27, 28, 29, 30, 31, 36, 40, 41, 43, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 79, 82, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93 }

B grade { 2, 3, 4, 5, 11, 12, 13, 19, 20, 21, 23, 42, 44, 52, 80, 81, 83 }

C grade { }

F normal fail { 32, 33, 34, 35, 37, 38, 39, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 96, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F(-1) timedout fail { 94, 95, 97, 98, 100, 101 }

F(-2) exception fail { }

Giac

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 54, 55, 56, 79, 80, 81, 82, 83, 85, 86, 87, 88, 92 }

B grade { 12, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 52, 53, 84, 89, 90, 91, 93 }

C grade { }

F normal fail { 57, 58, 59, 60, 61, 62, 63, 64, 66, 67, 68, 69, 71, 72, 73, 74, 75, 76, 77, 78, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 108, 109, 110, 111 }

F(-1) timedout fail { }

F(-2) exception fail { 65, 70, 106 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 10, 11, 12, 40, 41, 42, 43, 49, 50, 51, 79, 80, 81, 82, 87, 88 }

C grade { }

F normal fail { }

F(-1) timedout fail { 5, 6, 7, 8, 9, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 45, 46, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 84, 85, 86, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F(-2) exception fail { }

Sympy

A grade { 1, 2, 3, 4, 5, 10, 11, 13, 40, 41, 42, 43, 44, 49, 50, 51, 52, 79, 80, 81, 82, 83, 87, 88, 89 }

B grade { 12 }

C grade { }

F normal fail { 6, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 33, 34, 37, 45, 46, 47, 48, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 84, 85, 86, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100 }

F(-1) timedout fail { 7, 8, 9, 30, 32, 35, 36, 38, 39, 71, 72, 73, 74, 75, 76, 77, 78, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 6, 7, 8, 9, 10, 11, 12, 14, 15, 16, 17, 40, 41, 42, 43, 45, 46, 47, 48, 49, 50, 51, 53, 54, 55, 56, 79, 80, 81, 82, 84, 86, 87, 88, 90, 92, 93 }

C grade { }

F normal fail { 5, 13, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 52, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 83, 85, 89, 91, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	82	119	232	85	151	152	121	122
N.S.	1	1.00	0.66	0.96	1.87	0.69	1.22	1.23	0.98	0.98
time (sec)	N/A	0.545	0.126	0.625	0.041	0.077	0.320	0.117	0.152	2.039

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	65	94	196	67	117	116	91	92
N.S.	1	1.00	0.69	1.00	2.09	0.71	1.24	1.23	0.97	0.98
time (sec)	N/A	0.438	0.111	0.577	0.041	0.082	0.240	0.112	0.151	2.002

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	45	73	160	48	82	79	61	62
N.S.	1	1.00	0.70	1.14	2.50	0.75	1.28	1.23	0.95	0.97
time (sec)	N/A	0.313	0.094	0.563	0.049	0.081	0.196	0.124	0.148	1.978

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	27	30	68	30	46	46	33	35
N.S.	1	1.00	0.96	1.07	2.43	1.07	1.64	1.64	1.18	1.25
time (sec)	N/A	0.255	0.049	0.375	0.043	0.079	0.154	0.110	0.149	0.069

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	39	52	97	54	39	47	28	0
N.S.	1	1.00	1.39	1.86	3.46	1.93	1.39	1.68	1.00	0.00
time (sec)	N/A	0.332	0.052	0.496	0.103	0.085	1.462	0.111	0.150	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	59	75	82	76	0	72	92	0
N.S.	1	1.00	1.26	1.60	1.74	1.62	0.00	1.53	1.96	0.00
time (sec)	N/A	0.414	0.118	0.542	0.128	0.075	0.000	0.112	0.150	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	78	137	66	116	0	134	144	0
N.S.	1	1.00	0.89	1.56	0.75	1.32	0.00	1.52	1.64	0.00
time (sec)	N/A	0.501	0.137	0.561	0.126	0.086	0.000	0.121	0.150	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	110	204	78	143	0	199	201	0
N.S.	1	1.00	0.83	1.55	0.59	1.08	0.00	1.51	1.52	0.00
time (sec)	N/A	0.556	0.211	0.584	0.135	0.076	0.000	0.136	0.153	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	140	273	82	161	0	266	256	0
N.S.	1	1.00	0.84	1.64	0.49	0.97	0.00	1.60	1.54	0.00
time (sec)	N/A	0.659	0.240	0.607	0.135	0.077	0.000	0.136	0.151	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	100	134	329	127	228	236	181	168
N.S.	1	1.00	0.54	0.73	1.79	0.69	1.24	1.28	0.98	0.91
time (sec)	N/A	0.630	0.138	0.677	0.047	0.084	0.396	0.130	0.150	0.174

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	87	108	275	95	172	171	131	125
N.S.	1	1.00	0.65	0.81	2.05	0.71	1.28	1.28	0.98	0.93
time (sec)	N/A	0.509	0.114	0.564	0.057	0.084	0.308	0.131	0.148	1.982

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	56	77	135	64	112	112	83	82
N.S.	1	1.20	1.14	1.57	2.76	1.31	2.29	2.29	1.69	1.67
time (sec)	N/A	0.395	0.089	0.546	0.057	0.089	0.232	0.134	0.150	1.943

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	51	121	175	94	73	113	58	0
N.S.	1	1.00	0.82	1.95	2.82	1.52	1.18	1.82	0.94	0.00
time (sec)	N/A	0.437	0.151	0.515	0.116	0.097	1.826	0.139	0.155	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	62	124	136	122	0	119	136	0
N.S.	1	1.00	0.89	1.77	1.94	1.74	0.00	1.70	1.94	0.00
time (sec)	N/A	0.525	0.148	0.566	0.129	0.089	0.000	0.127	0.152	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	93	186	126	153	0	181	199	0
N.S.	1	1.00	0.77	1.54	1.04	1.26	0.00	1.50	1.64	0.00
time (sec)	N/A	0.625	0.225	0.536	0.161	0.082	0.000	0.121	0.151	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	154	292	117	194	0	285	290	0
N.S.	1	1.00	0.90	1.70	0.68	1.13	0.00	1.66	1.69	0.00
time (sec)	N/A	0.733	0.267	0.582	0.127	0.082	0.000	0.122	0.154	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	248	248	206	400	128	231	0	395	383	0
N.S.	1	1.00	0.83	1.61	0.52	0.93	0.00	1.59	1.54	0.00
time (sec)	N/A	0.777	0.315	0.623	0.139	0.080	0.000	0.134	0.152	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	159	442	437	236	0	407	179	0
N.S.	1	1.00	0.73	2.02	2.00	1.08	0.00	1.86	0.82	0.00
time (sec)	N/A	0.749	0.370	0.622	0.138	0.103	0.000	0.133	0.148	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	118	292	328	190	0	256	109	0
N.S.	1	1.00	0.79	1.95	2.19	1.27	0.00	1.71	0.73	0.00
time (sec)	N/A	0.558	0.263	0.499	0.113	0.089	0.000	0.130	0.145	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	100	89	184	233	156	0	148	60	0
N.S.	1	1.10	0.98	2.02	2.56	1.71	0.00	1.63	0.66	0.00
time (sec)	N/A	0.491	0.180	0.465	0.119	0.087	0.000	0.134	0.145	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	64	114	156	118	0	83	34	0
N.S.	1	1.00	0.94	1.68	2.29	1.74	0.00	1.22	0.50	0.00
time (sec)	N/A	0.374	0.086	0.425	0.090	0.090	0.000	0.129	0.144	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	49	81	57	95	0	56	16	0
N.S.	1	1.00	0.96	1.59	1.12	1.86	0.00	1.10	0.31	0.00
time (sec)	N/A	0.412	0.055	0.378	0.085	0.073	0.000	0.127	0.150	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	63	108	155	123	0	75	20	0
N.S.	1	1.00	0.86	1.48	2.12	1.68	0.00	1.03	0.27	0.00
time (sec)	N/A	0.452	0.091	0.442	0.116	0.094	0.000	0.117	0.151	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	101	155	192	179	0	129	22	0
N.S.	1	1.00	0.89	1.37	1.70	1.58	0.00	1.14	0.19	0.00
time (sec)	N/A	0.572	0.238	0.468	0.154	0.096	0.000	0.114	0.151	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	178	281	242	275	0	248	22	0
N.S.	1	1.00	0.94	1.48	1.27	1.45	0.00	1.31	0.12	0.00
time (sec)	N/A	0.760	0.277	0.517	0.208	0.081	0.000	0.117	0.153	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	231	173	669	406	371	0	2979	0	0
N.S.	1	1.00	0.75	2.90	1.76	1.61	0.00	12.90	0.00	0.00
time (sec)	N/A	0.823	0.687	0.684	0.138	0.091	0.000	0.173	0.164	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	182	156	549	312	333	0	1991	0	0
N.S.	1	1.04	0.89	3.14	1.78	1.90	0.00	11.38	0.00	0.00
time (sec)	N/A	0.767	0.553	0.588	0.123	0.094	0.000	0.165	0.157	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	115	436	236	274	0	1308	0	0
N.S.	1	1.00	0.78	2.97	1.61	1.86	0.00	8.90	0.00	0.00
time (sec)	N/A	0.707	0.435	0.536	0.134	0.083	0.000	0.149	0.159	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	97	294	178	200	0	994	79	0
N.S.	1	1.00	0.78	2.35	1.42	1.60	0.00	7.95	0.63	0.00
time (sec)	N/A	0.571	0.288	0.505	0.108	0.082	0.000	0.143	0.155	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	84	65	132	81	149	0	615	76	0
N.S.	1	1.18	0.92	1.86	1.14	2.10	0.00	8.66	1.07	0.00
time (sec)	N/A	0.573	0.136	0.461	0.074	0.073	0.000	0.129	0.152	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	139	254	227	270	0	1329	31	0
N.S.	1	1.00	0.93	1.69	1.51	1.80	0.00	8.86	0.21	0.00
time (sec)	N/A	0.742	0.718	0.531	0.143	0.083	0.000	0.147	0.149	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	183	312	0	377	0	3353	604	0
N.S.	1	1.00	0.98	1.68	0.00	2.03	0.00	18.03	3.25	0.00
time (sec)	N/A	0.903	0.811	0.599	0.000	0.092	0.000	0.169	0.160	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	236	1055	0	566	0	879	0	0
N.S.	1	1.00	0.89	4.00	0.00	2.14	0.00	3.33	0.00	0.00
time (sec)	N/A	0.895	0.552	0.759	0.000	0.144	0.000	0.118	0.168	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	153	917	0	475	0	741	0	0
N.S.	1	1.00	0.63	3.80	0.00	1.97	0.00	3.07	0.00	0.00
time (sec)	N/A	0.850	0.595	0.670	0.000	0.138	0.000	0.115	0.167	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	158	435	0	373	0	529	106	0
N.S.	1	1.00	0.89	2.44	0.00	2.10	0.00	2.97	0.60	0.00
time (sec)	N/A	0.614	0.362	0.624	0.000	0.122	0.000	0.115	0.152	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	111	88	276	95	253	0	298	103	0
N.S.	1	1.07	0.85	2.65	0.91	2.43	0.00	2.87	0.99	0.00
time (sec)	N/A	0.689	0.305	0.562	0.080	0.120	0.000	0.118	0.155	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	262	262	451	488	0	601	0	837	42	0
N.S.	1	1.00	1.72	1.86	0.00	2.29	0.00	3.19	0.16	0.00
time (sec)	N/A	0.959	0.812	0.651	0.000	0.151	0.000	0.109	0.150	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	298	298	541	643	0	762	0	1006	1170	0
N.S.	1	1.00	1.82	2.16	0.00	2.56	0.00	3.38	3.93	0.00
time (sec)	N/A	1.049	0.870	0.734	0.000	0.133	0.000	0.118	0.161	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	377	377	627	760	0	892	0	1169	0	0
N.S.	1	1.00	1.66	2.02	0.00	2.37	0.00	3.10	0.00	0.00
time (sec)	N/A	1.176	1.042	0.814	0.000	0.106	0.000	0.120	0.167	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	92	135	250	95	168	174	138	116
N.S.	1	1.00	0.66	0.97	1.80	0.68	1.21	1.25	0.99	0.83
time (sec)	N/A	0.529	0.108	0.668	0.041	0.117	0.564	0.113	0.150	1.948

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	74	70	214	78	134	138	108	93
N.S.	1	1.00	0.68	0.64	1.96	0.72	1.23	1.27	0.99	0.85
time (sec)	N/A	0.448	0.071	0.606	0.044	0.113	0.426	0.106	0.150	1.905

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	57	86	213	60	99	101	78	70
N.S.	1	1.00	0.72	1.09	2.70	0.76	1.25	1.28	0.99	0.89
time (sec)	N/A	0.376	0.059	0.530	0.047	0.118	0.318	0.111	0.151	1.932

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	40	39	86	42	65	70	50	47
N.S.	1	1.00	0.78	0.76	1.69	0.82	1.27	1.37	0.98	0.92
time (sec)	N/A	0.292	0.036	0.494	0.052	0.094	0.256	0.112	0.148	1.969

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	55	81	122	73	49	76	41	0
N.S.	1	1.00	1.34	1.98	2.98	1.78	1.20	1.85	1.00	0.00
time (sec)	N/A	0.342	0.064	0.485	0.103	0.087	2.009	0.110	0.149	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	83	80	82	0	80	89	0
N.S.	1	1.00	1.00	1.98	1.90	1.95	0.00	1.90	2.12	0.00
time (sec)	N/A	0.347	0.049	0.515	0.096	0.168	0.000	0.114	0.149	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	80	112	90	107	0	109	124	0
N.S.	1	1.00	1.08	1.51	1.22	1.45	0.00	1.47	1.68	0.00
time (sec)	N/A	0.398	0.087	0.528	0.121	0.196	0.000	0.114	0.152	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	95	175	73	127	0	170	177	0
N.S.	1	1.00	0.90	1.67	0.70	1.21	0.00	1.62	1.69	0.00
time (sec)	N/A	0.500	0.129	0.542	0.123	0.142	0.000	0.110	0.154	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	127	244	76	152	0	237	232	0
N.S.	1	1.00	0.85	1.64	0.51	1.02	0.00	1.59	1.56	0.00
time (sec)	N/A	0.607	0.155	0.595	0.126	0.178	0.000	0.113	0.153	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	138	129	383	155	286	304	233	182
N.S.	1	1.00	0.59	0.55	1.64	0.66	1.22	1.30	1.00	0.78
time (sec)	N/A	0.701	0.185	0.795	0.054	0.091	0.695	0.115	0.153	2.150

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	113	155	353	126	226	239	183	148
N.S.	1	1.00	0.61	0.84	1.92	0.68	1.23	1.30	0.99	0.80
time (sec)	N/A	0.562	0.137	0.720	0.066	0.090	0.688	0.114	0.150	2.018

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	85	72	189	98	172	180	135	114
N.S.	1	1.00	0.62	0.53	1.39	0.72	1.26	1.32	0.99	0.84
time (sec)	N/A	0.423	0.106	0.625	0.051	0.083	0.372	0.108	0.150	0.126

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	82	226	235	130	121	222	108	0
N.S.	1	1.00	0.75	2.05	2.14	1.18	1.10	2.02	0.98	0.00
time (sec)	N/A	0.407	0.239	0.603	0.128	0.094	2.205	0.114	0.153	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	95	200	179	127	0	197	191	0
N.S.	1	1.00	1.00	2.11	1.88	1.34	0.00	2.07	2.01	0.00
time (sec)	N/A	0.393	0.163	0.645	0.120	0.084	0.000	0.116	0.153	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	97	209	165	164	0	206	211	0
N.S.	1	1.00	0.85	1.83	1.45	1.44	0.00	1.81	1.85	0.00
time (sec)	N/A	0.435	0.233	0.701	0.119	0.087	0.000	0.123	0.150	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	241	135	171	0	236	238	0
N.S.	1	1.00	0.86	1.81	1.02	1.29	0.00	1.77	1.79	0.00
time (sec)	N/A	0.476	0.240	0.685	0.111	0.110	0.000	0.113	0.151	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	124	299	139	194	0	294	297	0
N.S.	1	1.00	0.71	1.71	0.79	1.11	0.00	1.68	1.70	0.00
time (sec)	N/A	0.602	0.286	0.747	0.165	0.108	0.000	0.121	0.155	0.000

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	276	369	0	605	0	0	79	0
N.S.	1	1.00	1.01	1.35	0.00	2.22	0.00	0.00	0.29	0.00
time (sec)	N/A	1.010	0.979	0.809	0.000	0.103	0.000	0.000	0.150	0.000

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	209	268	0	502	0	0	50	0
N.S.	1	1.00	1.00	1.28	0.00	2.40	0.00	0.00	0.24	0.00
time (sec)	N/A	0.642	1.182	0.619	0.000	0.098	0.000	0.000	0.152	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	221	259	0	496	0	0	36	0
N.S.	1	1.00	0.98	1.15	0.00	2.19	0.00	0.00	0.16	0.00
time (sec)	N/A	0.645	0.307	0.573	0.000	0.093	0.000	0.000	0.150	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	155	200	0	219	0	0	19	0
N.S.	1	1.00	0.88	1.13	0.00	1.24	0.00	0.00	0.11	0.00
time (sec)	N/A	0.570	0.175	0.509	0.000	0.098	0.000	0.000	0.151	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	166	212	0	316	0	0	18	0
N.S.	1	1.00	0.78	1.00	0.00	1.48	0.00	0.00	0.08	0.00
time (sec)	N/A	0.554	0.285	0.460	0.000	0.091	0.000	0.000	0.150	0.000

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	173	227	0	249	0	0	20	0
N.S.	1	1.00	0.88	1.15	0.00	1.26	0.00	0.00	0.10	0.00
time (sec)	N/A	0.715	0.264	0.492	0.000	0.100	0.000	0.000	0.153	0.000

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	243	288	0	599	0	0	22	0
N.S.	1	1.00	0.98	1.16	0.00	2.41	0.00	0.00	0.09	0.00
time (sec)	N/A	0.842	0.591	0.533	0.000	0.108	0.000	0.000	0.151	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	232	330	0	583	0	0	260	0
N.S.	1	1.00	0.86	1.22	0.00	2.16	0.00	0.00	0.96	0.00
time (sec)	N/A	0.847	0.706	0.566	0.000	0.109	0.000	0.000	0.160	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	449	476	307	1038	0	1179	0	0	463	0
N.S.	1	1.06	0.68	2.31	0.00	2.63	0.00	0.00	1.03	0.00
time (sec)	N/A	1.630	2.473	0.974	0.000	0.134	0.000	0.000	0.162	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	431	441	290	903	0	931	0	0	772	0
N.S.	1	1.02	0.67	2.10	0.00	2.16	0.00	0.00	1.79	0.00
time (sec)	N/A	1.329	1.380	0.821	0.000	0.104	0.000	0.000	0.166	0.000

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	416	429	287	901	0	1162	0	0	87	0
N.S.	1	1.03	0.69	2.17	0.00	2.79	0.00	0.00	0.21	0.00
time (sec)	N/A	1.202	2.244	0.745	0.000	0.099	0.000	0.000	0.157	0.000

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	244	236	449	0	641	0	0	163	0
N.S.	1	1.02	0.99	1.88	0.00	2.68	0.00	0.00	0.68	0.00
time (sec)	N/A	0.639	0.746	0.652	0.000	0.125	0.000	0.000	0.154	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	293	503	0	1162	0	0	29	0
N.S.	1	1.00	0.62	1.06	0.00	2.44	0.00	0.00	0.06	0.00
time (sec)	N/A	1.278	1.838	0.601	0.000	0.108	0.000	0.000	0.150	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	415	546	0	992	0	0	31	0
N.S.	1	1.00	0.95	1.26	0.00	2.28	0.00	0.00	0.07	0.00
time (sec)	N/A	1.256	0.788	0.663	0.000	0.142	0.000	0.000	0.149	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	500	500	334	595	0	1310	0	0	731	0
N.S.	1	1.00	0.67	1.19	0.00	2.62	0.00	0.00	1.46	0.00
time (sec)	N/A	1.892	2.222	0.787	0.000	0.133	0.000	0.000	0.165	0.000

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	714	332	1555	0	1620	0	0	1439	0
N.S.	1	1.50	0.70	3.27	0.00	3.40	0.00	0.00	3.02	0.00
time (sec)	N/A	2.610	0.930	1.310	0.000	0.149	0.000	0.000	0.179	0.000

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	746	759	380	2172	0	2047	0	0	114	0
N.S.	1	1.02	0.51	2.91	0.00	2.74	0.00	0.00	0.15	0.00
time (sec)	N/A	2.323	2.772	1.103	0.000	0.120	0.000	0.000	0.159	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	512	507	325	1503	0	1607	0	0	514	0
N.S.	1	0.99	0.63	2.94	0.00	3.14	0.00	0.00	1.00	0.00
time (sec)	N/A	1.293	0.879	0.974	0.000	0.144	0.000	0.000	0.157	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	856	856	392	1064	0	2116	0	0	586	0
N.S.	1	1.00	0.46	1.24	0.00	2.47	0.00	0.00	0.68	0.00
time (sec)	N/A	1.886	2.888	0.853	0.000	0.139	0.000	0.000	0.162	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	730	730	674	1090	0	2076	0	0	42	0
N.S.	1	1.00	0.92	1.49	0.00	2.84	0.00	0.00	0.06	0.00
time (sec)	N/A	2.513	1.445	0.959	0.000	0.152	0.000	0.000	0.152	0.000

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	874	874	613	1178	0	2346	0	0	0	0
N.S.	1	1.00	0.70	1.35	0.00	2.68	0.00	0.00	0.00	0.00
time (sec)	N/A	3.436	3.884	1.114	0.000	0.168	0.000	0.000	0.171	0.000

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	791	791	438	1294	0	2363	0	0	0	0
N.S.	1	1.00	0.55	1.64	0.00	2.99	0.00	0.00	0.00	0.00
time (sec)	N/A	2.446	1.660	1.300	0.000	0.149	0.000	0.000	0.174	0.000

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	100	153	268	105	185	192	153	152
N.S.	1	1.00	0.65	0.99	1.74	0.68	1.20	1.25	0.99	0.99
time (sec)	N/A	0.513	0.121	0.653	0.046	0.092	0.630	0.132	0.152	2.045

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	84	129	267	87	151	156	123	122
N.S.	1	1.00	0.68	1.04	2.15	0.70	1.22	1.26	0.99	0.98
time (sec)	N/A	0.451	0.078	0.610	0.042	0.090	0.465	0.127	0.152	1.945

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	66	104	196	68	116	119	93	92
N.S.	1	1.00	0.70	1.11	2.09	0.72	1.23	1.27	0.99	0.98
time (sec)	N/A	0.371	0.066	0.584	0.052	0.092	0.367	0.124	0.153	1.899

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	49	81	104	53	82	88	65	65
N.S.	1	1.00	0.74	1.23	1.58	0.80	1.24	1.33	0.98	0.98
time (sec)	N/A	0.307	0.054	0.497	0.034	0.085	0.299	0.125	0.149	0.106

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	49	113	140	86	66	109	56	0
N.S.	1	1.00	0.88	2.02	2.50	1.54	1.18	1.95	1.00	0.00
time (sec)	N/A	0.326	0.106	0.528	0.130	0.089	1.854	0.132	0.157	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	114	102	90	0	111	117	0
N.S.	1	1.00	1.00	2.07	1.85	1.64	0.00	2.02	2.13	0.00
time (sec)	N/A	0.345	0.069	0.552	0.098	0.091	0.000	0.120	0.152	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	86	119	87	101	0	118	64	0
N.S.	1	1.00	1.25	1.72	1.26	1.46	0.00	1.71	0.93	0.00
time (sec)	N/A	0.372	0.075	0.582	0.097	0.093	0.000	0.138	0.151	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	73	146	95	118	0	141	153	0
N.S.	1	1.00	0.80	1.60	1.04	1.30	0.00	1.55	1.68	0.00
time (sec)	N/A	0.455	0.133	0.604	0.128	0.097	0.000	0.128	0.151	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	139	206	383	161	284	303	233	301
N.S.	1	1.00	0.59	0.88	1.64	0.69	1.21	1.29	1.00	1.29
time (sec)	N/A	0.695	0.185	0.804	0.043	0.100	0.939	0.131	0.146	0.373

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	111	159	243	130	226	244	185	182
N.S.	1	1.00	0.60	0.85	1.31	0.70	1.22	1.31	0.99	0.98
time (sec)	N/A	0.596	0.130	0.712	0.046	0.096	0.841	0.126	0.149	0.249

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	A	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	303	289	161	168	331	158	0
N.S.	1	1.00	0.68	1.89	1.81	1.01	1.05	2.07	0.99	0.00
time (sec)	N/A	0.565	0.287	0.762	0.116	0.086	2.766	0.128	0.153	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	143	306	235	160	0	308	285	0
N.S.	1	1.00	1.00	2.14	1.64	1.12	0.00	2.15	1.99	0.00
time (sec)	N/A	0.523	0.228	0.751	0.105	0.078	0.000	0.128	0.150	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	136	281	203	161	0	280	139	0
N.S.	1	1.00	0.96	1.99	1.44	1.14	0.00	1.99	0.99	0.00
time (sec)	N/A	0.537	0.219	0.776	0.129	0.100	0.000	0.139	0.149	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	135	284	188	187	0	279	276	0
N.S.	1	1.00	0.90	1.89	1.25	1.25	0.00	1.86	1.84	0.00
time (sec)	N/A	0.520	0.347	0.827	0.144	0.090	0.000	0.133	0.152	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	150	322	154	196	0	315	305	0
N.S.	1	1.00	0.90	1.93	0.92	1.17	0.00	1.89	1.83	0.00
time (sec)	N/A	0.568	0.341	0.867	0.120	0.102	0.000	0.111	0.150	0.000

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	373	373	213	925	0	989	0	0	50	0
N.S.	1	1.00	0.57	2.48	0.00	2.65	0.00	0.00	0.13	0.00
time (sec)	N/A	1.162	0.140	0.687	0.000	0.137	0.000	0.000	0.160	0.000

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	358	358	198	671	0	977	0	0	36	0
N.S.	1	1.00	0.55	1.87	0.00	2.73	0.00	0.00	0.10	0.00
time (sec)	N/A	0.953	0.103	0.599	0.000	0.118	0.000	0.000	0.158	0.000

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	170	423	0	500	0	0	21	0
N.S.	1	1.00	0.60	1.49	0.00	1.77	0.00	0.00	0.07	0.00
time (sec)	N/A	0.743	5.039	0.582	0.000	0.102	0.000	0.000	0.162	0.000

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	180	280	0	671	0	0	19	0
N.S.	1	1.00	0.52	0.81	0.00	1.94	0.00	0.00	0.06	0.00
time (sec)	N/A	0.695	5.046	0.541	0.000	0.121	0.000	0.000	0.155	0.000

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	180	143	0	673	0	0	18	0
N.S.	1	1.00	0.52	0.41	0.00	1.95	0.00	0.00	0.05	0.00
time (sec)	N/A	0.867	5.045	0.516	0.000	0.108	0.000	0.000	0.155	0.000

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	303	303	186	138	0	530	0	0	20	0
N.S.	1	1.00	0.61	0.46	0.00	1.75	0.00	0.00	0.07	0.00
time (sec)	N/A	0.917	0.127	0.554	0.000	0.118	0.000	0.000	0.152	0.000

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	215	175	0	1154	0	0	22	0
N.S.	1	1.00	0.56	0.46	0.00	3.03	0.00	0.00	0.06	0.00
time (sec)	N/A	0.964	0.178	0.648	0.000	0.128	0.000	0.000	0.158	0.000

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F(-1)	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	410	410	237	226	0	1251	0	0	22	0
N.S.	1	1.00	0.58	0.55	0.00	3.05	0.00	0.00	0.05	0.00
time (sec)	N/A	1.000	0.190	0.698	0.000	0.138	0.000	0.000	0.158	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	718	729	363	877	0	2050	0	0	754	0
N.S.	1	1.02	0.51	1.22	0.00	2.86	0.00	0.00	1.05	0.00
time (sec)	N/A	1.768	0.160	0.980	0.000	0.125	0.000	0.000	0.213	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	373	376	203	594	0	1276	0	0	87	0
N.S.	1	1.01	0.54	1.59	0.00	3.42	0.00	0.00	0.23	0.00
time (sec)	N/A	0.879	0.121	0.869	0.000	0.143	0.000	0.000	0.186	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	695	725	387	395	0	2135	0	0	30	0
N.S.	1	1.04	0.56	0.57	0.00	3.07	0.00	0.00	0.04	0.00
time (sec)	N/A	1.983	0.131	0.834	0.000	0.174	0.000	0.000	0.160	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	739	832	387	226	0	2048	0	0	29	0
N.S.	1	1.13	0.52	0.31	0.00	2.77	0.00	0.00	0.04	0.00
time (sec)	N/A	2.315	0.139	0.764	0.000	0.136	0.000	0.000	0.161	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	697	846	411	338	0	1773	0	0	31	0
N.S.	1	1.21	0.59	0.48	0.00	2.54	0.00	0.00	0.04	0.00
time (sec)	N/A	2.178	0.365	0.878	0.000	0.142	0.000	0.000	0.155	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	784	1146	397	6251	0	2980	0	0	0	0
N.S.	1	1.46	0.51	7.97	0.00	3.80	0.00	0.00	0.00	0.00
time (sec)	N/A	3.452	0.382	2.064	0.000	0.164	0.000	0.000	0.319	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1105	1142	675	4708	0	4691	0	0	114	0
N.S.	1	1.03	0.61	4.26	0.00	4.25	0.00	0.00	0.10	0.00
time (sec)	N/A	3.630	0.367	1.762	0.000	0.173	0.000	0.000	0.210	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	B	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	776	1596	429	3281	0	2962	0	0	1448	0
N.S.	1	2.06	0.55	4.23	0.00	3.82	0.00	0.00	1.87	0.00
time (sec)	N/A	3.901	0.429	1.632	0.000	0.163	0.000	0.000	0.262	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	781	863	423	2238	0	2972	0	0	114	0
N.S.	1	1.10	0.54	2.87	0.00	3.81	0.00	0.00	0.15	0.00
time (sec)	N/A	2.100	0.305	1.526	0.000	0.158	0.000	0.000	0.209	0.000

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	1147	1813	669	1416	0	4691	0	0	110	0
N.S.	1	1.58	0.58	1.23	0.00	4.09	0.00	0.00	0.10	0.00
time (sec)	N/A	4.975	0.307	1.382	0.000	0.185	0.000	0.000	0.214	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [36] had the largest ratio of [.9285710000000000035]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	15	0.133
2	A	2	2	1.00	15	0.133
3	A	2	2	1.00	13	0.154
4	A	6	6	1.00	12	0.500
5	A	2	2	1.00	15	0.133
6	A	2	2	1.00	15	0.133
7	A	2	2	1.00	15	0.133
8	A	2	2	1.00	15	0.133
9	A	2	2	1.00	15	0.133
10	A	2	2	1.00	17	0.118
11	A	2	2	1.00	15	0.133
12	C	8	8	1.20	14	0.571
13	A	2	2	1.00	17	0.118
14	A	2	2	1.00	17	0.118
15	A	2	2	1.00	17	0.118
16	A	2	2	1.00	17	0.118
17	A	2	2	1.00	17	0.118
18	A	2	2	1.00	17	0.118
19	A	2	2	1.00	17	0.118
20	A	2	2	1.10	17	0.118
21	A	2	2	1.00	15	0.133

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	7	7	1.00	14	0.500
23	A	2	2	1.00	17	0.118
24	A	2	2	1.00	17	0.118
25	A	2	2	1.00	17	0.118
26	A	2	2	1.00	17	0.118
27	A	2	2	1.04	17	0.118
28	A	2	2	1.00	17	0.118
29	A	2	2	1.00	15	0.133
30	C	11	11	1.18	14	0.786
31	A	2	2	1.00	17	0.118
32	A	2	2	1.00	17	0.118
33	A	2	2	1.00	17	0.118
34	A	2	2	1.00	17	0.118
35	A	2	2	1.00	15	0.133
36	C	13	13	1.07	14	0.929
37	A	2	2	1.00	17	0.118
38	A	2	2	1.00	17	0.118
39	A	2	2	1.00	17	0.118
40	A	2	2	1.00	17	0.118
41	A	2	2	1.00	17	0.118
42	A	2	2	1.00	15	0.133
43	A	2	2	1.00	14	0.143
44	A	2	2	1.00	17	0.118
45	A	2	2	1.00	17	0.118
46	A	2	2	1.00	17	0.118
47	A	2	2	1.00	17	0.118
48	A	2	2	1.00	17	0.118
49	A	2	2	1.00	19	0.105
50	A	2	2	1.00	17	0.118
51	A	2	2	1.00	16	0.125
52	A	2	2	1.00	19	0.105
53	A	2	2	1.00	19	0.105

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	2	2	1.00	19	0.105
55	A	2	2	1.00	19	0.105
56	A	2	2	1.00	19	0.105
57	A	2	2	1.00	19	0.105
58	A	2	2	1.00	19	0.105
59	A	2	2	1.00	19	0.105
60	A	2	2	1.00	17	0.118
61	A	2	2	1.00	16	0.125
62	A	2	2	1.00	19	0.105
63	A	2	2	1.00	19	0.105
64	A	2	2	1.00	19	0.105
65	A	5	5	1.06	19	0.263
66	A	5	5	1.02	19	0.263
67	A	5	5	1.03	19	0.263
68	A	3	3	1.02	17	0.176
69	A	2	2	1.00	16	0.125
70	A	2	2	1.00	19	0.105
71	A	2	2	1.00	19	0.105
72	A	9	9	1.50	19	0.474
73	A	6	6	1.02	19	0.316
74	A	3	3	0.99	17	0.176
75	A	2	2	1.00	16	0.125
76	A	2	2	1.00	19	0.105
77	A	2	2	1.00	19	0.105
78	A	2	2	1.00	19	0.105
79	A	2	2	1.00	17	0.118
80	A	2	2	1.00	17	0.118
81	A	2	2	1.00	15	0.133
82	A	2	2	1.00	14	0.143
83	A	2	2	1.00	17	0.118
84	A	2	2	1.00	17	0.118
85	A	2	2	1.00	17	0.118

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	2	2	1.00	17	0.118
87	A	2	2	1.00	17	0.118
88	A	2	2	1.00	16	0.125
89	A	2	2	1.00	19	0.105
90	A	2	2	1.00	19	0.105
91	A	2	2	1.00	19	0.105
92	A	2	2	1.00	19	0.105
93	A	2	2	1.00	19	0.105
94	A	2	2	1.00	19	0.105
95	A	2	2	1.00	19	0.105
96	A	2	2	1.00	19	0.105
97	A	2	2	1.00	17	0.118
98	A	2	2	1.00	16	0.125
99	A	2	2	1.00	19	0.105
100	A	2	2	1.00	19	0.105
101	A	2	2	1.00	19	0.105
102	A	5	5	1.02	19	0.263
103	A	3	3	1.01	19	0.158
104	A	5	5	1.04	17	0.294
105	A	5	5	1.13	16	0.312
106	A	5	5	1.21	19	0.263
107	A	9	9	1.46	19	0.474
108	A	9	9	1.03	19	0.474
109	B	7	7	2.06	19	0.368
110	A	6	6	1.10	19	0.316
111	A	7	7	1.58	17	0.412

CHAPTER 3

Listing of integrals

3.1 $\int x^3(a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 124

$$\int x^3(a+bx)\cosh(c+dx)\,dx = -\frac{6a\cosh(c+dx)}{d^4}-\frac{24bx\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{6ax\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}$$

output
```
-6*a*cosh(d*x+c)/d^4-24*b*x*cosh(d*x+c)/d^4-3*a*x^2*cosh(d*x+c)/d^2-4*b*x^
3*cosh(d*x+c)/d^2+24*b*sinh(d*x+c)/d^5+6*a*x*sinh(d*x+c)/d^3+12*b*x^2*sinh
(d*x+c)/d^3+a*x^3*sinh(d*x+c)/d+b*x^4*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.66

$$\int x^3(a+bx)\cosh(c+dx)\,dx$$
$$=\frac{-d(3a(2+d^2x^2)+4bx(6+d^2x^2))\cosh(c+dx)+(ad^2x(6+d^2x^2)+b(24+12d^2x^2+d^4x^4))\sinh(c+d}{d^5}$$

input
```
Integrate[x^3*(a + b*x)*Cosh[c + d*x],x]
```

output
```
(-(d*(3*a*(2 + d^2*x^2) + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^2*x*(
6 + d^2*x^2) + b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx)\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(ax^3\cosh(c+dx)+bx^4\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

input
```
Int[x^3*(a + b*x)*Cosh[c + d*x],x]
```

output
```
(-6*a*Cosh[c + d*x])/d^4 - (24*b*x*Cosh[c + d*x])/d^4 - (3*a*x^2*Cosh[c + 
d*x])/d^2 - (4*b*x^3*Cosh[c + d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (6*a*
x*Sinh[c + d*x])/d^3 + (12*b*x^2*Sinh[c + d*x])/d^3 + (a*x^3*Sinh[c + d*x]
)/d + (b*x^4*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{3\left(x\left(\frac{4bx}{3}+a\right)d^2+8b\right)dx\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\left(\left(-b\,x^4-a\,x^3\right)d^4-6x(2bx+a)d^2-24b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+3d\left(x^2\left(\frac{4bx}{3}+a\right)d^2+\ldots\right)}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,x^4d^4+a\,d^4x^3-4b\,d^3x^3-3a\,d^3x^2+12b\,d^2x^2+6a\,d^2x-24dxb-6ad+24b\right)\mathrm{e}^{dx+c}}{2d^5}-\frac{\left(b\,x^4d^4+a\,d^4x^3+4b\,d^3x^3+3a\,d^3x^2\ldots\right)}{\ldots}$
orering	$-\frac{2\left(4b^2d^4x^5+7ab\,d^4x^4+3a^2d^4x^3+36b^2d^2x^3+45ab\,d^2x^2+12a^2d^2x+48b^2x+36ab\right)\cosh(dx+c)}{d^6x(bx+a)}+\frac{\left(b\,x^4d^4+a\,d^4x^3+\ldots\right)}{\ldots}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4+\ldots\right)}{\ldots}\right)}{\ldots}$
parts	$\frac{b\,x^4\sinh(dx+c)}{d}+\frac{a\,x^3\sinh(dx+c)}{d}-\frac{-\frac{4b\,c^3\cosh(dx+c)}{d^3}+\frac{12b\,c^2((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^3}-\frac{12bc\left((dx+c)^2\cosh(d\ldots\right)}{\ldots}}{\ldots}$
derivativedivides	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{4bc\left((dx+c)^3\sinh(dx\ldots\right)}{\ldots}}{\ldots}$
default	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{4bc\left((dx+c)^3\sinh(dx\ldots\right)}{\ldots}}{\ldots}$

input
```
int(x^3*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(3*(x*(4/3*b*x+a)*d^2+8*b)*d*x*tanh(1/2*d*x+1/2*c)^2+2*((-b*x^4-a*x^3)*d^4
-6*x*(2*b*x+a)*d^2-24*b)*tanh(1/2*d*x+1/2*c)+3*d*(x^2*(4/3*b*x+a)*d^2+8*b*
x+4*a))/d^5/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.69

$$\int x^3(a+bx)\cosh(c+dx)\,dx = -\frac{(4\,bd^3x^3+3\,ad^3x^2+24\,bdx+6\,ad)\cosh\left(dx+c\right)-(bd^4x^4+ad^4x^3+12\,bd^2x^2+6\,ad^2x+24\,b)\sinh\left(\right.}{d^5}$$

input
```
integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((4*b*d^3*x^3 + 3*a*d^3*x^2 + 24*b*d*x + 6*a*d)*cosh(d*x + c) - (b*d^4*x^
4 + a*d^4*x^3 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int x^3(a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^3\sinh(c+dx)}{d} - \frac{3ax^2\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} - \frac{6a\cosh(c+dx)}{d^4} + \frac{bx^4\sinh(c+dx)}{d} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{12bx^2\sinh(c+}{d^3} \\ \left(\frac{ax^4}{4}+\frac{bx^5}{5}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**3*(b*x+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*si
nh(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**4*sinh(c + d*x)/d - 4*b*x
**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*
x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**4/4 + b*x**5/5)*cosh(
c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.87

$$\int x^3(a+bx)\cosh(c+dx)\,dx =$$
$$-\frac{1}{40}\,d\left(\frac{5\left(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c\right)ae^{(dx)}}{d^5}+\frac{5\left(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx\right.}{d^5}\right.$$
$$+\frac{1}{20}\left(4\,bx^5+5\,ax^4\right)\cosh\left(dx+c\right)$$

input
```
integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/40*d*(5*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24
*e^c)*a*e^(d*x)/d^5 + 5*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a
*e^(-d*x - c)/d^5 + 4*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d
^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 4*(d^5*x^5 + 5*d^4*x^4
 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6) + 1/20*(4*
b*x^5 + 5*a*x^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.23

$$\int x^3(a+bx)\cosh(c+dx)\,dx$$
$$=\frac{\left(bd^4x^4+ad^4x^3-4\,bd^3x^3-3\,ad^3x^2+12\,bd^2x^2+6\,ad^2x-24\,bdx-6\,ad+24\,b\right)e^{(dx+c)}}{2\,d^5}$$
$$-\frac{\left(bd^4x^4+ad^4x^3+4\,bd^3x^3+3\,ad^3x^2+12\,bd^2x^2+6\,ad^2x+24\,bdx+6\,ad+24\,b\right)e^{(-dx-c)}}{2\,d^5}$$

input
```
integrate(x^3*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^4*x^4 + a*d^4*x^3 - 4*b*d^3*x^3 - 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*
a*d^2*x - 24*b*d*x - 6*a*d + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^
4*x^3 + 4*b*d^3*x^3 + 3*a*d^3*x^2 + 12*b*d^2*x^2 + 6*a*d^2*x + 24*b*d*x +
6*a*d + 24*b)*e^(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int x^3(a+bx)\cosh(c+dx)\,dx = \frac{12\,b\,x^2\sinh(c+d\,x)+6\,a\,x\sinh(c+d\,x)}{d^3} - \frac{6\,a\cosh(c+d\,x)+24\,b\,x\cosh(c+d\,x)}{d^4} - \frac{3\,a\,x^2\cosh(c+d\,x)+4\,b\,x^3\cosh(c+d\,x)}{d^2} + \frac{a\,x^3\sinh(c+d\,x)+b\,x^4\sinh(c+d\,x)}{d} + \frac{24\,b\sinh(c+d\,x)}{d^5}$$

input
```
int(x^3*cosh(c + d*x)*(a + b*x),x)
```

output
```
(12*b*x^2*sinh(c + d*x) + 6*a*x*sinh(c + d*x))/d^3 - (6*a*cosh(c + d*x) +
24*b*x*cosh(c + d*x))/d^4 - (3*a*x^2*cosh(c + d*x) + 4*b*x^3*cosh(c + d*x)
)/d^2 + (a*x^3*sinh(c + d*x) + b*x^4*sinh(c + d*x))/d + (24*b*sinh(c + d*x
))/d^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.98

$$\int x^3(a+bx)\cosh(c+dx)\,dx = \frac{-3\cosh(dx+c)\,a\,d^3x^2-6\cosh(dx+c)\,ad-4\cosh(dx+c)\,b\,d^3x^3-24\cosh(dx+c)\,bdx+\sinh(dx+}{d^5}$$

input
```
int(x^3*(b*x+a)*cosh(d*x+c),x)
```

output
```
( - 3*cosh(c + d*x)*a*d**3*x**2 - 6*cosh(c + d*x)*a*d - 4*cosh(c + d*x)*b*
d**3*x**3 - 24*cosh(c + d*x)*b*d*x + sinh(c + d*x)*a*d**4*x**3 + 6*sinh(c
+ d*x)*a*d**2*x + sinh(c + d*x)*b*d**4*x**4 + 12*sinh(c + d*x)*b*d**2*x**2
 + 24*sinh(c + d*x)*b)/d**5
```

3.2 $\int x^2(a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int x^2(a+bx)\cosh(c+dx)\,dx = -\frac{6b\cosh(c+dx)}{d^4} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{ax^2\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d}$$

output
```
-6*b*cosh(d*x+c)/d^4-2*a*x*cosh(d*x+c)/d^2-3*b*x^2*cosh(d*x+c)/d^2+2*a*sin
h(d*x+c)/d^3+6*b*x*sinh(d*x+c)/d^3+a*x^2*sinh(d*x+c)/d+b*x^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.69

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \frac{-((2ad^2x+3b(2+d^2x^2))\cosh(c+dx)) + d(a(2+d^2x^2)+bx(6+d^2x^2))\sinh(c+dx)}{d^4}$$

input
```
Integrate[x^2*(a + b*x)*Cosh[c + d*x],x]
```

output
```
(-((2*a*d^2*x + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a*(2 + d^2*x^2) + b
*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(ax^2\cosh(c+dx)+bx^3\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{6b\cosh(c+dx)}{d^4}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{bx^3\sinh(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x)*Cosh[c + d*x],x]
```

output
```
(-6*b*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d
*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x^2*Si
nh[c + d*x])/d + (b*x^3*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{2\left(\frac{3bx}{2}+a\right)d^2x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2d\left(d^2x^2(bx+a)+6bx+2a\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3b\,x^2+2ax\right)d^2+12b}{d^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,d^3x^3+a\,d^3x^2-3b\,d^2x^2-2a\,d^2x+6dxb+2ad-6b\right)e^{dx+c}}{2d^4}-\frac{\left(b\,d^3x^3+a\,d^3x^2+3b\,d^2x^2+2a\,d^2x+6dxb+2ad+6b\right)e^{-dx-}}{2d^4}$
orering	$-\frac{2\left(3b^2d^2x^4+5ab\,d^2x^3+2a^2d^2x^2+12x^2b^2+12abx+2a^2\right)\cosh(dx+c)}{d^4x(bx+a)}+\frac{\left(b\,d^2x^3+a\,d^2x^2+6bx+2a\right)\left(2x(bx+a)\cosh(\right.}{d^4x}$
parts	$\frac{b\,x^3\sinh(dx+c)}{d}+\frac{a\,x^2\sinh(dx+c)}{d}-\frac{\frac{3b\,c^2\cosh(dx+c)}{d^2}-\frac{6bc((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^2}+\frac{3b\left((dx+c)^2\cosh(dx+c)-\right.}{}}{}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i(}{}\right.}{d^4}$
derivativedivides	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}+\frac{b\left((dx+c)^3\sinh(dx+c)-3\right.}{}}{}$
default	$\frac{\frac{3b\,c^2((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{3bc\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}+\frac{b\left((dx+c)^3\sinh(dx+c)-3\right.}{}}{}$

input
```
int(x^2*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(2*(3/2*b*x+a)*d^2*x*tanh(1/2*d*x+1/2*c)^2-2*d*(d^2*x^2*(b*x+a)+6*b*x+2*a)
*tanh(1/2*d*x+1/2*c)+(3*b*x^2+2*a*x)*d^2+12*b)/d^4/(tanh(1/2*d*x+1/2*c)^2-
1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.71

$$\int x^2(a+bx)\cosh(c+dx)\,dx = -\frac{(3\,bd^2x^2+2\,ad^2x+6\,b)\cosh(dx+c)-(bd^3x^3+ad^3x^2+6\,bdx+2\,ad)\sinh(dx+c)}{d^4}$$

input
```
integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((3*b*d^2*x^2 + 2*a*d^2*x + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3*x^2 +
 6*b*d*x + 2*a*d)*sinh(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.24

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^2\sinh(c+dx)}{d}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{bx^3\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4} \\ \left(\frac{ax^3}{3}+\frac{bx^4}{4}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**2*(b*x+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c
+ d*x)/d**3 + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x
*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)), ((a*x**3/3 + b*x*
*4/4)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(94) = 188$.

Time = 0.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.09

$$\int x^2(a+bx)\cosh(c+dx)\,dx = -\frac{1}{24}d\left(\frac{4\,(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)ae^{(dx)}}{d^4} + \frac{4\,(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)ae^{(-dx-c)}}{d^4} + \frac{3\,(d^4x^4e^c \right. \\ + \frac{1}{12}\left(3\,bx^4 + 4\,ax^3\right)\cosh\left(dx + c\right)$$

input
```
integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/24*d*(4*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
 + 4*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^4*x^4*e^c
 - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b*e^(d*x)/d^5 + 3
*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b*e^(-d*x - c)/d^5) + 1/
12*(3*b*x^4 + 4*a*x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \frac{(bd^3x^3 + ad^3x^2 - 3\,bd^2x^2 - 2\,ad^2x + 6\,bdx + 2\,ad - 6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3 + ad^3x^2 + 3\,bd^2x^2 + 2\,ad^2x + 6\,bdx + 2\,ad + 6\,b)e^{(-dx-c)}}{2\,d^4}$$

input
```
integrate(x^2*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^3*x^3 + a*d^3*x^2 - 3*b*d^2*x^2 - 2*a*d^2*x + 6*b*d*x + 2*a*d - 6
*b)*e^(d*x + c)/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x^2 + 3*b*d^2*x^2 + 2*a*d^2*x
 + 6*b*d*x + 2*a*d + 6*b)*e^(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int x^2(a+bx)\cosh(c+dx)\,dx = \frac{2\,a\sinh(c+d\,x)+6\,b\,x\sinh(c+d\,x)}{d^3} - \frac{2\,a\,x\cosh(c+d\,x)+3\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{a\,x^2\sinh(c+d\,x)+b\,x^3\sinh(c+d\,x)}{d} - \frac{6\,b\cosh(c+d\,x)}{d^4}$$

input
```
int(x^2*cosh(c + d*x)*(a + b*x),x)
```

output
```
(2*a*sinh(c + d*x) + 6*b*x*sinh(c + d*x))/d^3 - (2*a*x*cosh(c + d*x) + 3*b
*x^2*cosh(c + d*x))/d^2 + (a*x^2*sinh(c + d*x) + b*x^3*sinh(c + d*x))/d -
(6*b*cosh(c + d*x))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

$$\int x^2(a+bx)\cosh(c+dx)\,dx$$
$$= \frac{-2\cosh\left(dx+c\right)a\,d^2x-3\cosh\left(dx+c\right)b\,d^2x^2-6\cosh\left(dx+c\right)b+\sinh\left(dx+c\right)a\,d^3x^2+2\sinh\left(dx+\right.}{d^4}$$

input
```
int(x^2*(b*x+a)*cosh(d*x+c),x)
```

output
```
( - 2*cosh(c + d*x)*a*d**2*x - 3*cosh(c + d*x)*b*d**2*x**2 - 6*cosh(c + d*
x)*b + sinh(c + d*x)*a*d**3*x**2 + 2*sinh(c + d*x)*a*d + sinh(c + d*x)*b*d
**3*x**3 + 6*sinh(c + d*x)*b*d*x)/d**4
```

3.3 $\int x(a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int x(a+bx)\cosh(c+dx)\,dx = -\frac{a\cosh(c+dx)}{d^2}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}$$

output
```
-a*cosh(d*x+c)/d^2-2*b*x*cosh(d*x+c)/d^2+2*b*sinh(d*x+c)/d^3+a*x*sinh(d*x+
c)/d+b*x^2*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.70

$$\int x(a+bx)\cosh(c+dx)\,dx = \frac{-d(a+2bx)\cosh(c+dx)+(ad^2x+b(2+d^2x^2))\sinh(c+dx)}{d^3}$$

input
```
Integrate[x*(a + b*x)*Cosh[c + d*x],x]
```

output
```
(-(d*(a + 2*b*x)*Cosh[c + d*x]) + (a*d^2*x + b*(2 + d^2*x^2))*Sinh[c + d*x
])/d^3
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(ax\cosh(c+dx)+bx^2\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

input
```
Int[x*(a + b*x)*Cosh[c + d*x],x]
```

output
```
-((a*Cosh[c + d*x])/d^2) - (2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])
/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^2*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result
parallelrisch	$\frac{2x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}bd+2\left(\left(-b\,x^{2}-ax\right)d^{2}-2b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2(bx+a)d}{d^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b\,d^{2}x^{2}+a\,d^{2}x-2dxb-ad+2b\right){\mathrm e}^{dx+c}}{2d^{3}}-\frac{\left(b\,d^{2}x^{2}+a\,d^{2}x+2dxb+ad+2b\right){\mathrm e}^{-dx-c}}{2d^{3}}$
parts	$\frac{b\,x^{2}\sinh(dx+c)}{d}+\frac{ax\sinh(dx+c)}{d}-\frac{\frac{2b((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d}-\frac{2bc\cosh(dx+c)}{d}+a\cosh(dx+c)}{d^{2}}$
orering	$-\frac{2(2bx+a)\left(b\,d^{2}x^{2}+a\,d^{2}x+b\right)\cosh(dx+c)}{d^{4}x(bx+a)}+\frac{\left(b\,d^{2}x^{2}+a\,d^{2}x+2b\right)\left((bx+a)\cosh(dx+c)+xb\cosh(dx+c)+x(bx+a)d\,\mathrm{si}\right.}{d^{4}x(bx+a)}$
derivativedivides	$\frac{\frac{b\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+a((dx+c)\sinh(dx+c)-\mathrm{cos}}{d^{2}}$
default	$\frac{\frac{b\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}+a((dx+c)\sinh(dx+c)-\mathrm{cos}}{d^{2}}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^{2}d^{2}}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^{3}}+\frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^{2}d^{2}}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^{3}}+$

input
```
int(x*(b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*(x*tanh(1/2*d*x+1/2*c)^2*b*d+((-b*x^2-a*x)*d^2-2*b)*tanh(1/2*d*x+1/2*c)+
(b*x+a)*d)/d^3/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.75

$$\int x(a+bx)\cosh(c+dx)\,dx = -\frac{(2\,bdx+ad)\cosh\left(dx+c\right)-\left(bd^{2}x^{2}+ad^{2}x+2\,b\right)\sinh\left(dx+c\right)}{d^{3}}$$

input
```
integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((2*b*d*x + a*d)*cosh(d*x + c) - (b*d^2*x^2 + a*d^2*x + 2*b)*sinh(d*x + c
))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.28

$$\int x(a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{ax\sinh(c+dx)}{d} - \frac{a\cosh(c+dx)}{d^2} + \frac{bx^2\sinh(c+dx)}{d} - \frac{2bx\cosh(c+dx)}{d^2} + \frac{2b\sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(\frac{ax^2}{2} + \frac{bx^3}{3}\right)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**2*sinh(c + d*
x)/d - 2*b*x*cosh(c + d*x)/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x
**2/2 + b*x**3/3)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(64) = 128.

Time = 0.05 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.50

$$\int x(a+bx)\cosh(c+dx)\,dx = -\frac{1}{12}d\left(\frac{3\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)ae^{(dx)}}{d^3} + \frac{3\,(d^2x^2 + 2\,dx + 2)ae^{(-dx-c)}}{d^3} + \frac{2\,(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c}{d^4}\right. + \frac{1}{6}\left(2\,bx^3 + 3\,ax^2\right)\cosh(dx+c)$$

input
```
integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/12*d*(3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 3*(d^2*x^2 +
2*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c
 - 6*e^c)*b*e^(d*x)/d^4 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*b*e^(-d*x -
c)/d^4) + 1/6*(2*b*x^3 + 3*a*x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.23

$$\int x(a+bx)\cosh(c+dx)\,dx = \frac{(bd^2x^2+ad^2x-2\,bdx-ad+2\,b)e^{(dx+c)}}{2\,d^3} - \frac{(bd^2x^2+ad^2x+2\,bdx+ad+2\,b)e^{(-dx-c)}}{2\,d^3}$$

input
```
integrate(x*(b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^2*x^2 + a*d^2*x - 2*b*d*x - a*d + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d
^2*x^2 + a*d^2*x + 2*b*d*x + a*d + 2*b)*e^(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.97

$$\int x(a+bx)\cosh(c+dx)\,dx = \frac{b\,x^2\sinh(c+d\,x)+a\,x\sinh(c+d\,x)}{d} - \frac{a\cosh(c+d\,x)+2\,b\,x\cosh(c+d\,x)}{d^2} + \frac{2\,b\sinh(c+d\,x)}{d^3}$$

input
```
int(x*cosh(c + d*x)*(a + b*x),x)
```

output
```
(b*x^2*sinh(c + d*x) + a*x*sinh(c + d*x))/d - (a*cosh(c + d*x) + 2*b*x*cos
h(c + d*x))/d^2 + (2*b*sinh(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int x(a+bx)\cosh(c+dx)\,dx = \frac{-\cosh(dx+c)\,ad-2\cosh(dx+c)\,bdx+\sinh(dx+c)\,a\,d^2x+\sinh(dx+c)\,b\,d^2x^2+2b\sinh(dx+c)}{d^3}$$

input
```
int(x*(b*x+a)*cosh(d*x+c),x)
```

output
```
( - cosh(c + d*x)*a*d - 2*cosh(c + d*x)*b*d*x + sinh(c + d*x)*a*d**2*x + s
inh(c + d*x)*b*d**2*x**2 + 2*sinh(c + d*x)*b)/d**3
```

3.4 $\int (a+bx)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 28

$$\int (a+bx)\cosh(c+dx)\,dx = -\frac{b\cosh(c+dx)}{d^2} + \frac{(a+bx)\sinh(c+dx)}{d}$$

output
```
-b*cosh(d*x+c)/d^2+(b*x+a)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.96

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{-b\cosh(c+dx) + d(a+bx)\sinh(c+dx)}{d^2}$$

input
```
Integrate[(a + b*x)*Cosh[c + d*x],x]
```

output
```
(-(b*Cosh[c + d*x]) + d*(a + b*x)*Sinh[c + d*x])/d^2
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3777, 26, 3042, 26, 3118}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+bx)\sin\left(ic+idx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a+bx)\sinh(c+dx)}{d}-\frac{ib\int -i\sinh(c+dx)dx}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a+bx)\sinh(c+dx)}{d}-\frac{b\int \sinh(c+dx)dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+bx)\sinh(c+dx)}{d}-\frac{b\int -i\sin(ic+idx)dx}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a+bx)\sinh(c+dx)}{d}+\frac{ib\int \sin(ic+idx)dx}{d}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(a+bx)\sinh(c+dx)}{d}-\frac{b\cosh(c+dx)}{d^2}$$

input
```
Int[(a + b*x)*Cosh[c + d*x],x]
```

output
```
-((b*Cosh[c + d*x])/d^2) + ((a + b*x)*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{(bx+a)d\sinh(dx+c)-b(1+\cosh(dx+c))}{d^2}$
parts	$\frac{bx\sinh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}-\frac{b\cosh(dx+c)}{d^2}$
orering	$-\frac{2b\cosh(dx+c)}{d^2}+\frac{b\cosh(dx+c)+(bx+a)d\sinh(dx+c)}{d^2}$
risch	$\frac{(dxb+ad-b)e^{dx+c}}{2d^2}-\frac{(dxb+ad+b)e^{-dx-c}}{2d^2}$
derivativedivides	$\frac{\frac{b((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{bc\sinh(dx+c)}{d}+a\sinh(dx+c)}{d}$
default	$\frac{\frac{b((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d}-\frac{bc\sinh(dx+c)}{d}+a\sinh(dx+c)}{d}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+\frac{a\cosh(c)\sinh(dx)}{d}-\frac{a\sin$

input
```
int((b*x+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
((b*x+a)*d*sinh(d*x+c)-b*(1+cosh(d*x+c)))/d^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.07

$$\int (a+bx)\cosh(c+dx)\,dx = -\frac{b\cosh(dx+c)-(bdx+ad)\sinh(dx+c)}{d^2}$$

input
```
integrate((b*x+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(b*cosh(d*x + c) - (b*d*x + a*d)*sinh(d*x + c))/d^2
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (a+bx)\cosh(c+dx)\,dx = \begin{cases} \frac{a\sinh(c+dx)}{d}+\frac{bx\sinh(c+dx)}{d}-\frac{b\cosh(c+dx)}{d^2} & \text{for } d\neq 0 \\ \left(ax+\frac{bx^2}{2}\right)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*sinh(c + d*x)/d + b*x*sinh(c + d*x)/d - b*cosh(c + d*x)/d**2,
 Ne(d, 0)), ((a*x + b*x**2/2)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.43

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{ae^{(dx+c)}}{2\,d} + \frac{(dxe^c - e^c)be^{(dx)}}{2\,d^2} - \frac{(dx+1)be^{(-dx-c)}}{2\,d^2} - \frac{ae^{(-dx-c)}}{2\,d}$$

input
```
integrate((b*x+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a*e^(d*x + c)/d + 1/2*(d*x*e^c - e^c)*b*e^(d*x)/d^2 - 1/2*(d*x + 1)*b*
e^(-d*x - c)/d^2 - 1/2*a*e^(-d*x - c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.64

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{(bdx+ad-b)e^{(dx+c)}}{2\,d^2} - \frac{(bdx+ad+b)e^{(-dx-c)}}{2\,d^2}$$

input
```
integrate((b*x+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d*x + a*d - b)*e^(d*x + c)/d^2 - 1/2*(b*d*x + a*d + b)*e^(-d*x - c)
/d^2
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.25

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{a\sinh(c+d\,x) + b\,x\sinh(c+d\,x)}{d} - \frac{b\cosh(c+d\,x)}{d^2}$$

input
```
int(cosh(c + d*x)*(a + b*x),x)
```

output
```
(a*sinh(c + d*x) + b*x*sinh(c + d*x))/d - (b*cosh(c + d*x))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

$$\int (a+bx)\cosh(c+dx)\,dx = \frac{-\cosh(dx+c)\,b+\sinh(dx+c)\,ad+\sinh(dx+c)\,bdx}{d^2}$$

input
```
int((b*x+a)*cosh(d*x+c),x)
```

output
```
( - cosh(c + d*x)*b + sinh(c + d*x)*a*d + sinh(c + d*x)*b*d*x)/d**2
```

3.5 $\int \frac{(a+bx)\cosh(c+dx)}{x} dx$

Optimal result

Integrand size = 15, antiderivative size = 28

$$\int \frac{(a+bx)\cosh(c+dx)}{x} dx = a\cosh(c)\text{Chi}(dx) + \frac{b\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)$$

output
```
a*cosh(c)*Chi(d*x)+b*sinh(d*x+c)/d+a*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)\cosh(c+dx)}{x} dx = a\cosh(c)\text{Chi}(dx) + \frac{b\cosh(dx)\sinh(c)}{d} + \frac{b\cosh(c)\sinh(dx)}{d} + a\sinh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x)*Cosh[c + d*x])/x,x]
```

output
```
a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*
x])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{a\cosh(c+dx)}{x}+b\cosh(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$a\cosh(c)\text{Chi}(dx)+a\sinh(c)\text{Shi}(dx)+\frac{b\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x)*Cosh[c + d*x])/x,x]
```

output
```
a*Cosh[c]*CoshIntegral[d*x] + (b*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral
[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{a\,\mathrm{e}^{c}\,\mathrm{expIntegral}_{1}(-dx)}{2}-\frac{a\,\mathrm{e}^{-c}\,\mathrm{expIntegral}_{1}(dx)}{2}+\frac{\mathrm{e}^{dx+c}b}{2d}-\frac{\mathrm{e}^{-dx-c}b}{2d}$
meijerg	$\frac{b\cosh(c)\sinh(dx)}{d}-\frac{b\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}+\frac{a\cosh(c)\sqrt{\pi}\left(\frac{2\ \mathrm{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+a\sinh(c)$

input
```
int((b*x+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*exp(c)*Ei(1,-d*x)-1/2*a*exp(-c)*Ei(1,d*x)+1/2/d*exp(d*x+c)*b-1/2/d*
exp(-d*x-c)*b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.93

$$\int\frac{(a+bx)\cosh(c+dx)}{x}\,dx = \frac{(ad\mathrm{Ei}(dx)+ad\mathrm{Ei}(-dx))\cosh(c)+2\,b\sinh(dx+c)+(ad\mathrm{Ei}(dx)-ad\mathrm{Ei}(-dx))\sinh(c)}{2\,d}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

output
```
1/2*((a*d*Ei(d*x) + a*d*Ei(-d*x))*cosh(c) + 2*b*sinh(d*x + c) + (a*d*Ei(d*
x) - a*d*Ei(-d*x))*sinh(c))/d
```

Sympy [A] (verification not implemented)

Time = 1.46 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = -a(-\sinh{(c)}\operatorname{Shi}{(dx)} - \cosh{(c)}\operatorname{Chi}{(dx)}) - b\left(\begin{cases} -x\cosh{(c)} & \text{for}\: d = 0 \\ -\frac{\sinh{(c+dx)}}{d} & \text{otherwise} \end{cases}\right)$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x,x)
```

output
```
-a*(-sinh(c)*Shi(d*x) - cosh(c)*Chi(d*x)) - b*Piecewise((-x*cosh(c), Eq(d,
 0)), (-sinh(c + d*x)/d, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.46

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = -\frac{1}{2}\left(b\left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right) + \frac{2\,a\cosh{(dx+c)}\log{(x)}}{d} - \frac{\left(\operatorname{Ei}(-dx)\,e^{(-c)} + \operatorname{Ei}(dx)\,e^{c}\right)a}{d}\right)d + (bx + a\log{(x)})\cosh{(dx+c)}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/2*(b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + 2*a*c
osh(d*x + c)*log(x)/d - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + (b*x + a*
log(x))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = \frac{ad\mathrm{Ei}(-dx)\,e^{(-c)} + ad\mathrm{Ei}(dx)\,e^{c} + be^{(dx+c)} - be^{(-dx-c)}}{2\,d}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(a*d*Ei(-d*x)*e^(-c) + a*d*Ei(d*x)*e^c + b*e^(d*x + c) - b*e^(-d*x - c
))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = a\,\mathrm{coshint}(d\,x)\,\cosh(c) + a\,\mathrm{sinhint}(d\,x)\,\sinh(c) + \frac{b\sinh(c+d\,x)}{d}$$

input
```
int((cosh(c + d*x)*(a + b*x))/x,x)
```

output
```
a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) + (b*sinh(c + d*x))/d
```

Reduce [F]

$$\int \frac{(a+bx)\cosh(c+dx)}{x}\,dx = \frac{\left(\int \frac{\cosh(dx+c)}{x}dx\right)ad + b\sinh\left(dx+c\right)}{d}$$

input
```
int((b*x+a)*cosh(d*x+c)/x,x)
```

output
```
(int(cosh(c + d*x)/x,x)*a*d + sinh(c + d*x)*b)/d
```

3.6 $\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 47

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c+dx)}{x} + b\cosh(c)\text{Chi}(dx) + ad\text{Chi}(dx)\sinh(c) + ad\cosh(c)\text{Shi}(dx) + b\sinh(c)\text{Shi}(dx)$$

output
```
-a*cosh(d*x+c)/x+b*cosh(c)*Chi(d*x)+a*d*Chi(d*x)*sinh(c)+a*d*cosh(c)*Shi(d
*x)+b*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c)\cosh(dx)}{x} + b\cosh(c)\text{Chi}(dx) - \frac{a\sinh(c)\sinh(dx)}{x} + b\sinh(c)\text{Shi}(dx) + ad(\text{Chi}(dx)\sinh(c) + \cosh(c)\text{Shi}(dx))$$

input
```
Integrate[((a + b*x)*Cosh[c + d*x])/x^2,x]
```

output
```
-((a*Cosh[c]*Cosh[d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] - (a*Sinh[c]*Sinh
[d*x])/x + b*Sinh[c]*SinhIntegral[d*x] + a*d*(CoshIntegral[d*x]*Sinh[c] +
Cosh[c]*SinhIntegral[d*x])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^2}+\frac{b\cosh(c+dx)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\sinh(c)\text{Chi}(dx)+ad\cosh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{x}+b\cosh(c)\text{Chi}(dx)+b\sinh(c)\text{Shi}(dx)$$

input
```
Int[((a + b*x)*Cosh[c + d*x])/x^2,x]
```

output
```
-((a*Cosh[c + d*x])/x) + b*Cosh[c]*CoshIntegral[d*x] + a*d*CoshIntegral[d*
x]*Sinh[c] + a*d*Cosh[c]*SinhIntegral[d*x] + b*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{-e^{-c}\,\mathrm{expIntegral}_1(dx)adx+e^{c}\,\mathrm{expIntegral}_1(-dx)adx+e^{-c}\,\mathrm{expIntegral}_1(dx)bx+e^{c}\,\mathrm{expIntegral}_1(-dx)bx+e^{-dx-c}a+e^{dx+c}a}{2x}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\,\mathrm{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+b\sinh(c)\,\mathrm{Shi}(dx)+\frac{ia\cosh(c)\sqrt{\pi}\,d\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{Shi}(dx)}{\sqrt{\pi}}\right)}{4}$

input
```
int((b*x+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-exp(-c)*Ei(1,d*x)*a*d*x+exp(c)*Ei(1,-d*x)*a*d*x+exp(-c)*Ei(1,d*x)*b
*x+exp(c)*Ei(1,-d*x)*b*x+exp(-d*x-c)*a+exp(d*x+c)*a)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{2\,a\cosh(dx+c)-((ad+b)x\mathrm{Ei}(dx)-(ad-b)x\mathrm{Ei}(-dx))\cosh(c)-((ad+b)x\mathrm{Ei}(dx)+(ad-b)x\mathrm{Ei}}{2\,x}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*cosh(d*x + c) - ((a*d + b)*x*Ei(d*x) - (a*d - b)*x*Ei(-d*x))*cos
h(c) - ((a*d + b)*x*Ei(d*x) + (a*d - b)*x*Ei(-d*x))*sinh(c))/x
```

Sympy [F]

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx)\cosh{(c+dx)}}{x^2}\,dx$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x)*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{1}{2}\left(\left(\mathrm{Ei}(-dx)\,e^{(-c)} - \mathrm{Ei}(dx)\,e^{c}\right)a + \frac{2\,b\cosh{(dx+c)}\log{(x)}}{d} - \frac{\left(\mathrm{Ei}(-dx)\,e^{(-c)} + \mathrm{Ei}(dx)\,e^{c}\right)b}{d}\right)d + \left(b\log{(x)} - \frac{a}{x}\right)\cosh{(dx+c)}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
-1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a + 2*b*cosh(d*x + c)*log(x)/d - (Ei
(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + (b*log(x) - a/x)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.53

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = -\frac{adx\mathrm{Ei}(-dx)\,e^{(-c)} - adx\mathrm{Ei}(dx)\,e^{c} - bx\mathrm{Ei}(-dx)\,e^{(-c)} - bx\mathrm{Ei}(dx)\,e^{c} + ae^{(dx+c)} + ae^{(-dx-c)}}{2\,x}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c - b*x*Ei(-d*x)*e^(-c) - b*
x*Ei(d*x)*e^c + a*e^(d*x + c) + a*e^(-d*x - c))/x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,(a+b\,x)}{x^2}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x))/x^2,x)
```

output
```
int((cosh(c + d*x)*(a + b*x))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.96

$$\int \frac{(a+bx)\cosh(c+dx)}{x^2}\,dx = \frac{-e^{dx}ei(-dx)\,adx + e^{dx}ei(-dx)\,bx + e^{dx+2c}ei(dx)\,adx + e^{dx+2c}ei(dx)\,bx - e^{2dx+2c}a - a}{2e^{dx+c}x}$$

input
```
int((b*x+a)*cosh(d*x+c)/x^2,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a*d*x + e**(d*x)*ei( - d*x)*b*x + e**(2*c + d*x)*e
i(d*x)*a*d*x + e**(2*c + d*x)*ei(d*x)*b*x - e**(2*c + 2*d*x)*a - a)/(2*e**
(c + d*x)*x)
```

3.7 $\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 88

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = -\frac{a\cosh(c+dx)}{2x^2} - \frac{b\cosh(c+dx)}{x} + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + bd\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{2x} + bd\cosh(c)\text{Shi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

output
```
-1/2*a*cosh(d*x+c)/x^2-b*cosh(d*x+c)/x+1/2*a*d^2*cosh(c)*Chi(d*x)+b*d*Chi(
d*x)*sinh(c)-1/2*a*d*sinh(d*x+c)/x+b*d*cosh(c)*Shi(d*x)+1/2*a*d^2*sinh(c)*
Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \frac{-a\cosh(c+dx) - 2bx\cosh(c+dx) + dx^2\text{Chi}(dx)(ad\cosh(c) + 2b\sinh(c)) - adx\sinh(c+dx) + dx^2(2\cdots}{2x^2}$$

input
```
Integrate[((a + b*x)*Cosh[c + d*x])/x^3,x]
```

output
```
(-(a*Cosh[c + d*x]) - 2*b*x*Cosh[c + d*x] + d*x^2*CoshIntegral[d*x]*(a*d*C
osh[c] + 2*b*Sinh[c]) - a*d*x*Sinh[c + d*x] + d*x^2*(2*b*Cosh[c] + a*d*Sin
h[c])*SinhIntegral[d*x])/(2*x^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+bd\sinh(c)\text{Chi}(dx)+bd\cosh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{x}$$

input
```
Int[((a + b*x)*Cosh[c + d*x])/x^3,x]
```

output
```
-1/2*(a*Cosh[c + d*x])/x^2 - (b*Cosh[c + d*x])/x + (a*d^2*Cosh[c]*CoshInte
gral[d*x])/2 + b*d*CoshIntegral[d*x]*Sinh[c] - (a*d*Sinh[c + d*x])/(2*x) +
 b*d*Cosh[c]*SinhIntegral[d*x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.56

method	result
risch	$-\frac{e^{c} \operatorname{expIntegral}_{1}(-dx) a\, d^{2} x^{2}+e^{-c} \operatorname{expIntegral}_{1}(dx) a\, d^{2} x^{2}+2\, e^{c} \operatorname{expIntegral}_{1}(-dx) b d\, x^{2}-2\, e^{-c} \operatorname{expIntegral}_{1}(dx) b d\, x^{2}+a d x\, e^{dx+}}{4x^{2}}$
meijerg	$\frac{i d b \cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\operatorname{Shi}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{d b\sinh(c)\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}}-\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}+\frac{4\ \operatorname{Chi}(dx)-4\ln(dx)-4\gamma}{\sqrt{\pi}}+\frac{4\gamma-4+4\ln(x)+4\ln(id)}{\sqrt{\pi}}\right)}{4}-\frac{a\operatorname{co}}{}$

input
```
int((b*x+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(exp(c)*Ei(1,-d*x)*a*d^2*x^2+exp(-c)*Ei(1,d*x)*a*d^2*x^2+2*exp(c)*Ei(
1,-d*x)*b*d*x^2-2*exp(-c)*Ei(1,d*x)*b*d*x^2+a*d*x*exp(d*x+c)-a*d*x*exp(-d*
x-c)+2*exp(d*x+c)*b*x+2*exp(-d*x-c)*b*x+exp(d*x+c)*a+exp(-d*x-c)*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.32

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = -\frac{2\,adx\sinh\left(dx+c\right)+2\left(2\,bx+a\right)\cosh\left(dx+c\right)-\left((ad^2+2\,bd)x^2\mathrm{Ei}(dx)+(ad^2-2\,bd)x^2\mathrm{Ei}(-dx)\right)\cos}{4\,x^2}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*d*x*sinh(d*x + c) + 2*(2*b*x + a)*cosh(d*x + c) - ((a*d^2 + 2*b*
d)*x^2*Ei(d*x) + (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b*d)*
x^2*Ei(d*x) - (a*d^2 - 2*b*d)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \frac{1}{4}\left(ade^{(-c)}\Gamma(-1,dx) + ade^{c}\Gamma(-1,-dx) - 2\,b\mathrm{Ei}(-dx)\,e^{(-c)} + 2\,b\mathrm{Ei}(dx)\,e^{c}\right)d - \frac{(2\,bx+a)\cosh(dx+c)}{2\,x^2}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*b*Ei(-d*x)*e^
(-c) + 2*b*Ei(d*x)*e^c)*d - 1/2*(2*b*x + a)*cosh(d*x + c)/x^2
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{ad^2x^2\text{Ei}(-dx)\,e^{(-c)} + ad^2x^2\text{Ei}(dx)\,e^{c} - 2\,bdx^2\text{Ei}(-dx)\,e^{(-c)} + 2\,bdx^2\text{Ei}(dx)\,e^{c} - adxe^{(dx+c)} + adxe^{(-dx-}}{4\,x^2}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c - 2*b*d*x^2*Ei(-d*x
)*e^(-c) + 2*b*d*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c)
- 2*b*x*e^(d*x + c) - 2*b*x*e^(-d*x - c) - a*e^(d*x + c) - a*e^(-d*x - c))
/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^3}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x))/x^3,x)
```

output
```
int((cosh(c + d*x)*(a + b*x))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx)\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{e^{dx}\,ei(-dx)\,a\,d^2x^2 - 2e^{dx}\,ei(-dx)\,bd\,x^2 + e^{dx+2c}\,ei(dx)\,a\,d^2x^2 + 2e^{dx+2c}\,ei(dx)\,bd\,x^2 - e^{2dx+2c}adx - e^{2dx+}}{4e^{dx+c}x^2}$$

input
```
int((b*x+a)*cosh(d*x+c)/x^3,x)
```

output
```
(e**(d*x)*ei( - d*x)*a*d**2*x**2 - 2*e**(d*x)*ei( - d*x)*b*d*x**2 + e**(2*
c + d*x)*ei(d*x)*a*d**2*x**2 + 2*e**(2*c + d*x)*ei(d*x)*b*d*x**2 - e**(2*c
 + 2*d*x)*a*d*x - e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b*x + a*d*x - a
- 2*b*x)/(4*e**(c + d*x)*x**2)
```

3.8 $\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 132

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = -\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{2x^2}-\frac{ad^2\cosh(c+dx)}{6x}+\frac{1}{2}bd^2\cosh(c)\text{Chi}(dx)+\frac{1}{6}ad^3\text{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{6x^2}-\frac{bd\sinh(c+dx)}{2x}+\frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)+\frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)$$

output
```
-1/3*a*cosh(d*x+c)/x^3-1/2*b*cosh(d*x+c)/x^2-1/6*a*d^2*cosh(d*x+c)/x+1/2*b
*d^2*cosh(c)*Chi(d*x)+1/6*a*d^3*Chi(d*x)*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2-1
/2*b*d*sinh(d*x+c)/x+1/6*a*d^3*cosh(c)*Shi(d*x)+1/2*b*d^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{2a\cosh(c+dx)+3bx\cosh(c+dx)+ad^2x^2\cosh(c+dx)-d^2x^3\text{Chi}(dx)(3b\cosh(c)+ad\sinh(c))+a}{6x^3}$$

input
```
Integrate[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

output
```
-1/6*(2*a*Cosh[c + d*x] + 3*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] -
d^2*x^3*CoshIntegral[d*x]*(3*b*Cosh[c] + a*d*Sinh[c]) + a*d*x*Sinh[c + d*x
] + 3*b*d*x^2*Sinh[c + d*x] - d^2*x^3*(a*d*Cosh[c] + 3*b*Sinh[c])*SinhInte
gral[d*x])/x^3
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{6}ad^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{a\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{6x^2}+\frac{1}{2}bd^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{2x^2}-\frac{bd\sinh(c+dx)}{2x}$$

input
```
Int[((a + b*x)*Cosh[c + d*x])/x^4,x]
```

output
```
-1/3*(a*Cosh[c + d*x])/x^3 - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d
*x])/(6*x) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^3*CoshIntegral[d*x
]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(6*x^2) - (b*d*Sinh[c + d*x])/(2*x) + (
a*d^3*Cosh[c]*SinhIntegral[d*x])/6 + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 204, normalized size of antiderivative = 1.55

method	result
risch	$-\frac{-e^{-c}\,\text{expIntegral}_1(dx)a\,d^3x^3+e^{c}\,\text{expIntegral}_1(-dx)a\,d^3x^3+3\,e^{-c}\,\text{expIntegral}_1(dx)b\,d^2x^3+3\,e^{c}\,\text{expIntegral}_1(-dx)b\,d^2x^3+a\,d^2}{12}$
meijerg	$-\frac{d^2b\cosh(c)\sqrt{\pi}\left(-\frac{4\left(\frac{9x^2d^2}{2}+3\right)}{3\sqrt{\pi}\,x^2d^2}+\frac{4\cosh(dx)}{\sqrt{\pi}\,x^2d^2}+\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}-\frac{4(\text{Chi}(dx)-\ln(dx)-\gamma)}{\sqrt{\pi}}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}\,x^2d^2}\right)}{8}+\frac{id^2b\sin}{}$

input
```
int((b*x+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-exp(-c)*Ei(1,d*x)*a*d^3*x^3+exp(c)*Ei(1,-d*x)*a*d^3*x^3+3*exp(-c)*
Ei(1,d*x)*b*d^2*x^3+3*exp(c)*Ei(1,-d*x)*b*d^2*x^3+a*d^2*x^2*exp(-d*x-c)+a*
d^2*x^2*exp(d*x+c)-3*b*d*x^2*exp(-d*x-c)+3*b*d*x^2*exp(d*x+c)-a*d*x*exp(-d
*x-c)+a*d*x*exp(d*x+c)+3*exp(-d*x-c)*b*x+3*exp(d*x+c)*b*x+2*exp(-d*x-c)*a+
2*exp(d*x+c)*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = -\frac{2\,(ad^2x^2+3\,bx+2\,a)\cosh\left(dx+c\right)-((ad^3+3\,bd^2)x^3\mathrm{Ei}(dx)-(ad^3-3\,bd^2)x^3\mathrm{Ei}(-dx))\cosh\left(c\right)+2}{12\,x^3}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(2*(a*d^2*x^2 + 3*b*x + 2*a)*cosh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*
Ei(d*x) - (a*d^3 - 3*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2*(3*b*d*x^2 + a*d*x)*
sinh(d*x + c) - ((a*d^3 + 3*b*d^2)*x^3*Ei(d*x) + (a*d^3 - 3*b*d^2)*x^3*Ei(
-d*x))*sinh(c))/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.59

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx$$
$$= \frac{1}{12}\left(2\,ad^2e^{(-c)}\Gamma(-2,dx)-2\,ad^2e^{c}\Gamma(-2,-dx)+3\,bde^{(-c)}\Gamma(-1,dx)+3\,bde^{c}\Gamma(-1,-dx)\right)d - \frac{(3\,bx+2\,a)\cosh\left(dx+c\right)}{6\,x^3}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
1/12*(2*a*d^2*e^(-c)*gamma(-2, d*x) - 2*a*d^2*e^c*gamma(-2, -d*x) + 3*b*d*
e^(-c)*gamma(-1, d*x) + 3*b*d*e^c*gamma(-1, -d*x))*d - 1/6*(3*b*x + 2*a)*c
osh(d*x + c)/x^3
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.51

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = -\frac{ad^3x^3\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x^3\mathrm{Ei}(dx)\,e^{c} - 3\,bd^2x^3\mathrm{Ei}(-dx)\,e^{(-c)} - 3\,bd^2x^3\mathrm{Ei}(dx)\,e^{c} + ad^2x^2e^{(dx+c)} + ad^2}{}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

output
```
-1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c - 3*b*d^2*x^3*Ei(
-d*x)*e^(-c) - 3*b*d^2*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2
*e^(-d*x - c) + 3*b*d*x^2*e^(d*x + c) - 3*b*d*x^2*e^(-d*x - c) + a*d*x*e^(
d*x + c) - a*d*x*e^(-d*x - c) + 3*b*x*e^(d*x + c) + 3*b*x*e^(-d*x - c) + 2
*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^4}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x))/x^4,x)
```

output
```
int((cosh(c + d*x)*(a + b*x))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)\cosh(c+dx)}{x^4}\,dx$$
$$= \frac{-e^{dx}ei(-dx)\,a\,d^3x^3+3e^{dx}ei(-dx)\,b\,d^2x^3+e^{dx+2c}ei(dx)\,a\,d^3x^3+3e^{dx+2c}ei(dx)\,b\,d^2x^3-e^{2dx+2c}a\,d^2x^2-}{12e^{dx+c}x^3}$$

input
```
int((b*x+a)*cosh(d*x+c)/x^4,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a*d**3*x**3 + 3*e**(d*x)*ei( - d*x)*b*d**2*x**3 +
e**(2*c + d*x)*ei(d*x)*a*d**3*x**3 + 3*e**(2*c + d*x)*ei(d*x)*b*d**2*x**3
- e**(2*c + 2*d*x)*a*d**2*x**2 - e**(2*c + 2*d*x)*a*d*x - 2*e**(2*c + 2*d*
x)*a - 3*e**(2*c + 2*d*x)*b*d*x**2 - 3*e**(2*c + 2*d*x)*b*x - a*d**2*x**2
+ a*d*x - 2*a + 3*b*d*x**2 - 3*b*x)/(12*e**(c + d*x)*x**3)
```

3.9 $\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 166

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{3x^3} - \frac{ad^2\cosh(c+dx)}{24x^2} - \frac{bd^2\cosh(c+dx)}{6x} + \frac{1}{24}ad^4\cosh(c)\text{Chi}(dx) + \frac{1}{6}bd^3\text{Chi}(dx)\sinh(c) - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{6x^2} - \frac{ad^3\sinh(c+dx)}{24x} + \frac{1}{6}bd^3\cosh(c)\text{Shi}(dx) + \frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)$$

output
```
-1/4*a*cosh(d*x+c)/x^4-1/3*b*cosh(d*x+c)/x^3-1/24*a*d^2*cosh(d*x+c)/x^2-1/
6*b*d^2*cosh(d*x+c)/x+1/24*a*d^4*cosh(c)*Chi(d*x)+1/6*b*d^3*Chi(d*x)*sinh(
c)-1/12*a*d*sinh(d*x+c)/x^3-1/6*b*d*sinh(d*x+c)/x^2-1/24*a*d^3*sinh(d*x+c)
/x+1/6*b*d^3*cosh(c)*Shi(d*x)+1/24*a*d^4*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.84

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = -\frac{6a\cosh(c+dx)+8bx\cosh(c+dx)+ad^2x^2\cosh(c+dx)+4bd^2x^3\cosh(c+dx)-d^3x^4\text{Chi}(dx)(ad\,\text{co}}{}$$

input
```
Integrate[((a + b*x)*Cosh[c + d*x])/x^5,x]
```

output
```
-1/24*(6*a*Cosh[c + d*x] + 8*b*x*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x] +
 4*b*d^2*x^3*Cosh[c + d*x] - d^3*x^4*CoshIntegral[d*x]*(a*d*Cosh[c] + 4*b*
Sinh[c]) + 2*a*d*x*Sinh[c + d*x] + 4*b*d*x^2*Sinh[c + d*x] + a*d^3*x^3*Sin
h[c + d*x] - d^3*x^4*(4*b*Cosh[c] + a*d*Sinh[c])*SinhIntegral[d*x])/x^4
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{a\cosh(c+dx)}{x^5}+\frac{b\cosh(c+dx)}{x^4}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{a\cosh(c+dx)}{4x^4}-\frac{ad\sinh(c+dx)}{12x^3}+\frac{1}{6}bd^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}bd^3\cosh(c)\text{Shi}(dx)-\frac{bd^2\cosh(c+dx)}{6x}-\frac{b\cosh(c+dx)}{3x^3}-\frac{bd\sinh(c+dx)}{6x^2}$$

input
```
Int[((a + b*x)*Cosh[c + d*x])/x^5,x]
```

output
```
-1/4*(a*Cosh[c + d*x])/x^4 - (b*Cosh[c + d*x])/(3*x^3) - (a*d^2*Cosh[c + d
*x])/(24*x^2) - (b*d^2*Cosh[c + d*x])/(6*x) + (a*d^4*Cosh[c]*CoshIntegral[
d*x])/24 + (b*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(12*x
^3) - (b*d*Sinh[c + d*x])/(6*x^2) - (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^3*
Cosh[c]*SinhIntegral[d*x])/6 + (a*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.64

method	result
risch	$\frac{-\mathrm{e}^{-c}\operatorname{expIntegral}_1(dx)a\,d^4x^4-\mathrm{e}^{c}\operatorname{expIntegral}_1(-dx)a\,d^4x^4+4\,\mathrm{e}^{-c}\operatorname{expIntegral}_1(dx)b\,d^3x^4-4\,\mathrm{e}^{c}\operatorname{expIntegral}_1(-dx)b\,d^3x^4+a\,d^3x^3\mathrm{e}}{}$
meijerg	$-\frac{id^3b\cosh(c)\sqrt{\pi}\left(-\frac{8i\left(x^2d^2+2\right)\cosh(dx)}{3d^3x^3\sqrt{\pi}}-\frac{8i\sinh(dx)}{3x^2d^2\sqrt{\pi}}+\frac{8i\operatorname{Shi}(dx)}{3\sqrt{\pi}}\right)}{16}-\frac{d^3b\sinh(c)\sqrt{\pi}\left(-\frac{8\left(\frac{55x^2d^2}{2}+45\right)}{45\sqrt{\pi}\,x^2d^2}+\frac{8\cosh(dx)}{3\sqrt{\pi}\,x^2d^2}+\frac{16\left(\frac{5x^2d^2}{2}+\right.}{15\sqrt{\pi}}\right.}{}$

input
```
int((b*x+a)*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(-exp(-c)*Ei(1,d*x)*a*d^4*x^4-exp(c)*Ei(1,-d*x)*a*d^4*x^4+4*exp(-c)*E
i(1,d*x)*b*d^3*x^4-4*exp(c)*Ei(1,-d*x)*b*d^3*x^4+a*d^3*x^3*exp(-d*x-c)-a*d
^3*x^3*exp(d*x+c)-4*b*d^2*x^3*exp(-d*x-c)-4*b*d^2*x^3*exp(d*x+c)-a*d^2*x^2
*exp(-d*x-c)-a*d^2*x^2*exp(d*x+c)+4*b*d*x^2*exp(-d*x-c)-4*b*d*x^2*exp(d*x+
c)+2*a*d*x*exp(-d*x-c)-2*a*d*x*exp(d*x+c)-8*exp(-d*x-c)*b*x-8*exp(d*x+c)*b
*x-6*exp(-d*x-c)*a-6*exp(d*x+c)*a)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.97

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = -\frac{2\left(4\,bd^2x^3+ad^2x^2+8\,bx+6\,a\right)\cosh\left(dx+c\right)-\left((ad^4+4\,bd^3)x^4\mathrm{Ei}(dx)+(ad^4-4\,bd^3)x^4\mathrm{Ei}(-dx)\right)\mathrm{co}}{}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
-1/48*(2*(4*b*d^2*x^3 + a*d^2*x^2 + 8*b*x + 6*a)*cosh(d*x + c) - ((a*d^4 +
 4*b*d^3)*x^4*Ei(d*x) + (a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*cosh(c) + 2*(a*d^3
*x^3 + 4*b*d*x^2 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 4*b*d^3)*x^4*Ei(d*x)
 - (a*d^4 - 4*b*d^3)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x**5,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.49

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{1}{24}\left(3\,ad^3e^{(-c)}\Gamma(-3,dx)+3\,ad^3e^{c}\Gamma(-3,-dx)+4\,bd^2e^{(-c)}\Gamma(-2,dx)-4\,bd^2e^{c}\Gamma(-2,-dx)\right)d$$
$$-\frac{(4\,bx+3\,a)\cosh\left(dx+c\right)}{12\,x^4}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
1/24*(3*a*d^3*e^(-c)*gamma(-3, d*x) + 3*a*d^3*e^c*gamma(-3, -d*x) + 4*b*d^
2*e^(-c)*gamma(-2, d*x) - 4*b*d^2*e^c*gamma(-2, -d*x))*d - 1/12*(4*b*x + 3
*a)*cosh(d*x + c)/x^4
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.60

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{ad^4x^4\mathrm{Ei}(-dx)\,e^{(-c)} + ad^4x^4\mathrm{Ei}(dx)\,e^c - 4\,bd^3x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 4\,bd^3x^4\mathrm{Ei}(dx)\,e^c - ad^3x^3e^{(dx+c)} + ad^3x^3e}{}$$

input
```
integrate((b*x+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

output
```
1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c - 4*b*d^3*x^4*Ei(-
d*x)*e^(-c) + 4*b*d^3*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^3*
e^(-d*x - c) - 4*b*d^2*x^3*e^(d*x + c) - 4*b*d^2*x^3*e^(-d*x - c) - a*d^2*
x^2*e^(d*x + c) - a*d^2*x^2*e^(-d*x - c) - 4*b*d*x^2*e^(d*x + c) + 4*b*d*x
^2*e^(-d*x - c) - 2*a*d*x*e^(d*x + c) + 2*a*d*x*e^(-d*x - c) - 8*b*x*e^(d*
x + c) - 8*b*x*e^(-d*x - c) - 6*a*e^(d*x + c) - 6*a*e^(-d*x - c))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)}{x^5}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x))/x^5,x)
```

output
```
int((cosh(c + d*x)*(a + b*x))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{e^{dx}\,ei(-dx)\,a\,d^4x^4 - 4e^{dx}\,ei(-dx)\,b\,d^3x^4 + e^{dx+2c}\,ei(dx)\,a\,d^4x^4 + 4e^{dx+2c}\,ei(dx)\,b\,d^3x^4 - e^{2dx+2c}a\,d^3x^3 - e$$

input
```
int((b*x+a)*cosh(d*x+c)/x^5,x)
```

output
```
(e**(d*x)*ei( - d*x)*a*d**4*x**4 - 4*e**(d*x)*ei( - d*x)*b*d**3*x**4 + e**
(2*c + d*x)*ei(d*x)*a*d**4*x**4 + 4*e**(2*c + d*x)*ei(d*x)*b*d**3*x**4 - e
**(2*c + 2*d*x)*a*d**3*x**3 - e**(2*c + 2*d*x)*a*d**2*x**2 - 2*e**(2*c + 2
*d*x)*a*d*x - 6*e**(2*c + 2*d*x)*a - 4*e**(2*c + 2*d*x)*b*d**2*x**3 - 4*e*
*(2*c + 2*d*x)*b*d*x**2 - 8*e**(2*c + 2*d*x)*b*x + a*d**3*x**3 - a*d**2*x*
*2 + 2*a*d*x - 6*a - 4*b*d**2*x**3 + 4*b*d*x**2 - 8*b*x)/(48*e**(c + d*x)*
x**4)
```

3.10 $\int x^2(a+bx)^2\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 184

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = -\frac{12ab\cosh(c+dx)}{d^4}-\frac{24b^2x\cosh(c+dx)}{d^4} -\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{6abx^2\cosh(c+dx)}{d^2} -\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{24b^2\sinh(c+dx)}{d^5} +\frac{2a^2\sinh(c+dx)}{d^3}+\frac{12abx\sinh(c+dx)}{d^3} +\frac{12b^2x^2\sinh(c+dx)}{d^3}+\frac{a^2x^2\sinh(c+dx)}{d} +\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^4\sinh(c+dx)}{d}$$

output
```
-12*a*b*cosh(d*x+c)/d^4-24*b^2*x*cosh(d*x+c)/d^4-2*a^2*x*cosh(d*x+c)/d^2-6
*a*b*x^2*cosh(d*x+c)/d^2-4*b^2*x^3*cosh(d*x+c)/d^2+24*b^2*sinh(d*x+c)/d^5+
2*a^2*sinh(d*x+c)/d^3+12*a*b*x*sinh(d*x+c)/d^3+12*b^2*x^2*sinh(d*x+c)/d^3+
a^2*x^2*sinh(d*x+c)/d+2*a*b*x^3*sinh(d*x+c)/d+b^2*x^4*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.54

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$=\frac{-2d(a+2bx)\left(ad^2x+b(6+d^2x^2)\right)\cosh(c+dx)+\left(a^2d^2(2+d^2x^2)+2abd^2x(6+d^2x^2)+b^2(24+12d^2x\right.}{d^5}$$

input
```
Integrate[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

output
```
(-2*d*(a + 2*b*x)*(a*d^2*x + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^2*(2
+ d^2*x^2) + 2*a*b*d^2*x*(6 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*
Sinh[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(a^2x^2\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{24b^2\sinh(c+dx)}{d^5}-\frac{24b^2x\cosh(c+dx)}{d^4}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{b^2x^4\sinh(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x)^2*Cosh[c + d*x],x]
```

output
```
(-12*a*b*Cosh[c + d*x])/d^4 - (24*b^2*x*Cosh[c + d*x])/d^4 - (2*a^2*x*Cosh
[c + d*x])/d^2 - (6*a*b*x^2*Cosh[c + d*x])/d^2 - (4*b^2*x^3*Cosh[c + d*x])
/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (12*a*b*x*
Sinh[c + d*x])/d^3 + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*
x])/d + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{2\left(\left((2bx+a)(bx+a)d^2+12b^2\right)dx\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2\left(-x^2(bx+a)^2d^4+2\left(-6x^2b^2-6abx-a^2\right)d^2-24b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2a}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b^2x^4d^4+2ab\,d^4x^3+a^2d^4x^2-4b^2d^3x^3-6ab\,d^3x^2-2a^2d^3x+12b^2d^2x^2+12ab\,d^2x+2d^2a^2-24b^2dx-12abd+24b^2\right)e^{dx+c}}{2d^5}$
orering	$-\frac{4\left(2b^3d^4x^5+5a\,b^2d^4x^4+4a^2b\,d^4x^3+a^3d^4x^2+18d^2x^3b^3+27a\,b^2d^2x^2+11a^2b\,d^2x+a^3d^2+24b^3x+12b^2a\right)\cosh(dx+c)}{d^6x(bx+a)}$
parts	$\frac{b^2x^4\sinh(dx+c)}{d}+\frac{2ab\,x^3\sinh(dx+c)}{d}+\frac{a^2x^2\sinh(dx+c)}{d}-\frac{2\left(\frac{2b^2\left((dx+c)^3\cosh(dx+c)-3(dx+c)^2\sinh(dx+c)+6(dx\right.}{d^3}\right.}{}$
meijerg	$-\frac{16ib^2\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b^2\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x\right.}{}\right.}{}$
derivativedivides	$\frac{\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4b^2c\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh\right.}{d^2}}{}$
default	$\frac{\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}-\frac{4b^2c\left((dx+c)^3\sinh(dx+c)-3(dx+c)^2\cosh(dx+c)+6(dx+c)\sinh\right.}{d^2}}{}$

input
```
int(x^2*(b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*(((2*b*x+a)*(b*x+a)*d^2+12*b^2)*d*x*tanh(1/2*d*x+1/2*c)^2+(-x^2*(b*x+a)^
2*d^4+2*(-6*b^2*x^2-6*a*b*x-a^2)*d^2-24*b^2)*tanh(1/2*d*x+1/2*c)+d*(x*(2*b
*x+a)*d^2+12*b)*(b*x+a))/d^5/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.69

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = -\frac{2\left(2\,b^2d^3x^3+3\,abd^3x^2+6\,abd+(a^2d^3+12\,b^2d)x\right)\cosh\left(dx+c\right)-\left(b^2d^4x^4+2\,abd^4x^3+12\,abd^2x+2\right.}{d^5}$$

input
```
integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(2*(2*b^2*d^3*x^3 + 3*a*b*d^3*x^2 + 6*a*b*d + (a^2*d^3 + 12*b^2*d)*x)*cos
h(d*x + c) - (b^2*d^4*x^4 + 2*a*b*d^4*x^3 + 12*a*b*d^2*x + 2*a^2*d^2 + (a^
2*d^4 + 12*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 228, normalized size of antiderivative = 1.24

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$= \begin{cases} \frac{a^2x^2\sinh(c+dx)}{d} - \frac{2a^2x\cosh(c+dx)}{d^2} + \frac{2a^2\sinh(c+dx)}{d^3} + \frac{2abx^3\sinh(c+dx)}{d} - \frac{6abx^2\cosh(c+dx)}{d^2} + \frac{12abx\sinh(c+dx)}{d^3} - \frac{12ab\,\mathrm{co}}{} \\ \left(\frac{a^2x^3}{3}+\frac{abx^4}{2}+\frac{b^2x^5}{5}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**2*(b*x+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**
2*sinh(c + d*x)/d**3 + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*
x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x
**4*sinh(c + d*x)/d - 4*b**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c
 + d*x)/d**3 - 24*b**2*x*cosh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5,
Ne(d, 0)), ((a**2*x**3/3 + a*b*x**4/2 + b**2*x**5/5)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.79

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx = -\frac{1}{60}\,d\left(\frac{10\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)a^2e^{(dx)}}{d^4}+\frac{10\,(d^3x^3+3\,d^2x^2+6\,dx+6)a^2e^{(-dx-c)}}{d^4}+\frac{15\,(d\right.$$
$$+\frac{1}{30}\left(6\,b^2x^5+15\,abx^4+10\,a^2x^3\right)\cosh\left(dx+c\right)$$

input
```
integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/60*d*(10*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)/
d^4 + 10*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 15*(d^4*
x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b*e^(d*x
)/d^5 + 15*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*b*e^(-d*x -
c)/d^5 + 6*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c
+ 120*d*x*e^c - 120*e^c)*b^2*e^(d*x)/d^6 + 6*(d^5*x^5 + 5*d^4*x^4 + 20*d^3
*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6) + 1/30*(6*b^2*x^5
 + 15*a*b*x^4 + 10*a^2*x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.28

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$=\frac{(b^2d^4x^4+2\,abd^4x^3+a^2d^4x^2-4\,b^2d^3x^3-6\,abd^3x^2-2\,a^2d^3x+12\,b^2d^2x^2+12\,abd^2x+2\,a^2d^2-24\,b^2dx}{2\,d^5}$$
$$-\frac{(b^2d^4x^4+2\,abd^4x^3+a^2d^4x^2+4\,b^2d^3x^3+6\,abd^3x^2+2\,a^2d^3x+12\,b^2d^2x^2+12\,abd^2x+2\,a^2d^2+24\,b^2c}{2\,d^5}$$

input
```
integrate(x^2*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3 + a^2*d^4*x^2 - 4*b^2*d^3*x^3 - 6*a*b*d^3
*x^2 - 2*a^2*d^3*x + 12*b^2*d^2*x^2 + 12*a*b*d^2*x + 2*a^2*d^2 - 24*b^2*d*
x - 12*a*b*d + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^3
+ a^2*d^4*x^2 + 4*b^2*d^3*x^3 + 6*a*b*d^3*x^2 + 2*a^2*d^3*x + 12*b^2*d^2*x
^2 + 12*a*b*d^2*x + 2*a^2*d^2 + 24*b^2*d*x + 12*a*b*d + 24*b^2)*e^(-d*x -
c)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.91

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx=\frac{2\sinh(c+d\,x)\,\left(a^2\,d^2+12\,b^2\right)}{d^5}-\frac{4\,b^2\,x^3\cosh(c+d\,x)}{d^2}+\frac{b^2\,x^4\sinh(c+d\,x)}{d}-\frac{12\,a\,b\cosh(c+d\,x)}{d^4}-\frac{2\,x\cosh(c+d\,x)\,\left(a^2\,d^2+12\,b^2\right)}{d^4}+\frac{x^2\sinh(c+d\,x)\,\left(a^2\,d^2+12\,b^2\right)}{d^3}-\frac{6\,a\,b\,x^2\cosh(c+d\,x)}{d^2}+\frac{2\,a\,b\,x^3\sinh(c+d\,x)}{d}+\frac{12\,a\,b\,x\sinh(c+d\,x)}{d^3}$$

input
```
int(x^2*cosh(c + d*x)*(a + b*x)^2,x)
```

output
```
(2*sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^5 - (4*b^2*x^3*cosh(c + d*x))/d^2 +
 (b^2*x^4*sinh(c + d*x))/d - (12*a*b*cosh(c + d*x))/d^4 - (2*x*cosh(c + d*
x)*(12*b^2 + a^2*d^2))/d^4 + (x^2*sinh(c + d*x)*(12*b^2 + a^2*d^2))/d^3 -
(6*a*b*x^2*cosh(c + d*x))/d^2 + (2*a*b*x^3*sinh(c + d*x))/d + (12*a*b*x*si
nh(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 181, normalized size of antiderivative = 0.98

$$\int x^2(a+bx)^2\cosh(c+dx)\,dx$$
$$= \frac{-2\cosh(dx+c)\,a^2d^3x-6\cosh(dx+c)\,ab\,d^3x^2-12\cosh(dx+c)\,abd-4\cosh(dx+c)\,b^2d^3x^3-24\cos}{}$$

input
```
int(x^2*(b*x+a)^2*cosh(d*x+c),x)
```

output
```
( - 2*cosh(c + d*x)*a**2*d**3*x - 6*cosh(c + d*x)*a*b*d**3*x**2 - 12*cosh(
c + d*x)*a*b*d - 4*cosh(c + d*x)*b**2*d**3*x**3 - 24*cosh(c + d*x)*b**2*d*
x + sinh(c + d*x)*a**2*d**4*x**2 + 2*sinh(c + d*x)*a**2*d**2 + 2*sinh(c +
d*x)*a*b*d**4*x**3 + 12*sinh(c + d*x)*a*b*d**2*x + sinh(c + d*x)*b**2*d**4
*x**4 + 12*sinh(c + d*x)*b**2*d**2*x**2 + 24*sinh(c + d*x)*b**2)/d**5
```

3.11 $\int x(a+bx)^2\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 134

$$\begin{aligned}\int x(a+bx)^2\cosh(c+dx)\,dx = & -\frac{6b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{d^2} \\ & -\frac{4abx\cosh(c+dx)}{d^2} \\ & -\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3} \\ & +\frac{6b^2x\sinh(c+dx)}{d^3}+\frac{a^2x\sinh(c+dx)}{d} \\ & +\frac{2abx^2\sinh(c+dx)}{d}+\frac{b^2x^3\sinh(c+dx)}{d}\end{aligned}$$

output
```
-6*b^2*cosh(d*x+c)/d^4-a^2*cosh(d*x+c)/d^2-4*a*b*x*cosh(d*x+c)/d^2-3*b^2*x
^2*cosh(d*x+c)/d^2+4*a*b*sinh(d*x+c)/d^3+6*b^2*x*sinh(d*x+c)/d^3+a^2*x*sin
h(d*x+c)/d+2*a*b*x^2*sinh(d*x+c)/d+b^2*x^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.65

$$\int x(a+bx)^2\cosh(c+dx)\,dx$$
$$= \frac{-((a^2d^2+4abd^2x+3b^2(2+d^2x^2))\cosh(c+dx))+d(a^2d^2x+2ab(2+d^2x^2)+b^2x(6+d^2x^2))\sinh(c+}{d^4}$$

input
```
Integrate[x*(a + b*x)^2*Cosh[c + d*x],x]
```

output
```
(-((a^2*d^2 + 4*a*b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x]) + d*(a^2*d
^2*x + 2*a*b*(2 + d^2*x^2) + b^2*x*(6 + d^2*x^2))*Sinh[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx)^2\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(a^2x\cosh(c+dx)+2abx^2\cosh(c+dx)+b^2x^3\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{6b^2\cosh(c+dx)}{d^4}+\frac{6b^2x\sinh(c+dx)}{d^3}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{b^2x^3\sinh(c+dx)}{d}$$

input
```
Int[x*(a + b*x)^2*Cosh[c + d*x],x]
```

output
```
(-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/d^2 - (4*a*b*x*Cosh[c + d
*x])/d^2 - (3*b^2*x^2*Cosh[c + d*x])/d^2 + (4*a*b*Sinh[c + d*x])/d^3 + (6*
b^2*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (2*a*b*x^2*Sinh[c + d
*x])/d + (b^2*x^3*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.81

method	result
parallelrisch	$\frac{4b\left(\frac{3bx}{4}+a\right)d^{2}x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\left(x(bx+a)^{2}d^{2}+6b^{2}x+4ab\right)d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3x^{2}b^{2}+4abx+2a^{2}\right)d^{2}+12b^{2}}{d^{4}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b^{2}d^{3}x^{3}+2ab\,d^{3}x^{2}+a^{2}d^{3}x-3b^{2}d^{2}x^{2}-4ab\,d^{2}x-d^{2}a^{2}+6b^{2}dx+4abd-6b^{2}\right)\mathrm{e}^{dx+c}}{2d^{4}}-\frac{\left(b^{2}d^{3}x^{3}+2ab\,d^{3}x^{2}+a^{2}d^{3}x+3b^{2}d^{2}\cdots\right.}{}$
orering	$-\frac{2\left(3b^{3}d^{2}x^{4}+7a\,b^{2}d^{2}x^{3}+5a^{2}b\,d^{2}x^{2}+a^{3}d^{2}x+12x^{2}b^{3}+12a\,b^{2}x+2a^{2}b\right)\cosh(dx+c)}{d^{4}x(bx+a)}+\frac{\left(b^{2}d^{2}x^{3}+2ab\,d^{2}x^{2}+a^{2}d^{2}x+6b\cdots\right.}{}$
parts	$\frac{b^{2}x^{3}\sinh(dx+c)}{d}+\frac{2ab\,x^{2}\sinh(dx+c)}{d}+\frac{a^{2}x\sinh(dx+c)}{d}-\frac{\frac{3b^{2}\left((dx+c)^{2}\cosh(dx+c)-2(dx+c)\sinh(dx+c)+2\cosh(dx+\cdots\right.}{d^{2}}}{}$
meijerg	$\frac{8b^{2}\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^{2}d^{2}}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^{2}d^{2}}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^{4}}-\frac{8ib^{2}\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^{2}d^{2}}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\cdots\right.}{d^{4}}$
derivativedivides	$\frac{\frac{b^{2}\left((dx+c)^{3}\sinh(dx+c)-3(dx+c)^{2}\cosh(dx+c)+6(dx+c)\sinh(dx+c)-6\cosh(dx+c)\right)}{d^{2}}-\frac{3b^{2}c\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx\cdots\right.}{d^{2}}}{}$
default	$\frac{\frac{b^{2}\left((dx+c)^{3}\sinh(dx+c)-3(dx+c)^{2}\cosh(dx+c)+6(dx+c)\sinh(dx+c)-6\cosh(dx+c)\right)}{d^{2}}-\frac{3b^{2}c\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx\cdots\right.}{d^{2}}}{}$

input
```
int(x*(b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(4*b*(3/4*b*x+a)*d^2*x*tanh(1/2*d*x+1/2*c)^2-2*(x*(b*x+a)^2*d^2+6*b^2*x+4*
a*b)*d*tanh(1/2*d*x+1/2*c)+(3*b^2*x^2+4*a*b*x+2*a^2)*d^2+12*b^2)/d^4/(tanh
(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.71

$$\int x(a+bx)^2\cosh(c+dx)\,dx = -\frac{(3\,b^2d^2x^2+4\,abd^2x+a^2d^2+6\,b^2)\cosh(dx+c)-(b^2d^3x^3+2\,abd^3x^2+4\,abd+(a^2d^3+6\,b^2d)x)\sinh}{d^4}$$

input
```
integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6*b^2)*cosh(d*x + c) - (b^2*d^3
*x^3 + 2*a*b*d^3*x^2 + 4*a*b*d + (a^2*d^3 + 6*b^2*d)*x)*sinh(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.28

$$\int x(a+bx)^2\cosh(c+dx)\,dx = \begin{cases} \frac{a^2x\sinh(c+dx)}{d}-\frac{a^2\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3}+\frac{b^2x^3\sinh(c+dx)}{d}-\frac{3b^2x^2\cosh(c}{d^2} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^3}{3}+\frac{b^2x^4}{4}\right)\cosh(c) \end{cases}$$

input
```
integrate(x*(b*x+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**2*s
inh(c + d*x)/d - 4*a*b*x*cosh(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b
**2*x**3*sinh(c + d*x)/d - 3*b**2*x**2*cosh(c + d*x)/d**2 + 6*b**2*x*sinh(
c + d*x)/d**3 - 6*b**2*cosh(c + d*x)/d**4, Ne(d, 0)), ((a**2*x**2/2 + 2*a*
b*x**3/3 + b**2*x**4/4)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(134) = 268$.

Time = 0.06 (sec) , antiderivative size = 275, normalized size of antiderivative = 2.05

$$\int x(a+bx)^2\cosh(c+dx)\,dx = -\frac{1}{24}\,d\left(\frac{6\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2e^{(dx)}}{d^3}+\frac{6\,(d^2x^2+2\,dx+2)a^2e^{(-dx-c)}}{d^3}+\frac{8\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe\right. \\ +\frac{1}{12}\left(3\,b^2x^4+8\,abx^3+6\,a^2x^2\right)\cosh\left(dx+c\right)$$

input
```
integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/24*d*(6*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 6*(d^2*x^2
+ 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 8*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x
*e^c - 6*e^c)*a*b*e^(d*x)/d^4 + 8*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^
(-d*x - c)/d^4 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*
e^c + 24*e^c)*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d
*x + 24)*b^2*e^(-d*x - c)/d^5) + 1/12*(3*b^2*x^4 + 8*a*b*x^3 + 6*a^2*x^2)*
cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.28

$$\int x(a+bx)^2\cosh(c+dx)\,dx \\ = \frac{(b^2d^3x^3+2\,abd^3x^2+a^2d^3x-3\,b^2d^2x^2-4\,abd^2x-a^2d^2+6\,b^2dx+4\,abd-6\,b^2)e^{(dx+c)}}{2\,d^4} \\ -\frac{(b^2d^3x^3+2\,abd^3x^2+a^2d^3x+3\,b^2d^2x^2+4\,abd^2x+a^2d^2+6\,b^2dx+4\,abd+6\,b^2)e^{(-dx-c)}}{2\,d^4}$$

input
```
integrate(x*(b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^3*x^3 + 2*a*b*d^3*x^2 + a^2*d^3*x - 3*b^2*d^2*x^2 - 4*a*b*d^2*x
 - a^2*d^2 + 6*b^2*d*x + 4*a*b*d - 6*b^2)*e^(d*x + c)/d^4 - 1/2*(b^2*d^3*x
^3 + 2*a*b*d^3*x^2 + a^2*d^3*x + 3*b^2*d^2*x^2 + 4*a*b*d^2*x + a^2*d^2 + 6
*b^2*d*x + 4*a*b*d + 6*b^2)*e^(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.93

$$\int x(a+bx)^2\cosh(c+dx)\,dx = \frac{b^2\,x^3\sinh(c+d\,x)}{d} - \frac{3\,b^2\,x^2\cosh(c+d\,x)}{d^2} - \frac{\cosh(c+d\,x)\,\left(a^2\,d^2+6\,b^2\right)}{d^4} + \frac{4\,a\,b\sinh(c+d\,x)}{d^3} + \frac{x\sinh(c+d\,x)\,\left(a^2\,d^2+6\,b^2\right)}{d^3} + \frac{2\,a\,b\,x^2\sinh(c+d\,x)}{d} - \frac{4\,a\,b\,x\cosh(c+d\,x)}{d^2}$$

input
```
int(x*cosh(c + d*x)*(a + b*x)^2,x)
```

output
```
(b^2*x^3*sinh(c + d*x))/d - (3*b^2*x^2*cosh(c + d*x))/d^2 - (cosh(c + d*x)
*(6*b^2 + a^2*d^2))/d^4 + (4*a*b*sinh(c + d*x))/d^3 + (x*sinh(c + d*x)*(6*
b^2 + a^2*d^2))/d^3 + (2*a*b*x^2*sinh(c + d*x))/d - (4*a*b*x*cosh(c + d*x)
)/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.98

$$\int x(a+bx)^2\cosh(c+dx)\,dx = \frac{-\cosh(dx+c)\,a^2d^2-4\cosh(dx+c)\,ab\,d^2x-3\cosh(dx+c)\,b^2d^2x^2-6\cosh(dx+c)\,b^2+\sinh(dx+c}{d^4}$$

input
```
int(x*(b*x+a)^2*cosh(d*x+c),x)
```

output
```
( - cosh(c + d*x)*a**2*d**2 - 4*cosh(c + d*x)*a*b*d**2*x - 3*cosh(c + d*x)
*b**2*d**2*x**2 - 6*cosh(c + d*x)*b**2 + sinh(c + d*x)*a**2*d**3*x + 2*sin
h(c + d*x)*a*b*d**3*x**2 + 4*sinh(c + d*x)*a*b*d + sinh(c + d*x)*b**2*d**3
*x**3 + 6*sinh(c + d*x)*b**2*d*x)/d**4
```

3.12 $\int (a+bx)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 49

$$\int (a+bx)^2 \cosh(c+dx)\,dx = -\frac{2b(a+bx)\cosh(c+dx)}{d^2} + \frac{2b^2\sinh(c+dx)}{d^3} + \frac{(a+bx)^2\sinh(c+dx)}{d}$$

output
```
-2*b*(b*x+a)*cosh(d*x+c)/d^2+2*b^2*sinh(d*x+c)/d^3+(b*x+a)^2*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

$$\int (a+bx)^2 \cosh(c+dx)\,dx = \frac{-2bd(a+bx)\cosh(c+dx) + (a^2d^2+2abd^2x+b^2(2+d^2x^2))\sinh(c+dx)}{d^3}$$

input
```
Integrate[(a + b*x)^2*Cosh[c + d*x],x]
```

output
```
(-2*b*d*(a + b*x)*Cosh[c + d*x] + (a^2*d^2 + 2*a*b*d^2*x + b^2*(2 + d^2*x^
2))*Sinh[c + d*x])/d^3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 3777, 26, 3042, 26, 3777, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx)^2\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int (a+bx)^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a+bx)^2\sinh(c+dx)}{d}-\frac{2ib\int -i(a+bx)\sinh(c+dx)dx}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a+bx)^2\sinh(c+dx)}{d}-\frac{2b\int (a+bx)\sinh(c+dx)dx}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+bx)^2\sinh(c+dx)}{d}-\frac{2b\int -i(a+bx)\sin(ic+idx)dx}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{2ib\int (a+bx)\sin(ic+idx)dx}{d}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{2ib\left(\frac{i(a+bx)\cosh(c+dx)}{d}-\frac{ib\int\cosh(c+dx)dx}{d}\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{2ib\left(\frac{i(a+bx)\cosh(c+dx)}{d}-\frac{ib\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}$$

$$\Big\downarrow \text{3117}$$

$$\frac{(a+bx)^2\sinh(c+dx)}{d}+\frac{2ib\left(\frac{i(a+bx)\cosh(c+dx)}{d}-\frac{ib\sinh(c+dx)}{d^2}\right)}{d}$$

input
```
Int[(a + b*x)^2*Cosh[c + d*x],x]
```

output
```
((a + b*x)^2*Sinh[c + d*x])/d + ((2*I)*b*((I*(a + b*x)*Cosh[c + d*x])/d -
(I*b*Sinh[c + d*x])/d^2))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.57

method	result
parallelrisch	$\frac{2x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}b^{2}d+2\left(-(bx+a)^{2}d^{2}-2b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+4\left(\frac{bx}{2}+a\right)bd}{d^{3}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
parts	$\frac{b^{2}x^{2}\sinh(dx+c)}{d}+\frac{2abx\sinh(dx+c)}{d}+\frac{a^{2}\sinh(dx+c)}{d}-\frac{2b\left(\frac{b((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d}-\frac{bc\cosh(dx+c)}{d}+a\,\mathrm{co}\right.}{d^{2}}$
risch	$\frac{(b^{2}d^{2}x^{2}+2ab\,d^{2}x+d^{2}a^{2}-2b^{2}dx-2abd+2b^{2})\,\mathrm{e}^{dx+c}}{2d^{3}}-\frac{(b^{2}d^{2}x^{2}+2ab\,d^{2}x+d^{2}a^{2}+2b^{2}dx+2abd+2b^{2})\,\mathrm{e}^{-dx-c}}{2d^{3}}$
orering	$-\frac{4b(b^{2}d^{2}x^{2}+2ab\,d^{2}x+d^{2}a^{2}+b^{2})\cosh(dx+c)}{d^{4}(bx+a)}+\frac{(b^{2}d^{2}x^{2}+2ab\,d^{2}x+d^{2}a^{2}+2b^{2})\left(2(bx+a)\cosh(dx+c)b+(bx+a)^{2}d\,\mathrm{si}\right.}{d^{4}(bx+a)^{2}}$
derivativedivides	$\frac{\frac{b^{2}\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}-\frac{2b^{2}c((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}+\frac{2ba((dx+c)\sinh(dx+c)-\mathrm{c}}{d}}{d}$
default	$\frac{\frac{b^{2}\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}-\frac{2b^{2}c((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}+\frac{2ba((dx+c)\sinh(dx+c)-\mathrm{c}}{d}}{d}$
meijerg	$\frac{4ib^{2}\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^{2}d^{2}}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^{3}}+\frac{4b^{2}\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^{2}d^{2}}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^{3}}$

input
```
int((b*x+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*(x*tanh(1/2*d*x+1/2*c)^2*b^2*d+(-(b*x+a)^2*d^2-2*b^2)*tanh(1/2*d*x+1/2*c
)+2*(1/2*b*x+a)*b*d)/d^3/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.31

$$\int (a+bx)^2\cosh(c+dx)\,dx$$
$$= -\frac{2\,(b^2dx+abd)\cosh{(dx+c)}-(b^2d^2x^2+2\,abd^2x+a^2d^2+2\,b^2)\sinh{(dx+c)}}{d^3}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(2*(b^2*d*x + a*b*d)*cosh(d*x + c) - (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2
 + 2*b^2)*sinh(d*x + c))/d^3
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(48) = 96$.

Time = 0.23 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (a+bx)^2 \cosh(c+dx)\,dx = \begin{cases} \frac{a^2\sinh(c+dx)}{d} + \frac{2abx\sinh(c+dx)}{d} - \frac{2ab\cosh(c+dx)}{d^2} + \frac{b^2x^2\sinh(c+dx)}{d} - \frac{2b^2x\cosh(c+dx)}{d^2} + \frac{2b^2\sinh(c+dx)}{d^3} & \text{for } d \neq 0 \\ \left(a^2x + abx^2 + \frac{b^2x^3}{3}\right)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x*sinh(c + d*x)/d - 2*a*b*cosh(c +
 d*x)/d**2 + b**2*x**2*sinh(c + d*x)/d - 2*b**2*x*cosh(c + d*x)/d**2 + 2*b
**2*sinh(c + d*x)/d**3, Ne(d, 0)), ((a**2*x + a*b*x**2 + b**2*x**3/3)*cosh
(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(49) = 98$.

Time = 0.06 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.76

$$\int (a+bx)^2 \cosh(c+dx)\,dx = \frac{a^2e^{(dx+c)}}{2\,d} + \frac{(dxe^c - e^c)abe^{(dx)}}{d^2} - \frac{(dx+1)abe^{(-dx-c)}}{d^2} - \frac{a^2e^{(-dx-c)}}{2\,d} + \frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)b^2e^{(dx)}}{2\,d^3} - \frac{(d^2x^2 + 2\,dx + 2)b^2e^{(-dx-c)}}{2\,d^3}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a^2*e^(d*x + c)/d + (d*x*e^c - e^c)*a*b*e^(d*x)/d^2 - (d*x + 1)*a*b*e^
(-d*x - c)/d^2 - 1/2*a^2*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c + 2
*e^c)*b^2*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. 2(49) = 98.

Time = 0.13 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.29

$$\int (a+bx)^2 \cosh(c+dx)\,dx = \frac{(b^2d^2x^2 + 2\,abd^2x + a^2d^2 - 2\,b^2dx - 2\,abd + 2\,b^2)e^{(dx+c)}}{2\,d^3} - \frac{(b^2d^2x^2 + 2\,abd^2x + a^2d^2 + 2\,b^2dx + 2\,abd + 2\,b^2)e^{(-dx-c)}}{2\,d^3}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 - 2*b^2*d*x - 2*a*b*d + 2*b^2)*e^
(d*x + c)/d^3 - 1/2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2 + 2*b^2*d*x + 2*a
*b*d + 2*b^2)*e^(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.67

$$\int (a+bx)^2 \cosh(c+dx)\,dx = \frac{\sinh(c+d\,x)\,\left(a^2\,d^2+2\,b^2\right)}{d^3} + \frac{b^2\,x^2\,\sinh(c+d\,x)}{d} - \frac{2\,a\,b\cosh(c+d\,x)}{d^2} - \frac{2\,b^2\,x\cosh(c+d\,x)}{d^2} + \frac{2\,a\,b\,x\sinh(c+d\,x)}{d}$$

input
```
int(cosh(c + d*x)*(a + b*x)^2,x)
```

output
```
(sinh(c + d*x)*(2*b^2 + a^2*d^2))/d^3 + (b^2*x^2*sinh(c + d*x))/d - (2*a*b
*cosh(c + d*x))/d^2 - (2*b^2*x*cosh(c + d*x))/d^2 + (2*a*b*x*sinh(c + d*x)
)/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int (a+bx)^2 \cosh(c+dx)\,dx$$
$$= \frac{-2\cosh\left(dx+c\right)abd - 2\cosh\left(dx+c\right)b^2dx + \sinh\left(dx+c\right)a^2d^2 + 2\sinh\left(dx+c\right)ab\,d^2x + \sinh\left(dx+c\right)}{d^3}$$

input
```
int((b*x+a)^2*cosh(d*x+c),x)
```

output
```
( - 2*cosh(c + d*x)*a*b*d - 2*cosh(c + d*x)*b**2*d*x + sinh(c + d*x)*a**2*
d**2 + 2*sinh(c + d*x)*a*b*d**2*x + sinh(c + d*x)*b**2*d**2*x**2 + 2*sinh(
c + d*x)*b**2)/d**3
```

3.13 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 62

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x} \, dx = -\frac{b^2 \cosh(c+dx)}{d^2} + a^2 \cosh(c)\text{Chi}(dx) + \frac{2ab \sinh(c+dx)}{d} + \frac{b^2 x \sinh(c+dx)}{d} + a^2 \sinh(c)\text{Shi}(dx)$$

output
```
-b^2*cosh(d*x+c)/d^2+a^2*cosh(c)*Chi(d*x)+2*a*b*sinh(d*x+c)/d+b^2*x*sinh(d
*x+c)/d+a^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.82

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x} \, dx = a^2 \cosh(c)\text{Chi}(dx) + \frac{b(-b \cosh(c+dx) + d(2a+bx) \sinh(c+dx))}{d^2} + a^2 \sinh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

output
```
a^2*Cosh[c]*CoshIntegral[d*x] + (b*(-(b*Cosh[c + d*x]) + d*(2*a + b*x)*Sin
h[c + d*x]))/d^2 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x}+2ab\cosh(c+dx)+b^2x\cosh(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$a^2\cosh(c)\text{Chi}(dx)+a^2\sinh(c)\text{Shi}(dx)+\frac{2ab\sinh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d^2}+\frac{b^2x\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x)^2*Cosh[c + d*x])/x,x]
```

output
```
-((b^2*Cosh[c + d*x])/d^2) + a^2*Cosh[c]*CoshIntegral[d*x] + (2*a*b*Sinh[c
 + d*x])/d + (b^2*x*Sinh[c + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.95

method	result
risch	$-\frac{a^2 e^c \operatorname{expIntegral}_1(-dx)}{2} - \frac{a^2 e^{-c} \operatorname{expIntegral}_1(dx)}{2} - \frac{e^{-dx-c} b^2 x}{2d} + \frac{e^{dx+c} b^2 x}{2d} - \frac{e^{-dx-c} ab}{d} + \frac{e^{dx+c} ab}{d} - \frac{e^{-dx-c} b^2}{2d^2} -$
meijerg	$-\frac{2b^2 \cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\cosh(dx)}{2\sqrt{\pi}} - \frac{dx \sinh(dx)}{2\sqrt{\pi}}\right)}{d^2} + \frac{b^2 \sinh(c)(\cosh(dx)xd - \sinh(dx))}{d^2} + \frac{2ab\cosh(c)\sinh(dx)}{d} - \frac{2ba\sinh(c)\sqrt{}}{}$

input
```
int((b*x+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a^2*exp(c)*Ei(1,-d*x)-1/2*a^2*exp(-c)*Ei(1,d*x)-1/2/d*exp(-d*x-c)*b^2
*x+1/2/d*exp(d*x+c)*b^2*x-1/d*exp(-d*x-c)*a*b+1/d*exp(d*x+c)*a*b-1/2/d^2*e
xp(-d*x-c)*b^2-1/2/d^2*exp(d*x+c)*b^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x} \, dx = -\frac{2\,b^2 \cosh\left(dx+c\right) - \left(a^2 d^2 \operatorname{Ei}(dx) + a^2 d^2 \operatorname{Ei}(-dx)\right)\cosh\left(c\right) - 2\left(b^2 dx + 2\,abd\right)\sinh\left(dx+c\right) - \left(a^2 d^2 \operatorname{Ei}(\right.}{2\,d^2}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei(d*x) + a^2*d^2*Ei(-d*x))*cosh(c) -
 2*(b^2*d*x + 2*a*b*d)*sinh(d*x + c) - (a^2*d^2*Ei(d*x) - a^2*d^2*Ei(-d*x)
)*sinh(c))/d^2
```

Sympy [A] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = a^2\sinh(c)\,\mathrm{Shi}(dx) + a^2\cosh(c)\,\mathrm{Chi}(dx) + 2ab\left(\begin{cases} x\cosh(c) & \text{for } d=0 \\ \frac{\sinh(c+dx)}{d} & \text{otherwise}\end{cases}\right) + b^2\left(\begin{cases} \frac{x\sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d\neq 0 \\ \frac{x^2\cosh(c)}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x+a)**2*cosh(d*x+c)/x,x)
```

output
```
a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x*cosh(c)
, Eq(d, 0)), (sinh(c + d*x)/d, True)) + b**2*Piecewise((x*sinh(c + d*x)/d
- cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(62) = 124.

