Computer Algebra Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.2-Hyperbolic-cosine/303-6.2.5

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**337**]. This is test number [303].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (337)	0.00 (0)
Mathematica	100.00 (337)	0.00 (0)
Fricas	96.74 (326)	3.26 (11)
Maple	91.10 (307)	8.90 (30)
Giac	75.37 (254)	24.63 (83)
Maxima	61.72 (208)	38.28 (129)
Reduce	59.94 (202)	40.06 (135)
Mupad	56.38 (190)	43.62 (147)
Sympy	30.86 (104)	69.14 (233)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

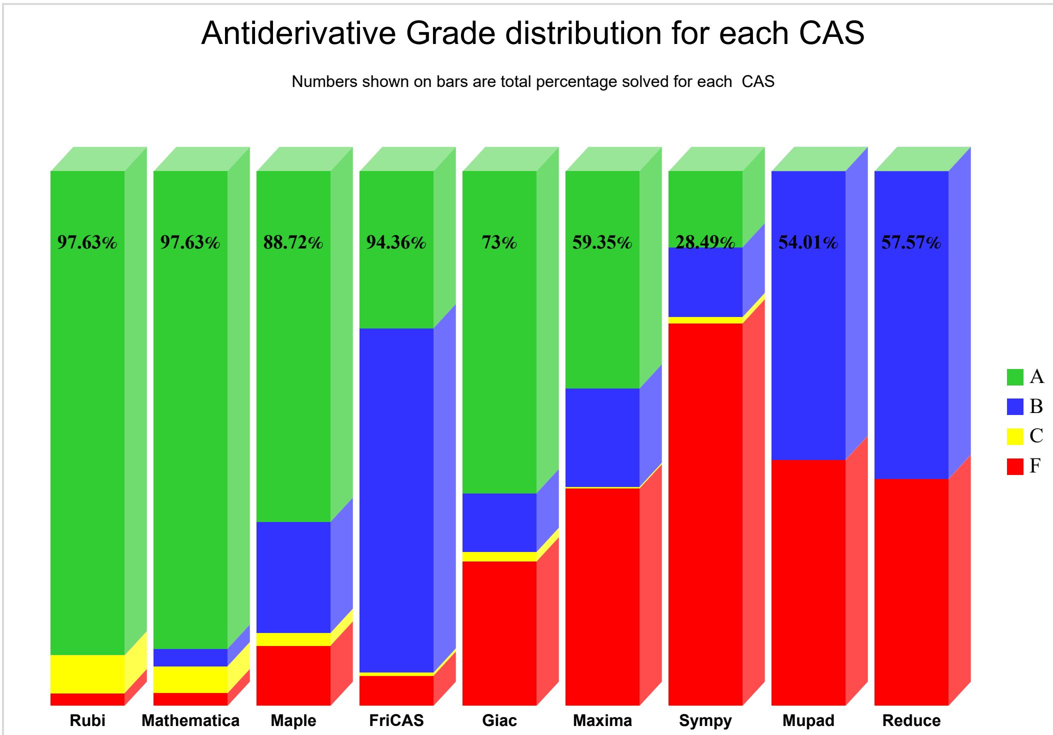
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

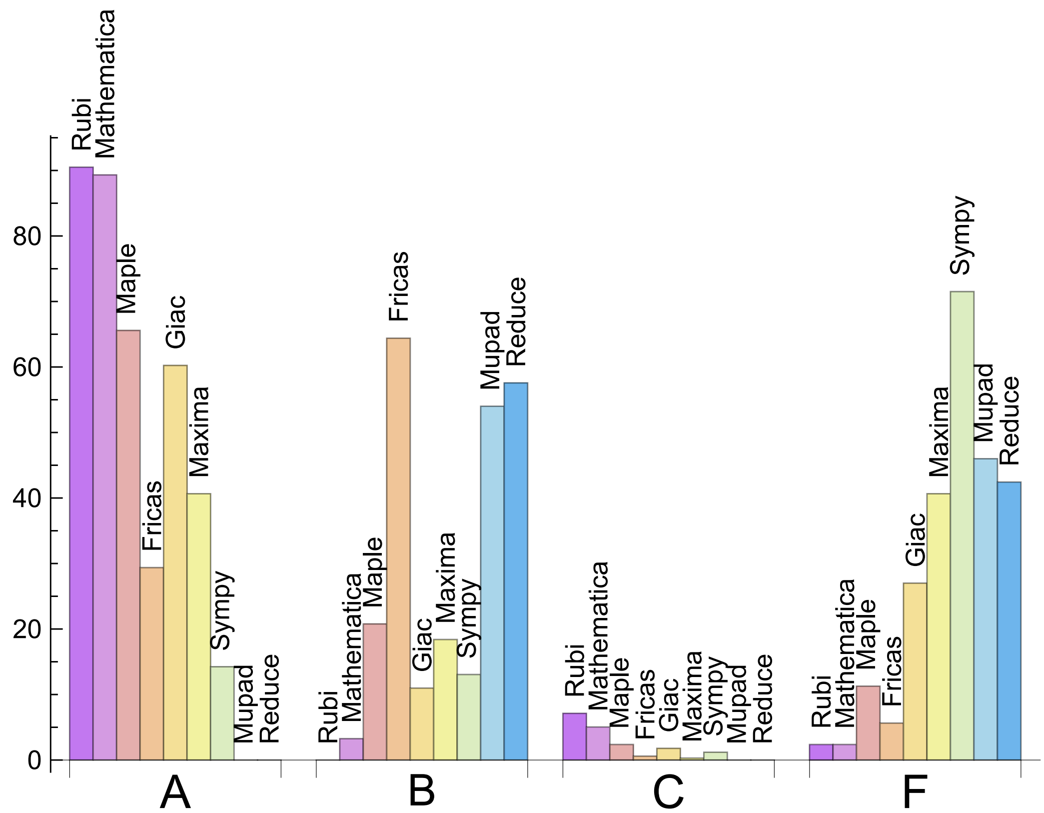
System	% A grade	% B grade	% C grade	% F grade
Rubi	90.504	0.000	7.122	2.374
Mathematica	89.318	3.264	5.045	2.374
Maple	65.579	20.772	2.374	11.276
Giac	60.237	10.979	1.780	27.003
Maxima	40.653	18.398	0.297	40.653
Fricas	29.377	64.392	0.593	5.638
Sympy	14.243	13.056	1.187	71.513
Mupad	0.000	54.006	0.000	45.994
Reduce	0.000	57.567	0.000	42.433

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	0	0.00	0.00	0.00
Fricas	11	72.73	0.00	27.27
Maple	30	100.00	0.00	0.00
Giac	83	89.16	0.00	10.84
Maxima	129	63.57	0.00	36.43
Reduce	135	100.00	0.00	0.00
Mupad	147	0.00	100.00	0.00
Sympy	233	72.10	27.90	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.08
Fricas	0.11
Giac	0.17
Reduce	0.25
Rubi	0.44
Mathematica	0.59
Mupad	1.94
Sympy	4.13
Maple	5.16

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	91.03	1.03	65.00	1.00
Mathematica	96.61	1.02	55.00	0.94
Maxima	108.00	1.72	81.50	1.29
Maple	126.00	1.47	71.00	1.01
Mupad	138.87	2.10	67.50	1.31
Giac	154.66	1.54	64.50	1.14
Reduce	180.79	2.88	76.00	1.66
Sympy	201.88	3.48	53.50	1.54
Fricas	496.49	4.75	201.00	2.68

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

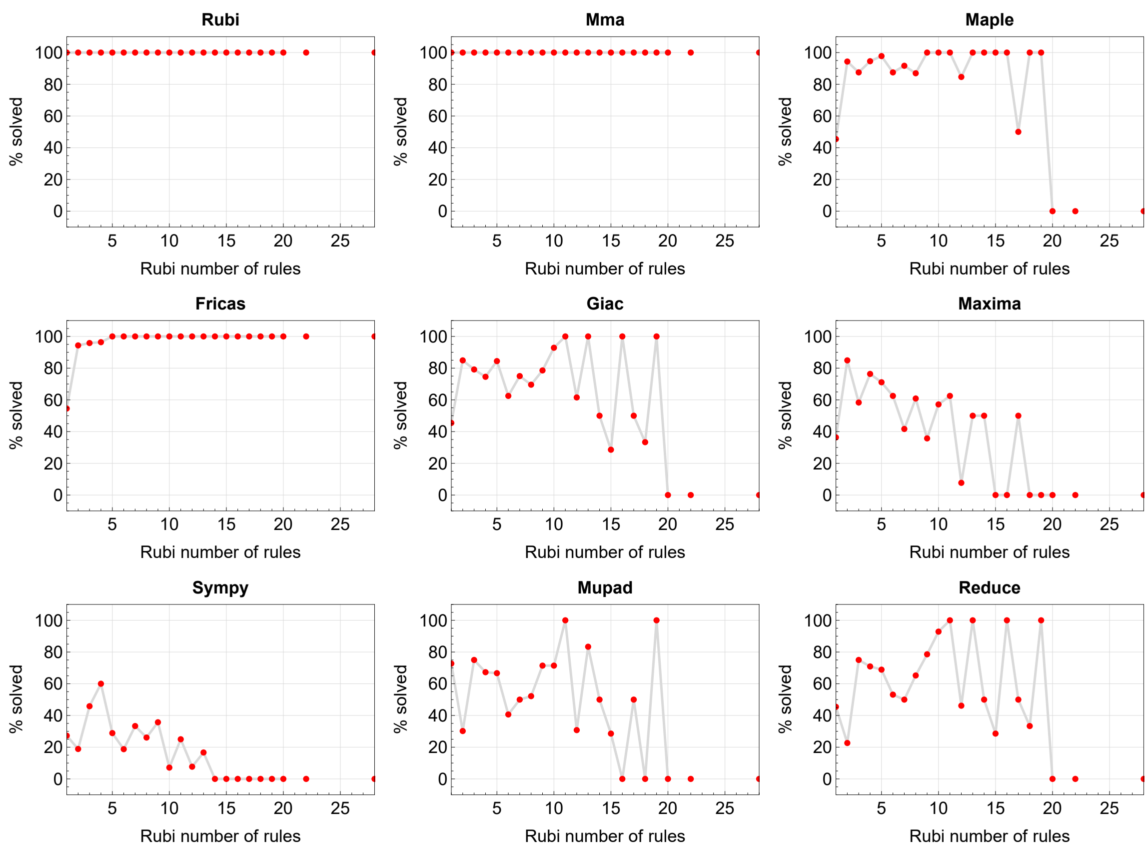


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

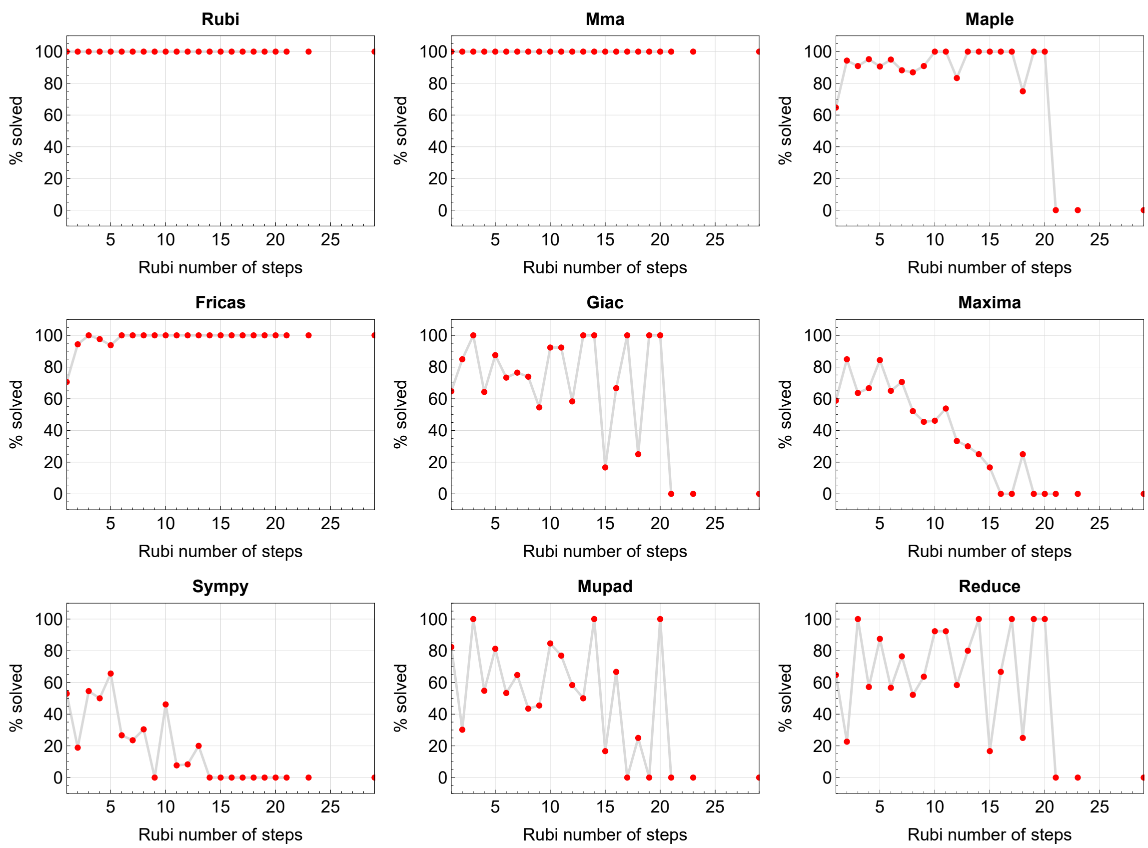


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

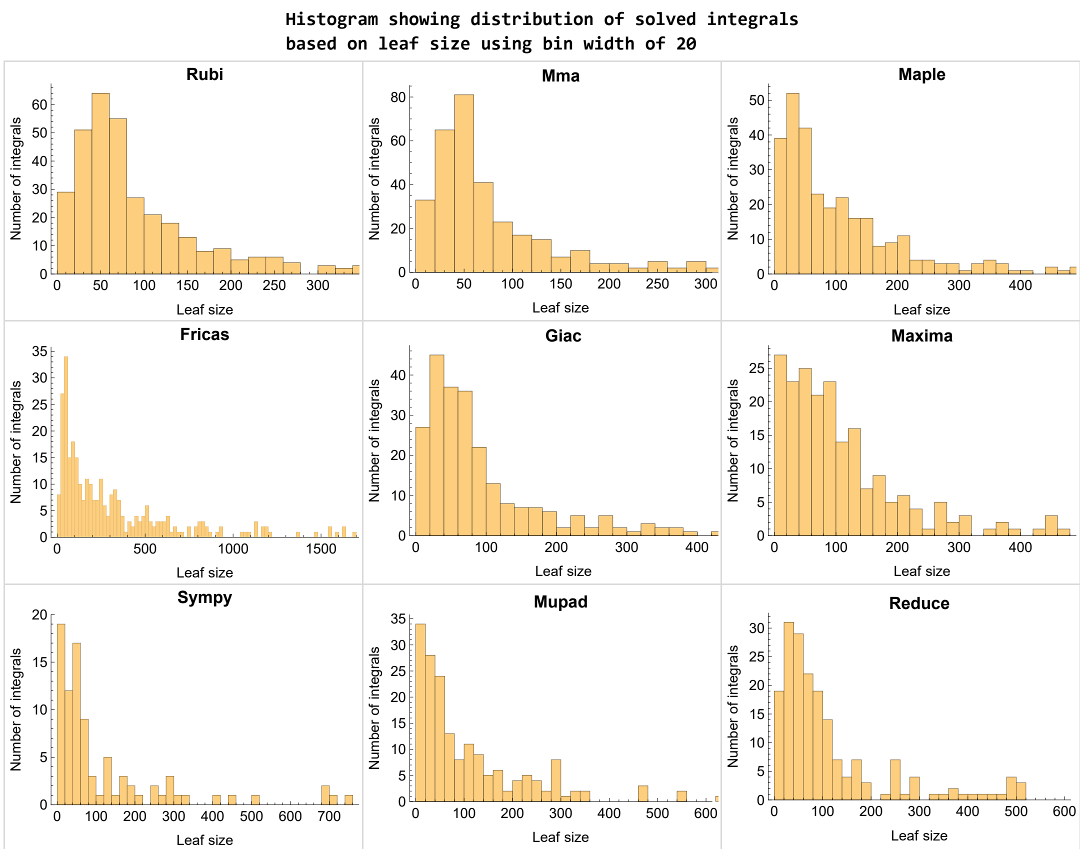


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

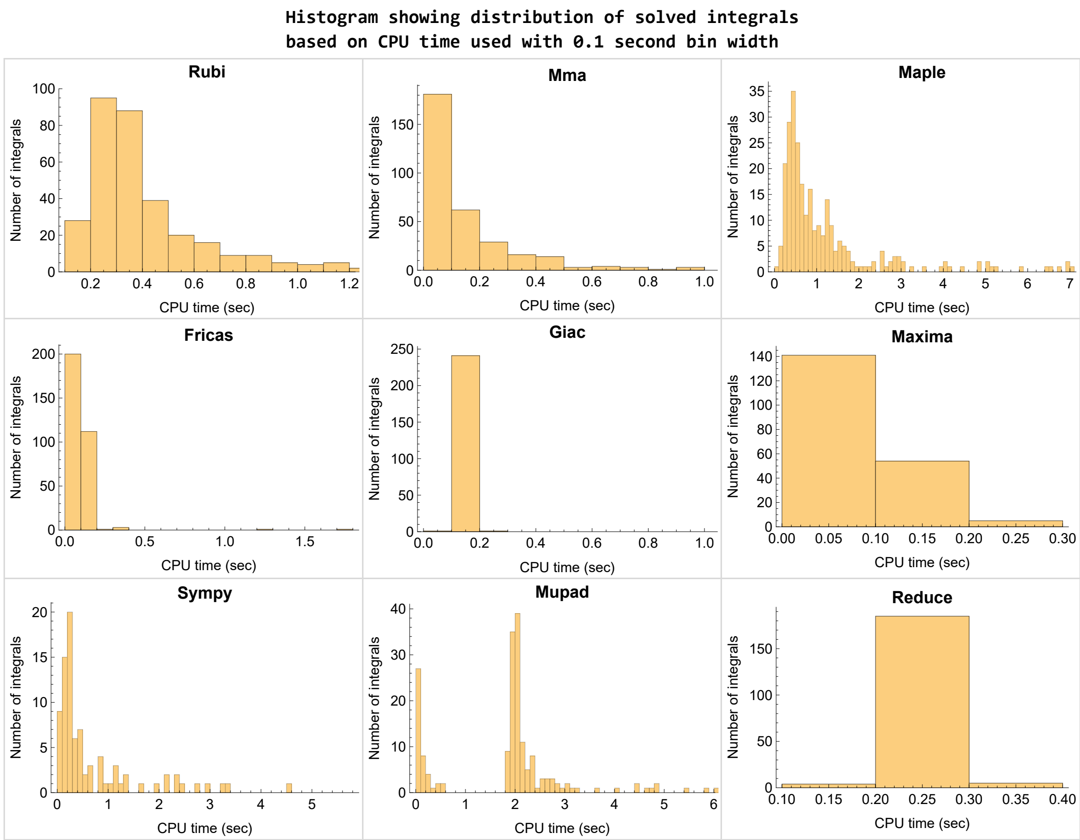


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

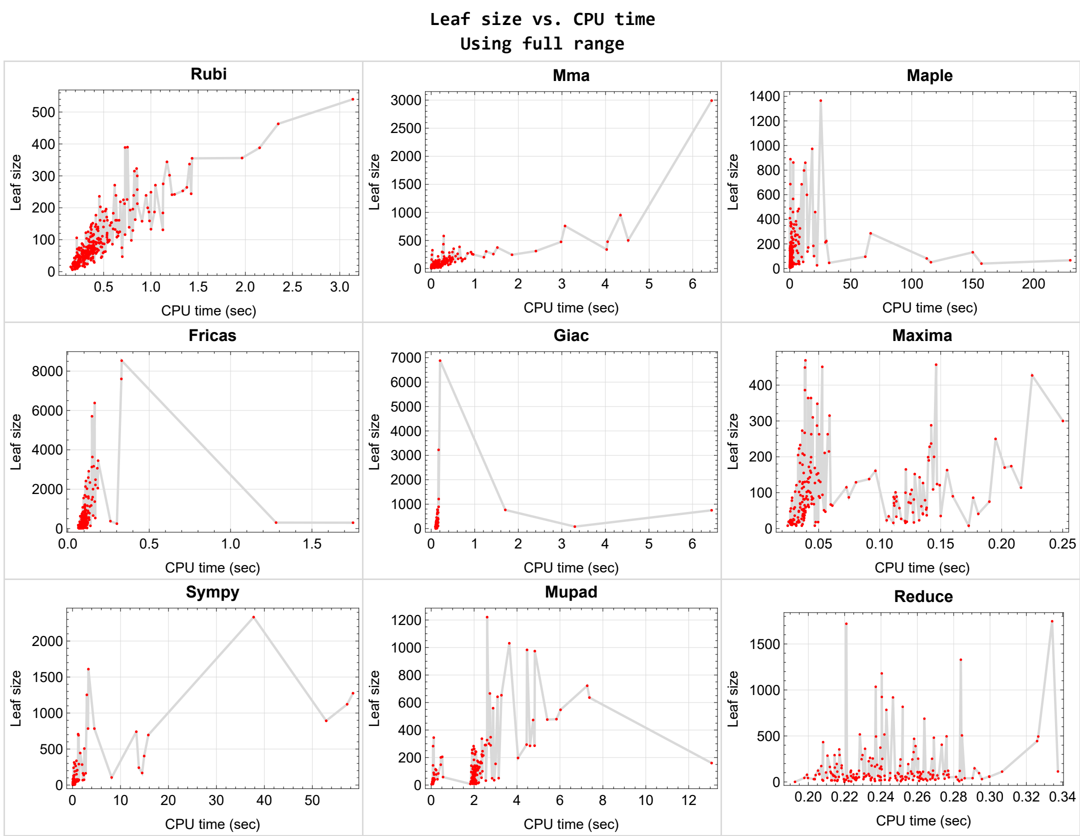


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

$\{216, 217, 221, 226, 227, 232, 233, 238\}$

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi $\{\}$

Mathematica $\{\}$

Maple $\{\}$

Maxima $\{\}$

Fricas $\{\}$

Sympy $\{\}$

Giac $\{\}$

Reduce $\{\}$

Mupad $\{\}$

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi $\{258, 259, 260, 266, 280, 293, 294\}$

Mathematica $\{234, 235, 320, 326, 327, 328, 330, 332\}$

Maple $\{296\}$

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

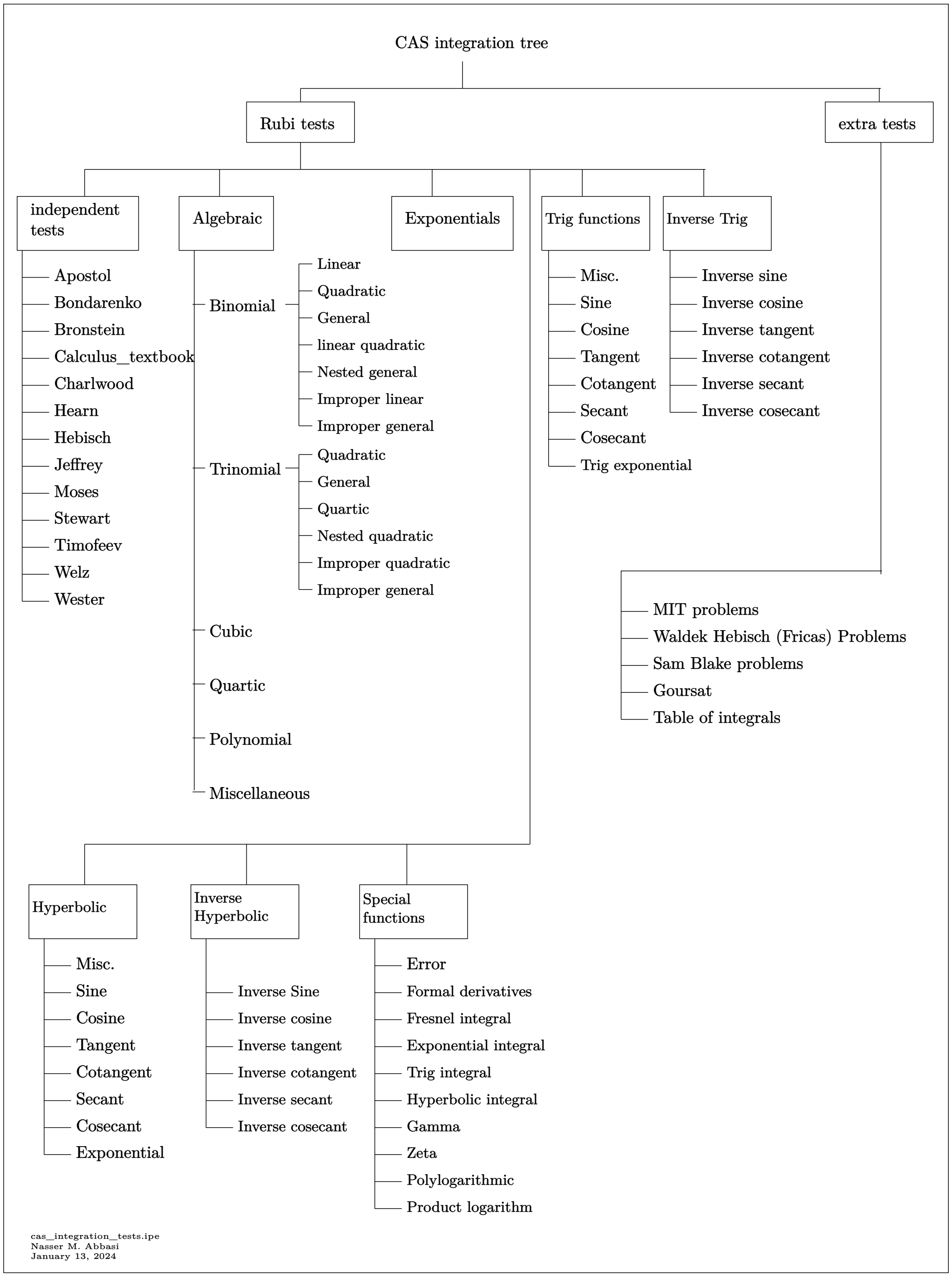
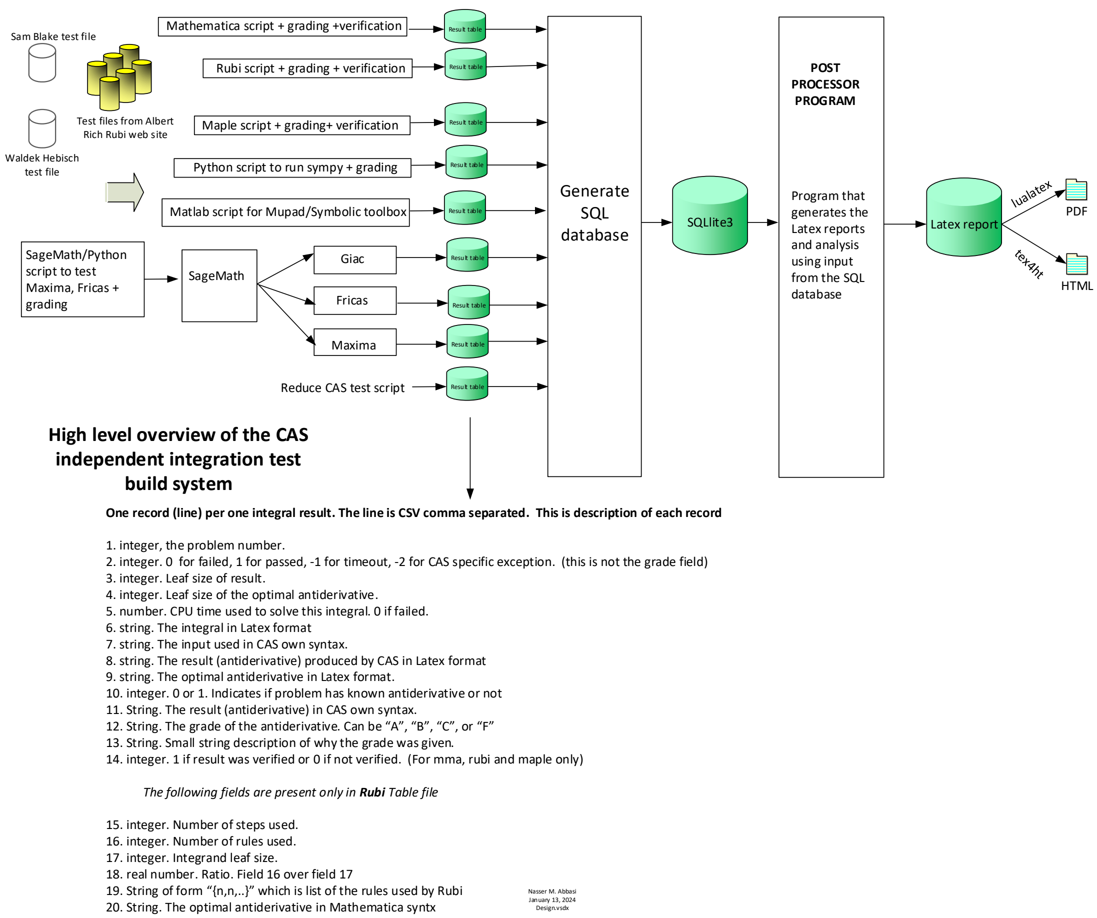


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 4, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 25, 26, 27, 28, 29, 30, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 187, 189, 191, 192, 197, 198, 199, 200, 201, 202, 203, 204, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 222, 223, 224, 225, 230, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337 }

B grade { }

C grade { 3, 5, 24, 31, 138, 139, 163, 186, 188, 190, 193, 194, 195, 196, 205, 228, 229, 234, 235, 236, 249, 251, 263, 264 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 2, 3, 4, 5, 6, 7, 8, 10, 11, 12, 14, 15, 16, 18, 19, 20, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 218, 219, 220, 222, 223, 224, 225, 228, 229, 230, 231, 234, 235, 236, 237, 239, 240, 241, 242, 243, 244, 245, 246, 248, 249, 250, 251, 252, 254, 255, 256, 260, 261, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 277, 278, 279, 282, 283, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 331, 332, 334, 335, 337 }

B grade { 1, 75, 76, 77, 78, 97, 247, 263, 264, 329, 330 }

C grade { 9, 13, 17, 21, 130, 143, 253, 257, 258, 259, 276, 280, 281, 284, 285, 333, 336 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 48, 49, 50, 51, 52, 53, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 104, 105, 106, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 153, 155, 157, 158, 159, 160, 161, 162, 163, 164, 165, 167, 169, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 200, 201, 202, 203, 204, 205, 207, 208, 209, 220, 225, 231, 237, 239, 240, 242, 243, 244, 245, 247, 248, 249, 250, 251, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 278, 279, 282, 283, 286, 287, 288, 293, 294, 295, 297, 298, 299, 300, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 334, 335, 336, 337 }

B grade { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 40, 45, 46, 47, 54, 55, 79, 80, 81, 82, 83, 84, 85, 101, 102, 103, 107, 108, 109, 118, 119, 120, 125, 126, 127, 136, 137, 152, 154, 156, 166, 168, 170, 197, 198, 199, 206, 210, 211, 212, 218, 219, 224, 230, 236, 241, 252, 253, 254, 255, 256, 257, 263, 264 }

C grade { 276, 277, 280, 281, 284, 285, 296, 303 }

F normal fail { 23, 128, 129, 130, 131, 132, 133, 213, 214, 215, 222, 223, 228, 229, 234, 235, 246, 258, 259, 260, 261, 262, 289, 290, 291, 292, 330, 331, 332, 333 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 2, 3, 4, 5, 6, 10, 11, 17, 18, 19, 25, 26, 27, 28, 32, 36, 40, 44, 45, 50, 51, 57, 58, 62, 63, 64, 65, 66, 67, 71, 75, 82, 86, 93, 94, 97, 98, 101, 110, 114, 115, 117, 118, 130, 142, 143, 154, 155, 156, 157, 158, 159, 172, 182, 183, 192, 203, 204, 205, 206, 220, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 255, 258, 259, 260, 261, 262, 263, 265, 267, 269, 275, 276, 280, 293, 294, 295, 296, 300, 301, 302, 303, 304, 305, 313, 334, 335 }

B grade { 7, 8, 9, 12, 13, 14, 15, 16, 20, 21, 22, 24, 29, 30, 31, 33, 34, 35, 37, 38, 39, 41, 42, 43, 46, 47, 48, 49, 52, 53, 54, 55, 56, 59, 60, 61, 68, 69, 70, 72, 73, 74, 76, 77, 78, 79, 80, 81, 83, 84, 85, 87, 88, 89, 90, 91, 92, 95, 96, 99, 100, 102, 103, 104, 105, 106, 107, 108, 109, 111, 112, 113, 116, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 174, 175, 176, 177, 178, 179, 180, 181, 184, 185, 186, 187, 188, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 207, 208, 209, 210, 211, 212, 218, 219, 222, 223, 224, 225, 228, 229, 230, 231, 234, 235, 236, 237, 245, 246, 252, 253, 254, 256, 257, 264, 266, 268, 270, 271, 272, 273, 274, 277, 278, 279, 281, 282, 283, 286, 287, 288, 297, 298, 299, 306, 307, 308, 309, 310, 311, 312, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 331, 336, 337 }

C grade { 284, 285 }

F normal fail { 23, 213, 214, 215, 289, 290, 291, 292 }

F(-1) timedout fail { }

F(-2) exception fail { 330, 332, 333 }

Maxima

A grade { 1, 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 32, 36, 42, 43, 44, 62, 63, 64, 65, 66, 71, 72, 73, 74, 75, 76, 77, 78, 93, 97, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 140, 141, 142, 143, 145, 146, 147, 151, 153, 155, 158, 159, 164, 171, 172, 174, 180, 182, 183, 185, 191, 192, 193, 200, 201, 220, 225, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 250, 265, 266, 267, 268, 269, 270, 272, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 286, 287, 288, 293, 294, 295, 296, 297, 298, 301, 302, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 334, 335 }

B grade { 3, 5, 31, 33, 34, 35, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 87, 88, 89, 90, 91, 92, 94, 95, 96, 98, 99, 100, 101, 102, 103, 138, 139, 144, 148, 149, 150, 152, 154, 156, 157, 160, 161, 162, 163, 165, 167, 169, 176, 187, 188, 189, 190, 194, 195, 196, 237, 249, 251, 271, 273, 299 }

C grade { 303 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 41, 51, 52, 53, 79, 80, 81, 82, 83, 84, 85, 86, 104, 105, 106, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 222, 223, 224, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 284, 285, 289, 290, 291, 292, 330, 331, 332, 333, 336 }

F(-1) timedout fail { }

F(-2) exception fail { 54, 55, 56, 57, 58, 59, 60, 61, 67, 68, 69, 70, 110, 111, 112, 113, 114, 115, 116, 166, 168, 170, 173, 175, 177, 178, 179, 181, 184, 186, 199, 202, 203, 204, 205, 206, 207, 208, 209, 218, 219, 228, 229, 230, 231, 300, 337 }

Giac

A grade { 2, 4, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 87, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 110, 111, 114, 115, 117, 121, 122, 123, 124, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 175, 178, 179, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 220, 225, 231, 237, 239, 240, 250, 251, 262, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 304, 305, 306, 308, 309, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 334, 335 }

B grade { 1, 3, 5, 48, 49, 50, 66, 88, 89, 90, 91, 92, 105, 106, 112, 113, 116, 144, 160, 174, 176, 177, 180, 195, 208, 209, 241, 242, 243, 244, 245, 246, 247, 248, 249, 263, 264 }

C grade { 286, 287, 288, 303, 307, 310 }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 79, 80, 81, 82, 83, 84, 85, 86, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 218, 219, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 260, 261, 289, 290, 291, 292, 330, 331, 332, 333, 336, 337 }

F(-1) timedout fail { }

F(-2) exception fail { 42, 43, 44, 45, 46, 47, 125, 258, 259 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 44, 50, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 110, 111, 114, 115, 116, 117, 124, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 218, 225, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 295, 297, 298, 299, 300, 330, 331, 332, 334, 335 }

C grade { }

F normal fail { }

F(-1) timedout fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 40, 41, 42, 43, 45, 46, 47, 48, 49, 51, 52, 53, 69, 70, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 112, 113, 118, 119, 120, 121, 122, 123, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 197, 198, 208, 209, 210, 211, 212, 213, 214, 215, 219, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 289, 290, 291, 292, 293, 294, 296, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 333, 336, 337 }

F(-2) exception fail { }

Sympy

A grade { 1, 3, 5, 27, 32, 33, 34, 35, 36, 37, 38, 39, 62, 63, 64, 65, 66, 71, 75, 93, 94, 95, 96, 97, 98, 99, 100, 115, 117, 124, 140, 141, 142, 143, 146, 147, 148, 149, 159, 171, 200, 201, 251, 275, 279, 283, 334, 335 }

B grade { 2, 4, 6, 24, 25, 26, 56, 57, 67, 68, 76, 77, 78, 110, 114, 144, 145, 150, 151, 152, 153, 154, 155, 156, 157, 158, 170, 199, 206, 225, 231, 247, 249, 265, 266, 267, 268, 274, 278, 282, 286, 287, 288, 300 }

C grade { 72, 73, 74, 295 }

F normal fail { 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 23, 28, 29, 30, 31, 40, 41, 43, 44, 45, 46, 47, 49, 50, 51, 52, 53, 58, 59, 60, 61, 80, 81, 82, 83, 84, 86, 88, 89, 91, 92, 101, 102, 104, 105, 108, 109, 118, 125, 126, 131, 160, 161, 162, 163, 164, 172, 173, 174, 175, 176, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 202, 203, 204, 205, 210, 211, 212, 213, 214, 215, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 239, 240, 241, 242, 243, 244, 245, 248, 250, 253, 254, 255, 256, 259, 260, 261, 263, 269, 270, 271, 272, 273, 276, 277, 280, 281, 284, 285, 289, 290, 291, 292, 296, 297, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 330, 331, 333, 336, 337 }

F(-1) timedout fail { 7, 8, 14, 15, 16, 22, 42, 48, 54, 55, 69, 70, 79, 85, 87, 90, 103, 106, 107, 111, 112, 113, 116, 119, 120, 121, 122, 123, 127, 128, 129, 130, 132, 133, 134, 135, 136, 137, 138, 139, 165, 166, 167, 168, 169, 177, 178, 207, 208, 209, 218, 219, 237, 246, 252, 257, 258, 262, 264, 293, 294, 298, 299, 329, 332 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 93, 94, 95, 96, 97, 98, 99, 100, 110, 111, 112, 113, 114, 115, 116, 117, 121, 122, 123, 124, 125, 126, 127, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 218, 219, 225, 231, 237, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 288, 293, 294, 295, 296, 297, 298, 299, 300, 334, 335 }

C grade { }

F normal fail { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 101, 102, 103, 104, 105, 106, 107, 108, 109, 118, 119, 120, 128, 129, 130, 131, 132, 133, 197, 198, 210, 211, 212, 213, 214, 215, 220, 222, 223, 224, 228, 229, 230, 234, 235, 236, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 286, 287, 289, 290, 291, 292, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 336, 337 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	21	11	10	10	12	26	10	10
N.S.	1	1.00	2.10	1.10	1.00	1.00	1.20	2.60	1.00	1.00
time (sec)	N/A	0.164	0.003	0.253	0.028	0.094	0.062	0.108	0.222	0.052

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	23	20	32	22	46	32	43	18
N.S.	1	1.00	0.92	0.80	1.28	0.88	1.84	1.28	1.72	0.72
time (sec)	N/A	0.171	0.011	0.450	0.038	0.089	0.109	0.109	0.200	1.992

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	32	26	23	54	32	36	54	53	22
N.S.	1	1.23	1.00	0.88	2.08	1.23	1.38	2.08	2.04	0.85
time (sec)	N/A	0.192	0.001	0.873	0.038	0.083	0.131	0.108	0.215	1.983

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	51	33	31	60	49	95	60	67	31
N.S.	1	1.11	0.72	0.67	1.30	1.07	2.07	1.30	1.46	0.67
time (sec)	N/A	0.239	0.032	1.244	0.042	0.082	0.192	0.113	0.238	0.088

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	41	33	82	66	58	82	79	31
N.S.	1	1.12	1.00	0.80	2.00	1.61	1.41	2.00	1.93	0.76
time (sec)	N/A	0.206	0.011	2.140	0.038	0.085	0.260	0.096	0.213	1.964

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	77	43	42	86	90	139	88	91	42
N.S.	1	1.15	0.64	0.63	1.28	1.34	2.07	1.31	1.36	0.63
time (sec)	N/A	0.370	0.035	2.827	0.028	0.085	0.380	0.113	0.231	2.041

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	74	55	201	0	326	0	0	18	0
N.S.	1	1.07	0.80	2.91	0.00	4.72	0.00	0.00	0.26	0.00
time (sec)	N/A	0.355	0.080	7.474	0.000	0.100	0.000	0.000	0.289	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	44	188	0	203	0	0	18	0
N.S.	1	1.00	0.96	4.09	0.00	4.41	0.00	0.00	0.39	0.00
time (sec)	N/A	0.268	0.043	4.830	0.000	0.106	0.000	0.000	0.206	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	81	174	0	102	0	0	16	0
N.S.	1	1.00	1.76	3.78	0.00	2.22	0.00	0.00	0.35	0.00
time (sec)	N/A	0.284	0.074	2.565	0.000	0.101	0.000	0.000	0.203	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	37	0	0	9	0
N.S.	1	1.00	1.00	6.75	0.00	1.85	0.00	0.00	0.45	0.00
time (sec)	N/A	0.182	0.024	1.690	0.000	0.094	0.000	0.000	0.234	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	20	135	0	24	0	0	18	0
N.S.	1	1.00	1.00	6.75	0.00	1.20	0.00	0.00	0.90	0.00
time (sec)	N/A	0.182	0.025	0.559	0.000	0.089	0.000	0.000	0.215	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	103	0	148	0	0	18	0
N.S.	1	1.00	1.00	2.45	0.00	3.52	0.00	0.00	0.43	0.00
time (sec)	N/A	0.249	0.046	0.813	0.000	0.097	0.000	0.000	0.239	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	84	190	0	310	0	0	18	0
N.S.	1	1.00	1.83	4.13	0.00	6.74	0.00	0.00	0.39	0.00
time (sec)	N/A	0.253	0.052	1.130	0.000	0.102	0.000	0.000	0.228	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	70	63	363	0	613	0	0	18	0
N.S.	1	1.01	0.91	5.26	0.00	8.88	0.00	0.00	0.26	0.00
time (sec)	N/A	0.334	0.100	1.462	0.000	0.108	0.000	0.000	0.207	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	71	53	145	0	247	0	0	16	0
N.S.	1	1.09	0.82	2.23	0.00	3.80	0.00	0.00	0.25	0.00
time (sec)	N/A	0.413	0.046	5.847	0.000	0.095	0.000	0.000	0.215	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	184	0	155	0	0	16	0
N.S.	1	1.00	0.85	3.83	0.00	3.23	0.00	0.00	0.33	0.00
time (sec)	N/A	0.332	0.043	3.986	0.000	0.097	0.000	0.000	0.226	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	57	130	0	65	0	0	12	0
N.S.	1	1.00	1.19	2.71	0.00	1.35	0.00	0.00	0.25	0.00
time (sec)	N/A	0.340	0.054	2.502	0.000	0.084	0.000	0.000	0.227	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	118	0	28	0	0	8	0
N.S.	1	1.00	1.00	4.37	0.00	1.04	0.00	0.00	0.30	0.00
time (sec)	N/A	0.219	0.019	1.951	0.000	0.096	0.000	0.000	0.216	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	27	100	0	16	0	0	16	0
N.S.	1	1.00	1.00	3.70	0.00	0.59	0.00	0.00	0.59	0.00
time (sec)	N/A	0.220	0.021	0.762	0.000	0.079	0.000	0.000	0.208	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	159	0	90	0	0	16	0
N.S.	1	1.00	0.74	3.46	0.00	1.96	0.00	0.00	0.35	0.00
time (sec)	N/A	0.300	0.030	0.988	0.000	0.104	0.000	0.000	0.214	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	56	177	0	186	0	0	16	0
N.S.	1	1.00	1.12	3.54	0.00	3.72	0.00	0.00	0.32	0.00
time (sec)	N/A	0.291	0.038	1.144	0.000	0.104	0.000	0.000	0.220	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	71	43	254	0	375	0	0	16	0
N.S.	1	1.06	0.64	3.79	0.00	5.60	0.00	0.00	0.24	0.00
time (sec)	N/A	0.381	0.045	1.456	0.000	0.096	0.000	0.000	0.210	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	65	0	0	0	0	0	14	0
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	0.20	0.00
time (sec)	N/A	0.220	0.048	0.000	0.000	0.000	0.000	0.000	0.228	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	62	53	31	66	100	337	70	75	70
N.S.	1	1.15	0.98	0.57	1.22	1.85	6.24	1.30	1.39	1.30
time (sec)	N/A	0.429	0.256	0.558	0.041	0.088	0.813	0.109	0.204	2.062

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	45	24	56	70	189	51	83	52
N.S.	1	1.00	1.05	0.56	1.30	1.63	4.40	1.19	1.93	1.21
time (sec)	N/A	0.287	0.208	0.365	0.027	0.082	0.436	0.111	0.206	2.006

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	24	32	15	41	47	63	35	34	34
N.S.	1	0.96	1.28	0.60	1.64	1.88	2.52	1.40	1.36	1.36
time (sec)	N/A	0.329	0.167	0.256	0.038	0.087	0.228	0.106	0.234	1.987

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	29	13	18	24	8	17	23	17
N.S.	1	1.00	1.61	0.72	1.00	1.33	0.44	0.94	1.28	0.94
time (sec)	N/A	0.244	0.119	0.175	0.050	0.080	0.167	0.111	0.208	1.967

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	19	23	29	0	20	30	31
N.S.	1	1.00	1.10	0.95	1.15	1.45	0.00	1.00	1.50	1.55
time (sec)	N/A	0.290	0.100	0.487	0.106	0.083	0.000	0.110	0.204	1.946

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	29	43	33	45	127	0	36	66	58
N.S.	1	1.04	1.54	1.18	1.61	4.54	0.00	1.29	2.36	2.07
time (sec)	N/A	0.401	0.150	0.513	0.127	0.076	0.000	0.115	0.214	1.915

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	44	49	46	73	325	0	48	122	73
N.S.	1	1.02	1.14	1.07	1.70	7.56	0.00	1.12	2.84	1.70
time (sec)	N/A	0.474	0.161	0.757	0.124	0.088	0.000	0.108	0.228	1.976

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	63	60	52	101	600	0	57	173	107
N.S.	1	1.12	1.07	0.93	1.80	10.71	0.00	1.02	3.09	1.91
time (sec)	N/A	0.519	0.221	0.838	0.113	0.097	0.000	0.116	0.207	2.091

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	14	14	18	22	17	15	23	15
N.S.	1	1.00	0.70	0.70	0.90	1.10	0.85	0.75	1.15	0.75
time (sec)	N/A	0.184	0.013	0.215	0.048	0.087	0.295	0.112	0.221	2.087

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	34	26	90	113	36	25	51	25
N.S.	1	1.00	0.72	0.55	1.91	2.40	0.77	0.53	1.09	0.53
time (sec)	N/A	0.253	0.022	0.264	0.037	0.071	0.475	0.108	0.225	0.064

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	75	44	37	205	174	51	36	87	36
N.S.	1	1.07	0.63	0.53	2.93	2.49	0.73	0.51	1.24	0.51
time (sec)	N/A	0.334	0.038	0.323	0.038	0.081	1.001	0.106	0.206	2.001

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	103	54	48	364	347	68	47	123	283
N.S.	1	1.11	0.58	0.52	3.91	3.73	0.73	0.51	1.32	3.04
time (sec)	N/A	0.436	0.057	0.314	0.041	0.080	2.124	0.112	0.223	1.979

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	14	14	18	24	19	15	23	15
N.S.	1	1.00	0.61	0.61	0.78	1.04	0.83	0.65	1.00	0.65
time (sec)	N/A	0.223	0.078	0.274	0.032	0.091	0.386	0.111	0.242	0.062

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	31	26	90	117	39	25	51	25
N.S.	1	1.00	0.61	0.51	1.76	2.29	0.76	0.49	1.00	0.49
time (sec)	N/A	0.312	0.058	0.342	0.040	0.066	0.662	0.109	0.228	0.064

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	81	41	37	205	174	56	36	87	36
N.S.	1	1.07	0.54	0.49	2.70	2.29	0.74	0.47	1.14	0.47
time (sec)	N/A	0.397	0.126	0.378	0.033	0.113	1.177	0.106	0.227	1.986

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	111	51	48	364	347	73	47	123	283
N.S.	1	1.10	0.50	0.48	3.60	3.44	0.72	0.47	1.22	2.80
time (sec)	N/A	0.486	0.087	0.419	0.044	0.073	2.399	0.110	0.205	0.090

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	34	92	114	56	0	37	22	0
N.S.	1	1.00	0.67	1.80	2.24	1.10	0.00	0.73	0.43	0.00
time (sec)	N/A	0.323	0.016	0.394	0.216	0.083	0.000	0.116	0.229	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	44	40	0	92	0	74	25	0
N.S.	1	1.00	0.83	0.75	0.00	1.74	0.00	1.40	0.47	0.00
time (sec)	N/A	0.327	0.114	0.494	0.000	0.094	0.000	0.116	0.205	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	93	71	73	121	327	0	0	58	0
N.S.	1	1.04	0.80	0.82	1.36	3.67	0.00	0.00	0.65	0.00
time (sec)	N/A	0.412	0.092	0.432	0.149	0.080	0.000	0.000	0.237	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	58	81	140	0	0	34	0
N.S.	1	1.00	0.93	0.98	1.37	2.37	0.00	0.00	0.58	0.00
time (sec)	N/A	0.287	0.049	0.374	0.128	0.099	0.000	0.000	0.245	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	F(-2)	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	29	43	40	41	0	0	14	26
N.S.	1	1.00	1.12	1.65	1.54	1.58	0.00	0.00	0.54	1.00
time (sec)	N/A	0.210	0.062	0.452	0.135	0.076	0.000	0.000	0.234	0.121

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	40	103	86	136	0	0	28	0
N.S.	1	1.00	0.87	2.24	1.87	2.96	0.00	0.00	0.61	0.00
time (sec)	N/A	0.229	0.013	0.389	0.177	0.110	0.000	0.000	0.223	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	63	144	170	232	0	0	38	0
N.S.	1	1.00	0.82	1.87	2.21	3.01	0.00	0.00	0.49	0.00
time (sec)	N/A	0.317	0.061	0.435	0.202	0.115	0.000	0.000	0.238	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	112	91	178	250	535	0	0	48	0
N.S.	1	1.05	0.85	1.66	2.34	5.00	0.00	0.00	0.45	0.00
time (sec)	N/A	0.381	0.180	0.412	0.195	0.169	0.000	0.000	0.228	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	96	72	71	190	328	0	194	64	0
N.S.	1	1.04	0.78	0.77	2.07	3.57	0.00	2.11	0.70	0.00
time (sec)	N/A	0.392	0.188	0.522	0.140	0.118	0.000	0.132	0.227	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	56	56	124	139	0	127	40	0
N.S.	1	1.00	0.92	0.92	2.03	2.28	0.00	2.08	0.66	0.00
time (sec)	N/A	0.282	0.145	0.424	0.147	0.139	0.000	0.124	0.223	0.000

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	30	41	58	42	0	63	16	27
N.S.	1	1.00	1.11	1.52	2.15	1.56	0.00	2.33	0.59	1.00
time (sec)	N/A	0.191	0.090	0.461	0.138	0.108	0.000	0.112	0.236	2.043

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	41	41	0	141	0	40	31	0
N.S.	1	1.00	0.85	0.85	0.00	2.94	0.00	0.83	0.65	0.00
time (sec)	N/A	0.214	0.014	0.410	0.000	0.085	0.000	0.117	0.209	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	98	87	0	274	0	115	40	0
N.S.	1	1.00	1.24	1.10	0.00	3.47	0.00	1.46	0.51	0.00
time (sec)	N/A	0.291	0.257	0.435	0.000	0.090	0.000	0.128	0.224	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	115	129	137	0	580	0	177	51	0
N.S.	1	1.05	1.17	1.25	0.00	5.27	0.00	1.61	0.46	0.00
time (sec)	N/A	0.392	0.263	0.439	0.000	0.098	0.000	0.139	0.219	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	129	99	203	0	1625	0	133	247	209
N.S.	1	1.15	0.88	1.81	0.00	14.51	0.00	1.19	2.21	1.87
time (sec)	N/A	0.809	0.159	0.858	0.000	0.126	0.000	0.117	0.229	2.381

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	95	78	153	0	903	0	92	148	167
N.S.	1	1.12	0.92	1.80	0.00	10.62	0.00	1.08	1.74	1.96
time (sec)	N/A	0.550	0.104	0.528	0.000	0.124	0.000	0.112	0.291	2.198

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	65	57	94	0	449	1275	62	78	139
N.S.	1	1.05	0.92	1.52	0.00	7.24	20.56	1.00	1.26	2.24
time (sec)	N/A	0.385	0.086	0.358	0.000	0.132	58.481	0.114	0.199	2.134

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	48	64	0	218	241	42	61	109
N.S.	1	1.00	0.92	1.23	0.00	4.19	4.63	0.81	1.17	2.10
time (sec)	N/A	0.283	0.049	0.223	0.000	0.115	13.828	0.115	0.232	0.233

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	51	0	227	0	45	67	286
N.S.	1	1.00	1.00	0.94	0.00	4.20	0.00	0.83	1.24	5.30
time (sec)	N/A	0.343	0.068	0.506	0.000	0.106	0.000	0.113	0.214	4.818

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	67	63	73	0	515	0	61	179	294
N.S.	1	1.05	0.98	1.14	0.00	8.05	0.00	0.95	2.80	4.59
time (sec)	N/A	0.445	0.114	0.758	0.000	0.118	0.000	0.115	0.218	4.446

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	98	82	109	0	1370	0	89	361	476
N.S.	1	1.13	0.94	1.25	0.00	15.75	0.00	1.02	4.15	5.47
time (sec)	N/A	0.793	0.184	1.124	0.000	0.166	0.000	0.108	0.231	5.409

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	131	101	145	0	2483	0	123	517	547
N.S.	1	1.15	0.89	1.27	0.00	21.78	0.00	1.08	4.54	4.80
time (sec)	N/A	1.127	0.309	1.592	0.000	0.170	0.000	0.119	0.228	6.027

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	196	133	132	273	190	314	263	354	160
N.S.	1	1.07	0.73	0.72	1.49	1.04	1.72	1.44	1.93	0.87
time (sec)	N/A	0.809	0.707	149.968	0.036	0.083	0.331	0.116	0.217	2.295

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	145	104	101	183	123	240	196	251	114
N.S.	1	1.06	0.76	0.74	1.34	0.90	1.75	1.43	1.83	0.83
time (sec)	N/A	0.567	0.371	19.839	0.035	0.081	0.220	0.115	0.241	0.204

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	93	80	67	116	78	128	131	163	73
N.S.	1	1.03	0.89	0.74	1.29	0.87	1.42	1.46	1.81	0.81
time (sec)	N/A	0.368	0.197	229.858	0.038	0.089	0.201	0.115	0.212	2.068

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	46	43	55	40	78	75	96	41
N.S.	1	1.00	0.92	0.86	1.10	0.80	1.56	1.50	1.92	0.82
time (sec)	N/A	0.224	0.123	1.005	0.030	0.081	0.111	0.113	0.213	2.032

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	26	16	15	17	17	32	17	15
N.S.	1	1.00	1.73	1.07	1.00	1.13	1.13	2.13	1.13	1.00
time (sec)	N/A	0.151	0.003	0.306	0.027	0.078	0.087	0.112	0.215	0.060

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	48	44	0	237	163	39	51	53
N.S.	1	1.00	0.98	0.90	0.00	4.84	3.33	0.80	1.04	1.08
time (sec)	N/A	0.210	0.046	0.282	0.000	0.101	2.738	0.116	0.226	2.345

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	97	84	118	0	743	2332	99	286	215
N.S.	1	1.13	0.98	1.37	0.00	8.64	27.12	1.15	3.33	2.50
time (sec)	N/A	0.341	0.189	0.412	0.000	0.115	37.760	0.111	0.211	2.429

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	159	113	186	0	2591	0	195	1035	0
N.S.	1	1.20	0.85	1.40	0.00	19.48	0.00	1.47	7.78	0.00
time (sec)	N/A	0.505	0.309	0.631	0.000	0.126	0.000	0.120	0.237	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	226	160	284	0	5705	0	329	1720	0
N.S.	1	1.23	0.87	1.54	0.00	31.01	0.00	1.79	9.35	0.00
time (sec)	N/A	0.748	0.725	1.116	0.000	0.149	0.000	0.122	0.221	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	20	18	19	24	24	16	17	34
N.S.	1	1.00	0.91	0.82	0.86	1.09	1.09	0.73	0.77	1.55
time (sec)	N/A	0.190	0.033	0.512	0.122	0.083	0.466	0.112	0.213	2.076

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	43	46	64	147	291	54	103	74
N.S.	1	1.00	0.90	0.96	1.33	3.06	6.06	1.12	2.15	1.54
time (sec)	N/A	0.275	0.079	0.536	0.112	0.077	1.317	0.115	0.210	2.005

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	78	53	62	108	408	507	76	196	137
N.S.	1	1.07	0.73	0.85	1.48	5.59	6.95	1.04	2.68	1.88
time (sec)	N/A	0.395	0.130	0.756	0.126	0.105	2.463	0.116	0.216	2.068

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	108	63	75	152	793	784	98	292	223
N.S.	1	1.10	0.64	0.77	1.55	8.09	8.00	1.00	2.98	2.28
time (sec)	N/A	0.534	0.165	0.809	0.129	0.090	4.556	0.114	0.214	2.035

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	65	30	37	42	41	28	30	40
N.S.	1	1.00	2.10	0.97	1.19	1.35	1.32	0.90	0.97	1.29
time (sec)	N/A	0.194	0.032	0.395	0.038	0.094	0.388	0.109	0.230	2.033

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	144	64	81	212	199	65	156	77
N.S.	1	1.09	2.57	1.14	1.45	3.79	3.55	1.16	2.79	1.38
time (sec)	N/A	0.272	0.102	0.468	0.044	0.112	0.838	0.112	0.215	2.036

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	91	217	90	125	563	445	87	293	141
N.S.	1	1.12	2.68	1.11	1.54	6.95	5.49	1.07	3.62	1.74
time (sec)	N/A	0.392	0.159	0.623	0.037	0.130	1.604	0.109	0.217	2.022

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	121	296	112	169	1078	784	109	433	226
N.S.	1	1.14	2.79	1.06	1.59	10.17	7.40	1.03	4.08	2.13
time (sec)	N/A	0.536	0.250	0.753	0.047	0.092	3.231	0.113	0.208	2.041

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	159	150	685	0	452	0	0	48	0
N.S.	1	1.04	0.98	4.48	0.00	2.95	0.00	0.00	0.31	0.00
time (sec)	N/A	0.985	0.373	9.737	0.000	0.104	0.000	0.000	0.231	0.000

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	125	111	466	0	259	0	0	26	0
N.S.	1	1.01	0.90	3.76	0.00	2.09	0.00	0.00	0.21	0.00
time (sec)	N/A	0.671	0.169	6.500	0.000	0.121	0.000	0.000	0.226	0.000

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	276	0	197	0	0	13	0
N.S.	1	1.00	1.00	4.52	0.00	3.23	0.00	0.00	0.21	0.00
time (sec)	N/A	0.304	0.103	4.829	0.000	0.119	0.000	0.000	0.230	0.000

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	146	0	61	0	0	18	0
N.S.	1	1.00	1.00	3.17	0.00	1.33	0.00	0.00	0.39	0.00
time (sec)	N/A	0.269	0.031	1.250	0.000	0.099	0.000	0.000	0.228	0.000

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	68	298	0	381	0	0	30	0
N.S.	1	1.00	0.81	3.55	0.00	4.54	0.00	0.00	0.36	0.00
time (sec)	N/A	0.390	0.094	1.515	0.000	0.098	0.000	0.000	0.217	0.000

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	187	135	459	0	1187	0	0	42	0
N.S.	1	1.06	0.76	2.59	0.00	6.71	0.00	0.00	0.24	0.00
time (sec)	N/A	0.980	0.387	2.644	0.000	0.104	0.000	0.000	0.218	0.000

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	253	165	566	0	3129	0	0	54	0
N.S.	1	1.11	0.73	2.49	0.00	13.78	0.00	0.00	0.24	0.00
time (sec)	N/A	1.337	0.499	2.711	0.000	0.148	0.000	0.000	0.268	0.000

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	73	181	0	174	0	0	20	0
N.S.	1	1.00	0.73	1.81	0.00	1.74	0.00	0.00	0.20	0.00
time (sec)	N/A	0.541	0.262	4.051	0.000	0.081	0.000	0.000	0.198	0.000

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	89	60	71	237	563	0	156	84	0
N.S.	1	0.95	0.64	0.76	2.52	5.99	0.00	1.66	0.89	0.00
time (sec)	N/A	0.457	0.086	1.585	0.142	0.087	0.000	0.124	0.228	0.000

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	66	46	57	163	279	0	113	52	0
N.S.	1	0.97	0.68	0.84	2.40	4.10	0.00	1.66	0.76	0.00
time (sec)	N/A	0.369	0.063	0.828	0.155	0.109	0.000	0.119	0.242	0.000

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	31	39	90	100	0	68	25	0
N.S.	1	1.00	0.78	0.98	2.25	2.50	0.00	1.70	0.62	0.00
time (sec)	N/A	0.284	0.029	0.823	0.160	0.088	0.000	0.115	0.221	0.000

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	93	61	69	288	564	0	295	96	0
N.S.	1	0.95	0.62	0.70	2.94	5.76	0.00	3.01	0.98	0.00
time (sec)	N/A	0.469	0.239	1.707	0.142	0.104	0.000	0.132	0.226	0.000

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	69	47	55	199	279	0	212	62	0
N.S.	1	0.97	0.66	0.77	2.80	3.93	0.00	2.99	0.87	0.00
time (sec)	N/A	0.369	0.193	0.903	0.140	0.102	0.000	0.124	0.221	0.000

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	32	39	109	107	0	131	29	0
N.S.	1	1.00	0.73	0.89	2.48	2.43	0.00	2.98	0.66	0.00
time (sec)	N/A	0.287	0.142	0.790	0.144	0.090	0.000	0.113	0.235	0.000

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	32	15	26	29	15	17	30	19
N.S.	1	1.00	1.78	0.83	1.44	1.61	0.83	0.94	1.67	1.06
time (sec)	N/A	0.245	0.168	0.243	0.029	0.118	0.174	0.113	0.220	0.051

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	25	26	129	50	36	30	45	30
N.S.	1	1.00	0.71	0.74	3.69	1.43	1.03	0.86	1.29	0.86
time (sec)	N/A	0.277	0.049	0.253	0.037	0.083	0.248	0.110	0.222	0.082

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	54	41	35	263	127	46	46	82	141
N.S.	1	0.96	0.73	0.62	4.70	2.27	0.82	0.82	1.46	2.52
time (sec)	N/A	0.354	0.058	0.334	0.044	0.076	0.454	0.112	0.225	1.872

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	71	55	52	449	175	78	60	112	231
N.S.	1	0.95	0.73	0.69	5.99	2.33	1.04	0.80	1.49	3.08
time (sec)	N/A	0.432	0.066	0.410	0.039	0.064	0.872	0.115	0.223	1.927

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	42	14	27	31	15	16	31	19
N.S.	1	1.00	2.10	0.70	1.35	1.55	0.75	0.80	1.55	0.95
time (sec)	N/A	0.286	0.164	0.262	0.036	0.075	0.266	0.106	0.203	0.054

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	25	26	131	48	36	32	44	32
N.S.	1	1.00	0.68	0.70	3.54	1.30	0.97	0.86	1.19	0.86
time (sec)	N/A	0.296	0.044	0.296	0.034	0.078	0.412	0.111	0.198	1.888

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	58	41	39	267	127	46	46	82	143
N.S.	1	0.97	0.68	0.65	4.45	2.12	0.77	0.77	1.37	2.38
time (sec)	N/A	0.372	0.059	0.332	0.039	0.072	0.667	0.112	0.219	0.087

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	77	55	55	451	175	78	60	112	233
N.S.	1	0.95	0.68	0.68	5.57	2.16	0.96	0.74	1.38	2.88
time (sec)	N/A	0.481	0.057	0.405	0.053	0.082	1.126	0.114	0.216	1.909

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	41	128	174	68	0	44	41	0
N.S.	1	1.00	0.73	2.29	3.11	1.21	0.00	0.79	0.73	0.00
time (sec)	N/A	0.329	0.027	0.913	0.208	0.104	0.000	0.116	0.213	0.000

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	44	159	300	197	0	78	53	0
N.S.	1	1.00	0.68	2.45	4.62	3.03	0.00	1.20	0.82	0.00
time (sec)	N/A	0.336	0.058	0.992	0.250	0.105	0.000	0.114	0.218	0.000

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	91	57	209	427	514	0	118	65	0
N.S.	1	0.98	0.61	2.25	4.59	5.53	0.00	1.27	0.70	0.00
time (sec)	N/A	0.444	0.103	1.020	0.225	0.097	0.000	0.122	0.225	0.000

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	51	63	0	99	0	84	46	0
N.S.	1	1.00	0.89	1.11	0.00	1.74	0.00	1.47	0.81	0.00
time (sec)	N/A	0.307	0.184	0.857	0.000	0.081	0.000	0.119	0.213	0.000

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	81	83	0	217	0	111	57	0
N.S.	1	1.00	1.25	1.28	0.00	3.34	0.00	1.71	0.88	0.00
time (sec)	N/A	0.320	0.282	0.945	0.000	0.097	0.000	0.121	0.214	0.000

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	92	118	118	0	548	0	189	70	0
N.S.	1	0.98	1.26	1.26	0.00	5.83	0.00	2.01	0.74	0.00
time (sec)	N/A	0.420	0.448	0.962	0.000	0.086	0.000	0.135	0.251	0.000

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	244	203	1365	0	1123	0	0	70	0
N.S.	1	1.05	0.87	5.86	0.00	4.82	0.00	0.00	0.30	0.00
time (sec)	N/A	1.426	0.412	25.397	0.000	0.103	0.000	0.000	0.224	0.000

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	187	124	973	0	623	0	0	48	0
N.S.	1	1.03	0.69	5.38	0.00	3.44	0.00	0.00	0.27	0.00
time (sec)	N/A	1.038	0.470	18.359	0.000	0.102	0.000	0.000	0.220	0.000

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	139	123	599	0	319	0	0	26	0
N.S.	1	1.01	0.89	4.34	0.00	2.31	0.00	0.00	0.19	0.00
time (sec)	N/A	0.770	0.250	14.028	0.000	0.084	0.000	0.000	0.246	0.000

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	73	0	240	403	50	1	242
N.S.	1	1.00	0.98	1.22	0.00	4.00	6.72	0.83	0.02	4.03
time (sec)	N/A	0.304	0.085	0.306	0.000	0.106	14.935	0.112	0.193	2.090

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	93	81	108	0	828	0	107	44	246
N.S.	1	1.13	0.99	1.32	0.00	10.10	0.00	1.30	0.54	3.00
time (sec)	N/A	0.381	0.140	0.400	0.000	0.098	0.000	0.115	0.210	2.393

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	161	134	207	0	3166	0	249	225	0
N.S.	1	1.19	0.99	1.53	0.00	23.45	0.00	1.84	1.67	0.00
time (sec)	N/A	0.639	0.288	0.644	0.000	0.159	0.000	0.119	0.233	0.000

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	239	196	342	0	7603	0	453	817	0
N.S.	1	1.21	0.99	1.74	0.00	38.59	0.00	2.30	4.15	0.00
time (sec)	N/A	0.949	0.486	1.071	0.000	0.329	0.000	0.124	0.252	0.000

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	63	56	81	0	190	168	57	41	205
N.S.	1	1.12	1.00	1.45	0.00	3.39	3.00	1.02	0.73	3.66
time (sec)	N/A	0.350	0.071	0.382	0.000	0.102	14.514	0.110	0.245	0.515

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	6	3	6	1	6
N.S.	1	1.00	1.00	1.17	0.00	1.00	0.50	1.00	0.17	1.00
time (sec)	N/A	0.169	0.000	0.123	0.000	0.070	0.083	0.108	0.220	0.021

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	0	54	0	26	25	51
N.S.	1	1.00	1.00	1.09	0.00	4.91	0.00	2.36	2.27	4.64
time (sec)	N/A	0.220	0.012	0.324	0.000	0.075	0.000	0.117	0.270	1.952

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	38	24	32	34	45	44	37	30	48
N.S.	1	1.06	0.67	0.89	0.94	1.25	1.22	1.03	0.83	1.33
time (sec)	N/A	0.295	0.046	0.253	0.107	0.092	0.356	0.114	0.226	0.114

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	80	218	0	184	0	0	9	0
N.S.	1	1.00	0.74	2.02	0.00	1.70	0.00	0.00	0.08	0.00
time (sec)	N/A	0.640	0.319	6.933	0.000	0.085	0.000	0.000	0.218	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	152	163	133	485	0	601	0	0	18	0
N.S.	1	1.07	0.88	3.19	0.00	3.95	0.00	0.00	0.12	0.00
time (sec)	N/A	0.834	0.279	7.425	0.000	0.092	0.000	0.000	0.248	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	231	241	172	797	0	2053	0	0	30	0
N.S.	1	1.04	0.74	3.45	0.00	8.89	0.00	0.00	0.13	0.00
time (sec)	N/A	1.223	0.618	11.669	0.000	0.184	0.000	0.000	0.262	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	78	39	38	71	817	0	79	65	0
N.S.	1	1.08	0.54	0.53	0.99	11.35	0.00	1.10	0.90	0.00
time (sec)	N/A	0.470	0.018	0.896	0.125	0.125	0.000	0.111	0.224	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	56	33	32	53	501	0	41	51	0
N.S.	1	1.06	0.62	0.60	1.00	9.45	0.00	0.77	0.96	0.00
time (sec)	N/A	0.386	0.014	0.809	0.138	0.094	0.000	0.111	0.240	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	23	24	35	222	0	29	33	0
N.S.	1	1.00	0.68	0.71	1.03	6.53	0.00	0.85	0.97	0.00
time (sec)	N/A	0.286	0.008	0.297	0.150	0.098	0.000	0.112	0.262	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	17	69	15	14	5	17
N.S.	1	1.00	1.00	1.15	1.31	5.31	1.15	1.08	0.38	1.31
time (sec)	N/A	0.218	0.016	0.269	0.128	0.079	0.200	0.112	0.224	0.057

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	55	8	186	0	0	11	0
N.S.	1	1.00	1.06	3.44	0.50	11.62	0.00	0.00	0.69	0.00
time (sec)	N/A	0.230	0.005	0.296	0.173	0.081	0.000	0.000	0.216	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	26	82	41	299	0	56	58	0
N.S.	1	1.00	0.62	1.95	0.98	7.12	0.00	1.33	1.38	0.00
time (sec)	N/A	0.311	0.012	0.375	0.181	0.082	0.000	0.118	0.251	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	69	36	102	75	837	0	72	114	0
N.S.	1	1.13	0.59	1.67	1.23	13.72	0.00	1.18	1.87	0.00
time (sec)	N/A	0.411	0.025	0.374	0.190	0.093	0.000	0.114	0.256	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	99	65	0	0	802	0	0	16	0
N.S.	1	0.82	0.54	0.00	0.00	6.63	0.00	0.00	0.13	0.00
time (sec)	N/A	0.561	0.095	0.000	0.000	0.107	0.000	0.000	0.247	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	63	54	0	0	305	0	0	14	0
N.S.	1	0.89	0.76	0.00	0.00	4.30	0.00	0.00	0.20	0.00
time (sec)	N/A	0.393	0.064	0.000	0.000	0.087	0.000	0.000	0.258	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	45	59	0	0	54	0	0	11	0
N.S.	1	0.94	1.23	0.00	0.00	1.12	0.00	0.00	0.23	0.00
time (sec)	N/A	0.315	0.053	0.000	0.000	0.090	0.000	0.000	0.218	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	36	0	0	82	0	0	16	0
N.S.	1	0.89	0.78	0.00	0.00	1.78	0.00	0.00	0.35	0.00
time (sec)	N/A	0.337	0.029	0.000	0.000	0.082	0.000	0.000	0.238	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	75	65	48	0	0	554	0	0	16	0
N.S.	1	0.87	0.64	0.00	0.00	7.39	0.00	0.00	0.21	0.00
time (sec)	N/A	0.374	0.056	0.000	0.000	0.127	0.000	0.000	0.274	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	95	61	0	0	1473	0	0	16	0
N.S.	1	0.79	0.50	0.00	0.00	12.17	0.00	0.00	0.13	0.00
time (sec)	N/A	0.555	0.085	0.000	0.000	0.148	0.000	0.000	0.235	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	92	53	171	100	1597	0	76	87	0
N.S.	1	0.70	0.40	1.30	0.76	12.10	0.00	0.58	0.66	0.00
time (sec)	N/A	0.542	0.097	15.123	0.121	0.153	0.000	0.112	0.250	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	60	38	125	62	659	0	52	55	0
N.S.	1	0.77	0.49	1.60	0.79	8.45	0.00	0.67	0.71	0.00
time (sec)	N/A	0.405	0.062	0.572	0.134	0.156	0.000	0.117	0.217	0.000

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	29	25	83	27	180	0	28	26	0
N.S.	1	0.81	0.69	2.31	0.75	5.00	0.00	0.78	0.72	0.00
time (sec)	N/A	0.240	0.024	0.564	0.118	0.095	0.000	0.112	0.263	0.000

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	15	29	16	116	0	13	21	39
N.S.	1	1.00	1.00	1.93	1.07	7.73	0.00	0.87	1.40	2.60
time (sec)	N/A	0.232	0.005	0.491	0.121	0.083	0.000	0.112	0.216	0.068

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	48	30	48	165	1137	0	27	60	48
N.S.	1	0.72	0.45	0.72	2.46	16.97	0.00	0.40	0.90	0.72
time (sec)	N/A	0.252	0.024	0.528	0.121	0.105	0.000	0.118	0.300	0.133

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	68	47	60	457	3065	0	39	102	256
N.S.	1	0.58	0.40	0.51	3.91	26.20	0.00	0.33	0.87	2.19
time (sec)	N/A	0.279	0.041	0.549	0.146	0.181	0.000	0.117	0.260	1.911

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	9	8
N.S.	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.12	1.00
time (sec)	N/A	0.190	0.009	0.277	0.032	0.082	0.181	0.107	0.251	0.078

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	8	8	12	9	8	31	7	10	10	8
N.S.	1	1.00	1.50	1.12	1.00	3.88	0.88	1.25	1.25	1.00
time (sec)	N/A	0.199	0.009	0.310	0.025	0.065	0.169	0.109	0.237	1.828

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	18	11	12	20	7	10	20	10
N.S.	1	1.00	1.50	0.92	1.00	1.67	0.58	0.83	1.67	0.83
time (sec)	N/A	0.203	0.001	0.404	0.037	0.075	0.211	0.105	0.226	1.830

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	24	11	12	22	7	10	22	10
N.S.	1	1.00	1.71	0.79	0.86	1.57	0.50	0.71	1.57	0.71
time (sec)	N/A	0.203	0.001	0.457	0.031	0.081	0.345	0.110	0.220	0.048

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	13	11	23	48	58	21	40	10
N.S.	1	1.00	1.30	1.10	2.30	4.80	5.80	2.10	4.00	1.00
time (sec)	N/A	0.221	0.015	1.264	0.033	0.090	0.209	0.105	0.286	2.018

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	13	11	23	54	58	22	40	10
N.S.	1	1.00	1.08	0.92	1.92	4.50	4.83	1.83	3.33	0.83
time (sec)	N/A	0.221	0.014	1.245	0.027	0.081	0.213	0.104	0.227	2.041

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	9	8	55	15	12	16	8
N.S.	1	1.00	1.20	0.90	0.80	5.50	1.50	1.20	1.60	0.80
time (sec)	N/A	0.199	0.008	0.477	0.036	0.075	0.262	0.110	0.265	2.089

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	12	11	8	55	14	12	14	8
N.S.	1	1.00	1.00	0.92	0.67	4.58	1.17	1.00	1.17	0.67
time (sec)	N/A	0.193	0.009	0.563	0.047	0.070	0.251	0.106	0.208	0.083

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	12	9	49	33	7	16	35	16
N.S.	1	1.00	0.86	0.64	3.50	2.36	0.50	1.14	2.50	1.14
time (sec)	N/A	0.208	0.076	0.689	0.039	0.071	0.296	0.109	0.275	0.094

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	12	9	49	33	8	16	35	16
N.S.	1	1.00	0.75	0.56	3.06	2.06	0.50	1.00	2.19	1.00
time (sec)	N/A	0.205	0.069	0.872	0.038	0.071	0.502	0.107	0.252	2.096

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	18	15	31	89	126	21	56	14
N.S.	1	1.00	1.29	1.07	2.21	6.36	9.00	1.50	4.00	1.00
time (sec)	N/A	0.223	0.009	1.056	0.033	0.101	0.269	0.114	0.253	2.183

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	17	35	90	126	20	52	16
N.S.	1	1.00	0.90	0.85	1.75	4.50	6.30	1.00	2.60	0.80
time (sec)	N/A	0.224	0.008	1.270	0.037	0.078	0.259	0.114	0.260	1.976

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	60	51	165	102	101	1253	90	111	131
N.S.	1	1.05	0.89	2.89	1.79	1.77	21.98	1.58	1.95	2.30
time (sec)	N/A	0.374	0.154	0.145	0.034	0.079	2.983	0.111	0.263	2.248

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	47	27	40	84	94	284	75	89	107
N.S.	1	1.02	0.59	0.87	1.83	2.04	6.17	1.63	1.93	2.33
time (sec)	N/A	0.269	0.021	157.084	0.045	0.080	1.995	0.117	0.257	2.169

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	46	39	96	78	57	692	66	83	95
N.S.	1	1.05	0.89	2.18	1.77	1.30	15.73	1.50	1.89	2.16
time (sec)	N/A	0.330	0.167	61.892	0.028	0.092	1.248	0.109	0.267	2.066

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	32	21	26	60	52	150	51	61	71
N.S.	1	0.97	0.64	0.79	1.82	1.58	4.55	1.55	1.85	2.15
time (sec)	N/A	0.241	0.015	22.314	0.039	0.080	0.835	0.109	0.239	2.027

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	30	25	60	54	27	294	40	53	59
N.S.	1	0.97	0.81	1.94	1.74	0.87	9.48	1.29	1.71	1.90
time (sec)	N/A	0.257	0.113	7.096	0.037	0.075	0.471	0.117	0.231	1.954

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	13	16	36	18	49	27	31	35
N.S.	1	1.00	0.81	1.00	2.25	1.12	3.06	1.69	1.94	2.19
time (sec)	N/A	0.222	0.011	2.323	0.030	0.079	0.296	0.114	0.236	2.001

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	17	24	23	11	46	17	23	23
N.S.	1	1.00	1.31	1.85	1.77	0.85	3.54	1.31	1.77	1.77
time (sec)	N/A	0.213	0.039	0.567	0.038	0.080	0.188	0.111	0.254	1.974

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	9	11	12	12	11	16	7	17	9	9
N.S.	1	1.22	1.33	1.33	1.22	1.78	0.78	1.89	1.00	1.00
time (sec)	N/A	0.214	0.007	0.181	0.032	0.085	0.061	0.106	0.283	1.943

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	29	42	20	47	103	0	52	89	51
N.S.	1	1.26	1.83	0.87	2.04	4.48	0.00	2.26	3.87	2.22
time (sec)	N/A	0.289	0.022	0.582	0.034	0.082	0.000	0.112	0.262	1.987

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	30	24	59	94	0	35	33	89
N.S.	1	1.00	1.25	1.00	2.46	3.92	0.00	1.46	1.38	3.71
time (sec)	N/A	0.324	0.170	0.973	0.035	0.075	0.000	0.110	0.265	1.994

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	61	60	38	103	631	0	94	245	114
N.S.	1	1.11	1.09	0.69	1.87	11.47	0.00	1.71	4.45	2.07
time (sec)	N/A	0.322	0.102	2.277	0.031	0.084	0.000	0.108	0.257	2.017

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	43	38	36	233	250	0	59	75	263
N.S.	1	1.16	1.03	0.97	6.30	6.76	0.00	1.59	2.03	7.11
time (sec)	N/A	0.337	0.124	5.173	0.035	0.076	0.000	0.113	0.276	1.995

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	93	89	54	155	1551	0	116	405	244
N.S.	1	0.99	0.95	0.57	1.65	16.50	0.00	1.23	4.31	2.60
time (sec)	N/A	0.351	0.187	10.963	0.038	0.097	0.000	0.111	0.273	2.069

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	144	148	310	2134	0	229	444	289
N.S.	1	1.00	1.03	1.06	2.21	15.24	0.00	1.64	3.17	2.06
time (sec)	N/A	0.438	0.157	0.092	0.045	0.108	0.000	0.122	0.326	2.771

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	184	154	409	0	2913	0	266	373	348
N.S.	1	1.19	1.00	2.66	0.00	18.92	0.00	1.73	2.42	2.26
time (sec)	N/A	1.127	0.188	0.109	0.000	0.129	0.000	0.112	0.240	2.772

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	82	84	82	178	866	0	124	255	169
N.S.	1	0.99	1.01	0.99	2.14	10.43	0.00	1.49	3.07	2.04
time (sec)	N/A	0.348	0.115	112.233	0.037	0.096	0.000	0.117	0.235	2.311

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	125	95	223	0	1099	0	146	194	222
N.S.	1	1.20	0.91	2.14	0.00	10.57	0.00	1.40	1.87	2.13
time (sec)	N/A	0.642	0.138	30.005	0.000	0.099	0.000	0.111	0.255	2.355

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	40	39	84	234	0	56	115	79
N.S.	1	1.00	1.00	0.98	2.10	5.85	0.00	1.40	2.88	1.98
time (sec)	N/A	0.255	0.066	6.921	0.040	0.091	0.000	0.113	0.275	2.077

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	73	54	100	0	279	892	68	64	139
N.S.	1	1.24	0.92	1.69	0.00	4.73	15.12	1.15	1.08	2.36
time (sec)	N/A	0.413	0.089	1.334	0.000	0.111	52.895	0.116	0.258	2.071

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	11	12	11	27	14	19	11	11
N.S.	1	1.00	1.00	1.09	1.00	2.45	1.27	1.73	1.00	1.00
time (sec)	N/A	0.201	0.028	0.250	0.027	0.091	0.084	0.111	0.261	0.063

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	66	49	52	59	58	0	67	66	160
N.S.	1	1.25	0.92	0.98	1.11	1.09	0.00	1.26	1.25	3.02
time (sec)	N/A	0.311	0.071	0.680	0.030	0.103	0.000	0.114	0.248	2.357

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	78	77	78	0	470	0	76	167	327
N.S.	1	1.16	1.15	1.16	0.00	7.01	0.00	1.13	2.49	4.88
time (sec)	N/A	0.337	0.175	1.240	0.000	0.084	0.000	0.117	0.237	2.605

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	117	100	90	154	818	0	179	494	291
N.S.	1	1.18	1.01	0.91	1.56	8.26	0.00	1.81	4.99	2.94
time (sec)	N/A	0.394	0.334	2.883	0.050	0.107	0.000	0.109	0.237	2.547

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	134	141	127	0	2339	0	156	481	642
N.S.	1	1.22	1.28	1.15	0.00	21.26	0.00	1.42	4.37	5.84
time (sec)	N/A	0.600	0.388	6.718	0.000	0.136	0.000	0.114	0.269	3.093

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	196	165	138	348	3450	0	338	1180	559
N.S.	1	1.17	0.99	0.83	2.08	20.66	0.00	2.02	7.07	3.35
time (sec)	N/A	0.526	0.471	14.531	0.049	0.186	0.000	0.124	0.240	2.886

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	200	201	213	0	6381	0	303	919	1031
N.S.	1	1.26	1.26	1.34	0.00	40.13	0.00	1.91	5.78	6.48
time (sec)	N/A	0.966	1.210	29.243	0.000	0.165	0.000	0.124	0.247	3.641

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	72	61	99	0	700	0	68	252	139
N.S.	1	1.07	0.91	1.48	0.00	10.45	0.00	1.01	3.76	2.07
time (sec)	N/A	0.396	0.099	1.328	0.000	0.102	0.000	0.122	0.268	2.106

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	133	100	154	0	2003	0	144	516	722
N.S.	1	1.18	0.88	1.36	0.00	17.73	0.00	1.27	4.57	6.39
time (sec)	N/A	1.000	0.308	1.379	0.000	0.155	0.000	0.115	0.242	7.269

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	59	46	95	96	450	0	115	292	1221
N.S.	1	1.04	0.81	1.67	1.68	7.89	0.00	2.02	5.12	21.42
time (sec)	N/A	0.344	0.098	0.766	0.125	0.102	0.000	0.114	0.230	2.607

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	75	61	78	0	326	0	67	111	285
N.S.	1	1.23	1.00	1.28	0.00	5.34	0.00	1.10	1.82	4.67
time (sec)	N/A	0.693	0.112	0.582	0.000	0.115	0.000	0.121	0.212	4.605

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	22	20	33	33	40	0	33	31	201
N.S.	1	1.10	1.00	1.65	1.65	2.00	0.00	1.65	1.55	10.05
time (sec)	N/A	0.235	0.008	0.336	0.114	0.084	0.000	0.108	0.296	0.452

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	59	50	53	59	60	0	67	65	148
N.S.	1	1.09	0.93	0.98	1.09	1.11	0.00	1.24	1.20	2.74
time (sec)	N/A	0.286	0.073	0.523	0.039	0.083	0.000	0.107	0.221	0.446

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	88	77	78	0	470	0	76	167	337
N.S.	1	1.14	1.00	1.01	0.00	6.10	0.00	0.99	2.17	4.38
time (sec)	N/A	0.487	0.172	0.820	0.000	0.113	0.000	0.112	0.240	2.363

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	135	101	91	156	839	0	178	494	291
N.S.	1	1.44	1.07	0.97	1.66	8.93	0.00	1.89	5.26	3.10
time (sec)	N/A	0.439	0.281	1.292	0.044	0.105	0.000	0.119	0.327	2.510

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	158	131	127	0	2417	0	172	496	666
N.S.	1	1.15	0.96	0.93	0.00	17.64	0.00	1.26	3.62	4.86
time (sec)	N/A	0.906	0.370	2.085	0.000	0.111	0.000	0.122	0.276	2.736

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	47	58	64	89	750	0	58	151	183
N.S.	1	1.02	1.26	1.39	1.93	16.30	0.00	1.26	3.28	3.98
time (sec)	N/A	0.481	0.219	1.257	0.114	0.087	0.000	0.116	0.218	1.993

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	38	25	43	223	174	0	48	77	117
N.S.	1	1.27	0.83	1.43	7.43	5.80	0.00	1.60	2.57	3.90
time (sec)	N/A	0.365	0.022	0.844	0.034	0.072	0.000	0.114	0.269	1.933

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	46	48	57	315	0	39	94	95
N.S.	1	1.00	1.39	1.45	1.73	9.55	0.00	1.18	2.85	2.88
time (sec)	N/A	0.394	0.141	0.634	0.116	0.094	0.000	0.113	0.255	1.868

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	27	17	26	70	66	0	22	41	25
N.S.	1	1.42	0.89	1.37	3.68	3.47	0.00	1.16	2.16	1.32
time (sec)	N/A	0.286	0.017	0.426	0.035	0.076	0.000	0.107	0.248	1.849

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	18	30	23	50	0	22	36	33
N.S.	1	1.00	1.20	2.00	1.53	3.33	0.00	1.47	2.40	2.20
time (sec)	N/A	0.299	0.167	0.352	0.132	0.088	0.000	0.118	0.255	1.923

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	22	12	14	24	28	0	22	21	26
N.S.	1	1.22	0.67	0.78	1.33	1.56	0.00	1.22	1.17	1.44
time (sec)	N/A	0.217	0.015	0.274	0.027	0.078	0.000	0.106	0.258	0.075

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	42	42	20	48	103	0	52	87	51
N.S.	1	1.27	1.27	0.61	1.45	3.12	0.00	1.58	2.64	1.55
time (sec)	N/A	0.385	0.029	0.272	0.027	0.076	0.000	0.108	0.233	1.882

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	36	25	29	121	91	0	35	47	92
N.S.	1	1.20	0.83	0.97	4.03	3.03	0.00	1.17	1.57	3.07
time (sec)	N/A	0.366	0.180	0.317	0.032	0.081	0.000	0.111	0.224	1.893

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	59	60	38	103	631	0	94	245	132
N.S.	1	1.28	1.30	0.83	2.24	13.72	0.00	2.04	5.33	2.87
time (sec)	N/A	0.493	0.075	0.385	0.042	0.076	0.000	0.110	0.251	1.944

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	46	41	45	469	224	0	59	103	263
N.S.	1	1.12	1.00	1.10	11.44	5.46	0.00	1.44	2.51	6.41
time (sec)	N/A	0.386	0.148	0.487	0.039	0.075	0.000	0.111	0.266	2.005

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	37	125	0	376	0	0	12	0
N.S.	1	1.00	1.00	3.38	0.00	10.16	0.00	0.00	0.32	0.00
time (sec)	N/A	0.246	0.017	0.461	0.000	0.262	0.000	0.000	0.213	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	24	104	0	356	0	0	20	0
N.S.	1	1.00	1.00	4.33	0.00	14.83	0.00	0.00	0.83	0.00
time (sec)	N/A	0.238	0.010	0.433	0.000	0.118	0.000	0.000	0.244	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	55	112	0	291	741	60	99	197
N.S.	1	1.00	0.98	2.00	0.00	5.20	13.23	1.07	1.77	3.52
time (sec)	N/A	0.336	0.076	0.614	0.000	0.107	13.298	0.110	0.249	4.046

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	23	19	46	20	22	47	22
N.S.	1	1.00	1.06	1.28	1.06	2.56	1.11	1.22	2.61	1.22
time (sec)	N/A	0.276	0.024	0.316	0.025	0.091	0.145	0.104	0.235	0.059

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	19	22	20	48	31	22	46	21
N.S.	1	1.00	0.79	0.92	0.83	2.00	1.29	0.92	1.92	0.88
time (sec)	N/A	0.318	0.220	0.358	0.028	0.089	0.232	0.109	0.261	1.913

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	61	100	0	315	0	66	120	160
N.S.	1	1.00	0.94	1.54	0.00	4.85	0.00	1.02	1.85	2.46
time (sec)	N/A	0.390	0.198	0.637	0.000	0.122	0.000	0.120	0.283	13.066

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	134	101	0	303	0	90	106	974
N.S.	1	1.00	1.34	1.01	0.00	3.03	0.00	0.90	1.06	9.74
time (sec)	N/A	0.449	0.313	0.673	0.000	1.279	0.000	0.115	0.223	4.829

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	63	59	0	249	0	53	45	636
N.S.	1	1.00	1.02	0.95	0.00	4.02	0.00	0.85	0.73	10.26
time (sec)	N/A	0.504	0.188	0.616	0.000	0.302	0.000	0.113	0.225	7.373

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F(-2)	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	112	116	100	0	298	0	90	108	983
N.S.	1	1.13	1.17	1.01	0.00	3.01	0.00	0.91	1.09	9.93
time (sec)	N/A	0.653	0.251	1.309	0.000	1.752	0.000	0.117	0.260	4.465

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	81	156	0	405	695	94	25	653
N.S.	1	1.00	0.94	1.81	0.00	4.71	8.08	1.09	0.29	7.59
time (sec)	N/A	0.593	0.440	1.115	0.000	0.103	15.802	0.123	0.282	3.274

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	132	115	144	0	1044	0	155	253	301
N.S.	1	1.09	0.95	1.19	0.00	8.63	0.00	1.28	2.09	2.49
time (sec)	N/A	0.622	0.548	1.864	0.000	0.104	0.000	0.138	0.260	2.695

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	213	175	273	0	3636	0	370	688	0
N.S.	1	1.14	0.94	1.46	0.00	19.44	0.00	1.98	3.68	0.00
time (sec)	N/A	0.857	0.776	6.407	0.000	0.151	0.000	0.150	0.264	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	302	245	459	0	8531	0	657	1747	0
N.S.	1	1.16	0.94	1.77	0.00	32.81	0.00	2.53	6.72	0.00
time (sec)	N/A	1.197	1.854	20.825	0.000	0.331	0.000	0.162	0.334	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	193	284	487	0	780	0	0	14	0
N.S.	1	1.01	1.49	2.55	0.00	4.08	0.00	0.00	0.07	0.00
time (sec)	N/A	0.781	0.417	0.514	0.000	0.133	0.000	0.000	0.256	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	291	275	434	686	0	1162	0	0	16	0
N.S.	1	0.95	1.49	2.36	0.00	3.99	0.00	0.00	0.05	0.00
time (sec)	N/A	1.130	0.281	0.482	0.000	0.120	0.000	0.000	0.224	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	391	355	580	889	0	1542	0	0	16	0
N.S.	1	0.91	1.48	2.27	0.00	3.94	0.00	0.00	0.04	0.00
time (sec)	N/A	1.436	0.288	0.537	0.000	0.127	0.000	0.000	0.252	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	55	0	0	0	0	0	35	0
N.S.	1	0.98	0.95	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.419	0.051	0.000	0.000	0.000	0.000	0.000	0.473	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	58	57	57	0	0	0	0	0	35	0
N.S.	1	0.98	0.98	0.00	0.00	0.00	0.00	0.00	0.60	0.00
time (sec)	N/A	0.381	0.027	0.000	0.000	0.000	0.000	0.000	0.284	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	26	0	0	0	0	0	33	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.27	0.00
time (sec)	N/A	0.303	0.023	0.000	0.000	0.000	0.000	0.000	0.297	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	36	30	33	32	29	32	33	35
N.S.	1	1.00	1.06	0.88	0.97	0.94	0.85	0.94	0.97	1.03
time (sec)	N/A	0.331	8.027	0.177	0.246	0.090	4.412	0.210	0.317	2.114

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	32	121	34	31	34	35	35
N.S.	1	1.00	1.06	0.89	3.36	0.94	0.86	0.94	0.97	0.97
time (sec)	N/A	0.374	27.737	0.215	0.243	0.080	4.823	0.276	0.392	2.238

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	59	138	0	480	0	0	112	110
N.S.	1	1.00	0.98	2.30	0.00	8.00	0.00	0.00	1.87	1.83
time (sec)	N/A	0.338	0.096	1.364	0.000	0.115	0.000	0.000	0.307	2.122

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F(-1)	F	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	103	87	231	0	1692	0	0	924	0
N.S.	1	1.18	1.00	2.66	0.00	19.45	0.00	0.00	10.62	0.00
time (sec)	N/A	0.434	0.174	5.083	0.000	0.104	0.000	0.000	0.240	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	41	47	42	67	0	45	35	0
N.S.	1	1.00	0.87	1.00	0.89	1.43	0.00	0.96	0.74	0.00
time (sec)	N/A	0.697	0.052	2.994	0.135	0.072	0.000	0.111	0.278	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	202	24	20	24	173	24
N.S.	1	1.00	1.09	1.00	9.18	1.09	0.91	1.09	7.86	1.09
time (sec)	N/A	0.266	2.674	0.141	0.975	0.085	0.765	0.144	0.313	1.959

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	327	337	326	0	0	624	0	0	79	0
N.S.	1	1.03	1.00	0.00	0.00	1.91	0.00	0.00	0.24	0.00
time (sec)	N/A	1.409	0.029	0.000	0.000	0.114	0.000	0.000	0.269	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	245	249	244	0	0	497	0	0	79	0
N.S.	1	1.02	1.00	0.00	0.00	2.03	0.00	0.00	0.32	0.00
time (sec)	N/A	1.000	0.017	0.000	0.000	0.089	0.000	0.000	0.280	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	160	368	0	354	0	0	75	0
N.S.	1	1.00	0.99	2.29	0.00	2.20	0.00	0.00	0.47	0.00
time (sec)	N/A	0.627	0.010	0.736	0.000	0.093	0.000	0.000	0.285	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	18	44	41	31	18	18
N.S.	1	1.00	1.00	1.06	1.00	2.44	2.28	1.72	1.00	1.00
time (sec)	N/A	0.215	0.141	0.352	0.027	0.099	0.573	0.117	0.245	1.914

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	24	22	56	24	19	24	86	24
N.S.	1	1.00	1.09	1.00	2.55	1.09	0.86	1.09	3.91	1.09
time (sec)	N/A	0.227	8.019	0.200	0.139	0.075	4.690	0.136	0.278	1.926

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	1291	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	53.79	1.08
time (sec)	N/A	0.242	6.934	0.407	0.228	0.077	1.276	0.141	0.329	1.947

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	495	463	386	0	0	1174	0	0	377	0
N.S.	1	0.94	0.78	0.00	0.00	2.37	0.00	0.00	0.76	0.00
time (sec)	N/A	2.348	0.651	0.000	0.000	0.144	0.000	0.000	0.286	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	370	356	293	0	0	937	0	0	334	0
N.S.	1	0.96	0.79	0.00	0.00	2.53	0.00	0.00	0.90	0.00
time (sec)	N/A	1.965	0.488	0.000	0.000	0.112	0.000	0.000	0.297	0.000

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	242	187	862	0	669	0	0	282	0
N.S.	1	0.99	0.77	3.53	0.00	2.74	0.00	0.00	1.16	0.00
time (sec)	N/A	1.250	0.681	2.917	0.000	0.099	0.000	0.000	0.285	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	87	69	129	0	415	1122	89	89	176
N.S.	1	1.19	0.95	1.77	0.00	5.68	15.37	1.22	1.22	2.41
time (sec)	N/A	0.505	0.263	1.592	0.000	0.104	57.259	0.128	0.257	2.159

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	92	26	20	26	26	26
N.S.	1	1.00	1.08	1.00	3.83	1.08	0.83	1.08	1.08	1.08
time (sec)	N/A	0.267	18.127	0.512	0.186	0.075	39.362	0.157	0.254	1.991

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	26	26	22	26	2616	26
N.S.	1	1.00	1.08	1.00	1.08	1.08	0.92	1.08	109.00	1.08
time (sec)	N/A	0.248	9.360	0.646	0.263	0.087	2.563	0.149	0.398	2.016

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	586	540	951	0	0	2025	0	0	829	0
N.S.	1	0.92	1.62	0.00	0.00	3.46	0.00	0.00	1.41	0.00
time (sec)	N/A	3.138	4.342	0.000	0.000	0.118	0.000	0.000	0.302	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	F	F	B	F	F	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	432	388	758	0	0	1622	0	0	712	0
N.S.	1	0.90	1.75	0.00	0.00	3.75	0.00	0.00	1.65	0.00
time (sec)	N/A	2.151	3.071	0.000	0.000	0.112	0.000	0.000	0.318	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	264	374	860	0	1196	0	0	583	0
N.S.	1	0.92	1.30	2.99	0.00	4.15	0.00	0.00	2.02	0.00
time (sec)	N/A	1.381	1.519	12.735	0.000	0.104	0.000	0.000	0.305	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	55	55	55	130	340	0	88	177	122
N.S.	1	0.90	0.90	0.90	2.13	5.57	0.00	1.44	2.90	2.00
time (sec)	N/A	0.318	0.253	7.570	0.051	0.089	0.000	0.124	0.235	2.215

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	26	24	143	26	20	26	183	26
N.S.	1	1.00	1.08	1.00	5.96	1.08	0.83	1.08	7.62	1.08
time (sec)	N/A	0.271	34.173	0.706	0.348	0.075	23.283	0.192	0.309	2.184

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	41	42	51	44	0	47	41	44
N.S.	1	1.00	0.76	0.78	0.94	0.81	0.00	0.87	0.76	0.81
time (sec)	N/A	0.227	0.044	0.465	0.054	0.083	0.000	0.115	0.290	1.963

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	56	59	67	90	0	169	123	53
N.S.	1	1.00	0.64	0.67	0.76	1.02	0.00	1.92	1.40	0.60
time (sec)	N/A	0.270	0.062	1.794	0.060	0.107	0.000	0.128	0.269	1.951

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	143	117	357	115	199	0	665	324	94
N.S.	1	0.96	0.79	2.40	0.77	1.34	0.00	4.46	2.17	0.63
time (sec)	N/A	0.367	0.342	5.254	0.073	0.098	0.000	0.145	0.258	2.011

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	177	167	184	129	293	0	777	390	102
N.S.	1	0.93	0.87	0.96	0.68	1.53	0.00	4.07	2.04	0.53
time (sec)	N/A	0.418	0.289	19.321	0.081	0.113	0.000	0.143	0.259	1.989

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	74	54	68	64	99	0	235	66	55
N.S.	1	1.01	0.74	0.93	0.88	1.36	0.00	3.22	0.90	0.75
time (sec)	N/A	0.262	0.085	1.171	0.061	0.102	0.000	0.117	0.234	1.991

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	122	87	94	87	250	0	759	262	73
N.S.	1	1.02	0.72	0.78	0.72	2.08	0.00	6.32	2.18	0.61
time (sec)	N/A	0.311	0.185	9.788	0.075	0.103	0.000	0.151	0.234	2.040

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	188	292	286	138	584	0	3225	784	117
N.S.	1	0.93	1.44	1.41	0.68	2.88	0.00	15.89	3.86	0.58
time (sec)	N/A	0.489	0.914	66.283	0.091	0.098	0.000	0.170	0.243	2.118

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	A	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	266	236	311	0	161	1123	0	6880	1329	134
N.S.	1	0.89	1.17	0.00	0.61	4.22	0.00	25.86	5.00	0.50
time (sec)	N/A	0.455	2.402	0.000	0.097	0.135	0.000	0.204	0.284	2.163

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	37	19	18	19	34	42	18	18
N.S.	1	1.00	2.06	1.06	1.00	1.06	1.89	2.33	1.00	1.00
time (sec)	N/A	0.221	0.007	0.472	0.034	0.083	0.237	0.112	0.239	2.068

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	44	36	30	49	39	0	82	71	32
N.S.	1	1.13	0.92	0.77	1.26	1.00	0.00	2.10	1.82	0.82
time (sec)	N/A	0.234	0.016	1.777	0.034	0.085	0.000	0.119	0.271	2.548

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	45	42	36	86	53	71	81	85	35
N.S.	1	1.07	1.00	0.86	2.05	1.26	1.69	1.93	2.02	0.83
time (sec)	N/A	0.277	0.004	7.796	0.051	0.095	1.340	0.105	0.274	2.937

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	80	51	46	93	84	0	116	107	50
N.S.	1	1.10	0.70	0.63	1.27	1.15	0.00	1.59	1.47	0.68
time (sec)	N/A	0.338	0.028	32.264	0.035	0.090	0.000	0.119	0.261	3.146

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	64	65	51	130	105	105	116	122	49
N.S.	1	0.98	1.00	0.78	2.00	1.62	1.62	1.78	1.88	0.75
time (sec)	N/A	0.268	0.007	115.650	0.039	0.096	8.155	0.117	0.277	2.839

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	65	62	256	0	332	0	0	31	0
N.S.	1	0.97	0.93	3.82	0.00	4.96	0.00	0.00	0.46	0.00
time (sec)	N/A	0.305	0.033	7.938	0.000	0.082	0.000	0.000	0.241	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	65	114	237	0	170	0	0	29	0
N.S.	1	0.97	1.70	3.54	0.00	2.54	0.00	0.00	0.43	0.00
time (sec)	N/A	0.303	0.089	2.944	0.000	0.088	0.000	0.000	0.270	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	58	0	0	18	0
N.S.	1	1.00	1.00	6.54	0.00	2.07	0.00	0.00	0.64	0.00
time (sec)	N/A	0.232	0.011	1.897	0.000	0.082	0.000	0.000	0.279	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	183	0	39	0	0	31	0
N.S.	1	1.00	1.00	6.54	0.00	1.39	0.00	0.00	1.11	0.00
time (sec)	N/A	0.230	0.009	1.003	0.000	0.088	0.000	0.000	0.265	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	61	58	141	0	243	0	0	31	0
N.S.	1	0.97	0.92	2.24	0.00	3.86	0.00	0.00	0.49	0.00
time (sec)	N/A	0.300	0.035	1.260	0.000	0.090	0.000	0.000	0.280	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	65	122	258	0	501	0	0	31	0
N.S.	1	0.97	1.82	3.85	0.00	7.48	0.00	0.00	0.46	0.00
time (sec)	N/A	0.299	0.050	1.605	0.000	0.115	0.000	0.000	0.290	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F(-1)	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	206	203	85	0	0	187	0	0	40	0
N.S.	1	0.99	0.41	0.00	0.00	0.91	0.00	0.00	0.19	0.00
time (sec)	N/A	0.465	0.236	0.000	0.000	0.097	0.000	0.000	0.246	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	A	F	F(-2)	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	170	152	75	0	0	177	0	0	38	0
N.S.	1	0.89	0.44	0.00	0.00	1.04	0.00	0.00	0.22	0.00
time (sec)	N/A	0.385	0.214	0.000	0.000	0.113	0.000	0.000	0.237	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	102	109	74	0	0	141	0	0	80	0
N.S.	1	1.07	0.73	0.00	0.00	1.38	0.00	0.00	0.78	0.00
time (sec)	N/A	0.360	0.213	0.000	0.000	0.091	0.000	0.000	0.254	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	61	0	0	68	0	0	40	0
N.S.	1	1.00	1.45	0.00	0.00	1.62	0.00	0.00	0.95	0.00
time (sec)	N/A	0.366	0.085	0.000	0.000	0.091	0.000	0.000	0.266	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	87	141	121	0	0	128	0	80	40	0
N.S.	1	1.62	1.39	0.00	0.00	1.47	0.00	0.92	0.46	0.00
time (sec)	N/A	0.412	0.149	0.000	0.000	0.094	0.000	3.298	0.244	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	116	305	347	0	171	0	764	1338	0
N.S.	1	1.15	3.02	3.44	0.00	1.69	0.00	7.56	13.25	0.00
time (sec)	N/A	0.726	1.261	0.734	0.000	0.097	0.000	1.702	0.299	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	F	B	F(-1)	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	122	475	358	0	366	0	749	1575	0
N.S.	1	1.14	4.44	3.35	0.00	3.42	0.00	7.00	14.72	0.00
time (sec)	N/A	0.656	2.979	3.539	0.000	0.109	0.000	6.438	0.296	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	71	62	45	68	113	139	60	67	58
N.S.	1	0.86	0.75	0.54	0.82	1.36	1.67	0.72	0.81	0.70
time (sec)	N/A	0.226	0.031	3.088	0.046	0.079	2.347	0.115	0.242	0.553

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	47	53	47	53	95	207	57	55	42
N.S.	1	0.82	0.93	0.82	0.93	1.67	3.63	1.00	0.96	0.74
time (sec)	N/A	0.217	0.021	1.648	0.036	0.087	0.994	0.116	0.268	0.294

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	43	39	35	40	54	78	34	38	34
N.S.	1	0.88	0.80	0.71	0.82	1.10	1.59	0.69	0.78	0.69
time (sec)	N/A	0.198	0.017	0.865	0.037	0.078	0.475	0.111	0.272	0.117

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	23	30	23	19	24	50	63	22	38	18
N.S.	1	1.30	1.00	0.83	1.04	2.17	2.74	0.96	1.65	0.78
time (sec)	N/A	0.192	0.010	0.393	0.030	0.081	0.229	0.107	0.262	2.219

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	17	17	16	30	0	16	17	16
N.S.	1	1.00	1.00	1.00	0.94	1.76	0.00	0.94	1.00	0.94
time (sec)	N/A	0.177	0.011	0.309	0.111	0.076	0.000	0.105	0.213	0.073

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	38	25	37	105	0	35	56	48
N.S.	1	1.08	0.95	0.62	0.92	2.62	0.00	0.88	1.40	1.20
time (sec)	N/A	0.189	0.044	0.674	0.139	0.086	0.000	0.107	0.300	0.093

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	22	68	86	0	31	41	31
N.S.	1	1.00	1.00	0.76	2.34	2.97	0.00	1.07	1.41	1.07
time (sec)	N/A	0.178	0.015	1.083	0.035	0.077	0.000	0.107	0.268	2.096

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	106	64	37	83	513	0	60	147	130
N.S.	1	1.12	0.67	0.39	0.87	5.40	0.00	0.63	1.55	1.37
time (sec)	N/A	0.214	0.057	4.105	0.112	0.089	0.000	0.112	0.239	2.043

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	44	35	172	233	0	42	81	42
N.S.	1	1.00	0.73	0.58	2.87	3.88	0.00	0.70	1.35	0.70
time (sec)	N/A	0.217	0.026	3.250	0.042	0.089	0.000	0.115	0.277	2.001

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	47	42	17	25	17
N.S.	1	1.08	1.00	0.65	0.65	1.81	1.62	0.65	0.96	0.65
time (sec)	N/A	0.196	0.011	0.572	0.028	0.075	0.154	0.110	0.270	0.087

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	21	16	14	13	26	20	13	18	12
N.S.	1	1.11	0.84	0.74	0.68	1.37	1.05	0.68	0.95	0.63
time (sec)	N/A	0.180	0.008	0.262	0.027	0.088	0.101	0.107	0.249	2.006

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	104	24	25	76	90	0	76	70	77
N.S.	1	1.60	0.37	0.38	1.17	1.38	0.00	1.17	1.08	1.18
time (sec)	N/A	0.319	0.009	0.645	0.114	0.081	0.000	0.109	0.232	0.235

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	125	106	36	88	345	0	88	186	85
N.S.	1	1.45	1.23	0.42	1.02	4.01	0.00	1.02	2.16	0.99
time (sec)	N/A	0.335	0.061	1.348	0.111	0.102	0.000	0.116	0.238	2.284

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	67	42	17	25	17
N.S.	1	1.08	1.00	0.65	0.65	2.58	1.62	0.65	0.96	0.65
time (sec)	N/A	0.187	0.013	0.596	0.026	0.085	0.155	0.119	0.263	1.967

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	16	14	13	38	20	13	18	12
N.S.	1	1.21	0.84	0.74	0.68	2.00	1.05	0.68	0.95	0.63
time (sec)	N/A	0.182	0.008	0.407	0.028	0.081	0.099	0.133	0.234	1.893

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	A	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	35	24	79	71	83	0	44	72	65
N.S.	1	0.64	0.44	1.44	1.29	1.51	0.00	0.80	1.31	1.18
time (sec)	N/A	0.250	0.010	0.653	0.112	0.080	0.000	0.114	0.268	2.013

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	121	34	59	79	481	0	79	175	84
N.S.	1	1.42	0.40	0.69	0.93	5.66	0.00	0.93	2.06	0.99
time (sec)	N/A	0.364	0.017	1.462	0.137	0.088	0.000	0.122	0.251	0.334

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	28	26	17	17	87	42	17	25	17
N.S.	1	1.08	1.00	0.65	0.65	3.35	1.62	0.65	0.96	0.65
time (sec)	N/A	0.209	0.012	0.681	0.033	0.092	0.155	0.117	0.252	1.920

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	23	19	14	13	46	20	13	18	14
N.S.	1	1.21	1.00	0.74	0.68	2.42	1.05	0.68	0.95	0.74
time (sec)	N/A	0.199	0.008	0.489	0.029	0.076	0.100	0.119	0.240	0.054

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	389	24	25	0	167	0	249	469	479
N.S.	1	1.35	0.08	0.09	0.00	0.58	0.00	0.86	1.62	1.66
time (sec)	N/A	0.727	0.010	0.756	0.000	0.098	0.000	0.161	0.258	5.838

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	390	34	36	0	1215	0	261	506	473
N.S.	1	1.33	0.12	0.12	0.00	4.15	0.00	0.89	1.73	1.61
time (sec)	N/A	0.752	0.015	1.364	0.000	0.106	0.000	0.136	0.285	4.750

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	190	159	148	134	2218	1610	1211	23	154
N.S.	1	0.94	0.79	0.73	0.66	10.98	7.97	6.00	0.11	0.76
time (sec)	N/A	0.517	0.427	2.319	0.050	0.170	3.305	0.174	0.232	3.002

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	85	90	94	699	707	889	23	100
N.S.	1	1.00	0.64	0.68	0.71	5.30	5.36	6.73	0.17	0.76
time (sec)	N/A	0.370	0.137	1.041	0.053	0.089	1.165	0.172	0.274	2.388

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	C	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	50	52	63	246	265	597	51	74
N.S.	1	1.00	0.67	0.69	0.84	3.28	3.53	7.96	0.68	0.99
time (sec)	N/A	0.240	0.063	0.373	0.041	0.105	0.655	0.138	0.254	2.141

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	70	0	0	0	0	0	21	0
N.S.	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	0.31	0.00
time (sec)	N/A	0.244	0.016	0.000	0.000	0.000	0.000	0.000	0.271	0.000

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	70	0	0	0	0	0	23	0
N.S.	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.33	0.00
time (sec)	N/A	0.228	0.013	0.000	0.000	0.000	0.000	0.000	0.252	0.000

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	124	139	96	0	0	0	0	0	23	0
N.S.	1	1.12	0.77	0.00	0.00	0.00	0.00	0.00	0.19	0.00
time (sec)	N/A	0.361	0.151	0.000	0.000	0.000	0.000	0.000	0.240	0.000

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	133	148	101	0	0	0	0	0	23	0
N.S.	1	1.11	0.76	0.00	0.00	0.00	0.00	0.00	0.17	0.00
time (sec)	N/A	0.370	0.128	0.000	0.000	0.000	0.000	0.000	0.280	0.000

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	102	109	326	112	218	0	99	97	0
N.S.	1	0.41	0.44	1.30	0.45	0.87	0.00	0.40	0.39	0.00
time (sec)	N/A	0.467	0.073	8.348	0.036	0.098	0.000	0.122	0.245	0.000

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	74	81	216	74	126	0	71	67	0
N.S.	1	0.46	0.50	1.33	0.46	0.78	0.00	0.44	0.41	0.00
time (sec)	N/A	0.394	0.047	1.206	0.037	0.076	0.000	0.129	0.262	0.000

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	C	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	59	48	106	29	66	163	25	55	76
N.S.	1	0.80	0.65	1.43	0.39	0.89	2.20	0.34	0.74	1.03
time (sec)	N/A	0.357	0.025	1.251	0.038	0.081	2.183	0.121	0.281	0.134

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	A	A	F	A	B	F(-1)
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	42	29	21	42	0	21	22	0
N.S.	1	1.00	0.95	0.66	0.48	0.95	0.00	0.48	0.50	0.00
time (sec)	N/A	0.338	0.037	0.622	0.121	0.081	0.000	0.128	0.284	0.000

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	46	69	84	120	0	38	50	76
N.S.	1	1.00	0.82	1.23	1.50	2.14	0.00	0.68	0.89	1.36
time (sec)	N/A	0.345	0.050	1.275	0.045	0.073	0.000	0.130	0.277	1.939

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	87	72	80	209	315	0	51	96	89
N.S.	1	0.62	0.51	0.57	1.48	2.23	0.00	0.36	0.68	0.63
time (sec)	N/A	0.419	0.048	1.264	0.037	0.090	0.000	0.132	0.294	0.116

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	191	105	84	91	386	589	0	64	138	345
N.S.	1	0.55	0.44	0.48	2.02	3.08	0.00	0.34	0.72	1.81
time (sec)	N/A	0.446	0.057	1.331	0.039	0.083	0.000	0.129	0.240	0.120

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	A	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	28	28	0	45	99	32	28	45
N.S.	1	1.00	0.68	0.68	0.00	1.10	2.41	0.78	0.68	1.10
time (sec)	N/A	0.200	0.035	0.575	0.000	0.075	0.239	0.116	0.267	1.962

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	79	72	65	104	0	73	14	0
N.S.	1	1.00	0.93	0.85	0.76	1.22	0.00	0.86	0.16	0.00
time (sec)	N/A	0.279	0.057	0.806	0.038	0.094	0.000	0.125	0.260	0.000

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	91	97	81	130	0	91	17	0
N.S.	1	1.00	0.90	0.96	0.80	1.29	0.00	0.90	0.17	0.00
time (sec)	N/A	0.344	0.106	1.254	0.041	0.080	0.000	0.122	0.262	0.000

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	C	A	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	51	52	45	72	0	45	14	0
N.S.	1	1.00	0.78	0.80	0.69	1.11	0.00	0.69	0.22	0.00
time (sec)	N/A	0.269	0.050	0.757	0.039	0.080	0.000	0.131	0.238	0.000

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	71	48	47	76	0	49	16	0
N.S.	1	1.00	1.09	0.74	0.72	1.17	0.00	0.75	0.25	0.00
time (sec)	N/A	0.286	0.068	1.518	0.055	0.077	0.000	0.113	0.261	0.000

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	122	105	89	165	0	101	19	0
N.S.	1	1.00	1.06	0.91	0.77	1.43	0.00	0.88	0.17	0.00
time (sec)	N/A	0.384	0.244	1.549	0.044	0.079	0.000	0.134	0.251	0.000

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	102	100	90	211	0	106	20	0
N.S.	1	1.00	0.93	0.91	0.82	1.92	0.00	0.96	0.18	0.00
time (sec)	N/A	0.364	0.087	0.492	0.050	0.091	0.000	0.159	0.246	0.000

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	149	126	127	278	0	355	22	0
N.S.	1	1.00	1.01	0.85	0.86	1.88	0.00	2.40	0.15	0.00
time (sec)	N/A	0.418	0.462	1.170	0.136	0.106	0.000	0.139	0.280	0.000

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	286	207	200	443	0	223	22	0
N.S.	1	1.00	1.20	0.87	0.84	1.85	0.00	0.93	0.09	0.00
time (sec)	N/A	0.630	0.275	2.759	0.145	0.102	0.000	0.145	0.231	0.000

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	123	126	102	251	0	132	23	0
N.S.	1	1.00	1.07	1.10	0.89	2.18	0.00	1.15	0.20	0.00
time (sec)	N/A	0.535	0.177	0.567	0.040	0.089	0.000	0.132	0.264	0.000

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	220	158	143	334	0	387	25	0
N.S.	1	1.00	1.37	0.98	0.89	2.07	0.00	2.40	0.16	0.00
time (sec)	N/A	0.539	0.423	1.681	0.133	0.084	0.000	0.162	0.224	0.000

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	353	265	228	539	0	281	25	0
N.S.	1	1.00	1.37	1.03	0.89	2.10	0.00	1.09	0.10	0.00
time (sec)	N/A	0.856	0.516	4.444	0.141	0.099	0.000	0.138	0.264	0.000

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	104	117	105	216	0	132	20	0
N.S.	1	1.00	0.78	0.88	0.79	1.62	0.00	0.99	0.15	0.00
time (sec)	N/A	0.429	0.101	0.410	0.041	0.091	0.000	0.140	0.260	0.000

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	131	139	131	242	0	150	22	0
N.S.	1	1.00	0.81	0.86	0.81	1.50	0.00	0.93	0.14	0.00
time (sec)	N/A	0.462	0.154	0.998	0.040	0.087	0.000	0.137	0.244	0.000

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	214	234	211	426	0	264	22	0
N.S.	1	1.00	0.79	0.86	0.78	1.57	0.00	0.97	0.08	0.00
time (sec)	N/A	0.617	0.297	2.537	0.055	0.094	0.000	0.145	0.274	0.000

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	75	70	69	145	0	75	22	0
N.S.	1	1.00	0.93	0.86	0.85	1.79	0.00	0.93	0.27	0.00
time (sec)	N/A	0.369	0.213	0.447	0.044	0.088	0.000	0.134	0.259	0.000

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	179	101	100	254	0	107	24	0
N.S.	1	1.00	1.40	0.79	0.78	1.98	0.00	0.84	0.19	0.00
time (sec)	N/A	0.441	0.357	1.004	0.043	0.097	0.000	0.146	0.243	0.000

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	270	144	143	491	0	155	24	0
N.S.	1	1.00	1.58	0.84	0.84	2.87	0.00	0.91	0.14	0.00
time (sec)	N/A	0.532	0.836	2.547	0.043	0.110	0.000	0.136	0.277	0.000

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	165	147	127	321	0	172	25	0
N.S.	1	1.00	1.18	1.05	0.91	2.29	0.00	1.23	0.18	0.00
time (sec)	N/A	0.498	0.432	0.600	0.047	0.104	0.000	0.131	0.242	0.000

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	258	177	161	420	0	198	27	0
N.S.	1	1.00	1.41	0.97	0.88	2.30	0.00	1.08	0.15	0.00
time (sec)	N/A	0.602	0.939	1.608	0.043	0.097	0.000	0.136	0.257	0.000

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	300	300	478	302	263	847	0	352	27	0
N.S.	1	1.00	1.59	1.01	0.88	2.82	0.00	1.17	0.09	0.00
time (sec)	N/A	0.856	4.047	4.000	0.050	0.107	0.000	0.147	0.234	0.000

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	134	162	129	262	0	167	24	0
N.S.	1	1.00	0.88	1.06	0.84	1.71	0.00	1.09	0.16	0.00
time (sec)	N/A	0.578	0.183	0.487	0.042	0.083	0.000	0.131	0.240	0.000

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	183	210	185	341	0	223	26	0
N.S.	1	1.00	0.84	0.96	0.84	1.56	0.00	1.02	0.12	0.00
time (sec)	N/A	0.677	0.315	1.093	0.043	0.105	0.000	0.139	0.251	0.000

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	262	326	263	526	0	339	26	0
N.S.	1	1.00	0.83	1.03	0.83	1.67	0.00	1.08	0.08	0.00
time (sec)	N/A	0.825	0.606	2.821	0.057	0.087	0.000	0.142	0.260	0.000

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	185	160	139	324	0	181	26	0
N.S.	1	1.00	1.20	1.04	0.90	2.10	0.00	1.18	0.17	0.00
time (sec)	N/A	0.584	0.438	0.540	0.041	0.113	0.000	0.138	0.271	0.000

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	225	225	257	217	199	466	0	239	28	0
N.S.	1	1.00	1.14	0.96	0.88	2.07	0.00	1.06	0.12	0.00
time (sec)	N/A	0.709	1.427	1.308	0.044	0.096	0.000	0.139	0.271	0.000

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	323	323	501	326	287	851	0	369	28	0
N.S.	1	1.00	1.55	1.01	0.89	2.63	0.00	1.14	0.09	0.00
time (sec)	N/A	0.849	4.519	3.086	0.049	0.125	0.000	0.150	0.242	0.000

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	251	186	151	362	0	207	29	0
N.S.	1	1.00	1.56	1.16	0.94	2.25	0.00	1.29	0.18	0.00
time (sec)	N/A	0.662	0.965	0.636	0.048	0.120	0.000	0.138	0.229	0.000

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	339	249	215	516	0	271	31	0
N.S.	1	1.00	1.42	1.04	0.90	2.16	0.00	1.13	0.13	0.00
time (sec)	N/A	0.818	4.025	1.779	0.058	0.104	0.000	0.150	0.264	0.000

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F(-1)	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	2991	384	315	939	0	427	31	0
N.S.	1	1.00	8.69	1.12	0.92	2.73	0.00	1.24	0.09	0.00
time (sec)	N/A	1.170	6.433	5.049	0.059	0.106	0.000	0.154	0.277	0.000

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	F	F	F(-2)	F	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	46	0	0	0	0	0	19	39
N.S.	1	1.00	2.30	0.00	0.00	0.00	0.00	0.00	0.95	1.95
time (sec)	N/A	0.196	0.216	0.000	0.000	0.000	0.000	0.000	0.227	0.166

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	F	F	B
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	24	16	0	0	109	0	0	25	42
N.S.	1	1.00	0.67	0.00	0.00	4.54	0.00	0.00	1.04	1.75
time (sec)	N/A	0.199	0.031	0.000	0.000	0.089	0.000	0.000	0.261	2.020

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	F(-2)	F(-1)	F	F	B
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	64	0	0	0	0	0	21	110
N.S.	1	1.00	1.36	0.00	0.00	0.00	0.00	0.00	0.45	2.34
time (sec)	N/A	0.222	0.394	0.000	0.000	0.000	0.000	0.000	0.300	2.172

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	76	0	0	0	0	0	23	0
N.S.	1	1.00	2.11	0.00	0.00	0.00	0.00	0.00	0.64	0.00
time (sec)	N/A	0.233	0.099	0.000	0.000	0.000	0.000	0.000	0.230	0.000

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	26	25	36	23	41	36	62	24
N.S.	1	1.00	0.87	0.83	1.20	0.77	1.37	1.20	2.07	0.80
time (sec)	N/A	0.222	0.042	1.458	0.037	0.079	0.065	0.127	0.252	0.052

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	51	52	81	54	85	75	114	48
N.S.	1	1.00	0.91	0.93	1.45	0.96	1.52	1.34	2.04	0.86
time (sec)	N/A	0.268	0.059	0.288	0.038	0.080	0.093	0.125	0.337	0.074

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	166	212	0	316	0	0	18	0
N.S.	1	1.00	0.78	1.00	0.00	1.48	0.00	0.00	0.08	0.00
time (sec)	N/A	0.721	0.349	0.522	0.000	0.092	0.000	0.000	0.215	0.000

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F(-2)	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	271	271	220	376	0	671	0	0	21	0
N.S.	1	1.00	0.81	1.39	0.00	2.48	0.00	0.00	0.08	0.00
time (sec)	N/A	1.048	0.561	0.826	0.000	0.105	0.000	0.000	0.231	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [85] had the largest ratio of [1.8000000000000004]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	2	2	1.00	6	0.333
2	A	3	3	1.00	8	0.375
3	C	4	3	1.23	8	0.375
4	A	5	5	1.11	8	0.625
5	C	4	3	1.12	8	0.375
6	A	7	7	1.15	8	0.875
7	A	6	6	1.07	10	0.600
8	A	4	4	1.00	10	0.400
9	A	4	4	1.00	10	0.400
10	A	2	2	1.00	10	0.200
11	A	2	2	1.00	10	0.200
12	A	4	4	1.00	10	0.400
13	A	4	4	1.00	10	0.400
14	A	6	6	1.01	10	0.600
15	A	8	8	1.09	8	1.000
16	A	6	6	1.00	8	0.750
17	A	6	6	1.00	8	0.750
18	A	4	4	1.00	8	0.500
19	A	4	4	1.00	8	0.500
20	A	6	6	1.00	8	0.750
21	A	6	6	1.00	8	0.750

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	8	8	1.06	8	1.000
23	A	2	2	1.00	10	0.200
24	C	10	9	1.15	13	0.692
25	A	4	4	1.00	13	0.308
26	A	7	7	0.96	13	0.538
27	A	4	4	1.00	11	0.364
28	A	5	5	1.00	11	0.455
29	A	10	9	1.04	13	0.692
30	A	12	11	1.02	13	0.846
31	C	12	11	1.12	13	0.846
32	A	2	2	1.00	10	0.200
33	A	4	4	1.00	10	0.400
34	A	6	6	1.07	10	0.600
35	A	8	8	1.11	10	0.800
36	A	2	2	1.00	12	0.167
37	A	4	4	1.00	12	0.333
38	A	6	6	1.07	12	0.500
39	A	8	8	1.10	12	0.667
40	A	6	5	1.00	13	0.385
41	A	6	5	1.00	14	0.357
42	A	6	6	1.04	14	0.429
43	A	4	4	1.00	14	0.286
44	A	2	2	1.00	14	0.143
45	A	4	3	1.00	14	0.214
46	A	6	5	1.00	14	0.357
47	A	8	7	1.05	14	0.500
48	A	6	6	1.04	15	0.400
49	A	4	4	1.00	15	0.267
50	A	2	2	1.00	15	0.133
51	A	4	3	1.00	15	0.200
52	A	6	5	1.00	15	0.333
53	A	8	7	1.05	15	0.467

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	14	13	1.15	13	1.000
55	A	10	9	1.12	13	0.692
56	A	10	9	1.05	13	0.692
57	A	6	5	1.00	11	0.455
58	A	7	6	1.00	11	0.545
59	A	11	10	1.05	13	0.769
60	A	13	12	1.13	13	0.923
61	A	16	15	1.15	13	1.154
62	A	8	8	1.07	12	0.667
63	A	6	6	1.06	12	0.500
64	A	4	4	1.03	12	0.333
65	A	2	2	1.00	12	0.167
66	A	1	1	1.00	10	0.100
67	A	4	3	1.00	12	0.250
68	A	8	7	1.13	12	0.583
69	A	11	10	1.20	12	0.833
70	A	13	12	1.23	12	1.000
71	A	4	3	1.00	12	0.250
72	A	7	6	1.00	12	0.500
73	A	10	9	1.07	12	0.750
74	A	13	12	1.10	12	1.000
75	A	2	2	1.00	12	0.167
76	A	5	5	1.09	12	0.417
77	A	8	8	1.12	12	0.667
78	A	11	11	1.14	12	0.917
79	A	15	15	1.04	10	1.500
80	A	12	12	1.01	10	1.200
81	A	4	4	1.00	14	0.286
82	A	4	4	1.00	10	0.400
83	A	7	7	1.00	10	0.700
84	A	15	15	1.06	10	1.500
85	A	18	18	1.11	10	1.800

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	9	9	1.00	13	0.692
87	A	8	8	0.95	17	0.471
88	A	6	6	0.97	17	0.353
89	A	4	4	1.00	17	0.235
90	A	8	8	0.95	18	0.444
91	A	6	6	0.97	18	0.333
92	A	4	4	1.00	18	0.222
93	A	4	4	1.00	13	0.308
94	A	4	4	1.00	13	0.308
95	A	6	6	0.96	13	0.462
96	A	8	8	0.95	13	0.615
97	A	4	4	1.00	15	0.267
98	A	4	4	1.00	15	0.267
99	A	6	6	0.97	15	0.400
100	A	8	8	0.95	15	0.533
101	A	6	5	1.00	17	0.294
102	A	6	5	1.00	17	0.294
103	A	8	7	0.98	17	0.412
104	A	6	5	1.00	18	0.278
105	A	6	5	1.00	18	0.278
106	A	8	7	0.98	18	0.389
107	A	18	18	1.05	17	1.059
108	A	15	15	1.03	17	0.882
109	A	12	12	1.01	17	0.706
110	A	6	5	1.00	15	0.333
111	A	8	7	1.13	15	0.467
112	A	11	10	1.19	15	0.667
113	A	13	12	1.21	15	0.800
114	A	6	5	1.12	20	0.250
115	A	2	2	1.00	20	0.100
116	A	3	3	1.00	15	0.200
117	A	4	4	1.06	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	9	9	1.00	17	0.529
119	A	12	12	1.07	17	0.706
120	A	15	15	1.04	17	0.882
121	A	10	10	1.08	10	1.000
122	A	8	8	1.06	10	0.800
123	A	6	6	1.00	10	0.600
124	A	4	4	1.00	10	0.400
125	A	4	4	1.00	10	0.400
126	A	6	6	1.00	10	0.600
127	A	8	8	1.13	10	0.800
128	A	12	12	0.82	10	1.200
129	A	8	8	0.89	10	0.800
130	A	6	6	0.94	10	0.600
131	A	6	6	0.89	10	0.600
132	A	8	8	0.87	10	0.800
133	A	12	12	0.79	10	1.200
134	A	13	13	0.70	10	1.300
135	A	9	9	0.77	10	0.900
136	A	5	5	0.81	10	0.500
137	A	6	5	1.00	10	0.500
138	C	6	5	0.72	10	0.500
139	C	6	5	0.58	10	0.500
140	A	5	4	1.00	9	0.444
141	A	5	4	1.00	11	0.364
142	A	4	4	1.00	11	0.364
143	A	4	4	1.00	13	0.308
144	A	6	5	1.00	11	0.455
145	A	6	5	1.00	13	0.385
146	A	5	4	1.00	9	0.444
147	A	5	4	1.00	11	0.364
148	A	3	3	1.00	11	0.273
149	A	3	3	1.00	13	0.231

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	6	5	1.00	11	0.455
151	A	6	5	1.00	13	0.385
152	A	13	13	1.05	13	1.000
153	A	6	5	1.02	13	0.385
154	A	10	10	1.05	13	0.769
155	A	6	5	0.97	13	0.385
156	A	7	7	0.97	13	0.538
157	A	5	4	1.00	13	0.308
158	A	4	4	1.00	13	0.308
159	A	5	4	1.22	11	0.364
160	A	6	5	1.26	11	0.455
161	A	9	8	1.00	13	0.615
162	A	6	5	1.11	13	0.385
163	C	6	5	1.16	13	0.385
164	A	6	5	0.99	13	0.385
165	A	6	5	1.00	13	0.385
166	A	16	15	1.19	13	1.154
167	A	6	5	0.99	13	0.385
168	A	12	11	1.20	13	0.846
169	A	6	5	1.00	13	0.385
170	A	10	9	1.24	13	0.692
171	A	5	4	1.00	11	0.364
172	A	6	5	1.25	11	0.455
173	A	8	7	1.16	13	0.538
174	A	6	5	1.18	13	0.385
175	A	10	9	1.22	13	0.692
176	A	6	5	1.17	13	0.385
177	A	14	13	1.26	13	1.000
178	A	10	9	1.07	13	0.692
179	A	12	11	1.18	13	0.846
180	A	6	5	1.04	13	0.385
181	A	13	12	1.23	13	0.923

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	7	6	1.10	11	0.545
183	A	9	8	1.09	11	0.727
184	A	13	12	1.14	13	0.923
185	A	9	8	1.44	13	0.615
186	C	20	19	1.15	13	1.462
187	A	15	14	1.02	13	1.077
188	C	11	10	1.27	13	0.769
189	A	11	10	1.00	13	0.769
190	C	10	9	1.42	13	0.692
191	A	9	8	1.00	13	0.615
192	A	7	6	1.22	11	0.545
193	C	14	13	1.27	11	1.182
194	C	11	10	1.20	13	0.769
195	C	18	17	1.28	13	1.308
196	C	11	10	1.12	13	0.769
197	A	7	6	1.00	13	0.462
198	A	6	5	1.00	13	0.385
199	A	3	3	1.00	15	0.200
200	A	3	3	1.00	13	0.231
201	A	3	3	1.00	15	0.200
202	A	3	3	1.00	15	0.200
203	A	3	3	1.00	15	0.200
204	A	9	8	1.00	15	0.533
205	C	8	8	1.13	15	0.533
206	A	12	11	1.00	31	0.355
207	A	14	13	1.09	31	0.419
208	A	17	16	1.14	31	0.516
209	A	19	18	1.16	31	0.581
210	A	9	8	1.01	12	0.667
211	A	10	9	0.95	14	0.643
212	A	11	10	0.91	14	0.714
213	A	5	4	0.98	36	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	5	4	0.98	36	0.111
215	A	4	3	1.00	34	0.088
216	N/A	4	0	1.00	34	0.000
217	N/A	4	0	1.00	36	0.000
218	A	5	4	1.00	12	0.333
219	A	9	8	1.18	12	0.667
220	A	2	2	1.00	20	0.100
221	N/A	1	0	1.00	22	0.000
222	A	7	6	1.03	22	0.273
223	A	6	5	1.02	22	0.227
224	A	5	4	1.00	20	0.200
225	A	5	4	1.00	19	0.211
226	N/A	1	0	1.00	22	0.000
227	N/A	1	0	1.00	24	0.000
228	C	23	22	0.94	24	0.917
229	C	18	17	0.96	24	0.708
230	A	15	14	0.99	22	0.636
231	A	10	9	1.19	21	0.429
232	N/A	1	0	1.00	24	0.000
233	N/A	1	0	1.00	24	0.000
234	C	29	28	0.92	24	1.167
235	C	21	20	0.90	24	0.833
236	C	16	15	0.92	22	0.682
237	A	6	5	0.90	21	0.238
238	N/A	1	0	1.00	24	0.000
239	A	1	1	1.00	11	0.091
240	A	2	2	1.00	13	0.154
241	A	2	2	0.96	13	0.154
242	A	3	3	0.93	13	0.231
243	A	1	1	1.01	15	0.067
244	A	2	2	1.02	17	0.118

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
245	A	2	2	0.93	17	0.118
246	A	3	3	0.89	17	0.176
247	A	4	3	1.00	15	0.200
248	A	5	4	1.13	17	0.235
249	C	5	4	1.07	17	0.235
250	A	7	6	1.10	17	0.353
251	C	5	4	0.98	17	0.235
252	A	6	5	0.97	19	0.263
253	A	6	5	0.97	19	0.263
254	A	4	3	1.00	19	0.158
255	A	4	3	1.00	19	0.158
256	A	6	5	0.97	19	0.263
257	A	6	5	0.97	19	0.263
258	A	9	8	0.99	18	0.444
259	A	8	7	0.89	18	0.389
260	A	7	6	1.07	18	0.333
261	A	4	3	1.00	18	0.167
262	A	5	4	1.62	18	0.222
263	C	13	12	1.15	14	0.857
264	C	13	12	1.14	16	0.750
265	A	5	4	0.86	16	0.250
266	A	6	5	0.82	16	0.312
267	A	5	4	0.88	16	0.250
268	A	5	4	1.30	14	0.286
269	A	4	3	1.00	14	0.214
270	A	5	4	1.08	16	0.250
271	A	4	3	1.00	16	0.188
272	A	7	6	1.12	16	0.375
273	A	6	5	1.00	16	0.312
274	A	5	4	1.08	10	0.400
275	A	5	4	1.11	8	0.500
276	A	11	10	1.60	8	1.250

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
277	A	12	11	1.45	10	1.100
278	A	5	4	1.08	10	0.400
279	A	5	4	1.21	8	0.500
280	A	11	10	0.64	8	1.250
281	A	12	11	1.42	10	1.100
282	A	5	4	1.08	10	0.400
283	A	5	4	1.21	8	0.500
284	A	11	10	1.35	8	1.250
285	A	11	10	1.33	10	1.000
286	A	2	2	0.94	18	0.111
287	A	2	2	1.00	18	0.111
288	A	1	1	1.00	16	0.062
289	A	1	1	1.00	16	0.062
290	A	1	1	1.00	18	0.056
291	A	2	2	1.12	18	0.111
292	A	2	2	1.11	18	0.111
293	A	7	6	0.41	25	0.240
294	A	7	6	0.46	25	0.240
295	A	6	5	0.80	25	0.200
296	A	5	4	1.00	25	0.160
297	A	5	4	1.00	25	0.160
298	A	7	6	0.62	25	0.240
299	A	7	6	0.55	25	0.240
300	A	1	1	1.00	10	0.100
301	A	2	2	1.00	12	0.167
302	A	2	2	1.00	15	0.133
303	A	2	2	1.00	12	0.167
304	A	2	2	1.00	14	0.143
305	A	2	2	1.00	17	0.118
306	A	2	2	1.00	16	0.125
307	A	2	2	1.00	18	0.111
308	A	2	2	1.00	18	0.111

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	2	2	1.00	19	0.105
310	A	2	2	1.00	21	0.095
311	A	2	2	1.00	21	0.095
312	A	2	2	1.00	16	0.125
313	A	2	2	1.00	18	0.111
314	A	2	2	1.00	18	0.111
315	A	2	2	1.00	18	0.111
316	A	2	2	1.00	20	0.100
317	A	2	2	1.00	20	0.100
318	A	2	2	1.00	21	0.095
319	A	2	2	1.00	23	0.087
320	A	2	2	1.00	23	0.087
321	A	2	2	1.00	19	0.105
322	A	2	2	1.00	21	0.095
323	A	2	2	1.00	21	0.095
324	A	2	2	1.00	21	0.095
325	A	2	2	1.00	23	0.087
326	A	2	2	1.00	23	0.087
327	A	2	2	1.00	24	0.083
328	A	2	2	1.00	26	0.077
329	A	2	2	1.00	26	0.077
330	A	1	1	1.00	17	0.059
331	A	1	1	1.00	20	0.050
332	A	1	1	1.00	20	0.050
333	A	1	1	1.00	21	0.048
334	A	2	2	1.00	6	0.333
335	A	2	2	1.00	6	0.333
336	A	2	2	1.00	16	0.125
337	A	2	2	1.00	19	0.105

CHAPTER 3

Listing of integrals

3.1 $\int \cosh(a+bx)\,dx$

Optimal result

Integrand size = 6, antiderivative size = 10

$$\int \cosh(a+bx)\,dx=\frac{\sinh(a+bx)}{b}$$

output
```
sinh(b*x+a)/b
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 21 vs. 2(10) = 20.

Time = 0.00 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \cosh(a+bx)\,dx=\frac{\cosh(bx)\sinh(a)}{b}+\frac{\cosh(a)\sinh(bx)}{b}$$

input
```
Integrate[Cosh[a + b*x],x]
```

output
```
(Cosh[b*x]*Sinh[a])/b + (Cosh[a]*Sinh[b*x])/b
```

Rubi [A] (verified)

Time = 0.16 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sinh(a+bx)}{b}$$

input
```
Int[Cosh[a + b*x],x]
```

output
```
Sinh[a + b*x]/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\frac{\sinh(bx+a)}{b}$	11
default	$\frac{\sinh(bx+a)}{b}$	11
parallelrisch	$\frac{\sinh(bx+a)}{b}$	11
orering	$\frac{\sinh(bx+a)}{b}$	11
risch	$\frac{e^{bx+a}}{2b}-\frac{e^{-bx-a}}{2b}$	27
meijerg	$\frac{\cosh(a)\sinh(bx)}{b}-\frac{\sinh(a)\sqrt{\pi}\left(\frac{1}{\sqrt{\pi}}-\frac{\cosh(bx)}{\sqrt{\pi}}\right)}{b}$	35

input
```
int(cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
sinh(b*x+a)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\,dx=\frac{\sinh\left(bx+a\right)}{b}$$

input
```
integrate(cosh(b*x+a),x, algorithm="fricas")
```

output
```
sinh(b*x + a)/b
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \cosh(a+bx)\,dx = \begin{cases} \frac{\sinh{(a+bx)}}{b} & \text{for } b \neq 0 \\ x\cosh{(a)} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a),x)
```

output
```
Piecewise((sinh(a + b*x)/b, Ne(b, 0)), (x*cosh(a), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\,dx = \frac{\sinh{(bx+a)}}{b}$$

input
```
integrate(cosh(b*x+a),x, algorithm="maxima")
```

output
```
sinh(b*x + a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. 2(10) = 20.

Time = 0.11 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.60

$$\int \cosh(a+bx)\,dx = \frac{e^{(bx+a)}}{2\,b} - \frac{e^{(-bx-a)}}{2\,b}$$

input
```
integrate(cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*e^(b*x + a)/b - 1/2*e^(-b*x - a)/b
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\,dx=\frac{\sinh(a+b\,x)}{b}$$

input
```
int(cosh(a + b*x),x)
```

output
```
sinh(a + b*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \cosh(a+bx)\,dx=\frac{\sinh\left(bx+a\right)}{b}$$

input
```
int(cosh(b*x+a),x)
```

output
```
sinh(a + b*x)/b
```

3.2 $\int \cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 25

$$\int \cosh^2(a+bx)\,dx=\frac{x}{2}+\frac{\cosh(a+bx)\sinh(a+bx)}{2b}$$

output
```
1/2*x+1/2*cosh(b*x+a)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.92

$$\int \cosh^2(a+bx)\,dx=\frac{2(a+bx)+\sinh(2(a+bx))}{4b}$$

input
```
Integrate[Cosh[a + b*x]^2,x]
```

output
```
(2*(a + b*x) + Sinh[2*(a + b*x)])/(4*b)
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}$$

input
```
Int[Cosh[a + b*x]^2,x]
```

output
```
x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.80

method	result	size
parallelrisch	$\frac{2bx+\sinh(2bx+2a)}{4b}$	20
derivativedivides	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
default	$\frac{\frac{\cosh(bx+a)\sinh(bx+a)}{2}+\frac{bx}{2}+\frac{a}{2}}{b}$	27
risch	$\frac{x}{2}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}$	33
orering	$x\cosh(bx+a)^2+\frac{\cosh(bx+a)\sinh(bx+a)}{2b}-\frac{x\left(2b^2\sinh(bx+a)^2+2\cosh(bx+a)^2b^2\right)}{4b^2}$	62

input
```
int(cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*b*x+sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.88

$$\int \cosh^2(a+bx)\,dx=\frac{bx+\cosh(bx+a)\sinh(bx+a)}{2\,b}$$

input
```
integrate(cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
1/2*(b*x + cosh(b*x + a)*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(19) = 38$.

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \cosh^2(a+bx)\,dx = \begin{cases} -\frac{x\sinh^2(a+bx)}{2} + \frac{x\cosh^2(a+bx)}{2} + \frac{\sinh(a+bx)\cosh(a+bx)}{2b} & \text{for } b \neq 0 \\ x\cosh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**2,x)
```

output
```
Piecewise((-x*sinh(a + b*x)**2/2 + x*cosh(a + b*x)**2/2 + sinh(a + b*x)*co
sh(a + b*x)/(2*b), Ne(b, 0)), (x*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\,dx = \frac{1}{2}x + \frac{e^{(2bx+2a)}}{8b} - \frac{e^{(-2bx-2a)}}{8b}$$

input
```
integrate(cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \cosh^2(a+bx)\,dx = \frac{1}{2}x + \frac{e^{(2bx+2a)}}{8b} - \frac{e^{(-2bx-2a)}}{8b}$$

input
```
integrate(cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/2*x + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.72

$$\int \cosh^2(a+bx)\,dx = \frac{x}{2} + \frac{\sinh(2\,a+2\,b\,x)}{4\,b}$$

input
```
int(cosh(a + b*x)^2,x)
```

output
```
x/2 + sinh(2*a + 2*b*x)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.72

$$\int \cosh^2(a+bx)\,dx = \frac{e^{4bx+4a} + 4e^{2bx+2a}bx - 1}{8e^{2bx+2a}b}$$

input
```
int(cosh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x) + 4*e**(2*a + 2*b*x)*b*x - 1)/(8*e**(2*a + 2*b*x)*b)
```

3.3 $\int \cosh^3(a + bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 26

$$\int \cosh^3(a + bx)\,dx = \frac{\sinh(a + bx)}{b} + \frac{\sinh^3(a + bx)}{3b}$$

output
```
sinh(b*x+a)/b+1/3*sinh(b*x+a)^3/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \cosh^3(a + bx)\,dx = \frac{\sinh(a + bx)}{b} + \frac{\sinh^3(a + bx)}{3b}$$

input
```
Integrate[Cosh[a + b*x]^3,x]
```

output
```
Sinh[a + b*x]/b + Sinh[a + b*x]^3/(3*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.19 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^3 dx$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int \left(\sinh^2(a+bx)+1\right)d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^3,x]
```

output
```
(I*((-I)*Sinh[a + b*x] - (I/3)*Sinh[a + b*x]^3))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$	23
default	$\frac{\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$	23
parallelrisch	$\frac{\sinh(3bx+3a)+9\sinh(bx+a)}{12b}$	24
risch	$\frac{e^{3bx+3a}}{24b}+\frac{3\,e^{bx+a}}{8b}-\frac{3\,e^{-bx-a}}{8b}-\frac{e^{-3bx-3a}}{24b}$	55
orering	$\frac{10\cosh(bx+a)^2\sinh(bx+a)}{3b}-\frac{6b^3\sinh(bx+a)^3+21\cosh(bx+a)^2b^3\sinh(bx+a)}{9b^4}$	59

input
```
int(cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.23

$$\int \cosh^3(a+bx)\,dx=\frac{\sinh\left(bx+a\right)^3+3\left(\cosh\left(bx+a\right)^2+3\right)\sinh\left(bx+a\right)}{12\,b}$$

input
```
integrate(cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
1/12*(sinh(b*x + a)^3 + 3*(cosh(b*x + a)^2 + 3)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.38

$$\int \cosh^3(a+bx)\,dx = \begin{cases} -\frac{2\sinh^3(a+bx)}{3b} + \frac{\sinh(a+bx)\cosh^2(a+bx)}{b} & \text{for } b \neq 0 \\ x\cosh^3(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**3,x)
```

output
```
Piecewise((-2*sinh(a + b*x)**3/(3*b) + sinh(a + b*x)*cosh(a + b*x)**2/b, N
e(b, 0)), (x*cosh(a)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(
-3*b*x - 3*a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(24) = 48$.

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.08

$$\int \cosh^3(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{24\,b} + \frac{3\,e^{(bx+a)}}{8\,b} - \frac{3\,e^{(-bx-a)}}{8\,b} - \frac{e^{(-3\,bx-3\,a)}}{24\,b}$$

input
```
integrate(cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/24*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 3/8*e^(-b*x - a)/b - 1/24*e^(
-3*b*x - 3*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.85

$$\int \cosh^3(a+bx)\,dx = \frac{\sinh(a+b\,x)^3+3\sinh(a+b\,x)}{3\,b}$$

input
```
int(cosh(a + b*x)^3,x)
```

output
```
(3*sinh(a + b*x) + sinh(a + b*x)^3)/(3*b)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 53, normalized size of antiderivative = 2.04

$$\int \cosh^3(a+bx)\,dx = \frac{e^{6bx+6a}+9e^{4bx+4a}-9e^{2bx+2a}-1}{24e^{3bx+3a}b}$$

input
```
int(cosh(b*x+a)^3,x)
```

output
```
(e**(6*a + 6*b*x) + 9*e**(4*a + 4*b*x) - 9*e**(2*a + 2*b*x) - 1)/(24*e**(3
*a + 3*b*x)*b)
```

3.4 $\int \cosh^4(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \cosh^4(a+bx)\,dx=\frac{3x}{8}+\frac{3\cosh(a+bx)\sinh(a+bx)}{8b}+\frac{\cosh^3(a+bx)\sinh(a+bx)}{4b}$$

output
```
3/8*x+3/8*cosh(b*x+a)*sinh(b*x+a)/b+1/4*cosh(b*x+a)^3*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.72

$$\int \cosh^4(a+bx)\,dx=\frac{12(a+bx)+8\sinh(2(a+bx))+\sinh(4(a+bx))}{32b}$$

input
```
Integrate[Cosh[a + b*x]^4,x]
```

output
```
(12*(a + b*x) + 8*Sinh[2*(a + b*x)] + Sinh[4*(a + b*x)])/(32*b)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.625, Rules used = {3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)$$

input
```
Int[Cosh[a + b*x]^4,x]
```

output
```
(Cosh[a + b*x]^3*Sinh[a + b*x])/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a +
b*x])/(2*b)))/4
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

method	result
parallelrisch	$\frac{12bx+\sinh(4bx+4a)+8\sinh(2bx+2a)}{32b}$
derivativedivides	$\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{\left(\frac{\cosh(bx+a)^3}{4}+\frac{3\cosh(bx+a)}{8}\right)\sinh(bx+a)+\frac{3bx}{8}+\frac{3a}{8}}{b}$
risch	$\frac{3x}{8}+\frac{e^{4bx+4a}}{64b}+\frac{e^{2bx+2a}}{8b}-\frac{e^{-2bx-2a}}{8b}-\frac{e^{-4bx-4a}}{64b}$
orering	$x\cosh(bx+a)^4+\frac{5\cosh(bx+a)^3\sinh(bx+a)}{4b}-\frac{5x\left(12\sinh(bx+a)^2\cosh(bx+a)^2b^2+4\cosh(bx+a)^4b^2\right)}{16b^2}-$

input
```
int(cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*b*x+sinh(4*b*x+4*a)+8*sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

$$\int \cosh^4(a+bx)\,dx = \frac{\cosh(bx+a)\sinh(bx+a)^3 + 3\,bx + \left(\cosh(bx+a)^3 + 4\,\cosh(bx+a)\right)\sinh(bx+a)}{8\,b}$$

input
```
integrate(cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*x + a)*sinh(b*x + a)^3 + 3*b*x + (cosh(b*x + a)^3 + 4*cosh(b*x
 + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(41) = 82.

Time = 0.19 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.07

$$\int \cosh^4(a+bx)\,dx = \begin{cases} \frac{3x\sinh^4(a+bx)}{8} - \frac{3x\sinh^2(a+bx)\cosh^2(a+bx)}{4} + \frac{3x\cosh^4(a+bx)}{8} - \frac{3\sinh^3(a+bx)\cosh(a+bx)}{8b} + \frac{5\sinh(a+bx)\cosh^3(a+bx)}{8b} & \text{fo} \\ x\cosh^4(a) & \text{o} \end{cases}$$

input
```
integrate(cosh(b*x+a)**4,x)
```

output
```
Piecewise((3*x*sinh(a + b*x)**4/8 - 3*x*sinh(a + b*x)**2*cosh(a + b*x)**2/
4 + 3*x*cosh(a + b*x)**4/8 - 3*sinh(a + b*x)**3*cosh(a + b*x)/(8*b) + 5*si
nh(a + b*x)*cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*cosh(a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \cosh^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2
*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \cosh^4(a+bx)\,dx = \frac{3}{8}\,x + \frac{e^{(4\,bx+4\,a)}}{64\,b} + \frac{e^{(2\,bx+2\,a)}}{8\,b} - \frac{e^{(-2\,bx-2\,a)}}{8\,b} - \frac{e^{(-4\,bx-4\,a)}}{64\,b}$$

input
```
integrate(cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
3/8*x + 1/64*e^(4*b*x + 4*a)/b + 1/8*e^(2*b*x + 2*a)/b - 1/8*e^(-2*b*x - 2
*a)/b - 1/64*e^(-4*b*x - 4*a)/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.67

$$\int \cosh^4(a+bx)\,dx = \frac{3\,x}{8} + \frac{\frac{\sinh(2\,a+2\,b\,x)}{4} + \frac{\sinh(4\,a+4\,b\,x)}{32}}{b}$$

input
```
int(cosh(a + b*x)^4,x)
```

output
```
(3*x)/8 + (sinh(2*a + 2*b*x)/4 + sinh(4*a + 4*b*x)/32)/b
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.46

$$\int \cosh^4(a+bx)\,dx = \frac{e^{8bx+8a}+8e^{6bx+6a}+24e^{4bx+4a}bx-8e^{2bx+2a}-1}{64e^{4bx+4a}b}$$

input
```
int(cosh(b*x+a)^4,x)
```

output
```
(e**(8*a + 8*b*x) + 8*e**(6*a + 6*b*x) + 24*e**(4*a + 4*b*x)*b*x - 8*e**(2
*a + 2*b*x) - 1)/(64*e**(4*a + 4*b*x)*b)
```

3.5 $\int \cosh^5(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 41

$$\int \cosh^5(a+bx)\,dx=\frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}$$

output
```
sinh(b*x+a)/b+2/3*sinh(b*x+a)^3/b+1/5*sinh(b*x+a)^5/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \cosh^5(a+bx)\,dx=\frac{\sinh(a+bx)}{b}+\frac{2\sinh^3(a+bx)}{3b}+\frac{\sinh^5(a+bx)}{5b}$$

input
```
Integrate[Cosh[a + b*x]^5,x]
```

output
```
Sinh[a + b*x]/b + (2*Sinh[a + b*x]^3)/(3*b) + Sinh[a + b*x]^5/(5*b)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.21 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^5(a+bx)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^5 dx$$

$$\Big\downarrow \; 3113$$

$$\frac{i\int \left(\sinh^4(a+bx)+2\sinh^2(a+bx)+1\right) d(-i\sinh(a+bx))}{b}$$

$$\Big\downarrow \; 2009$$

$$\frac{i\left(-\frac{1}{5}i\sinh^5(a+bx)-\frac{2}{3}i\sinh^3(a+bx)-i\sinh(a+bx)\right)}{b}$$

input
```
Int[Cosh[a + b*x]^5,x]
```

output
```
(I*((-I)*Sinh[a + b*x] - ((2*I)/3)*Sinh[a + b*x]^3 - (I/5)*Sinh[a + b*x]^5
))/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 2.14 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.80

method	result
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$
default	$\frac{\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$
parallelrisch	$\frac{3\sinh(5bx+5a)+25\sinh(3bx+3a)+150\sinh(bx+a)}{240b}$
risch	$\frac{e^{5bx+5a}}{160b}+\frac{5\,e^{3bx+3a}}{96b}+\frac{5\,e^{bx+a}}{16b}-\frac{5\,e^{-bx-a}}{16b}-\frac{5\,e^{-3bx-3a}}{96b}-\frac{e^{-5bx-5a}}{160b}$
orering	$\frac{259\cosh(bx+a)^4\sinh(bx+a)}{45b}-\frac{7\left(60\cosh(bx+a)^2b^3\sinh(bx+a)^3+65\cosh(bx+a)^4b^3\sinh(bx+a)\right)}{45b^4}+\frac{120b^5\sinh(b$

input
```
int(cosh(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.61

$$\int \cosh^5(a+bx)\,dx = \frac{3\,\sinh\left(bx+a\right)^5+5\,\left(6\,\cosh\left(bx+a\right)^2+5\right)\sinh\left(bx+a\right)^3+15\,\left(\cosh\left(bx+a\right)^4+5\,\cosh\left(bx+a\right)^2+10\right)}{240\,b}$$

input
```
integrate(cosh(b*x+a)^5,x, algorithm="fricas")
```

output
```
1/240*(3*sinh(b*x + a)^5 + 5*(6*cosh(b*x + a)^2 + 5)*sinh(b*x + a)^3 + 15*
(cosh(b*x + a)^4 + 5*cosh(b*x + a)^2 + 10)*sinh(b*x + a))/b
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.41

$$\int \cosh^5(a+bx)\,dx = \begin{cases} \frac{8\sinh^5(a+bx)}{15b} - \frac{4\sinh^3(a+bx)\cosh^2(a+bx)}{3b} + \frac{\sinh(a+bx)\cosh^4(a+bx)}{b} & \text{for } b \neq 0 \\ x\cosh^5(a) & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(b*x+a)**5,x)
```

output
```
Piecewise((8*sinh(a + b*x)**5/(15*b) - 4*sinh(a + b*x)**3*cosh(a + b*x)**2
/(3*b) + sinh(a + b*x)*cosh(a + b*x)**4/b, Ne(b, 0)), (x*cosh(a)**5, True)
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \cosh^5(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(cosh(b*x+a)^5,x, algorithm="maxima")
```

output
```
1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/
16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.00

$$\int \cosh^5(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{160\,b} + \frac{5\,e^{(3\,bx+3\,a)}}{96\,b} + \frac{5\,e^{(bx+a)}}{16\,b} - \frac{5\,e^{(-bx-a)}}{16\,b} - \frac{5\,e^{(-3\,bx-3\,a)}}{96\,b} - \frac{e^{(-5\,bx-5\,a)}}{160\,b}$$

input
```
integrate(cosh(b*x+a)^5,x, algorithm="giac")
```

output
```
1/160*e^(5*b*x + 5*a)/b + 5/96*e^(3*b*x + 3*a)/b + 5/16*e^(b*x + a)/b - 5/
16*e^(-b*x - a)/b - 5/96*e^(-3*b*x - 3*a)/b - 1/160*e^(-5*b*x - 5*a)/b
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.76

$$\int \cosh^5(a+bx)\,dx = \frac{\frac{\sinh(a+b\,x)^5}{5}+\frac{2\sinh(a+b\,x)^3}{3}+\sinh(a+b\,x)}{b}$$

input
```
int(cosh(a + b*x)^5,x)
```

output
```
(sinh(a + b*x) + (2*sinh(a + b*x)^3)/3 + sinh(a + b*x)^5/5)/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.93

$$\int \cosh^5(a+bx)\,dx = \frac{3e^{10bx+10a}+25e^{8bx+8a}+150e^{6bx+6a}-150e^{4bx+4a}-25e^{2bx+2a}-3}{480e^{5bx+5a}b}$$

input
```
int(cosh(b*x+a)^5,x)
```

output
```
(3*e**(10*a + 10*b*x) + 25*e**(8*a + 8*b*x) + 150*e**(6*a + 6*b*x) - 150*e
**(4*a + 4*b*x) - 25*e**(2*a + 2*b*x) - 3)/(480*e**(5*a + 5*b*x)*b)
```

3.6 $\int \cosh^6(a+bx)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \cosh^6(a+bx)\,dx = \frac{5x}{16} + \frac{5\cosh(a+bx)\sinh(a+bx)}{16b} + \frac{5\cosh^3(a+bx)\sinh(a+bx)}{24b} + \frac{\cosh^5(a+bx)\sinh(a+bx)}{6b}$$

output
```
5/16*x+5/16*cosh(b*x+a)*sinh(b*x+a)/b+5/24*cosh(b*x+a)^3*sinh(b*x+a)/b+1/6
*cosh(b*x+a)^5*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \cosh^6(a+bx)\,dx = \frac{60a+60bx+45\sinh(2(a+bx))+9\sinh(4(a+bx))+\sinh(6(a+bx))}{192b}$$

input
```
Integrate[Cosh[a + b*x]^6,x]
```

output
```
(60*a + 60*b*x + 45*Sinh[2*(a + b*x)] + 9*Sinh[4*(a + b*x)] + Sinh[6*(a +
b*x)])/(192*b)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.875$, Rules used = {3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^6(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\int \cosh^4(a+bx)dx+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^4dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(a+bx)dx+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^2dx\right)$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{\sinh(a+bx)\cosh(a+bx)}{2b}\right)+\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}\right)+\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(a+bx)\cosh^5(a+bx)}{6b}+\frac{5}{6}\left(\frac{\sinh(a+bx)\cosh^3(a+bx)}{4b}+\frac{3}{4}\left(\frac{\sinh(a+bx)\cosh(a+bx)}{2b}+\frac{x}{2}\right)\right)$$

input
```
Int[Cosh[a + b*x]^6,x]
```

output
```
(Cosh[a + b*x]^5*Sinh[a + b*x])/(6*b) + (5*((Cosh[a + b*x]^3*Sinh[a + b*x]
)/(4*b) + (3*(x/2 + (Cosh[a + b*x]*Sinh[a + b*x])/(2*b)))/4))/6
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

method	result
parallelrisch	$\frac{60bx+\sinh(6bx+6a)+9\sinh(4bx+4a)+45\sinh(2bx+2a)}{192b}$
derivativedivides	$\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)+\frac{5bx}{16}+\frac{5a}{16}}{b}$
default	$\frac{\left(\frac{\cosh(bx+a)^5}{6}+\frac{5\cosh(bx+a)^3}{24}+\frac{5\cosh(bx+a)}{16}\right)\sinh(bx+a)+\frac{5bx}{16}+\frac{5a}{16}}{b}$
risch	$\frac{5x}{16}+\frac{e^{6bx+6a}}{384b}+\frac{3\,e^{4bx+4a}}{128b}+\frac{15\,e^{2bx+2a}}{128b}-\frac{15\,e^{-2bx-2a}}{128b}-\frac{3\,e^{-4bx-4a}}{128b}-\frac{e^{-6bx-6a}}{384b}$
orering	$x\cosh\left(bx+a\right)^6+\frac{49\cosh(bx+a)^5\sinh(bx+a)}{24b}-\frac{49x\left(30\cosh(bx+a)^4b^2\sinh(bx+a)^2+6\cosh(bx+a)^6b^2\right)}{144b^2}-$

input
```
int(cosh(b*x+a)^6,x,method=_RETURNVERBOSE)
```

output
```
1/192*(60*b*x+sinh(6*b*x+6*a)+9*sinh(4*b*x+4*a)+45*sinh(2*b*x+2*a))/b
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.34

$$\int \cosh^6(a+bx)\,dx$$
$$=\frac{3\cosh\left(bx+a\right)\sinh\left(bx+a\right)^5+2\left(5\cosh\left(bx+a\right)^3+9\cosh\left(bx+a\right)\right)\sinh\left(bx+a\right)^3+30\,bx+3\left(\cosh\right.}{96\,b}$$

input
```
integrate(cosh(b*x+a)^6,x, algorithm="fricas")
```

output
```
1/96*(3*cosh(b*x + a)*sinh(b*x + a)^5 + 2*(5*cosh(b*x + a)^3 + 9*cosh(b*x
+ a))*sinh(b*x + a)^3 + 30*b*x + 3*(cosh(b*x + a)^5 + 6*cosh(b*x + a)^3 +
15*cosh(b*x + a))*sinh(b*x + a))/b
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(61) = 122$.

Time = 0.38 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \cosh^6(a+bx)\,dx$$
$$= \begin{cases} -\frac{5x\sinh^6(a+bx)}{16} + \frac{15x\sinh^4(a+bx)\cosh^2(a+bx)}{16} - \frac{15x\sinh^2(a+bx)\cosh^4(a+bx)}{16} + \frac{5x\cosh^6(a+bx)}{16} + \frac{5\sinh^5(a+bx)\cosh(a+b}{16b} \\ x\cosh^6(a) \end{cases}$$

input
```
integrate(cosh(b*x+a)**6,x)
```

output
```
Piecewise((-5*x*sinh(a + b*x)**6/16 + 15*x*sinh(a + b*x)**4*cosh(a + b*x)*
*2/16 - 15*x*sinh(a + b*x)**2*cosh(a + b*x)**4/16 + 5*x*cosh(a + b*x)**6/1
6 + 5*sinh(a + b*x)**5*cosh(a + b*x)/(16*b) - 5*sinh(a + b*x)**3*cosh(a +
b*x)**3/(6*b) + 11*sinh(a + b*x)*cosh(a + b*x)**5/(16*b), Ne(b, 0)), (x*co
sh(a)**6, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.28

$$\int \cosh^6(a+bx)\,dx = \frac{\left(9\,e^{(-2\,bx-2\,a)} + 45\,e^{(-4\,bx-4\,a)} + 1\right)e^{(6\,bx+6\,a)}}{384\,b} + \frac{5\,(bx+a)}{16\,b} - \frac{45\,e^{(-2\,bx-2\,a)} + 9\,e^{(-4\,bx-4\,a)} + e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^6,x, algorithm="maxima")
```

output
```
1/384*(9*e^(-2*b*x - 2*a) + 45*e^(-4*b*x - 4*a) + 1)*e^(6*b*x + 6*a)/b + 5
/16*(b*x + a)/b - 1/384*(45*e^(-2*b*x - 2*a) + 9*e^(-4*b*x - 4*a) + e^(-6*
b*x - 6*a))/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.31

$$\int \cosh^6(a+bx)\,dx = \frac{5}{16}x + \frac{e^{(6\,bx+6\,a)}}{384\,b} + \frac{3\,e^{(4\,bx+4\,a)}}{128\,b} + \frac{15\,e^{(2\,bx+2\,a)}}{128\,b} - \frac{15\,e^{(-2\,bx-2\,a)}}{128\,b} - \frac{3\,e^{(-4\,bx-4\,a)}}{128\,b} - \frac{e^{(-6\,bx-6\,a)}}{384\,b}$$

input
```
integrate(cosh(b*x+a)^6,x, algorithm="giac")
```

output
```
5/16*x + 1/384*e^(6*b*x + 6*a)/b + 3/128*e^(4*b*x + 4*a)/b + 15/128*e^(2*b
*x + 2*a)/b - 15/128*e^(-2*b*x - 2*a)/b - 3/128*e^(-4*b*x - 4*a)/b - 1/384
*e^(-6*b*x - 6*a)/b
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.63

$$\int \cosh^6(a+bx)\,dx = \frac{5\,x}{16} + \frac{\frac{15\sinh(2\,a+2\,b\,x)}{64} + \frac{3\sinh(4\,a+4\,b\,x)}{64} + \frac{\sinh(6\,a+6\,b\,x)}{192}}{b}$$

input
```
int(cosh(a + b*x)^6,x)
```

output
```
(5*x)/16 + ((15*sinh(2*a + 2*b*x))/64 + (3*sinh(4*a + 4*b*x))/64 + sinh(6*
a + 6*b*x)/192)/b
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.36

$$\int \cosh^6(a+bx)\,dx = \frac{e^{12bx+12a} + 9e^{10bx+10a} + 45e^{8bx+8a} + 120e^{6bx+6a}bx - 45e^{4bx+4a} - 9e^{2bx+2a} - 1}{384e^{6bx+6a}b}$$

input
```
int(cosh(b*x+a)^6,x)
```

output
```
(e**(12*a + 12*b*x) + 9*e**(10*a + 10*b*x) + 45*e**(8*a + 8*b*x) + 120*e**
(6*a + 6*b*x)*b*x - 45*e**(4*a + 4*b*x) - 9*e**(2*a + 2*b*x) - 1)/(384*e**
(6*a + 6*b*x)*b)
```

3.7 $\int \cosh^{\frac{7}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = -\frac{10i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{21b}+\frac{10\sqrt{\cosh(a+bx)}\sinh(a+bx)}{21b}+\frac{2\cosh^{\frac{5}{2}}(a+bx)\sinh(a+bx)}{7b}$$

output
```
-10/21*I*InverseJacobiAM(1/2*I*(b*x+a),2^(1/2))/b+10/21*cosh(b*x+a)^(1/2)*
sinh(b*x+a)/b+2/7*cosh(b*x+a)^(5/2)*sinh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.80

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \frac{-20i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)+\sqrt{\cosh(a+bx)}(23\sinh(a+bx)+3\sinh(3(a+bx)))}{42b}$$

input
```
Integrate[Cosh[a + b*x]^(7/2),x]
```

output
```
((-20*I)*EllipticF[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*(23*Sinh[a +
b*x] + 3*Sinh[3*(a + b*x)]))/(42*b)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5}{7}\int \cosh^{\frac{3}{2}}(a+bx)dx+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3115}$$

$$\frac{5}{7}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}\right)+\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{5}{2}}(a+bx)}{7b}+\frac{5}{7}\left(\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}\right)$$

input
```
Int[Cosh[a + b*x]^(7/2),x]
```

output
```
(2*Cosh[a + b*x]^(5/2)*Sinh[a + b*x])/(7*b) + (5*((((-2*I)/3)*EllipticF[(I
/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh[a + b*x])/(3*b)))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(57) = 114$.

Time = 7.47 (sec) , antiderivative size = 201, normalized size of antiderivative = 2.91

method	result
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\left(48\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^9-120\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^7+128\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^5-72\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^3+5\sqrt{-s}\right.}{21\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cos}}$

input
```
int(cosh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/21*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(48*cosh(1/
2*b*x+1/2*a)^9-120*cosh(1/2*b*x+1/2*a)^7+128*cosh(1/2*b*x+1/2*a)^5-72*cosh
(1/2*b*x+1/2*a)^3+5*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)
^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+16*cosh(1/2*b*x+1/2*a))
/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)
/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 326 vs. $2(56) = 112$.

Time = 0.10 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.72

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx$$
$$= \frac{40\left(\sqrt{2}\cosh(bx+a)^3+3\sqrt{2}\cosh(bx+a)^2\sinh(bx+a)+3\sqrt{2}\cosh(bx+a)\sinh(bx+a)^2+\sqrt{2}\sinh}{}$$

input
```
integrate(cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
1/84*(40*(sqrt(2)*cosh(b*x + a)^3 + 3*sqrt(2)*cosh(b*x + a)^2*sinh(b*x + a
) + 3*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^2 + sqrt(2)*sinh(b*x + a)^3)*wei
erstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a)) + (3*cosh(b*x + a)^
6 + 18*cosh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x +
a)^2 + 23)*sinh(b*x + a)^4 + 23*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 +
23*cosh(b*x + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 + 138*cosh(b*x + a
)^2 - 23)*sinh(b*x + a)^2 - 23*cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 + 46
*cosh(b*x + a)^3 - 23*cosh(b*x + a))*sinh(b*x + a) - 3)*sqrt(cosh(b*x + a)
))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a)^2*sinh(b*x + a) + 3*b*cosh(b*x +
 a)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^{\frac{7}{2}}(a + bx)\, dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^{\frac{7}{2}}(a + bx)\, dx = \int \cosh\left(bx + a\right)^{\frac{7}{2}}\, dx$$

input
```
integrate(cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(7/2), x)
```

Giac [F]

$$\int \cosh^{\frac{7}{2}}(a + bx)\, dx = \int \cosh\left(bx + a\right)^{\frac{7}{2}}\, dx$$

input
```
integrate(cosh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \int \cosh(a+b\,x)^{7/2}\,dx$$

input
```
int(cosh(a + b*x)^(7/2),x)
```

output
```
int(cosh(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \cosh^{\frac{7}{2}}(a+bx)\,dx = \int \sqrt{\cosh(bx+a)}\,\cosh(bx+a)^3\,dx$$

input
```
int(cosh(b*x+a)^(7/2),x)
```

output
```
int(sqrt(cosh(a + b*x))*cosh(a + b*x)**3,x)
```

3.8 $\int \cosh^{\frac{5}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = -\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\cosh^{\frac{3}{2}}(a+bx)\sinh(a+bx)}{5b}$$

output
```
-6/5*I*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b+2/5*cosh(b*x+a)^(3/2)*si
nh(b*x+a)/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \frac{-6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)+\sqrt{\cosh(a+bx)}\sinh(2(a+bx))}{5b}$$

input
```
Integrate[Cosh[a + b*x]^(5/2),x]
```

output
```
((-6*I)*EllipticE[(I/2)*(a + b*x), 2] + Sqrt[Cosh[a + b*x]]*Sinh[2*(a + b*
x)])/(5*b)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{3}{5}\int \sqrt{\cosh(a+bx)}dx+\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}+\frac{3}{5}\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$\frac{2\sinh(a+bx)\cosh^{\frac{3}{2}}(a+bx)}{5b}-\frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}$$

input
```
Int[Cosh[a + b*x]^(5/2),x]
```

output
```
(((-6*I)/5)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Cosh[a + b*x]^(3/2)*Sinh
[a + b*x])/(5*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 187 vs. $2(42) = 84$.

Time = 4.83 (sec) , antiderivative size = 188, normalized size of antiderivative = 4.09

method	result
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(8\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{7}-16\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}+10\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}-3\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cos\cdots}\right.}{5\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}\cdots}}$

input
```
int(cosh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(8*cosh(1/2*
b*x+1/2*a)^7-16*cosh(1/2*b*x+1/2*a)^5+10*cosh(1/2*b*x+1/2*a)^3-3*(-sinh(1/
2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/
2*b*x+1/2*a),2^(1/2))-2*cosh(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh
(1/2*b*x+1/2*a)^2)^(1/2)/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(
1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 203, normalized size of antiderivative = 4.41

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = -\frac{12\left(\sqrt{2}\cosh\left(bx+a\right)^2+2\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)^2\right)\text{weierstrassZeta}(-4,0,}{}$$

input
```
integrate(cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(12*(sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)
 + sqrt(2)*sinh(b*x + a)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cosh(b*x + a) + sinh(b*x + a))) - (cosh(b*x + a)^4 + 4*cosh(b*x + a)*s
inh(b*x + a)^3 + sinh(b*x + a)^4 + 6*(cosh(b*x + a)^2 - 2)*sinh(b*x + a)^2
 - 12*cosh(b*x + a)^2 + 4*(cosh(b*x + a)^3 - 6*cosh(b*x + a))*sinh(b*x + a
) - 1)*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*
x + a) + b*sinh(b*x + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \text{Timed out}$$

input
```
integrate(cosh(b*x+a)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(5/2), x)
```

Giac [F]

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{5}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \int \cosh(a+b\,x)^{5/2}\,dx$$

input
```
int(cosh(a + b*x)^(5/2),x)
```

output
```
int(cosh(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \cosh^{\frac{5}{2}}(a+bx)\,dx = \int \sqrt{\cosh(bx+a)}\,\cosh(bx+a)^2\,dx$$

input
```
int(cosh(b*x+a)^(5/2),x)
```

output
```
int(sqrt(cosh(a + b*x))*cosh(a + b*x)**2,x)
```

3.9 $\int \cosh^{\frac{3}{2}}(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx=-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}+\frac{2\sqrt{\cosh(a+bx)}\sinh(a+bx)}{3b}$$

output
```
-2/3*I*InverseJacobiAM(1/2*I*(b*x+a),2^(1/2))/b+2/3*cosh(b*x+a)^(1/2)*sinh
(b*x+a)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.07 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.76

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx =\frac{\sinh(2(a+bx))+2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{1+\cosh(2(a-}{3b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(3/2),x]
```

output
```
(Sinh[2*(a + b*x)] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(a + b*x)]
 - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a + b*x)]])/(3*
b*Sqrt[Cosh[a + b*x]])
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$\frac{2\sinh(a+bx)\sqrt{\cosh(a+bx)}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

input
```
Int[Cosh[a + b*x]^(3/2),x]
```

output
```
(((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sqrt[Cosh[a + b*x]]*Sinh
[a + b*x])/(3*b)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(38) = 76$.

Time = 2.56 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.78

method	result
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{5}-6\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{3}+\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\ \text{Ellipti}}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\, b}$

input
```
int(cosh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(4*cosh(1/2*
b*x+1/2*a)^5-6*cosh(1/2*b*x+1/2*a)^3+(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*co
sh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))+2*cosh
(1/2*b*x+1/2*a))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/sin
h(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.22

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx$$
$$= \frac{2\left(\sqrt{2}\cosh\left(bx+a\right)+\sqrt{2}\sinh\left(bx+a\right)\right)\text{weierstrassPInverse}(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))+(\text{co}}{3\left(b\cosh\left(bx+a\right)+b\sinh\left(b\text{a}\right.\right.}$$

input
```
integrate(cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
1/3*(2*(sqrt(2)*cosh(b*x + a) + sqrt(2)*sinh(b*x + a))*weierstrassPInverse
(-4, 0, cosh(b*x + a) + sinh(b*x + a)) + (cosh(b*x + a)^2 + 2*cosh(b*x + a
)*sinh(b*x + a) + sinh(b*x + a)^2 - 1)*sqrt(cosh(b*x + a)))/(b*cosh(b*x +
a) + b*sinh(b*x + a))
```

Sympy [F]

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh^{\frac{3}{2}}(a+bx)\,dx$$

input
```
integrate(cosh(b*x+a)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**(3/2), x)
```

Maxima [F]

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(3/2), x)
```

Giac [F]

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh\left(bx+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cosh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \cosh(a+b\,x)^{3/2}\,dx$$

input
```
int(cosh(a + b*x)^(3/2),x)
```

output
```
int(cosh(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \cosh^{\frac{3}{2}}(a+bx)\,dx = \int \sqrt{\cosh(bx+a)}\,\cosh(bx+a)\,dx$$

input
```
int(cosh(b*x+a)^(3/2),x)
```

output
```
int(sqrt(cosh(a + b*x))*cosh(a + b*x),x)
```

3.10 $\int \sqrt{\cosh(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \sqrt{\cosh(a+bx)}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

output
```
-2*I*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \sqrt{\cosh(a+bx)}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Integrate[Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cosh(a+bx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3119}$$

$$-\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Int[Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*x), 2])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(22) = 44$.

Time = 1.69 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.75

method	result
default	$-\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1}\,\mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}\,b}$
risch	$\frac{\sqrt{2}\sqrt{\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{-bx-a}}}{b}+\frac{\left(-\frac{2\left(1+\mathrm{e}^{2bx+2a}\right)}{\sqrt{\left(1+\mathrm{e}^{2bx+2a}\right)\mathrm{e}^{bx+a}}}+\frac{i\sqrt{-i\left(\mathrm{e}^{bx+a}+i\right)}\sqrt{2}\sqrt{i\left(\mathrm{e}^{bx+a}-i\right)}\sqrt{i\mathrm{e}^{bx+a}}\left(-2i\,\mathrm{EllipticE}\left(\sqrt{-i\left(\mathrm{e}^{bx+a}+i\right)},\right.\right.}{\sqrt{\mathrm{e}^{3bx+3a}+\mathrm{e}^{bx+a}}}}{b\left(1+\mathrm{e}^{2bx+2a}\right.}$

input
```
int(cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*
x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)*EllipticE(cosh(1/2*b*
x+1/2*a),2^(1/2))/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/si
nh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int\sqrt{\cosh(a+bx)}\,dx = -\frac{2\left(\sqrt{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right)))+\sqrt{\cosh\left(bx\right.}\right.}{b}$$

input
```
integrate(cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*x + a
) + sinh(b*x + a))) + sqrt(cosh(b*x + a)))/b
```

Sympy [F]

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh{(a+bx)}}\,dx$$

input
```
integrate(cosh(b*x+a)**(1/2),x)
```

output
```
Integral(sqrt(cosh(a + b*x)), x)
```

Maxima [F]

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh{(bx+a)}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(b*x + a)), x)
```

Giac [F]

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh{(bx+a)}}\,dx$$

input
```
integrate(cosh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh\left(a+b\,x\right)}\,dx$$

input
```
int(cosh(a + b*x)^(1/2),x)
```

output
```
int(cosh(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cosh(a+bx)}\,dx = \int \sqrt{\cosh\left(bx+a\right)}dx$$

input
```
int(cosh(b*x+a)^(1/2),x)
```

output
```
int(sqrt(cosh(a + b*x)),x)
```

3.11 $\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

output
```
-2*I*InverseJacobiAM(1/2*I*(b*x+a),2^(1/2))/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

input
```
Integrate[1/Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{b}$$

input
```
Int[1/Sqrt[Cosh[a + b*x]],x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*x), 2])/b
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. 2(18) = 36.

Time = 0.56 (sec) , antiderivative size = 135, normalized size of antiderivative = 6.75

method	result	size
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+1}\,\mathrm{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{4}+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$	135

input
```
int(1/cosh(b*x+a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-sinh(1/2*b*x
+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)
^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)*EllipticF(cosh(1/2*b*x+1/2*a),2^(1/2))/sin
h(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.20

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \frac{2\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bx+a\right)+\sinh\left(bx+a\right))}{b}$$

input
```
integrate(1/cosh(b*x+a)^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*x + a) + sinh(b*x + a))/b
```

Sympy [F]

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh{(a+bx)}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)**(1/2),x)
```

output
```
Integral(1/sqrt(cosh(a + b*x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh{(bx+a)}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(cosh(b*x + a)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh{(bx+a)}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(cosh(b*x + a)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{1}{\sqrt{\cosh\left(a+b\,x\right)}}\,dx$$

input
```
int(1/cosh(a + b*x)^(1/2),x)
```

output
```
int(1/cosh(a + b*x)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{\cosh(a+bx)}}\,dx = \int \frac{\sqrt{\cosh\left(bx+a\right)}}{\cosh\left(bx+a\right)}dx$$

input
```
int(1/cosh(b*x+a)^(1/2),x)
```

output
```
int(sqrt(cosh(a + b*x))/cosh(a + b*x),x)
```

3.12 $\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b} + \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

output
```
2*I*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b+2*sinh(b*x+a)/b/cosh(b*x+a)
^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b} + \frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}$$

input
```
Integrate[Cosh[a + b*x]^(-3/2),x]
```

output
```
((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
 + b*x]])
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\cosh(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}$$

input
```
Int[Cosh[a + b*x]^(-3/2),x]
```

output
```
((2*I)*EllipticE[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a
 + b*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(42) = 84$.

Time = 0.81 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.45

method	result	size
default	$\frac{4\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}+2\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\,\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\;\mathrm{EllipticE}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\,b}$	103

input
```
int(1/cosh(b*x+a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2*(2*cosh(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+(-sinh(1/2*b*x+1/2*a)^2)^(1
/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/
2)))/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2*a)^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(40) = 80$.

Time = 0.10 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.52

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \frac{2\left(\left(\sqrt{2}\cosh(bx+a)^2 + 2\sqrt{2}\cosh(bx+a)\sinh(bx+a) + \sqrt{2}\sinh(bx+a)^2 + \sqrt{2}\right)\mathrm{weierstrassZeta}(-4\right.}{b\cosh(bx+a}$$

input
```
integrate(1/cosh(b*x+a)^(3/2),x, algorithm="fricas")
```

output
```
2*((sqrt(2)*cosh(b*x + a)^2 + 2*sqrt(2)*cosh(b*x + a)*sinh(b*x + a) + sqrt
(2)*sinh(b*x + a)^2 + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(cosh(b*x + a)^2 + 2*cosh(b*x +
 a)*sinh(b*x + a) + sinh(b*x + a)^2)*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)
^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b)
```

Sympy [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx$$

input
```
integrate(1/cosh(b*x+a)**(3/2),x)
```

output
```
Integral(cosh(a + b*x)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(3/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(a+b\,x)^{3/2}}\,dx$$

input
```
int(1/cosh(a + b*x)^(3/2),x)
```

output
```
int(1/cosh(a + b*x)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\cosh(bx+a)}}{\cosh(bx+a)^{2}}dx$$

input
```
int(1/cosh(b*x+a)^(3/2),x)
```

output
```
int(sqrt(cosh(a + b*x))/cosh(a + b*x)**2,x)
```

3.13 $\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = -\frac{2i\operatorname{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}+\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

output
```
-2/3*I*InverseJacobiAM(1/2*I*(b*x+a),2^(1/2))/b+2/3*sinh(b*x+a)/b/cosh(b*x
+a)^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\Big(\sinh(a+bx)+\cosh(a+bx)\operatorname{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+bx))-\sinh(2(a+bx))\right)\sqrt{1+}}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(-5/2),x]
```

output
```
(2*(Sinh[a + b*x] + Cosh[a + b*x]*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2
*(a + b*x)] - Sinh[2*(a + b*x)]]*Sqrt[1 + Cosh[2*(a + b*x)] + Sinh[2*(a +
b*x)]]))/(3*b*Cosh[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+bx)}}dx+\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3120}$$

$$\frac{2\sinh(a+bx)}{3b\cosh^{\frac{3}{2}}(a+bx)}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+bx),2\right)}{3b}$$

input
```
Int[Cosh[a + b*x]^(-5/2),x]
```

output
```
(((-2*I)/3)*EllipticF[(I/2)*(a + b*x), 2])/b + (2*Sinh[a + b*x])/(3*b*Cosh
[a + b*x]^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(38) = 76$.

Time = 1.13 (sec) , antiderivative size = 190, normalized size of antiderivative = 4.13

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\left(\frac{\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}}{3\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-\frac{1}{2}\right)^2}+\frac{2\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}\sqrt{-2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+1}\,\operatorname{EllipticF}\left(\cosh\left(\frac{bx}{2}+\frac{a}{2}\right),\sqrt{2}\right)}{3\sqrt{2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^4+\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^2}}\right)}{\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)\sqrt{2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^2-1}\,b}$

input
```
int(1/cosh(b*x+a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)*(1/3*cosh(1/2*b*
x+1/2*a)*(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/(cosh(1/2*b
*x+1/2*a)^2-1/2)^2+2/3*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*cosh(1/2*b*x+1/2
*a)^2+1)^(1/2)/(2*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)*Ellip
ticF(cosh(1/2*b*x+1/2*a),2^(1/2)))/sinh(1/2*b*x+1/2*a)/(2*cosh(1/2*b*x+1/2
*a)^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 310, normalized size of antiderivative = 6.74

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \frac{2\left(\left(\sqrt{2}\cosh\left(bx+a\right)^4+4\sqrt{2}\cosh\left(bx+a\right)\sinh\left(bx+a\right)^3+\sqrt{2}\sinh\left(bx+a\right)^4+2\left(3\sqrt{2}\cosh\left(bx+a\right)^2\right.\right.\right.}{}$$

input
```
integrate(1/cosh(b*x+a)^(5/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*cosh(b*x + a)^4 + 4*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^3 +
sqrt(2)*sinh(b*x + a)^4 + 2*(3*sqrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x
 + a)^2 + 2*sqrt(2)*cosh(b*x + a)^2 + 4*(sqrt(2)*cosh(b*x + a)^3 + sqrt(2)
*cosh(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassPInverse(-4, 0, cosh(b
*x + a) + sinh(b*x + a)) + 2*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x +
 a)^2 + sinh(b*x + a)^3 + (3*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - cosh(b*x
 + a))*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^4 + 4*b*cosh(b*x + a)*sinh(b*
x + a)^3 + b*sinh(b*x + a)^4 + 2*b*cosh(b*x + a)^2 + 2*(3*b*cosh(b*x + a)^
2 + b)*sinh(b*x + a)^2 + 4*(b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x
+ a) + b)
```

Sympy [F]

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx$$

input
```
integrate(1/cosh(b*x+a)**(5/2),x)
```

output
```
Integral(cosh(a + b*x)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(5/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(a+b\,x)^{5/2}}\,dx$$

input
```
int(1/cosh(a + b*x)^(5/2),x)
```

output
```
int(1/cosh(a + b*x)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\cosh^{\frac{5}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\cosh(bx+a)}}{\cosh(bx+a)^3}dx$$

input

```
int(1/cosh(b*x+a)^(5/2),x)
```

output

```
int(sqrt(cosh(a + b*x))/cosh(a + b*x)**3,x)
```

3.14 $\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 69

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{6iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{5b}+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{6\sinh(a+bx)}{5b\sqrt{\cosh(a+bx)}}$$

output
```
6/5*I*EllipticE(I*sinh(1/2*a+1/2*b*x),2^(1/2))/b+2/5*sinh(b*x+a)/b/cosh(b*
x+a)^(5/2)+6/5*sinh(b*x+a)/b/cosh(b*x+a)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.91

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \frac{6i\cosh^{\frac{3}{2}}(a+bx)E\left(\frac{1}{2}i(a+bx)\middle|2\right)+3\sinh(2(a+bx))+2\tanh(a+bx)}{5b\cosh^{\frac{3}{2}}(a+bx)}$$

input
```
Integrate[Cosh[a + b*x]^(-7/2),x]
```

output
```
((6*I)*Cosh[a + b*x]^(3/2)*EllipticE[(I/2)*(a + b*x), 2] + 3*Sinh[2*(a + b
*x)] + 2*Tanh[a + b*x])/(5*b*Cosh[a + b*x]^(3/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.01, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{7/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{3}{5}\int \frac{1}{\cosh^{\frac{3}{2}}(a+bx)}dx+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\int \frac{1}{\sin\left(ia+ibx+\frac{\pi}{2}\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\cosh(a+bx)}dx\right)+\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}-\int \sqrt{\sin\left(ia+ibx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{ 3119}$$

$$\frac{2\sinh(a+bx)}{5b\cosh^{\frac{5}{2}}(a+bx)}+\frac{3}{5}\left(\frac{2\sinh(a+bx)}{b\sqrt{\cosh(a+bx)}}+\frac{2iE\left(\frac{1}{2}i(a+bx)\middle|2\right)}{b}\right)$$

input
```
Int[Cosh[a + b*x]^(-7/2),x]
```

output
```
(2*Sinh[a + b*x])/(5*b*Cosh[a + b*x]^(5/2)) + (3*(((2*I)*EllipticE[(I/2)*(
a + b*x), 2])/b + (2*Sinh[a + b*x])/(b*Sqrt[Cosh[a + b*x]])))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(61) = 122$.

Time = 1.46 (sec) , antiderivative size = 363, normalized size of antiderivative = 5.26

method	result
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1\right)\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\left(24\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{6}\cosh\left(\frac{bx}{2}+\frac{a}{2}\right)+12\sqrt{-\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}}\sqrt{-2\sinh\left(\frac{bx}{2}+\frac{a}{2}\right)^{2}-1}\ \text{Elliptic}}{}$

input
```
int(1/cosh(b*x+a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2/5*((2*cosh(1/2*b*x+1/2*a)^2-1)*sinh(1/2*b*x+1/2*a)^2)^(1/2)/(8*sinh(1/2*
b*x+1/2*a)^6+12*sinh(1/2*b*x+1/2*a)^4+6*sinh(1/2*b*x+1/2*a)^2+1)/sinh(1/2*
b*x+1/2*a)^3*(24*sinh(1/2*b*x+1/2*a)^6*cosh(1/2*b*x+1/2*a)+12*(-sinh(1/2*b
*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b
*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^4+24*sinh(1/2*b*x+1/2*a)^4*cosh(1/2
*b*x+1/2*a)+12*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2*sinh(1/2*b*x+1/2*a)^2-1)
^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2))*sinh(1/2*b*x+1/2*a)^2+8*cosh
(1/2*b*x+1/2*a)*sinh(1/2*b*x+1/2*a)^2+3*(-sinh(1/2*b*x+1/2*a)^2)^(1/2)*(-2
*sinh(1/2*b*x+1/2*a)^2-1)^(1/2)*EllipticE(cosh(1/2*b*x+1/2*a),2^(1/2)))*(2
*sinh(1/2*b*x+1/2*a)^4+sinh(1/2*b*x+1/2*a)^2)^(1/2)/(2*cosh(1/2*b*x+1/2*a)
^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 613 vs. $2(59) = 118$.

Time = 0.11 (sec) , antiderivative size = 613, normalized size of antiderivative = 8.88

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Too large to display}$$

input
```
integrate(1/cosh(b*x+a)^(7/2),x, algorithm="fricas")
```

output
```
2/5*(3*(sqrt(2)*cosh(b*x + a)^6 + 6*sqrt(2)*cosh(b*x + a)*sinh(b*x + a)^5
+ sqrt(2)*sinh(b*x + a)^6 + 3*(5*sqrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b
*x + a)^4 + 3*sqrt(2)*cosh(b*x + a)^4 + 4*(5*sqrt(2)*cosh(b*x + a)^3 + 3*s
qrt(2)*cosh(b*x + a))*sinh(b*x + a)^3 + 3*(5*sqrt(2)*cosh(b*x + a)^4 + 6*s
qrt(2)*cosh(b*x + a)^2 + sqrt(2))*sinh(b*x + a)^2 + 3*sqrt(2)*cosh(b*x + a
)^2 + 6*(sqrt(2)*cosh(b*x + a)^5 + 2*sqrt(2)*cosh(b*x + a)^3 + sqrt(2)*cos
h(b*x + a))*sinh(b*x + a) + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cosh(b*x + a) + sinh(b*x + a))) + 2*(3*cosh(b*x + a)^6 + 18*c
osh(b*x + a)*sinh(b*x + a)^5 + 3*sinh(b*x + a)^6 + (45*cosh(b*x + a)^2 + 8
)*sinh(b*x + a)^4 + 8*cosh(b*x + a)^4 + 4*(15*cosh(b*x + a)^3 + 8*cosh(b*x
 + a))*sinh(b*x + a)^3 + (45*cosh(b*x + a)^4 + 48*cosh(b*x + a)^2 + 1)*sin
h(b*x + a)^2 + cosh(b*x + a)^2 + 2*(9*cosh(b*x + a)^5 + 16*cosh(b*x + a)^3
 + cosh(b*x + a))*sinh(b*x + a))*sqrt(cosh(b*x + a)))/(b*cosh(b*x + a)^6 +
 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*x + a)^6 + 3*b*cosh(b*x + a)
^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^4 + 4*(5*b*cosh(b*x + a)^3
+ 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*cosh(b*x + a)^2 + 3*(5*b*cosh(b
*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*x + a)^2 + 6*(b*cosh(b*x + a)^
5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*sinh(b*x + a) + b)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \text{Timed out}$$

input
```
integrate(1/cosh(b*x+a)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(bx+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cosh(b*x+a)^(7/2),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{1}{\cosh(a+b\,x)^{7/2}}\,dx$$

input
```
int(1/cosh(a + b*x)^(7/2),x)
```

output
```
int(1/cosh(a + b*x)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}(a+bx)}\,dx = \int \frac{\sqrt{\cosh(bx+a)}}{\cosh(bx+a)^4}\,dx$$

input
```
int(1/cosh(b*x+a)^(7/2),x)
```

output
```
int(sqrt(cosh(a + b*x))/cosh(a + b*x)**4,x)
```

3.15 $\int (a\cosh(x))^{7/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 65

$$\int (a\cosh(x))^{7/2}\,dx = -\frac{10ia^4\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{21\sqrt{a\cosh(x)}} + \frac{10}{21}a^3\sqrt{a\cosh(x)}\sinh(x) + \frac{2}{7}a(a\cosh(x))^{5/2}\sinh(x)$$

output
```
-10/21*I*a^4*cosh(x)^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2))/(a*cosh(x))^(1
/2)+10/21*a^3*(a*cosh(x))^(1/2)*sinh(x)+2/7*a*(a*cosh(x))^(5/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.82

$$\int (a\cosh(x))^{7/2}\,dx = \frac{a^3\sqrt{a\cosh(x)}\left(-20i\,\text{EllipticF}\left(\frac{ix}{2},2\right)+\sqrt{\cosh(x)}(23\sinh(x)+3\sinh(3x))\right)}{42\sqrt{\cosh(x)}}$$

input
```
Integrate[(a*Cosh[x])^(7/2),x]
```

output
```
(a^3*Sqrt[a*Cosh[x]]*((-20*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(23*Si
nh[x] + 3*Sinh[3*x])))/(42*Sqrt[Cosh[x]])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3115, 3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x))^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{7/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}a^2\int (a\cosh(x))^{3/2}dx+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\int \left(a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{5}{7}a^2\left(\frac{1}{3}a^2\int \frac{1}{\sqrt{a\cosh(x)}}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}\right)+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{1}{3}a^2\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3121}$$

$$\frac{5}{7}a^2\left(\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{3\sqrt{a\cosh(x)}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}\right)+\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\cosh(x)}}\right)$$

$$\Big\downarrow \text{3120}$$

$$\frac{2}{7}a\sinh(x)(a\cosh(x))^{5/2}+\frac{5}{7}a^2\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\sqrt{a\cosh(x)}}\right)$$

input
```
Int[(a*Cosh[x])^(7/2),x]
```

output
```
(2*a*(a*Cosh[x])^(5/2)*Sinh[x])/7 + (5*a^2*((((-2*I)/3)*a^2*Sqrt[Cosh[x]]*
EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*a*Sqrt[a*Cosh[x]]*Sinh[x])/3))
/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(49) = 98$.

Time = 5.85 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.23

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a\sinh\left(\frac{x}{2}\right)^{2}}\,a^{4}\left(96\cosh\left(\frac{x}{2}\right)^{9}-240\cosh\left(\frac{x}{2}\right)^{7}+256\cosh\left(\frac{x}{2}\right)^{5}+5\sqrt{2}\,\sqrt{-2\cosh\left(\frac{x}{2}\right)^{2}+1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\ \operatorname{EllipticF}\left(\sqrt{\cdots}\right.\right.}{21\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\ \sinh\left(\frac{x}{2}\right)\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a}}$

input
```
int((cosh(x)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/21*((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1/2)*a^4*(96*cosh(1/2*x)^9-240
*cosh(1/2*x)^7+256*cosh(1/2*x)^5+5*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(1/2)*(-si
nh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-144*cosh(1/2
*x)^3+32*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x
)/((2*cosh(1/2*x)^2-1)*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 247, normalized size of antiderivative = 3.80

$$\int (a\cosh(x))^{7/2}\,dx = \frac{80\sqrt{\frac{1}{2}}\left(a^{3}\cosh\left(x\right)^{3}+3\,a^{3}\cosh\left(x\right)^{2}\sinh\left(x\right)+3\,a^{3}\cosh\left(x\right)\sinh\left(x\right)^{2}+a^{3}\sinh\left(x\right)^{3}\right)\sqrt{\cdots}}{\cdots}$$

input
```
integrate((a*cosh(x))^(7/2),x, algorithm="fricas")
```

output
```
1/84*(80*sqrt(1/2)*(a^3*cosh(x)^3 + 3*a^3*cosh(x)^2*sinh(x) + 3*a^3*cosh(x
)*sinh(x)^2 + a^3*sinh(x)^3)*sqrt(a)*weierstrassPInverse(-4, 0, cosh(x) +
sinh(x)) + (3*a^3*cosh(x)^6 + 18*a^3*cosh(x)*sinh(x)^5 + 3*a^3*sinh(x)^6 +
 23*a^3*cosh(x)^4 - 23*a^3*cosh(x)^2 + (45*a^3*cosh(x)^2 + 23*a^3)*sinh(x)
^4 + 4*(15*a^3*cosh(x)^3 + 23*a^3*cosh(x))*sinh(x)^3 - 3*a^3 + (45*a^3*cos
h(x)^4 + 138*a^3*cosh(x)^2 - 23*a^3)*sinh(x)^2 + 2*(9*a^3*cosh(x)^5 + 46*a
^3*cosh(x)^3 - 23*a^3*cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(cosh(x)^3 + 3*co
sh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a\cosh(x))^{7/2}\, dx = \text{Timed out}$$

input
```
integrate((a*cosh(x))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a\cosh(x))^{7/2}\, dx = \int (a\cosh(x))^{\frac{7}{2}}\, dx$$

input
```
integrate((a*cosh(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(7/2), x)
```

Giac [F]

$$\int (a\cosh(x))^{7/2}\,dx = \int (a\cosh(x))^{\frac{7}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x))^{7/2}\,dx = \int (a\cosh(x))^{7/2}\,dx$$

input
```
int((a*cosh(x))^(7/2),x)
```

output
```
int((a*cosh(x))^(7/2), x)
```

Reduce [F]

$$\int (a\cosh(x))^{7/2}\,dx = \sqrt{a}\left(\int \sqrt{\cosh(x)}\,\cosh(x)^3\,dx\right)a^3$$

input
```
int((a*cosh(x))^(7/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x))*cosh(x)**3,x)*a**3
```

3.16 $\int (a\cosh(x))^{5/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 48

$$\int (a\cosh(x))^{5/2}\,dx = -\frac{6ia^2\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{5\sqrt{\cosh(x)}} + \frac{2}{5}a(a\cosh(x))^{3/2}\sinh(x)$$

output
```
-6/5*I*a^2*(a*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2))/cosh(x)^(1/2
)+2/5*a*(a*cosh(x))^(3/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int (a\cosh(x))^{5/2}\,dx = \frac{2(a\cosh(x))^{5/2}\left(-3iE\left(\frac{ix}{2}\middle|2\right) + \cosh^{\frac{3}{2}}(x)\sinh(x)\right)}{5\cosh^{\frac{5}{2}}(x)}$$

input
```
Integrate[(a*Cosh[x])^(5/2),x]
```

output
```
(2*(a*Cosh[x])^(5/2)*((-3*I)*EllipticE[(I/2)*x, 2] + Cosh[x]^(3/2)*Sinh[x]
))/(5*Cosh[x]^(5/2))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x))^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}dx$$

$$\Big\downarrow \ 3115$$

$$\frac{3}{5}a^2\int \sqrt{a\cosh(x)}dx+\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}+\frac{3}{5}a^2\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3121$$

$$\frac{3a^2\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}dx}{5\sqrt{\cosh(x)}}+\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}$$

$$\Big\downarrow \ 3042$$

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}+\frac{3a^2\sqrt{a\cosh(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{5\sqrt{\cosh(x)}}$$

$$\Big\downarrow \ 3119$$

$$\frac{2}{5}a\sinh(x)(a\cosh(x))^{3/2}-\frac{6ia^2E\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{5\sqrt{\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(5/2),x]
```

output
```
(((-6*I)/5)*a^2*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]] + (2*
a*(a*Cosh[x])^(3/2)*Sinh[x])/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(39) = 78$.

Time = 3.99 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.83

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^2-1\right)a\sinh\left(\frac{x}{2}\right)^2}\,a^3\left(16\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)^6+16\sinh\left(\frac{x}{2}\right)^4\cosh\left(\frac{x}{2}\right)+3\sqrt{2}\sqrt{-2\sinh\left(\frac{x}{2}\right)^2-1}\sqrt{-\sinh\left(\frac{x}{2}\right)^2}\,\mathrm{EllipticF}\left(\sqrt{\right.\right.}}{5\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^4+\sinh\left(\frac{x}{2}\right)^2\right)}\sinh\left(\frac{x}{2}\right)\sqrt{}}$

input
```
int((cosh(x)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/5*((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1/2)*a^3*(16*cosh(1/2*x)*sinh(1
/2*x)^6+16*sinh(1/2*x)^4*cosh(1/2*x)+3*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*
(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-6*2^(1/2
)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(1/2)*cosh
(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*x)^4+sinh
(1/2*x)^2))^(1/2)/sinh(1/2*x)/((2*cosh(1/2*x)^2-1)*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(37) = 74$.

Time = 0.10 (sec) , antiderivative size = 155, normalized size of antiderivative = 3.23

$$\int (a\cosh(x))^{5/2}\,dx =$$
$$-\frac{24\sqrt{\frac{1}{2}}\left(a^2\cosh(x)^2+2a^2\cosh(x)\sinh(x)+a^2\sinh(x)^2\right)\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}}{}$$

input
```
integrate((a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/10*(24*sqrt(1/2)*(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh
(x))) - (a^2*cosh(x)^4 + 4*a^2*cosh(x)*sinh(x)^3 + a^2*sinh(x)^4 - 12*a^2*
cosh(x)^2 + 6*(a^2*cosh(x)^2 - 2*a^2)*sinh(x)^2 - a^2 + 4*(a^2*cosh(x)^3 -
 6*a^2*cosh(x))*sinh(x))*sqrt(a*cosh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) +
 sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a\cosh(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a\cosh(x))^{5/2}\,dx = \int (a\cosh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(5/2), x)
```

Giac [F]

$$\int (a\cosh(x))^{5/2}\,dx = \int (a\cosh(x))^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x))^{5/2}\,dx = \int (a\cosh(x))^{5/2}\,dx$$

input
```
int((a*cosh(x))^(5/2),x)
```

output
```
int((a*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int (a\cosh(x))^{5/2}\,dx = \sqrt{a}\left(\int \sqrt{\cosh(x)}\,\cosh(x)^2\,dx\right)a^2$$

input
```
int((a*cosh(x))^(5/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x))*cosh(x)**2,x)*a**2
```

3.17 $\int (a\cosh(x))^{3/2}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 48

$$\int (a\cosh(x))^{3/2}\,dx = -\frac{2ia^2\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\sqrt{a\cosh(x)}}+\frac{2}{3}a\sqrt{a\cosh(x)}\sinh(x)$$

output
```
-2/3*I*a^2*cosh(x)^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2))/(a*cosh(x))^(1/2
)+2/3*a*(a*cosh(x))^(1/2)*sinh(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.19

$$\int (a\cosh(x))^{3/2}\,dx = \frac{2}{3}(a\cosh(x))^{3/2}\left(\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2x)-\sinh(2x)\right)\text{sech}^2(x)\sqrt{1}\right.$$

input
```
Integrate[(a*Cosh[x])^(3/2),x]
```

output
```
(2*(a*Cosh[x])^(3/2)*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2
*x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3115, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3115}$$

$$\frac{1}{3}a^2\int \frac{1}{\sqrt{a\cosh(x)}}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{1}{3}a^2\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{3\sqrt{a\cosh(x)}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}+\frac{a^2\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3\sqrt{a\cosh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)}-\frac{2ia^2\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3\sqrt{a\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(3/2),x]
```

output
```
(((-2*I)/3)*a^2*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]] + (2*
a*Sqrt[a*Cosh[x]]*Sinh[x])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(36) = 72$.

Time = 2.50 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.71

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a\sinh\left(\frac{x}{2}\right)^{2}}\,a^{2}\left(8\sinh\left(\frac{x}{2}\right)^{4}\cosh\left(\frac{x}{2}\right)+\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\sqrt{2}\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)+4\sinh(\right.}{3\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a}}$

input
```
int((cosh(x)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1/2)*a^2*(8*sinh(1/2*x)^4*cosh(
1/2*x)+2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF
(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(
1/2*x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/((2*cosh(1/2*x)^2-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.35

$$\int (a\cosh(x))^{3/2}\,dx = \frac{4\sqrt{\frac{1}{2}}(a\cosh{(x)} + a\sinh{(x)})\sqrt{a}\text{weierstrassPInverse}(-4, 0, \cosh{(x)} + \sinh{(x)}) + (a}{3\left(\cosh{(x)} + \sinh{(x)}\right)}$$

input
```
integrate((a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/3*(4*sqrt(1/2)*(a*cosh(x) + a*sinh(x))*sqrt(a)*weierstrassPInverse(-4, 0
, cosh(x) + sinh(x)) + (a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 -
a)*sqrt(a*cosh(x)))/(cosh(x) + sinh(x))
```

Sympy [F]

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh{(x)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x))**(3/2),x)
```

output
```
Integral((a*cosh(x))**(3/2), x)
```

Maxima [F]

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(3/2), x)
```

Giac [F]

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a\cosh(x))^{3/2}\,dx = \int (a\cosh(x))^{3/2}\,dx$$

input
```
int((a*cosh(x))^(3/2),x)
```

output
```
int((a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int (a\cosh(x))^{3/2}\,dx = \sqrt{a}\left(\int \sqrt{\cosh(x)}\,\cosh(x)\,dx\right)a$$

input
```
int((a*cosh(x))^(3/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x))*cosh(x),x)*a
```

3.18 $\int \sqrt{a\cosh(x)}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \sqrt{a\cosh(x)}\,dx = -\frac{2i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{\cosh(x)}}$$

output
```
-2*I*(a*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2))/cosh(x)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sqrt{a\cosh(x)}\,dx = -\frac{2i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{\cosh(x)}}$$

input
```
Integrate[Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 3121$$

$$\frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}dx}{\sqrt{\cosh(x)}}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sqrt{a\cosh(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{\sqrt{\cosh(x)}}$$

$$\Big\downarrow \; 3119$$

$$-\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{\sqrt{\cosh(x)}}$$

input
```
Int[Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/Sqrt[Cosh[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(24) = 48$.

Time = 1.95 (sec) , antiderivative size = 118, normalized size of antiderivative = 4.37

method	result
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a\sinh\left(\frac{x}{2}\right)^{2}}\,a\sqrt{2}\,\sqrt{-2\cosh\left(\frac{x}{2}\right)^{2}+1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\left(\operatorname{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)-2\operatorname{EllipticE}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)\right.}{\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a}}$
risch	$\sqrt{2}\,\sqrt{\left(e^{2x}+1\right)a\,e^{-x}}+\frac{\left(-\frac{4\left(e^{2x}a+a\right)}{a\sqrt{e^{x}\left(e^{2x}a+a\right)}}+\frac{2i\sqrt{-i\left(e^{x}+i\right)}\,\sqrt{2}\,\sqrt{i\left(e^{x}-i\right)}\,\sqrt{i e^{x}}\left(-2i\operatorname{EllipticE}\left(\sqrt{-i\left(e^{x}+i\right)},\frac{\sqrt{2}}{2}\right)+i\operatorname{EllipticF}\left(\sqrt{-i(\right.\right.}{\sqrt{a\,e^{3x}+a\,e^{x}}}\right.}{2\,e^{2x}+2}$

input
```
int((cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1/2)*a*2^(1/2)*(-2*cosh(1/2*x)^2+1)
^(1/2)*(-sinh(1/2*x)^2)^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-
2*EllipticE(2^(1/2)*cosh(1/2*x),1/2*2^(1/2)))/(a*(2*sinh(1/2*x)^4+sinh(1/2
*x)^2))^(1/2)/sinh(1/2*x)/((2*cosh(1/2*x)^2-1)*a)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.04

$$\int \sqrt{a\cosh(x)}\,dx = -4\sqrt{\frac{1}{2}}\sqrt{a}\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cosh{(x)} + \sinh{(x)})) - 2\sqrt{a\cosh{(x)}}$$

input
```
integrate((a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
-4*sqrt(1/2)*sqrt(a)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
h(x) + sinh(x))) - 2*sqrt(a*cosh(x))
```

Sympy [F]

$$\int \sqrt{a\cosh(x)}\,dx = \int \sqrt{a\cosh{(x)}}\,dx$$

input
```
integrate((a*cosh(x))**(1/2),x)
```

output
```
Integral(sqrt(a*cosh(x)), x)
```

Maxima [F]

$$\int \sqrt{a\cosh(x)}\,dx = \int \sqrt{a\cosh{(x)}}\,dx$$

input
```
integrate((a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x)), x)
```

Giac [F]

$$\int \sqrt{a \cosh(x)}\, dx = \int \sqrt{a \cosh(x)}\, dx$$

input
```
integrate((a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a \cosh(x)}\, dx = \int \sqrt{a \cosh(x)}\, dx$$

input
```
int((a*cosh(x))^(1/2),x)
```

output
```
int((a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a \cosh(x)}\, dx = \sqrt{a} \left(\int \sqrt{\cosh(x)}dx\right)$$

input
```
int((a*cosh(x))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x)),x)
```

3.19 $\int \frac{1}{\sqrt{a\cosh(x)}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 27

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = -\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{\sqrt{a\cosh(x)}}$$

output
```
-2*I*cosh(x)^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2))/(a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = -\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{\sqrt{a\cosh(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3121}$$

$$\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{\sqrt{a\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{\sqrt{a\cosh(x)}}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{\sqrt{a\cosh(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/Sqrt[a*Cosh[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(21) = 42$.

Time = 0.76 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.70

method	result	size
default	$\frac{\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{2}\,\sqrt{-2\cosh\left(\frac{x}{2}\right)^{2}+1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\sqrt{2}\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)}{\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\sinh\left(\frac{x}{2}\right)\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a}}$	100

input
```
int(1/(cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1/2)*2^(1/2)*(-2*cosh(1/2*x)^2+1)^(
1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)*Elli
pticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))/sinh(1/2*x)/((2*cosh(1/2*x)^2-1)*a)
^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.59

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \frac{4\sqrt{\frac{1}{2}}\text{weierstrassPInverse}(-4,0,\cosh(x)+\sinh(x))}{\sqrt{a}}$$

input
```
integrate(1/(a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
4*sqrt(1/2)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x))/sqrt(a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cosh(x)), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a \cosh(x)}}\,dx = \int \frac{1}{\sqrt{a \cosh(x)}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cosh(x)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a \cosh(x)}}\,dx = \int \frac{1}{\sqrt{a \cosh(x)}}\,dx$$

input
```
int(1/(a*cosh(x))^(1/2),x)
```

output
```
int(1/(a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a \cosh(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)}}{\cosh(x)}dx\right)}{a}$$

input
```
int(1/(a*cosh(x))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x))/cosh(x),x))/a
```

3.20 $\int \frac{1}{(a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 46

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \frac{2i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{a^2\sqrt{\cosh(x)}} + \frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}$$

output
```
2*I*(a*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2))/a^2/cosh(x)^(1/2)+2
*sinh(x)/a/(a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.74

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \frac{2\cosh(x)\left(i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right) + \sinh(x)\right)}{(a\cosh(x))^{3/2}}$$

input
```
Integrate[(a*Cosh[x])^(-3/2),x]
```

output
```
(2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/(a*Cosh[x])^
(3/2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x))^{3/2}}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{ 3116}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\cosh(x)}dx}{a^2}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2}$$
$$\Big\downarrow \text{ 3121}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}dx}{a^2\sqrt{\cosh(x)}}$$
$$\Big\downarrow \text{ 3042}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2\sqrt{\cosh(x)}}$$
$$\Big\downarrow \text{ 3119}$$
$$\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}\middle|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(-3/2),x]
```

output
```
((2*I)*Sqrt[a*Cosh[x]]*EllipticE[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sin
h[x])/(a*Sqrt[a*Cosh[x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(41) = 82$.

Time = 0.99 (sec) , antiderivative size = 159, normalized size of antiderivative = 3.46

method	result
default	$\frac{\sqrt{2 \sinh\left(\frac{x}{2}\right)^{4} a+\sinh\left(\frac{x}{2}\right)^{2} a}\, \left(-\sqrt{2}\, \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2}-1}\, \sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\, \operatorname{EllipticF}\left(\sqrt{2}\, \cosh\left(\frac{x}{2}\right), \frac{\sqrt{2}}{2}\right)+2 \sqrt{2}\, \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2}-1}\, \sqrt{-\sinh}\right.}{a \sqrt{a \left(2 \sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\, \sinh\left(\frac{x}{2}\right) \sqrt{\left(2 \cosh\left(\frac{x}{2}\right)^{2}-1\right) a}}$

input
```
int(1/(cosh(x)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/a*(2*sinh(1/2*x)^4*a+sinh(1/2*x)^2*a)^(1/2)*(-2^(1/2)*(-2*sinh(1/2*x)^2-
1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))
+2*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(2^(
1/2)*cosh(1/2*x),1/2*2^(1/2))+4*sinh(1/2*x)^2*cosh(1/2*x))/(a*(2*sinh(1/2*
x)^4+sinh(1/2*x)^2))^(1/2)/sinh(1/2*x)/((2*cosh(1/2*x)^2-1)*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(39) = 78$.

Time = 0.10 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.96

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \frac{4\left(\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{2} + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^{2} + 1\right)\sqrt{a}\mathrm{weierstrassZeta}(-4, 0, \mathrm{we}\right.}{a^{2}\cosh\left(x\right)^{2} + 2}$$

input
```
integrate(1/(a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
4*(sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) + sqrt(a*
cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))/(a^2*cosh(x)^2 + 2*a
^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)
```

Sympy [F]

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))**(3/2),x)
```

output
```
Integral((a*cosh(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a\cosh(x))^{3/2}}\,dx$$

input
```
int(1/(a*cosh(x))^(3/2),x)
```

output
```
int(1/(a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a\cosh(x))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)}}{\cosh(x)^2}dx\right)}{a^2}$$

input
```
int(1/(a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x))/cosh(x)**2,x))/a**2
```

3.21 $\int \frac{1}{(a\cosh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 50

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = -\frac{2i\sqrt{\cosh(x)}\,\mathrm{EllipticF}\left(\frac{ix}{2},2\right)}{3a^2\sqrt{a\cosh(x)}} + \frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$

output
```
-2/3*I*cosh(x)^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2))/a^2/(a*cosh(x))^(1/2
)+2/3*sinh(x)/a/(a*cosh(x))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.04 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \frac{2\left(\mathrm{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2x)-\sinh(2x)\right)\sqrt{1+\cosh(2x)+\sinh(2x)}\right.}{3a^2\sqrt{a\cosh(x)}}$$

input
```
Integrate[(a*Cosh[x])^(-5/2),x]
```

output
```
(2*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2*x]]*Sqrt[1 + Cosh
[2*x] + Sinh[2*x]] + Tanh[x]))/(3*a^2*Sqrt[a*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 3116, 3042, 3121, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x))^{5/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$
$$\Big\downarrow \text{3116}$$
$$\frac{\int \frac{1}{\sqrt{a\cosh(x)}}dx}{3a^2}+\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3a^2}$$
$$\Big\downarrow \text{3121}$$
$$\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\cosh(x)}}dx}{3a^2\sqrt{a\cosh(x)}}+\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}+\frac{\sqrt{\cosh(x)}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx}{3a^2\sqrt{a\cosh(x)}}$$
$$\Big\downarrow \text{3120}$$
$$\frac{2\sinh(x)}{3a(a\cosh(x))^{3/2}}-\frac{2i\sqrt{\cosh(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{3a^2\sqrt{a\cosh(x)}}$$

input
```
Int[(a*Cosh[x])^(-5/2),x]
```

output
```
(((-2*I)/3)*Sqrt[Cosh[x]]*EllipticF[(I/2)*x, 2])/(a^2*Sqrt[a*Cosh[x]]) + (
2*Sinh[x])/(3*a*(a*Cosh[x])^(3/2))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(38) = 76$.

Time = 1.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.54

method	result
default	$\frac{\left(2\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\sqrt{2}\,\cosh\left(\frac{x}{2}\right),\frac{\sqrt{2}}{2}\right)\sinh\left(\frac{x}{2}\right)^{2}+\sqrt{2}\,\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}-1}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\sqrt{2}\right.\right.}{3a^{2}\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}\,\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)\sinh\left(\frac{x}{2}\right)\sqrt{\left(2\cosh\left(\frac{x}{2}\right.\right.}}$

input
```
int(1/(cosh(x)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(2*2^(1/2)*(-2*sinh(1/2*x)^2-1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF
(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))*sinh(1/2*x)^2+2^(1/2)*(-2*sinh(1/2*x)^2-
1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))
+4*sinh(1/2*x)^2*cosh(1/2*x))/a^2*((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1
/2)/(a*(2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(2*cosh(1/2*x)^2-1)/sinh(1/2
*x)/((2*cosh(1/2*x)^2-1)*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.72

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \frac{4\left(\sqrt{\frac{1}{2}}\left(\cosh\left(x\right)^{4}+4\,\cosh\left(x\right)\sinh\left(x\right)^{3}+\sinh\left(x\right)^{4}+2\left(3\,\cosh\left(x\right)^{2}+1\right)\sinh\left(x\right)^{2}+\right.\right.}{3\left(a^{3}\cosh\left(x\right)^{4}\right.}$$

input
```
integrate(1/(a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
4/3*(sqrt(1/2)*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)
^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqr
t(a)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + (cosh(x)^3 + 3*cosh(x
)*sinh(x)^2 + sinh(x)^3 + (3*cosh(x)^2 - 1)*sinh(x) - cosh(x))*sqrt(a*cosh
(x)))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*sinh(x)^4 + 2*a^3*cos
h(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 + a^
3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))**(5/2),x)
```

output
```
Integral((a*cosh(x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a\cosh(x))^{5/2}}\,dx$$

input
```
int(1/(a*cosh(x))^(5/2),x)
```

output
```
int(1/(a*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a\cosh(x))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)}}{\cosh(x)^3}dx\right)}{a^3}$$

input
```
int(1/(a*cosh(x))^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x))/cosh(x)**3,x))/a**3
```

3.22 $\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$

Optimal result

Integrand size = 8, antiderivative size = 67

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \frac{6i\sqrt{a\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)}{5a^4\sqrt{\cosh(x)}} + \frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}} + \frac{6\sinh(x)}{5a^3\sqrt{a\cosh(x)}}$$

output
```
6/5*I*(a*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2))/a^4/cosh(x)^(1/2)
+2/5*sinh(x)/a/(a*cosh(x))^(5/2)+6/5*sinh(x)/a^3/(a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \frac{2\left(3i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 3\cosh(x)\sinh(x) + \tanh(x)\right)}{5a^2(a\cosh(x))^{3/2}}$$

input
```
Integrate[(a*Cosh[x])^(-7/2),x]
```

output
```
(2*((3*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 3*Cosh[x]*Sinh[x] + Tanh[x
]))/(5*a^2*(a*Cosh[x])^(3/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3116, 3042, 3116, 3042, 3121, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\int \frac{1}{(a\cosh(x))^{3/2}}dx}{5a^2}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\int \frac{1}{\left(a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{5a^2}$$

$$\Big\downarrow \text{3116}$$

$$\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\cosh(x)}dx}{a^2}\right)}{5a^2}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2}\right)}{5a^2}$$

$$\Big\downarrow \text{3121}$$

$$\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int \sqrt{\cosh(x)}dx}{a^2\sqrt{\cosh(x)}}\right)}{5a^2}+\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}-\frac{\sqrt{a\cosh(x)}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2\sqrt{\cosh(x)}}\right)}{5a^2}$$

$\Big\downarrow$ 3119

$$\frac{2\sinh(x)}{5a(a\cosh(x))^{5/2}}+\frac{3\left(\frac{2\sinh(x)}{a\sqrt{a\cosh(x)}}+\frac{2iE\left(\frac{ix}{2}|2\right)\sqrt{a\cosh(x)}}{a^2\sqrt{\cosh(x)}}\right)}{5a^2}$$

input
```
Int[(a*Cosh[x])^(-7/2),x]
```

output
```
(2*Sinh[x])/(5*a*(a*Cosh[x])^(5/2)) + (3*(((2*I)*Sqrt[a*Cosh[x]]*EllipticE
[(I/2)*x, 2])/(a^2*Sqrt[Cosh[x]]) + (2*Sinh[x])/(a*Sqrt[a*Cosh[x]])))/(5*a
^2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3121
```
Int[((b_)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*Sin[c + d*x])
^n/Sin[c + d*x]^n   Int[Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] && Lt
Q[-1, n, 1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(54) = 108$.

Time = 1.46 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.79

method	result
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a\sinh\left(\frac{x}{2}\right)^{2}}\left(\frac{\cosh\left(\frac{x}{2}\right)\sqrt{a\left(2\sinh\left(\frac{x}{2}\right)^{4}+\sinh\left(\frac{x}{2}\right)^{2}\right)}}{20a\left(\cosh\left(\frac{x}{2}\right)^{2}-\frac{1}{2}\right)^{3}}+\frac{6\sinh\left(\frac{x}{2}\right)^{2}\cosh\left(\frac{x}{2}\right)}{5\sqrt{\left(2\cosh\left(\frac{x}{2}\right)^{2}-1\right)a\sinh\left(\frac{x}{2}\right)^{2}}}+\frac{3\sqrt{2}\,\sqrt{-2\cosh\left(\frac{x}{2}\right)^{2}+1}\,\sqrt{-\sin}}{10\sqrt{a\left(2\sinh\right.}}\right.}{a^{3}\sinh\left(\frac{x}{2}\right)\sqrt{\left(2\right.}}$

input
```
int(1/(cosh(x)*a)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1/2)/a^3*(1/20*cosh(1/2*x)/a*(a*(
2*sinh(1/2*x)^4+sinh(1/2*x)^2))^(1/2)/(cosh(1/2*x)^2-1/2)^3+6/5*sinh(1/2*x
)^2*cosh(1/2*x)/((2*cosh(1/2*x)^2-1)*a*sinh(1/2*x)^2)^(1/2)+3/10*2^(1/2)*(
-2*cosh(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(
1/2*x)^2))^(1/2)*EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-3/5*2^(1/2)*(-
2*cosh(1/2*x)^2+1)^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(a*(2*sinh(1/2*x)^4+sinh(1
/2*x)^2))^(1/2)*(EllipticF(2^(1/2)*cosh(1/2*x),1/2*2^(1/2))-EllipticE(2^(1
/2)*cosh(1/2*x),1/2*2^(1/2))))/sinh(1/2*x)/((2*cosh(1/2*x)^2-1)*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 375, normalized size of antiderivative = 5.60

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x))^(7/2),x, algorithm="fricas")
```

output
```
4/5*(3*sqrt(1/2)*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(
x)^2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3
+ 3*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5
 + 2*cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(a)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cosh(x) + sinh(x))) + (3*cosh(x)^6 + 18*cosh(x)*sin
h(x)^5 + 3*sinh(x)^6 + (45*cosh(x)^2 + 8)*sinh(x)^4 + 8*cosh(x)^4 + 4*(15*
cosh(x)^3 + 8*cosh(x))*sinh(x)^3 + (45*cosh(x)^4 + 48*cosh(x)^2 + 1)*sinh(
x)^2 + cosh(x)^2 + 2*(9*cosh(x)^5 + 16*cosh(x)^3 + cosh(x))*sinh(x))*sqrt(
a*cosh(x)))/(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 + 3*a
^4*cosh(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 + a^4)*sinh(x)^4 + a^4
 + 4*(5*a^4*cosh(x)^3 + 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 + 6*
a^4*cosh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 + 2*a^4*cosh(x)^3 + a^4*
cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x))^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a\cosh(x))^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x))^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a\cosh(x))^{7/2}}\,dx$$

input
```
int(1/(a*cosh(x))^(7/2),x)
```

output
```
int(1/(a*cosh(x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a\cosh(x))^{7/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)}}{\cosh(x)^4}dx\right)}{a^4}$$

input
```
int(1/(a*cosh(x))^(7/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x))/cosh(x)**4,x))/a**4
```

3.23 $\int (b\cosh(c+dx))^n\,dx$

Optimal result

Integrand size = 10, antiderivative size = 71

$$\int (b\cosh(c+dx))^n\,dx = -\frac{(b\cosh(c+dx))^{1+n}\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cosh^2(c+dx)\right)\sinh(c+dx)}{bd(1+n)\sqrt{-\sinh^2(c+dx)}}$$

output
```
-(b*cosh(d*x+c))^(1+n)*hypergeom([1/2, 1/2+1/2*n],[3/2+1/2*n],cosh(d*x+c)^
2)*sinh(d*x+c)/b/d/(1+n)/(-sinh(d*x+c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.92

$$\int (b\cosh(c+dx))^n\,dx = \frac{(b\cosh(c+dx))^n\coth(c+dx)\,\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{1+n}{2},\frac{3+n}{2},\cosh^2(c+dx)\right)\sqrt{-\sinh^2(c+dx)}}{d(1+n)}$$

input
```
Integrate[(b*Cosh[c + d*x])^n,x]
```

output
```
((b*Cosh[c + d*x])^n*Coth[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3 +
n)/2, Cosh[c + d*x]^2]*Sqrt[-Sinh[c + d*x]^2])/(d*(1 + n))
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{3042, 3122\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (b\cosh(c+dx))^n\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^n dx$$

$$\Big\downarrow \text{3122}$$

$$-\frac{\sinh(c+dx)(b\cosh(c+dx))^{n+1}\text{Hypergeometric2F1}\left(\frac{1}{2},\frac{n+1}{2},\frac{n+3}{2},\cosh^2(c+dx)\right)}{bd(n+1)\sqrt{-\sinh^2(c+dx)}}$$

input
```
Int[(b*Cosh[c + d*x])^n,x]
```

output
```
-(((b*Cosh[c + d*x])^(1 + n)*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2,
Cosh[c + d*x]^2]*Sinh[c + d*x])/(b*d*(1 + n)*Sqrt[-Sinh[c + d*x]^2]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3122
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Maple [F]

$$\int (b\cosh (dx+c))^n dx$$

input
```
int((b*cosh(d*x+c))^n,x)
```

output
```
int((b*cosh(d*x+c))^n,x)
```

Fricas [F]

$$\int (b\cosh(c+dx))^n dx = \int (b\cosh (dx+c))^n dx$$

input
```
integrate((b*cosh(d*x+c))^n,x, algorithm="fricas")
```

output
```
integral((b*cosh(d*x + c))^n, x)
```

Sympy [F]

$$\int (b\cosh(c+dx))^n\,dx = \int (b\cosh{(c+dx)})^n\,dx$$

input
```
integrate((b*cosh(d*x+c))**n,x)
```

output
```
Integral((b*cosh(c + d*x))**n, x)
```

Maxima [F]

$$\int (b\cosh(c+dx))^n\,dx = \int (b\cosh{(dx+c)})^n\,dx$$

input
```
integrate((b*cosh(d*x+c))^n,x, algorithm="maxima")
```

output
```
integrate((b*cosh(d*x + c))^n, x)
```

Giac [F]

$$\int (b\cosh(c+dx))^n\,dx = \int (b\cosh{(dx+c)})^n\,dx$$

input
```
integrate((b*cosh(d*x+c))^n,x, algorithm="giac")
```

output
```
integrate((b*cosh(d*x + c))^n, x)
```

Mupad [F(-1)]

Timed out.

$$\int (b\cosh(c+dx))^n\,dx = \int (b\cosh(c+d\,x))^n\,dx$$

input
```
int((b*cosh(c + d*x))^n,x)
```

output
```
int((b*cosh(c + d*x))^n, x)
```

Reduce [F]

$$\int (b\cosh(c+dx))^n\,dx = b^n\left(\int \cosh(dx+c)^n\,dx\right)$$

input
```
int((b*cosh(d*x+c))^n,x)
```

output
```
b**n*int(cosh(c + d*x)**n,x)
```

3.24 $\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 54

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = -\frac{3x}{2a}+\frac{4\sinh(x)}{a}-\frac{3\cosh(x)\sinh(x)}{2a} - \frac{\cosh^3(x)\sinh(x)}{a+a\cosh(x)}+\frac{4\sinh^3(x)}{3a}$$

output
```
-3/2*x/a+4*sinh(x)/a-3/2*cosh(x)*sinh(x)/a-cosh(x)^3*sinh(x)/(a+a*cosh(x))
+4/3*sinh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(-36x\cosh\left(\frac{x}{2}\right)+45\sinh\left(\frac{x}{2}\right)+18\sinh\left(\frac{3x}{2}\right)-2\sinh\left(\frac{5x}{2}\right)+\sinh\left(\frac{7x}{2}\right)\right)}{24a}$$

input
```
Integrate[Cosh[x]^4/(a + a*Cosh[x]),x]
```

output
```
(Sech[x/2]*(-36*x*Cosh[x/2] + 45*Sinh[x/2] + 18*Sinh[(3*x)/2] - 2*Sinh[(5*
x)/2] + Sinh[(7*x)/2]))/(24*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.43 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3246, 3042, 3227, 3042, 3113, 2009, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^4}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \cosh^2(x)(3a-4a\cosh(x))dx}{a^2}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{\int \sin\left(ix+\frac{\pi}{2}\right)^2\left(3a-4a\sin\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{3227}$$

$$-\frac{3a\int \cosh^2(x)dx-4a\int \cosh^3(x)dx}{a^2}-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx-4a\int \sin\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \text{3113}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx-4ia\int\left(\sinh^2(x)+1\right)d(-i\sinh(x))}{a^2}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\int \sin\left(ix+\frac{\pi}{2}\right)^2dx-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{ 3115}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\sinh(x)\cosh^3(x)}{a\cosh(x)+a}-\frac{3a\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)-4ia\left(-\frac{1}{3}i\sinh^3(x)-i\sinh(x)\right)}{a^2}$$

input
```
Int[Cosh[x]^4/(a + a*Cosh[x]),x]
```

output
```
-((Cosh[x]^3*Sinh[x])/(a + a*Cosh[x])) - (3*a*(x/2 + (Cosh[x]*Sinh[x])/2)
- (4*I)*a*((-I)*Sinh[x] - (I/3)*Sinh[x]^3))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.57

method	result
parallelrisch	$\frac{-18x+\tanh\left(\frac{x}{2}\right)\left(31+\cosh\left(3x\right)-\cosh\left(2x\right)+17\cosh\left(x\right)\right)}{12a}$
risch	$\frac{-18\,{\mathrm e}^{-x}+2\,{\mathrm e}^{-2x}+{\mathrm e}^{4x}-2\,{\mathrm e}^{3x}+18\,{\mathrm e}^{2x}-69-36x\,{\mathrm e}^{x}-{\mathrm e}^{-3x}+21\,{\mathrm e}^{x}-36x}{24\left({\mathrm e}^{x}+1\right)a}$
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{3\left(1+\tanh\left(\frac{x}{2}\right)\right)^{3}}+\frac{1}{\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{5}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{3\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{5}{2\left(\tanh\left(\frac{x}{2}\right.\right.}}{a}$

input
```
int(cosh(x)^4/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/12*(-18*x+tanh(1/2*x)*(31+cosh(3*x)-cosh(2*x)+17*cosh(x)))/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(48) = 96$.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.85

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{\cosh(x)^4 + (4\cosh(x)-1)\sinh(x)^3 + \sinh(x)^4 - 3\cosh(x)^3 + \left(6\cosh(x)^2 - 9\cosh(x)+20\right)\sinh(x}{24(a\cos}$$

input
```
integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/24*(cosh(x)^4 + (4*cosh(x) - 1)*sinh(x)^3 + sinh(x)^4 - 3*cosh(x)^3 + (6
*cosh(x)^2 - 9*cosh(x) + 20)*sinh(x)^2 - 3*(12*x - 1)*cosh(x) + 20*cosh(x)
^2 + (4*cosh(x)^3 - 3*cosh(x)^2 - 36*x + 32*cosh(x) + 39)*sinh(x) - 36*x -
 69)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(49) = 98$.

Time = 0.81 (sec) , antiderivative size = 337, normalized size of antiderivative = 6.24

$$\int \frac{\cosh^4(x)}{a+a\cosh(x)}\,dx = -\frac{9x\tanh^6\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} + \frac{27x\tanh^4\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} - \frac{27x\tanh^2\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} + \frac{9x}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} + \frac{6\tanh^7\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} - \frac{48\tanh^5\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} + \frac{50\tanh^3\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} - \frac{24\tanh\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a}$$

input
```
integrate(cosh(x)**4/(a+a*cosh(x)),x)
```

output
```
-9*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**
2 - 6*a) + 27*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*
tanh(x/2)**2 - 6*a) - 27*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)
**4 + 18*a*tanh(x/2)**2 - 6*a) + 9*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)**7/(6*a*tanh(x/2)**6 - 18*a*tanh
(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 48*tanh(x/2)**5/(6*a*tanh(x/2)**6 -
18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 50*tanh(x/2)**3/(6*a*tanh(x
/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 24*tanh(x/2)/(6*a*
tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.22

$$\int \frac{\cosh^4(x)}{a + a\cosh(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{21\,e^{(-x)} - 3\,e^{(-2\,x)} + e^{(-3\,x)}}{24\,a} - \frac{2\,e^{(-x)} - 18\,e^{(-2\,x)} - 69\,e^{(-3\,x)} - 1}{24\,(ae^{(-3\,x)} + ae^{(-4\,x)})}$$

input
```
integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-3/2*x/a - 1/24*(21*e^(-x) - 3*e^(-2*x) + e^(-3*x))/a - 1/24*(2*e^(-x) - 1
8*e^(-2*x) - 69*e^(-3*x) - 1)/(a*e^(-3*x) + a*e^(-4*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^4(x)}{a + a\cosh(x)}\,dx = -\frac{3\,x}{2\,a} - \frac{\left(69\,e^{(3\,x)} + 18\,e^{(2\,x)} - 2\,e^{x} + 1\right)e^{(-3\,x)}}{24\,a(e^{x} + 1)} + \frac{a^2e^{(3\,x)} - 3\,a^2e^{(2\,x)} + 21\,a^2e^{x}}{24\,a^3}$$

input
```
integrate(cosh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-3/2*x/a - 1/24*(69*e^(3*x) + 18*e^(2*x) - 2*e^x + 1)*e^(-3*x)/(a*(e^x + 1
)) + 1/24*(a^2*e^(3*x) - 3*a^2*e^(2*x) + 21*a^2*e^x)/a^3
```

Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^4(x)}{a + a\cosh(x)}\,dx = \frac{e^{-2x}}{8a} - \frac{7e^{-x}}{8a} - \frac{e^{2x}}{8a} - \frac{e^{-3x}}{24a} + \frac{e^{3x}}{24a} - \frac{3x}{2a} - \frac{2}{a(e^x + 1)} + \frac{7e^x}{8a}$$

input
```
int(cosh(x)^4/(a + a*cosh(x)),x)
```

output
```
exp(-2*x)/(8*a) - (7*exp(-x))/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) +
exp(3*x)/(24*a) - (3*x)/(2*a) - 2/(a*(exp(x) + 1)) + (7*exp(x))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.39

$$\int \frac{\cosh^4(x)}{a + a\cosh(x)}\,dx = \frac{e^{7x} - 2e^{6x} + 18e^{5x} - 36e^{4x}x + 90e^{4x} - 36e^{3x}x - 18e^{2x} + 2e^x - 1}{24e^{3x}a(e^x + 1)}$$

input
```
int(cosh(x)^4/(a+a*cosh(x)),x)
```

output
```
(e**(7*x) - 2*e**(6*x) + 18*e**(5*x) - 36*e**(4*x)*x + 90*e**(4*x) - 36*e*
*(3*x)*x - 18*e**(2*x) + 2*e**x - 1)/(24*e**(3*x)*a*(e**x + 1))
```

3.25 $\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = \frac{3x}{2a}-\frac{2\sinh(x)}{a}+\frac{3\cosh(x)\sinh(x)}{2a}-\frac{\cosh^2(x)\sinh(x)}{a+a\cosh(x)}$$

output
```
3/2*x/a-2*sinh(x)/a+3/2*cosh(x)*sinh(x)/a-cosh(x)^2*sinh(x)/(a+a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = \frac{\operatorname{sech}\left(\frac{x}{2}\right)\left(12x\cosh\left(\frac{x}{2}\right)-12\sinh\left(\frac{x}{2}\right)-3\sinh\left(\frac{3x}{2}\right)+\sinh\left(\frac{5x}{2}\right)\right)}{8a}$$

input
```
Integrate[Cosh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(Sech[x/2]*(12*x*Cosh[x/2] - 12*Sinh[x/2] - 3*Sinh[(3*x)/2] + Sinh[(5*x)/2
]))/(8*a)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3246, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^3}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3246}$$

$$-\frac{\int \cosh(x)(2a-3a\cosh(x))dx}{a^2}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}-\frac{\int \sin\left(ix+\frac{\pi}{2}\right)\left(2a-3a\sin\left(ix+\frac{\pi}{2}\right)\right)dx}{a^2}$$

$$\Big\downarrow \text{3213}$$

$$-\frac{-\frac{3ax}{2}+2a\sinh(x)-\frac{3}{2}a\sinh(x)\cosh(x)}{a^2}-\frac{\sinh(x)\cosh^2(x)}{a\cosh(x)+a}$$

input
```
Int[Cosh[x]^3/(a + a*Cosh[x]),x]
```

output
```
-((Cosh[x]^2*Sinh[x])/(a + a*Cosh[x])) - ((-3*a*x)/2 + 2*a*Sinh[x] - (3*a*
Cosh[x]*Sinh[x])/2)/a^2
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3246
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((c + d*Sin[e +
 f*x])^(n - 1)/(a*f*(a + b*Sin[e + f*x]))), x] - Simp[d/(a*b)   Int[(c + d*
Sin[e + f*x])^(n - 2)*Simp[b*d*(n - 1) - a*c*n + (b*c*(n - 1) - a*d*n)*Sin[
e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] &
& EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[n, 1] && (IntegerQ[2*n] ||
EqQ[c, 0])
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.56

method	result	siz
parallelrisch	$\frac{6x-4\sinh(x)+\sinh(2x)-4\tanh\left(\frac{x}{2}\right)}{4a}$	24
risch	$\frac{{\mathrm e}^{3x}-3\,{\mathrm e}^{2x}+20+3\,{\mathrm e}^{-x}+12x\,{\mathrm e}^{x}-4\,{\mathrm e}^{x}-{\mathrm e}^{-2x}+12x}{8\left({\mathrm e}^{x}+1\right)a}$	48
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2}-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}+\frac{3}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{3\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2}}{a}$	70

input
```
int(cosh(x)^3/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/4*(6*x-4*sinh(x)+sinh(2*x)-4*tanh(1/2*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.63

$$\int \frac{\cosh^3(x)}{a + a\cosh(x)}\,dx = \frac{\cosh(x)^3 + (3\cosh(x) - 4)\sinh(x)^2 + \sinh(x)^3 + (12x - 1)\cosh(x) - 4\cosh(x)^2 + (3\cosh(x)^2 + 12}{8(a\cosh(x) + a\sinh(x) + a)}$$

input
```
integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/8*(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 + (12*x - 1)*cosh(x
) - 4*cosh(x)^2 + (3*cosh(x)^2 + 12*x - 4*cosh(x) - 7)*sinh(x) + 12*x + 20
)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. 2(39) = 78.

Time = 0.44 (sec) , antiderivative size = 189, normalized size of antiderivative = 4.40

$$\int \frac{\cosh^3(x)}{a + a\cosh(x)}\,dx = \frac{3x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{6x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{3x}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{2\tanh^5\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} + \frac{10\tanh^3\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a} - \frac{4\tanh\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right) - 4a\tanh^2\left(\frac{x}{2}\right) + 2a}$$

input
```
integrate(cosh(x)**3/(a+a*cosh(x)),x)
```

output
```
3*x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 6*x*tanh(x/
2)**2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + 3*x/(2*a*tanh(x/2)**4
- 4*a*tanh(x/2)**2 + 2*a) - 2*tanh(x/2)**5/(2*a*tanh(x/2)**4 - 4*a*tanh(x/
2)**2 + 2*a) + 10*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
 - 4*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = \frac{3\,x}{2\,a} + \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a} - \frac{3\,e^{(-x)} + 20\,e^{(-2\,x)} - 1}{8\,(ae^{(-2\,x)} + ae^{(-3\,x)})}$$

input
```
integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
3/2*x/a + 1/8*(4*e^(-x) - e^(-2*x))/a - 1/8*(3*e^(-x) + 20*e^(-2*x) - 1)/(
a*e^(-2*x) + a*e^(-3*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^3(x)}{a+a\cosh(x)}\,dx = \frac{3\,x}{2\,a} + \frac{\left(20\,e^{(2\,x)} + 3\,e^{x} - 1\right)e^{(-2\,x)}}{8\,a(e^{x}+1)} + \frac{ae^{(2\,x)} - 4\,ae^{x}}{8\,a^2}$$

input
```
integrate(cosh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3/2*x/a + 1/8*(20*e^(2*x) + 3*e^x - 1)*e^(-2*x)/(a*(e^x + 1)) + 1/8*(a*e^(
2*x) - 4*a*e^x)/a^2
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.21

$$\int \frac{\cosh^3(x)}{a + a\cosh(x)}\,dx = \frac{e^{-x}}{2\,a} - \frac{e^{-2\,x}}{8\,a} + \frac{e^{2\,x}}{8\,a} + \frac{3\,x}{2\,a} + \frac{2}{a\,(e^x + 1)} - \frac{e^x}{2\,a}$$

input
```
int(cosh(x)^3/(a + a*cosh(x)),x)
```

output
```
exp(-x)/(2*a) - exp(-2*x)/(8*a) + exp(2*x)/(8*a) + (3*x)/(2*a) + 2/(a*(exp
(x) + 1)) - exp(x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.93

$$\int \frac{\cosh^3(x)}{a + a\cosh(x)}\,dx = \frac{e^x \cosh(x)^2 x + \cosh(x)^2 x + e^x \cosh(x)\sinh(x) + \cosh(x)\sinh(x) - e^x \sinh(x)^2 x - 2e^x \sinh(x) + 2e}{2a\,(e^x + 1)}$$

input
```
int(cosh(x)^3/(a+a*cosh(x)),x)
```

output
```
(e**x*cosh(x)**2*x + cosh(x)**2*x + e**x*cosh(x)*sinh(x) + cosh(x)*sinh(x)
 - e**x*sinh(x)**2*x - 2*e**x*sinh(x) + 2*e**x*x - 4*e**x - sinh(x)**2*x -
 2*sinh(x) + 2*x)/(2*a*(e**x + 1))
```

3.26 $\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 25

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = -\frac{x}{a}+\frac{\sinh(x)}{a}+\frac{\sinh(x)}{a(1+\cosh(x))}$$

output
```
-x/a+sinh(x)/a+sinh(x)/a/(1+cosh(x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.28

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = \frac{-2x+\operatorname{sech}\left(\frac{x}{2}\right)\sinh\left(\frac{3x}{2}\right)+3\tanh\left(\frac{x}{2}\right)}{2a}$$

input
```
Integrate[Cosh[x]^2/(a + a*Cosh[x]),x]
```

output
```
(-2*x + Sech[x/2]*Sinh[(3*x)/2] + 3*Tanh[x/2])/(2*a)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 3225, 25, 3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^2}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\int -\frac{\cosh(x)}{\cosh(x)+1}dx}{a}+\frac{\sinh(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh(x)}{a}-\frac{\int \frac{\cosh(x)}{\cosh(x)+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{a}-\frac{\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx}{a}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sinh(x)}{a}-\frac{x-\int \frac{1}{\cosh(x)+1}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{a}-\frac{x-\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx}{a}$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(x)}{a}-\frac{x-\frac{\sinh(x)}{\cosh(x)+1}}{a}$$

input
```
Int[Cosh[x]^2/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]/a - (x - Sinh[x]/(1 + Cosh[x]))/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.60

method	result	size
parallelrisch	$\frac{\sinh(x)-x+\tanh\left(\frac{x}{2}\right)}{a}$	15
risch	$-\frac{x}{a}+\frac{\mathrm{e}^{x}}{2a}-\frac{\mathrm{e}^{-x}}{2a}-\frac{2}{(\mathrm{e}^{x}+1)a}$	35
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{1}{1+\tanh\left(\frac{x}{2}\right)}-\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	46

input
```
int(cosh(x)^2/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
(sinh(x)-x+tanh(1/2*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.88

$$\int \frac{\cosh^2(x)}{a+a\cosh(x)}\,dx = -\frac{2\,x\cosh\left(x\right)-\cosh\left(x\right)^{2}+2\left(x-\cosh\left(x\right)-1\right)\sinh\left(x\right)-\sinh\left(x\right)^{2}+2\,x+5}{2\left(a\cosh\left(x\right)+a\sinh\left(x\right)+a\right)}$$

input
```
integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*x*cosh(x) - cosh(x)^2 + 2*(x - cosh(x) - 1)*sinh(x) - sinh(x)^2 +
2*x + 5)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(17) = 34$.

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x\tanh^2\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a} + \frac{x}{a\tanh^2\left(\frac{x}{2}\right) - a} + \frac{\tanh^3\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a} - \frac{3\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a}$$

input
```
integrate(cosh(x)**2/(a+a*cosh(x)),x)
```

output
```
-x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) + tanh(x/2)*
*3/(a*tanh(x/2)**2 - a) - 3*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.64

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x}{a} + \frac{5\,e^{(-x)} + 1}{2\,(ae^{(-x)} + ae^{(-2\,x)})} - \frac{e^{(-x)}}{2\,a}$$

input
```
integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-x/a + 1/2*(5*e^(-x) + 1)/(a*e^(-x) + a*e^(-2*x)) - 1/2*e^(-x)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x}{a} - \frac{(5\,e^x + 1)e^{(-x)}}{2\,a(e^x + 1)} + \frac{e^x}{2\,a}$$

input
```
integrate(cosh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-x/a - 1/2*(5*e^x + 1)*e^(-x)/(a*(e^x + 1)) + 1/2*e^x/a
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\,dx = \frac{e^x}{2\,a} - \frac{x}{a} - \frac{2}{a\,\left(e^x + 1\right)} - \frac{e^{-x}}{2\,a}$$

input
```
int(cosh(x)^2/(a + a*cosh(x)),x)
```

output
```
exp(x)/(2*a) - x/a - 2/(a*(exp(x) + 1)) - exp(-x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.36

$$\int \frac{\cosh^2(x)}{a + a\cosh(x)}\,dx = \frac{e^x \sinh(x) - e^x x + 2e^x + \sinh(x) - x}{a\,(e^x + 1)}$$

input
```
int(cosh(x)^2/(a+a*cosh(x)),x)
```

output
```
(e**x*sinh(x) - e**x*x + 2*e**x + sinh(x) - x)/(a*(e**x + 1))
```

3.27 $\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx = \frac{x}{a} - \frac{\sinh(x)}{a+a\cosh(x)}$$

output
```
x/a-sinh(x)/(a+a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{\cosh(x)}{a+a\cosh(x)}\,dx = \frac{\arcsin(\cosh(x))\operatorname{csch}(x)\sqrt{-\sinh^2(x)} - \tanh\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Cosh[x]/(a + a*Cosh[x]),x]
```

output
```
(ArcSin[Cosh[x]]*Csch[x]*Sqrt[-Sinh[x]^2] - Tanh[x/2])/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{a+a\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{x}{a}-\int \frac{1}{\cosh(x)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a}-\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{x}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

input
```
Int[Cosh[x]/(a + a*Cosh[x]),x]
```

output
```
x/a - Sinh[x]/(a + a*Cosh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.72

method	result	size
parallelrisch	$\frac{x-\tanh\left(\frac{x}{2}\right)}{a}$	13
risch	$\frac{x}{a}+\frac{2}{\left(e^{x}+1\right)a}$	18
default	$\frac{-\tanh\left(\frac{x}{2}\right)-\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{a}$	28

input
```
int(cosh(x)/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
(x-tanh(1/2*x))/a
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x\cosh{(x)} + x\sinh{(x)} + x + 2}{a\cosh{(x)} + a\sinh{(x)} + a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) + x + 2)/(a*cosh(x) + a*sinh(x) + a)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.44

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} - \frac{\tanh\left(\frac{x}{2}\right)}{a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x)
```

output
```
x/a - tanh(x/2)/a
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} - \frac{2}{ae^{(-x)} + a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
x/a - 2/(a*e^(-x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} + \frac{2}{a(e^x + 1)}$$

input
```
integrate(cosh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
x/a + 2/(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{x}{a} + \frac{2}{a\,(e^x + 1)}$$

input
```
int(cosh(x)/(a + a*cosh(x)),x)
```

output
```
x/a + 2/(a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

$$\int \frac{\cosh(x)}{a + a\cosh(x)}\,dx = \frac{e^x x - 2e^x + x}{a\,(e^x + 1)}$$

input
```
int(cosh(x)/(a+a*cosh(x)),x)
```

output
```
(e**x*x - 2*e**x + x)/(a*(e**x + 1))
```

3.28 $\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}-\frac{\sinh(x)}{a+a\cosh(x)}$$

output
```
arctan(sinh(x))/a-sinh(x)/(a+a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\text{sech}(x)}{a+a\cosh(x)}\,dx=\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sech[x]/(a + a*Cosh[x]),x]
```

output
```
(2*ArcTan[Tanh[x/2]] - Tanh[x/2])/a
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 3226, 3042, 3127, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3226}$$

$$\frac{\int \text{sech}(x)dx}{a}-\int \frac{1}{\cosh(x)a+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3127}$$

$$-\frac{\sinh(x)}{a\cosh(x)+a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{\sinh(x)}{a\cosh(x)+a}$$

input
```
Int[Sech[x]/(a + a*Cosh[x]),x]
```

output
```
ArcTan[Sinh[x]]/a - Sinh[x]/(a + a*Cosh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	19
parallelrisch	$-\frac{-i\left(-\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)\right)+\tanh\left(\frac{x}{2}\right)}{a}$	33
risch	$\frac{2}{\left(e^{x}+1\right)a}+\frac{i\ln\left(e^{x}+i\right)}{a}-\frac{i\ln\left(e^{x}-i\right)}{a}$	37

input
```
int(sech(x)/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-tanh(1/2*x)+2*arctan(tanh(1/2*x)))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx = \frac{2\left((\cosh(x)+\sinh(x)+1)\arctan(\cosh(x)+\sinh(x))+1\right)}{a\cosh(x)+a\sinh(x)+a}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
2*((cosh(x) + sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 1)/(a*cosh(x) + a*s
inh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{sech}(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)/(cosh(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx = -\frac{2\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-x)}+a}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x))/a - 2/(a*e^(-x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx = \frac{2\arctan\left(e^x\right)}{a} + \frac{2}{a(e^x+1)}$$

input
```
integrate(sech(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
2*arctan(e^x)/a + 2/(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx = \frac{2}{a\left(e^x+1\right)} + \frac{2\operatorname{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)*(a + a*cosh(x))),x)
```

output
```
2/(a*(exp(x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.50

$$\int \frac{\operatorname{sech}(x)}{a+a\cosh(x)}\,dx = \frac{2e^x atan(e^x)+2atan(e^x)-2e^x}{a\left(e^x+1\right)}$$

input
```
int(sech(x)/(a+a*cosh(x)),x)
```

output
```
(2*(e**x*atan(e**x) + atan(e**x) - e**x))/(a*(e**x + 1))
```

3.29 $\int \frac{\text{sech}^2(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 28

$$\int \frac{\text{sech}^2(x)}{a+a\cosh(x)}\,dx = -\frac{\arctan(\sinh(x))}{a}+\frac{2\tanh(x)}{a}-\frac{\tanh(x)}{a+a\cosh(x)}$$

output
```
-arctan(sinh(x))/a+2*tanh(x)/a-tanh(x)/(a+a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.54

$$\int \frac{\text{sech}^2(x)}{a+a\cosh(x)}\,dx = \frac{2\cosh\left(\frac{x}{2}\right)\left(\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\tanh(x)\right)\right)}{a(1+\cosh(x))}$$

input
```
Integrate[Sech[x]^2/(a + a*Cosh[x]),x]
```

output
```
(2*Cosh[x/2]*(Sinh[x/2] + Cosh[x/2]*(-2*ArcTan[Tanh[x/2]] + Tanh[x])))/(a*
(1 + Cosh[x]))
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\left((2a-a\cosh(x))\text{sech}^2(x)\right)dx}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (2a-a\cosh(x))\text{sech}^2(x)dx}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{\int \frac{2a-a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)^2}dx}{a^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{2a\int \text{sech}^2(x)dx-a\int \text{sech}(x)dx}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{2a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx-a\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{4254}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{2ia\int 1d(-i\tanh(x))-a\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \text{24}$$

$$-\frac{\tanh(x)}{a\cosh(x)+a}+\frac{2a\tanh(x)-a\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a^2}$$

$$\Big\downarrow \textcolor{blue}{4257}$$

$$\frac{2a\tanh(x)-a\arctan(\sinh(x))}{a^2}-\frac{\tanh(x)}{a\cosh(x)+a}$$

input
```
Int[Sech[x]^2/(a + a*Cosh[x]),x]
```

output
```
-(Tanh[x]/(a + a*Cosh[x])) + (-(a*ArcTan[Sinh[x]]) + 2*a*Tanh[x])/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.18

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)+\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}-2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	33
parallelrisch	$\frac{i\cosh(x)\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)-i\cosh(x)\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+2\tanh\left(\frac{x}{2}\right)\cosh(x)+\tanh\left(\frac{x}{2}\right)}{\cosh(x)a}$	48
risch	$-\frac{2(e^{2x}+e^{x}+2)}{(e^{2x}+1)a(e^{x}+1)}+\frac{i\ln(e^{x}-i)}{a}-\frac{i\ln(e^{x}+i)}{a}$	53

input
```
int(sech(x)^2/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(tanh(1/2*x)+2*tanh(1/2*x)/(tanh(1/2*x)^2+1)-2*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(28) = 56.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 4.54

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx =$$

$$-\frac{2\left(\left(\cosh(x)^{3}+(3\cosh(x)+1)\sinh(x)^{2}+\sinh(x)^{3}+\cosh(x)^{2}+\left(3\cosh(x)^{2}+2\cosh(x)+1\right)\sin\right.\right.}{a\cosh(x)^{3}+a\sinh(x)^{3}+a\cosh(x)^{2}+(3a\cosh(x)+a}$$

input
```
integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2*((cosh(x)^3 + (3*cosh(x) + 1)*sinh(x)^2 + sinh(x)^3 + cosh(x)^2 + (3*co
sh(x)^2 + 2*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x))
+ cosh(x)^2 + (2*cosh(x) + 1)*sinh(x) + sinh(x)^2 + cosh(x) + 2)/(a*cosh(x
)^3 + a*sinh(x)^3 + a*cosh(x)^2 + (3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x)
+ (3*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{sech}^2(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)**2/(cosh(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.61

$$\int \frac{\operatorname{sech}^2(x)}{a + a\cosh(x)}\,dx = \frac{2\left(e^{(-x)} + e^{(-2x)} + 2\right)}{ae^{(-x)} + ae^{(-2x)} + ae^{(-3x)} + a} + \frac{2\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
2*(e^(-x) + e^(-2*x) + 2)/(a*e^(-x) + a*e^(-2*x) + a*e^(-3*x) + a) + 2*arc
tan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = -\frac{2\arctan\left(e^x\right)}{a} - \frac{2\left(e^{(2x)}+e^x+2\right)}{a(e^{(3x)}+e^{(2x)}+e^x+1)}$$

input
```
integrate(sech(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-2*arctan(e^x)/a - 2*(e^(2*x) + e^x + 2)/(a*(e^(3*x) + e^(2*x) + e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = -\frac{\frac{2e^{2x}}{a}+\frac{4}{a}+\frac{2e^x}{a}}{e^{2x}+e^{3x}+e^x+1} - \frac{2\operatorname{atan}\left(\frac{e^x\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^2*(a + a*cosh(x))),x)
```

output
```
- ((2*exp(2*x))/a + 4/a + (2*exp(x))/a)/(exp(2*x) + exp(3*x) + exp(x) + 1)
 - (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.36

$$\int \frac{\operatorname{sech}^2(x)}{a+a\cosh(x)}\,dx = \frac{-2e^{3x}atan(e^x)-2e^{2x}atan(e^x)-2e^x atan(e^x)-2atan(e^x)+2e^{3x}-2}{a\left(e^{3x}+e^{2x}+e^x+1\right)}$$

input
```
int(sech(x)^2/(a+a*cosh(x)),x)
```

output
```
(2*( - e**(3*x)*atan(e**x) - e**(2*x)*atan(e**x) - e**x*atan(e**x) - atan(
e**x) + e**(3*x) - 1))/(a*(e**(3*x) + e**(2*x) + e**x + 1))
```

3.30 $\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 43

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx=\frac{3\arctan(\sinh(x))}{2a}-\frac{2\tanh(x)}{a}+\frac{3\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{a+a\cosh(x)}$$

output
```
3/2*arctan(sinh(x))/a-2*tanh(x)/a+3/2*sech(x)*tanh(x)/a-sech(x)*tanh(x)/(a
+a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.14

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{\cosh\left(\frac{x}{2}\right)\left(-2\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(6\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+(-2+\operatorname{sech}(x))\tanh(x)\right)\right)}{a(1+\cosh(x))}$$

input
```
Integrate[Sech[x]^3/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]*(-2*Sinh[x/2] + Cosh[x/2]*(6*ArcTan[Tanh[x/2]] + (-2 + Sech[x])
*Tanh[x])))/(a*(1 + Cosh[x]))
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.02, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.846, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 24, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\left((3a-2a\cosh(x))\text{sech}^3(x)\right)dx}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (3a-2a\cosh(x))\text{sech}^3(x)dx}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{\int \frac{3a-2a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)^3}dx}{a^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{3a\int \text{sech}^3(x)dx-2a\int \text{sech}^2(x)dx}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx-2a\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a^2}$$

$$\Big\downarrow \; 4254$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx-2ia\int 1d(-i\tanh(x))}{a^2}$$

$$\Big\downarrow \; 24$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{-2a\tanh(x)+3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \; 4255$$

$$\frac{3a\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-2a\tanh(x)}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}+\frac{-2a\tanh(x)+3a\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)}{a^2}$$

$$\Big\downarrow \; 4257$$

$$\frac{3a\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)-2a\tanh(x)}{a^2}-\frac{\tanh(x)\text{sech}(x)}{a\cosh(x)+a}$$

input
```
Int[Sech[x]^3/(a + a*Cosh[x]),x]
```

output
```
-((Sech[x]*Tanh[x])/(a + a*Cosh[x])) + (-2*a*Tanh[x] + 3*a*(ArcTan[Sinh[x]
]/2 + (Sech[x]*Tanh[x])/2))/a^2
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.07

method	result	size
default	$\frac{-\tanh\left(\frac{x}{2}\right)+\frac{-3\tanh\left(\frac{x}{2}\right)^{3}-\tanh\left(\frac{x}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	46
risch	$\frac{3\,e^{4x}+3\,e^{3x}+5\,e^{2x}+e^{x}+4}{\left(e^{2x}+1\right)^{2}a\left(e^{x}+1\right)}+\frac{3i\ln\left(e^{x}+i\right)}{2a}-\frac{3i\ln\left(e^{x}-i\right)}{2a}$	66
parallelrisch	$\frac{3i(-1-\cosh(2x))\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+3i(1+\cosh(2x))\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)-2\tanh\left(\frac{x}{2}\right)(\cosh(x)+2\cosh(2x)+1)}{2a(1+\cosh(2x))}$	67

input
```
int(sech(x)^3/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(-tanh(1/2*x)+2*(-3/2*tanh(1/2*x)^3-1/2*tanh(1/2*x))/(tanh(1/2*x)^2+1)
^2+3*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(39) = 78$.

Time = 0.09 (sec) , antiderivative size = 325, normalized size of antiderivative = 7.56

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\cosh(x)^4+3(4\cosh(x)+1)\sinh(x)^3+3\sinh(x)^4+3\cosh(x)^3+\left(18\cosh(x)^2+9\cosh(x)+5\right)s}{}$$

input
```
integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
(3*cosh(x)^4 + 3*(4*cosh(x) + 1)*sinh(x)^3 + 3*sinh(x)^4 + 3*cosh(x)^3 + (
18*cosh(x)^2 + 9*cosh(x) + 5)*sinh(x)^2 + 3*(cosh(x)^5 + (5*cosh(x) + 1)*s
inh(x)^4 + sinh(x)^5 + cosh(x)^4 + 2*(5*cosh(x)^2 + 2*cosh(x) + 1)*sinh(x)
^3 + 2*cosh(x)^3 + 2*(5*cosh(x)^3 + 3*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2
 + 2*cosh(x)^2 + (5*cosh(x)^4 + 4*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 1)
*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)) + 5*cosh(x)^2 + (12*cosh
(x)^3 + 9*cosh(x)^2 + 10*cosh(x) + 1)*sinh(x) + cosh(x) + 4)/(a*cosh(x)^5
+ a*sinh(x)^5 + a*cosh(x)^4 + (5*a*cosh(x) + a)*sinh(x)^4 + 2*a*cosh(x)^3
+ 2*(5*a*cosh(x)^2 + 2*a*cosh(x) + a)*sinh(x)^3 + 2*a*cosh(x)^2 + 2*(5*a*c
osh(x)^3 + 3*a*cosh(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (5*a*c
osh(x)^4 + 4*a*cosh(x)^3 + 6*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{sech}^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)**3/(cosh(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = -\frac{e^{(-x)}+5\,e^{(-2\,x)}+3\,e^{(-3\,x)}+3\,e^{(-4\,x)}+4}{ae^{(-x)}+2\,ae^{(-2\,x)}+2\,ae^{(-3\,x)}+ae^{(-4\,x)}+ae^{(-5\,x)}+a} - \frac{3\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-(e^(-x) + 5*e^(-2*x) + 3*e^(-3*x) + 3*e^(-4*x) + 4)/(a*e^(-x) + 2*a*e^(-2
*x) + 2*a*e^(-3*x) + a*e^(-4*x) + a*e^(-5*x) + a) - 3*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\arctan\left(e^{x}\right)}{a} + \frac{e^{(3\,x)}+2\,e^{(2\,x)}-e^{x}+2}{a\left(e^{(2\,x)}+1\right)^2} + \frac{2}{a(e^{x}+1)}$$

input
```
integrate(sech(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3*arctan(e^x)/a + (e^(3*x) + 2*e^(2*x) - e^x + 2)/(a*(e^(2*x) + 1)^2) + 2/
(a*(e^x + 1))
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx = \frac{2}{a\,\left(\mathrm{e}^x+1\right)} + \frac{\frac{2}{a}+\frac{\mathrm{e}^x}{a}}{\mathrm{e}^{2\,x}+1} + \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}} - \frac{2\,\mathrm{e}^x}{a\,\left(2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1\right)}$$

input
```
int(1/(cosh(x)^3*(a + a*cosh(x))),x)
```

output
```
2/(a*(exp(x) + 1)) + (2/a + exp(x)/a)/(exp(2*x) + 1) + (3*atan((exp(x)*(a^
2)^(1/2))/a))/(a^2)^(1/2) - (2*exp(x))/(a*(2*exp(2*x) + exp(4*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.84

$$\int \frac{\operatorname{sech}^3(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{3e^{5x}atan(e^x) + 3e^{4x}atan(e^x) + 6e^{3x}atan(e^x) + 6e^{2x}atan(e^x) + 3e^{x}atan(e^x) + 3atan(e^x) - 3e^{5x} - 3e^{3x} -}{a\left(e^{5x}+e^{4x}+2e^{3x}+2e^{2x}+e^{x}+1\right)}$$

input
```
int(sech(x)^3/(a+a*cosh(x)),x)
```

output
```
(3*e**(5*x)*atan(e**x) + 3*e**(4*x)*atan(e**x) + 6*e**(3*x)*atan(e**x) + 6
*e**(2*x)*atan(e**x) + 3*e**x*atan(e**x) + 3*atan(e**x) - 3*e**(5*x) - 3*e
**(3*x) - e**(2*x) - 2*e**x + 1)/(a*(e**(5*x) + e**(4*x) + 2*e**(3*x) + 2*
e**(2*x) + e**x + 1))
```

3.31 $\int \frac{\text{sech}^4(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{\text{sech}^4(x)}{a+a\cosh(x)}\,dx=-\frac{3\arctan(\sinh(x))}{2a}+\frac{4\tanh(x)}{a}-\frac{3\text{sech}(x)\tanh(x)}{2a}-\frac{\text{sech}^2(x)\tanh(x)}{a+a\cosh(x)}-\frac{4\tanh^3(x)}{3a}$$

output
```
-3/2*arctan(sinh(x))/a+4*tanh(x)/a-3/2*sech(x)*tanh(x)/a-sech(x)^2*tanh(x)
/(a+a*cosh(x))-4/3*tanh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{\text{sech}^4(x)}{a+a\cosh(x)}\,dx=\frac{\cosh\left(\frac{x}{2}\right)\left(6\sinh\left(\frac{x}{2}\right)+\cosh\left(\frac{x}{2}\right)\left(-18\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\left(10-3\text{sech}(x)+2\text{sech}^2(x)\right)\tanh(x)\right)\right)}{3a(1+\cosh(x))}$$

input
```
Integrate[Sech[x]^4/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]*(6*Sinh[x/2] + Cosh[x/2]*(-18*ArcTan[Tanh[x/2]] + (10 - 3*Sech[
x] + 2*Sech[x]^2)*Tanh[x])))/(3*a*(1 + Cosh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 3247, 25, 3042, 3227, 3042, 4254, 2009, 4255, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^4\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3247}$$

$$-\frac{\int -\left((4a-3a\cosh(x))\text{sech}^4(x)\right)dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int (4a-3a\cosh(x))\text{sech}^4(x)dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{\int \frac{4a-3a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)^4}dx}{a^2}$$

$$\Big\downarrow \text{3227}$$

$$\frac{4a\int \text{sech}^4(x)dx-3a\int \text{sech}^3(x)dx}{a^2}-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4a\int \csc\left(ix+\frac{\pi}{2}\right)^4dx-3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \; 4254$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4ia\int \left(1-\tanh^2(x)\right)d(-i\tanh(x))-3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)-3a\int \csc\left(ix+\frac{\pi}{2}\right)^3dx}{a^2}$$

$$\Big\downarrow \; 4255$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{-3a\left(\frac{\int \text{sech}(x)dx}{2}+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)}{a^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)-3a\left(\frac{1}{2}\tanh(x)\text{sech}(x)+\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)}{a^2}$$

$$\Big\downarrow \; 4257$$

$$-\frac{\tanh(x)\text{sech}^2(x)}{a\cosh(x)+a}+\frac{-3a\left(\frac{1}{2}\arctan(\sinh(x))+\frac{1}{2}\tanh(x)\text{sech}(x)\right)+4ia\left(\frac{1}{3}i\tanh^3(x)-i\tanh(x)\right)}{a^2}$$

input

```
Int[Sech[x]^4/(a + a*Cosh[x]),x]
```

output

```
-((Sech[x]^2*Tanh[x])/(a + a*Cosh[x])) + (-3*a*(ArcTan[Sinh[x]]/2 + (Sech[
x]*Tanh[x])/2) + (4*I)*a*((-I)*Tanh[x] + (I/3)*Tanh[x]^3))/a^2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3227
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[c   Int[(b*Sin[e + f*x])^m, x], x] + Simp[d/b   Int
[(b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

rule 3247
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_)/((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(b*c - a*d)*(a + b*Sin[e + f*x]))), x] + Simp[d/(a*(b*c - a*d))
   Int[(c + d*Sin[e + f*x])^n*(a*n - b*(n + 1)*Sin[e + f*x]), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[
c^2 - d^2, 0] && LtQ[n, 0] && (IntegerQ[2*n] || EqQ[c, 0])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4255
```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Simp[b^2*((n - 2)/(n - 1))
  Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
&& IntegerQ[2*n]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

method	result
default	$\frac{\tanh\left(\frac{x}{2}\right)-\frac{8\left(-\frac{5\tanh\left(\frac{x}{2}\right)^{5}}{8}-\frac{2\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{3\tanh\left(\frac{x}{2}\right)}{8}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}-3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$
risch	$-\frac{9\,\mathrm{e}^{6x}+9\,\mathrm{e}^{5x}+24\,\mathrm{e}^{4x}+24\,\mathrm{e}^{3x}+39\,\mathrm{e}^{2x}+7\,\mathrm{e}^{x}+16}{3(\mathrm{e}^{2x}+1)^{3}a(\mathrm{e}^{x}+1)}+\frac{3i\ln(\mathrm{e}^{x}-i)}{2a}-\frac{3i\ln(\mathrm{e}^{x}+i)}{2a}$
parallelrisch	$\frac{9i(\cosh(3x)+3\cosh(x))\ln\left(\tanh\left(\frac{x}{2}\right)-i\right)+9i(-\cosh(3x)-3\cosh(x))\ln\left(\tanh\left(\frac{x}{2}\right)+i\right)+44\left(\cosh(x)+\frac{7\cosh(2x)}{22}+\frac{4\cosh(3x)}{11}+\frac{1}{2}\right)}{6a(\cosh(3x)+3\cosh(x))}$

input
```
int(sech(x)^4/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(tanh(1/2*x)-8*(-5/8*tanh(1/2*x)^5-2/3*tanh(1/2*x)^3-3/8*tanh(1/2*x))/
(tanh(1/2*x)^2+1)^3-3*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(50) = 100.

Time = 0.10 (sec) , antiderivative size = 600, normalized size of antiderivative = 10.71

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/3*(9*cosh(x)^6 + 9*(6*cosh(x) + 1)*sinh(x)^5 + 9*sinh(x)^6 + 9*cosh(x)^
5 + 3*(45*cosh(x)^2 + 15*cosh(x) + 8)*sinh(x)^4 + 24*cosh(x)^4 + 6*(30*cos
h(x)^3 + 15*cosh(x)^2 + 16*cosh(x) + 4)*sinh(x)^3 + 24*cosh(x)^3 + 3*(45*c
osh(x)^4 + 30*cosh(x)^3 + 48*cosh(x)^2 + 24*cosh(x) + 13)*sinh(x)^2 + 9*(c
osh(x)^7 + (7*cosh(x) + 1)*sinh(x)^6 + sinh(x)^7 + cosh(x)^6 + 3*(7*cosh(x
)^2 + 2*cosh(x) + 1)*sinh(x)^5 + 3*cosh(x)^5 + (35*cosh(x)^3 + 15*cosh(x)^
2 + 15*cosh(x) + 3)*sinh(x)^4 + 3*cosh(x)^4 + (35*cosh(x)^4 + 20*cosh(x)^3
 + 30*cosh(x)^2 + 12*cosh(x) + 3)*sinh(x)^3 + 3*cosh(x)^3 + 3*(7*cosh(x)^5
 + 5*cosh(x)^4 + 10*cosh(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 3
*cosh(x)^2 + (7*cosh(x)^6 + 6*cosh(x)^5 + 15*cosh(x)^4 + 12*cosh(x)^3 + 9*
cosh(x)^2 + 6*cosh(x) + 1)*sinh(x) + cosh(x) + 1)*arctan(cosh(x) + sinh(x)
) + 39*cosh(x)^2 + (54*cosh(x)^5 + 45*cosh(x)^4 + 96*cosh(x)^3 + 72*cosh(x
)^2 + 78*cosh(x) + 7)*sinh(x) + 7*cosh(x) + 16)/(a*cosh(x)^7 + a*sinh(x)^7
 + a*cosh(x)^6 + (7*a*cosh(x) + a)*sinh(x)^6 + 3*a*cosh(x)^5 + 3*(7*a*cosh
(x)^2 + 2*a*cosh(x) + a)*sinh(x)^5 + 3*a*cosh(x)^4 + (35*a*cosh(x)^3 + 15*
a*cosh(x)^2 + 15*a*cosh(x) + 3*a)*sinh(x)^4 + 3*a*cosh(x)^3 + (35*a*cosh(x
)^4 + 20*a*cosh(x)^3 + 30*a*cosh(x)^2 + 12*a*cosh(x) + 3*a)*sinh(x)^3 + 3*
a*cosh(x)^2 + 3*(7*a*cosh(x)^5 + 5*a*cosh(x)^4 + 10*a*cosh(x)^3 + 6*a*cosh
(x)^2 + 3*a*cosh(x) + a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 6*a*cosh
(x)^5 + 15*a*cosh(x)^4 + 12*a*cosh(x)^3 + 9*a*cosh(x)^2 + 6*a*cosh(x) +...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx=\frac{\int \frac{\operatorname{sech}^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(sech(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(sech(x)**4/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(50) = 100$.

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^4(x)}{a + a\cosh(x)}\,dx = \frac{7\,e^{(-x)} + 39\,e^{(-2\,x)} + 24\,e^{(-3\,x)} + 24\,e^{(-4\,x)} + 9\,e^{(-5\,x)} + 9\,e^{(-6\,x)} + 16}{3\,(ae^{(-x)} + 3\,ae^{(-2\,x)} + 3\,ae^{(-3\,x)} + 3\,ae^{(-4\,x)} + 3\,ae^{(-5\,x)} + ae^{(-6\,x)} + ae^{(-7\,x)} + a)} + \frac{3\,\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
1/3*(7*e^(-x) + 39*e^(-2*x) + 24*e^(-3*x) + 24*e^(-4*x) + 9*e^(-5*x) + 9*e
^(-6*x) + 16)/(a*e^(-x) + 3*a*e^(-2*x) + 3*a*e^(-3*x) + 3*a*e^(-4*x) + 3*a
*e^(-5*x) + a*e^(-6*x) + a*e^(-7*x) + a) + 3*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^4(x)}{a + a\cosh(x)}\,dx = -\frac{3\,\arctan\left(e^{x}\right)}{a} - \frac{2}{a(e^{x} + 1)} - \frac{3\,e^{(5\,x)} + 6\,e^{(4\,x)} + 24\,e^{(2\,x)} - 3\,e^{x} + 10}{3\,a\left(e^{(2\,x)} + 1\right)^3}$$

input
```
integrate(sech(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-3*arctan(e^x)/a - 2/(a*(e^x + 1)) - 1/3*(3*e^(5*x) + 6*e^(4*x) + 24*e^(2*
x) - 3*e^x + 10)/(a*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = \frac{8}{3\,a\,\left(3\,{\mathrm{e}}^{2\,x}+3\,{\mathrm{e}}^{4\,x}+{\mathrm{e}}^{6\,x}+1\right)}-\frac{\frac{4}{a}-\frac{2\,{\mathrm{e}}^x}{a}}{2\,{\mathrm{e}}^{2\,x}+{\mathrm{e}}^{4\,x}+1}-\frac{2}{a\,\left({\mathrm{e}}^x+1\right)}-\frac{\frac{2}{a}+\frac{{\mathrm{e}}^x}{a}}{{\mathrm{e}}^{2\,x}+1}-\frac{3\,\mathrm{atan}\left(\frac{{\mathrm{e}}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^4*(a + a*cosh(x))),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2
*exp(2*x) + exp(4*x) + 1) - 2/(a*(exp(x) + 1)) - (2/a + exp(x)/a)/(exp(2*x
) + 1) - (3*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.09

$$\int \frac{\operatorname{sech}^4(x)}{a+a\cosh(x)}\,dx = \frac{-9e^{7x}atan(e^x)-9e^{6x}atan(e^x)-27e^{5x}atan(e^x)-27e^{4x}atan(e^x)-27e^{3x}atan(e^x)-27e^{2x}atan(e^x)-9e}{3a\left(e^{7x}+e^{6x}+3e^{5x}+3e^{4x}+3e^{3x}+3e^{2x}\right.}$$

input
```
int(sech(x)^4/(a+a*cosh(x)),x)
```

output
```
( - 9*e**(7*x)*atan(e**x) - 9*e**(6*x)*atan(e**x) - 27*e**(5*x)*atan(e**x)
 - 27*e**(4*x)*atan(e**x) - 27*e**(3*x)*atan(e**x) - 27*e**(2*x)*atan(e**x
) - 9*e**x*atan(e**x) - 9*atan(e**x) + 9*e**(7*x) + 18*e**(5*x) + 3*e**(4*
x) + 3*e**(3*x) - 12*e**(2*x) + 2*e**x - 7)/(3*a*(e**(7*x) + e**(6*x) + 3*
e**(5*x) + 3*e**(4*x) + 3*e**(3*x) + 3*e**(2*x) + e**x + 1))
```

3.32 $\int \frac{1}{1+\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 20

$$\int \frac{1}{1+\cosh(c+dx)}\,dx=\frac{\sinh(c+dx)}{d(1+\cosh(c+dx))}$$

output
```
sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

$$\int \frac{1}{1+\cosh(c+dx)}\,dx=\frac{\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-1),x]
```

output
```
Tanh[(c + d*x)/2]/d
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh(c+dx)+1}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{1}{1+\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3127}$$
$$\frac{\sinh(c+dx)}{d(\cosh(c+dx)+1)}$$

input
```
Int[(1 + Cosh[c + d*x])^(-1),x]
```

output
```
Sinh[c + d*x]/(d*(1 + Cosh[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
derivativedivides	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
default	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
parallelrisch	$\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
risch	$-\frac{2}{d\left(e^{dx+c}+1\right)}$	16

input
```
int(1/(1+cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*tanh(1/2*d*x+1/2*c)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = -\frac{2}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)+d}$$

input
```
integrate(1/(1+cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-2/(d*cosh(d*x + c) + d*sinh(d*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = \begin{cases} \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{d} & \text{for } d \neq 0 \\ \frac{x}{\cosh(c)+1} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c)),x)
```

output
```
Piecewise((tanh(c/2 + d*x/2)/d, Ne(d, 0)), (x/(cosh(c) + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = \frac{2}{d(e^{(-dx-c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c)),x, algorithm="maxima")
```

output
```
2/(d*(e^(-d*x - c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = -\frac{2}{d(e^{(dx+c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c)),x, algorithm="giac")
```

output
```
-2/(d*(e^(d*x + c) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = -\frac{2}{d\,\left(\mathrm{e}^{c+d\,x}+1\right)}$$

input
```
int(1/(cosh(c + d*x) + 1),x)
```

output
```
-2/(d*(exp(c + d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.15

$$\int \frac{1}{1+\cosh(c+dx)}\,dx = \frac{2e^{dx+c}}{d\,(e^{dx+c}+1)}$$

input
```
int(1/(1+cosh(d*x+c)),x)
```

output
```
(2*e**(c + d*x))/(d*(e**(c + d*x) + 1))
```

3.33 $\int \frac{1}{(1+\cosh(c+dx))^2} dx$

Optimal result

Integrand size = 10, antiderivative size = 47

$$\int \frac{1}{(1+\cosh(c+dx))^2} dx = \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))^2} + \frac{\sinh(c+dx)}{3d(1+\cosh(c+dx))}$$

output
```
1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+1/3*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1+\cosh(c+dx))^2} dx = \frac{4\sinh(c+dx)+\sinh(2(c+dx))}{6d(1+\cosh(c+dx))^2}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-2),x]
```

output
```
(4*Sinh[c + d*x] + Sinh[2*(c + d*x)])/(6*d*(1 + Cosh[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(c+dx)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1+\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{3}\int \frac{1}{\cosh(c+dx)+1}dx+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ic+idx+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}$$

input
```
Int[(1 + Cosh[c + d*x])^(-2),x]
```

output
```
Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c
 + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.55

method	result	size
risch	$-\frac{2(3\,\mathrm{e}^{dx+c}+1)}{3d(\mathrm{e}^{dx+c}+1)^{3}}$	26
parallelrisch	$-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{6d}$	29
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{d}$	30
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{d}$	30

input
```
int(1/(1+cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*exp(d*x+c)+1)/d/(exp(d*x+c)+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.40

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = -\frac{2\,(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx\right.}{3\left(d\cosh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)^2+3\left(d\cosh\left(dx+c\right)+d\right)\sinh\left(dx+c\right)^2+3\right.}$$

input
```
integrate(1/(1+cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1)/(d*cosh(d*x + c)^3 + d*sinh(d
*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) + d)*sinh(d*x + c)^2
+ 3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c) + d)*sinh(d
*x + c) + d)
```

Sympy [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.77

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \begin{cases} -\frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d}+\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d} & \text{for } d\neq 0 \\ \frac{x}{(\cosh(c)+1)^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c))**2,x)
```

output
```
Piecewise((-tanh(c/2 + d*x/2)**3/(6*d) + tanh(c/2 + d*x/2)/(2*d), Ne(d, 0)
), (x/(cosh(c) + 1)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(43) = 86$.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.91

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1)} + \frac{2}{3\,d(3\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c)
+ 1)) + 2/3/(d*(3*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) + 1
))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = -\frac{2\left(3\,e^{(dx+c)}+1\right)}{3\,d\left(e^{(dx+c)}+1\right)^3}$$

input
```
integrate(1/(1+cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.53

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = -\frac{2\,\left(3\,\mathrm{e}^{c+d\,x}+1\right)}{3\,d\,\left(\mathrm{e}^{c+d\,x}+1\right)^3}$$

input
```
int(1/(cosh(c + d*x) + 1)^2,x)
```

output
```
-(2*(3*exp(c + d*x) + 1))/(3*d*(exp(c + d*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{(1+\cosh(c+dx))^2}\,dx = \frac{-2e^{dx+c}-\frac{2}{3}}{d\,(e^{3dx+3c}+3e^{2dx+2c}+3e^{dx+c}+1)}$$

input
```
int(1/(1+cosh(d*x+c))^2,x)
```

output
```
(2*( - 3*e**(c + d*x) - 1))/(3*d*(e**(3*c + 3*d*x) + 3*e**(2*c + 2*d*x) +
3*e**(c + d*x) + 1))
```

3.34 $\int \frac{1}{(1+\cosh(c+dx))^3}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 70

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \frac{\sinh(c+dx)}{5d(1+\cosh(c+dx))^3}+\frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))^2}+\frac{2\sinh(c+dx)}{15d(1+\cosh(c+dx))}$$

output
```
1/5*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2/15*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2
/15*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.63

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \frac{15\sinh(c+dx)+6\sinh(2(c+dx))+\sinh(3(c+dx))}{30d(1+\cosh(c+dx))^3}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-3),x]
```

output
```
(15*Sinh[c + d*x] + 6*Sinh[2*(c + d*x)] + Sinh[3*(c + d*x)])/(30*d*(1 + Co
sh[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(c+dx)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1+\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\int \frac{1}{(\cosh(c+dx)+1)^2}dx+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{\cosh(c+dx)+1}dx+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ic+idx+\frac{\pi}{2}\right)+1}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)$$

input
```
Int[(1 + Cosh[c + d*x])^(-3),x]
```

output
```
Sinh[c + d*x]/(5*d*(1 + Cosh[c + d*x])^3) + (2*(Sinh[c + d*x]/(3*d*(1 + Co
sh[c + d*x])^2) + Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x]))))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.53

method	result	size
risch	$-\frac{4\left(10\,{\mathrm e}^{2 d x +2 c}+5\,{\mathrm e}^{d x +c}+1\right)}{15 d \left({\mathrm e}^{d x +c}+1\right)^{5}}$	37
derivativedivides	$\frac{\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{20}-\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{4}}{d}$	43
default	$\frac{\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{20}-\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{6}+\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{4}}{d}$	43
parallelrisch	$\frac{3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}-10 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}+15 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{60 d}$	44

input
```
int(1/(1+cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
-4/15*(10*exp(2*d*x+2*c)+5*exp(d*x+c)+1)/d/(exp(d*x+c)+1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(64) = 128$.

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.49

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx =$$

$$-\frac{}{15\left(d\cosh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^4+5\,d\cosh\left(dx+c\right)^3+\left(4\,d\cosh\left(dx+c\right)+5\,d\right)\sinh\left(dx+c\right)^3+\right.}$$

input
```
integrate(1/(1+cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
-4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) + 5)/(d*cosh(d*x + c)^4 + d*sinh
(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) + 5*d)*sinh(d*x + c
)^3 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 + 15*d*cosh(d*x + c) + 1
0*d)*sinh(d*x + c)^2 + 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 + 15*d*co
sh(d*x + c)^2 + 20*d*cosh(d*x + c) + 9*d)*sinh(d*x + c) + 5*d)
```

Sympy [A] (verification not implemented)

Time = 1.00 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \begin{cases} \frac{\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{20d} - \frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{6d} + \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{(\cosh(c)+1)^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c))**3,x)
```

output
```
Piecewise((tanh(c/2 + d*x/2)**5/(20*d) - tanh(c/2 + d*x/2)**3/(6*d) + tanh
(c/2 + d*x/2)/(4*d), Ne(d, 0)), (x/(cosh(c) + 1)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(64) = 128$.

Time = 0.04 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.93

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \frac{4\,e^{(-dx-c)}}{3\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)} + \frac{8\,e^{(-2\,dx-2\,c)}}{3\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)} + \frac{4}{15\,d(5\,e^{(-dx-c)}+10\,e^{(-2\,dx-2\,c)}+10\,e^{(-3\,dx-3\,c)}+5\,e^{(-4\,dx-4\,c)}+e^{(-5\,dx-5\,c)}+1)}$$

input
```
integrate(1/(1+cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
 3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) + 1)) + 8/3*e^(-2*d*x - 2*c)
/(d*(5*e^(-d*x - c) + 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*
d*x - 4*c) + e^(-5*d*x - 5*c) + 1)) + 4/15/(d*(5*e^(-d*x - c) + 10*e^(-2*d
*x - 2*c) + 10*e^(-3*d*x - 3*c) + 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) +
1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = -\frac{4\left(10\,e^{(2\,dx+2\,c)}+5\,e^{(dx+c)}+1\right)}{15\,d\left(e^{(dx+c)}+1\right)^5}$$

input
```
integrate(1/(1+cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
-4/15*(10*e^(2*d*x + 2*c) + 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.51

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = -\frac{4\left(5\,\mathrm{e}^{c+d\,x}+10\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}{15\,d\left(\mathrm{e}^{c+d\,x}+1\right)^5}$$

input
```
int(1/(cosh(c + d*x) + 1)^3,x)
```

output
```
-(4*(5*exp(c + d*x) + 10*exp(2*c + 2*d*x) + 1))/(15*d*(exp(c + d*x) + 1)^5
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.24

$$\int \frac{1}{(1+\cosh(c+dx))^3}\,dx = \frac{-\frac{8e^{2dx+2c}}{3}-\frac{4e^{dx+c}}{3}-\frac{4}{15}}{d\left(e^{5dx+5c}+5e^{4dx+4c}+10e^{3dx+3c}+10e^{2dx+2c}+5e^{dx+c}+1\right)}$$

input
```
int(1/(1+cosh(d*x+c))^3,x)
```

output
```
(4*( - 10*e**(2*c + 2*d*x) - 5*e**(c + d*x) - 1))/(15*d*(e**(5*c + 5*d*x)
+ 5*e**(4*c + 4*d*x) + 10*e**(3*c + 3*d*x) + 10*e**(2*c + 2*d*x) + 5*e**(c
 + d*x) + 1))
```

3.35 $\int \frac{1}{(1+\cosh(c+dx))^4}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 93

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \frac{\sinh(c+dx)}{7d(1+\cosh(c+dx))^4} + \frac{3\sinh(c+dx)}{35d(1+\cosh(c+dx))^3} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))^2} + \frac{2\sinh(c+dx)}{35d(1+\cosh(c+dx))}$$

output
```
1/7*sinh(d*x+c)/d/(1+cosh(d*x+c))^4+3/35*sinh(d*x+c)/d/(1+cosh(d*x+c))^3+2
/35*sinh(d*x+c)/d/(1+cosh(d*x+c))^2+2/35*sinh(d*x+c)/d/(1+cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.58

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \frac{56\sinh(c+dx) + 28\sinh(2(c+dx)) + 8\sinh(3(c+dx)) + \sinh(4(c+dx))}{140d(1+\cosh(c+dx))^4}$$

input
```
Integrate[(1 + Cosh[c + d*x])^(-4),x]
```

output
```
(56*Sinh[c + d*x] + 28*Sinh[2*(c + d*x)] + 8*Sinh[3*(c + d*x)] + Sinh[4*(c
 + d*x)])/(140*d*(1 + Cosh[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(\cosh(c+dx)+1)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1+\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\int \frac{1}{(\cosh(c+dx)+1)^3}dx+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(\cosh(c+dx)+1)^2}dx+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}\right)+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\left(\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)+1\right)^2}dx\right)$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{\cosh(c+dx)+1}dx+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)+\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}\right)+\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\left(\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ic+idx+\frac{\pi}{2}\right)+1}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{\sinh(c+dx)}{7d(\cosh(c+dx)+1)^4}+\frac{3}{7}\left(\frac{\sinh(c+dx)}{5d(\cosh(c+dx)+1)^3}+\frac{2}{5}\left(\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)}+\frac{\sinh(c+dx)}{3d(\cosh(c+dx)+1)^2}\right)\right)$$

input
```
Int[(1 + Cosh[c + d*x])^(-4),x]
```

output
```
Sinh[c + d*x]/(7*d*(1 + Cosh[c + d*x])^4) + (3*(Sinh[c + d*x]/(5*d*(1 + Co
sh[c + d*x])^3) + (2*(Sinh[c + d*x]/(3*d*(1 + Cosh[c + d*x])^2) + Sinh[c +
 d*x]/(3*d*(1 + Cosh[c + d*x]))))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.52

method	result	size
risch	$-\frac{4\left(35\,{\mathrm e}^{3 d x +3 c}+21\,{\mathrm e}^{2 d x +2 c}+7\,{\mathrm e}^{d x +c}+1\right)}{35 d \left({\mathrm e}^{d x +c}+1\right)^{7}}$	48
parallelrisch	$-\frac{\left(\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{6}-\frac{21 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{4}}{5}+7 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{2}-7\right) \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{56 d}$	54
derivativedivides	$\frac{-\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{56}+\frac{3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{40}-\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{8}+\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{8}}{d}$	56
default	$\frac{-\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{7}}{56}+\frac{3 \tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{5}}{40}-\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}{8}+\frac{\tanh \left(\frac{d x}{2}+\frac{c}{2}\right)}{8}}{d}$	56

input
```
int(1/(1+cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/35*(35*exp(3*d*x+3*c)+21*exp(2*d*x+2*c)+7*exp(d*x+c)+1)/d/(exp(d*x+c)+1
)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.73

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx =$$

$$-\frac{}{35\,\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}+7\,d\cosh\left(dx+c\right)^{5}+\left(6\,d\cosh\left(dx+c\right)+7\,d\right)\sinh\left(dx+c\right)^{5}+\right.}$$

input
```
integrate(1/(1+cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) + 2)*sinh(d*x + c) + 35*si
nh(d*x + c)^2 + 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)
^6 + 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) + 7*d)*sinh(d*x + c)^5 + 21*
d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 + 35*d*cosh(d*x + c) + 21*d)*sin
h(d*x + c)^4 + 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 + 70*d*cosh(d*
x + c)^2 + 84*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)
^2 + (15*d*cosh(d*x + c)^4 + 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2
+ 105*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 + 22*d*cosh(d*x + c) + (6*d*
cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 + 105*d*cosh
(d*x + c)^2 + 70*d*cosh(d*x + c) + 20*d)*sinh(d*x + c) + 7*d)
```

Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.73

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \begin{cases} -\frac{\tanh^7\left(\frac{c}{2}+\frac{dx}{2}\right)}{56d} + \frac{3\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{40d} - \frac{\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} + \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{8d} & \text{for } d \neq 0 \\ \frac{x}{(\cosh(c)+1)^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1+cosh(d*x+c))**4,x)
```

output
```
Piecewise((-tanh(c/2 + d*x/2)**7/(56*d) + 3*tanh(c/2 + d*x/2)**5/(40*d) -
tanh(c/2 + d*x/2)**3/(8*d) + tanh(c/2 + d*x/2)/(8*d), Ne(d, 0)), (x/(cosh(
c) + 1)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(85) = 170$.

Time = 0.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.91

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx$$
$$= \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)}+21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}+35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}}$$
$$+\frac{12\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)}+21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}+35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7}}$$
$$+\frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)}+21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}+35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-7\,c)}}$$
$$+\frac{4}{35\,d(7\,e^{(-dx-c)}+21\,e^{(-2\,dx-2\,c)}+35\,e^{(-3\,dx-3\,c)}+35\,e^{(-4\,dx-4\,c)}+21\,e^{(-5\,dx-5\,c)}+7\,e^{(-6\,dx-6\,c)}+e^{(-7\,dx-}}$$

input
```
integrate(1/(1+cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e
^(-7*d*x - 7*c) + 1)) + 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) + 21*e^(-
2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x -
5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1)) + 4*e^(-3*d*x - 3*c)/(d
*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) + 35*e^(-4*d*
x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) + 1
)) + 4/35/(d*(7*e^(-d*x - c) + 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) +
 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) + 7*e^(-6*d*x - 6*c) + e^(-7*d*
x - 7*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.51

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)}+21\,e^{(2\,dx+2\,c)}+7\,e^{(dx+c)}+1\right)}{35\,d(e^{(dx+c)}+1)^7}$$

input
```
integrate(1/(1+cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) + 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) + 1)/(d*(e^
(d*x + c) + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 283, normalized size of antiderivative = 3.04

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = -\frac{4}{35\,d\,\left(4\,\mathrm{e}^{c+d\,x}+6\,\mathrm{e}^{2\,c+2\,d\,x}+4\,\mathrm{e}^{3\,c+3\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} - \frac{16\,\mathrm{e}^{c+d\,x}}{35\,d\,\left(5\,\mathrm{e}^{c+d\,x}+10\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{3\,c+3\,d\,x}+5\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{5\,c+5\,d\,x}+1\right)} - \frac{8\,\mathrm{e}^{2\,c+2\,d\,x}}{7\,d\,\left(6\,\mathrm{e}^{c+d\,x}+15\,\mathrm{e}^{2\,c+2\,d\,x}+20\,\mathrm{e}^{3\,c+3\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+6\,\mathrm{e}^{5\,c+5\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} - \frac{16\,\mathrm{e}^{3\,c+3\,d\,x}}{7\,d\,\left(7\,\mathrm{e}^{c+d\,x}+21\,\mathrm{e}^{2\,c+2\,d\,x}+35\,\mathrm{e}^{3\,c+3\,d\,x}+35\,\mathrm{e}^{4\,c+4\,d\,x}+21\,\mathrm{e}^{5\,c+5\,d\,x}+7\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{7\,c+7\,d\,x}+1\right)}$$

input
```
int(1/(cosh(c + d*x) + 1)^4,x)
```

output
```
- 4/(35*d*(4*exp(c + d*x) + 6*exp(2*c + 2*d*x) + 4*exp(3*c + 3*d*x) + exp(
4*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) + 10*exp(2*c
+ 2*d*x) + 10*exp(3*c + 3*d*x) + 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) + 1
)) - (8*exp(2*c + 2*d*x))/(7*d*(6*exp(c + d*x) + 15*exp(2*c + 2*d*x) + 20*
exp(3*c + 3*d*x) + 15*exp(4*c + 4*d*x) + 6*exp(5*c + 5*d*x) + exp(6*c + 6*
d*x) + 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) + 21*exp(2*c + 2*d
*x) + 35*exp(3*c + 3*d*x) + 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) + 7*
exp(6*c + 6*d*x) + exp(7*c + 7*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.32

$$\int \frac{1}{(1+\cosh(c+dx))^4}\,dx = \frac{-4e^{3dx+3c}-\frac{12e^{2dx+2c}}{5}-\frac{4e^{dx+c}}{5}-\frac{4}{35}}{d\left(e^{7dx+7c}+7e^{6dx+6c}+21e^{5dx+5c}+35e^{4dx+4c}+35e^{3dx+3c}+21e^{2dx+2c}+7e^{dx+c}+1\right)}$$

input
```
int(1/(1+cosh(d*x+c))^4,x)
```

output
```
(4*( - 35*e**(3*c + 3*d*x) - 21*e**(2*c + 2*d*x) - 7*e**(c + d*x) - 1))/(3
5*d*(e**(7*c + 7*d*x) + 7*e**(6*c + 6*d*x) + 21*e**(5*c + 5*d*x) + 35*e**(
4*c + 4*d*x) + 35*e**(3*c + 3*d*x) + 21*e**(2*c + 2*d*x) + 7*e**(c + d*x)
+ 1))
```

3.36 $\int \frac{1}{1-\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 23

$$\int \frac{1}{1-\cosh(c+dx)}\,dx=-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

output
```
-sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

$$\int \frac{1}{1-\cosh(c+dx)}\,dx=\frac{\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-1),x]
```

output
```
Coth[(c + d*x)/2]/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{1-\cosh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3127}$$

$$-\frac{\sinh(c+dx)}{d(1-\cosh(c+dx))}$$

input
```
Int[(1 - Cosh[c + d*x])^(-1),x]
```

output
```
-(Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.61

method	result	size
parallelrisch	$\frac{\coth\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}$	14
derivativedivides	$\frac{1}{d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$	16
default	$\frac{1}{d\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}$	16
risch	$\frac{2}{d\left(e^{dx+c}-1\right)}$	16

input
```
int(1/(1-cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
coth(1/2*d*x+1/2*c)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{2}{d\cosh\left(dx+c\right)+d\sinh\left(dx+c\right)-d}$$

input
```
integrate(1/(1-cosh(d*x+c)),x, algorithm="fricas")
```

output
```
2/(d*cosh(d*x + c) + d*sinh(d*x + c) - d)
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \begin{cases} \frac{1}{d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d\neq 0 \\ \frac{x}{1-\cosh(c)} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c)),x)
```

output
```
Piecewise((1/(d*tanh(c/2 + d*x/2)), Ne(d, 0)), (x/(1 - cosh(c)), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = -\frac{2}{d(e^{(-dx-c)}-1)}$$

input
```
integrate(1/(1-cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-2/(d*(e^(-d*x - c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{2}{d(e^{(dx+c)}-1)}$$

input
```
integrate(1/(1-cosh(d*x+c)),x, algorithm="giac")
```

output
```
2/(d*(e^(d*x + c) - 1))
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.65

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{2}{d\,(\mathrm{e}^{c+d\,x}-1)}$$

input
```
int(-1/(cosh(c + d*x) - 1),x)
```

output
```
2/(d*(exp(c + d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int \frac{1}{1-\cosh(c+dx)}\,dx = \frac{2e^{dx+c}}{d\,(e^{dx+c}-1)}$$

input
```
int(1/(1-cosh(d*x+c)),x)
```

output
```
(2*e**(c + d*x))/(d*(e**(c + d*x) - 1))
```

3.37 $\int \frac{1}{(1-\cosh(c+dx))^2} dx$

Optimal result

Integrand size = 12, antiderivative size = 51

$$\int \frac{1}{(1-\cosh(c+dx))^2} dx = -\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2} - \frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}$$

output
```
-1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-1/3*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.61

$$\int \frac{1}{(1-\cosh(c+dx))^2} dx = \frac{(-2+\cosh(c+dx))\sinh(c+dx)}{3d(-1+\cosh(c+dx))^2}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-2),x]
```

output
```
((-2 + Cosh[c + d*x])*Sinh[c + d*x])/(3*d*(-1 + Cosh[c + d*x])^2)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}dx-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3127}$$

$$-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}$$

input
```
Int[(1 - Cosh[c + d*x])^(-2),x]
```

output
```
-1/3*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cos
h[c + d*x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{2\left(3\,{\mathrm e}^{d x +c}-1\right)}{3 d \left({\mathrm e}^{d x +c}-1\right)^{3}}$	26
parallelrisch	$-\frac{\coth\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}-3\coth\left(\frac{d x}{2}+\frac{c}{2}\right)}{6 d}$	29
derivativedivides	$\frac{\frac{1}{2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{1}{6\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}}{d}$	32
default	$\frac{\frac{1}{2\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)}-\frac{1}{6\tanh\left(\frac{d x}{2}+\frac{c}{2}\right)^{3}}}{d}$	32

input
```
int(1/(1-cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*(3*exp(d*x+c)-1)/d/(exp(d*x+c)-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(43) = 86$.

Time = 0.07 (sec) , antiderivative size = 117, normalized size of antiderivative = 2.29

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = -\frac{2\left(3\cosh\left(dx+c\right)+3\sinh\left(dx\right.\right.}{3\left(d\cosh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^3-3\,d\cosh\left(dx+c\right)^2+3\left(d\cosh\left(dx+c\right)-d\right)\sinh\left(dx+c\right)^2+3\right.}$$

input
```
integrate(1/(1-cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(d*x + c) + 3*sinh(d*x + c) - 1)/(d*cosh(d*x + c)^3 + d*sinh(d
*x + c)^3 - 3*d*cosh(d*x + c)^2 + 3*(d*cosh(d*x + c) - d)*sinh(d*x + c)^2
+ 3*d*cosh(d*x + c) + 3*(d*cosh(d*x + c)^2 - 2*d*cosh(d*x + c) + d)*sinh(d
*x + c) - d)
```

Sympy [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.76

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = \begin{cases} \frac{1}{2d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{6d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d \neq 0 \\ \frac{x}{(1-\cosh(c))^2} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c))**2,x)
```

output
```
Piecewise((1/(2*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3), Ne(d,
 0)), (x/(1 - cosh(c))**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. 2(43) = 86.

Time = 0.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.76

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = \frac{2\,e^{(-dx-c)}}{d(3\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}-1)} - \frac{2}{3\,d(3\,e^{(-dx-c)}-3\,e^{(-2\,dx-2\,c)}+e^{(-3\,dx-3\,c)}-1)}$$

input
```
integrate(1/(1-cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
2*e^(-d*x - c)/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c)
- 1)) - 2/3/(d*(3*e^(-d*x - c) - 3*e^(-2*d*x - 2*c) + e^(-3*d*x - 3*c) - 1
))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = -\frac{2\left(3\,e^{(dx+c)}-1\right)}{3\,d\left(e^{(dx+c)}-1\right)^3}$$

input
```
integrate(1/(1-cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
-2/3*(3*e^(d*x + c) - 1)/(d*(e^(d*x + c) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.49

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = -\frac{2\left(3\,\mathrm{e}^{c+d\,x}-1\right)}{3\,d\,\left(\mathrm{e}^{c+d\,x}-1\right)^3}$$

input
```
int(1/(cosh(c + d*x) - 1)^2,x)
```

output
```
-(2*(3*exp(c + d*x) - 1))/(3*d*(exp(c + d*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{(1-\cosh(c+dx))^2}\,dx = \frac{-2e^{dx+c}+\frac{2}{3}}{d\left(e^{3dx+3c}-3e^{2dx+2c}+3e^{dx+c}-1\right)}$$

input
```
int(1/(1-cosh(d*x+c))^2,x)
```

output
```
(2*( - 3*e**(c + d*x) + 1))/(3*d*(e**(3*c + 3*d*x) - 3*e**(2*c + 2*d*x) +
3*e**(c + d*x) - 1))
```

3.38 $\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 76

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = -\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))^2} - \frac{2\sinh(c+dx)}{15d(1-\cosh(c+dx))}$$

output
```
-1/5*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-2/15*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-
2/15*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.54

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{(8-6\cosh(c+dx)+\cosh(2(c+dx)))\sinh(c+dx)}{15d(-1+\cosh(c+dx))^3}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-3),x]
```

output
```
((8 - 6*Cosh[c + d*x] + Cosh[2*(c + d*x)])*Sinh[c + d*x])/(15*d*(-1 + Cosh
[c + d*x])^3)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\int \frac{1}{(1-\cosh(c+dx))^2}dx-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}dx-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}$$

input
```
Int[(1 - Cosh[c + d*x])^(-3),x]
```

output
```
-1/5*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^3) + (2*(-1/3*Sinh[c + d*x]/(d*(
1 - Cosh[c + d*x])^2) - Sinh[c + d*x]/(3*d*(1 - Cosh[c + d*x]))))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.49

method	result	size
risch	$\frac{\frac{8\,{\rm e}^{2dx+2c}}{3}-\frac{4\,{\rm e}^{dx+c}}{3}+\frac{4}{15}}{d\left({\rm e}^{dx+c}-1\right)^{5}}$	37
parallelrisch	$\frac{3\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}-10\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+15\coth\left(\frac{dx}{2}+\frac{c}{2}\right)}{60d}$	44
derivativedivides	$\frac{\frac{1}{20\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{1}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{d}$	45
default	$\frac{\frac{1}{20\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}+\frac{1}{4\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{1}{6\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}}{d}$	45

input
```
int(1/(1-cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
4/15*(10*exp(2*d*x+2*c)-5*exp(d*x+c)+1)/d/(exp(d*x+c)-1)^5
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(64) = 128$.

Time = 0.11 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.29

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx$$

$$= \frac{}{15\left(d\cosh\left(dx+c\right)^4 + d\sinh\left(dx+c\right)^4 - 5\,d\cosh\left(dx+c\right)^3 + \left(4\,d\cosh\left(dx+c\right) - 5\,d\right)\sinh\left(dx+c\right)^3 + 1}$$

input
```
integrate(1/(1-cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
4/15*(11*cosh(d*x + c) + 9*sinh(d*x + c) - 5)/(d*cosh(d*x + c)^4 + d*sinh(
d*x + c)^4 - 5*d*cosh(d*x + c)^3 + (4*d*cosh(d*x + c) - 5*d)*sinh(d*x + c)
^3 + 10*d*cosh(d*x + c)^2 + (6*d*cosh(d*x + c)^2 - 15*d*cosh(d*x + c) + 10
*d)*sinh(d*x + c)^2 - 11*d*cosh(d*x + c) + (4*d*cosh(d*x + c)^3 - 15*d*cos
h(d*x + c)^2 + 20*d*cosh(d*x + c) - 9*d)*sinh(d*x + c) + 5*d)
```

Sympy [A] (verification not implemented)

Time = 1.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.74

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \begin{cases} \frac{1}{4d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{6d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{1}{20d\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d \neq 0 \\ \frac{x}{(1-\cosh\left(c\right))^3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c))**3,x)
```

output
```
Piecewise((1/(4*d*tanh(c/2 + d*x/2)) - 1/(6*d*tanh(c/2 + d*x/2)**3) + 1/(2
0*d*tanh(c/2 + d*x/2)**5), Ne(d, 0)), (x/(1 - cosh(c))**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(64) = 128$.

Time = 0.03 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.70

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{4\,e^{(-dx-c)}}{3\,d(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} - 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} - 1)} - \frac{8\,e^{(-2\,dx-2\,c)}}{3\,d(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} - 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} - 1)} - \frac{4}{15\,d(5\,e^{(-dx-c)} - 10\,e^{(-2\,dx-2\,c)} + 10\,e^{(-3\,dx-3\,c)} - 5\,e^{(-4\,dx-4\,c)} + e^{(-5\,dx-5\,c)} - 1)}$$

input
```
integrate(1/(1-cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
4/3*e^(-d*x - c)/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x -
 3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) - 1)) - 8/3*e^(-2*d*x - 2*c)
/(d*(5*e^(-d*x - c) - 10*e^(-2*d*x - 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*
d*x - 4*c) + e^(-5*d*x - 5*c) - 1)) - 4/15/(d*(5*e^(-d*x - c) - 10*e^(-2*d
*x - 2*c) + 10*e^(-3*d*x - 3*c) - 5*e^(-4*d*x - 4*c) + e^(-5*d*x - 5*c) -
1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{4\left(10\,e^{(2\,dx+2\,c)} - 5\,e^{(dx+c)} + 1\right)}{15\,d(e^{(dx+c)} - 1)^5}$$

input
```
integrate(1/(1-cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
4/15*(10*e^(2*d*x + 2*c) - 5*e^(d*x + c) + 1)/(d*(e^(d*x + c) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{4\,\left(10\,{\mathrm{e}}^{2\,c+2\,d\,x}-5\,{\mathrm{e}}^{c+d\,x}+1\right)}{15\,d\,{\left({\mathrm{e}}^{c+d\,x}-1\right)}^5}$$

input
```
int(-1/(cosh(c + d*x) - 1)^3,x)
```

output
```
(4*(10*exp(2*c + 2*d*x) - 5*exp(c + d*x) + 1))/(15*d*(exp(c + d*x) - 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.14

$$\int \frac{1}{(1-\cosh(c+dx))^3}\,dx = \frac{\frac{8e^{2dx+2c}}{3}-\frac{4e^{dx+c}}{3}+\frac{4}{15}}{d\left(e^{5dx+5c}-5e^{4dx+4c}+10e^{3dx+3c}-10e^{2dx+2c}+5e^{dx+c}-1\right)}$$

input
```
int(1/(1-cosh(d*x+c))^3,x)
```

output
```
(4*(10*e**(2*c + 2*d*x) - 5*e**(c + d*x) + 1))/(15*d*(e**(5*c + 5*d*x) - 5
*e**(4*c + 4*d*x) + 10*e**(3*c + 3*d*x) - 10*e**(2*c + 2*d*x) + 5*e**(c +
d*x) - 1))
```

3.39 $\int \frac{1}{(1-\cosh(c+dx))^4} dx$

Optimal result

Integrand size = 12, antiderivative size = 101

$$\int \frac{1}{(1-\cosh(c+dx))^4} dx = -\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4} - \frac{3\sinh(c+dx)}{35d(1-\cosh(c+dx))^3} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))^2} - \frac{2\sinh(c+dx)}{35d(1-\cosh(c+dx))}$$

output
```
-1/7*sinh(d*x+c)/d/(1-cosh(d*x+c))^4-3/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^3-
2/35*sinh(d*x+c)/d/(1-cosh(d*x+c))^2-2/35*sinh(d*x+c)/d/(1-cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.50

$$\int \frac{1}{(1-\cosh(c+dx))^4} dx = \frac{(-32+29\cosh(c+dx)-8\cosh(2(c+dx))+\cosh(3(c+dx)))\sinh(c+dx)}{70d(-1+\cosh(c+dx))^4}$$

input
```
Integrate[(1 - Cosh[c + d*x])^(-4),x]
```

output
```
((-32 + 29*Cosh[c + d*x] - 8*Cosh[2*(c + d*x)] + Cosh[3*(c + d*x)])*Sinh[c
 + d*x])/(70*d*(-1 + Cosh[c + d*x])^4)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3129, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(1-\cosh(c+dx))^4} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\int \frac{1}{(1-\cosh(c+dx))^3}dx-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}+\frac{3}{7}\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\int \frac{1}{(1-\cosh(c+dx))^2}dx-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}\right)-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}+\frac{3}{7}\left(-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\int \frac{1}{\left(1-\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx\right)$$

$$\Big\downarrow \text{3129}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(\frac{1}{3}\int \frac{1}{1-\cosh(c+dx)}dx-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}\right)-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}+\frac{3}{7}\left(-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}+\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ic+idx+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{3}{7}\left(\frac{2}{5}\left(-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))}-\frac{\sinh(c+dx)}{3d(1-\cosh(c+dx))^2}\right)-\frac{\sinh(c+dx)}{5d(1-\cosh(c+dx))^3}\right)-\frac{\sinh(c+dx)}{7d(1-\cosh(c+dx))^4}$$

input
```
Int[(1 - Cosh[c + d*x])^(-4),x]
```

output
```
-1/7*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^4) + (3*(-1/5*Sinh[c + d*x]/(d*(
1 - Cosh[c + d*x])^3) + (2*(-1/3*Sinh[c + d*x]/(d*(1 - Cosh[c + d*x])^2) -
 Sinh[c + d*x]/(3*d*(1 - Cosh[c + d*x]))))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{4\left(35\,\mathrm{e}^{3dx+3c}-21\,\mathrm{e}^{2dx+2c}+7\,\mathrm{e}^{dx+c}-1\right)}{35d\left(\mathrm{e}^{dx+c}-1\right)^{7}}$	48
parallelrisch	$-\frac{\left(\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{6}-\frac{21\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{5}+7\coth\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-7\right)\coth\left(\frac{dx}{2}+\frac{c}{2}\right)}{56d}$	54
derivativedivides	$\frac{-\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{3}{40\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{1}{56\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}+\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	58
default	$\frac{-\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}+\frac{3}{40\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}-\frac{1}{56\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}+\frac{1}{8\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}}{d}$	58

input
```
int(1/(1-cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
-4/35*(35*exp(3*d*x+3*c)-21*exp(2*d*x+2*c)+7*exp(d*x+c)-1)/d/(exp(d*x+c)-1
)^7
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(85) = 170$.

Time = 0.07 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.44

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx =$$

$$-\frac{}{35\left(d\cosh\left(dx+c\right)^{6}+d\sinh\left(dx+c\right)^{6}-7\,d\cosh\left(dx+c\right)^{5}+\left(6\,d\cosh\left(dx+c\right)-7\,d\right)\sinh\left(dx+c\right)^{5}+\right.}$$

input
```
integrate(1/(1-cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-4/35*(35*cosh(d*x + c)^2 + 10*(7*cosh(d*x + c) - 2)*sinh(d*x + c) + 35*si
nh(d*x + c)^2 - 22*cosh(d*x + c) + 7)/(d*cosh(d*x + c)^6 + d*sinh(d*x + c)
^6 - 7*d*cosh(d*x + c)^5 + (6*d*cosh(d*x + c) - 7*d)*sinh(d*x + c)^5 + 21*
d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c)^2 - 35*d*cosh(d*x + c) + 21*d)*sin
h(d*x + c)^4 - 35*d*cosh(d*x + c)^3 + (20*d*cosh(d*x + c)^3 - 70*d*cosh(d*
x + c)^2 + 84*d*cosh(d*x + c) - 35*d)*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)
^2 + (15*d*cosh(d*x + c)^4 - 70*d*cosh(d*x + c)^3 + 126*d*cosh(d*x + c)^2
- 105*d*cosh(d*x + c) + 35*d)*sinh(d*x + c)^2 - 22*d*cosh(d*x + c) + (6*d*
cosh(d*x + c)^5 - 35*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^3 - 105*d*cosh
(d*x + c)^2 + 70*d*cosh(d*x + c) - 20*d)*sinh(d*x + c) + 7*d)
```

Sympy [A] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.72

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = \begin{cases} \frac{1}{8d\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{8d\tanh^3\left(\frac{c}{2}+\frac{dx}{2}\right)} + \frac{3}{40d\tanh^5\left(\frac{c}{2}+\frac{dx}{2}\right)} - \frac{1}{56d\tanh^7\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } d \neq 0 \\ \frac{x}{(1-\cosh(c))^4} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(1-cosh(d*x+c))**4,x)
```

output
```
Piecewise((1/(8*d*tanh(c/2 + d*x/2)) - 1/(8*d*tanh(c/2 + d*x/2)**3) + 3/(4
0*d*tanh(c/2 + d*x/2)**5) - 1/(56*d*tanh(c/2 + d*x/2)**7), Ne(d, 0)), (x/(
1 - cosh(c))**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(85) = 170$.

Time = 0.04 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.60

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx$$
$$= \frac{4\,e^{(-dx-c)}}{5\,d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c)}}$$
$$- \frac{12\,e^{(-2\,dx-2\,c)}}{5\,d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-}}$$
$$+ \frac{4\,e^{(-3\,dx-3\,c)}}{d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-7\,c}}$$
$$- \frac{4}{35\,d(7\,e^{(-dx-c)} - 21\,e^{(-2\,dx-2\,c)} + 35\,e^{(-3\,dx-3\,c)} - 35\,e^{(-4\,dx-4\,c)} + 21\,e^{(-5\,dx-5\,c)} - 7\,e^{(-6\,dx-6\,c)} + e^{(-7\,dx-}}$$

input
```
integrate(1/(1-cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
4/5*e^(-d*x - c)/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x -
 3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e
^(-7*d*x - 7*c) - 1)) - 12/5*e^(-2*d*x - 2*c)/(d*(7*e^(-d*x - c) - 21*e^(-
2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x -
5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1)) + 4*e^(-3*d*x - 3*c)/(d
*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) - 35*e^(-4*d*
x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*x - 7*c) - 1
)) - 4/35/(d*(7*e^(-d*x - c) - 21*e^(-2*d*x - 2*c) + 35*e^(-3*d*x - 3*c) -
 35*e^(-4*d*x - 4*c) + 21*e^(-5*d*x - 5*c) - 7*e^(-6*d*x - 6*c) + e^(-7*d*
x - 7*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.47

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = -\frac{4\left(35\,e^{(3\,dx+3\,c)} - 21\,e^{(2\,dx+2\,c)} + 7\,e^{(dx+c)} - 1\right)}{35\,d\left(e^{(dx+c)} - 1\right)^7}$$

input
```
integrate(1/(1-cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
-4/35*(35*e^(3*d*x + 3*c) - 21*e^(2*d*x + 2*c) + 7*e^(d*x + c) - 1)/(d*(e^
(d*x + c) - 1)^7)
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 283, normalized size of antiderivative = 2.80

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = -\frac{4}{35\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}-4\,\mathrm{e}^{c+d\,x}-4\,\mathrm{e}^{3\,c+3\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} - \frac{16\,\mathrm{e}^{c+d\,x}}{35\,d\,\left(5\,\mathrm{e}^{c+d\,x}-10\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{3\,c+3\,d\,x}-5\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{5\,c+5\,d\,x}-1\right)} - \frac{8\,\mathrm{e}^{2\,c+2\,d\,x}}{7\,d\,\left(15\,\mathrm{e}^{2\,c+2\,d\,x}-6\,\mathrm{e}^{c+d\,x}-20\,\mathrm{e}^{3\,c+3\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}-6\,\mathrm{e}^{5\,c+5\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} - \frac{16\,\mathrm{e}^{3\,c+3\,d\,x}}{7\,d\,\left(7\,\mathrm{e}^{c+d\,x}-21\,\mathrm{e}^{2\,c+2\,d\,x}+35\,\mathrm{e}^{3\,c+3\,d\,x}-35\,\mathrm{e}^{4\,c+4\,d\,x}+21\,\mathrm{e}^{5\,c+5\,d\,x}-7\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{7\,c+7\,d\,x}-1\right)}$$

input
```
int(1/(cosh(c + d*x) - 1)^4,x)
```

output
```
- 4/(35*d*(6*exp(2*c + 2*d*x) - 4*exp(c + d*x) - 4*exp(3*c + 3*d*x) + exp(
4*c + 4*d*x) + 1)) - (16*exp(c + d*x))/(35*d*(5*exp(c + d*x) - 10*exp(2*c
+ 2*d*x) + 10*exp(3*c + 3*d*x) - 5*exp(4*c + 4*d*x) + exp(5*c + 5*d*x) - 1
)) - (8*exp(2*c + 2*d*x))/(7*d*(15*exp(2*c + 2*d*x) - 6*exp(c + d*x) - 20*
exp(3*c + 3*d*x) + 15*exp(4*c + 4*d*x) - 6*exp(5*c + 5*d*x) + exp(6*c + 6*
d*x) + 1)) - (16*exp(3*c + 3*d*x))/(7*d*(7*exp(c + d*x) - 21*exp(2*c + 2*d
*x) + 35*exp(3*c + 3*d*x) - 35*exp(4*c + 4*d*x) + 21*exp(5*c + 5*d*x) - 7*
exp(6*c + 6*d*x) + exp(7*c + 7*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.22

$$\int \frac{1}{(1-\cosh(c+dx))^4}\,dx = \frac{-4e^{3dx+3c}+\frac{12e^{2dx+2c}}{5}-\frac{4e^{dx+c}}{5}+\frac{4}{35}}{d\left(e^{7dx+7c}-7e^{6dx+6c}+21e^{5dx+5c}-35e^{4dx+4c}+35e^{3dx+3c}-21e^{2dx+2c}+7e^{dx+c}-1\right)}$$

input
```
int(1/(1-cosh(d*x+c))^4,x)
```

output
```
(4*( - 35*e**(3*c + 3*d*x) + 21*e**(2*c + 2*d*x) - 7*e**(c + d*x) + 1))/(3
5*d*(e**(7*c + 7*d*x) - 7*e**(6*c + 6*d*x) + 21*e**(5*c + 5*d*x) - 35*e**(
4*c + 4*d*x) + 35*e**(3*c + 3*d*x) - 21*e**(2*c + 2*d*x) + 7*e**(c + d*x)
- 1))
```

3.40 $\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 51

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}} + \frac{2\sinh(x)}{\sqrt{a+a\cosh(x)}}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a+a*cosh(x))^(1/2))/a^(1/2)+2
*sinh(x)/(a+a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.67

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = -\frac{2\cosh\left(\frac{x}{2}\right)\left(\arctan\left(\sinh\left(\frac{x}{2}\right)\right)-2\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(1+\cosh(x))}}$$

input
```
Integrate[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

output
```
(-2*Cosh[x/2]*(ArcTan[Sinh[x/2]] - 2*Sinh[x/2]))/Sqrt[a*(1 + Cosh[x])]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{a\cosh(x)+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{ 3230}$$

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\int \frac{1}{\sqrt{\cosh(x)a+a}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-2i\int \frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sinh(x)}{\sqrt{a\cosh(x)+a}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}$$

input
```
Int[Cosh[x]/Sqrt[a + a*Cosh[x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/Sqrt[a
]) + (2*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(40) = 80$.

Time = 0.39 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.80

method	result	size
default	$\frac{\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right) a+2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}\right)\sqrt{2}}{\sqrt{-a}\,a\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	92

input
```
int(cosh(x)/(a+cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(
1/2)*(-a)^(1/2)-a))*a+2*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/(-a)^(1/2)/a/s
inh(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.10

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \frac{2\left(\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(x)+\sinh(x)-1)+\sqrt{2}\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{\sqrt{a}}\right)\right)}{a}$$

input
```
integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
2*(sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) - 1) + sqrt(2)
*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))/sqrt(a)))/a
```

Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a(\cosh(x)+1)}}\,dx$$

input
```
integrate(cosh(x)/(a+a*cosh(x))**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(a*(cosh(x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(40) = 80$.

Time = 0.22 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.24

$$\int \frac{\cosh(x)}{\sqrt{a + a\cosh(x)}}\,dx = -\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}} - \frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x} + \sqrt{a}}\right) + \frac{1}{3}\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}} - \frac{2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}} - \frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}e^{(-x)} + \sqrt{a}}\right) + \frac{3\sqrt{2}\sqrt{a}e^{\left(\frac{3}{2}x\right)} - \sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}x\right)}}{3\,(ae^{x} + a)}$$

input
```
integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) +
 1/3*sqrt(2)*(3*arctan(e^(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/
2*x)/(sqrt(a)*e^(-x) + sqrt(a))) + 1/3*(3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt
(2)*sqrt(a)*e^(-1/2*x))/(a*e^x + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.73

$$\int \frac{\cosh(x)}{\sqrt{a + a\cosh(x)}}\,dx = -\frac{2\sqrt{2}\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}} + \frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}} - \frac{\sqrt{2}e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}$$

input
```
integrate(cosh(x)/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(e^(1/2*x))/sqrt(a) + sqrt(2)*e^(1/2*x)/sqrt(a) - sqrt(2)
*e^(-1/2*x)/sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{a + a\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a + a\cosh\left(x\right)}}\,dx$$

input
```
int(cosh(x)/(a + a*cosh(x))^(1/2),x)
```

output
```
int(cosh(x)/(a + a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh(x)}{\sqrt{a + a\cosh(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)+1}\,\cosh(x)}{\cosh(x)+1}dx\right)}{a}$$

input
```
int(cosh(x)/(a+a*cosh(x))^(1/2),x)
```

output
```
(sqrt(a)*int((sqrt(cosh(x) + 1)*cosh(x))/(cosh(x) + 1),x))/a
```

3.41 $\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 53

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a-a*cosh(x))^(1/2))/a^(1/2)+2
*sinh(x)/(a-a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \frac{2\left(2\cosh\left(\frac{x}{2}\right)-\log\left(\cosh\left(\frac{x}{4}\right)\right)+\log\left(\sinh\left(\frac{x}{4}\right)\right)\right)\sinh\left(\frac{x}{2}\right)}{\sqrt{a-a\cosh(x)}}$$

input
```
Integrate[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

output
```
(2*(2*Cosh[x/2] - Log[Cosh[x/4]] + Log[Sinh[x/4]])*Sinh[x/2])/Sqrt[a - a*C
osh[x]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a-a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{ 3230}$$

$$\int \frac{1}{\sqrt{a-a\cosh(x)}}dx+\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+\int \frac{1}{\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3128}$$

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}+2i\int \frac{1}{\frac{a^2\sinh^2(x)}{a-a\cosh(x)}+2a}d\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{2\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

input
```
Int[Cosh[x]/Sqrt[a - a*Cosh[x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/Sqrt[a
]) + (2*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.75

method	result	size
default	$\frac{\sinh\left(\frac{x}{2}\right)\left(4\cosh\left(\frac{x}{2}\right)+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	40

input
```
int(cosh(x)/(a-cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sinh(1/2*x)*(4*cosh(1/2*x)+ln(cosh(1/2*x)-1)-ln(cosh(1/2*x)+1))/(-2*sinh(1
/2*x)^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.74

$$\int \frac{\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx = \frac{\sqrt{2}a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right) - 2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}(\cosh(}{a}$$

input
```
integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x))
)*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x) + sinh(
x) - 1)) - 2*sqrt(1/2)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x) + 1
))/a
```

Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{-a(\cosh(x) - 1)}}\,dx$$

input
```
integrate(cosh(x)/(a-a*cosh(x))**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(-a*(cosh(x) - 1)), x)
```

Maxima [F]

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{-a\cosh(x)+a}}\,dx$$

input
```
integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/sqrt(-a*cosh(x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.40

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = -\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{\sqrt{a}\,\mathrm{sgn}\left(-e^x+1\right)} - \frac{\sqrt{2}}{\sqrt{-ae^x}\,\mathrm{sgn}\left(-e^x+1\right)} + \frac{\sqrt{2}\sqrt{-ae^x}}{a\,\mathrm{sgn}\left(-e^x+1\right)}$$

input
```
integrate(cosh(x)/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x + 1)) - sqrt(2)/
(sqrt(-a*e^x)*sgn(-e^x + 1)) + sqrt(2)*sqrt(-a*e^x)/(a*sgn(-e^x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$$

input
```
int(cosh(x)/(a - a*cosh(x))^(1/2),x)
```

output
```
int(cosh(x)/(a - a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\cosh(x)+1}\,\cosh(x)}{\cosh(x)-1}dx\right)}{a}$$

input
```
int(cosh(x)/(a-a*cosh(x))^(1/2),x)
```

output
```
( - sqrt(a)*int((sqrt( - cosh(x) + 1)*cosh(x))/(cosh(x) - 1),x))/a
```

3.42 $\int (a+a\cosh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int (a+a\cosh(c+dx))^{5/2}\,dx=\frac{64a^3\sinh(c+dx)}{15d\sqrt{a+a\cosh(c+dx)}}+\frac{16a^2\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{15d}+\frac{2a(a+a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}$$

output
```
64/15*a^3*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)+16/15*a^2*(a+a*cosh(d*x+c)
)^(1/2)*sinh(d*x+c)/d+2/5*a*(a+a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.80

$$\int (a+a\cosh(c+dx))^{5/2}\,dx=\frac{a^2\sqrt{a(1+\cosh(c+dx))}\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(150\sinh\left(\frac{1}{2}(c+dx)\right)+25\sinh\left(\frac{3}{2}(c+dx)\right)+3\sin\right.}{30d}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(150*Sinh[(c + d*x)/2]
+ 25*Sinh[(3*(c + d*x))/2] + 3*Sinh[(5*(c + d*x))/2]))/(30*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)+a)^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (\cosh(c+dx)a+a)^{3/2}dx+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}+\frac{8}{5}a\int \left(\sin\left(ic+idx+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{\cosh(c+dx)a+a}dx+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}\right)+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}+\frac{8}{5}a\left(\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}+\frac{4}{3}a\int \sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{5}a\left(\frac{8a^2\sinh(c+dx)}{3d\sqrt{a\cosh(c+dx)+a}}+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}\right)+\frac{2a\sinh(c+dx)(a\cosh(c+dx)+a)^{3/2}}{5d}$$

input
```
Int[(a + a*Cosh[c + d*x])^(5/2),x]
```

output
```
(2*a*(a + a*Cosh[c + d*x])^(3/2)*Sinh[c + d*x])/(5*d) + (8*a*((8*a^2*Sinh[
c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Cosh[c + d*x]]
*Sinh[c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.82

method	result	size
default	$\frac{8a^3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^4+4\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+8\right)\sqrt{2}}{15\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}\,d}$	73

input
```
int((a+a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
8/15*a^3*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(3*cosh(1/2*d*x+1/2*c)^4+
4*cosh(1/2*d*x+1/2*c)^2+8)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(77) = 154$.

Time = 0.08 (sec) , antiderivative size = 327, normalized size of antiderivative = 3.67

$$\int (a + a\cosh(c + dx))^{5/2}\, dx = \frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5 + 3\,a^2\sinh\left(dx+c\right)^5 + 25\,a^2\cosh\left(dx+c\right)^4 + 150\,a^2\cosh\left(dx+c\right)^3 + \cdots\right.}{\cdots}$$

input
```
integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 + 25*a^2*cos
h(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) + 5*a^2)*s
inh(d*x + c)^4 - 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 + 10*
a^2*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a
^2*cosh(d*x + c)^3 + 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) - 5*a^2)
*sinh(d*x + c)^2 - 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 + 20*a^2*cosh(d*x + c)
^3 + 90*a^2*cosh(d*x + c)^2 - 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))
*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x
 + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a+a\cosh(c+dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+a*cosh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.36

$$\int (a+a\cosh(c+dx))^{5/2}\,dx = \frac{\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}}{20\,d} + \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{12\,d} + \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d} - \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,d} - \frac{5\,\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{12\,d} - \frac{\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{5}{2}\,dx-\frac{5}{2}\,c\right)}}{20\,d}$$

input
```
integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/20*sqrt(2)*a^(5/2)*e^(5/2*d*x + 5/2*c)/d + 5/12*sqrt(2)*a^(5/2)*e^(3/2*d
*x + 3/2*c)/d + 5/2*sqrt(2)*a^(5/2)*e^(1/2*d*x + 1/2*c)/d - 5/2*sqrt(2)*a^
(5/2)*e^(-1/2*d*x - 1/2*c)/d - 5/12*sqrt(2)*a^(5/2)*e^(-3/2*d*x - 3/2*c)/d
 - 1/20*sqrt(2)*a^(5/2)*e^(-5/2*d*x - 5/2*c)/d
```

Giac [F(-2)]

Exception generated.

$$\int (a+a\cosh(c+dx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cosh(c+dx))^{5/2}\,dx=\int (a+a\cosh(c+d\,x))^{5/2}\,dx$$

input
```
int((a + a*cosh(c + d*x))^(5/2),x)
```

output
```
int((a + a*cosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int (a+a\cosh(c+dx))^{5/2}\,dx&=\sqrt{a}\,a^2\left(\int \sqrt{\cosh(dx+c)+1}dx\right.\\&+2\left(\int \sqrt{\cosh(dx+c)+1}\,\cosh(dx+c)\,dx\right)\\&\left.+\int \sqrt{\cosh(dx+c)+1}\,\cosh(dx+c)^2\,dx\right)\end{aligned}$$

input
```
int((a+a*cosh(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(cosh(c + d*x) + 1),x) + 2*int(sqrt(cosh(c + d*x) +
1)*cosh(c + d*x),x) + int(sqrt(cosh(c + d*x) + 1)*cosh(c + d*x)**2,x))
```

3.43 $\int (a+a\cosh(c+dx))^{3/2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 59

$$\int (a+a\cosh(c+dx))^{3/2}\,dx=\frac{8a^2\sinh(c+dx)}{3d\sqrt{a+a\cosh(c+dx)}}+\frac{2a\sqrt{a+a\cosh(c+dx)}\sinh(c+dx)}{3d}$$

output
```
8/3*a^2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)+2/3*a*(a+a*cosh(d*x+c))^(1/2
)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.93

$$\int (a+a\cosh(c+dx))^{3/2}\,dx=\frac{a\sqrt{a(1+\cosh(c+dx))}\operatorname{sech}\left(\frac{1}{2}(c+dx)\right)\left(9\sinh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{3}{2}(c+dx)\right)\right)}{3d}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(3/2),x]
```

output
```
(a*Sqrt[a*(1 + Cosh[c + d*x])]*Sech[(c + d*x)/2]*(9*Sinh[(c + d*x)/2] + Si
nh[(3*(c + d*x))/2]))/(3*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(c+dx)+a)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{4}{3}a\int \sqrt{\cosh(c+dx)a+a}dx+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}+\frac{4}{3}a\int \sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{8a^2\sinh(c+dx)}{3d\sqrt{a\cosh(c+dx)+a}}+\frac{2a\sinh(c+dx)\sqrt{a\cosh(c+dx)+a}}{3d}$$

input
```
Int[(a + a*Cosh[c + d*x])^(3/2),x]
```

output
```
(8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a + a*Cosh[c + d*x]]) + (2*a*Sqrt[a + a*Co
sh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{4a^{2} \cosh\left(\frac{dx}{2}+\frac{c}{2}\right) \sinh\left(\frac{dx}{2}+\frac{c}{2}\right) \left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+2\right) \sqrt{2}}{3 \sqrt{a \cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\, d}$	58

input
```
int((a+a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
4/3*a^2*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2+2)*
2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(51) = 102$.

Time = 0.10 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.37

$$\int (a + a\cosh(c + dx))^{3/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^{3} + a\sinh\left(dx+c\right)^{3} + 9\,a\cosh\left(dx+c\right)^{2} + 3\left(a\cosh\left(dx+c\right) + 3\,a\right)\sinh(\ldots}{3\,(d\,\mathrm{co}\ldots}$$

input
```
integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 + 9*a*cosh(d*x + c)^2
 + 3*(a*cosh(d*x + c) + 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*co
sh(d*x + c)^2 + 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) - a)*sqrt(a/(cosh(d
*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int (a + a\cosh(c + dx))^{3/2}\,dx = \int \left(a\cosh\left(c+dx\right) + a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((a*cosh(c + d*x) + a)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int (a+a\cosh(c+dx))^{3/2}\,dx = \frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{6\,d} + \frac{3\,\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d} - \frac{3\,\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{2\,d} - \frac{\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,dx-\frac{3}{2}\,c\right)}}{6\,d}$$

input
```
integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*a^(3/2)*e^(3/2*d*x + 3/2*c)/d + 3/2*sqrt(2)*a^(3/2)*e^(1/2*d*x
 + 1/2*c)/d - 3/2*sqrt(2)*a^(3/2)*e^(-1/2*d*x - 1/2*c)/d - 1/6*sqrt(2)*a^(
3/2)*e^(-3/2*d*x - 3/2*c)/d
```

Giac [F(-2)]

Exception generated.

$$\int (a+a\cosh(c+dx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int (a + a\cosh(c + dx))^{3/2}\,dx = \int (a + a\cosh(c + d\,x))^{3/2}\,dx$$

input
```
int((a + a*cosh(c + d*x))^(3/2),x)
```

output
```
int((a + a*cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a + a\cosh(c + dx))^{3/2}\,dx = \sqrt{a}\,a\left(\int \sqrt{\cosh(dx + c) + 1}dx + \int \sqrt{\cosh(dx + c) + 1}\,\cosh(dx + c)\,dx\right)$$

input
```
int((a+a*cosh(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt(cosh(c + d*x) + 1),x) + int(sqrt(cosh(c + d*x) + 1)*co
sh(c + d*x),x))
```

3.44 $\int \sqrt{a+a\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 26

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\frac{2a\sinh(c+dx)}{d\sqrt{a+a\cosh(c+dx)}}$$

output
```
2*a*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.12

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\frac{2\sqrt{a(1+\cosh(c+dx))}\tanh\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*Sqrt[a*(1 + Cosh[c + d*x])]*Tanh[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh(c+dx)+a}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{2a\sinh(c+dx)}{d\sqrt{a\cosh(c+dx)+a}}$$

input
```
Int[Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(2*a*Sinh[c + d*x])/(d*Sqrt[a + a*Cosh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.65

method	result	size
default	$\frac{2a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2}}{\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$	43
risch	$\frac{\sqrt{2}\,\sqrt{a\left(e^{dx+c}+1\right)^{2}e^{-dx-c}}\,\left(e^{dx+c}-1\right)}{\left(e^{dx+c}+1\right)d}$	49

input
```
int((a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*a*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)
^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.58

$$\int\sqrt{a+a\cosh(c+dx)}\,dx = \frac{2\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)-1)}{d}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(
d*x + c) - 1)/d
```

Sympy [F]

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\int \sqrt{a\cosh\left(c+dx\right)+a}\,dx$$

input
```
integrate((a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a*cosh(c + d*x) + a), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\frac{\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{d}-\frac{\sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}\,dx-\frac{1}{2}\,c\right)}}{d}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
sqrt(2)*sqrt(a)*e^(1/2*d*x + 1/2*c)/d - sqrt(2)*sqrt(a)*e^(-1/2*d*x - 1/2*
c)/d
```

Giac [F(-2)]

Exception generated.

$$\int \sqrt{a+a\cosh(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate((a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \frac{2\tanh\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a+a\cosh\left(c+d\,x\right)}}{d}$$

input
```
int((a + a*cosh(c + d*x))^(1/2),x)
```

output
```
(2*tanh(c/2 + (d*x)/2)*(a + a*cosh(c + d*x))^(1/2))/d
```

Reduce [F]

$$\int \sqrt{a+a\cosh(c+dx)}\,dx = \sqrt{a}\left(\int \sqrt{\cosh\left(dx+c\right)+1}dx\right)$$

input
```
int((a+a*cosh(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(c + d*x) + 1),x)
```

3.45 $\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
2^(1/2)*arctan(1/2*a^(1/2)*sinh(d*x+c)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))/a^
(1/2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = -\frac{2\cot^{-1}\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a(1+\cosh(c+dx))}}$$

input
```
Integrate[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(-2*ArcCot[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2])/(d*Sqrt[a*(1 + Cosh[c + d
*x])])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh(c+dx)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{\cosh(c+dx)a+a}+2a}d\left(-\frac{ia\sinh(c+dx)}{\sqrt{\cosh(c+dx)a+a}}\right)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a + a*Cosh[c + d*x]],x]
```

output
```
(Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]]
)])/(Sqrt[a]*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(37) = 74$.

Time = 0.39 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.24

method	result	size
default	$-\frac{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\sqrt{2}}{\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$	103

input
```
int(1/(a+a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(1/2*d*x+1/2*c)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/(-a)^(1/2)*ln(2*((-a)
^(1/2)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))/sinh(1/2*d*
x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}(\cosh(dx+c)+\sinh(dx+c))+\cosh(dx+c)+\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)+1}\right)}{d}, -\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}d}\right]$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(2)*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh
(d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) + cosh(d*x + c) + s
inh(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) + 1))/d, -2*sqrt(2)*arcta
n(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c)))/sqrt(a))/(sqrt
(a)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a\cosh(c+dx)+a}}\,dx$$

input
```
integrate(1/(a+a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(c + d*x) + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(37) = 74$.

Time = 0.18 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.87

$$\int \frac{1}{\sqrt{a + a\cosh(c + dx)}}\, dx = 2\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}\right)}{\sqrt{a}d} + \frac{e^{\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}}{\left(\sqrt{a}e^{(dx+c)} + \sqrt{a}\right)d}\right) - \frac{2\sqrt{2}e^{\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}}{\sqrt{a}de^{(dx+c)} + \sqrt{a}d}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
2*sqrt(2)*(arctan(e^(1/2*d*x + 1/2*c))/(sqrt(a)*d) + e^(1/2*d*x + 1/2*c)/(
(sqrt(a)*e^(d*x + c) + sqrt(a))*d)) - 2*sqrt(2)*e^(1/2*d*x + 1/2*c)/(sqrt(
a)*d*e^(d*x + c) + sqrt(a)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a + a\cosh(c + dx)}}\, dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{a+a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a + a*cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a + a*cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+a\cosh(c+dx)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}}{\cosh(dx+c)+1}dx\right)}{a}$$

input
```
int(1/(a+a*cosh(d*x+c))^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(c + d*x) + 1)/(cosh(c + d*x) + 1),x))/a
```

3.46 $\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 77

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx=\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a+a\cosh(c+dx))^{3/2}}$$

output
```
1/4*arctan(1/2*a^(1/2)*sinh(d*x+c)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))*2^(1/2
)/a^(3/2)/d+1/2*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.82

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx=\frac{\cosh^2\left(\frac{1}{2}(c+dx)\right)\left(\arctan\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\cosh\left(\frac{1}{2}(c+dx)\right)+\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a(1+\cosh(c+dx)))^{3/2}}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(-3/2),x]
```

output
```
(Cosh[(c + d*x)/2]^2*(ArcTan[Sinh[(c + d*x)/2]]*Cosh[(c + d*x)/2] + Tanh[(
c + d*x)/2]))/(d*(a*(1 + Cosh[c + d*x]))^(3/2))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(c+dx)+a)^{3/2}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$
$$\Big\downarrow \text{3129}$$
$$\frac{\int \frac{1}{\sqrt{\cosh(c+dx)a+a}}dx}{4a}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a}$$
$$\Big\downarrow \text{3128}$$
$$\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{\cosh(c+dx)a+a}+2a}d\left(-\frac{ia\sinh(c+dx)}{\sqrt{\cosh(c+dx)a+a}}\right)}{2ad}$$
$$\Big\downarrow \text{219}$$
$$\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}$$

input
```
Int[(a + a*Cosh[c + d*x])^(-3/2),x]
```

output
```
ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqr
t[2]*a^(3/2)*d) + Sinh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(62) = 124$.

Time = 0.44 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.87

method	result	size
default	$-\frac{\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\,\left(\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a-\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\right)\sqrt{2}}{4a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$	144

input
```
int(1/(a+a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(ln(2*((-a)^(1/2)*(sinh(1/2*d*x+1/2*c
)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))*cosh(1/2*d*x+1/2*c)^2*a-(-a)^(1/2)*(s
inh(1/2*d*x+1/2*c)^2*a)^(1/2))/a^2/cosh(1/2*d*x+1/2*c)/(-a)^(1/2)/sinh(1/2
*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(62) = 124$.

Time = 0.12 (sec) , antiderivative size = 232, normalized size of antiderivative = 3.01

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{\sqrt{2}\left(\cosh\left(dx+c\right)^2+2\left(\cosh\left(dx+c\right)+1\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^2+\right.}{}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) + 1)*sinh(d*x + c) + sinh
(d*x + c)^2 + 2*cosh(d*x + c) + 1)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a
/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x + c))/sqrt(a))
 + 2*sqrt(1/2)*(cosh(d*x + c)^2 + (2*cosh(d*x + c) - 1)*sinh(d*x + c) + si
nh(d*x + c)^2 - cosh(d*x + c))*sqrt(a/(cosh(d*x + c) + sinh(d*x + c))))/(a
^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 + 2*a^2*d*cosh(d*x + c) + a^2
*d + 2*(a^2*d*cosh(d*x + c) + a^2*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((a*cosh(c + d*x) + a)**(-3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(62) = 124$.

Time = 0.20 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.21

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{1}{6}\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}+8\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}-3\,e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{\left(a^{\frac{3}{2}}e^{(3\,dx+3\,c)}+3\,a^{\frac{3}{2}}e^{(2\,dx+2\,c)}+3\,a^{\frac{3}{2}}e^{(dx+c)}+a^{\frac{3}{2}}\right)d}+\frac{3\arctan\left(e^{\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}\right)}{a^{\frac{3}{2}}d}\right) - \frac{4\sqrt{2}e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}}{3\left(a^{\frac{3}{2}}de^{(3\,dx+3\,c)}+3\,a^{\frac{3}{2}}de^{(2\,dx+2\,c)}+3\,a^{\frac{3}{2}}de^{(dx+c)}+a^{\frac{3}{2}}d\right)}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
1/6*sqrt(2)*((3*e^(5/2*d*x + 5/2*c) + 8*e^(3/2*d*x + 3/2*c) - 3*e^(1/2*d*x
 + 1/2*c))/((a^(3/2)*e^(3*d*x + 3*c) + 3*a^(3/2)*e^(2*d*x + 2*c) + 3*a^(3/
2)*e^(d*x + c) + a^(3/2))*d) + 3*arctan(e^(1/2*d*x + 1/2*c))/(a^(3/2)*d))
- 4/3*sqrt(2)*e^(3/2*d*x + 3/2*c)/(a^(3/2)*d*e^(3*d*x + 3*c) + 3*a^(3/2)*d
*e^(2*d*x + 2*c) + 3*a^(3/2)*d*e^(d*x + c) + a^(3/2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(a+a\cosh\left(c+d\,x\right)\right)^{3/2}}\,dx$$

input
```
int(1/(a + a*cosh(c + d*x))^(3/2),x)
```

output
```
int(1/(a + a*cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\cosh(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}}{\cosh(dx+c)^2+2\cosh(dx+c)+1}dx\right)}{a^2}$$

input
```
int(1/(a+a*cosh(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(c + d*x) + 1)/(cosh(c + d*x)**2 + 2*cosh(c + d*x) +
 1),x))/a**2
```

3.47 $\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 107

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{3\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a+a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} + \frac{\sinh(c+dx)}{4d(a+a\cosh(c+dx))^{5/2}} + \frac{3\sinh(c+dx)}{16ad(a+a\cosh(c+dx))^{3/2}}$$

output
```
3/32*arctan(1/2*a^(1/2)*sinh(d*x+c)*2^(1/2)/(a+a*cosh(d*x+c))^(1/2))*2^(1/
2)/a^(5/2)/d+1/4*sinh(d*x+c)/d/(a+a*cosh(d*x+c))^(5/2)+3/16*sinh(d*x+c)/a/
d/(a+a*cosh(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{\cosh^5\left(\frac{1}{2}(c+dx)\right)\left(32\operatorname{csch}^4(c+dx)\sinh^5\left(\frac{1}{2}(c+dx)\right)+3\left(\arctan\left(\sinh\left(\frac{1}{2}\right.\right.\right.\right.}{4d(a(1+\cosh(c+dx)))^{5/}}$$

input
```
Integrate[(a + a*Cosh[c + d*x])^(-5/2),x]
```

output
```
(Cosh[(c + d*x)/2]^5*(32*Csch[c + d*x]^4*Sinh[(c + d*x)/2]^5 + 3*(ArcTan[S
inh[(c + d*x)/2]] + Sech[(c + d*x)/2]*Tanh[(c + d*x)/2])))/(4*d*(a*(1 + Co
sh[c + d*x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3129, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh(c+dx)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\int \frac{1}{(\cosh(c+dx)a+a)^{3/2}}dx}{8a}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}+\frac{3\int \frac{1}{\left(\sin\left(ic+idx+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{\cosh(c+dx)a+a}}dx}{4a}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}+\frac{3\left(\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sin\left(ic+idx+\frac{\pi}{2}\right)a+a}}dx}{4a}\right)}{8a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}+\frac{3\left(\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{\cosh(c+dx)a+a}+2a}d\left(-\frac{ia\sinh(c+dx)}{\sqrt{\cosh(c+dx)a+a}}\right)}{2ad}\right)}{8a}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a\cosh(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{\sinh(c+dx)}{2d(a\cosh(c+dx)+a)^{3/2}}\right)}{8a}+\frac{\sinh(c+dx)}{4d(a\cosh(c+dx)+a)^{5/2}}$$

input
```
Int[(a + a*Cosh[c + d*x])^(-5/2),x]
```

output
```
Sinh[c + d*x]/(4*d*(a + a*Cosh[c + d*x])^(5/2)) + (3*(ArcTan[(Sqrt[a]*Sinh
[c + d*x])/(Sqrt[2]*Sqrt[a + a*Cosh[c + d*x]])]/(2*Sqrt[2]*a^(3/2)*d) + Si
nh[c + d*x]/(2*d*(a + a*Cosh[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. 2(88) = 176.

Time = 0.41 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.66

method	result
default	$-\frac{\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\,\left(3\ln\left(\frac{2\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}-2a}{\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}\right)a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}-3\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\,\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-2\sqrt{-a}\,\sqrt{\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2} a}\right)\sqrt{2}}{32a^{3}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}\sqrt{-a}\,\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}\,d}$

input
```
int(1/(a+a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*(3*ln(2*((-a)^(1/2)*(sinh(1/2*d*x+1/
2*c)^2*a)^(1/2)-a)/cosh(1/2*d*x+1/2*c))*a*cosh(1/2*d*x+1/2*c)^4-3*(-a)^(1/
2)*(sinh(1/2*d*x+1/2*c)^2*a)^(1/2)*cosh(1/2*d*x+1/2*c)^2-2*(-a)^(1/2)*(sin
h(1/2*d*x+1/2*c)^2*a)^(1/2))/a^3/cosh(1/2*d*x+1/2*c)^3/(-a)^(1/2)/sinh(1/2
*d*x+1/2*c)*2^(1/2)/(a*cosh(1/2*d*x+1/2*c)^2)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. 2(88) = 176.

Time = 0.17 (sec) , antiderivative size = 535, normalized size of antiderivative = 5.00

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/16*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) + 1)*sinh(d*x + c)^3 +
 sinh(d*x + c)^4 + 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 + 2*cosh(d*x + c
) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 + 3*cosh(d
*x + c)^2 + 3*cosh(d*x + c) + 1)*sinh(d*x + c) + 4*cosh(d*x + c) + 1)*sqrt
(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(d*x + c) + sinh(d*x + c))*(cosh
(d*x + c) + sinh(d*x + c))/sqrt(a)) + 2*sqrt(1/2)*(3*cosh(d*x + c)^4 + (12
*cosh(d*x + c) + 11)*sinh(d*x + c)^3 + 3*sinh(d*x + c)^4 + 11*cosh(d*x + c
)^3 + (18*cosh(d*x + c)^2 + 33*cosh(d*x + c) - 11)*sinh(d*x + c)^2 - 11*co
sh(d*x + c)^2 + (12*cosh(d*x + c)^3 + 33*cosh(d*x + c)^2 - 22*cosh(d*x + c
) - 3)*sinh(d*x + c) - 3*cosh(d*x + c))*sqrt(a/(cosh(d*x + c) + sinh(d*x +
 c))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*sinh(d*x + c)^4 + 4*a^3*d*cosh(d*x +
 c)^3 + 6*a^3*d*cosh(d*x + c)^2 + 4*a^3*d*cosh(d*x + c) + a^3*d + 4*(a^3*d
*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^3 + 6*(a^3*d*cosh(d*x + c)^2 + 2*a^3
*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c)^3 + 3*a
^3*d*cosh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx=\int \frac{1}{(a\cosh{(c+dx)}+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+a*cosh(d*x+c))**(5/2),x)
```

output
```
Integral((a*cosh(c + d*x) + a)**(-5/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(88) = 176.

Time = 0.19 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.34

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx=\frac{1}{80}\sqrt{2}\left(\frac{15\,e^{\left(\frac{9}{2}\,dx+\frac{9}{2}\,c\right)}+70\,e^{\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)}+128\,e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}-70\,e^{\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)}\cdots}{\left(a^{\frac{5}{2}}e^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}e^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}e^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}e^{(2\,dx+2\,c)}+\cdots}\right.$$

$$-\frac{8\sqrt{2}e^{\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)}}{5\left(a^{\frac{5}{2}}de^{(5\,dx+5\,c)}+5\,a^{\frac{5}{2}}de^{(4\,dx+4\,c)}+10\,a^{\frac{5}{2}}de^{(3\,dx+3\,c)}+10\,a^{\frac{5}{2}}de^{(2\,dx+2\,c)}+5\,a^{\frac{5}{2}}de^{(dx+c)}+a^{\frac{5}{2}}d\right)}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
1/80*sqrt(2)*((15*e^(9/2*d*x + 9/2*c) + 70*e^(7/2*d*x + 7/2*c) + 128*e^(5/
2*d*x + 5/2*c) - 70*e^(3/2*d*x + 3/2*c) - 15*e^(1/2*d*x + 1/2*c))/((a^(5/2
)*e^(5*d*x + 5*c) + 5*a^(5/2)*e^(4*d*x + 4*c) + 10*a^(5/2)*e^(3*d*x + 3*c)
 + 10*a^(5/2)*e^(2*d*x + 2*c) + 5*a^(5/2)*e^(d*x + c) + a^(5/2))*d) + 15*a
rctan(e^(1/2*d*x + 1/2*c))/(a^(5/2)*d)) - 8/5*sqrt(2)*e^(5/2*d*x + 5/2*c)/
(a^(5/2)*d*e^(5*d*x + 5*c) + 5*a^(5/2)*d*e^(4*d*x + 4*c) + 10*a^(5/2)*d*e^
(3*d*x + 3*c) + 10*a^(5/2)*d*e^(2*d*x + 2*c) + 5*a^(5/2)*d*e^(d*x + c) + a
^(5/2)*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a+a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \int \frac{1}{(a+a\cosh(c+d\,x))^{5/2}}\,dx$$

input
```
int(1/(a + a*cosh(c + d*x))^(5/2),x)
```

output
```
int(1/(a + a*cosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+a\cosh(c+dx))^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(dx+c)+1}}{\cosh(dx+c)^3+3\cosh(dx+c)^2+3\cosh(dx+c)+1}dx\right)}{a^3}$$

input
```
int(1/(a+a*cosh(d*x+c))^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(c + d*x) + 1)/(cosh(c + d*x)**3 + 3*cosh(c + d*x)**
2 + 3*cosh(c + d*x) + 1),x))/a**3
```

3.48 $\int (a - a\cosh(c+dx))^{5/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 92

$$\int (a-a\cosh(c+dx))^{5/2}\,dx = -\frac{64a^3\sinh(c+dx)}{15d\sqrt{a-a\cosh(c+dx)}} - \frac{16a^2\sqrt{a-a\cosh(c+dx)}\sinh(c+dx)}{15d} - \frac{2a(a-a\cosh(c+dx))^{3/2}\sinh(c+dx)}{5d}$$

output
```
-64/15*a^3*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)-16/15*a^2*(a-a*cosh(d*x+c
))^(1/2)*sinh(d*x+c)/d-2/5*a*(a-a*cosh(d*x+c))^(3/2)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.78

$$\int (a-a\cosh(c+dx))^{5/2}\,dx = \frac{a^2\sqrt{a-a\cosh(c+dx)}\left(150\cosh\left(\frac{1}{2}(c+dx)\right) - 25\cosh\left(\frac{3}{2}(c+dx)\right) + 3\cosh\left(\frac{5}{2}(c+dx)\right)\right)\text{cs}}{30d}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(5/2),x]
```

output
```
(a^2*Sqrt[a - a*Cosh[c + d*x]]*(150*Cosh[(c + d*x)/2] - 25*Cosh[(3*(c + d*
x))/2] + 3*Cosh[(5*(c + d*x))/2])*Csch[(c + d*x)/2])/(30*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(c+dx))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\int (a-a\cosh(c+dx))^{3/2}dx-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}+\frac{8}{5}a\int \left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{8}{5}a\left(\frac{4}{3}a\int \sqrt{a-a\cosh(c+dx)}dx-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}\right)-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}+\frac{8}{5}a\left(-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}+\frac{4}{3}a\int \sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{8}{5}a\left(-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}\right)-\frac{2a\sinh(c+dx)(a-a\cosh(c+dx))^{3/2}}{5d}$$

input
```
Int[(a - a*Cosh[c + d*x])^(5/2),x]
```

output
```
(-2*a*(a - a*Cosh[c + d*x])^(3/2)*Sinh[c + d*x])/(5*d) + (8*a*((-8*a^2*Sin
h[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*Cosh[c + d*x
]]*Sinh[c + d*x])/(3*d)))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.77

method	result	size
default	$-\frac{16\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a^3\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(3\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^4-4\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+8\right)}{15\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a}\,d}$	71

input
```
int((a-a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15*sinh(1/2*d*x+1/2*c)*a^3*cosh(1/2*d*x+1/2*c)*(3*sinh(1/2*d*x+1/2*c)^
4-4*sinh(1/2*d*x+1/2*c)^2+8)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 328 vs. 2(80) = 160.

Time = 0.12 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.57

$$\int (a - a\cosh(c+dx))^{5/2}\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\,a^2\cosh\left(dx+c\right)^5 + 3\,a^2\sinh\left(dx+c\right)^5 - 25\,a^2\cosh\left(dx+c\right)^4 + 150\,a^2\cosh\left(dx+c\right)^3 \cdots\right.}{}$$

input
```
integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*a^2*cosh(d*x + c)^5 + 3*a^2*sinh(d*x + c)^5 - 25*a^2*cos
h(d*x + c)^4 + 150*a^2*cosh(d*x + c)^3 + 5*(3*a^2*cosh(d*x + c) - 5*a^2)*s
inh(d*x + c)^4 + 150*a^2*cosh(d*x + c)^2 + 10*(3*a^2*cosh(d*x + c)^2 - 10*
a^2*cosh(d*x + c) + 15*a^2)*sinh(d*x + c)^3 - 25*a^2*cosh(d*x + c) + 30*(a
^2*cosh(d*x + c)^3 - 5*a^2*cosh(d*x + c)^2 + 15*a^2*cosh(d*x + c) + 5*a^2)
*sinh(d*x + c)^2 + 3*a^2 + 5*(3*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)
^3 + 90*a^2*cosh(d*x + c)^2 + 60*a^2*cosh(d*x + c) - 5*a^2)*sinh(d*x + c))
*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c)^2 + 2*d*cosh(d*
x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a - a\cosh(c + dx))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a-a*cosh(d*x+c))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. 2(80) = 160.

Time = 0.14 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.07

$$\int (a - a\cosh(c + dx))^{5/2}\,dx = \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-dx-c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-3\,dx-3\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} + \frac{5\sqrt{2}a^{\frac{5}{2}}e^{(-4\,dx-4\,c)}}{12\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{\sqrt{2}a^{\frac{5}{2}}e^{(-5\,dx-5\,c)}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}} - \frac{\sqrt{2}a^{\frac{5}{2}}}{20\,d\left(-e^{(-dx-c)}\right)^{\frac{5}{2}}}$$

input
```
integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
5/12*sqrt(2)*a^(5/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*
a^(5/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(5/2)) - 5/2*sqrt(2)*a^(5/2)*e
^(-3*d*x - 3*c)/(d*(-e^(-d*x - c))^(5/2)) + 5/12*sqrt(2)*a^(5/2)*e^(-4*d*x
 - 4*c)/(d*(-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)*e^(-5*d*x - 5*c)/
(d*(-e^(-d*x - c))^(5/2)) - 1/20*sqrt(2)*a^(5/2)/(d*(-e^(-d*x - c))^(5/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(80) = 160$.

Time = 0.13 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.11

$$\int (a - a\cosh(c + dx))^{5/2}\,dx =$$
$$-\frac{\sqrt{2}\left(3\sqrt{-ae^{(dx+c)}}a^2e^{(2\,dx+2\,c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right) - 25\sqrt{-ae^{(dx+c)}}a^2e^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right) + 150\sqrt{-ae^{\cdots}}\right.}{6\cdots}$$

input
```
integrate((a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*(3*sqrt(-a*e^(d*x + c))*a^2*e^(2*d*x + 2*c)*sgn(-e^(d*x + c)
 + 1) - 25*sqrt(-a*e^(d*x + c))*a^2*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 15
0*sqrt(-a*e^(d*x + c))*a^2*sgn(-e^(d*x + c) + 1) - (150*a^5*e^(2*d*x + 2*c
)*sgn(-e^(d*x + c) + 1) - 25*a^5*e^(d*x + c)*sgn(-e^(d*x + c) + 1) + 3*a^5
*sgn(-e^(d*x + c) + 1))*e^(-2*d*x - 2*c)/(sqrt(-a*e^(d*x + c))*a^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int (a - a\cosh(c + dx))^{5/2}\,dx = \int (a - a\cosh(c + d\,x))^{5/2}\,dx$$

input
```
int((a - a*cosh(c + d*x))^(5/2),x)
```

output
```
int((a - a*cosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int (a - a\cosh(c + dx))^{5/2}\,dx = \sqrt{a}\,a^2\left(\int \sqrt{-\cosh(dx + c) + 1}dx - 2\left(\int \sqrt{-\cosh(dx + c) + 1}\,\cosh(dx + c)\,dx\right) + \int \sqrt{-\cosh(dx + c) + 1}\,\cosh(dx + c)^2\,dx\right)$$

input
```
int((a-a*cosh(d*x+c))^(5/2),x)
```

output
```
sqrt(a)*a**2*(int(sqrt( - cosh(c + d*x) + 1),x) - 2*int(sqrt( - cosh(c + d
*x) + 1)*cosh(c + d*x),x) + int(sqrt( - cosh(c + d*x) + 1)*cosh(c + d*x)**
2,x))
```

3.49 $\int (a - a\cosh(c + dx))^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 61

$$\int (a - a\cosh(c + dx))^{3/2}\,dx = -\frac{8a^2\sinh(c + dx)}{3d\sqrt{a - a\cosh(c + dx)}} - \frac{2a\sqrt{a - a\cosh(c + dx)}\sinh(c + dx)}{3d}$$

output
```
-8/3*a^2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)-2/3*a*(a-a*cosh(d*x+c))^(1/
2)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

$$\int (a - a\cosh(c + dx))^{3/2}\,dx = -\frac{a\sqrt{a - a\cosh(c + dx)}\left(-9\cosh\left(\frac{1}{2}(c + dx)\right) + \cosh\left(\frac{3}{2}(c + dx)\right)\right)\operatorname{csch}\left(\frac{1}{2}(c + dx)\right)}{3d}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(3/2),x]
```

output
```
-1/3*(a*Sqrt[a - a*Cosh[c + d*x]]*(-9*Cosh[(c + d*x)/2] + Cosh[(3*(c + d*x
))/2])*Csch[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(c+dx))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{4}{3}a\int \sqrt{a-a\cosh(c+dx)}dx-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}+\frac{4}{3}a\int \sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3125}$$

$$-\frac{8a^2\sinh(c+dx)}{3d\sqrt{a-a\cosh(c+dx)}}-\frac{2a\sinh(c+dx)\sqrt{a-a\cosh(c+dx)}}{3d}$$

input
```
Int[(a - a*Cosh[c + d*x])^(3/2),x]
```

output
```
(-8*a^2*Sinh[c + d*x])/(3*d*Sqrt[a - a*Cosh[c + d*x]]) - (2*a*Sqrt[a - a*C
osh[c + d*x]]*Sinh[c + d*x])/(3*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

Maple [A] (verified)

Time = 0.42 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.92

method	result	size
default	$\frac{8\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a^{2}\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-3\right)}{3\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	56

input
```
int((a-a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
8/3*sinh(1/2*d*x+1/2*c)*a^2*cosh(1/2*d*x+1/2*c)*(cosh(1/2*d*x+1/2*c)^2-3)/
(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(53) = 106$.

Time = 0.14 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.28

$$\int (a - a\cosh(c + dx))^{3/2}\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(a\cosh\left(dx+c\right)^3 + a\sinh\left(dx+c\right)^3 - 9\,a\cosh\left(dx+c\right)^2 + 3\left(a\cosh\left(dx+c\right) - 3\,a\right)\sinh\left(dx+c\right)^2 - 9\right.}{3\left(d\cosh\left(dx+c\right)\right.}$$

input
```
integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/3*sqrt(1/2)*(a*cosh(d*x + c)^3 + a*sinh(d*x + c)^3 - 9*a*cosh(d*x + c)^
2 + 3*(a*cosh(d*x + c) - 3*a)*sinh(d*x + c)^2 - 9*a*cosh(d*x + c) + 3*(a*c
osh(d*x + c)^2 - 6*a*cosh(d*x + c) - 3*a)*sinh(d*x + c) + a)*sqrt(-a/(cosh
(d*x + c) + sinh(d*x + c)))/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int (a - a\cosh(c + dx))^{3/2}\,dx = \int \left(-a\cosh\left(c + dx\right) + a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a-a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((-a*cosh(c + d*x) + a)**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(53) = 106$.

Time = 0.15 (sec) , antiderivative size = 124, normalized size of antiderivative = 2.03

$$\int (a-a\cosh(c+dx))^{3/2}\,dx = \frac{3\sqrt{2}a^{\frac{3}{2}}e^{(-dx-c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}} + \frac{3\sqrt{2}a^{\frac{3}{2}}e^{(-2\,dx-2\,c)}}{2\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}} - \frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3\,dx-3\,c)}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}} - \frac{\sqrt{2}a^{\frac{3}{2}}}{6\,d\left(-e^{(-dx-c)}\right)^{\frac{3}{2}}}$$

input
```
integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
3/2*sqrt(2)*a^(3/2)*e^(-d*x - c)/(d*(-e^(-d*x - c))^(3/2)) + 3/2*sqrt(2)*a
^(3/2)*e^(-2*d*x - 2*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)*e^
(-3*d*x - 3*c)/(d*(-e^(-d*x - c))^(3/2)) - 1/6*sqrt(2)*a^(3/2)/(d*(-e^(-d*
x - c))^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(53) = 106$.

Time = 0.12 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.08

$$\int (a-a\cosh(c+dx))^{3/2}\,dx = \frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}ae^{(dx+c)}\mathrm{sgn}\left(-e^{(dx+c)}+1\right) - 9\sqrt{-ae^{(dx+c)}}a\mathrm{sgn}\left(-e^{(dx+c)}+1\right) + \frac{(9\,a^3e^{(dx+c)}\mathrm{sgn}}{}\right.}{6\,d}$$

input
```
integrate((a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*(sqrt(-a*e^(d*x + c))*a*e^(d*x + c)*sgn(-e^(d*x + c) + 1) - 9*
sqrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) + (9*a^3*e^(d*x + c)*sgn(-e^(
d*x + c) + 1) - a^3*sgn(-e^(d*x + c) + 1))*e^(-d*x - c)/(sqrt(-a*e^(d*x +
c))*a))/d
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\cosh(c+dx))^{3/2}\,dx=\int (a-a\cosh(c+d\,x))^{3/2}\,dx$$

input
```
int((a - a*cosh(c + d*x))^(3/2),x)
```

output
```
int((a - a*cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int (a-a\cosh(c+dx))^{3/2}\,dx=\sqrt{a}\,a\left(\int \sqrt{-\cosh(dx+c)+1}dx-\left(\int \sqrt{-\cosh(dx+c)+1}\,\cosh(dx+c)\,dx\right)\right)$$

input
```
int((a-a*cosh(d*x+c))^(3/2),x)
```

output
```
sqrt(a)*a*(int(sqrt( - cosh(c + d*x) + 1),x) - int(sqrt( - cosh(c + d*x) +
 1)*cosh(c + d*x),x))
```

3.50 $\int \sqrt{a-a\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 27

$$\int \sqrt{a-a\cosh(c+dx)}\,dx=-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

output
```
-2*a*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.11

$$\int \sqrt{a-a\cosh(c+dx)}\,dx=\frac{2\sqrt{a-a\cosh(c+dx)}\coth\left(\frac{1}{2}(c+dx)\right)}{d}$$

input
```
Integrate[Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
(2*Sqrt[a - a*Cosh[c + d*x]]*Coth[(c + d*x)/2])/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\cosh(c+dx)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{ 3125}$$
$$-\frac{2a\sinh(c+dx)}{d\sqrt{a-a\cosh(c+dx)}}$$

input
```
Int[Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
(-2*a*Sinh[c + d*x])/(d*Sqrt[a - a*Cosh[c + d*x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.52

method	result	size
default	$-\frac{4\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)a\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	41
risch	$\frac{\sqrt{2}\,\sqrt{-a\left(\mathrm{e}^{dx+c}-1\right)^{2}\mathrm{e}^{-dx-c}}\,\left(\mathrm{e}^{dx+c}+1\right)}{\left(\mathrm{e}^{dx+c}-1\right)d}$	50

input
```
int((a-a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-4*sinh(1/2*d*x+1/2*c)*a*cosh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^
(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.56

$$\int\sqrt{a-a\cosh(c+dx)}\,dx = \frac{2\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+1\right)}{d}$$

input
```
integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh
(d*x + c) + 1)/d
```

Sympy [F]

$$\int \sqrt{a - a\cosh(c + dx)}\, dx = \int \sqrt{-a\cosh{(c + dx)} + a}\, dx$$

input
```
integrate((a-a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(-a*cosh(c + d*x) + a), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(25) = 50$.

Time = 0.14 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \sqrt{a - a\cosh(c + dx)}\, dx = -\frac{\sqrt{2}\sqrt{a}e^{(-dx-c)}}{d\sqrt{-e^{(-dx-c)}}} - \frac{\sqrt{2}\sqrt{a}}{d\sqrt{-e^{(-dx-c)}}}$$

input
```
integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
-sqrt(2)*sqrt(a)*e^(-d*x - c)/(d*sqrt(-e^(-d*x - c))) - sqrt(2)*sqrt(a)/(d
*sqrt(-e^(-d*x - c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \sqrt{a - a\cosh(c + dx)}\, dx = -\frac{\sqrt{2}\left(\sqrt{-ae^{(dx+c)}}a\operatorname{sgn}\left(-e^{(dx+c)} + 1\right) - \frac{a^2\operatorname{sgn}\left(-e^{(dx+c)}+1\right)}{\sqrt{-ae^{(dx+c)}}}\right)}{ad}$$

input
```
integrate((a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-sqrt(2)*(sqrt(-a*e^(d*x + c))*a*sgn(-e^(d*x + c) + 1) - a^2*sgn(-e^(d*x +
 c) + 1)/sqrt(-a*e^(d*x + c)))/(a*d)
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00

$$\int \sqrt{a-a\cosh(c+dx)}\,dx=\frac{2\coth\left(\frac{c}{2}+\frac{dx}{2}\right)\,\sqrt{a-a\cosh\left(c+d\,x\right)}}{d}$$

input
```
int((a - a*cosh(c + d*x))^(1/2),x)
```

output
```
(2*coth(c/2 + (d*x)/2)*(a - a*cosh(c + d*x))^(1/2))/d
```

Reduce [F]

$$\int \sqrt{a-a\cosh(c+dx)}\,dx=\sqrt{a}\left(\int\sqrt{-\cosh\left(dx+c\right)+1}dx\right)$$

input
```
int((a-a*cosh(d*x+c))^(1/2),x)
```

output
```
sqrt(a)*int(sqrt( - cosh(c + d*x) + 1),x)
```

3.51 $\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 48

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx=-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

output
```
-2^(1/2)*arctan(1/2*a^(1/2)*sinh(d*x+c)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))/a
^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx=-\frac{2\text{arctanh}\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)\sinh\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{a-a\cosh(c+dx)}}$$

input
```
Integrate[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
(-2*ArcTanh[Cosh[(c + d*x)/2]]*Sinh[(c + d*x)/2])/(d*Sqrt[a - a*Cosh[c + d
*x]])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{a-a\cosh(c+dx)}+2a}d\frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{\sqrt{a}d}$$

input
```
Int[1/Sqrt[a - a*Cosh[c + d*x]],x]
```

output
```
-((Sqrt[2]*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x
]])])/(Sqrt[a]*d))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.85

method	result	size
default	$-\frac{2 \sinh \left(\frac{d x}{2}+\frac{c}{2}\right) \operatorname{arctanh}\left(\cosh \left(\frac{d x}{2}+\frac{c}{2}\right)\right)}{\sqrt{-2 \sinh \left(\frac{d x}{2}+\frac{c}{2}\right)^{2} a}\, d}$	41

input
```
int(1/(a-a*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2*sinh(1/2*d*x+1/2*c)*arctanh(cosh(1/2*d*x+1/2*c))/(-2*sinh(1/2*d*x+1/2*c
)^2*a)^(1/2)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.94

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx$$
$$= \left[\frac{\sqrt{2}\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(dx+c)+\sinh(dx+c)}}\sqrt{-\frac{1}{a}}(\cosh(dx+c)+\sinh(dx+c))-\cosh(dx+c)-\sinh(dx+c)-1}{\cosh(dx+c)+\sinh(dx+c)-1}\right)}{d}, \frac{2\sqrt{2}\arctan}{\quad}\right.$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
[sqrt(2)*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh
(d*x + c)))*sqrt(-1/a)*(cosh(d*x + c) + sinh(d*x + c)) - cosh(d*x + c) - s
inh(d*x + c) - 1)/(cosh(d*x + c) + sinh(d*x + c) - 1))/d, 2*sqrt(2)*arctan
(sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))/sqrt(a))/(sqrt
(a)*d)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx = \int \frac{1}{\sqrt{-a\cosh(c+dx)+a}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(1/sqrt(-a*cosh(c + d*x) + a), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx=\int \frac{1}{\sqrt{-a\cosh\left(dx+c\right)+a}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-a*cosh(d*x + c) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.83

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx=-\frac{2\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{\sqrt{a}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
-2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(sqrt(a)*d*sgn(-e^(d*x + c
) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}\,dx=\int \frac{1}{\sqrt{a-a\cosh\left(c+d\,x\right)}}\,dx$$

input
```
int(1/(a - a*cosh(c + d*x))^(1/2),x)
```

output
```
int(1/(a - a*cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a - a\cosh(c + dx)}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\cosh(dx+c)+1}}{\cosh(dx+c)-1}dx\right)}{a}$$

input
```
int(1/(a-a*cosh(d*x+c))^(1/2),x)
```

output
```
( - sqrt(a)*int(sqrt( - cosh(c + d*x) + 1)/(cosh(c + d*x) - 1),x))/a
```

3.52 $\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 79

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = -\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

output
```
-1/4*arctan(1/2*a^(1/2)*sinh(d*x+c)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))*2^(1/
2)/a^(3/2)/d-1/2*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.24

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \frac{\left(\operatorname{csch}^2\left(\frac{1}{4}(c+dx)\right) - 4\log\left(\cosh\left(\frac{1}{4}(c+dx)\right)\right) + 4\log\left(\sinh\left(\frac{1}{4}(c+dx)\right)\right) +}{4ad(-1+\cosh(c+dx))\sqrt{a-a\cosh(c+}}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(-3/2),x]
```

output
```
((Csch[(c + d*x)/4]^2 - 4*Log[Cosh[(c + d*x)/4]] + 4*Log[Sinh[(c + d*x)/4]
] + Sech[(c + d*x)/4]^2)*Sinh[(c + d*x)/2]^3)/(4*a*d*(-1 + Cosh[c + d*x])*
Sqrt[a - a*Cosh[c + d*x]])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx$$

$$\Big\downarrow \; 3129$$

$$\frac{\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}dx}{4a}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx}{4a}$$

$$\Big\downarrow \; 3128$$

$$-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{i\int \frac{1}{\frac{a^2\sinh^2(c+dx)}{a-a\cosh(c+dx)}+2a}d\frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}}{2ad}$$

$$\Big\downarrow \; 219$$

$$-\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}$$

input
```
Int[(a - a*Cosh[c + d*x])^(-3/2),x]
```

output
```
-1/2*ArcTan[(Sqrt[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(
Sqrt[2]*a^(3/2)*d) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.10

method	result	size
default	$-\frac{-2\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{4a\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	87

input
```
int(1/(a-a*cosh(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a*(-2*cosh(1/2*d*x+1/2*c)+(ln(cosh(1/2*d*x+1/2*c)+1)-ln(cosh(1/2*d*x+
1/2*c)-1))*sinh(1/2*d*x+1/2*c)^2)/sinh(1/2*d*x+1/2*c)/(-2*sinh(1/2*d*x+1/2
*c)^2*a)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.47

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx =$$

$$-\frac{\sqrt{2}\left(\cosh\left(dx+c\right)^{2}+2\left(\cosh\left(dx+c\right)-1\right)\sinh\left(dx+c\right)+\sinh\left(dx+c\right)^{2}-2\,\cosh\left(dx+c\right)+1\right)\sqrt{-a}\log}{4\left(a^{2}d\cosh\right.}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(2)*(cosh(d*x + c)^2 + 2*(cosh(d*x + c) - 1)*sinh(d*x + c) + sin
h(d*x + c)^2 - 2*cosh(d*x + c) + 1)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(1/2)*sqr
t(-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x + c)))*(cosh(d*x + c) + sinh(d*x +
 c)) + a*cosh(d*x + c) + a*sinh(d*x + c) + a)/(cosh(d*x + c) + sinh(d*x +
c) - 1)) + 4*sqrt(1/2)*(cosh(d*x + c)^2 + (2*cosh(d*x + c) + 1)*sinh(d*x +
 c) + sinh(d*x + c)^2 + cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + sinh(d*x +
 c))))/(a^2*d*cosh(d*x + c)^2 + a^2*d*sinh(d*x + c)^2 - 2*a^2*d*cosh(d*x +
 c) + a^2*d + 2*(a^2*d*cosh(d*x + c) - a^2*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{\left(-a\cosh\left(c+dx\right)+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))**(3/2),x)
```

output
```
Integral((-a*cosh(c + d*x) + a)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{(-a\cosh(dx+c)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="maxima")
```

output
```
integrate((-a*cosh(d*x + c) + a)^(-3/2), x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.46

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = -\frac{\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)} + \frac{\sqrt{2}\sqrt{-ae^{(dx+c)}}ae^{(dx+c)}+\sqrt{2}\sqrt{-ae^{(dx+c)}}a}{2\left(ae^{(dx+c)}-a\right)^{2}ad\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(3/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(3/2)*d*sgn(-e^(d*x +
 c) + 1)) + 1/2*(sqrt(2)*sqrt(-a*e^(d*x + c))*a*e^(d*x + c) + sqrt(2)*sqrt
(-a*e^(d*x + c))*a)/((a*e^(d*x + c) - a)^2*a*d*sgn(-e^(d*x + c) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \int \frac{1}{(a-a\cosh(c+d\,x))^{3/2}}\,dx$$

input
```
int(1/(a - a*cosh(c + d*x))^(3/2),x)
```

output
```
int(1/(a - a*cosh(c + d*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{-\cosh(dx+c)+1}}{\cosh(dx+c)^2-2\cosh(dx+c)+1}dx\right)}{a^2}$$

input
```
int(1/(a-a*cosh(d*x+c))^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt( - cosh(c + d*x) + 1)/(cosh(c + d*x)**2 - 2*cosh(c + d*x
) + 1),x))/a**2
```

3.53 $\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 110

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{3\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}} - \frac{3\sinh(c+dx)}{16ad(a-a\cosh(c+dx))^{3/2}}$$

output
```
-3/32*arctan(1/2*a^(1/2)*sinh(d*x+c)*2^(1/2)/(a-a*cosh(d*x+c))^(1/2))*2^(1
/2)/a^(5/2)/d-1/4*sinh(d*x+c)/d/(a-a*cosh(d*x+c))^(5/2)-3/16*sinh(d*x+c)/a
/d/(a-a*cosh(d*x+c))^(3/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.17

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = \frac{\left(6\mathrm{csch}^2\left(\frac{1}{4}(c+dx)\right) - \mathrm{csch}^4\left(\frac{1}{4}(c+dx)\right) + 24\left(-\log\left(\cosh\left(\frac{1}{4}(c+dx)\right)\right) + \right.\right.}{32a^2d(-1+\cosh(c}$$

input
```
Integrate[(a - a*Cosh[c + d*x])^(-5/2),x]
```

output
```
((6*Csch[(c + d*x)/4]^2 - Csch[(c + d*x)/4]^4 + 24*(-Log[Cosh[(c + d*x)/4]
] + Log[Sinh[(c + d*x)/4]]) + 6*Sech[(c + d*x)/4]^2 + Sech[(c + d*x)/4]^4)
*Sinh[(c + d*x)/2]^5)/(32*a^2*d*(-1 + Cosh[c + d*x])^2*Sqrt[a - a*Cosh[c +
 d*x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 3129, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\int \frac{1}{(a-a\cosh(c+dx))^{3/2}}dx}{8a}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}+\frac{3\int \frac{1}{\left(a-a\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^{3/2}}dx}{8a}$$

$$\Big\downarrow \text{3129}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a-a\cosh(c+dx)}}dx}{4a}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}\right)}{8a}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}+\frac{3\left(-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{\int \frac{1}{\sqrt{a-a\sin\left(ic+idx+\frac{\pi}{2}\right)}}dx}{4a}\right)}{8a}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}+\frac{3\left(-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}+\frac{i\int\frac{1}{\frac{a^2\sinh^2(c+dx)}{a-a\cosh(c+dx)}+2a}d\frac{ia\sinh(c+dx)}{\sqrt{a-a\cosh(c+dx)}}}{2ad}\right)}{8a}$$

$$\Big\downarrow \text{219}$$

$$\frac{3\left(-\frac{\arctan\left(\frac{\sqrt{a}\sinh(c+dx)}{\sqrt{2}\sqrt{a-a\cosh(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{\sinh(c+dx)}{2d(a-a\cosh(c+dx))^{3/2}}\right)}{8a}-\frac{\sinh(c+dx)}{4d(a-a\cosh(c+dx))^{5/2}}$$

input
```
Int[(a - a*Cosh[c + d*x])^(-5/2),x]
```

output
```
-1/4*Sinh[c + d*x]/(d*(a - a*Cosh[c + d*x])^(5/2)) + (3*(-1/2*ArcTan[(Sqrt
[a]*Sinh[c + d*x])/(Sqrt[2]*Sqrt[a - a*Cosh[c + d*x]])]/(Sqrt[2]*a^(3/2)*d
) - Sinh[c + d*x]/(2*d*(a - a*Cosh[c + d*x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.44 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.25

method	result	size
default	$-\frac{-6\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+\left(3\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-3\ln\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}}{32a^{2}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a}\,d}$	137

input
```
int(1/(a-a*cosh(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^2*(-6*cosh(1/2*d*x+1/2*c)*sinh(1/2*d*x+1/2*c)^2+4*cosh(1/2*d*x+1/2
*c)+(3*ln(cosh(1/2*d*x+1/2*c)+1)-3*ln(cosh(1/2*d*x+1/2*c)-1))*sinh(1/2*d*x
+1/2*c)^4)/(cosh(1/2*d*x+1/2*c)+1)/(cosh(1/2*d*x+1/2*c)-1)/sinh(1/2*d*x+1/
2*c)/(-2*sinh(1/2*d*x+1/2*c)^2*a)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(91) = 182.

Time = 0.10 (sec) , antiderivative size = 580, normalized size of antiderivative = 5.27

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="fricas")
```

output
```
-1/32*(3*sqrt(2)*(cosh(d*x + c)^4 + 4*(cosh(d*x + c) - 1)*sinh(d*x + c)^3
+ sinh(d*x + c)^4 - 4*cosh(d*x + c)^3 + 6*(cosh(d*x + c)^2 - 2*cosh(d*x +
c) + 1)*sinh(d*x + c)^2 + 6*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - 3*cosh(
d*x + c)^2 + 3*cosh(d*x + c) - 1)*sinh(d*x + c) - 4*cosh(d*x + c) + 1)*sqr
t(-a)*log(-(2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(d*x + c) + sinh(d*x
 + c)))*(cosh(d*x + c) + sinh(d*x + c)) + a*cosh(d*x + c) + a*sinh(d*x + c
) + a)/(cosh(d*x + c) + sinh(d*x + c) - 1)) + 4*sqrt(1/2)*(3*cosh(d*x + c)
^4 + (12*cosh(d*x + c) - 11)*sinh(d*x + c)^3 + 3*sinh(d*x + c)^4 - 11*cosh
(d*x + c)^3 + (18*cosh(d*x + c)^2 - 33*cosh(d*x + c) - 11)*sinh(d*x + c)^2
 - 11*cosh(d*x + c)^2 + (12*cosh(d*x + c)^3 - 33*cosh(d*x + c)^2 - 22*cosh
(d*x + c) + 3)*sinh(d*x + c) + 3*cosh(d*x + c))*sqrt(-a/(cosh(d*x + c) + s
inh(d*x + c))))/(a^3*d*cosh(d*x + c)^4 + a^3*d*sinh(d*x + c)^4 - 4*a^3*d*c
osh(d*x + c)^3 + 6*a^3*d*cosh(d*x + c)^2 - 4*a^3*d*cosh(d*x + c) + a^3*d +
 4*(a^3*d*cosh(d*x + c) - a^3*d)*sinh(d*x + c)^3 + 6*(a^3*d*cosh(d*x + c)^
2 - 2*a^3*d*cosh(d*x + c) + a^3*d)*sinh(d*x + c)^2 + 4*(a^3*d*cosh(d*x + c
)^3 - 3*a^3*d*cosh(d*x + c)^2 + 3*a^3*d*cosh(d*x + c) - a^3*d)*sinh(d*x +
c))
```

Sympy [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = \int \frac{1}{(-a\cosh(c+dx)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))**(5/2),x)
```

output
```
Integral((-a*cosh(c + d*x) + a)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = \int \frac{1}{(-a\cosh(dx+c)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="maxima")
```

output
```
integrate((-a*cosh(d*x + c) + a)^(-5/2), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.61

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{3\sqrt{2}\arctan\left(\frac{\sqrt{-ae^{(dx+c)}}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)} + \frac{3\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(3\,dx+3\,c)} - 11\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(2\,dx+2\,c)} - 11\sqrt{2}\sqrt{-ae^{(dx+c)}}a^3e^{(dx+c)} + 3\sqrt{2}\sqrt{-ae^{(dx}}}{16\left(ae^{(dx+c)}-a\right)^4a^2d\mathrm{sgn}\left(-e^{(dx+c)}+1\right)}$$

input
```
integrate(1/(a-a*cosh(d*x+c))^(5/2),x, algorithm="giac")
```

output
```
-3/16*sqrt(2)*arctan(sqrt(-a*e^(d*x + c))/sqrt(a))/(a^(5/2)*d*sgn(-e^(d*x
+ c) + 1)) + 1/16*(3*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3*e^(3*d*x + 3*c) - 11
*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3*e^(2*d*x + 2*c) - 11*sqrt(2)*sqrt(-a*e^(
d*x + c))*a^3*e^(d*x + c) + 3*sqrt(2)*sqrt(-a*e^(d*x + c))*a^3)/((a*e^(d*x
 + c) - a)^4*a^2*d*sgn(-e^(d*x + c) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = \int \frac{1}{\left(a-a\cosh\left(c+d\,x\right)\right)^{5/2}}\,dx$$

input
```
int(1/(a - a*cosh(c + d*x))^(5/2),x)
```

output
```
int(1/(a - a*cosh(c + d*x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a-a\cosh(c+dx))^{5/2}}\,dx = -\frac{\sqrt{a}\left(\int \frac{\sqrt{-\cosh(dx+c)+1}}{\cosh(dx+c)^3-3\cosh(dx+c)^2+3\cosh(dx+c)-1}dx\right)}{a^3}$$

input

```
int(1/(a-a*cosh(d*x+c))^(5/2),x)
```

output

```
( - sqrt(a)*int(sqrt( - cosh(c + d*x) + 1)/(cosh(c + d*x)**3 - 3*cosh(c +
d*x)**2 + 3*cosh(c + d*x) - 1),x))/a**3
```

3.54 $\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 112

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = -\frac{a(2a^2+b^2)\,x}{2b^4}+\frac{2a^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^4\sqrt{a+b}}+\frac{(3a^2+2b^2)\sinh(x)}{3b^3}-\frac{a\cosh(x)\sinh(x)}{2b^2}+\frac{\cosh^2(x)\sinh(x)}{3b}$$

output
```
-1/2*a*(2*a^2+b^2)*x/b^4+2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2)
)/(a-b)^(1/2)/b^4/(a+b)^(1/2)+1/3*(3*a^2+2*b^2)*sinh(x)/b^3-1/2*a*cosh(x)*
sinh(x)/b^2+1/3*cosh(x)^2*sinh(x)/b
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.88

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{-6a(2a^2+b^2)\,x-\frac{24a^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+3b(4a^2+3b^2)\sinh(x)-3ab^2\sinh(2x)+b^3\sinh(3x)}{12b^4}$$

input
```
Integrate[Cosh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(-6*a*(2*a^2 + b^2)*x - (24*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2
]])/Sqrt[-a^2 + b^2] + 3*b*(4*a^2 + 3*b^2)*Sinh[x] - 3*a*b^2*Sinh[2*x] + b
^3*Sinh[3*x])/(12*b^4)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.15, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3272, 3042, 3528, 25, 3042, 3502, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^4}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{\cosh(x)\left(-3a\cosh^2(x)+2b\cosh(x)+2a\right)}{a+b\cosh(x)}dx}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{\int \frac{\sin\left(ix+\frac{\pi}{2}\right)\left(-3a\sin\left(ix+\frac{\pi}{2}\right)^2+2b\sin\left(ix+\frac{\pi}{2}\right)+2a\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{3b}$$

$$\Big\downarrow \text{3528}$$

$$\frac{\frac{\int -\frac{3a^2-b\cosh(x)a-2\left(3a^2+2b^2\right)\cosh^2(x)}{a+b\cosh(x)}dx}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{-\frac{\int \frac{3a^2-b\cosh(x)a-2\left(3a^2+2b^2\right)\cosh^2(x)}{a+b\cosh(x)}dx}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{-\frac{3a\sinh(x)\cosh(x)}{2b}-\frac{\int \frac{3a^2-b\sin\left(ix+\frac{\pi}{2}\right)a-2\left(3a^2+2b^2\right)\sin\left(ix+\frac{\pi}{2}\right)^2}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b}}{3b}$$

$$\Big\downarrow \; 3502$$

$$\frac{-\frac{\frac{\int \frac{3\left(ba^2+\left(2a^2+b^2\right)\cosh(x)a\right)}{a+b\cosh(x)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \; 27$$

$$\frac{-\frac{\frac{3\int \frac{ba^2+\left(2a^2+b^2\right)\cosh(x)a}{a+b\cosh(x)}dx}{b}-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{-\frac{3a\sinh(x)\cosh(x)}{2b}-\frac{-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}+\frac{3\int \frac{ba^2+\left(2a^2+b^2\right)\sin\left(ix+\frac{\pi}{2}\right)a}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b}}{3b}$$

$$\Big\downarrow \; 3214$$

$$\frac{-\frac{\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{2a^4\int \frac{1}{a+b\cosh(x)}dx}{b}\right)}{b}-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh(x)\cosh^2(x)}{3b}+\frac{-\frac{3a\sinh(x)\cosh(x)}{2b}-\frac{-\frac{2\left(3a^2+2b^2\right)\sinh(x)}{b}+\frac{3\left(\frac{ax\left(2a^2+b^2\right)}{b}-\frac{2a^4\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}}{2b}}{3b}$$

$$\Big\downarrow \; 3138$$

$$-\frac{\frac{3\left(\frac{ax(2a^2+b^2)}{b}-\frac{4a^4\int \frac{1}{-((a-b)\tanh^2(\frac{x}{2}))+a+b}d\tanh(\frac{x}{2})}{b}\right)}{b}-\frac{2(3a^2+2b^2)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

$$\Big\downarrow \; 221$$

$$-\frac{\frac{3\left(\frac{ax(2a^2+b^2)}{b}-\frac{4a^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(\frac{x}{2})}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}\right)}{b}-\frac{2(3a^2+2b^2)\sinh(x)}{b}}{2b}-\frac{3a\sinh(x)\cosh(x)}{2b}}{3b}+\frac{\sinh(x)\cosh^2(x)}{3b}$$

input
```
Int[Cosh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(Cosh[x]^2*Sinh[x])/(3*b) + ((-3*a*Cosh[x]*Sinh[x])/(2*b) - ((3*((a*(2*a^2
 + b^2)*x)/b - (4*a^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[
a - b]*b*Sqrt[a + b])))/b - (2*(3*a^2 + 2*b^2)*Sinh[x])/b)/(2*b))/(3*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

rule 3528
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Simp[1/(d*(m +
n + 2))   Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*
d*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a
*c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
 n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[
m, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(94) = 188$.

Time = 0.86 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.81

method	result
default	$\frac{2a^4 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^4\sqrt{(a+b)(a-b)}} - \frac{1}{3b\left(\tanh\left(\frac{x}{2}\right)-1\right)^3} - \frac{a+b}{2b^2\left(\tanh\left(\frac{x}{2}\right)-1\right)^2} - \frac{2a^2+ab+2b^2}{2b^3\left(\tanh\left(\frac{x}{2}\right)-1\right)} + \frac{a\left(2a^2+b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b^4}$
risch	$-\frac{a^3x}{b^4} - \frac{ax}{2b^2} + \frac{e^{3x}}{24b} - \frac{a\,e^{2x}}{8b^2} + \frac{e^x a^2}{2b^3} + \frac{3\,e^x}{8b} - \frac{e^{-x}a^2}{2b^3} - \frac{3\,e^{-x}}{8b} + \frac{a\,e^{-2x}}{8b^2} - \frac{e^{-3x}}{24b} + \frac{a^4\ln\left(e^x + \frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^4} -$

input
```
int(cosh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*a^4/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/3/b/(tanh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*(2*a^2+a*b+
2*b^2)/b^3/(tanh(1/2*x)-1)+1/2*a*(2*a^2+b^2)/b^4*ln(tanh(1/2*x)-1)-1/3/b/(
1+tanh(1/2*x))^3-1/2*(-a-b)/b^2/(1+tanh(1/2*x))^2-1/2*(2*a^2+a*b+2*b^2)/b^
3/(1+tanh(1/2*x))-1/2*a*(2*a^2+b^2)/b^4*ln(1+tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(94) = 188$.

Time = 0.13 (sec) , antiderivative size = 1625, normalized size of antiderivative = 14.51

$$\int \frac{\cosh^4(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/24*((a^2*b^3 - b^5)*cosh(x)^6 + (a^2*b^3 - b^5)*sinh(x)^6 - 3*(a^3*b^2
- a*b^4)*cosh(x)^5 - 3*(a^3*b^2 - a*b^4 - 2*(a^2*b^3 - b^5)*cosh(x))*sinh(
x)^5 - a^2*b^3 + b^5 - 12*(2*a^5 - a^3*b^2 - a*b^4)*x*cosh(x)^3 + 3*(4*a^4
*b - a^2*b^3 - 3*b^5)*cosh(x)^4 + 3*(4*a^4*b - a^2*b^3 - 3*b^5 + 5*(a^2*b^
3 - b^5)*cosh(x)^2 - 5*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^4 + 2*(10*(a^2*b
^3 - b^5)*cosh(x)^3 - 15*(a^3*b^2 - a*b^4)*cosh(x)^2 - 6*(2*a^5 - a^3*b^2
- a*b^4)*x + 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x)^3 - 3*(4*a^4*b
 - a^2*b^3 - 3*b^5)*cosh(x)^2 - 3*(4*a^4*b - a^2*b^3 - 3*b^5 - 5*(a^2*b^3
- b^5)*cosh(x)^4 + 10*(a^3*b^2 - a*b^4)*cosh(x)^3 + 12*(2*a^5 - a^3*b^2 -
a*b^4)*x*cosh(x) - 6*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^2)*sinh(x)^2 + 24
*(a^4*cosh(x)^3 + 3*a^4*cosh(x)^2*sinh(x) + 3*a^4*cosh(x)*sinh(x)^2 + a^4*
sinh(x)^3)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh
(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*c
osh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*c
osh(x) + a)*sinh(x) + b)) + 3*(a^3*b^2 - a*b^4)*cosh(x) + 3*(2*(a^2*b^3 -
b^5)*cosh(x)^5 + a^3*b^2 - a*b^4 - 5*(a^3*b^2 - a*b^4)*cosh(x)^4 - 12*(2*a
^5 - a^3*b^2 - a*b^4)*x*cosh(x)^2 + 4*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x)^
3 - 2*(4*a^4*b - a^2*b^3 - 3*b^5)*cosh(x))*sinh(x))/((a^2*b^4 - b^6)*cosh(
x)^3 + 3*(a^2*b^4 - b^6)*cosh(x)^2*sinh(x) + 3*(a^2*b^4 - b^6)*cosh(x)*sin
h(x)^2 + (a^2*b^4 - b^6)*sinh(x)^3), 1/24*((a^2*b^3 - b^5)*cosh(x)^6 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^4(x)}{a + b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**4/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^4(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.19

$$\int \frac{\cosh^4(x)}{a + b\cosh(x)}\,dx = \frac{2\,a^4 \arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2} b^4} + \frac{b^2 e^{(3\,x)} - 3\,abe^{(2\,x)} + 12\,a^2 e^x + 9\,b^2 e^x}{24\,b^3} - \frac{(2\,a^3 + ab^2)x}{2\,b^4} + \frac{\left(3\,ab^2 e^x - b^3 - 3\,(4\,a^2 b + 3\,b^3)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4}$$

input
```
integrate(cosh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^4) + 1/24*(
b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x + 9*b^2*e^x)/b^3 - 1/2*(2*a^3 + a
*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b + 3*b^3)*e^(2*x))*e^(-3
*x)/b^4
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.87

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{e^{3x}}{24b} - \frac{e^{-3x}}{24b} - \frac{x(2a^3+ab^2)}{2b^4} + \frac{e^x(4a^2+3b^2)}{8b^3} + \frac{a\,e^{-2x}}{8b^2} - \frac{a\,e^{2x}}{8b^2} - \frac{e^{-x}(4a^2+3b^2)}{8b^3} + \frac{a^4\ln\left(-\frac{2a^4e^x}{b^5} - \frac{2a^4(b+a\,e^x)}{b^5\sqrt{a+b}\sqrt{a-b}}\right)}{b^4\sqrt{a+b}\sqrt{a-b}} - \frac{a^4\ln\left(\frac{2a^4(b+a\,e^x)}{b^5\sqrt{a+b}\sqrt{a-b}} - \frac{2a^4e^x}{b^5}\right)}{b^4\sqrt{a+b}\sqrt{a-b}}$$

input
```
int(cosh(x)^4/(a + b*cosh(x)),x)
```

output
```
exp(3*x)/(24*b) - exp(-3*x)/(24*b) - (x*(a*b^2 + 2*a^3))/(2*b^4) + (exp(x)
*(4*a^2 + 3*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2) -
 (exp(-x)*(4*a^2 + 3*b^2))/(8*b^3) + (a^4*log(- (2*a^4*exp(x))/b^5 - (2*a^
4*(b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)^(1/2))))/(b^4*(a + b)^(1/2)*(
a - b)^(1/2)) - (a^4*log((2*a^4*(b + a*exp(x)))/(b^5*(a + b)^(1/2)*(a - b)
^(1/2)) - (2*a^4*exp(x))/b^5))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.21

$$\int \frac{\cosh^4(x)}{a+b\cosh(x)}\,dx = \frac{-48e^{3x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^4 + e^{6x}a^2b^3 - e^{6x}b^5 - 3e^{5x}a^3b^2 + 3e^{5x}a\,b^4 + 12e^{4x}a^4b - 3e^{4x}a^2b^3 - 9}{24e^{3x}b}$$

input
```
int(cosh(x)^4/(a+b*cosh(x)),x)
```

output

```
( - 48*e**(3*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2
))*a**4 + e**(6*x)*a**2*b**3 - e**(6*x)*b**5 - 3*e**(5*x)*a**3*b**2 + 3*e*
*(5*x)*a*b**4 + 12*e**(4*x)*a**4*b - 3*e**(4*x)*a**2*b**3 - 9*e**(4*x)*b**
5 - 24*e**(3*x)*a**5*x + 12*e**(3*x)*a**3*b**2*x + 12*e**(3*x)*a*b**4*x -
12*e**(2*x)*a**4*b + 3*e**(2*x)*a**2*b**3 + 9*e**(2*x)*b**5 + 3*e**x*a**3*
b**2 - 3*e**x*a*b**4 - a**2*b**3 + b**5)/(24*e**(3*x)*b**4*(a**2 - b**2))
```

3.55 $\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 85

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx = \frac{(2a^2+b^2)\,x}{2b^3} - \frac{2a^3\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^3\sqrt{a+b}} - \frac{a\sinh(x)}{b^2} + \frac{\cosh(x)\sinh(x)}{2b}$$

output
```
1/2*(2*a^2+b^2)*x/b^3-2*a^3*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(
a-b)^(1/2)/b^3/(a+b)^(1/2)-a*sinh(x)/b^2+1/2*cosh(x)*sinh(x)/b
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx = \frac{4a^2x+2b^2x+\frac{8a^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-4ab\sinh(x)+b^2\sinh(2x)}{4b^3}$$

input
```
Integrate[Cosh[x]^3/(a + b*Cosh[x]),x]
```

output
```
(4*a^2*x + 2*b^2*x + (8*a^3*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/
Sqrt[-a^2 + b^2] - 4*a*b*Sinh[x] + b^2*Sinh[2*x])/(4*b^3)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.12, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3272, 3042, 3502, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^3}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3272}$$

$$\frac{\int \frac{-2a\cosh^2(x)+b\cosh(x)+a}{a+b\cosh(x)}dx}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2b}+\frac{\int \frac{-2a\sin\left(ix+\frac{\pi}{2}\right)^2+b\sin\left(ix+\frac{\pi}{2}\right)+a}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b}$$

$$\Big\downarrow \text{3502}$$

$$\frac{\frac{\int \frac{ab+\left(2a^2+b^2\right)\cosh(x)}{a+b\cosh(x)}dx}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2b}+\frac{-\frac{2a\sinh(x)}{b}+\frac{\int \frac{ab+\left(2a^2+b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{2a^3\int \frac{1}{a+b\cosh(x)}dx}{b}}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)\cosh(x)}{2b}+\frac{-\frac{2a\sinh(x)}{b}+\frac{\frac{x(2a^2+b^2)}{b}-\frac{2a^3\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}}{2b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\frac{\frac{x(2a^2+b^2)}{b}-\frac{4a^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{2a\sinh(x)}{b}}{2b}+\frac{\sinh(x)\cosh(x)}{2b}$$

input
```
Int[Cosh[x]^3/(a + b*Cosh[x]),x]
```

output
```
(Cosh[x]*Sinh[x])/(2*b) + ((((2*a^2 + b^2)*x)/b - (4*a^3*ArcTanh[(Sqrt[a -
 b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b - (2*a*Sinh[x]
)/b)/(2*b)
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3272
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m - 2)*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n))), x] + Simp[1/(d*(m
 + n))   Int[(a + b*Sin[e + f*x])^(m - 3)*(c + d*Sin[e + f*x])^n*Simp[a^3*d
*(m + n) + b^2*(b*c*(m - 2) + a*d*(n + 1)) - b*(a*b*c - b^2*d*(m + n - 1) -
 3*a^2*d*(m + n))*Sin[e + f*x] - b^2*(b*c*(m - 1) - a*d*(3*m + 2*n - 2))*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 2] && (IntegerQ[m
] || IntegersQ[2*m, 2*n]) &&  !(IGtQ[n, 2] && ( !IntegerQ[m] || (EqQ[a, 0]
&& NeQ[c, 0])))
```

rule 3502
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Simp[1/(b*(m
+ 2))   Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
 + 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
 &&  !LtQ[m, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(71) = 142$.

Time = 0.53 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

method	result
default	$-\frac{1}{2b\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{-2a-b}{2b^{2}\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{\left(2a^{2}+b^{2}\right)\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b^{3}}+\frac{1}{2b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{-2a-b}{2b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{\left(-2a^{2}-b\right.}{}$
risch	$\frac{x\,a^{2}}{b^{3}}+\frac{x}{2b}+\frac{\mathrm{e}^{2x}}{8b}-\frac{a\,\mathrm{e}^{x}}{2b^{2}}+\frac{a\,\mathrm{e}^{-x}}{2b^{2}}-\frac{\mathrm{e}^{-2x}}{8b}+\frac{a^{3}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{3}}-\frac{a^{3}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{3}}$

input
```
int(cosh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/b/(1+tanh(1/2*x))^2-1/2*(-2*a-b)/b^2/(1+tanh(1/2*x))+1/2*(2*a^2+b^2)/
b^3*ln(1+tanh(1/2*x))+1/2/b/(tanh(1/2*x)-1)^2-1/2*(-2*a-b)/b^2/(tanh(1/2*x
)-1)+1/2/b^3*(-2*a^2-b^2)*ln(tanh(1/2*x)-1)-2*a^3/b^3/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 417 vs. $2(71) = 142$.

Time = 0.12 (sec) , antiderivative size = 903, normalized size of antiderivative = 10.62

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2 - b^4)*sinh(x)^4 - a^2*b^2 + b^
4 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x)^3 -
4*(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x))*sinh(x)^3 + 2*(3*(a^2*b^2 - b^
4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x - 6*(a^3*b - a*b^3)*cosh(x))*si
nh(x)^2 + 8*(a^3*cosh(x)^2 + 2*a^3*cosh(x)*sinh(x) + a^3*sinh(x)^2)*sqrt(a
^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2
 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x
) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(
x) + b)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*
cosh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x) - 3*(a^3*b - a*b^3)*cosh(x
)^2)*sinh(x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2*(a^2*b^3 - b^5)*cosh(x)*sinh(
x) + (a^2*b^3 - b^5)*sinh(x)^2), 1/8*((a^2*b^2 - b^4)*cosh(x)^4 + (a^2*b^2
 - b^4)*sinh(x)^4 - a^2*b^2 + b^4 + 4*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)^2
- 4*(a^3*b - a*b^3)*cosh(x)^3 - 4*(a^3*b - a*b^3 - (a^2*b^2 - b^4)*cosh(x)
)*sinh(x)^3 + 2*(3*(a^2*b^2 - b^4)*cosh(x)^2 + 2*(2*a^4 - a^2*b^2 - b^4)*x
 - 6*(a^3*b - a*b^3)*cosh(x))*sinh(x)^2 + 16*(a^3*cosh(x)^2 + 2*a^3*cosh(x
)*sinh(x) + a^3*sinh(x)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
sh(x) + b*sinh(x) + a)/(a^2 - b^2)) + 4*(a^3*b - a*b^3)*cosh(x) + 4*(a^3*b
 - a*b^3 + (a^2*b^2 - b^4)*cosh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4)*x*cosh(x)
 - 3*(a^3*b - a*b^3)*cosh(x)^2)*sinh(x))/((a^2*b^3 - b^5)*cosh(x)^2 + 2...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^3(x)}{a + b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(cosh(x)**3/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.08

$$\int \frac{\cosh^3(x)}{a + b\cosh(x)}\,dx = -\frac{2\,a^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^3} + \frac{be^{(2\,x)} - 4\,ae^x}{8\,b^2} + \frac{(2\,a^2+b^2)x}{2\,b^3} + \frac{(4\,abe^x - b^2)e^{(-2\,x)}}{8\,b^3}$$

input
```
integrate(cosh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*a^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^3) + 1/8*(
b*e^(2*x) - 4*a*e^x)/b^2 + 1/2*(2*a^2 + b^2)*x/b^3 + 1/8*(4*a*b*e^x - b^2)
*e^(-2*x)/b^3
```

Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.96

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx = \frac{e^{2x}}{8b} - \frac{e^{-2x}}{8b} - \frac{a\,e^{x}}{2b^2} + \frac{a\,e^{-x}}{2b^2} + \frac{x\,(2a^2+b^2)}{2b^3} + \frac{a^3\,\ln\left(\frac{2a^3 e^{x}}{b^4} - \frac{2a^3\,(b+a\,e^{x})}{b^4\sqrt{a+b}\sqrt{a-b}}\right)}{b^3\sqrt{a+b}\sqrt{a-b}} - \frac{a^3\,\ln\left(\frac{2a^3 e^{x}}{b^4} + \frac{2a^3\,(b+a\,e^{x})}{b^4\sqrt{a+b}\sqrt{a-b}}\right)}{b^3\sqrt{a+b}\sqrt{a-b}}$$

input
```
int(cosh(x)^3/(a + b*cosh(x)),x)
```

output
```
exp(2*x)/(8*b) - exp(-2*x)/(8*b) - (a*exp(x))/(2*b^2) + (a*exp(-x))/(2*b^2
) + (x*(2*a^2 + b^2))/(2*b^3) + (a^3*log((2*a^3*exp(x))/b^4 - (2*a^3*(b +
a*exp(x)))/(b^4*(a + b)^(1/2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^
(1/2)) - (a^3*log((2*a^3*exp(x))/b^4 + (2*a^3*(b + a*exp(x)))/(b^4*(a + b)
^(1/2)*(a - b)^(1/2))))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.74

$$\int \frac{\cosh^3(x)}{a+b\cosh(x)}\,dx = \frac{4\sqrt{-a^2+b^2}\,atan\left(\frac{e^{x}b+a}{\sqrt{-a^2+b^2}}\right)a^3 + \cosh(x)^2 a^2b^2x - \cosh(x)^2 b^4x + \cosh(x)\sinh(x)a^2b^2 - \cosh(x)\sinh}{2b^3(a^2-b^2)}$$

input
```
int(cosh(x)^3/(a+b*cosh(x)),x)
```

output
```
(4*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**3 + cos
h(x)**2*a**2*b**2*x - cosh(x)**2*b**4*x + cosh(x)*sinh(x)*a**2*b**2 - cosh
(x)*sinh(x)*b**4 - sinh(x)**2*a**2*b**2*x + sinh(x)**2*b**4*x - 2*sinh(x)*
a**3*b + 2*sinh(x)*a*b**3 + 2*a**4*x - 2*a**2*b**2*x)/(2*b**3*(a**2 - b**2
))
```

3.56 $\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 62

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx=-\frac{ax}{b^2}+\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}}+\frac{\sinh(x)}{b}$$

output
```
-a*x/b^2+2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/b^
2/(a+b)^(1/2)+sinh(x)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx=\frac{a\left(-x-\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(x)}{b^2}$$

input
```
Integrate[Cosh[x]^2/(a + b*Cosh[x]),x]
```

output
```
(a*(-x - (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^
2]) + b*Sinh[x])/b^2
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 3225, 25, 27, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)^2}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3225}$$

$$\frac{\int -\frac{a\cosh(x)}{a+b\cosh(x)}dx}{b}+\frac{\sinh(x)}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh(x)}{b}-\frac{\int \frac{a\cosh(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sinh(x)}{b}-\frac{a\int \frac{\cosh(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{b}-\frac{a\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}\right)}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{2a\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}\right)}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sinh(x)}{b}-\frac{a\left(\frac{x}{b}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}\right)}{b}$$

input

```
Int[Cosh[x]^2/(a + b*Cosh[x]),x]
```

output

```
-((a*(x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b
]*b*Sqrt[a + b])))/b) + Sinh[x]/b
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27

```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3225
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2/((c_.) + (d_.)*sin[(e_.) + (f
_.)*(x_)]), x_Symbol] :> Simp[(-b^2)*(Cos[e + f*x]/(d*f)), x] + Simp[1/d
Int[Simp[a^2*d - b*(b*c - 2*a*d)*Sin[e + f*x], x]/(c + d*Sin[e + f*x]), x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.52

method	result	size
default	$\frac{2a^2 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^2}-\frac{1}{b\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{a\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{b^2}$	94
risch	$-\frac{ax}{b^2}+\frac{e^x}{2b}-\frac{e^{-x}}{2b}+\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2}-\frac{a^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,b^2}$	144

input
```
int(cosh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*a^2/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-1/b/(tanh(1/2*x)-1)+a/b^2*ln(tanh(1/2*x)-1)-1/b/(1+tanh(1/2*x))-a/b^2*l
n(1+tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 190 vs. $2(52) = 104$.

Time = 0.13 (sec) , antiderivative size = 449, normalized size of antiderivative = 7.24

$$\int \frac{\cosh^2(x)}{a + b\cosh(x)}\,dx$$
$$= \left[-\frac{a^2b - b^3 + 2\,(a^3 - ab^2)x\cosh(x) - (a^2b - b^3)\cosh(x)^2 - (a^2b - b^3)\sinh(x)^2 - 2\,(a^2\cosh(x) + a^2\,\mathrm{si}}{} \right.$$
$$-\frac{a^2b - b^3 + 2\,(a^3 - ab^2)x\cosh(x) - (a^2b - b^3)\cosh(x)^2 - (a^2b - b^3)\sinh(x)^2 + 4\,(a^2\cosh(x) + a^2\,\mathrm{sin}}{2\,((a^2b^2 - b^4)\cosh(x)}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh(x) - (a^2*b - b^3)*cosh(x)^2 -
 (a^2*b - b^3)*sinh(x)^2 - 2*(a^2*cosh(x) + a^2*sinh(x))*sqrt(a^2 - b^2)*l
og((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*c
osh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*
cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) +
2*((a^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(x))/((a^2*b^2 - b^4)*cosh
(x) + (a^2*b^2 - b^4)*sinh(x)), -1/2*(a^2*b - b^3 + 2*(a^3 - a*b^2)*x*cosh
(x) - (a^2*b - b^3)*cosh(x)^2 - (a^2*b - b^3)*sinh(x)^2 + 4*(a^2*cosh(x) +
 a^2*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sin
h(x) + a)/(a^2 - b^2)) + 2*((a^3 - a*b^2)*x - (a^2*b - b^3)*cosh(x))*sinh(
x))/((a^2*b^2 - b^4)*cosh(x) + (a^2*b^2 - b^4)*sinh(x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1275 vs. $2(53) = 106$.

Time = 58.48 (sec) , antiderivative size = 1275, normalized size of antiderivative = 20.56

$$\int \frac{\cosh^2(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(x)**2/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*sinh(x), Eq(a, 0) & Eq(b, 0)), (-x*tanh(x/2)**2/(b*tanh(x/2
)**2 - b) + x/(b*tanh(x/2)**2 - b) + tanh(x/2)**3/(b*tanh(x/2)**2 - b) - 3
*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b)), (x*tanh(x/2)**3/(b*tanh(x/2)**
3 - b*tanh(x/2)) - x*tanh(x/2)/(b*tanh(x/2)**3 - b*tanh(x/2)) - 3*tanh(x/2
)**2/(b*tanh(x/2)**3 - b*tanh(x/2)) + 1/(b*tanh(x/2)**3 - b*tanh(x/2)), Eq
(a, -b)), ((-x*sinh(x)**2/2 + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(b,
 0)), (-a**2*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(a*b**2*sqrt(a/(a
- b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b
))) + a**2*x*sqrt(a/(a - b) + b/(a - b))/(a*b**2*sqrt(a/(a - b) + b/(a - b
))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*sqrt(a/(a - b)
 + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b))) - a**2*log(
-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b**2*sqrt(a/(a -
 b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3*
sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b)
)) + a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b**2*sqrt(a/(a
- b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b/(a - b)) - b**3
*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 + b**3*sqrt(a/(a - b) + b/(a - b
))) + a**2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(a*b*
*2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - a*b**2*sqrt(a/(a - b) + b...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh^2(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,a^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2} - \frac{ax}{b^2} - \frac{e^{(-x)}}{2\,b} + \frac{e^x}{2\,b}$$

input
```
integrate(cosh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) - a*x/b^
2 - 1/2*e^(-x)/b + 1/2*e^x/b
```

Mupad [B] (verification not implemented)

Time = 2.13 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.24

$$\int \frac{\cosh^2(x)}{a+b\cosh(x)}\,dx = \frac{e^x}{2\,b} - \frac{e^{-x}}{2\,b} - \frac{a\,x}{b^2} + \frac{a^2\,\ln\left(-\frac{2\,a^2\,e^x}{b^3} - \frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{a^2\,\ln\left(\frac{2\,a^2\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{2\,a^2\,e^x}{b^3}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(cosh(x)^2/(a + b*cosh(x)),x)
```

output
```
exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (a^2*log(- (2*a^2*exp(x))/b^3 -
 (2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(
1/2)*(a - b)^(1/2)) - (a^2*log((2*a^2*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(
a - b)^(1/2)) - (2*a^2*exp(x))/b^3))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^2(x)}{a + b\cosh(x)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2 + \sinh(x)\,a^2 b - \sinh(x)\,b^3 - a^3 x + a\,b^2 x}{b^2\,(a^2-b^2)}$$

input
```
int(cosh(x)^2/(a+b*cosh(x)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**2 +
sinh(x)*a**2*b - sinh(x)*b**3 - a**3*x + a*b**2*x)/(b**2*(a**2 - b**2))
```

3.57 $\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 52

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{x}{b} - \frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}$$

output
```
x/b-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/b/(a+b)^(
1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.92

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{x + \frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}}{b}$$

input
```
Integrate[Cosh[x]/(a + b*Cosh[x]),x]
```

output
```
(x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2])/
b
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{b}-\frac{a\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{x}{b}-\frac{2a\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{x}{b}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Cosh[x]/(a + b*Cosh[x]),x]
```

output
```
x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sq
rt[a + b])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.23

method	result	size
default	$-\frac{2a \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{b}$	64
risch	$\frac{x}{b}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}$	122

input
```
int(cosh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/b*a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-
1/b*ln(tanh(1/2*x)-1)+1/b*ln(1+tanh(1/2*x))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 218, normalized size of antiderivative = 4.19

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \left[\frac{\sqrt{a^2-b^2}a\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+(}{a^2b-b^3}\right.$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) + (a^2 - b^2)*x)/(a^2*b - b^3), (2*sqrt(-a^2 + b^2)*a*a
rctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2 -
b^2)*x)/(a^2*b - b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(41) = 82.

Time = 13.83 (sec) , antiderivative size = 241, normalized size of antiderivative = 4.63

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \begin{cases} \tilde{\infty}x \\ \frac{x}{b}-\frac{\tanh\left(\frac{x}{2}\right)}{b} \\ \frac{x}{b}-\frac{1}{b\tanh\left(\frac{x}{2}\right)} \\ \frac{\sinh(x)}{a} \\ \frac{ax\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{a\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{a\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{bx\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-b^2\sqrt{\frac{a}{a-b}+\frac{}{}}} \end{cases}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0)), (x/b - tanh(x/2)/b, Eq(a, b)), (x/
b - 1/(b*tanh(x/2)), Eq(a, -b)), (sinh(x)/a, Eq(b, 0)), (a*x*sqrt(a/(a - b
) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/
(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a
 - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - a*log(sqrt(a/(a -
 b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt
(a/(a - b) + b/(a - b))) - b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a
- b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.81

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = -\frac{2\,a\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b} + \frac{x}{b}$$

input
```
integrate(cosh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b) + x/b
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.10

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{x}{b}+\frac{a\ \ln\left(\frac{2\,a\,\mathrm{e}^x}{b^2}-\frac{2\,a\,(b+a\,\mathrm{e}^x)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}-\frac{a\ \ln\left(\frac{2\,a\,\mathrm{e}^x}{b^2}+\frac{2\,a\,(b+a\,\mathrm{e}^x)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(cosh(x)/(a + b*cosh(x)),x)
```

output
```
x/b + (a*log((2*a*exp(x))/b^2 - (2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a
 - b)^(1/2))))/(b*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^2 +
 (2*a*(b + a*exp(x)))/(b^2*(a + b)^(1/2)*(a - b)^(1/2))))/(b*(a + b)^(1/2)
*(a - b)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.17

$$\int \frac{\cosh(x)}{a+b\cosh(x)}\,dx = \frac{2\sqrt{-a^2+b^2}\ atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a+a^2x-b^2x}{b\,(a^2-b^2)}$$

input
```
int(cosh(x)/(a+b*cosh(x)),x)
```

output
```
(2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a + a**2*x
 - b**2*x)/(b*(a**2 - b**2))
```

3.58 $\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}-\frac{2b\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

output
```
arctan(sinh(x))/a-2*b*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a/(a-b)
^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx=\frac{2\left(\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{b\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{a}$$

input
```
Integrate[Sech[x]/(a + b*Cosh[x]),x]
```

output
```
(2*(ArcTan[Tanh[x/2]] + (b*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/S
qrt[-a^2 + b^2]))/a
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.545, Rules used = {3042, 3226, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3226}$$

$$\frac{\int \text{sech}(x)dx}{a}-\frac{b\int \frac{1}{a+b\cosh(x)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{2b\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\arctan(\sinh(x))}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[Sech[x]/(a + b*Cosh[x]),x]
```

output
```
ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*
Sqrt[a - b]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

method	result	size
default	$\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2b\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$	51
risch	$\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{a}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{a}+\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,a}$	141

input
```
int(sech(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/a*arctan(tanh(1/2*x))-2*b/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x
)/((a+b)*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 227, normalized size of antiderivative = 4.20

$$\int \frac{\mathrm{sech}(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{\sqrt{a^2-b^2}b\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)+2}{a^3-ab^2}\right.$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2
*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x)
+ b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x)
+ a)*sinh(x) + b)) + 2*(a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2
), 2*(sqrt(-a^2 + b^2)*b*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) +
 a)/(a^2 - b^2)) + (a^2 - b^2)*arctan(cosh(x) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(x)}{a + b\cosh(x)}\,dx = \int \frac{\operatorname{sech}(x)}{a + b\cosh(x)}\,dx$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.83

$$\int \frac{\operatorname{sech}(x)}{a + b\cosh(x)}\,dx = -\frac{2\,b\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}a} + \frac{2\,\arctan\left(e^x\right)}{a}$$

input
```
integrate(sech(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*b*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a) + 2*arctan(
e^x)/a
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 286, normalized size of antiderivative = 5.30

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx = \frac{b\ln\left(64\,a^4\,b-64\,a^2\,b^3+128\,a^5\,e^x+32\,a\,b^3\,\sqrt{a^2-b^2}-64\,a^3\,b\,\sqrt{a^2-b^2}+32\,a\,b^4\,e^x-128\,a^4\,e^x\,\sqrt{a^2-b^2}\right.}{a\,\sqrt{a^2-b^2}}$$
$$-\frac{b\ln\left(64\,a^4\,b-64\,a^2\,b^3+128\,a^5\,e^x-32\,a\,b^3\,\sqrt{a^2-b^2}+64\,a^3\,b\,\sqrt{a^2-b^2}+32\,a\,b^4\,e^x+128\,a^4\,e^x\,\sqrt{a^2-\right.}{a\,\sqrt{a^2-b^2}}$$
$$-\frac{\ln\left(e^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(e^x+1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

input
```
int(1/(cosh(x)*(a + b*cosh(x))),x)
```

output
```
(b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) + 32*a*b^3*(a^2 - b^2)^(1/2)
 - 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) - 128*a^4*exp(x)*(a^2 - b^
2)^(1/2) - 160*a^3*b^2*exp(x) + 96*a^2*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(a*(
a^2 - b^2)^(1/2)) - (b*log(64*a^4*b - 64*a^2*b^3 + 128*a^5*exp(x) - 32*a*b
^3*(a^2 - b^2)^(1/2) + 64*a^3*b*(a^2 - b^2)^(1/2) + 32*a*b^4*exp(x) + 128*
a^4*exp(x)*(a^2 - b^2)^(1/2) - 160*a^3*b^2*exp(x) - 96*a^2*b^2*exp(x)*(a^2
 - b^2)^(1/2)))/(a*(a^2 - b^2)^(1/2)) - (log(exp(x) - 1i)*1i - log(exp(x)
+ 1i)*1i)/a
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}\,dx = \frac{2atan(e^x)\,a^2-2atan(e^x)\,b^2+2\sqrt{-a^2+b^2}\;atan\left(\frac{e^xb+a}{\sqrt{-a^2+b^2}}\right)b}{a\,(a^2-b^2)}$$

input
```
int(sech(x)/(a+b*cosh(x)),x)
```

output
```
(2*(atan(e**x)*a**2 - atan(e**x)*b**2 + sqrt( - a**2 + b**2)*atan((e**x*b
+ a)/sqrt( - a**2 + b**2))*b))/(a*(a**2 - b**2))
```

3.59 $\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 64

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx=-\frac{b\arctan(\sinh(x))}{a^2}+\frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}}+\frac{\tanh(x)}{a}$$

output
```
-b*arctan(sinh(x))/a^2+2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/
a^2/(a-b)^(1/2)/(a+b)^(1/2)+tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.98

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx=\frac{-2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{2b^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+a\tanh(x)}{a^2}$$

input
```
Integrate[Sech[x]^2/(a + b*Cosh[x]),x]
```

output
```
(-2*b*ArcTan[Tanh[x/2]] - (2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^
2]])/Sqrt[-a^2 + b^2] + a*Tanh[x])/a^2
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3281, 25, 27, 3042, 3226, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^2\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{b\operatorname{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)}{a}-\frac{\int \frac{b\operatorname{sech}(x)}{a+b\cosh(x)}dx}{a}$$

$$\Big\downarrow \text{27}$$

$$\frac{\tanh(x)}{a}-\frac{b\int \frac{\operatorname{sech}(x)}{a+b\cosh(x)}dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{a}-\frac{b\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3226}$$

$$\frac{\tanh(x)}{a}-\frac{b\left(\frac{\int \operatorname{sech}(x)dx}{a}-\frac{b\int \frac{1}{a+b\cosh(x)}dx}{a}\right)}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{a}-\frac{b\left(\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{b\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a}$$

$$\Big\downarrow \; 3138$$

$$\frac{\tanh(x)}{a}-\frac{b\left(-\frac{2b\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 221$$

$$\frac{\tanh(x)}{a}-\frac{b\left(-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}$$

$$\Big\downarrow \; 4257$$

$$\frac{\tanh(x)}{a}-\frac{b\left(\frac{\arctan(\sinh(x))}{a}-\frac{2b\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}$$

input
```
Int[Sech[x]^2/(a + b*Cosh[x]),x]
```

output
```
-((b*(ArcTan[Sinh[x]]/a - (2*b*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b])))/a) + Tanh[x]/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3226
```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (
f_.)*(x_)])), x_Symbol] :> Simp[b/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]),
 x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[
{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.14

method	result	size
default	$\frac{2b^2 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}} - \frac{2\left(-\frac{a\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^2+1}+b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)\right)}{a^2}$	73
risch	$-\frac{2}{a(e^{2x}+1)} + \frac{b^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2} - \frac{b^2\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^2} + \frac{ib\ln(e^x-i)}{a^2} - \frac{ib\ln(e^x+i)}{a^2}$	160

input
```
int(sech(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^2/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-2/a^2*(-a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+b*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 221 vs. 2(54) = 108.

Time = 0.12 (sec) , antiderivative size = 515, normalized size of antiderivative = 8.05

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-(2*a^3 - 2*a*b^2 - (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^
2 + b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x
) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cos
h(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cos
h(x) + a)*sinh(x) + b)) + 2*(a^2*b - b^3 + (a^2*b - b^3)*cosh(x)^2 + 2*(a^
2*b - b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*sinh(x)^2)*arctan(cosh(x) + sin
h(x)))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cosh(x)^2 + 2*(a^4 - a^2*b^2)*cosh
(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2), -2*(a^3 - a*b^2 + (b^2*cosh(x)^2
 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 + b^2)*sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3 +
(a^2*b - b^3)*cosh(x)^2 + 2*(a^2*b - b^3)*cosh(x)*sinh(x) + (a^2*b - b^3)*
sinh(x)^2)*arctan(cosh(x) + sinh(x)))/(a^4 - a^2*b^2 + (a^4 - a^2*b^2)*cos
h(x)^2 + 2*(a^4 - a^2*b^2)*cosh(x)*sinh(x) + (a^4 - a^2*b^2)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(sech(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)**2/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^2} - \frac{2\,b\arctan\left(e^x\right)}{a^2} - \frac{2}{a(e^{(2\,x)}+1)}$$

input
```
integrate(sech(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^2) - 2*b*ar
ctan(e^x)/a^2 - 2/(a*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 4.45 (sec) , antiderivative size = 294, normalized size of antiderivative = 4.59

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{b^2\ \ln\left(64\,a\,b^3-64\,a^3\,b+32\,b^3\,\sqrt{a^2-b^2}-128\,a^4\,\mathrm{e}^x-32\,b^4\,\mathrm{e}^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2-b^2}+\right.}{a^2\,\sqrt{a^2-b^2}}$$
$$+\frac{b\left(\ln\left(32\,\mathrm{e}^x-32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,\mathrm{e}^x+32\mathrm{i}\right)\,1\mathrm{i}\right)}{a^2}$$
$$-\frac{b^2\ \ln\left(64\,a^3\,b-64\,a\,b^3+32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2-b^2}\right.}{a^2\,\sqrt{a^2-b^2}}$$
$$-\frac{2}{a+a\,\mathrm{e}^{2\,x}}$$

input
```
int(1/(cosh(x)^2*(a + b*cosh(x))),x)
```

output
```
(b*(log(32*exp(x) - 32i)*1i - log(32*exp(x) + 32i)*1i))/a^2 - 2/(a + a*exp
(2*x)) - (b^2*log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^2)^(1/2) + 128*a^4
*exp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128*a^3*exp(x)*(a^2
 - b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2 - b^2)^(1/2)))/(
a^2*(a^2 - b^2)^(1/2)) + (b^2*log(64*a*b^3 - 64*a^3*b + 32*b^3*(a^2 - b^2)
^(1/2) - 128*a^4*exp(x) - 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 128
*a^3*exp(x)*(a^2 - b^2)^(1/2) + 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^2
- b^2)^(1/2)))/(a^2*(a^2 - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 179, normalized size of antiderivative = 2.80

$$\int \frac{\operatorname{sech}^2(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-2e^{2x}\,atan(e^x)\,a^2b + 2e^{2x}\,atan(e^x)\,b^3 - 2atan(e^x)\,a^2b + 2atan(e^x)\,b^3 - 2e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^xb+a}{\sqrt{-a^2+b^2}}\right)b^2}{a^2\left(e^{2x}a^2 - e^{2x}b^2 + a^2 - b^2\right)}$$

input
```
int(sech(x)^2/(a+b*cosh(x)),x)
```

output
```
(2*( - e**(2*x)*atan(e**x)*a**2*b + e**(2*x)*atan(e**x)*b**3 - atan(e**x)*
a**2*b + atan(e**x)*b**3 - e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)
/sqrt( - a**2 + b**2))*b**2 - sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt(
 - a**2 + b**2))*b**2 + e**(2*x)*a**3 - e**(2*x)*a*b**2))/(a**2*(e**(2*x)*
a**2 - e**(2*x)*b**2 + a**2 - b**2))
```

3.60 $\int \frac{\mathrm{sech}^3(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 87

$$\int \frac{\mathrm{sech}^3(x)}{a+b\cosh(x)}\,dx = \frac{(a^2+2b^2)\arctan(\sinh(x))}{2a^3} - \frac{2b^3\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\sqrt{a+b}} - \frac{b\tanh(x)}{a^2} + \frac{\mathrm{sech}(x)\tanh(x)}{2a}$$

output
```
1/2*(a^2+2*b^2)*arctan(sinh(x))/a^3-2*b^3*arctanh((a-b)^(1/2)*tanh(1/2*x)/
(a+b)^(1/2))/a^3/(a-b)^(1/2)/(a+b)^(1/2)-b*tanh(x)/a^2+1/2*sech(x)*tanh(x)
/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.94

$$\int \frac{\mathrm{sech}^3(x)}{a+b\cosh(x)}\,dx = \frac{2(a^2+2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) + \frac{4b^3\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + a(-2b+a\,\mathrm{sech}(x))\tanh(x)}{2a^3}$$

input
```
Integrate[Sech[x]^3/(a + b*Cosh[x]),x]
```

output
```
(2*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] + (4*b^3*ArcTan[((a - b)*Tanh[x/2])/Sqr
t[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(-2*b + a*Sech[x])*Tanh[x])/(2*a^3)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.923$, Rules used = {3042, 3281, 25, 3042, 3534, 25, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^3\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\left(-b\cosh^2(x)-a\cosh(x)+2b\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}+\frac{\tanh(x)\text{sech}(x)}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\int \frac{\left(-b\cosh^2(x)-a\cosh(x)+2b\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\int \frac{-b\sin\left(ix+\frac{\pi}{2}\right)^2-a\sin\left(ix+\frac{\pi}{2}\right)+2b}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{2a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{\int -\frac{\left(a^2+b\cosh(x)a+2b^2\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{2b\tanh(x)}{a}}{2a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\int\frac{(a^2+b\cosh(x)a+2b^2)\text{sech}(x)}{a+b\cosh(x)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\int\frac{a^2+b\sin\left(ix+\frac{\pi}{2}\right)a+2b^2}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\frac{(a^2+2b^2)\int\text{sech}(x)dx}{a}-\frac{2b^3\int\frac{1}{a+b\cosh(x)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\frac{(a^2+2b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{2b^3\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{-\frac{4b^3\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{(a^2+2b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{-\frac{4b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{(a^2+2b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}}{2a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2b\tanh(x)}{a}-\frac{\frac{(a^2+2b^2)\arctan(\sinh(x))}{a}-\frac{4b^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}}{a}}{2a}$$

input
```
Int[Sech[x]^3/(a + b*Cosh[x]),x]
```

output
```
(Sech[x]*Tanh[x])/(2*a) - (-((((a^2 + 2*b^2)*ArcTan[Sinh[x]])/a - (4*b^3*A
rcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/
a) + (2*b*Tanh[x])/a)/(2*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.25

method	result
default	$-\frac{2 b^{3} \operatorname{arctanh}\left(\frac{(a-b) \tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{3} \sqrt{(a+b)(a-b)}}+\frac{\frac{2\left(\left(-\frac{1}{2} a^{2}-a b\right) \tanh\left(\frac{x}{2}\right)^{3}+\left(\frac{1}{2} a^{2}-a b\right) \tanh\left(\frac{x}{2}\right)\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+\left(a^{2}+2 b^{2}\right) \arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{3}}$
risch	$\frac{a\,e^{3 x}+2\,e^{2 x} b-a\,e^{x}+2 b}{\left(e^{2 x}+1\right)^{2} a^{2}}+\frac{b^{3} \ln\left(e^{x}+\frac{a \sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\, a^{3}}-\frac{b^{3} \ln\left(e^{x}+\frac{a \sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b \sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\, a^{3}}+\frac{i \ln\left(e^{x}+i\right)}{2 a}+\frac{i \ln\left(e^{x}+i\right) b^{2}}{a^{3}}-\frac{i \ln}{}$

input
```
int(sech(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2*b^3/a^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/
2))+2/a^3*(((-1/2*a^2-a*b)*tanh(1/2*x)^3+(1/2*a^2-a*b)*tanh(1/2*x))/(tanh(
1/2*x)^2+1)^2+1/2*(a^2+2*b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(73) = 146$.

Time = 0.17 (sec) , antiderivative size = 1370, normalized size of antiderivative = 15.75

$$\int \frac{\operatorname{sech}^3(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(2*a^3*b - 2*a*b^3 + (a^4 - a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^
3 + 2*(a^3*b - a*b^3)*cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*c
osh(x))*sinh(x)^2 + (b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3*sinh(x)
^4 + 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 + b^3)*sinh(x)^2 + 4*(b^3*
cosh(x)^3 + b^3*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2
*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) +
 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + ((a^4 + a^2*b^2 - 2*b^4
)*cosh(x)^4 + 4*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)*sinh(x)^3 + (a^4 + a^2*b^2
 - 2*b^4)*sinh(x)^4 + a^4 + a^2*b^2 - 2*b^4 + 2*(a^4 + a^2*b^2 - 2*b^4)*co
sh(x)^2 + 2*(a^4 + a^2*b^2 - 2*b^4 + 3*(a^4 + a^2*b^2 - 2*b^4)*cosh(x)^2)*
sinh(x)^2 + 4*((a^4 + a^2*b^2 - 2*b^4)*cosh(x)^3 + (a^4 + a^2*b^2 - 2*b^4)
*cosh(x))*sinh(x))*arctan(cosh(x) + sinh(x)) - (a^4 - a^2*b^2)*cosh(x) - (
a^4 - a^2*b^2 - 3*(a^4 - a^2*b^2)*cosh(x)^2 - 4*(a^3*b - a*b^3)*cosh(x))*s
inh(x))/(a^5 - a^3*b^2 + (a^5 - a^3*b^2)*cosh(x)^4 + 4*(a^5 - a^3*b^2)*cos
h(x)*sinh(x)^3 + (a^5 - a^3*b^2)*sinh(x)^4 + 2*(a^5 - a^3*b^2)*cosh(x)^2 +
 2*(a^5 - a^3*b^2 + 3*(a^5 - a^3*b^2)*cosh(x)^2)*sinh(x)^2 + 4*((a^5 - a^3
*b^2)*cosh(x)^3 + (a^5 - a^3*b^2)*cosh(x))*sinh(x)), (2*a^3*b - 2*a*b^3 +
(a^4 - a^2*b^2)*cosh(x)^3 + (a^4 - a^2*b^2)*sinh(x)^3 + 2*(a^3*b - a*b^3)*
cosh(x)^2 + (2*a^3*b - 2*a*b^3 + 3*(a^4 - a^2*b^2)*cosh(x))*sinh(x)^2 +...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(sech(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)**3/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.02

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = -\frac{2\,b^3\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^3} + \frac{(a^2+2\,b^2)\arctan\left(e^x\right)}{a^3} + \frac{ae^{(3\,x)}+2\,be^{(2\,x)}-ae^x+2\,b}{a^2\left(e^{(2\,x)}+1\right)^2}$$

input
```
integrate(sech(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-2*b^3*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^3) + (a^2
+ 2*b^2)*arctan(e^x)/a^3 + (a*e^(3*x) + 2*b*e^(2*x) - a*e^x + 2*b)/(a^2*(e
^(2*x) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 5.41 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.47

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx = \frac{e^x}{a+a\,e^{2x}} - \frac{2\,e^x}{a+2\,a\,e^{2x}+a\,e^{4x}} + \frac{2\,b}{a^2\,e^{2x}+a^2} - \frac{\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i}{2\,a} - \frac{b^2\,\left(\ln\left(1+e^x\,1i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{a^3} - \frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5-24\,b^5\,\sqrt{a^2-b^2}+32\,a^3\,b^3+32\,a^6\,e^x+24\,b^6\,e^x+16\,a^4\,b\,\sqrt{a^2-b^2}+40\,a^2\,b^3\,\sqrt{\ }}{a^3\,\sqrt{a^2}} + \frac{b^3\,\ln\left(16\,a^5\,b-48\,a\,b^5+24\,b^5\,\sqrt{a^2-b^2}+32\,a^3\,b^3+32\,a^6\,e^x+24\,b^6\,e^x-16\,a^4\,b\,\sqrt{a^2-b^2}-40\,a^2\,b^3\,\sqrt{\ }}{a^3\,\sqrt{a^2}}$$

input
```
int(1/(cosh(x)^3*(a + b*cosh(x))),x)
```

output
```
exp(x)/(a + a*exp(2*x)) - (2*exp(x))/(a + 2*a*exp(2*x) + a*exp(4*x)) + (2*
b)/(a^2*exp(2*x) + a^2) - (log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i)/(2
*a) - (b^2*(log(exp(x)*1i + 1)*1i - log(exp(x) + 1i)*1i))/a^3 - (b^3*log(1
6*a^5*b - 48*a*b^5 - 24*b^5*(a^2 - b^2)^(1/2) + 32*a^3*b^3 + 32*a^6*exp(x)
 + 24*b^6*exp(x) + 16*a^4*b*(a^2 - b^2)^(1/2) + 40*a^2*b^3*(a^2 - b^2)^(1/
2) + 32*a^5*exp(x)*(a^2 - b^2)^(1/2) - 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp
(x) + 72*a^3*b^2*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^4*exp(x)*(a^2 - b^2)^(1
/2)))/(a^3*(a^2 - b^2)^(1/2)) + (b^3*log(16*a^5*b - 48*a*b^5 + 24*b^5*(a^2
 - b^2)^(1/2) + 32*a^3*b^3 + 32*a^6*exp(x) + 24*b^6*exp(x) - 16*a^4*b*(a^2
 - b^2)^(1/2) - 40*a^2*b^3*(a^2 - b^2)^(1/2) - 32*a^5*exp(x)*(a^2 - b^2)^(
1/2) - 112*a^2*b^4*exp(x) + 56*a^4*b^2*exp(x) - 72*a^3*b^2*exp(x)*(a^2 - b
^2)^(1/2) + 72*a*b^4*exp(x)*(a^2 - b^2)^(1/2)))/(a^3*(a^2 - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 361, normalized size of antiderivative = 4.15

$$\int \frac{\operatorname{sech}^3(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{e^{4x}atan(e^x)\,a^4 + e^{4x}atan(e^x)\,a^2b^2 - 2e^{4x}atan(e^x)\,b^4 + 2e^{2x}atan(e^x)\,a^4 + 2e^{2x}atan(e^x)\,a^2b^2 - 4e^{2x}atan(e}{}$$

input
```
int(sech(x)^3/(a+b*cosh(x)),x)
```

output
```
(e**(4*x)*atan(e**x)*a**4 + e**(4*x)*atan(e**x)*a**2*b**2 - 2*e**(4*x)*ata
n(e**x)*b**4 + 2*e**(2*x)*atan(e**x)*a**4 + 2*e**(2*x)*atan(e**x)*a**2*b**
2 - 4*e**(2*x)*atan(e**x)*b**4 + atan(e**x)*a**4 + atan(e**x)*a**2*b**2 -
2*atan(e**x)*b**4 + 2*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt
( - a**2 + b**2))*b**3 + 4*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)
/sqrt( - a**2 + b**2))*b**3 + 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqr
t( - a**2 + b**2))*b**3 - e**(4*x)*a**3*b + e**(4*x)*a*b**3 + e**(3*x)*a**
4 - e**(3*x)*a**2*b**2 - e**x*a**4 + e**x*a**2*b**2 + a**3*b - a*b**3)/(a*
*3*(e**(4*x)*a**2 - e**(4*x)*b**2 + 2*e**(2*x)*a**2 - 2*e**(2*x)*b**2 + a*
*2 - b**2))
```

3.61 $\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 114

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = -\frac{b(a^2+2b^2)\arctan(\sinh(x))}{2a^4} + \frac{2b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}} + \frac{(2a^2+3b^2)\tanh(x)}{3a^3} - \frac{b\operatorname{sech}(x)\tanh(x)}{2a^2} + \frac{\operatorname{sech}^2(x)\tanh(x)}{3a}$$

output
```
-1/2*b*(a^2+2*b^2)*arctan(sinh(x))/a^4+2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*
x)/(a+b)^(1/2))/a^4/(a-b)^(1/2)/(a+b)^(1/2)+1/3*(2*a^2+3*b^2)*tanh(x)/a^3-
1/2*b*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-6b(a^2+2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - \frac{12b^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} + a\left(4a^2+6b^2-3ab\operatorname{sech}(x)+2a^2\operatorname{sech}^2(x)\right)\operatorname{ta}}{6a^4}$$

input
```
Integrate[Sech[x]^4/(a + b*Cosh[x]),x]
```

output
```
(-6*b*(a^2 + 2*b^2)*ArcTan[Tanh[x/2]] - (12*b^4*ArcTan[((a - b)*Tanh[x/2])
/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + a*(4*a^2 + 6*b^2 - 3*a*b*Sech[x] +
2*a^2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.15, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.154$, Rules used = {3042, 3281, 25, 3042, 3534, 25, 3042, 3534, 27, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin\left(\frac{\pi}{2}+ix\right)^4\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3281}$$

$$\frac{\int -\frac{\left(-2b\cosh^2(x)-2a\cosh(x)+3b\right)\text{sech}^3(x)}{a+b\cosh(x)}dx}{3a}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\int \frac{\left(-2b\cosh^2(x)-2a\cosh(x)+3b\right)\text{sech}^3(x)}{a+b\cosh(x)}dx}{3a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\int \frac{-2b\sin\left(ix+\frac{\pi}{2}\right)^2-2a\sin\left(ix+\frac{\pi}{2}\right)+3b}{\sin\left(ix+\frac{\pi}{2}\right)^3\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{3a}$$

$$\Big\downarrow \text{3534}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{\int -\frac{\left(-3b^2\cosh^2(x)+ab\cosh(x)+2\left(2a^2+3b^2\right)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}+\frac{3b\tanh(x)\text{sech}(x)}{2a}}{3a}$$

$\big\downarrow$ 25

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\int \frac{\left(-3b^2\cosh^2(x)+ab\cosh(x)+2\left(2a^2+3b^2\right)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{2a}}{3a}$$

$\big\downarrow$ 3042

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\int \frac{-3b^2\sin\left(ix+\frac{\pi}{2}\right)^2+ab\sin\left(ix+\frac{\pi}{2}\right)+2\left(2a^2+3b^2\right)}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{2a}}{3a}$$

$\big\downarrow$ 3534

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{\int -\frac{3\left(a\cosh(x)b^2+\left(a^2+2b^2\right)b\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}}{2a}}{3a}$$

$\big\downarrow$ 27

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\int \frac{\left(a\cosh(x)b^2+\left(a^2+2b^2\right)b\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}}{2a}}{3a}$$

$\big\downarrow$ 3042

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\int \frac{a\sin\left(ix+\frac{\pi}{2}\right)b^2+\left(a^2+2b^2\right)b}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}}{2a}}{3a}$$

$\big\downarrow$ 3480

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int \text{sech}(x)dx}{a}-\frac{2b^4\int \frac{1}{a+b\cosh(x)}dx}{a}\right)}{a}}{2a}}{3a}$$

$\big\downarrow$ 3042

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2\left(2a^2+3b^2\right)\tanh(x)}{a}-\frac{3\left(\frac{b\left(a^2+2b^2\right)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{2b^4\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\tanh(x)}{a}-\frac{3\left(-\frac{4b^4\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{b(a^2+2b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{221}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\tanh(x)}{a}-\frac{3\left(-\frac{4b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b(a^2+2b^2)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{2a}}{3a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\tanh(x)\text{sech}^2(x)}{3a}-\frac{\frac{3b\tanh(x)\text{sech}(x)}{2a}-\frac{\frac{2(2a^2+3b^2)\tanh(x)}{a}-\frac{3\left(\frac{b(a^2+2b^2)\arctan(\sinh(x))}{a}-\frac{4b^4\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{2a}}{3a}$$

input
```
Int[Sech[x]^4/(a + b*Cosh[x]),x]
```

output
```
(Sech[x]^2*Tanh[x])/(3*a) - ((3*b*Sech[x]*Tanh[x])/(2*a) - ((-3*((b*(a^2 +
 2*b^2)*ArcTan[Sinh[x]])/a - (4*b^4*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a
 + b]])/(a*Sqrt[a - b]*Sqrt[a + b])))/a + (2*(2*a^2 + 3*b^2)*Tanh[x])/a)/(
2*a))/(3*a)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3281
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b^2)*Cos[e + f*x]*(a + b*Sin[e + f*
x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2
))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x
])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[a*(b*c - a*d)*(m + 1) + b^2*d*(m + n
 + 2) - (b^2*c + b*(b*c - a*d)*(m + 1))*Sin[e + f*x] - b^2*d*(m + n + 3)*Si
n[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*
d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -1] && IntegersQ[
2*m, 2*n] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(IntegerQ[2*
n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0])))
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.27

method	result
default	$\frac{2b^4 \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^4\sqrt{(a+b)(a-b)}} - \frac{2\left(\frac{\left(-a^3-\frac{1}{2}a^2b-b^2a\right)\tanh\left(\frac{x}{2}\right)^5+\left(-\frac{2}{3}a^3-2b^2a\right)\tanh\left(\frac{x}{2}\right)^3+\left(-a^3-b^2a+\frac{1}{2}a^2b\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^3}+\frac{b\left(a^2+2b^2\right)}{}\right.}{a^4}$
risch	$-\frac{3ab\,e^{5x}+6b^2e^{4x}+12a^2e^{2x}+12b^2e^{2x}-3b\,e^{x}a+4a^2+6b^2}{3a^3\left(e^{2x}+1\right)^3}+\frac{b^4\ln\left(e^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^4}-\frac{b^4\ln\left(e^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,a^4}+i$

input
```
int(sech(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*b^4/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2
))-2/a^4*(((-a^3-1/2*a^2*b-b^2*a)*tanh(1/2*x)^5+(-2/3*a^3-2*b^2*a)*tanh(1/
2*x)^3+(-a^3-b^2*a+1/2*a^2*b)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^3+1/2*b*(a^2+
2*b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1207 vs. $2(96) = 192$.

Time = 0.17 (sec) , antiderivative size = 2483, normalized size of antiderivative = 21.78

$$\int \frac{\operatorname{sech}^4(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/3*(3*(a^4*b - a^2*b^3)*cosh(x)^5 + 3*(a^4*b - a^2*b^3)*sinh(x)^5 + 4*a
^5 + 2*a^3*b^2 - 6*a*b^4 + 6*(a^3*b^2 - a*b^4)*cosh(x)^4 + 3*(2*a^3*b^2 -
2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x))*sinh(x)^4 + 6*(5*(a^4*b - a^2*b^3)*
cosh(x)^2 + 4*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a*b^4)*cosh
(x)^2 + 6*(2*a^5 - 2*a*b^4 + 5*(a^4*b - a^2*b^3)*cosh(x)^3 + 6*(a^3*b^2 -
a*b^4)*cosh(x)^2)*sinh(x)^2 - 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 +
 b^4*sinh(x)^6 + 3*b^4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 +
b^4)*sinh(x)^4 + b^4 + 4*(5*b^4*cosh(x)^3 + 3*b^4*cosh(x))*sinh(x)^3 + 3*(
5*b^4*cosh(x)^4 + 6*b^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 + 2*
b^4*cosh(x)^3 + b^4*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 +
 b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(
x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(
x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 3*((a^4*b + a^2*b^3
 - 2*b^5)*cosh(x)^6 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)*sinh(x)^5 + (a^4
*b + a^2*b^3 - 2*b^5)*sinh(x)^6 + a^4*b + a^2*b^3 - 2*b^5 + 3*(a^4*b + a^2
*b^3 - 2*b^5)*cosh(x)^4 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3
- 2*b^5)*cosh(x)^2)*sinh(x)^4 + 4*(5*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^3 +
 3*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x))*sinh(x)^3 + 3*(a^4*b + a^2*b^3 - 2*b
^5)*cosh(x)^2 + 3*(a^4*b + a^2*b^3 - 2*b^5 + 5*(a^4*b + a^2*b^3 - 2*b^5)*c
osh(x)^4 + 6*(a^4*b + a^2*b^3 - 2*b^5)*cosh(x)^2)*sinh(x)^2 + 6*((a^4*b...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(sech(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(sech(x)**4/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^4} - \frac{(a^2b+2\,b^3)\arctan(e^x)}{a^4} - \frac{3\,abe^{(5\,x)}+6\,b^2e^{(4\,x)}+12\,a^2e^{(2\,x)}+12\,b^2e^{(2\,x)}-3\,abe^x+4\,a^2+6\,b^2}{3\,a^3(e^{(2\,x)}+1)^3}$$

input
```
integrate(sech(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - (a^2*b
 + 2*b^3)*arctan(e^x)/a^4 - 1/3*(3*a*b*e^(5*x) + 6*b^2*e^(4*x) + 12*a^2*e^
(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x + 4*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 6.03 (sec) , antiderivative size = 547, normalized size of antiderivative = 4.80

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{8}{3\,\left(a+3\,a\,e^{2\,x}+3\,a\,e^{4\,x}+a\,e^{6\,x}\right)}-\frac{4}{a+2\,a\,e^{2\,x}+a\,e^{4\,x}}$$
$$-\frac{2\,b^2}{a^3\,e^{2\,x}+a^3}+\frac{b^3\,\left(\ln\left(e^x-i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{a^4}-\frac{b\,e^x}{a^2\,e^{2\,x}+a^2}$$
$$+\frac{2\,b\,e^x}{2\,a^2\,e^{2\,x}+a^2\,e^{4\,x}+a^2}+\frac{b\,\left(\ln\left(e^x-i\right)\,1i-\ln\left(e^x+1i\right)\,1i\right)}{2\,a^2}$$
$$+\frac{b^4\,\ln\left(32\,a^3\,b^4-24\,b^6\,\sqrt{a^2-b^2}-48\,a\,b^6+16\,a^5\,b^2+24\,b^7\,e^x+32\,a^6\,b\,e^x+40\,a^2\,b^4\,\sqrt{a^2-b^2}+16\,a^4\,b\right.}{a^4\,\sqrt{\right.}$$
$$-\frac{b^4\,\ln\left(24\,b^6\,\sqrt{a^2-b^2}-48\,a\,b^6+32\,a^3\,b^4+16\,a^5\,b^2+24\,b^7\,e^x+32\,a^6\,b\,e^x-40\,a^2\,b^4\,\sqrt{a^2-b^2}-16\,a^4\,b\right.}{a^4\,\sqrt{\right.}$$

input
```
int(1/(cosh(x)^4*(a + b*cosh(x))),x)
```

output
```
8/(3*(a + 3*a*exp(2*x) + 3*a*exp(4*x) + a*exp(6*x))) - 4/(a + 2*a*exp(2*x)
 + a*exp(4*x)) - (2*b^2)/(a^3*exp(2*x) + a^3) + (b^3*(log(exp(x) - 1i)*1i
- log(exp(x) + 1i)*1i))/a^4 - (b*exp(x))/(a^2*exp(2*x) + a^2) + (2*b*exp(x
))/(2*a^2*exp(2*x) + a^2*exp(4*x) + a^2) + (b*(log(exp(x) - 1i)*1i - log(e
xp(x) + 1i)*1i))/(2*a^2) + (b^4*log(32*a^3*b^4 - 24*b^6*(a^2 - b^2)^(1/2)
- 48*a*b^6 + 16*a^5*b^2 + 24*b^7*exp(x) + 32*a^6*b*exp(x) + 40*a^2*b^4*(a^
2 - b^2)^(1/2) + 16*a^4*b^2*(a^2 - b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^
4*b^3*exp(x) + 72*a^3*b^3*exp(x)*(a^2 - b^2)^(1/2) - 72*a*b^5*exp(x)*(a^2
- b^2)^(1/2) + 32*a^5*b*exp(x)*(a^2 - b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2))
 - (b^4*log(24*b^6*(a^2 - b^2)^(1/2) - 48*a*b^6 + 32*a^3*b^4 + 16*a^5*b^2
+ 24*b^7*exp(x) + 32*a^6*b*exp(x) - 40*a^2*b^4*(a^2 - b^2)^(1/2) - 16*a^4*
b^2*(a^2 - b^2)^(1/2) - 112*a^2*b^5*exp(x) + 56*a^4*b^3*exp(x) - 72*a^3*b^
3*exp(x)*(a^2 - b^2)^(1/2) + 72*a*b^5*exp(x)*(a^2 - b^2)^(1/2) - 32*a^5*b*
exp(x)*(a^2 - b^2)^(1/2)))/(a^4*(a^2 - b^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 517, normalized size of antiderivative = 4.54

$$\int \frac{\operatorname{sech}^4(x)}{a+b\cosh(x)}\,dx = \frac{-4a^5+6e^{6x}atan(e^x)\,b^5+18e^{4x}atan(e^x)\,b^5+18e^{2x}atan(e^x)\,b^5-3atan(e^x)\,a^4b-3atan(e^x)\,a^2b^3-6\sqrt{-}}{}$$

input
```
int(sech(x)^4/(a+b*cosh(x)),x)
```

output
```
( - 3*e**(6*x)*atan(e**x)*a**4*b - 3*e**(6*x)*atan(e**x)*a**2*b**3 + 6*e**
(6*x)*atan(e**x)*b**5 - 9*e**(4*x)*atan(e**x)*a**4*b - 9*e**(4*x)*atan(e**
x)*a**2*b**3 + 18*e**(4*x)*atan(e**x)*b**5 - 9*e**(2*x)*atan(e**x)*a**4*b
- 9*e**(2*x)*atan(e**x)*a**2*b**3 + 18*e**(2*x)*atan(e**x)*b**5 - 3*atan(e
**x)*a**4*b - 3*atan(e**x)*a**2*b**3 + 6*atan(e**x)*b**5 - 6*e**(6*x)*sqrt
( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*b**4 - 18*e**(4*x
)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*b**4 - 18*e
**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*b**4
- 6*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*b**4 + 2*
e**(6*x)*a**3*b**2 - 2*e**(6*x)*a*b**4 - 3*e**(5*x)*a**4*b + 3*e**(5*x)*a*
*2*b**3 - 12*e**(2*x)*a**5 + 6*e**(2*x)*a**3*b**2 + 6*e**(2*x)*a*b**4 + 3*
e**x*a**4*b - 3*e**x*a**2*b**3 - 4*a**5 + 4*a*b**4)/(3*a**4*(e**(6*x)*a**2
 - e**(6*x)*b**2 + 3*e**(4*x)*a**2 - 3*e**(4*x)*b**2 + 3*e**(2*x)*a**2 - 3
*e**(2*x)*b**2 + a**2 - b**2))
```

3.62 $\int (a+b\cosh(c+dx))^5\,dx$

Optimal result

Integrand size = 12, antiderivative size = 183

$$\begin{aligned}\int (a+b\cosh(c+dx))^5\,dx = {} & \frac{1}{8}a\left(8a^4+40a^2b^2+15b^4\right)x \\ & + \frac{b(107a^4+192a^2b^2+16b^4)\sinh(c+dx)}{30d} \\ & + \frac{7ab^2(22a^2+23b^2)\cosh(c+dx)\sinh(c+dx)}{120d} \\ & + \frac{b(47a^2+16b^2)\,(a+b\cosh(c+dx))^2\sinh(c+dx)}{60d} \\ & + \frac{9ab(a+b\cosh(c+dx))^3\sinh(c+dx)}{20d} \\ & + \frac{b(a+b\cosh(c+dx))^4\sinh(c+dx)}{5d}\end{aligned}$$

output
```
1/8*a*(8*a^4+40*a^2*b^2+15*b^4)*x+1/30*b*(107*a^4+192*a^2*b^2+16*b^4)*sinh
(d*x+c)/d+7/120*a*b^2*(22*a^2+23*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/60*b*(47
*a^2+16*b^2)*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d+9/20*a*b*(a+b*cosh(d*x+c))^
3*sinh(d*x+c)/d+1/5*b*(a+b*cosh(d*x+c))^4*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.73

$$\int (a+b\cosh(c+dx))^5\,dx$$
$$=\frac{60a(8a^4+40a^2b^2+15b^4)\,(c+dx)+300b(8a^4+12a^2b^2+b^4)\sinh(c+dx)+600ab^2(2a^2+b^2)\sinh(2(c-}{480d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^5,x]
```

output
```
(60*a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*(c + d*x) + 300*b*(8*a^4 + 12*a^2*b^2
+ b^4)*Sinh[c + d*x] + 600*a*b^2*(2*a^2 + b^2)*Sinh[2*(c + d*x)] + 50*b^3*
(8*a^2 + b^2)*Sinh[3*(c + d*x)] + 75*a*b^4*Sinh[4*(c + d*x)] + 6*b^5*Sinh[
5*(c + d*x)])/(480*d)
```

Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 3135, 3042, 3232, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^5\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^5dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{5}\int (a+b\cosh(c+dx))^3\left(5a^2+9b\cosh(c+dx)a+4b^2\right)dx+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}+$$
$$\frac{1}{5}\int\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3\left(5a^2+9b\sin\left(ic+idx+\frac{\pi}{2}\right)a+4b^2\right)dx$$

$\downarrow$ 3232

$$\frac{1}{5}\left(\frac{1}{4}\int(a+b\cosh(c+dx))^2\left(a\left(20a^2+43b^2\right)+b\left(47a^2+16b^2\right)\cosh(c+dx)\right)dx+\frac{9ab\sinh(c+dx)(a+b\cosh(\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

$\downarrow$ 3042

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}+$$
$$\frac{1}{5}\left(\frac{9ab\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+\frac{1}{4}\int\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2\left(a\left(20a^2+43b^2\right)+b\left(47a^2+16b^2\right)\right.\right.$$

$\downarrow$ 3232

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{1}{3}\int(a+b\cosh(c+dx))\left(60a^4+223b^2a^2+7b\left(22a^2+23b^2\right)\cosh(c+dx)a+32b^4\right)dx+\frac{b\left(47a^2+16b^2\right)}{}\right.\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

$\downarrow$ 3042

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}+$$
$$\frac{1}{5}\left(\frac{9ab\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+\frac{1}{4}\left(\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\int\left(a+b\right.\right.\right.$$

$\downarrow$ 3213

$$\frac{1}{5}\left(\frac{1}{4}\left(\frac{b\left(47a^2+16b^2\right)\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\left(\frac{7ab^2\left(22a^2+23b^2\right)\sinh(c+dx)\cosh(c+dx)}{2d}+\right.\right.\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^4}{5d}$$

input
```
Int[(a + b*Cosh[c + d*x])^5,x]
```

output
```
(b*(a + b*Cosh[c + d*x])^4*Sinh[c + d*x])/(5*d) + ((9*a*b*(a + b*Cosh[c +
d*x])^3*Sinh[c + d*x])/(4*d) + ((b*(47*a^2 + 16*b^2)*(a + b*Cosh[c + d*x])
^2*Sinh[c + d*x])/(3*d) + ((15*a*(8*a^4 + 40*a^2*b^2 + 15*b^4)*x)/2 + (2*b
*(107*a^4 + 192*a^2*b^2 + 16*b^4)*Sinh[c + d*x])/d + (7*a*b^2*(22*a^2 + 23
*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/3)/4)/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 149.97 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.72

method	result
parallelrisch	$\frac{600(2a^3b^2+a\,b^4)\sinh(2dx+2c)+50(8a^2b^3+b^5)\sinh(3dx+3c)+75a\,b^4\sinh(4dx+4c)+6b^5\sinh(5dx+5c)+300(8a^4b+1}{480d}$
derivativedivides	$\frac{b^5\left(\frac{8}{15}+\frac{\cosh(dx+c)^4}{5}+\frac{4\cosh(dx+c)^2}{15}\right)\sinh(dx+c)+5a\,b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2}{d}$
default	$\frac{b^5\left(\frac{8}{15}+\frac{\cosh(dx+c)^4}{5}+\frac{4\cosh(dx+c)^2}{15}\right)\sinh(dx+c)+5a\,b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+10a^2}{d}$
parts	$a^5x+\frac{b^5\left(\frac{8}{15}+\frac{\cosh(dx+c)^4}{5}+\frac{4\cosh(dx+c)^2}{15}\right)\sinh(dx+c)}{d}+\frac{5a^4b\sinh(dx+c)}{d}+\frac{10a^3b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}\right)}{d}$
risch	$a^5x+5a^3b^2x+\frac{15a\,b^4x}{8}+\frac{b^5e^{5dx+5c}}{160d}+\frac{5a\,b^4e^{4dx+4c}}{64d}+\frac{5b^3e^{3dx+3c}a^2}{12d}+\frac{5b^5e^{3dx+3c}}{96d}+\frac{5b^2a^3e^{2dx+2c}}{4d}+$
orering	Expression too large to display

input

```
int((a+b*cosh(d*x+c))^5,x,method=_RETURNVERBOSE)
```

output

```
1/480*(600*(2*a^3*b^2+a*b^4)*sinh(2*d*x+2*c)+50*(8*a^2*b^3+b^5)*sinh(3*d*x
+3*c)+75*a*b^4*sinh(4*d*x+4*c)+6*b^5*sinh(5*d*x+5*c)+300*(8*a^4*b+12*a^2*b
^3+b^5)*sinh(d*x+c)+480*a*d*(a^4+5*a^2*b^2+15/8*b^4)*x)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.04

$$\int (a+b\cosh(c+dx))^5\,dx$$
$$=\frac{3\,b^5\sinh(dx+c)^5+5\left(6\,b^5\cosh(dx+c)^2+30\,ab^4\cosh(dx+c)+40\,a^2b^3+5\,b^5\right)\sinh(dx+c)^3+30\,(8}{}$$

input

```
integrate((a+b*cosh(d*x+c))^5,x, algorithm="fricas")
```

output
```
1/240*(3*b^5*sinh(d*x + c)^5 + 5*(6*b^5*cosh(d*x + c)^2 + 30*a*b^4*cosh(d*
x + c) + 40*a^2*b^3 + 5*b^5)*sinh(d*x + c)^3 + 30*(8*a^5 + 40*a^3*b^2 + 15
*a*b^4)*d*x + 15*(b^5*cosh(d*x + c)^4 + 10*a*b^4*cosh(d*x + c)^3 + 80*a^4*
b + 120*a^2*b^3 + 10*b^5 + 5*(8*a^2*b^3 + b^5)*cosh(d*x + c)^2 + 40*(2*a^3
*b^2 + a*b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 314, normalized size of antiderivative = 1.72

$$\int (a+b\cosh(c+dx))^5\,dx$$
$$= \begin{cases} a^5x + \frac{5a^4b\sinh{(c+dx)}}{d} - 5a^3b^2x\sinh^2{(c+dx)} + 5a^3b^2x\cosh^2{(c+dx)} + \frac{5a^3b^2\sinh{(c+dx)}\cosh{(c+dx)}}{d} - \frac{20a^2b^3\,\text{s}}{} \\ x(a+b\cosh{(c)})^5 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**5,x)
```

output
```
Piecewise((a**5*x + 5*a**4*b*sinh(c + d*x)/d - 5*a**3*b**2*x*sinh(c + d*x)
**2 + 5*a**3*b**2*x*cosh(c + d*x)**2 + 5*a**3*b**2*sinh(c + d*x)*cosh(c +
d*x)/d - 20*a**2*b**3*sinh(c + d*x)**3/(3*d) + 10*a**2*b**3*sinh(c + d*x)*
cosh(c + d*x)**2/d + 15*a*b**4*x*sinh(c + d*x)**4/8 - 15*a*b**4*x*sinh(c +
 d*x)**2*cosh(c + d*x)**2/4 + 15*a*b**4*x*cosh(c + d*x)**4/8 - 15*a*b**4*s
inh(c + d*x)**3*cosh(c + d*x)/(8*d) + 25*a*b**4*sinh(c + d*x)*cosh(c + d*x
)**3/(8*d) + 8*b**5*sinh(c + d*x)**5/(15*d) - 4*b**5*sinh(c + d*x)**3*cosh
(c + d*x)**2/(3*d) + b**5*sinh(c + d*x)*cosh(c + d*x)**4/d, Ne(d, 0)), (x*
(a + b*cosh(c))**5, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.49

$$\int (a+b\cosh(c+dx))^5\,dx$$
$$= \frac{5}{64}ab^4\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{5}{4}a^3b^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+a^5x$$
$$+\frac{1}{480}b^5\left(\frac{3e^{(5dx+5c)}}{d}+\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}-\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}-\frac{3e^{(-5dx-5c)}}{d}\right)$$
$$+\frac{5}{12}a^2b^3\left(\frac{e^{(3dx+3c)}}{d}+\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}-\frac{e^{(-3dx-3c)}}{d}\right)+\frac{5a^4b\sinh(dx+c)}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^5,x, algorithm="maxima")
```

output
```
5/64*a*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x -
 2*c)/d - e^(-4*d*x - 4*c)/d) + 5/4*a^3*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(
-2*d*x - 2*c)/d) + a^5*x + 1/480*b^5*(3*e^(5*d*x + 5*c)/d + 25*e^(3*d*x +
3*c)/d + 150*e^(d*x + c)/d - 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d -
3*e^(-5*d*x - 5*c)/d) + 5/12*a^2*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d
- 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 5*a^4*b*sinh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.44

$$\int (a+b\cosh(c+dx))^5\,dx = \frac{b^5e^{(5dx+5c)}}{160d}+\frac{5ab^4e^{(4dx+4c)}}{64d}-\frac{5ab^4e^{(-4dx-4c)}}{64d}-\frac{b^5e^{(-5dx-5c)}}{160d}$$
$$+\frac{1}{8}\left(8a^5+40a^3b^2+15ab^4\right)x+\frac{5\left(8a^2b^3+b^5\right)e^{(3dx+3c)}}{96d}$$
$$+\frac{5\left(2a^3b^2+ab^4\right)e^{(2dx+2c)}}{8d}+\frac{5\left(8a^4b+12a^2b^3+b^5\right)e^{(dx+c)}}{16d}$$
$$-\frac{5\left(8a^4b+12a^2b^3+b^5\right)e^{(-dx-c)}}{16d}$$
$$-\frac{5\left(2a^3b^2+ab^4\right)e^{(-2dx-2c)}}{8d}-\frac{5\left(8a^2b^3+b^5\right)e^{(-3dx-3c)}}{96d}$$

input
```
integrate((a+b*cosh(d*x+c))^5,x, algorithm="giac")
```

output
```
1/160*b^5*e^(5*d*x + 5*c)/d + 5/64*a*b^4*e^(4*d*x + 4*c)/d - 5/64*a*b^4*e^
(-4*d*x - 4*c)/d - 1/160*b^5*e^(-5*d*x - 5*c)/d + 1/8*(8*a^5 + 40*a^3*b^2
+ 15*a*b^4)*x + 5/96*(8*a^2*b^3 + b^5)*e^(3*d*x + 3*c)/d + 5/8*(2*a^3*b^2
+ a*b^4)*e^(2*d*x + 2*c)/d + 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(d*x + c)
/d - 5/16*(8*a^4*b + 12*a^2*b^3 + b^5)*e^(-d*x - c)/d - 5/8*(2*a^3*b^2 + a
*b^4)*e^(-2*d*x - 2*c)/d - 5/96*(8*a^2*b^3 + b^5)*e^(-3*d*x - 3*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int (a + b\cosh(c + dx))^5 \, dx$$
$$= \frac{75\,b^5\sinh(c+d\,x) + \frac{25\,b^5\sinh(3\,c+3\,d\,x)}{2} + \frac{3\,b^5\sinh(5\,c+5\,d\,x)}{2} + 150\,a\,b^4\sinh(2\,c+2\,d\,x) + \frac{75\,a\,b^4\sinh(4\,c+4\,d\,x)}{4} + 9$$

input
```
int((a + b*cosh(c + d*x))^5,x)
```

output
```
(75*b^5*sinh(c + d*x) + (25*b^5*sinh(3*c + 3*d*x))/2 + (3*b^5*sinh(5*c + 5
*d*x))/2 + 150*a*b^4*sinh(2*c + 2*d*x) + (75*a*b^4*sinh(4*c + 4*d*x))/4 +
900*a^2*b^3*sinh(c + d*x) + 300*a^3*b^2*sinh(2*c + 2*d*x) + 100*a^2*b^3*si
nh(3*c + 3*d*x) + 600*a^4*b*sinh(c + d*x) + 120*a^5*d*x + 225*a*b^4*d*x +
600*a^3*b^2*d*x)/(120*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.93

$$\int (a + b\cosh(c + dx))^5 \, dx$$
$$= \frac{6e^{10dx+10c}b^5 + 75e^{9dx+9c}a\,b^4 + 400e^{8dx+8c}a^2b^3 + 50e^{8dx+8c}b^5 + 1200e^{7dx+7c}a^3b^2 + 600e^{7dx+7c}a\,b^4 + 2400e^{6}$$

input
```
int((a+b*cosh(d*x+c))^5,x)
```

output

```
(6*e**(10*c + 10*d*x)*b**5 + 75*e**(9*c + 9*d*x)*a*b**4 + 400*e**(8*c + 8*
d*x)*a**2*b**3 + 50*e**(8*c + 8*d*x)*b**5 + 1200*e**(7*c + 7*d*x)*a**3*b**
2 + 600*e**(7*c + 7*d*x)*a*b**4 + 2400*e**(6*c + 6*d*x)*a**4*b + 3600*e**(
6*c + 6*d*x)*a**2*b**3 + 300*e**(6*c + 6*d*x)*b**5 + 960*e**(5*c + 5*d*x)*
a**5*d*x + 4800*e**(5*c + 5*d*x)*a**3*b**2*d*x + 1800*e**(5*c + 5*d*x)*a*b
**4*d*x - 2400*e**(4*c + 4*d*x)*a**4*b - 3600*e**(4*c + 4*d*x)*a**2*b**3 -
 300*e**(4*c + 4*d*x)*b**5 - 1200*e**(3*c + 3*d*x)*a**3*b**2 - 600*e**(3*c
 + 3*d*x)*a*b**4 - 400*e**(2*c + 2*d*x)*a**2*b**3 - 50*e**(2*c + 2*d*x)*b*
*5 - 75*e**(c + d*x)*a*b**4 - 6*b**5)/(960*e**(5*c + 5*d*x)*d)
```

3.63 $\int (a+b\cosh(c+dx))^4\,dx$

Optimal result

Integrand size = 12, antiderivative size = 137

$$\int (a+b\cosh(c+dx))^4\,dx=\frac{1}{8}\left(8a^4+24a^2b^2+3b^4\right)x+\frac{ab(19a^2+16b^2)\sinh(c+dx)}{6d}+\frac{b^2(26a^2+9b^2)\cosh(c+dx)\sinh(c+dx)}{24d}+\frac{7ab(a+b\cosh(c+dx))^2\sinh(c+dx)}{12d}+\frac{b(a+b\cosh(c+dx))^3\sinh(c+dx)}{4d}$$

output
```
1/8*(8*a^4+24*a^2*b^2+3*b^4)*x+1/6*a*b*(19*a^2+16*b^2)*sinh(d*x+c)/d+1/24*
b^2*(26*a^2+9*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+7/12*a*b*(a+b*cosh(d*x+c))^2*
sinh(d*x+c)/d+1/4*b*(a+b*cosh(d*x+c))^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.76

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$=\frac{12(8a^4+24a^2b^2+3b^4)\,(c+dx)+96ab(4a^2+3b^2)\sinh(c+dx)+24b^2(6a^2+b^2)\sinh(2(c+dx))+32a}{96d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^4,x]
```

output
```
(12*(8*a^4 + 24*a^2*b^2 + 3*b^4)*(c + d*x) + 96*a*b*(4*a^2 + 3*b^2)*Sinh[c
 + d*x] + 24*b^2*(6*a^2 + b^2)*Sinh[2*(c + d*x)] + 32*a*b^3*Sinh[3*(c + d*
x)] + 3*b^4*Sinh[4*(c + d*x)])/(96*d)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.06, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3135, 3042, 3232, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4dx$$
$$\Big\downarrow \text{3135}$$
$$\frac{1}{4}\int (a+b\cosh(c+dx))^2\left(4a^2+7b\cosh(c+dx)a+3b^2\right)dx+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}$$
$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+$$
$$\frac{1}{4}\int\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2\left(4a^2+7b\sin\left(ic+idx+\frac{\pi}{2}\right)a+3b^2\right)dx$$

$\big\downarrow$ 3232

$$\frac{1}{4}\left(\frac{1}{3}\int(a+b\cosh(c+dx))\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\cosh(c+dx)\right)dx+\frac{7ab\sinh(c+dx)(a+b\cosh(c+\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}$$

$\big\downarrow$ 3042

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}+$$
$$\frac{1}{4}\left(\frac{7ab\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\int\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)\left(a\left(12a^2+23b^2\right)+b\left(26a^2+9b^2\right)\mathrm{si}\right.\right.$$

$\big\downarrow$ 3213

$$\frac{1}{4}\left(\frac{1}{3}\left(\frac{2ab\left(19a^2+16b^2\right)\sinh(c+dx)}{d}+\frac{b^2\left(26a^2+9b^2\right)\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3}{2}x\left(8a^4+24a^2b^2+3b^4\right)\right)\right.$$
$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^3}{4d}$$

input
```
Int[(a + b*Cosh[c + d*x])^4,x]
```

output
```
(b*(a + b*Cosh[c + d*x])^3*Sinh[c + d*x])/(4*d) + ((7*a*b*(a + b*Cosh[c +
d*x])^2*Sinh[c + d*x])/(3*d) + ((3*(8*a^4 + 24*a^2*b^2 + 3*b^4)*x)/2 + (2*
a*b*(19*a^2 + 16*b^2)*Sinh[c + d*x])/d + (b^2*(26*a^2 + 9*b^2)*Cosh[c + d*
x]*Sinh[c + d*x])/(2*d))/3)/4
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 19.84 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{24(6a^2b^2+b^4)\sinh(2dx+2c)+32ab^3\sinh(3dx+3c)+3b^4\sinh(4dx+4c)+96(4a^3b+3ab^3)\sinh(dx+c)+96(a^4+3a^2b^2+ \cdots}{96d}$
derivativedivides	$\frac{b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4ab^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+6a^2b^2\left(\frac{\cosh(dx+c)\sin \cdots}{2} \cdots\right.}{d}$
default	$\frac{b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+4ab^3\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+6a^2b^2\left(\frac{\cosh(dx+c)\sin \cdots}{2} \cdots\right.}{d}$
parts	$xa^4+\frac{b^4\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}+\frac{4a^3b\sinh(dx+c)}{d}+\frac{6a^2b^2\left(\frac{\cosh(dx+c)\sinh(dx \cdots}{2} \cdots\right.}{d} \cdots$
risch	$xa^4+3xa^2b^2+\frac{3xb^4}{8}+\frac{b^4e^{4dx+4c}}{64d}+\frac{ab^3e^{3dx+3c}}{6d}+\frac{3b^2e^{2dx+2c}a^2}{4d}+\frac{b^4e^{2dx+2c}}{8d}+\frac{2a^3be^{dx+c}}{d}+\frac{3ab^3 \cdots}{\cdots}$

input
```
int((a+b*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/96*(24*(6*a^2*b^2+b^4)*sinh(2*d*x+2*c)+32*a*b^3*sinh(3*d*x+3*c)+3*b^4*si
nh(4*d*x+4*c)+96*(4*a^3*b+3*a*b^3)*sinh(d*x+c)+96*(a^4+3*a^2*b^2+3/8*b^4)*
d*x)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.90

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$=\frac{(3\,b^4\cosh(dx+c)+8\,ab^3)\sinh(dx+c)^3+3\,(8\,a^4+24\,a^2b^2+3\,b^4)dx+3\,\left(b^4\cosh(dx+c)^3+8\,ab^3\cos\right.}{24\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/24*((3*b^4*cosh(d*x + c) + 8*a*b^3)*sinh(d*x + c)^3 + 3*(8*a^4 + 24*a^2*
b^2 + 3*b^4)*d*x + 3*(b^4*cosh(d*x + c)^3 + 8*a*b^3*cosh(d*x + c)^2 + 32*a
^3*b + 24*a*b^3 + 4*(6*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.75

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$=\begin{cases} a^4x+\frac{4a^3b\sinh(c+dx)}{d}-3a^2b^2x\sinh^2(c+dx)+3a^2b^2x\cosh^2(c+dx)+\frac{3a^2b^2\sinh(c+dx)\cosh(c+dx)}{d}-\frac{8ab^3\sin}{} \\ x(a+b\cosh(c))^4 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*sinh(c + d*x)/d - 3*a**2*b**2*x*sinh(c + d*x)
**2 + 3*a**2*b**2*x*cosh(c + d*x)**2 + 3*a**2*b**2*sinh(c + d*x)*cosh(c +
d*x)/d - 8*a*b**3*sinh(c + d*x)**3/(3*d) + 4*a*b**3*sinh(c + d*x)*cosh(c +
 d*x)**2/d + 3*b**4*x*sinh(c + d*x)**4/8 - 3*b**4*x*sinh(c + d*x)**2*cosh(
c + d*x)**2/4 + 3*b**4*x*cosh(c + d*x)**4/8 - 3*b**4*sinh(c + d*x)**3*cosh
(c + d*x)/(8*d) + 5*b**4*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)),
(x*(a + b*cosh(c))**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.34

$$\int (a+b\cosh(c+dx))^4\,dx = \frac{1}{64}b^4\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) + \frac{3}{4}a^2b^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)+a^4x + \frac{1}{6}ab^3\left(\frac{e^{(3dx+3c)}}{d}+\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}-\frac{e^{(-3dx-3c)}}{d}\right)+\frac{4a^3b\sinh(dx+c)}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
1/64*b^4*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/4*a^2*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2
*d*x - 2*c)/d) + a^4*x + 1/6*a*b^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d -
9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d) + 4*a^3*b*sinh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.43

$$\int (a+b\cosh(c+dx))^4\,dx = \frac{b^4e^{(4\,dx+4\,c)}}{64\,d} + \frac{ab^3e^{(3\,dx+3\,c)}}{6\,d} - \frac{ab^3e^{(-3\,dx-3\,c)}}{6\,d} - \frac{b^4e^{(-4\,dx-4\,c)}}{64\,d} + \frac{1}{8}\left(8\,a^4+24\,a^2b^2+3\,b^4\right)x + \frac{(6\,a^2b^2+b^4)e^{(2\,dx+2\,c)}}{8\,d} + \frac{(4\,a^3b+3\,ab^3)e^{(dx+c)}}{2\,d} - \frac{(4\,a^3b+3\,ab^3)e^{(-dx-c)}}{2\,d} - \frac{(6\,a^2b^2+b^4)e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/64*b^4*e^(4*d*x + 4*c)/d + 1/6*a*b^3*e^(3*d*x + 3*c)/d - 1/6*a*b^3*e^(-3
*d*x - 3*c)/d - 1/64*b^4*e^(-4*d*x - 4*c)/d + 1/8*(8*a^4 + 24*a^2*b^2 + 3*
b^4)*x + 1/8*(6*a^2*b^2 + b^4)*e^(2*d*x + 2*c)/d + 1/2*(4*a^3*b + 3*a*b^3)
*e^(d*x + c)/d - 1/2*(4*a^3*b + 3*a*b^3)*e^(-d*x - c)/d - 1/8*(6*a^2*b^2 +
 b^4)*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.83

$$\int (a+b\cosh(c+dx))^4\,dx = \frac{6\,b^4\sinh(2\,c+2\,d\,x) + \frac{3\,b^4\sinh(4\,c+4\,d\,x)}{4} + 8\,a\,b^3\sinh(3\,c+3\,d\,x) + 36\,a^2\,b^2\sinh(2\,c+2\,d\,x) + 72\,a\,b^3\sinh(\ldots}{24\,d}$$

input
```
int((a + b*cosh(c + d*x))^4,x)
```

output
```
(6*b^4*sinh(2*c + 2*d*x) + (3*b^4*sinh(4*c + 4*d*x))/4 + 8*a*b^3*sinh(3*c
+ 3*d*x) + 36*a^2*b^2*sinh(2*c + 2*d*x) + 72*a*b^3*sinh(c + d*x) + 96*a^3*
b*sinh(c + d*x) + 24*a^4*d*x + 9*b^4*d*x + 72*a^2*b^2*d*x)/(24*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.83

$$\int (a+b\cosh(c+dx))^4\,dx$$
$$= \frac{3e^{8dx+8c}b^4+32e^{7dx+7c}a\,b^3+144e^{6dx+6c}a^2b^2+24e^{6dx+6c}b^4+384e^{5dx+5c}a^3b+288e^{5dx+5c}a\,b^3+192e^{4dx+4c}}{}$$

input
```
int((a+b*cosh(d*x+c))^4,x)
```

output
```
(3*e**(8*c + 8*d*x)*b**4 + 32*e**(7*c + 7*d*x)*a*b**3 + 144*e**(6*c + 6*d*
x)*a**2*b**2 + 24*e**(6*c + 6*d*x)*b**4 + 384*e**(5*c + 5*d*x)*a**3*b + 28
8*e**(5*c + 5*d*x)*a*b**3 + 192*e**(4*c + 4*d*x)*a**4*d*x + 576*e**(4*c +
4*d*x)*a**2*b**2*d*x + 72*e**(4*c + 4*d*x)*b**4*d*x - 384*e**(3*c + 3*d*x)
*a**3*b - 288*e**(3*c + 3*d*x)*a*b**3 - 144*e**(2*c + 2*d*x)*a**2*b**2 - 2
4*e**(2*c + 2*d*x)*b**4 - 32*e**(c + d*x)*a*b**3 - 3*b**4)/(192*e**(4*c +
4*d*x)*d)
```

3.64 $\int (a+b\cosh(c+dx))^3\,dx$

Optimal result

Integrand size = 12, antiderivative size = 90

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{1}{2}a\left(2a^2+3b^2\right)x+\frac{2b(4a^2+b^2)\sinh(c+dx)}{3d}+\frac{5ab^2\cosh(c+dx)\sinh(c+dx)}{6d}+\frac{b(a+b\cosh(c+dx))^2\sinh(c+dx)}{3d}$$

output
```
1/2*a*(2*a^2+3*b^2)*x+2/3*b*(4*a^2+b^2)*sinh(d*x+c)/d+5/6*a*b^2*cosh(d*x+c
)*sinh(d*x+c)/d+1/3*b*(a+b*cosh(d*x+c))^2*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.89

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{12a^3c+18ab^2c+12a^3dx+18ab^2dx+9b(4a^2+b^2)\sinh(c+dx)+9ab^2\sinh(2(c+dx))+b^3\sinh(3(c+}{12d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^3,x]
```

output
```
(12*a^3*c + 18*a*b^2*c + 12*a^3*d*x + 18*a*b^2*d*x + 9*b*(4*a^2 + b^2)*Sin
h[c + d*x] + 9*a*b^2*Sinh[2*(c + d*x)] + b^3*Sinh[3*(c + d*x)])/(12*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3135, 3042, 3213}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3 dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{1}{3}\int (a+b\cosh(c+dx))\left(3a^2+5b\cosh(c+dx)a+2b^2\right)dx+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}$$

$$\Big\downarrow \text{3042}$$

$$\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}+\frac{1}{3}\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)\left(3a^2+5b\sin\left(ic+idx+\frac{\pi}{2}\right)a+2b^2\right)dx$$

$$\Big\downarrow \text{3213}$$

$$\frac{1}{3}\left(\frac{2b(4a^2+b^2)\sinh(c+dx)}{d}+\frac{3}{2}ax(2a^2+3b^2)+\frac{5ab^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)+\frac{b\sinh(c+dx)(a+b\cosh(c+dx))^2}{3d}$$

input
```
Int[(a + b*Cosh[c + d*x])^3,x]
```

output
```
(b*(a + b*Cosh[c + d*x])^2*Sinh[c + d*x])/(3*d) + ((3*a*(2*a^2 + 3*b^2)*x)
/2 + (2*b*(4*a^2 + b^2)*Sinh[c + d*x])/d + (5*a*b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d))/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3213
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)
*(x_)]), x_Symbol] :> Simp[(2*a*c + b*d)*(x/2), x] + (-Simp[(b*c + a*d)*(Co
s[e + f*x]/f), x] - Simp[b*d*Cos[e + f*x]*(Sin[e + f*x]/(2*f)), x]) /; Free
Q[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 229.86 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.74

method	result
parallelrisch	$\frac{9b^{2}a\sinh(2dx+2c)+b^{3}\sinh(3dx+3c)+9\left(4a^{2}b+b^{3}\right)\sinh(dx+c)+12\left(a^{2}+\frac{3b^{2}}{2}\right)adx}{12d}$
derivativedivides	$\frac{b^{3}\left(\frac{2}{3}+\frac{\cosh(dx+c)^{2}}{3}\right)\sinh(dx+c)+3b^{2}a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^{2}b\sinh(dx+c)+a^{3}(dx+c)}{d}$
default	$\frac{b^{3}\left(\frac{2}{3}+\frac{\cosh(dx+c)^{2}}{3}\right)\sinh(dx+c)+3b^{2}a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^{2}b\sinh(dx+c)+a^{3}(dx+c)}{d}$
parts	$a^{3}x+\frac{b^{3}\left(\frac{2}{3}+\frac{\cosh(dx+c)^{2}}{3}\right)\sinh(dx+c)}{d}+\frac{3a^{2}b\sinh(dx+c)}{d}+\frac{3b^{2}a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$
risch	$a^{3}x+\frac{3a\,b^{2}x}{2}+\frac{b^{3}\mathrm{e}^{3dx+3c}}{24d}+\frac{3\,\mathrm{e}^{2dx+2c}b^{2}a}{8d}+\frac{3b\,\mathrm{e}^{dx+c}a^{2}}{2d}+\frac{3\,\mathrm{e}^{dx+c}b^{3}}{8d}-\frac{3b\,\mathrm{e}^{-dx-c}a^{2}}{2d}-\frac{3b^{3}\mathrm{e}^{-dx-c}}{8d}-$ 3 e

input
```
int((a+b*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(9*b^2*a*sinh(2*d*x+2*c)+b^3*sinh(3*d*x+3*c)+9*(4*a^2*b+b^3)*sinh(d*x
+c)+12*(a^2+3/2*b^2)*a*d*x)/d
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.87

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{b^3\sinh(dx+c)^3+6\,(2\,a^3+3\,ab^2)dx+3\,\left(b^3\cosh(dx+c)^2+6\,ab^2\cosh(dx+c)+12\,a^2b+3\,b^3\right)\sinh(d}{12\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/12*(b^3*sinh(d*x + c)^3 + 6*(2*a^3 + 3*a*b^2)*d*x + 3*(b^3*cosh(d*x + c)
^2 + 6*a*b^2*cosh(d*x + c) + 12*a^2*b + 3*b^3)*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.42

$$\int (a+b\cosh(c+dx))^3\,dx = \begin{cases} a^3x+\frac{3a^2b\sinh(c+dx)}{d}-\frac{3ab^2x\sinh^2(c+dx)}{2}+\frac{3ab^2x\cosh^2(c+dx)}{2}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{2b^3\sinh^3(c+dx)}{3d}+\frac{b^3\sinh}{} \\ x(a+b\cosh(c))^3 \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)/d - 3*a*b**2*x*sinh(c + d*x)**2
/2 + 3*a*b**2*x*cosh(c + d*x)**2/2 + 3*a*b**2*sinh(c + d*x)*cosh(c + d*x)/
(2*d) - 2*b**3*sinh(c + d*x)**3/(3*d) + b**3*sinh(c + d*x)*cosh(c + d*x)**
2/d, Ne(d, 0)), (x*(a + b*cosh(c))**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.29

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{3}{8}ab^2\left(4x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^3x+\frac{1}{24}b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3\,a^2b\sinh\left(dx+c\right)}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
3/8*a*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^3*x + 1/24*b^
3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*
c)/d) + 3*a^2*b*sinh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.46

$$\int (a+b\cosh(c+dx))^3\,dx = \frac{b^3e^{(3\,dx+3\,c)}}{24\,d}+\frac{3\,ab^2e^{(2\,dx+2\,c)}}{8\,d}-\frac{3\,ab^2e^{(-2\,dx-2\,c)}}{8\,d}-\frac{b^3e^{(-3\,dx-3\,c)}}{24\,d}+\frac{1}{2}\left(2\,a^3+3\,ab^2\right)x+\frac{3\left(4\,a^2b+b^3\right)e^{(dx+c)}}{8\,d}-\frac{3\left(4\,a^2b+b^3\right)e^{(-dx-c)}}{8\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/24*b^3*e^(3*d*x + 3*c)/d + 3/8*a*b^2*e^(2*d*x + 2*c)/d - 3/8*a*b^2*e^(-2
*d*x - 2*c)/d - 1/24*b^3*e^(-3*d*x - 3*c)/d + 1/2*(2*a^3 + 3*a*b^2)*x + 3/
8*(4*a^2*b + b^3)*e^(d*x + c)/d - 3/8*(4*a^2*b + b^3)*e^(-d*x - c)/d
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.81

$$\int (a+b\cosh(c+dx))^3\,dx$$
$$=\frac{\frac{9\,b^3\sinh(c+d\,x)}{2}+\frac{b^3\sinh(3\,c+3\,d\,x)}{2}+\frac{9\,a\,b^2\sinh(2\,c+2\,d\,x)}{2}+18\,a^2\,b\sinh(c+d\,x)+6\,a^3\,d\,x+9\,a\,b^2\,d\,x}{6\,d}$$

input
```
int((a + b*cosh(c + d*x))^3,x)
```

output
```
((9*b^3*sinh(c + d*x))/2 + (b^3*sinh(3*c + 3*d*x))/2 + (9*a*b^2*sinh(2*c +
 2*d*x))/2 + 18*a^2*b*sinh(c + d*x) + 6*a^3*d*x + 9*a*b^2*d*x)/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.81

$$\int (a+b\cosh(c+dx))^3\,dx$$
$$=\frac{e^{6dx+6c}b^3+9e^{5dx+5c}a\,b^2+36e^{4dx+4c}a^2b+9e^{4dx+4c}b^3+24e^{3dx+3c}a^3dx+36e^{3dx+3c}a\,b^2dx-36e^{2dx+2c}a^2b-}{24e^{3dx+3c}d}$$

input
```
int((a+b*cosh(d*x+c))^3,x)
```

output
```
(e**(6*c + 6*d*x)*b**3 + 9*e**(5*c + 5*d*x)*a*b**2 + 36*e**(4*c + 4*d*x)*a
**2*b + 9*e**(4*c + 4*d*x)*b**3 + 24*e**(3*c + 3*d*x)*a**3*d*x + 36*e**(3*
c + 3*d*x)*a*b**2*d*x - 36*e**(2*c + 2*d*x)*a**2*b - 9*e**(2*c + 2*d*x)*b*
*3 - 9*e**(c + d*x)*a*b**2 - b**3)/(24*e**(3*c + 3*d*x)*d)
```

3.65 $\int (a+b\cosh(c+dx))^2\,dx$

Optimal result

Integrand size = 12, antiderivative size = 50

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{1}{2}\left(2a^2+b^2\right)x+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(2*a^2+b^2)*x+2*a*b*sinh(d*x+c)/d+1/2*b^2*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.92

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{2\left(2a^2+b^2\right)(c+dx)+8ab\sinh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^2,x]
```

output
```
(2*(2*a^2 + b^2)*(c + d*x) + 8*a*b*Sinh[c + d*x] + b^2*Sinh[2*(c + d*x)])/
(4*d)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3042, 3123}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(c+dx))^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2dx$$

$$\Big\downarrow \text{3123}$$

$$\frac{1}{2}x\left(2a^2+b^2\right)+\frac{2ab\sinh(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

input
```
Int[(a + b*Cosh[c + d*x])^2,x]
```

output
```
((2*a^2 + b^2)*x)/2 + (2*a*b*Sinh[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c
+ d*x])/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3123
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^2, x_Symbol] :> Simp[(2*a^2 + b^
2)*(x/2), x] + (-Simp[2*a*b*(Cos[c + d*x]/d), x] - Simp[b^2*Cos[c + d*x]*(S
in[c + d*x]/(2*d)), x]) /; FreeQ[{a, b, c, d}, x]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.86

method	result
parallelrisch	$\frac{b^{2}\sinh(2dx+2c)+8\sinh(dx+c)ab+4\left(a^{2}+\frac{b^{2}}{2}\right)dx}{4d}$
parts	$a^{2}x+\frac{b^{2}\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2ab\sinh(dx+c)}{d}$
derivativedivides	$\frac{b^{2}\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\sinh(dx+c)ab+a^{2}(dx+c)}{d}$
default	$\frac{b^{2}\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2\sinh(dx+c)ab+a^{2}(dx+c)}{d}$
risch	$a^{2}x+\frac{b^{2}x}{2}+\frac{e^{2dx+2c}b^{2}}{8d}+\frac{e^{dx+c}ab}{d}-\frac{e^{-dx-c}ab}{d}-\frac{e^{-2dx-2c}b^{2}}{8d}$
orering	$x(a+b\cosh(dx+c))^{2}+\frac{5(a+b\cosh(dx+c))b\sinh(dx+c)}{2d}-\frac{5x\left(2b^{2}d^{2}\sinh(dx+c)^{2}+2(a+b\cosh(dx+c))b\right.}{4d^{2}}$

input
```
int((a+b*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(b^2*sinh(2*d*x+2*c)+8*sinh(d*x+c)*a*b+4*(a^2+1/2*b^2)*d*x)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80

$$\int (a+b\cosh(c+dx))^{2}\,dx=\frac{(2\,a^{2}+b^{2})dx+(b^{2}\cosh{(dx+c)}+4\,ab)\sinh{(dx+c)}}{2\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*((2*a^2 + b^2)*d*x + (b^2*cosh(d*x + c) + 4*a*b)*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int (a + b\cosh(c + dx))^2 \, dx = \begin{cases} a^2x + \frac{2ab\sinh(c+dx)}{d} - \frac{b^2x\sinh^2(c+dx)}{2} + \frac{b^2x\cosh^2(c+dx)}{2} + \frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d} & \text{for } d \neq 0 \\ x(a + b\cosh(c))^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*cosh(d*x+c))**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*sinh(c + d*x)/d - b**2*x*sinh(c + d*x)**2/2 + b*
*2*x*cosh(c + d*x)**2/2 + b**2*sinh(c + d*x)*cosh(c + d*x)/(2*d), Ne(d, 0)
), (x*(a + b*cosh(c))**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.10

$$\int (a+b\cosh(c+dx))^2 \, dx = \frac{1}{8}b^2\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) + a^2x + \frac{2ab\sinh(dx+c)}{d}$$

input
```
integrate((a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/8*b^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) + a^2*x + 2*a*b*sin
h(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.50

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{1}{2}\left(2a^2+b^2\right)x + \frac{b^2e^{(2\,dx+2\,c)}}{8\,d} + \frac{abe^{(dx+c)}}{d} - \frac{abe^{(-dx-c)}}{d} - \frac{b^2e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate((a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/2*(2*a^2 + b^2)*x + 1/8*b^2*e^(2*d*x + 2*c)/d + a*b*e^(d*x + c)/d - a*b*
e^(-d*x - c)/d - 1/8*b^2*e^(-2*d*x - 2*c)/d
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.82

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{\frac{\sinh(2\,c+2\,d\,x)\,b^2}{4} + 2\,a\sinh(c+d\,x)\,b}{d} + a^2\,x + \frac{b^2\,x}{2}$$

input
```
int((a + b*cosh(c + d*x))^2,x)
```

output
```
((b^2*sinh(2*c + 2*d*x))/4 + 2*a*b*sinh(c + d*x))/d + a^2*x + (b^2*x)/2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.92

$$\int (a+b\cosh(c+dx))^2\,dx = \frac{e^{4dx+4c}b^2 + 8e^{3dx+3c}ab + 8e^{2dx+2c}a^2dx + 4e^{2dx+2c}b^2dx - 8e^{dx+c}ab - b^2}{8e^{2dx+2c}d}$$

input
```
int((a+b*cosh(d*x+c))^2,x)
```

output
```
(e**(4*c + 4*d*x)*b**2 + 8*e**(3*c + 3*d*x)*a*b + 8*e**(2*c + 2*d*x)*a**2*
d*x + 4*e**(2*c + 2*d*x)*b**2*d*x - 8*e**(c + d*x)*a*b - b**2)/(8*e**(2*c
+ 2*d*x)*d)
```

3.66 $\int (a+b\cosh(c+dx))\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int (a+b\cosh(c+dx))\,dx = ax+\frac{b\sinh(c+dx)}{d}$$

output
```
a*x+b*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.73

$$\int (a+b\cosh(c+dx))\,dx = ax+\frac{b\cosh(dx)\sinh(c)}{d}+\frac{b\cosh(c)\sinh(dx)}{d}$$

input
```
Integrate[a + b*Cosh[c + d*x],x]
```

output
```
a*x + (b*Cosh[d*x]*Sinh[c])/d + (b*Cosh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.15 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a + b\cosh(c + dx))\,dx$$

$$\Big\downarrow \text{2009}$$

$$ax + \frac{b\sinh(c + dx)}{d}$$

input
```
Int[a + b*Cosh[c + d*x],x]
```

output
```
a*x + (b*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

method	result	size
default	$ax+\frac{b\sinh(dx+c)}{d}$	16
parallelrisch	$ax+\frac{b\sinh(dx+c)}{d}$	16
parts	$ax+\frac{b\sinh(dx+c)}{d}$	16
derivativedivides	$\frac{(dx+c)a+b\sinh(dx+c)}{d}$	21
risch	$ax+\frac{e^{dx+c}b}{2d}-\frac{b\,e^{-dx-c}}{2d}$	32
orering	$x(a+b\cosh(dx+c))+\frac{b\sinh(dx+c)}{d}-xb\cosh(dx+c)$	35

input
```
int(a+b*cosh(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
a*x+b*sinh(d*x+c)/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\cosh(c+dx))\,dx=\frac{adx+b\sinh(dx+c)}{d}$$

input
```
integrate(a+b*cosh(d*x+c),x, algorithm="fricas")
```

output
```
(a*d*x + b*sinh(d*x + c))/d
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\cosh(c+dx))\,dx = ax + b\left(\begin{cases} \frac{\sinh(c+dx)}{d} & \text{for } d\neq 0 \\ x\cosh(c) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*cosh(d*x+c),x)
```

output
```
a*x + b*Piecewise((sinh(c + d*x)/d, Ne(d, 0)), (x*cosh(c), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(c+dx))\,dx = ax + \frac{b\sinh(dx+c)}{d}$$

input
```
integrate(a+b*cosh(d*x+c),x, algorithm="maxima")
```

output
```
a*x + b*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 32 vs. 2(15) = 30.

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 2.13

$$\int (a+b\cosh(c+dx))\,dx = ax + \frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right)$$

input
```
integrate(a+b*cosh(d*x+c),x, algorithm="giac")
```

output
```
a*x + 1/2*b*(e^(d*x + c)/d - e^(-d*x - c)/d)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int (a+b\cosh(c+dx))\,dx = a\,x+\frac{b\sinh(c+d\,x)}{d}$$

input
```
int(a + b*cosh(c + d*x),x)
```

output
```
a*x + (b*sinh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.13

$$\int (a+b\cosh(c+dx))\,dx = \frac{b\sinh(dx+c)+adx}{d}$$

input
```
int(a+b*cosh(d*x+c),x)
```

output
```
(sinh(c + d*x)*b + a*d*x)/d
```

3.67 $\int \frac{1}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 49

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = \frac{2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}d}$$

output
```
2*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(
1/2)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = -\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-1),x]
```

output
```
(-2*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(Sqrt[-a^2 + b^2
]*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = {3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3138}$$

$$-\frac{2i\int \frac{1}{-((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right))+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-1),x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt
[a + b]*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$\frac{2\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
default	$\frac{2\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{d\sqrt{(a+b)(a-b)}}$	44
risch	$\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}-\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}\,d}$	123

input
```
int(1/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
2/d/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 237, normalized size of antiderivative = 4.84

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx$$
$$=\left[\frac{\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,ab\cosh(dx+c)+2\,a^2-b^2+2\,\left(b^2\cosh(dx+c)+ab\right)\sinh(dx+c)-2\,\sqrt{a^2-b^2}(b\cosh(dx+c)+b\sinh(dx+}{b\cosh(dx+c)^2+b\sinh(dx+c)^2+2\,a\cosh(dx+c)+2\,(b\cosh(dx+c)+a)\sinh(dx+c)+b}\right.}{\sqrt{a^2-b^2}d}\right.$$
$$\left.-\frac{2\,\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(dx+c)+b\sinh(dx+c)+a)}{a^2-b^2}\right)}{(a^2-b^2)d}\right]$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
[log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*
a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*
(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x +
 c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b))/(s
qrt(a^2 - b^2)*d), -2*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*
x + c) + b*sinh(d*x + c) + a)/(a^2 - b^2))/((a^2 - b^2)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(41) = 82$.

Time = 2.74 (sec) , antiderivative size = 163, normalized size of antiderivative = 3.33

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = \begin{cases} \frac{\tilde{\infty}x}{\cosh(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{bd} & \text{for } a=b \\ -\frac{1}{bd\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)} & \text{for } a=-b \\ \frac{x}{a+b\cosh(c)} & \text{for } d=0 \\ -\frac{\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} + \frac{\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{ad\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}-bd\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x)
```

output
```
Piecewise((zoo*x/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (tanh(c/2 + d*x
/2)/(b*d), Eq(a, b)), (-1/(b*d*tanh(c/2 + d*x/2)), Eq(a, -b)), (x/(a + b*c
osh(c)), Eq(d, 0)), (-log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2)
)/(a*d*sqrt(a/(a - b) + b/(a - b)) - b*d*sqrt(a/(a - b) + b/(a - b))) + lo
g(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(a*d*sqrt(a/(a - b) + b
/(a - b)) - b*d*sqrt(a/(a - b) + b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = \frac{2\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}d}$$

input
```
integrate(1/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
2*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*d)
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\,d+b\,d\,\mathrm{e}^{d\,x}\,\mathrm{e}^c}{\sqrt{b^2\,d^2-a^2\,d^2}}\right)}{\sqrt{b^2\,d^2-a^2\,d^2}}$$

input
```
int(1/(a + b*cosh(c + d*x)),x)
```

output
```
(2*atan((a*d + b*d*exp(d*x)*exp(c))/(b^2*d^2 - a^2*d^2)^(1/2)))/(b^2*d^2 -
 a^2*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

$$\int \frac{1}{a+b\cosh(c+dx)}\,dx = -\frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{e^{dx+c}b+a}{\sqrt{-a^2+b^2}}\right)}{d\,(a^2-b^2)}$$

input
```
int(1/(a+b*cosh(d*x+c)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))
)/(d*(a**2 - b**2))
```

3.68 $\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 86

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d} - \frac{b\sinh(c+dx)}{(a^2-b^2)\,d(a+b\cosh(c+dx))}$$

output
```
2*a*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)
^(3/2)/d-b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \frac{\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} - \frac{b\sinh(c+dx)}{(a-b)(a+b)(a+b\cosh(c+dx))}}{d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-2),x]
```

output
```
((2*a*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(
3/2) - (b*Sinh[c + d*x])/((a - b)*(a + b)*(a + b*Cosh[c + d*x])))/d
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.583, Rules used = {3042, 3143, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(c+dx))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{a}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{a}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}+\frac{a\int \frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{b\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}-\frac{2ia\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{218}$$

$$\frac{2a\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}-\frac{b\sinh(c+dx)}{d\,(a^2-b^2)\,(a+b\cosh(c+dx))}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-2),x]
```

output
```
(2*a*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]*(a^2 - b^2)*d) - (b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c
+ d*x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.37

method	result	size
derivativedivides	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-a-b\right)}+\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$	118
default	$\frac{\frac{2b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^{2}-b^{2}\right)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a-b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-a-b\right)}+\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$	118
risch	$\frac{2\,e^{dx+c}a+2b}{d(a^{2}-b^{2})(e^{2dx+2c}b+2\,e^{dx+c}a+b)}+\frac{a\ln\left(e^{dx+c}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)d}-\frac{a\ln\left(e^{dx+c}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)d}$	199

input
```
int(1/(a+b*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a^2-b^2)*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2*a-b*tanh(1/2
*d*x+1/2*c)^2-a-b)+2*a/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(
1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(77) = 154$.

Time = 0.12 (sec) , antiderivative size = 743, normalized size of antiderivative = 8.64

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
[(2*a^2*b - 2*b^3 - (a*b*cosh(d*x + c)^2 + a*b*sinh(d*x + c)^2 + 2*a^2*cos
h(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)*sinh(d*x + c))*sqrt(a^2 - b
^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) +
 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) + 2*sqrt(a^2 - b^
2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*
x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b))
 + 2*(a^3 - a*b^2)*cosh(d*x + c) + 2*(a^3 - a*b^2)*sinh(d*x + c))/((a^4*b
- 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^4*b - 2*a^2*b^3 + b^5)*d*sinh(d*
x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*cosh(d*x + c) + (a^4*b - 2*a^2*b^
3 + b^5)*d + 2*((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c) + (a^5 - 2*a^3*b
^2 + a*b^4)*d)*sinh(d*x + c)), 2*(a^2*b - b^3 - (a*b*cosh(d*x + c)^2 + a*b
*sinh(d*x + c)^2 + 2*a^2*cosh(d*x + c) + a*b + 2*(a*b*cosh(d*x + c) + a^2)
*sinh(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x + c)
 + b*sinh(d*x + c) + a)/(a^2 - b^2)) + (a^3 - a*b^2)*cosh(d*x + c) + (a^3
- a*b^2)*sinh(d*x + c))/((a^4*b - 2*a^2*b^3 + b^5)*d*cosh(d*x + c)^2 + (a^
4*b - 2*a^2*b^3 + b^5)*d*sinh(d*x + c)^2 + 2*(a^5 - 2*a^3*b^2 + a*b^4)*d*c
osh(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d + 2*((a^4*b - 2*a^2*b^3 + b^5)*
d*cosh(d*x + c) + (a^5 - 2*a^3*b^2 + a*b^4)*d)*sinh(d*x + c))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2332 vs. $2(70) = 140$.

Time = 37.76 (sec) , antiderivative size = 2332, normalized size of antiderivative = 27.12

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c))**2,x)
```

output
```
Piecewise((zoo*x/cosh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-tanh(c/2 +
 d*x/2)**3/(6*b**2*d) + tanh(c/2 + d*x/2)/(2*b**2*d), Eq(a, b)), (1/(2*b**
2*d*tanh(c/2 + d*x/2)) - 1/(6*b**2*d*tanh(c/2 + d*x/2)**3), Eq(a, -b)), (x
/(a + b*cosh(c))**2, Eq(d, 0)), (-a**2*log(-sqrt(a/(a - b) + b/(a - b)) +
tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)**2/(a**4*d*sqrt(a/(a - b) + b/(a - b)
)*tanh(c/2 + d*x/2)**2 - a**4*d*sqrt(a/(a - b) + b/(a - b)) - 2*a**3*b*d*s
qrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 + 2*a**2*b**2*d*sqrt(a/(a
- b) + b/(a - b)) + 2*a*b**3*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/
2)**2 - b**4*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*s
qrt(a/(a - b) + b/(a - b))) + a**2*log(-sqrt(a/(a - b) + b/(a - b)) + tanh
(c/2 + d*x/2))/(a**4*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 -
a**4*d*sqrt(a/(a - b) + b/(a - b)) - 2*a**3*b*d*sqrt(a/(a - b) + b/(a - b)
)*tanh(c/2 + d*x/2)**2 + 2*a**2*b**2*d*sqrt(a/(a - b) + b/(a - b)) + 2*a*b
**3*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*sqrt(a/(a
- b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*sqrt(a/(a - b) + b/(a - b)
)) + a**2*log(sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c/2 +
d*x/2)**2/(a**4*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - a**4*
d*sqrt(a/(a - b) + b/(a - b)) - 2*a**3*b*d*sqrt(a/(a - b) + b/(a - b))*tan
h(c/2 + d*x/2)**2 + 2*a**2*b**2*d*sqrt(a/(a - b) + b/(a - b)) + 2*a*b**3*d
*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**4*d*sqrt(a/(a - ...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.15

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \frac{2\left(\frac{a\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} + \frac{ae^{(dx+c)}+b}{(a^2-b^2)\left(be^{(2\,dx+2\,c)}+2\,ae^{(dx+c)}+b\right)}\right)}{d}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
2*(a*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 +
 b^2)) + (a*e^(d*x + c) + b)/((a^2 - b^2)*(b*e^(2*d*x + 2*c) + 2*a*e^(d*x
+ c) + b)))/d
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.50

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx = \frac{\frac{2\,b^2}{d\,(a^2\,b-b^3)} + \frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d\,(a^2\,b-b^3)}}{b+2\,a\,\mathrm{e}^{c+d\,x}+b\,\mathrm{e}^{2\,c+2\,d\,x}} + \frac{a\,\ln\left(-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,(a^2-b^2)} - \frac{2\,a\,\left(b+a\,\mathrm{e}^{c+d\,x}\right)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{a\,\ln\left(\frac{2\,a\,\left(b+a\,\mathrm{e}^{c+d\,x}\right)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{2\,a\,\mathrm{e}^{c+d\,x}}{b\,(a^2-b^2)}\right)}{d\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int(1/(a + b*cosh(c + d*x))^2,x)
```

output
```
((2*b^2)/(d*(a^2*b - b^3)) + (2*a*b*exp(c + d*x))/(d*(a^2*b - b^3)))/(b +
2*a*exp(c + d*x) + b*exp(2*c + 2*d*x)) + (a*log(- (2*a*exp(c + d*x))/(b*(a
^2 - b^2)) - (2*a*(b + a*exp(c + d*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2))))/
(d*(a + b)^(3/2)*(a - b)^(3/2)) - (a*log((2*a*(b + a*exp(c + d*x)))/(b*(a
+ b)^(3/2)*(a - b)^(3/2)) - (2*a*exp(c + d*x))/(b*(a^2 - b^2))))/(d*(a + b
)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 286, normalized size of antiderivative = 3.33

$$\int \frac{1}{(a+b\cosh(c+dx))^2}\,dx$$
$$= \frac{-2e^{2dx+2c}\sqrt{-a^2+b^2}\,atan\left(\frac{e^{dx+c}b+a}{\sqrt{-a^2+b^2}}\right)ab-4e^{dx+c}\sqrt{-a^2+b^2}\,atan\left(\frac{e^{dx+c}b+a}{\sqrt{-a^2+b^2}}\right)a^2-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^{dx+c}}{\sqrt{-a^2}}\right.}{d\left(e^{2dx+2c}a^4b-2e^{2dx+2c}a^2b^3+e^{2dx+2c}b^5+2e^{dx+c}a^5-4e^{dx+c}a^3b^2+2e^{dx+c}a\,b^4+\right.}$$

input
```
int(1/(a+b*cosh(d*x+c))^2,x)
```

output
```
( - 2*e**(2*c + 2*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt
( - a**2 + b**2))*a*b - 4*e**(c + d*x)*sqrt( - a**2 + b**2)*atan((e**(c +
d*x)*b + a)/sqrt( - a**2 + b**2))*a**2 - 2*sqrt( - a**2 + b**2)*atan((e**(
c + d*x)*b + a)/sqrt( - a**2 + b**2))*a*b - e**(2*c + 2*d*x)*a**2*b + e**(
2*c + 2*d*x)*b**3 + a**2*b - b**3)/(d*(e**(2*c + 2*d*x)*a**4*b - 2*e**(2*c
 + 2*d*x)*a**2*b**3 + e**(2*c + 2*d*x)*b**5 + 2*e**(c + d*x)*a**5 - 4*e**(
c + d*x)*a**3*b**2 + 2*e**(c + d*x)*a*b**4 + a**4*b - 2*a**2*b**3 + b**5))
```

3.69 $\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 133

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \frac{(2a^2+b^2)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{b\sinh(c+dx)}{2\,(a^2-b^2)\,d(a+b\cosh(c+dx))^2} - \frac{3ab\sinh(c+dx)}{2\,(a^2-b^2)^2\,d(a+b\cosh(c+dx))}$$

output
```
(2*a^2+b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/
2)/(a+b)^(5/2)/d-1/2*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^2-3/2*a*b
*sinh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.85

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \frac{-\frac{2(2a^2+b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{b(-4a^2+b^2-3ab\cosh(c+dx))\sinh(c+dx)}{(a-b)^2(a+b)^2(a+b\cosh(c+dx))^2}}{2d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-3),x]
```

output
```
((-2*(2*a^2 + b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]])/(
-a^2 + b^2)^(5/2) + (b*(-4*a^2 + b^2 - 3*a*b*Cosh[c + d*x])*Sinh[c + d*x])
/((a - b)^2*(a + b)^2*(a + b*Cosh[c + d*x])^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{\int -\frac{2a-b\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2a-b\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{\int \frac{2a-b\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2a^2+b^2}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2a^2+b^2}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{(2a^2+b^2)\int \frac{1}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}+\frac{(2a^2+b^2)\int \frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}-\frac{2i(2a^2+b^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d(a^2-b^2)}}{2(a^2-b^2)}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2(2a^2+b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{3ab\sinh(c+dx)}{d(a^2-b^2)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{b\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-3),x]
```

output
```
-1/2*(b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x])^2) + ((2*(2*a^
2 + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b
]*Sqrt[a + b]*(a^2 - b^2)*d) - (3*a*b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b
*Cosh[c + d*x])))/(2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(4a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{b(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{(4a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{b(4a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a-b\right)^2}+\frac{\left(2a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{2a^2b\,\mathrm{e}^{3dx+3c}+b^3\mathrm{e}^{3dx+3c}+6\,\mathrm{e}^{2dx+2c}a^3+3a\,b^2\mathrm{e}^{2dx+2c}+10\,\mathrm{e}^{dx+c}a^2b-b^3\mathrm{e}^{dx+c}+3b^2a}{d(a^2-b^2)^2\left(\mathrm{e}^{2dx+2c}b+2\,\mathrm{e}^{dx+c}a+b\right)^2}+\frac{\ln\left(\mathrm{e}^{dx+c}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,(a+b)^2(a-b)^2d}$

input
```
int(1/(a+b*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(4*a+b)*b/(a-b)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/2*b*
(4*a-b)/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2*
a-b*tanh(1/2*d*x+1/2*c)^2-a-b)^2+(2*a^2+b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a
-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1239 vs. $2(120) = 240$.

Time = 0.13 (sec) , antiderivative size = 2591, normalized size of antiderivative = 19.48

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
[1/2*(6*a^3*b^2 - 6*a*b^4 + 2*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^3 +
2*(2*a^4*b - a^2*b^3 - b^5)*sinh(d*x + c)^3 + 6*(2*a^5 - a^3*b^2 - a*b^4)*
cosh(d*x + c)^2 + 6*(2*a^5 - a^3*b^2 - a*b^4 + (2*a^4*b - a^2*b^3 - b^5)*c
osh(d*x + c))*sinh(d*x + c)^2 + ((2*a^2*b^2 + b^4)*cosh(d*x + c)^4 + (2*a^
2*b^2 + b^4)*sinh(d*x + c)^4 + 2*a^2*b^2 + b^4 + 4*(2*a^3*b + a*b^3)*cosh(
d*x + c)^3 + 4*(2*a^3*b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*
x + c)^3 + 2*(4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 2*(4*a^4 + 4*a^2*
b^2 + b^4 + 3*(2*a^2*b^2 + b^4)*cosh(d*x + c)^2 + 6*(2*a^3*b + a*b^3)*cosh
(d*x + c))*sinh(d*x + c)^2 + 4*(2*a^3*b + a*b^3)*cosh(d*x + c) + 4*(2*a^3*
b + a*b^3 + (2*a^2*b^2 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b + a*b^3)*cosh(d
*x + c)^2 + (4*a^4 + 4*a^2*b^2 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a
^2 - b^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x
+ c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x + c) + a*b)*sinh(d*x + c) - 2*sqrt(a^
2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*x + c) + a))/(b*cosh(d*x + c)^2 + b*s
inh(d*x + c)^2 + 2*a*cosh(d*x + c) + 2*(b*cosh(d*x + c) + a)*sinh(d*x + c)
 + b)) + 2*(10*a^4*b - 11*a^2*b^3 + b^5)*cosh(d*x + c) + 2*(10*a^4*b - 11*
a^2*b^3 + b^5 + 3*(2*a^4*b - a^2*b^3 - b^5)*cosh(d*x + c)^2 + 6*(2*a^5 - a
^3*b^2 - a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^4*b^4 + 3*a^
2*b^6 - b^8)*d*cosh(d*x + c)^4 + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d
*sinh(d*x + c)^4 + 4*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cosh(d*x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(d*x+c))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.47

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \frac{\frac{(2a^2+b^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2a^2be^{(3dx+3c)}+b^3e^{(3dx+3c)}+6a^3e^{(2dx+2c)}+3ab^2e^{(2dx+2c)}+10a^2be^{(dx+c)}-b^3e^{(dx+c)}+3ab^2}{(a^4-2a^2b^2+b^4)\left(be^{(2dx+2c)}+2ae^{(dx+c)}+b\right)^2}}{d}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
((2*a^2 + b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*
b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*a^2*b*e^(3*d*x + 3*c) + b^3*e^(3*d*x + 3
*c) + 6*a^3*e^(2*d*x + 2*c) + 3*a*b^2*e^(2*d*x + 2*c) + 10*a^2*b*e^(d*x +
c) - b^3*e^(d*x + c) + 3*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(b*e^(2*d*x + 2*c
) + 2*a*e^(d*x + c) + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \int \frac{1}{\left(a+b\cosh\left(c+d\,x\right)\right)^3}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x))^3,x)
```

output
```
int(1/(a + b*cosh(c + d*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1035, normalized size of antiderivative = 7.78

$$\int \frac{1}{(a+b\cosh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*cosh(d*x+c))^3,x)
```

output
```
( - 8*e**(4*c + 4*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt
( - a**2 + b**2))*a**3*b**2 - 4*e**(4*c + 4*d*x)*sqrt( - a**2 + b**2)*atan
((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a*b**4 - 32*e**(3*c + 3*d*x)*s
qrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**4*b
 - 16*e**(3*c + 3*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt
( - a**2 + b**2))*a**2*b**3 - 32*e**(2*c + 2*d*x)*sqrt( - a**2 + b**2)*ata
n((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**5 - 32*e**(2*c + 2*d*x)*sq
rt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**3*b*
*2 - 8*e**(2*c + 2*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqr
t( - a**2 + b**2))*a*b**4 - 32*e**(c + d*x)*sqrt( - a**2 + b**2)*atan((e**
(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**4*b - 16*e**(c + d*x)*sqrt( - a*
*2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**3 - 8*s
qrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**3*b
**2 - 4*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2
))*a*b**4 - 2*e**(4*c + 4*d*x)*a**4*b**2 + e**(4*c + 4*d*x)*a**2*b**4 + e*
*(4*c + 4*d*x)*b**6 + 16*e**(2*c + 2*d*x)*a**6 - 12*e**(2*c + 2*d*x)*a**4*
b**2 - 6*e**(2*c + 2*d*x)*a**2*b**4 + 2*e**(2*c + 2*d*x)*b**6 + 32*e**(c +
 d*x)*a**5*b - 40*e**(c + d*x)*a**3*b**3 + 8*e**(c + d*x)*a*b**5 + 10*a**4
*b**2 - 11*a**2*b**4 + b**6)/(4*a*d*(e**(4*c + 4*d*x)*a**6*b**2 - 3*e**(4*
c + 4*d*x)*a**4*b**4 + 3*e**(4*c + 4*d*x)*a**2*b**6 - e**(4*c + 4*d*x)*...
```

3.70 $\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 184

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \frac{a(2a^2+3b^2)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \frac{b\sinh(c+dx)}{3(a^2-b^2)\,d(a+b\cosh(c+dx))^3} - \frac{5ab\sinh(c+dx)}{6(a^2-b^2)^2\,d(a+b\cosh(c+dx))^2} - \frac{b(11a^2+4b^2)\sinh(c+dx)}{6(a^2-b^2)^3\,d(a+b\cosh(c+dx))}$$

output
```
a*(2*a^2+3*b^2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)
^(7/2)/(a+b)^(7/2)/d-1/3*b*sinh(d*x+c)/(a^2-b^2)/d/(a+b*cosh(d*x+c))^3-5/6
*a*b*sinh(d*x+c)/(a^2-b^2)^2/d/(a+b*cosh(d*x+c))^2-1/6*b*(11*a^2+4*b^2)*si
nh(d*x+c)/(a^2-b^2)^3/d/(a+b*cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.87

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \frac{\frac{6a(2a^2+3b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}-\frac{b(36a^4+a^2b^2+8b^4+6ab(9a^2+b^2)\cosh(c+dx)+(11a^2b^2+4b^4)\cosh(2(c+dx)))\sinh(c+dx)}{2(a-b)^3(a+b)^3(a+b\cosh(c+dx))^3}}{6d}$$

input
```
Integrate[(a + b*Cosh[c + d*x])^(-4),x]
```

output
```
((6*a*(2*a^2 + 3*b^2)*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sqrt[-a^2 + b^2]]
)/(-a^2 + b^2)^(7/2) - (b*(36*a^4 + a^2*b^2 + 8*b^4 + 6*a*b*(9*a^2 + b^2)*
Cosh[c + d*x] + (11*a^2*b^2 + 4*b^4)*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(2*
(a - b)^3*(a + b)^3*(a + b*Cosh[c + d*x])^3))/(6*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \ 3143$$

$$-\frac{\int -\frac{3a-2b\cosh(c+dx)}{(a+b\cosh(c+dx))^3}dx}{3(a^2-b^2)}-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{3a-2b\cosh(c+dx)}{(a+b\cosh(c+dx))^3}dx}{3(a^2-b^2)}-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}+\frac{\int \frac{3a-2b\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx}{3(a^2-b^2)}$$

$\downarrow$ 3233

$$\frac{-\frac{\int -\frac{2\left(3a^2+2b^2\right)-5ab\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3(a^2-b^2)}-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{2\left(3a^2+2b^2\right)-5ab\cosh(c+dx)}{(a+b\cosh(c+dx))^2}dx}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3(a^2-b^2)}-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}+\frac{-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{\int \frac{2\left(3a^2+2b^2\right)-5ab\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{3(a^2-b^2)}$$

$\downarrow$ 3233

$$\frac{\frac{-\frac{\int -\frac{3a\left(2a^2+3b^2\right)}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3(a^2-b^2)}-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$\downarrow$ 27

$$\frac{\frac{\frac{3a\left(2a^2+3b^2\right)\int \frac{1}{a+b\cosh(c+dx)}dx}{a^2-b^2}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3(a^2-b^2)}-\frac{b\sinh(c+dx)}{3d(a^2-b^2)(a+b\cosh(c+dx))^3}$$

$\downarrow$ 3042

$$-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}+\frac{-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}+\frac{3a\left(2a^2+3b^2\right)\int\frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 3138

$$-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}+\frac{-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}+\frac{-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}-\frac{6ia\left(2a^2+3b^2\right)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d\left(a^2-b^2\right)}}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$\downarrow$ 218

$$\frac{\frac{\frac{6a\left(2a^2+3b^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{b\left(11a^2+4b^2\right)\sinh(c+dx)}{d\left(a^2-b^2\right)(a+b\cosh(c+dx))}}{2(a^2-b^2)}-\frac{5ab\sinh(c+dx)}{2d(a^2-b^2)(a+b\cosh(c+dx))^2}}{3\left(a^2-b^2\right)}-\frac{b\sinh(c+dx)}{3d\left(a^2-b^2\right)(a+b\cosh(c+dx))^3}$$

input
```
Int[(a + b*Cosh[c + d*x])^(-4),x]
```

output
```
-1/3*(b*Sinh[c + d*x])/((a^2 - b^2)*d*(a + b*Cosh[c + d*x])^3) + ((-5*a*b*
Sinh[c + d*x])/(2*(a^2 - b^2)*d*(a + b*Cosh[c + d*x])^2) + ((6*a*(2*a^2 +
3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*(a^2 - b^2)*d) - (b*(11*a^2 + 4*b^2)*Sinh[c + d*x])/((a^2 - b^
2)*d*(a + b*Cosh[c + d*x]))))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(6a^2+3ab+2b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)(a^3+3a^2b+3b^2a+b^3)}+\frac{2(9a^2+b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(6a^2-3ab+2b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3a^2b+3b^2a-b^3)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a-b\right)^3}+\frac{a(2a^2+3b^2)\,\mathrm{arcta}}{(a^6-3a^4b^2+3}}{d}$
default	$\frac{-\frac{2\left(-\frac{(6a^2+3ab+2b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{2(a-b)(a^3+3a^2b+3b^2a+b^3)}+\frac{2(9a^2+b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3(a^2+2ab+b^2)(a^2-2ab+b^2)}-\frac{(6a^2-3ab+2b^2)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)(a^3-3a^2b+3b^2a-b^3)}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a-b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2-a-b\right)^3}+\frac{a(2a^2+3b^2)\,\mathrm{arcta}}{(a^6-3a^4b^2+3}}{d}$
risch	$\frac{6\,e^{5dx+5c}a^3b^2+9a\,b^4e^{5dx+5c}+30a^4b\,e^{4dx+4c}+45a^2b^3e^{4dx+4c}+44a^5e^{3dx+3c}+82a^3b^2e^{3dx+3c}+24a\,b^4e^{3dx+3c}+102a^4b}{3d(a^2-b^2)^3(e^{2dx+2c}b+2\,e^{dx+c}a+b)}$

input
```
int(1/(a+b*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(6*a^2+3*a*b+2*b^2)*b/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1
/2*d*x+1/2*c)^5+2/3*(9*a^2+b^2)*b/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2
*d*x+1/2*c)^3-1/2*(6*a^2-3*a*b+2*b^2)*b/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*ta
nh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2*a-b*tanh(1/2*d*x+1/2*c)^2-a-b)^3
+a*(2*a^2+3*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh
((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2798 vs. $2(169) = 338$.

Time = 0.15 (sec) , antiderivative size = 5705, normalized size of antiderivative = 31.01

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(d*x+c))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 329, normalized size of antiderivative = 1.79

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \frac{\frac{3\,(2\,a^3+3\,ab^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} + \frac{6\,a^3b^2e^{(5\,dx+5\,c)}+9\,ab^4e^{(5\,dx+5\,c)}+30\,a^4be^{(4\,dx+4\,c)}+45\,a^2b^3e^{(4\,dx+4\,c)}+44\,a^5e^{(3\,dx+3\,c)}+82\,a^3b^2e^{(3\,\cdots}}{(a^6-3\,a^4b^2+3\cdots}}{3\,d}$$

input
```
integrate(1/(a+b*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/3*(3*(2*a^3 + 3*a*b^2)*arctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/((a^
6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + (6*a^3*b^2*e^(5*d*x +
 5*c) + 9*a*b^4*e^(5*d*x + 5*c) + 30*a^4*b*e^(4*d*x + 4*c) + 45*a^2*b^3*e^
(4*d*x + 4*c) + 44*a^5*e^(3*d*x + 3*c) + 82*a^3*b^2*e^(3*d*x + 3*c) + 24*a
*b^4*e^(3*d*x + 3*c) + 102*a^4*b*e^(2*d*x + 2*c) + 36*a^2*b^3*e^(2*d*x + 2
*c) + 12*b^5*e^(2*d*x + 2*c) + 60*a^3*b^2*e^(d*x + c) + 15*a*b^4*e^(d*x +
c) + 11*a^2*b^3 + 4*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(b*e^(2*d*x
+ 2*c) + 2*a*e^(d*x + c) + b)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \int \frac{1}{(a+b\cosh\left(c+d\,x\right))^4}\,dx$$

input
```
int(1/(a + b*cosh(c + d*x))^4,x)
```

output
```
int(1/(a + b*cosh(c + d*x))^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1720, normalized size of antiderivative = 9.35

$$\int \frac{1}{(a+b\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*cosh(d*x+c))^4,x)
```

output
```
( - 12*e**(6*c + 6*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqr
t( - a**2 + b**2))*a**3*b**3 - 18*e**(6*c + 6*d*x)*sqrt( - a**2 + b**2)*at
an((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a*b**5 - 72*e**(5*c + 5*d*x)
*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**4
*b**2 - 108*e**(5*c + 5*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a
)/sqrt( - a**2 + b**2))*a**2*b**4 - 144*e**(4*c + 4*d*x)*sqrt( - a**2 + b*
*2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**5*b - 252*e**(4*c +
 4*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2
))*a**3*b**3 - 54*e**(4*c + 4*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)
*b + a)/sqrt( - a**2 + b**2))*a*b**5 - 96*e**(3*c + 3*d*x)*sqrt( - a**2 +
b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**6 - 288*e**(3*c +
 3*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2
))*a**4*b**2 - 216*e**(3*c + 3*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x
)*b + a)/sqrt( - a**2 + b**2))*a**2*b**4 - 144*e**(2*c + 2*d*x)*sqrt( - a*
*2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**5*b - 252*e*
*(2*c + 2*d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2
 + b**2))*a**3*b**3 - 54*e**(2*c + 2*d*x)*sqrt( - a**2 + b**2)*atan((e**(c
 + d*x)*b + a)/sqrt( - a**2 + b**2))*a*b**5 - 72*e**(c + d*x)*sqrt( - a**2
 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2 + b**2))*a**4*b**2 - 108*e
**(c + d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( - a**2...
```

3.71 $\int \frac{1}{3+5\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

output
```
1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = -\frac{\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-1),x]
```

output
```
-1/2*ArcTan[2*Coth[(c + d*x)/2]]/d
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{5\cosh(c+dx)+3}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{3+5\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \text{3138}$$
$$-\frac{2i\int \frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$
$$\Big\downarrow \text{219}$$
$$\frac{\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-1),x]
```

output
```
ArcTan[Tanh[(c + d*x)/2]/2]/(2*d)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.51 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.82

method	result	size
derivativedivides	$\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
default	$\frac{\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{2d}$	18
risch	$\frac{i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{4d}-\frac{i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{4d}$	36
parallelrisch	$-\frac{i\left(\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)\right)}{4d}$	36

input
```
int(1/(3+5*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*arctan(1/2*tanh(1/2*d*x+1/2*c))/d
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx=\frac{\arctan\left(\frac{5}{4}\,\cosh\left(dx+c\right)+\frac{5}{4}\,\sinh\left(dx+c\right)+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c) + 3/4)/d
```

Sympy [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \begin{cases} \frac{\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{2d} & \text{for } d\neq 0 \\ \frac{x}{5\cosh(c)+3} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x)
```

output
```
Piecewise((atan(tanh(c/2 + d*x/2)/2)/(2*d), Ne(d, 0)), (x/(5*cosh(c) + 3),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = -\frac{\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/2*arctan(5/4*e^(-d*x - c) + 3/4)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.73

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{2\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c)),x, algorithm="giac")
```

output
```
1/2*arctan(5/4*e^(d*x + c) + 3/4)/d
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.55

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{3\sqrt{d^2}+5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{d^2}}{4\,d}\right)}{2\,\sqrt{d^2}}$$

input
```
int(1/(5*cosh(c + d*x) + 3),x)
```

output
```
atan((3*(d^2)^(1/2) + 5*exp(d*x)*exp(c)*(d^2)^(1/2))/(4*d))/(2*(d^2)^(1/2)
)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.77

$$\int \frac{1}{3+5\cosh(c+dx)}\,dx = \frac{atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)}{2d}$$

input
```
int(1/(3+5*cosh(d*x+c)),x)
```

output
```
atan((5*e**(c + d*x) + 3)/4)/(2*d)
```

3.72 $\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 48

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{3\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d} + \frac{5\sinh(c+dx)}{16d(3+5\cosh(c+dx))}$$

output
```
-3/32*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/16*sinh(d*x+c)/d/(3+5*cosh(d*x+c
))
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = \frac{3\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right) + \frac{10\sinh(c+dx)}{3+5\cosh(c+dx)}}{32d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-2),x]
```

output
```
(3*ArcTan[2*Coth[(c + d*x)/2]] + (10*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x]))
/(32*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3143, 27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\cosh(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3+5\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{16}\int -\frac{3}{5\cosh(c+dx)+3}dx+\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3}{16}\int \frac{1}{5\cosh(c+dx)+3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3}{16}\int \frac{1}{5\sin\left(ic+idx+\frac{\pi}{2}\right)+3}dx$$

$$\Big\downarrow \text{3138}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}+\frac{3i\int \frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

$$\Big\downarrow \text{219}$$

$$\frac{5\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{3\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-2),x]
```

output
```
(-3*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d) + (5*Sinh[c + d*x])/(16*d*(3 + 5*C
osh[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)}-\frac{3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
default	$\frac{\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{16\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)}-\frac{3\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{32}}{d}$	46
risch	$-\frac{3\,\mathrm{e}^{dx+c}+5}{8d\left(5\,\mathrm{e}^{2dx+2c}+6\,\mathrm{e}^{dx+c}+5\right)}+\frac{3i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{64d}-\frac{3i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{64d}$	74
parallelrisch	$\frac{(15i\cosh(dx+c)+9i)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+(-15i\cosh(dx+c)-9i)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2i\right)+20\sinh(dx+c)}{320d\cosh(dx+c)+192d}$	78

input
```
int(1/(3+5*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(5/16*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+4)-3/32*arctan(1/2*ta
nh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(41) = 82.

Time = 0.08 (sec) , antiderivative size = 147, normalized size of antiderivative = 3.06

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{3\left(5\cosh\left(dx+c\right)^{2}+2\left(5\cosh\left(dx+c\right)+3\right)\sinh\left(dx+c\right)+5\sinh\left(dx+c\right)^{2}+6\cosh\left(dx+c\right)+5\right)a}{32\left(5\,d\cosh\left(dx+c\right)^{2}+5\,d\sinh\left(dx+c\right)^{2}+6\,d\cosh\left(dx+c\right)\right.}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
-1/32*(3*(5*cosh(d*x + c)^2 + 2*(5*cosh(d*x + c) + 3)*sinh(d*x + c) + 5*si
nh(d*x + c)^2 + 6*cosh(d*x + c) + 5)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d
*x + c) + 3/4) + 12*cosh(d*x + c) + 12*sinh(d*x + c) + 20)/(5*d*cosh(d*x +
 c)^2 + 5*d*sinh(d*x + c)^2 + 6*d*cosh(d*x + c) + 2*(5*d*cosh(d*x + c) + 3
*d)*sinh(d*x + c) + 5*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.32 (sec) , antiderivative size = 291, normalized size of antiderivative = 6.06

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx$$
$$= \begin{cases} -\frac{\log\left(-3e^{-dx}-4ie^{-dx}\right)}{25d\cosh^2\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+30d\cosh\left(dx+\log\left(-3e^{-dx}-4ie^{-dx}\right)-\log(5)\right)+9d} & \text{for } c=\log\left((-3-4i)\,e^{\cdots}\right. \\ \frac{x}{25\cosh^2\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+30\cosh\left(dx+\log\left(-3e^{-dx}+4ie^{-dx}\right)-\log(5)\right)+9} & \text{for } c=\log\left((-3+4i)\,e^{\cdots}\right. \\ \frac{x}{(5\cosh(c)+3)^2} & \text{for } d=0 \\ -\frac{3\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}+\frac{10\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d}-\frac{12\operatorname{atan}\left(\frac{\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)}{2}\right)}{32d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)+128d} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(3+5*cosh(d*x+c))**2,x)
```

output
```
Piecewise((-log(-3*exp(-d*x) - 4*I*exp(-d*x))/(25*d*cosh(d*x + log(-3*exp(
-d*x) - 4*I*exp(-d*x)) - log(5))**2 + 30*d*cosh(d*x + log(-3*exp(-d*x) - 4
*I*exp(-d*x)) - log(5)) + 9*d), Eq(c, log((-3 - 4*I)*exp(-d*x)) - log(5)))
, (x/(25*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 30*co
sh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5)) + 9), Eq(c, log((-3 +
 4*I)*exp(-d*x)) - log(5))), (x/(5*cosh(c) + 3)**2, Eq(d, 0)), (-3*tanh(c/
2 + d*x/2)**2*atan(tanh(c/2 + d*x/2)/2)/(32*d*tanh(c/2 + d*x/2)**2 + 128*d
) + 10*tanh(c/2 + d*x/2)/(32*d*tanh(c/2 + d*x/2)**2 + 128*d) - 12*atan(tan
h(c/2 + d*x/2)/2)/(32*d*tanh(c/2 + d*x/2)**2 + 128*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.33

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = \frac{3\arctan\left(\frac{5}{4}e^{(-dx-c)}+\frac{3}{4}\right)}{32\,d} + \frac{3\,e^{(-dx-c)}+5}{8\,d(6\,e^{(-dx-c)}+5\,e^{(-2\,dx-2\,c)}+5)}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
3/32*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/8*(3*e^(-d*x - c) + 5)/(d*(6*e^(
-d*x - c) + 5*e^(-2*d*x - 2*c) + 5))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{\frac{4\,(3\,e^{(dx+c)}+5)}{5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5} + 3\arctan\left(\frac{5}{4}e^{(dx+c)}+\frac{3}{4}\right)}{32\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
-1/32*(4*(3*e^(d*x + c) + 5)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5) + 3*a
rctan(5/4*e^(d*x + c) + 3/4))/d
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.54

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = -\frac{\frac{3\,e^{c+d\,x}}{8\,d}+\frac{5}{8\,d}}{6\,e^{c+d\,x}+5\,e^{2\,c+2\,d\,x}+5} - \frac{3\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,e^{d\,x}\,e^{c}}{4\,d}\right)\sqrt{d^2}\right)}{32\,\sqrt{d^2}}$$

input
```
int(1/(5*cosh(c + d*x) + 3)^2,x)
```

output
```
- ((3*exp(c + d*x))/(8*d) + 5/(8*d))/(6*exp(c + d*x) + 5*exp(2*c + 2*d*x)
+ 5) - (3*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(32*(d^
2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.15

$$\int \frac{1}{(3+5\cosh(c+dx))^2}\,dx = \frac{-15e^{2dx+2c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)-18e^{dx+c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)-15atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)+10e^{2dx+2c}-10}{32d\left(5e^{2dx+2c}+6e^{dx+c}+5\right)}$$

input
```
int(1/(3+5*cosh(d*x+c))^2,x)
```

output
```
( - 15*e**(2*c + 2*d*x)*atan((5*e**(c + d*x) + 3)/4) - 18*e**(c + d*x)*ata
n((5*e**(c + d*x) + 3)/4) - 15*atan((5*e**(c + d*x) + 3)/4) + 10*e**(2*c +
 2*d*x) - 10)/(32*d*(5*e**(2*c + 2*d*x) + 6*e**(c + d*x) + 5))
```

3.73 $\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 73

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{43\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{1024d} + \frac{5\sinh(c+dx)}{32d(3+5\cosh(c+dx))^2} - \frac{45\sinh(c+dx)}{512d(3+5\cosh(c+dx))}$$

output
```
43/1024*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/32*sinh(d*x+c)/d/(3+5*cosh(d*x
+c))^2-45/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.73

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = -\frac{43\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right) + \frac{10(11+45\cosh(c+dx))\sinh(c+dx)}{(3+5\cosh(c+dx))^2}}{1024d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-3),x]
```

output
```
-1/1024*(43*ArcTan[2*Coth[(c + d*x)/2]] + (10*(11 + 45*Cosh[c + d*x])*Sinh
[c + d*x])/(3 + 5*Cosh[c + d*x])^2)/d
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\cosh(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3+5\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{32}\int -\frac{6-5\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}-\frac{1}{32}\int \frac{6-5\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}-\frac{1}{32}\int \frac{6-5\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(5\sin\left(ic+idx+\frac{\pi}{2}\right)+3\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{43}{5\cosh(c+dx)+3}dx-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{32}\left(\frac{43}{16}\int \frac{1}{5\cosh(c+dx)+3}dx-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}+\frac{43}{16}\int\frac{1}{5\sin\left(ic+idx+\frac{\pi}{2}\right)+3}dx\right)$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{43i\int\frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\right)$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{32}\left(\frac{43\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}-\frac{45\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)+\frac{5\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-3),x]
```

output
```
(5*Sinh[c + d*x])/(32*d*(3 + 5*Cosh[c + d*x])^2) + ((43*ArcTan[Tanh[(c + d
*x)/2]/2])/(32*d) - (45*Sinh[c + d*x])/(16*d*(3 + 5*Cosh[c + d*x])))/32
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{\frac{-\frac{85\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128}-\frac{35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{2}}+\frac{43\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
default	$\frac{\frac{-\frac{85\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{128}-\frac{35\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{32}}{4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{2}}+\frac{43\arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{1024}}{d}$
risch	$\frac{215\,\mathrm{e}^{3dx+3c}+387\,\mathrm{e}^{2dx+2c}+325\,\mathrm{e}^{dx+c}+225}{256d\left(5\,\mathrm{e}^{2dx+2c}+6\,\mathrm{e}^{dx+c}+5\right)^{2}}+\frac{43i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}+\frac{4i}{5}\right)}{2048d}-\frac{43i\ln\left(\mathrm{e}^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{2048d}$
parallelrisch	$\frac{43i(-25\cosh(2dx+2c)-43-60\cosh(dx+c))\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+43i(25\cosh(2dx+2c)+43+60\cosh(dx+c))\ln\left(\mathrm{ta}\right.}{2048d(25\cosh(2dx+2c)+43+60\cosh(dx+c))}$

input
```
int(1/(3+5*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4*(-85/128*tanh(1/2*d*x+1/2*c)^3-35/32*tanh(1/2*d*x+1/2*c))/(tanh(1
/2*d*x+1/2*c)^2+4)^2+43/1024*arctan(1/2*tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 408, normalized size of antiderivative = 5.59

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{860\,\cosh\left(dx+c\right)^3+516\left(5\,\cosh\left(dx+c\right)+3\right)\sinh\left(dx+c\right)^2+860\,\sinh\left(dx+c\right)^3+43\left(25\,\cosh\left(dx+c\right)\right)}{}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/1024*(860*cosh(d*x + c)^3 + 516*(5*cosh(d*x + c) + 3)*sinh(d*x + c)^2 +
860*sinh(d*x + c)^3 + 43*(25*cosh(d*x + c)^4 + 20*(5*cosh(d*x + c) + 3)*si
nh(d*x + c)^3 + 25*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(75*cosh(d*x +
 c)^2 + 90*cosh(d*x + c) + 43)*sinh(d*x + c)^2 + 86*cosh(d*x + c)^2 + 4*(2
5*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 43*cosh(d*x + c) + 15)*sinh(d*x +
 c) + 60*cosh(d*x + c) + 25)*arctan(5/4*cosh(d*x + c) + 5/4*sinh(d*x + c)
+ 3/4) + 1548*cosh(d*x + c)^2 + 4*(645*cosh(d*x + c)^2 + 774*cosh(d*x + c)
 + 325)*sinh(d*x + c) + 1300*cosh(d*x + c) + 900)/(25*d*cosh(d*x + c)^4 +
25*d*sinh(d*x + c)^4 + 60*d*cosh(d*x + c)^3 + 20*(5*d*cosh(d*x + c) + 3*d)
*sinh(d*x + c)^3 + 86*d*cosh(d*x + c)^2 + 2*(75*d*cosh(d*x + c)^2 + 90*d*c
osh(d*x + c) + 43*d)*sinh(d*x + c)^2 + 60*d*cosh(d*x + c) + 4*(25*d*cosh(d
*x + c)^3 + 45*d*cosh(d*x + c)^2 + 43*d*cosh(d*x + c) + 15*d)*sinh(d*x + c
) + 25*d)
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.46 (sec) , antiderivative size = 507, normalized size of antiderivative = 6.95

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3+5*cosh(d*x+c))**3,x)
```

output
```
Piecewise((-log(-3*exp(-d*x) - 4*I*exp(-d*x))/(125*d*cosh(d*x + log(-3*exp
(-d*x) - 4*I*exp(-d*x)) - log(5))**3 + 225*d*cosh(d*x + log(-3*exp(-d*x) -
 4*I*exp(-d*x)) - log(5))**2 + 135*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp
(-d*x)) - log(5)) + 27*d), Eq(c, log((-3 - 4*I)*exp(-d*x)) - log(5))), (x/
(125*cosh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**3 + 225*cosh(
d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 135*cosh(d*x + log(
-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5)) + 27), Eq(c, log((-3 + 4*I)*exp(-d
*x)) - log(5))), (x/(5*cosh(c) + 3)**3, Eq(d, 0)), (43*tanh(c/2 + d*x/2)**
4*atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2
 + d*x/2)**2 + 16384*d) - 170*tanh(c/2 + d*x/2)**3/(1024*d*tanh(c/2 + d*x/
2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) + 344*tanh(c/2 + d*x/2)**2*
atan(tanh(c/2 + d*x/2)/2)/(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 +
 d*x/2)**2 + 16384*d) - 280*tanh(c/2 + d*x/2)/(1024*d*tanh(c/2 + d*x/2)**4
 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d) + 688*atan(tanh(c/2 + d*x/2)/2)/
(1024*d*tanh(c/2 + d*x/2)**4 + 8192*d*tanh(c/2 + d*x/2)**2 + 16384*d), Tru
e))
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.48

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = -\frac{43\arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{1024\,d} - \frac{325\,e^{(-dx-c)}+387\,e^{(-2\,dx-2\,c)}+215\,e^{(-3\,dx-3\,c)}+225}{256\,d(60\,e^{(-dx-c)}+86\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+25\,e^{(-4\,dx-4\,c)}+25)}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-43/1024*arctan(5/4*e^(-d*x - c) + 3/4)/d - 1/256*(325*e^(-d*x - c) + 387*
e^(-2*d*x - 2*c) + 215*e^(-3*d*x - 3*c) + 225)/(d*(60*e^(-d*x - c) + 86*e^
(-2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 25*e^(-4*d*x - 4*c) + 25))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.04

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{\frac{4\left(215\,e^{(3\,dx+3\,c)}+387\,e^{(2\,dx+2\,c)}+325\,e^{(dx+c)}+225\right)}{\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)^2}+43\arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right)}{1024\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/1024*(4*(215*e^(3*d*x + 3*c) + 387*e^(2*d*x + 2*c) + 325*e^(d*x + c) + 2
25)/(5*e^(2*d*x + 2*c) + 6*e^(d*x + c) + 5)^2 + 43*arctan(5/4*e^(d*x + c)
+ 3/4))/d
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.88

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{\frac{43\,e^{c+d\,x}}{256\,d}+\frac{129}{1280\,d}}{6\,e^{c+d\,x}+5\,e^{2\,c+2\,d\,x}+5} - \frac{\frac{7\,e^{c+d\,x}}{40\,d}-\frac{3}{8\,d}}{60\,e^{c+d\,x}+86\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+25\,e^{4\,c+4\,d\,x}+25} + \frac{43\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,e^{d\,x}\,e^{c}}{4\,d}\right)\sqrt{d^2}\right)}{1024\,\sqrt{d^2}}$$

input
```
int(1/(5*cosh(c + d*x) + 3)^3,x)
```

output
```
((43*exp(c + d*x))/(256*d) + 129/(1280*d))/(6*exp(c + d*x) + 5*exp(2*c + 2
*d*x) + 5) - ((7*exp(c + d*x))/(40*d) - 3/(8*d))/(60*exp(c + d*x) + 86*exp
(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*c + 4*d*x) + 25) + (43*atan
((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(1/2)))/(1024*(d^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.68

$$\int \frac{1}{(3+5\cosh(c+dx))^3}\,dx = \frac{3225e^{4dx+4c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)+7740e^{3dx+3c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)+11094e^{2dx+2c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)+7740e^{d}}{3072d\,(25e^{4dx+4c}+60e^{3dx+3c}+86}$$

input
```
int(1/(3+5*cosh(d*x+c))^3,x)
```

output
```
(3225*e**(4*c + 4*d*x)*atan((5*e**(c + d*x) + 3)/4) + 7740*e**(3*c + 3*d*x
)*atan((5*e**(c + d*x) + 3)/4) + 11094*e**(2*c + 2*d*x)*atan((5*e**(c + d*
x) + 3)/4) + 7740*e**(c + d*x)*atan((5*e**(c + d*x) + 3)/4) + 3225*atan((5
*e**(c + d*x) + 3)/4) - 1075*e**(4*c + 4*d*x) + 946*e**(2*c + 2*d*x) + 132
0*e**(c + d*x) + 1625)/(3072*d*(25*e**(4*c + 4*d*x) + 60*e**(3*c + 3*d*x)
+ 86*e**(2*c + 2*d*x) + 60*e**(c + d*x) + 25))
```

3.74 $\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 98

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = -\frac{279\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{16384d} + \frac{5\sinh(c+dx)}{48d(3+5\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(3+5\cosh(c+dx))^2} + \frac{995\sinh(c+dx)}{24576d(3+5\cosh(c+dx))}$$

output
```
-279/16384*arctan(1/2*tanh(1/2*d*x+1/2*c))/d+5/48*sinh(d*x+c)/d/(3+5*cosh(
d*x+c))^3-25/512*sinh(d*x+c)/d/(3+5*cosh(d*x+c))^2+995/24576*sinh(d*x+c)/d
/(3+5*cosh(d*x+c))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.64

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \frac{837\arctan\left(2\coth\left(\frac{1}{2}(c+dx)\right)\right) + \frac{5(8141+9540\cosh(c+dx)+4975\cosh(2(c+dx)))\sinh(c+dx)}{(3+5\cosh(c+dx))^3}}{49152d}$$

input
```
Integrate[(3 + 5*Cosh[c + d*x])^(-4),x]
```

output
```
(837*ArcTan[2*Coth[(c + d*x)/2]] + (5*(8141 + 9540*Cosh[c + d*x] + 4975*Co
sh[2*(c + d*x)])*Sinh[c + d*x])/(3 + 5*Cosh[c + d*x])^3)/(49152*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.10, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 3143, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(5\cosh(c+dx)+3)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(3+5\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$\frac{1}{48}\int -\frac{9-10\cosh(c+dx)}{(5\cosh(c+dx)+3)^3}dx+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}-\frac{1}{48}\int \frac{9-10\cosh(c+dx)}{(5\cosh(c+dx)+3)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}-\frac{1}{48}\int \frac{9-10\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(5\sin\left(ic+idx+\frac{\pi}{2}\right)+3\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(-\frac{1}{32}\int -\frac{154-75\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{48}\left(\frac{1}{32}\int \frac{154-75\cosh(c+dx)}{(5\cosh(c+dx)+3)^2}dx-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}+\frac{1}{48}\left(-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\int \frac{154-75\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(5\sin\left(ic+idx+\frac{\pi}{2}\right)+3\right)^2}dx\right)$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{1}{16}\int -\frac{837}{5\cosh(c+dx)+3}dx+\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}\right)-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{837}{16}\int \frac{1}{5\cosh(c+dx)+3}dx\right)-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}+\frac{1}{48}\left(-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{837}{16}\int \frac{1}{5\sin\left(ic+idx+\frac{\pi}{2}\right)+3}dx\right)\right)$$

$$\Big\downarrow \text{3138}$$

$$\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}+\frac{1}{48}\left(-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}+\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}+\frac{837i\int \frac{1}{2\tanh^2\left(\frac{1}{2}(c+dx)\right)+8}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}\right.\right.$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{48}\left(\frac{1}{32}\left(\frac{995\sinh(c+dx)}{16d(5\cosh(c+dx)+3)}-\frac{837\arctan\left(\frac{1}{2}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{32d}\right)-\frac{75\sinh(c+dx)}{32d(5\cosh(c+dx)+3)^2}\right)+\frac{5\sinh(c+dx)}{48d(5\cosh(c+dx)+3)^3}$$

input
```
Int[(3 + 5*Cosh[c + d*x])^(-4),x]
```

output
```
(5*Sinh[c + d*x])/(48*d*(3 + 5*Cosh[c + d*x])^3) + ((-75*Sinh[c + d*x])/(3
2*d*(3 + 5*Cosh[c + d*x])^2) + ((-837*ArcTan[Tanh[(c + d*x)/2]/2])/(32*d)
+ (995*Sinh[c + d*x])/(16*d*(3 + 5*Cosh[c + d*x])))/32)/48
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.77

method	result
derivativedivides	$\frac{-\frac{-\frac{745 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1024}-\frac{265 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{96}-\frac{295 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{3}}-\frac{279 \arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
default	$\frac{-\frac{-\frac{745 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{1024}-\frac{265 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{96}-\frac{295 \tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{64}}{8\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+4\right)^{3}}-\frac{279 \arctan\left(\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}\right)}{16384}}{d}$
risch	$-\frac{20925 e^{5dx+5c}+62775 e^{4dx+4c}+111042 e^{3dx+3c}+119310 e^{2dx+2c}+68625 e^{dx+c}+24875}{12288 d \left(5 e^{2dx+2c}+6 e^{dx+c}+5\right)^{3}}+\frac{279 i \ln\left(e^{dx+c}+\frac{3}{5}-\frac{4i}{5}\right)}{32768 d}$
parallelrisch	$\frac{837 i (125 \cosh(3dx+3c)+915 \cosh(dx+c)+450 \cosh(2dx+2c)+558) \ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2i\right)+837 i (-125 \cosh(3dx+3c)}{98304 d (125 \cosh(3dx+3c)+}$

input
```
int(1/(3+5*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8*(-745/1024*tanh(1/2*d*x+1/2*c)^5-265/96*tanh(1/2*d*x+1/2*c)^3-29
5/64*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^2+4)^3-279/16384*arctan(1/2
*tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 793 vs. $2(87) = 174$.

Time = 0.09 (sec) , antiderivative size = 793, normalized size of antiderivative = 8.09

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
-1/49152*(83700*cosh(d*x + c)^5 + 83700*(5*cosh(d*x + c) + 3)*sinh(d*x + c
)^4 + 83700*sinh(d*x + c)^5 + 251100*cosh(d*x + c)^4 + 2232*(375*cosh(d*x
+ c)^2 + 450*cosh(d*x + c) + 199)*sinh(d*x + c)^3 + 444168*cosh(d*x + c)^3
 + 24*(34875*cosh(d*x + c)^3 + 62775*cosh(d*x + c)^2 + 55521*cosh(d*x + c)
 + 19885)*sinh(d*x + c)^2 + 837*(125*cosh(d*x + c)^6 + 150*(5*cosh(d*x + c
) + 3)*sinh(d*x + c)^5 + 125*sinh(d*x + c)^6 + 450*cosh(d*x + c)^5 + 15*(1
25*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 61)*sinh(d*x + c)^4 + 915*cosh(d*
x + c)^4 + 4*(625*cosh(d*x + c)^3 + 1125*cosh(d*x + c)^2 + 915*cosh(d*x +
c) + 279)*sinh(d*x + c)^3 + 1116*cosh(d*x + c)^3 + 3*(625*cosh(d*x + c)^4
+ 1500*cosh(d*x + c)^3 + 1830*cosh(d*x + c)^2 + 1116*cosh(d*x + c) + 305)*
sinh(d*x + c)^2 + 915*cosh(d*x + c)^2 + 6*(125*cosh(d*x + c)^5 + 375*cosh(
d*x + c)^4 + 610*cosh(d*x + c)^3 + 558*cosh(d*x + c)^2 + 305*cosh(d*x + c)
 + 75)*sinh(d*x + c) + 450*cosh(d*x + c) + 125)*arctan(5/4*cosh(d*x + c) +
 5/4*sinh(d*x + c) + 3/4) + 477240*cosh(d*x + c)^2 + 12*(34875*cosh(d*x +
c)^4 + 83700*cosh(d*x + c)^3 + 111042*cosh(d*x + c)^2 + 79540*cosh(d*x + c
) + 22875)*sinh(d*x + c) + 274500*cosh(d*x + c) + 99500)/(125*d*cosh(d*x +
 c)^6 + 125*d*sinh(d*x + c)^6 + 450*d*cosh(d*x + c)^5 + 150*(5*d*cosh(d*x
+ c) + 3*d)*sinh(d*x + c)^5 + 915*d*cosh(d*x + c)^4 + 15*(125*d*cosh(d*x +
 c)^2 + 150*d*cosh(d*x + c) + 61*d)*sinh(d*x + c)^4 + 1116*d*cosh(d*x + c)
^3 + 4*(625*d*cosh(d*x + c)^3 + 1125*d*cosh(d*x + c)^2 + 915*d*cosh(d*x...
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 4.56 (sec) , antiderivative size = 784, normalized size of antiderivative = 8.00

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(3+5*cosh(d*x+c))**4,x)
```

output
```
Piecewise((-log(-3*exp(-d*x) - 4*I*exp(-d*x))/(625*d*cosh(d*x + log(-3*exp
(-d*x) - 4*I*exp(-d*x)) - log(5))**4 + 1500*d*cosh(d*x + log(-3*exp(-d*x)
- 4*I*exp(-d*x)) - log(5))**3 + 1350*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*e
xp(-d*x)) - log(5))**2 + 540*d*cosh(d*x + log(-3*exp(-d*x) - 4*I*exp(-d*x)
) - log(5)) + 81*d), Eq(c, log((-3 - 4*I)*exp(-d*x)) - log(5))), (x/(625*c
osh(d*x + log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**4 + 1500*cosh(d*x +
 log(-3*exp(-d*x) + 4*I*exp(-d*x)) - log(5))**3 + 1350*cosh(d*x + log(-3*e
xp(-d*x) + 4*I*exp(-d*x)) - log(5))**2 + 540*cosh(d*x + log(-3*exp(-d*x) +
 4*I*exp(-d*x)) - log(5)) + 81), Eq(c, log((-3 + 4*I)*exp(-d*x)) - log(5))
), (x/(5*cosh(c) + 3)**4, Eq(d, 0)), (-837*tanh(c/2 + d*x/2)**6*atan(tanh(
c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x/2)
**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) + 4470*tanh(c/2 + d*x/2)
**5/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 235929
6*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 10044*tanh(c/2 + d*x/2)**4*atan(ta
nh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x
/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) + 16960*tanh(c/2 + d*
x/2)**3/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 + d*x/2)**4 + 23
59296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) - 40176*tanh(c/2 + d*x/2)**2*ata
n(tanh(c/2 + d*x/2)/2)/(49152*d*tanh(c/2 + d*x/2)**6 + 589824*d*tanh(c/2 +
 d*x/2)**4 + 2359296*d*tanh(c/2 + d*x/2)**2 + 3145728*d) + 28320*tanh(c...
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.55

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \frac{279 \arctan\left(\frac{5}{4}\,e^{(-dx-c)}+\frac{3}{4}\right)}{16384\,d} + \frac{68625\,e^{(-dx-c)}+119310\,e^{(-2\,dx-2\,c)}+111042\,e^{(-3\,dx-3\,c)}+62775\,e^{(-4\,dx-4\,c)}+20925\,e^{(-5\,dx-5\,c)}+}{12288\,d(450\,e^{(-dx-c)}+915\,e^{(-2\,dx-2\,c)}+1116\,e^{(-3\,dx-3\,c)}+915\,e^{(-4\,dx-4\,c)}+450\,e^{(-5\,dx-5\,c)}+125\,e^{(-6\,dx}}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
279/16384*arctan(5/4*e^(-d*x - c) + 3/4)/d + 1/12288*(68625*e^(-d*x - c) +
 119310*e^(-2*d*x - 2*c) + 111042*e^(-3*d*x - 3*c) + 62775*e^(-4*d*x - 4*c
) + 20925*e^(-5*d*x - 5*c) + 24875)/(d*(450*e^(-d*x - c) + 915*e^(-2*d*x -
 2*c) + 1116*e^(-3*d*x - 3*c) + 915*e^(-4*d*x - 4*c) + 450*e^(-5*d*x - 5*c
) + 125*e^(-6*d*x - 6*c) + 125))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.00

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = -\frac{\frac{4\,\left(20925\,e^{(5\,dx+5\,c)}+62775\,e^{(4\,dx+4\,c)}+111042\,e^{(3\,dx+3\,c)}+119310\,e^{(2\,dx+2\,c)}+68625\,e^{(dx+c)}+24875\right)}{\left(5\,e^{(2\,dx+2\,c)}+6\,e^{(dx+c)}+5\right)^3}+837 \arctan\left(\frac{5}{4}\,e^{(dx+c)}+\frac{3}{4}\right.}{49152\,d}$$

input
```
integrate(1/(3+5*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
-1/49152*(4*(20925*e^(5*d*x + 5*c) + 62775*e^(4*d*x + 4*c) + 111042*e^(3*d
*x + 3*c) + 119310*e^(2*d*x + 2*c) + 68625*e^(d*x + c) + 24875)/(5*e^(2*d*
x + 2*c) + 6*e^(d*x + c) + 5)^3 + 837*arctan(5/4*e^(d*x + c) + 3/4))/d
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.28

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \frac{\frac{39\,\mathrm{e}^{c+d\,x}}{50\,d}+\frac{7}{30\,d}}{450\,\mathrm{e}^{c+d\,x}+915\,\mathrm{e}^{2\,c+2\,d\,x}+1116\,\mathrm{e}^{3\,c+3\,d\,x}+915\,\mathrm{e}^{4\,c+4\,d\,x}+450\,\mathrm{e}^{5\,c+5\,d\,x}+125\,\mathrm{e}^{6\,c+6\,d\,x}+125} - \frac{\frac{93\,\mathrm{e}^{c+d\,x}}{640\,d}+\frac{791}{3200\,d}}{60\,\mathrm{e}^{c+d\,x}+86\,\mathrm{e}^{2\,c+2\,d\,x}+60\,\mathrm{e}^{3\,c+3\,d\,x}+25\,\mathrm{e}^{4\,c+4\,d\,x}+25} - \frac{279\,\mathrm{atan}\left(\left(\frac{3}{4\,d}+\frac{5\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}}{4\,d}\right)\sqrt{d^2}\right)}{16384\,\sqrt{d^2}} - \frac{\frac{279\,\mathrm{e}^{c+d\,x}}{4096\,d}+\frac{837}{20480\,d}}{6\,\mathrm{e}^{c+d\,x}+5\,\mathrm{e}^{2\,c+2\,d\,x}+5}$$

input
```
int(1/(5*cosh(c + d*x) + 3)^4,x)
```

output
```
((39*exp(c + d*x))/(50*d) + 7/(30*d))/(450*exp(c + d*x) + 915*exp(2*c + 2*
d*x) + 1116*exp(3*c + 3*d*x) + 915*exp(4*c + 4*d*x) + 450*exp(5*c + 5*d*x)
 + 125*exp(6*c + 6*d*x) + 125) - ((93*exp(c + d*x))/(640*d) + 791/(3200*d)
)/(60*exp(c + d*x) + 86*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 25*exp(4*
c + 4*d*x) + 25) - (279*atan((3/(4*d) + (5*exp(d*x)*exp(c))/(4*d))*(d^2)^(
1/2)))/(16384*(d^2)^(1/2)) - ((279*exp(c + d*x))/(4096*d) + 837/(20480*d))
/(6*exp(c + d*x) + 5*exp(2*c + 2*d*x) + 5)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 292, normalized size of antiderivative = 2.98

$$\int \frac{1}{(3+5\cosh(c+dx))^4}\,dx = \frac{-104625e^{6dx+6c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)-376650e^{5dx+5c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)-765855e^{4dx+4c}atan\left(\frac{5e^{dx+c}}{4}+\frac{3}{4}\right)-}{}$$

input
```
int(1/(3+5*cosh(d*x+c))^4,x)
```

output
```
( - 104625*e**(6*c + 6*d*x)*atan((5*e**(c + d*x) + 3)/4) - 376650*e**(5*c
+ 5*d*x)*atan((5*e**(c + d*x) + 3)/4) - 765855*e**(4*c + 4*d*x)*atan((5*e*
*(c + d*x) + 3)/4) - 934092*e**(3*c + 3*d*x)*atan((5*e**(c + d*x) + 3)/4)
- 765855*e**(2*c + 2*d*x)*atan((5*e**(c + d*x) + 3)/4) - 376650*e**(c + d*
x)*atan((5*e**(c + d*x) + 3)/4) - 104625*atan((5*e**(c + d*x) + 3)/4) + 23
250*e**(6*c + 6*d*x) - 80910*e**(4*c + 4*d*x) - 236592*e**(3*c + 3*d*x) -
307050*e**(2*c + 2*d*x) - 190800*e**(c + d*x) - 76250)/(49152*d*(125*e**(6
*c + 6*d*x) + 450*e**(5*c + 5*d*x) + 915*e**(4*c + 4*d*x) + 1116*e**(3*c +
 3*d*x) + 915*e**(2*c + 2*d*x) + 450*e**(c + d*x) + 125))
```

3.75 $\int \frac{1}{5+3\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx=\frac{x}{4}-\frac{\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{2d}$$

output
```
1/4*x-1/2*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 65 vs. 2(31) = 62.

Time = 0.03 (sec) , antiderivative size = 65, normalized size of antiderivative = 2.10

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx=-\frac{\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}+\frac{\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{4d}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-1),x]
```

output
```
-1/4*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]]/d + Log[2*Cosh[(c + d*x)
/2] + Sinh[(c + d*x)/2]]/(4*d)
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{3\cosh(c+dx)+5}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{5+3\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{x}{4}-\frac{\text{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-1),x]
```

output
```
x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{\ln(3+\mathrm{e}^{dx+c})}{4d}+\frac{\ln(\mathrm{e}^{dx+c}+\frac{1}{3})}{4d}$	30
parallelrisch	$\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)-\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4d}$	33
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{4}-\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{4}}{d}$	34

input
```
int(1/(5+3*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/4/d*ln(3+exp(d*x+c))+1/4/d*ln(exp(d*x+c)+1/3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.35

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \frac{\log\left(3\,\cosh\left(dx+c\right)+3\,\sinh\left(dx+c\right)+1\right)-\log\left(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)+3\right)}{4\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/4*(log(3*cosh(d*x + c) + 3*sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh
(d*x + c) + 3))/d
```

Sympy [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.32

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \begin{cases} -\frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{4d} + \frac{\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)}{4d} & \text{for } d \neq 0 \\ \frac{x}{3\cosh(c)+5} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x)
```

output
```
Piecewise((-log(tanh(c/2 + d*x/2) - 2)/(4*d) + log(tanh(c/2 + d*x/2) + 2)/
(4*d), Ne(d, 0)), (x/(3*cosh(c) + 5), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = -\frac{\log\left(3e^{(-dx-c)}+1\right)}{4d} + \frac{\log\left(e^{(-dx-c)}+3\right)}{4d}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/4*log(3*e^(-d*x - c) + 1)/d + 1/4*log(e^(-d*x - c) + 3)/d
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.90

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \frac{\log\left(3e^{(dx+c)}+1\right) - \log\left(e^{(dx+c)}+3\right)}{4d}$$

input
```
integrate(1/(5+3*cosh(d*x+c)),x, algorithm="giac")
```

output
```
1/4*(log(3*e^(d*x + c) + 1) - log(e^(d*x + c) + 3))/d
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = -\frac{\operatorname{atan}\left(\frac{5\sqrt{-d^2}+3\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{4\,d}\right)}{2\sqrt{-d^2}}$$

input
```
int(1/(3*cosh(c + d*x) + 5),x)
```

output
```
-atan((5*(-d^2)^(1/2) + 3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(4*d))/(2*(-d^2)^(
1/2))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97

$$\int \frac{1}{5+3\cosh(c+dx)}\,dx = \frac{-\log\left(e^{dx+c}+3\right)+\log\left(3e^{dx+c}+1\right)}{4d}$$

input
```
int(1/(5+3*cosh(d*x+c)),x)
```

output
```
( - log(e**(c + d*x) + 3) + log(3*e**(c + d*x) + 1))/(4*d)
```

3.76 $\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = \frac{5x}{64} - \frac{5\text{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{32d} - \frac{3\sinh(c+dx)}{16d(5+3\cosh(c+dx))}$$

output
```
5/64*x-5/32*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/16*sinh(d*x+c)/d/(5+3
*cosh(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 144 vs. 2(56) = 112.

Time = 0.10 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.57

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$$
$$= \frac{-15\cosh(c+dx)\left(\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+\right.\right.\right.}{64}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-2),x]
```

output
```
(-15*Cosh[c + d*x]*(Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] - Log[2*C
osh[(c + d*x)/2] + Sinh[(c + d*x)/2]]) + 25*(-Log[2*Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2]] + Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]]) - 12*Sinh
[c + d*x])/(64*d*(5 + 3*Cosh[c + d*x]))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 3143, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\cosh(c+dx)+5)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5+3\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{16}\int -\frac{5}{3\cosh(c+dx)+5}dx-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}$$

$$\Big\downarrow \text{27}$$

$$\frac{5}{16}\int \frac{1}{3\cosh(c+dx)+5}dx-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}+\frac{5}{16}\int \frac{1}{3\sin\left(ic+idx+\frac{\pi}{2}\right)+5}dx$$

$$\Big\downarrow \text{3136}$$

$$\frac{5}{16}\left(\frac{x}{4}-\frac{\text{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}\right)-\frac{3\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-2),x]
```

output
```
(5*(x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)))/16 - (3*Sinh[
c + d*x])/(16*d*(5 + 3*Cosh[c + d*x]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}+\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}}{d}$
default	$\frac{\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}-\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)}{64}+\frac{3}{32\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{5\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{64}}{d}$
risch	$\frac{5\,e^{dx+c}+3}{8d\left(3\,e^{2dx+2c}+10\,e^{dx+c}+3\right)}-\frac{5\ln\left(3+e^{dx+c}\right)}{64d}+\frac{5\ln\left(e^{dx+c}+\frac{1}{3}\right)}{64d}$
parallelrisch	$\frac{-15\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)\cosh(dx+c)+15\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)\cosh(dx+c)-25\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+25\ln\left(\tanh\left(\right.\right.}{64d(5+3\cosh(dx+c))}$

input
```
int(1/(5+3*cosh(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(3/32/(tanh(1/2*d*x+1/2*c)-2)-5/64*ln(tanh(1/2*d*x+1/2*c)-2)+3/32/(tan
h(1/2*d*x+1/2*c)+2)+5/64*ln(tanh(1/2*d*x+1/2*c)+2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(50) = 100$.

Time = 0.11 (sec) , antiderivative size = 212, normalized size of antiderivative = 3.79

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$$
$$= \frac{5\left(3\,\cosh\left(dx+c\right)^2+2\left(3\,\cosh\left(dx+c\right)+5\right)\sinh\left(dx+c\right)+3\,\sinh\left(dx+c\right)^2+10\,\cosh\left(dx+c\right)+3\right)\log}{}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/64*(5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*sin
h(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(3*cosh(d*x + c) + 3*sinh(d*x + c)
 + 1) - 5*(3*cosh(d*x + c)^2 + 2*(3*cosh(d*x + c) + 5)*sinh(d*x + c) + 3*s
inh(d*x + c)^2 + 10*cosh(d*x + c) + 3)*log(cosh(d*x + c) + sinh(d*x + c) +
 3) + 40*cosh(d*x + c) + 40*sinh(d*x + c) + 24)/(3*d*cosh(d*x + c)^2 + 3*d
*sinh(d*x + c)^2 + 10*d*cosh(d*x + c) + 2*(3*d*cosh(d*x + c) + 5*d)*sinh(d
*x + c) + 3*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(48) = 96$.

Time = 0.84 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.55

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx$$
$$= \begin{cases} -\frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{20\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)-2\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} + \frac{5\log\left(\tanh\left(\frac{c}{2}+\frac{dx}{2}\right)+2\right)\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{64d\tanh^2\left(\frac{c}{2}+\frac{dx}{2}\right)-256d} - \frac{20\log\left(\tanh\left(\frac{c}{2}\right.\right.}{64d\tanh^2\left(\frac{c}{2}+\right.} \\ \frac{x}{(3\cosh(c)+5)^2} \end{cases}$$

input
```
integrate(1/(5+3*cosh(d*x+c))**2,x)
```

output
```
Piecewise((-5*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(64*d*tanh(c
/2 + d*x/2)**2 - 256*d) + 20*log(tanh(c/2 + d*x/2) - 2)/(64*d*tanh(c/2 + d
*x/2)**2 - 256*d) + 5*log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(64*
d*tanh(c/2 + d*x/2)**2 - 256*d) - 20*log(tanh(c/2 + d*x/2) + 2)/(64*d*tanh
(c/2 + d*x/2)**2 - 256*d) + 12*tanh(c/2 + d*x/2)/(64*d*tanh(c/2 + d*x/2)**
2 - 256*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = -\frac{5\,\log\left(3\,e^{(-dx-c)}+1\right)}{64\,d} + \frac{5\,\log\left(e^{(-dx-c)}+3\right)}{64\,d} - \frac{5\,e^{(-dx-c)}+3}{8\,d(10\,e^{(-dx-c)}+3\,e^{(-2\,dx-2\,c)}+3)}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="maxima")
```

output
```
-5/64*log(3*e^(-d*x - c) + 1)/d + 5/64*log(e^(-d*x - c) + 3)/d - 1/8*(5*e^
(-d*x - c) + 3)/(d*(10*e^(-d*x - c) + 3*e^(-2*d*x - 2*c) + 3))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.16

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = \frac{\frac{8\,(5\,e^{(dx+c)}+3)}{3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3} + 5\,\log\left(3\,e^{(dx+c)}+1\right) - 5\,\log\left(e^{(dx+c)}+3\right)}{64\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^2,x, algorithm="giac")
```

output
```
1/64*(8*(5*e^(d*x + c) + 3)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3) + 5*l
og(3*e^(d*x + c) + 1) - 5*log(e^(d*x + c) + 3))/d
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.38

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = \frac{\frac{5\,e^{c+d\,x}}{8\,d}+\frac{3}{8\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3}-\frac{5\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\,\sqrt{-d^2}\right)}{32\,\sqrt{-d^2}}$$

input
```
int(1/(3*cosh(c + d*x) + 5)^2,x)
```

output
```
((5*exp(c + d*x))/(8*d) + 3/(8*d))/(10*exp(c + d*x) + 3*exp(2*c + 2*d*x) +
 3) - (5*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))/(32*(-d
^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.79

$$\int \frac{1}{(5+3\cosh(c+dx))^2}\,dx = \frac{-15e^{2dx+2c}\log\left(e^{dx+c}+3\right)+15e^{2dx+2c}\log\left(3e^{dx+c}+1\right)-12e^{2dx+2c}-50e^{dx+c}\log\left(e^{dx+c}+3\right)+50e^{dx+c}\log}{64d\left(3e^{2dx+2c}+10e^{dx+c}+3\right)}$$

input
```
int(1/(5+3*cosh(d*x+c))^2,x)
```

output
```
( - 15*e**(2*c + 2*d*x)*log(e**(c + d*x) + 3) + 15*e**(2*c + 2*d*x)*log(3*
e**(c + d*x) + 1) - 12*e**(2*c + 2*d*x) - 50*e**(c + d*x)*log(e**(c + d*x)
 + 3) + 50*e**(c + d*x)*log(3*e**(c + d*x) + 1) - 15*log(e**(c + d*x) + 3)
 + 15*log(3*e**(c + d*x) + 1) + 12)/(64*d*(3*e**(2*c + 2*d*x) + 10*e**(c +
 d*x) + 3))
```

3.77 $\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 81

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \frac{59x}{2048} - \frac{59\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{1024d} - \frac{3\sinh(c+dx)}{32d(5+3\cosh(c+dx))^2} - \frac{45\sinh(c+dx)}{512d(5+3\cosh(c+dx))}$$

output

```
59/2048*x-59/1024*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-3/32*sinh(d*x+c)/
d/(5+3*cosh(d*x+c))^2-45/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 217 vs. $2(81) = 162$.