Time = 0.12 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.82

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = -\frac{1}{4}\left(4\,ab\left(\frac{(dxe^c - e^c)e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right) + b^2\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2}{d^3}\right.\right. + \frac{1}{2}\left(b^2x^2 + 4\,abx + 2\,a^2\log(x)\right)\cosh(dx+c)$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/4*(4*a*b*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2) + b
^2*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*
e^(-d*x - c)/d^3) + 4*a^2*cosh(d*x + c)*log(x)/d - 2*(Ei(-d*x)*e^(-c) + Ei
(d*x)*e^c)*a^2/d)*d + 1/2*(b^2*x^2 + 4*a*b*x + 2*a^2*log(x))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.82

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = \frac{a^2d^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^2\mathrm{Ei}(dx)\,e^{c} + b^2dxe^{(dx+c)} - b^2dxe^{(-dx-c)} + 2\,abde^{(dx+c)} - 2\,abde^{(-dx-c)} - b^2e^{(dx+c}}{2\,d^2}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(a^2*d^2*Ei(-d*x)*e^(-c) + a^2*d^2*Ei(d*x)*e^c + b^2*d*x*e^(d*x + c) -
 b^2*d*x*e^(-d*x - c) + 2*a*b*d*e^(d*x + c) - 2*a*b*d*e^(-d*x - c) - b^2*e
^(d*x + c) - b^2*e^(-d*x - c))/d^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x)^2)/x,x)
```

output
```
int((cosh(c + d*x)*(a + b*x)^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x}\,dx$$
$$= \frac{-\cosh(dx+c)\,b^2+\left(\int \frac{\cosh(dx+c)}{x}dx\right)a^2d^2+2\sinh(dx+c)\,abd+\sinh(dx+c)\,b^2dx}{d^2}$$

input
```
int((b*x+a)^2*cosh(d*x+c)/x,x)
```

output
```
( - cosh(c + d*x)*b**2 + int(cosh(c + d*x)/x,x)*a**2*d**2 + 2*sinh(c + d*x
)*a*b*d + sinh(c + d*x)*b**2*d*x)/d**2
```

3.14 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} \, dx = -\frac{a^2 \cosh(c+dx)}{x} + 2ab \cosh(c)\text{Chi}(dx) + a^2 d\text{Chi}(dx) \sinh(c) + \frac{b^2 \sinh(c+dx)}{d} + a^2 d \cosh(c)\text{Shi}(dx) + 2ab \sinh(c)\text{Shi}(dx)$$

output
```
-a^2*cosh(d*x+c)/x+2*a*b*cosh(c)*Chi(d*x)+a^2*d*Chi(d*x)*sinh(c)+b^2*sinh(
d*x+c)/d+a^2*d*cosh(c)*Shi(d*x)+2*a*b*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2} \, dx = -\frac{a^2 \cosh(c+dx)}{x} + a\text{Chi}(dx)(2b \cosh(c) + ad \sinh(c)) + \frac{b^2 \sinh(c+dx)}{d} + a(ad \cosh(c) + 2b \sinh(c))\text{Shi}(dx)$$

input
```
Integrate[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

output
```
-((a^2*Cosh[c + d*x])/x) + a*CoshIntegral[d*x]*(2*b*Cosh[c] + a*d*Sinh[c])
 + (b^2*Sinh[c + d*x])/d + a*(a*d*Cosh[c] + 2*b*Sinh[c])*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^2}+\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2d\sinh(c)\text{Chi}(dx)+a^2d\cosh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{x}+2ab\cosh(c)\text{Chi}(dx)+2ab\sinh(c)\text{Shi}(dx)+\frac{b^2\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x)^2*Cosh[c + d*x])/x^2,x]
```

output
```
-((a^2*Cosh[c + d*x])/x) + 2*a*b*Cosh[c]*CoshIntegral[d*x] + a^2*d*CoshInt
egral[d*x]*Sinh[c] + (b^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*
x] + 2*a*b*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.77

method	result
risch	$-\frac{e^{c} \operatorname{expIntegral}_1(-dx)a^2d^2x - e^{-c}\operatorname{expIntegral}_1(dx)a^2d^2x + 2e^{c}\operatorname{expIntegral}_1(-dx)abdx + 2e^{-c}\operatorname{expIntegral}_1(dx)abdx + e^{dx+c}a^2d}{2dx}$
meijerg	$\frac{b^2\cosh(c)\sinh(dx)}{d} - \frac{b^2\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}} - \frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d} + ab\cosh(c)\sqrt{\pi}\left(\frac{2\operatorname{Chi}(dx) - 2\ln(dx) - 2\gamma}{\sqrt{\pi}} + \frac{2\gamma + 2\ln(x) + 2\ln(id)}{\sqrt{\pi}}\right.$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(exp(c)*Ei(1,-d*x)*a^2*d^2*x-exp(-c)*Ei(1,d*x)*a^2*d^2*x+2*exp(c)*E
i(1,-d*x)*a*b*d*x+2*exp(-c)*Ei(1,d*x)*a*b*d*x+exp(d*x+c)*a^2*d-exp(d*x+c)*
b^2*x+exp(-d*x-c)*a^2*d+exp(-d*x-c)*b^2*x)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.74

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx =$$
$$-\frac{2a^2d\cosh(dx+c) - 2b^2x\sinh(dx+c) - ((a^2d^2+2abd)x\operatorname{Ei}(dx) - (a^2d^2-2abd)x\operatorname{Ei}(-dx))\cosh(c}{2dx}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*d*cosh(d*x + c) - 2*b^2*x*sinh(d*x + c) - ((a^2*d^2 + 2*a*b*d)
*x*Ei(d*x) - (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*cosh(c) - ((a^2*d^2 + 2*a*b*d
)*x*Ei(d*x) + (a^2*d^2 - 2*a*b*d)*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F]

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2}\, dx = \int \frac{(a+bx)^2 \cosh(c+dx)}{x^2}\, dx$$

input
```
integrate((b*x+a)**2*cosh(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x)**2*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^2}\, dx =$$
$$-\frac{1}{2}\left(\left(\mathrm{Ei}(-dx)\, e^{(-c)} - \mathrm{Ei}(dx)\, e^{c}\right)a^2 + b^2\left(\frac{(dxe^{c} - e^{c})e^{(dx)}}{d^2} + \frac{(dx+1)e^{(-dx-c)}}{d^2}\right) + \frac{4\,ab\cosh\left(dx+c\right)\log}{d}\right.$$
$$+\left(b^2x + 2\,ab\log\left(x\right) - \frac{a^2}{x}\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
-1/2*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)*a^2 + b^2*((d*x*e^c - e^c)*e^(d*x)/d
^2 + (d*x + 1)*e^(-d*x - c)/d^2) + 4*a*b*cosh(d*x + c)*log(x)/d - 2*(Ei(-d
*x)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + (b^2*x + 2*a*b*log(x) - a^2/x)*cosh(d
*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx =$$
$$-\frac{a^2d^2x\mathrm{Ei}(-dx)\,e^{(-c)} - a^2d^2x\mathrm{Ei}(dx)\,e^c - 2\,abdx\mathrm{Ei}(-dx)\,e^{(-c)} - 2\,abdx\mathrm{Ei}(dx)\,e^c + a^2de^{(dx+c)} - b^2xe^{(dx+c)}}{2\,dx}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a^2*d^2*x*Ei(-d*x)*e^(-c) - a^2*d^2*x*Ei(d*x)*e^c - 2*a*b*d*x*Ei(-d*
x)*e^(-c) - 2*a*b*d*x*Ei(d*x)*e^c + a^2*d*e^(d*x + c) - b^2*x*e^(d*x + c)
+ a^2*d*e^(-d*x - c) + b^2*x*e^(-d*x - c))/(d*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^2}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x)^2)/x^2,x)
```

output
```
int((cosh(c + d*x)*(a + b*x)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.94

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^2}\,dx$$
$$= \frac{-e^{dx}\,ei(-dx)\,a^2d^2x + 2e^{dx}\,ei(-dx)\,abdx + e^{dx+2c}\,ei(dx)\,a^2d^2x + 2e^{dx+2c}\,ei(dx)\,abdx - e^{2dx+2c}a^2d + e^{2dx+}}{2e^{dx+c}dx}$$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^2,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a**2*d**2*x + 2*e**(d*x)*ei( - d*x)*a*b*d*x + e**(
2*c + d*x)*ei(d*x)*a**2*d**2*x + 2*e**(2*c + d*x)*ei(d*x)*a*b*d*x - e**(2*
c + 2*d*x)*a**2*d + e**(2*c + 2*d*x)*b**2*x - a**2*d - b**2*x)/(2*e**(c +
d*x)*d*x)
```

3.15 $\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = -\frac{a^2\cosh(c+dx)}{2x^2} - \frac{2ab\cosh(c+dx)}{x} + b^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx) + 2abd\text{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{2x} + 2abd\cosh(c)\text{Shi}(dx) + b^2\sinh(c)\text{Shi}(dx) + \frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)$$

output
```
-1/2*a^2*cosh(d*x+c)/x^2-2*a*b*cosh(d*x+c)/x+b^2*cosh(c)*Chi(d*x)+1/2*a^2*
d^2*cosh(c)*Chi(d*x)+2*a*b*d*Chi(d*x)*sinh(c)-1/2*a^2*d*sinh(d*x+c)/x+2*a*
b*d*cosh(c)*Shi(d*x)+b^2*sinh(c)*Shi(d*x)+1/2*a^2*d^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.77

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}\left(\operatorname{Chi}(dx)\left(\left(2b^2+a^2d^2\right)\cosh(c)+4abd\sinh(c)\right) - \frac{a((a+4bx)\cosh(c+dx)+adx\sinh(c+dx))}{x^2} + \left(4abd\cosh(c)+\left(2b^2+a^2d^2\right)\sinh(c)\right)\operatorname{Shi}(dx)\right)$$

input
```
Integrate[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

output
```
(CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cosh[c] + 4*a*b*d*Sinh[c]) - (a*((a
+ 4*b*x)*Cosh[c + d*x] + a*d*x*Sinh[c + d*x]))/x^2 + (4*a*b*d*Cosh[c] + (2
*b^2 + a^2*d^2)*Sinh[c])*SinhIntegral[d*x])/2
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^3}+\frac{2ab\cosh(c+dx)}{x^2}+\frac{b^2\cosh(c+dx)}{x}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{1}{2}a^2d^2\cosh(c)\operatorname{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\operatorname{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+2abd\sinh(c)\operatorname{Chi}(dx)+2abd\cosh(c)\operatorname{Shi}(dx)-\frac{2ab\cosh(c+dx)}{x}+b^2\cosh(c)\operatorname{Chi}(dx)+b^2\sinh(c)\operatorname{Shi}(dx)$$

input
```
Int[((a + b*x)^2*Cosh[c + d*x])/x^3,x]
```

output
```
-1/2*(a^2*Cosh[c + d*x])/x^2 - (2*a*b*Cosh[c + d*x])/x + b^2*Cosh[c]*CoshI
ntegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + 2*a*b*d*CoshIntegra
l[d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(2*x) + 2*a*b*d*Cosh[c]*SinhIntegra
l[d*x] + b^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^2*Sinh[c]*SinhIntegral[d*x
])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.54

method	result
risch	$-\frac{e^{c} \operatorname{expIntegral}_1(-dx) a^2 d^2 x^2 + e^{-c} \operatorname{expIntegral}_1(dx) a^2 d^2 x^2 + 4 e^{c} \operatorname{expIntegral}_1(-dx) abd\, x^2 - 4 e^{-c} \operatorname{expIntegral}_1(dx) abd\, x^2 + 2 e^{c}}{4}$
meijerg	$\frac{b^2 \cosh(c) \sqrt{\pi} \left(\frac{2\operatorname{Chi}(dx) - 2\ln(dx) - 2\gamma}{\sqrt{\pi}} + \frac{2\gamma + 2\ln(x) + 2\ln(id)}{\sqrt{\pi}}\right)}{2} + b^2 \sinh(c) \operatorname{Shi}(dx) + \frac{idab \cosh(c) \sqrt{\pi} \left(\frac{4i \cosh(dx)}{dx\sqrt{\pi}} - \frac{4i \operatorname{Shi}(dx}{\sqrt{\pi}}\right.}{2}$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(exp(c)*Ei(1,-d*x)*a^2*d^2*x^2+exp(-c)*Ei(1,d*x)*a^2*d^2*x^2+4*exp(c)
*Ei(1,-d*x)*a*b*d*x^2-4*exp(-c)*Ei(1,d*x)*a*b*d*x^2+2*exp(c)*Ei(1,-d*x)*b^
2*x^2+2*exp(-c)*Ei(1,d*x)*b^2*x^2+a^2*d*x*exp(d*x+c)-a^2*d*x*exp(-d*x-c)+4
*exp(d*x+c)*a*b*x+4*exp(-d*x-c)*a*b*x+exp(d*x+c)*a^2+exp(-d*x-c)*a^2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx =$$
$$-\frac{2\,a^2dx\sinh\left(dx+c\right)+2\left(4\,abx+a^2\right)\cosh\left(dx+c\right)-\left((a^2d^2+4\,abd+2\,b^2)x^2\mathrm{Ei}(dx)+(a^2d^2-4\,abd+\right.}{4}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*d*x*sinh(d*x + c) + 2*(4*a*b*x + a^2)*cosh(d*x + c) - ((a^2*d^
2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 - 4*a*b*d + 2*b^2)*x^2*Ei(-d*x
))*cosh(c) - ((a^2*d^2 + 4*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d^2 - 4*a*b*d
 + 2*b^2)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = \int \frac{\left(a+bx\right)^2\cosh\left(c+dx\right)}{x^3}\,dx$$

input
```
integrate((b*x+a)**2*cosh(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x)**2*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx$$
$$=\frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a^2-4\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)ab-\frac{4\,b^2\cosh\left(dx+c\right)\log\left(x\right)}{d}\right.$$
$$+\frac{1}{2}\left(2\,b^2\log\left(x\right)-\frac{4\,abx+a^2}{x^2}\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - 4*(Ei(-d*x)*e
^(-c) - Ei(d*x)*e^c)*a*b - 4*b^2*cosh(d*x + c)*log(x)/d + 2*(Ei(-d*x)*e^(-
c) + Ei(d*x)*e^c)*b^2/d)*d + 1/2*(2*b^2*log(x) - (4*a*b*x + a^2)/x^2)*cosh
(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.50

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = \frac{a^2d^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^2x^2\mathrm{Ei}(dx)\,e^{c} - 4\,abdx^2\mathrm{Ei}(-dx)\,e^{(-c)} + 4\,abdx^2\mathrm{Ei}(dx)\,e^{c} + 2\,b^2x^2\mathrm{Ei}(-dx)\,e^{(-c)}}{4\,x}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c - 4*a*b*d*x^2*E
i(-d*x)*e^(-c) + 4*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) + 2*b
^2*x^2*Ei(d*x)*e^c - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(-d*x - c) - 4*a*b*x*
e^(d*x + c) - 4*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) - a^2*e^(-d*x - c))/x
^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,{(a+b\,x)}^2}{x^3}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x)^2)/x^3,x)
```

output
```
int((cosh(c + d*x)*(a + b*x)^2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{e^{dx}\,ei(-dx)\,a^2d^2x^2 - 4e^{dx}\,ei(-dx)\,abd\,x^2 + 2e^{dx}\,ei(-dx)\,b^2x^2 + e^{dx+2c}ei(dx)\,a^2d^2x^2 + 4e^{dx+2c}ei(dx)\,abd}{4e^{dx+c}x^2}$$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^3,x)
```

output
```
(e**(d*x)*ei( - d*x)*a**2*d**2*x**2 - 4*e**(d*x)*ei( - d*x)*a*b*d*x**2 + 2
*e**(d*x)*ei( - d*x)*b**2*x**2 + e**(2*c + d*x)*ei(d*x)*a**2*d**2*x**2 + 4
*e**(2*c + d*x)*ei(d*x)*a*b*d*x**2 + 2*e**(2*c + d*x)*ei(d*x)*b**2*x**2 -
e**(2*c + 2*d*x)*a**2*d*x - e**(2*c + 2*d*x)*a**2 - 4*e**(2*c + 2*d*x)*a*b
*x + a**2*d*x - a**2 - 4*a*b*x)/(4*e**(c + d*x)*x**2)
```

3.16 $\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 172

$$\begin{aligned}\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = &-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{ab\cosh(c+dx)}{x^2}\\ &-\frac{b^2\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{6x}\\ &+abd^2\cosh(c)\text{Chi}(dx)+b^2d\text{Chi}(dx)\sinh(c)\\ &+\frac{1}{6}a^2d^3\text{Chi}(dx)\sinh(c)-\frac{a^2d\sinh(c+dx)}{6x^2}\\ &-\frac{abd\sinh(c+dx)}{x}+b^2d\cosh(c)\text{Shi}(dx)\\ &+\frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx)+abd^2\sinh(c)\text{Shi}(dx)\end{aligned}$$

output
```
-1/3*a^2*cosh(d*x+c)/x^3-a*b*cosh(d*x+c)/x^2-b^2*cosh(d*x+c)/x-1/6*a^2*d^2
*cosh(d*x+c)/x+a*b*d^2*cosh(c)*Chi(d*x)+b^2*d*Chi(d*x)*sinh(c)+1/6*a^2*d^3
*Chi(d*x)*sinh(c)-1/6*a^2*d*sinh(d*x+c)/x^2-a*b*d*sinh(d*x+c)/x+b^2*d*cosh
(c)*Shi(d*x)+1/6*a^2*d^3*cosh(c)*Shi(d*x)+a*b*d^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{2a^2\cosh(c+dx)+6abx\cosh(c+dx)+6b^2x^2\cosh(c+dx)+a^2d^2x^2\cosh(c+dx)-dx^3\text{Chi}(dx)\,(6ab$$

input
```
Integrate[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

output
```
-1/6*(2*a^2*Cosh[c + d*x] + 6*a*b*x*Cosh[c + d*x] + 6*b^2*x^2*Cosh[c + d*x
] + a^2*d^2*x^2*Cosh[c + d*x] - d*x^3*CoshIntegral[d*x]*(6*a*b*d*Cosh[c] +
 (6*b^2 + a^2*d^2)*Sinh[c]) + a^2*d*x*Sinh[c + d*x] + 6*a*b*d*x^2*Sinh[c +
 d*x] - d*x^3*(6*b^2*Cosh[c] + a^2*d^2*Cosh[c] + 6*a*b*d*Sinh[c])*SinhInte
gral[d*x])/x^3
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x^3}+\frac{b^2\cosh(c+dx)}{x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d\sinh(c+dx)}{6x^2}+abd^2\cosh(c)\text{Chi}(dx)+abd^2\sinh(c)\text{Shi}(dx)-\frac{ab\cosh(c+dx)}{x^2}-\frac{abd\sinh(c+dx)}{x}+b^2d\sinh(c)\text{Chi}(dx)+b^2d\cosh(c)\text{Shi}(dx)-\frac{b^2\cosh(c+dx)}{x}$$

input
```
Int[((a + b*x)^2*Cosh[c + d*x])/x^4,x]
```

output
```
-1/3*(a^2*Cosh[c + d*x])/x^3 - (a*b*Cosh[c + d*x])/x^2 - (b^2*Cosh[c + d*x
])/x - (a^2*d^2*Cosh[c + d*x])/(6*x) + a*b*d^2*Cosh[c]*CoshIntegral[d*x] +
 b^2*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d*x]*Sinh[c])/6 -
 (a^2*d*Sinh[c + d*x])/(6*x^2) - (a*b*d*Sinh[c + d*x])/x + b^2*d*Cosh[c]*S
inhIntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + a*b*d^2*Sinh[c]
*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.70

method	result
risch	$-\frac{e^{c}\,\text{expIntegral}_1(-dx)a^2d^3x^3-e^{-c}\,\text{expIntegral}_1(dx)a^2d^3x^3+6\,e^{c}\,\text{expIntegral}_1(-dx)ab\,d^2x^3+6\,e^{-c}\,\text{expIntegral}_1(dx)ab\,d^2x^3+6\,e}{}$
meijerg	$\frac{id\,b^2\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\text{Shi}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{d\,b^2\sinh(c)\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}}-\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}+\frac{4\ \text{Chi}(dx)-4\ln(dx)-4\gamma}{\sqrt{\pi}}+\frac{4\gamma-4+4\ln(x)+4\ln(id)}{\sqrt{\pi}}\right)}{4}-$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(exp(c)*Ei(1,-d*x)*a^2*d^3*x^3-exp(-c)*Ei(1,d*x)*a^2*d^3*x^3+6*exp(c
)*Ei(1,-d*x)*a*b*d^2*x^3+6*exp(-c)*Ei(1,d*x)*a*b*d^2*x^3+6*exp(c)*Ei(1,-d*
x)*b^2*d*x^3-6*exp(-c)*Ei(1,d*x)*b^2*d*x^3+a^2*d^2*x^2*exp(d*x+c)+a^2*d^2*
x^2*exp(-d*x-c)+6*a*b*d*x^2*exp(d*x+c)-6*a*b*d*x^2*exp(-d*x-c)+a^2*d*x*exp
(d*x+c)+6*exp(d*x+c)*b^2*x^2-a^2*d*x*exp(-d*x-c)+6*exp(-d*x-c)*b^2*x^2+6*e
xp(d*x+c)*a*b*x+6*exp(-d*x-c)*a*b*x+2*exp(d*x+c)*a^2+2*exp(-d*x-c)*a^2)/x^
3
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.13

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = -\frac{2\,(6\,abx+(a^2d^2+6\,b^2)x^2+2\,a^2)\cosh\,(dx+c)-((a^2d^3+6\,abd^2+6\,b^2d)x^3\mathrm{Ei}(dx)-(a^2d^3-6\,abd^2+$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(2*(6*a*b*x + (a^2*d^2 + 6*b^2)*x^2 + 2*a^2)*cosh(d*x + c) - ((a^2*d
^3 + 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) - (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^
3*Ei(-d*x))*cosh(c) + 2*(6*a*b*d*x^2 + a^2*d*x)*sinh(d*x + c) - ((a^2*d^3
+ 6*a*b*d^2 + 6*b^2*d)*x^3*Ei(d*x) + (a^2*d^3 - 6*a*b*d^2 + 6*b^2*d)*x^3*E
i(-d*x))*sinh(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx)^2\cosh\,(c+dx)}{x^4}\,dx$$

input
```
integrate((b*x+a)**2*cosh(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x)**2*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx$$
$$=\frac{1}{6}\left(a^2d^2e^{(-c)}\Gamma(-2,dx)-a^2d^2e^c\Gamma(-2,-dx)+3\,abde^{(-c)}\Gamma(-1,dx)+3\,abde^c\Gamma(-1,-dx)-3\,b^2\mathrm{Ei}(-dx)\,e\right.$$
$$-\frac{(3\,b^2x^2+3\,abx+a^2)\cosh(dx+c)}{3\,x^3}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) + 3*a*b*d
*e^(-c)*gamma(-1, d*x) + 3*a*b*d*e^c*gamma(-1, -d*x) - 3*b^2*Ei(-d*x)*e^(-
c) + 3*b^2*Ei(d*x)*e^c)*d - 1/3*(3*b^2*x^2 + 3*a*b*x + a^2)*cosh(d*x + c)/
x^3
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.66

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{a^2d^3x^3\mathrm{Ei}(-dx)\,e^{(-c)}-a^2d^3x^3\mathrm{Ei}(dx)\,e^c-6\,abd^2x^3\mathrm{Ei}(-dx)\,e^{(-c)}-6\,abd^2x^3\mathrm{Ei}(dx)\,e^c+6\,b^2dx^3\mathrm{Ei}(-dx}{}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

output
```
-1/12*(a^2*d^3*x^3*Ei(-d*x)*e^(-c) - a^2*d^3*x^3*Ei(d*x)*e^c - 6*a*b*d^2*x
^3*Ei(-d*x)*e^(-c) - 6*a*b*d^2*x^3*Ei(d*x)*e^c + 6*b^2*d*x^3*Ei(-d*x)*e^(-
c) - 6*b^2*d*x^3*Ei(d*x)*e^c + a^2*d^2*x^2*e^(d*x + c) + a^2*d^2*x^2*e^(-d
*x - c) + 6*a*b*d*x^2*e^(d*x + c) - 6*a*b*d*x^2*e^(-d*x - c) + a^2*d*x*e^(
d*x + c) + 6*b^2*x^2*e^(d*x + c) - a^2*d*x*e^(-d*x - c) + 6*b^2*x^2*e^(-d*
x - c) + 6*a*b*x*e^(d*x + c) + 6*a*b*x*e^(-d*x - c) + 2*a^2*e^(d*x + c) +
2*a^2*e^(-d*x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx=\int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^4}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x)^2)/x^4,x)
```

output
```
int((cosh(c + d*x)*(a + b*x)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 290, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^4}\,dx = \frac{-e^{dx}\,ei(-dx)\,a^2d^3x^3+6e^{dx}\,ei(-dx)\,ab\,d^2x^3-6e^{dx}\,ei(-dx)\,b^2d\,x^3+e^{dx+2c}\,ei(dx)\,a^2d^3x^3+6e^{dx+2c}\,ei(dx)}{}$$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^4,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a**2*d**3*x**3 + 6*e**(d*x)*ei( - d*x)*a*b*d**2*x*
*3 - 6*e**(d*x)*ei( - d*x)*b**2*d*x**3 + e**(2*c + d*x)*ei(d*x)*a**2*d**3*
x**3 + 6*e**(2*c + d*x)*ei(d*x)*a*b*d**2*x**3 + 6*e**(2*c + d*x)*ei(d*x)*b
**2*d*x**3 - e**(2*c + 2*d*x)*a**2*d**2*x**2 - e**(2*c + 2*d*x)*a**2*d*x -
 2*e**(2*c + 2*d*x)*a**2 - 6*e**(2*c + 2*d*x)*a*b*d*x**2 - 6*e**(2*c + 2*d
*x)*a*b*x - 6*e**(2*c + 2*d*x)*b**2*x**2 - a**2*d**2*x**2 + a**2*d*x - 2*a
**2 + 6*a*b*d*x**2 - 6*a*b*x - 6*b**2*x**2)/(12*e**(c + d*x)*x**3)
```

3.17 $\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 248

$$\begin{aligned}\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5} \, dx = & -\frac{a^2 \cosh(c+dx)}{4x^4} - \frac{2ab \cosh(c+dx)}{3x^3} - \frac{b^2 \cosh(c+dx)}{2x^2} \\ & - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{abd^2 \cosh(c+dx)}{3x} \\ & + \frac{1}{2} b^2 d^2 \cosh(c) \text{Chi}(dx) + \frac{1}{24} a^2 d^4 \cosh(c) \text{Chi}(dx) \\ & + \frac{1}{3} abd^3 \text{Chi}(dx) \sinh(c) - \frac{a^2 d \sinh(c+dx)}{12x^3} \\ & - \frac{abd \sinh(c+dx)}{3x^2} - \frac{b^2 d \sinh(c+dx)}{2x} \\ & - \frac{a^2 d^3 \sinh(c+dx)}{24x} + \frac{1}{3} abd^3 \cosh(c) \text{Shi}(dx) \\ & + \frac{1}{2} b^2 d^2 \sinh(c) \text{Shi}(dx) + \frac{1}{24} a^2 d^4 \sinh(c) \text{Shi}(dx)\end{aligned}$$

output
```
-1/4*a^2*cosh(d*x+c)/x^4-2/3*a*b*cosh(d*x+c)/x^3-1/2*b^2*cosh(d*x+c)/x^2-1
/24*a^2*d^2*cosh(d*x+c)/x^2-1/3*a*b*d^2*cosh(d*x+c)/x+1/2*b^2*d^2*cosh(c)*
Chi(d*x)+1/24*a^2*d^4*cosh(c)*Chi(d*x)+1/3*a*b*d^3*Chi(d*x)*sinh(c)-1/12*a
^2*d*sinh(d*x+c)/x^3-1/3*a*b*d*sinh(d*x+c)/x^2-1/2*b^2*d*sinh(d*x+c)/x-1/2
4*a^2*d^3*sinh(d*x+c)/x+1/3*a*b*d^3*cosh(c)*Shi(d*x)+1/2*b^2*d^2*sinh(c)*S
hi(d*x)+1/24*a^2*d^4*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.83

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx =$$
$$-\frac{6a^2\cosh(c+dx)+16abx\cosh(c+dx)+12b^2x^2\cosh(c+dx)+a^2d^2x^2\cosh(c+dx)+8abd^2x^3\cosh(c}{}$$

input
```
Integrate[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

output
```
-1/24*(6*a^2*Cosh[c + d*x] + 16*a*b*x*Cosh[c + d*x] + 12*b^2*x^2*Cosh[c +
d*x] + a^2*d^2*x^2*Cosh[c + d*x] + 8*a*b*d^2*x^3*Cosh[c + d*x] - d^2*x^4*C
oshIntegral[d*x]*((12*b^2 + a^2*d^2)*Cosh[c] + 8*a*b*d*Sinh[c]) + 2*a^2*d*
x*Sinh[c + d*x] + 8*a*b*d*x^2*Sinh[c + d*x] + 12*b^2*d*x^3*Sinh[c + d*x] +
 a^2*d^3*x^3*Sinh[c + d*x] - d^2*x^4*(8*a*b*d*Cosh[c] + 12*b^2*Sinh[c] + a
^2*d^2*Sinh[c])*SinhIntegral[d*x])/x^4
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^4}+\frac{b^2\cosh(c+dx)}{x^3}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d\sinh(c+dx)}{12x^3}+\frac{1}{3}abd^3\sinh(c)\text{Chi}(dx)+\frac{1}{3}abd^3\cosh(c)\text{Shi}(dx)-\frac{abd^2\cosh(c+dx)}{3x}-\frac{2ab\cosh(c+dx)}{3x^3}-\frac{abd\sinh(c+dx)}{3x^2}+\frac{1}{2}b^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}b^2d^2\sinh(c)\text{Shi}(dx)-\frac{b^2\cosh(c+dx)}{2x^2}-\frac{b^2d\sinh(c+dx)}{2x}$$

input
```
Int[((a + b*x)^2*Cosh[c + d*x])/x^5,x]
```

output
```
-1/4*(a^2*Cosh[c + d*x])/x^4 - (2*a*b*Cosh[c + d*x])/(3*x^3) - (b^2*Cosh[c
 + d*x])/(2*x^2) - (a^2*d^2*Cosh[c + d*x])/(24*x^2) - (a*b*d^2*Cosh[c + d*
x])/(3*x) + (b^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a^2*d^4*Cosh[c]*CoshI
ntegral[d*x])/24 + (a*b*d^3*CoshIntegral[d*x]*Sinh[c])/3 - (a^2*d*Sinh[c +
 d*x])/(12*x^3) - (a*b*d*Sinh[c + d*x])/(3*x^2) - (b^2*d*Sinh[c + d*x])/(2
*x) - (a^2*d^3*Sinh[c + d*x])/(24*x) + (a*b*d^3*Cosh[c]*SinhIntegral[d*x])
/3 + (b^2*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (a^2*d^4*Sinh[c]*SinhIntegral
[d*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 400, normalized size of antiderivative = 1.61

method	result
risch	$-\frac{e^{c}\operatorname{expIntegral}_1(-dx)a^2d^4x^4+e^{-c}\operatorname{expIntegral}_1(dx)a^2d^4x^4+8\,e^{c}\operatorname{expIntegral}_1(-dx)ab\,d^3x^4-8\,e^{-c}\operatorname{expIntegral}_1(dx)ab\,d^3x^4+12}{}$
meijerg	$-\frac{d^2b^2\cosh(c)\sqrt{\pi}\left(-\frac{4\left(\frac{9x^2d^2}{2}+3\right)}{3\sqrt{\pi}\,x^2d^2}+\frac{4\cosh(dx)}{\sqrt{\pi}\,x^2d^2}+\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}-\frac{4(\text{Chi}(dx)-\ln(dx)-\gamma)}{\sqrt{\pi}}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}\,x^2d^2}\right)}{8}+\frac{id^2b^2\,\mathrm{s}}{}$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(exp(c)*Ei(1,-d*x)*a^2*d^4*x^4+exp(-c)*Ei(1,d*x)*a^2*d^4*x^4+8*exp(c
)*Ei(1,-d*x)*a*b*d^3*x^4-8*exp(-c)*Ei(1,d*x)*a*b*d^3*x^4+12*exp(c)*Ei(1,-d
*x)*b^2*d^2*x^4+12*exp(-c)*Ei(1,d*x)*b^2*d^2*x^4-a^2*d^3*x^3*exp(-d*x-c)+a
^2*d^3*x^3*exp(d*x+c)+8*a*b*d^2*x^3*exp(-d*x-c)+8*a*b*d^2*x^3*exp(d*x+c)+a
^2*d^2*x^2*exp(-d*x-c)-12*b^2*d*x^3*exp(-d*x-c)+a^2*d^2*x^2*exp(d*x+c)+12*
b^2*d*x^3*exp(d*x+c)-8*a*b*d*x^2*exp(-d*x-c)+8*a*b*d*x^2*exp(d*x+c)-2*a^2*
d*x*exp(-d*x-c)+12*exp(-d*x-c)*b^2*x^2+2*a^2*d*x*exp(d*x+c)+12*exp(d*x+c)*
b^2*x^2+16*exp(-d*x-c)*a*b*x+16*exp(d*x+c)*a*b*x+6*exp(-d*x-c)*a^2+6*exp(d
*x+c)*a^2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 0.93

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx = -\frac{2\left(8\,abd^2x^3+16\,abx+(a^2d^2+12\,b^2)x^2+6\,a^2\right)\cosh\left(dx+c\right)-\left((a^2d^4+8\,abd^3+12\,b^2d^2)x^4\mathrm{Ei}(dx)+\right.}{}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
-1/48*(2*(8*a*b*d^2*x^3 + 16*a*b*x + (a^2*d^2 + 12*b^2)*x^2 + 6*a^2)*cosh(
d*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(d*x) + (a^2*d^4 - 8*
a*b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*(8*a*b*d*x^2 + 2*a^2*d*x +
 (a^2*d^3 + 12*b^2*d)*x^3)*sinh(d*x + c) - ((a^2*d^4 + 8*a*b*d^3 + 12*b^2*
d^2)*x^4*Ei(d*x) - (a^2*d^4 - 8*a*b*d^3 + 12*b^2*d^2)*x^4*Ei(-d*x))*sinh(c
))/x^4
```

Sympy [F]

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx)^2 \cosh{(c+dx)}}{x^5}\,dx$$

input
```
integrate((b*x+a)**2*cosh(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x)**2*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.52

$$\int \frac{(a+bx)^2 \cosh(c+dx)}{x^5}\,dx$$
$$= \frac{1}{24}\left(3\,a^2d^3e^{(-c)}\Gamma(-3,dx) + 3\,a^2d^3e^c\Gamma(-3,-dx) + 8\,abd^2e^{(-c)}\Gamma(-2,dx) - 8\,abd^2e^c\Gamma(-2,-dx) + 6\,b^2de^{(-\right.$$
$$- \frac{(6\,b^2x^2 + 8\,abx + 3\,a^2)\cosh{(dx+c)}}{12\,x^4}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
1/24*(3*a^2*d^3*e^(-c)*gamma(-3, d*x) + 3*a^2*d^3*e^c*gamma(-3, -d*x) + 8*
a*b*d^2*e^(-c)*gamma(-2, d*x) - 8*a*b*d^2*e^c*gamma(-2, -d*x) + 6*b^2*d*e^
(-c)*gamma(-1, d*x) + 6*b^2*d*e^c*gamma(-1, -d*x))*d - 1/12*(6*b^2*x^2 + 8
*a*b*x + 3*a^2)*cosh(d*x + c)/x^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 395, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{a^2d^4x^4\text{Ei}(-dx)\,e^{(-c)} + a^2d^4x^4\text{Ei}(dx)\,e^c - 8\,abd^3x^4\text{Ei}(-dx)\,e^{(-c)} + 8\,abd^3x^4\text{Ei}(dx)\,e^c + 12\,b^2d^2x^4\text{Ei}(-dx}{}$$

input
```
integrate((b*x+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

output
```
1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c - 8*a*b*d^3*x^
4*Ei(-d*x)*e^(-c) + 8*a*b*d^3*x^4*Ei(d*x)*e^c + 12*b^2*d^2*x^4*Ei(-d*x)*e^
(-c) + 12*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c) + a^2*d^3*x^3*
e^(-d*x - c) - 8*a*b*d^2*x^3*e^(d*x + c) - 8*a*b*d^2*x^3*e^(-d*x - c) - a^
2*d^2*x^2*e^(d*x + c) - 12*b^2*d*x^3*e^(d*x + c) - a^2*d^2*x^2*e^(-d*x - c
) + 12*b^2*d*x^3*e^(-d*x - c) - 8*a*b*d*x^2*e^(d*x + c) + 8*a*b*d*x^2*e^(-
d*x - c) - 2*a^2*d*x*e^(d*x + c) - 12*b^2*x^2*e^(d*x + c) + 2*a^2*d*x*e^(-
d*x - c) - 12*b^2*x^2*e^(-d*x - c) - 16*a*b*x*e^(d*x + c) - 16*a*b*x*e^(-d
*x - c) - 6*a^2*e^(d*x + c) - 6*a^2*e^(-d*x - c))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(a+b\,x\right)^2}{x^5}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x)^2)/x^5,x)
```

output
```
int((cosh(c + d*x)*(a + b*x)^2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx)^2\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{e^{dx}\,ei(-dx)\,a^2d^4x^4 - 8e^{dx}\,ei(-dx)\,ab\,d^3x^4 + 12e^{dx}\,ei(-dx)\,b^2d^2x^4 + e^{dx+2c}\,ei(dx)\,a^2d^4x^4 + 8e^{dx+2c}\,ei(dx)}{}$$

input
```
int((b*x+a)^2*cosh(d*x+c)/x^5,x)
```

output
```
(e**(d*x)*ei( - d*x)*a**2*d**4*x**4 - 8*e**(d*x)*ei( - d*x)*a*b*d**3*x**4
+ 12*e**(d*x)*ei( - d*x)*b**2*d**2*x**4 + e**(2*c + d*x)*ei(d*x)*a**2*d**4
*x**4 + 8*e**(2*c + d*x)*ei(d*x)*a*b*d**3*x**4 + 12*e**(2*c + d*x)*ei(d*x)
*b**2*d**2*x**4 - e**(2*c + 2*d*x)*a**2*d**3*x**3 - e**(2*c + 2*d*x)*a**2*
d**2*x**2 - 2*e**(2*c + 2*d*x)*a**2*d*x - 6*e**(2*c + 2*d*x)*a**2 - 8*e**(
2*c + 2*d*x)*a*b*d**2*x**3 - 8*e**(2*c + 2*d*x)*a*b*d*x**2 - 16*e**(2*c +
2*d*x)*a*b*x - 12*e**(2*c + 2*d*x)*b**2*d*x**3 - 12*e**(2*c + 2*d*x)*b**2*
x**2 + a**2*d**3*x**3 - a**2*d**2*x**2 + 2*a**2*d*x - 6*a**2 - 8*a*b*d**2*
x**3 + 8*a*b*d*x**2 - 16*a*b*x + 12*b**2*d*x**3 - 12*b**2*x**2)/(48*e**(c
+ d*x)*x**4)
```

3.18 $\int \frac{x^4 \cosh(c+dx)}{a+bx} dx$

Optimal result

Integrand size = 17, antiderivative size = 219

$$\int \frac{x^4 \cosh(c+dx)}{a+bx} dx = -\frac{6\cosh(c+dx)}{bd^4} - \frac{a^2\cosh(c+dx)}{b^3d^2} + \frac{2ax\cosh(c+dx)}{b^2d^2} - \frac{3x^2\cosh(c+dx)}{bd^2} + \frac{a^4\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^5} - \frac{2a\sinh(c+dx)}{b^2d^3} - \frac{a^3\sinh(c+dx)}{b^4d} + \frac{6x\sinh(c+dx)}{bd^3} + \frac{a^2x\sinh(c+dx)}{b^3d} - \frac{ax^2\sinh(c+dx)}{b^2d} + \frac{x^3\sinh(c+dx)}{bd} + \frac{a^4\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^5}$$

output
```
-6*cosh(d*x+c)/b/d^4-a^2*cosh(d*x+c)/b^3/d^2+2*a*x*cosh(d*x+c)/b^2/d^2-3*x
^2*cosh(d*x+c)/b/d^2+a^4*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^5-2*a*sinh(d*x+c)
/b^2/d^3-a^3*sinh(d*x+c)/b^4/d+6*x*sinh(d*x+c)/b/d^3+a^2*x*sinh(d*x+c)/b^3
/d-a*x^2*sinh(d*x+c)/b^2/d+x^3*sinh(d*x+c)/b/d-a^4*sinh(-c+a*d/b)*Shi(a*d/
b+d*x)/b^5
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.73

$$\int \frac{x^4 \cosh(c+dx)}{a+bx} \, dx$$
$$= \frac{a^4 d^4 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) - b(b(a^2d^2-2abd^2x+3b^2(2+d^2x^2))\cosh(c+dx)+d(a^3d^2-a^2bd^2x}{b^5d^4}$$

input
```
Integrate[(x^4*Cosh[c + d*x])/(a + b*x),x]
```

output
```
(a^4*d^4*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - b*(b*(a^2*d^2 - 2*a
*b*d^2*x + 3*b^2*(2 + d^2*x^2))*Cosh[c + d*x] + d*(a^3*d^2 - a^2*b*d^2*x +
 a*b^2*(2 + d^2*x^2) - b^3*x*(6 + d^2*x^2))*Sinh[c + d*x]) + a^4*d^4*Sinh[
c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^5*d^4)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx} \, dx$$
$$\Big\downarrow \text{ 7293}$$
$$\int \left(\frac{a^4 \cosh(c+dx)}{b^4(a+bx)} - \frac{a^3 \cosh(c+dx)}{b^4} + \frac{a^2 x \cosh(c+dx)}{b^3} - \frac{a x^2 \cosh(c+dx)}{b^2} + \frac{x^3 \cosh(c+dx)}{b}\right) dx$$
$$\Big\downarrow \text{ 2009}$$

$$\frac{a^4 \cosh\left(c-\frac{ad}{b}\right) \mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^4 \sinh\left(c-\frac{ad}{b}\right) \mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{a^3 \sinh(c+dx)}{b^4 d}-\frac{a^2 \cosh(c+dx)}{b^3 d^2}+\frac{a^2 x \sinh(c+dx)}{b^3 d}-\frac{2a \sinh(c+dx)}{b^2 d^3}+\frac{2ax \cosh(c+dx)}{b^2 d^2}-\frac{ax^2 \sinh(c+dx)}{b^2 d}-\frac{6 \cosh(c+dx)}{bd^4}+\frac{6x \sinh(c+dx)}{bd^3}-\frac{3x^2 \cosh(c+dx)}{bd^2}+\frac{x^3 \sinh(c+dx)}{bd}$$

input
```
Int[(x^4*Cosh[c + d*x])/(a + b*x),x]
```

output
```
(-6*Cosh[c + d*x])/(b*d^4) - (a^2*Cosh[c + d*x])/(b^3*d^2) + (2*a*x*Cosh[c
 + d*x])/(b^2*d^2) - (3*x^2*Cosh[c + d*x])/(b*d^2) + (a^4*Cosh[c - (a*d)/b
]*CoshIntegral[(a*d)/b + d*x])/b^5 - (2*a*Sinh[c + d*x])/(b^2*d^3) - (a^3*
Sinh[c + d*x])/(b^4*d) + (6*x*Sinh[c + d*x])/(b*d^3) + (a^2*x*Sinh[c + d*x
])/(b^3*d) - (a*x^2*Sinh[c + d*x])/(b^2*d) + (x^3*Sinh[c + d*x])/(b*d) + (
a^4*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 442, normalized size of antiderivative = 2.02

method	result
risch	$\frac{e^{dx+c} x^3}{2db}-\frac{e^{-dx-c} x^3}{2db}-\frac{e^{dx+c} a\, x^2}{2d\, b^2}-\frac{e^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right) a^4}{2b^5}+\frac{e^{-dx-c} a\, x^2}{2d\, b^2}-\frac{e^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(d\right.}{2b^5}$

input
```
int(x^4*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/d/b*exp(d*x+c)*x^3-1/2/d/b*exp(-d*x-c)*x^3-1/2/d/b^2*exp(d*x+c)*a*x^2-
1/2/b^5*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4+1/2/d/b^2*exp(-d*x-
c)*a*x^2-1/2/b^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4+1/2/d/b^3*ex
p(d*x+c)*a^2*x-3/2/d^2/b*exp(d*x+c)*x^2-1/2/d/b^3*exp(-d*x-c)*a^2*x-3/2/d^
2/b*exp(-d*x-c)*x^2-1/2/d/b^4*a^3*exp(d*x+c)+1/d^2/b^2*exp(d*x+c)*a*x+1/2/
d/b^4*exp(-d*x-c)*a^3+1/d^2/b^2*exp(-d*x-c)*a*x-1/2/d^2/b^3*a^2*exp(d*x+c)
+3/d^3/b*exp(d*x+c)*x-1/2/d^2/b^3*exp(-d*x-c)*a^2-3/d^3/b*exp(-d*x-c)*x-1/
d^3/b^2*a*exp(d*x+c)+1/d^3/b^2*exp(-d*x-c)*a-3/d^4/b*exp(d*x+c)-3/d^4/b*ex
p(-d*x-c)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.08

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx =$$
$$-\frac{2\,(3\,b^4d^2x^2-2\,ab^3d^2x+a^2b^2d^2+6\,b^4)\cosh\,(dx+c)-\left(a^4d^4\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+a^4d^4\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\,\left(-\frac{b}{}\right.}{}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*(3*b^4*d^2*x^2 - 2*a*b^3*d^2*x + a^2*b^2*d^2 + 6*b^4)*cosh(d*x + c
) - (a^4*d^4*Ei((b*d*x + a*d)/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b))*cosh(-(b*
c - a*d)/b) - 2*(b^4*d^3*x^3 - a*b^3*d^3*x^2 - a^3*b*d^3 - 2*a*b^3*d + (a^
2*b^2*d^3 + 6*b^4*d)*x)*sinh(d*x + c) + (a^4*d^4*Ei((b*d*x + a*d)/b) - a^4
*d^4*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*d^4)
```

Sympy [F]

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = \int \frac{x^4\cosh\,(c+dx)}{a+bx}\,dx$$

input
```
integrate(x**4*cosh(d*x+c)/(b*x+a),x)
```

output
```
Integral(x**4*cosh(c + d*x)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.00

$$\int \frac{x^4 \cosh(c+dx)}{a+bx}\,dx = -\frac{1}{24}\,d\left(\frac{12\,a^4\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^4d}-\frac{12\,a^3\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^4}+\frac{6\,a^2\left(\right.}{}\right.$$
$$+\frac{1}{12}\left(\frac{12\,a^4\log\left(bx+a\right)}{b^5}+\frac{3\,b^3x^4-4\,ab^2x^3+6\,a^2bx^2-12\,a^3x}{b^4}\right)\cosh\left(dx+c\right)$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/24*d*(12*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c
- a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^4*d) - 12*a^3*((d*x*e^c -
 e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^4 + 6*a^2*((d^2*x^2*e^c
- 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)
/b^3 - 4*a*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4
+ (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b^2 + 3*((d^4*x^4*e^
c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d
^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*e^(-d*x - c)/d^5)/b + 24*a^
4*cosh(d*x + c)*log(b*x + a)/(b^5*d)) + 1/12*(12*a^4*log(b*x + a)/b^5 + (3
*b^3*x^4 - 4*a*b^2*x^3 + 6*a^2*b*x^2 - 12*a^3*x)/b^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.86

$$\int \frac{x^4 \cosh(c+dx)}{a+bx}\,dx$$
$$= \frac{b^4d^3x^3e^{(dx+c)}-b^4d^3x^3e^{(-dx-c)}-ab^3d^3x^2e^{(dx+c)}+ab^3d^3x^2e^{(-dx-c)}+a^4d^4\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}+a^4d^4\mathrm{Ei}\left(-\right.}{}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^4*d^3*x^3*e^(d*x + c) - b^4*d^3*x^3*e^(-d*x - c) - a*b^3*d^3*x^2*e^
(d*x + c) + a*b^3*d^3*x^2*e^(-d*x - c) + a^4*d^4*Ei((b*d*x + a*d)/b)*e^(c
- a*d/b) + a^4*d^4*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^2*b^2*d^3*x*e^(
d*x + c) - 3*b^4*d^2*x^2*e^(d*x + c) - a^2*b^2*d^3*x*e^(-d*x - c) - 3*b^4*
d^2*x^2*e^(-d*x - c) - a^3*b*d^3*e^(d*x + c) + 2*a*b^3*d^2*x*e^(d*x + c) +
 a^3*b*d^3*e^(-d*x - c) + 2*a*b^3*d^2*x*e^(-d*x - c) - a^2*b^2*d^2*e^(d*x
+ c) + 6*b^4*d*x*e^(d*x + c) - a^2*b^2*d^2*e^(-d*x - c) - 6*b^4*d*x*e^(-d*
x - c) - 2*a*b^3*d*e^(d*x + c) + 2*a*b^3*d*e^(-d*x - c) - 6*b^4*e^(d*x + c
) - 6*b^4*e^(-d*x - c))/(b^5*d^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx=\int \frac{x^4\cosh(c+d\,x)}{a+b\,x}\,dx$$

input
```
int((x^4*cosh(c + d*x))/(a + b*x),x)
```

output
```
int((x^4*cosh(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x^4\cosh(c+dx)}{a+bx}\,dx = \frac{-\cosh(dx+c)\,a^2b\,d^2+2\cosh(dx+c)\,a\,b^2d^2x-3\cosh(dx+c)\,b^3d^2x^2-6\cosh(dx+c)\,b^3+\left(\int \frac{\cosh(dx}{bx+}\right.}{}$$

input
```
int(x^4*cosh(d*x+c)/(b*x+a),x)
```

output
```
( - cosh(c + d*x)*a**2*b*d**2 + 2*cosh(c + d*x)*a*b**2*d**2*x - 3*cosh(c +
 d*x)*b**3*d**2*x**2 - 6*cosh(c + d*x)*b**3 + int(cosh(c + d*x)/(a + b*x),
x)*a**4*d**4 - sinh(c + d*x)*a**3*d**3 + sinh(c + d*x)*a**2*b*d**3*x - sin
h(c + d*x)*a*b**2*d**3*x**2 - 2*sinh(c + d*x)*a*b**2*d + sinh(c + d*x)*b**
3*d**3*x**3 + 6*sinh(c + d*x)*b**3*d*x)/(b**4*d**4)
```

3.19 $\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\begin{aligned}\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx &= \frac{a\cosh(c+dx)}{b^2d^2}-\frac{2x\cosh(c+dx)}{bd^2}\\&\quad-\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{2\sinh(c+dx)}{bd^3}\\&\quad+\frac{a^2\sinh(c+dx)}{b^3d}-\frac{ax\sinh(c+dx)}{b^2d}\\&\quad+\frac{x^2\sinh(c+dx)}{bd}-\frac{a^3\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}\end{aligned}$$

output
```
a*cosh(d*x+c)/b^2/d^2-2*x*cosh(d*x+c)/b/d^2-a^3*cosh(-c+a*d/b)*Chi(a*d/b+d
*x)/b^4+2*sinh(d*x+c)/b/d^3+a^2*sinh(d*x+c)/b^3/d-a*x*sinh(d*x+c)/b^2/d+x^
2*sinh(d*x+c)/b/d+a^3*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b^4
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.79

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = \frac{-a^3d^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+b(bd(a-2bx)\cosh(c+dx)+(a^2d^2-abd^2x+b^2(2+d^2x^2))\sinh(}{b^4d^3}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

output
```
(-(a^3*d^3*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*(b*d*(a - 2*b*
x)*Cosh[c + d*x] + (a^2*d^2 - a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x]
) - a^3*d^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^4*d^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{a^3\cosh(c+dx)}{b^3(a+bx)}+\frac{a^2\cosh(c+dx)}{b^3}-\frac{ax\cosh(c+dx)}{b^2}+\frac{x^2\cosh(c+dx)}{b}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{a^2\sinh(c+dx)}{b^3d}+\frac{a\cosh(c+dx)}{b^2d^2}-\frac{ax\sinh(c+dx)}{b^2d}+\frac{2\sinh(c+dx)}{bd^3}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{x^2\sinh(c+dx)}{bd}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x),x]
```

output
```
(a*Cosh[c + d*x])/(b^2*d^2) - (2*x*Cosh[c + d*x])/(b*d^2) - (a^3*Cosh[c -
(a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 + (2*Sinh[c + d*x])/(b*d^3) + (a
^2*Sinh[c + d*x])/(b^3*d) - (a*x*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c + d*
x])/(b*d) - (a^3*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.95

method	result
risch	$\frac{\mathrm{e}^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right)a^3}{2b^4}-\frac{\mathrm{e}^{-dx-c}x^2}{2db}+\frac{\mathrm{e}^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)a^3}{2b^4}+\frac{\mathrm{e}^{dx+c}x^2}{2db}+\frac{\mathrm{e}^{-dx-c}ax}{2d\,b^2}$

input
```
int(x^3*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3-1/2/d/b*exp(-d*x-c)
*x^2+1/2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3+1/2/d/b*exp(d*x+
c)*x^2+1/2/d/b^2*exp(-d*x-c)*a*x-1/2/d/b^2*exp(d*x+c)*a*x-1/2/d/b^3*exp(-d
*x-c)*a^2-1/d^2/b*exp(-d*x-c)*x+1/2/d/b^3*a^2*exp(d*x+c)-1/d^2/b*exp(d*x+c
)*x+1/2/d^2/b^2*exp(-d*x-c)*a+1/2/d^2/b^2*a*exp(d*x+c)-1/d^3/b*exp(-d*x-c)
+1/d^3/b*exp(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.27

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = -\frac{2\,(2\,b^3dx - ab^2d)\cosh\left(dx+c\right) + \left(a^3d^3\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) + a^3d^3\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right) - 2\,(b^3d^2x^2 - a}{2\,b^4d^3}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*(2*b^3*d*x - a*b^2*d)*cosh(d*x + c) + (a^3*d^3*Ei((b*d*x + a*d)/b)
 + a^3*d^3*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*d^2*x^2 - a
*b^2*d^2*x + a^2*b*d^2 + 2*b^3)*sinh(d*x + c) - (a^3*d^3*Ei((b*d*x + a*d)/
b) - a^3*d^3*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*d^3)
```

Sympy [F]

$$\int \frac{x^3\cosh(c+dx)}{a+bx}\,dx = \int \frac{x^3\cosh\left(c+dx\right)}{a+bx}\,dx$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x+a),x)
```

output
```
Integral(x**3*cosh(c + d*x)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(151) = 302.

Time = 0.11 (sec) , antiderivative size = 328, normalized size of antiderivative = 2.19

$$\int \frac{x^3 \cosh(c+dx)}{a+bx}\,dx$$
$$= \frac{1}{12}\,d\left(\frac{6\,a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^3d}-\frac{6\,a^2\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^3}+\frac{3\,a\Big(\frac{(d^2x^2e}{}}{}\right.$$
$$-\frac{1}{6}\left(\frac{6\,a^3\log\left(bx+a\right)}{b^4}-\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{b^3}\right)\cosh\left(dx+c\right)$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
1/12*d*(6*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c -
a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^3*d) - 6*a^2*((d*x*e^c - e^
c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^3 + 3*a*((d^2*x^2*e^c - 2*d
*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^2
- 2*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*
x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4)/b + 12*a^3*cosh(d*x + c)*lo
g(b*x + a)/(b^4*d)) - 1/6*(6*a^3*log(b*x + a)/b^4 - (2*b^2*x^3 - 3*a*b*x^2
 + 6*a^2*x)/b^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 256, normalized size of antiderivative = 1.71

$$\int \frac{x^3 \cosh(c+dx)}{a+bx}\,dx$$
$$= \frac{b^3d^2x^2e^{(dx+c)}-b^3d^2x^2e^{(-dx-c)}-a^3d^3\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)}-a^3d^3\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}-ab^2d^2xe^{(dx+c)}+}{}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(b^3*d^2*x^2*e^(d*x + c) - b^3*d^2*x^2*e^(-d*x - c) - a^3*d^3*Ei((b*d*
x + a*d)/b)*e^(c - a*d/b) - a^3*d^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) -
a*b^2*d^2*x*e^(d*x + c) + a*b^2*d^2*x*e^(-d*x - c) + a^2*b*d^2*e^(d*x + c)
 - 2*b^3*d*x*e^(d*x + c) - a^2*b*d^2*e^(-d*x - c) - 2*b^3*d*x*e^(-d*x - c)
 + a*b^2*d*e^(d*x + c) + a*b^2*d*e^(-d*x - c) + 2*b^3*e^(d*x + c) - 2*b^3*
e^(-d*x - c))/(b^4*d^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{a+bx} \, dx = \int \frac{x^3 \cosh(c+d\,x)}{a+b\,x} \, dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x),x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx} \, dx$$
$$= \frac{\cosh(dx+c)\,abd - 2\cosh(dx+c)\,b^2dx - \left(\int \frac{\cosh(dx+c)}{bx+a}dx\right) a^3d^3 + \sinh(dx+c)\,a^2d^2 - \sinh(dx+c)\,ab}{b^3d^3}$$

input
```
int(x^3*cosh(d*x+c)/(b*x+a),x)
```

output
```
(cosh(c + d*x)*a*b*d - 2*cosh(c + d*x)*b**2*d*x - int(cosh(c + d*x)/(a + b
*x),x)*a**3*d**3 + sinh(c + d*x)*a**2*d**2 - sinh(c + d*x)*a*b*d**2*x + si
nh(c + d*x)*b**2*d**2*x**2 + 2*sinh(c + d*x)*b**2)/(b**3*d**3)
```

3.20 $\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx = -\frac{\cosh(c+dx)}{bd^2} + \frac{a^2 \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{(a-bx)\sinh(c+dx)}{b^2 d} + \frac{a^2 \sinh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}$$

output
```
-cosh(d*x+c)/b/d^2+a^2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^3-(-b*x+a)*sinh(d*x
+c)/b^2/d-a^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.98

$$\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx = \frac{a^2 d^2 \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + b(-b\cosh(c+dx) + d(-a+bx)\sinh(c+dx)) + a^2 d^2 \sinh\left(c-\frac{ad}{b}\right)}{b^3 d^2}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

output
```
(a^2*d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + b*(-(b*Cosh[c + d*x
]) + d*(-a + b*x)*Sinh[c + d*x]) + a^2*d^2*Sinh[c - (a*d)/b]*SinhIntegral[
d*(a/b + x)])/(b^3*d^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.10, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cosh(c+dx)}{a+bx} \, dx$$

$$\Big\downarrow \text{ 7293}$$

$$\int \left(\frac{a^2 \cosh(c+dx)}{b^2(a+bx)} - \frac{a \cosh(c+dx)}{b^2} + \frac{x \cosh(c+dx)}{b}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2 \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3} + \frac{a^2 \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3} - \frac{a \sinh(c+dx)}{b^2 d} - \frac{\cosh(c+dx)}{bd^2} + \frac{x \sinh(c+dx)}{bd}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x),x]
```

output
```
-(Cosh[c + d*x]/(b*d^2)) + (a^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d
*x])/b^3 - (a*Sinh[c + d*x])/(b^2*d) + (x*Sinh[c + d*x])/(b*d) + (a^2*Sinh
[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

method	result
risch	$-\frac{\mathrm{e}^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right) a^2}{2b^3} - \frac{\mathrm{e}^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right) a^2}{2b^3} - \frac{\mathrm{e}^{-dx-c} x}{2db} + \frac{\mathrm{e}^{dx+c} x}{2db} + \frac{\mathrm{e}^{-dx-c} a}{2d\, b^2}$

input
```
int(x^2*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2-1/2/b^3*exp((a*d-b
*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2-1/2/d/b*exp(-d*x-c)*x+1/2/d/b*exp(d*x+c
)*x+1/2/d/b^2*exp(-d*x-c)*a-1/2/d/b^2*a*exp(d*x+c)-1/2/d^2/b*exp(-d*x-c)-1
/2/d^2/b*exp(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.71

$$\int \frac{x^2 \cosh(c+dx)}{a+bx}\, dx = -\frac{2\, b^2 \cosh\left(dx+c\right) - \left(a^2 d^2 \operatorname{Ei}\left(\frac{bdx+ad}{b}\right) + a^2 d^2 \operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right) \cosh\left(-\frac{bc-ad}{b}\right) - 2\left(b^2 dx - abd\right) \sinh\left(dx+c\right.}{2\, b^3 d^2}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*cosh(d*x + c) - (a^2*d^2*Ei((b*d*x + a*d)/b) + a^2*d^2*Ei(-(b*
d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^2*d*x - a*b*d)*sinh(d*x + c) +
(a^2*d^2*Ei((b*d*x + a*d)/b) - a^2*d^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c -
a*d)/b))/(b^3*d^2)
```

Sympy [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx}\,dx = \int \frac{x^2 \cosh{(c+dx)}}{a+bx}\,dx$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x+a),x)
```

output
```
Integral(x**2*cosh(c + d*x)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. 2(94) = 188.

Time = 0.12 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.56

$$\int \frac{x^2 \cosh(c+dx)}{a+bx}\,dx = -\frac{1}{4}d\left(\frac{2a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^2d}-\frac{2a\left(\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}\right)}{b^2}+\frac{(d^2x^2e^c-2\,d\ldots}{\ldots}\right.$$
$$+\frac{1}{2}\left(\frac{2a^2\log{(bx+a)}}{b^3}+\frac{bx^2-2ax}{b^2}\right)\cosh{(dx+c)}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*d*(2*a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c -
a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b^2*d) - 2*a*((d*x*e^c - e^c)
*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/b^2 + ((d^2*x^2*e^c - 2*d*x*e^c
 + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b + 4*a^2*
cosh(d*x + c)*log(b*x + a)/(b^3*d)) + 1/2*(2*a^2*log(b*x + a)/b^3 + (b*x^2
 - 2*a*x)/b^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.63

$$\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx = \frac{a^2d^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + a^2d^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + b^2dxe^{(dx+c)} - b^2dxe^{(-dx-c)} - abde^{(dx+c)} + abde^{(-c\ldots}}{2\,b^3d^2}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(a^2*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)
/b)*e^(-c + a*d/b) + b^2*d*x*e^(d*x + c) - b^2*d*x*e^(-d*x - c) - a*b*d*e^
(d*x + c) + a*b*d*e^(-d*x - c) - b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^3*
d^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\cosh(c+dx)}{a+bx}\,dx = \int \frac{x^2\cosh(c+d\,x)}{a+b\,x}\,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x),x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx}\,dx$$
$$= \frac{-\cosh(dx+c)\,b + \left(\int \frac{\cosh(dx+c)}{bx+a}dx\right)a^2d^2 - \sinh(dx+c)\,ad + \sinh(dx+c)\,bdx}{b^2d^2}$$

input
```
int(x^2*cosh(d*x+c)/(b*x+a),x)
```

output
```
( - cosh(c + d*x)*b + int(cosh(c + d*x)/(a + b*x),x)*a**2*d**2 - sinh(c +
d*x)*a*d + sinh(c + d*x)*b*d*x)/(b**2*d**2)
```

3.21 $\int \frac{x\cosh(c+dx)}{a+bx}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 68

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = -\frac{a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} + \frac{\sinh(c+dx)}{bd} - \frac{a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

output
```
-a*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^2+sinh(d*x+c)/b/d+a*sinh(-c+a*d/b)*Shi(
a*d/b+d*x)/b^2
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.94

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \frac{-ad\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+b\sinh(c+dx)-ad\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2d}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x),x]
```

output
```
(-(a*d*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)]) + b*Sinh[c + d*x] - a*
d*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \cosh(c+dx)}{a+bx} \, dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\cosh(c+dx)}{b} - \frac{a \cosh(c+dx)}{b(a+bx)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a \cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2} - \frac{a \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2} + \frac{\sinh(c+dx)}{bd}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x),x]
```

output
```
-((a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^2) + Sinh[c + d*x]/(
b*d) - (a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.68

method	result	size
risch	$\frac{\mathrm{e}^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right) a}{2b^2}+\frac{\mathrm{e}^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right) a}{2b^2}-\frac{\mathrm{e}^{-dx-c}}{2db}+\frac{\mathrm{e}^{dx+c}}{2bd}$	114

input
```
int(x*cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/2/b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a+1/2/b^2*exp((a*d-b*c)
/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a-1/2/d/b*exp(-d*x-c)+1/2/b/d*exp(d*x+c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.74

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = -\frac{\left(ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-2\,b\sinh\left(dx+c\right)-\left(ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right.}{2\,b^2d}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*((a*d*Ei((b*d*x + a*d)/b) + a*d*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*
d)/b) - 2*b*sinh(d*x + c) - (a*d*Ei((b*d*x + a*d)/b) - a*d*Ei(-(b*d*x + a*
d)/b))*sinh(-(b*c - a*d)/b))/(b^2*d)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \int \frac{x\cosh{(c+dx)}}{a+bx}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a),x)
```

output
```
Integral(x*cosh(c + d*x)/(a + b*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(69) = 138.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.29

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx$$
$$= \frac{1}{2}d\left(\frac{a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{bd}-\frac{\frac{(dxe^c-e^c)e^{(dx)}}{d^2}+\frac{(dx+1)e^{(-dx-c)}}{d^2}}{b}+\frac{2\,a\cosh{(dx+c)}\log}{b^2d}\right.$$
$$+\left(\frac{x}{b}-\frac{a\log{(bx+a)}}{b^2}\right)\cosh{(dx+c)}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*d*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b
)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(b*d) - ((d*x*e^c - e^c)*e^(d*x)/d^
2 + (d*x + 1)*e^(-d*x - c)/d^2)/b + 2*a*cosh(d*x + c)*log(b*x + a)/(b^2*d)
) + (x/b - a*log(b*x + a)/b^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.22

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = -\frac{ad\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + ad\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} - be^{(dx+c)} + be^{(-dx-c)}}{2\,b^2d}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(a*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*d*Ei(-(b*d*x + a*d)/b)*e^(
-c + a*d/b) - b*e^(d*x + c) + b*e^(-d*x - c))/(b^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \int \frac{x\cosh(c+d\,x)}{a+b\,x}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x),x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x), x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{a+bx}\,dx = \frac{-\left(\int \frac{\cosh(dx+c)}{bx+a}dx\right)ad + \sinh(dx+c)}{bd}$$

input
```
int(x*cosh(d*x+c)/(b*x+a),x)
```

output
```
( - int(cosh(c + d*x)/(a + b*x),x)*a*d + sinh(c + d*x))/(b*d)
```

3.22 $\int \frac{\cosh(c+dx)}{a+bx} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int \frac{\cosh(c+dx)}{a+bx} \, dx = \frac{\cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right)}{b} + \frac{\sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b}$$

output
```
cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b-sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.96

$$\int \frac{\cosh(c+dx)}{a+bx} \, dx = \frac{\cosh\left(c-\frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b}+dx\right) + \sinh\left(c-\frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b}+dx\right)}{b}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x),x]
```

output
```
(Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x] + Sinh[c - (a*d)/b]*SinhInt
egral[(a*d)/b + d*x])/b
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{a+bx}dx$$

$$\Big\downarrow \text{3784}$$

$$\cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(xd+\frac{ad}{b}\right)}{a+bx}dx - i\sinh\left(c-\frac{ad}{b}\right)\int \frac{i\sinh\left(xd+\frac{ad}{b}\right)}{a+bx}dx$$

$$\Big\downarrow \text{26}$$

$$\sinh\left(c-\frac{ad}{b}\right)\int \frac{\sinh\left(xd+\frac{ad}{b}\right)}{a+bx}dx + \cosh\left(c-\frac{ad}{b}\right)\int \frac{\cosh\left(xd+\frac{ad}{b}\right)}{a+bx}dx$$

$$\Big\downarrow \text{3042}$$

$$\sinh\left(c-\frac{ad}{b}\right)\int -\frac{i\sin\left(ixd+\frac{iad}{b}\right)}{a+bx}dx + \cosh\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx$$

$$\Big\downarrow \text{26}$$

$$\cosh\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx - i\sinh\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(ixd+\frac{iad}{b}\right)}{a+bx}dx$$

$$\Big\downarrow \text{3779}$$

$$\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b} + \cosh\left(c-\frac{ad}{b}\right)\int \frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx$$

$$\Big\downarrow \text{3782}$$

$$\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b} + \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}$$

input
```
Int[Cosh[c + d*x]/(a + b*x),x]
```

output
```
(Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b + (Sinh[c - (a*d)/b]*Sin
hIntegral[(a*d)/b + d*x])/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.59

method	result	size
risch	$-\frac{e^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)}{2b}-\frac{e^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right)}{2b}$	81

input
```
int(cosh(d*x+c)/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2/b*exp(-(a*d-b*c)/b)*Ei
(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.86

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \frac{\left(\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)+\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)-\left(\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)-\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\sinh\left(-\frac{bc-ad}{b}\right)}{2\,b}$$

input
```
integrate(cosh(d*x+c)/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*((Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - (
Ei((b*d*x + a*d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/b
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \int \frac{\cosh\left(c+dx\right)}{a+bx}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x+a),x)
```

output
```
Integral(cosh(c + d*x)/(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.12

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = -\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{2\,b} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{2\,b}$$

input
```
integrate(cosh(d*x+c)/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - 1/2*e^(c - a*d/b)
*exp_integral_e(1, -(b*x + a)*d/b)/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \frac{\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + \mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,b}$$

input
```
integrate(cosh(d*x+c)/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/
b))/b
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \int \frac{\cosh(c+d\,x)}{a+b\,x}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x),x)
```

output
```
int(cosh(c + d*x)/(a + b*x), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{a+bx}\,dx = \int \frac{\cosh(dx+c)}{bx+a}dx$$

input
```
int(cosh(d*x+c)/(b*x+a),x)
```

output
```
int(cosh(c + d*x)/(a + b*x),x)
```

3.23 $\int \frac{\cosh(c+dx)}{x(a+bx)} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\cosh(c+dx)}{x(a+bx)} \, dx = \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a}$$

output
```
cosh(c)*Chi(d*x)/a-cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a+sinh(c)*Shi(d*x)/a+sinh
(-c+a*d/b)*Shi(a*d/b+d*x)/a
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(c+dx)}{x(a+bx)} \, dx = \frac{\cosh(c)\text{Chi}(dx) - \cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + \sinh(c)\text{Shi}(dx) - \sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{a}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x)),x]
```

output
```
(Cosh[c]*CoshIntegral[d*x] - Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] +
 Sinh[c]*SinhIntegral[d*x] - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/
a
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c+dx)}{a(a+bx)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a} - \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a} + \frac{\cosh(c)\text{Chi}(dx)}{a} + \frac{\sinh(c)\text{Shi}(dx)}{a}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x)),x]
```

output
```
(Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b +
d*x])/a + (Sinh[c]*SinhIntegral[d*x])/a - (Sinh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

method	result
risch	$-\frac{e^{-c}\,\text{expIntegral}_1(dx)}{2a}+\frac{e^{\frac{ad-cb}{b}}\,\text{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)}{2a}-\frac{e^{c}\,\text{expIntegral}_1(-dx)}{2a}+\frac{e^{-\frac{ad-cb}{b}}\,\text{expIntegral}_1\left(-dx-c-\right.}{2a}$

input
```
int(cosh(d*x+c)/x/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*exp(-c)*Ei(1,d*x)+1/2/a*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/
2/a*exp(c)*Ei(1,-d*x)+1/2/a*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.68

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$$
$$=\frac{(\mathrm{Ei}(dx)+\mathrm{Ei}(-dx))\cosh(c)-\left(\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+(\mathrm{Ei}(dx)-\mathrm{Ei}(-dx))\sinh(c}{2\,a}$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="fricas")
```

output
```
1/2*((Ei(d*x) + Ei(-d*x))*cosh(c) - (Ei((b*d*x + a*d)/b) + Ei(-(b*d*x + a*
d)/b))*cosh(-(b*c - a*d)/b) + (Ei(d*x) - Ei(-d*x))*sinh(c) + (Ei((b*d*x +
a*d)/b) - Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/a
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a),x)
```

output
```
Integral(cosh(c + d*x)/(x*(a + b*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. 2(74) = 148.

Time = 0.12 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.12

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$$
$$= \frac{1}{2}d\left(\frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{ad}+\frac{2\cosh(dx+c)\log(bx+a)}{ad}-\frac{2\cosh(dx+c)\log(\right.$$
$$-\left(\frac{\log(bx+a)}{a}-\frac{\log(x)}{a}\right)\cosh(dx+c)$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="maxima")
```

output
```
1/2*d*(b*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b
)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a*d) + 2*cosh(d*x + c)*log(b*x + a
)/(a*d) - 2*cosh(d*x + c)*log(x)/(a*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(
a*d)) - (log(b*x + a)/a - log(x)/a)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.03

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx$$
$$= \frac{\text{Ei}(-dx)\,e^{(-c)} - \text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + \text{Ei}(dx)\,e^{c} - \text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)}}{2\,a}$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a),x, algorithm="giac")
```

output
```
1/2*(Ei(-d*x)*e^(-c) - Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + Ei(d*x)*e^c - E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \int \frac{\cosh(c+d\,x)}{x\,(a+b\,x)}\,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x)),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)}\,dx = \int \frac{\cosh(dx+c)}{b\,x^2+ax}dx$$

input
```
int(cosh(d*x+c)/x/(b*x+a),x)
```

output
```
int(cosh(c + d*x)/(a*x + b*x**2),x)
```

3.24 $\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 113

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = -\frac{\cosh(c+dx)}{ax} - \frac{b\cosh(c)\mathrm{Chi}(dx)}{a^2} + \frac{b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{d\mathrm{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\mathrm{Shi}(dx)}{a} - \frac{b\sinh(c)\mathrm{Shi}(dx)}{a^2} + \frac{b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^2}$$

output
```
-cosh(d*x+c)/a/x-b*cosh(c)*Chi(d*x)/a^2+b*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a^
2+d*Chi(d*x)*sinh(c)/a+d*cosh(c)*Shi(d*x)/a-b*sinh(c)*Shi(d*x)/a^2-b*sinh(
-c+a*d/b)*Shi(a*d/b+d*x)/a^2
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = \frac{-a\cosh(c+dx) + bx\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + \mathrm{Chi}(dx)(-bx\cosh(c)+adx\sinh(c)) + adx\cosh(c)}{a^2x}$$

input
```
Integrate[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

output
```
(-(a*Cosh[c + d*x]) + b*x*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + Co
shIntegral[d*x]*(-(b*x*Cosh[c]) + a*d*x*Sinh[c]) + a*d*x*Cosh[c]*SinhInteg
ral[d*x] - b*x*Sinh[c]*SinhIntegral[d*x] + b*x*Sinh[c - (a*d)/b]*SinhInteg
ral[d*(a/b + x)])/(a^2*x)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{b^2\cosh(c+dx)}{a^2(a+bx)}-\frac{b\cosh(c+dx)}{a^2x}+\frac{\cosh(c+dx)}{ax^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}+\frac{b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\sinh(c)\text{Chi}(dx)}{a}+\frac{d\cosh(c)\text{Shi}(dx)}{a}-\frac{\cosh(c+dx)}{ax}$$

input
```
Int[Cosh[c + d*x]/(x^2*(a + b*x)),x]
```

output
```
-(Cosh[c + d*x]/(a*x)) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (b*Cosh[c - (
a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 + (d*CoshIntegral[d*x]*Sinh[c])/a
 + (d*Cosh[c]*SinhIntegral[d*x])/a - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (
b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.37

method	result
risch	$-\frac{e^{c} \operatorname{expIntegral}_1(-dx)adx - e^{-c} \operatorname{expIntegral}_1(dx)adx - e^{c} \operatorname{expIntegral}_1(-dx)bx + b\,e^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right)x - e^{-c}}{2a^2x}$

input
```
int(cosh(d*x+c)/x^2/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(exp(c)*Ei(1,-d*x)*a*d*x-exp(-c)*Ei(1,d*x)*a*d*x-exp(c)*Ei(1,-d*x)*b*
x+b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*x-exp(-c)*Ei(1,d*x)*b*x+b*e
xp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*x+exp(d*x+c)*a+exp(-d*x-c)*a)/a^2/
x
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.58

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = -\frac{2\,a\cosh\left(dx+c\right) - \left((ad-b)x\mathrm{Ei}(dx) - (ad+b)x\mathrm{Ei}(-dx)\right)\cosh\left(c\right) - \left(bx\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) + bx\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right.}{}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="fricas")
```

output
```
-1/2*(2*a*cosh(d*x + c) - ((a*d - b)*x*Ei(d*x) - (a*d + b)*x*Ei(-d*x))*cos
h(c) - (b*x*Ei((b*d*x + a*d)/b) + b*x*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a
*d)/b) - ((a*d - b)*x*Ei(d*x) + (a*d + b)*x*Ei(-d*x))*sinh(c) + (b*x*Ei((b
*d*x + a*d)/b) - b*x*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*x)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\cosh{(c+dx)}}{x^2\,(a+bx)}\,dx$$

input
```
integrate(cosh(d*x+c)/x**2/(b*x+a),x)
```

output
```
Integral(cosh(c + d*x)/(x**2*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.70

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = -\frac{1}{2}\,d\left(\frac{\mathrm{Ei}(-dx)\,e^{(-c)} - \mathrm{Ei}(dx)\,e^{c}}{a} + \frac{b^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d} + \frac{2\,b\cosh{(dx+c)}\,\mathrm{lo}}{a^2d}\right. + \left(\frac{b\log{(bx+a)}}{a^2} - \frac{b\log{(x)}}{a^2} - \frac{1}{ax}\right)\cosh{(dx+c)}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="maxima")
```

output
```
-1/2*d*((Ei(-d*x)*e^(-c) - Ei(d*x)*e^c)/a + b^2*(e^(-c + a*d/b)*exp_integr
al_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)
/b)/(a^2*d) + 2*b*cosh(d*x + c)*log(b*x + a)/(a^2*d) - 2*b*cosh(d*x + c)*l
og(x)/(a^2*d) + (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/(a^2*d)) + (b*log(b*x +
a)/a^2 - b*log(x)/a^2 - 1/(a*x))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx =$$

$$-\frac{adx\text{Ei}(-dx)\,e^{(-c)} - adx\text{Ei}(dx)\,e^{c} + bx\text{Ei}(-dx)\,e^{(-c)} - bx\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + bx\text{Ei}(dx)\,e^{c} - bx\text{Ei}\left(-\frac{b}{\ }\right.}{2\,a^2x}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a),x, algorithm="giac")
```

output
```
-1/2*(a*d*x*Ei(-d*x)*e^(-c) - a*d*x*Ei(d*x)*e^c + b*x*Ei(-d*x)*e^(-c) - b*
x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b*x*Ei(d*x)*e^c - b*x*Ei(-(b*d*x + a
*d)/b)*e^(-c + a*d/b) + a*e^(d*x + c) + a*e^(-d*x - c))/(a^2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,(a+b\,x)}\,dx$$

input
```
int(cosh(c + d*x)/(x^2*(a + b*x)),x)
```

output
```
int(cosh(c + d*x)/(x^2*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\,x^3+a\,x^2}dx$$

input
```
int(cosh(d*x+c)/x^2/(b*x+a),x)
```

output
```
int(cosh(c + d*x)/(a*x**2 + b*x**3),x)
```

3.25 $\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 190

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = & -\frac{\cosh(c+dx)}{2ax^2}+\frac{b\cosh(c+dx)}{a^2x}+\frac{b^2\cosh(c)\text{Chi}(dx)}{a^3}\\ & +\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}-\frac{b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}\\ & -\frac{bd\text{Chi}(dx)\sinh(c)}{a^2}-\frac{d\sinh(c+dx)}{2ax}\\ & -\frac{bd\cosh(c)\text{Shi}(dx)}{a^2}+\frac{b^2\sinh(c)\text{Shi}(dx)}{a^3}\\ & +\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}-\frac{b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}\end{aligned}$$

output

```
-1/2*cosh(d*x+c)/a/x^2+b*cosh(d*x+c)/a^2/x+b^2*cosh(c)*Chi(d*x)/a^3+1/2*d^
2*cosh(c)*Chi(d*x)/a-b^2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a^3-b*d*Chi(d*x)*si
nh(c)/a^2-1/2*d*sinh(d*x+c)/a/x-b*d*cosh(c)*Shi(d*x)/a^2+b^2*sinh(c)*Shi(d
*x)/a^3+1/2*d^2*sinh(c)*Shi(d*x)/a+b^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/a^3
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 178, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$$
$$= -a^2\cosh(c+dx)+2abx\cosh(c+dx)-2b^2x^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)+x^2\text{Chi}(dx)\left(\left(2b^2+a^2d^2\right)c \cdots\right.$$

input
```
Integrate[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```

output
```
(-(a^2*Cosh[c + d*x]) + 2*a*b*x*Cosh[c + d*x] - 2*b^2*x^2*Cosh[c - (a*d)/b
]*CoshIntegral[d*(a/b + x)] + x^2*CoshIntegral[d*x]*((2*b^2 + a^2*d^2)*Cos
h[c] - 2*a*b*d*Sinh[c]) - a^2*d*x*Sinh[c + d*x] - 2*a*b*d*x^2*Cosh[c]*Sinh
Integral[d*x] + 2*b^2*x^2*Sinh[c]*SinhIntegral[d*x] + a^2*d^2*x^2*Sinh[c]*
SinhIntegral[d*x] - 2*b^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])
/(2*a^3*x^2)
```

Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(-\frac{b^3\cosh(c+dx)}{a^3(a+bx)}+\frac{b^2\cosh(c+dx)}{a^3x}-\frac{b\cosh(c+dx)}{a^2x^2}+\frac{\cosh(c+dx)}{ax^3}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{b^2\cosh(c)\mathrm{Chi}(dx)}{a^3}-\frac{b^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{b^2\sinh(c)\mathrm{Shi}(dx)}{a^3}-\frac{b^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{bd\sinh(c)\mathrm{Chi}(dx)}{a^2}-\frac{bd\cosh(c)\mathrm{Shi}(dx)}{a^2}+\frac{b\cosh(c+dx)}{a^2x}+\frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a}+\frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a}-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}$$

input
```
Int[Cosh[c + d*x]/(x^3*(a + b*x)),x]
```

output
```
-1/2*Cosh[c + d*x]/(a*x^2) + (b*Cosh[c + d*x])/(a^2*x) + (b^2*Cosh[c]*Cosh
Integral[d*x])/a^3 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) - (b^2*Cosh[c -
 (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^3 - (b*d*CoshIntegral[d*x]*Sinh[c
])/a^2 - (d*Sinh[c + d*x])/(2*a*x) - (b*d*Cosh[c]*SinhIntegral[d*x])/a^2 +
 (b^2*Sinh[c]*SinhIntegral[d*x])/a^3 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*
a) - (b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.48

method	result
risch	$\frac{d\,\mathrm{e}^{-dx-c}}{4ax}+\frac{\mathrm{e}^{-dx-c}b}{2a^2x}-\frac{\mathrm{e}^{-dx-c}}{4a\,x^2}-\frac{d^2\mathrm{e}^{-c}\,\mathrm{expIntegral}_1(dx)}{4a}-\frac{d\,\mathrm{e}^{-c}\,\mathrm{expIntegral}_1(dx)b}{2a^2}-\frac{\mathrm{e}^{-c}\,\mathrm{expIntegral}_1(dx)b^2}{2a^3}+\frac{b^2\mathrm{e}^{\frac{ad}{}}}{}$

input
```
int(cosh(d*x+c)/x^3/(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*exp(-d*x-c)/a/x+1/2*exp(-d*x-c)/a^2/x*b-1/4*exp(-d*x-c)/a/x^2-1/4*d^
2/a*exp(-c)*Ei(1,d*x)-1/2*d/a^2*exp(-c)*Ei(1,d*x)*b-1/2/a^3*exp(-c)*Ei(1,d
*x)*b^2+1/2*b^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/2*b^2/a^3*e
xp(c)*Ei(1,-d*x)+1/2/a^3*b^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/
4/a/x^2*exp(d*x+c)-1/4*d/a/x*exp(d*x+c)-1/4*d^2/a*exp(c)*Ei(1,-d*x)+1/2*b/
a^2/x*exp(d*x+c)+1/2*d*b/a^2*exp(c)*Ei(1,-d*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 275, normalized size of antiderivative = 1.45

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = -\frac{2a^2dx\sinh(dx+c) - 2(2abx - a^2)\cosh(dx+c) - ((a^2d^2 - 2abd + 2b^2)x^2\mathrm{Ei}(dx) + (a^2d^2 + 2abd +$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(2*a^2*d*x*sinh(d*x + c) - 2*(2*a*b*x - a^2)*cosh(d*x + c) - ((a^2*d^
2 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) + (a^2*d^2 + 2*a*b*d + 2*b^2)*x^2*Ei(-d*x
))*cosh(c) + 2*(b^2*x^2*Ei((b*d*x + a*d)/b) + b^2*x^2*Ei(-(b*d*x + a*d)/b)
)*cosh(-(b*c - a*d)/b) - ((a^2*d^2 - 2*a*b*d + 2*b^2)*x^2*Ei(d*x) - (a^2*d
^2 + 2*a*b*d + 2*b^2)*x^2*Ei(-d*x))*sinh(c) - 2*(b^2*x^2*Ei((b*d*x + a*d)/
b) - b^2*x^2*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*x^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$$

input
```
integrate(cosh(d*x+c)/x**3/(b*x+a),x)
```

output
```
Integral(cosh(c + d*x)/(x**3*(a + b*x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.27

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$$
$$= \frac{1}{4}d\left(\frac{de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)}{a}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}-\mathrm{Ei}(dx)\,e^{c}\right)b}{a^2}+\frac{2\,b^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e}{}\right.}{a^3d}\right.$$
$$-\frac{1}{2}\left(\frac{2\,b^2\log\left(bx+a\right)}{a^3}-\frac{2\,b^2\log\left(x\right)}{a^3}-\frac{2\,bx-a}{a^2x^2}\right)\cosh\left(dx+c\right)$$

input

```
integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="maxima")
```

output

```
1/4*d*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))/a + 2*(Ei(-d*x)*e
^(-c) - Ei(d*x)*e^c)*b/a^2 + 2*b^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x
+ a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/(a^3*d) +
 4*b^2*cosh(d*x + c)*log(b*x + a)/(a^3*d) - 4*b^2*cosh(d*x + c)*log(x)/(a^
3*d) + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/(a^3*d)) - 1/2*(2*b^2*log(b*x
 + a)/a^3 - 2*b^2*log(x)/a^3 - (2*b*x - a)/(a^2*x^2))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.31

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx$$
$$= \frac{a^2d^2x^2\mathrm{Ei}(-dx)\,e^{(-c)}+a^2d^2x^2\mathrm{Ei}(dx)\,e^{c}+2\,abdx^2\mathrm{Ei}(-dx)\,e^{(-c)}-2\,abdx^2\mathrm{Ei}(dx)\,e^{c}+2\,b^2x^2\mathrm{Ei}(-dx)\,e^{(-c}}{}$$

input

```
integrate(cosh(d*x+c)/x^3/(b*x+a),x, algorithm="giac")
```

output
```
1/4*(a^2*d^2*x^2*Ei(-d*x)*e^(-c) + a^2*d^2*x^2*Ei(d*x)*e^c + 2*a*b*d*x^2*E
i(-d*x)*e^(-c) - 2*a*b*d*x^2*Ei(d*x)*e^c + 2*b^2*x^2*Ei(-d*x)*e^(-c) - 2*b
^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^2*x^2*Ei(d*x)*e^c - 2*b^2*x
^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*d*x*e^(d*x + c) + a^2*d*x*e^(
-d*x - c) + 2*a*b*x*e^(d*x + c) + 2*a*b*x*e^(-d*x - c) - a^2*e^(d*x + c) -
 a^2*e^(-d*x - c))/(a^3*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\,(a+b\,x)}\,dx$$

input
```
int(cosh(c + d*x)/(x^3*(a + b*x)),x)
```

output
```
int(cosh(c + d*x)/(x^3*(a + b*x)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)}\,dx = \int \frac{\cosh(dx+c)}{b\,x^4+a\,x^3}dx$$

input
```
int(cosh(d*x+c)/x^3/(b*x+a),x)
```

output
```
int(cosh(c + d*x)/(a*x**3 + b*x**4),x)
```

3.26 $\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 231

$$\begin{aligned}\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} \, dx &= \frac{2a \cosh(c+dx)}{b^3 d^2} - \frac{2x \cosh(c+dx)}{b^2 d^2} \\ &\quad - \frac{a^4 \cosh(c+dx)}{b^5 (a+bx)} - \frac{4a^3 \cosh\left(c - \frac{ad}{b}\right) \text{Chi}\left(\frac{ad}{b} + dx\right)}{b^5} \\ &\quad + \frac{a^4 d \text{Chi}\left(\frac{ad}{b} + dx\right) \sinh\left(c - \frac{ad}{b}\right)}{b^6} + \frac{2 \sinh(c+dx)}{b^2 d^3} \\ &\quad + \frac{3a^2 \sinh(c+dx)}{b^4 d} - \frac{2ax \sinh(c+dx)}{b^3 d} \\ &\quad + \frac{x^2 \sinh(c+dx)}{b^2 d} + \frac{a^4 d \cosh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^6} \\ &\quad - \frac{4a^3 \sinh\left(c - \frac{ad}{b}\right) \text{Shi}\left(\frac{ad}{b} + dx\right)}{b^5}\end{aligned}$$

output

```
2*a*cosh(d*x+c)/b^3/d^2-2*x*cosh(d*x+c)/b^2/d^2-a^4*cosh(d*x+c)/b^5/(b*x+a
)-4*a^3*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^5-a^4*d*Chi(a*d/b+d*x)*sinh(-c+a*d
/b)/b^6+2*sinh(d*x+c)/b^2/d^3+3*a^2*sinh(d*x+c)/b^4/d-2*a*x*sinh(d*x+c)/b^
3/d+x^2*sinh(d*x+c)/b^2/d+a^4*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^6+4*a^3*si
nh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.75

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} \, dx$$
$$= \frac{-\frac{b(-2a^2b^2+a^4d^2+2b^4x^2)\cosh(c+dx)}{d^2(a+bx)} + a^3\operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(-4b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)+\frac{b^2(3a^2d^2-2abd^2}{}}{b^6}$$

input
```
Integrate[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
(-((b*(-2*a^2*b^2 + a^4*d^2 + 2*b^4*x^2)*Cosh[c + d*x])/(d^2*(a + b*x))) +
 a^3*CoshIntegral[d*(a/b + x)]*(-4*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d
)/b]) + (b^2*(3*a^2*d^2 - 2*a*b*d^2*x + b^2*(2 + d^2*x^2))*Sinh[c + d*x])/
d^3 + a^3*(a*d*Cosh[c - (a*d)/b] - 4*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(
a/b + x)])/b^6
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2} \, dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(\frac{a^4\cosh(c+dx)}{b^4(a+bx)^2} - \frac{4a^3\cosh(c+dx)}{b^4(a+bx)} + \frac{3a^2\cosh(c+dx)}{b^4} - \frac{2ax\cosh(c+dx)}{b^3} + \frac{x^2\cosh(c+dx)}{b^2}\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{a^4 d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^6}+\frac{a^4 d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^6}-\frac{a^4\cosh(c+dx)}{b^5(a+bx)}-\frac{4a^3\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{4a^3\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{3a^2\sinh(c+dx)}{b^4d}+\frac{2a\cosh(c+dx)}{b^3d^2}-\frac{2ax\sinh(c+dx)}{b^3d}+\frac{2\sinh(c+dx)}{b^2d^3}-\frac{2x\cosh(c+dx)}{b^2d^2}+\frac{x^2\sinh(c+dx)}{b^2d}$$

input
```
Int[(x^4*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
(2*a*Cosh[c + d*x])/(b^3*d^2) - (2*x*Cosh[c + d*x])/(b^2*d^2) - (a^4*Cosh[
c + d*x])/(b^5*(a + b*x)) - (4*a^3*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b
+ d*x])/b^5 + (a^4*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^6 +
(2*Sinh[c + d*x])/(b^2*d^3) + (3*a^2*Sinh[c + d*x])/(b^4*d) - (2*a*x*Sinh[
c + d*x])/(b^3*d) + (x^2*Sinh[c + d*x])/(b^2*d) + (a^4*d*Cosh[c - (a*d)/b]
*SinhIntegral[(a*d)/b + d*x])/b^6 - (4*a^3*Sinh[c - (a*d)/b]*SinhIntegral[
(a*d)/b + d*x])/b^5
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. $2(236) = 472$.

Time = 0.68 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.90

method	result
risch	$-\frac{2\,\mathrm{e}^{-dx-c} b^5 x+2\,\mathrm{e}^{-dx-c} a\, b^4+3\,\mathrm{e}^{-dx-c} a^3 b^2 d^2-2\,\mathrm{e}^{-dx-c} a^2 b^3 d-\mathrm{e}^{\frac{ad-cb}{b}}\,\mathrm{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right) a^4 b\, d^4 x-4\,\mathrm{e}^{\frac{ad-cb}{b}}\,\mathrm{expInte}}{}$

input
```
int(x^4*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^3*(2*exp(-d*x-c)*b^5*x+2*exp(-d*x-c)*a*b^4+3*exp(-d*x-c)*a^3*b^2*d^
2-2*exp(-d*x-c)*a^2*b^3*d-exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*b*d
^4*x-4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b^2*d^3*x-exp(d*x+c)*b
^5*d^2*x^3+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^5*d^4+exp(d*x+c)*a
^4*b*d^3+2*exp(d*x+c)*b^5*d*x^2-3*exp(d*x+c)*a^3*b^2*d^2-2*exp(d*x+c)*a^2*
b^3*d-exp(-d*x-c)*a*b^4*d^2*x^2-2*exp(d*x+c)*b^5*x-2*exp(d*x+c)*a*b^4+exp(
-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4*b*d^4*x-4*exp(-(a*d-b*c)/b)*Ei(
1,-d*x-c-(a*d-b*c)/b)*a^3*b^2*d^3*x+exp(-d*x-c)*b^5*d^2*x^3-exp((a*d-b*c)/
b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^5*d^4+exp(-d*x-c)*a^4*b*d^3+2*exp(-d*x-c)*b^5
*d*x^2+exp(d*x+c)*a*b^4*d^2*x^2-4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/
b)*a^4*b*d^3-exp(d*x+c)*a^2*b^3*d^2*x-4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b
*c)/b)*a^4*b*d^3+exp(-d*x-c)*a^2*b^3*d^2*x)/b^6/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.61

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{2\left(a^4bd^3+2\,b^5dx^2-2\,a^2b^3d\right)\cosh\left(dx+c\right)-\left(\left(a^5d^4-4\,a^4bd^3+\left(a^4bd^4-4\,a^3b^2d^3\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^5\right.\right.}{}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^4*b*d^3 + 2*b^5*d*x^2 - 2*a^2*b^3*d)*cosh(d*x + c) - ((a^5*d^4
- 4*a^4*b*d^3 + (a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) - (a^5*
d^4 + 4*a^4*b*d^3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*c
osh(-(b*c - a*d)/b) - 2*(b^5*d^2*x^3 - a*b^4*d^2*x^2 + 3*a^3*b^2*d^2 + 2*a
*b^4 + (a^2*b^3*d^2 + 2*b^5)*x)*sinh(d*x + c) + ((a^5*d^4 - 4*a^4*b*d^3 +
(a^4*b*d^4 - 4*a^3*b^2*d^3)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^4 + 4*a^4*b*d^
3 + (a^4*b*d^4 + 4*a^3*b^2*d^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)
/b))/(b^7*d^3*x + a*b^6*d^3)
```

Sympy [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^4 \cosh{(c+dx)}}{(a+bx)^2}\,dx$$

input
```
integrate(x**4*cosh(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x**4*cosh(c + d*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 406, normalized size of antiderivative = 1.76

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{1}{6}\left(3\,a^4\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^6} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^6}\right) + \frac{12\,a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+}{b}\right.}{b}\right.}{b^4d}\right.$$
$$-\frac{1}{3}\left(\frac{3\,a^4}{b^6x+ab^5} + \frac{12\,a^3\log{(bx+a)}}{b^5} - \frac{b^2x^3-3\,abx^2+9\,a^2x}{b^4}\right)\cosh{(dx+c)}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/6*(3*a^4*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^6 - e^(c - a
*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^6) + 12*a^3*(e^(-c + a*d/b)*exp_
integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a
)*d/b)/b)/(b^4*d) - 9*a^2*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x
 - c)/d^2)/b^4 + 3*a*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2
*x^2 + 2*d*x + 2)*e^(-d*x - c)/d^3)/b^3 - ((d^3*x^3*e^c - 3*d^2*x^2*e^c +
6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x
 - c)/d^4)/b^2 + 24*a^3*cosh(d*x + c)*log(b*x + a)/(b^5*d))*d - 1/3*(3*a^4
/(b^6*x + a*b^5) + 12*a^3*log(b*x + a)/b^5 - (b^2*x^3 - 3*a*b*x^2 + 9*a^2*
x)/b^4)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2979 vs. $2(236) = 472$.

Time = 0.17 (sec) , antiderivative size = 2979, normalized size of antiderivative = 12.90

$$\int \frac{x^4 \cosh(c + dx)}{(a + bx)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*((b*x + a)*a^4*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^4*Ei(((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^4
*b*c*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
*e^((b*c - a*d)/b) + a^5*d^5*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^4*(b*c/(b*x + a) - a*
d/(b*x + a) + d)*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a^4*b*c*d^4*Ei(-((b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^5*d^5*Ei(-(
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a
*d)/b) - 4*(b*x + a)*a^3*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*
x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/
b) + 4*a^3*b^2*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b
*c + a*d)/b)*e^((b*c - a*d)/b) - 4*a^4*b*d^4*Ei(((b*x + a)*(b*c/(b*x + a)
- a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 4*(b*x + a)*a^3*b
*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 4*a^3*b^2*c*d^3*Ei(
-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c -
 a*d)/b) - 4*a^4*b*d^4*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
- b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^4*b*d^4*e^((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d)/b) - a^4*b*d^4*e^(-(b*x + a)*(b*c/(b*x + a) - a*d...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{(a+b\,x)^2}\,dx$$

input
```
int((x^4*cosh(c + d*x))/(a + b*x)^2,x)
```

output
```
int((x^4*cosh(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx)^2}\,dx = \text{too large to display}$$

input
```
int(x^4*cosh(d*x+c)/(b*x+a)^2,x)
```

output

```
(e**(2*c + 2*d*x)*a**4*d**4*x - e**(2*c + 2*d*x)*a**4*d**3 - e**(2*c + 2*d
*x)*a**3*b*d**4*x**2 - e**(2*c + 2*d*x)*a**3*b*d**2 + e**(2*c + 2*d*x)*a**
2*b**2*d**4*x**3 - 2*e**(2*c + 2*d*x)*a**2*b**2*d**3*x**2 + e**(2*c + 2*d*
x)*a**2*b**2*d**2*x - 2*e**(2*c + 2*d*x)*a**2*b**2*d + e**(2*c + 2*d*x)*a*
b**3*d**2*x**2 - 2*e**(2*c + 2*d*x)*a*b**3 - e**(2*c + 2*d*x)*b**4*d**2*x*
*3 + 2*e**(2*c + 2*d*x)*b**4*d*x**2 - 2*e**(2*c + 2*d*x)*b**4*x - e**(2*c
+ d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2
 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**8*d**7 - e**(2*c + d*x)*int((
e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2
 - 2*a*b**3*x - b**4*x**2),x)*a**7*b*d**7*x + 3*e**(2*c + d*x)*int((e**(d*
x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a
*b**3*x - b**4*x**2),x)*a**7*b*d**6 + 3*e**(2*c + d*x)*int((e**(d*x)*x)/(a
**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x
- b**4*x**2),x)*a**6*b**2*d**6*x + 5*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4
*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b
**4*x**2),x)*a**6*b**2*d**5 + 5*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2
 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x
**2),x)*a**5*b**3*d**5*x - 3*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 +
2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2
),x)*a**5*b**3*d**4 - 3*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*...
```

3.27 $\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 175

$$\begin{aligned}\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} \, dx = &-\frac{\cosh(c+dx)}{b^2d^2} + \frac{a^3\cosh(c+dx)}{b^4(a+bx)} \\ &+ \frac{3a^2 \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} \\ &- \frac{a^3 d \operatorname{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c-\frac{ad}{b}\right)}{b^5} - \frac{(2a-bx)\sinh(c+dx)}{b^3d} \\ &- \frac{a^3 d \cosh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} \\ &+ \frac{3a^2 \sinh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}\end{aligned}$$

output
```
-cosh(d*x+c)/b^2/d^2+a^3*cosh(d*x+c)/b^4/(b*x+a)+3*a^2*cosh(-c+a*d/b)*Chi(
a*d/b+d*x)/b^4+a^3*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^5-(-b*x+2*a)*sinh(d*x
+c)/b^3/d-a^3*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5-3*a^2*sinh(-c+a*d/b)*Shi
(a*d/b+d*x)/b^4
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 156, normalized size of antiderivative = 0.89

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$=\frac{a^2\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(3b\cosh\left(c-\frac{ad}{b}\right)-ad\sinh\left(c-\frac{ad}{b}\right)\right)+\frac{b\left(\left(-ab^2+a^3d^2-b^3x\right)\cosh(c+dx)+bd\left(-2a^2-abx+b^2x^2\right)\sinh(c}{d^2(a+bx)}}{b^5}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
(a^2*CoshIntegral[d*(a/b + x)]*(3*b*Cosh[c - (a*d)/b] - a*d*Sinh[c - (a*d)
/b]) + (b*((-(a*b^2) + a^3*d^2 - b^3*x)*Cosh[c + d*x] + b*d*(-2*a^2 - a*b*
x + b^2*x^2)*Sinh[c + d*x]))/(d^2*(a + b*x)) - a^2*(a*d*Cosh[c - (a*d)/b]
- 3*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^5
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.04, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(-\frac{a^3\cosh(c+dx)}{b^3(a+bx)^2}+\frac{3a^2\cosh(c+dx)}{b^3(a+bx)}-\frac{2a\cosh(c+dx)}{b^3}+\frac{x\cosh(c+dx)}{b^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{a^3 d \sinh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{a^3 d \cosh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{a^3\cosh(c+dx)}{b^4(a+bx)}+\frac{3a^2\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{3a^2\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{2a\sinh(c+dx)}{b^3 d}-\frac{\cosh(c+dx)}{b^2d^2}+\frac{x\sinh(c+dx)}{b^2 d}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
-(Cosh[c + d*x]/(b^2*d^2)) + (a^3*Cosh[c + d*x])/(b^4*(a + b*x)) + (3*a^2*
Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d*CoshIntegral[(
a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 - (2*a*Sinh[c + d*x])/(b^3*d) + (x*Si
nh[c + d*x])/(b^2*d) - (a^3*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x
])/b^5 + (3*a^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(178) = 356$.

Time = 0.59 (sec) , antiderivative size = 549, normalized size of antiderivative = 3.14

method	result
risch	$\frac{{\rm e}^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right) a^3 b\, d^3 x - {\rm e}^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right) a^3 b\, d^3 x + {\rm e}^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\right.}{}$

input
```
int(x^3*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2/d^2*(exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b*d^3*x-exp((a*d-b
*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b*d^3*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(
a*d-b*c)/b)*a^4*d^3-3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^2*d
^2*x-exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*d^3-3*exp((a*d-b*c)/b)*E
i(1,d*x+c+(a*d-b*c)/b)*a^2*b^2*d^2*x+exp(d*x+c)*b^4*d*x^2-3*exp(-(a*d-b*c)
/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b*d^2-exp(-d*x-c)*b^4*d*x^2-3*exp((a*d-b*
c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b*d^2+exp(d*x+c)*a^3*b*d^2-exp(d*x+c)*a*
b^3*d*x+exp(-d*x-c)*a^3*b*d^2+exp(-d*x-c)*a*b^3*d*x-2*exp(d*x+c)*a^2*b^2*d
-exp(d*x+c)*b^4*x+2*exp(-d*x-c)*a^2*b^2*d-exp(-d*x-c)*b^4*x-exp(d*x+c)*a*b
^3-exp(-d*x-c)*a*b^3)/b^5/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.90

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2} \, dx$$
$$= \frac{2 \left(a^3bd^2 - b^4x - ab^3\right) \cosh\left(dx+c\right) - \left(\left(a^4d^3 - 3\,a^3bd^2 + \left(a^3bd^3 - 3\,a^2b^2d^2\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) - \left(a^4d^3 + 3\,a^3b\right.\right.}{}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*(a^3*b*d^2 - b^4*x - a*b^3)*cosh(d*x + c) - ((a^4*d^3 - 3*a^3*b*d^2
 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^3 + 3*a^3*b
*d^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a
*d)/b) + 2*(b^4*d*x^2 - a*b^3*d*x - 2*a^2*b^2*d)*sinh(d*x + c) + ((a^4*d^3
 - 3*a^3*b*d^2 + (a^3*b*d^3 - 3*a^2*b^2*d^2)*x)*Ei((b*d*x + a*d)/b) + (a^4
*d^3 + 3*a^3*b*d^2 + (a^3*b*d^3 + 3*a^2*b^2*d^2)*x)*Ei(-(b*d*x + a*d)/b))*
sinh(-(b*c - a*d)/b))/(b^6*d^2*x + a*b^5*d^2)
```

Sympy [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3 \cosh{(c+dx)}}{(a+bx)^2}\,dx$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x**3*cosh(c + d*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.78

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx =$$

$$-\frac{1}{4}\left(2\,a^3\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^5}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^5}\right)+\frac{6\,a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx}{}\right.}{b}\right.}{b^3d}\right.$$

$$+\frac{1}{2}\left(\frac{2\,a^3}{b^5x+ab^4}+\frac{6\,a^2\log{(bx+a)}}{b^4}+\frac{bx^2-4\,ax}{b^3}\right)\cosh{(dx+c)}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*a^3*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^5 - e^(c -
a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^5) + 6*a^2*(e^(-c + a*d/b)*exp_
integral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a
)*d/b)/b)/(b^3*d) - 4*a*((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x -
 c)/d^2)/b^3 + ((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 +
 2*d*x + 2)*e^(-d*x - c)/d^3)/b^2 + 12*a^2*cosh(d*x + c)*log(b*x + a)/(b^4
*d))*d + 1/2*(2*a^3/(b^5*x + a*b^4) + 6*a^2*log(b*x + a)/b^4 + (b*x^2 - 4*
a*x)/b^3)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1991 vs. $2(176) = 352$.

Time = 0.16 (sec) , antiderivative size = 1991, normalized size of antiderivative = 11.38

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((b*x + a)*a^3*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x + a)*
(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^
3*b*c*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b
)*e^((b*c - a*d)/b) + a^4*d^4*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^3*(b*c/(b*x + a) - a
*d/(b*x + a) + d)*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
 b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a^3*b*c*d^3*Ei(-((b*x + a)*(b*c/(b*x +
 a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^4*d^4*Ei(-
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c -
a*d)/b) - 3*(b*x + a)*a^2*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b
*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)
/b) + 3*a^2*b^2*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b*c + a*d)/b)*e^((b*c - a*d)/b) - 3*a^3*b*d^3*Ei(((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 3*(b*x + a)*a^2*
b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) -
a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 3*a^2*b^2*c*d^2*Ei
(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c
- a*d)/b) - 3*a^3*b*d^3*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)
 - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^3*b*d^3*e^((b*x + a)*(b*c/(b*x + a
) - a*d/(b*x + a) + d)/b) - a^3*b*d^3*e^(-(b*x + a)*(b*c/(b*x + a) - a*...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{\left(a+b\,x\right)^2}\,dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x)^2,x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^2}\,dx = \text{too large to display}$$

input
```
int(x^3*cosh(d*x+c)/(b*x+a)^2,x)
```

output
```
( - e**(2*c + 2*d*x)*a**3*d**3*x + e**(2*c + 2*d*x)*a**3*d**2 + e**(2*c +
2*d*x)*a**2*b*d**3*x**2 - e**(2*c + 2*d*x)*a**2*b*d**2*x + 2*e**(2*c + 2*d
*x)*a**2*b*d + e**(2*c + 2*d*x)*a*b**2*d*x + e**(2*c + 2*d*x)*a*b**2 - e**
(2*c + 2*d*x)*b**3*d*x**2 + e**(2*c + 2*d*x)*b**3*x + e**(2*c + d*x)*int((
e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2
 - 2*a*b**3*x - b**4*x**2),x)*a**7*d**6 + e**(2*c + d*x)*int((e**(d*x)*x)/
(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*
x - b**4*x**2),x)*a**6*b*d**6*x - 2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*
d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b*
*4*x**2),x)*a**6*b*d**5 - 2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2
*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2)
,x)*a**5*b**2*d**5*x - 4*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a*
*3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)
*a**5*b**2*d**4 - 4*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*
d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**4
*b**3*d**4*x + 2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**
2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**4*b*
*3*d**3 + 2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x +
 a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**3*b**4*d*
*3*x + 3*e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x +...
```

3.28 $\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 147

$$\begin{aligned}\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} \, dx = &-\frac{a^2 \cosh(c+dx)}{b^3(a+bx)} - \frac{2a \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} \\ &+ \frac{a^2 d \operatorname{Chi}\left(\frac{ad}{b}+dx\right) \sinh\left(c-\frac{ad}{b}\right)}{b^4} + \frac{\sinh(c+dx)}{b^2 d} \\ &+ \frac{a^2 d \cosh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} \\ &- \frac{2a \sinh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^3}\end{aligned}$$

output

```
-a^2*cosh(d*x+c)/b^3/(b*x+a)-2*a*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^3-a^2*d*C
hi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4+sinh(d*x+c)/b^2/d+a^2*d*cosh(-c+a*d/b)*Sh
i(a*d/b+d*x)/b^4+2*a*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.78

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{a\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(-2b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)+b\left(-\frac{a^2\cosh(c+dx)}{a+bx}+\frac{b\sinh(c+dx)}{d}\right)+a\left(ad\cosh\left(c\right.\right.}{b^4}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
(a*CoshIntegral[d*(a/b + x)]*(-2*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/
b]) + b*(-((a^2*Cosh[c + d*x])/(a + b*x)) + (b*Sinh[c + d*x])/d) + a*(a*d*
Cosh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/b^4
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{a^2\cosh(c+dx)}{b^2(a+bx)^2}-\frac{2a\cosh(c+dx)}{b^2(a+bx)}+\frac{\cosh(c+dx)}{b^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{a^2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{a^2d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{a^2\cosh(c+dx)}{b^3(a+bx)}-\frac{2a\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{2a\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\sinh(c+dx)}{b^2d}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
-((a^2*Cosh[c + d*x])/(b^3*(a + b*x))) - (2*a*Cosh[c - (a*d)/b]*CoshIntegr
al[(a*d)/b + d*x])/b^3 + (a^2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)
/b])/b^4 + Sinh[c + d*x]/(b^2*d) + (a^2*d*Cosh[c - (a*d)/b]*SinhIntegral[(
a*d)/b + d*x])/b^4 - (2*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b
^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 435 vs. $2(152) = 304$.

Time = 0.54 (sec) , antiderivative size = 436, normalized size of antiderivative = 2.97

method	result
risch	$-\frac{e^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right) a^2 b d^2 x - e^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right) a^2 b d^2 x + e^{-\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(-dx-c\right.}{}$

input
```
int(x^2*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b*d^2*x-exp((a*d-b*
c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b*d^2*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a
*d-b*c)/b)*a^3*d^2-2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b^2*d*x-
exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*d^2-2*exp((a*d-b*c)/b)*Ei(1,d
*x+c+(a*d-b*c)/b)*a*b^2*d*x-2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a
^2*b*d-2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b*d+exp(-d*x-c)*a^2*
b*d+exp(-d*x-c)*b^3*x+exp(d*x+c)*a^2*b*d-exp(d*x+c)*b^3*x+exp(-d*x-c)*a*b^
2-exp(d*x+c)*a*b^2)/b^4/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 274, normalized size of antiderivative = 1.86

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\,dx =$$
$$-\frac{2\,a^2bd\cosh\left(dx+c\right)-\left(\left(a^3d^2-2\,a^2bd+\left(a^2bd^2-2\,ab^2d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-\left(a^3d^2+2\,a^2bd+\left(a^2bd^2+2\,a\right.\right.\right.}{}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*b*d*cosh(d*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b
^2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2
*d)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(b^3*x + a*b^2)*sinh
(d*x + c) + ((a^3*d^2 - 2*a^2*b*d + (a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x +
 a*d)/b) + (a^3*d^2 + 2*a^2*b*d + (a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x +
a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*d*x + a*b^4*d)
```

Sympy [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^2 \cosh\left(c+dx\right)}{\left(a+bx\right)^2}\,dx$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x**2*cosh(c + d*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.61

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$=\frac{1}{2}\left(a^2\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^4}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^4}\right)+\frac{2\,a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{b^2d}\right.$$
$$\left.-\left(\frac{a^2}{b^4x+ab^3}-\frac{x}{b^2}+\frac{2\,a\log\left(bx+a\right)}{b^3}\right)\cosh\left(dx+c\right)\right.$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*(a^2*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^4 - e^(c - a*d
/b)*exp_integral_e(1, -(b*x + a)*d/b)/b^4) + 2*a*(e^(-c + a*d/b)*exp_integ
ral_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b
)/b)/(b^2*d) - ((d*x*e^c - e^c)*e^(d*x)/d^2 + (d*x + 1)*e^(-d*x - c)/d^2)/
b^2 + 4*a*cosh(d*x + c)*log(b*x + a)/(b^3*d))*d - (a^2/(b^4*x + a*b^3) - x
/b^2 + 2*a*log(b*x + a)/b^3)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1308 vs. $2(152) = 304$.

Time = 0.15 (sec) , antiderivative size = 1308, normalized size of antiderivative = 8.90

$$\int \frac{x^2\cosh(c+dx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*((b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(
b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a^2
*b*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)
*e^((b*c - a*d)/b) + a^3*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a^2*(b*c/(b*x + a) - a*
d/(b*x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a^2*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x +
a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^3*d^3*Ei(-(
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a
*d)/b) - 2*(b*x + a)*a*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x +
a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) +
 2*a*b^2*c*d*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d
)/b)*e^((b*c - a*d)/b) - 2*a^2*b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - 2*(b*x + a)*a*b*(b*c/(b*x
 + a) - a*d/(b*x + a) + d)*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + 2*a*b^2*c*d*Ei(-((b*x + a)*(b*c
/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - 2*a^2
*b*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*
e^(-(b*c - a*d)/b) - a^2*b*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
 + d)/b) - a^2*b*d^2*e^(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{(a+b\,x)^2}\,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x)^2,x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^2} \, dx = \text{too large to display}$$

input
```
int(x^2*cosh(d*x+c)/(b*x+a)^2,x)
```

output
```
(e**(2*c + 2*d*x)*a**2*d**2*x - e**(2*c + 2*d*x)*a**2*d - e**(2*c + 2*d*x)
*a*b - e**(2*c + 2*d*x)*b**2*x - e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**
2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*
x**2),x)*a**6*d**5 - e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b
*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**
5*b*d**5*x + e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x
+ a**2*b**2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**5*b*d**4
 + e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**
2*d**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**4*b**2*d**4*x + 3*
e**(2*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d
**2*x**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**4*b**2*d**3 + 3*e**(2
*c + d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x
**2 - a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**3*b**3*d**3*x - e**(2*c +
d*x)*int((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 -
 a**2*b**2 - 2*a*b**3*x - b**4*x**2),x)*a**3*b**3*d**2 - e**(2*c + d*x)*in
t((e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b
**2 - 2*a*b**3*x - b**4*x**2),x)*a**2*b**4*d**2*x - 2*e**(2*c + d*x)*int((
e**(d*x)*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2
 - 2*a*b**3*x - b**4*x**2),x)*a**2*b**4*d - 2*e**(2*c + d*x)*int((e**(d*x)
*x)/(a**4*d**2 + 2*a**3*b*d**2*x + a**2*b**2*d**2*x**2 - a**2*b**2 - 2*...
```

3.29 $\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 125

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{a\cosh(c+dx)}{b^2(a+bx)} + \frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{b^2} - \frac{ad\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} - \frac{ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} + \frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

output
```
a*cosh(d*x+c)/b^2/(b*x+a)+cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^2+a*d*Chi(a*d/b+
d*x)*sinh(-c+a*d/b)/b^3-a*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3-sinh(-c+a*d/
b)*Shi(a*d/b+d*x)/b^2
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.78

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{\frac{ab\cosh(c+dx)}{a+bx} + \text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\cosh\left(c-\frac{ad}{b}\right) - ad\sinh\left(c-\frac{ad}{b}\right)\right) + \left(-ad\cosh\left(c-\frac{ad}{b}\right) + b\sinh\left(c-\frac{ad}{b}\right.\right.}{b^3}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
((a*b*Cosh[c + d*x])/(a + b*x) + CoshIntegral[d*(a/b + x)]*(b*Cosh[c - (a*
d)/b] - a*d*Sinh[c - (a*d)/b]) + (-(a*d*Cosh[c - (a*d)/b]) + b*Sinh[c - (a
*d)/b])*SinhIntegral[d*(a/b + x)])/b^3
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{\cosh(c+dx)}{b(a+bx)}-\frac{a\cosh(c+dx)}{b(a+bx)^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{ad\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}-\frac{ad\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^2}+\frac{a\cosh(c+dx)}{b^2(a+bx)}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x)^2,x]
```

output
```
(a*Cosh[c + d*x])/(b^2*(a + b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/
b + d*x])/b^2 - (a*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 -
(a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (Sinh[c - (a*d)/
b]*SinhIntegral[(a*d)/b + d*x])/b^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(129) = 258$.