Time = 0.16 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.68

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = -\frac{59\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} + \frac{59\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2048d} - \frac{3}{512d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sinh\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{3}{512d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{45\sinh\left(\frac{1}{2}(c+dx)\right)}{2048d\left(2\cosh\left(\frac{1}{2}(c+dx)\right)+\sinh\left(\frac{1}{2}(c+dx)\right)\right)}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-3),x]
```

output
```
(-59*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]])/(2048*d) + (59*Log[2*Co
sh[(c + d*x)/2] + Sinh[(c + d*x)/2]])/(2048*d) - 3/(512*d*(2*Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2])^2) - (45*Sinh[(c + d*x)/2])/(2048*d*(2*Cosh[(c +
 d*x)/2] - Sinh[(c + d*x)/2])) + 3/(512*d*(2*Cosh[(c + d*x)/2] + Sinh[(c +
 d*x)/2])^2) - (45*Sinh[(c + d*x)/2])/(2048*d*(2*Cosh[(c + d*x)/2] + Sinh[
(c + d*x)/2]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3143, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\cosh(c+dx)+5)^3}\,dx$$

$\Big\downarrow$ 3042

$$\int \frac{1}{\left(5+3\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^3}dx$$

$\Big\downarrow$ 3143

$$-\frac{1}{32}\int -\frac{10-3\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$\Big\downarrow$ 25

$$\frac{1}{32}\int \frac{10-3\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$\Big\downarrow$ 3042

$$-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\int \frac{10-3\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(3\sin\left(ic+idx+\frac{\pi}{2}\right)+5\right)^2}dx$$

$\Big\downarrow$ 3233

$$\frac{1}{32}\left(-\frac{1}{16}\int -\frac{59}{3\cosh(c+dx)+5}dx-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$\Big\downarrow$ 27

$$\frac{1}{32}\left(\frac{59}{16}\int \frac{1}{3\cosh(c+dx)+5}dx-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

$\Big\downarrow$ 3042

$$-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\left(-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}+\frac{59}{16}\int \frac{1}{3\sin\left(ic+idx+\frac{\pi}{2}\right)+5}dx\right)$$

$\Big\downarrow$ 3136

$$\frac{1}{32}\left(\frac{59}{16}\left(\frac{x}{4}-\frac{\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}\right)-\frac{45\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{3\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-3),x]
```

output
```
(-3*Sinh[c + d*x])/(32*d*(5 + 3*Cosh[c + d*x])^2) + ((59*(x/4 - ArcTanh[Si
nh[c + d*x]/(3 + Cosh[c + d*x])]/(2*d)))/16 - (45*Sinh[c + d*x])/(16*d*(5
+ 3*Cosh[c + d*x])))/32
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.11

method	result
risch	$\frac{\frac{177\,e^{3dx+3c}}{256}+\frac{885\,e^{2dx+2c}}{256}+\frac{723\,e^{dx+c}}{256}+\frac{135}{256}}{d\left(3\,e^{2dx+2c}+10\,e^{dx+c}+3\right)^{2}}-\frac{59\ln\left(3+e^{dx+c}\right)}{2048d}+\frac{59\ln\left(e^{dx+c}+\frac{1}{3}\right)}{2048d}$
derivativedivides	$\frac{-\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{59\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{2048}+\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
default	$\frac{-\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{59\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{2048}+\frac{9}{512\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)^{2}}+\frac{69}{1024\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{d}$
parallelrisch	$\frac{\left(-3540\cosh(dx+c)-531\cosh(2dx+2c)-3481\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+\left(3540\cosh(dx+c)+531\cosh(2dx+2c)+3481\right)\ln}{2048d\left(59+9\cosh(2dx+2c)+60\cosh(dx+c)\right)}$

input
```
int(1/(5+3*cosh(d*x+c))^3,x,method=_RETURNVERBOSE)
```

output
```
3/256*(59*exp(3*d*x+3*c)+295*exp(2*d*x+2*c)+241*exp(d*x+c)+45)/d/(3*exp(2*
d*x+2*c)+10*exp(d*x+c)+3)^2-59/2048/d*ln(3+exp(d*x+c))+59/2048/d*ln(exp(d*
x+c)+1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. 2(73) = 146.

Time = 0.13 (sec) , antiderivative size = 563, normalized size of antiderivative = 6.95

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="fricas")
```

output
```
1/2048*(1416*cosh(d*x + c)^3 + 1416*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^2
+ 1416*sinh(d*x + c)^3 + 7080*cosh(d*x + c)^2 + 59*(9*cosh(d*x + c)^4 + 12
*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x +
 c)^3 + 2*(27*cosh(d*x + c)^2 + 90*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 1
18*cosh(d*x + c)^2 + 4*(9*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d
*x + c) + 15)*sinh(d*x + c) + 60*cosh(d*x + c) + 9)*log(3*cosh(d*x + c) +
3*sinh(d*x + c) + 1) - 59*(9*cosh(d*x + c)^4 + 12*(3*cosh(d*x + c) + 5)*si
nh(d*x + c)^3 + 9*sinh(d*x + c)^4 + 60*cosh(d*x + c)^3 + 2*(27*cosh(d*x +
c)^2 + 90*cosh(d*x + c) + 59)*sinh(d*x + c)^2 + 118*cosh(d*x + c)^2 + 4*(9
*cosh(d*x + c)^3 + 45*cosh(d*x + c)^2 + 59*cosh(d*x + c) + 15)*sinh(d*x +
c) + 60*cosh(d*x + c) + 9)*log(cosh(d*x + c) + sinh(d*x + c) + 3) + 24*(17
7*cosh(d*x + c)^2 + 590*cosh(d*x + c) + 241)*sinh(d*x + c) + 5784*cosh(d*x
 + c) + 1080)/(9*d*cosh(d*x + c)^4 + 9*d*sinh(d*x + c)^4 + 60*d*cosh(d*x +
 c)^3 + 12*(3*d*cosh(d*x + c) + 5*d)*sinh(d*x + c)^3 + 118*d*cosh(d*x + c)
^2 + 2*(27*d*cosh(d*x + c)^2 + 90*d*cosh(d*x + c) + 59*d)*sinh(d*x + c)^2
+ 60*d*cosh(d*x + c) + 4*(9*d*cosh(d*x + c)^3 + 45*d*cosh(d*x + c)^2 + 59*
d*cosh(d*x + c) + 15*d)*sinh(d*x + c) + 9*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 445 vs. $2(71) = 142$.

Time = 1.60 (sec) , antiderivative size = 445, normalized size of antiderivative = 5.49

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*cosh(d*x+c))**3,x)
```

output
```
Piecewise((-59*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**4/(2048*d*tan
h(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 472*log(tanh
(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 163
84*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 944*log(tanh(c/2 + d*x/2) - 2)/(204
8*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 59*lo
g(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(2048*d*tanh(c/2 + d*x/2)**4
 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) - 472*log(tanh(c/2 + d*x/2) + 2
)*tanh(c/2 + d*x/2)**2/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d
*x/2)**2 + 32768*d) + 944*log(tanh(c/2 + d*x/2) + 2)/(2048*d*tanh(c/2 + d*
x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d) + 276*tanh(c/2 + d*x/2)*
*3/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 + d*x/2)**2 + 32768*d)
- 816*tanh(c/2 + d*x/2)/(2048*d*tanh(c/2 + d*x/2)**4 - 16384*d*tanh(c/2 +
d*x/2)**2 + 32768*d), Ne(d, 0)), (x/(3*cosh(c) + 5)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.54

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = -\frac{59\,\log\left(3\,e^{(-dx-c)}+1\right)}{2048\,d}+\frac{59\,\log\left(e^{(-dx-c)}+3\right)}{2048\,d} - \frac{3\left(241\,e^{(-dx-c)}+295\,e^{(-2\,dx-2\,c)}+59\,e^{(-3\,dx-3\,c)}+45\right)}{256\,d(60\,e^{(-dx-c)}+118\,e^{(-2\,dx-2\,c)}+60\,e^{(-3\,dx-3\,c)}+9\,e^{(-4\,dx-4\,c)}+9)}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="maxima")
```

output
```
-59/2048*log(3*e^(-d*x - c) + 1)/d + 59/2048*log(e^(-d*x - c) + 3)/d - 3/2
56*(241*e^(-d*x - c) + 295*e^(-2*d*x - 2*c) + 59*e^(-3*d*x - 3*c) + 45)/(d
*(60*e^(-d*x - c) + 118*e^(-2*d*x - 2*c) + 60*e^(-3*d*x - 3*c) + 9*e^(-4*d
*x - 4*c) + 9))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.07

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \frac{\frac{24\left(59\,e^{(3\,dx+3\,c)}+295\,e^{(2\,dx+2\,c)}+241\,e^{(dx+c)}+45\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^2}+59\,\log\left(3\,e^{(dx+c)}+1\right)-59\,\log\left(e^{(dx+c)}+3\right)}{2048\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^3,x, algorithm="giac")
```

output
```
1/2048*(24*(59*e^(3*d*x + 3*c) + 295*e^(2*d*x + 2*c) + 241*e^(d*x + c) + 4
5)/(3*e^(2*d*x + 2*c) + 10*e^(d*x + c) + 3)^2 + 59*log(3*e^(d*x + c) + 1)
- 59*log(e^(d*x + c) + 3))/d
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.74

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx = \frac{\frac{59\,e^{c+d\,x}}{256\,d}+\frac{295}{768\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3} - \frac{59\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\sqrt{-d^2}\right)}{1024\sqrt{-d^2}} - \frac{\frac{41\,e^{c+d\,x}}{24\,d}+\frac{5}{8\,d}}{60\,e^{c+d\,x}+118\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+9\,e^{4\,c+4\,d\,x}+9}$$

input
```
int(1/(3*cosh(c + d*x) + 5)^3,x)
```

output
```
((59*exp(c + d*x))/(256*d) + 295/(768*d))/(10*exp(c + d*x) + 3*exp(2*c + 2
*d*x) + 3) - (59*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(1/2)))
/(1024*(-d^2)^(1/2)) - ((41*exp(c + d*x))/(24*d) + 5/(8*d))/(60*exp(c + d*
x) + 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*c + 4*d*x) + 9)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 293, normalized size of antiderivative = 3.62

$$\int \frac{1}{(5+3\cosh(c+dx))^3}\,dx$$
$$= \frac{-2655e^{4dx+4c}\log\left(e^{dx+c}+3\right)+2655e^{4dx+4c}\log\left(3e^{dx+c}+1\right)-1062e^{4dx+4c}-17700e^{3dx+3c}\log\left(e^{dx+c}+3\right)\cdots}{}$$

input
```
int(1/(5+3*cosh(d*x+c))^3,x)
```

output
```
( - 2655*e**(4*c + 4*d*x)*log(e**(c + d*x) + 3) + 2655*e**(4*c + 4*d*x)*lo
g(3*e**(c + d*x) + 1) - 1062*e**(4*c + 4*d*x) - 17700*e**(3*c + 3*d*x)*log
(e**(c + d*x) + 3) + 17700*e**(3*c + 3*d*x)*log(3*e**(c + d*x) + 1) - 3481
0*e**(2*c + 2*d*x)*log(e**(c + d*x) + 3) + 34810*e**(2*c + 2*d*x)*log(3*e*
*(c + d*x) + 1) + 21476*e**(2*c + 2*d*x) - 17700*e**(c + d*x)*log(e**(c +
d*x) + 3) + 17700*e**(c + d*x)*log(3*e**(c + d*x) + 1) + 21840*e**(c + d*x
) - 2655*log(e**(c + d*x) + 3) + 2655*log(3*e**(c + d*x) + 1) + 4338)/(102
40*d*(9*e**(4*c + 4*d*x) + 60*e**(3*c + 3*d*x) + 118*e**(2*c + 2*d*x) + 60
*e**(c + d*x) + 9))
```

3.78 $\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 106

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \frac{385x}{32768} - \frac{385\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{3+\cosh(c+dx)}\right)}{16384d} - \frac{\sinh(c+dx)}{16d(5+3\cosh(c+dx))^3} - \frac{25\sinh(c+dx)}{512d(5+3\cosh(c+dx))^2} - \frac{311\sinh(c+dx)}{8192d(5+3\cosh(c+dx))}$$

output
```
385/32768*x-385/16384*arctanh(sinh(d*x+c)/(3+cosh(d*x+c)))/d-1/16*sinh(d*x
+c)/d/(5+3*cosh(d*x+c))^3-25/512*sinh(d*x+c)/d/(5+3*cosh(d*x+c))^2-311/819
2*sinh(d*x+c)/d/(5+3*cosh(d*x+c))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 296 vs. 2(106) = 212.

Time = 0.25 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.79

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = -\frac{296450\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)\right)+10395\cosh(3(c+dx))\log\left(2\cosh\left(\frac{1}{2}(c+dx)\right)-\right.}{}$$

input
```
Integrate[(5 + 3*Cosh[c + d*x])^(-4),x]
```

output
```
-1/131072*(296450*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] + 10395*Cos
h[3*(c + d*x)]*Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] + 377685*Cosh[
c + d*x]*(Log[2*Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]] - Log[2*Cosh[(c + d
*x)/2] + Sinh[(c + d*x)/2]]) + 103950*Cosh[2*(c + d*x)]*(Log[2*Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2]] - Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]]
) - 296450*Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]] - 10395*Cosh[3*(c
+ d*x)]*Log[2*Cosh[(c + d*x)/2] + Sinh[(c + d*x)/2]] + 175788*Sinh[c + d*x
] + 84240*Sinh[2*(c + d*x)] + 11196*Sinh[3*(c + d*x)])/(d*(5 + 3*Cosh[c +
d*x])^3)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.14, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {3042, 3143, 27, 3042, 3233, 25, 3042, 3233, 27, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(3\cosh(c+dx)+5)^4}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(5+3\sin\left(ic+idx+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{1}{48}\int -\frac{3(5-2\cosh(c+dx))}{(3\cosh(c+dx)+5)^3}dx-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\int \frac{5-2\cosh(c+dx)}{(3\cosh(c+dx)+5)^3}dx-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}+\frac{1}{16}\int \frac{5-2\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(3\sin\left(ic+idx+\frac{\pi}{2}\right)+5\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(-\frac{1}{32}\int -\frac{62-25\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{16}\left(\frac{1}{32}\int \frac{62-25\cosh(c+dx)}{(3\cosh(c+dx)+5)^2}dx-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}+\frac{1}{16}\left(-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\int \frac{62-25\sin\left(ic+idx+\frac{\pi}{2}\right)}{\left(3\sin\left(ic+idx+\frac{\pi}{2}\right)+5\right)^2}dx\right)$$

$$\Big\downarrow \text{3233}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(-\frac{1}{16}\int -\frac{385}{3\cosh(c+dx)+5}dx-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\int \frac{1}{3\cosh(c+dx)+5}dx-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}+\frac{1}{16}\left(-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}+\frac{1}{32}\left(-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}+\frac{385}{16}\int \frac{1}{3\sin\left(ic+idx+\frac{\pi}{2}\right)+5}dx\right)\right)$$

$$\Big\downarrow \text{3136}$$

$$\frac{1}{16}\left(\frac{1}{32}\left(\frac{385}{16}\left(\frac{x}{4}-\frac{\operatorname{arctanh}\left(\frac{\sinh(c+dx)}{\cosh(c+dx)+3}\right)}{2d}\right)-\frac{311\sinh(c+dx)}{16d(3\cosh(c+dx)+5)}\right)-\frac{25\sinh(c+dx)}{32d(3\cosh(c+dx)+5)^2}\right)-\frac{\sinh(c+dx)}{16d(3\cosh(c+dx)+5)^3}$$

input
```
Int[(5 + 3*Cosh[c + d*x])^(-4),x]
```

output
```
-1/16*Sinh[c + d*x]/(d*(5 + 3*Cosh[c + d*x])^3) + ((-25*Sinh[c + d*x])/(32
*d*(5 + 3*Cosh[c + d*x])^2) + ((385*(x/4 - ArcTanh[Sinh[c + d*x]/(3 + Cosh
[c + d*x])]/(2*d)))/16 - (311*Sinh[c + d*x])/(16*d*(5 + 3*Cosh[c + d*x])))
/32)/16
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.06

method	result
risch	$\frac{10395\,e^{5dx+5c}+86625\,e^{4dx+4c}+239470\,e^{3dx+3c}+218466\,e^{2dx+2c}+73575\,e^{dx+c}+8397}{12288d\left(3\,e^{2dx+2c}+10\,e^{dx+c}+3\right)^{3}}-\frac{385\ln\left(3+e^{dx+c}\right)}{32768d}+\frac{385\ln$
derivativedivides	$\frac{\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{3}}-\frac{81}{4096\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}+\frac{639}{16384\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{385\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{32768}+\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d}$
default	$\frac{\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{3}}-\frac{81}{4096\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)^{2}}+\frac{639}{16384\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}+\frac{385\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+2\right)}{32768}+\frac{9}{2048\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}{d}$
parallelrisch	$\frac{\left(-377685\cosh(dx+c)-103950\cosh(2dx+2c)-10395\cosh(3dx+3c)-296450\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-2\right)+\left(377685\cosh(dx\right.}{32768d(770+27\cosh}$

input
```
int(1/(5+3*cosh(d*x+c))^4,x,method=_RETURNVERBOSE)
```

output
```
1/12288*(10395*exp(5*d*x+5*c)+86625*exp(4*d*x+4*c)+239470*exp(3*d*x+3*c)+2
18466*exp(2*d*x+2*c)+73575*exp(d*x+c)+8397)/d/(3*exp(2*d*x+2*c)+10*exp(d*x
+c)+3)^3-385/32768/d*ln(3+exp(d*x+c))+385/32768/d*ln(exp(d*x+c)+1/3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. 2(96) = 192.

Time = 0.09 (sec) , antiderivative size = 1078, normalized size of antiderivative = 10.17

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="fricas")
```

output
```
1/98304*(83160*cosh(d*x + c)^5 + 138600*(3*cosh(d*x + c) + 5)*sinh(d*x + c
)^4 + 83160*sinh(d*x + c)^5 + 693000*cosh(d*x + c)^4 + 6160*(135*cosh(d*x
+ c)^2 + 450*cosh(d*x + c) + 311)*sinh(d*x + c)^3 + 1915760*cosh(d*x + c)^
3 + 48*(17325*cosh(d*x + c)^3 + 86625*cosh(d*x + c)^2 + 119735*cosh(d*x +
c) + 36411)*sinh(d*x + c)^2 + 1747728*cosh(d*x + c)^2 + 1155*(27*cosh(d*x
+ c)^6 + 54*(3*cosh(d*x + c) + 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 2
70*cosh(d*x + c)^5 + 9*(45*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh
(d*x + c)^4 + 981*cosh(d*x + c)^4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x
+ c)^2 + 981*cosh(d*x + c) + 385)*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 +
 3*(135*cosh(d*x + c)^4 + 900*cosh(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 154
0*cosh(d*x + c) + 327)*sinh(d*x + c)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(
d*x + c)^5 + 225*cosh(d*x + c)^4 + 654*cosh(d*x + c)^3 + 770*cosh(d*x + c)
^2 + 327*cosh(d*x + c) + 45)*sinh(d*x + c) + 270*cosh(d*x + c) + 27)*log(3
*cosh(d*x + c) + 3*sinh(d*x + c) + 1) - 1155*(27*cosh(d*x + c)^6 + 54*(3*c
osh(d*x + c) + 5)*sinh(d*x + c)^5 + 27*sinh(d*x + c)^6 + 270*cosh(d*x + c)
^5 + 9*(45*cosh(d*x + c)^2 + 150*cosh(d*x + c) + 109)*sinh(d*x + c)^4 + 98
1*cosh(d*x + c)^4 + 4*(135*cosh(d*x + c)^3 + 675*cosh(d*x + c)^2 + 981*cos
h(d*x + c) + 385)*sinh(d*x + c)^3 + 1540*cosh(d*x + c)^3 + 3*(135*cosh(d*x
 + c)^4 + 900*cosh(d*x + c)^3 + 1962*cosh(d*x + c)^2 + 1540*cosh(d*x + c)
+ 327)*sinh(d*x + c)^2 + 981*cosh(d*x + c)^2 + 6*(27*cosh(d*x + c)^5 + ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(94) = 188$.

Time = 3.23 (sec) , antiderivative size = 784, normalized size of antiderivative = 7.40

$$\int \frac{1}{(5 + 3\cosh(c + dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(5+3*cosh(d*x+c))**4,x)
```

output
```
Piecewise((-385*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2)**6/(32768*d*t
anh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 +
 d*x/2)**2 - 2097152*d) + 4620*log(tanh(c/2 + d*x/2) - 2)*tanh(c/2 + d*x/2
)**4/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 15728
64*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 18480*log(tanh(c/2 + d*x/2) - 2)*
tanh(c/2 + d*x/2)**2/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d
*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 24640*log(tanh(c/
2 + d*x/2) - 2)/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)
**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) + 385*log(tanh(c/2 + d*x
/2) + 2)*tanh(c/2 + d*x/2)**6/(32768*d*tanh(c/2 + d*x/2)**6 - 393216*d*tan
h(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152*d) - 4620*log
(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**4/(32768*d*tanh(c/2 + d*x/2)**6
 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 209715
2*d) + 18480*log(tanh(c/2 + d*x/2) + 2)*tanh(c/2 + d*x/2)**2/(32768*d*tanh
(c/2 + d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*
x/2)**2 - 2097152*d) - 24640*log(tanh(c/2 + d*x/2) + 2)/(32768*d*tanh(c/2
+ d*x/2)**6 - 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)*
*2 - 2097152*d) + 2556*tanh(c/2 + d*x/2)**5/(32768*d*tanh(c/2 + d*x/2)**6
- 393216*d*tanh(c/2 + d*x/2)**4 + 1572864*d*tanh(c/2 + d*x/2)**2 - 2097152
*d) - 14976*tanh(c/2 + d*x/2)**3/(32768*d*tanh(c/2 + d*x/2)**6 - 393216...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.59

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = -\frac{385\,\log\left(3\,e^{(-dx-c)}+1\right)}{32768\,d} + \frac{385\,\log\left(e^{(-dx-c)}+3\right)}{32768\,d} - \frac{73575\,e^{(-dx-c)} + 218466\,e^{(-2\,dx-2\,c)} + 239470\,e^{(-3\,dx-3\,c)} + 86625\,e^{(-4\,dx-4\,c)} + 10395\,e^{(-5\,dx-5\,c)} + \dots}{12288\,d(270\,e^{(-dx-c)} + 981\,e^{(-2\,dx-2\,c)} + 1540\,e^{(-3\,dx-3\,c)} + 981\,e^{(-4\,dx-4\,c)} + 270\,e^{(-5\,dx-5\,c)} + 27\,e^{(-6\,dx-\dots}}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="maxima")
```

output
```
-385/32768*log(3*e^(-d*x - c) + 1)/d + 385/32768*log(e^(-d*x - c) + 3)/d -
 1/12288*(73575*e^(-d*x - c) + 218466*e^(-2*d*x - 2*c) + 239470*e^(-3*d*x
- 3*c) + 86625*e^(-4*d*x - 4*c) + 10395*e^(-5*d*x - 5*c) + 8397)/(d*(270*e
^(-d*x - c) + 981*e^(-2*d*x - 2*c) + 1540*e^(-3*d*x - 3*c) + 981*e^(-4*d*x
 - 4*c) + 270*e^(-5*d*x - 5*c) + 27*e^(-6*d*x - 6*c) + 27))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.03

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \frac{\frac{8\left(10395\,e^{(5\,dx+5\,c)}+86625\,e^{(4\,dx+4\,c)}+239470\,e^{(3\,dx+3\,c)}+218466\,e^{(2\,dx+2\,c)}+73575\,e^{(dx+c)}+8397\right)}{\left(3\,e^{(2\,dx+2\,c)}+10\,e^{(dx+c)}+3\right)^3}+1155\log\left(3\,e^{(dx+c)}+1\right)-1}{98304\,d}$$

input
```
integrate(1/(5+3*cosh(d*x+c))^4,x, algorithm="giac")
```

output
```
1/98304*(8*(10395*e^(5*d*x + 5*c) + 86625*e^(4*d*x + 4*c) + 239470*e^(3*d*
x + 3*c) + 218466*e^(2*d*x + 2*c) + 73575*e^(d*x + c) + 8397)/(3*e^(2*d*x
+ 2*c) + 10*e^(d*x + c) + 3)^3 + 1155*log(3*e^(d*x + c) + 1) - 1155*log(e^
(d*x + c) + 3))/d
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.13

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \frac{\frac{385\,e^{c+d\,x}}{4096\,d}+\frac{1925}{12288\,d}}{10\,e^{c+d\,x}+3\,e^{2\,c+2\,d\,x}+3} - \frac{385\,\mathrm{atan}\left(\left(\frac{5}{4\,d}+\frac{3\,e^{d\,x}\,e^{c}}{4\,d}\right)\sqrt{-d^2}\right)}{16384\sqrt{-d^2}} - \frac{\frac{385\,e^{c+d\,x}}{1152\,d}+\frac{3461}{3456\,d}}{60\,e^{c+d\,x}+118\,e^{2\,c+2\,d\,x}+60\,e^{3\,c+3\,d\,x}+9\,e^{4\,c+4\,d\,x}+9} + \frac{\frac{365\,e^{c+d\,x}}{54\,d}+\frac{41}{18\,d}}{270\,e^{c+d\,x}+981\,e^{2\,c+2\,d\,x}+1540\,e^{3\,c+3\,d\,x}+981\,e^{4\,c+4\,d\,x}+270\,e^{5\,c+5\,d\,x}+27\,e^{6\,c+6\,d\,x}+27}$$

input
```
int(1/(3*cosh(c + d*x) + 5)^4,x)
```

output

```
((385*exp(c + d*x))/(4096*d) + 1925/(12288*d))/(10*exp(c + d*x) + 3*exp(2*
c + 2*d*x) + 3) - (385*atan((5/(4*d) + (3*exp(d*x)*exp(c))/(4*d))*(-d^2)^(
1/2)))/(16384*(-d^2)^(1/2)) - ((385*exp(c + d*x))/(1152*d) + 3461/(3456*d)
)/(60*exp(c + d*x) + 118*exp(2*c + 2*d*x) + 60*exp(3*c + 3*d*x) + 9*exp(4*
c + 4*d*x) + 9) + ((365*exp(c + d*x))/(54*d) + 41/(18*d))/(270*exp(c + d*x
) + 981*exp(2*c + 2*d*x) + 1540*exp(3*c + 3*d*x) + 981*exp(4*c + 4*d*x) +
270*exp(5*c + 5*d*x) + 27*exp(6*c + 6*d*x) + 27)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 433, normalized size of antiderivative = 4.08

$$\int \frac{1}{(5+3\cosh(c+dx))^4}\,dx = \frac{-10395e^{6dx+6c}\log\left(e^{dx+c}+3\right)+10395e^{6dx+6c}\log\left(3e^{dx+c}+1\right)-2772e^{6dx+6c}-103950e^{5dx+5c}\log\left(e^{dx+c}+\right.}{}$$

input

```
int(1/(5+3*cosh(d*x+c))^4,x)
```

output

```
( - 10395*e**(6*c + 6*d*x)*log(e**(c + d*x) + 3) + 10395*e**(6*c + 6*d*x)*
log(3*e**(c + d*x) + 1) - 2772*e**(6*c + 6*d*x) - 103950*e**(5*c + 5*d*x)*
log(e**(c + d*x) + 3) + 103950*e**(5*c + 5*d*x)*log(3*e**(c + d*x) + 1) -
377685*e**(4*c + 4*d*x)*log(e**(c + d*x) + 3) + 377685*e**(4*c + 4*d*x)*lo
g(3*e**(c + d*x) + 1) + 130284*e**(4*c + 4*d*x) - 592900*e**(3*c + 3*d*x)*
log(e**(c + d*x) + 3) + 592900*e**(3*c + 3*d*x)*log(3*e**(c + d*x) + 1) +
480480*e**(3*c + 3*d*x) - 377685*e**(2*c + 2*d*x)*log(e**(c + d*x) + 3) +
377685*e**(2*c + 2*d*x)*log(3*e**(c + d*x) + 1) + 481860*e**(2*c + 2*d*x)
- 103950*e**(c + d*x)*log(e**(c + d*x) + 3) + 103950*e**(c + d*x)*log(3*e*
*(c + d*x) + 1) + 168480*e**(c + d*x) - 10395*log(e**(c + d*x) + 3) + 1039
5*log(3*e**(c + d*x) + 1) + 19620)/(32768*d*(27*e**(6*c + 6*d*x) + 270*e**
(5*c + 5*d*x) + 981*e**(4*c + 4*d*x) + 1540*e**(3*c + 3*d*x) + 981*e**(2*c
 + 2*d*x) + 270*e**(c + d*x) + 27))
```

3.79 $\int (a+b\cosh(x))^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 153

$$\int (a+b\cosh(x))^{5/2}\,dx = -\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15\sqrt{a+b\cosh(x)}} + \frac{16}{15}ab\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}b(a+b\cosh(x))^{3/2}\sinh(x)$$

output
```
-2/15*I*(23*a^2+9*b^2)*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)
*(b/(a+b))^(1/2))/((a+b*cosh(x))/(a+b))^(1/2)+16/15*I*a*(a^2-b^2)*((a+b*co
sh(x))/(a+b))^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/(a+b*
cosh(x))^(1/2)+16/15*a*b*(a+b*cosh(x))^(1/2)*sinh(x)+2/5*b*(a+b*cosh(x))^(
3/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.98

$$\int (a+b\cosh(x))^{5/2}\,dx=\frac{-2i(23a^3+23a^2b+9ab^2+9b^3)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{Ell}}{15\sqrt{a+b\cosh(x}}$$

input
```
Integrate[(a + b*Cosh[x])^(5/2),x]
```

output
```
((-2*I)*(23*a^3 + 23*a^2*b + 9*a*b^2 + 9*b^3)*Sqrt[(a + b*Cosh[x])/(a + b)
]*EllipticE[(I/2)*x, (2*b)/(a + b)] + (16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cos
h[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)] + b*(22*a^2 + 3*b^2 + 28*
a*b*Cosh[x] + 3*b^2*Cosh[2*x])*Sinh[x])/(15*Sqrt[a + b*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.500, Rules used = {3042, 3135, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{5/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cosh(x)}\left(5a^2+8b\cosh(x)a+3b^2\right)dx+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\int \sqrt{a+b\cosh(x)}\left(5a^2+8b\cosh(x)a+3b^2\right)dx+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{1}{5}\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}\left(5a^2+8b\sin\left(ix+\frac{\pi}{2}\right)a+3b^2\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}\right)+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}\right)+\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{5}\left(\frac{1}{3}\left(\left(23a^2+9b^2\right)\int \sqrt{a+b\cosh(x)}dx-8a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx\right)+\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}\right)$$
$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\left(23a^2+9b^2\right)\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-8a\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(ix-\right.}}\right.\right.$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(23a^2+9b^2)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a(a^2-b^2)\int\frac{}{\sqrt{a\cdots}}\right.\right.$$

$\Big\downarrow$ 3042

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(23a^2+9b^2)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin(ix+\frac{\pi}{2})}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a(a^2-b^2)\int\frac{}{\sqrt{}}\right.\right.$$

$\Big\downarrow$ 3132

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-8a(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right.\right.$$

$\Big\downarrow$ 3142

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{8a(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh}}{\sqrt{\frac{a+b\cosh(}{a+b}}}\right.\right.$$

$\Big\downarrow$ 3042

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{8a(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+}}{\sqrt{\frac{a+b\,\mathrm{c}}{a}}}\right.\right.$$

$\Big\downarrow$ 3140

$$\frac{2}{5}b\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{16}{3}ab\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{16ia(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{2i(23a^2+9b^2)\sqrt{a+b\,\mathrm{c}}}{\sqrt{\frac{a+b\cosh}{a+b}}}\right.\right.$$

input
```
Int[(a + b*Cosh[x])^(5/2),x]
```

output
```
(2*b*(a + b*Cosh[x])^(3/2)*Sinh[x])/5 + ((((-2*I)*(23*a^2 + 9*b^2)*Sqrt[a
+ b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a + b*Cosh[x])/(a +
b)] + ((16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*
x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/3 + (16*a*b*Sqrt[a + b*Cosh[x]]*Si
nh[x])/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(138) = 276$.

Time = 9.74 (sec), antiderivative size = 685, normalized size of antiderivative = 4.48

method	result
default	$\frac{2\left(24\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{6}b^{3}+\left(56\sqrt{-\frac{2b}{a-b}}\,a\,b^{2}+24\sqrt{-\frac{2b}{a-b}}\,b^{3}\right)\sinh\left(\frac{x}{2}\right)^{4}\cosh\left(\frac{x}{2}\right)+\left(22\sqrt{-\frac{2b}{a-b}}\,a^{2}b+28\sqrt{-\frac{2b}{a-b}}\,a\,b^{2}+6\sqrt{-\right.\right.}{}$

input
```
int((a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
2/15*(24*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^6*b^3+(56*(-2*b/(a-b))
^(1/2)*a*b^2+24*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(22*(-2*
b/(a-b))^(1/2)*a^2*b+28*(-2*b/(a-b))^(1/2)*a*b^2+6*(-2*b/(a-b))^(1/2)*b^3)
*sinh(1/2*x)^2*cosh(1/2*x)+15*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1
/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-
2/b*(a-b))^(1/2))+23*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-s
inh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-
b))^(1/2))+17*b^2*a*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2
*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/
2))+9*b^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/
2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))-46*(2*
b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(
cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*a^2*b-18*(2*b/(a-b)
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/
2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*b^3)*((2*b*cosh(1/2*x)^2+a
-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(
1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 452 vs. $2(134) = 268$.

Time = 0.10 (sec) , antiderivative size = 452, normalized size of antiderivative = 2.95

$$\int (a + b\cosh(x))^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
-1/90*(8*sqrt(1/2)*((a^3 - 33*a*b^2)*cosh(x)^2 + 2*(a^3 - 33*a*b^2)*cosh(x
)*sinh(x) + (a^3 - 33*a*b^2)*sinh(x)^2)*sqrt(b)*weierstrassPInverse(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sin
h(x) + 2*a)/b) + 24*sqrt(1/2)*((23*a^2*b + 9*b^3)*cosh(x)^2 + 2*(23*a^2*b
+ 9*b^3)*cosh(x)*sinh(x) + (23*a^2*b + 9*b^3)*sinh(x)^2)*sqrt(b)*weierstra
ssZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh
(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(3*b^3*cosh(x)^4 + 3*b^3*sinh(x)^4 + 22*a
*b^2*cosh(x)^3 - 22*a*b^2*cosh(x) + 2*(6*b^3*cosh(x) + 11*a*b^2)*sinh(x)^3
 - 3*b^3 - 4*(23*a^2*b + 9*b^3)*cosh(x)^2 + 2*(9*b^3*cosh(x)^2 + 33*a*b^2*
cosh(x) - 46*a^2*b - 18*b^3)*sinh(x)^2 + 2*(6*b^3*cosh(x)^3 + 33*a*b^2*cos
h(x)^2 - 11*a*b^2 - 4*(23*a^2*b + 9*b^3)*cosh(x))*sinh(x))*sqrt(b*cosh(x)
+ a))/(b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int (a + b\cosh(x))^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a + b\cosh(x))^{5/2}\,dx = \int (b\cosh(x) + a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\cosh(x))^{5/2}\,dx = \int (b\cosh(x)+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{5/2}\,dx = \int (a+b\cosh(x))^{5/2}\,dx$$

input
```
int((a + b*cosh(x))^(5/2),x)
```

output
```
int((a + b*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int (a+b\cosh(x))^{5/2}\,dx = \left(\int \sqrt{\cosh(x)\,b+a}\,dx\right)a^2 + 2\left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)\,dx\right)ab + \left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)^2\,dx\right)b^2$$

input
```
int((a+b*cosh(x))^(5/2),x)
```

output
```
int(sqrt(cosh(x)*b + a),x)*a**2 + 2*int(sqrt(cosh(x)*b + a)*cosh(x),x)*a*b
 + int(sqrt(cosh(x)*b + a)*cosh(x)**2,x)*b**2
```

3.80 $\int (a+b\cosh(x))^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 124

$$\int (a+b\cosh(x))^{3/2}\,dx = -\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3\sqrt{a+b\cosh(x)}} + \frac{2}{3}b\sqrt{a+b\cosh(x)}\sinh(x)$$

output
```
-8/3*I*a*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/
2))/((a+b*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*((a+b*cosh(x))/(a+b))^(1/2
)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/(a+b*cosh(x))^(1/2)+2/3
*b*(a+b*cosh(x))^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.90

$$\int (a+b\cosh(x))^{3/2}\,dx = \frac{-8ia(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+2b(a}{3\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(a + b*Cosh[x])^(3/2),x]
```

output
```
((-8*I)*a*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(
a + b)] + (2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*
x, (2*b)/(a + b)] + 2*b*(a + b*Cosh[x])*Sinh[x])/(3*Sqrt[a + b*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3042, 3135, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3135}$$

$$\frac{2}{3}\int \frac{3a^2+4b\cosh(x)a+b^2}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3a^2+4b\cosh(x)a+b^2}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int \frac{3a^2+4b\sin\left(ix+\frac{\pi}{2}\right)a+b^2}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{1}{3}\left(4a\int \sqrt{a+b\cosh(x)}dx-\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx\right)+\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(4a\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3134}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)$$

$$\Big\downarrow \text{3132}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-(a^2-b^2)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3142}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3140}$$

$$\frac{2}{3}b\sinh(x)\sqrt{a+b\cosh(x)}+$$
$$\frac{1}{3}\left(\frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

input
```
Int[(a + b*Cosh[x])^(3/2),x]
```

output
```
(((-8*I)*a*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a
+ b*Cosh[x])/(a + b)] + ((2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*E
llipticF[(I/2)*x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/3 + (2*b*Sqrt[a + b
*Cosh[x]]*Sinh[x])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3135
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[1/n   Int[(a + b*
Sin[c + d*x])^(n - 2)*Simp[a^2*n + b^2*(n - 1) + a*b*(2*n - 1)*Sin[c + d*x]
, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] &&
 IntegerQ[2*n]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(113) = 226$.

Time = 6.50 (sec) , antiderivative size = 466, normalized size of antiderivative = 3.76

method	result
default	$\frac{2\left(4\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{4}b^{2}+2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}ab+2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}b^{2}+3a^{2}\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\,\sqrt{-\cdots}\right.}{\cdots}$

input
```
int((a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/3*(4*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^4*b^2+2*cosh(1/2*x)*(-2*
b/(a-b))^(1/2)*sinh(1/2*x)^2*a*b+2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2
*x)^2*b^2+3*a^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^
2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+
4*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*E
llipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+b^2*(2*b/(
a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cos
h(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))-8*(2*b/(a-b)*sinh(1/2*
x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b
/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*a*b)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1
/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^
(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. $2(111) = 222$.

Time = 0.12 (sec) , antiderivative size = 259, normalized size of antiderivative = 2.09

$$\int (a+b\cosh(x))^{3/2}\,dx = \frac{4\sqrt{\frac{1}{2}}\left((a^2+3\,b^2)\cosh{(x)}+(a^2+3\,b^2)\sinh{(x)}\right)\sqrt{b}\,\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8}{}\right.}{}$$

input
```
integrate((a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/9*(4*sqrt(1/2)*((a^2 + 3*b^2)*cosh(x) + (a^2 + 3*b^2)*sinh(x))*sqrt(b)*w
eierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1
/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) - 48*sqrt(1/2)*(a*b*cosh(x) + a*b*
sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*(b^2*cosh(x)^2
+ b^2*sinh(x)^2 - 8*a*b*cosh(x) - b^2 + 2*(b^2*cosh(x) - 4*a*b)*sinh(x))*s
qrt(b*cosh(x) + a))/(b*cosh(x) + b*sinh(x))
```

Sympy [F]

$$\int (a+b\cosh(x))^{3/2}\,dx=\int (a+b\cosh{(x)})^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))**(3/2),x)
```

output
```
Integral((a + b*cosh(x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\cosh(x))^{3/2}\,dx=\int (b\cosh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\cosh(x))^{3/2}\,dx=\int (b\cosh{(x)}+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{3/2}\,dx=\int (a+b\cosh(x))^{3/2}\,dx$$

input
```
int((a + b*cosh(x))^(3/2),x)
```

output
```
int((a + b*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int (a+b\cosh(x))^{3/2}\,dx=\left(\int \sqrt{\cosh(x)\,b+a}dx\right)a+\left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)\,dx\right)b$$

input
```
int((a+b*cosh(x))^(3/2),x)
```

output
```
int(sqrt(cosh(x)*b + a),x)*a + int(sqrt(cosh(x)*b + a)*cosh(x),x)*b
```

3.81 $\int \sqrt{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 61

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = -\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

output
```
-2*I*(a+b*cosh(d*x+c))^(1/2)*EllipticE(I*sinh(1/2*d*x+1/2*c),2^(1/2)*(b/(a
+b))^(1/2))/d/((a+b*cosh(d*x+c))/(a+b))^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = -\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

input
```
Integrate[Sqrt[a + b*Cosh[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)
])/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cosh(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(c+dx)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cosh(c+dx)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ic+idx+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2i\sqrt{a+b\cosh(c+dx)}E\left(\frac{1}{2}i(c+dx)|\frac{2b}{a+b}\right)}{d\sqrt{\frac{a+b\cosh(c+dx)}{a+b}}}$$

input
```
Int[Sqrt[a + b*Cosh[c + d*x]],x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[c + d*x]]*EllipticE[(I/2)*(c + d*x), (2*b)/(a + b)
])/(d*Sqrt[(a + b*Cosh[c + d*x])/(a + b)])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(62) = 124$.

Time = 4.83 (sec) , antiderivative size = 276, normalized size of antiderivative = 4.52

method	result
default	$\frac{2\left(a\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+b\,\mathrm{EllipticF}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2b\,\mathrm{EllipticE}\left(\cosh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}b+(a+b)\sinh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}\sin}}$
risch	$\frac{\sqrt{2}\sqrt{\left(e^{2dx+2c}b+2\,e^{dx+c}a+b\right)e^{-dx-c}}}{d}+\frac{\left(\frac{2a\left(a+\sqrt{a^{2}-b^{2}}\right)\sqrt{\frac{\left(e^{dx+c}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)b}{a+\sqrt{a^{2}-b^{2}}}}\sqrt{\frac{e^{dx+c}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}-b^{2}}}{b}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}}\sqrt{-\frac{b\,e^{dx+c}}{a+\sqrt{a^{2}-b^{2}}}}}{b\sqrt{b\,e^{3dx+3c}+2\,e^{2dx+2c}}}\right.}{}$

input
```
int((a+b*cosh(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(a*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/
2))+b*EllipticF(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1
/2))-2*b*EllipticE(cosh(1/2*d*x+1/2*c)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))
^(1/2)))*(-sinh(1/2*d*x+1/2*c)^2)^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)/(
a-b))^(1/2)*((2*b*cosh(1/2*d*x+1/2*c)^2+a-b)*sinh(1/2*d*x+1/2*c)^2)^(1/2)/
(-2*b/(a-b))^(1/2)/(2*sinh(1/2*d*x+1/2*c)^4*b+(a+b)*sinh(1/2*d*x+1/2*c)^2)
^(1/2)/sinh(1/2*d*x+1/2*c)/(2*b*sinh(1/2*d*x+1/2*c)^2+a+b)^(1/2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(60) = 120$.

Time = 0.12 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.23

$$\int \sqrt{a + b\cosh(c + dx)}\,dx = \frac{2\left(2\sqrt{\frac{1}{2}}a\sqrt{b}\,\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2}, -\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3}, \frac{3\,b\cosh(dx+c)+3\,b\sinh(dx+c)+2\,a}{3\,b}\right) - 6\sqrt{\frac{1}{2}}b^{\frac{3}{2}}\text{weiers}\right.}{}$$

input
```
integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="fricas")
```

output
```
2/3*(2*sqrt(1/2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8
/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(d*x + c) + 3*b*sinh(d*x + c) + 2*
a)/b) - 6*sqrt(1/2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(d*x + c) + 3*b*sinh(d*x + c) + 2*a)/
b)) - 3*sqrt(b*cosh(d*x + c) + a)*b)/(b*d)
```

Sympy [F]

$$\int \sqrt{a + b\cosh(c + dx)}\,dx = \int \sqrt{a + b\cosh{(c + dx)}}\,dx$$

input
```
integrate((a+b*cosh(d*x+c))**(1/2),x)
```

output
```
Integral(sqrt(a + b*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \sqrt{a + b\cosh(c + dx)}\, dx = \int \sqrt{b \cosh\left(dx + c\right) + a}\, dx$$

input
```
integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(d*x + c) + a), x)
```

Giac [F]

$$\int \sqrt{a + b\cosh(c + dx)}\, dx = \int \sqrt{b \cosh\left(dx + c\right) + a}\, dx$$

input
```
integrate((a+b*cosh(d*x+c))^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a + b\cosh(c + dx)}\, dx = \int \sqrt{a + b \cosh\left(c + d\,x\right)}\, dx$$

input
```
int((a + b*cosh(c + d*x))^(1/2),x)
```

output
```
int((a + b*cosh(c + d*x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\cosh(c+dx)}\,dx = \int \sqrt{\cosh\left(dx+c\right)b+a}dx$$

input
```
int((a+b*cosh(d*x+c))^(1/2),x)
```

output
```
int(sqrt(cosh(c + d*x)*b + a),x)
```

3.82 $\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*((a+b*cosh(x))/(a+b))^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))
^(1/2))/(a+b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[1/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/S
qrt[a + b*Cosh[x]]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3142}$$

$$\frac{\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}$$

input
```
Int[1/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/S
qrt[a + b*Cosh[x]]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. $2(43) = 86$.

Time = 1.25 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.17

method	result	size
default	$\frac{2\sqrt{\left(2b\cosh\left(\frac{x}{2}\right)^{2}+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{\frac{2b\cosh\left(\frac{x}{2}\right)^{2}+a-b}{a-b}}\,\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)}{\sqrt{-\frac{2b}{a-b}}\,\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\sinh\left(\frac{x}{2}\right)^{2}+a+b}}$	146

input
```
int(1/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)*((2*b*c
osh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(
a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2
/b*(a-b))^(1/2))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.33

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \frac{4\sqrt{\frac{1}{2}}\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)}{\sqrt{b}}$$

input
```
integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
4*sqrt(1/2)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*
a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)/sqrt(b)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{1}{\sqrt{a+b\cosh{(x)}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*cosh(x)), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{1}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*cosh(x) + a), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx=\int \frac{1}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(b*cosh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx=\int \frac{1}{\sqrt{a+b\cosh{(x)}}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(1/2),x)
```

output
```
int(1/(a + b*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\cosh(x)}}\,dx=\int \frac{\sqrt{\cosh{(x)}\,b+a}}{\cosh{(x)}\,b+a}dx$$

input
```
int(1/(a+b*cosh(x))^(1/2),x)
```

output
```
int(sqrt(cosh(x)*b + a)/(cosh(x)*b + a),x)
```

3.83 $\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 84

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = -\frac{2i\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))/
(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)-2*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^
(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = -\frac{2\left(i(a+b)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+b\sinh(x)\right)}{(a-b)(a+b)\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(a + b*Cosh[x])^(-3/2),x]
```

output
```
(-2*(I*(a + b)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2)*x, (2*b)/(a +
 b)] + b*Sinh[x]))/((a - b)*(a + b)*Sqrt[a + b*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.700$, Rules used = {3042, 3143, 27, 3042, 3134, 3042, 3132}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{2\int -\frac{1}{2}\sqrt{a+b\cosh(x)}dx}{a^2-b^2}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \sqrt{a+b\cosh(x)}dx}{a^2-b^2}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2b\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

input
```
Int[(a + b*Cosh[x])^(-3/2),x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/((a^2 - b^2
)*Sqrt[(a + b*Cosh[x])/(a + b)]) - (2*b*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*C
osh[x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(82) = 164$.

Time = 1.52 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.55

method	result
default	$-\frac{2\left(2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}b-\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\,\operatorname{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)a-\sqrt{-\sinh\left(\frac{x}{2}\right)\right.}{\sqrt{-\frac{2b}{a-b}}\,(a-b}$

input
```
int(1/(a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sinh(1/2*x)^2)^(1/2
)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/
(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*a-(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*s
inh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1
/2*(-2/b*(a-b))^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2
+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-
b))^(1/2))*b)/(-2*b/(a-b))^(1/2)/(a-b)/(a+b)/sinh(1/2*x)/(2*b*sinh(1/2*x)^
2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 381 vs. $2(80) = 160$.

Time = 0.10 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.54

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \frac{4\left(\sqrt{\frac{1}{2}}\left(ab\cosh\left(x\right)^{2}+ab\sinh\left(x\right)^{2}+2\,a^{2}\cosh\left(x\right)+ab+2\left(ab\cosh\left(x\right)+a^{2}\right)\sin\right.\right.}{}$$

input
```
integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
4/3*(sqrt(1/2)*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a
*b*cosh(x) + a^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)
/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b
) - 3*sqrt(1/2)*(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + b^2 + 2*(
b^2*cosh(x) + a*b)*sinh(x))*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b))
- 3*(b^2*cosh(x)^2 + b^2*sinh(x)^2 + a*b*cosh(x) + (2*b^2*cosh(x) + a*b)*s
inh(x))*sqrt(b*cosh(x) + a))/(a^2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 +
(a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b - a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 +
 (a^2*b^2 - b^4)*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx=\int \frac{1}{(a+b\cosh{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))**(3/2),x)
```

output
```
Integral((a + b*cosh(x))**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx=\int \frac{1}{(b\cosh{(x)}+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(3/2),x)
```

output
```
int(1/(a + b*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{\sqrt{\cosh(x)\,b+a}}{\cosh(x)^2 b^2+2\cosh(x)\,ab+a^2}dx$$

input
```
int(1/(a+b*cosh(x))^(3/2),x)
```

output
```
int(sqrt(cosh(x)*b + a)/(cosh(x)**2*b**2 + 2*cosh(x)*a*b + a**2),x)
```

3.84 $\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 177

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3(a^2-b^2)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3(a^2-b^2)\sqrt{a+b\cosh(x)}} - \frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}} - \frac{8ab\sinh(x)}{3(a^2-b^2)^2\sqrt{a+b\cosh(x)}}$$

output
```
-8/3*I*a*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/
2))/(a^2-b^2)^2/((a+b*cosh(x))/(a+b))^(1/2)+2/3*I*((a+b*cosh(x))/(a+b))^(1
/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/(a^2-b^2)/(a+b*cosh(x
))^(1/2)-2/3*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(3/2)-8/3*a*b*sinh(x)/(a^2-
b^2)^2/(a+b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.76

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \frac{-8ia(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2i(a-b)(a+b)^2\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\text{Ellip}}{3(a-b)^2(a+b)^2(a+b\cosh(x)}$$

input
```
Integrate[(a + b*Cosh[x])^(-5/2),x]
```

output
```
((-8*I)*a*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*EllipticE[(I/2)*x, (2*
b)/(a + b)] + (2*I)*(a - b)*(a + b)^2*((a + b*Cosh[x])/(a + b))^(3/2)*Elli
pticF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-5*a^2 + b^2 - 4*a*b*Cosh[x])*Sinh[x]
)/(3*(a - b)^2*(a + b)^2*(a + b*Cosh[x])^(3/2))
```

Rubi [A] (verified)

Time = 0.98 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.06, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 1.500$, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3143}$$

$$-\frac{2\int -\frac{3a-b\cosh(x)}{2(a+b\cosh(x))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{3a-b\cosh(x)}{(a+b\cosh(x))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{\int \frac{3a-b\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3233$$

$$\frac{-\frac{2\int -\frac{3a^2+4b\cosh(x)a+b^2}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\int \frac{3a^2+4b\cosh(x)a+b^2}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\int \frac{3a^2+4b\sin\left(ix+\frac{\pi}{2}\right)a+b^2}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3231$$

$$\frac{\frac{4a\int \sqrt{a+b\cosh(x)}dx-\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \; 3042$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{4a\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3134$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{3/2}}+\frac{-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{3/2}}+\frac{-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{4a\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{3/2}}+\frac{-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3142}$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{3/2}}+\frac{-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{3\left(a^2-b^2\right)\left(a+b\cosh(x)\right)^{3/2}}+\frac{-\frac{8ab\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2b\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{8ab\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{2i(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{8ia\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}$$

input
```
Int[(a + b*Cosh[x])^(-5/2),x]
```

output
```
(-2*b*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) + ((((-8*I)*a*Sqrt[a
+ b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a + b*Cosh[x])/(a +
b)] + ((2*I)*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x,
(2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/(a^2 - b^2) - (8*a*b*Sinh[x])/((a^2 -
 b^2)*Sqrt[a + b*Cosh[x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 458 vs. $2(162) = 324$.

Time = 2.64 (sec) , antiderivative size = 459, normalized size of antiderivative = 2.59

method	result
default	$\frac{\sqrt{\left(2b\cosh\left(\frac{x}{2}\right)^{2}+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}}{3b(a-b)(a+b)\left(\cosh\left(\frac{x}{2}\right)^{2}+\frac{a-b}{2b}\right)^{2}}-\frac{16\sinh\left(\frac{x}{2}\right)^{2}b\cosh\left(\frac{x}{2}\right)a}{3(a-b)^{2}(a+b)^{2}\sqrt{\left(2b\cosh\left(\frac{x}{2}\right)^{2}+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}}+\frac{2(3a-\cdots}{\cdots}\right.}{\cdots}$

input
```
int(1/(a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/3/b/(a-b)/(a+b)*cosh(1/2
*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2/b*(a-
b))^2-16/3*sinh(1/2*x)^2*b/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*b*cosh(1/2*x)
^2+a-b)*sinh(1/2*x)^2)^(1/2)+2*(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*b
/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)
/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b
/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-32/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(-
2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1
/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(cos
h(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*x
)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))))/sinh(1/2*x)/(2*b*sinh(1/2
*x)^2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1187 vs. $2(158) = 316$.

Time = 0.10 (sec) , antiderivative size = 1187, normalized size of antiderivative = 6.71

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
4/9*(sqrt(1/2)*((a^2*b^2 + 3*b^4)*cosh(x)^4 + (a^2*b^2 + 3*b^4)*sinh(x)^4
+ a^2*b^2 + 3*b^4 + 4*(a^3*b + 3*a*b^3)*cosh(x)^3 + 4*(a^3*b + 3*a*b^3 + (
a^2*b^2 + 3*b^4)*cosh(x))*sinh(x)^3 + 2*(2*a^4 + 7*a^2*b^2 + 3*b^4)*cosh(x
)^2 + 2*(2*a^4 + 7*a^2*b^2 + 3*b^4 + 3*(a^2*b^2 + 3*b^4)*cosh(x)^2 + 6*(a^
3*b + 3*a*b^3)*cosh(x))*sinh(x)^2 + 4*(a^3*b + 3*a*b^3)*cosh(x) + 4*(a^3*b
 + 3*a*b^3 + (a^2*b^2 + 3*b^4)*cosh(x)^3 + 3*(a^3*b + 3*a*b^3)*cosh(x)^2 +
 (2*a^4 + 7*a^2*b^2 + 3*b^4)*cosh(x))*sinh(x))*sqrt(b)*weierstrassPInverse
(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) +
3*b*sinh(x) + 2*a)/b) - 12*sqrt(1/2)*(a*b^3*cosh(x)^4 + a*b^3*sinh(x)^4 +
4*a^2*b^2*cosh(x)^3 + 4*a^2*b^2*cosh(x) + a*b^3 + 4*(a*b^3*cosh(x) + a^2*b
^2)*sinh(x)^3 + 2*(2*a^3*b + a*b^3)*cosh(x)^2 + 2*(3*a*b^3*cosh(x)^2 + 6*a
^2*b^2*cosh(x) + 2*a^3*b + a*b^3)*sinh(x)^2 + 4*(a*b^3*cosh(x)^3 + 3*a^2*b
^2*cosh(x)^2 + a^2*b^2 + (2*a^3*b + a*b^3)*cosh(x))*sinh(x))*sqrt(b)*weier
strassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstra
ssPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*
cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(4*a*b^3*cosh(x)^4 + 4*a*b^3*sinh(x)^
4 + (13*a^2*b^2 - b^4)*cosh(x)^3 + (16*a*b^3*cosh(x) + 13*a^2*b^2 - b^4)*s
inh(x)^3 + 4*(2*a^3*b + a*b^3)*cosh(x)^2 + (24*a*b^3*cosh(x)^2 + 8*a^3*b +
 4*a*b^3 + 3*(13*a^2*b^2 - b^4)*cosh(x))*sinh(x)^2 + (3*a^2*b^2 + b^4)*cos
h(x) + (16*a*b^3*cosh(x)^3 + 3*a^2*b^2 + b^4 + 3*(13*a^2*b^2 - b^4)*cos...
```

Sympy [F]

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{1}{(a+b\cosh{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))**(5/2),x)
```

output
```
Integral((a + b*cosh(x))**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx=\int \frac{1}{(b\cosh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx=\int \frac{1}{(b\cosh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx=\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(5/2),x)
```

output
```
int(1/(a + b*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{\sqrt{\cosh(x)\,b+a}}{\cosh(x)^3\,b^3+3\cosh(x)^2\,a\,b^2+3\cosh(x)\,a^2b+a^3}dx$$

input
```
int(1/(a+b*cosh(x))^(5/2),x)
```

output
```
int(sqrt(cosh(x)*b + a)/(cosh(x)**3*b**3 + 3*cosh(x)**2*a*b**2 + 3*cosh(x)
*a**2*b + a**3),x)
```

3.85 $\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 227

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = -\frac{2i(23a^2+9b^2)\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^3\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{16ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}} - \frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}} - \frac{16ab\sinh(x)}{15\left(a^2-b^2\right)^2(a+b\cosh(x))^{3/2}} - \frac{2b(23a^2+9b^2)\sinh(x)}{15\left(a^2-b^2\right)^3\sqrt{a+b\cosh(x)}}$$

output
```
-2/15*I*(23*a^2+9*b^2)*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)
*(b/(a+b))^(1/2))/(a^2-b^2)^3/((a+b*cosh(x))/(a+b))^(1/2)+16/15*I*a*((a+b*
cosh(x))/(a+b))^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/(a^
2-b^2)^2/(a+b*cosh(x))^(1/2)-2/5*b*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(5/2)-1
6/15*a*b*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^(3/2)-2/15*b*(23*a^2+9*b^2)*sin
h(x)/(a^2-b^2)^3/(a+b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.73

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \frac{2\left(-\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{5/2}\left((23a^2+9b^2)E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)+8a(-a+b)\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{(a-b)^3}+\frac{b(34a^4-5a^2b^2+3b^4+}{}\right.}{15(a+b\cosh(x))^{5/2}}$$

input
```
Integrate[(a + b*Cosh[x])^(-7/2),x]
```

output
```
(2*(((-I)*((a + b*Cosh[x])/(a + b))^(5/2)*((23*a^2 + 9*b^2)*EllipticE[(I/2
)*x, (2*b)/(a + b)] + 8*a*(-a + b)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(a
- b)^3 + (b*(34*a^4 - 5*a^2*b^2 + 3*b^4 + 2*a*b*(27*a^2 + 5*b^2)*Cosh[x] +
 b^2*(23*a^2 + 9*b^2)*Cosh[x]^2)*Sinh[x])/(-a^2 + b^2)^3))/(15*(a + b*Cosh
[x])^(5/2))
```

Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 253, normalized size of antiderivative = 1.11, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.800$, Rules used = {3042, 3143, 27, 3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{7/2}}dx$$

$$\Big\downarrow \text{3143}$$

$$-\frac{2\int -\frac{5a-3b\cosh(x)}{2(a+b\cosh(x))^{5/2}}dx}{5\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{5a-3b\cosh(x)}{(a+b\cosh(x))^{5/2}}dx}{5(a^2-b^2)}-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}+\frac{\int \frac{5a-3b\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{5/2}}dx}{5(a^2-b^2)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{2\int -\frac{3\left(5a^2+3b^2\right)-8ab\cosh(x)}{2(a+b\cosh(x))^{3/2}}dx}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5(a^2-b^2)}-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3\left(5a^2+3b^2\right)-8ab\cosh(x)}{(a+b\cosh(x))^{3/2}}dx}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5(a^2-b^2)}-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{\int \frac{3\left(5a^2+3b^2\right)-8ab\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{\frac{-\frac{2\int -\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5(a^2-b^2)}-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\frac{\int \frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5(a^2-b^2)}-\frac{2b\sinh(x)}{5(a^2-b^2)(a+b\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\int\frac{a\left(15a^2+17b^2\right)+b\left(23a^2+9b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3231

$$\frac{\frac{\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\cosh(x)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}}{5\left(a^2-b^2\right)}-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}$$

$\downarrow$ 3042

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\left(23a^2+9b^2\right)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3134

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3042

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\downarrow$ 3132

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-8a\left(a^2-b^2\right)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3142

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{8a\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3140

$$-\frac{2b\sinh(x)}{5\left(a^2-b^2\right)(a+b\cosh(x))^{5/2}}+\frac{-\frac{16ab\sinh(x)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2b\left(23a^2+9b^2\right)\sinh(x)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{16ia\left(a^2-b^2\right)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{\sqrt{a+b\cosh(x)}}-\frac{2i\left(23a^2+9b^2\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3(a^2-b^2)}}{5\left(a^2-b^2\right)}$$

input
```
Int[(a + b*Cosh[x])^(-7/2),x]
```

output
```
(-2*b*Sinh[x])/(5*(a^2 - b^2)*(a + b*Cosh[x])^(5/2)) + ((-16*a*b*Sinh[x])/
(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) + ((((-2*I)*(23*a^2 + 9*b^2)*Sqrt[a
+ b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/Sqrt[(a + b*Cosh[x])/(a +
b)] + ((16*I)*a*(a^2 - b^2)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*
x, (2*b)/(a + b)])/Sqrt[a + b*Cosh[x]])/(a^2 - b^2) - (2*b*(23*a^2 + 9*b^2
)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]]))/(3*(a^2 - b^2)))/(5*(a^2 - b
^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(208) = 416$.

Time = 2.71 (sec) , antiderivative size = 566, normalized size of antiderivative = 2.49

method	result
default	$\frac{\sqrt{\left(2b\cosh\left(\frac{x}{2}\right)^{2}+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\left(-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}}{10b^{2}(a-b)(a+b)\left(\cosh\left(\frac{x}{2}\right)^{2}+\frac{a-b}{2b}\right)^{3}}-\frac{8a\cosh\left(\frac{x}{2}\right)\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}}{15b(a+b)^{2}(a-b)^{2}\left(\cosh\left(\frac{x}{2}\right)^{2}+\frac{a-b}{2b}\right)^{2}}-\frac{4\sin\ldots}{15(a-b)^{3}(a\ldots}\right.}{\ldots}$

input
```
int(1/(a+b*cosh(x))^(7/2),x,method=_RETURNVERBOSE)
```

output
```
((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-1/10/b^2/(a-b)/(a+b)*cosh(
1/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2/b*
(a-b))^3-8/15*a/b/(a+b)^2/(a-b)^2*cosh(1/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sin
h(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2/b*(a-b))^2-4/15*sinh(1/2*x)^2*b/(a-b)
^3/(a+b)^3*cosh(1/2*x)*(23*a^2+9*b^2)/((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)
^2)^(1/2)+2*(15*a^2-8*a*b+9*b^2)/(15*a^5+15*a^4*b-30*a^3*b^2-30*a^2*b^3+15
*a*b^4+15*b^5)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-
sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*Ellipti
cF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-8/15*b*(23*a^2
+9*b^2)/(a+b)^3/(a-b)^3*(-a+b)/(-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)
/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^
2)^(1/2)/(2*a-2*b)*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*
b)/b)^(1/2))-EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(
1/2))))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3129 vs. $2(204) = 408$.

Time = 0.15 (sec) , antiderivative size = 3129, normalized size of antiderivative = 13.78

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*cosh(x))**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="maxima")
```

output
```
integrate((b*cosh(x) + a)^(-7/2), x)
```

Giac [F]

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \int \frac{1}{(b\cosh(x)+a)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/(a+b*cosh(x))^(7/2),x, algorithm="giac")
```

output
```
integrate((b*cosh(x) + a)^(-7/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx$$

input
```
int(1/(a + b*cosh(x))^(7/2),x)
```

output
```
int(1/(a + b*cosh(x))^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{(a+b\cosh(x))^{7/2}}\,dx = \int \frac{\sqrt{\cosh(x)\,b+a}}{\cosh(x)^4\,b^4+4\cosh(x)^3\,a\,b^3+6\cosh(x)^2\,a^2b^2+4\cosh(x)\,a^3b+a^4}dx$$

input
```
int(1/(a+b*cosh(x))^(7/2),x)
```

output
```
int(sqrt(cosh(x)*b + a)/(cosh(x)**4*b**4 + 4*cosh(x)**3*a*b**3 + 6*cosh(x)
**2*a**2*b**2 + 4*cosh(x)*a**3*b + a**4),x)
```

3.86 $\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 100

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))/
b/((a+b*cosh(x))/(a+b))^(1/2)+2*I*a*((a+b*cosh(x))/(a+b))^(1/2)*InverseJac
obiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/b/(a+b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.73

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-a\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*EllipticE[(I/2)*x, (2*b)/(a
 + b)] - a*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{\int \sqrt{a+b\cosh(x)}dx}{b}-\frac{a\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3134}$$

$$\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3132}$$

$$-\frac{a\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{ 3142}$$

$$-\frac{a\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{a\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{ 3140}$$

$$\frac{2ia\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

input
```
Int[Cosh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a
+ b*Cosh[x])/(a + b)]) + ((2*I)*a*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[
(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 4.05 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.81

method	result
default	$\frac{2\left(\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2\,\mathrm{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}, \frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)\right)\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\sqrt{\frac{2b\cosh\left(\frac{x}{2}\right)^{2}+a-b}{a-b}}\sqrt{(}}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\sinh\left(\frac{x}{2}\right)^{2}+a+b}}$
risch	$\frac{(e^{2x}b+2a\,e^{x}+b)\sqrt{2}\,e^{-x}}{b\sqrt{(e^{2x}b+2a\,e^{x}+b)e^{-x}}}+\frac{\left(-\frac{4\left(e^{2x}b+2a\,e^{x}+b\right)}{b\sqrt{\left(e^{2x}b+2a\,e^{x}+b\right)e^{x}}}+\frac{4\left(a+\sqrt{a^{2}-b^{2}}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)b}{a+\sqrt{a^{2}-b^{2}}}}\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}-b^{2}}}{b}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}}\sqrt{-\frac{e}{a+\sqrt{a}}}}{}\right.}{}$

input
```
int(cosh(x)/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))-2*Elli
pticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2)))*(-sinh(1/2*x
)^2)^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)*
sinh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*
x)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.74

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx =$$

$$-\frac{2\left(4\sqrt{\frac{1}{2}}a\sqrt{b}\mathrm{weierstrassPInverse}\left(\frac{4\,(4\,a^{2}-3\,b^{2})}{3\,b^{2}}, -\frac{8\,(8\,a^{3}-9\,ab^{2})}{27\,b^{3}}, \frac{3\,b\cosh(x)+3\,b\sinh(x)+2\,a}{3\,b}\right)+6\sqrt{\frac{1}{2}}b^{\frac{3}{2}}\mathrm{weierstras}\right.}{}$$

input
```
integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(4*sqrt(1/2)*a*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -
8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 6*s
qrt(1/2)*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9
*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 3*sqrt(b*cosh(x)
+ a)*b)/b^2
```

Sympy [F]

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{a+b\cosh{(x)}}}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral(cosh(x)/sqrt(a + b*cosh(x)), x)
```

Maxima [F]

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh{(x)}}{\sqrt{b\cosh{(x)}+a}}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

Giac [F]

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{b\cosh(x)+a}}\,dx$$

input
```
integrate(cosh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(cosh(x)/sqrt(b*cosh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$$

input
```
int(cosh(x)/(a + b*cosh(x))^(1/2),x)
```

output
```
int(cosh(x)/(a + b*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\sqrt{\cosh(x)b+a}\,\cosh(x)}{\cosh(x)b+a}dx$$

input
```
int(cosh(x)/(a+b*cosh(x))^(1/2),x)
```

output
```
int((sqrt(cosh(x)*b + a)*cosh(x))/(cosh(x)*b + a),x)
```

3.87 $\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 94

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{64a^3(7A+5B)\sinh(x)}{105\sqrt{a+a\cosh(x)}} + \frac{16}{105}a^2(7A+5B)\sqrt{a+a\cosh(x)}\sinh(x) + \frac{2}{35}a(7A+5B)(a+a\cosh(x))^{3/2}\sinh(x) + \frac{2}{7}B(a+a\cosh(x))^{5/2}\sinh(x)$$

output
```
64/105*a^3*(7*A+5*B)*sinh(x)/(a+a*cosh(x))^(1/2)+16/105*a^2*(7*A+5*B)*(a+a
*cosh(x))^(1/2)*sinh(x)+2/35*a*(7*A+5*B)*(a+a*cosh(x))^(3/2)*sinh(x)+2/7*B
*(a+a*cosh(x))^(5/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.64

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{1}{210}a^2\sqrt{a(1+\cosh(x))}(1246A+1040B + (392A+505B)\cosh(x)+6(7A+20B)\cosh(2x)+15B\cosh(3x))\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(a^2*Sqrt[a*(1 + Cosh[x])]*(1246*A + 1040*B + (392*A + 505*B)*Cosh[x] + 6*
(7*A + 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Tanh[x/2])/210
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+a)^{5/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A+5B)\int (\cosh(x)a+a)^{5/2}dx+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}+\frac{1}{7}(7A+5B)\int \left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\int (\cosh(x)a+a)^{3/2}dx+\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}\right)+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}+\frac{1}{7}(7A+5B)\left(\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{8}{5}a\int \left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{3/2}dx\right)$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{\cosh(x)a+a}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}\right)+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}+\frac{1}{7}(7A+5B)\left(\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{8}{5}a\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}+\frac{4}{3}a\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx\right)\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{7}(7A+5B)\left(\frac{8}{5}a\left(\frac{8a^2\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}a\sinh(x)(a\cosh(x)+a)^{3/2}\right)+\frac{2}{7}B\sinh(x)(a\cosh(x)+a)^{5/2}$$

input
```
Int[(a + a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + a*Cosh[x])^(5/2)*Sinh[x])/7 + ((7*A + 5*B)*((2*a*(a + a*Cosh[x])
^(3/2)*Sinh[x])/5 + (8*a*((8*a^2*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*a*S
qrt[a + a*Cosh[x]]*Sinh[x])/3))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.76

method	result
default	$\frac{8\cosh\left(\frac{x}{2}\right)a^{3}\sinh\left(\frac{x}{2}\right)\left(30B\sinh\left(\frac{x}{2}\right)^{6}+(21A+105B)\sinh\left(\frac{x}{2}\right)^{4}+(70A+140B)\sinh\left(\frac{x}{2}\right)^{2}+105A+105B\right)\sqrt{2}}{105\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$
parts	$\frac{8A\,a^{3}\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\left(3\cosh\left(\frac{x}{2}\right)^{4}+4\cosh\left(\frac{x}{2}\right)^{2}+8\right)\sqrt{2}}{15\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}+\frac{8B\cosh\left(\frac{x}{2}\right)a^{3}\sinh\left(\frac{x}{2}\right)\left(6\cosh\left(\frac{x}{2}\right)^{6}+3\cosh\left(\frac{x}{2}\right)^{4}+4\cosh\left(\frac{x}{2}\right)^{2}+8\right)\sqrt{2}}{21\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$

input
```
int((a+cosh(x)*a)^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
8/105*cosh(1/2*x)*a^3*sinh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A+105*B)*sinh(1/
2*x)^4+(70*A+140*B)*sinh(1/2*x)^2+105*A+105*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(
1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 563 vs. $2(78) = 156$.

Time = 0.09 (sec) , antiderivative size = 563, normalized size of antiderivative = 5.99

$$\int (a + a\cosh(x))^{5/2}(A + B\cosh(x))\, dx = \text{Too large to display}$$

input
```
integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A + 5*B)*
a^2*cosh(x)^6 + 35*(10*A + 11*B)*a^2*cosh(x)^5 + 525*(4*A + 3*B)*a^2*cosh(
x)^4 + 21*(5*B*a^2*cosh(x) + (2*A + 5*B)*a^2)*sinh(x)^6 - 525*(4*A + 3*B)*
a^2*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A + 5*B)*a^2*cosh(x) + 5*(10
*A + 11*B)*a^2)*sinh(x)^5 - 35*(10*A + 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*
cosh(x)^3 + 9*(2*A + 5*B)*a^2*cosh(x)^2 + 5*(10*A + 11*B)*a^2*cosh(x) + 15
*(4*A + 3*B)*a^2)*sinh(x)^4 - 21*(2*A + 5*B)*a^2*cosh(x) + 35*(15*B*a^2*co
sh(x)^4 + 12*(2*A + 5*B)*a^2*cosh(x)^3 + 10*(10*A + 11*B)*a^2*cosh(x)^2 +
60*(4*A + 3*B)*a^2*cosh(x) - 15*(4*A + 3*B)*a^2)*sinh(x)^3 - 15*B*a^2 + 35
*(9*B*a^2*cosh(x)^5 + 9*(2*A + 5*B)*a^2*cosh(x)^4 + 10*(10*A + 11*B)*a^2*c
osh(x)^3 + 90*(4*A + 3*B)*a^2*cosh(x)^2 - 45*(4*A + 3*B)*a^2*cosh(x) - (10
*A + 11*B)*a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A + 5*B)*a^2*cos
h(x)^5 + 25*(10*A + 11*B)*a^2*cosh(x)^4 + 300*(4*A + 3*B)*a^2*cosh(x)^3 -
225*(4*A + 3*B)*a^2*cosh(x)^2 - 10*(10*A + 11*B)*a^2*cosh(x) - 3*(2*A + 5*
B)*a^2)*sinh(x))*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sinh
(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a + a\cosh(x))^{5/2}(A + B\cosh(x))\, dx = \text{Timed out}$$

input
```
integrate((a+a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. 2(78) = 156.

Time = 0.14 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.52

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{1}{60}\left(3\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{5}{2}x\right)} + 25\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{3}{2}x\right)} + 150\sqrt{2}a^{\frac{5}{2}}e^{\left(\frac{1}{2}x\right)} - 150\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{1}{2}x\right)} - 25\sqrt{2}a^{\frac{5}{2}}e^{\left(-\frac{3}{2}x\right.}\right. + \frac{1}{168}\left(\left(3\sqrt{2}a^{\frac{5}{2}}e^{(-x)} + 21\sqrt{2}a^{\frac{5}{2}}e^{(-2x)} + 70\sqrt{2}a^{\frac{5}{2}}e^{(-3x)} + 210\sqrt{2}a^{\frac{5}{2}}e^{(-4x)} - 105\sqrt{2}a^{\frac{5}{2}}e^{(-5x)} - 7\sqrt{2}a^{\frac{5}{2}}e^{(-}\right.\right.$$

input
```
integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/60*(3*sqrt(2)*a^(5/2)*e^(5/2*x) + 25*sqrt(2)*a^(5/2)*e^(3/2*x) + 150*sqr
t(2)*a^(5/2)*e^(1/2*x) - 150*sqrt(2)*a^(5/2)*e^(-1/2*x) - 25*sqrt(2)*a^(5/
2)*e^(-3/2*x) - 3*sqrt(2)*a^(5/2)*e^(-5/2*x))*A + 1/168*((3*sqrt(2)*a^(5/2
)*e^(-x) + 21*sqrt(2)*a^(5/2)*e^(-2*x) + 70*sqrt(2)*a^(5/2)*e^(-3*x) + 210
*sqrt(2)*a^(5/2)*e^(-4*x) - 105*sqrt(2)*a^(5/2)*e^(-5*x) - 7*sqrt(2)*a^(5/
2)*e^(-6*x))*e^(9/2*x) + (7*sqrt(2)*a^(5/2)*e^(-x) + 105*sqrt(2)*a^(5/2)*e
^(-2*x) - 210*sqrt(2)*a^(5/2)*e^(-3*x) - 70*sqrt(2)*a^(5/2)*e^(-4*x) - 21*
sqrt(2)*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a^(5/2)*e^(-6*x))*e^(5/2*x))*B
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = -\frac{1}{840}\sqrt{2}a^4\left(\frac{\left(2100\,Aa^3e^{(3x)} + 1575\,Ba^3e^{(3x)} + 350\,Aa^3e^{(2x)} + 385\,Ba^3e^{(2x)} + 42\,Aa^3e^{x} + 105\,Ba^3e^{x} + 15\right.}{a^{\frac{9}{2}}}\right.$$

input
```
integrate((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
-1/840*sqrt(2)*a^4*((2100*A*a^3*e^(3*x) + 1575*B*a^3*e^(3*x) + 350*A*a^3*e
^(2*x) + 385*B*a^3*e^(2*x) + 42*A*a^3*e^x + 105*B*a^3*e^x + 15*B*a^3)*e^(-
7/2*x)/a^(9/2) - (15*B*a^(67/2)*e^(7/2*x) + 42*A*a^(67/2)*e^(5/2*x) + 105*
B*a^(67/2)*e^(5/2*x) + 350*A*a^(67/2)*e^(3/2*x) + 385*B*a^(67/2)*e^(3/2*x)
 + 2100*A*a^(67/2)*e^(1/2*x) + 1575*B*a^(67/2)*e^(1/2*x))/a^35)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx=\int (A+B\cosh(x))\,(a+a\cosh(x))^{5/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))*(a + a*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int (a+a\cosh(x))^{5/2}(A+B\cosh(x))\,dx=\sqrt{a}\,a^2\left(\left(\int\sqrt{\cosh(x)+1}dx\right)a+2\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)\,dx\right)a+\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)\,dx\right)b+\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)^3\,dx\right)b+\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)^2\,dx\right)a+2\left(\int\sqrt{\cosh(x)+1}\,\cosh(x)^2\,dx\right)b\right)$$

input
```
int((a+a*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt(cosh(x) + 1),x)*a + 2*int(sqrt(cosh(x) + 1)*cosh(x)
,x)*a + int(sqrt(cosh(x) + 1)*cosh(x),x)*b + int(sqrt(cosh(x) + 1)*cosh(x)
**3,x)*b + int(sqrt(cosh(x) + 1)*cosh(x)**2,x)*a + 2*int(sqrt(cosh(x) + 1)
*cosh(x)**2,x)*b)
```

3.88 $\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 68

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\frac{8a^2(5A+3B)\sinh(x)}{15\sqrt{a+a\cosh(x)}}+\frac{2}{15}a(5A+3B)\sqrt{a+a\cosh(x)}\sinh(x)+\frac{2}{5}B(a+a\cosh(x))^{3/2}\sinh(x)$$

output
```
8/15*a^2*(5*A+3*B)*sinh(x)/(a+a*cosh(x))^(1/2)+2/15*a*(5*A+3*B)*(a+a*cosh(
x))^(1/2)*sinh(x)+2/5*B*(a+a*cosh(x))^(3/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.68

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\frac{1}{15}a\sqrt{a(1+\cosh(x))}(50A+39B+2(5A+9B)\cosh(x)+3B\cosh(2x))\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(a*Sqrt[a*(1 + Cosh[x])]*(50*A + 39*B + 2*(5*A + 9*B)*Cosh[x] + 3*B*Cosh[2
*x])*Tanh[x/2])/15
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a\cosh(x)+a)^{3/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A+3B)\int (\cosh(x)a+a)^{3/2}dx+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{1}{5}(5A+3B)\int \left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{5}(5A+3B)\left(\frac{4}{3}a\int \sqrt{\cosh(x)a+a}dx+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}+\frac{1}{5}(5A+3B)\left(\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}+\frac{4}{3}a\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{5}(5A+3B)\left(\frac{8a^2\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}a\sinh(x)\sqrt{a\cosh(x)+a}\right)+\frac{2}{5}B\sinh(x)(a\cosh(x)+a)^{3/2}$$

input
```
Int[(a + a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + a*Cosh[x])^(3/2)*Sinh[x])/5 + ((5*A + 3*B)*((8*a^2*Sinh[x])/(3*S
qrt[a + a*Cosh[x]]) + (2*a*Sqrt[a + a*Cosh[x]]*Sinh[x])/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.84

method	result	size
default	$\frac{4\cosh\left(\frac{x}{2}\right)a^{2}\sinh\left(\frac{x}{2}\right)\left(6B\sinh\left(\frac{x}{2}\right)^{4}+(5A+15B)\sinh\left(\frac{x}{2}\right)^{2}+15A+15B\right)\sqrt{2}}{15\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	57
parts	$\frac{4A\,a^{2}\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\left(\cosh\left(\frac{x}{2}\right)^{2}+2\right)\sqrt{2}}{3\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}+\frac{4B\cosh\left(\frac{x}{2}\right)a^{2}\sinh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)^{4}+\cosh\left(\frac{x}{2}\right)^{2}+2\right)\sqrt{2}}{5\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	80

input
```
int((a+cosh(x)*a)^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
4/15*cosh(1/2*x)*a^2*sinh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A+15*B)*sinh(1/2*x)
^2+15*A+15*B)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(56) = 112$.

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.10

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh(x)^{5}+3\,Ba\sinh(x)^{5}+5\,(2\,A+3\,B)a\cosh(x)^{4}+30\,(3\,A+2\,B)a\cosh(x\right.}{}$$

input
```
integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A + 3*B)*a*cosh(x
)^4 + 30*(3*A + 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A + 3*B)*a)*sinh(
x)^4 - 30*(3*A + 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A + 3*B)*a*
cosh(x) + 3*(3*A + 2*B)*a)*sinh(x)^3 - 5*(2*A + 3*B)*a*cosh(x) + 30*(B*a*c
osh(x)^3 + (2*A + 3*B)*a*cosh(x)^2 + 3*(3*A + 2*B)*a*cosh(x) - (3*A + 2*B)
*a)*sinh(x)^2 - 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A + 3*B)*a*cosh(x)^3 + 1
8*(3*A + 2*B)*a*cosh(x)^2 - 12*(3*A + 2*B)*a*cosh(x) - (2*A + 3*B)*a)*sinh
(x))*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2)
```

Sympy [F]

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \int (a(\cosh{(x)}+1))^{\frac{3}{2}}\,(A+B\cosh{(x)})\,dx$$

input
```
integrate((a+a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

output
```
Integral((a*(cosh(x) + 1))**(3/2)*(A + B*cosh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. 2(56) = 112.

Time = 0.16 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.40

$$\int (a+a\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \frac{1}{6}\left(\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)} + 9\,\sqrt{2}a^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)} - 9\,\sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{1}{2}\,x\right)} - \sqrt{2}a^{\frac{3}{2}}e^{\left(-\frac{3}{2}\,x\right)}\right)A + \frac{1}{20}\left(\left(\sqrt{2}a^{\frac{3}{2}}e^{(-x)} + 5\,\sqrt{2}a^{\frac{3}{2}}e^{(-2\,x)} + 15\,\sqrt{2}a^{\frac{3}{2}}e^{(-3\,x)} - 5\,\sqrt{2}a^{\frac{3}{2}}e^{(-4\,x)}\right)e^{\left(\frac{7}{2}\,x\right)} + \left(5\,\sqrt{2}a^{\frac{3}{2}}e^{(-x)} - 15\,\sqrt{2}a^{\frac{3}{2}}e^{(-2}\right.\right.$$

input
```
integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/6*(sqrt(2)*a^(3/2)*e^(3/2*x) + 9*sqrt(2)*a^(3/2)*e^(1/2*x) - 9*sqrt(2)*a
^(3/2)*e^(-1/2*x) - sqrt(2)*a^(3/2)*e^(-3/2*x))*A + 1/20*((sqrt(2)*a^(3/2)
*e^(-x) + 5*sqrt(2)*a^(3/2)*e^(-2*x) + 15*sqrt(2)*a^(3/2)*e^(-3*x) - 5*sqr
t(2)*a^(3/2)*e^(-4*x))*e^(7/2*x) + (5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*
a^(3/2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2)*e^(-4*x))*
e^(3/2*x))*B
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.66

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx =$$

$$-\frac{1}{60}\sqrt{2}\left(\frac{\left(90\,Aa^4e^{(2\,x)}+60\,Ba^4e^{(2\,x)}+10\,Aa^4e^x+15\,Ba^4e^x+3\,Ba^4\right)e^{\left(-\frac{5}{2}\,x\right)}}{a^{\frac{5}{2}}}-\frac{3\,Ba^{\frac{13}{2}}e^{\left(\frac{5}{2}\,x\right)}+10\,Aa^{\frac{13}{2}}e^{(\frac{3}{2}}$$

input
```
integrate((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
-1/60*sqrt(2)*((90*A*a^4*e^(2*x) + 60*B*a^4*e^(2*x) + 10*A*a^4*e^x + 15*B*
a^4*e^x + 3*B*a^4)*e^(-5/2*x)/a^(5/2) - (3*B*a^(13/2)*e^(5/2*x) + 10*A*a^(
13/2)*e^(3/2*x) + 15*B*a^(13/2)*e^(3/2*x) + 90*A*a^(13/2)*e^(1/2*x) + 60*B
*a^(13/2)*e^(1/2*x))/a^5)
```

Mupad [F(-1)]

Timed out.

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,(a+a\cosh(x))^{3/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))*(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int (a+a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \sqrt{a}\,a\left(\left(\left(\int \sqrt{\cosh(x)+1}dx\right)a + \left(\int \sqrt{\cosh(x)+1}\,\cosh(x)\,dx\right)a + \left(\int \sqrt{\cosh(x)+1}\,\cosh(x)\,dx\right)b + \left(\int \sqrt{\cosh(x)+1}\,\cosh(x)^2\,dx\right)b\right)$$

input
```
int((a+a*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

output
```
sqrt(a)*a*(int(sqrt(cosh(x) + 1),x)*a + int(sqrt(cosh(x) + 1)*cosh(x),x)*a
 + int(sqrt(cosh(x) + 1)*cosh(x),x)*b + int(sqrt(cosh(x) + 1)*cosh(x)**2,x
)*b)
```

3.89 $\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 40

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx=\frac{2a(3A+B)\sinh(x)}{3\sqrt{a+a\cosh(x)}}+\frac{2}{3}B\sqrt{a+a\cosh(x)}\sinh(x)$$

output
```
2/3*a*(3*A+B)*sinh(x)/(a+a*cosh(x))^(1/2)+2/3*B*(a+a*cosh(x))^(1/2)*sinh(x
)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.78

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx=\frac{2}{3}\sqrt{a(1+\cosh(x))}(3A+2B+B\cosh(x))\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(2*Sqrt[a*(1 + Cosh[x])]*(3*A + 2*B + B*Cosh[x])*Tanh[x/2])/3
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh(x)+a}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{3}(3A+B)\int \sqrt{\cosh(x)a+a}dx+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}+\frac{1}{3}(3A+B)\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{2a(3A+B)\sinh(x)}{3\sqrt{a\cosh(x)+a}}+\frac{2}{3}B\sinh(x)\sqrt{a\cosh(x)+a}$$

input
```
Int[Sqrt[a + a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(2*a*(3*A + B)*Sinh[x])/(3*Sqrt[a + a*Cosh[x]]) + (2*B*Sqrt[a + a*Cosh[x]]
*Sinh[x])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{2\cosh\left(\frac{x}{2}\right) a \sinh\left(\frac{x}{2}\right)\left(2B\cosh\left(\frac{x}{2}\right)^{2}+3A+B\right)\sqrt{2}}{3\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	39
parts	$\frac{2Aa\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)\sqrt{2}}{\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}+\frac{2B\cosh\left(\frac{x}{2}\right) a \sinh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)^{2}+1\right)\sqrt{2}}{3\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$	62

input
```
int((a+cosh(x)*a)^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/3*cosh(1/2*x)*a*sinh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A+B)*2^(1/2)/(a*cosh(1/
2*x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 100 vs. $2(32) = 64$.

Time = 0.09 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.50

$$\int \sqrt{a + a\cosh(x)}(A + B\cosh(x))\,dx = \frac{\sqrt{\frac{1}{2}}\left(B\cosh\left(x\right)^{3} + B\sinh\left(x\right)^{3} + 3\,(2\,A + B)\cosh\left(x\right)^{2} + 3\,(B\cosh\left(x\right) + 2\,A + B)\sinh\left(x\right)^{2} - 3\,(2\,A + B\right.}{3\,(\cosh\left(x\right) + \sinh}$$

input
```
integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A + B)*cosh(x)^2 + 3*(B*co
sh(x) + 2*A + B)*sinh(x)^2 - 3*(2*A + B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A
 + B)*cosh(x) - 2*A - B)*sinh(x) - B)*sqrt(a/(cosh(x) + sinh(x)))/(cosh(x)
 + sinh(x))
```

Sympy [F]

$$\int \sqrt{a + a\cosh(x)}(A + B\cosh(x))\,dx = \int \sqrt{a\left(\cosh\left(x\right) + 1\right)}\left(A + B\cosh\left(x\right)\right)\,dx$$

input
```
integrate((a+a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

output
```
Integral(sqrt(a*(cosh(x) + 1))*(A + B*cosh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(32) = 64$.

Time = 0.16 (sec) , antiderivative size = 90, normalized size of antiderivative = 2.25

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx = \left(\sqrt{2}\sqrt{a}e^{\left(\frac{1}{2}x\right)} - \sqrt{2}\sqrt{a}e^{\left(-\frac{1}{2}x\right)}\right)A + \frac{1}{6}\left(\left(\sqrt{2}\sqrt{a}e^{(-x)} + 3\sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{5}{2}x\right)} - \left(3\sqrt{2}\sqrt{a}e^{(-x)} + \sqrt{2}\sqrt{a}e^{(-2x)}\right)e^{\left(\frac{1}{2}x\right)}\right)B$$

input
```
integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
(sqrt(2)*sqrt(a)*e^(1/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))*A + 1/6*((sqrt(2)
*sqrt(a)*e^(-x) + 3*sqrt(2)*sqrt(a)*e^(-2*x))*e^(5/2*x) - (3*sqrt(2)*sqrt(
a)*e^(-x) + sqrt(2)*sqrt(a)*e^(-2*x))*e^(1/2*x))*B
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(32) = 64$.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.70

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx = -\frac{1}{6}\sqrt{2}a^2\left(\frac{(6\,Aae^x + 3\,Bae^x + Ba)e^{\left(-\frac{3}{2}x\right)}}{a^{\frac{5}{2}}} - \frac{Ba^{\frac{15}{2}}e^{\left(\frac{3}{2}x\right)} + 6\,Aa^{\frac{15}{2}}e^{\left(\frac{1}{2}x\right)} + 3\,Ba^{\frac{15}{2}}e^{\left(\frac{1}{2}x\right)}}{a^9}\right)$$

input
```
integrate((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
-1/6*sqrt(2)*a^2*((6*A*a*e^x + 3*B*a*e^x + B*a)*e^(-3/2*x)/a^(5/2) - (B*a^
(15/2)*e^(3/2*x) + 6*A*a^(15/2)*e^(1/2*x) + 3*B*a^(15/2)*e^(1/2*x))/a^9)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,\sqrt{a+a\cosh\left(x\right)}\,dx$$

input
```
int((A + B*cosh(x))*(a + a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))*(a + a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+a\cosh(x)}(A+B\cosh(x))\,dx = \sqrt{a}\left(\left(\int \sqrt{\cosh\left(x\right)+1}dx\right)a + \left(\int \sqrt{\cosh\left(x\right)+1}\,\cosh\left(x\right)dx\right)b\right)$$

input
```
int((a+a*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

output
```
sqrt(a)*(int(sqrt(cosh(x) + 1),x)*a + int(sqrt(cosh(x) + 1)*cosh(x),x)*b)
```

3.90 $\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 98

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = -\frac{64a^3(7A-5B)\sinh(x)}{105\sqrt{a-a\cosh(x)}} - \frac{16}{105}a^2(7A-5B)\sqrt{a-a\cosh(x)}\sinh(x) - \frac{2}{35}a(7A-5B)(a-a\cosh(x))^{3/2}\sinh(x) + \frac{2}{7}B(a-a\cosh(x))^{5/2}\sinh(x)$$

output
```
-64/105*a^3*(7*A-5*B)*sinh(x)/(a-a*cosh(x))^(1/2)-16/105*a^2*(7*A-5*B)*(a-
a*cosh(x))^(1/2)*sinh(x)-2/35*a*(7*A-5*B)*(a-a*cosh(x))^(3/2)*sinh(x)+2/7*
B*(a-a*cosh(x))^(5/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.62

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{1}{210}a^2\sqrt{a-a\cosh(x)}(1246A-1040B + (-392A+505B)\cosh(x) + 6(7A-20B)\cosh(2x) + 15B\cosh(3x))\coth\left(\frac{x}{2}\right)$$

input
```
Integrate[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(a^2*Sqrt[a - a*Cosh[x]]*(1246*A - 1040*B + (-392*A + 505*B)*Cosh[x] + 6*(
7*A - 20*B)*Cosh[2*x] + 15*B*Cosh[3*x])*Coth[x/2])/210
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 3230, 3042, 3126, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{7}(7A-5B)\int (a-a\cosh(x))^{5/2}dx+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}+\frac{1}{7}(7A-5B)\int \left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{5/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A-5B)\left(\frac{8}{5}a\int (a-a\cosh(x))^{3/2}dx-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}\right)+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}+\frac{1}{7}(7A-5B)\left(-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}+\frac{8}{5}a\int \left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx\right)$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{7}(7A-5B)\left(\frac{8}{5}a\left(\frac{4}{3}a\int\sqrt{a-a\cosh(x)}dx-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}\right)+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}+\frac{1}{7}(7A-5B)\left(-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}+\frac{8}{5}a\left(-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}+\frac{4}{3}a\int\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{7}(7A-5B)\left(\frac{8}{5}a\left(-\frac{8a^2\sinh(x)}{3\sqrt{a-a\cosh(x)}}-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)-\frac{2}{5}a\sinh(x)(a-a\cosh(x))^{3/2}\right)+\frac{2}{7}B\sinh(x)(a-a\cosh(x))^{5/2}$$

input
```
Int[(a - a*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a - a*Cosh[x])^(5/2)*Sinh[x])/7 + ((7*A - 5*B)*((-2*a*(a - a*Cosh[x]
)^(3/2)*Sinh[x])/5 + (8*a*((-8*a^2*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) - (2*a
*Sqrt[a - a*Cosh[x]]*Sinh[x])/3))/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 1.71 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.70

method	result
default	$-\frac{16 \sinh\left(\frac{x}{2}\right) a^{3} \cosh\left(\frac{x}{2}\right) \left(30 B \sinh\left(\frac{x}{2}\right)^{6}+\left(21 A-15 B\right) \sinh\left(\frac{x}{2}\right)^{4}+\left(-28 A+20 B\right) \sinh\left(\frac{x}{2}\right)^{2}+56 A-40 B\right)}{105 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$
parts	$-\frac{16 A \sinh\left(\frac{x}{2}\right) a^{3} \cosh\left(\frac{x}{2}\right) \left(3 \sinh\left(\frac{x}{2}\right)^{4}-4 \sinh\left(\frac{x}{2}\right)^{2}+8\right)}{15 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}-\frac{16 B \sinh\left(\frac{x}{2}\right) a^{3} \cosh\left(\frac{x}{2}\right) \left(6 \sinh\left(\frac{x}{2}\right)^{6}-3 \sinh\left(\frac{x}{2}\right)^{4}+4 \sinh\left(\frac{x}{2}\right)^{2}-8\right)}{21 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$

input
```
int((a-cosh(x)*a)^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-16/105*sinh(1/2*x)*a^3*cosh(1/2*x)*(30*B*sinh(1/2*x)^6+(21*A-15*B)*sinh(1
/2*x)^4+(-28*A+20*B)*sinh(1/2*x)^2+56*A-40*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. 2(82) = 164.

Time = 0.10 (sec) , antiderivative size = 564, normalized size of antiderivative = 5.76

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \text{Too large to display}$$

input
```
integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/420*sqrt(1/2)*(15*B*a^2*cosh(x)^7 + 15*B*a^2*sinh(x)^7 + 21*(2*A - 5*B)*
a^2*cosh(x)^6 - 35*(10*A - 11*B)*a^2*cosh(x)^5 + 525*(4*A - 3*B)*a^2*cosh(
x)^4 + 21*(5*B*a^2*cosh(x) + (2*A - 5*B)*a^2)*sinh(x)^6 + 525*(4*A - 3*B)*
a^2*cosh(x)^3 + 7*(45*B*a^2*cosh(x)^2 + 18*(2*A - 5*B)*a^2*cosh(x) - 5*(10
*A - 11*B)*a^2)*sinh(x)^5 - 35*(10*A - 11*B)*a^2*cosh(x)^2 + 35*(15*B*a^2*
cosh(x)^3 + 9*(2*A - 5*B)*a^2*cosh(x)^2 - 5*(10*A - 11*B)*a^2*cosh(x) + 15
*(4*A - 3*B)*a^2)*sinh(x)^4 + 21*(2*A - 5*B)*a^2*cosh(x) + 35*(15*B*a^2*co
sh(x)^4 + 12*(2*A - 5*B)*a^2*cosh(x)^3 - 10*(10*A - 11*B)*a^2*cosh(x)^2 +
60*(4*A - 3*B)*a^2*cosh(x) + 15*(4*A - 3*B)*a^2)*sinh(x)^3 + 15*B*a^2 + 35
*(9*B*a^2*cosh(x)^5 + 9*(2*A - 5*B)*a^2*cosh(x)^4 - 10*(10*A - 11*B)*a^2*c
osh(x)^3 + 90*(4*A - 3*B)*a^2*cosh(x)^2 + 45*(4*A - 3*B)*a^2*cosh(x) - (10
*A - 11*B)*a^2)*sinh(x)^2 + 7*(15*B*a^2*cosh(x)^6 + 18*(2*A - 5*B)*a^2*cos
h(x)^5 - 25*(10*A - 11*B)*a^2*cosh(x)^4 + 300*(4*A - 3*B)*a^2*cosh(x)^3 +
225*(4*A - 3*B)*a^2*cosh(x)^2 - 10*(10*A - 11*B)*a^2*cosh(x) + 3*(2*A - 5*
B)*a^2)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^3 + 3*cosh(x)^2*sin
h(x) + 3*cosh(x)*sinh(x)^2 + sinh(x)^3)
```

Sympy [F(-1)]

Timed out.

$$\int (a - a\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \text{Timed out}$$

input
```
integrate((a-a*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(82) = 164$.

Time = 0.14 (sec) , antiderivative size = 288, normalized size of antiderivative = 2.94

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{1}{60}\left(\frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-2x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{150\sqrt{2}a^{\frac{5}{2}}e^{(-3x)}}{(-e^{(-x)})^{\frac{5}{2}}} + \frac{25\sqrt{2}a^{\frac{5}{2}}e^{(-4x)}}{(-e^{(-x)})^{\frac{5}{2}}} - \frac{3\sqrt{2}a^{\frac{5}{2}}e^{(-}}{(-e^{(-x)})}\right. + \frac{1}{168}B\left(\frac{\left(21\sqrt{2}a^{\frac{5}{2}}e^{(-x)} - 70\sqrt{2}a^{\frac{5}{2}}e^{(-2x)} + 210\sqrt{2}a^{\frac{5}{2}}e^{(-3x)} + 105\sqrt{2}a^{\frac{5}{2}}e^{(-4x)} - 7\sqrt{2}a^{\frac{5}{2}}e^{(-5x)} - 3\sqrt{2}a^{\frac{5}{2}}\right.}{(-e^{(-x)})^{\frac{5}{2}}}\right.$$

input
```
integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/60*(25*sqrt(2)*a^(5/2)*e^(-x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-
2*x)/(-e^(-x))^(5/2) - 150*sqrt(2)*a^(5/2)*e^(-3*x)/(-e^(-x))^(5/2) + 25*s
qrt(2)*a^(5/2)*e^(-4*x)/(-e^(-x))^(5/2) - 3*sqrt(2)*a^(5/2)*e^(-5*x)/(-e^(
-x))^(5/2) - 3*sqrt(2)*a^(5/2)/(-e^(-x))^(5/2))*A + 1/168*B*((21*sqrt(2)*a
^(5/2)*e^(-x) - 70*sqrt(2)*a^(5/2)*e^(-2*x) + 210*sqrt(2)*a^(5/2)*e^(-3*x)
 + 105*sqrt(2)*a^(5/2)*e^(-4*x) - 7*sqrt(2)*a^(5/2)*e^(-5*x) - 3*sqrt(2)*a
^(5/2))*e^x/(-e^(-x))^(5/2) - (7*sqrt(2)*a^(5/2)*e^(-x) - 105*sqrt(2)*a^(5
/2)*e^(-2*x) - 210*sqrt(2)*a^(5/2)*e^(-3*x) + 70*sqrt(2)*a^(5/2)*e^(-4*x)
- 21*sqrt(2)*a^(5/2)*e^(-5*x) + 3*sqrt(2)*a^(5/2)*e^(-6*x))/(-e^(-x))^(5/2
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(82) = 164$.

Time = 0.13 (sec) , antiderivative size = 295, normalized size of antiderivative = 3.01

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{1}{840}\sqrt{2}\left(\frac{(2100\,Aa^6e^{(3x)}\text{sgn}(-e^x+1) - 1575\,Ba^6e^{(3x)}\text{sgn}(-e^x+1) - 350\,Aa^6e^{(2x)}\text{sgn}(\right.$$

input
```
integrate((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
1/840*sqrt(2)*((2100*A*a^6*e^(3*x)*sgn(-e^x + 1) - 1575*B*a^6*e^(3*x)*sgn(
-e^x + 1) - 350*A*a^6*e^(2*x)*sgn(-e^x + 1) + 385*B*a^6*e^(2*x)*sgn(-e^x +
 1) + 42*A*a^6*e^x*sgn(-e^x + 1) - 105*B*a^6*e^x*sgn(-e^x + 1) + 15*B*a^6*
sgn(-e^x + 1))*e^(-3*x)/(sqrt(-a*e^x)*a^3) - (15*sqrt(-a*e^x)*B*a^9*e^(3*x
)*sgn(-e^x + 1) + 42*sqrt(-a*e^x)*A*a^9*e^(2*x)*sgn(-e^x + 1) - 105*sqrt(-
a*e^x)*B*a^9*e^(2*x)*sgn(-e^x + 1) - 350*sqrt(-a*e^x)*A*a^9*e^x*sgn(-e^x +
 1) + 385*sqrt(-a*e^x)*B*a^9*e^x*sgn(-e^x + 1) + 2100*sqrt(-a*e^x)*A*a^9*s
gn(-e^x + 1) - 1575*sqrt(-a*e^x)*B*a^9*sgn(-e^x + 1))/a^7)
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,\left(a-a\cosh(x)\right)^{5/2}\,dx$$

input
```
int((A + B*cosh(x))*(a - a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))*(a - a*cosh(x))^(5/2), x)
```

Reduce [F]

$$\begin{aligned}\int (a-a\cosh(x))^{5/2}(A+B\cosh(x))\,dx &= \sqrt{a}\,a^2\Bigg(\left(\left(\int \sqrt{-\cosh(x)+1}dx\right)a\right.\\ &\quad -2\left(\int \sqrt{-\cosh(x)+1}\,\cosh(x)\,dx\right)a+\left(\int \sqrt{-\cosh(x)+1}\,\cosh(x)\,dx\right)b\\ &\quad +\left(\int \sqrt{-\cosh(x)+1}\,\cosh(x)^3\,dx\right)b+\left(\int \sqrt{-\cosh(x)+1}\,\cosh(x)^2\,dx\right)a\\ &\quad -2\left(\int \sqrt{-\cosh(x)+1}\,\cosh(x)^2\,dx\right)b\Bigg)\end{aligned}$$

input
```
int((a-a*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

output
```
sqrt(a)*a**2*(int(sqrt( - cosh(x) + 1),x)*a - 2*int(sqrt( - cosh(x) + 1)*c
osh(x),x)*a + int(sqrt( - cosh(x) + 1)*cosh(x),x)*b + int(sqrt( - cosh(x)
+ 1)*cosh(x)**3,x)*b + int(sqrt( - cosh(x) + 1)*cosh(x)**2,x)*a - 2*int(sq
rt( - cosh(x) + 1)*cosh(x)**2,x)*b)
```

3.91 $\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 71

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=-\frac{8a^2(5A-3B)\sinh(x)}{15\sqrt{a-a\cosh(x)}}-\frac{2}{15}a(5A-3B)\sqrt{a-a\cosh(x)}\sinh(x)+\frac{2}{5}B(a-a\cosh(x))^{3/2}\sinh(x)$$

output
```
-8/15*a^2*(5*A-3*B)*sinh(x)/(a-a*cosh(x))^(1/2)-2/15*a*(5*A-3*B)*(a-a*cosh
(x))^(1/2)*sinh(x)+2/5*B*(a-a*cosh(x))^(3/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.66

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=-\frac{1}{15}a\sqrt{a-a\cosh(x)}(-50A+39B+2(5A-9B)\cosh(x)+3B\cosh(2x))\coth\left(\frac{x}{2}\right)$$

input
```
Integrate[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
-1/15*(a*Sqrt[a - a*Cosh[x]]*(-50*A + 39*B + 2*(5*A - 9*B)*Cosh[x] + 3*B*C
osh[2*x])*Coth[x/2])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3230, 3042, 3126, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{5}(5A-3B)\int (a-a\cosh(x))^{3/2}dx+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}+\frac{1}{5}(5A-3B)\int \left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}dx$$

$$\Big\downarrow \text{3126}$$

$$\frac{1}{5}(5A-3B)\left(\frac{4}{3}a\int \sqrt{a-a\cosh(x)}dx-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}+\frac{1}{5}(5A-3B)\left(-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}+\frac{4}{3}a\int \sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3125}$$

$$\frac{1}{5}(5A-3B)\left(-\frac{8a^2\sinh(x)}{3\sqrt{a-a\cosh(x)}}-\frac{2}{3}a\sinh(x)\sqrt{a-a\cosh(x)}\right)+\frac{2}{5}B\sinh(x)(a-a\cosh(x))^{3/2}$$

input
```
Int[(a - a*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a - a*Cosh[x])^(3/2)*Sinh[x])/5 + ((5*A - 3*B)*((-8*a^2*Sinh[x])/(3*
Sqrt[a - a*Cosh[x]]) - (2*a*Sqrt[a - a*Cosh[x]]*Sinh[x])/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3126
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[a*((2*n - 1)/n)
 Int[(a + b*Sin[c + d*x])^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[
a^2 - b^2, 0] && IGtQ[n - 1/2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.77

method	result	size
default	$\frac{8\sinh\left(\frac{x}{2}\right)a^{2}\cosh\left(\frac{x}{2}\right)\left(6B\sinh\left(\frac{x}{2}\right)^{4}+(5A-3B)\sinh\left(\frac{x}{2}\right)^{2}-10A+6B\right)}{15\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	55
parts	$\frac{8A\sinh\left(\frac{x}{2}\right)a^{2}\cosh\left(\frac{x}{2}\right)\left(\cosh\left(\frac{x}{2}\right)^{2}-3\right)}{3\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}+\frac{8B\sinh\left(\frac{x}{2}\right)a^{2}\cosh\left(\frac{x}{2}\right)\left(2\cosh\left(\frac{x}{2}\right)^{4}-5\cosh\left(\frac{x}{2}\right)^{2}+5\right)}{5\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	78

input
```
int((a-cosh(x)*a)^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
8/15*sinh(1/2*x)*a^2*cosh(1/2*x)*(6*B*sinh(1/2*x)^4+(5*A-3*B)*sinh(1/2*x)^
2-10*A+6*B)/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. 2(59) = 118.

Time = 0.10 (sec) , antiderivative size = 279, normalized size of antiderivative = 3.93

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx =$$

$$-\frac{\sqrt{\frac{1}{2}}\left(3\,Ba\cosh\left(x\right)^{5}+3\,Ba\sinh\left(x\right)^{5}+5\left(2\,A-3\,B\right)a\cosh\left(x\right)^{4}-30\left(3\,A-2\,B\right)a\cosh\left(x\right)^{3}+5\left(3\,Ba\cos\right.\right.}{}$$

input
```
integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/30*sqrt(1/2)*(3*B*a*cosh(x)^5 + 3*B*a*sinh(x)^5 + 5*(2*A - 3*B)*a*cosh(
x)^4 - 30*(3*A - 2*B)*a*cosh(x)^3 + 5*(3*B*a*cosh(x) + (2*A - 3*B)*a)*sinh
(x)^4 - 30*(3*A - 2*B)*a*cosh(x)^2 + 10*(3*B*a*cosh(x)^2 + 2*(2*A - 3*B)*a
*cosh(x) - 3*(3*A - 2*B)*a)*sinh(x)^3 + 5*(2*A - 3*B)*a*cosh(x) + 30*(B*a*
cosh(x)^3 + (2*A - 3*B)*a*cosh(x)^2 - 3*(3*A - 2*B)*a*cosh(x) - (3*A - 2*B
)*a)*sinh(x)^2 + 3*B*a + 5*(3*B*a*cosh(x)^4 + 4*(2*A - 3*B)*a*cosh(x)^3 -
18*(3*A - 2*B)*a*cosh(x)^2 - 12*(3*A - 2*B)*a*cosh(x) + (2*A - 3*B)*a)*sin
h(x))*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2)
```

Sympy [F]

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\int\left(-a(\cosh{(x)}-1)\right)^{\frac{3}{2}}\left(A+B\cosh{(x)}\right)\,dx$$

input
```
integrate((a-a*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

output
```
Integral((-a*(cosh(x) - 1))**(3/2)*(A + B*cosh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(59) = 118$.

Time = 0.14 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.80

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\frac{1}{6}\left(\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}+\frac{9\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}-\frac{\sqrt{2}a^{\frac{3}{2}}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}\right)A+\frac{1}{20}B\left(\frac{\left(5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}-15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}-5\sqrt{2}a^{\frac{3}{2}}e^{(-3x)}-\sqrt{2}a^{\frac{3}{2}}\right)e^{x}}{\left(-e^{(-x)}\right)^{\frac{3}{2}}}-\frac{5\sqrt{2}a^{\frac{3}{2}}e^{(-x)}+15\sqrt{2}a^{\frac{3}{2}}e^{(-2x)}-5}{\left(-e^{(-x)}\right)}\right.$$

input
```
integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
1/6*(9*sqrt(2)*a^(3/2)*e^(-x)/(-e^(-x))^(3/2) + 9*sqrt(2)*a^(3/2)*e^(-2*x)
/(-e^(-x))^(3/2) - sqrt(2)*a^(3/2)*e^(-3*x)/(-e^(-x))^(3/2) - sqrt(2)*a^(3
/2)/(-e^(-x))^(3/2))*A + 1/20*B*((5*sqrt(2)*a^(3/2)*e^(-x) - 15*sqrt(2)*a^
(3/2)*e^(-2*x) - 5*sqrt(2)*a^(3/2)*e^(-3*x) - sqrt(2)*a^(3/2))*e^x/(-e^(-x
))^(3/2) - (5*sqrt(2)*a^(3/2)*e^(-x) + 15*sqrt(2)*a^(3/2)*e^(-2*x) - 5*sqr
t(2)*a^(3/2)*e^(-3*x) + sqrt(2)*a^(3/2)*e^(-4*x))/(-e^(-x))^(3/2))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(59) = 118$.

Time = 0.12 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.99

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{1}{60}\sqrt{2}\left(\frac{(90\,Aa^4e^{(2\,x)}\mathrm{sgn}(-e^x+1) - 60\,Ba^4e^{(2\,x)}\mathrm{sgn}(-e^x+1) - 10\,Aa^4e^x\mathrm{sgn}(-e^x+1)}{\sqrt{-ae^x}a^2}\right.$$

input
```
integrate((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
1/60*sqrt(2)*((90*A*a^4*e^(2*x)*sgn(-e^x + 1) - 60*B*a^4*e^(2*x)*sgn(-e^x
+ 1) - 10*A*a^4*e^x*sgn(-e^x + 1) + 15*B*a^4*e^x*sgn(-e^x + 1) - 3*B*a^4*s
gn(-e^x + 1))*e^(-2*x)/(sqrt(-a*e^x)*a^2) + (3*sqrt(-a*e^x)*B*a^6*e^(2*x)*
sgn(-e^x + 1) + 10*sqrt(-a*e^x)*A*a^6*e^x*sgn(-e^x + 1) - 15*sqrt(-a*e^x)*
B*a^6*e^x*sgn(-e^x + 1) - 90*sqrt(-a*e^x)*A*a^6*sgn(-e^x + 1) + 60*sqrt(-a
*e^x)*B*a^6*sgn(-e^x + 1))/a^5)
```

Mupad [F(-1)]

Timed out.

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,\left(a-a\cosh(x)\right)^{3/2}\,dx$$

input
```
int((A + B*cosh(x))*(a - a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))*(a - a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int (a-a\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\sqrt{a}\,a\left(\left(\left(\int\sqrt{-\cosh(x)+1}dx\right)a-\left(\int\sqrt{-\cosh(x)+1}\cosh(x)\,dx\right)a+\left(\int\sqrt{-\cosh(x)+1}\cosh(x)\,dx\right)b-\left(\int\sqrt{-\cosh(x)+1}\cosh(x)^2\,dx\right)b\right)\right.$$

input
```
int((a-a*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

output
```
sqrt(a)*a*(int(sqrt( - cosh(x) + 1),x)*a - int(sqrt( - cosh(x) + 1)*cosh(x
),x)*a + int(sqrt( - cosh(x) + 1)*cosh(x),x)*b - int(sqrt( - cosh(x) + 1)*
cosh(x)**2,x)*b)
```

3.92 $\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 18, antiderivative size = 44

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx=-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}+\frac{2}{3}B\sqrt{a-a\cosh(x)}\sinh(x)$$

output
```
-2/3*a*(3*A-B)*sinh(x)/(a-a*cosh(x))^(1/2)+2/3*B*(a-a*cosh(x))^(1/2)*sinh(
x)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.73

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx=\frac{2}{3}\sqrt{a-a\cosh(x)}(3A-2B+B\cosh(x))\coth\left(\frac{x}{2}\right)$$

input
```
Integrate[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(2*Sqrt[a - a*Cosh[x]]*(3*A - 2*B + B*Cosh[x])*Coth[x/2])/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {3042, 3230, 3042, 3125}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-a\sin\left(\frac{\pi}{2}+ix\right)}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3230}$$

$$\frac{1}{3}(3A-B)\int \sqrt{a-a\cosh(x)}dx+\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}+\frac{1}{3}(3A-B)\int \sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3125}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a-a\cosh(x)}-\frac{2a(3A-B)\sinh(x)}{3\sqrt{a-a\cosh(x)}}$$

input
```
Int[Sqrt[a - a*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(-2*a*(3*A - B)*Sinh[x])/(3*Sqrt[a - a*Cosh[x]]) + (2*B*Sqrt[a - a*Cosh[x]
]*Sinh[x])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3125
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2*b*(Cos
[c + d*x]/(d*Sqrt[a + b*Sin[c + d*x]])), x] /; FreeQ[{a, b, c, d}, x] && Eq
Q[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

method	result	size
default	$-\frac{4 \sinh\left(\frac{x}{2}\right) a \cosh\left(\frac{x}{2}\right) \left(2 B \cosh\left(\frac{x}{2}\right)^{2}+3 A -3 B \right)}{3 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$	39
parts	$-\frac{4 A \sinh\left(\frac{x}{2}\right) a \cosh\left(\frac{x}{2}\right)}{\sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}-\frac{4 B \sinh\left(\frac{x}{2}\right) a \cosh\left(\frac{x}{2}\right) \left(2 \cosh\left(\frac{x}{2}\right)^{2}-3\right)}{3 \sqrt{-2 \sinh\left(\frac{x}{2}\right)^{2} a}}$	58

input
```
int((a-cosh(x)*a)^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-4/3*sinh(1/2*x)*a*cosh(1/2*x)*(2*B*cosh(1/2*x)^2+3*A-3*B)/(-2*sinh(1/2*x)
^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(36) = 72$.

Time = 0.09 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int \sqrt{a - a\cosh(x)}(A + B\cosh(x))\,dx$$
$$= \frac{\sqrt{\frac{1}{2}}\left(B\cosh\left(x\right)^{3} + B\sinh\left(x\right)^{3} + 3\left(2A - B\right)\cosh\left(x\right)^{2} + 3\left(B\cosh\left(x\right) + 2A - B\right)\sinh\left(x\right)^{2} + 3\left(2A - B\right.\right.}{3\left(\cosh\left(x\right) + \sin\right.}$$

input
```
integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
1/3*sqrt(1/2)*(B*cosh(x)^3 + B*sinh(x)^3 + 3*(2*A - B)*cosh(x)^2 + 3*(B*co
sh(x) + 2*A - B)*sinh(x)^2 + 3*(2*A - B)*cosh(x) + 3*(B*cosh(x)^2 + 2*(2*A
 - B)*cosh(x) + 2*A - B)*sinh(x) + B)*sqrt(-a/(cosh(x) + sinh(x)))/(cosh(x
) + sinh(x))
```

Sympy [F]

$$\int \sqrt{a - a\cosh(x)}(A + B\cosh(x))\,dx = \int \sqrt{-a\left(\cosh\left(x\right) - 1\right)}\left(A + B\cosh\left(x\right)\right)\,dx$$

input
```
integrate((a-a*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

output
```
Integral(sqrt(-a*(cosh(x) - 1))*(A + B*cosh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 109, normalized size of antiderivative = 2.48

$$
\begin{aligned}
&\int \sqrt{a - a\cosh(x)}(A + B\cosh(x))\,dx \\
&= -\left(\frac{\sqrt{2}\sqrt{a}e^{(-x)}}{\sqrt{-e^{(-x)}}} + \frac{\sqrt{2}\sqrt{a}}{\sqrt{-e^{(-x)}}}\right)A \\
&\quad + \frac{1}{6}\left(\frac{\left(3\sqrt{2}\sqrt{a}e^{(-x)} - \sqrt{2}\sqrt{a}\right)e^{x}}{\sqrt{-e^{(-x)}}} + \frac{3\sqrt{2}\sqrt{a}e^{(-x)} - \sqrt{2}\sqrt{a}e^{(-2x)}}{\sqrt{-e^{(-x)}}}\right)B
\end{aligned}
$$

input
```
integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
-(sqrt(2)*sqrt(a)*e^(-x)/sqrt(-e^(-x)) + sqrt(2)*sqrt(a)/sqrt(-e^(-x)))*A
+ 1/6*((3*sqrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a))*e^x/sqrt(-e^(-x)) + (3
*sqrt(2)*sqrt(a)*e^(-x) - sqrt(2)*sqrt(a)*e^(-2*x))/sqrt(-e^(-x)))*B
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. $2(36) = 72$.

Time = 0.11 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.98

$$
\begin{aligned}
&\int \sqrt{a - a\cosh(x)}(A + B\cosh(x))\,dx \\
&= \frac{1}{6}\sqrt{2}\left(\frac{\left(6\,Aa^{2}e^{x}\mathrm{sgn}(-e^{x}+1) - 3\,Ba^{2}e^{x}\mathrm{sgn}(-e^{x}+1) + Ba^{2}\mathrm{sgn}(-e^{x}+1)\right)e^{(-x)}}{\sqrt{-ae^{x}}a} - \frac{\sqrt{-ae^{x}}Ba^{3}e^{x}\mathrm{sgn}(-e\right.
\end{aligned}
$$

input
```
integrate((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
1/6*sqrt(2)*((6*A*a^2*e^x*sgn(-e^x + 1) - 3*B*a^2*e^x*sgn(-e^x + 1) + B*a^
2*sgn(-e^x + 1))*e^(-x)/(sqrt(-a*e^x)*a) - (sqrt(-a*e^x)*B*a^3*e^x*sgn(-e^
x + 1) + 6*sqrt(-a*e^x)*A*a^3*sgn(-e^x + 1) - 3*sqrt(-a*e^x)*B*a^3*sgn(-e^
x + 1))/a^3)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx=\int (A+B\cosh(x))\ \sqrt{a-a\cosh\left(x\right)}\,dx$$

input
```
int((A + B*cosh(x))*(a - a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))*(a - a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a-a\cosh(x)}(A+B\cosh(x))\,dx=\sqrt{a}\left(\left(\int \sqrt{-\cosh\left(x\right)+1}dx\right)a+\left(\int \sqrt{-\cosh\left(x\right)+1}\ \cosh\left(x\right)dx\right)b\right)$$

input
```
int((a-a*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

output
```
sqrt(a)*(int(sqrt( - cosh(x) + 1),x)*a + int(sqrt( - cosh(x) + 1)*cosh(x),
x)*b)
```

3.93 $\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = Bx+\frac{(A-B)\sinh(x)}{1+\cosh(x)}$$

output
```
B*x+(A-B)*sinh(x)/(1+cosh(x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = \sinh(x)\left(\frac{A-B}{1+\cosh(x)}-\frac{B\arcsin(\cosh(x))}{\sqrt{-\sinh^2(x)}}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

output
```
Sinh[x]*((A - B)/(1 + Cosh[x]) - (B*ArcSin[Cosh[x]])/Sqrt[-Sinh[x]^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\cosh(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{1+\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$(A-B)\int \frac{1}{\cosh(x)+1}dx+Bx$$

$$\Big\downarrow \text{3042}$$

$$Bx+(A-B)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A-B)\sinh(x)}{\cosh(x)+1}+Bx$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x]),x]
```

output
```
B*x + ((A - B)*Sinh[x])/(1 + Cosh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

method	result	size
parallelrisch	$Bx+\tanh\left(\frac{x}{2}\right)(A-B)$	15
risch	$Bx-\frac{2A}{e^{x}+1}+\frac{2B}{e^{x}+1}$	23
default	$A\tanh\left(\frac{x}{2}\right)-B\tanh\left(\frac{x}{2}\right)-B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)$	34

input
```
int((A+B*cosh(x))/(1+cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x+tanh(1/2*x)*(A-B)
```

Fricas [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.61

$$\int \frac{A + B\cosh(x)}{1 + \cosh(x)}\,dx = \frac{Bx\cosh(x) + Bx\sinh(x) + Bx - 2A + 2B}{\cosh(x) + \sinh(x) + 1}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
(B*x*cosh(x) + B*x*sinh(x) + B*x - 2*A + 2*B)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{A + B\cosh(x)}{1 + \cosh(x)}\,dx = A\tanh\left(\frac{x}{2}\right) + Bx - B\tanh\left(\frac{x}{2}\right)$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x)
```

output
```
A*tanh(x/2) + B*x - B*tanh(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{A + B\cosh(x)}{1 + \cosh(x)}\,dx = B\left(x - \frac{2}{e^{(-x)} + 1}\right) + \frac{2A}{e^{(-x)} + 1}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
B*(x - 2/(e^(-x) + 1)) + 2*A/(e^(-x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = Bx - \frac{2\,(A-B)}{e^x+1}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
B*x - 2*(A - B)/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = B\,x - \frac{2\,A-2\,B}{\mathrm{e}^x+1}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1),x)
```

output
```
B*x - (2*A - 2*B)/(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \frac{A+B\cosh(x)}{1+\cosh(x)}\,dx = \frac{2e^x a + e^x bx - 2e^x b + bx}{e^x+1}$$

input
```
int((A+B*cosh(x))/(1+cosh(x)),x)
```

output
```
(2*e**x*a + e**x*b*x - 2*e**x*b + b*x)/(e**x + 1)
```

3.94 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 35

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = \frac{(A-B)\sinh(x)}{3(1+\cosh(x))^2}+\frac{(A+2B)\sinh(x)}{3(1+\cosh(x))}$$

output
```
1/3*(A-B)*sinh(x)/(1+cosh(x))^2+(A+2*B)*sinh(x)/(3+3*cosh(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.71

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = \frac{(2A+B+(A+2B)\cosh(x))\sinh(x)}{3(1+\cosh(x))^2}$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

output
```
((2*A + B + (A + 2*B)*Cosh[x])*Sinh[x])/(3*(1 + Cosh[x])^2)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{1}{3}(A+2B)\int \frac{1}{\cosh(x)+1}dx+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}(A+2B)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx$$

$$\Big\downarrow \text{ 3127}$$

$$\frac{(A+2B)\sinh(x)}{3(\cosh(x)+1)}+\frac{(A-B)\sinh(x)}{3(\cosh(x)+1)^2}$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x])^2,x]
```

output
```
((A - B)*Sinh[x])/(3*(1 + Cosh[x])^2) + ((A + 2*B)*Sinh[x])/(3*(1 + Cosh[x
]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.74

method	result	size
parallelrisch	$\frac{\operatorname{sech}\left(\frac{x}{2}\right)^{2}\left(2A+B+\cosh\left(x\right)\left(A+2B\right)\right)\tanh\left(\frac{x}{2}\right)}{6}$	26
risch	$-\frac{2\left(3B\,{\mathrm e}^{2x}+3A\,{\mathrm e}^{x}+3B\,{\mathrm e}^{x}+A+2B\right)}{3\left({\mathrm e}^{x}+1\right)^{3}}$	31
default	$-\frac{A\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{B\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$	34

input
```
int((A+B*cosh(x))/(1+cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/6*sech(1/2*x)^2*(2*A+B+cosh(x)*(A+2*B))*tanh(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.43

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\left((A+5B)\cosh(x)-(A-B)\sinh(x)+3A+3B\right)}{3\left(\cosh(x)^2+2(\cosh(x)+1)\sinh(x)+\sinh(x)^2+4\cosh(x)+3\right)}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
-2/3*((A + 5*B)*cosh(x) - (A - B)*sinh(x) + 3*A + 3*B)/(cosh(x)^2 + 2*(cos
h(x) + 1)*sinh(x) + sinh(x)^2 + 4*cosh(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.03

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{A\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{2}+\frac{B\tanh^3\left(\frac{x}{2}\right)}{6}+\frac{B\tanh\left(\frac{x}{2}\right)}{2}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))**2,x)
```

output
```
-A*tanh(x/2)**3/6 + A*tanh(x/2)/2 + B*tanh(x/2)**3/6 + B*tanh(x/2)/2
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(31) = 62.

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 3.69

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = \frac{2}{3}B\left(\frac{3e^{(-x)}}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1}+\frac{3e^{(-2x)}}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1}+\frac{2}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1}\right)+\frac{2}{3}A\left(\frac{3e^{(-x)}}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1}+\frac{1}{3e^{(-x)}+3e^{(-2x)}+e^{(-3x)}+1}\right)$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
2/3*B*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 3*e^(-2*x)/(3*e^(
-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1
)) + 2/3*A*(3*e^(-x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 1/(3*e^(-x)
+ 3*e^(-2*x) + e^(-3*x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)}+3\,Ae^{x}+3\,Be^{x}+A+2\,B\right)}{3\,(e^{x}+1)^{3}}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*B*e^(2*x) + 3*A*e^x + 3*B*e^x + A + 2*B)/(e^x + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.86

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\,(A+2\,B+3\,A\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{x}+3\,B\,\mathrm{e}^{2\,x})}{3\,(\mathrm{e}^{x}+1)^{3}}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1)^2,x)
```

output
```
-(2*(A + 2*B + 3*A*exp(x) + 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.29

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^2}\,dx = \frac{2e^{3x}b-6e^{x}a-2a-2b}{3e^{3x}+9e^{2x}+9e^{x}+3}$$

input
```
int((A+B*cosh(x))/(1+cosh(x))^2,x)
```

output
```
(2*(e**(3*x)*b - 3*e**x*a - a - b))/(3*(e**(3*x) + 3*e**(2*x) + 3*e**x + 1
))
```

3.95 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 56

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx=\frac{(A-B)\sinh(x)}{5(1+\cosh(x))^3}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))^2}+\frac{(2A+3B)\sinh(x)}{15(1+\cosh(x))}$$

output
```
1/5*(A-B)*sinh(x)/(1+cosh(x))^3+1/15*(2*A+3*B)*sinh(x)/(1+cosh(x))^2+(2*A+
3*B)*sinh(x)/(15+15*cosh(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx=\frac{(7A+3B+(6A+9B)\cosh(x)+(2A+3B)\cosh^2(x))\sinh(x)}{15(1+\cosh(x))^3}$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

output
```
((7*A + 3*B + (6*A + 9*B)*Cosh[x] + (2*A + 3*B)*Cosh[x]^2)*Sinh[x])/(15*(1
 + Cosh[x])^3)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(\cosh(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{5}(2A+3B)\int \frac{1}{(\cosh(x)+1)^2}dx+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}+\frac{1}{5}(2A+3B)\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{5}(2A+3B)\left(\frac{1}{3}\int \frac{1}{\cosh(x)+1}dx+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)+\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}+\frac{1}{5}(2A+3B)\left(\frac{\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{(A-B)\sinh(x)}{5(\cosh(x)+1)^3}+\frac{1}{5}(2A+3B)\left(\frac{\sinh(x)}{3(\cosh(x)+1)}+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x])^3,x]
```

output
```
((A - B)*Sinh[x])/(5*(1 + Cosh[x])^3) + ((2*A + 3*B)*(Sinh[x]/(3*(1 + Cosh
[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.62

method	result	size
parallelrisch	$\frac{\left((A-B)\tanh\left(\frac{x}{2}\right)^{4}-\frac{10A\tanh\left(\frac{x}{2}\right)^{2}}{3}+5A+5B\right)\tanh\left(\frac{x}{2}\right)}{20}$	35
default	$\frac{(A-B)\tanh\left(\frac{x}{2}\right)^{5}}{20}-\frac{A\tanh\left(\frac{x}{2}\right)^{3}}{6}+\frac{A\tanh\left(\frac{x}{2}\right)}{4}+\frac{B\tanh\left(\frac{x}{2}\right)}{4}$	38
risch	$-\frac{2(15B\,{\mathrm e}^{3x}+20A\,{\mathrm e}^{2x}+15B\,{\mathrm e}^{2x}+10A\,{\mathrm e}^{x}+15B\,{\mathrm e}^{x}+2A+3B)}{15({\mathrm e}^{x}+1)^{5}}$	47

input
```
int((A+B*cosh(x))/(1+cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/20*((A-B)*tanh(1/2*x)^4-10/3*A*tanh(1/2*x)^2+5*A+5*B)*tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(50) = 100$.

Time = 0.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.27

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx =$$
$$-\frac{2\left(15\,B\cosh\left(x\right)^2+15\,B\sinh\left(x\right)^2+2\,(11\,A+9\,B)\cosh(\right.}{15\left(\cosh\left(x\right)^4+(4\,\cosh\left(x\right)+5)\sinh\left(x\right)^3+\sinh\left(x\right)^4+5\,\cosh\left(x\right)^3+\left(6\,\cosh\left(x\right)^2+15\,\cosh\left(x\right)+10\right.\right.}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A + 9*B)*cosh(x) + 6*(5*B*c
osh(x) + 3*A + 2*B)*sinh(x) + 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) + 5)*si
nh(x)^3 + sinh(x)^4 + 5*cosh(x)^3 + (6*cosh(x)^2 + 15*cosh(x) + 10)*sinh(x
)^2 + 10*cosh(x)^2 + (4*cosh(x)^3 + 15*cosh(x)^2 + 20*cosh(x) + 9)*sinh(x)
 + 11*cosh(x) + 5)
```

Sympy [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = \frac{A\tanh^5\left(\frac{x}{2}\right)}{20} - \frac{A\tanh^3\left(\frac{x}{2}\right)}{6} + \frac{A\tanh\left(\frac{x}{2}\right)}{4} - \frac{B\tanh^5\left(\frac{x}{2}\right)}{20} + \frac{B\tanh\left(\frac{x}{2}\right)}{4}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))**3,x)
```

output
```
A*tanh(x/2)**5/20 - A*tanh(x/2)**3/6 + A*tanh(x/2)/4 - B*tanh(x/2)**5/20 +
 B*tanh(x/2)/4
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 263 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 263, normalized size of antiderivative = 4.70

$$\int \frac{A + B\cosh(x)}{(1 + \cosh(x))^3}\,dx$$
$$= \frac{4}{15}\,A\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)} + 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} + 5\,e^{(-4\,x)} + e^{(-5\,x)} + 1} + \frac{10\,e^{(-2\,x)}}{5\,e^{(-x)} + 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} + 5\,e^{(-4\,x)}}\right.$$
$$+ \frac{2}{5}\,B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)} + 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} + 5\,e^{(-4\,x)} + e^{(-5\,x)} + 1} + \frac{5\,e^{(-2\,x)}}{5\,e^{(-x)} + 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} + 5\,e^{(-4\,x}}\right.$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
4/15*A*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) + e^(-
5*x) + 1) + 10*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x)
 + e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5*e^(-4*x) +
e^(-5*x) + 1)) + 2/5*B*(5*e^(-x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*x) + 5
*e^(-4*x) + e^(-5*x) + 1) + 5*e^(-2*x)/(5*e^(-x) + 10*e^(-2*x) + 10*e^(-3*
x) + 5*e^(-4*x) + e^(-5*x) + 1) + 5*e^(-3*x)/(5*e^(-x) + 10*e^(-2*x) + 10*
e^(-3*x) + 5*e^(-4*x) + e^(-5*x) + 1) + 1/(5*e^(-x) + 10*e^(-2*x) + 10*e^(
-3*x) + 5*e^(-4*x) + e^(-5*x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{A + B\cosh(x)}{(1 + \cosh(x))^3}\,dx$$
$$= -\frac{2\,\left(15\,Be^{(3\,x)} + 20\,Ae^{(2\,x)} + 15\,Be^{(2\,x)} + 10\,Ae^{x} + 15\,Be^{x} + 2\,A + 3\,B\right)}{15\,\left(e^{x} + 1\right)^{5}}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-2/15*(15*B*e^(3*x) + 20*A*e^(2*x) + 15*B*e^(2*x) + 10*A*e^x + 15*B*e^x +
2*A + 3*B)/(e^x + 1)^5
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.52

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = -\frac{\frac{4\,B\,e^x}{5}+\frac{8\,A\,e^{2\,x}}{5}+\frac{4\,B\,e^{3\,x}}{5}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^x+1} - \frac{\frac{B}{5}+\frac{4\,A\,e^x}{5}+\frac{3\,B\,e^{2\,x}}{5}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^x+1} - \frac{\frac{4\,A}{15}+\frac{2\,B\,e^x}{5}}{3\,e^{2\,x}+e^{3\,x}+3\,e^x+1} - \frac{B}{5\,\left(e^{2\,x}+2\,e^x+1\right)}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1)^3,x)
```

output
```
- ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(10*exp(2*x) + 10
*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1) - (B/5 + (4*A*exp(x))/5
+ (3*B*exp(2*x))/5)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1) -
((4*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - B/(5*
(exp(2*x) + 2*exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.46

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^3}\,dx = \frac{-30e^{3x}b-40e^{2x}a-30e^{2x}b-20e^{x}a-30e^{x}b-4a-6b}{15e^{5x}+75e^{4x}+150e^{3x}+150e^{2x}+75e^{x}+15}$$

input
```
int((A+B*cosh(x))/(1+cosh(x))^3,x)
```

output
```
(2*( - 15*e**(3*x)*b - 20*e**(2*x)*a - 15*e**(2*x)*b - 10*e**x*a - 15*e**x
*b - 2*a - 3*b))/(15*(e**(5*x) + 5*e**(4*x) + 10*e**(3*x) + 10*e**(2*x) +
5*e**x + 1))
```

3.96 $\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 75

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx=\frac{(A-B)\sinh(x)}{7(1+\cosh(x))^4}+\frac{(3A+4B)\sinh(x)}{35(1+\cosh(x))^3}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))^2}+\frac{2(3A+4B)\sinh(x)}{105(1+\cosh(x))}$$

output
```
1/7*(A-B)*sinh(x)/(1+cosh(x))^4+1/35*(3*A+4*B)*sinh(x)/(1+cosh(x))^3+2/105
*(3*A+4*B)*sinh(x)/(1+cosh(x))^2+2*(3*A+4*B)*sinh(x)/(105+105*cosh(x))
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx=\frac{(36A+13B+13(3A+4B)\cosh(x)+8(3A+4B)\cosh^2(x)+(6A+8B)\cosh^3(x))\sinh(x)}{105(1+\cosh(x))^4}$$

input
```
Integrate[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

output
```
((36*A + 13*B + 13*(3*A + 4*B)*Cosh[x] + 8*(3*A + 4*B)*Cosh[x]^2 + (6*A +
8*B)*Cosh[x]^3)*Sinh[x])/(105*(1 + Cosh[x])^4)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.615$, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(\cosh(x)+1)^4}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \ 3229$$

$$\frac{1}{7}(3A+4B)\int \frac{1}{(\cosh(x)+1)^3}dx+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \ 3129$$

$$\frac{1}{7}(3A+4B)\left(\frac{2}{5}\int \frac{1}{(\cosh(x)+1)^2}dx+\frac{\sinh(x)}{5(\cosh(x)+1)^3}\right)+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

$$\Big\downarrow \ 3042$$

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\left(\frac{\sinh(x)}{5(\cosh(x)+1)^3}+\frac{2}{5}\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)+1\right)^2}dx\right)$$

$$\Big\downarrow \ 3129$$

$$\frac{1}{7}(3A+4B)\left(\frac{2}{5}\left(\frac{1}{3}\int\frac{1}{\cosh(x)+1}dx+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)+\frac{\sinh(x)}{5(\cosh(x)+1)^3}\right)+\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}$$

$\big\downarrow$ 3042

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\left(\frac{\sinh(x)}{5(\cosh(x)+1)^3}+\frac{2}{5}\left(\frac{\sinh(x)}{3(\cosh(x)+1)^2}+\frac{1}{3}\int\frac{1}{\sin\left(ix+\frac{\pi}{2}\right)+1}dx\right)\right)$$

$\big\downarrow$ 3127

$$\frac{(A-B)\sinh(x)}{7(\cosh(x)+1)^4}+\frac{1}{7}(3A+4B)\left(\frac{\sinh(x)}{5(\cosh(x)+1)^3}+\frac{2}{5}\left(\frac{\sinh(x)}{3(\cosh(x)+1)}+\frac{\sinh(x)}{3(\cosh(x)+1)^2}\right)\right)$$

input
```
Int[(A + B*Cosh[x])/(1 + Cosh[x])^4,x]
```

output
```
((A - B)*Sinh[x])/(7*(1 + Cosh[x])^4) + ((3*A + 4*B)*(Sinh[x]/(5*(1 + Cosh
[x])^3) + (2*(Sinh[x]/(3*(1 + Cosh[x])^2) + Sinh[x]/(3*(1 + Cosh[x]))))/5)
)/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$-\frac{\tanh\left(\frac{x}{2}\right)\left((A-B)\tanh\left(\frac{x}{2}\right)^{6}+\frac{7(-3A+B)\tanh\left(\frac{x}{2}\right)^{4}}{5}+7\left(A+\frac{B}{3}\right)\tanh\left(\frac{x}{2}\right)^{2}-7A-7B\right)}{56}$	52
default	$-\frac{(A-B)\tanh\left(\frac{x}{2}\right)^{7}}{56}-\frac{(-3A+B)\tanh\left(\frac{x}{2}\right)^{5}}{40}-\frac{(3A+B)\tanh\left(\frac{x}{2}\right)^{3}}{24}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh\left(\frac{x}{2}\right)}{8}$	55
risch	$-\frac{4\left(70B\,e^{4x}+105A\,e^{3x}+70B\,e^{3x}+63A\,e^{2x}+84B\,e^{2x}+21A\,e^{x}+28B\,e^{x}+3A+4B\right)}{105\left(e^{x}+1\right)^{7}}$	61

input
```
int((A+B*cosh(x))/(1+cosh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-1/56*tanh(1/2*x)*((A-B)*tanh(1/2*x)^6+7/5*(-3*A+B)*tanh(1/2*x)^4+7*(A+1/3
*B)*tanh(1/2*x)^2-7*A-7*B)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(67) = 134.

Time = 0.06 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.33

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx =$$

$$-\frac{4\left((3\,A+74\,B)\cosh(\right.}{105\left(\cosh(x)^{5}+(5\,\cosh(x)+7)\sinh(x)^{4}+\sinh(x)^{5}+7\,\cosh(x)^{4}+\left(10\,\cosh(x)^{2}+28\,\cosh(x)+\right.\right.}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="fricas")
```

output
```
-4/105*((3*A + 74*B)*cosh(x)^2 + (3*A + 74*B)*sinh(x)^2 + 14*(9*A + 7*B)*c
osh(x) - 6*((A - 22*B)*cosh(x) - 14*A - 7*B)*sinh(x) + 63*A + 84*B)/(cosh(
x)^5 + (5*cosh(x) + 7)*sinh(x)^4 + sinh(x)^5 + 7*cosh(x)^4 + (10*cosh(x)^2
 + 28*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 + 42*cosh(x)^
2 + 63*cosh(x) + 36)*sinh(x)^2 + 36*cosh(x)^2 + (5*cosh(x)^4 + 28*cosh(x)^
3 + 63*cosh(x)^2 + 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) + 21)
```

Sympy [A] (verification not implemented)

Time = 0.87 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.04

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = -\frac{A\tanh^7\left(\frac{x}{2}\right)}{56}+\frac{3A\tanh^5\left(\frac{x}{2}\right)}{40}-\frac{A\tanh^3\left(\frac{x}{2}\right)}{8}+\frac{A\tanh\left(\frac{x}{2}\right)}{8}+\frac{B\tanh^7\left(\frac{x}{2}\right)}{56}-\frac{B\tanh^5\left(\frac{x}{2}\right)}{40}-\frac{B\tanh^3\left(\frac{x}{2}\right)}{24}+\frac{B\tanh\left(\frac{x}{2}\right)}{8}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))**4,x)
```

output
```
-A*tanh(x/2)**7/56 + 3*A*tanh(x/2)**5/40 - A*tanh(x/2)**3/8 + A*tanh(x/2)/
8 + B*tanh(x/2)**7/56 - B*tanh(x/2)**5/40 - B*tanh(x/2)**3/24 + B*tanh(x/2
)/8
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 449 vs. 2(67) = 134.

Time = 0.04 (sec) , antiderivative size = 449, normalized size of antiderivative = 5.99

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="maxima")
```

output
```
8/105*B*(14*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 2
1*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 42*e^(-2*x)/(7*e^(-x) + 21*e^(-2
*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1)
 + 35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^
(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 35*e^(-4*x)/(7*e^(-x) + 21*e^(-2*x)
+ 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 2
/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(
-6*x) + e^(-7*x) + 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(
-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 21*e^(-2*
x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e
^(-6*x) + e^(-7*x) + 1) + 35*e^(-3*x)/(7*e^(-x) + 21*e^(-2*x) + 35*e^(-3*x
) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-7*x) + 1) + 1/(7*e^(-x) +
 21*e^(-2*x) + 35*e^(-3*x) + 35*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) + e^(-
7*x) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.80

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}+70\,Be^{(3\,x)}+63\,Ae^{(2\,x)}+84\,Be^{(2\,x)}+21\,Ae^{x}+28\,Be^{x}+3\,A+4\,B\right)}{105\left(e^{x}+1\right)^{7}}$$

input
```
integrate((A+B*cosh(x))/(1+cosh(x))^4,x, algorithm="giac")
```

output
```
-4/105*(70*B*e^(4*x) + 105*A*e^(3*x) + 70*B*e^(3*x) + 63*A*e^(2*x) + 84*B*
e^(2*x) + 21*A*e^x + 28*B*e^x + 3*A + 4*B)/(e^x + 1)^7
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.08

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = -\frac{\frac{4A}{35}+\frac{8B\,e^{x}}{35}}{6\,e^{2x}+4\,e^{3x}+e^{4x}+4\,e^{x}+1} - \frac{8B}{105\,\left(3\,e^{2x}+e^{3x}+3\,e^{x}+1\right)} - \frac{\frac{16A\,e^{3x}}{7}+\frac{8B\,e^{2x}}{7}+\frac{8B\,e^{4x}}{7}}{21\,e^{2x}+35\,e^{3x}+35\,e^{4x}+21\,e^{5x}+7\,e^{6x}+e^{7x}+7\,e^{x}+1} - \frac{\frac{8B\,e^{x}}{21}+\frac{8A\,e^{2x}}{7}+\frac{16B\,e^{3x}}{21}}{15\,e^{2x}+20\,e^{3x}+15\,e^{4x}+6\,e^{5x}+e^{6x}+6\,e^{x}+1} - \frac{\frac{8B}{105}+\frac{16A\,e^{x}}{35}+\frac{16B\,e^{2x}}{35}}{10\,e^{2x}+10\,e^{3x}+5\,e^{4x}+e^{5x}+5\,e^{x}+1}$$

input
```
int((A + B*cosh(x))/(cosh(x) + 1)^4,x)
```

output
```
- ((4*A)/35 + (8*B*exp(x))/35)/(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp
(x) + 1) - (8*B)/(105*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1)) - ((16*A*exp
(3*x))/7 + (8*B*exp(2*x))/7 + (8*B*exp(4*x))/7)/(21*exp(2*x) + 35*exp(3*x)
 + 35*exp(4*x) + 21*exp(5*x) + 7*exp(6*x) + exp(7*x) + 7*exp(x) + 1) - ((8
*B*exp(x))/21 + (8*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) + 20*e
xp(3*x) + 15*exp(4*x) + 6*exp(5*x) + exp(6*x) + 6*exp(x) + 1) - ((8*B)/105
 + (16*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) + 10*exp(3*x) + 5*e
xp(4*x) + exp(5*x) + 5*exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.49

$$\int \frac{A+B\cosh(x)}{(1+\cosh(x))^4}\,dx = \frac{-280e^{4x}b-420e^{3x}a-280e^{3x}b-252e^{2x}a-336e^{2x}b-84e^{x}a-112e^{x}b-12a-16b}{105e^{7x}+735e^{6x}+2205e^{5x}+3675e^{4x}+3675e^{3x}+2205e^{2x}+735e^{x}+105}$$

input
```
int((A+B*cosh(x))/(1+cosh(x))^4,x)
```

output

```
(4*( - 70*e**(4*x)*b - 105*e**(3*x)*a - 70*e**(3*x)*b - 63*e**(2*x)*a - 84
*e**(2*x)*b - 21*e**x*a - 28*e**x*b - 3*a - 4*b))/(105*(e**(7*x) + 7*e**(6
*x) + 21*e**(5*x) + 35*e**(4*x) + 35*e**(3*x) + 21*e**(2*x) + 7*e**x + 1))
```

3.97 $\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 20

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx=-Bx-\frac{(A+B)\sinh(x)}{1-\cosh(x)}$$

output
```
-B*x-(A+B)*sinh(x)/(1-cosh(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 42 vs. 2(20) = 40.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.10

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx=\sinh(x)\left(\frac{A+B}{-1+\cosh(x)}-\frac{2B\arcsin\left(\sqrt{-\sinh^2\left(\frac{x}{2}\right)}\right)}{\sqrt{-\sinh^2(x)}}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

output
```
Sinh[x]*((A + B)/(-1 + Cosh[x]) - (2*B*ArcSin[Sqrt[-Sinh[x/2]^2]])/Sqrt[-S
inh[x]^2])
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3214, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{1-\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$(A+B)\int \frac{1}{1-\cosh(x)}dx-Bx$$

$$\Big\downarrow \text{3042}$$

$$-Bx+(A+B)\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3127}$$

$$-\frac{(A+B)\sinh(x)}{1-\cosh(x)}-Bx$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x]),x]
```

output
```
-(B*x) - ((A + B)*Sinh[x])/(1 - Cosh[x])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.70

method	result	size
parallelrisch	$-Bx+\coth\left(\frac{x}{2}\right)(A+B)$	14
risch	$-Bx+\frac{2A}{\mathrm{e}^{x}-1}+\frac{2B}{\mathrm{e}^{x}-1}$	24
default	$-\frac{-A-B}{\tanh\left(\frac{x}{2}\right)}-B\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$	36

input
```
int((A+B*cosh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-B*x+coth(1/2*x)*(A+B)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = -\frac{Bx\cosh(x)+Bx\sinh(x)-Bx-2A-2B}{\cosh(x)+\sinh(x)-1}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="fricas")
```

output
```
-(B*x*cosh(x) + B*x*sinh(x) - B*x - 2*A - 2*B)/(cosh(x) + sinh(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.75

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = \frac{A}{\tanh\left(\frac{x}{2}\right)} - Bx + \frac{B}{\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x)
```

output
```
A/tanh(x/2) - B*x + B/tanh(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = -B\left(x+\frac{2}{e^{(-x)}-1}\right) - \frac{2A}{e^{(-x)}-1}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="maxima")
```

output
```
-B*(x + 2/(e^(-x) - 1)) - 2*A/(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = -Bx + \frac{2\,(A+B)}{e^x-1}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x)),x, algorithm="giac")
```

output
```
-B*x + 2*(A + B)/(e^x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = \frac{2\,A+2\,B}{\mathrm{e}^x-1} - B\,x$$

input
```
int(-(A + B*cosh(x))/(cosh(x) - 1),x)
```

output
```
(2*A + 2*B)/(exp(x) - 1) - B*x
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{A+B\cosh(x)}{1-\cosh(x)}\,dx = \frac{2e^x a - e^x bx + 2e^x b + bx}{e^x-1}$$

input
```
int((A+B*cosh(x))/(1-cosh(x)),x)
```

output
```
(2*e**x*a - e**x*b*x + 2*e**x*b + b*x)/(e**x - 1)
```

3.98 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 37

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = -\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2} - \frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}$$

output
```
-1/3*(A+B)*sinh(x)/(1-cosh(x))^2-(A-2*B)*sinh(x)/(3-3*cosh(x))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = \frac{(-2A+B+(A-2B)\cosh(x))\sinh(x)}{3(-1+\cosh(x))^2}$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

output
```
((-2*A + B + (A - 2*B)*Cosh[x])*Sinh[x])/(3*(-1 + Cosh[x])^2)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3042, 3229, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{1}{3}(A-2B)\int \frac{1}{1-\cosh(x)}dx-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}(A-2B)\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3127}$$

$$-\frac{(A-2B)\sinh(x)}{3(1-\cosh(x))}-\frac{(A+B)\sinh(x)}{3(1-\cosh(x))^2}$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x])^2,x]
```

output
```
-1/3*((A + B)*Sinh[x])/(1 - Cosh[x])^2 - ((A - 2*B)*Sinh[x])/(3*(1 - Cosh[
x]))
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.70

method	result	size
default	$-\frac{A+B}{6\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-A+B}{2\tanh\left(\frac{x}{2}\right)}$	26
parallelrisch	$\frac{\coth\left(\frac{x}{2}\right)^{3}\left(-A-B+3\tanh\left(\frac{x}{2}\right)^{2}(A-B)\right)}{6}$	29
risch	$-\frac{2\left(3B\,\mathrm{e}^{2x}+3A\,\mathrm{e}^{x}-3B\,\mathrm{e}^{x}-A+2B\right)}{3\left(\mathrm{e}^{x}-1\right)^{3}}$	33

input
```
int((A+B*cosh(x))/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/6*(A+B)/tanh(1/2*x)^3-1/2*(-A+B)/tanh(1/2*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.30

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = \frac{2\,((A-5\,B)\cosh(x)-(A+B)\sinh(x)-3\,A+3\,B)}{3\,\left(\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-4\,\cosh(x)+3\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
2/3*((A - 5*B)*cosh(x) - (A + B)*sinh(x) - 3*A + 3*B)/(cosh(x)^2 + 2*(cosh
(x) - 1)*sinh(x) + sinh(x)^2 - 4*cosh(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = \frac{A}{2\tanh\left(\frac{x}{2}\right)}-\frac{A}{6\tanh^3\left(\frac{x}{2}\right)}-\frac{B}{2\tanh\left(\frac{x}{2}\right)}-\frac{B}{6\tanh^3\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))**2,x)
```

output
```
A/(2*tanh(x/2)) - A/(6*tanh(x/2)**3) - B/(2*tanh(x/2)) - B/(6*tanh(x/2)**3
)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 131 vs. 2(29) = 58.

Time = 0.03 (sec) , antiderivative size = 131, normalized size of antiderivative = 3.54

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2}{3}\,B\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{3\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{2}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-}\right. + \frac{2}{3}\,A\left(\frac{3\,e^{(-x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}-\frac{1}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1}\right)$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
-2/3*B*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 3*e^(-2*x)/(3*e^
(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) -
1)) + 2/3*A*(3*e^(-x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 1/(3*e^(-x)
 - 3*e^(-2*x) + e^(-3*x) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^2}\,dx = -\frac{2\left(3\,Be^{(2\,x)} + 3\,Ae^x - 3\,Be^x - A + 2\,B\right)}{3\,(e^x - 1)^3}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
-2/3*(3*B*e^(2*x) + 3*A*e^x - 3*B*e^x - A + 2*B)/(e^x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.86

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^2}\,dx = -\frac{2\,(2\,B - A + 3\,A\,\mathrm{e}^x - 3\,B\,\mathrm{e}^x + 3\,B\,\mathrm{e}^{2\,x})}{3\,(\mathrm{e}^x - 1)^3}$$

input
```
int((A + B*cosh(x))/(cosh(x) - 1)^2,x)
```

output
```
-(2*(2*B - A + 3*A*exp(x) - 3*B*exp(x) + 3*B*exp(2*x)))/(3*(exp(x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.19

$$\int \frac{A + B\cosh(x)}{(1-\cosh(x))^2}\,dx = \frac{-2e^{3x}b - 6e^{x}a + 2a - 2b}{3e^{3x} - 9e^{2x} + 9e^{x} - 3}$$

input
```
int((A+B*cosh(x))/(1-cosh(x))^2,x)
```

output
```
(2*( - e**(3*x)*b - 3*e***x*a + a - b))/(3*(e**(3*x) - 3*e**(2*x) + 3*e***x
- 1))
```

3.99 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx=-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))^2}-\frac{(2A-3B)\sinh(x)}{15(1-\cosh(x))}$$

output
```
-1/5*(A+B)*sinh(x)/(1-cosh(x))^3-1/15*(2*A-3*B)*sinh(x)/(1-cosh(x))^2-(2*A
-3*B)*sinh(x)/(15-15*cosh(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx=\frac{(7A-3B+(-6A+9B)\cosh(x)+(2A-3B)\cosh^2(x))\sinh(x)}{15(-1+\cosh(x))^3}$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

output
```
((7*A - 3*B + (-6*A + 9*B)*Cosh[x] + (2*A - 3*B)*Cosh[x]^2)*Sinh[x])/(15*(
-1 + Cosh[x])^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3229, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{5}(2A-3B)\int \frac{1}{(1-\cosh(x))^2}dx-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}+\frac{1}{5}(2A-3B)\int \frac{1}{\left(1-\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{5}(2A-3B)\left(\frac{1}{3}\int \frac{1}{1-\cosh(x)}dx-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}+\frac{1}{5}(2A-3B)\left(-\frac{\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}\int \frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{1}{5}(2A-3B)\left(-\frac{\sinh(x)}{3(1-\cosh(x))}-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{(A+B)\sinh(x)}{5(1-\cosh(x))^3}$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x])^3,x]
```

output
```
-1/5*((A + B)*Sinh[x])/(1 - Cosh[x])^3 + ((2*A - 3*B)*(-1/3*Sinh[x]/(1 - C
osh[x])^2 - Sinh[x]/(3*(1 - Cosh[x]))))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.33 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.65

method	result	size
default	$-\frac{A}{6\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-A-B}{20\tanh\left(\frac{x}{2}\right)^{5}}-\frac{-A+B}{4\tanh\left(\frac{x}{2}\right)}$	39
parallelrisch	$\frac{\coth\left(\frac{x}{2}\right)^{5}\left(15\tanh\left(\frac{x}{2}\right)^{4}A-15\tanh\left(\frac{x}{2}\right)^{4}B-10A\tanh\left(\frac{x}{2}\right)^{2}+3A+3B\right)}{60}$	43
risch	$\frac{2B\,{\mathrm e}^{3x}+\frac{8A\,{\mathrm e}^{2x}}{3}-2B\,{\mathrm e}^{2x}-\frac{4A\,{\mathrm e}^{x}}{3}+2B\,{\mathrm e}^{x}+\frac{4A}{15}-\frac{2B}{5}}{\left({\mathrm e}^{x}-1\right)^{5}}$	47

input
```
int((A+B*cosh(x))/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/6*A/tanh(1/2*x)^3-1/20*(-A-B)/tanh(1/2*x)^5-1/4*(-A+B)/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(48) = 96$.

Time = 0.07 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.12

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^3} dx = \frac{2\left(15 B \cosh(x)^2 + 15 B \sinh(x)^2 + 2(11 A - 9 B)\cosh(x)\right.}{15\left(\cosh(x)^4 + (4\cosh(x) - 5)\sinh(x)^3 + \sinh(x)^4 - 5\cosh(x)^3 + \left(6\cosh(x)^2 - 15\cosh(x) + 10\right)s\right.}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
2/15*(15*B*cosh(x)^2 + 15*B*sinh(x)^2 + 2*(11*A - 9*B)*cosh(x) + 6*(5*B*co
sh(x) + 3*A - 2*B)*sinh(x) - 10*A + 15*B)/(cosh(x)^4 + (4*cosh(x) - 5)*sin
h(x)^3 + sinh(x)^4 - 5*cosh(x)^3 + (6*cosh(x)^2 - 15*cosh(x) + 10)*sinh(x)
^2 + 10*cosh(x)^2 + (4*cosh(x)^3 - 15*cosh(x)^2 + 20*cosh(x) - 9)*sinh(x)
- 11*cosh(x) + 5)
```

Sympy [A] (verification not implemented)

Time = 0.67 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^3} dx = \frac{A}{4\tanh\left(\frac{x}{2}\right)} - \frac{A}{6\tanh^3\left(\frac{x}{2}\right)} + \frac{A}{20\tanh^5\left(\frac{x}{2}\right)} - \frac{B}{4\tanh\left(\frac{x}{2}\right)} + \frac{B}{20\tanh^5\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))**3,x)
```

output
```
A/(4*tanh(x/2)) - A/(6*tanh(x/2)**3) + A/(20*tanh(x/2)**5) - B/(4*tanh(x/2
)) + B/(20*tanh(x/2)**5)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(48) = 96$.

Time = 0.04 (sec) , antiderivative size = 267, normalized size of antiderivative = 4.45

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^3}\,dx = -\frac{2}{5}B\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)} - 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} - 5\,e^{(-4\,x)} + e^{(-5\,x)} - 1} - \frac{5\,e^{(-2\,x)}}{5\,e^{(-x)} - 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} - 5\,e^{(-4\,x)} \right. + \frac{4}{15}A\left(\frac{5\,e^{(-x)}}{5\,e^{(-x)} - 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} - 5\,e^{(-4\,x)} + e^{(-5\,x)} - 1} - \frac{10\,e^{(-2\,x)}}{5\,e^{(-x)} - 10\,e^{(-2\,x)} + 10\,e^{(-3\,x)} - 5\,e^{(-4} \right.$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
-2/5*B*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x) + e^(-
5*x) - 1) - 5*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*x)
+ e^(-5*x) - 1) + 5*e^(-3*x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(
-4*x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*x) - 5*e^(-4*
x) + e^(-5*x) - 1)) + 4/15*A*(5*e^(-x)/(5*e^(-x) - 10*e^(-2*x) + 10*e^(-3*
x) - 5*e^(-4*x) + e^(-5*x) - 1) - 10*e^(-2*x)/(5*e^(-x) - 10*e^(-2*x) + 10
*e^(-3*x) - 5*e^(-4*x) + e^(-5*x) - 1) - 1/(5*e^(-x) - 10*e^(-2*x) + 10*e^
(-3*x) - 5*e^(-4*x) + e^(-5*x) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.77

$$\int \frac{A + B\cosh(x)}{(1 - \cosh(x))^3}\,dx = \frac{2\left(15\,Be^{(3\,x)} + 20\,Ae^{(2\,x)} - 15\,Be^{(2\,x)} - 10\,Ae^{x} + 15\,Be^{x} + 2\,A - 3\,B\right)}{15\,(e^{x} - 1)^5}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
2/15*(15*B*e^(3*x) + 20*A*e^(2*x) - 15*B*e^(2*x) - 10*A*e^x + 15*B*e^x + 2
*A - 3*B)/(e^x - 1)^5
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.38

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = \frac{\frac{B}{5}+\frac{4\,A\,\mathrm{e}^{x}}{5}+\frac{3\,B\,\mathrm{e}^{2\,x}}{5}}{6\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}-4\,\mathrm{e}^{x}+1}-\frac{\frac{4\,A}{15}+\frac{2\,B\,\mathrm{e}^{x}}{5}}{3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}+1} - \frac{\frac{4\,B\,\mathrm{e}^{x}}{5}+\frac{8\,A\,\mathrm{e}^{2\,x}}{5}+\frac{4\,B\,\mathrm{e}^{3\,x}}{5}}{10\,\mathrm{e}^{2\,x}-10\,\mathrm{e}^{3\,x}+5\,\mathrm{e}^{4\,x}-\mathrm{e}^{5\,x}-5\,\mathrm{e}^{x}+1}+\frac{B}{5\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)}$$

input
```
int(-(A + B*cosh(x))/(cosh(x) - 1)^3,x)
```

output
```
(B/5 + (4*A*exp(x))/5 + (3*B*exp(2*x))/5)/(6*exp(2*x) - 4*exp(3*x) + exp(4
*x) - 4*exp(x) + 1) - ((4*A)/15 + (2*B*exp(x))/5)/(3*exp(2*x) - exp(3*x) -
 3*exp(x) + 1) - ((4*B*exp(x))/5 + (8*A*exp(2*x))/5 + (4*B*exp(3*x))/5)/(1
0*exp(2*x) - 10*exp(3*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) + B/(5*(e
xp(2*x) - 2*exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.37

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^3}\,dx = \frac{30e^{3x}b+40e^{2x}a-30e^{2x}b-20e^{x}a+30e^{x}b+4a-6b}{15e^{5x}-75e^{4x}+150e^{3x}-150e^{2x}+75e^{x}-15}$$

input
```
int((A+B*cosh(x))/(1-cosh(x))^3,x)
```

output
```
(2*(15*e**(3*x)*b + 20*e**(2*x)*a - 15*e**(2*x)*b - 10*e***x*a + 15*e***x*b
+ 2*a - 3*b))/(15*(e**(5*x) - 5*e**(4*x) + 10*e**(3*x) - 10*e**(2*x) + 5*e
**x - 1))
```

3.100 $\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 81

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = -\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4} - \frac{(3A-4B)\sinh(x)}{35(1-\cosh(x))^3} - \frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))^2} - \frac{2(3A-4B)\sinh(x)}{105(1-\cosh(x))}$$

output
```
-1/7*(A+B)*sinh(x)/(1-cosh(x))^4-1/35*(3*A-4*B)*sinh(x)/(1-cosh(x))^3-2/10
5*(3*A-4*B)*sinh(x)/(1-cosh(x))^2-2*(3*A-4*B)*sinh(x)/(105-105*cosh(x))
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \frac{\left(-36A+13B+13(3A-4B)\cosh(x)-8(3A-4B)\cosh^2(x)+(6A-8B)\cosh^3(x)\right)\sinh(x)}{105(-1+\cosh(x))^4}$$

input
```
Integrate[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

output
```
((-36*A + 13*B + 13*(3*A - 4*B)*Cosh[x] - 8*(3*A - 4*B)*Cosh[x]^2 + (6*A -
 8*B)*Cosh[x]^3)*Sinh[x])/(105*(-1 + Cosh[x])^4)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3229, 3042, 3129, 3042, 3129, 3042, 3127}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{1}{7}(3A-4B)\int \frac{1}{(1-\cosh(x))^3}dx-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\int \frac{1}{\left(1-\sin\left(ix+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{7}(3A-4B)\left(\frac{2}{5}\int \frac{1}{(1-\cosh(x))^2}dx-\frac{\sinh(x)}{5(1-\cosh(x))^3}\right)-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\left(-\frac{\sinh(x)}{5(1-\cosh(x))^3}+\frac{2}{5}\int \frac{1}{\left(1-\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx\right)$$

$$\Big\downarrow \text{3129}$$

$$\frac{1}{7}(3A-4B)\left(\frac{2}{5}\left(\frac{1}{3}\int\frac{1}{1-\cosh(x)}dx-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{\sinh(x)}{5(1-\cosh(x))^3}\right)-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}+\frac{1}{7}(3A-4B)\left(-\frac{\sinh(x)}{5(1-\cosh(x))^3}+\frac{2}{5}\left(-\frac{\sinh(x)}{3(1-\cosh(x))^2}+\frac{1}{3}\int\frac{1}{1-\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)$$

$$\Big\downarrow \text{3127}$$

$$\frac{1}{7}(3A-4B)\left(\frac{2}{5}\left(-\frac{\sinh(x)}{3(1-\cosh(x))}-\frac{\sinh(x)}{3(1-\cosh(x))^2}\right)-\frac{\sinh(x)}{5(1-\cosh(x))^3}\right)-\frac{(A+B)\sinh(x)}{7(1-\cosh(x))^4}$$

input
```
Int[(A + B*Cosh[x])/(1 - Cosh[x])^4,x]
```

output
```
-1/7*((A + B)*Sinh[x])/(1 - Cosh[x])^4 + ((3*A - 4*B)*(-1/5*Sinh[x]/(1 - C
osh[x])^3 + (2*(-1/3*Sinh[x]/(1 - Cosh[x])^2 - Sinh[x]/(3*(1 - Cosh[x]))))
/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3127
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> Simp[-Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{\coth\left(\frac{x}{2}\right)^{7}\left((A-B)\tanh\left(\frac{x}{2}\right)^{6}+\left(-A+\frac{B}{3}\right)\tanh\left(\frac{x}{2}\right)^{4}+\frac{(3A+B)\tanh\left(\frac{x}{2}\right)^{2}}{5}-\frac{A}{7}-\frac{B}{7}\right)}{8}$	55
default	$-\frac{-A+B}{8\tanh\left(\frac{x}{2}\right)}-\frac{A+B}{56\tanh\left(\frac{x}{2}\right)^{7}}-\frac{3A-B}{24\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-3A-B}{40\tanh\left(\frac{x}{2}\right)^{5}}$	56
risch	$-\frac{4\left(70B\,e^{4x}+105A\,e^{3x}-70B\,e^{3x}-63A\,e^{2x}+84B\,e^{2x}+21A\,e^{x}-28B\,e^{x}-3A+4B\right)}{105\left(e^{x}-1\right)^{7}}$	61

input
```
int((A+B*cosh(x))/(1-cosh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*coth(1/2*x)^7*((A-B)*tanh(1/2*x)^6+(-A+1/3*B)*tanh(1/2*x)^4+1/5*(3*A+B
)*tanh(1/2*x)^2-1/7*A-1/7*B)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. $2(65) = 130$.

Time = 0.08 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.16

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx$$
$$= \frac{4\left((3\,A-74\,B)\cosh\left(x\right)\right.}{105\left(\cosh\left(x\right)^{5}+\left(5\,\cosh\left(x\right)-7\right)\sinh\left(x\right)^{4}+\sinh\left(x\right)^{5}-7\,\cosh\left(x\right)^{4}+\left(10\,\cosh\left(x\right)^{2}-28\,\cosh\left(x\right)+21\right.\right.}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="fricas")
```

output
```
4/105*((3*A - 74*B)*cosh(x)^2 + (3*A - 74*B)*sinh(x)^2 - 14*(9*A - 7*B)*co
sh(x) - 6*((A + 22*B)*cosh(x) + 14*A - 7*B)*sinh(x) + 63*A - 84*B)/(cosh(x
)^5 + (5*cosh(x) - 7)*sinh(x)^4 + sinh(x)^5 - 7*cosh(x)^4 + (10*cosh(x)^2
- 28*cosh(x) + 21)*sinh(x)^3 + 21*cosh(x)^3 + (10*cosh(x)^3 - 42*cosh(x)^2
 + 63*cosh(x) - 36)*sinh(x)^2 - 36*cosh(x)^2 + (5*cosh(x)^4 - 28*cosh(x)^3
 + 63*cosh(x)^2 - 68*cosh(x) + 28)*sinh(x) + 42*cosh(x) - 21)
```

Sympy [A] (verification not implemented)

Time = 1.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.96

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \frac{A}{8\tanh\left(\frac{x}{2}\right)} - \frac{A}{8\tanh^3\left(\frac{x}{2}\right)} + \frac{3A}{40\tanh^5\left(\frac{x}{2}\right)} - \frac{A}{56\tanh^7\left(\frac{x}{2}\right)} - \frac{B}{8\tanh\left(\frac{x}{2}\right)} + \frac{B}{24\tanh^3\left(\frac{x}{2}\right)} + \frac{B}{40\tanh^5\left(\frac{x}{2}\right)} - \frac{B}{56\tanh^7\left(\frac{x}{2}\right)}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))**4,x)
```

output
```
A/(8*tanh(x/2)) - A/(8*tanh(x/2)**3) + 3*A/(40*tanh(x/2)**5) - A/(56*tanh(
x/2)**7) - B/(8*tanh(x/2)) + B/(24*tanh(x/2)**3) + B/(40*tanh(x/2)**5) - B
/(56*tanh(x/2)**7)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(65) = 130.

Time = 0.05 (sec) , antiderivative size = 451, normalized size of antiderivative = 5.57

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="maxima")
```

output
```
-8/105*B*(14*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) +
21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 42*e^(-2*x)/(7*e^(-x) - 21*e^(-
2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1
) + 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e
^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 35*e^(-4*x)/(7*e^(-x) - 21*e^(-2*x)
 + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) -
2/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^
(-6*x) + e^(-7*x) - 1)) + 4/35*A*(7*e^(-x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^
(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 21*e^(-2
*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*
e^(-6*x) + e^(-7*x) - 1) + 35*e^(-3*x)/(7*e^(-x) - 21*e^(-2*x) + 35*e^(-3*
x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(-7*x) - 1) - 1/(7*e^(-x)
- 21*e^(-2*x) + 35*e^(-3*x) - 35*e^(-4*x) + 21*e^(-5*x) - 7*e^(-6*x) + e^(
-7*x) - 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = -\frac{4\left(70\,Be^{(4\,x)}+105\,Ae^{(3\,x)}-70\,Be^{(3\,x)}-63\,Ae^{(2\,x)}+84\,Be^{(2\,x)}+21\,Ae^{x}-28\,Be^{x}-3\,A+4\,B\right)}{105\,(e^{x}-1)^{7}}$$

input
```
integrate((A+B*cosh(x))/(1-cosh(x))^4,x, algorithm="giac")
```

output
```
-4/105*(70*B*e^(4*x) + 105*A*e^(3*x) - 70*B*e^(3*x) - 63*A*e^(2*x) + 84*B*
e^(2*x) + 21*A*e^x - 28*B*e^x - 3*A + 4*B)/(e^x - 1)^7
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 233, normalized size of antiderivative = 2.88

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \frac{\frac{8B}{105}+\frac{16A\,e^{x}}{35}+\frac{16B\,e^{2x}}{35}}{10\,e^{2x}-10\,e^{3x}+5\,e^{4x}-e^{5x}-5\,e^{x}+1} - \frac{\frac{4A}{35}+\frac{8B\,e^{x}}{35}}{6\,e^{2x}-4\,e^{3x}+e^{4x}-4\,e^{x}+1} - \frac{\frac{8B\,e^{x}}{21}+\frac{8A\,e^{2x}}{7}+\frac{16B\,e^{3x}}{21}}{15\,e^{2x}-20\,e^{3x}+15\,e^{4x}-6\,e^{5x}+e^{6x}-6\,e^{x}+1} + \frac{8B}{105\,(3\,e^{2x}-e^{3x}-3\,e^{x}+1)} + \frac{\frac{16A\,e^{3x}}{7}+\frac{8B\,e^{2x}}{7}+\frac{8B\,e^{4x}}{7}}{21\,e^{2x}-35\,e^{3x}+35\,e^{4x}-21\,e^{5x}+7\,e^{6x}-e^{7x}-7\,e^{x}+1}$$

input

```
int((A + B*cosh(x))/(cosh(x) - 1)^4,x)
```

output

```
((8*B)/105 + (16*A*exp(x))/35 + (16*B*exp(2*x))/35)/(10*exp(2*x) - 10*exp(
3*x) + 5*exp(4*x) - exp(5*x) - 5*exp(x) + 1) - ((4*A)/35 + (8*B*exp(x))/35
)/(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1) - ((8*B*exp(x))/21 +
 (8*A*exp(2*x))/7 + (16*B*exp(3*x))/21)/(15*exp(2*x) - 20*exp(3*x) + 15*ex
p(4*x) - 6*exp(5*x) + exp(6*x) - 6*exp(x) + 1) + (8*B)/(105*(3*exp(2*x) -
exp(3*x) - 3*exp(x) + 1)) + ((16*A*exp(3*x))/7 + (8*B*exp(2*x))/7 + (8*B*e
xp(4*x))/7)/(21*exp(2*x) - 35*exp(3*x) + 35*exp(4*x) - 21*exp(5*x) + 7*exp
(6*x) - exp(7*x) - 7*exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.38

$$\int \frac{A+B\cosh(x)}{(1-\cosh(x))^4}\,dx = \frac{-280e^{4x}b-420e^{3x}a+280e^{3x}b+252e^{2x}a-336e^{2x}b-84e^{x}a+112e^{x}b+12a-16b}{105e^{7x}-735e^{6x}+2205e^{5x}-3675e^{4x}+3675e^{3x}-2205e^{2x}+735e^{x}-105}$$

input

```
int((A+B*cosh(x))/(1-cosh(x))^4,x)
```

output
```
(4*( - 70*e**(4*x)*b - 105*e**(3*x)*a + 70*e**(3*x)*b + 63*e**(2*x)*a - 84
*e**(2*x)*b - 21*e***x*a + 28*e***x*b + 3*a - 4*b))/(105*(e**(7*x) - 7*e**(6
*x) + 21*e**(5*x) - 35*e**(4*x) + 35*e**(3*x) - 21*e**(2*x) + 7*e***x - 1))
```

3.101 $\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 56

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx=\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a+a\cosh(x)}}$$

output
```
2^(1/2)*(A-B)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a+a*cosh(x))^(1/2))/a^(1
/2)+2*B*sinh(x)/(a+a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx=\frac{2\cosh\left(\frac{x}{2}\right)\left((A-B)\arctan\left(\sinh\left(\frac{x}{2}\right)\right)+2B\sinh\left(\frac{x}{2}\right)\right)}{\sqrt{a(1+\cosh(x))}}$$

input
```
Integrate[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

output
```
(2*Cosh[x/2]*((A - B)*ArcTan[Sinh[x/2]] + 2*B*Sinh[x/2]))/Sqrt[a*(1 + Cosh
[x])]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\sqrt{a\cosh(x)+a}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3230}$$

$$(A-B)\int \frac{1}{\sqrt{\cosh(x)a+a}}dx+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}+(A-B)\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}+2i(A-B)\int \frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{\sqrt{2}(A-B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a\cosh(x)+a}}$$

input
```
Int[(A + B*Cosh[x])/Sqrt[a + a*Cosh[x]],x]
```

output
```
(Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/
Sqrt[a] + (2*B*Sinh[x])/Sqrt[a + a*Cosh[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(45) = 90$.

Time = 0.91 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.29

method	result
default	$-\frac{\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\left(\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)aA-2B\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)aB\right)\sqrt{2}}{\sqrt{-a}\,a\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$
parts	$-\frac{A\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\sqrt{2}}{\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}+\frac{B\cosh\left(\frac{x}{2}\right)\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\left(\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2}a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)a+2\sqrt{\sinh\left(\frac{x}{2}\right.}\right.}{\sqrt{-a}\,a\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$

input
```
int((A+B*cosh(x))/(a+cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(1/2*x)*(sinh(1/2*x)^2*a)^(1/2)*(ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^
(1/2)*(-a)^(1/2)-a))*a*A-2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-ln(2/cosh(
1/2*x)*((sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)-a))*a*B)/(-a)^(1/2)/a/sinh(1/2*
x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.21

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = -\frac{2\left(\sqrt{2}(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}}{\sqrt{a}}\right)-\sqrt{\frac{1}{2}}(B\cosh{(x)}+B\sinh{(x)}-B)\sqrt{\frac{a}{\cosh(x)+\sinh(x)}}\right)}{a}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sin
h(x)))/sqrt(a)) - sqrt(1/2)*(B*cosh(x) + B*sinh(x) - B)*sqrt(a/(cosh(x) +
sinh(x))))/a
```

Sympy [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \int \frac{A+B\cosh{(x)}}{\sqrt{a\left(\cosh{(x)}+1\right)}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))**(1/2),x)
```

output
```
Integral((A + B*cosh(x))/sqrt(a*(cosh(x) + 1)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(45) = 90$.

Time = 0.21 (sec) , antiderivative size = 174, normalized size of antiderivative = 3.11

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = 2\left(\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\frac{\sqrt{2}e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)A - \frac{1}{3}\left(3\sqrt{2}\left(\frac{\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{e^{\left(\frac{1}{2}x\right)}}{\sqrt{a}e^{x}+\sqrt{a}}\right)-\sqrt{2}\left(\frac{3\arctan\left(e^{\left(-\frac{1}{2}x\right)}\right)}{\sqrt{a}}-\frac{2e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{e^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}e^{(-x)}+\sqrt{a}}\right)\right.$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
2*(sqrt(2)*(arctan(e^(1/2*x))/sqrt(a) + e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))
 - sqrt(2)*e^(1/2*x)/(sqrt(a)*e^x + sqrt(a)))*A - 1/3*(3*sqrt(2)*(arctan(e
^(1/2*x))/sqrt(a) - e^(1/2*x)/(sqrt(a)*e^x + sqrt(a))) - sqrt(2)*(3*arctan
(e^(-1/2*x))/sqrt(a) - 2*e^(-1/2*x)/sqrt(a) - e^(-1/2*x)/(sqrt(a)*e^(-x) +
 sqrt(a))) - (3*sqrt(2)*sqrt(a)*e^(3/2*x) - sqrt(2)*sqrt(a)*e^(-1/2*x))/(a
*e^x + a))*B
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.79

$$\int \frac{A+B\cosh(x)}{\sqrt{a+a\cosh(x)}}\,dx = \frac{2\sqrt{2}(A-B)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{\sqrt{a}}+\frac{\sqrt{2}Be^{\left(\frac{1}{2}x\right)}}{\sqrt{a}}-\frac{\sqrt{2}Be^{\left(-\frac{1}{2}x\right)}}{\sqrt{a}}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
2*sqrt(2)*(A - B)*arctan(e^(1/2*x))/sqrt(a) + sqrt(2)*B*e^(1/2*x)/sqrt(a)
- sqrt(2)*B*e^(-1/2*x)/sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{\sqrt{a + a\cosh(x)}}\, dx = \int \frac{A + B\cosh(x)}{\sqrt{a + a\cosh\left(x\right)}}\, dx$$

input
```
int((A + B*cosh(x))/(a + a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))/(a + a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{A + B\cosh(x)}{\sqrt{a + a\cosh(x)}}\, dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cosh(x)+1}}{\cosh(x)+1}dx\right)a + \left(\int \frac{\sqrt{\cosh(x)+1}\,\cosh(x)}{\cosh(x)+1}dx\right)b\right)}{a}$$

input
```
int((A+B*cosh(x))/(a+a*cosh(x))^(1/2),x)
```

output
```
(sqrt(a)*(int(sqrt(cosh(x) + 1)/(cosh(x) + 1),x)*a + int((sqrt(cosh(x) + 1
)*cosh(x))/(cosh(x) + 1),x)*b))/a
```

3.102 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx=\frac{(A+3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a+a\cosh(x))^{3/2}}$$

output
```
1/4*(A+3*B)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(1/2
)/a^(3/2)+1/2*(A-B)*sinh(x)/(a+a*cosh(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.68

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx=\frac{(A+3B)\arctan\left(\sinh\left(\frac{x}{2}\right)\right)\cosh^3\left(\frac{x}{2}\right)+\frac{1}{2}(A-B)\sinh(x)}{(a(1+\cosh(x)))^{3/2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

output
```
((A + 3*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^3 + ((A - B)*Sinh[x])/2)/(a*(1 + Co
sh[x]))^(3/2)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.294, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a\cosh(x)+a)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(A+3B)\int \frac{1}{\sqrt{\cosh(x)a+a}}dx}{4a}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{(A+3B)\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx}{4a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{i(A+3B)\int \frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)}{2a}$$

$$\Big\downarrow \text{219}$$

$$\frac{(A+3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{(A-B)\sinh(x)}{2(a\cosh(x)+a)^{3/2}}$$

input
```
Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(3/2),x]
```

output
```
((A + 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])])/(2*Sqr
t[2]*a^(3/2)) + ((A - B)*Sinh[x])/(2*(a + a*Cosh[x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(50) = 100$.

Time = 0.99 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.45

method	result
default	$-\frac{\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(A \ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\cosh\left(\frac{x}{2}\right)^{2} a+3B\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\cosh\left(\frac{x}{2}\right)^{2} a-A\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}+B}{4\cosh\left(\frac{x}{2}\right) a^{2}\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}$
parts	$-\frac{A\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}-2a}{\cosh\left(\frac{x}{2}\right)}\right)\cosh\left(\frac{x}{2}\right)^{2} a-\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}\right)\sqrt{2}}{4a^{2}\cosh\left(\frac{x}{2}\right)\sqrt{-a}\,\sinh\left(\frac{x}{2}\right)\sqrt{a\cosh\left(\frac{x}{2}\right)^{2}}}-\frac{B\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\left(3\ln\left(\frac{2\sqrt{\sinh\left(\frac{x}{2}\right)^{2} a}\,\sqrt{-a}}{\cosh\left(\frac{x}{2}\right)}\right.\right.}{4\cosh\left(\frac{x}{2}\right) a^{2}\sqrt{-a}\,\mathrm{s}}$

input
```
int((A+B*cosh(x))/(a+cosh(x)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*(sinh(1/2*x)^2*a)^(1/2)*(A*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/2)*
(-a)^(1/2)-a))*cosh(1/2*x)^2*a+3*B*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/
2)*(-a)^(1/2)-a))*cosh(1/2*x)^2*a-A*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)+B*(
sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/cosh(1/2*x)/a^2/(-a)^(1/2)/sinh(1/2*x)*
2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(50) = 100$.

Time = 0.11 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.03

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{3/2}}\, dx = \frac{\sqrt{2}\left((A + 3\,B)\cosh\left(x\right)^2 + (A + 3\,B)\sinh\left(x\right)^2 + 2\,(A + 3\,B)\cosh\left(x\right) + 2\,((A +\right.}{}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/2*(sqrt(2)*((A + 3*B)*cosh(x)^2 + (A + 3*B)*sinh(x)^2 + 2*(A + 3*B)*cosh
(x) + 2*((A + 3*B)*cosh(x) + A + 3*B)*sinh(x) + A + 3*B)*sqrt(a)*arctan(sq
rt(2)*sqrt(1/2)*sqrt(a)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x))/a)
 + 2*sqrt(1/2)*((A - B)*cosh(x)^2 + (A - B)*sinh(x)^2 - (A - B)*cosh(x) +
(2*(A - B)*cosh(x) - A + B)*sinh(x))*sqrt(a/(cosh(x) + sinh(x))))/(a^2*cos
h(x)^2 + a^2*sinh(x)^2 + 2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) + a^2)*sinh(
x))
```

Sympy [F]

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{3/2}}\, dx = \int \frac{A + B\cosh\left(x\right)}{\left(a\left(\cosh\left(x\right) + 1\right)\right)^{\frac{3}{2}}}\, dx$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))**(3/2),x)
```

output
```
Integral((A + B*cosh(x))/(a*(cosh(x) + 1))**(3/2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(50) = 100$.

Time = 0.25 (sec) , antiderivative size = 300, normalized size of antiderivative = 4.62

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{1}{6}\left(\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,x\right)}+8\,e^{\left(\frac{3}{2}\,x\right)}-3\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{3\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{3}{2}}}\right)-\frac{\quad}{a^{\frac{3}{2}}e^{(3\,x)}+\cdots}\right.$$

$$+\frac{1}{20}\left(\sqrt{2}\left(\frac{15\,e^{\left(\frac{5}{2}\,x\right)}+40\,e^{\left(\frac{3}{2}\,x\right)}+33\,e^{\left(\frac{1}{2}\,x\right)}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}e^{x}+a^{\frac{3}{2}}}+\frac{15\,\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{a^{\frac{3}{2}}}\right)+5\sqrt{2}\left(\frac{3\,e^{\left(\frac{5}{2}\,x\right)}-8\,e^{\left(\frac{3}{2}\,x\right)}-3\,e^{\cdots}}{a^{\frac{3}{2}}e^{(3\,x)}+3\,a^{\frac{3}{2}}e^{(2\,x)}+3\,a^{\frac{3}{2}}\cdots}\right.\right.$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
1/6*(sqrt(2)*((3*e^(5/2*x) + 8*e^(3/2*x) - 3*e^(1/2*x))/(a^(3/2)*e^(3*x) +
 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 3*arctan(e^(1/2*x))/a^(3/2
)) - 8*sqrt(2)*e^(3/2*x)/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*
e^x + a^(3/2)))*A + 1/20*(sqrt(2)*((15*e^(5/2*x) + 40*e^(3/2*x) + 33*e^(1/
2*x))/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) + 15
*arctan(e^(1/2*x))/a^(3/2)) + 5*sqrt(2)*((3*e^(5/2*x) - 8*e^(3/2*x) - 3*e^
(1/2*x))/(a^(3/2)*e^(3*x) + 3*a^(3/2)*e^(2*x) + 3*a^(3/2)*e^x + a^(3/2)) +
 3*arctan(e^(1/2*x))/a^(3/2)) - 8*(5*sqrt(2)*sqrt(a)*e^(5/2*x) + sqrt(2)*s
qrt(a)*e^(1/2*x))/(a^2*e^(3*x) + 3*a^2*e^(2*x) + 3*a^2*e^x + a^2))*B
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.20

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx = \frac{\left(\sqrt{2}A+3\,\sqrt{2}B\right)\arctan\left(e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,a^{\frac{3}{2}}} + \frac{\sqrt{2}\left(Aa^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Ba^{\frac{3}{2}}e^{\left(\frac{3}{2}\,x\right)}-Aa^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}+Ba^{\frac{3}{2}}e^{\left(\frac{1}{2}\,x\right)}\right)}{2\,(ae^{x}+a)^{2}a}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
1/2*(sqrt(2)*A + 3*sqrt(2)*B)*arctan(e^(1/2*x))/a^(3/2) + 1/2*sqrt(2)*(A*a
^(3/2)*e^(3/2*x) - B*a^(3/2)*e^(3/2*x) - A*a^(3/2)*e^(1/2*x) + B*a^(3/2)*e
^(1/2*x))/((a*e^x + a)^2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx=\int \frac{A+B\cosh(x)}{\left(a+a\cosh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))/(a + a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cosh(x)+1}}{\cosh(x)^2+2\cosh(x)+1}dx\right)a+\left(\int \frac{\sqrt{\cosh(x)+1}\,\cosh(x)}{\cosh(x)^2+2\cosh(x)+1}dx\right)b\right)}{a^2}$$

input
```
int((A+B*cosh(x))/(a+a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt(cosh(x) + 1)/(cosh(x)**2 + 2*cosh(x) + 1),x)*a + int((s
qrt(cosh(x) + 1)*cosh(x))/(cosh(x)**2 + 2*cosh(x) + 1),x)*b))/a**2
```

3.103 $\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 93

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \frac{(3A+5B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a+a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}} + \frac{(A-B)\sinh(x)}{4(a+a\cosh(x))^{5/2}} + \frac{(3A+5B)\sinh(x)}{16a(a+a\cosh(x))^{3/2}}$$

output
```
1/32*(3*A+5*B)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a+a*cosh(x))^(1/2))*2^(
1/2)/a^(5/2)+1/4*(A-B)*sinh(x)/(a+a*cosh(x))^(5/2)+1/16*(3*A+5*B)*sinh(x)/
a/(a+a*cosh(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.61

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \frac{4(3A+5B)\arctan\left(\sinh\left(\frac{x}{2}\right)\right)\cosh^5\left(\frac{x}{2}\right) + (7A+B+(3A+5B)\cosh(x))\sinh}{16(a(1+\cosh(x)))^{5/2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

output
```
(4*(3*A + 5*B)*ArcTan[Sinh[x/2]]*Cosh[x/2]^5 + (7*A + B + (3*A + 5*B)*Cosh
[x])*Sinh[x])/(16*(a*(1 + Cosh[x]))^(5/2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a\cosh(x)+a)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(3A+5B)\int \frac{1}{(\cosh(x)a+a)^{3/2}}dx}{8a}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}+\frac{(3A+5B)\int \frac{1}{\left(\sin\left(ix+\frac{\pi}{2}\right)a+a\right)^{3/2}}dx}{8a}$$

$$\Big\downarrow \text{3129}$$

$$\frac{(3A+5B)\left(\frac{\int \frac{1}{\sqrt{\cosh(x)a+a}}dx}{4a}+\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}\right)}{8a}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}+\frac{(3A+5B)\left(\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)a+a}}dx}{4a}\right)}{8a}$$

$$\Big\downarrow \text{3128}$$

$$\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}+\frac{(3A+5B)\left(\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}+\frac{i\int\frac{1}{\frac{a^2\sinh^2(x)}{\cosh(x)a+a}+2a}d\left(-\frac{ia\sinh(x)}{\sqrt{\cosh(x)a+a}}\right)}{2a}\right)}{8a}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{(3A+5B)\left(\frac{\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a\cosh(x)+a}}\right)}{2\sqrt{2}a^{3/2}}+\frac{\sinh(x)}{2(a\cosh(x)+a)^{3/2}}\right)}{8a}+\frac{(A-B)\sinh(x)}{4(a\cosh(x)+a)^{5/2}}$$

input
```
Int[(A + B*Cosh[x])/(a + a*Cosh[x])^(5/2),x]
```

output
```
((A - B)*Sinh[x])/(4*(a + a*Cosh[x])^(5/2)) + ((3*A + 5*B)*(ArcTan[(Sqrt[a
]*Sinh[x])/(Sqrt[2]*Sqrt[a + a*Cosh[x]])]/(2*Sqrt[2]*a^(3/2)) + Sinh[x]/(2
*(a + a*Cosh[x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(74) = 148$.

Time = 1.02 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.25

method	result
default	$-\frac{\sqrt{\sinh(\frac{x}{2})^2 a}\left(3A\ln\left(\frac{2\sqrt{\sinh(\frac{x}{2})^2 a}\,\sqrt{-a}-2a}{\cosh(\frac{x}{2})}\right)\cosh(\frac{x}{2})^4 a+5B\ln\left(\frac{2\sqrt{\sinh(\frac{x}{2})^2 a}\,\sqrt{-a}-2a}{\cosh(\frac{x}{2})}\right)\cosh(\frac{x}{2})^4 a-3A\cosh(\frac{x}{2})^2\sqrt{\sinh(\frac{x}{2}\cdots}{32\cosh(\frac{x}{2})^3 a^3\sqrt{-a}\,\sinh(\frac{x}{2})\sqrt{a\cosh(\cdots}$
parts	$-\frac{A\sqrt{\sinh(\frac{x}{2})^2 a}\left(3\ln\left(\frac{2\sqrt{\sinh(\frac{x}{2})^2 a}\,\sqrt{-a}-2a}{\cosh(\frac{x}{2})}\right)\cosh(\frac{x}{2})^4 a-3\sqrt{\sinh(\frac{x}{2})^2 a}\,\cosh(\frac{x}{2})^2\sqrt{-a}-2\sqrt{\sinh(\frac{x}{2})^2 a}\,\sqrt{-a}\right)\sqrt{2}}{32a^3\cosh(\frac{x}{2})^3\sqrt{-a}\,\sinh(\frac{x}{2})\sqrt{a\cosh(\frac{x}{2})^2}}-\frac{B\sqrt{\sin\cdots}}{\cdots}$

input
```
int((A+B*cosh(x))/(a+cosh(x)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32*(sinh(1/2*x)^2*a)^(1/2)*(3*A*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^(1/
2)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a+5*B*ln(2/cosh(1/2*x)*((sinh(1/2*x)^2*a)^
(1/2)*(-a)^(1/2)-a))*cosh(1/2*x)^4*a-3*A*cosh(1/2*x)^2*(sinh(1/2*x)^2*a)^(
1/2)*(-a)^(1/2)-5*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)*cosh(1/2*x)^2-2*A*(
sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2)+2*B*(sinh(1/2*x)^2*a)^(1/2)*(-a)^(1/2))/
cosh(1/2*x)^3/a^3/(-a)^(1/2)/sinh(1/2*x)*2^(1/2)/(a*cosh(1/2*x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(74) = 148$.

Time = 0.10 (sec) , antiderivative size = 514, normalized size of antiderivative = 5.53

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
1/16*(sqrt(2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)^4 + 4*(3*A + 5*
B)*cosh(x)^3 + 4*((3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x)^3 + 6*(3*A + 5*
B)*cosh(x)^2 + 6*((3*A + 5*B)*cosh(x)^2 + 2*(3*A + 5*B)*cosh(x) + 3*A + 5*
B)*sinh(x)^2 + 4*(3*A + 5*B)*cosh(x) + 4*((3*A + 5*B)*cosh(x)^3 + 3*(3*A +
 5*B)*cosh(x)^2 + 3*(3*A + 5*B)*cosh(x) + 3*A + 5*B)*sinh(x) + 3*A + 5*B)*
sqrt(a)*arctan(sqrt(2)*sqrt(1/2)*sqrt(a/(cosh(x) + sinh(x)))*(cosh(x) + si
nh(x))/sqrt(a)) + 2*sqrt(1/2)*((3*A + 5*B)*cosh(x)^4 + (3*A + 5*B)*sinh(x)
^4 + (11*A - 3*B)*cosh(x)^3 + (4*(3*A + 5*B)*cosh(x) + 11*A - 3*B)*sinh(x)
^3 - (11*A - 3*B)*cosh(x)^2 + (6*(3*A + 5*B)*cosh(x)^2 + 3*(11*A - 3*B)*co
sh(x) - 11*A + 3*B)*sinh(x)^2 - (3*A + 5*B)*cosh(x) + (4*(3*A + 5*B)*cosh(
x)^3 + 3*(11*A - 3*B)*cosh(x)^2 - 2*(11*A - 3*B)*cosh(x) - 3*A - 5*B)*sinh
(x))*sqrt(a/(cosh(x) + sinh(x))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 + 4*a^3*c
osh(x)^3 + 6*a^3*cosh(x)^2 + 4*a^3*cosh(x) + 4*(a^3*cosh(x) + a^3)*sinh(x)
^3 + a^3 + 6*(a^3*cosh(x)^2 + 2*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh
(x)^3 + 3*a^3*cosh(x)^2 + 3*a^3*cosh(x) + a^3)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(74) = 148$.

Time = 0.22 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.59

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
1/80*(sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) + 128*e^(5/2*x) - 70*e^(3/2*x)
 - 15*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x)
 + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x))/a^
(5/2)) - 128*sqrt(2)*e^(5/2*x)/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a
^(5/2)*e^(3*x) + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)))*A + 1/672*
(sqrt(2)*((105*e^(9/2*x) + 490*e^(7/2*x) + 896*e^(5/2*x) + 790*e^(3/2*x) -
 105*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^(3*x)
+ 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 105*arctan(e^(1/2*x))/a^
(5/2)) + 7*sqrt(2)*((15*e^(9/2*x) + 70*e^(7/2*x) - 128*e^(5/2*x) - 70*e^(3
/2*x) - 15*e^(1/2*x))/(a^(5/2)*e^(5*x) + 5*a^(5/2)*e^(4*x) + 10*a^(5/2)*e^
(3*x) + 10*a^(5/2)*e^(2*x) + 5*a^(5/2)*e^x + a^(5/2)) + 15*arctan(e^(1/2*x
))/a^(5/2)) - 128*(7*sqrt(2)*sqrt(a)*e^(7/2*x) + 3*sqrt(2)*sqrt(a)*e^(3/2*
x))/(a^3*e^(5*x) + 5*a^3*e^(4*x) + 10*a^3*e^(3*x) + 10*a^3*e^(2*x) + 5*a^3
*e^x + a^3))*B
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.27

$$\int \frac{A + B\cosh(x)}{(a + a\cosh(x))^{5/2}}\,dx = \frac{\sqrt{2}(3A + 5B)\arctan\left(e^{\left(\frac{1}{2}x\right)}\right)}{16\,a^{\frac{5}{2}}} + \frac{\sqrt{2}\left(3\,Aa^{\frac{7}{2}}e^{\left(\frac{7}{2}x\right)} + 5\,Ba^{\frac{7}{2}}e^{\left(\frac{7}{2}x\right)} + 11\,Aa^{\frac{7}{2}}e^{\left(\frac{5}{2}x\right)} - 3\,Ba^{\frac{7}{2}}e^{\left(\frac{5}{2}x\right)} - 11\,Aa^{\frac{7}{2}}e^{\left(\frac{3}{2}x\right)} + 3\,Ba^{\frac{7}{2}}e^{\left(\frac{3}{2}x\right)} - 3\,Aa^{\frac{7}{2}}e^{\left(\frac{1}{2}x\right)} -}{16\,(ae^{x} + a)^{4}a^{2}}$$

input
```
integrate((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
1/16*sqrt(2)*(3*A + 5*B)*arctan(e^(1/2*x))/a^(5/2) + 1/16*sqrt(2)*(3*A*a^(
7/2)*e^(7/2*x) + 5*B*a^(7/2)*e^(7/2*x) + 11*A*a^(7/2)*e^(5/2*x) - 3*B*a^(7
/2)*e^(5/2*x) - 11*A*a^(7/2)*e^(3/2*x) + 3*B*a^(7/2)*e^(3/2*x) - 3*A*a^(7/
2)*e^(1/2*x) - 5*B*a^(7/2)*e^(1/2*x))/((a*e^x + a)^4*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \int \frac{A+B\cosh(x)}{\left(a+a\cosh\left(x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))/(a + a*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A+B\cosh(x)}{(a+a\cosh(x))^{5/2}}\,dx = \frac{\sqrt{a}\left(\left(\int \frac{\sqrt{\cosh(x)+1}}{\cosh(x)^3+3\cosh(x)^2+3\cosh(x)+1}dx\right)a+\left(\int \frac{\sqrt{\cosh(x)+1}\,\cosh(x)}{\cosh(x)^3+3\cosh(x)^2+3\cosh(x)+1}dx\right)b\right)}{a^3}$$

input
```
int((A+B*cosh(x))/(a+a*cosh(x))^(5/2),x)
```

output
```
(sqrt(a)*(int(sqrt(cosh(x) + 1)/(cosh(x)**3 + 3*cosh(x)**2 + 3*cosh(x) + 1
),x)*a + int((sqrt(cosh(x) + 1)*cosh(x))/(cosh(x)**3 + 3*cosh(x)**2 + 3*co
sh(x) + 1),x)*b))/a**3
```

3.104 $\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 57

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = -\frac{\sqrt{2}(A+B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}+\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

output
```
-2^(1/2)*(A+B)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a-a*cosh(x))^(1/2))/a^(
1/2)+2*B*sinh(x)/(a-a*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \frac{2\left(2B\cosh\left(\frac{x}{2}\right)-(A+B)\left(\log\left(\cosh\left(\frac{x}{4}\right)\right)-\log\left(\sinh\left(\frac{x}{4}\right)\right)\right)\right)\sinh\left(\frac{x}{2}\right)}{\sqrt{a-a\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

output
```
(2*(2*B*Cosh[x/2] - (A + B)*(Log[Cosh[x/4]] - Log[Sinh[x/4]]))*Sinh[x/2])/
Sqrt[a - a*Cosh[x]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 3230, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a-a\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3230}$$

$$(A+B)\int \frac{1}{\sqrt{a-a\cosh(x)}}dx+\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+(A+B)\int \frac{1}{\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{3128}$$

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}+2i(A+B)\int \frac{1}{\frac{a^2\sinh^2(x)}{a-a\cosh(x)}+2a}d\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{2B\sinh(x)}{\sqrt{a-a\cosh(x)}}-\frac{\sqrt{2}(A+B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{\sqrt{a}}$$

input
```
Int[(A + B*Cosh[x])/Sqrt[a - a*Cosh[x]],x]
```

output
```
-((Sqrt[2]*(A + B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])]
)/Sqrt[a]) + (2*B*Sinh[x])/Sqrt[a - a*Cosh[x]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3230
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(b*(m + 1))   Int[(a + b*Sin[e
 + f*x])^m, x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b*c - a*d, 0]
&& EqQ[a^2 - b^2, 0] &&  !LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.11

method	result	size
default	$\frac{\sinh\left(\frac{x}{2}\right)\left(\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)B+4B\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	63
parts	$-\frac{2A\sinh\left(\frac{x}{2}\right)\operatorname{arctanh}\left(\cosh\left(\frac{x}{2}\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}+\frac{B\sinh\left(\frac{x}{2}\right)\left(4\cosh\left(\frac{x}{2}\right)+\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)-\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)\right)}{\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$	65

input
```
int((A+B*cosh(x))/(a-cosh(x)*a)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
sinh(1/2*x)*(ln(cosh(1/2*x)-1)*A-ln(cosh(1/2*x)+1)*A+ln(cosh(1/2*x)-1)*B-l
n(cosh(1/2*x)+1)*B+4*B*cosh(1/2*x))/(-2*sinh(1/2*x)^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(46) = 92$.

Time = 0.08 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.74

$$\int \frac{A + B\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx = \frac{\sqrt{2}(A+B)a\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{1}{2}}\sqrt{-\frac{a}{\cosh(x)+\sinh(x)}}\sqrt{-\frac{1}{a}}(\cosh(x)+\sinh(x))-\cosh(x)-\sinh(x)-1}{\cosh(x)+\sinh(x)-1}\right) - 2\sqrt{\frac{1}{2}}(B\cosh(x) + \dots}{a}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
(sqrt(2)*(A + B)*a*sqrt(-1/a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a/(cosh(x) +
sinh(x)))*sqrt(-1/a)*(cosh(x) + sinh(x)) - cosh(x) - sinh(x) - 1)/(cosh(x)
 + sinh(x) - 1)) - 2*sqrt(1/2)*(B*cosh(x) + B*sinh(x) + B)*sqrt(-a/(cosh(x
) + sinh(x))))/a
```

Sympy [F]

$$\int \frac{A + B\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx = \int \frac{A + B\cosh(x)}{\sqrt{-a(\cosh(x) - 1)}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))**(1/2),x)
```

output
```
Integral((A + B*cosh(x))/sqrt(-a*(cosh(x) - 1)), x)
```

Maxima [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{B\cosh\left(x\right)+A}{\sqrt{-a\cosh\left(x\right)+a}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/sqrt(-a*cosh(x) + a), x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.47

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = -\frac{2\left(\sqrt{2}A+\sqrt{2}B\right)\arctan\left(\frac{\sqrt{-ae^{x}}}{\sqrt{a}}\right)}{\sqrt{a}\mathrm{sgn}\left(-e^{x}+1\right)} - \frac{\sqrt{2}B}{\sqrt{-ae^{x}}\mathrm{sgn}\left(-e^{x}+1\right)} + \frac{\sqrt{2}\sqrt{-ae^{x}}B}{a\mathrm{sgn}\left(-e^{x}+1\right)}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
-2*(sqrt(2)*A + sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(sqrt(a)*sgn(-e^x
+ 1)) - sqrt(2)*B/(sqrt(-a*e^x)*sgn(-e^x + 1)) + sqrt(2)*sqrt(-a*e^x)*B/(a
*sgn(-e^x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh(x)}}\,dx = \int \frac{A+B\cosh(x)}{\sqrt{a-a\cosh\left(x\right)}}\,dx$$

input
```
int((A + B*cosh(x))/(a - a*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))/(a - a*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{A + B\cosh(x)}{\sqrt{a - a\cosh(x)}}\,dx = -\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{-\cosh(x)+1}}{\cosh(x)-1}dx\right)a + \left(\int \frac{\sqrt{-\cosh(x)+1}\cosh(x)}{\cosh(x)-1}dx\right)b\right)}{a}$$

input
```
int((A+B*cosh(x))/(a-a*cosh(x))^(1/2),x)
```

output
```
( - sqrt(a)*(int(sqrt( - cosh(x) + 1)/(cosh(x) - 1),x)*a + int((sqrt( - co
sh(x) + 1)*cosh(x))/(cosh(x) - 1),x)*b))/a
```

3.105 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 65

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = -\frac{(A-3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}} - \frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

output
```
-1/4*(A-3*B)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^(1/
2)/a^(3/2)-1/2*(A+B)*sinh(x)/(a-a*cosh(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.25

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \frac{\left((A+B)\text{csch}^2\left(\frac{x}{4}\right) - 4(A-3B)\left(\log\left(\cosh\left(\frac{x}{4}\right)\right) - \log\left(\sinh\left(\frac{x}{4}\right)\right)\right) + (A+B)\text{se}\right.}{4a(-1+\cosh(x))\sqrt{a-a\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

output
```
(((A + B)*Csch[x/4]^2 - 4*(A - 3*B)*(Log[Cosh[x/4]] - Log[Sinh[x/4]]) + (A
 + B)*Sech[x/4]^2)*Sinh[x/2]^3)/(4*a*(-1 + Cosh[x])*Sqrt[a - a*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.278, Rules used = {3042, 3229, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3229}$$

$$\frac{(A-3B)\int \frac{1}{\sqrt{a-a\cosh(x)}}dx}{4a}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{(A-3B)\int \frac{1}{\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}}dx}{4a}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{i(A-3B)\int \frac{1}{\frac{a^2\sinh^2(x)}{a-a\cosh(x)}+2a}d\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}}{2a}$$

$$\Big\downarrow \text{219}$$

$$-\frac{(A-3B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{(A+B)\sinh(x)}{2(a-a\cosh(x))^{3/2}}$$

input
```
Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(3/2),x]
```

output
```
-1/2*((A - 3*B)*ArcTan[(Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])])/(
Sqrt[2]*a^(3/2)) - ((A + B)*Sinh[x])/(2*(a - a*Cosh[x])^(3/2))
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.94 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.28

method	result
default	$-\frac{\cosh\left(\frac{x}{2}\right)(-2A-2B)+\left(\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)B+3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)B\right)\sinh\left(\frac{x}{2}\right)^{2}}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$
parts	$-\frac{A\left(-2\cosh\left(\frac{x}{2}\right)+\left(\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)-\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)\right)\sinh\left(\frac{x}{2}\right)^{2}\right)}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}+\frac{B\left(2\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)-3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)\right)\sinh(\right.}{4a\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$

input
```
int((A+B*cosh(x))/(a-cosh(x)*a)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4/a*(cosh(1/2*x)*(-2*A-2*B)+(ln(cosh(1/2*x)+1)*A-ln(cosh(1/2*x)-1)*A-3*
ln(cosh(1/2*x)+1)*B+3*ln(cosh(1/2*x)-1)*B)*sinh(1/2*x)^2)/sinh(1/2*x)/(-2*
sinh(1/2*x)^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(50) = 100$.

Time = 0.10 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.34

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \frac{\sqrt{2}\left((A-3\,B)\cosh\left(x\right)^{2}+(A-3\,B)\sinh\left(x\right)^{2}-2\,(A-3\,B)\cosh\left(x\right)+2\,((A-\right.}{}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
1/4*(sqrt(2)*((A - 3*B)*cosh(x)^2 + (A - 3*B)*sinh(x)^2 - 2*(A - 3*B)*cosh
(x) + 2*((A - 3*B)*cosh(x) - A + 3*B)*sinh(x) + A - 3*B)*sqrt(-a)*log((2*s
qrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(cosh(x) + sinh(x))
 - a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) - 4*sqrt(1/2)*((A +
 B)*cosh(x)^2 + (A + B)*sinh(x)^2 + (A + B)*cosh(x) + (2*(A + B)*cosh(x) +
 A + B)*sinh(x))*sqrt(-a/(cosh(x) + sinh(x))))/(a^2*cosh(x)^2 + a^2*sinh(x
)^2 - 2*a^2*cosh(x) + a^2 + 2*(a^2*cosh(x) - a^2)*sinh(x))
```

Sympy [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \int \frac{A+B\cosh\left(x\right)}{\left(-a\left(\cosh\left(x\right)-1\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))**(3/2),x)
```

output
```
Integral((A + B*cosh(x))/(-a*(cosh(x) - 1))**(3/2), x)
```

Maxima [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = \int \frac{B\cosh(x)+A}{(-a\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. 2(50) = 100.

Time = 0.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.71

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx = -\frac{\left(\sqrt{2}A-3\sqrt{2}B\right)\arctan\left(\frac{\sqrt{-ae^x}}{\sqrt{a}}\right)}{2\,a^{\frac{3}{2}}\mathrm{sgn}\left(-e^x+1\right)} + \frac{\sqrt{2}\left(\sqrt{-ae^x}Aae^x+\sqrt{-ae^x}Bae^x+\sqrt{-ae^x}Aa+\sqrt{-ae^x}Ba\right)}{2\left(ae^x-a\right)^2 a\mathrm{sgn}\left(-e^x+1\right)}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
-1/2*(sqrt(2)*A - 3*sqrt(2)*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(3/2)*sgn(-
e^x + 1)) + 1/2*sqrt(2)*(sqrt(-a*e^x)*A*a*e^x + sqrt(-a*e^x)*B*a*e^x + sqr
t(-a*e^x)*A*a + sqrt(-a*e^x)*B*a)/((a*e^x - a)^2*a*sgn(-e^x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx=\int \frac{A+B\cosh(x)}{\left(a-a\cosh\left(x\right)\right)^{3/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a - a*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))/(a - a*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{3/2}}\,dx=\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{-\cosh(x)+1}}{\cosh(x)^2-2\cosh(x)+1}dx\right)a+\left(\int \frac{\sqrt{-\cosh(x)+1}\,\cosh(x)}{\cosh(x)^2-2\cosh(x)+1}dx\right)b\right)}{a^2}$$

input
```
int((A+B*cosh(x))/(a-a*cosh(x))^(3/2),x)
```

output
```
(sqrt(a)*(int(sqrt( - cosh(x) + 1)/(cosh(x)**2 - 2*cosh(x) + 1),x)*a + int
((sqrt( - cosh(x) + 1)*cosh(x))/(cosh(x)**2 - 2*cosh(x) + 1),x)*b))/a**2
```

3.106 $\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 94

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = -\frac{(3A-5B)\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{16\sqrt{2}a^{5/2}} - \frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}} - \frac{(3A-5B)\sinh(x)}{16a(a-a\cosh(x))^{3/2}}$$

output
```
-1/32*(3*A-5*B)*arctan(1/2*a^(1/2)*sinh(x)*2^(1/2)/(a-a*cosh(x))^(1/2))*2^
(1/2)/a^(5/2)-1/4*(A+B)*sinh(x)/(a-a*cosh(x))^(5/2)-1/16*(3*A-5*B)*sinh(x)
/a/(a-a*cosh(x))^(3/2)
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = \frac{\left(2(3A-5B)\operatorname{csch}^2\left(\frac{x}{4}\right) - (A+B)\operatorname{csch}^4\left(\frac{x}{4}\right) - 8(3A-5B)\left(\log\left(\cosh\left(\frac{x}{4}\right)\right) - \log\right.\right.}{32a^2(-1+\cosh(x))^2\sqrt{a}}$$

input
```
Integrate[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

output
```
((2*(3*A - 5*B)*Csch[x/4]^2 - (A + B)*Csch[x/4]^4 - 8*(3*A - 5*B)*(Log[Cos
h[x/4]] - Log[Sinh[x/4]]) + 2*(3*A - 5*B)*Sech[x/4]^2 + (A + B)*Sech[x/4]^
4)*Sinh[x/2]^5)/(32*a^2*(-1 + Cosh[x])^2*Sqrt[a - a*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.98, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 3229, 3042, 3129, 3042, 3128, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a-a\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3229}$$

$$\frac{(3A-5B)\int \frac{1}{(a-a\cosh(x))^{3/2}}dx}{8a}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\int \frac{1}{\left(a-a\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{8a}$$

$$\Big\downarrow \text{ 3129}$$

$$\frac{(3A-5B)\left(\frac{\int \frac{1}{\sqrt{a-a\cosh(x)}}dx}{4a}-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}\right)}{8a}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\left(-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{\int \frac{1}{\sqrt{a-a\sin\left(ix+\frac{\pi}{2}\right)}}dx}{4a}\right)}{8a}$$

$$\Big\downarrow \text{3128}$$

$$-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}+\frac{(3A-5B)\left(-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}+\frac{i\int\frac{1}{\frac{a^2\sinh^2(x)}{a-a\cosh(x)}+2a}d\frac{ia\sinh(x)}{\sqrt{a-a\cosh(x)}}}{2a}\right)}{8a}$$

$$\Big\downarrow \text{219}$$

$$\frac{(3A-5B)\left(-\frac{\arctan\left(\frac{\sqrt{a}\sinh(x)}{\sqrt{2}\sqrt{a-a\cosh(x)}}\right)}{2\sqrt{2}a^{3/2}}-\frac{\sinh(x)}{2(a-a\cosh(x))^{3/2}}\right)}{8a}-\frac{(A+B)\sinh(x)}{4(a-a\cosh(x))^{5/2}}$$

input
```
Int[(A + B*Cosh[x])/(a - a*Cosh[x])^(5/2),x]
```

output
```
-1/4*((A + B)*Sinh[x])/(a - a*Cosh[x])^(5/2) + ((3*A - 5*B)*(-1/2*ArcTan[(
Sqrt[a]*Sinh[x])/(Sqrt[2]*Sqrt[a - a*Cosh[x]])]/(Sqrt[2]*a^(3/2)) - Sinh[x
]/(2*(a - a*Cosh[x])^(3/2))))/(8*a)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3128
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[-2/d
Subst[Int[1/(2*a - x^2), x], x, b*(Cos[c + d*x]/Sqrt[a + b*Sin[c + d*x]])],
 x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

rule 3129
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*Cos[c
+ d*x]*((a + b*Sin[c + d*x])^n/(a*d*(2*n + 1))), x] + Simp[(n + 1)/(a*(2*n
+ 1))   Int[(a + b*Sin[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d}, x]
&& EqQ[a^2 - b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 3229
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(b*c - a*d)*Cos[e + f*x]*((a + b*Sin[e + f*
x])^m/(a*f*(2*m + 1))), x] + Simp[(a*d*m + b*c*(m + 1))/(a*b*(2*m + 1))   I
nt[(a + b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{a, b, c, d, e, f}, x] &&
NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Maple [A] (verified)

Time = 0.96 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.26

method	result
default	$-\frac{(-6A+10B)\cosh\left(\frac{x}{2}\right)\sinh\left(\frac{x}{2}\right)^{2}+(4A+4B)\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)A-3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)A-5\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)B+5\ln\left(\cosh\left(\right.\right.\right.}{32a^{2}\left(\cosh\left(\frac{x}{2}\right)+1\right)\left(\cosh\left(\frac{x}{2}\right)-1\right)\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}$
parts	$-\frac{A\left(-6\sinh\left(\frac{x}{2}\right)^{2}\cosh\left(\frac{x}{2}\right)+4\cosh\left(\frac{x}{2}\right)+\left(3\ln\left(\cosh\left(\frac{x}{2}\right)+1\right)-3\ln\left(\cosh\left(\frac{x}{2}\right)-1\right)\right)\sinh\left(\frac{x}{2}\right)^{4}\right)}{32a^{2}\left(\cosh\left(\frac{x}{2}\right)+1\right)\left(\cosh\left(\frac{x}{2}\right)-1\right)\sinh\left(\frac{x}{2}\right)\sqrt{-2\sinh\left(\frac{x}{2}\right)^{2}a}}+\frac{B\left(-10\sinh\left(\frac{x}{2}\right)^{2}\cosh\left(\frac{x}{2}\right)-4\cosh\left(\right.\right.}{32a^{2}\left(\cosh\left(\frac{x}{2}\right)+1\right)}$

input
```
int((A+B*cosh(x))/(a-cosh(x)*a)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/32/a^2*((-6*A+10*B)*cosh(1/2*x)*sinh(1/2*x)^2+(4*A+4*B)*cosh(1/2*x)+(3*
ln(cosh(1/2*x)+1)*A-3*ln(cosh(1/2*x)-1)*A-5*ln(cosh(1/2*x)+1)*B+5*ln(cosh(
1/2*x)-1)*B)*sinh(1/2*x)^4)/(cosh(1/2*x)+1)/(cosh(1/2*x)-1)/sinh(1/2*x)/(-
2*sinh(1/2*x)^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(75) = 150$.

Time = 0.09 (sec) , antiderivative size = 548, normalized size of antiderivative = 5.83

$$\int \frac{A + B\cosh(x)}{(a - a\cosh(x))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
1/32*(sqrt(2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - 4*(3*A - 5*
B)*cosh(x)^3 + 4*((3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x)^3 + 6*(3*A - 5*
B)*cosh(x)^2 + 6*((3*A - 5*B)*cosh(x)^2 - 2*(3*A - 5*B)*cosh(x) + 3*A - 5*
B)*sinh(x)^2 - 4*(3*A - 5*B)*cosh(x) + 4*((3*A - 5*B)*cosh(x)^3 - 3*(3*A -
 5*B)*cosh(x)^2 + 3*(3*A - 5*B)*cosh(x) - 3*A + 5*B)*sinh(x) + 3*A - 5*B)*
sqrt(-a)*log((2*sqrt(2)*sqrt(1/2)*sqrt(-a)*sqrt(-a/(cosh(x) + sinh(x)))*(c
osh(x) + sinh(x)) - a*cosh(x) - a*sinh(x) - a)/(cosh(x) + sinh(x) - 1)) -
4*sqrt(1/2)*((3*A - 5*B)*cosh(x)^4 + (3*A - 5*B)*sinh(x)^4 - (11*A + 3*B)*
cosh(x)^3 + (4*(3*A - 5*B)*cosh(x) - 11*A - 3*B)*sinh(x)^3 - (11*A + 3*B)*
cosh(x)^2 + (6*(3*A - 5*B)*cosh(x)^2 - 3*(11*A + 3*B)*cosh(x) - 11*A - 3*B
)*sinh(x)^2 + (3*A - 5*B)*cosh(x) + (4*(3*A - 5*B)*cosh(x)^3 - 3*(11*A + 3
*B)*cosh(x)^2 - 2*(11*A + 3*B)*cosh(x) + 3*A - 5*B)*sinh(x))*sqrt(-a/(cosh
(x) + sinh(x))))/(a^3*cosh(x)^4 + a^3*sinh(x)^4 - 4*a^3*cosh(x)^3 + 6*a^3*
cosh(x)^2 - 4*a^3*cosh(x) + 4*(a^3*cosh(x) - a^3)*sinh(x)^3 + a^3 + 6*(a^3
*cosh(x)^2 - 2*a^3*cosh(x) + a^3)*sinh(x)^2 + 4*(a^3*cosh(x)^3 - 3*a^3*cos
h(x)^2 + 3*a^3*cosh(x) - a^3)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a - a\cosh(x))^{5/2}}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx=\int \frac{B\cosh\left(x\right)+A}{\left(-a\cosh\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(-a*cosh(x) + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 189 vs. $2(75) = 150$.

Time = 0.13 (sec) , antiderivative size = 189, normalized size of antiderivative = 2.01

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx=-\frac{\sqrt{2}(3\,A-5\,B)\arctan\left(\frac{\sqrt{-ae^{x}}}{\sqrt{a}}\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(-e^{x}+1\right)}+\frac{\sqrt{2}\big(3\,\sqrt{-ae^{x}}Aa^{3}e^{(3\,x)}-5\,\sqrt{-ae^{x}}Ba^{3}e^{(3\,x)}-11\,\sqrt{-ae^{x}}Aa^{3}e^{(2\,x)}-3\,\sqrt{-ae^{x}}Ba^{3}e^{(2\,x)}-11\,\sqrt{-ae^{x}}Aa^{3}e^{x}}{16\left(ae^{x}-a\right)^{4}a^{2}\mathrm{sgn}\left(-e^{x}+1\right)}$$

input
```
integrate((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*(3*A - 5*B)*arctan(sqrt(-a*e^x)/sqrt(a))/(a^(5/2)*sgn(-e^x +
 1)) + 1/16*sqrt(2)*(3*sqrt(-a*e^x)*A*a^3*e^(3*x) - 5*sqrt(-a*e^x)*B*a^3*e
^(3*x) - 11*sqrt(-a*e^x)*A*a^3*e^(2*x) - 3*sqrt(-a*e^x)*B*a^3*e^(2*x) - 11
*sqrt(-a*e^x)*A*a^3*e^x - 3*sqrt(-a*e^x)*B*a^3*e^x + 3*sqrt(-a*e^x)*A*a^3
- 5*sqrt(-a*e^x)*B*a^3)/((a*e^x - a)^4*a^2*sgn(-e^x + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx=\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a - a*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))/(a - a*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A+B\cosh(x)}{(a-a\cosh(x))^{5/2}}\,dx = -\frac{\sqrt{a}\left(\left(\int \frac{\sqrt{-\cosh(x)+1}}{\cosh(x)^3-3\cosh(x)^2+3\cosh(x)-1}dx\right)a+\left(\int \frac{\sqrt{-\cosh(x)+1}\,\cosh(x)}{\cosh(x)^3-3\cosh(x)^2+3\cosh(x)-1}dx\right)b\right)}{a^3}$$

input
```
int((A+B*cosh(x))/(a-a*cosh(x))^(5/2),x)
```

output
```
( - sqrt(a)*(int(sqrt( - cosh(x) + 1)/(cosh(x)**3 - 3*cosh(x)**2 + 3*cosh(
x) - 1),x)*a + int((sqrt( - cosh(x) + 1)*cosh(x))/(cosh(x)**3 - 3*cosh(x)*
*2 + 3*cosh(x) - 1),x)*b))/a**3
```

3.107 $\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 233

$$\begin{aligned}\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx &= \\ &-\frac{2i(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{105b\sqrt{\frac{a+b\cosh(x)}{a+b}}} \\ &+\frac{2i(a^2-b^2)(56aAb+15a^2B+25b^2B)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{105b\sqrt{a+b\cosh(x)}} \\ &+\frac{2}{105}(56aAb+15a^2B+25b^2B)\sqrt{a+b\cosh(x)}\sinh(x) \\ &+\frac{2}{35}(7Ab+5aB)(a+b\cosh(x))^{3/2}\sinh(x)+\frac{2}{7}B(a+b\cosh(x))^{5/2}\sinh(x)\end{aligned}$$

output
```
-2/105*I*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*(a+b*cosh(x))^(1/2)*E
llipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2))/b/((a+b*cosh(x))/(a+b))^(1
/2)+2/105*I*(a^2-b^2)*(56*A*a*b+15*B*a^2+25*B*b^2)*((a+b*cosh(x))/(a+b))^(
1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/b/(a+b*cosh(x))^(1/2
)+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*(a+b*cosh(x))^(1/2)*sinh(x)+2/35*(7*A
*b+5*B*a)*(a+b*cosh(x))^(3/2)*sinh(x)+2/7*B*(a+b*cosh(x))^(5/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.87

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \frac{-\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left(b(105a^3A+119aAb^2+135a^2bB+25b^3B)\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\left(\right.}{b}}{}$$

input
```
Integrate[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*(b*(105*a^3*A + 119*a*A*b^2 + 135*a
^2*b*B + 25*b^3*B)*EllipticF[(I/2)*x, (2*b)/(a + b)] + (161*a^2*A*b + 63*A
*b^3 + 15*a^3*B + 145*a*b^2*B)*((a + b)*EllipticE[(I/2)*x, (2*b)/(a + b)]
- a*EllipticF[(I/2)*x, (2*b)/(a + b)])))/b + (a + b*Cosh[x])*(154*a*A*b +
90*a^2*B + 65*b^2*B + 6*b*(7*A*b + 15*a*B)*Cosh[x] + 15*b^2*B*Cosh[2*x])*S
inh[x])/(105*Sqrt[a + b*Cosh[x]])
```

Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.05, number of steps used = 18, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 1.059$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \; 3232$$

$$\frac{2}{7}\int \frac{1}{2}(a+b\cosh(x))^{3/2}(7aA+5bB+(7Ab+5aB)\cosh(x))dx+\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 27

$$\frac{1}{7}\int (a+b\cosh(x))^{3/2}(7aA+5bB+(7Ab+5aB)\cosh(x))dx+\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+ \frac{1}{7}\int \left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}\left(7aA+5bB+(7Ab+5aB)\sin\left(ix+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3232

$$\frac{1}{7}\left(\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cosh(x)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56Aba+25b^2B\right)\cosh(x)\right)dx+\frac{2}{5}\sinh(x)(5aB\right. \frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\int \sqrt{a+b\cosh(x)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+56Aba+25b^2B\right)\cosh(x)\right)dx+\frac{2}{5}\sinh(x)(5aB+\right. \frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+ \frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}\left(35Aa^2+40bBa+21Ab^2+\left(15Ba^2+\right.\right.\right.$$

$\downarrow$ 3232

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{2}{3}\int \frac{105Aa^3+135bBa^2+119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2+145b^2Ba+63Ab^3\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\right.\right. \frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 27

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\int \frac{105Aa^3+135bBa^2+119Ab^2a+25b^3B+\left(15Ba^3+161Aba^2+145b^2Ba+63Ab^3\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\right.\right. \frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int\right.\right.$$

$\big\downarrow$ 3231

$$\frac{1}{7}\left(\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(15a^3B+161a^2Ab+145ab^2B+63Ab^3\right)\int\sqrt{a+b\cosh(x)}dx}{b}-\frac{\left(a^2-b^2\right)\left(15a^2B+56aAb+25b^2B\right)}{b}\right.\right.\right.$$
$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}$$

$\big\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\right.\right.\right.$$

$\big\downarrow$ 3134

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\right.\right.\right.$$

$\big\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\right.\right.\right.$$

$\big\downarrow$ 3132

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\right.\right.\right.$$

$\big\downarrow$ 3142

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\right.\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\right.\right.\right.$$

$\downarrow$ 3140

$$\frac{2}{7}B\sinh(x)(a+b\cosh(x))^{5/2}+$$
$$\frac{1}{7}\left(\frac{2}{5}\sinh(x)(5aB+7Ab)(a+b\cosh(x))^{3/2}+\frac{1}{5}\left(\frac{2}{3}\sinh(x)\left(15a^2B+56aAb+25b^2B\right)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{2i}{-}\right.\right.\right.$$

input
```
Int[(a + b*Cosh[x])^(5/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + b*Cosh[x])^(5/2)*Sinh[x])/7 + ((2*(7*A*b + 5*a*B)*(a + b*Cosh[x]
)^(3/2)*Sinh[x])/5 + (((((-2*I)*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*
b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*(56*a*A*b + 15*a^2*B + 25*b^2*B
)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt
[a + b*Cosh[x]]))/3 + (2*(56*a*A*b + 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Cosh[
x]]*Sinh[x])/3)/5)/7
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1364 vs. $2(214) = 428$.

Time = 25.40 (sec) , antiderivative size = 1365, normalized size of antiderivative = 5.86

method	result	size
default	Expression too large to display	1365
parts	Expression too large to display	1454

input
```
int((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/105*(240*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^8*b^3+(168*A*(-2*b
/(a-b))^(1/2)*b^3+480*B*(-2*b/(a-b))^(1/2)*a*b^2+360*B*(-2*b/(a-b))^(1/2)*
b^3)*sinh(1/2*x)^6*cosh(1/2*x)+(392*A*(-2*b/(a-b))^(1/2)*a*b^2+168*A*(-2*b
/(a-b))^(1/2)*b^3+360*B*(-2*b/(a-b))^(1/2)*a^2*b+480*B*(-2*b/(a-b))^(1/2)*
a*b^2+280*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^4*cosh(1/2*x)+(154*A*(-2*b
/(a-b))^(1/2)*a^2*b+196*A*(-2*b/(a-b))^(1/2)*a*b^2+42*A*(-2*b/(a-b))^(1/2)
*b^3+90*B*(-2*b/(a-b))^(1/2)*a^3+180*B*(-2*b/(a-b))^(1/2)*a^2*b+170*B*(-2*
b/(a-b))^(1/2)*a*b^2+80*B*(-2*b/(a-b))^(1/2)*b^3)*sinh(1/2*x)^2*cosh(1/2*x
)+105*A*a^3*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+161*
A*a^2*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)
*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+119*A*a*
b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*Ell
ipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+63*A*b^3*(2*
b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(
cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))-322*A*(2*b/(a-b)*si
nh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x
)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*a^2*b-126*A*(2*b/(a-b)*sinh(1
/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-
2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*b^3+15*a^3*B*(2*b/(a-b)*sinh(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. $2(212) = 424$.

Time = 0.10 (sec) , antiderivative size = 1123, normalized size of antiderivative = 4.82

$$\int (a + b\cosh(x))^{5/2}(A + B\cosh(x))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/1260*(16*sqrt(1/2)*((30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a*b^3
 - 75*B*b^4)*cosh(x)^3 + 3*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b^2 - 231*A*a
*b^3 - 75*B*b^4)*cosh(x)^2*sinh(x) + 3*(30*B*a^4 + 7*A*a^3*b - 115*B*a^2*b
^2 - 231*A*a*b^3 - 75*B*b^4)*cosh(x)*sinh(x)^2 + (30*B*a^4 + 7*A*a^3*b - 1
15*B*a^2*b^2 - 231*A*a*b^3 - 75*B*b^4)*sinh(x)^3)*sqrt(b)*weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x)
 + 3*b*sinh(x) + 2*a)/b) + 48*sqrt(1/2)*((15*B*a^3*b + 161*A*a^2*b^2 + 145
*B*a*b^3 + 63*A*b^4)*cosh(x)^3 + 3*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b
^3 + 63*A*b^4)*cosh(x)^2*sinh(x) + 3*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a
*b^3 + 63*A*b^4)*cosh(x)*sinh(x)^2 + (15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a
*b^3 + 63*A*b^4)*sinh(x)^3)*sqrt(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b))
- 3*(15*B*b^4*cosh(x)^6 + 15*B*b^4*sinh(x)^6 + 6*(15*B*a*b^3 + 7*A*b^4)*co
sh(x)^5 + 6*(15*B*b^4*cosh(x) + 15*B*a*b^3 + 7*A*b^4)*sinh(x)^5 - 15*B*b^4
 + (180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4)*cosh(x)^4 + (225*B*b^4*cosh(x
)^2 + 180*B*a^2*b^2 + 308*A*a*b^3 + 115*B*b^4 + 30*(15*B*a*b^3 + 7*A*b^4)*
cosh(x))*sinh(x)^4 - 8*(15*B*a^3*b + 161*A*a^2*b^2 + 145*B*a*b^3 + 63*A*b^
4)*cosh(x)^3 + 4*(75*B*b^4*cosh(x)^3 - 30*B*a^3*b - 322*A*a^2*b^2 - 290*B*
a*b^3 - 126*A*b^4 + 15*(15*B*a*b^3 + 7*A*b^4)*cosh(x)^2 + (180*B*a^2*b^...
```

Sympy [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(x))**(5/2)*(A+B*cosh(x)),x)
```

output
```
Timed out
```

Maxima [F]

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \int (B\cosh{(x)}+A)(b\cosh{(x)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

Giac [F]

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \int (B\cosh{(x)}+A)(b\cosh{(x)}+a)^{\frac{5}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,\left(a+b\cosh(x)\right)^{5/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + b*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))*(a + b*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int (a+b\cosh(x))^{5/2}(A+B\cosh(x))\,dx = \left(\int \sqrt{\cosh(x)\,b+a}dx\right)a^3 + 3\left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)\,dx\right)a^2b + \left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)^3\,dx\right)b^3 + 3\left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)^2\,dx\right)a\,b^2$$

input
```
int((a+b*cosh(x))^(5/2)*(A+B*cosh(x)),x)
```

output
```
int(sqrt(cosh(x)*b + a),x)*a**3 + 3*int(sqrt(cosh(x)*b + a)*cosh(x),x)*a**
2*b + int(sqrt(cosh(x)*b + a)*cosh(x)**3,x)*b**3 + 3*int(sqrt(cosh(x)*b +
a)*cosh(x)**2,x)*a*b**2
```

3.108 $\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 181

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = -\frac{2i(20aAb+3a^2B+9b^2B)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{15b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(a^2-b^2)(5Ab+3aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{15b\sqrt{a+b\cosh(x)}} + \frac{2}{15}(5Ab+3aB)\sqrt{a+b\cosh(x)}\sinh(x) + \frac{2}{5}B(a+b\cosh(x))^{3/2}\sinh(x)$$

output
```
-2/15*I*(20*A*a*b+3*B*a^2+9*B*b^2)*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/
2*x),2^(1/2)*(b/(a+b))^(1/2))/b/((a+b*cosh(x))/(a+b))^(1/2)+2/15*I*(a^2-b^
2)*(5*A*b+3*B*a)*((a+b*cosh(x))/(a+b))^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/
2)*(b/(a+b))^(1/2))/b/(a+b*cosh(x))^(1/2)+2/15*(5*A*b+3*B*a)*(a+b*cosh(x))
^(1/2)*sinh(x)+2/5*B*(a+b*cosh(x))^(3/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.69

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \frac{2}{15}\sqrt{a+b\cosh(x)}\left(-\frac{i\left((20aAb+3a^2B+9b^2B)\,E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)-(a-b)(5Ab+3aB)\,\text{Ellip}\right.}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + (5Ab+6aB+3bB\cosh(x))\sinh(x)\right)$$

input
```
Integrate[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*Sqrt[a + b*Cosh[x]]*(((-I)*((20*a*A*b + 3*a^2*B + 9*b^2*B)*EllipticE[(I
/2)*x, (2*b)/(a + b)] - (a - b)*(5*A*b + 3*a*B)*EllipticF[(I/2)*x, (2*b)/(
a + b)]))/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + (5*A*b + 6*a*B + 3*b*B*Cosh[
x])*Sinh[x]))/15
```

Rubi [A] (verified)

Time = 1.04 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.03, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 3232, 27, 3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \; 3232$$

$$\frac{2}{5}\int \frac{1}{2}\sqrt{a+b\cosh(x)}(5aA+3bB+(5Ab+3aB)\cosh(x))dx+\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 27

$$\frac{1}{5}\int \sqrt{a+b\cosh(x)}(5aA+3bB+(5Ab+3aB)\cosh(x))dx+\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+\frac{1}{5}\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}\left(5aA+3bB+(5Ab+3aB)\sin\left(ix+\frac{\pi}{2}\right)\right)dx$$

$\downarrow$ 3232

$$\frac{1}{5}\left(\frac{2}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}\right.$$
$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 27

$$\frac{1}{5}\left(\frac{1}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}\right.$$
$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int \frac{15Aa^2+12bBa+5Ab^2+\left(3Ba^2+20Aba+9b^2B\right)\sin\left(ix+\frac{\pi}{2}\right.}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}\right.$$

$\downarrow$ 3231

$$\frac{1}{5}\left(\frac{1}{3}\left(\frac{\left(3a^2B+20aAb+9b^2B\right)\int \sqrt{a+b\cosh(x)}dx}{b}-\frac{\left(a^2-b^2\right)(3aB+5Ab)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}\right)+\frac{2}{3}\sinh(x)(3\right.$$
$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(3a^2B+20aAb+9b^2B)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{(a^2-b^2)\,(\right.\right.$$

$\downarrow$ 3134

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(3a^2B+20aAb+9b^2B)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(3a^2B+20aAb+9b^2B)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right.\right.$$

$\downarrow$ 3132

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(a^2-b^2)\,(3aB+5Ab)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i(3a^2B+20aA}{}\right.\right.$$

$\downarrow$ 3142

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(a^2-b^2)\,(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(3a}{}\right.\right.$$

$\downarrow$ 3042

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{(a^2-b^2)\,(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i}{}\right.\right.$$

$$\Big\downarrow \text{3140}$$

$$\frac{2}{5}B\sinh(x)(a+b\cosh(x))^{3/2}+$$
$$\frac{1}{5}\left(\frac{2}{3}\sinh(x)(3aB+5Ab)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{2i(a^2-b^2)(3aB+5Ab)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i(3\right.\right.$$

input
```
Int[(a + b*Cosh[x])^(3/2)*(A + B*Cosh[x]),x]
```

output
```
(2*B*(a + b*Cosh[x])^(3/2)*Sinh[x])/5 + ((((-2*I)*(20*a*A*b + 3*a^2*B + 9*
b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(a +
 b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*(5*A*b + 3*a*B)*Sqrt[(a + b*Cos
h[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]))
/3 + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Cosh[x]]*Sinh[x])/3)/5
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 972 vs. $2(166) = 332$.

Time = 18.36 (sec) , antiderivative size = 973, normalized size of antiderivative = 5.38

method	result	size
default	Expression too large to display	973
parts	Expression too large to display	1066

input
```
int((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/15*(24*B*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^6*b^2+(20*A*(-2*b/(a
-b))^(1/2)*b^2+36*B*(-2*b/(a-b))^(1/2)*a*b+24*B*(-2*b/(a-b))^(1/2)*b^2)*si
nh(1/2*x)^4*cosh(1/2*x)+(10*A*(-2*b/(a-b))^(1/2)*a*b+10*A*(-2*b/(a-b))^(1/
2)*b^2+12*B*(-2*b/(a-b))^(1/2)*a^2+18*B*(-2*b/(a-b))^(1/2)*a*b+6*B*(-2*b/(
a-b))^(1/2)*b^2)*sinh(1/2*x)^2*cosh(1/2*x)+15*A*a^2*(2*b/(a-b)*sinh(1/2*x)
^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(
a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+20*A*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b
)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(
1/2),1/2*(-2/b*(a-b))^(1/2))+5*A*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))
^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2
*(-2/b*(a-b))^(1/2))-40*A*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-si
nh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b
))^(1/2))*a*b+3*B*a^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1
/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(
1/2))+12*B*a*b*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2
)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+9
*B*b^2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*
EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))-6*B*(2*b/
(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(co
sh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*a^2-18*B*(2*b/(a-b...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(164) = 328$.

Time = 0.10 (sec) , antiderivative size = 623, normalized size of antiderivative = 3.44

$$\int (a + b\cosh(x))^{3/2}(A + B\cosh(x))\,dx = \text{Too large to display}$$

input
```
integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/90*(8*sqrt(1/2)*((6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*cosh(x)^
2 + 2*(6*B*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*cosh(x)*sinh(x) + (6*B
*a^3 - 5*A*a^2*b - 18*B*a*b^2 - 15*A*b^3)*sinh(x)^2)*sqrt(b)*weierstrassPI
nverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh
(x) + 3*b*sinh(x) + 2*a)/b) + 24*sqrt(1/2)*((3*B*a^2*b + 20*A*a*b^2 + 9*B*
b^3)*cosh(x)^2 + 2*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*cosh(x)*sinh(x) + (3
*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*sinh(x)^2)*sqrt(b)*weierstrassZeta(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4
*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sin
h(x) + 2*a)/b)) - 3*(3*B*b^3*cosh(x)^4 + 3*B*b^3*sinh(x)^4 - 3*B*b^3 + 2*(
6*B*a*b^2 + 5*A*b^3)*cosh(x)^3 + 2*(6*B*b^3*cosh(x) + 6*B*a*b^2 + 5*A*b^3)
*sinh(x)^3 - 4*(3*B*a^2*b + 20*A*a*b^2 + 9*B*b^3)*cosh(x)^2 + 2*(9*B*b^3*c
osh(x)^2 - 6*B*a^2*b - 40*A*a*b^2 - 18*B*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)*cos
h(x))*sinh(x)^2 - 2*(6*B*a*b^2 + 5*A*b^3)*cosh(x) + 2*(6*B*b^3*cosh(x)^3 -
 6*B*a*b^2 - 5*A*b^3 + 3*(6*B*a*b^2 + 5*A*b^3)*cosh(x)^2 - 4*(3*B*a^2*b +
20*A*a*b^2 + 9*B*b^3)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(b^2*cosh(x)^
2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2)
```

Sympy [F]

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\,(a+b\cosh(x))^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))**(3/2)*(A+B*cosh(x)),x)
```

output
```
Integral((A + B*cosh(x))*(a + b*cosh(x))**(3/2), x)
```

Maxima [F]

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)(b\cosh(x)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

Giac [F]

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\int (B\cosh(x)+A)(b\cosh(x)+a)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)*(b*cosh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx=\int (A+B\cosh(x))\,\left(a+b\cosh(x)\right)^{3/2}\,dx$$

input
```
int((A + B*cosh(x))*(a + b*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))*(a + b*cosh(x))^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int (a+b\cosh(x))^{3/2}(A+B\cosh(x))\,dx&=\left(\int \sqrt{\cosh(x)\,b+a}dx\right)a^2\\&\quad+2\left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)\,dx\right)ab+\left(\int \sqrt{\cosh(x)\,b+a}\,\cosh(x)^2\,dx\right)b^2\end{aligned}$$

input
```
int((a+b*cosh(x))^(3/2)*(A+B*cosh(x)),x)
```

output
```
int(sqrt(cosh(x)*b + a),x)*a**2 + 2*int(sqrt(cosh(x)*b + a)*cosh(x),x)*a*b
 + int(sqrt(cosh(x)*b + a)*cosh(x)**2,x)*b**2
```

3.109 $\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 138

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = -\frac{2i(3Ab+aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\sqrt{\frac{a+b\cosh(x)}{a+b}}} + \frac{2i(a^2-b^2)\,B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3b\sqrt{a+b\cosh(x)}} + \frac{2}{3}B\sqrt{a+b\cosh(x)}\sinh(x)$$

output
```
-2/3*I*(3*A*b+B*a)*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/
(a+b))^(1/2))/b/((a+b*cosh(x))/(a+b))^(1/2)+2/3*I*(a^2-b^2)*B*((a+b*cosh(x
))/(a+b))^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/b/(a+b*co
sh(x))^(1/2)+2/3*B*(a+b*cosh(x))^(1/2)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.89

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \frac{-2i(a+b)(3Ab+aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+2i(a^2-b^2)\,B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)+2bB(a+b}{3b\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
((-2*I)*(a + b)*(3*A*b + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2
)*x, (2*b)/(a + b)] + (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*El
lipticF[(I/2)*x, (2*b)/(a + b)] + 2*b*B*(a + b*Cosh[x])*Sinh[x])/(3*b*Sqrt
[a + b*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.01, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 3232, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}\left(A+B\sin\left(\frac{\pi}{2}+ix\right)\right)dx$$

$$\Big\downarrow \text{3232}$$

$$\frac{2}{3}\int \frac{3aA+bB+(3Ab+aB)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{3}\int \frac{3aA+bB+(3Ab+aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\int \frac{3aA+bB+(3Ab+aB)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx$$

$$\Big\downarrow \text{ 3231}$$

$$\frac{1}{3}\left(\frac{(aB+3Ab)\int \sqrt{a+b\cosh(x)}dx}{b}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}\right)+\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{ 3134}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{(aB+3Ab)\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}\right)$$

$$\Big\downarrow \text{ 3132}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{B\left(a^2-b^2\right)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{ 3142}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3042}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(-\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

$$\Big\downarrow \text{3140}$$

$$\frac{2}{3}B\sinh(x)\sqrt{a+b\cosh(x)}+\frac{1}{3}\left(\frac{2iB(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i(aB+3Ab)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}\right)$$

input
```
Int[Sqrt[a + b*Cosh[x]]*(A + B*Cosh[x]),x]
```

output
```
(((-2*I)*(3*A*b + a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b
)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*B*Sqrt[(a + b*C
osh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]
))/3 + (2*B*Sqrt[a + b*Cosh[x]]*Sinh[x])/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3232
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-d)*Cos[e + f*x]*((a + b*Sin[e + f*x])^m/(
f*(m + 1))), x] + Simp[1/(m + 1)   Int[(a + b*Sin[e + f*x])^(m - 1)*Simp[b*
d*m + a*c*(m + 1) + (a*d*m + b*c*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ
[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && GtQ[m,
 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 598 vs. $2(127) = 254$.

Time = 14.03 (sec) , antiderivative size = 599, normalized size of antiderivative = 4.34

method	result
parts	$\frac{2A\left(a\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+b\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2b\,\mathrm{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\sqrt{-\right.\right.\right.}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\sinh(}}$
default	$2\left(4B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{4}b+2B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}a+2B\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\,\sinh\left(\frac{x}{2}\right)^{2}b+3Aa\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\frac{a+b}{a-b}}\right.$

input
```
int((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*A*(a*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+b*
EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))-2*b*Ellip
ticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2)))*(-sinh(1/2*x)
^2)^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)*s
inh(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x
)^2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)+2/3*B*(4*cosh(1/2*x)*
(-2*b/(a-b))^(1/2)*sinh(1/2*x)^4*b+2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1
/2*x)^2*a+2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b+(-sinh(1/2*x)^2
)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*
(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))*a+(-sinh(1/2*x)^2)^(1/2)*(2*b/(
a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(
1/2),1/2*(-2/b*(a-b))^(1/2))*b-2*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/
2)*(-sinh(1/2*x)^2)^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2
/b*(a-b))^(1/2))*a)*((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)/(-2*b/(a
-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/sinh(1/2*x)/(2*b*
sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(126) = 252$.

Time = 0.08 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.31

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx =$$
$$-\frac{4\sqrt{\frac{1}{2}}\left((2\,Ba^2-3\,Aab-3\,Bb^2)\cosh(x)+(2\,Ba^2-3\,Aab-3\,Bb^2)\sinh(x)\right)\sqrt{b}\text{weierstrassPInverse}\left(\frac{4}{}\right.}{}$$

input
```
integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="fricas")
```

output
```
-1/9*(4*sqrt(1/2)*((2*B*a^2 - 3*A*a*b - 3*B*b^2)*cosh(x) + (2*B*a^2 - 3*A*
a*b - 3*B*b^2)*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^
2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) +
 12*sqrt(1/2)*((B*a*b + 3*A*b^2)*cosh(x) + (B*a*b + 3*A*b^2)*sinh(x))*sqrt
(b)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3,
1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) - 3*(B*b^2*cosh(x)^2 + B*b^2*sin
h(x)^2 - B*b^2 - 2*(B*a*b + 3*A*b^2)*cosh(x) + 2*(B*b^2*cosh(x) - B*a*b -
3*A*b^2)*sinh(x))*sqrt(b*cosh(x) + a))/(b^2*cosh(x) + b^2*sinh(x))
```

Sympy [F]

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\sqrt{a+b\cosh(x)}\,dx$$

input
```
integrate((a+b*cosh(x))**(1/2)*(A+B*cosh(x)),x)
```

output
```
Integral((A + B*cosh(x))*sqrt(a + b*cosh(x)), x)
```

Maxima [F]

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)\sqrt{b\cosh(x)+a}\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

Giac [F]

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int (B\cosh(x)+A)\sqrt{b\cosh(x)+a}\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)*sqrt(b*cosh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \int (A+B\cosh(x))\ \sqrt{a+b\cosh(x)}\,dx$$

input
```
int((A + B*cosh(x))*(a + b*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))*(a + b*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\cosh(x)}(A+B\cosh(x))\,dx = \left(\int \sqrt{\cosh\left(x\right)b+a}dx\right)a + \left(\int \sqrt{\cosh\left(x\right)b+a}\,\cosh\left(x\right)dx\right)b$$

input
```
int((a+b*cosh(x))^(1/2)*(A+B*cosh(x)),x)
```

output
```
int(sqrt(cosh(x)*b + a),x)*a + int(sqrt(cosh(x)*b + a)*cosh(x),x)*b
```

3.110 $\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 60

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b} + \frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}}$$

output
```
B*x/b+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)
/b/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{Bx}{b} + \frac{2(-Ab+aB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/
(b*Sqrt[-a^2 + b^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{(Ab-aB)\int \frac{1}{a+b\cosh(x)}dx}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{Bx}{b}+\frac{(Ab-aB)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{2(Ab-aB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}+\frac{Bx}{b}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sq
rt[a - b]*b*Sqrt[a + b])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.22

method	result
default	$\frac{B\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}-\frac{2(-Ab+Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}$
risch	$\frac{Bx}{b}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Ba}{\sqrt{a^2-b^2}\,b}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2}{b\sqrt{a^2-b^2}}\right.}{\sqrt{a^2-b^2}\,b}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B/b*ln(1+tanh(1/2*x))-B/b*ln(tanh(1/2*x)-1)-2/b*(-A*b+B*a)/((a+b)*(a-b))^(
1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 240, normalized size of antiderivative = 4.00

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x)}\,dx$$
$$= \left[-\frac{(Ba - Ab)\sqrt{a^2 - b^2}\log\left(\frac{b^2\cosh(x)^2 + b^2\sinh(x)^2 + 2\,ab\cosh(x) + 2\,a^2 - b^2 + 2\,(b^2\cosh(x) + ab)\sinh(x) - 2\,\sqrt{a^2 - b^2}(b\cosh(x) + b\sin}{b\cosh(x)^2 + b\sinh(x)^2 + 2\,a\cosh(x) + 2\,(b\cosh(x) + a)\sinh(x) + b}\right.}{a^2b - b^3}\right.$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-((B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*
cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*
(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*
(b*cosh(x) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x)/(a^2*b - b^3), (2*(B*a
- A*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) +
a)/(a^2 - b^2)) + (B*a^2 - B*b^2)*x)/(a^2*b - b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(49) = 98.

Time = 14.94 (sec) , antiderivative size = 403, normalized size of antiderivative = 6.72

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x)}\,dx$$
$$= \begin{cases} \tilde{\infty}\left(2A\operatorname{atan}\left(\tanh\left(\frac{x}{2}\right)\right) + Bx\right) \\ \frac{A\tanh\left(\frac{x}{2}\right)}{b} + \frac{Bx}{b} - \frac{B\tanh\left(\frac{x}{2}\right)}{b} \\ -\frac{A}{b\tanh\left(\frac{x}{2}\right)} + \frac{Bx}{b} - \frac{B}{b\tanh\left(\frac{x}{2}\right)} \\ \frac{Ax + B\sinh(x)}{a} \\ -\frac{Ab\log\left(-\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{Ab\log\left(\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} + \tanh\left(\frac{x}{2}\right)\right)}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{Bax\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}} + \frac{Ba\log\left(-\sqrt{\frac{a}{a-b} + \frac{b}{a-b}}\right.}{ab\sqrt{\frac{a}{a-b} + \frac{b}{a-b}} - b^2\sqrt{}} \end{cases}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x), Eq(a, 0) & Eq(b, 0)), (A*tanh(
x/2)/b + B*x/b - B*tanh(x/2)/b, Eq(a, b)), (-A/(b*tanh(x/2)) + B*x/b - B/(
b*tanh(x/2)), Eq(a, -b)), ((A*x + B*sinh(x))/a, Eq(b, 0)), (-A*b*log(-sqrt
(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b*
*2*sqrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b
))) + B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) -
 b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*log(-sqrt(a/(a - b) + b/(a - b))
+ tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a
 - b))) - B*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a
- b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*x*sqrt(a/(a -
b) + b/(a - b))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b
/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.83

$$\int \frac{A + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{Bx}{b} - \frac{2\,(Ba - Ab)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}b}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*x/b - 2*(B*a - A*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^
2)*b)
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 4.03

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b^2\,e^x\,\sqrt{b^4-a^2\,b^2}\,\left(\frac{2\,\left(A\,b\,\sqrt{b^4-a^2\,b^2}-B\,a\,\sqrt{b^4-a^2\,b^2}\right)}{b^4\,\sqrt{b^4-a^2\,b^2}\,\sqrt{(A\,b-B\,a)^2}}+\frac{2\,a^2\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{b^2\,\left(b^4-a^2\,b^2\right)\,(A\,b-B\,a)}\right)}{2}+\frac{a\,b\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{\sqrt{b^4-a^2\,b^2}\,(A\,b-B\,a)}\right)\,\sqrt{A^2\,b^2}}{\sqrt{b^4-a^2\,b^2}} + \frac{B\,x}{b}$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x)),x)
```

output
```
(2*atan((b^2*exp(x)*(b^4 - a^2*b^2)^(1/2)*((2*(A*b*(b^4 - a^2*b^2)^(1/2) -
 B*a*(b^4 - a^2*b^2)^(1/2)))/(b^4*(b^4 - a^2*b^2)^(1/2)*((A*b - B*a)^2)^(1
/2)) + (2*a^2*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^2*(b^4 - a^2*b^2)*
(A*b - B*a))))/2 + (a*b*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/((b^4 - a^2
*b^2)^(1/2)*(A*b - B*a)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4 - a^
2*b^2)^(1/2) + (B*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.02

$$\int \frac{A+B\cosh(x)}{a+b\cosh(x)}\,dx = x$$

input
```
int((A+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
x
```

3.111 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 82

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx = \frac{2(aA-bB)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}} - \frac{(Ab-aB)\sinh(x)}{(a^2-b^2)\,(a+b\cosh(x))}$$

output
```
2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)
^(3/2)-(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.99

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx = \frac{2(aA-bB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\left(-a^2+b^2\right)^{3/2}} + \frac{(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^
(3/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b*Cosh[x]))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 3233, 25, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{aA-bB}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)\,(a+b\cosh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{aA-bB}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)\,(a+b\cosh(x))}$$

$$\Big\downarrow \text{27}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)\,(a+b\cosh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)\,(a+b\cosh(x))}+\frac{(aA-bB)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(aA-bB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)\,(a+b\cosh(x))}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}-\frac{\sinh(x)(Ab-aB)}{(a^2-b^2)\,(a+b\cosh(x))}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*
Sqrt[a + b]*(a^2 - b^2)) - ((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[
x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.32

method	result
default	$\frac{2(Ab-Ba)\tanh\left(\frac{x}{2}\right)}{(a^2-b^2)\left(\tanh\left(\frac{x}{2}\right)^2 a-b\tanh\left(\frac{x}{2}\right)^2-a-b\right)}+\frac{2(Aa-Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}$
risch	$\frac{2(Ab-Ba)(a\,\mathrm{e}^x+b)}{b(a^2-b^2)(\mathrm{e}^{2x}b+2a\,\mathrm{e}^x+b)}+\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Bb}{\sqrt{a^2-b^2}\,(a+b)(a-b)}-\frac{\ln\left(\mathrm{e}^x+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
2*(A*b-B*a)/(a^2-b^2)*tanh(1/2*x)/(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)+2*
(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)
*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(71) = 142$.

Time = 0.10 (sec) , antiderivative size = 828, normalized size of antiderivative = 10.10

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="fricas")
```

output
```
[-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a
*b^2 - B*b^3)*cosh(x)^2 + (A*a*b^2 - B*b^3)*sinh(x)^2 + 2*(A*a^2*b - B*a*b
^2)*cosh(x) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*cosh(x))*sinh(x))*s
qrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*
sinh(x) + b)) + 2*(B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*cosh(x) + 2*(B*a
^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(x))/(a^4*b^2 - 2*a^2*b^4 + b^6 +
(a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(x
)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cosh(x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5
 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x))*sinh(x)), -2*(B*a^3*b - A*a^2*b^2
- B*a*b^3 + A*b^4 + (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(x)^2 + (A*a*
b^2 - B*b^3)*sinh(x)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(x) + 2*(A*a^2*b - B*a*
b^2 + (A*a*b^2 - B*b^3)*cosh(x))*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^4 - A*a^3*b - B*
a^2*b^2 + A*a*b^3)*cosh(x) + (B*a^4 - A*a^3*b - B*a^2*b^2 + A*a*b^3)*sinh(
x))/(a^4*b^2 - 2*a^2*b^4 + b^6 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x)^2 + (
a^4*b^2 - 2*a^2*b^4 + b^6)*sinh(x)^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*cosh(
x) + 2*(a^5*b - 2*a^3*b^3 + a*b^5 + (a^4*b^2 - 2*a^2*b^4 + b^6)*cosh(x))*s
inh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.30

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^2}\,dx = \frac{2\,(Aa - Bb)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{(a^2 - b^2)\sqrt{-a^2 + b^2}} - \frac{2\,(Ba^2e^x - Aabe^x + Bab - Ab^2)}{(a^2b - b^3)(be^{(2\,x)} + 2\,ae^x + b)}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^2,x, algorithm="giac")
```

output
```
2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2
+ b^2)) - 2*(B*a^2*e^x - A*a*b*e^x + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2
*x) + 2*a*e^x + b))
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.00

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx = \frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,(a^2\,b-b^3)}-\frac{2\,e^x\,(B\,a^2\,b^2-A\,a\,b^3)}{b^2\,(a^2\,b-b^3)}}{b+2\,a\,e^x+b\,e^{2\,x}} + \frac{\ln\left(-\frac{2\,e^x\,(A\,a-B\,b)}{b\,(a^2-b^2)}-\frac{2\,(b+a\,e^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)\,(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{\ln\left(\frac{2\,(b+a\,e^x)\,(A\,a-B\,b)}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{2\,e^x\,(A\,a-B\,b)}{b\,(a^2-b^2)}\right)\,(A\,a-B\,b)}{(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^2,x)
```

output
```
((2*(A*b^3 - B*a*b^2))/(b*(a^2*b - b^3)) - (2*exp(x)*(B*a^2*b^2 - A*a*b^3)
)/(b^2*(a^2*b - b^3)))/(b + 2*a*exp(x) + b*exp(2*x)) + (log(- (2*exp(x)*(A
*a - B*b))/(b*(a^2 - b^2)) - (2*(b + a*exp(x))*(A*a - B*b))/(b*(a + b)^(3/
2)*(a - b)^(3/2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(b
 + a*exp(x))*(A*a - B*b))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(x)*(A*a
 - B*b))/(b*(a^2 - b^2)))*(A*a - B*b))/((a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.54

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^2}\,dx = -\frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)}{a^2-b^2}$$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^2,x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)))/(a**2
- b**2)
```

3.112 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 135

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3} \, dx = \frac{(2a^2A+Ab^2-3abB)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{(Ab-aB)\sinh(x)}{2(a^2-b^2)(a+b\cosh(x))^2} - \frac{(3aAb-a^2B-2b^2B)\sinh(x)}{2(a^2-b^2)^2(a+b\cosh(x))}$$

output
```
(2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)
^(5/2)/(a+b)^(5/2)-1/2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^2-1/2*(3*
A*a*b-B*a^2-2*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.99

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3} \, dx = \frac{1}{2}\left(-\frac{2(2a^2A+Ab^2-3abB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} + \frac{(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))^2} + \frac{(-3aAb+a^2B+2b^2B)\sinh(x)}{(a-b)^2(a+b)^2(a+b\cosh(x))}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

output
```
((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^
2]])/(-a^2 + b^2)^(5/2) + ((-(A*b) + a*B)*Sinh[x])/((a - b)*(a + b)*(a + b
*Cosh[x])^2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[x])/((a - b)^2*(a + b)^2
*(a + b*Cosh[x])))/2
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 3233, 25, 3042, 3233, 25, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{2(aA-bB)-(Ab-aB)\cosh(x)}{(a+b\cosh(x))^2}dx}{2\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cosh(x)}{(a+b\cosh(x))^2}dx}{2\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}+\frac{\int \frac{2(aA-bB)+(aB-Ab)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{(a^2-b^2)(a+b\cosh(x))}}{2\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\frac{\int \frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{(a^2-b^2)(a+b\cosh(x))}}{2\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\frac{\left(2a^2A-3abB+Ab^2\right)\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{(a^2-b^2)(a+b\cosh(x))}}{2\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}+\frac{-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{(a^2-b^2)(a+b\cosh(x))}+\frac{\left(2a^2A-3abB+Ab^2\right)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2\left(a^2-b^2\right)}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{\frac{2\left(2a^2A-3abB+Ab^2\right)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{(a^2-b^2)(a+b\cosh(x))}}{2\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{\frac{2\left(2a^2A-3abB+Ab^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\sinh(x)\left(a^2(-B)+3aAb-2b^2B\right)}{(a^2-b^2)(a+b\cosh(x))}}{2\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{2\left(a^2-b^2\right)(a+b\cosh(x))^2}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^3,x]
```

output
```
-1/2*((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])^2) + ((2*(2*a^2*A
+ A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a -
 b]*Sqrt[a + b]*(a^2 - b^2)) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[x])/((a^2
 - b^2)*(a + b*Cosh[x])))/(2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.53

method	result
default	$-\frac{2\left(-\frac{\left(4Aab+Ab^2-2Ba^2-Bab-2Bb^2\right)\tanh\left(\frac{x}{2}\right)^3}{2(a-b)\left(a^2+2ab+b^2\right)}+\frac{\left(4Aab-Ab^2-2Ba^2+Bab-2Bb^2\right)\tanh\left(\frac{x}{2}\right)}{2(a+b)\left(a^2-2ab+b^2\right)}\right)}{\left(\tanh\left(\frac{x}{2}\right)^2a-b\tanh\left(\frac{x}{2}\right)^2-a-b\right)^2}+\frac{\left(2Aa^2+Ab^2-3Bab\right)\operatorname{arctanh}\left(\right.}{\left(a^4-2a^2b^2+b^4\right)\sqrt{(a+b}}$
risch	$\frac{2Aa^2b^2e^{3x}+Ab^4e^{3x}-3Bab^3e^{3x}+6Aa^3be^{2x}+3Aab^3e^{2x}-2Ba^4e^{2x}-5Ba^2b^2e^{2x}-2Bb^4e^{2x}+10Aa^2b^2e^{x}-Ab^4e^{x}-4Ba^3be^{x}-5B}{b\left(a^2-b^2\right)^2\left(e^{2x}b+2ae^{x}+b\right)^2}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*tanh(
1/2*x)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*t
anh(1/2*x))/(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b
)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)
*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1545 vs. 2(120) = 240.

Time = 0.16 (sec) , antiderivative size = 3166, normalized size of antiderivative = 23.45

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^3}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. 2(120) = 240.

Time = 0.12 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.84

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^3}\, dx = \frac{(2\,Aa^2 - 3\,Bab + Ab^2)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4)\sqrt{-a^2 + b^2}} + \frac{2\,Aa^2b^2e^{(3\,x)} - 3\,Bab^3e^{(3\,x)} + Ab^4e^{(3\,x)} - 2\,Ba^4e^{(2\,x)} + 6\,Aa^3be^{(2\,x)} - 5\,Ba^2b^2e^{(2\,x)} + 3\,Aab^3e^{(2\,x)} - 2\,Bb}{(a^4b - 2\,a^2b^3 + b^5)(be^{(2\,x)} + 2\,a}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^3,x, algorithm="giac")
```

output
```
(2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2
*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*x) - 3*B*a*b^3*e^(3*
x) + A*b^4*e^(3*x) - 2*B*a^4*e^(2*x) + 6*A*a^3*b*e^(2*x) - 5*B*a^2*b^2*e^(
2*x) + 3*A*a*b^3*e^(2*x) - 2*B*b^4*e^(2*x) - 4*B*a^3*b*e^x + 10*A*a^2*b^2*
e^x - 5*B*a*b^3*e^x - A*b^4*e^x - B*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b
 - 2*a^2*b^3 + b^5)*(b*e^(2*x) + 2*a*e^x + b)^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^3,x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.67

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^3}\,dx = \frac{-2e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)ab-4e^x\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a}{e^{2x}a^4b-2e^{2x}a^2b^3+e^{2x}b^5+2e^x a^5-4e^x a^3b^2+2e^x a\,b^4+a^4b-2a^2b^3+b^5}$$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^3,x)
```

output
```
( - 2*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)
)*a*b - 4*e**x*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)
)*a**2 - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a*
b - e**(2*x)*a**2*b + e**(2*x)*b**3 + a**2*b - b**3)/(e**(2*x)*a**4*b - 2*
e**(2*x)*a**2*b**3 + e**(2*x)*b**5 + 2*e**x*a**5 - 4*e**x*a**3*b**2 + 2*e*
*x*a*b**4 + a**4*b - 2*a**2*b**3 + b**5)
```

3.113 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 197

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} - \frac{(Ab-aB)\sinh(x)}{3\,(a^2-b^2)\,(a+b\cosh(x))^3} - \frac{(5aAb-2a^2B-3b^2B)\sinh(x)}{6\,(a^2-b^2)^2\,(a+b\cosh(x))^2} - \frac{(11a^2Ab+4Ab^3-2a^3B-13ab^2B)\sinh(x)}{6\,(a^2-b^2)^3\,(a+b\cosh(x))}$$

output
```
(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^
(1/2))/(a-b)^(7/2)/(a+b)^(7/2)-1/3*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x
))^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+b*cosh(x))^2-1/6
*(11*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*sinh(x)/(a^2-b^2)^3/(a+b*cosh(x))
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.99

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \frac{1}{6}\left(\frac{6(2a^3A+3aAb^2-4a^2bB-b^3B)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} + \frac{2(-Ab+aB)\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))^3} + \frac{(-5aAb+2a^2B+3b^2B)\sinh(x)}{(a-b)^2(a+b)^2(a+b\cosh(x))^2} + \frac{(-11a^2Ab-4Ab^3+2a^3B+13ab^2B)\sinh(x)}{(a-b)^3(a+b)^3(a+b\cosh(x))}\right)$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

output
```
((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[x/2])/S
qrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + (2*(-(A*b) + a*B)*Sinh[x])/((a - b)
*(a + b)*(a + b*Cosh[x])^3) + ((-5*a*A*b + 2*a^2*B + 3*b^2*B)*Sinh[x])/((a
 - b)^2*(a + b)^2*(a + b*Cosh[x])^2) + ((-11*a^2*A*b - 4*A*b^3 + 2*a^3*B +
 13*a*b^2*B)*Sinh[x])/((a - b)^3*(a + b)^3*(a + b*Cosh[x])))/6
```

Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.21, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^4}dx$$

$$\Big\downarrow \; 3233$$

$$-\frac{\int -\frac{3(aA-bB)-2(Ab-aB)\cosh(x)}{(a+b\cosh(x))^3}dx}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\cosh(x)}{(a+b\cosh(x))^3}dx}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{\int \frac{3(aA-bB)-2(Ab-aB)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3233$$

$$\frac{-\frac{\int -\frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cosh(x)}{(a+b\cosh(x))^2}dx}{2(a^2-b^2)}-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)-\left(-2Ba^2+5Aba-3b^2B\right)\cosh(x)}{(a+b\cosh(x))^2}dx}{2(a^2-b^2)}-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{-\frac{\sinh(x)\left(-2a^2B+5aAb-3b^2B\right)}{2(a^2-b^2)(a+b\cosh(x))^2}+\frac{\int \frac{2\left(3Aa^2-5bBa+2Ab^2\right)+\left(2Ba^2-5Aba+3b^2B\right)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3233$$

$$\frac{\frac{-\frac{\int -\frac{3\left(2Aa^3-4bBa^2+3Ab^2a-b^3B\right)}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int\frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}+\frac{-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}+\frac{-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}+\frac{3\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}}{3\left(a^2-b^2\right)}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{\frac{6\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\sinh(x)\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)}{\left(a^2-b^2\right)(a+b\cosh(x))}}{2(a^2-b^2)}-\frac{\sinh(x)(-2a^2B+5aAb-3b^2B)}{2(a^2-b^2)(a+b\cosh(x))^2}}{3\left(a^2-b^2\right)}-\frac{\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^3}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^4,x]
```

output
```
-1/3*((A*b - a*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])^3) + (-1/2*((5*a*A
*b - 2*a^2*B - 3*b^2*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])^2) + ((6*(2*
a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqr
t[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - ((11*a^2*A*b + 4*A*b^3
- 2*a^3*B - 13*a*b^2*B)*Sinh[x])/((a^2 - b^2)*(a + b*Cosh[x])))/(2*(a^2 -
b^2)))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 1.07 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.74

method	result
default	$-\frac{2\left(-\frac{\left(6A\,a^2b+3Aa\,b^2+2A\,b^3-2a^3B-2B\,a^2b-6Ba\,b^2-B\,b^3\right)\tanh\left(\frac{x}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}+\frac{2\left(9A\,a^2b+A\,b^3-3a^3B-7Ba\,b^2\right)\tanh\left(\frac{x}{2}\right)^3}{3\left(a^2+2ab+b^2\right)\left(a^2-2ab+b^2\right)}-\frac{\left(6A\,a^2b-3Aa\,b^2+2A\right.}{2(}\right.}{\left(\tanh\left(\frac{x}{2}\right)^2a-b\tanh\left(\frac{x}{2}\right)^2-a-b\right)^3}$
risch	$9Aa\,b^5\mathrm{e}^{5x}-12B\,a^2b^4\mathrm{e}^{5x}+30A\,a^4b^2\mathrm{e}^{4x}+45A\,a^2b^4\mathrm{e}^{4x}-60B\,a^3b^3\mathrm{e}^{4x}-15Ba\,b^5\mathrm{e}^{4x}+44A\,a^5b\,\mathrm{e}^{3x}+82A\,a^3b^3\mathrm{e}^{3x}+24Aa\,b^5\mathrm{e}^{3x}-64B\,a^4$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^4,x,method=_RETURNVERBOSE)
```

output
```
-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^3)/(
a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*x)^5+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-
7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tanh(1/2*x)^3-1/2*(6*A*a^2*b-3*
A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a^3-3*a^2*b+3*a*
b^2-b^3)*tanh(1/2*x))/(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)^3+(2*A*a^3+3*A
*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*
arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3767 vs. 2(181) = 362.

Time = 0.33 (sec) , antiderivative size = 7603, normalized size of antiderivative = 38.59

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(181) = 362$.

Time = 0.12 (sec) , antiderivative size = 453, normalized size of antiderivative = 2.30

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \frac{(2\,Aa^3 - 4\,Ba^2b + 3\,Aab^2 - Bb^3)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\sqrt{-a^2+b^2}} + \frac{6\,Aa^3b^3e^{(5\,x)} - 12\,Ba^2b^4e^{(5\,x)} + 9\,Aab^5e^{(5\,x)} - 3\,Bb^6e^{(5\,x)} + 30\,Aa^4b^2e^{(4\,x)} - 60\,Ba^3b^3e^{(4\,x)} + 45\,Aa^2b^4e^{(}}{}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^4,x, algorithm="giac")
```

output
```
(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^x + a)/sqrt(-a^2 + b
^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) + 1/3*(6*A*a^3
*b^3*e^(5*x) - 12*B*a^2*b^4*e^(5*x) + 9*A*a*b^5*e^(5*x) - 3*B*b^6*e^(5*x)
+ 30*A*a^4*b^2*e^(4*x) - 60*B*a^3*b^3*e^(4*x) + 45*A*a^2*b^4*e^(4*x) - 15*
B*a*b^5*e^(4*x) - 8*B*a^6*e^(3*x) + 44*A*a^5*b*e^(3*x) - 64*B*a^4*b^2*e^(3
*x) + 82*A*a^3*b^3*e^(3*x) - 78*B*a^2*b^4*e^(3*x) + 24*A*a*b^5*e^(3*x) - 2
4*B*a^5*b*e^(2*x) + 102*A*a^4*b^2*e^(2*x) - 102*B*a^3*b^3*e^(2*x) + 36*A*a
^2*b^4*e^(2*x) - 24*B*a*b^5*e^(2*x) + 12*A*b^6*e^(2*x) - 12*B*a^4*b^2*e^x
+ 60*A*a^3*b^3*e^x - 66*B*a^2*b^4*e^x + 15*A*a*b^5*e^x + 3*B*b^6*e^x - 2*B
*a^3*b^3 + 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^6)/((a^6*b - 3*a^4*b^3 + 3*a^
2*b^5 - b^7)*(b*e^(2*x) + 2*a*e^x + b)^3)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)\right)^4}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^4,x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 817, normalized size of antiderivative = 4.15

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^4}\,dx = \frac{-8e^{4x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^3b^2 - 4e^{4x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a\,b^4 - 32e^{3x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x}{\sqrt{-}}\right.}{}$$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^4,x)
```

output

```
( - 8*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)
)*a**3*b**2 - 4*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a*
*2 + b**2))*a*b**4 - 32*e**(3*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sq
rt( - a**2 + b**2))*a**4*b - 16*e**(3*x)*sqrt( - a**2 + b**2)*atan((e**x*b
 + a)/sqrt( - a**2 + b**2))*a**2*b**3 - 32*e**(2*x)*sqrt( - a**2 + b**2)*a
tan((e**x*b + a)/sqrt( - a**2 + b**2))*a**5 - 32*e**(2*x)*sqrt( - a**2 + b
**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**3*b**2 - 8*e**(2*x)*sqrt(
- a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a*b**4 - 32*e**x*sq
rt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**4*b - 16*e**
x*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**2*b**3 -
 8*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**3*b**2
- 4*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a*b**4 -
2*e**(4*x)*a**4*b**2 + e**(4*x)*a**2*b**4 + e**(4*x)*b**6 + 16*e**(2*x)*a*
*6 - 12*e**(2*x)*a**4*b**2 - 6*e**(2*x)*a**2*b**4 + 2*e**(2*x)*b**6 + 32*e
**x*a**5*b - 40*e**x*a**3*b**3 + 8*e**x*a*b**5 + 10*a**4*b**2 - 11*a**2*b*
*4 + b**6)/(4*a*(e**(4*x)*a**6*b**2 - 3*e**(4*x)*a**4*b**4 + 3*e**(4*x)*a*
*2*b**6 - e**(4*x)*b**8 + 4*e**(3*x)*a**7*b - 12*e**(3*x)*a**5*b**3 + 12*e
**(3*x)*a**3*b**5 - 4*e**(3*x)*a*b**7 + 4*e**(2*x)*a**8 - 10*e**(2*x)*a**6
*b**2 + 6*e**(2*x)*a**4*b**4 + 2*e**(2*x)*a**2*b**6 - 2*e**(2*x)*b**8 + 4*
e**x*a**7*b - 12*e**x*a**5*b**3 + 12*e**x*a**3*b**5 - 4*e**x*a*b**7 + a...
```

3.114 $\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 56

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}-\frac{2\sqrt{a-b}\sqrt{a+b}B\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{ab}$$

output
```
B*x/b-2*(a-b)^(1/2)*(a+b)^(1/2)*B*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1
/2))/a/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{B\left(\frac{ax}{b}+\frac{2\sqrt{-a^2+b^2}\arctan\left(\frac{(-a+b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b}\right)}{a}$$

input
```
Integrate[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*((a*x)/b + (2*Sqrt[-a^2 + b^2]*ArcTan[((-a + b)*Tanh[x/2])/Sqrt[-a^2 +
b^2]])/b))/a
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.12, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\frac{bB}{a}+B\sin\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$\frac{Bx}{b}-\frac{B\left(a-\frac{b^2}{a}\right)\int \frac{1}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{Bx}{b}-\frac{B\left(a-\frac{b^2}{a}\right)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{Bx}{b}-\frac{2B\left(a-\frac{b^2}{a}\right)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{Bx}{b}-\frac{2B\left(a-\frac{b^2}{a}\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[((b*B)/a + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b - (2*(a - b^2/a)*B*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(
Sqrt[a - b]*b*Sqrt[a + b])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{2B\left(\frac{a\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2b}-\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{2b}-\frac{\left(a^{2}-b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}\right)}{a}$	81
risch	$\frac{Bx}{b}+\frac{\sqrt{a^{2}-b^{2}}\,B\ln\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)}{ba}-\frac{\sqrt{a^{2}-b^{2}}\,B\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}\right)}{ba}$	92

input
```
int((b*B/a+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*B/a*(1/2/b*a*ln(1+tanh(1/2*x))-1/2/b*a*ln(tanh(1/2*x)-1)-(a^2-b^2)/b/((a
+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 190, normalized size of antiderivative = 3.39

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx = \left[\frac{Bax+\sqrt{a^2-b^2}B\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.}{ab}\right.$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(B*a*x + sqrt(a^2 - b^2)*B*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)))/(a*b), (B*a*x + 2*sqrt(-a^2 + b^2)*B*arctan(-s
qrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a*b)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. 2(44) = 88.

Time = 14.51 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.00

$$\int \frac{\frac{bB}{a}+B\cosh(x)}{a+b\cosh(x)}\,dx = \begin{cases} \text{NaN} \\ \frac{B\sinh(x)}{a} \\ \frac{Bx}{b} \\ \frac{Bx}{b}+\frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{b\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}+\frac{B\log\left(-\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}+\tanh\left(\frac{x}{2}\right)\right)}{a\sqrt{\frac{a}{a-b}+\frac{b}{a-b}}}-\frac{B\log\left(\sqrt{\frac{a}{a-b}+\frac{a}{a}}\right.}{a\sqrt{\frac{a}{a-b}}} \end{cases}$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
Piecewise((nan, Eq(a, 0) & Eq(b, 0)), (B*sinh(x)/a, Eq(b, 0)), (B*x/b, Eq(
a, b) | Eq(a, -b)), (B*x/b + B*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2
))/(b*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) + b/(a - b)) + t
anh(x/2))/(b*sqrt(a/(a - b) + b/(a - b))) + B*log(-sqrt(a/(a - b) + b/(a -
 b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))) - B*log(sqrt(a/(a - b) +
 b/(a - b)) + tanh(x/2))/(a*sqrt(a/(a - b) + b/(a - b))), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.02

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{Bx}{b} - \frac{2\,(Ba^2 - Bb^2)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}ab}$$

input
```
integrate((b*B/a+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*x/b - 2*(B*a^2 - B*b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2
+ b^2)*a*b)
```

Mupad [B] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 205, normalized size of antiderivative = 3.66

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{b\sqrt{a^2\,b^2}\,\sqrt{B^2\,b^2-B^2\,a^2}}{B\,\left(b^4-a^2\,b^2\right)} + \frac{a\,b^2\,\mathrm{e}^x\,\left(\frac{2\,\sqrt{B^2\,b^2-B^2\,a^2}}{B\,b^2\,\left(b^4-a^2\,b^2\right)} - \frac{2\,\left(B\,a^2\,\sqrt{a^2\,b^2}-B\,b^2\,\sqrt{a^2\,b^2}\right)}{a^2\,b^4\,\sqrt{-B^2\,\left(a^2-b^2\right)}\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{2}\right)\,\sqrt{B^2\,b^2-B^2\,a^2}}{\sqrt{a^2\,b^2}} + \frac{B\,x}{b}$$

input
```
int((B*cosh(x) + (B*b)/a)/(a + b*cosh(x)),x)
```

output
```
(2*atan((b*(a^2*b^2)^(1/2)*(B^2*b^2 - B^2*a^2)^(1/2))/(B*(b^4 - a^2*b^2))
+ (a*b^2*exp(x)*((2*(B^2*b^2 - B^2*a^2)^(1/2))/(B*b^2*(b^4 - a^2*b^2)) - (
2*(B*a^2*(a^2*b^2)^(1/2) - B*b^2*(a^2*b^2)^(1/2)))/(a^2*b^4*(-B^2*(a^2 - b
^2))^(1/2)*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/2)*(B^2*b^2 - B^2*a^2)^(1/2)
)/(a^2*b^2)^(1/2) + (B*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.73

$$\int \frac{\frac{bB}{a} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right) + ax}{a}$$

input
```
int((b*B/a+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
(2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)) + a*x)/a
```

3.115 $\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 6

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}$$

output
```
B*x/b
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}$$

input
```
Integrate[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2011, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{ 2011}$$

$$\frac{B\int 1dx}{b}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{Bx}{b}$$

input
```
Int[((a*B)/b + B*Cosh[x])/(a + b*Cosh[x]),x]
```

output
```
(B*x)/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 2011
```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Simp[(b/d)^m   Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x
] && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x
, a + b*x])
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 7, normalized size of antiderivative = 1.17

method	result	size
default	$\frac{Bx}{b}$	7
risch	$\frac{Bx}{b}$	7
orering	$\frac{x\left(\frac{aB}{b}+B\cosh(x)\right)}{a+b\cosh(x)}$	22

input
```
int((a*B/b+B*cosh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x/b
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
B*x/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.50

$$\int \frac{\frac{aB}{b}+B\cosh(x)}{a+b\cosh(x)}\,dx=\frac{Bx}{b}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
B*x/b
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{Bx}{b}$$

input
```
integrate((a*B/b+B*cosh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*x/b
```

Mupad [B] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 6, normalized size of antiderivative = 1.00

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\,dx = \frac{B\,x}{b}$$

input
```
int((B*cosh(x) + (B*a)/b)/(a + b*cosh(x)),x)
```

output
```
(B*x)/b
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.17

$$\int \frac{\frac{aB}{b} + B\cosh(x)}{a + b\cosh(x)}\,dx = x$$

input
```
int((a*B/b+B*cosh(x))/(a+b*cosh(x)),x)
```

output
```
x
```

3.116 $\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 11

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx = \frac{\sinh(x)}{b+a\cosh(x)}$$

output
```
sinh(x)/(b+a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx = \frac{\sinh(x)}{b+a\cosh(x)}$$

input
```
Integrate[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

output
```
Sinh[x]/(b + a*Cosh[x])
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 3233, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+b\cosh(x)}{(a\cosh(x)+b)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a+b\sin\left(\frac{\pi}{2}+ix\right)}{\left(b+a\sin\left(\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{3233}$$

$$\frac{\int 0dx}{a^2-b^2}+\frac{\sinh(x)}{a\cosh(x)+b}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh(x)}{a\cosh(x)+b}$$

input
```
Int[(a + b*Cosh[x])/(b + a*Cosh[x])^2,x]
```

output
```
Sinh[x]/(b + a*Cosh[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
parallelrisch	$\frac{\sinh(x)}{b+\cosh(x)a}$	12
risch	$-\frac{2(e^{x}b+a)}{a(e^{2x}a+2\,e^{x}b+a)}$	27
default	$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}a-b\tanh\left(\frac{x}{2}\right)^{2}+a+b}$	29

input
```
int((a+b*cosh(x))/(b+cosh(x)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
sinh(x)/(b+cosh(x)*a)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(11) = 22.

Time = 0.07 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.91

$$\int \frac{a+b\cosh(x)}{(b+a\cosh(x))^2}\,dx$$
$$= -\frac{2\,(b\cosh{(x)}+b\sinh{(x)}+a)}{a^2\cosh{(x)}^2+a^2\sinh{(x)}^2+2\,ab\cosh{(x)}+a^2+2\,(a^2\cosh{(x)}+ab)\sinh{(x)}}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="fricas")
```

output
```
-2*(b*cosh(x) + b*sinh(x) + a)/(a^2*cosh(x)^2 + a^2*sinh(x)^2 + 2*a*b*cosh
(x) + a^2 + 2*(a^2*cosh(x) + a*b)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2} \, dx = \text{Timed out}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 26 vs. $2(11) = 22$.

Time = 0.12 (sec) , antiderivative size = 26, normalized size of antiderivative = 2.36

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2}\,dx = -\frac{2\,(be^x + a)}{(ae^{(2\,x)} + 2\,be^x + a)a}$$

input
```
integrate((a+b*cosh(x))/(b+a*cosh(x))^2,x, algorithm="giac")
```

output
```
-2*(b*e^x + a)/((a*e^(2*x) + 2*b*e^x + a)*a)
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 51, normalized size of antiderivative = 4.64

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2}\,dx = -\frac{\frac{2\,\mathrm{e}^x\,(a\,b^3 - a^3\,b)}{a\,(a\,b^2 - a^3)} + 2}{a + 2\,b\,\mathrm{e}^x + a\,\mathrm{e}^{2\,x}}$$

input
```
int((a + b*cosh(x))/(b + a*cosh(x))^2,x)
```

output
```
-((2*exp(x)*(a*b^3 - a^3*b))/(a*(a*b^2 - a^3)) + 2)/(a + 2*b*exp(x) + a*ex
p(2*x))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 2.27

$$\int \frac{a + b\cosh(x)}{(b + a\cosh(x))^2}\,dx = \frac{e^{2x} - 1}{e^{2x}a + 2e^x b + a}$$

input
```
int((a+b*cosh(x))/(b+a*cosh(x))^2,x)
```

output
```
(e**(2*x) - 1)/(e**(2*x)*a + 2*e***x*b + a)
```

3.117 $\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 36

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -x+\frac{5x}{\sqrt{3}}+\frac{10\operatorname{arctanh}\left(\frac{\sinh(x)}{2+\sqrt{3}-\cosh(x)}\right)}{\sqrt{3}}$$

output
```
-x+5/3*x*3^(1/2)+10/3*arctanh(sinh(x)/(2+3^(1/2)-cosh(x)))*3^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.67

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -x+\frac{10\operatorname{arctanh}\left(\sqrt{3}\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{3}}$$

input
```
Integrate[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

output
```
-x + (10*ArcTanh[Sqrt[3]*Tanh[x/2]])/Sqrt[3]
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 3214, 3042, 3136}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(x)+3}{2-\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{3+\sin\left(\frac{\pi}{2}+ix\right)}{2-\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3214}$$

$$5\int \frac{1}{2-\cosh(x)}dx-x$$

$$\Big\downarrow \text{3042}$$

$$-x+5\int \frac{1}{2-\sin\left(ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3136}$$

$$5\left(\frac{2\text{arctanh}\left(\frac{\sinh(x)}{-\cosh(x)+\sqrt{3}+2}\right)}{\sqrt{3}}+\frac{x}{\sqrt{3}}\right)-x$$

input
```
Int[(3 + Cosh[x])/(2 - Cosh[x]),x]
```

output
```
-x + 5*(x/Sqrt[3] + (2*ArcTanh[Sinh[x]/(2 + Sqrt[3] - Cosh[x])])/Sqrt[3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3136
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{q = Rt[
a^2 - b^2, 2]}, Simp[x/q, x] + Simp[(2/(d*q))*ArcTan[b*(Cos[c + d*x]/(a + q
 + b*Sin[c + d*x]))], x]] /; FreeQ[{a, b, c, d}, x] && GtQ[a^2 - b^2, 0] &&
 PosQ[a]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

method	result	size
default	$-\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+\frac{10\sqrt{3}\,\operatorname{arctanh}\left(\tanh\left(\frac{x}{2}\right)\sqrt{3}\right)}{3}$	32
risch	$-x+\frac{5\sqrt{3}\,\ln\left(e^{x}-2+\sqrt{3}\right)}{3}-\frac{5\sqrt{3}\,\ln\left(e^{x}-2-\sqrt{3}\right)}{3}$	33

input
```
int((3+cosh(x))/(2-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-ln(1+tanh(1/2*x))+ln(tanh(1/2*x)-1)+10/3*3^(1/2)*arctanh(tanh(1/2*x)*3^(1
/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = \frac{5}{3}\sqrt{3}\log\left(-\frac{2\left(\sqrt{3}-2\right)\cosh\left(x\right)-\left(2\sqrt{3}-3\right)\sinh\left(x\right)-\sqrt{3}+2}{\cosh\left(x\right)-2}\right) - x$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="fricas")
```

output
```
5/3*sqrt(3)*log(-(2*(sqrt(3) - 2)*cosh(x) - (2*sqrt(3) - 3)*sinh(x) - sqrt
(3) + 2)/(cosh(x) - 2)) - x
```

Sympy [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.22

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -x - \frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)-\frac{\sqrt{3}}{3}\right)}{3} + \frac{5\sqrt{3}\log\left(\tanh\left(\frac{x}{2}\right)+\frac{\sqrt{3}}{3}\right)}{3}$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x)
```

output
```
-x - 5*sqrt(3)*log(tanh(x/2) - sqrt(3)/3)/3 + 5*sqrt(3)*log(tanh(x/2) + sq
rt(3)/3)/3
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = \frac{5}{3}\sqrt{3}\log\left(-\frac{\sqrt{3}-e^{(-x)}+2}{\sqrt{3}+e^{(-x)}-2}\right) - x$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="maxima")
```

output
```
5/3*sqrt(3)*log(-(sqrt(3) - e^(-x) + 2)/(sqrt(3) + e^(-x) - 2)) - x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.03

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -\frac{5}{3}\sqrt{3}\log\left(\frac{\left|-2\sqrt{3}+2e^x-4\right|}{\left|2\sqrt{3}+2e^x-4\right|}\right)-x$$

input
```
integrate((3+cosh(x))/(2-cosh(x)),x, algorithm="giac")
```

output
```
-5/3*sqrt(3)*log(abs(-2*sqrt(3) + 2*e^x - 4)/abs(2*sqrt(3) + 2*e^x - 4)) -
 x
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.33

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = \frac{5\sqrt{3}\,\ln\left(10\,e^x+\frac{5\sqrt{3}\,(4\,e^x-2)}{3}\right)}{3}-\frac{5\sqrt{3}\,\ln\left(10\,e^x-\frac{5\sqrt{3}\,(4\,e^x-2)}{3}\right)}{3}-x$$

input
```
int(-(cosh(x) + 3)/(cosh(x) - 2),x)
```

output
```
(5*3^(1/2)*log(10*exp(x) + (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - (5*3^(1/2)*l
og(10*exp(x) - (5*3^(1/2)*(4*exp(x) - 2))/3))/3 - x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.83

$$\int \frac{3+\cosh(x)}{2-\cosh(x)}\,dx = -\frac{5\sqrt{3}\log\left(e^{x}-\sqrt{3}-2\right)}{3}+\frac{5\sqrt{3}\log\left(e^{x}+\sqrt{3}-2\right)}{3}-x$$

input
```
int((3+cosh(x))/(2-cosh(x)),x)
```

output
```
( - 5*sqrt(3)*log(e**x - sqrt(3) - 2) + 5*sqrt(3)*log(e**x + sqrt(3) - 2)
- 3*x)/3
```

3.118 $\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 108

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2iB\sqrt{a+b\cosh(x)}\,E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*B*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b))^(1/2)
)/b/((a+b*cosh(x))/(a+b))^(1/2)-2*I*(A*b-B*a)*((a+b*cosh(x))/(a+b))^(1/2)*
InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/b/(a+b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2i\sqrt{\frac{a+b\cosh(x)}{a+b}}\left((a+b)BE\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)+(Ab-aB)\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{b\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*Sqrt[(a + b*Cosh[x])/(a + b)]*((a + b)*B*EllipticE[(I/2)*x, (2*b)/
(a + b)] + (A*b - a*B)*EllipticF[(I/2)*x, (2*b)/(a + b)]))/(b*Sqrt[a + b*C
osh[x]])
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\sqrt{a+b\sin\left(\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{3231}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}+\frac{B\int \sqrt{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3134}$$

$$\frac{B\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{B\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3132}$$

$$\frac{(Ab-aB)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3142}$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

$$\Big\downarrow \text{3140}$$

$$-\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2iB\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$

input
```
Int[(A + B*Cosh[x])/Sqrt[a + b*Cosh[x]],x]
```

output
```
((-2*I)*B*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[(
a + b*Cosh[x])/(a + b)]) - ((2*I)*(A*b - a*B)*Sqrt[(a + b*Cosh[x])/(a + b)
]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(105) = 210$.

Time = 6.93 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.02

method	result
default	$\frac{2\left(A\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)+B\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)-2B\,\text{EllipticE}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\sqrt{-}\right.\right.}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\sinh(}}$
parts	$\frac{2A\sqrt{\left(2b\cosh\left(\frac{x}{2}\right)^{2}+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\sqrt{\frac{2b\cosh\left(\frac{x}{2}\right)^{2}+a-b}{a-b}}\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\,\text{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)}{\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\,\sinh\left(\frac{x}{2}\right)\sqrt{2b\sinh\left(\frac{x}{2}\right)^{2}+a+b}}+\frac{2B\left(\text{Elliptic}\right.}{}$
risch	$\frac{B\left(e^{2x}b+2a\,e^{x}+b\right)\sqrt{2}\,e^{-x}}{b\sqrt{\left(e^{2x}b+2a\,e^{x}+b\right)e^{-x}}}+\frac{\left(\frac{4A\left(a+\sqrt{a^{2}-b^{2}}\right)\sqrt{\frac{\left(e^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)b}{a+\sqrt{a^{2}-b^{2}}}}\sqrt{\frac{e^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}{-\frac{a+\sqrt{a^{2}-b^{2}}}{b}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}}}\sqrt{-\frac{e^{x}b}{a+\sqrt{a^{2}-b^{2}}}}\,\text{EllipticF}\left(\sqrt{\frac{(e^{x}}{}}\right.}{b\sqrt{e^{3x}b+2\,e^{2x}a+e^{x}b}}\right.}{}$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2*(A*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))+B*El
lipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2))-2*B*Ellipti
cE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*(-2/b*(a-b))^(1/2)))*(-sinh(1/2*x)^2
)^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)*sin
h(1/2*x)^2)^(1/2)/(-2*b/(a-b))^(1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^
2)^(1/2)/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.70

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx =$$
$$-\frac{2\left(6\sqrt{\frac{1}{2}}Bb^{\frac{3}{2}}\text{weierstrassZeta}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\text{weierstrassPInverse}\left(\frac{4\,(4\,a^2-3\,b^2)}{3\,b^2},-\frac{8\,(8\,a^3-9\,ab^2)}{27\,b^3},\right.\right.\right.}{}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
-2/3*(6*sqrt(1/2)*B*b^(3/2)*weierstrassZeta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27
*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b)) + 2*sqrt
(1/2)*(2*B*a - 3*A*b)*sqrt(b)*weierstrassPInverse(4/3*(4*a^2 - 3*b^2)/b^2,
 -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x) + 2*a)/b) + 3
*sqrt(b*cosh(x) + a)*B*b)/b^2
```

Sympy [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{A+B\cosh{(x)}}{\sqrt{a+b\cosh{(x)}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral((A + B*cosh(x))/sqrt(a + b*cosh(x)), x)
```

Maxima [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx=\int \frac{B\cosh\left(x\right)+A}{\sqrt{b\cosh\left(x\right)+a}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

Giac [F]

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx=\int \frac{B\cosh\left(x\right)+A}{\sqrt{b\cosh\left(x\right)+a}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)/sqrt(b*cosh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh(x)}}\,dx=\int \frac{A+B\cosh(x)}{\sqrt{a+b\cosh\left(x\right)}}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^(1/2),x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{A + B\cosh(x)}{\sqrt{a + b\cosh(x)}}\,dx = \int \sqrt{\cosh\left(x\right)b + a}dx$$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(1/2),x)
```

output
```
int(sqrt(cosh(x)*b + a),x)
```

3.119 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 152

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = -\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}} - \frac{2iB\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}} - \frac{2(Ab-aB)\sinh(x)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

output
```
-2*I*(A*b-B*a)*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2)*(b/(a+b
))^(1/2))/b/(a^2-b^2)/((a+b*cosh(x))/(a+b))^(1/2)-2*I*B*((a+b*cosh(x))/(a+
b))^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(a+b))^(1/2))/b/(a+b*cosh(x))
^(1/2)-2*(A*b-B*a)*sinh(x)/(a^2-b^2)/(a+b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.88

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = \frac{2i(a+b)(-Ab+aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right) - 2i(a^2-b^2)B\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{Elliptic}}{(a-b)b(a+b)\sqrt{a+b\cosh(x)}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

output
```
((2*I)*(a + b)*(-(A*b) + a*B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticE[(I/2
)*x, (2*b)/(a + b)] - (2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cosh[x])/(a + b)]*El
lipticF[(I/2)*x, (2*b)/(a + b)] + 2*b*(-(A*b) + a*B)*Sinh[x])/((a - b)*b*(
a + b)*Sqrt[a + b*Cosh[x]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{2\int -\frac{aA-bB+(Ab-aB)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{aA-bB+(Ab-aB)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\int \frac{aA-bB+(Ab-aB)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{B(a^2-b^2)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}+\frac{(Ab-aB)\int \sqrt{a+b\cosh(x)}dx}{b}}{a^2-b^2}-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}$$

$\big\downarrow$ 3042

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{(Ab-aB)\int \sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{a^2-b^2}$$

$\big\downarrow$ 3134

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{(Ab-aB)\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}+\frac{B(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}$$

$\big\downarrow$ 3042

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}+\frac{(Ab-aB)\sqrt{a+b\cosh(x)}\int \sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$\big\downarrow$ 3132

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\int \frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$\big\downarrow$ 3142

$$-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int \frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$\big\downarrow$ 3042

$$\frac{-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{B(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

$$\Big\downarrow \text{3140}$$

$$\frac{-\frac{2\sinh(x)(Ab-aB)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+-\frac{2iB(a^2-b^2)\sqrt{\frac{a+b\cosh(x)}{a+b}}\,\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i(Ab-aB)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(3/2),x]
```

output
```
(((-2*I)*(A*b - a*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/(a + b)]
)/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) - ((2*I)*(a^2 - b^2)*B*Sqrt[(a + b*Cos
h[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqrt[a + b*Cosh[x]]))
/(a^2 - b^2) - (2*(A*b - a*B)*Sinh[x])/((a^2 - b^2)*Sqrt[a + b*Cosh[x]])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 484 vs. $2(147) = 294$.

Time = 7.42 (sec) , antiderivative size = 485, normalized size of antiderivative = 3.19

method	result
default	$\frac{\sqrt{\left(2b\cosh\left(\frac{x}{2}\right)^2+a-b\right)\sinh\left(\frac{x}{2}\right)^2}\left(\frac{2B\sqrt{\frac{2b\cosh\left(\frac{x}{2}\right)^2+a-b}{a-b}}\sqrt{-\sinh\left(\frac{x}{2}\right)^2}\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{\frac{-2a+2b}{b}}}{2}\right)}{b\sqrt{-\frac{2b}{a-b}}\sqrt{2\sinh\left(\frac{x}{2}\right)^4b+(a+b)\sinh\left(\frac{x}{2}\right)^2}}-\frac{2(Ab-Ba)\sqrt{2\sinh\left(\right.}}{}\right.}{}$
parts	$-\frac{2A\left(2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\sinh\left(\frac{x}{2}\right)^2b-\sqrt{-\sinh\left(\frac{x}{2}\right)^2}\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^2}{a-b}+\frac{a+b}{a-b}}\,\mathrm{EllipticF}\left(\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}},\frac{\sqrt{-\frac{2(a-b)}{b}}}{2}\right)a-\sqrt{-\sinh\left(\right.}\right.}{\sqrt{-\frac{2b}{a-b}}\,(a-}$

input

```
int((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x,method=_RETURNVERBOSE)
```

output

```
((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(2*B/b/(-2*b/(a-b))^(1/2)*((
2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2)/(2*sinh(1/2*x)^
4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/
2*((-2*a+2*b)/b)^(1/2))-2*(A*b-B*a)/b/sinh(1/2*x)^2/(2*b*sinh(1/2*x)^2+a+b
)/(-2*b/(a-b))^(1/2)/(a^2-b^2)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/
2)*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sinh(1/2*x)^2)^(1/2
)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/
(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)
*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-2*b/(a-b))^(1/2)
,1/2*((-2*a+2*b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*
x)^2+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*
a+2*b)/b)^(1/2))*b))/sinh(1/2*x)/(2*b*sinh(1/2*x)^2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(144) = 288$.