Time = 0.50 (sec) , antiderivative size = 294, normalized size of antiderivative = 2.35

method	result
risch	$\frac{d\,e^{-dx-c}a}{2b^2(dxb+ad)} - \frac{d\,e^{\frac{ad-cb}{b}}\,\text{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)a}{2b^3} - \frac{e^{\frac{ad-cb}{b}}\,\text{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)}{2b^2} + \frac{e^{-\frac{ad-cb}{b}}\,\text{expIntegral}_1\left(-c\right.}{}$

input
```
int(x*cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*d*exp(-d*x-c)/b^2/(b*d*x+a*d)*a-1/2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+
(a*d-b*c)/b)*a-1/2/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/2*(exp(-
(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b*d*x+exp(-(a*d-b*c)/b)*Ei(1,-d*x-
c-(a*d-b*c)/b)*a^2*d-exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*b^2*x-exp(
-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b+exp(d*x+c)*a*b)/b^3/(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.60

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx$$
$$= \frac{2\,ab\cosh\left(dx+c\right) - \left(\left(a^2d - ab + (abd - b^2)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right) - \left(a^2d + ab + (abd + b^2)x\right)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh}{2\left(b^4x\right.}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*cosh(d*x + c) - ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*
d)/b) - (a^2*d + a*b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c -
 a*d)/b) + ((a^2*d - a*b + (a*b*d - b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d +
 a*b + (a*b*d + b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^4*x
 + a*b^3)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{x\cosh{(c+dx)}}{(a+bx)^2}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)**2,x)
```

output
```
Integral(x*cosh(c + d*x)/(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.42

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{1}{2}\left(a\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b^3}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b^3}\right)+\frac{\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}+\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}}{bd}+\right. \\ +\left(\frac{a}{b^3x+ab^2}+\frac{\log{(bx+a)}}{b^2}\right)\cosh{(dx+c)}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*(a*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b^3 - e^(c - a*d/
b)*exp_integral_e(1, -(b*x + a)*d/b)/b^3) + (e^(-c + a*d/b)*exp_integral_e
(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)/b)/
(b*d) + 2*cosh(d*x + c)*log(b*x + a)/(b^2*d))*d + (a/(b^3*x + a*b^2) + log
(b*x + a)/b^2)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 994 vs. $2(129) = 258$.

Time = 0.14 (sec) , antiderivative size = 994, normalized size of antiderivative = 7.95

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*
c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^
((b*c - a*d)/b) + a^2*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*
x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*e^(-(b*c - a*d)/b) + a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^2*d^3*Ei(-((b*x +
a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b)
- (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) + b^2*c*d*Ei
(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c -
a*d)/b) - a*b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
+ a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) +
 d)*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e
^(-(b*c - a*d)/b) + b^2*c*d*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a)
+ d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*Ei(-((b*x + a)*(b*c/(b*x
 + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a*b*d^2*e^
((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b) - a*b*d^2*e^(-(b*x + a)*
(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b/(((b*x + a)*b^4*(b*c/(b*x + a...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx=\int \frac{x\cosh(c+d\,x)}{\left(a+b\,x\right)^2}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x)^2,x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx)^2}\,dx=\frac{e^{2c}\left(\int \frac{e^{dx}x}{b^2x^2+2abx+a^2}dx\right)+\int \frac{x}{e^{dx}a^2+2e^{dx}abx+e^{dx}b^2x^2}dx}{2e^c}$$

input
```
int(x*cosh(d*x+c)/(b*x+a)^2,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x)/(a**2 + 2*a*b*x + b**2*x**2),x) + int(x/(e**(d*
x)*a**2 + 2*e**(d*x)*a*b*x + e**(d*x)*b**2*x**2),x))/(2*e**c)
```

3.30 $\int \frac{\cosh(c+dx)}{(a+bx)^2} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 71

$$\int \frac{\cosh(c+dx)}{(a+bx)^2} \, dx = -\frac{\cosh(c+dx)}{b(a+bx)} + \frac{d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^2} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^2}$$

output
```
-cosh(d*x+c)/b/(b*x+a)-d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^2+d*cosh(-c+a*d/b
)*Shi(a*d/b+d*x)/b^2
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(c+dx)}{(a+bx)^2} \, dx = \frac{-\frac{b\cosh(c+dx)}{a+bx} + d\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\sinh\left(c-\frac{ad}{b}\right) + d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{b^2}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x)^2,x]
```

output
```
(-((b*Cosh[c + d*x])/(a + b*x)) + d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*
d)/b] + d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/b^2
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.786$, Rules used = {3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{(a+bx)^2}dx$$

$$\Big\downarrow \text{3778}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}+\frac{id\int -\frac{i\sinh(c+dx)}{a+bx}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{d\int \frac{\sinh(c+dx)}{a+bx}dx}{b}-\frac{\cosh(c+dx)}{b(a+bx)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}+\frac{d\int -\frac{i\sin(ic+idx)}{a+bx}dx}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}-\frac{id\int \frac{\sin(ic+idx)}{a+bx}dx}{b}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}-\frac{id\left(i\sinh\left(c-\frac{ad}{b}\right)\int\frac{\cosh\left(xd+\frac{ad}{b}\right)}{a+bx}dx+\cosh\left(c-\frac{ad}{b}\right)\int\frac{i\sinh\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}-\frac{id\left(i\sinh\left(c-\frac{ad}{b}\right)\int\frac{\cosh\left(xd+\frac{ad}{b}\right)}{a+bx}dx+i\cosh\left(c-\frac{ad}{b}\right)\int\frac{\sinh\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}-\frac{id\left(i\sinh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx+i\cosh\left(c-\frac{ad}{b}\right)\int-\frac{i\sin\left(ixd+\frac{iad}{b}\right)}{a+bx}dx\right)}{b}$$

$$\Big\downarrow \text{26}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}-\frac{id\left(i\sinh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx+\cosh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}\right)}{a+bx}dx\right)}{b}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}-\frac{id\left(i\sinh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx+\frac{i\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\cosh(c+dx)}{b(a+bx)}-\frac{id\left(\frac{i\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b}+\frac{i\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}\right)}{b}$$

input
```
Int[Cosh[c + d*x]/(a + b*x)^2,x]
```

output
```
-(Cosh[c + d*x]/(b*(a + b*x))) - (I*d*((I*CoshIntegral[(a*d)/b + d*x]*Sinh
[c - (a*d)/b])/b + (I*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.86

method	result	siz
risch	$-\frac{d\,e^{-dx-c}}{2b(dxb+ad)}+\frac{d\,e^{\frac{ad-cb}{b}}\operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)}{2b^2}-\frac{d\,e^{dx+c}}{2b^2\left(\frac{da}{b}+dx\right)}-\frac{d\,e^{-\frac{ad-cb}{b}}\operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right)}{2b^2}$	132

input
```
int(cosh(d*x+c)/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*d*exp(-d*x-c)/b/(b*d*x+a*d)+1/2*d/b^2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*
d-b*c)/b)-1/2*d/b^2*exp(d*x+c)/(d/b*a+d*x)-1/2*d/b^2*exp(-(a*d-b*c)/b)*Ei(
1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(74) = 148$.

Time = 0.07 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.10

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = -\frac{2\,b\cosh\left(dx+c\right)-\left((bdx+ad)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)-(bdx+ad)\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)\right)\cosh\left(-\frac{bc-ad}{b}\right)+\left((bdx+ad)\mathrm{Ei}\left(\right.\right.}{2\left(b^3x+ab^2\right)}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*b*cosh(d*x + c) - ((b*d*x + a*d)*Ei((b*d*x + a*d)/b) - (b*d*x + a*
d)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b*d*x + a*d)*Ei((b*d*x +
 a*d)/b) + (b*d*x + a*d)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^3*
x + a*b^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.14

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b}-\frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{2\,b}-\frac{\cosh\left(dx+c\right)}{(bx+a)b}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/b - e^(c - a*d/b)*e
xp_integral_e(1, -(b*x + a)*d/b)/b)/b - cosh(d*x + c)/((b*x + a)*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. 2(74) = 148.

Time = 0.13 (sec) , antiderivative size = 615, normalized size of antiderivative = 8.66

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx$$

$$= \frac{\left((bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\mathrm{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(\frac{bc-ad}{b}\right)}-bcd^2\mathrm{Ei}\left(\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+\cdots}{b}\right.\right.}{2\left((bx+a)b^4\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right.\right.\cdots}$$

$$-\frac{\left((bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)d^2\mathrm{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}+d\right)-bc+ad}{b}\right)e^{\left(-\frac{bc-ad}{b}\right)}-bcd^2\mathrm{Ei}\left(-\frac{(bx+a)\left(\frac{bc}{bx+a}-\frac{ad}{bx+a}\cdots\right.}{b}\right.\right.}{2\left((bx+a)b^4\left(\frac{bc}{bx+a}-\cdots\right.\right.}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b*c/
(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - b*c*d^2
*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c
 - a*d)/b) + a*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c
 + a*d)/b)*e^((b*c - a*d)/b) - b*d^2*e^((b*x + a)*(b*c/(b*x + a) - a*d/(b*
x + a) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x + a) - a*d/(b*x + a) + d) -
b^5*c + a*b^4*d)*d) - 1/2*((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d
^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-
(b*c - a*d)/b) - b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + a*d^3*Ei(-((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) + b*d^2*e^(-(b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b))*b^2/(((b*x + a)*b^4*(b*c/(b*x
 + a) - a*d/(b*x + a) + d) - b^5*c + a*b^4*d)*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \int \frac{\cosh(c+d\,x)}{{(a+b\,x)}^2}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x)^2,x)
```

output
```
int(cosh(c + d*x)/(a + b*x)^2, x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{(a+bx)^2}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}}{b^2x^2+2abx+a^2}dx\right)+\int \frac{1}{e^{dx}a^2+2e^{dx}abx+e^{dx}b^2x^2}dx}{2e^c}$$

input
```
int(cosh(d*x+c)/(b*x+a)^2,x)
```

output
```
(e**(2*c)*int(e**(d*x)/(a**2 + 2*a*b*x + b**2*x**2),x) + int(1/(e**(d*x)*a
**2 + 2*e**(d*x)*a*b*x + e**(d*x)*b**2*x**2),x))/(2*e**c)
```

3.31 $\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 150

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx = \frac{\cosh(c+dx)}{a(a+bx)} + \frac{\cosh(c)\mathrm{Chi}(dx)}{a^2} - \frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{a^2} - \frac{d\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{ab} + \frac{\sinh(c)\mathrm{Shi}(dx)}{a^2} - \frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{ab} - \frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{a^2}$$

output
```
cosh(d*x+c)/a/(b*x+a)+cosh(c)*Chi(d*x)/a^2-cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a
^2+d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/a/b+sinh(c)*Shi(d*x)/a^2-d*cosh(-c+a*d/
b)*Shi(a*d/b+d*x)/a/b+sinh(-c+a*d/b)*Shi(a*d/b+d*x)/a^2
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.93

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2} dx = \frac{\frac{a\cosh(c)\cosh(dx)}{a+bx} + \cosh(c)\mathrm{Chi}(dx) - \frac{\mathrm{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left(b\cosh\left(c-\frac{ad}{b}\right)+ad\sinh\left(c-\frac{ad}{b}\right)\right)}{b} + \frac{a\sinh(c)\sinh(dx)}{a+bx} + \sinh(c)\mathrm{Shi}(d \ldots}{a^2}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

output
```
((a*Cosh[c]*Cosh[d*x])/(a + b*x) + Cosh[c]*CoshIntegral[d*x] - (CoshIntegr
al[d*(a/b + x)]*(b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b]))/b + (a*Sinh
[c]*Sinh[d*x])/(a + b*x) + Sinh[c]*SinhIntegral[d*x] - (a*d*Cosh[c - (a*d)
/b]*SinhIntegral[d*(a/b + x)])/b - Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b +
 x)])/a^2
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2} \, dx$$

$$\Big\downarrow \ 7293$$

$$\int \left(-\frac{b\cosh(c+dx)}{a^2(a+bx)}+\frac{\cosh(c+dx)}{a^2x}-\frac{b\cosh(c+dx)}{a(a+bx)^2}\right)dx$$

$$\Big\downarrow \ 2009$$

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{\cosh(c)\text{Chi}(dx)}{a^2}+\frac{\sinh(c)\text{Shi}(dx)}{a^2}-\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{ab}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{ab}+\frac{\cosh(c+dx)}{a(a+bx)}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x)^2),x]
```

output
```
Cosh[c + d*x]/(a*(a + b*x)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[c -
(a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^2 - (d*CoshIntegral[(a*d)/b + d*x]
*Sinh[c - (a*d)/b])/(a*b) + (Sinh[c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c -
(a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(a*b) - (Sinh[c - (a*d)/b]*SinhInteg
ral[(a*d)/b + d*x])/a^2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.69

method	result
risch	$\frac{e^{-dx-c}d}{2a(b(dx+c)+ad-cb)} - \frac{e^{-c}\operatorname{expIntegral}_1(dx)}{2a^2} - \frac{e^{\frac{ad-cb}{b}}\operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)d}{2ab} + \frac{e^{\frac{ad-cb}{b}}\operatorname{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)}{2a^2}$

input
```
int(cosh(d*x+c)/x/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(-d*x-c)*d/a/(b*(d*x+c)+a*d-c*b)-1/2/a^2*exp(-c)*Ei(1,d*x)-1/2/a/b*
exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1/2/a^2*exp((a*d-b*c)/b)*Ei(1,d
*x+c+(a*d-b*c)/b)+1/2*a*d/b*exp(d*x+c)/(d/b*a+d*x)+1/2*a*d/b*exp(-(a*d-b*c
)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/2/a^2*exp(-(a*d-
b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.80

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx$$
$$= \frac{2\,ab\cosh\left(dx+c\right)+\left(\left(b^2x+ab\right)\mathrm{Ei}(dx)+\left(b^2x+ab\right)\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(\left(a^2d+ab+\left(abd+b^2\right)x\right)\mathrm{Ei}\left(\frac{bdx}{\cdots}\right.\right.}{\cdots}$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(2*a*b*cosh(d*x + c) + ((b^2*x + a*b)*Ei(d*x) + (b^2*x + a*b)*Ei(-d*x)
)*cosh(c) - ((a^2*d + a*b + (a*b*d + b^2)*x)*Ei((b*d*x + a*d)/b) - (a^2*d
- a*b + (a*b*d - b^2)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + ((b^
2*x + a*b)*Ei(d*x) - (b^2*x + a*b)*Ei(-d*x))*sinh(c) + ((a^2*d + a*b + (a*
b*d + b^2)*x)*Ei((b*d*x + a*d)/b) + (a^2*d - a*b + (a*b*d - b^2)*x)*Ei(-(b
*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^2*b^2*x + a^3*b)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\cosh\left(c+dx\right)}{x\left(a+bx\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)**2,x)
```

output
```
Integral(cosh(c + d*x)/(x*(a + b*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.51

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = -\frac{1}{2}d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{ab} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{ab} - \frac{b\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_1\left(\frac{(bx+a)d}{b}\right)}{b} + \frac{e^{\left(c-\frac{ad}{b}\right)}E_1\left(-\frac{(bx+a)d}{b}\right)}{b}\right)}{a^2d} - \right. + \left(\frac{1}{abx+a^2} - \frac{\log(bx+a)}{a^2} + \frac{\log(x)}{a^2}\right)\cosh(dx+c)$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="maxima")
```

output
```
-1/2*d*(e^(-c + a*d/b)*exp_integral_e(1, (b*x + a)*d/b)/(a*b) - e^(c - a*d
/b)*exp_integral_e(1, -(b*x + a)*d/b)/(a*b) - b*(e^(-c + a*d/b)*exp_integr
al_e(1, (b*x + a)*d/b)/b + e^(c - a*d/b)*exp_integral_e(1, -(b*x + a)*d/b)
/b)/(a^2*d) - 2*cosh(d*x + c)*log(b*x + a)/(a^2*d) + 2*cosh(d*x + c)*log(x
)/(a^2*d) - (Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)/(a^2*d)) + (1/(a*b*x + a^2) -
log(b*x + a)/a^2 + log(x)/a^2)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1329 vs. 2(152) = 304.

Time = 0.15 (sec) , antiderivative size = 1329, normalized size of antiderivative = 8.86

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((b*x + a)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^2*Ei(((b*x + a)*(b
*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*
c*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^
((b*c - a*d)/b) + a^2*d^3*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d
) - b*c + a*d)/b)*e^((b*c - a*d)/b) - (b*x + a)*a*(b*c/(b*x + a) - a*d/(b*
x + a) + d)*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c +
 a*d)/b)*e^(-(b*c - a*d)/b) + a*b*c*d^2*Ei(-((b*x + a)*(b*c/(b*x + a) - a*
d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b) - a^2*d^3*Ei(-((b*x +
a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^(-(b*c - a*d)/b)
- (b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(-(b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) + b^2*c*d*Ei(-(b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) - a*b*d^2*Ei(-(b*x + a)*(b*c/(b*
x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c) - (b*x + a)*b*(b*c/(b*x + a) - a
*d/(b*x + a) + d)*d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c
)*e^c + b^2*c*d*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^
c - a*b*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c +
(b*x + a)*b*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d*Ei(((b*x + a)*(b*c/(b*x
+ a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - b^2*c*d*Ei((
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*
d)/b) + a*b*d^2*Ei(((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2} \, dx = \int \frac{\cosh(c+d\,x)}{x\,{(a+b\,x)}^2} \,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x)^2),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x)^2), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{b^2x^3+2ab\,x^2+a^2x}dx$$

input
```
int(cosh(d*x+c)/x/(b*x+a)^2,x)
```

output
```
int(cosh(c + d*x)/(a**2*x + 2*a*b*x**2 + b**2*x**3),x)
```

3.32 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx$

Optimal result

Integrand size = 17, antiderivative size = 186

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} dx = -\frac{\cosh(c+dx)}{a^2x} - \frac{b\cosh(c+dx)}{a^2(a+bx)} - \frac{2b\cosh(c)\text{Chi}(dx)}{a^3} + \frac{2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3} + \frac{d\text{Chi}(dx)\sinh(c)}{a^2} + \frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^2} + \frac{d\cosh(c)\text{Shi}(dx)}{a^2} - \frac{2b\sinh(c)\text{Shi}(dx)}{a^3} + \frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2} + \frac{2b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}$$

output
```
-cosh(d*x+c)/a^2/x-b*cosh(d*x+c)/a^2/(b*x+a)-2*b*cosh(c)*Chi(d*x)/a^3+2*b*
cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a^3+d*Chi(d*x)*sinh(c)/a^2-d*Chi(a*d/b+d*x)*
sinh(-c+a*d/b)/a^2+d*cosh(c)*Shi(d*x)/a^2-2*b*sinh(c)*Shi(d*x)/a^3+d*cosh(
-c+a*d/b)*Shi(a*d/b+d*x)/a^2-2*b*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/a^3
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.98

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx$$
$$= \frac{-\frac{a(a+2bx)\cosh(c)\cosh(dx)}{x(a+bx)} - 2b\cosh(c)\text{Chi}(dx) + 2b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right) + ad\text{Chi}(dx)\sinh(c) + ad\text{C}}{}$$

input
```
Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]
```

output
```
(-((a*(a + 2*b*x)*Cosh[c]*Cosh[d*x])/(x*(a + b*x))) - 2*b*Cosh[c]*CoshInte
gral[d*x] + 2*b*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] + a*d*CoshInte
gral[d*x]*Sinh[c] + a*d*CoshIntegral[d*(a/b + x)]*Sinh[c - (a*d)/b] - (a*(
a + 2*b*x)*Sinh[c]*Sinh[d*x])/(x*(a + b*x)) + a*d*Cosh[c]*SinhIntegral[d*x
] - 2*b*Sinh[c]*SinhIntegral[d*x] + a*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(
a/b + x)] + 2*b*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/a^3
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{2b^2\cosh(c+dx)}{a^3(a+bx)} - \frac{2b\cosh(c+dx)}{a^3x} + \frac{b^2\cosh(c+dx)}{a^2(a+bx)^2} + \frac{\cosh(c+dx)}{a^2x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{2b\cosh(c)\mathrm{Chi}(dx)}{a^3}+\frac{2b\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\sinh(c)\mathrm{Shi}(dx)}{a^3}+\frac{2b\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{a^2}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{a^2}-\frac{b\cosh(c+dx)}{a^2(a+bx)}+\frac{d\sinh(c)\mathrm{Chi}(dx)}{a^2}+\frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2}-\frac{\cosh(c+dx)}{a^2x}$$

input
```
Int[Cosh[c + d*x]/(x^2*(a + b*x)^2),x]
```

output
```
-(Cosh[c + d*x]/(a^2*x)) - (b*Cosh[c + d*x])/(a^2*(a + b*x)) - (2*b*Cosh[c
]*CoshIntegral[d*x])/a^3 + (2*b*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d
*x])/a^3 + (d*CoshIntegral[d*x]*Sinh[c])/a^2 + (d*CoshIntegral[(a*d)/b + d
*x]*Sinh[c - (a*d)/b])/a^2 + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (2*b*Sinh
[c]*SinhIntegral[d*x])/a^3 + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d
*x])/a^2 + (2*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 312, normalized size of antiderivative = 1.68

method	result
risch	$-\frac{d\,e^{-dx-c}b}{a^2(dxb+ad)}-\frac{d\,e^{-dx-c}}{2a(dxb+ad)x}+\frac{d\,e^{-c}\,\mathrm{expIntegral}_1(dx)}{2a^2}+\frac{e^{-c}\,\mathrm{expIntegral}_1(dx)b}{a^3}+\frac{d\,e^{\frac{ad-cb}{b}}\,\mathrm{expIntegral}_1\left(dx+c+\frac{ad-cb}{b}\right)}{2a^2}$

input
```
int(cosh(d*x+c)/x^2/(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-d*exp(-d*x-c)/a^2/(b*d*x+a*d)*b-1/2*d*exp(-d*x-c)/a/(b*d*x+a*d)/x+1/2*d/a
^2*exp(-c)*Ei(1,d*x)+1/a^3*exp(-c)*Ei(1,d*x)*b+1/2*d/a^2*exp((a*d-b*c)/b)*
Ei(1,d*x+c+(a*d-b*c)/b)-1/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b-1
/2*d/a^2*exp(d*x+c)/(d/b*a+d*x)-1/2*d/a^2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a
*d-b*c)/b)+1/a^3*b*exp(c)*Ei(1,-d*x)-b/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(
a*d-b*c)/b)-1/2/a^2/x*exp(d*x+c)-1/2*d/a^2*exp(c)*Ei(1,-d*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 377, normalized size of antiderivative = 2.03

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = -\frac{2\,(2\,abx+a^2)\cosh{(dx+c)} - (((abd-2\,b^2)x^2+(a^2d-2\,ab)x)\mathrm{Ei}(dx) - ((abd+2\,b^2)x^2+(a^2d+2\,a$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(2*a*b*x + a^2)*cosh(d*x + c) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2
*a*b)*x)*Ei(d*x) - ((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*cos
h(c) - (((a*b*d + 2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) - ((
a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c -
 a*d)/b) - (((a*b*d - 2*b^2)*x^2 + (a^2*d - 2*a*b)*x)*Ei(d*x) + ((a*b*d +
2*b^2)*x^2 + (a^2*d + 2*a*b)*x)*Ei(-d*x))*sinh(c) + (((a*b*d + 2*b^2)*x^2
+ (a^2*d + 2*a*b)*x)*Ei((b*d*x + a*d)/b) + ((a*b*d - 2*b^2)*x^2 + (a^2*d -
 2*a*b)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b*x^2 + a^4*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x**2/(b*x+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx+a\right)^2x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x + a)^2*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3353 vs. $2(191) = 382$.

Time = 0.17 (sec) , antiderivative size = 3353, normalized size of antiderivative = 18.03

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a)^2,x, algorithm="giac")
```

output
```
-1/2*((b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(-(b*x + a
)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(-c)/b - 2*(b*x + a)*a*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(
b*x + a) + d)/b + c)*e^(-c) + a*b*c^2*d^2*Ei(-(b*x + a)*(b*c/(b*x + a) - a
*d/(b*x + a) + d)/b + c)*e^(-c) + (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x
+ a) + d)*d^3*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)*e^(
-c)/b - a^2*c*d^3*Ei(-(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b + c)
*e^(-c) - (b*x + a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei((b*x
+ a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c/b + 2*(b*x + a)*a*(b*c
/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d)/b - c)*e^c - a*b*c^2*d^2*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b
*x + a) + d)/b - c)*e^c - (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d
)*d^3*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c/b + a^2*
c*d^3*Ei((b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d)/b - c)*e^c - (b*x +
 a)^2*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)^2*d^2*Ei(((b*x + a)*(b*c/(b*x
+ a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b)/b + 2*(b*x + a
)*a*(b*c/(b*x + a) - a*d/(b*x + a) + d)*c*d^2*Ei(((b*x + a)*(b*c/(b*x + a)
 - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*d)/b) - a*b*c^2*d^2*Ei((
(b*x + a)*(b*c/(b*x + a) - a*d/(b*x + a) + d) - b*c + a*d)/b)*e^((b*c - a*
d)/b) - (b*x + a)*a^2*(b*c/(b*x + a) - a*d/(b*x + a) + d)*d^3*Ei(((b*x ...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2} \, dx = \int \frac{\cosh(c+d\,x)}{x^2\,{(a+b\,x)}^2} \,dx$$

input
```
int(cosh(c + d*x)/(x^2*(a + b*x)^2),x)
```

output
```
int(cosh(c + d*x)/(x^2*(a + b*x)^2), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^2}\,dx$$
$$= \frac{-e^{2dx+2c} + e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^2+2abx+a^2}dx\right)abdx + e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^2+2abx+a^2}dx\right)b^2d\,x^2 + e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^3+2ab\,x^2+a^2x}d\right.}{}$$

input
```
int(cosh(d*x+c)/x^2/(b*x+a)^2,x)
```

output
```
( - e**(2*c + 2*d*x) + e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x + b**2*
x**2),x)*a*b*d*x + e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x + b**2*x**2
),x)*b**2*d*x**2 + e**(2*c + d*x)*int(e**(d*x)/(a**2*x + 2*a*b*x**2 + b**2
*x**3),x)*a**2*d*x + e**(2*c + d*x)*int(e**(d*x)/(a**2*x + 2*a*b*x**2 + b*
*2*x**3),x)*a*b*d*x**2 - 2*e**(2*c + d*x)*int(e**(d*x)/(a**2*x + 2*a*b*x**
2 + b**2*x**3),x)*a*b*x - 2*e**(2*c + d*x)*int(e**(d*x)/(a**2*x + 2*a*b*x*
*2 + b**2*x**3),x)*b**2*x**2 - e**(d*x)*int(1/(e**(d*x)*a**2*x + 2*e**(d*x
)*a*b*x**2 + e**(d*x)*b**2*x**3),x)*a**2*d*x - e**(d*x)*int(1/(e**(d*x)*a*
*2*x + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**3),x)*a*b*d*x**2 - 2*e**(d*x
)*int(1/(e**(d*x)*a**2*x + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**3),x)*a*
b*x - 2*e**(d*x)*int(1/(e**(d*x)*a**2*x + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b
**2*x**3),x)*b**2*x**2 - e**(d*x)*int(1/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x
+ e**(d*x)*b**2*x**2),x)*a*b*d*x - e**(d*x)*int(1/(e**(d*x)*a**2 + 2*e**(d
*x)*a*b*x + e**(d*x)*b**2*x**2),x)*b**2*d*x**2 - 1)/(2*e**(c + d*x)*a*x*(a
 + b*x))
```

3.33 $\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 264

$$\begin{aligned}\int \frac{x^3\cosh(c+dx)}{(a+bx)^3}\,dx &= \frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2}-\frac{3a^2\cosh(c+dx)}{b^4(a+bx)} \\ &\quad -\frac{3a\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{b^4} \\ &\quad -\frac{a^3d^2\cosh\left(c-\frac{ad}{b}\right)\operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{2b^6} \\ &\quad +\frac{3a^2d\operatorname{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^5}+\frac{\sinh(c+dx)}{b^3d} \\ &\quad +\frac{a^3d\sinh(c+dx)}{2b^5(a+bx)}+\frac{3a^2d\cosh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^5} \\ &\quad -\frac{3a\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{b^4} \\ &\quad -\frac{a^3d^2\sinh\left(c-\frac{ad}{b}\right)\operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{2b^6}\end{aligned}$$

output

```
1/2*a^3*cosh(d*x+c)/b^4/(b*x+a)^2-3*a^2*cosh(d*x+c)/b^4/(b*x+a)-3*a*cosh(-
c+a*d/b)*Chi(a*d/b+d*x)/b^4-1/2*a^3*d^2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^6-
3*a^2*d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^5+sinh(d*x+c)/b^3/d+1/2*a^3*d*sinh
(d*x+c)/b^5/(b*x+a)+3*a^2*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5+3*a*sinh(-c+
a*d/b)*Shi(a*d/b+d*x)/b^4+1/2*a^3*d^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b^6
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} \, dx =$$
$$-\frac{b\cosh(dx)\left(a^2bd(5a+6bx)\cosh(c)-(a+bx)\left(2ab^2+a^3d^2+2b^3x\right)\sinh(c)\right)-b\left((a+bx)\left(2ab^2+a^3d^2\right.\right.}{}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

output
```
-1/2*(b*Cosh[d*x]*(a^2*b*d*(5*a + 6*b*x)*Cosh[c] - (a + b*x)*(2*a*b^2 + a^
3*d^2 + 2*b^3*x)*Sinh[c]) - b*((a + b*x)*(2*a*b^2 + a^3*d^2 + 2*b^3*x)*Cos
h[c] - a^2*b*d*(5*a + 6*b*x)*Sinh[c])*Sinh[d*x] + a*d*(a + b*x)^2*(CoshInt
egral[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 6*a*b*d*Sinh[c -
 (a*d)/b]) + (-6*a*b*d*Cosh[c - (a*d)/b] + (6*b^2 + a^2*d^2)*Sinh[c - (a*d
)/b])*SinhIntegral[d*(a/b + x)]))/(b^6*d*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.90 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} \, dx$$
$$\Big\downarrow \; 7293$$
$$\int \left(-\frac{a^3\cosh(c+dx)}{b^3(a+bx)^3}+\frac{3a^2\cosh(c+dx)}{b^3(a+bx)^2}-\frac{3a\cosh(c+dx)}{b^3(a+bx)}+\frac{\cosh(c+dx)}{b^3}\right)dx$$
$$\Big\downarrow \; 2009$$

$$-\frac{a^3d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{2b^6}-\frac{a^3d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{2b^6}+\frac{a^3d\sinh(c+dx)}{2b^5(a+bx)}+\frac{a^3\cosh(c+dx)}{2b^4(a+bx)^2}+\frac{3a^2d\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^5}+\frac{3a^2d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^5}-\frac{3a^2\cosh(c+dx)}{b^4(a+bx)}-\frac{3a\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{3a\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{\sinh(c+dx)}{b^3d}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x)^3,x]
```

output
```
(a^3*Cosh[c + d*x])/(2*b^4*(a + b*x)^2) - (3*a^2*Cosh[c + d*x])/(b^4*(a +
b*x)) - (3*a*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^4 - (a^3*d^2
*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^6) + (3*a^2*d*CoshInt
egral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^5 + Sinh[c + d*x]/(b^3*d) + (a^3
*d*Sinh[c + d*x])/(2*b^5*(a + b*x)) + (3*a^2*d*Cosh[c - (a*d)/b]*SinhInteg
ral[(a*d)/b + d*x])/b^5 - (3*a*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*
x])/b^4 - (a^3*d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^6)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. $2(262) = 524$.

Time = 0.76 (sec) , antiderivative size = 1055, normalized size of antiderivative = 4.00

method	result	size
risch	Expression too large to display	1055

input
```
int(x^3*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^5*d^3-exp(-d*x-c)*a^4*b*d^
2-5*exp(-d*x-c)*a^3*b^2*d-4*exp(-d*x-c)*a*b^4*x+exp(-(a*d-b*c)/b)*Ei(1,-d*
x-c-(a*d-b*c)/b)*a^3*b^2*d^3*x^2+2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)
/b)*a^4*b*d^3*x-6*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^3*d^2*x
^2-12*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b^2*d^2*x+6*exp(-(a*d
-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b^4*d*x^2+12*exp(-(a*d-b*c)/b)*Ei(1,-d
*x-c-(a*d-b*c)/b)*a^2*b^3*d*x-2*exp(-d*x-c)*a^2*b^3+12*exp((a*d-b*c)/b)*Ei
(1,d*x+c+(a*d-b*c)/b)*a^3*b^2*d^2*x+6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c
)/b)*a*b^4*d*x^2+12*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b^3*d*x-e
xp(-d*x-c)*a^3*b^2*d^2*x+6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*b*
d^2-6*exp(-d*x-c)*a^2*b^3*d*x+6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a
^3*b^2*d-6*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4*b*d^2+exp(d*x+c)
*a^3*b^2*d^2*x+6*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b^2*d-6*ex
p(d*x+c)*a^2*b^3*d*x+exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b^2*d^3*
x^2+2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*b*d^3*x+exp(-(a*d-b*c)/
b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^5*d^3+exp(d*x+c)*a^4*b*d^2-5*exp(d*x+c)*a^3*
b^2*d+4*exp(d*x+c)*a*b^4*x+6*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*
b^3*d^2*x^2+2*exp(d*x+c)*b^5*x^2+2*exp(d*x+c)*a^2*b^3-2*exp(-d*x-c)*b^5*x^
2)/d/b^6/(b*x+a)^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(262) = 524$.

Time = 0.14 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.14

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} \, dx = -\frac{2 \left(6 a^2b^3dx + 5 a^3b^2d\right) \cosh\left(dx+c\right) + \left(\left(a^5d^3 - 6 a^4bd^2 + 6 a^3b^2d + \left(a^3b^2d^3 - 6 a^2b^3d^2 + 6 ab^4d\right)x^2 + 2\right.\right.}{}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(6*a^2*b^3*d*x + 5*a^3*b^2*d)*cosh(d*x + c) + ((a^5*d^3 - 6*a^4*b*
d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4
*b*d^3 - 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei((b*d*x + a*d)/b) + (a^5*d^3 +
6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2
+ 2*(a^4*b*d^3 + 6*a^3*b^2*d^2 + 6*a^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*cos
h(-(b*c - a*d)/b) - 2*(a^4*b*d^2 + 2*b^5*x^2 + 2*a^2*b^3 + (a^3*b^2*d^2 +
4*a*b^4)*x)*sinh(d*x + c) - ((a^5*d^3 - 6*a^4*b*d^2 + 6*a^3*b^2*d + (a^3*b
^2*d^3 - 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 - 6*a^3*b^2*d^2 + 6
*a^2*b^3*d)*x)*Ei((b*d*x + a*d)/b) - (a^5*d^3 + 6*a^4*b*d^2 + 6*a^3*b^2*d
+ (a^3*b^2*d^3 + 6*a^2*b^3*d^2 + 6*a*b^4*d)*x^2 + 2*(a^4*b*d^3 + 6*a^3*b^2
*d^2 + 6*a^2*b^3*d)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^8*d*
x^2 + 2*a*b^7*d*x + a^2*b^6*d)
```

Sympy [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^3 \cosh{(c+dx)}}{(a+bx)^3} \, dx$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x+a)**3,x)
```

output
```
Integral(x**3*cosh(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^3 \cosh{(dx+c)}}{(bx+a)^3} \, dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/2*a^2*d*integrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b
^3*d^2*x^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3/2*a^2*d*integrate(x/(b^5
*d^2*x^4*e^(d*x + c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(
d*x + c) + 4*a^3*b^2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) - 3*a*
b*integrate(x*e^(d*x + c)/(b^5*d^2*x^4 + 4*a*b^4*d^2*x^3 + 6*a^2*b^3*d^2*x
^2 + 4*a^3*b^2*d^2*x + a^4*b*d^2), x) - 3*a*b*integrate(x/(b^5*d^2*x^4*e^(
d*x + c) + 4*a*b^4*d^2*x^3*e^(d*x + c) + 6*a^2*b^3*d^2*x^2*e^(d*x + c) + 4
*a^3*b^2*d^2*x*e^(d*x + c) + a^4*b*d^2*e^(d*x + c)), x) + 1/2*((b*d*x^3*e^
(2*c) - 3*a*x*e^(2*c))*e^(d*x) - (b*d*x^3 + 3*a*x)*e^(-d*x))/(b^4*d^2*x^3*
e^c + 3*a*b^3*d^2*x^2*e^c + 3*a^2*b^2*d^2*x*e^c + a^3*b*d^2*e^c) - 3/2*a^2
*e^(-c + a*d/b)*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b^2*d^2) - 3
/2*a^2*e^(c - a*d/b)*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b^2*d^
2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. $2(262) = 524$.

Time = 0.12 (sec) , antiderivative size = 879, normalized size of antiderivative = 3.33

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(a^3*b^2*d^3*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^3*b^2*d^3*x^2*
Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^4*b*d^3*x*Ei((b*d*x + a*d)/b)*e^
(c - a*d/b) - 6*a^2*b^3*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^4*
b*d^3*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a^2*b^3*d^2*x^2*Ei(-(b*d*x
 + a*d)/b)*e^(-c + a*d/b) + a^5*d^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12
*a^3*b^2*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a*b^4*d*x^2*Ei((b*d*x
 + a*d)/b)*e^(c - a*d/b) + a^5*d^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 1
2*a^3*b^2*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 6*a*b^4*d*x^2*Ei(-(b
*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b^2*d^2*x*e^(d*x + c) + a^3*b^2*d^2*x*
e^(-d*x - c) - 6*a^4*b*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a^2*b^3*
d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a^4*b*d^2*Ei(-(b*d*x + a*d)/b)*e
^(-c + a*d/b) + 12*a^2*b^3*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^4*b
*d^2*e^(d*x + c) + 6*a^2*b^3*d*x*e^(d*x + c) - 2*b^5*x^2*e^(d*x + c) + a^4
*b*d^2*e^(-d*x - c) + 6*a^2*b^3*d*x*e^(-d*x - c) + 2*b^5*x^2*e^(-d*x - c)
+ 6*a^3*b^2*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 6*a^3*b^2*d*Ei(-(b*d*x +
 a*d)/b)*e^(-c + a*d/b) + 5*a^3*b^2*d*e^(d*x + c) - 4*a*b^4*x*e^(d*x + c)
+ 5*a^3*b^2*d*e^(-d*x - c) + 4*a*b^4*x*e^(-d*x - c) - 2*a^2*b^3*e^(d*x + c
) + 2*a^2*b^3*e^(-d*x - c))/(b^8*d*x^2 + 2*a*b^7*d*x + a^2*b^6*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{(a+b\,x)^3}\,dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x)^3,x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx)^3} \, dx = \text{too large to display}$$

input
```
int(x^3*cosh(d*x+c)/(b*x+a)^3,x)
```

output
```
( - e**(2*c + 2*d*x)*a**3*d**2*x + e**(2*c + 2*d*x)*a**3*d + e**(2*c + 2*d
*x)*a**2*b*d**2*x**2 + 2*e**(2*c + 2*d*x)*a**2*b + 4*e**(2*c + 2*d*x)*a*b*
*2*x - 4*e**(2*c + 2*d*x)*b**3*x**2 + e**(2*c + d*x)*int((e**(d*x)*x)/(a**
5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3
*d**2*x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**8*d**5 +
 2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b
**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12*a*
b**4*x**2 - 4*b**5*x**3),x)*a**7*b*d**5*x - 4*e**(2*c + d*x)*int((e**(d*x)
*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a
**2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**
7*b*d**4 + e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x +
3*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x
 - 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**6*b**2*d**5*x**2 - 8*e**(2*c + d*x)
*int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4
*a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**
5*x**3),x)*a**6*b**2*d**4*x - 10*e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**
2 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2
*x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**6*b**2*d**3 -
 4*e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b
**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12...
```

3.34 $\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 241

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{(a+bx)^3}\,dx =& -\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2}+\frac{2a\cosh(c+dx)}{b^3(a+bx)} \\ &+\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{b^3} \\ &+\frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2b^5} \\ &-\frac{2ad\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^4}-\frac{a^2d\sinh(c+dx)}{2b^4(a+bx)} \\ &-\frac{2ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^4}+\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} \\ &+\frac{a^2d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2b^5}\end{aligned}$$

output
```
-1/2*a^2*cosh(d*x+c)/b^3/(b*x+a)^2+2*a*cosh(d*x+c)/b^3/(b*x+a)+cosh(-c+a*d
/b)*Chi(a*d/b+d*x)/b^3+1/2*a^2*d^2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^5+2*a*d
*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^4-1/2*a^2*d*sinh(d*x+c)/b^4/(b*x+a)-2*a*d
*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^4-sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3-1/2*a
^2*d^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b^5
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.63

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx$$
$$= \frac{\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)\left((2b^2+a^2d^2)\cosh\left(c-\frac{ad}{b}\right)-4abd\sinh\left(c-\frac{ad}{b}\right)\right)-\frac{ab(-b(3a+4bx)\cosh(c+dx)+ad(a+bx)\sinh(c+dx}{(a+bx)^2}}{2b^5}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```

output
```
(CoshIntegral[d*(a/b + x)]*((2*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] - 4*a*b*d*
Sinh[c - (a*d)/b]) - (a*b*(-(b*(3*a + 4*b*x)*Cosh[c + d*x]) + a*d*(a + b*x
)*Sinh[c + d*x]))/(a + b*x)^2 + (-4*a*b*d*Cosh[c - (a*d)/b] + (2*b^2 + a^2
*d^2)*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])/(2*b^5)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{a^2 \cosh(c+dx)}{b^2(a+bx)^3}-\frac{2a \cosh(c+dx)}{b^2(a+bx)^2}+\frac{\cosh(c+dx)}{b^2(a+bx)}\right) dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{a^2d^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{2b^5}+\frac{a^2d^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{2b^5}-\frac{a^2d\sinh(c+dx)}{2b^4(a+bx)}-\frac{a^2\cosh(c+dx)}{2b^3(a+bx)^2}-\frac{2ad\sinh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^4}-\frac{2ad\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^4}+\frac{\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{2a\cosh(c+dx)}{b^3(a+bx)}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x)^3,x]
```

output
```
-1/2*(a^2*Cosh[c + d*x])/(b^3*(a + b*x)^2) + (2*a*Cosh[c + d*x])/(b^3*(a +
 b*x)) + (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Co
sh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^5) - (2*a*d*CoshIntegral
[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^4 - (a^2*d*Sinh[c + d*x])/(2*b^4*(a +
 b*x)) - (2*a*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^4 + (Sinh
[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 + (a^2*d^2*Sinh[c - (a*d)/b
]*SinhIntegral[(a*d)/b + d*x])/(2*b^5)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 916 vs. $2(240) = 480$.

Time = 0.67 (sec) , antiderivative size = 917, normalized size of antiderivative = 3.80

method	result
risch	$-\frac{e^{-\frac{ad-cb}{b}}\operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right)a^4d^2+2\,e^{-\frac{ad-cb}{b}}\operatorname{expIntegral}_1\left(-dx-c-\frac{ad-cb}{b}\right)b^4x^2+e^{dx+c}a^3bd-4\,e^{dx+c}a\,b^3x+2\,e^{-\ldots}}{\ldots}$

input
```
int(x^2*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^4*d^2+2*exp(-(a*d-b*c)/
b)*Ei(1,-d*x-c-(a*d-b*c)/b)*b^4*x^2+exp(d*x+c)*a^3*b*d-4*exp(d*x+c)*a*b^3*
x+2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^2+exp(d*x+c)*a^2*b^2*
d*x-4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^3*b*d+4*exp(-(a*d-b*c)/
b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b^3*x-3*exp(d*x+c)*a^2*b^2+exp((a*d-b*c)/b)*
Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b^2*d^2*x^2+2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d
-b*c)/b)*a^3*b*d^2*x+4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a*b^3*d*x^
2+8*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b^2*d*x+exp(-(a*d-b*c)/b)
*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^2*d^2*x^2+2*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-
(a*d-b*c)/b)*a^3*b*d^2*x-4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a*b^
3*d*x^2-8*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a^2*b^2*d*x-exp(-d*x-
c)*a^2*b^2*d*x+4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^3*b*d+4*exp((a
*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a*b^3*x-3*exp(-d*x-c)*a^2*b^2+exp((a*d-
b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^4*d^2+2*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d
-b*c)/b)*b^4*x^2-exp(-d*x-c)*a^3*b*d-4*exp(-d*x-c)*a*b^3*x+2*exp((a*d-b*c)
/b)*Ei(1,d*x+c+(a*d-b*c)/b)*a^2*b^2)/b^5/(b*x+a)^2
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.97

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3}\,dx = \frac{2\,(4\,ab^3x+3\,a^2b^2)\cosh{(dx+c)}+\left((a^4d^2-4\,a^3bd+2\,a^2b^2+(a^2b^2d^2-4\,ab^3d+2\,b^4)x^2+2\,(a^3bd^2-4\,a\right.}{}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(4*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + ((a^4*d^2 - 4*a^3*b*d + 2*a
^2*b^2 + (a^2*b^2*d^2 - 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*
d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (
a^2*b^2*d^2 + 4*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^
3)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a^2*b^2*d*x + a^3*b*
d)*sinh(d*x + c) - ((a^4*d^2 - 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 4*a*
b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x +
a*d)/b) - (a^4*d^2 + 4*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 4*a*b^3*d + 2*
b^4)*x^2 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*
sinh(-(b*c - a*d)/b))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Sympy [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^2 \cosh{(c+dx)}}{(a+bx)^3} \, dx$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x+a)**3,x)
```

output
```
Integral(x**2*cosh(c + d*x)/(a + b*x)**3, x)
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx = \int \frac{x^2 \cosh{(dx+c)}}{(bx+a)^3} \, dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
-3/2*a*d*integrate(x*e^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^
2*d^2*x^2 + 4*a^3*b*d^2*x + a^4*d^2), x) + 3/2*a*d*integrate(x/(b^4*d^2*x^
4*e^(d*x + c) + 4*a*b^3*d^2*x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c
) + 4*a^3*b*d^2*x*e^(d*x + c) + a^4*d^2*e^(d*x + c)), x) + b*integrate(x*e
^(d*x + c)/(b^4*d^2*x^4 + 4*a*b^3*d^2*x^3 + 6*a^2*b^2*d^2*x^2 + 4*a^3*b*d^
2*x + a^4*d^2), x) + b*integrate(x/(b^4*d^2*x^4*e^(d*x + c) + 4*a*b^3*d^2*
x^3*e^(d*x + c) + 6*a^2*b^2*d^2*x^2*e^(d*x + c) + 4*a^3*b*d^2*x*e^(d*x + c
) + a^4*d^2*e^(d*x + c)), x) + 1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) -
(d*x^2 - x)*e^(-d*x))/(b^3*d^2*x^3*e^c + 3*a*b^2*d^2*x^2*e^c + 3*a^2*b*d^2
*x*e^c + a^3*d^2*e^c) + 1/2*a*e^(-c + a*d/b)*exp_integral_e(4, (b*x + a)*d
/b)/((b*x + a)^3*b*d^2) + 1/2*a*e^(c - a*d/b)*exp_integral_e(4, -(b*x + a)
*d/b)/((b*x + a)^3*b*d^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(240) = 480$.

Time = 0.12 (sec) , antiderivative size = 741, normalized size of antiderivative = 3.07

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(
c - a*d/b) - 4*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2
*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/b
)*e^(-c + a*d/b) + a^4*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 8*a^2*b^2*d
*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*x^2*Ei((b*d*x + a*d)/b)*e^(c
- a*d/b) + a^4*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 8*a^2*b^2*d*x*Ei(
-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c +
a*d/b) - a^2*b^2*d*x*e^(d*x + c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a^3*b*d*Ei
((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(c - a*d
/b) + 4*a^3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 4*a*b^3*x*Ei(-(b*d*x
 + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*e^(d*x + c) + 4*a*b^3*x*e^(d*x + c) +
a^3*b*d*e^(-d*x - c) + 4*a*b^3*x*e^(-d*x - c) + 2*a^2*b^2*Ei((b*d*x + a*d)
/b)*e^(c - a*d/b) + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 3*a^2*
b^2*e^(d*x + c) + 3*a^2*b^2*e^(-d*x - c))/(b^7*x^2 + 2*a*b^6*x + a^2*b^5)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} dx = \int \frac{x^2 \cosh(c+d\,x)}{{(a+b\,x)}^3} \,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x)^3,x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx)^3} \, dx = \text{too large to display}$$

input
```
int(x^2*cosh(d*x+c)/(b*x+a)^3,x)
```

output
```
(e**(2*c + 2*d*x)*a**2*d**2*x - e**(2*c + 2*d*x)*a**2*d - 2*e**(2*c + 2*d*
x)*a*b - 4*e**(2*c + 2*d*x)*b**2*x - e**(2*c + d*x)*int((e**(d*x)*x)/(a**5
*d**2 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*
d**2*x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**7*d**5 -
2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b*
*2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12*a*b
**4*x**2 - 4*b**5*x**3),x)*a**6*b*d**5*x + 2*e**(2*c + d*x)*int((e**(d*x)*
x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a*
*2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**6
*b*d**4 - e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3
*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x
- 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**5*b**2*d**5*x**2 + 4*e**(2*c + d*x)*
int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4*
a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**5
*x**3),x)*a**5*b**2*d**4*x + 10*e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**2
 + 3*a**4*b*d**2*x + 3*a**3*b**2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2*
x**3 - 12*a**2*b**3*x - 12*a*b**4*x**2 - 4*b**5*x**3),x)*a**5*b**2*d**3 +
2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**5*d**2 + 3*a**4*b*d**2*x + 3*a**3*b*
*2*d**2*x**2 - 4*a**3*b**2 + a**2*b**3*d**2*x**3 - 12*a**2*b**3*x - 12*a*b
**4*x**2 - 4*b**5*x**3),x)*a**4*b**3*d**4*x**2 + 20*e**(2*c + d*x)*int(...
```

3.35 $\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 178

$$\begin{aligned}\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx &= \frac{a\cosh(c+dx)}{2b^2(a+bx)^2} - \frac{\cosh(c+dx)}{b^2(a+bx)} - \frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\mathrm{Chi}\left(\frac{ad}{b}+dx\right)}{2b^4} \\ &\quad + \frac{d\,\mathrm{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{b^3} + \frac{ad\sinh(c+dx)}{2b^3(a+bx)} \\ &\quad + \frac{d\cosh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{b^3} - \frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\mathrm{Shi}\left(\frac{ad}{b}+dx\right)}{2b^4}\end{aligned}$$

output
```
1/2*a*cosh(d*x+c)/b^2/(b*x+a)^2-cosh(d*x+c)/b^2/(b*x+a)-1/2*a*d^2*cosh(-c+
a*d/b)*Chi(a*d/b+d*x)/b^4-d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/b^3+1/2*a*d*sinh
(d*x+c)/b^3/(b*x+a)+d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3+1/2*a*d^2*sinh(-c+
a*d/b)*Shi(a*d/b+d*x)/b^4
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.89

$$\int \frac{x \cosh(c+dx)}{(a+bx)^3} dx = -\frac{b\cosh(dx)(b(a+2bx)\cosh(c)-ad(a+bx)\sinh(c))-b(ad(a+bx)\cosh(c)-b(a+2bx)\sinh(c))\sinh}{}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

output
```
-1/2*(b*Cosh[d*x]*(b*(a + 2*b*x)*Cosh[c] - a*d*(a + b*x)*Sinh[c]) - b*(a*d
*(a + b*x)*Cosh[c] - b*(a + 2*b*x)*Sinh[c])*Sinh[d*x] + d*(a + b*x)^2*(Cos
hIntegral[d*(a/b + x)]*(a*d*Cosh[c - (a*d)/b] - 2*b*Sinh[c - (a*d)/b]) + (
-2*b*Cosh[c - (a*d)/b] + a*d*Sinh[c - (a*d)/b])*SinhIntegral[d*(a/b + x)])
)/(b^4*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3} dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{\cosh(c+dx)}{b(a+bx)^2} - \frac{a\cosh(c+dx)}{b(a+bx)^3}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{ad^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2b^4}-\frac{ad^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2b^4}+\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b^3}+\frac{ad\sinh(c+dx)}{2b^3(a+bx)}-\frac{\cosh(c+dx)}{b^2(a+bx)}+\frac{a\cosh(c+dx)}{2b^2(a+bx)^2}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x)^3,x]
```

output
```
(a*Cosh[c + d*x])/(2*b^2*(a + b*x)^2) - Cosh[c + d*x]/(b^2*(a + b*x)) - (a
*d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*b^4) + (d*CoshInteg
ral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/b^3 + (a*d*Sinh[c + d*x])/(2*b^3*(a
+ b*x)) + (d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b^3 - (a*d^2*S
inh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*b^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(175) = 350$.

Time = 0.62 (sec) , antiderivative size = 435, normalized size of antiderivative = 2.44

method	result
risch	$-\frac{d^3 \mathrm{e}^{-dx-c} a x}{4b^2(b^2d^2x^2+2ab\,d^2x+d^2a^2)}-\frac{d^3\mathrm{e}^{-dx-c}a^2}{4b^3(b^2d^2x^2+2ab\,d^2x+d^2a^2)}-\frac{d^2\mathrm{e}^{-dx-c}x}{2b(b^2d^2x^2+2ab\,d^2x+d^2a^2)}-\frac{d^2\mathrm{e}^{-dx-c}a}{4b^2(b^2d^2x^2+2ab\,d^2x+d^2a^2)}$

input
```
int(x*cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*d^3*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a*x-1/4*d^3*exp
(-d*x-c)/b^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a^2-1/2*d^2*exp(-d*x-c)/b/(
b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x-1/4*d^2*exp(-d*x-c)/b^2/(b^2*d^2*x^2+2*
a*b*d^2*x+a^2*d^2)*a+1/4*d^2/b^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*
a+1/2*d/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/4*d^2/b^4*exp(d*x+c
)/(d/b*a+d*x)^2*a+1/4*d^2/b^4*exp(d*x+c)/(d/b*a+d*x)*a+1/4*d^2/b^4*exp(-(a
*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)*a-1/2*d/b^3*exp(d*x+c)/(d/b*a+d*x)-1/2
*d/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 373 vs. $2(175) = 350$.

Time = 0.12 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.10

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{2\left(2\,b^3x+ab^2\right)\cosh\left(dx+c\right)+\left(\left(a^3d^2-2\,a^2bd+\left(ab^2d^2-2\,b^3d\right)x^2+2\left(a^2bd^2-2\,ab^2d\right)x\right)\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)+\cdots}{\cdots}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(2*b^3*x + a*b^2)*cosh(d*x + c) + ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d
^2 - 2*b^3*d)*x^2 + 2*(a^2*b*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) + (a^
3*d^2 + 2*a^2*b*d + (a*b^2*d^2 + 2*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*
x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) - 2*(a*b^2*d*x + a^2*b*d)*si
nh(d*x + c) - ((a^3*d^2 - 2*a^2*b*d + (a*b^2*d^2 - 2*b^3*d)*x^2 + 2*(a^2*b
*d^2 - 2*a*b^2*d)*x)*Ei((b*d*x + a*d)/b) - (a^3*d^2 + 2*a^2*b*d + (a*b^2*d
^2 + 2*b^3*d)*x^2 + 2*(a^2*b*d^2 + 2*a*b^2*d)*x)*Ei(-(b*d*x + a*d)/b))*sin
h(-(b*c - a*d)/b))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\cosh\left(dx+c\right)}{\left(bx+a\right)^3}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
b*integrate(x*e^(d*x + c)/(b^4*d*x^4 + 4*a*b^3*d*x^3 + 6*a^2*b^2*d*x^2 + 4
*a^3*b*d*x + a^4*d), x) - b*integrate(x/(b^4*d*x^4*e^(d*x + c) + 4*a*b^3*d
*x^3*e^(d*x + c) + 6*a^2*b^2*d*x^2*e^(d*x + c) + 4*a^3*b*d*x*e^(d*x + c) +
 a^4*d*e^(d*x + c)), x) + 1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^3*e^
c + 3*a*b^2*d*x^2*e^c + 3*a^2*b*d*x*e^c + a^3*d*e^c) - 1/2*a*e^(-c + a*d/b
)*exp_integral_e(4, (b*x + a)*d/b)/((b*x + a)^3*b*d) + 1/2*a*e^(c - a*d/b)
*exp_integral_e(4, -(b*x + a)*d/b)/((b*x + a)^3*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(175) = 350$.

Time = 0.11 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.97

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = -\frac{ab^2d^2x^2\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + ab^2d^2x^2\mathrm{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2\,a^2bd^2x\mathrm{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} - 2\,b^3dx^2\mathrm{Ei}(}{}$$

input
```
integrate(x*cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(a*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a*b^2*d^2*x^2*Ei(-
(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^2*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c -
 a*d/b) - 2*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^2*b*d^2*x*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*b^3*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c
 + a*d/b) + a^3*d^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 4*a*b^2*d*x*Ei((b*
d*x + a*d)/b)*e^(c - a*d/b) + a^3*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b)
+ 4*a*b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a*b^2*d*x*e^(d*x + c)
+ a*b^2*d*x*e^(-d*x - c) - 2*a^2*b*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2
*a^2*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b*d*e^(d*x + c) + 2*b^3
*x*e^(d*x + c) + a^2*b*d*e^(-d*x - c) + 2*b^3*x*e^(-d*x - c) + a*b^2*e^(d*
x + c) + a*b^2*e^(-d*x - c))/(b^6*x^2 + 2*a*b^5*x + a^2*b^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{x\cosh(c+d\,x)}{(a+b\,x)^3}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x)^3,x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}x}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)+\int \frac{x}{e^{dx}a^3+3e^{dx}a^2bx+3e^{dx}a\,b^2x^2+e^{dx}b^3x^3}dx}{2e^c}$$

input
```
int(x*cosh(d*x+c)/(b*x+a)^3,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
,x) + int(x/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x + 3*e**(d*x)*a*b**2*x**2
+ e**(d*x)*b**3*x**3),x))/(2*e**c)
```

3.36 $\int \frac{\cosh(c+dx)}{(a+bx)^3} \, dx$

Optimal result

Integrand size = 14, antiderivative size = 104

$$\int \frac{\cosh(c+dx)}{(a+bx)^3} \, dx = -\frac{\cosh(c+dx)}{2b(a+bx)^2} + \frac{d^2 \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(\frac{ad}{b}+dx\right)}{2b^3} - \frac{d \sinh(c+dx)}{2b^2(a+bx)} + \frac{d^2 \sinh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(\frac{ad}{b}+dx\right)}{2b^3}$$

output
```
-1/2*cosh(d*x+c)/b/(b*x+a)^2+1/2*d^2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/b^3-1/2
*d*sinh(d*x+c)/b^2/(b*x+a)-1/2*d^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/b^3
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.85

$$\int \frac{\cosh(c+dx)}{(a+bx)^3} \, dx = \frac{d^2 \cosh\left(c-\frac{ad}{b}\right) \operatorname{Chi}\left(d\left(\frac{a}{b}+x\right)\right) - \frac{b(b\cosh(c+dx)+d(a+bx)\sinh(c+dx))}{(a+bx)^2} + d^2 \sinh\left(c-\frac{ad}{b}\right) \operatorname{Shi}\left(d\left(\frac{a}{b}+x\right)\right)}{2b^3}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x)^3,x]
```

output
```
(d^2*Cosh[c - (a*d)/b]*CoshIntegral[d*(a/b + x)] - (b*(b*Cosh[c + d*x] + d
*(a + b*x)*Sinh[c + d*x]))/(a + b*x)^2 + d^2*Sinh[c - (a*d)/b]*SinhIntegra
l[d*(a/b + x)])/(2*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.69 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.07, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.929$, Rules used = {3042, 3778, 26, 3042, 26, 3778, 3042, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{(a+bx)^3}dx$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}+\frac{id\int -\frac{i\sinh(c+dx)}{(a+bx)^2}dx}{2b}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{d\int \frac{\sinh(c+dx)}{(a+bx)^2}dx}{2b}-\frac{\cosh(c+dx)}{2b(a+bx)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}+\frac{d\int -\frac{i\sin(ic+idx)}{(a+bx)^2}dx}{2b}$$

$$\Big\downarrow \text{ 26}$$

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\int \frac{\sin(ic+idx)}{(a+bx)^2}dx}{2b}$$

$$\Big\downarrow \text{ 3778}$$

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\int\frac{\cosh(c+dx)}{a+bx}dx}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

$\Big\downarrow$ 3042

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\int\frac{\sin\left(ic+idx+\frac{\pi}{2}\right)}{a+bx}dx}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

$\Big\downarrow$ 3784

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\left(\cosh\left(c-\frac{ad}{b}\right)\int\frac{\cosh\left(xd+\frac{ad}{b}\right)}{a+bx}dx-i\sinh\left(c-\frac{ad}{b}\right)\int\frac{i\sinh\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

$\Big\downarrow$ 26

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\left(\sinh\left(c-\frac{ad}{b}\right)\int\frac{\sinh\left(xd+\frac{ad}{b}\right)}{a+bx}dx+\cosh\left(c-\frac{ad}{b}\right)\int\frac{\cosh\left(xd+\frac{ad}{b}\right)}{a+bx}dx\right)}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

$\Big\downarrow$ 3042

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\left(\sinh\left(c-\frac{ad}{b}\right)\int-\frac{i\sin\left(ixd+\frac{iad}{b}\right)}{a+bx}dx+\cosh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx\right)}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

$\Big\downarrow$ 26

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\left(\cosh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx-i\sinh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}\right)}{a+bx}dx\right)}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

$\Big\downarrow$ 3779

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\left(\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}+\cosh\left(c-\frac{ad}{b}\right)\int\frac{\sin\left(ixd+\frac{iad}{b}+\frac{\pi}{2}\right)}{a+bx}dx\right)}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\cosh(c+dx)}{2b(a+bx)^2}-\frac{id\left(\frac{id\left(\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{b}+\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{b}\right)}{b}-\frac{i\sinh(c+dx)}{b(a+bx)}\right)}{2b}$$

input
```
Int[Cosh[c + d*x]/(a + b*x)^3,x]
```

output
```
-1/2*Cosh[c + d*x]/(b*(a + b*x)^2) - ((I/2)*d*(((-I)*Sinh[c + d*x])/(b*(a
+ b*x)) + (I*d*((Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/b + (Sinh[
c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/b))/b))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(98) = 196$.

Time = 0.56 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.65

method	result
risch	$\frac{d^3 e^{-dx-c} x}{4b(b^2d^2x^2+2ab\,d^2x+d^2a^2)} + \frac{d^3 e^{-dx-c} a}{4b^2(b^2d^2x^2+2ab\,d^2x+d^2a^2)} - \frac{d^2 e^{-dx-c}}{4b(b^2d^2x^2+2ab\,d^2x+d^2a^2)} - \frac{d^2 e^{\frac{ad-cb}{b}} \operatorname{expIntegral}_1\left(dx+c+\frac{ac}{}\right.}{4b^3}$

input
```
int(cosh(d*x+c)/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^3*exp(-d*x-c)/b/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*x+1/4*d^3*exp(-d*x
-c)/b^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*a-1/4*d^2*exp(-d*x-c)/b/(b^2*d^2
*x^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/b^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c
)/b)-1/4*d^2/b^3*exp(d*x+c)/(d/b*a+d*x)^2-1/4*d^2/b^3*exp(d*x+c)/(d/b*a+d*
x)-1/4*d^2/b^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(98) = 196$.

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.43

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx =$$
$$-\frac{2\,b^2\cosh(dx+c) - \left(\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\operatorname{Ei}\left(\frac{bdx+ad}{b}\right)+\left(b^2d^2x^2+2\,abd^2x+a^2d^2\right)\operatorname{Ei}\left(-\frac{bdx+ad}{b}\right)\right) c}{}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*b^2*cosh(d*x + c) - ((b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei((b*d
*x + a*d)/b) + (b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*Ei(-(b*d*x + a*d)/b))
*cosh(-(b*c - a*d)/b) + 2*(b^2*d*x + a*b*d)*sinh(d*x + c) + ((b^2*d^2*x^2
+ 2*a*b*d^2*x + a^2*d^2)*Ei((b*d*x + a*d)/b) - (b^2*d^2*x^2 + 2*a*b*d^2*x
+ a^2*d^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(b^5*x^2 + 2*a*b^4*
x + a^2*b^3)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{d\left(\frac{e^{\left(-c+\frac{ad}{b}\right)}E_2\left(\frac{(bx+a)d}{b}\right)}{(bx+a)b} - \frac{e^{\left(c-\frac{ad}{b}\right)}E_2\left(-\frac{(bx+a)d}{b}\right)}{(bx+a)b}\right)}{4\,b} - \frac{\cosh\left(dx+c\right)}{2\,(bx+a)^2b}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/4*d*(e^(-c + a*d/b)*exp_integral_e(2, (b*x + a)*d/b)/((b*x + a)*b) - e^(
c - a*d/b)*exp_integral_e(2, -(b*x + a)*d/b)/((b*x + a)*b))/b - 1/2*cosh(d
*x + c)/((b*x + a)^2*b)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 298 vs. $2(98) = 196$.

Time = 0.12 (sec) , antiderivative size = 298, normalized size of antiderivative = 2.87

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx$$
$$= \frac{b^2d^2x^2\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + b^2d^2x^2\text{Ei}\left(-\frac{bdx+ad}{b}\right)e^{\left(-c+\frac{ad}{b}\right)} + 2\,abd^2x\text{Ei}\left(\frac{bdx+ad}{b}\right)e^{\left(c-\frac{ad}{b}\right)} + 2\,abd^2x\text{Ei}\left(-\frac{bdx+}{b}\right.}{}$$

input
```
integrate(cosh(d*x+c)/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + b^2*d^2*x^2*Ei(-(b*d*
x + a*d)/b)*e^(-c + a*d/b) + 2*a*b*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b)
 + 2*a*b*d^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^2*d^2*Ei((b*d*x + a
*d)/b)*e^(c - a*d/b) + a^2*d^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - b^2*d
*x*e^(d*x + c) + b^2*d*x*e^(-d*x - c) - a*b*d*e^(d*x + c) + a*b*d*e^(-d*x
- c) - b^2*e^(d*x + c) - b^2*e^(-d*x - c))/(b^5*x^2 + 2*a*b^4*x + a^2*b^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \int \frac{\cosh(c+d\,x)}{{(a+b\,x)}^3}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x)^3,x)
```

output
```
int(cosh(c + d*x)/(a + b*x)^3, x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{(a+bx)^3}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}}{b^3x^3+3a\,b^2x^2+3a^2bx+a^3}dx\right)+\int \frac{1}{e^{dx}a^3+3e^{dx}a^2bx+3e^{dx}a\,b^2x^2+e^{dx}b^3x^3}dx}{2e^c}$$

input
```
int(cosh(d*x+c)/(b*x+a)^3,x)
```

output
```
(e**(2*c)*int(e**(d*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)
+ int(1/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x + 3*e**(d*x)*a*b**2*x**2 + e*
*(d*x)*b**3*x**3),x))/(2*e**c)
```

3.37 $\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 262

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = {} & \frac{\cosh(c+dx)}{2a(a+bx)^2}+\frac{\cosh(c+dx)}{a^2(a+bx)}+\frac{\cosh(c)\text{Chi}(dx)}{a^3} \\ & -\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2ab^2} \\ & -\frac{d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^2b}+\frac{d\sinh(c+dx)}{2ab(a+bx)} \\ & +\frac{\sinh(c)\text{Shi}(dx)}{a^3}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^2b} \\ & -\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2ab^2}\end{aligned}$$

output
```
1/2*cosh(d*x+c)/a/(b*x+a)^2+cosh(d*x+c)/a^2/(b*x+a)+cosh(c)*Chi(d*x)/a^3-c
osh(-c+a*d/b)*Chi(a*d/b+d*x)/a^3-1/2*d^2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a/b
^2+d*Chi(a*d/b+d*x)*sinh(-c+a*d/b)/a^2/b+1/2*d*sinh(d*x+c)/a/b/(b*x+a)+sin
h(c)*Shi(d*x)/a^3-d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a^2/b+sinh(-c+a*d/b)*Shi
(a*d/b+d*x)/a^3+1/2*d^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/a/b^2
```

Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 451, normalized size of antiderivative = 1.72

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = -\frac{-3a^2b^2\cosh(c+dx)-2ab^3x\cosh(c+dx)-2b^2(a+bx)^2\cosh(c)\text{Chi}(dx)+(a+bx)^2\text{Chi}\left(d\left(\frac{a}{b}+x\right)\right)}{}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

output
```
-1/2*(-3*a^2*b^2*Cosh[c + d*x] - 2*a*b^3*x*Cosh[c + d*x] - 2*b^2*(a + b*x)
^2*Cosh[c]*CoshIntegral[d*x] + (a + b*x)^2*CoshIntegral[d*(a/b + x)]*((2*b
^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 2*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*Sin
h[c + d*x] - a^2*b^2*d*x*Sinh[c + d*x] - 2*a^2*b^2*Sinh[c]*SinhIntegral[d*
x] - 4*a*b^3*x*Sinh[c]*SinhIntegral[d*x] - 2*b^4*x^2*Sinh[c]*SinhIntegral[
d*x] + 2*a^3*b*d*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a^2*b^2*d
*x*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a*b^3*d*x^2*Cosh[c - (a
*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^2*b^2*Sinh[c - (a*d)/b]*SinhIntegra
l[d*(a/b + x)] + a^4*d^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 4*a
*b^3*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2*x*Sinh[c
- (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*b^4*x^2*Sinh[c - (a*d)/b]*SinhInt
egral[d*(a/b + x)] + a^2*b^2*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b
 + x)])/(a^3*b^2*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx$$

$$\Big\downarrow \; 7293$$

$$\int \left(-\frac{b\cosh(c+dx)}{a^3(a+bx)}+\frac{\cosh(c+dx)}{a^3x}-\frac{b\cosh(c+dx)}{a^2(a+bx)^2}-\frac{b\cosh(c+dx)}{a(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{\cosh(c)\text{Chi}(dx)}{a^3}+\frac{\sinh(c)\text{Shi}(dx)}{a^3}-\frac{d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^2b}-\frac{d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^2b}+\frac{\cosh(c+dx)}{a^2(a+bx)}-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2ab^2}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2ab^2}+\frac{d\sinh(c+dx)}{2ab(a+bx)}+\frac{\cosh(c+dx)}{2a(a+bx)^2}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x)^3),x]
```

output
```
Cosh[c + d*x]/(2*a*(a + b*x)^2) + Cosh[c + d*x]/(a^2*(a + b*x)) + (Cosh[c]
*CoshIntegral[d*x])/a^3 - (Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/
a^3 - (d^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a*b^2) - (d*C
oshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/(a^2*b) + (d*Sinh[c + d*x])/
(2*a*b*(a + b*x)) + (Sinh[c]*SinhIntegral[d*x])/a^3 - (d*Cosh[c - (a*d)/b]
*SinhIntegral[(a*d)/b + d*x])/(a^2*b) - (Sinh[c - (a*d)/b]*SinhIntegral[(a
*d)/b + d*x])/a^3 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2
*a*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 488, normalized size of antiderivative = 1.86

method	result
risch	$-\frac{e^{-dx-c} d\left((dx+c)abd+d^2a^2-bcda-2(dx+c)b^2-3abd+2c\,b^2\right)}{4a^2b\left((dx+c)^2b^2+2(dx+c)abd-2(dx+c)b^2c+d^2a^2-2bcda+b^2c^2\right)}-\frac{e^{-c}\,\mathrm{expIntegral}_1(dx)}{2a^3}+\frac{e^{\frac{ad-cb}{b}}\,\mathrm{expIntegral}_1\left(dx+c+\frac{ad}{}\right.}{4a\,b^2}$

input
```
int(cosh(d*x+c)/x/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*exp(-d*x-c)*d*((d*x+c)*a*b*d+d^2*a^2-b*c*d*a-2*(d*x+c)*b^2-3*a*b*d+2*
c*b^2)/a^2/b/((d*x+c)^2*b^2+2*(d*x+c)*a*b*d-2*(d*x+c)*b^2*c+d^2*a^2-2*b*c*
d*a+b^2*c^2)-1/2/a^3*exp(-c)*Ei(1,d*x)+1/4/a/b^2*exp((a*d-b*c)/b)*Ei(1,d*x
+c+(a*d-b*c)/b)*d^2-1/2/a^2/b*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*d+1
/2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+1/4/a/b^2*d^2*exp(d*x+c)/(
d/b*a+d*x)^2+1/4/a/b^2*d^2*exp(d*x+c)/(d/b*a+d*x)+1/4/a/b^2*d^2*exp(-(a*d-
b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+1/2/a^2*d/b*exp(d*x+c)/(d/b*a+d*x)+1/2/a^
2*d/b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^3*exp(c)*Ei(1,-d*x)
+1/2/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(258) = 516$.

Time = 0.15 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.29

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \frac{2\,(2\,ab^3x+3\,a^2b^2)\cosh\,(dx+c)+2\,((b^4x^2+2\,ab^3x+a^2b^2)\mathrm{Ei}(dx)+(b^4x^2+2\,ab^3x+a^2b^2)\mathrm{Ei}(-dx))\cos}{}$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(2*a*b^3*x + 3*a^2*b^2)*cosh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a
^2*b^2)*Ei(d*x) + (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*cosh(c) - ((a^
4*d^2 + 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*
(a^3*b*d^2 + 2*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) + (a^4*d^2 - 2*
a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2
 - 2*a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) +
2*(a^2*b^2*d*x + a^3*b*d)*sinh(d*x + c) + 2*((b^4*x^2 + 2*a*b^3*x + a^2*b^
2)*Ei(d*x) - (b^4*x^2 + 2*a*b^3*x + a^2*b^2)*Ei(-d*x))*sinh(c) + ((a^4*d^2
 + 2*a^3*b*d + 2*a^2*b^2 + (a^2*b^2*d^2 + 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*
b*d^2 + 2*a^2*b^2*d + 2*a*b^3)*x)*Ei((b*d*x + a*d)/b) - (a^4*d^2 - 2*a^3*b
*d + 2*a^2*b^2 + (a^2*b^2*d^2 - 2*a*b^3*d + 2*b^4)*x^2 + 2*(a^3*b*d^2 - 2*
a^2*b^2*d + 2*a*b^3)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^3*b
^4*x^2 + 2*a^4*b^3*x + a^5*b^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\cosh{(c+dx)}}{x\,(a+bx)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)**3,x)
```

output
```
Integral(cosh(c + d*x)/(x*(a + b*x)**3), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\cosh{(dx+c)}}{(bx+a)^3x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x + a)^3*x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(258) = 516$.

Time = 0.11 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.19

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x/(b*x+a)^3,x, algorithm="giac")
```

output
```
-1/4*(a^2*b^2*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^2*
Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x*Ei((b*d*x + a*d)/b)*e^
(c - a*d/b) + 2*a*b^3*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^
2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^2*Ei(-(b*d*x + a*d)/
b)*e^(-c + a*d/b) - 2*b^4*x^2*Ei(-d*x)*e^(-c) + a^4*d^2*Ei((b*d*x + a*d)/b
)*e^(c - a*d/b) + 4*a^2*b^2*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*b^4*
x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 2*b^4*x^2*Ei(d*x)*e^c + a^4*d^2*Ei
(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 4*a^2*b^2*d*x*Ei(-(b*d*x + a*d)/b)*e^(
-c + a*d/b) + 2*b^4*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x*
e^(d*x + c) + a^2*b^2*d*x*e^(-d*x - c) - 4*a*b^3*x*Ei(-d*x)*e^(-c) + 2*a^3
*b*d*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*x*Ei((b*d*x + a*d)/b)*e^(
c - a*d/b) - 4*a*b^3*x*Ei(d*x)*e^c - 2*a^3*b*d*Ei(-(b*d*x + a*d)/b)*e^(-c
+ a*d/b) + 4*a*b^3*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*e^(d*x
+ c) - 2*a*b^3*x*e^(d*x + c) + a^3*b*d*e^(-d*x - c) - 2*a*b^3*x*e^(-d*x -
c) - 2*a^2*b^2*Ei(-d*x)*e^(-c) + 2*a^2*b^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/
b) - 2*a^2*b^2*Ei(d*x)*e^c + 2*a^2*b^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b)
 - 3*a^2*b^2*e^(d*x + c) - 3*a^2*b^2*e^(-d*x - c))/(a^3*b^4*x^2 + 2*a^4*b^
3*x + a^5*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x\,{(a+b\,x)}^3}\,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x)^3),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x)^3), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx)^3}\,dx = \int \frac{\cosh(dx+c)}{b^3x^4+3a\,b^2x^3+3a^2b\,x^2+a^3x}dx$$

input
```
int(cosh(d*x+c)/x/(b*x+a)^3,x)
```

output
```
int(cosh(c + d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)
```

3.38 $\int \frac{\cosh(c+dx)}{x^2(a+bx)^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 298

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx)^3} dx = & -\frac{\cosh(c+dx)}{a^3x} - \frac{b\cosh(c+dx)}{2a^2(a+bx)^2} - \frac{2b\cosh(c+dx)}{a^3(a+bx)} \\ & - \frac{3b\cosh(c)\text{Chi}(dx)}{a^4} + \frac{3b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^4} \\ & + \frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2a^2b} \\ & + \frac{d\text{Chi}(dx)\sinh(c)}{a^3} + \frac{2d\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^3} \\ & - \frac{d\sinh(c+dx)}{2a^2(a+bx)} + \frac{d\cosh(c)\text{Shi}(dx)}{a^3} \\ & - \frac{3b\sinh(c)\text{Shi}(dx)}{a^4} + \frac{2d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^3} \\ & + \frac{3b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^4} + \frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2a^2b}\end{aligned}$$

output
```
-cosh(d*x+c)/a^3/x-1/2*b*cosh(d*x+c)/a^2/(b*x+a)^2-2*b*cosh(d*x+c)/a^3/(b*
x+a)-3*b*cosh(c)*Chi(d*x)/a^4+3*b*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a^4+1/2*d^
2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a^2/b+d*Chi(d*x)*sinh(c)/a^3-2*d*Chi(a*d/b
+d*x)*sinh(-c+a*d/b)/a^3-1/2*d*sinh(d*x+c)/a^2/(b*x+a)+d*cosh(c)*Shi(d*x)/
a^3-3*b*sinh(c)*Shi(d*x)/a^4+2*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a^3-3*b*sin
h(-c+a*d/b)*Shi(a*d/b+d*x)/a^4-1/2*d^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/a^2/b
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.82

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx$$
$$= \frac{-2a^3b\cosh(c+dx)-9a^2b^2x\cosh(c+dx)-6ab^3x^2\cosh(c+dx)+2bx(a+bx)^2\mathrm{Chi}(dx)(-3b\cosh(c)+$$

input
```
Integrate[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]
```

output
```
(-2*a^3*b*Cosh[c + d*x] - 9*a^2*b^2*x*Cosh[c + d*x] - 6*a*b^3*x^2*Cosh[c +
 d*x] + 2*b*x*(a + b*x)^2*CoshIntegral[d*x]*(-3*b*Cosh[c] + a*d*Sinh[c]) +
 x*(a + b*x)^2*CoshIntegral[d*(a/b + x)]*((6*b^2 + a^2*d^2)*Cosh[c - (a*d)
/b] + 4*a*b*d*Sinh[c - (a*d)/b]) - a^3*b*d*x*Sinh[c + d*x] - a^2*b^2*d*x^2
*Sinh[c + d*x] + 2*a^3*b*d*x*Cosh[c]*SinhIntegral[d*x] + 4*a^2*b^2*d*x^2*C
osh[c]*SinhIntegral[d*x] + 2*a*b^3*d*x^3*Cosh[c]*SinhIntegral[d*x] - 6*a^2
*b^2*x*Sinh[c]*SinhIntegral[d*x] - 12*a*b^3*x^2*Sinh[c]*SinhIntegral[d*x]
- 6*b^4*x^3*Sinh[c]*SinhIntegral[d*x] + 4*a^3*b*d*x*Cosh[c - (a*d)/b]*Sinh
Integral[d*(a/b + x)] + 8*a^2*b^2*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[d*(
a/b + x)] + 4*a*b^3*d*x^3*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*
a^2*b^2*x*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^4*d^2*x*Sinh[c -
 (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a*b^3*x^2*Sinh[c - (a*d)/b]*SinhI
ntegral[d*(a/b + x)] + 2*a^3*b*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a
/b + x)] + 6*b^4*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + a^2*b^2
*d^2*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)])/(2*a^4*b*x*(a + b*x)
^2)
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(\frac{3b^2\cosh(c+dx)}{a^4(a+bx)}-\frac{3b\cosh(c+dx)}{a^4x}+\frac{2b^2\cosh(c+dx)}{a^3(a+bx)^2}+\frac{\cosh(c+dx)}{a^3x^2}+\frac{b^2\cosh(c+dx)}{a^2(a+bx)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^4}-\frac{3b\sinh(c)\text{Shi}(dx)}{a^4}+\frac{3b\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{2d\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^3}+\frac{2d\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^3}-\frac{2b\cosh(c+dx)}{a^3(a+bx)}+\frac{d\sinh(c)\text{Chi}(dx)}{a^3}+\frac{d\cosh(c)\text{Shi}(dx)}{a^3}-\frac{\cosh(c+dx)}{a^3x}+\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2a^2b}+\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2a^2b}-\frac{d\sinh(c+dx)}{2a^2(a+bx)}-\frac{b\cosh(c+dx)}{2a^2(a+bx)^2}$$

input

```
Int[Cosh[c + d*x]/(x^2*(a + b*x)^3),x]
```

output

```
-(Cosh[c + d*x]/(a^3*x)) - (b*Cosh[c + d*x])/(2*a^2*(a + b*x)^2) - (2*b*Co
sh[c + d*x])/(a^3*(a + b*x)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 + (3*b*
Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^4 + (d^2*Cosh[c - (a*d)/b
]*CoshIntegral[(a*d)/b + d*x])/(2*a^2*b) + (d*CoshIntegral[d*x]*Sinh[c])/a
^3 + (2*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^3 - (d*Sinh[c +
 d*x])/(2*a^2*(a + b*x)) + (d*Cosh[c]*SinhIntegral[d*x])/a^3 - (3*b*Sinh[c
]*SinhIntegral[d*x])/a^4 + (2*d*Cosh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d
*x])/a^3 + (3*b*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/a^4 + (d^2*
Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*a^2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(296) = 592$.

Time = 0.73 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.16

method	result
risch	$\frac{e^{-dx-c}x\,d^3b}{4a^2(b^2d^2x^2+2ab\,d^2x+d^2a^2)}+\frac{e^{-dx-c}d^3}{4a(b^2d^2x^2+2ab\,d^2x+d^2a^2)}-\frac{3\,e^{-dx-c}x\,d^2b^2}{2a^3(b^2d^2x^2+2ab\,d^2x+d^2a^2)}-\frac{9\,e^{-dx-c}d^2b}{4a^2(b^2d^2x^2+2ab\,d^2x+d^2a^2)}-$

input
```
int(cosh(d*x+c)/x^2/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(-d*x-c)/a^2*x*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+1/4*exp(-d*x
-c)/a*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-3/2*exp(-d*x-c)/a^3*x*d^2/(b^2
*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^2-9/4*exp(-d*x-c)/a^2*d^2/(b^2*d^2*x^2+2*a
*b*d^2*x+a^2*d^2)*b-1/2*exp(-d*x-c)/a/x*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d
^2)+1/2*d/a^3*exp(-c)*Ei(1,d*x)+3/2/a^4*exp(-c)*Ei(1,d*x)*b-1/4/b/a^2*d^2*
exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)+d/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c
+(a*d-b*c)/b)-3/2*b/a^4*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-1/4/a^2*d
^2/b*exp(d*x+c)/(d/b*a+d*x)^2-1/4/a^2*d^2/b*exp(d*x+c)/(d/b*a+d*x)-1/4/a^2
*d^2/b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-d/a^3*exp(d*x+c)/(d/b*a+
d*x)-d/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+3/2/a^4*b*exp(c)*Ei(
1,-d*x)-3/2*b/a^4*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)-1/2/a^3/x*exp
(d*x+c)-1/2*d/a^3*exp(c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 762 vs. $2(296) = 592$.

Time = 0.13 (sec) , antiderivative size = 762, normalized size of antiderivative = 2.56

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(6*a*b^3*x^2 + 9*a^2*b^2*x + 2*a^3*b)*cosh(d*x + c) - 2*(((a*b^3*d
 - 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x^2 + (a^3*b*d - 3*a^2*b^2)*x)*Ei(
d*x) - ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^2*d + 3*a*b^3)*x^2 + (a^3*b*d + 3
*a^2*b^2)*x)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d^2 + 4*a*b^3*d + 6*b^4)*x^3 +
 2*(a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 + 4*a^3*b*d + 6*a^2*
b^2)*x)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*d^2 - 4*a*b^3*d + 6*b^4)*x^3 + 2*(
a^3*b*d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 + (a^4*d^2 - 4*a^3*b*d + 6*a^2*b^2)
*x)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*c - a*d)/b) + 2*(a^2*b^2*d*x^2 + a^3*b*
d*x)*sinh(d*x + c) - 2*(((a*b^3*d - 3*b^4)*x^3 + 2*(a^2*b^2*d - 3*a*b^3)*x
^2 + (a^3*b*d - 3*a^2*b^2)*x)*Ei(d*x) + ((a*b^3*d + 3*b^4)*x^3 + 2*(a^2*b^
2*d + 3*a*b^3)*x^2 + (a^3*b*d + 3*a^2*b^2)*x)*Ei(-d*x))*sinh(c) + (((a^2*b
^2*d^2 + 4*a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*d^2 + 4*a^2*b^2*d + 6*a*b^3)*x^
2 + (a^4*d^2 + 4*a^3*b*d + 6*a^2*b^2)*x)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d
^2 - 4*a*b^3*d + 6*b^4)*x^3 + 2*(a^3*b*d^2 - 4*a^2*b^2*d + 6*a*b^3)*x^2 +
(a^4*d^2 - 4*a^3*b*d + 6*a^2*b^2)*x)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*
d)/b))/(a^4*b^3*x^3 + 2*a^5*b^2*x^2 + a^6*b*x)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x**2/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx+a)^3x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x + a)^3*x^2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1006 vs. $2(296) = 592$.

Time = 0.12 (sec) , antiderivative size = 1006, normalized size of antiderivative = 3.38

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(a^2*b^2*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + a^2*b^2*d^2*x^3*E
i(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 2*a*b^3*d*x^3*Ei(-d*x)*e^(-c) + 2*a^3
*b*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a*b^3*d*x^3*Ei((b*d*x + a
*d)/b)*e^(c - a*d/b) + 2*a*b^3*d*x^3*Ei(d*x)*e^c + 2*a^3*b*d^2*x^2*Ei(-(b*
d*x + a*d)/b)*e^(-c + a*d/b) - 4*a*b^3*d*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c +
a*d/b) - 4*a^2*b^2*d*x^2*Ei(-d*x)*e^(-c) - 6*b^4*x^3*Ei(-d*x)*e^(-c) + a^4
*d^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 8*a^2*b^2*d*x^2*Ei((b*d*x + a*d
)/b)*e^(c - a*d/b) + 6*b^4*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 4*a^2*b
^2*d*x^2*Ei(d*x)*e^c - 6*b^4*x^3*Ei(d*x)*e^c + a^4*d^2*x*Ei(-(b*d*x + a*d)
/b)*e^(-c + a*d/b) - 8*a^2*b^2*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) +
 6*b^4*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^2*b^2*d*x^2*e^(d*x + c)
 + a^2*b^2*d*x^2*e^(-d*x - c) - 2*a^3*b*d*x*Ei(-d*x)*e^(-c) - 12*a*b^3*x^2
*Ei(-d*x)*e^(-c) + 4*a^3*b*d*x*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 12*a*b^
3*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d*x*Ei(d*x)*e^c - 12*a*b
^3*x^2*Ei(d*x)*e^c - 4*a^3*b*d*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*
a*b^3*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x*e^(d*x + c) - 6*
a*b^3*x^2*e^(d*x + c) + a^3*b*d*x*e^(-d*x - c) - 6*a*b^3*x^2*e^(-d*x - c)
- 6*a^2*b^2*x*Ei(-d*x)*e^(-c) + 6*a^2*b^2*x*Ei((b*d*x + a*d)/b)*e^(c - a*d
/b) - 6*a^2*b^2*x*Ei(d*x)*e^c + 6*a^2*b^2*x*Ei(-(b*d*x + a*d)/b)*e^(-c + a
*d/b) - 9*a^2*b^2*x*e^(d*x + c) - 9*a^2*b^2*x*e^(-d*x - c) - 2*a^3*b*e^...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,{\left(a+b\,x\right)}^3}\,dx$$

input
```
int(cosh(c + d*x)/(x^2*(a + b*x)^3),x)
```

output
```
int(cosh(c + d*x)/(x^2*(a + b*x)^3), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)/x^2/(b*x+a)^3,x)
```

output
```
( - e**(2*c + 2*d*x) + e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3),x)*a**2*b*d*x + 2*e**(2*c + d*x)*int(e**(d*x)/(a*
*3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a*b**2*d*x**2 + e**(2*c +
d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*b**3*
d*x**3 + e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x*
*3 + b**3*x**4),x)*a**3*d*x + 2*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a*
*2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*a**2*b*d*x**2 - 3*e**(2*c + d*x)
*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*a**2
*b*x + e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3
 + b**3*x**4),x)*a*b**2*d*x**3 - 6*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3
*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*a*b**2*x**2 - 3*e**(2*c + d*x
)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*b**
3*x**3 - e**(d*x)*int(1/(e**(d*x)*a**3*x + 3*e**(d*x)*a**2*b*x**2 + 3*e**(
d*x)*a*b**2*x**3 + e**(d*x)*b**3*x**4),x)*a**3*d*x - 2*e**(d*x)*int(1/(e**
(d*x)*a**3*x + 3*e**(d*x)*a**2*b*x**2 + 3*e**(d*x)*a*b**2*x**3 + e**(d*x)*
b**3*x**4),x)*a**2*b*d*x**2 - 3*e**(d*x)*int(1/(e**(d*x)*a**3*x + 3*e**(d*
x)*a**2*b*x**2 + 3*e**(d*x)*a*b**2*x**3 + e**(d*x)*b**3*x**4),x)*a**2*b*x
- e**(d*x)*int(1/(e**(d*x)*a**3*x + 3*e**(d*x)*a**2*b*x**2 + 3*e**(d*x)*a*
b**2*x**3 + e**(d*x)*b**3*x**4),x)*a*b**2*d*x**3 - 6*e**(d*x)*int(1/(e**(d
*x)*a**3*x + 3*e**(d*x)*a**2*b*x**2 + 3*e**(d*x)*a*b**2*x**3 + e**(d*x)...
```

3.39 $\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 377

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx =& -\frac{\cosh(c+dx)}{2a^3x^2}+\frac{3b\cosh(c+dx)}{a^4x}+\frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2}\\&+\frac{3b^2\cosh(c+dx)}{a^4(a+bx)}+\frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5}\\&+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3}-\frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{a^5}\\&-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(\frac{ad}{b}+dx\right)}{2a^3}-\frac{3bd\text{Chi}(dx)\sinh(c)}{a^4}\\&-\frac{3bd\text{Chi}\left(\frac{ad}{b}+dx\right)\sinh\left(c-\frac{ad}{b}\right)}{a^4}-\frac{d\sinh(c+dx)}{2a^3x}\\&+\frac{bd\sinh(c+dx)}{2a^3(a+bx)}-\frac{3bd\cosh(c)\text{Shi}(dx)}{a^4}+\frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5}\\&+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3}-\frac{3bd\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^4}\\&-\frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{a^5}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(\frac{ad}{b}+dx\right)}{2a^3}\end{aligned}$$

output

```
-1/2*cosh(d*x+c)/a^3/x^2+3*b*cosh(d*x+c)/a^4/x+1/2*b^2*cosh(d*x+c)/a^3/(b*
x+a)^2+3*b^2*cosh(d*x+c)/a^4/(b*x+a)+6*b^2*cosh(c)*Chi(d*x)/a^5+1/2*d^2*co
sh(c)*Chi(d*x)/a^3-6*b^2*cosh(-c+a*d/b)*Chi(a*d/b+d*x)/a^5-1/2*d^2*cosh(-c
+a*d/b)*Chi(a*d/b+d*x)/a^3-3*b*d*Chi(d*x)*sinh(c)/a^4+3*b*d*Chi(a*d/b+d*x)
*sinh(-c+a*d/b)/a^4-1/2*d*sinh(d*x+c)/a^3/x+1/2*b*d*sinh(d*x+c)/a^3/(b*x+a
)-3*b*d*cosh(c)*Shi(d*x)/a^4+6*b^2*sinh(c)*Shi(d*x)/a^5+1/2*d^2*sinh(c)*Sh
i(d*x)/a^3-3*b*d*cosh(-c+a*d/b)*Shi(a*d/b+d*x)/a^4+6*b^2*sinh(-c+a*d/b)*Sh
i(a*d/b+d*x)/a^5+1/2*d^2*sinh(-c+a*d/b)*Shi(a*d/b+d*x)/a^3
```

Mathematica [A] (verified)

Time = 1.04 (sec) , antiderivative size = 627, normalized size of antiderivative = 1.66

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3} dx =$$
$$-\frac{a^4\cosh(c+dx) - 4a^3bx\cosh(c+dx) - 18a^2b^2x^2\cosh(c+dx) - 12ab^3x^3\cosh(c+dx) - x^2(a+bx)^2\cdots}{}$$

input

```
Integrate[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```

output

```
-1/2*(a^4*Cosh[c + d*x] - 4*a^3*b*x*Cosh[c + d*x] - 18*a^2*b^2*x^2*Cosh[c
+ d*x] - 12*a*b^3*x^3*Cosh[c + d*x] - x^2*(a + b*x)^2*CoshIntegral[d*x]*((
12*b^2 + a^2*d^2)*Cosh[c] - 6*a*b*d*Sinh[c]) + x^2*(a + b*x)^2*CoshIntegra
l[d*(a/b + x)]*((12*b^2 + a^2*d^2)*Cosh[c - (a*d)/b] + 6*a*b*d*Sinh[c - (a
*d)/b]) + a^4*d*x*Sinh[c + d*x] + a^3*b*d*x^2*Sinh[c + d*x] + 6*a^3*b*d*x^
2*Cosh[c]*SinhIntegral[d*x] + 12*a^2*b^2*d*x^3*Cosh[c]*SinhIntegral[d*x] +
 6*a*b^3*d*x^4*Cosh[c]*SinhIntegral[d*x] - 12*a^2*b^2*x^2*Sinh[c]*SinhInte
gral[d*x] - a^4*d^2*x^2*Sinh[c]*SinhIntegral[d*x] - 24*a*b^3*x^3*Sinh[c]*S
inhIntegral[d*x] - 2*a^3*b*d^2*x^3*Sinh[c]*SinhIntegral[d*x] - 12*b^4*x^4*
Sinh[c]*SinhIntegral[d*x] - a^2*b^2*d^2*x^4*Sinh[c]*SinhIntegral[d*x] + 6*
a^3*b*d*x^2*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*a^2*b^2*d*x^3
*Cosh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 6*a*b^3*d*x^4*Cosh[c - (a*d
)/b]*SinhIntegral[d*(a/b + x)] + 12*a^2*b^2*x^2*Sinh[c - (a*d)/b]*SinhInte
gral[d*(a/b + x)] + a^4*d^2*x^2*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)
] + 24*a*b^3*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 2*a^3*b*d^2
*x^3*Sinh[c - (a*d)/b]*SinhIntegral[d*(a/b + x)] + 12*b^4*x^4*Sinh[c - (a*
d)/b]*SinhIntegral[d*(a/b + x)] + a^2*b^2*d^2*x^4*Sinh[c - (a*d)/b]*SinhIn
tegral[d*(a/b + x)])/(a^5*x^2*(a + b*x)^2)
```

Rubi [A] (verified)

Time = 1.18 (sec) , antiderivative size = 377, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(-\frac{6b^3\cosh(c+dx)}{a^5(a+bx)}+\frac{6b^2\cosh(c+dx)}{a^5x}-\frac{3b^3\cosh(c+dx)}{a^4(a+bx)^2}-\frac{3b\cosh(c+dx)}{a^4x^2}-\frac{b^3\cosh(c+dx)}{a^3(a+bx)^3}+\frac{\cosh(c+\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{6b^2\cosh(c)\text{Chi}(dx)}{a^5}-\frac{6b^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{6b^2\sinh(c)\text{Shi}(dx)}{a^5}-\frac{6b^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^5}+\frac{3b^2\cosh(c+dx)}{a^4(a+bx)}-\frac{3bd\sinh(c)\text{Chi}(dx)}{a^4}-\frac{3bd\sinh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{a^4}-\frac{3bd\cosh(c)\text{Shi}(dx)}{a^4}-\frac{3bd\cosh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{a^4}+\frac{3b\cosh(c+dx)}{a^4x}+\frac{b^2\cosh(c+dx)}{2a^3(a+bx)^2}-\frac{d^2\cosh\left(c-\frac{ad}{b}\right)\text{Chi}\left(xd+\frac{ad}{b}\right)}{2a^3}-\frac{d^2\sinh\left(c-\frac{ad}{b}\right)\text{Shi}\left(xd+\frac{ad}{b}\right)}{2a^3}+\frac{bd\sinh(c+dx)}{2a^3(a+bx)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3}-\frac{\cosh(c+dx)}{2a^3x^2}-\frac{d\sinh(c+dx)}{2a^3x}$$

input
```
Int[Cosh[c + d*x]/(x^3*(a + b*x)^3),x]
```

output
```
-1/2*Cosh[c + d*x]/(a^3*x^2) + (3*b*Cosh[c + d*x])/(a^4*x) + (b^2*Cosh[c +
 d*x])/(2*a^3*(a + b*x)^2) + (3*b^2*Cosh[c + d*x])/(a^4*(a + b*x)) + (6*b^
2*Cosh[c]*CoshIntegral[d*x])/a^5 + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3)
 - (6*b^2*Cosh[c - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/a^5 - (d^2*Cosh[c
 - (a*d)/b]*CoshIntegral[(a*d)/b + d*x])/(2*a^3) - (3*b*d*CoshIntegral[d*x
]*Sinh[c])/a^4 - (3*b*d*CoshIntegral[(a*d)/b + d*x]*Sinh[c - (a*d)/b])/a^4
 - (d*Sinh[c + d*x])/(2*a^3*x) + (b*d*Sinh[c + d*x])/(2*a^3*(a + b*x)) - (
3*b*d*Cosh[c]*SinhIntegral[d*x])/a^4 + (6*b^2*Sinh[c]*SinhIntegral[d*x])/a
^5 + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a^3) - (3*b*d*Cosh[c - (a*d)/b]*Si
nhIntegral[(a*d)/b + d*x])/a^4 - (6*b^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*
d)/b + d*x])/a^5 - (d^2*Sinh[c - (a*d)/b]*SinhIntegral[(a*d)/b + d*x])/(2*
a^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(367) = 734$.

Time = 0.81 (sec) , antiderivative size = 760, normalized size of antiderivative = 2.02

method	result
risch	$\frac{e^{-dx-c}d^3b}{4a^2(b^2d^2x^2+2ab\,d^2x+d^2a^2)}+\frac{3d^2e^{-dx-c}x\,b^3}{a^4(b^2d^2x^2+2ab\,d^2x+d^2a^2)}+\frac{d^3e^{-dx-c}}{4ax(b^2d^2x^2+2ab\,d^2x+d^2a^2)}+\frac{9\,e^{-dx-c}d^2b^2}{2a^3(b^2d^2x^2+2ab\,d^2x+d^2a^2)}+$

input
```
int(cosh(d*x+c)/x^3/(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(-d*x-c)/a^2*d^3/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b+3*d^2*exp(-d*x
-c)/a^4*x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)*b^3+1/4*d^3*exp(-d*x-c)/a/x/(b
^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)+9/2*exp(-d*x-c)/a^3*d^2/(b^2*d^2*x^2+2*a*b
*d^2*x+a^2*d^2)*b^2+d^2*exp(-d*x-c)/a^2/x/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2
)*b-1/4*exp(-d*x-c)/a/x^2*d^2/(b^2*d^2*x^2+2*a*b*d^2*x+a^2*d^2)-1/4*d^2/a^
3*exp(-c)*Ei(1,d*x)-3/2*d/a^4*exp(-c)*Ei(1,d*x)*b-3/a^5*exp(-c)*Ei(1,d*x)*
b^2+1/4*d^2/a^3*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)-3/2*d/a^4*exp((a*
d-b*c)/b)*Ei(1,d*x+c+(a*d-b*c)/b)*b+3/a^5*exp((a*d-b*c)/b)*Ei(1,d*x+c+(a*d
-b*c)/b)*b^2+1/4*d^2/a^3*exp(d*x+c)/(d/b*a+d*x)^2+1/4*d^2/a^3*exp(d*x+c)/(
d/b*a+d*x)+1/4*d^2/a^3*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-b*c)/b)+3/2*d/a^
4*b*exp(d*x+c)/(d/b*a+d*x)+3/2*d/a^4*b*exp(-(a*d-b*c)/b)*Ei(1,-d*x-c-(a*d-
b*c)/b)-3/a^5*b^2*exp(c)*Ei(1,-d*x)-1/4/a^3/x^2*exp(d*x+c)-1/4/a^3/x*d*exp
(d*x+c)-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)+3*b^2/a^5*exp(-(a*d-b*c)/b)*Ei(1,-d*
x-c-(a*d-b*c)/b)+3/2/a^4*b/x*exp(d*x+c)+3/2*d/a^4*b*exp(c)*Ei(1,-d*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. 2(367) = 734.

Time = 0.11 (sec) , antiderivative size = 892, normalized size of antiderivative = 2.37

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(12*a*b^3*x^3 + 18*a^2*b^2*x^2 + 4*a^3*b*x - a^4)*cosh(d*x + c) + (
((a^2*b^2*d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*
a*b^3)*x^3 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) + ((a^2*b^2*d
^2 + 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3
+ (a^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-d*x))*cosh(c) - (((a^2*b^2*d
^2 + 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3
+ (a^4*d^2 + 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei((b*d*x + a*d)/b) + ((a^2*b^2*
d^2 - 6*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3
 + (a^4*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(-(b*d*x + a*d)/b))*cosh(-(b*
c - a*d)/b) - 2*(a^3*b*d*x^2 + a^4*d*x)*sinh(d*x + c) + (((a^2*b^2*d^2 - 6
*a*b^3*d + 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4
*d^2 - 6*a^3*b*d + 12*a^2*b^2)*x^2)*Ei(d*x) - ((a^2*b^2*d^2 + 6*a*b^3*d +
12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^
3*b*d + 12*a^2*b^2)*x^2)*Ei(-d*x))*sinh(c) + (((a^2*b^2*d^2 + 6*a*b^3*d +
12*b^4)*x^4 + 2*(a^3*b*d^2 + 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 + 6*a^
3*b*d + 12*a^2*b^2)*x^2)*Ei((b*d*x + a*d)/b) - ((a^2*b^2*d^2 - 6*a*b^3*d +
 12*b^4)*x^4 + 2*(a^3*b*d^2 - 6*a^2*b^2*d + 12*a*b^3)*x^3 + (a^4*d^2 - 6*a
^3*b*d + 12*a^2*b^2)*x^2)*Ei(-(b*d*x + a*d)/b))*sinh(-(b*c - a*d)/b))/(a^5
*b^2*x^4 + 2*a^6*b*x^3 + a^7*x^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x**3/(b*x+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx+a)^3x^3}\,dx$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x + a)^3*x^3), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1169 vs. 2(367) = 734.

Time = 0.12 (sec) , antiderivative size = 1169, normalized size of antiderivative = 3.10

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x+a)^3,x, algorithm="giac")
```

output
```
1/4*(a^2*b^2*d^2*x^4*Ei(-d*x)*e^(-c) - a^2*b^2*d^2*x^4*Ei((b*d*x + a*d)/b)
*e^(c - a*d/b) + a^2*b^2*d^2*x^4*Ei(d*x)*e^c - a^2*b^2*d^2*x^4*Ei(-(b*d*x
+ a*d)/b)*e^(-c + a*d/b) + 2*a^3*b*d^2*x^3*Ei(-d*x)*e^(-c) + 6*a*b^3*d*x^4
*Ei(-d*x)*e^(-c) - 2*a^3*b*d^2*x^3*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a
*b^3*d*x^4*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) + 2*a^3*b*d^2*x^3*Ei(d*x)*e^c
 - 6*a*b^3*d*x^4*Ei(d*x)*e^c - 2*a^3*b*d^2*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c
+ a*d/b) + 6*a*b^3*d*x^4*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + a^4*d^2*x^2
*Ei(-d*x)*e^(-c) + 12*a^2*b^2*d*x^3*Ei(-d*x)*e^(-c) + 12*b^4*x^4*Ei(-d*x)*
e^(-c) - a^4*d^2*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*a^2*b^2*d*x^3*
Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 12*b^4*x^4*Ei((b*d*x + a*d)/b)*e^(c -
a*d/b) + a^4*d^2*x^2*Ei(d*x)*e^c - 12*a^2*b^2*d*x^3*Ei(d*x)*e^c + 12*b^4*x
^4*Ei(d*x)*e^c - a^4*d^2*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) + 12*a^2*
b^2*d*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 12*b^4*x^4*Ei(-(b*d*x + a*
d)/b)*e^(-c + a*d/b) + 6*a^3*b*d*x^2*Ei(-d*x)*e^(-c) + 24*a*b^3*x^3*Ei(-d*
x)*e^(-c) - 6*a^3*b*d*x^2*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 24*a*b^3*x^3
*Ei((b*d*x + a*d)/b)*e^(c - a*d/b) - 6*a^3*b*d*x^2*Ei(d*x)*e^c + 24*a*b^3*
x^3*Ei(d*x)*e^c + 6*a^3*b*d*x^2*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - 24*a
*b^3*x^3*Ei(-(b*d*x + a*d)/b)*e^(-c + a*d/b) - a^3*b*d*x^2*e^(d*x + c) + 1
2*a*b^3*x^3*e^(d*x + c) + a^3*b*d*x^2*e^(-d*x - c) + 12*a*b^3*x^3*e^(-d*x
- c) + 12*a^2*b^2*x^2*Ei(-d*x)*e^(-c) - 12*a^2*b^2*x^2*Ei((b*d*x + a*d)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\,{(a+b\,x)}^3}\,dx$$

input
```
int(cosh(c + d*x)/(x^3*(a + b*x)^3),x)
```

output
```
int(cosh(c + d*x)/(x^3*(a + b*x)^3), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx)^3}\,dx = \text{too large to display}$$

input
```
int(cosh(d*x+c)/x^3/(b*x+a)^3,x)
```

output
```
( - e**(2*c + 2*d*x)*a*d*x - e**(2*c + 2*d*x)*a + 4*e**(2*c + 2*d*x)*b*x +
 e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3),x)*a**3*b*d**2*x**2 + 2*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**2*d**2*x**3 - 4*e**(2*c + d*x)*int
(e**(d*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a**2*b**2*d*x
**2 + e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3),x)*a*b**3*d**2*x**4 - 8*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2
*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*a*b**3*d*x**3 - 4*e**(2*c + d*x)*int(
e**(d*x)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),x)*b**4*d*x**4 +
e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3
*x**4),x)*a**4*d**2*x**2 + 2*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*
b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*a**3*b*d**2*x**3 - 6*e**(2*c + d*x)
*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*a**3
*b*d*x**2 + e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2
*x**3 + b**3*x**4),x)*a**2*b**2*d**2*x**4 - 12*e**(2*c + d*x)*int(e**(d*x)
/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*a**2*b**2*d*x**3
+ 12*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 +
 b**3*x**4),x)*a**2*b**2*x**2 - 6*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*
a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x)*a*b**3*d*x**4 + 24*e**(2*c + d
*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**2 + 3*a*b**2*x**3 + b**3*x**4),x...
```

3.40 $\int x^3(a+bx^2)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 139

$$\begin{aligned}\int x^3(a+bx^2)\cosh(c+dx)\,dx = &-\frac{120b\cosh(c+dx)}{d^6}-\frac{6a\cosh(c+dx)}{d^4}\\&-\frac{60bx^2\cosh(c+dx)}{d^4}-\frac{3ax^2\cosh(c+dx)}{d^2}\\&-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{120bx\sinh(c+dx)}{d^5}\\&+\frac{6ax\sinh(c+dx)}{d^3}+\frac{20bx^3\sinh(c+dx)}{d^3}\\&+\frac{ax^3\sinh(c+dx)}{d}+\frac{bx^5\sinh(c+dx)}{d}\end{aligned}$$

output
```
-120*b*cosh(d*x+c)/d^6-6*a*cosh(d*x+c)/d^4-60*b*x^2*cosh(d*x+c)/d^4-3*a*x^
2*cosh(d*x+c)/d^2-5*b*x^4*cosh(d*x+c)/d^2+120*b*x*sinh(d*x+c)/d^5+6*a*x*si
nh(d*x+c)/d^3+20*b*x^3*sinh(d*x+c)/d^3+a*x^3*sinh(d*x+c)/d+b*x^5*sinh(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.66

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx = \frac{-((3ad^2(2+d^2x^2)+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx))+dx(ad^2(6+d^2x^2)+b(120+20d^2x^2+d^4}{d^6}$$

input
```
Integrate[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-((3*a*d^2*(2 + d^2*x^2) + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]
) + d*x*(a*d^2*(6 + d^2*x^2) + b*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*
x])/d^6
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int (ax^3\cosh(c+dx)+bx^5\cosh(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}-\frac{120b\cosh(c+dx)}{d^6}+\frac{120bx\sinh(c+dx)}{d^5}-\frac{60bx^2\cosh(c+dx)}{d^4}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{bx^5\sinh(c+dx)}{d}$$

input
```
Int[x^3*(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-120*b*Cosh[c + d*x])/d^6 - (6*a*Cosh[c + d*x])/d^4 - (60*b*x^2*Cosh[c +
d*x])/d^4 - (3*a*x^2*Cosh[c + d*x])/d^2 - (5*b*x^4*Cosh[c + d*x])/d^2 + (1
20*b*x*Sinh[c + d*x])/d^5 + (6*a*x*Sinh[c + d*x])/d^3 + (20*b*x^3*Sinh[c +
 d*x])/d^3 + (a*x^3*Sinh[c + d*x])/d + (b*x^5*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{3\left(\left(\frac{5b\,x^2}{3}+a\right)d^2+20b\right)d^2x^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\left(x^2\left(b\,x^2+a\right)d^4+2\left(10b\,x^2+3a\right)d^2+120b\right)dx\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(5b\,x^4+3a\,x\right.}{d^6\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,x^5d^5+a\,d^5x^3-5b\,x^4d^4-3a\,d^4x^2+20b\,d^3x^3+6a\,d^3x-60b\,d^2x^2-6a\,d^2+120dxb-120b\right)\mathrm{e}^{dx+c}}{2d^6}-\frac{\left(b\,x^5d^5+a\,d^5x^3+5b\right.}{}$
orering	$-\frac{2\left(5b^2d^4x^6+8ab\,d^4x^4+3a^2d^4x^2+80b^2d^2x^4+78ab\,d^2x^2+12d^2a^2+360x^2b^2+240ab\right)\cosh(dx+c)}{d^6\left(b\,x^2+a\right)}+\frac{\left(b\,x^4d^4+a\,d^4x^2\right.}{}$
meijerg	$-\frac{32b\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}x^2d^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}x^2d^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib\sinh(c)\sqrt{\pi}\left(-\right.}{}$
parts	$\frac{b\,x^5\sinh(dx+c)}{d}+\frac{a\,x^3\sinh(dx+c)}{d}-\frac{\frac{5b\,c^4\cosh(dx+c)}{d^4}-\frac{20b\,c^3((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^4}+\frac{30b\,c^2\left((dx+c)^2\cosh(d\right.}{}}{}$
derivativedivides	$\frac{-\frac{b\,c^5\sinh(dx+c)}{d^2}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+\frac{10}{}}{}$
default	$\frac{-\frac{b\,c^5\sinh(dx+c)}{d^2}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+\frac{10}{}}{}$

input
```
int(x^3*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(3*((5/3*b*x^2+a)*d^2+20*b)*d^2*x^2*tanh(1/2*d*x+1/2*c)^2-2*(x^2*(b*x^2+a)
*d^4+2*(10*b*x^2+3*a)*d^2+120*b)*d*x*tanh(1/2*d*x+1/2*c)+(5*b*x^4+3*a*x^2)
*d^4+12*(5*b*x^2+a)*d^2+240*b)/d^6/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.68

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx = -\frac{(5\,bd^4x^4+6\,ad^2+3\,(ad^4+20\,bd^2)x^2+120\,b)\cosh(dx+c)-(bd^5x^5+(ad^5+20\,bd^3)x^3+6\,(ad^3+20\,bd)x)\sinh(dx+c)}{d^6}$$

input
```
integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((5*b*d^4*x^4 + 6*a*d^2 + 3*(a*d^4 + 20*b*d^2)*x^2 + 120*b)*cosh(d*x + c)
 - (b*d^5*x^5 + (a*d^5 + 20*b*d^3)*x^3 + 6*(a*d^3 + 20*b*d)*x)*sinh(d*x +
c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.21

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^3\sinh(c+dx)}{d} - \frac{3ax^2\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} - \frac{6a\cosh(c+dx)}{d^4} + \frac{bx^5\sinh(c+dx)}{d} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{20bx^3\sinh(c+dx)}{d^3} \\ \left(\frac{ax^4}{4}+\frac{bx^6}{6}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**3*(b*x**2+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*si
nh(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**5*sinh(c + d*x)/d - 5*b*x
**4*cosh(c + d*x)/d**2 + 20*b*x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c +
 d*x)/d**4 + 120*b*x*sinh(c + d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d,
0)), ((a*x**4/4 + b*x**6/6)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.80

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx = -\frac{1}{24}\,d\left(\frac{3\,(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)ae^{(dx)}}{d^5}+\frac{3\,(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+\right.$$
$$+\frac{1}{12}\left(2\,bx^6+3\,ax^4\right)\cosh\left(dx+c\right)$$

input
```
integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/24*d*(3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24
*e^c)*a*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a
*e^(-d*x - c)/d^5 + 2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*
d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b*e^(d*x)/d^7 + 2*(
d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 7
20)*b*e^(-d*x - c)/d^7) + 1/12*(2*b*x^6 + 3*a*x^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.25

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx = \frac{(bd^5x^5+ad^5x^3-5\,bd^4x^4-3\,ad^4x^2+20\,bd^3x^3+6\,ad^3x-60\,bd^2x^2-6\,ad^2+120\,bdx-120\,b)e^{(dx+c)}}{2\,d^6} - \frac{(bd^5x^5+ad^5x^3+5\,bd^4x^4+3\,ad^4x^2+20\,bd^3x^3+6\,ad^3x+60\,bd^2x^2+6\,ad^2+120\,bdx+120\,b)e^{(-dx-c)}}{2\,d^6}$$

input
```
integrate(x^3*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^5*x^5 + a*d^5*x^3 - 5*b*d^4*x^4 - 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*
a*d^3*x - 60*b*d^2*x^2 - 6*a*d^2 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/
2*(b*d^5*x^5 + a*d^5*x^3 + 5*b*d^4*x^4 + 3*a*d^4*x^2 + 20*b*d^3*x^3 + 6*a*
d^3*x + 60*b*d^2*x^2 + 6*a*d^2 + 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.83

$$\int x^3 (a + bx^2) \cosh(c + dx) \, dx = \frac{x^3 \sinh(c + d\,x)\ (a\,d^2 + 20\,b)}{d^3} - \frac{3\,x^2 \cosh(c + d\,x)\ (a\,d^2 + 20\,b)}{d^4} - \frac{6\cosh(c + d\,x)\ (a\,d^2 + 20\,b)}{d^6} + \frac{6\,x \sinh(c + d\,x)\ (a\,d^2 + 20\,b)}{d^5} - \frac{5\,b\,x^4 \cosh(c + d\,x)}{d^2} + \frac{b\,x^5 \sinh(c + d\,x)}{d}$$

input
```
int(x^3*cosh(c + d*x)*(a + b*x^2),x)
```

output
```
(x^3*sinh(c + d*x)*(20*b + a*d^2))/d^3 - (3*x^2*cosh(c + d*x)*(20*b + a*d^
2))/d^4 - (6*cosh(c + d*x)*(20*b + a*d^2))/d^6 + (6*x*sinh(c + d*x)*(20*b
+ a*d^2))/d^5 - (5*b*x^4*cosh(c + d*x))/d^2 + (b*x^5*sinh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.99

$$\int x^3(a+bx^2)\cosh(c+dx)\,dx$$
$$= \frac{-3\cosh(dx+c)\,a\,d^4x^2-6\cosh(dx+c)\,a\,d^2-5\cosh(dx+c)\,b\,d^4x^4-60\cosh(dx+c)\,b\,d^2x^2-120\,\mathrm{co}}{}$$

input
```
int(x^3*(b*x^2+a)*cosh(d*x+c),x)
```

output
```
( - 3*cosh(c + d*x)*a*d**4*x**2 - 6*cosh(c + d*x)*a*d**2 - 5*cosh(c + d*x)
*b*d**4*x**4 - 60*cosh(c + d*x)*b*d**2*x**2 - 120*cosh(c + d*x)*b + sinh(c
 + d*x)*a*d**5*x**3 + 6*sinh(c + d*x)*a*d**3*x + sinh(c + d*x)*b*d**5*x**5
 + 20*sinh(c + d*x)*b*d**3*x**3 + 120*sinh(c + d*x)*b*d*x)/d**6
```

3.41 $\int x^2(a+bx^2)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 109

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = -\frac{24bx\cosh(c+dx)}{d^4}-\frac{2ax\cosh(c+dx)}{d^2}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{24b\sinh(c+dx)}{d^5}+\frac{2a\sinh(c+dx)}{d^3}+\frac{12bx^2\sinh(c+dx)}{d^3}+\frac{ax^2\sinh(c+dx)}{d}+\frac{bx^4\sinh(c+dx)}{d}$$

output
```
-24*b*x*cosh(d*x+c)/d^4-2*a*x*cosh(d*x+c)/d^2-4*b*x^3*cosh(d*x+c)/d^2+24*b
*sinh(d*x+c)/d^5+2*a*sinh(d*x+c)/d^3+12*b*x^2*sinh(d*x+c)/d^3+a*x^2*sinh(d
*x+c)/d+b*x^4*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.68

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx$$
$$=\frac{-2dx(ad^2+2b(6+d^2x^2))\cosh(c+dx)+(ad^2(2+d^2x^2)+b(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

input
```
Integrate[x^2*(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-2*d*x*(a*d^2 + 2*b*(6 + d^2*x^2))*Cosh[c + d*x] + (a*d^2*(2 + d^2*x^2) +
 b*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{5810}$$
$$\int \left(ax^2\cosh(c+dx)+bx^4\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-24*b*x*Cosh[c + d*x])/d^4 - (2*a*x*Cosh[c + d*x])/d^2 - (4*b*x^3*Cosh[c
+ d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (2*a*Sinh[c + d*x])/d^3 + (12*b*x
^2*Sinh[c + d*x])/d^3 + (a*x^2*Sinh[c + d*x])/d + (b*x^4*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.64

method	result
parallelrisch	$\frac{\left(x^{2}\left(b\,x^{2}+a\right)d^{4}+2\left(6b\,x^{2}+a\right)d^{2}+24b\right)\sinh\left(dx+c\right)-2\left(\left(2b\,x^{2}+a\right)d^{2}+12b\right)d\cosh\left(dx+c\right)x}{d^{5}}$
risch	$\frac{\left(b\,x^{4}d^{4}+a\,d^{4}x^{2}-4b\,d^{3}x^{3}-2a\,d^{3}x+12b\,d^{2}x^{2}+2a\,d^{2}-24dxb+24b\right)\mathrm{e}^{dx+c}}{2d^{5}}-$ $(b\,x^{4}d^{4}+a\,d^{4}x^{2}+4b\,d^{3}x^{3}+2a\,d^{3}x+12b\,d^{2}$ … / $2d^{5}$
orering	$-\frac{4\left(2b^{2}d^{4}x^{6}+3ab\,d^{4}x^{4}+a^{2}d^{4}x^{2}+18b^{2}d^{2}x^{4}+14ab\,d^{2}x^{2}+d^{2}a^{2}+24x^{2}b^{2}+12ab\right)\cosh\left(dx+c\right)}{d^{6}x\left(b\,x^{2}+a\right)}+$ $(b\,x^{4}d^{4}+a\,d^{4}x^{2}+12b$ …
parts	$\frac{b\,x^{4}\sinh\left(dx+c\right)}{d}+\frac{a\,x^{2}\sinh\left(dx+c\right)}{d}-$ $2\left(-\frac{2b\,c^{3}\cosh\left(dx+c\right)}{d^{3}}+\frac{6b\,c^{2}\left(\left(dx+c\right)\cosh\left(dx+c\right)-\sinh\left(dx+c\right)\right)}{d^{3}}-\right.$ $6bc\left(\left(dx+c\right)^{2}\cosh\right.$ …
meijerg	$-\frac{16ib\cosh\left(c\right)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^{2}d^{2}}{2}+15\right)\cosh\left(dx\right)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^{4}x^{4}+\frac{15}{2}x^{2}d^{2}+15\right)\sinh\left(dx\right)}{10\sqrt{\pi}}\right)}{d^{5}}-$ $16b\sinh\left(c\right)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\right.$ $\left(\frac{3}{8}d^{4}x^{4}+\right.$ …
derivativedivides	$\frac{b\,c^{4}\sinh\left(dx+c\right)}{d^{2}}-\frac{4b\,c^{3}\left(\left(dx+c\right)\sinh\left(dx+c\right)-\cosh\left(dx+c\right)\right)}{d^{2}}+\frac{6b\,c^{2}\left(\left(dx+c\right)^{2}\sinh\left(dx+c\right)-2\left(dx+c\right)\cosh\left(dx+c\right)+2\sinh\left(dx+c\right)\right)}{d^{2}}-$ $4bc\left(\right.$ …
default	$\frac{b\,c^{4}\sinh\left(dx+c\right)}{d^{2}}-\frac{4b\,c^{3}\left(\left(dx+c\right)\sinh\left(dx+c\right)-\cosh\left(dx+c\right)\right)}{d^{2}}+\frac{6b\,c^{2}\left(\left(dx+c\right)^{2}\sinh\left(dx+c\right)-2\left(dx+c\right)\cosh\left(dx+c\right)+2\sinh\left(dx+c\right)\right)}{d^{2}}-$ $4bc\left(\right.$ …

input
```
int(x^2*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
((x^2*(b*x^2+a)*d^4+2*(6*b*x^2+a)*d^2+24*b)*sinh(d*x+c)-2*((2*b*x^2+a)*d^2
+12*b)*d*cosh(d*x+c)*x)/d^5
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.72

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = -\frac{2\,(2\,bd^3x^3+(ad^3+12\,bd)x)\cosh(dx+c)-(bd^4x^4+2\,ad^2+(ad^4+12\,bd^2)x^2+24\,b)\sinh(dx+c)}{d^5}$$

input
```
integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(2*(2*b*d^3*x^3 + (a*d^3 + 12*b*d)*x)*cosh(d*x + c) - (b*d^4*x^4 + 2*a*d^
2 + (a*d^4 + 12*b*d^2)*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.23

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^2\sinh(c+dx)}{d} - \frac{2ax\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{bx^4\sinh(c+dx)}{d} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{12bx^2\sinh(c+dx)}{d^3} - \frac{24bx\cosh(c+}{d^4} \\ \left(\frac{ax^3}{3}+\frac{bx^5}{5}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c
+ d*x)/d**3 + b*x**4*sinh(c + d*x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*
x**2*sinh(c + d*x)/d**3 - 24*b*x*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d
**5, Ne(d, 0)), ((a*x**3/3 + b*x**5/5)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.96

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = -\frac{1}{30}d\left(\frac{5\,(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)ae^{(dx)}}{d^4}+\frac{5\,(d^3x^3+3\,d^2x^2+6\,dx+6)ae^{(-dx-c)}}{d^4}+\frac{3\,(d^5x^5e^c\right. + \frac{1}{15}\left(3\,bx^5+5\,ax^3\right)\cosh\left(dx+c\right)$$

input
```
integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/30*d*(5*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*e^(d*x)/d^4
 + 5*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*e^(-d*x - c)/d^4 + 3*(d^5*x^5*e^c
 - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c
)*b*e^(d*x)/d^6 + 3*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d
*x + 120)*b*e^(-d*x - c)/d^6) + 1/15*(3*b*x^5 + 5*a*x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.27

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \frac{(bd^4x^4+ad^4x^2-4\,bd^3x^3-2\,ad^3x+12\,bd^2x^2+2\,ad^2-24\,bdx+24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4+ad^4x^2+4\,bd^3x^3+2\,ad^3x+12\,bd^2x^2+2\,ad^2+24\,bdx+24\,b)e^{(-dx-c)}}{2\,d^5}$$

input
```
integrate(x^2*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^4*x^4 + a*d^4*x^2 - 4*b*d^3*x^3 - 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*
d^2 - 24*b*d*x + 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + a*d^4*x^2 + 4*b*
d^3*x^3 + 2*a*d^3*x + 12*b*d^2*x^2 + 2*a*d^2 + 24*b*d*x + 24*b)*e^(-d*x -
c)/d^5
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.85

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \frac{2\sinh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^5} + \frac{x^2\sinh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^3} - \frac{2\,x\cosh(c+d\,x)\,\left(a\,d^2+12\,b\right)}{d^4} - \frac{4\,b\,x^3\cosh(c+d\,x)}{d^2} + \frac{b\,x^4\sinh(c+d\,x)}{d}$$

input
```
int(x^2*cosh(c + d*x)*(a + b*x^2),x)
```

output
```
(2*sinh(c + d*x)*(12*b + a*d^2))/d^5 + (x^2*sinh(c + d*x)*(12*b + a*d^2))/
d^3 - (2*x*cosh(c + d*x)*(12*b + a*d^2))/d^4 - (4*b*x^3*cosh(c + d*x))/d^2
 + (b*x^4*sinh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^2)\cosh(c+dx)\,dx = \frac{-2\cosh(dx+c)\,a\,d^3x - 4\cosh(dx+c)\,b\,d^3x^3 - 24\cosh(dx+c)\,bdx + \sinh(dx+c)\,a\,d^4x^2 + 2\sinh(dx}{d^5}$$

input
```
int(x^2*(b*x^2+a)*cosh(d*x+c),x)
```

output
```
( - 2*cosh(c + d*x)*a*d**3*x - 4*cosh(c + d*x)*b*d**3*x**3 - 24*cosh(c + d
*x)*b*d*x + sinh(c + d*x)*a*d**4*x**2 + 2*sinh(c + d*x)*a*d**2 + sinh(c +
d*x)*b*d**4*x**4 + 12*sinh(c + d*x)*b*d**2*x**2 + 24*sinh(c + d*x)*b)/d**5
```

3.42 $\int x(a+bx^2)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int x(a+bx^2)\cosh(c+dx)\,dx = -\frac{6b\cosh(c+dx)}{d^4}-\frac{a\cosh(c+dx)}{d^2}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}+\frac{ax\sinh(c+dx)}{d}+\frac{bx^3\sinh(c+dx)}{d}$$

output
```
-6*b*cosh(d*x+c)/d^4-a*cosh(d*x+c)/d^2-3*b*x^2*cosh(d*x+c)/d^2+6*b*x*sinh(
d*x+c)/d^3+a*x*sinh(d*x+c)/d+b*x^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.72

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \frac{-((ad^2+3b(2+d^2x^2))\cosh(c+dx))+dx(ad^2+b(6+d^2x^2))\sinh(c+dx)}{d^4}$$

input
```
Integrate[x*(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-((a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x]) + d*x*(a*d^2 + b*(6 + d^2*x^
2))*Sinh[c + d*x])/d^4
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^2\right)\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{ 5810}$$
$$\int \left(ax\cosh(c+dx)+bx^3\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}-\frac{6b\cosh(c+dx)}{d^4}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{bx^3\sinh(c+dx)}{d}$$

input
```
Int[x*(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-6*b*Cosh[c + d*x])/d^4 - (a*Cosh[c + d*x])/d^2 - (3*b*x^2*Cosh[c + d*x])
/d^2 + (6*b*x*Sinh[c + d*x])/d^3 + (a*x*Sinh[c + d*x])/d + (b*x^3*Sinh[c +
 d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.09

method	result
parallelrisch	$\frac{3b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}x^{2}d^{2}-2dx\left(\left(b\,x^{2}+a\right)d^{2}+6b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3b\,x^{2}+2a\right)d^{2}+12b}{d^{4}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b\,d^{3}x^{3}+a\,d^{3}x-3b\,d^{2}x^{2}-a\,d^{2}+6dxb-6b\right){\mathrm e}^{dx+c}}{2d^{4}}-\frac{\left(b\,d^{3}x^{3}+a\,d^{3}x+3b\,d^{2}x^{2}+a\,d^{2}+6dxb+6b\right){\mathrm e}^{-dx-c}}{2d^{4}}$
parts	$\frac{b\,x^{3}\sinh(dx+c)}{d}+\frac{ax\sinh(dx+c)}{d}-\frac{\frac{3b\,c^{2}\cosh(dx+c)}{d^{2}}-\frac{6bc((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^{2}}+\frac{3b\left((dx+c)^{2}\cosh(dx+c)-2(\right.}{d^{2}}}{}$
orering	$-\frac{2\left(3b^{2}d^{2}x^{4}+4ab\,d^{2}x^{2}+d^{2}a^{2}+12x^{2}b^{2}+6ab\right)\cosh(dx+c)}{d^{4}\left(b\,x^{2}+a\right)}+\frac{\left(b\,d^{2}x^{2}+a\,d^{2}+6b\right)\left(\left(b\,x^{2}+a\right)\cosh(dx+c)+2x^{2}b\cosh(d\right.}{d^{4}\left(b\,x^{2}+a\right)}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^{2}d^{2}}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^{2}d^{2}}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^{4}}-\frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^{2}d^{2}}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i(}{}\right.}{d^{4}}$
derivativedivides	$\frac{\frac{3b\,c^{2}((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}-\frac{3bc\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}+\frac{b\left((dx+c)^{3}\sinh(dx+c)-3\right.}{}}{}$
default	$\frac{\frac{3b\,c^{2}((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{2}}-\frac{3bc\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{2}}+\frac{b\left((dx+c)^{3}\sinh(dx+c)-3\right.}{}}{}$

input
```
int(x*(b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(3*b*tanh(1/2*d*x+1/2*c)^2*x^2*d^2-2*d*x*((b*x^2+a)*d^2+6*b)*tanh(1/2*d*x+
1/2*c)+(3*b*x^2+2*a)*d^2+12*b)/d^4/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.76

$$\int x(a+bx^2)\cosh(c+dx)\,dx$$
$$= -\frac{(3\,bd^2x^2+ad^2+6\,b)\cosh(dx+c)-(bd^3x^3+(ad^3+6\,bd)x)\sinh(dx+c)}{d^4}$$

input
```
integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((3*b*d^2*x^2 + a*d^2 + 6*b)*cosh(d*x + c) - (b*d^3*x^3 + (a*d^3 + 6*b*d)
*x)*sinh(d*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.25

$$\int x(a+bx^2)\cosh(c+dx)\,dx$$
$$= \begin{cases} \frac{ax\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d^2}+\frac{bx^3\sinh(c+dx)}{d}-\frac{3bx^2\cosh(c+dx)}{d^2}+\frac{6bx\sinh(c+dx)}{d^3}-\frac{6b\cosh(c+dx)}{d^4} & \text{for } d\neq 0 \\ \left(\frac{ax^2}{2}+\frac{bx^4}{4}\right)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(x*(b*x**2+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**3*sinh(c + d*
x)/d - 3*b*x**2*cosh(c + d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c
 + d*x)/d**4, Ne(d, 0)), ((a*x**2/2 + b*x**4/4)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(79) = 158$.

Time = 0.05 (sec) , antiderivative size = 213, normalized size of antiderivative = 2.70

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \frac{(bx^2+a)^2\cosh(dx+c)}{4b} - \frac{\left(\frac{a^2e^{(dx+c)}}{d} + \frac{a^2e^{(-dx-c)}}{d} + \frac{2\,(d^2x^2e^c-2\,dxe^c+2\,e^c)abe^{(dx)}}{d^3} + \frac{2\,(d^2x^2+2\,dx+2)abe^{(-dx-c)}}{d^3} + \frac{(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,d}{d^5}\right.}{8\,b}$$

input
```
integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/4*(b*x^2 + a)^2*cosh(d*x + c)/b - 1/8*(a^2*e^(d*x + c)/d + a^2*e^(-d*x -
 c)/d + 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*b*e^(d*x)/d^3 + 2*(d^2*x^2 +
 2*d*x + 2)*a*b*e^(-d*x - c)/d^3 + (d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x
^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*
d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*d/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.28

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \frac{(bd^3x^3+ad^3x-3\,bd^2x^2-ad^2+6\,bdx-6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3+ad^3x+3\,bd^2x^2+ad^2+6\,bdx+6\,b)e^{(-dx-c)}}{2\,d^4}$$

input
```
integrate(x*(b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^3*x^3 + a*d^3*x - 3*b*d^2*x^2 - a*d^2 + 6*b*d*x - 6*b)*e^(d*x + c
)/d^4 - 1/2*(b*d^3*x^3 + a*d^3*x + 3*b*d^2*x^2 + a*d^2 + 6*b*d*x + 6*b)*e^
(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.89

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \frac{x\sinh(c+d\,x)\,\left(a\,d^2+6\,b\right)}{d^3} - \frac{\cosh(c+d\,x)\,\left(a\,d^2+6\,b\right)}{d^4} - \frac{3\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{b\,x^3\sinh(c+d\,x)}{d}$$

input
```
int(x*cosh(c + d*x)*(a + b*x^2),x)
```

output
```
(x*sinh(c + d*x)*(6*b + a*d^2))/d^3 - (cosh(c + d*x)*(6*b + a*d^2))/d^4 -
(3*b*x^2*cosh(c + d*x))/d^2 + (b*x^3*sinh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.99

$$\int x(a+bx^2)\cosh(c+dx)\,dx = \frac{-\cosh(dx+c)\,a\,d^2 - 3\cosh(dx+c)\,b\,d^2x^2 - 6\cosh(dx+c)\,b + \sinh(dx+c)\,a\,d^3x + \sinh(dx+c)\,b\,d^3}{d^4}$$

input
```
int(x*(b*x^2+a)*cosh(d*x+c),x)
```

output
```
( - cosh(c + d*x)*a*d**2 - 3*cosh(c + d*x)*b*d**2*x**2 - 6*cosh(c + d*x)*b
 + sinh(c + d*x)*a*d**3*x + sinh(c + d*x)*b*d**3*x**3 + 6*sinh(c + d*x)*b*
d*x)/d**4
```

3.43 $\int (a+bx^2)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 51

$$\int (a+bx^2)\cosh(c+dx)\,dx = -\frac{2bx\cosh(c+dx)}{d^2}+\frac{2b\sinh(c+dx)}{d^3}+\frac{a\sinh(c+dx)}{d}+\frac{bx^2\sinh(c+dx)}{d}$$

output
```
-2*b*x*cosh(d*x+c)/d^2+2*b*sinh(d*x+c)/d^3+a*sinh(d*x+c)/d+b*x^2*sinh(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.78

$$\int (a+bx^2)\cosh(c+dx)\,dx = \frac{-2bdx\cosh(c+dx)+(ad^2+b(2+d^2x^2))\sinh(c+dx)}{d^3}$$

input
```
Integrate[(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-2*b*d*x*Cosh[c + d*x] + (a*d^2 + b*(2 + d^2*x^2))*Sinh[c + d*x])/d^3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {5800, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)\cosh(c+dx)\,dx$$

$$\Big\downarrow \text{5800}$$

$$\int (a\cosh(c+dx)+bx^2\cosh(c+dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\sinh(c+dx)}{d}+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

input
```
Int[(a + b*x^2)*Cosh[c + d*x],x]
```

output
```
(-2*b*x*Cosh[c + d*x])/d^2 + (2*b*Sinh[c + d*x])/d^3 + (a*Sinh[c + d*x])/d
 + (b*x^2*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5800
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

method	result
parallelrisch	$\frac{((b\,x^2+a)d^2+2b)\sinh(dx+c)-2dxb\cosh(dx+c)}{d^3}$
parts	$\frac{b\,x^2\sinh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}-\frac{2b((dx+c)\cosh(dx+c)-\sinh(dx+c)-c\cosh(dx+c))}{d^3}$
risch	$\frac{(b\,d^2x^2+a\,d^2-2dxb+2b)\,\mathrm{e}^{dx+c}}{2d^3}-\frac{(b\,d^2x^2+a\,d^2+2dxb+2b)\,\mathrm{e}^{-dx-c}}{2d^3}$
orering	$-\frac{4bx(b\,d^2x^2+a\,d^2+b)\cosh(dx+c)}{d^4(b\,x^2+a)}+\frac{(b\,d^2x^2+a\,d^2+2b)(2xb\cosh(dx+c)+(b\,x^2+a)d\sinh(dx+c))}{d^4(b\,x^2+a)}$
derivativedivides	$\frac{\frac{b\,c^2\sinh(dx+c)}{d^2}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{b\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+a\sinh(dx}{d}$
default	$\frac{\frac{b\,c^2\sinh(dx+c)}{d^2}-\frac{2bc((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^2}+\frac{b\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^2}+a\sinh(dx}{d}$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^2d^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3}+\frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^2d^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^3}$

input
```
int((b*x^2+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(((b*x^2+a)*d^2+2*b)*sinh(d*x+c)-2*d*x*b*cosh(d*x+c))/d^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.82

$$\int (a+bx^2)\cosh(c+dx)\,dx=-\frac{2\,bdx\cosh(dx+c)-(bd^2x^2+ad^2+2\,b)\sinh(dx+c)}{d^3}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(2*b*d*x*cosh(d*x + c) - (b*d^2*x^2 + a*d^2 + 2*b)*sinh(d*x + c))/d^3
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.27

$$\int (a+bx^2)\cosh(c+dx)\,dx = \begin{cases} \frac{a\sinh{(c+dx)}}{d} + \frac{bx^2\sinh{(c+dx)}}{d} - \frac{2bx\cosh{(c+dx)}}{d^2} + \frac{2b\sinh{(c+dx)}}{d^3} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^3}{3}\right)\cosh{(c)} & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**2+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*sinh(c + d*x)/d + b*x**2*sinh(c + d*x)/d - 2*b*x*cosh(c + d*x
)/d**2 + 2*b*sinh(c + d*x)/d**3, Ne(d, 0)), ((a*x + b*x**3/3)*cosh(c), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.69

$$\int (a+bx^2)\cosh(c+dx)\,dx = \frac{ae^{(dx+c)}}{2\,d} - \frac{ae^{(-dx-c)}}{2\,d} + \frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)be^{(dx)}}{2\,d^3} - \frac{(d^2x^2 + 2\,dx + 2)be^{(-dx-c)}}{2\,d^3}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^2*x^2*e^c - 2*d*x*e^c
+ 2*e^c)*b*e^(d*x)/d^3 - 1/2*(d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.37

$$\int (a+bx^2)\cosh(c+dx)\,dx = \frac{(bd^2x^2+ad^2-2\,bdx+2\,b)e^{(dx+c)}}{2\,d^3} - \frac{(bd^2x^2+ad^2+2\,bdx+2\,b)e^{(-dx-c)}}{2\,d^3}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^2*x^2 + a*d^2 - 2*b*d*x + 2*b)*e^(d*x + c)/d^3 - 1/2*(b*d^2*x^2 +
 a*d^2 + 2*b*d*x + 2*b)*e^(-d*x - c)/d^3
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.92

$$\int (a+bx^2)\cosh(c+dx)\,dx = \frac{\sinh(c+d\,x)\,\left(a\,d^2+2\,b\right)}{d^3} - \frac{2\,b\,x\cosh(c+d\,x)}{d^2} + \frac{b\,x^2\sinh(c+d\,x)}{d}$$

input
```
int(cosh(c + d*x)*(a + b*x^2),x)
```

output
```
(sinh(c + d*x)*(2*b + a*d^2))/d^3 - (2*b*x*cosh(c + d*x))/d^2 + (b*x^2*sin
h(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

$$\int (a + bx^2) \cosh(c + dx) \, dx$$
$$= \frac{-2\cosh (dx + c) bdx + \sinh (dx + c) a d^2 + \sinh (dx + c) b d^2x^2 + 2b \sinh (dx + c)}{d^3}$$

input
```
int((b*x^2+a)*cosh(d*x+c),x)
```

output
```
( - 2*cosh(c + d*x)*b*d*x + sinh(c + d*x)*a*d**2 + sinh(c + d*x)*b*d**2*x*
*2 + 2*sinh(c + d*x)*b)/d**3
```

3.44 $\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 41

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = -\frac{b\cosh(c+dx)}{d^2} + a\cosh(c)\text{Chi}(dx) + \frac{bx\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)$$

output
```
-b*cosh(d*x+c)/d^2+a*cosh(c)*Chi(d*x)+b*x*sinh(d*x+c)/d+a*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx) + \frac{b\cosh(dx)(-\cosh(c)+dx\sinh(c))}{d^2} + \frac{b(dx\cosh(c)-\sinh(c))\sinh(dx)}{d^2} + a\sinh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

output
```
a*Cosh[c]*CoshIntegral[d*x] + (b*Cosh[d*x]*(-Cosh[c] + d*x*Sinh[c]))/d^2 +
 (b*(d*x*Cosh[c] - Sinh[c])*Sinh[d*x])/d^2 + a*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx$$
$$\Big\downarrow \text{5810}$$
$$\int \left(\frac{a\cosh(c+dx)}{x}+bx\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$a\cosh(c)\text{Chi}(dx)+a\sinh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^2)*Cosh[c + d*x])/x,x]
```

output
```
-((b*Cosh[c + d*x])/d^2) + a*Cosh[c]*CoshIntegral[d*x] + (b*x*Sinh[c + d*x
])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{a\,e^{-c}\,\text{expIntegral}_1(dx)}{2}-\frac{a\,e^{c}\,\text{expIntegral}_1(-dx)}{2}-\frac{e^{-dx-c}bx}{2d}+\frac{e^{dx+c}bx}{2d}-\frac{e^{-dx-c}b}{2d^2}-\frac{e^{dx+c}b}{2d^2}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+\frac{a\cosh(c)\sqrt{\pi}\left(\frac{2\ \text{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma}{}\right.}{2}$

input
```
int((b*x^2+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*exp(-c)*Ei(1,d*x)-1/2*a*exp(c)*Ei(1,-d*x)-1/2/d*exp(-d*x-c)*b*x+1/2
/d*exp(d*x+c)*b*x-1/2/d^2*exp(-d*x-c)*b-1/2/d^2*exp(d*x+c)*b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.78

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx$$
$$=\frac{2\,bdx\sinh{(dx+c)}-2\,b\cosh{(dx+c)}+(ad^2\text{Ei}(dx)+ad^2\text{Ei}(-dx))\cosh{(c)}+(ad^2\text{Ei}(dx)-ad^2\text{Ei}(-dx}{2\,d^2}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x*sinh(d*x + c) - 2*b*cosh(d*x + c) + (a*d^2*Ei(d*x) + a*d^2*Ei
(-d*x))*cosh(c) + (a*d^2*Ei(d*x) - a*d^2*Ei(-d*x))*sinh(c))/d^2
```

Sympy [A] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = a\sinh(c)\operatorname{Shi}(dx) + a\cosh(c)\operatorname{Chi}(dx) + b\left(\begin{cases}\frac{x\sinh(c+dx)}{d} - \frac{\cosh(c+dx)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\cosh(c)}{2} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)*cosh(d*x+c)/x,x)
```

output
```
a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x*sinh(c + d*x)/d -
 cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. 2(41) = 82.

Time = 0.10 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.98

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = -\frac{1}{4}\left(b\left(\frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)e^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2)e^{(-dx-c)}}{d^3}\right) + \frac{2\,a\cosh(dx+c)\log(x^2)}{d} - \frac{2\,(\mathrm{Ei}\right. + \frac{1}{2}\left(bx^2 + a\log(x^2)\right)\cosh(dx+c)$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/4*(b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x
+ 2)*e^(-d*x - c)/d^3) + 2*a*cosh(d*x + c)*log(x^2)/d - 2*(Ei(-d*x)*e^(-c)
 + Ei(d*x)*e^c)*a/d)*d + 1/2*(b*x^2 + a*log(x^2))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = \frac{ad^2\text{Ei}(-dx)\,e^{(-c)} + ad^2\text{Ei}(dx)\,e^c + bdxe^{(dx+c)} - bdxe^{(-dx-c)} - be^{(dx+c)} - be^{(-dx-c)}}{2\,d^2}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(a*d^2*Ei(-d*x)*e^(-c) + a*d^2*Ei(d*x)*e^c + b*d*x*e^(d*x + c) - b*d*x
*e^(-d*x - c) - b*e^(d*x + c) - b*e^(-d*x - c))/d^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx = a\,\text{coshint}(d\,x)\,\cosh(c) + a\,\text{sinhint}(d\,x)\,\sinh(c) - \frac{b\,(\cosh(c+d\,x) - d\,x\sinh(c+d\,x))}{d^2}$$

input
```
int((cosh(c + d*x)*(a + b*x^2))/x,x)
```

output
```
a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) - (b*(cosh(c + d*x) - d*x*
sinh(c + d*x)))/d^2
```

Reduce [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x}\,dx$$
$$=\frac{-\cosh(dx+c)\,b+\left(\int\frac{\cosh(dx+c)}{x}dx\right)a\,d^2+\sinh(dx+c)\,bdx}{d^2}$$

input
```
int((b*x^2+a)*cosh(d*x+c)/x,x)
```

output
```
( - cosh(c + d*x)*b + int(cosh(c + d*x)/x,x)*a*d**2 + sinh(c + d*x)*b*d*x)
/d**2
```

3.45 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c+dx)}{x} + ad\mathrm{Chi}(dx)\sinh(c) + \frac{b\sinh(c+dx)}{d} + ad\cosh(c)\mathrm{Shi}(dx)$$

output
```
-a*cosh(d*x+c)/x+a*d*Chi(d*x)*sinh(c)+b*sinh(d*x+c)/d+a*d*cosh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{a\cosh(c+dx)}{x} + ad\mathrm{Chi}(dx)\sinh(c) + \frac{b\sinh(c+dx)}{d} + ad\cosh(c)\mathrm{Shi}(dx)$$

input
```
Integrate[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

output
```
-((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])
/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^2}+b\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\sinh(c)\text{Chi}(dx)+ad\cosh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{x}+\frac{b\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^2)*Cosh[c + d*x])/x^2,x]
```

output
```
-((a*Cosh[c + d*x])/x) + a*d*CoshIntegral[d*x]*Sinh[c] + (b*Sinh[c + d*x])
/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.98

method	result
risch	$-\frac{e^{c} \operatorname{expIntegral}_{1}(-dx) a\, d^{2} x-e^{-c} \operatorname{expIntegral}_{1}(dx) a\, d^{2} x+d\, e^{dx+c} a-e^{dx+c} b x+d\, e^{-dx-c} a+e^{-dx-c} b x}{2 d x}$
meijerg	$\frac{b \cosh(c) \sinh(dx)}{d}-\frac{b \sinh(c) \sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}+\frac{i a \cosh(c) \sqrt{\pi}\, d\left(\frac{4 i \cosh(dx)}{d x \sqrt{\pi}}-\frac{4 i \operatorname{Shi}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{a \sinh(c) \sqrt{\pi}\, d\left(\frac{4}{\sqrt{\pi}}-\frac{4 \sinh(dx)}{\sqrt{\pi}\, x d}\right.}{}$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d*(exp(c)*Ei(1,-d*x)*a*d^2*x-exp(-c)*Ei(1,d*x)*a*d^2*x+d*exp(d*x+c)*a
-exp(d*x+c)*b*x+d*exp(-d*x-c)*a+exp(-d*x-c)*b*x)/x
```

Fricas [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{2\,ad\cosh\left(dx+c\right)-2\,bx\sinh\left(dx+c\right)-\left(ad^2x\mathrm{Ei}(dx)-ad^2x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-\left(ad^2x\mathrm{Ei}(dx)+ad^2x\right.}{2\,dx}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*cosh(d*x + c) - 2*b*x*sinh(d*x + c) - (a*d^2*x*Ei(d*x) - a*d^2
*x*Ei(-d*x))*cosh(c) - (a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c))/(d*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx$$

input
```
integrate((b*x**2+a)*cosh(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**2)*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{1}{2}\left(a\text{Ei}(-dx)\,e^{(-c)} - a\text{Ei}(dx)\,e^{c} + \frac{(dxe^{c}-e^{c})be^{(dx)}}{d^2} + \frac{(dx+1)be^{(-dx-c)}}{d^2}\right)d + \left(bx-\frac{a}{x}\right)\cosh(dx+c)$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
-1/2*(a*Ei(-d*x)*e^(-c) - a*Ei(d*x)*e^c + (d*x*e^c - e^c)*b*e^(d*x)/d^2 +
(d*x + 1)*b*e^(-d*x - c)/d^2)*d + (b*x - a/x)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.90

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = -\frac{ad^2x\text{Ei}(-dx)\,e^{(-c)} - ad^2x\text{Ei}(dx)\,e^{c} + ade^{(dx+c)} - bxe^{(dx+c)} + ade^{(-dx-c)} + bxe^{(-dx-c)}}{2\,dx}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a*d^2*x*Ei(-d*x)*e^(-c) - a*d^2*x*Ei(d*x)*e^c + a*d*e^(d*x + c) - b*
x*e^(d*x + c) + a*d*e^(-d*x - c) + b*x*e^(-d*x - c))/(d*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^2}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2))/x^2,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 2.12

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^2}\,dx = \frac{-e^{dx}\,ei(-dx)\,a\,d^2x+e^{dx+2c}ei(dx)\,a\,d^2x-e^{2dx+2c}ad+e^{2dx+2c}bx-ad-bx}{2e^{dx+c}dx}$$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^2,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a*d**2*x + e**(2*c + d*x)*ei(d*x)*a*d**2*x - e**(2
*c + 2*d*x)*a*d + e**(2*c + 2*d*x)*b*x - a*d - b*x)/(2*e**(c + d*x)*d*x)
```

3.46 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = -\frac{a\cosh(c+dx)}{2x^2} + b\cosh(c)\text{Chi}(dx) + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) - \frac{ad\sinh(c+dx)}{2x} + b\sinh(c)\text{Shi}(dx) + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

output
```
-1/2*a*cosh(d*x+c)/x^2+b*cosh(c)*Chi(d*x)+1/2*a*d^2*cosh(c)*Chi(d*x)-1/2*a
*d*sinh(d*x+c)/x+b*sinh(c)*Shi(d*x)+1/2*a*d^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.08

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = b\cosh(c)\text{Chi}(dx) - \frac{a\cosh(dx)(\cosh(c)+dx\sinh(c))}{2x^2} - \frac{a(dx\cosh(c)+\sinh(c))\sinh(dx)}{2x^2} + b\sinh(c)\text{Shi}(dx) + \frac{1}{2}ad^2(\cosh(c)\text{Chi}(dx)+\sinh(c)\text{Shi}(dx))$$

input
```
Integrate[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

output
```
b*Cosh[c]*CoshIntegral[d*x] - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2
) - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) + b*Sinh[c]*SinhIntegral
[d*x] + (a*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^3}+\frac{b\cosh(c+dx)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+b\cosh(c)\text{Chi}(dx)+b\sinh(c)\text{Shi}(dx)$$

input
```
Int[((a + b*x^2)*Cosh[c + d*x])/x^3,x]
```

output
```
-1/2*(a*Cosh[c + d*x])/x^2 + b*Cosh[c]*CoshIntegral[d*x] + (a*d^2*Cosh[c]*
CoshIntegral[d*x])/2 - (a*d*Sinh[c + d*x])/(2*x) + b*Sinh[c]*SinhIntegral[
d*x] + (a*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

method	result
risch	$-\frac{\mathrm{e}^{-c} \operatorname{expIntegral}_1(dx)a\,d^2x^2+\mathrm{e}^{c} \operatorname{expIntegral}_1(-dx)a\,d^2x^2+2\,\mathrm{e}^{-c} \operatorname{expIntegral}_1(dx)b\,x^2+2\,\mathrm{e}^{c} \operatorname{expIntegral}_1(-dx)b\,x^2+adx\,\mathrm{e}^{dx+c}-}{4x^2}$
meijerg	$\frac{b\cosh(c)\sqrt{\pi}\left(\frac{2\ \mathrm{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+b\sinh\left(c\right)\mathrm{Shi}\left(dx\right)-\frac{a\cosh(c)\sqrt{\pi}\,d^2\left(-\frac{4\left(\frac{9x^2d^2}{2}+3\right)}{3\sqrt{\pi}\,x^2d^2}+\frac{4\cosh(}{\sqrt{\pi}\,x^2}\right.}{}$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(exp(-c)*Ei(1,d*x)*a*d^2*x^2+exp(c)*Ei(1,-d*x)*a*d^2*x^2+2*exp(-c)*Ei
(1,d*x)*b*x^2+2*exp(c)*Ei(1,-d*x)*b*x^2+a*d*x*exp(d*x+c)-a*d*x*exp(-d*x-c)
+exp(d*x+c)*a+exp(-d*x-c)*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx =$$
$$-\frac{2\,adx\sinh\left(dx+c\right)+2\,a\cosh\left(dx+c\right)-\left((ad^2+2\,b)x^2\mathrm{Ei}(dx)+(ad^2+2\,b)x^2\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-((a}{4\,x^2}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((a*d^2 + 2*b)*x^2*Ei(d*
x) + (a*d^2 + 2*b)*x^2*Ei(-d*x))*cosh(c) - ((a*d^2 + 2*b)*x^2*Ei(d*x) - (a
*d^2 + 2*b)*x^2*Ei(-d*x))*sinh(c))/x^2
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \int \frac{(a+bx^2)\cosh{(c+dx)}}{x^3}\,dx$$

input
```
integrate((b*x**2+a)*cosh(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**2)*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.22

$$\begin{aligned}
&\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx \\
&= \frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a-\frac{2\,b\cosh{(dx+c)}\log{(x^2)}}{d}+\frac{2\left(\mathrm{Ei}(-dx)\,e^{(-c)}+\mathrm{Ei}(dx)\,e^{c}\right)b}{d}\right) \\
&\quad+\frac{1}{2}\left(b\log{(x^2)}-\frac{a}{x^2}\right)\cosh{(dx+c)}
\end{aligned}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a - 2*b*cosh(d*x +
c)*log(x^2)/d + 2*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/2*(b*log(x^2)
 - a/x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \frac{ad^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + ad^2x^2\mathrm{Ei}(dx)\,e^{c} + 2\,bx^2\mathrm{Ei}(-dx)\,e^{(-c)} + 2\,bx^2\mathrm{Ei}(dx)\,e^{c} - adxe^{(dx+c)} + adxe^{(-dx-c)}}{4\,x^2}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a*d^2*x^2*Ei(-d*x)*e^(-c) + a*d^2*x^2*Ei(d*x)*e^c + 2*b*x^2*Ei(-d*x)*
e^(-c) + 2*b*x^2*Ei(d*x)*e^c - a*d*x*e^(d*x + c) + a*d*x*e^(-d*x - c) - a*
e^(d*x + c) - a*e^(-d*x - c))/x^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^3}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2))/x^3,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2))/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^3}\,dx = \frac{e^{dx}ei(-dx)\,a\,d^2x^2 + 2e^{dx}ei(-dx)\,b\,x^2 + e^{dx+2c}ei(dx)\,a\,d^2x^2 + 2e^{dx+2c}ei(dx)\,b\,x^2 - e^{2dx+2c}adx - e^{2dx+2c}}{4e^{dx+c}x^2}$$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^3,x)
```

output

```
(e**(d*x)*ei( - d*x)*a*d**2*x**2 + 2*e**(d*x)*ei( - d*x)*b*x**2 + e**(2*c
+ d*x)*ei(d*x)*a*d**2*x**2 + 2*e**(2*c + d*x)*ei(d*x)*b*x**2 - e**(2*c + 2
*d*x)*a*d*x - e**(2*c + 2*d*x)*a + a*d*x - a)/(4*e**(c + d*x)*x**2)
```

3.47 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 105

$$\begin{aligned}\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = &-\frac{a\cosh(c+dx)}{3x^3}-\frac{b\cosh(c+dx)}{x}\\&-\frac{ad^2\cosh(c+dx)}{6x}+bd\text{Chi}(dx)\sinh(c)\\&+\frac{1}{6}ad^3\text{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{6x^2}\\&+bd\cosh(c)\text{Shi}(dx)+\frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)\end{aligned}$$

output
```
-1/3*a*cosh(d*x+c)/x^3-b*cosh(d*x+c)/x-1/6*a*d^2*cosh(d*x+c)/x+b*d*Chi(d*x
)*sinh(c)+1/6*a*d^3*Chi(d*x)*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2+b*d*cosh(c)*S
hi(d*x)+1/6*a*d^3*cosh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{2a\cosh(c+dx)+6bx^2\cosh(c+dx)+ad^2x^2\cosh(c+dx)-d(6b+ad^2)\,x^3\mathrm{Chi}(dx)\sinh(c)+adx\sin}{6x^3}$$

input
```
Integrate[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```

output
```
-1/6*(2*a*Cosh[c + d*x] + 6*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x]
- d*(6*b + a*d^2)*x^3*CoshIntegral[d*x]*Sinh[c] + a*d*x*Sinh[c + d*x] - d*
(6*b + a*d^2)*x^3*Cosh[c]*SinhIntegral[d*x])/x^3
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}dx$$
$$\Big\downarrow \text{5810}$$
$$\int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x^2}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{6}ad^3\sinh(c)\mathrm{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\mathrm{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{a\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{6x^2}+bd\sinh(c)\mathrm{Chi}(dx)+bd\cosh(c)\mathrm{Shi}(dx)-\frac{b\cosh(c+dx)}{x}$$

input
```
Int[((a + b*x^2)*Cosh[c + d*x])/x^4,x]
```

output
```
-1/3*(a*Cosh[c + d*x])/x^3 - (b*Cosh[c + d*x])/x - (a*d^2*Cosh[c + d*x])/(
6*x) + b*d*CoshIntegral[d*x]*Sinh[c] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6
 - (a*d*Sinh[c + d*x])/(6*x^2) + b*d*Cosh[c]*SinhIntegral[d*x] + (a*d^3*Co
sh[c]*SinhIntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.67

method	result
risch	$-\frac{e^{c} \operatorname{expIntegral}_{1}(-dx) a\, d^{3} x^{3}-e^{-c} \operatorname{expIntegral}_{1}(dx) a\, d^{3} x^{3}+6\, e^{c} \operatorname{expIntegral}_{1}(-dx) b d\, x^{3}-6\, e^{-c} \operatorname{expIntegral}_{1}(dx) b d\, x^{3}+a\, d^{2} x^{2} e^{d}}{12 x^{3}}$
meijerg	$\frac{i d b \cosh(c)\sqrt{\pi}\left(\frac{4 i \cosh(dx)}{dx\sqrt{\pi}}-\frac{4 i \operatorname{Shi}(dx)}{\sqrt{\pi}}\right)}{4}+\frac{d b \sinh(c)\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}}-\frac{4\sinh(dx)}{\sqrt{\pi}\, x d}+\frac{4\ \operatorname{Chi}(dx)-4\ln(dx)-4\gamma}{\sqrt{\pi}}+\frac{4\gamma-4+4\ln(x)+4\ln(id)}{\sqrt{\pi}}\right)}{4}-\frac{i a \operatorname{co}}{}$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(exp(c)*Ei(1,-d*x)*a*d^3*x^3-exp(-c)*Ei(1,d*x)*a*d^3*x^3+6*exp(c)*Ei
(1,-d*x)*b*d*x^3-6*exp(-c)*Ei(1,d*x)*b*d*x^3+a*d^2*x^2*exp(d*x+c)+a*d^2*x^
2*exp(-d*x-c)+a*d*x*exp(d*x+c)+6*exp(d*x+c)*b*x^2-a*d*x*exp(-d*x-c)+6*exp(
-d*x-c)*b*x^2+2*exp(d*x+c)*a+2*exp(-d*x-c)*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.21

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = -\frac{2\,adx\sinh\left(dx+c\right)+2\left((ad^2+6\,b)x^2+2\,a\right)\cosh\left(dx+c\right)-\left((ad^3+6\,bd)x^3\mathrm{Ei}(dx)-(ad^3+6\,bd)x^3\mathrm{E}\right.}{12\,x^3}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(2*a*d*x*sinh(d*x + c) + 2*((a*d^2 + 6*b)*x^2 + 2*a)*cosh(d*x + c) -
 ((a*d^3 + 6*b*d)*x^3*Ei(d*x) - (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*cosh(c) - ((
a*d^3 + 6*b*d)*x^3*Ei(d*x) + (a*d^3 + 6*b*d)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx^2)\cosh\left(c+dx\right)}{x^4}\,dx$$

input
```
integrate((b*x**2+a)*cosh(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**2)*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.70

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(ad^2e^{(-c)}\Gamma(-2,dx)-ad^2e^{c}\Gamma(-2,-dx)-3\,b\mathrm{Ei}(-dx)\,e^{(-c)}+3\,b\mathrm{Ei}(dx)\,e^{c}\right)d - \frac{\left(3\,bx^2+a\right)\cosh\left(dx+c\right)}{3\,x^3}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
1/6*(a*d^2*e^(-c)*gamma(-2, d*x) - a*d^2*e^c*gamma(-2, -d*x) - 3*b*Ei(-d*x
)*e^(-c) + 3*b*Ei(d*x)*e^c)*d - 1/3*(3*b*x^2 + a)*cosh(d*x + c)/x^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.62

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = -\frac{ad^3x^3\text{Ei}(-dx)\,e^{(-c)} - ad^3x^3\text{Ei}(dx)\,e^{c} + 6\,bdx^3\text{Ei}(-dx)\,e^{(-c)} - 6\,bdx^3\text{Ei}(dx)\,e^{c} + ad^2x^2e^{(dx+c)} + ad^2x^2}{12\,x^3}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

output
```
-1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + 6*b*d*x^3*Ei(-d
*x)*e^(-c) - 6*b*d*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*x + c) + a*d^2*x^2*e^(
-d*x - c) + a*d*x*e^(d*x + c) + 6*b*x^2*e^(d*x + c) - a*d*x*e^(-d*x - c) +
 6*b*x^2*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(-d*x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^4}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2))/x^4,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.69

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^4}\,dx$$
$$= \frac{-e^{dx}ei(-dx)\,a\,d^3x^3 - 6e^{dx}ei(-dx)\,bd\,x^3 + e^{dx+2c}ei(dx)\,a\,d^3x^3 + 6e^{dx+2c}ei(dx)\,bd\,x^3 - e^{2dx+2c}a\,d^2x^2 - e}{12e^{dx+c}x^3}$$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^4,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a*d**3*x**3 - 6*e**(d*x)*ei( - d*x)*b*d*x**3 + e**
(2*c + d*x)*ei(d*x)*a*d**3*x**3 + 6*e**(2*c + d*x)*ei(d*x)*b*d*x**3 - e**(
2*c + 2*d*x)*a*d**2*x**2 - e**(2*c + 2*d*x)*a*d*x - 2*e**(2*c + 2*d*x)*a -
 6*e**(2*c + 2*d*x)*b*x**2 - a*d**2*x**2 + a*d*x - 2*a - 6*b*x**2)/(12*e**
(c + d*x)*x**3)
```

3.48 $\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 149

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = -\frac{a\cosh(c+dx)}{4x^4} - \frac{b\cosh(c+dx)}{2x^2} - \frac{ad^2\cosh(c+dx)}{24x^2} + \frac{1}{2}bd^2\cosh(c)\text{Chi}(dx) + \frac{1}{24}ad^4\cosh(c)\text{Chi}(dx) - \frac{ad\sinh(c+dx)}{12x^3} - \frac{bd\sinh(c+dx)}{2x} - \frac{ad^3\sinh(c+dx)}{24x} + \frac{1}{2}bd^2\sinh(c)\text{Shi}(dx) + \frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)$$

output
```
-1/4*a*cosh(d*x+c)/x^4-1/2*b*cosh(d*x+c)/x^2-1/24*a*d^2*cosh(d*x+c)/x^2+1/
2*b*d^2*cosh(c)*Chi(d*x)+1/24*a*d^4*cosh(c)*Chi(d*x)-1/12*a*d*sinh(d*x+c)/
x^3-1/2*b*d*sinh(d*x+c)/x-1/24*a*d^3*sinh(d*x+c)/x+1/2*b*d^2*sinh(c)*Shi(d
*x)+1/24*a*d^4*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = -\frac{6a\cosh(c+dx)+12bx^2\cosh(c+dx)+ad^2x^2\cosh(c+dx)-d^2(12b+ad^2)\,x^4\cosh(c)\text{Chi}(dx)+2ad}{24x^4}$$

input
```
Integrate[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

output
```
-1/24*(6*a*Cosh[c + d*x] + 12*b*x^2*Cosh[c + d*x] + a*d^2*x^2*Cosh[c + d*x
] - d^2*(12*b + a*d^2)*x^4*Cosh[c]*CoshIntegral[d*x] + 2*a*d*x*Sinh[c + d*
x] + 12*b*d*x^3*Sinh[c + d*x] + a*d^3*x^3*Sinh[c + d*x] - d^2*(12*b + a*d^
2)*x^4*Sinh[c]*SinhIntegral[d*x])/x^4
```

Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^5}+\frac{b\cosh(c+dx)}{x^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{24}ad^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}ad^4\sinh(c)\text{Shi}(dx)-\frac{ad^3\sinh(c+dx)}{24x}-\frac{ad^2\cosh(c+dx)}{24x^2}-\frac{a\cosh(c+dx)}{4x^4}-\frac{ad\sinh(c+dx)}{12x^3}+\frac{1}{2}bd^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}bd^2\sinh(c)\text{Shi}(dx)-\frac{b\cosh(c+dx)}{2x^2}-\frac{bd\sinh(c+dx)}{2x}$$

input
```
Int[((a + b*x^2)*Cosh[c + d*x])/x^5,x]
```

output
```
-1/4*(a*Cosh[c + d*x])/x^4 - (b*Cosh[c + d*x])/(2*x^2) - (a*d^2*Cosh[c + d
*x])/(24*x^2) + (b*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (a*d^4*Cosh[c]*CoshI
ntegral[d*x])/24 - (a*d*Sinh[c + d*x])/(12*x^3) - (b*d*Sinh[c + d*x])/(2*x
) - (a*d^3*Sinh[c + d*x])/(24*x) + (b*d^2*Sinh[c]*SinhIntegral[d*x])/2 + (
a*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.64

method	result
risch	$\frac{-e^{-c}\,\mathrm{expIntegral}_1(dx)a\,d^4x^4-e^{c}\,\mathrm{expIntegral}_1(-dx)a\,d^4x^4-12\,e^{-c}\,\mathrm{expIntegral}_1(dx)b\,d^2x^4-12\,e^{c}\,\mathrm{expIntegral}_1(-dx)b\,d^2x^4+a\,d^3}{}$
meijerg	$-\frac{d^2b\cosh(c)\sqrt{\pi}\left(-\frac{4\left(\frac{9x^2d^2}{2}+3\right)}{3\sqrt{\pi}\,x^2d^2}+\frac{4\cosh(dx)}{\sqrt{\pi}\,x^2d^2}+\frac{4\sinh(dx)}{\sqrt{\pi}\,xd}-\frac{4(\mathrm{Chi}(dx)-\ln(dx)-\gamma)}{\sqrt{\pi}}-\frac{2(2\gamma-3+2\ln(x)+2\ln(id))}{\sqrt{\pi}}+\frac{4}{\sqrt{\pi}\,x^2d^2}\right)}{8}+\frac{id^2b\sinh}{}$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/48*(-exp(-c)*Ei(1,d*x)*a*d^4*x^4-exp(c)*Ei(1,-d*x)*a*d^4*x^4-12*exp(-c)*
Ei(1,d*x)*b*d^2*x^4-12*exp(c)*Ei(1,-d*x)*b*d^2*x^4+a*d^3*x^3*exp(-d*x-c)-a
*d^3*x^3*exp(d*x+c)-a*d^2*x^2*exp(-d*x-c)+12*b*d*x^3*exp(-d*x-c)-a*d^2*x^2
*exp(d*x+c)-12*b*d*x^3*exp(d*x+c)+2*a*d*x*exp(-d*x-c)-12*exp(-d*x-c)*b*x^2
-2*a*d*x*exp(d*x+c)-12*exp(d*x+c)*b*x^2-6*exp(-d*x-c)*a-6*exp(d*x+c)*a)/x^
4
```

Fricas [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = -\frac{2\,((ad^2+12\,b)x^2+6\,a)\cosh\left(dx+c\right)-((ad^4+12\,bd^2)x^4\mathrm{Ei}(dx)+(ad^4+12\,bd^2)x^4\mathrm{Ei}(-dx))\cosh\left(c\right)}{}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
-1/48*(2*((a*d^2 + 12*b)*x^2 + 6*a)*cosh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^
4*Ei(d*x) + (a*d^4 + 12*b*d^2)*x^4*Ei(-d*x))*cosh(c) + 2*((a*d^3 + 12*b*d)
*x^3 + 2*a*d*x)*sinh(d*x + c) - ((a*d^4 + 12*b*d^2)*x^4*Ei(d*x) - (a*d^4 +
 12*b*d^2)*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx^2)\cosh\left(c+dx\right)}{x^5}\,dx$$

input
```
integrate((b*x**2+a)*cosh(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x**2)*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.51

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{1}{8}\left(ad^3e^{(-c)}\Gamma(-3,dx) + ad^3e^c\Gamma(-3,-dx) + 2\,bde^{(-c)}\Gamma(-1,dx) + 2\,bde^c\Gamma(-1,-dx)\right)d - \frac{(2\,bx^2+a)\cosh(dx+c)}{4\,x^4}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
1/8*(a*d^3*e^(-c)*gamma(-3, d*x) + a*d^3*e^c*gamma(-3, -d*x) + 2*b*d*e^(-c
)*gamma(-1, d*x) + 2*b*d*e^c*gamma(-1, -d*x))*d - 1/4*(2*b*x^2 + a)*cosh(d
*x + c)/x^4
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.59

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{ad^4x^4\mathrm{Ei}(-dx)\,e^{(-c)} + ad^4x^4\mathrm{Ei}(dx)\,e^c + 12\,bd^2x^4\mathrm{Ei}(-dx)\,e^{(-c)} + 12\,bd^2x^4\mathrm{Ei}(dx)\,e^c - ad^3x^3e^{(dx+c)} + ad^3}{}$$

input
```
integrate((b*x^2+a)*cosh(d*x+c)/x^5,x, algorithm="giac")
```

output
```
1/48*(a*d^4*x^4*Ei(-d*x)*e^(-c) + a*d^4*x^4*Ei(d*x)*e^c + 12*b*d^2*x^4*Ei(
-d*x)*e^(-c) + 12*b*d^2*x^4*Ei(d*x)*e^c - a*d^3*x^3*e^(d*x + c) + a*d^3*x^
3*e^(-d*x - c) - a*d^2*x^2*e^(d*x + c) - 12*b*d*x^3*e^(d*x + c) - a*d^2*x^
2*e^(-d*x - c) + 12*b*d*x^3*e^(-d*x - c) - 2*a*d*x*e^(d*x + c) - 12*b*x^2*
e^(d*x + c) + 2*a*d*x*e^(-d*x - c) - 12*b*x^2*e^(-d*x - c) - 6*a*e^(d*x +
c) - 6*a*e^(-d*x - c))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)}{x^5}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2))/x^5,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2))/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 232, normalized size of antiderivative = 1.56

$$\int \frac{(a+bx^2)\cosh(c+dx)}{x^5}\,dx = \frac{e^{dx}\,ei(-dx)\,a\,d^4x^4+12e^{dx}\,ei(-dx)\,b\,d^2x^4+e^{dx+2c}ei(dx)\,a\,d^4x^4+12e^{dx+2c}ei(dx)\,b\,d^2x^4-e^{2dx+2c}a\,d^3x^3-}{}$$

input
```
int((b*x^2+a)*cosh(d*x+c)/x^5,x)
```

output
```
(e**(d*x)*ei( - d*x)*a*d**4*x**4 + 12*e**(d*x)*ei( - d*x)*b*d**2*x**4 + e*
*(2*c + d*x)*ei(d*x)*a*d**4*x**4 + 12*e**(2*c + d*x)*ei(d*x)*b*d**2*x**4 -
 e**(2*c + 2*d*x)*a*d**3*x**3 - e**(2*c + 2*d*x)*a*d**2*x**2 - 2*e**(2*c +
 2*d*x)*a*d*x - 6*e**(2*c + 2*d*x)*a - 12*e**(2*c + 2*d*x)*b*d*x**3 - 12*e
**(2*c + 2*d*x)*b*x**2 + a*d**3*x**3 - a*d**2*x**2 + 2*a*d*x - 6*a + 12*b*
d*x**3 - 12*b*x**2)/(48*e**(c + d*x)*x**4)
```

3.49 $\int x^2(a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 234

$$\begin{aligned}\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = &-\frac{720b^2x\cosh(c+dx)}{d^6}-\frac{48abx\cosh(c+dx)}{d^4}\\ &-\frac{2a^2x\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}\\ &-\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{6b^2x^5\cosh(c+dx)}{d^2}\\ &+\frac{720b^2\sinh(c+dx)}{d^7}+\frac{48ab\sinh(c+dx)}{d^5}\\ &+\frac{2a^2\sinh(c+dx)}{d^3}+\frac{360b^2x^2\sinh(c+dx)}{d^5}\\ &+\frac{24abx^2\sinh(c+dx)}{d^3}\\ &+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{30b^2x^4\sinh(c+dx)}{d^3}\\ &+\frac{2abx^4\sinh(c+dx)}{d}+\frac{b^2x^6\sinh(c+dx)}{d}\end{aligned}$$

output
```
-720*b^2*x*cosh(d*x+c)/d^6-48*a*b*x*cosh(d*x+c)/d^4-2*a^2*x*cosh(d*x+c)/d^
2-120*b^2*x^3*cosh(d*x+c)/d^4-8*a*b*x^3*cosh(d*x+c)/d^2-6*b^2*x^5*cosh(d*x
+c)/d^2+720*b^2*sinh(d*x+c)/d^7+48*a*b*sinh(d*x+c)/d^5+2*a^2*sinh(d*x+c)/d
^3+360*b^2*x^2*sinh(d*x+c)/d^5+24*a*b*x^2*sinh(d*x+c)/d^3+a^2*x^2*sinh(d*x
+c)/d+30*b^2*x^4*sinh(d*x+c)/d^3+2*a*b*x^4*sinh(d*x+c)/d+b^2*x^6*sinh(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.59

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx$$
$$= \frac{-2dx(a^2d^4+4abd^2(6+d^2x^2)+3b^2(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(a^2d^4(2+d^2x^2)+2abd^2(24}{d^7}$$

input
```
Integrate[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

output
```
(-2*d*x*(a^2*d^4 + 4*a*b*d^2*(6 + d^2*x^2) + 3*b^2*(120 + 20*d^2*x^2 + d^4
*x^4))*Cosh[c + d*x] + (a^2*d^4*(2 + d^2*x^2) + 2*a*b*d^2*(24 + 12*d^2*x^2
 + d^4*x^4) + b^2*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^6))*Sinh[c + d*x
])/d^7
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{5810}$$

$$\int \left(a^2x^2\cosh(c+dx)+2abx^4\cosh(c+dx)+b^2x^6\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2a^2\sinh(c+dx)}{d^3}-\frac{2a^2x\cosh(c+dx)}{d^2}+\frac{a^2x^2\sinh(c+dx)}{d}+\frac{48ab\sinh(c+dx)}{d^5}-\frac{48abx\cosh(c+dx)}{d^4}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{2abx^4\sinh(c+dx)}{d}+\frac{720b^2\sinh(c+dx)}{d^7}-\frac{720b^2x\cosh(c+dx)}{d^6}+\frac{360b^2x^2\sinh(c+dx)}{d^5}-\frac{120b^2x^3\cosh(c+dx)}{d^4}+\frac{30b^2x^4\sinh(c+dx)}{d^3}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{b^2x^6\sinh(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x^2)^2*Cosh[c + d*x],x]
```

output
```
(-720*b^2*x*Cosh[c + d*x])/d^6 - (48*a*b*x*Cosh[c + d*x])/d^4 - (2*a^2*x*C
osh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (8*a*b*x^3*Cosh[c +
d*x])/d^2 - (6*b^2*x^5*Cosh[c + d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 +
(48*a*b*Sinh[c + d*x])/d^5 + (2*a^2*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh
[c + d*x])/d^5 + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (a^2*x^2*Sinh[c + d*x])/
d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (2*a*b*x^4*Sinh[c + d*x])/d + (b^2*x^
6*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.55

method	result
parallelrisch	$\frac{\left(x^2(b\,x^2+a)^2 d^6+(30b^2x^4+24ab\,x^2+2a^2)d^4+(360x^2b^2+48ab)d^2+720b^2\right)\sinh(dx+c)-2\left((3b\,x^2+a)(b\,x^2+a)d^4+(60\right.}{d^7}$
risch	$\frac{(b^2x^6d^6+2ab\,d^6x^4-6b^2x^5d^5+a^2d^6x^2-8ab\,d^5x^3+30b^2x^4d^4-2a^2d^5x+24ab\,d^4x^2-120b^2d^3x^3+2a^2d^4-48ab\,d^3x+360b}{2d^7}$
orering	$-\frac{4(3b^3d^6x^8+7a\,b^2d^6x^6+5a^2b\,d^6x^4+75b^3d^4x^6+a^3d^6x^2+93a\,b^2d^4x^4+27a^2b\,d^4x^2+720b^3d^2x^4+a^3d^4+432a\,b^2d^2x^2+}{d^8x(b\,x^2+a)}$
meijerg	$\frac{64ib^2\cosh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{21}{8}d^4x^4+\frac{105}{2}x^2d^2+315\right)\cosh(dx)}{28\sqrt{\pi}}-\frac{i\left(\frac{7}{16}x^6d^6+\frac{105}{8}d^4x^4+\frac{315}{2}x^2d^2+315\right)\sinh(dx)}{28\sqrt{\pi}}\right)}{d^7}+\frac{64b^2\sinh(c)}{}$
parts	$\frac{b^2x^6\sinh(dx+c)}{d}+\frac{2ab\,x^4\sinh(dx+c)}{d}+\frac{a^2x^2\sinh(dx+c)}{d}-\frac{2\left(\frac{15b^2c^4((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^5}-\frac{30b^2c^3((d}{}\right.}{}$
derivativedivides	$\frac{a^2c^2\sinh(dx+c)+a^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)-\frac{8b\,c^3a((dx+c)\sinh(dx+c)-\cosh(dx+c}{d^2}}{}$
default	$\frac{a^2c^2\sinh(dx+c)+a^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)-\frac{8b\,c^3a((dx+c)\sinh(dx+c)-\cosh(dx+c}{d^2}}{}$

input
```
int(x^2*(b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
((x^2*(b*x^2+a)^2*d^6+(30*b^2*x^4+24*a*b*x^2+2*a^2)*d^4+(360*b^2*x^2+48*a*
b)*d^2+720*b^2)*sinh(d*x+c)-2*((3*b*x^2+a)*(b*x^2+a)*d^4+(60*b^2*x^2+24*a*
b)*d^2+360*b^2)*d*cosh(d*x+c)*x)/d^7
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.66

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = -\frac{2\,(3\,b^2d^5x^5+4\,(abd^5+15\,b^2d^3)x^3+(a^2d^5+24\,abd^3+360\,b^2d)x)\cosh(dx+c)-(b^2d^6x^6+2\,a^2d^4+2}{d^7}$$

input
```
integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(2*(3*b^2*d^5*x^5 + 4*(a*b*d^5 + 15*b^2*d^3)*x^3 + (a^2*d^5 + 24*a*b*d^3
+ 360*b^2*d)*x)*cosh(d*x + c) - (b^2*d^6*x^6 + 2*a^2*d^4 + 2*(a*b*d^6 + 15
*b^2*d^4)*x^4 + 48*a*b*d^2 + (a^2*d^6 + 24*a*b*d^4 + 360*b^2*d^2)*x^2 + 72
0*b^2)*sinh(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.22

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = \begin{cases} \frac{a^2x^2\sinh(c+dx)}{d} - \frac{2a^2x\cosh(c+dx)}{d^2} + \frac{2a^2\sinh(c+dx)}{d^3} + \frac{2abx^4\sinh(c+dx)}{d} - \frac{8abx^3\cosh(c+dx)}{d^2} + \frac{24abx^2\sinh(c+dx)}{d^3} - \frac{48abx \\ \left(\frac{a^2x^3}{3} + \frac{2abx^5}{5} + \frac{b^2x^7}{7}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**2*(b*x**2+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*x**2*sinh(c + d*x)/d - 2*a**2*x*cosh(c + d*x)/d**2 + 2*a**
2*sinh(c + d*x)/d**3 + 2*a*b*x**4*sinh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*
x)/d**2 + 24*a*b*x**2*sinh(c + d*x)/d**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 4
8*a*b*sinh(c + d*x)/d**5 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c
+ d*x)/d**2 + 30*b**2*x**4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x
)/d**4 + 360*b**2*x**2*sinh(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6
+ 720*b**2*sinh(c + d*x)/d**7, Ne(d, 0)), ((a**2*x**3/3 + 2*a*b*x**5/5 + b
**2*x**7/7)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.64

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = -\frac{1}{210}\,d\left(\frac{35\,(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)a^2e^{(dx)}}{d^4} + \frac{35\,(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)a^2e^{(-dx-c)}}{d^4} + \frac{42\,(\right. + \frac{1}{105}\left(15\,b^2x^7 + 42\,abx^5 + 35\,a^2x^3\right)\cosh(dx+c)$$

input
```
integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/210*d*(35*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a^2*e^(d*x)
/d^4 + 35*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a^2*e^(-d*x - c)/d^4 + 42*(d^5
*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c -
 120*e^c)*a*b*e^(d*x)/d^6 + 42*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*
x^2 + 120*d*x + 120)*a*b*e^(-d*x - c)/d^6 + 15*(d^7*x^7*e^c - 7*d^6*x^6*e^
c + 42*d^5*x^5*e^c - 210*d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c
+ 5040*d*x*e^c - 5040*e^c)*b^2*e^(d*x)/d^8 + 15*(d^7*x^7 + 7*d^6*x^6 + 42*
d^5*x^5 + 210*d^4*x^4 + 840*d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b^2*
e^(-d*x - c)/d^8) + 1/105*(15*b^2*x^7 + 42*a*b*x^5 + 35*a^2*x^3)*cosh(d*x
+ c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.30

$$\int x^2(a+bx^2)^2\cosh(c+dx)\,dx = \frac{(b^2d^6x^6+2\,abd^6x^4-6\,b^2d^5x^5+a^2d^6x^2-8\,abd^5x^3+30\,b^2d^4x^4-2\,a^2d^5x+24\,abd^4x^2-120\,b^2d^3x^3+2\,a}{2\,d^7} - \frac{(b^2d^6x^6+2\,abd^6x^4+6\,b^2d^5x^5+a^2d^6x^2+8\,abd^5x^3+30\,b^2d^4x^4+2\,a^2d^5x+24\,abd^4x^2+120\,b^2d^3x^3+}{2\,d^7}$$

input
```
integrate(x^2*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 - 6*b^2*d^5*x^5 + a^2*d^6*x^2 - 8*a*b*d^5
*x^3 + 30*b^2*d^4*x^4 - 2*a^2*d^5*x + 24*a*b*d^4*x^2 - 120*b^2*d^3*x^3 + 2
*a^2*d^4 - 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b*d^2 - 720*b^2*d*x + 720
*b^2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 2*a*b*d^6*x^4 + 6*b^2*d^5*x^5 +
 a^2*d^6*x^2 + 8*a*b*d^5*x^3 + 30*b^2*d^4*x^4 + 2*a^2*d^5*x + 24*a*b*d^4*x
^2 + 120*b^2*d^3*x^3 + 2*a^2*d^4 + 48*a*b*d^3*x + 360*b^2*d^2*x^2 + 48*a*b
*d^2 + 720*b^2*d*x + 720*b^2)*e^(-d*x - c)/d^7
```

Mupad [B] (verification not implemented)

Time = 2.15 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.78

$$\int x^2 (a+bx^2)^2 \cosh(c+dx)\,dx = \frac{2\sinh(c+d\,x)\,\left(a^2\,d^4+24\,a\,b\,d^2+360\,b^2\right)}{d^7} - \frac{6\,b^2\,x^5\cosh(c+d\,x)}{d^2} + \frac{b^2\,x^6\sinh(c+d\,x)}{d} - \frac{2\,x\cosh(c+d\,x)\,\left(a^2\,d^4+24\,a\,b\,d^2+360\,b^2\right)}{d^6} + \frac{x^2\sinh(c+d\,x)\,\left(a^2\,d^4+24\,a\,b\,d^2+360\,b^2\right)}{d^5} - \frac{8\,x^3\cosh(c+d\,x)\,\left(15\,b^2+a\,b\,d^2\right)}{d^4} + \frac{2\,x^4\sinh(c+d\,x)\,\left(15\,b^2+a\,b\,d^2\right)}{d^3}$$

input
```
int(x^2*cosh(c + d*x)*(a + b*x^2)^2,x)
```

output
```
(2*sinh(c + d*x)*(360*b^2 + a^2*d^4 + 24*a*b*d^2))/d^7 - (6*b^2*x^5*cosh(c
 + d*x))/d^2 + (b^2*x^6*sinh(c + d*x))/d - (2*x*cosh(c + d*x)*(360*b^2 + a
^2*d^4 + 24*a*b*d^2))/d^6 + (x^2*sinh(c + d*x)*(360*b^2 + a^2*d^4 + 24*a*b
*d^2))/d^5 - (8*x^3*cosh(c + d*x)*(15*b^2 + a*b*d^2))/d^4 + (2*x^4*sinh(c
+ d*x)*(15*b^2 + a*b*d^2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

$$\int x^2 (a+bx^2)^2 \cosh(c+dx)\,dx = \frac{-2\cosh(dx+c)\,a^2d^5x - 8\cosh(dx+c)\,ab\,d^5x^3 - 48\cosh(dx+c)\,ab\,d^3x - 6\cosh(dx+c)\,b^2d^5x^5 - 12}{}$$

input
```
int(x^2*(b*x^2+a)^2*cosh(d*x+c),x)
```

output
```
( - 2*cosh(c + d*x)*a**2*d**5*x - 8*cosh(c + d*x)*a*b*d**5*x**3 - 48*cosh(
c + d*x)*a*b*d**3*x - 6*cosh(c + d*x)*b**2*d**5*x**5 - 120*cosh(c + d*x)*b
**2*d**3*x**3 - 720*cosh(c + d*x)*b**2*d*x + sinh(c + d*x)*a**2*d**6*x**2
+ 2*sinh(c + d*x)*a**2*d**4 + 2*sinh(c + d*x)*a*b*d**6*x**4 + 24*sinh(c +
d*x)*a*b*d**4*x**2 + 48*sinh(c + d*x)*a*b*d**2 + sinh(c + d*x)*b**2*d**6*x
**6 + 30*sinh(c + d*x)*b**2*d**4*x**4 + 360*sinh(c + d*x)*b**2*d**2*x**2 +
 720*sinh(c + d*x)*b**2)/d**7
```

3.50 $\int x(a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 184

$$\begin{aligned}\int x(a+bx^2)^2\cosh(c+dx)\,dx = & -\frac{120b^2\cosh(c+dx)}{d^6}-\frac{12ab\cosh(c+dx)}{d^4} \\ & -\frac{a^2\cosh(c+dx)}{d^2}-\frac{60b^2x^2\cosh(c+dx)}{d^4} \\ & -\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{5b^2x^4\cosh(c+dx)}{d^2} \\ & +\frac{120b^2x\sinh(c+dx)}{d^5}+\frac{12abx\sinh(c+dx)}{d^3} \\ & +\frac{a^2x\sinh(c+dx)}{d}+\frac{20b^2x^3\sinh(c+dx)}{d^3} \\ & +\frac{2abx^3\sinh(c+dx)}{d}+\frac{b^2x^5\sinh(c+dx)}{d}\end{aligned}$$

output
```
-120*b^2*cosh(d*x+c)/d^6-12*a*b*cosh(d*x+c)/d^4-a^2*cosh(d*x+c)/d^2-60*b^2
*x^2*cosh(d*x+c)/d^4-6*a*b*x^2*cosh(d*x+c)/d^2-5*b^2*x^4*cosh(d*x+c)/d^2+1
20*b^2*x*sinh(d*x+c)/d^5+12*a*b*x*sinh(d*x+c)/d^3+a^2*x*sinh(d*x+c)/d+20*b
^2*x^3*sinh(d*x+c)/d^3+2*a*b*x^3*sinh(d*x+c)/d+b^2*x^5*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.61

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx$$
$$=\frac{-((a^2d^4+6abd^2(2+d^2x^2)+5b^2(24+12d^2x^2+d^4x^4))\cosh(c+dx))+dx(a^2d^4+2abd^2(6+d^2x^2)+b^2}{d^6}$$

input
```
Integrate[x*(a + b*x^2)^2*Cosh[c + d*x],x]
```

output
```
(-((a^2*d^4 + 6*a*b*d^2*(2 + d^2*x^2) + 5*b^2*(24 + 12*d^2*x^2 + d^4*x^4))
*Cosh[c + d*x]) + d*x*(a^2*d^4 + 2*a*b*d^2*(6 + d^2*x^2) + b^2*(120 + 20*d
^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{5810}$$
$$\int \left(a^2x\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^5\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{2abx^3\sinh(c+dx)}{d}-\frac{120b^2\cosh(c+dx)}{d^6}+\frac{120b^2x\sinh(c+dx)}{d^5}-\frac{60b^2x^2\cosh(c+dx)}{d^4}+\frac{20b^2x^3\sinh(c+dx)}{d^3}-\frac{5b^2x^4\cosh(c+dx)}{d^2}+\frac{b^2x^5\sinh(c+dx)}{d}$$

input
```
Int[x*(a + b*x^2)^2*Cosh[c + d*x],x]
```

output
```
(-120*b^2*Cosh[c + d*x])/d^6 - (12*a*b*Cosh[c + d*x])/d^4 - (a^2*Cosh[c +
d*x])/d^2 - (60*b^2*x^2*Cosh[c + d*x])/d^4 - (6*a*b*x^2*Cosh[c + d*x])/d^2
 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + (120*b^2*x*Sinh[c + d*x])/d^5 + (12*a*b
*x*Sinh[c + d*x])/d^3 + (a^2*x*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x
])/d^3 + (2*a*b*x^3*Sinh[c + d*x])/d + (b^2*x^5*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.72 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.84

method	result
parallelrisch	$\frac{6\left(\left(\frac{5b\,x^2}{6}+a\right)d^2+10b\right)b\,d^2x^2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2dx\left(\left(b\,x^2+a\right)^2d^4+4(5x^2b^2+3ab)d^2+120b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+(5b^2x^4+6}{d^6\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
orering	$-\frac{2(5b^3d^4x^6+11a\,b^2d^4x^4+7a^2b\,d^4x^2+80b^3d^2x^4+a^3d^4+76a\,b^2d^2x^2+12a^2b\,d^2+360x^2b^3+120b^2a)\cosh(dx+c)}{d^6(b\,x^2+a)}+\frac{(b}{}$
risch	$\frac{(b^2x^5d^5+2ab\,d^5x^3-5b^2x^4d^4+a^2d^5x-6ab\,d^4x^2+20b^2d^3x^3-a^2d^4+12ab\,d^3x-60b^2d^2x^2-12a\,d^2b+120b^2dx-120b^2)\mathrm{e}^{d}}{2d^6}$
meijerg	$-\frac{32b^2\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}x^2d^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}x^2d^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib^2\sinh(c)\sqrt{\pi}\,(}{}$
parts	$\frac{b^2x^5\sinh(dx+c)}{d}+\frac{2ab\,x^3\sinh(dx+c)}{d}+\frac{a^2x\sinh(dx+c)}{d}-\frac{5b^2c^4\cosh(dx+c)}{d^4}-\frac{20b^2c^3((dx+c)\cosh(dx+c)-\sinh(dx+c}{d^4}$
derivativedivides	$\frac{5b^2c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}-\frac{10b^2c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{10b^2c^2\left((dx+c)^3\sin\right.}{}$
default	$\frac{5b^2c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}-\frac{10b^2c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{10b^2c^2\left((dx+c)^3\sin\right.}{}$

input
```
int(x*(b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(6*((5/6*b*x^2+a)*d^2+10*b)*b*d^2*x^2*tanh(1/2*d*x+1/2*c)^2-2*d*x*((b*x^2+
a)^2*d^4+4*(5*b^2*x^2+3*a*b)*d^2+120*b^2)*tanh(1/2*d*x+1/2*c)+(5*b^2*x^4+6
*a*b*x^2+2*a^2)*d^4+12*(5*b^2*x^2+2*a*b)*d^2+240*b^2)/d^6/(tanh(1/2*d*x+1/
2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.68

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx =$$
$$-\frac{(5\,b^2d^4x^4+a^2d^4+12\,abd^2+6\,(abd^4+10\,b^2d^2)x^2+120\,b^2)\cosh(dx+c)-(b^2d^5x^5+2\,(abd^5+10\,b^2d}{d^6}$$

input
```
integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((5*b^2*d^4*x^4 + a^2*d^4 + 12*a*b*d^2 + 6*(a*b*d^4 + 10*b^2*d^2)*x^2 + 1
20*b^2)*cosh(d*x + c) - (b^2*d^5*x^5 + 2*(a*b*d^5 + 10*b^2*d^3)*x^3 + (a^2
*d^5 + 12*a*b*d^3 + 120*b^2*d)*x)*sinh(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.69 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx = \begin{cases} \frac{a^2x\sinh(c+dx)}{d} - \frac{a^2\cosh(c+dx)}{d^2} + \frac{2abx^3\sinh(c+dx)}{d} - \frac{6abx^2\cosh(c+dx)}{d^2} + \frac{12abx\sinh(c+dx)}{d^3} - \frac{12ab\cosh(c+dx)}{d^4} + \frac{b^2x^5\sinh}{d} \\ \left(\frac{a^2x^2}{2} + \frac{abx^4}{2} + \frac{b^2x^6}{6}\right)\cosh(c) \end{cases}$$

input
```
integrate(x*(b*x**2+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**3*s
inh(c + d*x)/d - 6*a*b*x**2*cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d*
*3 - 12*a*b*cosh(c + d*x)/d**4 + b**2*x**5*sinh(c + d*x)/d - 5*b**2*x**4*c
osh(c + d*x)/d**2 + 20*b**2*x**3*sinh(c + d*x)/d**3 - 60*b**2*x**2*cosh(c
+ d*x)/d**4 + 120*b**2*x*sinh(c + d*x)/d**5 - 120*b**2*cosh(c + d*x)/d**6,
 Ne(d, 0)), ((a**2*x**2/2 + a*b*x**4/2 + b**2*x**6/6)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.92

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx = \frac{(bx^2+a)^3\cosh(dx+c)}{6b} - \frac{\left(\frac{a^3e^{(dx+c)}}{d} + \frac{a^3e^{(-dx-c)}}{d} + \frac{3(d^2x^2e^c-2dxe^c+2e^c)a^2be^{(dx)}}{d^3} + \frac{3(d^2x^2+2dx+2)a^2be^{(-dx-c)}}{d^3} + \frac{3(d^4x^4e^c-4d^3x^3e^c+12d^2x^2e^c-2}{d^5}\right.}{}$$

input
```
integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/6*(b*x^2 + a)^3*cosh(d*x + c)/b - 1/12*(a^3*e^(d*x + c)/d + a^3*e^(-d*x
- c)/d + 3*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*b*e^(d*x)/d^3 + 3*(d^2*x^
2 + 2*d*x + 2)*a^2*b*e^(-d*x - c)/d^3 + 3*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 1
2*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*a*b^2*e^(d*x)/d^5 + 3*(d^4*x^4 + 4*d^
3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a*b^2*e^(-d*x - c)/d^5 + (d^6*x^6*e^c -
6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d
*x*e^c + 720*e^c)*b^3*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 12
0*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b^3*e^(-d*x - c)/d^7)*d/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.30

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx$$
$$=\frac{(b^2d^5x^5+2\,abd^5x^3-5\,b^2d^4x^4+a^2d^5x-6\,abd^4x^2+20\,b^2d^3x^3-a^2d^4+12\,abd^3x-60\,b^2d^2x^2-12\,abd^2}{2\,d^6}$$
$$-\frac{(b^2d^5x^5+2\,abd^5x^3+5\,b^2d^4x^4+a^2d^5x+6\,abd^4x^2+20\,b^2d^3x^3+a^2d^4+12\,abd^3x+60\,b^2d^2x^2+12\,abd}{2\,d^6}$$

input
```
integrate(x*(b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x^3 - 5*b^2*d^4*x^4 + a^2*d^5*x - 6*a*b*d^4*x
^2 + 20*b^2*d^3*x^3 - a^2*d^4 + 12*a*b*d^3*x - 60*b^2*d^2*x^2 - 12*a*b*d^2
 + 120*b^2*d*x - 120*b^2)*e^(d*x + c)/d^6 - 1/2*(b^2*d^5*x^5 + 2*a*b*d^5*x
^3 + 5*b^2*d^4*x^4 + a^2*d^5*x + 6*a*b*d^4*x^2 + 20*b^2*d^3*x^3 + a^2*d^4
+ 12*a*b*d^3*x + 60*b^2*d^2*x^2 + 12*a*b*d^2 + 120*b^2*d*x + 120*b^2)*e^(-
d*x - c)/d^6
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.80

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx = \frac{b^2\,x^5\sinh(c+d\,x)}{d}-\frac{5\,b^2\,x^4\cosh(c+d\,x)}{d^2}-\frac{\cosh(c+d\,x)\,\left(a^2\,d^4+12\,a\,b\,d^2+120\,b^2\right)}{d^6}+\frac{x\sinh(c+d\,x)\,\left(a^2\,d^4+12\,a\,b\,d^2+120\,b^2\right)}{d^5}-\frac{6\,x^2\cosh(c+d\,x)\,\left(10\,b^2+a\,b\,d^2\right)}{d^4}+\frac{2\,x^3\sinh(c+d\,x)\,\left(10\,b^2+a\,b\,d^2\right)}{d^3}$$

input
```
int(x*cosh(c + d*x)*(a + b*x^2)^2,x)
```

output
```
(b^2*x^5*sinh(c + d*x))/d - (5*b^2*x^4*cosh(c + d*x))/d^2 - (cosh(c + d*x)
*(120*b^2 + a^2*d^4 + 12*a*b*d^2))/d^6 + (x*sinh(c + d*x)*(120*b^2 + a^2*d
^4 + 12*a*b*d^2))/d^5 - (6*x^2*cosh(c + d*x)*(10*b^2 + a*b*d^2))/d^4 + (2*
x^3*sinh(c + d*x)*(10*b^2 + a*b*d^2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.99

$$\int x(a+bx^2)^2\cosh(c+dx)\,dx$$
$$= \frac{-\cosh(dx+c)\,a^2d^4-6\cosh(dx+c)\,ab\,d^4x^2-12\cosh(dx+c)\,ab\,d^2-5\cosh(dx+c)\,b^2d^4x^4-60\cosh}{}$$

input
```
int(x*(b*x^2+a)^2*cosh(d*x+c),x)
```

output
```
( - cosh(c + d*x)*a**2*d**4 - 6*cosh(c + d*x)*a*b*d**4*x**2 - 12*cosh(c +
d*x)*a*b*d**2 - 5*cosh(c + d*x)*b**2*d**4*x**4 - 60*cosh(c + d*x)*b**2*d**
2*x**2 - 120*cosh(c + d*x)*b**2 + sinh(c + d*x)*a**2*d**5*x + 2*sinh(c + d
*x)*a*b*d**5*x**3 + 12*sinh(c + d*x)*a*b*d**3*x + sinh(c + d*x)*b**2*d**5*
x**5 + 20*sinh(c + d*x)*b**2*d**3*x**3 + 120*sinh(c + d*x)*b**2*d*x)/d**6
```

3.51 $\int (a+bx^2)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 136

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = -\frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4abx\cosh(c+dx)}{d^2} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{24b^2\sinh(c+dx)}{d^5} + \frac{4ab\sinh(c+dx)}{d^3} + \frac{a^2\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} + \frac{2abx^2\sinh(c+dx)}{d} + \frac{b^2x^4\sinh(c+dx)}{d}$$

output

```
-24*b^2*x*cosh(d*x+c)/d^4-4*a*b*x*cosh(d*x+c)/d^2-4*b^2*x^3*cosh(d*x+c)/d^
2+24*b^2*sinh(d*x+c)/d^5+4*a*b*sinh(d*x+c)/d^3+a^2*sinh(d*x+c)/d+12*b^2*x^
2*sinh(d*x+c)/d^3+2*a*b*x^2*sinh(d*x+c)/d+b^2*x^4*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.11 (sec), antiderivative size = 85, normalized size of antiderivative = 0.62

$$\int (a+bx^2)^2 \cosh(c+dx)\, dx = \frac{-4bdx(ad^2+b(6+d^2x^2))\cosh(c+dx)+(a^2d^4+2abd^2(2+d^2x^2)+b^2(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

input
```
Integrate[(a + b*x^2)^2*Cosh[c + d*x],x]
```

output
```
(-4*b*d*x*(a*d^2 + b*(6 + d^2*x^2))*Cosh[c + d*x] + (a^2*d^4 + 2*a*b*d^2*(
2 + d^2*x^2) + b^2*(24 + 12*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.42 (sec), antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5800, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+bx^2)^2 \cosh(c+dx)\, dx$$

$$\Big\downarrow \text{5800}$$

$$\int \left(a^2\cosh(c+dx)+2abx^2\cosh(c+dx)+b^2x^4\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\sinh(c+dx)}{d}+\frac{4ab\sinh(c+dx)}{d^3}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{2abx^2\sinh(c+dx)}{d}+\frac{24b^2\sinh(c+dx)}{d^5}-\frac{24b^2x\cosh(c+dx)}{d^4}+\frac{12b^2x^2\sinh(c+dx)}{d^3}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{b^2x^4\sinh(c+dx)}{d}$$

input
```
Int[(a + b*x^2)^2*Cosh[c + d*x],x]
```

output
```
(-24*b^2*x*Cosh[c + d*x])/d^4 - (4*a*b*x*Cosh[c + d*x])/d^2 - (4*b^2*x^3*C
osh[c + d*x])/d^2 + (24*b^2*Sinh[c + d*x])/d^5 + (4*a*b*Sinh[c + d*x])/d^3
 + (a^2*Sinh[c + d*x])/d + (12*b^2*x^2*Sinh[c + d*x])/d^3 + (2*a*b*x^2*Sin
h[c + d*x])/d + (b^2*x^4*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5800
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.53

method	result
parallelrisch	$\frac{\left(\left(b x^2+a\right)^2 d^4+4 b\left(3 b x^2+a\right) d^2+24 b^2\right) \sinh(dx+c)-4 b d \cosh(dx+c) x\left(\left(b x^2+a\right) d^2+6 b\right)}{d^5}$
orering	$-\frac{8bx\left(b^2x^4d^4+2abd^4x^2+a^2d^4+9b^2d^2x^2+5ad^2b+12b^2\right)\cosh(dx+c)}{d^6(bx^2+a)}+\frac{(b^2x^4d^4+2abd^4x^2+a^2d^4+12b^2d^2x^2+4ad^2}{}$
risch	$\frac{\left(b^2x^4d^4+2abd^4x^2-4b^2d^3x^3+a^2d^4-4abd^3x+12b^2d^2x^2+4ad^2b-24b^2dx+24b^2\right)e^{dx+c}}{2d^5}-\frac{(b^2x^4d^4+2abd^4x^2+4b^2d^3}{}$
parts	$\frac{b^2x^4\sinh(dx+c)}{d}+\frac{2abx^2\sinh(dx+c)}{d}+\frac{a^2\sinh(dx+c)}{d}-\frac{4b\left(\frac{3bc^2((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^2}-\frac{bc^3\cosh(dx+c}{d^2}\right.}{}$
meijerg	$-\frac{16ib^2\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b^2\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x\right.}{}\right.}{}$
derivativedivides	$\frac{b^2c^4\sinh(dx+c)}{d^4}-\frac{4b^2c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}+\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{2b}{}$
default	$\frac{b^2c^4\sinh(dx+c)}{d^4}-\frac{4b^2c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^4}+\frac{6b^2c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^4}+\frac{2b}{}$

input
```
int((b*x^2+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(((b*x^2+a)^2*d^4+4*b*(3*b*x^2+a)*d^2+24*b^2)*sinh(d*x+c)-4*b*d*cosh(d*x+c
)*x*((b*x^2+a)*d^2+6*b))/d^5
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.72

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = -\frac{4\,(b^2d^3x^3+(abd^3+6\,b^2d)x)\cosh(dx+c)-(b^2d^4x^4+a^2d^4+4\,abd^2+2\,(abd^4+6\,b^2d^2)x^2+24\,b^2)\sin}{d^5}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(4*(b^2*d^3*x^3 + (a*b*d^3 + 6*b^2*d)*x)*cosh(d*x + c) - (b^2*d^4*x^4 + a
^2*d^4 + 4*a*b*d^2 + 2*(a*b*d^4 + 6*b^2*d^2)*x^2 + 24*b^2)*sinh(d*x + c))/
d^5
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int (a+bx^2)^2 \cosh(c+dx)\,dx = \begin{cases} \frac{a^2\sinh(c+dx)}{d}+\frac{2abx^2\sinh(c+dx)}{d}-\frac{4abx\cosh(c+dx)}{d^2}+\frac{4ab\sinh(c+dx)}{d^3}+\frac{b^2x^4\sinh(c+dx)}{d}-\frac{4b^2x^3\cosh(c+dx)}{d^2}+\frac{12b^2x^2\sinh}{d^3} \\ \left(a^2x+\frac{2abx^3}{3}+\frac{b^2x^5}{5}\right)\cosh(c) \end{cases}$$

input
```
integrate((b*x**2+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**2*sinh(c + d*x)/d - 4*a*b*x*cos
h(c + d*x)/d**2 + 4*a*b*sinh(c + d*x)/d**3 + b**2*x**4*sinh(c + d*x)/d - 4
*b**2*x**3*cosh(c + d*x)/d**2 + 12*b**2*x**2*sinh(c + d*x)/d**3 - 24*b**2*
x*cosh(c + d*x)/d**4 + 24*b**2*sinh(c + d*x)/d**5, Ne(d, 0)), ((a**2*x + 2
*a*b*x**3/3 + b**2*x**5/5)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.39

$$\int \left(a+bx^2\right)^2 \cosh(c+dx)\,dx$$
$$=\frac{a^2e^{(dx+c)}}{2\,d}-\frac{a^2e^{(-dx-c)}}{2\,d}+\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)abe^{(dx)}}{d^3}-\frac{(d^2x^2+2\,dx+2)abe^{(-dx-c)}}{d^3}+\frac{(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2x^2e^c-24\,dxe^c+24\,e^c)b^2e^{(dx)}}{2\,d^5}-\frac{(d^4x^4+4\,d^3x^3+12\,d^2x^2+24\,dx+24)b^2e^{(-dx-c)}}{2\,d^5}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^2*x^2*e^c - 2*d*x*e^c
+ 2*e^c)*a*b*e^(d*x)/d^3 - (d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^3 + 1/
2*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2
*e^(d*x)/d^5 - 1/2*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^
(-d*x - c)/d^5
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.32

$$\int \left(a+bx^2\right)^2 \cosh(c+dx)\,dx$$
$$=\frac{(b^2d^4x^4+2\,abd^4x^2-4\,b^2d^3x^3+a^2d^4-4\,abd^3x+12\,b^2d^2x^2+4\,abd^2-24\,b^2dx+24\,b^2)e^{(dx+c)}}{2\,d^5}-\frac{(b^2d^4x^4+2\,abd^4x^2+4\,b^2d^3x^3+a^2d^4+4\,abd^3x+12\,b^2d^2x^2+4\,abd^2+24\,b^2dx+24\,b^2)e^{(-dx-c)}}{2\,d^5}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^4*x^4 + 2*a*b*d^4*x^2 - 4*b^2*d^3*x^3 + a^2*d^4 - 4*a*b*d^3*x +
 12*b^2*d^2*x^2 + 4*a*b*d^2 - 24*b^2*d*x + 24*b^2)*e^(d*x + c)/d^5 - 1/2*(
b^2*d^4*x^4 + 2*a*b*d^4*x^2 + 4*b^2*d^3*x^3 + a^2*d^4 + 4*a*b*d^3*x + 12*b
^2*d^2*x^2 + 4*a*b*d^2 + 24*b^2*d*x + 24*b^2)*e^(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.84

$$\int \left(a+bx^2\right)^2 \cosh(c+dx)\,dx = \frac{\sinh(c+d\,x)\,\left(a^2\,d^4+4\,a\,b\,d^2+24\,b^2\right)}{d^5} - \frac{4\,b^2\,x^3\cosh(c+d\,x)}{d^2} + \frac{b^2\,x^4\sinh(c+d\,x)}{d} - \frac{4\,x\cosh(c+d\,x)\,\left(6\,b^2+a\,b\,d^2\right)}{d^4} + \frac{2\,x^2\sinh(c+d\,x)\,\left(6\,b^2+a\,b\,d^2\right)}{d^3}$$

input
```
int(cosh(c + d*x)*(a + b*x^2)^2,x)
```

output
```
(sinh(c + d*x)*(24*b^2 + a^2*d^4 + 4*a*b*d^2))/d^5 - (4*b^2*x^3*cosh(c + d
*x))/d^2 + (b^2*x^4*sinh(c + d*x))/d - (4*x*cosh(c + d*x)*(6*b^2 + a*b*d^2
))/d^4 + (2*x^2*sinh(c + d*x)*(6*b^2 + a*b*d^2))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \left(a+bx^2\right)^2 \cosh(c+dx)\,dx$$
$$= \frac{-4\cosh\left(dx+c\right)ab\,d^3x-4\cosh\left(dx+c\right)b^2d^3x^3-24\cosh\left(dx+c\right)b^2dx+\sinh\left(dx+c\right)a^2d^4+2\sinh\left(d\right.}{}$$

input
```
int((b*x^2+a)^2*cosh(d*x+c),x)
```

output
```
( - 4*cosh(c + d*x)*a*b*d**3*x - 4*cosh(c + d*x)*b**2*d**3*x**3 - 24*cosh(
c + d*x)*b**2*d*x + sinh(c + d*x)*a**2*d**4 + 2*sinh(c + d*x)*a*b*d**4*x**
2 + 4*sinh(c + d*x)*a*b*d**2 + sinh(c + d*x)*b**2*d**4*x**4 + 12*sinh(c +
d*x)*b**2*d**2*x**2 + 24*sinh(c + d*x)*b**2)/d**5
```

3.52 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x} dx$

Optimal result

Integrand size = 19, antiderivative size = 110

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x} dx = -\frac{6b^2 \cosh(c+dx)}{d^4} - \frac{2ab \cosh(c+dx)}{d^2} - \frac{3b^2x^2 \cosh(c+dx)}{d^2} + a^2 \cosh(c)\text{Chi}(dx) + \frac{6b^2x \sinh(c+dx)}{d^3} + \frac{2abx \sinh(c+dx)}{d} + \frac{b^2x^3 \sinh(c+dx)}{d} + a^2 \sinh(c)\text{Shi}(dx)$$

output

```
-6*b^2*cosh(d*x+c)/d^4-2*a*b*cosh(d*x+c)/d^2-3*b^2*x^2*cosh(d*x+c)/d^2+a^2
*cosh(c)*Chi(d*x)+6*b^2*x*sinh(d*x+c)/d^3+2*a*b*x*sinh(d*x+c)/d+b^2*x^3*si
nh(d*x+c)/d+a^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.75

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx = -\frac{b(2ad^2+3b(2+d^2x^2))\cosh(c+dx)}{d^4} + a^2\cosh(c)\text{Chi}(dx) + \frac{bx(2ad^2+b(6+d^2x^2))\sinh(c+dx)}{d^3} + a^2\sinh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

output
```
-((b*(2*a*d^2 + 3*b*(2 + d^2*x^2))*Cosh[c + d*x])/d^4) + a^2*Cosh[c]*CoshI
ntegral[d*x] + (b*x*(2*a*d^2 + b*(6 + d^2*x^2))*Sinh[c + d*x])/d^3 + a^2*S
inh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x}+2abx\cosh(c+dx)+b^2x^3\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2\cosh(c)\text{Chi}(dx)+a^2\sinh(c)\text{Shi}(dx)-\frac{2ab\cosh(c+dx)}{d^2}+\frac{2abx\sinh(c+dx)}{d}-\frac{6b^2\cosh(c+dx)}{d^4}+\frac{6b^2x\sinh(c+dx)}{d^3}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{b^2x^3\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Cosh[c + d*x])/x,x]
```

output
```
(-6*b^2*Cosh[c + d*x])/d^4 - (2*a*b*Cosh[c + d*x])/d^2 - (3*b^2*x^2*Cosh[c
 + d*x])/d^2 + a^2*Cosh[c]*CoshIntegral[d*x] + (6*b^2*x*Sinh[c + d*x])/d^3
 + (2*a*b*x*Sinh[c + d*x])/d + (b^2*x^3*Sinh[c + d*x])/d + a^2*Sinh[c]*Sin
hIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(110) = 220$.

Time = 0.60 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.05

method	result
risch	$\frac{e^{dx+c}b^2x^3}{2d} - \frac{e^{-dx-c}b^2x^3}{2d} - \frac{a^2e^c \operatorname{expIntegral}_1(-dx)}{2} - \frac{a^2e^{-c}\operatorname{expIntegral}_1(dx)}{2} + \frac{e^{dx+c}abx}{d} - \frac{3\,e^{dx+c}b^2x^2}{2d^2} - \frac{e^{-dx-c}ab}{d}$
meijerg	$\frac{8b^2\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}} - \frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}} + \frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4} - \frac{8ib^2\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}} - \frac{i\left(\frac{15x^2d^2}{2}+15\right.}{20\sqrt{}}\right.}{d^4}$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
1/2/d*exp(d*x+c)*b^2*x^3-1/2/d*exp(-d*x-c)*b^2*x^3-1/2*a^2*exp(c)*Ei(1,-d*
x)-1/2*a^2*exp(-c)*Ei(1,d*x)+1/d*exp(d*x+c)*a*b*x-3/2/d^2*exp(d*x+c)*b^2*x
^2-1/d*exp(-d*x-c)*a*b*x-3/2/d^2*exp(-d*x-c)*b^2*x^2-1/d^2*exp(d*x+c)*a*b+
3/d^3*exp(d*x+c)*b^2*x-1/d^2*exp(-d*x-c)*a*b-3/d^3*exp(-d*x-c)*b^2*x-3/d^4
*exp(d*x+c)*b^2-3/d^4*exp(-d*x-c)*b^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx = -\frac{2\,(3\,b^2d^2x^2+2\,abd^2+6\,b^2)\cosh(dx+c)-(a^2d^4\mathrm{Ei}(dx)+a^2d^4\mathrm{Ei}(-dx))\cosh(c)-2\,(b^2d^3x^3+2\,(abd}{2\,d^4}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

output
```
-1/2*(2*(3*b^2*d^2*x^2 + 2*a*b*d^2 + 6*b^2)*cosh(d*x + c) - (a^2*d^4*Ei(d*
x) + a^2*d^4*Ei(-d*x))*cosh(c) - 2*(b^2*d^3*x^3 + 2*(a*b*d^3 + 3*b^2*d)*x)
*sinh(d*x + c) - (a^2*d^4*Ei(d*x) - a^2*d^4*Ei(-d*x))*sinh(c))/d^4
```

Sympy [A] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.10

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx = a^2\sinh(c)\,\mathrm{Shi}(dx)+a^2\cosh(c)\,\mathrm{Chi}(dx) + 2ab\left(\begin{cases}\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2} & \text{for } d\neq 0\\ \frac{x^2\cosh(c)}{2} & \text{otherwise}\end{cases}\right) + b^2\left(\begin{cases}\frac{x^3\sinh(c+dx)}{d}-\frac{3x^2\cosh(c+dx)}{d^2}+\frac{6x\sinh(c+dx)}{d^3}-\frac{6\cosh(c+dx)}{d^4} & \text{for } d\neq 0\\ \frac{x^4\cosh(c)}{4} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**2+a)**2*cosh(d*x+c)/x,x)
```

output
```
a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x*sinh(c
+ d*x)/d - cosh(c + d*x)/d**2, Ne(d, 0)), (x**2*cosh(c)/2, True)) + b**2*P
iecewise((x**3*sinh(c + d*x)/d - 3*x**2*cosh(c + d*x)/d**2 + 6*x*sinh(c +
d*x)/d**3 - 6*cosh(c + d*x)/d**4, Ne(d, 0)), (x**4*cosh(c)/4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(110) = 220$.

Time = 0.13 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.14

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx =$$
$$-\frac{1}{8}\left(4\,ab\left(\frac{(d^2x^2e^c-2\,dxe^c+2\,e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2\,dx+2)e^{(-dx-c)}}{d^3}\right)+b^2\left(\frac{(d^4x^4e^c-4\,d^3x^3e^c+12\,d^2\right.\right.$$
$$+\frac{1}{4}\left(b^2x^4+4\,abx^2+2\,a^2\log\left(x^2\right)\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/8*(4*a*b*((d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*
d*x + 2)*e^(-d*x - c)/d^3) + b^2*((d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^
2*e^c - 24*d*x*e^c + 24*e^c)*e^(d*x)/d^5 + (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x
^2 + 24*d*x + 24)*e^(-d*x - c)/d^5) + 4*a^2*cosh(d*x + c)*log(x^2)/d - 4*(
Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/4*(b^2*x^4 + 4*a*b*x^2 + 2*a^2
*log(x^2))*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(110) = 220$.

Time = 0.11 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx$$
$$= \frac{b^2d^3x^3e^{(dx+c)}-b^2d^3x^3e^{(-dx-c)}+a^2d^4\mathrm{Ei}(-dx)\,e^{(-c)}+a^2d^4\mathrm{Ei}(dx)\,e^c+2\,abd^3xe^{(dx+c)}-3\,b^2d^2x^2e^{(dx+c)}-}{}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(b^2*d^3*x^3*e^(d*x + c) - b^2*d^3*x^3*e^(-d*x - c) + a^2*d^4*Ei(-d*x)
*e^(-c) + a^2*d^4*Ei(d*x)*e^c + 2*a*b*d^3*x*e^(d*x + c) - 3*b^2*d^2*x^2*e^
(d*x + c) - 2*a*b*d^3*x*e^(-d*x - c) - 3*b^2*d^2*x^2*e^(-d*x - c) - 2*a*b*
d^2*e^(d*x + c) + 6*b^2*d*x*e^(d*x + c) - 2*a*b*d^2*e^(-d*x - c) - 6*b^2*d
*x*e^(-d*x - c) - 6*b^2*e^(d*x + c) - 6*b^2*e^(-d*x - c))/d^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx=\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x}\,dx
=\frac{-2\cosh(dx+c)\,ab\,d^2-3\cosh(dx+c)\,b^2d^2x^2-6\cosh(dx+c)\,b^2+\left(\int\frac{\cosh(dx+c)}{x}dx\right)a^2d^4+2\sinh(dx}{d^4}$$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x,x)
```

output
```
( - 2*cosh(c + d*x)*a*b*d**2 - 3*cosh(c + d*x)*b**2*d**2*x**2 - 6*cosh(c +
 d*x)*b**2 + int(cosh(c + d*x)/x,x)*a**2*d**4 + 2*sinh(c + d*x)*a*b*d**3*x
 + sinh(c + d*x)*b**2*d**3*x**3 + 6*sinh(c + d*x)*b**2*d*x)/d**4
```

3.53 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 95

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} \, dx = -\frac{a^2 \cosh(c+dx)}{x} - \frac{2b^2 x \cosh(c+dx)}{d^2} + a^2 d \text{Chi}(dx) \sinh(c) + \frac{2b^2 \sinh(c+dx)}{d^3} + \frac{2ab \sinh(c+dx)}{d} + \frac{b^2 x^2 \sinh(c+dx)}{d} + a^2 d \cosh(c) \text{Shi}(dx)$$

output

```
-a^2*cosh(d*x+c)/x-2*b^2*x*cosh(d*x+c)/d^2+a^2*d*Chi(d*x)*sinh(c)+2*b^2*si
nh(d*x+c)/d^3+2*a*b*sinh(d*x+c)/d+b^2*x^2*sinh(d*x+c)/d+a^2*d*cosh(c)*Shi(
d*x)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2}\,dx = -\frac{a^2\cosh(c+dx)}{x} - \frac{2b^2x\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)\sinh(c) + \frac{2b^2\sinh(c+dx)}{d^3} + \frac{2ab\sinh(c+dx)}{d} + \frac{b^2x^2\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

output
```
-((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegra
l[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^
2*x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^2} + 2ab\cosh(c+dx) + b^2x^2\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a^2 d \sinh(c)\mathrm{Chi}(dx) + a^2 d \cosh(c)\mathrm{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} + \frac{2ab \sinh(c+dx)}{d} + \frac{2b^2 \sinh(c+dx)}{d^3} - \frac{2b^2 x \cosh(c+dx)}{d^2} + \frac{b^2 x^2 \sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Cosh[c + d*x])/x^2,x]
```

output
```
-((a^2*Cosh[c + d*x])/x) - (2*b^2*x*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegra
l[d*x]*Sinh[c] + (2*b^2*Sinh[c + d*x])/d^3 + (2*a*b*Sinh[c + d*x])/d + (b^
2*x^2*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(95) = 190$.

Time = 0.64 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.11

method	result
risch	$-\frac{-e^{-c} \operatorname{expIntegral}_1(dx) a^2 d^4 x + e^{c} \operatorname{expIntegral}_1(-dx) a^2 d^4 x + e^{-dx-c} b^2 d^2 x^3 - e^{dx+c} b^2 d^2 x^3 + e^{-dx-c} a^2 d^3 + 2ab\, d^2 x\, e^{-dx-c} + 2\, e^{-}}{2d^3 x}$
meijerg	$\frac{4ib^2 \cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}} - \frac{i\left(\frac{3x^2d^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3} + \frac{4b^2 \sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}} + \frac{\left(\frac{x^2d^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}} - \frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^3} + \frac{2ab\cosh}{}$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^3*(-exp(-c)*Ei(1,d*x)*a^2*d^4*x+exp(c)*Ei(1,-d*x)*a^2*d^4*x+exp(-d*
x-c)*b^2*d^2*x^3-exp(d*x+c)*b^2*d^2*x^3+exp(-d*x-c)*a^2*d^3+2*a*b*d^2*x*ex
p(-d*x-c)+2*exp(-d*x-c)*b^2*d*x^2+exp(d*x+c)*a^2*d^3-2*a*b*d^2*x*exp(d*x+c
)+2*exp(d*x+c)*b^2*d*x^2+2*exp(-d*x-c)*b^2*x-2*exp(d*x+c)*b^2*x)/x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.34

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} \, dx = -\frac{2\,(a^2d^3+2\,b^2dx^2)\cosh\left(dx+c\right)-\left(a^2d^4x\mathrm{Ei}(dx)-a^2d^4x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-2\,(b^2d^2x^3+2\,(abd^2+b^2)}{2\,d^3x}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2*d^3 + 2*b^2*d*x^2)*cosh(d*x + c) - (a^2*d^4*x*Ei(d*x) - a^2*d
^4*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^2*x^3 + 2*(a*b*d^2 + b^2)*x)*sinh(d*x +
c) - (a^2*d^4*x*Ei(d*x) + a^2*d^4*x*Ei(-d*x))*sinh(c))/(d^3*x)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^2} \, dx = \int \frac{(a+bx^2)^2 \cosh\left(c+dx\right)}{x^2} \, dx$$

input
```
integrate((b*x**2+a)**2*cosh(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**2)**2*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.88

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx =$$
$$-\frac{1}{6}\left(3\,a^2\mathrm{Ei}(-dx)\,e^{(-c)} - 3\,a^2\mathrm{Ei}(dx)\,e^c + \frac{6\,(dxe^c - e^c)abe^{(dx)}}{d^2} + \frac{6\,(dx+1)abe^{(-dx-c)}}{d^2} + \frac{(d^3x^3e^c - 3\,d^2x\right.$$
$$+\frac{1}{3}\left(b^2x^3 + 6\,abx - \frac{3\,a^2}{x}\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
-1/6*(3*a^2*Ei(-d*x)*e^(-c) - 3*a^2*Ei(d*x)*e^c + 6*(d*x*e^c - e^c)*a*b*e^
(d*x)/d^2 + 6*(d*x + 1)*a*b*e^(-d*x - c)/d^2 + (d^3*x^3*e^c - 3*d^2*x^2*e^
c + 6*d*x*e^c - 6*e^c)*b^2*e^(d*x)/d^4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)
*b^2*e^(-d*x - c)/d^4)*d + 1/3*(b^2*x^3 + 6*a*b*x - 3*a^2/x)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(95) = 190.

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx =$$
$$-\frac{a^2d^4x\mathrm{Ei}(-dx)\,e^{(-c)} - a^2d^4x\mathrm{Ei}(dx)\,e^c - b^2d^2x^3e^{(dx+c)} + b^2d^2x^3e^{(-dx-c)} + a^2d^3e^{(dx+c)} - 2\,abd^2xe^{(dx+c)}}{2\,d^3x}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a^2*d^4*x*Ei(-d*x)*e^(-c) - a^2*d^4*x*Ei(d*x)*e^c - b^2*d^2*x^3*e^(d
*x + c) + b^2*d^2*x^3*e^(-d*x - c) + a^2*d^3*e^(d*x + c) - 2*a*b*d^2*x*e^(
d*x + c) + 2*b^2*d*x^2*e^(d*x + c) + a^2*d^3*e^(-d*x - c) + 2*a*b*d^2*x*e^
(-d*x - c) + 2*b^2*d*x^2*e^(-d*x - c) - 2*b^2*x*e^(d*x + c) + 2*b^2*x*e^(-
d*x - c))/(d^3*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^2}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^2,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.01

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^2}\,dx$$
$$= \frac{-e^{dx}ei(-dx)\,a^2d^4x + e^{dx+2c}ei(dx)\,a^2d^4x - e^{2dx+2c}a^2d^3 + 2e^{2dx+2c}ab\,d^2x + e^{2dx+2c}b^2d^2x^3 - 2e^{2dx+2c}b^2d\,x}{2e^{dx+c}d^3x}$$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^2,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a**2*d**4*x + e**(2*c + d*x)*ei(d*x)*a**2*d**4*x -
 e**(2*c + 2*d*x)*a**2*d**3 + 2*e**(2*c + 2*d*x)*a*b*d**2*x + e**(2*c + 2*
d*x)*b**2*d**2*x**3 - 2*e**(2*c + 2*d*x)*b**2*d*x**2 + 2*e**(2*c + 2*d*x)*
b**2*x - a**2*d**3 - 2*a*b*d**2*x - b**2*d**2*x**3 - 2*b**2*d*x**2 - 2*b**
2*x)/(2*e**(c + d*x)*d**3*x)
```

3.54 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 114

$$\begin{aligned}\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} \, dx = & -\frac{b^2 \cosh(c+dx)}{d^2} - \frac{a^2 \cosh(c+dx)}{2x^2} \\ & + 2ab \cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2 d^2 \cosh(c)\text{Chi}(dx) \\ & - \frac{a^2 d \sinh(c+dx)}{2x} + \frac{b^2 x \sinh(c+dx)}{d} \\ & + 2ab \sinh(c)\text{Shi}(dx) + \frac{1}{2}a^2 d^2 \sinh(c)\text{Shi}(dx)\end{aligned}$$

output
```
-b^2*cosh(d*x+c)/d^2-1/2*a^2*cosh(d*x+c)/x^2+2*a*b*cosh(c)*Chi(d*x)+1/2*a^
2*d^2*cosh(c)*Chi(d*x)-1/2*a^2*d*sinh(d*x+c)/x+b^2*x*sinh(d*x+c)/d+2*a*b*s
inh(c)*Shi(d*x)+1/2*a^2*d^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.85

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} \, dx = \frac{1}{2}\left(-\frac{2b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x^2} + a(4b+ad^2)\cosh(c)\text{Chi}(dx) - \frac{a^2 d\sinh(c+dx)}{x} + \frac{2b^2 x\sinh(c+dx)}{d} + a(4b+ad^2)\sinh(c)\text{Shi}(dx)\right)$$

input
```
Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

output
```
((-2*b^2*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x^2 + a*(4*b + a*d^2)*Co
sh[c]*CoshIntegral[d*x] - (a^2*d*Sinh[c + d*x])/x + (2*b^2*x*Sinh[c + d*x]
)/d + a*(4*b + a*d^2)*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} \, dx$$

$$\Big\downarrow \; 5810$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^3} + \frac{2ab\cosh(c+dx)}{x} + b^2 x\cosh(c+dx)\right) dx$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}a^2 d^2\cosh(c)\text{Chi}(dx) + \frac{1}{2}a^2 d^2\sinh(c)\text{Shi}(dx) - \frac{a^2\cosh(c+dx)}{2x^2} - \frac{a^2 d\sinh(c+dx)}{2x} + 2ab\cosh(c)\text{Chi}(dx) + 2ab\sinh(c)\text{Shi}(dx) - \frac{b^2\cosh(c+dx)}{d^2} + \frac{b^2 x\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Cosh[c + d*x])/x^3,x]
```

output
```
-((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(2*x^2) + 2*a*b*Cosh[c]*C
oshIntegral[d*x] + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 - (a^2*d*Sinh[c +
 d*x])/(2*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*Sinh[c]*SinhIntegral[d*x] +
 (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.83

method	result
risch	$-\frac{\mathrm{e}^{-c} \operatorname{expIntegral}_1(dx) a^2 d^4 x^2+\mathrm{e}^{c} \operatorname{expIntegral}_1(-dx) a^2 d^4 x^2+4\,\mathrm{e}^{-c} \operatorname{expIntegral}_1(dx) ab\, d^2 x^2+4\,\mathrm{e}^{c} \operatorname{expIntegral}_1(-dx) ab\, d^2 x^2+\mathrm{e}^{d}}{4d^2 x^2}$
meijerg	$-\frac{2b^2 \cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b^2\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+ab\cosh\left(c\right)\sqrt{\pi}\left(\frac{2\ \mathrm{Chi}(dx)-2\ln(}{\sqrt{\pi}}\right.$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d^2*(exp(-c)*Ei(1,d*x)*a^2*d^4*x^2+exp(c)*Ei(1,-d*x)*a^2*d^4*x^2+4*ex
p(-c)*Ei(1,d*x)*a*b*d^2*x^2+4*exp(c)*Ei(1,-d*x)*a*b*d^2*x^2+exp(d*x+c)*a^2
*d^3*x-2*b^2*d*x^3*exp(d*x+c)-exp(-d*x-c)*a^2*d^3*x+2*b^2*d*x^3*exp(-d*x-c
)+a^2*d^2*exp(d*x+c)+2*exp(d*x+c)*b^2*x^2+a^2*d^2*exp(-d*x-c)+2*exp(-d*x-c
)*b^2*x^2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx = -\frac{2(a^2d^2+2b^2x^2)\cosh(dx+c)-((a^2d^4+4abd^2)x^2\text{Ei}(dx)+(a^2d^4+4abd^2)x^2\text{Ei}(-dx))\cosh(c)+2(\cdots}{4d^2x^2}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(a^2*d^2 + 2*b^2*x^2)*cosh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*E
i(d*x) + (a^2*d^4 + 4*a*b*d^2)*x^2*Ei(-d*x))*cosh(c) + 2*(a^2*d^3*x - 2*b^
2*d*x^3)*sinh(d*x + c) - ((a^2*d^4 + 4*a*b*d^2)*x^2*Ei(d*x) - (a^2*d^4 + 4
*a*b*d^2)*x^2*Ei(-d*x))*sinh(c))/(d^2*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx = \int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx$$

input
```
integrate((b*x**2+a)**2*cosh(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**2)**2*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.45

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3} dx$$
$$= \frac{1}{4}\left(\left(de^{(-c)}\Gamma(-1,dx)+de^{c}\Gamma(-1,-dx)\right)a^2 - b^2\left(\frac{(d^2x^2e^c-2dxe^c+2e^c)e^{(dx)}}{d^3}+\frac{(d^2x^2+2dx+2)e^{(-dx-\cdots}}{d^3}\cdots\right.\right.$$
$$+\frac{1}{2}\left(b^2x^2+2ab\log(x^2)-\frac{a^2}{x^2}\right)\cosh(dx+c)$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/4*((d*e^(-c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a^2 - b^2*((d^2*x^2
*e^c - 2*d*x*e^c + 2*e^c)*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*e^(-d*x - c)
/d^3) - 4*a*b*cosh(d*x + c)*log(x^2)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)
*a*b/d)*d + 1/2*(b^2*x^2 + 2*a*b*log(x^2) - a^2/x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3}\,dx$$
$$= \frac{a^2d^4x^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^4x^2\mathrm{Ei}(dx)\,e^c + 4\,abd^2x^2\mathrm{Ei}(-dx)\,e^{(-c)} + 4\,abd^2x^2\mathrm{Ei}(dx)\,e^c - a^2d^3xe^{(dx+c)} + 2\,}{4}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a^2*d^4*x^2*Ei(-d*x)*e^(-c) + a^2*d^4*x^2*Ei(d*x)*e^c + 4*a*b*d^2*x^2
*Ei(-d*x)*e^(-c) + 4*a*b*d^2*x^2*Ei(d*x)*e^c - a^2*d^3*x*e^(d*x + c) + 2*b
^2*d*x^3*e^(d*x + c) + a^2*d^3*x*e^(-d*x - c) - 2*b^2*d*x^3*e^(-d*x - c) -
 a^2*d^2*e^(d*x + c) - 2*b^2*x^2*e^(d*x + c) - a^2*d^2*e^(-d*x - c) - 2*b^
2*x^2*e^(-d*x - c))/(d^2*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^3}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^3,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{e^{dx}ei(-dx)\,a^2d^4x^2+4e^{dx}ei(-dx)\,ab\,d^2x^2+e^{dx+2c}ei(dx)\,a^2d^4x^2+4e^{dx+2c}ei(dx)\,ab\,d^2x^2-e^{2dx+2c}a^2d^3x}{4e^{dx+c}d^2x^2}$$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^3,x)
```

output
```
(e**(d*x)*ei( - d*x)*a**2*d**4*x**2 + 4*e**(d*x)*ei( - d*x)*a*b*d**2*x**2
+ e**(2*c + d*x)*ei(d*x)*a**2*d**4*x**2 + 4*e**(2*c + d*x)*ei(d*x)*a*b*d**
2*x**2 - e**(2*c + 2*d*x)*a**2*d**3*x - e**(2*c + 2*d*x)*a**2*d**2 + 2*e**
(2*c + 2*d*x)*b**2*d*x**3 - 2*e**(2*c + 2*d*x)*b**2*x**2 + a**2*d**3*x - a
**2*d**2 - 2*b**2*d*x**3 - 2*b**2*x**2)/(4*e**(c + d*x)*d**2*x**2)
```

3.55 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 133

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4} \, dx = -\frac{a^2 \cosh(c+dx)}{3x^3} - \frac{2ab \cosh(c+dx)}{x} - \frac{a^2 d^2 \cosh(c+dx)}{6x} + 2abd\text{Chi}(dx)\sinh(c) + \frac{1}{6}a^2 d^3 \text{Chi}(dx)\sinh(c) + \frac{b^2 \sinh(c+dx)}{d} - \frac{a^2 d \sinh(c+dx)}{6x^2} + 2abd\cosh(c)\text{Shi}(dx) + \frac{1}{6}a^2 d^3 \cosh(c)\text{Shi}(dx)$$

output
```
-1/3*a^2*cosh(d*x+c)/x^3-2*a*b*cosh(d*x+c)/x-1/6*a^2*d^2*cosh(d*x+c)/x+2*a
*b*d*Chi(d*x)*sinh(c)+1/6*a^2*d^3*Chi(d*x)*sinh(c)+b^2*sinh(d*x+c)/d-1/6*a
^2*d*sinh(d*x+c)/x^2+2*a*b*d*cosh(c)*Shi(d*x)+1/6*a^2*d^3*cosh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(-\frac{2a^2\cosh(c+dx)}{x^3}-\frac{12ab\cosh(c+dx)}{x}-\frac{a^2d^2\cosh(c+dx)}{x}+ad(12b+ad^2)\operatorname{Chi}(dx)\sinh(c)+\frac{6b^2\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{x^2}+ad(12b+ad^2)\cosh(c)\operatorname{Shi}(dx)\right)$$

input
```
Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

output
```
((-2*a^2*Cosh[c + d*x])/x^3 - (12*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
 d*x])/x + a*d*(12*b + a*d^2)*CoshIntegral[d*x]*Sinh[c] + (6*b^2*Sinh[c +
d*x])/d - (a^2*d*Sinh[c + d*x])/x^2 + a*d*(12*b + a*d^2)*Cosh[c]*SinhInteg
ral[d*x])/6
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx$$

$$\Big\downarrow \text{ 5810}$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x^2}+b^2\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d\sinh(c+dx)}{6x^2}+2abd\sinh(c)\text{Chi}(dx)+2abd\cosh(c)\text{Shi}(dx)-\frac{2ab\cosh(c+dx)}{x}+\frac{b^2\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^2)^2*Cosh[c + d*x])/x^4,x]
```

output
```
-1/3*(a^2*Cosh[c + d*x])/x^3 - (2*a*b*Cosh[c + d*x])/x - (a^2*d^2*Cosh[c +
 d*x])/(6*x) + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] + (a^2*d^3*CoshIntegral[d
*x]*Sinh[c])/6 + (b^2*Sinh[c + d*x])/d - (a^2*d*Sinh[c + d*x])/(6*x^2) + 2
*a*b*d*Cosh[c]*SinhIntegral[d*x] + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

method	result
risch	$-\frac{-e^{-c}\operatorname{expIntegral}_1(dx)a^2d^4x^3+e^{c}\operatorname{expIntegral}_1(-dx)a^2d^4x^3-12\,e^{-c}\operatorname{expIntegral}_1(dx)ab\,d^2x^3+12\,e^{c}\operatorname{expIntegral}_1(-dx)ab\,d^2x^3}{}$
meijerg	$\frac{b^2\cosh(c)\sinh(dx)}{d}-\frac{b^2\sinh(c)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}+\frac{idab\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\operatorname{Shi}(dx)}{\sqrt{\pi}}\right)}{2}+\frac{dba\sinh(c)\sqrt{\pi}\left(\frac{4}{\sqrt{\pi}}-\frac{4\sinh}{\sqrt{\pi}}\right.}{}$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12/d*(-exp(-c)*Ei(1,d*x)*a^2*d^4*x^3+exp(c)*Ei(1,-d*x)*a^2*d^4*x^3-12*e
xp(-c)*Ei(1,d*x)*a*b*d^2*x^3+12*exp(c)*Ei(1,-d*x)*a*b*d^2*x^3+exp(-d*x-c)*
a^2*d^3*x^2+exp(d*x+c)*a^2*d^3*x^2-exp(-d*x-c)*a^2*d^2*x+12*a*b*d*x^2*exp(
-d*x-c)+6*exp(-d*x-c)*b^2*x^3+exp(d*x+c)*a^2*d^2*x+12*a*b*d*x^2*exp(d*x+c)
-6*exp(d*x+c)*b^2*x^3+2*exp(-d*x-c)*a^2*d+2*exp(d*x+c)*a^2*d)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.29

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx = -\frac{2\,(2\,a^2d+(a^2d^3+12\,abd)x^2)\cosh\left(dx+c\right)-((a^2d^4+12\,abd^2)x^3\mathrm{Ei}(dx)-(a^2d^4+12\,abd^2)x^3\mathrm{Ei}(-dx)$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(2*(2*a^2*d + (a^2*d^3 + 12*a*b*d)*x^2)*cosh(d*x + c) - ((a^2*d^4 +
12*a*b*d^2)*x^3*Ei(d*x) - (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*cosh(c) + 2
*(a^2*d^2*x - 6*b^2*x^3)*sinh(d*x + c) - ((a^2*d^4 + 12*a*b*d^2)*x^3*Ei(d*
x) + (a^2*d^4 + 12*a*b*d^2)*x^3*Ei(-d*x))*sinh(c))/(d*x^3)
```

Sympy [F]

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx^2)^2\cosh\left(c+dx\right)}{x^4}\,dx$$

input
```
integrate((b*x**2+a)**2*cosh(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**2)**2*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.02

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4}\,dx$$
$$= \frac{1}{6}\left(a^2d^2e^{(-c)}\Gamma(-2,dx) - a^2d^2e^c\Gamma(-2,-dx) - 6\,ab\mathrm{Ei}(-dx)\,e^{(-c)} + 6\,ab\mathrm{Ei}(dx)\,e^c - \frac{3\,(dxe^c - e^c)b^2e^{(dx)}}{d^2} \right.$$
$$+ \frac{1}{3}\left(3\,b^2x - \frac{6\,abx^2 + a^2}{x^3}\right)\cosh(dx+c)$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
1/6*(a^2*d^2*e^(-c)*gamma(-2, d*x) - a^2*d^2*e^c*gamma(-2, -d*x) - 6*a*b*E
i(-d*x)*e^(-c) + 6*a*b*Ei(d*x)*e^c - 3*(d*x*e^c - e^c)*b^2*e^(d*x)/d^2 - 3
*(d*x + 1)*b^2*e^(-d*x - c)/d^2)*d + 1/3*(3*b^2*x - (6*a*b*x^2 + a^2)/x^3)
*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.77

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{a^2d^4x^3\mathrm{Ei}(-dx)\,e^{(-c)} - a^2d^4x^3\mathrm{Ei}(dx)\,e^c + 12\,abd^2x^3\mathrm{Ei}(-dx)\,e^{(-c)} - 12\,abd^2x^3\mathrm{Ei}(dx)\,e^c + a^2d^3x^2e^{(dx+c)}}{}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

output
```
-1/12*(a^2*d^4*x^3*Ei(-d*x)*e^(-c) - a^2*d^4*x^3*Ei(d*x)*e^c + 12*a*b*d^2*
x^3*Ei(-d*x)*e^(-c) - 12*a*b*d^2*x^3*Ei(d*x)*e^c + a^2*d^3*x^2*e^(d*x + c)
 + a^2*d^3*x^2*e^(-d*x - c) + a^2*d^2*x*e^(d*x + c) + 12*a*b*d*x^2*e^(d*x
+ c) - 6*b^2*x^3*e^(d*x + c) - a^2*d^2*x*e^(-d*x - c) + 12*a*b*d*x^2*e^(-d
*x - c) + 6*b^2*x^3*e^(-d*x - c) + 2*a^2*d*e^(d*x + c) + 2*a^2*d*e^(-d*x -
 c))/(d*x^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4}\, dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^4}\, dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^4,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.79

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^4}\, dx = \frac{-e^{dx} ei(-dx)\, a^2 d^4 x^3 - 12 e^{dx} ei(-dx)\, ab\, d^2 x^3 + e^{dx+2c} ei(dx)\, a^2 d^4 x^3 + 12 e^{dx+2c} ei(dx)\, ab\, d^2 x^3 - e^{2dx+2c} a^2 \cdots}{1 \cdots}$$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^4,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a**2*d**4*x**3 - 12*e**(d*x)*ei( - d*x)*a*b*d**2*x
**3 + e**(2*c + d*x)*ei(d*x)*a**2*d**4*x**3 + 12*e**(2*c + d*x)*ei(d*x)*a*
b*d**2*x**3 - e**(2*c + 2*d*x)*a**2*d**3*x**2 - e**(2*c + 2*d*x)*a**2*d**2
*x - 2*e**(2*c + 2*d*x)*a**2*d - 12*e**(2*c + 2*d*x)*a*b*d*x**2 + 6*e**(2*
c + 2*d*x)*b**2*x**3 - a**2*d**3*x**2 + a**2*d**2*x - 2*a**2*d - 12*a*b*d*
x**2 - 6*b**2*x**3)/(12*e**(c + d*x)*d*x**3)
```

3.56 $\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 175

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx = -\frac{a^2\cosh(c+dx)}{4x^4} - \frac{ab\cosh(c+dx)}{x^2} - \frac{a^2d^2\cosh(c+dx)}{24x^2} + b^2\cosh(c)\text{Chi}(dx) + abd^2\cosh(c)\text{Chi}(dx) + \frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx) - \frac{a^2d\sinh(c+dx)}{12x^3} - \frac{abd\sinh(c+dx)}{x} - \frac{a^2d^3\sinh(c+dx)}{24x} + b^2\sinh(c)\text{Shi}(dx) + abd^2\sinh(c)\text{Shi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)$$

output
```
-1/4*a^2*cosh(d*x+c)/x^4-a*b*cosh(d*x+c)/x^2-1/24*a^2*d^2*cosh(d*x+c)/x^2+
b^2*cosh(c)*Chi(d*x)+a*b*d^2*cosh(c)*Chi(d*x)+1/24*a^2*d^4*cosh(c)*Chi(d*x
)-1/12*a^2*d*sinh(d*x+c)/x^3-a*b*d*sinh(d*x+c)/x-1/24*a^2*d^3*sinh(d*x+c)/
x+b^2*sinh(c)*Shi(d*x)+a*b*d^2*sinh(c)*Shi(d*x)+1/24*a^2*d^4*sinh(c)*Shi(d
*x)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.71

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx$$
$$= \frac{(24b^2+24abd^2+a^2d^4)\,x^4\cosh(c)\text{Chi}(dx)-a((6a+24bx^2+ad^2x^2)\cosh(c+dx)+dx(2a+24bx^2+ac}{24x^4}$$

input
```
Integrate[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

output
```
((24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Cosh[c]*CoshIntegral[d*x] - a*((6*a +
 24*b*x^2 + a*d^2*x^2)*Cosh[c + d*x] + d*x*(2*a + 24*b*x^2 + a*d^2*x^2)*Si
nh[c + d*x]) + (24*b^2 + 24*a*b*d^2 + a^2*d^4)*x^4*Sinh[c]*SinhIntegral[d*
x])/(24*x^4)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx$$
$$\Big\downarrow \text{ 5810}$$
$$\int \left(\frac{a^2\cosh(c+dx)}{x^5}+\frac{2ab\cosh(c+dx)}{x^3}+\frac{b^2\cosh(c+dx)}{x}\right)dx$$
$$\Big\downarrow \text{ 2009}$$
$$\frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx)+\frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)-\frac{a^2d^3\sinh(c+dx)}{24x}-\frac{a^2d^2\cosh(c+dx)}{24x^2}-\frac{a^2\cosh(c+dx)}{4x^4}-\frac{a^2d\sinh(c+dx)}{12x^3}+abd^2\cosh(c)\text{Chi}(dx)+abd^2\sinh(c)\text{Shi}(dx)-\frac{ab\cosh(c+dx)}{x^2}-\frac{abd\sinh(c+dx)}{x}+b^2\cosh(c)\text{Chi}(dx)+b^2\sinh(c)\text{Shi}(dx)$$

input
```
Int[((a + b*x^2)^2*Cosh[c + d*x])/x^5,x]
```

output
```
-1/4*(a^2*Cosh[c + d*x])/x^4 - (a*b*Cosh[c + d*x])/x^2 - (a^2*d^2*Cosh[c +
 d*x])/(24*x^2) + b^2*Cosh[c]*CoshIntegral[d*x] + a*b*d^2*Cosh[c]*CoshInte
gral[d*x] + (a^2*d^4*Cosh[c]*CoshIntegral[d*x])/24 - (a^2*d*Sinh[c + d*x])
/(12*x^3) - (a*b*d*Sinh[c + d*x])/x - (a^2*d^3*Sinh[c + d*x])/(24*x) + b^2
*Sinh[c]*SinhIntegral[d*x] + a*b*d^2*Sinh[c]*SinhIntegral[d*x] + (a^2*d^4*
Sinh[c]*SinhIntegral[d*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 299, normalized size of antiderivative = 1.71

method	result
risch	$-\frac{\mathrm{e}^{c} \operatorname{expIntegral}_{1}(-dx) a^{2} d^{4} x^{4}+\mathrm{e}^{-c} \operatorname{expIntegral}_{1}(dx) a^{2} d^{4} x^{4}+24\, \mathrm{e}^{c} \operatorname{expIntegral}_{1}(-dx) ab\, d^{2} x^{4}+24\, \mathrm{e}^{-c} \operatorname{expIntegral}_{1}(dx) ab\, d^{2} x^{4}+}{}$
meijerg	$\frac{b^{2} \cosh(c) \sqrt{\pi}\left(\frac{2\ \mathrm{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+b^{2} \sinh(c) \operatorname{Shi}(dx)-\frac{d^{2} ab \cosh(c) \sqrt{\pi}\left(-\frac{4\left(\frac{9x^{2} d^{2}}{2}+3\right)}{3\sqrt{\pi}\, x^{2} d^{2}}+\frac{4 c}{\sqrt{}}\right.}{}$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
-1/48*(exp(c)*Ei(1,-d*x)*a^2*d^4*x^4+exp(-c)*Ei(1,d*x)*a^2*d^4*x^4+24*exp(
c)*Ei(1,-d*x)*a*b*d^2*x^4+24*exp(-c)*Ei(1,d*x)*a*b*d^2*x^4+a^2*d^3*x^3*exp
(d*x+c)-a^2*d^3*x^3*exp(-d*x-c)+24*exp(c)*Ei(1,-d*x)*b^2*x^4+24*exp(-c)*Ei
(1,d*x)*b^2*x^4+a^2*d^2*x^2*exp(d*x+c)+24*a*b*d*x^3*exp(d*x+c)+a^2*d^2*x^2
*exp(-d*x-c)-24*a*b*d*x^3*exp(-d*x-c)+2*a^2*d*x*exp(d*x+c)+24*exp(d*x+c)*a
*b*x^2-2*a^2*d*x*exp(-d*x-c)+24*exp(-d*x-c)*a*b*x^2+6*exp(d*x+c)*a^2+6*exp
(-d*x-c)*a^2)/x^4
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.11

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx =$$
$$-\frac{2\left((a^2d^2+24\,ab)x^2+6\,a^2\right)\cosh\left(dx+c\right)-\left((a^2d^4+24\,abd^2+24\,b^2)x^4\mathrm{Ei}(dx)+(a^2d^4+24\,abd^2+24\,\ldots\right.}{\ldots}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
-1/48*(2*((a^2*d^2 + 24*a*b)*x^2 + 6*a^2)*cosh(d*x + c) - ((a^2*d^4 + 24*a
*b*d^2 + 24*b^2)*x^4*Ei(d*x) + (a^2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(-d*x
))*cosh(c) + 2*(2*a^2*d*x + (a^2*d^3 + 24*a*b*d)*x^3)*sinh(d*x + c) - ((a^
2*d^4 + 24*a*b*d^2 + 24*b^2)*x^4*Ei(d*x) - (a^2*d^4 + 24*a*b*d^2 + 24*b^2)
*x^4*Ei(-d*x))*sinh(c))/x^4
```

Sympy [F]

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx^2)^2\cosh\left(c+dx\right)}{x^5}\,dx$$

input
```
integrate((b*x**2+a)**2*cosh(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x**2)**2*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.79

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$$
$$= \frac{1}{8}\left(\left(d^3e^{(-c)}\Gamma(-3,dx) + d^3e^c\Gamma(-3,-dx)\right)a^2 + 4\left(de^{(-c)}\Gamma(-1,dx) + de^c\Gamma(-1,-dx)\right)ab - \frac{4\,b^2\cosh\left(dx+\right.}{d}\right.$$
$$+ \frac{1}{4}\left(2\,b^2\log\left(x^2\right) - \frac{4\,abx^2+a^2}{x^4}\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
1/8*((d^3*e^(-c)*gamma(-3, d*x) + d^3*e^c*gamma(-3, -d*x))*a^2 + 4*(d*e^(-
c)*gamma(-1, d*x) + d*e^c*gamma(-1, -d*x))*a*b - 4*b^2*cosh(d*x + c)*log(x
^2)/d + 4*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b^2/d)*d + 1/4*(2*b^2*log(x^2) -
 (4*a*b*x^2 + a^2)/x^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^2)^2 \cosh(c+dx)}{x^5} dx$$
$$= \frac{a^2d^4x^4\text{Ei}(-dx)\,e^{(-c)} + a^2d^4x^4\text{Ei}(dx)\,e^c + 24\,abd^2x^4\text{Ei}(-dx)\,e^{(-c)} + 24\,abd^2x^4\text{Ei}(dx)\,e^c - a^2d^3x^3e^{(dx+c)} -}{}$$

input
```
integrate((b*x^2+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

output
```
1/48*(a^2*d^4*x^4*Ei(-d*x)*e^(-c) + a^2*d^4*x^4*Ei(d*x)*e^c + 24*a*b*d^2*x
^4*Ei(-d*x)*e^(-c) + 24*a*b*d^2*x^4*Ei(d*x)*e^c - a^2*d^3*x^3*e^(d*x + c)
+ a^2*d^3*x^3*e^(-d*x - c) + 24*b^2*x^4*Ei(-d*x)*e^(-c) + 24*b^2*x^4*Ei(d*
x)*e^c - a^2*d^2*x^2*e^(d*x + c) - 24*a*b*d*x^3*e^(d*x + c) - a^2*d^2*x^2*
e^(-d*x - c) + 24*a*b*d*x^3*e^(-d*x - c) - 2*a^2*d*x*e^(d*x + c) - 24*a*b*
x^2*e^(d*x + c) + 2*a^2*d*x*e^(-d*x - c) - 24*a*b*x^2*e^(-d*x - c) - 6*a^2
*e^(d*x + c) - 6*a^2*e^(-d*x - c))/x^4
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx=\int \frac{\cosh(c+d\,x)\,\left(b\,x^2+a\right)^2}{x^5}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^5,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^2)^2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.70

$$\int \frac{(a+bx^2)^2\cosh(c+dx)}{x^5}\,dx = \frac{e^{dx}ei(-dx)\,a^2d^4x^4+24e^{dx}ei(-dx)\,ab\,d^2x^4+24e^{dx}ei(-dx)\,b^2x^4+e^{dx+2c}ei(dx)\,a^2d^4x^4+24e^{dx+2c}ei(dx)}{}$$

input
```
int((b*x^2+a)^2*cosh(d*x+c)/x^5,x)
```

output
```
(e**(d*x)*ei( - d*x)*a**2*d**4*x**4 + 24*e**(d*x)*ei( - d*x)*a*b*d**2*x**4
 + 24*e**(d*x)*ei( - d*x)*b**2*x**4 + e**(2*c + d*x)*ei(d*x)*a**2*d**4*x**
4 + 24*e**(2*c + d*x)*ei(d*x)*a*b*d**2*x**4 + 24*e**(2*c + d*x)*ei(d*x)*b*
*2*x**4 - e**(2*c + 2*d*x)*a**2*d**3*x**3 - e**(2*c + 2*d*x)*a**2*d**2*x**
2 - 2*e**(2*c + 2*d*x)*a**2*d*x - 6*e**(2*c + 2*d*x)*a**2 - 24*e**(2*c + 2
*d*x)*a*b*d*x**3 - 24*e**(2*c + 2*d*x)*a*b*x**2 + a**2*d**3*x**3 - a**2*d*
*2*x**2 + 2*a**2*d*x - 6*a**2 + 24*a*b*d*x**3 - 24*a*b*x**2)/(48*e**(c + d
*x)*x**4)
```

3.57 $\int \frac{x^4\cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 273

$$\begin{aligned}\int \frac{x^4\cosh(c+dx)}{a+bx^2}\,dx = &-\frac{2x\cosh(c+dx)}{bd^2}+\frac{(-a)^{3/2}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}\\&-\frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}\\&+\frac{2\sinh(c+dx)}{bd^3}-\frac{a\sinh(c+dx)}{b^2d}+\frac{x^2\sinh(c+dx)}{bd}\\&-\frac{(-a)^{3/2}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}\\&-\frac{(-a)^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{5/2}}\end{aligned}$$

output

```
-2*x*cosh(d*x+c)/b/d^2+1/2*(-a)^(3/2)*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a
)^(1/2)*d/b^(1/2)-d*x)/b^(5/2)-1/2*(-a)^(3/2)*cosh(c-(-a)^(1/2)*d/b^(1/2))
*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)+2*sinh(d*x+c)/b/d^3-a*sinh(d*x+c)/b
^2/d+x^2*sinh(d*x+c)/b/d+1/2*(-a)^(3/2)*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-
(-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)-1/2*(-a)^(3/2)*sinh(c-(-a)^(1/2)*d/b^(1/
2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.98 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.01

$$\int \frac{x^4\cosh(c+dx)}{a+bx^2}\,dx$$
$$= \frac{-ia^{3/2}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+ia^{3/2}e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\right.}{}$$

input
```
Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
((-I)*a^(3/2)*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])
*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralEi[d*((I*Sqrt[
a])/Sqrt[b] + x)]) + I*a^(3/2)*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*S
qrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpInte
gralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*Sqrt[b]*Cosh[d*x]*(-2*b*d*x*Cosh
[c] + (-(a*d^2) + b*(2 + d^2*x^2))*Sinh[c]))/d^3 + (4*Sqrt[b]*((-(a*d^2) +
 b*(2 + d^2*x^2))*Cosh[c] - 2*b*d*x*Sinh[c])*Sinh[d*x])/d^3)/(4*b^(5/2))
```

Rubi [A] (verified)

Time = 1.01 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\cosh(c+dx)}{a+bx^2}\,dx$$
$$\Big\downarrow \; 5816$$
$$\int \left(\frac{a^2\cosh(c+dx)}{b^2(a+bx^2)}-\frac{a\cosh(c+dx)}{b^2}+\frac{x^2\cosh(c+dx)}{b}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{(-a)^{3/2}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{5/2}}-\frac{(-a)^{3/2}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{5/2}}-\frac{a\sinh(c+dx)}{b^2d}+\frac{2\sinh(c+dx)}{bd^3}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{x^2\sinh(c+dx)}{bd}$$

input
```
Int[(x^4*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
(-2*x*Cosh[c + d*x])/(b*d^2) + ((-a)^(3/2)*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*
CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Cosh[c
 - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5
/2)) + (2*Sinh[c + d*x])/(b*d^3) - (a*Sinh[c + d*x])/(b^2*d) + (x^2*Sinh[c
 + d*x])/(b*d) - ((-a)^(3/2)*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(
Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(5/2)) - ((-a)^(3/2)*Sinh[c - (Sqrt[-a]*d
)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.35

method	result
risch	$-\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\operatorname{expIntegral}_1\left(\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)a^2}{4b^2\sqrt{-ab}}+\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\operatorname{expIntegral}_1\left(-\frac{d\sqrt{-ab}+b(dx+c)-cb}{b}\right)a^2}{4b^2\sqrt{-ab}}-\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\ \mathrm{ex}}{}$

input
```
int(x^4*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b^2/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(
d*x+c)+c*b)/b)*a^2+1/4/b^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,
-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2-1/4/b^2/(-a*b)^(1/2)*exp(-(d*(-a*b)
^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2+1/2/d/b*exp(d*x
+c)*x^2-1/2/d/b*exp(-d*x-c)*x^2+1/4/b^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)
+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2-1/2/d/b^2*exp(d*x+c)*a
-1/d^2/b*exp(d*x+c)*x+1/2/d/b^2*exp(-d*x-c)*a-1/d^2/b*exp(-d*x-c)*x+1/d^3/
b*exp(d*x+c)-1/d^3/b*exp(-d*x-c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 605 vs. $2(217) = 434$.