Time = 0.09 (sec) , antiderivative size = 601, normalized size of antiderivative = 3.95

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="fricas")
```

output
```
4/3*(sqrt(1/2)*(2*B*a^2*b + A*a*b^2 - 3*B*b^3 + (2*B*a^2*b + A*a*b^2 - 3*B
*b^3)*cosh(x)^2 + (2*B*a^2*b + A*a*b^2 - 3*B*b^3)*sinh(x)^2 + 2*(2*B*a^3 +
 A*a^2*b - 3*B*a*b^2)*cosh(x) + 2*(2*B*a^3 + A*a^2*b - 3*B*a*b^2 + (2*B*a^
2*b + A*a*b^2 - 3*B*b^3)*cosh(x))*sinh(x))*sqrt(b)*weierstrassPInverse(4/3
*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*
sinh(x) + 2*a)/b) + 3*sqrt(1/2)*(B*a*b^2 - A*b^3 + (B*a*b^2 - A*b^3)*cosh(
x)^2 + (B*a*b^2 - A*b^3)*sinh(x)^2 + 2*(B*a^2*b - A*a*b^2)*cosh(x) + 2*(B*
a^2*b - A*a*b^2 + (B*a*b^2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b)*weierstrassZ
eta(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, weierstrassPInve
rse(4/3*(4*a^2 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x)
 + 3*b*sinh(x) + 2*a)/b)) + 3*((B*a*b^2 - A*b^3)*cosh(x)^2 + (B*a*b^2 - A*
b^3)*sinh(x)^2 + (B*a^2*b - A*a*b^2)*cosh(x) + (B*a^2*b - A*a*b^2 + 2*(B*a
*b^2 - A*b^3)*cosh(x))*sinh(x))*sqrt(b*cosh(x) + a))/(a^2*b^3 - b^5 + (a^2
*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*co
sh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{B\cosh(x)+A}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

Giac [F]

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{B\cosh(x)+A}{(b\cosh(x)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx = \int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{3/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^(3/2),x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^(3/2), x)
```

Reduce [F]

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{3/2}}\,dx = \int \frac{\sqrt{\cosh(x)\,b + a}}{\cosh(x)\,b + a}dx$$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(3/2),x)
```

output
```
int(sqrt(cosh(x)*b + a)/(cosh(x)*b + a),x)
```

3.120 $\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 231

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = -\frac{2i(4aAb-a^2B-3b^2B)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)^2\sqrt{\frac{a+b\cosh(x)}{a+b}}}$$
$$+\frac{2i(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{3b\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}$$
$$-\frac{2(Ab-aB)\sinh(x)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}-\frac{2(4aAb-a^2B-3b^2B)\sinh(x)}{3\left(a^2-b^2\right)^2\sqrt{a+b\cosh(x)}}$$

output
```
-2/3*I*(4*A*a*b-B*a^2-3*B*b^2)*(a+b*cosh(x))^(1/2)*EllipticE(I*sinh(1/2*x)
,2^(1/2)*(b/(a+b))^(1/2))/b/(a^2-b^2)^2/((a+b*cosh(x))/(a+b))^(1/2)+2/3*I*
(A*b-B*a)*((a+b*cosh(x))/(a+b))^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2)*(b/(
a+b))^(1/2))/b/(a^2-b^2)/(a+b*cosh(x))^(1/2)-2/3*(A*b-B*a)*sinh(x)/(a^2-b^
2)/(a+b*cosh(x))^(3/2)-2/3*(4*A*a*b-B*a^2-3*B*b^2)*sinh(x)/(a^2-b^2)^2/(a+
b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.62 (sec) , antiderivative size = 172, normalized size of antiderivative = 0.74

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \frac{2\left(\frac{i\left(\frac{a+b\cosh(x)}{a+b}\right)^{3/2}\left(\left(-4aAb+a^2B+3b^2B\right)E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)-(a-b)(-Ab+aB)\operatorname{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)\right)}{(a-b)^2b}+\frac{(-5a^2\cdots}{\cdots}\right.}{3(a+b\cosh(x))^{3/2}}$$

input
```
Integrate[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

output
```
(2*((I*((a + b*Cosh[x])/(a + b))^(3/2)*((-4*a*A*b + a^2*B + 3*b^2*B)*Ellip
ticE[(I/2)*x, (2*b)/(a + b)] - (a - b)*(-(A*b) + a*B)*EllipticF[(I/2)*x, (
2*b)/(a + b)]))/((a - b)^2*b) + ((-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B
 + b*(-4*a*A*b + a^2*B + 3*b^2*B)*Cosh[x])*Sinh[x])/(a^2 - b^2)^2))/(3*(a
+ b*Cosh[x])^(3/2))
```

Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.04, number of steps used = 15, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.882$, Rules used = {3042, 3233, 27, 3042, 3233, 27, 3042, 3231, 3042, 3134, 3042, 3132, 3142, 3042, 3140}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(\frac{\pi}{2}+ix\right)}{\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{2\int -\frac{3(aA-bB)-(Ab-aB)\cosh(x)}{2(a+b\cosh(x))^{3/2}}dx}{3\left(a^2-b^2\right)}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{3(aA-bB)-(Ab-aB)\cosh(x)}{(a+b\cosh(x))^{3/2}}dx}{3(a^2-b^2)}-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{\int \frac{3(aA-bB)+(aB-Ab)\sin\left(ix+\frac{\pi}{2}\right)}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^{3/2}}dx}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\frac{2\int -\frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cosh(x)}{2\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\cosh(x)}{\sqrt{a+b\cosh(x)}}dx}{a^2-b^2}-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\int \frac{3Aa^2-4bBa+Ab^2+\left(-Ba^2+4Aba-3b^2B\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{a^2-b^2}}{3(a^2-b^2)}$$

$$\Big\downarrow \text{3231}$$

$$\frac{\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\int \sqrt{a+b\cosh(x)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-aB)\int \frac{1}{\sqrt{a+b\cosh(x)}}dx}{b}}{a^2-b^2}-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}}{3(a^2-b^2)}-\frac{2\sinh(x)(Ab-aB)}{3(a^2-b^2)(a+b\cosh(x))^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\int\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3134

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{\frac{\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}\int\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}dx}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3132

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)(Ab-aB)\int\frac{1}{\sqrt{a+b\sin\left(ix+\frac{\pi}{2}\right)}}dx}{b}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3142

$$-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{\left(a^2-b^2\right)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\cosh(x)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}$$

$\Big\downarrow$ 3042

$$\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{-\frac{\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\int\frac{1}{\sqrt{\frac{a}{a+b}+\frac{b\sin\left(ix+\frac{\pi}{2}\right)}{a+b}}}dx}{b\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+}\right.}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+$$

$\big\downarrow$ 3140

$$\frac{-\frac{2\sinh(x)\left(a^2(-B)+4aAb-3b^2B\right)}{(a^2-b^2)\sqrt{a+b\cosh(x)}}+\frac{\frac{2i\left(a^2-b^2\right)(Ab-aB)\sqrt{\frac{a+b\cosh(x)}{a+b}}\text{EllipticF}\left(\frac{ix}{2},\frac{2b}{a+b}\right)}{b\sqrt{a+b\cosh(x)}}-\frac{2i\left(a^2(-B)+4aAb-3b^2B\right)\sqrt{a+b\cosh(x)}E\left(\frac{ix}{2}\middle|\frac{2b}{a+b}\right)}{b\sqrt{\frac{a+b\cosh(x)}{a+b}}}}{a^2-b^2}}{3\left(a^2-b^2\right)}-\frac{2\sinh(x)(Ab-aB)}{3\left(a^2-b^2\right)(a+b\cosh(x))^{3/2}}+$$

input
```
Int[(A + B*Cosh[x])/(a + b*Cosh[x])^(5/2),x]
```

output
```
(-2*(A*b - a*B)*Sinh[x])/(3*(a^2 - b^2)*(a + b*Cosh[x])^(3/2)) + ((((-2*I)
*(4*a*A*b - a^2*B - 3*b^2*B)*Sqrt[a + b*Cosh[x]]*EllipticE[(I/2)*x, (2*b)/
(a + b)])/(b*Sqrt[(a + b*Cosh[x])/(a + b)]) + ((2*I)*(a^2 - b^2)*(A*b - a*
B)*Sqrt[(a + b*Cosh[x])/(a + b)]*EllipticF[(I/2)*x, (2*b)/(a + b)])/(b*Sqr
t[a + b*Cosh[x]]))/(a^2 - b^2) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Sinh[x])/(
(a^2 - b^2)*Sqrt[a + b*Cosh[x]]))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3132
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3134
```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)]   Int[Sqrt[a/(a + b) + (
b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2
, 0] &&  !GtQ[a + b, 0]
```

rule 3140
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

rule 3142
```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]]   Int[1/Sqrt[a/(a + b)
 + (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] &&  !GtQ[a + b, 0]
```

rule 3231
```
Int[((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])/Sqrt[(a_) + (b_.)*sin[(e_.) + (
f_.)*(x_)]], x_Symbol] :> Simp[(b*c - a*d)/b   Int[1/Sqrt[a + b*Sin[e + f*x
]], x], x] + Simp[d/b   Int[Sqrt[a + b*Sin[e + f*x]], x], x] /; FreeQ[{a, b
, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(216) = 432$.

Time = 11.67 (sec) , antiderivative size = 797, normalized size of antiderivative = 3.45

method	result
default	$\frac{\sqrt{\left(2b\cosh\left(\frac{x}{2}\right)^{2}+a-b\right)\sinh\left(\frac{x}{2}\right)^{2}}\left(-\frac{2B\sqrt{2\sinh\left(\frac{x}{2}\right)^{4}b+(a+b)\sinh\left(\frac{x}{2}\right)^{2}}\left(2\cosh\left(\frac{x}{2}\right)\sqrt{-\frac{2b}{a-b}}\sinh\left(\frac{x}{2}\right)^{2}b-\sqrt{-\sinh\left(\frac{x}{2}\right)^{2}}\sqrt{\frac{2b\sinh\left(\frac{x}{2}\right)^{2}}{a-b}+\cdots}\right.}{\cdots}\right.}{\cdots}$
parts	Expression too large to display

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
((2*b*cosh(1/2*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)*(-2*B/b/sinh(1/2*x)^2/(2*b*s
inh(1/2*x)^2+a+b)/(-2*b/(a-b))^(1/2)/(a^2-b^2)*(2*sinh(1/2*x)^4*b+(a+b)*si
nh(1/2*x)^2)^(1/2)*(2*cosh(1/2*x)*(-2*b/(a-b))^(1/2)*sinh(1/2*x)^2*b-(-sin
h(1/2*x)^2)^(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(co
sh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*a-(-sinh(1/2*x)^2)^
(1/2)*(2*b/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticF(cosh(1/2*x)*(-
2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*b+2*(-sinh(1/2*x)^2)^(1/2)*(2*b
/(a-b)*sinh(1/2*x)^2+(a+b)/(a-b))^(1/2)*EllipticE(cosh(1/2*x)*(-2*b/(a-b))
^(1/2),1/2*((-2*a+2*b)/b)^(1/2))*b)+2*(A*b-B*a)/b*(-1/6/b/(a-b)/(a+b)*cosh
(1/2*x)*(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(cosh(1/2*x)^2+1/2/b
*(a-b))^2-8/3*sinh(1/2*x)^2*b/(a-b)^2/(a+b)^2*cosh(1/2*x)*a/((2*b*cosh(1/2
*x)^2+a-b)*sinh(1/2*x)^2)^(1/2)+(3*a-b)/(3*a^3+3*a^2*b-3*a*b^2-3*b^3)/(-2*
b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(1/2
)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)*EllipticF(cosh(1/2*x)*(-2*
b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-16/3*a*b/(a+b)^2/(a-b)^2*(-a+b)/(
-2*b/(a-b))^(1/2)*((2*b*cosh(1/2*x)^2+a-b)/(a-b))^(1/2)*(-sinh(1/2*x)^2)^(
1/2)/(2*sinh(1/2*x)^4*b+(a+b)*sinh(1/2*x)^2)^(1/2)/(2*a-2*b)*(EllipticF(co
sh(1/2*x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2))-EllipticE(cosh(1/2*
x)*(-2*b/(a-b))^(1/2),1/2*((-2*a+2*b)/b)^(1/2)))))/sinh(1/2*x)/(2*b*sinh(1
/2*x)^2+a+b)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2053 vs. $2(212) = 424$.

Time = 0.18 (sec) , antiderivative size = 2053, normalized size of antiderivative = 8.89

$$\int \frac{A + B\cosh(x)}{(a + b\cosh(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="fricas")
```

output
```
4/9*(sqrt(1/2)*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5 + (2*B*a^3*b
^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^4 + (2*B*a^3*b^2 + A*a^2*b^3
 - 6*B*a*b^4 + 3*A*b^5)*sinh(x)^4 + 4*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3
 + 3*A*a*b^4)*cosh(x)^3 + 4*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b
^4 + (2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x))*sinh(x)^3 +
2*(4*B*a^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)
*cosh(x)^2 + 2*(4*B*a^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B*a*b
^4 + 3*A*b^5 + 3*(2*B*a^3*b^2 + A*a^2*b^3 - 6*B*a*b^4 + 3*A*b^5)*cosh(x)^2
 + 6*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh(x))*sinh(x)^2
+ 4*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4)*cosh(x) + 4*(2*B*a^4
*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*a*b^4 + (2*B*a^3*b^2 + A*a^2*b^3 - 6*B*
a*b^4 + 3*A*b^5)*cosh(x)^3 + 3*(2*B*a^4*b + A*a^3*b^2 - 6*B*a^2*b^3 + 3*A*
a*b^4)*cosh(x)^2 + (4*B*a^5 + 2*A*a^4*b - 10*B*a^3*b^2 + 7*A*a^2*b^3 - 6*B
*a*b^4 + 3*A*b^5)*cosh(x))*sinh(x))*sqrt(b)*weierstrassPInverse(4/3*(4*a^2
 - 3*b^2)/b^2, -8/27*(8*a^3 - 9*a*b^2)/b^3, 1/3*(3*b*cosh(x) + 3*b*sinh(x)
 + 2*a)/b) + 3*sqrt(1/2)*(B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5 + (B*a^2*b^3 - 4
*A*a*b^4 + 3*B*b^5)*cosh(x)^4 + (B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*sinh(x)^
4 + 4*(B*a^3*b^2 - 4*A*a^2*b^3 + 3*B*a*b^4)*cosh(x)^3 + 4*(B*a^3*b^2 - 4*A
*a^2*b^3 + 3*B*a*b^4 + (B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cosh(x))*sinh(x)^
3 + 2*(2*B*a^4*b - 8*A*a^3*b^2 + 7*B*a^2*b^3 - 4*A*a*b^4 + 3*B*b^5)*cos...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{B\cosh(x)+A}{(b\cosh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="maxima")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

Giac [F]

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{B\cosh(x)+A}{(b\cosh(x)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x, algorithm="giac")
```

output
```
integrate((B*cosh(x) + A)/(b*cosh(x) + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{A+B\cosh(x)}{\left(a+b\cosh\left(x\right)\right)^{5/2}}\,dx$$

input
```
int((A + B*cosh(x))/(a + b*cosh(x))^(5/2),x)
```

output
```
int((A + B*cosh(x))/(a + b*cosh(x))^(5/2), x)
```

Reduce [F]

$$\int \frac{A+B\cosh(x)}{(a+b\cosh(x))^{5/2}}\,dx = \int \frac{\sqrt{\cosh\left(x\right)b+a}}{\cosh\left(x\right)^{2}b^{2}+2\cosh\left(x\right)ab+a^{2}}dx$$

input
```
int((A+B*cosh(x))/(a+b*cosh(x))^(5/2),x)
```

output
```
int(sqrt(cosh(x)*b + a)/(cosh(x)**2*b**2 + 2*cosh(x)*a*b + a**2),x)
```

3.121 $\int \left(a\cosh^2(x)\right)^{7/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 72

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx=\frac{16}{35}a^3\sqrt{a\cosh^2(x)}\tanh(x)+\frac{8}{35}a^2\left(a\cosh^2(x)\right)^{3/2}\tanh(x)+\frac{6}{35}a\left(a\cosh^2(x)\right)^{5/2}\tanh(x)+\frac{1}{7}\left(a\cosh^2(x)\right)^{7/2}\tanh(x)$$

output
```
16/35*a^3*(a*cosh(x)^2)^(1/2)*tanh(x)+8/35*a^2*(a*cosh(x)^2)^(3/2)*tanh(x)
+6/35*a*(a*cosh(x)^2)^(5/2)*tanh(x)+1/7*(a*cosh(x)^2)^(7/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.54

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx=\frac{1}{35}a^3\sqrt{a\cosh^2(x)}(35+35\sinh^2(x)+21\sinh^4(x)+5\sinh^6(x))\tanh(x)$$

input
```
Integrate[(a*Cosh[x]^2)^(7/2),x]
```

output
```
(a^3*Sqrt[a*Cosh[x]^2]*(35 + 35*Sinh[x]^2 + 21*Sinh[x]^4 + 5*Sinh[x]^6)*Ta
nh[x])/35
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3682, 3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^2(x)\right)^{7/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{7/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\int \left(a\cosh^2(x)\right)^{5/2}dx+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{7}a\int \left(a\sin\left(ix+\frac{\pi}{2}\right)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\int \left(a\cosh^2(x)\right)^{3/2}dx+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}\right)+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\int \left(a\sin\left(ix+\frac{\pi}{2}\right)^2\right)^{3/2}dx\right)$$

$$\Big\downarrow \text{3682}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\cosh^2(x)}dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}\right)+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}\right)+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\int\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}dx\right)\right)$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{6}{7}a\left(\frac{4}{5}a\left(\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int\cosh(x)dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}\right)+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}\right)+\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int\sin\left(ix+\frac{\pi}{2}\right)dx\right)\right)$$

$$\Big\downarrow \text{ 3117}$$

$$\frac{1}{7}\tanh(x)\left(a\cosh^2(x)\right)^{7/2}+\frac{6}{7}a\left(\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}\right)\right)$$

input
```
Int[(a*Cosh[x]^2)^(7/2),x]
```

output
```
((a*Cosh[x]^2)^(7/2)*Tanh[x])/7 + (6*a*(((a*Cosh[x]^2)^(5/2)*Tanh[x])/5 +
(4*a*((2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3)
)/5))/7
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.53

method	result
default	$\frac{a^4 \cosh(x) \sinh(x) \left(5 \cosh(x)^6+6 \cosh(x)^4+8 \cosh(x)^2+16\right)}{35 \sqrt{a \cosh(x)^2}}$
risch	$\frac{a^3 e^{8x} \sqrt{a(e^{2x}+1)^2 e^{-2x}}}{896 e^{2x}+896} + \frac{7a^3 e^{6x} \sqrt{a(e^{2x}+1)^2 e^{-2x}}}{640(e^{2x}+1)} + \frac{7a^3 e^{4x} \sqrt{a(e^{2x}+1)^2 e^{-2x}}}{128(e^{2x}+1)} + \frac{35a^3 e^{2x} \sqrt{a(e^{2x}+1)^2 e^{-2x}}}{128(e^{2x}+1)} - \frac{35\sqrt{a(e^{2x}+}}{128(e^{}}$

input
```
int((a*cosh(x)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
1/35*a^4*cosh(x)*sinh(x)*(5*cosh(x)^6+6*cosh(x)^4+8*cosh(x)^2+16)/(a*cosh(
x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 817 vs. $2(56) = 112$.

Time = 0.13 (sec) , antiderivative size = 817, normalized size of antiderivative = 11.35

$$\int \left(a \cosh^2(x)\right)^{7/2} dx = \text{Too large to display}$$

input
```
integrate((a*cosh(x)^2)^(7/2),x, algorithm="fricas")
```

output
```
1/4480*(70*a^3*cosh(x)*e^x*sinh(x)^13 + 5*a^3*e^x*sinh(x)^14 + 7*(65*a^3*c
osh(x)^2 + 7*a^3)*e^x*sinh(x)^12 + 28*(65*a^3*cosh(x)^3 + 21*a^3*cosh(x))*
e^x*sinh(x)^11 + 7*(715*a^3*cosh(x)^4 + 462*a^3*cosh(x)^2 + 35*a^3)*e^x*si
nh(x)^10 + 70*(143*a^3*cosh(x)^5 + 154*a^3*cosh(x)^3 + 35*a^3*cosh(x))*e^x
*sinh(x)^9 + 35*(429*a^3*cosh(x)^6 + 693*a^3*cosh(x)^4 + 315*a^3*cosh(x)^2
 + 35*a^3)*e^x*sinh(x)^8 + 8*(2145*a^3*cosh(x)^7 + 4851*a^3*cosh(x)^5 + 36
75*a^3*cosh(x)^3 + 1225*a^3*cosh(x))*e^x*sinh(x)^7 + 7*(2145*a^3*cosh(x)^8
 + 6468*a^3*cosh(x)^6 + 7350*a^3*cosh(x)^4 + 4900*a^3*cosh(x)^2 - 175*a^3)
*e^x*sinh(x)^6 + 14*(715*a^3*cosh(x)^9 + 2772*a^3*cosh(x)^7 + 4410*a^3*cos
h(x)^5 + 4900*a^3*cosh(x)^3 - 525*a^3*cosh(x))*e^x*sinh(x)^5 + 35*(143*a^3
*cosh(x)^10 + 693*a^3*cosh(x)^8 + 1470*a^3*cosh(x)^6 + 2450*a^3*cosh(x)^4
- 525*a^3*cosh(x)^2 - 7*a^3)*e^x*sinh(x)^4 + 140*(13*a^3*cosh(x)^11 + 77*a
^3*cosh(x)^9 + 210*a^3*cosh(x)^7 + 490*a^3*cosh(x)^5 - 175*a^3*cosh(x)^3 -
 7*a^3*cosh(x))*e^x*sinh(x)^3 + 7*(65*a^3*cosh(x)^12 + 462*a^3*cosh(x)^10
+ 1575*a^3*cosh(x)^8 + 4900*a^3*cosh(x)^6 - 2625*a^3*cosh(x)^4 - 210*a^3*c
osh(x)^2 - 7*a^3)*e^x*sinh(x)^2 + 14*(5*a^3*cosh(x)^13 + 42*a^3*cosh(x)^11
 + 175*a^3*cosh(x)^9 + 700*a^3*cosh(x)^7 - 525*a^3*cosh(x)^5 - 70*a^3*cosh
(x)^3 - 7*a^3*cosh(x))*e^x*sinh(x) + (5*a^3*cosh(x)^14 + 49*a^3*cosh(x)^12
 + 245*a^3*cosh(x)^10 + 1225*a^3*cosh(x)^8 - 1225*a^3*cosh(x)^6 - 245*a^3*
cosh(x)^4 - 49*a^3*cosh(x)^2 - 5*a^3)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x)...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{7/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \frac{1}{896}\,a^{\frac{7}{2}}e^{(7\,x)} + \frac{7}{640}\,a^{\frac{7}{2}}e^{(5\,x)} + \frac{7}{128}\,a^{\frac{7}{2}}e^{(3\,x)} - \frac{35}{128}\,a^{\frac{7}{2}}e^{(-x)} - \frac{7}{128}\,a^{\frac{7}{2}}e^{(-3\,x)} - \frac{7}{640}\,a^{\frac{7}{2}}e^{(-5\,x)} - \frac{1}{896}\,a^{\frac{7}{2}}e^{(-7\,x)} + \frac{35}{128}\,a^{\frac{7}{2}}e^{x}$$

input
```
integrate((a*cosh(x)^2)^(7/2),x, algorithm="maxima")
```

output
```
1/896*a^(7/2)*e^(7*x) + 7/640*a^(7/2)*e^(5*x) + 7/128*a^(7/2)*e^(3*x) - 35
/128*a^(7/2)*e^(-x) - 7/128*a^(7/2)*e^(-3*x) - 7/640*a^(7/2)*e^(-5*x) - 1/
896*a^(7/2)*e^(-7*x) + 35/128*a^(7/2)*e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.10

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \frac{1}{4480}\left(5\,a^3e^{(7\,x)} + 49\,a^3e^{(5\,x)} + 245\,a^3e^{(3\,x)} + 1225\,a^3e^{x} - \left(1225\,a^3e^{(6\,x)} + 245\,a^3e^{(4\,x}\right.\right.$$

input
```
integrate((a*cosh(x)^2)^(7/2),x, algorithm="giac")
```

output
```
1/4480*(5*a^3*e^(7*x) + 49*a^3*e^(5*x) + 245*a^3*e^(3*x) + 1225*a^3*e^x -
(1225*a^3*e^(6*x) + 245*a^3*e^(4*x) + 49*a^3*e^(2*x) + 5*a^3)*e^(-7*x))*sq
rt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \int \left(a\cosh(x)^2\right)^{7/2}\,dx$$

input
```
int((a*cosh(x)^2)^(7/2),x)
```

output
```
int((a*cosh(x)^2)^(7/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.90

$$\int \left(a\cosh^2(x)\right)^{7/2}\,dx = \frac{\sqrt{a}\,a^3(5e^{14x}+49e^{12x}+245e^{10x}+1225e^{8x}-1225e^{6x}-245e^{4x}-49e^{2x}-5)}{4480e^{7x}}$$

input
```
int((a*cosh(x)^2)^(7/2),x)
```

output
```
(sqrt(a)*a**3*(5*e**(14*x) + 49*e**(12*x) + 245*e**(10*x) + 1225*e**(8*x)
- 1225*e**(6*x) - 245*e**(4*x) - 49*e**(2*x) - 5))/(4480*e**(7*x))
```

3.122 $\int \left(a\cosh^2(x)\right)^{5/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 53

$$\int \left(a\cosh^2(x)\right)^{5/2} dx = \frac{8}{15}a^2\sqrt{a\cosh^2(x)}\tanh(x) + \frac{4}{15}a\left(a\cosh^2(x)\right)^{3/2}\tanh(x) + \frac{1}{5}\left(a\cosh^2(x)\right)^{5/2}\tanh(x)$$

output
```
8/15*a^2*(a*cosh(x)^2)^(1/2)*tanh(x)+4/15*a*(a*cosh(x)^2)^(3/2)*tanh(x)+1/
5*(a*cosh(x)^2)^(5/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.62

$$\int \left(a\cosh^2(x)\right)^{5/2} dx = \frac{1}{15}a^2\sqrt{a\cosh^2(x)}\left(15+10\sinh^2(x)+3\sinh^4(x)\right)\tanh(x)$$

input
```
Integrate[(a*Cosh[x]^2)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Cosh[x]^2]*(15 + 10*Sinh[x]^2 + 3*Sinh[x]^4)*Tanh[x])/15
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3682, 3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^2(x)\right)^{5/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\int \left(a\cosh^2(x)\right)^{3/2}dx+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\int \left(a\sin\left(ix+\frac{\pi}{2}\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\int \sqrt{a\cosh^2(x)}dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}\right)+\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}dx\right)$$

$$\Big\downarrow \text{3686}$$

$$\frac{4}{5}a\left(\frac{2}{3}a\operatorname{sech}(x)\sqrt{a\cosh^2(x)}\int \cosh(x)dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}\right)+ \\ \frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+$$
$$\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int\sin\left(ix+\frac{\pi}{2}\right)dx\right)$$
$$\Big\downarrow \text{3117}$$
$$\frac{1}{5}\tanh(x)\left(a\cosh^2(x)\right)^{5/2}+\frac{4}{5}a\left(\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}\right)$$

input
```
Int[(a*Cosh[x]^2)^(5/2),x]
```

output
```
((a*Cosh[x]^2)^(5/2)*Tanh[x])/5 + (4*a*((2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3
+ ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3))/5
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.60

method	result
default	$\frac{a^3\cosh(x)\sinh(x)\left(3\cosh(x)^4+4\cosh(x)^2+8\right)}{15\sqrt{a\cosh(x)^2}}$
risch	$\frac{a^2 e^{6x}\sqrt{a(e^{2x}+1)^2 e^{-2x}}}{160\,e^{2x}+160}+\frac{5a^2 e^{4x}\sqrt{a(e^{2x}+1)^2 e^{-2x}}}{96(e^{2x}+1)}+\frac{5a^2 e^{2x}\sqrt{a(e^{2x}+1)^2 e^{-2x}}}{16(e^{2x}+1)}-\frac{5\sqrt{a(e^{2x}+1)^2 e^{-2x}}\,a^2}{16(e^{2x}+1)}-\frac{5a^2 e^{-2x}\sqrt{a(e^{2x}}{96(e^{2x}+}$

input
```
int((a*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/15*a^3*cosh(x)*sinh(x)*(3*cosh(x)^4+4*cosh(x)^2+8)/(a*cosh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. 2(41) = 82.

Time = 0.09 (sec) , antiderivative size = 501, normalized size of antiderivative = 9.45

$$\int \left(a\cosh^2(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/480*(30*a^2*cosh(x)*e^x*sinh(x)^9 + 3*a^2*e^x*sinh(x)^10 + 5*(27*a^2*cos
h(x)^2 + 5*a^2)*e^x*sinh(x)^8 + 40*(9*a^2*cosh(x)^3 + 5*a^2*cosh(x))*e^x*s
inh(x)^7 + 10*(63*a^2*cosh(x)^4 + 70*a^2*cosh(x)^2 + 15*a^2)*e^x*sinh(x)^6
 + 4*(189*a^2*cosh(x)^5 + 350*a^2*cosh(x)^3 + 225*a^2*cosh(x))*e^x*sinh(x)
^5 + 10*(63*a^2*cosh(x)^6 + 175*a^2*cosh(x)^4 + 225*a^2*cosh(x)^2 - 15*a^2
)*e^x*sinh(x)^4 + 40*(9*a^2*cosh(x)^7 + 35*a^2*cosh(x)^5 + 75*a^2*cosh(x)^
3 - 15*a^2*cosh(x))*e^x*sinh(x)^3 + 5*(27*a^2*cosh(x)^8 + 140*a^2*cosh(x)^
6 + 450*a^2*cosh(x)^4 - 180*a^2*cosh(x)^2 - 5*a^2)*e^x*sinh(x)^2 + 10*(3*a
^2*cosh(x)^9 + 20*a^2*cosh(x)^7 + 90*a^2*cosh(x)^5 - 60*a^2*cosh(x)^3 - 5*
a^2*cosh(x))*e^x*sinh(x) + (3*a^2*cosh(x)^10 + 25*a^2*cosh(x)^8 + 150*a^2*
cosh(x)^6 - 150*a^2*cosh(x)^4 - 25*a^2*cosh(x)^2 - 3*a^2)*e^x)*sqrt(a*e^(4
*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^5*e^(2*x) + (e^(2*x) + 1)*sinh(x)^5
 + cosh(x)^5 + 5*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^4 + 10*(cosh(x)^2*e^(
2*x) + cosh(x)^2)*sinh(x)^3 + 10*(cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^2
 + 5*(cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \left(a\cosh^2(x)\right)^{5/2} dx = \frac{1}{160}\,a^{\frac{5}{2}}e^{(5\,x)} + \frac{5}{96}\,a^{\frac{5}{2}}e^{(3\,x)} - \frac{5}{16}\,a^{\frac{5}{2}}e^{(-x)} - \frac{5}{96}\,a^{\frac{5}{2}}e^{(-3\,x)} - \frac{1}{160}\,a^{\frac{5}{2}}e^{(-5\,x)} + \frac{5}{16}\,a^{\frac{5}{2}}e^{x}$$

input
```
integrate((a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/160*a^(5/2)*e^(5*x) + 5/96*a^(5/2)*e^(3*x) - 5/16*a^(5/2)*e^(-x) - 5/96*
a^(5/2)*e^(-3*x) - 1/160*a^(5/2)*e^(-5*x) + 5/16*a^(5/2)*e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.77

$$\int \left(a\cosh^2(x)\right)^{5/2} dx = -\frac{1}{480}\left(\left(150\,e^{(4\,x)}+25\,e^{(2\,x)}+3\right)e^{(-5\,x)}-3\,e^{(5\,x)}-25\,e^{(3\,x)}-150\,e^{x}\right)a^{\frac{5}{2}}$$

input
```
integrate((a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/480*((150*e^(4*x) + 25*e^(2*x) + 3)*e^(-5*x) - 3*e^(5*x) - 25*e^(3*x) -
 150*e^x)*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{5/2} dx = \int \left(a\cosh(x)^2\right)^{5/2} dx$$

input
```
int((a*cosh(x)^2)^(5/2),x)
```

output
```
int((a*cosh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96

$$\int \left(a\cosh^2(x)\right)^{5/2} dx = \frac{\sqrt{a}\, a^2(3e^{10x} + 25e^{8x} + 150e^{6x} - 150e^{4x} - 25e^{2x} - 3)}{480e^{5x}}$$

input
```
int((a*cosh(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*a**2*(3*e**(10*x) + 25*e**(8*x) + 150*e**(6*x) - 150*e**(4*x) - 2
5*e**(2*x) - 3))/(480*e**(5*x))
```

3.123 $\int \left(a\cosh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 34

$$\int \left(a\cosh^2(x)\right)^{3/2} dx = \frac{2}{3}a\sqrt{a\cosh^2(x)}\tanh(x) + \frac{1}{3}\left(a\cosh^2(x)\right)^{3/2}\tanh(x)$$

output
```
2/3*a*(a*cosh(x)^2)^(1/2)*tanh(x)+1/3*(a*cosh(x)^2)^(3/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.68

$$\int \left(a\cosh^2(x)\right)^{3/2} dx = \frac{1}{3}a\sqrt{a\cosh^2(x)}\left(3+\sinh^2(x)\right)\tanh(x)$$

input
```
Integrate[(a*Cosh[x]^2)^(3/2),x]
```

output
```
(a*Sqrt[a*Cosh[x]^2]*(3 + Sinh[x]^2)*Tanh[x])/3
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 3682, 3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{3682}$$

$$\frac{2}{3}a\int \sqrt{a\cosh^2(x)}dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\int \sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int \cosh(x)dx+\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\text{sech}(x)\sqrt{a\cosh^2(x)}\int \sin\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \text{3117}$$

$$\frac{1}{3}\tanh(x)\left(a\cosh^2(x)\right)^{3/2}+\frac{2}{3}a\tanh(x)\sqrt{a\cosh^2(x)}$$

input
```
Int[(a*Cosh[x]^2)^(3/2),x]
```

output
```
(2*a*Sqrt[a*Cosh[x]^2]*Tanh[x])/3 + ((a*Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3682
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(-Cot[e + f*x
])*((b*Sin[e + f*x]^2)^p/(2*f*p)), x] + Simp[b*((2*p - 1)/(2*p))   Int[(b*S
in[e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && G
tQ[p, 1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.71

method	result	size
default	$\frac{a^{2} \cosh \left(x \right) \sinh \left(x \right) \left(\cosh \left(x \right)^{2}+2\right)}{3 \sqrt{a \cosh \left(x \right)^{2}}}$	24
risch	$\frac{a \,{\mathrm e}^{4 x} \sqrt{a \left({\mathrm e}^{2 x}+1\right)^{2} {\mathrm e}^{-2 x}}}{24 \,{\mathrm e}^{2 x}+24}+\frac{3 a \,{\mathrm e}^{2 x} \sqrt{a \left({\mathrm e}^{2 x}+1\right)^{2} {\mathrm e}^{-2 x}}}{8 \left({\mathrm e}^{2 x}+1\right)}-\frac{3 \sqrt{a \left({\mathrm e}^{2 x}+1\right)^{2} {\mathrm e}^{-2 x}}\, a}{8 \left({\mathrm e}^{2 x}+1\right)}-\frac{a \,{\mathrm e}^{-2 x} \sqrt{a \left({\mathrm e}^{2 x}+1\right)^{2} {\mathrm e}^{-2 x}}}{24 \left({\mathrm e}^{2 x}+1\right)}$	122

input
```
int((a*cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/3*a^2*cosh(x)*sinh(x)*(cosh(x)^2+2)/(a*cosh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. 2(26) = 52.

Time = 0.10 (sec) , antiderivative size = 222, normalized size of antiderivative = 6.53

$$\int \left(a\cosh^2(x)\right)^{3/2}\,dx = \frac{\left(6\,a\cosh\left(x\right)e^{x}\sinh\left(x\right)^{5} + ae^{x}\sinh\left(x\right)^{6} + 3\left(5\,a\cosh\left(x\right)^{2} + 3\,a\right)e^{x}\sinh\left(x\right)^{4} + 4\,(5}{}$$

input
```
integrate((a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/24*(6*a*cosh(x)*e^x*sinh(x)^5 + a*e^x*sinh(x)^6 + 3*(5*a*cosh(x)^2 + 3*a
)*e^x*sinh(x)^4 + 4*(5*a*cosh(x)^3 + 9*a*cosh(x))*e^x*sinh(x)^3 + 3*(5*a*c
osh(x)^4 + 18*a*cosh(x)^2 - 3*a)*e^x*sinh(x)^2 + 6*(a*cosh(x)^5 + 6*a*cosh
(x)^3 - 3*a*cosh(x))*e^x*sinh(x) + (a*cosh(x)^6 + 9*a*cosh(x)^4 - 9*a*cosh
(x)^2 - a)*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)^3*e^(2*x
) + (e^(2*x) + 1)*sinh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*si
nh(x)^2 + 3*(cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \left(a\cosh^2(x)\right)^{3/2} dx = \frac{1}{24} a^{\frac{3}{2}} e^{(3x)} - \frac{3}{8} a^{\frac{3}{2}} e^{(-x)} - \frac{1}{24} a^{\frac{3}{2}} e^{(-3x)} + \frac{3}{8} a^{\frac{3}{2}} e^{x}$$

input
```
integrate((a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
1/24*a^(3/2)*e^(3*x) - 3/8*a^(3/2)*e^(-x) - 1/24*a^(3/2)*e^(-3*x) + 3/8*a^
(3/2)*e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \left(a\cosh^2(x)\right)^{3/2} dx = -\frac{1}{24} \left(\left(9 e^{(2x)} + 1\right) e^{(-3x)} - e^{(3x)} - 9 e^{x}\right) a^{\frac{3}{2}}$$

input
```
integrate((a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^2(x)\right)^{3/2} dx = \int \left(a\cosh(x)^2\right)^{3/2} dx$$

input
```
int((a*cosh(x)^2)^(3/2),x)
```

output
```
int((a*cosh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \left(a\cosh^2(x)\right)^{3/2}\,dx=\frac{\sqrt{a}\,a(e^{6x}+9e^{4x}-9e^{2x}-1)}{24e^{3x}}$$

input
```
int((a*cosh(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*a*(e**(6*x) + 9*e**(4*x) - 9*e**(2*x) - 1))/(24*e**(3*x))
```

3.124 $\int \sqrt{a \cosh^2(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \sqrt{a \cosh^2(x)}\, dx = \sqrt{a \cosh^2(x)} \tanh(x)$$

output
```
(a*cosh(x)^2)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \sqrt{a \cosh^2(x)}\, dx = \sqrt{a \cosh^2(x)} \tanh(x)$$

input
```
Integrate[Sqrt[a*Cosh[x]^2],x]
```

output
```
Sqrt[a*Cosh[x]^2]*Tanh[x]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3042, 3686, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^2}dx$$

$$\Big\downarrow \; 3686$$

$$\text{sech}(x)\sqrt{a\cosh^2(x)}\int \cosh(x)dx$$

$$\Big\downarrow \; 3042$$

$$\text{sech}(x)\sqrt{a\cosh^2(x)}\int \sin\left(ix+\frac{\pi}{2}\right)dx$$

$$\Big\downarrow \; 3117$$

$$\tanh(x)\sqrt{a\cosh^2(x)}$$

input
```
Int[Sqrt[a*Cosh[x]^2],x]
```

output
```
Sqrt[a*Cosh[x]^2]*Tanh[x]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

method	result	size
default	$\frac{a\cosh(x)\sinh(x)}{\sqrt{a\cosh(x)^{2}}}$	15
risch	$\frac{\sqrt{a(\mathrm{e}^{2x}+1)^{2}\mathrm{e}^{-2x}}\,\mathrm{e}^{2x}}{2\,\mathrm{e}^{2x}+2}-\frac{\sqrt{a(\mathrm{e}^{2x}+1)^{2}\mathrm{e}^{-2x}}}{2(\mathrm{e}^{2x}+1)}$	58

input
```
int((a*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(a*cosh(x)^2)^(1/2)*a*cosh(x)*sinh(x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 69, normalized size of antiderivative = 5.31

$$\int \sqrt{a\cosh^2(x)}\,dx = \frac{\left(2\cosh(x)e^x\sinh(x) + e^x\sinh(x)^2 + \left(\cosh(x)^2 - 1\right)e^x\right)\sqrt{ae^{(4x)} + 2\,ae^{(2x)} + a}e^{(-x)}}{2\left(\cosh(x)e^{(2x)} + \left(e^{(2x)} + 1\right)\sinh(x) + \cosh(x)\right)}$$

input
```
integrate((a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/2*(2*cosh(x)*e^x*sinh(x) + e^x*sinh(x)^2 + (cosh(x)^2 - 1)*e^x)*sqrt(a*e
^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(cosh(x)*e^(2*x) + (e^(2*x) + 1)*sinh(x)
+ cosh(x))
```

Sympy [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.15

$$\int \sqrt{a\cosh^2(x)}\,dx = \frac{\sqrt{a\cosh^2(x)}\sinh(x)}{\cosh(x)}$$

input
```
integrate((a*cosh(x)**2)**(1/2),x)
```

output
```
sqrt(a*cosh(x)**2)*sinh(x)/cosh(x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sqrt{a\cosh^2(x)}\,dx = -\frac{1}{2}\sqrt{a}e^{(-x)} + \frac{1}{2}\sqrt{a}e^{x}$$

input
```
integrate((a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*sqrt(a)*e^(-x) + 1/2*sqrt(a)*e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \sqrt{a\cosh^2(x)}\,dx = -\frac{1}{2}\sqrt{a}\left(e^{(-x)} - e^{x}\right)$$

input
```
integrate((a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a)*(e^(-x) - e^x)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \sqrt{a\cosh^2(x)}\,dx = \sqrt{a}\tanh(x)\left(\frac{e^{-x}}{2} + \frac{e^{x}}{2}\right)$$

input
```
int((a*cosh(x)^2)^(1/2),x)
```

output
```
a^(1/2)*tanh(x)*(exp(-x)/2 + exp(x)/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.38

$$\int \sqrt{a \cosh^2(x)}\, dx = \sqrt{a} \sinh(x)$$

input
```
int((a*cosh(x)^2)^(1/2),x)
```

output
```
sqrt(a)*sinh(x)
```

3.125 $\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \frac{\arctan(\sinh(x))\cosh(x)}{\sqrt{a\cosh^2(x)}}$$

output
```
arctan(sinh(x))*cosh(x)/(a*cosh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = -\frac{\cot^{-1}(\sinh(x))\cosh(x)}{\sqrt{a\cosh^2(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]^2],x]
```

output
```
-((ArcCot[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh(x)\int \text{sech}(x)dx}{\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh(x)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh(x)\arctan(\sinh(x))}{\sqrt{a\cosh^2(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]^2],x]
```

output
```
(ArcTan[Sinh[x]]*Cosh[x])/Sqrt[a*Cosh[x]^2]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(14) = 28$.

Time = 0.30 (sec) , antiderivative size = 55, normalized size of antiderivative = 3.44

method	result	size
default	$-\frac{\cosh(x)\sqrt{a\sinh(x)^2}\,\ln\left(\frac{2\sqrt{-a}\,\sqrt{a\sinh(x)^2}-2a}{\cosh(x)}\right)}{\sqrt{-a}\,\sinh(x)\sqrt{a\cosh(x)^2}}$	55
risch	$\frac{i\mathrm{e}^{-x}\left(\mathrm{e}^{2x}+1\right)\ln\left(\mathrm{e}^{x}+i\right)}{\sqrt{a\left(\mathrm{e}^{2x}+1\right)^2\mathrm{e}^{-2x}}}-\frac{i\mathrm{e}^{-x}\left(\mathrm{e}^{2x}+1\right)\ln\left(\mathrm{e}^{x}-i\right)}{\sqrt{a\left(\mathrm{e}^{2x}+1\right)^2\mathrm{e}^{-2x}}}$	72

input
```
int(1/(a*cosh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-cosh(x)*(a*sinh(x)^2)^(1/2)/(-a)^(1/2)*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/
2)-a)/cosh(x))/sinh(x)/(a*cosh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 35 vs. $2(14) = 28$.

Time = 0.08 (sec) , antiderivative size = 186, normalized size of antiderivative = 11.62

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \left[-\frac{\sqrt{-a}\log\left(\frac{a\cosh(x)^2-2\sqrt{ae^{(4x)}+2ae^{(2x)}+a}(\cosh(x)e^x+e^x\sinh(x))\sqrt{-a}e^{(-x)}+(ae^{(2x)}+a)\sinh(x)^2+\left(a\cosh(x)^2-a\right)e^{(2x)}+2(a\cdots}{(e^{(2x)}+1)\sinh(x)^2+\cosh(x)^2+\left(\cosh(x)^2+1\right)e^{(2x)}+2(\cosh(x)e^{(2x)}+\cosh(x))\sinh(x)+\cdots}\right.}{a}\right.$$

input
```
integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[-sqrt(-a)*log((a*cosh(x)^2 - 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*(cosh(x)
*e^x + e^x*sinh(x))*sqrt(-a)*e^(-x) + (a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(
x)^2 - a)*e^(2*x) + 2*(a*cosh(x)*e^(2*x) + a*cosh(x))*sinh(x) - a)/((e^(2*
x) + 1)*sinh(x)^2 + cosh(x)^2 + (cosh(x)^2 + 1)*e^(2*x) + 2*(cosh(x)*e^(2*
x) + cosh(x))*sinh(x) + 1))/a, 2*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*arctan(
cosh(x) + sinh(x))/(a*e^(2*x) + a)]
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh^2{(x)}}}\,dx$$

input
```
integrate(1/(a*cosh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \frac{2\arctan\left(e^x\right)}{\sqrt{a}}$$

input
```
integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(e^x)/sqrt(a)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(a*cosh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:sym2poly/r2sym(const gen & e,const
index_m & i,const vecteur & l) Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)^2}}\,dx$$

input
```
int(1/(a*cosh(x)^2)^(1/2),x)
```

output
```
int(1/(a*cosh(x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.69

$$\int \frac{1}{\sqrt{a\cosh^2(x)}}\,dx = \frac{2\sqrt{a}\,atan(e^x)}{a}$$

input
```
int(1/(a*cosh(x)^2)^(1/2),x)
```

output
```
(2*sqrt(a)*atan(e**x))/a
```

3.126 $\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 42

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\arctan(\sinh(x))\cosh(x)}{2a\sqrt{a\cosh^2(x)}} + \frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

output
```
1/2*arctan(sinh(x))*cosh(x)/a/(a*cosh(x)^2)^(1/2)+1/2*tanh(x)/a/(a*cosh(x)
^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.62

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\arctan(\sinh(x))\cosh(x) + \tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

input
```
Integrate[(a*Cosh[x]^2)^(-3/2),x]
```

output
```
(ArcTan[Sinh[x]]*Cosh[x] + Tanh[x])/(2*a*Sqrt[a*Cosh[x]^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh^2(x))^{3/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{\int \frac{1}{\sqrt{a\cosh^2(x)}}dx}{2a}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}}dx}{2a}$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh(x)\int \operatorname{sech}(x)dx}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cosh^2(x)}}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\cosh(x)\arctan(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}$$

input
```
Int[(a*Cosh[x]^2)^(-3/2),x]
```

output
```
(ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqrt[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Co
sh[x]^2])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(34) = 68$.

Time = 0.38 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

method	result	size
default	$\frac{\sqrt{a\sinh(x)^2}\left(-\ln\left(\frac{2\sqrt{-a}\sqrt{a\sinh(x)^2}-2a}{\cosh(x)}\right)\cosh(x)^2 a+\sqrt{-a}\sqrt{a\sinh(x)^2}\right)}{2a^2\cosh(x)\sqrt{-a}\sinh(x)\sqrt{a\cosh(x)^2}}$	82
risch	$\frac{e^{2x}-1}{a(e^{2x}+1)\sqrt{a(e^{2x}+1)^2 e^{-2x}}}+\frac{i(e^{2x}+1)e^{-x}\ln(e^x+i)}{2a\sqrt{a(e^{2x}+1)^2 e^{-2x}}}-\frac{i(e^{2x}+1)e^{-x}\ln(e^x-i)}{2a\sqrt{a(e^{2x}+1)^2 e^{-2x}}}$	112

input
```
int(1/(a*cosh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/a^2/cosh(x)*(a*sinh(x)^2)^(1/2)*(-ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(1/2)
-a)/cosh(x))*cosh(x)^2*a+(-a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x
)/(a*cosh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 299 vs. $2(34) = 68$.

Time = 0.08 (sec) , antiderivative size = 299, normalized size of antiderivative = 7.12

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\left(3\cosh\left(x\right)e^{x}\sinh\left(x\right)^{2}+e^{x}\sinh\left(x\right)^{3}+\left(3\cosh\left(x\right)^{2}-1\right)e^{x}\sinh\left(x\right)+\left(4\cosh\left(x\right)\right.\right.}{a^{2}\cosh\left(x\right)^{4}+\left(a^{2}e^{(2x)}+a^{2}\right)\sinh\left(x\right)^{4}+2a^{2}\cosh\left(x\right)^{2}+4\left(a^{2}\cosh\left(x\right)e^{(2x)}\right.}$$

input
```
integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
(3*cosh(x)*e^x*sinh(x)^2 + e^x*sinh(x)^3 + (3*cosh(x)^2 - 1)*e^x*sinh(x) +
 (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x
)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*
e^x)*arctan(cosh(x) + sinh(x)) + (cosh(x)^3 - cosh(x))*e^x)*sqrt(a*e^(4*x)
 + 2*a*e^(2*x) + a)*e^(-x)/(a^2*cosh(x)^4 + (a^2*e^(2*x) + a^2)*sinh(x)^4
+ 2*a^2*cosh(x)^2 + 4*(a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^3 + 2*(3
*a^2*cosh(x)^2 + a^2 + (3*a^2*cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^2 + a^2 +
(a^2*cosh(x)^4 + 2*a^2*cosh(x)^2 + a^2)*e^(2*x) + 4*(a^2*cosh(x)^3 + a^2*c
osh(x) + (a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x))*sinh(x))
```

Sympy [F]

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)**2)**(3/2),x)
```

output
```
Integral((a*cosh(x)**2)**(-3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.98

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{e^{(3\,x)} - e^{x}}{a^{\frac{3}{2}}e^{(4\,x)} + 2\,a^{\frac{3}{2}}e^{(2\,x)} + a^{\frac{3}{2}}} + \frac{\arctan\left(e^{x}\right)}{a^{\frac{3}{2}}}$$

input
```
integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
(e^(3*x) - e^x)/(a^(3/2)*e^(4*x) + 2*a^(3/2)*e^(2*x) + a^(3/2)) + arctan(e
^x)/a^(3/2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.33

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\frac{\pi+2\,\arctan\left(\frac{1}{2}\,\left(e^{(2\,x)}-1\right)e^{(-x)}\right)}{\sqrt{a}} - \frac{4\,\left(e^{(-x)}-e^{x}\right)}{\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)\sqrt{a}}}{4\,a}$$

input
```
integrate(1/(a*cosh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*((pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/sqrt(a) - 4*(e^(-x) - e^x)/
(((e^(-x) - e^x)^2 + 4)*sqrt(a)))/a
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh(x)^2\right)^{3/2}}\,dx$$

input
```
int(1/(a*cosh(x)^2)^(3/2),x)
```

output
```
int(1/(a*cosh(x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\,\left(e^{4x}atan(e^x) + 2e^{2x}atan(e^x) + atan(e^x) + e^{3x} - e^x\right)}{a^2\left(e^{4x} + 2e^{2x} + 1\right)}$$

input
```
int(1/(a*cosh(x)^2)^(3/2),x)
```

output
```
(sqrt(a)*(e**(4*x)*atan(e**x) + 2*e**(2*x)*atan(e**x) + atan(e**x) + e**(3
*x) - e**x))/(a**2*(e**(4*x) + 2*e**(2*x) + 1))
```

3.127 $\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 61

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx=\frac{3\arctan(\sinh(x))\cosh(x)}{8a^2\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}$$

output
```
3/8*arctan(sinh(x))*cosh(x)/a^2/(a*cosh(x)^2)^(1/2)+1/4*tanh(x)/a/(a*cosh(
x)^2)^(3/2)+3/8*tanh(x)/a^2/(a*cosh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.59

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx=\frac{3\arctan(\sinh(x))\cosh(x)+\left(3+2\text{sech}^2(x)\right)\tanh(x)}{8a^2\sqrt{a\cosh^2(x)}}$$

input
```
Integrate[(a*Cosh[x]^2)^(-5/2),x]
```

output
```
(3*ArcTan[Sinh[x]]*Cosh[x] + (3 + 2*Sech[x]^2)*Tanh[x])/(8*a^2*Sqrt[a*Cosh
[x]^2])
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 3683, 3042, 3683, 3042, 3686, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\int \frac{1}{\left(a\cosh^2(x)\right)^{3/2}}dx}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\int \frac{1}{\left(a\sin\left(ix+\frac{\pi}{2}\right)^2\right)^{3/2}}dx}{4a}$$

$$\Big\downarrow \text{3683}$$

$$\frac{3\left(\frac{\int \frac{1}{\sqrt{a\cosh^2(x)}}dx}{2a}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\left(\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\int \frac{1}{\sqrt{a\sin\left(ix+\frac{\pi}{2}\right)^2}}dx}{2a}\right)}{4a}$$

$$\Big\downarrow \text{3686}$$

$$\frac{3\left(\frac{\cosh(x)\int \operatorname{sech}(x)dx}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}+\frac{3\left(\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}+\frac{\cosh(x)\int \csc\left(ix+\frac{\pi}{2}\right)dx}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{3\left(\frac{\cosh(x)\arctan(\sinh(x))}{2a\sqrt{a\cosh^2(x)}}+\frac{\tanh(x)}{2a\sqrt{a\cosh^2(x)}}\right)}{4a}+\frac{\tanh(x)}{4a\left(a\cosh^2(x)\right)^{3/2}}$$

input
```
Int[(a*Cosh[x]^2)^(-5/2),x]
```

output
```
Tanh[x]/(4*a*(a*Cosh[x]^2)^(3/2)) + (3*((ArcTan[Sinh[x]]*Cosh[x])/(2*a*Sqr
t[a*Cosh[x]^2]) + Tanh[x]/(2*a*Sqrt[a*Cosh[x]^2])))/(4*a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3683
```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[Cot[e + f*x]*
((b*Sin[e + f*x]^2)^(p + 1)/(b*f*(2*p + 1))), x] + Simp[2*((p + 1)/(b*(2*p
+ 1)))   Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&
!IntegerQ[p] && LtQ[p, -1]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. 2(49) = 98.

Time = 0.37 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\sqrt{a\sinh(x)^2}\left(-3\ln\left(\frac{2\sqrt{-a}\sqrt{a\sinh(x)^2}-2a}{\cosh(x)}\right)a\cosh(x)^4+3\cosh(x)^2\sqrt{a\sinh(x)^2}\sqrt{-a}+2\sqrt{-a}\sqrt{a\sinh(x)^2}\right)}{8a^3\cosh(x)^3\sqrt{-a}\sinh(x)\sqrt{a\cosh(x)^2}}$	102
risch	$\frac{3\,e^{6x}+11\,e^{4x}-11\,e^{2x}-3}{4a^2(e^{2x}+1)^3\sqrt{a(e^{2x}+1)^2e^{-2x}}}+\frac{3i(e^{2x}+1)e^{-x}\ln(e^x+i)}{8a^2\sqrt{a(e^{2x}+1)^2e^{-2x}}}-\frac{3i(e^{2x}+1)e^{-x}\ln(e^x-i)}{8a^2\sqrt{a(e^{2x}+1)^2e^{-2x}}}$	127

input
```
int(1/(a*cosh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/8/a^3/cosh(x)^3*(a*sinh(x)^2)^(1/2)*(-3*ln(2*((-a)^(1/2)*(a*sinh(x)^2)^(
1/2)-a)/cosh(x))*a*cosh(x)^4+3*cosh(x)^2*(a*sinh(x)^2)^(1/2)*(-a)^(1/2)+2*
(-a)^(1/2)*(a*sinh(x)^2)^(1/2))/(-a)^(1/2)/sinh(x)/(a*cosh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(49) = 98$.

Time = 0.09 (sec) , antiderivative size = 837, normalized size of antiderivative = 13.72

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
1/4*(21*cosh(x)*e^x*sinh(x)^6 + 3*e^x*sinh(x)^7 + (63*cosh(x)^2 + 11)*e^x*
sinh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*e^x*sinh(x)^4 + (105*cosh(x)^4 +
 110*cosh(x)^2 - 11)*e^x*sinh(x)^3 + (63*cosh(x)^5 + 110*cosh(x)^3 - 33*co
sh(x))*e^x*sinh(x)^2 + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*e^
x*sinh(x) + 3*(8*cosh(x)*e^x*sinh(x)^7 + e^x*sinh(x)^8 + 4*(7*cosh(x)^2 +
1)*e^x*sinh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*e^x*sinh(x)^5 + 2*(35*cosh(
x)^4 + 30*cosh(x)^2 + 3)*e^x*sinh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3
*cosh(x))*e^x*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)
*e^x*sinh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*e^x*s
inh(x) + (cosh(x)^8 + 4*cosh(x)^6 + 6*cosh(x)^4 + 4*cosh(x)^2 + 1)*e^x)*ar
ctan(cosh(x) + sinh(x)) + (3*cosh(x)^7 + 11*cosh(x)^5 - 11*cosh(x)^3 - 3*c
osh(x))*e^x)*sqrt(a*e^(4*x) + 2*a*e^(2*x) + a)*e^(-x)/(a^3*cosh(x)^8 + 4*a
^3*cosh(x)^6 + (a^3*e^(2*x) + a^3)*sinh(x)^8 + 8*(a^3*cosh(x)*e^(2*x) + a^
3*cosh(x))*sinh(x)^7 + 6*a^3*cosh(x)^4 + 4*(7*a^3*cosh(x)^2 + a^3 + (7*a^3
*cosh(x)^2 + a^3)*e^(2*x))*sinh(x)^6 + 8*(7*a^3*cosh(x)^3 + 3*a^3*cosh(x)
+ (7*a^3*cosh(x)^3 + 3*a^3*cosh(x))*e^(2*x))*sinh(x)^5 + 4*a^3*cosh(x)^2 +
 2*(35*a^3*cosh(x)^4 + 30*a^3*cosh(x)^2 + 3*a^3 + (35*a^3*cosh(x)^4 + 30*a
^3*cosh(x)^2 + 3*a^3)*e^(2*x))*sinh(x)^4 + 8*(7*a^3*cosh(x)^5 + 10*a^3*cos
h(x)^3 + 3*a^3*cosh(x) + (7*a^3*cosh(x)^5 + 10*a^3*cosh(x)^3 + 3*a^3*cosh(
x))*e^(2*x))*sinh(x)^3 + a^3 + 4*(7*a^3*cosh(x)^6 + 15*a^3*cosh(x)^4 + ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \frac{3\,e^{(7\,x)} + 11\,e^{(5\,x)} - 11\,e^{(3\,x)} - 3\,e^{x}}{4\left(a^{\frac{5}{2}}e^{(8\,x)} + 4\,a^{\frac{5}{2}}e^{(6\,x)} + 6\,a^{\frac{5}{2}}e^{(4\,x)} + 4\,a^{\frac{5}{2}}e^{(2\,x)} + a^{\frac{5}{2}}\right)} + \frac{3\,\arctan\left(e^{x}\right)}{4\,a^{\frac{5}{2}}}$$

input
```
integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
1/4*(3*e^(7*x) + 11*e^(5*x) - 11*e^(3*x) - 3*e^x)/(a^(5/2)*e^(8*x) + 4*a^(
5/2)*e^(6*x) + 6*a^(5/2)*e^(4*x) + 4*a^(5/2)*e^(2*x) + a^(5/2)) + 3/4*arct
an(e^x)/a^(5/2)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.18

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \frac{\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,x)}-1\right)e^{(-x)}\right)\right)}{\sqrt{a}} - \frac{4\left(3\left(e^{(-x)}-e^{x}\right)^{3}+20\,e^{(-x)}-20\,e^{x}\right)}{\left(\left(e^{(-x)}-e^{x}\right)^{2}+4\right)^{2}\sqrt{a}}}{16\,a^{2}}$$

input
```
integrate(1/(a*cosh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/16*(3*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/sqrt(a) - 4*(3*(e^(-x) -
 e^x)^3 + 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*sqrt(a)))/a^2
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^2\right)^{5/2}}\,dx$$

input
```
int(1/(a*cosh(x)^2)^(5/2),x)
```

output
```
int(1/(a*cosh(x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.87

$$\int \frac{1}{\left(a\cosh^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\,\left(3e^{8x}atan(e^{x}) + 12e^{6x}atan(e^{x}) + 18e^{4x}atan(e^{x}) + 12e^{2x}atan(e^{x}) + 3atan(e^{x}) +\right.}{4a^3\left(e^{8x}+4e^{6x}+6e^{4x}+4e^{2x}+1\right)}$$

input
```
int(1/(a*cosh(x)^2)^(5/2),x)
```

output
```
(sqrt(a)*(3*e**(8*x)*atan(e**x) + 12*e**(6*x)*atan(e**x) + 18*e**(4*x)*ata
n(e**x) + 12*e**(2*x)*atan(e**x) + 3*atan(e**x) + 3*e**(7*x) + 11*e**(5*x)
 - 11*e**(3*x) - 3*e**x))/(4*a**3*(e**(8*x) + 4*e**(6*x) + 6*e**(4*x) + 4*
e**(2*x) + 1))
```

3.128 $\int \left(a\cosh^3(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = -\frac{26ia^2\sqrt{a\cosh^3(x)}\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{77\cosh^{\frac{3}{2}}(x)} + \frac{78}{385}a^2\cosh(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{26}{165}a^2\cosh^3(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{2}{15}a^2\cosh^5(x)\sqrt{a\cosh^3(x)}\sinh(x) + \frac{26}{77}a^2\sqrt{a\cosh^3(x)}\tanh(x)$$

output
```
-26/77*I*a^2*(a*cosh(x)^3)^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2))/cosh(x)^
(3/2)+78/385*a^2*cosh(x)*(a*cosh(x)^3)^(1/2)*sinh(x)+26/165*a^2*cosh(x)^3*
(a*cosh(x)^3)^(1/2)*sinh(x)+2/15*a^2*cosh(x)^5*(a*cosh(x)^3)^(1/2)*sinh(x)
+26/77*a^2*(a*cosh(x)^3)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.54

$$\int \left(a \cosh^3(x)\right)^{5/2} dx = \frac{a\left(a\cosh^3(x)\right)^{3/2}\left(-12480i\,\text{EllipticF}\left(\frac{ix}{2},2\right)+\sqrt{\cosh(x)}(15465\sinh(x)+3657\sin\right.}{36960\cosh^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Cosh[x]^3)^(5/2),x]
```

output
```
(a*(a*Cosh[x]^3)^(3/2)*((-12480*I)*EllipticF[(I/2)*x, 2] + Sqrt[Cosh[x]]*(
15465*Sinh[x] + 3657*Sinh[3*x] + 749*Sinh[5*x] + 77*Sinh[7*x])))/(36960*Co
sh[x]^(9/2))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.82, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a \cosh^3(x)\right)^{5/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a \sin\left(\frac{\pi}{2}+ix\right)^3\right)^{5/2} dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\int \cosh^{\frac{15}{2}}(x)dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{15/2}dx}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\int\cosh^{\frac{11}{2}}(x)dx+\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\int\sin\left(ix+\frac{\pi}{2}\right)^{11/2}dx\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\int\cosh^{\frac{7}{2}}(x)dx+\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\int\sin\left(ix+\frac{\pi}{2}\right)^{7/2}dx\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\int\cosh^{\frac{3}{2}}(x)dx+\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)\right)+\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)\right)+\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\left(\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)+\frac{5}{7}\int\sin\left(ix+\frac{\pi}{2}\right)^{3/2}d\right.\right.\right.}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3115}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{13}{15}\left(\frac{9}{11}\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\cosh(x)}}dx+\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}\right)+\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)\right)+\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)\right)\right.}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\left(\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh}\right.\right.\right.\right.}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3120}$$

$$\frac{a^2\sqrt{a\cosh^3(x)}\left(\frac{2}{15}\sinh(x)\cosh^{\frac{13}{2}}(x)+\frac{13}{15}\left(\frac{2}{11}\sinh(x)\cosh^{\frac{9}{2}}(x)+\frac{9}{11}\left(\frac{2}{7}\sinh(x)\cosh^{\frac{5}{2}}(x)+\frac{5}{7}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh(x}\right.\right.\right.\right.}{\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Cosh[x]^3)^(5/2),x]
```

output
```
(a^2*Sqrt[a*Cosh[x]^3]*((2*Cosh[x]^(13/2)*Sinh[x])/15 + (13*((2*Cosh[x]^(9
/2)*Sinh[x])/11 + (9*((2*Cosh[x]^(5/2)*Sinh[x])/7 + (5*(((-2*I)/3)*Ellipti
cF[(I/2)*x, 2] + (2*Sqrt[Cosh[x]]*Sinh[x])/3))/7))/11))/15))/Cosh[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(a\cosh \left(x\right)^{3}\right)^{\frac{5}{2}}dx$$

input
```
int((a*cosh(x)^3)^(5/2),x)
```

output
```
int((a*cosh(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 802 vs. 2(95) = 190.

Time = 0.11 (sec) , antiderivative size = 802, normalized size of antiderivative = 6.63

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
1/73920*(49920*sqrt(1/2)*(a^2*cosh(x)^7 + 7*a^2*cosh(x)^6*sinh(x) + 21*a^2
*cosh(x)^5*sinh(x)^2 + 35*a^2*cosh(x)^4*sinh(x)^3 + 35*a^2*cosh(x)^3*sinh(
x)^4 + 21*a^2*cosh(x)^2*sinh(x)^5 + 7*a^2*cosh(x)*sinh(x)^6 + a^2*sinh(x)^
7)*sqrt(a)*weierstrassPInverse(-4, 0, cosh(x) + sinh(x)) + (77*a^2*cosh(x)
^14 + 1078*a^2*cosh(x)*sinh(x)^13 + 77*a^2*sinh(x)^14 + 749*a^2*cosh(x)^12
 + 7*(1001*a^2*cosh(x)^2 + 107*a^2)*sinh(x)^12 + 3657*a^2*cosh(x)^10 + 28*
(1001*a^2*cosh(x)^3 + 321*a^2*cosh(x))*sinh(x)^11 + (77077*a^2*cosh(x)^4 +
 49434*a^2*cosh(x)^2 + 3657*a^2)*sinh(x)^10 + 15465*a^2*cosh(x)^8 + 2*(770
77*a^2*cosh(x)^5 + 82390*a^2*cosh(x)^3 + 18285*a^2*cosh(x))*sinh(x)^9 + 3*
(77077*a^2*cosh(x)^6 + 123585*a^2*cosh(x)^4 + 54855*a^2*cosh(x)^2 + 5155*a
^2)*sinh(x)^8 - 15465*a^2*cosh(x)^6 + 24*(11011*a^2*cosh(x)^7 + 24717*a^2*
cosh(x)^5 + 18285*a^2*cosh(x)^3 + 5155*a^2*cosh(x))*sinh(x)^7 + 3*(77077*a
^2*cosh(x)^8 + 230692*a^2*cosh(x)^6 + 255990*a^2*cosh(x)^4 + 144340*a^2*co
sh(x)^2 - 5155*a^2)*sinh(x)^6 - 3657*a^2*cosh(x)^4 + 2*(77077*a^2*cosh(x)^
9 + 296604*a^2*cosh(x)^7 + 460782*a^2*cosh(x)^5 + 433020*a^2*cosh(x)^3 - 4
6395*a^2*cosh(x))*sinh(x)^5 + (77077*a^2*cosh(x)^10 + 370755*a^2*cosh(x)^8
 + 767970*a^2*cosh(x)^6 + 1082550*a^2*cosh(x)^4 - 231975*a^2*cosh(x)^2 - 3
657*a^2)*sinh(x)^4 - 749*a^2*cosh(x)^2 + 4*(7007*a^2*cosh(x)^11 + 41195*a^
2*cosh(x)^9 + 109710*a^2*cosh(x)^7 + 216510*a^2*cosh(x)^5 - 77325*a^2*cosh
(x)^3 - 3657*a^2*cosh(x))*sinh(x)^3 + (7007*a^2*cosh(x)^12 + 49434*a^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**3)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \int \left(a\cosh\left(x\right)^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(5/2), x)
```

Giac [F]

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \int \left(a\cosh\left(x\right)^3\right)^{\frac{5}{2}}\,dx$$

input
```
integrate((a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{5/2}\,dx = \int \left(a\cosh(x)^3\right)^{5/2}\,dx$$

input
```
int((a*cosh(x)^3)^(5/2),x)
```

output
```
int((a*cosh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \left(a\cosh^3(x)\right)^{5/2} dx = \sqrt{a}\left(\int \sqrt{\cosh(x)}\,\cosh(x)^7\,dx\right)a^2$$

input
```
int((a*cosh(x)^3)^(5/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x))*cosh(x)**7,x)*a**2
```

3.129 $\int \left(a\cosh^3(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 71

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx=-\frac{14ia\sqrt{a\cosh^3(x)}E\left(\frac{ix}{2}\middle|2\right)}{15\cosh^{\frac{3}{2}}(x)}+\frac{14}{45}a\sqrt{a\cosh^3(x)}\sinh(x)+\frac{2}{9}a\cosh^2(x)\sqrt{a\cosh^3(x)}\sinh(x)$$

output
```
-14/15*I*a*(a*cosh(x)^3)^(1/2)*EllipticE(I*sinh(1/2*x),2^(1/2))/cosh(x)^(3
/2)+14/45*a*(a*cosh(x)^3)^(1/2)*sinh(x)+2/9*a*cosh(x)^2*(a*cosh(x)^3)^(1/2
)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx=\frac{\left(a\cosh^3(x)\right)^{3/2}\left(-168iE\left(\frac{ix}{2}\middle|2\right)+\sqrt{\cosh(x)}(38\sinh(2x)+5\sinh(4x))\right)}{180\cosh^{\frac{9}{2}}(x)}$$

input
```
Integrate[(a*Cosh[x]^3)^(3/2),x]
```

output
```
((a*Cosh[x]^3)^(3/2)*((-168*I)*EllipticE[(I/2)*x, 2] + Sqrt[Cosh[x]]*(38*S
inh[2*x] + 5*Sinh[4*x])))/(180*Cosh[x]^(9/2))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^3(x)\right)^{3/2}dx$$
$$\Big\downarrow \ 3042$$
$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^3\right)^{3/2}dx$$
$$\Big\downarrow \ 3686$$
$$\frac{a\sqrt{a\cosh^3(x)}\int\cosh^{\frac{9}{2}}(x)dx}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \ 3042$$
$$\frac{a\sqrt{a\cosh^3(x)}\int\sin\left(ix+\frac{\pi}{2}\right)^{9/2}dx}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \ 3115$$
$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{7}{9}\int\cosh^{\frac{5}{2}}(x)dx+\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \ 3042$$
$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)+\frac{7}{9}\int\sin\left(ix+\frac{\pi}{2}\right)^{5/2}dx\right)}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \ 3115$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{7}{9}\left(\frac{3}{5}\int\sqrt{\cosh(x)}dx+\frac{2}{5}\sinh(x)\cosh^{\frac{3}{2}}(x)\right)+\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\cosh^{\frac{3}{2}}(x)+\frac{3}{5}\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

$$\Big\downarrow \text{3119}$$

$$\frac{a\sqrt{a\cosh^3(x)}\left(\frac{2}{9}\sinh(x)\cosh^{\frac{7}{2}}(x)+\frac{7}{9}\left(\frac{2}{5}\sinh(x)\cosh^{\frac{3}{2}}(x)-\frac{6}{5}iE\left(\frac{ix}{2}\middle|2\right)\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[(a*Cosh[x]^3)^(3/2),x]
```

output
```
(a*Sqrt[a*Cosh[x]^3]*((2*Cosh[x]^(7/2)*Sinh[x])/9 + (7*(((-6*I)/5)*Ellipti
cE[(I/2)*x, 2] + (2*Cosh[x]^(3/2)*Sinh[x])/5))/9))/Cosh[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \left(a \cosh \left(x \right)^{3}\right)^{\frac{3}{2}}d x$$

input
```
int((a*cosh(x)^3)^(3/2),x)
```

output
```
int((a*cosh(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 305, normalized size of antiderivative = 4.30

$$\int \left(a\cosh^3(x)\right)^{3/2} dx =$$
$$-\frac{672\sqrt{\frac{1}{2}}\left(a\cosh\left(x\right)^{4}+4\,a\cosh\left(x\right)^{3}\sinh\left(x\right)+6\,a\cosh\left(x\right)^{2}\sinh\left(x\right)^{2}+4\,a\cosh\left(x\right)\sinh\left(x\right)^{3}+a\sinh\left(x\right)^{4}\right.}{}$$

input
```
integrate((a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
-1/360*(672*sqrt(1/2)*(a*cosh(x)^4 + 4*a*cosh(x)^3*sinh(x) + 6*a*cosh(x)^2
*sinh(x)^2 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4)*sqrt(a)*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) - (5*a*cosh(x)^8 + 4
0*a*cosh(x)*sinh(x)^7 + 5*a*sinh(x)^8 + 38*a*cosh(x)^6 + 2*(70*a*cosh(x)^2
 + 19*a)*sinh(x)^6 + 4*(70*a*cosh(x)^3 + 57*a*cosh(x))*sinh(x)^5 - 336*a*c
osh(x)^4 + 2*(175*a*cosh(x)^4 + 285*a*cosh(x)^2 - 168*a)*sinh(x)^4 + 8*(35
*a*cosh(x)^5 + 95*a*cosh(x)^3 - 168*a*cosh(x))*sinh(x)^3 - 38*a*cosh(x)^2
+ 2*(70*a*cosh(x)^6 + 285*a*cosh(x)^4 - 1008*a*cosh(x)^2 - 19*a)*sinh(x)^2
 + 4*(10*a*cosh(x)^7 + 57*a*cosh(x)^5 - 336*a*cosh(x)^3 - 19*a*cosh(x))*si
nh(x) - 5*a)*sqrt(a*cosh(x)))/(cosh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)
^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4)
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**3)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(a\cosh^3(x)\right)^{3/2} dx = \int \left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}} dx$$

input
```
integrate((a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(3/2), x)
```

Giac [F]

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx=\int \left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx=\int \left(a\cosh(x)^3\right)^{3/2}\,dx$$

input
```
int((a*cosh(x)^3)^(3/2),x)
```

output
```
int((a*cosh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \left(a\cosh^3(x)\right)^{3/2}\,dx=\sqrt{a}\left(\int \sqrt{\cosh\left(x\right)}\,\cosh\left(x\right)^4dx\right)a$$

input
```
int((a*cosh(x)^3)^(3/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x))*cosh(x)**4,x)*a
```

3.130 $\int \sqrt{a \cosh^3(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 48

$$\int \sqrt{a \cosh^3(x)}\, dx = -\frac{2i\sqrt{a \cosh^3(x)}\, \text{EllipticF}\left(\frac{ix}{2}, 2\right)}{3\cosh^{\frac{3}{2}}(x)} + \frac{2}{3}\sqrt{a \cosh^3(x)} \tanh(x)$$

output
```
-2/3*I*(a*cosh(x)^3)^(1/2)*InverseJacobiAM(1/2*I*x,2^(1/2))/cosh(x)^(3/2)+
2/3*(a*cosh(x)^3)^(1/2)*tanh(x)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.23

$$\int \sqrt{a \cosh^3(x)}\, dx = \frac{2}{3}\sqrt{a \cosh^3(x)} \left(\text{Hypergeometric2F1}\left(\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\cosh(2x) - \sinh(2x)\right) \text{sech}^2(x) \sqrt{1 + \cosh(2x) + \sinh(2x)} + \tanh(x)\right)$$

input
```
Integrate[Sqrt[a*Cosh[x]^3],x]
```

output
```
(2*Sqrt[a*Cosh[x]^3]*(Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*x] - Sinh[2
*x]]*Sech[x]^2*Sqrt[1 + Cosh[2*x] + Sinh[2*x]] + Tanh[x]))/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a\cosh^3(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^3}dx$$
$$\Big\downarrow \text{3686}$$
$$\frac{\sqrt{a\cosh^3(x)}\int \cosh^{\frac{3}{2}}(x)dx}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{a\cosh^3(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{3/2}dx}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\sqrt{a\cosh^3(x)}\left(\frac{1}{3}\int \frac{1}{\sqrt{\cosh(x)}}dx+\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}\right)}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sqrt{a\cosh^3(x)}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)}{\cosh^{\frac{3}{2}}(x)}$$
$$\Big\downarrow \text{3120}$$

$$\frac{\sqrt{a\cosh^3(x)}\left(\frac{2}{3}\sinh(x)\sqrt{\cosh(x)}-\frac{2}{3}i\,\text{EllipticF}\left(\frac{ix}{2},2\right)\right)}{\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[Sqrt[a*Cosh[x]^3],x]
```

output
```
(Sqrt[a*Cosh[x]^3]*(((-2*I)/3)*EllipticF[(I/2)*x, 2] + (2*Sqrt[Cosh[x]]*Si
nh[x])/3))/Cosh[x]^(3/2)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \sqrt{a \cosh(x)^3} dx$$

input
```
int((a*cosh(x)^3)^(1/2),x)
```

output
```
int((a*cosh(x)^3)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.12

$$\int \sqrt{a \cosh^3(x)}\, dx = \frac{4 \sqrt{\frac{1}{2}} \sqrt{a}(\cosh(x) + \sinh(x)) \mathrm{weierstrassPInverse}(-4, 0, \cosh(x) + \sinh(x)) + \sqrt{a \cosh(x)}(\cosh(x)^2 +}{3(\cosh(x) + \sinh(x))}$$

input
```
integrate((a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
1/3*(4*sqrt(1/2)*sqrt(a)*(cosh(x) + sinh(x))*weierstrassPInverse(-4, 0, co
sh(x) + sinh(x)) + sqrt(a*cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x
)^2 - 1))/(cosh(x) + sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a \cosh^3(x)}\, dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**3)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \sqrt{a\cosh^3(x)}\,dx = \int \sqrt{a\cosh(x)^3}\,dx$$

input
```
integrate((a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(a*cosh(x)^3), x)
```

Giac [F]

$$\int \sqrt{a\cosh^3(x)}\,dx = \int \sqrt{a\cosh(x)^3}\,dx$$

input
```
integrate((a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(a*cosh(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cosh^3(x)}\,dx = \int \sqrt{a\cosh(x)^3}\,dx$$

input
```
int((a*cosh(x)^3)^(1/2),x)
```

output
```
int((a*cosh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a\cosh^3(x)}\,dx = \sqrt{a}\left(\int \sqrt{\cosh(x)}\,\cosh(x)\,dx\right)$$

input
```
int((a*cosh(x)^3)^(1/2),x)
```

output
```
sqrt(a)*int(sqrt(cosh(x))*cosh(x),x)
```

3.131 $\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 46

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \frac{2i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{\sqrt{a\cosh^3(x)}} + \frac{2\cosh(x)\sinh(x)}{\sqrt{a\cosh^3(x)}}$$

output
```
2*I*cosh(x)^(3/2)*EllipticE(I*sinh(1/2*x),2^(1/2))/(a*cosh(x)^3)^(1/2)+2*c
osh(x)*sinh(x)/(a*cosh(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.78

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \frac{2\cosh(x)\left(i\sqrt{\cosh(x)}E\left(\frac{ix}{2}\middle|2\right)+\sinh(x)\right)}{\sqrt{a\cosh^3(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]^3],x]
```

output
```
(2*Cosh[x]*(I*Sqrt[Cosh[x]]*EllipticE[(I/2)*x, 2] + Sinh[x]))/Sqrt[a*Cosh[
x]^3]
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 3686, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^3}}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{3}{2}}(x)}dx}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{3/2}}dx}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int \sqrt{\cosh(x)}dx\right)}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int \sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)}{\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}+2iE\left(\frac{ix}{2}\middle|2\right)\right)}{\sqrt{a\cosh^3(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]^3],x]
```

output
```
(Cosh[x]^(3/2)*((2*I)*EllipticE[(I/2)*x, 2] + (2*Sinh[x])/Sqrt[Cosh[x]]))/
Sqrt[a*Cosh[x]^3]
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\sqrt{a \cosh(x)^3}}\,dx$$

input
```
int(1/(a*cosh(x)^3)^(1/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(39) = 78$.

Time = 0.08 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{1}{\sqrt{a \cosh^3(x)}}\,dx$$

$$= \frac{4\left(\sqrt{\frac{1}{2}}\left(\cosh(x)^2 + 2\cosh(x)\sinh(x) + \sinh(x)^2 + 1\right)\sqrt{a}\text{weierstrassZeta}(-4, 0, \text{weierstrassPInverse}(-\right.}{a\cosh(x)^2 + 2a\cosh(x)\sinh(x)}$$

input
```
integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="fricas")
```

output
```
4*(sqrt(1/2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a)*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) + sqrt(a*
cosh(x))*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2))/(a*cosh(x)^2 + 2*a*c
osh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F]

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh^3{(x)}}}\,dx$$

input
```
integrate(1/(a*cosh(x)**3)**(1/2),x)
```

output
```
Integral(1/sqrt(a*cosh(x)**3), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh{(x)}^3}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(a*cosh(x)^3), x)
```

Giac [F]

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh{(x)}^3}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(1/2),x, algorithm="giac")
```

output
```
integrate(1/sqrt(a*cosh(x)^3), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \int \frac{1}{\sqrt{a\cosh(x)^3}}\,dx$$

input
```
int(1/(a*cosh(x)^3)^(1/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(1/2), x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a\cosh^3(x)}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)}}{\cosh(x)^2}dx\right)}{a}$$

input
```
int(1/(a*cosh(x)^3)^(1/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x))/cosh(x)**2,x))/a
```

3.132 $\int \frac{1}{(a\cosh^3(x))^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 75

$$\int \frac{1}{(a\cosh^3(x))^{3/2}}\,dx = -\frac{10i\cosh^{\frac{3}{2}}(x)\,\text{EllipticF}\left(\frac{ix}{2},2\right)}{21a\sqrt{a\cosh^3(x)}} + \frac{10\sinh(x)}{21a\sqrt{a\cosh^3(x)}} + \frac{2\text{sech}(x)\tanh(x)}{7a\sqrt{a\cosh^3(x)}}$$

output
```
-10/21*I*cosh(x)^(3/2)*InverseJacobiAM(1/2*I*x,2^(1/2))/a/(a*cosh(x)^3)^(1
/2)+10/21*sinh(x)/a/(a*cosh(x)^3)^(1/2)+2/7*sech(x)*tanh(x)/a/(a*cosh(x)^3
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.64

$$\int \frac{1}{(a\cosh^3(x))^{3/2}}\,dx = \frac{2\cosh^2(x)\left(-5i\cosh^{\frac{5}{2}}(x)\,\text{EllipticF}\left(\frac{ix}{2},2\right)+5\cosh(x)\sinh(x)+3\tanh(x)\right)}{21\left(a\cosh^3(x)\right)^{3/2}}$$

input
```
Integrate[(a*Cosh[x]^3)^(-3/2),x]
```

output
```
(2*Cosh[x]^2*((-5*I)*Cosh[x]^(5/2)*EllipticF[(I/2)*x, 2] + 5*Cosh[x]*Sinh[
x] + 3*Tanh[x]))/(21*(a*Cosh[x]^3)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.87, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh^3(x))^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^3\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3686}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{9}{2}}(x)}dx}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{9/2}}dx}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{ 3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{5}{7}\int \frac{1}{\cosh^{\frac{5}{2}}(x)}dx+\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}+\frac{5}{7}\int\frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{5/2}}dx\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{5}{7}\left(\frac{1}{3}\int\frac{1}{\sqrt{\cosh(x)}}dx+\frac{2\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}\right)+\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}+\frac{5}{7}\left(\frac{2\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}+\frac{1}{3}\int\frac{1}{\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}}dx\right)\right)}{a\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{7\cosh^{\frac{7}{2}}(x)}+\frac{5}{7}\left(\frac{2\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}-\frac{2}{3}i\,\text{EllipticF}\left(\frac{ix}{2},2\right)\right)\right)}{a\sqrt{a\cosh^3(x)}}$$

input
```
Int[(a*Cosh[x]^3)^(-3/2),x]
```

output
```
(Cosh[x]^(3/2)*((2*Sinh[x])/(7*Cosh[x]^(7/2)) + (5*(((-2*I)/3)*EllipticF[(
I/2)*x, 2] + (2*Sinh[x])/(3*Cosh[x]^(3/2))))/7))/(a*Sqrt[a*Cosh[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(a \cosh \left(x \right)^{3}\right)^{\frac{3}{2}}}d x$$

input
```
int(1/(a*cosh(x)^3)^(3/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(57) = 114$.

Time = 0.13 (sec) , antiderivative size = 554, normalized size of antiderivative = 7.39

$$\int \frac{1}{\left(a \cosh^3(x)\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="fricas")
```

output
```
4/21*(5*sqrt(1/2)*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh
(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3 + 3*cosh(x))*sinh(x)^5
 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*cosh(x)^4 + 8*(7*cosh
(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cosh(x)^6 + 15*cosh(x)^
4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(x)^7 + 3*cosh(x)^5
+ 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(a)*weierstrassPInverse(-4, 0, c
osh(x) + sinh(x)) + (5*cosh(x)^7 + 35*cosh(x)*sinh(x)^6 + 5*sinh(x)^7 + (1
05*cosh(x)^2 + 17)*sinh(x)^5 + 17*cosh(x)^5 + 5*(35*cosh(x)^3 + 17*cosh(x)
)*sinh(x)^4 + (175*cosh(x)^4 + 170*cosh(x)^2 - 17)*sinh(x)^3 - 17*cosh(x)^
3 + (105*cosh(x)^5 + 170*cosh(x)^3 - 51*cosh(x))*sinh(x)^2 + (35*cosh(x)^6
 + 85*cosh(x)^4 - 51*cosh(x)^2 - 5)*sinh(x) - 5*cosh(x))*sqrt(a*cosh(x)))/
(a^2*cosh(x)^8 + 8*a^2*cosh(x)*sinh(x)^7 + a^2*sinh(x)^8 + 4*a^2*cosh(x)^6
 + 4*(7*a^2*cosh(x)^2 + a^2)*sinh(x)^6 + 6*a^2*cosh(x)^4 + 8*(7*a^2*cosh(x
)^3 + 3*a^2*cosh(x))*sinh(x)^5 + 2*(35*a^2*cosh(x)^4 + 30*a^2*cosh(x)^2 +
3*a^2)*sinh(x)^4 + 4*a^2*cosh(x)^2 + 8*(7*a^2*cosh(x)^5 + 10*a^2*cosh(x)^3
 + 3*a^2*cosh(x))*sinh(x)^3 + 4*(7*a^2*cosh(x)^6 + 15*a^2*cosh(x)^4 + 9*a^
2*cosh(x)^2 + a^2)*sinh(x)^2 + a^2 + 8*(a^2*cosh(x)^7 + 3*a^2*cosh(x)^5 +
3*a^2*cosh(x)^3 + a^2*cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**3)**(3/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(3/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(-3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{3/2}}\,dx$$

input
```
int(1/(a*cosh(x)^3)^(3/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{3/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)}}{\cosh(x)^5}dx\right)}{a^2}$$

input
```
int(1/(a*cosh(x)^3)^(3/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x))/cosh(x)**5,x))/a**2
```

3.133 $\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 121

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \frac{154i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\cosh(x)\sinh(x)}{195a^2\sqrt{a\cosh^3(x)}} + \frac{154\tanh(x)}{585a^2\sqrt{a\cosh^3(x)}} + \frac{22\operatorname{sech}^2(x)\tanh(x)}{117a^2\sqrt{a\cosh^3(x)}} + \frac{2\operatorname{sech}^4(x)\tanh(x)}{13a^2\sqrt{a\cosh^3(x)}}$$

output
```
154/195*I*cosh(x)^(3/2)*EllipticE(I*sinh(1/2*x),2^(1/2))/a^2/(a*cosh(x)^3)
^(1/2)+154/195*cosh(x)*sinh(x)/a^2/(a*cosh(x)^3)^(1/2)+154/585*tanh(x)/a^2
/(a*cosh(x)^3)^(1/2)+22/117*sech(x)^2*tanh(x)/a^2/(a*cosh(x)^3)^(1/2)+2/13
*sech(x)^4*tanh(x)/a^2/(a*cosh(x)^3)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.50

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \frac{462i\cosh^{\frac{3}{2}}(x)E\left(\frac{ix}{2}\middle|2\right) + 462\cosh(x)\sinh(x) + 2\left(77 + 55\operatorname{sech}^2(x) + 45\operatorname{sech}^4(x)\right) t}{585a^2\sqrt{a\cosh^3(x)}}$$

input
```
Integrate[(a*Cosh[x]^3)^(-5/2),x]
```

output
```
((462*I)*Cosh[x]^(3/2)*EllipticE[(I/2)*x, 2] + 462*Cosh[x]*Sinh[x] + 2*(77
 + 55*Sech[x]^2 + 45*Sech[x]^4)*Tanh[x])/(585*a^2*Sqrt[a*Cosh[x]^3])
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.79, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 1.200$, Rules used = {3042, 3686, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh^3(x))^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^3\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\cosh^{\frac{15}{2}}(x)}dx}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\int \frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{15/2}}dx}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\int \frac{1}{\cosh^{\frac{11}{2}}(x)}dx+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\int\frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{11/2}}dx\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\int\frac{1}{\cosh^{\frac{7}{2}}(x)}dx+\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\int\frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{7/2}}dx\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\int\frac{1}{\cosh^{\frac{3}{2}}(x)}dx+\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}\right)+\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\int\frac{1}{\sin\left(ix+\frac{\pi}{2}\right)^{3/2}}dx\right)\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{11}{13}\left(\frac{7}{9}\left(\frac{3}{5}\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int\sqrt{\cosh(x)}dx\right)+\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}\right)+\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}\right)+\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}-\int\sqrt{\sin\left(ix+\frac{\pi}{2}\right)}dx\right)\right)\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\cosh^{\frac{3}{2}}(x)\left(\frac{2\sinh(x)}{13\cosh^{\frac{13}{2}}(x)}+\frac{11}{13}\left(\frac{2\sinh(x)}{9\cosh^{\frac{9}{2}}(x)}+\frac{7}{9}\left(\frac{2\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{3}{5}\left(\frac{2\sinh(x)}{\sqrt{\cosh(x)}}+2iE\left(\frac{ix}{2}\middle|2\right)\right)\right)\right)\right)}{a^2\sqrt{a\cosh^3(x)}}$$

input
```
Int[(a*Cosh[x]^3)^(-5/2),x]
```

output
```
(Cosh[x]^(3/2)*((2*Sinh[x])/(13*Cosh[x]^(13/2)) + (11*((2*Sinh[x])/(9*Cosh
[x]^(9/2)) + (7*((2*Sinh[x])/(5*Cosh[x]^(5/2)) + (3*((2*I)*EllipticE[(I/2)
*x, 2] + (2*Sinh[x])/Sqrt[Cosh[x]]))/5))/9))/13))/(a^2*Sqrt[a*Cosh[x]^3])
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [F]

$$\int \frac{1}{\left(a \cosh \left(x \right)^{3}\right)^{\frac{5}{2}}}d x$$

input
```
int(1/(a*cosh(x)^3)^(5/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(98) = 196$.

Time = 0.15 (sec) , antiderivative size = 1473, normalized size of antiderivative = 12.17

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="fricas")
```

output
```
4/585*(231*sqrt(1/2)*(cosh(x)^14 + 14*cosh(x)*sinh(x)^13 + sinh(x)^14 + 7*
(13*cosh(x)^2 + 1)*sinh(x)^12 + 7*cosh(x)^12 + 28*(13*cosh(x)^3 + 3*cosh(x
))*sinh(x)^11 + 7*(143*cosh(x)^4 + 66*cosh(x)^2 + 3)*sinh(x)^10 + 21*cosh(
x)^10 + 14*(143*cosh(x)^5 + 110*cosh(x)^3 + 15*cosh(x))*sinh(x)^9 + 7*(429
*cosh(x)^6 + 495*cosh(x)^4 + 135*cosh(x)^2 + 5)*sinh(x)^8 + 35*cosh(x)^8 +
 8*(429*cosh(x)^7 + 693*cosh(x)^5 + 315*cosh(x)^3 + 35*cosh(x))*sinh(x)^7
+ 7*(429*cosh(x)^8 + 924*cosh(x)^6 + 630*cosh(x)^4 + 140*cosh(x)^2 + 5)*si
nh(x)^6 + 35*cosh(x)^6 + 14*(143*cosh(x)^9 + 396*cosh(x)^7 + 378*cosh(x)^5
 + 140*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 7*(143*cosh(x)^10 + 495*cosh(x)
^8 + 630*cosh(x)^6 + 350*cosh(x)^4 + 75*cosh(x)^2 + 3)*sinh(x)^4 + 21*cosh
(x)^4 + 28*(13*cosh(x)^11 + 55*cosh(x)^9 + 90*cosh(x)^7 + 70*cosh(x)^5 + 2
5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 7*(13*cosh(x)^12 + 66*cosh(x)^10 + 13
5*cosh(x)^8 + 140*cosh(x)^6 + 75*cosh(x)^4 + 18*cosh(x)^2 + 1)*sinh(x)^2 +
 7*cosh(x)^2 + 14*(cosh(x)^13 + 6*cosh(x)^11 + 15*cosh(x)^9 + 20*cosh(x)^7
 + 15*cosh(x)^5 + 6*cosh(x)^3 + cosh(x))*sinh(x) + 1)*sqrt(a)*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cosh(x) + sinh(x))) + (231*cosh(x)^1
4 + 3234*cosh(x)*sinh(x)^13 + 231*sinh(x)^14 + 77*(273*cosh(x)^2 + 20)*sin
h(x)^12 + 1540*cosh(x)^12 + 924*(91*cosh(x)^3 + 20*cosh(x))*sinh(x)^11 + 1
1*(21021*cosh(x)^4 + 9240*cosh(x)^2 + 397)*sinh(x)^10 + 4367*cosh(x)^10 +
22*(21021*cosh(x)^5 + 15400*cosh(x)^3 + 1985*cosh(x))*sinh(x)^9 + (6936...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**3)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="maxima")
```

output
```
integrate((a*cosh(x)^3)^(-5/2), x)
```

Giac [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a*cosh(x)^3)^(5/2),x, algorithm="giac")
```

output
```
integrate((a*cosh(x)^3)^(-5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cosh\left(x\right)^3\right)^{5/2}}\,dx$$

input
```
int(1/(a*cosh(x)^3)^(5/2),x)
```

output
```
int(1/(a*cosh(x)^3)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a\cosh^3(x)\right)^{5/2}}\,dx = \frac{\sqrt{a}\left(\int \frac{\sqrt{\cosh(x)}}{\cosh(x)^8}dx\right)}{a^3}$$

input
```
int(1/(a*cosh(x)^3)^(5/2),x)
```

output
```
(sqrt(a)*int(sqrt(cosh(x))/cosh(x)**8,x))/a**3
```

3.134 $\int \left(a\cosh^4(x)\right)^{5/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 132

$$\begin{aligned}\int \left(a\cosh^4(x)\right)^{5/2}\,dx &= \frac{63}{256}a^2x\sqrt{a\cosh^4(x)}\operatorname{sech}^2(x)\\ &+\frac{21}{128}a^2\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x)\\ &+\frac{21}{160}a^2\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x)+\frac{9}{80}a^2\cosh^5(x)\sqrt{a\cosh^4(x)}\sinh(x)\\ &+\frac{1}{10}a^2\cosh^7(x)\sqrt{a\cosh^4(x)}\sinh(x)+\frac{63}{256}a^2\sqrt{a\cosh^4(x)}\tanh(x)\end{aligned}$$

output
```
63/256*a^2*x*(a*cosh(x)^4)^(1/2)*sech(x)^2+21/128*a^2*cosh(x)*(a*cosh(x)^4
)^(1/2)*sinh(x)+21/160*a^2*cosh(x)^3*(a*cosh(x)^4)^(1/2)*sinh(x)+9/80*a^2*
cosh(x)^5*(a*cosh(x)^4)^(1/2)*sinh(x)+1/10*a^2*cosh(x)^7*(a*cosh(x)^4)^(1/
2)*sinh(x)+63/256*a^2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.40

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx = \frac{a\left(a\cosh^4(x)\right)^{3/2}\operatorname{sech}^6(x)(2520x + 2100\sinh(2x) + 600\sinh(4x) + 150\sinh(6x) +}{10240}$$

input
```
Integrate[(a*Cosh[x]^4)^(5/2),x]
```

output
```
(a*(a*Cosh[x]^4)^(3/2)*Sech[x]^6*(2520*x + 2100*Sinh[2*x] + 600*Sinh[4*x]
+ 150*Sinh[6*x] + 25*Sinh[8*x] + 2*Sinh[10*x]))/10240
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.70, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.300$, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^4\right)^{5/2}dx$$

$$\Big\downarrow \ 3686$$

$$a^2\operatorname{sech}^2(x)\sqrt{a\cosh^4(x)}\int \cosh^{10}(x)dx$$

$$\Big\downarrow \ 3042$$

$$a^2\operatorname{sech}^2(x)\sqrt{a\cosh^4(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^{10}dx$$

$$\Big\downarrow \ 3115$$

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\int \cosh^8(x)dx+\frac{1}{10}\sinh(x)\cosh^9(x)\right)$$

$\Big\downarrow$ 3042

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\int \sin\left(ix+\frac{\pi}{2}\right)^8dx\right)$$

$\Big\downarrow$ 3115

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\int \cosh^6(x)dx+\frac{1}{8}\sinh(x)\cosh^7(x)\right)+\frac{1}{10}\sinh(x)\cosh^9(x)\right)$$

$\Big\downarrow$ 3042

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\int \sin\left(ix+\frac{\pi}{2}\right)^6dx\right)\right)$$

$\Big\downarrow$ 3115

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\int \cosh^4(x)dx+\frac{1}{6}\sinh(x)\cosh^5(x)\right)+\frac{1}{8}\sinh(x)\cosh^7(x)\right)+\frac{1}{10}\sinh(x)\cos$$

$\Big\downarrow$ 3042

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\int \sin\left(ix+\right.\right.\right.\right.$$

$\Big\downarrow$ 3115

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)+\frac{1}{8}\sinh(x)\right.\right.$$

$\Big\downarrow$ 3042

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\right.\right.\right.\right.$$

$\Big\downarrow$ 3115

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{9}{10}\left(\frac{7}{8}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x\right.\right.\right.$$

$\Big\downarrow$ 24

$$a^2\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{10}\sinh(x)\cosh^9(x)+\frac{9}{10}\left(\frac{1}{8}\sinh(x)\cosh^7(x)+\frac{7}{8}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\right.\right.\right.\right.$$

input
```
Int[(a*Cosh[x]^4)^(5/2),x]
```

output
```
a^2*Sqrt[a*Cosh[x]^4]*Sech[x]^2*((Cosh[x]^9*Sinh[x])/10 + (9*((Cosh[x]^7*S
inh[x])/8 + (7*((Cosh[x]^5*Sinh[x])/6 + (5*((Cosh[x]^3*Sinh[x])/4 + (3*(x/
2 + (Cosh[x]*Sinh[x])/2))/4))/6))/8))/10)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 15.12 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.30

method	result
default	$\frac{a^{\frac{3}{2}}(1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(8\sqrt{a\sinh(2x)^{2}}\sqrt{a}\sinh(2x)^{4}+50\sqrt{a\sinh(2x)^{2}}\sqrt{a}\cosh(2x)\sinh(2x)^{2}+160\sqrt{a}\right.}{2560\sinh(2x)\sqrt{(1+c}}$
risch	$\frac{63a^{2}e^{2x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}\,x}{256(e^{2x}+1)^{2}}+\frac{a^{2}e^{12x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}}{10240(e^{2x}+1)^{2}}+\frac{5a^{2}e^{10x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}}{4096(e^{2x}+1)^{2}}+\frac{15a^{2}e^{8x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}}{2048(e^{2x}+1)^{2}}+\frac{15a^{2}e^{6}}{}$

input
```
int((a*cosh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/2560*a^(3/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*(8*(a*
sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^4+50*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*co
sh(2*x)*sinh(2*x)^2+160*(a*sinh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2+325*cosh
(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+640*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+315*
ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)^2)^(1/2))*a)/sinh(2*x)/((1+cosh(2*x))^2*
a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1597 vs. $2(108) = 216$.

Time = 0.15 (sec) , antiderivative size = 1597, normalized size of antiderivative = 12.10

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx = \text{Too large to display}$$

input
```
integrate((a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
1/20480*(40*a^2*cosh(x)*e^(2*x)*sinh(x)^19 + 2*a^2*e^(2*x)*sinh(x)^20 + 5*
(76*a^2*cosh(x)^2 + 5*a^2)*e^(2*x)*sinh(x)^18 + 30*(76*a^2*cosh(x)^3 + 15*
a^2*cosh(x))*e^(2*x)*sinh(x)^17 + 15*(646*a^2*cosh(x)^4 + 255*a^2*cosh(x)^
2 + 10*a^2)*e^(2*x)*sinh(x)^16 + 48*(646*a^2*cosh(x)^5 + 425*a^2*cosh(x)^3
 + 50*a^2*cosh(x))*e^(2*x)*sinh(x)^15 + 60*(1292*a^2*cosh(x)^6 + 1275*a^2*
cosh(x)^4 + 300*a^2*cosh(x)^2 + 10*a^2)*e^(2*x)*sinh(x)^14 + 120*(1292*a^2
*cosh(x)^7 + 1785*a^2*cosh(x)^5 + 700*a^2*cosh(x)^3 + 70*a^2*cosh(x))*e^(2
*x)*sinh(x)^13 + 60*(4199*a^2*cosh(x)^8 + 7735*a^2*cosh(x)^6 + 4550*a^2*co
sh(x)^4 + 910*a^2*cosh(x)^2 + 35*a^2)*e^(2*x)*sinh(x)^12 + 80*(4199*a^2*co
sh(x)^9 + 9945*a^2*cosh(x)^7 + 8190*a^2*cosh(x)^5 + 2730*a^2*cosh(x)^3 + 3
15*a^2*cosh(x))*e^(2*x)*sinh(x)^11 + 2*(184756*a^2*cosh(x)^10 + 546975*a^2
*cosh(x)^8 + 600600*a^2*cosh(x)^6 + 300300*a^2*cosh(x)^4 + 69300*a^2*cosh(
x)^2 + 2520*a^2*x)*e^(2*x)*sinh(x)^10 + 20*(16796*a^2*cosh(x)^11 + 60775*a
^2*cosh(x)^9 + 85800*a^2*cosh(x)^7 + 60060*a^2*cosh(x)^5 + 23100*a^2*cosh(
x)^3 + 2520*a^2*x*cosh(x))*e^(2*x)*sinh(x)^9 + 30*(8398*a^2*cosh(x)^12 + 3
6465*a^2*cosh(x)^10 + 64350*a^2*cosh(x)^8 + 60060*a^2*cosh(x)^6 + 34650*a^
2*cosh(x)^4 + 7560*a^2*x*cosh(x)^2 - 70*a^2)*e^(2*x)*sinh(x)^8 + 240*(646*
a^2*cosh(x)^13 + 3315*a^2*cosh(x)^11 + 7150*a^2*cosh(x)^9 + 8580*a^2*cosh(
x)^7 + 6930*a^2*cosh(x)^5 + 2520*a^2*x*cosh(x)^3 - 70*a^2*cosh(x))*e^(2*x)
*sinh(x)^7 + 60*(1292*a^2*cosh(x)^14 + 7735*a^2*cosh(x)^12 + 20020*a^2*...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{5/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**4)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx = \frac{63}{256}\,a^{\frac{5}{2}}x + \frac{1}{20480}\left(25\,a^{\frac{5}{2}}e^{(-2\,x)} + 150\,a^{\frac{5}{2}}e^{(-4\,x)} + 600\,a^{\frac{5}{2}}e^{(-6\,x)} + 2100\,a^{\frac{5}{2}}e^{(-8\,x)} - 2100\,a^{\frac{5}{2}}e^{(-12\,x)} - 600\,a^{\frac{5}{2}}e^{(-14\,x)} - 1$$

input
```
integrate((a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
63/256*a^(5/2)*x + 1/20480*(25*a^(5/2)*e^(-2*x) + 150*a^(5/2)*e^(-4*x) + 6
00*a^(5/2)*e^(-6*x) + 2100*a^(5/2)*e^(-8*x) - 2100*a^(5/2)*e^(-12*x) - 600
*a^(5/2)*e^(-14*x) - 150*a^(5/2)*e^(-16*x) - 25*a^(5/2)*e^(-18*x) - 2*a^(5
/2)*e^(-20*x) + 2*a^(5/2))*e^(10*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.58

$$\int \left(a\cosh^4(x)\right)^{5/2}\,dx = -\frac{1}{20480}\left(\left(5754\,e^{(10\,x)} + 2100\,e^{(8\,x)} + 600\,e^{(6\,x)} + 150\,e^{(4\,x)} + 25\,e^{(2\,x)} + 2\right)e^{(-10\,x)} - 5040\,x - 2\,e^{(10\,x)} - 25\,e^{(}$$

input
```
integrate((a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-1/20480*((5754*e^(10*x) + 2100*e^(8*x) + 600*e^(6*x) + 150*e^(4*x) + 25*e
^(2*x) + 2)*e^(-10*x) - 5040*x - 2*e^(10*x) - 25*e^(8*x) - 150*e^(6*x) - 6
00*e^(4*x) - 2100*e^(2*x))*a^(5/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{5/2} dx = \int \left(a\cosh(x)^4\right)^{5/2} dx$$

input
```
int((a*cosh(x)^4)^(5/2),x)
```

output
```
int((a*cosh(x)^4)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.66

$$\int \left(a\cosh^4(x)\right)^{5/2} dx = \frac{\sqrt{a}\, a^2(2e^{20x} + 25e^{18x} + 150e^{16x} + 600e^{14x} + 2100e^{12x} + 5040e^{10x}x - 2100e^{8x} - 60}{20480e^{10x}}$$

input
```
int((a*cosh(x)^4)^(5/2),x)
```

output
```
(sqrt(a)*a**2*(2*e**(20*x) + 25*e**(18*x) + 150*e**(16*x) + 600*e**(14*x)
+ 2100*e**(12*x) + 5040*e**(10*x)*x - 2100*e**(8*x) - 600*e**(6*x) - 150*e
**(4*x) - 25*e**(2*x) - 2))/(20480*e**(10*x))
```

3.135 $\int \left(a\cosh^4(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 78

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = \frac{5}{16}ax\sqrt{a\cosh^4(x)}\operatorname{sech}^2(x) + \frac{5}{24}a\cosh(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{1}{6}a\cosh^3(x)\sqrt{a\cosh^4(x)}\sinh(x) + \frac{5}{16}a\sqrt{a\cosh^4(x)}\tanh(x)$$

output
```
5/16*a*x*(a*cosh(x)^4)^(1/2)*sech(x)^2+5/24*a*cosh(x)*(a*cosh(x)^4)^(1/2)*
sinh(x)+1/6*a*cosh(x)^3*(a*cosh(x)^4)^(1/2)*sinh(x)+5/16*a*(a*cosh(x)^4)^(
1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.49

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = \frac{1}{192}\left(a\cosh^4(x)\right)^{3/2}\operatorname{sech}^6(x)(60x+45\sinh(2x)+9\sinh(4x)+\sinh(6x))$$

input
```
Integrate[(a*Cosh[x]^4)^(3/2),x]
```

output
```
((a*Cosh[x]^4)^(3/2)*Sech[x]^6*(60*x + 45*Sinh[2*x] + 9*Sinh[4*x] + Sinh[6
*x]))/192
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.77, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.900, Rules used = {3042, 3686, 3042, 3115, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a\sin\left(\frac{\pi}{2}+ix\right)^4\right)^{3/2}dx$$

$$\Big\downarrow \text{3686}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\int \cosh^6(x)dx$$

$$\Big\downarrow \text{3042}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^6dx$$

$$\Big\downarrow \text{3115}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{5}{6}\int \cosh^4(x)dx+\frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\int \sin\left(ix+\frac{\pi}{2}\right)^4dx\right)$$

$$\Big\downarrow \text{3115}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\int \cosh^2(x)dx+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{3042}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\int\sin\left(ix+\frac{\pi}{2}\right)^2dx\right)\right)$$

$$\Big\downarrow \text{3115}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh(x)\cosh^3(x)\right)+\frac{1}{6}\sinh(x)\cosh^5(x)\right)$$

$$\Big\downarrow \text{24}$$

$$a\text{sech}^2(x)\sqrt{a\cosh^4(x)}\left(\frac{1}{6}\sinh(x)\cosh^5(x)+\frac{5}{6}\left(\frac{1}{4}\sinh(x)\cosh^3(x)+\frac{3}{4}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)\right)\right)$$

input
```
Int[(a*Cosh[x]^4)^(3/2),x]
```

output
```
a*Sqrt[a*Cosh[x]^4]*Sech[x]^2*((Cosh[x]^5*Sinh[x])/6 + (5*((Cosh[x]^3*Sinh
[x])/4 + (3*(x/2 + (Cosh[x]*Sinh[x])/2))/4))/6)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.60

method	result
default	$\frac{\sqrt{a}\,(1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(2\sqrt{a\sinh(2x)^{2}}\sqrt{a}\,\sinh(2x)^{2}+9\cosh(2x)\sqrt{a\sinh(2x)^{2}}\sqrt{a}+24\sqrt{a\sinh(2x)^{2}}\,\sqrt{\right.}{96\sinh(2x)\sqrt{(1+\cosh(2x))^{2}a}}$
risch	$\frac{5a\,e^{2x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}\,x}{16(e^{2x}+1)^{2}}+\frac{a\,e^{8x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}}{384(e^{2x}+1)^{2}}+\frac{3a\,e^{6x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}}{128(e^{2x}+1)^{2}}+\frac{15a\,e^{4x}\sqrt{a(e^{2x}+1)^{4}e^{-4x}}}{128(e^{2x}+1)^{2}}-\frac{15\sqrt{a(e^{2x}+1}}{128(e^{2x}-}$

input
```
int((a*cosh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/96*a^(1/2)*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*(2*(a*si
nh(2*x)^2)^(1/2)*a^(1/2)*sinh(2*x)^2+9*cosh(2*x)*(a*sinh(2*x)^2)^(1/2)*a^(
1/2)+24*(a*sinh(2*x)^2)^(1/2)*a^(1/2)+15*ln(cosh(2*x)*a^(1/2)+(a*sinh(2*x)
^2)^(1/2))*a)/sinh(2*x)/((1+cosh(2*x))^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 659 vs. $2(62) = 124$.

Time = 0.16 (sec) , antiderivative size = 659, normalized size of antiderivative = 8.45

$$\int \left(a\cosh^{4}(x)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
1/384*(12*a*cosh(x)*e^(2*x)*sinh(x)^11 + a*e^(2*x)*sinh(x)^12 + 3*(22*a*co
sh(x)^2 + 3*a)*e^(2*x)*sinh(x)^10 + 10*(22*a*cosh(x)^3 + 9*a*cosh(x))*e^(2
*x)*sinh(x)^9 + 45*(11*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*e^(2*x)*sinh(x)^8
+ 72*(11*a*cosh(x)^5 + 15*a*cosh(x)^3 + 5*a*cosh(x))*e^(2*x)*sinh(x)^7 + 6
*(154*a*cosh(x)^6 + 315*a*cosh(x)^4 + 210*a*cosh(x)^2 + 20*a*x)*e^(2*x)*si
nh(x)^6 + 36*(22*a*cosh(x)^7 + 63*a*cosh(x)^5 + 70*a*cosh(x)^3 + 20*a*x*co
sh(x))*e^(2*x)*sinh(x)^5 + 45*(11*a*cosh(x)^8 + 42*a*cosh(x)^6 + 70*a*cosh
(x)^4 + 40*a*x*cosh(x)^2 - a)*e^(2*x)*sinh(x)^4 + 20*(11*a*cosh(x)^9 + 54*
a*cosh(x)^7 + 126*a*cosh(x)^5 + 120*a*x*cosh(x)^3 - 9*a*cosh(x))*e^(2*x)*s
inh(x)^3 + 3*(22*a*cosh(x)^10 + 135*a*cosh(x)^8 + 420*a*cosh(x)^6 + 600*a*
x*cosh(x)^4 - 90*a*cosh(x)^2 - 3*a)*e^(2*x)*sinh(x)^2 + 6*(2*a*cosh(x)^11
+ 15*a*cosh(x)^9 + 60*a*cosh(x)^7 + 120*a*x*cosh(x)^5 - 30*a*cosh(x)^3 - 3
*a*cosh(x))*e^(2*x)*sinh(x) + (a*cosh(x)^12 + 9*a*cosh(x)^10 + 45*a*cosh(x
)^8 + 120*a*x*cosh(x)^6 - 45*a*cosh(x)^4 - 9*a*cosh(x)^2 - a)*e^(2*x))*sqr
t(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(
x)^6*e^(4*x) + 2*cosh(x)^6*e^(2*x) + (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^6 +
 cosh(x)^6 + 6*(cosh(x)*e^(4*x) + 2*cosh(x)*e^(2*x) + cosh(x))*sinh(x)^5 +
 15*(cosh(x)^2*e^(4*x) + 2*cosh(x)^2*e^(2*x) + cosh(x)^2)*sinh(x)^4 + 20*(
cosh(x)^3*e^(4*x) + 2*cosh(x)^3*e^(2*x) + cosh(x)^3)*sinh(x)^3 + 15*(cosh(
x)^4*e^(4*x) + 2*cosh(x)^4*e^(2*x) + cosh(x)^4)*sinh(x)^2 + 6*(cosh(x)^...
```

Sympy [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{3/2} dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**4)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.79

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = \frac{5}{16}\,a^{\frac{3}{2}}x + \frac{1}{384}\left(9\,a^{\frac{3}{2}}e^{(-2\,x)} + 45\,a^{\frac{3}{2}}e^{(-4\,x)} - 45\,a^{\frac{3}{2}}e^{(-8\,x)} - 9\,a^{\frac{3}{2}}e^{(-10\,x)} - a^{\frac{3}{2}}e^{(-12\,x)} + a^{\frac{3}{2}}\right)e^{(6\,x)}$$

input
```
integrate((a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
5/16*a^(3/2)*x + 1/384*(9*a^(3/2)*e^(-2*x) + 45*a^(3/2)*e^(-4*x) - 45*a^(3
/2)*e^(-8*x) - 9*a^(3/2)*e^(-10*x) - a^(3/2)*e^(-12*x) + a^(3/2))*e^(6*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.67

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = -\frac{1}{384}\left(\left(110\,e^{(6\,x)} + 45\,e^{(4\,x)} + 9\,e^{(2\,x)} + 1\right)e^{(-6\,x)} - 120\,x - e^{(6\,x)} - 9\,e^{(4\,x)} - 45\,e^{(2\,x)}\right)a^{\frac{3}{2}}$$

input
```
integrate((a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-1/384*((110*e^(6*x) + 45*e^(4*x) + 9*e^(2*x) + 1)*e^(-6*x) - 120*x - e^(6
*x) - 9*e^(4*x) - 45*e^(2*x))*a^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \left(a\cosh^4(x)\right)^{3/2}\,dx = \int {\left(a\,{\cosh\left(x\right)}^4\right)}^{3/2} \,d x$$

input
```
int((a*cosh(x)^4)^(3/2),x)
```

output
```
int((a*cosh(x)^4)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.71

$$\int \left(a\cosh^4(x)\right)^{3/2} dx = \frac{\sqrt{a}\, a(e^{12x} + 9e^{10x} + 45e^{8x} + 120e^{6x}x - 45e^{4x} - 9e^{2x} - 1)}{384e^{6x}}$$

input
```
int((a*cosh(x)^4)^(3/2),x)
```

output
```
(sqrt(a)*a*(e**(12*x) + 9*e**(10*x) + 45*e**(8*x) + 120*e**(6*x)*x - 45*e*
*(4*x) - 9*e**(2*x) - 1))/(384*e**(6*x))
```

3.136 $\int \sqrt{a \cosh^4(x)}\, dx$

Optimal result

Integrand size = 10, antiderivative size = 36

$$\int \sqrt{a \cosh^4(x)}\, dx = \frac{1}{2} x \sqrt{a \cosh^4(x)} \operatorname{sech}^2(x) + \frac{1}{2} \sqrt{a \cosh^4(x)} \tanh(x)$$

output
```
1/2*x*(a*cosh(x)^4)^(1/2)*sech(x)^2+1/2*(a*cosh(x)^4)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.69

$$\int \sqrt{a \cosh^4(x)}\, dx = \frac{1}{2} \sqrt{a \cosh^4(x)} \operatorname{sech}^2(x) (x + \cosh(x) \sinh(x))$$

input
```
Integrate[Sqrt[a*Cosh[x]^4],x]
```

output
```
(Sqrt[a*Cosh[x]^4]*Sech[x]^2*(x + Cosh[x]*Sinh[x]))/2
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a \cosh^4(x)}\, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a \sin\left(\frac{\pi}{2}+ix\right)^4}dx$$

$$\Big\downarrow \text{3686}$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\int \cosh^2(x)dx$$

$$\Big\downarrow \text{3042}$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\int \sin\left(ix+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{3115}$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{\int 1dx}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

$$\Big\downarrow \text{24}$$

$$\text{sech}^2(x)\sqrt{a \cosh^4(x)}\left(\frac{x}{2}+\frac{1}{2}\sinh(x)\cosh(x)\right)$$

input
```
Int[Sqrt[a*Cosh[x]^4],x]
```

output
```
Sqrt[a*Cosh[x]^4]*Sech[x]^2*(x/2 + (Cosh[x]*Sinh[x])/2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(28) = 56$.

Time = 0.56 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.31

method	result	size
default	$\frac{(1+\cosh(2x))\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\left(\ln\left(\cosh(2x)\sqrt{a}+\sqrt{a\sinh(2x)^2}\right)a+\sqrt{a\sinh(2x)^2}\,\sqrt{a}\right)}{4\sqrt{a}\,\sinh(2x)\sqrt{(1+\cosh(2x))^2 a}}$	83
risch	$\frac{\sqrt{a(e^{2x}+1)^4 e^{-4x}}\,e^{2x}x}{2(e^{2x}+1)^2}+\frac{\sqrt{a(e^{2x}+1)^4 e^{-4x}}\,e^{4x}}{8(e^{2x}+1)^2}-\frac{\sqrt{a(e^{2x}+1)^4 e^{-4x}}}{8(e^{2x}+1)^2}$	89

input
```
int((a*cosh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*(1+cosh(2*x))*(a*(-1+cosh(2*x))*(1+cosh(2*x)))^(1/2)*(ln(cosh(2*x)*a^(
1/2)+(a*sinh(2*x)^2)^(1/2))*a+(a*sinh(2*x)^2)^(1/2)*a^(1/2))/a^(1/2)/sinh(
2*x)/((1+cosh(2*x))^2*a)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(28) = 56.

Time = 0.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 5.00

$$\int \sqrt{a \cosh^4(x)}\, dx = \frac{\left(4 \cosh(x) e^{(2x)} \sinh(x)^3 + e^{(2x)} \sinh(x)^4 + 2 \left(3 \cosh(x)^2 + 2x\right) e^{(2x)} \sinh(x)^2 + 4 \left(\cosh(x)^3 + 2x \cos\right.\right.}{8 \left(\cosh(x)^2 e^{(4x)} + 2 \cosh(x)^2 e^{(2x)} + \left(e^{(4x)} + 2e^{(2x)} + 1\right) \sinh(x)^2\right.}$$

input
```
integrate((a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*cosh(x)*e^(2*x)*sinh(x)^3 + e^(2*x)*sinh(x)^4 + 2*(3*cosh(x)^2 + 2*
x)*e^(2*x)*sinh(x)^2 + 4*(cosh(x)^3 + 2*x*cosh(x))*e^(2*x)*sinh(x) + (cosh
(x)^4 + 4*x*cosh(x)^2 - 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(
4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(cosh(x)^2*e^(4*x) + 2*cosh(x)^2*e^(2*x)
+ (e^(4*x) + 2*e^(2*x) + 1)*sinh(x)^2 + cosh(x)^2 + 2*(cosh(x)*e^(4*x) + 2
*cosh(x)*e^(2*x) + cosh(x))*sinh(x))
```

Sympy [F(-1)]

Timed out.

$$\int \sqrt{a \cosh^4(x)}\, dx = \text{Timed out}$$

input
```
integrate((a*cosh(x)**4)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.75

$$\int \sqrt{a\cosh^4(x)}\,dx = -\frac{1}{8}\left(\sqrt{a}e^{(-4x)} - \sqrt{a}\right)e^{(2x)} + \frac{1}{2}\sqrt{a}x$$

input
```
integrate((a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
-1/8*(sqrt(a)*e^(-4*x) - sqrt(a))*e^(2*x) + 1/2*sqrt(a)*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.78

$$\int \sqrt{a\cosh^4(x)}\,dx = -\frac{1}{8}\left(\left(2\,e^{(2\,x)} + 1\right)e^{(-2\,x)} - 4\,x - e^{(2\,x)}\right)\sqrt{a}$$

input
```
integrate((a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))*sqrt(a)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a\cosh^4(x)}\,dx = \int \sqrt{a\cosh(x)^4}\,dx$$

input
```
int((a*cosh(x)^4)^(1/2),x)
```

output
```
int((a*cosh(x)^4)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.72

$$\int \sqrt{a\cosh^4(x)}\,dx = \frac{\sqrt{a}\,(e^{4x}+4e^{2x}x-1)}{8e^{2x}}$$

input
```
int((a*cosh(x)^4)^(1/2),x)
```

output
```
(sqrt(a)*(e**(4*x) + 4*e**(2*x)*x - 1))/(8*e**(2*x))
```

3.137 $\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 15

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{\cosh(x)\sinh(x)}{\sqrt{a\cosh^4(x)}}$$

output
```
cosh(x)*sinh(x)/(a*cosh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{\cosh(x)\sinh(x)}{\sqrt{a\cosh^4(x)}}$$

input
```
Integrate[1/Sqrt[a*Cosh[x]^4],x]
```

output
```
(Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{1}{\sqrt{a\sin\left(\frac{\pi}{2}+ix\right)^4}}dx$$

$$\Big\downarrow \ 3686$$

$$\frac{\cosh^2(x)\int \operatorname{sech}^2(x)dx}{\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \ 3042$$

$$\frac{\cosh^2(x)\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \ 4254$$

$$\frac{i\cosh^2(x)\int 1d(-i\tanh(x))}{\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \ 24$$

$$\frac{\sinh(x)\cosh(x)}{\sqrt{a\cosh^4(x)}}$$

input
```
Int[1/Sqrt[a*Cosh[x]^4],x]
```

output
```
(Cosh[x]*Sinh[x])/Sqrt[a*Cosh[x]^4]
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 28 vs. $2(13) = 26$.

Time = 0.49 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.93

method	result	size
risch	$-\frac{2\,\mathrm{e}^{-2x}\left(\mathrm{e}^{2x}+1\right)}{\sqrt{a\left(\mathrm{e}^{2x}+1\right)^{4}\mathrm{e}^{-4x}}}$	29
default	$\frac{\sqrt{a(-1+\cosh(2x))(1+\cosh(2x))}\,\sqrt{a\sinh(2x)^{2}}}{a\sinh(2x)\sqrt{(1+\cosh(2x))^{2}a}}$	49

input
```
int(1/(a*cosh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-2/(a*(exp(2*x)+1)^4*exp(-4*x))^(1/2)*exp(-2*x)*(exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 116, normalized size of antiderivative = 7.73

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx =$$
$$-\frac{2\sqrt{ae^{(8x)} + 4\,ae^{(6x)} + 6\,ae^{(4x)} + 4\,ae^{(2x)} + \cdots}}{a\cosh(x)^2 + \left(ae^{(4x)} + 2\,ae^{(2x)} + a\right)\sinh(x)^2 + \left(a\cosh(x)^2 + a\right)e^{(4x)} + 2\left(a\cosh(x)^2 + a\right)e^{(2x)} + 2(\cdots}$$

input
```
integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
-2*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6*a*e^(4*x) + 4*a*e^(2*x) + a)/(a*cosh(x
)^2 + (a*e^(4*x) + 2*a*e^(2*x) + a)*sinh(x)^2 + (a*cosh(x)^2 + a)*e^(4*x)
+ 2*(a*cosh(x)^2 + a)*e^(2*x) + 2*(a*cosh(x)*e^(4*x) + 2*a*cosh(x)*e^(2*x)
 + a*cosh(x))*sinh(x) + a)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**4)**(1/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.07

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{2}{\sqrt{a}e^{(-2x)} + \sqrt{a}}$$

input
```
integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
2/(sqrt(a)*e^(-2*x) + sqrt(a))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.87

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = -\frac{2}{\sqrt{a}(e^{(2x)} + 1)}$$

input
```
integrate(1/(a*cosh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
-2/(sqrt(a)*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 39, normalized size of antiderivative = 2.60

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = -\frac{e^{-x}\sqrt{a\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)^4}}{a\left(\frac{e^{-x}}{2}+\frac{e^x}{2}\right)^3}$$

input
```
int(1/(a*cosh(x)^4)^(1/2),x)
```

output
```
-(exp(-x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(a*(exp(-x)/2 + exp(x)/2)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{a\cosh^4(x)}}\,dx = \frac{2e^{2x}\sqrt{a}}{a\,(e^{2x}+1)}$$

input
```
int(1/(a*cosh(x)^4)^(1/2),x)
```

output
```
(2*e**(2*x)*sqrt(a))/(a*(e**(2*x) + 1))
```

3.138 $\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 67

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \frac{\cosh(x)\sinh(x)}{a\sqrt{a\cosh^4(x)}} - \frac{2\sinh^2(x)\tanh(x)}{3a\sqrt{a\cosh^4(x)}} + \frac{\sinh^2(x)\tanh^3(x)}{5a\sqrt{a\cosh^4(x)}}$$

output
```
cosh(x)*sinh(x)/a/(a*cosh(x)^4)^(1/2)-2/3*sinh(x)^2*tanh(x)/a/(a*cosh(x)^4
)^(1/2)+1/5*sinh(x)^2*tanh(x)^3/a/(a*cosh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.45

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \frac{\cosh(x)(8+6\cosh(2x)+\cosh(4x))\sinh(x)}{15\left(a\cosh^4(x)\right)^{3/2}}$$

input
```
Integrate[(a*Cosh[x]^4)^(-3/2),x]
```

output
```
(Cosh[x]*(8 + 6*Cosh[2*x] + Cosh[4*x])*Sinh[x])/(15*(a*Cosh[x]^4)^(3/2))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^2(x)\int \operatorname{sech}^6(x)dx}{a\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^2(x)\int \csc\left(ix+\frac{\pi}{2}\right)^6dx}{a\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\cosh^2(x)\int \left(\tanh^4(x)-2\tanh^2(x)+1\right)d(-i\tanh(x))}{a\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\cosh^2(x)\left(-\frac{1}{5}i\tanh^5(x)+\frac{2}{3}i\tanh^3(x)-i\tanh(x)\right)}{a\sqrt{a\cosh^4(x)}}$$

input
```
Int[(a*Cosh[x]^4)^(-3/2),x]
```

output
```
(I*Cosh[x]^2*((-I)*Tanh[x] + ((2*I)/3)*Tanh[x]^3 - (I/5)*Tanh[x]^5))/(a*Sq
rt[a*Cosh[x]^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

method	result	size
risch	$-\frac{16\,\mathrm{e}^{-2x}\left(10\,\mathrm{e}^{4x}+5\,\mathrm{e}^{2x}+1\right)}{15a\left(\mathrm{e}^{2x}+1\right)^{3}\sqrt{a\left(\mathrm{e}^{2x}+1\right)^{4}\mathrm{e}^{-4x}}}$	48
default	$\frac{4\left(2\cosh\left(2x\right)^{2}+6\cosh\left(2x\right)+7\right)\sqrt{a\sinh\left(2x\right)^{2}}\,\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}}{15a^{2}\left(1+\cosh\left(2x\right)\right)^{2}\sinh\left(2x\right)\sqrt{\left(1+\cosh\left(2x\right)\right)^{2}a}}$	74

input
```
int(1/(a*cosh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15/a/(exp(2*x)+1)^3*exp(-2*x)/(a*(exp(2*x)+1)^4*exp(-4*x))^(1/2)*(10*e
xp(4*x)+5*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1137 vs. $2(57) = 114$.

Time = 0.11 (sec) , antiderivative size = 1137, normalized size of antiderivative = 16.97

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
-16/15*(40*cosh(x)*e^(2*x)*sinh(x)^3 + 10*e^(2*x)*sinh(x)^4 + 5*(12*cosh(x
)^2 + 1)*e^(2*x)*sinh(x)^2 + 10*(4*cosh(x)^3 + cosh(x))*e^(2*x)*sinh(x) +
(10*cosh(x)^4 + 5*cosh(x)^2 + 1)*e^(2*x))*sqrt(a*e^(8*x) + 4*a*e^(6*x) + 6
*a*e^(4*x) + 4*a*e^(2*x) + a)*e^(-2*x)/(a^2*cosh(x)^10 + (a^2*e^(4*x) + 2*
a^2*e^(2*x) + a^2)*sinh(x)^10 + 5*a^2*cosh(x)^8 + 10*(a^2*cosh(x)*e^(4*x)
+ 2*a^2*cosh(x)*e^(2*x) + a^2*cosh(x))*sinh(x)^9 + 5*(9*a^2*cosh(x)^2 + a^
2 + (9*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(9*a^2*cosh(x)^2 + a^2)*e^(2*x))*s
inh(x)^8 + 10*a^2*cosh(x)^6 + 40*(3*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*c
osh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh(x)^3 + a^2*cosh(x))*e^(2*x
))*sinh(x)^7 + 10*(21*a^2*cosh(x)^4 + 14*a^2*cosh(x)^2 + a^2 + (21*a^2*cos
h(x)^4 + 14*a^2*cosh(x)^2 + a^2)*e^(4*x) + 2*(21*a^2*cosh(x)^4 + 14*a^2*co
sh(x)^2 + a^2)*e^(2*x))*sinh(x)^6 + 10*a^2*cosh(x)^4 + 4*(63*a^2*cosh(x)^5
 + 70*a^2*cosh(x)^3 + 15*a^2*cosh(x) + (63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^
3 + 15*a^2*cosh(x))*e^(4*x) + 2*(63*a^2*cosh(x)^5 + 70*a^2*cosh(x)^3 + 15*
a^2*cosh(x))*e^(2*x))*sinh(x)^5 + 10*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4
+ 15*a^2*cosh(x)^2 + a^2 + (21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*c
osh(x)^2 + a^2)*e^(4*x) + 2*(21*a^2*cosh(x)^6 + 35*a^2*cosh(x)^4 + 15*a^2*
cosh(x)^2 + a^2)*e^(2*x))*sinh(x)^4 + 5*a^2*cosh(x)^2 + 40*(3*a^2*cosh(x)^
7 + 7*a^2*cosh(x)^5 + 5*a^2*cosh(x)^3 + a^2*cosh(x) + (3*a^2*cosh(x)^7 + 7
*a^2*cosh(x)^5 + 5*a^2*cosh(x)^3 + a^2*cosh(x))*e^(4*x) + 2*(3*a^2*cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**4)**(3/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(57) = 114$.

Time = 0.12 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.46

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \frac{16\,e^{(-2\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} + 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} + 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} + a^{\frac{3}{2}}\right)} + \frac{32\,e^{(-4\,x)}}{3\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} + 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} + 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} + a^{\frac{3}{2}}\right)} + \frac{16}{15\left(5\,a^{\frac{3}{2}}e^{(-2\,x)} + 10\,a^{\frac{3}{2}}e^{(-4\,x)} + 10\,a^{\frac{3}{2}}e^{(-6\,x)} + 5\,a^{\frac{3}{2}}e^{(-8\,x)} + a^{\frac{3}{2}}e^{(-10\,x)} + a^{\frac{3}{2}}\right)}$$

input
```
integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
16/3*e^(-2*x)/(5*a^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6
*x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 32/3*e^(-4*x)/(5
*a^(3/2)*e^(-2*x) + 10*a^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*
e^(-8*x) + a^(3/2)*e^(-10*x) + a^(3/2)) + 16/15/(5*a^(3/2)*e^(-2*x) + 10*a
^(3/2)*e^(-4*x) + 10*a^(3/2)*e^(-6*x) + 5*a^(3/2)*e^(-8*x) + a^(3/2)*e^(-1
0*x) + a^(3/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = -\frac{16\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\,a^{\frac{3}{2}}\left(e^{(2\,x)}+1\right)^5}$$

input
```
integrate(1/(a*cosh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
-16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(a^(3/2)*(e^(2*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.72

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = -\frac{64\,\mathrm{e}^{2\,x}\,\sqrt{a\,{\left(\frac{\mathrm{e}^{-x}}{2}+\frac{\mathrm{e}^{x}}{2}\right)}^4}\,\left(5\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{4\,x}+1\right)}{15\,a^2\,{\left(\mathrm{e}^{2\,x}+1\right)}^7}$$

input
```
int(1/(a*cosh(x)^4)^(3/2),x)
```

output
```
-(64*exp(2*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2)*(5*exp(2*x) + 10*exp(4*x)
 + 1))/(15*a^2*(exp(2*x) + 1)^7)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.90

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{3/2}}\,dx = \frac{16\sqrt{a}\left(-10e^{4x}-5e^{2x}-1\right)}{15a^2\left(e^{10x}+5e^{8x}+10e^{6x}+10e^{4x}+5e^{2x}+1\right)}$$

input
```
int(1/(a*cosh(x)^4)^(3/2),x)
```

output
```
(16*sqrt(a)*( - 10*e**(4*x) - 5*e**(2*x) - 1))/(15*a**2*(e**(10*x) + 5*e**
(8*x) + 10*e**(6*x) + 10*e**(4*x) + 5*e**(2*x) + 1))
```

3.139 $\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 117

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \frac{\cosh(x)\sinh(x)}{a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh(x)}{3a^2\sqrt{a\cosh^4(x)}} + \frac{6\sinh^2(x)\tanh^3(x)}{5a^2\sqrt{a\cosh^4(x)}} - \frac{4\sinh^2(x)\tanh^5(x)}{7a^2\sqrt{a\cosh^4(x)}} + \frac{\sinh^2(x)\tanh^7(x)}{9a^2\sqrt{a\cosh^4(x)}}$$

output
```
cosh(x)*sinh(x)/a^2/(a*cosh(x)^4)^(1/2)-4/3*sinh(x)^2*tanh(x)/a^2/(a*cosh(
x)^4)^(1/2)+6/5*sinh(x)^2*tanh(x)^3/a^2/(a*cosh(x)^4)^(1/2)-4/7*sinh(x)^2*
tanh(x)^5/a^2/(a*cosh(x)^4)^(1/2)+1/9*sinh(x)^2*tanh(x)^7/a^2/(a*cosh(x)^4
)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.40

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \frac{(128+130\cosh(2x)+46\cosh(4x)+10\cosh(6x)+\cosh(8x))\text{sech}^6(x)\tanh(x)}{315a^2\sqrt{a\cosh^4(x)}}$$

input
```
Integrate[(a*Cosh[x]^4)^(-5/2),x]
```

output
```
((128 + 130*Cosh[2*x] + 46*Cosh[4*x] + 10*Cosh[6*x] + Cosh[8*x])*Sech[x]^6
*Tanh[x])/(315*a^2*Sqrt[a*Cosh[x]^4])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.58, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 3686, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a\cosh^4(x))^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a\sin\left(\frac{\pi}{2}+ix\right)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{3686}$$

$$\frac{\cosh^2(x)\int \operatorname{sech}^{10}(x)dx}{a^2\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\cosh^2(x)\int \csc\left(ix+\frac{\pi}{2}\right)^{10}dx}{a^2\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{4254}$$

$$\frac{i\cosh^2(x)\int \left(\tanh^8(x)-4\tanh^6(x)+6\tanh^4(x)-4\tanh^2(x)+1\right)d(-i\tanh(x))}{a^2\sqrt{a\cosh^4(x)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\cosh^2(x)\left(-\frac{1}{9}i\tanh^9(x)+\frac{4}{7}i\tanh^7(x)-\frac{6}{5}i\tanh^5(x)+\frac{4}{3}i\tanh^3(x)-i\tanh(x)\right)}{a^2\sqrt{a\cosh^4(x)}}$$

input
```
Int[(a*Cosh[x]^4)^(-5/2),x]
```

output
```
(I*Cosh[x]^2*((-I)*Tanh[x] + ((4*I)/3)*Tanh[x]^3 - ((6*I)/5)*Tanh[x]^5 + (
(4*I)/7)*Tanh[x]^7 - (I/9)*Tanh[x]^9))/(a^2*Sqrt[a*Cosh[x]^4])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3686
```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Simp[(b*ff^n)^IntPart[p]*((b*Sin[e + f*x]^
n)^FracPart[p]/(Sin[e + f*x]/ff)^(n*FracPart[p]))   Int[ActivateTrig[u]*(Si
n[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
 && IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /
; FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.55 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.51

method	result	size
risch	$-\frac{256\,e^{-2x}\left(126\,e^{8x}+84\,e^{6x}+36\,e^{4x}+9\,e^{2x}+1\right)}{315a^{2}\left(e^{2x}+1\right)^{7}\sqrt{a\left(e^{2x}+1\right)^{4}e^{-4x}}}$	60
default	$\frac{16\left(8\cosh\left(2x\right)^{4}+40\cosh\left(2x\right)^{3}+84\cosh\left(2x\right)^{2}+100\cosh\left(2x\right)+83\right)\sqrt{a\sinh\left(2x\right)^{2}}\,\sqrt{a\left(-1+\cosh\left(2x\right)\right)\left(1+\cosh\left(2x\right)\right)}}{315a^{3}\left(1+\cosh\left(2x\right)\right)^{4}\sinh\left(2x\right)\sqrt{\left(1+\cosh\left(2x\right)\right)^{2}a}}$	90

input
```
int(1/(a*cosh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-256/315/a^2/(exp(2*x)+1)^7*exp(-2*x)/(a*(exp(2*x)+1)^4*exp(-4*x))^(1/2)*(
126*exp(8*x)+84*exp(6*x)+36*exp(4*x)+9*exp(2*x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3065 vs. 2(99) = 198.

Time = 0.18 (sec) , antiderivative size = 3065, normalized size of antiderivative = 26.20

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*cosh(x)**4)**(5/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 457 vs. $2(99) = 198$.

Time = 0.15 (sec) , antiderivative size = 457, normalized size of antiderivative = 3.91

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
256/35*e^(-2*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5/2)*e^(
-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e^(-12*x
) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x) + a^(5/
2)) + 1024/35*e^(-4*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) + 84*a^(5
/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^(5/2)*e
^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(-18*x)
+ a^(5/2)) + 1024/15*e^(-6*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x) +
84*a^(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84*a^
(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*e^(
-18*x) + a^(5/2)) + 512/5*e^(-8*x)/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*
x) + 84*a^(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) +
84*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2
)*e^(-18*x) + a^(5/2)) + 256/315/(9*a^(5/2)*e^(-2*x) + 36*a^(5/2)*e^(-4*x)
 + 84*a^(5/2)*e^(-6*x) + 126*a^(5/2)*e^(-8*x) + 126*a^(5/2)*e^(-10*x) + 84
*a^(5/2)*e^(-12*x) + 36*a^(5/2)*e^(-14*x) + 9*a^(5/2)*e^(-16*x) + a^(5/2)*
e^(-18*x) + a^(5/2))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.33

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = -\frac{256\left(126\,e^{(8\,x)} + 84\,e^{(6\,x)} + 36\,e^{(4\,x)} + 9\,e^{(2\,x)} + 1\right)}{315\,a^{\frac{5}{2}}\left(e^{(2\,x)} + 1\right)^{9}}$$

input
```
integrate(1/(a*cosh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
-256/315*(126*e^(8*x) + 84*e^(6*x) + 36*e^(4*x) + 9*e^(2*x) + 1)/(a^(5/2)*
(e^(2*x) + 1)^9)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 256, normalized size of antiderivative = 2.19

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \frac{4096\,e^{4\,x}\,\sqrt{a\,{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}^4}}{3\,a^3\,{\left(e^{2\,x}+1\right)}^6\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} - \frac{2048\,e^{4\,x}\,\sqrt{a\,{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}^4}}{5\,a^3\,{\left(e^{2\,x}+1\right)}^5\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} - \frac{12288\,e^{4\,x}\,\sqrt{a\,{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}^4}}{7\,a^3\,{\left(e^{2\,x}+1\right)}^7\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} + \frac{1024\,e^{4\,x}\,\sqrt{a\,{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}^4}}{a^3\,{\left(e^{2\,x}+1\right)}^8\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)} - \frac{2048\,e^{4\,x}\,\sqrt{a\,{\left(\frac{e^{-x}}{2}+\frac{e^{x}}{2}\right)}^4}}{9\,a^3\,{\left(e^{2\,x}+1\right)}^9\,\left(e^{2\,x}+2\,e^{4\,x}+e^{6\,x}\right)}$$

input
```
int(1/(a*cosh(x)^4)^(5/2),x)
```

output
```
(4096*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(3*a^3*(exp(2*x) + 1)^6
*(exp(2*x) + 2*exp(4*x) + exp(6*x))) - (2048*exp(4*x)*(a*(exp(-x)/2 + exp(
x)/2)^4)^(1/2))/(5*a^3*(exp(2*x) + 1)^5*(exp(2*x) + 2*exp(4*x) + exp(6*x))
) - (12288*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(7*a^3*(exp(2*x) +
 1)^7*(exp(2*x) + 2*exp(4*x) + exp(6*x))) + (1024*exp(4*x)*(a*(exp(-x)/2 +
 exp(x)/2)^4)^(1/2))/(a^3*(exp(2*x) + 1)^8*(exp(2*x) + 2*exp(4*x) + exp(6*
x))) - (2048*exp(4*x)*(a*(exp(-x)/2 + exp(x)/2)^4)^(1/2))/(9*a^3*(exp(2*x)
 + 1)^9*(exp(2*x) + 2*exp(4*x) + exp(6*x)))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.87

$$\int \frac{1}{\left(a\cosh^4(x)\right)^{5/2}}\,dx = \frac{256\sqrt{a}\left(-126e^{8x}-84e^{6x}-36e^{4x}-9e^{2x}-1\right)}{315a^3\left(e^{18x}+9e^{16x}+36e^{14x}+84e^{12x}+126e^{10x}+126e^{8x}+84e^{6x}+36e^{4x}+9e^{2x}\right.}$$

input
```
int(1/(a*cosh(x)^4)^(5/2),x)
```

output

```
(256*sqrt(a)*( - 126*e**(8*x) - 84*e**(6*x) - 36*e**(4*x) - 9*e**(2*x) - 1
))/(315*a**3*(e**(18*x) + 9*e**(16*x) + 36*e**(14*x) + 84*e**(12*x) + 126*
e**(10*x) + 126*e**(8*x) + 84*e**(6*x) + 36*e**(4*x) + 9*e**(2*x) + 1))
```

3.140 $\int \frac{\sinh(x)}{(1+\cosh(x))^2} \, dx$

Optimal result

Integrand size = 9, antiderivative size = 8

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2} \, dx = -\frac{1}{1+\cosh(x)}$$

output
```
-1/(1+cosh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2} \, dx = -\frac{1}{2}\operatorname{sech}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 + Cosh[x])^2,x]
```

output
```
-1/2*Sech[x/2]^2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(\cosh(x)+1)^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i \cos\left(-\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{1}{(\cosh(x)+1)^2}d\cosh(x)$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{\cosh(x)+1}$$

input
```
Int[Sinh[x]/(1 + Cosh[x])^2,x]
```

output
```
-(1 + Cosh[x])^(-1)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$-\frac{1}{1+\cosh(x)}$	9
default	$-\frac{1}{1+\cosh(x)}$	9
risch	$-\frac{2\,e^{x}}{(e^{x}+1)^{2}}$	11

input
```
int(sinh(x)/(1+cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-1/(1+cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. 2(8) = 16.

Time = 0.08 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.88

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\,(\cosh(x)+\sinh(x))}{\cosh(x)^2+2\,(\cosh(x)+1)\sinh(x)+\sinh(x)^2+2\,\cosh(x)+1}$$

input
```
integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
-2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 +
2*cosh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)+1}$$

input
```
integrate(sinh(x)/(1+cosh(x))**2,x)
```

output
```
-1/(cosh(x) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)+1}$$

input
```
integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
-1/(cosh(x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\,e^x}{(e^x+1)^2}$$

input
```
integrate(sinh(x)/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
-2*e^x/(e^x + 1)^2
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)+1}$$

input
```
int(sinh(x)/(cosh(x) + 1)^2,x)
```

output
```
-1/(cosh(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

$$\int \frac{\sinh(x)}{(1+\cosh(x))^2}\,dx = \frac{\cosh(x)}{\cosh(x)+1}$$

input
```
int(sinh(x)/(1+cosh(x))^2,x)
```

output
```
cosh(x)/(cosh(x) + 1)
```

3.141 $\int \frac{\sinh(x)}{(1-\cosh(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 8

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2} \, dx = \frac{1}{1-\cosh(x)}$$

output
```
1/(1-cosh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2} \, dx = -\frac{1}{2}\text{csch}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 - Cosh[x])^2,x]
```

output
```
-1/2*Csch[x/2]^2
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.364$, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(1-\cosh(x))^2}d(-\cosh(x))$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{1-\cosh(x)}$$

input
```
Int[Sinh[x]/(1 - Cosh[x])^2,x]
```

output
```
(1 - Cosh[x])^(-1)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.12

method	result	size
derivativedivides	$\frac{1}{1-\cosh(x)}$	9
default	$\frac{1}{1-\cosh(x)}$	9
risch	$-\frac{2\,\mathrm{e}^{x}}{(\mathrm{e}^{x}-1)^{2}}$	11

input
```
int(sinh(x)/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/(1-cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(8) = 16$.

Time = 0.06 (sec) , antiderivative size = 31, normalized size of antiderivative = 3.88

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^2+2\left(\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^2-2\,\cosh\left(x\right)+1}$$

input
```
integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
-2*(cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2 -
2*cosh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.88

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{1}{\cosh\left(x\right)-1}$$

input
```
integrate(sinh(x)/(1-cosh(x))**2,x)
```

output
```
-1/(cosh(x) - 1)
```

Maxima [A] (verification not implemented)

Time = 0.02 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{1}{\cosh\left(x\right)-1}$$

input
```
integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
-1/(cosh(x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{2\,e^x}{(e^x-1)^2}$$

input
```
integrate(sinh(x)/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
-2*e^x/(e^x - 1)^2
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 8, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{1}{\cosh(x)-1}$$

input
```
int(sinh(x)/(cosh(x) - 1)^2,x)
```

output
```
-1/(cosh(x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.25

$$\int \frac{\sinh(x)}{(1-\cosh(x))^2}\,dx = -\frac{\cosh(x)}{\cosh(x)-1}$$

input
```
int(sinh(x)/(1-cosh(x))^2,x)
```

output
```
( - cosh(x))/(cosh(x) - 1)
```

3.142 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx = x - \frac{2\sinh(x)}{1+\cosh(x)}$$

output
```
x-2*sinh(x)/(1+cosh(x))
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2} \, dx = 2\text{arctanh}\left(\tanh\left(\frac{x}{2}\right)\right) - 2\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

output
```
2*ArcTanh[Tanh[x/2]] - 2*Tanh[x/2]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3159}$$

$$\int 1dx-\frac{2\sinh(x)}{\cosh(x)+1}$$

$$\Big\downarrow \text{24}$$

$$x-\frac{2\sinh(x)}{\cosh(x)+1}$$

input
```
Int[Sinh[x]^2/(1 + Cosh[x])^2,x]
```

output
```
x - (2*Sinh[x])/(1 + Cosh[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
risch	$x+\frac{4}{{\mathrm e}^{x}+1}$	11
default	$-2 \tanh \left(\frac{x}{2}\right)-\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)+\ln \left(1+\tanh \left(\frac{x}{2}\right)\right)$	24

input
```
int(sinh(x)^2/(1+cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x+4/(exp(x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = \frac{x\cosh(x) + x\sinh(x) + x + 4}{\cosh(x) + \sinh(x) + 1}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) + x + 4)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.58

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = x - 2\tanh\left(\frac{x}{2}\right)$$

input
```
integrate(sinh(x)**2/(1+cosh(x))**2,x)
```

output
```
x - 2*tanh(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = x - \frac{4}{e^{(-x)} + 1}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
x - 4/(e^(-x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = x + \frac{4}{e^x+1}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
x + 4/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 1.83 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = x + \frac{4}{e^x+1}$$

input
```
int(sinh(x)^2/(cosh(x) + 1)^2,x)
```

output
```
x + 4/(exp(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.67

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^2}\,dx = \frac{e^x x - 4e^x + x}{e^x+1}$$

input
```
int(sinh(x)^2/(1+cosh(x))^2,x)
```

output
```
(e**x*x - 4*e**x + x)/(e**x + 1)
```

3.143 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} \, dx$

Optimal result

Integrand size = 13, antiderivative size = 14

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} \, dx = x + \frac{2\sinh(x)}{1-\cosh(x)}$$

output
```
x+2*sinh(x)/(1-cosh(x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.00 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.71

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} \, dx = -2\coth\left(\frac{x}{2}\right) \text{Hypergeometric2F1}\left(-\frac{1}{2}, 1, \frac{1}{2}, \tanh^2\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

output
```
-2*Coth[x/2]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[x/2]^2]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 25, 3159, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$
$$\Big\downarrow \text{3159}$$
$$\int 1dx+\frac{2\sinh(x)}{1-\cosh(x)}$$
$$\Big\downarrow \text{24}$$
$$x+\frac{2\sinh(x)}{1-\cosh(x)}$$

input
```
Int[Sinh[x]^2/(1 - Cosh[x])^2,x]
```

output
```
x + (2*Sinh[x])/(1 - Cosh[x])
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3159
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[2*g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f
*x])^(m + 1)/(b*f*(2*m + p + 1))), x] + Simp[g^2*((p - 1)/(b^2*(2*m + p + 1
)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 2), x], x] /;
FreeQ[{a, b, e, f, g}, x] && EqQ[a^2 - b^2, 0] && LeQ[m, -2] && GtQ[p, 1] &
& NeQ[2*m + p + 1, 0] &&  !ILtQ[m + p + 1, 0] && IntegersQ[2*m, 2*p]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.79

method	result	size
risch	$x-\frac{4}{{\mathrm e}^{x}-1}$	11
default	$\ln \left(1+\tanh \left(\frac{x}{2}\right)\right)-\frac{2}{\tanh \left(\frac{x}{2}\right)}-\ln \left(\tanh \left(\frac{x}{2}\right)-1\right)$	26

input
```
int(sinh(x)^2/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
x-4/(exp(x)-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} dx = \frac{x\cosh(x) + x\sinh(x) - x - 4}{\cosh(x) + \sinh(x) - 1}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
(x*cosh(x) + x*sinh(x) - x - 4)/(cosh(x) + sinh(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} dx = x - \frac{2}{\tanh\left(\frac{x}{2}\right)}$$

input
```
integrate(sinh(x)**2/(1-cosh(x))**2,x)
```

output
```
x - 2/tanh(x/2)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2} dx = x + \frac{4}{e^{(-x)} - 1}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
x + 4/(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = x - \frac{4}{e^x - 1}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
x - 4/(e^x - 1)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.71

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = x - \frac{4}{\mathrm{e}^x - 1}$$

input
```
int(sinh(x)^2/(cosh(x) - 1)^2,x)
```

output
```
x - 4/(exp(x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.57

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^2}\,dx = \frac{e^x x - 4e^x - x}{e^x - 1}$$

input
```
int(sinh(x)^2/(1-cosh(x))^2,x)
```

output
```
(e**x*x - 4*e**x - x)/(e**x - 1)
```

3.144 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} dx$

Optimal result

Integrand size = 11, antiderivative size = 10

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} dx = \cosh(x) - 2\log(1+\cosh(x))$$

output
```
cosh(x)-2*ln(1+cosh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.30

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2} dx = -1 + \cosh(x) - 4\log\left(\cosh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

output
```
-1 + Cosh[x] - 4*Log[Cosh[x/2]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(\cosh(x)+1)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \ 3146$$

$$-\int \frac{1-\cosh(x)}{\cosh(x)+1}d\cosh(x)$$

$$\Big\downarrow \ 49$$

$$-\int \left(\frac{2}{\cosh(x)+1}-1\right)d\cosh(x)$$

$$\Big\downarrow \ 2009$$

$$\cosh(x)-2\log(\cosh(x)+1)$$

input
```
Int[Sinh[x]^3/(1 + Cosh[x])^2,x]
```

output
```
Cosh[x] - 2*Log[1 + Cosh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.10

method	result	size
derivativedivides	$\cosh(x) - 2\ln(1+\cosh(x))$	11
default	$\cosh(x) - 2\ln(1+\cosh(x))$	11
risch	$2x + \frac{e^{x}}{2} + \frac{e^{-x}}{2} - 4\ln(e^{x}+1)$	22

input
```
int(sinh(x)^3/(1+cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
cosh(x)-2*ln(1+cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(10) = 20$.

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 4.80

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = \frac{4\,x\cosh\left(x\right) + \cosh\left(x\right)^2 - 8\left(\cosh\left(x\right) + \sinh\left(x\right)\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) + 2\left(2\,x + \cosh\left(x\right)\right)\sinh\left(x\right.}{2\left(\cosh\left(x\right) + \sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="fricas")
```

output
```
1/2*(4*x*cosh(x) + cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x)
 + 1) + 2*(2*x + cosh(x))*sinh(x) + sinh(x)^2 + 1)/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(10) = 20$.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 5.80

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = -\frac{2\log\left(\cosh\left(x\right)+1\right)\cosh\left(x\right)}{\cosh\left(x\right)+1} - \frac{2\log\left(\cosh\left(x\right)+1\right)}{\cosh\left(x\right)+1} - \frac{\sinh^2\left(x\right)}{\cosh\left(x\right)+1} + \frac{2\cosh^2\left(x\right)}{\cosh\left(x\right)+1} - \frac{2}{\cosh\left(x\right)+1}$$

input
```
integrate(sinh(x)**3/(1+cosh(x))**2,x)
```

output
```
-2*log(cosh(x) + 1)*cosh(x)/(cosh(x) + 1) - 2*log(cosh(x) + 1)/(cosh(x) +
1) - sinh(x)**2/(cosh(x) + 1) + 2*cosh(x)**2/(cosh(x) + 1) - 2/(cosh(x) +
1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 23 vs. $2(10) = 20$.

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 2.30

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = -2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4\,\log\left(e^{(-x)} + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="maxima")
```

output
```
-2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^(-x) + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 21 vs. $2(10) = 20$.

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 2.10

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = 2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} - 4\,\log\left(e^{x} + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^2,x, algorithm="giac")
```

output
```
2*x + 1/2*e^(-x) + 1/2*e^x - 4*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx = \cosh(x) - 2\,\ln\left(\cosh(x) + 1\right)$$

input
```
int(sinh(x)^3/(cosh(x) + 1)^2,x)
```

output
```
cosh(x) - 2*log(cosh(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 4.00

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^2}\,dx$$
$$= \frac{2\cosh(x)^2 - 2\cosh(x)\log(\cosh(x)+1) + 2\cosh(x) - 2\log(\cosh(x)+1) - \sinh(x)^2}{\cosh(x)+1}$$

input
```
int(sinh(x)^3/(1+cosh(x))^2,x)
```

output
```
(2*cosh(x)**2 - 2*cosh(x)*log(cosh(x) + 1) + 2*cosh(x) - 2*log(cosh(x) + 1
) - sinh(x)**2)/(cosh(x) + 1)
```

3.145 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} dx$

Optimal result

Integrand size = 13, antiderivative size = 12

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} dx = \cosh(x) + 2\log(1-\cosh(x))$$

output
```
cosh(x)+2*ln(1-cosh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} dx = -1 + \cosh(x) + 4\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

output
```
-1 + Cosh[x] + 4*Log[Sinh[x/2]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i \cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^2}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{\cosh(x)+1}{1-\cosh(x)}d(-\cosh(x))$$

$$\Big\downarrow \text{49}$$

$$\int \left(\frac{2}{1-\cosh(x)}-1\right)d(-\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$\cosh(x)+2\log(1-\cosh(x))$$

input
```
Int[Sinh[x]^3/(1 - Cosh[x])^2,x]
```

output
```
Cosh[x] + 2*Log[1 - Cosh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\cosh(x)+2\ln(-1+\cosh(x))$	11
default	$\cosh(x)+2\ln(-1+\cosh(x))$	11
risch	$-2x+\frac{e^{x}}{2}+\frac{e^{-x}}{2}+4\ln(e^{x}-1)$	22

input
```
int(sinh(x)^3/(1-cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
cosh(x)+2*ln(-1+cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. 2(12) = 24.

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 4.50

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = -\frac{4\,x\cosh{(x)} - \cosh{(x)}^2 - 8\,(\cosh{(x)} + \sinh{(x)})\log{(\cosh{(x)} + \sinh{(x)} - 1)} + 2\,(2\,x - \cosh{(x)})\sinh}{2\,(\cosh{(x)} + \sinh{(x)})}$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="fricas")
```

output
```
-1/2*(4*x*cosh(x) - cosh(x)^2 - 8*(cosh(x) + sinh(x))*log(cosh(x) + sinh(x
) - 1) + 2*(2*x - cosh(x))*sinh(x) - sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. 2(10) = 20.

Time = 0.21 (sec) , antiderivative size = 58, normalized size of antiderivative = 4.83

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = \frac{2\log{(\cosh{(x)} - 1)}\cosh{(x)}}{\cosh{(x)} - 1} - \frac{2\log{(\cosh{(x)} - 1)}}{\cosh{(x)} - 1} - \frac{\sinh^2{(x)}}{\cosh{(x)} - 1} + \frac{2\cosh^2{(x)}}{\cosh{(x)} - 1} - \frac{2}{\cosh{(x)} - 1}$$

input
```
integrate(sinh(x)**3/(1-cosh(x))**2,x)
```

output
```
2*log(cosh(x) - 1)*cosh(x)/(cosh(x) - 1) - 2*log(cosh(x) - 1)/(cosh(x) - 1
) - sinh(x)**2/(cosh(x) - 1) + 2*cosh(x)**2/(cosh(x) - 1) - 2/(cosh(x) - 1
)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.92

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = 2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} + 4\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="maxima")
```

output
```
2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.83

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = -2\,x + \frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^{x} + 4\,\log\left(|e^{x} - 1|\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^2,x, algorithm="giac")
```

output
```
-2*x + 1/2*e^(-x) + 1/2*e^x + 4*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 10, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx = 2\,\ln\left(\cosh(x) - 1\right) + \cosh(x)$$

input
```
int(sinh(x)^3/(cosh(x) - 1)^2,x)
```

output
```
2*log(cosh(x) - 1) + cosh(x)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 3.33

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^2}\,dx$$
$$= \frac{2\cosh(x)^2 + 2\cosh(x)\log(\cosh(x)-1) - 2\cosh(x) - 2\log(\cosh(x)-1) - \sinh(x)^2}{\cosh(x)-1}$$

input
```
int(sinh(x)^3/(1-cosh(x))^2,x)
```

output
```
(2*cosh(x)**2 + 2*cosh(x)*log(cosh(x) - 1) - 2*cosh(x) - 2*log(cosh(x) - 1
) - sinh(x)**2)/(cosh(x) - 1)
```

3.146 $\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx$

Optimal result

Integrand size = 9, antiderivative size = 10

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2(1+\cosh(x))^2}$$

output
```
-1/2/(1+cosh(x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{8}\text{sech}^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 + Cosh[x])^3,x]
```

output
```
-1/8*Sech[x/2]^4
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(\cosh(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$\int \frac{1}{(\cosh(x)+1)^3}d\cosh(x)$$

$$\Big\downarrow \text{17}$$

$$-\frac{1}{2(\cosh(x)+1)^2}$$

input
```
Int[Sinh[x]/(1 + Cosh[x])^3,x]
```

output
```
-1/2*1/(1 + Cosh[x])^2
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.90

method	result	size
derivativedivides	$-\frac{1}{2(1+\cosh(x))^2}$	9
default	$-\frac{1}{2(1+\cosh(x))^2}$	9
risch	$-\frac{2\,e^{2x}}{(e^{x}+1)^4}$	13

input
```
int(sinh(x)/(1+cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2/(1+cosh(x))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 5.50

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\,(\cosh(x)+\sinh(x))}{\cosh(x)^3 + (3\,\cosh(x)+4)\sinh(x)^2 + \sinh(x)^3 + 4\,\cosh(x)^2 + \left(3\,\cosh(x)^2 + 8\,\cosh(x)+5\right)\sinh}$$

input
```
integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) + 4)*sinh(x)^2 + sinh(x)^3
+ 4*cosh(x)^2 + (3*cosh(x)^2 + 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) + 4)
```

Sympy [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.50

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2\cosh^2(x) + 4\cosh(x) + 2}$$

input
```
integrate(sinh(x)/(1+cosh(x))**3,x)
```

output
```
-1/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2\,(\cosh(x)+1)^2}$$

input
```
integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
-1/2/(cosh(x) + 1)^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\,e^{(2\,x)}}{(e^x+1)^4}$$

input
```
integrate(sinh(x)/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-2*e^(2*x)/(e^x + 1)^4
```

Mupad [B] (verification not implemented)

Time = 2.09 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.80

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2\,(\cosh{(x)}+1)^2}$$

input
```
int(sinh(x)/(cosh(x) + 1)^3,x)
```

output
```
-1/(2*(cosh(x) + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.60

$$\int \frac{\sinh(x)}{(1+\cosh(x))^3}\,dx = -\frac{1}{2\cosh{(x)}^2+4\cosh{(x)}+2}$$

input
```
int(sinh(x)/(1+cosh(x))^3,x)
```

output
```
( - 1)/(2*(cosh(x)**2 + 2*cosh(x) + 1))
```

3.147 $\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 12

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2(1-\cosh(x))^2}$$

output
```
1/2/(1-cosh(x))^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{8}\operatorname{csch}^4\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]/(1 - Cosh[x])^3,x]
```

output
```
Csch[x/2]^4/8
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3} \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1}{(1-\cosh(x))^3}d(-\cosh(x))$$

$$\Big\downarrow \text{17}$$

$$\frac{1}{2(1-\cosh(x))^2}$$

input
```
Int[Sinh[x]/(1 - Cosh[x])^3,x]
```

output
```
1/(2*(1 - Cosh[x])^2)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{1}{2(1-\cosh(x))^2}$	11
default	$\frac{1}{2(1-\cosh(x))^2}$	11
risch	$\frac{2\,e^{2x}}{(e^{x}-1)^4}$	13

input
```
int(sinh(x)/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/2/(1-cosh(x))^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(8) = 16$.

Time = 0.07 (sec) , antiderivative size = 55, normalized size of antiderivative = 4.58

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx$$
$$= \frac{2\left(\cosh\left(x\right)+\sinh\left(x\right)\right)}{\cosh\left(x\right)^{3}+\left(3\,\cosh\left(x\right)-4\right)\sinh\left(x\right)^{2}+\sinh\left(x\right)^{3}-4\,\cosh\left(x\right)^{2}+\left(3\,\cosh\left(x\right)^{2}-8\,\cosh\left(x\right)+5\right)\sinh\left(x\right)}$$

input
```
integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
2*(cosh(x) + sinh(x))/(cosh(x)^3 + (3*cosh(x) - 4)*sinh(x)^2 + sinh(x)^3 -
 4*cosh(x)^2 + (3*cosh(x)^2 - 8*cosh(x) + 5)*sinh(x) + 7*cosh(x) - 4)
```

Sympy [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2\cosh^{2}\left(x\right)-4\cosh\left(x\right)+2}$$

input
```
integrate(sinh(x)/(1-cosh(x))**3,x)
```

output
```
1/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2\left(\cosh\left(x\right)-1\right)^{2}}$$

input
```
integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
1/2/(cosh(x) - 1)^2
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{2\,e^{(2\,x)}}{(e^x-1)^4}$$

input
```
integrate(sinh(x)/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
2*e^(2*x)/(e^x - 1)^4
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.67

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2\,(\cosh{(x)}-1)^2}$$

input
```
int(-sinh(x)/(cosh(x) - 1)^3,x)
```

output
```
1/(2*(cosh(x) - 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int \frac{\sinh(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{2\cosh{(x)}^2-4\cosh{(x)}+2}$$

input
```
int(sinh(x)/(1-cosh(x))^3,x)
```

output
```
1/(2*(cosh(x)**2 - 2*cosh(x) + 1))
```

3.148 $\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} dx = \frac{\sinh^3(x)}{3(1+\cosh(x))^3}$$

output
```
1/3*sinh(x)^3/(1+cosh(x))^3
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3} dx = \frac{1}{3}\tanh^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

output
```
Tanh[x/2]^3/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3042, 25, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(\cosh(x)+1)^3} dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \; 3150$$

$$\frac{\sinh^3(x)}{3(\cosh(x)+1)^3}$$

input
```
Int[Sinh[x]^2/(1 + Cosh[x])^3,x]
```

output
```
Sinh[x]^3/(3*(1 + Cosh[x])^3)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Maple [A] (verified)

Time = 0.69 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.64

method	result	size
default	$\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}$	9
risch	$-\frac{2\left(3\,{\mathrm e}^{2x}+1\right)}{3\left({\mathrm e}^{x}+1\right)^{3}}$	17

input
```
int(sinh(x)^2/(1+cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*tanh(1/2*x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. 2(12) = 24.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.36

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx$$
$$= -\frac{4\,(2\,\cosh{(x)}+\sinh{(x)})}{3\,\left(\cosh{(x)}^{2}+2\,(\cosh{(x)}+1)\sinh{(x)}+\sinh{(x)}^{2}+4\,\cosh{(x)}+3\right)}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 4*cosh(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.50

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = \frac{\tanh^3\left(\frac{x}{2}\right)}{3}$$

input
```
integrate(sinh(x)**2/(1+cosh(x))**3,x)
```

output
```
tanh(x/2)**3/3
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.50

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = \frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1} + \frac{2}{3\,(3\,e^{(-x)}+3\,e^{(-2\,x)}+e^{(-3\,x)}+1)}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
2*e^(-2*x)/(3*e^(-x) + 3*e^(-2*x) + e^(-3*x) + 1) + 2/3/(3*e^(-x) + 3*e^(-
2*x) + e^(-3*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\left(3\,e^{(2\,x)}+1\right)}{3\,(e^x+1)^3}$$

input
```
integrate(sinh(x)^2/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-2/3*(3*e^(2*x) + 1)/(e^x + 1)^3
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = -\frac{2\,(3\,e^{2\,x}+1)}{3\,(e^x+1)^3}$$

input
```
int(sinh(x)^2/(cosh(x) + 1)^3,x)
```

output
```
-(2*(3*exp(2*x) + 1))/(3*(exp(x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.50

$$\int \frac{\sinh^2(x)}{(1+\cosh(x))^3}\,dx = \frac{2e^x(e^{2x}+3)}{3e^{3x}+9e^{2x}+9e^x+3}$$

input
```
int(sinh(x)^2/(1+cosh(x))^3,x)
```

output
```
(2*e**x*(e**(2*x) + 3))/(3*(e**(3*x) + 3*e**(2*x) + 3*e**x + 1))
```

3.149 $\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} dx = -\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

output
```
-1/3*sinh(x)^3/(1-cosh(x))^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.75

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3} dx = \frac{1}{3}\coth^3\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

output
```
Coth[x/2]^3/3
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3042, 25, 3150}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}dx$$
$$\Big\downarrow \ 3042$$
$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$
$$\Big\downarrow \ 25$$
$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^3}dx$$
$$\Big\downarrow \ 3150$$
$$-\frac{\sinh^3(x)}{3(1-\cosh(x))^3}$$

input
```
Int[Sinh[x]^2/(1 - Cosh[x])^3,x]
```

output
```
-1/3*Sinh[x]^3/(1 - Cosh[x])^3
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3150
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*m)), x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0]
 && EqQ[Simplify[m + p + 1], 0] && !ILtQ[p, 0]
```

Maple [A] (verified)

Time = 0.87 (sec) , antiderivative size = 9, normalized size of antiderivative = 0.56

method	result	size
default	$\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}$	9
risch	$\frac{2\,\mathrm{e}^{2x}+\frac{2}{3}}{\left(\mathrm{e}^{x}-1\right)^{3}}$	17

input
```
int(sinh(x)^2/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
1/3/tanh(1/2*x)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 33 vs. $2(12) = 24$.

Time = 0.07 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.06

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{4\,(2\,\cosh(x)+\sinh(x))}{3\,\left(\cosh(x)^2+2\,(\cosh(x)-1)\sinh(x)+\sinh(x)^2-4\,\cosh(x)+3\right)}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
4/3*(2*cosh(x) + sinh(x))/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + sinh(x)^2
 - 4*cosh(x) + 3)
```

Sympy [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 8, normalized size of antiderivative = 0.50

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{1}{3\tanh^3\left(\frac{x}{2}\right)}$$

input
```
integrate(sinh(x)**2/(1-cosh(x))**3,x)
```

output
```
1/(3*tanh(x/2)**3)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.

Time = 0.04 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = -\frac{2\,e^{(-2\,x)}}{3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1} - \frac{2}{3\left(3\,e^{(-x)}-3\,e^{(-2\,x)}+e^{(-3\,x)}-1\right)}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
-2*e^(-2*x)/(3*e^(-x) - 3*e^(-2*x) + e^(-3*x) - 1) - 2/3/(3*e^(-x) - 3*e^(
-2*x) + e^(-3*x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{2\left(3\,e^{(2\,x)}+1\right)}{3\left(e^x-1\right)^3}$$

input
```
integrate(sinh(x)^2/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
2/3*(3*e^(2*x) + 1)/(e^x - 1)^3
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{2\,(3\,e^{2\,x}+1)}{3\,(e^x-1)^3}$$

input
```
int(-sinh(x)^2/(cosh(x) - 1)^3,x)
```

output
```
(2*(3*exp(2*x) + 1))/(3*(exp(x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{\sinh^2(x)}{(1-\cosh(x))^3}\,dx = \frac{2e^x(e^{2x}+3)}{3e^{3x}-9e^{2x}+9e^x-3}$$

input
```
int(sinh(x)^2/(1-cosh(x))^3,x)
```

output
```
(2*e**x*(e**(2*x) + 3))/(3*(e**(3*x) - 3*e**(2*x) + 3*e**x - 1))
```

3.150 $\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 14

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} \, dx = \frac{2}{1+\cosh(x)} + \log(1+\cosh(x))$$

output
```
2/(1+cosh(x))+ln(1+cosh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3} \, dx = 2\log\left(\cosh\left(\frac{x}{2}\right)\right) + \operatorname{sech}^2\left(\frac{x}{2}\right)$$

input
```
Integrate[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

output
```
2*Log[Cosh[x/2]] + Sech[x/2]^2
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.455$, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(\cosh(x)+1)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1-\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(1-\sin\left(ix-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3146}$$

$$-\int \frac{1-\cosh(x)}{(\cosh(x)+1)^2}d\cosh(x)$$

$$\Big\downarrow \text{49}$$

$$-\int \left(\frac{2}{(\cosh(x)+1)^2}+\frac{1}{-\cosh(x)-1}\right)d\cosh(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{2}{\cosh(x)+1}+\log(\cosh(x)+1)$$

input
```
Int[Sinh[x]^3/(1 + Cosh[x])^3,x]
```

output
```
2/(1 + Cosh[x]) + Log[1 + Cosh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 1.06 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.07

method	result	size
derivativedivides	$\frac{2}{1+\cosh(x)}+\ln(1+\cosh(x))$	15
default	$\frac{2}{1+\cosh(x)}+\ln(1+\cosh(x))$	15
risch	$-x+\frac{4\,\mathrm{e}^{x}}{(\mathrm{e}^{x}+1)^{2}}+2\ln(\mathrm{e}^{x}+1)$	22

input
```
int(sinh(x)^3/(1+cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(1+cosh(x))+ln(1+cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(14) = 28$.

Time = 0.10 (sec) , antiderivative size = 89, normalized size of antiderivative = 6.36

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = -\frac{x\cosh{(x)}^2 + x\sinh{(x)}^2 + 2(x-2)\cosh{(x)} - 2\left(\cosh{(x)}^2 + 2(\cosh{(x)}+1)\sinh{(x)} + \sinh{(x)}^2 + 2\right.}{\cosh{(x)}^2 + 2(\cosh{(x)}+1)\sinh{(x)} + \sinh}$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="fricas")
```

output
```
-(x*cosh(x)^2 + x*sinh(x)^2 + 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x
) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) + 1) + 2
*(x*cosh(x) + x - 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + s
inh(x)^2 + 2*cosh(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(12) = 24$.

Time = 0.27 (sec) , antiderivative size = 126, normalized size of antiderivative = 9.00

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = \frac{2\log{(\cosh{(x)}+1)}\cosh^2{(x)}}{2\cosh^2{(x)}+4\cosh{(x)}+2} + \frac{4\log{(\cosh{(x)}+1)}\cosh{(x)}}{2\cosh^2{(x)}+4\cosh{(x)}+2} + \frac{2\log{(\cosh{(x)}+1)}}{2\cosh^2{(x)}+4\cosh{(x)}+2} - \frac{\sinh^2{(x)}}{2\cosh^2{(x)}+4\cosh{(x)}+2} + \frac{2\cosh{(x)}}{2\cosh^2{(x)}+4\cosh{(x)}+2} + \frac{2}{2\cosh^2{(x)}+4\cosh{(x)}+2}$$

input
```
integrate(sinh(x)**3/(1+cosh(x))**3,x)
```

output
```
2*log(cosh(x) + 1)*cosh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 4*log(cosh(
x) + 1)*cosh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*log(cosh(x) + 1)/(2*cos
h(x)**2 + 4*cosh(x) + 2) - sinh(x)**2/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2*c
osh(x)/(2*cosh(x)**2 + 4*cosh(x) + 2) + 2/(2*cosh(x)**2 + 4*cosh(x) + 2)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 31 vs. $2(14) = 28$.

Time = 0.03 (sec) , antiderivative size = 31, normalized size of antiderivative = 2.21

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = x + \frac{4\,e^{(-x)}}{2\,e^{(-x)} + e^{(-2\,x)} + 1} + 2\,\log\left(e^{(-x)} + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="maxima")
```

output
```
x + 4*e^(-x)/(2*e^(-x) + e^(-2*x) + 1) + 2*log(e^(-x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = -x + \frac{4\,e^{x}}{\left(e^{x} + 1\right)^2} + 2\,\log\left(e^{x} + 1\right)$$

input
```
integrate(sinh(x)^3/(1+cosh(x))^3,x, algorithm="giac")
```

output
```
-x + 4*e^x/(e^x + 1)^2 + 2*log(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 2.18 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = \ln\left(\cosh(x) + 1\right) + \frac{2}{\cosh\left(x\right) + 1}$$

input
```
int(sinh(x)^3/(cosh(x) + 1)^3,x)
```

output
```
log(cosh(x) + 1) + 2/(cosh(x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 56, normalized size of antiderivative = 4.00

$$\int \frac{\sinh^3(x)}{(1+\cosh(x))^3}\,dx = \frac{2\cosh(x)^2\log(\cosh(x)+1)-\cosh(x)^2+4\cosh(x)\log(\cosh(x)+1)+2\log(\cosh(x)+1)-\sinh(x)^2}{2\cosh(x)^2+4\cosh(x)+2}$$

input
```
int(sinh(x)^3/(1+cosh(x))^3,x)
```

output
```
(2*cosh(x)**2*log(cosh(x) + 1) - cosh(x)**2 + 4*cosh(x)*log(cosh(x) + 1) +
 2*log(cosh(x) + 1) - sinh(x)**2 + 1)/(2*(cosh(x)**2 + 2*cosh(x) + 1))
```

3.151 $\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} dx$

Optimal result

Integrand size = 13, antiderivative size = 20

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} dx = -\frac{2}{1-\cosh(x)} - \log(1-\cosh(x))$$

output
```
-2/(1-cosh(x))-ln(1-cosh(x))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} dx = \operatorname{csch}^2\left(\frac{x}{2}\right) - 2\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

output
```
Csch[x/2]^2 - 2*Log[Sinh[x/2]]
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3} \, dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i \cos\left(-\frac{\pi}{2}+ix\right)^3}{\left(1+\sin\left(-\frac{\pi}{2}+ix\right)\right)^3}dx$$
$$\Big\downarrow \text{26}$$
$$i \int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{\left(\sin\left(ix-\frac{\pi}{2}\right)+1\right)^3}dx$$
$$\Big\downarrow \text{3146}$$
$$\int \frac{\cosh(x)+1}{(1-\cosh(x))^2}d(-\cosh(x))$$
$$\Big\downarrow \text{49}$$
$$\int \left(\frac{2}{(1-\cosh(x))^2}+\frac{1}{\cosh(x)-1}\right)d(-\cosh(x))$$
$$\Big\downarrow \text{2009}$$
$$-\frac{2}{1-\cosh(x)}-\log(1-\cosh(x))$$

input
```
Int[Sinh[x]^3/(1 - Cosh[x])^3,x]
```

output
```
-2/(1 - Cosh[x]) - Log[1 - Cosh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 1.27 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

method	result	size
derivativedivides	$\frac{2}{-1+\cosh(x)}-\ln\left(-1+\cosh\left(x\right)\right)$	17
default	$\frac{2}{-1+\cosh(x)}-\ln\left(-1+\cosh\left(x\right)\right)$	17
risch	$x+\frac{4\,{\mathrm e}^{x}}{\left({\mathrm e}^{x}-1\right)^{2}}-2\ln\left({\mathrm e}^{x}-1\right)$	20

input
```
int(sinh(x)^3/(1-cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
2/(-1+cosh(x))-ln(-1+cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 90 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 4.50

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = \frac{x\cosh(x)^2 + x\sinh(x)^2 - 2(x-2)\cosh(x) - 2\left(\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x)^2 - 2\,\mathrm{c}\right.}{\cosh(x)^2 + 2(\cosh(x)-1)\sinh(x) + \sinh(x}$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="fricas")
```

output
```
(x*cosh(x)^2 + x*sinh(x)^2 - 2*(x - 2)*cosh(x) - 2*(cosh(x)^2 + 2*(cosh(x)
 - 1)*sinh(x) + sinh(x)^2 - 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*
(x*cosh(x) - x + 2)*sinh(x) + x)/(cosh(x)^2 + 2*(cosh(x) - 1)*sinh(x) + si
nh(x)^2 - 2*cosh(x) + 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 126 vs. $2(14) = 28$.

Time = 0.26 (sec) , antiderivative size = 126, normalized size of antiderivative = 6.30

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = -\frac{2\log(\cosh(x)-1)\cosh^2(x)}{2\cosh^2(x)-4\cosh(x)+2} + \frac{4\log(\cosh(x)-1)\cosh(x)}{2\cosh^2(x)-4\cosh(x)+2} - \frac{2\log(\cosh(x)-1)}{2\cosh^2(x)-4\cosh(x)+2} + \frac{\sinh^2(x)}{2\cosh^2(x)-4\cosh(x)+2} + \frac{2\cosh(x)}{2\cosh^2(x)-4\cosh(x)+2} - \frac{2}{2\cosh^2(x)-4\cosh(x)+2}$$

input
```
integrate(sinh(x)**3/(1-cosh(x))**3,x)
```

output
```
-2*log(cosh(x) - 1)*cosh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 4*log(cosh
(x) - 1)*cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2*log(cosh(x) - 1)/(2*co
sh(x)**2 - 4*cosh(x) + 2) + sinh(x)**2/(2*cosh(x)**2 - 4*cosh(x) + 2) + 2*
cosh(x)/(2*cosh(x)**2 - 4*cosh(x) + 2) - 2/(2*cosh(x)**2 - 4*cosh(x) + 2)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.75

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = -x - \frac{4\,e^{(-x)}}{2\,e^{(-x)} - e^{(-2\,x)} - 1} - 2\,\log\left(e^{(-x)} - 1\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="maxima")
```

output
```
-x - 4*e^(-x)/(2*e^(-x) - e^(-2*x) - 1) - 2*log(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = x + \frac{4\,e^{x}}{(e^{x}-1)^2} - 2\,\log\left(|e^{x}-1|\right)$$

input
```
integrate(sinh(x)^3/(1-cosh(x))^3,x, algorithm="giac")
```

output
```
x + 4*e^x/(e^x - 1)^2 - 2*log(abs(e^x - 1))
```

Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx = \frac{2}{\cosh(x)-1} - \ln\left(\cosh(x)-1\right)$$

input
```
int(-sinh(x)^3/(cosh(x) - 1)^3,x)
```

output
```
2/(cosh(x) - 1) - log(cosh(x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{\sinh^3(x)}{(1-\cosh(x))^3}\,dx$$
$$= \frac{-2\cosh(x)^2\log(\cosh(x)-1)+\cosh(x)^2+4\cosh(x)\log(\cosh(x)-1)-2\log(\cosh(x)-1)+\sinh(x)}{2\cosh(x)^2-4\cosh(x)+2}$$

input
```
int(sinh(x)^3/(1-cosh(x))^3,x)
```

output
```
( - 2*cosh(x)**2*log(cosh(x) - 1) + cosh(x)**2 + 4*cosh(x)*log(cosh(x) - 1
) - 2*log(cosh(x) - 1) + sinh(x)**2 - 1)/(2*(cosh(x)**2 - 2*cosh(x) + 1))
```

3.152 $\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx=\frac{5x}{16a}-\frac{5\cosh(x)\sinh(x)}{16a}+\frac{5\cosh(x)\sinh^3(x)}{24a}-\frac{\cosh(x)\sinh^5(x)}{6a}+\frac{\sinh^7(x)}{7a}$$

output
```
5/16*x/a-5/16*cosh(x)*sinh(x)/a+5/24*cosh(x)*sinh(x)^3/a-1/6*cosh(x)*sinh(
x)^5/a+1/7*sinh(x)^7/a
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx=\frac{420x-105\sinh(x)-315\sinh(2x)+63\sinh(3x)+63\sinh(4x)-21\sinh(5x)-7\sinh(6x)+3\sinh(7x)}{1344a}$$

input
```
Integrate[Sinh[x]^8/(a + a*Cosh[x]),x]
```

output
```
(420*x - 105*Sinh[x] - 315*Sinh[2*x] + 63*Sinh[3*x] + 63*Sinh[4*x] - 21*Si
nh[5*x] - 7*Sinh[6*x] + 3*Sinh[7*x])/(1344*a)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 3161, 25, 3042, 25, 3115, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^8(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^8}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3161}$$

$$\frac{\int -\sinh^6(x)dx}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^7(x)}{7a}-\frac{\int \sinh^6(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^7(x)}{7a}-\frac{\int -\sin(ix)^6dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^7(x)}{7a}+\frac{\int \sin(ix)^6dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{5}{6}\int \sinh^4(x)dx-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^7(x)}{7a}+\frac{-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\int \sin(ix)^4dx}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int \sinh^2(x)dx\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \; 3042$$

$$\frac{\sinh^7(x)}{7a}+\frac{-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int -\sin(ix)^2dx\right)}{a}$$

$$\Big\downarrow \; 25$$

$$\frac{\sinh^7(x)}{7a}+\frac{-\frac{1}{6}\sinh^5(x)\cosh(x)+\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\int \sin(ix)^2dx\right)}{a}$$

$$\Big\downarrow \; 3115$$

$$\frac{\frac{5}{6}\left(\frac{3}{4}\left(\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)+\frac{1}{4}\sinh^3(x)\cosh(x)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}+\frac{\sinh^7(x)}{7a}$$

$$\Big\downarrow \; 24$$

$$\frac{\sinh^7(x)}{7a}+\frac{\frac{5}{6}\left(\frac{1}{4}\sinh^3(x)\cosh(x)+\frac{3}{4}\left(\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)\right)\right)-\frac{1}{6}\sinh^5(x)\cosh(x)}{a}$$

input
```
Int[Sinh[x]^8/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]^7/(7*a) + (-1/6*(Cosh[x]*Sinh[x]^5) + (5*((Cosh[x]*Sinh[x]^3)/4 +
(3*(x/2 - (Cosh[x]*Sinh[x])/2))/4))/6)/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 164 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.89

$$-\frac{1}{7\left(1+\tanh\left(\frac{x}{2}\right)\right)^{7}}+\frac{2}{3\left(1+\tanh\left(\frac{x}{2}\right)\right)^{6}}-\frac{1}{\left(1+\tanh\left(\frac{x}{2}\right)\right)^{5}}+\frac{1}{4\left(1+\tanh\left(\frac{x}{2}\right)\right)^{4}}+\frac{11}{24\left(1+\tanh\left(\frac{x}{2}\right)\right)^{3}}+\frac{1}{8\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{5}{16\left(1+\tanh\left(\frac{x}{2}\right)\right)}$$

input
```
int(sinh(x)^8/(a+cosh(x)*a),x)
```

output
```
256/a*(-1/1792/(1+tanh(1/2*x))^7+1/384/(1+tanh(1/2*x))^6-1/256/(1+tanh(1/2
*x))^5+1/1024/(1+tanh(1/2*x))^4+11/6144/(1+tanh(1/2*x))^3+1/2048/(1+tanh(1
/2*x))^2-5/4096/(1+tanh(1/2*x))+5/4096*ln(1+tanh(1/2*x))-1/1792/(tanh(1/2*
x)-1)^7-1/384/(tanh(1/2*x)-1)^6-1/256/(tanh(1/2*x)-1)^5-1/1024/(tanh(1/2*x
)-1)^4+11/6144/(tanh(1/2*x)-1)^3-1/2048/(tanh(1/2*x)-1)^2-5/4096/(tanh(1/2
*x)-1)-5/4096*ln(tanh(1/2*x)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(47) = 94$.

Time = 0.08 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{3\,\sinh(x)^7 + 21\,(3\,\cosh(x)^2 - 2\,\cosh(x) - 1)\,\sinh(x)^5 + 7\,(15\,\cosh(x)^4 - 20\,\cosh(x)^3 - 30\,\cosh(x)^2}{}$$

input
```
integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/1344*(3*sinh(x)^7 + 21*(3*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^5 + 7*(15*c
osh(x)^4 - 20*cosh(x)^3 - 30*cosh(x)^2 + 36*cosh(x) + 9)*sinh(x)^3 + 21*(c
osh(x)^6 - 2*cosh(x)^5 - 5*cosh(x)^4 + 12*cosh(x)^3 + 9*cosh(x)^2 - 30*cos
h(x) - 5)*sinh(x) + 420*x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1253 vs. $2(51) = 102$.

Time = 2.98 (sec) , antiderivative size = 1253, normalized size of antiderivative = 21.98

$$\int \frac{\sinh^8(x)}{a+a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**8/(a+a*cosh(x)),x)
```

output
```
105*x*tanh(x/2)**14/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*t
anh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x
/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 735*x*tanh(x/2)**12/(336*a*tanh(x/
2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**
8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336
*a) + 2205*x*tanh(x/2)**10/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7
056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a
*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 3675*x*tanh(x/2)**8/(336*a*
tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh
(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**
2 - 336*a) + 3675*x*tanh(x/2)**6/(336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**
12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 -
7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 2205*x*tanh(x/2)**4/(
336*a*tanh(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*
a*tanh(x/2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(
x/2)**2 - 336*a) + 735*x*tanh(x/2)**2/(336*a*tanh(x/2)**14 - 2352*a*tanh(x
/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/2)**8 + 11760*a*tanh(x/2)*
*6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**2 - 336*a) - 105*x/(336*a*tan
h(x/2)**14 - 2352*a*tanh(x/2)**12 + 7056*a*tanh(x/2)**10 - 11760*a*tanh(x/
2)**8 + 11760*a*tanh(x/2)**6 - 7056*a*tanh(x/2)**4 + 2352*a*tanh(x/2)**...
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(47) = 94$.

Time = 0.03 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.79

$$\int \frac{\sinh^8(x)}{a + a\cosh(x)}\,dx = -\frac{\left(7\,e^{(-x)} + 21\,e^{(-2\,x)} - 63\,e^{(-3\,x)} - 63\,e^{(-4\,x)} + 315\,e^{(-5\,x)} + 105\,e^{(-6\,x)} - 3\right)e^{(7\,x)}}{2688\,a} + \frac{5\,x}{16\,a} + \frac{105\,e^{(-x)} + 315\,e^{(-2\,x)} - 63\,e^{(-3\,x)} - 63\,e^{(-4\,x)} + 21\,e^{(-5\,x)} + 7\,e^{(-6\,x)} - 3\,e^{(-7\,x)}}{2688\,a}$$

input
```
integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/2688*(7*e^(-x) + 21*e^(-2*x) - 63*e^(-3*x) - 63*e^(-4*x) + 315*e^(-5*x)
 + 105*e^(-6*x) - 3)*e^(7*x)/a + 5/16*x/a + 1/2688*(105*e^(-x) + 315*e^(-2
*x) - 63*e^(-3*x) - 63*e^(-4*x) + 21*e^(-5*x) + 7*e^(-6*x) - 3*e^(-7*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^8(x)}{a + a\cosh(x)}\,dx = \frac{\left(105\,e^{(6\,x)} + 315\,e^{(5\,x)} - 63\,e^{(4\,x)} - 63\,e^{(3\,x)} + 21\,e^{(2\,x)} + 7\,e^{x} - 3\right)e^{(-7\,x)} + 840\,x + 3\,e^{(7\,x)} - 7\,e^{(6\,x)} - 21\,e^{\cdots}}{2688\,a}$$

input
```
integrate(sinh(x)^8/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/2688*((105*e^(6*x) + 315*e^(5*x) - 63*e^(4*x) - 63*e^(3*x) + 21*e^(2*x)
+ 7*e^x - 3)*e^(-7*x) + 840*x + 3*e^(7*x) - 7*e^(6*x) - 21*e^(5*x) + 63*e^
(4*x) + 63*e^(3*x) - 315*e^(2*x) - 105*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.25 (sec) , antiderivative size = 131, normalized size of antiderivative = 2.30

$$\int \frac{\sinh^8(x)}{a + a\cosh(x)}\,dx = \frac{5\,e^{-x}}{128\,a} + \frac{15\,e^{-2\,x}}{128\,a} - \frac{15\,e^{2\,x}}{128\,a} - \frac{3\,e^{-3\,x}}{128\,a} + \frac{3\,e^{3\,x}}{128\,a} - \frac{3\,e^{-4\,x}}{128\,a} + \frac{3\,e^{4\,x}}{128\,a} + \frac{e^{-5\,x}}{128\,a} - \frac{e^{5\,x}}{128\,a} + \frac{e^{-6\,x}}{384\,a} - \frac{e^{6\,x}}{384\,a} - \frac{e^{-7\,x}}{896\,a} + \frac{e^{7\,x}}{896\,a} + \frac{5\,x}{16\,a} - \frac{5\,e^{x}}{128\,a}$$

input
```
int(sinh(x)^8/(a + a*cosh(x)),x)
```

output
```
(5*exp(-x))/(128*a) + (15*exp(-2*x))/(128*a) - (15*exp(2*x))/(128*a) - (3*
exp(-3*x))/(128*a) + (3*exp(3*x))/(128*a) - (3*exp(-4*x))/(128*a) + (3*exp
(4*x))/(128*a) + exp(-5*x)/(128*a) - exp(5*x)/(128*a) + exp(-6*x)/(384*a)
- exp(6*x)/(384*a) - exp(-7*x)/(896*a) + exp(7*x)/(896*a) + (5*x)/(16*a) -
 (5*exp(x))/(128*a)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.95

$$\int \frac{\sinh^8(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{3e^{14x} - 7e^{13x} - 21e^{12x} + 63e^{11x} + 63e^{10x} - 315e^{9x} - 105e^{8x} + 840e^{7x}x + 105e^{6x} + 315e^{5x} - 63e^{4x} - 63e}{2688e^{7x}a}$$

input
```
int(sinh(x)^8/(a+a*cosh(x)),x)
```

output
```
(3*e**(14*x) - 7*e**(13*x) - 21*e**(12*x) + 63*e**(11*x) + 63*e**(10*x) -
315*e**(9*x) - 105*e**(8*x) + 840*e**(7*x)*x + 105*e**(6*x) + 315*e**(5*x)
 - 63*e**(4*x) - 63*e**(3*x) + 21*e**(2*x) + 7*e**x - 3)/(2688*e**(7*x)*a)
```

3.153 $\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx=\frac{(a-a\cosh(x))^4}{a^5}-\frac{4(a-a\cosh(x))^5}{5a^6}+\frac{(a-a\cosh(x))^6}{6a^7}$$

output
```
(a-a*cosh(x))^4/a^5-4/5*(a-a*cosh(x))^5/a^6+1/6*(a-a*cosh(x))^6/a^7
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.59

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx=\frac{4(27+28\cosh(x)+5\cosh(2x))\sinh^8\left(\frac{x}{2}\right)}{15a}$$

input
```
Integrate[Sinh[x]^7/(a + a*Cosh[x]),x]
```

output
```
(4*(27 + 28*Cosh[x] + 5*Cosh[2*x])*Sinh[x/2]^8)/(15*a)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(x)}{a\cosh(x)+a}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^7}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \ 26$$
$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^7}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$
$$\Big\downarrow \ 3146$$
$$-\frac{\int (a-a\cosh(x))^3(\cosh(x)a+a)^2d(a\cosh(x))}{a^7}$$
$$\Big\downarrow \ 49$$
$$-\frac{\int \left((a-a\cosh(x))^5-4a(a-a\cosh(x))^4+4a^2(a-a\cosh(x))^3\right)d(a\cosh(x))}{a^7}$$
$$\Big\downarrow \ 2009$$
$$-\frac{-a^2(a-a\cosh(x))^4-\frac{1}{6}(a-a\cosh(x))^6+\frac{4}{5}a(a-a\cosh(x))^5}{a^7}$$

input
```
Int[Sinh[x]^7/(a + a*Cosh[x]),x]
```

output
```
-((-(a^2*(a - a*Cosh[x])^4) + (4*a*(a - a*Cosh[x])^5)/5 - (a - a*Cosh[x])^
6/6)/a^7)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 157.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

method	result	s
derivativedivides	$\frac{\frac{\cosh(x)^6}{6}-\frac{\cosh(x)^5}{5}-\frac{\cosh(x)^4}{2}+\frac{2\cosh(x)^3}{3}+\frac{\cosh(x)^2}{2}-\cosh(x)}{a}$	4
default	$\frac{\frac{\cosh(x)^6}{6}-\frac{\cosh(x)^5}{5}-\frac{\cosh(x)^4}{2}+\frac{2\cosh(x)^3}{3}+\frac{\cosh(x)^2}{2}-\cosh(x)}{a}$	4
risch	$\frac{e^{6x}}{384a}-\frac{e^{5x}}{160a}-\frac{e^{4x}}{64a}+\frac{5\,e^{3x}}{96a}+\frac{5\,e^{2x}}{128a}-\frac{5\,e^{x}}{16a}-\frac{5\,e^{-x}}{16a}+\frac{5\,e^{-2x}}{128a}+\frac{5\,e^{-3x}}{96a}-\frac{e^{-4x}}{64a}-\frac{e^{-5x}}{160a}+\frac{e^{-6x}}{384a}$	1

input
```
int(sinh(x)^7/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/6*cosh(x)^6-1/5*cosh(x)^5-1/2*cosh(x)^4+2/3*cosh(x)^3+1/2*cosh(x)^2
-cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(45) = 90$.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \frac{\sinh^7(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{5\,\cosh\left(x\right)^{6} + 5\,\sinh\left(x\right)^{6} - 12\,\cosh\left(x\right)^{5} + 15\,\left(5\,\cosh\left(x\right)^{2} - 4\,\cosh\left(x\right) - 2\right)\sinh\left(x\right)^{4} - 30\,\cosh\left(x\right)^{4} + 1}{}$$

input
```
integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/960*(5*cosh(x)^6 + 5*sinh(x)^6 - 12*cosh(x)^5 + 15*(5*cosh(x)^2 - 4*cosh
(x) - 2)*sinh(x)^4 - 30*cosh(x)^4 + 100*cosh(x)^3 + 15*(5*cosh(x)^4 - 8*co
sh(x)^3 - 12*cosh(x)^2 + 20*cosh(x) + 5)*sinh(x)^2 + 75*cosh(x)^2 - 600*co
sh(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(39) = 78$.

Time = 2.00 (sec) , antiderivative size = 284, normalized size of antiderivative = 6.17

$$\int \frac{\sinh^7(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{320\tanh^{6}\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^{8}\left(\frac{x}{2}\right) - 300a\tanh^{6}\left(\frac{x}{2}\right) + 225a\tanh^{4}\left(\frac{x}{2}\right) - 90a\tanh^{2}\left(\frac{x}{2}\right) -}$$
$$- \frac{240\tanh^{4}\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^{8}\left(\frac{x}{2}\right) - 300a\tanh^{6}\left(\frac{x}{2}\right) + 225a\tanh^{4}\left(\frac{x}{2}\right) - 90a\tanh^{2}\left(\frac{x}{2}\right.}$$
$$+ \frac{96\tanh^{2}\left(\frac{x}{2}\right)}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^{8}\left(\frac{x}{2}\right) - 300a\tanh^{6}\left(\frac{x}{2}\right) + 225a\tanh^{4}\left(\frac{x}{2}\right) - 90a\tanh^{2}\left(\frac{x}{2}\right.}$$
$$- \frac{16}{15a\tanh^{12}\left(\frac{x}{2}\right) - 90a\tanh^{10}\left(\frac{x}{2}\right) + 225a\tanh^{8}\left(\frac{x}{2}\right) - 300a\tanh^{6}\left(\frac{x}{2}\right) + 225a\tanh^{4}\left(\frac{x}{2}\right) - 90a\tanh^{2}\left(\frac{x}{2}\right.}$$

input
```
integrate(sinh(x)**7/(a+a*cosh(x)),x)
```

output
```
320*tanh(x/2)**6/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2
)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
 - 240*tanh(x/2)**4/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(
x/2)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15
*a) + 96*tanh(x/2)**2/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tan
h(x/2)**8 - 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 +
15*a) - 16/(15*a*tanh(x/2)**12 - 90*a*tanh(x/2)**10 + 225*a*tanh(x/2)**8 -
 300*a*tanh(x/2)**6 + 225*a*tanh(x/2)**4 - 90*a*tanh(x/2)**2 + 15*a)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.83

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = -\frac{\left(12\,e^{(-x)}+30\,e^{(-2\,x)}-100\,e^{(-3\,x)}-75\,e^{(-4\,x)}+600\,e^{(-5\,x)}-5\right)e^{(6\,x)}}{1920\,a} - \frac{600\,e^{(-x)}-75\,e^{(-2\,x)}-100\,e^{(-3\,x)}+30\,e^{(-4\,x)}+12\,e^{(-5\,x)}-5\,e^{(-6\,x)}}{1920\,a}$$

input
```
integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/1920*(12*e^(-x) + 30*e^(-2*x) - 100*e^(-3*x) - 75*e^(-4*x) + 600*e^(-5*
x) - 5)*e^(6*x)/a - 1/1920*(600*e^(-x) - 75*e^(-2*x) - 100*e^(-3*x) + 30*e
^(-4*x) + 12*e^(-5*x) - 5*e^(-6*x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.63

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = -\frac{\left(600\,e^{(5\,x)}-75\,e^{(4\,x)}-100\,e^{(3\,x)}+30\,e^{(2\,x)}+12\,e^{x}-5\right)e^{(-6\,x)}-5\,e^{(6\,x)}+12\,e^{(5\,x)}+30\,e^{(4\,x)}-100\,e^{(3\,x}}{1920\,a}$$

input
```
integrate(sinh(x)^7/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/1920*((600*e^(5*x) - 75*e^(4*x) - 100*e^(3*x) + 30*e^(2*x) + 12*e^x - 5
)*e^(-6*x) - 5*e^(6*x) + 12*e^(5*x) + 30*e^(4*x) - 100*e^(3*x) - 75*e^(2*x
) + 600*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.33

$$\int \frac{\sinh^7(x)}{a+a\cosh(x)}\,dx = \frac{5\,e^{-2\,x}}{128\,a}-\frac{5\,e^{-x}}{16\,a}+\frac{5\,e^{2\,x}}{128\,a}+\frac{5\,e^{-3\,x}}{96\,a}+\frac{5\,e^{3\,x}}{96\,a}-\frac{e^{-4\,x}}{64\,a}-\frac{e^{4\,x}}{64\,a}-\frac{e^{-5\,x}}{160\,a}-\frac{e^{5\,x}}{160\,a}+\frac{e^{-6\,x}}{384\,a}+\frac{e^{6\,x}}{384\,a}-\frac{5\,e^{x}}{16\,a}$$

input
```
int(sinh(x)^7/(a + a*cosh(x)),x)
```

output
```
(5*exp(-2*x))/(128*a) - (5*exp(-x))/(16*a) + (5*exp(2*x))/(128*a) + (5*exp
(-3*x))/(96*a) + (5*exp(3*x))/(96*a) - exp(-4*x)/(64*a) - exp(4*x)/(64*a)
- exp(-5*x)/(160*a) - exp(5*x)/(160*a) + exp(-6*x)/(384*a) + exp(6*x)/(384
*a) - (5*exp(x))/(16*a)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \frac{\sinh^7(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{5e^{12x} - 12e^{11x} - 30e^{10x} + 100e^{9x} + 75e^{8x} - 600e^{7x} - 600e^{5x} + 75e^{4x} + 100e^{3x} - 30e^{2x} - 12e^{x} + 5}{1920e^{6x}a}$$

input
```
int(sinh(x)^7/(a+a*cosh(x)),x)
```

output
```
(5*e**(12*x) - 12*e**(11*x) - 30*e**(10*x) + 100*e**(9*x) + 75*e**(8*x) -
600*e**(7*x) - 600*e**(5*x) + 75*e**(4*x) + 100*e**(3*x) - 30*e**(2*x) - 1
2*e**x + 5)/(1920*e**(6*x)*a)
```

3.154 $\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 44

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = -\frac{3x}{8a}+\frac{3\cosh(x)\sinh(x)}{8a}-\frac{\cosh(x)\sinh^3(x)}{4a}+\frac{\sinh^5(x)}{5a}$$

output
```
-3/8*x/a+3/8*cosh(x)*sinh(x)/a-1/4*cosh(x)*sinh(x)^3/a+1/5*sinh(x)^5/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{-60x+20\sinh(x)+40\sinh(2x)-10\sinh(3x)-5\sinh(4x)+2\sinh(5x)}{160a}$$

input
```
Integrate[Sinh[x]^6/(a + a*Cosh[x]),x]
```

output
```
(-60*x + 20*Sinh[x] + 40*Sinh[2*x] - 10*Sinh[3*x] - 5*Sinh[4*x] + 2*Sinh[5
*x])/(160*a)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 25, 3161, 3042, 3115, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^6}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^6}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3161$$

$$\frac{\sinh^5(x)}{5a}-\frac{\int \sinh^4(x)dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh^5(x)}{5a}-\frac{\int \sin(ix)^4dx}{a}$$

$$\Big\downarrow \ 3115$$

$$\frac{\sinh^5(x)}{5a}-\frac{\frac{3}{4}\int -\sinh^2(x)dx+\frac{1}{4}\sinh^3(x)\cosh(x)}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{\sinh^5(x)}{5a}-\frac{\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int \sinh^2(x)dx}{a}$$

$$\Big\downarrow \ 3042$$

$$\frac{\sinh^5(x)}{5a}-\frac{\frac{1}{4}\sinh^3(x)\cosh(x)-\frac{3}{4}\int -\sin(ix)^2dx}{a}$$

$$\Big\downarrow \ 25$$

$$\frac{\sinh^5(x)}{5a} - \frac{\frac{1}{4}\sinh^3(x)\cosh(x) + \frac{3}{4}\int \sin(ix)^2 dx}{a}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\sinh^5(x)}{5a} - \frac{\frac{3}{4}\left(\frac{\int 1dx}{2} - \frac{1}{2}\sinh(x)\cosh(x)\right) + \frac{1}{4}\sinh^3(x)\cosh(x)}{a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\sinh^5(x)}{5a} - \frac{\frac{1}{4}\sinh^3(x)\cosh(x) + \frac{3}{4}\left(\frac{x}{2} - \frac{1}{2}\sinh(x)\cosh(x)\right)}{a}$$

input
```
Int[Sinh[x]^6/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]^5/(5*a) - ((Cosh[x]*Sinh[x]^3)/4 + (3*(x/2 - (Cosh[x]*Sinh[x])/2))
/4)/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(36) = 72$.

Time = 61.89 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

method	result
risch	$-\frac{3x}{8a}+\frac{e^{5x}}{160a}-\frac{e^{4x}}{64a}-\frac{e^{3x}}{32a}+\frac{e^{2x}}{8a}+\frac{e^{x}}{16a}-\frac{e^{-x}}{16a}-\frac{e^{-2x}}{8a}+\frac{e^{-3x}}{32a}+\frac{e^{-4x}}{64a}-\frac{e^{-5x}}{160a}$
default	$\frac{-\frac{1}{5\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{3}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{3}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}+\frac{1}{4\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}+\frac{3}{8\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{3\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{8}-\frac{1}{5\left(1+\tanh\left(\frac{x}{2}\right)\right)^{5}}+\frac{}{4(1\cdots}}{a}$

input
```
int(sinh(x)^6/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
-3/8*x/a+1/160/a*exp(5*x)-1/64/a*exp(4*x)-1/32/a*exp(3*x)+1/8/a*exp(2*x)+1
/16/a*exp(x)-1/16/a*exp(-x)-1/8/a*exp(-2*x)+1/32/a*exp(-3*x)+1/64/a*exp(-4
*x)-1/160/a*exp(-5*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.30

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{\sinh(x)^5+5\left(2\cosh(x)^2-2\cosh(x)-1\right)\sinh(x)^3+5\left(\cosh(x)^4-2\cosh(x)^3-3\cosh(x)^2+8\cos\cdots}{80\,a}$$

input
```
integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/80*(sinh(x)^5 + 5*(2*cosh(x)^2 - 2*cosh(x) - 1)*sinh(x)^3 + 5*(cosh(x)^4
 - 2*cosh(x)^3 - 3*cosh(x)^2 + 8*cosh(x) + 2)*sinh(x) - 30*x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 692 vs. $2(37) = 74$.

Time = 1.25 (sec) , antiderivative size = 692, normalized size of antiderivative = 15.73

$$\int \frac{\sinh^6(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**6/(a+a*cosh(x)),x)
```

output
```
-15*x*tanh(x/2)**10/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(
x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 75*x*tanh(x/2)
**8/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*
tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 150*x*tanh(x/2)**6/(40*a*tanh(
x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 +
200*a*tanh(x/2)**2 - 40*a) + 150*x*tanh(x/2)**4/(40*a*tanh(x/2)**10 - 200*
a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)
**2 - 40*a) - 75*x*tanh(x/2)**2/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 +
 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 15
*x/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*t
anh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 30*tanh(x/2)**9/(40*a*tanh(x/2)
**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*
a*tanh(x/2)**2 - 40*a) - 140*tanh(x/2)**7/(40*a*tanh(x/2)**10 - 200*a*tanh
(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 -
40*a) - 256*tanh(x/2)**5/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*
tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) + 140*tanh(
x/2)**3/(40*a*tanh(x/2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 40
0*a*tanh(x/2)**4 + 200*a*tanh(x/2)**2 - 40*a) - 30*tanh(x/2)/(40*a*tanh(x/
2)**10 - 200*a*tanh(x/2)**8 + 400*a*tanh(x/2)**6 - 400*a*tanh(x/2)**4 + 20
0*a*tanh(x/2)**2 - 40*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(36) = 72$.

Time = 0.03 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^6(x)}{a + a\cosh(x)}\,dx = -\frac{\left(5\,e^{(-x)} + 10\,e^{(-2\,x)} - 40\,e^{(-3\,x)} - 20\,e^{(-4\,x)} - 2\right)e^{(5\,x)}}{320\,a} - \frac{3\,x}{8\,a} - \frac{20\,e^{(-x)} + 40\,e^{(-2\,x)} - 10\,e^{(-3\,x)} - 5\,e^{(-4\,x)} + 2\,e^{(-5\,x)}}{320\,a}$$

input
```
integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/320*(5*e^(-x) + 10*e^(-2*x) - 40*e^(-3*x) - 20*e^(-4*x) - 2)*e^(5*x)/a
- 3/8*x/a - 1/320*(20*e^(-x) + 40*e^(-2*x) - 10*e^(-3*x) - 5*e^(-4*x) + 2*
e^(-5*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

$$\int \frac{\sinh^6(x)}{a + a\cosh(x)}\,dx = -\frac{\left(20\,e^{(4\,x)} + 40\,e^{(3\,x)} - 10\,e^{(2\,x)} - 5\,e^{x} + 2\right)e^{(-5\,x)} + 120\,x - 2\,e^{(5\,x)} + 5\,e^{(4\,x)} + 10\,e^{(3\,x)} - 40\,e^{(2\,x)} - 20\,e^{x}}{320\,a}$$

input
```
integrate(sinh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/320*((20*e^(4*x) + 40*e^(3*x) - 10*e^(2*x) - 5*e^x + 2)*e^(-5*x) + 120*
x - 2*e^(5*x) + 5*e^(4*x) + 10*e^(3*x) - 40*e^(2*x) - 20*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.16

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{e^{2x}}{8a} - \frac{e^{-2x}}{8a} - \frac{e^{-x}}{16a} + \frac{e^{-3x}}{32a} - \frac{e^{3x}}{32a} + \frac{e^{-4x}}{64a} - \frac{e^{4x}}{64a} - \frac{e^{-5x}}{160a} + \frac{e^{5x}}{160a} - \frac{3x}{8a} + \frac{e^{x}}{16a}$$

input
```
int(sinh(x)^6/(a + a*cosh(x)),x)
```

output
```
exp(2*x)/(8*a) - exp(-2*x)/(8*a) - exp(-x)/(16*a) + exp(-3*x)/(32*a) - exp
(3*x)/(32*a) + exp(-4*x)/(64*a) - exp(4*x)/(64*a) - exp(-5*x)/(160*a) + ex
p(5*x)/(160*a) - (3*x)/(8*a) + exp(x)/(16*a)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.89

$$\int \frac{\sinh^6(x)}{a+a\cosh(x)}\,dx = \frac{2e^{10x} - 5e^{9x} - 10e^{8x} + 40e^{7x} + 20e^{6x} - 120e^{5x}x - 20e^{4x} - 40e^{3x} + 10e^{2x} + 5e^{x} - 2}{320e^{5x}a}$$

input
```
int(sinh(x)^6/(a+a*cosh(x)),x)
```

output
```
(2*e**(10*x) - 5*e**(9*x) - 10*e**(8*x) + 40*e**(7*x) + 20*e**(6*x) - 120*
e**(5*x)*x - 20*e**(4*x) - 40*e**(3*x) + 10*e**(2*x) + 5*e**x - 2)/(320*e*
*(5*x)*a)
```

3.155 $\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx=-\frac{2(a-a\cosh(x))^3}{3a^4}+\frac{(a-a\cosh(x))^4}{4a^5}$$

output
```
-2/3*(a-a*cosh(x))^3/a^4+1/4*(a-a*cosh(x))^4/a^5
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.64

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx=\frac{2(5+3\cosh(x))\sinh^6\left(\frac{x}{2}\right)}{3a}$$

input
```
Integrate[Sinh[x]^5/(a + a*Cosh[x]),x]
```

output
```
(2*(5 + 3*Cosh[x])*Sinh[x/2]^6)/(3*a)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3146, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)^5}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^5}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3146}$$

$$\frac{\int (a-a\cosh(x))^2(\cosh(x)a+a)d(a\cosh(x))}{a^5}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{\int \left(2a(a-a\cosh(x))^2-(a-a\cosh(x))^3\right)d(a\cosh(x))}{a^5}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{4}(a-a\cosh(x))^4-\frac{2}{3}a(a-a\cosh(x))^3}{a^5}$$

input
```
Int[Sinh[x]^5/(a + a*Cosh[x]),x]
```

output
```
((-2*a*(a - a*Cosh[x])^3)/3 + (a - a*Cosh[x])^4/4)/a^5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 22.31 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.79

method	result	size
derivativedivides	$\frac{\frac{\cosh(x)^4}{4}-\frac{\cosh(x)^3}{3}-\frac{\cosh(x)^2}{2}+\cosh(x)}{a}$	26
default	$\frac{\frac{\cosh(x)^4}{4}-\frac{\cosh(x)^3}{3}-\frac{\cosh(x)^2}{2}+\cosh(x)}{a}$	26
risch	$\frac{e^{4x}}{64a}-\frac{e^{3x}}{24a}-\frac{e^{2x}}{16a}+\frac{3\,e^{x}}{8a}+\frac{3\,e^{-x}}{8a}-\frac{e^{-2x}}{16a}-\frac{e^{-3x}}{24a}+\frac{e^{-4x}}{64a}$	72

input
```
int(sinh(x)^5/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/4*cosh(x)^4-1/3*cosh(x)^3-1/2*cosh(x)^2+cosh(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \frac{\sinh^5(x)}{a + a\cosh(x)}\,dx = \frac{3\,\cosh\left(x\right)^{4} + 3\,\sinh\left(x\right)^{4} - 8\,\cosh\left(x\right)^{3} + 6\,\left(3\,\cosh\left(x\right)^{2} - 4\,\cosh\left(x\right) - 2\right)\sinh\left(x\right)^{2} - 12\,\cosh\left(x\right)^{2} + 72}{96\,a}$$

input
```
integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/96*(3*cosh(x)^4 + 3*sinh(x)^4 - 8*cosh(x)^3 + 6*(3*cosh(x)^2 - 4*cosh(x)
 - 2)*sinh(x)^2 - 12*cosh(x)^2 + 72*cosh(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. 2(27) = 54.

Time = 0.83 (sec) , antiderivative size = 150, normalized size of antiderivative = 4.55

$$\int \frac{\sinh^5(x)}{a + a\cosh(x)}\,dx = \frac{24\tanh^4\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right) - 12a\tanh^6\left(\frac{x}{2}\right) + 18a\tanh^4\left(\frac{x}{2}\right) - 12a\tanh^2\left(\frac{x}{2}\right) + 3a} - \frac{16\tanh^2\left(\frac{x}{2}\right)}{3a\tanh^8\left(\frac{x}{2}\right) - 12a\tanh^6\left(\frac{x}{2}\right) + 18a\tanh^4\left(\frac{x}{2}\right) - 12a\tanh^2\left(\frac{x}{2}\right) + 3a} + \frac{4}{3a\tanh^8\left(\frac{x}{2}\right) - 12a\tanh^6\left(\frac{x}{2}\right) + 18a\tanh^4\left(\frac{x}{2}\right) - 12a\tanh^2\left(\frac{x}{2}\right) + 3a}$$

input
```
integrate(sinh(x)**5/(a+a*cosh(x)),x)
```

output
```
24*tanh(x/2)**4/(3*a*tanh(x/2)**8 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4
- 12*a*tanh(x/2)**2 + 3*a) - 16*tanh(x/2)**2/(3*a*tanh(x/2)**8 - 12*a*tanh
(x/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a) + 4/(3*a*tanh(x/2)
**8 - 12*a*tanh(x/2)**6 + 18*a*tanh(x/2)**4 - 12*a*tanh(x/2)**2 + 3*a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.82

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx = -\frac{\left(8\,e^{(-x)}+12\,e^{(-2\,x)}-72\,e^{(-3\,x)}-3\right)e^{(4\,x)}}{192\,a} + \frac{72\,e^{(-x)}-12\,e^{(-2\,x)}-8\,e^{(-3\,x)}+3\,e^{(-4\,x)}}{192\,a}$$

input
```
integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*e^(-x) + 12*e^(-2*x) - 72*e^(-3*x) - 3)*e^(4*x)/a + 1/192*(72*e^
(-x) - 12*e^(-2*x) - 8*e^(-3*x) + 3*e^(-4*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx = \frac{\left(72\,e^{(3\,x)}-12\,e^{(2\,x)}-8\,e^{x}+3\right)e^{(-4\,x)}+3\,e^{(4\,x)}-8\,e^{(3\,x)}-12\,e^{(2\,x)}+72\,e^{x}}{192\,a}$$

input
```
integrate(sinh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/192*((72*e^(3*x) - 12*e^(2*x) - 8*e^x + 3)*e^(-4*x) + 3*e^(4*x) - 8*e^(3
*x) - 12*e^(2*x) + 72*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.03 (sec) , antiderivative size = 71, normalized size of antiderivative = 2.15

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx = \frac{3\,\mathrm{e}^{-x}}{8\,a}-\frac{\mathrm{e}^{-2\,x}}{16\,a}-\frac{\mathrm{e}^{2\,x}}{16\,a}-\frac{\mathrm{e}^{-3\,x}}{24\,a}-\frac{\mathrm{e}^{3\,x}}{24\,a}+\frac{\mathrm{e}^{-4\,x}}{64\,a}+\frac{\mathrm{e}^{4\,x}}{64\,a}+\frac{3\,\mathrm{e}^{x}}{8\,a}$$

input
```
int(sinh(x)^5/(a + a*cosh(x)),x)
```

output
```
(3*exp(-x))/(8*a) - exp(-2*x)/(16*a) - exp(2*x)/(16*a) - exp(-3*x)/(24*a)
- exp(3*x)/(24*a) + exp(-4*x)/(64*a) + exp(4*x)/(64*a) + (3*exp(x))/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.85

$$\int \frac{\sinh^5(x)}{a+a\cosh(x)}\,dx = \frac{3e^{8x}-8e^{7x}-12e^{6x}+72e^{5x}+72e^{3x}-12e^{2x}-8e^{x}+3}{192e^{4x}a}$$

input
```
int(sinh(x)^5/(a+a*cosh(x)),x)
```

output
```
(3*e**(8*x) - 8*e**(7*x) - 12*e**(6*x) + 72*e**(5*x) + 72*e**(3*x) - 12*e*
*(2*x) - 8*e**x + 3)/(192*e**(4*x)*a)
```

3.156 $\int \frac{\sinh^4(x)}{a+a\cosh(x)} dx$

Optimal result

Integrand size = 13, antiderivative size = 31

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)} dx = \frac{x}{2a} - \frac{\cosh(x)\sinh(x)}{2a} + \frac{\sinh^3(x)}{3a}$$

output
```
1/2*x/a-1/2*cosh(x)*sinh(x)/a+1/3*sinh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)} dx = \frac{6x - 3\sinh(x) - 3\sinh(2x) + \sinh(3x)}{12a}$$

input
```
Integrate[Sinh[x]^4/(a + a*Cosh[x]),x]
```

output
```
(6*x - 3*Sinh[x] - 3*Sinh[2*x] + Sinh[3*x])/(12*a)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.97, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.538$, Rules used = {3042, 3161, 25, 3042, 25, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a\cosh(x)+a}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$
$$\Big\downarrow \text{3161}$$
$$\frac{\int -\sinh^2(x)dx}{a}+\frac{\sinh^3(x)}{3a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\sinh^3(x)}{3a}-\frac{\int \sinh^2(x)dx}{a}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\sinh^3(x)}{3a}-\frac{\int -\sin(ix)^2dx}{a}$$
$$\Big\downarrow \text{25}$$
$$\frac{\sinh^3(x)}{3a}+\frac{\int \sin(ix)^2dx}{a}$$
$$\Big\downarrow \text{3115}$$
$$\frac{\frac{\int 1dx}{2}-\frac{1}{2}\sinh(x)\cosh(x)}{a}+\frac{\sinh^3(x)}{3a}$$
$$\Big\downarrow \text{24}$$
$$\frac{\sinh^3(x)}{3a}+\frac{\frac{x}{2}-\frac{1}{2}\sinh(x)\cosh(x)}{a}$$

input
```
Int[Sinh[x]^4/(a + a*Cosh[x]),x]
```

output
```
Sinh[x]^3/(3*a) + (x/2 - (Cosh[x]*Sinh[x])/2)/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(25) = 50$.

Time = 7.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

method	result
risch	$\frac{x}{2a}+\frac{e^{3x}}{24a}-\frac{e^{2x}}{8a}-\frac{e^{x}}{8a}+\frac{e^{-x}}{8a}+\frac{e^{-2x}}{8a}-\frac{e^{-3x}}{24a}$
default	$\frac{-\frac{1}{3\left(1+\tanh\left(\frac{x}{2}\right)\right)^{3}}+\frac{1}{\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{1}{2\left(1+\tanh\left(\frac{x}{2}\right)\right)}+\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{2}-\frac{1}{3\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{1}{\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{1}{2\left(\tanh\left(\frac{x}{2}\right)-1\right)}-\frac{\ln(\tanh(}{2}}{a}$

input
```
int(sinh(x)^4/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*x/a+1/24/a*exp(3*x)-1/8/a*exp(2*x)-1/8/a*exp(x)+1/8/a*exp(-x)+1/8/a*ex
p(-2*x)-1/24/a*exp(-3*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{\sinh\left(x\right)^3+3\left(\cosh\left(x\right)^2-2\cosh\left(x\right)-1\right)\sinh\left(x\right)+6\,x}{12\,a}$$

input
```
integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/12*(sinh(x)^3 + 3*(cosh(x)^2 - 2*cosh(x) - 1)*sinh(x) + 6*x)/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 294 vs. $2(22) = 44$.

Time = 0.47 (sec) , antiderivative size = 294, normalized size of antiderivative = 9.48

$$\int \frac{\sinh^4(x)}{a+a\cosh(x)}\,dx = \frac{3x\tanh^6\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} - \frac{9x\tanh^4\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} + \frac{9x\tanh^2\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} - \frac{3x}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} - \frac{6\tanh^5\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} - \frac{16\tanh^3\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a} + \frac{6\tanh\left(\frac{x}{2}\right)}{6a\tanh^6\left(\frac{x}{2}\right)-18a\tanh^4\left(\frac{x}{2}\right)+18a\tanh^2\left(\frac{x}{2}\right)-6a}$$

input
```
integrate(sinh(x)**4/(a+a*cosh(x)),x)
```

output
```
3*x*tanh(x/2)**6/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2
 - 6*a) - 9*x*tanh(x/2)**4/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 + 18*a*ta
nh(x/2)**2 - 6*a) + 9*x*tanh(x/2)**2/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4
 + 18*a*tanh(x/2)**2 - 6*a) - 3*x/(6*a*tanh(x/2)**6 - 18*a*tanh(x/2)**4 +
18*a*tanh(x/2)**2 - 6*a) - 6*tanh(x/2)**5/(6*a*tanh(x/2)**6 - 18*a*tanh(x/
2)**4 + 18*a*tanh(x/2)**2 - 6*a) - 16*tanh(x/2)**3/(6*a*tanh(x/2)**6 - 18*
a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a) + 6*tanh(x/2)/(6*a*tanh(x/2)**6
- 18*a*tanh(x/2)**4 + 18*a*tanh(x/2)**2 - 6*a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 54 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.74

$$\int \frac{\sinh^4(x)}{a + a\cosh(x)}\, dx = -\frac{\left(3\, e^{(-x)} + 3\, e^{(-2\,x)} - 1\right)e^{(3\,x)}}{24\,a} + \frac{x}{2\,a} + \frac{3\, e^{(-x)} + 3\, e^{(-2\,x)} - e^{(-3\,x)}}{24\,a}$$

input
```
integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/24*(3*e^(-x) + 3*e^(-2*x) - 1)*e^(3*x)/a + 1/2*x/a + 1/24*(3*e^(-x) + 3
*e^(-2*x) - e^(-3*x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^4(x)}{a + a\cosh(x)}\, dx = \frac{\left(3\, e^{(2\,x)} + 3\, e^{x} - 1\right)e^{(-3\,x)} + 12\,x + e^{(3\,x)} - 3\, e^{(2\,x)} - 3\, e^{x}}{24\,a}$$

input
```
integrate(sinh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/24*((3*e^(2*x) + 3*e^x - 1)*e^(-3*x) + 12*x + e^(3*x) - 3*e^(2*x) - 3*e^
x)/a
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.90

$$\int \frac{\sinh^4(x)}{a + a\cosh(x)}\, dx = \frac{e^{-x}}{8\,a} + \frac{e^{-2\,x}}{8\,a} - \frac{e^{2\,x}}{8\,a} - \frac{e^{-3\,x}}{24\,a} + \frac{e^{3\,x}}{24\,a} + \frac{x}{2\,a} - \frac{e^{x}}{8\,a}$$

input
```
int(sinh(x)^4/(a + a*cosh(x)),x)
```

output
```
exp(-x)/(8*a) + exp(-2*x)/(8*a) - exp(2*x)/(8*a) - exp(-3*x)/(24*a) + exp(
3*x)/(24*a) + x/(2*a) - exp(x)/(8*a)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.71

$$\int \frac{\sinh^4(x)}{a + a\cosh(x)}\,dx = \frac{e^{6x} - 3e^{5x} - 3e^{4x} + 12e^{3x}x + 3e^{2x} + 3e^{x} - 1}{24e^{3x}a}$$

input
```
int(sinh(x)^4/(a+a*cosh(x)),x)
```

output
```
(e**(6*x) - 3*e**(5*x) - 3*e**(4*x) + 12*e**(3*x)*x + 3*e**(2*x) + 3*e***x
- 1)/(24*e**(3*x)*a)
```

3.157 $\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 16

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx=\frac{(a-a\cosh(x))^2}{2a^3}$$

output
```
1/2*(a-a*cosh(x))^2/a^3
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.81

$$\int \frac{\sinh^3(x)}{a+a\cosh(x)}\,dx=\frac{2\sinh^4\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Sinh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(2*Sinh[x/2]^4)/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3042, 26, 3146, 17}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3146$$

$$-\frac{\int (a-a\cosh(x))d(a\cosh(x))}{a^3}$$

$$\Big\downarrow \ 17$$

$$\frac{(a-a\cosh(x))^2}{2a^3}$$

input
```
Int[Sinh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(a - a*Cosh[x])^2/(2*a^3)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\frac{\cosh(x)^2}{2}-\cosh(x)}{a}$	16
default	$\frac{\frac{\cosh(x)^2}{2}-\cosh(x)}{a}$	16
risch	$\frac{e^{2x}}{8a}-\frac{e^{x}}{2a}-\frac{e^{-x}}{2a}+\frac{e^{-2x}}{8a}$	36

input
```
int(sinh(x)^3/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*(1/2*cosh(x)^2-cosh(x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\sinh^3(x)}{a + a\cosh(x)}\, dx = \frac{\cosh{(x)}^2 + \sinh{(x)}^2 - 4\cosh{(x)}}{4\,a}$$

input
```
integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/4*(cosh(x)^2 + sinh(x)^2 - 4*cosh(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 49 vs. 2(12) = 24.

Time = 0.30 (sec) , antiderivative size = 49, normalized size of antiderivative = 3.06

$$\int \frac{\sinh^3(x)}{a + a\cosh(x)}\, dx = \frac{4\tanh^2\left(\frac{x}{2}\right)}{a\tanh^4\left(\frac{x}{2}\right) - 2a\tanh^2\left(\frac{x}{2}\right) + a} - \frac{2}{a\tanh^4\left(\frac{x}{2}\right) - 2a\tanh^2\left(\frac{x}{2}\right) + a}$$

input
```
integrate(sinh(x)**3/(a+a*cosh(x)),x)
```

output
```
4*tanh(x/2)**2/(a*tanh(x/2)**4 - 2*a*tanh(x/2)**2 + a) - 2/(a*tanh(x/2)**4
 - 2*a*tanh(x/2)**2 + a)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. 2(15) = 30.

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.25

$$\int \frac{\sinh^3(x)}{a + a\cosh(x)}\, dx = -\frac{\left(4\,e^{(-x)} - 1\right)e^{(2\,x)}}{8\,a} - \frac{4\,e^{(-x)} - e^{(-2\,x)}}{8\,a}$$

input
```
integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*e^(-x) - 1)*e^(2*x)/a - 1/8*(4*e^(-x) - e^(-2*x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.69

$$\int \frac{\sinh^3(x)}{a + a\cosh(x)}\,dx = -\frac{(4\,e^x - 1)e^{(-2\,x)} - e^{(2\,x)} + 4\,e^x}{8\,a}$$

input
```
integrate(sinh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/8*((4*e^x - 1)*e^(-2*x) - e^(2*x) + 4*e^x)/a
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 35, normalized size of antiderivative = 2.19

$$\int \frac{\sinh^3(x)}{a + a\cosh(x)}\,dx = \frac{e^{-2\,x}}{8\,a} - \frac{e^{-x}}{2\,a} + \frac{e^{2\,x}}{8\,a} - \frac{e^{x}}{2\,a}$$

input
```
int(sinh(x)^3/(a + a*cosh(x)),x)
```

output
```
exp(-2*x)/(8*a) - exp(-x)/(2*a) + exp(2*x)/(8*a) - exp(x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.94

$$\int \frac{\sinh^3(x)}{a + a\cosh(x)}\,dx = \frac{e^{4x} - 4e^{3x} - 4e^{x} + 1}{8e^{2x}a}$$

input
```
int(sinh(x)^3/(a+a*cosh(x)),x)
```

output
```
(e**(4*x) - 4*e**(3*x) - 4*e**x + 1)/(8*e**(2*x)*a)
```

3.158 $\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 13

$$\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx = -\frac{x}{a}+\frac{\sinh(x)}{a}$$

output
```
-x/a+sinh(x)/a
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^2(x)}{a+a\cosh(x)}\,dx = \frac{2\left(-\frac{x}{2}+\frac{\sinh(x)}{2}\right)}{a}$$

input
```
Integrate[Sinh[x]^2/(a + a*Cosh[x]),x]
```

output
```
(2*(-1/2*x + Sinh[x]/2))/a
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.308$, Rules used = {3042, 25, 3161, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3161}$$

$$\frac{\sinh(x)}{a}-\frac{\int 1dx}{a}$$

$$\Big\downarrow \text{ 24}$$

$$\frac{\sinh(x)}{a}-\frac{x}{a}$$

input
```
Int[Sinh[x]^2/(a + a*Cosh[x]),x]
```

output
```
-(x/a) + Sinh[x]/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3161
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[g*((g*Cos[e + f*x])^(p - 1)/(b*f*(p - 1))), x] + Si
mp[g^2/a   Int[(g*Cos[e + f*x])^(p - 2), x], x] /; FreeQ[{a, b, e, f, g}, x
] && EqQ[a^2 - b^2, 0] && GtQ[p, 1] && IntegerQ[2*p]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.85

method	result	size
risch	$-\frac{x}{a}+\frac{{\mathrm e}^{x}}{2 a}-\frac{{\mathrm e}^{-x}}{2 a}$	24
default	$\frac{-\frac{1}{1+\tanh\left(\frac{x}{2}\right)}-\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)-1}+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{a}$	45

input
```
int(sinh(x)^2/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
-x/a+1/2/a*exp(x)-1/2/a*exp(-x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 11, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x - \sinh(x)}{a}$$

input
```
integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-(x - sinh(x))/a
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(7) = 14.

Time = 0.19 (sec) , antiderivative size = 46, normalized size of antiderivative = 3.54

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x\tanh^2\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a} + \frac{x}{a\tanh^2\left(\frac{x}{2}\right) - a} - \frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a}$$

input
```
integrate(sinh(x)**2/(a+a*cosh(x)),x)
```

output
```
-x*tanh(x/2)**2/(a*tanh(x/2)**2 - a) + x/(a*tanh(x/2)**2 - a) - 2*tanh(x/2
)/(a*tanh(x/2)**2 - a)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = -\frac{x}{a} - \frac{e^{(-x)}}{2a} + \frac{e^x}{2a}$$

input
```
integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-x/a - 1/2*e^(-x)/a + 1/2*e^x/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = -\frac{2\,x + e^{(-x)} - e^{x}}{2\,a}$$

input
```
integrate(sinh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/2*(2*x + e^(-x) - e^x)/a
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = \frac{e^{x}}{2\,a} - \frac{x}{a} - \frac{e^{-x}}{2\,a}$$

input
```
int(sinh(x)^2/(a + a*cosh(x)),x)
```

output
```
exp(x)/(2*a) - x/a - exp(-x)/(2*a)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.77

$$\int \frac{\sinh^2(x)}{a + a\cosh(x)}\,dx = \frac{e^{2x} - 2e^{x}x - 1}{2e^{x}a}$$

input
```
int(sinh(x)^2/(a+a*cosh(x)),x)
```

output
```
(e**(2*x) - 2*e**x*x - 1)/(2*e**x*a)
```

3.159 $\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 9

$$\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx = \frac{\log(1+\cosh(x))}{a}$$

output
```
ln(1+cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

$$\int \frac{\sinh(x)}{a+a\cosh(x)}\,dx = \frac{2\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a}$$

input
```
Integrate[Sinh[x]/(a + a*Cosh[x]),x]
```

output
```
(2*Log[Cosh[x/2]])/a
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3146, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$\frac{\int \frac{1}{\cosh(x)a+a}d(a\cosh(x))}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a\cosh(x)+a)}{a}$$

input
```
Int[Sinh[x]/(a + a*Cosh[x]),x]
```

output
```
Log[a + a*Cosh[x]]/a
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\frac{\ln(a+\cosh(x)a)}{a}$	12
default	$\frac{\ln(a+\cosh(x)a)}{a}$	12
risch	$-\frac{x}{a}+\frac{2\ln(e^{x}+1)}{a}$	18

input
```
int(sinh(x)/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
ln(a+cosh(x)*a)/a
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.78

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = -\frac{x - 2\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-(x - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 7, normalized size of antiderivative = 0.78

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = \frac{\log\left(\cosh\left(x\right) + 1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x)
```

output
```
log(cosh(x) + 1)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.22

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = \frac{\log\left(a\cosh\left(x\right) + a\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
log(a*cosh(x) + a)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.89

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = -\frac{x}{a} + \frac{2\,\log\left(e^{x} + 1\right)}{a}$$

input
```
integrate(sinh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-x/a + 2*log(e^x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = \frac{\ln\left(\cosh(x) + 1\right)}{a}$$

input
```
int(sinh(x)/(a + a*cosh(x)),x)
```

output
```
log(cosh(x) + 1)/a
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 9, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + a\cosh(x)}\,dx = \frac{\log(\cosh\left(x\right) + 1)}{a}$$

input
```
int(sinh(x)/(a+a*cosh(x)),x)
```

output
```
log(cosh(x) + 1)/a
```

3.160 $\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 23

$$\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))}{2a} + \frac{1}{2(a+a\cosh(x))}$$

output
```
-1/2*arctanh(cosh(x))/a+1/(2*a+2*a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{csch}(x)}{a+a\cosh(x)}\,dx = \frac{1-2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]/(a + a*Cosh[x]),x]
```

output
```
(1 - 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(2*a*(1 + Cosh[x]))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.26, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 26, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$-a\int \frac{1}{(a-a\cosh(x))(\cosh(x)a+a)^2}d(a\cosh(x))$$

$$\Big\downarrow \text{54}$$

$$-a\int \left(\frac{1}{2\left(a^2-a^2\cosh^2(x)\right)a}+\frac{1}{2(\cosh(x)a+a)^2a}\right)d(a\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$-a\left(\frac{\operatorname{arctanh}(\cosh(x))}{2a^2}-\frac{1}{2a(a\cosh(x)+a)}\right)$$

input
```
Int[Csch[x]/(a + a*Cosh[x]),x]
```

output
```
-(a*(ArcTanh[Cosh[x]]/(2*a^2) - 1/(2*a*(a + a*Cosh[x]))))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$\frac{e^{x}}{\left(e^{x}+1\right)^{2} a}+\frac{\ln\left(e^{x}-1\right)}{2a}-\frac{\ln\left(e^{x}+1\right)}{2a}$	34

input
```
int(csch(x)/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(-1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(19) = 38.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 4.48

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = -\frac{\left(\cosh\left(x\right)^2 + 2\left(\cosh\left(x\right) + 1\right)\sinh\left(x\right) + \sinh\left(x\right)^2 + 2\,\cosh\left(x\right) + 1\right)\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right) - (\mathrm{co}}{2\left(a\cosh\left(x\right)^2 + a\sinh\left(x\right)^2 + \right.}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) - 2*cosh(x) - 2*sinh(x))/(a*
cosh(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(19) = 38$.

Time = 0.03 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.04

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = \frac{e^{(-x)}}{2\,ae^{(-x)} + ae^{(-2\,x)} + a} - \frac{\log\left(e^{(-x)} + 1\right)}{2\,a} + \frac{\log\left(e^{(-x)} - 1\right)}{2\,a}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^(
-x) - 1)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 52 vs. $2(19) = 38$.

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.26

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = -\frac{\log\left(e^{(-x)} + e^{x} + 2\right)}{4\,a} + \frac{\log\left(e^{(-x)} + e^{x} - 2\right)}{4\,a} + \frac{e^{(-x)} + e^{x} + 6}{4\,a(e^{(-x)} + e^{x} + 2)}$$

input
```
integrate(csch(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
 e^x + 6)/(a*(e^(-x) + e^x + 2))
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.22

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx = \frac{1}{a\,\left(\mathrm{e}^{x} + 1\right)} - \frac{1}{a\,\left(\mathrm{e}^{2\,x} + 2\,\mathrm{e}^{x} + 1\right)} - \frac{\operatorname{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

input
```
int(1/(sinh(x)*(a + a*cosh(x))),x)
```

output
```
1/(a*(exp(x) + 1)) - 1/(a*(exp(2*x) + 2*exp(x) + 1)) - atan((exp(x)*(-a^2)
^(1/2))/a)/(-a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.87

$$\int \frac{\operatorname{csch}(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{e^{2x}\log(e^x-1) - e^{2x}\log(e^x+1) - e^{2x} + 2e^x\log(e^x-1) - 2e^x\log(e^x+1) + \log(e^x-1) - \log(e^x+1) - 1}{2a\,(e^{2x}+2e^x+1)}$$

input
```
int(csch(x)/(a+a*cosh(x)),x)
```

output
```
(e**(2*x)*log(e**x - 1) - e**(2*x)*log(e**x + 1) - e**(2*x) + 2*e**x*log(e
**x - 1) - 2*e**x*log(e**x + 1) + log(e**x - 1) - log(e**x + 1) - 1)/(2*a*
(e**(2*x) + 2*e**x + 1))
```

3.161 $\int \frac{\text{csch}^2(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\text{csch}^2(x)}{a+a\cosh(x)}\,dx=-\frac{2\coth(x)}{3a}+\frac{\text{csch}(x)}{3(a+a\cosh(x))}$$

output
```
-2/3*coth(x)/a+csch(x)/(3*a+3*a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.25

$$\int \frac{\text{csch}^2(x)}{a+a\cosh(x)}\,dx=-\frac{(2\cosh(x)+\cosh(2x))\text{csch}\left(\frac{x}{2}\right)\text{sech}^3\left(\frac{x}{2}\right)}{12a}$$

input
```
Integrate[Csch[x]^2/(a + a*Cosh[x]),x]
```

output
```
-1/12*((2*Cosh[x] + Cosh[2*x])*Csch[x/2]*Sech[x/2]^3)/a
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 25, 3151, 25, 3042, 25, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^2\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\int -\operatorname{csch}^2(x)dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{2\int \operatorname{csch}^2(x)dx}{3a}+\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}+\frac{2\int -\csc(ix)^2dx}{3a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\int \csc(ix)^2dx}{3a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2i\int 1d(-i\coth(x))}{3a}$$

$$\Big\downarrow \text{24}$$

$$\frac{\operatorname{csch}(x)}{3(a\cosh(x)+a)}-\frac{2\coth(x)}{3a}$$

input
```
Int[Csch[x]^2/(a + a*Cosh[x]),x]
```

output
```
(-2*Coth[x])/(3*a) + Csch[x]/(3*(a + a*Cosh[x]))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

method	result	size
risch	$-\frac{4(2\,\mathrm{e}^{x}+1)}{3(\mathrm{e}^{x}+1)^{3}a(\mathrm{e}^{x}-1)}$	24
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}-2\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	29

input
```
int(csch(x)^2/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
-4/3*(2*exp(x)+1)/(exp(x)+1)^3/a/(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(20) = 40$.

Time = 0.08 (sec) , antiderivative size = 94, normalized size of antiderivative = 3.92

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx =$$

$$-\frac{4\,(2\,\cosh{(x)}+2\,\sinh{(x)}+}{3\,\left(a\cosh{(x)}^{4}+a\sinh{(x)}^{4}+2\,a\cosh{(x)}^{3}+2\,(2\,a\cosh{(x)}+a)\sinh{(x)}^{3}+6\,\left(a\cosh{(x)}^{2}+a\cosh{(x)}\right.\right.}$$

input
```
integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-4/3*(2*cosh(x) + 2*sinh(x) + 1)/(a*cosh(x)^4 + a*sinh(x)^4 + 2*a*cosh(x)^
3 + 2*(2*a*cosh(x) + a)*sinh(x)^3 + 6*(a*cosh(x)^2 + a*cosh(x))*sinh(x)^2
- 2*a*cosh(x) + 2*(2*a*cosh(x)^3 + 3*a*cosh(x)^2 - a)*sinh(x) - a)
```

Sympy [F]

$$\int \frac{\text{csch}^2(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\text{csch}^2(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**2/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(20) = 40$.

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.46

$$\int \frac{\text{csch}^2(x)}{a + a\cosh(x)}\,dx = -\frac{8\,e^{(-x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)} - \frac{4}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)}$$

input
```
integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-8/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 4/3/(2*a*e^(-x)
 - 2*a*e^(-3*x) - a*e^(-4*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.46

$$\int \frac{\text{csch}^2(x)}{a + a\cosh(x)}\,dx = -\frac{1}{2\,a(e^x - 1)} + \frac{3\,e^{(2\,x)} + 12\,e^x + 5}{6\,a(e^x + 1)^3}$$

input
```
integrate(csch(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/2/(a*(e^x - 1)) + 1/6*(3*e^(2*x) + 12*e^x + 5)/(a*(e^x + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 89, normalized size of antiderivative = 3.71

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx=\frac{\frac{e^{2\,x}}{6\,a}+\frac{1}{6\,a}+\frac{e^x}{a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^x+1}+\frac{\frac{1}{2\,a}+\frac{e^x}{6\,a}}{e^{2\,x}+2\,e^x+1}-\frac{1}{2\,a\,\left(e^x-1\right)}+\frac{1}{6\,a\,\left(e^x+1\right)}$$

input
```
int(1/(sinh(x)^2*(a + a*cosh(x))),x)
```

output
```
(exp(2*x)/(6*a) + 1/(6*a) + exp(x)/a)/(3*exp(2*x) + exp(3*x) + 3*exp(x) +
1) + (1/(2*a) + exp(x)/(6*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(exp(x) -
 1)) + 1/(6*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{csch}^2(x)}{a+a\cosh(x)}\,dx=\frac{-\frac{8e^x}{3}-\frac{4}{3}}{a\left(e^{4x}+2e^{3x}-2e^x-1\right)}$$

input
```
int(csch(x)^2/(a+a*cosh(x)),x)
```

output
```
(4*( - 2*e**x - 1))/(3*a*(e**(4*x) + 2*e**(3*x) - 2*e**x - 1))
```

3.162 $\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 55

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\operatorname{arctanh}(\cosh(x))}{8a}+\frac{1}{8(a-a\cosh(x))}-\frac{1}{4(a+a\cosh(x))}-\frac{a^3}{8\left(a^2+a^2\cosh(x)\right)^2}$$

output
```
3/8*arctanh(cosh(x))/a+1/(8*a-8*a*cosh(x))-1/(4*a+4*a*cosh(x))-1/8*a^3/(a^
2+a^2*cosh(x))^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = -\frac{4+2\coth^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+\operatorname{sech}^2\left(\frac{x}{2}\right)}{16a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]^3/(a + a*Cosh[x]),x]
```

output
```
-1/16*(4 + 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]
) + Sech[x/2]^2)/(a*(1 + Cosh[x]))
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.11, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^3\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^3\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$a^3\int \frac{1}{(a-a\cosh(x))^2(\cosh(x)a+a)^3}d(a\cosh(x))$$

$$\Big\downarrow \text{54}$$

$$a^3\int \left(\frac{1}{8a^3(a-a\cosh(x))^2}+\frac{1}{4a^3(\cosh(x)a+a)^2}+\frac{1}{4a^2(\cosh(x)a+a)^3}+\frac{3}{8a^3\left(a^2-a^2\cosh^2(x)\right)}\right)d(a\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$a^3\left(\frac{3\text{arctanh}(\cosh(x))}{8a^4}+\frac{1}{8a^3(a-a\cosh(x))}-\frac{1}{4a^3(a\cosh(x)+a)}-\frac{1}{8a^2(a\cosh(x)+a)^2}\right)$$

input
```
Int[Csch[x]^3/(a + a*Cosh[x]),x]
```

output
```
a^3*((3*ArcTanh[Cosh[x]])/(8*a^4) + 1/(8*a^3*(a - a*Cosh[x])) - 1/(8*a^2*(
a + a*Cosh[x])^2) - 1/(4*a^3*(a + a*Cosh[x])))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 2.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.69

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{4}}{4}+\frac{3\tanh\left(\frac{x}{2}\right)^{2}}{2}-\frac{1}{2\tanh\left(\frac{x}{2}\right)^{2}}-3\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{8a}$	38
risch	$-\frac{e^{x}\left(3\,e^{4x}+6\,e^{3x}-2\,e^{2x}+6\,e^{x}+3\right)}{4\left(e^{x}+1\right)^{4}a\left(e^{x}-1\right)^{2}}-\frac{3\ln\left(e^{x}-1\right)}{8a}+\frac{3\ln\left(e^{x}+1\right)}{8a}$	65

input
```
int(csch(x)^3/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(-1/4*tanh(1/2*x)^4+3/2*tanh(1/2*x)^2-1/2/tanh(1/2*x)^2-3*ln(tanh(1/
2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(48) = 96$.

Time = 0.08 (sec) , antiderivative size = 631, normalized size of antiderivative = 11.47

$$\int \frac{\operatorname{csch}^3(x)}{a + a\cosh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/8*(6*cosh(x)^5 + 6*(5*cosh(x) + 2)*sinh(x)^4 + 6*sinh(x)^5 + 12*cosh(x)
^4 + 4*(15*cosh(x)^2 + 12*cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + 12*(5*cos
h(x)^3 + 6*cosh(x)^2 - cosh(x) + 1)*sinh(x)^2 + 12*cosh(x)^2 - 3*(cosh(x)^
6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2
+ 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - c
osh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cos
h(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x
)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) + 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + si
nh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x
)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3
+ (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 -
 cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - co
sh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 6*(5*cosh
(x)^4 + 8*cosh(x)^3 - 2*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x) + 6*cosh(x))/(a
*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)^5 -
 a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cosh(x)
^3 + 4*(5*a*cosh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*cosh(
x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x) - a
)*sinh(x)^2 + 2*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*cosh...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**3/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(48) = 96$.

Time = 0.03 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{csch}^3(x)}{a + a\cosh(x)}\,dx = -\frac{3\,e^{(-x)} + 6\,e^{(-2\,x)} - 2\,e^{(-3\,x)} + 6\,e^{(-4\,x)} + 3\,e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} + \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a} - \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a}$$

input
```
integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/4*(3*e^(-x) + 6*e^(-2*x) - 2*e^(-3*x) + 6*e^(-4*x) + 3*e^(-5*x))/(2*a*e
^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x)
 + a) + 3/8*log(e^(-x) + 1)/a - 3/8*log(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\log\left(e^{(-x)}+e^x+2\right)}{16\,a} - \frac{3\log\left(e^{(-x)}+e^x-2\right)}{16\,a} + \frac{3\,e^{(-x)}+3\,e^x-10}{16\,a(e^{(-x)}+e^x-2)} - \frac{9\left(e^{(-x)}+e^x\right)^2+52\,e^{(-x)}+52\,e^x+84}{32\,a(e^{(-x)}+e^x+2)^2}$$

input
```
integrate(csch(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3/16*log(e^(-x) + e^x + 2)/a - 3/16*log(e^(-x) + e^x - 2)/a + 1/16*(3*e^(-
x) + 3*e^x - 10)/(a*(e^(-x) + e^x - 2)) - 1/32*(9*(e^(-x) + e^x)^2 + 52*e^
(-x) + 52*e^x + 84)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.07

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx = \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}} - \frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^x+1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^x-1\right)} - \frac{1}{2\,a\,\left(\mathrm{e}^x+1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^x+1\right)} + \frac{1}{a\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^x+1\right)}$$

input
```
int(1/(sinh(x)^3*(a + a*cosh(x))),x)
```

output
```
(3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^(1/2)) - 1/(2*a*(6*exp(2*x) +
4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x) - 1)) - 1/(2*a*(ex
p(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/(a*(3*exp(2*x) + exp(3*
x) + 3*exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 245, normalized size of antiderivative = 4.45

$$\int \frac{\operatorname{csch}^3(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{-3e^{6x}\log(e^x-1)+3e^{6x}\log(e^x+1)+3e^{6x}-6e^{5x}\log(e^x-1)+6e^{5x}\log(e^x+1)+3e^{4x}\log(e^x-1)-3e^{4}}{}$$

input
```
int(csch(x)^3/(a+a*cosh(x)),x)
```

output
```
( - 3*e**(6*x)*log(e**x - 1) + 3*e**(6*x)*log(e**x + 1) + 3*e**(6*x) - 6*e
**(5*x)*log(e**x - 1) + 6*e**(5*x)*log(e**x + 1) + 3*e**(4*x)*log(e**x - 1
) - 3*e**(4*x)*log(e**x + 1) - 15*e**(4*x) + 12*e**(3*x)*log(e**x - 1) - 1
2*e**(3*x)*log(e**x + 1) - 8*e**(3*x) + 3*e**(2*x)*log(e**x - 1) - 3*e**(2
*x)*log(e**x + 1) - 15*e**(2*x) - 6*e**x*log(e**x - 1) + 6*e**x*log(e**x +
 1) - 3*log(e**x - 1) + 3*log(e**x + 1) + 3)/(8*a*(e**(6*x) + 2*e**(5*x) -
 e**(4*x) - 4*e**(3*x) - e**(2*x) + 2*e**x + 1))
```

3.163 $\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx=\frac{4\coth(x)}{5a}-\frac{4\coth^3(x)}{15a}+\frac{\operatorname{csch}^3(x)}{5(a+a\cosh(x))}$$

output
```
4/5*coth(x)/a-4/15*coth(x)^3/a+csch(x)^3/(5*a+5*a*cosh(x))
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx=\frac{(-6\cosh(x)-2\cosh(2x)+2\cosh(3x)+\cosh(4x))\operatorname{csch}^3(x)}{15a(1+\cosh(x))}$$

input
```
Integrate[Csch[x]^4/(a + a*Cosh[x]),x]
```

output
```
((-6*Cosh[x] - 2*Cosh[2*x] + 2*Cosh[3*x] + Cosh[4*x])*Csch[x]^3)/(15*a*(1
+ Cosh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.16, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 3151, 3042, 4254, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3151}$$

$$\frac{4\int \text{csch}^4(x)dx}{5a}+\frac{\text{csch}^3(x)}{5(a\cosh(x)+a)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{csch}^3(x)}{5(a\cosh(x)+a)}+\frac{4\int \csc(ix)^4dx}{5a}$$

$$\Big\downarrow \text{4254}$$

$$\frac{\text{csch}^3(x)}{5(a\cosh(x)+a)}+\frac{4i\int\left(1-\coth^2(x)\right)d(-i\coth(x))}{5a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\text{csch}^3(x)}{5(a\cosh(x)+a)}+\frac{4i\left(\frac{1}{3}i\coth^3(x)-i\coth(x)\right)}{5a}$$

input
```
Int[Csch[x]^4/(a + a*Cosh[x]),x]
```

output
```
(((4*I)/5)*((-I)*Coth[x] + (I/3)*Coth[x]^3))/a + Csch[x]^3/(5*(a + a*Cosh[
x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3151
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[b*(g*Cos[e + f*x])^(p + 1)*((a + b*Sin[e + f*x
])^m/(a*f*g*Simplify[2*m + p + 1])), x] + Simp[Simplify[m + p + 1]/(a*Simpl
ify[2*m + p + 1])   Int[(g*Cos[e + f*x])^p*(a + b*Sin[e + f*x])^(m + 1), x]
, x] /; FreeQ[{a, b, e, f, g, m, p}, x] && EqQ[a^2 - b^2, 0] && ILtQ[Simpli
fy[m + p + 1], 0] && NeQ[2*m + p + 1, 0] &&  !IGtQ[m, 0]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 5.17 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.97

method	result	size
risch	$-\frac{16\left(6\,{\mathrm e}^{3x}+2\,{\mathrm e}^{2x}-2\,{\mathrm e}^{x}-1\right)}{15\left({\mathrm e}^{x}+1\right)^{5} a \left({\mathrm e}^{x}-1\right)^{3}}$	36
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{5}}{5}-\frac{4\tanh\left(\frac{x}{2}\right)^{3}}{3}+6\tanh\left(\frac{x}{2}\right)-\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}+\frac{4}{\tanh\left(\frac{x}{2}\right)}}{16a}$	45

input
```
int(csch(x)^4/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
-16/15*(6*exp(3*x)+2*exp(2*x)-2*exp(x)-1)/(exp(x)+1)^5/a/(exp(x)-1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(31) = 62$.

Time = 0.08 (sec) , antiderivative size = 250, normalized size of antiderivative = 6.76

$$\int \frac{\operatorname{csch}^4(x)}{a + a\cosh(x)}\,dx =$$

$$-\frac{}{15\left(a\cosh\left(x\right)^{7} + a\sinh\left(x\right)^{7} + 2\,a\cosh\left(x\right)^{6} + \left(7\,a\cosh\left(x\right) + 2\,a\right)\sinh\left(x\right)^{6} - 2\,a\cosh\left(x\right)^{5} + \left(21\,a\cosh\right.\right.}$$

input
```
integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-16/15*(6*cosh(x)^2 + 3*(4*cosh(x) + 1)*sinh(x) + 6*sinh(x)^2 + cosh(x) -
2)/(a*cosh(x)^7 + a*sinh(x)^7 + 2*a*cosh(x)^6 + (7*a*cosh(x) + 2*a)*sinh(x
)^6 - 2*a*cosh(x)^5 + (21*a*cosh(x)^2 + 12*a*cosh(x) - 2*a)*sinh(x)^5 - 6*
a*cosh(x)^4 + (35*a*cosh(x)^3 + 30*a*cosh(x)^2 - 10*a*cosh(x) - 6*a)*sinh(
x)^4 + (35*a*cosh(x)^4 + 40*a*cosh(x)^3 - 20*a*cosh(x)^2 - 24*a*cosh(x))*s
inh(x)^3 + 6*a*cosh(x)^2 + (21*a*cosh(x)^5 + 30*a*cosh(x)^4 - 20*a*cosh(x)
^3 - 36*a*cosh(x)^2 + 6*a)*sinh(x)^2 + a*cosh(x) + (7*a*cosh(x)^6 + 12*a*c
osh(x)^5 - 10*a*cosh(x)^4 - 24*a*cosh(x)^3 + 12*a*cosh(x) + 3*a)*sinh(x) -
 2*a)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**4/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(31) = 62$.

Time = 0.03 (sec) , antiderivative size = 233, normalized size of antiderivative = 6.30

$$\int \frac{\operatorname{csch}^4(x)}{a + a\cosh(x)}\,dx = \frac{32\,e^{(-x)}}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{32\,e^{(-2\,x)}}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} - \frac{32\,e^{(-3\,x)}}{5\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)} + \frac{16}{15\,(2\,ae^{(-x)} - 2\,ae^{(-2\,x)} - 6\,ae^{(-3\,x)} + 6\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 2\,ae^{(-7\,x)} - ae^{(-8\,x)} + a)}$$

input
```
integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
32/15*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*
a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 32/15*e^(-2*x)/(2*a*e^(-x) -
 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x)
- a*e^(-8*x) + a) - 32/5*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x
) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 16/15/(
2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2
*a*e^(-7*x) - a*e^(-8*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.59

$$\int \frac{\operatorname{csch}^4(x)}{a + a\cosh(x)}\,dx = \frac{9\,e^{(2\,x)} - 24\,e^x + 11}{24\,a(e^x - 1)^3} - \frac{45\,e^{(4\,x)} + 240\,e^{(3\,x)} + 490\,e^{(2\,x)} + 320\,e^x + 73}{120\,a(e^x + 1)^5}$$

input
```
integrate(csch(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
1/24*(9*e^(2*x) - 24*e^x + 11)/(a*(e^x - 1)^3) - 1/120*(45*e^(4*x) + 240*e
^(3*x) + 490*e^(2*x) + 320*e^x + 73)/(a*(e^x + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 263, normalized size of antiderivative = 7.11

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{1}{6\,a\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}-\frac{\frac{3\,e^{2\,x}}{8\,a}+\frac{3\,e^{3\,x}}{40\,a}+\frac{1}{8\,a}+\frac{5\,e^{x}}{8\,a}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1} - \frac{\frac{3\,e^{2\,x}}{40\,a}+\frac{5}{24\,a}+\frac{e^{x}}{4\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{\frac{1}{8\,a}+\frac{3\,e^{x}}{40\,a}}{e^{2\,x}+2\,e^{x}+1} - \frac{\frac{5\,e^{2\,x}}{4\,a}+\frac{e^{3\,x}}{2\,a}+\frac{3\,e^{4\,x}}{40\,a}+\frac{3}{40\,a}+\frac{e^{x}}{2\,a}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^{x}+1} - \frac{1}{4\,a\,\left(e^{2\,x}-2\,e^{x}+1\right)}+\frac{3}{8\,a\,\left(e^{x}-1\right)}-\frac{3}{40\,a\,\left(e^{x}+1\right)}$$

input
```
int(1/(sinh(x)^4*(a + a*cosh(x))),x)
```

output
```
1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (3*
exp(3*x))/(40*a) + 1/(8*a) + (5*exp(x))/(8*a))/(6*exp(2*x) + 4*exp(3*x) +
exp(4*x) + 4*exp(x) + 1) - ((3*exp(2*x))/(40*a) + 5/(24*a) + exp(x)/(4*a))
/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (3*exp(x))/(40*a))/(e
xp(2*x) + 2*exp(x) + 1) - ((5*exp(2*x))/(4*a) + exp(3*x)/(2*a) + (3*exp(4*
x))/(40*a) + 3/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x) + 5*exp(4
*x) + exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 3/(8*
a*(exp(x) - 1)) - 3/(40*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 2.03

$$\int \frac{\operatorname{csch}^4(x)}{a+a\cosh(x)}\,dx = \frac{-\frac{32e^{3x}}{5}-\frac{32e^{2x}}{15}+\frac{32e^{x}}{15}+\frac{16}{15}}{a\left(e^{8x}+2e^{7x}-2e^{6x}-6e^{5x}+6e^{3x}+2e^{2x}-2e^{x}-1\right)}$$

input
```
int(csch(x)^4/(a+a*cosh(x)),x)
```

output
```
(16*( - 6*e**(3*x) - 2*e**(2*x) + 2*e**x + 1))/(15*a*(e**(8*x) + 2*e**(7*x
) - 2*e**(6*x) - 6*e**(5*x) + 6*e**(3*x) + 2*e**(2*x) - 2*e**x - 1))
```

3.164 $\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = -\frac{5\operatorname{arctanh}(\cosh(x))}{16a} - \frac{1}{8(a-a\cosh(x))} + \frac{3}{16(a+a\cosh(x))} + \frac{a^5}{24\left(a^2+a^2\cosh(x)\right)^3} - \frac{a^5}{32\left(a^3-a^3\cosh(x)\right)^2} + \frac{3a^5}{32\left(a^3+a^3\cosh(x)\right)^2}$$

output
```
-5/16*arctanh(cosh(x))/a-1/(8*a-8*a*cosh(x))+3/(16*a+16*a*cosh(x))+1/24*a^
5/(a^2+a^2*cosh(x))^3-1/32*a^5/(a^3-a^3*cosh(x))^2+3/32*a^5/(a^3+a^3*cosh(
x))^2
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = \frac{\cosh^2\left(\frac{x}{2}\right)\left(24\operatorname{csch}^2\left(\frac{x}{2}\right)-3\operatorname{csch}^4\left(\frac{x}{2}\right)-120\log\left(\cosh\left(\frac{x}{2}\right)\right)+120\log\left(\sinh\left(\frac{x}{2}\right)\right)+36\operatorname{sech}^2\left(\frac{x}{2}\right)+9\operatorname{sech}^4\left(\frac{x}{2}\right)\right.}{192(a+a\cosh(x))}$$

input
```
Integrate[Csch[x]^5/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]^2*(24*Csch[x/2]^2 - 3*Csch[x/2]^4 - 120*Log[Cosh[x/2]] + 120*Lo
g[Sinh[x/2]] + 36*Sech[x/2]^2 + 9*Sech[x/2]^4 + 2*Sech[x/2]^6))/(192*(a +
a*Cosh[x]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3146, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^5(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^5\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^5\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3146}$$

$$-a^5\int \frac{1}{(a-a\cosh(x))^3(\cosh(x)a+a)^4}d(a\cosh(x))$$

$$\Big\downarrow \text{54}$$

$$-a^5\int \left(\frac{1}{8a^5(a-a\cosh(x))^2}+\frac{3}{16a^5(\cosh(x)a+a)^2}+\frac{1}{16a^4(a-a\cosh(x))^3}+\frac{3}{16a^4(\cosh(x)a+a)^3}+\frac{}{8a^3(\cos$$

$$\Big\downarrow \text{2009}$$

$$-a^5\left(\frac{5\text{arctanh}(\cosh(x))}{16a^6}+\frac{1}{8a^5(a-a\cosh(x))}-\frac{3}{16a^5(a\cosh(x)+a)}+\frac{1}{32a^4(a-a\cosh(x))^2}-\frac{3}{32a^4(a\cosh(x)$$

input
```
Int[Csch[x]^5/(a + a*Cosh[x]),x]
```

output
```
-(a^5*((5*ArcTanh[Cosh[x]])/(16*a^6) + 1/(32*a^4*(a - a*Cosh[x])^2) + 1/(8
*a^5*(a - a*Cosh[x])) - 1/(24*a^3*(a + a*Cosh[x])^3) - 3/(32*a^4*(a + a*Co
sh[x])^2) - 3/(16*a^5*(a + a*Cosh[x]))))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3146
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^(m + (p - 1)/2)*(a - x
)^((p - 1)/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && I
ntegerQ[(p - 1)/2] && EqQ[a^2 - b^2, 0] && (GeQ[p, -1] ||  !IntegerQ[m + 1/
2])
```

Maple [A] (verified)

Time = 10.96 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.57

method	result	size
default	$\frac{-\frac{\tanh\left(\frac{x}{2}\right)^{6}}{6}+\frac{5\tanh\left(\frac{x}{2}\right)^{4}}{4}-5\tanh\left(\frac{x}{2}\right)^{2}+\frac{5}{2\tanh\left(\frac{x}{2}\right)^{2}}-\frac{1}{4\tanh\left(\frac{x}{2}\right)^{4}}+10\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{32a}$	54
risch	$\frac{e^{x}\left(15\,e^{8x}+30\,e^{7x}-40\,e^{6x}-110\,e^{5x}+18\,e^{4x}-110\,e^{3x}-40\,e^{2x}+30\,e^{x}+15\right)}{24(e^{x}+1)^{6}a(e^{x}-1)^{4}}+\frac{5\ln(e^{x}-1)}{16a}-\frac{5\ln(e^{x}+1)}{16a}$	89

input
```
int(csch(x)^5/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/32/a*(-1/6*tanh(1/2*x)^6+5/4*tanh(1/2*x)^4-5*tanh(1/2*x)^2+5/2/tanh(1/2*
x)^2-1/4/tanh(1/2*x)^4+10*ln(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1551 vs. 2(84) = 168.

Time = 0.10 (sec) , antiderivative size = 1551, normalized size of antiderivative = 16.50

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx=\text{Too large to display}$$

input
```
integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
1/48*(30*cosh(x)^9 + 30*(9*cosh(x) + 2)*sinh(x)^8 + 30*sinh(x)^9 + 60*cosh
(x)^8 + 40*(27*cosh(x)^2 + 12*cosh(x) - 2)*sinh(x)^7 - 80*cosh(x)^7 + 20*(
126*cosh(x)^3 + 84*cosh(x)^2 - 28*cosh(x) - 11)*sinh(x)^6 - 220*cosh(x)^6
+ 12*(315*cosh(x)^4 + 280*cosh(x)^3 - 140*cosh(x)^2 - 110*cosh(x) + 3)*sin
h(x)^5 + 36*cosh(x)^5 + 20*(189*cosh(x)^5 + 210*cosh(x)^4 - 140*cosh(x)^3
- 165*cosh(x)^2 + 9*cosh(x) - 11)*sinh(x)^4 - 220*cosh(x)^4 + 40*(63*cosh(
x)^6 + 84*cosh(x)^5 - 70*cosh(x)^4 - 110*cosh(x)^3 + 9*cosh(x)^2 - 22*cosh
(x) - 2)*sinh(x)^3 - 80*cosh(x)^3 + 60*(18*cosh(x)^7 + 28*cosh(x)^6 - 28*c
osh(x)^5 - 55*cosh(x)^4 + 6*cosh(x)^3 - 22*cosh(x)^2 - 4*cosh(x) + 1)*sinh
(x)^2 + 60*cosh(x)^2 - 15*(cosh(x)^10 + 2*(5*cosh(x) + 1)*sinh(x)^9 + sinh
(x)^10 + 2*cosh(x)^9 + 3*(15*cosh(x)^2 + 6*cosh(x) - 1)*sinh(x)^8 - 3*cosh
(x)^8 + 8*(15*cosh(x)^3 + 9*cosh(x)^2 - 3*cosh(x) - 1)*sinh(x)^7 - 8*cosh(
x)^7 + 2*(105*cosh(x)^4 + 84*cosh(x)^3 - 42*cosh(x)^2 - 28*cosh(x) + 1)*si
nh(x)^6 + 2*cosh(x)^6 + 12*(21*cosh(x)^5 + 21*cosh(x)^4 - 14*cosh(x)^3 - 1
4*cosh(x)^2 + cosh(x) + 1)*sinh(x)^5 + 12*cosh(x)^5 + 2*(105*cosh(x)^6 + 1
26*cosh(x)^5 - 105*cosh(x)^4 - 140*cosh(x)^3 + 15*cosh(x)^2 + 30*cosh(x) +
 1)*sinh(x)^4 + 2*cosh(x)^4 + 8*(15*cosh(x)^7 + 21*cosh(x)^6 - 21*cosh(x)^
5 - 35*cosh(x)^4 + 5*cosh(x)^3 + 15*cosh(x)^2 + cosh(x) - 1)*sinh(x)^3 - 8
*cosh(x)^3 + 3*(15*cosh(x)^8 + 24*cosh(x)^7 - 28*cosh(x)^6 - 56*cosh(x)^5
+ 10*cosh(x)^4 + 40*cosh(x)^3 + 4*cosh(x)^2 - 8*cosh(x) - 1)*sinh(x)^2 ...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = \frac{\int \frac{\operatorname{csch}^5(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(csch(x)**5/(a+a*cosh(x)),x)
```

output
```
Integral(csch(x)**5/(cosh(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx$$
$$= \frac{15\,e^{(-x)} + 30\,e^{(-2\,x)} - 40\,e^{(-3\,x)} - 110\,e^{(-4\,x)} + 18\,e^{(-5\,x)} - 110\,e^{(-6\,x)} - 40\,e^{(-7\,x)} + 30\,e^{(-8\,x)} +}{24\,(2\,ae^{(-x)} - 3\,ae^{(-2\,x)} - 8\,ae^{(-3\,x)} + 2\,ae^{(-4\,x)} + 12\,ae^{(-5\,x)} + 2\,ae^{(-6\,x)} - 8\,ae^{(-7\,x)} - 3\,ae^{(-8\,x)} + 2\,ae^{(-9}}$$
$$- \frac{5\,\log\left(e^{(-x)}+1\right)}{16\,a} + \frac{5\,\log\left(e^{(-x)}-1\right)}{16\,a}$$

input
```
integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
1/24*(15*e^(-x) + 30*e^(-2*x) - 40*e^(-3*x) - 110*e^(-4*x) + 18*e^(-5*x) -
 110*e^(-6*x) - 40*e^(-7*x) + 30*e^(-8*x) + 15*e^(-9*x))/(2*a*e^(-x) - 3*a
*e^(-2*x) - 8*a*e^(-3*x) + 2*a*e^(-4*x) + 12*a*e^(-5*x) + 2*a*e^(-6*x) - 8
*a*e^(-7*x) - 3*a*e^(-8*x) + 2*a*e^(-9*x) + a*e^(-10*x) + a) - 5/16*log(e^
(-x) + 1)/a + 5/16*log(e^(-x) - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx$$
$$= -\frac{5\,\log\left(e^{(-x)}+e^x+2\right)}{32\,a} + \frac{5\,\log\left(e^{(-x)}+e^x-2\right)}{32\,a}$$
$$- \frac{15\left(e^{(-x)}+e^x\right)^2 - 76\,e^{(-x)} - 76\,e^x + 100}{64\,a(e^{(-x)}+e^x-2)^2}$$
$$+ \frac{55\left(e^{(-x)}+e^x\right)^3 + 402\left(e^{(-x)}+e^x\right)^2 + 1020\,e^{(-x)} + 1020\,e^x + 936}{192\,a(e^{(-x)}+e^x+2)^3}$$

input
```
integrate(csch(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-5/32*log(e^(-x) + e^x + 2)/a + 5/32*log(e^(-x) + e^x - 2)/a - 1/64*(15*(e
^(-x) + e^x)^2 - 76*e^(-x) - 76*e^x + 100)/(a*(e^(-x) + e^x - 2)^2) + 1/19
2*(55*(e^(-x) + e^x)^3 + 402*(e^(-x) + e^x)^2 + 1020*e^(-x) + 1020*e^x + 9
36)/(a*(e^(-x) + e^x + 2)^3)
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 244, normalized size of antiderivative = 2.60

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = \frac{1}{a\,\left(10\,\mathrm{e}^{2\,x}+10\,\mathrm{e}^{3\,x}+5\,\mathrm{e}^{4\,x}+\mathrm{e}^{5\,x}+5\,\mathrm{e}^{x}+1\right)} + \frac{1}{4\,a\,\left(3\,\mathrm{e}^{2\,x}-\mathrm{e}^{3\,x}-3\,\mathrm{e}^{x}+1\right)} + \frac{1}{8\,a\,\left(\mathrm{e}^{2\,x}-2\,\mathrm{e}^{x}+1\right)} - \frac{1}{8\,a\,\left(6\,\mathrm{e}^{2\,x}-4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}-4\,\mathrm{e}^{x}+1\right)} - \frac{5}{8\,a\,\left(6\,\mathrm{e}^{2\,x}+4\,\mathrm{e}^{3\,x}+\mathrm{e}^{4\,x}+4\,\mathrm{e}^{x}+1\right)} + \frac{1}{4\,a\,\left(\mathrm{e}^{x}-1\right)} + \frac{3}{8\,a\,\left(\mathrm{e}^{x}+1\right)} - \frac{5\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{8\,\sqrt{-a^2}} - \frac{1}{3\,a\,\left(15\,\mathrm{e}^{2\,x}+20\,\mathrm{e}^{3\,x}+15\,\mathrm{e}^{4\,x}+6\,\mathrm{e}^{5\,x}+\mathrm{e}^{6\,x}+6\,\mathrm{e}^{x}+1\right)} - \frac{5}{12\,a\,\left(3\,\mathrm{e}^{2\,x}+\mathrm{e}^{3\,x}+3\,\mathrm{e}^{x}+1\right)}$$

input
```
int(1/(sinh(x)^5*(a + a*cosh(x))),x)
```

output
```
1/(a*(10*exp(2*x) + 10*exp(3*x) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1)) +
 1/(4*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) + 1/(8*a*(exp(2*x) - 2*exp
(x) + 1)) - 1/(8*a*(6*exp(2*x) - 4*exp(3*x) + exp(4*x) - 4*exp(x) + 1)) -
5/(8*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) + 1/(4*a*(exp(
x) - 1)) + 3/(8*a*(exp(x) + 1)) - (5*atan((exp(x)*(-a^2)^(1/2))/a))/(8*(-a
^2)^(1/2)) - 1/(3*a*(15*exp(2*x) + 20*exp(3*x) + 15*exp(4*x) + 6*exp(5*x)
+ exp(6*x) + 6*exp(x) + 1)) - 5/(12*a*(3*exp(2*x) + exp(3*x) + 3*exp(x) +
1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 405, normalized size of antiderivative = 4.31

$$\int \frac{\operatorname{csch}^5(x)}{a+a\cosh(x)}\,dx = \frac{-15-250e^{4x}+15\log(e^x-1)-15\log(e^x+1)+105e^{8x}-250e^{6x}+40e^{3x}-144e^{5x}+30e^{6x}\log(e^x-1)}{}$$

input
```
int(csch(x)^5/(a+a*cosh(x)),x)
```

output
```
(15*e**(10*x)*log(e**x - 1) - 15*e**(10*x)*log(e**x + 1) - 15*e**(10*x) +
30*e**(9*x)*log(e**x - 1) - 30*e**(9*x)*log(e**x + 1) - 45*e**(8*x)*log(e*
*x - 1) + 45*e**(8*x)*log(e**x + 1) + 105*e**(8*x) - 120*e**(7*x)*log(e**x
 - 1) + 120*e**(7*x)*log(e**x + 1) + 40*e**(7*x) + 30*e**(6*x)*log(e**x -
1) - 30*e**(6*x)*log(e**x + 1) - 250*e**(6*x) + 180*e**(5*x)*log(e**x - 1)
 - 180*e**(5*x)*log(e**x + 1) - 144*e**(5*x) + 30*e**(4*x)*log(e**x - 1) -
 30*e**(4*x)*log(e**x + 1) - 250*e**(4*x) - 120*e**(3*x)*log(e**x - 1) + 1
20*e**(3*x)*log(e**x + 1) + 40*e**(3*x) - 45*e**(2*x)*log(e**x - 1) + 45*e
**(2*x)*log(e**x + 1) + 105*e**(2*x) + 30*e**x*log(e**x - 1) - 30*e**x*log
(e**x + 1) + 15*log(e**x - 1) - 15*log(e**x + 1) - 15)/(48*a*(e**(10*x) +
2*e**(9*x) - 3*e**(8*x) - 8*e**(7*x) + 2*e**(6*x) + 12*e**(5*x) + 2*e**(4*
x) - 8*e**(3*x) - 3*e**(2*x) + 2*e**x + 1))
```

3.165 $\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 140

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = -\frac{a(a^4-3a^2b^2+3b^4)\cosh(x)}{b^6}+\frac{(a^4-3a^2b^2+3b^4)\cosh^2(x)}{2b^5} - \frac{a(a^2-3b^2)\cosh^3(x)}{3b^4}+\frac{(a^2-3b^2)\cosh^4(x)}{4b^3} - \frac{a\cosh^5(x)}{5b^2}+\frac{\cosh^6(x)}{6b}+\frac{(a^2-b^2)^3\log(a+b\cosh(x))}{b^7}$$

output
```
-a*(a^4-3*a^2*b^2+3*b^4)*cosh(x)/b^6+1/2*(a^4-3*a^2*b^2+3*b^4)*cosh(x)^2/b
^5-1/3*a*(a^2-3*b^2)*cosh(x)^3/b^4+1/4*(a^2-3*b^2)*cosh(x)^4/b^3-1/5*a*cos
h(x)^5/b^2+1/6*cosh(x)^6/b+(a^2-b^2)^3*ln(a+b*cosh(x))/b^7
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.03

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \frac{-120ab(8a^4-22a^2b^2+19b^4)\cosh(x)+15b^2(16a^4-40a^2b^2+29b^4)\cosh(2x)-20a(2a-3b)b^3(2a+3b}{9}$$

input
```
Integrate[Sinh[x]^7/(a + b*Cosh[x]),x]
```

output
```
(-120*a*b*(8*a^4 - 22*a^2*b^2 + 19*b^4)*Cosh[x] + 15*b^2*(16*a^4 - 40*a^2*
b^2 + 29*b^4)*Cosh[2*x] - 20*a*(2*a - 3*b)*b^3*(2*a + 3*b)*Cosh[3*x] - 30*
b^4*(-a^2 + 2*b^2)*Cosh[4*x] - 12*a*b^5*Cosh[5*x] + 5*b^6*Cosh[6*x] + 960*
(a^2 - b^2)^3*Log[a + b*Cosh[x]])/(960*b^7)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^7}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^7}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{\left(b^2-b^2\cosh^2(x)\right)^3}{a+b\cosh(x)}d(b\cosh(x))}{b^7}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(\left(\frac{3(b^2-a^2)b^2}{a^4}+1\right)a^5+b^4\cosh^4(x)a+b^2\left(a^2-3b^2\right)\cosh^2(x)a-b^5\cosh^5(x)-b^3\left(a^2-3b^2\right)\cosh^3(x)-b(a^4}{b^7}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-(a^2-b^2)^3\log(a+b\cosh(x))-\frac{1}{4}b^4(a^2-3b^2)\cosh^4(x)+\frac{1}{3}ab^3(a^2-3b^2)\cosh^3(x)-\frac{1}{2}b^2(a^4-3a^2b^2+3b^4)\cosh}{b^7}$$

input
```
Int[Sinh[x]^7/(a + b*Cosh[x]),x]
```

output
```
-((a*b*(a^4 - 3*a^2*b^2 + 3*b^4)*Cosh[x] - (b^2*(a^4 - 3*a^2*b^2 + 3*b^4)*
Cosh[x]^2)/2 + (a*b^3*(a^2 - 3*b^2)*Cosh[x]^3)/3 - (b^4*(a^2 - 3*b^2)*Cosh
[x]^4)/4 + (a*b^5*Cosh[x]^5)/5 - (b^6*Cosh[x]^6)/6 - (a^2 - b^2)^3*Log[a +
 b*Cosh[x]])/b^7)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.06

$$\frac{\frac{\cosh(x)^6 b^5}{6}-\frac{a\cosh(x)^5 b^4}{5}+\frac{b(a^2b^2-3b^4)\cosh(x)^4}{4}-\frac{a(a^2b^2-3b^4)\cosh(x)^3}{3}+\frac{(a^4-3a^2b^2+3b^4)\cosh(x)^2 b}{2}-a(a^4-3a^2b^2+3b^4)}{b^6}$$

input
```
int(sinh(x)^7/(a+b*cosh(x)),x)
```

output
```
1/b^6*(1/6*cosh(x)^6*b^5-1/5*a*cosh(x)^5*b^4+1/4*b*(a^2*b^2-3*b^4)*cosh(x)
^4-1/3*a*(a^2*b^2-3*b^4)*cosh(x)^3+1/2*(a^4-3*a^2*b^2+3*b^4)*cosh(x)^2*b-a
*(a^4-3*a^2*b^2+3*b^4)*cosh(x))+(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/b^7*ln(a+b*c
osh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2134 vs. 2(130) = 260.

Time = 0.11 (sec) , antiderivative size = 2134, normalized size of antiderivative = 15.24

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/1920*(5*b^6*cosh(x)^12 + 5*b^6*sinh(x)^12 - 12*a*b^5*cosh(x)^11 + 12*(5*
b^6*cosh(x) - a*b^5)*sinh(x)^11 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^10 + 6*(55*
b^6*cosh(x)^2 - 22*a*b^5*cosh(x) + 5*a^2*b^4 - 10*b^6)*sinh(x)^10 - 20*(4*
a^3*b^3 - 9*a*b^5)*cosh(x)^9 + 20*(55*b^6*cosh(x)^3 - 33*a*b^5*cosh(x)^2 -
 4*a^3*b^3 + 9*a*b^5 + 15*(a^2*b^4 - 2*b^6)*cosh(x))*sinh(x)^9 + 15*(16*a^
4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x)^8 + 15*(165*b^6*cosh(x)^4 - 132*a*b^5
*cosh(x)^3 + 16*a^4*b^2 - 40*a^2*b^4 + 29*b^6 + 90*(a^2*b^4 - 2*b^6)*cosh(
x)^2 - 12*(4*a^3*b^3 - 9*a*b^5)*cosh(x))*sinh(x)^8 - 1920*(a^6 - 3*a^4*b^2
 + 3*a^2*b^4 - b^6)*x*cosh(x)^6 - 120*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*co
sh(x)^7 + 120*(33*b^6*cosh(x)^5 - 33*a*b^5*cosh(x)^4 - 8*a^5*b + 22*a^3*b^
3 - 19*a*b^5 + 30*(a^2*b^4 - 2*b^6)*cosh(x)^3 - 6*(4*a^3*b^3 - 9*a*b^5)*co
sh(x)^2 + (16*a^4*b^2 - 40*a^2*b^4 + 29*b^6)*cosh(x))*sinh(x)^7 - 12*a*b^5
*cosh(x) + 12*(385*b^6*cosh(x)^6 - 462*a*b^5*cosh(x)^5 + 525*(a^2*b^4 - 2*
b^6)*cosh(x)^4 - 140*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^3 + 35*(16*a^4*b^2 - 40
*a^2*b^4 + 29*b^6)*cosh(x)^2 - 160*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x -
 70*(8*a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x))*sinh(x)^6 + 5*b^6 - 120*(8*
a^5*b - 22*a^3*b^3 + 19*a*b^5)*cosh(x)^5 + 24*(165*b^6*cosh(x)^7 - 231*a*b
^5*cosh(x)^6 - 40*a^5*b + 110*a^3*b^3 - 95*a*b^5 + 315*(a^2*b^4 - 2*b^6)*c
osh(x)^5 - 105*(4*a^3*b^3 - 9*a*b^5)*cosh(x)^4 + 35*(16*a^4*b^2 - 40*a^2*b
^4 + 29*b^6)*cosh(x)^3 - 480*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*x*cosh...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**7/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. 2(130) = 260.

Time = 0.05 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.21

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = -\frac{(12\,ab^4e^{(-x)} - 5\,b^5 - 30\,(a^2b^3 - 2\,b^5)e^{(-2\,x)} + 20\,(4\,a^3b^2 - 9\,ab^4)e^{(-3\,x)} - 15\,(16\,a^4b - 40\,a^2b^3 + 29\,b^5)e}{1920\,b^6} - \frac{12\,ab^4e^{(-5\,x)} - 5\,b^5e^{(-6\,x)} + 120\,(8\,a^5 - 22\,a^3b^2 + 19\,ab^4)e^{(-x)} - 15\,(16\,a^4b - 40\,a^2b^3 + 29\,b^5)e^{(-2\,x)} + 2}{1920\,b^6} + \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)x}{b^7} + \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{b^7}$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-1/1920*(12*a*b^4*e^(-x) - 5*b^5 - 30*(a^2*b^3 - 2*b^5)*e^(-2*x) + 20*(4*a
^3*b^2 - 9*a*b^4)*e^(-3*x) - 15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-4*x)
+ 120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-5*x))*e^(6*x)/b^6 - 1/1920*(12*a
*b^4*e^(-5*x) - 5*b^5*e^(-6*x) + 120*(8*a^5 - 22*a^3*b^2 + 19*a*b^4)*e^(-x
) - 15*(16*a^4*b - 40*a^2*b^3 + 29*b^5)*e^(-2*x) + 20*(4*a^3*b^2 - 9*a*b^4
)*e^(-3*x) - 30*(a^2*b^3 - 2*b^5)*e^(-4*x))/b^6 + (a^6 - 3*a^4*b^2 + 3*a^2
*b^4 - b^6)*x/b^7 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*log(2*a*e^(-x) + b
*e^(-2*x) + b)/b^7
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.64

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \frac{5\,b^5\left(e^{(-x)} + e^x\right)^6 - 12\,ab^4\left(e^{(-x)} + e^x\right)^5 + 30\,a^2b^3\left(e^{(-x)} + e^x\right)^4 - 90\,b^5\left(e^{(-x)} + e^x\right)^4 - 80\,a^3b^2\left(e^{(-x)} + e^x\right)}{} + \frac{(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\log\left(\left|b\left(e^{(-x)} + e^x\right) + 2\,a\right|\right)}{b^7}$$

input
```
integrate(sinh(x)^7/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/1920*(5*b^5*(e^(-x) + e^x)^6 - 12*a*b^4*(e^(-x) + e^x)^5 + 30*a^2*b^3*(e
^(-x) + e^x)^4 - 90*b^5*(e^(-x) + e^x)^4 - 80*a^3*b^2*(e^(-x) + e^x)^3 + 2
40*a*b^4*(e^(-x) + e^x)^3 + 240*a^4*b*(e^(-x) + e^x)^2 - 720*a^2*b^3*(e^(-
x) + e^x)^2 + 720*b^5*(e^(-x) + e^x)^2 - 960*a^5*(e^(-x) + e^x) + 2880*a^3
*b^2*(e^(-x) + e^x) - 2880*a*b^4*(e^(-x) + e^x))/b^6 + (a^6 - 3*a^4*b^2 +
3*a^2*b^4 - b^6)*log(abs(b*(e^(-x) + e^x) + 2*a))/b^7
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.06

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx = \frac{e^{-6x}}{384\,b}+\frac{e^{6x}}{384\,b}-\frac{x\,(a^2-b^2)^3}{b^7}-\frac{e^{-x}\,(8\,a^5-22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6}+\frac{e^{-3x}\,(9\,a\,b^2-4\,a^3)}{96\,b^4}+\frac{e^{3x}\,(9\,a\,b^2-4\,a^3)}{96\,b^4}+\frac{e^{-4x}\,(a^2-2\,b^2)}{64\,b^3}+\frac{e^{4x}\,(a^2-2\,b^2)}{64\,b^3}-\frac{a\,e^{-5x}}{160\,b^2}-\frac{a\,e^{5x}}{160\,b^2}+\frac{e^{-2x}\,(16\,a^4-40\,a^2\,b^2+29\,b^4)}{128\,b^5}+\frac{e^{2x}\,(16\,a^4-40\,a^2\,b^2+29\,b^4)}{128\,b^5}-\frac{e^{x}\,(8\,a^5-22\,a^3\,b^2+19\,a\,b^4)}{16\,b^6}+\frac{\ln\left(b+2\,a\,e^{x}+b\,e^{2x}\right)\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{b^7}$$

input
```
int(sinh(x)^7/(a + b*cosh(x)),x)
```

output
```
exp(-6*x)/(384*b) + exp(6*x)/(384*b) - (x*(a^2 - b^2)^3)/b^7 - (exp(-x)*(1
9*a*b^4 + 8*a^5 - 22*a^3*b^2))/(16*b^6) + (exp(-3*x)*(9*a*b^2 - 4*a^3))/(9
6*b^4) + (exp(3*x)*(9*a*b^2 - 4*a^3))/(96*b^4) + (exp(-4*x)*(a^2 - 2*b^2))
/(64*b^3) + (exp(4*x)*(a^2 - 2*b^2))/(64*b^3) - (a*exp(-5*x))/(160*b^2) -
(a*exp(5*x))/(160*b^2) + (exp(-2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b
^5) + (exp(2*x)*(16*a^4 + 29*b^4 - 40*a^2*b^2))/(128*b^5) - (exp(x)*(19*a*
b^4 + 8*a^5 - 22*a^3*b^2))/(16*b^6) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a
^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.17

$$\int \frac{\sinh^7(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{5b^6 - 12e^{11x}a\,b^5 + 30e^{10x}a^2b^4 - 80e^{9x}a^3b^3 + 180e^{9x}a\,b^5 + 240e^{8x}a^4b^2 - 600e^{8x}a^2b^4 - 960e^{7x}a^5b + 2640e^{7x}}{}$$

input
```
int(sinh(x)^7/(a+b*cosh(x)),x)
```

output
```
(5*e**(12*x)*b**6 - 12*e**(11*x)*a*b**5 + 30*e**(10*x)*a**2*b**4 - 60*e**(
10*x)*b**6 - 80*e**(9*x)*a**3*b**3 + 180*e**(9*x)*a*b**5 + 240*e**(8*x)*a*
*4*b**2 - 600*e**(8*x)*a**2*b**4 + 435*e**(8*x)*b**6 - 960*e**(7*x)*a**5*b
 + 2640*e**(7*x)*a**3*b**3 - 2280*e**(7*x)*a*b**5 + 1920*e**(6*x)*log(e**(
2*x)*b + 2*e**x*a + b)*a**6 - 5760*e**(6*x)*log(e**(2*x)*b + 2*e**x*a + b)
*a**4*b**2 + 5760*e**(6*x)*log(e**(2*x)*b + 2*e**x*a + b)*a**2*b**4 - 1920
*e**(6*x)*log(e**(2*x)*b + 2*e**x*a + b)*b**6 - 1920*e**(6*x)*a**6*x + 576
0*e**(6*x)*a**4*b**2*x - 5760*e**(6*x)*a**2*b**4*x + 1920*e**(6*x)*b**6*x
 - 960*e**(5*x)*a**5*b + 2640*e**(5*x)*a**3*b**3 - 2280*e**(5*x)*a*b**5 + 2
40*e**(4*x)*a**4*b**2 - 600*e**(4*x)*a**2*b**4 + 435*e**(4*x)*b**6 - 80*e*
*(3*x)*a**3*b**3 + 180*e**(3*x)*a*b**5 + 30*e**(2*x)*a**2*b**4 - 60*e**(2*
x)*b**6 - 12*e**x*a*b**5 + 5*b**6)/(1920*e**(6*x)*b**7)
```

3.166 $\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 154

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = -\frac{a(8a^4-20a^2b^2+15b^4)\,x}{8b^6} + \frac{2(a-b)^{5/2}(a+b)^{5/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^6} + \frac{\left(8(a^2-b^2)^2-ab(4a^2-7b^2)\cosh(x)\right)\sinh(x)}{8b^5} + \frac{(4(a^2-b^2)-3ab\cosh(x))\sinh^3(x)}{12b^3} + \frac{\sinh^5(x)}{5b}$$

output
```
-1/8*a*(8*a^4-20*a^2*b^2+15*b^4)*x/b^6+2*(a-b)^(5/2)*(a+b)^(5/2)*arctanh((
a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/b^6+1/8*(8*(a^2-b^2)^2-a*b*(4*a^2-7*b^
2)*cosh(x))*sinh(x)/b^5+1/12*(4*a^2-4*b^2-3*a*b*cosh(x))*sinh(x)^3/b^3+1/5
*sinh(x)^5/b
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = \frac{-60a(8a^4-20a^2b^2+15b^4)\,x+960(-a^2+b^2)^{5/2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+60b(8a^4-18a^2b^2+11b^4)\sinh}{480b^6}$$

input
```
Integrate[Sinh[x]^6/(a + b*Cosh[x]),x]
```

output
```
(-60*a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x + 960*(-a^2 + b^2)^(5/2)*ArcTan[((a
 - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + 60*b*(8*a^4 - 18*a^2*b^2 + 11*b^4)*Si
nh[x] - 120*a*b^2*(a^2 - 2*b^2)*Sinh[2*x] - 10*b^3*(-4*a^2 + 7*b^2)*Sinh[3
*x] - 15*a*b^4*Sinh[4*x] + 6*b^5*Sinh[5*x])/(480*b^6)
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.19, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.154, Rules used = {3042, 25, 3174, 25, 3042, 3344, 3042, 25, 3344, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^6}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^6}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3174$$

$$\frac{\int -\frac{(b+a\cosh(x))\sinh^4(x)}{a+b\cosh(x)}dx}{b}+\frac{\sinh^5(x)}{5b}$$

$\Big\downarrow$ 25

$$\frac{\sinh^5(x)}{5b}-\frac{\int \frac{(b+a\cosh(x))\sinh^4(x)}{a+b\cosh(x)}dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^5(x)}{5b}-\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^4\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$\Big\downarrow$ 3344

$$\frac{\sinh^5(x)}{5b}-\frac{\frac{\int \frac{\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\cosh(x)\right)\sinh^2(x)}{a+b\cosh(x)}dx}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}+\frac{\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{4b^2}}{b}$$

$\Big\downarrow$ 25

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}-\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b\left(a^2-4b^2\right)+a\left(4a^2-7b^2\right)\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{4b^2}}{b}$$

$\Big\downarrow$ 3344

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\int -\frac{b\left(4a^4-9b^2a^2+8b^4\right)+a\left(8a^4-20b^2a^2+15b^4\right)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}+\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}}{b}$$

$\Big\downarrow$ 25

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\int \frac{b\left(4a^4-9b^2a^2+8b^4\right)+a\left(8a^4-20b^2a^2+15b^4\right)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}}{4b^2}-\frac{\sinh^3(x)\left(4\left(a^2-b^2\right)-3ab\cosh(x)\right)}{12b^2}}{b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)(4(a^2-b^2)-3ab\cosh(x))}{12b^2}-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\int\frac{b\left(4a^4-9b^2a^2+8b^4\right)+a\left(8a^4-20b^2a^2+15b^4\right)\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b^2}}{4b^2}}{b}$$

$\downarrow$ 3214

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{8\left(a^2-b^2\right)^3\int\frac{1}{a+b\cosh(x)}dx}{b}}{2b^2}}{4b^2}-\frac{\sinh^3(x)(4(a^2-b^2)-3ab\cosh(x))}{12b^2}}{b}$$

$\downarrow$ 3042

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)(4(a^2-b^2)-3ab\cosh(x))}{12b^2}-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{8\left(a^2-b^2\right)^3\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b^2}}{4b^2}}{b}$$

$\downarrow$ 3138

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{16\left(a^2-b^2\right)^3\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{2b^2}}{4b^2}-\frac{\sinh^3(x)(4(a^2-b^2)-3ab\cosh(x))}{12b^2}}{b}$$

$\downarrow$ 221

$$\frac{\sinh^5(x)}{5b}-\frac{-\frac{\sinh^3(x)(4(a^2-b^2)-3ab\cosh(x))}{12b^2}-\frac{\frac{\sinh(x)\left(8\left(a^2-b^2\right)^2-ab\left(4a^2-7b^2\right)\cosh(x)\right)}{2b^2}-\frac{\frac{ax\left(8a^4-20a^2b^2+15b^4\right)}{b}-\frac{16\left(a^2-b^2\right)^3\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{2b^2}}{4b^2}}{b}$$

input

```
Int[Sinh[x]^6/(a + b*Cosh[x]),x]
```

output
```
Sinh[x]^5/(5*b) - (-1/12*((4*(a^2 - b^2) - 3*a*b*Cosh[x])*Sinh[x]^3)/b^2 -
 (-1/2*((a*(8*a^4 - 20*a^2*b^2 + 15*b^4)*x)/b - (16*(a^2 - b^2)^3*ArcTanh[
(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b^2 + (
(8*(a^2 - b^2)^2 - a*b*(4*a^2 - 7*b^2)*Cosh[x])*Sinh[x])/(2*b^2))/(4*b^2))
/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(135) = 270$.

Time = 0.11 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.66

$$-\frac{1}{5b\left(\tanh\left(\frac{x}{2}\right)-1\right)^{5}}-\frac{a+2b}{4b^{2}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{4}}-\frac{4a^{2}+6ab-b^{2}}{12b^{3}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{3}}-\frac{4a^{3}+4a^{2}b-5b^{2}a-5b^{3}}{8b^{4}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{2}}-\frac{8a^{4}+4a^{3}b}{8b^{5}}$$

input
```
int(sinh(x)^6/(a+b*cosh(x)),x)
```

output
```
-1/5/b/(tanh(1/2*x)-1)^5-1/4*(a+2*b)/b^2/(tanh(1/2*x)-1)^4-1/12*(4*a^2+6*a
*b-b^2)/b^3/(tanh(1/2*x)-1)^3-1/8*(4*a^3+4*a^2*b-5*a*b^2-5*b^3)/b^4/(tanh(
1/2*x)-1)^2-1/8*(8*a^4+4*a^3*b-16*a^2*b^2-7*a*b^3+8*b^4)/b^5/(tanh(1/2*x)-
1)+1/8*a*(8*a^4-20*a^2*b^2+15*b^4)/b^6*ln(tanh(1/2*x)-1)-1/5/b/(1+tanh(1/2
*x))^5-1/4*(-a-2*b)/b^2/(1+tanh(1/2*x))^4-1/12*(4*a^2+6*a*b-b^2)/b^3/(1+ta
nh(1/2*x))^3-1/8*(-4*a^3-4*a^2*b+5*a*b^2+5*b^3)/b^4/(1+tanh(1/2*x))^2-1/8*
(8*a^4+4*a^3*b-16*a^2*b^2-7*a*b^3+8*b^4)/b^5/(1+tanh(1/2*x))-1/8*a*(8*a^4-
20*a^2*b^2+15*b^4)/b^6*ln(1+tanh(1/2*x))-2/b^6*(-a^6+3*a^4*b^2-3*a^2*b^4+b
^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1422 vs. $2(134) = 268$.