Time = 0.10 (sec) , antiderivative size = 605, normalized size of antiderivative = 2.22

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2} \, dx =$$

$$-\frac{8\,bdx\cosh\left(dx+c\right)+\left(\left(ad^2\cosh\left(dx+c\right)^2-ad^2\sinh\left(dx+c\right)^2\right)\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\left(ad^2\cos\right.\right.}{}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(8*b*d*x*cosh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c
)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*
d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sq
rt(-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d
^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x +
 c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b))
 - 4*(b*d^2*x^2 - a*d^2 + 2*b)*sinh(d*x + c) + ((a*d^2*cosh(d*x + c)^2 - a
*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cos
h(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2
/b)))*sinh(c + sqrt(-a*d^2/b)) + ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 -
 a*d^2*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c
+ sqrt(-a*d^2/b)))/(b^2*d^3*cosh(d*x + c)^2 - b^2*d^3*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh{(c+dx)}}{a+bx^2}\,dx$$

input
```
integrate(x**4*cosh(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x**4*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh{(dx+c)}}{bx^2+a}\,dx$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*((b*d^2*x^4*e^(2*c) - 2*b*d*x^3*e^(2*c) - 2*a*d*x*e^(2*c) + 2*b*x^2*e^
(2*c))*e^(d*x) - (b*d^2*x^4 + 2*b*d*x^3 + 2*a*d*x + 2*b*x^2)*e^(-d*x))/(b^
2*d^3*x^2*e^c + a*b*d^3*e^c) + 1/2*integrate(2*(a*b*d*x^2*e^c + a^2*d*e^c
+ (a^2*d^2*e^c - 2*a*b*e^c)*x)*e^(d*x)/(b^3*d^3*x^4 + 2*a*b^2*d^3*x^2 + a^
2*b*d^3), x) + 1/2*integrate(2*(a*b*d*x^2 + a^2*d - (a^2*d^2 - 2*a*b)*x)*e
^(-d*x)/(b^3*d^3*x^4*e^c + 2*a*b^2*d^3*x^2*e^c + a^2*b*d^3*e^c), x)
```

Giac [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh\left(dx+c\right)}{bx^2+a}\,dx$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^4*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{b\,x^2+a}\,dx$$

input
```
int((x^4*cosh(c + d*x))/(a + b*x^2),x)
```

output
```
int((x^4*cosh(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^2}\,dx$$
$$= \frac{-2\cosh(dx+c)\,bdx + \left(\int \frac{\cosh(dx+c)}{b\,x^2+a}dx\right)a^2d^3 - \sinh(dx+c)\,a\,d^2 + \sinh(dx+c)\,b\,d^2x^2 + 2b\sinh(dx+}{b^2d^3}$$

input
```
int(x^4*cosh(d*x+c)/(b*x^2+a),x)
```

output
```
( - 2*cosh(c + d*x)*b*d*x + int(cosh(c + d*x)/(a + b*x**2),x)*a**2*d**3 -
sinh(c + d*x)*a*d**2 + sinh(c + d*x)*b*d**2*x**2 + 2*sinh(c + d*x)*b)/(b**
2*d**3)
```

3.58 $\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 209

$$\begin{aligned}\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx = &-\frac{\cosh(c+dx)}{bd^2}-\frac{a\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}\\&-\frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}\\&+\frac{x\sinh(c+dx)}{bd}+\frac{a\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}\\&-\frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}\end{aligned}$$

output
```
-cosh(d*x+c)/b/d^2-1/2*a*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(
1/2)-d*x)/b^2-1/2*a*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+
d*x)/b^2+x*sinh(d*x+c)/b/d-1/2*a*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1
/2)*d/b^(1/2)+d*x)/b^2-1/2*a*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d
/b^(1/2)+d*x)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.18 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00

$$\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx =$$
$$-\frac{ae^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+ae^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\right.}{}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
-1/4*(a*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIn
tegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ExpIntegralEi[d*((I*Sqrt[a])/Sq
rt[b] + x)]) + a*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt
[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt
[a]*d)/Sqrt[b] - d*x]) + (4*b*Cosh[d*x]*(Cosh[c] - d*x*Sinh[c]))/d^2 - (4*
b*(d*x*Cosh[c] - Sinh[c])*Sinh[d*x])/d^2)/b^2
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\cosh(c+dx)}{a+bx^2}\,dx$$
$$\Big\downarrow \ 5816$$
$$\int \left(\frac{x\cosh(c+dx)}{b}-\frac{ax\cosh(c+dx)}{b\,(a+bx^2)}\right)dx$$
$$\Big\downarrow \ 2009$$

$$-\frac{a\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}+\frac{a\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{x\sinh(c+dx)}{bd}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
-(Cosh[c + d*x]/(b*d^2)) - (a*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) - (a*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2) + (x*Sinh[c + d*x])/(b*d)
 + (a*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x])/(2*b^2) - (a*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(2*b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.28

method	result
risch	$\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\operatorname{expIntegral}_1\left(\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)a}{4b^2}+\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\operatorname{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)a}{4b^2}+\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\operatorname{expInte}$

input
```
int(x^3*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4/b^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)
*a+1/4/b^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*
b)/b)*a+1/4/b^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+
c)-c*b)/b)*a+1/4/b^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*
(d*x+c)-c*b)/b)*a-1/2/d/b*exp(-d*x-c)*x+1/2/d/b*exp(d*x+c)*x-1/2/d^2/b*exp
(-d*x-c)-1/2/d^2/b*exp(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 502 vs. $2(169) = 338$.

Time = 0.10 (sec) , antiderivative size = 502, normalized size of antiderivative = 2.40

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2} \, dx$$
$$= \frac{4\,bdx\sinh\left(dx+c\right) - 4\,b\cosh\left(dx+c\right) - \left(\left(ad^2\cosh\left(dx+c\right)^2 - ad^2\sinh\left(dx+c\right)^2\right)\mathrm{Ei}\left(dx - \sqrt{-\frac{ad^2}{b}}\right) + \cdots\right.}{}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(4*b*d*x*sinh(d*x + c) - 4*b*cosh(d*x + c) - ((a*d^2*cosh(d*x + c)^2 -
 a*d^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (a*d^2*cosh(d*x + c)^2
- a*d^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b
)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2
/b)) + (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d
^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - ((a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(d
*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) - (a*d^2*cosh(d*x + c)^2 - a*d^2*sinh(
d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + ((a*d^2*
cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (a*d^2
*cosh(d*x + c)^2 - a*d^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(
-c + sqrt(-a*d^2/b)))/(b^2*d^2*cosh(d*x + c)^2 - b^2*d^2*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2} dx = \int \frac{x^3 \cosh{(c+dx)}}{a+bx^2} dx$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x**3*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2} dx = \int \frac{x^3 \cosh{(dx+c)}}{bx^2+a} dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*((d*x^3*e^(2*c) - x^2*e^(2*c))*e^(d*x) - (d*x^3 + x^2)*e^(-d*x))/(b*d^
2*x^2*e^c + a*d^2*e^c) - 1/2*integrate(2*(a*d*x^2*e^c - a*x*e^c)*e^(d*x)/(
b^2*d^2*x^4 + 2*a*b*d^2*x^2 + a^2*d^2), x) + 1/2*integrate(2*(a*d*x^2 + a*
x)*e^(-d*x)/(b^2*d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c), x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2} dx = \int \frac{x^3 \cosh{(dx+c)}}{bx^2+a} dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^3*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{b\,x^2+a}\,dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x^2),x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^2}\,dx = \frac{-\cosh(dx+c) - \left(\int \frac{\cosh(dx+c)x}{b\,x^2+a}dx\right) a\,d^2 + \sinh(dx+c)\,dx}{b\,d^2}$$

input
```
int(x^3*cosh(d*x+c)/(b*x^2+a),x)
```

output
```
( - cosh(c + d*x) - int((cosh(c + d*x)*x)/(a + b*x**2),x)*a*d**2 + sinh(c
+ d*x)*d*x)/(b*d**2)
```

3.59 $\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 226

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx = \frac{\sqrt{-a}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}} + \frac{\sinh(c+dx)}{bd} - \frac{\sqrt{-a}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^{3/2}}$$

output
```
1/2*(-a)^(1/2)*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/
b^(3/2)-1/2*(-a)^(1/2)*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/
2)+d*x)/b^(3/2)+sinh(d*x+c)/b/d+1/2*(-a)^(1/2)*sinh(c+(-a)^(1/2)*d/b^(1/2)
)*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^(3/2)-1/2*(-a)^(1/2)*sinh(c-(-a)^(1/2)*
d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.98

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx = \frac{i\sqrt{a}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{4b^{3/2}} - \frac{i\sqrt{a}e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(-\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)-\text{ExpIntegralEi}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)\right)}{4b^{3/2}} + \frac{\cosh(dx)\sinh(c)}{bd} + \frac{\cosh(c)\sinh(dx)}{bd}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
((I/4)*Sqrt[a]*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b]
)*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralEi[d*((I*Sqrt
[a])/Sqrt[b] + x)]))/b^(3/2) - ((I/4)*Sqrt[a]*E^(-c - (I*Sqrt[a]*d)/Sqrt[b
])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] -
 d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/b^(3/2) + (Cosh[d*x]*
Sinh[c])/(b*d) + (Cosh[c]*Sinh[d*x])/(b*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx$$

$$\Big\downarrow \ 5816$$

$$\int \left(\frac{\cosh(c+dx)}{b} - \frac{a\cosh(c+dx)}{b(a+bx^2)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} - \frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}} - \frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}} + \frac{\sinh(c+dx)}{bd}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
(Sqrt[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
 - d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegr
al[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2)) + Sinh[c + d*x]/(b*d) - (Sqrt[
-a]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(2*b^(3/2)) - (Sqrt[-a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{e^{-\frac{-d\sqrt{-ab}+cb}{b}} \text{expIntegral}_1\left(\frac{d\sqrt{-ab}+b(dx+c)-cb}{b}\right)a}{4b\sqrt{-ab}}+\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}} \text{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)a}{4b\sqrt{-ab}}+\frac{e^{\frac{d\sqrt{-ab}+cb}{b}} \text{exp}$

input
```
int(x^2*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(
d*x+c)-c*b)/b)*a+1/4/b/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a+1/4/b/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*
b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a-1/4/b/(-a*b)^(1/2)*exp((-d*
(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a-1/2/d/b*exp
(-d*x-c)+1/2/b/d*exp(d*x+c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(170) = 340$.

Time = 0.09 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.19

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx = \frac{\left(\sqrt{-\frac{ad^2}{b}}\left(\cosh(dx+c)^2-\sinh(dx+c)^2\right)\text{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\left(\cosh(dx+c)^2-\sinh(dx+c)^2\right.\right.}{}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*((sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*
d^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqr
t(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*(cosh(d*x + c)^2
- sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*(cosh(d*x + c
)^2 - sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)
) + (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d
^2/b)) - sqrt(-a*d^2/b)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(-d*x + sqrt
(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*(cosh(d*x + c)^2 -
 sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*(cosh(d*x + c)
^2 - sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b))
 + 4*sinh(d*x + c))/(b*d*cosh(d*x + c)^2 - b*d*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2\cosh{(c+dx)}}{a+bx^2}\,dx$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x**2*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x^2\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2\cosh{(dx+c)}}{bx^2+a}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
-a*integrate(x*e^(d*x + c)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) + a*integ
rate(x*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c), x) + 1/2*(x
^2*e^(d*x + 2*c) - x^2*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c)
```

Giac [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \cosh(dx+c)}{bx^2+a}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x^2*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{b\,x^2+a}\,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x^2),x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^2}\,dx = \frac{-\left(\int \frac{\cosh(dx+c)}{b\,x^2+a}dx\right)ad+\sinh(dx+c)}{bd}$$

input
```
int(x^2*cosh(d*x+c)/(b*x^2+a),x)
```

output
```
( - int(cosh(c + d*x)/(a + b*x**2),x)*a*d + sinh(c + d*x))/(b*d)
```

3.60 $\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 177

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b}$$

output
```
1/2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/b+1/2*cosh(
c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/b+1/2*sinh(c+(-a)^(1
/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/b+1/2*sinh(c-(-a)^(1/2)*d/b^
(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.18 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.88

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \frac{e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{2c+\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+e^{2c}\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpInteg}\right.}{4b}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
(E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(2*c + ((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIn
tegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + E^(2*c)*ExpIntegralEi[d*((I*Sqr
t[a])/Sqrt[b] + x)] + E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sq
rt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/(4*
b)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 5816}$$

$$\int \left(\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x^2),x]
```

output
```
(Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(2*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x])/(2*b) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(2*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt
[-a]*d)/Sqrt[b] + d*x])/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.13

method	result
risch	$-\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\,\text{expIntegral}_1\left(\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)}{4b}-\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\,\text{expIntegral}_1\left(-\frac{d\sqrt{-ab}+b(dx+c)-cb}{b}\right)}{4b}-\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\,\text{expInte}}{}$

input
```
int(x*cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/b*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-
1/4/b*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)
-1/4/b*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b
)-1/4/b*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/
b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.24

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx$$
$$= \frac{\left(\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)+\left(\mathrm{Ei}\left(dx+\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx-\right.\right.}{}$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(
-a*d^2/b)) + (Ei(d*x + sqrt(-a*d^2/b)) + Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-
c + sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*d^2/b)) - Ei(-d*x + sqrt(-a*d^2/b)
))*sinh(c + sqrt(-a*d^2/b)) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-
a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/b
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh{(c+dx)}}{a+bx^2}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(x*cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh(dx+c)}{bx^2+a}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b*d*x^2*e^c + a*d*e^c) + 1/2*integrate
((b*x^2*e^c - a*e^c)*e^(d*x)/(b^2*d*x^4 + 2*a*b*d*x^2 + a^2*d), x) - 1/2*i
ntegrate((b*x^2 - a)*e^(-d*x)/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*d*e^c
), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh(dx+c)}{bx^2+a}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(x*cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{x\cosh(c+d\,x)}{b\,x^2+a}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x^2),x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{\cosh\left(dx+c\right)x}{b\,x^2+a}dx$$

input
```
int(x*cosh(d*x+c)/(b*x^2+a),x)
```

output
```
int((cosh(c + d*x)*x)/(a + b*x**2),x)
```

3.61 $\int \frac{\cosh(c+dx)}{a+bx^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2\sqrt{-a}\sqrt{b}}$$

output
```
1/2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(1/2)/
b^(1/2)-1/2*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a
)^(1/2)/b^(1/2)+1/2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)
+d*x)/(-a)^(1/2)/b^(1/2)-1/2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d
/b^(1/2)+d*x)/(-a)^(1/2)/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = -\frac{ie^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{2c+\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-e^{2c}\,\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpI}\right.}{4\sqrt{a}\sqrt{b}}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x^2),x]
```

output
```
((-1/4*I)*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(2*c + ((2*I)*Sqrt[a]*d)/Sqrt[
b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - E^(2*c)*ExpIntegralEi[
d*((I*Sqrt[a])/Sqrt[b] + x)] - E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi
[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d
*x]))/(Sqrt[a]*Sqrt[b])
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx$$

$$\Big\downarrow \text{ 5804}$$

$$\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}+\sqrt{b}x\right)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}$$

input
```
Int[Cosh[c + d*x]/(a + b*x^2),x]
```

output
```
(Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/
(2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[
-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[
b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sq
rt[-a]*Sqrt[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\,\text{expIntegral}_1\left(\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)}{4\sqrt{-ab}}+\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\,\text{expIntegral}_1\left(-\frac{d\sqrt{-ab}+b(dx+c)-cb}{b}\right)}{4\sqrt{-ab}}-\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\,\text{expInte}}{}$

input
```
int(cosh(d*x+c)/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+
c)+c*b)/b)+1/4/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(
1/2)+b*(d*x+c)-c*b)/b)-1/4/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,
-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/4/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = -\frac{\left(\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-\left(\sqrt{-\frac{ad^2}{b}}\operatorname{Ei}\left(dx+\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x +
sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (sqrt(-a*d^2/b)*Ei(d*x + sqrt(
-a*d^2/b)) + sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d
^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x
 + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (sqrt(-a*d^2/b)*Ei(d*x + sq
rt(-a*d^2/b)) - sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-
a*d^2/b)))/(a*d)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{\cosh(c+dx)}{a+bx^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x**2+a),x)
```

output
```
Integral(cosh(c + d*x)/(a + b*x**2), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx=\int \frac{\cosh\left(dx+c\right)}{bx^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx=\int \frac{\cosh\left(dx+c\right)}{bx^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*x^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx=\int \frac{\cosh(c+d\,x)}{b\,x^2+a}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x^2),x)
```

output
```
int(cosh(c + d*x)/(a + b*x^2), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{a+bx^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\,x^2+a}dx$$

input
```
int(cosh(d*x+c)/(b*x^2+a),x)
```

output
```
int(cosh(c + d*x)/(a + b*x**2),x)
```

3.62 $\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 197

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx = \frac{\cosh(c)\text{Chi}(dx)}{a} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a} + \frac{\sinh(c)\text{Shi}(dx)}{a} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a}$$

output
```
cosh(c)*Chi(d*x)/a-1/2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/
2)-d*x)/a-1/2*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/a
+sinh(c)*Shi(d*x)/a-1/2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(
1/2)+d*x)/a-1/2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)
/a
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.26 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx = -\frac{-4\cosh(c)\mathrm{Chi}(dx) + e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{2c+\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\mathrm{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right) + e^{2c}\,\mathrm{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}\right.\right.\right.}{4a}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

output
```
-1/4*(-4*Cosh[c]*CoshIntegral[d*x] + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(2*
c + ((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x
)] + E^(2*c)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)] + E^(((2*I)*Sqrt[a
]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralE
i[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) - 4*Sinh[c]*SinhIntegral[d*x])/a
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx$$

$$\Big\downarrow \; 5816$$

$$\int \left(\frac{\cosh(c+dx)}{ax} - \frac{bx\cosh(c+dx)}{a(a+bx^2)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cosh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Chi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\cosh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Chi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}+\frac{\sinh\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}+c\right)\operatorname{Shi}\left(\frac{\sqrt{-a}\,d}{\sqrt{b}}-dx\right)}{2a}-\frac{\sinh\left(c-\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)\operatorname{Shi}\left(xd+\frac{\sqrt{-a}\,d}{\sqrt{b}}\right)}{2a}+\frac{\cosh(c)\operatorname{Chi}(dx)}{a}+\frac{\sinh(c)\operatorname{Shi}(dx)}{a}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x^2)),x]
```

output
```
(Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegr
al[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Co
shIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a) + (Sinh[c]*SinhIntegral[d*x]
)/a + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
d*x])/(2*a) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x])/(2*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.15

method	result
risch	$-\frac{e^{-c}\operatorname{expIntegral}_1(dx)}{2a}-\frac{e^{c}\operatorname{expIntegral}_1(-dx)}{2a}+\frac{e^{\frac{d\sqrt{-ab}+cb}{b}}\operatorname{expIntegral}_1\left(\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)}{4a}+\frac{e^{\frac{-d\sqrt{-ab}+cb}{b}}\operatorname{expIntegra}}{}$

input
```
int(cosh(d*x+c)/x/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*exp(-c)*Ei(1,d*x)-1/2/a*exp(c)*Ei(1,-d*x)+1/4/a*exp((d*(-a*b)^(1/2)
+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/4/a*exp((-d*(-a*b)^(1/2)
+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)+1/4/a*exp(-(d*(-a*b)^(1/2
)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/4/a*exp(-(-d*(-a*b)^(1
/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.26

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx = -\frac{\left(\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\mathrm{Ei}\left(-dx+\sqrt{-\frac{ad^2}{b}}\right)\right)\cosh\left(c+\sqrt{-\frac{ad^2}{b}}\right)-2\left(\mathrm{Ei}(dx)+\mathrm{Ei}(-dx)\right)\cosh(c)+}{}$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*((Ei(d*x - sqrt(-a*d^2/b)) + Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt
(-a*d^2/b)) - 2*(Ei(d*x) + Ei(-d*x))*cosh(c) + (Ei(d*x + sqrt(-a*d^2/b)) +
 Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (Ei(d*x - sqrt(-a*
d^2/b)) - Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 2*(Ei(d*x)
 - Ei(-d*x))*sinh(c) - (Ei(d*x + sqrt(-a*d^2/b)) - Ei(-d*x - sqrt(-a*d^2/b
)))*sinh(-c + sqrt(-a*d^2/b)))/a
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx = \int \frac{\cosh(c+dx)}{x(a+bx^2)}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x**2+a),x)
```

output
```
Integral(cosh(c + d*x)/(x*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)}\,dx = \int \frac{\cosh\left(dx+c\right)}{(bx^2+a)x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)}\,dx = \int \frac{\cosh\left(dx+c\right)}{(bx^2+a)x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)}\,dx = \int \frac{\cosh(c+d\,x)}{x\,\left(b\,x^2+a\right)}\,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x^2)),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\,x^3+ax}dx$$

input
```
int(cosh(d*x+c)/x/(b*x^2+a),x)
```

output
```
int(cosh(c + d*x)/(a*x + b*x**3),x)
```

3.63 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 249

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = &-\frac{\cosh(c+dx)}{ax}+\frac{\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}\\ &-\frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}+\frac{d\text{Chi}(dx)\sinh(c)}{a}\\ &+\frac{d\cosh(c)\text{Shi}(dx)}{a}-\frac{\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}}\\ &-\frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2(-a)^{3/2}}\end{aligned}$$

output
```
-cosh(d*x+c)/a/x+1/2*b^(1/2)*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d
/b^(1/2)-d*x)/(-a)^(3/2)-1/2*b^(1/2)*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)
^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)+d*Chi(d*x)*sinh(c)/a+d*cosh(c)*Shi(d*x)/a
+1/2*b^(1/2)*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(
-a)^(3/2)-1/2*b^(1/2)*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2
)+d*x)/(-a)^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.59 (sec) , antiderivative size = 243, normalized size of antiderivative = 0.98

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = -\frac{\cosh(c)\cosh(dx)}{ax} + \frac{i\sqrt{b}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{4a^{3/2}} - \frac{i\sqrt{b}e^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(-\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)-\text{ExpIntegralEi}\left(\frac{i\sqrt{a}d}{\sqrt{b}}-dx\right)\right)}{4a^{3/2}} - \frac{\sinh(c)\sinh(dx)}{ax} + \frac{d(\text{Chi}(dx)\sinh(c)+\cosh(c)\text{Shi}(dx))}{a}$$

input
```
Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
```

output
```
-((Cosh[c]*Cosh[d*x])/(a*x)) + ((I/4)*Sqrt[b]*E^(c - (I*Sqrt[a]*d)/Sqrt[b]
)*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] +
 x)] - ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/a^(3/2) - ((I/4)*Sqrt[
b]*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegr
alEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b]
 - d*x]))/a^(3/2) - (Sinh[c]*Sinh[d*x])/(a*x) + (d*(CoshIntegral[d*x]*Sinh
[c] + Cosh[c]*SinhIntegral[d*x]))/a
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 5816}$$

$$\int \left(\frac{\cosh(c+dx)}{ax^2} - \frac{b\cosh(c+dx)}{a(a+bx^2)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{b}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2(-a)^{3/2}} - \frac{\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2(-a)^{3/2}} + \frac{d\sinh(c)\text{Chi}(dx)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} - \frac{\cosh(c+dx)}{ax}$$

input
```
Int[Cosh[c + d*x]/(x^2*(a + b*x^2)),x]
```

output
```
-(Cosh[c + d*x]/(a*x)) + (Sqrt[b]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*(-a)^(3/2)) - (Sqrt[b]*Cosh[c - (Sqrt[
-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2)) +
 (d*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - (Sqrt
[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(2*(-a)^(3/2)) - (Sqrt[b]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(
Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*(-a)^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 288, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{e^{-dx-c}}{2ax}+\frac{d\,e^{-c}\,\mathrm{expIntegral}_1(dx)}{2a}+\frac{b\,e^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)}{4a\sqrt{-ab}}-\frac{b\,e^{-\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_1\Big(}{4a\sqrt{-ab}}$

input
```
int(cosh(d*x+c)/x^2/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(-d*x-c)/a/x+1/2*d/a*exp(-c)*Ei(1,d*x)+1/4*b/a/(-a*b)^(1/2)*exp(-(
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-1/4*b/a/(-a
*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b
)/b)-1/2*exp(d*x+c)/a/x-1/2*d/a*exp(c)*Ei(1,-d*x)+1/4*b/a/(-a*b)^(1/2)*exp
((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-1/4*b/a/(-
a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*
b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 599 vs. 2(193) = 386.

Time = 0.11 (sec) , antiderivative size = 599, normalized size of antiderivative = 2.41

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)}\,dx =$$

$$-\frac{4\,ad\cosh\left(dx+c\right)-\left(\left(bx\cosh\left(dx+c\right)^2-bx\sinh\left(dx+c\right)^2\right)\sqrt{-\frac{ad^2}{b}}\,\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^2}{b}}\right)+\left(bx\cosh\left(d\right.\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(4*a*d*cosh(d*x + c) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*s
qrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x*cosh(d*x + c)^2 - b*x*sinh(d
*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/
b)) - 2*(a*d^2*x*Ei(d*x) - a*d^2*x*Ei(-d*x))*cosh(c) + ((b*x*cosh(d*x + c)
^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) + (b*x*c
osh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2
/b)))*cosh(-c + sqrt(-a*d^2/b)) - ((b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c
)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - (b*x*cosh(d*x + c)^2 - b*x*
sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-
a*d^2/b)) - 2*(a*d^2*x*Ei(d*x) + a*d^2*x*Ei(-d*x))*sinh(c) - ((b*x*cosh(d*
x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) -
(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x - sqrt(
-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/(a^2*d*x*cosh(d*x + c)^2 - a^2*d*x*
sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx$$

input
```
integrate(cosh(d*x+c)/x**2/(b*x**2+a),x)
```

output
```
Integral(cosh(c + d*x)/(x**2*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,(b\,x^2+a)}\,dx$$

input
```
int(cosh(c + d*x)/(x^2*(a + b*x^2)),x)
```

output
```
int(cosh(c + d*x)/(x^2*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)}\,dx = \int \frac{\cosh(dx+c)}{b\,x^4+a\,x^2}dx$$

input
```
int(cosh(d*x+c)/x^2/(b*x^2+a),x)
```

output
```
int(cosh(c + d*x)/(a*x**2 + b*x**4),x)
```

3.64 $\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 270

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx = &-\frac{\cosh(c+dx)}{2ax^2}-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}\\&+\frac{b\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\&+\frac{b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}-\frac{d\sinh(c+dx)}{2ax}\\&-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}\\&-\frac{b\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}\\&+\frac{b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}\end{aligned}$$

output
```
-1/2*cosh(d*x+c)/a/x^2-b*cosh(c)*Chi(d*x)/a^2+1/2*d^2*cosh(c)*Chi(d*x)/a+1
/2*b*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/a^2+1/2*b*
cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/a^2-1/2*d*sinh(
d*x+c)/a/x-b*sinh(c)*Shi(d*x)/a^2+1/2*d^2*sinh(c)*Shi(d*x)/a+1/2*b*sinh(c+
(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^2+1/2*b*sinh(c-(-a)
^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/a^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.71 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.86

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx = \frac{be^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\operatorname{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+be^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\operatorname{Ex}\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```

output
```
(b*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegra
lEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b]
 + x)]) + b*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*
ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d
)/Sqrt[b] - d*x]) - (2*a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/x^2 - (2*a*(d*
x*Cosh[c] + Sinh[c])*Sinh[d*x])/x^2 + 2*(-2*b + a*d^2)*(Cosh[c]*CoshIntegr
al[d*x] + Sinh[c]*SinhIntegral[d*x]))/(4*a^2)
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx$$

$$\Big\downarrow \text{ 5816}$$

$$\int \left(\frac{b^2x\cosh(c+dx)}{a^2(a+bx^2)}-\frac{b\cosh(c+dx)}{a^2x}+\frac{\cosh(c+dx)}{ax^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}-\frac{b\sinh(c)\text{Shi}(dx)}{a^2}-\frac{b\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}+\frac{b\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}$$

input
```
Int[Cosh[c + d*x]/(x^3*(a + b*x^2)),x]
```

output
```
-1/2*Cosh[c + d*x]/(a*x^2) - (b*Cosh[c]*CoshIntegral[d*x])/a^2 + (d^2*Cosh
[c]*CoshIntegral[d*x])/(2*a) + (b*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) + (b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b
]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^2) - (d*Sinh[c + d*x])/(
2*a*x) - (b*Sinh[c]*SinhIntegral[d*x])/a^2 + (d^2*Sinh[c]*SinhIntegral[d*x
])/(2*a) - (b*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqr
t[b] - d*x])/(2*a^2) + (b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqr
t[-a]*d)/Sqrt[b] + d*x])/(2*a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 330, normalized size of antiderivative = 1.22

method	result
risch	$\frac{d\,\mathrm{e}^{-dx-c}}{4ax}-\frac{\mathrm{e}^{-dx-c}}{4a\,x^{2}}-\frac{d^{2}\mathrm{e}^{-c}\,\mathrm{expIntegral}_{1}(dx)}{4a}+\frac{\mathrm{e}^{-c}\,\mathrm{expIntegral}_{1}(dx)b}{2a^{2}}-\frac{b\,\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_{1}\left(-\frac{d\sqrt{-ab}-b(dx+c)+c}{b}\right.}{4a^{2}}$

input
```
int(cosh(d*x+c)/x^3/(b*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*d*exp(-d*x-c)/a/x-1/4*exp(-d*x-c)/a/x^2-1/4*d^2/a*exp(-c)*Ei(1,d*x)+1/
2/a^2*exp(-c)*Ei(1,d*x)*b-1/4*b/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-1/4*b/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)-1/4*d*exp(d*x+c)/a/x-1/4*exp(d*x+c)/a/
x^2-1/4*d^2/a*exp(c)*Ei(1,-d*x)+1/2/a^2*exp(c)*Ei(1,-d*x)*b-1/4*b/a^2*exp(
(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-1/4*b/a^2*e
xp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(222) = 444.

Time = 0.11 (sec) , antiderivative size = 583, normalized size of antiderivative = 2.16

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)}\,dx =$$

$$-\frac{2\,adx\sinh\left(dx+c\right)+2\,a\cosh\left(dx+c\right)-\left(\left(bx^{2}\cosh\left(dx+c\right)^{2}-bx^{2}\sinh\left(dx+c\right)^{2}\right)\mathrm{Ei}\left(dx-\sqrt{-\frac{ad^{2}}{b}}\right.\right.}{}$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(2*a*d*x*sinh(d*x + c) + 2*a*cosh(d*x + c) - ((b*x^2*cosh(d*x + c)^2
- b*x^2*sinh(d*x + c)^2)*Ei(d*x - sqrt(-a*d^2/b)) + (b*x^2*cosh(d*x + c)^2
 - b*x^2*sinh(d*x + c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/
b)) - ((a*d^2 - 2*b)*x^2*Ei(d*x) + (a*d^2 - 2*b)*x^2*Ei(-d*x))*cosh(c) - (
(b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) +
 (b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b))
)*cosh(-c + sqrt(-a*d^2/b)) - ((b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c
)^2)*Ei(d*x - sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x +
c)^2)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - ((a*d^2 - 2*b)
*x^2*Ei(d*x) - (a*d^2 - 2*b)*x^2*Ei(-d*x))*sinh(c) + ((b*x^2*cosh(d*x + c)
^2 - b*x^2*sinh(d*x + c)^2)*Ei(d*x + sqrt(-a*d^2/b)) - (b*x^2*cosh(d*x + c
)^2 - b*x^2*sinh(d*x + c)^2)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*
d^2/b)))/(a^2*x^2*cosh(d*x + c)^2 - a^2*x^2*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx$$

input
```
integrate(cosh(d*x+c)/x**3/(b*x**2+a),x)
```

output
```
Integral(cosh(c + d*x)/(x**3*(a + b*x**2)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)x^3}\,dx$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx=\int \frac{\cosh(dx+c)}{(bx^2+a)x^3}\,dx$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^2+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx=\int \frac{\cosh(c+d\,x)}{x^3\,(b\,x^2+a)}\,dx$$

input
```
int(cosh(c + d*x)/(x^3*(a + b*x^2)),x)
```

output
```
int(cosh(c + d*x)/(x^3*(a + b*x^2)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)}\,dx = \frac{-e^{2dx+2c}dx-e^{2dx+2c}+e^{dx+2c}\left(\int \frac{e^{dx}}{b\,x^3+ax}dx\right)a\,d^2x^2-2e^{dx+2c}\left(\int \frac{e^{dx}}{b\,x^3+ax}dx\right)b\,x^2+e^{dx+2c}\left(\int \frac{e^{dx}x}{b\,x^2+a}dx\right)b\,d}{4e^{dx}}$$

input
```
int(cosh(d*x+c)/x^3/(b*x^2+a),x)
```

output
```
( - e**(2*c + 2*d*x)*d*x - e**(2*c + 2*d*x) + e**(2*c + d*x)*int(e**(d*x)/
(a*x + b*x**3),x)*a*d**2*x**2 - 2*e**(2*c + d*x)*int(e**(d*x)/(a*x + b*x**
3),x)*b*x**2 + e**(2*c + d*x)*int((e**(d*x)*x)/(a + b*x**2),x)*b*d**2*x**2
 + e**(d*x)*int(x/(e**(d*x)*a + e**(d*x)*b*x**2),x)*b*d**2*x**2 + e**(d*x)
*int(1/(e**(d*x)*a*x + e**(d*x)*b*x**3),x)*a*d**2*x**2 - 2*e**(d*x)*int(1/
(e**(d*x)*a*x + e**(d*x)*b*x**3),x)*b*x**2 + d*x - 1)/(4*e**(c + d*x)*a*x*
*2)
```

3.65 $\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 449

$$\begin{aligned}
\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} \, dx = {} & \frac{x \cosh(c+dx)}{2b^2} - \frac{x^3 \cosh(c+dx)}{2b(a+bx^2)} \\
& + \frac{3\sqrt{-a} \cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\
& - \frac{3\sqrt{-a} \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} \\
& - \frac{ad\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right) \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} \\
& - \frac{ad\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right) \sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^3} + \frac{\sinh(c+dx)}{b^2 d} \\
& + \frac{ad \cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^3} \\
& - \frac{3\sqrt{-a} \sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\
& - \frac{ad \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^3} \\
& - \frac{3\sqrt{-a} \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}}
\end{aligned}$$

output
```
1/2*x*cosh(d*x+c)/b^2-1/2*x^3*cosh(d*x+c)/b/(b*x^2+a)+3/4*(-a)^(1/2)*cosh(
c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/b^(5/2)-3/4*(-a)^(1/
2)*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)-1/4*
a*d*Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/b^3-1/4*a*d
*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/b^3+sinh(d*x+c
)/b^2/d-1/4*a*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x
)/b^3+3/4*(-a)^(1/2)*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2
)+d*x)/b^(5/2)-1/4*a*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/
2)+d*x)/b^3-3/4*(-a)^(1/2)*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b
^(1/2)+d*x)/b^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.47 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.68

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} \, dx$$
$$= \frac{-\sqrt{a}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-3i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(3i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(\right.\right.}{}$$

input
```
Integrate[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
(-(Sqrt[a]*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^(
((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] +
 ((3*I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))
+ Sqrt[a]*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^(
((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] +
 ((3*I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) +
 4*b*Cosh[d*x]*((a*x*Cosh[c])/(a + b*x^2) + (2*Sinh[c])/d) + 4*b*((2*Cosh[
c])/d + (a*x*Sinh[c])/(a + b*x^2))*Sinh[d*x])/(8*b^3)
```

Rubi [A] (verified)

Time = 1.63 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.06, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5814, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4\cosh(c+dx)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 5814}$$

$$\frac{3\int \frac{x^2\cosh(c+dx)}{bx^2+a}dx}{2b}+\frac{d\int \frac{x^3\sinh(c+dx)}{bx^2+a}dx}{2b}-\frac{x^3\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 5815}$$

$$\frac{d\int \left(\frac{x\sinh(c+dx)}{b}-\frac{ax\sinh(c+dx)}{b(bx^2+a)}\right)dx}{2b}+\frac{3\int \frac{x^2\cosh(c+dx)}{bx^2+a}dx}{2b}-\frac{x^3\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\int \frac{x^2\cosh(c+dx)}{bx^2+a}dx}{2b}+$$

$$\frac{d\left(-\frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}-\frac{a\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b\ldots}\right.}{2b}$$

$$\frac{x^3\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 5816}$$

$$\frac{3\int \left(\frac{\cosh(c+dx)}{b}-\frac{a\cosh(c+dx)}{b(bx^2+a)}\right)dx}{2b}+$$

$$\frac{d\left(-\frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}-\frac{a\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b\ldots}\right.}{2b}$$

$$\frac{x^3\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3\left(\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}-\frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\sqrt{-a}\sin\right.}{2b}$$

$$\frac{d\left(-\frac{a\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^2}-\frac{a\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}+\frac{a\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2}-\frac{a\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right.}{2b}\right.}{2b}$$

$$\frac{x^3\cosh(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^4*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^3*Cosh[c + d*x])/(b*(a + b*x^2)) + (d*((x*Cosh[c + d*x])/(b*d) - (
a*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])
/(2*b^2) - (a*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*
d)/Sqrt[b]])/(2*b^2) - Sinh[c + d*x]/(b*d^2) + (a*Cosh[c + (Sqrt[-a]*d)/Sq
rt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^2) - (a*Cosh[c - (Sq
rt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^2)))/(2*
b) + (3*((Sqrt[-a]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x])/(2*b^(3/2)) - (Sqrt[-a]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*C
oshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2)) + Sinh[c + d*x]/(b*d)
 - (Sqrt[-a]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(2*b^(3/2)) - (Sqrt[-a]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b^(3/2))))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(349) = 698$.

Time = 0.97 (sec) , antiderivative size = 1038, normalized size of antiderivative = 2.31

method	result	size
risch	Expression too large to display	1038

input
```
int(x^4*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8/d*(exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*
(d*x+c)+c*b)/b)*a*b*d^2*x^2+(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b*d^2*x^2-(-a*b)^(1/2)*exp((d*(-a*b)
^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a*b*d^2*x^2-(-a*b)^(
1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*
a*b*d^2*x^2-3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)
+c*b)/b)*a*b^2*d*x^2+3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+
b*(d*x+c)-c*b)/b)*a*b^2*d*x^2-3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)
^(1/2)-b*(d*x+c)+c*b)/b)*a*b^2*d*x^2+3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-
(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b^2*d*x^2+exp(-(d*(-a*b)^(1/2)+c*b)/b)
*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2*d^2+(-a*b)^(1/2)
*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*
d^2-(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c
)+c*b)/b)*a^2*d^2-(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b
)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*d^2-3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*
(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2*b*d-2*(-a*b)^(1/2)*exp(-d*x-c)*a*b*d*x+
4*(-a*b)^(1/2)*exp(-d*x-c)*b^2*x^2-2*(-a*b)^(1/2)*exp(d*x+c)*a*b*d*x-4*(-a
*b)^(1/2)*exp(d*x+c)*b^2*x^2+3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b
)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*b*d-3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-
a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2*b*d+3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1179 vs. $2(349) = 698$.

Time = 0.13 (sec) , antiderivative size = 1179, normalized size of antiderivative = 2.63

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
 (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c
)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^
2/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)
*sinh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*si
nh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a
*d^2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*
d^2)*sinh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b
)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^
2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3
*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + 8*(b^2*x
^2 + a*b)*sinh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b
*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 -
 (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b))
 + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh
(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*
x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/
b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*
sinh(d*x + c)^2 + 3*((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*...
```

Sympy [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4 \cosh{(c+dx)}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**4*cosh(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x**4*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4 \cosh(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((b*d*x^4*e^(2*c) - 4*a*x*e^(2*c))*e^(d*x) - (b*d*x^4 + 4*a*x)*e^(-d*x
))/(b^3*d^2*x^4*e^c + 2*a*b^2*d^2*x^2*e^c + a^2*b*d^2*e^c) - 1/2*integrate
(-4*(a^2*d*x*e^c - 3*a*b*x^2*e^c + a^2*e^c)*e^(d*x)/(b^4*d^2*x^6 + 3*a*b^3
*d^2*x^4 + 3*a^2*b^2*d^2*x^2 + a^3*b*d^2), x) - 1/2*integrate(4*(a^2*d*x +
 3*a*b*x^2 - a^2)*e^(-d*x)/(b^4*d^2*x^6*e^c + 3*a*b^3*d^2*x^4*e^c + 3*a^2*
b^2*d^2*x^2*e^c + a^3*b*d^2*e^c), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^4*cosh(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x^4*cosh(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{e^{2dx+2c}a + e^{2dx+2c}b\,x^2 - e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^3d - e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^2bd\,x^2 - 2e^{dx+2c}\left(\int\right.}{}$$

input
```
int(x^4*cosh(d*x+c)/(b*x^2+a)^2,x)
```

output
```
(e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b*x**2 - e**(2*c + d*x)*int(e**(d*x
)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**3*d - e**(2*c + d*x)*int(e**(d*x)/
(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2*b*d*x**2 - 2*e**(2*c + d*x)*int((e
**(d*x)*x**2)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2*b*d - 2*e**(2*c + d*
x)*int((e**(d*x)*x**2)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a*b**2*d*x**2 -
e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**
4),x)*a**2*b*d - e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 +
e**(d*x)*b**2*x**4),x)*a*b**2*d*x**2 + 2*e**(d*x)*int(x/(e**(d*x)*a**2 + 2
*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a**2*b + 2*e**(d*x)*int(x/(e**
(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*x**2 - b*
x**2)/(2*e**(c + d*x)*b**2*d*(a + b*x**2))
```

3.66 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 431

$$
\begin{aligned}
\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2} \, dx = {} & \frac{\cosh(c+dx)}{2b^2} - \frac{x^2 \cosh(c+dx)}{2b\,(a+bx^2)} \\
& + \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} \\
& + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2} \\
& - \frac{\sqrt{-a}d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right) \sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} \\
& + \frac{\sqrt{-a}d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right) \sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^{5/2}} \\
& - \frac{\sqrt{-a}d \cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^{5/2}} \\
& - \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^2} \\
& - \frac{\sqrt{-a}d \cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^{5/2}} \\
& + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right) \operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2b^2}
\end{aligned}
$$

output
```
1/2*cosh(d*x+c)/b^2-1/2*x^2*cosh(d*x+c)/b/(b*x^2+a)+1/2*cosh(c+(-a)^(1/2)*
d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/b^2+1/2*cosh(c-(-a)^(1/2)*d/b^(1/
2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/b^2-1/4*(-a)^(1/2)*d*Chi((-a)^(1/2)*d/b^
(1/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/b^(5/2)+1/4*(-a)^(1/2)*d*Chi((-a)^
(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/b^(5/2)+1/4*(-a)^(1/2)*d
*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)+1/2*s
inh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^2-1/4*(-a)^(1
/2)*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/b^(5/2)+1
/2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.67

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{\frac{4a\sqrt{b}\cosh(c)\cosh(dx)}{a+bx^2} + e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(2\sqrt{b}+i\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(2\sqrt{b}-i\sqrt{a}d\right)\text{Exp}}{}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
((4*a*Sqrt[b]*Cosh[c]*Cosh[d*x])/(a + b*x^2) + E^(c - (I*Sqrt[a]*d)/Sqrt[b
])*((2*Sqrt[b] + I*Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[
d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + (2*Sqrt[b] - I*Sqrt[a]*d)*ExpIntegralEi[
d*((I*Sqrt[a])/Sqrt[b] + x)]) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((2*Sqrt[b]
 + I*Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*
d)/Sqrt[b] - d*x] + (2*Sqrt[b] - I*Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/
Sqrt[b] - d*x]) + (4*a*Sqrt[b]*Sinh[c]*Sinh[d*x])/(a + b*x^2))/(8*b^(5/2))
```

Rubi [A] (verified)

Time = 1.33 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5814, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \text{ 5814}$$

$$\frac{d\int \frac{x^2\sinh(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \frac{x\cosh(c+dx)}{bx^2+a}dx}{b}-\frac{x^2\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 5815}$$

$$\frac{d\int \left(\frac{\sinh(c+dx)}{b}-\frac{a\sinh(c+dx)}{b(bx^2+a)}\right)dx}{2b}+\frac{\int \frac{x\cosh(c+dx)}{bx^2+a}dx}{b}-\frac{x^2\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\int \frac{x\cosh(c+dx)}{bx^2+a}dx}{b}+\frac{d\left(-\frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\,\text{c}}{}\right.}{2b}$$

$$\frac{x^2\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 5816}$$

$$\frac{\int \left(\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{b}+\frac{d\left(-\frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\,\text{c}}{}\right.}{2b}$$

$$\frac{x^2\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\left(-\frac{\sqrt{-a}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b^{3/2}}+\frac{\sqrt{-a}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b^{3/2}}-\frac{\sqrt{-a}\,c\cdots}{}\right.}{2b}$$

$$\frac{\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-}\cdots}{}\right)}{2b}}{b}$$

$$\frac{x^2\cosh(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x^2*Cosh[c + d*x])/(b*(a + b*x^2)) + (d*(Cosh[c + d*x]/(b*d) - (Sqrt
[-a]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b
]])/(2*b^(3/2)) + (Sqrt[-a]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[
c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b^(3/2)) - (Sqrt[-a]*Cosh[c + (Sqrt[-a]*d)/S
qrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b^(3/2)) - (Sqrt[-a]*
Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(
2*b^(3/2))))/(2*b) + ((Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-
a]*d)/Sqrt[b] - d*x])/(2*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*Sinh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[
b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 902 vs. $2(331) = 662$.

Time = 0.82 (sec) , antiderivative size = 903, normalized size of antiderivative = 2.10

method	result
risch	$-\frac{{\mathrm e}^{-\frac{d\sqrt{-ab}+cb}{b}} \operatorname{expIntegral}_{1}\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right) abd\,x^{2}-{\mathrm e}^{-\frac{-d\sqrt{-ab}+cb}{b}} \operatorname{expIntegral}_{1}\left(\frac{d\sqrt{-ab}+b(dx+c)-cb}{b}\right) abd\,x^{2}-\operatorname{expIntegra}}{}$

input
```
int(x^3*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)
*a*b*d*x^2-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*
b)/b)*a*b*d*x^2-Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)
+c*b)/b)*a*b*d*x^2+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d
*x+c)-c*b)/b)*a*b*d*x^2+2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-
(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b*x^2+2*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b*x^2+2*(-a*b)^(1/2)*Ei(
1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c*b)/b)*b*x^2+2*(-
a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*
b)/b)*b*x^2+exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c
*b)/b)*a^2*d-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-
c*b)/b)*a^2*d-Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c
*b)/b)*a^2*d+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-
c*b)/b)*a^2*d+2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^
(1/2)-b*(d*x+c)+c*b)/b)*a+2*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a+2*(-a*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-
b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c*b)/b)*a+2*(-a*b)^(1/2)*exp((-d*(-a
*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a-2*a*(-a*b)^(1/
2)*exp(d*x+c)-2*a*(-a*b)^(1/2)*exp(-d*x-c))/b^2/(b*x^2+a)/(-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 931 vs. $2(331) = 662$.

Time = 0.10 (sec) , antiderivative size = 931, normalized size of antiderivative = 2.16

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*cosh(d*x + c) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*s
inh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^
2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)
^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2
 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c +
 sqrt(-a*d^2/b)) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*
x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqr
t(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2
*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*
sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt
(-a*d^2/b)) + ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c
)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*
d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x
^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^
2/b)) - ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 -
((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)
)*Ei(d*x + sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a
)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/...
```

Sympy [F]

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{x^3\cosh{(c+dx)}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x**3*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3 \cosh(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((d^2*x^3*e^(2*c) + d*x^2*e^(2*c) + 2*x*e^(2*c))*e^(d*x) - (d^2*x^3 -
d*x^2 + 2*x)*e^(-d*x))/(b^2*d^3*x^4*e^c + 2*a*b*d^3*x^2*e^c + a^2*d^3*e^c)
 - 1/2*integrate(2*(2*a*d*x*e^c + (2*a*d^2*e^c - 3*b*e^c)*x^2 + a*e^c)*e^(
d*x)/(b^3*d^3*x^6 + 3*a*b^2*d^3*x^4 + 3*a^2*b*d^3*x^2 + a^3*d^3), x) + 1/2
*integrate(-2*(2*a*d*x - (2*a*d^2 - 3*b)*x^2 - a)*e^(-d*x)/(b^3*d^3*x^6*e^
c + 3*a*b^2*d^3*x^4*e^c + 3*a^2*b*d^3*x^2*e^c + a^3*d^3*e^c), x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3 \cosh(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \frac{e^{2dx+2c} a\,d^2 + 2e^{2dx+2c} bdx - e^{dx+2c} \left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right) a^3 d^3 - e^{dx+2c} \left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right) a^2 b\, d^3 x^2 - 2e^{dx}}{}$$

input
```
int(x^3*cosh(d*x+c)/(b*x^2+a)^2,x)
```

output
```
(e**(2*c + 2*d*x)*a*d**2 + 2*e**(2*c + 2*d*x)*b*d*x - e**(2*c + d*x)*int(e
**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**3*d**3 - e**(2*c + d*x)*int(
e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2*b*d**3*x**2 - 2*e**(2*c +
 d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2*b*d - 2*e**(2*c
 + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a*b**2*d*x**2 - e*
*(2*c + d*x)*int((e**(d*x)*x**2)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2*b
*d**3 - e**(2*c + d*x)*int((e**(d*x)*x**2)/(a**2 + 2*a*b*x**2 + b**2*x**4)
,x)*a*b**2*d**3*x**2 + 2*e**(2*c + d*x)*int((e**(d*x)*x**2)/(a**2 + 2*a*b*
x**2 + b**2*x**4),x)*a*b**2*d + 2*e**(2*c + d*x)*int((e**(d*x)*x**2)/(a**2
 + 2*a*b*x**2 + b**2*x**4),x)*b**3*d*x**2 - 4*e**(d*x)*int(x**2/(e**(d*x)*
a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*d - 4*e**(d*x)*
int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*b**
3*d*x**2 - 2*e**(d*x)*int(x/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x
)*b**2*x**4),x)*a**2*b*d**2 - 2*e**(d*x)*int(x/(e**(d*x)*a**2 + 2*e**(d*x)
*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*d**2*x**2 - 4*e**(d*x)*int(x/(e*
*(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2 - 4*e**(
d*x)*int(x/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*b
**3*x**2 - 2*b*d*x - 2*b)/(4*e**(c + d*x)*b**2*d**2*(a + b*x**2))
```

3.67 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 416

$$\begin{aligned}
\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx = &-\frac{x\cosh(c+dx)}{2b(a+bx^2)} + \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} \\
&- \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}} \\
&+ \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} \\
&+ \frac{d\operatorname{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4b^2} \\
&- \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4b^2} \\
&- \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} \\
&+ \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4b^2} \\
&- \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\operatorname{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}
\end{aligned}$$

output
```
-1/2*x*cosh(d*x+c)/b/(b*x^2+a)+1/4*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(
1/2)*d/b^(1/2)-d*x)/(-a)^(1/2)/b^(3/2)-1/4*cosh(c-(-a)^(1/2)*d/b^(1/2))*Ch
i((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)+1/4*d*Chi((-a)^(1/2)*d/b^(1
/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/b^2+1/4*d*Chi((-a)^(1/2)*d/b^(1/2)-d
*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/b^2+1/4*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Sh
i(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^2+1/4*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a
)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)+1/4*d*cosh(c-(-a)^(1/2)*d/b^(1/2
))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/b^2-1/4*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi(
(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.24 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.69

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2} \, dx = -\frac{\frac{4bx\cosh(c)\cosh(dx)}{a+bx^2} - \frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{a}} +}{8b^2}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/8*((4*b*x*Cosh[c]*Cosh[d*x])/(a + b*x^2) - (E^(c - (I*Sqrt[a]*d)/Sqrt[b
])*(((-I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi
[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + (I*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[d
*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[a] + (E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((
(-I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I
)*Sqrt[a]*d)/Sqrt[b] - d*x] + (I*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqr
t[a]*d)/Sqrt[b] - d*x]))/Sqrt[a] + (4*b*x*Sinh[c]*Sinh[d*x])/(a + b*x^2))/
b^2
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.03, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5814, 5804, 2009, 5815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}dx$$

$$\Big\downarrow \ 5814$$

$$\frac{d\int \frac{x\sinh(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \frac{\cosh(c+dx)}{bx^2+a}dx}{2b}-\frac{x\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \ 5804$$

$$\frac{d\int \frac{x\sinh(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{x\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{d\int \frac{x\sinh(c+dx)}{bx^2+a}dx}{2b}+\frac{\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-}}{}\right.}{2\sqrt{-a}\sqrt{b}}}{2b}$$
$$\frac{x\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \ 5815$$

$$\frac{d\int \left(\frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\sinh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{2b}+\frac{\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-}}{}\right.}{2\sqrt{-a}\sqrt{b}}}{2b}$$
$$\frac{x\cosh(c+dx)}{2b(a+bx^2)}$$

$$\Big\downarrow \ 2009$$

$$\frac{d\left(\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2b}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2b}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd-\right.}{2b}\right.}{2b}$$

$$\frac{\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-}}{\sqrt{}}\right.}{2\sqrt{-a}\sqrt{b}}}{2b}$$

$$\frac{x\cosh(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*(x*Cosh[c + d*x])/(b*(a + b*x^2)) + (d*((CoshIntegral[(Sqrt[-a]*d)/Sq
rt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(2*b) + (CoshIntegral[(Sqrt[-
a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*b) - (Cosh[c + (Sq
rt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Cosh
[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b)
))/(2*b) + ((Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshInt
egral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[
-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt
[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] +
 d*x])/(2*Sqrt[-a]*Sqrt[b]))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 900 vs. 2(318) = 636.

Time = 0.74 (sec) , antiderivative size = 901, normalized size of antiderivative = 2.17

method	result
risch	$-\frac{-\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\sqrt{-ab}\,\mathrm{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)bd\,x^2-\sqrt{-ab}\,\mathrm{e}^{-\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_1\left(\frac{d\sqrt{-ab}+b(dx+c)-cb}{b}\right)bd\,x^2\cdots}{}$

input
```
int(x^2*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*(-exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*(
d*x+c)+c*b)/b)*b*d*x^2-(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*
(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b*d*x^2+(-a*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-b
*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c*b)/b)*b*d*x^2+(-a*b)^(1/2)*exp((-d*
(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b*d*x^2+exp(-
(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b^2*x^2-ex
p(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b^2*x^2
+Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c*b)/b)*b^2*x^
2-exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b^2
*x^2-exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*
x+c)+c*b)/b)*a*d-(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)
^(1/2)+b*(d*x+c)-c*b)/b)*a*d+(-a*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c
*b)/b)*exp((d*(-a*b)^(1/2)+c*b)/b)*a*d+(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*d+2*exp(-d*x-c)*x*b*(-a*b
)^(1/2)+exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/
b)*a*b+2*exp(d*x+c)*x*b*(-a*b)^(1/2)-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d
*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b+Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)
*exp((d*(-a*b)^(1/2)+c*b)/b)*a*b-exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a
*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b)/b^2/(b*x^2+a)/(-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(318) = 636$.

Time = 0.10 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.79

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2
- (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)
^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2
/b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*
sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(
d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^
2/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2
)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sin
h(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a
^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*
x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/
b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2
+ a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b
^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d
^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2
 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c
)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d
^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)
^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x +
 c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(...
```

Sympy [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{x^2\cosh{(c+dx)}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x**2*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2\cosh(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*((d*x^2*e^(2*c) + 2*x*e^(2*c))*e^(d*x) - (d*x^2 - 2*x)*e^(-d*x))/(b^2*
d^2*x^4*e^c + 2*a*b*d^2*x^2*e^c + a^2*d^2*e^c) + 1/2*integrate(-2*(2*a*d*x
*e^c - 3*b*x^2*e^c + a*e^c)*e^(d*x)/(b^3*d^2*x^6 + 3*a*b^2*d^2*x^4 + 3*a^2
*b*d^2*x^2 + a^3*d^2), x) + 1/2*integrate(2*(2*a*d*x + 3*b*x^2 - a)*e^(-d*
x)/(b^3*d^2*x^6*e^c + 3*a*b^2*d^2*x^4*e^c + 3*a^2*b*d^2*x^2*e^c + a^3*d^2*
e^c), x)
```

Giac [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2\cosh(dx+c)}{(bx^2+a)^2}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x^2\cosh(c+d\,x)}{(b\,x^2+a)^2}\,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^2}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}x^2}{b^2x^4+2ab\,x^2+a^2}dx\right)+\int \frac{x^2}{e^{dx}a^2+2e^{dx}ab\,x^2+e^{dx}b^2x^4}dx}{2e^c}$$

input
```
int(x^2*cosh(d*x+c)/(b*x^2+a)^2,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x**2)/(a**2 + 2*a*b*x**2 + b**2*x**4),x) + int(x**
2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x))/(2*e**c)
```

3.68 $\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 239

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\cosh(c+dx)}{2b(a+bx^2)} - \frac{d\,\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} + \frac{d\,\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4\sqrt{-a}b^{3/2}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4\sqrt{-a}b^{3/2}}$$

output
```
-1/2*cosh(d*x+c)/b/(b*x^2+a)-1/4*d*Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(-
a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b^(3/2)+1/4*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)
*sinh(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b^(3/2)+1/4*d*cosh(c+(-a)^(1/2)*d
/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)-1/4*d*cosh(c-(
-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.99

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{-\frac{4\sqrt{b}\cosh(c)\cosh(dx)}{a+bx^2} - \frac{ide^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{a}} - \frac{ide^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{Exp}\right.}{}}{8b^{3/2}}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
((-4*Sqrt[b]*Cosh[c]*Cosh[d*x])/(a + b*x^2) - (I*d*E^(c - (I*Sqrt[a]*d)/Sq
rt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt
[b] + x)] - ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[a] - (I*d*E^
(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[
((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*
x]))/Sqrt[a] - (4*Sqrt[b]*Sinh[c]*Sinh[d*x])/(a + b*x^2))/(8*b^(3/2))
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.02, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5812, 5803, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx$$
$$\Big\downarrow \ 5812$$
$$\frac{d\int \frac{\sinh(c+dx)}{bx^2+a}dx}{2b} - \frac{\cosh(c+dx)}{2b(a+bx^2)}$$
$$\Big\downarrow \ 5803$$

$$\frac{d\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{\cosh(c+dx)}{2b\left(a+bx^2\right)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(x\right.}{2\sqrt{-a}\sqrt{b}}\right.}{2b}$$

$$\frac{\cosh(c+dx)}{2b\left(a+bx^2\right)}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x^2)^2,x]
```

output
```
-1/2*Cosh[c + d*x]/(b*(a + b*x^2)) + (d*(-1/2*(CoshIntegral[(Sqrt[-a]*d)/S
qrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(Sqrt[-a]*Sqrt[b]) + (CoshIn
tegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*Sqrt
[-a]*Sqrt[b]) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*Sin
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b])))/(2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(181) = 362$.

Time = 0.65 (sec) , antiderivative size = 449, normalized size of antiderivative = 1.88

method	result
risch	$\frac{\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)bd\,x^2-\mathrm{expIntegral}_1\left(\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)\mathrm{e}^{\frac{d\sqrt{-ab}+cb}{b}}bd\,x^2+\mathrm{e}^{\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{expIn}}{}$

input
```
int(x*cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*(exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*
b*d*x^2-Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c*b)/b)
*b*d*x^2+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)
/b)*b*d*x^2-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c
*b)/b)*b*d*x^2+exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c
)+c*b)/b)*a*d-Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c
*b)/b)*a*d+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*
b)/b)*a*d-exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b
)/b)*a*d-2*(-a*b)^(1/2)*exp(d*x+c)-2*(-a*b)^(1/2)*exp(-d*x-c))/(b*x^2+a)/b
/(-a*b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 641 vs. $2(181) = 362$.

Time = 0.12 (sec) , antiderivative size = 641, normalized size of antiderivative = 2.68

$$\int \frac{x\cosh(c+dx)}{\left(a+bx^2\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*a*cosh(d*x + c) + (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh
(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b)) - ((b*x^2 + a)*cosh(d
*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x + sqrt(-a*
d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - (((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2
 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-a*d^2/b)) - ((b*x^2 +
 a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(-d*x
- sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cosh(d*x + c)
^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x - sqrt(-a*d^2/b))
+ ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/
b)*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((b*x^2 + a)*cos
h(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b)*Ei(d*x + sqrt(-
a*d^2/b)) + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sq
rt(-a*d^2/b)*Ei(-d*x - sqrt(-a*d^2/b)))*sinh(-c + sqrt(-a*d^2/b)))/((a*b^2
*x^2 + a^2*b)*cosh(d*x + c)^2 - (a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\cosh{(c+dx)}}{(a+bx^2)^2}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(x*cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{x\cosh{(dx+c)}}{(bx^2+a)^2}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^2*d*x^4*e^c + 2*a*b*d*x^2*e^c + a^2*
d*e^c) + 1/2*integrate((3*b*x^2*e^c - a*e^c)*e^(d*x)/(b^3*d*x^6 + 3*a*b^2*
d*x^4 + 3*a^2*b*d*x^2 + a^3*d), x) - 1/2*integrate((3*b*x^2 - a)*e^(-d*x)/
(b^3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*a^2*b*d*x^2*e^c + a^3*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{x\cosh\left(dx+c\right)}{\left(bx^2+a\right)^2}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(x*cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{x\cosh(c+d\,x)}{\left(b\,x^2+a\right)^2}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x^2)^2,x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{-e^{2dx+2c}+e^{dx+2c}\left(\int \frac{e^{dx}}{b\,x^2+a}dx\right)ad+e^{dx+2c}\left(\int \frac{e^{dx}}{b\,x^2+a}dx\right)bd\,x^2-e^{dx}\left(\int \frac{1}{e^{dx}a+e^{dx}b\,x^2}dx\right)ad-e^{dx}\left(\int \frac{1}{e^{dx}a+e^{d}}\right.}{4e^{dx+c}b\,(b\,x^2+a)}$$

input
```
int(x*cosh(d*x+c)/(b*x^2+a)^2,x)
```

output
```
( - e**(2*c + 2*d*x) + e**(2*c + d*x)*int(e**(d*x)/(a + b*x**2),x)*a*d + e
**(2*c + d*x)*int(e**(d*x)/(a + b*x**2),x)*b*d*x**2 - e**(d*x)*int(1/(e**(
d*x)*a + e**(d*x)*b*x**2),x)*a*d - e**(d*x)*int(1/(e**(d*x)*a + e**(d*x)*b
*x**2),x)*b*d*x**2 - 1)/(4*e**(c + d*x)*b*(a + b*x**2))
```

3.69 $\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 476

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = -\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4ab} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}}$$

output

```
-1/4*cosh(d*x+c)/a/b^(1/2)/((-a)^(1/2)-b^(1/2)*x)+1/4*cosh(d*x+c)/a/b^(1/2
)/((-a)^(1/2)+b^(1/2)*x)-1/4*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d
/b^(1/2)-d*x)/(-a)^(3/2)/b^(1/2)+1/4*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)
^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(1/2)-1/4*d*Chi((-a)^(1/2)*d/b^(1/2)+d*
x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/a/b-1/4*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*si
nh(c+(-a)^(1/2)*d/b^(1/2))/a/b-1/4*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a
)^(1/2)*d/b^(1/2)+d*x)/a/b-1/4*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2
)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(1/2)-1/4*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi
((-a)^(1/2)*d/b^(1/2)+d*x)/a/b+1/4*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(
1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.84 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.62

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx$$
$$= \frac{\frac{4\sqrt{a}x\cosh(c)\cosh(dx)}{a+bx^2} - \frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{b} + \cdots}{8a^{3/2}}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x^2)^2,x]
```

output
```
((4*Sqrt[a]*x*Cosh[c]*Cosh[d*x])/(a + b*x^2) - (E^(c - (I*Sqrt[a]*d)/Sqrt[
b])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d
*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[
d*((I*Sqrt[a])/Sqrt[b] + x)]))/b + (E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((I*Sqr
t[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a
]*d)/Sqrt[b] - d*x] + ((-I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*
d)/Sqrt[b] - d*x]))/b + (4*Sqrt[a]*x*Sinh[c]*Sinh[d*x])/(a + b*x^2))/(8*a^
(3/2))
```

Rubi [A] (verified)

Time = 1.28 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx$$
$$\Big\downarrow \text{ 5804}$$

$$\int \left(-\frac{b\cosh(c+dx)}{2a\left(-ab-b^2x^2\right)}-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}+bx\right)^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{4a\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$

input
```
Int[Cosh[c + d*x]/(a + b*x^2)^2,x]
```

output
```
-1/4*Cosh[c + d*x]/(a*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a
*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshInt
egral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqr
t[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*
Sqrt[b]) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]])/(4*a*b) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c
+ (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhI
ntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt
[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d
*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/
(4*a*b) + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b
] + d*x])/(4*(-a)^(3/2)*Sqrt[b])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 503, normalized size of antiderivative = 1.06

method	result
risch	$\frac{d^2 \mathrm{e}^{-dx-c} x}{4a(b\,d^2x^2+a\,d^2)} - \frac{d\,\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)}{8ba} - \frac{d\,\mathrm{e}^{-\frac{-d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_1\left(\frac{d\sqrt{-ab}+b(dx+c)-cb}{b}\right)}{8ba}$

input
```
int(cosh(d*x+c)/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*d^2*exp(-d*x-c)*x/a/(b*d^2*x^2+a*d^2)-1/8*d/b/a*exp(-(d*(-a*b)^(1/2)+c
*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-1/8*d/b/a*exp(-(-d*(-a*b)^(
1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)-1/8/(-a*b)^(1/2)/a*exp
(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/8/(-a*
b)^(1/2)/a*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*
b)/b)+1/4*d^2*exp(d*x+c)*x/a/(b*d^2*x^2+a*d^2)+1/8*d/b/a*exp((d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/8*d/b/a*exp((-d*(-a*b)
^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)-1/8/(-a*b)^(1/2)/a*
exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/8/(-a
*b)^(1/2)/a*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c
*b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(365) = 730$.

Time = 0.11 (sec) , antiderivative size = 1162, normalized size of antiderivative = 2.44

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2} \, dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*b*d*x*cosh(d*x + c) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 -
 (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x^2 + a*b)*cosh(d*x + c)^
2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/
b)) - ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*s
inh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d
*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2
/b)) - (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)
*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh
(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^2 + a^
2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^2*x
^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b
))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^2 +
 a^2*d^2)*cosh(d*x + c)^2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 + ((b^
2*x^2 + a*b)*cosh(d*x + c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^2
- (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x + c)
^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^
2/b)))*sinh(c + sqrt(-a*d^2/b)) + (((a*b*d^2*x^2 + a^2*d^2)*cosh(d*x + c)^
2 - (a*b*d^2*x^2 + a^2*d^2)*sinh(d*x + c)^2 - ((b^2*x^2 + a*b)*cosh(d*x +
c)^2 - (b^2*x^2 + a*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-...
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\cosh{(c+dx)}}{(a+bx^2)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x**2+a)**2,x)
```

output
```
Integral(cosh(c + d*x)/(a + b*x**2)**2, x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\cosh{(dx+c)}}{(bx^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx = \int \frac{\cosh{(dx+c)}}{(bx^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*x^2 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\cosh(c+d\,x)}{\left(b\,x^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x^2)^2,x)
```

output
```
int(cosh(c + d*x)/(a + b*x^2)^2, x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^2}\,dx=\int \frac{\cosh\left(dx+c\right)}{b^2x^4+2ab\,x^2+a^2}dx$$

input
```
int(cosh(d*x+c)/(b*x^2+a)^2,x)
```

output
```
int(cosh(c + d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)
```

3.70 $\int \frac{\cosh(c+dx)}{x(a+bx^2)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 435

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^2}\,dx = \frac{\cosh(c+dx)}{2a(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2} - \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\sinh(c)\text{Shi}(dx)}{a^2} - \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{3/2}\sqrt{b}} - \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^2}$$

output

```
1/2*cosh(d*x+c)/a/(b*x^2+a)+cosh(c)*Chi(d*x)/a^2-1/2*cosh(c+(-a)^(1/2)*d/b
^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/a^2-1/2*cosh(c-(-a)^(1/2)*d/b^(1/2))
*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/a^2-1/4*d*Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sin
h(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(1/2)+1/4*d*Chi((-a)^(1/2)*d/b^(1/2
)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(1/2)+sinh(c)*Shi(d*x)/a^
2+1/4*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(
3/2)/b^(1/2)-1/2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*
x)/a^2-1/4*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/(-
a)^(3/2)/b^(1/2)-1/2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)
+d*x)/a^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 415, normalized size of antiderivative = 0.95

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)^2}\,dx = \frac{\frac{4a\cosh(c)\cosh(dx)}{a+bx^2} + \frac{i\sqrt{a}de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{b}} - 2e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{Exp}\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```

output
```
((4*a*Cosh[c]*Cosh[d*x])/(a + b*x^2) + (I*Sqrt[a]*d*E^(c - (I*Sqrt[a]*d)/S
qrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqr
t[b] + x)] - ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[b] - 2*E^(c
 - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(
((-I)*Sqrt[a])/Sqrt[b] + x)] + ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)])
 + (I*Sqrt[a]*d*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[
b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[
a]*d)/Sqrt[b] - d*x]))/Sqrt[b] - 2*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*
I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + Exp
IntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*a*Sinh[c]*Sinh[d*x])/(a + b*
x^2) + 8*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/(8*a^2)
```

Rubi [A] (verified)

Time = 1.26 (sec) , antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^2)^2}\,dx$$

$$\Big\downarrow \text{5816}$$

$$\int \left(-\frac{bx\cosh(c+dx)}{a^2(a+bx^2)}+\frac{\cosh(c+dx)}{a^2x}-\frac{bx\cosh(c+dx)}{a(a+bx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^2}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^2}+\frac{\cosh(c)\text{Chi}(dx)}{a^2}+\frac{\sinh(c)\text{Shi}(dx)}{a^2}-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh(c+dx)}{2a(a+bx^2)}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x^2)^2),x]
```

output
```
Cosh[c + d*x]/(2*a*(a + b*x^2)) + (Cosh[c]*CoshIntegral[d*x])/a^2 - (Cosh[
c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2
) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*
x])/(2*a^2) - (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-
a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt
[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[
c]*SinhIntegral[d*x])/a^2 - (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral
[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]
*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^2) - (d*Cosh[c
 - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)
^(3/2)*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(2*a^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 546, normalized size of antiderivative = 1.26

method	result
risch	$\frac{e^{-dx-c}d^2}{4a\left((dx+c)^2b-2b(dx+c)c+a\,d^2+b\,c^2\right)}-\frac{e^{-c}\operatorname{expIntegral}_1(dx)}{2a^2}-\frac{e^{-\frac{d\sqrt{-ab}+cb}{b}}\operatorname{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)d}{8a\sqrt{-ab}}+\frac{e^{-\frac{-c}{}}}{}$

input
```
int(cosh(d*x+c)/x/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(-d*x-c)*d^2/a/((d*x+c)^2*b-2*b*(d*x+c)*c+a*d^2+b*c^2)-1/2/a^2*exp(
-c)*Ei(1,d*x)-1/8/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a
*b)^(1/2)-b*(d*x+c)+c*b)/b)*d+1/8/a/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*d+1/4/a^2*exp(-(d*(-a*b)^(1/2)
+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/4/a^2*exp(-(-d*(-a*b)^(
1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)+1/4*exp(d*x+c)*d^2/a/(
(d*x+c)^2*b-2*b*(d*x+c)*c+a*d^2+b*c^2)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/8/a/(-a
*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)^(1/2)+c*b)/
b)*d-1/8/a/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)
+b*(d*x+c)-c*b)/b)*d+1/4/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1
/2)-b*(d*x+c)+c*b)/b)+1/4/a^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)
^(1/2)+b*(d*x+c)-c*b)/b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(337) = 674$.

Time = 0.14 (sec) , antiderivative size = 992, normalized size of antiderivative = 2.28

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
1/8*(4*a*cosh(d*x + c) - ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*s
inh(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^
2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)
^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2
 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c +
 sqrt(-a*d^2/b)) + 4*((b*x^2 + a)*Ei(d*x) + (b*x^2 + a)*Ei(-d*x))*cosh(c)
- ((2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^
2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d
*x + sqrt(-a*d^2/b)) + (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh
(d*x + c)^2 - ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*
sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - ((2
*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a
)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x -
sqrt(-a*d^2/b)) - (2*(b*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x
+ c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*x^2 + a)*sinh(d*x + c)^2)*sqrt(
-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) + 4*((b*x^2
 + a)*Ei(d*x) - (b*x^2 + a)*Ei(-d*x))*sinh(c) + ((2*(b*x^2 + a)*cosh(d*x +
 c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 + ((b*x^2 + a)*cosh(d*x + c)^2 - (b*
x^2 + a)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (2*(b
*x^2 + a)*cosh(d*x + c)^2 - 2*(b*x^2 + a)*sinh(d*x + c)^2 - ((b*x^2 + a...
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx=\int \frac{\cosh\left(c+dx\right)}{x\left(a+bx^2\right)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x**2+a)**2,x)
```

output
```
Integral(cosh(c + d*x)/(x*(a + b*x**2)**2), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx=\int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^2x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx=\text{Exception raised: AttributeError}$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx=\int \frac{\cosh(c+d\,x)}{x\,\left(b\,x^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x^2)^2),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^2}\,dx=\int \frac{\cosh\left(dx+c\right)}{b^2x^5+2ab\,x^3+a^2x}dx$$

input
```
int(cosh(d*x+c)/x/(b*x^2+a)^2,x)
```

output
```
int(cosh(c + d*x)/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)
```

3.71 $\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 500

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx = -\frac{\cosh(c+dx)}{a^2x} + \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{3\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} + \frac{3\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}} + \frac{d\text{Chi}(dx)\sinh(c)}{a^2} + \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2} + \frac{d\cosh(c)\text{Shi}(dx)}{a^2} - \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2} + \frac{3\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}} + \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4a^2} + \frac{3\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{4(-a)^{5/2}}$$

output
```
-cosh(d*x+c)/a^2/x+1/4*b^(1/2)*cosh(d*x+c)/a^2/((-a)^(1/2)-b^(1/2)*x)-1/4*
b^(1/2)*cosh(d*x+c)/a^2/((-a)^(1/2)+b^(1/2)*x)-3/4*b^(1/2)*cosh(c+(-a)^(1/
2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(5/2)+3/4*b^(1/2)*cosh(c-
(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)+d*Chi(d*x)*
sinh(c)/a^2+1/4*d*Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2
))/a^2+1/4*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/a^
2+d*cosh(c)*Shi(d*x)/a^2+1/4*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2
)*d/b^(1/2)+d*x)/a^2-3/4*b^(1/2)*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1
/2)*d/b^(1/2)+d*x)/(-a)^(5/2)+1/4*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^
(1/2)*d/b^(1/2)+d*x)/a^2+3/4*b^(1/2)*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)
^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.22 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.67

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx$$
$$= \frac{-\frac{4\sqrt{a}(2a+3bx^2)\cosh(c)\cosh(dx)}{x(a+bx^2)} + e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(3i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-3i\sqrt{b}+\right.\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

output
```
((-4*Sqrt[a]*(2*a + 3*b*x^2)*Cosh[c]*Cosh[d*x])/(x*(a + b*x^2)) + E^(c - (
I*Sqrt[a]*d)/Sqrt[b])*(((3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sq
rt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-3*I)*Sqrt[b] + S
qrt[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]) - E^(-c - (I*Sqrt[a]
*d)/Sqrt[b])*(((3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*Ex
pIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ((-3*I)*Sqrt[b] + Sqrt[a]*d)
*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) - (4*Sqrt[a]*(2*a + 3*b*x^2)*
Sinh[c]*Sinh[d*x])/(x*(a + b*x^2)) + 8*Sqrt[a]*d*(CoshIntegral[d*x]*Sinh[c
] + Cosh[c]*SinhIntegral[d*x]))/(8*a^(5/2))
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx$$
$$\Big\downarrow \ 5816$$
$$\int \left(-\frac{b\cosh(c+dx)}{a^2(a+bx^2)}+\frac{\cosh(c+dx)}{a^2x^2}-\frac{b\cosh(c+dx)}{a(a+bx^2)^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4a^2}+\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4a^2}+\frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{\sqrt{b}\cosh(c+dx)}{4a^2\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{d\sinh(c)\mathrm{Chi}(dx)}{a^2}+\frac{d\cosh(c)\mathrm{Shi}(dx)}{a^2}-\frac{\cosh(c+dx)}{a^2x}-\frac{3\sqrt{b}\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{5/2}}+\frac{3\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{5/2}}$$

input
```
Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^2),x]
```

output
```
-(Cosh[c + d*x]/(a^2*x)) + (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] - Sqrt
[b]*x)) - (Sqrt[b]*Cosh[c + d*x])/(4*a^2*(Sqrt[-a] + Sqrt[b]*x)) - (3*Sqrt
[b]*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x
])/(4*(-a)^(5/2)) + (3*Sqrt[b]*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2)) + (d*CoshIntegral[d*x]*Sinh[c
])/a^2 + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)
/Sqrt[b]])/(4*a^2) + (d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c +
(Sqrt[-a]*d)/Sqrt[b]])/(4*a^2) + (d*Cosh[c]*SinhIntegral[d*x])/a^2 - (d*Co
sh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*
a^2) + (3*Sqrt[b]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(4*(-a)^(5/2)) + (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIn
tegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a^2) + (3*Sqrt[b]*Sinh[c - (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(5/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 595, normalized size of antiderivative = 1.19

method	result
risch	$-\frac{3\,\mathrm{e}^{-dx-c} x\,d^2 b}{4a^2(b\,d^2x^2+a\,d^2)}-\frac{\mathrm{e}^{-dx-c} d^2}{2ax(b\,d^2x^2+a\,d^2)}+\frac{d\,\mathrm{e}^{-c}\,\mathrm{expIntegral}_1(dx)}{2a^2}+\frac{d\,\mathrm{e}^{-\frac{d\sqrt{-ab}+cb}{b}}\,\mathrm{expIntegral}_1\left(-\frac{d\sqrt{-ab}-b(dx+c)+cb}{b}\right)}{8a^2}+$

input
```
int(cosh(d*x+c)/x^2/(b*x^2+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-3/4*exp(-d*x-c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*exp(-d*x-c)/a/x*d^2/(b*
d^2*x^2+a*d^2)+1/2*d/a^2*exp(-c)*Ei(1,d*x)+1/8*d/a^2*exp(-(d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/8*d/a^2*exp(-(-d*(-a*b)^
(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)+3/8/a^2/(-a*b)^(1/2)*
exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b-3/8
/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*
x+c)-c*b)/b)*b-3/4*exp(d*x+c)/a^2*x*d^2/(b*d^2*x^2+a*d^2)*b-1/2*exp(d*x+c)
/a/x*d^2/(b*d^2*x^2+a*d^2)-1/2*d/a^2*exp(c)*Ei(1,-d*x)-1/8*d/a^2*exp((d*(-
a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-1/8*d/a^2*exp((-
d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)+3/8/a^2/(-a
*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/
b)*b-3/8/a^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/
2)+b*(d*x+c)-c*b)/b)*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1310 vs. $2(389) = 778$.

Time = 0.13 (sec) , antiderivative size = 1310, normalized size of antiderivative = 2.62

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="fricas")
```

output
```
-1/8*(4*(3*a*b*d*x^2 + 2*a^2*d)*cosh(d*x + c) - (((a*b*d^2*x^3 + a^2*d^2*x
)*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^
3 + a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^
2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x + c)^
2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*cosh(
d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x +
sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d^2*x)*E
i(d*x) - (a*b*d^2*x^3 + a^2*d^2*x)*Ei(-d*x))*cosh(c) - (((a*b*d^2*x^3 + a^
2*d^2*x)*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*(
(b^2*x^3 + a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqr
t(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*
x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^2*x^3 + a*b*x
)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(
-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - (((a*b*d^2*x^3 + a^2*d
^2*x)*cosh(d*x + c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 + 3*((b^
2*x^3 + a*b*x)*cosh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-
a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + ((a*b*d^2*x^3 + a^2*d^2*x)*cosh(d*x +
 c)^2 - (a*b*d^2*x^3 + a^2*d^2*x)*sinh(d*x + c)^2 - 3*((b^2*x^3 + a*b*x)*c
osh(d*x + c)^2 - (b^2*x^3 + a*b*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*
x + sqrt(-a*d^2/b)))*sinh(c + sqrt(-a*d^2/b)) - 4*((a*b*d^2*x^3 + a^2*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x**2/(b*x**2+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^2x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^2}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^2x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a)^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^2*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx=\int \frac{\cosh(c+d\,x)}{x^2\,\left(b\,x^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)/(x^2*(a + b*x^2)^2),x)
```

output
```
int(cosh(c + d*x)/(x^2*(a + b*x^2)^2), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^2}\,dx$$
$$=\frac{-e^{2dx+2c}a\,d^2x-2e^{2dx+2c}ad-6e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^2bdx-6e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right)a\,b^2d\,x^3+}{}$$

input
```
int(cosh(d*x+c)/x^2/(b*x^2+a)^2,x)
```

output
```
( - e**(2*c + 2*d*x)*a*d**2*x - 2*e**(2*c + 2*d*x)*a*d - 6*e**(2*c + d*x)*
int(e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2*b*d*x - 6*e**(2*c + d
*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a*b**2*d*x**3 + 2*e**(
2*c + d*x)*int(e**(d*x)/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a**3*d**2*x +
 2*e**(2*c + d*x)*int(e**(d*x)/(a**2*x + 2*a*b*x**3 + b**2*x**5),x)*a**2*b
*d**2*x**3 + e**(2*c + d*x)*int(e**(d*x)/(a + b*x**2),x)*a**2*d**3*x + e**
(2*c + d*x)*int(e**(d*x)/(a + b*x**2),x)*a*b*d**3*x**3 + 6*e**(d*x)*int(x*
*2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*d*
x + 6*e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b*
*2*x**4),x)*b**3*d*x**3 + 12*e**(d*x)*int(x/(e**(d*x)*a**2 + 2*e**(d*x)*a*
b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*x + 12*e**(d*x)*int(x/(e**(d*x)*a**
2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*b**3*x**3 - 2*e**(d*x)*in
t(1/(e**(d*x)*a**2*x + 2*e**(d*x)*a*b*x**3 + e**(d*x)*b**2*x**5),x)*a**3*d
**2*x - 2*e**(d*x)*int(1/(e**(d*x)*a**2*x + 2*e**(d*x)*a*b*x**3 + e**(d*x)
*b**2*x**5),x)*a**2*b*d**2*x**3 + e**(d*x)*int(1/(e**(d*x)*a + e**(d*x)*b*
x**2),x)*a**2*d**3*x + e**(d*x)*int(1/(e**(d*x)*a + e**(d*x)*b*x**2),x)*a*
b*d**3*x**3 + a*d**2*x - 2*a*d + 6*b*x)/(4*e**(c + d*x)*a**2*d*x*(a + b*x*
*2))
```

3.72 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 476

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^2)^3}\,dx = -\frac{x^2\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh(c+dx)}{4b^2(a+bx^2)} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} + \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3} - \frac{3d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} + \frac{3d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16\sqrt{-a}b^{5/2}} - \frac{dx\sinh(c+dx)}{8b^2(a+bx^2)} - \frac{3d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16b^3} - \frac{3d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16b^3}$$

output
```
-1/4*x^2*cosh(d*x+c)/b/(b*x^2+a)^2-1/4*cosh(d*x+c)/b^2/(b*x^2+a)+1/16*d^2*
cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/b^3+1/16*d^2*co
sh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/b^3-3/16*d*Chi((-
a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)/b^(5/2)+3/
16*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(1/2)
/b^(5/2)-1/8*d*x*sinh(d*x+c)/b^2/(b*x^2+a)+3/16*d*cosh(c+(-a)^(1/2)*d/b^(1
/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(5/2)+1/16*d^2*sinh(c+(-a
)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/b^3-3/16*d*cosh(c-(-a)^(
1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^(5/2)+1/16*d^2*
sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/b^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.93 (sec) , antiderivative size = 332, normalized size of antiderivative = 0.70

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3}\,dx = \frac{\frac{de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-3i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\operatorname{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(3i\sqrt{b}+\sqrt{a}d\right)\operatorname{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)}{\sqrt{a}}+\frac{de^{-c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(-3i\sqrt{b}+\sqrt{\right.\right.}{}}{}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

output
```
((d*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*
Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((3*I)
*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[a]
 + (d*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(((-3*I)*Sqrt[b] + Sqrt[a]*d)*E^(((2*
I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ((3
*I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/Sqrt
[a] - (4*b*Cosh[d*x]*(2*(a + 2*b*x^2)*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/
(a + b*x^2)^2 - (4*b*(d*x*(a + b*x^2)*Cosh[c] + 2*(a + 2*b*x^2)*Sinh[c])*S
inh[d*x])/(a + b*x^2)^2)/(32*b^3)
```

Rubi [A] (verified)

Time = 2.61 (sec) , antiderivative size = 714, normalized size of antiderivative = 1.50, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5814, 5812, 5803, 2009, 5813, 5803, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{ 5814}$$

$$\frac{d\int \frac{x^2\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}+\frac{\int \frac{x\cosh(c+dx)}{(bx^2+a)^2}dx}{2b}-\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 5812

$$\frac{d\int \frac{x^2\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}+\frac{\frac{d\int \frac{\sinh(c+dx)}{bx^2+a}dx}{2b}-\frac{\cosh(c+dx)}{2b(a+bx^2)}}{2b}-\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 5803

$$\frac{d\int \frac{x^2\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}+\frac{\frac{d\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{\cosh(c+dx)}{2b(a+bx^2)}}{2b}-\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 2009

$$\frac{d\int \frac{x^2\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}+$$
$$\frac{\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right)}{2b}}{2b}$$
$$\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 5813

$$\frac{d\left(\frac{\int \frac{\sinh(c+dx)}{bx^2+a}dx}{2b}+\frac{d\int \frac{x\cosh(c+dx)}{bx^2+a}dx}{2b}-\frac{x\sinh(c+dx)}{2b(a+bx^2)}\right)}{4b}+$$
$$\frac{\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right)}{2b}}{2b}$$
$$\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\big\downarrow$ 5803

$$\frac{\frac{d\left(\frac{d\int \frac{x\cosh(c+dx)}{bx^2+a}dx}{2b}+\frac{\int \left(\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\sinh(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{x\sinh(c+dx)}{2b(a+bx^2)}\right)}{4b}+\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right)}{2b}}{2b}$$

$$\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{d\left(\frac{d\int \frac{x\cosh(c+dx)}{bx^2+a}dx}{2b}+\frac{-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\sqrt{\ }\right.}{}}{2b}\right.}{4b}\quad\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right)}{2b}}{2b}$$

$$\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 5816$$

$$\frac{\frac{d\left(\frac{d\int \left(\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\cosh(c+dx)}{2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}\right)dx}{2b}+\frac{-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}}{2\sqrt{-a}\sqrt{\ }}}{2b}\right.}{4b}\quad\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right)}{2b}}{2b}$$

$$\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}\right)}{2b}}{2b}$$

$$d\left(\frac{-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}}{2b}\right.$$

$$\frac{x^2\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

output
```
-1/4*(x^2*Cosh[c + d*x])/(b*(a + b*x^2)^2) + (-1/2*Cosh[c + d*x]/(b*(a + b
*x^2)) + (d*(-1/2*(CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt
[-a]*d)/Sqrt[b]])/(Sqrt[-a]*Sqrt[b]) + (CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqrt[b]) - (Cosh[c + (S
qrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*
Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[
b] + d*x])/(2*Sqrt[-a]*Sqrt[b])))/(2*b))/(2*b) + (d*(-1/2*(x*Sinh[c + d*x]
)/(b*(a + b*x^2)) + (-1/2*(CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c
 - (Sqrt[-a]*d)/Sqrt[b]])/(Sqrt[-a]*Sqrt[b]) + (CoshIntegral[(Sqrt[-a]*d)/
Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(2*Sqrt[-a]*Sqrt[b]) - (Cos
h[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*S
qrt[-a]*Sqrt[b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*
d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b]))/(2*b) + (d*((Cosh[c + (Sqrt[-a]*d
)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b) + (Cosh[c - (Sq
rt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*b) - (Sinh
[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*b)
 + (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x
])/(2*b)))/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

rule 5813
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Sinh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1554 vs. $2(374) = 748$.

Time = 1.31 (sec) , antiderivative size = 1555, normalized size of antiderivative = 3.27

method	result	size
risch	Expression too large to display	1555

input
```
int(x^3*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32*(-2*(-a*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b)
^(1/2)+c*b)/b)*a*b*d^2*x^2-2*(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b*d^2*x^2+3*exp(-(d*(-a*b)^(1/2)+c*
b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b^3*d*x^4-3*exp(-(-d*(-a*b)^
(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b^3*d*x^4+2*exp(-d*x-
c)*(-a*b)^(1/2)*b^2*d*x^3-exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-
(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2*d^2-(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*d^2+3*exp(-(d*(-a*b)
^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2*b*d-3*exp(-(-d*
(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*b*d-2*exp(
-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)
/b)*a*b*d^2*x^2-2*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b
)^(1/2)+b*(d*x+c)-c*b)/b)*a*b*d^2*x^2-6*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b^2*d*x^2+6*exp(-(d*(-a*b)^(1/2)+c*b)
/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a*b^2*d*x^2-(-a*b)^(1/2)*exp(-
(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b^2*d^2*x^
4-exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c
)+c*b)/b)*b^2*d^2*x^4+3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)
+b*(d*x+c)-c*b)/b)*a^2*b*d-8*(-a*b)^(1/2)*exp(d*x+c)*b^2*x^2-8*(-a*b)^(1/2
)*exp(-d*x-c)*b^2*x^2-(-a*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1620 vs. $2(374) = 748$.

Time = 0.15 (sec) , antiderivative size = 1620, normalized size of antiderivative = 3.40

$$\int \frac{x^3 \cosh(c + dx)}{(a + bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(8*(2*a*b^2*x^2 + a^2*b)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*
d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^
3*d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^
2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*
x - sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*
x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + 3
*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2
 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh
(c + sqrt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(
d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 +
 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x
^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((
a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^
4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2
 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^
2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) +
4*(a*b^2*d*x^3 + a^2*b*d*x)*sinh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2
*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d
^2)*sinh(d*x + c)^2 - 3*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 -
 (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^2+a)^3} \, dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3
 - 3*d*x^2 + 12*x)*e^(-d*x))/(b^3*d^3*x^6*e^c + 3*a*b^2*d^3*x^4*e^c + 3*a^
2*b*d^3*x^2*e^c + a^3*d^3*e^c) - 1/2*integrate(6*(3*a*d*x*e^c + (a*d^2*e^c
 - 10*b*e^c)*x^2 + 2*a*e^c)*e^(d*x)/(b^4*d^3*x^8 + 4*a*b^3*d^3*x^6 + 6*a^2
*b^2*d^3*x^4 + 4*a^3*b*d^3*x^2 + a^4*d^3), x) + 1/2*integrate(-6*(3*a*d*x
- (a*d^2 - 10*b)*x^2 - 2*a)*e^(-d*x)/(b^4*d^3*x^8*e^c + 4*a*b^3*d^3*x^6*e^
c + 6*a^2*b^2*d^3*x^4*e^c + 4*a^3*b*d^3*x^2*e^c + a^4*d^3*e^c), x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^2+a)^3} \, dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3}\, dx = \int \frac{x^3 \cosh(c+d\,x)}{(b\,x^2+a)^3}\, dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x^2)^3,x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^2)^3}\, dx = \text{Too large to display}$$

input
```
int(x^3*cosh(d*x+c)/(b*x^2+a)^3,x)
```

output
```
(e**(2*c + 2*d*x)*a*d**2 + 4*e**(2*c + 2*d*x)*b*d*x - e**(2*c + d*x)*int(e
**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**4*d**3 -
2*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3
*x**6),x)*a**3*b*d**3*x**2 - 4*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*
b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**3*b*d - e**(2*c + d*x)*int(e**(d
*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**2*b**2*d**3*x
**4 - 8*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4
+ b**3*x**6),x)*a**2*b**2*d*x**2 - 4*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3
*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a*b**3*d*x**4 - e**(2*c + d*x
)*int((e**(d*x)*x**2)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x
)*a**3*b*d**3 - 2*e**(2*c + d*x)*int((e**(d*x)*x**2)/(a**3 + 3*a**2*b*x**2
 + 3*a*b**2*x**4 + b**3*x**6),x)*a**2*b**2*d**3*x**2 + 12*e**(2*c + d*x)*i
nt((e**(d*x)*x**2)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a
**2*b**2*d - e**(2*c + d*x)*int((e**(d*x)*x**2)/(a**3 + 3*a**2*b*x**2 + 3*
a*b**2*x**4 + b**3*x**6),x)*a*b**3*d**3*x**4 + 24*e**(2*c + d*x)*int((e**(
d*x)*x**2)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a*b**3*d*
x**2 + 12*e**(2*c + d*x)*int((e**(d*x)*x**2)/(a**3 + 3*a**2*b*x**2 + 3*a*b
**2*x**4 + b**3*x**6),x)*b**4*d*x**4 - 16*e**(d*x)*int(x**2/(e**(d*x)*a**3
 + 3*e**(d*x)*a**2*b*x**2 + 3*e**(d*x)*a*b**2*x**4 + e**(d*x)*b**3*x**6),x
)*a**2*b**2*d - 32*e**(d*x)*int(x**2/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b...
```

3.73 $\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 746

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx = -\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\cosh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)} - \frac{x\cosh(c+dx)}{4b(a+bx^2)^2} - \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} + \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} + \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\mathrm{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16ab^2} - \frac{d\sinh(c+dx)}{8b^2(a+bx^2)} + \frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2} + \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16\sqrt{-a}b^{5/2}} - \frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2} + \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}} - \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\mathrm{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16\sqrt{-a}b^{5/2}}$$

output
```
-1/16*cosh(d*x+c)/a/b^(3/2)/((-a)^(1/2)-b^(1/2)*x)+1/16*cosh(d*x+c)/a/b^(3
/2)/((-a)^(1/2)+b^(1/2)*x)-1/4*x*cosh(d*x+c)/b/(b*x^2+a)^2-1/16*cosh(c+(-a
)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(3/2)/b^(3/2)+1/16*d
^2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(1/2)/b
^(5/2)+1/16*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a
)^(3/2)/b^(3/2)-1/16*d^2*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(
1/2)+d*x)/(-a)^(1/2)/b^(5/2)-1/16*d*Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(
-a)^(1/2)*d/b^(1/2))/a/b^2-1/16*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a
)^(1/2)*d/b^(1/2))/a/b^2-1/8*d*sinh(d*x+c)/b^2/(b*x^2+a)-1/16*d*cosh(c+(-a
)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/a/b^2-1/16*sinh(c+(-a)^(
1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(3/2)+1/16*d^2
*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(1/2)/b^
(5/2)-1/16*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/a/
b^2+1/16*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(
3/2)/b^(3/2)-1/16*d^2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2
)+d*x)/(-a)^(1/2)/b^(5/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.77 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.51

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{-ie^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(b-i\sqrt{a}\sqrt{b}d+ad^2\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\,\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\left(b+i\sqrt{a}\sqrt{b}d+ad^2\right)\text{ExpInte}}{}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

output
```
((-I)*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*((b - I*Sqrt[a]*Sqrt[b]*d + a*d^2)*E^(
((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] -
 (b + I*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] +
x)]) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((I*b + Sqrt[a]*Sqrt[b]*d + I*a*d^2)
*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*
x] - I*(b + I*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[
b] - d*x]) - (4*Sqrt[a]*Sqrt[b]*Cosh[d*x]*(-(b*x*(-a + b*x^2)*Cosh[c]) + a
*d*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 - (4*Sqrt[a]*Sqrt[b]*(a*d*(a + b*x^
2)*Cosh[c] + b*x*(a - b*x^2)*Sinh[c])*Sinh[d*x])/(a + b*x^2)^2)/(32*a^(3/2
)*b^(5/2))
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 759, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5814, 5804, 2009, 5811, 5804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}dx$$

$$\Big\downarrow \text{5814}$$

$$\frac{d\int \frac{x\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}+\frac{\int \frac{\cosh(c+dx)}{(bx^2+a)^2}dx}{4b}-\frac{x\cosh(c+dx)}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{5804}$$

$$\frac{\int \left(-\frac{b\cosh(c+dx)}{2a(-b^2x^2-ab)}-\frac{b\cosh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\cosh(c+dx)}{4a\left(bx+\sqrt{-a}\sqrt{b}\right)^2}\right)dx}{4b}+\frac{d\int \frac{x\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}-\frac{x\cosh(c+dx)}{4b(a+bx^2)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{d\int \frac{x\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}+$$

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\cdots\right.}{4(-a)^{3/2}\sqrt{b}}\cdots$$

$$\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\downarrow$ 5811

$$\frac{d\left(\frac{d\int \frac{\cosh(c+dx)}{bx^2+a}dx}{2b}-\frac{\sinh(c+dx)}{2b(a+bx^2)}\right)}{4b}+$$

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\cdots\right.}{4(-a)^{3/2}\sqrt{b}}\cdots$$

$$\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\downarrow$ 5804

$$\frac{d\left(\frac{d\int\left(\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{-a}\cosh(c+dx)}{2a\left(\sqrt{b}x+\sqrt{-a}\right)}\right)dx}{2b}-\frac{\sinh(c+dx)}{2b(a+bx^2)}\right)}{4b}+$$

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\cdots\right.}{4(-a)^{3/2}\sqrt{b}}\cdots$$

$$\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$\downarrow$ 2009

$$\frac{d\left(\frac{d\left(\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2\sqrt{-a}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right.\cdots}{2\sqrt{-a}\sqrt{b}}\cdots\right.}{2b}\cdots\right.}{4b}$$

$$-\frac{d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4ab}-\frac{d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\cdots\right.}{4(-a)^{3/2}\sqrt{b}}\cdots$$

$$\frac{x\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

output
```
-1/4*(x*Cosh[c + d*x])/(b*(a + b*x^2)^2) + (-1/4*Cosh[c + d*x]/(a*Sqrt[b]*
(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x
)) - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d
*x])/(4*(-a)^(3/2)*Sqrt[b]) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral
[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*CoshIntegral[(Sq
rt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*a*b) - (d*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(4*a*
b) + (d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] -
 d*x])/(4*a*b) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] - d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]
]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) + (Sinh[c - (Sqrt[-a]*
d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b
]))/(4*b) + (d*(-1/2*Sinh[c + d*x]/(b*(a + b*x^2)) + (d*((Cosh[c + (Sqrt[-
a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[
b]) - (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] +
d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[
(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*Sqrt[-a]*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)
/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*Sqrt[-a]*Sqrt[b])))
/(2*b)))/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5811
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2171 vs. $2(574) = 1148$.

Time = 1.10 (sec) , antiderivative size = 2172, normalized size of antiderivative = 2.91

method	result	size
risch	Expression too large to display	2172

input
```
int(x^2*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/32/a*(2*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp(-(d*(-a*b)^(1/2)+c*
b)/b)*a*b^2*x^2-2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*
x+c)-c*b)/b)*a*b^2*x^2+Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp(-(d*(-a
*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*a^2*d+exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d
*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*(-a*b)^(1/2)*a^2*d+2*(-a*b)^(1/2)*exp(-d*x
-c)*a*b*x-2*exp(-d*x-c)*(-a*b)^(1/2)*a^2*d+Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)
+c*b)/b)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*b^3*x^4-exp(-(-d*(-a*b)^(1/2)+c*b)/b
)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b^3*x^4+Ei(1,-(d*(-a*b)^(1/2)-b*(
d*x+c)+c*b)/b)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*a^3*d^2-2*b^2*x^3*(-a*b)^(1/2)
*exp(-d*x-c)-2*(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/
2)-b*(d*x+c)+c*b)/b)*a*b*d*x^2-2*(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)
*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b*d*x^2+exp((d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a*b^2*d^2*x^4-exp((-d*(-a*b)^(
1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b^2*d^2*x^4-(-a*b)^
(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b
^2*d*x^4-(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b
*(d*x+c)-c*b)/b)*b^2*d*x^4+2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1
/2)-b*(d*x+c)+c*b)/b)*a^2*b*d^2*x^2-2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(
d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*b*d^2*x^2+Ei(1,-(d*(-a*b)^(1/2)-b*(d*
x+c)+c*b)/b)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*a*b^2*d^2*x^4-exp(-(-d*(-a*b)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2047 vs. $2(575) = 1150$.

Time = 0.12 (sec) , antiderivative size = 2047, normalized size of antiderivative = 2.74

$$\int \frac{x^2 \cosh(c + dx)}{(a + bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/32*(4*(a*b^2*d*x^3 - a^2*b*d*x)*cosh(d*x + c) - (((a*b^2*d^2*x^4 + 2*a^2
*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
 a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*
(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^
4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei
(d*x - sqrt(-a*d^2/b)) - ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh
(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2
- ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*c
osh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 +
 a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*c
osh(c + sqrt(-a*d^2/b)) - (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*co
sh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^
2 - ((a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)
*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2
 + a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) -
 ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2
*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2
 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^
2 + (a*b^2*d^2 + b^3)*x^4 + a^2*b + 2*(a^2*b*d^2 + a*b^2)*x^2)*sinh(d*x +
c)^2)*sqrt(-a*d^2/b))*Ei(-d*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^2\cosh(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*((d*x^2*e^(2*c) + 4*x*e^(2*c))*e^(d*x) - (d*x^2 - 4*x)*e^(-d*x))/(b^3*
d^2*x^6*e^c + 3*a*b^2*d^2*x^4*e^c + 3*a^2*b*d^2*x^2*e^c + a^3*d^2*e^c) + 1
/2*integrate(-2*(3*a*d*x*e^c - 10*b*x^2*e^c + 2*a*e^c)*e^(d*x)/(b^4*d^2*x^
8 + 4*a*b^3*d^2*x^6 + 6*a^2*b^2*d^2*x^4 + 4*a^3*b*d^2*x^2 + a^4*d^2), x) +
 1/2*integrate(2*(3*a*d*x + 10*b*x^2 - 2*a)*e^(-d*x)/(b^4*d^2*x^8*e^c + 4*
a*b^3*d^2*x^6*e^c + 6*a^2*b^2*d^2*x^4*e^c + 4*a^3*b*d^2*x^2*e^c + a^4*d^2*
e^c), x)
```

Giac [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x^2\cosh(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} \, dx = \int \frac{x^2 \cosh(c+d\,x)}{(b\,x^2+a)^3} \, dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x^2)^3,x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^2)^3} \, dx = \frac{e^{2c}\left(\int \frac{e^{dx}x^2}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)+\int \frac{x^2}{e^{dx}a^3+3e^{dx}a^2b\,x^2+3e^{dx}a\,b^2x^4+e^{dx}b^3x^6}dx}{2e^c}$$

input
```
int(x^2*cosh(d*x+c)/(b*x^2+a)^3,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x**2)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3
*x**6),x) + int(x**2/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**2 + 3*e**(d*x)*
a*b**2*x**4 + e**(d*x)*b**3*x**6),x))/(2*e**c)
```

3.74 $\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 512

$$
\begin{aligned}
\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = &-\frac{\cosh(c+dx)}{4b(a+bx^2)^2}-\frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}\\
&-\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}\\
&+\frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}\\
&-\frac{d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{3/2}b^{3/2}}-\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}-\sqrt{b}x\right)}\\
&+\frac{d\sinh(c+dx)}{16ab^{3/2}\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{3/2}b^{3/2}}\\
&+\frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16ab^2}\\
&+\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{3/2}b^{3/2}}\\
&-\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16ab^2}
\end{aligned}
$$

output
```
-1/4*cosh(d*x+c)/b/(b*x^2+a)^2-1/16*d^2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((
-a)^(1/2)*d/b^(1/2)-d*x)/a/b^2-1/16*d^2*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((
-a)^(1/2)*d/b^(1/2)+d*x)/a/b^2+1/16*d*Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sinh(c
-(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d*Chi((-a)^(1/2)*d/b^(1/2)-
d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(3/2)/b^(3/2)-1/16*d*sinh(d*x+c)/a/
b^(3/2)/((-a)^(1/2)-b^(1/2)*x)+1/16*d*sinh(d*x+c)/a/b^(3/2)/((-a)^(1/2)+b^
(1/2)*x)-1/16*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x
)/(-a)^(3/2)/b^(3/2)-1/16*d^2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)
*d/b^(1/2)+d*x)/a/b^2+1/16*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d
/b^(1/2)+d*x)/(-a)^(3/2)/b^(3/2)-1/16*d^2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi
((-a)^(1/2)*d/b^(1/2)+d*x)/a/b^2
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.88 (sec) , antiderivative size = 325, normalized size of antiderivative = 0.63

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{-de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(i\sqrt{b}+\sqrt{a}d\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-i\sqrt{b}+\sqrt{a}d\right)\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{}}{\sqrt{}}\right.\right.\right.}{}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

output
```
(-(d*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt
[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-I)*Sqrt
[b] + Sqrt[a]*d)*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)])) - d*E^(-c -
(I*Sqrt[a]*d)/Sqrt[b])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[
b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ((-I)*Sqrt[b] + Sqrt[a
]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*Sqrt[a]*b*Cosh[d*x]*
(-2*a*Cosh[c] + d*x*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 + (4*Sqrt[a]*b*(d*
x*(a + b*x^2)*Cosh[c] - 2*a*Sinh[c])*Sinh[d*x])/(a + b*x^2)^2)/(32*a^(3/2)
*b^2)
```

Rubi [A] (verified)

Time = 1.29 (sec) , antiderivative size = 507, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {5812, 5803, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \text{5812}$$

$$\frac{d\int \frac{\sinh(c+dx)}{(bx^2+a)^2}dx}{4b}-\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{5803}$$

$$\frac{d\int \left(-\frac{b\sinh(c+dx)}{2a(-b^2x^2-ab)}-\frac{b\sinh(c+dx)}{4a\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{b\sinh(c+dx)}{4a\left(bx+\sqrt{-a}\sqrt{b}\right)^2}\right)dx}{4b}-\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$d\left(-\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4(-a)^{3/2}\sqrt{b}}+\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{4(-a)^{3/2}\sqrt{b}}-\frac{d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{4ab}-\frac{d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Cl}}{4ab}\right.$$

$$\frac{\cosh(c+dx)}{4b\left(a+bx^2\right)^2}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x^2)^3,x]
```

output
```
-1/4*Cosh[c + d*x]/(b*(a + b*x^2)^2) + (d*(-1/4*(d*Cosh[c + (Sqrt[-a]*d)/S
qrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(a*b) - (d*Cosh[c - (Sqr
t[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b) + (Cos
hIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(4*(
-a)^(3/2)*Sqrt[b]) - (CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (S
qrt[-a]*d)/Sqrt[b]])/(4*(-a)^(3/2)*Sqrt[b]) - Sinh[c + d*x]/(4*a*Sqrt[b]*(
Sqrt[-a] - Sqrt[b]*x)) + Sinh[c + d*x]/(4*a*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)
) + (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*
x])/(4*(-a)^(3/2)*Sqrt[b]) + (d*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(4*a*b) + (Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*S
inhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*(-a)^(3/2)*Sqrt[b]) - (d*Sinh[
c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(4*a*b
)))/(4*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1502 vs. $2(398) = 796$.

Time = 0.97 (sec) , antiderivative size = 1503, normalized size of antiderivative = 2.94

method	result	size
risch	Expression too large to display	1503

input
```
int(x*cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/32/a*(2*(-a*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp((d*(-a*b
)^(1/2)+c*b)/b)*a*b*d^2*x^2+2*(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei
(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b*d^2*x^2+exp(-(d*(-a*b)^(1/2)+c*b
)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b^3*d*x^4-exp(-(-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b^3*d*x^4-2*exp(-d*x-c)*
(-a*b)^(1/2)*b^2*d*x^3+exp(-(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*
(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2*d^2+(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*d^2+exp(-(d*(-a*b)^(1/2
)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a^2*b*d-exp(-(-d*(-a*b)^
(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*a^2*b*d+2*exp(-(d*(-a
*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a*b
*d^2*x^2+2*(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)
+b*(d*x+c)-c*b)/b)*a*b*d^2*x^2-2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a
*b)^(1/2)+b*(d*x+c)-c*b)/b)*a*b^2*d*x^2+2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*a*b^2*d*x^2+(-a*b)^(1/2)*exp(-(-d*(-a
*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b^2*d^2*x^4+exp(-
(d*(-a*b)^(1/2)+c*b)/b)*(-a*b)^(1/2)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/
b)*b^2*d^2*x^4+exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c
)-c*b)/b)*a^2*b*d+(-a*b)^(1/2)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*exp(
(d*(-a*b)^(1/2)+c*b)/b)*b^2*d^2*x^4+(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1607 vs. $2(399) = 798$.

Time = 0.14 (sec) , antiderivative size = 1607, normalized size of antiderivative = 3.14

$$\int \frac{x \cosh(c+dx)}{(a+bx^2)^3} \, dx = \text{Too large to display}$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(8*a^2*b*cosh(d*x + c) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^
2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x
+ c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a
*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)
) + ((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*
d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2
*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x +
 c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b))
 + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*
d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((b^3*x^4 + 2*a*b^2
*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x +
 c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a*b^2*d^2*x^4 + 2*a^2*
b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 +
a^3*d^2)*sinh(d*x + c)^2 + ((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^
2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d
*x - sqrt(-a*d^2/b)))*cosh(-c + sqrt(-a*d^2/b)) - 4*(a*b^2*d*x^3 + a^2*b*d
*x)*sinh(d*x + c) + (((a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x
 + c)^2 - (a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((
b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - ((a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\cosh{(dx+c)}}{(bx^2+a)^3}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^6*e^c + 3*a*b^2*d*x^4*e^c + 3*
a^2*b*d*x^2*e^c + a^3*d*e^c) + 1/2*integrate((5*b*x^2*e^c - a*e^c)*e^(d*x)
/(b^4*d*x^8 + 4*a*b^3*d*x^6 + 6*a^2*b^2*d*x^4 + 4*a^3*b*d*x^2 + a^4*d), x)
 - 1/2*integrate((5*b*x^2 - a)*e^(-d*x)/(b^4*d*x^8*e^c + 4*a*b^3*d*x^6*e^c
 + 6*a^2*b^2*d*x^4*e^c + 4*a^3*b*d*x^2*e^c + a^4*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\cosh{(dx+c)}}{(bx^2+a)^3}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(x*cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{x\cosh(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x^2)^3,x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{-e^{2dx+2c}a + e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^3d + 2e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+a^2}dx\right)a^2bd\,x^2 + e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^4+2ab\,x^2+}\right.}{}$$

input
```
int(x*cosh(d*x+c)/(b*x^2+a)^3,x)
```

output
```
( - e**(2*c + 2*d*x)*a + e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 +
b**2*x**4),x)*a**3*d + 2*e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 +
b**2*x**4),x)*a**2*b*d*x**2 + e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x*
*2 + b**2*x**4),x)*a*b**2*d*x**4 + e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e*
*(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a**2*b*d + 2*e**(d*x)*int(x**2/(e
**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*d*x**2
+ e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x
**4),x)*b**3*d*x**4 + 2*e**(d*x)*int(x/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**
2 + e**(d*x)*b**2*x**4),x)*a**2*b + 4*e**(d*x)*int(x/(e**(d*x)*a**2 + 2*e*
*(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*x**2 + 2*e**(d*x)*int(x/(e
**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*b**3*x**4 + b*
x**2)/(8*e**(c + d*x)*a*b*(a**2 + 2*a*b*x**2 + b**2*x**4))
```

3.75 $\int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx$

Optimal result

Integrand size = 16, antiderivative size = 856

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3} dx = \text{Too large to display}$$

output
```
-1/16*cosh(d*x+c)/(-a)^(3/2)/b^(1/2)/((-a)^(1/2)-b^(1/2)*x)^2-3/16*cosh(d*
x+c)/a^2/b^(1/2)/((-a)^(1/2)-b^(1/2)*x)+1/16*cosh(d*x+c)/(-a)^(3/2)/b^(1/2
)/((-a)^(1/2)+b^(1/2)*x)^2+3/16*cosh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+b^(1/2
)*x)+3/16*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^
(5/2)/b^(1/2)+1/16*d^2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/
2)-d*x)/(-a)^(3/2)/b^(3/2)-3/16*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2
)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)-1/16*d^2*cosh(c-(-a)^(1/2)*d/b^(1/2))*
Chi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(3/2)-3/16*d*Chi((-a)^(1/2)*d/b
^(1/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/a^2/b-3/16*d*Chi((-a)^(1/2)*d/b^(
1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/a^2/b+1/16*d*sinh(d*x+c)/(-a)^(3/2)
/b/((-a)^(1/2)-b^(1/2)*x)+1/16*d*sinh(d*x+c)/(-a)^(3/2)/b/((-a)^(1/2)+b^(1
/2)*x)-3/16*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/
a^2/b+3/16*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a
)^(5/2)/b^(1/2)+1/16*d^2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^
(1/2)+d*x)/(-a)^(3/2)/b^(3/2)-3/16*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)
^(1/2)*d/b^(1/2)+d*x)/a^2/b-3/16*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/
2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)-1/16*d^2*sinh(c-(-a)^(1/2)*d/b^(1/2))
*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(3/2)/b^(3/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.89 (sec) , antiderivative size = 392, normalized size of antiderivative = 0.46

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \frac{-e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(3ib+3\sqrt{a}\sqrt{b}d-iad^2\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(-3ib+3\sqrt{a}\sqrt{b}d+iad^2\right)\text{E}\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

output
```
(-(E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((3*I)*b + 3*Sqrt[a]*Sqrt[b]*d - I*a*d^2
)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] +
x)] + ((-3*I)*b + 3*Sqrt[a]*Sqrt[b]*d + I*a*d^2)*ExpIntegralEi[d*((I*Sqrt[
a])/Sqrt[b] + x)])) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(((3*I)*b + 3*Sqrt[a]
*Sqrt[b]*d - I*a*d^2)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sq
rt[a]*d)/Sqrt[b] - d*x] + ((-3*I)*b + 3*Sqrt[a]*Sqrt[b]*d + I*a*d^2)*ExpIn
tegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) + (4*Sqrt[a]*Sqrt[b]*Cosh[d*x]*(b*x
*(5*a + 3*b*x^2)*Cosh[c] + a*d*(a + b*x^2)*Sinh[c]))/(a + b*x^2)^2 + (4*Sq
rt[a]*Sqrt[b]*(a*d*(a + b*x^2)*Cosh[c] + b*x*(5*a + 3*b*x^2)*Sinh[c])*Sinh
[d*x])/(a + b*x^2)^2)/(32*a^(5/2)*b^(3/2))
```

Rubi [A] (verified)

Time = 1.89 (sec) , antiderivative size = 856, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {5804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx$$

$$\Big\downarrow \ 5804$$

$$\int \left(-\frac{3b\cosh(c+dx)}{8a^2\left(-ab-b^2x^2\right)}-\frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}-bx\right)^2}-\frac{3b\cosh(c+dx)}{16a^2\left(\sqrt{-a}\sqrt{b}+bx\right)^2}-\frac{b^{3/2}\cosh(c+dx)}{8(-a)^{3/2}\left(\sqrt{-a}\sqrt{b}-bx\right)^3}-\frac{}{8(-}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{3/2}b^{3/2}}-\frac{3\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b}-\frac{3\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b}+\frac{\sinh(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sinh(c+dx)d}{16(-a)^{3/2}b\left(\sqrt{b}x+\sqrt{-a}\right)}+\frac{3\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^2b}-\frac{3\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^2b}-\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{3\cosh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)^2}+\frac{\cosh(c+dx)}{16(-a)^{3/2}\sqrt{b}\left(\sqrt{b}x+\sqrt{-a}\right)^2}+\frac{3\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{3\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{3\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{3\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$

input
```
Int[Cosh[c + d*x]/(a + b*x^2)^3,x]
```

output
```
-1/16*Cosh[c + d*x]/((-a)^(3/2)*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)^2) - (3*Cos
h[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) + Cosh[c + d*x]/(16*(-
a)^(3/2)*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)^2) + (3*Cosh[c + d*x])/(16*a^2*Sqr
t[b]*(Sqrt[-a] + Sqrt[b]*x)) + (3*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (d^2*Cosh[c + (
Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3
/2)*b^(3/2)) - (3*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt
[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(3/2)*b^(3/2)) - (
3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]
])/(16*a^2*b) - (3*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sq
rt[-a]*d)/Sqrt[b]])/(16*a^2*b) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[
-a] - Sqrt[b]*x)) + (d*Sinh[c + d*x])/(16*(-a)^(3/2)*b*(Sqrt[-a] + Sqrt[b]
*x)) + (3*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[
b] - d*x])/(16*a^2*b) - (3*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sq
rt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c + (Sqrt[-a
]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(3/2)*b^(
3/2)) - (3*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt
[b] + d*x])/(16*a^2*b) - (3*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(S
qrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (d^2*Sinh[c - (Sqr...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.85 (sec) , antiderivative size = 1064, normalized size of antiderivative = 1.24

method	result	size
risch	Expression too large to display	1064

input
```
int(cosh(d*x+c)/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*d^5*exp(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*e
xp(-d*x-c)/a^2*b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^3-1/16*d^5*exp(-d*x
-c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)+5/16*d^4*exp(-d*x-c)/a/(b^2*d^4*
x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/32*d^2/b/a/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2
)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)-1/32*d^2/b/a/(-a*b)^(1/2
)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)-3/3
2*d/b/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b
)/b)-3/32*d/b/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*
x+c)-c*b)/b)-3/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(
-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+3/32/a^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+
c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)+1/16*d^5*exp(d*x+c)/a/(b^2*
d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x^2+3/16*d^4*exp(d*x+c)/a^2*b/(b^2*d^4*x^4+
2*a*b*d^4*x^2+a^2*d^4)*x^3+1/16*d^5*exp(d*x+c)/b/(b^2*d^4*x^4+2*a*b*d^4*x^
2+a^2*d^4)+5/16*d^4*exp(d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*x+1/3
2*d^2/b/a/(-a*b)^(1/2)*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*
(d*x+c)+c*b)/b)-1/32*d^2/b/a/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)+3/32*d/b/a^2*exp((d*(-a*b)^(1/2)+c*b)
/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+3/32*d/b/a^2*exp((-d*(-a*b)^(1/
2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)-3/32/a^2/(-a*b)^(1/2)*e
xp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+3/32/...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(655) = 1310$.

Time = 0.14 (sec) , antiderivative size = 2116, normalized size of antiderivative = 2.47

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/32*(4*(3*a*b^2*d*x^3 + 5*a^2*b*d*x)*cosh(d*x + c) - ((3*(a*b^2*d^2*x^4 +
 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 -
3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2
*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2
)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2*a^2*b*d
^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2*x^2 + a
^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b +
2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 3*b
^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^2)*sqrt(-a*
d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - ((3*(a*b^2*d
^2*x^4 + 2*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2
*a^2*b*d^2*x^2 + a^3*d^2)*sinh(d*x + c)^2 + ((a^3*d^2 + (a*b^2*d^2 - 3*b^3
)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2
+ (a*b^2*d^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*
x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) - (3*(a*b^2*d^2*x^4 + 2
*a^2*b*d^2*x^2 + a^3*d^2)*cosh(d*x + c)^2 - 3*(a*b^2*d^2*x^4 + 2*a^2*b*d^2
*x^2 + a^3*d^2)*sinh(d*x + c)^2 - ((a^3*d^2 + (a*b^2*d^2 - 3*b^3)*x^4 - 3*
a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d
^2 - 3*b^3)*x^4 - 3*a^2*b + 2*(a^2*b*d^2 - 3*a*b^2)*x^2)*sinh(d*x + c)^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*x^2 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx = \int \frac{\cosh(c+d\,x)}{(b\,x^2+a)^3}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x^2)^3,x)
```

output
```
int(cosh(c + d*x)/(a + b*x^2)^3, x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^2)^3}\,dx$$
$$= \frac{e^{dx+2c}\left(\int \frac{e^{dx}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)a^3+e^{dx+2c}\left(\int \frac{e^{dx}}{b^3x^6+3a\,b^2x^4+3a^2b\,x^2+a^3}dx\right)a^2b\,x^2+e^{dx}\left(\int \frac{x^3}{e^{dx}a^2+2e^{dx}ab\,x^2+e}\right.}{}$$

input
```
int(cosh(d*x+c)/(b*x^2+a)^3,x)
```

output
```
(e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*
x**6),x)*a**3 + e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b*
*2*x**4 + b**3*x**6),x)*a**2*b*x**2 + e**(d*x)*int(x**3/(e**(d*x)*a**2 + 2
*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b*d + e**(d*x)*int(x**3/(e**
(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*b**2*d*x**2 - e*
*(d*x)*int(x**2/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**2 + 3*e**(d*x)*a*b**
2*x**4 + e**(d*x)*b**3*x**6),x)*a**2*b - e**(d*x)*int(x**2/(e**(d*x)*a**3
+ 3*e**(d*x)*a**2*b*x**2 + 3*e**(d*x)*a*b**2*x**4 + e**(d*x)*b**3*x**6),x)
*a*b**2*x**2 + e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e*
*(d*x)*b**2*x**4),x)*a*b + e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a
*b*x**2 + e**(d*x)*b**2*x**4),x)*b**2*x**2 + e**(d*x)*int(x/(e**(d*x)*a**2
 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a**2*d + e**(d*x)*int(x/(e
**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b*d*x**2 + x
)/(2*e**(c + d*x)*a**2*(a + b*x**2))
```

3.76 $\int \frac{\cosh(c+dx)}{x(a+bx^2)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 730

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx = \frac{\cosh(c+dx)}{4a(a+bx^2)^2} + \frac{\cosh(c+dx)}{2a^2(a+bx^2)} + \frac{\cosh(c)\text{Chi}(dx)}{a^3}$$
$$- \frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}$$
$$+ \frac{d^2\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}$$
$$- \frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3}$$
$$+ \frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$
$$+ \frac{5d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$- \frac{5d\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)} - \frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}$$
$$+ \frac{\sinh(c)\text{Shi}(dx)}{a^3} + \frac{5d\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$+ \frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}$$
$$- \frac{d^2\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}$$
$$+ \frac{5d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16(-a)^{5/2}\sqrt{b}}$$
$$- \frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{2a^3}$$
$$+ \frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}+dx\right)}{16a^2b}$$

output
```
1/4*cosh(d*x+c)/a/(b*x^2+a)^2+1/2*cosh(d*x+c)/a^2/(b*x^2+a)+cosh(c)*Chi(d*
x)/a^3-1/2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/a^3+
1/16*d^2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/a^2/b-
1/2*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/a^3+1/16*d^
2*cosh(c-(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/a^2/b+5/16*d*
Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(5/2)/b^(1
/2)-5/16*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/(-a)
^(5/2)/b^(1/2)+1/16*d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)-b^(1/2)*x)-1/16*
d*sinh(d*x+c)/a^2/b^(1/2)/((-a)^(1/2)+b^(1/2)*x)+sinh(c)*Shi(d*x)/a^3-5/16
*d*cosh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/
b^(1/2)-1/2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^
3+1/16*d^2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^2
/b+5/16*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^
(5/2)/b^(1/2)-1/2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*
x)/a^3+1/16*d^2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)
/a^2/b
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.44 (sec) , antiderivative size = 674, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx$$

$$= \frac{\frac{8a(3a+2bx^2)\cosh(c+dx)}{(a+bx^2)^2} + 32\cosh(c)\text{Chi}(dx) + \frac{4i\sqrt{a}de^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right.\right.\right.}{\sqrt{b}}}{}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```

output
```
((8*a*(3*a + 2*b*x^2)*Cosh[c + d*x])/(a + b*x^2)^2 + 32*Cosh[c]*CoshIntegr
al[d*x] + ((4*I)*Sqrt[a]*d*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a
]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralE
i[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[b] - 8*E^(c - (I*Sqrt[a]*d)/Sqrt[b])
*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] +
x)] + ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]) + (Sqrt[a]*d*E^(c - (I*S
qrt[a]*d)/Sqrt[b])*((I*Sqrt[b] + Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*
ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] + ((-I)*Sqrt[b] + Sqrt[a]*d)
*ExpIntegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/b + ((4*I)*Sqrt[a]*d*E^(-c -
 (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)
*Sqrt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]))/
Sqrt[b] - 8*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*
ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x] + ExpIntegralEi[(I*Sqrt[a]*d
)/Sqrt[b] - d*x]) + (Sqrt[a]*d*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((I*Sqrt[b]
+ Sqrt[a]*d)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/
Sqrt[b] - d*x] + ((-I)*Sqrt[b] + Sqrt[a]*d)*ExpIntegralEi[(I*Sqrt[a]*d)/Sq
rt[b] - d*x]))/b - (4*a*d*x*Sinh[c + d*x])/(a + b*x^2) + 32*Sinh[c]*SinhIn
tegral[d*x])/(32*a^3)
```

Rubi [A] (verified)

Time = 2.51 (sec) , antiderivative size = 730, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx$$

$$\Big\downarrow \text{ 5816}$$

$$\int \left(-\frac{bx\cosh(c+dx)}{a^3\left(a+bx^2\right)}+\frac{\cosh(c+dx)}{a^3x}-\frac{bx\cosh(c+dx)}{a^2\left(a+bx^2\right)^2}-\frac{bx\cosh(c+dx)}{a\left(a+bx^2\right)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}+\frac{\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{2a^3}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{2a^3}+\frac{\cosh(c)\text{Chi}(dx)}{a^3}+\frac{\sinh(c)\text{Shi}(dx)}{a^3}+\frac{d^2\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{d^2\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}-\frac{d^2\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16a^2b}+\frac{d^2\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16a^2b}+\frac{\cosh(c+dx)}{2a^2(a+bx^2)}+\frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{d\sinh(c+dx)}{16a^2\sqrt{b}\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{5d\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}-\frac{5d\sinh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{5d\cosh\left(\frac{\sqrt{-a}d}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{5d\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{5/2}\sqrt{b}}+\frac{\cosh(c+dx)}{4a\left(a+bx^2\right)^2}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x^2)^3),x]
```

output
```
Cosh[c + d*x]/(4*a*(a + b*x^2)^2) + Cosh[c + d*x]/(2*a^2*(a + b*x^2)) + (C
osh[c]*CoshIntegral[d*x])/a^3 - (Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegr
al[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[
b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) - (Cosh[c - (Sqrt
[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^3) + (d^2*
Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(
16*a^2*b) + (5*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-
a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) - (5*d*CoshIntegral[(Sqrt[-a]*d)/S
qrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(5/2)*Sqrt[b]) + (d
*Sinh[c + d*x])/(16*a^2*Sqrt[b]*(Sqrt[-a] - Sqrt[b]*x)) - (d*Sinh[c + d*x]
)/(16*a^2*Sqrt[b]*(Sqrt[-a] + Sqrt[b]*x)) + (Sinh[c]*SinhIntegral[d*x])/a^
3 + (5*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b]
- d*x])/(16*(-a)^(5/2)*Sqrt[b]) + (Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInte
gral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^3) - (d^2*Sinh[c + (Sqrt[-a]*d)/Sqr
t[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^2*b) + (5*d*Cosh[c -
 (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^
(5/2)*Sqrt[b]) - (Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)
/Sqrt[b] + d*x])/(2*a^3) + (d^2*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^2*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 1090, normalized size of antiderivative = 1.49

method	result	size
risch	Expression too large to display	1090

input
```
int(cosh(d*x+c)/x/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*exp(-d*x-c)*d^2*(b*(d*x+c)^3-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x+c)*
b*c^2-d^2*c*a-b*c^3+4*(d*x+c)^2*b-8*b*(d*x+c)*c+6*a*d^2+4*b*c^2)/a^2/((d*x
+c)^4*b^2-4*(d*x+c)^3*c*b^2+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*a*b*
(d*x+c)*c*d^2-4*b^2*(d*x+c)*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/2/a^3*exp
(-c)*Ei(1,d*x)-1/32/b/a^2*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/
2)-b*(d*x+c)+c*b)/b)*d^2-1/32/b/a^2*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*
(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*d^2-5/32/a^2/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1
/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*d+5/32/a^2/(-a*b)^(1/2
)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*d+1
/4/a^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b
)+1/4/a^3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b
)/b)-1/16*exp(d*x+c)*d^2*(b*(d*x+c)^3-3*(d*x+c)^2*b*c+(d*x+c)*a*d^2+3*(d*x
+c)*b*c^2-d^2*c*a-b*c^3-4*(d*x+c)^2*b+8*b*(d*x+c)*c-6*a*d^2-4*b*c^2)/a^2/(
(d*x+c)^4*b^2-4*(d*x+c)^3*c*b^2+2*(d*x+c)^2*a*b*d^2+6*(d*x+c)^2*c^2*b^2-4*
a*b*(d*x+c)*c*d^2-4*b^2*(d*x+c)*c^3+a^2*d^4+2*a*b*c^2*d^2+b^2*c^4)-1/2/a^3
*exp(c)*Ei(1,-d*x)-1/32/b/a^2*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(
1/2)-b*(d*x+c)+c*b)/b)*d^2-1/32/b/a^2*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(
d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*d^2+5/32/a^2/(-a*b)^(1/2)*exp((d*(-a*b)^(
1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*d-5/32/a^2/(-a*b)^(1/2
)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2076 vs. $2(575) = 1150$.

Time = 0.15 (sec) , antiderivative size = 2076, normalized size of antiderivative = 2.84

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
1/32*(8*(2*a*b^2*x^2 + 3*a^2*b)*cosh(d*x + c) + (((a^3*d^2 + (a*b^2*d^2 -
8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3
*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*si
nh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*
x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(
-a*d^2/b)) + ((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2
- 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a
^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((b^3*x^4 + 2*a*b^
2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x
+ c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)
) + 16*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*Ei(d*x) + (b^3*x^4 + 2*a*b^2*x^2 +
 a^2*b)*Ei(-d*x))*cosh(c) + (((a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b
 + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 -
8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*sinh(d*x + c)^2 - 5*((
b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 +
a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + ((a^3*d
^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2 - 8*a*b^2)*x^2)*cosh
(d*x + c)^2 - (a^3*d^2 + (a*b^2*d^2 - 8*b^3)*x^4 - 8*a^2*b + 2*(a^2*b*d^2
- 8*a*b^2)*x^2)*sinh(d*x + c)^2 + 5*((b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*cosh(
d*x + c)^2 - (b^3*x^4 + 2*a*b^2*x^2 + a^2*b)*sinh(d*x + c)^2)*sqrt(-a*d...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^3*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx^2)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x\,(b\,x^2+a)^3}\,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x^2)^3),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^2\right)^3}\,dx=\int \frac{\cosh\left(dx+c\right)}{b^3x^7+3a\,b^2x^5+3a^2b\,x^3+a^3x}dx$$

input
```
int(cosh(d*x+c)/x/(b*x^2+a)^3,x)
```

output
```
int(cosh(c + d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)
```

3.77 $\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 874

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx = \text{Too large to display}$$

output
```
-cosh(d*x+c)/a^3/x-1/16*b^(1/2)*cosh(d*x+c)/(-a)^(5/2)/((-a)^(1/2)-b^(1/2)
*x)^2+7/16*b^(1/2)*cosh(d*x+c)/a^3/((-a)^(1/2)-b^(1/2)*x)+1/16*b^(1/2)*cos
h(d*x+c)/(-a)^(5/2)/((-a)^(1/2)+b^(1/2)*x)^2-7/16*b^(1/2)*cosh(d*x+c)/a^3/
((-a)^(1/2)+b^(1/2)*x)+15/16*b^(1/2)*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)
^(1/2)*d/b^(1/2)-d*x)/(-a)^(7/2)+1/16*d^2*cosh(c+(-a)^(1/2)*d/b^(1/2))*Chi
((-a)^(1/2)*d/b^(1/2)-d*x)/(-a)^(5/2)/b^(1/2)-15/16*b^(1/2)*cosh(c-(-a)^(1
/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(7/2)-1/16*d^2*cosh(c-(-
a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)+d*Chi
(d*x)*sinh(c)/a^3+7/16*d*Chi((-a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(-a)^(1/2)*d
/b^(1/2))/a^3+7/16*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(
1/2))/a^3+1/16*d*sinh(d*x+c)/(-a)^(5/2)/((-a)^(1/2)-b^(1/2)*x)+1/16*d*sinh
(d*x+c)/(-a)^(5/2)/((-a)^(1/2)+b^(1/2)*x)+d*cosh(c)*Shi(d*x)/a^3+7/16*d*co
sh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^3+15/16*b^(1/2
)*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(7/2)+1
/16*d^2*sinh(c+(-a)^(1/2)*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(
5/2)/b^(1/2)+7/16*d*cosh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+
d*x)/a^3-15/16*b^(1/2)*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/
2)+d*x)/(-a)^(7/2)-1/16*d^2*sinh(c-(-a)^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/
b^(1/2)+d*x)/(-a)^(5/2)/b^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.88 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.70

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx$$

$$= \frac{8i\sqrt{b}e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)-\text{ExpIntegralEi}\left(d\left(\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)\right)+\frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(7ib+7\sqrt{a}\right.\right.}{}}{}$$

input
```
Integrate[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

output
```
((8*I)*Sqrt[b]*E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b]
)*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)] - ExpIntegralEi[d*((I*Sqrt
[a])/Sqrt[b] + x)]) + (E^(c - (I*Sqrt[a]*d)/Sqrt[b])*(((7*I)*b + 7*Sqrt[a]
*Sqrt[b]*d - I*a*d^2)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)
*Sqrt[a])/Sqrt[b] + x)] + ((-7*I)*b + 7*Sqrt[a]*Sqrt[b]*d + I*a*d^2)*ExpIn
tegralEi[d*((I*Sqrt[a])/Sqrt[b] + x)]))/Sqrt[b] - (8*I)*Sqrt[b]*E^(-c - (I
*Sqrt[a]*d)/Sqrt[b])*(E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sq
rt[a]*d)/Sqrt[b] - d*x] - ExpIntegralEi[(I*Sqrt[a]*d)/Sqrt[b] - d*x]) - (I
*E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((7*b - (7*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*E
^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d)/Sqrt[b] - d*x]
 + (-7*b - (7*I)*Sqrt[a]*Sqrt[b]*d + a*d^2)*ExpIntegralEi[(I*Sqrt[a]*d)/Sq
rt[b] - d*x]))/Sqrt[b] - (4*Sqrt[a]*Cosh[d*x]*((8*a^2 + 25*a*b*x^2 + 15*b^
2*x^4)*Cosh[c] + a*d*x*(a + b*x^2)*Sinh[c]))/(x*(a + b*x^2)^2) - (4*Sqrt[a
]*(a*d*x*(a + b*x^2)*Cosh[c] + (8*a^2 + 25*a*b*x^2 + 15*b^2*x^4)*Sinh[c])*
Sinh[d*x])/(x*(a + b*x^2)^2) + 32*Sqrt[a]*d*(CoshIntegral[d*x]*Sinh[c] + C
osh[c]*SinhIntegral[d*x]))/(32*a^(7/2))
```

Rubi [A] (verified)

Time = 3.44 (sec) , antiderivative size = 874, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^2 (a+bx^2)^3} \, dx$$

$$\Big\downarrow \text{ 5816}$$

$$\int \left(-\frac{b\cosh(c+dx)}{a^3 (a+bx^2)}+\frac{\cosh(c+dx)}{a^3x^2}-\frac{b\cosh(c+dx)}{a^2 (a+bx^2)^2}-\frac{b\cosh(c+dx)}{a (a+bx^2)^3}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d^2}{16(-a)^{5/2}\sqrt{b}}-\frac{\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d^2}{16(-a)^{5/2}\sqrt{b}}+\frac{\text{Chi}(dx)\sinh(c)d}{a^3}+\frac{7\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3}+\frac{7\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3}+\frac{\sinh(c+dx)d}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sinh(c+dx)d}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)}+\frac{\cosh(c)\text{Shi}(dx)d}{a^3}-\frac{7\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)d}{16a^3}+\frac{7\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)d}{16a^3}-\frac{\cosh(c+dx)}{a^3x}+\frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}-\frac{7\sqrt{b}\cosh(c+dx)}{16a^3\left(\sqrt{b}x+\sqrt{-a}\right)}-\frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{-a}-\sqrt{b}x\right)^2}+\frac{\sqrt{b}\cosh(c+dx)}{16(-a)^{5/2}\left(\sqrt{b}x+\sqrt{-a}\right)^2}+\frac{15\sqrt{b}\cosh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}-\frac{15\sqrt{b}\cosh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}-\frac{15\sqrt{b}\sinh\left(c+\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(\frac{\sqrt{-a}d}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}-\frac{15\sqrt{b}\sinh\left(c-\frac{\sqrt{-a}d}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-a}d}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

input
```
Int[Cosh[c + d*x]/(x^2*(a + b*x^2)^3),x]
```

output
```
-(Cosh[c + d*x]/(a^3*x)) - (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a
] - Sqrt[b]*x)^2) + (7*Sqrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] - Sqrt[b]*
x)) + (Sqrt[b]*Cosh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)^2) - (
7*Sqrt[b]*Cosh[c + d*x])/(16*a^3*(Sqrt[-a] + Sqrt[b]*x)) + (15*Sqrt[b]*Cos
h[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*
(-a)^(7/2)) + (d^2*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d
)/Sqrt[b] - d*x])/(16*(-a)^(5/2)*Sqrt[b]) - (15*Sqrt[b]*Cosh[c - (Sqrt[-a]
*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) - (
d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x
])/(16*(-a)^(5/2)*Sqrt[b]) + (d*CoshIntegral[d*x]*Sinh[c])/a^3 + (7*d*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] + d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*a
^3) + (7*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/
Sqrt[b]])/(16*a^3) + (d*Sinh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] - Sqrt[b]*
x)) + (d*Sinh[c + d*x])/(16*(-a)^(5/2)*(Sqrt[-a] + Sqrt[b]*x)) + (d*Cosh[c
]*SinhIntegral[d*x])/a^3 - (7*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegra
l[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c + (Sqrt[-a]*d
)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (d^
2*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])
/(16*(-a)^(5/2)*Sqrt[b]) + (7*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegra
l[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(16*a^3) - (15*Sqrt[b]*Sinh[c - (Sqrt[-a...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 1.11 (sec) , antiderivative size = 1178, normalized size of antiderivative = 1.35

method	result	size
risch	Expression too large to display	1178

input

```
int(cosh(d*x+c)/x^2/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/16*d^5*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x^2-15/16*e
xp(-d*x-c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+1/16*d^5*ex
p(-d*x-c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(-d*x-c)*d^4/a^2/
(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x-1/2*exp(-d*x-c)/a/x*d^4/(b^2*d^4*x
^4+2*a*b*d^4*x^2+a^2*d^4)+1/2*d/a^3*exp(-c)*Ei(1,d*x)-1/32/a^2*d^2/(-a*b)^
(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)
+1/32/a^2*d^2/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1
/2)+b*(d*x+c)-c*b)/b)+7/32*d/a^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a
*b)^(1/2)-b*(d*x+c)+c*b)/b)+7/32*d/a^3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,
(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)+15/32/a^3/(-a*b)^(1/2)*exp(-(d*(-a*b)^(1
/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b-15/32/a^3/(-a*b)^(1/
2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b-
1/16*d^5*exp(d*x+c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x^2-15/16*ex
p(d*x+c)/a^3*x^3*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2-1/16*d^5*exp(
d*x+c)/a/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-25/16*exp(d*x+c)*d^4/a^2/(b^2
*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x-1/2*exp(d*x+c)/a/x*d^4/(b^2*d^4*x^4+2*
a*b*d^4*x^2+a^2*d^4)-1/2*d/a^3*exp(c)*Ei(1,-d*x)-1/32/a^2*d^2/(-a*b)^(1/2)
*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)+1/32/a
^2*d^2/(-a*b)^(1/2)*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)+b*(
d*x+c)-c*b)/b)-7/32*d/a^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2346 vs. $2(673) = 1346$.

Time = 0.17 (sec) , antiderivative size = 2346, normalized size of antiderivative = 2.68

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(4*(15*a*b^2*d*x^4 + 25*a^2*b*d*x^2 + 8*a^3*d)*cosh(d*x + c) - ((7*(
a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(a*b^2*d^
2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 - (((a*b^2*d^2 - 15*b
^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*cosh(d*x
+ c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d
^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)
) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*x + c)^2 - 7*(
a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2 + (((a*b^2*d^
2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*c
osh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3
+ (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*Ei(-d*x + sqrt(
-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 16*((a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^
3 + a^3*d^2*x)*Ei(d*x) - (a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*Ei(
-d*x))*cosh(c) - ((7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*cosh(d*
x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh(d*x + c)^2
 + (((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 - 15*a*b^2)*x^3 + (a^3*d^2 -
15*a^2*b)*x)*cosh(d*x + c)^2 - ((a*b^2*d^2 - 15*b^3)*x^5 + 2*(a^2*b*d^2 -
15*a*b^2)*x^3 + (a^3*d^2 - 15*a^2*b)*x)*sinh(d*x + c)^2)*sqrt(-a*d^2/b))*E
i(d*x + sqrt(-a*d^2/b)) - (7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)
*cosh(d*x + c)^2 - 7*(a*b^2*d^2*x^5 + 2*a^2*b*d^2*x^3 + a^3*d^2*x)*sinh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x**2/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\cosh(dx+c)}{(bx^2+a)^3x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^2)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,(b\,x^2+a)^3}\,dx$$

input
```
int(cosh(c + d*x)/(x^2*(a + b*x^2)^3),x)
```

output
```
int(cosh(c + d*x)/(x^2*(a + b*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^2 (a+bx^2)^3} \, dx = \text{too large to display}$$

input
```
int(cosh(d*x+c)/x^2/(b*x^2+a)^3,x)
```

output
```
( - e**(2*c + 2*d*x)*a*d**2*x - 4*e**(2*c + 2*d*x)*a*d - 20*e**(2*c + d*x)
*int(e**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**3*b
*d*x - 40*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**
4 + b**3*x**6),x)*a**2*b**2*d*x**3 - 20*e**(2*c + d*x)*int(e**(d*x)/(a**3
+ 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a*b**3*d*x**5 + e**(2*c +
d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**3*d**3*x + 2*e**(2
*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a**2*b*d**3*x**3
 + e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**2 + b**2*x**4),x)*a*b**2*d
**3*x**5 + 4*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**
2*x**5 + b**3*x**7),x)*a**4*d**2*x + 8*e**(2*c + d*x)*int(e**(d*x)/(a**3*x
 + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**3*b*d**2*x**3 + 4*e**(
2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**
7),x)*a**2*b**2*d**2*x**5 + e**(d*x)*int(x**3/(e**(d*x)*a**2 + 2*e**(d*x)*
a*b*x**2 + e**(d*x)*b**2*x**4),x)*a**2*b*d**4*x + 2*e**(d*x)*int(x**3/(e**
(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*x**4),x)*a*b**2*d**4*x**3
 + e**(d*x)*int(x**3/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**2 + e**(d*x)*b**2*
x**4),x)*b**3*d**4*x**5 + 20*e**(d*x)*int(x**2/(e**(d*x)*a**3 + 3*e**(d*x)
*a**2*b*x**2 + 3*e**(d*x)*a*b**2*x**4 + e**(d*x)*b**3*x**6),x)*a**2*b**2*d
*x + 40*e**(d*x)*int(x**2/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**2 + 3*e**(
d*x)*a*b**2*x**4 + e**(d*x)*b**3*x**6),x)*a*b**3*d*x**3 + 20*e**(d*x)*i...
```

3.78 $\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 791

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^2)^3}\,dx = \text{Too large to display}$$

output
```
-1/2*cosh(d*x+c)/a^3/x^2-1/4*b*cosh(d*x+c)/a^2/(b*x^2+a)^2-b*cosh(d*x+c)/a
^3/(b*x^2+a)-3*b*cosh(c)*Chi(d*x)/a^4+1/2*d^2*cosh(c)*Chi(d*x)/a^3+3/2*b*c
osh(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/a^4-1/16*d^2*cos
h(c+(-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)-d*x)/a^3+3/2*b*cosh(c-(
-a)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/a^4-1/16*d^2*cosh(c-(-a
)^(1/2)*d/b^(1/2))*Chi((-a)^(1/2)*d/b^(1/2)+d*x)/a^3+9/16*b^(1/2)*d*Chi((-
a)^(1/2)*d/b^(1/2)+d*x)*sinh(c-(-a)^(1/2)*d/b^(1/2))/(-a)^(7/2)-9/16*b^(1/
2)*d*Chi((-a)^(1/2)*d/b^(1/2)-d*x)*sinh(c+(-a)^(1/2)*d/b^(1/2))/(-a)^(7/2)
-1/2*d*sinh(d*x+c)/a^3/x-1/16*b^(1/2)*d*sinh(d*x+c)/a^3/((-a)^(1/2)-b^(1/2
)*x)+1/16*b^(1/2)*d*sinh(d*x+c)/a^3/((-a)^(1/2)+b^(1/2)*x)-3*b*sinh(c)*Shi
(d*x)/a^4+1/2*d^2*sinh(c)*Shi(d*x)/a^3-9/16*b^(1/2)*d*cosh(c+(-a)^(1/2)*d/
b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(7/2)+3/2*b*sinh(c+(-a)^(1/2)
*d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^4-1/16*d^2*sinh(c+(-a)^(1/2)*
d/b^(1/2))*Shi(-(-a)^(1/2)*d/b^(1/2)+d*x)/a^3+9/16*b^(1/2)*d*cosh(c-(-a)^(
1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/(-a)^(7/2)+3/2*b*sinh(c-(-a)
^(1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/a^4-1/16*d^2*sinh(c-(-a)^(
1/2)*d/b^(1/2))*Shi((-a)^(1/2)*d/b^(1/2)+d*x)/a^3
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.66 (sec) , antiderivative size = 438, normalized size of antiderivative = 0.55

$$\int \frac{\cosh(c+dx)}{x^3 (a+bx^2)^3} dx$$
$$= \frac{e^{c-\frac{i\sqrt{a}d}{\sqrt{b}}}\left(\left(24b-9i\sqrt{a}\sqrt{b}d-ad^2\right)e^{\frac{2i\sqrt{a}d}{\sqrt{b}}}\text{ExpIntegralEi}\left(d\left(-\frac{i\sqrt{a}}{\sqrt{b}}+x\right)\right)+\left(24b+9i\sqrt{a}\sqrt{b}d-ad^2\right)\text{Exp}\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

output
```
(E^(c - (I*Sqrt[a]*d)/Sqrt[b])*((24*b - (9*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*E
^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[d*(((-I)*Sqrt[a])/Sqrt[b] + x)]
 + (24*b + (9*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*ExpIntegralEi[d*((I*Sqrt[a])/S
qrt[b] + x)]) + E^(-c - (I*Sqrt[a]*d)/Sqrt[b])*((24*b - (9*I)*Sqrt[a]*Sqrt
[b]*d - a*d^2)*E^(((2*I)*Sqrt[a]*d)/Sqrt[b])*ExpIntegralEi[((-I)*Sqrt[a]*d
)/Sqrt[b] - d*x] + (24*b + (9*I)*Sqrt[a]*Sqrt[b]*d - a*d^2)*ExpIntegralEi[
(I*Sqrt[a]*d)/Sqrt[b] - d*x]) - (4*a*Cosh[d*x]*(2*(2*a^2 + 9*a*b*x^2 + 6*b
^2*x^4)*Cosh[c] + d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Sinh[c]))/(x^2*(a +
b*x^2)^2) - (4*a*(d*x*(4*a^2 + 7*a*b*x^2 + 3*b^2*x^4)*Cosh[c] + 2*(2*a^2 +
 9*a*b*x^2 + 6*b^2*x^4)*Sinh[c])*Sinh[d*x])/(x^2*(a + b*x^2)^2) + 16*(-6*b
 + a*d^2)*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/(32*a^4
)
```

Rubi [A] (verified)

Time = 2.45 (sec) , antiderivative size = 791, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^3 (a+bx^2)^3} dx$$

$$\Big\downarrow \text{5816}$$

$$\int \left(\frac{3b^2x\cosh(c+dx)}{a^4(a+bx^2)}-\frac{3b\cosh(c+dx)}{a^4x}+\frac{2b^2x\cosh(c+dx)}{a^3(a+bx^2)^2}+\frac{\cosh(c+dx)}{a^3x^3}+\frac{b^2x\cosh(c+dx)}{a^2(a+bx^2)^3}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3b\cosh(c)\text{Chi}(dx)}{a^4}+\frac{3b\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2a^4}+\frac{3b\cosh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^4}-\frac{3b\sinh(c)\text{Shi}(dx)}{a^4}-\frac{3b\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{2a^4}+\frac{3b\sinh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{2a^4}-\frac{d^2\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{16a^3}-\frac{d^2\cosh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16a^3}+\frac{d^2\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{16a^3}-\frac{d^2\sinh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16a^3}-\frac{b\cosh(c+dx)}{a^3(a+bx^2)}-\frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}-\sqrt{b}x\right)}+\frac{\sqrt{b}d\sinh(c+dx)}{16a^3\left(\sqrt{-a}+\sqrt{b}x\right)}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a^3}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a^3}-\frac{\cosh(c+dx)}{2a^3x^2}-\frac{d\sinh(c+dx)}{2a^3x}-\frac{b\cosh(c+dx)}{4a^2(a+bx^2)^2}+\frac{9\sqrt{b}d\sinh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Chi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{7/2}}-\frac{9\sqrt{b}d\sinh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Chi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}+\frac{9\sqrt{b}d\cosh\left(\frac{\sqrt{-ad}}{\sqrt{b}}+c\right)\text{Shi}\left(\frac{\sqrt{-ad}}{\sqrt{b}}-dx\right)}{16(-a)^{7/2}}+\frac{9\sqrt{b}d\cosh\left(c-\frac{\sqrt{-ad}}{\sqrt{b}}\right)\text{Shi}\left(xd+\frac{\sqrt{-ad}}{\sqrt{b}}\right)}{16(-a)^{7/2}}$$

input
```
Int[Cosh[c + d*x]/(x^3*(a + b*x^2)^3),x]
```

output
```
-1/2*Cosh[c + d*x]/(a^3*x^2) - (b*Cosh[c + d*x])/(4*a^2*(a + b*x^2)^2) - (
b*Cosh[c + d*x])/(a^3*(a + b*x^2)) - (3*b*Cosh[c]*CoshIntegral[d*x])/a^4 +
 (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a^3) + (3*b*Cosh[c + (Sqrt[-a]*d)/Sqrt
[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) - (d^2*Cosh[c + (Sq
rt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a^3) + (3
*b*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x]
)/(2*a^4) - (d^2*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*CoshIntegral[(Sqrt[-a]*d)/
Sqrt[b] + d*x])/(16*a^3) + (9*Sqrt[b]*d*CoshIntegral[(Sqrt[-a]*d)/Sqrt[b]
+ d*x]*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]])/(16*(-a)^(7/2)) - (9*Sqrt[b]*d*Cosh
Integral[(Sqrt[-a]*d)/Sqrt[b] - d*x]*Sinh[c + (Sqrt[-a]*d)/Sqrt[b]])/(16*(
-a)^(7/2)) - (d*Sinh[c + d*x])/(2*a^3*x) - (Sqrt[b]*d*Sinh[c + d*x])/(16*a
^3*(Sqrt[-a] - Sqrt[b]*x)) + (Sqrt[b]*d*Sinh[c + d*x])/(16*a^3*(Sqrt[-a] +
 Sqrt[b]*x)) - (3*b*Sinh[c]*SinhIntegral[d*x])/a^4 + (d^2*Sinh[c]*SinhInte
gral[d*x])/(2*a^3) + (9*Sqrt[b]*d*Cosh[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhInteg
ral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*(-a)^(7/2)) - (3*b*Sinh[c + (Sqrt[-a]
*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(2*a^4) + (d^2*Sinh
[c + (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d)/Sqrt[b] - d*x])/(16*a
^3) + (9*Sqrt[b]*d*Cosh[c - (Sqrt[-a]*d)/Sqrt[b]]*SinhIntegral[(Sqrt[-a]*d
)/Sqrt[b] + d*x])/(16*(-a)^(7/2)) + (3*b*Sinh[c - (Sqrt[-a]*d)/Sqrt[b]]*Si
nhIntegral[(Sqrt[-a]*d)/Sqrt[b] + d*x])/(2*a^4) - (d^2*Sinh[c - (Sqrt[-...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1293 vs. 2(629) = 1258.

Time = 1.30 (sec) , antiderivative size = 1294, normalized size of antiderivative = 1.64

method	result	size
risch	Expression too large to display	1294

input
```
int(cosh(d*x+c)/x^3/(b*x^2+a)^3,x,method=_RETURNVERBOSE)
```

output
```
7/16*d^5*exp(-d*x-c)/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b*x-3/4*exp(-
d*x-c)/a^3*x^2*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b^2+9/32*d/a^3/(-a*
b)^(1/2)*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)
/b)*b-9/32*d/a^3/(-a*b)^(1/2)*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*b)
^(1/2)+b*(d*x+c)-c*b)/b)*b-1/4*d^2/a^3*exp(-c)*Ei(1,d*x)+3/2/a^4*exp(-c)*E
i(1,d*x)*b+1/32*d^2/a^3*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(-a*b)^(1/2)
-b*(d*x+c)+c*b)/b)+1/32*d^2/a^3*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*
b)^(1/2)+b*(d*x+c)-c*b)/b)-3/4/a^4*exp(-(d*(-a*b)^(1/2)+c*b)/b)*Ei(1,-(d*(
-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b-3/4/a^4*exp(-(-d*(-a*b)^(1/2)+c*b)/b)*Ei(1
,(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b-1/4*d^2/a^3*exp(c)*Ei(1,-d*x)+3/2/a^4
*exp(c)*Ei(1,-d*x)*b+1/32*d^2/a^3*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(1,(d*(-a*
b)^(1/2)-b*(d*x+c)+c*b)/b)+1/32*d^2/a^3*exp((-d*(-a*b)^(1/2)+c*b)/b)*Ei(1,
-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)-3/4/a^4*exp((d*(-a*b)^(1/2)+c*b)/b)*Ei(
1,(d*(-a*b)^(1/2)-b*(d*x+c)+c*b)/b)*b-3/4/a^4*exp((-d*(-a*b)^(1/2)+c*b)/b)
*Ei(1,-(d*(-a*b)^(1/2)+b*(d*x+c)-c*b)/b)*b-1/4*exp(-d*x-c)/a/x^2*d^4/(b^2*
d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)-7/16*d^5*exp(d*x+c)/a^2/(b^2*d^4*x^4+2*a*b*
d^4*x^2+a^2*d^4)*b*x-3/4*exp(d*x+c)/a^3*x^2*d^4/(b^2*d^4*x^4+2*a*b*d^4*x^2
+a^2*d^4)*b^2-3/16*d^5*exp(d*x+c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d
^4)*b^2+3/16*d^5*exp(-d*x-c)/a^3*x^3/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b
^2-9/8*exp(-d*x-c)*d^4/a^2/(b^2*d^4*x^4+2*a*b*d^4*x^2+a^2*d^4)*b-9/32*d...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2363 vs. $2(630) = 1260$.

Time $= 0.15$ (sec) , antiderivative size $= 2363$, normalized size of antiderivative $= 2.99$

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(8*(6*a*b^2*x^4 + 9*a^2*b*x^2 + 2*a^3)*cosh(d*x + c) + ((((a*b^2*d^2
 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*
cosh(d*x + c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4
 + (a^3*d^2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 + 9*((b^3*x^6 + 2*a*b^2*x^4 +
 a^2*b*x^2)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x
 + c)^2)*sqrt(-a*d^2/b))*Ei(d*x - sqrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)
*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x +
 c)^2 - ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^
2 - 24*a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2
)*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*s
qrt(-a*d^2/b))*Ei(-d*x + sqrt(-a*d^2/b)))*cosh(c + sqrt(-a*d^2/b)) - 8*(((
a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (a^3*d^2 - 6*a^2*b)
*x^2)*Ei(d*x) + ((a*b^2*d^2 - 6*b^3)*x^6 + 2*(a^2*b*d^2 - 6*a*b^2)*x^4 + (
a^3*d^2 - 6*a^2*b)*x^2)*Ei(-d*x))*cosh(c) + ((((a*b^2*d^2 - 24*b^3)*x^6 +
2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 -
 ((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*
a^2*b)*x^2)*sinh(d*x + c)^2 - 9*((b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*cosh(
d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^4 + a^2*b*x^2)*sinh(d*x + c)^2)*sqrt(-a*
d^2/b))*Ei(d*x + sqrt(-a*d^2/b)) + (((a*b^2*d^2 - 24*b^3)*x^6 + 2*(a^2*b*d
^2 - 24*a*b^2)*x^4 + (a^3*d^2 - 24*a^2*b)*x^2)*cosh(d*x + c)^2 - ((a*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x**3/(b*x**2+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^3x^3}\,dx$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx^2+a\right)^3x^3}\,dx$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^2+a)^3,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^2 + a)^3*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\left(b\,x^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)/(x^3*(a + b*x^2)^3),x)
```

output
```
int(cosh(c + d*x)/(x^3*(a + b*x^2)^3), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^3\left(a+bx^2\right)^3}\,dx = \text{too large to display}$$

input
```
int(cosh(d*x+c)/x^3/(b*x^2+a)^3,x)
```

output
```
( - e**(2*c + 2*d*x)*a*d*x - e**(2*c + 2*d*x)*a - 4*e**(2*c + d*x)*int(e**
(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**3*b*d*x**2
- 8*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b*
*3*x**6),x)*a**2*b**2*d*x**4 - 4*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**
2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a*b**3*d*x**6 + e**(2*c + d*x)*in
t(e**(d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**4*d*
*2*x**2 + 2*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2
*x**5 + b**3*x**7),x)*a**3*b*d**2*x**4 - 6*e**(2*c + d*x)*int(e**(d*x)/(a*
*3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**3*b*x**2 + e**(2*c
 + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),
x)*a**2*b**2*d**2*x**6 - 12*e**(2*c + d*x)*int(e**(d*x)/(a**3*x + 3*a**2*b
*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a**2*b**2*x**4 - 6*e**(2*c + d*x)*in
t(e**(d*x)/(a**3*x + 3*a**2*b*x**3 + 3*a*b**2*x**5 + b**3*x**7),x)*a*b**3*
x**6 + e**(2*c + d*x)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x*
*4 + b**3*x**6),x)*a**3*b*d**2*x**2 + 2*e**(2*c + d*x)*int((e**(d*x)*x)/(a
**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b**3*x**6),x)*a**2*b**2*d**2*x**4 +
e**(2*c + d*x)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x**2 + 3*a*b**2*x**4 + b*
*3*x**6),x)*a*b**3*d**2*x**6 + e**(d*x)*int(x**2/(e**(d*x)*a**3 + 3*e**(d*
x)*a**2*b*x**2 + 3*e**(d*x)*a*b**2*x**4 + e**(d*x)*b**3*x**6),x)*a**2*b**2
*d*x**2 + 2*e**(d*x)*int(x**2/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**2 +...
```

3.79 $\int x^3(a+bx^3)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 154

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = -\frac{6a\cosh(c+dx)}{d^4}-\frac{720bx\cosh(c+dx)}{d^6} -\frac{3ax^2\cosh(c+dx)}{d^2} -\frac{120bx^3\cosh(c+dx)}{d^4}-\frac{6bx^5\cosh(c+dx)}{d^2} +\frac{720b\sinh(c+dx)}{d^7}+\frac{6ax\sinh(c+dx)}{d^3} +\frac{360bx^2\sinh(c+dx)}{d^5}+\frac{ax^3\sinh(c+dx)}{d} +\frac{30bx^4\sinh(c+dx)}{d^3}+\frac{bx^6\sinh(c+dx)}{d}$$

output

```
-6*a*cosh(d*x+c)/d^4-720*b*x*cosh(d*x+c)/d^6-3*a*x^2*cosh(d*x+c)/d^2-120*b
*x^3*cosh(d*x+c)/d^4-6*b*x^5*cosh(d*x+c)/d^2+720*b*sinh(d*x+c)/d^7+6*a*x*s
inh(d*x+c)/d^3+360*b*x^2*sinh(d*x+c)/d^5+a*x^3*sinh(d*x+c)/d+30*b*x^4*sinh
(d*x+c)/d^3+b*x^6*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.65

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx$$
$$= \frac{-3d(ad^2(2+d^2x^2)+2bx(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(ad^4x(6+d^2x^2)+b(720+360d^2x^2+3}{d^7}$$

input
```
Integrate[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
(-3*d*(a*d^2*(2 + d^2*x^2) + 2*b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c +
d*x] + (a*d^4*x*(6 + d^2*x^2) + b*(720 + 360*d^2*x^2 + 30*d^4*x^4 + d^6*x^
6))*Sinh[c + d*x])/d^7
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{5810}$$
$$\int (ax^3\cosh(c+dx)+bx^6\cosh(c+dx))\,dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{6a\cosh(c+dx)}{d^4}+\frac{6ax\sinh(c+dx)}{d^3}-\frac{3ax^2\cosh(c+dx)}{d^2}+\frac{ax^3\sinh(c+dx)}{d}+\frac{720b\sinh(c+dx)}{d^7}-\frac{720bx\cosh(c+dx)}{d^6}+\frac{360bx^2\sinh(c+dx)}{d^5}-\frac{120bx^3\cosh(c+dx)}{d^4}+\frac{30bx^4\sinh(c+dx)}{d^3}-\frac{6bx^5\cosh(c+dx)}{d^2}+\frac{bx^6\sinh(c+dx)}{d}$$

input
```
Int[x^3*(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
(-6*a*Cosh[c + d*x])/d^4 - (720*b*x*Cosh[c + d*x])/d^6 - (3*a*x^2*Cosh[c +
 d*x])/d^2 - (120*b*x^3*Cosh[c + d*x])/d^4 - (6*b*x^5*Cosh[c + d*x])/d^2 +
 (720*b*Sinh[c + d*x])/d^7 + (6*a*x*Sinh[c + d*x])/d^3 + (360*b*x^2*Sinh[c
 + d*x])/d^5 + (a*x^3*Sinh[c + d*x])/d + (30*b*x^4*Sinh[c + d*x])/d^3 + (b
*x^6*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

method	result
parallelrisch	$\frac{3dx\left(x\left(2b\,x^{3}+a\right)d^{4}+40b\,d^{2}x^{2}+240b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\left(\left(-b\,x^{6}-a\,x^{3}\right)d^{6}-6x\left(5b\,x^{3}+a\right)d^{4}-360b\,d^{2}x^{2}-720b\right)\tanh\left(\frac{dx}{2}\right.}{d^{7}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b\,x^{6}d^{6}-6b\,x^{5}d^{5}+a\,d^{6}x^{3}+30b\,x^{4}d^{4}-3a\,d^{5}x^{2}-120b\,d^{3}x^{3}+6a\,d^{4}x+360b\,d^{2}x^{2}-6d^{3}a-720dxb+720b\right)\mathrm{e}^{dx+c}}{2d^{7}}-\frac{\left(b\,x^{6}d^{6}\right.}{}$
orering	$-\frac{6\left(2b^{2}d^{6}x^{9}+3ab\,d^{6}x^{6}+50b^{2}d^{4}x^{7}+a^{2}d^{6}x^{3}+42ab\,d^{4}x^{4}+480b^{2}d^{2}x^{5}+4a^{2}d^{4}x+300ab\,d^{2}x^{2}+720x^{3}b^{2}+360ab\right)\cosh(dx}{d^{8}x\left(b\,x^{3}+a\right)}$
meijerg	$\frac{64ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{21}{8}d^{4}x^{4}+\frac{105}{2}x^{2}d^{2}+315\right)\cosh(dx)}{28\sqrt{\pi}}-\frac{i\left(\frac{7}{16}x^{6}d^{6}+\frac{105}{8}d^{4}x^{4}+\frac{315}{2}x^{2}d^{2}+315\right)\sinh(dx)}{28\sqrt{\pi}}\right)}{d^{7}}+\frac{64b\sinh(c)\sqrt{}}{}$
parts	$\frac{b\,x^{6}\sinh(dx+c)}{d}+\frac{a\,x^{3}\sinh(dx+c)}{d}-\frac{3\left(-\frac{2b\,c^{5}\cosh(dx+c)}{d^{5}}+\frac{10b\,c^{4}\left((dx+c)\cosh(dx+c)-\sinh(dx+c)\right)}{d^{5}}-\frac{20b\,c^{3}\left((dx+c)^{2}\,\mathrm{c}\right.}{}\right.}{}$
derivativedivides	$\frac{b\,c^{6}\sinh(dx+c)}{d^{3}}-\frac{6b\,c^{5}\left((dx+c)\sinh(dx+c)-\cosh(dx+c)\right)}{d^{3}}+\frac{15b\,c^{4}\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{3}}-\frac{20b}{}$
default	$\frac{b\,c^{6}\sinh(dx+c)}{d^{3}}-\frac{6b\,c^{5}\left((dx+c)\sinh(dx+c)-\cosh(dx+c)\right)}{d^{3}}+\frac{15b\,c^{4}\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{3}}-\frac{20b}{}$

input
```
int(x^3*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(3*d*x*(x*(2*b*x^3+a)*d^4+40*b*d^2*x^2+240*b)*tanh(1/2*d*x+1/2*c)^2+2*((-b
*x^6-a*x^3)*d^6-6*x*(5*b*x^3+a)*d^4-360*b*d^2*x^2-720*b)*tanh(1/2*d*x+1/2*
c)+3*(x^2*(2*b*x^3+a)*d^4+4*(10*b*x^3+a)*d^2+240*b*x)*d)/d^7/(tanh(1/2*d*x
+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.68

$$\int x^3\left(a+bx^3\right)\cosh(c+dx)\,dx =$$
$$-\frac{3\left(2\,bd^5x^5+ad^5x^2+40\,bd^3x^3+2\,ad^3+240\,bdx\right)\cosh\left(dx+c\right)-\left(bd^6x^6+ad^6x^3+30\,bd^4x^4+6\,ad^4x\right.}{d^7}$$

input
```
integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(3*(2*b*d^5*x^5 + a*d^5*x^2 + 40*b*d^3*x^3 + 2*a*d^3 + 240*b*d*x)*cosh(d*
x + c) - (b*d^6*x^6 + a*d^6*x^3 + 30*b*d^4*x^4 + 6*a*d^4*x + 360*b*d^2*x^2
 + 720*b)*sinh(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.63 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx$$
$$= \begin{cases} \frac{ax^3\sinh(c+dx)}{d} - \frac{3ax^2\cosh(c+dx)}{d^2} + \frac{6ax\sinh(c+dx)}{d^3} - \frac{6a\cosh(c+dx)}{d^4} + \frac{bx^6\sinh(c+dx)}{d} - \frac{6bx^5\cosh(c+dx)}{d^2} + \frac{30bx^4\sinh(c+ \\ \left(\frac{ax^4}{4} + \frac{bx^7}{7}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**3*(b*x**3+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x**3*sinh(c + d*x)/d - 3*a*x**2*cosh(c + d*x)/d**2 + 6*a*x*si
nh(c + d*x)/d**3 - 6*a*cosh(c + d*x)/d**4 + b*x**6*sinh(c + d*x)/d - 6*b*x
**5*cosh(c + d*x)/d**2 + 30*b*x**4*sinh(c + d*x)/d**3 - 120*b*x**3*cosh(c
+ d*x)/d**4 + 360*b*x**2*sinh(c + d*x)/d**5 - 720*b*x*cosh(c + d*x)/d**6 +
 720*b*sinh(c + d*x)/d**7, Ne(d, 0)), ((a*x**4/4 + b*x**7/7)*cosh(c), True
))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.74

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx =$$
$$-\frac{1}{56}d\left(\frac{7(d^4x^4e^c - 4d^3x^3e^c + 12d^2x^2e^c - 24dxe^c + 24e^c)ae^{(dx)}}{d^5} + \frac{7(d^4x^4 + 4d^3x^3 + 12d^2x^2 + 24dx}{d^5}\right.$$
$$+ \frac{1}{28}(4bx^7 + 7ax^4)\cosh(dx+c)$$

input
```
integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/56*d*(7*(d^4*x^4*e^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24
*e^c)*a*e^(d*x)/d^5 + 7*(d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*a
*e^(-d*x - c)/d^5 + 4*(d^7*x^7*e^c - 7*d^6*x^6*e^c + 42*d^5*x^5*e^c - 210*
d^4*x^4*e^c + 840*d^3*x^3*e^c - 2520*d^2*x^2*e^c + 5040*d*x*e^c - 5040*e^c
)*b*e^(d*x)/d^8 + 4*(d^7*x^7 + 7*d^6*x^6 + 42*d^5*x^5 + 210*d^4*x^4 + 840*
d^3*x^3 + 2520*d^2*x^2 + 5040*d*x + 5040)*b*e^(-d*x - c)/d^8) + 1/28*(4*b*
x^7 + 7*a*x^4)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.25

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx$$
$$= \frac{(bd^6x^6 - 6\,bd^5x^5 + ad^6x^3 + 30\,bd^4x^4 - 3\,ad^5x^2 - 120\,bd^3x^3 + 6\,ad^4x + 360\,bd^2x^2 - 6\,ad^3 - 720\,bdx + 72}{2\,d^7}$$
$$- \frac{(bd^6x^6 + 6\,bd^5x^5 + ad^6x^3 + 30\,bd^4x^4 + 3\,ad^5x^2 + 120\,bd^3x^3 + 6\,ad^4x + 360\,bd^2x^2 + 6\,ad^3 + 720\,bdx +}{2\,d^7}$$

input
```
integrate(x^3*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^6*x^6 - 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 - 3*a*d^5*x^2 - 12
0*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 - 6*a*d^3 - 720*b*d*x + 720*b)*e^(
d*x + c)/d^7 - 1/2*(b*d^6*x^6 + 6*b*d^5*x^5 + a*d^6*x^3 + 30*b*d^4*x^4 + 3
*a*d^5*x^2 + 120*b*d^3*x^3 + 6*a*d^4*x + 360*b*d^2*x^2 + 6*a*d^3 + 720*b*d
*x + 720*b)*e^(-d*x - c)/d^7
```

Mupad [B] (verification not implemented)

Time = 2.05 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.99

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = \frac{30\,b\,x^4\sinh(c+d\,x)+6\,a\,x\sinh(c+d\,x)}{d^3} - \frac{3\,a\,x^2\cosh(c+d\,x)+6\,b\,x^5\cosh(c+d\,x)}{d^2} + \frac{a\,x^3\sinh(c+d\,x)+b\,x^6\sinh(c+d\,x)}{d} - \frac{6\,a\cosh(c+d\,x)+120\,b\,x^3\cosh(c+d\,x)}{d^4} + \frac{720\,b\sinh(c+d\,x)}{d^7} - \frac{720\,b\,x\cosh(c+d\,x)}{d^6} + \frac{360\,b\,x^2\sinh(c+d\,x)}{d^5}$$

input
```
int(x^3*cosh(c + d*x)*(a + b*x^3),x)
```

output
```
(30*b*x^4*sinh(c + d*x) + 6*a*x*sinh(c + d*x))/d^3 - (3*a*x^2*cosh(c + d*x
) + 6*b*x^5*cosh(c + d*x))/d^2 + (a*x^3*sinh(c + d*x) + b*x^6*sinh(c + d*x
))/d - (6*a*cosh(c + d*x) + 120*b*x^3*cosh(c + d*x))/d^4 + (720*b*sinh(c +
 d*x))/d^7 - (720*b*x*cosh(c + d*x))/d^6 + (360*b*x^2*sinh(c + d*x))/d^5
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.99

$$\int x^3(a+bx^3)\cosh(c+dx)\,dx = \frac{-3\cosh(dx+c)\,a\,d^5x^2-6\cosh(dx+c)\,a\,d^3-6\cosh(dx+c)\,b\,d^5x^5-120\cosh(dx+c)\,b\,d^3x^3-720\,\mathrm{c}}{}$$

input
```
int(x^3*(b*x^3+a)*cosh(d*x+c),x)
```

output
```
( - 3*cosh(c + d*x)*a*d**5*x**2 - 6*cosh(c + d*x)*a*d**3 - 6*cosh(c + d*x)
*b*d**5*x**5 - 120*cosh(c + d*x)*b*d**3*x**3 - 720*cosh(c + d*x)*b*d*x + s
inh(c + d*x)*a*d**6*x**3 + 6*sinh(c + d*x)*a*d**4*x + sinh(c + d*x)*b*d**6
*x**6 + 30*sinh(c + d*x)*b*d**4*x**4 + 360*sinh(c + d*x)*b*d**2*x**2 + 720
*sinh(c + d*x)*b)/d**7
```

3.80 $\int x^2(a+bx^3)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 124

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = -\frac{120b\cosh(c+dx)}{d^6} - \frac{2ax\cosh(c+dx)}{d^2} - \frac{60bx^2\cosh(c+dx)}{d^4} - \frac{5bx^4\cosh(c+dx)}{d^2} + \frac{2a\sinh(c+dx)}{d^3} + \frac{120bx\sinh(c+dx)}{d^5} + \frac{ax^2\sinh(c+dx)}{d} + \frac{20bx^3\sinh(c+dx)}{d^3} + \frac{bx^5\sinh(c+dx)}{d}$$

output

```
-120*b*cosh(d*x+c)/d^6-2*a*x*cosh(d*x+c)/d^2-60*b*x^2*cosh(d*x+c)/d^4-5*b*
x^4*cosh(d*x+c)/d^2+2*a*sinh(d*x+c)/d^3+120*b*x*sinh(d*x+c)/d^5+a*x^2*sinh
(d*x+c)/d+20*b*x^3*sinh(d*x+c)/d^3+b*x^5*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.68

$$\int x^2\left(a+bx^3\right)\cosh(c+dx)\,dx$$
$$=\frac{-((2ad^4x+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx))+d(ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\,\mathrm{sinh}}{d^6}$$

input
```
Integrate[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
(-((2*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x]) + d*(a*d^2
*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^6
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^2\left(a+bx^3\right)\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{5810}$$
$$\int \left(ax^2\cosh(c+dx)+bx^5\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2a\sinh(c+dx)}{d^3}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{ax^2\sinh(c+dx)}{d}-\frac{120b\cosh(c+dx)}{d^6}+\frac{120bx\sinh(c+dx)}{d^5}-\frac{60bx^2\cosh(c+dx)}{d^4}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{bx^5\sinh(c+dx)}{d}$$

input
```
Int[x^2*(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
(-120*b*Cosh[c + d*x])/d^6 - (2*a*x*Cosh[c + d*x])/d^2 - (60*b*x^2*Cosh[c
+ d*x])/d^4 - (5*b*x^4*Cosh[c + d*x])/d^2 + (2*a*Sinh[c + d*x])/d^3 + (120
*b*x*Sinh[c + d*x])/d^5 + (a*x^2*Sinh[c + d*x])/d + (20*b*x^3*Sinh[c + d*x
])/d^3 + (b*x^5*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.61 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{\left(\left(5b\,x^4+2ax\right)d^4+60b\,d^2x^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\left(x^2\left(b\,x^3+a\right)d^4+2\left(10b\,x^3+a\right)d^2+120bx\right)d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(5b\,x^4+2ax\right)\cdots}{d^6\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{\left(b\,x^5d^5-5b\,x^4d^4+a\,d^5x^2+20b\,d^3x^3-2a\,d^4x-60b\,d^2x^2+2d^3a+120dxb-120b\right)\mathrm{e}^{dx+c}}{2d^6}-\frac{\left(b\,x^5d^5+5b\,x^4d^4+a\,d^5x^2+20\cdots\right.}{\cdots}$
orering	$-\frac{2\left(5b^2d^4x^8+7ab\,d^4x^5+80b^2d^2x^6+2a^2d^4x^2+55ab\,d^2x^3+360b^2x^4+2d^2a^2+180abx\right)\cosh(dx+c)}{d^6x\left(b\,x^3+a\right)}+\frac{\left(b\,d^4x^5+a\,d^4x^2\cdots\right.}{\cdots}$
meijerg	$-\frac{32b\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}x^2d^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}x^2d^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib\sinh(c)\sqrt{\pi}\left(-\cdots\right.}{\cdots}$
parts	$\frac{b\,x^5\sinh(dx+c)}{d}+\frac{a\,x^2\sinh(dx+c)}{d}-\frac{\frac{5b\,c^4\cosh(dx+c)}{d^4}-\frac{20b\,c^3((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^4}+\frac{30b\,c^2\left((dx+c)^2\cosh(d\cdots\right.}{\cdots}}{\cdots}$
derivativedivides	$\frac{-\frac{b\,c^5\sinh(dx+c)}{d^3}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{10\cdots}{\cdots}}{\cdots}$
default	$\frac{-\frac{b\,c^5\sinh(dx+c)}{d^3}+\frac{5b\,c^4((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}-\frac{10b\,c^3\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}+\frac{10\cdots}{\cdots}}{\cdots}$

input
```
int(x^2*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(((5*b*x^4+2*a*x)*d^4+60*b*d^2*x^2)*tanh(1/2*d*x+1/2*c)^2-2*(x^2*(b*x^3+a)
*d^4+2*(10*b*x^3+a)*d^2+120*b*x)*d*tanh(1/2*d*x+1/2*c)+(5*b*x^4+2*a*x)*d^4
+60*b*d^2*x^2+240*b)/d^6/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.70

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = -\frac{(5\,bd^4x^4+2\,ad^4x+60\,bd^2x^2+120\,b)\cosh(dx+c)-(bd^5x^5+ad^5x^2+20\,bd^3x^3+2\,ad^3+120\,bdx)\,\mathrm{si}}{d^6}$$

input
```
integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((5*b*d^4*x^4 + 2*a*d^4*x + 60*b*d^2*x^2 + 120*b)*cosh(d*x + c) - (b*d^5*
x^5 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^3 + 120*b*d*x)*sinh(d*x + c))/d^6
```

Sympy [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.22

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = \begin{cases} \frac{ax^2\sinh(c+dx)}{d}-\frac{2ax\cosh(c+dx)}{d^2}+\frac{2a\sinh(c+dx)}{d^3}+\frac{bx^5\sinh(c+dx)}{d}-\frac{5bx^4\cosh(c+dx)}{d^2}+\frac{20bx^3\sinh(c+dx)}{d^3}-\frac{60bx^2\cosh(c+}{d^4} \\ \left(\frac{ax^3}{3}+\frac{bx^6}{6}\right)\cosh(c) \end{cases}$$

input
```
integrate(x**2*(b*x**3+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x**2*sinh(c + d*x)/d - 2*a*x*cosh(c + d*x)/d**2 + 2*a*sinh(c
+ d*x)/d**3 + b*x**5*sinh(c + d*x)/d - 5*b*x**4*cosh(c + d*x)/d**2 + 20*b*
x**3*sinh(c + d*x)/d**3 - 60*b*x**2*cosh(c + d*x)/d**4 + 120*b*x*sinh(c +
d*x)/d**5 - 120*b*cosh(c + d*x)/d**6, Ne(d, 0)), ((a*x**3/3 + b*x**6/6)*co
sh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(124) = 248$.

Time = 0.04 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.15

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = \frac{(bx^3+a)^2\cosh(dx+c)}{6\,b} - \frac{\left(\frac{a^2e^{(dx+c)}}{d} + \frac{a^2e^{(-dx-c)}}{d} + \frac{2\,(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)abe^{(dx)}}{d^4} + \frac{2\,(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)abe^{(-dx-c)}}{d^4} + \frac{(d^6x^6e^c - 6\,d^5x^5e^c}{}\right.}{}$$

input
```
integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/6*(b*x^3 + a)^2*cosh(d*x + c)/b - 1/12*(a^2*e^(d*x + c)/d + a^2*e^(-d*x
- c)/d + 2*(d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d
^4 + 2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*a*b*e^(-d*x - c)/d^4 + (d^6*x^6*e
^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c -
720*d*x*e^c + 720*e^c)*b^2*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4
 + 120*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*b^2*e^(-d*x - c)/d^7)*d/b
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.26

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx$$
$$= \frac{(bd^5x^5 - 5\,bd^4x^4 + ad^5x^2 + 20\,bd^3x^3 - 2\,ad^4x - 60\,bd^2x^2 + 2\,ad^3 + 120\,bdx - 120\,b)e^{(dx+c)}}{2\,d^6} - \frac{(bd^5x^5 + 5\,bd^4x^4 + ad^5x^2 + 20\,bd^3x^3 + 2\,ad^4x + 60\,bd^2x^2 + 2\,ad^3 + 120\,bdx + 120\,b)e^{(-dx-c)}}{2\,d^6}$$

input
```
integrate(x^2*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^5*x^5 - 5*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 - 2*a*d^4*x - 60*b
*d^2*x^2 + 2*a*d^3 + 120*b*d*x - 120*b)*e^(d*x + c)/d^6 - 1/2*(b*d^5*x^5 +
 5*b*d^4*x^4 + a*d^5*x^2 + 20*b*d^3*x^3 + 2*a*d^4*x + 60*b*d^2*x^2 + 2*a*d
^3 + 120*b*d*x + 120*b)*e^(-d*x - c)/d^6
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.98

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = \frac{a\,x^2\sinh(c+d\,x)+b\,x^5\sinh(c+d\,x)}{d} - \frac{2\,a\,x\cosh(c+d\,x)+5\,b\,x^4\cosh(c+d\,x)}{d^2} + \frac{2\,a\sinh(c+d\,x)+20\,b\,x^3\sinh(c+d\,x)}{d^3} - \frac{120\,b\cosh(c+d\,x)}{d^6} + \frac{120\,b\,x\sinh(c+d\,x)}{d^5} - \frac{60\,b\,x^2\cosh(c+d\,x)}{d^4}$$

input
```
int(x^2*cosh(c + d*x)*(a + b*x^3),x)
```

output
```
(a*x^2*sinh(c + d*x) + b*x^5*sinh(c + d*x))/d - (2*a*x*cosh(c + d*x) + 5*b
*x^4*cosh(c + d*x))/d^2 + (2*a*sinh(c + d*x) + 20*b*x^3*sinh(c + d*x))/d^3
 - (120*b*cosh(c + d*x))/d^6 + (120*b*x*sinh(c + d*x))/d^5 - (60*b*x^2*cos
h(c + d*x))/d^4
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.99

$$\int x^2(a+bx^3)\cosh(c+dx)\,dx = \frac{-2\cosh(dx+c)\,a\,d^4x-5\cosh(dx+c)\,b\,d^4x^4-60\cosh(dx+c)\,b\,d^2x^2-120\cosh(dx+c)\,b+\sinh(dx$$

input
```
int(x^2*(b*x^3+a)*cosh(d*x+c),x)
```

output
```
( - 2*cosh(c + d*x)*a*d**4*x - 5*cosh(c + d*x)*b*d**4*x**4 - 60*cosh(c + d
*x)*b*d**2*x**2 - 120*cosh(c + d*x)*b + sinh(c + d*x)*a*d**5*x**2 + 2*sinh
(c + d*x)*a*d**3 + sinh(c + d*x)*b*d**5*x**5 + 20*sinh(c + d*x)*b*d**3*x**
3 + 120*sinh(c + d*x)*b*d*x)/d**6
```

3.81 $\int x(a+bx^3)\cosh(c+dx)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 94

$$\int x(a+bx^3)\cosh(c+dx)\,dx = -\frac{a\cosh(c+dx)}{d^2} - \frac{24bx\cosh(c+dx)}{d^4} - \frac{4bx^3\cosh(c+dx)}{d^2} + \frac{24b\sinh(c+dx)}{d^5} + \frac{ax\sinh(c+dx)}{d} + \frac{12bx^2\sinh(c+dx)}{d^3} + \frac{bx^4\sinh(c+dx)}{d}$$

output
```
-a*cosh(d*x+c)/d^2-24*b*x*cosh(d*x+c)/d^4-4*b*x^3*cosh(d*x+c)/d^2+24*b*sin
h(d*x+c)/d^5+a*x*sinh(d*x+c)/d+12*b*x^2*sinh(d*x+c)/d^3+b*x^4*sinh(d*x+c)/
d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.70

$$\int x(a+bx^3)\cosh(c+dx)\,dx$$
$$=\frac{-d(ad^2+4bx(6+d^2x^2))\cosh(c+dx)+(ad^4x+b(24+12d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}$$

input
```
Integrate[x*(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
(-(d*(a*d^2 + 4*b*x*(6 + d^2*x^2))*Cosh[c + d*x]) + (a*d^4*x + b*(24 + 12*
d^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.133$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x(a+bx^3)\cosh(c+dx)\,dx$$
$$\Big\downarrow \text{5810}$$
$$\int \left(ax\cosh(c+dx)+bx^4\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$
$$-\frac{a\cosh(c+dx)}{d^2}+\frac{ax\sinh(c+dx)}{d}+\frac{24b\sinh(c+dx)}{d^5}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}$$

input
```
Int[x*(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
-((a*Cosh[c + d*x])/d^2) - (24*b*x*Cosh[c + d*x])/d^4 - (4*b*x^3*Cosh[c +
d*x])/d^2 + (24*b*Sinh[c + d*x])/d^5 + (a*x*Sinh[c + d*x])/d + (12*b*x^2*S
inh[c + d*x])/d^3 + (b*x^4*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.11

method	result
parallelrisch	$\frac{4dxb(x^2d^2+6)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+2((-b\,x^4-ax)d^4-12b\,d^2x^2-24b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2((2b\,x^3+a)d^2+12bx)d}{d^5\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-1\right)}$
risch	$\frac{(b\,x^4d^4-4b\,d^3x^3+a\,d^4x+12b\,d^2x^2-d^3a-24dxb+24b)\mathrm{e}^{dx+c}}{2d^5}-\frac{(b\,x^4d^4+4b\,d^3x^3+a\,d^4x+12b\,d^2x^2+d^3a+24dxb+24b}{2d^5}$
orering	$-\frac{2(4b^2d^4x^7+5ab\,d^4x^4+36b^2d^2x^5+a^2d^4x+18ab\,d^2x^2+48x^3b^2+12ab)\cosh(dx+c)}{d^6x(b\,x^3+a)}+\frac{(b\,x^4d^4+a\,d^4x+12b\,d^2x^2+24b}{}$
parts	$\frac{b\,x^4\sinh(dx+c)}{d}+\frac{ax\sinh(dx+c)}{d}-\frac{-\frac{4b\,c^3\cosh(dx+c)}{d^3}+\frac{12b\,c^2((dx+c)\cosh(dx+c)-\sinh(dx+c))}{d^3}-\frac{12bc\left((dx+c)^2\cosh(dx\right.}{}}{}$
meijerg	$-\frac{16ib\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5}-\frac{16b\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4+\right.}{}\right.}{}$
derivativedivides	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{4bc\left((dx+c)^3\sinh(dx\right.}{}}{}$
default	$\frac{-\frac{4b\,c^3((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^3}+\frac{6b\,c^2\left((dx+c)^2\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^3}-\frac{4bc\left((dx+c)^3\sinh(dx\right.}{}}{}$

input
```
int(x*(b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*(2*d*x*b*(d^2*x^2+6)*tanh(1/2*d*x+1/2*c)^2+((-b*x^4-a*x)*d^4-12*b*d^2*x^
2-24*b)*tanh(1/2*d*x+1/2*c)+((2*b*x^3+a)*d^2+12*b*x)*d)/d^5/(tanh(1/2*d*x+
1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.72

$$\int x(a+bx^3)\cosh(c+dx)\,dx = -\frac{(4\,bd^3x^3+ad^3+24\,bdx)\cosh(dx+c)-(bd^4x^4+ad^4x+12\,bd^2x^2+24\,b)\sinh(dx+c)}{d^5}$$

input
```
integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((4*b*d^3*x^3 + a*d^3 + 24*b*d*x)*cosh(d*x + c) - (b*d^4*x^4 + a*d^4*x +
12*b*d^2*x^2 + 24*b)*sinh(d*x + c))/d^5
```

Sympy [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \begin{cases} \frac{ax\sinh(c+dx)}{d}-\frac{a\cosh(c+dx)}{d^2}+\frac{bx^4\sinh(c+dx)}{d}-\frac{4bx^3\cosh(c+dx)}{d^2}+\frac{12bx^2\sinh(c+dx)}{d^3}-\frac{24bx\cosh(c+dx)}{d^4}+\frac{24b\sinh(c+dx)}{d^5} \\ \left(\frac{ax^2}{2}+\frac{bx^5}{5}\right)\cosh(c) \end{cases}$$

input
```
integrate(x*(b*x**3+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*x*sinh(c + d*x)/d - a*cosh(c + d*x)/d**2 + b*x**4*sinh(c + d*
x)/d - 4*b*x**3*cosh(c + d*x)/d**2 + 12*b*x**2*sinh(c + d*x)/d**3 - 24*b*x
*cosh(c + d*x)/d**4 + 24*b*sinh(c + d*x)/d**5, Ne(d, 0)), ((a*x**2/2 + b*x
**5/5)*cosh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(94) = 188$.

Time = 0.05 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.09

$$\int x(a+bx^3)\cosh(c+dx)\,dx = -\frac{1}{20}\,d\left(\frac{5\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)ae^{(dx)}}{d^3} + \frac{5\,(d^2x^2 + 2\,dx + 2)ae^{(-dx-c)}}{d^3} + \frac{2\,(d^5x^5e^c - 5\,d^4x^4e^c + 20\,d^3x\right. + \frac{1}{10}\left(2\,bx^5 + 5\,ax^2\right)\cosh\left(dx + c\right)$$

input
```
integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/20*d*(5*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a*e^(d*x)/d^3 + 5*(d^2*x^2 +
2*d*x + 2)*a*e^(-d*x - c)/d^3 + 2*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^
3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*b*e^(d*x)/d^6 + 2*(d^5*x^5
 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*b*e^(-d*x - c)/d^6
) + 1/10*(2*b*x^5 + 5*a*x^2)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.27

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \frac{(bd^4x^4 - 4\,bd^3x^3 + ad^4x + 12\,bd^2x^2 - ad^3 - 24\,bdx + 24\,b)e^{(dx+c)}}{2\,d^5} - \frac{(bd^4x^4 + 4\,bd^3x^3 + ad^4x + 12\,bd^2x^2 + ad^3 + 24\,bdx + 24\,b)e^{(-dx-c)}}{2\,d^5}$$

input
```
integrate(x*(b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^4*x^4 - 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2*x^2 - a*d^3 - 24*b*d*x +
 24*b)*e^(d*x + c)/d^5 - 1/2*(b*d^4*x^4 + 4*b*d^3*x^3 + a*d^4*x + 12*b*d^2
*x^2 + a*d^3 + 24*b*d*x + 24*b)*e^(-d*x - c)/d^5
```

Mupad [B] (verification not implemented)

Time = 1.90 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \frac{b\,x^4\sinh(c+d\,x)+a\,x\sinh(c+d\,x)}{d} - \frac{a\cosh(c+d\,x)+4\,b\,x^3\cosh(c+d\,x)}{d^2} + \frac{24\,b\sinh(c+d\,x)}{d^5} - \frac{24\,b\,x\cosh(c+d\,x)}{d^4} + \frac{12\,b\,x^2\sinh(c+d\,x)}{d^3}$$

input
```
int(x*cosh(c + d*x)*(a + b*x^3),x)
```

output
```
(b*x^4*sinh(c + d*x) + a*x*sinh(c + d*x))/d - (a*cosh(c + d*x) + 4*b*x^3*c
osh(c + d*x))/d^2 + (24*b*sinh(c + d*x))/d^5 - (24*b*x*cosh(c + d*x))/d^4
+ (12*b*x^2*sinh(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99

$$\int x(a+bx^3)\cosh(c+dx)\,dx = \frac{-\cosh(dx+c)\,a\,d^3-4\cosh(dx+c)\,b\,d^3x^3-24\cosh(dx+c)\,bdx+\sinh(dx+c)\,a\,d^4x+\sinh(dx+c)}{d^5}$$

input
```
int(x*(b*x^3+a)*cosh(d*x+c),x)
```

output
```
( - cosh(c + d*x)*a*d**3 - 4*cosh(c + d*x)*b*d**3*x**3 - 24*cosh(c + d*x)*
b*d*x + sinh(c + d*x)*a*d**4*x + sinh(c + d*x)*b*d**4*x**4 + 12*sinh(c + d
*x)*b*d**2*x**2 + 24*sinh(c + d*x)*b)/d**5
```

3.82 $\int (a+bx^3) \cosh(c+dx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 66

$$\int (a+bx^3) \cosh(c+dx) \, dx = -\frac{6b\cosh(c+dx)}{d^4} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{a\sinh(c+dx)}{d} + \frac{6bx\sinh(c+dx)}{d^3} + \frac{bx^3\sinh(c+dx)}{d}$$

output
```
-6*b*cosh(d*x+c)/d^4-3*b*x^2*cosh(d*x+c)/d^2+a*sinh(d*x+c)/d+6*b*x*sinh(d*
x+c)/d^3+b*x^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.74

$$\int (a+bx^3) \cosh(c+dx) \, dx = \frac{-3b(2+d^2x^2)\cosh(c+dx) + d(ad^2+bx(6+d^2x^2))\sinh(c+dx)}{d^4}$$

input
```
Integrate[(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
(-3*b*(2 + d^2*x^2)*Cosh[c + d*x] + d*(a*d^2 + b*x*(6 + d^2*x^2))*Sinh[c +
 d*x])/d^4
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = $\{5800, 2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + bx^3) \cosh(c + dx) \, dx$$

$$\Big\downarrow \text{5800}$$

$$\int (a \cosh(c + dx) + bx^3 \cosh(c + dx)) \, dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{a \sinh(c + dx)}{d} - \frac{6b \cosh(c + dx)}{d^4} + \frac{6bx \sinh(c + dx)}{d^3} - \frac{3bx^2 \cosh(c + dx)}{d^2} + \frac{bx^3 \sinh(c + dx)}{d}$$

input
```
Int[(a + b*x^3)*Cosh[c + d*x],x]
```

output
```
(-6*b*Cosh[c + d*x])/d^4 - (3*b*x^2*Cosh[c + d*x])/d^2 + (a*Sinh[c + d*x])
/d + (6*b*x*Sinh[c + d*x])/d^3 + (b*x^3*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5800
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.50 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.23

method	result
parallelrisch	$\frac{3b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}x^{2}d^{2}-2\left(\left(b\,x^{3}+a\right)d^{2}+6bx\right)d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+3b\,d^{2}x^{2}+12b}{d^{4}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-1\right)}$
risch	$\frac{\left(b\,d^{3}x^{3}-3b\,d^{2}x^{2}+d^{3}a+6dxb-6b\right)\mathrm{e}^{dx+c}}{2d^{4}}-\frac{\left(b\,d^{3}x^{3}+3b\,d^{2}x^{2}+d^{3}a+6dxb+6b\right)\mathrm{e}^{-dx-c}}{2d^{4}}$
parts	$\frac{b\,x^{3}\sinh(dx+c)}{d}+\frac{a\sinh(dx+c)}{d}-\frac{3b\left(c^{2}\cosh(dx+c)-2c((dx+c)\cosh(dx+c)-\sinh(dx+c))+(dx+c)^{2}\cosh(dx+c)\right.}{d^{4}}$
orering	$-\frac{6b\left(b\,d^{2}x^{5}+a\,d^{2}x^{2}+4b\,x^{3}+a\right)\cosh(dx+c)}{d^{4}\left(b\,x^{3}+a\right)}+\frac{\left(b\,d^{2}x^{3}+a\,d^{2}+6bx\right)\left(3x^{2}b\cosh(dx+c)+\left(b\,x^{3}+a\right)d\sinh(dx+c)\right)}{d^{4}\left(b\,x^{3}+a\right)}$
meijerg	$\frac{8b\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^{2}d^{2}}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^{2}d^{2}}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^{4}}-\frac{8ib\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^{2}d^{2}}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i(}{}\right.}{d^{4}}$
derivativedivides	$\frac{-\frac{b\,c^{3}\sinh(dx+c)}{d^{3}}+\frac{3b\,c^{2}((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{3}}-\frac{3bc\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{3}}+\frac{b((d}{}}{d}$
default	$\frac{-\frac{b\,c^{3}\sinh(dx+c)}{d^{3}}+\frac{3b\,c^{2}((dx+c)\sinh(dx+c)-\cosh(dx+c))}{d^{3}}-\frac{3bc\left((dx+c)^{2}\sinh(dx+c)-2(dx+c)\cosh(dx+c)+2\sinh(dx+c)\right)}{d^{3}}+\frac{b((d}{}}{d}$

input
```
int((b*x^3+a)*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(3*b*tanh(1/2*d*x+1/2*c)^2*x^2*d^2-2*((b*x^3+a)*d^2+6*b*x)*d*tanh(1/2*d*x+
1/2*c)+3*b*d^2*x^2+12*b)/d^4/(tanh(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.80

$$\int (a+bx^3)\cosh(c+dx)\,dx = -\frac{3\,(bd^2x^2+2\,b)\cosh\left(dx+c\right)-(bd^3x^3+ad^3+6\,bdx)\sinh\left(dx+c\right)}{d^4}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(3*(b*d^2*x^2 + 2*b)*cosh(d*x + c) - (b*d^3*x^3 + a*d^3 + 6*b*d*x)*sinh(d
*x + c))/d^4
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.24

$$\int (a+bx^3)\cosh(c+dx)\,dx = \begin{cases} \frac{a\sinh(c+dx)}{d} + \frac{bx^3\sinh(c+dx)}{d} - \frac{3bx^2\cosh(c+dx)}{d^2} + \frac{6bx\sinh(c+dx)}{d^3} - \frac{6b\cosh(c+dx)}{d^4} & \text{for } d \neq 0 \\ \left(ax + \frac{bx^4}{4}\right)\cosh(c) & \text{otherwise} \end{cases}$$

input
```
integrate((b*x**3+a)*cosh(d*x+c),x)
```

output
```
Piecewise((a*sinh(c + d*x)/d + b*x**3*sinh(c + d*x)/d - 3*b*x**2*cosh(c +
d*x)/d**2 + 6*b*x*sinh(c + d*x)/d**3 - 6*b*cosh(c + d*x)/d**4, Ne(d, 0)),
((a*x + b*x**4/4)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.58

$$\int (a+bx^3)\cosh(c+dx)\,dx = \frac{ae^{(dx+c)}}{2d} - \frac{ae^{(-dx-c)}}{2d} + \frac{(d^3x^3e^c - 3d^2x^2e^c + 6dxe^c - 6e^c)be^{(dx)}}{2d^4} - \frac{(d^3x^3 + 3d^2x^2 + 6dx + 6)be^{(-dx-c)}}{2d^4}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a*e^(d*x + c)/d - 1/2*a*e^(-d*x - c)/d + 1/2*(d^3*x^3*e^c - 3*d^2*x^2*
e^c + 6*d*x*e^c - 6*e^c)*b*e^(d*x)/d^4 - 1/2*(d^3*x^3 + 3*d^2*x^2 + 6*d*x
+ 6)*b*e^(-d*x - c)/d^4
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.33

$$\int (a+bx^3)\cosh(c+dx)\,dx = \frac{(bd^3x^3 - 3\,bd^2x^2 + ad^3 + 6\,bdx - 6\,b)e^{(dx+c)}}{2\,d^4} - \frac{(bd^3x^3 + 3\,bd^2x^2 + ad^3 + 6\,bdx + 6\,b)e^{(-dx-c)}}{2\,d^4}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b*d^3*x^3 - 3*b*d^2*x^2 + a*d^3 + 6*b*d*x - 6*b)*e^(d*x + c)/d^4 - 1/
2*(b*d^3*x^3 + 3*b*d^2*x^2 + a*d^3 + 6*b*d*x + 6*b)*e^(-d*x - c)/d^4
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int (a+bx^3)\cosh(c+dx)\,dx = \frac{a\sinh(c+d\,x) + b\,x^3\sinh(c+d\,x)}{d} - \frac{6\,b\cosh(c+d\,x)}{d^4} + \frac{6\,b\,x\sinh(c+d\,x)}{d^3} - \frac{3\,b\,x^2\cosh(c+d\,x)}{d^2}$$

input
```
int(cosh(c + d*x)*(a + b*x^3),x)
```

output
```
(a*sinh(c + d*x) + b*x^3*sinh(c + d*x))/d - (6*b*cosh(c + d*x))/d^4 + (6*b
*x*sinh(c + d*x))/d^3 - (3*b*x^2*cosh(c + d*x))/d^2
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.98

$$\int (a + bx^3) \cosh(c + dx)\, dx$$
$$= \frac{-3\cosh(dx+c)\, b\, d^2x^2 - 6\cosh(dx+c)\, b + \sinh(dx+c)\, a\, d^3 + \sinh(dx+c)\, b\, d^3x^3 + 6\sinh(dx+c)\, b\, d}{d^4}$$

input
```
int((b*x^3+a)*cosh(d*x+c),x)
```

output
```
( - 3*cosh(c + d*x)*b*d**2*x**2 - 6*cosh(c + d*x)*b + sinh(c + d*x)*a*d**3
 + sinh(c + d*x)*b*d**3*x**3 + 6*sinh(c + d*x)*b*d*x)/d**4
```

3.83 $\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = -\frac{2bx\cosh(c+dx)}{d^2} + a\cosh(c)\text{Chi}(dx) + \frac{2b\sinh(c+dx)}{d^3} + \frac{bx^2\sinh(c+dx)}{d} + a\sinh(c)\text{Shi}(dx)$$

output
```
-2*b*x*cosh(d*x+c)/d^2+a*cosh(c)*Chi(d*x)+2*b*sinh(d*x+c)/d^3+b*x^2*sinh(d
*x+c)/d+a*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.88

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = a\cosh(c)\text{Chi}(dx) + \frac{b(-2dx\cosh(c+dx) + (2+d^2x^2)\sinh(c+dx))}{d^3} + a\sinh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x^3)*Cosh[c + d*x])/x,x]
```

output
```
a*Cosh[c]*CoshIntegral[d*x] + (b*(-2*d*x*Cosh[c + d*x] + (2 + d^2*x^2)*Sin
h[c + d*x]))/d^3 + a*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a\cosh(c+dx)}{x}+bx^2\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$a\cosh(c)\text{Chi}(dx)+a\sinh(c)\text{Shi}(dx)+\frac{2b\sinh(c+dx)}{d^3}-\frac{2bx\cosh(c+dx)}{d^2}+\frac{bx^2\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)*Cosh[c + d*x])/x,x]
```

output
```
(-2*b*x*Cosh[c + d*x])/d^2 + a*Cosh[c]*CoshIntegral[d*x] + (2*b*Sinh[c + d
*x])/d^3 + (b*x^2*Sinh[c + d*x])/d + a*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.02

method	result
risch	$-\frac{a\,\mathrm{e}^{-c}\,\mathrm{expIntegral}_1(dx)}{2}-\frac{\mathrm{e}^{-dx-c}b\,x^2}{2d}-\frac{a\,\mathrm{e}^{c}\,\mathrm{expIntegral}_1(-dx)}{2}+\frac{\mathrm{e}^{dx+c}b\,x^2}{2d}-\frac{\mathrm{e}^{-dx-c}bx}{d^2}-\frac{\mathrm{e}^{dx+c}bx}{d^2}-\frac{\mathrm{e}^{-dx-c}b}{d^3}+$
meijerg	$\frac{4ib\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^2d^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3}+\frac{4b\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^2d^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^3}+\frac{a\cosh(c)\sqrt{}}{}$

input
```
int((b*x^3+a)*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*exp(-c)*Ei(1,d*x)-1/2/d*exp(-d*x-c)*b*x^2-1/2*a*exp(c)*Ei(1,-d*x)+1
/2/d*exp(d*x+c)*b*x^2-1/d^2*exp(-d*x-c)*b*x-1/d^2*exp(d*x+c)*b*x-1/d^3*exp
(-d*x-c)*b+1/d^3*exp(d*x+c)*b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.54

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = -\frac{4\,bdx\cosh\left(dx+c\right)-\left(ad^3\mathrm{Ei}(dx)+ad^3\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-2\left(bd^2x^2+2\,b\right)\sinh\left(dx+c\right)-\left(ad^3\mathrm{Ei}(dx)\right.}{2\,d^3}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="fricas")
```

output
```
-1/2*(4*b*d*x*cosh(d*x + c) - (a*d^3*Ei(d*x) + a*d^3*Ei(-d*x))*cosh(c) - 2
*(b*d^2*x^2 + 2*b)*sinh(d*x + c) - (a*d^3*Ei(d*x) - a*d^3*Ei(-d*x))*sinh(c
))/d^3
```

Sympy [A] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.18

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = a\sinh(c)\operatorname{Shi}(dx) + a\cosh(c)\operatorname{Chi}(dx) + b\left(\begin{cases}\frac{x^2\sinh(c+dx)}{d} - \frac{2x\cosh(c+dx)}{d^2} + \frac{2\sinh(c+dx)}{d^3} & \text{for } d\neq 0\\ \frac{x^3\cosh(c)}{3} & \text{otherwise}\end{cases}\right)$$

input
```
integrate((b*x**3+a)*cosh(d*x+c)/x,x)
```

output
```
a*sinh(c)*Shi(d*x) + a*cosh(c)*Chi(d*x) + b*Piecewise((x**2*sinh(c + d*x)/
d - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x**3*cosh(c
)/3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(56) = 112.

Time = 0.13 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.50

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx = -\frac{1}{6}\left(b\left(\frac{(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)e^{(dx)}}{d^4} + \frac{(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)e^{(-dx-c)}}{d^4}\right) + \frac{2\,a\cosh(dx \ldots}{d \ldots}\right. + \frac{1}{3}\left(bx^3 + a\log\left(x^3\right)\right)\cosh(dx+c)$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/6*(b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (
d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + 2*a*cosh(d*x + c)*log
(x^3)/d - 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a/d)*d + 1/3*(b*x^3 + a*log(x^
3))*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.95

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$$
$$= \frac{bd^2x^2e^{(dx+c)} - bd^2x^2e^{(-dx-c)} + ad^3\mathrm{Ei}(-dx)\,e^{(-c)} + ad^3\mathrm{Ei}(dx)\,e^{c} - 2\,bdxe^{(dx+c)} - 2\,bdxe^{(-dx-c)} + 2\,be^{(dx+}}{2\,d^3}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(b*d^2*x^2*e^(d*x + c) - b*d^2*x^2*e^(-d*x - c) + a*d^3*Ei(-d*x)*e^(-c
) + a*d^3*Ei(d*x)*e^c - 2*b*d*x*e^(d*x + c) - 2*b*d*x*e^(-d*x - c) + 2*b*e
^(d*x + c) - 2*b*e^(-d*x - c))/d^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$$
$$= a\,\mathrm{coshint}(d\,x)\,\cosh(c) + a\,\mathrm{sinhint}(d\,x)\,\sinh(c) + \frac{b\,(2\sinh(c+d\,x) + d^2\,x^2\sinh(c+d\,x) - 2\,d\,x\cosh(c+d\,x))}{d^3}$$

input
```
int((cosh(c + d*x)*(a + b*x^3))/x,x)
```

output
```
a*coshint(d*x)*cosh(c) + a*sinhint(d*x)*sinh(c) + (b*(2*sinh(c + d*x) + d^
2*x^2*sinh(c + d*x) - 2*d*x*cosh(c + d*x)))/d^3
```

Reduce [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x}\,dx$$
$$= \frac{-2\cosh\left(dx+c\right)bdx+\left(\int \frac{\cosh(dx+c)}{x}dx\right)a\,d^3+\sinh\left(dx+c\right)b\,d^2x^2+2b\sinh\left(dx+c\right)}{d^3}$$

input
```
int((b*x^3+a)*cosh(d*x+c)/x,x)
```

output
```
( - 2*cosh(c + d*x)*b*d*x + int(cosh(c + d*x)/x,x)*a*d**3 + sinh(c + d*x)*
b*d**2*x**2 + 2*sinh(c + d*x)*b)/d**3
```

3.84 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 55

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{b\cosh(c+dx)}{d^2} - \frac{a\cosh(c+dx)}{x} + ad\text{Chi}(dx)\sinh(c) + \frac{bx\sinh(c+dx)}{d} + ad\cosh(c)\text{Shi}(dx)$$

output
```
-b*cosh(d*x+c)/d^2-a*cosh(d*x+c)/x+a*d*Chi(d*x)*sinh(c)+b*x*sinh(d*x+c)/d+
a*d*cosh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{b\cosh(c+dx)}{d^2} - \frac{a\cosh(c+dx)}{x} + ad\text{Chi}(dx)\sinh(c) + \frac{bx\sinh(c+dx)}{d} + ad\cosh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

output
```
-((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sin
h[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^2}+bx\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$ad\sinh(c)\text{Chi}(dx)+ad\cosh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{x}-\frac{b\cosh(c+dx)}{d^2}+\frac{bx\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)*Cosh[c + d*x])/x^2,x]
```

output
```
-((b*Cosh[c + d*x])/d^2) - (a*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*Sin
h[c] + (b*x*Sinh[c + d*x])/d + a*d*Cosh[c]*SinhIntegral[d*x]
```


Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(55) = 110$.

Time = 0.55 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

method	result
risch	$-\frac{e^{c}\,\mathrm{expIntegral}_1(-dx)a\,d^3x-e^{-c}\,\mathrm{expIntegral}_1(dx)a\,d^3x-bd\,x^2e^{dx+c}+bd\,x^2e^{-dx-c}+e^{dx+c}a\,d^2+e^{-dx-c}a\,d^2+e^{dx+c}bx+e^{-dx-c}}{2d^2x}$
meijerg	$-\frac{2b\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+\frac{ia\cosh(c)\sqrt{\pi}\,d\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\mathrm{Shi}(dx)}{\sqrt{\pi}}\right)}{4}$

input
```
int((b*x^3+a)*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2/d^2*(exp(c)*Ei(1,-d*x)*a*d^3*x-exp(-c)*Ei(1,d*x)*a*d^3*x-b*d*x^2*exp(
d*x+c)+b*d*x^2*exp(-d*x-c)+exp(d*x+c)*a*d^2+exp(-d*x-c)*a*d^2+exp(d*x+c)*b
*x+exp(-d*x-c)*b*x)/x
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx$$
$$=\frac{2\,bdx^2\sinh\left(dx+c\right)-2\left(ad^2+bx\right)\cosh\left(dx+c\right)+\left(ad^3x\mathrm{Ei}(dx)-ad^3x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+\left(ad^3x\mathrm{Ei}(dx)\right.}{2\,d^2x}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
1/2*(2*b*d*x^2*sinh(d*x + c) - 2*(a*d^2 + b*x)*cosh(d*x + c) + (a*d^3*x*Ei
(d*x) - a*d^3*x*Ei(-d*x))*cosh(c) + (a*d^3*x*Ei(d*x) + a*d^3*x*Ei(-d*x))*s
inh(c))/(d^2*x)
```

Sympy [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx^3)\cosh{(c+dx)}}{x^2}\,dx$$

input
```
integrate((b*x**3+a)*cosh(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**3)*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.85

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{1}{4}\left(2\,a\mathrm{Ei}(-dx)\,e^{(-c)} - 2\,a\mathrm{Ei}(dx)\,e^{c} + \frac{(d^2x^2e^c - 2\,dxe^c + 2\,e^c)be^{(dx)}}{d^3} + \frac{(d^2x^2 + 2\,dx + 2)be^{(-dx-c)}}{d^3}\right)d + \frac{1}{2}\left(bx^2 - \frac{2\,a}{x}\right)\cosh{(dx+c)}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
-1/4*(2*a*Ei(-d*x)*e^(-c) - 2*a*Ei(d*x)*e^c + (d^2*x^2*e^c - 2*d*x*e^c + 2
*e^c)*b*e^(d*x)/d^3 + (d^2*x^2 + 2*d*x + 2)*b*e^(-d*x - c)/d^3)*d + 1/2*(b
*x^2 - 2*a/x)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(55) = 110.