Time = 0.13 (sec) , antiderivative size = 2913, normalized size of antiderivative = 18.92

$$\int \frac{\sinh^6(x)}{a + b\cosh(x)} \, dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/960*(6*b^5*cosh(x)^10 + 6*b^5*sinh(x)^10 - 15*a*b^4*cosh(x)^9 + 15*(4*b
^5*cosh(x) - a*b^4)*sinh(x)^9 + 10*(4*a^2*b^3 - 7*b^5)*cosh(x)^8 + 5*(54*b
^5*cosh(x)^2 - 27*a*b^4*cosh(x) + 8*a^2*b^3 - 14*b^5)*sinh(x)^8 - 120*(a^3
*b^2 - 2*a*b^4)*cosh(x)^7 + 20*(36*b^5*cosh(x)^3 - 27*a*b^4*cosh(x)^2 - 6*
a^3*b^2 + 12*a*b^4 + 4*(4*a^2*b^3 - 7*b^5)*cosh(x))*sinh(x)^7 - 120*(8*a^5
 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^5 + 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)
*cosh(x)^6 + 20*(63*b^5*cosh(x)^4 - 63*a*b^4*cosh(x)^3 + 24*a^4*b - 54*a^2
*b^3 + 33*b^5 + 14*(4*a^2*b^3 - 7*b^5)*cosh(x)^2 - 42*(a^3*b^2 - 2*a*b^4)*
cosh(x))*sinh(x)^6 + 15*a*b^4*cosh(x) + 2*(756*b^5*cosh(x)^5 - 945*a*b^4*c
osh(x)^4 + 280*(4*a^2*b^3 - 7*b^5)*cosh(x)^3 - 1260*(a^3*b^2 - 2*a*b^4)*co
sh(x)^2 - 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x + 180*(8*a^4*b - 18*a^2*b^3
 + 11*b^5)*cosh(x))*sinh(x)^5 - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)
*cosh(x)^4 + 10*(126*b^5*cosh(x)^6 - 189*a*b^4*cosh(x)^5 - 48*a^4*b + 108*
a^2*b^3 - 66*b^5 + 70*(4*a^2*b^3 - 7*b^5)*cosh(x)^4 - 420*(a^3*b^2 - 2*a*b
^4)*cosh(x)^3 - 60*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x) + 90*(8*a^4*b
 - 18*a^2*b^3 + 11*b^5)*cosh(x)^2)*sinh(x)^4 + 120*(a^3*b^2 - 2*a*b^4)*cos
h(x)^3 + 20*(36*b^5*cosh(x)^7 - 63*a*b^4*cosh(x)^6 + 28*(4*a^2*b^3 - 7*b^5
)*cosh(x)^5 + 6*a^3*b^2 - 12*a*b^4 - 210*(a^3*b^2 - 2*a*b^4)*cosh(x)^4 - 6
0*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x*cosh(x)^2 + 60*(8*a^4*b - 18*a^2*b^3 +
 11*b^5)*cosh(x)^3 - 12*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*cosh(x))*sinh(x...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**6/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.73

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{6\,b^4e^{(5\,x)} - 15\,ab^3e^{(4\,x)} + 40\,a^2b^2e^{(3\,x)} - 70\,b^4e^{(3\,x)} - 120\,a^3be^{(2\,x)} + 240\,ab^3e^{(2\,x)} + 480\,a^4e^x - 1080\,a^2b^2e^x}{960\,b^5}$$
$$- \frac{(8\,a^5 - 20\,a^3b^2 + 15\,ab^4)x}{8\,b^6}$$
$$+ \frac{\left(15\,ab^4e^x - 6\,b^5 - 60\,(8\,a^4b - 18\,a^2b^3 + 11\,b^5)e^{(4\,x)} + 120\,(a^3b^2 - 2\,ab^4)e^{(3\,x)} - 10\,(4\,a^2b^3 - 7\,b^5)e^{(2\,x)}\right)e}{960\,b^6}$$
$$+ \frac{2\,(a^6 - 3\,a^4b^2 + 3\,a^2b^4 - b^6)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^6}$$

input
```
integrate(sinh(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/960*(6*b^4*e^(5*x) - 15*a*b^3*e^(4*x) + 40*a^2*b^2*e^(3*x) - 70*b^4*e^(3
*x) - 120*a^3*b*e^(2*x) + 240*a*b^3*e^(2*x) + 480*a^4*e^x - 1080*a^2*b^2*e
^x + 660*b^4*e^x)/b^5 - 1/8*(8*a^5 - 20*a^3*b^2 + 15*a*b^4)*x/b^6 + 1/960*
(15*a*b^4*e^x - 6*b^5 - 60*(8*a^4*b - 18*a^2*b^3 + 11*b^5)*e^(4*x) + 120*(
a^3*b^2 - 2*a*b^4)*e^(3*x) - 10*(4*a^2*b^3 - 7*b^5)*e^(2*x))*e^(-5*x)/b^6
+ 2*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*arctan((b*e^x + a)/sqrt(-a^2 + b^2
))/(sqrt(-a^2 + b^2)*b^6)
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.26

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx = \frac{e^{5x}}{160\,b}-\frac{e^{-5x}}{160\,b}-\frac{e^{-2x}\,(2\,a\,b^2-a^3)}{8\,b^4}+\frac{e^{2x}\,(2\,a\,b^2-a^3)}{8\,b^4} - \frac{x\,(8\,a^5-20\,a^3\,b^2+15\,a\,b^4)}{8\,b^6}+\frac{e^{x}\,(8\,a^4-18\,a^2\,b^2+11\,b^4)}{16\,b^5}+\frac{a\,e^{-4x}}{64\,b^2}-\frac{a\,e^{4x}}{64\,b^2} - \frac{e^{-x}\,(8\,a^4-18\,a^2\,b^2+11\,b^4)}{16\,b^5}-\frac{e^{-3x}\,(4\,a^2-7\,b^2)}{96\,b^3}+\frac{e^{3x}\,(4\,a^2-7\,b^2)}{96\,b^3} + \frac{\ln\left(-\frac{2\,e^{x}\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{b^7}-\frac{2\,(a+b)^{5/2}\,(b+a\,e^{x})\,(a-b)^{5/2}}{b^7}\right)\,(a+b)^{5/2}\,(a-b)^{5/2}}{b^6} - \frac{\ln\left(\frac{2\,(a+b)^{5/2}\,(b+a\,e^{x})\,(a-b)^{5/2}}{b^7}-\frac{2\,e^{x}\,(a^6-3\,a^4\,b^2+3\,a^2\,b^4-b^6)}{b^7}\right)\,(a+b)^{5/2}\,(a-b)^{5/2}}{b^6}$$

input
```
int(sinh(x)^6/(a + b*cosh(x)),x)
```

output
```
exp(5*x)/(160*b) - exp(-5*x)/(160*b) - (exp(-2*x)*(2*a*b^2 - a^3))/(8*b^4)
 + (exp(2*x)*(2*a*b^2 - a^3))/(8*b^4) - (x*(15*a*b^4 + 8*a^5 - 20*a^3*b^2)
)/(8*b^6) + (exp(x)*(8*a^4 + 11*b^4 - 18*a^2*b^2))/(16*b^5) + (a*exp(-4*x)
)/(64*b^2) - (a*exp(4*x))/(64*b^2) - (exp(-x)*(8*a^4 + 11*b^4 - 18*a^2*b^2
))/(16*b^5) - (exp(-3*x)*(4*a^2 - 7*b^2))/(96*b^3) + (exp(3*x)*(4*a^2 - 7*
b^2))/(96*b^3) + (log(- (2*exp(x)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7
 - (2*(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7)*(a + b)^(5/2)*(a -
b)^(5/2))/b^6 - (log((2*(a + b)^(5/2)*(b + a*exp(x))*(a - b)^(5/2))/b^7 -
(2*exp(x)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2))/b^7)*(a + b)^(5/2)*(a - b)^
(5/2))/b^6
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 373, normalized size of antiderivative = 2.42

$$\int \frac{\sinh^6(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-1920e^{5x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^4+3840e^{5x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2b^2-1920e^{5x}\sqrt{-a^2+b^2}\,a}{}$$

input
```
int(sinh(x)^6/(a+b*cosh(x)),x)
```

output
```
( - 1920*e**(5*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b*
*2))*a**4 + 3840*e**(5*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a
**2 + b**2))*a**2*b**2 - 1920*e**(5*x)*sqrt( - a**2 + b**2)*atan((e**x*b +
 a)/sqrt( - a**2 + b**2))*b**4 + 6*e**(10*x)*b**5 - 15*e**(9*x)*a*b**4 + 4
0*e**(8*x)*a**2*b**3 - 70*e**(8*x)*b**5 - 120*e**(7*x)*a**3*b**2 + 240*e**
(7*x)*a*b**4 + 480*e**(6*x)*a**4*b - 1080*e**(6*x)*a**2*b**3 + 660*e**(6*x
)*b**5 - 960*e**(5*x)*a**5*x + 2400*e**(5*x)*a**3*b**2*x - 1800*e**(5*x)*a
*b**4*x - 480*e**(4*x)*a**4*b + 1080*e**(4*x)*a**2*b**3 - 660*e**(4*x)*b**
5 + 120*e**(3*x)*a**3*b**2 - 240*e**(3*x)*a*b**4 - 40*e**(2*x)*a**2*b**3 +
 70*e**(2*x)*b**5 + 15*e**x*a*b**4 - 6*b**5)/(960*e**(5*x)*b**6)
```

3.167 $\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 83

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = -\frac{a(a^2-2b^2)\cosh(x)}{b^4}+\frac{(a^2-2b^2)\cosh^2(x)}{2b^3} -\frac{a\cosh^3(x)}{3b^2}+\frac{\cosh^4(x)}{4b}+\frac{(a^2-b^2)^2\log(a+b\cosh(x))}{b^5}$$

output
```
-a*(a^2-2*b^2)*cosh(x)/b^4+1/2*(a^2-2*b^2)*cosh(x)^2/b^3-1/3*a*cosh(x)^3/b
^2+1/4*cosh(x)^4/b+(a^2-b^2)^2*ln(a+b*cosh(x))/b^5
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{-24ab(4a^2-7b^2)\cosh(x)-12b^2(-2a^2+3b^2)\cosh(2x)-8ab^3\cosh(3x)+3b^4\cosh(4x)+96(a^2-b^2)^2 \mathrm{l}}{96b^5}$$

input
```
Integrate[Sinh[x]^5/(a + b*Cosh[x]),x]
```

output
```
(-24*a*b*(4*a^2 - 7*b^2)*Cosh[x] - 12*b^2*(-2*a^2 + 3*b^2)*Cosh[2*x] - 8*a
*b^3*Cosh[3*x] + 3*b^4*Cosh[4*x] + 96*(a^2 - b^2)^2*Log[a + b*Cosh[x]])/(9
6*b^5)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)^5}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^5}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{\left(b^2-b^2\cosh^2(x)\right)^2}{a+b\cosh(x)}d(b\cosh(x))}{b^5}$$

$$\Big\downarrow \text{476}$$

$$\frac{\int \left(-\left(\left(1-\frac{2b^2}{a^2}\right)a^3\right)-b^2\cosh^2(x)a+b^3\cosh^3(x)+b\left(a^2-2b^2\right)\cosh(x)+\frac{\left(a^2-b^2\right)^2}{a+b\cosh(x)}\right)d(b\cosh(x))}{b^5}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}b^2\left(a^2-2b^2\right)\cosh^2(x)-ab\left(a^2-2b^2\right)\cosh(x)+\left(a^2-b^2\right)^2\log(a+b\cosh(x))-\frac{1}{3}ab^3\cosh^3(x)+\frac{1}{4}b^4\cosh^4(x)}{b^5}$$

input
```
Int[Sinh[x]^5/(a + b*Cosh[x]),x]
```

output
```
(-(a*b*(a^2 - 2*b^2)*Cosh[x]) + (b^2*(a^2 - 2*b^2)*Cosh[x]^2)/2 - (a*b^3*C
osh[x]^3)/3 + (b^4*Cosh[x]^4)/4 + (a^2 - b^2)^2*Log[a + b*Cosh[x]])/b^5
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 112.23 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.99

method	result
derivativedivides	$-\frac{-\frac{\cosh(x)^4 b^3}{4}+\frac{a\cosh(x)^3 b^2}{3}-\frac{\left(a^2-2b^2\right)\cosh(x)^2 b}{2}+a\left(a^2-2b^2\right)\cosh(x)}{b^4}+\frac{\left(a^4-2a^2b^2+b^4\right)\ln(a+b\cosh(x))}{b^5}$
default	$-\frac{-\frac{\cosh(x)^4 b^3}{4}+\frac{a\cosh(x)^3 b^2}{3}-\frac{\left(a^2-2b^2\right)\cosh(x)^2 b}{2}+a\left(a^2-2b^2\right)\cosh(x)}{b^4}+\frac{\left(a^4-2a^2b^2+b^4\right)\ln(a+b\cosh(x))}{b^5}$
risch	$-\frac{x\,a^4}{b^5}+\frac{2x\,a^2}{b^3}-\frac{x}{b}+\frac{e^{4x}}{64b}-\frac{a\,e^{3x}}{24b^2}+\frac{e^{2x}a^2}{8b^3}-\frac{3\,e^{2x}}{16b}-\frac{a^3e^{x}}{2b^4}+\frac{7a\,e^{x}}{8b^2}-\frac{a^3e^{-x}}{2b^4}+\frac{7a\,e^{-x}}{8b^2}+\frac{e^{-2x}a^2}{8b^3}-$

input
```
int(sinh(x)^5/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^4*(-1/4*cosh(x)^4*b^3+1/3*a*cosh(x)^3*b^2-1/2*(a^2-2*b^2)*cosh(x)^2*b
+a*(a^2-2*b^2)*cosh(x))+(a^4-2*a^2*b^2+b^4)/b^5*ln(a+b*cosh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 866 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 866, normalized size of antiderivative = 10.43

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/192*(3*b^4*cosh(x)^8 + 3*b^4*sinh(x)^8 - 8*a*b^3*cosh(x)^7 + 8*(3*b^4*co
sh(x) - a*b^3)*sinh(x)^7 + 12*(2*a^2*b^2 - 3*b^4)*cosh(x)^6 + 4*(21*b^4*co
sh(x)^2 - 14*a*b^3*cosh(x) + 6*a^2*b^2 - 9*b^4)*sinh(x)^6 - 192*(a^4 - 2*a
^2*b^2 + b^4)*x*cosh(x)^4 - 24*(4*a^3*b - 7*a*b^3)*cosh(x)^5 + 24*(7*b^4*c
osh(x)^3 - 7*a*b^3*cosh(x)^2 - 4*a^3*b + 7*a*b^3 + 3*(2*a^2*b^2 - 3*b^4)*c
osh(x))*sinh(x)^5 - 8*a*b^3*cosh(x) + 2*(105*b^4*cosh(x)^4 - 140*a*b^3*cos
h(x)^3 + 90*(2*a^2*b^2 - 3*b^4)*cosh(x)^2 - 96*(a^4 - 2*a^2*b^2 + b^4)*x -
 60*(4*a^3*b - 7*a*b^3)*cosh(x))*sinh(x)^4 + 3*b^4 - 24*(4*a^3*b - 7*a*b^3
)*cosh(x)^3 + 8*(21*b^4*cosh(x)^5 - 35*a*b^3*cosh(x)^4 - 12*a^3*b + 21*a*b
^3 + 30*(2*a^2*b^2 - 3*b^4)*cosh(x)^3 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(
x) - 30*(4*a^3*b - 7*a*b^3)*cosh(x)^2)*sinh(x)^3 + 12*(2*a^2*b^2 - 3*b^4)*
cosh(x)^2 + 12*(7*b^4*cosh(x)^6 - 14*a*b^3*cosh(x)^5 + 15*(2*a^2*b^2 - 3*b
^4)*cosh(x)^4 + 2*a^2*b^2 - 3*b^4 - 96*(a^4 - 2*a^2*b^2 + b^4)*x*cosh(x)^2
 - 20*(4*a^3*b - 7*a*b^3)*cosh(x)^3 - 6*(4*a^3*b - 7*a*b^3)*cosh(x))*sinh(
x)^2 + 192*((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*
cosh(x)^3*sinh(x) + 6*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2*sinh(x)^2 + 4*(a^4
 - 2*a^2*b^2 + b^4)*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4)
*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 8*(3*b^4*cosh(x)^7 - 7*a*b^3
*cosh(x)^6 + 9*(2*a^2*b^2 - 3*b^4)*cosh(x)^5 - 96*(a^4 - 2*a^2*b^2 + b^4)*
x*cosh(x)^3 - 15*(4*a^3*b - 7*a*b^3)*cosh(x)^4 - a*b^3 - 9*(4*a^3*b - 7...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^5(x)}{a + b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**5/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(77) = 154$.

Time = 0.04 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.14

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$$
$$= -\frac{\left(8\,ab^2e^{(-x)} - 3\,b^3 - 12\,(2\,a^2b - 3\,b^3)e^{(-2\,x)} + 24\,(4\,a^3 - 7\,ab^2)e^{(-3\,x)}\right)e^{(4\,x)}}{192\,b^4}$$
$$- \frac{8\,ab^2e^{(-3\,x)} - 3\,b^3e^{(-4\,x)} + 24\,(4\,a^3 - 7\,ab^2)e^{(-x)} - 12\,(2\,a^2b - 3\,b^3)e^{(-2\,x)}}{192\,b^4}$$
$$+ \frac{(a^4 - 2\,a^2b^2 + b^4)x}{b^5} + \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(2\,ae^{(-x)} + be^{(-2\,x)} + b\right)}{b^5}$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-1/192*(8*a*b^2*e^(-x) - 3*b^3 - 12*(2*a^2*b - 3*b^3)*e^(-2*x) + 24*(4*a^3
 - 7*a*b^2)*e^(-3*x))*e^(4*x)/b^4 - 1/192*(8*a*b^2*e^(-3*x) - 3*b^3*e^(-4*
x) + 24*(4*a^3 - 7*a*b^2)*e^(-x) - 12*(2*a^2*b - 3*b^3)*e^(-2*x))/b^4 + (a
^4 - 2*a^2*b^2 + b^4)*x/b^5 + (a^4 - 2*a^2*b^2 + b^4)*log(2*a*e^(-x) + b*e
^(-2*x) + b)/b^5
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.49

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{3\,b^3\left(e^{(-x)} + e^x\right)^4 - 8\,ab^2\left(e^{(-x)} + e^x\right)^3 + 24\,a^2b\left(e^{(-x)} + e^x\right)^2 - 48\,b^3\left(e^{(-x)} + e^x\right)^2 - 96\,a^3\left(e^{(-x)} + e^x\right) + 1}{192\,b^4}$$
$$+ \frac{(a^4 - 2\,a^2b^2 + b^4)\log\left(\left|b\left(e^{(-x)} + e^x\right) + 2\,a\right|\right)}{b^5}$$

input
```
integrate(sinh(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/192*(3*b^3*(e^(-x) + e^x)^4 - 8*a*b^2*(e^(-x) + e^x)^3 + 24*a^2*b*(e^(-x
) + e^x)^2 - 48*b^3*(e^(-x) + e^x)^2 - 96*a^3*(e^(-x) + e^x) + 192*a*b^2*(
e^(-x) + e^x))/b^4 + (a^4 - 2*a^2*b^2 + b^4)*log(abs(b*(e^(-x) + e^x) + 2*
a))/b^5
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 169, normalized size of antiderivative = 2.04

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{e^{-4x}}{64\,b}+\frac{e^{4x}}{64\,b}-\frac{x\,{\left(a^2-b^2\right)}^2}{b^5}+\frac{e^{-x}\,\left(7\,a\,b^2-4\,a^3\right)}{8\,b^4} + \frac{\ln\left(b+2\,a\,e^{x}+b\,e^{2\,x}\right)\,\left(a^4-2\,a^2\,b^2+b^4\right)}{b^5}-\frac{a\,e^{-3\,x}}{24\,b^2}-\frac{a\,e^{3\,x}}{24\,b^2} + \frac{e^{-2\,x}\,\left(2\,a^2-3\,b^2\right)}{16\,b^3}+\frac{e^{2\,x}\,\left(2\,a^2-3\,b^2\right)}{16\,b^3}+\frac{e^{x}\,\left(7\,a\,b^2-4\,a^3\right)}{8\,b^4}$$

input
```
int(sinh(x)^5/(a + b*cosh(x)),x)
```

output
```
exp(-4*x)/(64*b) + exp(4*x)/(64*b) - (x*(a^2 - b^2)^2)/b^5 + (exp(-x)*(7*a
*b^2 - 4*a^3))/(8*b^4) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^4 + b^4 - 2*
a^2*b^2))/b^5 - (a*exp(-3*x))/(24*b^2) - (a*exp(3*x))/(24*b^2) + (exp(-2*x
)*(2*a^2 - 3*b^2))/(16*b^3) + (exp(2*x)*(2*a^2 - 3*b^2))/(16*b^3) + (exp(x
)*(7*a*b^2 - 4*a^3))/(8*b^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 255, normalized size of antiderivative = 3.07

$$\int \frac{\sinh^5(x)}{a+b\cosh(x)}\,dx = \frac{3e^{8x}b^4-8e^{7x}a\,b^3+24e^{6x}a^2b^2-36e^{6x}b^4-96e^{5x}a^3b+168e^{5x}a\,b^3+192e^{4x}\log(e^{2x}b+2e^{x}a+b)\,a^4-384e}{}$$

input
```
int(sinh(x)^5/(a+b*cosh(x)),x)
```

output
```
(3*e**(8*x)*b**4 - 8*e**(7*x)*a*b**3 + 24*e**(6*x)*a**2*b**2 - 36*e**(6*x)
*b**4 - 96*e**(5*x)*a**3*b + 168*e**(5*x)*a*b**3 + 192*e**(4*x)*log(e**(2*
x)*b + 2*e**x*a + b)*a**4 - 384*e**(4*x)*log(e**(2*x)*b + 2*e**x*a + b)*a*
*2*b**2 + 192*e**(4*x)*log(e**(2*x)*b + 2*e**x*a + b)*b**4 - 192*e**(4*x)*
a**4*x + 384*e**(4*x)*a**2*b**2*x - 192*e**(4*x)*b**4*x - 96*e**(3*x)*a**3
*b + 168*e**(3*x)*a*b**3 + 24*e**(2*x)*a**2*b**2 - 36*e**(2*x)*b**4 - 8*e*
*x*a*b**3 + 3*b**4)/(192*e**(4*x)*b**5)
```

3.168 $\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 104

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = -\frac{a(2a^2-3b^2)\,x}{2b^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^4} + \frac{(2(a^2-b^2)-ab\cosh(x))\sinh(x)}{2b^3} + \frac{\sinh^3(x)}{3b}$$

output
```
-1/2*a*(2*a^2-3*b^2)*x/b^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh((a-b)^(1/2)*t
anh(1/2*x)/(a+b)^(1/2))/b^4+1/2*(2*a^2-2*b^2-a*b*cosh(x))*sinh(x)/b^3+1/3*
sinh(x)^3/b
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{-12a^3x+18ab^2x-24(-a^2+b^2)^{3/2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+12a^2b\sinh(x)-15b^3\sinh(x)-3ab^2\sinh(2}{12b^4}$$

input
```
Integrate[Sinh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(-12*a^3*x + 18*a*b^2*x - 24*(-a^2 + b^2)^(3/2)*ArcTan[((a - b)*Tanh[x/2])
/Sqrt[-a^2 + b^2]] + 12*a^2*b*Sinh[x] - 15*b^3*Sinh[x] - 3*a*b^2*Sinh[2*x]
 + b^3*Sinh[3*x])/(12*b^4)
```

Rubi [A] (verified)

Time = 0.64 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.20, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 3174, 3042, 25, 3344, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\cos\left(-\frac{\pi}{2}+ix\right)^4}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\sinh^3(x)}{3b}-\frac{\int \frac{(b+a\cosh(x))\sinh^2(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh^3(x)}{3b}-\frac{\int -\frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh^3(x)}{3b}+\frac{\int \frac{\cos\left(ix+\frac{\pi}{2}\right)^2\left(b+a\sin\left(ix+\frac{\pi}{2}\right)\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3344}$$

$$\frac{\frac{\int -\frac{b\left(a^2-2b^2\right)+a\left(2a^2-3b^2\right)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}+\frac{\sinh(x)\left(2\left(a^2-b^2\right)-ab\cosh(x)\right)}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\int \frac{b(a^2-2b^2)+a(2a^2-3b^2)\cosh(x)}{a+b\cosh(x)}dx}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^3(x)}{3b}+\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\int \frac{b(a^2-2b^2)+a(2a^2-3b^2)\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{2b^2}}{b}$$

$\Big\downarrow$ 3214

$$\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{2(a^2-b^2)^2\int \frac{1}{a+b\cosh(x)}dx}{b}}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$\Big\downarrow$ 3042

$$\frac{\sinh^3(x)}{3b}+\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{2(a^2-b^2)^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{2b^2}}{b}$$

$\Big\downarrow$ 3138

$$\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{4(a^2-b^2)^2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

$\Big\downarrow$ 221

$$\frac{\frac{\sinh(x)(2(a^2-b^2)-ab\cosh(x))}{2b^2}-\frac{\frac{ax(2a^2-3b^2)}{b}-\frac{4(a^2-b^2)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{2b^2}}{b}+\frac{\sinh^3(x)}{3b}$$

input

```
Int[Sinh[x]^4/(a + b*Cosh[x]),x]
```

output

```
Sinh[x]^3/(3*b) + (-1/2*((a*(2*a^2 - 3*b^2)*x)/b - (4*(a^2 - b^2)^2*ArcTan
h[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]))/b^2 +
 ((2*(a^2 - b^2) - a*b*Cosh[x])*Sinh[x])/(2*b^2))/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 3344
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[g*(g*
Cos[e + f*x])^(p - 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c*(m + p + 1) - a*d*
p + b*d*(m + p)*Sin[e + f*x])/(b^2*f*(m + p)*(m + p + 1))), x] + Simp[g^2*(
(p - 1)/(b^2*(m + p)*(m + p + 1)))   Int[(g*Cos[e + f*x])^(p - 2)*(a + b*Si
n[e + f*x])^m*Simp[b*(a*d*m + b*c*(m + p + 1)) + (a*b*c*(m + p + 1) - d*(a^
2*p - b^2*(m + p)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p, 0] && NeQ[m + p + 1
, 0] && IntegerQ[2*m]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 222 vs. $2(87) = 174$.

Time = 30.00 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.14

method	result
default	$-\frac{2\left(-a^{4}+2a^{2}b^{2}-b^{4}\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{4}\sqrt{(a+b)(a-b)}}-\frac{1}{3b\left(1+\tanh\left(\frac{x}{2}\right)\right)^{3}}-\frac{-a-b}{2b^{2}\left(1+\tanh\left(\frac{x}{2}\right)\right)^{2}}-\frac{2a^{2}+ab-2b^{2}}{2b^{3}\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{a\left(2a^{2}-3b\right.}{}$
risch	$-\frac{a^{3}x}{b^{4}}+\frac{3ax}{2b^{2}}+\frac{e^{3x}}{24b}-\frac{a\,e^{2x}}{8b^{2}}+\frac{e^{x}a^{2}}{2b^{3}}-\frac{5\,e^{x}}{8b}-\frac{e^{-x}a^{2}}{2b^{3}}+\frac{5\,e^{-x}}{8b}+\frac{a\,e^{-2x}}{8b^{2}}-\frac{e^{-3x}}{24b}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b}{b\sqrt{a^{2}-b^{2}}}\right.}{b^{4}}$

input
```
int(sinh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-2/b^4*(-a^4+2*a^2*b^2-b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/
((a+b)*(a-b))^(1/2))-1/3/b/(1+tanh(1/2*x))^3-1/2*(-a-b)/b^2/(1+tanh(1/2*x)
)^2-1/2*(2*a^2+a*b-2*b^2)/b^3/(1+tanh(1/2*x))-1/2*a*(2*a^2-3*b^2)/b^4*ln(1
+tanh(1/2*x))-1/3/b/(tanh(1/2*x)-1)^3-1/2*(a+b)/b^2/(tanh(1/2*x)-1)^2-1/2*
(2*a^2+a*b-2*b^2)/b^3/(tanh(1/2*x)-1)+1/2*a*(2*a^2-3*b^2)/b^4*ln(tanh(1/2*
x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 515 vs. $2(86) = 172$.

Time = 0.10 (sec) , antiderivative size = 1099, normalized size of antiderivative = 10.57

$$\int \frac{\sinh^4(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5 + 3*(2*b^3*cosh(x
) - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3 + 3*(4*a^2*b - 5*b
^3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4*a^2*b - 5*b^3)*si
nh(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b^2*cosh(x)^2 - 6*(
2*a^3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sinh(x)^3 - b^3 - 3*(4*a
^2*b - 5*b^3)*cosh(x)^2 + 3*(5*b^3*cosh(x)^4 - 10*a*b^2*cosh(x)^3 - 4*a^2*
b + 5*b^3 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x) + 6*(4*a^2*b - 5*b^3)*cosh(x)^2
)*sinh(x)^2 - 24*((a^2 - b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2*sinh(x)
+ 3*(a^2 - b^2)*cosh(x)*sinh(x)^2 + (a^2 - b^2)*sinh(x)^3)*sqrt(a^2 - b^2)
*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2
*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(
b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b))
+ 3*(2*b^3*cosh(x)^5 - 5*a*b^2*cosh(x)^4 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^
2 + 4*(4*a^2*b - 5*b^3)*cosh(x)^3 + a*b^2 - 2*(4*a^2*b - 5*b^3)*cosh(x))*s
inh(x))/(b^4*cosh(x)^3 + 3*b^4*cosh(x)^2*sinh(x) + 3*b^4*cosh(x)*sinh(x)^2
 + b^4*sinh(x)^3), 1/24*(b^3*cosh(x)^6 + b^3*sinh(x)^6 - 3*a*b^2*cosh(x)^5
 + 3*(2*b^3*cosh(x) - a*b^2)*sinh(x)^5 - 12*(2*a^3 - 3*a*b^2)*x*cosh(x)^3
+ 3*(4*a^2*b - 5*b^3)*cosh(x)^4 + 3*(5*b^3*cosh(x)^2 - 5*a*b^2*cosh(x) + 4
*a^2*b - 5*b^3)*sinh(x)^4 + 3*a*b^2*cosh(x) + 2*(10*b^3*cosh(x)^3 - 15*a*b
^2*cosh(x)^2 - 6*(2*a^3 - 3*a*b^2)*x + 6*(4*a^2*b - 5*b^3)*cosh(x))*sin...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**4/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.40

$$\begin{aligned}\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx &= \frac{b^2e^{(3\,x)}-3\,abe^{(2\,x)}+12\,a^2e^x-15\,b^2e^x}{24\,b^3}-\frac{(2\,a^3-3\,ab^2)x}{2\,b^4}\\ &\quad+\frac{\left(3\,ab^2e^x-b^3-3\,(4\,a^2b-5\,b^3)e^{(2\,x)}\right)e^{(-3\,x)}}{24\,b^4}\\ &\quad+\frac{2\,(a^4-2\,a^2b^2+b^4)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^4}\end{aligned}$$

input
```
integrate(sinh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/24*(b^2*e^(3*x) - 3*a*b*e^(2*x) + 12*a^2*e^x - 15*b^2*e^x)/b^3 - 1/2*(2*
a^3 - 3*a*b^2)*x/b^4 + 1/24*(3*a*b^2*e^x - b^3 - 3*(4*a^2*b - 5*b^3)*e^(2*
x))*e^(-3*x)/b^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^x + a)/sqrt(-a^2
+ b^2))/(sqrt(-a^2 + b^2)*b^4)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 222, normalized size of antiderivative = 2.13

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx = \frac{e^{3x}}{24b}-\frac{e^{-3x}}{24b}+\frac{x\,(3\,a\,b^2-2\,a^3)}{2\,b^4}+\frac{e^{x}\,(4\,a^2-5\,b^2)}{8\,b^3}+\frac{a\,e^{-2x}}{8\,b^2}-\frac{a\,e^{2x}}{8\,b^2}-\frac{e^{-x}\,(4\,a^2-5\,b^2)}{8\,b^3}+\frac{\ln\left(-\frac{2\,e^{x}\,(a^4-2\,a^2\,b^2+b^4)}{b^5}-\frac{2\,(a+b)^{3/2}\,(b+a\,e^{x})\,(a-b)^{3/2}}{b^5}\right)\,(a+b)^{3/2}\,(a-b)^{3/2}}{b^4}-\frac{\ln\left(\frac{2\,(a+b)^{3/2}\,(b+a\,e^{x})\,(a-b)^{3/2}}{b^5}-\frac{2\,e^{x}\,(a^4-2\,a^2\,b^2+b^4)}{b^5}\right)\,(a+b)^{3/2}\,(a-b)^{3/2}}{b^4}$$

input
```
int(sinh(x)^4/(a + b*cosh(x)),x)
```

output
```
exp(3*x)/(24*b) - exp(-3*x)/(24*b) + (x*(3*a*b^2 - 2*a^3))/(2*b^4) + (exp(
x)*(4*a^2 - 5*b^2))/(8*b^3) + (a*exp(-2*x))/(8*b^2) - (a*exp(2*x))/(8*b^2)
 - (exp(-x)*(4*a^2 - 5*b^2))/(8*b^3) + (log(- (2*exp(x)*(a^4 + b^4 - 2*a^2
*b^2))/b^5 - (2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/2))/b^5)*(a + b)^(
3/2)*(a - b)^(3/2))/b^4 - (log((2*(a + b)^(3/2)*(b + a*exp(x))*(a - b)^(3/
2))/b^5 - (2*exp(x)*(a^4 + b^4 - 2*a^2*b^2))/b^5)*(a + b)^(3/2)*(a - b)^(3
/2))/b^4
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.87

$$\int \frac{\sinh^4(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-48e^{3x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2+48e^{3x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)b^2+e^{6x}b^3-3e^{5x}a\,b^2+12e^{4x}a^2b}{24e^{3x}b^4}$$

input
```
int(sinh(x)^4/(a+b*cosh(x)),x)
```

output
```
( - 48*e**(3*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2
))*a**2 + 48*e**(3*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2
+ b**2))*b**2 + e**(6*x)*b**3 - 3*e**(5*x)*a*b**2 + 12*e**(4*x)*a**2*b - 1
5*e**(4*x)*b**3 - 24*e**(3*x)*a**3*x + 36*e**(3*x)*a*b**2*x - 12*e**(2*x)*
a**2*b + 15*e**(2*x)*b**3 + 3*e**x*a*b**2 - b**3)/(24*e**(3*x)*b**4)
```

3.169 $\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 40

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{a\cosh(x)}{b^2}+\frac{\cosh^2(x)}{2b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}$$

output
```
-a*cosh(x)/b^2+1/2*cosh(x)^2/b+(a^2-b^2)*ln(a+b*cosh(x))/b^3
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{a\cosh(x)}{b^2}+\frac{\cosh(2x)}{4b}+\frac{(a^2-b^2)\log(a+b\cosh(x))}{b^3}$$

input
```
Integrate[Sinh[x]^3/(a + b*Cosh[x]),x]
```

output
```
-((a*Cosh[x])/b^2) + Cosh[2*x]/(4*b) + ((a^2 - b^2)*Log[a + b*Cosh[x]])/b^
3
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(-\frac{\pi}{2}+ix\right)^3}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^3}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{b^2-b^2\cosh^2(x)}{a+b\cosh(x)}d(b\cosh(x))}{b^3}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(a-b\cosh(x)+\frac{b^2-a^2}{a+b\cosh(x)}\right)d(b\cosh(x))}{b^3}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-(a^2-b^2)\log(a+b\cosh(x))+ab\cosh(x)-\frac{1}{2}b^2\cosh^2(x)}{b^3}$$

input
```
Int[Sinh[x]^3/(a + b*Cosh[x]),x]
```

output
```
-((a*b*Cosh[x] - (b^2*Cosh[x]^2)/2 - (a^2 - b^2)*Log[a + b*Cosh[x]])/b^3)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 6.92 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.98

method	result	size
derivativedivides	$-\frac{-\frac{b\cosh(x)^2}{2}+\cosh(x)a}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(x))}{b^3}$	39
default	$-\frac{-\frac{b\cosh(x)^2}{2}+\cosh(x)a}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(x))}{b^3}$	39
risch	$-\frac{x\,a^2}{b^3}+\frac{x}{b}+\frac{e^{2x}}{8b}-\frac{a\,e^{x}}{2b^2}-\frac{a\,e^{-x}}{2b^2}+\frac{e^{-2x}}{8b}+\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)a^2}{b^3}-\frac{\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{b}$	94

input
```
int(sinh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b^2*(-1/2*b*cosh(x)^2+cosh(x)*a)+(a^2-b^2)*ln(a+b*cosh(x))/b^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 234 vs. $2(38) = 76$.

Time = 0.09 (sec) , antiderivative size = 234, normalized size of antiderivative = 5.85

$$\int \frac{\sinh^3(x)}{a + b\cosh(x)}\,dx$$
$$= \frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 - 4\,ab\cosh(x)^3 - 8\,(a^2 - b^2)x\cosh(x)^2 + 4\,(b^2\cosh(x) - ab)\sinh(x)^3 - 4\,ab}{}$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(x)^4 + b^2*sinh(x)^4 - 4*a*b*cosh(x)^3 - 8*(a^2 - b^2)*x*cos
h(x)^2 + 4*(b^2*cosh(x) - a*b)*sinh(x)^3 - 4*a*b*cosh(x) + 2*(3*b^2*cosh(x
)^2 - 6*a*b*cosh(x) - 4*(a^2 - b^2)*x)*sinh(x)^2 + b^2 + 8*((a^2 - b^2)*co
sh(x)^2 + 2*(a^2 - b^2)*cosh(x)*sinh(x) + (a^2 - b^2)*sinh(x)^2)*log(2*(b*
cosh(x) + a)/(cosh(x) - sinh(x))) + 4*(b^2*cosh(x)^3 - 3*a*b*cosh(x)^2 - 4
*(a^2 - b^2)*x*cosh(x) - a*b)*sinh(x))/(b^3*cosh(x)^2 + 2*b^3*cosh(x)*sinh
(x) + b^3*sinh(x)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(x)}{a + b\cosh(x)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**3/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = -\frac{\left(4\,ae^{(-x)}-b\right)e^{(2\,x)}}{8\,b^2} - \frac{4\,ae^{(-x)}-be^{(-2\,x)}}{8\,b^2} + \frac{(a^2-b^2)x}{b^3} + \frac{(a^2-b^2)\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{b^3}$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-1/8*(4*a*e^(-x) - b)*e^(2*x)/b^2 - 1/8*(4*a*e^(-x) - b*e^(-2*x))/b^2 + (a
^2 - b^2)*x/b^3 + (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/b^3
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = \frac{b\left(e^{(-x)}+e^x\right)^2-4\,a\left(e^{(-x)}+e^x\right)}{8\,b^2} + \frac{(a^2-b^2)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{b^3}$$

input
```
integrate(sinh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
1/8*(b*(e^(-x) + e^x)^2 - 4*a*(e^(-x) + e^x))/b^2 + (a^2 - b^2)*log(abs(b*
(e^(-x) + e^x) + 2*a))/b^3
```

Mupad [B] (verification not implemented)

Time = 2.08 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.98

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = \frac{\mathrm{e}^{-2\,x}}{8\,b}+\frac{\mathrm{e}^{2\,x}}{8\,b}+\frac{\ln\left(b+2\,a\,\mathrm{e}^{x}+b\,\mathrm{e}^{2\,x}\right)\,\left(a^2-b^2\right)}{b^3} - \frac{a\,\mathrm{e}^{x}}{2\,b^2}-\frac{a\,\mathrm{e}^{-x}}{2\,b^2}-\frac{x\,\left(a^2-b^2\right)}{b^3}$$

input
```
int(sinh(x)^3/(a + b*cosh(x)),x)
```

output
```
exp(-2*x)/(8*b) + exp(2*x)/(8*b) + (log(b + 2*a*exp(x) + b*exp(2*x))*(a^2
- b^2))/b^3 - (a*exp(x))/(2*b^2) - (a*exp(-x))/(2*b^2) - (x*(a^2 - b^2))/b
^3
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.88

$$\int \frac{\sinh^3(x)}{a+b\cosh(x)}\,dx = \frac{e^{4x}b^2-4e^{3x}ab+8e^{2x}\log(e^{2x}b+2e^{x}a+b)\,a^2-8e^{2x}\log(e^{2x}b+2e^{x}a+b)\,b^2-8e^{2x}a^2x+8e^{2x}b^2x-4e^{x}ab}{8e^{2x}b^3}$$

input
```
int(sinh(x)^3/(a+b*cosh(x)),x)
```

output
```
(e**(4*x)*b**2 - 4*e**(3*x)*a*b + 8*e**(2*x)*log(e**(2*x)*b + 2*e**x*a + b
)*a**2 - 8*e**(2*x)*log(e**(2*x)*b + 2*e**x*a + b)*b**2 - 8*e**(2*x)*a**2*
x + 8*e**(2*x)*b**2*x - 4*e**x*a*b + b**2)/(8*e**(2*x)*b**3)
```

3.170 $\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 59

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx=-\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b^2}+\frac{\sinh(x)}{b}$$

output
```
-a*x/b^2+2*(a-b)^(1/2)*(a+b)^(1/2)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(
1/2))/b^2+sinh(x)/b
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.92

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx=\frac{-ax+2\sqrt{-a^2+b^2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(x)}{b^2}$$

input
```
Integrate[Sinh[x]^2/(a + b*Cosh[x]),x]
```

output
```
(-(a*x) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]]
+ b*Sinh[x])/b^2
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.24, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 25, 3174, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3174}$$

$$\frac{\int -\frac{b+a\cosh(x)}{a+b\cosh(x)}dx}{b}+\frac{\sinh(x)}{b}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\sinh(x)}{b}-\frac{\int \frac{b+a\cosh(x)}{a+b\cosh(x)}dx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(x)}{b}-\frac{\int \frac{b+a\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{\sinh(x)}{b}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\cosh(x)}dx}{b}}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\sinh(x)}{b}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sinh(x)}{b} - \frac{\frac{ax}{b} - \frac{2(a^2-b^2)\int \frac{1}{-((a-b)\tanh^2(\frac{x}{2}))+a+b}d\tanh(\frac{x}{2})}{b}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\sinh(x)}{b} - \frac{\frac{ax}{b} - \frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh(\frac{x}{2})}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}$$

input
```
Int[Sinh[x]^2/(a + b*Cosh[x]),x]
```

output
```
-(((a*x)/b - (2*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/
(Sqrt[a - b]*b*Sqrt[a + b]))/b) + Sinh[x]/b
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(49) = 98$.

Time = 1.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.69

method	result	si
default	$-\frac{1}{b\left(1+\tanh\left(\frac{x}{2}\right)\right)}-\frac{a\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{b^{2}}-\frac{2\left(-a^{2}+b^{2}\right)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^{2}\sqrt{(a+b)(a-b)}}-\frac{1}{b\left(\tanh\left(\frac{x}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}$	10
risch	$-\frac{ax}{b^{2}}+\frac{e^{x}}{2b}-\frac{e^{-x}}{2b}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(e^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}\right)}{b^{2}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(e^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)}{b^{2}}$	10

input
```
int(sinh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/b/(1+tanh(1/2*x))-a/b^2*ln(1+tanh(1/2*x))-2/b^2*(-a^2+b^2)/((a+b)*(a-b)
)^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/b/(tanh(1/2*x)-1)
+a/b^2*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(49) = 98$.

Time = 0.11 (sec) , antiderivative size = 279, normalized size of antiderivative = 4.73

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx$$
$$= \left[-\frac{2\,ax\cosh\left(x\right) - b\cosh\left(x\right)^2 - b\sinh\left(x\right)^2 - 2\,\sqrt{a^2-b^2}(\cosh\left(x\right)+\sinh\left(x\right))\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,a\right.}{b\cos}\right.}{2\left(b^2\cosh\left(x\right)+\right.}\right.$$
$$-\frac{2\,ax\cosh\left(x\right) - b\cosh\left(x\right)^2 - b\sinh\left(x\right)^2 + 4\,\sqrt{-a^2+b^2}(\cosh\left(x\right)+\sinh\left(x\right))\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)}{a^2-b^2}\right.}{2\left(b^2\cosh\left(x\right)+b^2\sinh\left(x\right)\right)}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 - 2*sqrt(a^2 - b^2)*(cosh
(x) + sinh(x))*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2
- b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*s
inh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*
sinh(x) + b)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x
)), -1/2*(2*a*x*cosh(x) - b*cosh(x)^2 - b*sinh(x)^2 + 4*sqrt(-a^2 + b^2)*(
cosh(x) + sinh(x))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a
^2 - b^2)) + 2*(a*x - b*cosh(x))*sinh(x) + b)/(b^2*cosh(x) + b^2*sinh(x))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(49) = 98$.

Time = 52.90 (sec) , antiderivative size = 892, normalized size of antiderivative = 15.12

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)**2/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(-2*tanh(x/2)**2*atan(tanh(x/2))/(tanh(x/2)**2 - 1) - 2*tan
h(x/2)/(tanh(x/2)**2 - 1) + 2*atan(tanh(x/2))/(tanh(x/2)**2 - 1)), Eq(a, 0
) & Eq(b, 0)), (-x*tanh(x/2)**2/(b*tanh(x/2)**2 - b) + x/(b*tanh(x/2)**2 -
 b) - 2*tanh(x/2)/(b*tanh(x/2)**2 - b), Eq(a, b)), (x*tanh(x/2)**2/(b*tanh
(x/2)**2 - b) - x/(b*tanh(x/2)**2 - b) - 2*tanh(x/2)/(b*tanh(x/2)**2 - b),
 Eq(a, -b)), ((x*sinh(x)**2/2 - x*cosh(x)**2/2 + sinh(x)*cosh(x)/2)/a, Eq(
b, 0)), (-a*x*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2/(b**2*sqrt(a/(a - b
) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) + a*x*sqrt
(a/(a - b) + b/(a - b))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b
**2*sqrt(a/(a - b) + b/(a - b))) - a*log(-sqrt(a/(a - b) + b/(a - b)) + ta
nh(x/2))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**
2*sqrt(a/(a - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh
(x/2))/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b
) + b/(a - b))) + a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))*tanh(x/2)
**2/(b**2*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) +
 b/(a - b))) - a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(b**2*sqrt(a
/(a - b) + b/(a - b))*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) - 2
*b*sqrt(a/(a - b) + b/(a - b))*tanh(x/2)/(b**2*sqrt(a/(a - b) + b/(a - b))
*tanh(x/2)**2 - b**2*sqrt(a/(a - b) + b/(a - b))) - b*log(-sqrt(a/(a - b)
+ b/(a - b)) + tanh(x/2))*tanh(x/2)**2/(b**2*sqrt(a/(a - b) + b/(a - b)...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.15

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = -\frac{ax}{b^2} - \frac{e^{(-x)}}{2b} + \frac{e^x}{2b} + \frac{2(a^2-b^2)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-a*x/b^2 - 1/2*e^(-x)/b + 1/2*e^x/b + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqr
t(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2)
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.36

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = \frac{e^x}{2b} - \frac{e^{-x}}{2b} - \frac{a\,x}{b^2} + \frac{\ln\left(-\frac{2\,e^x\,(a^2-b^2)}{b^3} - \frac{2\sqrt{a+b}\,(b+a\,e^x)\,\sqrt{a-b}}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2} - \frac{\ln\left(\frac{2\sqrt{a+b}\,(b+a\,e^x)\,\sqrt{a-b}}{b^3} - \frac{2\,e^x\,(a^2-b^2)}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2}$$

input
```
int(sinh(x)^2/(a + b*cosh(x)),x)
```

output
```
exp(x)/(2*b) - exp(-x)/(2*b) - (a*x)/b^2 + (log(- (2*exp(x)*(a^2 - b^2))/b
^3 - (2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3)*(a + b)^(1/2)*(a
- b)^(1/2))/b^2 - (log((2*(a + b)^(1/2)*(b + a*exp(x))*(a - b)^(1/2))/b^3
- (2*exp(x)*(a^2 - b^2))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/b^2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(x)}{a+b\cosh(x)}\,dx = \frac{-4e^x\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)+e^{2x}b-2e^x ax-b}{2e^x b^2}$$

input
```
int(sinh(x)^2/(a+b*cosh(x)),x)
```

output
```
( - 4*e**x*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)) +
e**(2*x)*b - 2*e**x*a*x - b)/(2*e**x*b**2)
```

3.171 $\int \frac{\sinh(x)}{a+b\cosh(x)} \, dx$

Optimal result

Integrand size = 11, antiderivative size = 11

$$\int \frac{\sinh(x)}{a+b\cosh(x)} \, dx = \frac{\log(a+b\cosh(x))}{b}$$

output
```
ln(a+b*cosh(x))/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a+b\cosh(x)} \, dx = \frac{\log(a+b\cosh(x))}{b}$$

input
```
Integrate[Sinh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[a + b*Cosh[x]]/b
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3042, 26, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\cos\left(-\frac{\pi}{2}+ix\right)}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\cos\left(ix-\frac{\pi}{2}\right)}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{b}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a+b\cosh(x))}{b}$$

input
```
Int[Sinh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[a + b*Cosh[x]]/b
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.25 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.09

method	result	size
derivativedivides	$\frac{\ln(a+b\cosh(x))}{b}$	12
default	$\frac{\ln(a+b\cosh(x))}{b}$	12
risch	$-\frac{x}{b}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)}{b}$	27

input
```
int(sinh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*cosh(x))/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 27 vs. $2(11) = 22$.

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 2.45

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = -\frac{x-\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)}{b}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(x - log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/b
```

Sympy [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = \begin{cases} \frac{\log\left(\frac{a}{b}+\cosh(x)\right)}{b} & \text{for } b \neq 0 \\ \frac{\cosh(x)}{a} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x)
```

output
```
Piecewise((log(a/b + cosh(x))/b, Ne(b, 0)), (cosh(x)/a, True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a+b\cosh(x)}\,dx = \frac{\log\left(b\cosh(x)+a\right)}{b}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
log(b*cosh(x) + a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.73

$$\int \frac{\sinh(x)}{a + b\cosh(x)}\,dx = \frac{\log\left(\left|b\left(e^{(-x)} + e^{x}\right) + 2\,a\right|\right)}{b}$$

input
```
integrate(sinh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
log(abs(b*(e^(-x) + e^x) + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + b\cosh(x)}\,dx = \frac{\ln\left(a + b\cosh(x)\right)}{b}$$

input
```
int(sinh(x)/(a + b*cosh(x)),x)
```

output
```
log(a + b*cosh(x))/b
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(x)}{a + b\cosh(x)}\,dx = \frac{\log(\cosh\left(x\right)b + a)}{b}$$

input
```
int(sinh(x)/(a+b*cosh(x)),x)
```

output
```
log(cosh(x)*b + a)/b
```

3.172 $\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 53

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx=\frac{\log(1-\cosh(x))}{2(a+b)}-\frac{\log(1+\cosh(x))}{2(a-b)}+\frac{b\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
ln(1-cosh(x))/(2*a+2*b)-ln(1+cosh(x))/(2*a-2*b)+b*ln(a+b*cosh(x))/(a^2-b^2
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx=\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{-a+b}+\frac{b\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a+b}$$

input
```
Integrate[Csch[x]/(a + b*Cosh[x]),x]
```

output
```
Log[Cosh[x/2]]/(-a + b) + (b*Log[a + b*Cosh[x]])/(a^2 - b^2) + Log[Sinh[x/
2]]/(a + b)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.455, Rules used = {3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-b\int \frac{1}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))$$

$$\Big\downarrow \text{477}$$

$$-\frac{\int \left(-\frac{b^2}{(a^2-b^2)(a+b\cosh(x))}+\frac{b}{2(a+b)(b-b\cosh(x))}+\frac{b}{2(a-b)(\cosh(x)b+b)}\right)d(b\cosh(x))}{b}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\frac{b^2\log(a+b\cosh(x))}{a^2-b^2}-\frac{b\log(b-b\cosh(x))}{2(a+b)}+\frac{b\log(b\cosh(x)+b)}{2(a-b)}}{b}$$

input
```
Int[Csch[x]/(a + b*Cosh[x]),x]
```

output
```
-((-1/2*(b*Log[b - b*Cosh[x]])/(a + b) - (b^2*Log[a + b*Cosh[x]])/(a^2 - b
^2) + (b*Log[b + b*Cosh[x]])/(2*(a - b)))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}+\frac{b\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-b\tanh\left(\frac{x}{2}\right)^{2}-a-b\right)}{(a+b)(a-b)}$	52
risch	$-\frac{x}{a+b}+\frac{x}{a-b}-\frac{2xb}{a^{2}-b^{2}}+\frac{\ln(e^{x}-1)}{a+b}-\frac{\ln(e^{x}+1)}{a-b}+\frac{b\ln\left(e^{2x}+\frac{2a\,e^{x}}{b}+1\right)}{a^{2}-b^{2}}$	87

input
```
int(csch(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)*ln(tanh(1/2*x))+b/(a+b)/(a-b)*ln(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a
-b)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.09

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{b\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)+(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
(b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sinh
(x) + 1) + (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F]

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{csch}\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)/(a + b*cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.11

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{b\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^2-b^2} - \frac{\log\left(e^{(-x)}+1\right)}{a-b} + \frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
b*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) - log(e^(-x) + 1)/(a - b) +
 log(e^(-x) - 1)/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{b^2\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^2b-b^3} - \frac{\log\left(e^{(-x)}+e^x+2\right)}{2\,(a-b)} + \frac{\log\left(e^{(-x)}+e^x-2\right)}{2\,(a+b)}$$

input
```
integrate(csch(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
b^2*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) - 1/2*log(e^(-x) + e^x
+ 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.02

$$\begin{aligned}\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx &= \frac{\ln\left(128\,a\,b^2-128\,a^2\,b+32\,a^3-32\,a^3\,\mathrm{e}^x-128\,a\,b^2\,\mathrm{e}^x+128\,a^2\,b\,\mathrm{e}^x\right)}{a+b} \\ &\quad - \frac{\ln\left(128\,a\,b^2+128\,a^2\,b+32\,a^3+32\,a^3\,\mathrm{e}^x+128\,a\,b^2\,\mathrm{e}^x+128\,a^2\,b\,\mathrm{e}^x\right)}{a-b} \\ &\quad + \frac{b\,\ln\left(16\,b^3\,\mathrm{e}^{2\,x}-4\,a^2\,b+16\,b^3-8\,a^3\,\mathrm{e}^x+32\,a\,b^2\,\mathrm{e}^x-4\,a^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}\end{aligned}$$

input
```
int(1/(sinh(x)*(a + b*cosh(x))),x)
```

output
```
log(128*a*b^2 - 128*a^2*b + 32*a^3 - 32*a^3*exp(x) - 128*a*b^2*exp(x) + 12
8*a^2*b*exp(x))/(a + b) - log(128*a*b^2 + 128*a^2*b + 32*a^3 + 32*a^3*exp(
x) + 128*a*b^2*exp(x) + 128*a^2*b*exp(x))/(a - b) + (b*log(16*b^3*exp(2*x)
 - 4*a^2*b + 16*b^3 - 8*a^3*exp(x) + 32*a*b^2*exp(x) - 4*a^2*b*exp(2*x)))/
(a^2 - b^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.25

$$\int \frac{\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{\log(e^x-1)\,a-\log(e^x-1)\,b-\log(e^x+1)\,a-\log(e^x+1)\,b+\log(e^{2x}b+2e^xa+b)\,b}{a^2-b^2}$$

input
```
int(csch(x)/(a+b*cosh(x)),x)
```

output
```
(log(e**x - 1)*a - log(e**x - 1)*b - log(e**x + 1)*a - log(e**x + 1)*b + l
og(e**(2*x)*b + 2*e**x*a + b)*b)/(a**2 - b**2)
```

3.173 $\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx = \frac{2b^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}+\frac{(b-a\cosh(x))\operatorname{csch}(x)}{a^2-b^2}$$

output
```
2*b^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)
+(b-a*cosh(x))*csch(x)/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.15

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx = \frac{2b^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}$$

input
```
Integrate[Csch[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*b^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) -
Coth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.538, Rules used = {3042, 25, 3175, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^2\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\int \frac{b^2}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{b^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}+\frac{b^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2b^2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{221}$$

$$\frac{2b^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}+\frac{\operatorname{csch}(x)(b-a\cosh(x))}{a^2-b^2}$$

input
```
Int[Csch[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*b^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a +
 b]*(a^2 - b^2)) + ((b - a*Cosh[x])*Csch[x])/(a^2 - b^2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.16

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}+\frac{2b^{2}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}$	78
risch	$-\frac{2(-e^{x}b+a)}{(e^{2x}-1)(a^{2}-b^{2})}+\frac{b^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}-\frac{b^{2}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}$	167

input
```
int(csch(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a-b)*tanh(1/2*x)+2/(a+b)/(a-b)*b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tanh(1/2*x)/((a+b)*(a-b))^(1/2))-1/2/(a+b)/tanh(1/2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. 2(59) = 118.

Time = 0.08 (sec) , antiderivative size = 470, normalized size of antiderivative = 7.01

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{2a^{3}-2ab^{2}+\left(b^{2}\cosh\left(x\right)^{2}+2b^{2}\cosh\left(x\right)\sinh\left(x\right)+b^{2}\sinh\left(x\right)^{2}-b^{2}\right)\sqrt{a^{2}-b^{2}}\log\left(\frac{b^{2}\cosh(x)^{2}+b^{2}\sinh(x)^{2}}{}\right.}{a^{4}-2a^{2}b^{2}+b^{4}-\left(a^{4}-2a^{2}b^{2}+b^{4}\right)\cosh\left(x\right)^{2}-2(a}\right.$$

input
```
integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(2*a^3 - 2*a*b^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2
 - b^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
 + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh
(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh
(x) + a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x)
)/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^
3 - a*b^2 + (b^2*cosh(x)^2 + 2*b^2*cosh(x)*sinh(x) + b^2*sinh(x)^2 - b^2)*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2
 - b^2)) - (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2
 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos
h(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(csch(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**2/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx=\frac{2\,b^2\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

input
```
integrate(csch(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2))
+ 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.88

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx=-\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,\mathrm{e}^x}{a^2-b^2}}{\mathrm{e}^{2\,x}-1}$$

$$-\frac{2\,\mathrm{atan}\left(\left(\mathrm{e}^x\left(\frac{2}{{\left(a^2-b^2\right)}^2\,\sqrt{b^4}}+\frac{2\,a\,\left(a^3\,\sqrt{b^4}-a\,b^2\,\sqrt{b^4}\right)}{b^4\,\left(a^2-b^2\right)\,\sqrt{-{\left(a^2-b^2\right)}^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)-\frac{2\,a\,\left(b^3\,\sqrt{b^4}-a^2\,b\,\sqrt{b^4}\right)}{b^4\,\left(a^2-b^2\right)\,\sqrt{-{\left(a^2-b^2\right)}^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\cdots}}\cdots}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

input
```
int(1/(sinh(x)^2*(a + b*cosh(x))),x)
```

output
```
- ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan(
(exp(x)*(2/((a^2 - b^2)^2*(b^4)^(1/2)) + (2*a*(a^3*(b^4)^(1/2) - a*b^2*(b^
4)^(1/2)))/(b^4*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4
+ 3*a^4*b^2)^(1/2))) - (2*a*(b^3*(b^4)^(1/2) - a^2*b*(b^4)^(1/2)))/(b^4*(a
^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2)
))*((b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(b^6 - a^6
- 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2))*(b^4)^(1/2))/(b^6 - a^6 - 3*a^2*b^4 +
3*a^4*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.49

$$\int \frac{\operatorname{csch}^2(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-2e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)b^2+2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)b^2-2e^{2x}a^3+2e^{2x}a\,b^2+2e^x a^2b-2e^x}{e^{2x}a^4-2e^{2x}a^2b^2+e^{2x}b^4-a^4+2a^2b^2-b^4}$$

input
```
int(csch(x)^2/(a+b*cosh(x)),x)
```

output
```
(2*( - e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2
))*b**2 + sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*b**
2 - e**(2*x)*a**3 + e**(2*x)*a*b**2 + e**x*a**2*b - e**x*b**3))/(e**(2*x)*
a**4 - 2*e**(2*x)*a**2*b**2 + e**(2*x)*b**4 - a**4 + 2*a**2*b**2 - b**4)
```

3.174 $\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 99

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{1}{4(a+b)(1-\cosh(x))} - \frac{1}{4(a-b)(1+\cosh(x))} - \frac{(a+2b)\log(1-\cosh(x))}{4(a+b)^2} + \frac{(a-2b)\log(1+\cosh(x))}{4(a-b)^2} + \frac{b^3\log(a+b\cosh(x))}{(a^2-b^2)^2}$$

output
```
1/4/(a+b)/(1-cosh(x))-1/4/(a-b)/(1+cosh(x))-1/4*(a+2*b)*ln(1-cosh(x))/(a+b
)^2+1/4*(a-2*b)*ln(1+cosh(x))/(a-b)^2+b^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{1}{8}\left(-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b} + \frac{4(a-2b)\log\left(\cosh\left(\frac{x}{2}\right)\right)}{(a-b)^2} + \frac{8b^3\log(a+b\cosh(x))}{(a^2-b^2)^2} - \frac{4(a+2b)\log\left(\sinh\left(\frac{x}{2}\right)\right)}{(a+b)^2} - \frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Csch[x]^3/(a + b*Cosh[x]),x]
```

output
```
(-(Csch[x/2]^2/(a + b)) + (4*(a - 2*b)*Log[Cosh[x/2]])/(a - b)^2 + (8*b^3*
Log[a + b*Cosh[x]])/(a^2 - b^2)^2 - (4*(a + 2*b)*Log[Sinh[x/2]])/(a + b)^2
 - Sech[x/2]^2/(a - b))/8
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.18, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^3\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^3\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$b^3\int \frac{1}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)^2}d(b\cosh(x))$$

$$\Big\downarrow \text{477}$$

$$\frac{\int \left(\frac{b^4}{(a^2-b^2)^2(a+b\cosh(x))}+\frac{b^2}{4(a+b)(b-b\cosh(x))^2}+\frac{b^2}{4(a-b)(\cosh(x)b+b)^2}+\frac{(a+2b)b}{4(a+b)^2(b-b\cosh(x))}+\frac{(a-2b)b}{4(a-b)^2(\cosh(x)b+b)}\right)d(b\cos}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^4\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{b^2}{4(a+b)(b-b\cosh(x))}-\frac{b^2}{4(a-b)(b\cosh(x)+b)}-\frac{b(a+2b)\log(b-b\cosh(x))}{4(a+b)^2}+\frac{b(a-2b)\log(b\cosh(x)+b)}{4(a-b)^2}}{b}$$

input
```
Int[Csch[x]^3/(a + b*Cosh[x]),x]
```

output
```
(b^2/(4*(a + b)*(b - b*Cosh[x])) - b^2/(4*(a - b)*(b + b*Cosh[x])) - (b*(a
 + 2*b)*Log[b - b*Cosh[x]])/(4*(a + b)^2) + (b^4*Log[a + b*Cosh[x]])/(a^2
- b^2)^2 + ((a - 2*b)*b*Log[b + b*Cosh[x]])/(4*(a - b)^2))/b
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

method	result
default	$\frac{b^{3} \ln \left(\tanh \left(\frac{x}{2}\right)^{2} a-b \tanh \left(\frac{x}{2}\right)^{2}-a-b\right)}{\left(a+b\right)^{2} \left(a-b\right)^{2}}+\frac{\tanh \left(\frac{x}{2}\right)^{2}}{8 a-8 b}-\frac{1}{8 \left(a+b\right) \tanh \left(\frac{x}{2}\right)^{2}}+\frac{\left(-2 a-4 b\right) \ln \left(\tanh \left(\frac{x}{2}\right)\right)}{4 \left(a+b\right)^{2}}$
risch	$\frac{a x}{2 a^{2}+4 a b+2 b^{2}}+\frac{x b}{a^{2}+2 a b+b^{2}}-\frac{x a}{2 \left(a^{2}-2 a b+b^{2}\right)}+\frac{x b}{a^{2}-2 a b+b^{2}}-\frac{2 x \,b^{3}}{a^{4}-2 a^{2} b^{2}+b^{4}}-\frac{{\mathrm e}^{x} \left({\mathrm e}^{2 x} a-2 \,{\mathrm e}^{x} b+a\right)}{\left({\mathrm e}^{2 x}-1\right)^{2} \left(a^{2}-b^{2}\right)}-\frac{\ln \left({\mathrm e}^{x}-1\right) a}{2 \left(a^{2}+2 a b+b^{2}\right)}-$

input
```
int(csch(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
b^3/(a+b)^2/(a-b)^2*ln(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)+1/8*tanh(1/2*x
)^2/(a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(-2*a-4*b)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(89) = 178.

Time = 0.11 (sec) , antiderivative size = 818, normalized size of antiderivative = 8.26

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 - a*b^2)*cosh(x)^3 + 2*(a^3 - a*b^2)*sinh(x)^3 - 4*(a^2*b - b
^3)*cosh(x)^2 - 2*(2*a^2*b - 2*b^3 - 3*(a^3 - a*b^2)*cosh(x))*sinh(x)^2 +
2*(a^3 - a*b^2)*cosh(x) - 2*(b^3*cosh(x)^4 + 4*b^3*cosh(x)*sinh(x)^3 + b^3
*sinh(x)^4 - 2*b^3*cosh(x)^2 + b^3 + 2*(3*b^3*cosh(x)^2 - b^3)*sinh(x)^2 +
 4*(b^3*cosh(x)^3 - b^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) -
 sinh(x))) - ((a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 - 2*b^3
)*cosh(x)*sinh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 -
2*b^3 - 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 - 2*b^3 - 3
*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 - 2*b^3)
*cosh(x)^3 - (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(
x) + 1) + ((a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^4 + 4*(a^3 - 3*a*b^2 + 2*b^3)*c
osh(x)*sinh(x)^3 + (a^3 - 3*a*b^2 + 2*b^3)*sinh(x)^4 + a^3 - 3*a*b^2 + 2*b
^3 - 2*(a^3 - 3*a*b^2 + 2*b^3)*cosh(x)^2 - 2*(a^3 - 3*a*b^2 + 2*b^3 - 3*(a
^3 - 3*a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^2 + 4*((a^3 - 3*a*b^2 + 2*b^3)*co
sh(x)^3 - (a^3 - 3*a*b^2 + 2*b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x)
- 1) + 2*(a^3 - a*b^2 + 3*(a^3 - a*b^2)*cosh(x)^2 - 4*(a^2*b - b^3)*cosh(x
))*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)
*cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
 b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*
(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx=\int \frac{\operatorname{csch}^3\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

input
```
integrate(csch(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**3/(a + b*cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.56

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{b^3\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^4-2\,a^2b^2+b^4} + \frac{(a-2\,b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)} - \frac{(a+2\,b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)} - \frac{ae^{(-x)}-2\,be^{(-2\,x)}+ae^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}}$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
b^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(a - 2*
b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) - 1/2*(a + 2*b)*log(e^(-x) - 1)/(a^
2 + 2*a*b + b^2) - (a*e^(-x) - 2*b*e^(-2*x) + a*e^(-3*x))/(a^2 - b^2 - 2*(
a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 179 vs. 2(89) = 178.

Time = 0.11 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.81

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{b^4\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^4b-2\,a^2b^3+b^5} + \frac{(a-2\,b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)} - \frac{(a+2\,b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)} + \frac{b^3\left(e^{(-x)}+e^x\right)^2-2\,a^3\left(e^{(-x)}+e^x\right)+2\,ab^2\left(e^{(-x)}+e^x\right)+4\,a^2b-8\,b^3}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

input
```
integrate(csch(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
b^4*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(a -
2*b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) - 1/4*(a + 2*b)*log(e^(-x)
+ e^x - 2)/(a^2 + 2*a*b + b^2) + 1/2*(b^3*(e^(-x) + e^x)^2 - 2*a^3*(e^(-x)
 + e^x) + 2*a*b^2*(e^(-x) + e^x) + 4*a^2*b - 8*b^3)/((a^4 - 2*a^2*b^2 + b^
4)*((e^(-x) + e^x)^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 291, normalized size of antiderivative = 2.94

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{\frac{2\,(a^2\,b-b^3)}{(a^2-b^2)^2}+\frac{\mathrm{e}^x\,(a\,b^2-a^3)}{(a^2-b^2)^2}}{\mathrm{e}^{2\,x}-1}+\frac{\frac{2\,b}{a^2-b^2}-\frac{2\,a\,\mathrm{e}^x}{a^2-b^2}}{\mathrm{e}^{4\,x}-2\,\mathrm{e}^{2\,x}+1} + \frac{b^3\,\ln\left(16\,b^7\,\mathrm{e}^{2\,x}-a^6\,b+16\,b^7-9\,a^2\,b^5+6\,a^4\,b^3-2\,a^7\,\mathrm{e}^x-9\,a^2\,b^5\,\mathrm{e}^{2\,x}+6\,a^4\,b^3\,\mathrm{e}^{2\,x}+32\,a\,b^6\,\mathrm{e}^x-a^6\,b\,\mathrm{e}\right.}{a^4-2\,a^2\,b^2+b^4} - \frac{\ln(\mathrm{e}^x-1)\,(a+2\,b)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{\ln(\mathrm{e}^x+1)\,(a-2\,b)}{2\,a^2-4\,a\,b+2\,b^2}$$

input
```
int(1/(sinh(x)^3*(a + b*cosh(x))),x)
```

output
```
((2*(a^2*b - b^3))/(a^2 - b^2)^2 + (exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(
exp(2*x) - 1) + ((2*b)/(a^2 - b^2) - (2*a*exp(x))/(a^2 - b^2))/(exp(4*x) -
 2*exp(2*x) + 1) + (b^3*log(16*b^7*exp(2*x) - a^6*b + 16*b^7 - 9*a^2*b^5 +
 6*a^4*b^3 - 2*a^7*exp(x) - 9*a^2*b^5*exp(2*x) + 6*a^4*b^3*exp(2*x) + 32*a
*b^6*exp(x) - a^6*b*exp(2*x) - 18*a^3*b^4*exp(x) + 12*a^5*b^2*exp(x)))/(a^
4 + b^4 - 2*a^2*b^2) - (log(exp(x) - 1)*(a + 2*b))/(4*a*b + 2*a^2 + 2*b^2)
 + (log(exp(x) + 1)*(a - 2*b))/(2*a^2 - 4*a*b + 2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 494, normalized size of antiderivative = 4.99

$$\int \frac{\operatorname{csch}^3(x)}{a+b\cosh(x)}\,dx = \frac{-2b^3-\log(e^x-1)\,a^3+\log(e^x+1)\,a^3-2e^{3x}a^3-2e^xa^3-2\log(e^x-1)\,b^3-2\log(e^x+1)\,b^3+2\log(e^{2x}}{}$$

input
```
int(csch(x)^3/(a+b*cosh(x)),x)
```

output
```
( - e**(4*x)*log(e**x - 1)*a**3 + 3*e**(4*x)*log(e**x - 1)*a*b**2 - 2*e**(
4*x)*log(e**x - 1)*b**3 + e**(4*x)*log(e**x + 1)*a**3 - 3*e**(4*x)*log(e**
x + 1)*a*b**2 - 2*e**(4*x)*log(e**x + 1)*b**3 + 2*e**(4*x)*log(e**(2*x)*b
+ 2*e**x*a + b)*b**3 + 2*e**(4*x)*a**2*b - 2*e**(4*x)*b**3 - 2*e**(3*x)*a*
*3 + 2*e**(3*x)*a*b**2 + 2*e**(2*x)*log(e**x - 1)*a**3 - 6*e**(2*x)*log(e*
*x - 1)*a*b**2 + 4*e**(2*x)*log(e**x - 1)*b**3 - 2*e**(2*x)*log(e**x + 1)*
a**3 + 6*e**(2*x)*log(e**x + 1)*a*b**2 + 4*e**(2*x)*log(e**x + 1)*b**3 - 4
*e**(2*x)*log(e**(2*x)*b + 2*e**x*a + b)*b**3 - 2*e**x*a**3 + 2*e**x*a*b**
2 - log(e**x - 1)*a**3 + 3*log(e**x - 1)*a*b**2 - 2*log(e**x - 1)*b**3 + l
og(e**x + 1)*a**3 - 3*log(e**x + 1)*a*b**2 - 2*log(e**x + 1)*b**3 + 2*log(
e**(2*x)*b + 2*e**x*a + b)*b**3 + 2*a**2*b - 2*b**3)/(2*(e**(4*x)*a**4 - 2
*e**(4*x)*a**2*b**2 + e**(4*x)*b**4 - 2*e**(2*x)*a**4 + 4*e**(2*x)*a**2*b*
*2 - 2*e**(2*x)*b**4 + a**4 - 2*a**2*b**2 + b**4))
```

3.175 $\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 110

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{2b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} + \frac{(3b^3+a(2a^2-5b^2)\cosh(x))\operatorname{csch}(x)}{3(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^3(x)}{3(a^2-b^2)}$$

output
```
2*b^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)
+1/3*(3*b^3+a*(2*a^2-5*b^2)*cosh(x))*csch(x)/(a^2-b^2)^2+(b-a*cosh(x))*csc
h(x)^3/(3*a^2-3*b^2)
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.28

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{1}{24}\left(-\frac{48b^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{2(4a+7b)\coth\left(\frac{x}{2}\right)}{(a+b)^2}+\frac{8\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b}-\frac{\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)}+\frac{8a\tanh\left(\frac{x}{2}\right)}{(a-b)^2}-\frac{14b\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)$$

input
```
Integrate[Csch[x]^4/(a + b*Cosh[x]),x]
```

output
```
((-48*b^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
 + (2*(4*a + 7*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b)
 - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (8*a*Tanh[x/2])/(a - b)^2 - (14*b*T
anh[x/2])/(a - b)^2)/24
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.22, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 3175, 3042, 25, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^4\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 3175}$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}-\frac{\int\frac{(2a^2+2b\cosh(x)a-3b^2)\operatorname{csch}^2(x)}{a+b\cosh(x)}dx}{3(a^2-b^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}-\frac{\int-\frac{2a^2-2b\sin\left(ix-\frac{\pi}{2}\right)a-3b^2}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3(a^2-b^2)}$$

$$\Big\downarrow \; 25$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}+\frac{\int\frac{2a^2-2b\sin\left(ix-\frac{\pi}{2}\right)a-3b^2}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3(a^2-b^2)}$$

$$\Big\downarrow \; 3345$$

$$\frac{\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}-\frac{\int-\frac{3b^4}{a+b\cosh(x)}dx}{a^2-b^2}}{3(a^2-b^2)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{3b^4\int\frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}}{3(a^2-b^2)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}$$

$$\Big\downarrow \; 3042$$

$$\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}+\frac{\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}+\frac{3b^4\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{3(a^2-b^2)}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{6b^4\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}}{3(a^2-b^2)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{6b^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}+\frac{\operatorname{csch}(x)\left(a\left(2a^2-5b^2\right)\cosh(x)+3b^3\right)}{a^2-b^2}}{3(a^2-b^2)}+\frac{\operatorname{csch}^3(x)(b-a\cosh(x))}{3(a^2-b^2)}$$

input
```
Int[Csch[x]^4/(a + b*Cosh[x]),x]
```

output
```
((b - a*Cosh[x])*Csch[x]^3)/(3*(a^2 - b^2)) + ((6*b^4*ArcTanh[(Sqrt[a - b]
*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) + ((3*b^3
+ a*(2*a^2 - 5*b^2)*Cosh[x])*Csch[x])/(a^2 - b^2))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 6.72 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.15

method	result
default	$-\frac{\frac{a\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{b\tanh\left(\frac{x}{2}\right)^{3}}{3}-3a\tanh\left(\frac{x}{2}\right)+5b\tanh\left(\frac{x}{2}\right)}{8(a-b)^{2}}-\frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^{3}}-\frac{-3a-5b}{8(a+b)^{2}\tanh\left(\frac{x}{2}\right)}+\frac{2b^{4}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{(a+b)(a-b)}}$
risch	$-\frac{2\left(-3b^{3}\mathrm{e}^{5x}+3a\,b^{2}\mathrm{e}^{4x}-4a^{2}b\,\mathrm{e}^{3x}+10b^{3}\mathrm{e}^{3x}+6a^{3}\mathrm{e}^{2x}-12a\,b^{2}\mathrm{e}^{2x}-3b^{3}\mathrm{e}^{x}-2a^{3}+5b^{2}a\right)}{3\left(a^{2}-b^{2}\right)^{2}\left(\mathrm{e}^{2x}-1\right)^{3}}+\frac{b^{4}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}}-\frac{b^{4}\ln\left(\mathrm{e}\ldots\right.}{\sqrt{a^{2}\ldots}}$

input
```
int(csch(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3-3*a*tanh(1/2*x)+5*b*
tanh(1/2*x))-1/24/(a+b)/tanh(1/2*x)^3-1/8/(a+b)^2*(-3*a-5*b)/tanh(1/2*x)+2
/(a-b)^2/(a+b)^2*b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*
(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1135 vs. $2(97) = 194$.

Time = 0.14 (sec) , antiderivative size = 2339, normalized size of antiderivative = 21.26

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/3*(6*(a^2*b^3 - b^5)*cosh(x)^5 + 6*(a^2*b^3 - b^5)*sinh(x)^5 + 4*a^5 -
14*a^3*b^2 + 10*a*b^4 - 6*(a^3*b^2 - a*b^4)*cosh(x)^4 - 6*(a^3*b^2 - a*b^4
 - 5*(a^2*b^3 - b^5)*cosh(x))*sinh(x)^4 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5)*
cosh(x)^3 + 4*(2*a^4*b - 7*a^2*b^3 + 5*b^5 + 15*(a^2*b^3 - b^5)*cosh(x)^2
- 6*(a^3*b^2 - a*b^4)*cosh(x))*sinh(x)^3 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4)*
cosh(x)^2 - 12*(a^5 - 3*a^3*b^2 + 2*a*b^4 - 5*(a^2*b^3 - b^5)*cosh(x)^3 +
3*(a^3*b^2 - a*b^4)*cosh(x)^2 - (2*a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x))*sin
h(x)^2 + 3*(b^4*cosh(x)^6 + 6*b^4*cosh(x)*sinh(x)^5 + b^4*sinh(x)^6 - 3*b^
4*cosh(x)^4 + 3*b^4*cosh(x)^2 + 3*(5*b^4*cosh(x)^2 - b^4)*sinh(x)^4 - b^4
+ 4*(5*b^4*cosh(x)^3 - 3*b^4*cosh(x))*sinh(x)^3 + 3*(5*b^4*cosh(x)^4 - 6*b
^4*cosh(x)^2 + b^4)*sinh(x)^2 + 6*(b^4*cosh(x)^5 - 2*b^4*cosh(x)^3 + b^4*c
osh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*
b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2
)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) +
2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(a^2*b^3 - b^5)*cosh(x) + 6*(a^2*b^3 -
 b^5 + 5*(a^2*b^3 - b^5)*cosh(x)^4 - 4*(a^3*b^2 - a*b^4)*cosh(x)^3 + 2*(2*
a^4*b - 7*a^2*b^3 + 5*b^5)*cosh(x)^2 - 4*(a^5 - 3*a^3*b^2 + 2*a*b^4)*cosh(
x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)^6 + 6*(a^6 - 3*a
^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 + (a^6 - 3*a^4*b^2 + 3*a^2*b^4
 - b^6)*sinh(x)^6 - a^6 + 3*a^4*b^2 - 3*a^2*b^4 + b^6 - 3*(a^6 - 3*a^4*...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(csch(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**4/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2\,(3\,b^3e^{(5\,x)}-3\,ab^2e^{(4\,x)}+4\,a^2be^{(3\,x)}-10\,b^3e^{(3\,x)}-6\,a^3e^{(2\,x)}+12\,ab^2e^{(2\,x)}+3\,b^3e^x+2\,a^3-5\,ab^2)}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

input
```
integrate(csch(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-
a^2 + b^2)) + 2/3*(3*b^3*e^(5*x) - 3*a*b^2*e^(4*x) + 4*a^2*b*e^(3*x) - 10*
b^3*e^(3*x) - 6*a^3*e^(2*x) + 12*a*b^2*e^(2*x) + 3*b^3*e^x + 2*a^3 - 5*a*b
^2)/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 3.09 (sec) , antiderivative size = 642, normalized size of antiderivative = 5.84

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx = \frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,e^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{e^{4\,x}-2\,e^{2\,x}+1}-\frac{\frac{2\,a\,b^2}{(a^2-b^2)^2}-\frac{2\,b^3\,e^x}{(a^2-b^2)^2}}{e^{2\,x}-1}-\frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,e^x}{3\,(a^2-b^2)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} + \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,b^2}{(a^2-b^2)^2\,\sqrt{b^8}\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\,a\,\left(a^5\,\sqrt{b^8}-2\,a^3\,b^2\,\sqrt{b^8}+a\,b^4\,\sqrt{b^8}\right)}{b^6\,\sqrt{-(a^2-b^2)^5}\,(a^4-2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b^{10}}}\right)\right.\right.}{}$$

input
```
int(1/(sinh(x)^4*(a + b*cosh(x))),x)
```

output
```
((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)
^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((2*a*b^2)/(a^2 - b^2)^2 - (2*b^3*exp(x
))/(a^2 - b^2)^2)/(exp(2*x) - 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(
3*(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) + (2*atan((exp(x)
*((2*b^2)/((a^2 - b^2)^2*(b^8)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)) + (2*a*(a^5*
(b^8)^(1/2) - 2*a^3*b^2*(b^8)^(1/2) + a*b^4*(b^8)^(1/2)))/(b^6*(-(a^2 - b^
2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6
- 10*a^6*b^4 + 5*a^8*b^2)^(1/2))) + (2*a*(b^5*(b^8)^(1/2) - 2*a^2*b^3*(b^8
)^(1/2) + a^4*b*(b^8)^(1/2)))/(b^6*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a
^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1
/2)))*((b^5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2
)^(1/2))/2 - a^2*b^3*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 +
5*a^8*b^2)^(1/2) + (a^4*b*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b
^4 + 5*a^8*b^2)^(1/2))/2))*(b^8)^(1/2))/(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*
b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 481, normalized size of antiderivative = 4.37

$$\int \frac{\operatorname{csch}^4(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-6e^{6x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^xb+a}{\sqrt{-a^2+b^2}}\right)b^4+18e^{4x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^xb+a}{\sqrt{-a^2+b^2}}\right)b^4-18e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^xb+}{\sqrt{-a^2-}}\right.}{3e^{6x}a^6-9e^{6x}a^4b^2+9e^{6x}a^2b^4-3}$$

input
```
int(csch(x)^4/(a+b*cosh(x)),x)
```

output
```
(2*( - 3*e**(6*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b*
*2))*b**4 + 9*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2
 + b**2))*b**4 - 9*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( -
 a**2 + b**2))*b**4 + 3*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**
2 + b**2))*b**4 - e**(6*x)*a**3*b**2 + e**(6*x)*a*b**4 + 3*e**(5*x)*a**2*b
**3 - 3*e**(5*x)*b**5 + 4*e**(3*x)*a**4*b - 14*e**(3*x)*a**2*b**3 + 10*e**
(3*x)*b**5 - 6*e**(2*x)*a**5 + 15*e**(2*x)*a**3*b**2 - 9*e**(2*x)*a*b**4 +
 3*e**x*a**2*b**3 - 3*e**x*b**5 + 2*a**5 - 6*a**3*b**2 + 4*a*b**4))/(3*(e*
*(6*x)*a**6 - 3*e**(6*x)*a**4*b**2 + 3*e**(6*x)*a**2*b**4 - e**(6*x)*b**6
- 3*e**(4*x)*a**6 + 9*e**(4*x)*a**4*b**2 - 9*e**(4*x)*a**2*b**4 + 3*e**(4*
x)*b**6 + 3*e**(2*x)*a**6 - 9*e**(2*x)*a**4*b**2 + 9*e**(2*x)*a**2*b**4 -
3*e**(2*x)*b**6 - a**6 + 3*a**4*b**2 - 3*a**2*b**4 + b**6))
```

3.176 $\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 167

$$\begin{aligned}\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = &-\frac{1}{16(a+b)(1-\cosh(x))^2}-\frac{3a+5b}{16(a+b)^2(1-\cosh(x))}\\ &+\frac{1}{16(a-b)(1+\cosh(x))^2}+\frac{3a-5b}{16(a-b)^2(1+\cosh(x))}\\ &+\frac{(3a^2+9ab+8b^2)\log(1-\cosh(x))}{16(a+b)^3}\\ &-\frac{(3a^2-9ab+8b^2)\log(1+\cosh(x))}{16(a-b)^3}+\frac{b^5\log(a+b\cosh(x))}{(a^2-b^2)^3}\end{aligned}$$

output
```
-1/16/(a+b)/(1-cosh(x))^2-1/16*(3*a+5*b)/(a+b)^2/(1-cosh(x))+1/16/(a-b)/(1
+cosh(x))^2+1/16*(3*a-5*b)/(a-b)^2/(1+cosh(x))+1/16*(3*a^2+9*a*b+8*b^2)*ln
(1-cosh(x))/(a+b)^3-1/16*(3*a^2-9*a*b+8*b^2)*ln(1+cosh(x))/(a-b)^3+b^5*ln(
a+b*cosh(x))/(a^2-b^2)^3
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{1}{64}\left(\frac{2(3a+5b)\operatorname{csch}^2\left(\frac{x}{2}\right)}{(a+b)^2}-\frac{\operatorname{csch}^4\left(\frac{x}{2}\right)}{a+b} - \frac{8(3a^2-9ab+8b^2)\log\left(\cosh\left(\frac{x}{2}\right)\right)}{(a-b)^3}+\frac{64b^5\log(a+b\cosh(x))}{(a^2-b^2)^3} + \frac{8(3a^2+9ab+8b^2)\log\left(\sinh\left(\frac{x}{2}\right)\right)}{(a+b)^3}+\frac{2(3a-5b)\operatorname{sech}^2\left(\frac{x}{2}\right)}{(a-b)^2} + \frac{\operatorname{sech}^4\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Csch[x]^5/(a + b*Cosh[x]),x]
```

output
```
((2*(3*a + 5*b)*Csch[x/2]^2)/(a + b)^2 - Csch[x/2]^4/(a + b) - (8*(3*a^2 -
 9*a*b + 8*b^2)*Log[Cosh[x/2]])/(a - b)^3 + (64*b^5*Log[a + b*Cosh[x]])/(a
^2 - b^2)^3 + (8*(3*a^2 + 9*a*b + 8*b^2)*Log[Sinh[x/2]])/(a + b)^3 + (2*(3
*a - 5*b)*Sech[x/2]^2)/(a - b)^2 + Sech[x/2]^4/(a - b))/64
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.17, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3147, 477, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i}{\cos\left(-\frac{\pi}{2}+ix\right)^5\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^5\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \; 3147$$

$$-b^5\int \frac{1}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)^3}d(b\cosh(x))$$

$$\Big\downarrow \; 477$$

$$-\frac{\int \left(-\frac{b^6}{(a^2-b^2)^3(a+b\cosh(x))}+\frac{b^3}{8(a+b)(b-b\cosh(x))^3}+\frac{b^3}{8(a-b)(\cosh(x)b+b)^3}+\frac{(3a+5b)b^2}{16(a+b)^2(b-b\cosh(x))^2}+\frac{(3a-5b)b^2}{16(a-b)^2(\cosh(x)b+b)^2}+\right.}{b}$$

$$\Big\downarrow \; 2009$$

$$-\frac{-\frac{b(3a^2+9ab+8b^2)\log(b-b\cosh(x))}{16(a+b)^3}+\frac{b(3a^2-9ab+8b^2)\log(b\cosh(x)+b)}{16(a-b)^3}-\frac{b^6\log(a+b\cosh(x))}{(a^2-b^2)^3}+\frac{b^3}{16(a+b)(b-b\cosh(x))^2}-\frac{}{16(a-b)(b}}{b}$$

input
```
Int[Csch[x]^5/(a + b*Cosh[x]),x]
```

output
```
-((b^3/(16*(a + b)*(b - b*Cosh[x])^2) + (b^2*(3*a + 5*b))/(16*(a + b)^2*(b
 - b*Cosh[x])) - b^3/(16*(a - b)*(b + b*Cosh[x])^2) - ((3*a - 5*b)*b^2)/(1
6*(a - b)^2*(b + b*Cosh[x])) - (b*(3*a^2 + 9*a*b + 8*b^2)*Log[b - b*Cosh[x
]])/(16*(a + b)^3) - (b^6*Log[a + b*Cosh[x]])/(a^2 - b^2)^3 + (b*(3*a^2 -
9*a*b + 8*b^2)*Log[b + b*Cosh[x]])/(16*(a - b)^3))/b)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 477
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
a^p   Int[ExpandIntegrand[(c + d*x)^n*(1 - Rt[-b/a, 2]*x)^p*(1 + Rt[-b/a, 2
]*x)^p, x], x], x] /; FreeQ[{a, b, c, d}, x] && ILtQ[p, 0] && IntegerQ[n] &
& NiceSqrtQ[-b/a] &&  !FractionalPowerFactorQ[Rt[-b/a, 2]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 14.53 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.83

method	result
default	$\frac{\left(\tanh\left(\frac{x}{2}\right)^{2} a-b\tanh\left(\frac{x}{2}\right)^{2}-4a+6b\right)^{2}}{64(a-b)^{3}}-\frac{1}{64(a+b)\tanh\left(\frac{x}{2}\right)^{4}}-\frac{-4a-6b}{32(a+b)^{2}\tanh\left(\frac{x}{2}\right)^{2}}+\frac{(6a^{2}+18ab+16b^{2})\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{16(a+b)^{3}}+\frac{b^{5}\ln}{}$
risch	$-\frac{3x\,a^{2}}{8(a^{3}+3a^{2}b+3b^{2}a+b^{3})}-\frac{9xab}{8(a^{3}+3a^{2}b+3b^{2}a+b^{3})}-\frac{x\,b^{2}}{a^{3}+3a^{2}b+3b^{2}a+b^{3}}+\frac{3x\,a^{2}}{8(a^{3}-3a^{2}b+3b^{2}a-b^{3})}-\frac{9xab}{8(a^{3}-3a^{2}b+3b^{2}a-b^{3})}$

input
```
int(csch(x)^5/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/64*(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-4*a+6*b)^2/(a-b)^3-1/64/(a+b)/tanh(1
/2*x)^4-1/32*(-4*a-6*b)/(a+b)^2/tanh(1/2*x)^2+1/16/(a+b)^3*(6*a^2+18*a*b+1
6*b^2)*ln(tanh(1/2*x))+1/(a-b)^3*b^5/(a+b)^3*ln(tanh(1/2*x)^2*a-b*tanh(1/2
*x)^2-a-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3450 vs. $2(151) = 302$.

Time = 0.19 (sec) , antiderivative size = 3450, normalized size of antiderivative = 20.66

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(csch(x)**5/(a+b*cosh(x)),x)
```

output
```
Integral(csch(x)**5/(a + b*cosh(x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(151) = 302$.

Time = 0.05 (sec) , antiderivative size = 348, normalized size of antiderivative = 2.08

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{b^5\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^6-3\,a^4b^2+3\,a^2b^4-b^6} - \frac{(3\,a^2-9\,ab+8\,b^2)\log\left(e^{(-x)}+1\right)}{8\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{(3\,a^2+9\,ab+8\,b^2)\log\left(e^{(-x)}-1\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{8\,b^3e^{(-2\,x)}+8\,b^3e^{(-6\,x)}+(3\,a^3-7\,ab^2)e^{(-x)}-(11\,a^3-15\,ab^2)e^{(-3\,x)}+16\,(a^2b-2\,b^3)e^{(-4\,x)}-(11\,a^3-}{4\,(a^4-2\,a^2b^2+b^4-4\,(a^4-2\,a^2b^2+b^4)e^{(-2\,x)}+6\,(a^4-2\,a^2b^2+b^4)e^{(-4\,x)}-4\,(a^4-2\,a^2b^2+b^4)e}$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
b^5*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) -
 1/8*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^
3) + 1/8*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2
+ b^3) + 1/4*(8*b^3*e^(-2*x) + 8*b^3*e^(-6*x) + (3*a^3 - 7*a*b^2)*e^(-x) -
 (11*a^3 - 15*a*b^2)*e^(-3*x) + 16*(a^2*b - 2*b^3)*e^(-4*x) - (11*a^3 - 15
*a*b^2)*e^(-5*x) + (3*a^3 - 7*a*b^2)*e^(-7*x))/(a^4 - 2*a^2*b^2 + b^4 - 4*
(a^4 - 2*a^2*b^2 + b^4)*e^(-2*x) + 6*(a^4 - 2*a^2*b^2 + b^4)*e^(-4*x) - 4*
(a^4 - 2*a^2*b^2 + b^4)*e^(-6*x) + (a^4 - 2*a^2*b^2 + b^4)*e^(-8*x))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 338 vs. $2(151) = 302$.

Time = 0.12 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.02

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{b^6\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^6b-3\,a^4b^3+3\,a^2b^5-b^7} - \frac{(3\,a^2-9\,ab+8\,b^2)\log\left(e^{(-x)}+e^x+2\right)}{16\,(a^3-3\,a^2b+3\,ab^2-b^3)} + \frac{(3\,a^2+9\,ab+8\,b^2)\log\left(e^{(-x)}+e^x-2\right)}{16\,(a^3+3\,a^2b+3\,ab^2+b^3)} + \frac{3\,b^5\left(e^{(-x)}+e^x\right)^4+3\,a^5\left(e^{(-x)}+e^x\right)^3-10\,a^3b^2\left(e^{(-x)}+e^x\right)^3+7\,ab^4\left(e^{(-x)}+e^x\right)^3+8\,a^2b^3\left(e^{(-x)}+e^x\right)^2}{4\,(a^6-3\,a^4b^2+3\,a^2}$$

input
```
integrate(csch(x)^5/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
b^6*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)
 - 1/16*(3*a^2 - 9*a*b + 8*b^2)*log(e^(-x) + e^x + 2)/(a^3 - 3*a^2*b + 3*a
*b^2 - b^3) + 1/16*(3*a^2 + 9*a*b + 8*b^2)*log(e^(-x) + e^x - 2)/(a^3 + 3*
a^2*b + 3*a*b^2 + b^3) + 1/4*(3*b^5*(e^(-x) + e^x)^4 + 3*a^5*(e^(-x) + e^x
)^3 - 10*a^3*b^2*(e^(-x) + e^x)^3 + 7*a*b^4*(e^(-x) + e^x)^3 + 8*a^2*b^3*(
e^(-x) + e^x)^2 - 32*b^5*(e^(-x) + e^x)^2 - 20*a^5*(e^(-x) + e^x) + 56*a^3
*b^2*(e^(-x) + e^x) - 36*a*b^4*(e^(-x) + e^x) + 16*a^4*b - 64*a^2*b^3 + 96
*b^5)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*((e^(-x) + e^x)^2 - 4)^2)
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.35

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \frac{\frac{4b}{a^2-b^2}-\frac{4a\,e^x}{a^2-b^2}}{6\,e^{4x}-4\,e^{2x}-4\,e^{6x}+e^{8x}+1}-\frac{\frac{2\,(b^5-a^2\,b^3)}{(a^2-b^2)^3}-\frac{e^x\,(3\,a^5-10\,a^3\,b^2+7\,a\,b^4)}{4\,(a^2-b^2)^3}}{e^{2x}-1}$$
$$+\frac{\frac{8\,(a^2\,b-b^3)}{(a^2-b^2)^2}+\frac{6\,e^x\,(a\,b^2-a^3)}{(a^2-b^2)^2}}{3\,e^{2x}-3\,e^{4x}+e^{6x}-1}+\frac{\frac{2\,(2\,a^2\,b-b^3)}{(a^2-b^2)^2}-\frac{e^x\,(a^3+3\,a\,b^2)}{2\,(a^2-b^2)^2}}{e^{4x}-2\,e^{2x}+1}$$
$$+\frac{b^5\,\ln\left(256\,b^{11}\,e^{2x}-9\,a^{10}\,b+256\,b^{11}-225\,a^2\,b^9+300\,a^4\,b^7-190\,a^6\,b^5+60\,a^8\,b^3-18\,a^{11}\,e^x-225\,a^2\,b\right.}{}$$
$$+\frac{\ln(e^x-1)\,(3\,a^2+9\,a\,b+8\,b^2)}{8\,a^3+24\,a^2\,b+24\,a\,b^2+8\,b^3}-\frac{\ln(e^x+1)\,(3\,a^2-9\,a\,b+8\,b^2)}{8\,a^3-24\,a^2\,b+24\,a\,b^2-8\,b^3}$$

input

```
int(1/(sinh(x)^5*(a + b*cosh(x))),x)
```

output

```
((4*b)/(a^2 - b^2) - (4*a*exp(x))/(a^2 - b^2))/(6*exp(4*x) - 4*exp(2*x) -
4*exp(6*x) + exp(8*x) + 1) - ((2*(b^5 - a^2*b^3))/(a^2 - b^2)^3 - (exp(x)*
(7*a*b^4 + 3*a^5 - 10*a^3*b^2))/(4*(a^2 - b^2)^3))/(exp(2*x) - 1) + ((8*(a
^2*b - b^3))/(a^2 - b^2)^2 + (6*exp(x)*(a*b^2 - a^3))/(a^2 - b^2)^2)/(3*ex
p(2*x) - 3*exp(4*x) + exp(6*x) - 1) + ((2*(2*a^2*b - b^3))/(a^2 - b^2)^2 -
 (exp(x)*(3*a*b^2 + a^3))/(2*(a^2 - b^2)^2))/(exp(4*x) - 2*exp(2*x) + 1) +
 (b^5*log(256*b^11*exp(2*x) - 9*a^10*b + 256*b^11 - 225*a^2*b^9 + 300*a^4*
b^7 - 190*a^6*b^5 + 60*a^8*b^3 - 18*a^11*exp(x) - 225*a^2*b^9*exp(2*x) + 3
00*a^4*b^7*exp(2*x) - 190*a^6*b^5*exp(2*x) + 60*a^8*b^3*exp(2*x) + 512*a*b
^10*exp(x) - 9*a^10*b*exp(2*x) - 450*a^3*b^8*exp(x) + 600*a^5*b^6*exp(x) -
 380*a^7*b^4*exp(x) + 120*a^9*b^2*exp(x)))/(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*
b^2) + (log(exp(x) - 1)*(9*a*b + 3*a^2 + 8*b^2))/(24*a*b^2 + 24*a^2*b + 8*
a^3 + 8*b^3) - (log(exp(x) + 1)*(3*a^2 - 9*a*b + 8*b^2))/(24*a*b^2 - 24*a^
2*b + 8*a^3 - 8*b^3)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1180, normalized size of antiderivative = 7.07

$$\int \frac{\operatorname{csch}^5(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
int(csch(x)^5/(a+b*cosh(x)),x)
```

output
```
(3*e**(8*x)*log(e**x - 1)*a**5 - 10*e**(8*x)*log(e**x - 1)*a**3*b**2 + 15*
e**(8*x)*log(e**x - 1)*a*b**4 - 8*e**(8*x)*log(e**x - 1)*b**5 - 3*e**(8*x)
*log(e**x + 1)*a**5 + 10*e**(8*x)*log(e**x + 1)*a**3*b**2 - 15*e**(8*x)*lo
g(e**x + 1)*a*b**4 - 8*e**(8*x)*log(e**x + 1)*b**5 + 8*e**(8*x)*log(e**(2*
x)*b + 2*e**x*a + b)*b**5 + 4*e**(8*x)*a**2*b**3 - 4*e**(8*x)*b**5 + 6*e**
(7*x)*a**5 - 20*e**(7*x)*a**3*b**2 + 14*e**(7*x)*a*b**4 - 12*e**(6*x)*log(
e**x - 1)*a**5 + 40*e**(6*x)*log(e**x - 1)*a**3*b**2 - 60*e**(6*x)*log(e**
x - 1)*a*b**4 + 32*e**(6*x)*log(e**x - 1)*b**5 + 12*e**(6*x)*log(e**x + 1)
*a**5 - 40*e**(6*x)*log(e**x + 1)*a**3*b**2 + 60*e**(6*x)*log(e**x + 1)*a*
b**4 + 32*e**(6*x)*log(e**x + 1)*b**5 - 32*e**(6*x)*log(e**(2*x)*b + 2*e**
x*a + b)*b**5 - 22*e**(5*x)*a**5 + 52*e**(5*x)*a**3*b**2 - 30*e**(5*x)*a*b
**4 + 18*e**(4*x)*log(e**x - 1)*a**5 - 60*e**(4*x)*log(e**x - 1)*a**3*b**2
 + 90*e**(4*x)*log(e**x - 1)*a*b**4 - 48*e**(4*x)*log(e**x - 1)*b**5 - 18*
e**(4*x)*log(e**x + 1)*a**5 + 60*e**(4*x)*log(e**x + 1)*a**3*b**2 - 90*e**
(4*x)*log(e**x + 1)*a*b**4 - 48*e**(4*x)*log(e**x + 1)*b**5 + 48*e**(4*x)*
log(e**(2*x)*b + 2*e**x*a + b)*b**5 + 32*e**(4*x)*a**4*b - 72*e**(4*x)*a**
2*b**3 + 40*e**(4*x)*b**5 - 22*e**(3*x)*a**5 + 52*e**(3*x)*a**3*b**2 - 30*
e**(3*x)*a*b**4 - 12*e**(2*x)*log(e**x - 1)*a**5 + 40*e**(2*x)*log(e**x -
1)*a**3*b**2 - 60*e**(2*x)*log(e**x - 1)*a*b**4 + 32*e**(2*x)*log(e**x - 1
)*b**5 + 12*e**(2*x)*log(e**x + 1)*a**5 - 40*e**(2*x)*log(e**x + 1)*a**...
```

3.177 $\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 159

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{2b^6\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}} + \frac{(15b^5-a(8a^4-26a^2b^2+33b^4)\cosh(x))\operatorname{csch}(x)}{15\,(a^2-b^2)^3} + \frac{(5b^3+a(4a^2-9b^2)\cosh(x))\operatorname{csch}^3(x)}{15\,(a^2-b^2)^2} + \frac{(b-a\cosh(x))\operatorname{csch}^5(x)}{5\,(a^2-b^2)}$$

output
```
2*b^6*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)
+1/15*(15*b^5-a*(8*a^4-26*a^2*b^2+33*b^4)*cosh(x))*csch(x)/(a^2-b^2)^3+1/1
5*(5*b^3+a*(4*a^2-9*b^2)*cosh(x))*csch(x)^3/(a^2-b^2)^2+(b-a*cosh(x))*csch
(x)^5/(5*a^2-5*b^2)
```

Mathematica [A] (verified)

Time = 1.21 (sec) , antiderivative size = 201, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{1}{480}\left(\frac{960b^6\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}} - \frac{2(64a^2+183ab+149b^2)\coth\left(\frac{x}{2}\right)}{(a+b)^3} - \frac{8(19a-29b)\operatorname{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{(a-b)^2} - \frac{96\operatorname{csch}^5(x)\sinh^6\left(\frac{x}{2}\right)}{a-b} + \frac{(19a+29b)\operatorname{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)^2} - \frac{3\operatorname{csch}^6\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} - \frac{2(64a^2-183ab+149b^2)\tanh\left(\frac{x}{2}\right)}{(a-b)^3}\right)$$

input
```
Integrate[Csch[x]^6/(a + b*Cosh[x]),x]
```

output
```
((960*b^6*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2)
 - (2*(64*a^2 + 183*a*b + 149*b^2)*Coth[x/2])/(a + b)^3 - (8*(19*a - 29*b)
*Csch[x]^3*Sinh[x/2]^4)/(a - b)^2 - (96*Csch[x]^5*Sinh[x/2]^6)/(a - b) + (
(19*a + 29*b)*Csch[x/2]^4*Sinh[x])/(2*(a + b)^2) - (3*Csch[x/2]^6*Sinh[x])
/(2*(a + b)) - (2*(64*a^2 - 183*a*b + 149*b^2)*Tanh[x/2])/(a - b)^3)/480
```

Rubi [A] (verified)

Time = 0.97 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.26, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {3042, 25, 3175, 25, 3042, 3345, 3042, 25, 3345, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\cos\left(-\frac{\pi}{2}+ix\right)^6\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\cos\left(ix-\frac{\pi}{2}\right)^6\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{3175}$$

$$\frac{\int -\frac{(4a^2+4b\cosh(x)a-5b^2)\text{csch}^4(x)}{a+b\cosh(x)}dx}{5(a^2-b^2)}+\frac{\text{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\text{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\int \frac{(4a^2+4b\cosh(x)a-5b^2)\text{csch}^4(x)}{a+b\cosh(x)}dx}{5(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\int \frac{4a^2-4b\sin\left(ix-\frac{\pi}{2}\right)a-5b^2}{\cos\left(ix-\frac{\pi}{2}\right)^4\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{5(a^2-b^2)}$$

$$\Big\downarrow \text{3345}$$

$$\frac{\text{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{-\frac{\int \frac{\left(8a^4-18b^2a^2+2b\left(4a^2-9b^2\right)\cosh(x)a+15b^4\right)\text{csch}^2(x)}{a+b\cosh(x)}dx}{3(a^2-b^2)}-\frac{\text{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\text{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{-\frac{\text{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}-\frac{\int -\frac{8a^4-18b^2a^2-2b\left(4a^2-9b^2\right)\sin\left(ix-\frac{\pi}{2}\right)a+15b^4}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}+\frac{\int \frac{8a^4-18b^2a^2-2b\left(4a^2-9b^2\right)\sin\left(ix-\frac{\pi}{2}\right)a+15b^4}{\cos\left(ix-\frac{\pi}{2}\right)^2\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)}dx}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \ 3345$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\frac{-\frac{\int \frac{15b^6}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3(a^2-b^2)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \ 27$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\frac{-\frac{15b^6\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3(a^2-b^2)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \ 3042$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}+\frac{-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}-\frac{15b^6\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \ 3138$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\frac{-\frac{30b^6\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3(a^2-b^2)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

$$\Big\downarrow \ 221$$

$$\frac{\operatorname{csch}^5(x)(b-a\cosh(x))}{5(a^2-b^2)}-\frac{\frac{-\frac{30b^6\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{\operatorname{csch}(x)\left(15b^5-a\left(8a^4-26a^2b^2+33b^4\right)\cosh(x)\right)}{a^2-b^2}}{3(a^2-b^2)}-\frac{\operatorname{csch}^3(x)\left(a\left(4a^2-9b^2\right)\cosh(x)+5b^3\right)}{3(a^2-b^2)}}{5(a^2-b^2)}$$

input
```
Int[Csch[x]^6/(a + b*Cosh[x]),x]
```

output
```
((b - a*Cosh[x])*Csch[x]^5)/(5*(a^2 - b^2)) - (-1/3*((5*b^3 + a*(4*a^2 - 9
*b^2)*Cosh[x])*Csch[x]^3)/(a^2 - b^2) + ((-30*b^6*ArcTanh[(Sqrt[a - b]*Tan
h[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - ((15*b^5 - a
*(8*a^4 - 26*a^2*b^2 + 33*b^4)*Cosh[x])*Csch[x])/(a^2 - b^2))/(3*(a^2 - b^
2)))/(5*(a^2 - b^2))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3175
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[(g*Cos[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^
(m + 1)*((b - a*Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*
(a^2 - b^2)*(p + 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*
(a^2*(p + 2) - b^2*(m + p + 2) + a*b*(m + p + 3)*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && LtQ[p, -1] && IntegersQ
[2*m, 2*p]
```

rule 3345
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_.)*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Simp[(g*Co
s[e + f*x])^(p + 1)*(a + b*Sin[e + f*x])^(m + 1)*((b*c - a*d - (a*c - b*d)*
Sin[e + f*x])/(f*g*(a^2 - b^2)*(p + 1))), x] + Simp[1/(g^2*(a^2 - b^2)*(p +
 1))   Int[(g*Cos[e + f*x])^(p + 2)*(a + b*Sin[e + f*x])^m*Simp[c*(a^2*(p +
 2) - b^2*(m + p + 2)) + a*b*d*m + b*(a*c - b*d)*(m + p + 3)*Sin[e + f*x],
x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && Lt
Q[p, -1] && IntegerQ[2*m]
```

Maple [A] (verified)

Time = 29.24 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.34

method	result
default	$-\frac{\frac{\tanh\left(\frac{x}{2}\right)^{5} a^{2}}{5}-\frac{2\tanh\left(\frac{x}{2}\right)^{5} a b}{5}+\frac{b^{2}\tanh\left(\frac{x}{2}\right)^{5}}{5}-\frac{5\tanh\left(\frac{x}{2}\right)^{3} a^{2}}{3}+4\tanh\left(\frac{x}{2}\right)^{3} a b-\frac{7b^{2}\tanh\left(\frac{x}{2}\right)^{3}}{3}+10a^{2}\tanh\left(\frac{x}{2}\right)-28ab\tanh\left(\frac{x}{2}\right)+22b^{2}\mathrm{ta}}{32(a-b)^{3}}$
risch	$-\frac{2\left(-15b^{5}\mathrm{e}^{9x}+15a\,b^{4}\mathrm{e}^{8x}-20a^{2}b^{3}\mathrm{e}^{7x}+80b^{5}\mathrm{e}^{7x}+30a^{3}b^{2}\mathrm{e}^{6x}-90a\,b^{4}\mathrm{e}^{6x}-48a^{4}b\,\mathrm{e}^{5x}+136a^{2}b^{3}\mathrm{e}^{5x}-178b^{5}\mathrm{e}^{5x}+80a^{5}\mathrm{e}^{4x}-230a^{3}b^{2}\mathrm{e}^{4x}\right.}{15(a^{2}-b^{2})^{3}(\mathrm{e}^{2x}-1)^{5}}$

input
```
int(csch(x)^6/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/32/(a-b)^3*(1/5*tanh(1/2*x)^5*a^2-2/5*tanh(1/2*x)^5*a*b+1/5*b^2*tanh(1/
2*x)^5-5/3*tanh(1/2*x)^3*a^2+4*tanh(1/2*x)^3*a*b-7/3*b^2*tanh(1/2*x)^3+10*
a^2*tanh(1/2*x)-28*a*b*tanh(1/2*x)+22*b^2*tanh(1/2*x))-1/160/(a+b)/tanh(1/
2*x)^5-1/96*(-5*a-7*b)/(a+b)^2/tanh(1/2*x)^3-1/32/(a+b)^3*(10*a^2+28*a*b+2
2*b^2)/tanh(1/2*x)+2/(a-b)^3/(a+b)^3*b^6/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3156 vs. $2(144) = 288$.

Time = 0.16 (sec) , antiderivative size = 6381, normalized size of antiderivative = 40.13

$$\int \frac{\operatorname{csch}^6(x)}{a + b\cosh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^6(x)}{a + b\cosh(x)}\, dx = \text{Timed out}$$

input
```
integrate(csch(x)**6/(a+b*cosh(x)),x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^6(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(144) = 288$.

Time = 0.12 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{2\,b^6\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}} + \frac{2\left(15\,b^5e^{(9\,x)}-15\,ab^4e^{(8\,x)}+20\,a^2b^3e^{(7\,x)}-80\,b^5e^{(7\,x)}-30\,a^3b^2e^{(6\,x)}+90\,ab^4e^{(6\,x)}+48\,a^4be^{(5\,x)}-136\,a\right.}{}$$

input
```
integrate(csch(x)^6/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b^6*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 -
 b^6)*sqrt(-a^2 + b^2)) + 2/15*(15*b^5*e^(9*x) - 15*a*b^4*e^(8*x) + 20*a^2
*b^3*e^(7*x) - 80*b^5*e^(7*x) - 30*a^3*b^2*e^(6*x) + 90*a*b^4*e^(6*x) + 48
*a^4*b*e^(5*x) - 136*a^2*b^3*e^(5*x) + 178*b^5*e^(5*x) - 80*a^5*e^(4*x) +
230*a^3*b^2*e^(4*x) - 240*a*b^4*e^(4*x) + 20*a^2*b^3*e^(3*x) - 80*b^5*e^(3
*x) + 40*a^5*e^(2*x) - 130*a^3*b^2*e^(2*x) + 150*a*b^4*e^(2*x) + 15*b^5*e^
x - 8*a^5 + 26*a^3*b^2 - 33*a*b^4)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(e
^(2*x) - 1)^5)
```

Mupad [B] (verification not implemented)

Time = 3.64 (sec) , antiderivative size = 1031, normalized size of antiderivative = 6.48

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(x)^6*(a + b*cosh(x))),x)
```

output
```
((16*(a*b^2 - a^3))/(a^2 - b^2)^2 + (64*exp(x)*(a^2*b - b^3))/(5*(a^2 - b^
2)^2))/(6*exp(4*x) - 4*exp(2*x) - 4*exp(6*x) + exp(8*x) + 1) - ((2*a*b^4)/
(a^2 - b^2)^3 - (2*b^5*exp(x))/(a^2 - b^2)^3)/(exp(2*x) - 1) - ((32*a)/(5*
(a^2 - b^2)) - (32*b*exp(x))/(5*(a^2 - b^2)))/(5*exp(2*x) - 10*exp(4*x) +
10*exp(6*x) - 5*exp(8*x) + exp(10*x) - 1) + ((8*(3*a*b^2 - 4*a^3))/(3*(a^2
 - b^2)^2) + (8*exp(x)*(12*a^2*b - 7*b^3))/(15*(a^2 - b^2)^2))/(3*exp(2*x)
 - 3*exp(4*x) + exp(6*x) - 1) + ((4*(a*b^4 - a^3*b^2))/(a^2 - b^2)^3 - (8*
exp(x)*(b^5 - a^2*b^3))/(3*(a^2 - b^2)^3))/(exp(4*x) - 2*exp(2*x) + 1) - (
2*atan((exp(x)*((2*b^4)/((a^2 - b^2)^3*(b^12)^(1/2)*(a^6 - b^6 + 3*a^2*b^4
 - 3*a^4*b^2)) + (2*a*(a^7*(b^12)^(1/2) + 3*a^3*b^4*(b^12)^(1/2) - 3*a^5*b
^2*(b^12)^(1/2) - a*b^6*(b^12)^(1/2)))/(b^8*(-(a^2 - b^2)^7)^(1/2)*(a^6 -
b^6 + 3*a^2*b^4 - 3*a^4*b^2)*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*
a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))) - (2*a*(b^7*(b^12
)^(1/2) - 3*a^2*b^5*(b^12)^(1/2) + 3*a^4*b^3*(b^12)^(1/2) - a^6*b*(b^12)^(
1/2)))/(b^8*(-(a^2 - b^2)^7)^(1/2)*(a^6 - b^6 + 3*a^2*b^4 - 3*a^4*b^2)*(b^
14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b
^4 + 7*a^12*b^2)^(1/2)))*((b^7*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 3
5*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4 + 7*a^12*b^2)^(1/2))/2 - (a^6*b*(b^14
 - a^14 - 7*a^2*b^12 + 21*a^4*b^10 - 35*a^6*b^8 + 35*a^8*b^6 - 21*a^10*b^4
 + 7*a^12*b^2)^(1/2))/2 - (3*a^2*b^5*(b^14 - a^14 - 7*a^2*b^12 + 21*a^4...
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 919, normalized size of antiderivative = 5.78

$$\int \frac{\operatorname{csch}^6(x)}{a+b\cosh(x)}\,dx = \frac{30\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right) b^6 - 6e^{10x}a^3b^4 + 6e^{10x}a\,b^6 + 30e^{9x}a^2b^5 + 40e^{7x}a^4b^3 - 200e^{7x}a^2b^5 - 60e^{6x}a^{\cdots}}{}$$

input
```
int(csch(x)^6/(a+b*cosh(x)),x)
```

output
```
(2*( - 15*e**(10*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 +
b**2))*b**6 + 75*e**(8*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a
**2 + b**2))*b**6 - 150*e**(6*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sq
rt( - a**2 + b**2))*b**6 + 150*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b
+ a)/sqrt( - a**2 + b**2))*b**6 - 75*e**(2*x)*sqrt( - a**2 + b**2)*atan((e
**x*b + a)/sqrt( - a**2 + b**2))*b**6 + 15*sqrt( - a**2 + b**2)*atan((e**x
*b + a)/sqrt( - a**2 + b**2))*b**6 - 3*e**(10*x)*a**3*b**4 + 3*e**(10*x)*a
*b**6 + 15*e**(9*x)*a**2*b**5 - 15*e**(9*x)*b**7 + 20*e**(7*x)*a**4*b**3 -
 100*e**(7*x)*a**2*b**5 + 80*e**(7*x)*b**7 - 30*e**(6*x)*a**5*b**2 + 90*e*
*(6*x)*a**3*b**4 - 60*e**(6*x)*a*b**6 + 48*e**(5*x)*a**6*b - 184*e**(5*x)*
a**4*b**3 + 314*e**(5*x)*a**2*b**5 - 178*e**(5*x)*b**7 - 80*e**(4*x)*a**7
+ 310*e**(4*x)*a**5*b**2 - 440*e**(4*x)*a**3*b**4 + 210*e**(4*x)*a*b**6 +
20*e**(3*x)*a**4*b**3 - 100*e**(3*x)*a**2*b**5 + 80*e**(3*x)*b**7 + 40*e**
(2*x)*a**7 - 170*e**(2*x)*a**5*b**2 + 265*e**(2*x)*a**3*b**4 - 135*e**(2*x
)*a*b**6 + 15*e**x*a**2*b**5 - 15*e**x*b**7 - 8*a**7 + 34*a**5*b**2 - 56*a
**3*b**4 + 30*a*b**6))/(15*(e**(10*x)*a**8 - 4*e**(10*x)*a**6*b**2 + 6*e**
(10*x)*a**4*b**4 - 4*e**(10*x)*a**2*b**6 + e**(10*x)*b**8 - 5*e**(8*x)*a**
8 + 20*e**(8*x)*a**6*b**2 - 30*e**(8*x)*a**4*b**4 + 20*e**(8*x)*a**2*b**6
- 5*e**(8*x)*b**8 + 10*e**(6*x)*a**8 - 40*e**(6*x)*a**6*b**2 + 60*e**(6*x)
*a**4*b**4 - 40*e**(6*x)*a**2*b**6 + 10*e**(6*x)*b**8 - 10*e**(4*x)*a**...
```

3.178 $\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 67

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \frac{x}{b^2} - \frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b^2\sqrt{a+b}} - \frac{\sinh(x)}{b(a+b\cosh(x))}$$

output
```
x/b^2-2*a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/b^2/(a+
b)^(1/2)-sinh(x)/b/(a+b*cosh(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.91

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \frac{x + \frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \frac{b\sinh(x)}{a+b\cosh(x)}}{b^2}$$

input
```
Integrate[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

output
```
(x + (2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] -
 (b*Sinh[x])/(a + b*Cosh[x]))/b^2
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.07, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.692, Rules used = {3042, 25, 3172, 25, 3042, 3214, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(-\frac{\pi}{2}+ix\right)^2}{\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(ix-\frac{\pi}{2}\right)^2}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{3172}$$

$$-\frac{\int -\frac{\cosh(x)}{a+b\cosh(x)}dx}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\cosh(x)}{a+b\cosh(x)}dx}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{\int \frac{\sin\left(ix+\frac{\pi}{2}\right)}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\frac{x}{b}-\frac{a\int \frac{1}{a+b\cosh(x)}dx}{b}}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\sinh(x)}{b(a+b\cosh(x))}+\frac{\frac{x}{b}-\frac{a\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}}{b}$$

$$\Big\downarrow \; 3138$$

$$\frac{\frac{x}{b}-\frac{2a\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

$$\Big\downarrow \; 221$$

$$\frac{\frac{x}{b}-\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}}{b}-\frac{\sinh(x)}{b(a+b\cosh(x))}$$

input
```
Int[Sinh[x]^2/(a + b*Cosh[x])^2,x]
```

output
```
(x/b - (2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*S
qrt[a + b]))/b - Sinh[x]/(b*(a + b*Cosh[x]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3172
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + 1))), x] + Simp[g^2*((p - 1)/(b*(m + 1)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^(m + 1)*Sin[e + f*x], x], x] /; Fre
eQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[p, 1] && I
ntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.48

method	result	size
default	$\frac{\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{b^{2}}+\frac{\frac{2b\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}a-b\tanh\left(\frac{x}{2}\right)^{2}-a-b}-\frac{2a\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b^{2}}-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b^{2}}$	99
risch	$\frac{x}{b^{2}}+\frac{2a\,\mathrm{e}^{x}+2b}{b^{2}\left(\mathrm{e}^{2x}b+2a\,\mathrm{e}^{x}+b\right)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{2}}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b^{2}}$	148

input
```
int(sinh(x)^2/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/b^2*ln(1+tanh(1/2*x))+2/b^2*(b*tanh(1/2*x)/(tanh(1/2*x)^2*a-b*tanh(1/2*x
)^2-a-b)-a/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/
2)))-1/b^2*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(57) = 114$.

Time = 0.10 (sec) , antiderivative size = 700, normalized size of antiderivative = 10.45

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="fricas")
```

output
```
[((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh(x)^2 + 2*a^2*b - 2*b^3
+ (a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) +
a^2)*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*c
osh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(
b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(
b*cosh(x) + a)*sinh(x) + b)) + (a^2*b - b^3)*x + 2*(a^3 - a*b^2 + (a^3 - a
*b^2)*x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*cosh(x) + (a^3 - a*b^2
)*x)*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)^2 + (a^2*b^3 - b^5)
*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2 - a*b^4 + (a^2*b^3 -
 b^5)*cosh(x))*sinh(x)), ((a^2*b - b^3)*x*cosh(x)^2 + (a^2*b - b^3)*x*sinh
(x)^2 + 2*a^2*b - 2*b^3 + 2*(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x)
 + a*b + 2*(a*b*cosh(x) + a^2)*sinh(x))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (a^2*b - b^3)*x + 2*(a^
3 - a*b^2 + (a^3 - a*b^2)*x)*cosh(x) + 2*(a^3 - a*b^2 + (a^2*b - b^3)*x*co
sh(x) + (a^3 - a*b^2)*x)*sinh(x))/(a^2*b^3 - b^5 + (a^2*b^3 - b^5)*cosh(x)
^2 + (a^2*b^3 - b^5)*sinh(x)^2 + 2*(a^3*b^2 - a*b^4)*cosh(x) + 2*(a^3*b^2
- a*b^4 + (a^2*b^3 - b^5)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(x)**2/(a+b*cosh(x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.01

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = -\frac{2\,a\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2} + \frac{x}{b^2} + \frac{2\,(ae^x+b)}{(be^{(2\,x)}+2\,ae^x+b)b^2}$$

input
```
integrate(sinh(x)^2/(a+b*cosh(x))^2,x, algorithm="giac")
```

output
```
-2*a*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2) + x/b^2 +
 2*(a*e^x + b)/((b*e^(2*x) + 2*a*e^x + b)*b^2)
```

Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.07

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \frac{x}{b^2} + \frac{\frac{2}{b}+\frac{2\,a\,e^x}{b^2}}{b+2\,a\,e^x+b\,e^{2\,x}} + \frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}-\frac{2\,a\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}} - \frac{a\,\ln\left(\frac{2\,a\,e^x}{b^3}+\frac{2\,a\,(b+a\,e^x)}{b^3\,\sqrt{a+b}\,\sqrt{a-b}}\right)}{b^2\,\sqrt{a+b}\,\sqrt{a-b}}$$

input
```
int(sinh(x)^2/(a + b*cosh(x))^2,x)
```

output
```
x/b^2 + (2/b + (2*a*exp(x))/b^2)/(b + 2*a*exp(x) + b*exp(2*x)) + (a*log((2
*a*exp(x))/b^3 - (2*a*(b + a*exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/
(b^2*(a + b)^(1/2)*(a - b)^(1/2)) - (a*log((2*a*exp(x))/b^3 + (2*a*(b + a*
exp(x)))/(b^3*(a + b)^(1/2)*(a - b)^(1/2))))/(b^2*(a + b)^(1/2)*(a - b)^(1
/2))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.76

$$\int \frac{\sinh^2(x)}{(a+b\cosh(x))^2}\,dx = \frac{2e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)ab + 4e^x\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2 + 2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)ab}{b^2\left(e^{2x}a^2b - e^{2x}b^3 + 2e^x a^3 - 2e^x a b\right.}$$

input
```
int(sinh(x)^2/(a+b*cosh(x))^2,x)
```

output
```
(2*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a
*b + 4*e**x*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a
**2 + 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a*b +
 e**(2*x)*a**2*b*x - e**(2*x)*a**2*b - e**(2*x)*b**3*x + e**(2*x)*b**3 + 2
*e**x*a**3*x - 2*e**x*a*b**2*x + a**2*b*x + a**2*b - b**3*x - b**3)/(b**2*
(e**(2*x)*a**2*b - e**(2*x)*b**3 + 2*e**x*a**3 - 2*e**x*a*b**2 + a**2*b -
b**3))
```

3.179 $\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 113

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \frac{b(3a^2-2b^2)\arctan(\sinh(x))}{2a^4} + \frac{2(a-b)^{3/2}(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^4} - \frac{(4a^2-3b^2)\tanh(x)}{3a^3} - \frac{b\text{sech}(x)\tanh(x)}{2a^2} + \frac{\text{sech}^2(x)\tanh(x)}{3a}$$

output
```
1/2*b*(3*a^2-2*b^2)*arctan(sinh(x))/a^4+2*(a-b)^(3/2)*(a+b)^(3/2)*arctanh(
(a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/a^4-1/3*(4*a^2-3*b^2)*tanh(x)/a^3-1/2
*b*sech(x)*tanh(x)/a^2+1/3*sech(x)^2*tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.88

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx = \frac{6b(3a^2-2b^2)\arctan\left(\tanh\left(\frac{x}{2}\right)\right) - 12(-a^2+b^2)^{3/2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right) + a\left(-8a^2+6b^2-3ab\text{sech}(x)\right.}{6a^4}$$

input
```
Integrate[Tanh[x]^4/(a + b*Cosh[x]),x]
```

output
```
(6*b*(3*a^2 - 2*b^2)*ArcTan[Tanh[x/2]] - 12*(-a^2 + b^2)^(3/2)*ArcTan[((a
- b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] + a*(-8*a^2 + 6*b^2 - 3*a*b*Sech[x] + 2*
a^2*Sech[x]^2)*Tanh[x])/(6*a^4)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.18, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.846$, Rules used = {3042, 3204, 3042, 3534, 27, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^4\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3204}$$

$$-\frac{\int \frac{\left(-3\left(2a^2-b^2\right)\cosh^2(x)-ab\cosh(x)+2\left(4a^2-3b^2\right)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{-3\left(2a^2-b^2\right)\sin\left(ix+\frac{\pi}{2}\right)^2-ab\sin\left(ix+\frac{\pi}{2}\right)+2\left(4a^2-3b^2\right)}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{3534}$$

$$-\frac{\frac{\int -\frac{3\left(b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\cosh(x)\right)\text{sech}(x)}{a+b\cosh(x)}dx}{a}+\frac{2\left(4a^2-3b^2\right)\tanh(x)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\int\frac{\left(b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\cosh(x)\right)\operatorname{sech}(x)}{a+b\cosh(x)}dx}{a}}{6a^2}-\frac{b\tanh(x)\operatorname{sech}(x)}{2a^2}+\frac{\tanh(x)\operatorname{sech}^2(x)}{3a}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\int\frac{b\left(3a^2-2b^2\right)+a\left(2a^2-b^2\right)\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}}{6a^2}-\frac{b\tanh(x)\operatorname{sech}(x)}{2a^2}+\frac{\tanh(x)\operatorname{sech}^2(x)}{3a}$$

$\Big\downarrow$ 3480

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{2\left(a^2-b^2\right)^2\int\frac{1}{a+b\cosh(x)}dx}{a}+\frac{b\left(3a^2-2b^2\right)\int\operatorname{sech}(x)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\operatorname{sech}(x)}{2a^2}+\frac{\tanh(x)\operatorname{sech}^2(x)}{3a}$$

$\Big\downarrow$ 3042

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{2\left(a^2-b^2\right)^2\int\frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}+\frac{b\left(3a^2-2b^2\right)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\operatorname{sech}(x)}{2a^2}+\frac{\tanh(x)\operatorname{sech}^2(x)}{3a}$$

$\Big\downarrow$ 3138

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{4\left(a^2-b^2\right)^2\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{b\left(3a^2-2b^2\right)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\operatorname{sech}(x)}{2a^2}+\frac{\tanh(x)\operatorname{sech}^2(x)}{3a}$$

$\Big\downarrow$ 221

$$-\frac{\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{4\left(a^2-b^2\right)^2\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b\left(3a^2-2b^2\right)\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}\right)}{a}}{6a^2}-\frac{b\tanh(x)\operatorname{sech}(x)}{2a^2}+\frac{\tanh(x)\operatorname{sech}^2(x)}{3a}$$

$\Big\downarrow$ 4257

$$-\frac{-\frac{2(4a^2-3b^2)\tanh(x)}{a}-\frac{3\left(\frac{b(3a^2-2b^2)\arctan(\sinh(x))}{a}+\frac{4(a^2-b^2)^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}\right)}{a}}{6a^2}-\frac{b\tanh(x)\text{sech}(x)}{2a^2}+\frac{\tanh(x)\text{sech}^2(x)}{3a}$$

input
```
Int[Tanh[x]^4/(a + b*Cosh[x]),x]
```

output
```
-1/2*(b*Sech[x]*Tanh[x])/a^2 + (Sech[x]^2*Tanh[x])/(3*a) - ((-3*((b*(3*a^2
 - 2*b^2)*ArcTan[Sinh[x]])/a + (4*(a^2 - b^2)^2*ArcTanh[(Sqrt[a - b]*Tanh[
x/2])/Sqrt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])))/a + (2*(4*a^2 - 3*b^2)*T
anh[x])/a)/(6*a^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3204
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^4,
x_Symbol] :> Simp[(-Cos[e + f*x])*((a + b*Sin[e + f*x])^(m + 1)/(3*a*f*Sin[
e + f*x]^3)), x] + (-Simp[b*(m - 2)*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m +
 1)/(6*a^2*f*Sin[e + f*x]^2)), x] - Simp[1/(6*a^2)   Int[((a + b*Sin[e + f*
x])^m/Sin[e + f*x]^2)*Simp[8*a^2 - b^2*(m - 1)*(m - 2) + a*b*m*Sin[e + f*x]
 - (6*a^2 - b^2*m*(m - 2))*Sin[e + f*x]^2, x], x], x]) /; FreeQ[{a, b, e, f
, m}, x] && NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1] && IntegerQ[2*m]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3534
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(b*c - a*d)*(a^2 - b^2))   Int
[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*
d)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 -
a*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A
*b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ
[n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) |
| EqQ[a, 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.38 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.36

method	result
default	$\frac{2(a+b)^{2}(a-b)^{2}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{4}\sqrt{(a+b)(a-b)}}+\frac{\frac{2\left(\left(-a^{3}+\frac{1}{2}a^{2}b+b^{2}a\right)\tanh\left(\frac{x}{2}\right)^{5}+\left(-\frac{10}{3}a^{3}+2b^{2}a\right)\tanh\left(\frac{x}{2}\right)^{3}+\left(-a^{3}+b^{2}a-\frac{1}{2}a^{2}b\right)\tanh\left(\frac{x}{2}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}}{a^{4}}$
risch	$\frac{-3ab\,\mathrm{e}^{5x}+12a^{2}\mathrm{e}^{4x}-6b^{2}\mathrm{e}^{4x}+12a^{2}\mathrm{e}^{2x}-12b^{2}\mathrm{e}^{2x}+3b\,\mathrm{e}^{x}a+8a^{2}-6b^{2}}{3a^{3}\left(\mathrm{e}^{2x}+1\right)^{3}}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}\right)}{a^{2}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{}}{}\right)}{a^{4}}$

input
```
int(tanh(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*(a+b)^2*(a-b)^2/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)
*(a-b))^(1/2))+2/a^4*(((-a^3+1/2*a^2*b+b^2*a)*tanh(1/2*x)^5+(-10/3*a^3+2*b
^2*a)*tanh(1/2*x)^3+(-a^3+b^2*a-1/2*a^2*b)*tanh(1/2*x))/(tanh(1/2*x)^2+1)^
3+1/2*b*(3*a^2-2*b^2)*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. 2(95) = 190.

Time = 0.15 (sec) , antiderivative size = 2003, normalized size of antiderivative = 17.73

$$\int \frac{\tanh^{4}(x)}{a+b\cosh(x)}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-1/3*(3*a^2*b*cosh(x)^5 + 3*a^2*b*sinh(x)^5 - 6*(2*a^3 - a*b^2)*cosh(x)^4
 + 3*(5*a^2*b*cosh(x) - 4*a^3 + 2*a*b^2)*sinh(x)^4 - 3*a^2*b*cosh(x) + 6*(
5*a^2*b*cosh(x)^2 - 4*(2*a^3 - a*b^2)*cosh(x))*sinh(x)^3 - 8*a^3 + 6*a*b^2
 - 12*(a^3 - a*b^2)*cosh(x)^2 + 6*(5*a^2*b*cosh(x)^3 - 2*a^3 + 2*a*b^2 - 6
*(2*a^3 - a*b^2)*cosh(x)^2)*sinh(x)^2 + 3*((a^2 - b^2)*cosh(x)^6 + 6*(a^2
- b^2)*cosh(x)*sinh(x)^5 + (a^2 - b^2)*sinh(x)^6 + 3*(a^2 - b^2)*cosh(x)^4
 + 3*(5*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^4 + 4*(5*(a^2 - b^2)*co
sh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^3 + 3*(a^2 - b^2)*cosh(x)^2 + 3*(
5*(a^2 - b^2)*cosh(x)^4 + 6*(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 +
 a^2 - b^2 + 6*((a^2 - b^2)*cosh(x)^5 + 2*(a^2 - b^2)*cosh(x)^3 + (a^2 - b
^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 +
 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2
- b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(
x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - 3*((3*a^2*b - 2*b^3)*cosh(x)^6 + 6*
(3*a^2*b - 2*b^3)*cosh(x)*sinh(x)^5 + (3*a^2*b - 2*b^3)*sinh(x)^6 + 3*(3*a
^2*b - 2*b^3)*cosh(x)^4 + 3*(3*a^2*b - 2*b^3 + 5*(3*a^2*b - 2*b^3)*cosh(x)
^2)*sinh(x)^4 + 4*(5*(3*a^2*b - 2*b^3)*cosh(x)^3 + 3*(3*a^2*b - 2*b^3)*cos
h(x))*sinh(x)^3 + 3*a^2*b - 2*b^3 + 3*(3*a^2*b - 2*b^3)*cosh(x)^2 + 3*(5*(
3*a^2*b - 2*b^3)*cosh(x)^4 + 3*a^2*b - 2*b^3 + 6*(3*a^2*b - 2*b^3)*cosh(x)
^2)*sinh(x)^2 + 6*((3*a^2*b - 2*b^3)*cosh(x)^5 + 2*(3*a^2*b - 2*b^3)*co...
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a+b\cosh(x)}\,dx=\int \frac{\tanh^4\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)**4/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^4(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.27

$$\begin{aligned}\int \frac{\tanh^4(x)}{a + b\cosh(x)}\, dx &= \frac{(3\,a^2b - 2\,b^3)\arctan(e^x)}{a^4} + \frac{2\,(a^4 - 2\,a^2b^2 + b^4)\arctan\left(\frac{be^x + a}{\sqrt{-a^2 + b^2}}\right)}{\sqrt{-a^2 + b^2}a^4} \\ &\quad - \frac{3\,abe^{(5\,x)} - 12\,a^2e^{(4\,x)} + 6\,b^2e^{(4\,x)} - 12\,a^2e^{(2\,x)} + 12\,b^2e^{(2\,x)} - 3\,abe^x - 8\,a^2 + 6\,b^2}{3\,a^3(e^{(2\,x)} + 1)^3}\end{aligned}$$

input
```
integrate(tanh(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
(3*a^2*b - 2*b^3)*arctan(e^x)/a^4 + 2*(a^4 - 2*a^2*b^2 + b^4)*arctan((b*e^
x + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*a^4) - 1/3*(3*a*b*e^(5*x) - 12*
a^2*e^(4*x) + 6*b^2*e^(4*x) - 12*a^2*e^(2*x) + 12*b^2*e^(2*x) - 3*a*b*e^x
- 8*a^2 + 6*b^2)/(a^3*(e^(2*x) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 7.27 (sec) , antiderivative size = 722, normalized size of antiderivative = 6.39

$$\int \frac{\tanh^4(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^4/(a + b*cosh(x)),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*b*exp(x))/a^2
)/(2*exp(2*x) + exp(4*x) + 1) + ((2*(2*a^2 - b^2))/a^3 - (b*exp(x))/a^2)/(
exp(2*x) + 1) + (log((((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456*a^4*b^4 - 272*
a^6*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*exp(x) + 600*a^
5*b^3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)*(3*a^2*b - 2*b
^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) + (16*(a^2 - b^2)*(4*a^2*b
- 4*b^3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)^3*(a - b)^3)^(
1/2))/a^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2)^2*(3*a^2 - 2*
b^2)*(6*a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a^9*b^3))*((a +
b)^3*(a - b)^3)^(1/2))/a^4 - (log(- (((32*a^8 + 64*b^8 - 288*a^2*b^6 + 456
*a^4*b^4 - 272*a^6*b^2 + 96*a*b^7*exp(x) - 288*a^7*b*exp(x) - 416*a^3*b^5*
exp(x) + 600*a^5*b^3*exp(x))/(a^6*b^4) - (((32*((a + b)^3*(a - b)^3)^(1/2)
*(3*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(a^2*b^5) - (16*(a^2 -
 b^2)*(4*a^2*b - 4*b^3 + 8*a^3*exp(x) - 7*a*b^2*exp(x)))/(a*b^5))*((a + b)
^3*(a - b)^3)^(1/2))/a^4)*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (8*(a^2 - b^2
)^2*(3*a^2 - 2*b^2)*(6*a^2*b - 4*b^3 + 10*a^3*exp(x) - 7*a*b^2*exp(x)))/(a
^9*b^3))*((a + b)^3*(a - b)^3)^(1/2))/a^4 - (b*log(exp(x) - 1i)*(3*a^2 - 2
*b^2)*1i)/(2*a^4) + (b*log(exp(x) + 1i)*(3*a^2 - 2*b^2)*1i)/(2*a^4)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 516, normalized size of antiderivative = 4.57

$$\int \frac{\tanh^4(x)}{a + b\cosh(x)}\,dx = \frac{9e^{6x}atan(e^x)\,a^2b - 6e^{6x}atan(e^x)\,b^3 + 27e^{4x}atan(e^x)\,a^2b - 18e^{4x}atan(e^x)\,b^3 + 27e^{2x}atan(e^x)\,a^2b - 18e^{2x}}{}$$

input
```
int(tanh(x)^4/(a+b*cosh(x)),x)
```

output
```
(9*e**(6*x)*atan(e**x)*a**2*b - 6*e**(6*x)*atan(e**x)*b**3 + 27*e**(4*x)*a
tan(e**x)*a**2*b - 18*e**(4*x)*atan(e**x)*b**3 + 27*e**(2*x)*atan(e**x)*a*
*2*b - 18*e**(2*x)*atan(e**x)*b**3 + 9*atan(e**x)*a**2*b - 6*atan(e**x)*b*
*3 - 6*e**(6*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2
))*a**2 + 6*e**(6*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 +
 b**2))*b**2 - 18*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( -
a**2 + b**2))*a**2 + 18*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sq
rt( - a**2 + b**2))*b**2 - 18*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b +
 a)/sqrt( - a**2 + b**2))*a**2 + 18*e**(2*x)*sqrt( - a**2 + b**2)*atan((e*
*x*b + a)/sqrt( - a**2 + b**2))*b**2 - 6*sqrt( - a**2 + b**2)*atan((e**x*b
 + a)/sqrt( - a**2 + b**2))*a**2 + 6*sqrt( - a**2 + b**2)*atan((e**x*b + a
)/sqrt( - a**2 + b**2))*b**2 - 4*e**(6*x)*a**3 + 2*e**(6*x)*a*b**2 - 3*e**
(5*x)*a**2*b - 6*e**(2*x)*a*b**2 + 3*e**x*a**2*b + 4*a**3 - 4*a*b**2)/(3*a
**4*(e**(6*x) + 3*e**(4*x) + 3*e**(2*x) + 1))
```

3.180 $\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 57

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{(a^2-b^2)\log(\cosh(x))}{a^3} - \frac{(a^2-b^2)\log(a+b\cosh(x))}{a^3} - \frac{b\operatorname{sech}(x)}{a^2} + \frac{\operatorname{sech}^2(x)}{2a}$$

output
```
(a^2-b^2)*ln(cosh(x))/a^3-(a^2-b^2)*ln(a+b*cosh(x))/a^3-b*sech(x)/a^2+1/2*
sech(x)^2/a
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.81

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{2(a^2-b^2)(\log(\cosh(x))-\log(a+b\cosh(x)))-2ab\operatorname{sech}(x)+a^2\operatorname{sech}^2(x)}{2a^3}$$

input
```
Integrate[Tanh[x]^3/(a + b*Cosh[x]),x]
```

output
```
(2*(a^2 - b^2)*(Log[Cosh[x]] - Log[a + b*Cosh[x]]) - 2*a*b*Sech[x] + a^2*S
ech[x]^2)/(2*a^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.385$, Rules used = {3042, 26, 3200, 522, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)^3\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int \frac{\left(b^2-b^2\cosh^2(x)\right)\text{sech}^3(x)}{b^3(a+b\cosh(x))}d(b\cosh(x))$$

$$\Big\downarrow \text{522}$$

$$-\int \left(\frac{\text{sech}^3(x)}{ab}-\frac{\text{sech}^2(x)}{a^2}+\frac{\left(b^2-a^2\right)\text{sech}(x)}{a^3b}+\frac{a^2-b^2}{a^3(a+b\cosh(x))}\right)d(b\cosh(x))$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b\text{sech}(x)}{a^2}+\frac{\left(a^2-b^2\right)\log(b\cosh(x))}{a^3}-\frac{\left(a^2-b^2\right)\log(a+b\cosh(x))}{a^3}+\frac{\text{sech}^2(x)}{2a}$$

input
```
Int[Tanh[x]^3/(a + b*Cosh[x]),x]
```

output
```
((a^2 - b^2)*Log[b*Cosh[x]])/a^3 - ((a^2 - b^2)*Log[a + b*Cosh[x]])/a^3 -
(b*Sech[x])/a^2 + Sech[x]^2/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 522
```
Int[((e_.)*(x_))^(m_.)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_.
), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(c + d*x)^n*(a + b*x^2)^p, x],
x] /; FreeQ[{a, b, c, d, e, m, n}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

method	result	size
default	$\frac{\frac{2a^2}{\left(\tanh\left(\frac{x}{2}\right)^2+1\right)^2}-\frac{2a(a+b)}{\tanh\left(\frac{x}{2}\right)^2+1}+\left(a^2-b^2\right)\ln\left(\tanh\left(\frac{x}{2}\right)^2+1\right)}{a^3}-\frac{(a+b)(a-b)\ln\left(\tanh\left(\frac{x}{2}\right)^2 a-b\tanh\left(\frac{x}{2}\right)^2-a-b\right)}{a^3}$	95
risch	$\frac{2\,\mathrm{e}^{x}\left(-\mathrm{e}^{2x}b+a\,\mathrm{e}^{x}-b\right)}{\left(\mathrm{e}^{2x}+1\right)^2 a^2}+\frac{\ln\left(\mathrm{e}^{2x}+1\right)}{a}-\frac{\ln\left(\mathrm{e}^{2x}+1\right)b^2}{a^3}-\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)}{a}+\frac{\ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)b^2}{a^3}$	100

input
```
int(tanh(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a^3*(2*a^2/(tanh(1/2*x)^2+1)^2-2*a*(a+b)/(tanh(1/2*x)^2+1)+(a^2-b^2)*ln(
tanh(1/2*x)^2+1))-(a+b)*(a-b)/a^3*ln(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(55) = 110$.

Time = 0.10 (sec) , antiderivative size = 450, normalized size of antiderivative = 7.89

$$\int \frac{\tanh^3(x)}{a + b\cosh(x)}\,dx =$$
$$-\frac{2\,ab\cosh\left(x\right)^{3} + 2\,ab\sinh\left(x\right)^{3} - 2\,a^{2}\cosh\left(x\right)^{2} + 2\,ab\cosh\left(x\right) + 2\left(3\,ab\cosh\left(x\right) - a^{2}\right)\sinh\left(x\right)^{2} + \left(\left(a^{2}\right.\right.}{}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(2*a*b*cosh(x)^3 + 2*a*b*sinh(x)^3 - 2*a^2*cosh(x)^2 + 2*a*b*cosh(x) + 2*
(3*a*b*cosh(x) - a^2)*sinh(x)^2 + ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*c
osh(x)*sinh(x)^3 + (a^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*
(a^2 - b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*
cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) -
 sinh(x))) - ((a^2 - b^2)*cosh(x)^4 + 4*(a^2 - b^2)*cosh(x)*sinh(x)^3 + (a
^2 - b^2)*sinh(x)^4 + 2*(a^2 - b^2)*cosh(x)^2 + 2*(3*(a^2 - b^2)*cosh(x)^2
 + a^2 - b^2)*sinh(x)^2 + a^2 - b^2 + 4*((a^2 - b^2)*cosh(x)^3 + (a^2 - b^
2)*cosh(x))*sinh(x))*log(2*cosh(x)/(cosh(x) - sinh(x))) + 2*(3*a*b*cosh(x)
^2 - 2*a^2*cosh(x) + a*b)*sinh(x))/(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^
3 + a^3*sinh(x)^4 + 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 + a^3)*sinh
(x)^2 + 4*(a^3*cosh(x)^3 + a^3*cosh(x))*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)**3/(a + b*cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.68

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = -\frac{2\left(be^{(-x)} - ae^{(-2x)} + be^{(-3x)}\right)}{2a^2e^{(-2x)} + a^2e^{(-4x)} + a^2} - \frac{(a^2-b^2)\log\left(2ae^{(-x)} + be^{(-2x)} + b\right)}{a^3} + \frac{(a^2-b^2)\log\left(e^{(-2x)}+1\right)}{a^3}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-2*(b*e^(-x) - a*e^(-2*x) + b*e^(-3*x))/(2*a^2*e^(-2*x) + a^2*e^(-4*x) + a
^2) - (a^2 - b^2)*log(2*a*e^(-x) + b*e^(-2*x) + b)/a^3 + (a^2 - b^2)*log(e
^(-2*x) + 1)/a^3
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 115 vs. $2(55) = 110$.

Time = 0.11 (sec) , antiderivative size = 115, normalized size of antiderivative = 2.02

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{(a^2-b^2)\log\left(e^{(-x)}+e^x\right)}{a^3} - \frac{(a^2b-b^3)\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^3b} - \frac{3\,a^2\left(e^{(-x)}+e^x\right)^2-3\,b^2\left(e^{(-x)}+e^x\right)^2+4\,ab\left(e^{(-x)}+e^x\right)-4\,a^2}{2\,a^3\left(e^{(-x)}+e^x\right)^2}$$

input
```
integrate(tanh(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
(a^2 - b^2)*log(e^(-x) + e^x)/a^3 - (a^2*b - b^3)*log(abs(b*(e^(-x) + e^x)
 + 2*a))/(a^3*b) - 1/2*(3*a^2*(e^(-x) + e^x)^2 - 3*b^2*(e^(-x) + e^x)^2 +
4*a*b*(e^(-x) + e^x) - 4*a^2)/(a^3*(e^(-x) + e^x)^2)
```

Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 1221, normalized size of antiderivative = 21.42

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
int(tanh(x)^3/(a + b*cosh(x)),x)
```

output
```
(2/a - (2*b*exp(x))/a^2)/(exp(2*x) + 1) - 2/(a*(2*exp(2*x) + exp(4*x) + 1)
) + ((2*atan((4*a^4*b^3*(a^2 - b^2)^2*(-a^6)^(1/2) - 4*a^6*b*(a^2 - b^2)^2
*(-a^6)^(1/2))*(exp(x)*(1/(16*a^4*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2))
 - (a^2 - 2*b^2)^2/(16*a^8*b^2*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2))) + 1/(
8*a^5*b*(a^2 - b^2)^3*((a^2 - b^2)^2)^(1/2)) + (a^2 - 2*b^2)/(8*a^7*b*(a^2
 - b^2)^3*((a^2 - b^2)^2)^(1/2)))) + 2*atan((a^2*(-a^6)^(1/2)*(a^4 + b^4 -
 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(2*a
^3*(a^2 - b^2)^2) + ((a^7 - a^5*b^2)*(-a^6)^(1/2))/(2*a^6*(a^2 - b^2)*((a^
2 - b^2)^2)^(1/2)) + (a^6*b^2*exp(3*x)*((2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2*
a^2*b^2)^(1/2))/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) - (2*(a^2 - 2
*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2)
*(a^4 + b^4 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*(
a^2 - b^2)^2*(-a^6)^(1/2)))*(-a^6)^(1/2))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)
) - (a^6*b^2*exp(x)*(-a^6)^(1/2)*((8*(a^4 + b^4 - 2*a^2*b^2))/(a^8*b*(a^2
- b^2)^2) - (4*(2*a^6*b - 2*a^4*b^3)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^12*
b^2*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) - (2*(a^7 - a^5*b^2)*(a^4 + b^4 - 2
*a^2*b^2)^(1/2))/(a^11*b^3*(a^2 - b^2)*((a^2 - b^2)^2)^(1/2)) + (2*(a^2 -
2*b^2)*(a^2*(-a^6)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)^(1/2) - 2*b^2*(-a^6)^(1/2
)*(a^4 + b^4 - 2*a^2*b^2)^(1/2))*(a^4 + b^4 - 2*a^2*b^2)^(1/2))/(a^10*b^3*
(a^2 - b^2)^2*(-a^6)^(1/2))))/(8*(a^4 + b^4 - 2*a^2*b^2)^(1/2)) + (a^6*...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 292, normalized size of antiderivative = 5.12

$$\int \frac{\tanh^3(x)}{a+b\cosh(x)}\,dx = \frac{e^{4x}\log(e^{2x}+1)\,a^2 - e^{4x}\log(e^{2x}+1)\,b^2 - e^{4x}\log(e^{2x}b+2e^xa+b)\,a^2 + e^{4x}\log(e^{2x}b+2e^xa+b)\,b^2 - e^{4x}a^2}{}$$

input
```
int(tanh(x)^3/(a+b*cosh(x)),x)
```

output
```
(e**(4*x)*log(e**(2*x) + 1)*a**2 - e**(4*x)*log(e**(2*x) + 1)*b**2 - e**(4
*x)*log(e**(2*x)*b + 2*e**x*a + b)*a**2 + e**(4*x)*log(e**(2*x)*b + 2*e**x
*a + b)*b**2 - e**(4*x)*a**2 - 2*e**(3*x)*a*b + 2*e**(2*x)*log(e**(2*x) +
1)*a**2 - 2*e**(2*x)*log(e**(2*x) + 1)*b**2 - 2*e**(2*x)*log(e**(2*x)*b +
2*e**x*a + b)*a**2 + 2*e**(2*x)*log(e**(2*x)*b + 2*e**x*a + b)*b**2 - 2*e*
*x*a*b + log(e**(2*x) + 1)*a**2 - log(e**(2*x) + 1)*b**2 - log(e**(2*x)*b
+ 2*e**x*a + b)*a**2 + log(e**(2*x)*b + 2*e**x*a + b)*b**2 - a**2)/(a**3*(
e**(4*x) + 2*e**(2*x) + 1))
```

3.181 $\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 61

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \frac{b\arctan(\sinh(x))}{a^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a^2}-\frac{\tanh(x)}{a}$$

output
```
b*arctan(sinh(x))/a^2+2*(a-b)^(1/2)*(a+b)^(1/2)*arctanh((a-b)^(1/2)*tanh(1
/2*x)/(a+b)^(1/2))/a^2-tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \frac{2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+2\sqrt{-a^2+b^2}\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)-a\tanh(x)}{a^2}$$

input
```
Integrate[Tanh[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*b*ArcTan[Tanh[x/2]] + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[x/2])/Sqr
t[-a^2 + b^2]] - a*Tanh[x])/a^2
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.23, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 25, 3202, 3042, 3535, 25, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^2\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3202}$$

$$-\int \frac{\left(1-\cosh^2(x)\right)\text{sech}^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$-\int \frac{1-\sin\left(ix+\frac{\pi}{2}\right)^2}{\sin\left(ix+\frac{\pi}{2}\right)^2\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx$$

$$\Big\downarrow \text{ 3535}$$

$$-\frac{\int -\frac{(b+a\cosh(x))\text{sech}(x)}{a+b\cosh(x)}dx}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{(b+a\cosh(x))\text{sech}(x)}{a+b\cosh(x)}dx}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a}+\frac{\int \frac{b+a\sin\left(ix+\frac{\pi}{2}\right)}{\sin\left(ix+\frac{\pi}{2}\right)\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)}dx}{a}$$

$$\Big\downarrow \text{3480}$$

$$\frac{\frac{(a^2-b^2)\int \frac{1}{a+b\cosh(x)}dx}{a}+\frac{b\int \text{sech}(x)dx}{a}}{a}-\frac{\tanh(x)}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh(x)}{a}+\frac{\frac{(a^2-b^2)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}$$

$$\Big\downarrow \text{3138}$$

$$-\frac{\tanh(x)}{a}+\frac{\frac{2(a^2-b^2)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\tanh(x)}{a}+\frac{\frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{\frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{b\arctan(\sinh(x))}{a}}{a}-\frac{\tanh(x)}{a}$$

input
```
Int[Tanh[x]^2/(a + b*Cosh[x]),x]
```

output
```
((b*ArcTan[Sinh[x]])/a + (2*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sq
rt[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b]))/a - Tanh[x]/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3202
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)/tan[(e_.) + (f_.)*(x_)]^2,
x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((1 - Sin[e + f*x]^2)/Sin[e + f*x]^
2), x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b^2, 0]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 3535
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :>
Simp[(-(A*b^2 + a^2*C))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*S
in[e + f*x])^(n + 1)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Simp[1/((m
+ 1)*(b*c - a*d)*(a^2 - b^2))   Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin
[e + f*x])^n*Simp[a*(m + 1)*(b*c - a*d)*(A + C) + d*(A*b^2 + a^2*C)*(m + n
+ 2) - (c*(A*b^2 + a^2*C) + b*(m + 1)*(b*c - a*d)*(A + C))*Sin[e + f*x] - d
*(A*b^2 + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c,
d, e, f, A, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2
- d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||
 !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a,
 0])))
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.28

method	result	size
default	$\frac{-\frac{2a\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}+2b\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a^{2}}+\frac{2(a+b)(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^{2}\sqrt{(a+b)(a-b)}}$	78
risch	$\frac{2}{a(\mathrm{e}^{2x}+1)}+\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}-\frac{-a+\sqrt{a^{2}-b^{2}}}{b}\right)}{a^{2}}-\frac{\sqrt{a^{2}-b^{2}}\,\ln\left(\mathrm{e}^{x}+\frac{a+\sqrt{a^{2}-b^{2}}}{b}\right)}{a^{2}}+\frac{ib\ln(\mathrm{e}^{x}+i)}{a^{2}}-\frac{ib\ln(\mathrm{e}^{x}-i)}{a^{2}}$	117

input
```
int(tanh(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/a^2*(-a*tanh(1/2*x)/(tanh(1/2*x)^2+1)+b*arctan(tanh(1/2*x)))+2*(a+b)*(a-
b)/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 129 vs. $2(51) = 102$.

Time = 0.12 (sec) , antiderivative size = 326, normalized size of antiderivative = 5.34

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}\left(\cosh{(x)}^2+2\,\cosh{(x)}\sinh{(x)}+\sinh{(x)}^2+1\right)\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cos}\right.}{a^2\cosh{(x)}^2} \right.$$
$$-\frac{2\left(\sqrt{-a^2+b^2}\left(\cosh{(x)}^2+2\,\cosh{(x)}\sinh{(x)}+\sinh{(x)}^2+1\right)\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)\right.}{a^2\cosh{(x)}^2+2\,a^2\cosh{(x)}\sinh{(x}}$$

input
```
integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*log((b^2
*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x)
+ a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)
^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 2*(b*co
sh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x))
 + 2*a)/(a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2), -2*
(sqrt(-a^2 + b^2)*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(-
sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (b*cosh(x)^2 +
 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*arctan(cosh(x) + sinh(x)) - a)/(a^
2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 + a^2)]
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \int \frac{\tanh^2{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)**2/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.10

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,b\arctan\left(e^x\right)}{a^2} + \frac{2\left(a^2-b^2\right)\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}a^2} + \frac{2}{a(e^{(2\,x)}+1)}$$

input
```
integrate(tanh(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*b*arctan(e^x)/a^2 + 2*(a^2 - b^2)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(
sqrt(-a^2 + b^2)*a^2) + 2/(a*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 285, normalized size of antiderivative = 4.67

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \frac{2}{a+a\,\mathrm{e}^{2\,x}} + \frac{\ln\left(64\,a^3\,b-64\,a\,b^3-32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x+64\,a^2\,b\,\sqrt{a^2-b^2}+128\,a^3\,\mathrm{e}^x\,\sqrt{a^2-b^2}-\right.}{a^2} - \frac{\ln\left(64\,a^3\,b-64\,a\,b^3+32\,b^3\,\sqrt{a^2-b^2}+128\,a^4\,\mathrm{e}^x+32\,b^4\,\mathrm{e}^x-64\,a^2\,b\,\sqrt{a^2-b^2}-128\,a^3\,\mathrm{e}^x\,\sqrt{a^2-b^2}-\right.}{a^2} - \frac{b\left(\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}-\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}\right)}{a^2}$$

input
```
int(tanh(x)^2/(a + b*cosh(x)),x)
```

output
```
2/(a + a*exp(2*x)) + (log(64*a^3*b - 64*a*b^3 - 32*b^3*(a^2 - b^2)^(1/2) +
 128*a^4*exp(x) + 32*b^4*exp(x) + 64*a^2*b*(a^2 - b^2)^(1/2) + 128*a^3*exp
(x)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) - 96*a*b^2*exp(x)*(a^2 - b^2)^(
1/2))*(a^2 - b^2)^(1/2))/a^2 - (log(64*a^3*b - 64*a*b^3 + 32*b^3*(a^2 - b^
2)^(1/2) + 128*a^4*exp(x) + 32*b^4*exp(x) - 64*a^2*b*(a^2 - b^2)^(1/2) - 1
28*a^3*exp(x)*(a^2 - b^2)^(1/2) - 160*a^2*b^2*exp(x) + 96*a*b^2*exp(x)*(a^
2 - b^2)^(1/2))*(a^2 - b^2)^(1/2))/a^2 - (b*(log(exp(x) - 1i)*1i - log(exp
(x) + 1i)*1i))/a^2
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.82

$$\int \frac{\tanh^2(x)}{a+b\cosh(x)}\,dx = \frac{2e^{2x}\,atan(e^x)\,b+2atan(e^x)\,b-2e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)-2e^{2x}a}{a^2\left(e^{2x}+1\right)}$$

input
```
int(tanh(x)^2/(a+b*cosh(x)),x)
```

output
```
(2*(e**(2*x)*atan(e**x)*b + atan(e**x)*b - e**(2*x)*sqrt( - a**2 + b**2)*a
tan((e**x*b + a)/sqrt( - a**2 + b**2)) - sqrt( - a**2 + b**2)*atan((e**x*b
 + a)/sqrt( - a**2 + b**2)) - e**(2*x)*a))/(a**2*(e**(2*x) + 1))
```

3.182 $\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 20

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

output
```
ln(cosh(x))/a-ln(a+b*cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{\log(\cosh(x))}{a} - \frac{\log(a+b\cosh(x))}{a}$$

input
```
Integrate[Tanh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 3200, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)\left(a-b\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{1}{\left(a-b\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3200$$

$$\int \frac{\text{sech}(x)}{b(a+b\cosh(x))}d(b\cosh(x))$$

$$\Big\downarrow \ 47$$

$$\frac{\int \frac{\text{sech}(x)}{b}d(b\cosh(x))}{a}-\frac{\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{a}$$

$$\Big\downarrow \ 14$$

$$\frac{\log(b\cosh(x))}{a}-\frac{\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{a}$$

$$\Big\downarrow \ 16$$

$$\frac{\log(b\cosh(x))}{a}-\frac{\log(a+b\cosh(x))}{a}$$

input
```
Int[Tanh[x]/(a + b*Cosh[x]),x]
```

output
```
Log[b*Cosh[x]]/a - Log[a + b*Cosh[x]]/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

method	result	size
risch	$\frac{\ln\left({\mathrm e}^{2 x}+1\right)}{a}-\frac{\ln\left({\mathrm e}^{2 x}+\frac{2 a \,{\mathrm e}^{x}}{b}+1\right)}{a}$	33
default	$-\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2} a -b \tanh\left(\frac{x}{2}\right)^{2}-a -b \right)}{a}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{a}$	45

input
```
int(tanh(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(exp(2*x)+1)-1/a*ln(exp(2*x)+2/b*a*exp(x)+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = -\frac{\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\log\left(\frac{2\,\cosh(x)}{\cosh(x)-\sinh(x)}\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - log(2*cosh(x)/(cosh(x) - si
nh(x))))/a
```

Sympy [F]

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \int \frac{\tanh(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x)
```

output
```
Integral(tanh(x)/(a + b*cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = -\frac{\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a}+\frac{\log\left(e^{(-2\,x)}+1\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-log(2*a*e^(-x) + b*e^(-2*x) + b)/a + log(e^(-2*x) + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.65

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{\log\left(e^{(-x)}+e^{x}\right)}{a} - \frac{\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{a}$$

input
```
integrate(tanh(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
log(e^(-x) + e^x)/a - log(abs(b*(e^(-x) + e^x) + 2*a))/a
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 201, normalized size of antiderivative = 10.05

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{-a^2}+b\,e^{x}\,\sqrt{-a^2}+2\,a\,e^{2\,x}\,\sqrt{-a^2}+b\,e^{3\,x}\,\sqrt{-a^2}}{a^2}\right)}{\sqrt{-a^2}} - \frac{2\,\mathrm{atan}\left(\left(4\,a^4\,b\,\sqrt{-a^2}-4\,a^2\,b^3\,\sqrt{-a^2}\right)\,\left(e^{x}\,\left(\frac{1}{16\,b^2\,(a^2-b^2)^2}-\frac{(a^2-2\,b^2)^2}{16\,a^4\,b^2\,(a^2-b^2)^2}\right)+\frac{1}{8\,a\,b\,(a^2-b^2)^2}+\frac{a^2-2\,b^2}{8\,a^3\,b\,(a^2-b^2)^2}\right)}{\sqrt{-a^2}}$$

input
```
int(tanh(x)/(a + b*cosh(x)),x)
```

output
```
(2*atan((a*(-a^2)^(1/2) + b*exp(x)*(-a^2)^(1/2) + 2*a*exp(2*x)*(-a^2)^(1/2
) + b*exp(3*x)*(-a^2)^(1/2))/a^2))/(-a^2)^(1/2) - (2*atan((4*a^4*b*(-a^2)^
(1/2) - 4*a^2*b^3*(-a^2)^(1/2))*(exp(x)*(1/(16*b^2*(a^2 - b^2)^2) - (a^2 -
 2*b^2)^2/(16*a^4*b^2*(a^2 - b^2)^2)) + 1/(8*a*b*(a^2 - b^2)^2) + (a^2 - 2
*b^2)/(8*a^3*b*(a^2 - b^2)^2))))/(-a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.55

$$\int \frac{\tanh(x)}{a+b\cosh(x)}\,dx = \frac{\log(e^{2x}+1)-\log(e^{2x}b+2e^{x}a+b)}{a}$$

input
```
int(tanh(x)/(a+b*cosh(x)),x)
```

output
```
(log(e**(2*x) + 1) - log(e**(2*x)*b + 2*e**x*a + b))/a
```

3.183 $\int \frac{\coth(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx=\frac{\log(1-\cosh(x))}{2(a+b)}+\frac{\log(1+\cosh(x))}{2(a-b)}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
ln(1-cosh(x))/(2*a+2*b)+ln(1+cosh(x))/(2*a-2*b)-a*ln(a+b*cosh(x))/(a^2-b^2
)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx=\frac{\log\left(\cosh\left(\frac{x}{2}\right)\right)}{a-b}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}+\frac{\log\left(\sinh\left(\frac{x}{2}\right)\right)}{a+b}$$

input
```
Integrate[Coth[x]/(a + b*Cosh[x]),x]
```

output
```
Log[Cosh[x/2]]/(a - b) - (a*Log[a + b*Cosh[x]])/(a^2 - b^2) + Log[Sinh[x/2
]]/(a + b)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.727, Rules used = {3042, 26, 3200, 587, 16, 452, 219, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(-\frac{\pi}{2}+ix\right)}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3200}$$

$$-\int \frac{b\cosh(x)}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))$$

$$\Big\downarrow \text{587}$$

$$\frac{\int \frac{b^2-ab\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\int \frac{1}{a+b\cosh(x)}d(b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{16}$$

$$\frac{\int \frac{b^2-ab\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{452}$$

$$\frac{b^2\int \frac{1}{b^2-b^2\cosh^2(x)}d(b\cosh(x))-a\int \frac{b\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\text{arctanh}(\cosh(x))-a\int \frac{b\cosh(x)}{b^2-b^2\cosh^2(x)}d(b\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

$$\Big\downarrow \text{240}$$

$$\frac{\frac{1}{2}a\log\left(b^2-b^2\cosh^2(x)\right)+b\text{arctanh}(\cosh(x))}{a^2-b^2}-\frac{a\log(a+b\cosh(x))}{a^2-b^2}$$

input
```
Int[Coth[x]/(a + b*Cosh[x]),x]
```

output
```
-((a*Log[a + b*Cosh[x]])/(a^2 - b^2)) + (b*ArcTanh[Cosh[x]] + (a*Log[b^2 -
 b^2*Cosh[x]^2])/2)/(a^2 - b^2)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 452
```
Int[((c_) + (d_.)*(x_))/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[c   Int[1/
(a + b*x^2), x], x] + Simp[d   Int[x/(a + b*x^2), x], x] /; FreeQ[{a, b, c,
 d}, x] && NeQ[b*c^2 + a*d^2, 0]
```

rule 587
```
Int[(x_.)/(((c_) + (d_.)*(x_))*((a_) + (b_.)*(x_)^2)), x_Symbol] :> Simp[(-
c)*(d/(b*c^2 + a*d^2))   Int[1/(c + d*x), x], x] + Simp[1/(b*c^2 + a*d^2)
 Int[(a*d + b*c*x)/(a + b*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c
^2 + a*d^2, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.98

method	result	size
default	$-\frac{a \ln\left(\tanh\left(\frac{x}{2}\right)^{2} a-b \tanh\left(\frac{x}{2}\right)^{2}-a-b\right)}{(a+b)(a-b)}+\frac{\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}$	53
risch	$-\frac{x}{a+b}-\frac{x}{a-b}+\frac{2xa}{a^{2}-b^{2}}+\frac{\ln\left(\mathrm{e}^{x}-1\right)}{a+b}+\frac{\ln\left(\mathrm{e}^{x}+1\right)}{a-b}-\frac{a \ln\left(\mathrm{e}^{2x}+\frac{2a\,\mathrm{e}^{x}}{b}+1\right)}{a^{2}-b^{2}}$	88

input
```
int(coth(x)/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-a/(a+b)/(a-b)*ln(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)+1/(a+b)*ln(tanh(1/2
*x))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.11

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = -\frac{a\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(a+b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-(a-b)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)}{a^2-b^2}$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
-(a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - (a + b)*log(cosh(x) + sin
h(x) + 1) - (a - b)*log(cosh(x) + sinh(x) - 1))/(a^2 - b^2)
```

Sympy [F]

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = \int \frac{\coth\left(x\right)}{a+b\cosh\left(x\right)}\,dx$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)/(a + b*cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.09

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = -\frac{a\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^2-b^2}+\frac{\log\left(e^{(-x)}+1\right)}{a-b}+\frac{\log\left(e^{(-x)}-1\right)}{a+b}$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-a*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^2 - b^2) + log(e^(-x) + 1)/(a - b)
+ log(e^(-x) - 1)/(a + b)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = -\frac{ab\log\left(\left|b\left(e^{(-x)}+e^{x}\right)+2\,a\right|\right)}{a^2b-b^3} + \frac{\log\left(e^{(-x)}+e^{x}+2\right)}{2\,(a-b)} + \frac{\log\left(e^{(-x)}+e^{x}-2\right)}{2\,(a+b)}$$

input
```
integrate(coth(x)/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-a*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^2*b - b^3) + 1/2*log(e^(-x) + e^x
 + 2)/(a - b) + 1/2*log(e^(-x) + e^x - 2)/(a + b)
```

Mupad [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 148, normalized size of antiderivative = 2.74

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = \frac{\ln\left(128\,a\,b-128\,a^2-32\,b^2+128\,a^2\,\mathrm{e}^{x}+32\,b^2\,\mathrm{e}^{x}-128\,a\,b\,\mathrm{e}^{x}\right)}{a+b} + \frac{\ln\left(-128\,a\,b-128\,a^2-32\,b^2-128\,a^2\,\mathrm{e}^{x}-32\,b^2\,\mathrm{e}^{x}-128\,a\,b\,\mathrm{e}^{x}\right)}{a-b} - \frac{a\,\ln\left(16\,a^2\,b-4\,b^3\,\mathrm{e}^{2\,x}-4\,b^3+32\,a^3\,\mathrm{e}^{x}-8\,a\,b^2\,\mathrm{e}^{x}+16\,a^2\,b\,\mathrm{e}^{2\,x}\right)}{a^2-b^2}$$

input
```
int(coth(x)/(a + b*cosh(x)),x)
```

output
```
log(128*a*b - 128*a^2 - 32*b^2 + 128*a^2*exp(x) + 32*b^2*exp(x) - 128*a*b*
exp(x))/(a + b) + log(- 128*a*b - 128*a^2 - 32*b^2 - 128*a^2*exp(x) - 32*b
^2*exp(x) - 128*a*b*exp(x))/(a - b) - (a*log(16*a^2*b - 4*b^3*exp(2*x) - 4
*b^3 + 32*a^3*exp(x) - 8*a*b^2*exp(x) + 16*a^2*b*exp(2*x)))/(a^2 - b^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.20

$$\int \frac{\coth(x)}{a+b\cosh(x)}\,dx = \frac{\log(e^x-1)\,a-\log(e^x-1)\,b+\log(e^x+1)\,a+\log(e^x+1)\,b-\log(e^{2x}b+2e^xa+b)\,a}{a^2-b^2}$$

input
```
int(coth(x)/(a+b*cosh(x)),x)
```

output
```
(log(e**x - 1)*a - log(e**x - 1)*b + log(e**x + 1)*a + log(e**x + 1)*b - l
og(e**(2*x)*b + 2*e**x*a + b)*a)/(a**2 - b**2)
```

3.184 $\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 77

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx=\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

output
```
2*a^2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/(a+b)^(3/2)
-a*coth(x)/(a^2-b^2)+b*csch(x)/(a^2-b^2)
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx=\frac{2a^2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{\coth\left(\frac{x}{2}\right)}{2(a+b)}-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}$$

input
```
Integrate[Coth[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*a^2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) -
Coth[x/2]/(2*(a + b)) - Tanh[x/2]/(2*(a - b))
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.923, Rules used = {3042, 25, 3206, 25, 3042, 25, 3086, 24, 3138, 221, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(-\frac{\pi}{2}+ix\right)^2}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3206}$$

$$\frac{a^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{a\int -\text{csch}^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{a\int \text{csch}^2(x)dx}{a^2-b^2}-\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int -\csc(ix)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3086}$$

$$\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \csc(ix)^2dx}{a^2-b^2}+\frac{ib\int 1d(-i\text{csch}(x))}{a^2-b^2}$$

$$\Big\downarrow \ 24$$

$$\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int \csc(ix)^2dx}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \ 3138$$

$$-\frac{a\int \csc(ix)^2dx}{a^2-b^2}+\frac{2a^2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \ 221$$

$$-\frac{a\int \csc(ix)^2dx}{a^2-b^2}+\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \ 4254$$

$$-\frac{ia\int 1d(-i\coth(x))}{a^2-b^2}+\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}+\frac{b\text{csch}(x)}{a^2-b^2}$$

$$\Big\downarrow \ 24$$

$$\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}\,(a^2-b^2)}-\frac{a\coth(x)}{a^2-b^2}+\frac{b\text{csch}(x)}{a^2-b^2}$$

input
```
Int[Coth[x]^2/(a + b*Cosh[x]),x]
```

output
```
(2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a +
 b]*(a^2 - b^2)) - (a*Coth[x])/(a^2 - b^2) + (b*Csch[x])/(a^2 - b^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 0.82 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.01

method	result	size
default	$-\frac{\tanh\left(\frac{x}{2}\right)}{2(a-b)}-\frac{1}{2(a+b)\tanh\left(\frac{x}{2}\right)}+\frac{2a^{2}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)(a+b)\sqrt{(a+b)(a-b)}}$	78
risch	$-\frac{2(-\mathrm{e}^{x}b+a)}{(\mathrm{e}^{2x}-1)(a^{2}-b^{2})}+\frac{a^{2}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}-\frac{a^{2}\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)(a-b)}$	167

input
```
int(coth(x)^2/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a-b)*tanh(1/2*x)-1/2/(a+b)/tanh(1/2*x)+2/(a-b)*a^2/(a+b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 200 vs. $2(67) = 134$.

Time = 0.11 (sec) , antiderivative size = 470, normalized size of antiderivative = 6.10

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$$
$$= \left[\frac{2\,a^3-2\,ab^2+\left(a^2\cosh\left(x\right)^2+2\,a^2\cosh\left(x\right)\sinh\left(x\right)+a^2\sinh\left(x\right)^2-a^2\right)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)}{}\right.}{a^4-2\,a^2b^2+b^4-\left(a^4-2\,a^2b^2+b^4\right)\cosh\left(x\right)^2-2\,(a}\right.$$

input
```
integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(2*a^3 - 2*a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2
 - a^2)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x)
 + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*(b*cosh
(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh
(x) + a)*sinh(x) + b)) - 2*(a^2*b - b^3)*cosh(x) - 2*(a^2*b - b^3)*sinh(x)
)/(a^4 - 2*a^2*b^2 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*
a^2*b^2 + b^4)*cosh(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2), 2*(a^
3 - a*b^2 + (a^2*cosh(x)^2 + 2*a^2*cosh(x)*sinh(x) + a^2*sinh(x)^2 - a^2)*
sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2
 - b^2)) - (a^2*b - b^3)*cosh(x) - (a^2*b - b^3)*sinh(x))/(a^4 - 2*a^2*b^2
 + b^4 - (a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4)*cos
h(x)*sinh(x) - (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^2)]
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx=\int \frac{\coth^2{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)**2/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.99

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx = \frac{2\,a^2 \arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}} + \frac{2\,(be^x-a)}{(a^2-b^2)(e^{(2\,x)}-1)}$$

input
```
integrate(coth(x)^2/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^2*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*sqrt(-a^2 + b^2))
+ 2*(b*e^x - a)/((a^2 - b^2)*(e^(2*x) - 1))
```

Mupad [B] (verification not implemented)

Time = 2.36 (sec) , antiderivative size = 337, normalized size of antiderivative = 4.38

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx = -\frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{2\,x}-1} - \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,a^2}{b^2\,(a^2-b^2)^2\,\sqrt{a^4}}+\frac{2\left(a^3\,\sqrt{a^4}-a\,b^2\,\sqrt{a^4}\right)}{a\,b^2\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}\right)-\frac{2\left(b^3\,\sqrt{a^4}-a^2\,b\,\sqrt{a^4}\right)}{a\,b^2\,(a^2-b^2)\,\sqrt{-(a^2-b^2)^3}\,\sqrt{-a^6+3\,a}}\right.\right.}{\sqrt{-a^6+3\,a^4\,b^2-3\,a^2\,b^4+b^6}}$$

input
```
int(coth(x)^2/(a + b*cosh(x)),x)
```

output
```
- ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(2*x) - 1) - (2*atan(
(exp(x)*((2*a^2)/(b^2*(a^2 - b^2)^2*(a^4)^(1/2)) + (2*(a^3*(a^4)^(1/2) - a
*b^2*(a^4)^(1/2)))/(a*b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 -
3*a^2*b^4 + 3*a^4*b^2)^(1/2))) - (2*(b^3*(a^4)^(1/2) - a^2*b*(a^4)^(1/2)))
/(a*b^2*(a^2 - b^2)*(-(a^2 - b^2)^3)^(1/2)*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*
b^2)^(1/2)))*((b^3*(b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2 - (a^2*b*(
b^6 - a^6 - 3*a^2*b^4 + 3*a^4*b^2)^(1/2))/2))*(a^4)^(1/2))/(b^6 - a^6 - 3*
a^2*b^4 + 3*a^4*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.17

$$\int \frac{\coth^2(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-2e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2+2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2-2e^{2x}a^3+2e^{2x}a\,b^2+2e^x a^2 b-2e}{e^{2x}a^4-2e^{2x}a^2b^2+e^{2x}b^4-a^4+2a^2b^2-b^4}$$

input
```
int(coth(x)^2/(a+b*cosh(x)),x)
```

output
```
(2*( - e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2
))*a**2 + sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**
2 - e**(2*x)*a**3 + e**(2*x)*a*b**2 + e**x*a**2*b - e**x*b**3))/(e**(2*x)*
a**4 - 2*e**(2*x)*a**2*b**2 + e**(2*x)*b**4 - a**4 + 2*a**2*b**2 - b**4)
```

3.185 $\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 94

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = -\frac{(a-b\cosh(x))\operatorname{csch}^2(x)}{2(a^2-b^2)}+\frac{(2a+b)\log(1-\cosh(x))}{4(a+b)^2}+\frac{(2a-b)\log(1+\cosh(x))}{4(a-b)^2}-\frac{a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}$$

output
```
-1/2*(a-b*cosh(x))*csch(x)^2/(a^2-b^2)+1/4*(2*a+b)*ln(1-cosh(x))/(a+b)^2+1
/4*(2*a-b)*ln(1+cosh(x))/(a-b)^2-a^3*ln(a+b*cosh(x))/(a^2-b^2)^2
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.07

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \frac{1}{8}\left(-\frac{\operatorname{csch}^2\left(\frac{x}{2}\right)}{a+b}+\frac{4(2a-b)\log\left(\cosh\left(\frac{x}{2}\right)\right)}{(a-b)^2}-\frac{8a^3\log(a+b\cosh(x))}{(a^2-b^2)^2}+\frac{4(2a+b)\log\left(\sinh\left(\frac{x}{2}\right)\right)}{(a+b)^2}+\frac{\operatorname{sech}^2\left(\frac{x}{2}\right)}{a-b}\right)$$

input
```
Integrate[Coth[x]^3/(a + b*Cosh[x]),x]
```

output
```
(-(Csch[x/2]^2/(a + b)) + (4*(2*a - b)*Log[Cosh[x/2]])/(a - b)^2 - (8*a^3*
Log[a + b*Cosh[x]])/(a^2 - b^2)^2 + (4*(2*a + b)*Log[Sinh[x/2]])/(a + b)^2
 + Sech[x/2]^2/(a - b))/8
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.44, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 26, 3200, 601, 25, 27, 657, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(-\frac{\pi}{2}+ix\right)^3}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^3}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{b^3\cosh^3(x)}{\left(b^2-b^2\cosh^2(x)\right)^2(a+b\cosh(x))}d(b\cosh(x))$$

$$\Big\downarrow \text{601}$$

$$\frac{b^2(a-b\cosh(x))}{2\left(a^2-b^2\right)\left(b^2-b^2\cosh^2(x)\right)}-\frac{\int -\frac{b^2\left(\frac{ab^2}{a^2-b^2}-\frac{b\left(2a^2-b^2\right)\cosh(x)}{a^2-b^2}\right)}{(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))}{2b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{b^2\left(ab^2-b\left(2a^2-b^2\right)\cosh(x)\right)}{\left(a^2-b^2\right)(a+b\cosh(x))\left(b^2-b^2\cosh^2(x)\right)}d(b\cosh(x))}{2b^2}+\frac{b^2(a-b\cosh(x))}{2\left(a^2-b^2\right)\left(b^2-b^2\cosh^2(x)\right)}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{ab^2-b(2a^2-b^2)\cosh(x)}{(a+b\cosh(x))(b^2-b^2\cosh^2(x))}d(b\cosh(x))}{2(a^2-b^2)}+\frac{b^2(a-b\cosh(x))}{2(a^2-b^2)(b^2-b^2\cosh^2(x))}$$

$$\Big\downarrow \text{657}$$

$$\frac{\int \left(-\frac{2a^3}{(a-b)(a+b)(a+b\cosh(x))}+\frac{-2a^2+ba+b^2}{2(a+b)(b-b\cosh(x))}+\frac{(2a-b)(a+b)}{2(a-b)(\cosh(x)b+b)}\right)d(b\cosh(x))}{2(a^2-b^2)}+\frac{b^2(a-b\cosh(x))}{2(a^2-b^2)(b^2-b^2\cosh^2(x))}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2(a-b\cosh(x))}{2(a^2-b^2)(b^2-b^2\cosh^2(x))}+\frac{-\frac{2a^3\log(a+b\cosh(x))}{a^2-b^2}+\frac{(a-b)(2a+b)\log(b-b\cosh(x))}{2(a+b)}+\frac{(2a-b)(a+b)\log(b\cosh(x)+b)}{2(a-b)}}{2(a^2-b^2)}$$

input
```
Int[Coth[x]^3/(a + b*Cosh[x]),x]
```

output
```
(b^2*(a - b*Cosh[x]))/(2*(a^2 - b^2)*(b^2 - b^2*Cosh[x]^2)) + (((a - b)*(2
*a + b)*Log[b - b*Cosh[x]])/(2*(a + b)) - (2*a^3*Log[a + b*Cosh[x]])/(a^2
- b^2) + ((2*a - b)*(a + b)*Log[b + b*Cosh[x]])/(2*(a - b)))/(2*(a^2 - b^2
))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 601
```
Int[(x_)^(m_)*((c_) + (d_.)*(x_))^(n_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbo
l] :> With[{Qx = PolynomialQuotient[x^m*(c + d*x)^n, a + b*x^2, x], e = Coe
ff[PolynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 0], f = Coeff[Pol
ynomialRemainder[x^m*(c + d*x)^n, a + b*x^2, x], x, 1]}, Simp[(a*f - b*e*x)
*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] + Simp[1/(2*a*(p + 1))   Int[(c
+ d*x)^n*(a + b*x^2)^(p + 1)*ExpandToSum[(2*a*(p + 1)*Qx)/(c + d*x)^n + (e*
(2*p + 3))/(c + d*x)^n, x], x], x]] /; FreeQ[{a, b, c, d}, x] && IGtQ[m, 1]
 && LtQ[p, -1] && ILtQ[n, 0] && NeQ[b*c^2 + a*d^2, 0]
```

rule 657
```
Int[(((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.))/((a_) + (c_.)*(
x_)^2), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*((f + g*x)^n/(a + c*x^
2)), x], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && IntegersQ[n]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 1.29 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97

method	result
default	$-\frac{\tanh\left(\frac{x}{2}\right)^{2}}{8(a-b)}-\frac{a^{3}\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-b\tanh\left(\frac{x}{2}\right)^{2}-a-b\right)}{(a-b)^{2}(a+b)^{2}}-\frac{1}{8(a+b)\tanh\left(\frac{x}{2}\right)^{2}}+\frac{(4a+2b)\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{4(a+b)^{2}}$
risch	$-\frac{xa}{a^{2}+2ab+b^{2}}-\frac{xb}{2(a^{2}+2ab+b^{2})}-\frac{xa}{a^{2}-2ab+b^{2}}+\frac{bx}{2a^{2}-4ab+2b^{2}}+\frac{2x\,a^{3}}{a^{4}-2a^{2}b^{2}+b^{4}}-\frac{e^{x}\left(-e^{2x}b+2a\,e^{x}-b\right)}{\left(e^{2x}-1\right)^{2}(a^{2}-b^{2})}+\frac{\ln(e^{x}-1)a}{a^{2}+2ab+b^{2}}$

input
```
int(coth(x)^3/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8*tanh(1/2*x)^2/(a-b)-1/(a-b)^2*a^3/(a+b)^2*ln(tanh(1/2*x)^2*a-b*tanh(1
/2*x)^2-a-b)-1/8/(a+b)/tanh(1/2*x)^2+1/4/(a+b)^2*(4*a+2*b)*ln(tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 839 vs. $2(89) = 178$.

Time = 0.10 (sec) , antiderivative size = 839, normalized size of antiderivative = 8.93

$$\int \frac{\coth^3(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
1/2*(2*(a^2*b - b^3)*cosh(x)^3 + 2*(a^2*b - b^3)*sinh(x)^3 - 4*(a^3 - a*b^
2)*cosh(x)^2 - 2*(2*a^3 - 2*a*b^2 - 3*(a^2*b - b^3)*cosh(x))*sinh(x)^2 + 2
*(a^2*b - b^3)*cosh(x) - 2*(a^3*cosh(x)^4 + 4*a^3*cosh(x)*sinh(x)^3 + a^3*
sinh(x)^4 - 2*a^3*cosh(x)^2 + a^3 + 2*(3*a^3*cosh(x)^2 - a^3)*sinh(x)^2 +
4*(a^3*cosh(x)^3 - a^3*cosh(x))*sinh(x))*log(2*(b*cosh(x) + a)/(cosh(x) -
sinh(x))) + ((2*a^3 + 3*a^2*b - b^3)*cosh(x)^4 + 4*(2*a^3 + 3*a^2*b - b^3)
*cosh(x)*sinh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*sinh(x)^4 + 2*a^3 + 3*a^2*b -
 b^3 - 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 - 2*(2*a^3 + 3*a^2*b - b^3 - 3*
(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 + 3*a^2*b - b^3)*
cosh(x)^3 - (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x
) + 1) + ((2*a^3 - 3*a^2*b + b^3)*cosh(x)^4 + 4*(2*a^3 - 3*a^2*b + b^3)*co
sh(x)*sinh(x)^3 + (2*a^3 - 3*a^2*b + b^3)*sinh(x)^4 + 2*a^3 - 3*a^2*b + b^
3 - 2*(2*a^3 - 3*a^2*b + b^3)*cosh(x)^2 - 2*(2*a^3 - 3*a^2*b + b^3 - 3*(2*
a^3 - 3*a^2*b + b^3)*cosh(x)^2)*sinh(x)^2 + 4*((2*a^3 - 3*a^2*b + b^3)*cos
h(x)^3 - (2*a^3 - 3*a^2*b + b^3)*cosh(x))*sinh(x))*log(cosh(x) + sinh(x) -
 1) + 2*(a^2*b - b^3 + 3*(a^2*b - b^3)*cosh(x)^2 - 4*(a^3 - a*b^2)*cosh(x)
)*sinh(x))/((a^4 - 2*a^2*b^2 + b^4)*cosh(x)^4 + 4*(a^4 - 2*a^2*b^2 + b^4)*
cosh(x)*sinh(x)^3 + (a^4 - 2*a^2*b^2 + b^4)*sinh(x)^4 + a^4 - 2*a^2*b^2 +
b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2 - 2*(a^4 - 2*a^2*b^2 + b^4 - 3*(
a^4 - 2*a^2*b^2 + b^4)*cosh(x)^2)*sinh(x)^2 + 4*((a^4 - 2*a^2*b^2 + b^4...
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx$$

input
```
integrate(coth(x)**3/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)**3/(a + b*cosh(x)), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.66

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = -\frac{a^3\log\left(2\,ae^{(-x)}+be^{(-2\,x)}+b\right)}{a^4-2\,a^2b^2+b^4} + \frac{(2\,a-b)\log\left(e^{(-x)}+1\right)}{2\,(a^2-2\,ab+b^2)} + \frac{(2\,a+b)\log\left(e^{(-x)}-1\right)}{2\,(a^2+2\,ab+b^2)} + \frac{be^{(-x)}-2\,ae^{(-2\,x)}+be^{(-3\,x)}}{a^2-b^2-2\,(a^2-b^2)e^{(-2\,x)}+(a^2-b^2)e^{(-4\,x)}}$$

input
```
integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
-a^3*log(2*a*e^(-x) + b*e^(-2*x) + b)/(a^4 - 2*a^2*b^2 + b^4) + 1/2*(2*a -
 b)*log(e^(-x) + 1)/(a^2 - 2*a*b + b^2) + 1/2*(2*a + b)*log(e^(-x) - 1)/(a
^2 + 2*a*b + b^2) + (b*e^(-x) - 2*a*e^(-2*x) + b*e^(-3*x))/(a^2 - b^2 - 2*
(a^2 - b^2)*e^(-2*x) + (a^2 - b^2)*e^(-4*x))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.89

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = -\frac{a^3b\log\left(\left|b\left(e^{(-x)}+e^x\right)+2\,a\right|\right)}{a^4b-2\,a^2b^3+b^5} + \frac{(2\,a-b)\log\left(e^{(-x)}+e^x+2\right)}{4\,(a^2-2\,ab+b^2)} + \frac{(2\,a+b)\log\left(e^{(-x)}+e^x-2\right)}{4\,(a^2+2\,ab+b^2)} - \frac{a^3\left(e^{(-x)}+e^x\right)^2-2\,a^2b\left(e^{(-x)}+e^x\right)+2\,b^3\left(e^{(-x)}+e^x\right)-4\,ab^2}{2\,(a^4-2\,a^2b^2+b^4)\left(\left(e^{(-x)}+e^x\right)^2-4\right)}$$

input
```
integrate(coth(x)^3/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-a^3*b*log(abs(b*(e^(-x) + e^x) + 2*a))/(a^4*b - 2*a^2*b^3 + b^5) + 1/4*(2
*a - b)*log(e^(-x) + e^x + 2)/(a^2 - 2*a*b + b^2) + 1/4*(2*a + b)*log(e^(-
x) + e^x - 2)/(a^2 + 2*a*b + b^2) - 1/2*(a^3*(e^(-x) + e^x)^2 - 2*a^2*b*(e
^(-x) + e^x) + 2*b^3*(e^(-x) + e^x) - 4*a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*((
e^(-x) + e^x)^2 - 4))
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 291, normalized size of antiderivative = 3.10

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \frac{\frac{2\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{e^x\,(a^2\,b-b^3)}{(a^2-b^2)^2}}{e^{2\,x}-1} - \frac{\frac{2\,a}{a^2-b^2}-\frac{2\,b\,e^x}{a^2-b^2}}{e^{4\,x}-2\,e^{2\,x}+1} + \frac{\ln\left(e^x+1\right)\,(2\,a-b)}{2\,a^2-4\,a\,b+2\,b^2} - \frac{a^3\,\ln\left(b^7\,e^{2\,x}-16\,a^6\,b+b^7-6\,a^2\,b^5+9\,a^4\,b^3-32\,a^7\,e^x-6\,a^2\,b^5\,e^{2\,x}+9\,a^4\,b^3\,e^{2\,x}+2\,a\,b^6\,e^x-16\,a^6\,b\,e\right.}{a^4-2\,a^2\,b^2+b^4} + \frac{\ln\left(e^x-1\right)\,(2\,a+b)}{2\,a^2+4\,a\,b+2\,b^2}$$

input
```
int(coth(x)^3/(a + b*cosh(x)),x)
```

output
```
((2*(a*b^2 - a^3))/(a^2 - b^2)^2 + (exp(x)*(a^2*b - b^3))/(a^2 - b^2)^2)/(
exp(2*x) - 1) - ((2*a)/(a^2 - b^2) - (2*b*exp(x))/(a^2 - b^2))/(exp(4*x) -
 2*exp(2*x) + 1) + (log(exp(x) + 1)*(2*a - b))/(2*a^2 - 4*a*b + 2*b^2) - (
a^3*log(b^7*exp(2*x) - 16*a^6*b + b^7 - 6*a^2*b^5 + 9*a^4*b^3 - 32*a^7*exp
(x) - 6*a^2*b^5*exp(2*x) + 9*a^4*b^3*exp(2*x) + 2*a*b^6*exp(x) - 16*a^6*b*
exp(2*x) - 12*a^3*b^4*exp(x) + 18*a^5*b^2*exp(x)))/(a^4 + b^4 - 2*a^2*b^2)
 + (log(exp(x) - 1)*(2*a + b))/(4*a*b + 2*a^2 + 2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.26

$$\int \frac{\coth^3(x)}{a+b\cosh(x)}\,dx = \frac{2a\,b^2+2e^x a^2 b-3e^{4x}\log(e^x-1)\,a^2 b+3e^{4x}\log(e^x+1)\,a^2 b+6e^{2x}\log(e^x-1)\,a^2 b-6e^{2x}\log(e^x+1)\,a^2 b+}{}$$

input
```
int(coth(x)^3/(a+b*cosh(x)),x)
```

output
```
(2*e**(4*x)*log(e**x - 1)*a**3 - 3*e**(4*x)*log(e**x - 1)*a**2*b + e**(4*x
)*log(e**x - 1)*b**3 + 2*e**(4*x)*log(e**x + 1)*a**3 + 3*e**(4*x)*log(e**x
 + 1)*a**2*b - e**(4*x)*log(e**x + 1)*b**3 - 2*e**(4*x)*log(e**(2*x)*b + 2
*e**x*a + b)*a**3 - 2*e**(4*x)*a**3 + 2*e**(4*x)*a*b**2 + 2*e**(3*x)*a**2*
b - 2*e**(3*x)*b**3 - 4*e**(2*x)*log(e**x - 1)*a**3 + 6*e**(2*x)*log(e**x
- 1)*a**2*b - 2*e**(2*x)*log(e**x - 1)*b**3 - 4*e**(2*x)*log(e**x + 1)*a**
3 - 6*e**(2*x)*log(e**x + 1)*a**2*b + 2*e**(2*x)*log(e**x + 1)*b**3 + 4*e*
*(2*x)*log(e**(2*x)*b + 2*e**x*a + b)*a**3 + 2*e**x*a**2*b - 2*e**x*b**3 +
 2*log(e**x - 1)*a**3 - 3*log(e**x - 1)*a**2*b + log(e**x - 1)*b**3 + 2*lo
g(e**x + 1)*a**3 + 3*log(e**x + 1)*a**2*b - log(e**x + 1)*b**3 - 2*log(e**
(2*x)*b + 2*e**x*a + b)*a**3 - 2*a**3 + 2*a*b**2)/(2*(e**(4*x)*a**4 - 2*e*
*(4*x)*a**2*b**2 + e**(4*x)*b**4 - 2*e**(2*x)*a**4 + 4*e**(2*x)*a**2*b**2
- 2*e**(2*x)*b**4 + a**4 - 2*a**2*b**2 + b**4))
```

3.186 $\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 137

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{2a^4\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}} - \frac{a^3\coth(x)}{(a^2-b^2)^2} - \frac{a\coth^3(x)}{3(a^2-b^2)} + \frac{a^2b\operatorname{csch}(x)}{(a^2-b^2)^2} + \frac{b\operatorname{csch}(x)}{a^2-b^2} + \frac{b\operatorname{csch}^3(x)}{3(a^2-b^2)}$$

output
```
2*a^4*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(5/2)/(a+b)^(5/2)
-a^3*coth(x)/(a^2-b^2)^2-a*coth(x)^3/(3*a^2-3*b^2)+a^2*b*csch(x)/(a^2-b^2)
^2+b*csch(x)/(a^2-b^2)+b*csch(x)^3/(3*a^2-3*b^2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.96

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{1}{24}\left(-\frac{48a^4\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}} - \frac{2(8a+5b)\coth\left(\frac{x}{2}\right)}{(a+b)^2} + \frac{8\text{csch}^3(x)\sinh^4\left(\frac{x}{2}\right)}{a-b} - \frac{\text{csch}^4\left(\frac{x}{2}\right)\sinh(x)}{2(a+b)} + \frac{2(-8a+5b)\tanh\left(\frac{x}{2}\right)}{(a-b)^2}\right)$$

input

```
Integrate[Coth[x]^4/(a + b*Cosh[x]),x]
```

output

```
((-48*a^4*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(5/2)
 - (2*(8*a + 5*b)*Coth[x/2])/(a + b)^2 + (8*Csch[x]^3*Sinh[x/2]^4)/(a - b)
 - (Csch[x/2]^4*Sinh[x])/(2*(a + b)) + (2*(-8*a + 5*b)*Tanh[x/2])/(a - b)^
2)/24
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.91 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.15, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.462$, Rules used = {3042, 3206, 25, 3042, 25, 3086, 2009, 3087, 15, 3206, 25, 3042, 25, 3086, 24, 3138, 221, 4254, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan\left(-\frac{\pi}{2}+ix\right)^4}{a-b\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3206}$$

$$-\frac{a^2 \int -\frac{\coth^2(x)}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{b\int -\coth^3(x)\text{csch}(x)dx}{a^2-b^2}+\frac{a\int \coth^2(x)\text{csch}^2(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{a^2 \int \frac{\coth^2(x)}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\int \coth^3(x)\text{csch}(x)dx}{a^2-b^2}+\frac{a\int \coth^2(x)\text{csch}^2(x)dx}{a^2-b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{a^2 \int -\frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{b\int -\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a^2-b^2}$$

$$\Big\downarrow \text{25}$$

$$-\frac{a^2 \int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}+\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a^2-b^2}$$

$$\Big\downarrow \text{3086}$$

$$-\frac{ib\int \left(-\text{csch}^2(x)-1\right)d(-i\text{csch}(x))}{a^2-b^2}-\frac{a^2 \int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2 \int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3087}$$

$$-\frac{ia\int -\coth^2(x)d(i\coth(x))}{a^2-b^2}-\frac{a^2 \int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{15}$$

$$-\frac{a^2 \int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-b\sin\left(ix-\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \text{3206}$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}+\frac{a\int -\text{csch}^2(x)dx}{a^2-b^2}+\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{a\int \text{csch}^2(x)dx}{a^2-b^2}+\frac{b\int \coth(x)\text{csch}(x)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3042$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{a\int -\csc(ix)^2dx}{a^2-b^2}+\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 25$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \csc(ix)^2dx}{a^2-b^2}+\frac{b\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)dx}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3086$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{ib\int 1d(-i\text{csch}(x))}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 24$$

$$-\frac{a^2\left(-\frac{a^2\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}+\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3(a^2-b^2)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 3138$$

$$-\frac{a^2\left(\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{2a^2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 221$$

$$-\frac{a^2\left(\frac{a\int \csc(ix)^2dx}{a^2-b^2}-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 4254$$

$$-\frac{a^2\left(\frac{ia\int 1d(-i\coth(x))}{a^2-b^2}-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

$$\Big\downarrow \; 24$$

$$-\frac{a^2\left(-\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}+\frac{a\coth(x)}{a^2-b^2}-\frac{b\text{csch}(x)}{a^2-b^2}\right)}{a^2-b^2}-\frac{a\coth^3(x)}{3\left(a^2-b^2\right)}-\frac{ib\left(\frac{1}{3}i\text{csch}^3(x)+i\text{csch}(x)\right)}{a^2-b^2}$$

input
```
Int[Coth[x]^4/(a + b*Cosh[x]),x]
```

output
```
-1/3*(a*Coth[x]^3)/(a^2 - b^2) - (a^2*((-2*a^2*ArcTanh[(Sqrt[a - b]*Tanh[x
/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) + (a*Coth[x])/(a^
2 - b^2) - (b*Csch[x])/(a^2 - b^2)))/(a^2 - b^2) - (I*b*(I*Csch[x] + (I/3)
*Csch[x]^3))/(a^2 - b^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3206
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Simp[a/(a^2 - b^2)   Int[(g*Tan[e + f*x])^p/Sin[e + f*x]
^2, x], x] + (-Simp[b*(g/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 1)/Cos[e
+ f*x], x], x] - Simp[a^2*(g^2/(a^2 - b^2))   Int[(g*Tan[e + f*x])^(p - 2)/
(a + b*Sin[e + f*x]), x], x]) /; FreeQ[{a, b, e, f, g}, x] && NeQ[a^2 - b^2
, 0] && IntegersQ[2*p] && GtQ[p, 1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.93

method	result
default	$-\frac{\frac{a\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{b\tanh\left(\frac{x}{2}\right)^{3}}{3}+5a\tanh\left(\frac{x}{2}\right)-3b\tanh\left(\frac{x}{2}\right)}{8(a-b)^{2}}-\frac{1}{24(a+b)\tanh\left(\frac{x}{2}\right)^{3}}-\frac{5a+3b}{8(a+b)^{2}\tanh\left(\frac{x}{2}\right)}+\frac{2a^{4}\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a-b)^{2}(a+b)^{2}\sqrt{(a+b)(a-b)}}$
risch	$-\frac{2\left(-6a^{2}b\,e^{5x}+3b^{3}e^{5x}+6a^{3}e^{4x}-3a\,b^{2}e^{4x}+8a^{2}b\,e^{3x}-2b^{3}e^{3x}-6a^{3}e^{2x}-6a^{2}b\,e^{x}+3b^{3}e^{x}+4a^{3}-b^{2}a\right)}{3(a^{4}-2a^{2}b^{2}+b^{4})(e^{2x}-1)^{3}}+\frac{a^{4}\ln\left(e^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}\cdots}{b\sqrt{a^{2}-b^{2}}}\cdots\right)}{\sqrt{a^{2}-b^{2}}\,(a+b)^{2}(a-b)^{2}}\cdots$

input
```
int(coth(x)^4/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-1/8/(a-b)^2*(1/3*a*tanh(1/2*x)^3-1/3*b*tanh(1/2*x)^3+5*a*tanh(1/2*x)-3*b*
tanh(1/2*x))-1/24/(a+b)/tanh(1/2*x)^3-1/8*(5*a+3*b)/(a+b)^2/tanh(1/2*x)+2/
(a-b)^2*a^4/(a+b)^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(
a-b))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(123) = 246$.

Time = 0.11 (sec) , antiderivative size = 2417, normalized size of antiderivative = 17.64

$$\int \frac{\coth^4(x)}{a + b\cosh(x)}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[1/3*(6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^5 + 6*(2*a^4*b - 3*a^2*b^3 + b
^5)*sinh(x)^5 - 8*a^5 + 10*a^3*b^2 - 2*a*b^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^
4)*cosh(x)^4 - 6*(2*a^5 - 3*a^3*b^2 + a*b^4 - 5*(2*a^4*b - 3*a^2*b^3 + b^5
)*cosh(x))*sinh(x)^4 - 4*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^3 - 4*(4*a^4*
b - 5*a^2*b^3 + b^5 - 15*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^2 + 6*(2*a^5
- 3*a^3*b^2 + a*b^4)*cosh(x))*sinh(x)^3 + 12*(a^5 - a^3*b^2)*cosh(x)^2 + 1
2*(a^5 - a^3*b^2 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^3 - 3*(2*a^5 - 3*
a^3*b^2 + a*b^4)*cosh(x)^2 - (4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x))*sinh(x)^
2 + 3*(a^4*cosh(x)^6 + 6*a^4*cosh(x)*sinh(x)^5 + a^4*sinh(x)^6 - 3*a^4*cos
h(x)^4 + 3*a^4*cosh(x)^2 + 3*(5*a^4*cosh(x)^2 - a^4)*sinh(x)^4 - a^4 + 4*(
5*a^4*cosh(x)^3 - 3*a^4*cosh(x))*sinh(x)^3 + 3*(5*a^4*cosh(x)^4 - 6*a^4*co
sh(x)^2 + a^4)*sinh(x)^2 + 6*(a^4*cosh(x)^5 - 2*a^4*cosh(x)^3 + a^4*cosh(x
))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cos
h(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*
cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*
cosh(x) + a)*sinh(x) + b)) + 6*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x) + 6*(2*
a^4*b - 3*a^2*b^3 + b^5 + 5*(2*a^4*b - 3*a^2*b^3 + b^5)*cosh(x)^4 - 4*(2*a
^5 - 3*a^3*b^2 + a*b^4)*cosh(x)^3 - 2*(4*a^4*b - 5*a^2*b^3 + b^5)*cosh(x)^
2 + 4*(a^5 - a^3*b^2)*cosh(x))*sinh(x))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6)*cosh(x)^6 + 6*(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*cosh(x)*sinh(x)^5 ...
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \int \frac{\coth^4{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate(coth(x)**4/(a+b*cosh(x)),x)
```

output
```
Integral(coth(x)**4/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.26

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{2\,a^4\arctan\left(\frac{be^x+a}{\sqrt{-a^2+b^2}}\right)}{(a^4-2\,a^2b^2+b^4)\sqrt{-a^2+b^2}} + \frac{2\left(6\,a^2be^{(5\,x)}-3\,b^3e^{(5\,x)}-6\,a^3e^{(4\,x)}+3\,ab^2e^{(4\,x)}-8\,a^2be^{(3\,x)}+2\,b^3e^{(3\,x)}+6\,a^3e^{(2\,x)}+6\,a^2be^x-3\,b^3e^x\right.}{3\,(a^4-2\,a^2b^2+b^4)(e^{(2\,x)}-1)^3}$$

input
```
integrate(coth(x)^4/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*a^4*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-
a^2 + b^2)) + 2/3*(6*a^2*b*e^(5*x) - 3*b^3*e^(5*x) - 6*a^3*e^(4*x) + 3*a*b
^2*e^(4*x) - 8*a^2*b*e^(3*x) + 2*b^3*e^(3*x) + 6*a^3*e^(2*x) + 6*a^2*b*e^x
 - 3*b^3*e^x - 4*a^3 + a*b^2)/((a^4 - 2*a^2*b^2 + b^4)*(e^(2*x) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 666, normalized size of antiderivative = 4.86

$$\int \frac{\coth^4(x)}{a+b\cosh(x)}\,dx = \frac{\frac{4\,(a\,b^2-a^3)}{(a^2-b^2)^2}+\frac{8\,e^x\,(a^2\,b-b^3)}{3\,(a^2-b^2)^2}}{e^{4\,x}-2\,e^{2\,x}+1} - \frac{\frac{8\,a}{3\,(a^2-b^2)}-\frac{8\,b\,e^x}{3\,(a^2-b^2)}}{3\,e^{2\,x}-3\,e^{4\,x}+e^{6\,x}-1} - \frac{\frac{2\,a\,(2\,a^2-b^2)}{(a^2-b^2)^2}-\frac{2\,b\,e^x\,(2\,a^2-b^2)}{(a^2-b^2)^2}}{e^{2\,x}-1} + \frac{2\,\mathrm{atan}\left(\left(e^x\left(\frac{2\,a^4}{b^2\,(a^2-b^2)^2\,\sqrt{a^8}\,(a^4-2\,a^2\,b^2+b^4)}+\frac{2\,\left(a^5\,\sqrt{a^8}-2\,a^3\,b^2\,\sqrt{a^8}+a\,b^4\,\sqrt{a^8}\right)}{a^3\,b^2\,\sqrt{-(a^2-b^2)^5}\,(a^4-2\,a^2\,b^2+b^4)\,\sqrt{-a^{10}+5\,a^8\,b^2-10\,a^6\,b^4+10\,a^4\,b^6-5\,a^2\,b^8+b}}\right.\right.\right.}{}$$

input
```
int(coth(x)^4/(a + b*cosh(x)),x)
```

output
```
((4*(a*b^2 - a^3))/(a^2 - b^2)^2 + (8*exp(x)*(a^2*b - b^3))/(3*(a^2 - b^2)
^2))/(exp(4*x) - 2*exp(2*x) + 1) - ((8*a)/(3*(a^2 - b^2)) - (8*b*exp(x))/(
3*(a^2 - b^2)))/(3*exp(2*x) - 3*exp(4*x) + exp(6*x) - 1) - ((2*a*(2*a^2 -
b^2))/(a^2 - b^2)^2 - (2*b*exp(x)*(2*a^2 - b^2))/(a^2 - b^2)^2)/(exp(2*x)
- 1) + (2*atan((exp(x)*((2*a^4)/(b^2*(a^2 - b^2)^2*(a^8)^(1/2)*(a^4 + b^4
- 2*a^2*b^2)) + (2*(a^5*(a^8)^(1/2) - 2*a^3*b^2*(a^8)^(1/2) + a*b^4*(a^8)^
(1/2)))/(a^3*b^2*(-(a^2 - b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^
10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))) + (2*(b^5*(a
^8)^(1/2) - 2*a^2*b^3*(a^8)^(1/2) + a^4*b*(a^8)^(1/2)))/(a^3*b^2*(-(a^2 -
b^2)^5)^(1/2)*(a^4 + b^4 - 2*a^2*b^2)*(b^10 - a^10 - 5*a^2*b^8 + 10*a^4*b^
6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)))*((b^5*(b^10 - a^10 - 5*a^2*b^8 + 10*a^
4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2 - a^2*b^3*(b^10 - a^10 - 5*a^2*b^
8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2) + (a^4*b*(b^10 - a^10 - 5*a
^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2))/2))*(a^8)^(1/2))/(b^1
0 - a^10 - 5*a^2*b^8 + 10*a^4*b^6 - 10*a^6*b^4 + 5*a^8*b^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 496, normalized size of antiderivative = 3.62

$$\int \frac{\coth^4(x)}{a + b\cosh(x)}\,dx$$
$$= \frac{-6e^{6x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^4 + 18e^{4x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^4 - 18e^{2x}\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+}{\sqrt{-a^2}}\right.}{3e^{6x}a^6 - 9e^{6x}a^4b^2 + 9e^{6x}a}$$

input
```
int(coth(x)^4/(a+b*cosh(x)),x)
```

output
```
(2*( - 3*e**(6*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b*
*2))*a**4 + 9*e**(4*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2
 + b**2))*a**4 - 9*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( -
 a**2 + b**2))*a**4 + 3*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**
2 + b**2))*a**4 - 2*e**(6*x)*a**5 + 3*e**(6*x)*a**3*b**2 - e**(6*x)*a*b**4
 + 6*e**(5*x)*a**4*b - 9*e**(5*x)*a**2*b**3 + 3*e**(5*x)*b**5 - 8*e**(3*x)
*a**4*b + 10*e**(3*x)*a**2*b**3 - 2*e**(3*x)*b**5 + 3*e**(2*x)*a**3*b**2 -
 3*e**(2*x)*a*b**4 + 6*e**x*a**4*b - 9*e**x*a**2*b**3 + 3*e**x*b**5 - 2*a*
*5 + 2*a**3*b**2))/(3*(e**(6*x)*a**6 - 3*e**(6*x)*a**4*b**2 + 3*e**(6*x)*a
**2*b**4 - e**(6*x)*b**6 - 3*e**(4*x)*a**6 + 9*e**(4*x)*a**4*b**2 - 9*e**(
4*x)*a**2*b**4 + 3*e**(4*x)*b**6 + 3*e**(2*x)*a**6 - 9*e**(2*x)*a**4*b**2
+ 9*e**(2*x)*a**2*b**4 - 3*e**(2*x)*b**6 - a**6 + 3*a**4*b**2 - 3*a**2*b**
4 + b**6))
```

3.187 $\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx = \frac{3\arctan(\sinh(x))}{8a} - \frac{3\text{sech}(x)\tanh(x)}{8a} - \frac{\text{sech}(x)\tanh^3(x)}{4a} - \frac{\tanh^5(x)}{5a}$$

output
```
3/8*arctan(sinh(x))/a-3/8*sech(x)*tanh(x)/a-1/4*sech(x)*tanh(x)^3/a-1/5*ta
nh(x)^5/a
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx = \frac{\cosh^2\left(\frac{x}{2}\right)\left(30\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\left(-8-25\text{sech}(x)+16\text{sech}^2(x)+10\text{sech}^3(x)-8\text{sech}^4(x)\right)\tanh(x)\right)}{20a(1+\cosh(x))}$$

input
```
Integrate[Tanh[x]^6/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]^2*(30*ArcTan[Tanh[x/2]] + (-8 - 25*Sech[x] + 16*Sech[x]^2 + 10*
Sech[x]^3 - 8*Sech[x]^4)*Tanh[x]))/(20*a*(1 + Cosh[x]))
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.077$, Rules used = {3042, 25, 3185, 25, 3042, 3087, 15, 3091, 25, 3042, 25, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^6\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^6}dx$$

$$\Big\downarrow \text{ 3185}$$

$$-\frac{\int \text{sech}^2(x)\tanh^4(x)dx}{a}-\frac{\int -\text{sech}(x)\tanh^4(x)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \text{sech}(x)\tanh^4(x)dx}{a}-\frac{\int \text{sech}^2(x)\tanh^4(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sec(ix)\tan(ix)^4dx}{a}-\frac{\int \sec(ix)^2\tan(ix)^4dx}{a}$$

$$\Big\downarrow \text{ 3087}$$

$$\frac{i\int \tanh^4(x)d(i\tanh(x))}{a}+\frac{\int \sec(ix)\tan(ix)^4dx}{a}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\tanh^5(x)}{5a}+\frac{\int \sec(ix)\tan(ix)^4dx}{a}$$

$$\Big\downarrow \textbf{3091}$$

$$\frac{-\frac{3}{4}\int -\text{sech}(x)\tanh^2(x)dx-\frac{1}{4}\tanh^3(x)\text{sech}(x)}{a}-\frac{\tanh^5(x)}{5a}$$

$$\Big\downarrow \textbf{25}$$

$$\frac{\frac{3}{4}\int \text{sech}(x)\tanh^2(x)dx-\frac{1}{4}\tanh^3(x)\text{sech}(x)}{a}-\frac{\tanh^5(x)}{5a}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\tanh^5(x)}{5a}+\frac{-\frac{1}{4}\tanh^3(x)\text{sech}(x)+\frac{3}{4}\int -\sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{\tanh^5(x)}{5a}+\frac{-\frac{1}{4}\tanh^3(x)\text{sech}(x)-\frac{3}{4}\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \textbf{3091}$$

$$\frac{-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{\int \text{sech}(x)dx}{2}\right)-\frac{1}{4}\tanh^3(x)\text{sech}(x)}{a}-\frac{\tanh^5(x)}{5a}$$

$$\Big\downarrow \textbf{3042}$$

$$-\frac{\tanh^5(x)}{5a}+\frac{-\frac{1}{4}\tanh^3(x)\text{sech}(x)-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx\right)}{a}$$

$$\Big\downarrow \textbf{4257}$$

$$\frac{-\frac{3}{4}\left(\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{1}{2}\arctan(\sinh(x))\right)-\frac{1}{4}\tanh^3(x)\text{sech}(x)}{a}-\frac{\tanh^5(x)}{5a}$$

input
```
Int[Tanh[x]^6/(a + a*Cosh[x]),x]
```

output
```
-1/5*Tanh[x]^5/a + (-1/4*(Sech[x]*Tanh[x]^3) - (3*(-1/2*ArcTan[Sinh[x]] +
(Sech[x]*Tanh[x])/2))/4)/a
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.39

method	result	size
default	$\frac{\frac{64\left(\frac{3\tanh\left(\frac{x}{2}\right)^{9}}{256}+\frac{7\tanh\left(\frac{x}{2}\right)^{7}}{128}-\frac{\tanh\left(\frac{x}{2}\right)^{5}}{10}-\frac{7\tanh\left(\frac{x}{2}\right)^{3}}{128}-\frac{3\tanh\left(\frac{x}{2}\right)}{256}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{5}}+\frac{3\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{4}}{a}$	64
risch	$-\frac{25\,{\rm e}^{9x}-40\,{\rm e}^{8x}+10\,{\rm e}^{7x}-80\,{\rm e}^{4x}-10\,{\rm e}^{3x}-25\,{\rm e}^{x}-8}{20({\rm e}^{2x}+1)^{5}a}+\frac{3i\ln({\rm e}^{x}+i)}{8a}-\frac{3i\ln({\rm e}^{x}-i)}{8a}$	75

input
```
int(tanh(x)^6/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
64/a*((3/256*tanh(1/2*x)^9+7/128*tanh(1/2*x)^7-1/10*tanh(1/2*x)^5-7/128*ta
nh(1/2*x)^3-3/256*tanh(1/2*x))/(tanh(1/2*x)^2+1)^5+3/256*arctan(tanh(1/2*x
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. 2(38) = 76.

Time = 0.09 (sec) , antiderivative size = 750, normalized size of antiderivative = 16.30

$$\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/20*(25*cosh(x)^9 + 5*(45*cosh(x) - 8)*sinh(x)^8 + 25*sinh(x)^9 - 40*cos
h(x)^8 + 10*(90*cosh(x)^2 - 32*cosh(x) + 1)*sinh(x)^7 + 10*cosh(x)^7 + 70*
(30*cosh(x)^3 - 16*cosh(x)^2 + cosh(x))*sinh(x)^6 + 70*(45*cosh(x)^4 - 32*
cosh(x)^3 + 3*cosh(x)^2)*sinh(x)^5 + 10*(315*cosh(x)^5 - 280*cosh(x)^4 + 3
5*cosh(x)^3 - 8)*sinh(x)^4 - 80*cosh(x)^4 + 10*(210*cosh(x)^6 - 224*cosh(x
)^5 + 35*cosh(x)^4 - 32*cosh(x) - 1)*sinh(x)^3 - 10*cosh(x)^3 + 10*(90*cos
h(x)^7 - 112*cosh(x)^6 + 21*cosh(x)^5 - 48*cosh(x)^2 - 3*cosh(x))*sinh(x)^
2 - 15*(cosh(x)^10 + 10*cosh(x)*sinh(x)^9 + sinh(x)^10 + 5*(9*cosh(x)^2 +
1)*sinh(x)^8 + 5*cosh(x)^8 + 40*(3*cosh(x)^3 + cosh(x))*sinh(x)^7 + 10*(21
*cosh(x)^4 + 14*cosh(x)^2 + 1)*sinh(x)^6 + 10*cosh(x)^6 + 4*(63*cosh(x)^5
+ 70*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 10*(21*cosh(x)^6 + 35*cosh(x)^4 +
 15*cosh(x)^2 + 1)*sinh(x)^4 + 10*cosh(x)^4 + 40*(3*cosh(x)^7 + 7*cosh(x)^
5 + 5*cosh(x)^3 + cosh(x))*sinh(x)^3 + 5*(9*cosh(x)^8 + 28*cosh(x)^6 + 30*
cosh(x)^4 + 12*cosh(x)^2 + 1)*sinh(x)^2 + 5*cosh(x)^2 + 10*(cosh(x)^9 + 4*
cosh(x)^7 + 6*cosh(x)^5 + 4*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(
x) + sinh(x)) + 5*(45*cosh(x)^8 - 64*cosh(x)^7 + 14*cosh(x)^6 - 64*cosh(x)
^3 - 6*cosh(x)^2 - 5)*sinh(x) - 25*cosh(x) - 8)/(a*cosh(x)^10 + 10*a*cosh(
x)*sinh(x)^9 + a*sinh(x)^10 + 5*a*cosh(x)^8 + 5*(9*a*cosh(x)^2 + a)*sinh(x
)^8 + 40*(3*a*cosh(x)^3 + a*cosh(x))*sinh(x)^7 + 10*a*cosh(x)^6 + 10*(21*a
*cosh(x)^4 + 14*a*cosh(x)^2 + a)*sinh(x)^6 + 4*(63*a*cosh(x)^5 + 70*a*c...
```

Sympy [F]

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^6(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**6/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**6/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 89 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.93

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = -\frac{25\,e^{(-x)} + 10\,e^{(-3\,x)} + 80\,e^{(-4\,x)} - 10\,e^{(-7\,x)} + 40\,e^{(-8\,x)} - 25\,e^{(-9\,x)} + 8}{20\,(5\,ae^{(-2\,x)} + 10\,ae^{(-4\,x)} + 10\,ae^{(-6\,x)} + 5\,ae^{(-8\,x)} + ae^{(-10\,x)} + a)} - \frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

input
```
integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/20*(25*e^(-x) + 10*e^(-3*x) + 80*e^(-4*x) - 10*e^(-7*x) + 40*e^(-8*x) -
 25*e^(-9*x) + 8)/(5*a*e^(-2*x) + 10*a*e^(-4*x) + 10*a*e^(-6*x) + 5*a*e^(-
8*x) + a*e^(-10*x) + a) - 3/4*arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.26

$$\int \frac{\tanh^6(x)}{a + a\cosh(x)}\,dx = \frac{3\,\arctan\left(e^{x}\right)}{4\,a} - \frac{25\,e^{(9\,x)} - 40\,e^{(8\,x)} + 10\,e^{(7\,x)} - 80\,e^{(4\,x)} - 10\,e^{(3\,x)} - 25\,e^{x} - 8}{20\,a(e^{(2\,x)} + 1)^5}$$

input
```
integrate(tanh(x)^6/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
3/4*arctan(e^x)/a - 1/20*(25*e^(9*x) - 40*e^(8*x) + 10*e^(7*x) - 80*e^(4*x
) - 10*e^(3*x) - 25*e^x - 8)/(a*(e^(2*x) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 183, normalized size of antiderivative = 3.98

$$\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx = \frac{\frac{16}{a}-\frac{6\,e^x}{a}}{3\,e^{2\,x}+3\,e^{4\,x}+e^{6\,x}+1}-\frac{\frac{8}{a}-\frac{9\,e^x}{2\,a}}{2\,e^{2\,x}+e^{4\,x}+1} + \frac{32}{5\,a\,\left(5\,e^{2\,x}+10\,e^{4\,x}+10\,e^{6\,x}+5\,e^{8\,x}+e^{10\,x}+1\right)} - \frac{\frac{16}{a}-\frac{4\,e^x}{a}}{4\,e^{2\,x}+6\,e^{4\,x}+4\,e^{6\,x}+e^{8\,x}+1}+\frac{\frac{2}{a}-\frac{5\,e^x}{4\,a}}{e^{2\,x}+1}+\frac{3\,\mathrm{atan}\left(\frac{e^x\,\sqrt{a^2}}{a}\right)}{4\,\sqrt{a^2}}$$

input
```
int(tanh(x)^6/(a + a*cosh(x)),x)
```

output
```
(16/a - (6*exp(x))/a)/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (8/a - (9
*exp(x))/(2*a))/(2*exp(2*x) + exp(4*x) + 1) + 32/(5*a*(5*exp(2*x) + 10*exp
(4*x) + 10*exp(6*x) + 5*exp(8*x) + exp(10*x) + 1)) - (16/a - (4*exp(x))/a)
/(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1) + (2/a - (5*exp(x))
/(4*a))/(exp(2*x) + 1) + (3*atan((exp(x)*(a^2)^(1/2))/a))/(4*(a^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.28

$$\int \frac{\tanh^6(x)}{a+a\cosh(x)}\,dx = \frac{15e^{10x}atan(e^x)+75e^{8x}atan(e^x)+150e^{6x}atan(e^x)+150e^{4x}atan(e^x)+75e^{2x}atan(e^x)+15atan(e^x)-8e}{20a\left(e^{10x}+5e^{8x}+10e^{6x}+10e^{4x}+5e^{2x}+1\right)}$$

input
```
int(tanh(x)^6/(a+a*cosh(x)),x)
```

output
```
(15*e**(10*x)*atan(e**x) + 75*e**(8*x)*atan(e**x) + 150*e**(6*x)*atan(e**x
) + 150*e**(4*x)*atan(e**x) + 75*e**(2*x)*atan(e**x) + 15*atan(e**x) - 8*e
**(10*x) - 25*e**(9*x) - 10*e**(7*x) - 80*e**(6*x) + 10*e**(3*x) - 40*e**(
2*x) + 25*e**x)/(20*a*(e**(10*x) + 5*e**(8*x) + 10*e**(6*x) + 10*e**(4*x)
+ 5*e**(2*x) + 1))
```

3.188 $\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx = -\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^3(x)}{3a}-\frac{\tanh^4(x)}{4a}$$

output
```
-sech(x)/a+1/3*sech(x)^3/a-1/4*tanh(x)^4/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx = \frac{2(3+5\cosh(x))\operatorname{sech}^4(x)\sinh^6\left(\frac{x}{2}\right)}{3a}$$

input
```
Integrate[Tanh[x]^5/(a + a*Cosh[x]),x]
```

output
```
(2*(3 + 5*Cosh[x])*Sech[x]^4*Sinh[x/2]^6)/(3*a)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.36 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.27, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 26, 3185, 26, 3042, 26, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)^5\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^5}dx$$

$$\Big\downarrow \text{3185}$$

$$i\left(\frac{\int i\text{sech}^2(x)\tanh^3(x)dx}{a}+\frac{\int -i\text{sech}(x)\tanh^3(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\int \text{sech}^2(x)\tanh^3(x)dx}{a}-\frac{i\int \text{sech}(x)\tanh^3(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int i\sec(ix)^2\tan(ix)^3dx}{a}-\frac{i\int i\sec(ix)\tan(ix)^3dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \sec(ix)\tan(ix)^3dx}{a}-\frac{\int \sec(ix)^2\tan(ix)^3dx}{a}\right)$$

$$\Big\downarrow \text{3086}$$

$$i\left(-\frac{i\int \left(\text{sech}^2(x)-1\right)d\text{sech}(x)}{a}-\frac{\int \sec(ix)^2\tan(ix)^3dx}{a}\right)$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{\int \sec(ix)^2\tan(ix)^3dx}{a}-\frac{i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a}\right)$$

$$\Big\downarrow \text{3087}$$

$$i\left(\frac{i\int -i\tanh^3(x)d(i\tanh(x))}{a}-\frac{i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{i\tanh^4(x)}{4a}-\frac{i\left(\frac{\text{sech}^3(x)}{3}-\text{sech}(x)\right)}{a}\right)$$

input
```
Int[Tanh[x]^5/(a + a*Cosh[x]),x]
```

output
```
I*(((-I)*(-Sech[x] + Sech[x]^3/3))/a + ((I/4)*Tanh[x]^4)/a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.84 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.43

method	result	size
default	$\frac{-\frac{4}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{4}}-\frac{8}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}+\frac{32}{3\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}}{a}$	43
risch	$-\frac{2\,{\mathrm e}^{x}\left(3\,{\mathrm e}^{6 x}-3\,{\mathrm e}^{5 x}+5\,{\mathrm e}^{4 x}+5\,{\mathrm e}^{2 x}-3\,{\mathrm e}^{x}+3\right)}{3\left({\mathrm e}^{2 x}+1\right)^{4} a}$	46

input
```
int(tanh(x)^5/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
32/a*(-1/8/(tanh(1/2*x)^2+1)^4-1/4/(tanh(1/2*x)^2+1)^2+1/3/(tanh(1/2*x)^2+
1)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(26) = 52$.

Time = 0.07 (sec) , antiderivative size = 174, normalized size of antiderivative = 5.80

$$\int \frac{\tanh^5(x)}{a + a\cosh(x)}\,dx = -\frac{2\left(3\cosh(x)^4 + 3(4\cosh(x) - 1)\sinh(x)^3 + 3\sinh(x)^4 - 3\cosh(x)^3 + \left(18\cosh(x)^2 - 9\cosh(x)\right.\right.}{3\left(a\cosh(x)^5 + 5a\cosh(x)\sinh(x)^4 + a\sinh(x)^5 + 5a\cosh(x)^3 + \left(10a\cosh(x)^2 + 3a\right)\sinh(x)^3 + \right.}$$

input
```
integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(x)^4 + 3*(4*cosh(x) - 1)*sinh(x)^3 + 3*sinh(x)^4 - 3*cosh(x)^
3 + (18*cosh(x)^2 - 9*cosh(x) + 8)*sinh(x)^2 + 8*cosh(x)^2 + (12*cosh(x)^3
 - 9*cosh(x)^2 + 4*cosh(x) + 3)*sinh(x) - 3*cosh(x) + 5)/(a*cosh(x)^5 + 5*
a*cosh(x)*sinh(x)^4 + a*sinh(x)^5 + 5*a*cosh(x)^3 + (10*a*cosh(x)^2 + 3*a)
*sinh(x)^3 + 5*(2*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^2 + 10*a*cosh(x) + (5
*a*cosh(x)^4 + 9*a*cosh(x)^2 + 2*a)*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^5(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**5/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**5/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 223, normalized size of antiderivative = 7.43

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx = -\frac{2\,e^{(-x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a} + \frac{2\,e^{(-2\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a} - \frac{10\,e^{(-3\,x)}}{3\,(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a)} - \frac{10\,e^{(-5\,x)}}{3\,(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a)} + \frac{2\,e^{(-6\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a} - \frac{2\,e^{(-7\,x)}}{4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a}$$

input
```
integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*e^(-x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) +
2*e^(-2*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) -
 10/3*e^(-3*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) +
a) - 10/3*e^(-5*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x
) + a) + 2*e^(-6*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*
x) + a) - 2*e^(-7*x)/(4*a*e^(-2*x) + 6*a*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8
*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.60

$$\int \frac{\tanh^5(x)}{a+a\cosh(x)}\,dx = -\frac{2\left(3\left(e^{(-x)}+e^{x}\right)^3-3\left(e^{(-x)}+e^{x}\right)^2-4\,e^{(-x)}-4\,e^{x}+6\right)}{3\,a\left(e^{(-x)}+e^{x}\right)^4}$$

input
```
integrate(tanh(x)^5/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-2/3*(3*(e^(-x) + e^x)^3 - 3*(e^(-x) + e^x)^2 - 4*e^(-x) - 4*e^x + 6)/(a*(
e^(-x) + e^x)^4)
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 117, normalized size of antiderivative = 3.90

$$\int \frac{\tanh^5(x)}{a + a\cosh(x)}\,dx = \frac{\frac{8}{a} - \frac{8\,\mathrm{e}^{x}}{3\,a}}{3\,\mathrm{e}^{2\,x} + 3\,\mathrm{e}^{4\,x} + \mathrm{e}^{6\,x} + 1} - \frac{\frac{6}{a} - \frac{8\,\mathrm{e}^{x}}{3\,a}}{2\,\mathrm{e}^{2\,x} + \mathrm{e}^{4\,x} + 1} + \frac{\frac{2}{a} - \frac{2\,\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x} + 1} - \frac{4}{a\,\left(4\,\mathrm{e}^{2\,x} + 6\,\mathrm{e}^{4\,x} + 4\,\mathrm{e}^{6\,x} + \mathrm{e}^{8\,x} + 1\right)}$$

input
```
int(tanh(x)^5/(a + a*cosh(x)),x)
```

output
```
(8/a - (8*exp(x))/(3*a))/(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1) - (6/a -
 (8*exp(x))/(3*a))/(2*exp(2*x) + exp(4*x) + 1) + (2/a - (2*exp(x))/a)/(exp
(2*x) + 1) - 4/(a*(4*exp(2*x) + 6*exp(4*x) + 4*exp(6*x) + exp(8*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 77, normalized size of antiderivative = 2.57

$$\int \frac{\tanh^5(x)}{a + a\cosh(x)}\,dx = \frac{-3e^{8x} - 12e^{7x} - 20e^{5x} - 18e^{4x} - 20e^{3x} - 12e^{x} - 3}{6a\,(e^{8x} + 4e^{6x} + 6e^{4x} + 4e^{2x} + 1)}$$

input
```
int(tanh(x)^5/(a+a*cosh(x)),x)
```

output
```
( - 3*e**(8*x) - 12*e**(7*x) - 20*e**(5*x) - 18*e**(4*x) - 20*e**(3*x) - 1
2*e**x - 3)/(6*a*(e**(8*x) + 4*e**(6*x) + 6*e**(4*x) + 4*e**(2*x) + 1))
```

3.189 $\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 33

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{2a}-\frac{\operatorname{sech}(x)\tanh(x)}{2a}-\frac{\tanh^3(x)}{3a}$$

output
```
1/2*arctan(sinh(x))/a-1/2*sech(x)*tanh(x)/a-1/3*tanh(x)^3/a
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.39

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx =\frac{\cosh^2\left(\frac{x}{2}\right)\left(6\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\left(-2-3\operatorname{sech}(x)+2\operatorname{sech}^2(x)\right)\tanh(x)\right)}{3a(1+\cosh(x))}$$

input
```
Integrate[Tanh[x]^4/(a + a*Cosh[x]),x]
```

output
```
(Cosh[x/2]^2*(6*ArcTan[Tanh[x/2]] + (-2 - 3*Sech[x] + 2*Sech[x]^2)*Tanh[x]
))/(3*a*(1 + Cosh[x]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3185, 25, 3042, 25, 3087, 15, 3091, 3042, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^4\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 3185}$$

$$\frac{\int -\text{sech}^2(x)\tanh^2(x)dx}{a}+\frac{\int \text{sech}(x)\tanh^2(x)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \text{sech}(x)\tanh^2(x)dx}{a}-\frac{\int \text{sech}^2(x)\tanh^2(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int -\sec(ix)\tan(ix)^2dx}{a}-\frac{\int -\sec(ix)^2\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \sec(ix)^2\tan(ix)^2dx}{a}-\frac{\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{ 3087}$$

$$-\frac{i\int -\tanh^2(x)d(i\tanh(x))}{a}-\frac{\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{\tanh^3(x)}{3a}-\frac{\int \sec(ix)\tan(ix)^2dx}{a}$$

$$\Big\downarrow \text{ 3091}$$

$$-\frac{\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{\int \text{sech}(x)dx}{2}}{a}-\frac{\tanh^3(x)}{3a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\tanh^3(x)}{3a}-\frac{\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{1}{2}\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$-\frac{\frac{1}{2}\tanh(x)\text{sech}(x)-\frac{1}{2}\arctan(\sinh(x))}{a}-\frac{\tanh^3(x)}{3a}$$

input
```
Int[Tanh[x]^4/(a + a*Cosh[x]),x]
```

output
```
-1/3*Tanh[x]^3/a - (-1/2*ArcTan[Sinh[x]] + (Sech[x]*Tanh[x])/2)/a
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

method	result	size
default	$\frac{\frac{16\left(\frac{\tanh\left(\frac{x}{2}\right)^{5}}{16}-\frac{\tanh\left(\frac{x}{2}\right)^{3}}{6}-\frac{\tanh\left(\frac{x}{2}\right)}{16}\right)}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{3}}+\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	48
risch	$-\frac{3\,\mathrm{e}^{5x}-6\,\mathrm{e}^{4x}-3\,\mathrm{e}^{x}-2}{3\left(\mathrm{e}^{2x}+1\right)^{3}a}+\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{2a}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{2a}$	57

input
```
int(tanh(x)^4/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
16/a*((1/16*tanh(1/2*x)^5-1/6*tanh(1/2*x)^3-1/16*tanh(1/2*x))/(tanh(1/2*x)
^2+1)^3+1/16*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. $2(27) = 54$.

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 9.55

$$\int \frac{\tanh^4(x)}{a + a\cosh(x)}\,dx =$$
$$-\frac{3\cosh(x)^5 + 3(5\cosh(x) - 2)\sinh(x)^4 + 3\sinh(x)^5 - 6\cosh(x)^4 + 6(5\cosh(x)^2 - 4\cosh(x))\,\mathrm{si}}{}$$

input
```
integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/3*(3*cosh(x)^5 + 3*(5*cosh(x) - 2)*sinh(x)^4 + 3*sinh(x)^5 - 6*cosh(x)^
4 + 6*(5*cosh(x)^2 - 4*cosh(x))*sinh(x)^3 + 6*(5*cosh(x)^3 - 6*cosh(x)^2)*
sinh(x)^2 - 3*(cosh(x)^6 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 3*(5*cosh(x)^
2 + 1)*sinh(x)^4 + 3*cosh(x)^4 + 4*(5*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 3
*(5*cosh(x)^4 + 6*cosh(x)^2 + 1)*sinh(x)^2 + 3*cosh(x)^2 + 6*(cosh(x)^5 +
2*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + 3*(5*cosh(
x)^4 - 8*cosh(x)^3 - 1)*sinh(x) - 3*cosh(x) - 2)/(a*cosh(x)^6 + 6*a*cosh(x
)*sinh(x)^5 + a*sinh(x)^6 + 3*a*cosh(x)^4 + 3*(5*a*cosh(x)^2 + a)*sinh(x)^
4 + 4*(5*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cos
h(x)^4 + 6*a*cosh(x)^2 + a)*sinh(x)^2 + 6*(a*cosh(x)^5 + 2*a*cosh(x)^3 + a
*cosh(x))*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**4/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 57 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.73

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx = -\frac{3\,e^{(-x)}+6\,e^{(-4\,x)}-3\,e^{(-5\,x)}+2}{3\,(3\,ae^{(-2\,x)}+3\,ae^{(-4\,x)}+ae^{(-6\,x)}+a)}-\frac{\arctan\left(e^{(-x)}\right)}{a}$$

input
```
integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/3*(3*e^(-x) + 6*e^(-4*x) - 3*e^(-5*x) + 2)/(3*a*e^(-2*x) + 3*a*e^(-4*x)
 + a*e^(-6*x) + a) - arctan(e^(-x))/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.18

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx = \frac{\arctan\left(e^{x}\right)}{a}-\frac{3\,e^{(5\,x)}-6\,e^{(4\,x)}-3\,e^{x}-2}{3\,a(e^{(2\,x)}+1)^3}$$

input
```
integrate(tanh(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
arctan(e^x)/a - 1/3*(3*e^(5*x) - 6*e^(4*x) - 3*e^x - 2)/(a*(e^(2*x) + 1)^3
)
```

Mupad [B] (verification not implemented)

Time = 1.87 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.88

$$\int \frac{\tanh^4(x)}{a+a\cosh(x)}\,dx = \frac{8}{3\,a\,\left(3\,\mathrm{e}^{2\,x}+3\,\mathrm{e}^{4\,x}+\mathrm{e}^{6\,x}+1\right)} - \frac{\frac{4}{a}-\frac{2\,\mathrm{e}^{x}}{a}}{2\,\mathrm{e}^{2\,x}+\mathrm{e}^{4\,x}+1}+\frac{\frac{2}{a}-\frac{\mathrm{e}^{x}}{a}}{\mathrm{e}^{2\,x}+1}+\frac{\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(tanh(x)^4/(a + a*cosh(x)),x)
```

output
```
8/(3*a*(3*exp(2*x) + 3*exp(4*x) + exp(6*x) + 1)) - (4/a - (2*exp(x))/a)/(2
*exp(2*x) + exp(4*x) + 1) + (2/a - exp(x)/a)/(exp(2*x) + 1) + atan((exp(x)
*(a^2)^(1/2))/a)/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.85

$$\int \frac{\tanh^4(x)}{a + a\cosh(x)}\,dx = \frac{3e^{6x}atan(e^x) + 9e^{4x}atan(e^x) + 9e^{2x}atan(e^x) + 3atan(e^x) - 2e^{6x} - 3e^{5x} - 6e^{2x} + 3e^x}{3a\left(e^{6x} + 3e^{4x} + 3e^{2x} + 1\right)}$$

input
```
int(tanh(x)^4/(a+a*cosh(x)),x)
```

output
```
(3*e**(6*x)*atan(e**x) + 9*e**(4*x)*atan(e**x) + 9*e**(2*x)*atan(e**x) + 3
*atan(e**x) - 2*e**(6*x) - 3*e**(5*x) - 6*e**(2*x) + 3*e**x)/(3*a*(e**(6*x
) + 3*e**(4*x) + 3*e**(2*x) + 1))
```

3.190 $\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 19

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx = -\frac{\operatorname{sech}(x)}{a}+\frac{\operatorname{sech}^2(x)}{2a}$$

output
```
-sech(x)/a+1/2*sech(x)^2/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.89

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx = \frac{2\operatorname{sech}^2(x)\sinh^4\left(\frac{x}{2}\right)}{a}$$

input
```
Integrate[Tanh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(2*Sech[x]^2*Sinh[x/2]^4)/a
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.42, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.692$, Rules used = {3042, 26, 3185, 26, 3042, 26, 3086, 15, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)^3\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^3}dx$$

$$\Big\downarrow \text{3185}$$

$$-i\left(\frac{\int -i\text{sech}^2(x)\tanh(x)dx}{a}+\frac{\int i\text{sech}(x)\tanh(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \text{sech}(x)\tanh(x)dx}{a}-\frac{i\int \text{sech}^2(x)\tanh(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -i\sec(ix)\tan(ix)dx}{a}-\frac{i\int -i\sec(ix)^2\tan(ix)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \sec(ix)\tan(ix)dx}{a}-\frac{\int \sec(ix)^2\tan(ix)dx}{a}\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(\frac{i\int \text{sech}(x)d\text{sech}(x)}{a}-\frac{i\int 1d\text{sech}(x)}{a}\right)$$

$$\Big\downarrow \; 15$$

$$-i\left(\frac{i\text{sech}^2(x)}{2a}-\frac{i\int 1d\text{sech}(x)}{a}\right)$$

$$\Big\downarrow \; 24$$

$$-i\left(\frac{i\text{sech}^2(x)}{2a}-\frac{i\text{sech}(x)}{a}\right)$$

input
```
Int[Tanh[x]^3/(a + a*Cosh[x]),x]
```

output
```
(-I)*(((-I)*Sech[x])/a + ((I/2)*Sech[x]^2)/a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

method	result	size
risch	$-\frac{2\,\mathrm{e}^{x}\left(\mathrm{e}^{2x}-\mathrm{e}^{x}+1\right)}{\left(\mathrm{e}^{2x}+1\right)^{2}a}$	26
default	$\frac{-\frac{4}{\tanh\left(\frac{x}{2}\right)^{2}+1}+\frac{2}{\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)^{2}}}{a}$	31

input
```
int(tanh(x)^3/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
-2*exp(x)*(exp(2*x)-exp(x)+1)/(exp(2*x)+1)^2/a
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. 2(17) = 34.

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 3.47

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx = -\frac{2\left(\cosh\left(x\right)^{2}+\left(2\,\cosh\left(x\right)-1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}-\cosh\left(x\right)+1\right)}{a\cosh\left(x\right)^{3}+3\,a\cosh\left(x\right)\sinh\left(x\right)^{2}+a\sinh\left(x\right)^{3}+3\,a\cosh\left(x\right)+\left(3\,a\cosh\left(x\right)^{2}+a\right)\sinh\left(x\right)}$$

input
```
integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2*(cosh(x)^2 + (2*cosh(x) - 1)*sinh(x) + sinh(x)^2 - cosh(x) + 1)/(a*cosh
(x)^3 + 3*a*cosh(x)*sinh(x)^2 + a*sinh(x)^3 + 3*a*cosh(x) + (3*a*cosh(x)^2
 + a)*sinh(x))
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**3/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. 2(17) = 34.

Time = 0.03 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.68

$$\int \frac{\tanh^3(x)}{a + a\cosh(x)}\,dx = -\frac{2\,e^{(-x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} + \frac{2\,e^{(-2\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a} - \frac{2\,e^{(-3\,x)}}{2\,ae^{(-2\,x)} + ae^{(-4\,x)} + a}$$

input
```
integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*e^(-x)/(2*a*e^(-2*x) + a*e^(-4*x) + a) + 2*e^(-2*x)/(2*a*e^(-2*x) + a*e
^(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-2*x) + a*e^(-4*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \frac{\tanh^3(x)}{a + a\cosh(x)}\,dx = -\frac{2\left(e^{(-x)} + e^{x} - 1\right)}{a\left(e^{(-x)} + e^{x}\right)^2}$$

input
```
integrate(tanh(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-2*(e^(-x) + e^x - 1)/(a*(e^(-x) + e^x)^2)
```

Mupad [B] (verification not implemented)

Time = 1.85 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.32

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx = -\frac{2\,\mathrm{e}^x\,\left(\mathrm{e}^{2\,x}-\mathrm{e}^x+1\right)}{a\,{\left(\mathrm{e}^{2\,x}+1\right)}^2}$$

input
```
int(tanh(x)^3/(a + a*cosh(x)),x)
```

output
```
-(2*exp(x)*(exp(2*x) - exp(x) + 1))/(a*(exp(2*x) + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.16

$$\int \frac{\tanh^3(x)}{a+a\cosh(x)}\,dx = \frac{-e^{4x}-2e^{3x}-2e^{x}-1}{a\,(e^{4x}+2e^{2x}+1)}$$

input
```
int(tanh(x)^3/(a+a*cosh(x)),x)
```

output
```
( - e**(4*x) - 2*e**(3*x) - 2*e**x - 1)/(a*(e**(4*x) + 2*e**(2*x) + 1))
```

3.191 $\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 15

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx=\frac{\arctan(\sinh(x))}{a}-\frac{\tanh(x)}{a}$$

output
```
arctan(sinh(x))/a-tanh(x)/a
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.20

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx=\frac{2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)-\tanh(x)}{a}$$

input
```
Integrate[Tanh[x]^2/(a + a*Cosh[x]),x]
```

output
```
(2*ArcTan[Tanh[x/2]] - Tanh[x])/a
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.615, Rules used = {3042, 25, 3185, 25, 3042, 4254, 24, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{1}{\tan\left(-\frac{\pi}{2}+ix\right)^2\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)^2}dx$$

$$\Big\downarrow \text{ 3185}$$

$$-\frac{\int \text{sech}^2(x)dx}{a}-\frac{\int -\text{sech}(x)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \text{sech}(x)dx}{a}-\frac{\int \text{sech}^2(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{\int \csc\left(ix+\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{ 4254}$$

$$\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}-\frac{i\int 1d(-i\tanh(x))}{a}$$

$$\Big\downarrow \text{ 24}$$

$$-\frac{\tanh(x)}{a}+\frac{\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{ 4257}$$

$$\frac{\arctan(\sinh(x))}{a} - \frac{\tanh(x)}{a}$$

input
```
Int[Tanh[x]^2/(a + a*Cosh[x]),x]
```

output
```
ArcTan[Sinh[x]]/a - Tanh[x]/a
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4254
```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
andIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
 d}, x] && IGtQ[n/2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 30, normalized size of antiderivative = 2.00

method	result	size
default	$\frac{-\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh\left(\frac{x}{2}\right)^{2}+1}+2\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$	30
risch	$\frac{2}{a\left(\mathrm{e}^{2x}+1\right)}+\frac{i\ln\left(\mathrm{e}^{x}+i\right)}{a}-\frac{i\ln\left(\mathrm{e}^{x}-i\right)}{a}$	39

input
```
int(tanh(x)^2/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
4/a*(-1/2*tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/2*arctan(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(15) = 30$.

Time = 0.09 (sec) , antiderivative size = 50, normalized size of antiderivative = 3.33

$$\int \frac{\tanh^2(x)}{a+a\cosh(x)}\,dx = \frac{2\left(\left(\cosh\left(x\right)^{2}+2\cosh\left(x\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+1\right)\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)+1\right)}{a\cosh\left(x\right)^{2}+2\,a\cosh\left(x\right)\sinh\left(x\right)+a\sinh\left(x\right)^{2}+a}$$

input
```
integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
2*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*arctan(cosh(x) + sinh(x
)) + 1)/(a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 + a)
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh^2(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(tanh(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)**2/(cosh(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.53

$$\int \frac{\tanh^2(x)}{a + a\cosh(x)}\,dx = -\frac{2\arctan\left(e^{(-x)}\right)}{a} - \frac{2}{ae^{(-2x)} + a}$$

input
```
integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*arctan(e^(-x))/a - 2/(a*e^(-2*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^2(x)}{a + a\cosh(x)}\,dx = \frac{2\arctan\left(e^{x}\right)}{a} + \frac{2}{a(e^{(2x)} + 1)}$$

input
```
integrate(tanh(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
2*arctan(e^x)/a + 2/(a*(e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 33, normalized size of antiderivative = 2.20

$$\int \frac{\tanh^2(x)}{a + a\cosh(x)}\,dx = \frac{2}{a\,\left(\mathrm{e}^{2\,x}+1\right)} + \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,\sqrt{a^2}}{a}\right)}{\sqrt{a^2}}$$

input
```
int(tanh(x)^2/(a + a*cosh(x)),x)
```

output
```
2/(a*(exp(2*x) + 1)) + (2*atan((exp(x)*(a^2)^(1/2))/a))/(a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.40

$$\int \frac{\tanh^2(x)}{a + a\cosh(x)}\,dx = \frac{2e^{2x}atan(e^x) + 2atan(e^x) - 2e^{2x}}{a\,(e^{2x}+1)}$$

input
```
int(tanh(x)^2/(a+a*cosh(x)),x)
```

output
```
(2*(e**(2*x)*atan(e**x) + atan(e**x) - e**(2*x)))/(a*(e**(2*x) + 1))
```

3.192 $\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 18

$$\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx=\frac{\log(\cosh(x))}{a}-\frac{\log(1+\cosh(x))}{a}$$

output
```
ln(cosh(x))/a-ln(1+cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.67

$$\int \frac{\tanh(x)}{a+a\cosh(x)}\,dx=-\frac{2\text{arctanh}(1+2\cosh(x))}{a}$$

input
```
Integrate[Tanh[x]/(a + a*Cosh[x]),x]
```

output
```
(-2*ArcTanh[1 + 2*Cosh[x]])/a
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.545$, Rules used = {3042, 26, 3186, 47, 14, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)\left(a-a\sin\left(-\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\left(a-a\sin\left(ix-\frac{\pi}{2}\right)\right)\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3186}$$

$$\int \frac{\operatorname{sech}(x)}{a(a\cosh(x)+a)}d(a\cosh(x))$$

$$\Big\downarrow \text{47}$$

$$\frac{\int \frac{\operatorname{sech}(x)}{a}d(a\cosh(x))}{a}-\frac{\int \frac{1}{\cosh(x)a+a}d(a\cosh(x))}{a}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log(a\cosh(x))}{a}-\frac{\int \frac{1}{\cosh(x)a+a}d(a\cosh(x))}{a}$$

$$\Big\downarrow \text{16}$$

$$\frac{\log(a\cosh(x))}{a}-\frac{\log(a\cosh(x)+a)}{a}$$

input
```
Int[Tanh[x]/(a + a*Cosh[x]),x]
```

output
```
Log[a*Cosh[x]]/a - Log[a + a*Cosh[x]]/a
```

Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3186
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[x^p*((a + x)^(m - (p + 1)/2)/(a - x)
^((p + 1)/2)), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && E
qQ[a^2 - b^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.78

method	result	size
default	$\frac{\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{a}$	14
risch	$-\frac{2\ln\left(e^{x}+1\right)}{a}+\frac{\ln\left(e^{2x}+1\right)}{a}$	23

input
```
int(tanh(x)/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/a*ln(tanh(1/2*x)^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.56

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = \frac{\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right) - 2\log\left(\cosh\left(x\right) + \sinh\left(x\right) + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
(log(2*cosh(x)/(cosh(x) - sinh(x))) - 2*log(cosh(x) + sinh(x) + 1))/a
```

Sympy [F]

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\tanh\left(x\right)}{\cosh\left(x\right)+1}\,dx}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x)
```

output
```
Integral(tanh(x)/(cosh(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = -\frac{2\log\left(e^{(-x)} + 1\right)}{a} + \frac{\log\left(e^{(-2x)} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2*log(e^(-x) + 1)/a + log(e^(-2*x) + 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = \frac{\log\left(e^{(2x)} + 1\right)}{a} - \frac{2\,\log\left(e^{x} + 1\right)}{a}$$

input
```
integrate(tanh(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
log(e^(2*x) + 1)/a - 2*log(e^x + 1)/a
```

Mupad [B] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.44

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = -\frac{2\,\ln\left(36\,e^{x} + 36\right) - \ln\left(3\,e^{2\,x} + 3\right)}{a}$$

input
```
int(tanh(x)/(a + a*cosh(x)),x)
```

output
```
-(2*log(36*exp(x) + 36) - log(3*exp(2*x) + 3))/a
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

$$\int \frac{\tanh(x)}{a + a\cosh(x)}\,dx = \frac{\log(e^{2x} + 1) - 2\log(e^{x} + 1)}{a}$$

input
```
int(tanh(x)/(a+a*cosh(x)),x)
```

output
```
(log(e**(2*x) + 1) - 2*log(e**x + 1))/a
```

3.193 $\int \frac{\coth(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 11, antiderivative size = 33

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx = -\frac{\operatorname{arctanh}(\cosh(x))}{2a} - \frac{\coth(x)\operatorname{csch}(x)}{2a} + \frac{\operatorname{csch}^2(x)}{2a}$$

output
```
-1/2*arctanh(cosh(x))/a-1/2*coth(x)*csch(x)/a+1/2*csch(x)^2/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx = -\frac{1+2\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)}{2a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]/(a + a*Cosh[x]),x]
```

output
```
-1/2*(1 + 2*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]))/(a*(1 + Cosh[x]
))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 1.182$, Rules used = {3042, 26, 3185, 26, 3042, 26, 3086, 15, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan\left(-\frac{\pi}{2}+ix\right)}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$-i\left(\frac{\int i\coth^2(x)\text{csch}(x)dx}{a}+\frac{\int -i\coth(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{i\int \coth^2(x)\text{csch}(x)dx}{a}-\frac{i\int \coth(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}\right)$$

$$\Big\downarrow \text{3086}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\int -i\text{csch}(x)d(-i\text{csch}(x))}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$-i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3091}$$

$$-i\left(\frac{-\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(\frac{\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(\frac{-\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

$$\Big\downarrow \text{4257}$$

$$-i\left(\frac{-\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)}{a}+\frac{i\text{csch}^2(x)}{2a}\right)$$

input
```
Int[Coth[x]/(a + a*Cosh[x]),x]
```

output
```
(-I)*(((I/2)*Csch[x]^2)/a + ((-1/2*I)*ArcTanh[Cosh[x]] - (I/2)*Coth[x]*Csc
h[x])/a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.61

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{2}}{2}+\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{2a}$	20
risch	$-\frac{e^{x}}{\left(e^{x}+1\right)^{2} a}+\frac{\ln\left(e^{x}-1\right)}{2a}-\frac{\ln\left(e^{x}+1\right)}{2a}$	35

input
```
int(coth(x)/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/2/a*(1/2*tanh(1/2*x)^2+ln(tanh(1/2*x)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(27) = 54.

Time = 0.08 (sec) , antiderivative size = 103, normalized size of antiderivative = 3.12

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx = -\frac{\left(\cosh\left(x\right)^{2}+2\left(\cosh\left(x\right)+1\right)\sinh\left(x\right)+\sinh\left(x\right)^{2}+2\,\cosh\left(x\right)+1\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)+1\right)-\left(\mathrm{co}\right.}{2\left(a\cosh\left(x\right)^{2}+a\sinh\left(x\right)^{2}+\right.}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/2*((cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^2 + 2*cosh(x) + 1)*lo
g(cosh(x) + sinh(x) + 1) - (cosh(x)^2 + 2*(cosh(x) + 1)*sinh(x) + sinh(x)^
2 + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*cosh(x) + 2*sinh(x))/(a*
cosh(x)^2 + a*sinh(x)^2 + 2*a*cosh(x) + 2*(a*cosh(x) + a)*sinh(x) + a)
```

Sympy [F]

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx=\frac{\int \frac{\coth(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)/(cosh(x) + 1), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx=-\frac{e^{(-x)}}{2\,ae^{(-x)}+ae^{(-2\,x)}+a}-\frac{\log\left(e^{(-x)}+1\right)}{2\,a}+\frac{\log\left(e^{(-x)}-1\right)}{2\,a}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-e^(-x)/(2*a*e^(-x) + a*e^(-2*x) + a) - 1/2*log(e^(-x) + 1)/a + 1/2*log(e^
(-x) - 1)/a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.58

$$\int \frac{\coth(x)}{a+a\cosh(x)}\,dx=-\frac{\log\left(e^{(-x)}+e^{x}+2\right)}{4\,a}+\frac{\log\left(e^{(-x)}+e^{x}-2\right)}{4\,a}+\frac{e^{(-x)}+e^{x}-2}{4\,a(e^{(-x)}+e^{x}+2)}$$

input
```
integrate(coth(x)/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/4*log(e^(-x) + e^x + 2)/a + 1/4*log(e^(-x) + e^x - 2)/a + 1/4*(e^(-x) +
 e^x - 2)/(a*(e^(-x) + e^x + 2))
```

Mupad [B] (verification not implemented)

Time = 1.88 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.55

$$\int \frac{\coth(x)}{a + a\cosh(x)}\,dx = \frac{1}{a\,\left(e^{2\,x} + 2\,e^x + 1\right)} - \frac{1}{a\,\left(e^x + 1\right)} - \frac{\operatorname{atan}\left(\frac{e^x\,\sqrt{-a^2}}{a}\right)}{\sqrt{-a^2}}$$

input
```
int(coth(x)/(a + a*cosh(x)),x)
```

output
```
1/(a*(exp(2*x) + 2*exp(x) + 1)) - 1/(a*(exp(x) + 1)) - atan((exp(x)*(-a^2)
^(1/2))/a)/(-a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 87, normalized size of antiderivative = 2.64

$$\int \frac{\coth(x)}{a + a\cosh(x)}\,dx = \frac{e^{2x}\log(e^x-1) - e^{2x}\log(e^x+1) + e^{2x} + 2e^x\log(e^x-1) - 2e^x\log(e^x+1) + \log(e^x-1) - \log(e^x+1) + 1}{2a\left(e^{2x}+2e^x+1\right)}$$

input
```
int(coth(x)/(a+a*cosh(x)),x)
```

output
```
(e**(2*x)*log(e**x - 1) - e**(2*x)*log(e**x + 1) + e**(2*x) + 2*e**x*log(e
**x - 1) - 2*e**x*log(e**x + 1) + log(e**x - 1) - log(e**x + 1) + 1)/(2*a*
(e**(2*x) + 2*e**x + 1))
```

3.194 $\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 30

$$\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx=\frac{\coth^3(x)}{3a}-\frac{\operatorname{csch}(x)}{a}-\frac{\operatorname{csch}^3(x)}{3a}$$

output
```
1/3*coth(x)^3/a-csch(x)/a-1/3*csch(x)^3/a
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

$$\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx=\frac{(-3-4\cosh(x)+\cosh(2x))\operatorname{csch}(x)}{6a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]^2/(a + a*Cosh[x]),x]
```

output
```
((-3 - 4*Cosh[x] + Cosh[2*x])*Csch[x])/(6*a*(1 + Cosh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.769, Rules used = {3042, 25, 3185, 25, 3042, 25, 3086, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan\left(-\frac{\pi}{2}+ix\right)^2}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^2}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$-\frac{\int -\coth^3(x)\text{csch}(x)dx}{a}-\frac{\int \coth^2(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \coth^3(x)\text{csch}(x)dx}{a}-\frac{\int \coth^2(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int -\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}$$

$$\Big\downarrow \text{3086}$$

$$\frac{i\int \left(-\text{csch}^2(x)-1\right)d(-i\text{csch}(x))}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(\frac{1}{3}i\operatorname{csch}^3(x)+i\operatorname{csch}(x)\right)}{a}-\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}$$

$$\Big\downarrow \text{3087}$$

$$\frac{i\int -\coth^2(x)d(i\coth(x))}{a}+\frac{i\left(\frac{1}{3}i\operatorname{csch}^3(x)+i\operatorname{csch}(x)\right)}{a}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^3(x)}{3a}+\frac{i\left(\frac{1}{3}i\operatorname{csch}^3(x)+i\operatorname{csch}(x)\right)}{a}$$

input
```
Int[Coth[x]^2/(a + a*Cosh[x]),x]
```

output
```
Coth[x]^3/(3*a) + (I*(I*Csch[x] + (I/3)*Csch[x]^3))/a
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.97

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{3}}{3}+2\tanh\left(\frac{x}{2}\right)-\frac{1}{\tanh\left(\frac{x}{2}\right)}}{4a}$	29
risch	$-\frac{2\left(3\,\mathrm{e}^{3x}+3\,\mathrm{e}^{2x}+\mathrm{e}^{x}-1\right)}{3\left(\mathrm{e}^{x}+1\right)^{3}a\left(\mathrm{e}^{x}-1\right)}$	34

input
```
int(coth(x)^2/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*(1/3*tanh(1/2*x)^3+2*tanh(1/2*x)-1/tanh(1/2*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. 2(26) = 52.

Time = 0.08 (sec) , antiderivative size = 91, normalized size of antiderivative = 3.03

$$\int \frac{\coth^2(x)}{a+a\cosh(x)}\,dx = -\frac{2\left(3\cosh(x)^2+2\left(3\cosh(x)+2\right)\sinh(x)+3\sinh(x)^2+2\cosh(x)\right.}{3\left(a\cosh(x)^3+a\sinh(x)^3+2\,a\cosh(x)^2+(3\,a\cosh(x)+2\,a)\sinh(x)^2-a\cosh(x)+\left(3\,a\cosh(x)^2\right.\right.}$$

input
```
integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2/3*(3*cosh(x)^2 + 2*(3*cosh(x) + 2)*sinh(x) + 3*sinh(x)^2 + 2*cosh(x) +
1)/(a*cosh(x)^3 + a*sinh(x)^3 + 2*a*cosh(x)^2 + (3*a*cosh(x) + 2*a)*sinh(x
)^2 - a*cosh(x) + (3*a*cosh(x)^2 + 4*a*cosh(x) + a)*sinh(x) - 2*a)
```

Sympy [F]

$$\int \frac{\coth^2(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\coth^2(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**2/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)**2/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 121 vs. $2(26) = 52$.

Time = 0.03 (sec) , antiderivative size = 121, normalized size of antiderivative = 4.03

$$\begin{aligned}\int \frac{\coth^2(x)}{a + a\cosh(x)}\,dx = &-\frac{2\,e^{(-x)}}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)} \\ &- \frac{2\,e^{(-2\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} \\ &- \frac{2\,e^{(-3\,x)}}{2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a} \\ &+ \frac{2}{3\,(2\,ae^{(-x)} - 2\,ae^{(-3\,x)} - ae^{(-4\,x)} + a)}\end{aligned}$$

input
```
integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-2/3*e^(-x)/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-2*x)/(2*a
*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a) - 2*e^(-3*x)/(2*a*e^(-x) - 2*a*e^
(-3*x) - a*e^(-4*x) + a) + 2/3/(2*a*e^(-x) - 2*a*e^(-3*x) - a*e^(-4*x) + a
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.17

$$\int \frac{\coth^2(x)}{a + a\cosh(x)}\, dx = -\frac{1}{2\, a(e^x - 1)} - \frac{9\, e^{(2\, x)} + 12\, e^x + 7}{6\, a(e^x + 1)^3}$$

input
```
integrate(coth(x)^2/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/2/(a*(e^x - 1)) - 1/6*(9*e^(2*x) + 12*e^x + 7)/(a*(e^x + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.07

$$\int \frac{\coth^2(x)}{a + a\cosh(x)}\, dx = -\frac{\frac{e^{2\,x}}{2\,a} + \frac{1}{2\,a} + \frac{e^x}{3\,a}}{3\, e^{2\,x} + e^{3\,x} + 3\, e^x + 1} - \frac{\frac{1}{6\,a} + \frac{e^x}{2\,a}}{e^{2\,x} + 2\, e^x + 1} - \frac{1}{2\,a\,\left(e^x - 1\right)} - \frac{1}{2\,a\,\left(e^x + 1\right)}$$

input
```
int(coth(x)^2/(a + a*cosh(x)),x)
```

output
```
- (exp(2*x)/(2*a) + 1/(2*a) + exp(x)/(3*a))/(3*exp(2*x) + exp(3*x) + 3*exp
(x) + 1) - (1/(6*a) + exp(x)/(2*a))/(exp(2*x) + 2*exp(x) + 1) - 1/(2*a*(ex
p(x) - 1)) - 1/(2*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.57

$$\int \frac{\coth^2(x)}{a + a\cosh(x)}\,dx = \frac{3e^{4x} - 6e^{2x} - 8e^{x} - 1}{3a\left(e^{4x} + 2e^{3x} - 2e^{x} - 1\right)}$$

input
```
int(coth(x)^2/(a+a*cosh(x)),x)
```

output
```
(3*e**(4*x) - 6*e**(2*x) - 8*e**x - 1)/(3*a*(e**(4*x) + 2*e**(3*x) - 2*e**
x - 1))
```

3.195 $\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 46

$$\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx=-\frac{3\text{arctanh}(\cosh(x))}{8a}+\frac{\coth^4(x)}{4a}-\frac{3\coth(x)\text{csch}(x)}{8a}-\frac{\coth^3(x)\text{csch}(x)}{4a}$$

output
```
-3/8*arctanh(cosh(x))/a+1/4*coth(x)^4/a-3/8*coth(x)*csch(x)/a-1/4*coth(x)^
3*csch(x)/a
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.30

$$\int \frac{\coth^3(x)}{a+a\cosh(x)}\,dx=\frac{-8-2\coth^2\left(\frac{x}{2}\right)-12\cosh^2\left(\frac{x}{2}\right)\left(\log\left(\cosh\left(\frac{x}{2}\right)\right)-\log\left(\sinh\left(\frac{x}{2}\right)\right)\right)+\text{sech}^2\left(\frac{x}{2}\right)}{16a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]^3/(a + a*Cosh[x]),x]
```

output
```
(-8 - 2*Coth[x/2]^2 - 12*Cosh[x/2]^2*(Log[Cosh[x/2]] - Log[Sinh[x/2]]) + S
ech[x/2]^2)/(16*a*(1 + Cosh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.49 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.28, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 1.308$, Rules used = {3042, 26, 3185, 26, 3042, 26, 3087, 15, 3091, 26, 3042, 26, 3091, 26, 3042, 26, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan\left(-\frac{\pi}{2}+ix\right)^3}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan\left(ix-\frac{\pi}{2}\right)^3}{a-a\sin\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3185}$$

$$i\left(\frac{\int -i\coth^4(x)\text{csch}(x)dx}{a}+\frac{\int i\coth^3(x)\text{csch}^2(x)dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{i\int \coth^3(x)\text{csch}^2(x)dx}{a}-\frac{i\int \coth^4(x)\text{csch}(x)dx}{a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{i\int -i\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}-\frac{i\int i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^3dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}\right)$$

$$\Big\downarrow \text{3087}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int -i\coth^3(x)d(i\coth(x))}{a}\right)$$

$$\Big\downarrow \text{15}$$

$$i\left(\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \text{3091}$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\int i\coth^2(x)\text{csch}(x)dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}i\int \coth^2(x)\text{csch}(x)dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}i\int -i\sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^2dx}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \text{3091}$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(-\frac{1}{2}\int -i\text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(\frac{1}{2}i\int \text{csch}(x)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(\frac{1}{2}i\int i\csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 26$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(-\frac{1}{2}\int \csc(ix)dx-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

$$\Big\downarrow \; 4257$$

$$i\left(\frac{\frac{1}{4}i\coth^3(x)\text{csch}(x)-\frac{3}{4}\left(-\frac{1}{2}i\text{arctanh}(\cosh(x))-\frac{1}{2}i\coth(x)\text{csch}(x)\right)}{a}-\frac{i\coth^4(x)}{4a}\right)$$

input
```
Int[Coth[x]^3/(a + a*Cosh[x]),x]
```

output
```
I*(((-1/4*I)*Coth[x]^4)/a + ((I/4)*Coth[x]^3*Csch[x] - (3*((-1/2*I)*ArcTan
h[Cosh[x]] - (I/2)*Coth[x]*Csch[x]))/4)/a)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3091
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[b*(a*Sec[e + f*x])^m*((b*Tan[e + f*x])^(n - 1)/(f*(m
 + n - 1))), x] - Simp[b^2*((n - 1)/(m + n - 1))   Int[(a*Sec[e + f*x])^m*(
b*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &
& NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^4}{4}+\frac{3\tanh\left(\frac{x}{2}\right)^2}{2}+3\ln\left(\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{2\tanh\left(\frac{x}{2}\right)^2}}{8a}$	38
risch	$-\frac{e^x\left(5\,e^{4x}+2\,e^{3x}+2\,e^{2x}+2\,e^x+5\right)}{4(e^x+1)^4a(e^x-1)^2}+\frac{3\ln(e^x-1)}{8a}-\frac{3\ln(e^x+1)}{8a}$	65

input
```
int(coth(x)^3/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*(1/4*tanh(1/2*x)^4+3/2*tanh(1/2*x)^2+3*ln(tanh(1/2*x))-1/2/tanh(1/2*
x)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 631 vs. $2(38) = 76$.

Time = 0.08 (sec) , antiderivative size = 631, normalized size of antiderivative = 13.72

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-1/8*(10*cosh(x)^5 + 2*(25*cosh(x) + 2)*sinh(x)^4 + 10*sinh(x)^5 + 4*cosh(
x)^4 + 4*(25*cosh(x)^2 + 4*cosh(x) + 1)*sinh(x)^3 + 4*cosh(x)^3 + 4*(25*co
sh(x)^3 + 6*cosh(x)^2 + 3*cosh(x) + 1)*sinh(x)^2 + 4*cosh(x)^2 + 3*(cosh(x
)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 + sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^
2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 -
 cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*c
osh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh
(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 - cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*
log(cosh(x) + sinh(x) + 1) - 3*(cosh(x)^6 + 2*(3*cosh(x) + 1)*sinh(x)^5 +
sinh(x)^6 + 2*cosh(x)^5 + (15*cosh(x)^2 + 10*cosh(x) - 1)*sinh(x)^4 - cosh
(x)^4 + 4*(5*cosh(x)^3 + 5*cosh(x)^2 - cosh(x) - 1)*sinh(x)^3 - 4*cosh(x)^
3 + (15*cosh(x)^4 + 20*cosh(x)^3 - 6*cosh(x)^2 - 12*cosh(x) - 1)*sinh(x)^2
 - cosh(x)^2 + 2*(3*cosh(x)^5 + 5*cosh(x)^4 - 2*cosh(x)^3 - 6*cosh(x)^2 -
cosh(x) + 1)*sinh(x) + 2*cosh(x) + 1)*log(cosh(x) + sinh(x) - 1) + 2*(25*c
osh(x)^4 + 8*cosh(x)^3 + 6*cosh(x)^2 + 4*cosh(x) + 5)*sinh(x) + 10*cosh(x)
)/(a*cosh(x)^6 + a*sinh(x)^6 + 2*a*cosh(x)^5 + 2*(3*a*cosh(x) + a)*sinh(x)
^5 - a*cosh(x)^4 + (15*a*cosh(x)^2 + 10*a*cosh(x) - a)*sinh(x)^4 - 4*a*cos
h(x)^3 + 4*(5*a*cosh(x)^3 + 5*a*cosh(x)^2 - a*cosh(x) - a)*sinh(x)^3 - a*c
osh(x)^2 + (15*a*cosh(x)^4 + 20*a*cosh(x)^3 - 6*a*cosh(x)^2 - 12*a*cosh(x)
 - a)*sinh(x)^2 + 2*a*cosh(x) + 2*(3*a*cosh(x)^5 + 5*a*cosh(x)^4 - 2*a*...
```

Sympy [F]

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\coth^3(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**3/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)**3/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(38) = 76$.

Time = 0.04 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.24

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = -\frac{5\,e^{(-x)} + 2\,e^{(-2\,x)} + 2\,e^{(-3\,x)} + 2\,e^{(-4\,x)} + 5\,e^{(-5\,x)}}{4\,(2\,ae^{(-x)} - ae^{(-2\,x)} - 4\,ae^{(-3\,x)} - ae^{(-4\,x)} + 2\,ae^{(-5\,x)} + ae^{(-6\,x)} + a)} - \frac{3\,\log\left(e^{(-x)} + 1\right)}{8\,a} + \frac{3\,\log\left(e^{(-x)} - 1\right)}{8\,a}$$

input
```
integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-1/4*(5*e^(-x) + 2*e^(-2*x) + 2*e^(-3*x) + 2*e^(-4*x) + 5*e^(-5*x))/(2*a*e
^(-x) - a*e^(-2*x) - 4*a*e^(-3*x) - a*e^(-4*x) + 2*a*e^(-5*x) + a*e^(-6*x)
 + a) - 3/8*log(e^(-x) + 1)/a + 3/8*log(e^(-x) - 1)/a
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 94 vs. $2(38) = 76$.

Time = 0.11 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.04

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = -\frac{3\log\left(e^{(-x)} + e^x + 2\right)}{16\,a} + \frac{3\log\left(e^{(-x)} + e^x - 2\right)}{16\,a} - \frac{3\,e^{(-x)} + 3\,e^x - 2}{16\,a(e^{(-x)} + e^x - 2)} + \frac{9\left(e^{(-x)} + e^x\right)^2 + 4\,e^{(-x)} + 4\,e^x - 12}{32\,a(e^{(-x)} + e^x + 2)^2}$$

input
```
integrate(coth(x)^3/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-3/16*log(e^(-x) + e^x + 2)/a + 3/16*log(e^(-x) + e^x - 2)/a - 1/16*(3*e^(
-x) + 3*e^x - 2)/(a*(e^(-x) + e^x - 2)) + 1/32*(9*(e^(-x) + e^x)^2 + 4*e^(
-x) + 4*e^x - 12)/(a*(e^(-x) + e^x + 2)^2)
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.87

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx = \frac{3}{2\,a\,\left(\mathrm{e}^{2\,x} + 2\,\mathrm{e}^{x} + 1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{2\,x} - 2\,\mathrm{e}^{x} + 1\right)} + \frac{1}{2\,a\,\left(6\,\mathrm{e}^{2\,x} + 4\,\mathrm{e}^{3\,x} + \mathrm{e}^{4\,x} + 4\,\mathrm{e}^{x} + 1\right)} - \frac{1}{4\,a\,\left(\mathrm{e}^{x} - 1\right)} - \frac{1}{a\,\left(\mathrm{e}^{x} + 1\right)} - \frac{3\,\mathrm{atan}\left(\frac{\mathrm{e}^{x}\,\sqrt{-a^2}}{a}\right)}{4\,\sqrt{-a^2}} - \frac{1}{a\,\left(3\,\mathrm{e}^{2\,x} + \mathrm{e}^{3\,x} + 3\,\mathrm{e}^{x} + 1\right)}$$

input
```
int(coth(x)^3/(a + a*cosh(x)),x)
```

output
```
3/(2*a*(exp(2*x) + 2*exp(x) + 1)) - 1/(4*a*(exp(2*x) - 2*exp(x) + 1)) + 1/
(2*a*(6*exp(2*x) + 4*exp(3*x) + exp(4*x) + 4*exp(x) + 1)) - 1/(4*a*(exp(x)
 - 1)) - 1/(a*(exp(x) + 1)) - (3*atan((exp(x)*(-a^2)^(1/2))/a))/(4*(-a^2)^
(1/2)) - 1/(a*(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 245, normalized size of antiderivative = 5.33

$$\int \frac{\coth^3(x)}{a + a\cosh(x)}\,dx$$
$$= \frac{3e^{6x}\log(e^x-1) - 3e^{6x}\log(e^x+1) + 5e^{6x} + 6e^{5x}\log(e^x-1) - 6e^{5x}\log(e^x+1) - 3e^{4x}\log(e^x-1) + 3e^{4x}\mathrm{l}}{}$$

input
```
int(coth(x)^3/(a+a*cosh(x)),x)
```

output
```
(3*e**(6*x)*log(e**x - 1) - 3*e**(6*x)*log(e**x + 1) + 5*e**(6*x) + 6*e**(
5*x)*log(e**x - 1) - 6*e**(5*x)*log(e**x + 1) - 3*e**(4*x)*log(e**x - 1) +
 3*e**(4*x)*log(e**x + 1) - 9*e**(4*x) - 12*e**(3*x)*log(e**x - 1) + 12*e*
*(3*x)*log(e**x + 1) - 24*e**(3*x) - 3*e**(2*x)*log(e**x - 1) + 3*e**(2*x)
*log(e**x + 1) - 9*e**(2*x) + 6*e**x*log(e**x - 1) - 6*e**x*log(e**x + 1)
+ 3*log(e**x - 1) - 3*log(e**x + 1) + 5)/(8*a*(e**(6*x) + 2*e**(5*x) - e**
(4*x) - 4*e**(3*x) - e**(2*x) + 2*e**x + 1))
```

3.196 $\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 41

$$\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx = \frac{\coth^5(x)}{5a} - \frac{\operatorname{csch}(x)}{a} - \frac{2\operatorname{csch}^3(x)}{3a} - \frac{\operatorname{csch}^5(x)}{5a}$$

output
```
1/5*coth(x)^5/a-csch(x)/a-2/3*csch(x)^3/a-1/5*csch(x)^5/a
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx = -\frac{(-25+8\cosh(x)+36\cosh(2x)+24\cosh(3x)-3\cosh(4x))\operatorname{csch}^3(x)}{120a(1+\cosh(x))}$$

input
```
Integrate[Coth[x]^4/(a + a*Cosh[x]),x]
```

output
```
-1/120*((-25 + 8*Cosh[x] + 36*Cosh[2*x] + 24*Cosh[3*x] - 3*Cosh[4*x])*Csch
[x]^3)/(a*(1 + Cosh[x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.39 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.12, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.769$, Rules used = {3042, 3185, 25, 3042, 25, 3086, 210, 2009, 3087, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(x)}{a\cosh(x)+a}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan\left(-\frac{\pi}{2}+ix\right)^4}{a-a\sin\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{ 3185}$$

$$\frac{\int \coth^5(x)\text{csch}(x)dx}{a}+\frac{\int -\coth^4(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \coth^5(x)\text{csch}(x)dx}{a}-\frac{\int \coth^4(x)\text{csch}^2(x)dx}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^5dx}{a}-\frac{\int -\sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}+\frac{\int \sec\left(ix-\frac{\pi}{2}\right)\tan\left(ix-\frac{\pi}{2}\right)^5dx}{a}$$

$$\Big\downarrow \text{ 3086}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int \left(-\text{csch}^2(x)-1\right)^2d(-i\text{csch}(x))}{a}$$

$$\Big\downarrow \text{ 210}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2\tan\left(ix-\frac{\pi}{2}\right)^4dx}{a}-\frac{i\int \left(\text{csch}^4(x)+2\text{csch}^2(x)+1\right)d(-i\text{csch}(x))}{a}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\int \sec\left(ix-\frac{\pi}{2}\right)^2 \tan\left(ix-\frac{\pi}{2}\right)^4 dx}{a} - \frac{i\left(-\frac{1}{5}i\text{csch}^5(x)-\frac{2}{3}i\text{csch}^3(x)-i\text{csch}(x)\right)}{a}$$

$$\Big\downarrow \text{3087}$$

$$-\frac{i\int \coth^4(x)d(i\coth(x))}{a} - \frac{i\left(-\frac{1}{5}i\text{csch}^5(x)-\frac{2}{3}i\text{csch}^3(x)-i\text{csch}(x)\right)}{a}$$

$$\Big\downarrow \text{15}$$

$$\frac{\coth^5(x)}{5a} - \frac{i\left(-\frac{1}{5}i\text{csch}^5(x)-\frac{2}{3}i\text{csch}^3(x)-i\text{csch}(x)\right)}{a}$$

input
```
Int[Coth[x]^4/(a + a*Cosh[x]),x]
```

output
```
Coth[x]^5/(5*a) - (I*((-I)*Csch[x] - ((2*I)/3)*Csch[x]^3 - (I/5)*Csch[x]^5
))/a
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3086
```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Simp[a/f   Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2
), x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2
] &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

rule 3087
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Simp[1/f   Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e +
f*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)
/2] && LtQ[0, n, m - 1])
```

rule 3185
```
Int[((g_.)*tan[(e_.) + (f_.)*(x_)])^(p_.)/((a_) + (b_.)*sin[(e_.) + (f_.)*(
x_)]), x_Symbol] :> Simp[1/a   Int[Sec[e + f*x]^2*(g*Tan[e + f*x])^p, x], x
] - Simp[1/(b*g)   Int[Sec[e + f*x]*(g*Tan[e + f*x])^(p + 1), x], x] /; Fre
eQ[{a, b, e, f, g, p}, x] && EqQ[a^2 - b^2, 0] && NeQ[p, -1]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

method	result	size
default	$\frac{\frac{\tanh\left(\frac{x}{2}\right)^{5}}{5}+\frac{4\tanh\left(\frac{x}{2}\right)^{3}}{3}+6\tanh\left(\frac{x}{2}\right)-\frac{4}{\tanh\left(\frac{x}{2}\right)}-\frac{1}{3\tanh\left(\frac{x}{2}\right)^{3}}}{16a}$	45
risch	$-\frac{2\left(15\,{\mathrm e}^{7x}+15\,{\mathrm e}^{6x}-5\,{\mathrm e}^{5x}-25\,{\mathrm e}^{4x}+13\,{\mathrm e}^{3x}+21\,{\mathrm e}^{2x}+9\,{\mathrm e}^{x}-3\right)}{15\left({\mathrm e}^{x}-1\right)^{3}a\left({\mathrm e}^{x}+1\right)^{5}}$	60

input
```
int(coth(x)^4/(a+cosh(x)*a),x,method=_RETURNVERBOSE)
```

output
```
1/16/a*(1/5*tanh(1/2*x)^5+4/3*tanh(1/2*x)^3+6*tanh(1/2*x)-4/tanh(1/2*x)-1/
3/tanh(1/2*x)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 224 vs. $2(35) = 70$.

Time = 0.07 (sec) , antiderivative size = 224, normalized size of antiderivative = 5.46

$$\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx =$$

$$-\frac{2\left(15\,\cosh\left(x\right)^{4} + 6\left(10\,\cosh\left(x\right) + 3\right)\sinh\left(x\right)^{3} + 15\,\sin\right.}{15\left(a\cosh\left(x\right)^{5} + a\sinh\left(x\right)^{5} + 2\,a\cosh\left(x\right)^{4} + \left(5\,a\cosh\left(x\right) + 2\,a\right)\sinh\left(x\right)^{4} - 3\,a\cosh\left(x\right)^{3} + \left(10\,a\cosh\right.\right.}$$

input
```
integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="fricas")
```

output
```
-2/15*(15*cosh(x)^4 + 6*(10*cosh(x) + 3)*sinh(x)^3 + 15*sinh(x)^4 + 12*cos
h(x)^3 + 2*(45*cosh(x)^2 + 18*cosh(x) + 2)*sinh(x)^2 + 4*cosh(x)^2 + 2*(30
*cosh(x)^3 + 27*cosh(x)^2 - 14*cosh(x) - 23)*sinh(x) - 4*cosh(x) + 13)/(a*
cosh(x)^5 + a*sinh(x)^5 + 2*a*cosh(x)^4 + (5*a*cosh(x) + 2*a)*sinh(x)^4 -
3*a*cosh(x)^3 + (10*a*cosh(x)^2 + 8*a*cosh(x) - a)*sinh(x)^3 - 8*a*cosh(x)
^2 + (10*a*cosh(x)^3 + 12*a*cosh(x)^2 - 9*a*cosh(x) - 8*a)*sinh(x)^2 + 2*a
*cosh(x) + (5*a*cosh(x)^4 + 8*a*cosh(x)^3 - 3*a*cosh(x)^2 - 8*a*cosh(x) -
2*a)*sinh(x) + 6*a)
```

Sympy [F]

$$\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx = \frac{\int \frac{\coth^4(x)}{\cosh(x)+1}\,dx}{a}$$

input
```
integrate(coth(x)**4/(a+a*cosh(x)),x)
```

output
```
Integral(coth(x)**4/(cosh(x) + 1), x)/a
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. $2(35) = 70$.

Time = 0.04 (sec) , antiderivative size = 469, normalized size of antiderivative = 11.44

$$\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="maxima")
```

output
```
-6/5*e^(-x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a
*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 14/5*e^(-2*x)/(2*a*e^(-x) - 2
*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) -
a*e^(-8*x) + a) - 26/15*e^(-3*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x)
 + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 10/3*e^(
-4*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6
*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) + 2/3*e^(-5*x)/(2*a*e^(-x) - 2*a*e^(-
2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8
*x) + a) - 2*e^(-6*x)/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-
5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a) - 2*e^(-7*x)/(2*a*e^(
-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-
7*x) - a*e^(-8*x) + a) + 2/5/(2*a*e^(-x) - 2*a*e^(-2*x) - 6*a*e^(-3*x) + 6
*a*e^(-5*x) + 2*a*e^(-6*x) - 2*a*e^(-7*x) - a*e^(-8*x) + a)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.44

$$\int \frac{\coth^4(x)}{a + a\cosh(x)}\,dx = -\frac{15\,e^{(2\,x)} - 24\,e^{x} + 13}{24\,a(e^{x} - 1)^{3}} - \frac{165\,e^{(4\,x)} + 480\,e^{(3\,x)} + 650\,e^{(2\,x)} + 400\,e^{x} + 113}{120\,a(e^{x} + 1)^{5}}$$

input
```
integrate(coth(x)^4/(a+a*cosh(x)),x, algorithm="giac")
```

output
```
-1/24*(15*e^(2*x) - 24*e^x + 13)/(a*(e^x - 1)^3) - 1/120*(165*e^(4*x) + 48
0*e^(3*x) + 650*e^(2*x) + 400*e^x + 113)/(a*(e^x + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 263, normalized size of antiderivative = 6.41

$$\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx = \frac{1}{6\,a\,\left(3\,e^{2\,x}-e^{3\,x}-3\,e^{x}+1\right)}-\frac{\frac{3\,e^{2\,x}}{8\,a}+\frac{11\,e^{3\,x}}{40\,a}+\frac{1}{8\,a}+\frac{17\,e^{x}}{40\,a}}{6\,e^{2\,x}+4\,e^{3\,x}+e^{4\,x}+4\,e^{x}+1}-\frac{\frac{11\,e^{2\,x}}{40\,a}+\frac{17}{120\,a}+\frac{e^{x}}{4\,a}}{3\,e^{2\,x}+e^{3\,x}+3\,e^{x}+1}-\frac{\frac{1}{8\,a}+\frac{11\,e^{x}}{40\,a}}{e^{2\,x}+2\,e^{x}+1}-\frac{\frac{17\,e^{2\,x}}{20\,a}+\frac{e^{3\,x}}{2\,a}+\frac{11\,e^{4\,x}}{40\,a}+\frac{11}{40\,a}+\frac{e^{x}}{2\,a}}{10\,e^{2\,x}+10\,e^{3\,x}+5\,e^{4\,x}+e^{5\,x}+5\,e^{x}+1}-\frac{1}{4\,a\,\left(e^{2\,x}-2\,e^{x}+1\right)}-\frac{5}{8\,a\,\left(e^{x}-1\right)}-\frac{11}{40\,a\,\left(e^{x}+1\right)}$$

input
```
int(coth(x)^4/(a + a*cosh(x)),x)
```

output
```
1/(6*a*(3*exp(2*x) - exp(3*x) - 3*exp(x) + 1)) - ((3*exp(2*x))/(8*a) + (11
*exp(3*x))/(40*a) + 1/(8*a) + (17*exp(x))/(40*a))/(6*exp(2*x) + 4*exp(3*x)
 + exp(4*x) + 4*exp(x) + 1) - ((11*exp(2*x))/(40*a) + 17/(120*a) + exp(x)/
(4*a))/(3*exp(2*x) + exp(3*x) + 3*exp(x) + 1) - (1/(8*a) + (11*exp(x))/(40
*a))/(exp(2*x) + 2*exp(x) + 1) - ((17*exp(2*x))/(20*a) + exp(3*x)/(2*a) +
(11*exp(4*x))/(40*a) + 11/(40*a) + exp(x)/(2*a))/(10*exp(2*x) + 10*exp(3*x
) + 5*exp(4*x) + exp(5*x) + 5*exp(x) + 1) - 1/(4*a*(exp(2*x) - 2*exp(x) +
1)) - 5/(8*a*(exp(x) - 1)) - 11/(40*a*(exp(x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.51

$$\int \frac{\coth^4(x)}{a+a\cosh(x)}\,dx = \frac{15e^{8x}-60e^{6x}-80e^{5x}+50e^{4x}+64e^{3x}-12e^{2x}-48e^{x}-9}{15a\left(e^{8x}+2e^{7x}-2e^{6x}-6e^{5x}+6e^{3x}+2e^{2x}-2e^{x}-1\right)}$$

input
```
int(coth(x)^4/(a+a*cosh(x)),x)
```

output
```
(15*e**(8*x) - 60*e**(6*x) - 80*e**(5*x) + 50*e**(4*x) + 64*e**(3*x) - 12*
e**(2*x) - 48*e**x - 9)/(15*a*(e**(8*x) + 2*e**(7*x) - 2*e**(6*x) - 6*e**(
5*x) + 6*e**(3*x) + 2*e**(2*x) - 2*e**x - 1))
```

3.197 $\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx$

Optimal result

Integrand size = 13, antiderivative size = 37

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx=-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh(x)}$$

output
```
-2*a^(1/2)*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))+2*(a+b*cosh(x))^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx=-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)+2\sqrt{a+b\cosh(x)}$$

input
```
Integrate[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

output
```
-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.462$, Rules used = {3042, 26, 3200, 60, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sqrt{a-b\sin\left(-\frac{\pi}{2}+ix\right)}}{\tan\left(-\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sqrt{a-b\sin\left(ix-\frac{\pi}{2}\right)}}{\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 3200$$

$$\int \frac{\operatorname{sech}(x)\sqrt{a+b\cosh(x)}}{b}d(b\cosh(x))$$

$$\Big\downarrow \; 60$$

$$a\int \frac{\operatorname{sech}(x)}{b\sqrt{a+b\cosh(x)}}d(b\cosh(x))+2\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \; 73$$

$$2a\int \frac{1}{b^2\cosh^2(x)-a}d\sqrt{a+b\cosh(x)}+2\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \; 220$$

$$2\sqrt{a+b\cosh(x)}-2\sqrt{a}\,\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)$$

input
```
Int[Sqrt[a + b*Cosh[x]]*Tanh[x],x]
```

output
```
-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]] + 2*Sqrt[a + b*Cosh[x]]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(29) = 58$.

Time = 0.46 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.38

method	result
default	$2\sqrt{2b\sinh\left(\frac{x}{2}\right)^{2}+a+b}-\sqrt{a}\,\ln\left(\frac{4\cosh\left(\frac{x}{2}\right)b\sqrt{2}+4\sqrt{a}\,\sqrt{2b\sinh\left(\frac{x}{2}\right)^{2}+a+b}+4a-4b}{2\cosh\left(\frac{x}{2}\right)-\sqrt{2}}\right)-\sqrt{a}\,\ln\left(-\frac{4\left(\cosh\left(\frac{x}{2}\right)\right.}{}\right.$

input
```
int((a+b*cosh(x))^(1/2)*tanh(x),x,method=_RETURNVERBOSE)
```

output
```
2*(2*b*sinh(1/2*x)^2+a+b)^(1/2)-a^(1/2)*ln(4/(2*cosh(1/2*x)-2^(1/2))*(cosh
(1/2*x)*b*2^(1/2)+a^(1/2)*(2*b*sinh(1/2*x)^2+a+b)^(1/2)+a-b))-a^(1/2)*ln(-
4/(2*cosh(1/2*x)+2^(1/2))*(cosh(1/2*x)*b*2^(1/2)-a^(1/2)*(2*b*sinh(1/2*x)^
2+a+b)^(1/2)-a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(29) = 58$.