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 2.02

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = -\frac{ad^3x\mathrm{Ei}(-dx)\,e^{(-c)} - ad^3x\mathrm{Ei}(dx)\,e^{c} - bdx^2e^{(dx+c)} + bdx^2e^{(-dx-c)} + ad^2e^{(dx+c)} + ad^2e^{(-dx-c)} + bxe^{(dx+}}{2\,d^2x}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^2,x, algorithm="giac")
```

output
```
-1/2*(a*d^3*x*Ei(-d*x)*e^(-c) - a*d^3*x*Ei(d*x)*e^c - b*d*x^2*e^(d*x + c)
+ b*d*x^2*e^(-d*x - c) + a*d^2*e^(d*x + c) + a*d^2*e^(-d*x - c) + b*x*e^(d
*x + c) + b*x*e^(-d*x - c))/(d^2*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^2}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3))/x^2,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3))/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.13

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^2}\,dx = \frac{-e^{dx}ei(-dx)\,a\,d^3x+e^{dx+2c}ei(dx)\,a\,d^3x-e^{2dx+2c}a\,d^2+e^{2dx+2c}bd\,x^2-e^{2dx+2c}bx-a\,d^2-bd\,x^2-bx}{2e^{dx+c}d^2x}$$

input
```
int((b*x^3+a)*cosh(d*x+c)/x^2,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a*d**3*x + e**(2*c + d*x)*ei(d*x)*a*d**3*x - e**(2
*c + 2*d*x)*a*d**2 + e**(2*c + 2*d*x)*b*d*x**2 - e**(2*c + 2*d*x)*b*x - a*
d**2 - b*d*x**2 - b*x)/(2*e**(c + d*x)*d**2*x)
```

3.85 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^3} dx$

Optimal result

Integrand size = 17, antiderivative size = 69

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3} dx = -\frac{a\cosh(c+dx)}{2x^2} + \frac{1}{2}ad^2\cosh(c)\text{Chi}(dx) + \frac{b\sinh(c+dx)}{d} - \frac{ad\sinh(c+dx)}{2x} + \frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)$$

output
```
-1/2*a*cosh(d*x+c)/x^2+1/2*a*d^2*cosh(c)*Chi(d*x)+b*sinh(d*x+c)/d-1/2*a*d*
sinh(d*x+c)/x+1/2*a*d^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3} dx = \frac{b\cosh(dx)\sinh(c)}{d} - \frac{a\cosh(dx)(\cosh(c)+dx\sinh(c))}{2x^2} + \frac{b\cosh(c)\sinh(dx)}{d} - \frac{a(dx\cosh(c)+\sinh(c))\sinh(dx)}{2x^2} + \frac{1}{2}ad^2(\cosh(c)\text{Chi}(dx)+\sinh(c)\text{Shi}(dx))$$

input
```
Integrate[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

output
```
(b*Cosh[d*x]*Sinh[c])/d - (a*Cosh[d*x]*(Cosh[c] + d*x*Sinh[c]))/(2*x^2) +
(b*Cosh[c]*Sinh[d*x])/d - (a*(d*x*Cosh[c] + Sinh[c])*Sinh[d*x])/(2*x^2) +
(a*d^2*(Cosh[c]*CoshIntegral[d*x] + Sinh[c]*SinhIntegral[d*x]))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^3}+b\cosh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}ad^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}ad^2\sinh(c)\text{Shi}(dx)-\frac{a\cosh(c+dx)}{2x^2}-\frac{ad\sinh(c+dx)}{2x}+\frac{b\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)*Cosh[c + d*x])/x^3,x]
```

output
```
-1/2*(a*Cosh[c + d*x])/x^2 + (a*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (b*Sinh
[c + d*x])/d - (a*d*Sinh[c + d*x])/(2*x) + (a*d^2*Sinh[c]*SinhIntegral[d*x
])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.72

method	result
risch	$-\frac{e^{c} \operatorname{expIntegral}_{1}(-dx) a\, d^{3} x^{2}+e^{-c} \operatorname{expIntegral}_{1}(dx) a\, d^{3} x^{2}+e^{dx+c} a\, d^{2} x-e^{-dx-c} a\, d^{2} x-2\, e^{dx+c} b\, x^{2}+2\, e^{-dx-c} b\, x^{2}+d\, e^{dx+c} a+d}{4 d\, x^{2}}$
meijerg	$\frac{b \cosh(c) \sinh(dx)}{d}-\frac{b \sinh(c) \sqrt{\pi}\, \left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(dx)}{\sqrt{\pi}}\right)}{d}-\frac{a \cosh(c) \sqrt{\pi}\, d^{2} \left(-\frac{4 \left(\frac{9 x^{2} d^{2}}{2}+3\right)}{3 \sqrt{\pi}\, x^{2} d^{2}}+\frac{4 \cosh(dx)}{\sqrt{\pi}\, x^{2} d^{2}}+\frac{4 \sinh(dx)}{\sqrt{\pi}\, x d}-\frac{4(\mathrm{Chi}(dx)-\ln(dx)-}{\sqrt{\pi}}\right.}{8}$

input
```
int((b*x^3+a)*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*(exp(c)*Ei(1,-d*x)*a*d^3*x^2+exp(-c)*Ei(1,d*x)*a*d^3*x^2+exp(d*x+c)
*a*d^2*x-exp(-d*x-c)*a*d^2*x-2*exp(d*x+c)*b*x^2+2*exp(-d*x-c)*b*x^2+d*exp(
d*x+c)*a+d*exp(-d*x-c)*a)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.46

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = -\frac{2\,ad\cosh\left(dx+c\right)-\left(ad^3x^2\mathrm{Ei}(dx)+ad^3x^2\mathrm{Ei}(-dx)\right)\cosh\left(c\right)+2\left(ad^2x-2\,bx^2\right)\sinh\left(dx+c\right)-\left(ad^3x\right.}{4\,dx^2}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*a*d*cosh(d*x + c) - (a*d^3*x^2*Ei(d*x) + a*d^3*x^2*Ei(-d*x))*cosh(
c) + 2*(a*d^2*x - 2*b*x^2)*sinh(d*x + c) - (a*d^3*x^2*Ei(d*x) - a*d^3*x^2*
Ei(-d*x))*sinh(c))/(d*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = \int \frac{(a+bx^3)\cosh{(c+dx)}}{x^3}\,dx$$

input
```
integrate((b*x**3+a)*cosh(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**3)*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.26

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{1}{4}\left(ade^{(-c)}\Gamma(-1,dx) + ade^{c}\Gamma(-1,-dx) - \frac{2\,(dxe^{c}-e^{c})be^{(dx)}}{d^2} - \frac{2\,(dx+1)be^{(-dx-c)}}{d^2}\right)d$$
$$+ \frac{1}{2}\left(2\,bx - \frac{a}{x^2}\right)\cosh{(dx+c)}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/4*(a*d*e^(-c)*gamma(-1, d*x) + a*d*e^c*gamma(-1, -d*x) - 2*(d*x*e^c - e^
c)*b*e^(d*x)/d^2 - 2*(d*x + 1)*b*e^(-d*x - c)/d^2)*d + 1/2*(2*b*x - a/x^2)
*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.71

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{ad^3x^2\text{Ei}(-dx)\,e^{(-c)} + ad^3x^2\text{Ei}(dx)\,e^c - ad^2xe^{(dx+c)} + ad^2xe^{(-dx-c)} + 2\,bx^2e^{(dx+c)} - 2\,bx^2e^{(-dx-c)} - ade^{(}}{4\,dx^2}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a*d^3*x^2*Ei(-d*x)*e^(-c) + a*d^3*x^2*Ei(d*x)*e^c - a*d^2*x*e^(d*x +
c) + a*d^2*x*e^(-d*x - c) + 2*b*x^2*e^(d*x + c) - 2*b*x^2*e^(-d*x - c) - a
*d*e^(d*x + c) - a*d*e^(-d*x - c))/(d*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^3}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3))/x^3,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3))/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^3}\,dx$$
$$= \frac{-\cosh(dx+c)\,ad + \left(\int \frac{\cosh(dx+c)}{x}dx\right) a\,d^3x^2 - \sinh(dx+c)\,a\,d^2x + 2\sinh(dx+c)\,b\,x^2}{2d\,x^2}$$

input
```
int((b*x^3+a)*cosh(d*x+c)/x^3,x)
```

output
```
( - cosh(c + d*x)*a*d + int(cosh(c + d*x)/x,x)*a*d**3*x**2 - sinh(c + d*x)
*a*d**2*x + 2*sinh(c + d*x)*b*x**2)/(2*d*x**2)
```

3.86 $\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 91

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = -\frac{a\cosh(c+dx)}{3x^3}-\frac{ad^2\cosh(c+dx)}{6x}+b\cosh(c)\text{Chi}(dx)+\frac{1}{6}ad^3\text{Chi}(dx)\sinh(c)-\frac{ad\sinh(c+dx)}{6x^2}+\frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)+b\sinh(c)\text{Shi}(dx)$$

output
```
-1/3*a*cosh(d*x+c)/x^3-1/6*a*d^2*cosh(d*x+c)/x+b*cosh(c)*Chi(d*x)+1/6*a*d^
3*Chi(d*x)*sinh(c)-1/6*a*d*sinh(d*x+c)/x^2+1/6*a*d^3*cosh(c)*Shi(d*x)+b*si
nh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.80

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(\text{Chi}(dx)\left(6b\cosh(c)+ad^3\sinh(c)\right)-\frac{a((2+d^2x^2)\cosh(c+dx)+dx\sinh(c+dx))}{x^3}+\left(ad^3\cosh(c)+6b\sinh(c)\right)\text{Shi}(dx)\right)$$

input
```
Integrate[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

output
```
(CoshIntegral[d*x]*(6*b*Cosh[c] + a*d^3*Sinh[c]) - (a*((2 + d^2*x^2)*Cosh[
c + d*x] + d*x*Sinh[c + d*x]))/x^3 + (a*d^3*Cosh[c] + 6*b*Sinh[c])*SinhInt
egral[d*x])/6
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.118, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a\cosh(c+dx)}{x^4}+\frac{b\cosh(c+dx)}{x}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{6}ad^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}ad^3\cosh(c)\text{Shi}(dx)-\frac{ad^2\cosh(c+dx)}{6x}-\frac{a\cosh(c+dx)}{3x^3}-\frac{ad\sinh(c+dx)}{6x^2}+b\cosh(c)\text{Chi}(dx)+b\sinh(c)\text{Shi}(dx)$$

input
```
Int[((a + b*x^3)*Cosh[c + d*x])/x^4,x]
```

output
```
-1/3*(a*Cosh[c + d*x])/x^3 - (a*d^2*Cosh[c + d*x])/(6*x) + b*Cosh[c]*CoshI
ntegral[d*x] + (a*d^3*CoshIntegral[d*x]*Sinh[c])/6 - (a*d*Sinh[c + d*x])/(
6*x^2) + (a*d^3*Cosh[c]*SinhIntegral[d*x])/6 + b*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.60

method	result
risch	$-\frac{-\mathrm{e}^{-c} \operatorname{expIntegral}_{1}(dx) a\, d^{3} x^{3}+\mathrm{e}^{c} \operatorname{expIntegral}_{1}(-dx) a\, d^{3} x^{3}+a\, d^{2} x^{2} \mathrm{e}^{dx+c}+a\, d^{2} x^{2} \mathrm{e}^{-dx-c}+6\, \mathrm{e}^{-c} \operatorname{expIntegral}_{1}(dx) b\, x^{3}+6\, \mathrm{e}^{c} \operatorname{exp}}{12x^{3}}$
meijerg	$\frac{b \cosh(c) \sqrt{\pi}\, \left(\frac{2\ \operatorname{Chi}(dx)-2\ln(dx)-2\gamma}{\sqrt{\pi}}+\frac{2\gamma+2\ln(x)+2\ln(id)}{\sqrt{\pi}}\right)}{2}+b\sinh\left(c\right)\operatorname{Shi}\left(dx\right)-\frac{ia\cosh(c)\sqrt{\pi}\, d^{3}\left(-\frac{8i\left(x^{2}d^{2}+2\right)\cosh(dx)}{3d^{3}x^{3}\sqrt{\pi}}\right.}{16}$

input
```
int((b*x^3+a)*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12*(-exp(-c)*Ei(1,d*x)*a*d^3*x^3+exp(c)*Ei(1,-d*x)*a*d^3*x^3+a*d^2*x^2*
exp(d*x+c)+a*d^2*x^2*exp(-d*x-c)+6*exp(-c)*Ei(1,d*x)*b*x^3+6*exp(c)*Ei(1,-
d*x)*b*x^3+a*d*x*exp(d*x+c)-a*d*x*exp(-d*x-c)+2*exp(d*x+c)*a+2*exp(-d*x-c)
*a)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.30

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{2\,adx\sinh\left(dx+c\right)+2\left(ad^2x^2+2\,a\right)\cosh\left(dx+c\right)-\left(\left(ad^3+6\,b\right)x^3\mathrm{Ei}(dx)-\left(ad^3-6\,b\right)x^3\mathrm{Ei}(-dx)\right)\mathrm{c}}{12\,x^3}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(2*a*d*x*sinh(d*x + c) + 2*(a*d^2*x^2 + 2*a)*cosh(d*x + c) - ((a*d^3
 + 6*b)*x^3*Ei(d*x) - (a*d^3 - 6*b)*x^3*Ei(-d*x))*cosh(c) - ((a*d^3 + 6*b)
*x^3*Ei(d*x) + (a*d^3 - 6*b)*x^3*Ei(-d*x))*sinh(c))/x^3
```

Sympy [F]

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx$$

input
```
integrate((b*x**3+a)*cosh(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**3)*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.04

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma(-2,dx) - d^2e^c\Gamma(-2,-dx)\right)a - \frac{2\,b\cosh(dx+c)\log(x^3)}{d} + \frac{3\left(\mathrm{Ei}(-dx)\,e^{(-c)} + \mathrm{Ei}(dx)\,e^c\right)b}{d} \right. + \frac{1}{3}\left(b\log(x^3) - \frac{a}{x^3}\right)\cosh(dx+c)$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a - 2*b*cosh(d*
x + c)*log(x^3)/d + 3*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*b/d)*d + 1/3*(b*log(
x^3) - a/x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.55

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = -\frac{ad^3x^3\text{Ei}(-dx)\,e^{(-c)} - ad^3x^3\text{Ei}(dx)\,e^{c} + ad^2x^2e^{(dx+c)} + ad^2x^2e^{(-dx-c)} - 6\,bx^3\text{Ei}(-dx)\,e^{(-c)} - 6\,bx^3\text{Ei}(}{12\,x^3}$$

input
```
integrate((b*x^3+a)*cosh(d*x+c)/x^4,x, algorithm="giac")
```

output
```
-1/12*(a*d^3*x^3*Ei(-d*x)*e^(-c) - a*d^3*x^3*Ei(d*x)*e^c + a*d^2*x^2*e^(d*
x + c) + a*d^2*x^2*e^(-d*x - c) - 6*b*x^3*Ei(-d*x)*e^(-c) - 6*b*x^3*Ei(d*x
)*e^c + a*d*x*e^(d*x + c) - a*d*x*e^(-d*x - c) + 2*a*e^(d*x + c) + 2*a*e^(
-d*x - c))/x^3
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)}{x^4}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3))/x^4,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3))/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.68

$$\int \frac{(a+bx^3)\cosh(c+dx)}{x^4}\,dx = \frac{-e^{dx}\,ei(-dx)\,a\,d^3x^3 + 6e^{dx}\,ei(-dx)\,b\,x^3 + e^{dx+2c}\,ei(dx)\,a\,d^3x^3 + 6e^{dx+2c}\,ei(dx)\,b\,x^3 - e^{2dx+2c}a\,d^2x^2 - e^{2dx}}{12e^{dx+c}x^3}$$

input
```
int((b*x^3+a)*cosh(d*x+c)/x^4,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a*d**3*x**3 + 6*e**(d*x)*ei( - d*x)*b*x**3 + e**(2
*c + d*x)*ei(d*x)*a*d**3*x**3 + 6*e**(2*c + d*x)*ei(d*x)*b*x**3 - e**(2*c
+ 2*d*x)*a*d**2*x**2 - e**(2*c + 2*d*x)*a*d*x - 2*e**(2*c + 2*d*x)*a - a*d
**2*x**2 + a*d*x - 2*a)/(12*e**(c + d*x)*x**3)
```

3.87 $\int x(a+bx^3)^2\cosh(c+dx)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 234

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx = -\frac{5040b^2\cosh(c+dx)}{d^8}-\frac{a^2\cosh(c+dx)}{d^2} -\frac{48abx\cosh(c+dx)}{d^4}-\frac{2520b^2x^2\cosh(c+dx)}{d^6} -\frac{8abx^3\cosh(c+dx)}{d^2}-\frac{210b^2x^4\cosh(c+dx)}{d^4} -\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{48ab\sinh(c+dx)}{d^5} +\frac{5040b^2x\sinh(c+dx)}{d^7} +\frac{a^2x\sinh(c+dx)}{d}+\frac{24abx^2\sinh(c+dx)}{d^3} +\frac{840b^2x^3\sinh(c+dx)}{d^5}+\frac{2abx^4\sinh(c+dx)}{d} +\frac{42b^2x^5\sinh(c+dx)}{d^3}+\frac{b^2x^7\sinh(c+dx)}{d}$$

output
```
-5040*b^2*cosh(d*x+c)/d^8-a^2*cosh(d*x+c)/d^2-48*a*b*x*cosh(d*x+c)/d^4-252
0*b^2*x^2*cosh(d*x+c)/d^6-8*a*b*x^3*cosh(d*x+c)/d^2-210*b^2*x^4*cosh(d*x+c
)/d^4-7*b^2*x^6*cosh(d*x+c)/d^2+48*a*b*sinh(d*x+c)/d^5+5040*b^2*x*sinh(d*x
+c)/d^7+a^2*x*sinh(d*x+c)/d+24*a*b*x^2*sinh(d*x+c)/d^3+840*b^2*x^3*sinh(d*
x+c)/d^5+2*a*b*x^4*sinh(d*x+c)/d+42*b^2*x^5*sinh(d*x+c)/d^3+b^2*x^7*sinh(d
*x+c)/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.59

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx = \frac{-((a^2d^6+8abd^4x(6+d^2x^2)+7b^2(720+360d^2x^2+30d^4x^4+d^6x^6))\cosh(c+dx))+d(a^2d^6x+2abd^2(2}{d^8}$$

input
```
Integrate[x*(a + b*x^3)^2*Cosh[c + d*x],x]
```

output
```
(-((a^2*d^6 + 8*a*b*d^4*x*(6 + d^2*x^2) + 7*b^2*(720 + 360*d^2*x^2 + 30*d^
4*x^4 + d^6*x^6))*Cosh[c + d*x]) + d*(a^2*d^6*x + 2*a*b*d^2*(24 + 12*d^2*x
^2 + d^4*x^4) + b^2*x*(5040 + 840*d^2*x^2 + 42*d^4*x^4 + d^6*x^6))*Sinh[c
+ d*x])/d^8
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx$$

$$\Big\downarrow \; 5810$$

$$\int \left(a^2 x \cosh(c+dx) + 2abx^4 \cosh(c+dx) + b^2 x^7 \cosh(c+dx)\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{a^2\cosh(c+dx)}{d^2}+\frac{a^2x\sinh(c+dx)}{d}+\frac{48ab\sinh(c+dx)}{d^5}-\frac{48abx\cosh(c+dx)}{d^4}+\frac{24abx^2\sinh(c+dx)}{d^3}-\frac{8abx^3\cosh(c+dx)}{d^2}+\frac{2abx^4\sinh(c+dx)}{d}-\frac{5040b^2\cosh(c+dx)}{d^8}+\frac{5040b^2x\sinh(c+dx)}{d^7}-\frac{2520b^2x^2\cosh(c+dx)}{d^6}+\frac{840b^2x^3\sinh(c+dx)}{d^5}-\frac{210b^2x^4\cosh(c+dx)}{d^4}+\frac{42b^2x^5\sinh(c+dx)}{d^3}-\frac{7b^2x^6\cosh(c+dx)}{d^2}+\frac{b^2x^7\sinh(c+dx)}{d}$$

input
```
Int[x*(a + b*x^3)^2*Cosh[c + d*x],x]
```

output
```
(-5040*b^2*Cosh[c + d*x])/d^8 - (a^2*Cosh[c + d*x])/d^2 - (48*a*b*x*Cosh[c
 + d*x])/d^4 - (2520*b^2*x^2*Cosh[c + d*x])/d^6 - (8*a*b*x^3*Cosh[c + d*x]
)/d^2 - (210*b^2*x^4*Cosh[c + d*x])/d^4 - (7*b^2*x^6*Cosh[c + d*x])/d^2 +
(48*a*b*Sinh[c + d*x])/d^5 + (5040*b^2*x*Sinh[c + d*x])/d^7 + (a^2*x*Sinh[
c + d*x])/d + (24*a*b*x^2*Sinh[c + d*x])/d^3 + (840*b^2*x^3*Sinh[c + d*x])
/d^5 + (2*a*b*x^4*Sinh[c + d*x])/d + (42*b^2*x^5*Sinh[c + d*x])/d^3 + (b^2
*x^7*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.80 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.88

method	result
parallelrisch	$\frac{8bd^2\left(x^2\left(\frac{7bx^3}{8}+a\right)d^4+3\left(\frac{35bx^3}{4}+2a\right)d^2+315bx\right)x\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-2\left(x\left(bx^3+a\right)^2d^6+6\left(7b^2x^5+4abx^2\right)d^4+24\left(35x^3\right.\right.}{d^8\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}$
orering	$-\frac{2\left(7b^3d^6x^{10}+15ab^2d^6x^7+252b^3d^4x^8+9a^2bd^6x^4+234ab^2d^4x^5+4200b^3d^2x^6+a^3d^6x+36a^2bd^4x^2+1848ab^2d^2x^3+2\right.}{d^8x\left(bx^3+a\right)}$
risch	$\frac{\left(b^2x^7d^7-7b^2x^6d^6+2abd^7x^4+42b^2x^5d^5-8abd^6x^3+a^2d^7x-210b^2x^4d^4+24abd^5x^2-a^2d^6+840b^2d^3x^3-48abd^4x-252\right.}{2d^8}$
meijerg	$\frac{128b^2\cosh(c)\sqrt{\pi}\left(\frac{315}{8\sqrt{\pi}}-\frac{\left(\frac{7}{16}x^6d^6+\frac{105}{8}d^4x^4+\frac{315}{2}x^2d^2+315\right)\cosh(dx)}{8\sqrt{\pi}}+\frac{xd\left(\frac{1}{16}x^6d^6+\frac{21}{8}d^4x^4+\frac{105}{2}x^2d^2+315\right)\sinh(dx)}{8\sqrt{\pi}}\right)}{d^8}-$
parts	$\frac{b^2x^7\sinh(dx+c)}{d}+\frac{2abx^4\sinh(dx+c)}{d}+\frac{a^2x\sinh(dx+c)}{d}-\frac{\frac{7b^2c^6\cosh(dx+c)}{d^6}+\frac{7b^2\left((dx+c)^6\cosh(dx+c)-6(dx+c)^5\mathrm{si}\right.}{}}{}$
derivativedivides	Expression too large to display
default	Expression too large to display

input
```
int(x*(b*x^3+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
(8*b*d^2*(x^2*(7/8*b*x^3+a)*d^4+3*(35/4*b*x^3+2*a)*d^2+315*b*x)*x*tanh(1/2
*d*x+1/2*c)^2-2*(x*(b*x^3+a)^2*d^6+6*(7*b^2*x^5+4*a*b*x^2)*d^4+24*(35*b^2*
x^3+2*a*b)*d^2+5040*b^2*x)*d*tanh(1/2*d*x+1/2*c)+(7*b^2*x^6+8*a*b*x^3+2*a^
2)*d^6+6*(35*b^2*x^4+8*a*b*x)*d^4+2520*b^2*d^2*x^2+10080*b^2)/d^8/(tanh(1/
2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.69

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx = -\frac{\left(7\,b^2d^6x^6+8\,abd^6x^3+210\,b^2d^4x^4+a^2d^6+48\,abd^4x+2520\,b^2d^2x^2+5040\,b^2\right)\cosh\left(dx+c\right)-\left(b^2d^7x^7\right.}{d^8}$$

input
```
integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-((7*b^2*d^6*x^6 + 8*a*b*d^6*x^3 + 210*b^2*d^4*x^4 + a^2*d^6 + 48*a*b*d^4*
x + 2520*b^2*d^2*x^2 + 5040*b^2)*cosh(d*x + c) - (b^2*d^7*x^7 + 2*a*b*d^7*
x^4 + 42*b^2*d^5*x^5 + 24*a*b*d^5*x^2 + 840*b^2*d^3*x^3 + 48*a*b*d^3 + (a^
2*d^7 + 5040*b^2*d)*x)*sinh(d*x + c))/d^8
```

Sympy [A] (verification not implemented)

Time = 0.94 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.21

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx$$
$$=\begin{cases}\frac{a^2x\sinh{(c+dx)}}{d}-\frac{a^2\cosh{(c+dx)}}{d^2}+\frac{2abx^4\sinh{(c+dx)}}{d}-\frac{8abx^3\cosh{(c+dx)}}{d^2}+\frac{24abx^2\sinh{(c+dx)}}{d^3}-\frac{48abx\cosh{(c+dx)}}{d^4}+\frac{48ab\sinh}{} \\ \left(\frac{a^2x^2}{2}+\frac{2abx^5}{5}+\frac{b^2x^8}{8}\right)\cosh{(c)}\end{cases}$$

input
```
integrate(x*(b*x**3+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*x*sinh(c + d*x)/d - a**2*cosh(c + d*x)/d**2 + 2*a*b*x**4*s
inh(c + d*x)/d - 8*a*b*x**3*cosh(c + d*x)/d**2 + 24*a*b*x**2*sinh(c + d*x)
/d**3 - 48*a*b*x*cosh(c + d*x)/d**4 + 48*a*b*sinh(c + d*x)/d**5 + b**2*x**
7*sinh(c + d*x)/d - 7*b**2*x**6*cosh(c + d*x)/d**2 + 42*b**2*x**5*sinh(c +
 d*x)/d**3 - 210*b**2*x**4*cosh(c + d*x)/d**4 + 840*b**2*x**3*sinh(c + d*x
)/d**5 - 2520*b**2*x**2*cosh(c + d*x)/d**6 + 5040*b**2*x*sinh(c + d*x)/d**
7 - 5040*b**2*cosh(c + d*x)/d**8, Ne(d, 0)), ((a**2*x**2/2 + 2*a*b*x**5/5
+ b**2*x**8/8)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 383, normalized size of antiderivative = 1.64

$$\int x\left(a+bx^3\right)^2\cosh(c+dx)\,dx =$$
$$-\frac{1}{80}\,d\left(\frac{20\,(d^2x^2e^c-2\,dxe^c+2\,e^c)a^2e^{(dx)}}{d^3}+\frac{20\,(d^2x^2+2\,dx+2)a^2e^{(-dx-c)}}{d^3}+\frac{16\,(d^5x^5e^c-5\,d^4x^4e^c+2}{}\right.$$
$$+\frac{1}{40}\left(5\,b^2x^8+16\,abx^5+20\,a^2x^2\right)\cosh\left(dx+c\right)$$

input
```
integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
-1/80*d*(20*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*a^2*e^(d*x)/d^3 + 20*(d^2*x^
2 + 2*d*x + 2)*a^2*e^(-d*x - c)/d^3 + 16*(d^5*x^5*e^c - 5*d^4*x^4*e^c + 20
*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d*x*e^c - 120*e^c)*a*b*e^(d*x)/d^6 + 1
6*(d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60*d^2*x^2 + 120*d*x + 120)*a*b*e^(-
d*x - c)/d^6 + 5*(d^8*x^8*e^c - 8*d^7*x^7*e^c + 56*d^6*x^6*e^c - 336*d^5*x
^5*e^c + 1680*d^4*x^4*e^c - 6720*d^3*x^3*e^c + 20160*d^2*x^2*e^c - 40320*d
*x*e^c + 40320*e^c)*b^2*e^(d*x)/d^9 + 5*(d^8*x^8 + 8*d^7*x^7 + 56*d^6*x^6
+ 336*d^5*x^5 + 1680*d^4*x^4 + 6720*d^3*x^3 + 20160*d^2*x^2 + 40320*d*x +
40320)*b^2*e^(-d*x - c)/d^9) + 1/40*(5*b^2*x^8 + 16*a*b*x^5 + 20*a^2*x^2)*
cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.29

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx$$
$$= \frac{(b^2d^7x^7 - 7\,b^2d^6x^6 + 2\,abd^7x^4 + 42\,b^2d^5x^5 - 8\,abd^6x^3 + a^2d^7x - 210\,b^2d^4x^4 + 24\,abd^5x^2 - a^2d^6 + 840\,b^2 \cdots}{2\,d^8}$$
$$- \frac{(b^2d^7x^7 + 7\,b^2d^6x^6 + 2\,abd^7x^4 + 42\,b^2d^5x^5 + 8\,abd^6x^3 + a^2d^7x + 210\,b^2d^4x^4 + 24\,abd^5x^2 + a^2d^6 + 840 \cdots}{2\,d^8}$$

input
```
integrate(x*(b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^7*x^7 - 7*b^2*d^6*x^6 + 2*a*b*d^7*x^4 + 42*b^2*d^5*x^5 - 8*a*b*
d^6*x^3 + a^2*d^7*x - 210*b^2*d^4*x^4 + 24*a*b*d^5*x^2 - a^2*d^6 + 840*b^2
*d^3*x^3 - 48*a*b*d^4*x - 2520*b^2*d^2*x^2 + 48*a*b*d^3 + 5040*b^2*d*x - 5
040*b^2)*e^(d*x + c)/d^8 - 1/2*(b^2*d^7*x^7 + 7*b^2*d^6*x^6 + 2*a*b*d^7*x^
4 + 42*b^2*d^5*x^5 + 8*a*b*d^6*x^3 + a^2*d^7*x + 210*b^2*d^4*x^4 + 24*a*b*
d^5*x^2 + a^2*d^6 + 840*b^2*d^3*x^3 + 48*a*b*d^4*x + 2520*b^2*d^2*x^2 + 48
*a*b*d^3 + 5040*b^2*d*x + 5040*b^2)*e^(-d*x - c)/d^8
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 301, normalized size of antiderivative = 1.29

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx = -e^{c+dx}\left(\frac{a^2d^6-48abd^3+5040b^2}{2d^8}-\frac{x(a^2d^7-48abd^4+5040b^2d)}{2d^8}-\frac{b^2x^7}{2d}+\frac{7b^2x^6}{2d^2}-\frac{21b^2x^5}{d^3}+\frac{bx^4(105b-ad^3)}{d^4}-\frac{4bx^3(105b-ad^3)}{d^5}+\frac{12bx^2(105b-ad^3)}{d^6}\right)-e^{-c-dx}\left(\frac{a^2d^6+48abd^3+5040b^2}{2d^8}+\frac{x(a^2d^7+48abd^4+5040b^2d)}{2d^8}+\frac{b^2x^7}{2d}+\frac{7b^2x^6}{2d^2}+\frac{21b^2x^5}{d^3}+\frac{bx^4(ad^3+105b)}{d^4}+\frac{4bx^3(ad^3+105b)}{d^5}+\frac{12bx^2(ad^3+105b)}{d^6}\right)$$

input
```
int(x*cosh(c + d*x)*(a + b*x^3)^2,x)
```

output
```
- exp(c + d*x)*((5040*b^2 + a^2*d^6 - 48*a*b*d^3)/(2*d^8) - (x*(5040*b^2*d
 + a^2*d^7 - 48*a*b*d^4))/(2*d^8) - (b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2)
- (21*b^2*x^5)/d^3 + (b*x^4*(105*b - a*d^3))/d^4 - (4*b*x^3*(105*b - a*d^3
))/d^5 + (12*b*x^2*(105*b - a*d^3))/d^6) - exp(- c - d*x)*((5040*b^2 + a^2
*d^6 + 48*a*b*d^3)/(2*d^8) + (x*(5040*b^2*d + a^2*d^7 + 48*a*b*d^4))/(2*d^
8) + (b^2*x^7)/(2*d) + (7*b^2*x^6)/(2*d^2) + (21*b^2*x^5)/d^3 + (b*x^4*(10
5*b + a*d^3))/d^4 + (4*b*x^3*(105*b + a*d^3))/d^5 + (12*b*x^2*(105*b + a*d
^3))/d^6)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.00

$$\int x(a+bx^3)^2\cosh(c+dx)\,dx$$
$$=\frac{-\cosh(dx+c)\,a^2d^6-8\cosh(dx+c)\,ab\,d^6x^3-48\cosh(dx+c)\,ab\,d^4x-7\cosh(dx+c)\,b^2d^6x^6-210\,\mathrm{c}}{}$$

input
```
int(x*(b*x^3+a)^2*cosh(d*x+c),x)
```

output
```
( - cosh(c + d*x)*a**2*d**6 - 8*cosh(c + d*x)*a*b*d**6*x**3 - 48*cosh(c +
d*x)*a*b*d**4*x - 7*cosh(c + d*x)*b**2*d**6*x**6 - 210*cosh(c + d*x)*b**2*
d**4*x**4 - 2520*cosh(c + d*x)*b**2*d**2*x**2 - 5040*cosh(c + d*x)*b**2 +
sinh(c + d*x)*a**2*d**7*x + 2*sinh(c + d*x)*a*b*d**7*x**4 + 24*sinh(c + d*
x)*a*b*d**5*x**2 + 48*sinh(c + d*x)*a*b*d**3 + sinh(c + d*x)*b**2*d**7*x**
7 + 42*sinh(c + d*x)*b**2*d**5*x**5 + 840*sinh(c + d*x)*b**2*d**3*x**3 + 5
040*sinh(c + d*x)*b**2*d*x)/d**8
```

3.88 $\int (a+bx^3)^2 \cosh(c+dx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 186

$$\begin{aligned}\int (a+bx^3)^2 \cosh(c+dx)\,dx = &-\frac{12ab\cosh(c+dx)}{d^4}-\frac{720b^2x\cosh(c+dx)}{d^6}\\ &-\frac{6abx^2\cosh(c+dx)}{d^2}-\frac{120b^2x^3\cosh(c+dx)}{d^4}\\ &-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{720b^2\sinh(c+dx)}{d^7}\\ &+\frac{a^2\sinh(c+dx)}{d}+\frac{12abx\sinh(c+dx)}{d^3}\\ &+\frac{360b^2x^2\sinh(c+dx)}{d^5}+\frac{2abx^3\sinh(c+dx)}{d}\\ &+\frac{30b^2x^4\sinh(c+dx)}{d^3}+\frac{b^2x^6\sinh(c+dx)}{d}\end{aligned}$$

output

```
-12*a*b*cosh(d*x+c)/d^4-720*b^2*x*cosh(d*x+c)/d^6-6*a*b*x^2*cosh(d*x+c)/d^
2-120*b^2*x^3*cosh(d*x+c)/d^4-6*b^2*x^5*cosh(d*x+c)/d^2+720*b^2*sinh(d*x+c
)/d^7+a^2*sinh(d*x+c)/d+12*a*b*x*sinh(d*x+c)/d^3+360*b^2*x^2*sinh(d*x+c)/d
^5+2*a*b*x^3*sinh(d*x+c)/d+30*b^2*x^4*sinh(d*x+c)/d^3+b^2*x^6*sinh(d*x+c)/
d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.60

$$\int \left(a+bx^3\right)^2\cosh(c+dx)\,dx$$
$$=\frac{-6bd(ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\cosh(c+dx)+(a^2d^6+2abd^4x(6+d^2x^2)+b^2(720+3}{d^7}$$

input
```
Integrate[(a + b*x^3)^2*Cosh[c + d*x],x]
```

output
```
(-6*b*d*(a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d^2*x^2 + d^4*x^4))*Cosh[c +
d*x] + (a^2*d^6 + 2*a*b*d^4*x*(6 + d^2*x^2) + b^2*(720 + 360*d^2*x^2 + 30*
d^4*x^4 + d^6*x^6))*Sinh[c + d*x])/d^7
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5800, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+bx^3\right)^2\cosh(c+dx)\,dx$$
$$\Big\downarrow \; 5800$$
$$\int \left(a^2\cosh(c+dx)+2abx^3\cosh(c+dx)+b^2x^6\cosh(c+dx)\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{a^2\sinh(c+dx)}{d}-\frac{12ab\cosh(c+dx)}{d^4}+\frac{12abx\sinh(c+dx)}{d^3}-\frac{6abx^2\cosh(c+dx)}{d^2}+\frac{2abx^3\sinh(c+dx)}{d}+\frac{720b^2\sinh(c+dx)}{d^7}-\frac{720b^2x\cosh(c+dx)}{d^6}+\frac{360b^2x^2\sinh(c+dx)}{d^5}-\frac{120b^2x^3\cosh(c+dx)}{d^4}+\frac{30b^2x^4\sinh(c+dx)}{d^3}-\frac{6b^2x^5\cosh(c+dx)}{d^2}+\frac{b^2x^6\sinh(c+dx)}{d}$$

input
```
Int[(a + b*x^3)^2*Cosh[c + d*x],x]
```

output
```
(-12*a*b*Cosh[c + d*x])/d^4 - (720*b^2*x*Cosh[c + d*x])/d^6 - (6*a*b*x^2*C
osh[c + d*x])/d^2 - (120*b^2*x^3*Cosh[c + d*x])/d^4 - (6*b^2*x^5*Cosh[c +
d*x])/d^2 + (720*b^2*Sinh[c + d*x])/d^7 + (a^2*Sinh[c + d*x])/d + (12*a*b*
x*Sinh[c + d*x])/d^3 + (360*b^2*x^2*Sinh[c + d*x])/d^5 + (2*a*b*x^3*Sinh[c
 + d*x])/d + (30*b^2*x^4*Sinh[c + d*x])/d^3 + (b^2*x^6*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5800
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c,
d, n}, x] && IGtQ[p, 0]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.85

method	result
parallelrisch	$\frac{6(x(bx^3+a)d^4+20bd^2x^2+120b)bdx\tanh(\frac{dx}{2}+\frac{c}{2})^2+2(-(bx^3+a)^2d^6+6(-5b^2x^4-2abx)d^4-360b^2d^2x^2-720b^2)\tan}{d^7(\tanh(\frac{dx}{2}+\frac{c}{2})^2-1)}$
orering	$-\frac{12b(b^2d^6x^8+2abd^6x^5+25b^2d^4x^6+a^2d^6x^2+17abd^4x^3+240b^2x^4d^2+a^2d^4+60abd^2x+360x^2b^2)\cosh(dx+c)}{d^8(bx^3+a)}+\frac{(b^2}{}$
risch	$\frac{(b^2x^6d^6-6b^2x^5d^5+2abd^6x^3+30b^2x^4d^4-6abd^5x^2+a^2d^6-120b^2d^3x^3+12abd^4x+360b^2d^2x^2-12abd^3-720b^2dx+720}{2d^7}$
meijerg	$\frac{64ib^2\cosh(c)\sqrt{\pi}\left(\frac{ixd(\frac{21}{8}d^4x^4+\frac{105}{2}x^2d^2+315)\cosh(dx)}{28\sqrt{\pi}}-\frac{i(\frac{7}{16}x^6d^6+\frac{105}{8}d^4x^4+\frac{315}{2}x^2d^2+315)\sinh(dx)}{28\sqrt{\pi}}\right)}{d^7}+\frac{64b^2\sinh(c}{}$
parts	$\frac{b^2x^6\sinh(dx+c)}{d}+\frac{2abx^3\sinh(dx+c)}{d}+\frac{a^2\sinh(dx+c)}{d}-\frac{6b\left(-\frac{bc^5\cosh(dx+c)}{d^3}+\frac{5bc^4((dx+c)\cosh(dx+c)-\sinh(dx+}{d^3}\right.}{}$
derivativedivides	$\frac{b^2((dx+c)^6\sinh(dx+c)-6(dx+c)^5\cosh(dx+c)+30(dx+c)^4\sinh(dx+c)-120(dx+c)^3\cosh(dx+c)+360(dx+c)^2\sinh(dx+c)-720(dx+}{d^6}$
default	$\frac{b^2((dx+c)^6\sinh(dx+c)-6(dx+c)^5\cosh(dx+c)+30(dx+c)^4\sinh(dx+c)-120(dx+c)^3\cosh(dx+c)+360(dx+c)^2\sinh(dx+c)-720(dx+}{d^6}$

input
```
int((b*x^3+a)^2*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
2*(3*(x*(b*x^3+a)*d^4+20*b*d^2*x^2+120*b)*b*d*x*tanh(1/2*d*x+1/2*c)^2+(-(b
*x^3+a)^2*d^6+6*(-5*b^2*x^4-2*a*b*x)*d^4-360*b^2*d^2*x^2-720*b^2)*tanh(1/2
*d*x+1/2*c)+3*(x^2*(b*x^3+a)*d^4+4*(5*b*x^3+a)*d^2+120*b*x)*b*d)/d^7/(tanh
(1/2*d*x+1/2*c)^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.70

$$\int (a+bx^3)^2\cosh(c+dx)\,dx = -\frac{6(b^2d^5x^5+abd^5x^2+20b^2d^3x^3+2abd^3+120b^2dx)\cosh(dx+c)-(b^2d^6x^6+2abd^6x^3+30b^2d^4x^4+}{d^7}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="fricas")
```

output
```
-(6*(b^2*d^5*x^5 + a*b*d^5*x^2 + 20*b^2*d^3*x^3 + 2*a*b*d^3 + 120*b^2*d*x)
*cosh(d*x + c) - (b^2*d^6*x^6 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + a^2*d^6 +
 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 720*b^2)*sinh(d*x + c))/d^7
```

Sympy [A] (verification not implemented)

Time = 0.84 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.22

$$\int \left(a+bx^3\right)^2 \cosh(c+dx)\,dx$$
$$=\begin{cases} \frac{a^2\sinh\left(c+dx\right)}{d}+\frac{2abx^3\sinh\left(c+dx\right)}{d}-\frac{6abx^2\cosh\left(c+dx\right)}{d^2}+\frac{12abx\sinh\left(c+dx\right)}{d^3}-\frac{12ab\cosh\left(c+dx\right)}{d^4}+\frac{b^2x^6\sinh\left(c+dx\right)}{d}-\frac{6b^2x^5\cosh\ldots}{d\ldots} \\ \left(a^2x+\frac{abx^4}{2}+\frac{b^2x^7}{7}\right)\cosh\left(c\right) \end{cases}$$

input
```
integrate((b*x**3+a)**2*cosh(d*x+c),x)
```

output
```
Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*x**3*sinh(c + d*x)/d - 6*a*b*x**2*
cosh(c + d*x)/d**2 + 12*a*b*x*sinh(c + d*x)/d**3 - 12*a*b*cosh(c + d*x)/d*
*4 + b**2*x**6*sinh(c + d*x)/d - 6*b**2*x**5*cosh(c + d*x)/d**2 + 30*b**2*
x**4*sinh(c + d*x)/d**3 - 120*b**2*x**3*cosh(c + d*x)/d**4 + 360*b**2*x**2
*sinh(c + d*x)/d**5 - 720*b**2*x*cosh(c + d*x)/d**6 + 720*b**2*sinh(c + d*
x)/d**7, Ne(d, 0)), ((a**2*x + a*b*x**4/2 + b**2*x**7/7)*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 243, normalized size of antiderivative = 1.31

$$\int \left(a+bx^3\right)^2 \cosh(c+dx)\,dx=\frac{a^2e^{(dx+c)}}{2\,d}-\frac{a^2e^{(-dx-c)}}{2\,d}+\frac{(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)abe^{(dx)}}{d^4}-\frac{(d^3x^3+3\,d^2x^2+6\,dx+6)abe^{(-dx-c)}}{d^4}+\frac{(d^6x^6e^c-6\,d^5x^5e^c+30\,d^4x^4e^c-120\,d^3x^3e^c+360\,d^2x^2e^c-720\,dxe^c+720\,e^c)b^2e^{(dx)}}{2\,d^7}-\frac{(d^6x^6+6\,d^5x^5+30\,d^4x^4+120\,d^3x^3+360\,d^2x^2+720\,dx+720)b^2e^{(-dx-c)}}{2\,d^7}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="maxima")
```

output
```
1/2*a^2*e^(d*x + c)/d - 1/2*a^2*e^(-d*x - c)/d + (d^3*x^3*e^c - 3*d^2*x^2*
e^c + 6*d*x*e^c - 6*e^c)*a*b*e^(d*x)/d^4 - (d^3*x^3 + 3*d^2*x^2 + 6*d*x +
6)*a*b*e^(-d*x - c)/d^4 + 1/2*(d^6*x^6*e^c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^
c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 720*d*x*e^c + 720*e^c)*b^2*e^(d*x)
/d^7 - 1/2*(d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 120*d^3*x^3 + 360*d^2*x^2 +
 720*d*x + 720)*b^2*e^(-d*x - c)/d^7
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.31

$$\int \left(a+bx^3\right)^2 \cosh(c+dx)\,dx$$
$$= \frac{(b^2d^6x^6 - 6\,b^2d^5x^5 + 2\,abd^6x^3 + 30\,b^2d^4x^4 - 6\,abd^5x^2 + a^2d^6 - 120\,b^2d^3x^3 + 12\,abd^4x + 360\,b^2d^2x^2 - 12}{2\,d^7}$$
$$- \frac{(b^2d^6x^6 + 6\,b^2d^5x^5 + 2\,abd^6x^3 + 30\,b^2d^4x^4 + 6\,abd^5x^2 + a^2d^6 + 120\,b^2d^3x^3 + 12\,abd^4x + 360\,b^2d^2x^2 +}{2\,d^7}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c),x, algorithm="giac")
```

output
```
1/2*(b^2*d^6*x^6 - 6*b^2*d^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 - 6*a*b*
d^5*x^2 + a^2*d^6 - 120*b^2*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 - 12*
a*b*d^3 - 720*b^2*d*x + 720*b^2)*e^(d*x + c)/d^7 - 1/2*(b^2*d^6*x^6 + 6*b^
2*d^5*x^5 + 2*a*b*d^6*x^3 + 30*b^2*d^4*x^4 + 6*a*b*d^5*x^2 + a^2*d^6 + 120
*b^2*d^3*x^3 + 12*a*b*d^4*x + 360*b^2*d^2*x^2 + 12*a*b*d^3 + 720*b^2*d*x +
 720*b^2)*e^(-d*x - c)/d^7
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.98

$$\int (a+bx^3)^2 \cosh(c+dx)\,dx = \frac{\sinh(c+d\,x)\,\left(a^2\,d^6+720\,b^2\right)}{d^7} - \frac{6\,b^2\,x^5\cosh(c+d\,x)}{d^2} - \frac{120\,b^2\,x^3\cosh(c+d\,x)}{d^4} + \frac{b^2\,x^6\sinh(c+d\,x)}{d} + \frac{30\,b^2\,x^4\sinh(c+d\,x)}{d^3} + \frac{360\,b^2\,x^2\sinh(c+d\,x)}{d^5} - \frac{12\,a\,b\cosh(c+d\,x)}{d^4} - \frac{720\,b^2\,x\cosh(c+d\,x)}{d^6} - \frac{6\,a\,b\,x^2\cosh(c+d\,x)}{d^2} + \frac{2\,a\,b\,x^3\sinh(c+d\,x)}{d} + \frac{12\,a\,b\,x\sinh(c+d\,x)}{d^3}$$

input
```
int(cosh(c + d*x)*(a + b*x^3)^2,x)
```

output
```
(sinh(c + d*x)*(720*b^2 + a^2*d^6))/d^7 - (6*b^2*x^5*cosh(c + d*x))/d^2 -
(120*b^2*x^3*cosh(c + d*x))/d^4 + (b^2*x^6*sinh(c + d*x))/d + (30*b^2*x^4*
sinh(c + d*x))/d^3 + (360*b^2*x^2*sinh(c + d*x))/d^5 - (12*a*b*cosh(c + d*
x))/d^4 - (720*b^2*x*cosh(c + d*x))/d^6 - (6*a*b*x^2*cosh(c + d*x))/d^2 +
(2*a*b*x^3*sinh(c + d*x))/d + (12*a*b*x*sinh(c + d*x))/d^3
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.99

$$\int (a+bx^3)^2 \cosh(c+dx)\,dx = \frac{-6\cosh(dx+c)\,ab\,d^5x^2 - 12\cosh(dx+c)\,ab\,d^3 - 6\cosh(dx+c)\,b^2d^5x^5 - 120\cosh(dx+c)\,b^2d^3x^3 - 7}{}$$

input
```
int((b*x^3+a)^2*cosh(d*x+c),x)
```

output
```
( - 6*cosh(c + d*x)*a*b*d**5*x**2 - 12*cosh(c + d*x)*a*b*d**3 - 6*cosh(c +
 d*x)*b**2*d**5*x**5 - 120*cosh(c + d*x)*b**2*d**3*x**3 - 720*cosh(c + d*x
)*b**2*d*x + sinh(c + d*x)*a**2*d**6 + 2*sinh(c + d*x)*a*b*d**6*x**3 + 12*
sinh(c + d*x)*a*b*d**4*x + sinh(c + d*x)*b**2*d**6*x**6 + 30*sinh(c + d*x)
*b**2*d**4*x**4 + 360*sinh(c + d*x)*b**2*d**2*x**2 + 720*sinh(c + d*x)*b**
2)/d**7
```

3.89 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 160

$$\begin{aligned}\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x} \, dx = &-\frac{120b^2\cosh(c+dx)}{d^6} - \frac{4abx\cosh(c+dx)}{d^2} \\ &- \frac{60b^2x^2\cosh(c+dx)}{d^4} \\ &- \frac{5b^2x^4\cosh(c+dx)}{d^2} + a^2\cosh(c)\text{Chi}(dx) \\ &+ \frac{4ab\sinh(c+dx)}{d^3} + \frac{120b^2x\sinh(c+dx)}{d^5} \\ &+ \frac{2abx^2\sinh(c+dx)}{d} + \frac{20b^2x^3\sinh(c+dx)}{d^3} \\ &+ \frac{b^2x^5\sinh(c+dx)}{d} + a^2\sinh(c)\text{Shi}(dx)\end{aligned}$$

output
```
-120*b^2*cosh(d*x+c)/d^6-4*a*b*x*cosh(d*x+c)/d^2-60*b^2*x^2*cosh(d*x+c)/d^
4-5*b^2*x^4*cosh(d*x+c)/d^2+a^2*cosh(c)*Chi(d*x)+4*a*b*sinh(d*x+c)/d^3+120
*b^2*x*sinh(d*x+c)/d^5+2*a*b*x^2*sinh(d*x+c)/d+20*b^2*x^3*sinh(d*x+c)/d^3+
b^2*x^5*sinh(d*x+c)/d+a^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.68

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx$$
$$= -\frac{b(4ad^4x+5b(24+12d^2x^2+d^4x^4))\cosh(c+dx)}{d^6}+a^2\cosh(c)\text{Chi}(dx)$$
$$+\frac{b(2ad^2(2+d^2x^2)+bx(120+20d^2x^2+d^4x^4))\sinh(c+dx)}{d^5}+a^2\sinh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```

output
```
-((b*(4*a*d^4*x + 5*b*(24 + 12*d^2*x^2 + d^4*x^4))*Cosh[c + d*x])/d^6) + a
^2*Cosh[c]*CoshIntegral[d*x] + (b*(2*a*d^2*(2 + d^2*x^2) + b*x*(120 + 20*d
^2*x^2 + d^4*x^4))*Sinh[c + d*x])/d^5 + a^2*Sinh[c]*SinhIntegral[d*x]
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx$$
$$\Big\downarrow \text{5810}$$
$$\int \left(\frac{a^2\cosh(c+dx)}{x}+2abx^2\cosh(c+dx)+b^2x^5\cosh(c+dx)\right)dx$$
$$\Big\downarrow \text{2009}$$

$$a^2\cosh(c)\mathrm{Chi}(dx) + a^2\sinh(c)\mathrm{Shi}(dx) + \frac{4ab\sinh(c+dx)}{d^3} - \frac{4abx\cosh(c+dx)}{d^2} + \frac{2abx^2\sinh(c+dx)}{d} - \frac{120b^2\cosh(c+dx)}{d^6} + \frac{120b^2x\sinh(c+dx)}{d^5} - \frac{60b^2x^2\cosh(c+dx)}{d^4} + \frac{20b^2x^3\sinh(c+dx)}{d^3} - \frac{5b^2x^4\cosh(c+dx)}{d^2} + \frac{b^2x^5\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Cosh[c + d*x])/x,x]
```

output
```
(-120*b^2*Cosh[c + d*x])/d^6 - (4*a*b*x*Cosh[c + d*x])/d^2 - (60*b^2*x^2*C
osh[c + d*x])/d^4 - (5*b^2*x^4*Cosh[c + d*x])/d^2 + a^2*Cosh[c]*CoshIntegr
al[d*x] + (4*a*b*Sinh[c + d*x])/d^3 + (120*b^2*x*Sinh[c + d*x])/d^5 + (2*a
*b*x^2*Sinh[c + d*x])/d + (20*b^2*x^3*Sinh[c + d*x])/d^3 + (b^2*x^5*Sinh[c
 + d*x])/d + a^2*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.89

method	result
meijerg	$-\frac{32b^2\cosh(c)\sqrt{\pi}\left(-\frac{15}{4\sqrt{\pi}}+\frac{\left(\frac{15}{8}d^4x^4+\frac{45}{2}x^2d^2+45\right)\cosh(dx)}{12\sqrt{\pi}}-\frac{xd\left(\frac{3}{8}d^4x^4+\frac{15}{2}x^2d^2+45\right)\sinh(dx)}{12\sqrt{\pi}}\right)}{d^6}+\frac{32ib^2\sinh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{7}{8}d^4x^4\right.}{}\right.}{}$
risch	$\frac{e^{dx+c}b^2x^5}{2d}-\frac{e^{-dx-c}b^2x^5}{2d}-\frac{5\,e^{dx+c}b^2x^4}{2d^2}-\frac{5\,e^{-dx-c}b^2x^4}{2d^2}+\frac{e^{dx+c}ab\,x^2}{d}-\frac{e^{-dx-c}ab\,x^2}{d}-\frac{a^2e^{-c}\,\mathrm{expIntegral}_1(dx)}{2}-a$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x,x,method=_RETURNVERBOSE)
```

output
```
-32/d^6*b^2*cosh(c)*Pi^(1/2)*(-15/4/Pi^(1/2)+1/12/Pi^(1/2)*(15/8*d^4*x^4+4
5/2*x^2*d^2+45)*cosh(d*x)-1/12/Pi^(1/2)*x*d*(3/8*d^4*x^4+15/2*x^2*d^2+45)*
sinh(d*x))+32*I/d^6*b^2*sinh(c)*Pi^(1/2)*(-1/28*I/Pi^(1/2)*x*d*(7/8*d^4*x^
4+35/2*x^2*d^2+105)*cosh(d*x)+1/28*I/Pi^(1/2)*(35/8*d^4*x^4+105/2*x^2*d^2+
105)*sinh(d*x))+8*I/d^3*a*b*cosh(c)*Pi^(1/2)*(1/2*I/Pi^(1/2)*x*d*cosh(d*x)
-1/6*I/Pi^(1/2)*(3/2*x^2*d^2+3)*sinh(d*x))+8/d^3*b*a*sinh(c)*Pi^(1/2)*(-1/
2/Pi^(1/2)+1/2/Pi^(1/2)*(1/2*x^2*d^2+1)*cosh(d*x)-1/2/Pi^(1/2)*d*x*sinh(d*
x))+1/2*a^2*cosh(c)*Pi^(1/2)*(2/Pi^(1/2)*(Chi(d*x)-ln(d*x)-gamma)+(2*gamma
+2*ln(x)+2*ln(I*d))/Pi^(1/2))+a^2*sinh(c)*Shi(d*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.01

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = -\frac{2\,(5\,b^2d^4x^4+4\,abd^4x+60\,b^2d^2x^2+120\,b^2)\cosh\left(dx+c\right)-\left(a^2d^6\mathrm{Ei}(dx)+a^2d^6\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-2\,(\ldots}{2\ldots}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="fricas")
```

output
```
-1/2*(2*(5*b^2*d^4*x^4 + 4*a*b*d^4*x + 60*b^2*d^2*x^2 + 120*b^2)*cosh(d*x
+ c) - (a^2*d^6*Ei(d*x) + a^2*d^6*Ei(-d*x))*cosh(c) - 2*(b^2*d^5*x^5 + 2*a
*b*d^5*x^2 + 20*b^2*d^3*x^3 + 4*a*b*d^3 + 120*b^2*d*x)*sinh(d*x + c) - (a^
2*d^6*Ei(d*x) - a^2*d^6*Ei(-d*x))*sinh(c))/d^6
```

Sympy [A] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.05

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = a^2\sinh(c)\,\mathrm{Shi}(dx) + a^2\cosh(c)\,\mathrm{Chi}(dx) + 2ab\left(\begin{cases}\frac{x^2\sinh(c+dx)}{d} - \frac{2x\cosh(c+dx)}{d^2} + \frac{2\sinh(c+dx)}{d^3} & \text{for } d\neq 0\\ \frac{x^3\cosh(c)}{3} & \text{otherwise}\end{cases}\right) + b^2\left(\begin{cases}\frac{x^5\sinh(c+dx)}{d} - \frac{5x^4\cosh(c+dx)}{d^2} + \frac{20x^3\sinh(c+dx)}{d^3} - \frac{60x^2\cosh(c+dx)}{d^4} + \frac{120x\sinh(c+dx)}{d^5} - \frac{120\cosh(c+dx)}{d^6} & \text{for } a \\ \frac{x^6\cosh(c)}{6} & \text{othe}\end{cases}\right.$$

input
```
integrate((b*x**3+a)**2*cosh(d*x+c)/x,x)
```

output
```
a**2*sinh(c)*Shi(d*x) + a**2*cosh(c)*Chi(d*x) + 2*a*b*Piecewise((x**2*sinh
(c + d*x)/d - 2*x*cosh(c + d*x)/d**2 + 2*sinh(c + d*x)/d**3, Ne(d, 0)), (x
**3*cosh(c)/3, True)) + b**2*Piecewise((x**5*sinh(c + d*x)/d - 5*x**4*cosh
(c + d*x)/d**2 + 20*x**3*sinh(c + d*x)/d**3 - 60*x**2*cosh(c + d*x)/d**4 +
 120*x*sinh(c + d*x)/d**5 - 120*cosh(c + d*x)/d**6, Ne(d, 0)), (x**6*cosh(
c)/6, True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.81

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x}\,dx = -\frac{1}{12}\left(4\,ab\left(\frac{(d^3x^3e^c - 3\,d^2x^2e^c + 6\,dxe^c - 6\,e^c)e^{(dx)}}{d^4} + \frac{(d^3x^3 + 3\,d^2x^2 + 6\,dx + 6)e^{(-dx-c)}}{d^4}\right) + b^2\left(\frac{(d^6x^6\right.\right.$$

$$+ \frac{1}{6}\left(b^2x^6 + 4\,abx^3 + 2\,a^2\log\left(x^3\right)\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="maxima")
```

output
```
-1/12*(4*a*b*((d^3*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^
4 + (d^3*x^3 + 3*d^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) + b^2*((d^6*x^6*e^
c - 6*d^5*x^5*e^c + 30*d^4*x^4*e^c - 120*d^3*x^3*e^c + 360*d^2*x^2*e^c - 7
20*d*x*e^c + 720*e^c)*e^(d*x)/d^7 + (d^6*x^6 + 6*d^5*x^5 + 30*d^4*x^4 + 12
0*d^3*x^3 + 360*d^2*x^2 + 720*d*x + 720)*e^(-d*x - c)/d^7) + 4*a^2*cosh(d*
x + c)*log(x^3)/d - 6*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a^2/d)*d + 1/6*(b^2*
x^6 + 4*a*b*x^3 + 2*a^2*log(x^3))*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(160) = 320$.

Time = 0.13 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.07

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x}\,dx = \frac{b^2d^5x^5e^{(dx+c)} - b^2d^5x^5e^{(-dx-c)} - 5\,b^2d^4x^4e^{(dx+c)} - 5\,b^2d^4x^4e^{(-dx-c)} + 2\,abd^5x^2e^{(dx+c)} - 2\,abd^5x^2e^{(-dx-c)}}{}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x,x, algorithm="giac")
```

output
```
1/2*(b^2*d^5*x^5*e^(d*x + c) - b^2*d^5*x^5*e^(-d*x - c) - 5*b^2*d^4*x^4*e^
(d*x + c) - 5*b^2*d^4*x^4*e^(-d*x - c) + 2*a*b*d^5*x^2*e^(d*x + c) - 2*a*b
*d^5*x^2*e^(-d*x - c) + a^2*d^6*Ei(-d*x)*e^(-c) + a^2*d^6*Ei(d*x)*e^c + 20
*b^2*d^3*x^3*e^(d*x + c) - 20*b^2*d^3*x^3*e^(-d*x - c) - 4*a*b*d^4*x*e^(d*
x + c) - 4*a*b*d^4*x*e^(-d*x - c) - 60*b^2*d^2*x^2*e^(d*x + c) - 60*b^2*d^
2*x^2*e^(-d*x - c) + 4*a*b*d^3*e^(d*x + c) - 4*a*b*d^3*e^(-d*x - c) + 120*
b^2*d*x*e^(d*x + c) - 120*b^2*d*x*e^(-d*x - c) - 120*b^2*e^(d*x + c) - 120
*b^2*e^(-d*x - c))/d^6
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x}\,dx$$
$$= \frac{-4\cosh(dx+c)\,ab\,d^4x - 5\cosh(dx+c)\,b^2d^4x^4 - 60\cosh(dx+c)\,b^2d^2x^2 - 120\cosh(dx+c)\,b^2 + \left(\int \mathrm{c}\right.}{}$$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x,x)
```

output
```
( - 4*cosh(c + d*x)*a*b*d**4*x - 5*cosh(c + d*x)*b**2*d**4*x**4 - 60*cosh(
c + d*x)*b**2*d**2*x**2 - 120*cosh(c + d*x)*b**2 + int(cosh(c + d*x)/x,x)*
a**2*d**6 + 2*sinh(c + d*x)*a*b*d**5*x**2 + 4*sinh(c + d*x)*a*b*d**3 + sin
h(c + d*x)*b**2*d**5*x**5 + 20*sinh(c + d*x)*b**2*d**3*x**3 + 120*sinh(c +
 d*x)*b**2*d*x)/d**6
```

3.90 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 143

$$\begin{aligned}\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx = &-\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} \\ &- \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(c+dx)}{d^2} \\ &+ a^2 d\text{Chi}(dx)\sinh(c) + \frac{24b^2\sinh(c+dx)}{d^5} \\ &+ \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} \\ &+ \frac{b^2x^4\sinh(c+dx)}{d} + a^2 d\cosh(c)\text{Shi}(dx)\end{aligned}$$

output

```
-2*a*b*cosh(d*x+c)/d^2-a^2*cosh(d*x+c)/x-24*b^2*x*cosh(d*x+c)/d^4-4*b^2*x^
3*cosh(d*x+c)/d^2+a^2*d*Chi(d*x)*sinh(c)+24*b^2*sinh(d*x+c)/d^5+2*a*b*x*si
nh(d*x+c)/d+12*b^2*x^2*sinh(d*x+c)/d^3+b^2*x^4*sinh(d*x+c)/d+a^2*d*cosh(c)
*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx = -\frac{2ab\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{x} - \frac{24b^2x\cosh(c+dx)}{d^4} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + a^2d\text{Chi}(dx)\sinh(c) + \frac{24b^2\sinh(c+dx)}{d^5} + \frac{2abx\sinh(c+dx)}{d} + \frac{12b^2x^2\sinh(c+dx)}{d^3} + \frac{b^2x^4\sinh(c+dx)}{d} + a^2d\cosh(c)\text{Shi}(dx)$$

input
```
Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```

output
```
(-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*
x])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c]
+ (24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sin
h[c + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d
*x]
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2} dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^2} + 2abx\cosh(c+dx) + b^2x^4\cosh(c+dx)\right) dx$$

$$\Big\downarrow \text{2009}$$

$$a^2 d \sinh(c)\text{Chi}(dx) + a^2 d \cosh(c)\text{Shi}(dx) - \frac{a^2 \cosh(c+dx)}{x} - \frac{2ab\cosh(c+dx)}{d^2} + \frac{2abx\sinh(c+dx)}{d} + \frac{24b^2\sinh(c+dx)}{d^5} - \frac{24b^2x\cosh(c+dx)}{d^4} + \frac{12b^2x^2\sinh(c+dx)}{d^3} - \frac{4b^2x^3\cosh(c+dx)}{d^2} + \frac{b^2x^4\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Cosh[c + d*x])/x^2,x]
```

output
```
(-2*a*b*Cosh[c + d*x])/d^2 - (a^2*Cosh[c + d*x])/x - (24*b^2*x*Cosh[c + d*
x])/d^4 - (4*b^2*x^3*Cosh[c + d*x])/d^2 + a^2*d*CoshIntegral[d*x]*Sinh[c]
+ (24*b^2*Sinh[c + d*x])/d^5 + (2*a*b*x*Sinh[c + d*x])/d + (12*b^2*x^2*Sin
h[c + d*x])/d^3 + (b^2*x^4*Sinh[c + d*x])/d + a^2*d*Cosh[c]*SinhIntegral[d
*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.75 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.14

method	result
meijerg	$-\frac{16ib^2\cosh(c)\sqrt{\pi}\left(-\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{10\sqrt{\pi}}+\frac{i\left(\frac{5}{8}d^4x^4+\frac{15}{2}x^2d^2+15\right)\sinh(dx)}{10\sqrt{\pi}}\right)}{d^5} - \frac{16b^2\sinh(c)\sqrt{\pi}\left(\frac{3}{2\sqrt{\pi}}-\frac{\left(\frac{3}{8}d^4x^4+\frac{9}{2}x^2d^2+9\right)}{6\sqrt{\pi}}\right.}{d^5}$
risch	$-\frac{-e^{dx+c}b^2d^4x^5+e^{-dx-c}b^2d^4x^5+e^c\,\text{expIntegral}_1(-dx)a^2d^6x-e^{-c}\,\text{expIntegral}_1(dx)a^2d^6x+4\,e^{dx+c}b^2d^3x^4+4\,e^{-dx-c}b^2d^3x^4-2}{}$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^2,x,method=_RETURNVERBOSE)
```

output
```
-16*I/d^5*b^2*cosh(c)*Pi^(1/2)*(-1/10*I/Pi^(1/2)*x*d*(5/2*x^2*d^2+15)*cosh
(d*x)+1/10*I/Pi^(1/2)*(5/8*d^4*x^4+15/2*x^2*d^2+15)*sinh(d*x))-16/d^5*b^2*
sinh(c)*Pi^(1/2)*(3/2/Pi^(1/2)-1/6/Pi^(1/2)*(3/8*d^4*x^4+9/2*x^2*d^2+9)*co
sh(d*x)+1/6/Pi^(1/2)*x*d*(3/2*x^2*d^2+9)*sinh(d*x))-4/d^2*a*b*cosh(c)*Pi^(
1/2)*(-1/2/Pi^(1/2)+1/2/Pi^(1/2)*cosh(d*x)-1/2/Pi^(1/2)*d*x*sinh(d*x))+2/d
^2*b*a*sinh(c)*(cosh(d*x)*x*d-sinh(d*x))+1/4*I*a^2*cosh(c)*Pi^(1/2)*d*(4*I
/d/x*cosh(d*x)/Pi^(1/2)-4*I/Pi^(1/2)*Shi(d*x))+1/4*a^2*sinh(c)*Pi^(1/2)*d*
(4/Pi^(1/2)-4/Pi^(1/2)/x/d*sinh(d*x)+4/Pi^(1/2)*(Chi(d*x)-ln(d*x)-gamma)+2
*(2*gamma-2+2*ln(x)+2*ln(I*d))/Pi^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.12

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^2}\,dx = -\frac{2\left(4\,b^2d^3x^4+a^2d^5+2\,abd^3x+24\,b^2dx^2\right)\cosh\left(dx+c\right)-\left(a^2d^6x\mathrm{Ei}(dx)-a^2d^6x\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-2\,(}{2\,d^5x}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(4*b^2*d^3*x^4 + a^2*d^5 + 2*a*b*d^3*x + 24*b^2*d*x^2)*cosh(d*x +
c) - (a^2*d^6*x*Ei(d*x) - a^2*d^6*x*Ei(-d*x))*cosh(c) - 2*(b^2*d^4*x^5 + 2
*a*b*d^4*x^2 + 12*b^2*d^2*x^3 + 24*b^2*x)*sinh(d*x + c) - (a^2*d^6*x*Ei(d*
x) + a^2*d^6*x*Ei(-d*x))*sinh(c))/(d^5*x)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2}\,dx = \int \frac{(a+bx^3)^2 \cosh{(c+dx)}}{x^2}\,dx$$

input
```
integrate((b*x**3+a)**2*cosh(d*x+c)/x**2,x)
```

output
```
Integral((a + b*x**3)**2*cosh(c + d*x)/x**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 235, normalized size of antiderivative = 1.64

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^2}\,dx = -\frac{1}{10}\left(5\,a^2\mathrm{Ei}(-dx)\,e^{(-c)} - 5\,a^2\mathrm{Ei}(dx)\,e^{c} + \frac{5\,(d^2x^2e^c - 2\,dxe^c + 2\,e^c)abe^{(dx)}}{d^3} + \frac{5\,(d^2x^2 + 2\,dx + 2)abe^{(-c}}{d^3} \right. + \frac{1}{5}\left(b^2x^5 + 5\,abx^2 - \frac{5\,a^2}{x}\right)\cosh{(dx+c)}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="maxima")
```

output
```
-1/10*(5*a^2*Ei(-d*x)*e^(-c) - 5*a^2*Ei(d*x)*e^c + 5*(d^2*x^2*e^c - 2*d*x*
e^c + 2*e^c)*a*b*e^(d*x)/d^3 + 5*(d^2*x^2 + 2*d*x + 2)*a*b*e^(-d*x - c)/d^
3 + (d^5*x^5*e^c - 5*d^4*x^4*e^c + 20*d^3*x^3*e^c - 60*d^2*x^2*e^c + 120*d
*x*e^c - 120*e^c)*b^2*e^(d*x)/d^6 + (d^5*x^5 + 5*d^4*x^4 + 20*d^3*x^3 + 60
*d^2*x^2 + 120*d*x + 120)*b^2*e^(-d*x - c)/d^6)*d + 1/5*(b^2*x^5 + 5*a*b*x
^2 - 5*a^2/x)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. 2(143) = 286.

Time = 0.13 (sec) , antiderivative size = 308, normalized size of antiderivative = 2.15

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^2}\,dx = \frac{b^2d^4x^5e^{(dx+c)} - b^2d^4x^5e^{(-dx-c)} - a^2d^6x\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^6x\mathrm{Ei}(dx)\,e^{c} - 4\,b^2d^3x^4e^{(dx+c)} - 4\,b^2d^3x^4e^{(-dx-}}{}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^2,x, algorithm="giac")
```

output
```
1/2*(b^2*d^4*x^5*e^(d*x + c) - b^2*d^4*x^5*e^(-d*x - c) - a^2*d^6*x*Ei(-d*
x)*e^(-c) + a^2*d^6*x*Ei(d*x)*e^c - 4*b^2*d^3*x^4*e^(d*x + c) - 4*b^2*d^3*
x^4*e^(-d*x - c) + 2*a*b*d^4*x^2*e^(d*x + c) - 2*a*b*d^4*x^2*e^(-d*x - c)
- a^2*d^5*e^(d*x + c) + 12*b^2*d^2*x^3*e^(d*x + c) - a^2*d^5*e^(-d*x - c)
- 12*b^2*d^2*x^3*e^(-d*x - c) - 2*a*b*d^3*x*e^(d*x + c) - 2*a*b*d^3*x*e^(-
d*x - c) - 24*b^2*d*x^2*e^(d*x + c) - 24*b^2*d*x^2*e^(-d*x - c) + 24*b^2*x
*e^(d*x + c) - 24*b^2*x*e^(-d*x - c))/(d^5*x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^2}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^2}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^2,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 285, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^2}\,dx$$
$$= \frac{-e^{dx}ei(-dx)\,a^2d^6x + e^{dx+2c}ei(dx)\,a^2d^6x - e^{2dx+2c}a^2d^5 + 2e^{2dx+2c}ab\,d^4x^2 - 2e^{2dx+2c}ab\,d^3x + e^{2dx+2c}b^2d^4}{}$$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^2,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a**2*d**6*x + e**(2*c + d*x)*ei(d*x)*a**2*d**6*x -
 e**(2*c + 2*d*x)*a**2*d**5 + 2*e**(2*c + 2*d*x)*a*b*d**4*x**2 - 2*e**(2*c
 + 2*d*x)*a*b*d**3*x + e**(2*c + 2*d*x)*b**2*d**4*x**5 - 4*e**(2*c + 2*d*x
)*b**2*d**3*x**4 + 12*e**(2*c + 2*d*x)*b**2*d**2*x**3 - 24*e**(2*c + 2*d*x
)*b**2*d*x**2 + 24*e**(2*c + 2*d*x)*b**2*x - a**2*d**5 - 2*a*b*d**4*x**2 -
 2*a*b*d**3*x - b**2*d**4*x**5 - 4*b**2*d**3*x**4 - 12*b**2*d**2*x**3 - 24
*b**2*d*x**2 - 24*b**2*x)/(2*e**(c + d*x)*d**5*x)
```

3.91 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 141

$$\begin{aligned}\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3} \, dx = & -\frac{6b^2 \cosh(c+dx)}{d^4} - \frac{a^2 \cosh(c+dx)}{2x^2} \\ & - \frac{3b^2x^2 \cosh(c+dx)}{d^2} + \frac{1}{2}a^2d^2 \cosh(c)\text{Chi}(dx) \\ & + \frac{2ab \sinh(c+dx)}{d} - \frac{a^2 d \sinh(c+dx)}{2x} \\ & + \frac{6b^2 x \sinh(c+dx)}{d^3} + \frac{b^2x^3 \sinh(c+dx)}{d} \\ & + \frac{1}{2}a^2d^2 \sinh(c)\text{Shi}(dx)\end{aligned}$$

output
```
-6*b^2*cosh(d*x+c)/d^4-1/2*a^2*cosh(d*x+c)/x^2-3*b^2*x^2*cosh(d*x+c)/d^2+1
/2*a^2*d^2*cosh(c)*Chi(d*x)+2*a*b*sinh(d*x+c)/d-1/2*a^2*d*sinh(d*x+c)/x+6*
b^2*x*sinh(d*x+c)/d^3+b^2*x^3*sinh(d*x+c)/d+1/2*a^2*d^2*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.96

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = \frac{1}{2}\left(-\frac{12b^2\cosh(c+dx)}{d^4}-\frac{a^2\cosh(c+dx)}{x^2}-\frac{6b^2x^2\cosh(c+dx)}{d^2}+a^2d^2\cosh(c)\text{Chi}(dx)+\frac{4ab\sinh(c+dx)}{d}-\frac{a^2d\sinh(c+dx)}{x}+\frac{12b^2x\sinh(c+dx)}{d^3}+\frac{2b^2x^3\sinh(c+dx)}{d}+a^2d^2\sinh(c)\text{Shi}(dx)\right)$$

input
```
Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```

output
```
((-12*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/x^2 - (6*b^2*x^2*Cosh[c
 + d*x])/d^2 + a^2*d^2*Cosh[c]*CoshIntegral[d*x] + (4*a*b*Sinh[c + d*x])/d
 - (a^2*d*Sinh[c + d*x])/x + (12*b^2*x*Sinh[c + d*x])/d^3 + (2*b^2*x^3*Sin
h[c + d*x])/d + a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx$$

$$\Big\downarrow \text{5810}$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^3}+2ab\cosh(c+dx)+b^2x^3\cosh(c+dx)\right)dx$$

$\downarrow$ 2009

$$\frac{1}{2}a^2d^2\cosh(c)\text{Chi}(dx)+\frac{1}{2}a^2d^2\sinh(c)\text{Shi}(dx)-\frac{a^2\cosh(c+dx)}{2x^2}-\frac{a^2d\sinh(c+dx)}{2x}+\frac{2ab\sinh(c+dx)}{d}-\frac{6b^2\cosh(c+dx)}{d^4}+\frac{6b^2x\sinh(c+dx)}{d^3}-\frac{3b^2x^2\cosh(c+dx)}{d^2}+\frac{b^2x^3\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Cosh[c + d*x])/x^3,x]
```

output
```
(-6*b^2*Cosh[c + d*x])/d^4 - (a^2*Cosh[c + d*x])/(2*x^2) - (3*b^2*x^2*Cosh
[c + d*x])/d^2 + (a^2*d^2*Cosh[c]*CoshIntegral[d*x])/2 + (2*a*b*Sinh[c + d
*x])/d - (a^2*d*Sinh[c + d*x])/(2*x) + (6*b^2*x*Sinh[c + d*x])/d^3 + (b^2*
x^3*Sinh[c + d*x])/d + (a^2*d^2*Sinh[c]*SinhIntegral[d*x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(133) = 266$.

Time = 0.78 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.99

method	result
risch	$-\frac{e^{-c}\,\text{expIntegral}_1(dx)a^2d^6x^2+e^{c}\,\text{expIntegral}_1(-dx)a^2d^6x^2-2\,e^{dx+c}b^2d^3x^5+2\,e^{-dx-c}b^2d^3x^5+e^{dx+c}a^2d^5x+6b^2d^2x^4e^{dx+c}-e^{-}}{}$
meijerg	$\frac{8b^2\cosh(c)\sqrt{\pi}\left(\frac{3}{4\sqrt{\pi}}-\frac{\left(\frac{3x^2d^2}{2}+3\right)\cosh(dx)}{4\sqrt{\pi}}+\frac{dx\left(\frac{x^2d^2}{2}+3\right)\sinh(dx)}{4\sqrt{\pi}}\right)}{d^4}-\frac{8ib^2\sinh(c)\sqrt{\pi}\left(\frac{ixd\left(\frac{5x^2d^2}{2}+15\right)\cosh(dx)}{20\sqrt{\pi}}-\frac{i\left(\frac{15x^2d^2}{2}+15\right.}{20\sqrt{}}\right.}{d^4}$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4/d^4*(exp(-c)*Ei(1,d*x)*a^2*d^6*x^2+exp(c)*Ei(1,-d*x)*a^2*d^6*x^2-2*ex
p(d*x+c)*b^2*d^3*x^5+2*exp(-d*x-c)*b^2*d^3*x^5+exp(d*x+c)*a^2*d^5*x+6*b^2*
d^2*x^4*exp(d*x+c)-exp(-d*x-c)*a^2*d^5*x+6*b^2*d^2*x^4*exp(-d*x-c)-4*exp(d
*x+c)*a*b*d^3*x^2+4*exp(-d*x-c)*a*b*d^3*x^2+exp(d*x+c)*a^2*d^4-12*b^2*d*x^
3*exp(d*x+c)+exp(-d*x-c)*a^2*d^4+12*b^2*d*x^3*exp(-d*x-c)+12*exp(d*x+c)*b^
2*x^2+12*exp(-d*x-c)*b^2*x^2)/x^2
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.14

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = -\frac{2\,(6\,b^2d^2x^4+a^2d^4+12\,b^2x^2)\cosh\left(dx+c\right)-\left(a^2d^6x^2\mathrm{Ei}(dx)+a^2d^6x^2\mathrm{Ei}(-dx)\right)\cosh\left(c\right)-2\,(2\,b^2d^3x^5}{4\,d^4x^2}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="fricas")
```

output
```
-1/4*(2*(6*b^2*d^2*x^4 + a^2*d^4 + 12*b^2*x^2)*cosh(d*x + c) - (a^2*d^6*x^
2*Ei(d*x) + a^2*d^6*x^2*Ei(-d*x))*cosh(c) - 2*(2*b^2*d^3*x^5 - a^2*d^5*x +
 4*a*b*d^3*x^2 + 12*b^2*d*x^3)*sinh(d*x + c) - (a^2*d^6*x^2*Ei(d*x) - a^2*
d^6*x^2*Ei(-d*x))*sinh(c))/(d^4*x^2)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^3}\,dx = \int \frac{(a+bx^3)^2\cosh\left(c+dx\right)}{x^3}\,dx$$

input
```
integrate((b*x**3+a)**2*cosh(d*x+c)/x**3,x)
```

output
```
Integral((a + b*x**3)**2*cosh(c + d*x)/x**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.44

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx$$
$$= \frac{1}{8}\left(2\,a^2de^{(-c)}\Gamma(-1,dx) + 2\,a^2de^{c}\Gamma(-1,-dx) - \frac{8\,(dxe^c - e^c)abe^{(dx)}}{d^2} - \frac{8\,(dx+1)abe^{(-dx-c)}}{d^2} - \frac{(d^4x^4e^c -\right.$$
$$+ \frac{1}{4}\left(b^2x^4 + 8\,abx - \frac{2\,a^2}{x^2}\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="maxima")
```

output
```
1/8*(2*a^2*d*e^(-c)*gamma(-1, d*x) + 2*a^2*d*e^c*gamma(-1, -d*x) - 8*(d*x*
e^c - e^c)*a*b*e^(d*x)/d^2 - 8*(d*x + 1)*a*b*e^(-d*x - c)/d^2 - (d^4*x^4*e
^c - 4*d^3*x^3*e^c + 12*d^2*x^2*e^c - 24*d*x*e^c + 24*e^c)*b^2*e^(d*x)/d^5
 - (d^4*x^4 + 4*d^3*x^3 + 12*d^2*x^2 + 24*d*x + 24)*b^2*e^(-d*x - c)/d^5)*
d + 1/4*(b^2*x^4 + 8*a*b*x - 2*a^2/x^2)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(133) = 266$.

Time = 0.14 (sec) , antiderivative size = 280, normalized size of antiderivative = 1.99

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx$$
$$= \frac{a^2d^6x^2\mathrm{Ei}(-dx)\,e^{(-c)} + a^2d^6x^2\mathrm{Ei}(dx)\,e^{c} + 2\,b^2d^3x^5e^{(dx+c)} - 2\,b^2d^3x^5e^{(-dx-c)} - a^2d^5xe^{(dx+c)} - 6\,b^2d^2x^4e^{(d}}{}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^3,x, algorithm="giac")
```

output
```
1/4*(a^2*d^6*x^2*Ei(-d*x)*e^(-c) + a^2*d^6*x^2*Ei(d*x)*e^c + 2*b^2*d^3*x^5
*e^(d*x + c) - 2*b^2*d^3*x^5*e^(-d*x - c) - a^2*d^5*x*e^(d*x + c) - 6*b^2*
d^2*x^4*e^(d*x + c) + a^2*d^5*x*e^(-d*x - c) - 6*b^2*d^2*x^4*e^(-d*x - c)
+ 4*a*b*d^3*x^2*e^(d*x + c) - 4*a*b*d^3*x^2*e^(-d*x - c) - a^2*d^4*e^(d*x
+ c) + 12*b^2*d*x^3*e^(d*x + c) - a^2*d^4*e^(-d*x - c) - 12*b^2*d*x^3*e^(-
d*x - c) - 12*b^2*x^2*e^(d*x + c) - 12*b^2*x^2*e^(-d*x - c))/(d^4*x^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^3}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^3,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^3, x)
```

Reduce [F]

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^3}\,dx = \frac{-\cosh(dx+c)\,a^2d^4-6\cosh(dx+c)\,b^2d^2x^4-12\cosh(dx+c)\,b^2x^2+\left(\int \frac{\cosh(dx+c)}{x}dx\right)a^2d^6x^2-\sinh(\ldots}{2d^4x^2}$$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^3,x)
```

output
```
( - cosh(c + d*x)*a**2*d**4 - 6*cosh(c + d*x)*b**2*d**2*x**4 - 12*cosh(c +
 d*x)*b**2*x**2 + int(cosh(c + d*x)/x,x)*a**2*d**6*x**2 - sinh(c + d*x)*a*
*2*d**5*x + 4*sinh(c + d*x)*a*b*d**3*x**2 + 2*sinh(c + d*x)*b**2*d**3*x**5
 + 12*sinh(c + d*x)*b**2*d*x**3)/(2*d**4*x**2)
```

3.92 $\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 150

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx = -\frac{a^2\cosh(c+dx)}{3x^3} - \frac{a^2d^2\cosh(c+dx)}{6x} - \frac{2b^2x\cosh(c+dx)}{d^2} + 2ab\cosh(c)\text{Chi}(dx) + \frac{1}{6}a^2d^3\text{Chi}(dx)\sinh(c) + \frac{2b^2\sinh(c+dx)}{d^3} - \frac{a^2d\sinh(c+dx)}{6x^2} + \frac{b^2x^2\sinh(c+dx)}{d} + \frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx) + 2ab\sinh(c)\text{Shi}(dx)$$

output
```
-1/3*a^2*cosh(d*x+c)/x^3-1/6*a^2*d^2*cosh(d*x+c)/x-2*b^2*x*cosh(d*x+c)/d^2
+2*a*b*cosh(c)*Chi(d*x)+1/6*a^2*d^3*Chi(d*x)*sinh(c)+2*b^2*sinh(d*x+c)/d^3
-1/6*a^2*d*sinh(d*x+c)/x^2+b^2*x^2*sinh(d*x+c)/d+1/6*a^2*d^3*cosh(c)*Shi(d
*x)+2*a*b*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx = \frac{1}{6}\left(-\frac{2a^2\cosh(c+dx)}{x^3}-\frac{a^2d^2\cosh(c+dx)}{x}-\frac{12b^2x\cosh(c+dx)}{d^2}+a\mathrm{Chi}(dx)\left(12b\cosh(c)+ad^3\sinh(c)\right)+\frac{12b^2\sinh(c+dx)}{d^3}-\frac{a^2d\sinh(c+dx)}{x^2}+\frac{6b^2x^2\sinh(c+dx)}{d}+a\left(ad^3\cosh(c)+12b\sinh(c)\right)\mathrm{Shi}(dx)\right)$$

input
```
Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

output
```
((-2*a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/x - (12*b^2*x*Cosh[c
 + d*x])/d^2 + a*CoshIntegral[d*x]*(12*b*Cosh[c] + a*d^3*Sinh[c]) + (12*b^
2*Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/x^2 + (6*b^2*x^2*Sinh[c + d*x
])/d + a*(a*d^3*Cosh[c] + 12*b*Sinh[c])*SinhIntegral[d*x])/6
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx$$

$$\Big\downarrow \; 5810$$

$$\int \left(\frac{a^2\cosh(c+dx)}{x^4}+\frac{2ab\cosh(c+dx)}{x}+b^2x^2\cosh(c+dx)\right)dx$$

$$\big\downarrow \text{2009}$$

$$\frac{1}{6}a^2d^3\sinh(c)\text{Chi}(dx)+\frac{1}{6}a^2d^3\cosh(c)\text{Shi}(dx)-\frac{a^2d^2\cosh(c+dx)}{6x}-\frac{a^2\cosh(c+dx)}{3x^3}-\frac{a^2d\sinh(c+dx)}{6x^2}+2ab\cosh(c)\text{Chi}(dx)+2ab\sinh(c)\text{Shi}(dx)+\frac{2b^2\sinh(c+dx)}{d^3}-\frac{2b^2x\cosh(c+dx)}{d^2}+\frac{b^2x^2\sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Cosh[c + d*x])/x^4,x]
```

output
```
-1/3*(a^2*Cosh[c + d*x])/x^3 - (a^2*d^2*Cosh[c + d*x])/(6*x) - (2*b^2*x*Co
sh[c + d*x])/d^2 + 2*a*b*Cosh[c]*CoshIntegral[d*x] + (a^2*d^3*CoshIntegral
[d*x]*Sinh[c])/6 + (2*b^2*Sinh[c + d*x])/d^3 - (a^2*d*Sinh[c + d*x])/(6*x^
2) + (b^2*x^2*Sinh[c + d*x])/d + (a^2*d^3*Cosh[c]*SinhIntegral[d*x])/6 + 2
*a*b*Sinh[c]*SinhIntegral[d*x]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. 2(140) = 280.

Time = 0.83 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.89

method	result
risch	$-\frac{e^{c}\,\text{expIntegral}_1(-dx)a^2d^6x^3-e^{-c}\,\text{expIntegral}_1(dx)a^2d^6x^3+e^{dx+c}a^2d^5x^2-6\,e^{dx+c}b^2d^2x^5+12\,e^{c}\,\text{expIntegral}_1(-dx)ab\,d^3x^3+e}{}$
meijerg	$\frac{4ib^2\cosh(c)\sqrt{\pi}\left(\frac{ixd\cosh(dx)}{2\sqrt{\pi}}-\frac{i\left(\frac{3x^2d^2}{2}+3\right)\sinh(dx)}{6\sqrt{\pi}}\right)}{d^3}+\frac{4b^2\sinh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\left(\frac{x^2d^2}{2}+1\right)\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^3}+ab\cos$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^4,x,method=_RETURNVERBOSE)
```

output
```
-1/12/d^3*(exp(c)*Ei(1,-d*x)*a^2*d^6*x^3-exp(-c)*Ei(1,d*x)*a^2*d^6*x^3+exp
(d*x+c)*a^2*d^5*x^2-6*exp(d*x+c)*b^2*d^2*x^5+12*exp(c)*Ei(1,-d*x)*a*b*d^3*
x^3+exp(-d*x-c)*a^2*d^5*x^2+6*exp(-d*x-c)*b^2*d^2*x^5+12*exp(-c)*Ei(1,d*x)
*a*b*d^3*x^3+exp(d*x+c)*a^2*d^4*x+12*exp(d*x+c)*b^2*d*x^4-exp(-d*x-c)*a^2*
d^4*x+12*exp(-d*x-c)*b^2*d*x^4+2*exp(d*x+c)*a^2*d^3-12*exp(d*x+c)*b^2*x^3+
2*exp(-d*x-c)*a^2*d^3+12*exp(-d*x-c)*b^2*x^3)/x^3
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{2\,(a^2d^5x^2 + 12\,b^2dx^4 + 2\,a^2d^3)\cosh\,(dx+c) - ((a^2d^6 + 12\,abd^3)x^3\mathrm{Ei}(dx) - (a^2d^6 - 12\,abd^3)x^3\mathrm{Ei}(-dx$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="fricas")
```

output
```
-1/12*(2*(a^2*d^5*x^2 + 12*b^2*d*x^4 + 2*a^2*d^3)*cosh(d*x + c) - ((a^2*d^
6 + 12*a*b*d^3)*x^3*Ei(d*x) - (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*cosh(c)
 - 2*(6*b^2*d^2*x^5 - a^2*d^4*x + 12*b^2*x^3)*sinh(d*x + c) - ((a^2*d^6 +
12*a*b*d^3)*x^3*Ei(d*x) + (a^2*d^6 - 12*a*b*d^3)*x^3*Ei(-d*x))*sinh(c))/(d
^3*x^3)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4}\,dx = \int \frac{(a+bx^3)^2 \cosh\,(c+dx)}{x^4}\,dx$$

input
```
integrate((b*x**3+a)**2*cosh(d*x+c)/x**4,x)
```

output
```
Integral((a + b*x**3)**2*cosh(c + d*x)/x**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.25

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx$$
$$=\frac{1}{6}\left(\left(d^2e^{(-c)}\Gamma(-2,dx)-d^2e^c\Gamma(-2,-dx)\right)a^2-b^2\left(\frac{(d^3x^3e^c-3\,d^2x^2e^c+6\,dxe^c-6\,e^c)e^{(dx)}}{d^4}+\frac{(d^3x^3+3\,d\right.\right.$$
$$+\frac{1}{3}\left(b^2x^3+2\,ab\log\left(x^3\right)-\frac{a^2}{x^3}\right)\cosh\left(dx+c\right)$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="maxima")
```

output
```
1/6*((d^2*e^(-c)*gamma(-2, d*x) - d^2*e^c*gamma(-2, -d*x))*a^2 - b^2*((d^3
*x^3*e^c - 3*d^2*x^2*e^c + 6*d*x*e^c - 6*e^c)*e^(d*x)/d^4 + (d^3*x^3 + 3*d
^2*x^2 + 6*d*x + 6)*e^(-d*x - c)/d^4) - 4*a*b*cosh(d*x + c)*log(x^3)/d + 6
*(Ei(-d*x)*e^(-c) + Ei(d*x)*e^c)*a*b/d)*d + 1/3*(b^2*x^3 + 2*a*b*log(x^3)
- a^2/x^3)*cosh(d*x + c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.86

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^4}\,dx =$$
$$-\frac{a^2d^6x^3\mathrm{Ei}(-dx)\,e^{(-c)}-a^2d^6x^3\mathrm{Ei}(dx)\,e^c+a^2d^5x^2e^{(dx+c)}-6\,b^2d^2x^5e^{(dx+c)}+a^2d^5x^2e^{(-dx-c)}+6\,b^2d^2x^5e}{}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^4,x, algorithm="giac")
```

output
```
-1/12*(a^2*d^6*x^3*Ei(-d*x)*e^(-c) - a^2*d^6*x^3*Ei(d*x)*e^c + a^2*d^5*x^2
*e^(d*x + c) - 6*b^2*d^2*x^5*e^(d*x + c) + a^2*d^5*x^2*e^(-d*x - c) + 6*b^
2*d^2*x^5*e^(-d*x - c) - 12*a*b*d^3*x^3*Ei(-d*x)*e^(-c) - 12*a*b*d^3*x^3*E
i(d*x)*e^c + a^2*d^4*x*e^(d*x + c) + 12*b^2*d*x^4*e^(d*x + c) - a^2*d^4*x*
e^(-d*x - c) + 12*b^2*d*x^4*e^(-d*x - c) + 2*a^2*d^3*e^(d*x + c) - 12*b^2*
x^3*e^(d*x + c) + 2*a^2*d^3*e^(-d*x - c) + 12*b^2*x^3*e^(-d*x - c))/(d^3*x
^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4}\,dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^4}\,dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^4,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.84

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^4}\,dx$$
$$= \frac{-e^{dx} ei(-dx)\, a^2 d^6 x^3 + 12 e^{dx} ei(-dx)\, ab\, d^3 x^3 + e^{dx+2c} ei(dx)\, a^2 d^6 x^3 + 12 e^{dx+2c} ei(dx)\, ab\, d^3 x^3 - e^{2dx+2c} a^2}{}$$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^4,x)
```

output
```
( - e**(d*x)*ei( - d*x)*a**2*d**6*x**3 + 12*e**(d*x)*ei( - d*x)*a*b*d**3*x
**3 + e**(2*c + d*x)*ei(d*x)*a**2*d**6*x**3 + 12*e**(2*c + d*x)*ei(d*x)*a*
b*d**3*x**3 - e**(2*c + 2*d*x)*a**2*d**5*x**2 - e**(2*c + 2*d*x)*a**2*d**4
*x - 2*e**(2*c + 2*d*x)*a**2*d**3 + 6*e**(2*c + 2*d*x)*b**2*d**2*x**5 - 12
*e**(2*c + 2*d*x)*b**2*d*x**4 + 12*e**(2*c + 2*d*x)*b**2*x**3 - a**2*d**5*
x**2 + a**2*d**4*x - 2*a**2*d**3 - 6*b**2*d**2*x**5 - 12*b**2*d*x**4 - 12*
b**2*x**3)/(12*e**(c + d*x)*d**3*x**3)
```

3.93 $\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx$

Optimal result

Integrand size = 19, antiderivative size = 167

$$\begin{aligned}\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx = &-\frac{b^2\cosh(c+dx)}{d^2} - \frac{a^2\cosh(c+dx)}{4x^4} - \frac{a^2d^2\cosh(c+dx)}{24x^2} \\ &- \frac{2ab\cosh(c+dx)}{x} + \frac{1}{24}a^2d^4\cosh(c)\text{Chi}(dx) \\ &+ 2abd\text{Chi}(dx)\sinh(c) - \frac{a^2d\sinh(c+dx)}{12x^3} \\ &- \frac{a^2d^3\sinh(c+dx)}{24x} + \frac{b^2x\sinh(c+dx)}{d} \\ &+ 2abd\cosh(c)\text{Shi}(dx) + \frac{1}{24}a^2d^4\sinh(c)\text{Shi}(dx)\end{aligned}$$

output
```
-b^2*cosh(d*x+c)/d^2-1/4*a^2*cosh(d*x+c)/x^4-1/24*a^2*d^2*cosh(d*x+c)/x^2-
2*a*b*cosh(d*x+c)/x+1/24*a^2*d^4*cosh(c)*Chi(d*x)+2*a*b*d*Chi(d*x)*sinh(c)
-1/12*a^2*d*sinh(d*x+c)/x^3-1/24*a^2*d^3*sinh(d*x+c)/x+b^2*x*sinh(d*x+c)/d
+2*a*b*d*cosh(c)*Shi(d*x)+1/24*a^2*d^4*sinh(c)*Shi(d*x)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.90

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx = \frac{1}{24}\Bigg(-\frac{24b^2\cosh(c+dx)}{d^2}-\frac{6a^2\cosh(c+dx)}{x^4}-\frac{a^2d^2\cosh(c+dx)}{x^2}-\frac{48ab\cosh(c+dx)}{x}+ad\text{Chi}(dx)\left(ad^3\cosh(c)+48b\sinh(c)\right)-\frac{2a^2d\sinh(c+dx)}{x^3}-\frac{a^2d^3\sinh(c+dx)}{x}+\frac{24b^2x\sinh(c+dx)}{d}+ad\left(48b\cosh(c)+ad^3\sinh(c)\right)\text{Shi}(dx)\Bigg)$$

input
```
Integrate[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

output
```
((-24*b^2*Cosh[c + d*x])/d^2 - (6*a^2*Cosh[c + d*x])/x^4 - (a^2*d^2*Cosh[c
 + d*x])/x^2 - (48*a*b*Cosh[c + d*x])/x + a*d*CoshIntegral[d*x]*(a*d^3*Cos
h[c] + 48*b*Sinh[c]) - (2*a^2*d*Sinh[c + d*x])/x^3 - (a^2*d^3*Sinh[c + d*x
])/x + (24*b^2*x*Sinh[c + d*x])/d + a*d*(48*b*Cosh[c] + a*d^3*Sinh[c])*Sin
hIntegral[d*x])/24
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5810, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx$$

$$\Big\downarrow \text{ 5810}$$

$$\int \left(\frac{a^2 \cosh(c+dx)}{x^5} + \frac{2ab \cosh(c+dx)}{x^2} + b^2 x \cosh(c+dx) \right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{24} a^2 d^4 \cosh(c) \text{Chi}(dx) + \frac{1}{24} a^2 d^4 \sinh(c) \text{Shi}(dx) - \frac{a^2 d^3 \sinh(c+dx)}{24x} - \frac{a^2 d^2 \cosh(c+dx)}{24x^2} - \frac{a^2 \cosh(c+dx)}{4x^4} - \frac{a^2 d \sinh(c+dx)}{12x^3} + 2abd \sinh(c) \text{Chi}(dx) + 2abd \cosh(c) \text{Shi}(dx) - \frac{2ab \cosh(c+dx)}{x} - \frac{b^2 \cosh(c+dx)}{d^2} + \frac{b^2 x \sinh(c+dx)}{d}$$

input
```
Int[((a + b*x^3)^2*Cosh[c + d*x])/x^5,x]
```

output
```
-((b^2*Cosh[c + d*x])/d^2) - (a^2*Cosh[c + d*x])/(4*x^4) - (a^2*d^2*Cosh[c
 + d*x])/(24*x^2) - (2*a*b*Cosh[c + d*x])/x + (a^2*d^4*Cosh[c]*CoshIntegra
l[d*x])/24 + 2*a*b*d*CoshIntegral[d*x]*Sinh[c] - (a^2*d*Sinh[c + d*x])/(12
*x^3) - (a^2*d^3*Sinh[c + d*x])/(24*x) + (b^2*x*Sinh[c + d*x])/d + 2*a*b*d
*Cosh[c]*SinhIntegral[d*x] + (a^2*d^4*Sinh[c]*SinhIntegral[d*x])/24
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5810
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_.), x_Symbol] :> Int[ExpandIntegrand[Cosh[c + d*x], (e*x)^m*(a + b*x^n)^p,
 x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(155) = 310$.

Time = 0.87 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.93

method	result
risch	$\frac{-e^{-c}\,\text{expIntegral}_1(dx)a^2d^6x^4-e^c\,\text{expIntegral}_1(-dx)a^2d^6x^4+e^{-dx-c}a^2d^5x^3+48\,e^{-c}\,\text{expIntegral}_1(dx)ab\,d^3x^4-48\,e^c\,\text{expIntegral}_1}{}$
meijerg	$-\frac{2b^2\cosh(c)\sqrt{\pi}\left(-\frac{1}{2\sqrt{\pi}}+\frac{\cosh(dx)}{2\sqrt{\pi}}-\frac{dx\sinh(dx)}{2\sqrt{\pi}}\right)}{d^2}+\frac{b^2\sinh(c)(\cosh(dx)xd-\sinh(dx))}{d^2}+\frac{idab\cosh(c)\sqrt{\pi}\left(\frac{4i\cosh(dx)}{dx\sqrt{\pi}}-\frac{4i\,\text{Shi}(dx}{\sqrt{\pi}}\right.}{2}$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^5,x,method=_RETURNVERBOSE)
```

output
```
1/48/d^2*(-exp(-c)*Ei(1,d*x)*a^2*d^6*x^4-exp(c)*Ei(1,-d*x)*a^2*d^6*x^4+exp
(-d*x-c)*a^2*d^5*x^3+48*exp(-c)*Ei(1,d*x)*a*b*d^3*x^4-48*exp(c)*Ei(1,-d*x)
*a*b*d^3*x^4-exp(d*x+c)*a^2*d^5*x^3-exp(-d*x-c)*a^2*d^4*x^2-24*exp(-d*x-c)
*b^2*d*x^5-exp(d*x+c)*a^2*d^4*x^2+24*exp(d*x+c)*b^2*d*x^5-48*a*b*d^2*x^3*e
xp(-d*x-c)-48*a*b*d^2*x^3*exp(d*x+c)+2*exp(-d*x-c)*a^2*d^3*x-24*exp(-d*x-c
)*b^2*x^4-2*exp(d*x+c)*a^2*d^3*x-24*exp(d*x+c)*b^2*x^4-6*a^2*d^2*exp(-d*x-
c)-6*a^2*d^2*exp(d*x+c))/x^4
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx^3)^2\cosh(c+dx)}{x^5}\,dx = -\frac{2\,(a^2d^4x^2+48\,abd^2x^3+24\,b^2x^4+6\,a^2d^2)\cosh{(dx+c)}-((a^2d^6+48\,abd^3)x^4\text{Ei}(dx)+(a^2d^6-48\,abd}{}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="fricas")
```

output
```
-1/48*(2*(a^2*d^4*x^2 + 48*a*b*d^2*x^3 + 24*b^2*x^4 + 6*a^2*d^2)*cosh(d*x
+ c) - ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) + (a^2*d^6 - 48*a*b*d^3)*x^4*Ei
(-d*x))*cosh(c) + 2*(a^2*d^5*x^3 - 24*b^2*d*x^5 + 2*a^2*d^3*x)*sinh(d*x +
c) - ((a^2*d^6 + 48*a*b*d^3)*x^4*Ei(d*x) - (a^2*d^6 - 48*a*b*d^3)*x^4*Ei(-
d*x))*sinh(c))/(d^2*x^4)
```

Sympy [F]

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5}\,dx = \int \frac{(a+bx^3)^2 \cosh{(c+dx)}}{x^5}\,dx$$

input
```
integrate((b*x**3+a)**2*cosh(d*x+c)/x**5,x)
```

output
```
Integral((a + b*x**3)**2*cosh(c + d*x)/x**5, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5}\,dx$$
$$= \frac{1}{8}\left(a^2d^3e^{(-c)}\Gamma(-3,dx) + a^2d^3e^{c}\Gamma(-3,-dx) - 8\,ab\mathrm{Ei}(-dx)\,e^{(-c)} + 8\,ab\mathrm{Ei}(dx)\,e^{c} - \frac{2\,(d^2x^2e^c - 2\,dxe^c +}{d^3}\right.$$
$$+ \frac{1}{4}\left(2\,b^2x^2 - \frac{8\,abx^3 + a^2}{x^4}\right)\cosh{(dx+c)}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="maxima")
```

output
```
1/8*(a^2*d^3*e^(-c)*gamma(-3, d*x) + a^2*d^3*e^c*gamma(-3, -d*x) - 8*a*b*E
i(-d*x)*e^(-c) + 8*a*b*Ei(d*x)*e^c - 2*(d^2*x^2*e^c - 2*d*x*e^c + 2*e^c)*b
^2*e^(d*x)/d^3 - 2*(d^2*x^2 + 2*d*x + 2)*b^2*e^(-d*x - c)/d^3)*d + 1/4*(2*
b^2*x^2 - (8*a*b*x^3 + a^2)/x^4)*cosh(d*x + c)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(155) = 310$.

Time = 0.11 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.89

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx$$
$$= \frac{a^2d^6x^4\text{Ei}(-dx)\,e^{(-c)} + a^2d^6x^4\text{Ei}(dx)\,e^c - a^2d^5x^3e^{(dx+c)} + a^2d^5x^3e^{(-dx-c)} - 48\,abd^3x^4\text{Ei}(-dx)\,e^{(-c)} + 48}{}$$

input
```
integrate((b*x^3+a)^2*cosh(d*x+c)/x^5,x, algorithm="giac")
```

output
```
1/48*(a^2*d^6*x^4*Ei(-d*x)*e^(-c) + a^2*d^6*x^4*Ei(d*x)*e^c - a^2*d^5*x^3*
e^(d*x + c) + a^2*d^5*x^3*e^(-d*x - c) - 48*a*b*d^3*x^4*Ei(-d*x)*e^(-c) +
48*a*b*d^3*x^4*Ei(d*x)*e^c - a^2*d^4*x^2*e^(d*x + c) + 24*b^2*d*x^5*e^(d*x
 + c) - a^2*d^4*x^2*e^(-d*x - c) - 24*b^2*d*x^5*e^(-d*x - c) - 48*a*b*d^2*
x^3*e^(d*x + c) - 48*a*b*d^2*x^3*e^(-d*x - c) - 2*a^2*d^3*x*e^(d*x + c) -
24*b^2*x^4*e^(d*x + c) + 2*a^2*d^3*x*e^(-d*x - c) - 24*b^2*x^4*e^(-d*x - c
) - 6*a^2*d^2*e^(d*x + c) - 6*a^2*d^2*e^(-d*x - c))/(d^2*x^4)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5} dx = \int \frac{\cosh(c+d\,x)\,\left(b\,x^3+a\right)^2}{x^5} dx$$

input
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^5,x)
```

output
```
int((cosh(c + d*x)*(a + b*x^3)^2)/x^5, x)
```

Reduce [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 305, normalized size of antiderivative = 1.83

$$\int \frac{(a+bx^3)^2 \cosh(c+dx)}{x^5}\,dx$$
$$= \frac{e^{dx} ei(-dx)\, a^2 d^6 x^4 - 48 e^{dx} ei(-dx)\, ab\, d^3 x^4 + e^{dx+2c} ei(dx)\, a^2 d^6 x^4 + 48 e^{dx+2c} ei(dx)\, ab\, d^3 x^4 - e^{2dx+2c} a^2 d^5}{}$$

input
```
int((b*x^3+a)^2*cosh(d*x+c)/x^5,x)
```

output
```
(e**(d*x)*ei( - d*x)*a**2*d**6*x**4 - 48*e**(d*x)*ei( - d*x)*a*b*d**3*x**4
 + e**(2*c + d*x)*ei(d*x)*a**2*d**6*x**4 + 48*e**(2*c + d*x)*ei(d*x)*a*b*d
**3*x**4 - e**(2*c + 2*d*x)*a**2*d**5*x**3 - e**(2*c + 2*d*x)*a**2*d**4*x*
*2 - 2*e**(2*c + 2*d*x)*a**2*d**3*x - 6*e**(2*c + 2*d*x)*a**2*d**2 - 48*e*
*(2*c + 2*d*x)*a*b*d**2*x**3 + 24*e**(2*c + 2*d*x)*b**2*d*x**5 - 24*e**(2*
c + 2*d*x)*b**2*x**4 + a**2*d**5*x**3 - a**2*d**4*x**2 + 2*a**2*d**3*x - 6
*a**2*d**2 - 48*a*b*d**2*x**3 - 24*b**2*d*x**5 - 24*b**2*x**4)/(48*e**(c +
 d*x)*d**2*x**4)
```

3.94 $\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 373

$$\begin{aligned}\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx = &-\frac{\cosh(c+dx)}{bd^2}\\ &+\frac{(-1)^{2/3}a^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}\\ &-\frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}\\ &+\frac{a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}+\frac{x\sinh(c+dx)}{bd}\\ &-\frac{(-1)^{2/3}a^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}}\\ &+\frac{a^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}\\ &-\frac{\sqrt[3]{-1}a^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{5/3}}\end{aligned}$$

output
```
-cosh(d*x+c)/b/d^2+1/3*(-1)^(2/3)*a^(2/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1
/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/b^(5/3)-1/3*(-1)^(1/3)*a^(2/3)*
cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x
)/b^(5/3)+1/3*a^(2/3)*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)
/b^(5/3)+x*sinh(d*x+c)/b/d+1/3*(-1)^(2/3)*a^(2/3)*sinh(c+(-1)^(1/3)*a^(1/3
)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)+1/3*a^(2/3)*si
nh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/b^(5/3)-1/3*(-1)^(1/3)*
a^(2/3)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1
/3)+d*x)/b^(5/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 213, normalized size of antiderivative = 0.57

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3} \, dx =$$

$$-\frac{ad^2\text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\text{si}}{\#1}\right.}{}$$

input
```
Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
-1/6*(a*d^2*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)
] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[
d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]])/#1 & ] + a*d^2*Root
Sum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral
[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Si
nh[c + d*#1]*SinhIntegral[d*(x - #1)]])/#1 & ] + 6*b*(Cosh[c + d*x] - d*x*S
inh[c + d*x]))/(b^2*d^2)
```

Rubi [A] (verified)

Time = 1.16 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3} \, dx$$

$$\Big\downarrow \text{5816}$$

$$\int \left(\frac{x\cosh(c+dx)}{b} - \frac{ax\cosh(c+dx)}{b\left(a+bx^3\right)}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{(-1)^{2/3}a^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} + \frac{a^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} - \frac{(-1)^{2/3}a^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{5/3}} + \frac{a^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} - \frac{\sqrt[3]{-1}a^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{5/3}} - \frac{\cosh(c+dx)}{bd^2} + \frac{x\sinh(c+dx)}{bd}$$

input
```
Int[(x^4*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
-(Cosh[c + d*x]/(b*d^2)) + ((-1)^(2/3)*a^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(5/
3)) - ((-1)^(1/3)*a^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b^(5/3)) + (a^(2/3)*Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(
5/3)) + (x*Sinh[c + d*x])/(b*d) - ((-1)^(2/3)*a^(2/3)*Sinh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(
3*b^(5/3)) + (a^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*
d)/b^(1/3) + d*x])/(3*b^(5/3)) - ((-1)^(1/3)*a^(2/3)*Sinh[c - ((-1)^(2/3)*
a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3
*b^(5/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.69 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.48

method	result	size
risch	Expression too large to display	925

input
```
int(x^4*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/d^2/b*c^4*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=Roo
tOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2/b*c^4*sum(1/(_R1^2-
2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))+2/3/d^2/b*c^3*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1
,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+2/3/d^2
/b*c^3*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d/b*exp(-d*x-c)*x-1/d^2/b*c^2
*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b
-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*exp(d*x+c)*x-1/d^2/b*c^2*sum(
_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z
^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d^2/b*exp(-d*x-c)+2/3/d^2/b^2*sum((3*_
R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_
R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*c+2/3/d^2/b^2*
sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(
1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*c-1/2/
d^2/b*exp(d*x+c)-1/6/d^2/b^2*sum((6*_R1^2*b*c^2-_R1*a*d^3-8*_R1*b*c^3-3*a*
c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d^2/b^2*sum((6*_R1^2*b*c^2-_R
1*a*d^3-8*_R1*b*c^3-3*a*c*d^3+3*b*c^4)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,
d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 989 vs. $2(265) = 530$.

Time = 0.14 (sec) , antiderivative size = 989, normalized size of antiderivative = 2.65

$$\int \frac{x^4 \cosh(c + dx)}{a + bx^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*((a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sin
h(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x
+ c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(s
qrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2
/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d
*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1) - c) - (-a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3)
 + 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh
(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*(cosh(d*x +
 c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(
1/3)) + 2*(a*d^3/b)^(2/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*
d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(2/3)*((sqrt(-3) - 1)
*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^
(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d
^3/b)^(2/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)
^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(2/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 -
 (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) -
1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(2/3)*((s...
```

Sympy [F]

$$\int \frac{x^4\cosh(c+dx)}{a+bx^3}\,dx=\int \frac{x^4\cosh{(c+dx)}}{a+bx^3}\,dx$$

input
```
integrate(x**4*cosh(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x**4*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3} \, dx = \text{Timed out}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3} \, dx = \int \frac{x^4 \cosh(dx+c)}{bx^3+a} \, dx$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^4*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3} \, dx = \int \frac{x^4 \cosh(c+d\,x)}{b\,x^3+a} \, dx$$

input
```
int((x^4*cosh(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^4*cosh(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^4 \cosh(c+dx)}{a+bx^3}\,dx = \frac{-\cosh(dx+c) - \left(\int \frac{\cosh(dx+c)x}{b\,x^3+a}dx\right) a\,d^2 + \sinh(dx+c)\,dx}{b\,d^2}$$

input
```
int(x^4*cosh(d*x+c)/(b*x^3+a),x)
```

output
```
( - cosh(c + d*x) - int((cosh(c + d*x)*x)/(a + b*x**3),x)*a*d**2 + sinh(c
+ d*x)*d*x)/(b*d**2)
```

3.95 $\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 358

$$\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx = \frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}} + \frac{\sinh(c+dx)}{bd} - \frac{\sqrt[3]{-1}\sqrt[3]{a}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}} - \frac{\sqrt[3]{a}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}} - \frac{(-1)^{2/3}\sqrt[3]{a}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b^{4/3}}$$

output
```
1/3*(-1)^(1/3)*a^(1/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)
*a^(1/3)*d/b^(1/3)-d*x)/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*cosh(c-(-1)^(2/3)*a
^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/b^(4/3)-1/3*a^(1/
3)*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)+sinh(d*x+c
)/b/d+1/3*(-1)^(1/3)*a^(1/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1
)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*a^(1/3)*sinh(c-a^(1/3)*d/b^(1/3
))*Shi(a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)-1/3*(-1)^(2/3)*a^(1/3)*sinh(c-(-1)^(
2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.55

$$\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx =$$
$$-\frac{ad\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sin}{\#1^2}\right.}{}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
-1/6*(a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
- CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*
(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + a*d*RootSu
m[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d
*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh
[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] - 6*b*Sinh[c + d*x])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 358, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3\cosh(c+dx)}{a+bx^3}dx$$

$$\Big\downarrow \text{ 5816}$$

$$\int \left(\frac{\cosh(c+dx)}{b}-\frac{a\cosh(c+dx)}{b\,(a+bx^3)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt[3]{-1}\sqrt[3]{a}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{a}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b^{4/3}}-\frac{\sqrt[3]{a}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{a}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b^{4/3}}+\frac{\sinh(c+dx)}{bd}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
((-1)^(1/3)*a^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - ((-1)^(2/3)*a^(1/3)*C
osh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)
*d)/b^(1/3)) - d*x])/(3*b^(4/3)) - (a^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*
CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3)) + Sinh[c + d*x]/(b*d)
 - ((-1)^(1/3)*a^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegr
al[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*b^(4/3)) - (a^(1/3)*Sinh[c -
(a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3))
- ((-1)^(2/3)*a^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.60 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.87

method	result
risch	$\frac{c^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+d^3a-bc^3\right)}\frac{e^{\_R1}\mathrm{expIntegral}_1\left(-dx+\_R1-c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6db}+\frac{c^3\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+\right.}\right.}{}$

input
```
int(x^3*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6/d/b*c^3*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d/b*c^3*sum(1/(_R1^2-2*_R1
*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+
a*d^3-b*c^3))-1/2/d/b*c^2*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_
R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/d/b*c^2*su
m(_R1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z
^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/d/b*c*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*ex
p(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))+1/2/d/b*c*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=
RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/2/b/d*exp(d*x+c)-1/2/d
/b*exp(-d*x-c)-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R1^2-
2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))-1/6/d/b^2*sum((3*_R1^2*b*c-3*_R1*b*c^2-a*d^3+b*c^3)/(_R
1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 977 vs. $2(250) = 500$.

Time = 0.12 (sec) , antiderivative size = 977, normalized size of antiderivative = 2.73

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/12*((a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sin
h(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x
+ c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(s
qrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1
/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d
*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1) - c) + (-a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 - (sqrt(-3)
 - 1)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh
(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*(cosh(d*x +
 c)^2 - sinh(d*x + c)^2)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(
1/3)) - 2*(a*d^3/b)^(1/3)*(cosh(d*x + c)^2 - sinh(d*x + c)^2)*Ei(d*x + (a*
d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((sqrt(-3) + 1)
*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)^
(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d
^3/b)^(1/3)*((sqrt(-3) + 1)*cosh(d*x + c)^2 - (sqrt(-3) + 1)*sinh(d*x + c)
^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(1/3)*((sqrt(-3) - 1)*cosh(d*x + c)^2 -
 (sqrt(-3) - 1)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) -
1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((s...
```

Sympy [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x^3 \cosh{(c+dx)}}{a+bx^3}\,dx$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x**3*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3} dx = \text{Timed out}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3} dx = \int \frac{x^3 \cosh(dx+c)}{bx^3+a} dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^3*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{a+bx^3} dx = \int \frac{x^3 \cosh(c+d\,x)}{b\,x^3+a} dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^3\cosh(c+dx)}{a+bx^3}\,dx = \frac{-\left(\int \frac{\cosh(dx+c)}{b\,x^3+a}dx\right)ad+\sinh\left(dx+c\right)}{bd}$$

input
```
int(x^3*cosh(d*x+c)/(b*x^3+a),x)
```

output
```
( - int(cosh(c + d*x)/(a + b*x**3),x)*a*d + sinh(c + d*x))/(b*d)
```

3.96 $\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 283

$$\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx = \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b} - \frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b} + \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b} + \frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3b}$$

output
```
1/3*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-
d*x)/b+1/3*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/
b^(1/3)-d*x)/b+1/3*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/b+
1/3*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)
+d*x)/b+1/3*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/b+1/3*sin
h(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.04 (sec) , antiderivative size = 170, normalized size of antiderivative = 0.60

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3} \, dx = \frac{\text{RootSum}\left[a+b\#1^3\&, \cosh(c+d\#1)\text{Chi}(d(x-\#1)) - \text{Chi}(d(x-\#1))\sinh(c+d\#1) - \cosh(c+d\#1\right.}{}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
(RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInte
gral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
+ Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] & ] + RootSum[a + b*#1^3 & , Cos
h[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d
*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegr
al[d*(x - #1)] & ])/(6*b)
```

Rubi [A] (verified)

Time = 0.74 (sec) , antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3} \, dx$$

$$\Big\downarrow \text{5816}$$

$$\int \left(\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left(\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right)}+\frac{\cosh(c+dx)}{3b^{2/3}\left((-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}-\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3b}+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}+\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3b}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
(Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)
*d)/b^(1/3) - d*x])/(3*b) + (Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Cosh
Integral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*b) + (Cosh[c - (a^(1
/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*b) - (Sinh[c +
 ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/
3) - d*x])/(3*b) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)
/b^(1/3) + d*x])/(3*b) + (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhInt
egral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.58 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.49

method	result
risch	$-\frac{c^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+d^3a-bc^3\right)}\frac{e^{\_R1}\,\text{expIntegral}_1\left(-dx+\_R1-c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}-\frac{c^2\left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z\right.}\right.}{}$

input
```
int(x^2*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6/b*c^2*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*c^2*sum(1/(_R1^2-2*_R1*c+
c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))+1/3/b*c*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_
R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/3/b*c*sum(_R1/(_R1^
2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))-1/6/b*sum(_R1^2/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*
x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/b*sum(_
R1^2/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(207) = 414$.

Time = 0.10 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.77

$$\int \frac{x^2 \cosh(c + dx)}{a + bx^3} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)
*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh
(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1/
2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1
) + c) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + Ei(d*x +
 (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) + Ei(d*x - 1/2*(a*d^3/b)^(1/3
)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + Ei(-d*x -
 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3)
+ 1) - c) - Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b
)^(1/3)*(sqrt(-3) - 1) - c) - Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1
))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - Ei(-d*x + (-a*d^3/b)^(1
/3))*sinh(c + (-a*d^3/b)^(1/3)) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d
^3/b)^(1/3)))/b
```

Sympy [F]

$$\int \frac{x^2 \cosh(c + dx)}{a + bx^3} \, dx = \int \frac{x^2 \cosh(c + dx)}{a + bx^3} \, dx$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x**2*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F]

$$\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx=\int \frac{x^2\cosh\left(dx+c\right)}{bx^3+a}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
1/2*((d*x^2*e^(2*c) + x*e^(2*c))*e^(d*x) - (d*x^2 - x)*e^(-d*x))/(b*d^2*x^
3*e^c + a*d^2*e^c) + 1/2*integrate((2*b*x^3*e^c - 3*a*d*x*e^c - a*e^c)*e^(
d*x)/(b^2*d^2*x^6 + 2*a*b*d^2*x^3 + a^2*d^2), x) + 1/2*integrate((2*b*x^3
+ 3*a*d*x - a)*e^(-d*x)/(b^2*d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c
), x)
```

Giac [F]

$$\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx=\int \frac{x^2\cosh\left(dx+c\right)}{bx^3+a}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x^2*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\cosh(c+dx)}{a+bx^3}\,dx=\int \frac{x^2\cosh(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x^3),x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{a+bx^3}\, dx = \int \frac{\cosh\left(dx+c\right) x^2}{b\, x^3+a}dx$$

input
```
int(x^2*cosh(d*x+c)/(b*x^3+a),x)
```

output
```
int((cosh(c + d*x)*x**2)/(a + b*x**3),x)
```

3.97 $\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 345

$$\begin{aligned}\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx =& -\frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}\\
&+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}\\
&-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\\
&+\frac{(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}\\
&-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\\
&+\frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3\sqrt[3]{a}b^{2/3}}\end{aligned}$$

output
```
-1/3*(-1)^(2/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*cosh(c-(-1)^(2/3)*a^(1/3)*
d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(1/3)/b^(2/3)-1/3*cosh
(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(2/3)-1/3*(-1)^
(2/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/
3)+d*x)/a^(1/3)/b^(2/3)-1/3*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3
)+d*x)/a^(1/3)/b^(2/3)+1/3*(-1)^(1/3)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.05 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \frac{\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d\#1)\text{Shi}(d(x-\#1))}{\#1}\right.}{}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
(RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInt
egral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
 + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] + RootSum[a + b*#1^3 &
, (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh
[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*Sinh
Integral[d*(x - #1)])/#1 & ])/(6*b)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{a+bx^3}dx$$

$$\Big\downarrow \text{5816}$$

$$\int \left(-\frac{\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}x\right)}-\frac{(-1)^{2/3}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}\cosh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{(-1)^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{(-1)^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x^3),x]
```

output
```
-1/3*((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c
 - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b
^(1/3)) - d*x])/(3*a^(1/3)*b^(2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Sinh
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b
^(1/3) - d*x])/(3*a^(1/3)*b^(2/3)) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Sinh[
c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x])/(3*a^(1/3)*b^(2/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.54 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.81

method	result
risch	$\frac{dc\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+d^3a-b\,c^3\right)}\frac{e^{\_R1}\,\mathrm{expIntegral}_1\left(-dx+\_R1-c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}+\frac{dc\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+\right.}\right.}{}$

input
```
int(x*cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
1/6*d/b*c*sum(1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_
Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*d/b*c*sum(1/(_R1^2-2*_R1*c+c
^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))-1/6*d/b*sum(_R1/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R
1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d/b*sum(_R1/(_R1^2
-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*
b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(237) = 474$.

Time = 0.12 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.94

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*((a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(
sqrt(-3) - 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x +
 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) -
1) - c) - (-a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(
sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(
2/3)*(sqrt(-3) - 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-a*d^3/b)^(2/3)*(sqrt(-3) - 1)*Ei(-
d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt
(-3) + 1) - c) + (a*d^3/b)^(2/3)*(sqrt(-3) + 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/
3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/
b)^(2/3)*(sqrt(-3) + 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sin
h(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(2/3)*Ei(-d*x +
(-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(2/3)*Ei(d*x +
(a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(2/3)*Ei(-d*x +
 (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(2/3)*Ei(d*x +
 (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/(a*d^2)
```

Sympy [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x\cosh(c+dx)}{a+bx^3}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(x*cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{x\cosh(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(x*cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx=\int \frac{x\cosh(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x^3),x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{a+bx^3}\,dx=\int \frac{\cosh(dx+c)\,x}{b\,x^3+a}dx$$

input
```
int(x*cosh(d*x+c)/(b*x^3+a),x)
```

output
```
int((cosh(c + d*x)*x)/(a + b*x**3),x)
```

3.98 $\int \frac{\cosh(c+dx)}{a+bx^3}\,dx$

Optimal result

Integrand size = 16, antiderivative size = 345

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+bx^3}\,dx =& -\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}\\ &+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}\\ &+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\\ &+\frac{\sqrt[3]{-1}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}\\ &+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\\ &+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{2/3}\sqrt[3]{b}}\end{aligned}$$

output
```
-1/3*(-1)^(1/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3
)*d/b^(1/3)-d*x)/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*cosh(c-(-1)^(2/3)*a^(1/3)*
d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(2/3)/b^(1/3)+1/3*cosh
(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(1/3)-1/3*(-1)^
(1/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/
3)+d*x)/a^(2/3)/b^(1/3)+1/3*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3
)+d*x)/a^(2/3)/b^(1/3)+1/3*(-1)^(2/3)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 5.04 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.52

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx$$
$$= \frac{\text{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\text{Shi}(d(x-\#1))+\sinh(c+d}{\#1^2}\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x^3),x]
```

output
```
(RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshInt
egral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
 + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + RootSum[a + b*#1^3
& , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Si
nh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*Si
nhIntegral[d*(x - #1)]])/#1^2 & ])/(6*b)
```

Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+bx^3}dx$$

$$\Big\downarrow \text{5804}$$

$$\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}x\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}x\right)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}$$

input
```
Int[Cosh[c + d*x]/(a + b*x^3),x]
```

output
```
-1/3*((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-
1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cosh[c
 - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b
^(1/3)) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Sinh
[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b
^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIn
tegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Sinh[
c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^
(1/3) + d*x])/(3*a^(2/3)*b^(1/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.52 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.41

method	result
risch	$-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+d^3a-bc^3\right)}\frac{e^{-\_R1}\,\mathrm{expIntegral}_1\left(dx-\_R1+c\right)}{\_R1^2-2\_R1c+c^2}\right)}{6b}-\frac{d^2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z\right.}\right.}{}$

input
```
int(cosh(d*x+c)/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6*d^2/b*sum(1/(_R1^2-2*_R1*c+
c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d
^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 673 vs. $2(237) = 474$.

Time = 0.11 (sec) , antiderivative size = 673, normalized size of antiderivative = 1.95

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*((a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-
3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(
sqrt(-3) + 1)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x +
 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) -
1) - c) + (-a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(
sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + (a*d^3/b)^(
1/3)*(sqrt(-3) + 1)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(1/3)*(sqrt(-3) + 1)*Ei(-
d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt
(-3) + 1) - c) + (a*d^3/b)^(1/3)*(sqrt(-3) - 1)*Ei(d*x + 1/2*(a*d^3/b)^(1/
3)*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/
b)^(1/3)*(sqrt(-3) - 1)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sin
h(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*(-a*d^3/b)^(1/3)*Ei(-d*x +
(-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*Ei(d*x +
(a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - 2*(-a*d^3/b)^(1/3)*Ei(-d*x +
 (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) + 2*(a*d^3/b)^(1/3)*Ei(d*x +
 (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(1/3)))/(a*d)
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{\cosh(c+dx)}{a+bx^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x**3+a),x)
```

output
```
Integral(cosh(c + d*x)/(a + b*x**3), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{\cosh(dx+c)}{bx^3+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*x^3 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{\cosh(c+d\,x)}{b\,x^3+a}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x^3),x)
```

output
```
int(cosh(c + d*x)/(a + b*x^3), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{a+bx^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\,x^3+a}dx$$

input
```
int(cosh(d*x+c)/(b*x^3+a),x)
```

output
```
int(cosh(c + d*x)/(a + b*x**3),x)
```

3.99 $\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 303

$$
\begin{aligned}
\int \frac{\cosh(c+dx)}{x\,(a+bx^3)}\,dx &= \frac{\cosh(c)\operatorname{Chi}(dx)}{a} - \frac{\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\
&\quad - \frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\
&\quad - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a} + \frac{\sinh(c)\operatorname{Shi}(dx)}{a} \\
&\quad + \frac{\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a} \\
&\quad - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a} \\
&\quad - \frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a}
\end{aligned}
$$

output
```
cosh(c)*Chi(d*x)/a-1/3*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)
*a^(1/3)*d/b^(1/3)-d*x)/a-1/3*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-
1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a-1/3*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3
)*d/b^(1/3)+d*x)/a+sinh(c)*Shi(d*x)/a-1/3*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1
/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a-1/3*sinh(c-a^(1/3)*d/b^(1/3)
)*Shi(a^(1/3)*d/b^(1/3)+d*x)/a-1/3*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Sh
i((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 186, normalized size of antiderivative = 0.61

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx =$$
$$-\frac{-6\cosh(c)\text{Chi}(dx) + \text{RootSum}\left[a+b\#1^3\&, \cosh(c+d\#1)\text{Chi}(d(x-\#1)) - \text{Chi}(d(x-\#1))\sinh(c\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x^3)),x]
```

output
```
-1/6*(-6*Cosh[c]*CoshIntegral[d*x] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh
[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #
1)] & ] + RootSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] +
 CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(
x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] & ] - 6*Sinh[c]*SinhInt
egral[d*x])/a
```

Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)}\,dx$$

$$\Big\downarrow \; 5816$$

$$\int \left(\frac{\cosh(c+dx)}{ax}-\frac{bx^2\cosh(c+dx)}{a\left(a+bx^3\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}+\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}+\frac{\cosh(c)\text{Chi}(dx)}{a}+\frac{\sinh(c)\text{Shi}(dx)}{a}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x^3)),x]
```

output
```
(Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*
CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
 - d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b
^(1/3) + d*x])/(3*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + ((-1)^(1/
3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])
/(3*a) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) +
 d*x])/(3*a) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1
)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.55 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{e^{-c}\,\text{expIntegral}_1(dx)}{2a}+\frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+d^3a-bc^3\right)} e^{-\_R1}\,\text{expIntegral}_1\left(dx-\_R1+c\right)}{6a}-\frac{e^c\,\text{expInte}}{2}$

input
```
int(cosh(d*x+c)/x/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/a*exp(-c)*Ei(1,d*x)+1/6/a*sum(exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/2/a*exp(c)*Ei(1,-d*x)+1/6/a*sum
(exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b
*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. 2(227) = 454.

Time = 0.12 (sec) , antiderivative size = 530, normalized size of antiderivative = 1.75

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/6*(Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3
)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cos
h(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + Ei(d*x + 1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + Ei(-d*x + 1
/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
1) + c) + Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(d
*x) + Ei(-d*x))*cosh(c) + Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1
/3)) + Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/
3)*(sqrt(-3) + 1) + c) + Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*si
nh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - Ei(d*x + 1/2*(a*d^3/b)^(1/3)
*(sqrt(-3) - 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) - Ei(-d*x +
1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) -
 1) + c) - Ei(-d*x + (-a*d^3/b)^(1/3))*sinh(c + (-a*d^3/b)^(1/3)) - 3*(Ei(
d*x) - Ei(-d*x))*sinh(c) - Ei(d*x + (a*d^3/b)^(1/3))*sinh(-c + (a*d^3/b)^(
1/3)))/a
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\cosh{(c+dx)}}{x\,(a+bx^3)}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x**3+a),x)
```

output
```
Integral(cosh(c + d*x)/(x*(a + b*x**3)), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)}\,dx = \int \frac{\cosh{(dx+c)}}{(bx^3+a)x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\cosh(dx+c)}{(bx^3+a)x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^3 + a)*x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\cosh(c+d\,x)}{x\,(b\,x^3+a)}\,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x^3)),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)}\,dx = \int \frac{\cosh(dx+c)}{b\,x^4+ax}dx$$

input
```
int(cosh(d*x+c)/x/(b*x^3+a),x)
```

output
```
int(cosh(c + d*x)/(a*x + b*x**4),x)
```

3.100 $\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 381

$$\begin{aligned}\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx = & -\frac{\cosh(c+dx)}{ax} \\ & + \frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\ & - \frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\ & + \frac{\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}} \\ & + \frac{d\text{Chi}(dx)\sinh(c)}{a} + \frac{d\cosh(c)\text{Shi}(dx)}{a} \\ & - \frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}} \\ & + \frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}} \\ & - \frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{4/3}}\end{aligned}$$

output
```
-cosh(d*x+c)/a/x+1/3*(-1)^(2/3)*b^(1/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3
))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*co
sh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/
a^(4/3)+1/3*b^(1/3)*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a
^(4/3)+d*Chi(d*x)*sinh(c)/a+d*cosh(c)*Shi(d*x)/a+1/3*(-1)^(2/3)*b^(1/3)*si
nh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/
a^(4/3)+1/3*b^(1/3)*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a
^(4/3)-1/3*(-1)^(1/3)*b^(1/3)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1
)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.18 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.56

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx =$$
$$-\frac{6\cosh(c+dx)+x\text{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1}{\#1}\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

output
```
-1/6*(6*Cosh[c + d*x] + x*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshInteg
ral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]
*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 &
] + x*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + Co
shIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x -
 #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1 & ] - 6*d*x*CoshIntegr
al[d*x]*Sinh[c] - 6*d*x*Cosh[c]*SinhIntegral[d*x])/(a*x)
```

Rubi [A] (verified)

Time = 0.96 (sec) , antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx$$

$$\Big\downarrow \; 5816$$

$$\int \left(\frac{\cosh(c+dx)}{ax^2}-\frac{bx\cosh(c+dx)}{a(a+bx^3)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{d\sinh(c)\text{Chi}(dx)}{a}+\frac{d\cosh(c)\text{Shi}(dx)}{a}-\frac{\cosh(c+dx)}{ax}$$

input
```
Int[Cosh[c + d*x]/(x^2*(a + b*x^3)),x]
```

output
```
-(Cosh[c + d*x]/(a*x)) + ((-1)^(2/3)*b^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*
d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)
) - ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshInteg
ral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cosh[
c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/
3)) + (d*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a -
((-1)^(2/3)*b^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Sinh[c - (a^
(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - (
(-1)^(1/3)*b^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
(-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.65 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.46

method	result
risch	$-\frac{3\,\mathrm{e}^{c}\,\mathrm{expIntegral}_1(-dx)dx-3\,\mathrm{e}^{-c}\,\mathrm{expIntegral}_1(dx)dx-\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+d^3a-b\,c^3\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{expIntegral}_1\left(-c\right.}{\_R1-}\right.}{}$

input
```
int(cosh(d*x+c)/x^2/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(3*exp(c)*Ei(1,-d*x)*d*x-3*exp(-c)*Ei(1,d*x)*d*x-sum(1/(_R1-c)*exp(_R
1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*
d*x-sum(1/(_R1-c)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3
*_Z*b*c^2+a*d^3-b*c^3))*d*x+3*exp(-d*x-c)+3*exp(d*x+c))/x/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1154 vs. $2(273) = 546$.

Time = 0.13 (sec) , antiderivative size = 1154, normalized size of antiderivative = 3.03

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*(12*a*d^2*cosh(d*x + c) - (a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/b)
^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) - (-a*
d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*
sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(
-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x
)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*
d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c)
+ (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x +
 b*x)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh
(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(b*x*cosh(d*x + c)^2 - b*x*s
inh(d*x + c)^2)*(-a*d^3/b)^(2/3)*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*
d^3/b)^(1/3)) - 2*(b*x*cosh(d*x + c)^2 - b*x*sinh(d*x + c)^2)*(a*d^3/b)^(2
/3)*Ei(d*x + (a*d^3/b)^(1/3))*cosh(-c + (a*d^3/b)^(1/3)) - (a*d^3/b)^(2/3)
*((sqrt(-3)*b*x - b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c
)^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)
*(sqrt(-3) + 1) + c) - (-a*d^3/b)^(2/3)*((sqrt(-3)*b*x - b*x)*cosh(d*x + c
)^2 - (sqrt(-3)*b*x - b*x)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)
*(sqrt(-3) + 1))*sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)
^(2/3)*((sqrt(-3)*b*x + b*x)*cosh(d*x + c)^2 - (sqrt(-3)*b*x + b*x)*sin...
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx$$

input
```
integrate(cosh(d*x+c)/x**2/(b*x**3+a),x)
```

output
```
Integral(cosh(c + d*x)/(x**2*(a + b*x**3)), x)
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^2(a+bx^3)}\,dx = \int \frac{\cosh(dx+c)}{(bx^3+a)x^2}\,dx$$

input
```
integrate(cosh(d*x+c)/x^2/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^3 + a)*x^2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^2\,(a+bx^3)}\,dx = \int \frac{\cosh(c+d\,x)}{x^2\,\left(b\,x^3+a\right)}\,dx$$

input
```
int(cosh(c + d*x)/(x^2*(a + b*x^3)),x)
```

output
```
int(cosh(c + d*x)/(x^2*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^2\,(a+bx^3)}\,dx = \int \frac{\cosh\left(dx+c\right)}{b\,x^5+a\,x^2}dx$$

input
```
int(cosh(d*x+c)/x^2/(b*x^3+a),x)
```

output
```
int(cosh(c + d*x)/(a*x**2 + b*x**5),x)
```

3.101 $\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 410

$$\begin{aligned}
\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx &= -\frac{\cosh(c+dx)}{2ax^2}+\frac{d^2\cosh(c)\mathrm{Chi}(dx)}{2a} \\
&+\frac{\sqrt[3]{-1}b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} \\
&-\frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} \\
&-\frac{b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} \\
&-\frac{d\sinh(c+dx)}{2ax}+\frac{d^2\sinh(c)\mathrm{Shi}(dx)}{2a} \\
&-\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}} \\
&-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}} \\
&-\frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{3a^{5/3}}
\end{aligned}$$

output
```
-1/2*cosh(d*x+c)/a/x^2+1/2*d^2*cosh(c)*Chi(d*x)/a+1/3*(-1)^(1/3)*b^(2/3)*c
osh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/
a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(
-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(5/3)-1/3*b^(2/3)*cosh(c-a^(1/3)*d/b^(1
/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)-1/2*d*sinh(d*x+c)/a/x+1/2*d^2*sinh
(c)*Shi(d*x)/a+1/3*(-1)^(1/3)*b^(2/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))
*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)-1/3*b^(2/3)*sinh(c-a^(1/3)
*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)-1/3*(-1)^(2/3)*b^(2/3)*sinh
(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(
5/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.19 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.58

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx =$$

$$-\frac{3\cosh(c+dx)-3d^2x^2\cosh(c)\mathrm{Chi}(dx)+x^2\mathrm{RootSum}\left[a+b\#1^3\&,\frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#\ldots}{\ldots}\right.}{\ldots}$$

input
```
Integrate[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```

output
```
-1/6*(3*Cosh[c + d*x] - 3*d^2*x^2*Cosh[c]*CoshIntegral[d*x] + x^2*RootSum[
a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(
x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c
 + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ] + x^2*RootSum[a + b*#1^3 & , (
Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c
+ d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhInt
egral[d*(x - #1)])/#1^2 & ] + 3*d*x*Sinh[c + d*x] - 3*d^2*x^2*Sinh[c]*Sinh
Integral[d*x])/(a*x^2)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 410, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}dx$$

$$\Big\downarrow \ 5816$$

$$\int \left(\frac{\cosh(c+dx)}{ax^3}-\frac{b\cosh(c+dx)}{a(a+bx^3)}\right)dx$$

$$\Big\downarrow \ 2009$$

$$\frac{\sqrt[3]{-1}b^{2/3}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{b^{2/3}\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Shi}\left(xd+\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{d^2\cosh(c)\text{Chi}(dx)}{2a}+\frac{d^2\sinh(c)\text{Shi}(dx)}{2a}-\frac{\cosh(c+dx)}{2ax^2}-\frac{d\sinh(c+dx)}{2ax}$$

input
```
Int[Cosh[c + d*x]/(x^3*(a + b*x^3)),x]
```

output
```
-1/2*Cosh[c + d*x]/(a*x^2) + (d^2*Cosh[c]*CoshIntegral[d*x])/(2*a) + ((-1)
^(1/3)*b^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)
^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c
 - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b
^(1/3)) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshI
ntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - (d*Sinh[c + d*x])/(2*a*x
) + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Sinh[c + (
(-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
 - d*x])/(3*a^(5/3)) - (b^(2/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral
[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Sinh[c - ((
-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3)
+ d*x])/(3*a^(5/3))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.70 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.55

method	result
risch	$-\frac{3\,\mathrm{e}^{c}\,\mathrm{expIntegral}_1(-dx)d^2x^2+3\,\mathrm{e}^{-c}\,\mathrm{expIntegral}_1(dx)d^2x^2-2\left(\sum_{\_R1=\mathrm{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+d^3a-b\,c^3\right)}\frac{\mathrm{e}^{\_R1}\,\mathrm{expIntegra}}{\_R1^2-}\right.}{}$

input
```
int(cosh(d*x+c)/x^3/(b*x^3+a),x,method=_RETURNVERBOSE)
```

output
```
-1/12*(3*exp(c)*Ei(1,-d*x)*d^2*x^2+3*exp(-c)*Ei(1,d*x)*d^2*x^2-2*sum(1/(_R
1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))*d^2*x^2-2*sum(1/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,
d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*d^2*x^2-3
*exp(-d*x-c)*d*x+3*d*x*exp(d*x+c)+3*exp(-d*x-c)+3*exp(d*x+c))/a/x^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. $2(294) = 588$.

Time = 0.14 (sec) , antiderivative size = 1251, normalized size of antiderivative = 3.05

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="fricas")
```

output
```
-1/12*(6*a*d^2*x*sinh(d*x + c) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)
*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*
(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) +
c) + (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3
)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3)
 + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^3/b)^(1/3)*((s
qrt(-3)*b*x^2 - b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x
 + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(
1/3)*(sqrt(-3) - 1) - c) - (-a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 - b*x^2)*cosh
(d*x + c)^2 - (sqrt(-3)*b*x^2 - b*x^2)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c)
 - 2*(b*x^2*cosh(d*x + c)^2 - b*x^2*sinh(d*x + c)^2)*(-a*d^3/b)^(1/3)*Ei(-
d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) + 2*(b*x^2*cosh(d*x + c
)^2 - b*x^2*sinh(d*x + c)^2)*(a*d^3/b)^(1/3)*Ei(d*x + (a*d^3/b)^(1/3))*cos
h(-c + (a*d^3/b)^(1/3)) - (a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d
*x + c)^2 - (sqrt(-3)*b*x^2 + b*x^2)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3/
b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (-
a*d^3/b)^(1/3)*((sqrt(-3)*b*x^2 + b*x^2)*cosh(d*x + c)^2 - (sqrt(-3)*b*x^2
 + b*x^2)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*
sinh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - (a*d^3/b)^(1/3)*((sqrt(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3\,(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x**3/(b*x**3+a),x)
```

output
```
Timed out
```

Maxima [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3\,(a+bx^3)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="maxima")
```

output
```
Timed out
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{x^3\,(a+bx^3)}\,dx = \int \frac{\cosh\,(dx+c)}{(bx^3+a)x^3}\,dx$$

input
```
integrate(cosh(d*x+c)/x^3/(b*x^3+a),x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/((b*x^3 + a)*x^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\cosh(c+d\,x)}{x^3\,(b\,x^3+a)}\,dx$$

input
```
int(cosh(c + d*x)/(x^3*(a + b*x^3)),x)
```

output
```
int(cosh(c + d*x)/(x^3*(a + b*x^3)), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x^3(a+bx^3)}\,dx = \int \frac{\cosh(dx+c)}{b\,x^6+a\,x^3}dx$$

input
```
int(cosh(d*x+c)/x^3/(b*x^3+a),x)
```

output
```
int(cosh(c + d*x)/(a*x**3 + b*x**6),x)
```

3.102 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 718

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} dx = \text{Too large to display}$$

output
```
-1/3*x*cosh(d*x+c)/b/(b*x^3+a)-1/9*(-1)^(1/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/
b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(2/3)/b^(4/3)+1/9*(-1)^(2
/3)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)
-d*x)/a^(2/3)/b^(4/3)+1/9*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+
d*x)/a^(2/3)/b^(4/3)-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(
1/3))/a^(1/3)/b^(5/3)-1/9*(-1)^(2/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*
x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(1/3)/b^(5/3)+1/9*(-1)^(1/3)*d*C
hi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
/a^(1/3)/b^(5/3)-1/9*(-1)^(2/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi
(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)-1/9*(-1)^(1/3)*sinh(c+
(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/
3)/b^(4/3)-1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(1
/3)/b^(5/3)+1/9*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(2/
3)/b^(4/3)+1/9*(-1)^(1/3)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^
(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(5/3)+1/9*(-1)^(2/3)*sinh(c-(-1)^(2
/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/
3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.16 (sec) , antiderivative size = 363, normalized size of antiderivative = 0.51

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx$$
$$= \frac{-\frac{6bx\cosh(c+dx)}{a+bx^3} - \text{RootSum}\left[a+b\#1^3\&, \frac{-\cosh(c+d\#1)\text{Chi}(d(x-\#1))+\text{Chi}(d(x-\#1))\sinh(c+d\#1)+\cosh(c+d\#1)\text{Sh}}{}\right.}{}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

output
```
((-6*b*x*Cosh[c + d*x])/(a + b*x^3) - RootSum[a + b*#1^3 & , (-(Cosh[c + d
*#1]*CoshIntegral[d*(x - #1)]) + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] +
 Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - Sinh[c + d*#1]*SinhIntegral[d*(
x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d
*(x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#
1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] + RootSum[a + b
*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #
1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*
#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#
1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIn
tegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2
 & ])/(18*b^2)
```

Rubi [A] (verified)

Time = 1.77 (sec) , antiderivative size = 729, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5814, 5804, 2009, 5815, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx$$

$\downarrow$ 5814

$$\frac{d\int \frac{x\sinh(c+dx)}{bx^3+a}dx}{3b}+\frac{\int \frac{\cosh(c+dx)}{bx^3+a}dx}{3b}-\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)}$$

$\downarrow$ 5804

$$\frac{\int \left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-(-1)^{2/3}\sqrt[3]{b}x-\sqrt[3]{a}\right)}\right)dx}{3b}+\frac{d\int \frac{x\sinh(c+dx)}{bx^3+a}dx}{3b}-\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)}$$

$\downarrow$ 2009

$$\frac{d\int \frac{x\sinh(c+dx)}{bx^3+a}dx}{3b}+\frac{-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{3}}{}$$

$$\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)}$$

$\downarrow$ 5815

$$\frac{d\int \left(-\frac{\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{b}x+\sqrt[3]{a}\right)}-\frac{(-1)^{2/3}\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}x\right)}+\frac{\sqrt[3]{-1}\sinh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}\left((-1)^{2/3}\sqrt[3]{b}x+\sqrt[3]{a}\right)}\right)dx}{3b}+\frac{-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{3}}{}$$

$$\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)}$$

$\downarrow$ 2009

$$-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}}{\sqrt[3]{b}}\right.}{3}$$

$$d\left(-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3\sqrt[3]{a}b^{2/3}}-\frac{(-1)^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3\sqrt[3]{a}b^{2/3}}+\frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{C}}{3\sqrt[3]{a}b^{2/3}}\right.$$

$$\frac{x\cosh(c+dx)}{3b\left(a+bx^3\right)}$$

input

```
Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

output

```
-1/3*(x*Cosh[c + d*x])/(b*(a + b*x^3)) + (d*(-1/3*(CoshIntegral[(a^(1/3)*d
)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(a^(1/3)*b^(2/3)) - ((-1)^
(2/3)*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*CoshIntegral[-(
((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(
1/3)])/(3*a^(1/3)*b^(2/3)) + ((-1)^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(1/3)*b^(
2/3)) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) +
d*x])/(3*a^(1/3)*b^(2/3)) + ((-1)^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(1/3)*b^(2
/3))))/(3*b) + (-1/3*((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*
CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(2/3)*b^(1/3)) + ((
-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d
)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) +
((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + (Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + (
(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)))/(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.98 (sec) , antiderivative size = 877, normalized size of antiderivative = 1.22

method	result	size
risch	Expression too large to display	877

input
```
int(x^3*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)*x-1/18/d/a/b^2*sum((3*_R2^2*b*c^2
-_R2*a*d^3-5*_R2*b*c^3-2*a*c*d^3+2*b*c^4+3*_R2*b*c^2+a*d^3-b*c^3)/(_R2^2-2
*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*
c^2+a*d^3-b*c^3))+1/18/d*c^3/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R
1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1
/6/d*c^2/a/b*sum((_R1^2-_R1*c+_R1+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*
x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6/d*c/a/b
^2*sum((2*_R2^2*b*c-3*_R2*b*c^2-a*d^3+b*c^3+2*_R2*b*c)/(_R2^2-2*_R2*c+c^2)
*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b
*c^3))-1/6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)*x+1/18/d/a/b^2*sum((3*_R2^2*
b*c^2-_R2*a*d^3-5*_R2*b*c^3-2*a*c*d^3+2*b*c^4-3*_R2*b*c^2-a*d^3+b*c^3)/(_R
2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*
_Z*b*c^2+a*d^3-b*c^3))-1/18/d*c^3/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*ex
p(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^
3))+1/6/d*c^2/a/b*sum((_R1^2-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(
1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/6/d*
c/a/b^2*sum((2*_R2^2*b*c-3*_R2*b*c^2-a*d^3+b*c^3-2*_R2*b*c)/(_R2^2-2*_R2*c
+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*
d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2050 vs. $2(500) = 1000$.

Time = 0.12 (sec) , antiderivative size = 2050, normalized size of antiderivative = 2.86

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x +
 c)^2) - (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)
^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*
d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
+ ((-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 -
(b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (-a*d^3/b)^(1/3)*((b
*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^
3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1
))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b*x^
3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 +
 a) + a)*sinh(d*x + c)^2) - (a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a)
 + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2
))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(
sqrt(-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)
*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) - (
-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x
^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^
(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*((
-a*d^3/b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c + dx)}{\left(a + bx^3\right)^2} \, dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/2*((d^2*x^3*e^(2*c) + 3*d*x^2*e^(2*c) + 12*x*e^(2*c))*e^(d*x) - (d^2*x^3
 - 3*d*x^2 + 12*x)*e^(-d*x))/(b^2*d^3*x^6*e^c + 2*a*b*d^3*x^3*e^c + a^2*d^
3*e^c) + 1/2*integrate(-6*(a*d^2*x^2*e^c - 10*b*x^3*e^c + 3*a*d*x*e^c + 2*
a*e^c)*e^(d*x)/(b^3*d^3*x^9 + 3*a*b^2*d^3*x^6 + 3*a^2*b*d^3*x^3 + a^3*d^3)
, x) - 1/2*integrate(-6*(a*d^2*x^2 - 10*b*x^3 - 3*a*d*x + 2*a)*e^(-d*x)/(b
^3*d^3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^2*b*d^3*x^3*e^c + a^3*d^3*e^c),
 x)
```

Giac [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^3 \cosh(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x^3)^2,x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^2}\,dx$$
$$= \frac{-e^{2dx+2c}dx + e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^6+2ab\,x^3+a^2}dx\right)a^2d + e^{dx+2c}\left(\int \frac{e^{dx}}{b^2x^6+2ab\,x^3+a^2}dx\right)abd\,x^3 + e^{dx+2c}\left(\int \frac{e^{dx}x^4}{b^2x^6+2ab\,x^3+a}\right.}{}$$

input
```
int(x^3*cosh(d*x+c)/(b*x^3+a)^2,x)
```

output
```
( - e**(2*c + 2*d*x)*d*x + e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**3
+ b**2*x**6),x)*a**2*d + e**(2*c + d*x)*int(e**(d*x)/(a**2 + 2*a*b*x**3 +
b**2*x**6),x)*a*b*d*x**3 + e**(2*c + d*x)*int((e**(d*x)*x**4)/(a**2 + 2*a*
b*x**3 + b**2*x**6),x)*a*b*d**2 + e**(2*c + d*x)*int((e**(d*x)*x**4)/(a**2
 + 2*a*b*x**3 + b**2*x**6),x)*b**2*d**2*x**3 + e**(2*c + d*x)*int((e**(d*x
)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a**2*d**2 + e**(2*c + d*x)*int((e*
*(d*x)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)*a*b*d**2*x**3 - e**(d*x)*int(
x**4/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**3 + e**(d*x)*b**2*x**6),x)*a*b*d**
2 - e**(d*x)*int(x**4/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**3 + e**(d*x)*b**2
*x**6),x)*b**2*d**2*x**3 - e**(d*x)*int(x**3/(e**(d*x)*a**2 + 2*e**(d*x)*a
*b*x**3 + e**(d*x)*b**2*x**6),x)*a*b*d - e**(d*x)*int(x**3/(e**(d*x)*a**2
+ 2*e**(d*x)*a*b*x**3 + e**(d*x)*b**2*x**6),x)*b**2*d*x**3 - 3*e**(d*x)*in
t(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**3 + e**(d*x)*b**2*x**6),x)*a*b -
 3*e**(d*x)*int(x**2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**3 + e**(d*x)*b**2*
x**6),x)*b**2*x**3 - e**(d*x)*int(x/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**3 +
 e**(d*x)*b**2*x**6),x)*a**2*d**2 - e**(d*x)*int(x/(e**(d*x)*a**2 + 2*e**(
d*x)*a*b*x**3 + e**(d*x)*b**2*x**6),x)*a*b*d**2*x**3 - d*x - 1)/(4*e**(c +
 d*x)*b*d*(a + b*x**3))
```

3.103 $\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 373

$$\begin{aligned}\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^2}\,dx = &-\frac{\cosh(c+dx)}{3b(a+bx^3)}+\frac{d\operatorname{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}\\&-\frac{\sqrt[3]{-1}d\operatorname{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}\\&+\frac{(-1)^{2/3}d\operatorname{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9a^{2/3}b^{4/3}}\\&+\frac{\sqrt[3]{-1}d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{2/3}b^{4/3}}\\&+\frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}\\&+\frac{(-1)^{2/3}d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\operatorname{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{2/3}b^{4/3}}\end{aligned}$$

output
```
-1/3*cosh(d*x+c)/b/(b*x^3+a)+1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/
3)*d/b^(1/3))/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^
(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(4/3)+1/9*(-1)^(
2/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/
b^(1/3))/a^(2/3)/b^(4/3)-1/9*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1
/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*d*cosh(c-a
^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(4/3)+1/9*(-1)^(2/3
)*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+
d*x)/a^(2/3)/b^(4/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.12 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.54

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^2}\,dx = \frac{-\frac{6b\cosh(c+dx)}{a+bx^3} - d\,\mathrm{RootSum}\left[a+b\#1^3\&, \frac{\cosh(c+d\#1)\mathrm{Chi}(d(x-\#1))-\mathrm{Chi}(d(x-\#1))\sinh(c+d\#1)-\cosh(c+d\#1)\mathrm{Shi}}{\#1^2}\right.}{}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

output
```
((-6*b*Cosh[c + d*x])/(a + b*x^3) - d*RootSum[a + b*#1^3 & , (Cosh[c + d*#
1]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Co
sh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x -
 #1)])/#1^2 & ] + d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*
(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhI
ntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/(
18*b^2)
```

Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.01, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {5812, 5803, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} \, dx$$

$$\Big\downarrow \ 5812$$

$$\frac{d \int \frac{\sinh(c+dx)}{bx^3+a} dx}{3b} - \frac{\cosh(c+dx)}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 5803$$

$$\frac{d \int \left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{b}x-\sqrt[3]{a}\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)} - \frac{\sinh(c+dx)}{3a^{2/3}\left(-(-1)^{2/3}\sqrt[3]{b}x-\sqrt[3]{a}\right)}\right) dx}{3b} - \frac{\cosh(c+dx)}{3b\,(a+bx^3)}$$

$$\Big\downarrow \ 2009$$

$$\frac{d\left(\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}} - \frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}} + \frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}}{3a^{2/3}\sqrt[3]{b}}\right.}{} \quad \frac{\cosh(c+dx)}{3b\,(a+bx^3)}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

output
```
-1/3*Cosh[c + d*x]/(b*(a + b*x^3)) + (d*((CoshIntegral[(a^(1/3)*d)/b^(1/3)
 + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) - ((-1)^(1/3)*C
oshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(
1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*CoshIntegral[-(((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/
(3*a^(2/3)*b^(1/3)) + ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]
*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) +
 (Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(
3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*
SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3))))/
(3*b)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.87 (sec) , antiderivative size = 594, normalized size of antiderivative = 1.59

method	result
risch	$-\frac{d^{3} \mathrm{e}^{-d x -c}}{6 b \left(b \,d^{3} x^{3}+d^{3} a \right)}-\frac{\sum_{\textit{\_R2} =\operatorname{RootOf}\left(b \textit{\_Z}^{3}-3 c b \textit{\_Z}^{2}+3 b \,c^{2} \textit{\_Z} +d^{3} a -b \,c^{3}\right)}\frac{\left(2 \textit{\_R2}^{2} b c -3 \textit{\_R2} b \,c^{2}-d^{3} a +b \,c^{3}+2 \textit{\_R2} b c \right) \mathrm{e}^{-\textit{\_R2}} \operatorname{exp}}{\textit{\_R2}^{2}-2 \textit{\_R2} c +c^{2}}}{18 a \,b^{2}}$

input
```
int(x^2*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*d^3*exp(-d*x-c)/b/(b*d^3*x^3+a*d^3)-1/18/a/b^2*sum((2*_R2^2*b*c-3*_R2
*b*c^2-a*d^3+b*c^3+2*_R2*b*c)/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c
),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*c^2/a/b*sum((
_R1-c+2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3
*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/9*c/a/b*sum((_R1^2-_R1*c+_R1+c)/(_R1^
2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))-1/6*d^3*exp(d*x+c)/b/(b*d^3*x^3+a*d^3)+1/18/a/b^2*sum
((2*_R2^2*b*c-3*_R2*b*c^2-a*d^3+b*c^3-2*_R2*b*c)/(_R2^2-2*_R2*c+c^2)*exp(_
R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
+1/18*c^2/a/b*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),
_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/9*c/a/b*sum((_R1^2
-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. $2(263) = 526$.

Time = 0.14 (sec) , antiderivative size = 1276, normalized size of antiderivative = 3.42

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2} \, dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*((a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x - 1/2*(a*d^3
/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (
-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x
^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x - 1/2*(-a*d^3/b)^(
1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (a*d^
3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 -
sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (-a*d^3/b)^(1
/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3
)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) - 2*(-a*d^3/b)^(1/3
)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(-d*x + (-
a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3)) - 2*(a*d^3/b)^(1/3)*((b*x^3 + a
)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^2)*Ei(d*x + (a*d^3/b)^(1/3))
*cosh(-c + (a*d^3/b)^(1/3)) + (a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 +
a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)
^2)*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*sinh(1/2*(a*d^3/b)^(1/3)*
(sqrt(-3) + 1) + c) + (-a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)
*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2)*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2}\, dx = \text{Timed out}$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^2 \cosh(dx+c)}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/2*((d*x^2*e^(2*c) + 4*x*e^(2*c))*e^(d*x) - (d*x^2 - 4*x)*e^(-d*x))/(b^2*
d^2*x^6*e^c + 2*a*b*d^2*x^3*e^c + a^2*d^2*e^c) + 1/2*integrate(2*(10*b*x^3
*e^c - 3*a*d*x*e^c - 2*a*e^c)*e^(d*x)/(b^3*d^2*x^9 + 3*a*b^2*d^2*x^6 + 3*a
^2*b*d^2*x^3 + a^3*d^2), x) + 1/2*integrate(2*(10*b*x^3 + 3*a*d*x - 2*a)*e
^(-d*x)/(b^3*d^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3
*d^2*e^c), x)
```

Giac [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^2 \cosh(dx+c)}{(bx^3+a)^2}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x^2 \cosh(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x^3)^2,x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^2}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}x^2}{b^2x^6+2ab\,x^3+a^2}dx\right)+\int \frac{x^2}{e^{dx}a^2+2e^{dx}ab\,x^3+e^{dx}b^2x^6}dx}{2e^c}$$

input
```
int(x^2*cosh(d*x+c)/(b*x^3+a)^2,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x**2)/(a**2 + 2*a*b*x**3 + b**2*x**6),x) + int(x**
2/(e**(d*x)*a**2 + 2*e**(d*x)*a*b*x**3 + e**(d*x)*b**2*x**6),x))/(2*e**c)
```

3.104 $\int \frac{x \cosh(c+dx)}{(a+bx^3)^2} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 695

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \frac{\cosh(c+dx)}{3abx} - \frac{\cosh(c+dx)}{3bx(a+bx^3)} - \frac{(-1)^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} + \frac{\sqrt[3]{-1}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\,\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} - \frac{d\,\mathrm{Chi}\left(-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{9ab} + \frac{d\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9ab} + \frac{(-1)^{2/3}\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} - \frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}} - \frac{d\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9ab} + \frac{\sqrt[3]{-1}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\mathrm{Shi}\left(\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}+dx\right)}{9a^{4/3}b^{2/3}}$$

output
```
1/3*cosh(d*x+c)/a/b/x-1/3*cosh(d*x+c)/b/x/(b*x^3+a)-1/9*(-1)^(2/3)*cosh(c+
(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(4/3
)/b^(2/3)+1/9*(-1)^(1/3)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2
/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(4/3)/b^(2/3)-1/9*cosh(c-a^(1/3)*d/b^(1/3))*C
hi(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)
*sinh(c-a^(1/3)*d/b^(1/3))/a/b-1/9*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)
*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a/b-1/9*d*Chi(-(-1)^(2/3)*a^(1/3)*d/
b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b-1/9*d*cosh(c+(-1)^(1
/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b-1/9*(-1)
^(2/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1
/3)+d*x)/a^(4/3)/b^(2/3)-1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(
1/3)+d*x)/a/b-1/9*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(
4/3)/b^(2/3)-1/9*d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(
1/3)*d/b^(1/3)+d*x)/a/b+1/9*(-1)^(1/3)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3)
)*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.13 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.56

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx$$

$$= \frac{6bx^2\cosh(c+dx) + (a+bx^3)\,\text{RootSum}\Big[a+b\#1^3\&, \frac{\cosh(c+d\#1)\text{Chi}(d(x-\#1))-\text{Chi}(d(x-\#1))\sinh(c+d\#1)-\text{c}}{}}{}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

output
```
(6*b*x^2*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cos
h[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x -
#1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x
- #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 +
d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1 & ] - (a + b*x^3)*RootSum
[a + b*#1^3 & , (-(Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) - CoshIntegral
[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - Si
nh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x
 - #1)]*#1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#
1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]
*#1)/#1 & ])/(18*a*b*(a + b*x^3))
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 725, normalized size of antiderivative = 1.04, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.294$, Rules used = {5814, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{5814}$$

$$\frac{d\int \frac{\sinh(c+dx)}{x(bx^3+a)}dx}{3b}-\frac{\int \frac{\cosh(c+dx)}{x^2(bx^3+a)}dx}{3b}-\frac{\cosh(c+dx)}{3bx\,(a+bx^3)}$$

$$\Big\downarrow \text{5815}$$

$$\frac{d\int \left(\frac{\sinh(c+dx)}{ax}-\frac{bx^2\sinh(c+dx)}{a(bx^3+a)}\right)dx}{3b}-\frac{\int \frac{\cosh(c+dx)}{x^2(bx^3+a)}dx}{3b}-\frac{\cosh(c+dx)}{3bx\,(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\int \frac{\cosh(c+dx)}{x^2(bx^3+a)}dx}{3b}+$$

$$d\left(-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^2}{}\right.}{3a}\right.$$

$$\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)}$$

$\Big\downarrow$ 5816

$$-\frac{\int \left(\frac{\cosh(c+dx)}{ax^2}-\frac{bx\cosh(c+dx)}{a(bx^3+a)}\right)dx}{3b}+$$

$$d\left(-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^2}{}\right.}{3a}\right.$$

$$\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)}$$

$\Big\downarrow$ 2009

$$-\frac{\frac{(-1)^{2/3}\sqrt[3]{b}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{\sqrt[3]{b}\,c}{}}{}$$

$$d\left(-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^2}{}\right.}{3a}\right.$$

$$\frac{\cosh(c+dx)}{3bx\left(a+bx^3\right)}$$

input

```
Int[(x*Cosh[c + d*x])/(a + b*x^3)^2,x]
```

output
```
-1/3*Cosh[c + d*x]/(b*x*(a + b*x^3)) + (d*((CoshIntegral[d*x]*Sinh[c])/a -
 (CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(
3*a) - (CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a) - (CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/
b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a) + (Cosh[c]
*SinhIntegral[d*x])/a + (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhInte
gral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/
b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (Cosh[c - ((-1)^
(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*
x])/(3*a)))/(3*b) - (-(Cosh[c + d*x]/(a*x)) + ((-1)^(2/3)*b^(1/3)*Cosh[c +
 ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/
3) - d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(4
/3)) + (b^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(
1/3) + d*x])/(3*a^(4/3)) + (d*CoshIntegral[d*x]*Sinh[c])/a + (d*Cosh[c]*Si
nhIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b
^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) +
(b^(1/3)*Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) +
d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^
(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)))...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.83 (sec) , antiderivative size = 395, normalized size of antiderivative = 0.57

method	result
risch	$\frac{d^3 e^{-dx-c} x^2}{6a(b d^3 x^3+d^3 a)} - \frac{d \left(\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3bc^2\_Z+d^3 a-bc^3\right)} \frac{\left(\_R1^2-\_R1c+\_R1+c\right) e^{-\_R1} \text{expIntegral}_1\left(dx-\_\right.}{\_R1^2-2\_R1c+c^2}\right.}{18ab}$

input
```
int(x*cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*d^3*exp(-d*x-c)*x^2/a/(b*d^3*x^3+a*d^3)-1/18*d/a/b*sum((_R1^2-_R1*c+_R
1+c)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^
2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/18*d*c/a/b*sum((_R1-c+2)/(_R1^2-2*_R1*c+c
^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^
3-b*c^3))+1/6*d^3*exp(d*x+c)*x^2/a/(b*d^3*x^3+a*d^3)+1/18*d/a/b*sum((_R1^2
-_R1*c-_R1-c)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^
3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))-1/18*d*c/a/b*sum((_R1-c-2)/(_R1^2-
2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2135 vs. 2(507) = 1014.

Time = 0.17 (sec) , antiderivative size = 2135, normalized size of antiderivative = 3.07

$$\int \frac{x \cosh(c+dx)}{(a+bx^3)^2}\, dx = \text{Too large to display}$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*a*b*d^2*x^2*cosh(d*x + c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x +
 c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*
x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b - s
qrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(s
qrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (2*(a*b*d^3*x
^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2
+ (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x +
c)^2 - (b^2*x^3 + a*b - sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*
x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-
3) + 1) - c) - (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - 2*(a*b*d^3*x^3
 + a^2*d^3)*sinh(d*x + c)^2 - (a*d^3/b)^(2/3)*((b^2*x^3 + a*b + sqrt(-3)*(
b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b
))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2
*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (2*(a*b*d^3*x^3 + a^2*d^3)*cosh(d*x
 + c)^2 - 2*(a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c)^2 + (-a*d^3/b)^(2/3)*((b
^2*x^3 + a*b + sqrt(-3)*(b^2*x^3 + a*b))*cosh(d*x + c)^2 - (b^2*x^3 + a*b
+ sqrt(-3)*(b^2*x^3 + a*b))*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c) + 2*((a*b
*d^3*x^3 + a^2*d^3)*cosh(d*x + c)^2 - (a*b*d^3*x^3 + a^2*d^3)*sinh(d*x + c
)^2 - (-a*d^3/b)^(2/3)*((b^2*x^3 + a*b)*cosh(d*x + c)^2 - (b^2*x^3 + a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\cosh\left(dx+c\right)}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^2*d*x^6*e^c + 2*a*b*d*x^3*e^c + a^2*
d*e^c) + 1/2*integrate((5*b*x^3*e^c - a*e^c)*e^(d*x)/(b^3*d*x^9 + 3*a*b^2*
d*x^6 + 3*a^2*b*d*x^3 + a^3*d), x) - 1/2*integrate((5*b*x^3 - a)*e^(-d*x)/
(b^3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*a^2*b*d*x^3*e^c + a^3*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\cosh\left(dx+c\right)}{(bx^3+a)^2}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(x*cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{x\cosh(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x^3)^2,x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\cosh\left(dx+c\right)x}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(x*cosh(d*x+c)/(b*x^3+a)^2,x)
```

output
```
int((cosh(c + d*x)*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.105 $\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 739

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2} dx = \text{Too large to display}$$

output
```
1/3*cosh(d*x+c)/a/b/x^2-1/3*cosh(d*x+c)/b/x^2/(b*x^3+a)-2/9*(-1)^(1/3)*cos
h(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^
(5/3)/b^(1/3)+2/9*(-1)^(2/3)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1
)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(5/3)/b^(1/3)+2/9*cosh(c-a^(1/3)*d/b^(1/3
))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+1/9*d*Chi(a^(1/3)*d/b^(1/3)+
d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*(-1)^(2/3)*d*Chi((-1)^(
1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b
^(2/3)-1/9*(-1)^(1/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)
^(2/3)*a^(1/3)*d/b^(1/3))/a^(4/3)/b^(2/3)+1/9*(-1)^(2/3)*d*cosh(c+(-1)^(1/
3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/
3)-2/9*(-1)^(1/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(
1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)+1/9*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^
(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)+2/9*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(
1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/9*(-1)^(1/3)*d*cosh(c-(-1)^(2/3)*a^(
1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(2/3)+2/9*
(-1)^(2/3)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b
^(1/3)+d*x)/a^(5/3)/b^(1/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.14 (sec) , antiderivative size = 387, normalized size of antiderivative = 0.52

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \frac{6bx\cosh(c+dx)+(a+bx^3)\,\text{RootSum}\left[a+b\#1^3\&,\frac{2\cosh(c+d\#1)\text{Chi}(d(x-\#1))-2\text{Chi}(d(x-\#1))\sinh(c+d\#1)-}{}\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*x^3)^2,x]
```

output
```
(6*b*x*Cosh[c + d*x] + (a + b*x^3)*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1
]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2
*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d
*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral
[d*(x - #1)]*Sinh[c + d*#1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]
*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (a + b*x^3)
*RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*Co
shIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x
 - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*Cos
hIntegral[d*(x - #1)]*#1 + d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 +
d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegr
al[d*(x - #1)]*#1)/#1^2 & ])/(18*a*b*(a + b*x^3))
```

Rubi [A] (verified)

Time = 2.32 (sec) , antiderivative size = 832, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {5802, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx$$

$$\Big\downarrow \text{5802}$$

$$-\frac{2\int \frac{\cosh(c+dx)}{x^3(bx^3+a)}dx}{3b}+\frac{d\int \frac{\sinh(c+dx)}{x^2(bx^3+a)}dx}{3b}-\frac{\cosh(c+dx)}{3bx^2(a+bx^3)}$$

$$\Big\downarrow \text{5815}$$

$$-\frac{2\int \frac{\cosh(c+dx)}{x^3(bx^3+a)}dx}{3b}+\frac{d\int \left(\frac{\sinh(c+dx)}{ax^2}-\frac{bx\sinh(c+dx)}{a(bx^3+a)}\right)dx}{3b}-\frac{\cosh(c+dx)}{3bx^2(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\int \frac{\cosh(c+dx)}{x^3(bx^3+a)}dx}{3b}+$$

$$d\left(\frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}}{\sqrt[3]{}}\right.}{}\right.$$

$$\frac{\cosh(c+dx)}{3bx^2(a+bx^3)}$$

$$\Big\downarrow \text{5816}$$

$$-\frac{2\int \left(\frac{\cosh(c+dx)}{ax^3}-\frac{b\cosh(c+dx)}{a(bx^3+a)}\right)dx}{3b}+$$

$$d\left(\frac{\sqrt[3]{b}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{4/3}}-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh\left(c-\frac{(-1)^{2/3}}{\sqrt[3]{}}\right.}{}\right.$$

$$\frac{\cosh(c+dx)}{3bx^2(a+bx^3)}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cosh(c+dx)}{3bx^2(bx^3+a)}+$$

$$d\left(\frac{d\cosh(c)\text{Chi}(dx)}{a}+\frac{\sqrt[3]{b}\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\frac{\sqrt[3]{-}}{}\right.$$

$$2\left(\frac{\cosh(c)\text{Chi}(dx)d^2}{2a}+\frac{\sinh(c)\text{Shi}(dx)d^2}{2a}-\frac{\sinh(c+dx)d}{2ax}-\frac{\cosh(c+dx)}{2ax^2}+\frac{\sqrt[3]{-1}b^{2/3}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}\right.$$

input
```
Int[Cosh[c + d*x]/(a + b*x^3)^2,x]
```

output
```
-1/3*Cosh[c + d*x]/(b*x^2*(a + b*x^3)) + (d*((d*Cosh[c]*CoshIntegral[d*x])
/a + (b^(1/3)*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)
/b^(1/3)])/(3*a^(4/3)) + ((-1)^(2/3)*b^(1/3)*CoshIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)
) - ((-1)^(1/3)*b^(1/3)*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d
*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - Sinh[c + d*x]/
(a*x) + (d*Sinh[c]*SinhIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Cosh[c + ((-
1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) -
 d*x])/(3*a^(4/3)) + (b^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(
a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1
)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) +
d*x])/(3*a^(4/3))))/(3*b) - (2*(-1/2*Cosh[c + d*x]/(a*x^2) + (d^2*Cosh[c]*
CoshIntegral[d*x])/(2*a) + ((-1)^(1/3)*b^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3
)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/
3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cos
h[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(
5/3)) - (d*Sinh[c + d*x])/(2*a*x) + (d^2*Sinh[c]*SinhIntegral[d*x])/(2*a)
- ((-1)^(1/3)*b^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegra
l[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b^(2/3)*Sinh[c ...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5802
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x] + (-Sim
p[(-n + 1)/(b*n*(p + 1))   Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
 x] - Simp[d/(b*n*(p + 1))   Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*
x], x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p
, -1] && GtQ[n, 2]
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.76 (sec) , antiderivative size = 226, normalized size of antiderivative = 0.31

method	result
risch	$\frac{d^{3} e^{-d x -c} x}{6 a \left(b \,d^{3} x^{3}+d^{3} a \right)}-\frac{d^{2} \left(\sum_{\_R1 =\operatorname{RootOf}\left(b \_Z^{3}-3 c b \_Z^{2}+3 b \,c^{2} \_Z +d^{3} a -b \,c^{3}\right)}\frac{\left(\_R1 -c +2\right) e^{-\_R1} \operatorname{expIntegral}_{1}\left(d x -\_R1 +c \right)}{\_R1^{2}-2 \_R1 c +c^{2}}\right)}{18 a b}+$

input
```
int(cosh(d*x+c)/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*d^3*exp(-d*x-c)*x/a/(b*d^3*x^3+a*d^3)-1/18*d^2/a/b*sum((_R1-c+2)/(_R1^
2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z
*b*c^2+a*d^3-b*c^3))+1/6*d^3*exp(d*x+c)*x/a/(b*d^3*x^3+a*d^3)+1/18*d^2/a/b
*sum((_R1-c-2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z
^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2048 vs. $2(519) = 1038$.

Time = 0.14 (sec) , antiderivative size = 2048, normalized size of antiderivative = 2.77

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
1/36*(12*a*d*x*cosh(d*x + c) - ((a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3
+ a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x +
c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c
)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a
*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c)
 + ((-a*d^3/b)^(2/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 -
 (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*
((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b
*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3)
+ 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) - ((a*d^3/b)^(2/3)*((b
*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^
3 + a) + a)*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3
 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x +
 c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1
/3)*(sqrt(-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a)
 + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2
) + 2*(-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2
 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x + 1/2*(-a*
d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1) + c)
 - 2*((-a*d^3/b)^(2/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\cosh(dx+c)}{(bx^3+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Giac [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\cosh(dx+c)}{(bx^3+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
integrate(cosh(d*x + c)/(b*x^3 + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\cosh(c+d\,x)}{(b\,x^3+a)^2}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*x^3)^2,x)
```

output
```
int(cosh(c + d*x)/(a + b*x^3)^2, x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{(a+bx^3)^2}\,dx = \int \frac{\cosh(dx+c)}{b^2x^6+2ab\,x^3+a^2}dx$$

input
```
int(cosh(d*x+c)/(b*x^3+a)^2,x)
```

output
```
int(cosh(c + d*x)/(a**2 + 2*a*b*x**3 + b**2*x**6),x)
```

3.106 $\int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx$

Optimal result

Integrand size = 19, antiderivative size = 697

$$\int \frac{\cosh(c+dx)}{x(a+bx^3)^2} dx = \text{Too large to display}$$

output
```
1/3*cosh(d*x+c)/a/b/x^3-1/3*cosh(d*x+c)/b/x^3/(b*x^3+a)+cosh(c)*Chi(d*x)/a
^2-1/3*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/
3)-d*x)/a^2-1/3*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/
3)*d/b^(1/3)-d*x)/a^2-1/3*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+
d*x)/a^2-1/9*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(5/3
)/b^(1/3)+1/9*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-
1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(1/3)-1/9*(-1)^(2/3)*d*Chi(-(-1)^(2/
3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(
1/3)+sinh(c)*Shi(d*x)/a^2+1/9*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(
1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/3*sinh(c+(-
1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2-1/9
*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(1/3)-1/
3*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^2-1/9*(-1)^(2/3)*
d*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*
x)/a^(5/3)/b^(1/3)-1/3*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)
*a^(1/3)*d/b^(1/3)+d*x)/a^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.36 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.59

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)^2}\,dx$$

$$= \frac{\frac{6a\cosh(c)\cosh(dx)}{a+bx^3} + 18\cosh(c)\text{Chi}(dx) - 3\text{RootSum}\left[a+b\#1^3\&,\cosh(c+d\#1)\text{Chi}(d(x-\#1)) - \text{Chi}(d(x\right.}{}$$

input
```
Integrate[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

output
```
((6*a*Cosh[c]*Cosh[d*x])/(a + b*x^3) + 18*Cosh[c]*CoshIntegral[d*x] - 3*Ro
otSum[a + b*#1^3 & , Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - CoshIntegra
l[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + S
inh[c + d*#1]*SinhIntegral[d*(x - #1)] & ] - 3*RootSum[a + b*#1^3 & , Cosh
[c + d*#1]*CoshIntegral[d*(x - #1)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*
#1] + Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegra
l[d*(x - #1)] & ] + (a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegr
al[d*(x - #1)] - CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - Cosh[c + d*#1]*
SinhIntegral[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 &
 ])/b - (a*d*RootSum[a + b*#1^3 & , (Cosh[c + d*#1]*CoshIntegral[d*(x - #1
)] + CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + Cosh[c + d*#1]*SinhIntegral
[d*(x - #1)] + Sinh[c + d*#1]*SinhIntegral[d*(x - #1)])/#1^2 & ])/b + (6*a
*Sinh[c]*Sinh[d*x])/(a + b*x^3) + 18*Sinh[c]*SinhIntegral[d*x])/(18*a^2)
```

Rubi [A] (verified)

Time = 2.18 (sec) , antiderivative size = 846, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {5814, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}dx$$

$\Big\downarrow$ 5814

$$\frac{d\int \frac{\sinh(c+dx)}{x^3(bx^3+a)}dx}{3b}-\frac{\int \frac{\cosh(c+dx)}{x^4(bx^3+a)}dx}{b}-\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$\Big\downarrow$ 5815

$$\frac{d\int \left(\frac{\sinh(c+dx)}{ax^3}-\frac{b\sinh(c+dx)}{a(bx^3+a)}\right)dx}{3b}-\frac{\int \frac{\cosh(c+dx)}{x^4(bx^3+a)}dx}{b}-\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$\Big\downarrow$ 2009

$$-\frac{\int \frac{\cosh(c+dx)}{x^4(bx^3+a)}dx}{b}+$$

$$d\left(-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$\Big\downarrow$ 5816

$$-\frac{\int \left(\frac{b^2\cosh(c+dx)x^2}{a^2(bx^3+a)}-\frac{b\cosh(c+dx)}{a^2x}+\frac{\cosh(c+dx)}{ax^4}\right)dx}{b}+$$

$$d\left(-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\cdots}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\cosh(c+dx)}{3bx^3\left(a+bx^3\right)}$$

$\Big\downarrow$ 2009

$$-\frac{\cosh(c+dx)}{3bx^3\left(bx^3+a\right)}+$$

$$d\left(\frac{\text{Chi}(dx)\sinh(c)d^2}{2a}+\frac{\cosh(c)\text{Shi}(dx)d^2}{2a}-\frac{\cosh(c+dx)d}{2ax}-\frac{b^{2/3}\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\cdots\right.}{3a^{5\cdots}}\right.$$

$$\frac{\text{Chi}(dx)\sinh(c)d^3}{6a}+\frac{\cosh(c)\text{Shi}(dx)d^3}{6a}-\frac{\cosh(c+dx)d^2}{6ax}-\frac{\sinh(c+dx)d}{6ax^2}-\frac{\cosh(c+dx)}{3ax^3}-\frac{b\cosh(c)\text{Chi}(dx)}{a^2}+\frac{b\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\cdots\right.}{\cdots}$$

input
```
Int[Cosh[c + d*x]/(x*(a + b*x^3)^2),x]
```

output
```
-1/3*Cosh[c + d*x]/(b*x^3*(a + b*x^3)) + (d*(-1/2*(d*Cosh[c + d*x])/(a*x)
+ (d^2*CoshIntegral[d*x]*Sinh[c])/(2*a) - (b^(2/3)*CoshIntegral[(a^(1/3)*d
)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) + ((-1)^(1/3)*
b^(2/3)*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^
(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*CoshIntegral[
-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b
^(1/3)])/(3*a^(5/3)) - Sinh[c + d*x]/(2*a*x^2) + (d^2*Cosh[c]*SinhIntegral
[d*x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)
]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b^(2/
3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/
(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]
*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3))))/(3*b) -
 (-1/3*Cosh[c + d*x]/(a*x^3) - (d^2*Cosh[c + d*x])/(6*a*x) - (b*Cosh[c]*Co
shIntegral[d*x])/a^2 + (b*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + (b*Cosh[c - ((-1)^(
2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) -
d*x])/(3*a^2) + (b*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/
b^(1/3) + d*x])/(3*a^2) + (d^3*CoshIntegral[d*x]*Sinh[c])/(6*a) - (d*Sinh[
c + d*x])/(6*a*x^2) + (d^3*Cosh[c]*SinhIntegral[d*x])/(6*a) - (b*Sinh[c]*S
inhIntegral[d*x])/a^2 - (b*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Sin...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 0.88 (sec) , antiderivative size = 338, normalized size of antiderivative = 0.48

method	result
risch	$\frac{e^{-dx-c} d^3}{6a\left(b(dx+c)^3-3(dx+c)^2bc+3(dx+c)b\,c^2+d^3a-b\,c^3\right)} - \frac{e^{-c}\,\text{expIntegral}_1(dx)}{2a^2} + \frac{\sum_{\_R1=\text{RootOf}\left(b\_Z^3-3cb\_Z^2+3b\,c^2\_Z+d^3a-b\,c\ldots\right.}}{}$

input
```
int(cosh(d*x+c)/x/(b*x^3+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(-d*x-c)*d^3/a/(b*(d*x+c)^3-3*(d*x+c)^2*b*c+3*(d*x+c)*b*c^2+d^3*a-b
*c^3)-1/2/a^2*exp(-c)*Ei(1,d*x)+1/18/a^2/b*sum((-a*d^3+3*_R1^2*b-6*_R1*b*c
+3*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-
3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))+1/6*exp(d*x+c)*d^3/a/(b*(d*x+c)^3-3*(d
*x+c)^2*b*c+3*(d*x+c)*b*c^2+d^3*a-b*c^3)-1/2/a^2*exp(c)*Ei(1,-d*x)+1/18/a^
2/b*sum((a*d^3+3*_R1^2*b-6*_R1*b*c+3*b*c^2)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*E
i(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1773 vs. $2(509) = 1018$.

Time = 0.14 (sec) , antiderivative size = 1773, normalized size of antiderivative = 2.54

$$\int \frac{\cosh(c+dx)}{x\,(a+bx^3)^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="fricas")
```

output
```
-1/36*((6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a
*d^3/b)^(1/3)*((b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3
 + sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3
)*(sqrt(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + (6*(b*x^3
 + a)*cosh(d*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*(
(b*x^3 + sqrt(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 + sqrt(-3)*(b*
x^3 + a) + a)*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) +
 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + (6*(b*x^3 + a)*cosh(d
*x + c)^2 - 6*(b*x^3 + a)*sinh(d*x + c)^2 - (a*d^3/b)^(1/3)*((b*x^3 - sqrt
(-3)*(b*x^3 + a) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)
*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(
a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + (6*(b*x^3 + a)*cosh(d*x + c)^2 - 6*(b
*x^3 + a)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b*x^3 - sqrt(-3)*(b*x^3 + a
) + a)*cosh(d*x + c)^2 - (b*x^3 - sqrt(-3)*(b*x^3 + a) + a)*sinh(d*x + c)^
2))*Ei(-d*x + 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(-a*d^3/b)^(1/
3)*(sqrt(-3) - 1) + c) + 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*
sinh(d*x + c)^2 + (-a*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 +
 a)*sinh(d*x + c)^2))*Ei(-d*x + (-a*d^3/b)^(1/3))*cosh(c + (-a*d^3/b)^(1/3
)) + 2*(3*(b*x^3 + a)*cosh(d*x + c)^2 - 3*(b*x^3 + a)*sinh(d*x + c)^2 + (a
*d^3/b)^(1/3)*((b*x^3 + a)*cosh(d*x + c)^2 - (b*x^3 + a)*sinh(d*x + c)^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/x/(b*x**3+a)**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(bx^3+a\right)^2x}\,dx$$

input
```
integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="maxima")
```

output
```
integrate(cosh(d*x + c)/((b*x^3 + a)^2*x), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(cosh(d*x+c)/x/(b*x^3+a)^2,x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\cosh(c+d\,x)}{x\,\left(b\,x^3+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)/(x*(a + b*x^3)^2),x)
```

output
```
int(cosh(c + d*x)/(x*(a + b*x^3)^2), x)
```

Reduce [F]

$$\int \frac{\cosh(c+dx)}{x\left(a+bx^3\right)^2}\,dx=\int \frac{\cosh\left(dx+c\right)}{b^2x^7+2ab\,x^4+a^2x}dx$$

input
```
int(cosh(d*x+c)/x/(b*x^3+a)^2,x)
```

output
```
int(cosh(c + d*x)/(a**2*x + 2*a*b*x**4 + b**2*x**7),x)
```

3.107 $\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 784

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx = \text{Too large to display}$$

output

```
-1/6*x^3*cosh(d*x+c)/b/(b*x^3+a)^2-1/6*cosh(d*x+c)/b^2/(b*x^3+a)-1/54*(-1)
^(2/3)*d^2*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b
^(1/3)-d*x)/a^(1/3)/b^(8/3)+1/54*(-1)^(1/3)*d^2*cosh(c-(-1)^(2/3)*a^(1/3)*
d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(1/3)/b^(8/3)-1/54*d^2
*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(8/3)+2/27
*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)-2/
27*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^
(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)+2/27*(-1)^(2/3)*d*Chi(-(-1)^(2/3)*a^(1/3)
*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^(2/3)/b^(7/3)-1/18*
d*x*sinh(d*x+c)/b^2/(b*x^3+a)-2/27*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*
d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(7/3)-1/54*(-1
)^(2/3)*d^2*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d
/b^(1/3)+d*x)/a^(1/3)/b^(8/3)+2/27*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)
*d/b^(1/3)+d*x)/a^(2/3)/b^(7/3)-1/54*d^2*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(
1/3)*d/b^(1/3)+d*x)/a^(1/3)/b^(8/3)+2/27*(-1)^(2/3)*d*cosh(c-(-1)^(2/3)*a^
(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^(7/3)+1/5
4*(-1)^(1/3)*d^2*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/
3)*d/b^(1/3)+d*x)/a^(1/3)/b^(8/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.38 (sec) , antiderivative size = 397, normalized size of antiderivative = 0.51

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{d\text{RootSum}\left[a+b\#1^3\&, \frac{-4\cosh(c+d\#1)\text{Chi}(d(x-\#1))+4\text{Chi}(d(x-\#1))\sinh(c+d\#1)+4\cosh(c+d\#1)\text{Shi}(d(x-\#1))-4}{}\right.}{}$$

input
```
Integrate[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
(d*RootSum[a + b*#1^3 & , (-4*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + 4*
CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*Cosh[c + d*#1]*SinhIntegral[d*
(x - #1)] - 4*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d*#1]*C
oshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1
- d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*SinhInte
gral[d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (4*Cosh[c + d*#1]
*CoshIntegral[d*(x - #1)] + 4*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 4*
Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + 4*Sinh[c + d*#1]*SinhIntegral[d*
(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIntegral[
d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*
#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*(3*(a +
 2*b*x^3)*Cosh[c + d*x] + d*x*(a + b*x^3)*Sinh[c + d*x]))/(a + b*x^3)^2)/(
108*b^3)
```

Rubi [A] (verified)

Time = 3.45 (sec) , antiderivative size = 1146, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5814, 5812, 5803, 2009, 5813, 5803, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^5 \cosh(c+dx)}{\left(a+bx^3\right)^3} \, dx$$

$$\Big\downarrow \text{ 5814}$$

$$\frac{d\int \frac{x^3 \sinh(c+dx)}{\left(bx^3+a\right)^2}dx}{6b}+\frac{\int \frac{x^2 \cosh(c+dx)}{\left(bx^3+a\right)^2}dx}{2b}-\frac{x^3 \cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 5812}$$

$$\frac{d\int \frac{x^3 \sinh(c+dx)}{\left(bx^3+a\right)^2}dx}{6b}+\frac{\frac{d\int \frac{\sinh(c+dx)}{bx^3+a}dx}{3b}-\frac{\cosh(c+dx)}{3b(a+bx^3)}}{2b}-\frac{x^3 \cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 5803}$$

$$\frac{\frac{d\int \left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\sinh(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\sinh(c+dx)}{3a^{2/3}\left(-(-1)^{2/3}\sqrt[3]{b}x-\sqrt[3]{a}\right)}\right)dx}{3b}-\frac{\cosh(c+dx)}{3b(a+bx^3)}}{2b}+$$

$$\frac{d\int \frac{x^3 \sinh(c+dx)}{\left(bx^3+a\right)^2}dx}{6b}-\frac{x^3 \cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{d\int \frac{x^3 \sinh(c+dx)}{\left(bx^3+a\right)^2}dx}{6b}+$$

$$d\left(\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1\cdots\right.}{3a^{2/3}\sqrt[3]{b}}\right.$$

$$\frac{x^3 \cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 5813}$$

$$\frac{d\left(\frac{\int \frac{\sinh(c+dx)}{bx^3+a}dx}{3b}+\frac{d\int \frac{x\cosh(c+dx)}{bx^3+a}dx}{3b}-\frac{x\sinh(c+dx)}{3b(a+bx^3)}\right)}{6b}+$$

$$d\left(\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1\cdots}{}\right.}{3a^{2/3}\sqrt[3]{b}}\cdots\right.$$

$$\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 5803$$

$$\frac{d\left(\frac{\int\left(-\frac{\sinh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\sinh(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\sinh(c+dx)}{3a^{2/3}\left(-(-1)^{2/3}\sqrt[3]{b}x-\sqrt[3]{a}\right)}\right)dx}{3b}+\frac{d\int\frac{x\cosh(c+dx)}{bx^3+a}dx}{3b}-\frac{x\sinh(c+dx)}{3b(a+bx^3)}\right)}{6b}+$$

$$d\left(\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1\cdots}{}\right.}{3a^{2/3}\sqrt[3]{b}}\cdots\right.$$

$$\frac{x^3\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\cosh(c+dx)x^3}{6b\left(bx^3+a\right)^2}+$$

$$d\left(\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{(-1\cdots}{}\right.}{3a^{2/3}\sqrt[3]{b}}\cdots\right.$$

$$d\left(-\frac{x\sinh(c+dx)}{3b(bx^3+a)}+\frac{\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\text{Chi}\left(-xd-\frac{(-1)^{2/\cdots}}{}\right.}{3\cdots}}{\cdots}\right.$$

$$\Big\downarrow \; 5816$$

$$-\frac{\cosh(c+dx)x^3}{6b\left(bx^3+a\right)^2}+$$

$$\frac{d\left(\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{(-1\cdots}{}\right)}{3a^{2/3}\sqrt[3]{b}}\cdots\right.}{\cdots}$$

$$d\left(-\frac{x\sinh(c+dx)}{3b\left(bx^3+a\right)}+\frac{\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\text{Chi}\left(-xd-\frac{(-1)^{2/\cdots}}{\cdots}\right)\cdots}{3\cdots}}{\cdots}\cdots\right.$$

$\downarrow$ 2009

$$-\frac{\cosh(c+dx)x^3}{6b\left(bx^3+a\right)^2}+$$

$$\frac{d\left(\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{(-1\cdots}{}\right)}{3a^{2/3}\sqrt[3]{b}}\cdots\right.}{\cdots}$$

$$d\left(-\frac{x\sinh(c+dx)}{3b\left(bx^3+a\right)}+\frac{\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}-\frac{\sqrt[3]{-1}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\text{Chi}\left(-xd-\frac{(-1)^{2/\cdots}}{\cdots}\right)\cdots}{3\cdots}}{\cdots}\cdots\right.$$

input
```
Int[(x^5*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*(x^3*Cosh[c + d*x])/(b*(a + b*x^3)^2) + (-1/3*Cosh[c + d*x]/(b*(a + b
*x^3)) + (d*((CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)
/b^(1/3)])/(3*a^(2/3)*b^(1/3)) - ((-1)^(1/3)*CoshIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)
*b^(1/3)) + ((-1)^(2/3)*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d
*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^
(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^
(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + ((-1)^(
2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(
1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3))))/(3*b))/(2*b) + (d*(-1/3*(x*S
inh[c + d*x])/(b*(a + b*x^3)) + ((CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*
Sinh[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(2/3)*b^(1/3)) - ((-1)^(1/3)*CoshInteg
ral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/
b^(1/3)])/(3*a^(2/3)*b^(1/3)) + ((-1)^(2/3)*CoshIntegral[-(((-1)^(2/3)*a^(
1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(2/
3)*b^(1/3)) + ((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhInt
egral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c
 - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3
)*b^(1/3)) + ((-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhI...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5803
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> In
t[ExpandIntegrand[Sinh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

rule 5813
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Sinh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 2.06 (sec) , antiderivative size = 6251, normalized size of antiderivative = 7.97

method	result	size
risch	Expression too large to display	6251

input
```
int(x^5*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
result too large to display
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2980 vs. $2(562) = 1124$.

Time = 0.16 (sec) , antiderivative size = 2980, normalized size of antiderivative = 3.80

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3} dx = \text{Too large to display}$$

input
```
integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
1/216*(((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 +
2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*
(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^
6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^
2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sin
h(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(a*d^
3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 +
 a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 +
2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) -
 4*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*
b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*
(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*d^3/b)
^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))
*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*
x^3 + a^2))*sinh(d*x + c)^2) - 4*(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a
^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*
a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei
(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(
-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^5 \cosh(c + dx)}{(a + bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**5*cosh(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^5 \cosh(c + dx)}{(a + bx^3)^3} \, dx = \int \frac{x^5 \cosh(dx + c)}{(bx^3 + a)^3} \, dx$$

input
```
integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/2*((b*d^4*x^5*e^(2*c) + 4*b*d^3*x^4*e^(2*c) + 20*b*d^2*x^3*e^(2*c) + 120
*b*d*x^2*e^(2*c) - 3*(3*a*d^3*e^(2*c) - 280*b*e^(2*c))*x)*e^(d*x) - (b*d^4
*x^5 - 4*b*d^3*x^4 + 20*b*d^2*x^3 - 120*b*d*x^2 + 3*(3*a*d^3 + 280*b)*x)*e
^(-d*x))/(b^4*d^5*x^9*e^c + 3*a*b^3*d^5*x^6*e^c + 3*a^2*b^2*d^5*x^3*e^c +
a^3*b*d^5*e^c) - 1/2*integrate(3*(60*a*b*d^2*x^2*e^c - 3*a^2*d^3*e^c + 4*(
9*a*b*d^3*e^c - 560*b^2*e^c)*x^3 + 280*a*b*e^c - 3*(a^2*d^4*e^c - 120*a*b*
d*e^c)*x)*e^(d*x)/(b^5*d^5*x^12 + 4*a*b^4*d^5*x^9 + 6*a^2*b^3*d^5*x^6 + 4*
a^3*b^2*d^5*x^3 + a^4*b*d^5), x) - 1/2*integrate(-3*(60*a*b*d^2*x^2 + 3*a^
2*d^3 - 4*(9*a*b*d^3 + 560*b^2)*x^3 + 280*a*b - 3*(a^2*d^4 + 120*a*b*d)*x)
*e^(-d*x)/(b^5*d^5*x^12*e^c + 4*a*b^4*d^5*x^9*e^c + 6*a^2*b^3*d^5*x^6*e^c
+ 4*a^3*b^2*d^5*x^3*e^c + a^4*b*d^5*e^c), x)
```

Giac [F]

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^5 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^5*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x^5*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^5 \cosh(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x^5*cosh(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x^5*cosh(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^5 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{too large to display}$$

input
```
int(x^5*cosh(d*x+c)/(b*x^3+a)^3,x)
```

output

```
(e**(2*c + 2*d*x)*a*d**2*x + 5*e**(2*c + 2*d*x)*a*d + 30*e**(2*c + 2*d*x)*
b*x**2 - 6*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x*
*6 + b**3*x**9),x)*a**4*d**2 - 12*e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a*
*2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**3*b*d**2*x**3 - 6*e**(2*c + d
*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**
2*b**2*d**2*x**6 - e**(2*c + d*x)*int((e**(d*x)*x**4)/(a**3 + 3*a**2*b*x**
3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**3*b*d**3 - 2*e**(2*c + d*x)*int((e**(
d*x)*x**4)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**2*b**2
*d**3*x**3 + 120*e**(2*c + d*x)*int((e**(d*x)*x**4)/(a**3 + 3*a**2*b*x**3
+ 3*a*b**2*x**6 + b**3*x**9),x)*a**2*b**2 - e**(2*c + d*x)*int((e**(d*x)*x
**4)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a*b**3*d**3*x**
6 + 240*e**(2*c + d*x)*int((e**(d*x)*x**4)/(a**3 + 3*a**2*b*x**3 + 3*a*b**
2*x**6 + b**3*x**9),x)*a*b**3*x**3 + 120*e**(2*c + d*x)*int((e**(d*x)*x**4
)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*b**4*x**6 - e**(2*
c + d*x)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**
9),x)*a**4*d**3 - 2*e**(2*c + d*x)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x**3
+ 3*a*b**2*x**6 + b**3*x**9),x)*a**3*b*d**3*x**3 - 60*e**(2*c + d*x)*int((
e**(d*x)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**3*b -
 e**(2*c + d*x)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b
**3*x**9),x)*a**2*b**2*d**3*x**6 - 120*e**(2*c + d*x)*int((e**(d*x)*x)/...
```

3.108 $\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 1105

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

output
```
1/9*cosh(d*x+c)/a/b^2/x-1/6*x^2*cosh(d*x+c)/b/(b*x^3+a)^2-1/9*cosh(d*x+c)/
b^2/x/(b*x^3+a)-1/27*(-1)^(2/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((
-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(4/3)/b^(5/3)-1/54*(-1)^(1/3)*d^2*cosh(
c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(2
/3)/b^(7/3)+1/27*(-1)^(1/3)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)
^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(4/3)/b^(5/3)+1/54*(-1)^(2/3)*d^2*cosh(c-(
-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(2/3
)/b^(7/3)-1/27*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3
)/b^(5/3)+1/54*d^2*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^
(2/3)/b^(7/3)-1/27*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh(c-a^(1/3)*d/b^(1/3))/
a/b^2-1/27*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))/a/b^2-1/27*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(
-1)^(2/3)*a^(1/3)*d/b^(1/3))/a/b^2-1/18*d*sinh(d*x+c)/b^2/(b*x^3+a)-1/27*d
*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*
x)/a/b^2-1/27*(-1)^(2/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1
/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)-1/54*(-1)^(1/3)*d^2*sinh(c+(-1)
^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b
^(7/3)-1/27*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a/b^2-1
/27*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)+1
/54*d^2*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(2/3)/b^...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.37 (sec) , antiderivative size = 675, normalized size of antiderivative = 0.61

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
(RootSum[a + b*#1^3 & , (a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - a
*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - a*d^2*Cosh[c + d*#1]*SinhIn
tegral[d*(x - #1)] + a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 2*b*C
osh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 2*b*CoshIntegral[d*(x - #1)]*S
inh[c + d*#1]*#1 - 2*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*Si
nh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 2*b*d*Cosh[c + d*#1]*CoshIntegr
al[d*(x - #1)]*#1^2 - 2*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 -
 2*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 2*b*d*Sinh[c + d*#1]
*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (-(a*d^2
*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]) - a*d^2*CoshIntegral[d*(x - #1)]
*Sinh[c + d*#1] - a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - a*d^2*Si
nh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*b*Cosh[c + d*#1]*CoshIntegral[d*
(x - #1)]*#1 - 2*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 2*b*Cosh[c
 + d*#1]*SinhIntegral[d*(x - #1)]*#1 - 2*b*Sinh[c + d*#1]*SinhIntegral[d*(
x - #1)]*#1 + 2*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 2*b*d*C
oshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + 2*b*d*Cosh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1^2 + 2*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^
2)/#1^2 & ] + (6*b*Cosh[d*x]*(b*x^2*(-a + 2*b*x^3)*Cosh[c] - a*d*(a + b*x^
3)*Sinh[c]))/(a + b*x^3)^2 + (6*b*(-(a*d*(a + b*x^3)*Cosh[c]) + b*x^2*(-a
+ 2*b*x^3)*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(108*a*b^3)
```

Rubi [A] (verified)

Time = 3.63 (sec) , antiderivative size = 1142, normalized size of antiderivative = 1.03, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.474$, Rules used = {5814, 5811, 5804, 2009, 5814, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}dx$$

$$\Big\downarrow \text{ 5814}$$

$$\frac{\int \frac{x\cosh(c+dx)}{(bx^3+a)^2}dx}{3b}+\frac{d\int \frac{x^2\sinh(c+dx)}{(bx^3+a)^2}dx}{6b}-\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 5811}$$

$$\frac{\int \frac{x\cosh(c+dx)}{(bx^3+a)^2}dx}{3b}+\frac{d\left(\frac{d\int\frac{\cosh(c+dx)}{bx^3+a}dx}{3b}-\frac{\sinh(c+dx)}{3b(a+bx^3)}\right)}{6b}-\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2}$$

$\Big\downarrow$ 5804

$$\frac{d\left(\frac{d\int\left(-\frac{\cosh(c+dx)}{3a^{2/3}\left(-\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}\right)}-\frac{\cosh(c+dx)}{3a^{2/3}\left(-(-1)^{2/3}\sqrt[3]{b}x-\sqrt[3]{a}\right)}\right)dx}{3b}-\frac{\sinh(c+dx)}{3b(a+bx^3)}\right)}{6b}+\frac{\int\frac{x\cosh(c+dx)}{(bx^3+a)^2}dx}{3b}-\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2}$$

$\Big\downarrow$ 2009

$$\frac{\int \frac{x\cosh(c+dx)}{(bx^3+a)^2}dx}{3b}+\frac{d\left(\frac{d\left(-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\cdots}{3a^{2}\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2}$$

$\Big\downarrow$ 5814

$$\frac{\frac{d\int\frac{\sinh(c+dx)}{x\left(bx^3+a\right)}dx}{3b}-\frac{\int\frac{\cosh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\cosh(c+dx)}{3bx(a+bx^3)}}{3b}+\frac{d\left(\frac{d\left(-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\cdots}{3a^{2}\cdots}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$$

$$\frac{x^2\cosh(c+dx)}{6b(a+bx^3)^2}$$

$\downarrow$ 5815

$$\frac{\frac{d\int\left(\frac{\sinh(c+dx)}{ax}-\frac{bx^2\sinh(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{\int\frac{\cosh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\cosh(c+dx)}{3bx(a+bx^3)}}{3b}+$$

$$\frac{d\left(\frac{d\left(-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\cdots}{3a^{2/\cdots}}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 2009

$$\frac{-\frac{\int\frac{\cosh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}+\frac{d\left(-\frac{\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a}-\frac{\sinh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\cdots\right.}{3a}\cdots\right.}{\cdots}}{\cdots}$$

$$\frac{d\left(\frac{d\left(-\frac{\sqrt[3]{-1}\cosh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\cdots}{3a^{2/\cdots}}\right.}{\cdots}\right.}{\cdots}$$

$$\frac{x^2\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 5816

$$-\frac{\cosh(c+dx)x^2}{6b\left(bx^3+a\right)^2}+$$

$$\frac{d\left(\frac{d\left(-\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/}}\right.}{}\right.}{-\frac{\cosh(c+dx)}{3bx\left(bx^3+a\right)}+\frac{d\left(\frac{\text{Chi}(dx)\sinh(c)}{a}-\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{Chi}\left(-xd-\frac{(-1)^{2}}{}\right.}{}\right.}{}}$$

$$\Big\downarrow \ 2009$$

$$-\frac{\cosh(c+dx)x^2}{6b\left(bx^3+a\right)^2}+$$

$$\frac{d\left(\frac{d\left(-\frac{\sqrt[3]{-1}\cosh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{(-1)^{2/3}\cosh\left(c-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(-xd-\frac{(-1)^{2/3}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/3}\sqrt[3]{b}}+\frac{\cosh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{2/}}\right.}{}\right.}{-\frac{\cosh(c+dx)}{3bx\left(bx^3+a\right)}+\frac{d\left(\frac{\text{Chi}(dx)\sinh(c)}{a}-\frac{\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a}-\frac{\text{Chi}\left(-xd-\frac{(-1)^{2}}{}\right.}{}\right.}{}}$$

input
```
Int[(x^4*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*(x^2*Cosh[c + d*x])/(b*(a + b*x^3)^2) + (d*(-1/3*Sinh[c + d*x]/(b*(a
+ b*x^3)) + (d*(-1/3*((-1)^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*
CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(a^(2/3)*b^(1/3)) + ((
-1)^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2
/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(2/3)*b^(1/3)) + (Cosh[c - (a^(1/3)*d
)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) +
((-1)^(1/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1
/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(2/3)*b^(1/3)) + (Sinh[c - (a^(1/3)*d)
/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3)) + (
(-1)^(2/3)*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/
3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(2/3)*b^(1/3))))/(3*b)))/(6*b) + (-1/3*
Cosh[c + d*x]/(b*x*(a + b*x^3)) + (d*((CoshIntegral[d*x]*Sinh[c])/a - (Cos
hIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(3*a)
- (CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)
*a^(1/3)*d)/b^(1/3)])/(3*a) - (CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/
3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a) + (Cosh[c]*Sinh
Integral[d*x])/a + (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/
3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a) - (Cosh[c - ((-1)^(2/3)
*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x]...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

rule 5811
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_
)], x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Cosh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.76 (sec) , antiderivative size = 4708, normalized size of antiderivative = 4.26

method	result	size
risch	Expression too large to display	4708

input
```
int(x^4*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/108/d^2*(-sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*
b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-2*_R1^2*a*b*d^3-16*_R1^2*b^2*c^3+4*_
R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*c^5-2*_R1*a*b*d^3-16*_R1
*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1
+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2-sum((4*_R1^2
*a*b*c*d^3-_R1^2*b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3
*d^3-b^2*c^6+2*_R1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c
^4-10*a*b*c^2*d^3+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^
2*c^4)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2-sum((4*_R1^2*a*b*c*d^3-_R1^2*b^2*c^4-
2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6-2*_R1^2*a*b*
d^3-16*_R1^2*b^2*c^3+4*_R1*a*b*c*d^3+26*_R1*b^2*c^4+10*a*b*c^2*d^3-10*b^2*
c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_R1*c+c^2
)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-
b*c^3))*b^2*x^6+3*exp(d*x+c)*a^2*b^2*d^3*x^3-sum((4*_R1^2*a*b*c*d^3-_R1^2*
b^2*c^4-2*_R1*a*b*c^2*d^3+2*_R1*b^2*c^5-a^2*d^6+2*a*b*c^3*d^3-b^2*c^6+2*_R
1^2*a*b*d^3+16*_R1^2*b^2*c^3-4*_R1*a*b*c*d^3-26*_R1*b^2*c^4-10*a*b*c^2*d^3
+10*b^2*c^5-2*_R1*a*b*d^3-16*_R1*b^2*c^3-6*a*b*c*d^3+6*b^2*c^4)/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^
2+a*d^3-b*c^3))*b^2*x^6+sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4691 vs. $2(805) = 1610$.

Time = 0.17 (sec) , antiderivative size = 4691, normalized size of antiderivative = 4.25

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**4*cosh(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^4 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/2*((d^3*x^4*e^(2*c) + 5*d^2*x^3*e^(2*c) + 30*d*x^2*e^(2*c) + 210*x*e^(2*
c))*e^(d*x) - (d^3*x^4 - 5*d^2*x^3 + 30*d*x^2 - 210*x)*e^(-d*x))/(b^3*d^4*
x^9*e^c + 3*a*b^2*d^4*x^6*e^c + 3*a^2*b*d^4*x^3*e^c + a^3*d^4*e^c) - 1/2*i
ntegrate(3*(15*a*d^2*x^2*e^c + (3*a*d^3*e^c - 560*b*e^c)*x^3 + 90*a*d*x*e^
c + 70*a*e^c)*e^(d*x)/(b^4*d^4*x^12 + 4*a*b^3*d^4*x^9 + 6*a^2*b^2*d^4*x^6
+ 4*a^3*b*d^4*x^3 + a^4*d^4), x) + 1/2*integrate(-3*(15*a*d^2*x^2 - (3*a*d
^3 + 560*b)*x^3 - 90*a*d*x + 70*a)*e^(-d*x)/(b^4*d^4*x^12*e^c + 4*a*b^3*d^
4*x^9*e^c + 6*a^2*b^2*d^4*x^6*e^c + 4*a^3*b*d^4*x^3*e^c + a^4*d^4*e^c), x)
```

Giac [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^4 \cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^4*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x^4*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^4 \cosh(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x^4*cosh(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x^4*cosh(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^4 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}x^4}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx\right)+\int \frac{x^4}{e^{dx}a^3+3e^{dx}a^2b\,x^3+3e^{dx}a\,b^2x^6+e^{dx}b^3x^9}dx}{2e^c}$$

input
```
int(x^4*cosh(d*x+c)/(b*x^3+a)^3,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x**4)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3
*x**9),x) + int(x**4/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**3 + 3*e**(d*x)*
a*b**2*x**6 + e**(d*x)*b**3*x**9),x))/(2*e**c)
```

3.109 $\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 776

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx = \text{Too large to display}$$

output

```
1/18*cosh(d*x+c)/a/b^2/x^2-1/6*x*cosh(d*x+c)/b/(b*x^3+a)^2-1/18*cosh(d*x+c
)/b^2/x^2/(b*x^3+a)-1/27*(-1)^(1/3)*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*C
hi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(5/3)/b^(4/3)-1/54*d^2*cosh(c+(-1)^
(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a/b^2+1/27*
(-1)^(2/3)*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/
b^(1/3)-d*x)/a^(5/3)/b^(4/3)-1/54*d^2*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))
*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a/b^2+1/27*cosh(c-a^(1/3)*d/b^(1/3
))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-1/54*d^2*cosh(c-a^(1/3)*d/b^
(1/3))*Chi(a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1/18*d*sinh(d*x+c)/a/b^2/x-1/18*d*
sinh(d*x+c)/b^2/x/(b*x^3+a)-1/27*(-1)^(1/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^
(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-1/54*d^2*sin
h(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a
/b^2+1/27*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(
4/3)-1/54*d^2*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a/b^2+1
/27*(-1)^(2/3)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)
*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-1/54*d^2*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/
3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.43 (sec) , antiderivative size = 429, normalized size of antiderivative = 0.55

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3}\,dx = -\frac{\text{RootSum}\left[a+b\#1^3\&,\frac{-2\cosh(c+d\#1)\text{Chi}(d(x-\#1))+2\text{Chi}(d(x-\#1))\sinh(c+d\#1)+2\cosh(c+d\#1)\text{Shi}(d(x-\#1))-\ldots}{}\right.}{}$$

input
```
Integrate[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/108*(RootSum[a + b*#1^3 & , (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]
 + 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + 2*Cosh[c + d*#1]*SinhIntegr
al[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d^2*Cosh[c +
d*#1]*CoshIntegral[d*(x - #1)]*#1^2 - d^2*CoshIntegral[d*(x - #1)]*Sinh[c
+ d*#1]*#1^2 - d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + d^2*Sinh
[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] + RootSum[a + b*#1^3 &
, (-2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]
*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d
*#1]*SinhIntegral[d*(x - #1)] + d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)
]*#1^2 + d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + d^2*Cosh[c + d
*#1]*SinhIntegral[d*(x - #1)]*#1^2 + d^2*Sinh[c + d*#1]*SinhIntegral[d*(x
- #1)]*#1^2)/#1^2 & ] - (6*b*x*((-2*a + b*x^3)*Cosh[c + d*x] + d*x*(a + b*
x^3)*Sinh[c + d*x]))/(a + b*x^3)^2)/(a*b^2)
```

Rubi [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1596 vs. 2(776) = 1552.

Time = 3.90 (sec) , antiderivative size = 1596, normalized size of antiderivative = 2.06, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {5814, 5802, 5813, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{ 5814}$$

$$\frac{d\int \frac{x\sinh(c+dx)}{(bx^3+a)^2}dx}{6b}+\frac{\int \frac{\cosh(c+dx)}{(bx^3+a)^2}dx}{6b}-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 5802}$$

$$\frac{d\int \frac{x\sinh(c+dx)}{(bx^3+a)^2}dx}{6b}+\frac{-\frac{2\int \frac{\cosh(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\sinh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\cosh(c+dx)}{3bx^2(a+bx^3)}}{6b}-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 5813}$$

$$\frac{d\left(\frac{d\int \frac{\cosh(c+dx)}{x\left(bx^3+a\right)}dx}{3b}-\frac{\int \frac{\sinh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\sinh(c+dx)}{3bx(a+bx^3)}\right)}{6b}+\frac{-\frac{2\int \frac{\cosh(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\sinh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\cosh(c+dx)}{3bx^2(a+bx^3)}}{6b}-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 5815}$$

$$\frac{d\left(\frac{d\int \frac{\cosh(c+dx)}{x\left(bx^3+a\right)}dx}{3b}-\frac{\int \left(\frac{\sinh(c+dx)}{ax^2}-\frac{bx\sinh(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{\sinh(c+dx)}{3bx(a+bx^3)}\right)}{6b}+\frac{-\frac{2\int \frac{\cosh(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}+\frac{d\int \left(\frac{\sinh(c+dx)}{ax^2}-\frac{bx\sinh(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{\cosh(c+dx)}{3bx^2(a+bx^3)}}{6b}-\frac{x\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{\cosh(c+dx)}{3bx^2(bx^3+a)}+\frac{d\left(\frac{d\cosh(c)\text{Chi}(dx)}{a}+\frac{\sqrt[3]{b}\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}\cdots\right.}{\cdots}}{d\left(-\frac{\sinh(c+dx)}{3bx(bx^3+a)}-\frac{\frac{d\cosh(c)\text{Chi}(dx)}{a}+\frac{\sqrt[3]{b}\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}\cdots}{\cdots}\right.\cdots}$$

$$-\frac{x\cosh(c+dx)}{6b\left(bx^3+a\right)^2}+$$

$\downarrow$ 5816

$$-\frac{x\cosh(c+dx)}{6b\left(bx^3+a\right)^2}+$$

$$\frac{-\frac{\cosh(c+dx)}{3bx^2(bx^3+a)}+\frac{d\left(\frac{d\cosh(c)\text{Chi}(dx)}{a}+\frac{\sqrt[3]{b}\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}\cdots\right.}{\cdots}}{d\left(-\frac{\sinh(c+dx)}{3bx(bx^3+a)}-\frac{\frac{d\cosh(c)\text{Chi}(dx)}{a}+\frac{\sqrt[3]{b}\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}\cdots}{\cdots}\right.\cdots}$$

$\downarrow$ 2009

$$-\frac{x\cosh(c+dx)}{6b\left(bx^3+a\right)^2}+$$

$$\frac{d\left(-\frac{\sinh(c+dx)}{3bx(bx^3+a)}-\frac{\frac{d\cosh(c)\mathrm{Chi}(dx)}{a}+\frac{\sqrt[3]{b}\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-}{}\right.}{-\frac{\cosh(c+dx)}{3bx^2(bx^3+a)}+\frac{d\left(\frac{d\cosh(c)\mathrm{Chi}(dx)}{a}+\frac{\sqrt[3]{b}\mathrm{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}+\frac{(-1)^{2/3}\sqrt[3]{b}\mathrm{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)\sinh\left(c+\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{4/3}}-\right.}{}}$$

input
```
Int[(x^3*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*(x*Cosh[c + d*x])/(b*(a + b*x^3)^2) + (d*(-1/3*Sinh[c + d*x]/(b*x*(a
+ b*x^3)) - ((d*Cosh[c]*CoshIntegral[d*x])/a + (b^(1/3)*CoshIntegral[(a^(1
/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) + ((-1)^(
2/3)*b^(1/3)*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + (
(-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*CoshInte
gral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)
*d)/b^(1/3)])/(3*a^(4/3)) - Sinh[c + d*x]/(a*x) + (d*Sinh[c]*SinhIntegral[
d*x])/a - ((-1)^(2/3)*b^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*Sin
hIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) + (b^(1/3)*Co
sh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^
(4/3)) - ((-1)^(1/3)*b^(1/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*Sinh
Integral[((-1)^(2/3)*a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)))/(3*b) + (d*((
Cosh[c]*CoshIntegral[d*x])/a - (Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*C
oshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a) - (Cosh[c - ((-1)
^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3))
- d*x])/(3*a) - (Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^
(1/3) + d*x])/(3*a) + (Sinh[c]*SinhIntegral[d*x])/a + (Sinh[c + ((-1)^(1/3
)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/
(3*a) - (Sinh[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) +
d*x])/(3*a) - (Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5802
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1))), x] + (-Sim
p[(-n + 1)/(b*n*(p + 1))   Int[((a + b*x^n)^(p + 1)*Cosh[c + d*x])/x^n, x],
 x] - Simp[d/(b*n*(p + 1))   Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Sinh[c + d*
x], x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p
, -1] && GtQ[n, 2]
```

rule 5813
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Sinh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.63 (sec) , antiderivative size = 3281, normalized size of antiderivative = 4.23

method	result	size
risch	Expression too large to display	3281

input
```
int(x^3*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108/d*(3*sum((_R2^2*b*c^2-_R2*a*d^3-2*_R2*b*c^3-a*c*d^3+b*c^4+8*_R2^2*b*
c-10*_R2*b*c^2-2*a*d^3+2*b*c^3+8*_R2*b*c+2*b*c^2)/(_R2^2-2*_R2*c+c^2)*exp(
-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)
)*a^2*c+3*exp(d*x+c)*a*b^2*d^2*x^5+2*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/
(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c
+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^3*x^3+3*sum((_R2^2*b*c^2-_R2*a*d^3-2*_R2
*b*c^3-a*c*d^3+b*c^4-8*_R2^2*b*c+10*_R2*b*c^2+2*a*d^3-2*b*c^3+8*_R2*b*c+2*
b*c^2)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_
Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*c+sum((_R2^2*a*d^3-_R2^2*b*c^3+_R2*a*
c*d^3+2*_R2*b*c^4+a*c^2*d^3-b*c^5+12*_R2^2*b*c^2-18*_R2*b*c^3-6*a*c*d^3+6*
b*c^4-12*_R2*b*c^2-2*a*d^3+2*b*c^3)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x
+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*x^6-3*su
m((_R2^2*b*c-2*_R2*b*c^2-a*d^3+b*c^3-4*_R2^2*b+2*_R2*b*c+2*b*c^2+4*_R2*b+6
*b*c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z
^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2*c^2+sum((_R2^2*a*d^3-_R2^2*b*c^3+_R2*a
*c*d^3+2*_R2*b*c^4+a*c^2*d^3-b*c^5+12*_R2^2*b*c^2-18*_R2*b*c^3-6*a*c*d^3+6
*b*c^4-12*_R2*b*c^2-2*a*d^3+2*b*c^3)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*
x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a^2+sum((_R
2^2*a*d^3-_R2^2*b*c^3+_R2*a*c*d^3+2*_R2*b*c^4+a*c^2*d^3-b*c^5-12*_R2^2*b*c
^2+18*_R2*b*c^3+6*a*c*d^3-6*b*c^4-12*_R2*b*c^2-2*a*d^3+2*b*c^3)/(_R2^2-...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2962 vs. 2(582) = 1164.

Time = 0.16 (sec) , antiderivative size = 2962, normalized size of antiderivative = 3.82

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} dx = \text{Too large to display}$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/108*(((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*
b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 + (a*d^3/b)^(1/3)
*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b
))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 +
2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt
(-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((a*b^2*d^3*x^6
+ 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^
3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^
3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b
^3*x^6 + 2*a*b^2*x^3 + a^2*b + sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*s
inh(d*x + c)^2))*Ei(-d*x - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(
-a*d^3/b)^(1/3)*(sqrt(-3) + 1) - c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 +
a^3*d^3)*cosh(d*x + c)^2 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sin
h(d*x + c)^2 + (a*d^3/b)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*
(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*cosh(d*x + c)^2 - (b^3*x^6 + 2*a*b^2*x^3
+ a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x^3 + a^2*b))*sinh(d*x + c)^2))*Ei(d
*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3
) - 1) - c) + ((a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*cosh(d*x + c)^2
 - (a*b^2*d^3*x^6 + 2*a^2*b*d^3*x^3 + a^3*d^3)*sinh(d*x + c)^2 - (-a*d^3/b
)^(1/3)*((b^3*x^6 + 2*a*b^2*x^3 + a^2*b - sqrt(-3)*(b^3*x^6 + 2*a*b^2*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \text{Timed out}$$

input
```
integrate(x**3*cosh(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^3 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \int \frac{x^3 \cosh(dx+c)}{(bx^3+a)^3} \, dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/2*((d^2*x^3*e^(2*c) + 6*d*x^2*e^(2*c) + 42*x*e^(2*c))*e^(d*x) - (d^2*x^3
 - 6*d*x^2 + 42*x)*e^(-d*x))/(b^3*d^3*x^9*e^c + 3*a*b^2*d^3*x^6*e^c + 3*a^
2*b*d^3*x^3*e^c + a^3*d^3*e^c) + 1/2*integrate(-3*(3*a*d^2*x^2*e^c - 112*b
*x^3*e^c + 18*a*d*x*e^c + 14*a*e^c)*e^(d*x)/(b^4*d^3*x^12 + 4*a*b^3*d^3*x^
9 + 6*a^2*b^2*d^3*x^6 + 4*a^3*b*d^3*x^3 + a^4*d^3), x) - 1/2*integrate(-3*
(3*a*d^2*x^2 - 112*b*x^3 - 18*a*d*x + 14*a)*e^(-d*x)/(b^4*d^3*x^12*e^c + 4
*a*b^3*d^3*x^9*e^c + 6*a^2*b^2*d^3*x^6*e^c + 4*a^3*b*d^3*x^3*e^c + a^4*d^3
*e^c), x)
```

Giac [F]

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^3\cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^3*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x^3*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^3\cosh(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x^3*cosh(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x^3*cosh(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^3\cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
int(x^3*cosh(d*x+c)/(b*x^3+a)^3,x)
```

output
```
( - e**(2*c + 2*d*x)*d*x + e**(2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x*
*3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**3*d + 2*e**(2*c + d*x)*int(e**(d*x)/
(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**2*b*d*x**3 + e**(
2*c + d*x)*int(e**(d*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9)
,x)*a*b**2*d*x**6 + e**(2*c + d*x)*int((e**(d*x)*x**4)/(a**3 + 3*a**2*b*x*
*3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**2*b*d**2 + 2*e**(2*c + d*x)*int((e**
(d*x)*x**4)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a*b**2*d
**2*x**3 + e**(2*c + d*x)*int((e**(d*x)*x**4)/(a**3 + 3*a**2*b*x**3 + 3*a*
b**2*x**6 + b**3*x**9),x)*b**3*d**2*x**6 + e**(2*c + d*x)*int((e**(d*x)*x)
/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a**3*d**2 + 2*e**(2
*c + d*x)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x*
*9),x)*a**2*b*d**2*x**3 + e**(2*c + d*x)*int((e**(d*x)*x)/(a**3 + 3*a**2*b
*x**3 + 3*a*b**2*x**6 + b**3*x**9),x)*a*b**2*d**2*x**6 - e**(d*x)*int(x**4
/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**3 + 3*e**(d*x)*a*b**2*x**6 + e**(d*
x)*b**3*x**9),x)*a**2*b*d**2 - 2*e**(d*x)*int(x**4/(e**(d*x)*a**3 + 3*e**(
d*x)*a**2*b*x**3 + 3*e**(d*x)*a*b**2*x**6 + e**(d*x)*b**3*x**9),x)*a*b**2*
d**2*x**3 - e**(d*x)*int(x**4/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**3 + 3*
e**(d*x)*a*b**2*x**6 + e**(d*x)*b**3*x**9),x)*b**3*d**2*x**6 - e**(d*x)*in
t(x**3/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**3 + 3*e**(d*x)*a*b**2*x**6 +
e**(d*x)*b**3*x**9),x)*a**2*b*d - 2*e**(d*x)*int(x**3/(e**(d*x)*a**3 + ...
```

3.110 $\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} dx$

Optimal result

Integrand size = 19, antiderivative size = 781

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

output
```
-1/6*cosh(d*x+c)/b/(b*x^3+a)^2+1/54*(-1)^(2/3)*d^2*cosh(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(4/3)/b^(5/3)-1/54*(
-1)^(1/3)*d^2*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)
*d/b^(1/3)-d*x)/a^(4/3)/b^(5/3)+1/54*d^2*cosh(c-a^(1/3)*d/b^(1/3))*Chi(a^(
1/3)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)+1/27*d*Chi(a^(1/3)*d/b^(1/3)+d*x)*sinh
(c-a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)-1/27*(-1)^(1/3)*d*Chi((-1)^(1/3)*a^(
1/3)*d/b^(1/3)-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1
/27*(-1)^(2/3)*d*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*
a^(1/3)*d/b^(1/3))/a^(5/3)/b^(4/3)+1/18*d*sinh(d*x+c)/a/b^2/x^2-1/18*d*sin
h(d*x+c)/b^2/x^2/(b*x^3+a)-1/27*(-1)^(1/3)*d*cosh(c+(-1)^(1/3)*a^(1/3)*d/b
^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*(-1)^(
2/3)*d^2*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^
(1/3)+d*x)/a^(4/3)/b^(5/3)+1/27*d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/
b^(1/3)+d*x)/a^(5/3)/b^(4/3)+1/54*d^2*sinh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3
)*d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)+1/27*(-1)^(2/3)*d*cosh(c-(-1)^(2/3)*a^(1/
3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-1/54*(
-1)^(1/3)*d^2*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Shi((-1)^(2/3)*a^(1/3)*
d/b^(1/3)+d*x)/a^(4/3)/b^(5/3)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 423, normalized size of antiderivative = 0.54

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^3}\,dx =$$

$$-\frac{d\text{RootSum}\left[a+b\#1^3\&,\frac{2\cosh(c+d\#1)\text{Chi}(d(x-\#1))-2\text{Chi}(d(x-\#1))\sinh(c+d\#1)-2\cosh(c+d\#1)\text{Shi}(d(x-\#1))+}{}\right.}{}$$

input
```
Integrate[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/108*(d*RootSum[a + b*#1^3 & , (2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)
] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] - 2*Cosh[c + d*#1]*SinhInteg
ral[d*(x - #1)] + 2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + d*Cosh[c + d
*#1]*CoshIntegral[d*(x - #1)]*#1 - d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#
1]*#1 - d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + d*Sinh[c + d*#1]*Si
nhIntegral[d*(x - #1)]*#1)/#1^2 & ] + d*RootSum[a + b*#1^3 & , (-2*Cosh[c
+ d*#1]*CoshIntegral[d*(x - #1)] - 2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#
1] - 2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] - 2*Sinh[c + d*#1]*SinhInte
gral[d*(x - #1)] + d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 + d*CoshIn
tegral[d*(x - #1)]*Sinh[c + d*#1]*#1 + d*Cosh[c + d*#1]*SinhIntegral[d*(x
- #1)]*#1 + d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1)/#1^2 & ] - (6*b*
Cosh[d*x]*(-3*a*Cosh[c] + d*x*(a + b*x^3)*Sinh[c]))/(a + b*x^3)^2 - (6*b*(
d*x*(a + b*x^3)*Cosh[c] - 3*a*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(a*b^2)
```

Rubi [A] (verified)

Time = 2.10 (sec) , antiderivative size = 863, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.316$, Rules used = {5812, 5801, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^3}\,dx$$

$\big\downarrow$ 5812

$$\frac{d\int \frac{\sinh(c+dx)}{(bx^3+a)^2}dx}{6b}-\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2}$$

$\big\downarrow$ 5801

$$\frac{d\left(-\frac{2\int \frac{\sinh(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\cosh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\sinh(c+dx)}{3bx^2(a+bx^3)}\right)}{6b}-\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2}$$

$\big\downarrow$ 5815

$$\frac{d\left(-\frac{2\int \left(\frac{\sinh(c+dx)}{ax^3}-\frac{b\sinh(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}+\frac{d\int \frac{\cosh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{\sinh(c+dx)}{3bx^2(a+bx^3)}\right)}{6b}-\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2}$$

$\big\downarrow$ 2009

$$d\left(\frac{d\int \frac{\cosh(c+dx)}{x^2\left(bx^3+a\right)}dx}{3b}-\frac{2\left(-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}-dx\right)}{3a^{5/3}}-\frac{(-1)^{2/3}b^{2/}{\cdots}}{\cdots}\right.}{\cdots}\right.$$

$$\frac{\cosh(c+dx)}{6b\,(a+bx^3)^2}$$

$\big\downarrow$ 5816

$$d\left(\frac{d\int\left(\frac{\cosh(c+dx)}{ax^2}-\frac{bx\cosh(c+dx)}{a\left(bx^3+a\right)}\right)dx}{3b}-\frac{2\left(-\frac{b^{2/3}\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\sinh\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}+c\right)\text{Chi}\left(\frac{\sqrt[3]{-1}\sqrt[3]{a}d}{\sqrt[3]{b}}\right.}{3a^{5/3}}\right.}{}\right.$$

$$\frac{\cosh(c+dx)}{6b\left(a+bx^3\right)^2}$$

$\downarrow$ 2009

$$d\left(-\frac{\sinh(c+dx)}{3bx^2(bx^3+a)}-\frac{2\left(\frac{\text{Chi}(dx)\sinh(c)d^2}{2a}+\frac{\cosh(c)\text{Shi}(dx)d^2}{2a}-\frac{\cosh(c+dx)d}{2ax}-\frac{b^{2/3}\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^{5/3}}+\frac{\sqrt[3]{-1}b^{2/3}\text{Chi}\left(\frac{\sqrt[3]{-}}{}\right.}{}\right.}{}\right.$$

$$\frac{\cosh(c+dx)}{6b\left(bx^3+a\right)^2}$$

input
```
Int[(x^2*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*Cosh[c + d*x]/(b*(a + b*x^3)^2) + (d*(-1/3*Sinh[c + d*x]/(b*x^2*(a +
b*x^3)) - (2*(-1/2*(d*Cosh[c + d*x])/(a*x) + (d^2*CoshIntegral[d*x]*Sinh[c
])/(2*a) - (b^(2/3)*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*x]*Sinh[c - (a^(1
/3)*d)/b^(1/3)])/(3*a^(5/3)) + ((-1)^(1/3)*b^(2/3)*CoshIntegral[((-1)^(1/3
)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)])/(3*a
^(5/3)) - ((-1)^(2/3)*b^(2/3)*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)) - d*x]*Sinh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^(5/3)) - Sinh[c +
 d*x]/(2*a*x^2) + (d^2*Cosh[c]*SinhIntegral[d*x])/(2*a) - ((-1)^(1/3)*b^(2
/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1
/3)*d)/b^(1/3) - d*x])/(3*a^(5/3)) - (b^(2/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)
]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/
3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^(1/
3)*d)/b^(1/3) + d*x])/(3*a^(5/3))))/(3*b) + (d*(-(Cosh[c + d*x]/(a*x)) + (
(-1)^(2/3)*b^(1/3)*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[(
(-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^(4/3)) - ((-1)^(1/3)*b^(1/3)*Co
sh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*
d)/b^(1/3)) - d*x])/(3*a^(4/3)) + (b^(1/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*C
oshIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^(4/3)) + (d*CoshIntegral[d*x]
*Sinh[c])/a + (d*Cosh[c]*SinhIntegral[d*x])/a - ((-1)^(2/3)*b^(1/3)*Sinh[c
 + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5801
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Si
mp[x^(-n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1))), x] + (-Sim
p[(-n + 1)/(b*n*(p + 1))   Int[((a + b*x^n)^(p + 1)*Sinh[c + d*x])/x^n, x],
 x] - Simp[d/(b*n*(p + 1))   Int[x^(-n + 1)*(a + b*x^n)^(p + 1)*Cosh[c + d*
x], x], x]) /; FreeQ[{a, b, c, d}, x] && IntegerQ[p] && IGtQ[n, 0] && LtQ[p
, -1] && GtQ[n, 2]
```

rule 5812
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p
_), x_Symbol] :> Simp[e^m*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)))
, x] - Simp[d*(e^m/(b*n*(p + 1)))   Int[(a + b*x^n)^(p + 1)*Sinh[c + d*x],
x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IntegerQ[p] && EqQ[m - n + 1,
0] && LtQ[p, -1] && (IntegerQ[n] || GtQ[e, 0])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.53 (sec) , antiderivative size = 2238, normalized size of antiderivative = 2.87

method	result	size
risch	Expression too large to display	2238

input
```
int(x^2*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/108*(-sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*
Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^3
*c^2*x^6-sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1
)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^
3*c^2*x^6+2*sum((_R2^2*b*c-2*_R2*b*c^2-a*d^3+b*c^3-4*_R2^2*b+2*_R2*b*c+2*b
*c^2+4*_R2*b+6*b*c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=Root
Of(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*c*x^6+2*sum((_R2^2*b*c-2
*_R2*b*c^2-a*d^3+b*c^3+4*_R2^2*b-2*_R2*b*c-2*b*c^2+4*_R2*b+6*b*c)/(_R2^2-2
*_R2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*
c^2+a*d^3-b*c^3))*b^2*c*x^6+3*a*b^2*d*x^4*exp(d*x+c)-sum((_R2^2*b*c^2-_R2*
a*d^3-2*_R2*b*c^3-a*c*d^3+b*c^4-8*_R2^2*b*c+10*_R2*b*c^2+2*a*d^3-2*b*c^3+8
*_R2*b*c+2*b*c^2)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*x^6-2*sum((_R1^2-2*_R1*c+c
^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(
_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c^2*x^3-3*a*b^2*d*x^4*exp
(-d*x-c)-sum((_R2^2*b*c^2-_R2*a*d^3-2*_R2*b*c^3-a*c*d^3+b*c^4+8*_R2^2*b*c-
10*_R2*b*c^2-2*a*d^3+2*b*c^3+8*_R2*b*c+2*b*c^2)/(_R2^2-2*_R2*c+c^2)*exp(-_
R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*
b^2*x^6-2*sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R
1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3)...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2972 vs. $2(559) = 1118$.

Time = 0.16 (sec) , antiderivative size = 2972, normalized size of antiderivative = 3.81

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
-1/216*(36*a^2*cosh(d*x + c) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^
2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a
*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*
(a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^
3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2*x^6
 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(d*x - 1/2*(a*d^3/b)^(1/3)*(sqrt(
-3) + 1))*cosh(1/2*(a*d^3/b)^(1/3)*(sqrt(-3) + 1) + c) + ((-a*d^3/b)^(2/3)
*((b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(
d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 +
a^2))*sinh(d*x + c)^2) + 2*(-a*d^3/b)^(1/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 +
sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x
^3 + a^2 + sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2))*Ei(-d*x
 - 1/2*(-a*d^3/b)^(1/3)*(sqrt(-3) + 1))*cosh(1/2*(-a*d^3/b)^(1/3)*(sqrt(-3
) + 1) - c) + ((a*d^3/b)^(2/3)*((b^2*x^6 + 2*a*b*x^3 + a^2 + sqrt(-3)*(b^2
*x^6 + 2*a*b*x^3 + a^2))*cosh(d*x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 + sq
rt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*sinh(d*x + c)^2) + 2*(a*d^3/b)^(1/3)*(
(b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^2))*cosh(d*
x + c)^2 - (b^2*x^6 + 2*a*b*x^3 + a^2 - sqrt(-3)*(b^2*x^6 + 2*a*b*x^3 + a^
2))*sinh(d*x + c)^2))*Ei(d*x + 1/2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1))*cosh(1/
2*(a*d^3/b)^(1/3)*(sqrt(-3) - 1) - c) + ((-a*d^3/b)^(2/3)*((b^2*x^6 + 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x**2*cosh(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2\cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/2*((d*x^2*e^(2*c) + 7*x*e^(2*c))*e^(d*x) - (d*x^2 - 7*x)*e^(-d*x))/(b^3*
d^2*x^9*e^c + 3*a*b^2*d^2*x^6*e^c + 3*a^2*b*d^2*x^3*e^c + a^3*d^2*e^c) + 1
/2*integrate((56*b*x^3*e^c - 9*a*d*x*e^c - 7*a*e^c)*e^(d*x)/(b^4*d^2*x^12
+ 4*a*b^3*d^2*x^9 + 6*a^2*b^2*d^2*x^6 + 4*a^3*b*d^2*x^3 + a^4*d^2), x) + 1
/2*integrate((56*b*x^3 + 9*a*d*x - 7*a)*e^(-d*x)/(b^4*d^2*x^12*e^c + 4*a*b
^3*d^2*x^9*e^c + 6*a^2*b^2*d^2*x^6*e^c + 4*a^3*b*d^2*x^3*e^c + a^4*d^2*e^c
), x)
```

Giac [F]

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2\cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x^2*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x^2*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x^2\cosh(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x^2*cosh(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x^2*cosh(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x^2 \cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}x^2}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx\right)+\int \frac{x^2}{e^{dx}a^3+3e^{dx}a^2b\,x^3+3e^{dx}a\,b^2x^6+e^{dx}b^3x^9}dx}{2e^c}$$

input
```
int(x^2*cosh(d*x+c)/(b*x^3+a)^3,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x**2)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3
*x**9),x) + int(x**2/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**3 + 3*e**(d*x)*
a*b**2*x**6 + e**(d*x)*b**3*x**9),x))/(2*e**c)
```

3.111 $\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 1147

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

output
```
-1/18*cosh(d*x+c)/a/b^2/x^4+2/9*cosh(d*x+c)/a^2/b/x-1/6*cosh(d*x+c)/b/x/(b
*x^3+a)^2+1/18*cosh(d*x+c)/b^2/x^4/(b*x^3+a)-2/27*(-1)^(2/3)*cosh(c+(-1)^(
1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(7/3)/b^(2
/3)+1/54*(-1)^(1/3)*d^2*cosh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Chi((-1)^(1/3
)*a^(1/3)*d/b^(1/3)-d*x)/a^(5/3)/b^(4/3)+2/27*(-1)^(1/3)*cosh(c-(-1)^(2/3)
*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*a^(1/3)*d/b^(1/3)-d*x)/a^(7/3)/b^(2/3)
-1/54*(-1)^(2/3)*d^2*cosh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))*Chi(-(-1)^(2/3)*
a^(1/3)*d/b^(1/3)-d*x)/a^(5/3)/b^(4/3)-2/27*cosh(c-a^(1/3)*d/b^(1/3))*Chi(
a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-1/54*d^2*cosh(c-a^(1/3)*d/b^(1/3))*
Chi(a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-2/27*d*Chi(a^(1/3)*d/b^(1/3)+d*
x)*sinh(c-a^(1/3)*d/b^(1/3))/a^2/b-2/27*d*Chi((-1)^(1/3)*a^(1/3)*d/b^(1/3)
-d*x)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))/a^2/b-2/27*d*Chi(-(-1)^(2/3)*a^
(1/3)*d/b^(1/3)-d*x)*sinh(c-(-1)^(2/3)*a^(1/3)*d/b^(1/3))/a^2/b+1/18*d*sin
h(d*x+c)/a/b^2/x^3-1/18*d*sinh(d*x+c)/b^2/x^3/(b*x^3+a)-2/27*d*cosh(c+(-1)
^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/2
7*(-1)^(2/3)*sinh(c+(-1)^(1/3)*a^(1/3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*
d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)+1/54*(-1)^(1/3)*d^2*sinh(c+(-1)^(1/3)*a^(1/
3)*d/b^(1/3))*Shi(-(-1)^(1/3)*a^(1/3)*d/b^(1/3)+d*x)/a^(5/3)/b^(4/3)-2/27*
d*cosh(c-a^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^2/b-2/27*sinh(c-a
^(1/3)*d/b^(1/3))*Shi(a^(1/3)*d/b^(1/3)+d*x)/a^(7/3)/b^(2/3)-1/54*d^2*s...
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 4 in optimal.

Time = 0.31 (sec) , antiderivative size = 669, normalized size of antiderivative = 0.58

$$\int \frac{x \cosh(c+dx)}{(a+bx^3)^3} \, dx = \text{Too large to display}$$

input
```
Integrate[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
(RootSum[a + b*#1^3 & , (-(a*d^2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)])
+ a*d^2*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1] + a*d^2*Cosh[c + d*#1]*Sin
hIntegral[d*(x - #1)] - a*d^2*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)] + 4*
b*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1 - 4*b*CoshIntegral[d*(x - #1)
]*Sinh[c + d*#1]*#1 - 4*b*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b
*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1 + 4*b*d*Cosh[c + d*#1]*CoshInt
egral[d*(x - #1)]*#1^2 - 4*b*d*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^
2 - 4*b*d*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^2 + 4*b*d*Sinh[c + d*
#1]*SinhIntegral[d*(x - #1)]*#1^2)/#1^2 & ] - RootSum[a + b*#1^3 & , (a*d^
2*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)] + a*d^2*CoshIntegral[d*(x - #1)]
*Sinh[c + d*#1] + a*d^2*Cosh[c + d*#1]*SinhIntegral[d*(x - #1)] + a*d^2*Si
nh[c + d*#1]*SinhIntegral[d*(x - #1)] - 4*b*Cosh[c + d*#1]*CoshIntegral[d*
(x - #1)]*#1 - 4*b*CoshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1 - 4*b*Cosh[c
 + d*#1]*SinhIntegral[d*(x - #1)]*#1 - 4*b*Sinh[c + d*#1]*SinhIntegral[d*(
x - #1)]*#1 + 4*b*d*Cosh[c + d*#1]*CoshIntegral[d*(x - #1)]*#1^2 + 4*b*d*C
oshIntegral[d*(x - #1)]*Sinh[c + d*#1]*#1^2 + 4*b*d*Cosh[c + d*#1]*SinhInt
egral[d*(x - #1)]*#1^2 + 4*b*d*Sinh[c + d*#1]*SinhIntegral[d*(x - #1)]*#1^
2)/#1^2 & ] + (6*b*Cosh[d*x]*(b*x^2*(7*a + 4*b*x^3)*Cosh[c] + a*d*(a + b*x
^3)*Sinh[c]))/(a + b*x^3)^2 + (6*b*(a*d*(a + b*x^3)*Cosh[c] + b*x^2*(7*a +
 4*b*x^3)*Sinh[c])*Sinh[d*x])/(a + b*x^3)^2)/(108*a^2*b^2)
```

Rubi [A] (verified)

Time = 4.97 (sec) , antiderivative size = 1813, normalized size of antiderivative = 1.58, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {5814, 5813, 5814, 5815, 2009, 5816, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx$$

$$\Big\downarrow \text{5814}$$

$$\frac{d\int \frac{\sinh(c+dx)}{x(bx^3+a)^2}dx}{6b}-\frac{\int \frac{\cosh(c+dx)}{x^2(bx^3+a)^2}dx}{6b}-\frac{\cosh(c+dx)}{6bx\,(a+bx^3)^2}$$

$$\Big\downarrow \text{5813}$$

$$\frac{d\left(\frac{d\int \frac{\cosh(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}-\frac{\int \frac{\sinh(c+dx)}{x^4\left(bx^3+a\right)}dx}{b}-\frac{\sinh(c+dx)}{3bx^3(a+bx^3)}\right)}{6b}-\frac{\int \frac{\cosh(c+dx)}{x^2(bx^3+a)^2}dx}{6b}-\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{5814}$$

$$\frac{d\left(\frac{d\int \frac{\cosh(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}-\frac{\int \frac{\sinh(c+dx)}{x^4\left(bx^3+a\right)}dx}{b}-\frac{\sinh(c+dx)}{3bx^3(a+bx^3)}\right)}{6b}-\frac{-\frac{4\int \frac{\cosh(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b}+\frac{d\int \frac{\sinh(c+dx)}{x^4\left(bx^3+a\right)}dx}{3b}-\frac{\cosh(c+dx)}{3bx^4(a+bx^3)}}{6b}-\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{5815}$$

$$\frac{d\left(-\frac{\int\left(\frac{b^2\sinh(c+dx)x^2}{a^2\left(bx^3+a\right)}-\frac{b\sinh(c+dx)}{a^2x}+\frac{\sinh(c+dx)}{ax^4}\right)dx}{b}+\frac{d\int \frac{\cosh(c+dx)}{x^3\left(bx^3+a\right)}dx}{3b}-\frac{\sinh(c+dx)}{3bx^3(a+bx^3)}\right)}{6b}-\frac{\frac{d\int\left(\frac{b^2\sinh(c+dx)x^2}{a^2\left(bx^3+a\right)}-\frac{b\sinh(c+dx)}{a^2x}+\frac{\sinh(c+dx)}{ax^4}\right)dx}{3b}-\frac{4\int \frac{\cosh(c+dx)}{x^5\left(bx^3+a\right)}dx}{3b}-\frac{\cosh(c+dx)}{3bx^4(a+bx^3)}}{6b}-\frac{\cosh(c+dx)}{6bx\left(a+bx^3\right)^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\cosh(c+dx)}{6bx\left(bx^3+a\right)^2}-$$

$$\frac{-\frac{\cosh(c+dx)}{3bx^4(bx^3+a)}+\frac{d\left(\frac{\cosh(c)\text{Chi}(dx)d^3}{6a}+\frac{\sinh(c)\text{Shi}(dx)d^3}{6a}-\frac{\sinh(c+dx)d^2}{6ax}-\frac{\cosh(c+dx)d}{6ax^2}-\frac{b\text{Chi}(dx)\sinh(c)}{a^2}+\frac{b\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\cdots\right.}{\cdots}}{\cdots}$$

$$d\left(-\frac{\sinh(c+dx)}{3bx^3(bx^3+a)}-\frac{\frac{\cosh(c)\text{Chi}(dx)d^3}{6a}+\frac{\sinh(c)\text{Shi}(dx)d^3}{6a}-\frac{\sinh(c+dx)d^2}{6ax}-\frac{\cosh(c+dx)d}{6ax^2}-\frac{b\text{Chi}(dx)\sinh(c)}{a^2}+\frac{b\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\cdots}{\cdots}\right.$$

$$\Big\downarrow \text{5816}$$

$$-\frac{\cosh(c+dx)}{6bx\left(bx^3+a\right)^2}+\frac{d\left(-\frac{\sinh(c+dx)}{3bx^3(bx^3+a)}-\frac{\frac{\cosh(c)\text{Chi}(dx)d^3}{6a}+\frac{\sinh(c)\text{Shi}(dx)d^3}{6a}-\frac{\sinh(c+dx)d^2}{6ax}-\frac{\cosh(c+dx)d}{6ax^2}-\frac{b\text{Chi}(dx)\sinh(c)}{a^2}+\frac{b\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\cdots}{\cdots}\right.}{-\frac{\cosh(c+dx)}{3bx^4(bx^3+a)}+\frac{d\left(\frac{\cosh(c)\text{Chi}(dx)d^3}{6a}+\frac{\sinh(c)\text{Shi}(dx)d^3}{6a}-\frac{\sinh(c+dx)d^2}{6ax}-\frac{\cosh(c+dx)d}{6ax^2}-\frac{b\text{Chi}(dx)\sinh(c)}{a^2}+\frac{b\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\cdots\right.}{\cdots}}$$

$\downarrow$ 2009

$$-\frac{\cosh(c+dx)}{6bx\left(bx^3+a\right)^2}+\frac{d\left(-\frac{\sinh(c+dx)}{3bx^3(bx^3+a)}-\frac{\frac{\cosh(c)\text{Chi}(dx)d^3}{6a}+\frac{\sinh(c)\text{Shi}(dx)d^3}{6a}-\frac{\sinh(c+dx)d^2}{6ax}-\frac{\cosh(c+dx)d}{6ax^2}-\frac{b\text{Chi}(dx)\sinh(c)}{a^2}+\frac{b\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\cdots}{\cdots}\right.}{-\frac{\cosh(c+dx)}{3bx^4(bx^3+a)}+\frac{d\left(\frac{\cosh(c)\text{Chi}(dx)d^3}{6a}+\frac{\sinh(c)\text{Shi}(dx)d^3}{6a}-\frac{\sinh(c+dx)d^2}{6ax}-\frac{\cosh(c+dx)d}{6ax^2}-\frac{b\text{Chi}(dx)\sinh(c)}{a^2}+\frac{b\text{Chi}\left(xd+\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)\sinh\left(c-\frac{\sqrt[3]{a}d}{\sqrt[3]{b}}\right)}{3a^2}\cdots\right.}{\cdots}}$$

input
```
Int[(x*Cosh[c + d*x])/(a + b*x^3)^3,x]
```

output
```
-1/6*Cosh[c + d*x]/(b*x*(a + b*x^3)^2) + (d*(-1/3*Sinh[c + d*x]/(b*x^3*(a
+ b*x^3)) - (-1/6*(d*Cosh[c + d*x])/(a*x^2) + (d^3*Cosh[c]*CoshIntegral[d*
x])/(6*a) - (b*CoshIntegral[d*x]*Sinh[c])/a^2 + (b*CoshIntegral[(a^(1/3)*d
)/b^(1/3) + d*x]*Sinh[c - (a^(1/3)*d)/b^(1/3)])/(3*a^2) + (b*CoshIntegral[
((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)])/(3*a^2) + (b*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x]*S
inh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)])/(3*a^2) - Sinh[c + d*x]/(3*a*x^3)
 - (d^2*Sinh[c + d*x])/(6*a*x) - (b*Cosh[c]*SinhIntegral[d*x])/a^2 + (d^3*
Sinh[c]*SinhIntegral[d*x])/(6*a) - (b*Cosh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1
/3)]*SinhIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x])/(3*a^2) + (b*Cosh
[c - (a^(1/3)*d)/b^(1/3)]*SinhIntegral[(a^(1/3)*d)/b^(1/3) + d*x])/(3*a^2)
 + (b*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3)]*SinhIntegral[((-1)^(2/3)*a^
(1/3)*d)/b^(1/3) + d*x])/(3*a^2))/b + (d*(-1/2*Cosh[c + d*x]/(a*x^2) + (d^
2*Cosh[c]*CoshIntegral[d*x])/(2*a) + ((-1)^(1/3)*b^(2/3)*Cosh[c + ((-1)^(1
/3)*a^(1/3)*d)/b^(1/3)]*CoshIntegral[((-1)^(1/3)*a^(1/3)*d)/b^(1/3) - d*x]
)/(3*a^(5/3)) - ((-1)^(2/3)*b^(2/3)*Cosh[c - ((-1)^(2/3)*a^(1/3)*d)/b^(1/3
)]*CoshIntegral[-(((-1)^(2/3)*a^(1/3)*d)/b^(1/3)) - d*x])/(3*a^(5/3)) - (b
^(2/3)*Cosh[c - (a^(1/3)*d)/b^(1/3)]*CoshIntegral[(a^(1/3)*d)/b^(1/3) + d*
x])/(3*a^(5/3)) - (d*Sinh[c + d*x])/(2*a*x) + (d^2*Sinh[c]*SinhIntegral[d*
x])/(2*a) - ((-1)^(1/3)*b^(2/3)*Sinh[c + ((-1)^(1/3)*a^(1/3)*d)/b^(1/3)...
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5813
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Sinh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Sinh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Cosh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5814
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Simp[x^(m - n + 1)*(a + b*x^n)^(p + 1)*(Cosh[c + d*x]/(b*n*(p + 1)
)), x] + (-Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*(a + b*x^n)^(p +
1)*Cosh[c + d*x], x], x] - Simp[d/(b*n*(p + 1))   Int[x^(m - n + 1)*(a + b*
x^n)^(p + 1)*Sinh[c + d*x], x], x]) /; FreeQ[{a, b, c, d}, x] && ILtQ[p, -1
] && IGtQ[n, 0] && RationalQ[m] && (GtQ[m - n + 1, 0] || GtQ[n, 2])
```

rule 5815
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*Sinh[(c_.) + (d_.)*(x_)], x_Sy
mbol] :> Int[ExpandIntegrand[Sinh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

rule 5816
```
Int[Cosh[(c_.) + (d_.)*(x_)]*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Sy
mbol] :> Int[ExpandIntegrand[Cosh[c + d*x], x^m*(a + b*x^n)^p, x], x] /; Fr
eeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[m] && IGtQ[n, 0] && (EqQ[n,
2] || EqQ[p, -1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 1.38 (sec) , antiderivative size = 1416, normalized size of antiderivative = 1.23

method	result	size
risch	Expression too large to display	1416

input
```
int(x*cosh(d*x+c)/(b*x^3+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/108*(-sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(_R1)
*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^
3*c*d*x^6-sum((_R1^2-2*_R1*c+c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R
1)*Ei(1,d*x-_R1+c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b
^3*c*d*x^6+sum((_R2^2*b*c-2*_R2*b*c^2-a*d^3+b*c^3-4*_R2^2*b+2*_R2*b*c+2*b*
c^2+4*_R2*b+6*b*c)/(_R2^2-2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootO
f(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*b^2*d*x^6+sum((_R2^2*b*c-2*_R
2*b*c^2-a*d^3+b*c^3+4*_R2^2*b-2*_R2*b*c-2*b*c^2+4*_R2*b+6*b*c)/(_R2^2-2*_R
2*c+c^2)*exp(-_R2)*Ei(1,d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2
+a*d^3-b*c^3))*b^2*d*x^6-2*sum((_R1^2-2*_R1*c+c^2-6*_R1+6*c+10)/(_R1^2-2*_
R1*c+c^2)*exp(_R1)*Ei(1,-d*x+_R1-c),_R1=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^
2+a*d^3-b*c^3))*a*b^2*c*d*x^3-12*exp(-d*x-c)*b^3*x^5-2*sum((_R1^2-2*_R1*c+
c^2+6*_R1-6*c+10)/(_R1^2-2*_R1*c+c^2)*exp(-_R1)*Ei(1,d*x-_R1+c),_R1=RootOf
(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b^2*c*d*x^3-12*exp(d*x+c)*b^
3*x^5+3*a*b^2*d*x^3*exp(-d*x-c)-3*a*b^2*d*x^3*exp(d*x+c)+2*sum((_R2^2*b*c-
2*_R2*b*c^2-a*d^3+b*c^3-4*_R2^2*b+2*_R2*b*c+2*b*c^2+4*_R2*b+6*b*c)/(_R2^2-
2*_R2*c+c^2)*exp(_R2)*Ei(1,-d*x+_R2-c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b
*c^2+a*d^3-b*c^3))*a*b*d*x^3+2*sum((_R2^2*b*c-2*_R2*b*c^2-a*d^3+b*c^3+4*_R
2^2*b-2*_R2*b*c-2*b*c^2+4*_R2*b+6*b*c)/(_R2^2-2*_R2*c+c^2)*exp(-_R2)*Ei(1,
d*x-_R2+c),_R2=RootOf(_Z^3*b-3*_Z^2*b*c+3*_Z*b*c^2+a*d^3-b*c^3))*a*b*d*...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4691 vs. $2(843) = 1686$.

Time = 0.19 (sec) , antiderivative size = 4691, normalized size of antiderivative = 4.09

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \text{Timed out}$$

input
```
integrate(x*cosh(d*x+c)/(b*x**3+a)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="maxima")
```

output
```
1/2*(x*e^(d*x + 2*c) - x*e^(-d*x))/(b^3*d*x^9*e^c + 3*a*b^2*d*x^6*e^c + 3*
a^2*b*d*x^3*e^c + a^3*d*e^c) + 1/2*integrate((8*b*x^3*e^c - a*e^c)*e^(d*x)
/(b^4*d*x^12 + 4*a*b^3*d*x^9 + 6*a^2*b^2*d*x^6 + 4*a^3*b*d*x^3 + a^4*d), x
) - 1/2*integrate((8*b*x^3 - a)*e^(-d*x)/(b^4*d*x^12*e^c + 4*a*b^3*d*x^9*e
^c + 6*a^2*b^2*d*x^6*e^c + 4*a^3*b*d*x^3*e^c + a^4*d*e^c), x)
```

Giac [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\cosh(dx+c)}{(bx^3+a)^3}\,dx$$

input
```
integrate(x*cosh(d*x+c)/(b*x^3+a)^3,x, algorithm="giac")
```

output
```
integrate(x*cosh(d*x + c)/(b*x^3 + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \int \frac{x\cosh(c+d\,x)}{(b\,x^3+a)^3}\,dx$$

input
```
int((x*cosh(c + d*x))/(a + b*x^3)^3,x)
```

output
```
int((x*cosh(c + d*x))/(a + b*x^3)^3, x)
```

Reduce [F]

$$\int \frac{x\cosh(c+dx)}{(a+bx^3)^3}\,dx = \frac{e^{2c}\left(\int \frac{e^{dx}x}{b^3x^9+3a\,b^2x^6+3a^2b\,x^3+a^3}dx\right)+\int \frac{x}{e^{dx}a^3+3e^{dx}a^2b\,x^3+3e^{dx}a\,b^2x^6+e^{dx}b^3x^9}dx}{2e^c}$$

input
```
int(x*cosh(d*x+c)/(b*x^3+a)^3,x)
```

output
```
(e**(2*c)*int((e**(d*x)*x)/(a**3 + 3*a**2*b*x**3 + 3*a*b**2*x**6 + b**3*x*
*9),x) + int(x/(e**(d*x)*a**3 + 3*e**(d*x)*a**2*b*x**3 + 3*e**(d*x)*a*b**2
*x**6 + e**(d*x)*b**3*x**9),x))/(2*e**c)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
      1
    else
      max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
        1
      else
        max(2,ExpnType(op(1,expn)))
      end if
    else
        max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
         return True
      else:
         return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
      return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):    #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
         return expnType(expn.args[0])    #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#            Albert Rich to use with Sagemath. This is used to
#            grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#            'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#            issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
       return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
       if expn.parent() is SR:
          return expn.operator() is None
       if expn.parent() in (ZZ, QQ, AA, QQbar):
          return expn in expn.parent() # Should always return True
       if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
          return expn in expn.parent().base_ring() or expn in expn.parent().gens()

       return False

    except AttributeError as error:
       print("Exception,AttributeError in  is_atom")
       print ("cought exception" , type(error).__name__ )
       return False


def expnType(expn):

    if debug:
       print (">>>>>Enter expnType, expn=", expn)
       print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
       return 1
    elif type(expn)==list:   #isinstance(expn,list):
       return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
       if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
          return 1
       else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
       if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])    #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))  #max(3,expnType(expr
  elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
    m1 = expnType(expn.operands()[0])  #expnType(expn.args[0])
    m2 = expnType(expn.operands()[1:])  #expnType(list(expn.args[1:]))
    return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif is_elementary_function(expn.operator()):   #is_elementary_function(expn.func)
    return max(3,expnType(expn.operands()[0]))
  elif is_special_function(expn.operator()): #is_special_function(expn.func)
    m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
    return max(4,m1)    #max(4,m1)
  elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(5,m1)    #max(5,m1)
  elif is_appell_function(expn.operator()):
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(6,m1)    #max(6,m1)
  elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
    m1 = max(map(expnType, expn.operands()))        #max(map(expnType, list(expn.args)))
    return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
  else:
    return 9

#main function
def grade_antiderivative(result,optimal):


  if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file