Time = 0.26 (sec) , antiderivative size = 376, normalized size of antiderivative = 10.16

$$\int\sqrt{a+b\cosh(x)}\tanh(x)\,dx$$

$$=\left[\frac{1}{2}\sqrt{a}\log\left(-\frac{b^{2}\cosh\left(x\right)^{4}+b^{2}\sinh\left(x\right)^{4}+16\,ab\cosh\left(x\right)^{3}+4\left(b^{2}\cosh\left(x\right)+4\,ab\right)\sinh\left(x\right)^{3}+16\,ab\cosh\left(\right.}{}\right.\right.$$

$$+2\sqrt{b\cosh\left(x\right)+a},\sqrt{-a}\arctan\left(\frac{\left(b\cosh\left(x\right)^{2}+b\sinh\left(x\right)^{2}+4\,a\cosh\left(x\right)+2\left(b\cosh\left(x\right)+2\,a\right)\sinh\left(x\right)\right.}{2\left(ab\cosh\left(x\right)^{2}+ab\sinh\left(x\right)^{2}+2\,a^{2}\cosh\left(x\right)+ab+2\left(ab\cosh\right.\right.}\right.$$

$$\left.+2\sqrt{b\cosh\left(x\right)+a}\right]$$

input
```
integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="fricas")
```

output
```
[1/2*sqrt(a)*log(-(b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b
^2*cosh(x) + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^
2 + 2*(3*b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*c
osh(x)^3 + b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b
*cosh(x) + (3*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*
sqrt(a) + b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^
2)*cosh(x))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*c
osh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) +
1)) + 2*sqrt(b*cosh(x) + a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^
2 + 4*a*cosh(x) + 2*(b*cosh(x) + 2*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqr
t(-a)/(a*b*cosh(x)^2 + a*b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x
) + a^2)*sinh(x))) + 2*sqrt(b*cosh(x) + a)]
```

Sympy [F]

$$\int \sqrt{a + b\cosh(x)}\tanh(x)\,dx = \int \sqrt{a + b\cosh{(x)}}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x))**(1/2)*tanh(x),x)
```

output
```
Integral(sqrt(a + b*cosh(x))*tanh(x), x)
```

Maxima [F]

$$\int \sqrt{a + b\cosh(x)}\tanh(x)\,dx = \int \sqrt{b\cosh{(x)} + a}\tanh{(x)}\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

Giac [F]

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx = \int \sqrt{b\cosh(x)+a}\tanh(x)\,dx$$

input
```
integrate((a+b*cosh(x))^(1/2)*tanh(x),x, algorithm="giac")
```

output
```
integrate(sqrt(b*cosh(x) + a)*tanh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx = \int \tanh(x)\,\sqrt{a+b\cosh(x)}\,dx$$

input
```
int(tanh(x)*(a + b*cosh(x))^(1/2),x)
```

output
```
int(tanh(x)*(a + b*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\cosh(x)}\tanh(x)\,dx = \int \sqrt{\cosh(x)\,b+a}\,\tanh(x)\,dx$$

input
```
int((a+b*cosh(x))^(1/2)*tanh(x),x)
```

output
```
int(sqrt(cosh(x)*b + a)*tanh(x),x)
```

3.198 $\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 24

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

output
```
-2*arctanh((a+b*cosh(x))^(1/2)/a^(1/2))/a^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = -\frac{2\operatorname{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3042, 26, 3200, 73, 220}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan\left(-\frac{\pi}{2}+ix\right)\sqrt{a-b\sin\left(-\frac{\pi}{2}+ix\right)}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sqrt{a-b\sin\left(ix-\frac{\pi}{2}\right)}\tan\left(ix-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3200}$$

$$\int \frac{\text{sech}(x)}{b\sqrt{a+b\cosh(x)}}d(b\cosh(x))$$

$$\Big\downarrow \text{73}$$

$$2\int \frac{1}{b^2\cosh^2(x)-a}d\sqrt{a+b\cosh(x)}$$

$$\Big\downarrow \text{220}$$

$$-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\cosh(x)}}{\sqrt{a}}\right)}{\sqrt{a}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Cosh[x]],x]
```

output
```
(-2*ArcTanh[Sqrt[a + b*Cosh[x]]/Sqrt[a]])/Sqrt[a]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 220
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3200
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(p
_.), x_Symbol] :> Simp[1/f   Subst[Int[(x^p*(a + x)^m)/(b^2 - x^2)^((p + 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && NeQ[a^2 - b
^2, 0] && IntegerQ[(p + 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(18) = 36$.

Time = 0.43 (sec) , antiderivative size = 104, normalized size of antiderivative = 4.33

method	result	size
default	$-\frac{\ln\left(\frac{4\cosh\left(\frac{x}{2}\right)b\sqrt{2}+4\sqrt{a}\,\sqrt{2b\sinh\left(\frac{x}{2}\right)^{2}+a+b}+4a-4b}{2\cosh\left(\frac{x}{2}\right)-\sqrt{2}}\right)+\ln\left(-\frac{4\left(\cosh\left(\frac{x}{2}\right)b\sqrt{2}-\sqrt{a}\,\sqrt{2b\sinh\left(\frac{x}{2}\right)^{2}+a+b}-a+b\right)}{2\cosh\left(\frac{x}{2}\right)+\sqrt{2}}\right)}{\sqrt{a}}$	104

input
```
int(tanh(x)/(a+b*cosh(x))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/a^(1/2)*(ln(4/(2*cosh(1/2*x)-2^(1/2))*(cosh(1/2*x)*b*2^(1/2)+a^(1/2)*(2
*b*sinh(1/2*x)^2+a+b)^(1/2)+a-b))+ln(-4/(2*cosh(1/2*x)+2^(1/2))*(cosh(1/2*
x)*b*2^(1/2)-a^(1/2)*(2*b*sinh(1/2*x)^2+a+b)^(1/2)-a+b)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(18) = 36$.

Time = 0.12 (sec) , antiderivative size = 356, normalized size of antiderivative = 14.83

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh(x)}}\,dx$$
$$= \left[\frac{\log\left(\frac{b^2\cosh(x)^4 + b^2\sinh(x)^4 + 16\,ab\cosh(x)^3 + 4\left(b^2\cosh(x) + 4\,ab\right)\sinh(x)^3 + 16\,ab\cosh(x) + 2\left(16\,a^2 + b^2\right)\cosh(x)^2 + 2\left(3\,b^2\cosh(x)^2 + 24\,a\right.}{\cosh}\right.}{} \right.$$

input
```
integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="fricas")
```

output
```
[1/2*log((b^2*cosh(x)^4 + b^2*sinh(x)^4 + 16*a*b*cosh(x)^3 + 4*(b^2*cosh(x
) + 4*a*b)*sinh(x)^3 + 16*a*b*cosh(x) + 2*(16*a^2 + b^2)*cosh(x)^2 + 2*(3*
b^2*cosh(x)^2 + 24*a*b*cosh(x) + 16*a^2 + b^2)*sinh(x)^2 - 8*(b*cosh(x)^3
+ b*sinh(x)^3 + 4*a*cosh(x)^2 + (3*b*cosh(x) + 4*a)*sinh(x)^2 + b*cosh(x)
+ (3*b*cosh(x)^2 + 8*a*cosh(x) + b)*sinh(x))*sqrt(b*cosh(x) + a)*sqrt(a) +
 b^2 + 4*(b^2*cosh(x)^3 + 12*a*b*cosh(x)^2 + 4*a*b + (16*a^2 + b^2)*cosh(x
))*sinh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2
+ 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1))/sqrt(
a), sqrt(-a)*arctan(1/2*(b*cosh(x)^2 + b*sinh(x)^2 + 4*a*cosh(x) + 2*(b*co
sh(x) + 2*a)*sinh(x) + b)*sqrt(b*cosh(x) + a)*sqrt(-a)/(a*b*cosh(x)^2 + a*
b*sinh(x)^2 + 2*a^2*cosh(x) + a*b + 2*(a*b*cosh(x) + a^2)*sinh(x)))/a]
```

Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh(x)}}\,dx = \int \frac{\tanh\left(x\right)}{\sqrt{a + b\cosh\left(x\right)}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x))**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*cosh(x)), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh(x)}}\,dx = \int \frac{\tanh\left(x\right)}{\sqrt{b\cosh\left(x\right) + a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a + b\cosh(x)}}\,dx = \int \frac{\tanh\left(x\right)}{\sqrt{b\cosh\left(x\right) + a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*cosh(x))^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/sqrt(b*cosh(x) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{a+b\cosh\left(x\right)}}\,dx$$

input
```
int(tanh(x)/(a + b*cosh(x))^(1/2),x)
```

output
```
int(tanh(x)/(a + b*cosh(x))^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\cosh(x)}}\,dx = \int \frac{\sqrt{\cosh\left(x\right)b+a}\,\tanh\left(x\right)}{\cosh\left(x\right)b+a}dx$$

input
```
int(tanh(x)/(a+b*cosh(x))^(1/2),x)
```

output
```
int((sqrt(cosh(x)*b + a)*tanh(x))/(cosh(x)*b + a),x)
```

3.199 $\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx=\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

output
```
2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)+B
*ln(a+b*cosh(x))/b
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.98

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx=-\frac{2A\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{B\log(a+b\cosh(x))}{b}$$

input
```
Integrate[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

output
```
(-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*
Log[a + b*Cosh[x]])/b
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A-iB\sin(ix)}{a+b\cos(ix)}dx$$
$$\Big\downarrow \text{4901}$$
$$\int \left(\frac{A}{a+b\cosh(x)}+\frac{B\sinh(x)}{a+b\cosh(x)}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(a+b\cosh(x))}{b}$$

input
```
Int[(A + B*Sinh[x])/(a + b*Cosh[x]),x]
```

output
```
(2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b
]) + (B*Log[a + b*Cosh[x]])/b
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(46) = 92$.

Time = 0.61 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.00

method	result
default	$\frac{\frac{2(Ba-Bb)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-b\tanh\left(\frac{x}{2}\right)^{2}-a-b\right)}{2a-2b}+\frac{2Ab\,\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{b}-\frac{B\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)}{b}-\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)}{b}$
risch	$\frac{Bx}{b}+\frac{2xB\,a^{2}b}{-a^{2}b^{2}+b^{4}}-\frac{2xB\,b^{3}}{-a^{2}b^{2}+b^{4}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{Aab-\sqrt{A^{2}a^{2}b^{2}-A^{2}b^{4}}}{A\,b^{2}}\right)B\,a^{2}}{(a^{2}-b^{2})b}-\frac{b\ln\left(\mathrm{e}^{x}+\frac{Aab-\sqrt{A^{2}a^{2}b^{2}-A^{2}b^{4}}}{A\,b^{2}}\right)B}{a^{2}-b^{2}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{Aab-}{}\right.}{}$

input
```
int((A+B*sinh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/b*(1/2*(B*a-B*b)/(a-b)*ln(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)+A*b/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))-B/b*ln(1+ta
nh(1/2*x))-B/b*ln(tanh(1/2*x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 112 vs. $2(46) = 92$.

Time = 0.11 (sec) , antiderivative size = 291, normalized size of antiderivative = 5.20

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx$$

$$= \left[\frac{\sqrt{a^2-b^2}Ab\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-}{a^2b-b^3}\right.$$

$$-\frac{2\sqrt{-a^2+b^2}Ab\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(Ba^2-Bb^2)x-(Ba^2-Bb^2)\log\left(\frac{2\,(b\cosh(x)+}{\cosh(x)-\sinh}\right.}{a^2b-b^3}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*A*b*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x
) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x
) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*x + (B*a^2 - B*b^2)*log(2*(b*cosh(x
) + a)/(cosh(x) - sinh(x))))/(a^2*b - b^3), -(2*sqrt(-a^2 + b^2)*A*b*arcta
n(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) + (B*a^2 - B*
b^2)*x - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))))/(a^2*
b - b^3)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 741 vs. $2(48) = 96$.

Time = 13.30 (sec) , antiderivative size = 741, normalized size of antiderivative = 13.23

$$\int \frac{A + B\sinh(x)}{a + b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x)
```

output
```
Piecewise((zoo*(2*A*atan(tanh(x/2)) + B*x - 2*B*log(tanh(x/2) + 1) + B*log
(tanh(x/2)**2 + 1)), Eq(a, 0) & Eq(b, 0)), (A*tanh(x/2)/b + B*x/b - 2*B*lo
g(tanh(x/2) + 1)/b, Eq(a, b)), (-A/(b*tanh(x/2)) + B*x/b - 2*B*log(tanh(x/
2) + 1)/b + 2*B*log(tanh(x/2))/b, Eq(a, -b)), ((A*x + B*cosh(x))/a, Eq(b,
0)), (-A*b*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a -
b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + A*b*log(sqrt(a/(a -
b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(
a/(a - b) + b/(a - b))) + B*a*x*sqrt(a/(a - b) + b/(a - b))/(a*b*sqrt(a/(a
 - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a - b
) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(
a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + B*a*sqrt(a/(a -
b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*sqrt(a/(
a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - 2*B*a*sqrt(a/(a
- b) + b/(a - b))*log(tanh(x/2) + 1)/(a*b*sqrt(a/(a - b) + b/(a - b)) - b*
*2*sqrt(a/(a - b) + b/(a - b))) - B*b*x*sqrt(a/(a - b) + b/(a - b))/(a*b*s
qrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt(
a/(a - b) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*
sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) - B*b*sqrt
(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(x/2))/(a*b*
sqrt(a/(a - b) + b/(a - b)) - b**2*sqrt(a/(a - b) + b/(a - b))) + 2*B*b...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\sinh(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.07

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx=\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}-\frac{Bx}{b}+\frac{B\log\left(be^{(2\,x)}+2\,ae^{x}+b\right)}{b}$$

input
```
integrate((A+B*sinh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*x/b + B*log(
b*e^(2*x) + 2*a*e^x + b)/b
```

Mupad [B] (verification not implemented)

Time = 4.05 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.52

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx=\frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^{x}\,\sqrt{b^2-a^2}}{(A\,b^3-A\,a^2\,b)\,\sqrt{A^2}}+\frac{A^2\,a\,b\,\sqrt{b^2-a^2}}{(A\,b^3-A\,a^2\,b)\,\sqrt{A^2}}\right)\sqrt{A^2}}{\sqrt{b^2-a^2}}-\frac{B\,x}{b}+\frac{B\,b^3\,\ln\left(4\,A^2\,b+8\,A^2\,a\,\mathrm{e}^{x}+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{b^4-a^2\,b^2}-\frac{B\,a^2\,b\,\ln\left(4\,A^2\,b+8\,A^2\,a\,\mathrm{e}^{x}+4\,A^2\,b\,\mathrm{e}^{2\,x}\right)}{b^4-a^2\,b^2}$$

input
```
int((A + B*sinh(x))/(a + b*cosh(x)),x)
```

output
```
(2*atan((A^2*b^2*exp(x)*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2))
 + (A^2*a*b*(b^2 - a^2)^(1/2))/((A*b^3 - A*a^2*b)*(A^2)^(1/2)))*(A^2)^(1/2
))/(b^2 - a^2)^(1/2) - (B*x)/b + (B*b^3*log(4*A^2*b + 8*A^2*a*exp(x) + 4*A
^2*b*exp(2*x)))/(b^4 - a^2*b^2) - (B*a^2*b*log(4*A^2*b + 8*A^2*a*exp(x) +
4*A^2*b*exp(2*x)))/(b^4 - a^2*b^2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.77

$$\int \frac{A+B\sinh(x)}{a+b\cosh(x)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a+\log(e^{2x}b+2e^x a+b)\,a^2-\log(e^{2x}b+2e^x a+b)\,b^2-a^2x+b^2x}{a^2-b^2}$$

input
```
int((A+B*sinh(x))/(a+b*cosh(x)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a + log
(e**(2*x)*b + 2*e**x*a + b)*a**2 - log(e**(2*x)*b + 2*e**x*a + b)*b**2 - a
**2*x + b**2*x)/(a**2 - b**2)
```

3.200 $\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx$

Optimal result

Integrand size = 13, antiderivative size = 18

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = B\log(1+\cosh(x)) + \frac{A\sinh(x)}{1+\cosh(x)}$$

output
```
B*ln(1+cosh(x))+A*sinh(x)/(1+cosh(x))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = 2B\log\left(\cosh\left(\frac{x}{2}\right)\right) + A\tanh\left(\frac{x}{2}\right)$$

input
```
Integrate[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

output
```
2*B*Log[Cosh[x/2]] + A*Tanh[x/2]
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{\cosh(x)+1}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{A-iB\sin(ix)}{1+\cos(ix)}dx$$
$$\Big\downarrow \ 4901$$
$$\int \left(\frac{A}{\cosh(x)+1}+\frac{B\sinh(x)}{\cosh(x)+1}\right)dx$$
$$\Big\downarrow \ 2009$$
$$\frac{A\sinh(x)}{\cosh(x)+1}+B\log(\cosh(x)+1)$$

input
```
Int[(A + B*Sinh[x])/(1 + Cosh[x]),x]
```

output
```
B*Log[1 + Cosh[x]] + (A*Sinh[x])/(1 + Cosh[x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.28

method	result	size
risch	$-Bx - \frac{2A}{e^x+1} + 2B \ln(e^x + 1)$	23
default	$A \tanh\left(\frac{x}{2}\right) - B \ln\left(\tanh\left(\frac{x}{2}\right) - 1\right) - B \ln\left(1 + \tanh\left(\frac{x}{2}\right)\right)$	28

input
```
int((A+B*sinh(x))/(1+cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
-B*x-2*A/(exp(x)+1)+2*B*ln(exp(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. 2(18) = 36.

Time = 0.09 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.56

$$\int \frac{A + B\sinh(x)}{1 + \cosh(x)}\, dx = -\frac{Bx\cosh(x) + Bx\sinh(x) + Bx - 2\left(B\cosh(x) + B\sinh(x) + B\right)\log\left(\cosh(x) + \sinh(x) + 1\right) + 2A}{\cosh(x) + \sinh(x) + 1}$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="fricas")
```

output
```
-(B*x*cosh(x) + B*x*sinh(x) + B*x - 2*(B*cosh(x) + B*sinh(x) + B)*log(cosh
(x) + sinh(x) + 1) + 2*A)/(cosh(x) + sinh(x) + 1)
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = A\tanh\left(\frac{x}{2}\right)+Bx-2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x)
```

output
```
A*tanh(x/2) + B*x - 2*B*log(tanh(x/2) + 1)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = B\log\left(\cosh\left(x\right)+1\right)+\frac{2\,A}{e^{(-x)}+1}$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="maxima")
```

output
```
B*log(cosh(x) + 1) + 2*A/(e^(-x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{A+B\sinh(x)}{1+\cosh(x)}\,dx = -Bx+2\,B\log\left(e^{x}+1\right)-\frac{2\,A}{e^{x}+1}$$

input
```
integrate((A+B*sinh(x))/(1+cosh(x)),x, algorithm="giac")
```

output
```
-B*x + 2*B*log(e^x + 1) - 2*A/(e^x + 1)
```

Mupad [B] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.22

$$\int \frac{A + B\sinh(x)}{1 + \cosh(x)}\, dx = 2\,B\,\ln\left(\mathrm{e}^x + 1\right) - \frac{2\,A}{\mathrm{e}^x + 1} - B\,x$$

input
```
int((A + B*sinh(x))/(cosh(x) + 1),x)
```

output
```
2*B*log(exp(x) + 1) - (2*A)/(exp(x) + 1) - B*x
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.61

$$\int \frac{A + B\sinh(x)}{1 + \cosh(x)}\, dx = \frac{2e^x \log(e^x + 1)\,b + 2e^x a - e^x bx + 2\log(e^x + 1)\,b - bx}{e^x + 1}$$

input
```
int((A+B*sinh(x))/(1+cosh(x)),x)
```

output
```
(2*e**x*log(e**x + 1)*b + 2*e**x*a - e**x*b*x + 2*log(e**x + 1)*b - b*x)/(
e**x + 1)
```

3.201 $\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 24

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx = -B\log(1-\cosh(x)) - \frac{A\sinh(x)}{1-\cosh(x)}$$

output
```
-B*ln(1-cosh(x))-A*sinh(x)/(1-cosh(x))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.79

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx = A\coth\left(\frac{x}{2}\right) - 2B\log\left(\sinh\left(\frac{x}{2}\right)\right)$$

input
```
Integrate[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

output
```
A*Coth[x/2] - 2*B*Log[Sinh[x/2]]
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A-iB\sin(ix)}{1-\cos(ix)}dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(-\frac{A}{\cosh(x)-1}-\frac{B\sinh(x)}{\cosh(x)-1}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{A\sinh(x)}{1-\cosh(x)}-B\log(1-\cosh(x))$$

input
```
Int[(A + B*Sinh[x])/(1 - Cosh[x]),x]
```

output
```
-(B*Log[1 - Cosh[x]]) - (A*Sinh[x])/(1 - Cosh[x])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

method	result	size
risch	$Bx+\frac{2A}{\mathrm{e}^{x}-1}-2B\ln\left(\mathrm{e}^{x}-1\right)$	22
default	$B\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)+B\ln\left(1+\tanh\left(\frac{x}{2}\right)\right)+\frac{A}{\tanh\left(\frac{x}{2}\right)}-2B\ln\left(\tanh\left(\frac{x}{2}\right)\right)$	36

input
```
int((A+B*sinh(x))/(1-cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B*x+2*A/(exp(x)-1)-2*B*ln(exp(x)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 48 vs. $2(21) = 42$.

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 2.00

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx = \frac{Bx\cosh\left(x\right)+Bx\sinh\left(x\right)-Bx-2\left(B\cosh\left(x\right)+B\sinh\left(x\right)-B\right)\log\left(\cosh\left(x\right)+\sinh\left(x\right)-1\right)+2A}{\cosh\left(x\right)+\sinh\left(x\right)-1}$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="fricas")
```

output
```
(B*x*cosh(x) + B*x*sinh(x) - B*x - 2*(B*cosh(x) + B*sinh(x) - B)*log(cosh(
x) + sinh(x) - 1) + 2*A)/(cosh(x) + sinh(x) - 1)
```

Sympy [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.29

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx=\frac{A}{\tanh\left(\frac{x}{2}\right)}-Bx+2B\log\left(\tanh\left(\frac{x}{2}\right)+1\right)-2B\log\left(\tanh\left(\frac{x}{2}\right)\right)$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x)
```

output
```
A/tanh(x/2) - B*x + 2*B*log(tanh(x/2) + 1) - 2*B*log(tanh(x/2))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx=-B\log\left(\cosh\left(x\right)-1\right)-\frac{2\,A}{e^{(-x)}-1}$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="maxima")
```

output
```
-B*log(cosh(x) - 1) - 2*A/(e^(-x) - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{A+B\sinh(x)}{1-\cosh(x)}\,dx=Bx-2\,B\log\left(|e^{x}-1|\right)+\frac{2\,A}{e^{x}-1}$$

input
```
integrate((A+B*sinh(x))/(1-cosh(x)),x, algorithm="giac")
```

output
```
B*x - 2*B*log(abs(e^x - 1)) + 2*A/(e^x - 1)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.88

$$\int \frac{A + B\sinh(x)}{1 - \cosh(x)}\,dx = B\,x + \frac{2\,A}{e^x - 1} - 2\,B\,\ln\left(e^x - 1\right)$$

input
```
int(-(A + B*sinh(x))/(cosh(x) - 1),x)
```

output
```
B*x + (2*A)/(exp(x) - 1) - 2*B*log(exp(x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.92

$$\int \frac{A + B\sinh(x)}{1 - \cosh(x)}\,dx = \frac{-2e^x\log(e^x - 1)\,b + 2e^x a + e^x bx + 2\log(e^x - 1)\,b - bx}{e^x - 1}$$

input
```
int((A+B*sinh(x))/(1-cosh(x)),x)
```

output
```
( - 2*e**x*log(e**x - 1)*b + 2*e**x*a + e**x*b*x + 2*log(e**x - 1)*b - b*x
)/(e**x - 1)
```

3.202 $\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 65

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(\cosh(x))}{a}-\frac{B\log(a+b\cosh(x))}{a}$$

output
```
2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)+B
*ln(cosh(x))/a-B*ln(a+b*cosh(x))/a
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.94

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = -\frac{2A\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{B(\log(\cosh(x))-\log(a+b\cosh(x)))}{a}$$

input
```
Integrate[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

output
```
(-2*A*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/Sqrt[-a^2 + b^2] + (B*
(Log[Cosh[x]] - Log[a + b*Cosh[x]]))/a
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A + B\tanh(x)}{a + b\cosh(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A - iB\tan(ix)}{a + b\cos(ix)}dx$$

$$\Big\downarrow \text{4901}$$

$$\int \left(\frac{A}{a + b\cosh(x)} + \frac{B\tanh(x)}{a + b\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} - \frac{B\log(a + b\cosh(x))}{a} + \frac{B\log(\cosh(x))}{a}$$

input
```
Int[(A + B*Tanh[x])/(a + b*Cosh[x]),x]
```

output
```
(2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b
]) + (B*Log[Cosh[x]])/a - (B*Log[a + b*Cosh[x]])/a
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.54

method	result
default	$\frac{\frac{2(-Ba+Bb)\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-b\tanh\left(\frac{x}{2}\right)^{2}-a-b\right)}{2a-2b}+\frac{2Aa\ \operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+1\right)}{a}$
risch	$-\frac{2xB}{a}-\frac{2x\,a^{3}B}{-a^{4}+a^{2}b^{2}}+\frac{2xBa\,b^{2}}{-a^{4}+a^{2}b^{2}}+\frac{B\ln\left(\mathrm{e}^{2x}+1\right)}{a}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{A\,a^{2}-\sqrt{A^{2}a^{4}-A^{2}a^{2}b^{2}}}{Aab}\right)B}{a^{2}-b^{2}}+\frac{\ln\left(\mathrm{e}^{x}+\frac{A\,a^{2}-\sqrt{A^{2}a^{4}-A^{2}a^{2}b^{2}}}{Aab}\right.}{(a^{2}-b^{2})a}$

input
```
int((A+B*tanh(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2/a*(1/2*(-B*a+B*b)/(a-b)*ln(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)+A*a/((a+
b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))+B/a*ln(tan
h(1/2*x)^2+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 124 vs. 2(55) = 110.

Time = 0.12 (sec) , antiderivative size = 315, normalized size of antiderivative = 4.85

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx$$
$$= \left[\frac{\sqrt{a^2-b^2}Aa\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sinh(x)+a)}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right)-}{a^3-ab^2}\right.$$
$$-\frac{2\sqrt{-a^2+b^2}Aa\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)+(Ba^2-Bb^2)\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-(Ba^2-B}{a^3-ab^2}$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(sqrt(a^2 - b^2)*A*a*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) +
 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x
) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x
) + a)*sinh(x) + b)) - (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - si
nh(x))) + (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2
), -(2*sqrt(-a^2 + b^2)*A*a*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x
) + a)/(a^2 - b^2)) + (B*a^2 - B*b^2)*log(2*(b*cosh(x) + a)/(cosh(x) - sin
h(x))) - (B*a^2 - B*b^2)*log(2*cosh(x)/(cosh(x) - sinh(x))))/(a^3 - a*b^2)
]
```

Sympy [F]

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \int \frac{A+B\tanh{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*tanh(x))/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\tanh(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.02

$$\int \frac{A + B\tanh(x)}{a + b\cosh(x)}\, dx = \frac{2\, A \arctan\left(\frac{be^{x}+a}{\sqrt{-a^{2}+b^{2}}}\right)}{\sqrt{-a^{2}+b^{2}}} - \frac{B \log\left(be^{(2\,x)} + 2\, ae^{x} + b\right)}{a} + \frac{B \log\left(e^{(2\,x)} + 1\right)}{a}$$

input
```
integrate((A+B*tanh(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*A*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(b*e^(2*x
) + 2*a*e^x + b)/a + B*log(e^(2*x) + 1)/a
```

Mupad [B] (verification not implemented)

Time = 13.07 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.46

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \frac{B\,\ln\left(16\,B^2\,b^2-16\,B^2\,a^2-16\,B^2\,a^2\,\mathrm{e}^{2\,x}+16\,B^2\,b^2\,\mathrm{e}^{2\,x}\right)}{a} - \frac{B\,\ln\left(16\,B^2\,b+32\,B^2\,a\,\mathrm{e}^{x}+16\,B^2\,b\,\mathrm{e}^{2\,x}\right)}{a} - \frac{2\,\mathrm{atan}\left(\frac{A^2\,b^2\,\mathrm{e}^{x}\,\sqrt{b^2-a^2}+A^2\,a\,b\,\sqrt{b^2-a^2}}{A\,b\,(a^2-b^2)\,\sqrt{A^2}}\right)\,\sqrt{A^2}}{\sqrt{b^2-a^2}}$$

input
```
int((A + B*tanh(x))/(a + b*cosh(x)),x)
```

output
```
(B*log(16*B^2*b^2 - 16*B^2*a^2 - 16*B^2*a^2*exp(2*x) + 16*B^2*b^2*exp(2*x)
))/a - (B*log(16*B^2*b + 32*B^2*a*exp(x) + 16*B^2*b*exp(2*x)))/a - (2*atan
((A^2*b^2*exp(x)*(b^2 - a^2)^(1/2) + A^2*a*b*(b^2 - a^2)^(1/2))/(A*b*(a^2
- b^2)*(A^2)^(1/2)))*(A^2)^(1/2))/(b^2 - a^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.85

$$\int \frac{A+B\tanh(x)}{a+b\cosh(x)}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a^2+\log(e^{2x}+1)\,a^2b-\log(e^{2x}+1)\,b^3-\log(e^{2x}b+2e^x a+b)\,a^2b+\log(e^{2}}{a\,(a^2-b^2)}$$

input
```
int((A+B*tanh(x))/(a+b*cosh(x)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a**2 +
log(e**(2x) + 1)*a**2*b - log(e**(2x) + 1)*b**3 - log(e**(2x)*b + 2*e**
x*a + b)*a**2*b + log(e**(2x)*b + 2*e**x*a + b)*b**3)/(a*(a**2 - b**2))
```

3.203 $\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 100

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \frac{2A\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}} + \frac{B\log(1-\cosh(x))}{2(a+b)} + \frac{B\log(1+\cosh(x))}{2(a-b)} - \frac{aB\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)+B
*ln(1-cosh(x))/(2*a+2*b)+B*ln(1+cosh(x))/(2*a-2*b)-a*B*ln(a+b*cosh(x))/(a^
2-b^2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.34

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \frac{(A+B\coth(x))\left(-2A(a^2-b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)+\sqrt{-a^2+b^2}B\left((a+b)\log\left(\cosh\left(\frac{x}{2}\right)\right)-a\log(a+\right.\right.}{(a-b)(a+b)\sqrt{-a^2+b^2}(B\cosh(x)+A\sinh(x))}$$

input
```
Integrate[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

output
```
((A + B*Coth[x])*(-2*A*(a^2 - b^2)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 +
b^2]] + Sqrt[-a^2 + b^2]*B*((a + b)*Log[Cosh[x/2]] - a*Log[a + b*Cosh[x]]
+ (a - b)*Log[Sinh[x/2]]))*Sinh[x])/((a - b)*(a + b)*Sqrt[-a^2 + b^2]*(B*C
osh[x] + A*Sinh[x]))
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3042, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{A+iB\cot(ix)}{a+b\cos(ix)}dx$$

$$\Big\downarrow \text{ 4901}$$

$$\int \left(\frac{A}{a+b\cosh(x)}+\frac{B\coth(x)}{a+b\cosh(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{bB\text{arctanh}(\cosh(x))}{a^2-b^2}+\frac{aB\log(\sinh(x))}{a^2-b^2}-\frac{aB\log(a+b\cosh(x))}{a^2-b^2}+\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}$$

input
```
Int[(A + B*Coth[x])/(a + b*Cosh[x]),x]
```

output
```
(b*B*ArcTanh[Cosh[x]])/(a^2 - b^2) + (2*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/
Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]) - (a*B*Log[a + b*Cosh[x]])/(a^2 -
b^2) + (a*B*Log[Sinh[x]])/(a^2 - b^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.01

method	result
default	$\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}+\frac{-\frac{Ba\ln\left(\tanh\left(\frac{x}{2}\right)^{2}a-b\tanh\left(\frac{x}{2}\right)^{2}-a-b\right)}{a-b}-\frac{(-2Aa-2Ab)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a+b}$
risch	$-\frac{xB}{a-b}-\frac{xB}{a+b}-\frac{2xa^{3}B}{-a^{4}+2a^{2}b^{2}-b^{4}}+\frac{2xBab^{2}}{-a^{4}+2a^{2}b^{2}-b^{4}}+\frac{B\ln(\mathrm{e}^{x}+1)}{a-b}+\frac{B\ln(\mathrm{e}^{x}-1)}{a+b}-\frac{\ln\left(\mathrm{e}^{x}+\frac{Aa-\sqrt{A^{2}a^{2}-A^{2}b^{2}}}{Ab}\right)Ba}{(a+b)(a-b)}+$

input
```
int((A+B*coth(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
B/(a+b)*ln(tanh(1/2*x))+1/(a+b)*(-B*a/(a-b)*ln(tanh(1/2*x)^2*a-b*tanh(1/2*
x)^2-a-b)-(-2*A*a-2*A*b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a
+b)*(a-b))^(1/2)))
```

Fricas [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.03

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx$$
$$= \left[-\frac{Ba\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)-\sqrt{a^2-b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\surd}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\,\mathrm{sinl}}\right.}{}\right.$$
$$-\frac{Ba\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+2\sqrt{-a^2+b^2}A\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-(Ba+Bb)\log(\cosh}{a^2-b^2}$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - sqrt(a^2 - b^2)*A*log(
(b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh
(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cos
h(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*
a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x) + sinh(x) -
1))/(a^2 - b^2), -(B*a*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + 2*sqrt
(-a^2 + b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 -
 b^2)) - (B*a + B*b)*log(cosh(x) + sinh(x) + 1) - (B*a - B*b)*log(cosh(x)
+ sinh(x) - 1))/(a^2 - b^2)]
```

Sympy [F]

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \int \frac{A+B\coth{(x)}}{a+b\cosh{(x)}}\,dx$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*coth(x))/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.90

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = -\frac{Ba\log\left(be^{(2x)}+2\,ae^{x}+b\right)}{a^2-b^2}+\frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+\frac{B\log\left(e^{x}+1\right)}{a-b}+\frac{B\log\left(|e^{x}-1|\right)}{a+b}$$

input
```
integrate((A+B*coth(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
-B*a*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqr
t(-a^2 + b^2))/sqrt(-a^2 + b^2) + B*log(e^x + 1)/(a - b) + B*log(abs(e^x -
 1))/(a + b)
```

Mupad [B] (verification not implemented)

Time = 4.83 (sec) , antiderivative size = 974, normalized size of antiderivative = 9.74

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
int((A + B*coth(x))/(a + b*cosh(x)),x)
```

output
```
(B*log(exp(x) + 1))/(a - b) + (log((((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b +
3*B^2*a^2*b + 4*B^2*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*
a^3*exp(x) + 2*A^2*a*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 + ((A*((a + b)^
3*(a - b)^3)^(1/2) - B*a^3 + B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B)
+ a*b^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) -
a^3*b^3*(128*A - 256*B) - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)
*(A - 3*B) + 128*A*a^3*exp(x)*((a^2 - b^2)^3)^(1/2) + 96*A*a^2*b*((a^2 - b
^2)^3)^(1/2) - 32*A*a*b^2*exp(x)*((a^2 - b^2)^3)^(1/2)))/((b^7 - a^2*b^5)*
(a^2 - b^2)^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*a^3 + B*a*b^2))/(a^2 -
b^2)^2 - (32*B*(A^2*b^2*exp(x) + 4*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*
A*B*a^2*exp(x) - A*B*b^2*exp(x) + 2*A*B*a*b))/b^5)*(A*((a + b)^3*(a - b)^3
)^(1/2) - B*a^3 + B*a*b^2))/(a^4 + b^4 - 2*a^2*b^2) - (log(- (32*B*(A^2*b^
2*exp(x) + 4*B^2*a^2*exp(x) + A^2*a*b + B^2*a*b + 4*A*B*a^2*exp(x) - A*B*b
^2*exp(x) + 2*A*B*a*b))/b^5 - (((32*(A^2*b^3 + B^2*b^3 + A^2*a^2*b + 3*B^2
*a^2*b + 4*B^2*a^3*exp(x) + 5*B^2*a*b^2*exp(x) + 4*A*B*a^2*b + 8*A*B*a^3*e
xp(x) + 2*A^2*a*b^2*exp(x) - 2*A*B*a*b^2*exp(x)))/b^5 - ((B*a^3 + A*((a +
b)^3*(a - b)^3)^(1/2) - B*a*b^2)*(128*exp(x)*(a^2 - b^2)^3*(A - 2*B) + a*b
^5*(64*A - 128*B) + a^5*b*(64*A - 128*B) + 96*b^6*exp(x)*(A - 3*B) - a^3*b
^3*(128*A - 256*B) - 192*a^2*b^4*exp(x)*(A - 3*B) + 96*a^4*b^2*exp(x)*(A -
 3*B) - 128*A*a^3*exp(x)*((a^2 - b^2)^3)^(1/2) - 96*A*a^2*b*((a^2 - b^2...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.06

$$\int \frac{A+B\coth(x)}{a+b\cosh(x)}\,dx = \frac{-2\sqrt{-a^2+b^2}\, atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right) a+\log(e^x-1)\, ab-\log(e^x-1)\, b^2+\log(e^x+1)\, ab+\log(e^x+1)\, b^2-\log}{a^2-b^2}$$

input
```
int((A+B*coth(x))/(a+b*cosh(x)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a + log
(e**x - 1)*a*b - log(e**x - 1)*b**2 + log(e**x + 1)*a*b + log(e**x + 1)*b*
*2 - log(e**(2*x)*b + 2*e**x*a + b)*a*b)/(a**2 - b**2)
```

3.204 $\int \frac{A+B\,\text{sech}(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 62

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx=\frac{B\arctan(\sinh(x))}{a}+\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}$$

output
```
B*arctan(sinh(x))/a+2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2
))/a/(a-b)^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.02

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx=\frac{2\left(B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{(-aA+bB)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}\right)}{a}$$

input
```
Integrate[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

output
```
(2*(B*ArcTan[Tanh[x/2]] + ((-(a*A) + b*B)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[
-a^2 + b^2]])/Sqrt[-a^2 + b^2]))/a
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 3307, 3042, 3480, 3042, 3138, 221, 4257}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\csc\left(\frac{\pi}{2}+ix\right)}{a+b\sin\left(\frac{\pi}{2}+ix\right)}dx$$

$$\Big\downarrow \text{3307}$$

$$\int \frac{\text{sech}(x)(A\cosh(x)+B)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{B+A\sin\left(\frac{\pi}{2}+ix\right)}{\sin\left(\frac{\pi}{2}+ix\right)\left(a+b\sin\left(\frac{\pi}{2}+ix\right)\right)}dx$$

$$\Big\downarrow \text{3480}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cosh(x)}dx}{a}+\frac{B\int \text{sech}(x)dx}{a}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(aA-bB)\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a}+\frac{B\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{3138}$$

$$\frac{2(aA-bB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a}+\frac{B\int \csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{221}$$

$$\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\int\csc\left(ix+\frac{\pi}{2}\right)dx}{a}$$

$$\Big\downarrow \text{4257}$$

$$\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\sqrt{a+b}}+\frac{B\arctan(\sinh(x))}{a}$$

input
```
Int[(A + B*Sech[x])/(a + b*Cosh[x]),x]
```

output
```
(B*ArcTan[Sinh[x]])/a + (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqr
t[a + b]])/(a*Sqrt[a - b]*Sqrt[a + b])
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3307
```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((a_) + (b_.)*sin[(e_.) +
(f_.)*(x_)])^(m_.), x_Symbol] :> Int[(a + b*Sin[e + f*x])^m*((d + c*Sin[e +
 f*x])^n/Sin[e + f*x]^n), x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IntegerQ
[n]
```

rule 3480
```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)])/(((a_.) + (b_.)*sin[(e_.) + (f_
.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])), x_Symbol] :> Simp[(A*b
- a*B)/(b*c - a*d)   Int[1/(a + b*Sin[e + f*x]), x], x] + Simp[(B*c - A*d)/
(b*c - a*d)   Int[1/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f
, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

rule 4257
```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.95

method	result
default	$\frac{2B\arctan\left(\tanh\left(\frac{x}{2}\right)\right)}{a}-\frac{2(-Aa+Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}$
risch	$\frac{iB\ln(\mathrm{e}^{x}+i)}{a}-\frac{iB\ln(\mathrm{e}^{x}-i)}{a}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Bb}{\sqrt{a^2-b^2}\,a}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}}$

input
```
int((A+B*sech(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
2*B/a*arctan(tanh(1/2*x))-2*(-A*a+B*b)/a/((a+b)*(a-b))^(1/2)*arctanh((a-b)
*tanh(1/2*x)/((a+b)*(a-b))^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 4.02

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx = \left[-\frac{(Aa-Bb)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)+2\,\sqrt{a^2-b^2}(b\cosh(x)+b\sin}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x)+b}\right.}{a^3-ab^2}\right.$$

$$-\frac{2\left((Aa-Bb)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cosh(x)+b\sinh(x)+a)}{a^2-b^2}\right)-(Ba^2-Bb^2)\arctan(\cosh(x)+\sinh\right.}{a^3-ab^2}$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[-((A*a - B*b)*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*
cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) + 2*sqrt(a^2 - b^2)*
(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*
(b*cosh(x) + a)*sinh(x) + b)) - 2*(B*a^2 - B*b^2)*arctan(cosh(x) + sinh(x)
))/(a^3 - a*b^2), -2*((A*a - B*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)) - (B*a^2 - B*b^2)*arctan(cosh(x
) + sinh(x)))/(a^3 - a*b^2)]
```

Sympy [F]

$$\int \frac{A+B\text{sech}(x)}{a+b\cosh(x)}\,dx = \int \frac{A+B\,\text{sech}\,(x)}{a+b\cosh\,(x)}\,dx$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*sech(x))/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\text{sech}(x)}{a + b\cosh(x)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.85

$$\int \frac{A + B\text{sech}(x)}{a + b\cosh(x)}\, dx = \frac{2\, B \arctan\left(e^{x}\right)}{a} + \frac{2\, (Aa - Bb) \arctan\left(\frac{be^{x} + a}{\sqrt{-a^{2} + b^{2}}}\right)}{\sqrt{-a^{2} + b^{2}} a}$$

input
```
integrate((A+B*sech(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
2*B*arctan(e^x)/a + 2*(A*a - B*b)*arctan((b*e^x + a)/sqrt(-a^2 + b^2))/(sq
rt(-a^2 + b^2)*a)
```

Mupad [B] (verification not implemented)

Time = 7.37 (sec) , antiderivative size = 636, normalized size of antiderivative = 10.26

$$\int \frac{A+B\mathrm{sech}(x)}{a+b\cosh(x)}\,dx$$

$$= \frac{\ln\left(\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,\mathrm{e}^x\,B^2\,a^3-2\,B^2\,a^2\,b+3\,\mathrm{e}^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}+\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,a^2\,\left(2\,B\,b^2-4\,A\,a^2\right.}{}\right.}{}\right.}{a\,b^2-a^3}\right.}{}$$

$$-\frac{\ln\left(-\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,\left(A^2\,a^2\,b-2\,A\,B\,a\,b^2-4\,\mathrm{e}^x\,B^2\,a^3-2\,B^2\,a^2\,b+3\,\mathrm{e}^x\,B^2\,a\,b^2+2\,B^2\,b^3\right)}{b^5}-\frac{\sqrt{(a+b)\,(a-b)}\,(A\,a-B\,b)\left(\frac{32\,a^2\,\left(2\,B\,b^2-\right.}{}\right.}{}\right.}{a\,b^2-a^3}\right.}{}$$

$$-\frac{B\,\ln\left(\mathrm{e}^x-\mathrm{i}\right)\,1\mathrm{i}}{a}+\frac{B\,\ln\left(\mathrm{e}^x+1\mathrm{i}\right)\,1\mathrm{i}}{a}$$

input
```
int((A + B/cosh(x))/(a + b*cosh(x)),x)
```

output
```
(B*log(exp(x) + 1i)*1i)/a - (B*log(exp(x) - 1i)*1i)/a + (log((((a + b)*(a
- b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*a^2*b - 4*B^2*
a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 + (((a + b)*(a - b))^(
1/2)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*exp(x) - 2*A*a
*b + 3*B*a*b*exp(x)))/b^5 - (32*a^2*((a + b)*(a - b))^(1/2)*(A*a - B*b)*(3
*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2 - a^3))))/(a*
b^2 - a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*exp(x) + 4*B*a
*exp(x)))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 - a^3) - (log(-
 (((a + b)*(a - b))^(1/2)*(A*a - B*b)*((32*(2*B^2*b^3 + A^2*a^2*b - 2*B^2*
a^2*b - 4*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 2*A*B*a*b^2))/b^5 - (((a +
 b)*(a - b))^(1/2)*(A*a - B*b)*((32*a^2*(2*B*b^2 - 4*A*a^2*exp(x) + A*b^2*
exp(x) - 2*A*a*b + 3*B*a*b*exp(x)))/b^5 + (32*a^2*((a + b)*(a - b))^(1/2)*
(A*a - B*b)*(3*a^2*b - 2*b^3 + 4*a^3*exp(x) - 3*a*b^2*exp(x)))/(b^5*(a*b^2
 - a^3))))/(a*b^2 - a^3)))/(a*b^2 - a^3) - (32*B*(A*a - B*b)*(2*B*b - A*b*
exp(x) + 4*B*a*exp(x)))/b^5)*((a + b)*(a - b))^(1/2)*(A*a - B*b))/(a*b^2 -
 a^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.73

$$\int \frac{A + B\text{sech}(x)}{a + b\cosh(x)}\,dx = \frac{2atan(e^x)\,b - 2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)}{a}$$

input
```
int((A+B*sech(x))/(a+b*cosh(x)),x)
```

output
```
(2*(atan(e**x)*b - sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b
**2))))/a
```

3.205 $\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 99

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx=\frac{2A\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}+\frac{B\log(1-\cosh(x))}{2(a+b)}-\frac{B\log(1+\cosh(x))}{2(a-b)}+\frac{bB\log(a+b\cosh(x))}{a^2-b^2}$$

output
```
2*A*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/(a+b)^(1/2)+B
*ln(1-cosh(x))/(2*a+2*b)-B*ln(1+cosh(x))/(2*a-2*b)+b*B*ln(a+b*cosh(x))/(a^
2-b^2)
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.17

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx=\frac{-2A(a^2-b^2)\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)-\sqrt{-a^2+b^2}B\left((a+b)\log\left(\cosh\left(\frac{x}{2}\right)\right)-b\log(a+b\cosh(x))+(-a\right.}{(a-b)(a+b)\sqrt{-a^2+b^2}}$$

input
```
Integrate[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

output
```
(-2*A*(a^2 - b^2)*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]] - Sqrt[-a^2
 + b^2]*B*((a + b)*Log[Cosh[x/2]] - b*Log[a + b*Cosh[x]] + (-a + b)*Log[Si
nh[x/2]]))/((a - b)*(a + b)*Sqrt[-a^2 + b^2])
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 4713, 26, 26, 3042, 26, 4901, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+iB\csc(ix)}{a+b\cos(ix)}dx$$

$$\Big\downarrow \text{4713}$$

$$\int -\frac{i\text{csch}(x)(iA\sinh(x)+iB)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{i\text{csch}(x)(B+A\sinh(x))}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\text{csch}(x)(A\sinh(x)+B)}{a+b\cosh(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i(B-iA\sin(ix))}{\sin(ix)(a+b\cos(ix))}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{B-iA\sin(ix)}{(a+b\cos(ix))\sin(ix)}dx$$

$$\Big\downarrow \text{4901}$$

$$i\int \left(-\frac{iA}{a+b\cosh(x)}-\frac{iB\text{csch}(x)}{a+b\cosh(x)}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{ibB\log(a+b\cosh(x))}{a^2-b^2}-\frac{2iA\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}}-\frac{iB\log(1-\cosh(x))}{2(a+b)}+\frac{iB\log(\cosh(x)+1)}{2(a-b)}\right)$$

input

```
Int[(A + B*Csch[x])/(a + b*Cosh[x]),x]
```

output

```
I*(((-2*I)*A*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sq
rt[a + b]) - ((I/2)*B*Log[1 - Cosh[x]])/(a + b) + ((I/2)*B*Log[1 + Cosh[x]
])/(a - b) - (I*b*B*Log[a + b*Cosh[x]])/(a^2 - b^2))
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4713

```
Int[(csc[(a_.) + (b_.)*(x_)]*(B_.) + (A_))*(u_), x_Symbol] :> Int[ActivateT
rig[u]*((B + A*Sin[a + b*x])/Sin[a + b*x]), x] /; FreeQ[{a, b, A, B}, x] &&
 KnownSineIntegrandQ[u, x]
```

rule 4901
```
Int[u_, x_Symbol] :> With[{v = ExpandTrig[u, x]}, Int[v, x] /; SumQ[v]] /;
 !InertTrigFreeQ[u]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.01

method	result
default	$\frac{\frac{Bb\ln\left(\tanh\left(\frac{x}{2}\right)^2a-b\tanh\left(\frac{x}{2}\right)^2-a-b\right)}{a-b}-\frac{(-2Aa-2Ab)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}}{a+b}+\frac{B\ln\left(\tanh\left(\frac{x}{2}\right)\right)}{a+b}$
risch	$\frac{xB}{a-b}-\frac{xB}{a+b}+\frac{2xB\,a^2b}{-a^4+2a^2b^2-b^4}-\frac{2xB\,b^3}{-a^4+2a^2b^2-b^4}-\frac{B\ln(\mathrm{e}^x+1)}{a-b}+\frac{B\ln(\mathrm{e}^x-1)}{a+b}+\frac{\ln\left(\mathrm{e}^x+\frac{Aa-\sqrt{A^2a^2-A^2b^2}}{Ab}\right)Bb}{(a+b)(a-b)}+\frac{\ln\Big(}{}$

input
```
int((A+B*csch(x))/(a+b*cosh(x)),x,method=_RETURNVERBOSE)
```

output
```
1/(a+b)*(B*b/(a-b)*ln(tanh(1/2*x)^2*a-b*tanh(1/2*x)^2-a-b)-(-2*A*a-2*A*b)/
((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*x)/((a+b)*(a-b))^(1/2)))+B/(a+b
)*ln(tanh(1/2*x))
```

Fricas [A] (verification not implemented)

Time = 1.75 (sec) , antiderivative size = 298, normalized size of antiderivative = 3.01

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx$$
$$=\left[\frac{Bb\log\left(\frac{2\,(b\cosh(x)+a)}{\cosh(x)-\sinh(x)}\right)+\sqrt{a^2-b^2}A\log\left(\frac{b^2\cosh(x)^2+b^2\sinh(x)^2+2\,ab\cosh(x)+2\,a^2-b^2+2\,(b^2\cosh(x)+ab)\sinh(x)-2\,\sqrt{a^2}}{b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)+a)\sinh(x}\right.}{a}\right.$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="fricas")
```

output
```
[(B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) + sqrt(a^2 - b^2)*A*log((
b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(
x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh
(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) - (B*a
 + B*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) + sinh(x) - 1
))/(a^2 - b^2), (B*b*log(2*(b*cosh(x) + a)/(cosh(x) - sinh(x))) - 2*sqrt(-
a^2 + b^2)*A*arctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b
^2)) - (B*a + B*b)*log(cosh(x) + sinh(x) + 1) + (B*a - B*b)*log(cosh(x) +
sinh(x) - 1))/(a^2 - b^2)]
```

Sympy [F]

$$\int \frac{A + B\text{csch}(x)}{a + b\cosh(x)}\,dx = \int \frac{A + B\operatorname{csch}(x)}{a + b\cosh(x)}\,dx$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x)
```

output
```
Integral((A + B*csch(x))/(a + b*cosh(x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\text{csch}(x)}{a + b\cosh(x)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.91

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \frac{Bb\log\left(be^{(2x)}+2\,ae^{x}+b\right)}{a^2-b^2} + \frac{2\,A\arctan\left(\frac{be^{x}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}} - \frac{B\log\left(e^{x}+1\right)}{a-b} + \frac{B\log\left(\left|e^{x}-1\right|\right)}{a+b}$$

input
```
integrate((A+B*csch(x))/(a+b*cosh(x)),x, algorithm="giac")
```

output
```
B*b*log(b*e^(2*x) + 2*a*e^x + b)/(a^2 - b^2) + 2*A*arctan((b*e^x + a)/sqrt
(-a^2 + b^2))/sqrt(-a^2 + b^2) - B*log(e^x + 1)/(a - b) + B*log(abs(e^x -
1))/(a + b)
```

Mupad [B] (verification not implemented)

Time = 4.47 (sec) , antiderivative size = 983, normalized size of antiderivative = 9.93

$$\int \frac{A+B\operatorname{csch}(x)}{a+b\cosh(x)}\,dx = \text{Too large to display}$$

input
```
int((A + B/sinh(x))/(a + b*cosh(x)),x)
```

output
```
(log((((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b + 4*B^2*a^3*exp(x) + 3*B^2*a
*b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x) + 2*A^2*a*b^2*exp(x) - 8*A*B*
a^2*b*exp(x)))/b^5 + (((32*(2*B*b^4 + B*a^2*b^2 - 4*A*a^4*exp(x) - A*b^4*e
xp(x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp(x) - 3*B*a^3*b*exp(x) + 5*A*
a^2*b^2*exp(x)))/b^5 - (32*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*
b)*(3*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^4*exp(x) - 5*a^3*b^2*exp(x)))
/(b^5*(a^4 + b^4 - 2*a^2*b^2)))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B
*a^2*b))/(a^4 + b^4 - 2*a^2*b^2))*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 +
 B*a^2*b))/(a^4 + b^4 - 2*a^2*b^2) - (32*(2*B^3*b^2 + A^2*B*b^2 - 2*A*B^2*
a*b + 4*B^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*exp(x) + A^2*B*a*b
*exp(x)))/b^5)*(A*((a + b)^3*(a - b)^3)^(1/2) - B*b^3 + B*a^2*b))/(a^4 + b
^4 - 2*a^2*b^2) - (B*log(exp(x) + 1))/(a - b) - (log(- (32*(2*B^3*b^2 + A^
2*B*b^2 - 2*A*B^2*a*b + 4*B^3*a*b*exp(x) - 4*A*B^2*a^2*exp(x) + A*B^2*b^2*
exp(x) + A^2*B*a*b*exp(x)))/b^5 - (((32*(A^2*b^3 + A^2*a^2*b + 2*B^2*a^2*b
 + 4*B^2*a^3*exp(x) + 3*B^2*a*b^2*exp(x) - 4*A*B*a*b^2 + 2*A*B*b^3*exp(x)
+ 2*A^2*a*b^2*exp(x) - 8*A*B*a^2*b*exp(x)))/b^5 - (((32*(2*B*b^4 + B*a^2*b
^2 - 4*A*a^4*exp(x) - A*b^4*exp(x) + 2*A*a*b^3 - 2*A*a^3*b + 6*B*a*b^3*exp
(x) - 3*B*a^3*b*exp(x) + 5*A*a^2*b^2*exp(x)))/b^5 + (32*(B*b^3 + A*((a + b
)^3*(a - b)^3)^(1/2) - B*a^2*b)*(3*a^4*b - 3*a^2*b^3 + 4*a^5*exp(x) + a*b^
4*exp(x) - 5*a^3*b^2*exp(x)))/(b^5*(a^4 + b^4 - 2*a^2*b^2)))*(B*b^3 + A...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.09

$$\int \frac{A+B\text{csch}(x)}{a+b\cosh(x)}\,dx$$
$$= \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a+\log(e^x-1)\,ab-\log(e^x-1)\,b^2-\log(e^x+1)\,ab-\log(e^x+1)\,b^2+\log}{a^2-b^2}$$

input
```
int((A+B*csch(x))/(a+b*cosh(x)),x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a + log
(e**x - 1)*a*b - log(e**x - 1)*b**2 - log(e**x + 1)*a*b - log(e**x + 1)*b*
*2 + log(e**(2*x)*b + 2*e**x*a + b)*b**2)/(a**2 - b**2)
```

3.206 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 86

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{Bx}{b}+\frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}e}+\frac{C\log(a+b\cosh(d+ex))}{be}$$

output
```
B*x/b+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2))/(a-
b)^(1/2)/b/(a+b)^(1/2)/e+C*ln(a+b*cosh(e*x+d))/b/e
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{B(d+ex)+\frac{2(-Ab+aB)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}}+C\log(a+b\cosh(d+ex))}{be}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

output
```
(B*(d + e*x) + (2*(-(A*b) + a*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-
a^2 + b^2]])/Sqrt[-a^2 + b^2] + C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.355, Rules used = {3042, 4877, 26, 3042, 26, 3147, 16, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{a+b\cos(id+iex)}dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{A+B\cosh(d+ex)}{a+b\cosh(d+ex)}dx-iC\int \frac{i\sinh(d+ex)}{a+b\cosh(d+ex)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\cosh(d+ex)}{a+b\cosh(d+ex)}dx+C\int \frac{\sinh(d+ex)}{a+b\cosh(d+ex)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{a-b\sin\left(id+iex-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{C\int \frac{1}{a+b\cosh(d+ex)}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{16}$$

$$\frac{C\log(a+b\cosh(d+ex))}{be}+\int\frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{ 3214}$$

$$\frac{(Ab-aB)\int\frac{1}{a+b\cosh(d+ex)}dx}{b}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(Ab-aB)\int\frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{b}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 3138}$$

$$-\frac{2i(Ab-aB)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(d+ex)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{be}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{2(Ab-aB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{be\sqrt{a-b}\sqrt{a+b}}+\frac{C\log(a+b\cosh(d+ex))}{be}+\frac{Bx}{b}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x]),x]
```

output
```
(B*x)/b + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a +
b]])/(Sqrt[a - b]*b*Sqrt[a + b]*e) + (C*Log[a + b*Cosh[d + e*x]])/(b*e)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(77) = 154$.

Time = 1.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{b}+\frac{(B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{b}+\frac{\frac{2(aC-bC)\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-a-b\right)}{2a-2b}-\frac{2(-A}{}}{b}}{e}$
default	$\frac{\frac{(-B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)-1\right)}{b}+\frac{(B-C)\ln\left(\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)+1\right)}{b}+\frac{\frac{2(aC-bC)\ln\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-a-b\right)}{2a-2b}-\frac{2(-A}{}}{b}}{e}$
risch	$\frac{Bx}{b}+\frac{xC}{b}+\frac{2C\,a^{2}b\,e^{2}x}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}-\frac{2C\,b^{3}e^{2}x}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}+\frac{2C\,a^{2}bde}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}-\frac{2C\,b^{3}de}{-a^{2}b^{2}e^{2}+b^{4}e^{2}}+\frac{\ln\left(\mathrm{e}^{ex+d}+\frac{Aab-B\,a^{2}-}{}\right.}{}$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x,method=_RETURNVERB
OSE)
```

output
```
1/e*((-B-C)/b*ln(tanh(1/2*e*x+1/2*d)-1)+(B-C)/b*ln(tanh(1/2*e*x+1/2*d)+1)+
2/b*(1/2*(C*a-C*b)/(a-b)*ln(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^
2-a-b)-(-A*b+B*a)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((
a+b)*(a-b))^(1/2))))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 405, normalized size of antiderivative = 4.71

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx$$
$$=\left[\frac{((B-C)a^2-(B-C)b^2)ex-(Ba-Ab)\sqrt{a^2-b^2}\log\left(\frac{b^2\cosh(ex+d)^2+b^2\sinh(ex+d)^2+2\,ab\cosh(ex+d)+2\,a^2-b^2+}{b\cosh(ex+d)^2+b\sinh(ex+d)^2+2}\right.}{(a^2b-}\right.$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="
fricas")
```

output
```
[(((B - C)*a^2 - (B - C)*b^2)*e*x - (B*a - A*b)*sqrt(a^2 - b^2)*log((b^2*c
osh(e*x + d)^2 + b^2*sinh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 +
 2*(b^2*cosh(e*x + d) + a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x
 + d) + b*sinh(e*x + d) + a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a
*cosh(e*x + d) + 2*(b*cosh(e*x + d) + a)*sinh(e*x + d) + b)) + (C*a^2 - C*
b^2)*log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sinh(e*x + d))))/((a^2*b
 - b^3)*e), (((B - C)*a^2 - (B - C)*b^2)*e*x + 2*(B*a - A*b)*sqrt(-a^2 + b
^2)*arctan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2
- b^2)) + (C*a^2 - C*b^2)*log(2*(b*cosh(e*x + d) + a)/(cosh(e*x + d) - sin
h(e*x + d))))/((a^2*b - b^3)*e)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 695 vs. $2(73) = 146$.

Time = 15.80 (sec) , antiderivative size = 695, normalized size of antiderivative = 8.08

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{a + b\cosh(d+ex)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

output
```
Piecewise((zoo*x*(A + B*cosh(d) + C*sinh(d))/cosh(d), Eq(a, 0) & Eq(b, 0)
& Eq(e, 0)), (A*tanh(d/2 + e*x/2)/(b*e) + B*x/b - B*tanh(d/2 + e*x/2)/(b*e
) + C*x/b - 2*C*log(tanh(d/2 + e*x/2) + 1)/(b*e), Eq(a, b)), (-A/(b*e*tanh
(d/2 + e*x/2)) + B*x/b - B/(b*e*tanh(d/2 + e*x/2)) + C*x/b - 2*C*log(tanh(
d/2 + e*x/2) + 1)/(b*e) + 2*C*log(tanh(d/2 + e*x/2))/(b*e), Eq(a, -b)), ((
A*x + B*sinh(d + e*x)/e + C*cosh(d + e*x)/e)/a, Eq(b, 0)), (x*(A + B*cosh(
d) + C*sinh(d))/(a + b*cosh(d)), Eq(e, 0)), (-A*b*sqrt(a/(a - b) + b/(a -
b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e)
 + A*b*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a - b)) + tanh(
d/2 + e*x/2))/(a*b*e + b**2*e) + B*a*e*x/(a*b*e + b**2*e) + B*a*sqrt(a/(a
- b) + b/(a - b))*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2 + e*x/2))/(a
*b*e + b**2*e) - B*a*sqrt(a/(a - b) + b/(a - b))*log(sqrt(a/(a - b) + b/(a
 - b)) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e) + B*b*e*x/(a*b*e + b**2*e) +
C*a*e*x/(a*b*e + b**2*e) + C*a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(d/2
 + e*x/2))/(a*b*e + b**2*e) + C*a*log(sqrt(a/(a - b) + b/(a - b)) + tanh(d
/2 + e*x/2))/(a*b*e + b**2*e) - 2*C*a*log(tanh(d/2 + e*x/2) + 1)/(a*b*e +
b**2*e) + C*b*e*x/(a*b*e + b**2*e) + C*b*log(-sqrt(a/(a - b) + b/(a - b))
+ tanh(d/2 + e*x/2))/(a*b*e + b**2*e) + C*b*log(sqrt(a/(a - b) + b/(a - b)
) + tanh(d/2 + e*x/2))/(a*b*e + b**2*e) - 2*C*b*log(tanh(d/2 + e*x/2) + 1)
/(a*b*e + b**2*e), True))
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{a + b\cosh(d + ex)}\, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="
maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{\frac{(ex+d)(B-C)}{b}+\frac{C\log\left(be^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}+b\right)}{b}-\frac{2\,(Ba-Ab)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b}}{e}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x, algorithm="
giac")
```

output
```
((e*x + d)*(B - C)/b + C*log(b*e^(2*e*x + 2*d) + 2*a*e^(e*x + d) + b)/b -
2*(B*a - A*b)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^
2)*b))/e
```

Mupad [B] (verification not implemented)

Time = 3.27 (sec) , antiderivative size = 653, normalized size of antiderivative = 7.59

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{2\,\mathrm{atan}\left(\frac{a\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b-A\,e\,a^2\,b^2-B\,e\,a\,b^3+A\,e\,b^4}+\frac{a^2\,b^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{b^4\,e^2-a^2\,b^2\,e^2}\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B^2\,a^2}}{B\,e\,a^3\,b^4-A\,e\,a^2\,b^5-B\,e\,a\,b^6+A\,e\,b^7}+\frac{A\,\mathrm{e}^{e\,x}\,\mathrm{e}^d\,\sqrt{b^4\,e^2-a^2\,b^2}}{b\,e\,\sqrt{A^2\,b^2-2\,A\,B\,a\,b+B}}\right.}{\sqrt{b^4\,e^2-a^2\,b^2\,e^2}}$$
$$+\frac{B\,x}{b}-\frac{C\,x}{b}$$
$$+\frac{C\,b^3\,e\,\ln\left(4\,A^2\,b^3+4\,B^2\,a^2\,b-8\,A\,B\,a\,b^2+8\,B^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,A^2\,b^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,b^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,B^2\,a^2\right.}{b^4\,e^2-a^2\,b^2\,e^2}$$
$$-\frac{C\,a^2\,b\,e\,\ln\left(4\,A^2\,b^3+4\,B^2\,a^2\,b-8\,A\,B\,a\,b^2+8\,B^2\,a^3\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,A^2\,b^3\,\mathrm{e}^{2\,d}\,\mathrm{e}^{2\,e\,x}+8\,A^2\,a\,b^2\,\mathrm{e}^{e\,x}\,\mathrm{e}^d+4\,B^2\right.}{b^4\,e^2-a^2\,b^2\,e^2}$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x)),x)
```

output
```
(2*atan((a*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(
1/2))/(A*b^4*e - B*a*b^3*e + B*a^3*b*e - A*a^2*b^2*e) + (a^2*b^2*exp(e*x)*
exp(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2)*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2)
)/(A*b^7*e - B*a*b^6*e - A*a^2*b^5*e + B*a^3*b^4*e) + (A*exp(e*x)*exp(d)*(
b^4*e^2 - a^2*b^2*e^2)^(1/2))/(b*e*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))
- (B*a*exp(e*x)*exp(d)*(b^4*e^2 - a^2*b^2*e^2)^(1/2))/(b^2*e*(A^2*b^2 + B^
2*a^2 - 2*A*B*a*b)^(1/2)))*(A^2*b^2 + B^2*a^2 - 2*A*B*a*b)^(1/2))/(b^4*e^2
 - a^2*b^2*e^2)^(1/2) + (B*x)/b - (C*x)/b + (C*b^3*e*log(4*A^2*b^3 + 4*B^2
*a^2*b - 8*A*B*a*b^2 + 8*B^2*a^3*exp(e*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(
2*e*x) + 8*A^2*a*b^2*exp(e*x)*exp(d) + 4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 1
6*A*B*a^2*b*exp(e*x)*exp(d) - 8*A*B*a*b^2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 -
 a^2*b^2*e^2) - (C*a^2*b*e*log(4*A^2*b^3 + 4*B^2*a^2*b - 8*A*B*a*b^2 + 8*B
^2*a^3*exp(e*x)*exp(d) + 4*A^2*b^3*exp(2*d)*exp(2*e*x) + 8*A^2*a*b^2*exp(e
*x)*exp(d) + 4*B^2*a^2*b*exp(2*d)*exp(2*e*x) - 16*A*B*a^2*b*exp(e*x)*exp(d
) - 8*A*B*a*b^2*exp(2*d)*exp(2*e*x)))/(b^4*e^2 - a^2*b^2*e^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.29

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{a+b\cosh(d+ex)}\,dx = \frac{\log(\cosh(ex+d)\,b+a)\,c+bex}{be}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d)),x)
```

output
```
(log(cosh(d + e*x)*b + a)*c + b*e*x)/(b*e)
```

3.207 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 121

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}e} - \frac{C}{be(a+b\cosh(d+ex))} - \frac{(Ab-aB)\sinh(d+ex)}{(a^2-b^2)\,e(a+b\cosh(d+ex))}$$

output
```
2*(A*a-B*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2))/(a-b)^(3/
2)/(a+b)^(3/2)/e-C/b/e/(a+b*cosh(e*x+d))-(A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e
/(a+b*cosh(e*x+d))
```

Mathematica [A] (verified)

Time = 0.55 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.95

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \frac{\frac{2(aA-bB)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}} + \frac{(-a^2+b^2)C-b(Ab-aB)\sinh(d+ex)}{(a-b)b(a+b)(a+b\cosh(d+ex))}}{e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,
x]
```

output
```
((2*(a*A - b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-a^2 + b^2]])/(-a^
2 + b^2)^(3/2) + ((-a^2 + b^2)*C - b*(A*b - a*B)*Sinh[d + e*x])/((a - b)*b
*(a + b)*(a + b*Cosh[d + e*x])))/e
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.419$, Rules used = {3042, 4877, 26, 3042, 26, 3147, 17, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a+b\cos(id+iex))^2}dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx-iC\int \frac{i\sinh(d+ex)}{(a+b\cosh(d+ex))^2}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx+C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{\left(a-b\sin\left(id+iex-\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{\left(a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)\right)^2}dx$$

$$\Big\downarrow \; 3147$$

$$\frac{C\int \frac{1}{(a+b\cosh(d+ex))^2}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 17$$

$$-\frac{C}{be(a+b\cosh(d+ex))}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx$$

$$\Big\downarrow \; 3233$$

$$-\frac{\int -\frac{aA-bB}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{aA-bB}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \; 27$$

$$\frac{(aA-bB)\int \frac{1}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \; 3042$$

$$\frac{(aA-bB)\int \frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{a^2-b^2}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \; 3138$$

$$-\frac{2i(aA-bB)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(d+ex)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{e\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

$$\Big\downarrow \; 218$$

$$\frac{2(aA-bB)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e\sqrt{a-b}\sqrt{a+b}\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{e\left(a^2-b^2\right)(a+b\cosh(d+ex))}-\frac{C}{be(a+b\cosh(d+ex))}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^2,x]
```

output
```
(2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/(Sqrt
[a - b]*Sqrt[a + b]*(a^2 - b^2)*e) - C/(b*e*(a + b*Cosh[d + e*x])) - ((A*b
 - a*B)*Sinh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.19

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(Ab-Ba)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^2-b^2}+\frac{C}{a-b}\right)}{a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-a-b}+\frac{2(Aa-Bb)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{e}$
default	$\frac{-\frac{2\left(-\frac{(Ab-Ba)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{a^2-b^2}+\frac{C}{a-b}\right)}{a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^2-a-b}+\frac{2(Aa-Bb)\operatorname{arctanh}\left(\frac{(a-b)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{e}$
risch	$\frac{2Aab\,e^{ex+d}-2B\,a^2e^{ex+d}-2C\,a^2e^{ex+d}+2C\,b^2e^{ex+d}+2A\,b^2-2Bab}{be(a^2-b^2)\left(b\,e^{2ex+2d}+2a\,e^{ex+d}+b\right)}+\frac{\ln\left(e^{ex+d}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)Aa}{\sqrt{a^2-b^2}\,(a+b)(a-b)e}-$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-(A*b-B*a)/(a^2-b^2)*tanh(1/2*e*x+1/2*d)+C/(a-b))/(a*tanh(1/2*e*x
+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)+2*(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b
))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(111) = 222$.

Time = 0.10 (sec) , antiderivative size = 1044, normalized size of antiderivative = 8.63

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm
="fricas")
```

output
```
[-(2*B*a^3*b - 2*A*a^2*b^2 - 2*B*a*b^3 + 2*A*b^4 - (A*a*b^2 - B*b^3 + (A*a
*b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)*sinh(e*x + d)^2 + 2*(A*a
^2*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*b^2 + (A*a*b^2 - B*b^3)*c
osh(e*x + d))*sinh(e*x + d))*sqrt(a^2 - b^2)*log((b^2*cosh(e*x + d)^2 + b^
2*sinh(e*x + d)^2 + 2*a*b*cosh(e*x + d) + 2*a^2 - b^2 + 2*(b^2*cosh(e*x +
d) + a*b)*sinh(e*x + d) - 2*sqrt(a^2 - b^2)*(b*cosh(e*x + d) + b*sinh(e*x
+ d) + a))/(b*cosh(e*x + d)^2 + b*sinh(e*x + d)^2 + 2*a*cosh(e*x + d) + 2*
(b*cosh(e*x + d) + a)*sinh(e*x + d) + b)) + 2*((B + C)*a^4 - A*a^3*b - (B
+ 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x + d) + 2*((B + C)*a^4 - A*a^3*b
 - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*sinh(e*x + d))/((a^4*b^2 - 2*a^2*b
^4 + b^6)*e*cosh(e*x + d)^2 + (a^4*b^2 - 2*a^2*b^4 + b^6)*e*sinh(e*x + d)^
2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*e*cosh(e*x + d) + (a^4*b^2 - 2*a^2*b^4 +
 b^6)*e + 2*((a^4*b^2 - 2*a^2*b^4 + b^6)*e*cosh(e*x + d) + (a^5*b - 2*a^3*
b^3 + a*b^5)*e)*sinh(e*x + d)), -2*(B*a^3*b - A*a^2*b^2 - B*a*b^3 + A*b^4
+ (A*a*b^2 - B*b^3 + (A*a*b^2 - B*b^3)*cosh(e*x + d)^2 + (A*a*b^2 - B*b^3)
*sinh(e*x + d)^2 + 2*(A*a^2*b - B*a*b^2)*cosh(e*x + d) + 2*(A*a^2*b - B*a*
b^2 + (A*a*b^2 - B*b^3)*cosh(e*x + d))*sinh(e*x + d))*sqrt(-a^2 + b^2)*arc
tan(-sqrt(-a^2 + b^2)*(b*cosh(e*x + d) + b*sinh(e*x + d) + a)/(a^2 - b^2))
 + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*cosh(e*x
+ d) + ((B + C)*a^4 - A*a^3*b - (B + 2*C)*a^2*b^2 + A*a*b^3 + C*b^4)*si...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.28

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$$
$$=\frac{2\left(\frac{(Aa-Bb)\arctan\left(\frac{be^{(ex+d)}+a}{\sqrt{-a^2+b^2}}\right)}{(a^2-b^2)\sqrt{-a^2+b^2}}-\frac{Ba^2e^{(ex+d)}+Ca^2e^{(ex+d)}-Aabe^{(ex+d)}-Cb^2e^{(ex+d)}+Bab-Ab^2}{(a^2b-b^3)\left(be^{(2\,ex+2\,d)}+2\,ae^{(ex+d)}+b\right)}\right)}{e}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x, algorithm
="giac")
```

output
```
2*((A*a - B*b)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/((a^2 - b^2)*s
qrt(-a^2 + b^2)) - (B*a^2*e^(e*x + d) + C*a^2*e^(e*x + d) - A*a*b*e^(e*x +
 d) - C*b^2*e^(e*x + d) + B*a*b - A*b^2)/((a^2*b - b^3)*(b*e^(2*e*x + 2*d)
 + 2*a*e^(e*x + d) + b)))/e
```

Mupad [B] (verification not implemented)

Time = 2.69 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.49

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx = \frac{\frac{2\,(A\,b^3-B\,a\,b^2)}{b\,e\,(a^2\,b-b^3)}+\frac{2\,\mathrm{e}^{d+e\,x}\,(C\,b^4-B\,a^2\,b^2-C\,a^2\,b^2+A\,a\,b^3)}{b^2\,e\,(a^2\,b-b^3)}}{b+2\,a\,\mathrm{e}^{d+e\,x}+b\,\mathrm{e}^{2\,d+2\,e\,x}} + \frac{\ln\left(-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a-B\,b)}{b\,(a^2-b^2)}-\frac{2\,(A\,a-B\,b)\,(b+a\,\mathrm{e}^{d+e\,x})}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}\right)(A\,a-B\,b)}{e\,(a+b)^{3/2}\,(a-b)^{3/2}} - \frac{\ln\left(\frac{2\,(A\,a-B\,b)\,(b+a\,\mathrm{e}^{d+e\,x})}{b\,(a+b)^{3/2}\,(a-b)^{3/2}}-\frac{2\,\mathrm{e}^{d+e\,x}\,(A\,a-B\,b)}{b\,(a^2-b^2)}\right)(A\,a-B\,b)}{e\,(a+b)^{3/2}\,(a-b)^{3/2}}$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^2,x)
```

output
```
((2*(A*b^3 - B*a*b^2))/(b*e*(a^2*b - b^3)) + (2*exp(d + e*x)*(C*b^4 - B*a^
2*b^2 - C*a^2*b^2 + A*a*b^3))/(b^2*e*(a^2*b - b^3)))/(b + 2*a*exp(d + e*x)
 + b*exp(2*d + 2*e*x)) + (log(- (2*exp(d + e*x)*(A*a - B*b))/(b*(a^2 - b^2
)) - (2*(A*a - B*b)*(b + a*exp(d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)))
*(A*a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2)) - (log((2*(A*a - B*b)*(b + a
*exp(d + e*x)))/(b*(a + b)^(3/2)*(a - b)^(3/2)) - (2*exp(d + e*x)*(A*a - B
*b))/(b*(a^2 - b^2)))*(A*a - B*b))/(e*(a + b)^(3/2)*(a - b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.09

$$\int \frac{A + B\cosh(d+ex) + C\sinh(d+ex)}{(a+b\cosh(d+ex))^2}\,dx$$
$$= \frac{-2e^{2ex+2d}\sqrt{-a^2+b^2}\,atan\left(\frac{e^{ex+d}b+a}{\sqrt{-a^2+b^2}}\right)ab - 4e^{ex+d}\sqrt{-a^2+b^2}\,atan\left(\frac{e^{ex+d}b+a}{\sqrt{-a^2+b^2}}\right)a^2 - 2\sqrt{-a^2+b^2}\,atan\left(\frac{e^{ex+d}}{\sqrt{-a^2}}\right.}{ae\left(e^{2ex+2d}a^2b - e^{2ex+2d}b^3 + 2e^{ex+d}a^3 - 2e^{ex+d}a\,b^2 + a^2b - b\right.}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^2,x)
```

output
```
( - 2*e**(2*d + 2*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt
( - a**2 + b**2))*a*b - 4*e**(d + e*x)*sqrt( - a**2 + b**2)*atan((e**(d +
e*x)*b + a)/sqrt( - a**2 + b**2))*a**2 - 2*sqrt( - a**2 + b**2)*atan((e**(
d + e*x)*b + a)/sqrt( - a**2 + b**2))*a*b + e**(2*d + 2*e*x)*a**2*c - e**(
2*d + 2*e*x)*b**2*c + a**2*c - b**2*c)/(a*e*(e**(2*d + 2*e*x)*a**2*b - e**
(2*d + 2*e*x)*b**3 + 2*e**(d + e*x)*a**3 - 2*e**(d + e*x)*a*b**2 + a**2*b
- b**3))
```

3.208 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 187

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \frac{(2a^2A+Ab^2-3abB)\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}e} - \frac{C}{2be(a+b\cosh(d+ex))^2} - \frac{(Ab-aB)\sinh(d+ex)}{2\left(a^2-b^2\right)e(a+b\cosh(d+ex))^2} - \frac{(3aAb-a^2B-2b^2B)\sinh(d+ex)}{2\left(a^2-b^2\right)^2e(a+b\cosh(d+ex))}$$

output
```
(2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d)/(a+b)^(1/2
))/(a-b)^(5/2)/(a+b)^(5/2)/e-1/2*C/b/e/(a+b*cosh(e*x+d))^2-1/2*(A*b-B*a)*s
inh(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*sin
h(e*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 175, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \frac{-\frac{2(2a^2A+Ab^2-3abB)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{5/2}}+\frac{(-3aAb+a^2B+2b^2B)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))}+\frac{(-a^2+b^2)C-b(Ab-aB)\sinh(d+ex)}{(a-b)b(a+b)(a+b\cosh(d+ex))^2}}{2e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,
x]
```

output
```
((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTan[((a - b)*Tanh[(d + e*x)/2])/Sqrt[-
a^2 + b^2]])/(-a^2 + b^2)^(5/2) + ((-3*a*A*b + a^2*B + 2*b^2*B)*Sinh[d + e
*x])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])) + ((-a^2 + b^2)*C - b*(A*
b - a*B)*Sinh[d + e*x])/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^2))/(2*e)
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.14, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.516, Rules used = {3042, 4877, 26, 3042, 26, 3147, 17, 3233, 25, 3042, 3233, 25, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a+b\cos(id+iex))^3}dx$$

$$\Big\downarrow \text{4877}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx-iC\int \frac{i\sinh(d+ex)}{(a+b\cosh(d+ex))^3}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx+C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{\left(a-b\sin\left(id+iex-\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{\left(a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)\right)^3}dx$$

$$\Big\downarrow \text{3147}$$

$$\frac{C\int \frac{1}{(a+b\cosh(d+ex))^3}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{17}$$

$$-\frac{C}{2be(a+b\cosh(d+ex))^2}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx$$

$$\Big\downarrow \text{3233}$$

$$-\frac{\int -\frac{2(aA-bB)-(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{2(aA-bB)-(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{2(aA-bB)+(aB-Ab)\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3233}$$

$$\frac{-\frac{\int -\frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{2Aa^2-3bBa+Ab^2}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\left(2a^2A-3abB+Ab^2\right)\int \frac{1}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}+\frac{\left(2a^2A-3abB+Ab^2\right)\int \frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{3138}$$

$$\frac{-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}-\frac{2i\left(2a^2A-3abB+Ab^2\right)\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(d+ex)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(d+ex)\right)\right)}{e(a^2-b^2)}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{2\left(2a^2A-3abB+Ab^2\right)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{\left(a^2(-B)+3aAb-2b^2B\right)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{2e\left(a^2-b^2\right)(a+b\cosh(d+ex))^2}-\frac{C}{2be(a+b\cosh(d+ex))^2}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^3,x]
```

output
```
-1/2*C/(b*e*(a + b*Cosh[d + e*x])^2) - ((A*b - a*B)*Sinh[d + e*x])/(2*(a^2
 - b^2)*e*(a + b*Cosh[d + e*x])^2) + ((2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTa
nh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*Sqrt[a + b]*
(a^2 - b^2)*e) - ((3*a*A*b - a^2*B - 2*b^2*B)*Sinh[d + e*x])/((a^2 - b^2)*
e*(a + b*Cosh[d + e*x])))/(2*(a^2 - b^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 6.41 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.46

method	result
derivativedivides	$-\frac{\frac{2\left(-\frac{\left(4Aab+A\,b^{2}-2B\,a^{2}-Bab-2B\,b^{2}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{a-b}+\frac{\left(4Aab-A\,b^{2}-2B\,a^{2}+Bab-2B\,b^{2}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)} \cdots\right)}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-a-b\right)^{2}} \cdots}{e}$
default	$-\frac{\frac{2\left(-\frac{\left(4Aab+A\,b^{2}-2B\,a^{2}-Bab-2B\,b^{2}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{3}}{2(a-b)\left(a^{2}+2ab+b^{2}\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}}{a-b}+\frac{\left(4Aab-A\,b^{2}-2B\,a^{2}+Bab-2B\,b^{2}\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)}{2(a+b)\left(a^{2}-2ab+b^{2}\right)} \cdots\right)}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-b\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^{2}-a-b\right)^{2}} \cdots}{e}$
risch	$\frac{2A\,a^{2}b^{2}\mathrm{e}^{3ex+3d}+A\,b^{4}\mathrm{e}^{3ex+3d}-3Ba\,b^{3}\mathrm{e}^{3ex+3d}+6A\,a^{3}b\,\mathrm{e}^{2ex+2d}+3Aa\,b^{3}\mathrm{e}^{2ex+2d}-2B\,a^{4}\mathrm{e}^{2ex+2d}-5B\,a^{2}b^{2}\mathrm{e}^{2ex+2d} \cdots}{be(a^{2} \cdots}$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*
tanh(1/2*e*x+1/2*d)^3+C/(a-b)*tanh(1/2*e*x+1/2*d)^2+1/2*(4*A*a*b-A*b^2-2*B
*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)-a*C/(a^2-2*a
*b+b^2))/(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)^2+(2*A*a^2+
A*b^2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tanh(
1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(171) = 342$.

Time = 0.15 (sec) , antiderivative size = 3636, normalized size of antiderivative = 19.44

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^3}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^3}\, dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^3} \, dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(171) = 342$.

Time = 0.15 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.98

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^3} \, dx$$
$$= \frac{\frac{(2\,Aa^2 - 3\,Bab + Ab^2)\arctan\left(\frac{be^{(ex+d)} + a}{\sqrt{-a^2+b^2}}\right)}{(a^4 - 2\,a^2b^2 + b^4)\sqrt{-a^2+b^2}} + \frac{2\,Aa^2b^2e^{(3\,ex+3\,d)} - 3\,Bab^3e^{(3\,ex+3\,d)} + Ab^4e^{(3\,ex+3\,d)} - 2\,Ba^4e^{(2\,ex+2\,d)} - 2\,Ca^4e^{(2\,ex+2\,d)} + 6\,A}{}}{}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x, algorithm
="giac")
```

output
```
((2*A*a^2 - 3*B*a*b + A*b^2)*arctan((b*e^(e*x + d) + a)/sqrt(-a^2 + b^2))/
((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) + (2*A*a^2*b^2*e^(3*e*x + 3*d)
- 3*B*a*b^3*e^(3*e*x + 3*d) + A*b^4*e^(3*e*x + 3*d) - 2*B*a^4*e^(2*e*x + 2
*d) - 2*C*a^4*e^(2*e*x + 2*d) + 6*A*a^3*b*e^(2*e*x + 2*d) - 5*B*a^2*b^2*e^
(2*e*x + 2*d) + 4*C*a^2*b^2*e^(2*e*x + 2*d) + 3*A*a*b^3*e^(2*e*x + 2*d) -
2*B*b^4*e^(2*e*x + 2*d) - 2*C*b^4*e^(2*e*x + 2*d) - 4*B*a^3*b*e^(e*x + d)
+ 10*A*a^2*b^2*e^(e*x + d) - 5*B*a*b^3*e^(e*x + d) - A*b^4*e^(e*x + d) - B
*a^2*b^2 + 3*A*a*b^3 - 2*B*b^4)/((a^4*b - 2*a^2*b^3 + b^5)*(b*e^(2*e*x + 2
*d) + 2*a*e^(e*x + d) + b)^2))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+b\cosh\left(d+e\,x\right)\right)^3}\,dx$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3,x)
```

output
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 688, normalized size of antiderivative = 3.68

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^3}\,dx = \frac{-4e^{4ex+4d}\sqrt{-a^2+b^2}\,atan\left(\frac{e^{ex+d}b+a}{\sqrt{-a^2+b^2}}\right)a\,b^3-16e^{3ex+3d}\sqrt{-a^2+b^2}\,atan\left(\frac{e^{ex+d}b+a}{\sqrt{-a^2+b^2}}\right)a^2b^2-16e^{2ex+2d}\sqrt{-a^2}}{}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^3,x)
```

output
```
( - 4*e**(4*d + 4*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt
( - a**2 + b**2))*a*b**3 - 16*e**(3*d + 3*e*x)*sqrt( - a**2 + b**2)*atan((
e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2 - 16*e**(2*d + 2*e*x)*
sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**3*
b - 8*e**(2*d + 2*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt
( - a**2 + b**2))*a*b**3 - 16*e**(d + e*x)*sqrt( - a**2 + b**2)*atan((e**(
d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**2 - 4*sqrt( - a**2 + b**2)*a
tan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a*b**3 - e**(4*d + 4*e*x)*a
**2*b**3 + e**(4*d + 4*e*x)*b**5 + 4*e**(2*d + 2*e*x)*a**4*b - 4*e**(2*d +
 2*e*x)*a**4*c - 2*e**(2*d + 2*e*x)*a**2*b**3 + 8*e**(2*d + 2*e*x)*a**2*b*
*2*c - 2*e**(2*d + 2*e*x)*b**5 - 4*e**(2*d + 2*e*x)*b**4*c + 8*e**(d + e*x
)*a**3*b**2 - 8*e**(d + e*x)*a*b**4 + 3*a**2*b**3 - 3*b**5)/(2*b*e*(e**(4*
d + 4*e*x)*a**4*b**2 - 2*e**(4*d + 4*e*x)*a**2*b**4 + e**(4*d + 4*e*x)*b**
6 + 4*e**(3*d + 3*e*x)*a**5*b - 8*e**(3*d + 3*e*x)*a**3*b**3 + 4*e**(3*d +
 3*e*x)*a*b**5 + 4*e**(2*d + 2*e*x)*a**6 - 6*e**(2*d + 2*e*x)*a**4*b**2 +
2*e**(2*d + 2*e*x)*b**6 + 4*e**(d + e*x)*a**5*b - 8*e**(d + e*x)*a**3*b**3
 + 4*e**(d + e*x)*a*b**5 + a**4*b**2 - 2*a**2*b**4 + b**6))
```

3.209 $\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$

Optimal result

Integrand size = 31, antiderivative size = 260

$$\begin{aligned}\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx &= \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\,\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}e} \\ &\quad - \frac{C}{3be(a+b\cosh(d+ex))^3} - \frac{(Ab-aB)\sinh(d+ex)}{3(a^2-b^2)\,e(a+b\cosh(d+ex))^3} \\ &\quad - \frac{(5aAb-2a^2B-3b^2B)\sinh(d+ex)}{6(a^2-b^2)^2\,e(a+b\cosh(d+ex))^2} \\ &\quad - \frac{(11a^2Ab+4Ab^3-2a^3B-13ab^2B)\sinh(d+ex)}{6(a^2-b^2)^3\,e(a+b\cosh(d+ex))}\end{aligned}$$

output

```
(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tanh(1/2*e*x+1/2*d
)/(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/e-1/3*C/b/e/(a+b*cosh(e*x+d))^3-1/3
*(A*b-B*a)*sinh(e*x+d)/(a^2-b^2)/e/(a+b*cosh(e*x+d))^3-1/6*(5*A*a*b-2*B*a^
2-3*B*b^2)*sinh(e*x+d)/(a^2-b^2)^2/e/(a+b*cosh(e*x+d))^2-1/6*(11*A*a^2*b+4
*A*b^3-2*B*a^3-13*B*a*b^2)*sinh(e*x+d)/(a^2-b^2)^3/e/(a+b*cosh(e*x+d))
```

Mathematica [A] (verified)

Time = 1.85 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.94

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx$$
$$= \frac{\frac{6(2a^3A+3aAb^2-4a^2bB-b^3B)\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{7/2}}+\frac{(-5aAb+2a^2B+3b^2B)\sinh(d+ex)}{(a-b)^2(a+b)^2(a+b\cosh(d+ex))^2}+\frac{(-11a^2Ab-4Ab^3+2a^3B+13ab^2B)}{(a-b)^3(a+b)^3(a+b\cosh(d+}}{6e}$$

input
```
Integrate[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,
x]
```

output
```
((6*(2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTan[((a - b)*Tanh[(d + e*
x)/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(7/2) + ((-5*a*A*b + 2*a^2*B + 3*b^
2*B)*Sinh[d + e*x])/((a - b)^2*(a + b)^2*(a + b*Cosh[d + e*x])^2) + ((-11*
a^2*A*b - 4*A*b^3 + 2*a^3*B + 13*a*b^2*B)*Sinh[d + e*x])/((a - b)^3*(a + b
)^3*(a + b*Cosh[d + e*x])) + (2*(-a^2 + b^2)*C - 2*b*(A*b - a*B)*Sinh[d +
e*x])/((a - b)*b*(a + b)*(a + b*Cosh[d + e*x])^3))/(6*e)
```

Rubi [A] (verified)

Time = 1.20 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.16, number of steps used = 19, number of rules used = 18, $\frac{\text{number of rules}}{\text{integrand size}} = 0.581$, Rules used = {3042, 4877, 26, 3042, 26, 3147, 17, 3233, 25, 3042, 3233, 25, 3042, 3233, 27, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{A+B\cos(id+iex)-iC\sin(id+iex)}{(a+b\cos(id+iex))^4}dx$$
$$\Big\downarrow \text{4877}$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^4}dx-iC\int \frac{i\sinh(d+ex)}{(a+b\cosh(d+ex))^4}dx$$

$$\Big\downarrow 26$$

$$\int \frac{A+B\cosh(d+ex)}{(a+b\cosh(d+ex))^4}dx+C\int \frac{\sinh(d+ex)}{(a+b\cosh(d+ex))^4}dx$$

$$\Big\downarrow 3042$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx+C\int -\frac{i\cos\left(id+iex-\frac{\pi}{2}\right)}{\left(a-b\sin\left(id+iex-\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow 26$$

$$\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx-iC\int \frac{\cos\left(\frac{1}{2}(2id-\pi)+iex\right)}{\left(a-b\sin\left(\frac{1}{2}(2id-\pi)+iex\right)\right)^4}dx$$

$$\Big\downarrow 3147$$

$$\frac{C\int \frac{1}{(a+b\cosh(d+ex))^4}d(b\cosh(d+ex))}{be}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow 17$$

$$-\frac{C}{3be(a+b\cosh(d+ex))^3}+\int \frac{A+B\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^4}dx$$

$$\Big\downarrow 3233$$

$$-\frac{\int -\frac{3(aA-bB)-2(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow 25$$

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\cosh(d+ex)}{(a+b\cosh(d+ex))^3}dx}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow 3042$$

$$\frac{\int \frac{3(aA-bB)-2(Ab-aB)\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^3}dx}{3\left(a^2-b^2\right)}-\frac{(Ab-aB)\sinh(d+ex)}{3e\left(a^2-b^2\right)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \; 3233$$

$$\frac{-\frac{\int -\frac{2(3Aa^2-5bBa+2Ab^2)-(-2Ba^2+5Aba-3b^2B)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\int \frac{2(3Aa^2-5bBa+2Ab^2)-(-2Ba^2+5Aba-3b^2B)\cosh(d+ex)}{(a+b\cosh(d+ex))^2}dx}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}+\frac{\int \frac{2(3Aa^2-5bBa+2Ab^2)+(2Ba^2-5Aba+3b^2B)\sin\left(id+iex+\frac{\pi}{2}\right)}{\left(a+b\sin\left(id+iex+\frac{\pi}{2}\right)\right)^2}dx}{2(a^2-b^2)}}{3(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \; 3233$$

$$\frac{\frac{-\frac{\int -\frac{3(2Aa^3-4bBa^2+3Ab^2a-b^3B)}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(-2a^3B+11a^2Ab-13ab^2B+4Ab^3)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{3(2a^3A-4a^2bB+3aAb^2-b^3B)\int \frac{1}{a+b\cosh(d+ex)}dx}{a^2-b^2}-\frac{(-2a^3B+11a^2Ab-13ab^2B+4Ab^3)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$$\Big\downarrow \; 3042$$

$$\frac{-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}+\frac{-\frac{(-2a^3B+11a^2Ab-13ab^2B+4Ab^3)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}+\frac{3(2a^3A-4a^2bB+3aAb^2-b^3B)\int\frac{1}{a+b\sin\left(id+iex+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2(a^2-b^2)}}{3(a^2-b^2)}-\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\Big\downarrow$ 3138

$$\frac{-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+ex)}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}+\frac{-\frac{(-2a^3B+11a^2Ab-13ab^2B+4Ab^3)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}-\frac{6i(2a^3A-4a^2bB+3aAb^2-b^3B)\int\frac{1}{-((a-b)\tanh^2(\frac{1}{2}(d+ex))}}{e(a^2-b^2)}}{2(a^2-b^2)}}{3(a^2-b^2)}\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}-\frac{C}{3be(a+b\cosh(d+ex))^3}$$

$\Big\downarrow$ 218

$$-\frac{(Ab-aB)\sinh(d+ex)}{3e(a^2-b^2)(a+b\cosh(d+ex))^3}+\frac{\frac{\frac{6(2a^3A-4a^2bB+3aAb^2-b^3B)\operatorname{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(d+ex)\right)}{\sqrt{a+b}}\right)}{e\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{(-2a^3B+11a^2Ab-13ab^2B+4Ab^3)\sinh(d+ex)}{e(a^2-b^2)(a+b\cosh(d+ex))}}{2(a^2-b^2)}-\frac{(-2a^2B+5aAb-3b^2B)\sinh(d+}{2e(a^2-b^2)(a+b\cosh(d+ex))^2}}{3(a^2-b^2)}\frac{C}{3be(a+b\cosh(d+ex))^3}$$

input
```
Int[(A + B*Cosh[d + e*x] + C*Sinh[d + e*x])/(a + b*Cosh[d + e*x])^4,x]
```

output
```
-1/3*C/(b*e*(a + b*Cosh[d + e*x])^3) - ((A*b - a*B)*Sinh[d + e*x])/(3*(a^2
 - b^2)*e*(a + b*Cosh[d + e*x])^3) + (-1/2*((5*a*A*b - 2*a^2*B - 3*b^2*B)*
Sinh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x])^2) + ((6*(2*a^3*A + 3*
a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tanh[(d + e*x)/2])/Sqrt[
a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)*e) - ((11*a^2*A*b + 4*A*b^3
 - 2*a^3*B - 13*a*b^2*B)*Sinh[d + e*x])/((a^2 - b^2)*e*(a + b*Cosh[d + e*x]
)))/(2*(a^2 - b^2)))/(3*(a^2 - b^2))
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

rule 3233
```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Simp[(-(b*c - a*d))*Cos[e + f*x]*((a + b*Sin[e +
 f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Simp[1/((m + 1)*(a^2 - b^2))
  Int[(a + b*Sin[e + f*x])^(m + 1)*Simp[(a*c - b*d)*(m + 1) - (b*c - a*d)*(
m + 2)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && IntegerQ[2*m]
```

rule 4877
```
Int[(u_)*((v_) + (d_.)*(F_)[(c_.)*((a_.) + (b_.)*(x_))]^(n_.)), x_Symbol] :
> With[{e = FreeFactors[Cos[c*(a + b*x)], x]}, Int[ActivateTrig[u*v], x] +
Simp[d   Int[ActivateTrig[u]*Sin[c*(a + b*x)]^n, x], x] /; FunctionOfQ[Cos[
c*(a + b*x)]/e, u, x]] /; FreeQ[{a, b, c, d}, x] &&  !FreeQ[v, x] && Intege
rQ[(n - 1)/2] && NonsumQ[u] && (EqQ[F, Sin] || EqQ[F, sin])
```

Maple [A] (verified)

Time = 20.82 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.77

method	result
derivativedivides	$-\frac{2\left(-\frac{\left(6A\,a^2b+3Aa\,b^2+2A\,b^3-2a^3B-2B\,a^2b-6Ba\,b^2-B\,b^3\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{a-b}+\frac{2\left(9A\,a^2b+A\,b^3-3a^3B-7Ba\ldots\right.}{3\left(a^2+2ab+b^2\right)\left(a^2\ldots\right.}\ldots\right.}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)\ldots\right.}$
default	$-\frac{2\left(-\frac{\left(6A\,a^2b+3Aa\,b^2+2A\,b^3-2a^3B-2B\,a^2b-6Ba\,b^2-B\,b^3\right)\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^5}{2(a-b)\left(a^3+3a^2b+3b^2a+b^3\right)}+\frac{C\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)^4}{a-b}+\frac{2\left(9A\,a^2b+A\,b^3-3a^3B-7Ba\ldots\right.}{3\left(a^2+2ab+b^2\right)\left(a^2\ldots\right.}\ldots\right.}{\left(a\tanh\left(\frac{ex}{2}+\frac{d}{2}\right)\ldots\right.}$
risch	Expression too large to display

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x,method=_RETURNVE
RBOSE)
```

output
```
1/e*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b
^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tanh(1/2*e*x+1/2*d)^5+C/(a-b)*tanh(1/2
*e*x+1/2*d)^4+2/3*(9*A*a^2*b+A*b^3-3*B*a^3-7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2
-2*a*b+b^2)*tanh(1/2*e*x+1/2*d)^3-2*a*C/(a^2-2*a*b+b^2)*tanh(1/2*e*x+1/2*d
)^2-1/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tanh(1/2*e*x+1/2*d)+1/3*C*(3*a^2+b^2)/(a^3-3
*a^2*b+3*a*b^2-b^3))/(a*tanh(1/2*e*x+1/2*d)^2-b*tanh(1/2*e*x+1/2*d)^2-a-b)
^3+(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*e*x+1/2*d)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4211 vs. $2(243) = 486$.

Time = 0.33 (sec) , antiderivative size = 8531, normalized size of antiderivative = 32.81

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm
="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = \text{Timed out}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))**4,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^4}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm
="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 657 vs. 2(243) = 486.

Time = 0.16 (sec) , antiderivative size = 657, normalized size of antiderivative = 2.53

$$\int \frac{A + B\cosh(d + ex) + C\sinh(d + ex)}{(a + b\cosh(d + ex))^4}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x, algorithm
="giac")
```

output
```
1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*arctan((b*e^(e*x + d) + a
)/sqrt(-a^2 + b^2))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 + b^2))
 + (6*A*a^3*b^3*e^(5*e*x + 5*d) - 12*B*a^2*b^4*e^(5*e*x + 5*d) + 9*A*a*b^5
*e^(5*e*x + 5*d) - 3*B*b^6*e^(5*e*x + 5*d) + 30*A*a^4*b^2*e^(4*e*x + 4*d)
- 60*B*a^3*b^3*e^(4*e*x + 4*d) + 45*A*a^2*b^4*e^(4*e*x + 4*d) - 15*B*a*b^5
*e^(4*e*x + 4*d) - 8*B*a^6*e^(3*e*x + 3*d) - 8*C*a^6*e^(3*e*x + 3*d) + 44*
A*a^5*b*e^(3*e*x + 3*d) - 64*B*a^4*b^2*e^(3*e*x + 3*d) + 24*C*a^4*b^2*e^(3
*e*x + 3*d) + 82*A*a^3*b^3*e^(3*e*x + 3*d) - 78*B*a^2*b^4*e^(3*e*x + 3*d)
- 24*C*a^2*b^4*e^(3*e*x + 3*d) + 24*A*a*b^5*e^(3*e*x + 3*d) + 8*C*b^6*e^(3
*e*x + 3*d) - 24*B*a^5*b*e^(2*e*x + 2*d) + 102*A*a^4*b^2*e^(2*e*x + 2*d) -
 102*B*a^3*b^3*e^(2*e*x + 2*d) + 36*A*a^2*b^4*e^(2*e*x + 2*d) - 24*B*a*b^5
*e^(2*e*x + 2*d) + 12*A*b^6*e^(2*e*x + 2*d) - 12*B*a^4*b^2*e^(e*x + d) + 6
0*A*a^3*b^3*e^(e*x + d) - 66*B*a^2*b^4*e^(e*x + d) + 15*A*a*b^5*e^(e*x + d
) + 3*B*b^6*e^(e*x + d) - 2*B*a^3*b^3 + 11*A*a^2*b^4 - 13*B*a*b^5 + 4*A*b^
6)/((a^6*b - 3*a^4*b^3 + 3*a^2*b^5 - b^7)*(b*e^(2*e*x + 2*d) + 2*a*e^(e*x
+ d) + b)^3))/e
```

Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = \int \frac{A+B\cosh(d+e\,x)+C\sinh(d+e\,x)}{\left(a+b\cosh\left(d+e\,x\right)\right)^4}\,dx$$

input
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4,x)
```

output
```
int((A + B*cosh(d + e*x) + C*sinh(d + e*x))/(a + b*cosh(d + e*x))^4, x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1747, normalized size of antiderivative = 6.72

$$\int \frac{A+B\cosh(d+ex)+C\sinh(d+ex)}{(a+b\cosh(d+ex))^4}\,dx = \text{Too large to display}$$

input
```
int((A+B*cosh(e*x+d)+C*sinh(e*x+d))/(a+b*cosh(e*x+d))^4,x)
```

output
```
( - 12*e**(6*d + 6*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqr
t( - a**2 + b**2))*a**3*b**4 - 6*e**(6*d + 6*e*x)*sqrt( - a**2 + b**2)*ata
n((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a*b**6 - 72*e**(5*d + 5*e*x)*
sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**4*
b**3 - 36*e**(5*d + 5*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/
sqrt( - a**2 + b**2))*a**2*b**5 - 144*e**(4*d + 4*e*x)*sqrt( - a**2 + b**2
)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**5*b**2 - 108*e**(4*d
+ 4*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**
2))*a**3*b**4 - 18*e**(4*d + 4*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x
)*b + a)/sqrt( - a**2 + b**2))*a*b**6 - 96*e**(3*d + 3*e*x)*sqrt( - a**2 +
 b**2)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**6*b - 192*e**(3*
d + 3*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b
**2))*a**4*b**3 - 72*e**(3*d + 3*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e
*x)*b + a)/sqrt( - a**2 + b**2))*a**2*b**5 - 144*e**(2*d + 2*e*x)*sqrt( -
a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**5*b**2 - 1
08*e**(2*d + 2*e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( -
 a**2 + b**2))*a**3*b**4 - 18*e**(2*d + 2*e*x)*sqrt( - a**2 + b**2)*atan((
e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a*b**6 - 72*e**(d + e*x)*sqrt( -
 a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( - a**2 + b**2))*a**4*b**3 -
36*e**(d + e*x)*sqrt( - a**2 + b**2)*atan((e**(d + e*x)*b + a)/sqrt( - ...
```

3.210 $\int \frac{x}{a+b\cosh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 191

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \frac{x\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} + \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}} - \frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}$$

output
```
1/2*x*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)-1
/2*x*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)+1/
4*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
-1/4*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1
/2)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.49

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \frac{x\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)+x\log\left(1+\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)-x\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b+2\sqrt{a(a+b)}}{b}}}\right)-x\log\left(1+\right.}{}$$

input
```
Integrate[x/(a + b*Cosh[x]^2),x]
```

output
```
(x*Log[1 - E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + x*Log[1 + E^x/S
qrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - x*Log[1 - E^x/Sqrt[-((2*a + b +
 2*Sqrt[a*(a + b)])/b)]] - x*Log[1 + E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b
)])/b)]] + PolyLog[2, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)])] + P
olyLog[2, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - PolyLog[2, -(E^x
/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - PolyLog[2, E^x/Sqrt[-((2*a +
 b + 2*Sqrt[a*(a + b)])/b)]])/(2*Sqrt[a*(a + b)])
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {6164, 3042, 3801, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x}{a+b\cosh^2(x)}\,dx$$

$$\Big\downarrow \text{6164}$$

$$2\int \frac{x}{2a+b+b\cosh(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x}{2a+b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$\Big\downarrow$ 3801

$$4\int \frac{e^{2x}x}{e^{4x}b+b+2(2a+b)e^{2x}}dx$$

$\Big\downarrow$ 2694

$$4\left(\frac{b\int \frac{e^{2x}x}{2\left(2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x}{2\left(2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}\right)$$

$\Big\downarrow$ 27

$$4\left(\frac{b\int \frac{e^{2x}x}{2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x}{2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}\right)$$

$\Big\downarrow$ 2620

$$4\left(\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int \log\left(\frac{e^{2x}b}{2a-2\sqrt{a+b}\sqrt{a}+b}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int \log\left(\frac{e^{2x}b}{2a+2\sqrt{a+b}\sqrt{a}+b}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}\right)$$

$\Big\downarrow$ 2715

$$4\left(\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int e^{-2x}\log\left(\frac{e^{2x}b}{2a-2\sqrt{a+b}\sqrt{a}+b}+1\right)de^{2x}}{4b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int e^{-2x}\log\left(\frac{e^{2x}b}{2a+2\sqrt{a+b}\sqrt{a}\cdots}\right)}{4b}\right)}{4\sqrt{a}\sqrt{a+b}}\right)$$

$\Big\downarrow$ 2838

$$4\left(\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{4b}+\frac{x\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+2\sqrt{a+b}\sqrt{a}+b}\right)}{4b}+\frac{x\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}\right)$$

input
```
Int[x/(a + b*Cosh[x]^2),x]
```

output
```
4*((b*((x*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*b) +
PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/(4*b)))/(4*Sq
rt[a]*Sqrt[a + b]) - (b*((x*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[
a + b])])/(2*b) + PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a + b
]))]/(4*b)))/(4*Sqrt[a]*Sqrt[a + b]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6164
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(147) = 294$.

Time = 0.51 (sec) , antiderivative size = 487, normalized size of antiderivative = 2.55

method	result
risch	$\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)x}{-2\sqrt{a(a+b)}-2a-b}-\frac{x^{2}}{-2\sqrt{a(a+b)}-2a-b}+\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)ax}{\sqrt{a(a+b)}\left(-2\sqrt{a(a+b)}-2a-b\right)}+\frac{\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)bx}{2\sqrt{a(a+b)}\left(-2\sqrt{a(a+b)}-2a-b\right)}-$

input
```
int(x/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x
-1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*
a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x+1/2/(a*(a+b))^(1/2)/(
-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b*x-
1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^2-1/2/(a*(a+b))^(1/2)/(-2
*(a*(a+b))^(1/2)-2*a-b)*b*x^2+1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*e
xp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2
)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+1/4/(a*(a+b))^
(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-
2*a-b))*b+1/2/(a*(a+b))^(1/2)*x*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))
-1/2/(a*(a+b))^(1/2)*x^2+1/4/(a*(a+b))^(1/2)*polylog(2,b*exp(2*x)/(2*(a*(a
+b))^(1/2)-2*a-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 780 vs. $2(149) = 298$.

Time = 0.13 (sec) , antiderivative size = 780, normalized size of antiderivative = 4.08

$$\int \frac{x}{a + b\cosh^2(x)}\, dx = \text{Too large to display}$$

input
```
integrate(x/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x
) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2
+ a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a
+ b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a
*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x*sqr
t((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) -
2*a - b)/b) + b)/b) - b*x*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*cosh(x) +
 (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt
((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + b*sqrt((a^2 + a*b)/b^2
)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x
))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) +
 b)/b + 1) + b*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)
*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqr
t((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - b*sqrt((a^2 + a*b)/b^2)*dil
og(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sq
rt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) + b)/b
+ 1) - b*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cosh(x) + (2*a + b)*sinh(
x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2
+ a*b)/b^2) - 2*a - b)/b) - b)/b + 1))/(a^2 + a*b)
```

Sympy [F]

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \int \frac{x}{a+b\cosh^2(x)}\,dx$$

input
```
integrate(x/(a+b*cosh(x)**2),x)
```

output
```
Integral(x/(a + b*cosh(x)**2), x)
```

Maxima [F]

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \int \frac{x}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(x/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x/(b*cosh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x}{a+b\cosh^2(x)}\,dx = \int \frac{x}{b\cosh(x)^2+a}\,dx$$

input
```
integrate(x/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
integrate(x/(b*cosh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x}{a+b\cosh^2(x)}\,dx=\int \frac{x}{b\cosh(x)^2+a}\,dx$$

input
```
int(x/(a + b*cosh(x)^2),x)
```

output
```
int(x/(a + b*cosh(x)^2), x)
```

Reduce [F]

$$\int \frac{x}{a+b\cosh^2(x)}\,dx=\int \frac{x}{\cosh(x)^2 b+a}dx$$

input
```
int(x/(a+b*cosh(x)^2),x)
```

output
```
int(x/(cosh(x)**2*b + a),x)
```

3.211 $\int \frac{x^2}{a+b\cosh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 291

$$\begin{aligned}\int \frac{x^2}{a+b\cosh^2(x)}\,dx = {} & \frac{x^2\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}-\frac{x^2\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}\\ & +\frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}\\ & -\frac{x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}\\ & -\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}+\frac{\text{PolyLog}\left(3,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\end{aligned}$$

output
```
1/2*x^2*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
-1/2*x^2*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2
)+1/2*x*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)
^(1/2)-1/2*x*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/
(a+b)^(1/2)-1/4*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/
2)/(a+b)^(1/2)+1/4*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^
(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.49

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx$$
$$= \frac{x^2\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)+x^2\log\left(1+\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)-x^2\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b+2\sqrt{a(a+b)}}{b}}}\right)-x^2\log\left(\right.}{}$$

input
```
Integrate[x^2/(a + b*Cosh[x]^2),x]
```

output
```
(x^2*Log[1 - E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + x^2*Log[1 + E
^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - x^2*Log[1 - E^x/Sqrt[-((2*a
 + b + 2*Sqrt[a*(a + b)])/b)]] - x^2*Log[1 + E^x/Sqrt[-((2*a + b + 2*Sqrt[
a*(a + b)])/b)]] + 2*x*PolyLog[2, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)
])/b)])] + 2*x*PolyLog[2, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] -
2*x*PolyLog[2, -(E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - 2*x*Poly
Log[2, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)]] - 2*PolyLog[3, -(E^x/
Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)])] - 2*PolyLog[3, E^x/Sqrt[-((2*a
+ b - 2*Sqrt[a*(a + b)])/b)]] + 2*PolyLog[3, -(E^x/Sqrt[-((2*a + b + 2*Sqr
t[a*(a + b)])/b)])] + 2*PolyLog[3, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)]
)/b)]])/(2*Sqrt[a*(a + b)])
```

Rubi [A] (verified)

Time = 1.13 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.643$, Rules used = {6164, 3042, 3801, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx$$
$$\Big\downarrow \text{6164}$$

$$2\int \frac{x^2}{2a+b+b\cosh(2x)}dx$$

$$\Big\downarrow \text{3042}$$

$$2\int \frac{x^2}{2a+b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{3801}$$

$$4\int \frac{e^{2x}x^2}{e^{4x}b+b+2(2a+b)e^{2x}}dx$$

$$\Big\downarrow \text{2694}$$

$$4\left(\frac{b\int \frac{e^{2x}x^2}{2\left(2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^2}{2\left(2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{b\int \frac{e^{2x}x^2}{2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^2}{2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \text{2620}$$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a-2\sqrt{a+b}\sqrt{a}+b}+1\right)dx}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\int x\log\left(\frac{e^{2x}b}{2a+2\sqrt{a+b}\sqrt{a}+b}+1\right)}{b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

$$\Big\downarrow \text{3011}$$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\frac{1}{2}\int \text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)dx-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b+}}\right.}{2b}\right.}{}\right.$$

$$\Big\downarrow \text{2720}$$

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)de^{2x}-\frac{1}{2}x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a}+b}\right)}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{}{2\sqrt{}}\right.}{}\right.}{}\right.$$

$\downarrow$ 7143

$$4\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{\frac{1}{4}\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)-\frac{1}{2}x\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)}{b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}\right.}{2b}\right.}{}\right.$$

input
```
Int[x^2/(a + b*Cosh[x]^2),x]
```

output
```
4*((b*((x^2*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*b)
- (-1/2*(x*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]) +
 PolyLog[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/4)/b))/(4*Sq
rt[a]*Sqrt[a + b]) - (b*((x^2*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqr
t[a + b])])/(2*b) - (-1/2*(x*PolyLog[2, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]
*Sqrt[a + b]))]) + PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt[a +
b]))]/4)/b))/(4*Sqrt[a]*Sqrt[a + b]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6164
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 685 vs. $2(225) = 450$.

Time = 0.48 (sec) , antiderivative size = 686, normalized size of antiderivative = 2.36

method	result
risch	$-\frac{2x^3}{3\left(-2\sqrt{a(a+b)}-2a-b\right)}+\frac{x^2\ln\left(1-\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)}{-2\sqrt{a(a+b)}-2a-b}+\frac{x\,\mathrm{polylog}\left(2,\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right)}{-2\sqrt{a(a+b)}-2a-b}-\frac{\mathrm{polylog}\left(3,\frac{b\,e^{2x}}{-2\sqrt{a(a+b)}-2a-b}\right.}{2\left(-2\sqrt{a(a+b)}-2a-b\right)}$

input
```
int(x^2/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
-2/3/(-2*(a*(a+b))^(1/2)-2*a-b)*x^3+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x^2*ln(1-
b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(-2*(a*(a+b))^(1/2)-2*a-b)*x*poly
log(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/2/(-2*(a*(a+b))^(1/2)-2*a-b
)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-2/3/(a*(a+b))^(1/2)/(-2
*(a*(a+b))^(1/2)-2*a-b)*a*x^3+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)
*a*x^2*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))+1/(a*(a+b))^(1/2)/(-2*(
a*(a+b))^(1/2)-2*a-b)*a*x*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))
-1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*polylog(3,b*exp(2*x)/(-2
*(a*(a+b))^(1/2)-2*a-b))-1/3/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*
x^3+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^2*ln(1-b*exp(2*x)/(
-2*(a*(a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*
b*x*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)/(
-2*(a*(a+b))^(1/2)-2*a-b)*b*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b
))-1/3/(a*(a+b))^(1/2)*x^3+1/2/(a*(a+b))^(1/2)*x^2*ln(1-b*exp(2*x)/(2*(a*(
a+b))^(1/2)-2*a-b))+1/2/(a*(a+b))^(1/2)*x*polylog(2,b*exp(2*x)/(2*(a*(a+b)
)^(1/2)-2*a-b))-1/4/(a*(a+b))^(1/2)*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1/2
)-2*a-b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1162 vs. $2(228) = 456$.

Time = 0.12 (sec) , antiderivative size = 1162, normalized size of antiderivative = 3.99

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x^2*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh
(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((
2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2
 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x
^2*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(
b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) - 2*a - b)/b) + b)/b) - b*x^2*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*c
osh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 2*b*x*sqrt((a^
2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x
) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2
*a + b)/b) + b)/b + 1) + 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*cos
h(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2)
)*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 2*b*x*sqrt(
(a^2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cos
h(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) -
 2*a - b)/b) + b)/b + 1) - 2*b*x*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)*c
osh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) - 2*b*sqr...
```

Sympy [F]

$$\int \frac{x^2}{a+b\cosh^2(x)}\,dx=\int \frac{x^2}{a+b\cosh^2{(x)}}\,dx$$

input
```
integrate(x**2/(a+b*cosh(x)**2),x)
```

output
```
Integral(x**2/(a + b*cosh(x)**2), x)
```

Maxima [F]

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x^2/(b*cosh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

input
```
integrate(x^2/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
integrate(x^2/(b*cosh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{b\cosh(x)^2 + a}\,dx$$

input
```
int(x^2/(a + b*cosh(x)^2),x)
```

output
```
int(x^2/(a + b*cosh(x)^2), x)
```

Reduce [F]

$$\int \frac{x^2}{a + b\cosh^2(x)}\,dx = \int \frac{x^2}{\cosh(x)^2 b + a}dx$$

input
```
int(x^2/(a+b*cosh(x)^2),x)
```

output
```
int(x**2/(cosh(x)**2*b + a),x)
```

3.212 $\int \frac{x^3}{a+b\cosh^2(x)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 391

$$\begin{aligned}\int \frac{x^3}{a+b\cosh^2(x)}\,dx &= \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}} - \frac{x^3\log\left(1+\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{2\sqrt{a}\sqrt{a+b}}\\
&+ \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\\
&- \frac{3x^2\operatorname{PolyLog}\left(2,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\\
&- \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\\
&+ \frac{3x\operatorname{PolyLog}\left(3,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{4\sqrt{a}\sqrt{a+b}}\\
&+ \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+b-2\sqrt{a}\sqrt{a+b}}\right)}{8\sqrt{a}\sqrt{a+b}}\\
&- \frac{3\operatorname{PolyLog}\left(4,-\frac{be^{2x}}{2a+b+2\sqrt{a}\sqrt{a+b}}\right)}{8\sqrt{a}\sqrt{a+b}}\end{aligned}$$

output
```
1/2*x^3*ln(1+b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
-1/2*x^3*ln(1+b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+b)^(1/2
)+3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))/a^(1/2)/(a+
b)^(1/2)-3/4*x^2*polylog(2,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1/2)))/a^(1
/2)/(a+b)^(1/2)-3/4*x*polylog(3,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)^(1/2)))
/a^(1/2)/(a+b)^(1/2)+3/4*x*polylog(3,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a+b)^(1
/2)))/a^(1/2)/(a+b)^(1/2)+3/8*polylog(4,-b*exp(2*x)/(2*a+b-2*a^(1/2)*(a+b)
^(1/2)))/a^(1/2)/(a+b)^(1/2)-3/8*polylog(4,-b*exp(2*x)/(2*a+b+2*a^(1/2)*(a
+b)^(1/2)))/a^(1/2)/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.48

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx = \frac{x^3\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)+x^3\log\left(1+\frac{e^x}{\sqrt{-\frac{2a+b-2\sqrt{a(a+b)}}{b}}}\right)-x^3\log\left(1-\frac{e^x}{\sqrt{-\frac{2a+b+2\sqrt{a(a+b)}}{b}}}\right)-x^3\log\left(\right.}{}$$

input
```
Integrate[x^3/(a + b*Cosh[x]^2),x]
```

output
```
(x^3*Log[1 - E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + x^3*Log[1 + E
^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] - x^3*Log[1 - E^x/Sqrt[-((2*a
 + b + 2*Sqrt[a*(a + b)])/b)]] - x^3*Log[1 + E^x/Sqrt[-((2*a + b + 2*Sqrt[
a*(a + b)])/b)]] + 3*x^2*PolyLog[2, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a +
b)])/b)])] + 3*x^2*PolyLog[2, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]
] - 3*x^2*PolyLog[2, -(E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - 3*
x^2*PolyLog[2, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)]] - 6*x*PolyLog
[3, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)])] - 6*x*PolyLog[3, E^x/
Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]] + 6*x*PolyLog[3, -(E^x/Sqrt[-((2
*a + b + 2*Sqrt[a*(a + b)])/b)])] + 6*x*PolyLog[3, E^x/Sqrt[-((2*a + b + 2
*Sqrt[a*(a + b)])/b)]] + 6*PolyLog[4, -(E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a
+ b)])/b)])] + 6*PolyLog[4, E^x/Sqrt[-((2*a + b - 2*Sqrt[a*(a + b)])/b)]]
- 6*PolyLog[4, -(E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)])] - 6*PolyLo
g[4, E^x/Sqrt[-((2*a + b + 2*Sqrt[a*(a + b)])/b)]])/(2*Sqrt[a*(a + b)])
```

Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 355, normalized size of antiderivative = 0.91, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.714, Rules used = {6164, 3042, 3801, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx$$

$$\Big\downarrow \ 6164$$

$$2\int \frac{x^3}{2a+b+b\cosh(2x)}dx$$

$$\Big\downarrow \ 3042$$

$$2\int \frac{x^3}{2a+b+b\sin\left(2ix+\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \ 3801$$

$$4\int \frac{e^{2x}x^3}{e^{4x}b+b+2(2a+b)e^{2x}}dx$$

$$\Big\downarrow \ 2694$$

$$4\left(\frac{b\int \frac{e^{2x}x^3}{2\left(2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^3}{2\left(2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b\right)}dx}{2\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \ 27$$

$$4\left(\frac{b\int \frac{e^{2x}x^3}{2a-2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}-\frac{b\int \frac{e^{2x}x^3}{2a+2\sqrt{a+b}\sqrt{a}+be^{2x}+b}dx}{4\sqrt{a}\sqrt{a+b}}\right)$$

$$\Big\downarrow \ 2620$$

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a-2\sqrt{a+b}\sqrt{a}+b}+1\right)dx}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\int x^2\log\left(\frac{e^{2x}b}{2a+2\sqrt{a+b}\sqrt{a}+\right.}\right.}{2b}}{4\sqrt{a}\sqrt{a+b}}\right.$$

$\Big\downarrow$ 3011

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(\int x\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)\right)}{2b}\right)}{4\sqrt{a}\sqrt{a+b}}-\frac{b\left(\frac{x^3\log\left(\frac{}{2\sqrt{a}\sqrt{}}\right.}{2}\right.}{}\right.$$

$\Big\downarrow$ 7163

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}\int\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)dx-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,-\frac{}{2a-2}\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

$\Big\downarrow$ 2720

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(-\frac{1}{4}\int e^{-2x}\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)de^{2x}-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

$\Big\downarrow$ 7143

$$4\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{2x}}{-2\sqrt{a}\sqrt{a+b}+2a+b}+1\right)}{2b}-\frac{3\left(-\frac{1}{2}x^2\,\text{PolyLog}\left(2,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)+\frac{1}{2}x\,\text{PolyLog}\left(3,-\frac{be^{2x}}{2a-2\sqrt{a+b}\sqrt{a+b}}\right)-\frac{1}{4}\,\text{PolyLog}\left(4,-\frac{be^{2x}}{2a-2\sqrt{a+}}\right.\right.}{2b}\right.}{4\sqrt{a}\sqrt{a+b}}\right.$$

input
```
Int[x^3/(a + b*Cosh[x]^2),x]
```

output
```
4*((b*((x^3*Log[1 + (b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])])/(2*b)
- (3*(-1/2*(x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b])
)]) + (x*PolyLog[3, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))])/2 -
 PolyLog[4, -((b*E^(2*x))/(2*a + b - 2*Sqrt[a]*Sqrt[a + b]))]/4))/(2*b)))/
(4*Sqrt[a]*Sqrt[a + b]) - (b*((x^3*Log[1 + (b*E^(2*x))/(2*a + b + 2*Sqrt[a
]*Sqrt[a + b])])/(2*b) - (3*(-1/2*(x^2*PolyLog[2, -((b*E^(2*x))/(2*a + b +
 2*Sqrt[a]*Sqrt[a + b]))]) + (x*PolyLog[3, -((b*E^(2*x))/(2*a + b + 2*Sqrt
[a]*Sqrt[a + b]))])/2 - PolyLog[4, -((b*E^(2*x))/(2*a + b + 2*Sqrt[a]*Sqrt
[a + b]))]/4))/(2*b)))/(4*Sqrt[a]*Sqrt[a + b]))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6164
```
Int[(Cosh[(c_.) + (d_.)*(x_)]^2*(b_.) + (a_))^(n_)*(x_)^(m_.), x_Symbol] :>
 Simp[1/2^n   Int[x^m*(2*a + b + b*Cosh[2*c + 2*d*x])^n, x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[a - b, 0] && IGtQ[m, 0] && ILtQ[n, 0] && (EqQ[n, -1]
|| (EqQ[m, 1] && EqQ[n, -2]))
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(303) = 606$.

Time = 0.54 (sec) , antiderivative size = 889, normalized size of antiderivative = 2.27

method	result	size
risch	Expression too large to display	889

input
```
int(x^3/(a+b*cosh(x)^2),x,method=_RETURNVERBOSE)
```

output
```
1/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x
^3-1/2/(-2*(a*(a+b))^(1/2)-2*a-b)*x^4+1/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2
)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x^3+1/2/(a*(a+b))^(
1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*ln(1-b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b)
)*b*x^3-1/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*a*x^4-1/4/(a*(a+b))
^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*b*x^4+3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*pol
ylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x^2+3/2/(a*(a+b))^(1/2)/(-2*
(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*
x^2+3/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(2,b*exp(2*x)/(-
2*(a*(a+b))^(1/2)-2*a-b))*b*x^2-3/2/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b
*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*x-3/2/(a*(a+b))^(1/2)/(-2*(a*(a+b))^
(1/2)-2*a-b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a*x-3/4/(a*(
a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(3,b*exp(2*x)/(-2*(a*(a+b))^
(1/2)-2*a-b))*b*x+3/4/(-2*(a*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*x)/(-2*
(a*(a+b))^(1/2)-2*a-b))+3/4/(a*(a+b))^(1/2)/(-2*(a*(a+b))^(1/2)-2*a-b)*pol
ylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*a+3/8/(a*(a+b))^(1/2)/(-2*(a
*(a+b))^(1/2)-2*a-b)*polylog(4,b*exp(2*x)/(-2*(a*(a+b))^(1/2)-2*a-b))*b+1/
2/(a*(a+b))^(1/2)*x^3*ln(1-b*exp(2*x)/(2*(a*(a+b))^(1/2)-2*a-b))-1/4/(a*(a
+b))^(1/2)*x^4+3/4/(a*(a+b))^(1/2)*x^2*polylog(2,b*exp(2*x)/(2*(a*(a+b))^(
1/2)-2*a-b))-3/4/(a*(a+b))^(1/2)*x*polylog(3,b*exp(2*x)/(2*(a*(a+b))^(1...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. $2(307) = 614$.

Time = 0.13 (sec) , antiderivative size = 1542, normalized size of antiderivative = 3.94

$$\int \frac{x^3}{a + b\cosh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3/(a+b*cosh(x)^2),x, algorithm="fricas")
```

output
```
-1/2*(b*x^3*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh
(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^
2 + a*b)/b^2) + 2*a + b)/b) + b)/b) + b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((
2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2
 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b) - b*x
^3*sqrt((a^2 + a*b)/b^2)*log((((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(
b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b
^2) - 2*a - b)/b) + b)/b) - b*x^3*sqrt((a^2 + a*b)/b^2)*log(-(((2*a + b)*c
osh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^
2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b) + 3*b*x^2*sqrt((
a^2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh
(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) +
 2*a + b)/b) + b)/b + 1) + 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*a + b)
*cosh(x) + (2*a + b)*sinh(x) - 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/
b^2))*sqrt(-(2*b*sqrt((a^2 + a*b)/b^2) + 2*a + b)/b) - b)/b + 1) - 3*b*x^2
*sqrt((a^2 + a*b)/b^2)*dilog(-(((2*a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*
(b*cosh(x) + b*sinh(x))*sqrt((a^2 + a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/
b^2) - 2*a - b)/b) + b)/b + 1) - 3*b*x^2*sqrt((a^2 + a*b)/b^2)*dilog((((2*
a + b)*cosh(x) + (2*a + b)*sinh(x) + 2*(b*cosh(x) + b*sinh(x))*sqrt((a^2 +
 a*b)/b^2))*sqrt((2*b*sqrt((a^2 + a*b)/b^2) - 2*a - b)/b) - b)/b + 1) -...
```

Sympy [F]

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx=\int \frac{x^3}{a+b\cosh^2{(x)}}\,dx$$

input
```
integrate(x**3/(a+b*cosh(x)**2),x)
```

output
```
Integral(x**3/(a + b*cosh(x)**2), x)
```

Maxima [F]

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx=\int \frac{x^3}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(x^3/(a+b*cosh(x)^2),x, algorithm="maxima")
```

output
```
integrate(x^3/(b*cosh(x)^2 + a), x)
```

Giac [F]

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx=\int \frac{x^3}{b\cosh{(x)}^2+a}\,dx$$

input
```
integrate(x^3/(a+b*cosh(x)^2),x, algorithm="giac")
```

output
```
integrate(x^3/(b*cosh(x)^2 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx=\int \frac{x^3}{b\cosh\left(x\right)^2+a}\,dx$$

input
```
int(x^3/(a + b*cosh(x)^2),x)
```

output
```
int(x^3/(a + b*cosh(x)^2), x)
```

Reduce [F]

$$\int \frac{x^3}{a+b\cosh^2(x)}\,dx=\int \frac{x^3}{\cosh\left(x\right)^2 b+a}dx$$

input
```
int(x^3/(a+b*cosh(x)^2),x)
```

output
```
int(x**3/(cosh(x)**2*b + a),x)
```

3.213 $\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a} - \frac{\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

output
```
-3/4*Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/4*Chi(3*(-a*x+1)^(1/2)/(a*x+1)^
(1/2))/a
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.95

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \frac{-3\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right) - \text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{4a}$$

input
```
Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
(-3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]] - CoshIntegral[(3*Sqrt[1 - a
*x])/Sqrt[1 + a*x]])/(4*a)
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{ 7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^3}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{ 3793}$$

$$-\frac{\int \left(\frac{3\sqrt{ax+1}\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}+\frac{\sqrt{ax+1}\cosh\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{4\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{\frac{3}{4}\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{4}\text{Chi}\left(\frac{3\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^3/(1 - a^2*x^2),x]
```

output
```
-(((3*CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]])/4 + CoshIntegral[(3*Sqrt[
1 - a*x])/Sqrt[1 + a*x]]/4)/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{3}}{-a^{2}x^{2}+1}dx$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

output
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh^3\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**3/(-a**2*x**2+1),x)
```

output
```
-Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
-integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^3/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^3}{a^2\,x^2-1}\,dx$$

input
```
int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1),x)
```

output
```
-int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^3/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\cosh^3\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^3}{a^2x^2-1}dx\right)$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^3/(-a^2*x^2+1),x)
```

output
```
- int(cosh(sqrt( - a*x + 1)/sqrt(a*x + 1))**3/(a**2*x**2 - 1),x)
```

3.214 $\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 58

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a}$$

output
```
-1/2*Chi(2*(-a*x+1)^(1/2)/(a*x+1)^(1/2))/a-1/2*ln((-a*x+1)^(1/2)/(a*x+1)^(
1/2))/a
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\frac{\operatorname{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{2a} - \frac{\log(1-ax)}{4a} + \frac{\log(1+ax)}{4a}$$

input
```
Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-1/2*CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/a - Log[1 - a*x]/(4*a)
+ Log[1 + a*x]/(4*a)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {7232, 3042, 3793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \; 7232$$

$$-\frac{\int \frac{\sqrt{ax+1}\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \; 3042$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \; 3793$$

$$-\frac{\int \left(\frac{\sqrt{ax+1}\cosh\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{2\sqrt{1-ax}}+\frac{\sqrt{ax+1}}{2\sqrt{1-ax}}\right)d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{2}\text{Chi}\left(\frac{2\sqrt{1-ax}}{\sqrt{ax+1}}\right)+\frac{1}{2}\log\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
-((CoshIntegral[(2*Sqrt[1 - a*x])/Sqrt[1 + a*x]]/2 + Log[Sqrt[1 - a*x]/Sqr
t[1 + a*x]]/2)/a)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3793
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> In
t[ExpandTrigReduce[(c + d*x)^m, Sin[e + f*x]^n, x], x] /; FreeQ[{c, d, e, f
, m}, x] && IGtQ[n, 1] && ( !RationalQ[m] || (GeQ[m, -1] && LtQ[m, 1]))
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{-a^2x^2+1}dx$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh^2\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

output
```
-Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
1/4*log(a*x + 1)/a - 1/4*log(a*x - 1)/a - 1/4*integrate(e^(2*sqrt(-a*x + 1
)/sqrt(a*x + 1))/(a^2*x^2 - 1), x) - 1/4*integrate(e^(-2*sqrt(-a*x + 1)/sq
rt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2}{a^2\,x^2-1}\,dx$$

input
```
int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1),x)
```

output
```
-int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\cosh^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}dx\right)$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
- int(cosh(sqrt( - a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1),x)
```

3.215 $\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 34, antiderivative size = 26

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\operatorname{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

output
```
-Chi((-a*x+1)^(1/2)/(a*x+1)^(1/2))/a
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\frac{\operatorname{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{a}$$

input
```
Integrate[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.088, Rules used = {7232, 3042, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\sin\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3782}$$

$$-\frac{\text{Chi}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{a}$$

input
```
Int[Cosh[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
-(CoshIntegral[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/a)
```

Defintions of rubi rules used

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 7232
```
Int[((a_.) + (b_.)*(F_)[((c_.)*Sqrt[(d_.) + (e_.)*(x_)])/Sqrt[(f_.) + (g_.)
*(x_)]])^(n_.)/((A_.) + (C_.)*(x_)^2), x_Symbol] :> Simp[2*e*(g/(C*(e*f - d
*g)))   Subst[Int[(a + b*F[c*x])^n/x, x], x, Sqrt[d + e*x]/Sqrt[f + g*x]],
x] /; FreeQ[{a, b, c, d, e, f, g, A, C, F}, x] && EqQ[C*d*f - A*e*g, 0] &&
EqQ[e*f + d*g, 0] && IGtQ[n, 0]
```

Maple [F]

$$\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="ma
xima")
```

output
```
-integrate(cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gi
ac")
```

output
```
integrate(-cosh(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\int \frac{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)}{a^2\,x^2-1}\,dx$$

input
```
int(-cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1),x)
```

output
```
-int(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))/(a^2*x^2 - 1), x)
```

Reduce [F]

$$\int \frac{\cosh\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\left(\int \frac{\cosh\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}dx\right)$$

input
```
int(cosh((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
- int(cosh(sqrt( - a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.216 $\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$

Optimal result

Integrand size = 34, antiderivative size = 34

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \operatorname{Int}\left(\frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)}, x\right)$$

output
```
Defer(Int)(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a*x+1)/(a*x+1),x)
```

Mathematica [N/A]

Not integrable

Time = 8.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx = \int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2} \, dx$$

input
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$
$$\Big\downarrow \text{ 7232}$$
$$-\frac{\int \frac{\sqrt{ax+1}\text{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{ 3042}$$
$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$
$$\Big\downarrow \text{ 4680}$$
$$-\frac{\int \frac{\sqrt{ax+1}\text{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.88

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{-a^2x^2+1}dx$$

input
```
int(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
int(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int -\frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="fr
icas")
```

output
```
integral(-sech(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 4.41 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sech((-a*x+1)**(1/2)/(a*x+1)**(1/2))/(-a**2*x**2+1),x)
```

output
```
-Integral(sech(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="ma
xima")
```

output
```
-integrate(sech(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}\,dx$$

input
```
integrate(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x, algorithm="gi
ac")
```

output
```
integrate(-sech(sqrt(-a*x + 1)/sqrt(a*x + 1))/(a^2*x^2 - 1), x)
```

Mupad [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.03

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 33, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = -\left(\int \frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^2x^2-1}dx\right)$$

input
```
int(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))/(-a^2*x^2+1),x)
```

output
```
- int(sech(sqrt( - a*x + 1)/sqrt(a*x + 1))/(a**2*x**2 - 1),x)
```

3.217 $\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$

Optimal result

Integrand size = 36, antiderivative size = 36

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \operatorname{Int}\left(\frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{(1-ax)(1+ax)},x\right)$$

output
```
Defer(Int)(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a*x+1)/(a*x+1),x)
```

Mathematica [N/A]

Not integrable

Time = 27.74 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx = \int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx$$

input
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
Integrate[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{1-a^2x^2}dx$$

$$\Big\downarrow \text{7232}$$

$$-\frac{\int \frac{\sqrt{ax+1}\text{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int \frac{\sqrt{ax+1}\csc\left(\frac{i\sqrt{1-ax}}{\sqrt{ax+1}}+\frac{\pi}{2}\right)^2}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

$$\Big\downarrow \text{4680}$$

$$-\frac{\int \frac{\sqrt{ax+1}\text{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{ax+1}}\right)}{\sqrt{1-ax}}d\frac{\sqrt{1-ax}}{\sqrt{ax+1}}}{a}$$

input
```
Int[Sech[Sqrt[1 - a*x]/Sqrt[1 + a*x]]^2/(1 - a^2*x^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.89

$$\int \frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{-a^{2}x^{2}+1}dx$$

input
```
int(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
int(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^{2}}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
fricas")
```

output
```
integral(-sech(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Sympy [N/A]

Not integrable

Time = 4.82 (sec) , antiderivative size = 31, normalized size of antiderivative = 0.86

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\int \frac{\operatorname{sech}^{2}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)}{a^{2}x^{2}-1}\,dx$$

input
```
integrate(sech((-a*x+1)**(1/2)/(a*x+1)**(1/2))**2/(-a**2*x**2+1),x)
```

output
```
-Integral(sech(sqrt(-a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 121, normalized size of antiderivative = 3.36

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
maxima")
```

output
```
2*sqrt(a*x + 1)/(sqrt(-a*x + 1)*a*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + sqr
t(-a*x + 1)*a) + 4*integrate(1/2*sqrt(a*x + 1)/((a^2*x^2 - 1)*sqrt(-a*x +
1)*e^(2*sqrt(-a*x + 1)/sqrt(a*x + 1)) + (a^2*x^2 - 1)*sqrt(-a*x + 1)), x)
```

Giac [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=\int -\frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}\,dx$$

input
```
integrate(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x, algorithm="
giac")
```

output
```
integrate(-sech(sqrt(-a*x + 1)/sqrt(a*x + 1))^2/(a^2*x^2 - 1), x)
```

Mupad [N/A]

Not integrable

Time = 2.24 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\int \frac{1}{\cosh\left(\frac{\sqrt{1-a\,x}}{\sqrt{a\,x+1}}\right)^2\,\left(a^2\,x^2-1\right)}\,dx$$

input
```
int(-1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)),x)
```

output
```
-int(1/(cosh((1 - a*x)^(1/2)/(a*x + 1)^(1/2))^2*(a^2*x^2 - 1)), x)
```

Reduce [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{sech}^2\left(\frac{\sqrt{1-ax}}{\sqrt{1+ax}}\right)}{1-a^2x^2}\,dx=-\left(\int \frac{\operatorname{sech}\left(\frac{\sqrt{-ax+1}}{\sqrt{ax+1}}\right)^2}{a^2x^2-1}dx\right)$$

input
```
int(sech((-a*x+1)^(1/2)/(a*x+1)^(1/2))^2/(-a^2*x^2+1),x)
```

output
```
- int(sech(sqrt( - a*x + 1)/sqrt(a*x + 1))**2/(a**2*x**2 - 1),x)
```

3.218 $\int \frac{x \sinh(x)}{(a+b\cosh(x))^2} \, dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \frac{x \sinh(x)}{(a+b\cosh(x))^2} \, dx = \frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}b\sqrt{a+b}} - \frac{x}{b(a+b\cosh(x))}$$

output
```
2*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(1/2)/b/(a+b)^(1/2)-x
/b/(a+b*cosh(x))
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.98

$$\int \frac{x \sinh(x)}{(a+b\cosh(x))^2} \, dx = -\frac{2\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{b\sqrt{-a^2+b^2}} - \frac{x}{b(a+b\cosh(x))}$$

input
```
Integrate[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(-2*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(b*Sqrt[-a^2 + b^2]) - x
/(b*(a + b*Cosh[x]))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {5988, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}dx$$

$$\Big\downarrow \text{ 5988}$$

$$\frac{\int \frac{1}{a+b\cosh(x)}dx}{b}-\frac{x}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x}{b(a+b\cosh(x))}+\frac{\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{ 3138}$$

$$\frac{2\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{b}-\frac{x}{b(a+b\cosh(x))}$$

$$\Big\downarrow \text{ 221}$$

$$\frac{2\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{b\sqrt{a-b}\sqrt{a+b}}-\frac{x}{b(a+b\cosh(x))}$$

input
```
Int[(x*Sinh[x])/(a + b*Cosh[x])^2,x]
```

output
```
(2*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b
]) - x/(b*(a + b*Cosh[x]))
```

Defintions of rubi rules used

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 5988
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*((a + b*Cosh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 137 vs. $2(50) = 100$.

Time = 1.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.30

method	result	size
risch	$-\frac{2x\,\mathrm{e}^{x}}{b(\mathrm{e}^{2x}b+2a\,\mathrm{e}^{x}+b)}+\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}-a^{2}+b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}-\frac{\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^{2}-b^{2}}+a^{2}-b^{2}}{b\sqrt{a^{2}-b^{2}}}\right)}{\sqrt{a^{2}-b^{2}}\,b}$	138

input
```
int(x*sinh(x)/(a+b*cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
-2*x/b*exp(x)/(exp(x)^2*b+2*a*exp(x)+b)+1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(a*(
a^2-b^2)^(1/2)-a^2+b^2)/b/(a^2-b^2)^(1/2))-1/(a^2-b^2)^(1/2)/b*ln(exp(x)+(
a*(a^2-b^2)^(1/2)+a^2-b^2)/b/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(50) = 100$.

Time = 0.11 (sec) , antiderivative size = 480, normalized size of antiderivative = 8.00

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx$$
$$= \left[-\frac{2\,(a^2-b^2)x\cosh(x)+2\,(a^2-b^2)x\sinh(x)-\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)\right.}{a^2b^2-b^4+(a^2b^2-b^4)\cosh(x)^2+(a^2b^2-b^4)\sinh(x)^2}\right.$$
$$-\frac{2\left((a^2-b^2)x\cosh(x)+(a^2-b^2)x\sinh(x)+\left(b\cosh(x)^2+b\sinh(x)^2+2\,a\cosh(x)+2\,(b\cosh(x)\right.\right.}{a^2b^2-b^4+(a^2b^2-b^4)\cosh(x)^2+(a^2b^2-b^4)\sinh(x)^2+2\,(a^3b-ab^3)\cosh(x)}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="fricas")
```

output
```
[-(2*(a^2 - b^2)*x*cosh(x) + 2*(a^2 - b^2)*x*sinh(x) - (b*cosh(x)^2 + b*si
nh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(a^2 - b^2)*log
((b^2*cosh(x)^2 + b^2*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cos
h(x) + a*b)*sinh(x) - 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*co
sh(x)^2 + b*sinh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)))/(a^
2*b^2 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a
^3*b - a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x
)), -2*((a^2 - b^2)*x*cosh(x) + (a^2 - b^2)*x*sinh(x) + (b*cosh(x)^2 + b*s
inh(x)^2 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)*sqrt(-a^2 + b^2)*a
rctan(-sqrt(-a^2 + b^2)*(b*cosh(x) + b*sinh(x) + a)/(a^2 - b^2)))/(a^2*b^2
 - b^4 + (a^2*b^2 - b^4)*cosh(x)^2 + (a^2*b^2 - b^4)*sinh(x)^2 + 2*(a^3*b
- a*b^3)*cosh(x) + 2*(a^3*b - a*b^3 + (a^2*b^2 - b^4)*cosh(x))*sinh(x))]
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \text{Timed out}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))**2,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^2}\,dx = \int \frac{x\sinh{(x)}}{(b\cosh{(x)}+a)^2}\,dx$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^2,x, algorithm="giac")
```

output
```
integrate(x*sinh(x)/(b*cosh(x) + a)^2, x)
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.83

$$\int \frac{x \sinh(x)}{(a+b\cosh(x))^2}\,dx = \frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^x\,(b^4-a^2\,b^2)+a\,b^3+a^2\,b^2\,\mathrm{e}^x}{b^2\,\sqrt{b^4-a^2\,b^2}}\right)}{\sqrt{b^4-a^2\,b^2}} - \frac{2\,\mathrm{e}^x\,(a^2\,x-b^2\,x)}{(a^2\,b-b^3)\,\left(b+2\,a\,\mathrm{e}^x+b\,\mathrm{e}^{2\,x}\right)}$$

input
```
int((x*sinh(x))/(a + b*cosh(x))^2,x)
```

output
```
(2*atan((exp(x)*(b^4 - a^2*b^2) + a*b^3 + a^2*b^2*exp(x))/(b^2*(b^4 - a^2*
b^2)^(1/2))))/(b^4 - a^2*b^2)^(1/2) - (2*exp(x)*(a^2*x - b^2*x))/((a^2*b -
 b^3)*(b + 2*a*exp(x) + b*exp(2*x)))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.87

$$\int \frac{x \sinh(x)}{(a+b\cosh(x))^2}\,dx = \frac{-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)\cosh(x)\,b-2\sqrt{-a^2+b^2}\,atan\left(\frac{e^x b+a}{\sqrt{-a^2+b^2}}\right)a-a^2x+b^2x}{b\left(\cosh(x)\,a^2b-\cosh(x)\,b^3+a^3-a\,b^2\right)}$$

input
```
int(x*sinh(x)/(a+b*cosh(x))^2,x)
```

output
```
( - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*cosh(x)
*b - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*a - a*
*2*x + b**2*x)/(b*(cosh(x)*a**2*b - cosh(x)*b**3 + a**3 - a*b**2))
```

3.219 $\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 87

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \frac{a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b(a+b)^{3/2}} - \frac{x}{2b(a+b\cosh(x))^2} - \frac{\sinh(x)}{2\left(a^2-b^2\right)(a+b\cosh(x))}$$

output
```
a*arctanh((a-b)^(1/2)*tanh(1/2*x)/(a+b)^(1/2))/(a-b)^(3/2)/b/(a+b)^(3/2)-1
/2*x/b/(a+b*cosh(x))^2-1/2*sinh(x)/(a^2-b^2)/(a+b*cosh(x))
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.00

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx=\frac{1}{2}\left(\frac{\frac{2a\arctan\left(\frac{(a-b)\tanh\left(\frac{x}{2}\right)}{\sqrt{-a^2+b^2}}\right)}{(-a^2+b^2)^{3/2}}-\frac{x}{(a+b\cosh(x))^2}}{b}-\frac{\sinh(x)}{(a-b)(a+b)(a+b\cosh(x))}\right)$$

input
```
Integrate[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

output
```
(((2*a*ArcTan[((a - b)*Tanh[x/2])/Sqrt[-a^2 + b^2]])/(-a^2 + b^2)^(3/2) -
x/(a + b*Cosh[x])^2)/b - Sinh[x]/((a - b)*(a + b)*(a + b*Cosh[x])))/2
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {5988, 3042, 3143, 25, 27, 3042, 3138, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}dx$$

$$\Big\downarrow \text{ 5988}$$

$$\frac{\int \frac{1}{(a+b\cosh(x))^2}dx}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$$\Big\downarrow \text{ 3042}$$

$$-\frac{x}{2b(a+b\cosh(x))^2}+\frac{\int \frac{1}{\left(a+b\sin\left(ix+\frac{\pi}{2}\right)\right)^2}dx}{2b}$$

$\Big\downarrow$ 3143

$$\frac{-\frac{\int -\frac{a}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{a}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$\Big\downarrow$ 27

$$\frac{\frac{a\int \frac{1}{a+b\cosh(x)}dx}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$\Big\downarrow$ 3042

$$-\frac{x}{2b(a+b\cosh(x))^2}+\frac{-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}+\frac{a\int \frac{1}{a+b\sin\left(ix+\frac{\pi}{2}\right)}dx}{a^2-b^2}}{2b}$$

$\Big\downarrow$ 3138

$$\frac{\frac{2a\int \frac{1}{-\left((a-b)\tanh^2\left(\frac{x}{2}\right)\right)+a+b}d\tanh\left(\frac{x}{2}\right)}{a^2-b^2}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

$\Big\downarrow$ 221

$$\frac{\frac{2a\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{x}{2}\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\sqrt{a+b}(a^2-b^2)}-\frac{b\sinh(x)}{(a^2-b^2)(a+b\cosh(x))}}{2b}-\frac{x}{2b(a+b\cosh(x))^2}$$

input
```
Int[(x*Sinh[x])/(a + b*Cosh[x])^3,x]
```

output
```
-1/2*x/(b*(a + b*Cosh[x])^2) + ((2*a*ArcTanh[(Sqrt[a - b]*Tanh[x/2])/Sqrt[
a + b]])/(Sqrt[a - b]*Sqrt[a + b]*(a^2 - b^2)) - (b*Sinh[x])/((a^2 - b^2)*
(a + b*Cosh[x])))/(2*b)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3143
```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos
[c + d*x]*((a + b*Sin[c + d*x])^(n + 1)/(d*(n + 1)*(a^2 - b^2))), x] + Simp
[1/((n + 1)*(a^2 - b^2))   Int[(a + b*Sin[c + d*x])^(n + 1)*Simp[a*(n + 1)
- b*(n + 2)*Sin[c + d*x], x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
 b^2, 0] && LtQ[n, -1] && IntegerQ[2*n]
```

rule 5988
```
Int[(Cosh[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_.)*((e_.) + (f_.)*(x_))^(m_.
)*Sinh[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[(e + f*x)^m*((a + b*Cosh[c +
d*x])^(n + 1)/(b*d*(n + 1))), x] - Simp[f*(m/(b*d*(n + 1)))   Int[(e + f*x)
^(m - 1)*(a + b*Cosh[c + d*x])^(n + 1), x], x] /; FreeQ[{a, b, c, d, e, f,
n}, x] && IGtQ[m, 0] && NeQ[n, -1]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(73) = 146$.

Time = 5.08 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.66

method	result	si
risch	$-\frac{2a^2x\,\mathrm{e}^{2x}-ab\,\mathrm{e}^{3x}-2b^2x\,\mathrm{e}^{2x}-2a^2\mathrm{e}^{2x}-b^2\mathrm{e}^{2x}-3b\,\mathrm{e}^{x}a-b^2}{b(\mathrm{e}^{2x}b+2a\,\mathrm{e}^{x}+b)^2(a^2-b^2)}+\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}-a^2+b^2}{b\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}-\frac{a\ln\left(\mathrm{e}^{x}+\frac{a\sqrt{a^2-b^2}+a^2-b^2}{b\sqrt{a^2-b^2}}\right)}{2\sqrt{a^2-b^2}\,(a+b)(a-b)b}$	23

input
```
int(x*sinh(x)/(a+b*cosh(x))^3,x,method=_RETURNVERBOSE)
```

output
```
-1/b*(2*a^2*x*exp(x)^2-a*b*exp(x)^3-2*b^2*x*exp(x)^2-2*a^2*exp(x)^2-b^2*ex
p(x)^2-3*b*exp(x)*a-b^2)/(exp(x)^2*b+2*a*exp(x)+b)^2/(a^2-b^2)+1/2/(a^2-b^
2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)-a^2+b^2)/b/(a^2-b^2)
^(1/2))-1/2/(a^2-b^2)^(1/2)*a/(a+b)/(a-b)/b*ln(exp(x)+(a*(a^2-b^2)^(1/2)+a
^2-b^2)/b/(a^2-b^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(73) = 146$.

Time = 0.10 (sec) , antiderivative size = 1692, normalized size of antiderivative = 19.45

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \text{Too large to display}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="fricas")
```

output
```
[1/2*(2*a^2*b^2 - 2*b^4 + 2*(a^3*b - a*b^3)*cosh(x)^3 + 2*(a^3*b - a*b^3)*
sinh(x)^3 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x
)^2 + 2*(2*a^4 - a^2*b^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x + 3*(a^3*b -
a*b^3)*cosh(x))*sinh(x)^2 - (a*b^2*cosh(x)^4 + a*b^2*sinh(x)^4 + 4*a^2*b*c
osh(x)^3 + 4*a^2*b*cosh(x) + 4*(a*b^2*cosh(x) + a^2*b)*sinh(x)^3 + a*b^2 +
 2*(2*a^3 + a*b^2)*cosh(x)^2 + 2*(3*a*b^2*cosh(x)^2 + 6*a^2*b*cosh(x) + 2*
a^3 + a*b^2)*sinh(x)^2 + 4*(a*b^2*cosh(x)^3 + 3*a^2*b*cosh(x)^2 + a^2*b +
(2*a^3 + a*b^2)*cosh(x))*sinh(x))*sqrt(a^2 - b^2)*log((b^2*cosh(x)^2 + b^2
*sinh(x)^2 + 2*a*b*cosh(x) + 2*a^2 - b^2 + 2*(b^2*cosh(x) + a*b)*sinh(x) +
 2*sqrt(a^2 - b^2)*(b*cosh(x) + b*sinh(x) + a))/(b*cosh(x)^2 + b*sinh(x)^2
 + 2*a*cosh(x) + 2*(b*cosh(x) + a)*sinh(x) + b)) + 6*(a^3*b - a*b^3)*cosh(
x) + 2*(3*a^3*b - 3*a*b^3 + 3*(a^3*b - a*b^3)*cosh(x)^2 + 2*(2*a^4 - a^2*b
^2 - b^4 - 2*(a^4 - 2*a^2*b^2 + b^4)*x)*cosh(x))*sinh(x))/(a^4*b^3 - 2*a^2
*b^5 + b^7 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^4 + (a^4*b^3 - 2*a^2*b^5
+ b^7)*sinh(x)^4 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x)^3 + 4*(a^5*b^2
- 2*a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x))*sinh(x)^3 + 2*(
2*a^6*b - 3*a^4*b^3 + b^7)*cosh(x)^2 + 2*(2*a^6*b - 3*a^4*b^3 + b^7 + 3*(a
^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^2 + 6*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh
(x))*sinh(x)^2 + 4*(a^5*b^2 - 2*a^3*b^4 + a*b^6)*cosh(x) + 4*(a^5*b^2 - 2*
a^3*b^4 + a*b^6 + (a^4*b^3 - 2*a^2*b^5 + b^7)*cosh(x)^3 + 3*(a^5*b^2 - ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{x \sinh(x)}{(a + b\cosh(x))^3}\, dx = \text{Timed out}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [F]

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \int \frac{x\sinh(x)}{(b\cosh(x)+a)^3}\,dx$$

input
```
integrate(x*sinh(x)/(a+b*cosh(x))^3,x, algorithm="giac")
```

output
```
integrate(x*sinh(x)/(b*cosh(x) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx$$

input
```
int((x*sinh(x))/(a + b*cosh(x))^3,x)
```

output
```
int((x*sinh(x))/(a + b*cosh(x))^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 924, normalized size of antiderivative = 10.62

$$\int \frac{x\sinh(x)}{(a+b\cosh(x))^3}\,dx = \text{Too large to display}$$

input
```
int(x*sinh(x)/(a+b*cosh(x))^3,x)
```

output
```
( - 2*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2)
)*cosh(x)**2*a*b**3 - 4*e**x*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt(
- a**2 + b**2))*cosh(x)**2*a**2*b**2 - 2*sqrt( - a**2 + b**2)*atan((e**x*b
 + a)/sqrt( - a**2 + b**2))*cosh(x)**2*a*b**3 - 4*e**(2*x)*sqrt( - a**2 +
b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*cosh(x)*a**2*b**2 - 8*e**x*s
qrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*cosh(x)*a**3*b
 - 4*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2 + b**2))*cosh(x)*
a**2*b**2 - 2*e**(2*x)*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2
 + b**2))*a**3*b - 4*e**x*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a
**2 + b**2))*a**4 - 2*sqrt( - a**2 + b**2)*atan((e**x*b + a)/sqrt( - a**2
+ b**2))*a**3*b - e**(2*x)*cosh(x)**2*a**2*b**3 + e**(2*x)*cosh(x)**2*b**5
 + cosh(x)**2*a**2*b**3 - cosh(x)**2*b**5 - 2*e**(2*x)*cosh(x)*a**3*b**2 +
 2*e**(2*x)*cosh(x)*a*b**4 + 2*cosh(x)*a**3*b**2 - 2*cosh(x)*a*b**4 - e**(
2*x)*a**4*b*x - e**(2*x)*a**4*b + 2*e**(2*x)*a**2*b**3*x + e**(2*x)*a**2*b
**3 - e**(2*x)*b**5*x - 2*e**x*a**5*x + 4*e**x*a**3*b**2*x - 2*e**x*a*b**4
*x - a**4*b*x + a**4*b + 2*a**2*b**3*x - a**2*b**3 - b**5*x)/(2*b*(e**(2*x
)*cosh(x)**2*a**4*b**3 - 2*e**(2*x)*cosh(x)**2*a**2*b**5 + e**(2*x)*cosh(x
)**2*b**7 + 2*e**x*cosh(x)**2*a**5*b**2 - 4*e**x*cosh(x)**2*a**3*b**4 + 2*
e**x*cosh(x)**2*a*b**6 + cosh(x)**2*a**4*b**3 - 2*cosh(x)**2*a**2*b**5 + c
osh(x)**2*b**7 + 2*e**(2*x)*cosh(x)*a**5*b**2 - 4*e**(2*x)*cosh(x)*a**3...
```

3.220 $\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{9}{4}\text{Chi}(bx)\sinh(a)+\frac{1}{4}\text{Chi}(3bx)\sinh(3a) + \frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

output
```
9/4*Chi(b*x)*sinh(a)+1/4*Chi(3*b*x)*sinh(3*a)+9/4*cosh(a)*Shi(b*x)+1/4*cos
h(3*a)*Shi(3*b*x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.87

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{4}(9\text{Chi}(bx)\sinh(a)+\text{Chi}(3bx)\sinh(3a) + 9\cosh(a)\text{Shi}(bx)+\cosh(3a)\text{Shi}(3bx))$$

input
```
Integrate[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

output
```
(9*CoshIntegral[b*x]*Sinh[a] + CoshIntegral[3*b*x]*Sinh[3*a] + 9*Cosh[a]*S
inhIntegral[b*x] + Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(a+bx)\left(\cosh^2(a+bx)+2\right)}{x}\,dx$$
$$\Big\downarrow \text{7293}$$
$$\int \left(\frac{2\sinh(a+bx)}{x}+\frac{\sinh(a+bx)\cosh^2(a+bx)}{x}\right)dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{9}{4}\sinh(a)\text{Chi}(bx)+\frac{1}{4}\sinh(3a)\text{Chi}(3bx)+\frac{9}{4}\cosh(a)\text{Shi}(bx)+\frac{1}{4}\cosh(3a)\text{Shi}(3bx)$$

input
```
Int[((2 + Cosh[a + b*x]^2)*Sinh[a + b*x])/x,x]
```

output
```
(9*CoshIntegral[b*x]*Sinh[a])/4 + (CoshIntegral[3*b*x]*Sinh[3*a])/4 + (9*C
osh[a]*SinhIntegral[b*x])/4 + (Cosh[3*a]*SinhIntegral[3*b*x])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 2.99 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

method	result	size
risch	$\frac{e^{-3a}\,\text{expIntegral}_1(3bx)}{8}+\frac{9\,e^{-a}\,\text{expIntegral}_1(bx)}{8}-\frac{9\,e^{a}\,\text{expIntegral}_1(-bx)}{8}-\frac{e^{3a}\,\text{expIntegral}_1(-3bx)}{8}$	47

input
```
int((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(-3*a)*Ei(1,3*b*x)+9/8*exp(-a)*Ei(1,b*x)-9/8*exp(a)*Ei(1,-b*x)-1/8*
exp(3*a)*Ei(1,-3*b*x)
```

Fricas [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.43

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{8}\left(\text{Ei}(3\,bx)-\text{Ei}(-3\,bx)\right)\cosh(3\,a) + \frac{9}{8}\left(\text{Ei}(bx)-\text{Ei}(-bx)\right)\cosh(a) + \frac{1}{8}\left(\text{Ei}(3\,bx)+\text{Ei}(-3\,bx)\right)\sinh(3\,a) + \frac{9}{8}\left(\text{Ei}(bx)+\text{Ei}(-bx)\right)\sinh(a)$$

input
```
integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="fricas")
```

output
```
1/8*(Ei(3*b*x) - Ei(-3*b*x))*cosh(3*a) + 9/8*(Ei(b*x) - Ei(-b*x))*cosh(a)
+ 1/8*(Ei(3*b*x) + Ei(-3*b*x))*sinh(3*a) + 9/8*(Ei(b*x) + Ei(-b*x))*sinh(a
)
```

Sympy [F]

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \int \frac{(\cosh^2(a+bx)+2)\sinh(a+bx)}{x}\,dx$$

input
```
integrate((2+cosh(b*x+a)**2)*sinh(b*x+a)/x,x)
```

output
```
Integral((cosh(a + b*x)**2 + 2)*sinh(a + b*x)/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{8}\,\mathrm{Ei}(3\,bx)\,e^{(3\,a)} - \frac{9}{8}\,\mathrm{Ei}(-bx)\,e^{(-a)} - \frac{1}{8}\,\mathrm{Ei}(-3\,bx)\,e^{(-3\,a)} + \frac{9}{8}\,\mathrm{Ei}(bx)\,e^{a}$$

input
```
integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="maxima")
```

output
```
1/8*Ei(3*b*x)*e^(3*a) - 9/8*Ei(-b*x)*e^(-a) - 1/8*Ei(-3*b*x)*e^(-3*a) + 9/
8*Ei(b*x)*e^a
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \frac{1}{8}\left(\mathrm{Ei}(3\,bx)\,e^{(6\,a)} + 9\,\mathrm{Ei}(bx)\,e^{(4\,a)} - 9\,\mathrm{Ei}(-bx)\,e^{(2\,a)} - \mathrm{Ei}(-3\,bx)\right)e^{(-3\,a)}$$

input
```
integrate((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x, algorithm="giac")
```

output
```
1/8*(Ei(3*b*x)*e^(6*a) + 9*Ei(b*x)*e^(4*a) - 9*Ei(-b*x)*e^(2*a) - Ei(-3*b*
x))*e^(-3*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = \int \frac{\sinh(a+b\,x)\,\left(\cosh(a+b\,x)^2+2\right)}{x}\,dx$$

input
```
int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x,x)
```

output
```
int((sinh(a + b*x)*(cosh(a + b*x)^2 + 2))/x, x)
```

Reduce [F]

$$\int \frac{(2+\cosh^2(a+bx))\sinh(a+bx)}{x}\,dx = 2\left(\int \frac{\sinh(bx+a)}{x}dx\right) + \int \frac{\cosh(bx+a)^2\sinh(bx+a)}{x}dx$$

input
```
int((2+cosh(b*x+a)^2)*sinh(b*x+a)/x,x)
```

output
```
2*int(sinh(a + b*x)/x,x) + int((cosh(a + b*x)**2*sinh(a + b*x))/x,x)
```

3.221 $\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)},x\right)$$

output
```
Defer(Int)(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 2.67 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
Integrate[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)} \, dx$$

$$\Big\downarrow \text{ 6112}$$

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}dx$$

input
```
Int[(x^m*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sinh(dx+c)}{a+b\cosh(dx+c)}dx$$

input
```
int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(dx+c)}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.76 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
integrate(x**m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**m*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.98 (sec) , antiderivative size = 202, normalized size of antiderivative = 9.18

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(dx+c)}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
x*e^(2*d*x + m*log(x) + 2*c)/(b*(m + 1)*e^(2*d*x + 2*c) + 2*a*(m + 1)*e^(d
*x + c) + b*(m + 1)) - 1/2*integrate(2*(2*a*d*x*e^(3*d*x + 3*c) + 2*a*(m +
 1)*e^(d*x + c) + b*(m + 1) + (2*b*d*x*e^(2*c) + b*(m + 1)*e^(2*c))*e^(2*d
*x))*x^m/(b^2*(m + 1)*e^(4*d*x + 4*c) + 4*a*b*(m + 1)*e^(3*d*x + 3*c) + 4*
a*b*(m + 1)*e^(d*x + c) + b^2*(m + 1) + 2*(2*a^2*(m + 1)*e^(2*c) + b^2*(m
+ 1)*e^(2*c))*e^(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.96 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^m*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 173, normalized size of antiderivative = 7.86

$$\int \frac{x^m \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{-2e^c\left(\int \frac{x^m e^{dx}}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)am - 2e^c\left(\int \frac{x^m e^{dx}}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)a + x^m x - 2\left(\int \frac{x^m}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)bm}{b\,(m+1)}$$

input
```
int(x^m*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
( - 2*e**c*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b)
,x)*a*m - 2*e**c*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*
a + b),x)*a + x**m*x - 2*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a +
 b),x)*b*m - 2*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)*b)/
(b*(m + 1))
```

3.222 $\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 327

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^4}{4b} + \frac{x^3 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^3 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{3x^2 \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2} - \frac{6x \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3} - \frac{6x \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3} + \frac{6 \operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^4} + \frac{6 \operatorname{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}$$

output
```
-1/4*x^4/b+x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^3*ln(1+b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))/b/d+3*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^
(1/2)))/b/d^2+3*x^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-6*x
*polylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-6*x*polylog(3,-b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^3+6*polylog(4,-b*exp(d*x+c)/(a-(a^2-b^2)^(1
/2)))/b/d^4+6*polylog(4,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^4
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00

$$\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^4}{4b}+\frac{x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd}+\frac{x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd}+\frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{3x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}-\frac{6x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3}-\frac{6x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}+\frac{6\,\text{PolyLog}\left(4,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^4}+\frac{6\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^4}$$

input
```
Integrate[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (3*x^2*PolyLo
g[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (3*x^2*PolyLog[2
, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (6*x*PolyLog[3, -((
b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^3) - (6*x*PolyLog[3, -((b*E^(
c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^3) + (6*PolyLog[4, (b*E^(c + d*x))
/(-a + Sqrt[a^2 - b^2])])/(b*d^4) + (6*PolyLog[4, -((b*E^(c + d*x))/(a + S
qrt[a^2 - b^2]))])/(b*d^4)
```

Rubi [A] (verified)

Time = 1.41 (sec) , antiderivative size = 337, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {6096, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6096}$$

$$\int \frac{e^{c+dx}x^3}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x^3}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^4}{4b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$$\Big\downarrow \text{ 3011}$$

$$-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$$\Big\downarrow \text{ 7163}$$

$$-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$$\Big\downarrow \text{ 2720}$$

$$-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

$\Big\downarrow$ 7143

$$-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}$$

input
```
Int[(x^3*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) - (3*(-((x^2*Po
lyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + (2*((x*PolyLog[3,
 -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d - PolyLog[4, -((b*E^(c + d*x
))/(a - Sqrt[a^2 - b^2]))]/d^2))/d))/(b*d) - (3*(-((x^2*PolyLog[2, -((b*E^
(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x
))/(a + Sqrt[a^2 - b^2]))])/d - PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2
 - b^2]))]/d^2))/d))/(b*d)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3 \sinh (dx + c)}{a + b \cosh (dx + c)} dx$$

input
```
int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(299) = 598$.

Time = 0.11 (sec) , antiderivative size = 624, normalized size of antiderivative = 1.91

$$\int \frac{x^3 \sinh(c + dx)}{a + b \cosh(c + dx)} dx = \text{Too large to display}$$

input
```
integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(d^4*x^4 - 12*d^2*x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*
cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 12*d^
2*x^2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 4*c^3*log(2*b*cosh(d*x + c
) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*c^3*log(2*b*c
osh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 24*d
*x*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*d*x*polylog(3, -(a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2))/b) - 4*(d^3*x^3 + c^3)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 4*(d^3*
x^3 + c^3)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*s
inh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 24*polylog(4, -(a*cosh(d*x +
 c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2))/b) - 24*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b))/(b*d^4)
```

Sympy [F]

$$\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3\sinh\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

input
```
integrate(x**3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**3*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3\sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*x^4/b - 1/2*integrate(4*(a*x^3*e^(d*x + c) + b*x^3)/(b^2*e^(2*d*x + 2*
c) + 2*a*b*e^(d*x + c) + b^2), x)
```

Giac [F]

$$\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3\sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^3*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3\sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^3*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = e^{2c}\left(\int \frac{e^{2dx}x^3}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right) - \left(\int \frac{x^3}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)$$

input
```
int(x^3*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
e**(2*c)*int((e**(2*d*x)*x**3)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b)
,x) - int(x**3/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)
```

3.223 $\int \frac{x^2 \sinh(c+dx)}{a+b \cosh(c+dx)} \, dx$

Optimal result

Integrand size = 22, antiderivative size = 245

$$\int \frac{x^2 \sinh(c+dx)}{a+b \cosh(c+dx)} \, dx = -\frac{x^3}{3b} + \frac{x^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{2x \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{2x \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2} - \frac{2 \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^3} - \frac{2 \operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}$$

output
```
-1/3*x^3/b+x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x^2*ln(1+b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))/b/d+2*x*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1
/2)))/b/d^2+2*x*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2-2*polyl
og(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d^3-2*polylog(3,-b*exp(d*x+c)/(a
+(a^2-b^2)^(1/2)))/b/d^3
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.00

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^3}{3b} + \frac{x^2 \log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x^2 \log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{2x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2} - \frac{2\,\text{PolyLog}\left(3,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^3} - \frac{2\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^3}$$

input
```
Integrate[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + (2*x*PolyLog[
2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/(b*d^2) + (2*x*PolyLog[2, -(
(b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/(b*d^2) - (2*PolyLog[3, (b*E^(c +
 d*x))/(-a + Sqrt[a^2 - b^2])])/(b*d^3) - (2*PolyLog[3, -((b*E^(c + d*x))/
(a + Sqrt[a^2 - b^2]))])/(b*d^3)
```

Rubi [A] (verified)

Time = 1.00 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.02, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.227$, Rules used = {6096, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{6096}$$

$$\int \frac{e^{c+dx}x^2}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx + \int \frac{e^{c+dx}x^2}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx - \frac{x^3}{3b}$$

$$\Big\downarrow \text{2620}$$

$$-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

$$\Big\downarrow \text{3011}$$

$$-\frac{2\left(\frac{\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int \operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

$$\Big\downarrow \text{2720}$$

$$-\frac{2\left(\frac{\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int e^{-c-dx}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

$$\Big\downarrow \text{7143}$$

$$-\frac{2\left(\frac{\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\operatorname{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}$$

input
```
Int[(x^2*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) +
(x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) - (2*(-((x*Poly
Log[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(
c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/(b*d) - (2*(-((x*PolyLog[2, -((b*E
^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(c + d*x))/(a
+ Sqrt[a^2 - b^2]))]/d^2))/(b*d)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2 \sinh (dx + c)}{a + b \cosh (dx + c)} dx$$

input
```
int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 497 vs. $2(223) = 446$.

Time = 0.09 (sec) , antiderivative size = 497, normalized size of antiderivative = 2.03

$$\int \frac{x^2 \sinh(c + dx)}{a + b \cosh(c + dx)} dx =$$
$$- \frac{d^3x^3 - 6\, dx \mathrm{Li}_2\left(-\frac{a \cosh(dx+c)+a \sinh(dx+c)+(b \cosh(dx+c)+b \sinh(dx+c))\sqrt{\frac{a^2-b^2}{b^2}}+b}{b} + 1\right) - 6\, dx \mathrm{Li}_2\left(-\frac{a \cosh(dx+c)+\cdots}{}\right.}{}$$

input
```
integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/3*(d^3*x^3 - 6*d*x*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(
d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 6*d*x*dilo
g(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c)
)*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 3*c^2*log(2*b*cosh(d*x + c) + 2*b*si
nh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*c^2*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 3*(d^2*x^2 - c
^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 3*(d^2*x^2 - c^2)*log((a*cosh(d*x +
 c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2) + b)/b) + 6*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*co
sh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 6*polylog(3, -(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2))/b))/(b*d^3)
```

Sympy [F]

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

input
```
integrate(x**2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**2*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/3*x^3/b - 1/2*integrate(4*(a*x^2*e^(d*x + c) + b*x^2)/(b^2*e^(2*d*x + 2*
c) + 2*a*b*e^(d*x + c) + b^2), x)
```

Giac [F]

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh{(dx+c)}}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(c+d\,x)}{a+b\cosh{(c+d\,x)}}\,dx$$

input
```
int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^2*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2 \sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = e^{2c}\left(\int \frac{e^{2dx}x^2}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right) - \left(\int \frac{x^2}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)$$

input
```
int(x^2*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
e**(2*c)*int((e**(2*d*x)*x**2)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b)
,x) - int(x**2/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)
```

3.224 $\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 20, antiderivative size = 161

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^2}{2b} + \frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}$$

output
```
-1/2*x^2/b+x*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/d+x*ln(1+b*exp(d*x+c
)/(a+(a^2-b^2)^(1/2)))/b/d+polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b/
d^2+polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b/d^2
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.99

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{x^2}{2b} + \frac{x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd} + \frac{x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd} + \frac{\text{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)}{bd^2} + \frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}$$

input
```
Integrate[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, (b*E^(
c + d*x))/(-a + Sqrt[a^2 - b^2])]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))/(
a + Sqrt[a^2 - b^2]))]/(b*d^2)
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {6096, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x \sinh(c+dx)}{a+b\cosh(c+dx)} \, dx$$

$$\Big\downarrow \text{ 6096}$$

$$\int \frac{e^{c+dx}x}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^2}{2b}$$

$$\Big\downarrow \text{ 2620}$$

$$-\frac{\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

$$\Big\downarrow \text{ 2715}$$

$$-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

$$\Big\downarrow \text{ 2838}$$

$$\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}$$

input
```
Int[(x*Sinh[c + d*x])/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])])/(b*d) + (x
*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E
^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((b*E^(c + d*x))
/(a + Sqrt[a^2 - b^2]))]/(b*d^2)
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(147) = 294$.

Time = 0.74 (sec) , antiderivative size = 368, normalized size of antiderivative = 2.29

method	result
risch	$-\frac{x^2}{2b}+\frac{\ln\left(\frac{-e^{dx+c}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)x}{db}+\frac{\ln\left(\frac{-e^{dx+c}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)c}{d^2b}+\frac{\ln\left(\frac{e^{dx+c}b+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)x}{db}+\frac{\ln\left(\frac{e^{dx+c}b+\sqrt{a^2-b^2}+a}{a+\sqrt{a^2-b^2}}\right)}{d^2b}$

input
```
int(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*x^2/b+1/d/b*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2))
)*x+1/d^2/b*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c+1
/d/b*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x+1/d^2/b*ln
((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d^2/b*dilog((-e
xp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/d^2/b*dilog((exp(d*
x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/d/b*c*x-1/d^2/b*c^2-1/d^2
/b*c*ln(exp(2*d*x+2*c)*b+2*exp(d*x+c)*a+b)+2/d^2/b*c*ln(exp(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(145) = 290$.

Time = 0.09 (sec) , antiderivative size = 354, normalized size of antiderivative = 2.20

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx =$$
$$-\frac{d^2x^2+2c\log\left(2b\cosh(dx+c)+2b\sinh(dx+c)+2b\sqrt{\frac{a^2-b^2}{b^2}}+2a\right)+2c\log\left(2b\cosh(dx+c)+\right.}{}$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(d^2*x^2 + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((
a^2 - b^2)/b^2) + 2*a) + 2*c*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2
*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sin
h(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b
)/b) - 2*(d*x + c)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) - 2*dilog(-(a*cosh(d*x
 + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 -
b^2)/b^2) + b)/b + 1) - 2*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*c
osh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1))/(b*d^2)
```

Sympy [F]

$$\int \frac{x \sinh(c + dx)}{a + b \cosh(c + dx)}\, dx = \int \frac{x \sinh(c + dx)}{a + b \cosh(c + dx)}\, dx$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x*sinh(c + d*x)/(a + b*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x \sinh(c + dx)}{a + b \cosh(c + dx)}\, dx = \int \frac{x \sinh(dx + c)}{b \cosh(dx + c) + a}\, dx$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/2*x^2/b - 1/2*integrate(4*(a*x*e^(d*x + c) + b*x)/(b^2*e^(2*d*x + 2*c) +
 2*a*b*e^(d*x + c) + b^2), x)
```

Giac [F]

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\sinh\left(dx+c\right)}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*sinh(d*x + c)/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\sinh(c+d\,x)}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)),x)
```

output
```
int((x*sinh(c + d*x))/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx=e^{2c}\left(\int \frac{e^{2dx}x}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)-\left(\int \frac{x}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)$$

input
```
int(x*sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
e**(2*c)*int((e**(2*d*x)*x)/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)
 - int(x/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)
```

3.225 $\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 18

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log(a+b\cosh(c+dx))}{bd}$$

output
```
ln(a+b*cosh(d*x+c))/b/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log(a+b\cosh(c+dx))}{bd}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

output
```
Log[a + b*Cosh[c + d*x]]/(b*d)
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.211, Rules used = {3042, 26, 3147, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\cos\left(ic+idx-\frac{\pi}{2}\right)}{a-b\sin\left(ic+idx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\cos\left(\frac{1}{2}(2ic-\pi)+idx\right)}{a-b\sin\left(\frac{1}{2}(2ic-\pi)+idx\right)}dx$$

$$\Big\downarrow \; 3147$$

$$\frac{\int \frac{1}{a+b\cosh(c+dx)}d(b\cosh(c+dx))}{bd}$$

$$\Big\downarrow \; 16$$

$$\frac{\log(a+b\cosh(c+dx))}{bd}$$

input
```
Int[Sinh[c + d*x]/(a + b*Cosh[c + d*x]),x]
```

output
```
Log[a + b*Cosh[c + d*x]]/(b*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 0.35 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\ln(a+b\cosh(dx+c))}{bd}$	19
default	$\frac{\ln(a+b\cosh(dx+c))}{bd}$	19
risch	$-\frac{x}{b}-\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{b}+1\right)}{bd}$	48

input
```
int(sinh(d*x+c)/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
ln(a+b*cosh(d*x+c))/b/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 44 vs. $2(18) = 36$.

Time = 0.10 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.44

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{dx - \log\left(\frac{2\,(b\cosh(dx+c)+a)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-(d*x - log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))))/(b*d
)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 41 vs. $2(14) = 28$.

Time = 0.57 (sec) , antiderivative size = 41, normalized size of antiderivative = 2.28

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \begin{cases} \frac{x\sinh(c)}{a} & \text{for } b=0 \wedge d=0 \\ \frac{\cosh(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\sinh(c)}{a+b\cosh(c)} & \text{for } d=0 \\ \frac{\log\left(\frac{a}{b}+\cosh(c+dx)\right)}{bd} & \text{otherwise} \end{cases}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
Piecewise((x*sinh(c)/a, Eq(b, 0) & Eq(d, 0)), (cosh(c + d*x)/(a*d), Eq(b,
0)), (x*sinh(c)/(a + b*cosh(c)), Eq(d, 0)), (log(a/b + cosh(c + d*x))/(b*d
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log\left(b\cosh\left(dx+c\right)+a\right)}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
log(b*cosh(d*x + c) + a)/(b*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log\left(\left|b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,a\right|\right)}{bd}$$

input
```
integrate(sinh(d*x+c)/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/(b*d)
```

Mupad [B] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\ln\left(a+b\cosh(c+d\,x)\right)}{b\,d}$$

input
```
int(sinh(c + d*x)/(a + b*cosh(c + d*x)),x)
```

output
```
log(a + b*cosh(c + d*x))/(b*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\sinh(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\log(\cosh(dx+c)\,b+a)}{bd}$$

input
```
int(sinh(d*x+c)/(a+b*cosh(d*x+c)),x)
```

output
```
log(cosh(c + d*x)*b + a)/(b*d)
```

3.226 $\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 22

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \text{Int}\left(\frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.02 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))} \, dx$$
$$\Big\downarrow \text{6112}$$
$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(dx+c\right)}{x\left(a+b\cosh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 4.69 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.86

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(c+dx\right)}{x\left(a+b\cosh\left(c+dx\right)\right)}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 56, normalized size of antiderivative = 2.55

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
log(x)/b - 1/2*integrate(4*(a*e^(d*x + c) + b)/(b^2*x*e^(2*d*x + 2*c) + 2*
a*b*x*e^(d*x + c) + b^2*x), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)}{(b\cosh\left(dx+c\right)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)/((b*cosh(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.09

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)/(x*(a + b*cosh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 86, normalized size of antiderivative = 3.91

$$\int \frac{\sinh(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \frac{-2e^c\left(\int \frac{e^{dx}}{e^{2dx+2c}bx+2e^{dx+c}ax+bx}dx\right)a-2\left(\int \frac{1}{e^{2dx+2c}bx+2e^{dx+c}ax+bx}dx\right)b+\log(x)}{b}$$

input
```
int(sinh(d*x+c)/x/(a+b*cosh(d*x+c)),x)
```

output
```
( - 2*e**c*int(e**(d*x)/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),
x)*a - 2*int(1/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*b + lo
g(x))/b
```

3.227 $\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)},x\right)$$

output
```
Defer(Int)(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 6.93 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
Integrate[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$\Big\downarrow \; 6112$$
$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}dx$$

input
```
Int[(x^m*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sinh\left(dx+c\right)^2}{a+b\cosh\left(dx+c\right)}dx$$

input
```
int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 1.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^2\left(c+dx\right)}{a+b\cosh\left(c+dx\right)}\,dx$$

input
```
integrate(x**m*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**m*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 1.95 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+d\,x)^2}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^m*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 1291, normalized size of antiderivative = 53.79

$$\int \frac{x^m \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(x^m*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
(x**m*e**(2*c + 2*d*x)*b**2*m + x**m*e**(2*c + 2*d*x)*b**2 - e**(4*c + d*x
)*int((x**m*e**(3*d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),
x)*b**3*m**2 - e**(4*c + d*x)*int((x**m*e**(3*d*x))/(e**(2*c + 2*d*x)*b*x
+ 2*e**(c + d*x)*a*x + b*x),x)*b**3*m + 8*e**(2*c + d*x)*int((x**m*e**(d*x
))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*a**2*b*m**2 + 8*e*
*(2*c + d*x)*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*
x + b*x),x)*a**2*b*m - 4*e**(2*c + d*x)*int((x**m*e**(d*x))/(e**(2*c + 2*d
*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*b**3*m**2 - 4*e**(2*c + d*x)*int((x
**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*b**3*m
- 2*x**m*e**(c + d*x)*a*b*d*x - 2*x**m*e**(c + d*x)*a*b*m - 2*x**m*e**(c +
 d*x)*a*b + 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)
*a*x + b*x),x)*a**3*m**2 + 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b*x +
 2*e**(c + d*x)*a*x + b*x),x)*a**3*m - 4*e**(c + d*x)*int(x**m/(e**(2*c +
2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*a*b**2*m**2 - 4*e**(c + d*x)*int
(x**m/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*a*b**2*m - 8*e*
*(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)*a**3*d*
m - 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)
*a**3*d + 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a +
 b),x)*a*b**2*d*m + 8*e**(c + d*x)*int(x**m/(e**(2*c + 2*d*x)*b + 2*e**(c
+ d*x)*a + b),x)*a*b**2*d + 4*e**(d*x)*int(x**m/(e**(2*c + 3*d*x)*b*x +...
```

3.228 $\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 495

$$\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{ax^4}{4b^2}-\frac{6\cosh(c+dx)}{bd^4}-\frac{3x^2\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}-\frac{\sqrt{a^2-b^2}x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}+\frac{3\sqrt{a^2-b^2}x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{3\sqrt{a^2-b^2}x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}-\frac{6\sqrt{a^2-b^2}x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2-b^2}x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}+\frac{6\sqrt{a^2-b^2}\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^4}-\frac{6\sqrt{a^2-b^2}\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^4}+\frac{6x\sinh(c+dx)}{bd^3}+\frac{x^3\sinh(c+dx)}{bd}$$

output

```
-1/4*a*x^4/b^2-6*cosh(d*x+c)/b/d^4-3*x^2*cosh(d*x+c)/b/d^2+(a^2-b^2)^(1/2)
*x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^2/d-(a^2-b^2)^(1/2)*x^3*ln(1
+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^2/d+3*(a^2-b^2)^(1/2)*x^2*polylog(2,-
b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^2/d^2-3*(a^2-b^2)^(1/2)*x^2*polylog(2,
-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^2/d^2-6*(a^2-b^2)^(1/2)*x*polylog(3,-
b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^2/d^3+6*(a^2-b^2)^(1/2)*x*polylog(3,-b
*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^2/d^3+6*(a^2-b^2)^(1/2)*polylog(4,-b*ex
p(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^2/d^4-6*(a^2-b^2)^(1/2)*polylog(4,-b*exp(d
*x+c)/(a+(a^2-b^2)^(1/2)))/b^2/d^4+6*x*sinh(d*x+c)/b/d^3+x^3*sinh(d*x+c)/b
/d
```

Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 386, normalized size of antiderivative = 0.78

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{-ad^4x^4 + 4\sqrt{a^2-b^2}\left(d^3x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right) - d^3x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right) + 3d^2x^2\operatorname{PolyLog}\left(2, \frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right.\right.}{}$$

input
```
Integrate[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
(-(a*d^4*x^4) + 4*Sqrt[a^2 - b^2]*(d^3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 - b^2])] - d^3*x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] +
 3*d^2*x^2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 3*d^2*x^2*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 6*d*x*PolyLog[3, (b
*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] + 6*d*x*PolyLog[3, -((b*E^(c + d*x))
/(a + Sqrt[a^2 - b^2]))] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b
^2])] - 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 4*b*Cosh
[d*x]*(-3*(2 + d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c]) + 4*b*(d*x*(6
 + d^2*x^2)*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/(4*b^2*d^4)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.35 (sec) , antiderivative size = 463, normalized size of antiderivative = 0.94, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.917$, Rules used = {6100, 15, 3042, 3777, 26, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 3801, 2694, 27, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$\Big\downarrow \; 6100$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x^3dx}{b^2}+\frac{\int x^3\cosh(c+dx)dx}{b}$$

$$\Big\downarrow \text{15}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x^3\cosh(c+dx)dx}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int x^3\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}-\frac{3i\int -ix^2\sinh(c+dx)dx}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}-\frac{3\int x^2\sinh(c+dx)dx}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}-\frac{3\int -ix^2\sin(ic+idx)dx}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\int x^2\sin(ic+idx)dx}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\cosh(c+dx)dx}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3777}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{i\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{26}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}+\frac{i\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{d}}{b}-\frac{ax^4}{4b^2}$$

$$\Big\downarrow \text{3118}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3801}$$

$$\frac{2\left(a^2-b^2\right)\int \frac{e^{c+dx}x^3}{2e^{c+dx}a+be^{2(c+dx)}+b}dx}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{2694}$$

$$\frac{2(a^2-b^2)\left(\frac{b\int \frac{e^{c+dx}x^3}{2\left(a+be^{c+dx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^3}{2\left(a+be^{c+dx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{2(a^2-b^2)\left(\frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^3}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\cdots\right.}{\cdots}\right.}{b^2}$$

$$\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\downarrow$ 7163

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\cdots\right.}{b^2}$$

$$\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\downarrow$ 2720

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c\cdots}}{a-\sqrt{\cdots}}\right)}{d}\cdots\right)}{bd}\cdots\right)}{2\sqrt{a^2-b^2}}\cdots\right)}{\cdots}$$

$$\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

$\downarrow$ 7143

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\cdots\right)}{b^2}$$

$$\frac{ax^4}{4b^2}+\frac{\frac{x^3\sinh(c+dx)}{d}+\frac{3i\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{d}}{b}$$

input
```
Int[(x^3*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/4*(a*x^4)/b^2 + (2*(a^2 - b^2)*((b*((x^3*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 - b^2])])/(b*d) - (3*(-((x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2]))])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 -
b^2]))])/d - PolyLog[4, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/d)
)/(b*d)))/(2*Sqrt[a^2 - b^2]) - (b*((x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt
[a^2 - b^2])])/(b*d) - (3*(-((x^2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))])/d) + (2*((x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2
]))])/d - PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]/d^2))/d))/(
b*d)))/(2*Sqrt[a^2 - b^2])))/b^2 + ((x^3*Sinh[c + d*x])/d + ((3*I)*((I*x^2
*Cosh[c + d*x])/d - ((2*I)*(-(Cosh[c + d*x]/d^2) + (x*Sinh[c + d*x])/d))/d
))/d)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3 \sinh (dx + c)^2}{a + b \cosh (dx + c)} dx$$

input
```
int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1174 vs. $2(451) = 902$.

Time = 0.14 (sec) , antiderivative size = 1174, normalized size of antiderivative = 2.37

$$\int \frac{x^3 \sinh^2(c + dx)}{a + b \cosh(c + dx)} \, dx = \text{Too large to display}$$

input
```
integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/4*(a*d^4*x^4*cosh(d*x + c) + 2*b*d^3*x^3 + 6*b*d^2*x^2 + 12*b*d*x - 2*(
b*d^3*x^3 - 3*b*d^2*x^2 + 6*b*d*x - 6*b)*cosh(d*x + c)^2 - 2*(b*d^3*x^3 -
3*b*d^2*x^2 + 6*b*d*x - 6*b)*sinh(d*x + c)^2 - 12*(b*d^2*x^2*cosh(d*x + c)
 + b*d^2*x^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c)
+ a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b
^2) + b)/b + 1) + 12*(b*d^2*x^2*cosh(d*x + c) + b*d^2*x^2*sinh(d*x + c))*s
qrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d
*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 4*(b*c^3*co
sh(d*x + c) + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x
+ c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 4*(b*c^3*cos
h(d*x + c) + b*c^3*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b*cosh(d*x +
 c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 4*((b*d^3*x^3
 + b*c^3)*cosh(d*x + c) + (b*d^3*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 4*((b*d^3*x^3 + b*c^3)*cosh(d*
x + c) + (b*d^3*x^3 + b*c^3)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*c
osh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt(
(a^2 - b^2)/b^2) + b)/b) - 24*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^
2 - b^2)/b^2)*polylog(4, -(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x
 + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) + 24*(b*cosh(d*x + c...
```

Sympy [F]

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh^2{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x**3*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**3*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^3*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(c+d\,x)^2}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^3*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^3\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{2e^{2dx+2c}b^2d^3x^3 - 6e^{2dx+2c}b^2d^2x^2 + 12e^{2dx+2c}b^2dx - 12e^{2dx+2c}b^2 - 16e^{dx+c}\left(\int \frac{x^3}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)a^3d^4 +}{}$$

input
```
int(x^3*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
(2*e**(2*c + 2*d*x)*b**2*d**3*x**3 - 6*e**(2*c + 2*d*x)*b**2*d**2*x**2 + 1
2*e**(2*c + 2*d*x)*b**2*d*x - 12*e**(2*c + 2*d*x)*b**2 - 16*e**(c + d*x)*i
nt(x**3/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)*a**3*d**4 + 16*e**(
c + d*x)*int(x**3/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)*a*b**2*d*
*4 - e**(c + d*x)*a*b*d**4*x**4 - 8*e**(d*x)*int(x**3/(e**(2*c + 3*d*x)*b
+ 2*e**(c + 2*d*x)*a + e**(d*x)*b),x)*a**2*b*d**4 + 8*e**(d*x)*int(x**3/(e
**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x)*b),x)*b**3*d**4 - 8*a**2
*d**3*x**3 - 24*a**2*d**2*x**2 - 48*a**2*d*x - 48*a**2 + 6*b**2*d**3*x**3
+ 18*b**2*d**2*x**2 + 36*b**2*d*x + 36*b**2)/(4*e**(c + d*x)*b**3*d**4)
```

3.229 $\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 370

$$\begin{aligned}\int \frac{x^2\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx &= -\frac{ax^3}{3b^2}-\frac{2x\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}\\ &\quad-\frac{\sqrt{a^2-b^2}x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}\\ &\quad+\frac{2\sqrt{a^2-b^2}x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}\\ &\quad-\frac{2\sqrt{a^2-b^2}x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}\\ &\quad-\frac{2\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^3}\\ &\quad+\frac{2\sqrt{a^2-b^2}\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^3}\\ &\quad+\frac{2\sinh(c+dx)}{bd^3}+\frac{x^2\sinh(c+dx)}{bd}\end{aligned}$$

output
```
-1/3*a*x^3/b^2-2*x*cosh(d*x+c)/b/d^2+(a^2-b^2)^(1/2)*x^2*ln(1+b*exp(d*x+c)
/(a-(a^2-b^2)^(1/2)))/b^2/d-(a^2-b^2)^(1/2)*x^2*ln(1+b*exp(d*x+c)/(a+(a^2-
b^2)^(1/2)))/b^2/d+2*(a^2-b^2)^(1/2)*x*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2
)^(1/2)))/b^2/d^2-2*(a^2-b^2)^(1/2)*x*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)
^(1/2)))/b^2/d^2-2*(a^2-b^2)^(1/2)*polylog(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1
/2)))/b^2/d^3+2*(a^2-b^2)^(1/2)*polylog(3,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)
))/b^2/d^3+2*sinh(d*x+c)/b/d^3+x^2*sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.79

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{-ad^3x^3+3\sqrt{a^2-b^2}\left(d^2x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)-d^2x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)+2dx\,\mathrm{PolyLog}\left(2,\frac{be^{c+dx}}{-a+\sqrt{a^2-b^2}}\right)\right.}{}$$

input
```
Integrate[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
(-(a*d^3*x^3) + 3*Sqrt[a^2 - b^2]*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sq
rt[a^2 - b^2])] - d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] +
 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 2*d*x*PolyLog[
2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, (b*E^(c + d*x)
)/(-a + Sqrt[a^2 - b^2])] + 2*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 -
 b^2]))]) + 3*b*Cosh[d*x]*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c]) + 3*b*(
(2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c])*Sinh[d*x])/(3*b^2*d^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.97 (sec) , antiderivative size = 356, normalized size of antiderivative = 0.96, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.708$, Rules used = {6100, 15, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 3801, 2694, 27, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{ 6100}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x^2dx}{b^2}+\frac{\int x^2\cosh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x^2\cosh(c+dx)dx}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int x^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}-\frac{2i\int -ix\sinh(c+dx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}-\frac{2\int x\sinh(c+dx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}-\frac{2\int -ix\sin(ic+idx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\int x\sin(ic+idx)dx}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{ 3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\cosh(c+dx)dx}{d}\right)}{d}}{b}-\frac{ax^3}{3b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int \sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}}{b}-\frac{ax^3}{3b^2}$$

$\Big\downarrow$ 3117

$$\frac{\left(a^2-b^2\right)\int \frac{x^2}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 3801

$$\frac{2\left(a^2-b^2\right)\int \frac{e^{c+dx}x^2}{2e^{c+dx}a+be^{2(c+dx)}+b}dx}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 2694

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x^2}{2\left(a+be^{c+dx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^2}{2\left(a+be^{c+dx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 27

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x^2}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 2620

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\Big\downarrow$ 3011

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{2\left(\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b^2}$$

$$\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 2720

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\left(\frac{\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}\cdots}\right.}{bd}\right.}{\cdots}\right.}{b^2}$$

$$\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

$\downarrow$ 7143

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{2\left(\frac{\text{Poly}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\right.}{b^2}$$

$$\frac{ax^3}{3b^2}+\frac{\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}}{b}$$

input
```
Int[(x^2*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/3*(a*x^3)/b^2 + (2*(a^2 - b^2)*((b*((x^2*Log[1 + (b*E^(c + d*x))/(a - S
qrt[a^2 - b^2])])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[
a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d
^2))/(b*d)))/(2*Sqrt[a^2 - b^2]) - (b*((x^2*Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 - b^2])])/(b*d) - (2*(-((x*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[
a^2 - b^2]))])/d) + PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]/d
^2))/(b*d)))/(2*Sqrt[a^2 - b^2])))/b^2 + ((x^2*Sinh[c + d*x])/d + ((2*I)*(
(I*x*Cosh[c + d*x])/d - (I*Sinh[c + d*x])/d^2))/d)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] && !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] && !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2 \sinh (dx+c)^2}{a+b\cosh (dx+c)}dx$$

input
```
int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 937 vs. $2(336) = 672$.

Time = 0.11 (sec) , antiderivative size = 937, normalized size of antiderivative = 2.53

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/6*(2*a*d^3*x^3*cosh(d*x + c) + 3*b*d^2*x^2 + 6*b*d*x - 3*(b*d^2*x^2 - 2
*b*d*x + 2*b)*cosh(d*x + c)^2 - 3*(b*d^2*x^2 - 2*b*d*x + 2*b)*sinh(d*x + c
)^2 - 12*(b*d*x*cosh(d*x + c) + b*d*x*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)
*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x
 + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 12*(b*d*x*cosh(d*x + c) + b*d*x
*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x
 + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b +
 1) + 6*(b*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*
log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*
a) - 6*(b*c^2*cosh(d*x + c) + b*c^2*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*l
og(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a
) - 6*((b*d^2*x^2 - b*c^2)*cosh(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x +
c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh
(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 6*((b*d^2*x^2
 - b*c^2)*cosh(d*x + c) + (b*d^2*x^2 - b*c^2)*sinh(d*x + c))*sqrt((a^2 - b
^2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sin
h(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 12*(b*cosh(d*x + c) + b*sinh(d
*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a*cosh(d*x + c) + a*sinh(d*x +
 c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2))/b) - 12*(
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*polylog(3, -(a...
```

Sympy [F]

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
integrate(x**2*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**2*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh\left(dx+c\right)^2}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh(c+d\,x)^2}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^2*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2 \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{3e^{2dx+2c}b^2d^2x^2 - 6e^{2dx+2c}b^2dx + 6e^{2dx+2c}b^2 - 24e^{dx+c}\left(\int \frac{x^2}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)a^3d^3 + 24e^{dx+c}\left(\int \frac{x^2}{e^{2dx+2c}b+\cdots}\right.}{}$$

input
```
int(x^2*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
(3*e**(2*c + 2*d*x)*b**2*d**2*x**2 - 6*e**(2*c + 2*d*x)*b**2*d*x + 6*e**(2
*c + 2*d*x)*b**2 - 24*e**(c + d*x)*int(x**2/(e**(2*c + 2*d*x)*b + 2*e**(c
+ d*x)*a + b),x)*a**3*d**3 + 24*e**(c + d*x)*int(x**2/(e**(2*c + 2*d*x)*b
+ 2*e**(c + d*x)*a + b),x)*a*b**2*d**3 - 2*e**(c + d*x)*a*b*d**3*x**3 - 12
*e**(d*x)*int(x**2/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x)*b),
x)*a**2*b*d**3 + 12*e**(d*x)*int(x**2/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*
x)*a + e**(d*x)*b),x)*b**3*d**3 - 12*a**2*d**2*x**2 - 24*a**2*d*x - 24*a**
2 + 9*b**2*d**2*x**2 + 18*b**2*d*x + 18*b**2)/(6*e**(c + d*x)*b**3*d**3)
```

3.230 $\int \frac{x \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 244

$$\begin{aligned}\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = &-\frac{ax^2}{2b^2}-\frac{\cosh(c+dx)}{bd^2}+\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d}\\&-\frac{\sqrt{a^2-b^2}\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d}\\&+\frac{\sqrt{a^2-b^2}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^2d^2}\\&-\frac{\sqrt{a^2-b^2}\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^2d^2}+\frac{x\sinh(c+dx)}{bd}\end{aligned}$$

output
```
-1/2*a*x^2/b^2-cosh(d*x+c)/b/d^2+(a^2-b^2)^(1/2)*x*ln(1+b*exp(d*x+c)/(a-(a
^2-b^2)^(1/2)))/b^2/d-(a^2-b^2)^(1/2)*x*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/
2)))/b^2/d+(a^2-b^2)^(1/2)*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^
2/d^2-(a^2-b^2)^(1/2)*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^2/d^2
+x*sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.68 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.77

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{a(c-dx)(c+dx)-2b\cosh(c+dx)+2\sqrt{a^2-b^2}\left(dx\left(\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)-\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)\right)+\text{Po}\right.}{2b^2d^2}$$

input
```
Integrate[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
(a*(c - d*x)*(c + d*x) - 2*b*Cosh[c + d*x] + 2*Sqrt[a^2 - b^2]*(d*x*(Log[1
 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] - Log[1 + (b*E^(c + d*x))/(a + S
qrt[a^2 - b^2])]) + PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - P
olyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 2*b*d*x*Sinh[c + d*
x])/(2*b^2*d^2)
```

Rubi [A] (verified)

Time = 1.25 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.99, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.636$, Rules used = {6100, 15, 3042, 3777, 26, 3042, 26, 3118, 3801, 2694, 27, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6100}$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int xdx}{b^2}+\frac{\int x\cosh(c+dx)dx}{b}$$

$$\Big\downarrow \text{ 15}$$

$$\frac{(a^2-b^2)\int \frac{x}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x\cosh(c+dx)dx}{b}-\frac{ax^2}{2b^2}$$

$$\Big\downarrow \text{ 3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\int x\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{b}-\frac{ax^2}{2b^2}$$

$\Big\downarrow$ 3777

$$\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{i\int -i\sinh(c+dx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$\Big\downarrow$ 26

$$\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\int \sinh(c+dx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$\Big\downarrow$ 3042

$$\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\int -i\sin(ic+idx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$\Big\downarrow$ 26

$$\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}+\frac{\frac{x\sinh(c+dx)}{d}+\frac{i\int \sin(ic+idx)dx}{d}}{b}-\frac{ax^2}{2b^2}$$

$\Big\downarrow$ 3118

$$\frac{\left(a^2-b^2\right)\int \frac{x}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\Big\downarrow$ 3801

$$\frac{2\left(a^2-b^2\right)\int \frac{e^{c+dx}x}{2e^{c+dx}a+be^{2(c+dx)}+b}dx}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\Big\downarrow$ 2694

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x}{2\left(a+be^{c+dx}-\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x}{2\left(a+be^{c+dx}+\sqrt{a^2-b^2}\right)}dx}{\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\Big\downarrow$ 27

$$\frac{2\left(a^2-b^2\right)\left(\frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}-\frac{b\int \frac{e^{c+dx}x}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\Big\downarrow$ 2620

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{\int\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\downarrow$ 2715

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{c}}{bd^2}\right.}{2\sqrt{a^2-b^2}}}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

$\downarrow$ 2838

$$\frac{2(a^2-b^2)\left(\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}\right)}{2\sqrt{a^2-b^2}}-\frac{b\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}\right)}{2\sqrt{a^2-b^2}}\right)}{b^2}-\frac{ax^2}{2b^2}+\frac{\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}}{b}$$

input
```
Int[(x*Sinh[c + d*x]^2)/(a + b*Cosh[c + d*x]),x]
```

output
```
-1/2*(a*x^2)/b^2 + (2*(a^2 - b^2)*((b*((x*Log[1 + (b*E^(c + d*x))/(a - Sqr
t[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])
)]/(b*d^2)))/(2*Sqrt[a^2 - b^2]) - (b*((x*Log[1 + (b*E^(c + d*x))/(a + Sqr
t[a^2 - b^2])])/(b*d) + PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])
)]/(b*d^2)))/(2*Sqrt[a^2 - b^2])))/b^2 + (-(Cosh[c + d*x]/d^2) + (x*Sinh[c
 + d*x])/d)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2694
```
Int[((F_)^(u_)*((f_.) + (g_.)*(x_))^(m_.))/((a_.) + (b_.)*(F_)^(u_) + (c_.)
*(F_)^(v_)), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Simp[2*(c/q)   Int
[(f + g*x)^m*(F^u/(b - q + 2*c*F^u)), x], x] - Simp[2*(c/q)   Int[(f + g*x)
^m*(F^u/(b + q + 2*c*F^u)), x], x]] /; FreeQ[{F, a, b, c, f, g}, x] && EqQ[
v, 2*u] && LinearQ[u, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3801
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*sin[(e_.) + Pi*(k_.) + (Comple
x[0, fz_])*(f_.)*(x_)]), x_Symbol] :> Simp[2   Int[((c + d*x)^m*(E^((-I)*e
+ f*fz*x)/(b + (2*a*E^((-I)*e + f*fz*x))/E^(I*Pi*(k - 1/2)) - (b*E^(2*((-I)
*e + f*fz*x)))/E^(2*I*k*Pi))))/E^(I*Pi*(k - 1/2)), x], x] /; FreeQ[{a, b, c
, d, e, f, fz}, x] && IntegerQ[2*k] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(222) = 444$.

Time = 2.92 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.53

method	result
risch	$-\frac{a\,x^{2}}{2b^{2}}+\frac{(dx-1)\mathrm{e}^{dx+c}}{2b\,d^{2}}-\frac{(dx+1)\mathrm{e}^{-dx-c}}{2b\,d^{2}}+\frac{\ln\left(\frac{-\mathrm{e}^{dx+c}b+\sqrt{a^{2}-b^{2}}-a}{-a+\sqrt{a^{2}-b^{2}}}\right)x\,a^{2}}{d\,b^{2}\sqrt{a^{2}-b^{2}}}-\frac{\ln\left(\frac{-\mathrm{e}^{dx+c}b+\sqrt{a^{2}-b^{2}}-a}{-a+\sqrt{a^{2}-b^{2}}}\right)x}{d\sqrt{a^{2}-b^{2}}}-\frac{\ln\left(\frac{\mathrm{e}^{dx+c}b+\sqrt{}}{a+\sqrt{}}\right.}{d\,b^{2}\sqrt{}}$

input
```
int(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x^2/b^2+1/2*(d*x-1)/b/d^2*exp(d*x+c)-1/2*(d*x+1)/b/d^2*exp(-d*x-c)+
1/d/b^2/(a^2-b^2)^(1/2)*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)
^(1/2)))*x*a^2-1/d/(a^2-b^2)^(1/2)*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-
a+(a^2-b^2)^(1/2)))*x-1/d/b^2/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a^2-b^2)^(
1/2)+a)/(a+(a^2-b^2)^(1/2)))*x*a^2+1/d/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a
^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x+1/d^2/b^2/(a^2-b^2)^(1/2)*ln((-exp
(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c*a^2-1/d^2/(a^2-b^2)^(
1/2)*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*c-1/d^2/b^
2/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))
*c*a^2+1/d^2/(a^2-b^2)^(1/2)*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b
^2)^(1/2)))*c+1/d^2/b^2/(a^2-b^2)^(1/2)*dilog((-exp(d*x+c)*b+(a^2-b^2)^(1/
2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2-1/d^2/(a^2-b^2)^(1/2)*dilog((-exp(d*x+c)*b
+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))-1/d^2/b^2/(a^2-b^2)^(1/2)*dilog(
(exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2+1/d^2/(a^2-b^2)^
(1/2)*dilog((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-2/d^2/b^
2*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*exp(d*x+c)*b+2*a)/(-a^2+b^2)^(1/2))*a^2
+2/d^2*c/(-a^2+b^2)^(1/2)*arctan(1/2*(2*exp(d*x+c)*b+2*a)/(-a^2+b^2)^(1/2)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 669 vs. $2(220) = 440$.

Time = 0.10 (sec) , antiderivative size = 669, normalized size of antiderivative = 2.74

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
-1/2*(a*d^2*x^2*cosh(d*x + c) + b*d*x - (b*d*x - b)*cosh(d*x + c)^2 - (b*d
*x - b)*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2
- b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 2*(b*cosh(d*x + c) +
 b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d
*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b
 + 1) - 2*(b*c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*lo
g(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a)
 + 2*(b*c*cosh(d*x + c) + b*c*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log(2*b
*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 2*
((b*d*x + b*c)*cosh(d*x + c) + (b*d*x + b*c)*sinh(d*x + c))*sqrt((a^2 - b^
2)/b^2)*log((a*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh
(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b) + 2*((b*d*x + b*c)*cosh(d*x + c)
+ (b*d*x + b*c)*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2)*log((a*cosh(d*x + c)
+ a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b
^2) + b)/b) + (a*d^2*x^2 - 2*(b*d*x - b)*cosh(d*x + c))*sinh(d*x + c) + b)
/(b^2*d^2*cosh(d*x + c) + b^2*d^2*sinh(d*x + c))
```

Sympy [F]

$$\int \frac{x \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x \sinh^2{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x*sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x*sinh(c + d*x)**2/(a + b*cosh(c + d*x)), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{x \sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(a-b>0)', see `assume?` for more
details)Is
```

Giac [F]

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\sinh{(dx+c)}^2}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*sinh(d*x + c)^2/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\,{\mathrm{sinh}\left(c+d\,x\right)}^2}{a+b\,\mathrm{cosh}\left(c+d\,x\right)}\,dx$$

input
```
int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x*sinh(c + d*x)^2)/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{e^{2dx+2c}b^2dx - e^{2dx+2c}b^2 - 8e^{dx+c}\left(\int \frac{x}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)a^3d^2 + 8e^{dx+c}\left(\int \frac{x}{e^{2dx+2c}b+2e^{dx+c}a+b}dx\right)a\,b^2d^2 - e}{}$$

input
```
int(x*sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output
```
(e**(2*c + 2*d*x)*b**2*d*x - e**(2*c + 2*d*x)*b**2 - 8*e**(c + d*x)*int(x/
(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)*a**3*d**2 + 8*e**(c + d*x)*
int(x/(e**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b),x)*a*b**2*d**2 - e**(c +
 d*x)*a*b*d**2*x**2 - 4*e**(d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d
*x)*a + e**(d*x)*b),x)*a**2*b*d**2 + 4*e**(d*x)*int(x/(e**(2*c + 3*d*x)*b
+ 2*e**(c + 2*d*x)*a + e**(d*x)*b),x)*b**3*d**2 - 4*a**2*d*x - 4*a**2 + 3*
b**2*d*x + 3*b**2)/(2*e**(c + d*x)*b**3*d**2)
```

3.231 $\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{ax}{b^2}+\frac{2\sqrt{a-b}\sqrt{a+b}\,\mathrm{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{\sinh(c+dx)}{bd}$$

output
```
-a*x/b^2+2*(a-b)^(1/2)*(a+b)^(1/2)*arctanh((a-b)^(1/2)*tanh(1/2*d*x+1/2*c)
/(a+b)^(1/2))/b^2/d+sinh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.95

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{-a(c+dx)+2\sqrt{-a^2+b^2}\arctan\left(\frac{(a-b)\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)+b\sinh(c+dx)}{b^2d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

output
```
(-(a*(c + d*x)) + 2*Sqrt[-a^2 + b^2]*ArcTan[((a - b)*Tanh[(c + d*x)/2])/Sq
rt[-a^2 + b^2]] + b*Sinh[c + d*x])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.19, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 25, 3174, 25, 3042, 3214, 3042, 3138, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\cos\left(ic+idx-\frac{\pi}{2}\right)^2}{a-b\sin\left(ic+idx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\cos\left(\frac{1}{2}(2ic-\pi)+idx\right)^2}{a-b\sin\left(\frac{1}{2}(2ic-\pi)+idx\right)}dx$$

$$\Big\downarrow \text{3174}$$

$$\frac{\int -\frac{b+a\cosh(c+dx)}{a+b\cosh(c+dx)}dx}{b}+\frac{\sinh(c+dx)}{bd}$$

$$\Big\downarrow \text{25}$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\int \frac{b+a\cosh(c+dx)}{a+b\cosh(c+dx)}dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\int \frac{b+a\sin\left(ic+idx+\frac{\pi}{2}\right)}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b}$$

$$\Big\downarrow \text{3214}$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int \frac{1}{a+b\cosh(c+dx)}dx}{b}}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\frac{ax}{b}-\frac{(a^2-b^2)\int\frac{1}{a+b\sin\left(ic+idx+\frac{\pi}{2}\right)}dx}{b}}{b}$$

$$\Big\downarrow \text{3138}$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\frac{ax}{b}+\frac{2i(a^2-b^2)\int\frac{1}{-\left((a-b)\tanh^2\left(\frac{1}{2}(c+dx)\right)\right)+a+b}d\left(i\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}}{b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sinh(c+dx)}{bd}-\frac{\frac{ax}{b}-\frac{2(a^2-b^2)\text{arctanh}\left(\frac{\sqrt{a-b}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\sqrt{a+b}}}{b}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Cosh[c + d*x]),x]
```

output
```
-(((a*x)/b - (2*(a^2 - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[(c + d*x)/2])/Sqrt[a
 + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d))/b) + Sinh[c + d*x]/(b*d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3138
```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Simp[2*(e/d)   Subst[Int[1/(a + b +
(a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
 && NeQ[a^2 - b^2, 0]
```

rule 3174
```
Int[(cos[(e_.) + (f_.)*(x_)]*(g_.))^(p_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)])^(m_), x_Symbol] :> Simp[g*(g*Cos[e + f*x])^(p - 1)*((a + b*Sin[e + f*x
])^(m + 1)/(b*f*(m + p))), x] + Simp[g^2*((p - 1)/(b*(m + p)))   Int[(g*Cos
[e + f*x])^(p - 2)*(a + b*Sin[e + f*x])^m*(b + a*Sin[e + f*x]), x], x] /; F
reeQ[{a, b, e, f, g, m}, x] && NeQ[a^2 - b^2, 0] && GtQ[p, 1] && NeQ[m + p,
 0] && IntegersQ[2*m, 2*p]
```

rule 3214
```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])/((c_.) + (d_.)*sin[(e_.) + (f_.
)*(x_)]), x_Symbol] :> Simp[b*(x/d), x] - Simp[(b*c - a*d)/d   Int[1/(c + d
*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0]
```

Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.77

method	result
derivativedivides	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\,\mathrm{ta}}{\sqrt{(a-}}\right)}{b^2\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{1}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{a\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^2}-\frac{2\left(-a^2+b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\,\mathrm{ta}}{\sqrt{(a-}}\right)}{b^2\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{ax}{b^2}+\frac{\mathrm{e}^{dx+c}}{2bd}-\frac{\mathrm{e}^{-dx-c}}{2db}+\frac{\sqrt{a^2-b^2}\,\ln\left(\mathrm{e}^{dx+c}-\frac{-a+\sqrt{a^2-b^2}}{b}\right)}{d\,b^2}-\frac{\sqrt{a^2-b^2}\,\ln\left(\mathrm{e}^{dx+c}+\frac{a+\sqrt{a^2-b^2}}{b}\right)}{d\,b^2}$

input
```
int(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b/(tanh(1/2*d*x+1/2*c)+1)-a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/b/(tan
h(1/2*d*x+1/2*c)-1)+a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-2/b^2*(-a^2+b^2)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tanh(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(64) = 128$.

Time = 0.10 (sec) , antiderivative size = 415, normalized size of antiderivative = 5.68

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \left[-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^2-b\sinh\left(dx+c\right)^2-2\sqrt{a^2-b^2}(\cosh\left(dx+c\right)+\sinh\left(dx+c\right))}{}\right.$$
$$-\frac{2\,adx\cosh\left(dx+c\right)-b\cosh\left(dx+c\right)^2-b\sinh\left(dx+c\right)^2+4\sqrt{-a^2+b^2}(\cosh\left(dx+c\right)+\sinh\left(dx+c\right)}{2\left(b^2d\cosh\left(dx+c\right)+b^2d\right.}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 - 2*s
qrt(a^2 - b^2)*(cosh(d*x + c) + sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 +
b^2*sinh(d*x + c)^2 + 2*a*b*cosh(d*x + c) + 2*a^2 - b^2 + 2*(b^2*cosh(d*x
+ c) + a*b)*sinh(d*x + c) - 2*sqrt(a^2 - b^2)*(b*cosh(d*x + c) + b*sinh(d*
x + c) + a))/(b*cosh(d*x + c)^2 + b*sinh(d*x + c)^2 + 2*a*cosh(d*x + c) +
2*(b*cosh(d*x + c) + a)*sinh(d*x + c) + b)) + 2*(a*d*x - b*cosh(d*x + c))*
sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c)), -1/2*(2*a*
d*x*cosh(d*x + c) - b*cosh(d*x + c)^2 - b*sinh(d*x + c)^2 + 4*sqrt(-a^2 +
b^2)*(cosh(d*x + c) + sinh(d*x + c))*arctan(-sqrt(-a^2 + b^2)*(b*cosh(d*x
+ c) + b*sinh(d*x + c) + a)/(a^2 - b^2)) + 2*(a*d*x - b*cosh(d*x + c))*sin
h(d*x + c) + b)/(b^2*d*cosh(d*x + c) + b^2*d*sinh(d*x + c))]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1122 vs. $2(61) = 122$.

Time = 57.26 (sec) , antiderivative size = 1122, normalized size of antiderivative = 15.37

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*cosh(d*x+c)),x)
```

output
```
Piecewise((zoo*x*sinh(c)**2/cosh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-d*
x*tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) + d*x/(b*d*tanh(c/
2 + d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2 + d*x/2)**2 - b*d
), Eq(a, b)), (d*x*tanh(c/2 + d*x/2)**2/(b*d*tanh(c/2 + d*x/2)**2 - b*d) -
 d*x/(b*d*tanh(c/2 + d*x/2)**2 - b*d) - 2*tanh(c/2 + d*x/2)/(b*d*tanh(c/2
+ d*x/2)**2 - b*d), Eq(a, -b)), ((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**
2/2 + sinh(c + d*x)*cosh(c + d*x)/(2*d))/a, Eq(b, 0)), (x*sinh(c)**2/(a +
b*cosh(c)), Eq(d, 0)), (-a*d*x*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/
2)**2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sq
rt(a/(a - b) + b/(a - b))) + a*d*x*sqrt(a/(a - b) + b/(a - b))/(b**2*d*sqr
t(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/
(a - b))) - a*log(-sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))*tanh(c
/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)**2 -
b**2*d*sqrt(a/(a - b) + b/(a - b))) + a*log(-sqrt(a/(a - b) + b/(a - b)) +
 tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a - b) + b/(a - b))*tanh(c/2 + d*x/2)*
*2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) + a*log(sqrt(a/(a - b) + b/(a - b
)) + tanh(c/2 + d*x/2))*tanh(c/2 + d*x/2)**2/(b**2*d*sqrt(a/(a - b) + b/(a
 - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) - a*log(
sqrt(a/(a - b) + b/(a - b)) + tanh(c/2 + d*x/2))/(b**2*d*sqrt(a/(a - b) +
b/(a - b))*tanh(c/2 + d*x/2)**2 - b**2*d*sqrt(a/(a - b) + b/(a - b))) -...
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` f
or more de
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{\frac{2\,(dx+c)a}{b^2} - \frac{e^{(dx+c)}}{b} + \frac{e^{(-dx-c)}}{b} - \frac{4\,(a^2-b^2)\arctan\left(\frac{be^{(dx+c)}+a}{\sqrt{-a^2+b^2}}\right)}{\sqrt{-a^2+b^2}b^2}}{2\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*a/b^2 - e^(d*x + c)/b + e^(-d*x - c)/b - 4*(a^2 - b^2)*a
rctan((b*e^(d*x + c) + a)/sqrt(-a^2 + b^2))/(sqrt(-a^2 + b^2)*b^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.41

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{e^{c+d\,x}}{2\,b\,d} - \frac{e^{-c-d\,x}}{2\,b\,d} - \frac{a\,x}{b^2} + \frac{\ln\left(-\frac{2\,e^{c+d\,x}\,(a^2-b^2)}{b^3} - \frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,(b+a\,e^{c+d\,x})}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2\,d} - \frac{\ln\left(\frac{2\,\sqrt{a+b}\,\sqrt{a-b}\,(b+a\,e^{c+d\,x})}{b^3} - \frac{2\,e^{c+d\,x}\,(a^2-b^2)}{b^3}\right)\sqrt{a+b}\,\sqrt{a-b}}{b^2\,d}$$

input
```
int(sinh(c + d*x)^2/(a + b*cosh(c + d*x)),x)
```

output
```
exp(c + d*x)/(2*b*d) - exp(- c - d*x)/(2*b*d) - (a*x)/b^2 + (log(- (2*exp(
c + d*x)*(a^2 - b^2))/b^3 - (2*(a + b)^(1/2)*(a - b)^(1/2)*(b + a*exp(c +
d*x)))/b^3)*(a + b)^(1/2)*(a - b)^(1/2))/(b^2*d) - (log((2*(a + b)^(1/2)*(
a - b)^(1/2)*(b + a*exp(c + d*x)))/b^3 - (2*exp(c + d*x)*(a^2 - b^2))/b^3)
*(a + b)^(1/2)*(a - b)^(1/2))/(b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.22

$$\int \frac{\sinh^2(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{-4e^{dx+c}\sqrt{-a^2+b^2}\,atan\left(\frac{e^{dx+c}b+a}{\sqrt{-a^2+b^2}}\right)+e^{2dx+2c}b-2e^{dx+c}adx-b}{2e^{dx+c}b^2d}$$

input

```
int(sinh(d*x+c)^2/(a+b*cosh(d*x+c)),x)
```

output

```
( - 4*e**(c + d*x)*sqrt( - a**2 + b**2)*atan((e**(c + d*x)*b + a)/sqrt( -
a**2 + b**2)) + e**(2*c + 2*d*x)*b - 2*e**(c + d*x)*a*d*x - b)/(2*e**(c +
d*x)*b**2*d)
```

3.232 $\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 18.13 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))} \, dx$$

$$\Big\downarrow \; 6112$$

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^2/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.51 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sinh \left(dx+c\right)^{2}}{x \left(a+b\cosh \left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(b\cosh(dx+c)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^2/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 39.36 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

input
```
integrate(sinh(d*x+c)**2/x/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**2/(x*(a + b*cosh(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 92, normalized size of antiderivative = 3.83

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{(b\cosh(dx+c)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
2*(a^2*e^c - b^2*e^c)*integrate(e^(d*x)/(b^3*x*e^(2*d*x + 2*c) + 2*a*b^2*x
*e^(d*x + c) + b^3*x), x) + 1/2*Ei(-d*x)*e^(-c)/b + 1/2*Ei(d*x)*e^c/b - a*
log(x)/b^2
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^2}{\left(b\cosh\left(dx+c\right)+a\right)x}\,dx$$

input
```
integrate(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^2/((b*cosh(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 1.99 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^2}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^2/(x*(a + b*cosh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^2(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(dx+c)^2}{\cosh(dx+c)\,bx+ax}dx$$

input
```
int(sinh(d*x+c)^2/x/(a+b*cosh(d*x+c)),x)
```

output
```
int(sinh(c + d*x)**2/(cosh(c + d*x)*b*x + a*x),x)
```

3.233 $\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Int}\left(\frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)},x\right)$$

output
```
Defer(Int)(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 9.36 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

input
```
Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
Integrate[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)} \, dx$$

$$\Big\downarrow \text{ 6112}$$

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}dx$$

input
```
Int[(x^m*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.65 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{x^m \sinh(dx+c)^3}{a+b\cosh(dx+c)}dx$$

input
```
int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh{(dx+c)}^3}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Sympy [N/A]

Not integrable

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh^3{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x**m*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**m*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh{(dx+c)}^3}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh\left(dx+c\right)^3}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^m*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^m \sinh(c+d\,x)^3}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^m*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 2616, normalized size of antiderivative = 109.00

$$\int \frac{x^m \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input

```
int(x^m*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output

```
(x**m*e**(4*c + 4*d*x)*b**4*m + x**m*e**(4*c + 4*d*x)*b**4 - 4*x**m*e**(3*
c + 3*d*x)*a*b**3*m - 4*x**m*e**(3*c + 3*d*x)*a*b**3 - e**(6*c + 2*d*x)*in
t((x**m*e**(4*d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*b
**5*m**2 - e**(6*c + 2*d*x)*int((x**m*e**(4*d*x))/(e**(2*c + 2*d*x)*b*x +
2*e**(c + d*x)*a*x + b*x),x)*b**5*m + 2*e**(5*c + 2*d*x)*int((x**m*e**(3*d
*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*a*b**4*m**2 + 2*
e**(5*c + 2*d*x)*int((x**m*e**(3*d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d
*x)*a*x + b*x),x)*a*b**4*m - 32*e**(3*c + 2*d*x)*int((x**m*e**(d*x))/(e**(
2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*a**3*b**2*m**2 - 32*e**(3*
c + 2*d*x)*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x
+ b*x),x)*a**3*b**2*m + 26*e**(3*c + 2*d*x)*int((x**m*e**(d*x))/(e**(2*c +
 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x)*a*b**4*m**2 + 26*e**(3*c + 2*d*
x)*int((x**m*e**(d*x))/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x),x
)*a*b**4*m + 8*x**m*e**(2*c + 2*d*x)*a**2*b**2*d*x + 8*x**m*e**(2*c + 2*d*
x)*a**2*b**2*m + 8*x**m*e**(2*c + 2*d*x)*a**2*b**2 - 8*x**m*e**(2*c + 2*d*
x)*b**4*d*x - x**m*e**(2*c + 2*d*x)*b**4*m - x**m*e**(2*c + 2*d*x)*b**4 +
16*e**(2*c + 2*d*x)*int(x**m/(e**(4*c + 4*d*x)*b*x + 2*e**(3*c + 3*d*x)*a*
x + e**(2*c + 2*d*x)*b*x),x)*a**4*b*m**2 + 16*e**(2*c + 2*d*x)*int(x**m/(e
**(4*c + 4*d*x)*b*x + 2*e**(3*c + 3*d*x)*a*x + e**(2*c + 2*d*x)*b*x),x)*a*
*4*b*m - 24*e**(2*c + 2*d*x)*int(x**m/(e**(4*c + 4*d*x)*b*x + 2*e**(3*c...
```

3.234 $\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 586

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{3x}{8bd^3}+\frac{x^3}{4bd}-\frac{(a^2-b^2)\,x^4}{4b^3}-\frac{6ax\cosh(c+dx)}{b^2d^3}-\frac{ax^3\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x^3\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{(a^2-b^2)\,x^3\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d}+\frac{3(a^2-b^2)\,x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{3(a^2-b^2)\,x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}-\frac{6(a^2-b^2)\,x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3}-\frac{6(a^2-b^2)\,x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3}+\frac{6(a^2-b^2)\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^4}+\frac{6(a^2-b^2)\,\text{PolyLog}\left(4,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^4}+\frac{6a\sinh(c+dx)}{b^2d^4}+\frac{3ax^2\sinh(c+dx)}{b^2d^2}-\frac{3\cosh(c+dx)\sinh(c+dx)}{8bd^4}-\frac{3x^2\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{3x\sinh^2(c+dx)}{4bd^3}+\frac{x^3\sinh^2(c+dx)}{2bd}$$

output
```
3/8*x/b/d^3+1/4*x^3/b/d-1/4*(a^2-b^2)*x^4/b^3-6*a*x*cosh(d*x+c)/b^2/d^3-a*
x^3*cosh(d*x+c)/b^2/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))
/b^3/d+(a^2-b^2)*x^3*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+3*(a^2-b
^2)*x^2*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+3*(a^2-b^2)*x
^2*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-6*(a^2-b^2)*x*poly
log(3,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^3-6*(a^2-b^2)*x*polylog(3,-
b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^3+6*(a^2-b^2)*polylog(4,-b*exp(d*x
+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^4+6*(a^2-b^2)*polylog(4,-b*exp(d*x+c)/(a+(a
^2-b^2)^(1/2)))/b^3/d^4+6*a*sinh(d*x+c)/b^2/d^4+3*a*x^2*sinh(d*x+c)/b^2/d^
2-3/8*cosh(d*x+c)*sinh(d*x+c)/b/d^4-3/4*x^2*cosh(d*x+c)*sinh(d*x+c)/b/d^2+
3/4*x*sinh(d*x+c)^2/b/d^3+1/2*x^3*sinh(d*x+c)^2/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 4.34 (sec) , antiderivative size = 951, normalized size of antiderivative = 1.62

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((8*(-a^2 + b^2)*(-x^4 + (2*b^2*(1 + E^(2*c))*(d^3*x^3*Log[1 + ((a - Sqrt[
a^2 - b^2])*E^(-c - d*x))/b] - 3*d^2*x^2*PolyLog[2, ((-a + Sqrt[a^2 - b^2]
)*E^(-c - d*x))/b] - 6*d*x*PolyLog[3, ((-a + Sqrt[a^2 - b^2])*E^(-c - d*x)
)/b] - 6*PolyLog[4, ((-a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b]))/(Sqrt[a^2 -
 b^2]*(-a + Sqrt[a^2 - b^2])*d^4) + (2*b^2*(1 + E^(2*c))*(d^3*x^3*Log[1 +
((a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b] - 3*d^2*x^2*PolyLog[2, -(((a + Sqr
t[a^2 - b^2])*E^(-c - d*x))/b)] - 6*d*x*PolyLog[3, -(((a + Sqrt[a^2 - b^2]
)*E^(-c - d*x))/b)] - 6*PolyLog[4, -(((a + Sqrt[a^2 - b^2])*E^(-c - d*x))/
b)]))/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d^4) + (2*a*(1 + E^(2*c))*(d^
3*x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] + 3*d^2*x^2*PolyLog[2
, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 6*d*x*PolyLog[3, (b*E^(c + d*x
))/(-a + Sqrt[a^2 - b^2])] + 6*PolyLog[4, (b*E^(c + d*x))/(-a + Sqrt[a^2 -
 b^2])]))/(Sqrt[a^2 - b^2]*d^4) - (2*a*(1 + E^(2*c))*(d^3*x^3*Log[1 + (b*E
^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 3*d^2*x^2*PolyLog[2, -((b*E^(c + d*x)
)/(a + Sqrt[a^2 - b^2]))] - 6*d*x*PolyLog[3, -((b*E^(c + d*x))/(a + Sqrt[a
^2 - b^2]))] + 6*PolyLog[4, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]))/(S
qrt[a^2 - b^2]*d^4)))/(1 + E^(2*c)) - (16*a*b*Cosh[d*x]*(d*x*(6 + d^2*x^2)
*Cosh[c] - 3*(2 + d^2*x^2)*Sinh[c]))/d^4 + (b^2*Cosh[2*d*x]*(2*d*x*(3 + 2*
d^2*x^2)*Cosh[2*c] - 3*(1 + 2*d^2*x^2)*Sinh[2*c]))/d^4 - (16*a*b*(-3*(2 +
d^2*x^2)*Cosh[c] + d*x*(6 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^4 + (b^2*(-3...
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 3.14 (sec) , antiderivative size = 540, normalized size of antiderivative = 0.92, number of steps used = 29, number of rules used = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 1.167$, Rules used = {6100, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3777, 3042, 3117, 5895, 3042, 25, 3792, 15, 25, 3042, 25, 3115, 24, 6096, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$

$$\Big\downarrow \text{ 6100}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x^3\sinh(c+dx)dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int -ix^3\sin(ic+idx)dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\int x^3\sin(ic+idx)dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\int x^2\cosh(c+dx)dx}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\int x^2\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}-\frac{2i\int -ix\sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}-\frac{2\int x\sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}-\frac{2\int -ix\sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 26$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\int x\sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3777$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\cosh(c+dx)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \ 3117$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x^3\cosh(c+dx)\sinh(c+dx)dx}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 5895$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}-\frac{3\int x^2\sinh^2(c+dx)dx}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 3042$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}-\frac{3\int -x^2\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 25$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}+\frac{3\int x^2\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 3792$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(\frac{\int -\sinh^2(c+dx)dx}{2d^2}+\frac{\int x^2dx}{2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 15$$

$$\frac{\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(\frac{\int -\sinh^2(c+dx)dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}}{}$$

$$\Big\downarrow \text{25}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(-\frac{\int \sinh^2(c+dx)dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}+\frac{3\left(-\frac{\int -\sin(ic+idx)^2dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^3\sinh^2(c+dx)}{2d}+\frac{3\left(\frac{\int \sin(ic+idx)^2dx}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 3115$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{3\left(\frac{\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}+\frac{x\sinh^2(c+dx)}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \; 24$$

$$\frac{(a^2-b^2)\int \frac{x^3\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \; 6096$$

$$\frac{(a^2-b^2)\left(\int \frac{e^{c+dx}x^3}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x^3}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^4}{4b}\right)}{b^2}+\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(a^2-b^2)\left(-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{3\int x^2\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^3\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^4}{4b}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{3011}$$

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\int x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{b}{a+}\right.}{d}\right.}{bd}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{7163}$$

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int \text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{\cdots}}\right)}{d}\cdots\right.}{\cdots}\right.}{\cdots}\cdots\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{2720}$$

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\left(\frac{x\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\int e^{-c-dx}\,\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}\right)}{d}-\frac{x^2\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\text{Poly}\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.}{\cdots}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \text{7143}$$

$$\frac{(a^2-b^2)\left(-\frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}-\frac{\mathrm{PolyLog}\left(4,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}\right)}{d}-\frac{x^2\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{3\left(\frac{2\left(\frac{x\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-\cdots}}\right)}{d}\cdots\right.}{\cdots}\right.}{\cdots}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix^3\cosh(c+dx)}{d}-\frac{3i\left(\frac{x^2\sinh(c+dx)}{d}+\frac{2i\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{d}\right)}{d}\right)}{b^2}+$$

$$\frac{\frac{3\left(\frac{x\sinh^2(c+dx)}{2d^2}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d^2}-\frac{x^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^3}{6}\right)}{2d}+\frac{x^3\sinh^2(c+dx)}{2d}}{b}$$

input

```
Int[(x^3*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output

```
((a^2 - b^2)*(-1/4*x^4/b + (x^3*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^
2])])/(b*d) + (x^3*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) -
 (3*(-((x^2*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + (2*
((x*PolyLog[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d - PolyLog[4, -
((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/d))/(b*d) - (3*(-((x^2*Poly
Log[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + (2*((x*PolyLog[3, -
((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d - PolyLog[4, -((b*E^(c + d*x))
/(a + Sqrt[a^2 - b^2]))]/d^2))/d))/(b*d)))/b^2 + (I*a*((I*x^3*Cosh[c + d*x
])/d - ((3*I)*((x^2*Sinh[c + d*x])/d + ((2*I)*((I*x*Cosh[c + d*x])/d - (I*
Sinh[c + d*x])/d^2))/d))/d))/b^2 + ((x^3*Sinh[c + d*x]^2)/(2*d) + (3*(x^3/
6 - (x^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + (x*Sinh[c + d*x]^2)/(2*d^2)
+ (x/2 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/(2*d^2)))/(2*d))/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3792
```
Int[((c_.) + (d_.)*(x_))^(m_)*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbo
l] :> Simp[d*m*(c + d*x)^(m - 1)*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Sim
p[b*(c + d*x)^m*Cos[e + f*x]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^
2*((n - 1)/n)   Int[(c + d*x)^m*(b*Sin[e + f*x])^(n - 2), x], x] - Simp[d^2
*m*((m - 1)/(f^2*n^2))   Int[(c + d*x)^(m - 2)*(b*Sin[e + f*x])^n, x], x])
/; FreeQ[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [F]

$$\int \frac{x^3 \sinh\left(dx+c\right)^3}{a+b\cosh\left(dx+c\right)}dx$$

input
```
int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2025 vs. $2(546) = 1092$.

Time = 0.12 (sec) , antiderivative size = 2025, normalized size of antiderivative = 3.46

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/32*(4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b
^2*d*x - 3*b^2)*cosh(d*x + c)^4 + (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d
*x - 3*b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 16*(a*b*d^3*x^3 - 3*a*b*d^2*x^2
+ 6*a*b*d*x - 6*a*b)*cosh(d*x + c)^3 - 4*(4*a*b*d^3*x^3 - 12*a*b*d^2*x^2 +
 24*a*b*d*x - 24*a*b - (4*b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)
*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^4*x^4 - 2*(a^2 - b^2)*c
^4)*cosh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^4*x^4 - 8*(a^2 - b^2)*c^4 - 3*(4*
b^2*d^3*x^3 - 6*b^2*d^2*x^2 + 6*b^2*d*x - 3*b^2)*cosh(d*x + c)^2 + 24*(a*b
*d^3*x^3 - 3*a*b*d^2*x^2 + 6*a*b*d*x - 6*a*b)*cosh(d*x + c))*sinh(d*x + c)
^2 + 3*b^2 - 16*(a*b*d^3*x^3 + 3*a*b*d^2*x^2 + 6*a*b*d*x + 6*a*b)*cosh(d*x
 + c) + 96*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d^2*x^2*co
sh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*sinh(d*x + c)^2)*dilog(-(a
*cosh(d*x + c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqr
t((a^2 - b^2)/b^2) + b)/b + 1) + 96*((a^2 - b^2)*d^2*x^2*cosh(d*x + c)^2 +
 2*(a^2 - b^2)*d^2*x^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d^2*x^2*s
inh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x +
c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 32*((a^2 - b^2)*
c^3*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^3*cosh(d*x + c)*sinh(d*x + c) + (a^2
 - b^2)*c^3*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2
*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 32*((a^2 - b^2)*c^3*cosh(d*x + c)^2 +...
```

Sympy [F]

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh^3{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x**3*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**3*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/32*(8*(a^2*d^4*e^(2*c) - b^2*d^4*e^(2*c))*x^4 + (4*b^2*d^3*x^3*e^(4*c) -
 6*b^2*d^2*x^2*e^(4*c) + 6*b^2*d*x*e^(4*c) - 3*b^2*e^(4*c))*e^(2*d*x) - 16
*(a*b*d^3*x^3*e^(3*c) - 3*a*b*d^2*x^2*e^(3*c) + 6*a*b*d*x*e^(3*c) - 6*a*b*
e^(3*c))*e^(d*x) - 16*(a*b*d^3*x^3*e^c + 3*a*b*d^2*x^2*e^c + 6*a*b*d*x*e^c
 + 6*a*b*e^c)*e^(-d*x) + (4*b^2*d^3*x^3 + 6*b^2*d^2*x^2 + 6*b^2*d*x + 3*b^
2)*e^(-2*d*x))*e^(-2*c)/(b^3*d^4) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c
)*x^3*e^(d*x) + (a^2*b - b^3)*x^3)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x +
 c) + b^4), x)
```

Giac [F]

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(dx+c)^3}{b\cosh(dx+c)+a}\,dx$$

input
```
integrate(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^3*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^3 \sinh(c+d\,x)^3}{a+b\cosh(c+d\,x)}\,dx$$

input
```
int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^3*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^3 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{384e^{dx+c}a^3b-64a^4d^3x^3-96a^4d^2x^2-96a^4dx-28b^4d^3x^3-42b^4d^2x^2-42b^4dx-16e^{3dx+3c}a\,b^3d^3x^3+4}{}$$

input
```
int(x^3*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
(4*e**(4*c + 4*d*x)*b**4*d**3*x**3 - 6*e**(4*c + 4*d*x)*b**4*d**2*x**2 + 6
*e**(4*c + 4*d*x)*b**4*d*x - 3*e**(4*c + 4*d*x)*b**4 - 16*e**(3*c + 3*d*x)
*a*b**3*d**3*x**3 + 48*e**(3*c + 3*d*x)*a*b**3*d**2*x**2 - 96*e**(3*c + 3*
d*x)*a*b**3*d*x + 96*e**(3*c + 3*d*x)*a*b**3 - 128*e**(2*c + 2*d*x)*int(x*
*3/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b),x)*a**
4*b*d**4 + 192*e**(2*c + 2*d*x)*int(x**3/(e**(4*c + 4*d*x)*b + 2*e**(3*c +
 3*d*x)*a + e**(2*c + 2*d*x)*b),x)*a**2*b**3*d**4 - 64*e**(2*c + 2*d*x)*in
t(x**3/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b),x)
*b**5*d**4 + 8*e**(2*c + 2*d*x)*a**2*b**2*d**4*x**4 - 8*e**(2*c + 2*d*x)*b
**4*d**4*x**4 - 256*e**(c + 2*d*x)*int(x**3/(e**(2*c + 3*d*x)*b + 2*e**(c
+ 2*d*x)*a + e**(d*x)*b),x)*a**5*d**4 + 448*e**(c + 2*d*x)*int(x**3/(e**(2
*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x)*b),x)*a**3*b**2*d**4 - 192*e
**(c + 2*d*x)*int(x**3/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x)
*b),x)*a*b**4*d**4 + 64*e**(c + d*x)*a**3*b*d**3*x**3 + 192*e**(c + d*x)*a
**3*b*d**2*x**2 + 384*e**(c + d*x)*a**3*b*d*x + 384*e**(c + d*x)*a**3*b -
80*e**(c + d*x)*a*b**3*d**3*x**3 - 240*e**(c + d*x)*a*b**3*d**2*x**2 - 480
*e**(c + d*x)*a*b**3*d*x - 480*e**(c + d*x)*a*b**3 - 64*a**4*d**3*x**3 - 9
6*a**4*d**2*x**2 - 96*a**4*d*x - 48*a**4 + 96*a**2*b**2*d**3*x**3 + 144*a*
*2*b**2*d**2*x**2 + 144*a**2*b**2*d*x + 72*a**2*b**2 - 28*b**4*d**3*x**3 -
 42*b**4*d**2*x**2 - 42*b**4*d*x - 21*b**4)/(32*e**(2*c + 2*d*x)*b**5*d...
```

3.235 $\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 432

$$\begin{aligned}\int \frac{x^2\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{x^2}{4bd}-\frac{(a^2-b^2)\,x^3}{3b^3}-\frac{2a\cosh(c+dx)}{b^2d^3} \\ &\quad-\frac{ax^2\cosh(c+dx)}{b^2d}+\frac{(a^2-b^2)\,x^2\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d} \\ &\quad+\frac{(a^2-b^2)\,x^2\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d} \\ &\quad+\frac{2(a^2-b^2)\,x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2} \\ &\quad+\frac{2(a^2-b^2)\,x\,\mathrm{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2} \\ &\quad-\frac{2(a^2-b^2)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^3} \\ &\quad-\frac{2(a^2-b^2)\,\mathrm{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^3} \\ &\quad+\frac{2ax\sinh(c+dx)}{b^2d^2}-\frac{x\cosh(c+dx)\sinh(c+dx)}{2bd^2} \\ &\quad+\frac{\sinh^2(c+dx)}{4bd^3}+\frac{x^2\sinh^2(c+dx)}{2bd}\end{aligned}$$

output
```
1/4*x^2/b/d-1/3*(a^2-b^2)*x^3/b^3-2*a*cosh(d*x+c)/b^2/d^3-a*x^2*cosh(d*x+c
)/b^2/d+(a^2-b^2)*x^2*ln(1+b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^
2)*x^2*ln(1+b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d+2*(a^2-b^2)*x*polylog(
2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d^2+2*(a^2-b^2)*x*polylog(2,-b*ex
p(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/
(a-(a^2-b^2)^(1/2)))/b^3/d^3-2*(a^2-b^2)*polylog(3,-b*exp(d*x+c)/(a+(a^2-b
^2)^(1/2)))/b^3/d^3+2*a*x*sinh(d*x+c)/b^2/d^2-1/2*x*cosh(d*x+c)*sinh(d*x+c
)/b/d^2+1/4*sinh(d*x+c)^2/b/d^3+1/2*x^2*sinh(d*x+c)^2/b/d
```

Mathematica [A] (warning: unable to verify)

Time = 3.07 (sec) , antiderivative size = 758, normalized size of antiderivative = 1.75

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{8(-a^2+b^2)\left(-2x^3+\frac{3b^2\left(1+e^{2c}\right)\left(d^2x^2\log\left(1+\frac{\left(a-\sqrt{a^2-b^2}\right)e^{-c-dx}}{b}\right)-2dx\,\text{PolyLog}\left(2,\frac{\left(-a+\sqrt{a^2-b^2}\right)e^{-c-dx}}{b}\right)-2\,\text{PolyLog}\left(3,\frac{\left(-a+\sqrt{a^2-b^2}\right)e^{-c-d}}{b}\right.\right.}{\sqrt{a^2-b^2}\left(-a+\sqrt{a^2-b^2}\right)d^3}\right.}{}$$

input
```
Integrate[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((8*(-a^2 + b^2)*(-2*x^3 + (3*b^2*(1 + E^(2*c))*(d^2*x^2*Log[1 + ((a - Sqr
t[a^2 - b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, ((-a + Sqrt[a^2 - b^2])*
E^(-c - d*x))/b] - 2*PolyLog[3, ((-a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b]))
/(Sqrt[a^2 - b^2]*(-a + Sqrt[a^2 - b^2])*d^3) + (3*b^2*(1 + E^(2*c))*(d^2*
x^2*Log[1 + ((a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b] - 2*d*x*PolyLog[2, -((
(a + Sqrt[a^2 - b^2])*E^(-c - d*x))/b)] - 2*PolyLog[3, -(((a + Sqrt[a^2 -
b^2])*E^(-c - d*x))/b)]))/(Sqrt[a^2 - b^2]*(a + Sqrt[a^2 - b^2])*d^3) + (3
*a*(1 + E^(2*c))*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] +
 2*d*x*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])] - 2*PolyLog[3, (
b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])]))/(Sqrt[a^2 - b^2]*d^3) - (3*a*(1 +
 E^(2*c))*(d^2*x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] + 2*d*x*
PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))] - 2*PolyLog[3, -((b*E
^(c + d*x))/(a + Sqrt[a^2 - b^2]))]))/(Sqrt[a^2 - b^2]*d^3)))/(1 + E^(2*c)
) - (24*a*b*Cosh[d*x]*((2 + d^2*x^2)*Cosh[c] - 2*d*x*Sinh[c]))/d^3 + (3*b^
2*Cosh[2*d*x]*((1 + 2*d^2*x^2)*Cosh[2*c] - 2*d*x*Sinh[2*c]))/d^3 - (24*a*b
*(-2*d*x*Cosh[c] + (2 + d^2*x^2)*Sinh[c])*Sinh[d*x])/d^3 + (3*b^2*(-2*d*x*
Cosh[2*c] + (1 + 2*d^2*x^2)*Sinh[2*c])*Sinh[2*d*x])/d^3 + 8*(a^2 - b^2)*x^
3*Tanh[c])/(24*b^3)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 2.15 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.90, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.833, Rules used = {6100, 3042, 26, 3777, 3042, 3777, 26, 3042, 26, 3118, 5895, 3042, 25, 3791, 15, 6096, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{6100}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x^2\sinh(c+dx)dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int -ix^2\sin(ic+idx)dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\int x^2\sin(ic+idx)dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\cosh(c+dx)dx}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\int x\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3777}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{i\int -i\sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int \sinh(c+dx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\int -i\sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{26}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}+\frac{i\int \sin(ic+idx)dx}{d}\right)}{d}\right)}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$$\Big\downarrow \text{3118}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x^2\cosh(c+dx)\sinh(c+dx)dx}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{5895}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^2\sinh^2(c+dx)}{2d}-\frac{\int x\sinh^2(c+dx)dx}{d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3042}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^2\sinh^2(c+dx)}{2d}-\frac{\int -x\sin(ic+idx)^2dx}{d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x^2\sinh^2(c+dx)}{2d}+\frac{\int x\sin(ic+idx)^2dx}{d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \text{3791}$$

$$\frac{(a^2-b^2)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{\int \frac{x}{2}dx+\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}$$

$$\Big\downarrow \ 15$$

$$\frac{\left(a^2-b^2\right)\int \frac{x^2\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 6096$$

$$\frac{\left(a^2-b^2\right)\left(\int \frac{e^{c+dx}x^2}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x^2}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^3}{3b}\right)}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 2620$$

$$\frac{\left(a^2-b^2\right)\left(-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{2\int x\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x^2\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^3}{3b}\right)}{b^2}+\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$$\Big\downarrow \ 3011$$

$$\frac{(a^2-b^2)\left(-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int \text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)dx}{d}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right.}{d}\right.}{bd}\right.}{b^2}$$

$$\frac{\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 2720

$$\frac{(a^2-b^2)\left(-\frac{2\left(\frac{\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\int e^{-c-dx}\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)de^{c+dx}}{d^2}-\right.}{bd}\right.}{b^2}$$

$$\frac{\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

$\downarrow$ 7143

$$\frac{(a^2-b^2)\left(-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}-\frac{2\left(\frac{\text{PolyLog}\left(3,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d^2}-\frac{x\,\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{d}\right)}{bd}+\right.}{b^2}$$

$$\frac{\frac{ia\left(\frac{ix^2\cosh(c+dx)}{d}-\frac{2i\left(\frac{x\sinh(c+dx)}{d}-\frac{\cosh(c+dx)}{d^2}\right)}{d}\right)}{b^2}+\frac{\frac{\frac{\sinh^2(c+dx)}{4d^2}-\frac{x\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{x^2}{4}}{d}+\frac{x^2\sinh^2(c+dx)}{2d}}{b}$$

input
```
Int[(x^2*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((a^2 - b^2)*(-1/3*x^3/b + (x^2*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^
2])])/(b*d) + (x^2*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) -
 (2*(-((x*PolyLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))])/d) + PolyL
og[3, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/d^2))/(b*d) - (2*(-((x*Pol
yLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))])/d) + PolyLog[3, -((b*E^
(c + d*x))/(a + Sqrt[a^2 - b^2]))]/d^2))/(b*d)))/b^2 + (I*a*((I*x^2*Cosh[c
 + d*x])/d - ((2*I)*(-(Cosh[c + d*x]/d^2) + (x*Sinh[c + d*x])/d))/d))/b^2
+ ((x^2*Sinh[c + d*x]^2)/(2*d) + (x^2/4 - (x*Cosh[c + d*x]*Sinh[c + d*x])/
(2*d) + Sinh[c + d*x]^2/(4*d^2))/d)/b
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3118
```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 3791
```
Int[((c_.) + (d_.)*(x_))*((b_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :>
 Simp[d*((b*Sin[e + f*x])^n/(f^2*n^2)), x] + (-Simp[b*(c + d*x)*Cos[e + f*x
]*((b*Sin[e + f*x])^(n - 1)/(f*n)), x] + Simp[b^2*((n - 1)/n)   Int[(c + d*
x)*(b*Sin[e + f*x])^(n - 2), x], x]) /; FreeQ[{b, c, d, e, f}, x] && GtQ[n,
 1]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [F]

$$\int \frac{x^2 \sinh{(dx+c)}^3}{a+b\cosh{(dx+c)}}dx$$

input
```
int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1622 vs. $2(402) = 804$.

Time = 0.11 (sec) , antiderivative size = 1622, normalized size of antiderivative = 3.75

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/48*(6*b^2*d^2*x^2 + 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c)^4
+ 3*(2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*sinh(d*x + c)^4 + 6*b^2*d*x - 24*(a*
b*d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c)^3 - 12*(2*a*b*d^2*x^2 - 4*a*b
*d*x + 4*a*b - (2*b^2*d^2*x^2 - 2*b^2*d*x + b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 - 16*((a^2 - b^2)*d^3*x^3 + 2*(a^2 - b^2)*c^3)*cosh(d*x + c)^2 - 2*(
8*(a^2 - b^2)*d^3*x^3 + 16*(a^2 - b^2)*c^3 - 9*(2*b^2*d^2*x^2 - 2*b^2*d*x
+ b^2)*cosh(d*x + c)^2 + 36*(a*b*d^2*x^2 - 2*a*b*d*x + 2*a*b)*cosh(d*x + c
))*sinh(d*x + c)^2 + 3*b^2 - 24*(a*b*d^2*x^2 + 2*a*b*d*x + 2*a*b)*cosh(d*x
 + c) + 96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x*cosh(d*x +
 c)*sinh(d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x +
c) + a*sinh(d*x + c) + (b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2
)/b^2) + b)/b + 1) + 96*((a^2 - b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*d*x*sinh(d*x + c)^2)*dilog(-(
a*cosh(d*x + c) + a*sinh(d*x + c) - (b*cosh(d*x + c) + b*sinh(d*x + c))*sq
rt((a^2 - b^2)/b^2) + b)/b + 1) + 48*((a^2 - b^2)*c^2*cosh(d*x + c)^2 + 2*
(a^2 - b^2)*c^2*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c^2*sinh(d*x + c
)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) + 2*b*sqrt((a^2 - b^2)/b^2)
 + 2*a) + 48*((a^2 - b^2)*c^2*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c^2*cosh(d*x
 + c)*sinh(d*x + c) + (a^2 - b^2)*c^2*sinh(d*x + c)^2)*log(2*b*cosh(d*x +
c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) + 48*(((a^2 -...
```

Sympy [F]

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh^3{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x**2*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x**2*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh\left(dx+c\right)^3}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/48*(16*(a^2*d^3*e^(2*c) - b^2*d^3*e^(2*c))*x^3 + 3*(2*b^2*d^2*x^2*e^(4*c
) - 2*b^2*d*x*e^(4*c) + b^2*e^(4*c))*e^(2*d*x) - 24*(a*b*d^2*x^2*e^(3*c) -
 2*a*b*d*x*e^(3*c) + 2*a*b*e^(3*c))*e^(d*x) - 24*(a*b*d^2*x^2*e^c + 2*a*b*
d*x*e^c + 2*a*b*e^c)*e^(-d*x) + 3*(2*b^2*d^2*x^2 + 2*b^2*d*x + b^2)*e^(-2*
d*x))*e^(-2*c)/(b^3*d^3) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x^2*e^(
d*x) + (a^2*b - b^3)*x^2)/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4
), x)
```

Giac [F]

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2 \sinh\left(dx+c\right)^3}{b\cosh\left(dx+c\right)+a}\,dx$$

input
```
integrate(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x^2*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x^2\,{\mathrm{sinh}\left(c+d\,x\right)}^3}{a+b\,\mathrm{cosh}\left(c+d\,x\right)}\,dx$$

input
```
int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x^2*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x^2 \sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{192e^{dx+c}a^3b - 96a^4d^2x^2 - 96a^4dx - 42b^4d^2x^2 - 42b^4dx - 24e^{3dx+3c}a\,b^3d^2x^2 + 48e^{3dx+3c}a\,b^3dx - 21b^4 -}{}$$

input
```
int(x^2*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
(6*e**(4*c + 4*d*x)*b**4*d**2*x**2 - 6*e**(4*c + 4*d*x)*b**4*d*x + 3*e**(4
*c + 4*d*x)*b**4 - 24*e**(3*c + 3*d*x)*a*b**3*d**2*x**2 + 48*e**(3*c + 3*d
*x)*a*b**3*d*x - 48*e**(3*c + 3*d*x)*a*b**3 - 192*e**(2*c + 2*d*x)*int(x**
2/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b),x)*a**4
*b*d**3 + 288*e**(2*c + 2*d*x)*int(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c +
3*d*x)*a + e**(2*c + 2*d*x)*b),x)*a**2*b**3*d**3 - 96*e**(2*c + 2*d*x)*int
(x**2/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b),x)*
b**5*d**3 + 16*e**(2*c + 2*d*x)*a**2*b**2*d**3*x**3 - 16*e**(2*c + 2*d*x)*
b**4*d**3*x**3 - 384*e**(c + 2*d*x)*int(x**2/(e**(2*c + 3*d*x)*b + 2*e**(c
 + 2*d*x)*a + e**(d*x)*b),x)*a**5*d**3 + 672*e**(c + 2*d*x)*int(x**2/(e**(
2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x)*b),x)*a**3*b**2*d**3 - 288*
e**(c + 2*d*x)*int(x**2/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x
)*b),x)*a*b**4*d**3 + 96*e**(c + d*x)*a**3*b*d**2*x**2 + 192*e**(c + d*x)*
a**3*b*d*x + 192*e**(c + d*x)*a**3*b - 120*e**(c + d*x)*a*b**3*d**2*x**2 -
 240*e**(c + d*x)*a*b**3*d*x - 240*e**(c + d*x)*a*b**3 - 96*a**4*d**2*x**2
 - 96*a**4*d*x - 48*a**4 + 144*a**2*b**2*d**2*x**2 + 144*a**2*b**2*d*x + 7
2*a**2*b**2 - 42*b**4*d**2*x**2 - 42*b**4*d*x - 21*b**4)/(48*e**(2*c + 2*d
*x)*b**5*d**3)
```

3.236 $\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 22, antiderivative size = 288

$$\begin{aligned}\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx &= \frac{x}{4bd}-\frac{(a^2-b^2)\,x^2}{2b^3}-\frac{ax\cosh(c+dx)}{b^2d}\\ &\quad+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d}\\ &\quad+\frac{(a^2-b^2)\,x\log\left(1+\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d}\\ &\quad+\frac{(a^2-b^2)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{b^3d^2}\\ &\quad+\frac{(a^2-b^2)\operatorname{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{b^3d^2}+\frac{a\sinh(c+dx)}{b^2d^2}\\ &\quad-\frac{\cosh(c+dx)\sinh(c+dx)}{4bd^2}+\frac{x\sinh^2(c+dx)}{2bd}\end{aligned}$$

output

```
1/4*x/b/d-1/2*(a^2-b^2)*x^2/b^3-a*x*cosh(d*x+c)/b^2/d+(a^2-b^2)*x*ln(1+b*e
xp(d*x+c)/(a-(a^2-b^2)^(1/2)))/b^3/d+(a^2-b^2)*x*ln(1+b*exp(d*x+c)/(a+(a^2
-b^2)^(1/2)))/b^3/d+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a-(a^2-b^2)^(1/2)))
/b^3/d^2+(a^2-b^2)*polylog(2,-b*exp(d*x+c)/(a+(a^2-b^2)^(1/2)))/b^3/d^2+a*
sinh(d*x+c)/b^2/d^2-1/4*cosh(d*x+c)*sinh(d*x+c)/b/d^2+1/2*x*sinh(d*x+c)^2/
b/d
```

Mathematica [A] (verified)

Time = 1.52 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.30

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$= \frac{-8abdx\cosh(c+dx)+2b^2dx\cosh(2(c+dx))+4(a^2-b^2)\left(2c(c+dx)-(c+dx)^2+\frac{4a\sqrt{-(a^2-b^2)^2}\,c\,\mathrm{arcta}}{(a^2-b^2)^{3/}}\right.}{}$$

input
```
Integrate[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
(-8*a*b*d*x*Cosh[c + d*x] + 2*b^2*d*x*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*(2
*c*(c + d*x) - (c + d*x)^2 + (4*a*Sqrt[-(a^2 - b^2)^2]*c*ArcTan[(a + b*E^(
c + d*x))/Sqrt[-a^2 + b^2]])/(a^2 - b^2)^(3/2) + (4*a*Sqrt[-(a^2 - b^2)^2]
*c*ArcTanh[(a + b*E^(c + d*x))/Sqrt[a^2 - b^2]])/(-a^2 + b^2)^(3/2) + 2*(c
 + d*x)*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2])] + 2*(c + d*x)*Log[1
 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])] - 2*c*Log[b + 2*a*E^(c + d*x) +
b*E^(2*(c + d*x))] + 2*PolyLog[2, (b*E^(c + d*x))/(-a + Sqrt[a^2 - b^2])]
+ 2*PolyLog[2, -((b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]) + 8*a*b*Sinh[c +
 d*x] - b^2*Sinh[2*(c + d*x)])/(8*b^3*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.92, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.682$, Rules used = {6100, 3042, 26, 3777, 3042, 3117, 5895, 3042, 25, 3115, 24, 6096, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$$
$$\Big\downarrow \; 6100$$

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int x\sinh(c+dx)dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}-\frac{a\int -ix\sin(ic+idx)dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 26

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\int x\sin(ic+idx)dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3777

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\cosh(c+dx)dx}{d}\right)}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3042

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\int\sin\left(ic+idx+\frac{\pi}{2}\right)dx}{d}\right)}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}$$

$\Big\downarrow$ 3117

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\int x\cosh(c+dx)\sinh(c+dx)dx}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 5895

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}-\frac{\int\sinh^2(c+dx)dx}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 3042

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}-\frac{\int-\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 25

$$\frac{(a^2-b^2)\int\frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\int\sin(ic+idx)^2dx}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$\Big\downarrow$ 3115

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{\frac{\frac{\int 1dx}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}+\frac{x\sinh^2(c+dx)}{2d}}{b}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{(a^2-b^2)\int \frac{x\sinh(c+dx)}{a+b\cosh(c+dx)}dx}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

$$\Big\downarrow \text{6096}$$

$$\frac{(a^2-b^2)\left(\int \frac{e^{c+dx}x}{a+be^{c+dx}-\sqrt{a^2-b^2}}dx+\int \frac{e^{c+dx}x}{a+be^{c+dx}+\sqrt{a^2-b^2}}dx-\frac{x^2}{2b}\right)}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

$$\Big\downarrow \text{2620}$$

$$\frac{(a^2-b^2)\left(-\frac{\int \log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)dx}{bd}-\frac{\int \log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)dx}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}\right)}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

$$\Big\downarrow \text{2715}$$

$$\frac{(a^2-b^2)\left(-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a-\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}-\frac{\int e^{-c-dx}\log\left(\frac{e^{c+dx}b}{a+\sqrt{a^2-b^2}}+1\right)de^{c+dx}}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}}\right.}{bd}\right.}{b^2}$$

$$\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

$$\Big\downarrow \text{2838}$$

$$\frac{(a^2-b^2)\left(\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{\text{PolyLog}\left(2,-\frac{be^{c+dx}}{a+\sqrt{a^2-b^2}}\right)}{bd^2}+\frac{x\log\left(\frac{be^{c+dx}}{a-\sqrt{a^2-b^2}}+1\right)}{bd}+\frac{x\log\left(\frac{be^{c+dx}}{\sqrt{a^2-b^2}+a}+1\right)}{bd}-\frac{x^2}{2b}\right)}{b^2}+\frac{ia\left(\frac{ix\cosh(c+dx)}{d}-\frac{i\sinh(c+dx)}{d^2}\right)}{b^2}+\frac{\frac{x\sinh^2(c+dx)}{2d}+\frac{\frac{x}{2}-\frac{\sinh(c+dx)\cosh(c+dx)}{2d}}{2d}}{b}$$

input
```
Int[(x*Sinh[c + d*x]^3)/(a + b*Cosh[c + d*x]),x]
```

output
```
((a^2 - b^2)*(-1/2*x^2/b + (x*Log[1 + (b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]
)])/(b*d) + (x*Log[1 + (b*E^(c + d*x))/(a + Sqrt[a^2 - b^2])])/(b*d) + Pol
yLog[2, -((b*E^(c + d*x))/(a - Sqrt[a^2 - b^2]))]/(b*d^2) + PolyLog[2, -((
b*E^(c + d*x))/(a + Sqrt[a^2 - b^2]))]/(b*d^2)))/b^2 + (I*a*((I*x*Cosh[c +
 d*x])/d - (I*Sinh[c + d*x])/d^2))/b^2 + ((x*Sinh[c + d*x]^2)/(2*d) + (x/2
 - (Cosh[c + d*x]*Sinh[c + d*x])/(2*d))/(2*d))/b
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

rule 3777
```
Int[((c_.) + (d_.)*(x_))^(m_.)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(
-(c + d*x)^m)*(Cos[e + f*x]/f), x] + Simp[d*(m/f)   Int[(c + d*x)^(m - 1)*C
os[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && GtQ[m, 0]
```

rule 5895
```
Int[Cosh[(a_.) + (b_.)*(x_)^(n_.)]*(x_)^(m_.)*Sinh[(a_.) + (b_.)*(x_)^(n_.)
]^(p_.), x_Symbol] :> Simp[x^(m - n + 1)*(Sinh[a + b*x^n]^(p + 1)/(b*n*(p +
 1))), x] - Simp[(m - n + 1)/(b*n*(p + 1))   Int[x^(m - n)*Sinh[a + b*x^n]^
(p + 1), x], x] /; FreeQ[{a, b, p}, x] && LtQ[0, n, m + 1] && NeQ[p, -1]
```

rule 6096
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)])/(Cosh[(c_.) + (d_
.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-(e + f*x)^(m + 1)/(b*f*(m + 1)),
 x] + (Int[(e + f*x)^m*(E^(c + d*x)/(a - Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x] + Int[(e + f*x)^m*(E^(c + d*x)/(a + Rt[a^2 - b^2, 2] + b*E^(c + d*x)))
, x]) /; FreeQ[{a, b, c, d, e, f}, x] && IGtQ[m, 0] && NeQ[a^2 - b^2, 0]
```

rule 6100
```
Int[(((e_.) + (f_.)*(x_))^(m_.)*Sinh[(c_.) + (d_.)*(x_)]^(n_))/(Cosh[(c_.)
+ (d_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[-a/b^2   Int[(e + f*x)^m*Sin
h[c + d*x]^(n - 2), x], x] + (Simp[1/b   Int[(e + f*x)^m*Sinh[c + d*x]^(n -
 2)*Cosh[c + d*x], x], x] + Simp[(a^2 - b^2)/b^2   Int[(e + f*x)^m*(Sinh[c
+ d*x]^(n - 2)/(a + b*Cosh[c + d*x])), x], x]) /; FreeQ[{a, b, c, d, e, f},
 x] && IGtQ[n, 1] && NeQ[a^2 - b^2, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(268) = 536$.

Time = 12.74 (sec) , antiderivative size = 860, normalized size of antiderivative = 2.99

method	result
risch	$\frac{(2dx-1)e^{2dx+2c}}{16b\,d^2}+\frac{c^2}{d^2b}-\frac{\operatorname{dilog}\left(\frac{-e^{dx+c}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)}{d^2b}+\frac{(2dx+1)e^{-2dx-2c}}{16b\,d^2}-\frac{\ln\left(\frac{-e^{dx+c}b+\sqrt{a^2-b^2}-a}{-a+\sqrt{a^2-b^2}}\right)c}{d^2b}-$

input
```
int(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/16*(2*d*x-1)/b/d^2*exp(2*d*x+2*c)+1/d^2/b*c^2-1/d^2/b*dilog((-exp(d*x+c)
*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))+1/16*(2*d*x+1)/b/d^2*exp(-2*d*
x-2*c)-1/d^2/b*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*
c-1/d/b*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*x-1/d^2/b
*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*c+1/d^2/b*c*ln(e
xp(2*d*x+2*c)*b+2*exp(d*x+c)*a+b)-2/d^2/b*c*ln(exp(d*x+c))+2/d/b*c*x-1/d/b
*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*x-1/d^2/b*dilo
g((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))-1/2*a*(d*x+1)/b^2/
d^2*exp(-d*x-c)-2/d/b^3*a^2*c*x+1/d^2/b^3*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2
)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*c+1/d^2/b^3*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2
)+a)/(a+(a^2-b^2)^(1/2)))*a^2*c-1/d^2/b^3*c*a^2*ln(exp(2*d*x+2*c)*b+2*exp(
d*x+c)*a+b)+2/d^2/b^3*c*a^2*ln(exp(d*x+c))-1/2*a*(d*x-1)/b^2/d^2*exp(d*x+c
)+1/d/b^3*ln((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+(a^2-b^2)^(1/2)))*a^2*x
+1/d/b^3*ln((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a^2-b^2)^(1/2)))*a^2*x-1/
d^2/b^3*a^2*c^2+1/d^2/b^3*a^2*dilog((exp(d*x+c)*b+(a^2-b^2)^(1/2)+a)/(a+(a
^2-b^2)^(1/2)))+1/d^2/b^3*a^2*dilog((-exp(d*x+c)*b+(a^2-b^2)^(1/2)-a)/(-a+
(a^2-b^2)^(1/2)))-1/2*x^2/b^3*a^2+1/2*x^2/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1196 vs. $2(266) = 532$.

Time = 0.10 (sec) , antiderivative size = 1196, normalized size of antiderivative = 4.15

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/16*((2*b^2*d*x - b^2)*cosh(d*x + c)^4 + (2*b^2*d*x - b^2)*sinh(d*x + c)^
4 + 2*b^2*d*x - 8*(a*b*d*x - a*b)*cosh(d*x + c)^3 - 4*(2*a*b*d*x - 2*a*b -
 (2*b^2*d*x - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*((a^2 - b^2)*d^2*x^2
 - 2*(a^2 - b^2)*c^2)*cosh(d*x + c)^2 - 2*(4*(a^2 - b^2)*d^2*x^2 - 8*(a^2
- b^2)*c^2 - 3*(2*b^2*d*x - b^2)*cosh(d*x + c)^2 + 12*(a*b*d*x - a*b)*cosh
(d*x + c))*sinh(d*x + c)^2 + b^2 - 8*(a*b*d*x + a*b)*cosh(d*x + c) + 16*((
a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (
a^2 - b^2)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) + (b
*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) + 16*(
(a^2 - b^2)*cosh(d*x + c)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) +
(a^2 - b^2)*sinh(d*x + c)^2)*dilog(-(a*cosh(d*x + c) + a*sinh(d*x + c) - (
b*cosh(d*x + c) + b*sinh(d*x + c))*sqrt((a^2 - b^2)/b^2) + b)/b + 1) - 16*
((a^2 - b^2)*c*cosh(d*x + c)^2 + 2*(a^2 - b^2)*c*cosh(d*x + c)*sinh(d*x +
c) + (a^2 - b^2)*c*sinh(d*x + c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x +
 c) + 2*b*sqrt((a^2 - b^2)/b^2) + 2*a) - 16*((a^2 - b^2)*c*cosh(d*x + c)^2
 + 2*(a^2 - b^2)*c*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*c*sinh(d*x +
c)^2)*log(2*b*cosh(d*x + c) + 2*b*sinh(d*x + c) - 2*b*sqrt((a^2 - b^2)/b^2
) + 2*a) + 16*(((a^2 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*x + c)^2 + 2*((a^2
 - b^2)*d*x + (a^2 - b^2)*c)*cosh(d*x + c)*sinh(d*x + c) + ((a^2 - b^2)*d*
x + (a^2 - b^2)*c)*sinh(d*x + c)^2)*log((a*cosh(d*x + c) + a*sinh(d*x +...
```

Sympy [F]

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh^3{(c+dx)}}{a+b\cosh{(c+dx)}}\,dx$$

input
```
integrate(x*sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(x*sinh(c + d*x)**3/(a + b*cosh(c + d*x)), x)
```

Maxima [F]

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh{(dx+c)}^3}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/16*(8*(a^2*d^2*e^(2*c) - b^2*d^2*e^(2*c))*x^2 + (2*b^2*d*x*e^(4*c) - b^2
*e^(4*c))*e^(2*d*x) - 8*(a*b*d*x*e^(3*c) - a*b*e^(3*c))*e^(d*x) - 8*(a*b*d
*x*e^c + a*b*e^c)*e^(-d*x) + (2*b^2*d*x + b^2)*e^(-2*d*x))*e^(-2*c)/(b^3*d
^2) - 1/8*integrate(16*((a^3*e^c - a*b^2*e^c)*x*e^(d*x) + (a^2*b - b^3)*x)
/(b^4*e^(2*d*x + 2*c) + 2*a*b^3*e^(d*x + c) + b^4), x)
```

Giac [F]

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \int \frac{x\sinh{(dx+c)}^3}{b\cosh{(dx+c)}+a}\,dx$$

input
```
integrate(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(x*sinh(d*x + c)^3/(b*cosh(d*x + c) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx=\int \frac{x\,\sinh(c+d\,x)^3}{a+b\cosh\left(c+d\,x\right)}\,dx$$

input
```
int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)),x)
```

output
```
int((x*sinh(c + d*x)^3)/(a + b*cosh(c + d*x)), x)
```

Reduce [F]

$$\int \frac{x\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{2e^{4dx+4c}b^4dx-e^{4dx+4c}b^4-8e^{3dx+3c}a\,b^3dx+8e^{3dx+3c}a\,b^3-64e^{2dx+2c}\left(\int \frac{x}{e^{4dx+4c}b+2e^{3dx+3c}a+e^{2dx+2c}b}dx\right)a^4b\,d}{}$$

input
```
int(x*sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
(2*e**(4*c + 4*d*x)*b**4*d*x - e**(4*c + 4*d*x)*b**4 - 8*e**(3*c + 3*d*x)*
a*b**3*d*x + 8*e**(3*c + 3*d*x)*a*b**3 - 64*e**(2*c + 2*d*x)*int(x/(e**(4*
c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b),x)*a**4*b*d**2 +
 96*e**(2*c + 2*d*x)*int(x/(e**(4*c + 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + e*
*(2*c + 2*d*x)*b),x)*a**2*b**3*d**2 - 32*e**(2*c + 2*d*x)*int(x/(e**(4*c +
 4*d*x)*b + 2*e**(3*c + 3*d*x)*a + e**(2*c + 2*d*x)*b),x)*b**5*d**2 + 8*e*
*(2*c + 2*d*x)*a**2*b**2*d**2*x**2 - 8*e**(2*c + 2*d*x)*b**4*d**2*x**2 - 1
28*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x
)*b),x)*a**5*d**2 + 224*e**(c + 2*d*x)*int(x/(e**(2*c + 3*d*x)*b + 2*e**(c
 + 2*d*x)*a + e**(d*x)*b),x)*a**3*b**2*d**2 - 96*e**(c + 2*d*x)*int(x/(e**
(2*c + 3*d*x)*b + 2*e**(c + 2*d*x)*a + e**(d*x)*b),x)*a*b**4*d**2 + 32*e**
(c + d*x)*a**3*b*d*x + 32*e**(c + d*x)*a**3*b - 40*e**(c + d*x)*a*b**3*d*x
 - 40*e**(c + d*x)*a*b**3 - 32*a**4*d*x - 16*a**4 + 48*a**2*b**2*d*x + 24*
a**2*b**2 - 14*b**4*d*x - 7*b**4)/(16*e**(2*c + 2*d*x)*b**5*d**2)
```

3.237 $\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 61

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{a\cosh(c+dx)}{b^2d}+\frac{\cosh^2(c+dx)}{2bd}+\frac{(a^2-b^2)\log(a+b\cosh(c+dx))}{b^3d}$$

output
```
-a*cosh(d*x+c)/b^2/d+1/2*cosh(d*x+c)^2/b/d+(a^2-b^2)*ln(a+b*cosh(d*x+c))/b
^3/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{-4ab\cosh(c+dx)+b^2\cosh(2(c+dx))+4(a^2-b^2)\log(a+b\cosh(c+dx))}{4b^3d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

output
```
(-4*a*b*Cosh[c + d*x] + b^2*Cosh[2*(c + d*x)] + 4*(a^2 - b^2)*Log[a + b*Co
sh[c + d*x]])/(4*b^3*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 26, 3147, 476, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\cos\left(ic+idx-\frac{\pi}{2}\right)^3}{a-b\sin\left(ic+idx-\frac{\pi}{2}\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\cos\left(\frac{1}{2}(2ic-\pi)+idx\right)^3}{a-b\sin\left(\frac{1}{2}(2ic-\pi)+idx\right)}dx$$

$$\Big\downarrow \text{3147}$$

$$-\frac{\int \frac{b^2-b^2\cosh^2(c+dx)}{a+b\cosh(c+dx)}d(b\cosh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{476}$$

$$-\frac{\int \left(a-b\cosh(c+dx)+\frac{b^2-a^2}{a+b\cosh(c+dx)}\right)d(b\cosh(c+dx))}{b^3d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-\left(a^2-b^2\right)\log(a+b\cosh(c+dx))+ab\cosh(c+dx)-\frac{1}{2}b^2\cosh^2(c+dx)}{b^3d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Cosh[c + d*x]),x]
```

output
```
-((a*b*Cosh[c + d*x] - (b^2*Cosh[c + d*x]^2)/2 - (a^2 - b^2)*Log[a + b*Cos
h[c + d*x]])/(b^3*d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 476
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c + d*x)^n*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, n},
 x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3147
```
Int[cos[(e_.) + (f_.)*(x_)]^(p_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m
_.), x_Symbol] :> Simp[1/(b^p*f)   Subst[Int[(a + x)^m*(b^2 - x^2)^((p - 1)
/2), x], x, b*Sin[e + f*x]], x] /; FreeQ[{a, b, e, f, m}, x] && IntegerQ[(p
 - 1)/2] && NeQ[a^2 - b^2, 0]
```

Maple [A] (verified)

Time = 7.57 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{-\frac{-\frac{b\cosh(dx+c)^2}{2}+a\cosh(dx+c)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(dx+c))}{b^3}}{d}$
default	$\frac{-\frac{-\frac{b\cosh(dx+c)^2}{2}+a\cosh(dx+c)}{b^2}+\frac{(a^2-b^2)\ln(a+b\cosh(dx+c))}{b^3}}{d}$
risch	$-\frac{x a^2}{b^3}+\frac{x}{b}+\frac{e^{2dx+2c}}{8db}-\frac{a e^{dx+c}}{2d b^2}-\frac{a e^{-dx-c}}{2d b^2}+\frac{e^{-2dx-2c}}{8db}-\frac{2a^2 c}{d b^3}+\frac{2c}{bd}+\frac{\ln\left(e^{2dx+2c}+\frac{2a e^{dx+c}}{b}+1\right)a^2}{d b^3}$

input
```
int(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b^2*(-1/2*b*cosh(d*x+c)^2+a*cosh(d*x+c))+(a^2-b^2)/b^3*ln(a+b*cosh
(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. 2(59) = 118.

Time = 0.09 (sec) , antiderivative size = 340, normalized size of antiderivative = 5.57

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{b^2\cosh{(dx+c)}^4 + b^2\sinh{(dx+c)}^4 - 8\,(a^2-b^2)dx\cosh{(dx+c)}^2 - 4\,ab\cosh{(dx+c)}^3 + 4\,(b^2\cosh{(dx}}{}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
1/8*(b^2*cosh(d*x + c)^4 + b^2*sinh(d*x + c)^4 - 8*(a^2 - b^2)*d*x*cosh(d*
x + c)^2 - 4*a*b*cosh(d*x + c)^3 + 4*(b^2*cosh(d*x + c) - a*b)*sinh(d*x +
c)^3 - 4*a*b*cosh(d*x + c) + 2*(3*b^2*cosh(d*x + c)^2 - 4*(a^2 - b^2)*d*x
- 6*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + b^2 + 8*((a^2 - b^2)*cosh(d*x + c
)^2 + 2*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a^2 - b^2)*sinh(d*x + c
)^2)*log(2*(b*cosh(d*x + c) + a)/(cosh(d*x + c) - sinh(d*x + c))) + 4*(b^2
*cosh(d*x + c)^3 - 4*(a^2 - b^2)*d*x*cosh(d*x + c) - 3*a*b*cosh(d*x + c)^2
 - a*b)*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^2 + 2*b^3*d*cosh(d*x + c)*sinh
(d*x + c) + b^3*d*sinh(d*x + c)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*cosh(d*x+c)),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(59) = 118$.

Time = 0.05 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.13

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = -\frac{\left(4\,ae^{(-dx-c)}-b\right)e^{(2\,dx+2\,c)}}{8\,b^2d} + \frac{(a^2-b^2)(dx+c)}{b^3d} - \frac{4\,ae^{(-dx-c)}-be^{(-2\,dx-2\,c)}}{8\,b^2d} + \frac{(a^2-b^2)\log\left(2\,ae^{(-dx-c)}+be^{(-2\,dx-2\,c)}+b\right)}{b^3d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
-1/8*(4*a*e^(-d*x - c) - b)*e^(2*d*x + 2*c)/(b^2*d) + (a^2 - b^2)*(d*x + c
)/(b^3*d) - 1/8*(4*a*e^(-d*x - c) - b*e^(-2*d*x - 2*c))/(b^2*d) + (a^2 - b
^2)*log(2*a*e^(-d*x - c) + b*e^(-2*d*x - 2*c) + b)/(b^3*d)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.44

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{\frac{b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{b^2}+\frac{8\,(a^2-b^2)\log\left(\left|b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+2\,a\right|\right)}{b^3}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
1/8*((b*(e^(d*x + c) + e^(-d*x - c))^2 - 4*a*(e^(d*x + c) + e^(-d*x - c)))
/b^2 + 8*(a^2 - b^2)*log(abs(b*(e^(d*x + c) + e^(-d*x - c)) + 2*a))/b^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.00

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{e^{-2\,c-2\,d\,x}}{8\,b\,d}+\frac{e^{2\,c+2\,d\,x}}{8\,b\,d}-\frac{x\,(a^2-b^2)}{b^3}+\frac{\ln\left(b+2\,a\,e^{d\,x}\,e^{c}+b\,e^{2\,c}\,e^{2\,d\,x}\right)\,(a^2-b^2)}{b^3\,d}-\frac{a\,e^{-c-d\,x}}{2\,b^2\,d}-\frac{a\,e^{c+d\,x}}{2\,b^2\,d}$$

input
```
int(sinh(c + d*x)^3/(a + b*cosh(c + d*x)),x)
```

output
```
exp(- 2*c - 2*d*x)/(8*b*d) + exp(2*c + 2*d*x)/(8*b*d) - (x*(a^2 - b^2))/b^
3 + (log(b + 2*a*exp(d*x)*exp(c) + b*exp(2*c)*exp(2*d*x))*(a^2 - b^2))/(b^
3*d) - (a*exp(- c - d*x))/(2*b^2*d) - (a*exp(c + d*x))/(2*b^2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.90

$$\int \frac{\sinh^3(c+dx)}{a+b\cosh(c+dx)}\,dx = \frac{e^{4dx+4c}b^2-4e^{3dx+3c}ab+8e^{2dx+2c}\log\left(e^{2dx+2c}b+2e^{dx+c}a+b\right)a^2-8e^{2dx+2c}\log\left(e^{2dx+2c}b+2e^{dx+c}a+b\right)b^2}{8e^{2dx+2c}b^3d}$$

input
```
int(sinh(d*x+c)^3/(a+b*cosh(d*x+c)),x)
```

output
```
(e**(4*c + 4*d*x)*b**2 - 4*e**(3*c + 3*d*x)*a*b + 8*e**(2*c + 2*d*x)*log(e
**(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b)*a**2 - 8*e**(2*c + 2*d*x)*log(e*
*(2*c + 2*d*x)*b + 2*e**(c + d*x)*a + b)*b**2 - 8*e**(2*c + 2*d*x)*a**2*d*
x + 8*e**(2*c + 2*d*x)*b**2*d*x - 4*e**(c + d*x)*a*b + b**2)/(8*e**(2*c +
2*d*x)*b**3*d)
```

3.238 $\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$

Optimal result

Integrand size = 24, antiderivative size = 24

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \operatorname{Int}\left(\frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))},x\right)$$

output
```
Defer(Int)(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

Mathematica [N/A]

Not integrable

Time = 34.17 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$

input
```
Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
Integrate[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))} \, dx$$

$$\Big\downarrow \text{ 6112}$$

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}dx$$

input
```
Int[Sinh[c + d*x]^3/(x*(a + b*Cosh[c + d*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \frac{\sinh\left(dx+c\right)^{3}}{x\left(a+b\cosh\left(dx+c\right)\right)}dx$$

input
```
int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

output
```
int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(b\cosh\left(dx+c\right)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="fricas")
```

output
```
integral(sinh(d*x + c)^3/(b*x*cosh(d*x + c) + a*x), x)
```

Sympy [N/A]

Not integrable

Time = 23.28 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.83

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh^3\left(c+dx\right)}{x\left(a+b\cosh\left(c+dx\right)\right)}\,dx$$

input
```
integrate(sinh(d*x+c)**3/x/(a+b*cosh(d*x+c)),x)
```

output
```
Integral(sinh(c + d*x)**3/(x*(a + b*cosh(c + d*x))), x)
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 5.96

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{(b\cosh\left(dx+c\right)+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="maxima")
```

output
```
1/4*Ei(2*d*x)*e^(2*c)/b + 1/2*a*Ei(-d*x)*e^(-c)/b^2 - 1/4*Ei(-2*d*x)*e^(-2
*c)/b - 1/2*a*Ei(d*x)*e^c/b^2 + (a^2 - b^2)*log(x)/b^3 - 1/8*integrate(16*
(a^2*b - b^3 + (a^3*e^c - a*b^2*e^c)*e^(d*x))/(b^4*x*e^(2*d*x + 2*c) + 2*a
*b^3*x*e^(d*x + c) + b^4*x), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh{(dx+c)}^3}{(b\cosh{(dx+c)}+a)x}\,dx$$

input
```
integrate(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x, algorithm="giac")
```

output
```
integrate(sinh(d*x + c)^3/((b*cosh(d*x + c) + a)*x), x)
```

Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.08

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx = \int \frac{\sinh(c+d\,x)^3}{x\,\left(a+b\cosh\left(c+d\,x\right)\right)}\,dx$$

input
```
int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))),x)
```

output
```
int(sinh(c + d*x)^3/(x*(a + b*cosh(c + d*x))), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 7.62

$$\int \frac{\sinh^3(c+dx)}{x(a+b\cosh(c+dx))}\,dx$$
$$= \frac{e^{4c}\left(\int \frac{e^{4dx}}{e^{2dx+2c}bx+2e^{dx+c}ax+bx}dx\right)b+6e^{c}\left(\int \frac{e^{dx}}{e^{2dx+2c}bx+2e^{dx+c}ax+bx}dx\right)a-\left(\int \frac{1}{e^{4dx+4c}bx+2e^{3dx+3c}ax+e^{2dx+2c}bx}dx\right)b}{4b}$$

input
```
int(sinh(d*x+c)^3/x/(a+b*cosh(d*x+c)),x)
```

output
```
(e**(4*c)*int(e**(4*d*x)/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b*x)
,x)*b + 6*e**c*int(e**(d*x)/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x + b
*x),x)*a - int(1/(e**(4*c + 4*d*x)*b*x + 2*e**(3*c + 3*d*x)*a*x + e**(2*c
+ 2*d*x)*b*x),x)*b + 6*int(1/(e**(2*c + 2*d*x)*b*x + 2*e**(c + d*x)*a*x +
b*x),x)*b - 3*log(x))/(4*b)
```

3.239 $\int \cosh(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 11, antiderivative size = 54

$$\int \cosh(a+b\log(cx^n))\,dx = \frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2} - \frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}$$

output
```
x*cosh(a+b*ln(c*x^n))/(-b^2*n^2+1)-b*n*x*sinh(a+b*ln(c*x^n))/(-b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \cosh(a+b\log(cx^n))\,dx = \frac{x(-\cosh(a+b\log(cx^n))+bn\sinh(a+b\log(cx^n)))}{-1+b^2n^2}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]],x]
```

output
```
(x*(-Cosh[a + b*Log[c*x^n]] + b*n*Sinh[a + b*Log[c*x^n]]))/(-1 + b^2*n^2)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {6044}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \text{ 6044}$$

$$\frac{x\cosh(a + b\log(cx^n))}{1 - b^2n^2} - \frac{bnx\sinh(a + b\log(cx^n))}{1 - b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]],x]
```

output
```
(x*Cosh[a + b*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/
(1 - b^2*n^2)
```

Defintions of rubi rules used

rule 6044
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$\frac{(-\cosh(a+b\ln(c\,x^n))+bn\sinh(a+b\ln(c\,x^n)))x}{b^2n^2-1}$	42

input
```
int(cosh(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
(-cosh(a+b*ln(c*x^n))+b*n*sinh(a+b*ln(c*x^n)))*x/(b^2*n^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cosh (a+b\log (cx^n))\, dx = \frac{bnx\sinh (bn\log (x)+b\log (c)+a)-x\cosh (bn\log (x)+b\log (c)+a)}{b^2n^2-1}$$

input
```
integrate(cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
(b*n*x*sinh(b*n*log(x) + b*log(c) + a) - x*cosh(b*n*log(x) + b*log(c) + a)
)/(b^2*n^2 - 1)
```

Sympy [F]

$$\int \cosh (a+b\log (cx^n))\, dx = \begin{cases} \int \cosh \left(a-\frac{\log (cx^n)}{n}\right) dx & \text{for } b=-\frac{1}{n} \\ \int \cosh \left(a+\frac{\log (cx^n)}{n}\right) dx & \text{for } b=\frac{1}{n} \\ \frac{bnx\sinh (a+b\log (cx^n))}{b^2n^2-1}-\frac{x\cosh (a+b\log (cx^n))}{b^2n^2-1} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/n), x), Eq(b, -1/n)), (Integral(c
osh(a + log(c*x**n)/n), x), Eq(b, 1/n)), (b*n*x*sinh(a + b*log(c*x**n))/(b
**2*n**2 - 1) - x*cosh(a + b*log(c*x**n))/(b**2*n**2 - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.94

$$\int \cosh\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{c^b x e^{(b\log(x^n)+a)}}{2\,(bn+1)} - \frac{x e^{(-a)}}{2\,(bc^b n - c^b)(x^n)^b}$$

input
```
integrate(cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*c^b*x*e^(b*log(x^n) + a)/(b*n + 1) - 1/2*x*e^(-a)/((b*c^b*n - c^b)*(x^
n)^b)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.87

$$\int \cosh\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{c^b x x^{bn} e^a}{2\,(bn+1)} - \frac{x e^{(-a)}}{2\,(bn-1)c^b x^{bn}}$$

input
```
integrate(cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*c^b*x*x^(b*n)*e^a/(b*n + 1) - 1/2*x*e^(-a)/((b*n - 1)*c^b*x^(b*n))
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.81

$$\int \cosh\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{x\,\mathrm{e}^a\,{\left(c\,x^n\right)}^b}{2\,b\,n+2} - \frac{x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^b\,\left(2\,b\,n-2\right)}$$

input
```
int(cosh(a + b*log(c*x^n)),x)
```

output
```
(x*exp(a)*(c*x^n)^b)/(2*b*n + 2) - (x*exp(-a))/((c*x^n)^b*(2*b*n - 2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.76

$$\int \cosh\left(a + b\log\left(cx^n\right)\right)\, dx = \frac{x(-\cosh\left(\log(x^n c)\, b + a\right) + \sinh\left(\log(x^n c)\, b + a\right) bn)}{b^2n^2 - 1}$$

input
```
int(cosh(a+b*log(c*x^n)),x)
```

output
```
(x*( - cosh(log(x**n*c)*b + a) + sinh(log(x**n*c)*b + a)*b*n))/(b**2*n**2
- 1)
```

3.240 $\int \cosh^2(a + b\log(cx^n))\, dx$

Optimal result

Integrand size = 13, antiderivative size = 88

$$\int \cosh^2(a + b\log(cx^n))\, dx = -\frac{2b^2n^2x}{1-4b^2n^2} + \frac{x\cosh^2(a + b\log(cx^n))}{1-4b^2n^2} - \frac{2bnx\cosh(a + b\log(cx^n))\sinh(a + b\log(cx^n))}{1-4b^2n^2}$$

output
```
-2*b^2*n^2*x/(-4*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^2/(-4*b^2*n^2+1)-2*b*n*x
*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(-4*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.64

$$\int \cosh^2(a + b\log(cx^n))\, dx = \frac{x(-1 + 4b^2n^2 - \cosh(2(a + b\log(cx^n))) + 2bn\sinh(2(a + b\log(cx^n))))}{-2 + 8b^2n^2}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(x*(-1 + 4*b^2*n^2 - Cosh[2*(a + b*Log[c*x^n])] + 2*b*n*Sinh[2*(a + b*Log[
c*x^n])]))/(-2 + 8*b^2*n^2)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6046, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(a+b\log(cx^n))\,dx$$
$$\Big\downarrow \text{6046}$$
$$-\frac{2b^2n^2\int 1dx}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}$$
$$\Big\downarrow \text{24}$$
$$\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(-2*b^2*n^2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n
^2) - (2*b*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n
^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6046
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2
*d^2*n^2*p^2 - 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Maple [A] (verified)

Time = 1.79 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.67

method	result	size
parallelrisch	$\frac{x\left(-\cosh\left(2b\ln\left(c\,x^{n}\right)+2a\right)-1+2bn\sinh\left(2b\ln\left(c\,x^{n}\right)+2a\right)+4b^{2}n^{2}\right)}{8b^{2}n^{2}-2}$	59

input
```
int(cosh(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
x*(-cosh(2*b*ln(c*x^n)+2*a)-1+2*b*n*sinh(2*b*ln(c*x^n)+2*a)+4*b^2*n^2)/(8*
b^2*n^2-2)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.02

$$\int \cosh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$=\frac{4\,bnx\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)-x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)^2}{2\left(4\,b^2n^2-1\right)}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/2*(4*b*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) +
a) - x*cosh(b*n*log(x) + b*log(c) + a)^2 - x*sinh(b*n*log(x) + b*log(c) +
a)^2 + (4*b^2*n^2 - 1)*x)/(4*b^2*n^2 - 1)
```

Sympy [F]

$$\int \cosh^2(a+b\log(cx^n))\,dx$$
$$=\begin{cases}\int \cosh^2\left(a-\frac{\log(cx^n)}{2n}\right)dx\\ \int \cosh^2\left(a+\frac{\log(cx^n)}{2n}\right)dx\\ -\frac{2b^2n^2x\sinh^2(a+b\log(cx^n))}{4b^2n^2-1}+\frac{2b^2n^2x\cosh^2(a+b\log(cx^n))}{4b^2n^2-1}+\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2n^2-1}-\frac{x\cosh^2(a+b\log(cx}{4b^2n^2-1}\end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**2, x), Eq(b, -1/(2*n))),
(Integral(cosh(a + log(c*x**n)/(2*n))**2, x), Eq(b, 1/(2*n))), (-2*b**2*n*
*2*x*sinh(a + b*log(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b**2*n**2*x*cosh(a +
 b*log(c*x**n))**2/(4*b**2*n**2 - 1) + 2*b*n*x*sinh(a + b*log(c*x**n))*cos
h(a + b*log(c*x**n))/(4*b**2*n**2 - 1) - x*cosh(a + b*log(c*x**n))**2/(4*b
**2*n**2 - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.76

$$\int \cosh^2(a+b\log(cx^n))\,dx=\frac{c^{2b}xe^{(2b\log(x^n)+2a)}}{4(2bn+1)}+\frac{1}{2}x-\frac{xe^{(-2a)}}{4(2bc^{2b}n-c^{2b})(x^n)^{2b}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*c^(2*b)*x*e^(2*b*log(x^n) + 2*a)/(2*b*n + 1) + 1/2*x - 1/4*x*e^(-2*a)/
((2*b*c^(2*b)*n - c^(2*b))*(x^n)^(2*b))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.92

$$\int \cosh^2(a+b\log(cx^n))\,dx = \frac{bc^{2\,b}nxx^{2\,bn}e^{(2\,a)}}{2\,(4\,b^2n^2-1)} + \frac{2\,b^2n^2x}{4\,b^2n^2-1} - \frac{c^{2\,b}xx^{2\,bn}e^{(2\,a)}}{4\,(4\,b^2n^2-1)} - \frac{bnxe^{(-2\,a)}}{2\,(4\,b^2n^2-1)c^{2\,b}x^{2\,bn}} - \frac{x}{2\,(4\,b^2n^2-1)} - \frac{xe^{(-2\,a)}}{4\,(4\,b^2n^2-1)c^{2\,b}x^{2\,bn}}$$

input

```
integrate(cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output

```
1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) + 2*b^2*n^2*x/(4*b^2*n
^2 - 1) - 1/4*c^(2*b)*x*x^(2*b*n)*e^(2*a)/(4*b^2*n^2 - 1) - 1/2*b*n*x*e^(-
2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x/(4*b^2*n^2 - 1) - 1/4*x*e
^(-2*a)/((4*b^2*n^2 - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.95 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.60

$$\int \cosh^2(a+b\log(cx^n))\,dx = \frac{x}{2} - \frac{x\,e^{-2\,a}}{{(c\,x^n)}^{2\,b}\,(8\,b\,n-4)} + \frac{x\,e^{2\,a}\,{(c\,x^n)}^{2\,b}}{8\,b\,n+4}$$

input

```
int(cosh(a + b*log(c*x^n))^2,x)
```

output

```
x/2 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^(2*b
))/(8*b*n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.40

$$\int \cosh^2(a+b\log(cx^n))\,dx = \frac{x\left(2x^{4bn}e^{4a}c^{4b}bn - x^{4bn}e^{4a}c^{4b} + 8x^{2bn}e^{2a}c^{2b}b^2n^2 - 2x^{2bn}e^{2a}c^{2b} - 2bn - 1\right)}{4x^{2bn}e^{2a}c^{2b}\left(4b^2n^2-1\right)}$$

input
```
int(cosh(a+b*log(c*x^n))^2,x)
```

output
```
(x*(2*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n - x**(4*b*n)*e**(4*a)*c**(4*b) + 8*
x**(2*b*n)*e**(2*a)*c**(2*b)*b**2*n**2 - 2*x**(2*b*n)*e**(2*a)*c**(2*b) -
2*b*n - 1))/(4*x**(2*b*n)*e**(2*a)*c**(2*b)*(4*b**2*n**2 - 1))
```

3.241 $\int \cosh^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 149

$$\int \cosh^3(a+b\log(cx^n))\,dx = -\frac{6b^2n^2x\cosh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} + \frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2} + \frac{6b^3n^3x\sinh(a+b\log(cx^n))}{1-10b^2n^2+9b^4n^4} - \frac{3bnx\cosh^2(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-9b^2n^2}$$

output
```
-6*b^2*n^2*x*cosh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n^2+1)+x*cosh(a+b*ln(c*
x^n))^3/(-9*b^2*n^2+1)+6*b^3*n^3*x*sinh(a+b*ln(c*x^n))/(9*b^4*n^4-10*b^2*n
^2+1)-3*b*n*x*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/(-9*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.79

$$\int \cosh^3(a+b\log(cx^n))\,dx = \frac{x((3-27b^2n^2)\cosh(a+b\log(cx^n))+(1-b^2n^2)\cosh(3(a+b\log(cx^n)))+6bn(-1+5b^2n^2+(-1+b}{4-40b^2n^2+36b^4n^4}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*((3 - 27*b^2*n^2)*Cosh[a + b*Log[c*x^n]] + (1 - b^2*n^2)*Cosh[3*(a + b*
Log[c*x^n])] + 6*b*n*(-1 + 5*b^2*n^2 + (-1 + b^2*n^2)*Cosh[2*(a + b*Log[c*
x^n])])*Sinh[a + b*Log[c*x^n]]))/(4 - 40*b^2*n^2 + 36*b^4*n^4)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.96, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {6046, 6044}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6046}$$

$$-\frac{6b^2n^2\int \cosh(a+b\log(cx^n))\,dx}{1-9b^2n^2}+\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{1-9b^2n^2}$$

$$\Big\downarrow \text{6044}$$

$$\frac{x\cosh^3(a+b\log(cx^n))}{1-9b^2n^2}-\frac{3bnx\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{1-9b^2n^2}-\frac{6b^2n^2\left(\frac{x\cosh(a+b\log(cx^n))}{1-b^2n^2}-\frac{bnx\sinh(a+b\log(cx^n))}{1-b^2n^2}\right)}{1-9b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x*Cosh[a + b*Log[c*x^n]]^3)/(1 - 9*b^2*n^2) - (3*b*n*x*Cosh[a + b*Log[c*x
^n]]^2*Sinh[a + b*Log[c*x^n]])/(1 - 9*b^2*n^2) - (6*b^2*n^2*((x*Cosh[a + b
*Log[c*x^n]])/(1 - b^2*n^2) - (b*n*x*Sinh[a + b*Log[c*x^n]])/(1 - b^2*n^2)
))/(1 - 9*b^2*n^2)
```

Defintions of rubi rules used

rule 6044
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)], x_Symbol] :> Simp[(-
x)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] + Simp[b*d*n*x*(Sinh[
d*(a + b*Log[c*x^n])]/(b^2*d^2*n^2 - 1)), x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b^2*d^2*n^2 - 1, 0]
```

rule 6046
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2
*d^2*n^2*p^2 - 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(149) = 298$.

Time = 5.25 (sec) , antiderivative size = 357, normalized size of antiderivative = 2.40

method	result
parallelrisch	$-\frac{x\left(1+3\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^{n}}\right)\right)^{2}-6bn\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^{n}}\right)\right)-12bn\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^{n}}\right)\right)^{3}+3\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c x^{n}}\right)\right)^{4}b^{2}n^{2}+\right.}{}$

input
```
int(cosh(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
-x/(9*b^4*n^4-10*b^2*n^2+1)*(1+3*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^2-6*b*n*t
anh(1/2*a+b*ln((c*x^n)^(1/2)))-12*b*n*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^3+3*
tanh(1/2*a+b*ln((c*x^n)^(1/2)))^4*b^2*n^2+3*tanh(1/2*a+b*ln((c*x^n)^(1/2))
)^2*b^2*n^2-7*b^2*n^2+3*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^4+tanh(1/2*a+b*ln(
(c*x^n)^(1/2)))^6+18*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^5*b^3*n^3-7*tanh(1/2*
a+b*ln((c*x^n)^(1/2)))^6*b^2*n^2-12*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^3*b^3*
n^3+18*tanh(1/2*a+b*ln((c*x^n)^(1/2)))*b^3*n^3-6*b*n*tanh(1/2*a+b*ln((c*x^
n)^(1/2)))^5)/(tanh(1/2*a+b*ln((c*x^n)^(1/2)))^6-3*tanh(1/2*a+b*ln((c*x^n)
^(1/2)))^4+3*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 199, normalized size of antiderivative = 1.34

$$\int \cosh^3(a+b\log(cx^n))\,dx =$$
$$-\frac{(b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^3+3(b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log$$

input
```
integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
-1/4*((b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^3 + 3*(b^2*n^2 - 1)*
x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 - 3*(b
^3*n^3 - b*n)*x*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(9*b^2*n^2 - 1)*x*co
sh(b*n*log(x) + b*log(c) + a) - 3*(3*(b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b
*log(c) + a)^2 + (9*b^3*n^3 - b*n)*x)*sinh(b*n*log(x) + b*log(c) + a))/(9*
b^4*n^4 - 10*b^2*n^2 + 1)
```

Sympy [F]

$$\int \cosh^3(a+b\log(cx^n))\,dx$$
$$= \begin{cases} \int \cosh^3\left(a-\frac{\log(cx^n)}{n}\right)dx \\ \int \cosh^3\left(a-\frac{\log(cx^n)}{3n}\right)dx \\ \int \cosh^3\left(a+\frac{\log(cx^n)}{3n}\right)dx \\ \int \cosh^3\left(a+\frac{\log(cx^n)}{n}\right)dx \\ -\frac{6b^3n^3x\sinh^3(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}+\frac{9b^3n^3x\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1}+\frac{6b^2n^2x\sinh^2(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{9b^4n^4-10b^2n^2+1} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/n)**3, x), Eq(b, -1/n)), (Integra
l(cosh(a - log(c*x**n)/(3*n))**3, x), Eq(b, -1/(3*n))), (Integral(cosh(a +
 log(c*x**n)/(3*n))**3, x), Eq(b, 1/(3*n))), (Integral(cosh(a + log(c*x**n
)/n)**3, x), Eq(b, 1/n)), (-6*b**3*n**3*x*sinh(a + b*log(c*x**n))**3/(9*b*
*4*n**4 - 10*b**2*n**2 + 1) + 9*b**3*n**3*x*sinh(a + b*log(c*x**n))*cosh(a
 + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*n**2 + 1) + 6*b**2*n**2*x*sinh
(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*n**2
 + 1) - 7*b**2*n**2*x*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*n*
*2 + 1) - 3*b*n*x*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b*
*4*n**4 - 10*b**2*n**2 + 1) + x*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 -
10*b**2*n**2 + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.77

$$\int \cosh^3(a + b\log(cx^n))\,dx = \frac{c^{3b}xe^{(3b\log(x^n)+3a)}}{8(3bn+1)} + \frac{3c^bxe^{(b\log(x^n)+a)}}{8(bn+1)} - \frac{3xe^{(-b\log(x^n)-a)}}{8(bc^bn-c^b)} - \frac{xe^{(-3a)}}{8(3bc^{3b}n-c^{3b})(x^n)^{3b}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*c^(3*b)*x*e^(3*b*log(x^n) + 3*a)/(3*b*n + 1) + 3/8*c^b*x*e^(b*log(x^n)
 + a)/(b*n + 1) - 3/8*x*e^(-b*log(x^n) - a)/(b*c^b*n - c^b) - 1/8*x*e^(-3*
a)/((3*b*c^(3*b)*n - c^(3*b))*(x^n)^(3*b))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. 2(150) = 300.

Time = 0.14 (sec) , antiderivative size = 665, normalized size of antiderivative = 4.46

$$\int \cosh^3(a+b\log(cx^n))\,dx = \frac{3\,b^3c^{3\,b}n^3xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{27\,b^3c^bn^3xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{b^2c^{3\,b}n^2xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^2c^bn^2xx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{3\,bc^{3\,b}nxx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^3n^3xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} - \frac{3\,b^3n^3xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} - \frac{3\,bc^bnxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{c^{3\,b}xx^{3\,bn}e^{(3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)} - \frac{27\,b^2n^2xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} - \frac{b^2n^2xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} + \frac{3\,c^bxx^{bn}e^a}{8\,(9\,b^4n^4-10\,b^2n^2+1)} + \frac{3\,bnxe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} + \frac{3\,bnxe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}} + \frac{3\,xe^{(-a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^bx^{bn}} + \frac{xe^{(-3\,a)}}{8\,(9\,b^4n^4-10\,b^2n^2+1)c^{3\,b}x^{3\,bn}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 27/
8*b^3*c^b*n^3*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 1/8*b^2*c^(3*b)
*n^2*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^2*c^b*n^2*x
*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 3/8*b*c^(3*b)*n*x*x^(3*b*n)*e^
(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1) - 27/8*b^3*n^3*x*e^(-a)/((9*b^4*n^4 - 1
0*b^2*n^2 + 1)*c^b*x^(b*n)) - 3/8*b^3*n^3*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*
n^2 + 1)*c^(3*b)*x^(3*b*n)) - 3/8*b*c^b*n*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^
2*n^2 + 1) + 1/8*c^(3*b)*x*x^(3*b*n)*e^(3*a)/(9*b^4*n^4 - 10*b^2*n^2 + 1)
- 27/8*b^2*n^2*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) - 1/8*b
^2*n^2*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*c
^b*x*x^(b*n)*e^a/(9*b^4*n^4 - 10*b^2*n^2 + 1) + 3/8*b*n*x*e^(-a)/((9*b^4*n
^4 - 10*b^2*n^2 + 1)*c^b*x^(b*n)) + 3/8*b*n*x*e^(-3*a)/((9*b^4*n^4 - 10*b^
2*n^2 + 1)*c^(3*b)*x^(3*b*n)) + 3/8*x*e^(-a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)
*c^b*x^(b*n)) + 1/8*x*e^(-3*a)/((9*b^4*n^4 - 10*b^2*n^2 + 1)*c^(3*b)*x^(3*
b*n))
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.63

$$\int \cosh^3(a + b\log(cx^n))\, dx = \frac{x\,\mathrm{e}^{3\,a}\,{\left(c\,x^n\right)}^{3\,b}}{24\,b\,n+8} - \frac{x\,\mathrm{e}^{-3\,a}}{{\left(c\,x^n\right)}^{3\,b}\,\left(24\,b\,n-8\right)} - \frac{3\,x\,\mathrm{e}^{-a}}{{\left(c\,x^n\right)}^{b}\,\left(8\,b\,n-8\right)} + \frac{3\,x\,\mathrm{e}^{a}\,{\left(c\,x^n\right)}^{b}}{8\,b\,n+8}$$

input
```
int(cosh(a + b*log(c*x^n))^3,x)
```

output
```
(x*exp(3*a)*(c*x^n)^(3*b))/(24*b*n + 8) - (x*exp(-3*a))/((c*x^n)^(3*b)*(24
*b*n - 8)) - (3*x*exp(-a))/((c*x^n)^b*(8*b*n - 8)) + (3*x*exp(a)*(c*x^n)^b
)/(8*b*n + 8)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.17

$$\int \cosh^3(a + b\log(cx^n))\, dx$$
$$= \frac{x(3x^{6bn}e^{6a}c^{6b}b^3n^3 - x^{6bn}e^{6a}c^{6b}b^2n^2 - 3x^{6bn}e^{6a}c^{6b}bn + x^{6bn}e^{6a}c^{6b} + 27x^{4bn}e^{4a}c^{4b}b^3n^3 - 27x^{4bn}e^{4a}c^{4b}b^2n^2 -}{8x^{3bn}}$$

input
```
int(cosh(a+b*log(c*x^n))^3,x)
```

output
```
(x*(3*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 - x**(6*b*n)*e**(6*a)*c**(6*b
)*b**2*n**2 - 3*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n + x**(6*b*n)*e**(6*a)*c**
(6*b) + 27*x**(4*b*n)*e**(4*a)*c**(4*b)*b**3*n**3 - 27*x**(4*b*n)*e**(4*a)
*c**(4*b)*b**2*n**2 - 3*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n + 3*x**(4*b*n)*e*
*(4*a)*c**(4*b) - 27*x**(2*b*n)*e**(2*a)*c**(2*b)*b**3*n**3 - 27*x**(2*b*n
)*e**(2*a)*c**(2*b)*b**2*n**2 + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n + 3*x**
(2*b*n)*e**(2*a)*c**(2*b) - 3*b**3*n**3 - b**2*n**2 + 3*b*n + 1))/(8*x**(3
*b*n)*e**(3*a)*c**(3*b)*(9*b**4*n**4 - 10*b**2*n**2 + 1))
```

3.242 $\int \cosh^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 13, antiderivative size = 191

$$\begin{aligned}\int \cosh^4(a+b\log(cx^n))\,dx =& \frac{24b^4n^4x}{1-20b^2n^2+64b^4n^4}-\frac{12b^2n^2x\cosh^2(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}\\&+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}\\&+\frac{24b^3n^3x\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-20b^2n^2+64b^4n^4}\\&-\frac{4bnx\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{1-16b^2n^2}\end{aligned}$$

output
```
24*b^4*n^4*x/(64*b^4*n^4-20*b^2*n^2+1)-12*b^2*n^2*x*cosh(a+b*ln(c*x^n))^2/
(64*b^4*n^4-20*b^2*n^2+1)+x*cosh(a+b*ln(c*x^n))^4/(-16*b^2*n^2+1)+24*b^3*n
^3*x*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/(64*b^4*n^4-20*b^2*n^2+1)-4*b
*n*x*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/(-16*b^2*n^2+1)
```

Mathematica [A] (verified)

Time = 0.29 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.87

$$\int \cosh^4(a+b\log(cx^n))\,dx = \frac{x(3-60b^2n^2+192b^4n^4+(4-64b^2n^2)\cosh(2(a+b\log(cx^n)))+(1-4b^2n^2)\cosh(4(a+b\log(cx^n)))$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*(3 - 60*b^2*n^2 + 192*b^4*n^4 + (4 - 64*b^2*n^2)*Cosh[2*(a + b*Log[c*x^
n])] + (1 - 4*b^2*n^2)*Cosh[4*(a + b*Log[c*x^n])] - 8*b*n*Sinh[2*(a + b*Lo
g[c*x^n])] + 128*b^3*n^3*Sinh[2*(a + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a + b*
Log[c*x^n])] + 16*b^3*n^3*Sinh[4*(a + b*Log[c*x^n])]))/(8*(1 - 20*b^2*n^2
+ 64*b^4*n^4))
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.93, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {6046, 6046, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6046}$$

$$-\frac{12b^2n^2\int \cosh^2(a+b\log(cx^n))\,dx}{1-16b^2n^2}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}$$

$$\Big\downarrow \text{6046}$$

$$-\frac{12b^2n^2\left(-\frac{2b^2n^2\int 1dx}{1-4b^2n^2}+\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}\right)}{1-16b^2n^2}+\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}$$

$$\Big\downarrow \text{24}$$

$$\frac{x\cosh^4(a+b\log(cx^n))}{1-16b^2n^2}-\frac{4bnx\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{1-16b^2n^2}-\frac{12b^2n^2\left(\frac{x\cosh^2(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2bnx\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{1-4b^2n^2}-\frac{2b^2n^2x}{1-4b^2n^2}\right)}{1-16b^2n^2}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x*Cosh[a + b*Log[c*x^n]]^4)/(1 - 16*b^2*n^2) - (4*b*n*x*Cosh[a + b*Log[c*
x^n]]^3*Sinh[a + b*Log[c*x^n]])/(1 - 16*b^2*n^2) - (12*b^2*n^2*((-2*b^2*n^
2*x)/(1 - 4*b^2*n^2) + (x*Cosh[a + b*Log[c*x^n]]^2)/(1 - 4*b^2*n^2) - (2*b
*n*x*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(1 - 4*b^2*n^2)))/(1 -
 16*b^2*n^2)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 6046
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_), x_Symbol] :> Si
mp[(-x)*(Cosh[d*(a + b*Log[c*x^n])]^p/(b^2*d^2*n^2*p^2 - 1)), x] + (Simp[b*
d*n*p*x*Cosh[d*(a + b*Log[c*x^n])]^(p - 1)*(Sinh[d*(a + b*Log[c*x^n])]/(b^2
*d^2*n^2*p^2 - 1)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - 1))
   Int[Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ[{a, b, c, d, n}
, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - 1, 0]
```

Maple [A] (verified)

Time = 19.32 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.96

method	result
parallelrisch	$\frac{x(4(-16b^2n^2+1)\cosh(2b\ln(cx^n)+2a)+192b^4n^4+128b^3n^3\sinh(2b\ln(cx^n)+2a)+16b^3n^3\sinh(4b\ln(cx^n)+4a)-4b^2n^2\cosh}{512b^4n^4-160b^2n^2+}$

input
```
int(cosh(a+b*ln(c*x^n))^4,x,method=_RETURNVERBOSE)
```

output
```
1/8*x*(4*(-16*b^2*n^2+1)*cosh(2*b*ln(c*x^n)+2*a)+192*b^4*n^4+128*b^3*n^3*s
inh(2*b*ln(c*x^n)+2*a)+16*b^3*n^3*sinh(4*b*ln(c*x^n)+4*a)-4*b^2*n^2*cosh(4
*b*ln(c*x^n)+4*a)-60*b^2*n^2-8*b*n*sinh(2*b*ln(c*x^n)+2*a)-4*b*n*sinh(4*b*
ln(c*x^n)+4*a)+cosh(4*b*ln(c*x^n)+4*a)+3)/(64*b^4*n^4-20*b^2*n^2+1)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.53

$$\int \cosh^4(a+b\log(cx^n))\,dx = -\frac{(4\,b^2n^2-1)x\cosh(bn\log(x)+b\log(c)+a)^4-16\,(4\,b^3n^3-bn)x\cosh(bn\log(x)+b\log(c)+a)\sinh}{}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
-1/8*((4*b^2*n^2 - 1)*x*cosh(b*n*log(x) + b*log(c) + a)^4 - 16*(4*b^3*n^3
- b*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3
 + (4*b^2*n^2 - 1)*x*sinh(b*n*log(x) + b*log(c) + a)^4 + 4*(16*b^2*n^2 - 1
)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*(4*b^2*n^2 - 1)*x*cosh(b*n*lo
g(x) + b*log(c) + a)^2 + 2*(16*b^2*n^2 - 1)*x)*sinh(b*n*log(x) + b*log(c)
+ a)^2 - 3*(64*b^4*n^4 - 20*b^2*n^2 + 1)*x - 16*((4*b^3*n^3 - b*n)*x*cosh(
b*n*log(x) + b*log(c) + a)^3 + (16*b^3*n^3 - b*n)*x*cosh(b*n*log(x) + b*lo
g(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(64*b^4*n^4 - 20*b^2*n^2 + 1)
```

Sympy [F]

$$\int \cosh^4(a+b\log(cx^n))\,dx$$
$$= \begin{cases} \int \cosh^4\left(a-\frac{\log(cx^n)}{2n}\right)dx \\ \int \cosh^4\left(a-\frac{\log(cx^n)}{4n}\right)dx \\ \int \cosh^4\left(a+\frac{\log(cx^n)}{4n}\right)dx \\ \int \cosh^4\left(a+\frac{\log(cx^n)}{2n}\right)dx \\ \frac{24b^4n^4x\sinh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} - \frac{48b^4n^4x\sinh^2(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} + \frac{24b^4n^4x\cosh^4(a+b\log(cx^n))}{64b^4n^4-20b^2n^2+1} - \frac{24b^3n^3x\sinh^3}{} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**4,x)
```

output
```
Piecewise((Integral(cosh(a - log(c*x**n)/(2*n))**4, x), Eq(b, -1/(2*n))),
(Integral(cosh(a - log(c*x**n)/(4*n))**4, x), Eq(b, -1/(4*n))), (Integral(
cosh(a + log(c*x**n)/(4*n))**4, x), Eq(b, 1/(4*n))), (Integral(cosh(a + lo
g(c*x**n)/(2*n))**4, x), Eq(b, 1/(2*n))), (24*b**4*n**4*x*sinh(a + b*log(c
*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2 + 1) - 48*b**4*n**4*x*sinh(a + b*l
og(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 - 20*b**2*n**2 + 1
) + 24*b**4*n**4*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n**4 - 20*b**2*n**2
 + 1) - 24*b**3*n**3*x*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))/
(64*b**4*n**4 - 20*b**2*n**2 + 1) + 40*b**3*n**3*x*sinh(a + b*log(c*x**n))
*cosh(a + b*log(c*x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) + 12*b**2*n*
*2*x*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))**2/(64*b**4*n**4 -
 20*b**2*n**2 + 1) - 16*b**2*n**2*x*cosh(a + b*log(c*x**n))**4/(64*b**4*n*
*4 - 20*b**2*n**2 + 1) - 4*b*n*x*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*
x**n))**3/(64*b**4*n**4 - 20*b**2*n**2 + 1) + x*cosh(a + b*log(c*x**n))**4
/(64*b**4*n**4 - 20*b**2*n**2 + 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.68

$$\int \cosh^4(a+b\log(cx^n))\,dx = \frac{c^{4b}xe^{(4b\log(x^n)+4a)}}{16(4bn+1)} + \frac{c^{2b}xe^{(2b\log(x^n)+2a)}}{4(2bn+1)} + \frac{3}{8}x - \frac{xe^{(-2b\log(x^n)-2a)}}{4(2bc^{2b}n-c^{2b})} - \frac{xe^{(-4a)}}{16(4bc^{4b}n-c^{4b})(x^n)^{4b}}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*c^(4*b)*x*e^(4*b*log(x^n) + 4*a)/(4*b*n + 1) + 1/4*c^(2*b)*x*e^(2*b*l
og(x^n) + 2*a)/(2*b*n + 1) + 3/8*x - 1/4*x*e^(-2*b*log(x^n) - 2*a)/(2*b*c^
(2*b)*n - c^(2*b)) - 1/16*x*e^(-4*a)/((4*b*c^(4*b)*n - c^(4*b))*(x^n)^(4*b
))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 777 vs. 2(192) = 384.

Time = 0.14 (sec) , antiderivative size = 777, normalized size of antiderivative = 4.07

$$\int \cosh^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
b^3*c^(4*b)*n^3*x*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 8*b^3*
c^(2*b)*n^3*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 24*b^4*n^4
*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b^2*c^(4*b)*n^2*x*x^(4*b*n)*e^(4*a)
/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*c^(2*b)*n^2*x*x^(2*b*n)*e^(2*a)/(64
*b^4*n^4 - 20*b^2*n^2 + 1) - 1/4*b*c^(4*b)*n*x*x^(4*b*n)*e^(4*a)/(64*b^4*n
^4 - 20*b^2*n^2 + 1) - 1/2*b*c^(2*b)*n*x*x^(2*b*n)*e^(2*a)/(64*b^4*n^4 - 2
0*b^2*n^2 + 1) - 8*b^3*n^3*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*
b)*x^(2*b*n)) - b^3*n^3*x*e^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*
x^(4*b*n)) - 15/2*b^2*n^2*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/16*c^(4*b)*x
*x^(4*b*n)*e^(4*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1/4*c^(2*b)*x*x^(2*b*n)
*e^(2*a)/(64*b^4*n^4 - 20*b^2*n^2 + 1) - 4*b^2*n^2*x*e^(-2*a)/((64*b^4*n^4
 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) - 1/4*b^2*n^2*x*e^(-4*a)/((64*b^4*n^
4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n)) + 1/2*b*n*x*e^(-2*a)/((64*b^4*n^4 -
 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/4*b*n*x*e^(-4*a)/((64*b^4*n^4 - 20
*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n)) + 3/8*x/(64*b^4*n^4 - 20*b^2*n^2 + 1) + 1
/4*x*e^(-2*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(2*b)*x^(2*b*n)) + 1/16*x*e
^(-4*a)/((64*b^4*n^4 - 20*b^2*n^2 + 1)*c^(4*b)*x^(4*b*n))
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.53

$$\int \cosh^4(a+b\log(cx^n))\,dx = \frac{3x}{8}-\frac{x\,\mathrm{e}^{-2a}}{(c\,x^n)^{2b}\,(8\,b\,n-4)}+\frac{x\,\mathrm{e}^{2a}\,(c\,x^n)^{2b}}{8\,b\,n+4} - \frac{x\,\mathrm{e}^{-4a}}{(c\,x^n)^{4b}\,(64\,b\,n-16)}+\frac{x\,\mathrm{e}^{4a}\,(c\,x^n)^{4b}}{64\,b\,n+16}$$

input
```
int(cosh(a + b*log(c*x^n))^4,x)
```

output
```
(3*x)/8 - (x*exp(-2*a))/((c*x^n)^(2*b)*(8*b*n - 4)) + (x*exp(2*a)*(c*x^n)^
(2*b))/(8*b*n + 4) - (x*exp(-4*a))/((c*x^n)^(4*b)*(64*b*n - 16)) + (x*exp(
4*a)*(c*x^n)^(4*b))/(64*b*n + 16)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 390, normalized size of antiderivative = 2.04

$$\int \cosh^4(a + b\log(cx^n)) \, dx$$
$$= \frac{x(16x^{8bn}e^{8a}c^{8b}b^3n^3 - 4x^{8bn}e^{8a}c^{8b}b^2n^2 - 4x^{8bn}e^{8a}c^{8b}bn + x^{8bn}e^{8a}c^{8b} + 128x^{6bn}e^{6a}c^{6b}b^3n^3 - 64x^{6bn}e^{6a}c^{6b}b^2}{}$$

input
```
int(cosh(a+b*log(c*x^n))^4,x)
```

output
```
(x*(16*x**(8*b*n)*e**(8*a)*c**(8*b)*b**3*n**3 - 4*x**(8*b*n)*e**(8*a)*c**(
8*b)*b**2*n**2 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*n + x**(8*b*n)*e**(8*a)*
c**(8*b) + 128*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 - 64*x**(6*b*n)*e**(
6*a)*c**(6*b)*b**2*n**2 - 8*x**(6*b*n)*e**(6*a)*c**(6*b)*b*n + 4*x**(6*b*n
)*e**(6*a)*c**(6*b) + 384*x**(4*b*n)*e**(4*a)*c**(4*b)*b**4*n**4 - 120*x**
(4*b*n)*e**(4*a)*c**(4*b)*b**2*n**2 + 6*x**(4*b*n)*e**(4*a)*c**(4*b) - 128
*x**(2*b*n)*e**(2*a)*c**(2*b)*b**3*n**3 - 64*x**(2*b*n)*e**(2*a)*c**(2*b)*
b**2*n**2 + 8*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n + 4*x**(2*b*n)*e**(2*a)*c**
(2*b) - 16*b**3*n**3 - 4*b**2*n**2 + 4*b*n + 1))/(16*x**(4*b*n)*e**(4*a)*c
**(4*b)*(64*b**4*n**4 - 20*b**2*n**2 + 1))
```

3.243 $\int x^m \cosh(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 15, antiderivative size = 73

$$\int x^m \cosh(a+b\log(cx^n))\,dx = \frac{(1+m)x^{1+m}\cosh(a+b\log(cx^n))}{(1+m)^2-b^2n^2} - \frac{bnx^{1+m}\sinh(a+b\log(cx^n))}{(1+m)^2-b^2n^2}$$

output
```
(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-b^2*n^2)-b*n*x^(1+m)*sinh(a+b*l
n(c*x^n))/((1+m)^2-b^2*n^2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.74

$$\int x^m \cosh(a+b\log(cx^n))\,dx = \frac{x^{1+m}((1+m)\cosh(a+b\log(cx^n))-bn\sinh(a+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]],x]
```

output
```
(x^(1 + m)*((1 + m)*Cosh[a + b*Log[c*x^n]] - b*n*Sinh[a + b*Log[c*x^n]]))/
((1 + m - b*n)*(1 + m + b*n))
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.01, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {6054}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh (a+b \log (c x^n)) \, dx$$

$$\Big\downarrow \; 6054$$

$$\frac{(m+1) x^{m+1} \cosh (a+b \log (c x^n))}{(-b n+m+1)(b n+m+1)}-\frac{b n x^{m+1} \sinh (a+b \log (c x^n))}{(m+1)^2-b^2 n^2}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]],x]
```

output
```
((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/((1 + m - b*n)*(1 + m + b*n)) -
 (b*n*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - b^2*n^2)
```

Defintions of rubi rules used

rule 6054
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sinh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.93

method	result	size
parallelrisch	$\frac{x^{1+m}(bn\sinh(a+b\ln(c\,x^{n}))-\cosh(a+b\ln(c\,x^{n}))m-\cosh(a+b\ln(c\,x^{n})))}{b^{2}n^{2}-m^{2}-2m-1}$	68

input
```
int(x^m*cosh(a+b*ln(c*x^n)),x,method=_RETURNVERBOSE)
```

output
```
x^(1+m)/(b^2*n^2-m^2-2*m-1)*(b*n*sinh(a+b*ln(c*x^n))-cosh(a+b*ln(c*x^n))*m
-cosh(a+b*ln(c*x^n)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.36

$$\int x^m \cosh\left(a+b\log\left(cx^n\right)\right)\,dx =$$
$$-\frac{(m+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)\cosh\left(m\log\left(x\right)\right)+(m+1)x\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)s}{b^2n^2-m^2-2}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="fricas")
```

output
```
-((m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (m + 1)*x*cos
h(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)) - (b*n*x*cosh(m*log(x)) + b*n*
x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a))/(b^2*n^2 - m^2 - 2*m -
1)
```

Sympy [F]

$$\int x^m \cosh (a + b \log (cx^n))\, dx$$
$$= \begin{cases} \log (x) \cosh (a) & \text{for } b = 0 \wedge m = -1 \\ \int x^m \cosh \left(-a + \frac{m \log (cx^n)}{n} + \frac{\log (cx^n)}{n}\right) dx & \text{for } b = -\frac{m+1}{n} \\ \int x^m \cosh \left(a + \frac{m \log (cx^n)}{n} + \frac{\log (cx^n)}{n}\right) dx & \text{for } b = \frac{m+1}{n} \\ \frac{bnxx^m \sinh (a+b \log (cx^n))}{b^2n^2-m^2-2m-1} - \frac{mxx^m \cosh (a+b \log (cx^n))}{b^2n^2-m^2-2m-1} - \frac{xx^m \cosh (a+b \log (cx^n))}{b^2n^2-m^2-2m-1} & \text{otherwise} \end{cases}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n)),x)
```

output
```
Piecewise((log(x)*cosh(a), Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-a +
 m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, -(m + 1)/n)), (Integral(x**m*
cosh(a + m*log(c*x**n)/n + log(c*x**n)/n), x), Eq(b, (m + 1)/n)), (b*n*x*x
**m*sinh(a + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - m*x*x**m*cosh(a
 + b*log(c*x**n))/(b**2*n**2 - m**2 - 2*m - 1) - x*x**m*cosh(a + b*log(c*x
**n))/(b**2*n**2 - m**2 - 2*m - 1), True))
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.88

$$\int x^m \cosh (a + b \log (cx^n))\, dx = \frac{c^b x e^{(b \log (x^n) + m \log (x) + a)}}{2\,(bn + m + 1)} - \frac{x e^{(-b \log (x^n) + m \log (x) - a)}}{2\,(bc^b n - c^b (m + 1))}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="maxima")
```

output
```
1/2*c^b*x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 1/2*x*e^(-b*log(x^
n) + m*log(x) - a)/(b*c^b*n - c^b*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(75) = 150$.

Time = 0.12 (sec) , antiderivative size = 235, normalized size of antiderivative = 3.22

$$\int x^m \cosh\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{bc^b n x x^{bn} x^m e^a}{2\left(b^2n^2 - m^2 - 2m - 1\right)} - \frac{c^b m x x^{bn} x^m e^a}{2\left(b^2n^2 - m^2 - 2m - 1\right)} - \frac{c^b x x^{bn} x^m e^a}{2\left(b^2n^2 - m^2 - 2m - 1\right)} - \frac{bnxx^m e^{(-a)}}{2\left(b^2n^2 - m^2 - 2m - 1\right)c^b x^{bn}} - \frac{mxx^m e^{(-a)}}{2\left(b^2n^2 - m^2 - 2m - 1\right)c^b x^{bn}} - \frac{xx^m e^{(-a)}}{2\left(b^2n^2 - m^2 - 2m - 1\right)c^b x^{bn}}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n)),x, algorithm="giac")
```

output
```
1/2*b*c^b*n*x*x^(b*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*m*x*x^(b
*n)*x^m*e^a/(b^2*n^2 - m^2 - 2*m - 1) - 1/2*c^b*x*x^(b*n)*x^m*e^a/(b^2*n^2
 - m^2 - 2*m - 1) - 1/2*b*n*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^
(b*n)) - 1/2*m*x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n)) - 1/2*
x*x^m*e^(-a)/((b^2*n^2 - m^2 - 2*m - 1)*c^b*x^(b*n))
```

Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.75

$$\int x^m \cosh\left(a + b\log\left(cx^n\right)\right)\,dx = \frac{x\,x^m\,\mathrm{e}^{-a}}{\left(c\,x^n\right)^b\,\left(2\,m - 2\,b\,n + 2\right)} + \frac{x\,x^m\,\mathrm{e}^{a}\,\left(c\,x^n\right)^b}{2\,m + 2\,b\,n + 2}$$

input
```
int(x^m*cosh(a + b*log(c*x^n)),x)
```

output
```
(x*x^m*exp(-a))/((c*x^n)^b*(2*m - 2*b*n + 2)) + (x*x^m*exp(a)*(c*x^n)^b)/(
2*m + 2*b*n + 2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.90

$$\int x^m \cosh (a + b \log (cx^n)) \, dx$$
$$= \frac{x^m x(-\cosh (\log(x^n c) b + a) m - \cosh (\log(x^n c) b + a) + \sinh (\log(x^n c) b + a) bn)}{b^2 n^2 - m^2 - 2m - 1}$$

input
```
int(x^m*cosh(a+b*log(c*x^n)),x)
```

output
```
(x**m*x*( - cosh(log(x**n*c)*b + a)*m - cosh(log(x**n*c)*b + a) + sinh(log
(x**n*c)*b + a)*b*n))/(b**2*n**2 - m**2 - 2*m - 1)
```

3.244 $\int x^m \cosh^2(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 120

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = -\frac{2b^2n^2x^{1+m}}{(1+m)\left((1+m)^2-4b^2n^2\right)} + \frac{(1+m)x^{1+m}\cosh^2(a+b\log(cx^n))}{(1+m)^2-4b^2n^2} - \frac{2bnx^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-4b^2n^2}$$

output
```
-2*b^2*n^2*x^(1+m)/(1+m)/((1+m)^2-4*b^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x
^n))^2/((1+m)^2-4*b^2*n^2)-2*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c
*x^n))/((1+m)^2-4*b^2*n^2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = \frac{x^{1+m}\left(1+2m+m^2-4b^2n^2+(1+m)^2\cosh(2(a+b\log(cx^n)))-2b(1+m)n\sinh(2(a+b\log(cx^n)))\right)}{2(1+m)(1+m-2bn)(1+m+2bn)}$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(x^(1 + m)*(1 + 2*m + m^2 - 4*b^2*n^2 + (1 + m)^2*Cosh[2*(a + b*Log[c*x^n]
)] - 2*b*(1 + m)*n*Sinh[2*(a + b*Log[c*x^n])]))/(2*(1 + m)*(1 + m - 2*b*n)
*(1 + m + 2*b*n))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.02, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6056, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \text{6056}$$

$$-\frac{2b^2n^2\int x^m dx}{(m+1)^2-4b^2n^2}+\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)\left((m+1)^2-4b^2n^2\right)}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]]^2,x]
```

output
```
(-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 +
 m)*Cosh[a + b*Log[c*x^n]]^2)/(1 + 2*m + m^2 - 4*b^2*n^2) - (2*b*n*x^(1 +
m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6056
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sinh
[d*(a + b*Log[c*x^n])]*(Cosh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Maple [A] (verified)

Time = 9.79 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.78

method	result	size
parallelrisch	$-\frac{\left(-4b^{2}n^{2}-2bn(1+m)\sinh(2b\ln(c\,x^{n})+2a)+(1+m)^{2}(\cosh(2b\ln(c\,x^{n})+2a)+1)\right)x^{1+m}}{8b^{2}m\,n^{2}+8b^{2}n^{2}-2m^{3}-6m^{2}-6m-2}$	94

input
```
int(x^m*cosh(a+b*ln(c*x^n))^2,x,method=_RETURNVERBOSE)
```

output
```
-(-4*b^2*n^2-2*b*n*(1+m)*sinh(2*b*ln(c*x^n)+2*a)+(1+m)^2*(cosh(2*b*ln(c*x^
n)+2*a)+1))*x^(1+m)/(8*b^2*m*n^2+8*b^2*n^2-2*m^3-6*m^2-6*m-2)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.08

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx$$
$$= \frac{(m^2+2\,m+1)x\cosh(bn\log(x)+b\log(c)+a)^2\cosh(m\log(x))-(4\,b^2n^2-m^2-2\,m-1)x\cosh(m\,\mathrm{l}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="fricas")
```

output
```
1/2*((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) -
(4*b^2*n^2 - m^2 - 2*m - 1)*x*cosh(m*log(x)) + ((m^2 + 2*m + 1)*x*cosh(m*l
og(x)) + (m^2 + 2*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)
^2 - 4*((b*m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (b*
m + b)*n*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)))*sinh(b*n*log(x)
 + b*log(c) + a) + ((m^2 + 2*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)^2 -
(4*b^2*n^2 - m^2 - 2*m - 1)*x)*sinh(m*log(x)))/(m^3 - 4*(b^2*m + b^2)*n^2
+ 3*m^2 + 3*m + 1)
```

Sympy [F]

$$\int x^m \cosh^2(a + b\log(cx^n))\,dx$$

$$= \begin{cases} \log(x)\cosh^2(a) \\ \int x^m \cosh^2\left(-a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right)dx \\ \int x^m \cosh^2\left(a + \frac{m\log(cx^n)}{2n} + \frac{\log(cx^n)}{2n}\right)dx \\ \int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx \\ -\frac{2b^2n^2xx^m\sinh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2b^2n^2xx^m\cosh^2(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + \frac{2bmnxx^m\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{4b^2mn^2+4b^2n^2-m^3-3m^2-3m-1} + 2b \end{cases}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n))**2,x)
```

output
```
Piecewise((log(x)*cosh(a)**2, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-
a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x), Eq(b, -(m + 1)/(2*n))
), (Integral(x**m*cosh(a + m*log(c*x**n)/(2*n) + log(c*x**n)/(2*n))**2, x)
, Eq(b, (m + 1)/(2*n))), (Integral(cosh(a + b*log(c*x**n))**2/x, x), Eq(m,
 -1)), (-2*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*
b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b**2*n**2*x*x**m*cosh(a + b*log(c
*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*m
*n*x*x**m*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 +
 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) + 2*b*n*x*x**m*sinh(a + b*log(c*x*
*n))*cosh(a + b*log(c*x**n))/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2
- 3*m - 1) - m**2*x*x**m*cosh(a + b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**
2*n**2 - m**3 - 3*m**2 - 3*m - 1) - 2*m*x*x**m*cosh(a + b*log(c*x**n))**2/
(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1) - x*x**m*cosh(a +
b*log(c*x**n))**2/(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m**2 - 3*m - 1),
 True))
```

Maxima [A] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.72

$$\int x^m \cosh^2(a+b\log(cx^n))\,dx = \frac{c^{2b}xe^{(2b\log(x^n)+m\log(x)+2a)}}{4(2bn+m+1)} - \frac{xe^{(-2b\log(x^n)+m\log(x)-2a)}}{4(2bc^{2b}n-c^{2b}(m+1))} + \frac{x^{m+1}}{2(m+1)}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="maxima")
```

output
```
1/4*c^(2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^
(-2*b*log(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) + 1/2*x
^(m + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 759 vs. $2(127) = 254$.

Time = 0.15 (sec) , antiderivative size = 759, normalized size of antiderivative = 6.32

$$\int x^m \cosh^2(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^2,x, algorithm="giac")
```

output
```
1/2*b*c^(2*b)*m*n*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 -
 3*m^2 - 3*m - 1) - 1/4*c^(2*b)*m^2*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 +
 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 1/2*b*c^(2*b)*n*x*x^(2*b*n)*x^m*e^(2
*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) + 2*b^2*n^2*x*x^m/(4
*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*c^(2*b)*m*x*x^(2*b*n
)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/4*c^(2
*b)*x*x^(2*b*n)*x^m*e^(2*a)/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m -
 1) - 1/2*m^2*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/
2*b*m*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*
c^(2*b)*x^(2*b*n)) - m*x*x^m/(4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m
- 1) - 1/4*m^2*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*
m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*b*n*x*x^m*e^(-2*a)/((4*b^2*m*n^2 + 4*b^2*n
^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/2*x*x^m/(4*b^2*m*n^2 +
4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1) - 1/2*m*x*x^m*e^(-2*a)/((4*b^2*m*n^2 +
4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n)) - 1/4*x*x^m*e^(-2*a)
/((4*b^2*m*n^2 + 4*b^2*n^2 - m^3 - 3*m^2 - 3*m - 1)*c^(2*b)*x^(2*b*n))
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.61

$$\int x^m \cosh^2(a+b\log(cx^n))\, dx = \frac{x\,x^m}{2\,m+2} + \frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{(c\,x^n)}^{2\,b}\,(4\,m-8\,b\,n+4)} + \frac{x\,x^m\,\mathrm{e}^{2\,a}\,{(c\,x^n)}^{2\,b}}{4\,m+8\,b\,n+4}$$

input
```
int(x^m*cosh(a + b*log(c*x^n))^2,x)
```

output
```
(x*x^m)/(2*m + 2) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4)) +
(x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 262, normalized size of antiderivative = 2.18

$$\int x^m \cosh^2\left(a+b\log\left(cx^n\right)\right)\,dx$$
$$= \frac{x^m x\left(2x^{4bn}e^{4a}c^{4b}bmn + 2x^{4bn}e^{4a}c^{4b}bn - x^{4bn}e^{4a}c^{4b}m^2 - 2x^{4bn}e^{4a}c^{4b}m - x^{4bn}e^{4a}c^{4b} + 8x^{2bn}e^{2a}c^{2b}b^2n^2 - 2\right.}{4x^{2bn}e^{2a}c^{2b}\left(4b^2m\,n^2 + 4b^2n^2 - m^3 - 3m\right.}$$

input
```
int(x^m*cosh(a+b*log(c*x^n))^2,x)
```

output
```
(x**m*x*(2*x**(4*b*n)*e**(4*a)*c**(4*b)*b*m*n + 2*x**(4*b*n)*e**(4*a)*c**(
4*b)*b*n - x**(4*b*n)*e**(4*a)*c**(4*b)*m**2 - 2*x**(4*b*n)*e**(4*a)*c**(4
*b)*m - x**(4*b*n)*e**(4*a)*c**(4*b) + 8*x**(2*b*n)*e**(2*a)*c**(2*b)*b**2
*n**2 - 2*x**(2*b*n)*e**(2*a)*c**(2*b)*m**2 - 4*x**(2*b*n)*e**(2*a)*c**(2*
b)*m - 2*x**(2*b*n)*e**(2*a)*c**(2*b) - 2*b*m*n - 2*b*n - m**2 - 2*m - 1))
/(4*x**(2*b*n)*e**(2*a)*c**(2*b)*(4*b**2*m*n**2 + 4*b**2*n**2 - m**3 - 3*m
**2 - 3*m - 1))
```

3.245 $\int x^m \cosh^3(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 203

$$\begin{aligned}\int x^m \cosh^3(a+b\log(cx^n))\,dx &= -\frac{6b^2(1+m)n^2x^{1+m}\cosh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)\,((1+m)^2-b^2n^2)} \\ &\quad + \frac{(1+m)x^{1+m}\cosh^3(a+b\log(cx^n))}{(1+m)^2-9b^2n^2} + \frac{6b^3n^3x^{1+m}\sinh(a+b\log(cx^n))}{((1+m)^2-9b^2n^2)\,((1+m)^2-b^2n^2)} \\ &\quad - \frac{3bnx^{1+m}\cosh^2(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-9b^2n^2}\end{aligned}$$

output
```
-6*b^2*(1+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-
b^2*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^3/((1+m)^2-9*b^2*n^2)+6*b^3*n^3
*x^(1+m)*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)/((1+m)^2-b^2*n^2)-3*b*n*x
^(1+m)*cosh(a+b*ln(c*x^n))^2*sinh(a+b*ln(c*x^n))/((1+m)^2-9*b^2*n^2)
```

Mathematica [A] (verified)

Time = 0.91 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.44

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx$$

$$= \frac{1}{4}x^{1+m}\Bigg(\frac{3\sinh(bn\log(x))\,(-bn\cosh(a-bn\log(x)+b\log(cx^n))+(1+m)\sinh(a-bn\log(x)+b\log(c}{(1+m-bn)(1+m+bn)}$$

$$+\frac{3\cosh(bn\log(x))\,((1+m)\cosh(a-bn\log(x)+b\log(cx^n))-bn\sinh(a-bn\log(x)+b\log(cx^n)))}{(1+m-bn)(1+m+bn)}$$

$$+\frac{\sinh(3bn\log(x))\,(-3bn\cosh(3(a-bn\log(x)+b\log(cx^n)))+(1+m)\sinh(3(a-bn\log(x)+b\log(c}{(1+m-3bn)(1+m+3bn)}$$

$$+\frac{\cosh(3bn\log(x))\,((1+m)\cosh(3(a-bn\log(x)+b\log(cx^n)))-3bn\sinh(3(a-bn\log(x)+b\log(cx}{(1+m-3bn)(1+m+3bn)}$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
(x^(1 + m)*((3*Sinh[b*n*Log[x]]*(-(b*n*Cosh[a - b*n*Log[x] + b*Log[c*x^n]]
) + (1 + m)*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m +
b*n)) + (3*Cosh[b*n*Log[x]]*((1 + m)*Cosh[a - b*n*Log[x] + b*Log[c*x^n]] -
 b*n*Sinh[a - b*n*Log[x] + b*Log[c*x^n]]))/((1 + m - b*n)*(1 + m + b*n)) +
 (Sinh[3*b*n*Log[x]]*(-3*b*n*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n])] + (1
+ m)*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1 + m + 3
*b*n)) + (Cosh[3*b*n*Log[x]]*((1 + m)*Cosh[3*(a - b*n*Log[x] + b*Log[c*x^n
])] - 3*b*n*Sinh[3*(a - b*n*Log[x] + b*Log[c*x^n])]))/((1 + m - 3*b*n)*(1
+ m + 3*b*n))))/4
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 188, normalized size of antiderivative = 0.93, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6056, 6054}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^3(a + b\log(cx^n))\,dx$$

$$\Big\downarrow \; 6056$$

$$-\frac{6b^2n^2\int x^m\cosh(a+b\log(cx^n))\,dx}{(m+1)^2-9b^2n^2}+\frac{(m+1)x^{m+1}\cosh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}-\frac{3bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}$$

$$\Big\downarrow \; 6054$$

$$\frac{(m+1)x^{m+1}\cosh^3(a+b\log(cx^n))}{-9b^2n^2+m^2+2m+1}-\frac{3bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{(m+1)^2-9b^2n^2}-\frac{6b^2n^2\left(\frac{(m+1)x^{m+1}\cosh(a+b\log(cx^n))}{(-bn+m+1)(bn+m+1)}-\frac{bnx^{m+1}\sinh(a+b\log(cx^n))}{(m+1)^2-b^2n^2}\right)}{(m+1)^2-9b^2n^2}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]]^3,x]
```

output
```
((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3)/(1 + 2*m + m^2 - 9*b^2*n^2) -
 (3*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2*Sinh[a + b*Log[c*x^n]])/((1 + m
)^2 - 9*b^2*n^2) - (6*b^2*n^2*(((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]])/
((1 + m - b*n)*(1 + m + b*n)) - (b*n*x^(1 + m)*Sinh[a + b*Log[c*x^n]])/((1
 + m)^2 - b^2*n^2)))/((1 + m)^2 - 9*b^2*n^2)
```

Defintions of rubi rules used

rule 6054
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]*((e_.)*(x_))^(m_.), x
_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]/(b^2*
d^2*e*n^2 - e*(m + 1)^2)), x] + Simp[b*d*n*(e*x)^(m + 1)*(Sinh[d*(a + b*Log
[c*x^n])]/(b^2*d^2*e*n^2 - e*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, e, m, n}
, x] && NeQ[b^2*d^2*n^2 - (m + 1)^2, 0]
```

rule 6056
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sinh
[d*(a + b*Log[c*x^n])]*(Cosh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Maple [A] (verified)

Time = 66.28 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.41

method	result
parallelrisch	$\frac{7\left(-\frac{18\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{4}-\frac{2\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{2}}{3}+1\right)n^{3}b^{3}}{7}+(1+m)\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)^{4}-\frac{10}{}\right.}{9\left(\tanh\left(\frac{a}{2}+b\ln\left(\sqrt{c\,x^{n}}\right)\right)-1\right)^{3}}$

input
```
int(x^m*cosh(a+b*ln(c*x^n))^3,x,method=_RETURNVERBOSE)
```

output
```
7/9*(-18/7*tanh(1/2*a+b*ln((c*x^n)^(1/2)))*(tanh(1/2*a+b*ln((c*x^n)^(1/2))
)^4-2/3*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^2+1)*n^3*b^3+(1+m)*(tanh(1/2*a+b*l
n((c*x^n)^(1/2)))^4-10/7*tanh(1/2*a+b*ln((c*x^n)^(1/2)))^2+1)*(tanh(1/2*a+
b*ln((c*x^n)^(1/2)))^2+1)*n^2*b^2+6/7*tanh(1/2*a+b*ln((c*x^n)^(1/2)))*(1+m
)^2*(tanh(1/2*a+b*ln((c*x^n)^(1/2)))^2+1)^2*n*b-1/7*(1+m)^3*(tanh(1/2*a+b*
ln((c*x^n)^(1/2)))^2+1)^3)*x^(1+m)/(tanh(1/2*a+b*ln((c*x^n)^(1/2)))-1)^3/(
b*n+1/3*m+1/3)/(b*n+m+1)/(b*n-m-1)/(b*n-1/3*m-1/3)/(tanh(1/2*a+b*ln((c*x^n
)^(1/2)))+1)^3
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(214) = 428$.

Time = 0.10 (sec) , antiderivative size = 584, normalized size of antiderivative = 2.88

$$\int x^m \cosh^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="fricas")
```

output
```
1/4*((m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log
(c) + a)^3*cosh(m*log(x)) + 3*(m^3 - 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1
)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + 3*((b^3*n^3 - (b*m^2
+ 2*b*m + b)*n)*x*cosh(m*log(x)) + (b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*sin
h(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*((m^3 - (b^2*m + b^2)*n
^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (
m^3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) +
a)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 3*(3*(b^3*n^3 - (b*
m^2 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2*cosh(m*log(x)) + (
9*b^3*n^3 - (b*m^2 + 2*b*m + b)*n)*x*cosh(m*log(x)) + (3*(b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)^2 + (9*b^3*n^3 - (b*m^2
 + 2*b*m + b)*n)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a) + ((m^
3 - (b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b*log(c) + a)
^3 + 3*(m^3 - 9*(b^2*m + b^2)*n^2 + 3*m^2 + 3*m + 1)*x*cosh(b*n*log(x) + b
*log(c) + a))*sinh(m*log(x)))/(9*b^4*n^4 + m^4 + 4*m^3 - 10*(b^2*m^2 + 2*b
^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)
```

Sympy [F]

$$\int x^m \cosh^3(a + b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n))**3,x)
```

output
```
Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & Eq(m, -1)), (Integral(x**m*cosh(-
a + m*log(c*x**n)/(3*n) + log(c*x**n)/(3*n))**3, x), Eq(b, (-m - 1)/(3*n))
), (Integral(x**m*cosh(-a + m*log(c*x**n)/n + log(c*x**n)/n)**3, x), Eq(b,
 (-m - 1)/n)), (Integral(x**m*cosh(a + m*log(c*x**n)/(3*n) + log(c*x**n)/(
3*n))**3, x), Eq(b, (m + 1)/(3*n))), (Integral(x**m*cosh(a + m*log(c*x**n)
/n + log(c*x**n)/n)**3, x), Eq(b, (m + 1)/n)), (-6*b**3*n**3*x*x**m*sinh(a
 + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 1
0*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) + 9*b**3*n**3*x*x**m*sinh(
a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b**2*m**2*
n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) +
 6*b**2*m*n**2*x*x**m*sinh(a + b*log(c*x**n))**2*cosh(a + b*log(c*x**n))/(
9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4
*m**3 + 6*m**2 + 4*m + 1) - 7*b**2*m*n**2*x*x**m*cosh(a + b*log(c*x**n))**
3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4
+ 4*m**3 + 6*m**2 + 4*m + 1) + 6*b**2*n**2*x*x**m*sinh(a + b*log(c*x**n))*
*2*cosh(a + b*log(c*x**n))/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n*
*2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 7*b**2*n**2*x*x**m
*cosh(a + b*log(c*x**n))**3/(9*b**4*n**4 - 10*b**2*m**2*n**2 - 20*b**2*m*n
**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 + 4*m + 1) - 3*b*m**2*n*x*x**m
*sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(9*b**4*n**4 - 10*b...
```

Maxima [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.68

$$\int x^m \cosh^3(a + b\log(cx^n)) \, dx = \frac{c^{3b} x e^{(3b\log(x^n) + m\log(x) + 3a)}}{8(3bn + m + 1)} + \frac{3c^b x e^{(b\log(x^n) + m\log(x) + a)}}{8(bn + m + 1)} - \frac{3xe^{(-b\log(x^n) + m\log(x) - a)}}{8(bc^b n - c^b(m + 1))} - \frac{xe^{(-3b\log(x^n) + m\log(x) - 3a)}}{8(3bc^{3b} n - c^{3b}(m + 1))}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="maxima")
```

output
```
1/8*c^(3*b)*x*e^(3*b*log(x^n) + m*log(x) + 3*a)/(3*b*n + m + 1) + 3/8*c^b*
x*e^(b*log(x^n) + m*log(x) + a)/(b*n + m + 1) - 3/8*x*e^(-b*log(x^n) + m*l
og(x) - a)/(b*c^b*n - c^b*(m + 1)) - 1/8*x*e^(-3*b*log(x^n) + m*log(x) - 3
*a)/(3*b*c^(3*b)*n - c^(3*b)*(m + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3225 vs. $2(214) = 428$.

Time = 0.17 (sec) , antiderivative size = 3225, normalized size of antiderivative = 15.89

$$\int x^m \cosh^3(a + b\log(cx^n))\, dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^3,x, algorithm="giac")
```

output
```
3/8*b^3*c^(3*b)*n^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 -
20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 27/8*b^3*c^b*
n^3*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 1
0*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b^2*c^(3*b)*m*n^2*x*x^(3*b*n)*x
^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 +
 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*m*n^2*x*x^(b*n)*x^m*e^a/(9*b^4*n^
4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m
 + 1) - 3/8*b*c^(3*b)*m^2*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^
2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 1/8*b
^2*c^(3*b)*n^2*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^
2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/8*b*c^b*m^2*n*x*
x^(b*n)*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*
n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 27/8*b^2*c^b*n^2*x*x^(b*n)*x^m*e^a/(9*b^4
*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 +
4*m + 1) + 1/8*c^(3*b)*m^3*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2
*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*
c^(3*b)*m*n*x*x^(3*b*n)*x^m*e^(3*a)/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m
*n^2 + m^4 - 10*b^2*n^2 + 4*m^3 + 6*m^2 + 4*m + 1) + 3/8*c^b*m^3*x*x^(b*n)
*x^m*e^a/(9*b^4*n^4 - 10*b^2*m^2*n^2 - 20*b^2*m*n^2 + m^4 - 10*b^2*n^2 + 4
*m^3 + 6*m^2 + 4*m + 1) - 3/4*b*c^b*m*n*x*x^(b*n)*x^m*e^a/(9*b^4*n^4 - ...
```

Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.58

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m\,\mathrm{e}^{-a}}{(c\,x^n)^b\,(8\,m-8\,b\,n+8)}+\frac{x\,x^m\,\mathrm{e}^{-3\,a}}{(c\,x^n)^{3\,b}\,(8\,m-24\,b\,n+8)}+\frac{x\,x^m\,\mathrm{e}^{3\,a}\,(c\,x^n)^{3\,b}}{8\,m+24\,b\,n+8}+\frac{3\,x\,x^m\,\mathrm{e}^{a}\,(c\,x^n)^{b}}{8\,m+8\,b\,n+8}$$

input
```
int(x^m*cosh(a + b*log(c*x^n))^3,x)
```

output
```
(3*x*x^m*exp(-a))/((c*x^n)^b*(8*m - 8*b*n + 8)) + (x*x^m*exp(-3*a))/((c*x^
n)^(3*b)*(8*m - 24*b*n + 8)) + (x*x^m*exp(3*a)*(c*x^n)^(3*b))/(8*m + 24*b*
n + 8) + (3*x*x^m*exp(a)*(c*x^n)^b)/(8*m + 8*b*n + 8)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 784, normalized size of antiderivative = 3.86

$$\int x^m \cosh^3(a+b\log(cx^n))\,dx$$
$$= \frac{x^m x\left(1+3m+x^{6bn}e^{6a}c^{6b}m^3+3x^{6bn}e^{6a}c^{6b}m^2+3x^{6bn}e^{6a}c^{6b}m+3x^{4bn}e^{4a}c^{4b}m^3-3b^3n^3+3x^{6bn}e^{6a}c^{6b}b^3n\right.}{}$$

input
```
int(x^m*cosh(a+b*log(c*x^n))^3,x)
```

output

```
(x**m*x*(3*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 - x**(6*b*n)*e**(6*a)*c*
*(6*b)*b**2*m*n**2 - x**(6*b*n)*e**(6*a)*c**(6*b)*b**2*n**2 - 3*x**(6*b*n)
*e**(6*a)*c**(6*b)*b*m**2*n - 6*x**(6*b*n)*e**(6*a)*c**(6*b)*b*m*n - 3*x**
(6*b*n)*e**(6*a)*c**(6*b)*b*n + x**(6*b*n)*e**(6*a)*c**(6*b)*m**3 + 3*x**(
6*b*n)*e**(6*a)*c**(6*b)*m**2 + 3*x**(6*b*n)*e**(6*a)*c**(6*b)*m + x**(6*b
*n)*e**(6*a)*c**(6*b) + 27*x**(4*b*n)*e**(4*a)*c**(4*b)*b**3*n**3 - 27*x**
(4*b*n)*e**(4*a)*c**(4*b)*b**2*m*n**2 - 27*x**(4*b*n)*e**(4*a)*c**(4*b)*b*
*2*n**2 - 3*x**(4*b*n)*e**(4*a)*c**(4*b)*b*m**2*n - 6*x**(4*b*n)*e**(4*a)*
c**(4*b)*b*m*n - 3*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n + 3*x**(4*b*n)*e**(4*a
)*c**(4*b)*m**3 + 9*x**(4*b*n)*e**(4*a)*c**(4*b)*m**2 + 9*x**(4*b*n)*e**(4
*a)*c**(4*b)*m + 3*x**(4*b*n)*e**(4*a)*c**(4*b) - 27*x**(2*b*n)*e**(2*a)*c
**(2*b)*b**3*n**3 - 27*x**(2*b*n)*e**(2*a)*c**(2*b)*b**2*m*n**2 - 27*x**(2
*b*n)*e**(2*a)*c**(2*b)*b**2*n**2 + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*b*m**2*
n + 6*x**(2*b*n)*e**(2*a)*c**(2*b)*b*m*n + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*
b*n + 3*x**(2*b*n)*e**(2*a)*c**(2*b)*m**3 + 9*x**(2*b*n)*e**(2*a)*c**(2*b)
*m**2 + 9*x**(2*b*n)*e**(2*a)*c**(2*b)*m + 3*x**(2*b*n)*e**(2*a)*c**(2*b)
- 3*b**3*n**3 - b**2*m*n**2 - b**2*n**2 + 3*b*m**2*n + 6*b*m*n + 3*b*n + m
**3 + 3*m**2 + 3*m + 1))/(8*x**(3*b*n)*e**(3*a)*c**(3*b)*(9*b**4*n**4 - 10
*b**2*m**2*n**2 - 20*b**2*m*n**2 - 10*b**2*n**2 + m**4 + 4*m**3 + 6*m**2 +
 4*m + 1))
```

3.246 $\int x^m \cosh^4(a+b\log(cx^n))\,dx$

Optimal result

Integrand size = 17, antiderivative size = 266

$$\begin{aligned}\int x^m \cosh^4(a+b\log(cx^n))\,dx &= \frac{24b^4n^4x^{1+m}}{(1+m)\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad - \frac{12b^2(1+m)n^2x^{1+m}\cosh^2(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} + \frac{(1+m)x^{1+m}\cosh^4(a+b\log(cx^n))}{(1+m)^2-16b^2n^2} \\ &\quad + \frac{24b^3n^3x^{1+m}\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{\left((1+m)^2-16b^2n^2\right)\left((1+m)^2-4b^2n^2\right)} \\ &\quad - \frac{4bnx^{1+m}\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{(1+m)^2-16b^2n^2}\end{aligned}$$

output
```
24*b^4*n^4*x^(1+m)/(1+m)/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2*n^2)-12*b^2*(
1+m)*n^2*x^(1+m)*cosh(a+b*ln(c*x^n))^2/((1+m)^2-16*b^2*n^2)/((1+m)^2-4*b^2
*n^2)+(1+m)*x^(1+m)*cosh(a+b*ln(c*x^n))^4/((1+m)^2-16*b^2*n^2)+24*b^3*n^3*
x^(1+m)*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/((1+m)^2-16*b^2*n^2)/((1+m
)^2-4*b^2*n^2)-4*b*n*x^(1+m)*cosh(a+b*ln(c*x^n))^3*sinh(a+b*ln(c*x^n))/((1
+m)^2-16*b^2*n^2)
```

Mathematica [A] (verified)

Time = 2.40 (sec) , antiderivative size = 311, normalized size of antiderivative = 1.17

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \frac{1}{8}x^{1+m}\left(\frac{3}{1+m}\right.$$
$$+\frac{4\sinh(2bn\log(x))\left(-2bn\cosh\left(2(a-bn\log(x)+b\log(cx^n))\right)+(1+m)\sinh\left(2(a-bn\log(x)+b\log\right.\right.}{(1+m-2bn)(1+m+2bn)}$$
$$+\frac{4\cosh(2bn\log(x))\left((1+m)\cosh\left(2(a-bn\log(x)+b\log(cx^n))\right)-2bn\sinh\left(2(a-bn\log(x)+b\log(c\right.\right.}{(1+m-2bn)(1+m+2bn)}$$
$$+\frac{\sinh(4bn\log(x))\left(-4bn\cosh\left(4(a-bn\log(x)+b\log(cx^n))\right)+(1+m)\sinh\left(4(a-bn\log(x)+b\log(c\right.\right.}{(1+m-4bn)(1+m+4bn)}$$
$$+\frac{\cosh(4bn\log(x))\left((1+m)\cosh\left(4(a-bn\log(x)+b\log(cx^n))\right)-4bn\sinh\left(4(a-bn\log(x)+b\log(cx\right.\right.}{(1+m-4bn)(1+m+4bn)}$$

input
```
Integrate[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
(x^(1 + m)*(3/(1 + m) + (4*Sinh[2*b*n*Log[x]]*(-2*b*n*Cosh[2*(a - b*n*Log[
x] + b*Log[c*x^n])] + (1 + m)*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^n])]))/((
1 + m - 2*b*n)*(1 + m + 2*b*n)) + (4*Cosh[2*b*n*Log[x]]*((1 + m)*Cosh[2*(a
 - b*n*Log[x] + b*Log[c*x^n])] - 2*b*n*Sinh[2*(a - b*n*Log[x] + b*Log[c*x^
n])]))/((1 + m - 2*b*n)*(1 + m + 2*b*n)) + (Sinh[4*b*n*Log[x]]*(-4*b*n*Cos
h[4*(a - b*n*Log[x] + b*Log[c*x^n])] + (1 + m)*Sinh[4*(a - b*n*Log[x] + b*
Log[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n)) + (Cosh[4*b*n*Log[x]]*((1
 + m)*Cosh[4*(a - b*n*Log[x] + b*Log[c*x^n])] - 4*b*n*Sinh[4*(a - b*n*Log[
x] + b*Log[c*x^n])]))/((1 + m - 4*b*n)*(1 + m + 4*b*n))))/8
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.89, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.176, Rules used = {6056, 6056, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx$$

$$\Big\downarrow \; 6056$$

$$-\frac{12b^2n^2\int x^m\cosh^2(a+b\log(cx^n))\,dx}{(m+1)^2-16b^2n^2}+\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

$$\Big\downarrow \; 6056$$

$$-\frac{12b^2n^2\left(-\frac{2b^2n^2\int x^m dx}{(m+1)^2-4b^2n^2}+\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}\right)}{(m+1)^2-16b^2n^2}+\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

$$\Big\downarrow \; 15$$

$$\frac{(m+1)x^{m+1}\cosh^4(a+b\log(cx^n))}{-16b^2n^2+m^2+2m+1}-\frac{12b^2n^2\left(\frac{(m+1)x^{m+1}\cosh^2(a+b\log(cx^n))}{-4b^2n^2+m^2+2m+1}-\frac{2bnx^{m+1}\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{(m+1)^2-4b^2n^2}-\frac{2b^2n^2x^{m+1}}{(m+1)((m+1)^2-4b^2n^2)}\right)}{(m+1)^2-16b^2n^2}-\frac{4bnx^{m+1}\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{(m+1)^2-16b^2n^2}$$

input
```
Int[x^m*Cosh[a + b*Log[c*x^n]]^4,x]
```

output
```
((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^4)/(1 + 2*m + m^2 - 16*b^2*n^2)
- (4*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/((1 +
m)^2 - 16*b^2*n^2) - (12*b^2*n^2*((-2*b^2*n^2*x^(1 + m))/((1 + m)*((1 + m)
^2 - 4*b^2*n^2)) + ((1 + m)*x^(1 + m)*Cosh[a + b*Log[c*x^n]]^2)/(1 + 2*m +
 m^2 - 4*b^2*n^2) - (2*b*n*x^(1 + m)*Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log
[c*x^n]])/((1 + m)^2 - 4*b^2*n^2)))/((1 + m)^2 - 16*b^2*n^2)
```

Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 6056
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_
.), x_Symbol] :> Simp[(-(m + 1))*(e*x)^(m + 1)*(Cosh[d*(a + b*Log[c*x^n])]^
p/(b^2*d^2*e*n^2*p^2 - e*(m + 1)^2)), x] + (Simp[b*d*n*p*(e*x)^(m + 1)*Sinh
[d*(a + b*Log[c*x^n])]*(Cosh[d*(a + b*Log[c*x^n])]^(p - 1)/(b^2*d^2*e*n^2*p
^2 - e*(m + 1)^2)), x] + Simp[b^2*d^2*n^2*p*((p - 1)/(b^2*d^2*n^2*p^2 - (m
+ 1)^2))   Int[(e*x)^m*Cosh[d*(a + b*Log[c*x^n])]^(p - 2), x], x]) /; FreeQ
[{a, b, c, d, e, m, n}, x] && IGtQ[p, 1] && NeQ[b^2*d^2*n^2*p^2 - (m + 1)^2
, 0]
```

Maple [F]

$$\int x^m \cosh\left(a+b\ln\left(c\,x^n\right)\right)^4 dx$$

input
```
int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

output
```
int(x^m*cosh(a+b*ln(c*x^n))^4,x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1123 vs. 2(283) = 566.

Time = 0.13 (sec) , antiderivative size = 1123, normalized size of antiderivative = 4.22

$$\int x^m \cosh^4\left(a+b\log\left(cx^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="fricas")
```

output
```
1/8*((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*c
osh(b*n*log(x) + b*log(c) + a)^4*cosh(m*log(x)) + 4*(m^4 + 4*m^3 - 16*(b^2
*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*log(c)
+ a)^2*cosh(m*log(x)) + ((m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 +
6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (m^4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m +
b^2)*n^2 + 6*m^2 + 4*m + 1)*x*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) +
 a)^4 + 16*((4*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh
(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + (4*(b^3*m + b^3)*n^3 - (b*m^3
 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*sinh(m*log(x)
))*sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(64*b^4*n^4 + m^4 + 4*m^3 - 20*(b
^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + 2*(3*(m^
4 + 4*m^3 - 4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*
log(x) + b*log(c) + a)^2*cosh(m*log(x)) + 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2
*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(m*log(x)) + (3*(m^4 + 4*m^3 -
4*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 + 4*m + 1)*x*cosh(b*n*log(x) + b*l
og(c) + a)^2 + 2*(m^4 + 4*m^3 - 16*(b^2*m^2 + 2*b^2*m + b^2)*n^2 + 6*m^2 +
 4*m + 1)*x)*sinh(m*log(x)))*sinh(b*n*log(x) + b*log(c) + a)^2 + 16*((4*(b
^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3*b*m + b)*n)*x*cosh(b*n*log(x) + b*l
og(c) + a)^3*cosh(m*log(x)) + (16*(b^3*m + b^3)*n^3 - (b*m^3 + 3*b*m^2 + 3
*b*m + b)*n)*x*cosh(b*n*log(x) + b*log(c) + a)*cosh(m*log(x)) + ((4*(b^...
```

Sympy [F(-1)]

Timed out.

$$\int x^m \cosh^4(a + b\log(cx^n))\,dx = \text{Timed out}$$

input
```
integrate(x**m*cosh(a+b*ln(c*x**n))**4,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.61

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \frac{c^{4b}xe^{(4b\log(x^n)+m\log(x)+4a)}}{16(4bn+m+1)} + \frac{c^{2b}xe^{(2b\log(x^n)+m\log(x)+2a)}}{4(2bn+m+1)} - \frac{xe^{(-2b\log(x^n)+m\log(x)-2a)}}{4(2bc^{2b}n-c^{2b}(m+1))} - \frac{xe^{(-4b\log(x^n)+m\log(x)-4a)}}{16(4bc^{4b}n-c^{4b}(m+1))} + \frac{3x^{m+1}}{8(m+1)}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="maxima")
```

output
```
1/16*c^(4*b)*x*e^(4*b*log(x^n) + m*log(x) + 4*a)/(4*b*n + m + 1) + 1/4*c^(
2*b)*x*e^(2*b*log(x^n) + m*log(x) + 2*a)/(2*b*n + m + 1) - 1/4*x*e^(-2*b*l
og(x^n) + m*log(x) - 2*a)/(2*b*c^(2*b)*n - c^(2*b)*(m + 1)) - 1/16*x*e^(-4
*b*log(x^n) + m*log(x) - 4*a)/(4*b*c^(4*b)*n - c^(4*b)*(m + 1)) + 3/8*x^(m
 + 1)/(m + 1)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6880 vs. 2(283) = 566.

Time = 0.20 (sec) , antiderivative size = 6880, normalized size of antiderivative = 25.86

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
integrate(x^m*cosh(a+b*log(c*x^n))^4,x, algorithm="giac")
```

output
```
b^3*c^(4*b)*m*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*
b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 1
0*m^3 + 10*m^2 + 5*m + 1) + 8*b^3*c^(2*b)*m*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(6
4*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*
m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b^2*c^(4*b)*
m^2*n^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^
2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10
*m^2 + 5*m + 1) + b^3*c^(4*b)*n^3*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 +
64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4
- 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 4*b^2*c^(2*b)*m^2*n^2*x*x^(2*b
*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n
^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1)
+ 8*b^3*c^(2*b)*n^3*x*x^(2*b*n)*x^m*e^(2*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 2
0*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 +
 10*m^3 + 10*m^2 + 5*m + 1) + 24*b^4*n^4*x*x^m/(64*b^4*m*n^4 + 64*b^4*n^4
- 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60*b^2*m*n^2 + 5*m^4 - 20*b^2*n^
2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/4*b*c^(4*b)*m^3*n*x*x^(4*b*n)*x^m*e^(4*
a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*m^3*n^2 - 60*b^2*m^2*n^2 + m^5 - 60
*b^2*m*n^2 + 5*m^4 - 20*b^2*n^2 + 10*m^3 + 10*m^2 + 5*m + 1) - 1/2*b^2*c^(
4*b)*m*n^2*x*x^(4*b*n)*x^m*e^(4*a)/(64*b^4*m*n^4 + 64*b^4*n^4 - 20*b^2*...
```

Mupad [B] (verification not implemented)

Time = 2.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.50

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \frac{3\,x\,x^m}{8\,m+8}+\frac{x\,x^m\,\mathrm{e}^{-2\,a}}{{\left(c\,x^n\right)}^{2\,b}\,\left(4\,m-8\,b\,n+4\right)}+\frac{x\,x^m\,\mathrm{e}^{2\,a}\,{\left(c\,x^n\right)}^{2\,b}}{4\,m+8\,b\,n+4}+\frac{x\,x^m\,\mathrm{e}^{-4\,a}}{{\left(c\,x^n\right)}^{4\,b}\,\left(16\,m-64\,b\,n+16\right)}+\frac{x\,x^m\,\mathrm{e}^{4\,a}\,{\left(c\,x^n\right)}^{4\,b}}{16\,m+64\,b\,n+16}$$

input
```
int(x^m*cosh(a + b*log(c*x^n))^4,x)
```

output
```
(3*x*x^m)/(8*m + 8) + (x*x^m*exp(-2*a))/((c*x^n)^(2*b)*(4*m - 8*b*n + 4))
+ (x*x^m*exp(2*a)*(c*x^n)^(2*b))/(4*m + 8*b*n + 4) + (x*x^m*exp(-4*a))/((c
*x^n)^(4*b)*(16*m - 64*b*n + 16)) + (x*x^m*exp(4*a)*(c*x^n)^(4*b))/(16*m +
 64*b*n + 16)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1329, normalized size of antiderivative = 5.00

$$\int x^m \cosh^4(a+b\log(cx^n))\,dx = \text{Too large to display}$$

input
```
int(x^m*cosh(a+b*log(c*x^n))^4,x)
```

output
```
(x**m*x*(16*x**(8*b*n)*e**(8*a)*c**(8*b)*b**3*m*n**3 + 16*x**(8*b*n)*e**(8
*a)*c**(8*b)*b**3*n**3 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)*b**2*m**2*n**2 - 8
*x**(8*b*n)*e**(8*a)*c**(8*b)*b**2*m*n**2 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)
*b**2*n**2 - 4*x**(8*b*n)*e**(8*a)*c**(8*b)*b*m**3*n - 12*x**(8*b*n)*e**(8
*a)*c**(8*b)*b*m**2*n - 12*x**(8*b*n)*e**(8*a)*c**(8*b)*b*m*n - 4*x**(8*b*
n)*e**(8*a)*c**(8*b)*b*n + x**(8*b*n)*e**(8*a)*c**(8*b)*m**4 + 4*x**(8*b*n
)*e**(8*a)*c**(8*b)*m**3 + 6*x**(8*b*n)*e**(8*a)*c**(8*b)*m**2 + 4*x**(8*b
*n)*e**(8*a)*c**(8*b)*m + x**(8*b*n)*e**(8*a)*c**(8*b) + 128*x**(6*b*n)*e*
*(6*a)*c**(6*b)*b**3*m*n**3 + 128*x**(6*b*n)*e**(6*a)*c**(6*b)*b**3*n**3 -
 64*x**(6*b*n)*e**(6*a)*c**(6*b)*b**2*m**2*n**2 - 128*x**(6*b*n)*e**(6*a)*
c**(6*b)*b**2*m*n**2 - 64*x**(6*b*n)*e**(6*a)*c**(6*b)*b**2*n**2 - 8*x**(6
*b*n)*e**(6*a)*c**(6*b)*b*m**3*n - 24*x**(6*b*n)*e**(6*a)*c**(6*b)*b*m**2*
n - 24*x**(6*b*n)*e**(6*a)*c**(6*b)*b*m*n - 8*x**(6*b*n)*e**(6*a)*c**(6*b)
*b*n + 4*x**(6*b*n)*e**(6*a)*c**(6*b)*m**4 + 16*x**(6*b*n)*e**(6*a)*c**(6*
b)*m**3 + 24*x**(6*b*n)*e**(6*a)*c**(6*b)*m**2 + 16*x**(6*b*n)*e**(6*a)*c*
*(6*b)*m + 4*x**(6*b*n)*e**(6*a)*c**(6*b) + 384*x**(4*b*n)*e**(4*a)*c**(4*
b)*b**4*n**4 - 120*x**(4*b*n)*e**(4*a)*c**(4*b)*b**2*m**2*n**2 - 240*x**(4
*b*n)*e**(4*a)*c**(4*b)*b**2*m*n**2 - 120*x**(4*b*n)*e**(4*a)*c**(4*b)*b**
2*n**2 + 6*x**(4*b*n)*e**(4*a)*c**(4*b)*m**4 + 24*x**(4*b*n)*e**(4*a)*c**(
4*b)*m**3 + 36*x**(4*b*n)*e**(4*a)*c**(4*b)*m**2 + 24*x**(4*b*n)*e**(4*...
```

3.247 $\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 18

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(a+b\log(cx^n))}{bn}$$

output
```
sinh(a+b*ln(c*x^n))/b/n
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 37 vs. 2(18) = 36.

Time = 0.01 (sec) , antiderivative size = 37, normalized size of antiderivative = 2.06

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\cosh(b\log(cx^n))\sinh(a)}{bn} + \frac{\cosh(a)\sinh(b\log(cx^n))}{bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]/x,x]
```

output
```
(Cosh[b*Log[c*x^n]]*Sinh[a])/(b*n) + (Cosh[a]*Sinh[b*Log[c*x^n]])/(b*n)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3039, 3042, 3117}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh (a+b \log (cx^n))}{x} \, dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh (a+b \log (cx^n))d\log (cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin \left(ia+ib \log (cx^n)+\frac{\pi}{2}\right)d\log (cx^n)}{n}$$

$$\Big\downarrow \text{3117}$$

$$\frac{\sinh (a+b \log (cx^n))}{bn}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]/x,x]
```

output
```
Sinh[a + b*Log[c*x^n]]/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3117
```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

method	result	size
derivativedivides	$\frac{\sinh(a+b\ln(c\,x^n))}{bn}$	19
default	$\frac{\sinh(a+b\ln(c\,x^n))}{bn}$	19
parallelrisch	$\frac{\sinh(a+b\ln(c\,x^n))}{bn}$	19

input
```
int(cosh(a+b*ln(c*x^n))/x,x,method=_RETURNVERBOSE)
```

output
```
sinh(a+b*ln(c*x^n))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.06

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(bn\log(x)+b\log(c)+a)}{bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="fricas")
```

output
```
sinh(b*n*log(x) + b*log(c) + a)/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 34 vs. $2(14) = 28$.

Time = 0.24 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.89

$$\int \frac{\cosh(a + b\log(cx^n))}{x}\, dx = \begin{cases} \log(x)\cosh(a) & \text{for } b = 0 \wedge (b = 0 \vee n = 0) \\ \log(x)\cosh(a + b\log(c)) & \text{for } n = 0 \\ \frac{\sinh(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))/x,x)
```

output
```
Piecewise((log(x)*cosh(a), Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*cosh
(a + b*log(c)), Eq(n, 0)), (sinh(a + b*log(c*x**n))/(b*n), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(a + b\log(cx^n))}{x}\, dx = \frac{\sinh(b\log(cx^n) + a)}{bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="maxima")
```

output
```
sinh(b*log(c*x^n) + a)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(18) = 36$.

Time = 0.11 (sec) , antiderivative size = 42, normalized size of antiderivative = 2.33

$$\int \frac{\cosh(a + b\log(cx^n))}{x}\, dx = \frac{\left(c^{2b}x^{bn}e^{(2a)} - \frac{1}{x^{bn}}\right)e^{(-a)}}{2bc^{b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))/x,x, algorithm="giac")
```

output
```
1/2*(c^(2*b)*x^(b*n)*e^(2*a) - 1/x^(b*n))*e^(-a)/(b*c^b*n)
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(a+b\,\ln(c\,x^n))}{b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))/x,x)
```

output
```
sinh(a + b*log(c*x^n))/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(\log(x^n c)\,b+a)}{bn}$$

input
```
int(cosh(a+b*log(c*x^n))/x,x)
```

output
```
sinh(log(x**n*c)*b + a)/(b*n)
```

3.248 $\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 39

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx=\frac{\log(x)}{2}+\frac{\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{2bn}$$

output
```
1/2*ln(x)+1/2*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.92

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx=\frac{2(a+b\log(cx^n))+\sinh(2(a+b\log(cx^n)))}{4bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(2*(a + b*Log[c*x^n]) + Sinh[2*(a + b*Log[c*x^n])])/(4*b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.13, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \ 3039$$

$$\frac{\int \cosh^2(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3042$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \ 3115$$

$$\frac{\frac{1}{2}\int 1d\log(cx^n)+\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}}{n}$$

$$\Big\downarrow \ 24$$

$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}+\frac{1}{2}\log(cx^n)}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^2/x,x]
```

output
```
(Log[c*x^n]/2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b))/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.77

method	result	size
parallelrisch	$\frac{2\ln(x)bn+\sinh(2b\ln(c\,x^{n})+2a)}{4bn}$	30
derivativedivides	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45
default	$\frac{\frac{\cosh(a+b\ln(c\,x^{n}))\sinh(a+b\ln(c\,x^{n}))}{2}+\frac{b\ln(c\,x^{n})}{2}+\frac{a}{2}}{nb}$	45

input
```
int(cosh(a+b*ln(c*x^n))^2/x,x,method=_RETURNVERBOSE)
```

output
```
1/4*(2*ln(x)*b*n+sinh(2*b*ln(c*x^n)+2*a))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{bn\log(x)+\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)}{2\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="fricas")
```

output
```
1/2*(b*n*log(x) + cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(
c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**2/x,x)
```

output
```
Integral(cosh(a + b*log(c*x**n))**2/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} + \frac{1}{2}\log(x)$$

input
```
integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="maxima")
```

output
```
1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) - 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) +
 1/2*log(x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. $2(35) = 70$.

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.10

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{\left(c^{4\,b}x^{2\,bn}e^{(4\,a)} + 4\,c^{2\,b}e^{(2\,a)}\log\left(x^{bn}\right) - \frac{2\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{2\,bn}}\right)e^{(-2\,a)}}{8\,bc^{2\,b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^2/x,x, algorithm="giac")
```

output
```
1/8*(c^(4*b)*x^(2*b*n)*e^(4*a) + 4*c^(2*b)*e^(2*a)*log(x^(b*n)) - (2*c^(2*
b)*x^(2*b*n)*e^(2*a) + 1)/x^(2*b*n))*e^(-2*a)/(b*c^(2*b)*n)
```

Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.82

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{\ln(x^n)}{2\,n} + \frac{\sinh(2\,a+2\,b\,\ln(c\,x^n))}{4\,b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))^2/x,x)
```

output
```
log(x^n)/(2*n) + sinh(2*a + 2*b*log(c*x^n))/(4*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.82

$$\int \frac{\cosh^2(a+b\log(cx^n))}{x}\,dx = \frac{x^{4bn}e^{4a}c^{4b} + 4x^{2bn}e^{2a}c^{2b}\log(x)\,bn - 1}{8x^{2bn}e^{2a}c^{2b}bn}$$

input
```
int(cosh(a+b*log(c*x^n))^2/x,x)
```

output
```
(x**(4*b*n)*e**(4*a)*c**(4*b) + 4*x**(2*b*n)*e**(2*a)*c**(2*b)*log(x)*b*n
- 1)/(8*x**(2*b*n)*e**(2*a)*c**(2*b)*b*n)
```

3.249 $\int \frac{\cosh^3(a+b\log(cx^n))}{x} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 42

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x} \, dx = \frac{\sinh(a+b\log(cx^n))}{bn} + \frac{\sinh^3(a+b\log(cx^n))}{3bn}$$

output
```
sinh(a+b*ln(c*x^n))/b/n+1/3*sinh(a+b*ln(c*x^n))^3/b/n
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x} \, dx = \frac{\sinh(a+b\log(cx^n))}{bn} + \frac{\sinh^3(a+b\log(cx^n))}{3bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

output
```
Sinh[a + b*Log[c*x^n]]/(b*n) + Sinh[a + b*Log[c*x^n]]^3/(3*b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.07, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.235, Rules used = {3039, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh^3(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^3d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3113}$$

$$\frac{i\int \left(\sinh^2(a+b\log(cx^n))+1\right)d(-i\sinh(a+b\log(cx^n)))}{bn}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{1}{3}i\sinh^3(a+b\log(cx^n))-i\sinh(a+b\log(cx^n))\right)}{bn}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^3/x,x]
```

output
```
(I*((-I)*Sinh[a + b*Log[c*x^n]] - (I/3)*Sinh[a + b*Log[c*x^n]]^3))/(b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 7.80 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.86

method	result	size
derivativedivides	$\frac{\left(\frac{2}{3}+\frac{\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{3}\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$	36
default	$\frac{\left(\frac{2}{3}+\frac{\cosh\left(a+b\ln\left(c\,x^{n}\right)\right)^{2}}{3}\right)\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{nb}$	36
parallelrisch	$\frac{\sinh\left(3b\ln\left(c\,x^{n}\right)+3a\right)+9\sinh\left(a+b\ln\left(c\,x^{n}\right)\right)}{12bn}$	37

input
```
int(cosh(a+b*ln(c*x^n))^3/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(2/3+1/3*cosh(a+b*ln(c*x^n))^2)*sinh(a+b*ln(c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.26

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(bn\log(x)+b\log(c)+a)^3+3\left(\cosh(bn\log(x)+b\log(c)+a)^2+3\right)\sinh(bn\log(x)+b\log(c)+a)}{12\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="fricas")
```

output
```
1/12*(sinh(b*n*log(x) + b*log(c) + a)^3 + 3*(cosh(b*n*log(x) + b*log(c) +
a)^2 + 3)*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 71 vs. 2(32) = 64.

Time = 1.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.69

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx = \begin{cases} \log(x)\cosh^3(a) & \text{for } b=0 \wedge (b=0 \vee n=0) \\ \log(x)\cosh^3(a+b\log(c)) & \text{for } n=0 \\ -\frac{2\sinh^3(a+b\log(cx^n))}{3bn}+\frac{\sinh(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**3/x,x)
```

output
```
Piecewise((log(x)*cosh(a)**3, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*c
osh(a + b*log(c))**3, Eq(n, 0)), (-2*sinh(a + b*log(c*x**n))**3/(3*b*n) +
sinh(a + b*log(c*x**n))*cosh(a + b*log(c*x**n))**2/(b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(40) = 80$.

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.05

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx = \frac{e^{(3\,b\log(cx^n)+3\,a)}}{24\,bn} + \frac{3\,e^{(b\log(cx^n)+a)}}{8\,bn} - \frac{3\,e^{(-b\log(cx^n)-a)}}{8\,bn} - \frac{e^{(-3\,b\log(cx^n)-3\,a)}}{24\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="maxima")
```

output
```
1/24*e^(3*b*log(c*x^n) + 3*a)/(b*n) + 3/8*e^(b*log(c*x^n) + a)/(b*n) - 3/8
*e^(-b*log(c*x^n) - a)/(b*n) - 1/24*e^(-3*b*log(c*x^n) - 3*a)/(b*n)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. $2(40) = 80$.

Time = 0.11 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.93

$$\int \frac{\cosh^3(a+b\log(cx^n))}{x}\,dx = \frac{\left(c^{6\,b}x^{3\,bn}e^{(6\,a)} + 9\,c^{4\,b}x^{bn}e^{(4\,a)} - \frac{9\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{3\,bn}}\right)e^{(-3\,a)}}{24\,bc^{3\,b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^3/x,x, algorithm="giac")
```

output
```
1/24*(c^(6*b)*x^(3*b*n)*e^(6*a) + 9*c^(4*b)*x^(b*n)*e^(4*a) - (9*c^(2*b)*x
^(2*b*n)*e^(2*a) + 1)/x^(3*b*n))*e^(-3*a)/(b*c^(3*b)*n)
```

Mupad [B] (verification not implemented)

Time = 2.94 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^3(a + b\log(cx^n))}{x} dx = \frac{\sinh(a + b\ln(c x^n))^3 + 3\sinh(a + b\ln(c x^n))}{3 b n}$$

input
```
int(cosh(a + b*log(c*x^n))^3/x,x)
```

output
```
(3*sinh(a + b*log(c*x^n)) + sinh(a + b*log(c*x^n))^3)/(3*b*n)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.02

$$\int \frac{\cosh^3(a + b\log(cx^n))}{x} dx = \frac{x^{6bn}e^{6a}c^{6b} + 9x^{4bn}e^{4a}c^{4b} - 9x^{2bn}e^{2a}c^{2b} - 1}{24x^{3bn}e^{3a}c^{3b}bn}$$

input
```
int(cosh(a+b*log(c*x^n))^3/x,x)
```

output
```
(x**(6*b*n)*e**(6*a)*c**(6*b) + 9*x**(4*b*n)*e**(4*a)*c**(4*b) - 9*x**(2*b
*n)*e**(2*a)*c**(2*b) - 1)/(24*x**(3*b*n)*e**(3*a)*c**(3*b)*b*n)
```

3.250 $\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 73

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{3\log(x)}{8} + \frac{3\cosh(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{8bn} + \frac{\cosh^3(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{4bn}$$

output
```
3/8*ln(x)+3/8*cosh(a+b*ln(c*x^n))*sinh(a+b*ln(c*x^n))/b/n+1/4*cosh(a+b*ln(
c*x^n))^3*sinh(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.70

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{12(a+b\log(cx^n)) + 8\sinh(2(a+b\log(cx^n))) + \sinh(4(a+b\log(cx^n)))}{32bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

output
```
(12*(a + b*Log[c*x^n]) + 8*Sinh[2*(a + b*Log[c*x^n])] + Sinh[4*(a + b*Log[
c*x^n])])/(32*b*n)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.10, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.353, Rules used = {3039, 3042, 3115, 3042, 3115, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh^4(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^4d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\int \cosh^2(a+b\log(cx^n))d\log(cx^n)+\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}+\frac{3}{4}\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^2d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{4}\left(\frac{1}{2}\int 1d\log(cx^n)+\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}\right)+\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}}{n}$$

$$\Big\downarrow \text{24}$$

$$\frac{\frac{\sinh(a+b\log(cx^n))\cosh^3(a+b\log(cx^n))}{4b}+\frac{3}{4}\left(\frac{\sinh(a+b\log(cx^n))\cosh(a+b\log(cx^n))}{2b}+\frac{1}{2}\log(cx^n)\right)}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^4/x,x]
```

output
```
((Cosh[a + b*Log[c*x^n]]^3*Sinh[a + b*Log[c*x^n]])/(4*b) + (3*(Log[c*x^n]/
2 + (Cosh[a + b*Log[c*x^n]]*Sinh[a + b*Log[c*x^n]])/(2*b)))/4)/n
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

Maple [A] (verified)

Time = 32.26 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.63

method	result	size
parallelrisch	$\frac{12 \ln \left(x \right) b n +\sinh \left(4 b \ln \left(c \,x^{n}\right)+4 a \right)+8 \sinh \left(2 b \ln \left(c \,x^{n}\right)+2 a \right)}{32 b n}$	46
derivativedivides	$\frac{\left(\frac{\cosh \left(a +b \ln \left(c \,x^{n}\right)\right)^{3}}{4}+\frac{3 \cosh \left(a +b \ln \left(c \,x^{n}\right)\right)}{8}\right) \sinh \left(a +b \ln \left(c \,x^{n}\right)\right)+\frac{3 b \ln \left(c \,x^{n}\right)}{8}+\frac{3 a}{8}}{n b}$	62
default	$\frac{\left(\frac{\cosh \left(a +b \ln \left(c \,x^{n}\right)\right)^{3}}{4}+\frac{3 \cosh \left(a +b \ln \left(c \,x^{n}\right)\right)}{8}\right) \sinh \left(a +b \ln \left(c \,x^{n}\right)\right)+\frac{3 b \ln \left(c \,x^{n}\right)}{8}+\frac{3 a}{8}}{n b}$	62

input
```
int(cosh(a+b*ln(c*x^n))^4/x,x,method=_RETURNVERBOSE)
```

output
```
1/32*(12*ln(x)*b*n+sinh(4*b*ln(c*x^n)+4*a)+8*sinh(2*b*ln(c*x^n)+2*a))/b/n
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.15

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c)+a)^3+3\,bn\log(x)+\left(\cosh(bn\log(x)+b\log\right.}{8\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="fricas")
```

output
```
1/8*(cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + 3
*b*n*log(x) + (cosh(b*n*log(x) + b*log(c) + a)^3 + 4*cosh(b*n*log(x) + b*l
og(c) + a))*sinh(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**4/x,x)
```

output
```
Integral(cosh(a + b*log(c*x**n))**4/x, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.27

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{e^{(4\,b\log(cx^n)+4\,a)}}{64\,bn} + \frac{e^{(2\,b\log(cx^n)+2\,a)}}{8\,bn} - \frac{e^{(-2\,b\log(cx^n)-2\,a)}}{8\,bn} - \frac{e^{(-4\,b\log(cx^n)-4\,a)}}{64\,bn} + \frac{3}{8}\log(x)$$

input
```
integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="maxima")
```

output
```
1/64*e^(4*b*log(c*x^n) + 4*a)/(b*n) + 1/8*e^(2*b*log(c*x^n) + 2*a)/(b*n) -
 1/8*e^(-2*b*log(c*x^n) - 2*a)/(b*n) - 1/64*e^(-4*b*log(c*x^n) - 4*a)/(b*n
) + 3/8*log(x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.59

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{\left(c^{8\,b}x^{4\,bn}e^{(8\,a)} + 8\,c^{6\,b}x^{2\,bn}e^{(6\,a)} + 24\,c^{4\,b}e^{(4\,a)}\log\left(x^{bn}\right) - \frac{18\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+8\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+1}{x^{4\,bn}}\right)e^{(-4\,a)}}{64\,bc^{4\,b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^4/x,x, algorithm="giac")
```

output
```
1/64*(c^(8*b)*x^(4*b*n)*e^(8*a) + 8*c^(6*b)*x^(2*b*n)*e^(6*a) + 24*c^(4*b)
*e^(4*a)*log(x^(b*n)) - (18*c^(4*b)*x^(4*b*n)*e^(4*a) + 8*c^(2*b)*x^(2*b*n
)*e^(2*a) + 1)/x^(4*b*n))*e^(-4*a)/(b*c^(4*b)*n)
```

Mupad [B] (verification not implemented)

Time = 3.15 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.68

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{3\,\ln(x^n)}{8\,n}+\frac{\frac{\sinh(2\,a+2\,b\,\ln(c\,x^n))}{4}+\frac{\sinh(4\,a+4\,b\,\ln(c\,x^n))}{32}}{b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))^4/x,x)
```

output
```
(3*log(x^n))/(8*n) + (sinh(2*a + 2*b*log(c*x^n))/4 + sinh(4*a + 4*b*log(c*
x^n))/32)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.47

$$\int \frac{\cosh^4(a+b\log(cx^n))}{x}\,dx = \frac{x^{8bn}e^{8a}c^{8b}+8x^{6bn}e^{6a}c^{6b}+24x^{4bn}e^{4a}c^{4b}\log(x)\,bn-8x^{2bn}e^{2a}c^{2b}-1}{64x^{4bn}e^{4a}c^{4b}bn}$$

input
```
int(cosh(a+b*log(c*x^n))^4/x,x)
```

output
```
(x**(8*b*n)*e**(8*a)*c**(8*b) + 8*x**(6*b*n)*e**(6*a)*c**(6*b) + 24*x**(4*
b*n)*e**(4*a)*c**(4*b)*log(x)*b*n - 8*x**(2*b*n)*e**(2*a)*c**(2*b) - 1)/(6
4*x**(4*b*n)*e**(4*a)*c**(4*b)*b*n)
```

3.251 $\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 65

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(a+b\log(cx^n))}{bn} + \frac{2\sinh^3(a+b\log(cx^n))}{3bn} + \frac{\sinh^5(a+b\log(cx^n))}{5bn}$$

output
```
sinh(a+b*ln(c*x^n))/b/n+2/3*sinh(a+b*ln(c*x^n))^3/b/n+1/5*sinh(a+b*ln(c*x^
n))^5/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(a+b\log(cx^n))}{bn} + \frac{2\sinh^3(a+b\log(cx^n))}{3bn} + \frac{\sinh^5(a+b\log(cx^n))}{5bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

output
```
Sinh[a + b*Log[c*x^n]]/(b*n) + (2*Sinh[a + b*Log[c*x^n]]^3)/(3*b*n) + Sinh
[a + b*Log[c*x^n]]^5/(5*b*n)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.235$, Rules used = {3039, 3042, 3113, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \cosh^5(a+b\log(cx^n))d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^5d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3113}$$
$$\frac{i\int \left(\sinh^4(a+b\log(cx^n))+2\sinh^2(a+b\log(cx^n))+1\right)d(-i\sinh(a+b\log(cx^n)))}{bn}$$
$$\Big\downarrow \text{2009}$$
$$\frac{i\left(-\frac{1}{5}i\sinh^5(a+b\log(cx^n))-\frac{2}{3}i\sinh^3(a+b\log(cx^n))-i\sinh(a+b\log(cx^n))\right)}{bn}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^5/x,x]
```

output
```
(I*((-I)*Sinh[a + b*Log[c*x^n]] - ((2*I)/3)*Sinh[a + b*Log[c*x^n]]^3 - (I/
5)*Sinh[a + b*Log[c*x^n]]^5))/(b*n)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3113
```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Simp[-d^(-1)   Subst[Int[Exp
and[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
 && IGtQ[(n - 1)/2, 0]
```

Maple [A] (verified)

Time = 115.65 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

method	result	size
derivativedivides	$\frac{\left(\frac{8}{15}+\frac{\cosh(a+b\ln(c\,x^{n}))^{4}}{5}+\frac{4\cosh(a+b\ln(c\,x^{n}))^{2}}{15}\right)\sinh(a+b\ln(c\,x^{n}))}{nb}$	51
default	$\frac{\left(\frac{8}{15}+\frac{\cosh(a+b\ln(c\,x^{n}))^{4}}{5}+\frac{4\cosh(a+b\ln(c\,x^{n}))^{2}}{15}\right)\sinh(a+b\ln(c\,x^{n}))}{nb}$	51
parallelrisch	$\frac{3\sinh(5b\ln(c\,x^{n})+5a)+25\sinh(3b\ln(c\,x^{n})+3a)+150\sinh(a+b\ln(c\,x^{n}))}{240bn}$	55

input
```
int(cosh(a+b*ln(c*x^n))^5/x,x,method=_RETURNVERBOSE)
```

output
```
1/n/b*(8/15+1/5*cosh(a+b*ln(c*x^n))^4+4/15*cosh(a+b*ln(c*x^n))^2)*sinh(a+b
*ln(c*x^n))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{3\,\sinh(bn\log(x)+b\log(c)+a)^5+5\left(6\,\cosh(bn\log(x)+b\log(c)+a)^2+5\right)\sinh(bn\log(x)+b\log(c$$

input
```
integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="fricas")
```

output
```
1/240*(3*sinh(b*n*log(x) + b*log(c) + a)^5 + 5*(6*cosh(b*n*log(x) + b*log(
c) + a)^2 + 5)*sinh(b*n*log(x) + b*log(c) + a)^3 + 15*(cosh(b*n*log(x) + b
*log(c) + a)^4 + 5*cosh(b*n*log(x) + b*log(c) + a)^2 + 10)*sinh(b*n*log(x)
 + b*log(c) + a))/(b*n)
```

Sympy [A] (verification not implemented)

Time = 8.16 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.62

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx$$
$$= \begin{cases} \log(x)\cosh^5(a) & \text{for } b=0 \wedge (b \\ \log(x)\cosh^5(a+b\log(c)) & \text{for } n=0 \\ \frac{8\sinh^5(a+b\log(cx^n))}{15bn} - \frac{4\sinh^3(a+b\log(cx^n))\cosh^2(a+b\log(cx^n))}{3bn} + \frac{\sinh(a+b\log(cx^n))\cosh^4(a+b\log(cx^n))}{bn} & \text{otherwise} \end{cases}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**5/x,x)
```

output
```
Piecewise((log(x)*cosh(a)**5, Eq(b, 0) & (Eq(b, 0) | Eq(n, 0))), (log(x)*c
osh(a + b*log(c))**5, Eq(n, 0)), (8*sinh(a + b*log(c*x**n))**5/(15*b*n) -
4*sinh(a + b*log(c*x**n))**3*cosh(a + b*log(c*x**n))**2/(3*b*n) + sinh(a +
 b*log(c*x**n))*cosh(a + b*log(c*x**n))**4/(b*n), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 130 vs. $2(61) = 122$.

Time = 0.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.00

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{e^{(5\,b\log(cx^n)+5\,a)}}{160\,bn}+\frac{5\,e^{(3\,b\log(cx^n)+3\,a)}}{96\,bn}+\frac{5\,e^{(b\log(cx^n)+a)}}{16\,bn}-\frac{5\,e^{(-b\log(cx^n)-a)}}{16\,bn}-\frac{5\,e^{(-3\,b\log(cx^n)-3\,a)}}{96\,bn}-\frac{e^{(-5\,b\log(cx^n)-5\,a)}}{160\,bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="maxima")
```

output
```
1/160*e^(5*b*log(c*x^n) + 5*a)/(b*n) + 5/96*e^(3*b*log(c*x^n) + 3*a)/(b*n)
 + 5/16*e^(b*log(c*x^n) + a)/(b*n) - 5/16*e^(-b*log(c*x^n) - a)/(b*n) - 5/
96*e^(-3*b*log(c*x^n) - 3*a)/(b*n) - 1/160*e^(-5*b*log(c*x^n) - 5*a)/(b*n)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.78

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{\left(3\,c^{10\,b}x^{5\,bn}e^{(10\,a)}+25\,c^{8\,b}x^{3\,bn}e^{(8\,a)}+150\,c^{6\,b}x^{bn}e^{(6\,a)}-\frac{150\,c^{4\,b}x^{4\,bn}e^{(4\,a)}+25\,c^{2\,b}x^{2\,bn}e^{(2\,a)}+3}{x^{5\,bn}}\right)e^{(-5\,a)}}{480\,bc^{5\,b}n}$$

input
```
integrate(cosh(a+b*log(c*x^n))^5/x,x, algorithm="giac")
```

output
```
1/480*(3*c^(10*b)*x^(5*b*n)*e^(10*a) + 25*c^(8*b)*x^(3*b*n)*e^(8*a) + 150*
c^(6*b)*x^(b*n)*e^(6*a) - (150*c^(4*b)*x^(4*b*n)*e^(4*a) + 25*c^(2*b)*x^(2
*b*n)*e^(2*a) + 3)/x^(5*b*n))*e^(-5*a)/(b*c^(5*b)*n)
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{\frac{\sinh(a+b\ln(c\,x^n))^5}{5}+\frac{2\sinh(a+b\ln(c\,x^n))^3}{3}+\sinh(a+b\ln(c\,x^n))}{b\,n}$$

input
```
int(cosh(a + b*log(c*x^n))^5/x,x)
```

output
```
(sinh(a + b*log(c*x^n)) + (2*sinh(a + b*log(c*x^n))^3)/3 + sinh(a + b*log(
c*x^n))^5/5)/(b*n)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.88

$$\int \frac{\cosh^5(a+b\log(cx^n))}{x}\,dx = \frac{3x^{10bn}e^{10a}c^{10b}+25x^{8bn}e^{8a}c^{8b}+150x^{6bn}e^{6a}c^{6b}-150x^{4bn}e^{4a}c^{4b}-25x^{2bn}e^{2a}c^{2b}-3}{480x^{5bn}e^{5a}c^{5b}bn}$$

input
```
int(cosh(a+b*log(c*x^n))^5/x,x)
```

output
```
(3*x**(10*b*n)*e**(10*a)*c**(10*b) + 25*x**(8*b*n)*e**(8*a)*c**(8*b) + 150
*x**(6*b*n)*e**(6*a)*c**(6*b) - 150*x**(4*b*n)*e**(4*a)*c**(4*b) - 25*x**(
2*b*n)*e**(2*a)*c**(2*b) - 3)/(480*x**(5*b*n)*e**(5*a)*c**(5*b)*b*n)
```

3.252 $\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{5bn} + \frac{2\cosh^{\frac{3}{2}}(a+b\log(cx^n))\sinh(a+b\log(cx^n))}{5bn}$$

output
```
-6/5*I*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2/5*cosh(a+b*l
n(c*x^n))^(3/2)*sinh(a+b*ln(c*x^n))/b/n
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.93

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{-6iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)+\sqrt{\cosh(a+b\log(cx^n))}\sinh(2(a+b\log(cx^n)))}{5bn}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((-6*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sqrt[Cosh[a + b*Log[c*x^n
]]]*Sinh[2*(a + b*Log[c*x^n])])/(5*b*n)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3115, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh^{\frac{5}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{5/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{3}{5}\int \sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5b}+\frac{3}{5}\int \sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{5b}-\frac{6iE\left(\frac{1}{2}i(a+b\log(cx^n))|2\right)}{5b}}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^(5/2)/x,x]
```

output
```
((((-6*I)/5)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Cosh[a + b*Log
[c*x^n]]^(3/2)*Sinh[a + b*Log[c*x^n]])/(5*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(63) = 126$.

Time = 7.94 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.82

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(}{}\right.}}$
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\left(8\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^7-16\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^5+10\cosh\left(\frac{a}{2}+\right.\right.}{5n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(}{}\right.}}$

input
```
int(cosh(a+b*ln(c*x^n))^(5/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/5/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*(8*cosh(1/2*a+1/2*b*ln(c*x^n))^7-16*cosh(1/2*a+1/2*b*ln(c*x^n))^5+10
*cosh(1/2*a+1/2*b*ln(c*x^n))^3-3*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-
2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x
^n)),2^(1/2))-2*cosh(1/2*a+1/2*b*ln(c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n)
)^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*co
sh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(61) = 122$.

Time = 0.08 (sec) , antiderivative size = 332, normalized size of antiderivative = 4.96

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{12\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^2+2\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log$$

input
```
integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="fricas")
```

output
```
-1/10*(12*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*
log(x) + b*log(c) + a)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) - (c
osh(b*n*log(x) + b*log(c) + a)^4 + 4*cosh(b*n*log(x) + b*log(c) + a)*sinh(
b*n*log(x) + b*log(c) + a)^3 + sinh(b*n*log(x) + b*log(c) + a)^4 + 6*(cosh
(b*n*log(x) + b*log(c) + a)^2 - 2)*sinh(b*n*log(x) + b*log(c) + a)^2 - 12*
cosh(b*n*log(x) + b*log(c) + a)^2 + 4*(cosh(b*n*log(x) + b*log(c) + a)^3 -
 6*cosh(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) - 1)*s
qrt(cosh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)
^2 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)
 + b*n*sinh(b*n*log(x) + b*log(c) + a)^2)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \text{Timed out}$$

input
```
integrate(cosh(a+b*ln(c*x**n))**(5/2)/x,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="maxima")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx = \int \frac{\cosh(b\log(cx^n)+a)^{\frac{5}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(5/2)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(5/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{\cosh(a+b\,\ln(c\,x^n))^{5/2}}{x}\,dx$$

input
```
int(cosh(a + b*log(c*x^n))^(5/2)/x,x)
```

output
```
int(cosh(a + b*log(c*x^n))^(5/2)/x, x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}{x}\,dx=\int \frac{\sqrt{\cosh(\log(x^nc)\,b+a)}\,\cosh(\log(x^nc)\,b+a)^2}{x}dx$$

input
```
int(cosh(a+b*log(c*x^n))^(5/2)/x,x)
```

output
```
int((sqrt(cosh(log(x**n*c)*b + a))*cosh(log(x**n*c)*b + a)**2)/x,x)
```

3.253 $\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn} + \frac{2\sqrt{\cosh(a+b\log(cx^n))}\sinh(a+b\log(cx^n))}{3bn}$$

output
```
-2/3*I*InverseJacobiAM(1/2*I*(a+b*ln(c*x^n)),2^(1/2))/b/n+2/3*cosh(a+b*ln(
c*x^n))^(1/2)*sinh(a+b*ln(c*x^n))/b/n
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.09 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.70

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx = \frac{\sinh(2(a+b\log(cx^n))) + 2\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+b\log(cx^n))) - \sinh(2(a+b\log(\right.}{3bn\sqrt{\cosh(a+b\log(cx^n))}}$$

input
```
Integrate[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
(Sinh[2*(a + b*Log[c*x^n])] + 2*Hypergeometric2F1[1/4, 1/2, 5/4, -Cosh[2*(
a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[1 + Cosh[2*(a + b*Lo
g[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]])/(3*b*n*Sqrt[Cosh[a + b*Log[c*x^n
]]])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3039, 3042, 3115, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \cosh^{\frac{3}{2}}(a+b\log(cx^n))d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{3/2}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3115}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))\sqrt{\cosh(a+b\log(cx^n))}}{3b}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\cosh(a+b\log(cx^n))}}{3b}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))\sqrt{\cosh(a+b\log(cx^n))}}{3b}-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[Cosh[a + b*Log[c*x^n]]^(3/2)/x,x]
```

output
```
((((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Sqrt[Cosh[a +
b*Log[c*x^n]]]*Sinh[a + b*Log[c*x^n]])/(3*b))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3115
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Simp[b^2*((n - 1)/n)   Int[(b*Sin
[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[
2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(59) = 118$.

Time = 2.94 (sec) , antiderivative size = 237, normalized size of antiderivative = 3.54

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{5}-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{3}+\sqrt{-\sinh\left(\frac{a}{2}+\right.}}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sinh\left(\right.}$
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\left(4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{5}-6\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{3}+\sqrt{-\sinh\left(\frac{a}{2}+\right.}}{3n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sinh\left(\right.}$

input
```
int(cosh(a+b*ln(c*x^n))^(3/2)/x,x,method=_RETURNVERBOSE)
```

output
```
2/3/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^
(1/2)*(4*cosh(1/2*a+1/2*b*ln(c*x^n))^5-6*cosh(1/2*a+1/2*b*ln(c*x^n))^3+(-s
inh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+1)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))+2*cosh(1/2*a+1/2*b*ln(
c*x^n)))/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)/sinh(1/2*a+1/2*b*ln(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)
/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(58) = 116$.

Time = 0.09 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.54

$$\int \frac{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}{x}\,dx$$
$$= \frac{2\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)+\sqrt{2}\sinh(bn\log(x)+b\log(c)+a)\right)\text{weierstrassPInverse}(-4,0,c}{}$$

input
```
integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="fricas")
```

output
```
1/3*(2*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
+ b*log(c) + a))*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a
) + sinh(b*n*log(x) + b*log(c) + a)) + (cosh(b*n*log(x) + b*log(c) + a)^2
+ 2*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sinh
(b*n*log(x) + b*log(c) + a)^2 - 1)*sqrt(cosh(b*n*log(x) + b*log(c) + a)))/
(b*n*cosh(b*n*log(x) + b*log(c) + a) + b*n*sinh(b*n*log(x) + b*log(c) + a)
)
```

Sympy [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\cosh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**(3/2)/x,x)
```

output
```
Integral(cosh(a + b*log(c*x**n))**(3/2)/x, x)
```

Maxima [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\cosh(b\log(cx^n) + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="maxima")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Giac [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\cosh(b\log(cx^n) + a)^{\frac{3}{2}}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(3/2)/x,x, algorithm="giac")
```

output
```
integrate(cosh(b*log(c*x^n) + a)^(3/2)/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\cosh(a + b\ln(c\,x^n))^{3/2}}{x}\,dx$$

input
```
int(cosh(a + b*log(c*x^n))^(3/2)/x,x)
```

output
```
int(cosh(a + b*log(c*x^n))^(3/2)/x, x)
```

Reduce [F]

$$\int \frac{\cosh^{\frac{3}{2}}(a + b\log(cx^n))}{x}\,dx = \int \frac{\sqrt{\cosh(\log(x^n c)\,b + a)}\,\cosh(\log(x^n c)\,b + a)}{x}dx$$

input
```
int(cosh(a+b*log(c*x^n))^(3/2)/x,x)
```

output
```
int((sqrt(cosh(log(x**n*c)*b + a))*cosh(log(x**n*c)*b + a))/x,x)
```

3.254 $\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

output
```
-2*I*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx = -\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Integrate[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3039, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}dx$$
$$\Big\downarrow \text{3039}$$
$$\frac{\int \sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3042}$$
$$\frac{\int \sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$
$$\Big\downarrow \text{3119}$$
$$-\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle|2\right)}{bn}$$

input
```
Int[Sqrt[Cosh[a + b*Log[c*x^n]]]/x,x]
```

output
```
((-2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(30) = 60$.

Time = 1.90 (sec) , antiderivative size = 183, normalized size of antiderivative = 6.54

method	result
derivativedivides	$-\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$
default	$-\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\ \mathrm{Ellip}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\ \sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right.}}$

input
```
int(cosh(a+b*ln(c*x^n))^(1/2)/x,x,method=_RETURNVERBOSE)
```

output
```
-2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n)
)^2+1)^(1/2)*EllipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/(2*sinh(1/2*a+
1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/sinh(1/2*a+1/2*b*l
n(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(28) = 56$.

Time = 0.08 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.07

$$\int \frac{\sqrt{\cosh(a+b\log(cx^n))}}{x}\,dx =$$

$$-\frac{2\left(\sqrt{2}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cosh(bn\log(x)+b\log(c)+a)+\sinh(bn\log\right.}{bn}$$

input
```
integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="fricas")
```

output
```
-2*(sqrt(2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cosh(b*n*log
(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) + sqrt(cosh(b*n*lo
g(x) + b*log(c) + a)))/(b*n)
```

Sympy [F]

$$\int \frac{\sqrt{\cosh\left(a + b\log\left(cx^{n}\right)\right)}}{x}\,dx = \int \frac{\sqrt{\cosh\left(a + b\log\left(cx^{n}\right)\right)}}{x}\,dx$$

input
```
integrate(cosh(a+b*ln(c*x**n))**(1/2)/x,x)
```

output
```
Integral(sqrt(cosh(a + b*log(c*x**n)))/x, x)
```

Maxima [F]

$$\int \frac{\sqrt{\cosh\left(a + b\log\left(cx^{n}\right)\right)}}{x}\,dx = \int \frac{\sqrt{\cosh\left(b\log\left(cx^{n}\right) + a\right)}}{x}\,dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

Giac [F]

$$\int \frac{\sqrt{\cosh (a + b \log (cx^n))}}{x} \, dx = \int \frac{\sqrt{\cosh (b \log (cx^n) + a)}}{x} \, dx$$

input
```
integrate(cosh(a+b*log(c*x^n))^(1/2)/x,x, algorithm="giac")
```

output
```
integrate(sqrt(cosh(b*log(c*x^n) + a))/x, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sqrt{\cosh (a + b \log (cx^n))}}{x} \, dx = \int \frac{\sqrt{\cosh (a + b \ln (c x^n))}}{x} \, dx$$

input
```
int(cosh(a + b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(cosh(a + b*log(c*x^n))^(1/2)/x, x)
```

Reduce [F]

$$\int \frac{\sqrt{\cosh (a + b \log (cx^n))}}{x} \, dx = \int \frac{\sqrt{\cosh (\log (x^n c) b + a)}}{x} dx$$

input
```
int(cosh(a+b*log(c*x^n))^(1/2)/x,x)
```

output
```
int(sqrt(cosh(log(x**n*c)*b + a))/x,x)
```

3.255 $\int \frac{1}{x\sqrt{\cosh(a+b\log(cx^n))}}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 28

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx=-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log\left(cx^n\right)),2\right)}{bn}$$

output
```
-2*I*InverseJacobiAM(1/2*I*(a+b*ln(c*x^n)),2^(1/2))/b/n
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx=-\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log\left(cx^n\right)),2\right)}{bn}$$

input
```
Integrate[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.158$, Rules used = {3039, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\sqrt{\cosh (a+b\log (cx^n))}}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log (cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log (cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$-\frac{2i \operatorname{EllipticF}\left(\frac{1}{2}i(a+b\log (cx^n)),2\right)}{bn}$$

input
```
Int[1/(x*Sqrt[Cosh[a + b*Log[c*x^n]]]),x]
```

output
```
((-2*I)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/(b*n)
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. 2(26) = 52.

Time = 1.00 (sec) , antiderivative size = 183, normalized size of antiderivative = 6.54

method	result
derivativedivides	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\,\mathrm{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}}$
default	$\frac{2\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sqrt{-2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2+1}\,\mathrm{Elliptic}}{n\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^4+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}\,\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^n)}{2}\right)^2}}$

input
```
int(1/x/cosh(a+b*ln(c*x^n))^(1/2),x,method=_RETURNVERBOSE)
```

output
```
2/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))
^2+1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2
)^(1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/sinh(1/2*a+1/2*b*ln
(c*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.39

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx$$
$$=\frac{2\sqrt{2}\mathrm{weierstrassPInverse}(-4,0,\cosh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right)+\sinh\left(bn\log\left(x\right)+b\log\left(c\right)+a\right))}{bn}$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="fricas")
```

output
```
2*sqrt(2)*weierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sin
h(b*n*log(x) + b*log(c) + a))/(b*n)
```

Sympy [F]

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx$$

input
```
integrate(1/x/cosh(a+b*ln(c*x**n))**(1/2),x)
```

output
```
Integral(1/(x*sqrt(cosh(a + b*log(c*x**n)))), x)
```

Maxima [F]

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\cosh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

Giac [F]

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx = \int \frac{1}{x\sqrt{\cosh\left(b\log\left(cx^n\right)+a\right)}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/(x*sqrt(cosh(b*log(c*x^n) + a))), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{1}{x\,\sqrt{\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)}}\,dx$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^(1/2)),x)
```

output
```
int(1/(x*cosh(a + b*log(c*x^n))^(1/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\sqrt{\cosh\left(a+b\log\left(cx^n\right)\right)}}\,dx=\int \frac{\sqrt{\cosh\left(\log\left(x^n c\right)b+a\right)}}{\cosh\left(\log\left(x^n c\right)b+a\right)x}dx$$

input
```
int(1/x/cosh(a+b*log(c*x^n))^(1/2),x)
```

output
```
int(sqrt(cosh(log(x**n*c)*b + a))/(cosh(log(x**n*c)*b + a)*x),x)
```

3.256 $\int \frac{1}{x \cosh^{\frac{3}{2}}(a+b\log(cx^n))} \, dx$

Optimal result

Integrand size = 19, antiderivative size = 63

$$\int \frac{1}{x \cosh^{\frac{3}{2}}(a+b\log(cx^n))} \, dx = \frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle| 2\right)}{bn} + \frac{2\sinh(a+b\log(cx^n))}{bn\sqrt{\cosh(a+b\log(cx^n))}}$$

output
```
2*I*EllipticE(I*sinh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2))/b/n+2*sinh(a+b*ln(c*x
^n))/b/n/cosh(a+b*ln(c*x^n))^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

$$\int \frac{1}{x \cosh^{\frac{3}{2}}(a+b\log(cx^n))} \, dx = \frac{2\left(iE\left(\frac{1}{2}i(a+b\log(cx^n))\middle| 2\right) + \frac{\sinh(a+b\log(cx^n))}{\sqrt{\cosh(a+b\log(cx^n))}}\right)}{bn}$$

input
```
Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(2*(I*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2] + Sinh[a + b*Log[c*x^n]]/Sqrt
[Cosh[a + b*Log[c*x^n]]]))/(b*n)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3116, 3042, 3119}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x \cosh^{\frac{3}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\cosh^{\frac{3}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{3/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{b\sqrt{\cosh(a+b\log(cx^n))}}-\int \sqrt{\cosh(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{b\sqrt{\cosh(a+b\log(cx^n))}}-\int \sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3119}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{b\sqrt{\cosh(a+b\log(cx^n))}}+\frac{2iE\left(\frac{1}{2}i(a+b\log(cx^n))|2\right)}{b}}{n}$$

input
```
Int[1/(x*Cosh[a + b*Log[c*x^n]]^(3/2)),x]
```

output
```
(((2*I)*EllipticE[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Sinh[a + b*Log[c*x^
n]])/(b*Sqrt[Cosh[a + b*Log[c*x^n]]]))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3119
```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(63) = 126$.

Time = 1.26 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.24

method	result
derivativedivides	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}\,\mathrm{EllipticE}\left(\cosh\left(\right.\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}\,b}$
default	$\frac{4\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}+2\sqrt{-\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\,\sqrt{-2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}\,\mathrm{EllipticE}\left(\cosh\left(\right.\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1}\,b}$

input
```
int(1/x/cosh(a+b*ln(c*x^n))^(3/2),x,method=_RETURNVERBOSE)
```

output
```
2/n*(2*cosh(1/2*a+1/2*b*ln(c*x^n))*sinh(1/2*a+1/2*b*ln(c*x^n))^2+(-sinh(1/
2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*sinh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)*E
llipticE(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/sinh(1/2*a+1/2*b*ln(c*x^n))
/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(61) = 122$.

Time = 0.09 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.86

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \frac{2\left(\left(\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)^2+2\sqrt{2}\cosh(bn\log(x)+b\log(c)+a)\sinh(bn\log(x)+b\log(c\right.\right.}{}$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="fricas")
```

output
```
2*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x)
+ b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2)*sinh(b*n*log(x)
+ b*log(c) + a)^2 + sqrt(2))*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a))) +
 2*(cosh(b*n*log(x) + b*log(c) + a)^2 + 2*cosh(b*n*log(x) + b*log(c) + a)*
sinh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x) + b*log(c) + a)^2)*sqrt(
cosh(b*n*log(x) + b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^2 +
 2*b*n*cosh(b*n*log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a) + b
*n*sinh(b*n*log(x) + b*log(c) + a)^2 + b*n)
```

Sympy [F]

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx$$

input
```
integrate(1/x/cosh(a+b*ln(c*x**n))**(3/2),x)
```

output
```
Integral(1/(x*cosh(a + b*log(c*x**n))**(3/2)), x)
```

Maxima [F]

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

Giac [F]

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh(b\log(cx^n)+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(3/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{1}{x\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{3/2}}\,dx$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^(3/2)),x)
```

output
```
int(1/(x*cosh(a + b*log(c*x^n))^(3/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\cosh^{\frac{3}{2}}(a+b\log(cx^n))}\,dx = \int \frac{\sqrt{\cosh(\log(x^n c)\,b+a)}}{\cosh(\log(x^n c)\,b+a)^2 x}dx$$

input
```
int(1/x/cosh(a+b*log(c*x^n))^(3/2),x)
```

output
```
int(sqrt(cosh(log(x**n*c)*b + a))/(cosh(log(x**n*c)*b + a)**2*x),x)
```

3.257 $\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 67

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = -\frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3bn} + \frac{2\sinh(a+b\log(cx^n))}{3bn\cosh^{\frac{3}{2}}(a+b\log(cx^n))}$$

output
```
-2/3*I*InverseJacobiAM(1/2*I*(a+b*ln(c*x^n)),2^(1/2))/b/n+2/3*sinh(a+b*ln(
c*x^n))/b/n/cosh(a+b*ln(c*x^n))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.05 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.82

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \frac{2\Big(\sinh(a+b\log(cx^n)) + \cosh(a+b\log(cx^n))\,\text{Hypergeometric2F1}\left(\frac{1}{4},\frac{1}{2},\frac{5}{4},-\cosh(2(a+b\log(cx^n)))\right)}{3bn\cosh^{\frac{3}{2}}(a+b\log(}$$

input
```
Integrate[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
(2*(Sinh[a + b*Log[c*x^n]] + Cosh[a + b*Log[c*x^n]]*Hypergeometric2F1[1/4,
 1/2, 5/4, -Cosh[2*(a + b*Log[c*x^n])] - Sinh[2*(a + b*Log[c*x^n])]]*Sqrt[
1 + Cosh[2*(a + b*Log[c*x^n])] + Sinh[2*(a + b*Log[c*x^n])]]))/(3*b*n*Cosh
[a + b*Log[c*x^n]]^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.97, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.263, Rules used = {3039, 3042, 3116, 3042, 3120}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}dx$$

$$\Big\downarrow \text{3039}$$

$$\frac{\int \frac{1}{\cosh^{\frac{5}{2}}(a+b\log(cx^n))}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\int \frac{1}{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)^{5/2}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3116}$$

$$\frac{\frac{1}{3}\int \frac{1}{\sqrt{\cosh(a+b\log(cx^n))}}d\log(cx^n)+\frac{2\sinh(a+b\log(cx^n))}{3b\cosh^{\frac{3}{2}}(a+b\log(cx^n))}}{n}$$

$$\Big\downarrow \text{3042}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{3b\cosh^{\frac{3}{2}}(a+b\log(cx^n))}+\frac{1}{3}\int \frac{1}{\sqrt{\sin\left(ia+ib\log(cx^n)+\frac{\pi}{2}\right)}}d\log(cx^n)}{n}$$

$$\Big\downarrow \text{3120}$$

$$\frac{\frac{2\sinh(a+b\log(cx^n))}{3b\cosh^{\frac{3}{2}}(a+b\log(cx^n))} - \frac{2i\,\text{EllipticF}\left(\frac{1}{2}i(a+b\log(cx^n)),2\right)}{3b}}{n}$$

input
```
Int[1/(x*Cosh[a + b*Log[c*x^n]]^(5/2)),x]
```

output
```
((((-2*I)/3)*EllipticF[(I/2)*(a + b*Log[c*x^n]), 2])/b + (2*Sinh[a + b*Log
[c*x^n]])/(3*b*Cosh[a + b*Log[c*x^n]]^(3/2)))/n
```

Defintions of rubi rules used

rule 3039
```
Int[u_, x_Symbol] :> With[{lst = FunctionOfLog[Cancel[x*u], x]}, Simp[1/lst
[[3]]   Subst[Int[lst[[1]], x], x, Log[lst[[2]]]], x] /;  !FalseQ[lst]] /;
NonsumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3116
```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Simp[(n + 2)/(b^2*(n + 1))   I
nt[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
 IntegerQ[2*n]
```

rule 3120
```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(59) = 118$.

Time = 1.60 (sec) , antiderivative size = 258, normalized size of antiderivative = 3.85

method	result
derivativedivides	$\frac{\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\left(\frac{\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)}}{3\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-\frac{1}{2}\right)^{2}}\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln}{}\right.}}$
default	$\frac{\sqrt{\left(2\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-1\right)\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}}\left(\frac{\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{4}+\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)}}{3\left(\cosh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)^{2}-\frac{1}{2}\right)^{2}}\right.}{n\sinh\left(\frac{a}{2}+\frac{b\ln(c\,x^{n})}{2}\right)\sqrt{2\cosh\left(\frac{a}{2}+\frac{b\ln}{}\right.}}$

input
```
int(1/x/cosh(a+b*ln(c*x^n))^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/n*((2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)*sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1
/2)*(1/3*cosh(1/2*a+1/2*b*ln(c*x^n))*(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh
(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)/(cosh(1/2*a+1/2*b*ln(c*x^n))^2-1/2)^2+2/3
*(-sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(1/2)*(-2*cosh(1/2*a+1/2*b*ln(c*x^n))^2+
1)^(1/2)/(2*sinh(1/2*a+1/2*b*ln(c*x^n))^4+sinh(1/2*a+1/2*b*ln(c*x^n))^2)^(
1/2)*EllipticF(cosh(1/2*a+1/2*b*ln(c*x^n)),2^(1/2)))/sinh(1/2*a+1/2*b*ln(c
*x^n))/(2*cosh(1/2*a+1/2*b*ln(c*x^n))^2-1)^(1/2)/b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 501 vs. $2(58) = 116$.

Time = 0.12 (sec) , antiderivative size = 501, normalized size of antiderivative = 7.48

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx = \text{Too large to display}$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="fricas")
```

output
```
2/3*((sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*sqrt(2)*cosh(b*n*log(x
) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*sinh(b*n*log
(x) + b*log(c) + a)^4 + 2*(3*sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^2 + s
qrt(2))*sinh(b*n*log(x) + b*log(c) + a)^2 + 2*sqrt(2)*cosh(b*n*log(x) + b*
log(c) + a)^2 + 4*(sqrt(2)*cosh(b*n*log(x) + b*log(c) + a)^3 + sqrt(2)*cos
h(b*n*log(x) + b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a) + sqrt(2))*w
eierstrassPInverse(-4, 0, cosh(b*n*log(x) + b*log(c) + a) + sinh(b*n*log(x
) + b*log(c) + a)) + 2*(cosh(b*n*log(x) + b*log(c) + a)^3 + 3*cosh(b*n*log
(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^2 + sinh(b*n*log(x) +
b*log(c) + a)^3 + (3*cosh(b*n*log(x) + b*log(c) + a)^2 - 1)*sinh(b*n*log(x
) + b*log(c) + a) - cosh(b*n*log(x) + b*log(c) + a))*sqrt(cosh(b*n*log(x)
+ b*log(c) + a)))/(b*n*cosh(b*n*log(x) + b*log(c) + a)^4 + 4*b*n*cosh(b*n*
log(x) + b*log(c) + a)*sinh(b*n*log(x) + b*log(c) + a)^3 + b*n*sinh(b*n*lo
g(x) + b*log(c) + a)^4 + 2*b*n*cosh(b*n*log(x) + b*log(c) + a)^2 + 2*(3*b*
n*cosh(b*n*log(x) + b*log(c) + a)^2 + b*n)*sinh(b*n*log(x) + b*log(c) + a)
^2 + b*n + 4*(b*n*cosh(b*n*log(x) + b*log(c) + a)^3 + b*n*cosh(b*n*log(x)
+ b*log(c) + a))*sinh(b*n*log(x) + b*log(c) + a))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{x\cosh^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/x/cosh(a+b*ln(c*x**n))**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{x\cosh^{\frac{5}{2}}\left(a+b\log\left(cx^n\right)\right)}\,dx=\int \frac{1}{x\cosh\left(b\log\left(cx^n\right)+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="maxima")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

Giac [F]

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{1}{x\cosh(b\log(cx^n)+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/x/cosh(a+b*log(c*x^n))^(5/2),x, algorithm="giac")
```

output
```
integrate(1/(x*cosh(b*log(c*x^n) + a)^(5/2)), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{1}{x\cosh\left(a+b\,\ln\left(c\,x^n\right)\right)^{5/2}}\,dx$$

input
```
int(1/(x*cosh(a + b*log(c*x^n))^(5/2)),x)
```

output
```
int(1/(x*cosh(a + b*log(c*x^n))^(5/2)), x)
```

Reduce [F]

$$\int \frac{1}{x\cosh^{\frac{5}{2}}(a+b\log(cx^n))}\,dx=\int \frac{\sqrt{\cosh(\log(x^nc)\,b+a)}}{\cosh(\log(x^nc)\,b+a)^3\,x}\,dx$$

input
```
int(1/x/cosh(a+b*log(c*x^n))^(5/2),x)
```

output
```
int(sqrt(cosh(log(x**n*c)*b + a))/(cosh(log(x**n*c)*b + a)**3*x),x)
```

3.258 $\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 206

$$\begin{aligned}
&\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx \\
&= -\frac{1}{4}x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) \\
&\quad+\frac{5e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}+\frac{5x\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{12\left(1+e^{-2a}(cx^n)^{-4/n}\right)} \\
&\quad-\frac{5e^{-3a}x(cx^n)^{-6/n}\operatorname{csch}^{-1}\left(e^a(cx^n)^{2/n}\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{5/2}}
\end{aligned}$$

output

```
-1/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)+5/4*x*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(2
*a)/((c*x^n)^(4/n))/(1+1/exp(2*a)/((c*x^n)^(4/n)))^2+5*x*cosh(a+2*ln(c*x^n
)/n)^(5/2)/(12+12/exp(2*a)/((c*x^n)^(4/n)))-5/4*x*arccsch(exp(a)*(c*x^n)^(
2/n))*cosh(a+2*ln(c*x^n)/n)^(5/2)/exp(3*a)/((c*x^n)^(6/n))/(1+1/exp(2*a)/(
(c*x^n)^(4/n)))^(5/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.41

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx=\frac{1}{14}e^{2a}x(cx^n)^{4/n}\left(1+e^{2a}(cx^n)^{4/n}\right)\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\text{Hypergeometric2F1}\left(2,\frac{7}{2},\frac{9}{2},1+e^{2a}(cx^n)^{4/n}\right)$$

input
```
Integrate[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

output
```
(E^(2*a)*x*(c*x^n)^(4/n)*(1 + E^(2*a)*(c*x^n)^(4/n))*Cosh[a + (2*Log[c*x^n
])/n]^(5/2)*Hypergeometric2F1[2, 7/2, 9/2, 1 + E^(2*a)*(c*x^n)^(4/n)])/14
```

Rubi [A] (warning: unable to verify)

Time = 0.46 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.444, Rules used = {6052, 6060, 876, 872, 868, 773, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx$$

$$\Big\downarrow \text{ 6052}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6060}$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\int (cx^n)^{\frac{6}{n}-1}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}d(cx^n)}{n\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^{5/2}}$$

$$\Big\downarrow \text{ 876}$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\int (cx^n)^{\frac{6}{n}-1}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}d(cx^n)-\frac{1}{4}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+\right.\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$$\Big\downarrow \; 872$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(e^{-2a}\int (cx^n)^{\frac{2}{n}-1}\sqrt{e^{-2a}(cx^n)^{-4/n}+1}d(cx^n)+\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/}\right.\right.\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$$\Big\downarrow \; 868$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{2}e^{-2a}n\int \sqrt{\frac{e^{-2a}x^{-2n}}{c^2}+1}d(cx^n)^{2/n}+\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}\right.\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$$\Big\downarrow \; 773$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}-\frac{1}{2}e^{-2a}n\int \frac{x^{-2n}\sqrt{c^2e^{-2a}x^{2n}+1}}{c^2}d\frac{x^{-n}}{c}\right)-\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$$\Big\downarrow \; 247$$

$$\frac{x(cx^n)^{-6/n}\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}-\frac{1}{2}e^{-2a}n\left(e^{-2a}\int \frac{1}{\sqrt{c^2e^{-2a}x^{2n}+1}}d\frac{x^{-n}}{c}-\right.\right.\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

$$\Big\downarrow \; 222$$

$$\frac{x(cx^n)^{-6/n}\left(\frac{5}{2}\left(\frac{1}{6}n(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}-\frac{1}{2}e^{-2a}n\left(e^{-a}\text{arcsinh}\left(\frac{e^{-a}x^{-n}}{c}\right)-\frac{x^{-n}\sqrt{e^{-2a}c^2x^{2n}+1}}{c}\right)\right)-\frac{1}{4}n\right.}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}$$

input
```
Int[Cosh[a + (2*Log[c*x^n])/n]^(5/2),x]
```

output
```
(x*(-1/4*(n*(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2)) + (5*((n*
(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(3/2))/6 - (n*(-(Sqrt[1 + (c
^2*x^(2*n))/E^(2*a)]/(c*x^n)) + ArcSinh[1/(c*E^a*x^n)]/E^a))/(2*E^(2*a))))
/2)*Cosh[a + (2*Log[c*x^n])/n]^(5/2))/(n*(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*
x^n)^(4/n)))^(5/2))
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 872
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*(
(a + b*x^n)^p/(m + 1)), x] - Simp[b*n*(p/(m + 1))   Int[x^(m + n)*(a + b*x^
n)^(p - 1), x], x] /; FreeQ[{a, b, m, n}, x] && EqQ[(m + 1)/n + p, 0] && Gt
Q[p, 0]
```

rule 876
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^p/(c*(m + n*p + 1))), x] + Simp[a*n*(p/(m + n*p + 1
))   Int[(c*x)^m*(a + b*x^n)^(p - 1), x], x] /; FreeQ[{a, b, c, m, n}, x] &
& IntegerQ[p + Simplify[(m + 1)/n]] && GtQ[p, 0] && NeQ[m + n*p + 1, 0]
```

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \cosh\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)^{\frac{5}{2}}dx$$

input
```
int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

output
```
int(cosh(a+2*ln(c*x^n)/n)^(5/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.91

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)\,dx = \frac{\left(15\,\sqrt{2}x^{3}e^{\left(\frac{3\,(an+2\,\log(c))}{2\,n}\right)}\log\left(\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^{4}e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^{2}}}+2}{x^{4}}\right)+4\,\sqrt{\frac{1}{2}}\left(2\,x^{8}e^{\left(\frac{4\,(an+2\,\log(c)}{n}\right.}\right.\right.}{192\,x^{3}}$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="fricas")
```

output
```
1/192*(15*sqrt(2)*x^3*e^(3/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n + 2*lo
g(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x
^2) + 2)/x^4) + 4*sqrt(1/2)*(2*x^8*e^(4*(a*n + 2*log(c))/n) + 14*x^4*e^(2*
(a*n + 2*log(c))/n) - 3)*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-
1/2*(a*n + 2*log(c))/n))*e^(-2*(a*n + 2*log(c))/n)/x^3
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = \text{Timed out}$$

input
```
integrate(cosh(a+2*ln(c*x**n)/n)**(5/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)dx = \int \cosh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{5}{2}}dx$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="maxima")
```

output
```
integrate(cosh(a + 2*log(c*x^n)/n)^(5/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx=\text{Exception raised: AttributeError}$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(5/2),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx=\int \cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{5/2}\,dx$$

input
```
int(cosh(a + (2*log(c*x^n))/n)^(5/2),x)
```

output
```
int(cosh(a + (2*log(c*x^n))/n)^(5/2), x)
```

Reduce [F]

$$\int \cosh^{\frac{5}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx$$
$$=\int \sqrt{\cosh\left(\frac{2\log\left(x^n c\right)+an}{n}\right)}\cosh\left(\frac{2\log(x^n c)+an}{n}\right)^2 dx$$

input
```
int(cosh(a+2*log(c*x^n)/n)^(5/2),x)
```

output
```
int(sqrt(cosh((2*log(x**n*c) + a*n)/n))*cosh((2*log(x**n*c) + a*n)/n)**2,x
)
```

3.259 $\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 170

$$\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\,dx = \frac{1}{4}x\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)-\frac{3e^{-2a}x(cx^n)^{-4/n}\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)} + \frac{3e^{-2a}x(cx^n)^{-4/n}\operatorname{arctanh}\left(\sqrt{1+e^{-2a}(cx^n)^{-4/n}}\right)\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}{4\left(1+e^{-2a}(cx^n)^{-4/n}\right)^{3/2}}$$

output
```
1/4*x*cosh(a+2*ln(c*x^n)/n)^(3/2)-3/4*x*cosh(a+2*ln(c*x^n)/n)^(3/2)/exp(2*
a)/((c*x^n)^(4/n))/(1+1/exp(2*a)/((c*x^n)^(4/n)))+3/4*x*arctanh((1+1/exp(2
*a)/((c*x^n)^(4/n)))^(1/2))*cosh(a+2*ln(c*x^n)/n)^(3/2)/exp(2*a)/((c*x^n)^
(4/n))/(1+1/exp(2*a)/((c*x^n)^(4/n)))^(3/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.44

$$\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) dx = -\frac{x\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\text{Hypergeometric2F1}\left(-\frac{3}{2},-\frac{1}{2},\frac{1}{2},-e^{2a+\frac{4\log(cx^n)}{n}}\right)}{2\left(1+e^{2a+\frac{4\log(cx^n)}{n}}\right)^{3/2}}$$

input
```
Integrate[Cosh[a + (2*Log[c*x^n])/n]^(3/2),x]
```

output
```
-1/2*(x*Cosh[a + (2*Log[c*x^n])/n]^(3/2)*Hypergeometric2F1[-3/2, -1/2, 1/2
, -E^(2*a + (4*Log[c*x^n])/n)])/(1 + E^(2*a + (4*Log[c*x^n])/n))^(3/2)
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.389, Rules used = {6052, 6060, 798, 51, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right) dx$$

$$\Big\downarrow \text{ 6052}$$

$$\frac{x(cx^n)^{-1/n}\int (cx^n)^{\frac{1}{n}-1}\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)d(cx^n)}{n}$$

$$\Big\downarrow \text{ 6060}$$

$$\frac{x(cx^n)^{-4/n}\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\int (cx^n)^{\frac{4}{n}-1}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}d(cx^n)}{n\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}}$$

$$\Big\downarrow \text{798}$$

$$-\frac{x(cx^n)^{-4/n}\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\int \frac{x^{-2n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}}{c^2}d(cx^n)^{-4/n}}{4\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^{3/2}}$$

$$\Big\downarrow \text{51}$$

$$-\frac{x(cx^n)^{-4/n}\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{3}{2}e^{-2a}\int \frac{x^{-n}\sqrt{e^{-2a}(cx^n)^{-4/n}+1}}{c}d(cx^n)^{-4/n}-\frac{x^{-n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}}{c}\right)}{4\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^{3/2}}$$

$$\Big\downarrow \text{60}$$

$$-\frac{x(cx^n)^{-4/n}\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{3}{2}e^{-2a}\left(\int \frac{x^{-n}}{c\sqrt{e^{-2a}(cx^n)^{-4/n}+1}}d(cx^n)^{-4/n}+2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}\right)-\frac{x^{-n}\left(e^{-}\right.}{}\right.}{4\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^{3/2}}$$

$$\Big\downarrow \text{73}$$

$$-\frac{x(cx^n)^{-4/n}\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)\left(\frac{3}{2}e^{-2a}\left(2e^{2a}\int \frac{1}{c^2e^{2a}x^{2n}-e^{2a}}d\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}+2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}\right)\right.}{4\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^{3/2}}$$

$$\Big\downarrow \text{221}$$

$$-\frac{x(cx^n)^{-4/n}\left(\frac{3}{2}e^{-2a}\left(2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}-2\text{arctanh}\left(\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}\right)\right)-\frac{x^{-n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}}{c}\right)}{4\left(e^{-2a}\left(cx^n\right)^{-4/n}+1\right)^{3/2}}$$

input

```
Int[Cosh[a + (2*Log[c*x^n])/n]^(3/2),x]
```

output
```
-1/4*(x*(-((1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(3/2)/(c*x^n)) + (3*(2*Sqrt[1 +
 1/(E^(2*a)*(c*x^n)^(4/n))] - 2*ArcTanh[Sqrt[1 + 1/(E^(2*a)*(c*x^n)^(4/n))
]]))/(2*E^(2*a)))*Cosh[a + (2*Log[c*x^n])/n]^(3/2))/((c*x^n)^(4/n)*(1 + 1/
(E^(2*a)*(c*x^n)^(4/n)))^(3/2))
```

Defintions of rubi rules used

rule 51
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + 1))), x] - Simp[d*(n/(b*(m + 1)))
Int[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d, n}, x
] && ILtQ[m, -1] && FractionQ[n] && GtQ[n, 0]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 798
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/n   Subst
[Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a,
b, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] && !IntegerQ[p]
```

Maple [F]

$$\int \cosh\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)^{\frac{3}{2}}dx$$

input
```
int(cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

output
```
int(cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.04

$$\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)\,dx = \frac{\left(3\sqrt{2}xe^{\left(\frac{an+2\log(c)}{2n}\right)}\log\left(-2\sqrt{2}\sqrt{\frac{1}{2}}x^{3}\sqrt{\frac{x^{4}e^{\left(\frac{2(an+2\log(c))}{n}\right)}+1}{x^{2}}}e^{\left(\frac{an+2\log(c)}{n}\right)}-2x^{4}e^{\left(\frac{2(an+2\log(c))}{n}\right)}-1\right)+4\sqrt{}\right.}{32x}$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

output
```
1/32*(3*sqrt(2)*x*e^(1/2*(a*n + 2*log(c))/n)*log(-2*sqrt(2)*sqrt(1/2)*x^3*
sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^((a*n + 2*log(c))/n) - 2*x^
4*e^(2*(a*n + 2*log(c))/n) - 1) + 4*sqrt(1/2)*(x^4*e^(2*(a*n + 2*log(c))/n
) - 2)*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n + 2*log(c
))/n))*e^(-(a*n + 2*log(c))/n)/x
```

Sympy [F]

$$\int \cosh^{\frac{3}{2}}\left(a + \frac{2\log\left(cx^{n}\right)}{n}\right)\,dx = \int \cosh^{\frac{3}{2}}\left(a + \frac{2\log\left(cx^{n}\right)}{n}\right)\,dx$$

input
```
integrate(cosh(a+2*ln(c*x**n)/n)**(3/2),x)
```

output
```
Integral(cosh(a + 2*log(c*x**n)/n)**(3/2), x)
```

Maxima [F]

$$\int \cosh^{\frac{3}{2}}\left(a + \frac{2\log\left(cx^{n}\right)}{n}\right)\,dx = \int \cosh\left(a + \frac{2\,\log\left(cx^{n}\right)}{n}\right)^{\frac{3}{2}}\,dx$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(a + 2*log(c*x^n)/n)^(3/2), x)
```

Giac [F(-2)]

Exception generated.

$$\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx = \text{Exception raised: NotImplementedError}$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

output
```
Exception raised: NotImplementedError >> unable to parse Giac output: sqrt
(2)/2*2/16*sageVARx*sqrt(sageVARx^2*exp(sageVARa)*exp(ln(sageVARc)/sageVAR
n)^2+sageVARx^6*exp(sageVARa)^3*exp(ln(sageVARc)/sageVARn)^6)+sqrt(2)/2/ex
p(ln(sageVARc)/sage
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx = \int \cosh\left(a+\frac{2\,\ln\left(c\,x^n\right)}{n}\right)^{3/2}\,dx$$

input
```
int(cosh(a + (2*log(c*x^n))/n)^(3/2),x)
```

output
```
int(cosh(a + (2*log(c*x^n))/n)^(3/2), x)
```

Reduce [F]

$$\int \cosh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)\,dx = \int \sqrt{\cosh\left(\frac{2\log\left(x^n c\right)+an}{n}\right)}\cosh\left(\frac{2\log(x^n c)+an}{n}\right)dx$$

input
```
int(cosh(a+2*log(c*x^n)/n)^(3/2),x)
```

output
```
int(sqrt(cosh((2*log(x**n*c) + a*n)/n))*cosh((2*log(x**n*c) + a*n)/n),x)
```

3.260 $\int \sqrt{\cosh \left(a+\frac{2 \log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 102

$$\int \sqrt{\cosh \left(a+\frac{2 \log (cx^n)}{n}\right)}\,dx = \frac{1}{2}x\sqrt{\cosh \left(a+\frac{2 \log (cx^n)}{n}\right)} - \frac{e^{-a}x(cx^n)^{-2/n}\operatorname{csch}^{-1}\left(e^a(cx^n)^{2/n}\right)\sqrt{\cosh \left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{1+e^{-2a}\left(cx^n\right)^{-4/n}}}$$

output
```
1/2*x*cosh(a+2*ln(c*x^n)/n)^(1/2)-1/2*x*arccsch(exp(a)*(c*x^n)^(2/n))*cosh
(a+2*ln(c*x^n)/n)^(1/2)/exp(a)/((c*x^n)^(2/n))/(1+1/exp(2*a)/((c*x^n)^(4/n
)))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.73

$$\int \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)} \, dx = \frac{1}{2} x \left(1-\frac{\operatorname{arctanh}\left(\sqrt{1+e^{2 a} (c x^n)^{4/n}}\right)}{\sqrt{1+e^{2 a} (c x^n)^{4/n}}}\right) \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)}$$

input
```
Integrate[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

output
```
(x*(1 - ArcTanh[Sqrt[1 + E^(2*a)*(c*x^n)^(4/n)]]/Sqrt[1 + E^(2*a)*(c*x^n)^
(4/n)])*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {6052, 6060, 868, 773, 247, 222}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)} \, dx$$

$$\Big\downarrow \text{ 6052}$$

$$\frac{x (c x^n)^{-1/n} \int (c x^n)^{\frac{1}{n}-1} \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)} d(c x^n)}{n}$$

$$\Big\downarrow \text{ 6060}$$

$$\frac{x (c x^n)^{-2/n} \sqrt{\cosh \left(a+\frac{2 \log (c x^n)}{n}\right)} \int (c x^n)^{\frac{2}{n}-1} \sqrt{e^{-2 a} (c x^n)^{-4/n}+1} d(c x^n)}{n \sqrt{e^{-2 a} (c x^n)^{-4/n}+1}}$$

$$\Big\downarrow \text{868}$$

$$\frac{x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\int\sqrt{\frac{e^{-2a}x^{-2n}}{c^2}+1}d(cx^n)^{2/n}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

$$\Big\downarrow \text{773}$$

$$-\frac{x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\int\frac{x^{-2n}\sqrt{c^2e^{-2a}x^{2n}+1}}{c^2}d\frac{x^{-n}}{c}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

$$\Big\downarrow \text{247}$$

$$-\frac{x(cx^n)^{-2/n}\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\left(e^{-2a}\int\frac{1}{\sqrt{c^2e^{-2a}x^{2n}+1}}d\frac{x^{-n}}{c}-\frac{x^{-n}\sqrt{e^{-2a}c^2x^{2n}+1}}{c}\right)}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

$$\Big\downarrow \text{222}$$

$$-\frac{x(cx^n)^{-2/n}\left(e^{-a}\text{arcsinh}\left(\frac{e^{-a}x^{-n}}{c}\right)-\frac{x^{-n}\sqrt{e^{-2a}c^2x^{2n}+1}}{c}\right)\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}{2\sqrt{e^{-2a}\left(cx^n\right)^{-4/n}+1}}$$

input
```
Int[Sqrt[Cosh[a + (2*Log[c*x^n])/n]],x]
```

output
```
-1/2*(x*(-(Sqrt[1 + (c^2*x^(2*n))/E^(2*a)]/(c*x^n)) + ArcSinh[1/(c*E^a*x^n
)]/E^a)*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])/((c*x^n)^(2/n)*Sqrt[1 + 1/(E^(2*
a)*(c*x^n)^(4/n))])
```

Defintions of rubi rules used

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 247
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^p/(c*(m + 1))), x] - Simp[2*b*(p/(c^2*(m + 1)))   Int[
(c*x)^(m + 2)*(a + b*x^2)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && GtQ[p,
0] && LtQ[m, -1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomialQ[a, b, c, 2,
m, p, x]
```

rule 773
```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a + b/x^n)^p/x^
2, x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] &&  !IntegerQ[p]
```

rule 868
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[1/(m + 1)
 Subst[Int[(a + b*x^Simplify[n/(m + 1)])^p, x], x, x^(m + 1)], x] /; FreeQ[
{a, b, m, n, p}, x] && IntegerQ[Simplify[n/(m + 1)]] &&  !IntegerQ[n]
```

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \sqrt{\cosh\left(a+\frac{2\ln\left(c\,x^{n}\right)}{n}\right)}dx$$

input
```
int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

output
```
int(cosh(a+2*ln(c*x^n)/n)^(1/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.38

$$\int \sqrt{\cosh\left(a+\frac{2\log\left(cx^{n}\right)}{n}\right)}\,dx$$
$$=\frac{1}{8}\left(4\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}+\sqrt{2}e^{\left(\frac{an+2\,\log(c)}{2\,n}\right)}\log\left(\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}-2\,\sqrt{2}\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e}{}}}{x^4}\right.\right.$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="fricas")
```

output
```
1/8*(4*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(1/2*(a*
n + 2*log(c))/n) + sqrt(2)*e^(1/2*(a*n + 2*log(c))/n)*log((x^4*e^(2*(a*n +
 2*log(c))/n) - 2*sqrt(2)*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) +
 1)/x^2) + 2)/x^4))*e^(-(a*n + 2*log(c))/n)
```

Sympy [F]

$$\int \sqrt{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx = \int \sqrt{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx$$

input
```
integrate(cosh(a+2*ln(c*x**n)/n)**(1/2),x)
```

output
```
Integral(sqrt(cosh(a + 2*log(c*x**n)/n)), x)
```

Maxima [F]

$$\int \sqrt{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx = \int \sqrt{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(cosh(a + 2*log(c*x^n)/n)), x)
```

Giac [F]

$$\int \sqrt{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx = \int \sqrt{\cosh\left(a + \frac{2\log(cx^n)}{n}\right)}\,dx$$

input
```
integrate(cosh(a+2*log(c*x^n)/n)^(1/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \sqrt{\cosh\left(a+\frac{2\,\ln(c\,x^n)}{n}\right)}\,dx$$

input
```
int(cosh(a + (2*log(c*x^n))/n)^(1/2),x)
```

output
```
int(cosh(a + (2*log(c*x^n))/n)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \sqrt{\cosh\left(\frac{2\log(x^nc)+an}{n}\right)}\,x - \left(\int \frac{\sqrt{\cosh\left(\frac{2\log(x^nc)+an}{n}\right)}\sinh\left(\frac{2\log(x^nc)+an}{n}\right)}{\cosh\left(\frac{2\log(x^nc)+an}{n}\right)}dx\right)$$

input
```
int(cosh(a+2*log(c*x^n)/n)^(1/2),x)
```

output
```
sqrt(cosh((2*log(x**n*c) + a*n)/n))*x - int((sqrt(cosh((2*log(x**n*c) + a*
n)/n))*sinh((2*log(x**n*c) + a*n)/n))/cosh((2*log(x**n*c) + a*n)/n),x)
```

3.261 $\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 42

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=-\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

output
```
-1/2*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(3/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx =\frac{-\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)}{x\sqrt{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)}}$$

input
```
Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

output
```
(-Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + Sinh[a - 2*Log[x] + (2*Log[c*x^n
])/n])/(x*Sqrt[Cosh[a + (2*Log[c*x^n])/n]])
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6052, 6060, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}dx$$

$$\Big\downarrow \; 6052$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}d(cx^n)}{n}$$

$$\Big\downarrow \; 6060$$

$$\frac{x(cx^n)^{2/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}\int \frac{(cx^n)^{-1-\frac{2}{n}}}{\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{3/2}}d(cx^n)}{n\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \; 796$$

$$-\frac{x\left(e^{-2a}(cx^n)^{-4/n}+1\right)}{2\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Int[Cosh[a + (2*Log[c*x^n])/n]^(-3/2),x]
```

output
```
-1/2*(x*(1 + 1/(E^(2*a)*(c*x^n)^(4/n))))/Cosh[a + (2*Log[c*x^n])/n]^(3/2)
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^{n})}{n}\right)^{\frac{3}{2}}}dx$$

input
```
int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

output
```
int(1/cosh(a+2*ln(c*x^n)/n)^(3/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.62

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{2\sqrt{\frac{1}{2}}x\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="fricas")
```

output
```
-2*sqrt(1/2)*x*sqrt((x^4*e^(2*(a*n + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n +
 2*log(c))/n)/(x^4*e^(2*(a*n + 2*log(c))/n) + 1)
```

Sympy [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log\left(cx^n\right)}{n}\right)}\,dx$$

input
```
integrate(1/cosh(a+2*ln(c*x**n)/n)**(3/2),x)
```

output
```
Integral(cosh(a + 2*log(c*x**n)/n)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="maxima")
```

output
```
integrate(cosh(a + 2*log(c*x^n)/n)^(-3/2), x)
```

Giac [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(3/2),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{3/2}}\,dx$$

input
```
int(1/cosh(a + (2*log(c*x^n))/n)^(3/2),x)
```

output
```
int(1/cosh(a + (2*log(c*x^n))/n)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\cosh^{\frac{3}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{\sqrt{\cosh\left(\frac{2\log(x^nc)+an}{n}\right)}}{\cosh\left(\frac{2\log(x^nc)+an}{n}\right)^2}\,dx$$

input
```
int(1/cosh(a+2*log(c*x^n)/n)^(3/2),x)
```

output
```
int(sqrt(cosh((2*log(x**n*c) + a*n)/n))/cosh((2*log(x**n*c) + a*n)/n)**2,x
)
```

3.262 $\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx$

Optimal result

Integrand size = 18, antiderivative size = 87

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)}{10\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)} - \frac{x\left(1+e^{-2a}(cx^n)^{-4/n}\right)^2}{15\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

output
```
-1/10*x*(1+1/exp(2*a)/((c*x^n)^(4/n)))/cosh(a+2*ln(c*x^n)/n)^(7/2)-1/15*x*
(1+1/exp(2*a)/((c*x^n)^(4/n)))^2/cosh(a+2*ln(c*x^n)/n)^(7/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.39

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \frac{\left((2+5x^4)\cosh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)+(-2+5x^4)\sinh\left(a-2\log(x)+\frac{2\log(cx^n)}{n}\right)\right)\left(-\cosh\left(2a-\right.\right.}{15x^5\cosh^{\frac{5}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Integrate[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

output
```
(((2 + 5*x^4)*Cosh[a - 2*Log[x] + (2*Log[c*x^n])/n] + (-2 + 5*x^4)*Sinh[a
- 2*Log[x] + (2*Log[c*x^n])/n])*(-Cosh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]
+ Sinh[2*a - 4*Log[x] + (4*Log[c*x^n])/n]))/(15*x^5*Cosh[a + (2*Log[c*x^n]
)/n]^(5/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.62, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {6052, 6060, 803, 796}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}dx$$

$$\Big\downarrow \text{6052}$$

$$\frac{x(cx^n)^{-1/n}\int \frac{(cx^n)^{\frac{1}{n}-1}}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}d(cx^n)}{n}$$

$$\Big\downarrow \text{6060}$$

$$\frac{x(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}\int \frac{(cx^n)^{-1-\frac{6}{n}}}{\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}}d(cx^n)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{803}$$

$$\frac{x(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}\left(\frac{2}{3}e^{-2a}\int \frac{(cx^n)^{-1-\frac{10}{n}}}{\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}}d(cx^n)-\frac{n(cx^n)^{-6/n}}{6\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}\right)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

$$\Big\downarrow \text{796}$$

$$\frac{x(cx^n)^{6/n}\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{7/2}\left(-\frac{e^{-2a}n(cx^n)^{-10/n}}{15\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}-\frac{n(cx^n)^{-6/n}}{6\left(e^{-2a}(cx^n)^{-4/n}+1\right)^{5/2}}\right)}{n\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}$$

input
```
Int[Cosh[a + (2*Log[c*x^n])/n]^(-7/2),x]
```

output
```
(x*(c*x^n)^(6/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(7/2)*(-1/15*n/(E^(2*a)*(
c*x^n)^(10/n)*(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2)) - n/(6*(c*x^n)^(6/n)*
(1 + 1/(E^(2*a)*(c*x^n)^(4/n)))^(5/2))))/(n*Cosh[a + (2*Log[c*x^n])/n]^(7/
2))
```

Defintions of rubi rules used

rule 796
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*
x)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, n,
 p}, x] && EqQ[(m + 1)/n + p + 1, 0] && NeQ[m, -1]
```

rule 803
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[x^(m + 1)*((
a + b*x^n)^(p + 1)/(a*(m + 1))), x] - Simp[b*((m + n*(p + 1) + 1)/(a*(m + 1
)))   Int[x^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, m, n, p}, x] && I
LtQ[Simplify[(m + 1)/n + p + 1], 0] && NeQ[m, -1]
```

rule 6052
```
Int[Cosh[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(d_.)]^(p_.), x_Symbol] :> S
imp[x/(n*(c*x^n)^(1/n))   Subst[Int[x^(1/n - 1)*Cosh[d*(a + b*Log[x])]^p, x
], x, c*x^n], x] /; FreeQ[{a, b, c, d, n, p}, x] && (NeQ[c, 1] || NeQ[n, 1]
)
```

rule 6060
```
Int[Cosh[((a_.) + Log[x_]*(b_.))*(d_.)]^(p_)*((e_.)*(x_))^(m_.), x_Symbol]
:> Simp[Cosh[d*(a + b*Log[x])]^p/(x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p
)   Int[(e*x)^m*x^(b*d*p)*(1 + 1/(E^(2*a*d)*x^(2*b*d)))^p, x], x] /; FreeQ[
{a, b, d, e, m, p}, x] &&  !IntegerQ[p]
```

Maple [F]

$$\int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{\frac{7}{2}}}dx$$

input
```
int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

output
```
int(1/cosh(a+2*ln(c*x^n)/n)^(7/2),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.47

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{8\,\sqrt{\frac{1}{2}}\left(5\,x^5e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+2\,x\right)\sqrt{\frac{x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1}{x^2}}\,e^{\left(-\frac{an+2\,\log(c)}{2\,n}\right)}}{15\left(x^{12}e^{\left(\frac{6\,(an+2\,\log(c))}{n}\right)}+3\,x^8e^{\left(\frac{4\,(an+2\,\log(c))}{n}\right)}+3\,x^4e^{\left(\frac{2\,(an+2\,\log(c))}{n}\right)}+1\right)}$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="fricas")
```

output
```
-8/15*sqrt(1/2)*(5*x^5*e^(2*(a*n + 2*log(c))/n) + 2*x)*sqrt((x^4*e^(2*(a*n
 + 2*log(c))/n) + 1)/x^2)*e^(-1/2*(a*n + 2*log(c))/n)/(x^12*e^(6*(a*n + 2*
log(c))/n) + 3*x^8*e^(4*(a*n + 2*log(c))/n) + 3*x^4*e^(2*(a*n + 2*log(c))/
n) + 1)
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/cosh(a+2*ln(c*x**n)/n)**(7/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = \int \frac{1}{\cosh\left(a+\frac{2\log(cx^n)}{n}\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="maxima")
```

output
```
integrate(cosh(a + 2*log(c*x^n)/n)^(-7/2), x)
```

Giac [A] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.92

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx = -\frac{4\sqrt{2}c^{\frac{7}{n}}\left(\frac{5e^a}{c^{\frac{4}{n}}\operatorname{sgn}(x)}+\frac{2e^{(-a)}}{c^{\frac{8}{n}}x^4\operatorname{sgn}(x)}\right)e^{(3a)}}{15\left(c^{\frac{4}{n}}e^{(3a)}+\frac{e^a}{x^4}\right)^{\frac{5}{2}}x^6}$$

input
```
integrate(1/cosh(a+2*log(c*x^n)/n)^(7/2),x, algorithm="giac")
```

output
```
-4/15*sqrt(2)*c^(7/n)*(5*e^a/(c^(4/n)*sgn(x)) + 2*e^(-a)/(c^(8/n)*x^4*sgn(
x)))*e^(3*a)/((c^(4/n)*e^(3*a) + e^a/x^4)^(5/2)*x^6)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{1}{\cosh\left(a+\frac{2\ln(c\,x^n)}{n}\right)^{7/2}}\,dx$$

input
```
int(1/cosh(a + (2*log(c*x^n))/n)^(7/2),x)
```

output
```
int(1/cosh(a + (2*log(c*x^n))/n)^(7/2), x)
```

Reduce [F]

$$\int \frac{1}{\cosh^{\frac{7}{2}}\left(a+\frac{2\log(cx^n)}{n}\right)}\,dx=\int \frac{\sqrt{\cosh\left(\frac{2\log(x^n c)+an}{n}\right)}}{\cosh\left(\frac{2\log(x^n c)+an}{n}\right)^4}dx$$

input
```
int(1/cosh(a+2*log(c*x^n)/n)^(7/2),x)
```

output
```
int(sqrt(cosh((2*log(x**n*c) + a*n)/n))/cosh((2*log(x**n*c) + a*n)/n)**4,x
)
```

3.263 $\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 101

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx=\frac{(c+dx)\cosh\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{(bc-ad)\operatorname{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)\sinh\left(\frac{b}{d}\right)}{d^2}-\frac{(bc-ad)\cosh\left(\frac{b}{d}\right)\operatorname{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)}{d^2}$$

output
```
(d*x+c)*cosh((b*x+a)/(d*x+c))/d+(-a*d+b*c)*Chi((-a*d+b*c)/d/(d*x+c))*sinh(
b/d)/d^2-(-a*d+b*c)*cosh(b/d)*Shi((-a*d+b*c)/d/(d*x+c))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 305 vs. $2(101) = 202$.

Time = 1.26 (sec) , antiderivative size = 305, normalized size of antiderivative = 3.02

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{cde^{-\frac{a+bx}{c+dx}}+cde^{\frac{a+bx}{c+dx}}+2d^2x\cosh\left(\frac{b}{d}\right)\cosh\left(\frac{-bc+ad}{d(c+dx)}\right)+2d^2x\sinh\left(\frac{b}{d}\right)\sinh\left(\frac{-bc+ad}{d(c+dx)}\right)-(bc-ad)\left(\operatorname{Chi}\left(\frac{bc-}{cd+}\right.\right.}{}$$

input
```
Integrate[Cosh[(a + b*x)/(c + d*x)],x]
```

output
```
((c*d)/E^((a + b*x)/(c + d*x)) + c*d*E^((a + b*x)/(c + d*x)) + 2*d^2*x*Cos
h[b/d]*Cosh[(-(b*c) + a*d)/(d*(c + d*x))] + 2*d^2*x*Sinh[b/d]*Sinh[(-(b*c)
 + a*d)/(d*(c + d*x))] - (b*c - a*d)*(CoshIntegral[(b*c - a*d)/(c*d + d^2*
x)]*(Cosh[b/d] - Sinh[b/d]) - CoshIntegral[(-(b*c) + a*d)/(d*(c + d*x))]*(
Cosh[b/d] + Sinh[b/d]) - Cosh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x
))] - Sinh[b/d]*SinhIntegral[(-(b*c) + a*d)/(d*(c + d*x))] + Cosh[b/d]*Sin
hIntegral[(b*c - a*d)/(c*d + d^2*x)] - Sinh[b/d]*SinhIntegral[(b*c - a*d)/
(c*d + d^2*x)]))/(2*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.15, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.857$, Rules used = {6142, 3042, 3778, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh \left(\frac{a+bx}{c+dx}\right) dx$$

$$\Big\downarrow \text{6142}$$

$$-\frac{\int (c+dx)^2 \cosh \left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (c+dx)^2 \sin \left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right) d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3778}$$

$$-\frac{-\left((c+dx) \cosh \left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{i(bc-ad) \int -i(c+dx) \sinh \left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right) d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sinh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$\big\downarrow$ 3042

$$-\frac{-\left((c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{(bc-ad)\int -i(c+dx)\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$\big\downarrow$ 26

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\int (c+dx)\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$\big\downarrow$ 3784

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int (c+dx)\cosh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{b}{d}\right)\int -i(c+dx)\sinh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$\big\downarrow$ 26

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int (c+dx)\cosh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{b}{d}\right)\int (c+dx)\sinh\left(\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$\big\downarrow$ 3042

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{b}{d}\right)\int -i(c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right.}{d}}{d}$$

$\big\downarrow$ 26

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cosh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$\big\downarrow$ 3779

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\int (c+dx)\sin\left(\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}}{d}$$

$\big\downarrow$ 3782

$$-\frac{-(c+dx)\cosh\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{b}{d}\right)\text{Chi}\left(\frac{bc-ad}{d(c+dx)}\right)-i\cosh\left(\frac{b}{d}\right)\text{Shi}\left(\frac{bc-ad}{d(c+dx)}\right)\right)}{d}}{d}$$

input
```
Int[Cosh[(a + b*x)/(c + d*x)],x]
```

output
```
-((-((c + d*x)*Cosh[b/d - (b*c - a*d)/(d*(c + d*x))]) + (I*(b*c - a*d)*(I*
CoshIntegral[(b*c - a*d)/(d*(c + d*x))]*Sinh[b/d] - I*Cosh[b/d]*SinhIntegr
al[(b*c - a*d)/(d*(c + d*x))]))/d)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3778
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)], x_Symbol] :> Simp[(c
 + d*x)^(m + 1)*(Sin[e + f*x]/(d*(m + 1))), x] - Simp[f/(d*(m + 1))   Int[(
c + d*x)^(m + 1)*Cos[e + f*x], x], x] /; FreeQ[{c, d, e, f}, x] && LtQ[m, -
1]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 6142
```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Cosh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(101) = 202$.

Time = 0.73 (sec) , antiderivative size = 347, normalized size of antiderivative = 3.44

method	result
risch	$\frac{e^{-\frac{bx+a}{dx+c}} a}{\frac{2da}{dx+c}-\frac{2cb}{dx+c}} - \frac{e^{-\frac{bx+a}{dx+c}} cb}{2d\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)} - \frac{e^{-\frac{b}{d}} \operatorname{expIntegral}_1\left(\frac{ad-cb}{d(dx+c)}\right) a}{2d} + \frac{e^{-\frac{b}{d}} \operatorname{expIntegral}_1\left(\frac{ad-cb}{d(dx+c)}\right) bc}{2d^2} + \frac{d\, e^{\frac{bx+a}{dx+c}} xa}{2ad-2cb} - \frac{e^{\frac{bx+}{dx+}}}{2(ad}$

input
```
int(cosh((b*x+a)/(d*x+c)),x,method=_RETURNVERBOSE)
```

output
```
1/2*exp(-(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*a-1/2/d*exp(-(b*x+a)
/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*c*b-1/2/d*exp(-b/d)*Ei(1,(a*d-b*c)/d
/(d*x+c))*a+1/2/d^2*exp(-b/d)*Ei(1,(a*d-b*c)/d/(d*x+c))*b*c+1/2*d*exp((b*x
+a)/(d*x+c))/(a*d-b*c)*x*a-1/2*exp((b*x+a)/(d*x+c))/(a*d-b*c)*x*c*b+1/2*ex
p((b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/2/d*exp((b*x+a)/(d*x+c))/(a*d-b*c)*c^2*
b+1/2/d*exp(b/d)*Ei(1,-(a*d-b*c)/d/(d*x+c))*a-1/2/d^2*exp(b/d)*Ei(1,-(a*d-
b*c)/d/(d*x+c))*b*c
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.69

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{2\left(d^2x+cd\right)\cosh\left(\frac{bx+a}{dx+c}\right) - \left((bc-ad)\mathrm{Ei}\left(\frac{bc-ad}{d^2x+cd}\right) - (bc-ad)\mathrm{Ei}\left(-\frac{bc-ad}{d^2x+cd}\right)\right)\cosh\left(\frac{b}{d}\right) + \left((bc-ad)\mathrm{Ei}\left(\frac{bc-a}{d^2x+}\right.\right.}{2\,d^2}$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="fricas")
```

output
```
1/2*(2*(d^2*x + c*d)*cosh((b*x + a)/(d*x + c)) - ((b*c - a*d)*Ei((b*c - a*
d)/(d^2*x + c*d)) - (b*c - a*d)*Ei(-(b*c - a*d)/(d^2*x + c*d)))*cosh(b/d)
+ ((b*c - a*d)*Ei((b*c - a*d)/(d^2*x + c*d)) + (b*c - a*d)*Ei(-(b*c - a*d)
/(d^2*x + c*d)))*sinh(b/d))/d^2
```

Sympy [F]

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x)
```

output
```
Integral(cosh((a + b*x)/(c + d*x)), x)
```

Maxima [F]

$$\int \cosh\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh\left(\frac{bx+a}{dx+c}\right)\,dx$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="maxima")
```

output
```
integrate(cosh((b*x + a)/(d*x + c)), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(101) = 202$.

Time = 1.70 (sec) , antiderivative size = 764, normalized size of antiderivative = 7.56

$$\int \cosh\left(\frac{a+bx}{c+dx}\right) dx = \text{Too large to display}$$

input
```
integrate(cosh((b*x+a)/(d*x+c)),x, algorithm="giac")
```

output
```
1/2*(b^3*c^2*Ei(-(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - 2*a*b^2*c*d*Ei(-
(b - (b*x + a)*d/(d*x + c))/d)*e^(b/d) - (b*x + a)*b^2*c^2*d*Ei(-(b - (b*x
 + a)*d/(d*x + c))/d)*e^(b/d)/(d*x + c) + a^2*b*d^2*Ei(-(b - (b*x + a)*d/(
d*x + c))/d)*e^(b/d) + 2*(b*x + a)*a*b*c*d^2*Ei(-(b - (b*x + a)*d/(d*x + c
))/d)*e^(b/d)/(d*x + c) - (b*x + a)*a^2*d^3*Ei(-(b - (b*x + a)*d/(d*x + c)
)/d)*e^(b/d)/(d*x + c) + b^2*c^2*d*e^((b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e
^((b*x + a)/(d*x + c)) + a^2*d^3*e^((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)
^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c)) - 1/2*(b^3*c^2*E
i((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d) - 2*a*b^2*c*d*Ei((b - (b*x + a)*
d/(d*x + c))/d)*e^(-b/d) - (b*x + a)*b^2*c^2*d*Ei((b - (b*x + a)*d/(d*x +
c))/d)*e^(-b/d)/(d*x + c) + a^2*b*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^
(-b/d) + 2*(b*x + a)*a*b*c*d^2*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/
(d*x + c) - (b*x + a)*a^2*d^3*Ei((b - (b*x + a)*d/(d*x + c))/d)*e^(-b/d)/(
d*x + c) - b^2*c^2*d*e^(-(b*x + a)/(d*x + c)) + 2*a*b*c*d^2*e^(-(b*x + a)/
(d*x + c)) - a^2*d^3*e^(-(b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b
*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \cosh\left(\frac{a+bx}{c+dx}\right) dx = \int \cosh\left(\frac{a+b\,x}{c+d\,x}\right) dx$$

input
```
int(cosh((a + b*x)/(c + d*x)),x)
```

output
```
int(cosh((a + b*x)/(c + d*x)), x)
```

Reduce [F]

$$\int \cosh \left(\frac{a+bx}{c+dx}\right) dx = \text{Too large to display}$$

input
```
int(cosh((b*x+a)/(d*x+c)),x)
```

output
```
(e**((2*a + 2*b*x)/(c + d*x))*a*d**2*x**2 - e**((2*a + 2*b*x)/(c + d*x))*b
*c*d*x**2 - e**((2*a + 2*b*x)/(c + d*x))*c**3 - e**((2*a + 2*b*x)/(c + d*x
))*c**2*d*x - e**((a + b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/(c + d*x))*
c**3 + 3*e**((a + b*x)/(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*
d**2*x**2 + e**((a + b*x)/(c + d*x))*d**3*x**3),x)*a**2*c*d**3 - e**((a +
b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e**((a + b*x)/
(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 + e**((a + b*
x)/(c + d*x))*d**3*x**3),x)*a**2*d**4*x + 2*e**((a + b*x)/(c + d*x))*int(x
**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e**((a + b*x)/(c + d*x))*c**2*d*x +
 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 + e**((a + b*x)/(c + d*x))*d**3*x*
*3),x)*a*b*c**2*d**2 + 2*e**((a + b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/
(c + d*x))*c**3 + 3*e**((a + b*x)/(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c
 + d*x))*c*d**2*x**2 + e**((a + b*x)/(c + d*x))*d**3*x**3),x)*a*b*c*d**3*x
 - e**((a + b*x)/(c + d*x))*int(x**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e*
*((a + b*x)/(c + d*x))*c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 +
 e**((a + b*x)/(c + d*x))*d**3*x**3),x)*b**2*c**3*d - e**((a + b*x)/(c + d
*x))*int(x**2/(e**((a + b*x)/(c + d*x))*c**3 + 3*e**((a + b*x)/(c + d*x))*
c**2*d*x + 3*e**((a + b*x)/(c + d*x))*c*d**2*x**2 + e**((a + b*x)/(c + d*x
))*d**3*x**3),x)*b**2*c**2*d**2*x + e**((a + b*x)/(c + d*x))*int((e**((a +
 b*x)/(c + d*x))*x**2)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),...
```

3.264 $\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 107

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx=\frac{(c+dx)\cosh^2\left(\frac{a+bx}{c+dx}\right)}{d}+\frac{(bc-ad)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\sinh\left(\frac{2b}{d}\right)}{d^2}-\frac{(bc-ad)\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)}{d^2}$$

output
```
(d*x+c)*cosh((b*x+a)/(d*x+c))^2/d+(-a*d+b*c)*Chi(2*(-a*d+b*c)/d/(d*x+c))*s
inh(2*b/d)/d^2-(-a*d+b*c)*cosh(2*b/d)*Shi(2*(-a*d+b*c)/d/(d*x+c))/d^2
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 475 vs. 2(107) = 214.

Time = 2.98 (sec) , antiderivative size = 475, normalized size of antiderivative = 4.44

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$=\frac{cde^{-\frac{2(a+bx)}{c+dx}}+cde^{\frac{2(a+bx)}{c+dx}}+2d^2x+2d^2x\cosh\left(\frac{2b}{d}\right)\cosh\left(\frac{2(-bc+ad)}{d(c+dx)}\right)-2(bc-ad)\text{Chi}\left(\frac{2bc-2ad}{cd+d^2x}\right)\left(\cosh\left(\frac{2b}{d}\right)-\right.}{}$$

input
```
Integrate[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

output
```
((c*d)/E^((2*(a + b*x))/(c + d*x)) + c*d*E^((2*(a + b*x))/(c + d*x)) + 2*d
^2*x + 2*d^2*x*Cosh[(2*b)/d]*Cosh[(2*(-(b*c) + a*d))/(d*(c + d*x))] - 2*(b
*c - a*d)*CoshIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)]*(Cosh[(2*b)/d] - Sin
h[(2*b)/d]) + 2*(b*c - a*d)*CoshIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))]
*(Cosh[(2*b)/d] + Sinh[(2*b)/d]) + 2*d^2*x*Sinh[(2*b)/d]*Sinh[(2*(-(b*c) +
 a*d))/(d*(c + d*x))] + 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d)
)/(d*(c + d*x))] - 2*a*d*Cosh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*
(c + d*x))] + 2*b*c*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c +
d*x))] - 2*a*d*Sinh[(2*b)/d]*SinhIntegral[(2*(-(b*c) + a*d))/(d*(c + d*x))
] - 2*b*c*Cosh[(2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*a*
d*Cosh[(2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] + 2*b*c*Sinh[(
2*b)/d]*SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)] - 2*a*d*Sinh[(2*b)/d]*
SinhIntegral[(2*b*c - 2*a*d)/(c*d + d^2*x)])/(4*d^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.66 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {6142, 3042, 3794, 27, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)dx$$

$$\Big\downarrow \text{6142}$$

$$-\frac{\int (c+dx)^2\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{\int (c+dx)^2\sin\left(\frac{ib}{d}-\frac{i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)^2d\frac{1}{c+dx}}{d}$$

$$\Big\downarrow \text{3794}$$

$$-\frac{-\left((c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{2i(bc-ad)\int -\frac{1}{2}i(c+dx)\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{27}$$

$$-\frac{-\frac{(bc-ad)\int (c+dx)\sinh\left(\frac{2b}{d}-\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}-\left((c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-\left((c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)\right)-\frac{(bc-ad)\int -i(c+dx)\sin\left(\frac{2ib}{d}-\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\int (c+dx)\sin\left(\frac{2ib}{d}-\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}}{d}}{d}$$

$$\Big\downarrow \text{3784}$$

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}+\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sinh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\cosh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int (c+dx)\sinh\left(\frac{2(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\int -i(c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{26}$$

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-\cosh\left(\frac{2b}{d}\right)\int (c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}\right)d\frac{1}{c+dx}\right)}{d}}{d}$$

$$\Big\downarrow \text{3779}$$

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\int(c+dx)\sin\left(\frac{2i(bc-ad)}{d(c+dx)}+\frac{\pi}{2}\right)d\frac{1}{c+dx}-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}}{d}$$

$$\Big\downarrow \ 3782$$

$$-\frac{-(c+dx)\cosh^2\left(\frac{b}{d}-\frac{bc-ad}{d(c+dx)}\right)+\frac{i(bc-ad)\left(i\sinh\left(\frac{2b}{d}\right)\text{Chi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)-i\cosh\left(\frac{2b}{d}\right)\text{Shi}\left(\frac{2(bc-ad)}{d(c+dx)}\right)\right)}{d}}{d}$$

input
```
Int[Cosh[(a + b*x)/(c + d*x)]^2,x]
```

output
```
-((-((c + d*x)*Cosh[b/d - (b*c - a*d)/(d*(c + d*x))]^2) + (I*(b*c - a*d)*(
I*CoshIntegral[(2*(b*c - a*d))/(d*(c + d*x))]*Sinh[(2*b)/d] - I*Cosh[(2*b)
/d]*SinhIntegral[(2*(b*c - a*d))/(d*(c + d*x))]))/d)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 3794
```
Int[((c_.) + (d_.)*(x_))^(m_)*sin[(e_.) + (f_.)*(x_)]^(n_), x_Symbol] :> Si
mp[(c + d*x)^(m + 1)*(Sin[e + f*x]^n/(d*(m + 1))), x] - Simp[f*(n/(d*(m + 1
)))   Int[ExpandTrigReduce[(c + d*x)^(m + 1), Cos[e + f*x]*Sin[e + f*x]^(n
- 1), x], x], x] /; FreeQ[{c, d, e, f, m}, x] && IGtQ[n, 1] && GeQ[m, -2] &
& LtQ[m, -1]
```

rule 6142
```
Int[Cosh[((e_.)*((a_.) + (b_.)*(x_)))/((c_.) + (d_.)*(x_))]^(n_.), x_Symbol
] :> Simp[-d^(-1)   Subst[Int[Cosh[b*(e/d) - e*(b*c - a*d)*(x/d)]^n/x^2, x]
, x, 1/(c + d*x)], x] /; FreeQ[{a, b, c, d}, x] && IGtQ[n, 0] && NeQ[b*c -
a*d, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(107) = 214$.

Time = 3.54 (sec) , antiderivative size = 358, normalized size of antiderivative = 3.35

method	result
risch	$\frac{x}{2}+\frac{\mathrm{e}^{-\frac{2(bx+a)}{dx+c}} a}{\frac{4da}{dx+c}-\frac{4cb}{dx+c}}-\frac{\mathrm{e}^{-\frac{2(bx+a)}{dx+c}} cb}{4d\left(\frac{da}{dx+c}-\frac{cb}{dx+c}\right)}-\frac{\mathrm{e}^{-\frac{2b}{d}} \operatorname{expIntegral}_{1}\left(\frac{2ad-2cb}{(dx+c)d}\right) a}{2d}+\frac{\mathrm{e}^{-\frac{2b}{d}} \operatorname{expIntegral}_{1}\left(\frac{2ad-2cb}{(dx+c)d}\right) bc}{2d^{2}}+\frac{d\,\mathrm{e}^{\frac{2bx+2a}{dx+c}} xa}{4ad-4cb}$

input
```
int(cosh((b*x+a)/(d*x+c))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/4*exp(-2*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*a-1/4/d*exp(
-2*(b*x+a)/(d*x+c))/(d/(d*x+c)*a-1/(d*x+c)*c*b)*c*b-1/2/d*exp(-2*b/d)*Ei(1
,2*(a*d-b*c)/d/(d*x+c))*a+1/2/d^2*exp(-2*b/d)*Ei(1,2*(a*d-b*c)/d/(d*x+c))*
b*c+1/4*d*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*x*a-1/4*exp(2*(b*x+a)/(d*x+c))/
(a*d-b*c)*x*c*b+1/4*exp(2*(b*x+a)/(d*x+c))/(a*d-b*c)*c*a-1/4/d*exp(2*(b*x+
a)/(d*x+c))/(a*d-b*c)*c^2*b+1/2/d*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*
a-1/2/d^2*exp(2*b/d)*Ei(1,-2*(a*d-b*c)/d/(d*x+c))*b*c
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(107) = 214$.

Time = 0.11 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.42

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx$$
$$= \frac{d^2x + (d^2x+cd)\cosh\left(\frac{bx+a}{dx+c}\right)^2 + \left(d^2x - (bc-ad)\mathrm{Ei}\left(-\frac{2\,(bc-ad)}{d^2x+cd}\right)\cosh\left(\frac{2\,b}{d}\right) + cd\right)\sinh\left(\frac{bx+a}{dx+c}\right)^2 + \left((bc-\right.}{}$$

input
```
integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="fricas")
```

output
```
1/2*(d^2*x + (d^2*x + c*d)*cosh((b*x + a)/(d*x + c))^2 + (d^2*x - (b*c - a
*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh(2*b/d) + c*d)*sinh((b*x + a)/(d*
x + c))^2 + ((b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(
d*x + c))^2 - (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*cosh(2*b/d) + (
(b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*cosh((b*x + a)/(d*x + c))^2 -
 (b*c - a*d)*Ei(-2*(b*c - a*d)/(d^2*x + c*d))*sinh((b*x + a)/(d*x + c))^2
+ (b*c - a*d)*Ei(2*(b*c - a*d)/(d^2*x + c*d)))*sinh(2*b/d))/(d^2*cosh((b*x
 + a)/(d*x + c))^2 - d^2*sinh((b*x + a)/(d*x + c))^2)
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Timed out}$$

input
```
integrate(cosh((b*x+a)/(d*x+c))**2,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx = \int \cosh\left(\frac{bx+a}{dx+c}\right)^2\,dx$$

input
```
integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="maxima")
```

output
```
1/2*x + 1/4*integrate(e^(2*b*c/(d^2*x + c*d) - 2*a/(d*x + c) - 2*b/d), x)
+ 1/4*integrate(e^(-2*b*c/(d^2*x + c*d) + 2*a/(d*x + c) + 2*b/d), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(107) = 214$.

Time = 6.44 (sec) , antiderivative size = 749, normalized size of antiderivative = 7.00

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx = \text{Too large to display}$$

input
```
integrate(cosh((b*x+a)/(d*x+c))^2,x, algorithm="giac")
```

output
```
1/4*(2*b^3*c^2*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 4*a*b^2*c*
d*Ei(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) - 2*(b*x + a)*b^2*c^2*d*E
i(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) + 2*a^2*b*d^2*Ei(-
2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d) + 4*(b*x + a)*a*b*c*d^2*Ei(-2*(
b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*(b*x + a)*a^2*d^3*Ei
(-2*(b - (b*x + a)*d/(d*x + c))/d)*e^(2*b/d)/(d*x + c) - 2*b^3*c^2*Ei(2*(b
 - (b*x + a)*d/(d*x + c))/d)*e^(-2*b/d) + 4*a*b^2*c*d*Ei(2*(b - (b*x + a)*
d/(d*x + c))/d)*e^(-2*b/d) + 2*(b*x + a)*b^2*c^2*d*Ei(2*(b - (b*x + a)*d/(
d*x + c))/d)*e^(-2*b/d)/(d*x + c) - 2*a^2*b*d^2*Ei(2*(b - (b*x + a)*d/(d*x
 + c))/d)*e^(-2*b/d) - 4*(b*x + a)*a*b*c*d^2*Ei(2*(b - (b*x + a)*d/(d*x +
c))/d)*e^(-2*b/d)/(d*x + c) + 2*(b*x + a)*a^2*d^3*Ei(2*(b - (b*x + a)*d/(d
*x + c))/d)*e^(-2*b/d)/(d*x + c) + b^2*c^2*d*e^(2*(b*x + a)/(d*x + c)) - 2
*a*b*c*d^2*e^(2*(b*x + a)/(d*x + c)) + a^2*d^3*e^(2*(b*x + a)/(d*x + c)) +
 b^2*c^2*d*e^(-2*(b*x + a)/(d*x + c)) - 2*a*b*c*d^2*e^(-2*(b*x + a)/(d*x +
 c)) + a^2*d^3*e^(-2*(b*x + a)/(d*x + c)) + 2*b^2*c^2*d - 4*a*b*c*d^2 + 2*
a^2*d^3)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(b*d^2 - (b*x + a)*d^3/(d
*x + c))
```

Mupad [F(-1)]

Timed out.

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx=\int \cosh\left(\frac{a+b\,x}{c+d\,x}\right)^2\,dx$$

input
```
int(cosh((a + b*x)/(c + d*x))^2,x)
```

output
```
int(cosh((a + b*x)/(c + d*x))^2, x)
```

Reduce [F]

$$\int \cosh^2\left(\frac{a+bx}{c+dx}\right)\,dx=\text{Too large to display}$$

input
```
int(cosh((b*x+a)/(d*x+c))^2,x)
```

output

```
(2*e**((4*a + 4*b*x)/(c + d*x))*a*d**2*x**2 - 2*e**((4*a + 4*b*x)/(c + d*x
))*b*c*d*x**2 - e**((4*a + 4*b*x)/(c + d*x))*c**3 - e**((4*a + 4*b*x)/(c +
 d*x))*c**2*d*x - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*
x)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a
+ 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),
x)*a**2*c*d**3 - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x
)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x
)*a**2*d**4*x + 8*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x)
/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x)
*a*b*c**2*d**2 + 8*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x
)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x
)*a*b*c*d**3*x - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x
)/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
 2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3),x
)*b**2*c**3*d - 4*e**((2*a + 2*b*x)/(c + d*x))*int(x**2/(e**((2*a + 2*b*x)
/(c + d*x))*c**3 + 3*e**((2*a + 2*b*x)/(c + d*x))*c**2*d*x + 3*e**((2*a +
2*b*x)/(c + d*x))*c*d**2*x**2 + e**((2*a + 2*b*x)/(c + d*x))*d**3*x**3)...
```

3.265 $\int e^{a+bx} \cosh^4(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 83

$$\int e^{a+bx} \cosh^4(a+bx)\,dx = -\frac{e^{-3a-3bx}}{48b} - \frac{e^{-a-bx}}{4b} + \frac{3e^{a+bx}}{8b} + \frac{e^{3a+3bx}}{12b} + \frac{e^{5a+5bx}}{80b}$$

output
```
-1/48*exp(-3*b*x-3*a)/b-1/4*exp(-b*x-a)/b+3/8*exp(b*x+a)/b+1/12*exp(3*b*x+
3*a)/b+1/80*exp(5*b*x+5*a)/b
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.75

$$\int e^{a+bx} \cosh^4(a+bx)\,dx = \frac{e^{-3(a+bx)}\left(-5-60e^{2(a+bx)}+90e^{4(a+bx)}+20e^{6(a+bx)}+3e^{8(a+bx)}\right)}{240b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

output
```
(-5 - 60*E^(2*(a + b*x)) + 90*E^(4*(a + b*x)) + 20*E^(6*(a + b*x)) + 3*E^(
8*(a + b*x)))/(240*b*E^(3*(a + b*x)))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^4(a+bx)\,dx$$

$$\Big\downarrow \; 2720$$

$$\frac{\int \frac{1}{16}e^{-4a-4bx}\left(1+e^{2a+2bx}\right)^4de^{a+bx}}{b}$$

$$\Big\downarrow \; 27$$

$$\frac{\int e^{-4a-4bx}\left(1+e^{2a+2bx}\right)^4de^{a+bx}}{16b}$$

$$\Big\downarrow \; 244$$

$$\frac{\int \left(6+e^{-4a-4bx}+4e^{-2a-2bx}+4e^{2a+2bx}+e^{4a+4bx}\right)de^{a+bx}}{16b}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{3}e^{-3a-3bx}-4e^{-a-bx}+6e^{a+bx}+\frac{4}{3}e^{3a+3bx}+\frac{1}{5}e^{5a+5bx}}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^4,x]
```

output
```
(-1/3*E^(-3*a - 3*b*x) - 4*E^(-a - b*x) + 6*E^(a + b*x) + (4*E^(3*a + 3*b*
x))/3 + E^(5*a + 5*b*x)/5)/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.54

method	result
derivativedivides	$\frac{\frac{\cosh(bx+a)^5}{5}+\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$
default	$\frac{\frac{\cosh(bx+a)^5}{5}+\left(\frac{8}{15}+\frac{\cosh(bx+a)^4}{5}+\frac{4\cosh(bx+a)^2}{15}\right)\sinh(bx+a)}{b}$
risch	$-\frac{e^{-3bx-3a}}{48b}-\frac{e^{-bx-a}}{4b}+\frac{3\,e^{bx+a}}{8b}+\frac{e^{3bx+3a}}{12b}+\frac{e^{5bx+5a}}{80b}$
parallelrisch	$\frac{2\,e^{bx+a}\left(15\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^7-15\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^6-5\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^5+25\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^4+13\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3-21\tanh\left(\frac{bx}{2}\right.\right.}{15b\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^4\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^4}$
orering	$\frac{e^{bx+a}\cosh(bx+a)^4}{5b}+\frac{\frac{10b\,e^{bx+a}\cosh(bx+a)^4}{9}+\frac{40\,e^{bx+a}\cosh(bx+a)^3 b\sinh(bx+a)}{9}}{b^2}-\frac{2\left(5b^2 e^{bx+a}\cosh(bx+a)^4+8b^2 e^{bx}\right.}{}$

input
```
int(exp(b*x+a)*cosh(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/5*cosh(b*x+a)^5+(8/15+1/5*cosh(b*x+a)^4+4/15*cosh(b*x+a)^2)*sinh(b*
x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.36

$$\int e^{a+bx}\cosh^4(a+bx)\,dx =$$
$$-\frac{\cosh(bx+a)^4-16\cosh(bx+a)\sinh(bx+a)^3+\sinh(bx+a)^4+2\left(3\cosh(bx+a)^2+10\right)\sinh(bx}{120\left(b\cosh(bx+a)-b\sin\right.}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="fricas")
```

output
```
-1/120*(cosh(b*x + a)^4 - 16*cosh(b*x + a)*sinh(b*x + a)^3 + sinh(b*x + a)
^4 + 2*(3*cosh(b*x + a)^2 + 10)*sinh(b*x + a)^2 + 20*cosh(b*x + a)^2 - 16*
(cosh(b*x + a)^3 + 5*cosh(b*x + a))*sinh(b*x + a) - 45)/(b*cosh(b*x + a) -
 b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. 2(65) = 130.

Time = 2.35 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\cosh^4(a+bx)\,dx$$
$$=\begin{cases}\frac{8e^ae^{bx}\sinh^4(a+bx)}{15b}-\frac{8e^ae^{bx}\sinh^3(a+bx)\cosh(a+bx)}{15b}-\frac{4e^ae^{bx}\sinh^2(a+bx)\cosh^2(a+bx)}{5b}+\frac{4e^ae^{bx}\sinh(a+bx)\cosh^3(a+bx)}{5b}+e^{\cdots} \\ xe^a\cosh^4(a)\end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**4,x)
```

output
```
Piecewise((8*exp(a)*exp(b*x)*sinh(a + b*x)**4/(15*b) - 8*exp(a)*exp(b*x)*s
inh(a + b*x)**3*cosh(a + b*x)/(15*b) - 4*exp(a)*exp(b*x)*sinh(a + b*x)**2*
cosh(a + b*x)**2/(5*b) + 4*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**3/
(5*b) + exp(a)*exp(b*x)*cosh(a + b*x)**4/(5*b), Ne(b, 0)), (x*exp(a)*cosh(
a)**4, True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^4(a+bx)\,dx = \frac{e^{(5\,bx+5\,a)}}{80\,b}+\frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{3\,e^{(bx+a)}}{8\,b}-\frac{e^{(-bx-a)}}{4\,b}-\frac{e^{(-3\,bx-3\,a)}}{48\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="maxima")
```

output
```
1/80*e^(5*b*x + 5*a)/b + 1/12*e^(3*b*x + 3*a)/b + 3/8*e^(b*x + a)/b - 1/4*
e^(-b*x - a)/b - 1/48*e^(-3*b*x - 3*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.72

$$\int e^{a+bx}\cosh^4(a+bx)\,dx = -\frac{5\left(12\,e^{(2\,bx+2\,a)}+1\right)e^{(-3\,bx-3\,a)}-3\,e^{(5\,bx+5\,a)}-20\,e^{(3\,bx+3\,a)}-90\,e^{(bx+a)}}{240\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^4,x, algorithm="giac")
```

output
```
-1/240*(5*(12*e^(2*b*x + 2*a) + 1)*e^(-3*b*x - 3*a) - 3*e^(5*b*x + 5*a) -
20*e^(3*b*x + 3*a) - 90*e^(b*x + a))/b
```

Mupad [B] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.70

$$\int e^{a+bx} \cosh^4(a+bx)\, dx = \frac{90\, e^{a+b\,x} - 60\, e^{-a-b\,x} - 5\, e^{-3\,a-3\,b\,x} + 20\, e^{3\,a+3\,b\,x} + 3\, e^{5\,a+5\,b\,x}}{240\, b}$$

input
```
int(cosh(a + b*x)^4*exp(a + b*x),x)
```

output
```
(90*exp(a + b*x) - 60*exp(- a - b*x) - 5*exp(- 3*a - 3*b*x) + 20*exp(3*a +
 3*b*x) + 3*exp(5*a + 5*b*x))/(240*b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int e^{a+bx} \cosh^4(a+bx)\, dx = \frac{3e^{8bx+8a} + 20e^{6bx+6a} + 90e^{4bx+4a} - 60e^{2bx+2a} - 5}{240e^{3bx+3a}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^4,x)
```

output
```
(3*e**(8*a + 8*b*x) + 20*e**(6*a + 6*b*x) + 90*e**(4*a + 4*b*x) - 60*e**(2
*a + 2*b*x) - 5)/(240*e**(3*a + 3*b*x)*b)
```

3.266 $\int e^{a+bx} \cosh^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 57

$$\int e^{a+bx} \cosh^3(a+bx)\,dx = -\frac{e^{-2a-2bx}}{16b} + \frac{3e^{2a+2bx}}{16b} + \frac{e^{4a+4bx}}{32b} + \frac{3x}{8}$$

output
```
-1/16*exp(-2*b*x-2*a)/b+3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int e^{a+bx} \cosh^3(a+bx)\,dx = \frac{-\frac{1}{16}e^{-2a-2bx} + \frac{3}{16}e^{2a+2bx} + \frac{1}{32}e^{4a+4bx} + \frac{3bx}{8}}{b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^3,x]
```

output
```
(-1/16*E^(-2*a - 2*b*x) + (3*E^(2*a + 2*b*x))/16 + E^(4*a + 4*b*x)/32 + (3
*b*x)/8)/b
```

Rubi [A] (warning: unable to verify)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.312$, Rules used = {2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{8}e^{-3a-3bx}\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-3a-3bx}\left(1+e^{2a+2bx}\right)^3de^{a+bx}}{8b}$$

$$\Big\downarrow \text{243}$$

$$\frac{\int e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^3de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(3+e^{-2a-2bx}+3e^{-a-bx}+e^{2a+2bx}\right)de^{2a+2bx}}{16b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-e^{-a-bx}+\frac{7}{2}e^{2a+2bx}+3\log\left(e^{2a+2bx}\right)}{16b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^3,x]
```

output
```
(-E^(-a - b*x) + (7*E^(2*a + 2*b*x))/2 + 3*Log[E^(2*a + 2*b*x)])/(16*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.65 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.82

method	result
risch	$-\frac{{\rm e}^{-2bx-2a}}{16b}+\frac{3\,{\rm e}^{2bx+2a}}{16b}+\frac{{\rm e}^{4bx+4a}}{32b}+\frac{3x}{8}$
derivativedivides	$\frac{\frac{\cosh\left(bx+a\right)^{4}}{4}+\left(\frac{\cosh\left(bx+a\right)^{3}}{4}+\frac{3\cosh\left(bx+a\right)}{8}\right)\sinh\left(bx+a\right)+\frac{3bx}{8}+\frac{3a}{8}}{b}$
default	$\frac{\frac{\cosh\left(bx+a\right)^{4}}{4}+\left(\frac{\cosh\left(bx+a\right)^{3}}{4}+\frac{3\cosh\left(bx+a\right)}{8}\right)\sinh\left(bx+a\right)+\frac{3bx}{8}+\frac{3a}{8}}{b}$
parallelrisch	$\frac{{\rm e}^{bx+a}\left(12bx\cosh\left(bx+a\right)-12bx\sinh\left(bx+a\right)+\cosh\left(bx+a\right)+11\sinh\left(bx+a\right)-\cosh\left(3bx+3a\right)+3\sinh\left(3bx+3a\right)\right)}{32b}$
orering	$\frac{\left(4bx+1\right){\rm e}^{bx+a}\cosh\left(bx+a\right)^{3}}{4b}-\frac{\left(bx-1\right)\left(b\,{\rm e}^{bx+a}\cosh\left(bx+a\right)^{3}+3\,{\rm e}^{bx+a}\cosh\left(bx+a\right)^{2}b\sinh\left(bx+a\right)\right)}{4b^{2}}-\frac{\left(4bx+1\right)\left(4\,{\rm e}^{bx}\right.}{}$

input
```
int(exp(b*x+a)*cosh(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*exp(-2*b*x-2*a)/b+3/16*exp(2*b*x+2*a)/b+1/32*exp(4*b*x+4*a)/b+3/8*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. 2(46) = 92.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.67

$$\int e^{a+bx}\cosh^3(a+bx)\,dx = -\frac{\cosh(bx+a)^3 + 3\cosh(bx+a)\sinh(bx+a)^2 - 3\sinh(bx+a)^3 - 6(2bx+1)\cosh(bx+a) + 3(4b}{32(b\cosh(bx+a) - b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*x + a)^3 + 3*cosh(b*x + a)*sinh(b*x + a)^2 - 3*sinh(b*x + a)
^3 - 6*(2*b*x + 1)*cosh(b*x + a) + 3*(4*b*x - 3*cosh(b*x + a)^2 - 2)*sinh(
b*x + a))/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 207 vs. 2(48) = 96.

Time = 0.99 (sec) , antiderivative size = 207, normalized size of antiderivative = 3.63

$$\int e^{a+bx}\cosh^3(a+bx)\,dx = \begin{cases} \frac{3xe^{a}e^{bx}\sinh^3(a+bx)}{8} - \frac{3xe^{a}e^{bx}\sinh^2(a+bx)\cosh(a+bx)}{8} - \frac{3xe^{a}e^{bx}\sinh(a+bx)\cosh^2(a+bx)}{8} + \frac{3xe^{a}e^{bx}\cosh^3(a+bx)}{8} - \frac{5e^{a}e^{bx}\sin}{8} \\ xe^{a}\cosh^3(a) \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**3,x)
```

output
```
Piecewise((3*x*exp(a)*exp(b*x)*sinh(a + b*x)**3/8 - 3*x*exp(a)*exp(b*x)*si
nh(a + b*x)**2*cosh(a + b*x)/8 - 3*x*exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a
+ b*x)**2/8 + 3*x*exp(a)*exp(b*x)*cosh(a + b*x)**3/8 - 5*exp(a)*exp(b*x)*s
inh(a + b*x)**3/(8*b) + exp(a)*exp(b*x)*sinh(a + b*x)**2*cosh(a + b*x)/(4*
b) + exp(a)*exp(b*x)*sinh(a + b*x)*cosh(a + b*x)**2/b - 3*exp(a)*exp(b*x)*
cosh(a + b*x)**3/(8*b), Ne(b, 0)), (x*exp(a)*cosh(a)**3, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.93

$$\int e^{a+bx}\cosh^3(a+bx)\,dx=\frac{3\,(bx+a)}{8\,b}+\frac{e^{(4\,bx+4\,a)}}{32\,b}+\frac{3\,e^{(2\,bx+2\,a)}}{16\,b}-\frac{e^{(-2\,bx-2\,a)}}{16\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="maxima")
```

output
```
3/8*(b*x + a)/b + 1/32*e^(4*b*x + 4*a)/b + 3/16*e^(2*b*x + 2*a)/b - 1/16*e
^(-2*b*x - 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\cosh^3(a+bx)\,dx=\frac{12\,bx-2\left(3\,e^{(2\,bx+2\,a)}+1\right)e^{(-2\,bx-2\,a)}+12\,a+e^{(4\,bx+4\,a)}+6\,e^{(2\,bx+2\,a)}}{32\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^3,x, algorithm="giac")
```

output
```
1/32*(12*b*x - 2*(3*e^(2*b*x + 2*a) + 1)*e^(-2*b*x - 2*a) + 12*a + e^(4*b*
x + 4*a) + 6*e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.74

$$\int e^{a+bx}\cosh^3(a+bx)\,dx=\frac{3\,x}{8}+\frac{\frac{3\,\mathrm{e}^{2\,a+2\,b\,x}}{16}-\frac{\mathrm{e}^{-2\,a-2\,b\,x}}{16}+\frac{\mathrm{e}^{4\,a+4\,b\,x}}{32}}{b}$$

input
```
int(cosh(a + b*x)^3*exp(a + b*x),x)
```

output
```
(3*x)/8 + ((3*exp(2*a + 2*b*x))/16 - exp(- 2*a - 2*b*x)/16 + exp(4*a + 4*b
*x)/32)/b
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\cosh^3(a+bx)\,dx=\frac{e^{6bx+6a}+6e^{4bx+4a}+12e^{2bx+2a}bx-2}{32e^{2bx+2a}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^3,x)
```

output
```
(e**(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) + 12*e**(2*a + 2*b*x)*b*x - 2)/(32*
e**(2*a + 2*b*x)*b)
```

3.267 $\int e^{a+bx}\cosh^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 49

$$\int e^{a+bx}\cosh^2(a+bx)\,dx=-\frac{e^{-a-bx}}{4b}+\frac{e^{a+bx}}{2b}+\frac{e^{3a+3bx}}{12b}$$

output
```
-1/4*exp(-b*x-a)/b+1/2*exp(b*x+a)/b+1/12*exp(3*b*x+3*a)/b
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.80

$$\int e^{a+bx}\cosh^2(a+bx)\,dx=\frac{e^{-a-bx}\left(-3+6e^{2(a+bx)}+e^{4(a+bx)}\right)}{12b}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

output
```
(E^(-a - b*x)*(-3 + 6*E^(2*(a + b*x)) + E^(4*(a + b*x))))/(12*b)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{1}{4}e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int e^{-2a-2bx}\left(1+e^{2a+2bx}\right)^2de^{a+bx}}{4b}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(2+e^{-2a-2bx}+e^{2a+2bx}\right)de^{a+bx}}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-e^{-a-bx}+2e^{a+bx}+\frac{1}{3}e^{3a+3bx}}{4b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x]^2,x]
```

output
```
(-E^(-a - b*x) + 2*E^(a + b*x) + E^(3*a + 3*b*x)/3)/(4*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.71

method	result
derivativedivides	$\frac{\frac{\cosh(bx+a)^3}{3}+\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$
default	$\frac{\frac{\cosh(bx+a)^3}{3}+\left(\frac{2}{3}+\frac{\cosh(bx+a)^2}{3}\right)\sinh(bx+a)}{b}$
risch	$-\frac{e^{-bx-a}}{4b}+\frac{e^{bx+a}}{2b}+\frac{e^{3bx+3a}}{12b}$
parallelrisch	$\frac{2\,e^{bx+a}\left(3\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^3-3\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)}{3b\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)^2\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^2}$
orering	$\frac{e^{bx+a}\cosh(bx+a)^2}{3b}+\frac{b\,e^{bx+a}\cosh(bx+a)^2+2\,e^{bx+a}\cosh(bx+a)b\sinh(bx+a)}{b^2}-\frac{3b^2e^{bx+a}\cosh(bx+a)^2+4b^2e^{bx+a}}{}$

input
```
int(exp(b*x+a)*cosh(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(1/3*cosh(b*x+a)^3+(2/3+1/3*cosh(b*x+a)^2)*sinh(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = -\frac{\cosh(bx+a)^2-4\cosh(bx+a)\sinh(bx+a)+\sinh(bx+a)^2-3}{6\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="fricas")
```

output
```
-1/6*(cosh(b*x + a)^2 - 4*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 -
3)/(b*cosh(b*x + a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. 2(34) = 68.

Time = 0.47 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.59

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \begin{cases} -\frac{2e^ae^{bx}\sinh^2(a+bx)}{3b}+\frac{2e^ae^{bx}\sinh(a+bx)\cosh(a+bx)}{3b}+\frac{e^ae^{bx}\cosh^2(a+bx)}{3b} & \text{for } b\neq 0 \\ xe^a\cosh^2(a) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)**2,x)
```

output
```
Piecewise((-2*exp(a)*exp(b*x)*sinh(a + b*x)**2/(3*b) + 2*exp(a)*exp(b*x)*s
inh(a + b*x)*cosh(a + b*x)/(3*b) + exp(a)*exp(b*x)*cosh(a + b*x)**2/(3*b),
 Ne(b, 0)), (x*exp(a)*cosh(a)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.82

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}}{12\,b}+\frac{e^{(bx+a)}}{2\,b}-\frac{e^{(-bx-a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="maxima")
```

output
```
1/12*e^(3*b*x + 3*a)/b + 1/2*e^(b*x + a)/b - 1/4*e^(-b*x - a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{e^{(3\,bx+3\,a)}+6\,e^{(bx+a)}-3\,e^{(-bx-a)}}{12\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a)^2,x, algorithm="giac")
```

output
```
1/12*(e^(3*b*x + 3*a) + 6*e^(b*x + a) - 3*e^(-b*x - a))/b
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.69

$$\int e^{a+bx}\cosh^2(a+bx)\,dx = \frac{6\,\mathrm{e}^{a+b\,x}-3\,\mathrm{e}^{-a-b\,x}+\mathrm{e}^{3\,a+3\,b\,x}}{12\,b}$$

input
```
int(cosh(a + b*x)^2*exp(a + b*x),x)
```

output
```
(6*exp(a + b*x) - 3*exp(- a - b*x) + exp(3*a + 3*b*x))/(12*b)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh^2(a+bx)\,dx=\frac{e^{4bx+4a}+6e^{2bx+2a}-3}{12e^{bx+a}b}$$

input
```
int(exp(b*x+a)*cosh(b*x+a)^2,x)
```

output
```
(e**(4*a + 4*b*x) + 6*e**(2*a + 2*b*x) - 3)/(12*e**(a + b*x)*b)
```

3.268 $\int e^{a+bx} \cosh(a+bx) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 23

$$\int e^{a+bx} \cosh(a+bx) \, dx = \frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

output
```
1/4*exp(2*b*x+2*a)/b+1/2*x
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00

$$\int e^{a+bx} \cosh(a+bx) \, dx = \frac{e^{2a+2bx}}{4b} + \frac{x}{2}$$

input
```
Integrate[E^(a + b*x)*Cosh[a + b*x],x]
```

output
```
E^(2*a + 2*b*x)/(4*b) + x/2
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\frac{\int \frac{1}{2}e^{-a-bx}\left(1+e^{2a+2bx}\right)de^{a+bx}}{b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int e^{-a-bx}\left(1+e^{2a+2bx}\right)de^{a+bx}}{2b}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(e^{-a-bx}+e^{a+bx}\right)de^{a+bx}}{2b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}e^{2a+2bx}+\log\left(e^{a+bx}\right)}{2b}$$

input
```
Int[E^(a + b*x)*Cosh[a + b*x],x]
```

output
```
(E^(2*a + 2*b*x)/2 + Log[E^(a + b*x)])/(2*b)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.39 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.83

method	result	size
risch	$\frac{{\mathrm e}^{2 b x +2 a}}{4 b}+\frac{x}{2}$	19
derivativedivides	$\frac{\frac{\cosh \left(b x +a \right)^{2}}{2}+\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}}{b}$	37
default	$\frac{\frac{\cosh \left(b x +a \right)^{2}}{2}+\frac{\cosh \left(b x +a \right) \sinh \left(b x +a \right)}{2}+\frac{b x}{2}+\frac{a}{2}}{b}$	37
parallelrisch	$\frac{{\mathrm e}^{b x +a} \left(b x \cosh \left(b x +a \right)-b x \sinh \left(b x +a \right)+\sinh \left(b x +a \right)\right)}{2 b}$	38
orering	$\frac{\left(2 b x +1\right) {\mathrm e}^{b x +a} \cosh \left(b x +a \right)}{2 b}-\frac{x \left(b \,{\mathrm e}^{b x +a} \cosh \left(b x +a \right)+{\mathrm e}^{b x +a} b \sinh \left(b x +a \right)\right)}{2 b}$	60

input
```
int(exp(b*x+a)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*exp(2*b*x+2*a)/b+1/2*x
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 50 vs. $2(18) = 36$.

Time = 0.08 (sec) , antiderivative size = 50, normalized size of antiderivative = 2.17

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{(2\,bx+1)\cosh(bx+a)-(2\,bx-1)\sinh(bx+a)}{4\,(b\cosh(bx+a)-b\sinh(bx+a))}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*((2*b*x + 1)*cosh(b*x + a) - (2*b*x - 1)*sinh(b*x + a))/(b*cosh(b*x +
a) - b*sinh(b*x + a))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(15) = 30$.

Time = 0.23 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.74

$$\int e^{a+bx}\cosh(a+bx)\,dx = \begin{cases} -\frac{xe^ae^{bx}\sinh(a+bx)}{2}+\frac{xe^ae^{bx}\cosh(a+bx)}{2}+\frac{e^ae^{bx}\sinh(a+bx)}{2b} & \text{for } b\neq 0 \\ xe^a\cosh(a) & \text{otherwise} \end{cases}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x)
```

output
```
Piecewise((-x*exp(a)*exp(b*x)*sinh(a + b*x)/2 + x*exp(a)*exp(b*x)*cosh(a +
 b*x)/2 + exp(a)*exp(b*x)*sinh(a + b*x)/(2*b), Ne(b, 0)), (x*exp(a)*cosh(a
), True))
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.04

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bx+2\,a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*a/b + 1/4*e^(2*b*x + 2*a)/b
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.96

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{2\,bx + 2\,a + e^{(2\,bx+2\,a)}}{4\,b}$$

input
```
integrate(exp(b*x+a)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/4*(2*b*x + 2*a + e^(2*b*x + 2*a))/b
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.78

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{x}{2} + \frac{\mathrm{e}^{2\,a+2\,b\,x}}{4\,b}$$

input
```
int(cosh(a + b*x)*exp(a + b*x),x)
```

output
```
x/2 + exp(2*a + 2*b*x)/(4*b)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.65

$$\int e^{a+bx}\cosh(a+bx)\,dx = \frac{e^{bx+a}(\cosh(bx+a)\,bx+\cosh(bx+a)-\sinh(bx+a)\,bx)}{2b}$$

input
```
int(exp(b*x+a)*cosh(b*x+a),x)
```

output
```
(e**(a + b*x)*(cosh(a + b*x)*b*x + cosh(a + b*x) - sinh(a + b*x)*b*x))/(2*
b)
```

3.269 $\int e^{a+bx}\text{sech}(a+bx)\,dx$

Optimal result

Integrand size = 14, antiderivative size = 17

$$\int e^{a+bx}\text{sech}(a+bx)\,dx=\frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

output
```
ln(1+exp(2*b*x+2*a))/b
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\text{sech}(a+bx)\,dx=\frac{\log\left(1+e^{2a+2bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x],x]
```

output
```
Log[1 + E^(2*a + 2*b*x)]/b
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}(a+bx)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\frac{\int \frac{2e^{a+bx}}{1+e^{2a+2bx}}de^{a+bx}}{b}$$
$$\Big\downarrow \text{27}$$
$$\frac{2\int \frac{e^{a+bx}}{1+e^{2a+2bx}}de^{a+bx}}{b}$$
$$\Big\downarrow \text{240}$$
$$\frac{\log\left(e^{2a+2bx}+1\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x],x]
```

output
```
Log[1 + E^(2*a + 2*b*x)]/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.31 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

method	result	size
derivativedivides	$\frac{\ln(\cosh(bx+a))+bx+a}{b}$	17
default	$\frac{\ln(\cosh(bx+a))+bx+a}{b}$	17
risch	$-\frac{2a}{b}+\frac{\ln\left(1+e^{2bx+2a}\right)}{b}$	24

input
```
int(exp(b*x+a)*sech(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/b*(ln(cosh(b*x+a))+b*x+a)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.76

$$\int e^{a+bx}\text{sech}(a+bx)\,dx=\frac{\log\left(\frac{2\cosh(bx+a)}{\cosh(bx+a)-\sinh(bx+a)}\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="fricas")
```

output
```
log(2*cosh(b*x + a)/(cosh(b*x + a) - sinh(b*x + a)))/b
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}(a+bx)\,dx = e^{a}\int e^{bx}\operatorname{sech}(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="maxima")
```

output
```
log(e^(2*b*x + 2*a) + 1)/b
```

Giac [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\operatorname{sech}(a+bx)\,dx = \frac{\log\left(e^{(2\,bx+2\,a)}+1\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a),x, algorithm="giac")
```

output
```
log(e^(2*b*x + 2*a) + 1)/b
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.94

$$\int e^{a+bx}\mathrm{sech}(a+bx)\,dx=\frac{\ln\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{b}$$

input
```
int(exp(a + b*x)/cosh(a + b*x),x)
```

output
```
log(exp(2*a + 2*b*x) + 1)/b
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\mathrm{sech}(a+bx)\,dx=\frac{\log\left(e^{2bx+2a}+1\right)}{b}$$

input
```
int(exp(b*x+a)*sech(b*x+a),x)
```

output
```
log(e**(2*a + 2*b*x) + 1)/b
```

3.270 $\int e^{a+bx}\text{sech}^2(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 40

$$\int e^{a+bx}\text{sech}^2(a+bx)\,dx=-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{2\arctan\left(e^{a+bx}\right)}{b}$$

output
```
-2*exp(b*x+a)/b/(1+exp(2*b*x+2*a))+2*arctan(exp(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.95

$$\int e^{a+bx}\text{sech}^2(a+bx)\,dx=\frac{-\frac{2e^{a+bx}}{1+e^{2a+2bx}}+2\arctan\left(e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^2,x]
```

output
```
((-2*E^(a + b*x))/(1 + E^(2*a + 2*b*x)) + 2*ArcTan[E^(a + b*x)])/b
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2720, 27, 252, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^2(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{4e^{2a+2bx}}{\left(1+e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{4\int \frac{e^{2a+2bx}}{\left(1+e^{2a+2bx}\right)^2}de^{a+bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{4\left(\frac{1}{2}\int \frac{1}{1+e^{2a+2bx}}de^{a+bx}-\frac{e^{a+bx}}{2\left(e^{2a+2bx}+1\right)}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{4\left(\frac{1}{2}\arctan\left(e^{a+bx}\right)-\frac{e^{a+bx}}{2\left(e^{2a+2bx}+1\right)}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^2,x]
```

output
```
(4*(-1/2*E^(a + b*x)/(1 + E^(2*a + 2*b*x)) + ArcTan[E^(a + b*x)]/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.62

method	result	size
derivativedivides	$\frac{-\frac{1}{\cosh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	25
default	$\frac{-\frac{1}{\cosh(bx+a)}+2\arctan\left(e^{bx+a}\right)}{b}$	25
risch	$-\frac{2\,e^{bx+a}}{b\left(1+e^{2bx+2a}\right)}+\frac{i\ln\left(e^{bx+a}+i\right)}{b}-\frac{i\ln\left(e^{bx+a}-i\right)}{b}$	58

input
```
int(exp(b*x+a)*sech(b*x+a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/cosh(b*x+a)+2*arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(37) = 74$.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.62

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx$$
$$= \frac{2\left(\left(\cosh\left(bx+a\right)^2+2\,\cosh\left(bx+a\right)\sinh\left(bx+a\right)+\sinh\left(bx+a\right)^2+1\right)\arctan\left(\cosh\left(bx+a\right)+\sinh\left(bx\right.\right.}{b\cosh\left(bx+a\right)^2+2\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)+b\sinh\left(bx+a\right)^2}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="fricas")
```

output
```
2*((cosh(b*x + a)^2 + 2*cosh(b*x + a)*sinh(b*x + a) + sinh(b*x + a)^2 + 1)
*arctan(cosh(b*x + a) + sinh(b*x + a)) - cosh(b*x + a) - sinh(b*x + a))/(b
*cosh(b*x + a)^2 + 2*b*cosh(b*x + a)*sinh(b*x + a) + b*sinh(b*x + a)^2 + b
)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx = e^a \int e^{bx}\operatorname{sech}^2(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**2,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx=\frac{2\arctan\left(e^{(bx+a)}\right)}{b}-\frac{2\,e^{(bx+a)}}{b(e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="maxima")
```

output
```
2*arctan(e^(b*x + a))/b - 2*e^(b*x + a)/(b*(e^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.88

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx=-\frac{2\left(\frac{e^{(bx+a)}}{e^{(2\,bx+2\,a)}+1}-\arctan\left(e^{(bx+a)}\right)\right)}{b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^2,x, algorithm="giac")
```

output
```
-2*(e^(b*x + a)/(e^(2*b*x + 2*a) + 1) - arctan(e^(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.20

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx=\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{b\,x}\,\mathrm{e}^{a}\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,\mathrm{e}^{a+b\,x}}{b\,\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^2,x)
```

output
```
(2*atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b))/(b^2)^(1/2) - (2*exp(a + b*x))/(
b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

$$\int e^{a+bx}\operatorname{sech}^2(a+bx)\,dx=\frac{2e^{2bx+2a}atan\left(e^{bx+a}\right)+2atan\left(e^{bx+a}\right)-2e^{bx+a}}{b\left(e^{2bx+2a}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+a)^2,x)
```

output
```
(2*(e**(2*a + 2*b*x)*atan(e**(a + b*x)) + atan(e**(a + b*x)) - e**(a + b*x
)))/(b*(e**(2*a + 2*b*x) + 1))
```

3.271 $\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 29

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx=\frac{2e^{4a+4bx}}{b\,(1+e^{2a+2bx})^2}$$

output
```
2*exp(4*b*x+4*a)/b/(1+exp(2*b*x+2*a))^2
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx=\frac{2e^{4a+4bx}}{b\,(1+e^{2a+2bx})^2}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^3,x]
```

output
```
(2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{8e^{3a+3bx}}{\left(1+e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\int \frac{e^{3a+3bx}}{\left(1+e^{2a+2bx}\right)^3}de^{a+bx}}{b}$$

$$\Big\downarrow \text{242}$$

$$\frac{2e^{4a+4bx}}{b\left(e^{2a+2bx}+1\right)^2}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^3,x]
```

output
```
(2*E^(4*a + 4*b*x))/(b*(1 + E^(2*a + 2*b*x))^2)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.76

method	result	size
derivativedivides	$\frac{-\frac{1}{2\cosh(bx+a)^2}+\tanh(bx+a)}{b}$	22
default	$\frac{-\frac{1}{2\cosh(bx+a)^2}+\tanh(bx+a)}{b}$	22
risch	$-\frac{2\left(2\,\mathrm{e}^{2bx+2a}+1\right)}{b\left(1+\mathrm{e}^{2bx+2a}\right)^2}$	32
parallelrisch	$-\frac{\mathrm{e}^{bx+a}\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)-1\right)\left(\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)+1\right)^3}{2b\left(1+\tanh\left(\frac{bx}{2}+\frac{a}{2}\right)^2\right)^2}$	51

input
```
int(exp(b*x+a)*sech(b*x+a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/2/cosh(b*x+a)^2+tanh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(27) = 54$.

Time = 0.08 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.97

$$\int e^{a+bx}\mathrm{sech}^3(a+bx)\,dx =$$
$$-\frac{2\,(3\,\cosh(bx+a)+\sinh(bx+a))}{b\cosh(bx+a)^3+3\,b\cosh(bx+a)\sinh(bx+a)^2+b\sinh(bx+a)^3+3\,b\cosh(bx+a)+\left(3\,b\cosh(bx\right.}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*x + a) + sinh(b*x + a))/(b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a
)*sinh(b*x + a)^2 + b*sinh(b*x + a)^3 + 3*b*cosh(b*x + a) + (3*b*cosh(b*x
+ a)^2 + b)*sinh(b*x + a))
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx = e^a\int e^{bx}\operatorname{sech}^3(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**3,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. $2(27) = 54$.

Time = 0.03 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int e^{a+bx}\operatorname{sech}^3(a+bx)\,dx = -\frac{4\,e^{(2\,bx+2\,a)}}{b(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1)} - \frac{2}{b(e^{(4\,bx+4\,a)}+2\,e^{(2\,bx+2\,a)}+1)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="maxima")
```

output
```
-4*e^(2*b*x + 2*a)/(b*(e^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1)) - 2/(b*(e
^(4*b*x + 4*a) + 2*e^(2*b*x + 2*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx=-\frac{2\left(2\,e^{(2\,bx+2\,a)}+1\right)}{b\left(e^{(2\,bx+2\,a)}+1\right)^2}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^3,x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 2.10 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.07

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx=-\frac{2\left(2\,\mathrm{e}^{2\,a+2\,b\,x}+1\right)}{b\left(\mathrm{e}^{2\,a+2\,b\,x}+1\right)^2}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^3,x)
```

output
```
-(2*(2*exp(2*a + 2*b*x) + 1))/(b*(exp(2*a + 2*b*x) + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.41

$$\int e^{a+bx}\text{sech}^3(a+bx)\,dx=\frac{2e^{4bx+4a}}{b\left(e^{4bx+4a}+2e^{2bx+2a}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+a)^3,x)
```

output
```
(2*e**(4*a + 4*b*x))/(b*(e**(4*a + 4*b*x) + 2*e**(2*a + 2*b*x) + 1))
```

3.272 $\int e^{a+bx}\mathbf{sech}^4(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 95

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx=-\frac{8e^{3a+3bx}}{3b\left(1+e^{2a+2bx}\right)^3}-\frac{2e^{a+bx}}{b\left(1+e^{2a+2bx}\right)^2}+\frac{e^{a+bx}}{b\left(1+e^{2a+2bx}\right)}+\frac{\arctan\left(e^{a+bx}\right)}{b}$$

output
```
-8/3*exp(3*b*x+3*a)/b/(1+exp(2*b*x+2*a))^3-2*exp(b*x+a)/b/(1+exp(2*b*x+2*a
))^2+exp(b*x+a)/b/(1+exp(2*b*x+2*a))+arctan(exp(b*x+a))/b
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.67

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx=\frac{e^{a+bx}\left(-3-8e^{2(a+bx)}+3e^{4(a+bx)}\right)}{3b\left(1+e^{2(a+bx)}\right)^3}+\frac{\arctan\left(e^{a+bx}\right)}{b}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^4,x]
```

output
```
(E^(a + b*x)*(-3 - 8*E^(2*(a + b*x)) + 3*E^(4*(a + b*x))))/(3*b*(1 + E^(2*
(a + b*x)))^3) + ArcTan[E^(a + b*x)]/b
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.12, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {2720, 27, 252, 252, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{16e^{4a+4bx}}{(1+e^{2a+2bx})^4}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{16\int \frac{e^{4a+4bx}}{(1+e^{2a+2bx})^4}de^{a+bx}}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16\left(\frac{1}{2}\int \frac{e^{2a+2bx}}{(1+e^{2a+2bx})^3}de^{a+bx}-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{252}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\int \frac{1}{(1+e^{2a+2bx})^2}de^{a+bx}-\frac{e^{a+bx}}{4(e^{2a+2bx}+1)^2}\right)-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{215}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1}{1+e^{2a+2bx}}de^{a+bx}+\frac{e^{a+bx}}{2(e^{2a+2bx}+1)}\right)-\frac{e^{a+bx}}{4(e^{2a+2bx}+1)^2}\right)-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

$$\Big\downarrow \text{216}$$

$$\frac{16\left(\frac{1}{2}\left(\frac{1}{4}\left(\frac{1}{2}\arctan\left(e^{a+bx}\right)+\frac{e^{a+bx}}{2(e^{2a+2bx}+1)}\right)-\frac{e^{a+bx}}{4(e^{2a+2bx}+1)^2}\right)-\frac{e^{3a+3bx}}{6(e^{2a+2bx}+1)^3}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^4,x]
```

output
```
(16*(-1/6*E^(3*a + 3*b*x)/(1 + E^(2*a + 2*b*x))^3 + (-1/4*E^(a + b*x)/(1 +
 E^(2*a + 2*b*x))^2 + (E^(a + b*x)/(2*(1 + E^(2*a + 2*b*x))) + ArcTan[E^(a
 + b*x)]/2)/4)/2))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 252
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x
)^(m - 1)*((a + b*x^2)^(p + 1)/(2*b*(p + 1))), x] - Simp[c^2*((m - 1)/(2*b*
(p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c
}, x] && LtQ[p, -1] && GtQ[m, 1] &&  !ILtQ[(m + 2*p + 3)/2, 0] && IntBinomi
alQ[a, b, c, 2, m, p, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 4.10 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.39

method	result	size
derivativedivides	$\frac{-\frac{1}{3\cosh(bx+a)^3}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	37
default	$\frac{-\frac{1}{3\cosh(bx+a)^3}+\frac{\operatorname{sech}(bx+a)\tanh(bx+a)}{2}+\arctan\left(e^{bx+a}\right)}{b}$	37
risch	$\frac{e^{bx+a}\left(3\,e^{4bx+4a}-8\,e^{2bx+2a}-3\right)}{3b\left(1+e^{2bx+2a}\right)^3}+\frac{i\ln\left(e^{bx+a}+i\right)}{2b}-\frac{i\ln\left(e^{bx+a}-i\right)}{2b}$	82

input
```
int(exp(b*x+a)*sech(b*x+a)^4,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/3/cosh(b*x+a)^3+1/2*sech(b*x+a)*tanh(b*x+a)+arctan(exp(b*x+a)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 513 vs. 2(86) = 172.

Time = 0.09 (sec) , antiderivative size = 513, normalized size of antiderivative = 5.40

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = \text{Too large to display}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="fricas")
```

output
```
1/3*(3*cosh(b*x + a)^5 + 15*cosh(b*x + a)*sinh(b*x + a)^4 + 3*sinh(b*x + a
)^5 + 2*(15*cosh(b*x + a)^2 - 4)*sinh(b*x + a)^3 - 8*cosh(b*x + a)^3 + 6*(
5*cosh(b*x + a)^3 - 4*cosh(b*x + a))*sinh(b*x + a)^2 + 3*(cosh(b*x + a)^6
+ 6*cosh(b*x + a)*sinh(b*x + a)^5 + sinh(b*x + a)^6 + 3*(5*cosh(b*x + a)^2
 + 1)*sinh(b*x + a)^4 + 3*cosh(b*x + a)^4 + 4*(5*cosh(b*x + a)^3 + 3*cosh(
b*x + a))*sinh(b*x + a)^3 + 3*(5*cosh(b*x + a)^4 + 6*cosh(b*x + a)^2 + 1)*
sinh(b*x + a)^2 + 3*cosh(b*x + a)^2 + 6*(cosh(b*x + a)^5 + 2*cosh(b*x + a)
^3 + cosh(b*x + a))*sinh(b*x + a) + 1)*arctan(cosh(b*x + a) + sinh(b*x + a
)) + 3*(5*cosh(b*x + a)^4 - 8*cosh(b*x + a)^2 - 1)*sinh(b*x + a) - 3*cosh(
b*x + a))/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(
b*x + a)^6 + 3*b*cosh(b*x + a)^4 + 3*(5*b*cosh(b*x + a)^2 + b)*sinh(b*x +
a)^4 + 4*(5*b*cosh(b*x + a)^3 + 3*b*cosh(b*x + a))*sinh(b*x + a)^3 + 3*b*c
osh(b*x + a)^2 + 3*(5*b*cosh(b*x + a)^4 + 6*b*cosh(b*x + a)^2 + b)*sinh(b*
x + a)^2 + 6*(b*cosh(b*x + a)^5 + 2*b*cosh(b*x + a)^3 + b*cosh(b*x + a))*s
inh(b*x + a) + b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = e^a\int e^{bx}\operatorname{sech}^4(a+bx)\,dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**4,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = \frac{\arctan\left(e^{(bx+a)}\right)}{b} + \frac{3\,e^{(5\,bx+5\,a)} - 8\,e^{(3\,bx+3\,a)} - 3\,e^{(bx+a)}}{3\,b\left(e^{(6\,bx+6\,a)} + 3\,e^{(4\,bx+4\,a)} + 3\,e^{(2\,bx+2\,a)} + 1\right)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="maxima")
```

output
```
arctan(e^(b*x + a))/b + 1/3*(3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(
b*x + a))/(b*(e^(6*b*x + 6*a) + 3*e^(4*b*x + 4*a) + 3*e^(2*b*x + 2*a) + 1)
)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.63

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx=\frac{\frac{3\,e^{(5\,bx+5\,a)}-8\,e^{(3\,bx+3\,a)}-3\,e^{(bx+a)}}{\left(e^{(2\,bx+2\,a)}+1\right)^3}+3\,\arctan\left(e^{(bx+a)}\right)}{3\,b}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^4,x, algorithm="giac")
```

output
```
1/3*((3*e^(5*b*x + 5*a) - 8*e^(3*b*x + 3*a) - 3*e^(b*x + a))/(e^(2*b*x + 2
*a) + 1)^3 + 3*arctan(e^(b*x + a)))/b
```

Mupad [B] (verification not implemented)

Time = 2.04 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.37

$$\int e^{a+bx}\text{sech}^4(a+bx)\,dx=\frac{\text{atan}\left(\frac{e^{b\,x}\,e^a\,\sqrt{b^2}}{b}\right)}{\sqrt{b^2}}-\frac{2\,e^{a+b\,x}}{b\,\left(2\,e^{2\,a+2\,b\,x}+e^{4\,a+4\,b\,x}+1\right)}-\frac{8\,e^{3\,a+3\,b\,x}}{3\,b\,\left(3\,e^{2\,a+2\,b\,x}+3\,e^{4\,a+4\,b\,x}+e^{6\,a+6\,b\,x}+1\right)}+\frac{e^{a+b\,x}}{b\,\left(e^{2\,a+2\,b\,x}+1\right)}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^4,x)
```

output
```
atan((exp(b*x)*exp(a)*(b^2)^(1/2))/b)/(b^2)^(1/2) - (2*exp(a + b*x))/(b*(2
*exp(2*a + 2*b*x) + exp(4*a + 4*b*x) + 1)) - (8*exp(3*a + 3*b*x))/(3*b*(3*
exp(2*a + 2*b*x) + 3*exp(4*a + 4*b*x) + exp(6*a + 6*b*x) + 1)) + exp(a + b
*x)/(b*(exp(2*a + 2*b*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.55

$$\int e^{a+bx}\operatorname{sech}^4(a+bx)\,dx = \frac{3e^{6bx+6a}atan\left(e^{bx+a}\right)+9e^{4bx+4a}atan\left(e^{bx+a}\right)+9e^{2bx+2a}atan\left(e^{bx+a}\right)+3atan\left(e^{bx+a}\right)+3e^{5bx+5a}-8e^{3bx+3a}}{3b\left(e^{6bx+6a}+3e^{4bx+4a}+3e^{2bx+2a}+1\right)}$$

input
```
int(exp(b*x+a)*sech(b*x+a)^4,x)
```

output
```
(3*e**(6*a + 6*b*x)*atan(e**(a + b*x)) + 9*e**(4*a + 4*b*x)*atan(e**(a + b
*x)) + 9*e**(2*a + 2*b*x)*atan(e**(a + b*x)) + 3*atan(e**(a + b*x)) + 3*e*
*(5*a + 5*b*x) - 8*e**(3*a + 3*b*x) - 3*e**(a + b*x))/(3*b*(e**(6*a + 6*b*
x) + 3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1))
```

3.273 $\int e^{a+bx}\mathbf{sech}^5(a+bx)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 60

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx = -\frac{4}{b(1+e^{2a+2bx})^4}+\frac{32}{3b(1+e^{2a+2bx})^3}-\frac{8}{b(1+e^{2a+2bx})^2}$$

output
```
-4/b/(1+exp(2*b*x+2*a))^4+32/3/b/(1+exp(2*b*x+2*a))^3-8/b/(1+exp(2*b*x+2*a
))^2
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.73

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx = -\frac{4(1+4e^{2(a+bx)}+6e^{4(a+bx)})}{3b(1+e^{2(a+bx)})^4}$$

input
```
Integrate[E^(a + b*x)*Sech[a + b*x]^5,x]
```

output
```
(-4*(1 + 4*E^(2*(a + b*x)) + 6*E^(4*(a + b*x))))/(3*b*(1 + E^(2*(a + b*x))
)^4)
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.312, Rules used = {2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{a+bx}\text{sech}^5(a+bx)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\int \frac{32e^{5a+5bx}}{\left(1+e^{2a+2bx}\right)^5}de^{a+bx}}{b}$$

$$\Big\downarrow \text{27}$$

$$\frac{32\int \frac{e^{5a+5bx}}{\left(1+e^{2a+2bx}\right)^5}de^{a+bx}}{b}$$

$$\Big\downarrow \text{243}$$

$$\frac{16\int \frac{e^{2a+2bx}}{\left(1+e^{2a+2bx}\right)^5}de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{53}$$

$$\frac{16\int \left(\frac{1}{\left(1+e^{2a+2bx}\right)^3}-\frac{2}{\left(1+e^{2a+2bx}\right)^4}+\frac{1}{\left(1+e^{2a+2bx}\right)^5}\right)de^{2a+2bx}}{b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(-\frac{1}{2\left(e^{2a+2bx}+1\right)^2}+\frac{2}{3\left(e^{2a+2bx}+1\right)^3}-\frac{1}{4\left(e^{2a+2bx}+1\right)^4}\right)}{b}$$

input
```
Int[E^(a + b*x)*Sech[a + b*x]^5,x]
```

output
```
(16*(-1/4*1/(1 + E^(2*a + 2*b*x))^4 + 2/(3*(1 + E^(2*a + 2*b*x))^3) - 1/(2
*(1 + E^(2*a + 2*b*x))^2)))/b
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 3.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.58

method	result	size
derivativedivides	$\frac{-\frac{1}{4\cosh(bx+a)^4}+\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	35
default	$\frac{-\frac{1}{4\cosh(bx+a)^4}+\left(\frac{2}{3}+\frac{\operatorname{sech}(bx+a)^2}{3}\right)\tanh(bx+a)}{b}$	35
risch	$-\frac{4\left(6\,\mathrm{e}^{4bx+4a}+4\,\mathrm{e}^{2bx+2a}+1\right)}{3b\left(1+\mathrm{e}^{2bx+2a}\right)^4}$	43
parallelrisch	$\frac{2\,\mathrm{e}^{bx+a}(\cosh(3bx+3a)+\sinh(3bx+3a)+4\cosh(bx+a)+4\sinh(bx+a))}{3b(\cosh(4bx+4a)+4\cosh(2bx+2a)+3)}$	71

input
```
int(exp(b*x+a)*sech(b*x+a)^5,x,method=_RETURNVERBOSE)
```

output
```
1/b*(-1/4/cosh(b*x+a)^4+(2/3+1/3*sech(b*x+a)^2)*tanh(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 233 vs. $2(55) = 110$.

Time = 0.09 (sec) , antiderivative size = 233, normalized size of antiderivative = 3.88

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx =$$

$$-\frac{}{3\left(b\cosh\left(bx+a\right)^{6}+6\,b\cosh\left(bx+a\right)\sinh\left(bx+a\right)^{5}+b\sinh\left(bx+a\right)^{6}+4\,b\cosh\left(bx+a\right)^{4}+\left(15\,b\cos\right.\right.}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*x + a)^2 + 10*cosh(b*x + a)*sinh(b*x + a) + 7*sinh(b*x + a)
^2 + 4)/(b*cosh(b*x + a)^6 + 6*b*cosh(b*x + a)*sinh(b*x + a)^5 + b*sinh(b*
x + a)^6 + 4*b*cosh(b*x + a)^4 + (15*b*cosh(b*x + a)^2 + 4*b)*sinh(b*x + a
)^4 + 4*(5*b*cosh(b*x + a)^3 + 4*b*cosh(b*x + a))*sinh(b*x + a)^3 + 7*b*co
sh(b*x + a)^2 + (15*b*cosh(b*x + a)^4 + 24*b*cosh(b*x + a)^2 + 7*b)*sinh(b
*x + a)^2 + 2*(3*b*cosh(b*x + a)^5 + 8*b*cosh(b*x + a)^3 + 5*b*cosh(b*x +
a))*sinh(b*x + a) + 4*b)
```

Sympy [F]

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx = e^{a}\int e^{bx}\operatorname{sech}^{5}\left(a+bx\right)dx$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)**5,x)
```

output
```
exp(a)*Integral(exp(b*x)*sech(a + b*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(55) = 110$.

Time = 0.04 (sec) , antiderivative size = 172, normalized size of antiderivative = 2.87

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx = -\frac{8\,e^{(4\,bx+4\,a)}}{b\big(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\big)} - \frac{16\,e^{(2\,bx+2\,a)}}{3\,b\big(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\big)} - \frac{4}{3\,b\big(e^{(8\,bx+8\,a)}+4\,e^{(6\,bx+6\,a)}+6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\big)}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="maxima")
```

output
```
-8*e^(4*b*x + 4*a)/(b*(e^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x +
4*a) + 4*e^(2*b*x + 2*a) + 1)) - 16/3*e^(2*b*x + 2*a)/(b*(e^(8*b*x + 8*a)
+ 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)) - 4/3/(b
*(e^(8*b*x + 8*a) + 4*e^(6*b*x + 6*a) + 6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2
*a) + 1))
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx = -\frac{4\left(6\,e^{(4\,bx+4\,a)}+4\,e^{(2\,bx+2\,a)}+1\right)}{3\,b\big(e^{(2\,bx+2\,a)}+1\big)^4}$$

input
```
integrate(exp(b*x+a)*sech(b*x+a)^5,x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*x + 4*a) + 4*e^(2*b*x + 2*a) + 1)/(b*(e^(2*b*x + 2*a) + 1)^
4)
```

Mupad [B] (verification not implemented)

Time = 2.00 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.70

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx=-\frac{4\,\left(4\,{\mathrm{e}}^{2\,a+2\,b\,x}+6\,{\mathrm{e}}^{4\,a+4\,b\,x}+1\right)}{3\,b\,{\left({\mathrm{e}}^{2\,a+2\,b\,x}+1\right)}^4}$$

input
```
int(exp(a + b*x)/cosh(a + b*x)^5,x)
```

output
```
-(4*(4*exp(2*a + 2*b*x) + 6*exp(4*a + 4*b*x) + 1))/(3*b*(exp(2*a + 2*b*x)
+ 1)^4)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int e^{a+bx}\operatorname{sech}^5(a+bx)\,dx=\frac{-8e^{4bx+4a}-\frac{16e^{2bx+2a}}{3}-\frac{4}{3}}{b\,(e^{8bx+8a}+4e^{6bx+6a}+6e^{4bx+4a}+4e^{2bx+2a}+1)}$$

input
```
int(exp(b*x+a)*sech(b*x+a)^5,x)
```

output
```
(4*( - 6*e**(4*a + 4*b*x) - 4*e**(2*a + 2*b*x) - 1))/(3*b*(e**(8*a + 8*b*x
) + 4*e**(6*a + 6*b*x) + 6*e**(4*a + 4*b*x) + 4*e**(2*a + 2*b*x) + 1))
```

3.274 $\int e^x \cosh^2(2x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \cosh^2(2x)\,dx = -\frac{1}{12}e^{-3x} + \frac{e^x}{2} + \frac{e^{5x}}{20}$$

output
```
-1/12/exp(3*x)+1/2*exp(x)+1/20*exp(5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \cosh^2(2x)\,dx = -\frac{1}{12}e^{-3x} + \frac{e^x}{2} + \frac{e^{5x}}{20}$$

input
```
Integrate[E^x*Cosh[2*x]^2,x]
```

output
```
-1/12*1/E^(3*x) + E^x/2 + E^(5*x)/20
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh^2(2x) \, dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{1}{4} e^{-4x} \left(e^{4x}+1\right)^2 de^x$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4} \int e^{-4x} \left(1+e^{4x}\right)^2 de^x$$

$$\Big\downarrow \text{802}$$

$$\frac{1}{4} \int \left(2+e^{-4x}+e^{4x}\right) de^x$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4} \left(-\frac{1}{3} e^{-3x}+2 e^x+\frac{e^{5x}}{5}\right)$$

input
```
Int[E^x*Cosh[2*x]^2,x]
```

output
```
(-1/3*1/E^(3*x) + 2*E^x + E^(5*x)/5)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{{\mathrm e}^{x} \left(-15+\cosh \left(4 x \right)-4 \sinh \left(4 x \right)\right)}{30}$	17
risch	$\frac{{\mathrm e}^{5 x}}{20}+\frac{{\mathrm e}^{x}}{2}-\frac{{\mathrm e}^{-3 x}}{12}$	18
default	$\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(3 x \right)}{12}+\frac{\sinh \left(5 x \right)}{20}+\frac{\cosh \left(x \right)}{2}-\frac{\cosh \left(3 x \right)}{12}+\frac{\cosh \left(5 x \right)}{20}$	34
orering	$\frac{7 \,{\mathrm e}^{x} \cosh \left(2 x \right)^{2}}{15}+\frac{4 \,{\mathrm e}^{x} \cosh \left(2 x \right) \sinh \left(2 x \right)}{15}-\frac{8 \,{\mathrm e}^{x} \sinh \left(2 x \right)^{2}}{15}$	34

input
```
int(exp(x)*cosh(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/30*exp(x)*(-15+cosh(4*x)-4*sinh(4*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 47 vs. $2(17) = 34$.

Time = 0.08 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.81

$$\int e^x \cosh^2(2x)\,dx = -\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4 - 15}{30(\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*cosh(2*x)^2,x, algorithm="fricas")
```

output
```
-1/30*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(
x)*sinh(x)^3 + sinh(x)^4 - 15)/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.15 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \cosh^2(2x)\,dx = -\frac{8e^x\sinh^2(2x)}{15} + \frac{4e^x\sinh(2x)\cosh(2x)}{15} + \frac{7e^x\cosh^2(2x)}{15}$$

input
```
integrate(exp(x)*cosh(2*x)**2,x)
```

output
```
-8*exp(x)*sinh(2*x)**2/15 + 4*exp(x)*sinh(2*x)*cosh(2*x)/15 + 7*exp(x)*cos
h(2*x)**2/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(2x)\,dx = \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(2*x)^2,x, algorithm="maxima")
```

output
```
1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(2x)\,dx = \frac{1}{20}\,e^{(5\,x)} - \frac{1}{12}\,e^{(-3\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(2*x)^2,x, algorithm="giac")
```

output
```
1/20*e^(5*x) - 1/12*e^(-3*x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(2x)\,dx = \frac{\mathrm{e}^{5\,x}}{20} - \frac{\mathrm{e}^{-3\,x}}{12} + \frac{\mathrm{e}^x}{2}$$

input
```
int(cosh(2*x)^2*exp(x),x)
```

output
```
exp(5*x)/20 - exp(-3*x)/12 + exp(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^x \cosh^2(2x)\,dx = \frac{3e^{8x} + 30e^{4x} - 5}{60e^{3x}}$$

input
```
int(exp(x)*cosh(2*x)^2,x)
```

output
```
(3*e**(8*x) + 30*e**(4*x) - 5)/(60*e**(3*x))
```

3.275 $\int e^x \cosh(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \cosh(2x)\,dx = -\frac{e^{-x}}{2} + \frac{e^{3x}}{6}$$

output
```
-1/2/exp(x)+1/6*exp(3*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \cosh(2x)\,dx = \frac{1}{6}e^{-x}\left(-3 + e^{4x}\right)$$

input
```
Integrate[E^x*Cosh[2*x],x]
```

output
```
(-3 + E^(4*x))/(6*E^x)
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.11, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(2x) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{1}{2} e^{-2x} \left(e^{4x}+1\right) de^x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2} \int e^{-2x} \left(1+e^{4x}\right) de^x$$

$$\Big\downarrow \text{ 802}$$

$$\frac{1}{2} \int \left(e^{-2x}+e^{2x}\right) de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{e^{3x}}{3} - e^{-x}\right)$$

input
```
Int[E^x*Cosh[2*x],x]
```

output
```
(-E^(-x) + E^(3*x)/3)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{\rm e}^{3 x}}{6}-\frac{{\rm e}^{-x}}{2}$	14
parallelrisch	$-\frac{{\rm e}^{x} \left(\cosh \left(2 x \right)-2 \sinh \left(2 x \right)\right)}{3}$	16
orering	$-\frac{{\rm e}^{x} \cosh \left(2 x \right)}{3}+\frac{2 \,{\rm e}^{x} \sinh \left(2 x \right)}{3}$	18
default	$\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(3 x \right)}{6}-\frac{\cosh \left(x \right)}{2}+\frac{\cosh \left(3 x \right)}{6}$	22

input
```
int(exp(x)*cosh(2*x),x,method=_RETURNVERBOSE)
```

output
```
1/6*exp(3*x)-1/2*exp(-x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.37

$$\int e^x \cosh(2x)\,dx = -\frac{\cosh(x)^2 - 4\cosh(x)\sinh(x) + \sinh(x)^2}{3(\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*cosh(2*x),x, algorithm="fricas")
```

output
```
-1/3*(cosh(x)^2 - 4*cosh(x)*sinh(x) + sinh(x)^2)/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \cosh(2x)\,dx = \frac{2e^x\sinh(2x)}{3} - \frac{e^x\cosh(2x)}{3}$$

input
```
integrate(exp(x)*cosh(2*x),x)
```

output
```
2*exp(x)*sinh(2*x)/3 - exp(x)*cosh(2*x)/3
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(2x)\,dx = \frac{1}{6}\,e^{(3\,x)} - \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(exp(x)*cosh(2*x),x, algorithm="maxima")
```

output
```
1/6*e^(3*x) - 1/2*e^(-x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(2x)\,dx = \frac{1}{6}\,e^{(3\,x)} - \frac{1}{2}\,e^{(-x)}$$

input
```
integrate(exp(x)*cosh(2*x),x, algorithm="giac")
```

output
```
1/6*e^(3*x) - 1/2*e^(-x)
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \cosh(2x)\,dx = \frac{\mathrm{e}^{-x}\,\left(\mathrm{e}^{4\,x}-3\right)}{6}$$

input
```
int(cosh(2*x)*exp(x),x)
```

output
```
(exp(-x)*(exp(4*x) - 3))/6
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^x \cosh(2x)\,dx = \frac{e^x(-\cosh\left(2x\right)+2\sinh\left(2x\right))}{3}$$

input
```
int(exp(x)*cosh(2*x),x)
```

output
```
(e**x*( - cosh(2*x) + 2*sinh(2*x)))/3
```

3.276 $\int e^x \text{sech}(2x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 65

$$\int e^x \text{sech}(2x)\,dx = -\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{\sqrt{2}} - \frac{\text{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{\sqrt{2}}$$

output
```
1/2*arctan(-1+2^(1/2)*exp(x))*2^(1/2)+1/2*arctan(1+2^(1/2)*exp(x))*2^(1/2)
-1/2*arctanh(2^(1/2)*exp(x)/(1+exp(2*x)))*2^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.37

$$\int e^x \text{sech}(2x)\,dx = \frac{2}{3}e^{3x}\,\text{Hypergeometric2F1}\left(\frac{3}{4},1,\frac{7}{4},-e^{4x}\right)$$

input
```
Integrate[E^x*Sech[2*x],x]
```

output
```
(2*E^(3*x)*Hypergeometric2F1[3/4, 1, 7/4, -E^(4*x)])/3
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.60, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {2720, 27, 826, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{2x}}{e^{4x}+1}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{2x}}{1+e^{4x}}de^x$$

$$\Big\downarrow \text{826}$$

$$2\left(\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{1476}$$

$$2\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{1082}$$

$$2\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{217}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)$$

$$\Big\downarrow \text{1479}$$

$$2\left(\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{ 25}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{ 27}$$

$$2\left(\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)$$

$$\Big\downarrow \text{ 1103}$$

$$2\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right)$$

input
```
Int[E^x*Sech[2*x],x]
```

output
```
2*((-(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[1 + Sqrt[2]*E^x]/Sqrt[2])/
2 + (Log[1 - Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]) - Log[1 + Sqrt[2]*E^x + E^
(2*x)]/(2*Sqrt[2]))/2)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 826
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[a/b,
2]], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*s)   Int[(r + s*x^2)/(a + b*x^
4), x], x] - Simp[1/(2*s)   Int[(r - s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{
a, b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]]
 && AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.64 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.38

method	result	size
risch	$2\left(\sum_{\_R=\operatorname{RootOf}(256\_Z^4+1)} \_R\ln\left(64\_R^3+e^x\right)\right)$	25

input
```
int(exp(x)*sech(2*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(64*_R^3+exp(x)),_R=RootOf(256*_Z^4+1))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.38

$$\int e^x \operatorname{sech}(2x)\,dx = \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)+1\right) + \frac{1}{2}\sqrt{2}\arctan\left(\sqrt{2}\cosh(x)+\sqrt{2}\sinh(x)-1\right) - \frac{1}{4}\sqrt{2}\log\left(\frac{\sqrt{2}+2\cosh(x)}{\cosh(x)-\sinh(x)}\right) + \frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(exp(x)*sech(2*x),x, algorithm="fricas")
```

output
```
1/2*sqrt(2)*arctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) + 1) + 1/2*sqrt(2)*ar
ctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) - 1) - 1/4*sqrt(2)*log((sqrt(2) + 2
*cosh(x))/(cosh(x) - sinh(x))) + 1/4*sqrt(2)*log(-(sqrt(2) - 2*cosh(x))/(c
osh(x) - sinh(x)))
```

Sympy [F]

$$\int e^x \operatorname{sech}(2x)\,dx = \int e^x \operatorname{sech}(2x)\,dx$$

input
```
integrate(exp(x)*sech(2*x),x)
```

output
```
Integral(exp(x)*sech(2*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\begin{aligned}\int e^x \operatorname{sech}(2x)\,dx =& \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\ &+ \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)\\ &- \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)\end{aligned}$$

input
```
integrate(exp(x)*sech(2*x),x, algorithm="maxima")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\begin{aligned}\int e^x \operatorname{sech}(2x)\,dx =& \frac{1}{2}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right)\\ &+ \frac{1}{2}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right)\\ &- \frac{1}{4}\sqrt{2}\log\left(\sqrt{2}e^x+e^{(2x)}+1\right)+\frac{1}{4}\sqrt{2}\log\left(-\sqrt{2}e^x+e^{(2x)}+1\right)\end{aligned}$$

input
```
integrate(exp(x)*sech(2*x),x, algorithm="giac")
```

output
```
1/2*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/2*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) - 1/4*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
+ 1/4*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.18

$$\begin{aligned}\int e^x \operatorname{sech}(2x)\,dx =& \sqrt{2}\,\ln\left(4+\sqrt{2}\,\mathrm{e}^x\,(-2-2\mathrm{i})\right)\,\left(\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)\\&+\sqrt{2}\,\ln\left(4+\sqrt{2}\,\mathrm{e}^x\,(-2+2\mathrm{i})\right)\,\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)\\&+\sqrt{2}\,\ln\left(4+\sqrt{2}\,\mathrm{e}^x\,(2-2\mathrm{i})\right)\,\left(-\frac{1}{4}+\frac{1}{4}\mathrm{i}\right)\\&+\sqrt{2}\,\ln\left(4+\sqrt{2}\,\mathrm{e}^x\,(2+2\mathrm{i})\right)\,\left(-\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)\end{aligned}$$

input
```
int(exp(x)/cosh(2*x),x)
```

output
```
2^(1/2)*log(4 - 2^(1/2)*exp(x)*(2 + 2i))*(1/4 + 1i/4) + 2^(1/2)*log(4 - 2^
(1/2)*exp(x)*(2 - 2i))*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 - 2i)
+ 4)*(1/4 - 1i/4) - 2^(1/2)*log(2^(1/2)*exp(x)*(2 + 2i) + 4)*(1/4 + 1i/4)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.08

$$\begin{aligned}&\int e^x \operatorname{sech}(2x)\,dx\\&=\frac{\sqrt{2}\left(2atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right)+2atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right)+\log\left(e^{2x}-e^x\sqrt{2}+1\right)-\log\left(e^{2x}+e^x\sqrt{2}+1\right)\right)}{4}\end{aligned}$$

input
```
int(exp(x)*sech(2*x),x)
```

output
```
(sqrt(2)*(2*atan((2*e**x - sqrt(2))/sqrt(2)) + 2*atan((2*e**x + sqrt(2))/s
qrt(2)) + log(e**(2*x) - e**x*sqrt(2) + 1) - log(e**(2*x) + e**x*sqrt(2) +
 1)))/4
```

3.277 $\int e^x \mathbf{sech}^2(2x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 86

$$\int e^x \operatorname{sech}^2(2x)\,dx = -\frac{e^x}{1+e^{4x}} - \frac{\arctan\left(1-\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\arctan\left(1+\sqrt{2}e^x\right)}{2\sqrt{2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}e^x}{1+e^{2x}}\right)}{2\sqrt{2}}$$

output
```
-exp(x)/(1+exp(4*x))+1/4*arctan(-1+2^(1/2)*exp(x))*2^(1/2)+1/4*arctan(1+2^
(1/2)*exp(x))*2^(1/2)+1/4*arctanh(2^(1/2)*exp(x)/(1+exp(2*x)))*2^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.23

$$\int e^x \operatorname{sech}^2(2x)\,dx = \frac{1}{8}\left(-\frac{8e^x}{1+e^{4x}} - 2\sqrt{2}\arctan\left(1-\sqrt{2}e^x\right) + 2\sqrt{2}\arctan\left(1+\sqrt{2}e^x\right) - \sqrt{2}\log\left(1-\sqrt{2}e^x+e^{2x}\right) + \sqrt{2}\log\left(1+\sqrt{2}e^x+e^{2x}\right)\right)$$

input
```
Integrate[E^x*Sech[2*x]^2,x]
```

output
```
((-8*E^x)/(1 + E^(4*x)) - 2*Sqrt[2]*ArcTan[1 - Sqrt[2]*E^x] + 2*Sqrt[2]*Ar
cTan[1 + Sqrt[2]*E^x] - Sqrt[2]*Log[1 - Sqrt[2]*E^x + E^(2*x)] + Sqrt[2]*L
og[1 + Sqrt[2]*E^x + E^(2*x)])/8
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.45, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.100, Rules used = {2720, 27, 817, 755, 1476, 1082, 217, 1479, 25, 27, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}^2(2x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{4x}}{(e^{4x}+1)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{4x}}{(1+e^{4x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{4}\int \frac{1}{1+e^{4x}}de^x-\frac{e^x}{4(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{755}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\int \frac{1+e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1476}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2}e^x+e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4(e^{4x}+1)}\right)$$

$$\Big\downarrow \text{1082}$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{1}{-1-e^{2x}}d\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}-\frac{\int \frac{1}{-1-e^{2x}}d\left(1+\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)$$

$$\Big\downarrow \; 217$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\int \frac{1-e^{2x}}{1+e^{4x}}de^x+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{e^x}{4\left(e^{4x}+1\right)}\right)$$

$$\Big\downarrow \; 1479$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(-\frac{\int -\frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}-\frac{\int -\frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{}{4}$$

$$\Big\downarrow \; 25$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}\left(1+\sqrt{2}e^x\right)}{1+\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-\frac{e^x}{4\left(e^{4x}+\right.}$$

$$\Big\downarrow \; 27$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\int \frac{\sqrt{2}-2e^x}{1-\sqrt{2}e^x+e^{2x}}de^x}{2\sqrt{2}}+\frac{1}{2}\int \frac{1+\sqrt{2}e^x}{1+\sqrt{2}e^x+e^{2x}}de^x\right)+\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)\right)-$$

$$\Big\downarrow \; 1103$$

$$4\left(\frac{1}{4}\left(\frac{1}{2}\left(\frac{\arctan\left(\sqrt{2}e^x+1\right)}{\sqrt{2}}-\frac{\arctan\left(1-\sqrt{2}e^x\right)}{\sqrt{2}}\right)+\frac{1}{2}\left(\frac{\log\left(\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}-\frac{\log\left(-\sqrt{2}e^x+e^{2x}+1\right)}{2\sqrt{2}}\right)\right)-$$

input
```
Int[E^x*Sech[2*x]^2,x]
```

output
```
4*(-1/4*E^x/(1 + E^(4*x)) + ((-(ArcTan[1 - Sqrt[2]*E^x]/Sqrt[2]) + ArcTan[
1 + Sqrt[2]*E^x]/Sqrt[2])/2 + (-1/2*Log[1 - Sqrt[2]*E^x + E^(2*x)]/Sqrt[2]
 + Log[1 + Sqrt[2]*E^x + E^(2*x)]/(2*Sqrt[2]))/2)/4)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 755
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b, 2]
], s = Denominator[Rt[a/b, 2]]}, Simp[1/(2*r)   Int[(r - s*x^2)/(a + b*x^4)
, x], x] + Simp[1/(2*r)   Int[(r + s*x^2)/(a + b*x^4), x], x]] /; FreeQ[{a,
 b}, x] && (GtQ[a/b, 0] || (PosQ[a/b] && AtomQ[SplitProduct[SumBaseQ, a]] &
& AtomQ[SplitProduct[SumBaseQ, b]]))
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1476
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
2*(d/e), 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^2, x], x], x] + Simp[
e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x]
 && EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

rule 1479
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-2*(d/e), 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
 x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; F
reeQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.35 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.42

method	result
risch	$-\frac{\mathrm{e}^{x}}{1+\mathrm{e}^{4x}}+4\left(\sum_{\_R=\operatorname{RootOf}\left(65536\_Z^{4}+1\right)}\_R\ln\left(\mathrm{e}^{x}+16\_R\right)\right)$
default	$\frac{-\tanh\left(\frac{x}{2}\right)^{3}-3\tanh\left(\frac{x}{2}\right)^{2}+\tanh\left(\frac{x}{2}\right)-1}{\tanh\left(\frac{x}{2}\right)^{4}+6\tanh\left(\frac{x}{2}\right)^{2}+1}+\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+3+2\sqrt{2}\right)}{8}+\frac{\left(2+\sqrt{2}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{2+2\sqrt{2}}\right)}{4+4\sqrt{2}}-\frac{\sqrt{2}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+3\right.}{8}$

input
```
int(exp(x)*sech(2*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-exp(x)/(1+exp(4*x))+4*sum(_R*ln(exp(x)+16*_R),_R=RootOf(65536*_Z^4+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(61) = 122$.

Time = 0.10 (sec) , antiderivative size = 345, normalized size of antiderivative = 4.01

$$\int e^x \text{sech}^2(2x) \, dx = \frac{2 \left(\sqrt{2}\cosh(x)^4 + 4\sqrt{2}\cosh(x)^3\sinh(x) + 6\sqrt{2}\cosh(x)^2\sinh(x)^2 + 4\sqrt{2}\cosh(x)\sinh(x)^3 + \sqrt{2}\sinh(\right.}{}$$

input
```
integrate(exp(x)*sech(2*x)^2,x, algorithm="fricas")
```

output
```
1/8*(2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)^3*sinh(x) + 6*sqrt(2)*cosh(x
)^2*sinh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + sqrt(2))
*arctan(sqrt(2)*cosh(x) + sqrt(2)*sinh(x) + 1) + 2*(sqrt(2)*cosh(x)^4 + 4*
sqrt(2)*cosh(x)^3*sinh(x) + 6*sqrt(2)*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*cosh
(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + sqrt(2))*arctan(sqrt(2)*cosh(x) + sqrt
(2)*sinh(x) - 1) + (sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)^3*sinh(x) + 6*sq
rt(2)*cosh(x)^2*sinh(x)^2 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^
4 + sqrt(2))*log((sqrt(2) + 2*cosh(x))/(cosh(x) - sinh(x))) - (sqrt(2)*cos
h(x)^4 + 4*sqrt(2)*cosh(x)^3*sinh(x) + 6*sqrt(2)*cosh(x)^2*sinh(x)^2 + 4*s
qrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^4 + sqrt(2))*log(-(sqrt(2) - 2*
cosh(x))/(cosh(x) - sinh(x))) - 8*cosh(x) - 8*sinh(x))/(cosh(x)^4 + 4*cosh
(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 +
1)
```

Sympy [F]

$$\int e^x \text{sech}^2(2x) \, dx = \int e^x \operatorname{sech}^2(2x) \, dx$$

input
```
integrate(exp(x)*sech(2*x)**2,x)
```

output
```
Integral(exp(x)*sech(2*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int e^x \operatorname{sech}^2(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^x}{e^{(4x)} + 1}$$

input
```
integrate(exp(x)*sech(2*x)^2,x, algorithm="maxima")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.02

$$\int e^x \operatorname{sech}^2(2x)\,dx = \frac{1}{4}\sqrt{2}\arctan\left(\frac{1}{2}\sqrt{2}\left(\sqrt{2}+2e^x\right)\right) + \frac{1}{4}\sqrt{2}\arctan\left(-\frac{1}{2}\sqrt{2}\left(\sqrt{2}-2e^x\right)\right) + \frac{1}{8}\sqrt{2}\log\left(\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{1}{8}\sqrt{2}\log\left(-\sqrt{2}e^x + e^{(2x)} + 1\right) - \frac{e^x}{e^{(4x)} + 1}$$

input
```
integrate(exp(x)*sech(2*x)^2,x, algorithm="giac")
```

output
```
1/4*sqrt(2)*arctan(1/2*sqrt(2)*(sqrt(2) + 2*e^x)) + 1/4*sqrt(2)*arctan(-1/
2*sqrt(2)*(sqrt(2) - 2*e^x)) + 1/8*sqrt(2)*log(sqrt(2)*e^x + e^(2*x) + 1)
- 1/8*sqrt(2)*log(-sqrt(2)*e^x + e^(2*x) + 1) - e^x/(e^(4*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.99

$$\int e^x \operatorname{sech}^2(2x)\,dx = \frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\left(\mathrm{e}^x - \frac{\sqrt{2}}{2}\right)\right)}{4} - \frac{\mathrm{e}^x}{\mathrm{e}^{4x}+1} + \frac{\sqrt{2}\,\operatorname{atan}\left(\sqrt{2}\left(\mathrm{e}^x + \frac{\sqrt{2}}{2}\right)\right)}{4} - \frac{\sqrt{2}\,\ln\left(\left(\mathrm{e}^x - \frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2}\right)}{8} + \frac{\sqrt{2}\,\ln\left(\left(\mathrm{e}^x + \frac{\sqrt{2}}{2}\right)^2 + \frac{1}{2}\right)}{8}$$

input
```
int(exp(x)/cosh(2*x)^2,x)
```

output
```
(2^(1/2)*atan(2^(1/2)*(exp(x) - 2^(1/2)/2)))/4 - exp(x)/(exp(4*x) + 1) + (
2^(1/2)*atan(2^(1/2)*(exp(x) + 2^(1/2)/2)))/4 - (2^(1/2)*log((exp(x) - 2^(
1/2)/2)^2 + 1/2))/8 + (2^(1/2)*log((exp(x) + 2^(1/2)/2)^2 + 1/2))/8
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.16

$$\int e^x \operatorname{sech}^2(2x)\,dx = \frac{2e^{4x}\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{2e^x-\sqrt{2}}{\sqrt{2}}\right) + 2e^{4x}\sqrt{2}\,atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right) + 2\sqrt{2}\,atan\left(\frac{2e^x+\sqrt{2}}{\sqrt{2}}\right) - e^{4x}\sqrt{2}\log}{8e}$$

input
```
int(exp(x)*sech(2*x)^2,x)
```

output
```
(2*e**(4*x)*sqrt(2)*atan((2*e**x - sqrt(2))/sqrt(2)) + 2*sqrt(2)*atan((2*e
**x - sqrt(2))/sqrt(2)) + 2*e**(4*x)*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(
2)) + 2*sqrt(2)*atan((2*e**x + sqrt(2))/sqrt(2)) - e**(4*x)*sqrt(2)*log(e*
*(2*x) - e**x*sqrt(2) + 1) + e**(4*x)*sqrt(2)*log(e**(2*x) + e**x*sqrt(2)
+ 1) - 8*e**x - sqrt(2)*log(e**(2*x) - e**x*sqrt(2) + 1) + sqrt(2)*log(e**
(2*x) + e**x*sqrt(2) + 1))/(8*(e**(4*x) + 1))
```

3.278 $\int e^x \cosh^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \cosh^2(3x)\,dx = -\frac{1}{20}e^{-5x} + \frac{e^x}{2} + \frac{e^{7x}}{28}$$

output
```
-1/20/exp(5*x)+1/2*exp(x)+1/28*exp(7*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \cosh^2(3x)\,dx = -\frac{1}{20}e^{-5x} + \frac{e^x}{2} + \frac{e^{7x}}{28}$$

input
```
Integrate[E^x*Cosh[3*x]^2,x]
```

output
```
-1/20*1/E^(5*x) + E^x/2 + E^(7*x)/28
```

Rubi [A] (verified)

Time = 0.19 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh^2(3x)\,dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{1}{4}e^{-6x}\left(e^{6x}+1\right)^2 de^x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{4}\int e^{-6x}\left(1+e^{6x}\right)^2 de^x$$

$$\Big\downarrow \text{ 802}$$

$$\frac{1}{4}\int \left(2+e^{-6x}+e^{6x}\right) de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\left(-\frac{1}{5}e^{-5x}+2e^x+\frac{e^{7x}}{7}\right)$$

input
```
Int[E^x*Cosh[3*x]^2,x]
```

output
```
(-1/5*1/E^(5*x) + 2*E^x + E^(7*x)/7)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{e^{x} \left(-35+\cosh \left(6 x \right)-6 \sinh \left(6 x \right)\right)}{70}$	17
risch	$\frac{e^{7 x}}{28}+\frac{e^{x}}{2}-\frac{e^{-5 x}}{20}$	18
default	$\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(5 x \right)}{20}+\frac{\sinh \left(7 x \right)}{28}+\frac{\cosh \left(x \right)}{2}-\frac{\cosh \left(5 x \right)}{20}+\frac{\cosh \left(7 x \right)}{28}$	34
orering	$\frac{17 e^{x} \cosh \left(3 x \right)^{2}}{35}+\frac{6 e^{x} \cosh \left(3 x \right) \sinh \left(3 x \right)}{35}-\frac{18 e^{x} \sinh \left(3 x \right)^{2}}{35}$	34

input
```
int(exp(x)*cosh(3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/70*exp(x)*(-35+cosh(6*x)-6*sinh(6*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.58

$$\int e^x \cosh^2(3x)\,dx =$$
$$-\frac{\cosh(x)^6 - 36\cosh(x)^5\sinh(x) + 15\cosh(x)^4\sinh(x)^2 - 120\cosh(x)^3\sinh(x)^3 + 15\cosh(x)^2\sinh}{70(\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*cosh(3*x)^2,x, algorithm="fricas")
```

output
```
-1/70*(cosh(x)^6 - 36*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 - 120*cos
h(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 - 36*cosh(x)*sinh(x)^5 + sinh(x)
^6 - 35)/(cosh(x) - sinh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \cosh^2(3x)\,dx = -\frac{18e^x\sinh^2(3x)}{35} + \frac{6e^x\sinh(3x)\cosh(3x)}{35} + \frac{17e^x\cosh^2(3x)}{35}$$

input
```
integrate(exp(x)*cosh(3*x)**2,x)
```

output
```
-18*exp(x)*sinh(3*x)**2/35 + 6*exp(x)*sinh(3*x)*cosh(3*x)/35 + 17*exp(x)*c
osh(3*x)**2/35
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(3x)\,dx = \frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(3*x)^2,x, algorithm="maxima")
```

output
```
1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(3x)\,dx = \frac{1}{28}\,e^{(7\,x)} - \frac{1}{20}\,e^{(-5\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(3*x)^2,x, algorithm="giac")
```

output
```
1/28*e^(7*x) - 1/20*e^(-5*x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(3x)\,dx = \frac{e^{7\,x}}{28} - \frac{e^{-5\,x}}{20} + \frac{e^x}{2}$$

input
```
int(cosh(3*x)^2*exp(x),x)
```

output
```
exp(7*x)/28 - exp(-5*x)/20 + exp(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^x \cosh^2(3x)\,dx = \frac{5e^{12x} + 70e^{6x} - 7}{140e^{5x}}$$

input
```
int(exp(x)*cosh(3*x)^2,x)
```

output
```
(5*e**(12*x) + 70*e**(6*x) - 7)/(140*e**(5*x))
```

3.279 $\int e^x \cosh(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \cosh(3x)\,dx = -\frac{1}{4}e^{-2x} + \frac{e^{4x}}{8}$$

output
```
-1/4/exp(2*x)+1/8*exp(4*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.84

$$\int e^x \cosh(3x)\,dx = \frac{1}{8}e^{-2x}\left(-2 + e^{6x}\right)$$

input
```
Integrate[E^x*Cosh[3*x],x]
```

output
```
(-2 + E^(6*x))/(8*E^(2*x))
```

Rubi [A] (verified)

Time = 0.18 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(3x)\, dx$$

$$\Big\downarrow \; 2720$$

$$\int \frac{1}{2}e^{-3x}\left(e^{6x}+1\right) de^x$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\int e^{-3x}\left(1+e^{6x}\right) de^x$$

$$\Big\downarrow \; 802$$

$$\frac{1}{2}\int \left(e^{-3x}+e^{3x}\right) de^x$$

$$\Big\downarrow \; 2009$$

$$\frac{1}{2}\left(\frac{e^{4x}}{4}-\frac{1}{2}e^{-2x}\right)$$

input
```
Int[E^x*Cosh[3*x],x]
```

output
```
(-1/2*1/E^(2*x) + E^(4*x)/4)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{\rm e}^{4 x}}{8}-\frac{{\rm e}^{-2 x}}{4}$	14
parallelrisch	$-\frac{{\rm e}^{x} \left(\cosh \left(3 x \right)-3 \sinh \left(3 x \right)\right)}{8}$	16
orering	$-\frac{{\rm e}^{x} \cosh \left(3 x \right)}{8}+\frac{3 \,{\rm e}^{x} \sinh \left(3 x \right)}{8}$	18
default	$\frac{\sinh \left(4 x \right)}{8}+\frac{\sinh \left(2 x \right)}{4}-\frac{\cosh \left(2 x \right)}{4}+\frac{\cosh \left(4 x \right)}{8}$	26

input
```
int(exp(x)*cosh(3*x),x,method=_RETURNVERBOSE)
```

output
```
1/8*exp(4*x)-1/4*exp(-2*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. 2(13) = 26.

Time = 0.08 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.00

$$\int e^x \cosh(3x)\,dx = -\frac{\cosh(x)^3 - 9\cosh(x)^2\sinh(x) + 3\cosh(x)\sinh(x)^2 - 3\sinh(x)^3}{8(\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*cosh(3*x),x, algorithm="fricas")
```

output
```
-1/8*(cosh(x)^3 - 9*cosh(x)^2*sinh(x) + 3*cosh(x)*sinh(x)^2 - 3*sinh(x)^3)
/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \cosh(3x)\,dx = \frac{3e^x\sinh(3x)}{8} - \frac{e^x\cosh(3x)}{8}$$

input
```
integrate(exp(x)*cosh(3*x),x)
```

output
```
3*exp(x)*sinh(3*x)/8 - exp(x)*cosh(3*x)/8
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(3x)\,dx = \frac{1}{8}e^{(4x)} - \frac{1}{4}e^{(-2x)}$$

input
```
integrate(exp(x)*cosh(3*x),x, algorithm="maxima")
```

output
```
1/8*e^(4*x) - 1/4*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(3x)\,dx = \frac{1}{8}\,e^{(4\,x)} - \frac{1}{4}\,e^{(-2\,x)}$$

input
```
integrate(exp(x)*cosh(3*x),x, algorithm="giac")
```

output
```
1/8*e^(4*x) - 1/4*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 1.89 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.63

$$\int e^x \cosh(3x)\,dx = \frac{\mathrm{e}^{-2\,x}\,\left(\mathrm{e}^{6\,x}-2\right)}{8}$$

input
```
int(cosh(3*x)*exp(x),x)
```

output
```
(exp(-2*x)*(exp(6*x) - 2))/8
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^x \cosh(3x)\,dx = \frac{e^x(-\cosh(3x) + 3\sinh(3x))}{8}$$

input
```
int(exp(x)*cosh(3*x),x)
```

output
```
(e**x*( - cosh(3*x) + 3*sinh(3*x)))/8
```

3.280 $\int e^x \text{sech}(3x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 55

$$\int e^x \text{sech}(3x)\,dx = -\frac{\arctan\left(\frac{1-2e^{2x}}{\sqrt{3}}\right)}{\sqrt{3}} - \frac{1}{3}\log\left(1+e^{2x}\right) + \frac{1}{6}\log\left(1-e^{2x}+e^{4x}\right)$$

output
```
-1/3*arctan(1/3*(1-2*exp(2*x))*3^(1/2))*3^(1/2)-1/3*ln(1+exp(2*x))+1/6*ln(
1-exp(2*x)+exp(4*x))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.44

$$\int e^x \text{sech}(3x)\,dx = \frac{1}{2}e^{4x}\,\text{Hypergeometric2F1}\left(\frac{2}{3},1,\frac{5}{3},-e^{6x}\right)$$

input
```
Integrate[E^x*Sech[3*x],x]
```

output
```
(E^(4*x)*Hypergeometric2F1[2/3, 1, 5/3, -E^(6*x)])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.64, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {2720, 27, 807, 821, 16, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \operatorname{sech}(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{3x}}{e^{6x}+1}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{3x}}{1+e^{6x}}de^x$$

$$\Big\downarrow \text{807}$$

$$\int \frac{e^{2x}}{e^{3x}+1}de^{2x}$$

$$\Big\downarrow \text{821}$$

$$\frac{1}{3}\int \left(1+e^{2x}\right)de^{2x}-\frac{1}{3}\int \frac{1}{1+e^{2x}}de^{2x}$$

$$\Big\downarrow \text{16}$$

$$\frac{1}{3}\int \left(1+e^{2x}\right)de^{2x}-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \text{1142}$$

$$\frac{1}{3}\left(\frac{3\int 1de^{2x}}{2}+\frac{1}{2}\int \left(-1+2e^{2x}\right)de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{3}\left(\frac{3\int 1de^{2x}}{2}-\frac{1}{2}\int \left(1-2e^{2x}\right)de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \text{1083}$$

$$\frac{1}{3}\left(-3\int \frac{1}{-2-2e^{2x}}d(-1+2e^{2x})-\frac{1}{2}\int (1-2e^{2x})\,de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \; 217$$

$$\frac{1}{3}\left(\sqrt{3}\arctan\left(\frac{2e^{2x}-1}{\sqrt{3}}\right)-\frac{1}{2}\int (1-2e^{2x})\,de^{2x}\right)-\frac{1}{3}\log\left(e^{2x}+1\right)$$

$$\Big\downarrow \; 1103$$

$$\frac{\arctan\left(\frac{2e^{2x}-1}{\sqrt{3}}\right)}{\sqrt{3}}-\frac{1}{3}\log\left(e^{2x}+1\right)$$

input
```
Int[E^x*Sech[3*x],x]
```

output
```
ArcTan[(-1 + 2*E^(2*x))/Sqrt[3]]/Sqrt[3] - Log[1 + E^(2*x)]/3
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 807
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Simp[1/k   Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x,
x^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.44

method	result	size
risch	$-\frac{\ln\left(\mathrm{e}^{2x}+1\right)}{3}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)}{6}+\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{2}+\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}+\frac{\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)}{6}-\frac{i\ln\left(\mathrm{e}^{2x}-\frac{1}{2}-\frac{i\sqrt{3}}{2}\right)\sqrt{3}}{6}$	79

input
```
int(exp(x)*sech(3*x),x,method=_RETURNVERBOSE)
```

output
```
-1/3*ln(exp(2*x)+1)+1/6*ln(exp(2*x)-1/2+1/2*I*3^(1/2))+1/6*I*ln(exp(2*x)-1
/2+1/2*I*3^(1/2))*3^(1/2)+1/6*ln(exp(2*x)-1/2-1/2*I*3^(1/2))-1/6*I*ln(exp(
2*x)-1/2-1/2*I*3^(1/2))*3^(1/2)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.51

$$\int e^x \operatorname{sech}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(-\frac{\sqrt{3}\cosh(x)+3\sqrt{3}\sinh(x)}{3(\cosh(x)-\sinh(x))}\right) + \frac{1}{6}\log\left(\frac{2\cosh(x)^2+2\sinh(x)^2-1}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right) - \frac{1}{3}\log\left(\frac{2\cosh(x)}{\cosh(x)-\sinh(x)}\right)$$

input
```
integrate(exp(x)*sech(3*x),x, algorithm="fricas")
```

output
```
-1/3*sqrt(3)*arctan(-1/3*(sqrt(3)*cosh(x) + 3*sqrt(3)*sinh(x))/(cosh(x) -
sinh(x))) + 1/6*log((2*cosh(x)^2 + 2*sinh(x)^2 - 1)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 1/3*log(2*cosh(x)/(cosh(x) - sinh(x)))
```

Sympy [F]

$$\int e^x \operatorname{sech}(3x)\,dx = \int e^x \operatorname{sech}(3x)\,dx$$

input
```
integrate(exp(x)*sech(3*x),x)
```

output
```
Integral(exp(x)*sech(3*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.29

$$\int e^x \operatorname{sech}(3x)\,dx = -\frac{1}{3}\sqrt{3}\arctan\left(\sqrt{3}+2\,e^x\right)+\frac{1}{3}\sqrt{3}\arctan\left(-\sqrt{3}+2\,e^x\right)+\frac{1}{6}\log\left(\sqrt{3}e^x+e^{(2\,x)}+1\right)+\frac{1}{6}\log\left(-\sqrt{3}e^x+e^{(2\,x)}+1\right)-\frac{1}{3}\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(exp(x)*sech(3*x),x, algorithm="maxima")
```

output
```
-1/3*sqrt(3)*arctan(sqrt(3) + 2*e^x) + 1/3*sqrt(3)*arctan(-sqrt(3) + 2*e^x
) + 1/6*log(sqrt(3)*e^x + e^(2*x) + 1) + 1/6*log(-sqrt(3)*e^x + e^(2*x) +
1) - 1/3*log(e^(2*x) + 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.80

$$\int e^x \operatorname{sech}(3x)\,dx = \frac{1}{3}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\,e^{(2\,x)}-1)\right)+\frac{1}{6}\log\left(e^{(4\,x)}-e^{(2\,x)}+1\right)-\frac{1}{3}\log\left(e^{(2\,x)}+1\right)$$

input
```
integrate(exp(x)*sech(3*x),x, algorithm="giac")
```

output
```
1/3*sqrt(3)*arctan(1/3*sqrt(3)*(2*e^(2*x) - 1)) + 1/6*log(e^(4*x) - e^(2*x
) + 1) - 1/3*log(e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.18

$$\int e^x \operatorname{sech}(3x)\,dx = -\frac{\ln\left(8\,\mathrm{e}^{2\,x}+8\right)}{3} - \ln\left(24\,\mathrm{e}^{2\,x}\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)+8\right)\left(-\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right) + \ln\left(8-24\,\mathrm{e}^{2\,x}\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)\right)\left(\frac{1}{6}+\frac{\sqrt{3}\,1\mathrm{i}}{6}\right)$$

input
```
int(exp(x)/cosh(3*x),x)
```

output
```
log(8 - 24*exp(2*x)*((3^(1/2)*1i)/6 + 1/6))*((3^(1/2)*1i)/6 + 1/6) - log(2
4*exp(2*x)*((3^(1/2)*1i)/6 - 1/6) + 8)*((3^(1/2)*1i)/6 - 1/6) - log(8*exp(
2*x) + 8)/3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.31

$$\int e^x \operatorname{sech}(3x)\,dx = \frac{\sqrt{3}\,atan\left(2e^x-\sqrt{3}\right)}{3} - \frac{\sqrt{3}\,atan\left(2e^x+\sqrt{3}\right)}{3} + \frac{\log\left(e^{2x}-e^x\sqrt{3}+1\right)}{6} + \frac{\log\left(e^{2x}+e^x\sqrt{3}+1\right)}{6} - \frac{\log\left(e^{2x}+1\right)}{3}$$

input
```
int(exp(x)*sech(3*x),x)
```

output
```
(2*sqrt(3)*atan(2*e**x - sqrt(3)) - 2*sqrt(3)*atan(2*e**x + sqrt(3)) + log
(e**(2*x) - e**x*sqrt(3) + 1) + log(e**(2*x) + e**x*sqrt(3) + 1) - 2*log(e
**(2*x) + 1))/6
```

3.281 $\int e^x \mathrm{sech}^2(3x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 85

$$\int e^x \mathrm{sech}^2(3x)\,dx = -\frac{2e^x}{3(1+e^{6x})} + \frac{2\arctan(e^x)}{9} - \frac{1}{9}\arctan\left(\sqrt{3}-2e^x\right) + \frac{1}{9}\arctan\left(\sqrt{3}+2e^x\right) + \frac{\operatorname{arctanh}\left(\frac{\sqrt{3}e^x}{1+e^{2x}}\right)}{3\sqrt{3}}$$

output
```
-2*exp(x)/(3+3*exp(6*x))+2/9*arctan(exp(x))+1/9*arctan(-3^(1/2)+2*exp(x))+
1/9*arctan(3^(1/2)+2*exp(x))+1/9*arctanh(3^(1/2)*exp(x)/(1+exp(2*x)))*3^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.40

$$\int e^x \mathrm{sech}^2(3x)\,dx = \frac{2}{3}e^x\left(-\frac{1}{1+e^{6x}} + \text{Hypergeometric2F1}\left(\frac{1}{6},1,\frac{7}{6},-e^{6x}\right)\right)$$

input
```
Integrate[E^x*Sech[3*x]^2,x]
```

output
```
(2*E^x*(-(1 + E^(6*x))^(-1) + Hypergeometric2F1[1/6, 1, 7/6, -E^(6*x)]))/3
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 121, normalized size of antiderivative = 1.42, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 1.100$, Rules used = {2720, 27, 817, 753, 27, 216, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \text{sech}^2(3x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{4e^{6x}}{(e^{6x}+1)^2}de^x$$

$$\Big\downarrow \text{27}$$

$$4\int \frac{e^{6x}}{(1+e^{6x})^2}de^x$$

$$\Big\downarrow \text{817}$$

$$4\left(\frac{1}{6}\int \frac{1}{1+e^{6x}}de^x - \frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \text{753}$$

$$4\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{2x}}de^x + \frac{1}{3}\int \frac{2-\sqrt{3}e^x}{2\left(1-\sqrt{3}e^x+e^{2x}\right)}de^x + \frac{1}{3}\int \frac{2+\sqrt{3}e^x}{2\left(1+\sqrt{3}e^x+e^{2x}\right)}de^x\right) - \frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \text{27}$$

$$4\left(\frac{1}{6}\left(\frac{1}{3}\int \frac{1}{1+e^{2x}}de^x + \frac{1}{6}\int \frac{2-\sqrt{3}e^x}{1-\sqrt{3}e^x+e^{2x}}de^x + \frac{1}{6}\int \frac{2+\sqrt{3}e^x}{1+\sqrt{3}e^x+e^{2x}}de^x\right) - \frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \text{216}$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\int \frac{2-\sqrt{3}e^x}{1-\sqrt{3}e^x+e^{2x}}de^x + \frac{1}{6}\int \frac{2+\sqrt{3}e^x}{1+\sqrt{3}e^x+e^{2x}}de^x + \frac{\arctan(e^x)}{3}\right) - \frac{e^x}{6(e^{6x}+1)}\right)$$

$$\Big\downarrow \ 1142$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{3}e^x+e^{2x}}de^x-\frac{1}{2}\sqrt{3}\int-\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1+\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{2}\sqrt{3}\int\frac{}{1+}\right.\right.\right.$$

$$\Big\downarrow \ 25$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1-\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x\right)+\frac{1}{6}\left(\frac{1}{2}\int\frac{1}{1+\sqrt{3}e^x+e^{2x}}de^x+\frac{1}{2}\sqrt{3}\int\frac{\sqrt{}}{1+}\right.\right.\right.$$

$$\Big\downarrow \ 1083$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x-\int\frac{1}{-1-e^{2x}}d\left(-\sqrt{3}+2e^x\right)\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}+2e^x}{1+\sqrt{3}e^x+e^{2x}}de^x-\int\frac{}{-1}\right.\right.\right.$$

$$\Big\downarrow \ 217$$

$$4\left(\frac{1}{6}\left(\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}-2e^x}{1-\sqrt{3}e^x+e^{2x}}de^x-\arctan\left(\sqrt{3}-2e^x\right)\right)+\frac{1}{6}\left(\frac{1}{2}\sqrt{3}\int\frac{\sqrt{3}+2e^x}{1+\sqrt{3}e^x+e^{2x}}de^x+\arctan\left(2e^x+\right.\right.\right.\right.$$

$$\Big\downarrow \ 1103$$

$$4\left(\frac{1}{6}\left(\frac{\arctan\left(e^x\right)}{3}+\frac{1}{6}\left(-\arctan\left(\sqrt{3}-2e^x\right)-\frac{1}{2}\sqrt{3}\log\left(-\sqrt{3}e^x+e^{2x}+1\right)\right)+\frac{1}{6}\left(\arctan\left(2e^x+\sqrt{3}\right)+\frac{1}{2}\sqrt{3}\right.\right.\right.$$

input
```
Int[E^x*Sech[3*x]^2,x]
```

output
```
4*(-1/6*E^x/(1 + E^(6*x)) + (ArcTan[E^x]/3 + (-ArcTan[Sqrt[3] - 2*E^x] - (
Sqrt[3]*Log[1 - Sqrt[3]*E^x + E^(2*x)])/2)/6 + (ArcTan[Sqrt[3] + 2*E^x] +
(Sqrt[3]*Log[1 + Sqrt[3]*E^x + E^(2*x)])/2)/6)/6)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 753
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> Module[{r = Numerator[Rt[a/
b, n]], s = Denominator[Rt[a/b, n]], k, u, v}, Simp[u = Int[(r - s*Cos[(2*k
 - 1)*(Pi/n)]*x)/(r^2 - 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*x^2), x] + Int[
(r + s*Cos[(2*k - 1)*(Pi/n)]*x)/(r^2 + 2*r*s*Cos[(2*k - 1)*(Pi/n)]*x + s^2*
x^2), x]; 2*(r^2/(a*n))   Int[1/(r^2 + s^2*x^2), x] + 2*(r/(a*n))   Sum[u,
{k, 1, (n - 2)/4}], x]] /; FreeQ[{a, b}, x] && IGtQ[(n - 2)/4, 0] && PosQ[a
/b]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.69

method	result
risch	$-\frac{2\,\mathrm{e}^{x}}{3(\mathrm{e}^{6x}+1)}+\frac{i\ln(\mathrm{e}^{x}+i)}{9}-\frac{i\ln(\mathrm{e}^{x}-i)}{9}+4\left(\sum_{\_R=\mathrm{RootOf}(1679616\_Z^{4}-1296\_Z^{2}+1)}\_R\ln\left(\mathrm{e}^{x}+36\_R\right)\right)$
default	$\frac{-\frac{4\tanh\left(\frac{x}{2}\right)^{3}}{3}-\frac{28\tanh\left(\frac{x}{2}\right)^{2}}{9}+\frac{4\tanh\left(\frac{x}{2}\right)}{3}-\frac{4}{9}}{\tanh\left(\frac{x}{2}\right)^{4}+14\tanh\left(\frac{x}{2}\right)^{2}+1}+\frac{\sqrt{3}\,\ln\left(\tanh\left(\frac{x}{2}\right)^{2}+7+4\sqrt{3}\right)}{18}+\frac{2\left(2+\sqrt{3}\right)\arctan\left(\frac{2\tanh\left(\frac{x}{2}\right)}{4+2\sqrt{3}}\right)}{9\left(4+2\sqrt{3}\right)}-\frac{\sqrt{3}\,\ln\left(\tanh\left(\frac{x}{2}\right.\right.}{18}$

input
```
int(exp(x)*sech(3*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-2/3*exp(x)/(exp(6*x)+1)+1/9*I*ln(exp(x)+I)-1/9*I*ln(exp(x)-I)+4*sum(_R*ln
(exp(x)+36*_R),_R=RootOf(1679616*_Z^4-1296*_Z^2+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 481, normalized size of antiderivative = 5.66

$$\int e^x \operatorname{sech}^2(3x)\,dx = \text{Too large to display}$$

input
```
integrate(exp(x)*sech(3*x)^2,x, algorithm="fricas")
```

output
```
1/18*(2*(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cos
h(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^
6 + 1)*arctan(sqrt(3) + 2*cosh(x) + 2*sinh(x)) + 2*(cosh(x)^6 + 6*cosh(x)^
5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2
*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 1)*arctan(-sqrt(3) + 2*cosh
(x) + 2*sinh(x)) + 4*(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)
^5 + sinh(x)^6 + 1)*arctan(cosh(x) + sinh(x)) + (sqrt(3)*cosh(x)^6 + 6*sqr
t(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*cosh(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(
x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*sinh(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x
)^5 + sqrt(3)*sinh(x)^6 + sqrt(3))*log((sqrt(3) + 2*cosh(x))/(cosh(x) - si
nh(x))) - (sqrt(3)*cosh(x)^6 + 6*sqrt(3)*cosh(x)^5*sinh(x) + 15*sqrt(3)*co
sh(x)^4*sinh(x)^2 + 20*sqrt(3)*cosh(x)^3*sinh(x)^3 + 15*sqrt(3)*cosh(x)^2*
sinh(x)^4 + 6*sqrt(3)*cosh(x)*sinh(x)^5 + sqrt(3)*sinh(x)^6 + sqrt(3))*log
(-(sqrt(3) - 2*cosh(x))/(cosh(x) - sinh(x))) - 12*cosh(x) - 12*sinh(x))/(c
osh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sin
h(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6 + 1)
```

Sympy [F]

$$\int e^x \operatorname{sech}^2(3x)\,dx = \int e^x \operatorname{sech}^2(3x)\,dx$$

input
```
integrate(exp(x)*sech(3*x)**2,x)
```

output
```
Integral(exp(x)*sech(3*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int e^x \operatorname{sech}^2(3x)\,dx = \frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{2\,e^x}{3\left(e^{(6x)} + 1\right)} + \frac{1}{9}\arctan\left(\sqrt{3} + 2\,e^x\right) + \frac{1}{9}\arctan\left(-\sqrt{3} + 2\,e^x\right) + \frac{2}{9}\arctan\left(e^x\right)$$

input
```
integrate(exp(x)*sech(3*x)^2,x, algorithm="maxima")
```

output
```
1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^
x + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1
/9*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int e^x \operatorname{sech}^2(3x)\,dx = \frac{1}{18}\sqrt{3}\log\left(\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{1}{18}\sqrt{3}\log\left(-\sqrt{3}e^x + e^{(2x)} + 1\right) - \frac{2\,e^x}{3\left(e^{(6x)} + 1\right)} + \frac{1}{9}\arctan\left(\sqrt{3} + 2\,e^x\right) + \frac{1}{9}\arctan\left(-\sqrt{3} + 2\,e^x\right) + \frac{2}{9}\arctan\left(e^x\right)$$

input
```
integrate(exp(x)*sech(3*x)^2,x, algorithm="giac")
```

output
```
1/18*sqrt(3)*log(sqrt(3)*e^x + e^(2*x) + 1) - 1/18*sqrt(3)*log(-sqrt(3)*e^
x + e^(2*x) + 1) - 2/3*e^x/(e^(6*x) + 1) + 1/9*arctan(sqrt(3) + 2*e^x) + 1
/9*arctan(-sqrt(3) + 2*e^x) + 2/9*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.99

$$\int e^x \operatorname{sech}^2(3x)\,dx = \frac{2\,\mathrm{atan}(\mathrm{e}^x)}{9} + \frac{\mathrm{atan}\left(2\,\mathrm{e}^x+\sqrt{3}\right)}{9} + \frac{\mathrm{atan}\left(2\,\mathrm{e}^x-\sqrt{3}\right)}{9} - \frac{2\,\mathrm{e}^x}{3\,\left(\mathrm{e}^{6\,x}+1\right)} - \frac{\sqrt{3}\,\ln\left(\left(\frac{2\,\mathrm{e}^x}{3}-\frac{\sqrt{3}}{3}\right)^2+\frac{1}{9}\right)}{18} + \frac{\sqrt{3}\,\ln\left(\left(\frac{2\,\mathrm{e}^x}{3}+\frac{\sqrt{3}}{3}\right)^2+\frac{1}{9}\right)}{18}$$

input
```
int(exp(x)/cosh(3*x)^2,x)
```

output
```
(2*atan(exp(x)))/9 + atan(2*exp(x) + 3^(1/2))/9 + atan(2*exp(x) - 3^(1/2))
/9 - (2*exp(x))/(3*(exp(6*x) + 1)) - (3^(1/2)*log(((2*exp(x))/3 - 3^(1/2)/
3)^2 + 1/9))/18 + (3^(1/2)*log(((2*exp(x))/3 + 3^(1/2)/3)^2 + 1/9))/18
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 175, normalized size of antiderivative = 2.06

$$\int e^x \operatorname{sech}^2(3x)\,dx = \frac{4e^{6x}atan(e^x)+4atan(e^x)+2e^{6x}atan\left(2e^x-\sqrt{3}\right)+2atan\left(2e^x-\sqrt{3}\right)+2e^{6x}atan\left(2e^x+\sqrt{3}\right)+2atan(2}{}$$

input
```
int(exp(x)*sech(3*x)^2,x)
```

output
```
(4*e**(6*x)*atan(e**x) + 4*atan(e**x) + 2*e**(6*x)*atan(2*e**x - sqrt(3))
+ 2*atan(2*e**x - sqrt(3)) + 2*e**(6*x)*atan(2*e**x + sqrt(3)) + 2*atan(2*
e**x + sqrt(3)) - e**(6*x)*sqrt(3)*log(e**(2*x) - e**x*sqrt(3) + 1) + e**(
6*x)*sqrt(3)*log(e**(2*x) + e**x*sqrt(3) + 1) - 12*e**x - sqrt(3)*log(e**(
2*x) - e**x*sqrt(3) + 1) + sqrt(3)*log(e**(2*x) + e**x*sqrt(3) + 1))/(18*(
e**(6*x) + 1))
```

3.282 $\int e^x \cosh^2(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 26

$$\int e^x \cosh^2(4x)\,dx = -\frac{1}{28}e^{-7x} + \frac{e^x}{2} + \frac{e^{9x}}{36}$$

output
```
-1/28/exp(7*x)+1/2*exp(x)+1/36*exp(9*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int e^x \cosh^2(4x)\,dx = -\frac{1}{28}e^{-7x} + \frac{e^x}{2} + \frac{e^{9x}}{36}$$

input
```
Integrate[E^x*Cosh[4*x]^2,x]
```

output
```
-1/28*1/E^(7*x) + E^x/2 + E^(9*x)/36
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.08, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh^2(4x)\,dx$$
$$\Big\downarrow \text{2720}$$
$$\int \frac{1}{4}e^{-8x}\left(e^{8x}+1\right)^2de^x$$
$$\Big\downarrow \text{27}$$
$$\frac{1}{4}\int e^{-8x}\left(1+e^{8x}\right)^2de^x$$
$$\Big\downarrow \text{802}$$
$$\frac{1}{4}\int \left(2+e^{-8x}+e^{8x}\right)de^x$$
$$\Big\downarrow \text{2009}$$
$$\frac{1}{4}\left(-\frac{1}{7}e^{-7x}+2e^x+\frac{e^{9x}}{9}\right)$$

input
```
Int[E^x*Cosh[4*x]^2,x]
```

output
```
(-1/7*1/E^(7*x) + 2*E^x + E^(9*x)/9)/4
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

method	result	size
parallelrisch	$-\frac{{\mathrm e}^{x} \left(-63+\cosh \left(8 x \right)-8 \sinh \left(8 x \right)\right)}{126}$	17
risch	$\frac{{\mathrm e}^{9 x}}{36}+\frac{{\mathrm e}^{x}}{2}-\frac{{\mathrm e}^{-7 x}}{28}$	18
default	$\frac{\sinh \left(x \right)}{2}+\frac{\sinh \left(7 x \right)}{28}+\frac{\sinh \left(9 x \right)}{36}+\frac{\cosh \left(x \right)}{2}-\frac{\cosh \left(7 x \right)}{28}+\frac{\cosh \left(9 x \right)}{36}$	34
orering	$\frac{31 \,{\mathrm e}^{x} \cosh \left(4 x \right)^{2}}{63}+\frac{8 \,{\mathrm e}^{x} \cosh \left(4 x \right) \sinh \left(4 x \right)}{63}-\frac{32 \,{\mathrm e}^{x} \sinh \left(4 x \right)^{2}}{63}$	34

input
```
int(exp(x)*cosh(4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/126*exp(x)*(-63+cosh(8*x)-8*sinh(8*x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(17) = 34$.

Time = 0.09 (sec) , antiderivative size = 87, normalized size of antiderivative = 3.35

$$\int e^x \cosh^2(4x)\,dx =$$

$$-\frac{\cosh(x)^8 - 64\cosh(x)^7\sinh(x) + 28\cosh(x)^6\sinh(x)^2 - 448\cosh(x)^5\sinh(x)^3 + 70\cosh(x)^4\sinh}{126(\cosh(x) -}$$

input
```
integrate(exp(x)*cosh(4*x)^2,x, algorithm="fricas")
```

output
```
-1/126*(cosh(x)^8 - 64*cosh(x)^7*sinh(x) + 28*cosh(x)^6*sinh(x)^2 - 448*co
sh(x)^5*sinh(x)^3 + 70*cosh(x)^4*sinh(x)^4 - 448*cosh(x)^3*sinh(x)^5 + 28*
cosh(x)^2*sinh(x)^6 - 64*cosh(x)*sinh(x)^7 + sinh(x)^8 - 63)/(cosh(x) - si
nh(x))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 42 vs. $2(19) = 38$.

Time = 0.16 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.62

$$\int e^x \cosh^2(4x)\,dx = -\frac{32e^x\sinh^2(4x)}{63} + \frac{8e^x\sinh(4x)\cosh(4x)}{63} + \frac{31e^x\cosh^2(4x)}{63}$$

input
```
integrate(exp(x)*cosh(4*x)**2,x)
```

output
```
-32*exp(x)*sinh(4*x)**2/63 + 8*exp(x)*sinh(4*x)*cosh(4*x)/63 + 31*exp(x)*c
osh(4*x)**2/63
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(4x)\,dx = \frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(4*x)^2,x, algorithm="maxima")
```

output
```
1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(4x)\,dx = \frac{1}{36}\,e^{(9\,x)} - \frac{1}{28}\,e^{(-7\,x)} + \frac{1}{2}\,e^x$$

input
```
integrate(exp(x)*cosh(4*x)^2,x, algorithm="giac")
```

output
```
1/36*e^(9*x) - 1/28*e^(-7*x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 1.92 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.65

$$\int e^x \cosh^2(4x)\,dx = \frac{e^{9\,x}}{36} - \frac{e^{-7\,x}}{28} + \frac{e^x}{2}$$

input
```
int(cosh(4*x)^2*exp(x),x)
```

output
```
exp(9*x)/36 - exp(-7*x)/28 + exp(x)/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.96

$$\int e^x \cosh^2(4x)\,dx = \frac{7e^{16x} + 126e^{8x} - 9}{252e^{7x}}$$

input
```
int(exp(x)*cosh(4*x)^2,x)
```

output
```
(7*e**(16*x) + 126*e**(8*x) - 9)/(252*e**(7*x))
```

3.283 $\int e^x \cosh(4x) \, dx$

Optimal result

Integrand size = 8, antiderivative size = 19

$$\int e^x \cosh(4x) \, dx = -\frac{1}{6}e^{-3x} + \frac{e^{5x}}{10}$$

output
```
-1/6/exp(3*x)+1/10*exp(5*x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00

$$\int e^x \cosh(4x) \, dx = -\frac{1}{6}e^{-3x} + \frac{e^{5x}}{10}$$

input
```
Integrate[E^x*Cosh[4*x],x]
```

output
```
-1/6*1/E^(3*x) + E^(5*x)/10
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.21, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {2720, 27, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(4x) \, dx$$

$$\Big\downarrow \text{ 2720}$$

$$\int \frac{1}{2} e^{-4x} \left(e^{8x}+1\right) de^x$$

$$\Big\downarrow \text{ 27}$$

$$\frac{1}{2} \int e^{-4x} \left(1+e^{8x}\right) de^x$$

$$\Big\downarrow \text{ 802}$$

$$\frac{1}{2} \int \left(e^{-4x}+e^{4x}\right) de^x$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{2} \left(\frac{e^{5x}}{5}-\frac{1}{3} e^{-3x}\right)$$

input
```
Int[E^x*Cosh[4*x],x]
```

output
```
(-1/3*1/E^(3*x) + E^(5*x)/5)/2
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{{\mathrm e}^{5 x}}{10}-\frac{{\mathrm e}^{-3 x}}{6}$	14
parallelrisch	$-\frac{{\mathrm e}^{x} \left(\cosh \left(4 x \right)-4 \sinh \left(4 x \right)\right)}{15}$	16
orering	$-\frac{{\mathrm e}^{x} \cosh \left(4 x \right)}{15}+\frac{4 \,{\mathrm e}^{x} \sinh \left(4 x \right)}{15}$	18
default	$\frac{\sinh \left(3 x \right)}{6}+\frac{\sinh \left(5 x \right)}{10}-\frac{\cosh \left(3 x \right)}{6}+\frac{\cosh \left(5 x \right)}{10}$	26

input
```
int(exp(x)*cosh(4*x),x,method=_RETURNVERBOSE)
```

output
```
1/10*exp(5*x)-1/6*exp(-3*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 46 vs. $2(13) = 26$.

Time = 0.08 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.42

$$\int e^x \cosh(4x)\,dx = -\frac{\cosh(x)^4 - 16\cosh(x)^3\sinh(x) + 6\cosh(x)^2\sinh(x)^2 - 16\cosh(x)\sinh(x)^3 + \sinh(x)^4}{15\,(\cosh(x) - \sinh(x))}$$

input
```
integrate(exp(x)*cosh(4*x),x, algorithm="fricas")
```

output
```
-1/15*(cosh(x)^4 - 16*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 16*cosh(
x)*sinh(x)^3 + sinh(x)^4)/(cosh(x) - sinh(x))
```

Sympy [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int e^x \cosh(4x)\,dx = \frac{4e^x\sinh(4x)}{15} - \frac{e^x\cosh(4x)}{15}$$

input
```
integrate(exp(x)*cosh(4*x),x)
```

output
```
4*exp(x)*sinh(4*x)/15 - exp(x)*cosh(4*x)/15
```

Maxima [A] (verification not implemented)

Time = 0.03 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(4x)\,dx = \frac{1}{10}\,e^{(5\,x)} - \frac{1}{6}\,e^{(-3\,x)}$$

input
```
integrate(exp(x)*cosh(4*x),x, algorithm="maxima")
```

output
```
1/10*e^(5*x) - 1/6*e^(-3*x)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 13, normalized size of antiderivative = 0.68

$$\int e^x \cosh(4x)\,dx = \frac{1}{10}\,e^{(5\,x)} - \frac{1}{6}\,e^{(-3\,x)}$$

input
```
integrate(exp(x)*cosh(4*x),x, algorithm="giac")
```

output
```
1/10*e^(5*x) - 1/6*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.74

$$\int e^x \cosh(4x)\,dx = \frac{e^{-3\,x}\,\left(3\,e^{8\,x}-5\right)}{30}$$

input
```
int(cosh(4*x)*exp(x),x)
```

output
```
(exp(-3*x)*(3*exp(8*x) - 5))/30
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int e^x \cosh(4x)\,dx = \frac{e^x(-\cosh(4x)+4\sinh(4x))}{15}$$

input
```
int(exp(x)*cosh(4*x),x)
```

output
```
(e**x*( - cosh(4*x) + 4*sinh(4*x)))/15
```

3.284 $\int e^x \mathbf{sech}(4x)\,dx$

Optimal result

Integrand size = 8, antiderivative size = 289

$$\int e^x \operatorname{sech}(4x)\,dx = \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} - \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}} + \frac{\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}e^x}{1+e^{2x}}\right)}{2\sqrt{2\left(2-\sqrt{2}\right)}} - \frac{\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}e^x}{1+e^{2x}}\right)}{2\sqrt{2\left(2+\sqrt{2}\right)}}$$

output
```
1/2*arctan(((2-2^(1/2))^(1/2)-2*exp(x))/(2+2^(1/2))^(1/2))/(4+2*2^(1/2))^(
1/2)-1/2*arctan(((2+2^(1/2))^(1/2)-2*exp(x))/(2-2^(1/2))^(1/2))/(4-2*2^(1/
2))^(1/2)-1/2*arctan(((2-2^(1/2))^(1/2)+2*exp(x))/(2+2^(1/2))^(1/2))/(4+2*
2^(1/2))^(1/2)+1/2*arctan(((2+2^(1/2))^(1/2)+2*exp(x))/(2-2^(1/2))^(1/2))/
(4-2*2^(1/2))^(1/2)+1/2*arctanh((2-2^(1/2))^(1/2)*exp(x)/(1+exp(2*x)))/(4-
2*2^(1/2))^(1/2)-1/2*arctanh((2+2^(1/2))^(1/2)*exp(x)/(1+exp(2*x)))/(4+2*2
^(1/2))^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.08

$$\int e^x \operatorname{sech}(4x)\,dx=\frac{2}{5}e^{5x}\,\text{Hypergeometric2F1}\left(\frac{5}{8},1,\frac{13}{8},-e^{8x}\right)$$

input
```
Integrate[E^x*Sech[4*x],x]
```

output
```
(2*E^(5*x)*Hypergeometric2F1[5/8, 1, 13/8, -E^(8*x)])/5
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 389, normalized size of antiderivative = 1.35, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.250, Rules used = {2720, 27, 828, 1447, 1475, 1083, 217, 1478, 25, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \operatorname{sech}(4x)\,dx$$

$$\Big\downarrow \text{2720}$$

$$\int \frac{2e^{4x}}{e^{8x}+1}de^x$$

$$\Big\downarrow \text{27}$$

$$2\int \frac{e^{4x}}{1+e^{8x}}de^x$$

$$\Big\downarrow \text{828}$$

$$2\left(\frac{\int \frac{e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\int \frac{e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}\right)$$

$$\Big\downarrow \text{1447}$$

$$2\left(\frac{\frac{1}{2}\int \frac{1+e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\int \frac{1+e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x-\frac{1}{2}\int \frac{1-e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}\right)$$

$$\Big\downarrow \textbf{1475}$$

$$2\left(\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x+\frac{1}{2}\int \frac{1}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{1}{2}\int \frac{1}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x+\frac{1}{2}\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \textbf{1083}$$

$$2\left(\frac{\frac{1}{2}\left(-\int \frac{1}{-2+\sqrt{2}-e^{2x}}d\left(-\sqrt{2+\sqrt{2}}+2e^x\right)-\int \frac{1}{-2+\sqrt{2}-e^{2x}}d\left(\sqrt{2+\sqrt{2}}+2e^x\right)\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\left(-\int\cdots\right.}{\cdots}\right.$$

$$\Big\downarrow \textbf{217}$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)-\frac{1}{2}\int \frac{1-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)}{\sqrt{2+\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2-\cdots}}{\sqrt{2+\cdots}}\right)}{\sqrt{2+\sqrt{2}}}\cdots\right.}{2\sqrt{2}}\right.$$

$$\Big\downarrow \textbf{1478}$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{2+\sqrt{2}}-2e^x}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}+\frac{\int -\frac{\sqrt{2+\sqrt{2}}+2e^x}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{\int -\frac{\sqrt{\cdots}}{1-\sqrt{\cdots}}}{2\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \textbf{25}$$

$$2\left(\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2+\sqrt{2}}-2e^x}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}-\frac{\int \frac{\sqrt{2+\sqrt{2}}+2e^x}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)}{2\sqrt{2}}-\frac{\frac{1}{2}\left(-\frac{\int \frac{\sqrt{2\cdots}}{1-\sqrt{2\cdots}}}{2\cdots}\right.}{\cdots}\right.$$

$$\Big\downarrow \textbf{1103}$$

$$2\left(\frac{\frac{1}{2}\left(\frac{\arctan\left(\frac{2e^x-\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}}\right)}{\sqrt{2-\sqrt{2}}}\right)+\frac{1}{2}\left(\frac{\log\left(-\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2+\sqrt{2}}}-\frac{\log\left(\sqrt{2+\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2+\sqrt{2}}}\right)}{2\sqrt{2}}-\frac{\frac{1}{2}\left(\frac{\arctan\cdots}{}\right.}{}\right.$$

input
```
Int[E^x*Sech[4*x],x]
```

output
```
2*(-1/2*((ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 +
Sqrt[2]] + ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[2]]]/Sqrt[2 +
Sqrt[2]])/2 + (Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 - Sqrt[2
]]) - Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 - Sqrt[2]]))/2)/S
qrt[2] + ((ArcTan[(-Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/Sqrt[2 -
 Sqrt[2]] + ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt[2]]]/Sqrt[2 -
 Sqrt[2]])/2 + (Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 + Sqrt[
2]]) - Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)]/(2*Sqrt[2 + Sqrt[2]]))/2)/
(2*Sqrt[2]))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 828
```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{r = Numerator[R
t[a/b, 4]], s = Denominator[Rt[a/b, 4]]}, Simp[s^3/(2*Sqrt[2]*b*r)   Int[x^
(m - n/4)/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] - Simp[s^3/(2*S
qrt[2]*b*r)   Int[x^(m - n/4)/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x]
, x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 0] && IGtQ[m, 0] && LtQ[m, n - 1] &&
 GtQ[a/b, 0]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1447
```
Int[(x_)^2/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a/c, 2]}, Simp[1/2   Int[(q + x^2)/(a + b*x^2 + c*x^4), x], x] - Simp[1/2
 Int[(q - x^2)/(a + b*x^2 + c*x^4), x], x]] /; FreeQ[{a, b, c}, x] && LtQ[b
^2 - 4*a*c, 0] && PosQ[a*c]
```

rule 1475
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[2*(d/e) - b/c, 2]}, Simp[e/(2*c)   Int[1/Simp[d/e + q*x + x^
2, x], x], x] + Simp[e/(2*c)   Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; F
reeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ[c*d^2 - a*e^2, 0] &&
 (GtQ[2*(d/e) - b/c, 0] || ( !LtQ[2*(d/e) - b/c, 0] && EqQ[d - e*Rt[a/c, 2]
, 0]))
```

rule 1478
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[-2*(d/e) - b/c, 2]}, Simp[e/(2*c*q)   Int[(q - 2*x)/Simp[d/e
 + q*x - x^2, x], x], x] + Simp[e/(2*c*q)   Int[(q + 2*x)/Simp[d/e - q*x -
x^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && EqQ
[c*d^2 - a*e^2, 0] &&  !GtQ[b^2 - 4*a*c, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.76 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.09

method	result	size
risch	$2\left(\sum_{\_R=\mathrm{RootOf}\left(16777216\_Z^{8}+1\right)}\_R\ln\left(-32768\_R^{5}+\mathrm{e}^{x}\right)\right)$	25

input
```
int(exp(x)*sech(4*x),x,method=_RETURNVERBOSE)
```

output
```
2*sum(_R*ln(-32768*_R^5+exp(x)),_R=RootOf(16777216*_Z^8+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.58

$$\int e^x \operatorname{sech}(4x)\,dx = \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i+1)\sqrt{2}(-1)^{\frac{5}{8}}+2\cosh(x)+2\sinh(x)\right) - \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i-1)\sqrt{2}(-1)^{\frac{5}{8}}+2\cosh(x) + 2\sinh(x)\right) + \left(\frac{1}{8}i-\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left((i-1)\sqrt{2}(-1)^{\frac{5}{8}} + 2\cosh(x)+2\sinh(x)\right) - \left(\frac{1}{8}i+\frac{1}{8}\right)\sqrt{2}(-1)^{\frac{1}{8}}\log\left(-(i+1)\sqrt{2}(-1)^{\frac{5}{8}}+2\cosh(x) + 2\sinh(x)\right) - \frac{1}{4}(-1)^{\frac{1}{8}}\log\left((-1)^{\frac{5}{8}}+\cosh(x)+\sinh(x)\right) - \frac{1}{4}i(-1)^{\frac{1}{8}}\log\left(i(-1)^{\frac{5}{8}}+\cosh(x)+\sinh(x)\right) + \frac{1}{4}i(-1)^{\frac{1}{8}}\log\left(-i(-1)^{\frac{5}{8}}+\cosh(x)+\sinh(x)\right) + \frac{1}{4}(-1)^{\frac{1}{8}}\log\left(-(-1)^{\frac{5}{8}}+\cosh(x)+\sinh(x)\right)$$

input
```
integrate(exp(x)*sech(4*x),x, algorithm="fricas")
```

output
```
(1/8*I + 1/8)*sqrt(2)*(-1)^(1/8)*log((I + 1)*sqrt(2)*(-1)^(5/8) + 2*cosh(x
) + 2*sinh(x)) - (1/8*I - 1/8)*sqrt(2)*(-1)^(1/8)*log(-(I - 1)*sqrt(2)*(-1
)^(5/8) + 2*cosh(x) + 2*sinh(x)) + (1/8*I - 1/8)*sqrt(2)*(-1)^(1/8)*log((I
 - 1)*sqrt(2)*(-1)^(5/8) + 2*cosh(x) + 2*sinh(x)) - (1/8*I + 1/8)*sqrt(2)*
(-1)^(1/8)*log(-(I + 1)*sqrt(2)*(-1)^(5/8) + 2*cosh(x) + 2*sinh(x)) - 1/4*
(-1)^(1/8)*log((-1)^(5/8) + cosh(x) + sinh(x)) - 1/4*I*(-1)^(1/8)*log(I*(-
1)^(5/8) + cosh(x) + sinh(x)) + 1/4*I*(-1)^(1/8)*log(-I*(-1)^(5/8) + cosh(
x) + sinh(x)) + 1/4*(-1)^(1/8)*log(-(-1)^(5/8) + cosh(x) + sinh(x))
```

Sympy [F]

$$\int e^x \text{sech}(4x)\, dx = \int e^{x} \operatorname{sech}{\left(4x \right)}\, dx$$

input
```
integrate(exp(x)*sech(4*x),x)
```

output
```
Integral(exp(x)*sech(4*x), x)
```

Maxima [F]

$$\int e^x \text{sech}(4x)\, dx = \int e^{x} \operatorname{sech}\left(4\, x\right)\, dx$$

input
```
integrate(exp(x)*sech(4*x),x, algorithm="maxima")
```

output
```
integrate(e^x*sech(4*x), x)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 249, normalized size of antiderivative = 0.86

$$\begin{aligned}\int e^x \operatorname{sech}(4x)\,dx = {}& \frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) \\ & +\frac{1}{4}\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) \\ & -\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^x}{\sqrt{\sqrt{2}+2}}\right) \\ & -\frac{1}{4}\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^x}{\sqrt{\sqrt{2}+2}}\right) \\ & -\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) \\ & +\frac{1}{8}\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) \\ & +\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) \\ & -\frac{1}{8}\sqrt{\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right)\end{aligned}$$

input
```
integrate(exp(x)*sech(4*x),x, algorithm="giac")
```

output
```
1/4*sqrt(sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) + 2
)) + 1/4*sqrt(sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-sqrt(
2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/sqrt
(sqrt(2) + 2)) - 1/4*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) - 2*e^
x)/sqrt(sqrt(2) + 2)) - 1/8*sqrt(-sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e^x +
 e^(2*x) + 1) + 1/8*sqrt(-sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^(2*x
) + 1) + 1/8*sqrt(sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1) -
 1/8*sqrt(sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 5.84 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.66

$$\int e^x \text{sech}(4x)\,dx = \text{Too large to display}$$

input
```
int(exp(x)/cosh(4*x),x)
```

output
```
log(32768*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 +
512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(
x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3 - 512)*((2^(1/2)
 + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8) - log(32768*exp(x)*(((2^(1/2)
+ 2)^(1/2)*1i)/8 - (2 - 2^(1/2))^(1/2)/8)^3 - 512)*(((2^(1/2) + 2)^(1/2)*1
i)/8 - (2 - 2^(1/2))^(1/2)/8) + log(32768*exp(x)*(((2^(1/2) + 2)^(1/2)*1i)
/8 - (2 - 2^(1/2))^(1/2)/8)^3 + 512)*(((2^(1/2) + 2)^(1/2)*1i)/8 - (2 - 2^
(1/2))^(1/2)/8) + 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2
- 2^(1/2))^(1/2)*1i)/8)^3*(16384 - 16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 +
 ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2
^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 - 16384i) + 512
)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 + 1i/2) + 2^(1
/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*1i)/8
)^3*(16384 + 16384i) - 512)*((2^(1/2) + 2)^(1/2)/8 + ((2 - 2^(1/2))^(1/2)*
1i)/8)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*exp(x)*((2^(1/2) + 2)^(1/2)/8 +
((2 - 2^(1/2))^(1/2)*1i)/8)^3*(16384 + 16384i) + 512)*((2^(1/2) + 2)^(1/2)
/8 + ((2 - 2^(1/2))^(1/2)*1i)/8)*(1/2 - 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 469, normalized size of antiderivative = 1.62

$$\int e^{x}\operatorname{sech}(4x)\,dx = \frac{\sqrt{\sqrt{2}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}-2e^{x}}{\sqrt{\sqrt{2}+2}}\right)}{4} - \frac{\sqrt{\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}-2e^{x}}{\sqrt{\sqrt{2}+2}}\right)}{4} - \frac{\sqrt{\sqrt{2}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}+2e^{x}}{\sqrt{\sqrt{2}+2}}\right)}{4} + \frac{\sqrt{\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}+2e^{x}}{\sqrt{\sqrt{2}+2}}\right)}{4} - \frac{\sqrt{-\sqrt{2}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)}{4} - \frac{\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}-2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{2}+2}\,\sqrt{2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)}{4} + \frac{\sqrt{-\sqrt{2}+2}\,atan\left(\frac{\sqrt{\sqrt{2}+2}+2e^{x}}{\sqrt{-\sqrt{2}+2}}\right)}{4} - \frac{\sqrt{-\sqrt{2}+2}\,\sqrt{2}\log\left(-e^{x}\sqrt{-\sqrt{2}+2}+e^{2x}+1\right)}{8} + \frac{\sqrt{-\sqrt{2}+2}\,\sqrt{2}\log\left(e^{x}\sqrt{-\sqrt{2}+2}+e^{2x}+1\right)}{8} - \frac{\sqrt{-\sqrt{2}+2}\log\left(-e^{x}\sqrt{-\sqrt{2}+2}+e^{2x}+1\right)}{8} + \frac{\sqrt{-\sqrt{2}+2}\log\left(e^{x}\sqrt{-\sqrt{2}+2}+e^{2x}+1\right)}{8} + \frac{\sqrt{\sqrt{2}+2}\,\sqrt{2}\log\left(-e^{x}\sqrt{\sqrt{2}+2}+e^{2x}+1\right)}{8} - \frac{\sqrt{\sqrt{2}+2}\,\sqrt{2}\log\left(e^{x}\sqrt{\sqrt{2}+2}+e^{2x}+1\right)}{8} - \frac{\sqrt{\sqrt{2}+2}\log\left(-e^{x}\sqrt{\sqrt{2}+2}+e^{2x}+1\right)}{8} + \frac{\sqrt{\sqrt{2}+2}\log\left(e^{x}\sqrt{\sqrt{2}+2}+e^{2x}+1\right)}{8}$$

input
```
int(exp(x)*sech(4*x),x)
```

output
```
(2*sqrt(sqrt(2) + 2)*sqrt(2)*atan((sqrt( - sqrt(2) + 2) - 2*e**x)/sqrt(sqr
t(2) + 2)) - 2*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2) - 2*e**x)/sqrt
(sqrt(2) + 2)) - 2*sqrt(sqrt(2) + 2)*sqrt(2)*atan((sqrt( - sqrt(2) + 2) +
2*e**x)/sqrt(sqrt(2) + 2)) + 2*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2
) + 2*e**x)/sqrt(sqrt(2) + 2)) - 2*sqrt( - sqrt(2) + 2)*sqrt(2)*atan((sqrt
(sqrt(2) + 2) - 2*e**x)/sqrt( - sqrt(2) + 2)) - 2*sqrt( - sqrt(2) + 2)*ata
n((sqrt(sqrt(2) + 2) - 2*e**x)/sqrt( - sqrt(2) + 2)) + 2*sqrt( - sqrt(2) +
 2)*sqrt(2)*atan((sqrt(sqrt(2) + 2) + 2*e**x)/sqrt( - sqrt(2) + 2)) + 2*sq
rt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) + 2*e**x)/sqrt( - sqrt(2) + 2))
 - sqrt( - sqrt(2) + 2)*sqrt(2)*log( - e**x*sqrt( - sqrt(2) + 2) + e**(2*x
) + 1) + sqrt( - sqrt(2) + 2)*sqrt(2)*log(e**x*sqrt( - sqrt(2) + 2) + e**(
2*x) + 1) - sqrt( - sqrt(2) + 2)*log( - e**x*sqrt( - sqrt(2) + 2) + e**(2*
x) + 1) + sqrt( - sqrt(2) + 2)*log(e**x*sqrt( - sqrt(2) + 2) + e**(2*x) +
1) + sqrt(sqrt(2) + 2)*sqrt(2)*log( - e**x*sqrt(sqrt(2) + 2) + e**(2*x) +
1) - sqrt(sqrt(2) + 2)*sqrt(2)*log(e**x*sqrt(sqrt(2) + 2) + e**(2*x) + 1)
- sqrt(sqrt(2) + 2)*log( - e**x*sqrt(sqrt(2) + 2) + e**(2*x) + 1) + sqrt(s
qrt(2) + 2)*log(e**x*sqrt(sqrt(2) + 2) + e**(2*x) + 1))/8
```

3.285 $\int e^x \mathbf{sech}^2(4x)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 293

$$\begin{aligned}\int e^x \operatorname{sech}^2(4x)\,dx = &-\frac{e^x}{2(1+e^{8x})}-\frac{1}{16}\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}-2e^x}{\sqrt{2+\sqrt{2}}}\right)\\&-\frac{1}{16}\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}-2e^x}{\sqrt{2-\sqrt{2}}}\right)\\&+\frac{1}{16}\sqrt{2+\sqrt{2}}\arctan\left(\frac{\sqrt{2-\sqrt{2}}+2e^x}{\sqrt{2+\sqrt{2}}}\right)\\&+\frac{1}{16}\sqrt{2-\sqrt{2}}\arctan\left(\frac{\sqrt{2+\sqrt{2}}+2e^x}{\sqrt{2-\sqrt{2}}}\right)\\&+\frac{1}{16}\sqrt{2-\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2-\sqrt{2}}e^x}{1+e^{2x}}\right)\\&+\frac{1}{16}\sqrt{2+\sqrt{2}}\operatorname{arctanh}\left(\frac{\sqrt{2+\sqrt{2}}e^x}{1+e^{2x}}\right)\end{aligned}$$

output
```
-1/2*exp(x)/(1+exp(8*x))-1/16*(2+2^(1/2))^(1/2)*arctan(((2-2^(1/2))^(1/2)-
2*exp(x))/(2+2^(1/2))^(1/2))-1/16*(2-2^(1/2))^(1/2)*arctan(((2+2^(1/2))^(1
/2)-2*exp(x))/(2-2^(1/2))^(1/2))+1/16*(2+2^(1/2))^(1/2)*arctan(((2-2^(1/2)
)^(1/2)+2*exp(x))/(2+2^(1/2))^(1/2))+1/16*(2-2^(1/2))^(1/2)*arctan(((2+2^(
1/2))^(1/2)+2*exp(x))/(2-2^(1/2))^(1/2))+1/16*(2-2^(1/2))^(1/2)*arctanh((2
-2^(1/2))^(1/2)*exp(x)/(1+exp(2*x)))+1/16*(2+2^(1/2))^(1/2)*arctanh((2+2^(
1/2))^(1/2)*exp(x)/(1+exp(2*x)))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.12

$$\int e^x \operatorname{sech}^2(4x)\,dx = \frac{1}{2}e^x\left(-\frac{1}{1+e^{8x}} + \text{Hypergeometric2F1}\left(\frac{1}{8},1,\frac{9}{8},-e^{8x}\right)\right)$$

input
```
Integrate[E^x*Sech[4*x]^2,x]
```

output
```
(E^x*(-(1 + E^(8*x))^(-1) + Hypergeometric2F1[1/8, 1, 9/8, -E^(8*x)]))/2
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.33, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1.000, Rules used = {2720, 27, 817, 757, 1483, 1142, 25, 1083, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \operatorname{sech}^2(4x)\,dx$$

$$\Big\downarrow \; 2720$$

$$\int \frac{4e^{8x}}{(e^{8x}+1)^2}de^x$$

$$\Big\downarrow \; 27$$

$$4\int \frac{e^{8x}}{(1+e^{8x})^2}de^x$$

$$\Big\downarrow \; 817$$

$$4\left(\frac{1}{8}\int \frac{1}{1+e^{8x}}de^x-\frac{e^x}{8(e^{8x}+1)}\right)$$

$$\Big\downarrow \; 757$$

$$4\left(\frac{1}{8}\left(\frac{\int \frac{\sqrt{2}-e^{2x}}{1-\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}+\frac{\int \frac{\sqrt{2}+e^{2x}}{1+\sqrt{2}e^{2x}+e^{4x}}de^x}{2\sqrt{2}}\right)-\frac{e^x}{8(e^{8x}+1)}\right)$$

$$\Big\downarrow \; 1483$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}+\left(1-\sqrt{2}\right)e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2-\sqrt{2}\right)}-\left(1-\sqrt{2}\right)e^x}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}-\left(1+\sqrt{2}\right)e^x}{1-\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}+\frac{\int \frac{\sqrt{2\left(2+\sqrt{2}\right)}+\left(1+\sqrt{2}\right)e^x}{1+\sqrt{2+\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2+\sqrt{2}}}}{2\sqrt{2}}\right)-\frac{}{8}\right.$$

$$\Big\downarrow \; 1142$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x+\frac{1}{2}\left(1-\sqrt{2}\right)\int -\frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}-}}{1+\sqrt{2-\sqrt{2}}e}}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}\right.\right.$$

$$\Big\downarrow \; 25$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\frac{1}{2}\sqrt{2+\sqrt{2}}\int \frac{1}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}+2}{1+\sqrt{2-\sqrt{2}}e^x}}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}\right.\right.$$

$$\Big\downarrow \; 1083$$

$$4\left(\frac{1}{8}\left(\frac{\frac{-\sqrt{2+\sqrt{2}}\int \frac{1}{-2-\sqrt{2}-e^{2x}}d\left(-\sqrt{2-\sqrt{2}}+2e^x\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int \frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{-\sqrt{2+\sqrt{2}}\int \frac{1}{-2-\sqrt{2}-e^{2x}}d\left(\sqrt{2-\sqrt{2}}+2e^x\right)-\frac{1}{2}\left(1-\right.}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}\right.\right.$$

$$\Big\downarrow \ 217$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\arctan\left(\frac{2e^x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int\frac{\sqrt{2-\sqrt{2}}-2e^x}{1-\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\int\frac{\sqrt{2-\sqrt{2}}+2e^x}{1+\sqrt{2-\sqrt{2}}e^x+e^{2x}}de^x}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\frac{1}{2}\left(1+\sqrt{\right.}}{}\right.\right.$$

$$\Big\downarrow \ 1103$$

$$4\left(\frac{1}{8}\left(\frac{\frac{\arctan\left(\frac{2e^x-\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)+\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(-\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2-\sqrt{2}}}+\frac{\arctan\left(\frac{2e^x+\sqrt{2-\sqrt{2}}}{\sqrt{2+\sqrt{2}}}\right)-\frac{1}{2}\left(1-\sqrt{2}\right)\log\left(\sqrt{2-\sqrt{2}}e^x+e^{2x}+1\right)}{2\sqrt{2-\sqrt{2}}}}{2\sqrt{2}}+\frac{\text{arc}}{}\right.\right.$$

input
```
Int[E^x*Sech[4*x]^2,x]
```

output
```
4*(-1/8*E^x/(1 + E^(8*x)) + (((ArcTan[(-Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2
+ Sqrt[2]]] + ((1 - Sqrt[2])*Log[1 - Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/2)/
(2*Sqrt[2 - Sqrt[2]]) + (ArcTan[(Sqrt[2 - Sqrt[2]] + 2*E^x)/Sqrt[2 + Sqrt[
2]]] - ((1 - Sqrt[2])*Log[1 + Sqrt[2 - Sqrt[2]]*E^x + E^(2*x)])/2)/(2*Sqrt
[2 - Sqrt[2]]))/(2*Sqrt[2]) + ((ArcTan[(-Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2
 - Sqrt[2]]] - ((1 + Sqrt[2])*Log[1 - Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/2)
/(2*Sqrt[2 + Sqrt[2]]) + (ArcTan[(Sqrt[2 + Sqrt[2]] + 2*E^x)/Sqrt[2 - Sqrt
[2]]] + ((1 + Sqrt[2])*Log[1 + Sqrt[2 + Sqrt[2]]*E^x + E^(2*x)])/2)/(2*Sqr
t[2 + Sqrt[2]]))/(2*Sqrt[2]))/8)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 757
```
Int[((a_) + (b_.)*(x_)^(n_))^(-1), x_Symbol] :> With[{r = Numerator[Rt[a/b,
 4]], s = Denominator[Rt[a/b, 4]]}, Simp[r/(2*Sqrt[2]*a)   Int[(Sqrt[2]*r -
 s*x^(n/4))/(r^2 - Sqrt[2]*r*s*x^(n/4) + s^2*x^(n/2)), x], x] + Simp[r/(2*S
qrt[2]*a)   Int[(Sqrt[2]*r + s*x^(n/4))/(r^2 + Sqrt[2]*r*s*x^(n/4) + s^2*x^
(n/2)), x], x]] /; FreeQ[{a, b}, x] && IGtQ[n/4, 1] && GtQ[a/b, 0]
```

rule 817
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[c^(
n - 1)*(c*x)^(m - n + 1)*((a + b*x^n)^(p + 1)/(b*n*(p + 1))), x] - Simp[c^n
*((m - n + 1)/(b*n*(p + 1)))   Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x
] /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !
ILtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

rule 1083
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Simp[-2   Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 1483
```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Simp[1/(2*c*q*r)   In
t[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Simp[1/(2*c*q*r)   Int[(d*r
 + (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.36 (sec) , antiderivative size = 36, normalized size of antiderivative = 0.12

method	result
risch	$-\frac{e^{x}}{2(1+e^{8x})}+4\left(\sum_{\_R=\mathrm{RootOf}(281474976710656\_Z^{8}+1)}\_R\ln\left(e^{x}+64\_R\right)\right)$
default	$\frac{-\frac{3\tanh\left(\frac{x}{2}\right)^{7}}{2}-\frac{7\tanh\left(\frac{x}{2}\right)^{6}}{2}-\frac{7\tanh\left(\frac{x}{2}\right)^{5}}{2}-\frac{35\tanh\left(\frac{x}{2}\right)^{4}}{2}+\frac{7\tanh\left(\frac{x}{2}\right)^{3}}{2}-\frac{21\tanh\left(\frac{x}{2}\right)^{2}}{2}+\frac{3\tanh\left(\frac{x}{2}\right)}{2}-\frac{1}{2}}{\tanh\left(\frac{x}{2}\right)^{8}+28\tanh\left(\frac{x}{2}\right)^{6}+70\tanh\left(\frac{x}{2}\right)^{4}+28\tanh\left(\frac{x}{2}\right)^{2}+1}+\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+28\_Z^{6}+\right.}\right.}{}$

input
```
int(exp(x)*sech(4*x)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*exp(x)/(1+exp(8*x))+4*sum(_R*ln(exp(x)+64*_R),_R=RootOf(2814749767106
56*_Z^8+1))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1215, normalized size of antiderivative = 4.15

$$\int e^{x}\mathrm{sech}^{2}(4x)\,dx=\text{Too large to display}$$

input
```
integrate(exp(x)*sech(4*x)^2,x, algorithm="fricas")
```

output
```
1/32*(((I + 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 + (8*I + 8)*sqrt(2)*(-1)^(1/8)
*cosh(x)^7*sinh(x) + (28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*sinh(x)^2 +
(56*I + 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 + (70*I + 70)*sqrt(2)*(
-1)^(1/8)*cosh(x)^4*sinh(x)^4 + (56*I + 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^3*s
inh(x)^5 + (28*I + 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 + (8*I + 8)*
sqrt(2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 + (I + 1)*sqrt(2)*(-1)^(1/8)*sinh(x)^
8 + (I + 1)*sqrt(2)*(-1)^(1/8))*log((I + 1)*sqrt(2)*(-1)^(1/8) + 2*cosh(x)
 + 2*sinh(x)) + (-(I - 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 - (8*I - 8)*sqrt(2)
*(-1)^(1/8)*cosh(x)^7*sinh(x) - (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^6*s
inh(x)^2 - (56*I - 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 - (70*I - 70
)*sqrt(2)*(-1)^(1/8)*cosh(x)^4*sinh(x)^4 - (56*I - 56)*sqrt(2)*(-1)^(1/8)*
cosh(x)^3*sinh(x)^5 - (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*sinh(x)^6 -
 (8*I - 8)*sqrt(2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 - (I - 1)*sqrt(2)*(-1)^(1/
8)*sinh(x)^8 - (I - 1)*sqrt(2)*(-1)^(1/8))*log(-(I - 1)*sqrt(2)*(-1)^(1/8)
 + 2*cosh(x) + 2*sinh(x)) + ((I - 1)*sqrt(2)*(-1)^(1/8)*cosh(x)^8 + (8*I -
 8)*sqrt(2)*(-1)^(1/8)*cosh(x)^7*sinh(x) + (28*I - 28)*sqrt(2)*(-1)^(1/8)*
cosh(x)^6*sinh(x)^2 + (56*I - 56)*sqrt(2)*(-1)^(1/8)*cosh(x)^5*sinh(x)^3 +
 (70*I - 70)*sqrt(2)*(-1)^(1/8)*cosh(x)^4*sinh(x)^4 + (56*I - 56)*sqrt(2)*
(-1)^(1/8)*cosh(x)^3*sinh(x)^5 + (28*I - 28)*sqrt(2)*(-1)^(1/8)*cosh(x)^2*
sinh(x)^6 + (8*I - 8)*sqrt(2)*(-1)^(1/8)*cosh(x)*sinh(x)^7 + (I - 1)*sq...
```

Sympy [F]

$$\int e^x \text{sech}^2(4x)\,dx = \int e^x \operatorname{sech}^2(4x)\,dx$$

input
```
integrate(exp(x)*sech(4*x)**2,x)
```

output
```
Integral(exp(x)*sech(4*x)**2, x)
```

Maxima [F]

$$\int e^x \operatorname{sech}^2(4x)\,dx = \int e^x \operatorname{sech}\left(4\,x\right)^2\,dx$$

input
```
integrate(exp(x)*sech(4*x)^2,x, algorithm="maxima")
```

output
```
-1/2*e^x/(e^(8*x) + 1) + 4*integrate(1/8*e^x/(e^(8*x) + 1), x)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.89

$$\begin{aligned}\int e^x \operatorname{sech}^2(4x)\,dx = {} & \frac{1}{16}\,\sqrt{-\sqrt{2}+2}\arctan\left(\frac{\sqrt{\sqrt{2}+2}+2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) \\ & + \frac{1}{16}\,\sqrt{-\sqrt{2}+2}\arctan\left(-\frac{\sqrt{\sqrt{2}+2}-2\,e^x}{\sqrt{-\sqrt{2}+2}}\right) \\ & + \frac{1}{16}\,\sqrt{\sqrt{2}+2}\arctan\left(\frac{\sqrt{-\sqrt{2}+2}+2\,e^x}{\sqrt{\sqrt{2}+2}}\right) \\ & + \frac{1}{16}\,\sqrt{\sqrt{2}+2}\arctan\left(-\frac{\sqrt{-\sqrt{2}+2}-2\,e^x}{\sqrt{\sqrt{2}+2}}\right) \\ & + \frac{1}{32}\,\sqrt{\sqrt{2}+2}\log\left(\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) \\ & - \frac{1}{32}\,\sqrt{\sqrt{2}+2}\log\left(-\sqrt{\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) \\ & + \frac{1}{32}\,\sqrt{-\sqrt{2}+2}\log\left(\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) \\ & - \frac{1}{32}\,\sqrt{-\sqrt{2}+2}\log\left(-\sqrt{-\sqrt{2}+2}e^x+e^{(2\,x)}+1\right) - \frac{e^x}{2\,\left(e^{(8\,x)}+1\right)}\end{aligned}$$

input
```
integrate(exp(x)*sech(4*x)^2,x, algorithm="giac")
```

output
```
1/16*sqrt(-sqrt(2) + 2)*arctan((sqrt(sqrt(2) + 2) + 2*e^x)/sqrt(-sqrt(2) +
 2)) + 1/16*sqrt(-sqrt(2) + 2)*arctan(-(sqrt(sqrt(2) + 2) - 2*e^x)/sqrt(-s
qrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan((sqrt(-sqrt(2) + 2) + 2*e^x)/
sqrt(sqrt(2) + 2)) + 1/16*sqrt(sqrt(2) + 2)*arctan(-(sqrt(-sqrt(2) + 2) -
2*e^x)/sqrt(sqrt(2) + 2)) + 1/32*sqrt(sqrt(2) + 2)*log(sqrt(sqrt(2) + 2)*e
^x + e^(2*x) + 1) - 1/32*sqrt(sqrt(2) + 2)*log(-sqrt(sqrt(2) + 2)*e^x + e^
(2*x) + 1) + 1/32*sqrt(-sqrt(2) + 2)*log(sqrt(-sqrt(2) + 2)*e^x + e^(2*x)
+ 1) - 1/32*sqrt(-sqrt(2) + 2)*log(-sqrt(-sqrt(2) + 2)*e^x + e^(2*x) + 1)
- 1/2*e^x/(e^(8*x) + 1)
```

Mupad [B] (verification not implemented)

Time = 4.75 (sec) , antiderivative size = 473, normalized size of antiderivative = 1.61

$$\int e^x \operatorname{sech}^2(4x)\,dx = \text{Too large to display}$$

input
```
int(exp(x)/cosh(4*x)^2,x)
```

output
```
log(- exp(x)/2 - (2^(1/2) + 2)^(1/2)/4 - ((2 - 2^(1/2))^(1/2)*1i)/4)*((2^(
1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) - exp(x)/(2*(exp(8*x) +
1)) - log((2^(1/2) + 2)^(1/2)/4 - exp(x)/2 + ((2 - 2^(1/2))^(1/2)*1i)/4)*(
(2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32) + log((2 - 2^(1/2))^
(1/2)/4 - ((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2)*(((2^(1/2) + 2)^(1/2)*1i)
/32 - (2 - 2^(1/2))^(1/2)/32) - log(((2^(1/2) + 2)^(1/2)*1i)/4 - exp(x)/2
- (2 - 2^(1/2))^(1/2)/4)*(((2^(1/2) + 2)^(1/2)*1i)/32 - (2 - 2^(1/2))^(1/2
)/32) + 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2
^(1/2))^(1/2)*1i)/32)*(4 + 4i))*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(
1/2)*1i)/32)*(1/2 + 1i/2) + 2^(1/2)*log(- exp(x)/2 - 2^(1/2)*((2^(1/2) + 2
)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i))*((2^(1/2) + 2)^(1/2)/3
2 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*log(2^(1/2)*((2^(1
/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 - 4i) - exp(x)/2)*((2^
(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2 - 1i/2) - 2^(1/2)*
log(2^(1/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(4 + 4i
) - exp(x)/2)*((2^(1/2) + 2)^(1/2)/32 + ((2 - 2^(1/2))^(1/2)*1i)/32)*(1/2
+ 1i/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.73

$$\int e^x \text{sech}^2(4x)\,dx$$
$$= \frac{-2e^{8x}\sqrt{\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}-2e^x}{\sqrt{\sqrt{2}+2}}\right)-2\sqrt{\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}-2e^x}{\sqrt{\sqrt{2}+2}}\right)+2e^{8x}\sqrt{\sqrt{2}+2}\,atan\left(\frac{\sqrt{-\sqrt{2}+2}+2e^x}{\sqrt{\sqrt{2}+2}}\right.}{}$$

input
```
int(exp(x)*sech(4*x)^2,x)
```

output
```
( - 2*e**(8*x)*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2) - 2*e**x)/sqrt
(sqrt(2) + 2)) - 2*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2) + 2) - 2*e**x)/
sqrt(sqrt(2) + 2)) + 2*e**(8*x)*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2) +
2) + 2*e**x)/sqrt(sqrt(2) + 2)) + 2*sqrt(sqrt(2) + 2)*atan((sqrt( - sqrt(2
) + 2) + 2*e**x)/sqrt(sqrt(2) + 2)) - 2*e**(8*x)*sqrt( - sqrt(2) + 2)*atan
((sqrt(sqrt(2) + 2) - 2*e**x)/sqrt( - sqrt(2) + 2)) - 2*sqrt( - sqrt(2) +
2)*atan((sqrt(sqrt(2) + 2) - 2*e**x)/sqrt( - sqrt(2) + 2)) + 2*e**(8*x)*sq
rt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) + 2*e**x)/sqrt( - sqrt(2) + 2))
 + 2*sqrt( - sqrt(2) + 2)*atan((sqrt(sqrt(2) + 2) + 2*e**x)/sqrt( - sqrt(2
) + 2)) - e**(8*x)*sqrt( - sqrt(2) + 2)*log( - e**x*sqrt( - sqrt(2) + 2) +
 e**(2*x) + 1) + e**(8*x)*sqrt( - sqrt(2) + 2)*log(e**x*sqrt( - sqrt(2) +
2) + e**(2*x) + 1) - sqrt( - sqrt(2) + 2)*log( - e**x*sqrt( - sqrt(2) + 2)
 + e**(2*x) + 1) + sqrt( - sqrt(2) + 2)*log(e**x*sqrt( - sqrt(2) + 2) + e*
*(2*x) + 1) - e**(8*x)*sqrt(sqrt(2) + 2)*log( - e**x*sqrt(sqrt(2) + 2) + e
**(2*x) + 1) + e**(8*x)*sqrt(sqrt(2) + 2)*log(e**x*sqrt(sqrt(2) + 2) + e**
(2*x) + 1) - sqrt(sqrt(2) + 2)*log( - e**x*sqrt(sqrt(2) + 2) + e**(2*x) +
1) + sqrt(sqrt(2) + 2)*log(e**x*sqrt(sqrt(2) + 2) + e**(2*x) + 1) - 16*e**
x)/(32*(e**(8*x) + 1))
```

3.286 $\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 202

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh^3(d+ex)\log(F)}{9e^2-b^2c^2\log^2(F)} - \frac{6bce^2F^{c(a+bx)}\cosh(d+ex)\log(F)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)} + \frac{3eF^{c(a+bx)}\cosh^2(d+ex)\sinh(d+ex)}{9e^2-b^2c^2\log^2(F)} + \frac{6e^3F^{c(a+bx)}\sinh(d+ex)}{9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\log^4(F)}$$

output
```
-b*c*F^(c*(b*x+a))*cosh(e*x+d)^3*ln(F)/(9*e^2-b^2*c^2*ln(F)^2)-6*b*c*e^2*F
^(c*(b*x+a))*cosh(e*x+d)*ln(F)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)
^4)+3*e*F^(c*(b*x+a))*cosh(e*x+d)^2*sinh(e*x+d)/(9*e^2-b^2*c^2*ln(F)^2)+6*
e^3*F^(c*(b*x+a))*sinh(e*x+d)/(9*e^4-10*b^2*c^2*e^2*ln(F)^2+b^4*c^4*ln(F)^
4)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 159, normalized size of antiderivative = 0.79

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx$$
$$=\frac{F^{c(a+bx)}\left(3\cosh(d+ex)\left(-9bce^2\log(F)+b^3c^3\log^3(F)\right)+\cosh(3(d+ex))\left(-bce^2\log(F)+b^3c^3\log^3(F)\right.\right.}{4\left(9e^4-10b^2c^2e^2\log^2(F)+b^4c^4\right.}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(3*Cosh[d + e*x]*(-9*b*c*e^2*Log[F] + b^3*c^3*Log[F]^3) +
 Cosh[3*(d + e*x)]*(-(b*c*e^2*Log[F]) + b^3*c^3*Log[F]^3) + 6*e*(5*e^2 - b
^2*c^2*Log[F]^2 + Cosh[2*(d + e*x)]*(e^2 - b^2*c^2*Log[F]^2))*Sinh[d + e*x
]))/(4*(9*e^4 - 10*b^2*c^2*e^2*Log[F]^2 + b^4*c^4*Log[F]^4))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.94, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}=0.111$, Rules used = {6000, 5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(d+ex)F^{c(a+bx)}\,dx$$
$$\Big\downarrow \ 6000$$
$$\frac{6e^2\int F^{c(a+bx)}\cosh(d+ex)dx}{9e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}$$
$$\Big\downarrow \ 5998$$

$$-\frac{bc\log(F)\cosh^3(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{3e\sinh(d+ex)\cosh^2(d+ex)F^{c(a+bx)}}{9e^2-b^2c^2\log^2(F)}+\frac{6e^2\left(\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}\right)}{9e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^3,x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]^3*Log[F])/(9*e^2 - b^2*c^2*Log[F]^2))
 + (3*e*F^(c*(a + b*x))*Cosh[d + e*x]^2*Sinh[d + e*x])/(9*e^2 - b^2*c^2*Lo
g[F]^2) + (6*e^2*(-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*
c^2*Log[F]^2)) + (e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2
)))/(9*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 2.32 (sec) , antiderivative size = 148, normalized size of antiderivative = 0.73

method	result
parallelrisch	$\frac{F^{c(bx+a)}\left(\left(\ln(F)^{3}b^{3}c^{3}-\ln(F)bc\,e^{2}\right)\cosh(3ex+3d)+\left(-3\ln(F)^{2}b^{2}c^{2}e+3e^{3}\right)\sinh(3ex+3d)-3(bc\ln(F)-3e)(bc\ln(F)+3e)(-}{36e^{4}-40b^{2}c^{2}e^{2}\ln(F)^{2}+4b^{4}c^{4}\ln(F)^{4}}$
risch	$\frac{\left(\ln(F)^{3}b^{3}c^{3}\mathrm{e}^{6ex+6d}+3\ln(F)^{3}b^{3}c^{3}\mathrm{e}^{4ex+4d}-3\ln(F)^{2}b^{2}c^{2}e\,\mathrm{e}^{6ex+6d}+3\ln(F)^{3}b^{3}c^{3}\mathrm{e}^{2ex+2d}-3\ln(F)^{2}b^{2}c^{2}e\,\mathrm{e}^{4ex+4d}-\ln(F)bc}{}$
orering	$\frac{4\ln(F)bc\left(b^{2}c^{2}\ln(F)^{2}-5e^{2}\right)F^{c(bx+a)}\cosh(ex+d)^{3}}{9e^{4}-10b^{2}c^{2}e^{2}\ln(F)^{2}+b^{4}c^{4}\ln(F)^{4}}-\frac{2\left(3b^{2}c^{2}\ln(F)^{2}-5e^{2}\right)\left(F^{c(bx+a)}bc\ln(F)\cosh(ex+d)^{3}+3F^{c(bx+a)}c\right.}{9e^{4}-10b^{2}c^{2}e^{2}\ln(F)^{2}+b^{4}c^{4}\ln(F)^{4}}$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
F^(c*(b*x+a))*((ln(F)^3*b^3*c^3-ln(F)*b*c*e^2)*cosh(3*e*x+3*d)+(-3*ln(F)^2
*b^2*c^2*e+3*e^3)*sinh(3*e*x+3*d)-3*(b*c*ln(F)-3*e)*(b*c*ln(F)+3*e)*(-ln(F
)*cosh(e*x+d)*b*c+e*sinh(e*x+d)))/(36*e^4-40*b^2*c^2*e^2*ln(F)^2+4*b^4*c^4
*ln(F)^4)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(199) = 398$.

Time = 0.17 (sec) , antiderivative size = 2218, normalized size of antiderivative = 10.98

$$\int F^{c(a+bx)}\cosh^{3}(d+ex)\,dx=\text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
1/8*((3*e^3*cosh(e*x + d)^6 + 27*e^3*cosh(e*x + d)^4 + (b^3*c^3*log(F)^3 -
 3*b^2*c^2*e*log(F)^2 - b*c*e^2*log(F) + 3*e^3)*sinh(e*x + d)^6 + 6*(b^3*c
^3*cosh(e*x + d)*log(F)^3 - 3*b^2*c^2*e*cosh(e*x + d)*log(F)^2 - b*c*e^2*c
osh(e*x + d)*log(F) + 3*e^3*cosh(e*x + d))*sinh(e*x + d)^5 - 27*e^3*cosh(e
*x + d)^2 + 3*(15*e^3*cosh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^2 + b^3*c
^3)*log(F)^3 + 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^2 + b^2*c^2*e)*log(F)^2
 - (5*b*c*e^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^4 + (b^3*
c^3*cosh(e*x + d)^6 + 3*b^3*c^3*cosh(e*x + d)^4 + 3*b^3*c^3*cosh(e*x + d)^
2 + b^3*c^3)*log(F)^3 + 4*(15*e^3*cosh(e*x + d)^3 + 27*e^3*cosh(e*x + d) +
 (5*b^3*c^3*cosh(e*x + d)^3 + 3*b^3*c^3*cosh(e*x + d))*log(F)^3 - 3*(5*b^2
*c^2*e*cosh(e*x + d)^3 + b^2*c^2*e*cosh(e*x + d))*log(F)^2 - (5*b*c*e^2*co
sh(e*x + d)^3 + 27*b*c*e^2*cosh(e*x + d))*log(F))*sinh(e*x + d)^3 - 3*e^3
- 3*(b^2*c^2*e*cosh(e*x + d)^6 + b^2*c^2*e*cosh(e*x + d)^4 - b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 + 3*(15*e^3*cosh(e*x + d)^4 + 54*e^3*co
sh(e*x + d)^2 + (5*b^3*c^3*cosh(e*x + d)^4 + 6*b^3*c^3*cosh(e*x + d)^2 + b
^3*c^3)*log(F)^3 - 9*e^3 - (15*b^2*c^2*e*cosh(e*x + d)^4 + 6*b^2*c^2*e*cos
h(e*x + d)^2 - b^2*c^2*e)*log(F)^2 - (5*b*c*e^2*cosh(e*x + d)^4 + 54*b*c*e
^2*cosh(e*x + d)^2 + 9*b*c*e^2)*log(F))*sinh(e*x + d)^2 - (b*c*e^2*cosh(e*
x + d)^6 + 27*b*c*e^2*cosh(e*x + d)^4 + 27*b*c*e^2*cosh(e*x + d)^2 + b*c*e
^2)*log(F) + 6*(3*e^3*cosh(e*x + d)^5 + 18*e^3*cosh(e*x + d)^3 - 9*e^3*...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. $2(199) = 398$.

Time = 3.31 (sec) , antiderivative size = 1610, normalized size of antiderivative = 7.97

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**3,x)
```

output
```
Piecewise((x*cosh(d)**3, Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d)**3, Eq(
b, 0) & Eq(e, 0)), (x*cosh(d)**3, Eq(c, 0) & Eq(e, 0)), (3*F**(a*c + b*c*x
)*x*sinh(b*c*x*log(F) - d)**3/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)**2*cosh(b*c*x*log(F) - d)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F) -
 d)*cosh(b*c*x*log(F) - d)**2/8 + 3*F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) -
 d)**3/8 - 5*F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**3/(8*b*c*log(F)) + F
**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)**2*cosh(b*c*x*log(F) - d)/(4*b*c*lo
g(F)) + F**(a*c + b*c*x)*sinh(b*c*x*log(F) - d)*cosh(b*c*x*log(F) - d)**2/
(b*c*log(F)) - 3*F**(a*c + b*c*x)*cosh(b*c*x*log(F) - d)**3/(8*b*c*log(F))
, Eq(e, -b*c*log(F))), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**3/8
+ 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d
)/8 - 3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 -
d)**2/8 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/3 - d)**3/8 + 11*F**(a*c +
b*c*x)*sinh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)) - 15*F**(a*c + b*c*x)*si
nh(b*c*x*log(F)/3 - d)**2*cosh(b*c*x*log(F)/3 - d)/(4*b*c*log(F)) + 3*F**(
a*c + b*c*x)*sinh(b*c*x*log(F)/3 - d)*cosh(b*c*x*log(F)/3 - d)**2/(b*c*log
(F)) - F**(a*c + b*c*x)*cosh(b*c*x*log(F)/3 - d)**3/(8*b*c*log(F)), Eq(e,
-b*c*log(F)/3)), (-F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**3/8 + 3*F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)**2*cosh(b*c*x*log(F)/3 + d)/8 -
3*F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/3 + d)*cosh(b*c*x*log(F)/3 + d)*...
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.66

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+3\,ex+3\,d)}}{8\,(bc\log(F)+3\,e)} + \frac{3\,F^{ac}e^{(bcx\log(F)+ex+d)}}{8\,(bc\log(F)+e)} + \frac{3\,F^{ac}e^{(bcx\log(F)-ex)}}{8\,(bce^{d}\log(F)-ee^{d})} + \frac{F^{ac}e^{(bcx\log(F)-3\,ex)}}{8\,(bce^{(3\,d)}\log(F)-3\,ee^{(3\,d)})}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/8*F^(a*c)*e^(b*c*x*log(F) + 3*e*x + 3*d)/(b*c*log(F) + 3*e) + 3/8*F^(a*c
)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 3/8*F^(a*c)*e^(b*c*x*log(F
) - e*x)/(b*c*e^d*log(F) - e*e^d) + 1/8*F^(a*c)*e^(b*c*x*log(F) - 3*e*x)/(
b*c*e^(3*d)*log(F) - 3*e*e^(3*d))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 1211, normalized size of antiderivative = 6.00

$$\int F^{c(a+bx)}\cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
1/4*(2*(b*c*log(abs(F)) + 3*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1
/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(ab
s(F)) + 3*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*
pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4
*(b*c*log(abs(F)) + 3*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*
x + 3*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*s
gn(F) - 1/2*I*pi*a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F))
 + 48*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F))
+ 48*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 3*e)*x + 3*d) + 3/4*(2*(b
*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*s
gn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) + e)^
2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/
2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs
(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3*I*(I*e
^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*
a*c)/(8*I*pi*b*c*sgn(F) - 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e) - I*e^(-
1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*
c)/(-8*I*pi*b*c*sgn(F) + 8*I*pi*b*c + 16*b*c*log(abs(F)) + 16*e))*e^(a*c*l
og(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 3/4*(2*(b*c*log(abs(F)) - e...
```

Mupad [B] (verification not implemented)

Time = 3.00 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx$$
$$= \frac{F^{a\,c+b\,c\,x}\,\left(6\,e^3\,\sinh(d+e\,x)+3\,e^3\,\cosh(d+e\,x)^2\,\sinh(d+e\,x)+b^3\,c^3\,\cosh(d+e\,x)^3\,\ln\left(F\right)^3-b\,c\,e^2\,\cosh}{b^4\,c^4\,\ln\left(F\right)^4-10\,b^2\,c^2\,e^2}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^3,x)
```

output
```
(F^(a*c + b*c*x)*(6*e^3*sinh(d + e*x) + 3*e^3*cosh(d + e*x)^2*sinh(d + e*x
) + b^3*c^3*cosh(d + e*x)^3*log(F)^3 - b*c*e^2*cosh(d + e*x)^3*log(F) - 6*
b*c*e^2*cosh(d + e*x)*log(F) - 3*b^2*c^2*e*cosh(d + e*x)^2*sinh(d + e*x)*l
og(F)^2))/(9*e^4 + b^4*c^4*log(F)^4 - 10*b^2*c^2*e^2*log(F)^2)
```

Reduce [F]

$$\int F^{c(a+bx)} \cosh^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh\left(ex+d\right)^3 dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**3,x)
```

3.287 $\int F^{c(a+bx)}\cosh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 132

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx=\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}-\frac{bcF^{c(a+bx)}\cosh^2(d+ex)\log(F)}{4e^2-b^2c^2\log^2(F)}+\frac{2eF^{c(a+bx)}\cosh(d+ex)\sinh(d+ex)}{4e^2-b^2c^2\log^2(F)}$$

output
```
2*e^2*F^(c*(b*x+a))/b/c/ln(F)/(4*e^2-b^2*c^2*ln(F)^2)-b*c*F^(c*(b*x+a))*co
sh(e*x+d)^2*ln(F)/(4*e^2-b^2*c^2*ln(F)^2)+2*e*F^(c*(b*x+a))*cosh(e*x+d)*si
nh(e*x+d)/(4*e^2-b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.64

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx=\frac{F^{c(a+bx)}\left(-4e^2+b^2c^2\log^2(F)+b^2c^2\cosh(2(d+ex))\log^2(F)-2bce\log(F)\sinh(2(d+ex))\right)}{-8bce^2\log(F)+2b^3c^3\log^3(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(F^(c*(a + b*x))*(-4*e^2 + b^2*c^2*Log[F]^2 + b^2*c^2*Cosh[2*(d + e*x)]*Lo
g[F]^2 - 2*b*c*e*Log[F]*Sinh[2*(d + e*x)]))/(-8*b*c*e^2*Log[F] + 2*b^3*c^3
*Log[F]^3)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6000, 2624}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6000}$$

$$\frac{2e^2\int F^{c(a+bx)}dx}{4e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}$$

$$\Big\downarrow \text{2624}$$

$$-\frac{bc\log(F)\cosh^2(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e\sinh(d+ex)\cosh(d+ex)F^{c(a+bx)}}{4e^2-b^2c^2\log^2(F)}+\frac{2e^2F^{c(a+bx)}}{bc\log(F)\left(4e^2-b^2c^2\log^2(F)\right)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x]^2,x]
```

output
```
(2*e^2*F^(c*(a + b*x)))/(b*c*Log[F]*(4*e^2 - b^2*c^2*Log[F]^2)) - (b*c*F^(
c*(a + b*x))*Cosh[d + e*x]^2*Log[F])/(4*e^2 - b^2*c^2*Log[F]^2) + (2*e*F^(
c*(a + b*x))*Cosh[d + e*x]*Sinh[d + e*x])/(4*e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 2624
```
Int[((F_)^(v_))^(n_.), x_Symbol] :> Simp[(F^v)^n/(n*Log[F]*D[v, x]), x] /;
FreeQ[{F, n}, x] && LinearQ[v, x]
```

rule 6000
```
Int[Cosh[(d_.) + (e_.)*(x_)]^(n_)*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symb
ol] :> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]^n/(e^2*n^2 - b^2*c
^2*Log[F]^2)), x] + (Simp[e*n*F^(c*(a + b*x))*Sinh[d + e*x]*(Cosh[d + e*x]^
(n - 1)/(e^2*n^2 - b^2*c^2*Log[F]^2)), x] + Simp[n*(n - 1)*(e^2/(e^2*n^2 -
b^2*c^2*Log[F]^2))   Int[F^(c*(a + b*x))*Cosh[d + e*x]^(n - 2), x], x]) /;
FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2*n^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n
, 1]
```

Maple [A] (verified)

Time = 1.04 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.68

method	result
parallelrisch	$-\frac{2 F^{c(bx+a)} \left(-\frac{c^{2} b^{2} \ln(F)^{2} \cosh(2ex+2d)}{2}-\frac{b^{2} c^{2} \ln(F)^{2}}{2}+\ln(F) bce \sinh(2ex+2d)+2e^{2}\right)}{2\ln(F)^{3} b^{3} c^{3}-8\ln(F) bc\, e^{2}}$
risch	$\frac{\left(\ln(F)^{2} b^{2} c^{2} e^{4ex+4d}+2\ln(F)^{2} b^{2} c^{2} e^{2ex+2d}-2\ln(F) bce\, e^{4ex+4d}+b^{2} c^{2} \ln(F)^{2}+2bc\ln(F) e-8e^{2} e^{2ex+2d}\right) e^{-2ex-2d} F^{c(bx+a)}}{4bc\ln(F)(bc\ln(F)-2e)(2e+bc\ln(F))}$
orering	$\frac{\left(3b^{2}c^{2}\ln(F)^{2}-4e^{2}\right) F^{c(bx+a)} \cosh(ex+d)^{2}}{\left(b^{2}c^{2}\ln(F)^{2}-4e^{2}\right) bc\ln(F)}-\frac{3\left(F^{c(bx+a)} bc\ln(F)\cosh(ex+d)^{2}+2F^{c(bx+a)}\cosh(ex+d) e\sinh(ex+d)\right)}{b^{2}c^{2}\ln(F)^{2}-4e^{2}}+$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-2*F^(c*(b*x+a))*(-1/2*c^2*b^2*ln(F)^2*cosh(2*e*x+2*d)-1/2*b^2*c^2*ln(F)^2
+ln(F)*b*c*e*sinh(2*e*x+2*d)+2*e^2)/(2*ln(F)^3*b^3*c^3-8*ln(F)*b*c*e^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(128) = 256$.

Time = 0.09 (sec) , antiderivative size = 699, normalized size of antiderivative = 5.30

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
1/4*(((b^2*c^2*log(F)^2 - 2*b*c*e*log(F))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x
 + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(F)^2 - 2*b*c*e*cosh(e*x + d)*log(F)
)*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x + d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 +
 b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*x + d)^2*log(F) - (3*b^2*c^2*cosh(e
*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x
 + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*cosh(e*x + d)^3*log(F) + 4*e^2*cosh(e
*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^2*c^2*cosh(e*x + d))*log(F)^2)*sinh
(e*x + d))*cosh((b*c*x + a*c)*log(F)) + ((b^2*c^2*log(F)^2 - 2*b*c*e*log(F
))*sinh(e*x + d)^4 - 8*e^2*cosh(e*x + d)^2 + 4*(b^2*c^2*cosh(e*x + d)*log(
F)^2 - 2*b*c*e*cosh(e*x + d)*log(F))*sinh(e*x + d)^3 + (b^2*c^2*cosh(e*x +
 d)^4 + 2*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 - 2*(6*b*c*e*cosh(e*
x + d)^2*log(F) - (3*b^2*c^2*cosh(e*x + d)^2 + b^2*c^2)*log(F)^2 + 4*e^2)*
sinh(e*x + d)^2 - 2*(b*c*e*cosh(e*x + d)^4 - b*c*e)*log(F) - 4*(2*b*c*e*co
sh(e*x + d)^3*log(F) + 4*e^2*cosh(e*x + d) - (b^2*c^2*cosh(e*x + d)^3 + b^
2*c^2*cosh(e*x + d))*log(F)^2)*sinh(e*x + d))*sinh((b*c*x + a*c)*log(F)))/
(b^3*c^3*cosh(e*x + d)^2*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)^2*log(F) + (b^
3*c^3*log(F)^3 - 4*b*c*e^2*log(F))*sinh(e*x + d)^2 + 2*(b^3*c^3*cosh(e*x +
 d)*log(F)^3 - 4*b*c*e^2*cosh(e*x + d)*log(F))*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 707 vs. $2(119) = 238$.

Time = 1.16 (sec) , antiderivative size = 707, normalized size of antiderivative = 5.36

$$\int F^{c(a+bx)} \cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d)**2,x)
```

output
```
Piecewise((x*cosh(d)**2, Eq(F, 1) & Eq(b, 0) & Eq(c, 0) & Eq(e, 0)), (-x*s
inh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*
e), Eq(F, 1)), (F**(a*c)*(-x*sinh(d + e*x)**2/2 + x*cosh(d + e*x)**2/2 + s
inh(d + e*x)*cosh(d + e*x)/(2*e)), Eq(b, 0)), (-x*sinh(d + e*x)**2/2 + x*c
osh(d + e*x)**2/2 + sinh(d + e*x)*cosh(d + e*x)/(2*e), Eq(c, 0)), (F**(a*c
 + b*c*x)*x*sinh(b*c*x*log(F)/2 - d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*
log(F)/2 - d)*cosh(b*c*x*log(F)/2 - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*l
og(F)/2 - d)**2/4 - F**(a*c + b*c*x)*sinh(b*c*x*log(F)/2 - d)*cosh(b*c*x*l
og(F)/2 - d)/(2*b*c*log(F)) + F**(a*c + b*c*x)*cosh(b*c*x*log(F)/2 - d)**2
/(b*c*log(F)), Eq(e, -b*c*log(F)/2)), (F**(a*c + b*c*x)*x*sinh(b*c*x*log(F
)/2 + d)**2/4 - F**(a*c + b*c*x)*x*sinh(b*c*x*log(F)/2 + d)*cosh(b*c*x*log
(F)/2 + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F)/2 + d)**2/4 - F**(a*c
+ b*c*x)*sinh(b*c*x*log(F)/2 + d)**2/(b*c*log(F)) + 3*F**(a*c + b*c*x)*sin
h(b*c*x*log(F)/2 + d)*cosh(b*c*x*log(F)/2 + d)/(2*b*c*log(F)), Eq(e, b*c*l
og(F)/2)), (F**(a*c + b*c*x)*b**2*c**2*log(F)**2*cosh(d + e*x)**2/(b**3*c*
*3*log(F)**3 - 4*b*c*e**2*log(F)) - 2*F**(a*c + b*c*x)*b*c*e*log(F)*sinh(d
 + e*x)*cosh(d + e*x)/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F)) + 2*F**(a*
c + b*c*x)*e**2*sinh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e**2*log(F))
 - 2*F**(a*c + b*c*x)*e**2*cosh(d + e*x)**2/(b**3*c**3*log(F)**3 - 4*b*c*e
**2*log(F)), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.71

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+2\,ex+2\,d)}}{4\,(bc\log(F)+2\,e)} + \frac{F^{ac}e^{(bcx\log(F)-2\,ex)}}{4\,(bce^{(2\,d)}\log(F)-2\,ee^{(2\,d)})} + \frac{F^{bcx+ac}}{2\,bc\log(F)}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/4*F^(a*c)*e^(b*c*x*log(F) + 2*e*x + 2*d)/(b*c*log(F) + 2*e) + 1/4*F^(a*c
)*e^(b*c*x*log(F) - 2*e*x)/(b*c*e^(2*d)*log(F) - 2*e*e^(2*d)) + 1/2*F^(b*c
*x + a*c)/(b*c*log(F))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.17 (sec) , antiderivative size = 889, normalized size of antiderivative = 6.73

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d)^2,x, algorithm="giac")
```

output
```
(2*b*c*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*p
i*a*c)*log(abs(F))/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*b*c)^2)
- (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*p
i*a*c*sgn(F) + 1/2*pi*a*c)/(4*b^2*c^2*log(abs(F))^2 + (pi*b*c*sgn(F) - pi*
b*c)^2))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) + I*(I*e^(1/2*I*pi*b*c*x*
sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(2*I*pi*b*c*
sgn(F) - 2*I*pi*b*c + 4*b*c*log(abs(F))) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1
/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-2*I*pi*b*c*sgn(F) +
2*I*pi*b*c + 4*b*c*log(abs(F))))*e^(b*c*x*log(abs(F)) + a*c*log(abs(F))) +
 1/2*(2*(b*c*log(abs(F)) + 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x -
1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(a
bs(F)) + 2*e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2
*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 +
4*(b*c*log(abs(F)) + 2*e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)
*x + 2*d) + I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*
sgn(F) - 1/2*I*pi*a*c)/(4*I*pi*b*c*sgn(F) - 4*I*pi*b*c + 8*b*c*log(abs(F))
 + 16*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn
(F) + 1/2*I*pi*a*c)/(-4*I*pi*b*c*sgn(F) + 4*I*pi*b*c + 8*b*c*log(abs(F)) +
 16*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + 2*e)*x + 2*d) + 1/2*(2*(b*
c*log(abs(F)) - 2*e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a...
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = -\frac{2\,F^{a\,c+b\,c\,x}\,e^2-F^{a\,c+b\,c\,x}\,b^2\,c^2\cosh(d+e\,x)^2\ln(F)^2+2\,F^{a\,c+b\,c\,x}\,b\,c\,e\cosh(d+e\,x)\,\sinh(d+e\,x)\,\ln(F)}{b^3\,c^3\ln(F)^3-4\,b\,c\,e^2\,\ln(F)}$$

input
```
int(F^(c*(a + b*x))*cosh(d + e*x)^2,x)
```

output
```
-(2*F^(a*c + b*c*x)*e^2 - F^(a*c + b*c*x)*b^2*c^2*cosh(d + e*x)^2*log(F)^2
 + 2*F^(a*c + b*c*x)*b*c*e*cosh(d + e*x)*sinh(d + e*x)*log(F))/(b^3*c^3*lo
g(F)^3 - 4*b*c*e^2*log(F))
```

Reduce [F]

$$\int F^{c(a+bx)}\cosh^2(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\cosh(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*cosh(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*cosh(d + e*x)**2,x)
```

3.288 $\int F^{c(a+bx)}\cosh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 75

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = -\frac{bcF^{c(a+bx)}\cosh(d+ex)\log(F)}{e^2-b^2c^2\log^2(F)}+\frac{eF^{c(a+bx)}\sinh(d+ex)}{e^2-b^2c^2\log^2(F)}$$

output
```
-b*c*F^(c*(b*x+a))*cosh(e*x+d)*ln(F)/(e^2-b^2*c^2*ln(F)^2)+e*F^(c*(b*x+a))
*sinh(e*x+d)/(e^2-b^2*c^2*ln(F)^2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.67

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \frac{F^{c(a+bx)}(-bc\cosh(d+ex)\log(F)+e\sinh(d+ex))}{(e-bc\log(F))(e+bc\log(F))}$$

input
```
Integrate[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
(F^(c*(a + b*x))*(-(b*c*Cosh[d + e*x]*Log[F]) + e*Sinh[d + e*x]))/((e - b*
c*Log[F])*(e + b*c*Log[F]))
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \; 5998$$

$$\frac{e\sinh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}-\frac{bc\log(F)\cosh(d+ex)F^{c(a+bx)}}{e^2-b^2c^2\log^2(F)}$$

input
```
Int[F^(c*(a + b*x))*Cosh[d + e*x],x]
```

output
```
-((b*c*F^(c*(a + b*x))*Cosh[d + e*x]*Log[F])/(e^2 - b^2*c^2*Log[F]^2)) + (
e*F^(c*(a + b*x))*Sinh[d + e*x])/(e^2 - b^2*c^2*Log[F]^2)
```

Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.37 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.69

method	result	size
parallelrisch	$-\frac{(-\ln(F)\cosh(ex+d)bc+e\sinh(ex+d))F^{c(bx+a)}}{b^2c^2\ln(F)^2-e^2}$	52
risch	$\frac{(\ln(F)bc\,e^{2ex+2d}+bc\ln(F)-e\,e^{2ex+2d}+e)e^{-ex-d}F^{c(bx+a)}}{2(bc\ln(F)-e)(e+bc\ln(F))}$	74
orering	$\frac{2bc\ln(F)F^{c(bx+a)}\cosh(ex+d)}{b^2c^2\ln(F)^2-e^2}-\frac{F^{c(bx+a)}bc\ln(F)\cosh(ex+d)+F^{c(bx+a)}e\sinh(ex+d)}{b^2c^2\ln(F)^2-e^2}$	101

input
```
int(F^(c*(b*x+a))*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-(-ln(F)*cosh(e*x+d)*b*c+e*sinh(e*x+d))*F^(c*(b*x+a))/(b^2*c^2*ln(F)^2-e^2
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 246 vs. $2(77) = 154$.

Time = 0.10 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.28

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx =$$
$$-\frac{\left(e\cosh\left(ex+d\right)^2-\left(bc\log\left(F\right)-e\right)\sinh\left(ex+d\right)^2-\left(bc\cosh\left(ex+d\right)^2+bc\right)\log\left(F\right)-2\left(bc\cosh\left(ex\right.\right.}{}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/2*((e*cosh(e*x + d)^2 - (b*c*log(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*
x + d)^2 + b*c)*log(F) - 2*(b*c*cosh(e*x + d)*log(F) - e*cosh(e*x + d))*si
nh(e*x + d) - e)*cosh((b*c*x + a*c)*log(F)) + (e*cosh(e*x + d)^2 - (b*c*lo
g(F) - e)*sinh(e*x + d)^2 - (b*c*cosh(e*x + d)^2 + b*c)*log(F) - 2*(b*c*co
sh(e*x + d)*log(F) - e*cosh(e*x + d))*sinh(e*x + d) - e)*sinh((b*c*x + a*c
)*log(F)))/(b^2*c^2*cosh(e*x + d)*log(F)^2 - e^2*cosh(e*x + d) + (b^2*c^2*
log(F)^2 - e^2)*sinh(e*x + d))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(68) = 136$.

Time = 0.65 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.53

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \begin{cases} x\cosh(d) & \text{for } F=1\wedge e=0 \\ F^{ac}x\cosh(d) & \text{for } b=0\wedge e=0 \\ x\cosh(d) & \text{for } c=0\wedge e=0 \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}x\cosh(bcx\log(F)-d)}{2}+\frac{F^{ac+bcx}\sinh(bcx\log(F)-d)}{2bc\log(F)} & \text{for } e=-bc\log(F) \\ -\frac{F^{ac+bcx}x\sinh(bcx\log(F)+d)}{2}+\frac{F^{ac+bcx}x\cosh(bcx\log(F)+d)}{2}+\frac{F^{ac+bcx}\sinh(bcx\log(F)+d)}{2bc\log(F)} & \text{for } e=bc\log(F) \\ \frac{F^{ac+bcx}bc\log(F)\cosh(d+ex)}{b^2c^2\log(F)^2-e^2}-\frac{F^{ac+bcx}e\sinh(d+ex)}{b^2c^2\log(F)^2-e^2} & \text{otherwise} \end{cases}$$

input
```
integrate(F**(c*(b*x+a))*cosh(e*x+d),x)
```

output
```
Piecewise((x*cosh(d), Eq(F, 1) & Eq(e, 0)), (F**(a*c)*x*cosh(d), Eq(b, 0)
& Eq(e, 0)), (x*cosh(d), Eq(c, 0) & Eq(e, 0)), (-F**(a*c + b*c*x)*x*sinh(b
*c*x*log(F) - d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*log(F) - d)/2 + F**(a*c
 + b*c*x)*sinh(b*c*x*log(F) - d)/(2*b*c*log(F)), Eq(e, -b*c*log(F))), (-F*
*(a*c + b*c*x)*x*sinh(b*c*x*log(F) + d)/2 + F**(a*c + b*c*x)*x*cosh(b*c*x*
log(F) + d)/2 + F**(a*c + b*c*x)*sinh(b*c*x*log(F) + d)/(2*b*c*log(F)), Eq
(e, b*c*log(F))), (F**(a*c + b*c*x)*b*c*log(F)*cosh(d + e*x)/(b**2*c**2*lo
g(F)**2 - e**2) - F**(a*c + b*c*x)*e*sinh(d + e*x)/(b**2*c**2*log(F)**2 -
e**2), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.84

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \frac{F^{ac}e^{(bcx\log(F)+ex+d)}}{2\,(bc\log(F)+e)}+\frac{F^{ac}e^{(bcx\log(F)-ex)}}{2\,(bce^d\log(F)-ee^d)}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + e) + 1/2*F^(a*c)*e^(b
*c*x*log(F) - e*x)/(b*c*e^d*log(F) - e*e^d)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 597, normalized size of antiderivative = 7.96

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(F^(c*(b*x+a))*cosh(e*x+d),x, algorithm="giac")
```

output
```
(2*(b*c*log(abs(F)) + e)*cos(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*
a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F))
+ e)^2) - (pi*b*c*sgn(F) - pi*b*c)*sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x
 - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*lo
g(abs(F)) + e)^2))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) + e)*x + d) + 1/2
*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/
2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi*b*c + 2*b*c*log(abs(F)) + 2*e) - I*e^(
-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a
*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c + 2*b*c*log(abs(F)) + 2*e))*e^(a*c*log(ab
s(F)) + (b*c*log(abs(F)) + e)*x + d) + (2*(b*c*log(abs(F)) - e)*cos(-1/2*p
i*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/((pi*b*c*s
gn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2) - (pi*b*c*sgn(F) - pi*b*c)*
sin(-1/2*pi*b*c*x*sgn(F) + 1/2*pi*b*c*x - 1/2*pi*a*c*sgn(F) + 1/2*pi*a*c)/
((pi*b*c*sgn(F) - pi*b*c)^2 + 4*(b*c*log(abs(F)) - e)^2))*e^(a*c*log(abs(F
)) + (b*c*log(abs(F)) - e)*x - d) + 1/2*I*(I*e^(1/2*I*pi*b*c*x*sgn(F) - 1/
2*I*pi*b*c*x + 1/2*I*pi*a*c*sgn(F) - 1/2*I*pi*a*c)/(I*pi*b*c*sgn(F) - I*pi
*b*c + 2*b*c*log(abs(F)) - 2*e) - I*e^(-1/2*I*pi*b*c*x*sgn(F) + 1/2*I*pi*b
*c*x - 1/2*I*pi*a*c*sgn(F) + 1/2*I*pi*a*c)/(-I*pi*b*c*sgn(F) + I*pi*b*c +
2*b*c*log(abs(F)) - 2*e))*e^(a*c*log(abs(F)) + (b*c*log(abs(F)) - e)*x - d
)
```

Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.99

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = -\frac{F^{a\,c+b\,c\,x}\,\mathrm{e}^{-d-e\,x}\,\left(e-e\,\mathrm{e}^{2\,d+2\,e\,x}+b\,c\,\ln\left(F\right)+b\,c\,\mathrm{e}^{2\,d+2\,e\,x}\,\ln\left(F\right)\right)}{2\,\left(e^2-b^2\,c^2\,\ln\left(F\right)^2\right)}$$

input

```
int(F^(c*(a + b*x))*cosh(d + e*x),x)
```

output

```
-(F^(a*c + b*c*x)*exp(- d - e*x)*(e - e*exp(2*d + 2*e*x) + b*c*log(F) + b*
c*exp(2*d + 2*e*x)*log(F)))/(2*(e^2 - b^2*c^2*log(F)^2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.68

$$\int F^{c(a+bx)}\cosh(d+ex)\,dx = \frac{f^{bcx+ac}(\cosh(ex+d)\log(f)\,bc-\sinh(ex+d)\,e)}{\log(f)^2\,b^2c^2-e^2}$$

input

```
int(F^(c*(b*x+a))*cosh(e*x+d),x)
```

output

```
(f**(a*c + b*c*x)*(cosh(d + e*x)*log(f)*b*c - sinh(d + e*x)*e))/(log(f)**2
*b**2*c**2 - e**2)
```

3.289 $\int F^{c(a+bx)}\mathbf{sech}(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 68

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

output
```
2*exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.03

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = \frac{2e^{d+ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{1}{2}+\frac{bc\log(F)}{2e},\frac{3}{2}+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, 1/2 + (b*c*Log[F])/(2*
e), 3/2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.062$, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6015}$$

$$\frac{2e^{d+ex}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x],x]
```

output
```
(2*E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e)
, (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))])/(e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = \int F^{(bx+a)c} \operatorname{sech}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d), x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = \int F^{c(a+bx)} \operatorname{sech}(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d),x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x), x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="maxima")
```

output
```
-4*F^(a*c)*e*integrate(e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e^(4*
d)*log(F) - e*e^(4*d))*e^(4*e*x) + 2*(b*c*e^(2*d)*log(F) - e*e^(2*d))*e^(2
*e*x) - e), x) + 2*F^(a*c)*e^(b*c*x*log(F) + e*x + d)/(b*c*log(F) + (b*c*e
^(2*d)*log(F) - e*e^(2*d))*e^(2*e*x) - e)
```

Giac [F]

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d),x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d), x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{sech}(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh(d+e\,x)}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x),x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x), x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d),x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x),x)
```

3.290 $\int F^{c(a+bx)}\mathbf{sech}^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 70

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

output
```
4*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b*c
*ln(F)/e],-exp(2*e*x+2*d))/(2*e+b*c*ln(F))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \frac{4e^{2(d+ex)}F^{c(a+bx)}\,\text{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)}{2e+bc\log(F)}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.056$, Rules used = {6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^2(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6015}$$

$$\frac{4e^{2(d+ex)}F^{c(a+bx)}\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,-e^{2(d+ex)}\right)}{bc\log(F)+2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^2,x]
```

output
```
(4*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))])/(2*e + b*c*Log[F])
```

Defintions of rubi rules used

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^2 \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}(ex+d)^2 \, dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^2, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{sech}^2(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**2,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**2, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="maxima")
```

output
```
16*F^(a*c)*b*c*e*integrate(F^(b*c*x)/(b^2*c^2*log(F)^2 - 6*b*c*e*log(F) +
8*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 6*b*c*e*e^(6*d)*log(F) + 8*e^2*e^(6*d)
)*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*log(F) + 8*e^2
*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^(2*d)*log(F)
 + 8*e^2*e^(2*d))*e^(2*e*x)), x)*log(F) - 4*(4*F^(a*c)*e - (F^(a*c)*b*c*e^
(2*d)*log(F) - 4*F^(a*c)*e*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2
 - 6*b*c*e*log(F) + 8*e^2 + (b^2*c^2*e^(4*d)*log(F)^2 - 6*b*c*e*e^(4*d)*lo
g(F) + 8*e^2*e^(4*d))*e^(4*e*x) + 2*(b^2*c^2*e^(2*d)*log(F)^2 - 6*b*c*e*e^
(2*d)*log(F) + 8*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^2\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^2,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^2}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^2, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\text{sech}^2(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\text{sech}(ex+d)^2\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^2,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**2,x)
```

3.291 $\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 124

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{e^{d+ex}F^{c(a+bx)}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)(e-bc\log(F))}{e^2} + \frac{bcF^{c(a+bx)}\log(F)\text{sech}(d+ex)}{2e^2} + \frac{F^{c(a+bx)}\text{sech}(d+ex)\tanh(d+ex)}{2e}$$

output
```
exp(e*x+d)*F^(c*(b*x+a))*hypergeom([1, 1/2*(e+b*c*ln(F))/e],[3/2+1/2*b*c*l
n(F)/e],-exp(2*e*x+2*d))*(e-b*c*ln(F))/e^2+1/2*b*c*F^(c*(b*x+a))*ln(F)*sec
h(e*x+d)/e^2+1/2*F^(c*(b*x+a))*sech(e*x+d)*tanh(e*x+d)/e
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.77

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \frac{F^{c(a+bx)}\left(2e^{d+ex}\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(3+\frac{bc\log(F)}{e}\right),-e^{2(d+ex)}\right)(e-bc\log(F))+\text{sech}(d\right.}{2e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(F^(c*(a + b*x))*(2*E^(d + e*x)*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e
), (3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(e - b*c*Log[F]) + Sech[d + e
*x]*(b*c*Log[F] + e*Tanh[d + e*x])))/(2*e^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 139, normalized size of antiderivative = 1.12, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{6013}$$

$$\frac{1}{2}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{sech}(d+ex)dx+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

$$\Big\downarrow \text{6015}$$

$$\frac{e^{d+ex}F^{c(a+bx)}\left(1-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(1,\frac{e+bc\log(F)}{2e},\frac{1}{2}\left(\frac{bc\log(F)}{e}+3\right),-e^{2(d+ex)}\right)}{bc\log(F)+e}+\frac{bc\log(F)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e^2}+\frac{\tanh(d+ex)\operatorname{sech}(d+ex)F^{c(a+bx)}}{2e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^3,x]
```

output
```
(E^(d + e*x)*F^(c*(a + b*x))*Hypergeometric2F1[1, (e + b*c*Log[F])/(2*e),
(3 + (b*c*Log[F])/e)/2, -E^(2*(d + e*x))]*(1 - (b^2*c^2*Log[F]^2)/e^2))/(e
 + b*c*Log[F]) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x])/(2*e^2) + (F^(
c*(a + b*x))*Sech[d + e*x]*Tanh[d + e*x])/(2*e)
```

Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)}\operatorname{sech}(ex+d)^3\,dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^3, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int F^{c(a+bx)}\,\text{sech}^3\,(d+ex)\,dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**3,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**3, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx=\int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\,\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="maxima")
```

output
```
48*(F^(a*c)*b*c*e*e^d*log(F) + F^(a*c)*e^2*e^d)*integrate(e^(b*c*x*log(F)
+ e*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*log(F) + 15*e^2 + (b^2*c^2*e^(8*d)*log(
F)^2 - 8*b*c*e*e^(8*d)*log(F) + 15*e^2*e^(8*d))*e^(8*e*x) + 4*(b^2*c^2*e^(
6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e^2*e^(6*d))*e^(6*e*x) + 6*(b^
2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(F) + 15*e^2*e^(4*d))*e^(4*e*x
) + 4*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(2*d)*log(F) + 15*e^2*e^(2*d))
*e^(2*e*x)), x) - 8*(6*F^(a*c)*e*e^(e*x + d) - (F^(a*c)*b*c*e^(3*d)*log(F)
 - 5*F^(a*c)*e*e^(3*d))*e^(3*e*x))*F^(b*c*x)/(b^2*c^2*log(F)^2 - 8*b*c*e*l
og(F) + 15*e^2 + (b^2*c^2*e^(6*d)*log(F)^2 - 8*b*c*e*e^(6*d)*log(F) + 15*e
^2*e^(6*d))*e^(6*e*x) + 3*(b^2*c^2*e^(4*d)*log(F)^2 - 8*b*c*e*e^(4*d)*log(
F) + 15*e^2*e^(4*d))*e^(4*e*x) + 3*(b^2*c^2*e^(2*d)*log(F)^2 - 8*b*c*e*e^(
2*d)*log(F) + 15*e^2*e^(2*d))*e^(2*e*x))
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^3\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^3,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\text{sech}^3(d+ex)\,dx = \int \frac{F^{c\,(a+b\,x)}}{\cosh\left(d+e\,x\right)^3}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^3, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{sech}^3(d+ex)\,dx = f^{ac}\left(\int f^{bcx}\operatorname{sech}(ex+d)^3\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^3,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**3,x)
```

3.292 $\int F^{c(a+bx)}\mathrm{sech}^4(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int F^{c(a+bx)}\mathrm{sech}^4(d+ex)\,dx = \frac{2e^{2(d+ex)}F^{c(a+bx)}\,\mathrm{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)(2e-bc\log(F))}{3e^2} + \frac{bcF^{c(a+bx)}\log(F)\mathrm{sech}^2(d+ex)}{6e^2} + \frac{F^{c(a+bx)}\mathrm{sech}^2(d+ex)\tanh(d+ex)}{3e}$$

output
```
2/3*exp(2*e*x+2*d)*F^(c*(b*x+a))*hypergeom([2, 1+1/2*b*c*ln(F)/e],[2+1/2*b
*c*ln(F)/e],-exp(2*e*x+2*d))*(2*e-b*c*ln(F))/e^2+1/6*b*c*F^(c*(b*x+a))*ln(
F)*sech(e*x+d)^2/e^2+1/3*F^(c*(b*x+a))*sech(e*x+d)^2*tanh(e*x+d)/e
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.76

$$\int F^{c(a+bx)}\mathrm{sech}^4(d+ex)\,dx = \frac{F^{c(a+bx)}\left(4e^{2(d+ex)}\,\mathrm{Hypergeometric2F1}\left(2,1+\frac{bc\log(F)}{2e},2+\frac{bc\log(F)}{2e},-e^{2(d+ex)}\right)(2e-bc\log(F))+\mathrm{sech}^2(\right.}{6e^2}$$

input
```
Integrate[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

output
```
(F^(c*(a + b*x))*(4*E^(2*(d + e*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/
(2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(2*e - b*c*Log[F]) + Sech
[d + e*x]^2*(b*c*Log[F] + 2*e*Tanh[d + e*x])))/(6*e^2)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6013, 6015}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(d+ex)F^{c(a+bx)}\,dx$$

$$\Big\downarrow \text{ 6013}$$

$$\frac{1}{6}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right)\int F^{c(a+bx)}\operatorname{sech}^2(d+ex)dx+\frac{bc\log(F)\operatorname{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh(d+ex)\operatorname{sech}^2(d+ex)F^{c(a+bx)}}{3e}$$

$$\Big\downarrow \text{ 6015}$$

$$\frac{2e^{2(d+ex)}F^{c(a+bx)}\left(4-\frac{b^2c^2\log^2(F)}{e^2}\right)\text{Hypergeometric2F1}\left(2,\frac{bc\log(F)}{2e}+1,\frac{bc\log(F)}{2e}+2,-e^{2(d+ex)}\right)}{3(bc\log(F)+2e)}+\frac{bc\log(F)\operatorname{sech}^2(d+ex)F^{c(a+bx)}}{6e^2}+\frac{\tanh(d+ex)\operatorname{sech}^2(d+ex)F^{c(a+bx)}}{3e}$$

input
```
Int[F^(c*(a + b*x))*Sech[d + e*x]^4,x]
```

output
```
(2*E^(2*(d + e*x))*F^(c*(a + b*x))*Hypergeometric2F1[2, 1 + (b*c*Log[F])/(
2*e), 2 + (b*c*Log[F])/(2*e), -E^(2*(d + e*x))]*(4 - (b^2*c^2*Log[F]^2)/e^
2))/(3*(2*e + b*c*Log[F])) + (b*c*F^(c*(a + b*x))*Log[F]*Sech[d + e*x]^2)/
(6*e^2) + (F^(c*(a + b*x))*Sech[d + e*x]^2*Tanh[d + e*x])/(3*e)
```

Defintions of rubi rules used

rule 6013
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_), x_Symb
ol] :> Simp[b*c*Log[F]*F^(c*(a + b*x))*(Sech[d + e*x]^(n - 2)/(e^2*(n - 1)*
(n - 2))), x] + (Simp[F^(c*(a + b*x))*Sech[d + e*x]^(n - 1)*(Sinh[d + e*x]/
(e*(n - 1))), x] + Simp[(e^2*(n - 2)^2 - b^2*c^2*Log[F]^2)/(e^2*(n - 1)*(n
- 2))   Int[F^(c*(a + b*x))*Sech[d + e*x]^(n - 2), x], x]) /; FreeQ[{F, a,
b, c, d, e}, x] && NeQ[e^2*(n - 2)^2 - b^2*c^2*Log[F]^2, 0] && GtQ[n, 1] &&
 NeQ[n, 2]
```

rule 6015
```
Int[(F_)^((c_.)*((a_.) + (b_.)*(x_)))*Sech[(d_.) + (e_.)*(x_)]^(n_.), x_Sym
bol] :> Simp[2^n*E^(n*(d + e*x))*(F^(c*(a + b*x))/(e*n + b*c*Log[F]))*Hyper
geometric2F1[n, n/2 + b*c*(Log[F]/(2*e)), 1 + n/2 + b*c*(Log[F]/(2*e)), -E^
(2*(d + e*x))], x] /; FreeQ[{F, a, b, c, d, e}, x] && IntegerQ[n]
```

Maple [F]

$$\int F^{c(bx+a)} \operatorname{sech}(ex+d)^4 \, dx$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

output
```
int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

Fricas [F]

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="fricas")
```

output
```
integral(F^(b*c*x + a*c)*sech(e*x + d)^4, x)
```

Sympy [F]

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx = \int F^{c(a+bx)}\operatorname{sech}^4\left(d+ex\right)dx$$

input
```
integrate(F**(c*(b*x+a))*sech(e*x+d)**4,x)
```

output
```
Integral(F**(c*(a + b*x))*sech(d + e*x)**4, x)
```

Maxima [F]

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx = \int F^{(bx+a)c}\operatorname{sech}\left(ex+d\right)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="maxima")
```

output
```
-128*(F^(a*c)*b^2*c^2*e*log(F)^2 + 2*F^(a*c)*b*c*e^2*log(F))*integrate(F^(
b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^2 + 104*b*c*e^2*log(F) - 19
2*e^3 + (b^3*c^3*e^(10*d)*log(F)^3 - 18*b^2*c^2*e*e^(10*d)*log(F)^2 + 104*
b*c*e^2*e^(10*d)*log(F) - 192*e^3*e^(10*d))*e^(10*e*x) + 5*(b^3*c^3*e^(8*d
)*log(F)^3 - 18*b^2*c^2*e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) -
192*e^3*e^(8*d))*e^(8*e*x) + 10*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e
^(6*d)*log(F)^2 + 104*b*c*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x)
+ 10*(b^3*c^3*e^(4*d)*log(F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e
^2*e^(4*d)*log(F) - 192*e^3*e^(4*d))*e^(4*e*x) + 5*(b^3*c^3*e^(2*d)*log(F)
^3 - 18*b^2*c^2*e*e^(2*d)*log(F)^2 + 104*b*c*e^2*e^(2*d)*log(F) - 192*e^3*
e^(2*d))*e^(2*e*x)), x) + 16*(8*F^(a*c)*b*c*e*log(F) + 16*F^(a*c)*e^2 + (F
^(a*c)*b^2*c^2*e^(4*d)*log(F)^2 - 14*F^(a*c)*b*c*e*e^(4*d)*log(F) + 48*F^(
a*c)*e^2*e^(4*d))*e^(4*e*x) - 8*(F^(a*c)*b*c*e*e^(2*d)*log(F) - 8*F^(a*c)*
e^2*e^(2*d))*e^(2*e*x))*F^(b*c*x)/(b^3*c^3*log(F)^3 - 18*b^2*c^2*e*log(F)^
2 + 104*b*c*e^2*log(F) - 192*e^3 + (b^3*c^3*e^(8*d)*log(F)^3 - 18*b^2*c^2*
e*e^(8*d)*log(F)^2 + 104*b*c*e^2*e^(8*d)*log(F) - 192*e^3*e^(8*d))*e^(8*e*
x) + 4*(b^3*c^3*e^(6*d)*log(F)^3 - 18*b^2*c^2*e*e^(6*d)*log(F)^2 + 104*b*c
*e^2*e^(6*d)*log(F) - 192*e^3*e^(6*d))*e^(6*e*x) + 6*(b^3*c^3*e^(4*d)*log(
F)^3 - 18*b^2*c^2*e*e^(4*d)*log(F)^2 + 104*b*c*e^2*e^(4*d)*log(F) - 192*e^
3*e^(4*d))*e^(4*e*x) + 4*(b^3*c^3*e^(2*d)*log(F)^3 - 18*b^2*c^2*e*e^(2*...
```

Giac [F]

$$\int F^{c(a+bx)}\text{sech}^4(d+ex)\,dx = \int F^{(bx+a)c}\,\text{sech}\,(ex+d)^4\,dx$$

input
```
integrate(F^(c*(b*x+a))*sech(e*x+d)^4,x, algorithm="giac")
```

output
```
integrate(F^((b*x + a)*c)*sech(e*x + d)^4, x)
```

Mupad [F(-1)]

Timed out.

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx=\int \frac{F^{c\,(a+b\,x)}}{{\cosh\left(d+e\,x\right)}^4}\,dx$$

input
```
int(F^(c*(a + b*x))/cosh(d + e*x)^4,x)
```

output
```
int(F^(c*(a + b*x))/cosh(d + e*x)^4, x)
```

Reduce [F]

$$\int F^{c(a+bx)}\operatorname{sech}^4(d+ex)\,dx=f^{ac}\left(\int f^{bcx}\operatorname{sech}(ex+d)^4\,dx\right)$$

input
```
int(F^(c*(b*x+a))*sech(e*x+d)^4,x)
```

output
```
f**(a*c)*int(f**(b*c*x)*sech(d + e*x)**4,x)
```

3.293 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 250

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2}\,dx = -\frac{e^{-4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{128bc} - \frac{5e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{64bc} + \frac{5e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{32bc} + \frac{5e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{128bc} + \frac{e^{6c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{192bc} + \frac{5}{16}x\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)$$

output
```
-1/128*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c/exp(4*c*(b*x+a))-5/64
*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))+5/32*exp(2
*c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c+5/128*exp(4*c*(b
*x+a))*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c+1/192*exp(6*c*(b*x+a)
)*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c+5/16*x*(cosh(b*c*x+a*c)^2)
^(1/2)*sech(b*c*x+a*c)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.44

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2} dx = \frac{\left(-\frac{1}{128}e^{-4c(a+bx)} - \frac{5}{64}e^{-2c(a+bx)} + \frac{5}{32}e^{2c(a+bx)} + \frac{5}{128}e^{4c(a+bx)} + \frac{1}{192}e^{6c(a+bx)} + \frac{5bcx}{16}\right) \cosh^2(c(a+\cdots}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
((-1/128*1/E^(4*c*(a + b*x)) - 5/(64*E^(2*c*(a + b*x))) + (5*E^(2*c*(a + b
*x)))/32 + (5*E^(4*c*(a + b*x)))/128 + E^(6*c*(a + b*x))/192 + (5*b*c*x)/1
6)*(Cosh[c*(a + b*x)]^2)^(5/2)*Sech[c*(a + b*x)]^5)/(b*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.41, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2} dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int e^{c(a+bx)} \cosh^5(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int \frac{1}{32}e^{-5c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5 de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx) \int e^{-5c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5 de^{c(a+bx)}}{32bc}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)\int e^{-3c(a+bx)}\left(1+e^{2c(a+bx)}\right)^5de^{2c(a+bx)}}{64bc}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)\int \left(10+e^{-3c(a+bx)}+5e^{-2c(a+bx)}+10e^{-c(a+bx)}+6e^{2c(a+bx)}\right)de^{2c(a+bx)}}{64bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-\frac{1}{2}e^{-2c(a+bx)}-5e^{-c(a+bx)}+\frac{25}{2}e^{2c(a+bx)}+\frac{1}{3}e^{3c(a+bx)}+10\log\left(e^{2c(a+bx)}\right)\right)\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{64bc}$$

input
```
Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(Sqrt[Cosh[a*c + b*c*x]^2]*(-1/2*1/E^(2*c*(a + b*x)) - 5/E^(c*(a + b*x)) +
 (25*E^(2*c*(a + b*x)))/2 + E^(3*c*(a + b*x))/3 + 10*Log[E^(2*c*(a + b*x))
])*Sech[a*c + b*c*x])/(64*b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 8.35 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.30

method	result
risch	$\frac{5x\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{c(bx+a)}}{16\left(1+{\mathrm e}^{2c(bx+a)}\right)}+\frac{\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{7c(bx+a)}}{192bc\left(1+{\mathrm e}^{2c(bx+a)}\right)}+\frac{5\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{5c(bx+a)}}{128bc\left(1+{\mathrm e}^{2c(bx+a)}\right)}$

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
5/16*x*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)
))*exp(c*(b*x+a))+1/192/b/c*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/
2)/(1+exp(2*c*(b*x+a)))*exp(7*c*(b*x+a))+5/128/b/c*((1+exp(2*c*(b*x+a)))^2
*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(5*c*(b*x+a))+5/32/b/c*(
(1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(3
*c*(b*x+a))-5/64/b/c*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+e
xp(2*c*(b*x+a)))*exp(-c*(b*x+a))-1/128/b/c*((1+exp(2*c*(b*x+a)))^2*exp(-2*
c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(-3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.87

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx =$$
$$-\frac{\cosh\left(bcx+ac\right)^5+5\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^4-5\sinh\left(bcx+ac\right)^5-5\left(10\cosh\left(bcx+ac\right)^2+9\right)}{}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-1/384*(cosh(b*c*x + a*c)^5 + 5*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^4 - 5*
sinh(b*c*x + a*c)^5 - 5*(10*cosh(b*c*x + a*c)^2 + 9)*sinh(b*c*x + a*c)^3 +
 15*cosh(b*c*x + a*c)^3 + 5*(2*cosh(b*c*x + a*c)^3 + 9*cosh(b*c*x + a*c))*
sinh(b*c*x + a*c)^2 - 60*(2*b*c*x + 1)*cosh(b*c*x + a*c) - 5*(5*cosh(b*c*x
 + a*c)^4 - 24*b*c*x + 27*cosh(b*c*x + a*c)^2 + 12)*sinh(b*c*x + a*c))/(b*
c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 112, normalized size of antiderivative = 0.45

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx=\frac{5\,(bcx+ac)}{16\,bc}+\frac{e^{(6\,bcx+6\,ac)}}{192\,bc}+\frac{5\,e^{(4\,bcx+4\,ac)}}{128\,bc}+\frac{5\,e^{(2\,bcx+2\,ac)}}{32\,bc}-\frac{5\,e^{(-2\,bcx-2\,ac)}}{64\,bc}-\frac{e^{(-4\,bcx-4\,ac)}}{128\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
5/16*(b*c*x + a*c)/(b*c) + 1/192*e^(6*b*c*x + 6*a*c)/(b*c) + 5/128*e^(4*b*
c*x + 4*a*c)/(b*c) + 5/32*e^(2*b*c*x + 2*a*c)/(b*c) - 5/64*e^(-2*b*c*x - 2
*a*c)/(b*c) - 1/128*e^(-4*b*c*x - 4*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 99, normalized size of antiderivative = 0.40

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{5/2}\,dx=\frac{\left(120\,bcxe^{(4\,ac)}-3\left(30\,e^{(4\,bcx+4\,ac)}+10\,e^{(2\,bcx+2\,ac)}+1\right)e^{(-4\,bcx)}+2\,e^{(6\,bcx+10\,ac)}+15\,e^{(4\,bcx+8\,ac)}}{384\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
1/384*(120*b*c*x*e^(4*a*c) - 3*(30*e^(4*b*c*x + 4*a*c) + 10*e^(2*b*c*x + 2
*a*c) + 1)*e^(-4*b*c*x) + 2*e^(6*b*c*x + 10*a*c) + 15*e^(4*b*c*x + 8*a*c)
+ 60*e^(2*b*c*x + 6*a*c))*e^(-4*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2} dx = \int e^{c(a+bx)} \left(\cosh(ac+bcx)^2\right)^{5/2} dx$$

input
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(5/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.39

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{5/2} dx = \frac{2e^{10bcx+10ac} + 15e^{8bcx+8ac} + 60e^{6bcx+6ac} + 120e^{4bcx+4ac}bcx - 30e^{2bcx+2ac} - 3}{384e^{4bcx+4ac}bc}$$

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(2*e**(10*a*c + 10*b*c*x) + 15*e**(8*a*c + 8*b*c*x) + 60*e**(6*a*c + 6*b*c
*x) + 120*e**(4*a*c + 4*b*c*x)*b*c*x - 30*e**(2*a*c + 2*b*c*x) - 3)/(384*e
**(4*a*c + 4*b*c*x)*b*c)
```

3.294 $\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 162

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2}\,dx = -\frac{e^{-2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{16bc} + \frac{3e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{16bc} + \frac{e^{4c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{32bc} + \frac{3}{8}x\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)$$

output
```
-1/16*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c/exp(2*c*(b*x+a))+3/16*
exp(2*c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c+1/32*exp(4*
c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c+3/8*x*(cosh(b*c*x
+a*c)^2)^(1/2)*sech(b*c*x+a*c)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.50

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx=\frac{\left(-\frac{1}{16}e^{-2c(a+bx)}+\frac{3}{16}e^{2c(a+bx)}+\frac{1}{32}e^{4c(a+bx)}+\frac{3bcx}{8}\right)\cosh^2(c(a+bx))^{3/2}\text{sech}^3(c(a+bx))}{bc}$$

input
```
Integrate[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
((-1/16*1/E^(2*c*(a + b*x)) + (3*E^(2*c*(a + b*x)))/16 + E^(4*c*(a + b*x))
/32 + (3*b*c*x)/8)*(Cosh[c*(a + b*x)]^2)^(3/2)*Sech[c*(a + b*x)]^3)/(b*c)
```

Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{c(a+bx)}\cosh^3(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int \frac{1}{8}e^{-3c(a+bx)}\left(1+e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{-3c(a+bx)}\left(1+e^{2c(a+bx)}\right)^3de^{c(a+bx)}}{8bc}$$

$$\Big\downarrow \text{243}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{-2c(a+bx)}\left(1+e^{2c(a+bx)}\right)^3 de^{2c(a+bx)}}{16bc}$$

$$\Big\downarrow \text{49}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int \left(3+e^{-2c(a+bx)}+3e^{-c(a+bx)}+e^{2c(a+bx)}\right) de^{2c(a+bx)}}{16bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(-e^{-c(a+bx)}+\frac{7}{2}e^{2c(a+bx)}+3\log\left(e^{2c(a+bx)}\right)\right)\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{16bc}$$

input
```
Int[E^(c*(a + b*x))*(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(Sqrt[Cosh[a*c + b*c*x]^2]*(-E^(-(c*(a + b*x))) + (7*E^(2*c*(a + b*x)))/2
+ 3*Log[E^(2*c*(a + b*x))])*Sech[a*c + b*c*x])/(16*b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 1.21 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.33

method	result
risch	$\frac{3x\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{c(bx+a)}}{8\left(1+{\mathrm e}^{2c(bx+a)}\right)}+\frac{\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{5c(bx+a)}}{32bc\left(1+{\mathrm e}^{2c(bx+a)}\right)}+\frac{3\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,{\mathrm e}^{3c(bx+a)}}{16bc\left(1+{\mathrm e}^{2c(bx+a)}\right)}$

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
3/8*x*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a))
)*exp(c*(b*x+a))+1/32/b/c*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)
/(1+exp(2*c*(b*x+a)))*exp(5*c*(b*x+a))+3/16/b/c*((1+exp(2*c*(b*x+a)))^2*ex
p(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(3*c*(b*x+a))-1/16/b/c*((1+
exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))*exp(-c*(
b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.78

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx =$$
$$-\frac{\cosh\left(bcx+ac\right)^3+3\,\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2-3\,\sinh\left(bcx+ac\right)^3-6\left(2\,bcx+1\right)\cosh\left(bcx+ac\right.}{32\left(bc\cosh\left(bcx+ac\right)-bc\sinh\left(bcx+ac\right)\right)}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-1/32*(cosh(b*c*x + a*c)^3 + 3*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 - 3*s
inh(b*c*x + a*c)^3 - 6*(2*b*c*x + 1)*cosh(b*c*x + a*c) + 3*(4*b*c*x - 3*co
sh(b*c*x + a*c)^2 - 2)*sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c) - b*c*sin
h(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(3/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.46

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx = \frac{3\,(bcx+ac)}{8\,bc}+\frac{e^{(4\,bcx+4\,ac)}}{32\,bc}+\frac{3\,e^{(2\,bcx+2\,ac)}}{16\,bc}-\frac{e^{(-2\,bcx-2\,ac)}}{16\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
3/8*(b*c*x + a*c)/(b*c) + 1/32*e^(4*b*c*x + 4*a*c)/(b*c) + 3/16*e^(2*b*c*x
 + 2*a*c)/(b*c) - 1/16*e^(-2*b*c*x - 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.44

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx = \frac{\left(12\,bcxe^{(2\,ac)} - 2\left(3\,e^{(2\,bcx+2\,ac)} + 1\right)e^{(-2\,bcx)} + e^{(4\,bcx+6\,ac)} + 6\,e^{(2\,bcx+4\,ac)}\right)e^{(-2\,ac)}}{32\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
1/32*(12*b*c*x*e^(2*a*c) - 2*(3*e^(2*b*c*x + 2*a*c) + 1)*e^(-2*b*c*x) + e^
(4*b*c*x + 6*a*c) + 6*e^(2*b*c*x + 4*a*c))*e^(-2*a*c)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int e^{c(a+bx)}\cosh^2(ac+bcx)^{3/2}\,dx = \int e^{c\,(a+b\,x)}\,{\left({\mathrm{cosh}\left(a\,c+b\,c\,x\right)}^2\right)}^{3/2}\,dx$$

input
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(3/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.41

$$\int e^{c(a+bx)} \cosh^2(ac+bcx)^{3/2}\,dx = \frac{e^{6bcx+6ac}+6e^{4bcx+4ac}+12e^{2bcx+2ac}bcx-2}{32e^{2bcx+2ac}bc}$$

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(3/2),x)
```

output
```
(e**(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) + 12*e**(2*a*c + 2*b*c*x)*b
*c*x - 2)/(32*e**(2*a*c + 2*b*c*x)*b*c)
```

3.295 $\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 74

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx=\frac{e^{2c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)}{4bc}+\frac{1}{2}x\sqrt{\cosh^2(ac+bcx)}\operatorname{sech}(ac+bcx)$$

output
```
1/4*exp(2*c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)/b/c+1/2*x*(
cosh(b*c*x+a*c)^2)^(1/2)*sech(b*c*x+a*c)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.65

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx=\frac{\left(e^{2c(a+bx)}+2bcx\right)\sqrt{\cosh^2(c(a+bx))}\operatorname{sech}(c(a+bx))}{4bc}$$

input
```
Integrate[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
((E^(2*c*(a + b*x)) + 2*b*c*x)*Sqrt[Cosh[c*(a + b*x)]^2]*Sech[c*(a + b*x)]
)/(4*b*c)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.80, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {7271, 2720, 27, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{c(a+bx)}\cosh(ac+bxc)dx$$

$$\Big\downarrow \text{2720}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int \frac{1}{2}e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)de^{c(a+bx)}}{bc}$$

$$\Big\downarrow \text{27}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int e^{-c(a+bx)}\left(1+e^{2c(a+bx)}\right)de^{c(a+bx)}}{2bc}$$

$$\Big\downarrow \text{244}$$

$$\frac{\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)\int \left(e^{-c(a+bx)}+e^{c(a+bx)}\right)de^{c(a+bx)}}{2bc}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(\frac{1}{2}e^{2c(a+bx)}+\log\left(e^{c(a+bx)}\right)\right)\sqrt{\cosh^2(ac+bcx)}\text{sech}(ac+bcx)}{2bc}$$

input
```
Int[E^(c*(a + b*x))*Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
(Sqrt[Cosh[a*c + b*c*x]^2]*(E^(2*c*(a + b*x))/2 + Log[E^(c*(a + b*x))])*Se
ch[a*c + b*c*x])/(2*b*c)
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.43

method	result	size
risch	$\frac{x\sqrt{\left(1+{\rm e}^{2c(bx+a)}\right)^{2} {\rm e}^{-2c(bx+a)}}\,{\rm e}^{c(bx+a)}}{2+2\,{\rm e}^{2c(bx+a)}}+\frac{\sqrt{\left(1+{\rm e}^{2c(bx+a)}\right)^{2} {\rm e}^{-2c(bx+a)}}\,{\rm e}^{3c(bx+a)}}{4bc\left(1+{\rm e}^{2c(bx+a)}\right)}$	106

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*x*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a))
)*exp(c*(b*x+a))+1/4/b/c*((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/
(1+exp(2*c*(b*x+a)))*exp(3*c*(b*x+a))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.89

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{(2\,bcx+1)\cosh(bcx+ac) - (2\,bcx-1)\sinh(bcx+ac)}{4\,(bc\cosh(bcx+ac) - bc\sinh(bcx+ac))}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*((2*b*c*x + 1)*cosh(b*c*x + a*c) - (2*b*c*x - 1)*sinh(b*c*x + a*c))/(b
*c*cosh(b*c*x + a*c) - b*c*sinh(b*c*x + a*c))
```

Sympy [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 2.18 (sec) , antiderivative size = 163, normalized size of antiderivative = 2.20

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \begin{cases} x\sqrt{\cosh^2(ac)}e^{ac} & \text{for } b=0 \\ x & \text{for } c=0 \\ 0 & \text{for } a=-\frac{2bcx}{2} \\ -\frac{x\sqrt{\cosh^2(ac+bcx)}e^{ac}e^{bcx}\sinh(ac+bcx)}{2\cosh(ac+bcx)} + \frac{x\sqrt{\cosh^2(ac+bcx)}e^{ac}e^{bcx}}{2} + \frac{\sqrt{\cosh^2(ac+bcx)}e^{ac}e^{bcx}\sinh(ac+bcx)}{2bc\cosh(ac+bcx)} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)**2)**(1/2),x)
```

output
```
Piecewise((x*sqrt(cosh(a*c)**2)*exp(a*c), Eq(b, 0)), (x, Eq(c, 0)), (0, Eq
(a, -(2*b*c*x - I*pi)/(2*c))), (-x*sqrt(cosh(a*c + b*c*x)**2)*exp(a*c)*exp
(b*c*x)*sinh(a*c + b*c*x)/(2*cosh(a*c + b*c*x)) + x*sqrt(cosh(a*c + b*c*x)
**2)*exp(a*c)*exp(b*c*x)/2 + sqrt(cosh(a*c + b*c*x)**2)*exp(a*c)*exp(b*c*x
)*sinh(a*c + b*c*x)/(2*b*c*cosh(a*c + b*c*x)), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.39

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{1}{2}\,x + \frac{a}{2\,b} + \frac{e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
1/2*x + 1/2*a/b + 1/4*e^(2*b*c*x + 2*a*c)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.34

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{2\,bcx + e^{(2\,bcx+2\,ac)}}{4\,bc}$$

input
```
integrate(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*(2*b*c*x + e^(2*b*c*x + 2*a*c))/(b*c)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{\left(x\,\mathrm{e}^{a\,c+b\,c\,x}+\frac{\mathrm{e}^{3\,a\,c+3\,b\,c\,x}}{2\,b\,c}\right)\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1}$$

input
```
int(exp(c*(a + b*x))*(cosh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
((x*exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x)/(2*b*c))*((exp(a*c + b*c*x)/2
+ exp(- a*c - b*c*x)/2)^2)^(1/2))/(exp(2*a*c + 2*b*c*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.74

$$\int e^{c(a+bx)}\sqrt{\cosh^2(ac+bcx)}\,dx = \frac{e^{bcx+ac}(\cosh(bcx+ac)\,bcx+\cosh(bcx+ac)-\sinh(bcx+ac)\,bcx)}{2bc}$$

input
```
int(exp(c*(b*x+a))*(cosh(b*c*x+a*c)^2)^(1/2),x)
```

output
```
(e**(a*c + b*c*x)*(cosh(a*c + b*c*x)*b*c*x + cosh(a*c + b*c*x) - sinh(a*c
+ b*c*x)*b*c*x))/(2*b*c)
```

3.296 $\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 44

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\cosh(ac+bcx)\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

output
```
cosh(b*c*x+a*c)*ln(1+exp(2*c*(b*x+a)))/b/c/(cosh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\cosh(c(a+bx))\log\left(1+e^{2c(a+bx)}\right)}{bc\sqrt{\cosh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
(Cosh[c*(a + b*x)]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[c*(a + b*x)]
^2])
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {7271, 2720, 27, 240}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{2e^{c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{2\cosh(ac+bcx)\int \frac{e^{c(a+bx)}}{1+e^{2c(a+bx)}}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{240}$$

$$\frac{\log\left(e^{2c(a+bx)}+1\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/Sqrt[Cosh[a*c + b*c*x]^2],x]
```

output
```
(Cosh[a*c + b*c*x]*Log[1 + E^(2*c*(a + b*x))])/(b*c*Sqrt[Cosh[a*c + b*c*x]
^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 240
```
Int[(x_)/((a_) + (b_.)*(x_)^2), x_Symbol] :> Simp[Log[RemoveContent[a + b*x
^2, x]]/(2*b), x] /; FreeQ[{a, b}, x]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 0.62 (sec) , antiderivative size = 29, normalized size of antiderivative = 0.66

method	result	size
default	$\operatorname{csgn}\left(\cosh\left(c(bx+a)\right)\right)\left(x+\frac{\ln\left(\cosh\left(c(bx+a)\right)\right)}{cb}\right)$	29
risch	$\frac{\ln\left(\mathrm{e}^{2bcx}+\mathrm{e}^{-2ac}\right)\left(1+\mathrm{e}^{2c(bx+a)}\right)\mathrm{e}^{-c(bx+a)}}{bc\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}\mathrm{e}^{-2c(bx+a)}}}$	66

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
csgn(cosh(c*(b*x+a)))*(x+1/c/b*ln(cosh(c*(b*x+a))))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\log\left(\frac{2\cosh(bcx+ac)}{\cosh(bcx+ac)-\sinh(bcx+ac)}\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="fricas")
```

output
```
log(2*cosh(b*c*x + a*c)/(cosh(b*c*x + a*c) - sinh(b*c*x + a*c)))/(b*c)
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = e^{ac}\int \frac{e^{bcx}}{\sqrt{\cosh^2(ac+bcx)}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(1/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/sqrt(cosh(a*c + b*c*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="maxima")
```

output
```
log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.48

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{(2\,bcx+2\,ac)}+1\right)}{bc}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x, algorithm="giac")
```

output
```
log(e^(2*b*c*x + 2*a*c) + 1)/(b*c)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \int \frac{e^{c\,(a+b\,x)}}{\sqrt{\cosh\left(a\,c+b\,c\,x\right)^2}}\,dx$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2),x)
```

output
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(1/2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.50

$$\int \frac{e^{c(a+bx)}}{\sqrt{\cosh^2(ac+bcx)}}\,dx = \frac{\log\left(e^{2bcx+2ac}+1\right)}{bc}$$

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(1/2),x)
```

output
```
log(e**(2*a*c + 2*b*c*x) + 1)/(b*c)
```

3.297 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 56

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = \frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

output
```
2*exp(4*c*(b*x+a))*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+
a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = \frac{4e^{5c(a+bx)}\sqrt{\cosh^2(c(a+bx))}}{bc\left(1+e^{2c(a+bx)}\right)^3}$$

input
```
Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(4*E^(5*c*(a + b*x))*Sqrt[Cosh[c*(a + b*x)]^2])/(b*c*(1 + E^(2*c*(a + b*x)
))^3)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {7271, 2720, 27, 242}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^3(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{8e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{8\cosh(ac+bcx)\int \frac{e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^3}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{242}$$

$$\frac{2e^{4c(a+bx)}\cosh(ac+bcx)}{bc\left(e^{2c(a+bx)}+1\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(3/2),x]
```

output
```
(2*E^(4*c*(a + b*x))*Cosh[a*c + b*c*x])/(b*c*(1 + E^(2*c*(a + b*x)))^2*Sqr
t[Cosh[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 242
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^
(m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] /; FreeQ[{a, b, c, m, p}, x
] && EqQ[m + 2*p + 3, 0] && NeQ[m, -1]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 1.28 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.23

method	result	size
risch	$-\frac{2\left(2\,{\mathrm e}^{2c(bx+a)}+1\right){\mathrm e}^{-c(bx+a)}}{bc\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,\left(1+{\mathrm e}^{2c(bx+a)}\right)}$	69

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-2/b/c*(2*exp(2*c*(b*x+a))+1)/((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(
1/2)/(1+exp(2*c*(b*x+a)))*exp(-c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 120 vs. 2(52) = 104.

Time = 0.07 (sec) , antiderivative size = 120, normalized size of antiderivative = 2.14

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{2\,(3\,\cosh\left(bcx+ac\right)+\sinh\left(bcx+ac\right))}{bc\cosh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^2+bc\sinh\left(bcx+ac\right)^3+3\,bc\cosh\left(bcx+ac\right)+ (}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="fricas")
```

output
```
-2*(3*cosh(b*c*x + a*c) + sinh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^3 + 3*
b*c*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2 + b*c*sinh(b*c*x + a*c)^3 + 3*b*
c*cosh(b*c*x + a*c) + (3*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c))
```

Sympy [F]

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = e^{ac}\int \frac{e^{bcx}}{\left(\cosh^2\left(ac+bcx\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(3/2),x)
```

output
```
exp(a*c)*Integral(exp(b*c*x)/(cosh(a*c + b*c*x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.50

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,e^{(2\,bcx+2\,ac)}}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)} - \frac{2}{bc(e^{(4\,bcx+4\,ac)}+2\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="maxima")
```

output
```
-4*e^(2*b*c*x + 2*a*c)/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) +
 1)) - 2/(b*c*(e^(4*b*c*x + 4*a*c) + 2*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.68

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{2\left(2\,e^{(2\,bcx+2\,ac)}+1\right)}{bc(e^{(2\,bcx+2\,ac)}+1)^2}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*(2*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^2)
```

Mupad [B] (verification not implemented)

Time = 1.94 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.36

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = -\frac{4\,\mathrm{e}^{a\,c+b\,c\,x}\,\left(2\,\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)\,\sqrt{{\left(\frac{\mathrm{e}^{a\,c+b\,c\,x}}{2}+\frac{\mathrm{e}^{-a\,c-b\,c\,x}}{2}\right)}^2}}{b\,c\,{\left(\mathrm{e}^{2\,a\,c+2\,b\,c\,x}+1\right)}^3}$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(3/2),x)
```

output
```
-(4*exp(a*c + b*c*x)*(2*exp(2*a*c + 2*b*c*x) + 1)*((exp(a*c + b*c*x)/2 + e
xp(- a*c - b*c*x)/2)^2)^(1/2))/(b*c*(exp(2*a*c + 2*b*c*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.89

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{3/2}}\,dx = \frac{2e^{4bcx+4ac}}{bc\left(e^{4bcx+4ac}+2e^{2bcx+2ac}+1\right)}$$

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(3/2),x)
```

output
```
(2*e**(4*a*c + 4*b*c*x))/(b*c*(e**(4*a*c + 4*b*c*x) + 2*e**(2*a*c + 2*b*c*
x) + 1))
```

3.298 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx$

Optimal result

Integrand size = 25, antiderivative size = 141

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}} + \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}} - \frac{8\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^2\sqrt{\cosh^2(ac+bcx)}}$$

output
```
-4*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)+32
/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)-8*
cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^2/(cosh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.51

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(1+4e^{2c(a+bx)}+6e^{4c(a+bx)}\right)\cosh(c(a+bx))}{3bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(-4*(1 + 4*E^(2*c*(a + b*x)) + 6*E^(4*c*(a + b*x)))*Cosh[c*(a + b*x)])/(3*
b*c*(1 + E^(2*c*(a + b*x)))^4*Sqrt[Cosh[c*(a + b*x)]^2])
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.62, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^5(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{32e^{5c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{32\cosh(ac+bcx)\int \frac{e^{5c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{243}$$

$$\frac{16\cosh(ac+bcx)\int \frac{e^{2c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^5}de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{53}$$

$$\frac{16\cosh(ac+bcx)\int \left(\frac{1}{\left(1+e^{2c(a+bx)}\right)^3}-\frac{2}{\left(1+e^{2c(a+bx)}\right)^4}+\frac{1}{\left(1+e^{2c(a+bx)}\right)^5}\right)de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2009}$$

$$\frac{16\left(-\frac{1}{2(e^{2c(a+bx)}+1)^2}+\frac{2}{3(e^{2c(a+bx)}+1)^3}-\frac{1}{4(e^{2c(a+bx)}+1)^4}\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(5/2),x]
```

output
```
(16*(-1/4*1/(1 + E^(2*c*(a + b*x)))^4 + 2/(3*(1 + E^(2*c*(a + b*x)))^3) -
1/(2*(1 + E^(2*c*(a + b*x)))^2))*Cosh[a*c + b*c*x])/(b*c*Sqrt[Cosh[a*c + b
*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.57

method	result	size
risch	$-\frac{4\left(6\,\mathrm{e}^{4c(bx+a)}+4\,\mathrm{e}^{2c(bx+a)}+1\right)\mathrm{e}^{-c(bx+a)}}{3bc\sqrt{\left(1+\mathrm{e}^{2c(bx+a)}\right)^{2}\mathrm{e}^{-2c(bx+a)}}\,\left(1+\mathrm{e}^{2c(bx+a)}\right)^{3}}$	80

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-4/3/b/c*(6*exp(4*c*(b*x+a))+4*exp(2*c*(b*x+a))+1)/((1+exp(2*c*(b*x+a)))^2
*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^3*exp(-c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(130) = 260.

Time = 0.09 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.23

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx =$$

$$-\frac{}{3\left(bc\cosh\left(bcx+ac\right)^{6}+6\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^{5}+bc\sinh\left(bcx+ac\right)^{6}+4\,bc\cosh\left(bcx+ac\right)^{4}\right.}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="fricas")
```

output
```
-4/3*(7*cosh(b*c*x + a*c)^2 + 10*cosh(b*c*x + a*c)*sinh(b*c*x + a*c) + 7*s
inh(b*c*x + a*c)^2 + 4)/(b*c*cosh(b*c*x + a*c)^6 + 6*b*c*cosh(b*c*x + a*c)
*sinh(b*c*x + a*c)^5 + b*c*sinh(b*c*x + a*c)^6 + 4*b*c*cosh(b*c*x + a*c)^4
 + (15*b*c*cosh(b*c*x + a*c)^2 + 4*b*c)*sinh(b*c*x + a*c)^4 + 7*b*c*cosh(b
*c*x + a*c)^2 + 4*(5*b*c*cosh(b*c*x + a*c)^3 + 4*b*c*cosh(b*c*x + a*c))*si
nh(b*c*x + a*c)^3 + (15*b*c*cosh(b*c*x + a*c)^4 + 24*b*c*cosh(b*c*x + a*c)
^2 + 7*b*c)*sinh(b*c*x + a*c)^2 + 4*b*c + 2*(3*b*c*cosh(b*c*x + a*c)^5 + 8
*b*c*cosh(b*c*x + a*c)^3 + 5*b*c*cosh(b*c*x + a*c))*sinh(b*c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(5/2),x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.48

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\,e^{(4\,bcx+4\,ac)}}{bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{16\,e^{(2\,bcx+2\,ac)}}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)} - \frac{4}{3\,bc(e^{(8\,bcx+8\,ac)}+4\,e^{(6\,bcx+6\,ac)}+6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="maxima")
```

output
```
-8*e^(4*b*c*x + 4*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) +
 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 16/3*e^(2*b*c*x + 2
*a*c)/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4
*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)) - 4/3/(b*c*(e^(8*b*c*x + 8*a*c) + 4*e^
(6*b*c*x + 6*a*c) + 6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.36

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{4\left(6\,e^{(4\,bcx+4\,ac)}+4\,e^{(2\,bcx+2\,ac)}+1\right)}{3\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^4}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*(6*e^(4*b*c*x + 4*a*c) + 4*e^(2*b*c*x + 2*a*c) + 1)/(b*c*(e^(2*b*c*x
+ 2*a*c) + 1)^4)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.63

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = -\frac{8\,e^{a\,c+b\,c\,x}\,\sqrt{{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)}^2}\,\left(4\,e^{2\,a\,c+2\,b\,c\,x}+6\,e^{4\,a\,c+4\,b\,c\,x}+1\right)}{3\,b\,c\,{\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)}^5}$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(5/2),x)
```

output
```
-(8*exp(a*c + b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2)
*(4*exp(2*a*c + 2*b*c*x) + 6*exp(4*a*c + 4*b*c*x) + 1))/(3*b*c*(exp(2*a*c
+ 2*b*c*x) + 1)^5)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.68

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{5/2}}\,dx = \frac{-8e^{4bcx+4ac}-\frac{16e^{2bcx+2ac}}{3}-\frac{4}{3}}{bc\left(e^{8bcx+8ac}+4e^{6bcx+6ac}+6e^{4bcx+4ac}+4e^{2bcx+2ac}+1\right)}$$

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(5/2),x)
```

output
```
(4*( - 6*e**(4*a*c + 4*b*c*x) - 4*e**(2*a*c + 2*b*c*x) - 1))/(3*b*c*(e**(8
*a*c + 8*b*c*x) + 4*e**(6*a*c + 6*b*c*x) + 6*e**(4*a*c + 4*b*c*x) + 4*e**(
2*a*c + 2*b*c*x) + 1))
```

3.299 $\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}} \, dx$

Optimal result

Integrand size = 25, antiderivative size = 191

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}} \, dx = \frac{32\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(ac+bcx)}} - \frac{192\cosh(ac+bcx)}{5bc\left(1+e^{2c(a+bx)}\right)^5\sqrt{\cosh^2(ac+bcx)}} + \frac{48\cosh(ac+bcx)}{bc\left(1+e^{2c(a+bx)}\right)^4\sqrt{\cosh^2(ac+bcx)}} - \frac{64\cosh(ac+bcx)}{3bc\left(1+e^{2c(a+bx)}\right)^3\sqrt{\cosh^2(ac+bcx)}}$$

output
```
32/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^6/(cosh(b*c*x+a*c)^2)^(1/2)-
192/5*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^5/(cosh(b*c*x+a*c)^2)^(1/2)
+48*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^4/(cosh(b*c*x+a*c)^2)^(1/2)-6
4/3*cosh(b*c*x+a*c)/b/c/(1+exp(2*c*(b*x+a)))^3/(cosh(b*c*x+a*c)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.44

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\left(1+6e^{2c(a+bx)}+15e^{4c(a+bx)}+20e^{6c(a+bx)}\right)\cosh(c(a+bx))}{15bc\left(1+e^{2c(a+bx)}\right)^6\sqrt{\cosh^2(c(a+bx))}}$$

input
```
Integrate[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(-16*(1 + 6*E^(2*c*(a + b*x)) + 15*E^(4*c*(a + b*x)) + 20*E^(6*c*(a + b*x)
))*Cosh[c*(a + b*x)])/(15*b*c*(1 + E^(2*c*(a + b*x)))^6*Sqrt[Cosh[c*(a + b
*x)]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.55, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {7271, 2720, 27, 243, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx$$

$$\Big\downarrow \text{7271}$$

$$\frac{\cosh(ac+bcx)\int e^{c(a+bx)}\text{sech}^7(ac+bxc)dx}{\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{2720}$$

$$\frac{\cosh(ac+bcx)\int \frac{128e^{7c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{128\cosh(ac+bcx)\int \frac{e^{7c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 243}$$

$$\frac{64\cosh(ac+bcx)\int \frac{e^{3c(a+bx)}}{\left(1+e^{2c(a+bx)}\right)^7}de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 53}$$

$$\frac{64\cosh(ac+bcx)\int \left(\frac{1}{\left(1+e^{2c(a+bx)}\right)^4}-\frac{3}{\left(1+e^{2c(a+bx)}\right)^5}+\frac{3}{\left(1+e^{2c(a+bx)}\right)^6}-\frac{1}{\left(1+e^{2c(a+bx)}\right)^7}\right)de^{2c(a+bx)}}{bc\sqrt{\cosh^2(ac+bcx)}}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{64\left(-\frac{1}{3\left(e^{2c(a+bx)}+1\right)^3}+\frac{3}{4\left(e^{2c(a+bx)}+1\right)^4}-\frac{3}{5\left(e^{2c(a+bx)}+1\right)^5}+\frac{1}{6\left(e^{2c(a+bx)}+1\right)^6}\right)\cosh(ac+bcx)}{bc\sqrt{\cosh^2(ac+bcx)}}$$

input
```
Int[E^(c*(a + b*x))/(Cosh[a*c + b*c*x]^2)^(7/2),x]
```

output
```
(64*(1/(6*(1 + E^(2*c*(a + b*x)))^6) - 3/(5*(1 + E^(2*c*(a + b*x)))^5) + 3
/(4*(1 + E^(2*c*(a + b*x)))^4) - 1/(3*(1 + E^(2*c*(a + b*x)))^3))*Cosh[a*c
 + b*c*x])/(b*c*Sqrt[Cosh[a*c + b*c*x]^2])
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 243
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[1/2   Subst[In
t[x^((m - 1)/2)*(a + b*x)^p, x], x, x^2], x] /; FreeQ[{a, b, m, p}, x] && I
ntegerQ[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 7271
```
Int[(u_.)*((a_.)*(v_)^(m_.))^(p_), x_Symbol] :> Simp[a^IntPart[p]*((a*v^m)^
FracPart[p]/v^(m*FracPart[p]))   Int[u*v^(m*p), x], x] /; FreeQ[{a, m, p},
x] &&  !IntegerQ[p] &&  !FreeQ[v, x] &&  !(EqQ[a, 1] && EqQ[m, 1]) &&  !(Eq
Q[v, x] && EqQ[m, 1])
```

Maple [A] (verified)

Time = 1.33 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.48

method	result	size
risch	$-\frac{16\left(20\,{\mathrm e}^{6c(bx+a)}+15\,{\mathrm e}^{4c(bx+a)}+6\,{\mathrm e}^{2c(bx+a)}+1\right){\mathrm e}^{-c(bx+a)}}{15bc\sqrt{\left(1+{\mathrm e}^{2c(bx+a)}\right)^{2}{\mathrm e}^{-2c(bx+a)}}\,\left(1+{\mathrm e}^{2c(bx+a)}\right)^{5}}$	91

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x,method=_RETURNVERBOSE)
```

output
```
-16/15/b/c*(20*exp(6*c*(b*x+a))+15*exp(4*c*(b*x+a))+6*exp(2*c*(b*x+a))+1)/
((1+exp(2*c*(b*x+a)))^2*exp(-2*c*(b*x+a)))^(1/2)/(1+exp(2*c*(b*x+a)))^5*ex
p(-c*(b*x+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 589 vs. $2(173) = 346$.

Time = 0.08 (sec) , antiderivative size = 589, normalized size of antiderivative = 3.08

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx =$$

$$-\frac{}{15\left(bc\cosh\left(bcx+ac\right)^9+9\,bc\cosh\left(bcx+ac\right)\sinh\left(bcx+ac\right)^8+bc\sinh\left(bcx+ac\right)^9+6\,bc\cosh\left(bcx+ac\right)\right.}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="fricas")
```

output
```
-16/15*(21*cosh(b*c*x + a*c)^3 + 63*cosh(b*c*x + a*c)*sinh(b*c*x + a*c)^2
+ 19*sinh(b*c*x + a*c)^3 + 3*(19*cosh(b*c*x + a*c)^2 + 3)*sinh(b*c*x + a*c
) + 21*cosh(b*c*x + a*c))/(b*c*cosh(b*c*x + a*c)^9 + 9*b*c*cosh(b*c*x + a*
c)*sinh(b*c*x + a*c)^8 + b*c*sinh(b*c*x + a*c)^9 + 6*b*c*cosh(b*c*x + a*c)
^7 + 6*(6*b*c*cosh(b*c*x + a*c)^2 + b*c)*sinh(b*c*x + a*c)^7 + 15*b*c*cosh
(b*c*x + a*c)^5 + 42*(2*b*c*cosh(b*c*x + a*c)^3 + b*c*cosh(b*c*x + a*c))*s
inh(b*c*x + a*c)^6 + 3*(42*b*c*cosh(b*c*x + a*c)^4 + 42*b*c*cosh(b*c*x + a
*c)^2 + 5*b*c)*sinh(b*c*x + a*c)^5 + 21*b*c*cosh(b*c*x + a*c)^3 + 3*(42*b*
c*cosh(b*c*x + a*c)^5 + 70*b*c*cosh(b*c*x + a*c)^3 + 25*b*c*cosh(b*c*x + a
*c))*sinh(b*c*x + a*c)^4 + (84*b*c*cosh(b*c*x + a*c)^6 + 210*b*c*cosh(b*c*
x + a*c)^4 + 150*b*c*cosh(b*c*x + a*c)^2 + 19*b*c)*sinh(b*c*x + a*c)^3 + 2
1*b*c*cosh(b*c*x + a*c) + 3*(12*b*c*cosh(b*c*x + a*c)^7 + 42*b*c*cosh(b*c*
x + a*c)^5 + 50*b*c*cosh(b*c*x + a*c)^3 + 21*b*c*cosh(b*c*x + a*c))*sinh(b
*c*x + a*c)^2 + 3*(3*b*c*cosh(b*c*x + a*c)^8 + 14*b*c*cosh(b*c*x + a*c)^6
+ 25*b*c*cosh(b*c*x + a*c)^4 + 19*b*c*cosh(b*c*x + a*c)^2 + 3*b*c)*sinh(b*
c*x + a*c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \text{Timed out}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)**2)**(7/2),x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(173) = 346$.

Time = 0.04 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.02

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx =$$
$$-\frac{64\,e^{(6\,bcx+6\,ac)}}{3\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$
$$-\frac{16\,e^{(4\,bcx+4\,ac)}}{bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$
$$-\frac{32\,e^{(2\,bcx+2\,ac)}}{5\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$
$$-\frac{16}{15\,bc(e^{(12\,bcx+12\,ac)}+6\,e^{(10\,bcx+10\,ac)}+15\,e^{(8\,bcx+8\,ac)}+20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1)}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="maxima")
```

output
```
-64/3*e^(6*b*c*x + 6*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16*e^(4*b*c*x + 4*a*c)/(b*c*(e^(12*
b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^
(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) -
 32/5*e^(2*b*c*x + 2*a*c)/(b*c*(e^(12*b*c*x + 12*a*c) + 6*e^(10*b*c*x + 10
*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x +
4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1)) - 16/15/(b*c*(e^(12*b*c*x + 12*a*c) +
 6*e^(10*b*c*x + 10*a*c) + 15*e^(8*b*c*x + 8*a*c) + 20*e^(6*b*c*x + 6*a*c)
 + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2*a*c) + 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.34

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = -\frac{16\left(20\,e^{(6\,bcx+6\,ac)}+15\,e^{(4\,bcx+4\,ac)}+6\,e^{(2\,bcx+2\,ac)}+1\right)}{15\,bc\left(e^{(2\,bcx+2\,ac)}+1\right)^6}$$

input
```
integrate(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x, algorithm="giac")
```

output
```
-16/15*(20*e^(6*b*c*x + 6*a*c) + 15*e^(4*b*c*x + 4*a*c) + 6*e^(2*b*c*x + 2
*a*c) + 1)/(b*c*(e^(2*b*c*x + 2*a*c) + 1)^6)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.81

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \frac{96\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^4} - \frac{128\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^3} - \frac{384\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{5\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^5} + \frac{64\,e^{2\,a\,c+2\,b\,c\,x}\sqrt{\left(\frac{e^{a\,c+b\,c\,x}}{2}+\frac{e^{-a\,c-b\,c\,x}}{2}\right)^2}}{3\,b\,c\,\left(e^{a\,c+b\,c\,x}+e^{3\,a\,c+3\,b\,c\,x}\right)\left(e^{2\,a\,c+2\,b\,c\,x}+1\right)^6}$$

input
```
int(exp(c*(a + b*x))/(cosh(a*c + b*c*x)^2)^(7/2),x)
```

output
```
(96*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(
1/2))/(b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x)
 + 1)^4) - (128*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 + exp(- a*c - b*
c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b*c*x))*(exp(2*
a*c + 2*b*c*x) + 1)^3) - (384*exp(2*a*c + 2*b*c*x)*((exp(a*c + b*c*x)/2 +
exp(- a*c - b*c*x)/2)^2)^(1/2))/(5*b*c*(exp(a*c + b*c*x) + exp(3*a*c + 3*b
*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^5) + (64*exp(2*a*c + 2*b*c*x)*((exp(a*c
+ b*c*x)/2 + exp(- a*c - b*c*x)/2)^2)^(1/2))/(3*b*c*(exp(a*c + b*c*x) + ex
p(3*a*c + 3*b*c*x))*(exp(2*a*c + 2*b*c*x) + 1)^6)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.72

$$\int \frac{e^{c(a+bx)}}{\cosh^2(ac+bcx)^{7/2}}\,dx = \frac{-\frac{64e^{6bcx+6ac}}{3}-16e^{4bcx+4ac}-\frac{32e^{2bcx+2ac}}{5}-\frac{16}{15}}{bc\left(e^{12bcx+12ac}+6e^{10bcx+10ac}+15e^{8bcx+8ac}+20e^{6bcx+6ac}+15e^{4bcx+4ac}+6e^{2bcx+2}\right.}$$

input
```
int(exp(c*(b*x+a))/(cosh(b*c*x+a*c)^2)^(7/2),x)
```

output
```
(16*( - 20*e**(6*a*c + 6*b*c*x) - 15*e**(4*a*c + 4*b*c*x) - 6*e**(2*a*c +
2*b*c*x) - 1))/(15*b*c*(e**(12*a*c + 12*b*c*x) + 6*e**(10*a*c + 10*b*c*x)
+ 15*e**(8*a*c + 8*b*c*x) + 20*e**(6*a*c + 6*b*c*x) + 15*e**(4*a*c + 4*b*c
*x) + 6*e**(2*a*c + 2*b*c*x) + 1))
```

3.300 $\int e^x \cosh(a+bx)\,dx$

Optimal result

Integrand size = 10, antiderivative size = 41

$$\int e^x \cosh(a+bx)\,dx = \frac{e^x \cosh(a+bx)}{1-b^2} - \frac{be^x \sinh(a+bx)}{1-b^2}$$

output
```
exp(x)*cosh(b*x+a)/(-b^2+1)-b*exp(x)*sinh(b*x+a)/(-b^2+1)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^x \cosh(a+bx)\,dx = \frac{e^x(-\cosh(a+bx)+b\sinh(a+bx))}{-1+b^2}$$

input
```
Integrate[E^x*Cosh[a + b*x],x]
```

output
```
(E^x*(-Cosh[a + b*x] + b*Sinh[a + b*x]))/(-1 + b^2)
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {5998}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh(a+bx)\,dx$$

$$\Big\downarrow \text{ 5998}$$

$$\frac{e^x\cosh(a+bx)}{1-b^2}-\frac{be^x\sinh(a+bx)}{1-b^2}$$

input
```
Int[E^x*Cosh[a + b*x],x]
```

output
```
(E^x*Cosh[a + b*x])/(1 - b^2) - (b*E^x*Sinh[a + b*x])/(1 - b^2)
```

Defintions of rubi rules used

rule 5998
```
Int[Cosh[(d_.) + (e_.)*(x_)]*(F_)^((c_.)*((a_.) + (b_.)*(x_))), x_Symbol] :
> Simp[(-b)*c*Log[F]*F^(c*(a + b*x))*(Cosh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2
)), x] + Simp[e*F^(c*(a + b*x))*(Sinh[d + e*x]/(e^2 - b^2*c^2*Log[F]^2)), x
] /; FreeQ[{F, a, b, c, d, e}, x] && NeQ[e^2 - b^2*c^2*Log[F]^2, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

method	result	size
parallelrisch	$\frac{(b\sinh(bx+a)-\cosh(bx+a))\mathrm{e}^{x}}{b^{2}-1}$	28
risch	$\frac{\mathrm{e}^{bx+a+x}}{2+2b}-\frac{\mathrm{e}^{-bx-a+x}}{2(b-1)}$	33
orering	$-\frac{2\,\mathrm{e}^{x}\cosh(bx+a)}{b^{2}-1}+\frac{\mathrm{e}^{x}\cosh(bx+a)+\mathrm{e}^{x}b\sinh(bx+a)}{b^{2}-1}$	47
default	$\frac{\sinh(x(b-1)+a)}{2b-2}+\frac{\sinh((1+b)x+a)}{2+2b}-\frac{\cosh(x(b-1)+a)}{2(b-1)}+\frac{\cosh((1+b)x+a)}{2+2b}$	62

input
```
int(exp(x)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
(b*sinh(b*x+a)-cosh(b*x+a))*exp(x)/(b^2-1)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^{x}\cosh(a+bx)\,dx = -\frac{\cosh(bx+a)\cosh(x)-(b\cosh(x)+b\sinh(x))\sinh(bx+a)+\cosh(bx+a)\sinh(x)}{b^{2}-1}$$

input
```
integrate(exp(x)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
-(cosh(b*x + a)*cosh(x) - (b*cosh(x) + b*sinh(x))*sinh(b*x + a) + cosh(b*x
 + a)*sinh(x))/(b^2 - 1)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(31) = 62.

Time = 0.24 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.41

$$\int e^{x}\cosh(a+bx)\,dx = \begin{cases} \frac{xe^{x}\sinh(a-x)}{2}+\frac{xe^{x}\cosh(a-x)}{2}+\frac{e^{x}\cosh(a-x)}{2} & \text{for } b=-1 \\ -\frac{xe^{x}\sinh(a+x)}{2}+\frac{xe^{x}\cosh(a+x)}{2}+\frac{e^{x}\cosh(a+x)}{2} & \text{for } b=1 \\ \frac{be^{x}\sinh(a+bx)}{b^{2}-1}-\frac{e^{x}\cosh(a+bx)}{b^{2}-1} & \text{otherwise} \end{cases}$$

input
```
integrate(exp(x)*cosh(b*x+a),x)
```

output
```
Piecewise((x*exp(x)*sinh(a - x)/2 + x*exp(x)*cosh(a - x)/2 + exp(x)*cosh(a
 - x)/2, Eq(b, -1)), (-x*exp(x)*sinh(a + x)/2 + x*exp(x)*cosh(a + x)/2 + e
xp(x)*cosh(a + x)/2, Eq(b, 1)), (b*exp(x)*sinh(a + b*x)/(b**2 - 1) - exp(x
)*cosh(a + b*x)/(b**2 - 1), True))
```

Maxima [F(-2)]

Exception generated.

$$\int e^x \cosh(a+bx)\,dx = \text{Exception raised: ValueError}$$

input
```
integrate(exp(x)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(-b>0)', see `assume?` for more d
etails)Is
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.78

$$\int e^x \cosh(a+bx)\,dx = \frac{e^{(bx+a+x)}}{2\,(b+1)} - \frac{e^{(-bx-a+x)}}{2\,(b-1)}$$

input
```
integrate(exp(x)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/2*e^(b*x + a + x)/(b + 1) - 1/2*e^(-b*x - a + x)/(b - 1)
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int e^x \cosh(a+bx)\,dx = -\frac{e^{x-a-bx}\left(b+e^{2a+2bx}-b\,e^{2a+2bx}+1\right)}{2\,(b^2-1)}$$

input
```
int(cosh(a + b*x)*exp(x),x)
```

output
```
-(exp(x - a - b*x)*(b + exp(2*a + 2*b*x) - b*exp(2*a + 2*b*x) + 1))/(2*(b^
2 - 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.68

$$\int e^x \cosh(a+bx)\,dx = \frac{e^x(-\cosh(bx+a)+\sinh(bx+a)\,b)}{b^2-1}$$

input
```
int(exp(x)*cosh(b*x+a),x)
```

output
```
(e**x*( - cosh(a + b*x) + sinh(a + b*x)*b))/(b**2 - 1)
```

3.301 $\int e^x \cosh(a+cx^2)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 85

$$\int e^x \cosh(a+cx^2)\,dx = -\frac{e^{-a+\frac{1}{4c}}\sqrt{\pi}\,\text{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}} + \frac{e^{a-\frac{1}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
-1/4*exp(-a+1/4/c)*Pi^(1/2)*erf(1/2*(-2*c*x+1)/c^(1/2))/c^(1/2)+1/4*exp(a-
1/4/c)*Pi^(1/2)*erfi(1/2*(2*c*x+1)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.93

$$\int e^x \cosh(a+cx^2)\,dx = \frac{e^{-\frac{1}{4}/c}\sqrt{\pi}\left(e^{\frac{1}{2}/c}\text{erf}\left(\frac{-1+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\text{erfi}\left(\frac{1+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Cosh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*(E^(1/(2*c))*Erf[(-1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] - Sinh[a]) +
 Erfi[(1 + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sqrt[c]*E^(1/(4*c)
))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh\left(a+cx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-a-cx^2+x}+\frac{1}{2}e^{a+cx^2+x}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{a-\frac{1}{4c}}\text{erfi}\left(\frac{2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{1}{4c}-a}\text{erf}\left(\frac{1-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Cosh[a + c*x^2],x]
```

output
```
-1/4*(E^(-a + 1/(4*c))*Sqrt[Pi]*Erf[(1 - 2*c*x)/(2*Sqrt[c])])/Sqrt[c] + (E
^(a - 1/(4*c))*Sqrt[Pi]*Erfi[(1 + 2*c*x)/(2*Sqrt[c])])/(4*Sqrt[c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-1}{4c}}\,\mathrm{erf}\left(\sqrt{c}\,x-\frac{1}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-1}{4c}}\,\mathrm{erf}\left(\sqrt{-c}\,x-\frac{1}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	72

input
```
int(exp(x)*cosh(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*(4*a*c-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2/c^(1/2))+1/4*
Pi^(1/2)*exp(1/4*(4*a*c-1)/c)/(-c)^(1/2)*erf((-c)^(1/2)*x-1/2/(-c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.22

$$\int e^x \cosh\left(a+cx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)+\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{(2\,cx+1)\sqrt{-c}}{2\,c}\right)-\sqrt{\pi}\sqrt{c}\left(\cosh\left(\frac{4\,ac-1}{4\,c}\right)-\sinh\left(\frac{4\,ac-1}{4\,c}\right)\right)\mathrm{erf}\left(\frac{2}{2}\right.}{4\,c}$$

input
```
integrate(exp(x)*cosh(c*x^2+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c)*(cosh(1/4*(4*a*c - 1)/c) + sinh(1/4*(4*a*c - 1)/c)
)*erf(1/2*(2*c*x + 1)*sqrt(-c)/c) - sqrt(pi)*sqrt(c)*(cosh(1/4*(4*a*c - 1)
/c) - sinh(1/4*(4*a*c - 1)/c))*erf(1/2*(2*c*x - 1)/sqrt(c)))/c
```

Sympy [F]

$$\int e^x \cosh\left(a + cx^2\right) \, dx = \int e^x \cosh\left(a + cx^2\right) dx$$

input
```
integrate(exp(x)*cosh(c*x**2+a),x)
```

output
```
Integral(exp(x)*cosh(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.76

$$\int e^x \cosh\left(a + cx^2\right) \, dx = \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x - \frac{1}{2\sqrt{-c}}\right)e^{\left(a - \frac{1}{4c}\right)}}{4\sqrt{-c}} + \frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x - \frac{1}{2\sqrt{c}}\right)e^{\left(-a + \frac{1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2/sqrt(-c))*e^(a - 1/4/c)/sqrt(-c) + 1/4*s
qrt(pi)*erf(sqrt(c)*x - 1/2/sqrt(c))*e^(-a + 1/4/c)/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.86

$$\int e^x \cosh\left(a+cx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{1}{c}\right)\right)e^{\left(\frac{4ac-1}{4c}\right)}}{4\sqrt{-c}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x-\frac{1}{c}\right)\right)e^{\left(-\frac{4ac-1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + 1/c))*e^(1/4*(4*a*c - 1)/c)/sqrt(-c
) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x - 1/c))*e^(-1/4*(4*a*c - 1)/c)/sqrt
(c)
```

Mupad [F(-1)]

Timed out.

$$\int e^x \cosh\left(a+cx^2\right)\,dx = \int e^x \cosh\left(c\,x^2+a\right)\,dx$$

input
```
int(exp(x)*cosh(a + c*x^2),x)
```

output
```
int(exp(x)*cosh(a + c*x^2), x)
```

Reduce [F]

$$\int e^x \cosh\left(a+cx^2\right)\,dx = \int e^x \cosh\left(c\,x^2+a\right)dx$$

input
```
int(exp(x)*cosh(c*x^2+a),x)
```

output
```
int(e**x*cosh(a + c*x**2),x)
```

3.302 $\int e^x \cosh(a+bx+cx^2)\,dx$

Optimal result

Integrand size = 15, antiderivative size = 101

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx = -\frac{e^{-a+\frac{(1-b)^2}{4c}}\sqrt{\pi}\,\mathrm{erf}\left(\frac{1-b-2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}+\frac{e^{a-\frac{(1+b)^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

output
```
-1/4*exp(-a+1/4*(1-b)^2/c)*Pi^(1/2)*erf(1/2*(-2*c*x-b+1)/c^(1/2))/c^(1/2)+
1/4*exp(a-1/4*(1+b)^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b+1)/c^(1/2))/c^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx$$
$$=\frac{e^{-\frac{(1+b)^2}{4c}}\sqrt{\pi}\left(e^{\frac{1+b^2}{2c}}\mathrm{erf}\left(\frac{-1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)-\sinh(a))+\mathrm{erfi}\left(\frac{1+b+2cx}{2\sqrt{c}}\right)(\cosh(a)+\sinh(a))\right)}{4\sqrt{c}}$$

input
```
Integrate[E^x*Cosh[a + b*x + c*x^2],x]
```

output
```
(Sqrt[Pi]*(E^((1 + b^2)/(2*c))*Erf[(-1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a]
- Sinh[a]) + Erfi[(1 + b + 2*c*x)/(2*Sqrt[c])]*(Cosh[a] + Sinh[a])))/(4*Sq
rt[c]*E^((1 + b)^2/(4*c)))
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-a+(1-b)x-cx^2}+\frac{1}{2}e^{a+(b+1)x+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{a-\frac{(b+1)^2}{4c}}\text{erfi}\left(\frac{b+2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}e^{\frac{(1-b)^2}{4c}-a}\text{erf}\left(\frac{-b-2cx+1}{2\sqrt{c}}\right)}{4\sqrt{c}}$$

input
```
Int[E^x*Cosh[a + b*x + c*x^2],x]
```

output
```
-1/4*(E^(-a + (1 - b)^2/(4*c))*Sqrt[Pi]*Erf[(1 - b - 2*c*x)/(2*Sqrt[c])])/
Sqrt[c] + (E^(a - (1 + b)^2/(4*c))*Sqrt[Pi]*Erfi[(1 + b + 2*c*x)/(2*Sqrt[c
])])/(4*Sqrt[c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.25 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.96

method	result	size
risch	$\frac{\sqrt{\pi}\, e^{-\frac{4ac-b^2+2b-1}{4c}} \operatorname{erf}\left(\sqrt{c}\,x-\frac{1-b}{2\sqrt{c}}\right)}{4\sqrt{c}}-\frac{\sqrt{\pi}\, e^{\frac{4ac-b^2-2b-1}{4c}} \operatorname{erf}\left(-\sqrt{-c}\,x+\frac{1+b}{2\sqrt{-c}}\right)}{4\sqrt{-c}}$	97

input
```
int(exp(x)*cosh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2+2*b-1)/c)/c^(1/2)*erf(c^(1/2)*x-1/2*(1-b)
/c^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2-2*b-1)/c)/(-c)^(1/2)*erf(-(-c)^(
1/2)*x+1/2*(1+b)/(-c)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.29

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{-c}\left(\cosh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)+\sinh\left(-\frac{b^2-4\,ac+2\,b+1}{4\,c}\right)\right)\operatorname{erf}\left(\frac{(2\,cx+b+1)\sqrt{-c}}{2\,c}\right)-\sqrt{\pi}\sqrt{c}\left(\cosh\left(-\frac{b^2-4\,ac-}{4\,c}\right.\right.}{4\,c}$$

input
```
integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-c)*(cosh(-1/4*(b^2 - 4*a*c + 2*b + 1)/c) + sinh(-1/4*
(b^2 - 4*a*c + 2*b + 1)/c))*erf(1/2*(2*c*x + b + 1)*sqrt(-c)/c) - sqrt(pi)
*sqrt(c)*(cosh(-1/4*(b^2 - 4*a*c - 2*b + 1)/c) - sinh(-1/4*(b^2 - 4*a*c -
2*b + 1)/c))*erf(1/2*(2*c*x + b - 1)/sqrt(c)))/c
```

Sympy [F]

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx=\int e^x \cosh\left(a+bx+cx^2\right)dx$$

input
```
integrate(exp(x)*cosh(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x)*cosh(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.80

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c}x-\frac{b+1}{2\sqrt{-c}}\right)e^{\left(a-\frac{(b+1)^2}{4c}\right)}}{4\sqrt{-c}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c}x+\frac{b-1}{2\sqrt{c}}\right)e^{\left(-a+\frac{(b-1)^2}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c)*x - 1/2*(b + 1)/sqrt(-c))*e^(a - 1/4*(b + 1)^2/c
)/sqrt(-c) + 1/4*sqrt(pi)*erf(sqrt(c)*x + 1/2*(b - 1)/sqrt(c))*e^(-a + 1/4
*(b - 1)^2/c)/sqrt(c)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.90

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c}\left(2x+\frac{b+1}{c}\right)\right)e^{\left(-\frac{b^2-4ac+2b+1}{4c}\right)}}{4\sqrt{-c}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{c}\left(2x+\frac{b-1}{c}\right)\right)e^{\left(\frac{b^2-4ac-2b+1}{4c}\right)}}{4\sqrt{c}}$$

input
```
integrate(exp(x)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c)*(2*x + (b + 1)/c))*e^(-1/4*(b^2 - 4*a*c +
2*b + 1)/c)/sqrt(-c) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c)*(2*x + (b - 1)/c))*e^
(1/4*(b^2 - 4*a*c - 2*b + 1)/c)/sqrt(c)
```

Mupad [F(-1)]

Timed out.

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx = \int e^x \cosh\left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int(exp(x)*cosh(a + b*x + c*x^2),x)
```

output
```
int(exp(x)*cosh(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int e^x \cosh\left(a+bx+cx^2\right)\,dx = \int e^x \cosh\left(c\,x^2+bx+a\right)dx$$

input
```
int(exp(x)*cosh(c*x^2+b*x+a),x)
```

output
```
int(e**x*cosh(a + b*x + c*x**2),x)
```

3.303 $\int e^{x^2} \cosh(a+bx)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 65

$$\int e^{x^2} \cosh(a+bx)\,dx = \frac{1}{4}e^{-a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(-b+2x)\right)+\frac{1}{4}e^{a-\frac{b^2}{4}}\sqrt{\pi}\text{erfi}\left(\frac{1}{2}(b+2x)\right)$$

output
```
-1/4*exp(-a-1/4*b^2)*Pi^(1/2)*erfi(1/2*b-x)+1/4*exp(a-1/4*b^2)*Pi^(1/2)*er
fi(1/2*b+x)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.78

$$\int e^{x^2} \cosh(a+bx)\,dx = \frac{1}{4}e^{-\frac{b^2}{4}}\sqrt{\pi}\left(\text{erfi}\left(\frac{b}{2}-x\right)(-\cosh(a)+\sinh(a))+\text{erfi}\left(\frac{b}{2}+x\right)(\cosh(a)+\sinh(a))\right)$$

input
```
Integrate[E^x^2*Cosh[a + b*x],x]
```

output
```
(Sqrt[Pi]*(Erfi[b/2 - x]*(-Cosh[a] + Sinh[a]) + Erfi[b/2 + x]*(Cosh[a] + S
inh[a])))/(4*E^(b^2/4))
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \cosh(a+bx) \, dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-a-bx+x^2}+\frac{1}{2}e^{a+bx+x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{4}\sqrt{\pi}e^{-a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(2x-b)\right)+\frac{1}{4}\sqrt{\pi}e^{a-\frac{b^2}{4}}\text{erfi}\left(\frac{1}{2}(b+2x)\right)$$

input
```
Int[E^x^2*Cosh[a + b*x],x]
```

output
```
(E^(-a - b^2/4)*Sqrt[Pi]*Erfi[(-b + 2*x)/2])/4 + (E^(a - b^2/4)*Sqrt[Pi]*E
rfi[(b + 2*x)/2])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 0.76 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.80

method	result	size
risch	$\frac{i\sqrt{\pi}\,\mathrm{e}^{-a-\frac{b^2}{4}}\operatorname{erf}\left(-ix+\frac{1}{2}ib\right)}{4}-\frac{i\sqrt{\pi}\,\mathrm{e}^{a-\frac{b^2}{4}}\operatorname{erf}\left(ix+\frac{1}{2}ib\right)}{4}$	52

input
```
int(exp(x^2)*cosh(b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*I*Pi^(1/2)*exp(-a-1/4*b^2)*erf(-I*x+1/2*I*b)-1/4*I*Pi^(1/2)*exp(a-1/4*
b^2)*erf(I*x+1/2*I*b)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.11

$$\int e^{x^2}\cosh(a+bx)\,dx$$
$$=\frac{1}{4}\sqrt{\pi}\left(\cosh\left(\frac{1}{4}b^2-a\right)\operatorname{erfi}\left(\frac{1}{2}b+x\right)+\cosh\left(\frac{1}{4}b^2+a\right)\operatorname{erfi}\left(-\frac{1}{2}b+x\right)-\operatorname{erfi}\left(-\frac{1}{2}b+x\right)\sinh\left(\frac{1}{4}b\right.\right.$$

input
```
integrate(exp(x^2)*cosh(b*x+a),x, algorithm="fricas")
```

output
```
1/4*sqrt(pi)*(cosh(1/4*b^2 - a)*erfi(1/2*b + x) + cosh(1/4*b^2 + a)*erfi(-
1/2*b + x) - erfi(-1/2*b + x)*sinh(1/4*b^2 + a) - erfi(1/2*b + x)*sinh(1/4
*b^2 - a))
```

Sympy [F]

$$\int e^{x^2}\cosh(a+bx)\,dx=\int e^{x^2}\cosh\left(a+bx\right)dx$$

input
```
integrate(exp(x**2)*cosh(b*x+a),x)
```

output
```
Integral(exp(x**2)*cosh(a + b*x), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.04 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2}\cosh(a+bx)\,dx=-\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}-\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\,b+i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

input
```
integrate(exp(x^2)*cosh(b*x+a),x, algorithm="maxima")
```

output
```
-1/4*I*sqrt(pi)*erf(1/2*I*b + I*x)*e^(-1/4*b^2 + a) - 1/4*I*sqrt(pi)*erf(-
1/2*I*b + I*x)*e^(-1/4*b^2 - a)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.69

$$\int e^{x^2}\cosh(a+bx)\,dx=\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2+a\right)}+\frac{1}{4}i\sqrt{\pi}\operatorname{erf}\left(\frac{1}{2}i\,b-i\,x\right)e^{\left(-\frac{1}{4}\,b^2-a\right)}$$

input
```
integrate(exp(x^2)*cosh(b*x+a),x, algorithm="giac")
```

output
```
1/4*I*sqrt(pi)*erf(-1/2*I*b - I*x)*e^(-1/4*b^2 + a) + 1/4*I*sqrt(pi)*erf(1
/2*I*b - I*x)*e^(-1/4*b^2 - a)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2} \cosh(a+bx)\,dx = \int \cosh(a+b\,x)\,\mathrm{e}^{x^2}\,dx$$

input
```
int(cosh(a + b*x)*exp(x^2),x)
```

output
```
int(cosh(a + b*x)*exp(x^2), x)
```

Reduce [F]

$$\int e^{x^2} \cosh(a+bx)\,dx = \int e^{x^2} \cosh(bx+a)\,dx$$

input
```
int(exp(x^2)*cosh(b*x+a),x)
```

output
```
int(e**(x**2)*cosh(a + b*x),x)
```

3.304 $\int e^{x^2} \cosh\left(a + cx^2\right) \, dx$

Optimal result

Integrand size = 14, antiderivative size = 65

$$\int e^{x^2} \cosh\left(a + cx^2\right) \, dx = \frac{e^{-a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1-c}\,x\right)}{4\sqrt{1-c}} + \frac{e^{a}\sqrt{\pi}\operatorname{erfi}\left(\sqrt{1+c}\,x\right)}{4\sqrt{1+c}}$$

output
```
1/4*Pi^(1/2)*erfi((1-c)^(1/2)*x)/(1-c)^(1/2)/exp(a)+1/4*exp(a)*Pi^(1/2)*er
fi((1+c)^(1/2)*x)/(1+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.09

$$\int e^{x^2} \cosh\left(a + cx^2\right) \, dx$$
$$= \frac{\sqrt{\pi}\left(\sqrt{-1+c}(1+c)\operatorname{erf}\left(\sqrt{-1+c}\,x\right)(\cosh(a)-\sinh(a)) + (-1+c)\sqrt{1+c}\operatorname{erfi}\left(\sqrt{1+c}\,x\right)(\cosh(a)+\sin\right.}{4\left(-1+c^2\right)}$$

input
```
Integrate[E^x^2*Cosh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*Erf[Sqrt[-1 + c]*x]*(Cosh[a] - Sinh[a]) +
(-1 + c)*Sqrt[1 + c]*Erfi[Sqrt[1 + c]*x]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^
2))
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \cosh\left(a + cx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{(1-c)x^2-a} + \frac{1}{2}e^{a+(c+1)x^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}e^{-a}\text{erfi}\left(\sqrt{1-c}x\right)}{4\sqrt{1-c}} + \frac{\sqrt{\pi}e^{a}\text{erfi}\left(\sqrt{c+1}x\right)}{4\sqrt{c+1}}$$

input
```
Int[E^x^2*Cosh[a + c*x^2],x]
```

output
```
(Sqrt[Pi]*Erfi[Sqrt[1 - c]*x])/(4*Sqrt[1 - c]*E^a) + (E^a*Sqrt[Pi]*Erfi[Sq
rt[1 + c]*x])/(4*Sqrt[1 + c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.74

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-a}\,\mathrm{erf}\left(\sqrt{c-1}\,x\right)}{4\sqrt{c-1}}+\frac{\sqrt{\pi}\,\mathrm{e}^{a}\,\mathrm{erf}\left(\sqrt{-c-1}\,x\right)}{4\sqrt{-c-1}}$	48

input
```
int(exp(x^2)*cosh(c*x^2+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-a)/(c-1)^(1/2)*erf((c-1)^(1/2)*x)+1/4*Pi^(1/2)*exp(a)/(-
c-1)^(1/2)*erf((-c-1)^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.17

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx = \frac{\sqrt{\pi}((c+1)\cosh(a)-(c+1)\sinh(a))\sqrt{c-1}\,\mathrm{erf}\left(\sqrt{c-1}x\right)-\sqrt{\pi}((c-1)\cosh(a)+(c-1)\sinh(a))\sqrt{}}{4\left(c^2-1\right)}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*((c + 1)*cosh(a) - (c + 1)*sinh(a))*sqrt(c - 1)*erf(sqrt(c -
 1)*x) - sqrt(pi)*((c - 1)*cosh(a) + (c - 1)*sinh(a))*sqrt(-c - 1)*erf(sqr
t(-c - 1)*x))/(c^2 - 1)
```

Sympy [F]

$$\int e^{x^2} \cosh\left(a + cx^2\right) \, dx = \int e^{x^2} \cosh\left(a + cx^2\right) dx$$

input
```
integrate(exp(x**2)*cosh(c*x**2+a),x)
```

output
```
Integral(exp(x**2)*cosh(a + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.72

$$\int e^{x^2} \cosh\left(a + cx^2\right) \, dx = \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{c - 1}x\right) e^{(-a)}}{4 \sqrt{c - 1}} + \frac{\sqrt{\pi} \operatorname{erf}\left(\sqrt{-c - 1}x\right) e^{a}}{4 \sqrt{-c - 1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) + 1/4*sqrt(pi)*erf(sqrt
(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.75

$$\int e^{x^2} \cosh\left(a + cx^2\right) \, dx = -\frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{c - 1}x\right) e^{(-a)}}{4 \sqrt{c - 1}} - \frac{\sqrt{\pi} \operatorname{erf}\left(-\sqrt{-c - 1}x\right) e^{a}}{4 \sqrt{-c - 1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-sqrt(c - 1)*x)*e^(-a)/sqrt(c - 1) - 1/4*sqrt(pi)*erf(-s
qrt(-c - 1)*x)*e^a/sqrt(-c - 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=\int \mathrm{e}^{x^2}\cosh\left(c\,x^2+a\right)\,dx$$

input
```
int(exp(x^2)*cosh(a + c*x^2),x)
```

output
```
int(exp(x^2)*cosh(a + c*x^2), x)
```

Reduce [F]

$$\int e^{x^2}\cosh\left(a+cx^2\right)\,dx=\int e^{x^2}\cosh\left(c\,x^2+a\right)dx$$

input
```
int(exp(x^2)*cosh(c*x^2+a),x)
```

output
```
int(e**(x**2)*cosh(a + c*x**2),x)
```

3.305 $\int e^{x^2} \cosh\left(a+bx+cx^2\right) dx$

Optimal result

Integrand size = 17, antiderivative size = 115

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx = -\frac{e^{-a-\frac{b^2}{4(1-c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}+\frac{e^{a-\frac{b^2}{4(1+c)}}\sqrt{\pi}\,\text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right)}{4\sqrt{1+c}}$$

output
```
-1/4*exp(-a-b^2/(4-4*c))*Pi^(1/2)*erfi(1/2*(b-2*(1-c)*x)/(1-c)^(1/2))/(1-c
)^(1/2)+1/4*exp(a-b^2/(4+4*c))*Pi^(1/2)*erfi(1/2*(b+2*(1+c)*x)/(1+c)^(1/2)
)/(1+c)^(1/2)
```

Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 122, normalized size of antiderivative = 1.06

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx = \frac{e^{-\frac{b^2}{4+4c}}\sqrt{\pi}\left(\sqrt{-1+c}(1+c)e^{\frac{b^2c}{2(-1+c^2)}}\text{erf}\left(\frac{b+2(-1+c)x}{2\sqrt{-1+c}}\right)(\cosh(a)-\sinh(a))+(-1+c)\sqrt{1+c}\,\text{erfi}\left(\frac{b+2(1+c)x}{2\sqrt{1+c}}\right.\right.}{4\left(-1+c^2\right)}$$

input
```
Integrate[E^x^2*Cosh[a + b*x + c*x^2],x]
```

output
```
(Sqrt[Pi]*(Sqrt[-1 + c]*(1 + c)*E^((b^2*c)/(2*(-1 + c^2)))*Erf[(b + 2*(-1
+ c)*x)/(2*Sqrt[-1 + c])]*(Cosh[a] - Sinh[a]) + (-1 + c)*Sqrt[1 + c]*Erfi[
(b + 2*(1 + c)*x)/(2*Sqrt[1 + c])]*(Cosh[a] + Sinh[a])))/(4*(-1 + c^2)*E^(
b^2/(4 + 4*c)))
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int e^{x^2} \cosh \left(a+bx+cx^2\right) \, dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{2} e^{-a-bx+(1-c)x^2}+\frac{1}{2} e^{a+bx+(c+1)x^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi} e^{a-\frac{b^2}{4(c+1)}} \text{erfi}\left(\frac{b+2(c+1)x}{2\sqrt{c+1}}\right)}{4\sqrt{c+1}}-\frac{\sqrt{\pi} e^{-a-\frac{b^2}{4(1-c)}} \text{erfi}\left(\frac{b-2(1-c)x}{2\sqrt{1-c}}\right)}{4\sqrt{1-c}}$$

input
```
Int[E^x^2*Cosh[a + b*x + c*x^2],x]
```

output
```
-1/4*(E^(-a - b^2/(4*(1 - c)))*Sqrt[Pi]*Erfi[(b - 2*(1 - c)*x)/(2*Sqrt[1 -
 c])])/Sqrt[1 - c] + (E^(a - b^2/(4*(1 + c)))*Sqrt[Pi]*Erfi[(b + 2*(1 + c)
*x)/(2*Sqrt[1 + c])])/(4*Sqrt[1 + c])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.55 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.91

method	result	size
risch	$\frac{\sqrt{\pi}\,\mathrm{e}^{-\frac{4ac-b^2-4a}{4(c-1)}}\,\mathrm{erf}\left(\sqrt{c-1}\,x+\frac{b}{2\sqrt{c-1}}\right)}{4\sqrt{c-1}}-\frac{\sqrt{\pi}\,\mathrm{e}^{\frac{4ac-b^2+4a}{4+4c}}\,\mathrm{erf}\left(-\sqrt{-c-1}\,x+\frac{b}{2\sqrt{-c-1}}\right)}{4\sqrt{-c-1}}$	105

input
```
int(exp(x^2)*cosh(c*x^2+b*x+a),x,method=_RETURNVERBOSE)
```

output
```
1/4*Pi^(1/2)*exp(-1/4*(4*a*c-b^2-4*a)/(c-1))/(c-1)^(1/2)*erf((c-1)^(1/2)*x
+1/2*b/(c-1)^(1/2))-1/4*Pi^(1/2)*exp(1/4*(4*a*c-b^2+4*a)/(c+1))/(-c-1)^(1/
2)*erf(-(-c-1)^(1/2)*x+1/2*b/(-c-1)^(1/2))
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.43

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}\left((c+1)\cosh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)-(c+1)\sinh\left(-\frac{b^2-4\,ac+4\,a}{4\,(c-1)}\right)\right)\sqrt{c-1}\,\mathrm{erf}\left(\frac{2\,(c-1)x+b}{2\,\sqrt{c-1}}\right)-\sqrt{\pi}\Big((c-1)\cosh}{4\,(c^2-1)}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="fricas")
```

output
```
1/4*(sqrt(pi)*((c + 1)*cosh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)) - (c + 1)*si
nh(-1/4*(b^2 - 4*a*c + 4*a)/(c - 1)))*sqrt(c - 1)*erf(1/2*(2*(c - 1)*x + b
)/sqrt(c - 1)) - sqrt(pi)*((c - 1)*cosh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1))
+ (c - 1)*sinh(-1/4*(b^2 - 4*a*c - 4*a)/(c + 1)))*sqrt(-c - 1)*erf(1/2*(2*
(c + 1)*x + b)*sqrt(-c - 1)/(c + 1)))/(c^2 - 1)
```

Sympy [F]

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx=\int e^{x^2}\cosh\left(a+bx+cx^2\right)dx$$

input
```
integrate(exp(x**2)*cosh(c*x**2+b*x+a),x)
```

output
```
Integral(exp(x**2)*cosh(a + b*x + c*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.77

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx=\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{-c-1}x-\frac{b}{2\sqrt{-c-1}}\right)e^{\left(a-\frac{b^2}{4\,(c+1)}\right)}}{4\sqrt{-c-1}}+\frac{\sqrt{\pi}\,\mathrm{erf}\left(\sqrt{c-1}x+\frac{b}{2\sqrt{c-1}}\right)e^{\left(-a+\frac{b^2}{4\,(c-1)}\right)}}{4\sqrt{c-1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*erf(sqrt(-c - 1)*x - 1/2*b/sqrt(-c - 1))*e^(a - 1/4*b^2/(c +
1))/sqrt(-c - 1) + 1/4*sqrt(pi)*erf(sqrt(c - 1)*x + 1/2*b/sqrt(c - 1))*e^(
-a + 1/4*b^2/(c - 1))/sqrt(c - 1)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.88

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c-1}\left(2x+\frac{b}{c+1}\right)\right)e^{\left(-\frac{b^2-4ac-4a}{4(c+1)}\right)}}{4\sqrt{-c-1}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{c-1}\left(2x+\frac{b}{c-1}\right)\right)e^{\left(\frac{b^2-4ac+4a}{4(c-1)}\right)}}{4\sqrt{c-1}}$$

input
```
integrate(exp(x^2)*cosh(c*x^2+b*x+a),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c - 1)*(2*x + b/(c + 1)))*e^(-1/4*(b^2 - 4*a*
c - 4*a)/(c + 1))/sqrt(-c - 1) - 1/4*sqrt(pi)*erf(-1/2*sqrt(c - 1)*(2*x +
b/(c - 1)))*e^(1/4*(b^2 - 4*a*c + 4*a)/(c - 1))/sqrt(c - 1)
```

Mupad [F(-1)]

Timed out.

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx = \int e^{x^2}\cosh\left(c\,x^2+b\,x+a\right)\,dx$$

input
```
int(exp(x^2)*cosh(a + b*x + c*x^2),x)
```

output
```
int(exp(x^2)*cosh(a + b*x + c*x^2), x)
```

Reduce [F]

$$\int e^{x^2}\cosh\left(a+bx+cx^2\right)\,dx = \int e^{x^2}\cosh\left(c\,x^2+bx+a\right)dx$$

input
```
int(exp(x^2)*cosh(c*x^2+b*x+a),x)
```

output
```
int(e**(x**2)*cosh(a + b*x + c*x**2),x)
```

3.306 $\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 110

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=\frac{1}{4}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)$$

output
```
1/4*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(2*f*x-b*ln(f))/
f^(1/2))+1/4*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*Pi^(1/2)*erfi(1/2*(2*f*x+
b*ln(f))/f^(1/2))
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=\frac{1}{4}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\left(e^{\frac{b^2\log^2(f)}{2f}}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + f*x^2],x]
```

output
```
(f^(-1/2 + a)*Sqrt[Pi]*(E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])/(2
*Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])]*(Cos
h[d] + Sinh[d])))/(4*E^((b^2*Log[f]^2)/(4*f)))
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{2}e^{-d-fx^2}f^{a+bx}+\frac{1}{2}e^{d+fx^2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + f*x^2],x]
```

output
```
(E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[f]
)/(2*Sqrt[f])])/4 + (E^(d - (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Er
fi[(2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result	size
risch	$-\frac{\operatorname{erf}\left(-\sqrt{f}\, x+\frac{\ln(f) b}{2\sqrt{f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{b^{2}\ln(f)^{2}-4 d f}{4 f}}}{4\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-f}\, x+\frac{\ln(f) b}{2\sqrt{-f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-4 d f}{4 f}}}{4\sqrt{-f}}$	100

input
```
int(f^(b*x+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2
*ln(f)^2-4*d*f)/f)-1/4*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))/(-f)^(1/2
)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(84) = 168$.

Time = 0.09 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.92

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f))\sqrt{-f}}{2\,f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)\operatorname{er}}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*
erf(1/2*(2*f*x + b*log(f))*sqrt(-f)/f) + sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*lo
g(f)^2 + 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + s
qrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^
2 + 4*a*f*log(f) - 4*d*f)/f) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f)
)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=\int f^{a+bx}\cosh\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x)*cosh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.82

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx=\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2*
log(f)^2/f - d) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*b*log(f)/sqrt(-f))
*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.96

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{4\sqrt{f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - b*log(f)/f))*e^(1/4*(b^2*log(f)^2 +
4*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b
*log(f)/f))*e^(-1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx = \int f^{a+b\,x}\cosh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x)*cosh(d + f*x^2),x)
```

output
```
int(f^(a + b*x)*cosh(d + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx}\cosh\left(d+fx^2\right)\,dx = f^a\left(\int f^{bx}\cosh\left(f\,x^2+d\right)dx\right)$$

input
```
int(f^(b*x+a)*cosh(f*x^2+d),x)
```

output
```
f**a*int(f**(b*x)*cosh(d + f*x**2),x)
```

3.307 $\int f^{a+bx} \cosh^2(d+fx^2)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 148

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx = \frac{1}{8}e^{-2d+\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{1}{8}e^{2d-\frac{b^2\log^2(f)}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{f^{a+bx}}{2b\log(f)}$$

output
```
1/16*exp(-2*d+1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*2^(1/2)*Pi^(1/2)*erf(1/4*(4*f*
x-b*ln(f))*2^(1/2)/f^(1/2))+1/16*exp(2*d-1/8*b^2*ln(f)^2/f)*f^(-1/2+a)*2^(
1/2)*Pi^(1/2)*erfi(1/4*(4*f*x+b*ln(f))*2^(1/2)/f^(1/2))+1/2*f^(b*x+a)/b/ln
(f)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 149, normalized size of antiderivative = 1.01

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=\frac{1}{16}f^a\left(\frac{8f^{bx}}{b\log(f)}+\frac{e^{\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)-\sinh(2d))}{\sqrt{f}}+\frac{e^{-\frac{b^2\log^2(f)}{8f}}\sqrt{2\pi}\,\mathrm{erfi}\left(\frac{4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right)(\cosh(2d)+\sinh(2d))}{\sqrt{f}}\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*((8*f^(b*x))/(b*Log[f]) + (E^((b^2*Log[f]^2)/(8*f))*Sqrt[2*Pi]*Erf[(4
*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] - Sinh[2*d]))/Sqrt[f] + (
Sqrt[2*Pi]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*(Cosh[2*d] + Sinh[
2*d]))/(E^((b^2*Log[f]^2)/(8*f))*Sqrt[f])))/16
```

Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{4}e^{-2d-2fx^2}f^{a+bx}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx}+\frac{1}{2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{8f}-2d}\mathrm{erf}\left(\frac{4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{b^2\log^2(f)}{8f}}\mathrm{erfi}\left(\frac{b\log(f)+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Cosh[d + f*x^2]^2,x]
```

output
```
(E^(-2*d + (b^2*Log[f]^2)/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(4*f*x - b*Lo
g[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (b^2*Log[f]^2)/(8*f))*f^(-1/2 +
a)*Sqrt[Pi/2]*Erfi[(4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + f^(a + b*x
)/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.85

method	result	size
risch	$-\frac{\mathrm{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{b\ln(f)\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{16\sqrt{f}}-\frac{\mathrm{erf}\left(-\sqrt{-2f}\,x+\frac{b\ln(f)}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-16df}{8f}}}{8\sqrt{-2f}}+\frac{f^{a}f^{bx}}{2b\ln(f)}$	126

input
```
int(f^(b*x+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*b*ln(f)*2^(1/2)/f^(1/2))/f^(1/2)*2^(1/2)*
Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-16*d*f)/f)-1/8*erf(-(-2*f)^(1/2)*x+1/2*b
*ln(f)/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/8*(b^2*ln(f)^2-16*d*
f)/f)+1/2*f^a*f^(b*x)/b/ln(f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 278 vs. $2(114) = 228$.

Time = 0.11 (sec) , antiderivative size = 278, normalized size of antiderivative = 1.88

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-8\,af\log(f)-16\,df}{8\,f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4\,fx+b\log(f))\sqrt{-f}}{4\,f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}\cosh\left(\frac{b^2\log(f}{}\right.}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 - 8*a*f*log(f) -
 16*d*f)/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f) + sqrt(2
)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)*er
f(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f) + sqrt(2)*sqrt(pi)*b*sqr
t(f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f))/sqrt(f))*log(f)*sinh(1/8*(b^2*log
(f)^2 + 8*a*f*log(f) - 16*d*f)/f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sq
rt(2)*(4*f*x + b*log(f))*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 - 8*a*f
*log(f) - 16*d*f)/f) - 8*f*cosh((b*x + a)*log(f)) - 8*f*sinh((b*x + a)*log
(f)))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx = \int f^{a+bx}\cosh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.86

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=\frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{8f}-2d\right)}}{16\sqrt{f}}+\frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}b\log(f)}{4\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{8f}+2d\right)}}{16\sqrt{-f}}+\frac{f^{bx+a}}{2b\log(f)}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*b*log(f)/sqr
t(f))*e^(1/8*b^2*log(f)^2/f - 2*d)/sqrt(f) + 1/16*sqrt(2)*sqrt(pi)*f^a*erf
(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*b*log(f)/sqrt(-f))*e^(-1/8*b^2*log(f)^2/
f + 2*d)/sqrt(-f) + 1/2*f^(b*x + a)/(b*log(f))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 355, normalized size of antiderivative = 2.40

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - b*log(f)/f))*e^(1/8
*(b^2*log(f)^2 + 8*a*f*log(f) - 16*d*f)/f)/sqrt(f) - 1/16*sqrt(2)*sqrt(pi)
*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + b*log(f)/f))*e^(-1/8*(b^2*log(f)^2 - 8*a
*f*log(f) - 16*d*f)/f)/sqrt(-f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x
 - 1/2*pi*a*sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sg
n(f) - pi*b)^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x
 - 1/2*pi*a*sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)
^2))*e^(b*x*log(abs(f)) + a*log(abs(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1
/2*I*pi*b*x + 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b
+ 4*b*log(abs(f))) - I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a
*sgn(f) + 1/2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(
b*x*log(abs(f)) + a*log(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,{\mathrm{cosh}\left(f\,x^2+d\right)}^2\,dx$$

input
```
int(f^(a + b*x)*cosh(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*cosh(d + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx}\cosh^2\left(d+fx^2\right)\,dx=f^a\left(\int f^{bx}\cosh\left(f\,x^2+d\right)^2dx\right)$$

input
```
int(f^(b*x+a)*cosh(f*x^2+d)^2,x)
```

output
```
f**a*int(f**(b*x)*cosh(d + f*x**2)**2,x)
```

3.308 $\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 239

$$\begin{aligned}\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx &= \frac{3}{16}e^{-d+\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)\\&\quad+\frac{1}{16}e^{-3d+\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\&\quad+\frac{3}{16}e^{d-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\operatorname{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\\&\quad+\frac{1}{16}e^{3d-\frac{b^2\log^2(f)}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\operatorname{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\end{aligned}$$

output
```
3/16*exp(-d+1/4*b^2*ln(f)^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(2*f*x-b*ln(f))
/f^(1/2))+1/48*exp(-3*d+1/12*b^2*ln(f)^2/f)*f^(-1/2+a)*3^(1/2)*Pi^(1/2)*er
f(1/6*(6*f*x-b*ln(f))*3^(1/2)/f^(1/2))+3/16*exp(d-1/4*b^2*ln(f)^2/f)*f^(-1
/2+a)*Pi^(1/2)*erfi(1/2*(2*f*x+b*ln(f))/f^(1/2))+1/48*exp(3*d-1/12*b^2*ln(
f)^2/f)*f^(-1/2+a)*3^(1/2)*Pi^(1/2)*erfi(1/6*(6*f*x+b*ln(f))*3^(1/2)/f^(1/
2))
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.20

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx$$

$$=\frac{1}{16}e^{-\frac{b^2\log^2(f)}{4f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}\cosh(d)\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}e^{\frac{b^2\log^2(f)}{2f}}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+3\sqrt{3}\text{erfi}\left(\frac{2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)+e^{\frac{b^2\log^2(f)}{3f}}\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\text{erfi}\left(\frac{6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

output
```
(f^(-1/2 + a)*Sqrt[Pi/3]*(3*Sqrt[3]*Cosh[d]*Erfi[(2*f*x + b*Log[f])/(2*Sqr
t[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*Erfi[(6*f*x + b*Log[f])/(2*Sqr
t[3]*Sqrt[f])] + 3*Sqrt[3]*E^((b^2*Log[f]^2)/(2*f))*Erf[(2*f*x - b*Log[f])
/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) + 3*Sqrt[3]*Erfi[(2*f*x + b*Log[f])/(2*S
qrt[f])]*Sinh[d] + E^((b^2*Log[f]^2)/(3*f))*Erf[(6*f*x - b*Log[f])/(2*Sqrt
[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Erfi[(6*f
*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((b^2*Log[f]^2)/(4*f
)))
```

Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{8}e^{-3d-3fx^2}f^{a+bx}+\frac{3}{8}e^{-d-fx^2}f^{a+bx}+\frac{3}{8}e^{d+fx^2}f^{a+bx}+\frac{1}{8}e^{3d+3fx^2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{4f}-d}\text{erf}\left(\frac{2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{b^2\log^2(f)}{12f}-3d}\text{erf}\left(\frac{6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{b^2\log^2(f)}{4f}}\text{erfi}\left(\frac{b\log(f)+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{b^2\log^2(f)}{12f}}\text{erfi}\left(\frac{b\log(f)+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + f*x^2]^3,x]
```

output
```
(3*E^(-d + (b^2*Log[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(2*f*x - b*Log[
f])/(2*Sqrt[f])])/16 + (E^(-3*d + (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt
[Pi/3]*Erf[(6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (3*E^(d - (b^2*Lo
g[f]^2)/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(2*f*x + b*Log[f])/(2*Sqrt[f])])
/16 + (E^(3*d - (b^2*Log[f]^2)/(12*f))*f^(-1/2 + a)*Sqrt[Pi/3]*Erfi[(6*f*x
 + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.76 (sec) , antiderivative size = 207, normalized size of antiderivative = 0.87

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{3}\,\sqrt{f}\,x+\frac{\ln(f) b\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-36 d f}{12 f}}}{48\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-3 f}\,x+\frac{\ln(f) b}{2\sqrt{-3 f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-36 d f}{12 f}}}{16\sqrt{-3 f}}-\frac{3\operatorname{erf}\left(-\sqrt{f}\,x+\frac{\ln(}{2}\right.}{}$

input
```
int(f^(b*x+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*erf(-3^(1/2)*f^(1/2)*x+1/6*ln(f)*b*3^(1/2)/f^(1/2))/f^(1/2)*3^(1/2)*
Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-36*d*f)/f)-1/16*erf(-(-3*f)^(1/2)*x+1/2
*ln(f)*b/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/12*(b^2*ln(f)^2-36
*d*f)/f)-3/16*erf(-f^(1/2)*x+1/2*ln(f)*b/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp
(1/4*(b^2*ln(f)^2-4*d*f)/f)-3/16*erf(-(-f)^(1/2)*x+1/2*ln(f)*b/(-f)^(1/2))
/(-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*f)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 443 vs. 2(181) = 362.

Time = 0.10 (sec) , antiderivative size = 443, normalized size of antiderivative = 1.85

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx = -\frac{\sqrt{3}\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)\operatorname{erf}\left(\frac{\sqrt{3}(6\,fx+b\log(f))\sqrt{-f}}{6\,f}\right)+\sqrt{3}\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+12\,af\log}{12\,f}\right.}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) -
 36*d*f)/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f))*sqrt(-f)/f) + sqrt(3)*sqrt(
pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)*erf(-1/6*
sqrt(3)*(6*f*x - b*log(f))/sqrt(f)) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sq
rt(3)*(6*f*x - b*log(f))/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 12*a*f*log(f)
- 36*d*f)/f) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f)
)*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 - 12*a*f*log(f) - 36*d*f)/f) + 9*sqr
t(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f)*erf(1/2*(
2*f*x + b*log(f))*sqrt(-f)/f) + 9*sqrt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2
+ 4*a*f*log(f) - 4*d*f)/f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f)) + 9*sqrt(p
i)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f))/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + 4
*a*f*log(f) - 4*d*f)/f) - 9*sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x + b*log(f))*s
qrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 - 4*a*f*log(f) - 4*d*f)/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx=\int f^{a+bx}\cosh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 200, normalized size of antiderivative = 0.84

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx=\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)}{2\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{4f}-d\right)}+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{f}}\right)e^{\left(\frac{b^2\log(f)^2}{12f}-3d\right)}}{48\sqrt{f}}+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}b\log(f)}{6\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{12f}+3d\right)}}{48\sqrt{-f}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)}{2\sqrt{-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4f}+d\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
3/16*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*b*log(f)/sqrt(f))*e^(1/4*b^2
*log(f)^2/f - d) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*s
qrt(3)*b*log(f)/sqrt(f))*e^(1/12*b^2*log(f)^2/f - 3*d)/sqrt(f) + 1/48*sqrt
(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*b*log(f)/sqrt(-f))*e
^(-1/12*b^2*log(f)^2/f + 3*d)/sqrt(-f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x
- 1/2*b*log(f)/sqrt(-f))*e^(-1/4*b^2*log(f)^2/f + d)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.93

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx$$
$$= -\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+12\,af\log(f)-36\,df}{12\,f}\right)}}{48\sqrt{f}}$$
$$-\frac{\sqrt{3}\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-12\,af\log(f)-36\,df}{12\,f}\right)}}{48\sqrt{-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2+4\,af\log(f)-4\,df}{4\,f}\right)}}{16\sqrt{f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,af\log(f)-4\,df}{4\,f}\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - b*log(f)/f))*e^(1/1
2*(b^2*log(f)^2 + 12*a*f*log(f) - 36*d*f)/f)/sqrt(f) - 1/48*sqrt(3)*sqrt(p
i)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + b*log(f)/f))*e^(-1/12*(b^2*log(f)^2 -
12*a*f*log(f) - 36*d*f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x
- b*log(f)/f))*e^(1/4*(b^2*log(f)^2 + 4*a*f*log(f) - 4*d*f)/f)/sqrt(f) - 3
/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + b*log(f)/f))*e^(-1/4*(b^2*log(f)^2 -
 4*a*f*log(f) - 4*d*f)/f)/sqrt(-f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx=\int f^{a+b\,x}\,\cosh\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*cosh(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*cosh(d + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx}\cosh^3\left(d+fx^2\right)\,dx = f^a\left(\int f^{bx}\cosh\left(f\,x^2+d\right)^3dx\right)$$

input
```
int(f^(b*x+a)*cosh(f*x^2+d)^3,x)
```

output
```
f**a*int(f**(b*x)*cosh(d + f*x**2)**3,x)
```

3.309 $\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 115

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\frac{1}{4}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)+\frac{1}{4}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)$$

output
```
1/4*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(e+2*f*x-b*ln(
f))/f^(1/2))+1/4*exp(d-1/4*(e+b*ln(f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erfi(1/2*(
e+2*f*x+b*ln(f))/f^(1/2))
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.07

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{4}e^{-\frac{e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\pi}\left(e^{\frac{e^2+b^2\log^2(f)}{2f}}\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)(\cosh(d)+\sinh(d))\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

output
```
(f^(a - (b*e + f)/(2*f))*Sqrt[Pi]*(E^((e^2 + b^2*Log[f]^2)/(2*f))*Erf[(e +
 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Sinh[d]) + Erfi[(e + 2*f*x + b*
Log[f])/(2*Sqrt[f])]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*f
)))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx} \cosh \left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{2} f^{a+bx} e^{-d-ex-fx^2}+\frac{1}{2} f^{a+bx} e^{d+ex+fx^2}\right) dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{1}{4} \sqrt{\pi } f^{a-\frac{1}{2}} e^{\frac{(e-b \log (f))^2}{4 f}-d} \text{erf}\left(\frac{-b \log (f)+e+2 f x}{2 \sqrt{f}}\right)+\frac{1}{4} \sqrt{\pi } f^{a-\frac{1}{2}} e^{d-\frac{(b \log (f)+e)^2}{4 f}} \text{erfi}\left(\frac{b \log (f)+e+2 f x}{2 \sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + e*x + f*x^2],x]
```

output
```
(E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x - b*
Log[f])/(2*Sqrt[f])])/4 + (E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqr
t[Pi]*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])])/4
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.10

method	result	s
risch	$-\frac{\operatorname{erf}\left(-\sqrt{f}\, x+\frac{b \ln\left(f\right)-e}{2 \sqrt{f}}\right) \sqrt{\pi}\, f^{a} {\mathrm e}^{\frac{b^{2} \ln\left(f\right)^{2}-2 \ln\left(f\right) b e-4 d f+e^{2}}{4 f}}}{4 \sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-f}\, x+\frac{e+b \ln\left(f\right)}{2 \sqrt{-f}}\right) \sqrt{\pi}\, f^{a} {\mathrm e}^{-\frac{b^{2} \ln\left(f\right)^{2}+2 \ln\left(f\right) b e-4 d f+e^{2}}{4 f}}}{4 \sqrt{-f}}$	1

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-f^(1/2)*x+1/2*(b*ln(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*
(b^2*ln(f)^2-2*ln(f)*b*e-4*d*f+e^2)/f)-1/4*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f
))/(-f)^(1/2))/(-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4
*d*f+e^2)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. 2(90) = 180.

Time = 0.09 (sec) , antiderivative size = 251, normalized size of antiderivative = 2.18

$$\int f^{a+bx} \cosh\left(d+ex+fx^2\right)\, dx = -\frac{\sqrt{\pi}\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df+2\,(be-2\,af)\log(f)}{4\,f}\right)\operatorname{erf}\left(\frac{(2\,fx+b\log(f)+e)\sqrt{-f}}{2\,f}\right)+\sqrt{\pi}\sqrt{f}\cosh\left(\frac{b^2\log(f)^2+e^2-4\,df}{4}\right.}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e - 2*
a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + sqrt(pi)*sqrt
(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f)*erf(
-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) - sqrt(pi)*sqrt(-f)*erf(1/2*(2*f*x +
b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*(b*e -
2*a*f)*log(f))/f) + sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(
f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(f))/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cosh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x)*cosh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=\frac{1}{4}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log\left(f\right)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{4\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d
 + 1/4*(b*log(f) - e)^2/f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(
f) + e)/sqrt(-f))*e^(d - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.15

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{-f}}$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{4\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f) + e)/f))*e^(-1/4*(b^2*log
(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(-f) - 1/4*sqrt(
pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*e^(1/4*(b^2*log(f)^2 - 2*b*
e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx}\cosh\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{bx}\cosh\left(f\,x^2+ex+d\right)dx\right)$$

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d),x)
```

output
```
f**a*int(f**(b*x)*cosh(d + e*x + f*x**2),x)
```

3.310 $\int f^{a+bx}\cosh^2(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 161

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx = \frac{1}{8}e^{-2d+\frac{(2e-b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{1}{8}e^{2d-\frac{(2e+b\log(f))^2}{8f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{2}}\operatorname{erfi}\left(\frac{2e+4fx+b\log(f)}{2\sqrt{2}\sqrt{f}}\right) + \frac{f^{a+bx}}{2b\log(f)}$$

output
```
1/16*exp(-2*d+1/8*(2*e-b*ln(f))^2/f)*f^(-1/2+a)*2^(1/2)*Pi^(1/2)*erf(1/4*(
2*e+4*f*x-b*ln(f))*2^(1/2)/f^(1/2))+1/16*exp(2*d-1/8*(2*e+b*ln(f))^2/f)*f^
(-1/2+a)*2^(1/2)*Pi^(1/2)*erfi(1/4*(2*e+4*f*x+b*ln(f))*2^(1/2)/f^(1/2))+1/
2*f^(b*x+a)/b/ln(f)
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.37

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx$$

$$= \frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f}}f^{a-\frac{be+f}{2f}}\left(4\sqrt{2}e^{\frac{4e^2+b^2\log^2(f)}{8f}}f^{\frac{1}{2}+b\left(\frac{e}{2f}+x\right)}+be^{\frac{4e^2+b^2\log^2(f)}{4f}}\sqrt{\pi}\operatorname{erf}\left(\frac{2e+4fx-b\log(f)}{2\sqrt{2}\sqrt{f}}\right)\log(f)(\cosh(2d)\right.}{8\sqrt{2}b\log(f)}$$

input
```
Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*(4*Sqrt[2]*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*f^(1/
2 + b*(e/(2*f) + x)) + b*E^((4*e^2 + b^2*Log[f]^2)/(4*f))*Sqrt[Pi]*Erf[(2*
e + 4*f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Cosh[2*d] - Sinh[2*d])
+ b*Sqrt[Pi]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[f])]*Log[f]*(Co
sh[2*d] + Sinh[2*d])))/(8*Sqrt[2]*b*E^((4*e^2 + b^2*Log[f]^2)/(8*f))*Log[f
])
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{4}f^{a+bx}e^{-2d-2ex-2fx^2}+\frac{1}{4}f^{a+bx}e^{2d+2ex+2fx^2}+\frac{1}{2}f^{a+bx}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{\frac{(2e-b\log(f))^2}{8f}-2d}\operatorname{erf}\left(\frac{-b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{1}{8}\sqrt{\frac{\pi}{2}}f^{a-\frac{1}{2}}e^{2d-\frac{(b\log(f)+2e)^2}{8f}}\operatorname{erfi}\left(\frac{b\log(f)+2e+4fx}{2\sqrt{2}\sqrt{f}}\right)+\frac{f^{a+bx}}{2b\log(f)}$$

input
```
Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(E^(-2*d + (2*e - b*Log[f])^2/(8*f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erf[(2*e + 4*
f*x - b*Log[f])/(2*Sqrt[2]*Sqrt[f])])/8 + (E^(2*d - (2*e + b*Log[f])^2/(8*
f))*f^(-1/2 + a)*Sqrt[Pi/2]*Erfi[(2*e + 4*f*x + b*Log[f])/(2*Sqrt[2]*Sqrt[
f])])/8 + f^(a + b*x)/(2*b*Log[f])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.98

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{2}\,\sqrt{f}\,x+\frac{(b\ln(f)-2e)\sqrt{2}}{4\sqrt{f}}\right)\sqrt{2}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-4\ln(f)be-16df+4e^{2}}{8f}}}{16\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-2f}\,x+\frac{2e+b\ln(f)}{2\sqrt{-2f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+4\ln(}{8}}}{8\sqrt{-2f}}$

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-2^(1/2)*f^(1/2)*x+1/4*(b*ln(f)-2*e)*2^(1/2)/f^(1/2))/f^(1/2)*2^
(1/2)*Pi^(1/2)*f^a*exp(1/8*(b^2*ln(f)^2-4*ln(f)*b*e-16*d*f+4*e^2)/f)-1/8*e
rf(-(-2*f)^(1/2)*x+1/2*(2*e+b*ln(f))/(-2*f)^(1/2))/(-2*f)^(1/2)*Pi^(1/2)*f
^a*exp(-1/8*(b^2*ln(f)^2+4*ln(f)*b*e-16*d*f+4*e^2)/f)+1/2*f^a*f^(b*x)/b/ln
(f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 334 vs. $2(126) = 252$.

Time = 0.08 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.07

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx =$$

$$-\frac{\sqrt{2}\sqrt{\pi}b\sqrt{-f}\cosh\left(\frac{b^2\log(f)^2+4e^2-16df+4(be-2af)\log(f)}{8f}\right)\operatorname{erf}\left(\frac{\sqrt{2}(4fx+b\log(f)+2e)\sqrt{-f}}{4f}\right)\log(f)+\sqrt{2}\sqrt{\pi}b\sqrt{f}}{}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(2)*sqrt(pi)*b*sqrt(-f)*cosh(1/8*(b^2*log(f)^2 + 4*e^2 - 16*d*f
 + 4*(b*e - 2*a*f)*log(f))/f)*erf(1/4*sqrt(2)*(4*f*x + b*log(f) + 2*e)*sqr
t(-f)/f)*log(f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*cosh(1/8*(b^2*log(f)^2 + 4*e^
2 - 16*d*f - 4*(b*e - 2*a*f)*log(f))/f)*erf(-1/4*sqrt(2)*(4*f*x - b*log(f)
 + 2*e)/sqrt(f))*log(f) - sqrt(2)*sqrt(pi)*b*sqrt(-f)*erf(1/4*sqrt(2)*(4*f
*x + b*log(f) + 2*e)*sqrt(-f)/f)*log(f)*sinh(1/8*(b^2*log(f)^2 + 4*e^2 - 1
6*d*f + 4*(b*e - 2*a*f)*log(f))/f) + sqrt(2)*sqrt(pi)*b*sqrt(f)*erf(-1/4*s
qrt(2)*(4*f*x - b*log(f) + 2*e)/sqrt(f))*log(f)*sinh(1/8*(b^2*log(f)^2 + 4
*e^2 - 16*d*f - 4*(b*e - 2*a*f)*log(f))/f) - 8*f*cosh((b*x + a)*log(f)) -
8*f*sinh((b*x + a)*log(f)))/(b*f*log(f))
```

Sympy [F]

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{-f}x-\frac{\sqrt{2}(b\log(f)+2e)}{4\sqrt{-f}}\right)e^{\left(2d-\frac{(b\log(f)+2e)^2}{8f}\right)}}{16\sqrt{-f}} + \frac{\sqrt{2}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{2}\sqrt{f}x-\frac{\sqrt{2}(b\log(f)-2e)}{4\sqrt{f}}\right)e^{\left(-2d+\frac{(b\log(f)-2e)^2}{8f}\right)}}{16\sqrt{f}} + \frac{f^{bx+a}}{2b\log(f)}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/16*sqrt(2)*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(-f)*x - 1/4*sqrt(2)*(b*log(f) +
 2*e)/sqrt(-f))*e^(2*d - 1/8*(b*log(f) + 2*e)^2/f)/sqrt(-f) + 1/16*sqrt(2)
*sqrt(pi)*f^a*erf(sqrt(2)*sqrt(f)*x - 1/4*sqrt(2)*(b*log(f) - 2*e)/sqrt(f)
)*e^(-2*d + 1/8*(b*log(f) - 2*e)^2/f)/sqrt(f) + 1/2*f^(b*x + a)/(b*log(f))
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 387, normalized size of antiderivative = 2.40

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(-f)*(4*x + (b*log(f) + 2*e)/f
))*e^(-1/8*(b^2*log(f)^2 + 4*b*e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/f
)/sqrt(-f) - 1/16*sqrt(2)*sqrt(pi)*erf(-1/4*sqrt(2)*sqrt(f)*(4*x - (b*log(
f) - 2*e)/f))*e^(1/8*(b^2*log(f)^2 - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 -
 16*d*f)/f)/sqrt(f) + (2*b*cos(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*
sgn(f) + 1/2*pi*a)*log(abs(f))/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)
^2) - (pi*b*sgn(f) - pi*b)*sin(-1/2*pi*b*x*sgn(f) + 1/2*pi*b*x - 1/2*pi*a*
sgn(f) + 1/2*pi*a)/(4*b^2*log(abs(f))^2 + (pi*b*sgn(f) - pi*b)^2))*e^(b*x*
log(abs(f)) + a*log(abs(f))) + I*(I*e^(1/2*I*pi*b*x*sgn(f) - 1/2*I*pi*b*x
+ 1/2*I*pi*a*sgn(f) - 1/2*I*pi*a)/(2*I*pi*b*sgn(f) - 2*I*pi*b + 4*b*log(ab
s(f))) - I*e^(-1/2*I*pi*b*x*sgn(f) + 1/2*I*pi*b*x - 1/2*I*pi*a*sgn(f) + 1/
2*I*pi*a)/(-2*I*pi*b*sgn(f) + 2*I*pi*b + 4*b*log(abs(f))))*e^(b*x*log(abs(
f)) + a*log(abs(f)))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^2\left(d+ex+fx^2\right)\,dx = \int f^{a+b\,x}\,{\mathrm{cosh}\left(f\,x^2+e\,x+d\right)}^2\,dx$$

input
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx} \cosh^2 \left(d + ex + fx^2\right) \, dx = f^a \left(\int f^{bx} \cosh \left(f\,x^2 + ex + d\right)^2 dx\right)$$

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d)^2,x)
```

output
```
f**a*int(f**(b*x)*cosh(d + e*x + f*x**2)**2,x)
```

3.311 $\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 257

$$\begin{aligned}\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx &= \frac{3}{16}e^{-d+\frac{(e-b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)\\ &\quad+\frac{1}{16}e^{-3d+\frac{(3e-b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\\ &\quad+\frac{3}{16}e^{d-\frac{(e+b\log(f))^2}{4f}}f^{-\frac{1}{2}+a}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\\ &\quad+\frac{1}{16}e^{3d-\frac{(3e+b\log(f))^2}{12f}}f^{-\frac{1}{2}+a}\sqrt{\frac{\pi}{3}}\,\mathrm{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\end{aligned}$$

output
```
3/16*exp(-d+1/4*(e-b*ln(f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erf(1/2*(e+2*f*x-b*ln
(f))/f^(1/2))+1/48*exp(-3*d+1/12*(3*e-b*ln(f))^2/f)*f^(-1/2+a)*3^(1/2)*Pi^
(1/2)*erf(1/6*(3*e+6*f*x-b*ln(f))*3^(1/2)/f^(1/2))+3/16*exp(d-1/4*(e+b*ln(
f))^2/f)*f^(-1/2+a)*Pi^(1/2)*erfi(1/2*(e+2*f*x+b*ln(f))/f^(1/2))+1/48*exp(
3*d-1/12*(3*e+b*ln(f))^2/f)*f^(-1/2+a)*3^(1/2)*Pi^(1/2)*erfi(1/6*(3*e+6*f*
x+b*ln(f))*3^(1/2)/f^(1/2))
```

Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 353, normalized size of antiderivative = 1.37

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{1}{16}e^{-\frac{3e^2+b^2\log^2(f)}{4f}}f^{a-\frac{be+f}{2f}}\sqrt{\frac{\pi}{3}}\left(3\sqrt{3}e^{\frac{e^2}{2f}}\cosh(d)\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)+e^{\frac{b^2\log^2(f)}{6f}}\cosh(3d)\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)+3\sqrt{3}e^{\frac{2e^2+b^2\log^2(f)}{2f}}\text{erf}\left(\frac{e+2fx-b\log(f)}{2\sqrt{f}}\right)(\cosh(d)-\sinh(d))+3\sqrt{3}e^{\frac{e^2}{2f}}\text{erfi}\left(\frac{e+2fx+b\log(f)}{2\sqrt{f}}\right)\sinh(d)+e^{\frac{9e^2+2b^2\log^2(f)}{6f}}\text{erf}\left(\frac{3e+6fx-b\log(f)}{2\sqrt{3}\sqrt{f}}\right)(\cosh(3d)-\sinh(3d))+e^{\frac{b^2\log^2(f)}{6f}}\text{erfi}\left(\frac{3e+6fx+b\log(f)}{2\sqrt{3}\sqrt{f}}\right)\sinh(3d)\right)$$

input
```
Integrate[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(f^(a - (b*e + f)/(2*f))*Sqrt[Pi/3]*(3*Sqrt[3]*E^(e^2/(2*f))*Cosh[d]*Erfi[
(e + 2*f*x + b*Log[f])/(2*Sqrt[f])] + E^((b^2*Log[f]^2)/(6*f))*Cosh[3*d]*E
rfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])] + 3*Sqrt[3]*E^((2*e^2 +
b^2*Log[f]^2)/(2*f))*Erf[(e + 2*f*x - b*Log[f])/(2*Sqrt[f])]*(Cosh[d] - Si
nh[d]) + 3*Sqrt[3]*E^(e^2/(2*f))*Erfi[(e + 2*f*x + b*Log[f])/(2*Sqrt[f])]*
Sinh[d] + E^((9*e^2 + 2*b^2*Log[f]^2)/(6*f))*Erf[(3*e + 6*f*x - b*Log[f])/
(2*Sqrt[3]*Sqrt[f])]*(Cosh[3*d] - Sinh[3*d]) + E^((b^2*Log[f]^2)/(6*f))*Er
fi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])]*Sinh[3*d]))/(16*E^((3*e^2
 + b^2*Log[f]^2)/(4*f)))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{3}{8}f^{a+bx}\exp\left(-3\left(d+ex+fx^2\right)+2d+2ex+2fx^2\right)+\frac{3}{8}f^{a+bx}\exp\left(-3\left(d+ex+fx^2\right)+4d+4ex+4fx^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{\frac{(e-b\log(f))^2}{4f}-d}\text{erf}\left(\frac{-b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{\frac{(3e-b\log(f))^2}{12f}-3d}\text{erf}\left(\frac{-b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}e^{d-\frac{(b\log(f)+e)^2}{4f}}\text{erfi}\left(\frac{b\log(f)+e+2fx}{2\sqrt{f}}\right)+\frac{1}{16}\sqrt{\frac{\pi}{3}}f^{a-\frac{1}{2}}e^{3d-\frac{(b\log(f)+3e)^2}{12f}}\text{erfi}\left(\frac{b\log(f)+3e+6fx}{2\sqrt{3}\sqrt{f}}\right)$$

input
```
Int[f^(a + b*x)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + (e - b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erf[(e + 2*f*x -
b*Log[f])/(2*Sqrt[f])])/16 + (E^(-3*d + (3*e - b*Log[f])^2/(12*f))*f^(-1/2
 + a)*Sqrt[Pi/3]*Erf[(3*e + 6*f*x - b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16 + (
3*E^(d - (e + b*Log[f])^2/(4*f))*f^(-1/2 + a)*Sqrt[Pi]*Erfi[(e + 2*f*x + b
*Log[f])/(2*Sqrt[f])])/16 + (E^(3*d - (3*e + b*Log[f])^2/(12*f))*f^(-1/2 +
 a)*Sqrt[Pi/3]*Erfi[(3*e + 6*f*x + b*Log[f])/(2*Sqrt[3]*Sqrt[f])])/16
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 4.44 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{3}\,\sqrt{f}\,x+\frac{(b\ln(f)-3e)\sqrt{3}}{6\sqrt{f}}\right)\sqrt{3}\,\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{b^{2}\ln(f)^{2}-6\ln(f)be-36df+9e^{2}}{12f}}}{48\sqrt{f}}-\frac{\operatorname{erf}\left(-\sqrt{-3f}\,x+\frac{3e+b\ln(f)}{2\sqrt{-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+6\ln(}{12}}}{16\sqrt{-3f}}$

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/48*erf(-3^(1/2)*f^(1/2)*x+1/6*(b*ln(f)-3*e)*3^(1/2)/f^(1/2))/f^(1/2)*3^
(1/2)*Pi^(1/2)*f^a*exp(1/12*(b^2*ln(f)^2-6*ln(f)*b*e-36*d*f+9*e^2)/f)-1/16
*erf(-(-3*f)^(1/2)*x+1/2*(3*e+b*ln(f))/(-3*f)^(1/2))/(-3*f)^(1/2)*Pi^(1/2)
*f^a*exp(-1/12*(b^2*ln(f)^2+6*ln(f)*b*e-36*d*f+9*e^2)/f)-3/16*erf(-f^(1/2)
*x+1/2*(b*ln(f)-e)/f^(1/2))/f^(1/2)*Pi^(1/2)*f^a*exp(1/4*(b^2*ln(f)^2-2*ln
(f)*b*e-4*d*f+e^2)/f)-3/16*erf(-(-f)^(1/2)*x+1/2*(e+b*ln(f))/(-f)^(1/2))/(
-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d*f+e^2)/f)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(199) = 398$.

Time = 0.10 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.10

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/48*(sqrt(3)*sqrt(pi)*sqrt(-f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f
+ 6*(b*e - 2*a*f)*log(f))/f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*e)*sqrt
(-f)/f) + sqrt(3)*sqrt(pi)*sqrt(f)*cosh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*
f - 6*(b*e - 2*a*f)*log(f))/f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) + 3*e)/s
qrt(f)) - sqrt(3)*sqrt(pi)*sqrt(-f)*erf(1/6*sqrt(3)*(6*f*x + b*log(f) + 3*
e)*sqrt(-f)/f)*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f + 6*(b*e - 2*a*f)*
log(f))/f) + sqrt(3)*sqrt(pi)*sqrt(f)*erf(-1/6*sqrt(3)*(6*f*x - b*log(f) +
 3*e)/sqrt(f))*sinh(1/12*(b^2*log(f)^2 + 9*e^2 - 36*d*f - 6*(b*e - 2*a*f)*
log(f))/f) + 9*sqrt(pi)*sqrt(-f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f + 2*
(b*e - 2*a*f)*log(f))/f)*erf(1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f) + 9*sq
rt(pi)*sqrt(f)*cosh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*log(
f))/f)*erf(-1/2*(2*f*x - b*log(f) + e)/sqrt(f)) - 9*sqrt(pi)*sqrt(-f)*erf(
1/2*(2*f*x + b*log(f) + e)*sqrt(-f)/f)*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*
f + 2*(b*e - 2*a*f)*log(f))/f) + 9*sqrt(pi)*sqrt(f)*erf(-1/2*(2*f*x - b*lo
g(f) + e)/sqrt(f))*sinh(1/4*(b^2*log(f)^2 + e^2 - 4*d*f - 2*(b*e - 2*a*f)*
log(f))/f))/f
```

Sympy [F]

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x)*cosh(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 228, normalized size of antiderivative = 0.89

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{-f}x-\frac{\sqrt{3}(b\log(f)+3e)}{6\sqrt{-f}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{12f}\right)}}{48\sqrt{-f}}$$
$$+\frac{3}{16}\sqrt{\pi}f^{a-\frac{1}{2}}\operatorname{erf}\left(\sqrt{f}x-\frac{b\log(f)-e}{2\sqrt{f}}\right)e^{\left(-d+\frac{(b\log(f)-e)^2}{4f}\right)}$$
$$+\frac{\sqrt{3}\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{3}\sqrt{f}x-\frac{\sqrt{3}(b\log(f)-3e)}{6\sqrt{f}}\right)e^{\left(-3d+\frac{(b\log(f)-3e)^2}{12f}\right)}}{48\sqrt{f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-f}x-\frac{b\log(f)+e}{2\sqrt{-f}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4f}\right)}}{16\sqrt{-f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(-f)*x - 1/6*sqrt(3)*(b*log(f) +
 3*e)/sqrt(-f))*e^(3*d - 1/12*(b*log(f) + 3*e)^2/f)/sqrt(-f) + 3/16*sqrt(p
i)*f^(a - 1/2)*erf(sqrt(f)*x - 1/2*(b*log(f) - e)/sqrt(f))*e^(-d + 1/4*(b*
log(f) - e)^2/f) + 1/48*sqrt(3)*sqrt(pi)*f^a*erf(sqrt(3)*sqrt(f)*x - 1/6*s
qrt(3)*(b*log(f) - 3*e)/sqrt(f))*e^(-3*d + 1/12*(b*log(f) - 3*e)^2/f)/sqrt
(f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-f)*x - 1/2*(b*log(f) + e)/sqrt(-f))*e^(d
 - 1/4*(b*log(f) + e)^2/f)/sqrt(-f)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.09

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{-f}\left(6x+\frac{b\log(f)+3e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+6be\log(f)-12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{-f}}$$
$$-\frac{\sqrt{3}\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{6}\sqrt{3}\sqrt{f}\left(6x-\frac{b\log(f)-3e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-6be\log(f)+12af\log(f)+9e^2-36df}{12f}\right)}}{48\sqrt{f}}$$
$$-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-f}\left(2x+\frac{b\log(f)+e}{f}\right)\right)e^{\left(-\frac{b^2\log(f)^2+2be\log(f)-4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{-f}}$$
$$-\frac{3\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{f}\left(2x-\frac{b\log(f)-e}{f}\right)\right)e^{\left(\frac{b^2\log(f)^2-2be\log(f)+4af\log(f)+e^2-4df}{4f}\right)}}{16\sqrt{f}}$$

input
```
integrate(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(-f)*(6*x + (b*log(f) + 3*e)/f
))*e^(-1/12*(b^2*log(f)^2 + 6*b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)
/f)/sqrt(-f) - 1/48*sqrt(3)*sqrt(pi)*erf(-1/6*sqrt(3)*sqrt(f)*(6*x - (b*lo
g(f) - 3*e)/f))*e^(1/12*(b^2*log(f)^2 - 6*b*e*log(f) + 12*a*f*log(f) + 9*e
^2 - 36*d*f)/f)/sqrt(f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-f)*(2*x + (b*log(f)
 + e)/f))*e^(-1/4*(b^2*log(f)^2 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*
f)/f)/sqrt(-f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(f)*(2*x - (b*log(f) - e)/f))*
e^(1/4*(b^2*log(f)^2 - 2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/f)/sqrt(
f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+b\,x}\,\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x)*cosh(d + e*x + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx}\cosh^3\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{bx}\cosh\left(f\,x^2+ex+d\right)^3dx\right)$$

input
```
int(f^(b*x+a)*cosh(f*x^2+e*x+d)^3,x)
```

output
```
f**a*int(f**(b*x)*cosh(d + e*x + f*x**2)**3,x)
```

3.312 $\int f^{a+cx^2}\cosh(d+ex)\,dx$

Optimal result

Integrand size = 16, antiderivative size = 133

$$\int f^{a+cx^2}\cosh(d+ex)\,dx = -\frac{e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/4*exp(-d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-2*c*x*ln(f))/c^(1/2)
/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*
erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\cosh(d+ex)\,dx = \frac{e^{-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\left(\mathrm{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\mathrm{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sinh(d))\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x],x]
```

output
```
(f^a*Sqrt[Pi]*(Erfi[(-e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d]
 - Sinh[d]) + Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] +
 Sinh[d])))/(4*Sqrt[c]*E^(e^2/(4*c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh(d+ex)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-ex}f^{a+cx^2}+\frac{1}{2}e^{d+ex}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\text{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\text{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x],x]
```

output
```
-1/4*(E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sq
rt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^(d - e^2/(4*c*Log[f]))*f
^a*Sqrt[Pi]*Erfi[(e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*
Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.41 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.88

method	result	size
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{4d\ln(f)c+e^{2}}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{4d\ln(f)c-e^{2}}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}$	117

input
```
int(f^(c*x^2+a)*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/
2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2
*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(1/4*(4*d*ln(f)*c-e^
2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. 2(101) = 202.

Time = 0.09 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.62

$$\int f^{a+cx^2}\cosh(d+ex)\,dx =$$
$$-\frac{\sqrt{-c\log\left(f\right)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)+e)\sqrt{}}{2\,c\log(f)}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) -
e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^2)
/(c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqr
t(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*l
og(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*log(f
))))*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cosh(d+ex)\,dx=\int f^{a+cx^2}\cosh\left(d+ex\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\cosh(d+ex)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log\left(f\right)}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4
*e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x
+ 1/2*e/sqrt(-c*log(f)))*e^(-d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.99

$$\int f^{a+cx^2} \cosh(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-e^2}{4\,c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c
*log(f)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)
*erf(-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4
*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2} \cosh(d+ex)\,dx = \int f^{c\,x^2+a}\cosh(d+e\,x)\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x),x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x), x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh(d+ex)\,dx = f^a\left(\int f^{c\,x^2}\cosh(ex+d)\,dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(e*x+d),x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + e*x),x)
```

3.313 $\int f^{a+cx^2}\cosh^2(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 161

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = \frac{f^a\sqrt{\pi}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}} + \frac{e^{2d-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*f^a*Pi^(1/2)*erfi(c^(1/2)*x*ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)-1/8*exp(-
2*d-e^2/c/ln(f))*f^a*Pi^(1/2)*erfi((e-c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1
/2)/ln(f)^(1/2)+1/8*exp(2*d-e^2/c/ln(f))*f^a*Pi^(1/2)*erfi((e+c*x*ln(f))/c
^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.15 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = \frac{e^{-\frac{e^2}{c\log(f)}}f^a\sqrt{\pi}\left(2e^{\frac{e^2}{c\log(f)}}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)+\text{erfi}\left(\frac{-e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))+\text{erfi}\left(\frac{e+cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)(\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(2*E^(e^2/(c*Log[f]))*Erfi[Sqrt[c]*x*Sqrt[Log[f]]] + Erfi[(-
e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(e
+ c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(8*Sqrt[c]
*E^(e^2/(c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.111$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{4}e^{-2d-2ex}f^{a+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+cx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\sqrt{\pi}f^{a}e^{-\frac{e^{2}}{c\log(f)}-2d}\text{erfi}\left(\frac{e-cx\log(f)}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^{a}e^{2d-\frac{e^{2}}{c\log(f)}}\text{erfi}\left(\frac{cx\log(f)+e}{\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^{a}\text{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) - (E^
(-2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - c*x*Log[f])/(Sqrt[c]*Sqrt[L
og[f]])])/(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - e^2/(c*Log[f]))*f^a*Sqrt[Pi
]*Erfi[(e + c*x*Log[f])/(Sqrt[c]*Sqrt[Log[f]])])/(8*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{2d\ln(f)c+e^{2}}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e}{\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{\frac{2d\ln(f)c-e^{2}}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}+\frac{f^{a}\sqrt{\pi}\,\operatorname{erf}\left(\sqrt{-c}\right.}{4\sqrt{-c\ln(}}$

input
```
int(f^(c*x^2+a)*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*erf((-c*ln(f))^(1/2)*x+e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f
^a*exp(-(2*d*ln(f)*c+e^2)/ln(f)/c)-1/8*erf(-(-c*ln(f))^(1/2)*x+e/(-c*ln(f)
)^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp((2*d*ln(f)*c-e^2)/ln(f)/c)+1/4*
f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 242, normalized size of antiderivative = 1.50

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx =$$
$$-\frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh(a\log(f))+\sqrt{\pi}\sinh(a\log(f))\right)\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)+\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(a*log(f)) + sqrt(pi)*sinh(a*log(f))
)*erf(sqrt(-c*log(f))*x) + sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 +
2*c*d*log(f) - e^2)/(c*log(f))) + sqrt(pi)*sinh((a*c*log(f)^2 + 2*c*d*log(
f) - e^2)/(c*log(f))))*erf((c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) +
sqrt(-c*log(f))*(sqrt(pi)*cosh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(
f))) + sqrt(pi)*sinh((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f))))*erf(
(c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx=\int f^{a+cx^2}\cosh^2(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+a)*cosh(e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 131, normalized size of antiderivative = 0.81

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(2d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{\sqrt{-c\log(f)}}\right)e^{\left(-2d-\frac{e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - e/sqrt(-c*log(f)))*e^(2*d - e^2/(
c*log(f)))/sqrt(-c*log(f)) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + e/sq
rt(-c*log(f)))*e^(-2*d - e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*f^
a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 150, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx = -\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2+2cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log(f)}\left(x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{ac\log(f)^2-2cd\log(f)-e^2}{c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*e
rf(-sqrt(-c*log(f))*(x + e/(c*log(f))))*e^((a*c*log(f)^2 + 2*c*d*log(f) -
e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f))*(x -
e/(c*log(f))))*e^((a*c*log(f)^2 - 2*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*
log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx=\int f^{c\,x^2+a}\cosh(d+e\,x)^2\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x)^2,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x)^2, x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh^2(d+ex)\,dx=f^a\left(\int f^{c\,x^2}\cosh\left(ex+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(e*x+d)^2,x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + e*x)**2,x)
```

3.314 $\int f^{a+cx^2}\cosh^3(d+ex)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 271

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3e^{d-\frac{e^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\text{erfi}\left(\frac{3e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
-3/16*exp(-d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-2*c*x*ln(f))/c^(1/2
)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-9/4*e^2/c/ln(f))*f^a*Pi^(
1/2)*erfi(1/2*(3*e-2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+3
/16*exp(d-1/4*e^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e+2*c*x*ln(f))/c^(1/2)/l
n(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/16*exp(3*d-9/4*e^2/c/ln(f))*f^a*Pi^(1/2)
*erfi(1/2*(3*e+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 214, normalized size of antiderivative = 0.79

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$
$$=\frac{e^{-\frac{9e^2}{4c\log(f)}}f^a\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(3e^{\frac{2e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e^2}{c\log(f)}}\operatorname{erfi}\left(\frac{-e+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\mathrm{s}\right.\right.}{16\sqrt{c}\sqrt{\mathrm{lo}}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

output
```
(f^a*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((2*e^2)/(c*Log[f]))*Erfi[(e + 2*c
*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*E^((2*e^2)/(c*Log[f]))*Erfi[(-e +
 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3
*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + Er
fi[(-3*e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3*d])
))/(16*Sqrt[c]*E^((9*e^2)/(4*c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$
$$\Big\downarrow \text{6039}$$
$$\int \left(\frac{1}{8}e^{-3d-3ex}f^{a+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+cx^2}+\frac{3}{8}e^{d+ex}f^{a+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\pi}f^a e^{-\frac{e^2}{4c\log(f)}-d}\operatorname{erfi}\left(\frac{e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{9e^2}{4c\log(f)}-3d}\operatorname{erfi}\left(\frac{3e-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{9e^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input

```
Int[f^(a + c*x^2)*Cosh[d + e*x]^3,x]
```

output

```
(-3*E^(-d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - 2*c*x*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E^(-3*d - (9*e^2)/(4*c*Lo
g[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(
16*Sqrt[c]*Sqrt[Log[f]]) + (3*E^(d - e^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(
e + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (
E^(3*d - (9*e^2)/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + 2*c*x*Log[f])/(2*S
qrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039

```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.86

method	result
risch	$\frac{\operatorname{erf}\left(\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a \mathrm{e}^{-\frac{3\left(4d\ln(f)c+3e^2\right)}{4\ln(f)c}}}{16\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a \mathrm{e}^{\frac{3d\ln(f)c-\frac{9e^2}{4}}{c\ln(f)}}}{16\sqrt{-c\ln(f)}}+\frac{3\operatorname{erf}\left(\sqrt{\right.}}{}$

input

```
int(f^(c*x^2+a)*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*erf((-c*ln(f))^(1/2)*x+3/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1
/2)*f^a*exp(-3/4*(4*d*ln(f)*c+3*e^2)/ln(f)/c)-1/16*erf(-(-c*ln(f))^(1/2)*x
+3/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*
c-3*e^2)/ln(f)/c)+3/16*erf((-c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*
ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c+e^2)/ln(f)/c)-3/16*erf(-(-
c*ln(f))^(1/2)*x+1/2*e/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp
(1/4*(4*d*ln(f)*c-e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(205) = 410$.

Time = 0.09 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.57

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx =$$

$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)+\sqrt{\pi}\sinh\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)-9\,e^2}{4\,c\log(f)}\right)\right)\operatorname{erf}\left(\frac{(2\,cx\log(f)+}{2\,c}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f)
- 9*e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) -
 9*e^2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f
))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f)
- e^2)/(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f) - e^
2)/(c*log(f))))*erf(1/2*(2*c*x*log(f) + e)*sqrt(-c*log(f))/(c*log(f))) + 3
*sqrt(-c*log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/
(c*log(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - e^2)/(c*l
og(f))))*erf(1/2*(2*c*x*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*
log(f))*(sqrt(pi)*cosh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log
(f))) + sqrt(pi)*sinh(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 9*e^2)/(c*log(
f))))*erf(1/2*(2*c*x*log(f) - 3*e)*sqrt(-c*log(f))/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx=\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+a)*cosh(e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.78

$$\begin{aligned}\int f^{a+cx^2}\cosh^3(d+ex)\,dx=&\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}\\&+\frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}\\&+\frac{3\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x+\frac{e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}\\&+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x+\frac{3e}{2\sqrt{-c\log(f)}}\right)e^{\left(-3d-\frac{9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}\end{aligned}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 3/2*e/sqrt(-c*log(f)))*e^(3*d -
9/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f)
)*x - 1/2*e/sqrt(-c*log(f)))*e^(d - 1/4*e^2/(c*log(f)))/sqrt(-c*log(f)) +
3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x + 1/2*e/sqrt(-c*log(f)))*e^(-d - 1
/4*e^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))
*x + 3/2*e/sqrt(-c*log(f)))*e^(-3*d - 9/4*e^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.97

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x-\frac{3e}{c\log(f)}\right)\right)e^{\left(\frac{4ac\log(f)^2-12cd\log(f)-9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + 3*e/(c*log(f))))*e^(1/4*(4*
a*c*log(f)^2 + 12*c*d*log(f) - 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*s
qrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + e/(c*log(f))))*e^(1/4*(4*a*c*log(f
)^2 + 4*c*d*log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(
-1/2*sqrt(-c*log(f))*(2*x - e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*
log(f) - e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c
*log(f))*(2*x - 3*e/(c*log(f))))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) -
9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = \int f^{c\,x^2+a}\cosh(d+e\,x)^3\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x)^3,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x)^3, x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh^3(d+ex)\,dx = f^a\left(\int f^{c\,x^2}\cosh(ex+d)^3\,dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(e*x+d)^3,x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + e*x)**3,x)
```

3.315 $\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 18, antiderivative size = 81

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\frac{e^{-d}f^a\sqrt{\pi}\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^d f^a\sqrt{\pi}\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
1/4*f^a*Pi^(1/2)*erf(x*(f-c*ln(f))^(1/2))/exp(d)/(f-c*ln(f))^(1/2)+1/4*exp
(d)*f^a*Pi^(1/2)*erfi(x*(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\frac{1}{4}f^a\sqrt{\pi}\left(\frac{\operatorname{erf}\left(x\sqrt{f-c\log(f)}\right)(\cosh(d)-\sinh(d))}{\sqrt{f-c\log(f)}}+\frac{\operatorname{erfi}\left(x\sqrt{f+c\log(f)}\right)(\cosh(d)+\sinh(d))}{\sqrt{f+c\log(f)}}\right)$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*((Erf[x*Sqrt[f - c*Log[f]]]*(Cosh[d] - Sinh[d]))/Sqrt[f - c*
Log[f]] + (Erfi[x*Sqrt[f + c*Log[f]]]*(Cosh[d] + Sinh[d]))/Sqrt[f + c*Log[
f]]))/4
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.111, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{2}e^{-d-fx^2}f^{a+cx^2}+\frac{1}{2}e^{d+fx^2}f^{a+cx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}e^{-d}f^a\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^{d}f^a\text{erfi}\left(x\sqrt{c\log(f)+f}\right)}{4\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(4*E^d*Sqrt[f - c*Log[f]]) + (E^d
*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.45 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

method	result	size
risch	$\frac{f^{a} {\mathrm e}^{-d} \sqrt{\pi}\, \operatorname{erf}\left(x \sqrt{f -c \ln \left(f \right)}\right)}{4 \sqrt{f -c \ln \left(f \right)}}+\frac{f^{a} {\mathrm e}^{d} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-f}\, x \right)}{4 \sqrt{-c \ln \left(f \right)-f}}$	70

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*f^a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+1/4*f^
a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 145 vs. 2(63) = 126.

Time = 0.09 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.79

$$\int f^{a+cx^2} \cosh\left(d+fx^2\right)\, dx =$$

$$-\frac{\left(\sqrt{\pi}(c\log\left(f\right)+f)\cosh\left(a\log\left(f\right)-d\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(a\log\left(f\right)-d\right)\right)\sqrt{-c\log\left(f\right)+f}\operatorname{erf}\left(\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(a*log(f) - d) + sqrt(pi)*(c*log(f) + f
)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-c*log(f) + f)*x) + (sq
rt(pi)*(c*log(f) - f)*cosh(a*log(f) + d) + sqrt(pi)*(c*log(f) - f)*sinh(a*
log(f) + d))*sqrt(-c*log(f) - f)*erf(sqrt(-c*log(f) - f)*x))/(c^2*log(f)^2
 - f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cosh\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+d),x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.85

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x\right)e^{(-d)}}{4\sqrt{-c\log(f)+f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x\right)e^{d}}{4\sqrt{-c\log(f)-f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) + f) + 1
/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f) - f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.93

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{(a\log(f)+d)}}{4\sqrt{-c\log\left(f\right)-f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{(a\log(f)-d)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/sqrt(-c*log(f)
- f) - 1/4*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f) - d)/sqrt(-c*l
og(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx = \int f^{c\,x^2+a}\,\cosh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*cosh(d + f*x^2), x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh\left(d+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2}\cosh\left(f\,x^2+d\right)dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d),x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + f*x**2),x)
```

3.316 $\int f^{a+cx^2} \cosh^2(d+fx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 128

$$\int f^{a+cx^2}\cosh^2(d+fx^2)\,dx = \frac{f^a\sqrt{\pi}\,\mathrm{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}} + \frac{e^{-2d}f^a\sqrt{\pi}\,\mathrm{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}} + \frac{e^{2d}f^a\sqrt{\pi}\,\mathrm{erfi}\left(x\sqrt{2f+c\log(f)}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^a*Pi^(1/2)*erfi(c^(1/2)*x*ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/8*f^a*P
i^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))/exp(2*d)/(2*f-c*ln(f))^(1/2)+1/8*exp(2*
d)*f^a*Pi^(1/2)*erfi(x*(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.40

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx$$
$$=\frac{f^a\sqrt{\pi}\left(\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(-8f^2+2c^2\log^2(f)\right)+\sqrt{c}\sqrt{\log(f)}\left(\operatorname{erf}\left(x\sqrt{2f-c\log(f)}\right)\sqrt{2f-c\log(f)}(\right.\right.}{8\sqrt{c}\sqrt{\mathrm{lo}}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*(Erfi[Sqrt[c]*x*Sqrt[Log[f]]]*(-8*f^2 + 2*c^2*Log[f]^2) + Sq
rt[c]*Sqrt[Log[f]]*(Erf[x*Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f
+ c*Log[f])*(-Cosh[2*d] + Sinh[2*d]) - Erfi[x*Sqrt[2*f + c*Log[f]]]*(2*f -
 c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt
[Log[f]]*(-4*f^2 + c^2*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \; 6039$$
$$\int\left(\frac{1}{4}e^{-2d-2fx^2}f^{a+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+cx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \; 2009$$
$$\frac{\sqrt{\pi}e^{-2d}f^a\operatorname{erf}\left(x\sqrt{2f-c\log(f)}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi}e^{2d}f^a\operatorname{erfi}\left(x\sqrt{c\log(f)+2f}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (f^
a*Sqrt[Pi]*Erf[x*Sqrt[2*f - c*Log[f]]])/(8*E^(2*d)*Sqrt[2*f - c*Log[f]]) +
 (E^(2*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[2*f + c*Log[f]]])/(8*Sqrt[2*f + c*Log[f
]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.79

method	result	size
risch	$\frac{f^{a} {\mathrm e}^{-2 d} \sqrt{\pi}\, \operatorname{erf}\left(x \sqrt{2 f -c \ln \left(f \right)}\right)}{8 \sqrt{2 f -c \ln \left(f \right)}}+\frac{f^{a} {\mathrm e}^{2 d} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)-2 f}\, x \right)}{8 \sqrt{-c \ln \left(f \right)-2 f}}+\frac{f^{a} \sqrt{\pi}\, \operatorname{erf}\left(\sqrt{-c \ln \left(f \right)}\, x \right)}{4 \sqrt{-c \ln \left(f \right)}}$	101

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*f^a*exp(-2*d)*Pi^(1/2)/(2*f-c*ln(f))^(1/2)*erf(x*(2*f-c*ln(f))^(1/2))+
1/8*f^a*exp(2*d)*Pi^(1/2)/(-c*ln(f)-2*f)^(1/2)*erf((-c*ln(f)-2*f)^(1/2)*x)
+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)*erf((-c*ln(f))^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. $2(98) = 196$.

Time = 0.10 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.98

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx =$$

$$-\frac{\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\cosh\left(a\log\left(f\right)-2\,d\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2+2\,cf\log\left(f\right)\right)\sinh\left(a\log\left(f\right)-\right.\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(a*log(f) - 2*d) + sqrt(
pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(a*log(f) - 2*d))*sqrt(-c*log(f) + 2
*f)*erf(sqrt(-c*log(f) + 2*f)*x) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))
*cosh(a*log(f) + 2*d) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(a*log(
f) + 2*d))*sqrt(-c*log(f) - 2*f)*erf(sqrt(-c*log(f) - 2*f)*x) + 2*(sqrt(pi
)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*
sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x))/(c^3*log(f)^3 - 4*
c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \int f^{a+cx^2}\cosh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.78

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{(2\,d)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}} + \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{(-2\,d)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}} + \frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)}x\right)}{4\,\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x)*e^(2*d)/sqrt(-c*log(f) - 2*f
) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x)*e^(-2*d)/sqrt(-c*log(f)
+ 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 107, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\,\sqrt{-c\log\left(f\right)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-2\,f}x\right)e^{(a\log(f)+2\,d)}}{8\,\sqrt{-c\log\left(f\right)-2\,f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+2\,f}x\right)e^{(a\log(f)-2\,d)}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*e
rf(-sqrt(-c*log(f) - 2*f)*x)*e^(a*log(f) + 2*d)/sqrt(-c*log(f) - 2*f) - 1/
8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*x)*e^(a*log(f) - 2*d)/sqrt(-c*log(f)
 + 2*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\int f^{c\,x^2+a}\cosh\left(f\,x^2+d\right)^2\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh^2\left(d+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2}\cosh\left(f\,x^2+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d)^2,x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + f*x**2)**2,x)
```

3.317 $\int f^{a+cx^2} \cosh^3(d+fx^2)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 171

$$\int f^{a+cx^2}\cosh^3(d+fx^2)\,dx = \frac{3e^{-d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}} + \frac{e^{-3d}f^a\sqrt{\pi}\,\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}} + \frac{3e^{d}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{f+c\log(f)}\right)}{16\sqrt{f+c\log(f)}} + \frac{e^{3d}f^a\sqrt{\pi}\,\text{erfi}\left(x\sqrt{3f+c\log(f)}\right)}{16\sqrt{3f+c\log(f)}}$$

output

```
3/16*f^a*Pi^(1/2)*erf(x*(f-c*ln(f))^(1/2))/exp(d)/(f-c*ln(f))^(1/2)+1/16*f
^a*Pi^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))/exp(3*d)/(3*f-c*ln(f))^(1/2)+3/16*e
xp(d)*f^a*Pi^(1/2)*erfi(x*(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)+1/16*exp(3*
d)*f^a*Pi^(1/2)*erfi(x*(3*f+c*ln(f))^(1/2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.58

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx$$
$$= \frac{f^a\sqrt{\pi}\Big(3\mathrm{erf}\Big(x\sqrt{f-c\log(f)}\Big)\sqrt{f-c\log(f)}\left(9f^3+9cf^2\log(f)-c^2f\log^2(f)-c^3\log^3(f)\right)(\cosh(d)-\mathrm{s}}{}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*Erf[x*Sqrt[f - c*Log[f]]]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c
*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*
Log[f])*(Erf[x*Sqrt[3*f - c*Log[f]]]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*L
og[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*Erfi[x
*Sqrt[f + c*Log[f]]]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d
]) + Erfi[x*Sqrt[3*f + c*Log[f]]]*(f + c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cos
h[3*d] + Sinh[3*d]))))/(16*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Log[f]^4))
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx$$
$$\Big\downarrow \text{6039}$$
$$\int \left(\frac{1}{8}e^{-3d-3fx^2}f^{a+cx^2}+\frac{3}{8}e^{-d-fx^2}f^{a+cx^2}+\frac{3}{8}e^{d+fx^2}f^{a+cx^2}+\frac{1}{8}e^{3d+3fx^2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}e^{-d}f^a\text{erf}\left(x\sqrt{f-c\log(f)}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}e^{-3d}f^a\text{erf}\left(x\sqrt{3f-c\log(f)}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}e^{d}f^a\text{erfi}\left(x\sqrt{c\log(f)+f}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}e^{3d}f^a\text{erfi}\left(x\sqrt{c\log(f)+3f}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(3*f^a*Sqrt[Pi]*Erf[x*Sqrt[f - c*Log[f]]])/(16*E^d*Sqrt[f - c*Log[f]]) + (
f^a*Sqrt[Pi]*Erf[x*Sqrt[3*f - c*Log[f]]])/(16*E^(3*d)*Sqrt[3*f - c*Log[f]]
) + (3*E^d*f^a*Sqrt[Pi]*Erfi[x*Sqrt[f + c*Log[f]]])/(16*Sqrt[f + c*Log[f]]
) + (E^(3*d)*f^a*Sqrt[Pi]*Erfi[x*Sqrt[3*f + c*Log[f]]])/(16*Sqrt[3*f + c*L
og[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.55 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.84

method	result
risch	$\frac{f^a e^{-3d}\sqrt{\pi}\,\text{erf}\left(x\sqrt{3f-c\ln(f)}\right)}{16\sqrt{3f-c\ln(f)}}+\frac{f^a e^{3d}\sqrt{\pi}\,\text{erf}\left(\sqrt{-c\ln(f)-3f}\,x\right)}{16\sqrt{-c\ln(f)-3f}}+\frac{3f^a e^{-d}\sqrt{\pi}\,\text{erf}\left(x\sqrt{f-c\ln(f)}\right)}{16\sqrt{f-c\ln(f)}}+\frac{3f^a e^{d}\sqrt{\pi}\,\text{erf}\left(\sqrt{-c}\right.}{16\sqrt{-c\ln(f}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*f^a*exp(-3*d)*Pi^(1/2)/(3*f-c*ln(f))^(1/2)*erf(x*(3*f-c*ln(f))^(1/2))
+1/16*f^a*exp(3*d)*Pi^(1/2)/(-c*ln(f)-3*f)^(1/2)*erf((-c*ln(f)-3*f)^(1/2)*
x)+3/16*f^a*exp(-d)*Pi^(1/2)/(f-c*ln(f))^(1/2)*erf(x*(f-c*ln(f))^(1/2))+3/
16*f^a*exp(d)*Pi^(1/2)/(-c*ln(f)-f)^(1/2)*erf((-c*ln(f)-f)^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(135) = 270$.

Time = 0.11 (sec) , antiderivative size = 491, normalized size of antiderivative = 2.87

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(a*log(f) - 3*d) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*l
og(f) - 3*f^3)*sinh(a*log(f) - 3*d))*sqrt(-c*log(f) + 3*f)*erf(sqrt(-c*log
(f) + 3*f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f
) - 9*f^3)*cosh(a*log(f) - d) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 -
9*c*f^2*log(f) - 9*f^3)*sinh(a*log(f) - d))*sqrt(-c*log(f) + f)*erf(sqrt(-
c*log(f) + f)*x) + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*lo
g(f) + 9*f^3)*cosh(a*log(f) + d) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2
 - 9*c*f^2*log(f) + 9*f^3)*sinh(a*log(f) + d))*sqrt(-c*log(f) - f)*erf(sqr
t(-c*log(f) - f)*x) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*l
og(f) + 3*f^3)*cosh(a*log(f) + 3*d) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log
(f)^2 - c*f^2*log(f) + 3*f^3)*sinh(a*log(f) + 3*d))*sqrt(-c*log(f) - 3*f)*
erf(sqrt(-c*log(f) - 3*f)*x))/(c^4*log(f)^4 - 10*c^2*f^2*log(f)^2 + 9*f^4)
```

Sympy [F]

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx=\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.84

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-3f}x\right)e^{(3d)}}{16\sqrt{-c\log(f)-3f}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x\right)e^{(-d)}}{16\sqrt{-c\log(f)+f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+3f}x\right)e^{(-3d)}}{16\sqrt{-c\log(f)+3f}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x\right)e^{d}}{16\sqrt{-c\log(f)-f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x)*e^(3*d)/sqrt(-c*log(f) - 3*
f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x)*e^(-d)/sqrt(-c*log(f) +
f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x)*e^(-3*d)/sqrt(-c*log(f
) + 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x)*e^d/sqrt(-c*log(f)
 - f)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.91

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-3\,f}x\right)e^{\left(a\log\left(f\right)+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)-f}x\right)e^{\left(a\log\left(f\right)+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}} - \frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+f}x\right)e^{\left(a\log\left(f\right)-d\right)}}{16\,\sqrt{-c\log\left(f\right)+f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\sqrt{-c\log\left(f\right)+3\,f}x\right)e^{\left(a\log\left(f\right)-3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-sqrt(-c*log(f) - 3*f)*x)*e^(a*log(f) + 3*d)/sqrt(-c*lo
g(f) - 3*f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) - f)*x)*e^(a*log(f) + d)/s
qrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-sqrt(-c*log(f) + f)*x)*e^(a*log(f)
 - d)/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*erf(-sqrt(-c*log(f) + 3*f)*x)*e^
(a*log(f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx = \int f^{c\,x^2+a}\,{\mathrm{cosh}\left(f\,x^2+d\right)}^3\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh^3\left(d+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2}\cosh\left(f\,x^2+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+d)^3,x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + f*x**2)**3,x)
```

3.318 $\int f^{a+cx^2} \cosh \left(d+ex+fx^2\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int f^{a+cx^2} \cosh \left(d+ex+fx^2\right) \, dx = \frac{e^{-d+\frac{e^2}{4f-4c\log(f)}} f^a \sqrt{\pi}\operatorname{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}} + \frac{e^{d-\frac{e^2}{4(f+c\log(f))}} f^a \sqrt{\pi}\operatorname{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
1/4*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e+2*x*(f-c*ln(f)))/(
f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/4*exp(d-e^2/(4*f+4*c*ln(f)))*f^a*Pi^
(1/2)*erfi(1/2*(e+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.18

$$\int f^{a+cx^2} \cosh \left(d+ex+fx^2\right) \, dx = \frac{e^{-\frac{e^2}{4(f+c\log(f))}} f^a \sqrt{\pi}\left(e^{\frac{e^2 f}{2f^2-2c^2\log^2(f)}}\operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f+c\log(f)}(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{e+2fx+2cx\log}{2\sqrt{f+c\log}}\right.\right.}{4\sqrt{f-c\log(f)}\sqrt{f+c\log(f)}}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*(E^((e^2*f)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(e + 2*f*x - 2*c*x
*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f + c*Log[f]]*(Cosh[d] - Sinh[d]) +
Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f - c*Log[f]]
*(Cosh[d] + Sinh[d])))/(4*E^(e^2/(4*(f + c*Log[f])))*Sqrt[f - c*Log[f]]*Sq
rt[f + c*Log[f]])
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \cosh\left(d+ex+fx^2\right) \, dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2} f^{a+cx^2} e^{-d-ex-fx^2}+\frac{1}{2} f^{a+cx^2} e^{d+ex+fx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{4f-4c\log(f)}-d} \text{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{d-\frac{e^2}{4(c\log(f)+f)}} \text{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]))
/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) + (E^(d - e^2/(4*(f + c*L
og[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])
])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.05

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{f-c\ln(f)}+\frac{e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{4d\ln(f)c-4df+e^{2}}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\, x+\frac{e}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{\frac{4d\ln(f)c+4df-e^{2}}{4f+4c\ln(f)}}}{4\sqrt{-c\ln(f)-f}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
1/4*erf(x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^
(1/2)*f^a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))-1/4*erf(-(-c*ln(f)
-f)^(1/2)*x+1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(
1/4*(4*d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(119) = 238.

Time = 0.10 (sec) , antiderivative size = 321, normalized size of antiderivative = 2.29

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx = -\frac{\left(\sqrt{\pi}(c\log\left(f\right)+f)\cosh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-4\,(cd+af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log\left(f\right)+f)\sinh\left(\frac{4\,ac\log(f)^2-e^2+4\,df-\cdots}{4\,(c\log(f)\cdots}\right.\right.}{\cdots}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f - 4*
(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(1/4*(4*
a*c*log(f)^2 - e^2 + 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-
c*log(f) + f)*erf(1/2*(2*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*lo
g(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d
*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(
1/4*(4*a*c*log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))
*sqrt(-c*log(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f
)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.91

$$\begin{aligned}\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx &= \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}\\&\quad+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}\end{aligned}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^
(d - 1/4*e^2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 1/4*sqrt(pi)*f^a*erf(sq
rt(-c*log(f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f)
 - f))/sqrt(-c*log(f) + f)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.23

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4ac\log(f)^2+4cd\log(f)+4af\log(f)-e^2+4df}{4(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4ac\log(f)^2-4cd\log(f)-4af\log(f)-e^2+4df}{4(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/
4*(4*a*c*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) +
 f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x
- e/(c*log(f) - f)))*e^(1/4*(4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f)
- e^2 + 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+a}\cosh\left(f\,x^2+e\,x+d\right)\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh\left(d+ex+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2}\cosh\left(f\,x^2+ex+d\right)dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d),x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + e*x + f*x**2),x)
```

3.319 $\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 183

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\frac{f^a\sqrt{\pi}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e+x(2f-c\log(f))}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{e^2}{2f+c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+x(2f+c\log(f))}{\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^a*Pi^(1/2)*erfi(c^(1/2)*x*ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/8*exp(-
2*d+e^2/(2*f-c*ln(f)))*f^a*Pi^(1/2)*erf((e+x*(2*f-c*ln(f)))/(2*f-c*ln(f))^
(1/2))/(2*f-c*ln(f))^(1/2)+1/8*exp(2*d-e^2/(2*f+c*ln(f)))*f^a*Pi^(1/2)*erf
i((e+x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.41

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{e^{\frac{e^2}{2f-c\log(f)}}f^a\sqrt{\pi}\left(-2e^{\frac{e^2}{-2f+c\log(f)}}\operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)\left(4f^2-c^2\log^2(f)\right)-\sqrt{c}\sqrt{\log(f)}\left(\operatorname{erf}\left(\frac{e+2fx-cx\log(f)}{\sqrt{2f-c\log(f)}}\right)\right.\right.}{}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(E^(e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*(-2*E^(e^2/(-2*f + c*Log[f]))*Erfi[
Sqrt[c]*x*Sqrt[Log[f]]]*(4*f^2 - c^2*Log[f]^2) - Sqrt[c]*Sqrt[Log[f]]*(Erf
[(e + 2*f*x - c*x*Log[f])/Sqrt[2*f - c*Log[f]]]*Sqrt[2*f - c*Log[f]]*(2*f
+ c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + E^((4*e^2*f)/(-4*f^2 + c^2*Log[f]^2)
)*Erfi[(e + 2*f*x + c*x*Log[f])/Sqrt[2*f + c*Log[f]]]*(2*f - c*Log[f])*Sqr
t[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2*d]))))/(8*Sqrt[c]*Sqrt[Log[f]]*(-4*f
^2 + c^2*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$\Big\downarrow \; 6039$$
$$\int \left(\frac{1}{4}f^{a+cx^2}e^{-2d-2ex-2fx^2}+\frac{1}{4}f^{a+cx^2}e^{2d+2ex+2fx^2}+\frac{1}{2}f^{a+cx^2}\right)dx$$
$$\Big\downarrow \; 2009$$

$$\frac{\sqrt{\pi} f^a e^{\frac{e^2}{2f-c\log(f)}-2d}\operatorname{erf}\left(\frac{x(2f-c\log(f))+e}{\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}} + \frac{\sqrt{\pi} f^a e^{2d-\frac{e^2}{c\log(f)+2f}}\operatorname{erfi}\left(\frac{x(c\log(f)+2f)+e}{\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}} + \frac{\sqrt{\pi} f^a \operatorname{erfi}\left(\sqrt{c}x\sqrt{\log(f)}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*Erfi[Sqrt[c]*x*Sqrt[Log[f]]])/(4*Sqrt[c]*Sqrt[Log[f]]) + (E^
(-2*d + e^2/(2*f - c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + x*(2*f - c*Log[f]))/Sq
rt[2*f - c*Log[f]]])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - e^2/(2*f + c*Log
[f]))*f^a*Sqrt[Pi]*Erfi[(e + x*(2*f + c*Log[f]))/Sqrt[2*f + c*Log[f]]])/(8
*Sqrt[2*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.61 (sec) , antiderivative size = 177, normalized size of antiderivative = 0.97

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{2f-c\ln(f)}+\frac{e}{\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{2d\ln(f)c-4df+e^2}{c\ln(f)-2f}}}{8\sqrt{2f-c\ln(f)}} - \frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\, x+\frac{e}{\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{\frac{2d\ln(f)c+4df-e^2}{2f+c\ln(f)}}}{8\sqrt{-c\ln(f)-2f}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*erf(x*(2*f-c*ln(f))^(1/2)+e/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f))^(1/2)*P
i^(1/2)*f^a*exp(-(2*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)
-2*f)^(1/2)*x+e/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*ex
p((2*d*ln(f)*c+4*d*f-e^2)/(2*f+c*ln(f)))+1/4*f^a*Pi^(1/2)/(-c*ln(f))^(1/2)
*erf((-c*ln(f))^(1/2)*x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(155) = 310$.

Time = 0.10 (sec) , antiderivative size = 420, normalized size of antiderivative = 2.30

$$\int f^{a+cx^2} \cosh^2\left(d+ex+fx^2\right) dx =$$

$$-\frac{2\left(\sqrt{\pi}\left(c^2\log\left(f\right)^2-4\,f^2\right)\cosh\left(a\log\left(f\right)\right)+\sqrt{\pi}\left(c^2\log\left(f\right)^2-4\,f^2\right)\sinh\left(a\log\left(f\right)\right)\right)\sqrt{-c\log\left(f\right)}\,\mathrm{erf}\left(\sqrt{\right.}\right.}{}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*(sqrt(pi)*(c^2*log(f)^2 - 4*f^2)*cosh(a*log(f)) + sqrt(pi)*(c^2*lo
g(f)^2 - 4*f^2)*sinh(a*log(f)))*sqrt(-c*log(f))*erf(sqrt(-c*log(f))*x) + (
sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh((a*c*log(f)^2 - e^2 + 4*d*f -
2*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*l
og(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f - 2*(c*d + a*f)*log(f))/(c*log(f)
- 2*f)))*sqrt(-c*log(f) + 2*f)*erf((c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f)
 + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*cosh((
a*c*log(f)^2 - e^2 + 4*d*f + 2*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqr
t(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh((a*c*log(f)^2 - e^2 + 4*d*f + 2*(
c*d + a*f)*log(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf((c*x*log(f
) + 2*f*x + e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)))/(c^3*log(f)^3 - 4*
c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 161, normalized size of antiderivative = 0.88

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-2f}x-\frac{e}{\sqrt{-c\log(f)-2f}}\right)e^{\left(2d-\frac{e^2}{c\log(f)+2f}\right)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+2f}x+\frac{e}{\sqrt{-c\log(f)+2f}}\right)e^{\left(-2d-\frac{e^2}{c\log(f)-2f}\right)}}{8\sqrt{-c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x\right)}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - e/sqrt(-c*log(f) - 2*f))*e^
(2*d - e^2/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(
sqrt(-c*log(f) + 2*f)*x + e/sqrt(-c*log(f) + 2*f))*e^(-2*d - e^2/(c*log(f)
 - 2*f))/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x)/s
qrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.08

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$= -\frac{\sqrt{\pi}f^a\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)}x\right)}{4\sqrt{-c\log\left(f\right)}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)-2f}\left(x+\frac{e}{c\log(f)+2f}\right)\right)e^{\left(\frac{ac\log(f)^2+2cd\log(f)+2af\log(f)-e^2+4df}{c\log(f)+2f}\right)}}{8\sqrt{-c\log\left(f\right)-2f}} - \frac{\sqrt{\pi}\operatorname{erf}\left(-\sqrt{-c\log\left(f\right)+2f}\left(x-\frac{e}{c\log(f)-2f}\right)\right)e^{\left(\frac{ac\log(f)^2-2cd\log(f)-2af\log(f)-e^2+4df}{c\log(f)-2f}\right)}}{8\sqrt{-c\log\left(f\right)+2f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*f^a*erf(-sqrt(-c*log(f))*x)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*e
rf(-sqrt(-c*log(f) - 2*f)*(x + e/(c*log(f) + 2*f)))*e^((a*c*log(f)^2 + 2*c
*d*log(f) + 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) -
 2*f) - 1/8*sqrt(pi)*erf(-sqrt(-c*log(f) + 2*f)*(x - e/(c*log(f) - 2*f)))*
e^((a*c*log(f)^2 - 2*c*d*log(f) - 2*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) -
2*f))/sqrt(-c*log(f) + 2*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx = \int f^{c\,x^2+a}\,{\cosh\left(f\,x^2+e\,x+d\right)}^2\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2}\cosh\left(f\,x^2+ex+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^2,x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + e*x + f*x**2)**2,x)
```

3.320 $\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 300

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\frac{3e^{-d+\frac{e^2}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{e^{-3d+\frac{9e^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{3e+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3e^{d-\frac{e^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}+\frac{e^{3d-\frac{9e^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{3e+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

output

```
3/16*exp(-d+e^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e+2*x*(f-c*ln(f)))/
(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+9*e^2/(12*f-4*c*ln(f)))
*f^a*Pi^(1/2)*erf(1/2*(3*e+2*x*(3*f-c*ln(f)))/(3*f-c*ln(f))^(1/2))/(3*f-c*
ln(f))^(1/2)+3/16*exp(d-e^2/(4*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(e+2*x*
(f+c*ln(f)))/(f+c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)+1/16*exp(3*d-9*e^2/(12*f
+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(3*e+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(
1/2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.05 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.59

$$\int f^{a+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$$
$$= \frac{e^{-\frac{1}{4}e^2\left(\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)} f^a\sqrt{\pi}\left(3e^{\frac{1}{4}e^2\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{9}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{e+2fx-2cx\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(9f^3}{}$$

input
```
Integrate[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*E^((e^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + 9/(3
*f + c*Log[f])))/4)*Erf[(e + 2*f*x - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]
*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2 - c^3*Log[f]^
3)*(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((e^2*(9/(3*f - c*Log[f]) + (f
+ c*Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*Erf[(3*e + 6*f*x - 2*c*x*Log[f]
)/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]]*(3*f^2 + 4*c*f*Log[f] + c
^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f - c*Log[f])*(3*E^((9*e^2)/(4*(
3*f + c*Log[f])))*Erfi[(e + 2*f*x + 2*c*x*Log[f])/(2*Sqrt[f + c*Log[f]])]*
Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(Cosh[d] + Sinh[d]) + E^(e^2/(4*(f + c
*Log[f])))*Erfi[(3*e + 6*f*x + 2*c*x*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f
+ c*Log[f])*Sqrt[3*f + c*Log[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((e^2*(
(f + c*Log[f])^(-1) + 9/(3*f + c*Log[f])))/4)*(9*f^4 - 10*c^2*f^2*Log[f]^2
 + c^4*Log[f]^4))
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 300, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{3}{8}f^{a+cx^2}\exp\left(-3(d+ex+fx^2)+2d+2ex+2fx^2\right)+\frac{3}{8}f^{a+cx^2}\exp\left(-3(d+ex+fx^2)+4d+4ex+4fx^2\right)\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{3\sqrt{\pi}f^a e^{\frac{e^2}{4f-4c\log(f)}-d}\operatorname{erf}\left(\frac{2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{\frac{9e^2}{12f-4c\log(f)}-3d}\operatorname{erf}\left(\frac{2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{e^2}{4(c\log(f)+f)}}\operatorname{erfi}\left(\frac{2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi}f^a e^{3d-\frac{9e^2}{4(c\log(f)+3f)}}\operatorname{erfi}\left(\frac{2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + e^2/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(e + 2*x*(f - c*Log[f]
))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) + (E^(-3*d + (9*e^2)/(
12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3*e + 2*x*(3*f - c*Log[f]))/(2*Sqrt[
3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]) + (3*E^(d - e^2/(4*(f + c*Log
[f])))*f^a*Sqrt[Pi]*Erfi[(e + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])
/(16*Sqrt[f + c*Log[f]]) + (E^(3*d - (9*e^2)/(4*(3*f + c*Log[f])))*f^a*Sqr
t[Pi]*Erfi[(3*e + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqr
t[3*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 4.00 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.01

method	result
risch	$\frac{\operatorname{erf}\left(x\sqrt{3f-c\ln(f)}+\frac{3e}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a}\mathrm{e}^{-\frac{3\left(4d\ln(f)c-12df+3e^{2}\right)}{4(c\ln(f)-3f)}}}{16\sqrt{3f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\, f^{a}\mathrm{e}^{\frac{3d\ln(f)c-}{3f+c}}}{16\sqrt{-c\ln(f)-3f}}$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*erf(x*(3*f-c*ln(f))^(1/2)+3/2*e/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1
/2)*Pi^(1/2)*f^a*exp(-3/4*(4*d*ln(f)*c-12*d*f+3*e^2)/(c*ln(f)-3*f))-1/16*e
rf(-(-c*ln(f)-3*f)^(1/2)*x+3/2*e/(-c*ln(f)-3*f)^(1/2))/(-c*ln(f)-3*f)^(1/2
)*Pi^(1/2)*f^a*exp(3/4*(4*d*ln(f)*c+12*d*f-3*e^2)/(3*f+c*ln(f)))+3/16*erf(
x*(f-c*ln(f))^(1/2)+1/2*e/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^
a*exp(-1/4*(4*d*ln(f)*c-4*d*f+e^2)/(c*ln(f)-f))-3/16*erf(-(-c*ln(f)-f)^(1/
2)*x+1/2*e/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(1/4*(4*
d*ln(f)*c+4*d*f-e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 847 vs. 2(253) = 506.

Time = 0.11 (sec) , antiderivative size = 847, normalized size of antiderivative = 2.82

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c*log(
f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*
f^3)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f - 12*(c*d + a*f)*log(f))/(c
*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(1/2*(2*c*x*log(f) - 6*f*x - 3*e
)*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^
2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(1/4*(4*a*c*log(f)^2 - e^2 + 4*
d*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2
*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(1/4*(4*a*c*log(f)^2 - e^2 + 4*d
*f - 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(1/2*(2
*c*x*log(f) - 2*f*x - e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + 3*(sqrt(pi)
*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh(1/4*(4*a*c*
log(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*
(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*sinh(1/4*(4*a*c*l
og(f)^2 - e^2 + 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log
(f) - f)*erf(1/2*(2*c*x*log(f) + 2*f*x + e)*sqrt(-c*log(f) - f)/(c*log(f)
+ f)) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)
*cosh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f))/(c*log
(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3
*f^3)*sinh(1/4*(4*a*c*log(f)^2 - 9*e^2 + 36*d*f + 12*(c*d + a*f)*log(f)...
```

Sympy [F]

$$\int f^{a+cx^2} \cosh^3\left(d+ex+fx^2\right)\,dx=\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+a)*cosh(f*x**2+e*x+d)**3,x)
```

output
```
Integral(f**(a + c*x**2)*cosh(d + e*x + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.88

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-3f}x-\frac{3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(3d-\frac{9e^2}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}} + \frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x-\frac{e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(d-\frac{e^2}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}} + \frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x+\frac{e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-d-\frac{e^2}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+3f}x+\frac{3e}{2\sqrt{-c\log(f)+3f}}\right)e^{\left(-3d-\frac{9e^2}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 3/2*e/sqrt(-c*log(f) - 3*f
))*e^(3*d - 9/4*e^2/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) + 3/16*sqrt(pi
)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*e/sqrt(-c*log(f) - f))*e^(d - 1/4*e^
2/(c*log(f) + f))/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(
f) + f)*x + 1/2*e/sqrt(-c*log(f) + f))*e^(-d - 1/4*e^2/(c*log(f) - f))/sqr
t(-c*log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x + 3/2*e/s
qrt(-c*log(f) + 3*f))*e^(-3*d - 9/4*e^2/(c*log(f) - 3*f))/sqrt(-c*log(f) +
 3*f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 352, normalized size of antiderivative = 1.17

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-3\,f}\left(2\,x+\frac{3\,e}{c\log(f)+3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+12\,cd\log(f)+12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)+3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}$$
$$-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{e}{c\log(f)+f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)+f)}\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{3\,\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x-\frac{e}{c\log(f)-f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-e^2+4\,df}{4\,(c\log(f)-f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+3\,f}\left(2\,x-\frac{3\,e}{c\log(f)-3\,f}\right)\right)e^{\left(\frac{4\,ac\log(f)^2-12\,cd\log(f)-12\,af\log(f)-9\,e^2+36\,df}{4\,(c\log(f)-3\,f)}\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}$$

input
```
integrate(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + 3*e/(c*log(f) + 3*f))
)*e^(1/4*(4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) - 9*e^2 + 36*d*f)
/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*
log(f) - f)*(2*x + e/(c*log(f) + f)))*e^(1/4*(4*a*c*log(f)^2 + 4*c*d*log(f
) + 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 3/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x - e/(c*log(f) - f)))*e^(1/4*(4
*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - e^2 + 4*d*f)/(c*log(f) - f))
/sqrt(-c*log(f) + f) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x -
 3*e/(c*log(f) - 3*f)))*e^(1/4*(4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*lo
g(f) - 9*e^2 + 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+a}\,\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2}\cosh\left(f\,x^2+ex+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+a)*cosh(f*x^2+e*x+d)^3,x)
```

output
```
f**a*int(f**(c*x**2)*cosh(d + e*x + f*x**2)**3,x)
```

3.321 $\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 153

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx=-\frac{e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

output
```
-1/4*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-b*ln(f)-2*
c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/4*exp(d-1/4*(e+b*ln(
f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)
^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$$
$$=\frac{e^{-\frac{e(e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(e^{\frac{be}{c}}\operatorname{erfi}\left(\frac{-e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)-\sinh(d))+\operatorname{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(d)+\sin\right.}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*(E^((b*e)/c)*Erfi[(-e + (b + 2*c*x)*Log[f])/(2
*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] - Sinh[d]) + Erfi[(e + (b + 2*c*x)*Log[f]
)/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[d] + Sinh[d])))/(4*Sqrt[c]*E^((e*(e + 2*
b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(d+ex) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{ 6039}$$

$$\int \left(\frac{1}{2}e^{-d-ex}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x],x]
```

output
```
-1/4*(E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(Sqrt[c]*Sqrt[Log[f]]) + (E^
(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] + 2*c*
x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(4*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.49 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.06

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{2\ln(f)be-4d\ln(f)c-e^{2}}{4\ln(f)c}}}{4\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{2\ln(f)}{}}}{4\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*ln(f))^(1/2))/(-c*ln(f))^
(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)*b*e-4*d*ln(f)*c-e^2)/ln
(f)/c)-1/4*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*l
n(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*(2*ln(f)*b*e-4*d*ln(f)*c+
e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(121) = 242.

Time = 0.08 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.71

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx =$$
$$-\frac{\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(f)}{4c\log(f)}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)^2+e^2-2(2cd-be)\log(}{4c\log(f)}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="fricas")
```

output
```
-1/4*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 -
2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*lo
g(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*l
og(f) + e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1
/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + s
qrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/
(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))))
/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx = \int f^{a+bx+cx^2}\cosh(d+ex)\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx = \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}} + \frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x - \frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)
))*e^(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*
f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/
4*(b*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.09

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx$$
$$= -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)+e^2}{4c\log(f)}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - e)/(c*log(f))))*
e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2
)/(c*log(f)))/sqrt(-c*log(f)) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
 + (b*log(f) + e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*
c*d*log(f) + 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\cosh(d+e\,x)\;dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x),x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x), x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\cosh(d+ex)\,dx = f^a\left(\int f^{c\,x^2+bx}\cosh\left(ex+d\right)dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d),x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + e*x),x)
```

3.322 $\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d-\frac{(2e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{2e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*d-1/4*(2*e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi
(1/2*(2*e-b*ln(f)-2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+1/
8*exp(2*d-1/4*(2*e+b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(2*e+b*ln(f)+
2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.84

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx$$
$$=\frac{e^{-\frac{e(e+b\log(f))}{c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left(2e^{\frac{e(e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)+e^{\frac{2be}{c}}\operatorname{erfi}\left(\frac{-2e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)(\cosh(2d)-\sinh(2d))\right.}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*(2*E^((e*(e + b*Log[f]))/(c*Log[f]))*Erfi[((b
+ 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])] + E^((2*b*e)/c)*Erfi[(-2*e + (b + 2*c*
x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(2*e +
 (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])))/(
8*Sqrt[c]*E^((e*(e + b*Log[f]))/(c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(d+ex)f^{a+bx+cx^2}\,dx$$
$$\Big\downarrow \text{6039}$$
$$\int \left(\frac{1}{4}e^{-2d-2ex}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^{a-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^a e^{-\frac{(2e-b\log(f))^2}{4c\log(f)}-2d}\operatorname{erfi}\left(\frac{-b\log(f)-2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^a e^{2d-\frac{(b\log(f)+2e)^2}{4c\log(f)}}\operatorname{erfi}\left(\frac{b\log(f)+2cx\log(f)+2e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{8\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d - (2*e - b*Log[f])^2/(4*c*Log[f]))*f^a
*Sqrt[Pi]*Erfi[(2*e - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/
(8*Sqrt[c]*Sqrt[Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(4*c*Log[f]))*f^a*
Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(
8*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.09 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-2e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{\frac{\ln(f)be-2d\ln(f)c-e^2}{\ln(f)c}}}{8\sqrt{-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\,f^a f^{-\frac{b^2}{4c}}\,\mathrm{e}^{-\frac{\ln(f)be\cdots}{}}}{8\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-2*e)/(-c*ln(f))^(1/2))/(-c*ln(f)
)^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp((ln(f)*b*e-2*d*ln(f)*c-e^2)/ln(f)/
c)-1/8*erf(-(-c*ln(f))^(1/2)*x+1/2*(2*e+b*ln(f))/(-c*ln(f))^(1/2))/(-c*ln(
f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-(ln(f)*b*e-2*d*ln(f)*c+e^2)/ln(
f)/c)-1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/
2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(167) = 334$.

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx =$$

$$-\frac{2\sqrt{-c\log(f)}\left(\sqrt{\pi}\cosh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)+\sqrt{\pi}\sinh\left(-\frac{(b^2-4ac)\log(f)}{4c}\right)\right)\operatorname{erf}\left(\frac{(2cx+b)\sqrt{-c\log(f)}}{2c}\right)+\sqrt{-c}}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*(2*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt
(pi)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*erf(1/2*(2*c*x + b)*sqrt(-c*log(f)
)/c) + sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2
 - 4*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)
*log(f)^2 + 4*e^2 - 4*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x +
 b)*log(f) + 2*e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 + 4*(2*c*d - b*
e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 2*e)*sqrt(-c*log(f))
/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx=\int f^{a+bx+cx^2}\cosh^2{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.84

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx\\ &=\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log{(f)}}x-\frac{b\log(f)+2\,e}{2\sqrt{-c\log(f)}}\right)e^{\left(2\,d-\frac{(b\log(f)+2\,e)^2}{4\,c\log(f)}\right)}}{8\sqrt{-c\log{(f)}}}\\ &\quad+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log{(f)}}x-\frac{b\log(f)-2\,e}{2\sqrt{-c\log(f)}}\right)e^{\left(-2\,d-\frac{(b\log(f)-2\,e)^2}{4\,c\log(f)}\right)}}{8\sqrt{-c\log{(f)}}}\\ &\quad+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log{(f)}}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log{(f)}}f^{\frac{b^2}{4\,c}}}\end{aligned}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 2*e)/sqrt(-c*log(
f)))*e^(2*d - 1/4*(b*log(f) + 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/8*sqr
t(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - 2*e)/sqrt(-c*log(f)))*e^
(-2*d - 1/4*(b*log(f) - 2*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/4*sqrt(pi)*
f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))
*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.02

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{4\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-2\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+8\,cd\log(f)-4\,be\log(f)+4\,e^2}{4\,c\log(f)}\right)}}{8\sqrt{-c\log(f)}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+2\,e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)+4\,be\log(f)+4\,e^2}{4\,c\log(f)}\right)}}{8\sqrt{-c\log(f)}}$$

input

```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x, algorithm="giac")
```

output

```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-1/4*(b^2*log(f) -
4*a*c*log(f))/c)/sqrt(-c*log(f)) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(
2*x + (b*log(f) - 2*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2
 + 8*c*d*log(f) - 4*b*e*log(f) + 4*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 1/8*
sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + 2*e)/(c*log(f))))*e^(
-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*e*log(f) + 4*e^2)
/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^2(d+ex)\,dx = \int f^{c\,x^2+b\,x+a}\cosh(d+e\,x)^2\,dx$$

input

```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2,x)
```

output

```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^2, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2} \cosh^2(d+ex)\,dx = f^a\left(\int f^{c\,x^2+bx}\cosh(ex+d)^2\,dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + e*x)**2,x)
```

3.323 $\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 315

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx = -\frac{3e^{-d-\frac{(e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-3d-\frac{(3e-b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3e-b\log(f)-2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{3e^{d-\frac{(e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}} + \frac{e^{3d-\frac{(3e+b\log(f))^2}{4c\log(f)}}f^a\sqrt{\pi}\,\text{erfi}\left(\frac{3e+b\log(f)+2cx\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

output
```
-3/16*exp(-d-1/4*(e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(e-b*ln(f)-2
*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)-1/16*exp(-3*d-1/4*(3*
e-b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(1/2*(3*e-b*ln(f)-2*c*x*ln(f))/c^(1
/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)+3/16*exp(d-1/4*(e+b*ln(f))^2/c/ln(f))
*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2
)/ln(f)^(1/2)+1/16*exp(3*d-1/4*(3*e+b*ln(f))^2/c/ln(f))*f^a*Pi^(1/2)*erfi(
1/2*(3*e+b*ln(f)+2*c*x*ln(f))/c^(1/2)/ln(f)^(1/2))/c^(1/2)/ln(f)^(1/2)
```

Mathematica [A] (verified)

Time = 0.61 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx = \frac{e^{-\frac{3e(3e+2b\log(f))}{4c\log(f)}}f^{a-\frac{b^2}{4c}}\sqrt{\pi}\left((\cosh(d)+\sinh(d))\left(3e^{\frac{e(2e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{e+(b+2cx)\log(f)}{2\sqrt{c}\sqrt{\log(f)}}\right)+3e^{\frac{2e(e+b\log(f))}{c\log(f)}}\operatorname{erfi}\left(\frac{-e+(b+\dots}{2\sqrt{c}\dots}\right.\right.\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*((Cosh[d] + Sinh[d])*(3*E^((e*(2*e + b*Log[f])
)/(c*Log[f]))*Erfi[(e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])] + 3*
E^((2*e*(e + b*Log[f]))/(c*Log[f]))*Erfi[(-e + (b + 2*c*x)*Log[f])/(2*Sqrt
[c]*Sqrt[Log[f]])]*(Cosh[2*d] - Sinh[2*d]) + Erfi[(3*e + (b + 2*c*x)*Log[f
])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[2*d] + Sinh[2*d])) + E^((3*b*e)/c)*Erfi
[(-3*e + (b + 2*c*x)*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])]*(Cosh[3*d] - Sinh[3
*d])))/(16*Sqrt[c]*E^((3*e*(3*e + 2*b*Log[f]))/(4*c*Log[f]))*Sqrt[Log[f]])
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(d+ex)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{8}e^{-3d-3ex}f^{a+bx+cx^2}+\frac{3}{8}e^{-d-ex}f^{a+bx+cx^2}+\frac{3}{8}e^{d+ex}f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3ex}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\pi}f^ae^{-\frac{(e-b\log(f))^2}{4c\log(f)}-d}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}-\frac{\sqrt{\pi}f^ae^{-\frac{(3e-b\log(f))^2}{4c\log(f)}-3d}\text{erfi}\left(\frac{-b\log(f)-2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{3\sqrt{\pi}f^ae^{d-\frac{(b\log(f)+e)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi}f^ae^{3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}}\text{erfi}\left(\frac{b\log(f)+2cx\log(f)+3e}{2\sqrt{c}\sqrt{\log(f)}}\right)}{16\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x]^3,x]
```

output
```
(-3*E^(-d - (e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) - (E
^(-3*d - (3*e - b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e - b*Log[f
] - 2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (
3*E^(d - (e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(e + b*Log[f] +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]]) + (E^(3
*d - (3*e + b*Log[f])^2/(4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] +
2*c*x*Log[f])/(2*Sqrt[c]*Sqrt[Log[f]])])/(16*Sqrt[c]*Sqrt[Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 2.82 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.03

method	result
risch	$-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{b\ln(f)-3e}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{\frac{\frac{3\ln(f)be}{2}-3d\ln(f)c-\frac{9e^{2}}{4}}{c\ln(f)}}}{16\sqrt{-c\ln(f)}} - \frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)}\,x+\frac{3e+b\ln(f)}{2\sqrt{-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} f^{-\frac{b^{2}}{4c}} \mathrm{e}^{-\frac{3(2\cdots}{\cdots}}}{16\sqrt{-c\ln(f)}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-3*e)/(-c*ln(f))^(1/2))/(-c*ln(f
))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(3/4*(2*ln(f)*b*e-4*d*ln(f)*c-3*e^
2)/ln(f)/c)-1/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(3*e+b*ln(f))/(-c*ln(f))^(1/2
))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-3/4*(2*ln(f)*b*e-4*d*
ln(f)*c+3*e^2)/ln(f)/c)-3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(b*ln(f)-e)/(-c*l
n(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(1/4*(2*ln(f)
*b*e-4*d*ln(f)*c-e^2)/ln(f)/c)-3/16*erf(-(-c*ln(f))^(1/2)*x+1/2*(e+b*ln(f)
)/(-c*ln(f))^(1/2))/(-c*ln(f))^(1/2)*Pi^(1/2)*f^a*f^(-1/4*b^2/c)*exp(-1/4*
(2*ln(f)*b*e-4*d*ln(f)*c+e^2)/ln(f)/c)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 526 vs. 2(247) = 494.

Time = 0.09 (sec) , antiderivative size = 526, normalized size of antiderivative = 1.67

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*(sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2
 - 6*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)
*log(f)^2 + 9*e^2 - 6*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x +
 b)*log(f) + 3*e)*sqrt(-c*log(f))/(c*log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi
)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 2*(2*c*d - b*e)
*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) + e)*sqrt(-c*log(f))/(c*
log(f))) + 3*sqrt(-c*log(f))*(sqrt(pi)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 +
 e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*
a*c)*log(f)^2 + e^2 + 2*(2*c*d - b*e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x
 + b)*log(f) - e)*sqrt(-c*log(f))/(c*log(f))) + sqrt(-c*log(f))*(sqrt(pi)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*e)*log(f))/(c*log
(f))) + sqrt(pi)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 + 6*(2*c*d - b*
e)*log(f))/(c*log(f))))*erf(1/2*((2*c*x + b)*log(f) - 3*e)*sqrt(-c*log(f))
/(c*log(f))))/(c*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2} \cosh^3(d+ex)\,dx = \int f^{a+bx+cx^2} \cosh^3{(d+ex)}\,dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(e*x+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 263, normalized size of antiderivative = 0.83

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)}}\right)e^{\left(3d-\frac{(b\log(f)+3e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)}}\right)e^{\left(d-\frac{(b\log(f)+e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)}}\right)e^{\left(-d-\frac{(b\log(f)-e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)-3e}{2\sqrt{-c\log(f)}}\right)e^{\left(-3d-\frac{(b\log(f)-3e)^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + 3*e)/sqrt(-c*log
(f)))*e^(3*d - 1/4*(b*log(f) + 3*e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*s
qrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) + e)/sqrt(-c*log(f)))*e^
(d - 1/4*(b*log(f) + e)^2/(c*log(f)))/sqrt(-c*log(f)) + 3/16*sqrt(pi)*f^a*
erf(sqrt(-c*log(f))*x - 1/2*(b*log(f) - e)/sqrt(-c*log(f)))*e^(-d - 1/4*(b
*log(f) - e)^2/(c*log(f)))/sqrt(-c*log(f)) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c
*log(f))*x - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f)))*e^(-3*d - 1/4*(b*log(f)
 - 3*e)^2/(c*log(f)))/sqrt(-c*log(f))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.08

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx =$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)-e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)+e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b\log(f)+3e}{c\log(f)}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)+9e^2}{4c\log(f)}\right)}}{16\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) - 3*e)/(c*log(f))
))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) - 6*b*e*log(f) +
 9*e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f
))*(2*x + (b*log(f) - e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)
^2 + 4*c*d*log(f) - 2*b*e*log(f) + e^2)/(c*log(f)))/sqrt(-c*log(f)) - 3/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + (b*log(f) + e)/(c*log(f))))*e^(-
1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(f) + e^2)/(c
*log(f)))/sqrt(-c*log(f)) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x +
(b*log(f) + 3*e)/(c*log(f))))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*
c*d*log(f) + 6*b*e*log(f) + 9*e^2)/(c*log(f)))/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx=\int f^{c\,x^2+b\,x+a}\cosh(d+e\,x)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x)^3, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\cosh^3(d+ex)\,dx=f^a\left(\int f^{c\,x^2+bx}\cosh\left(ex+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(e*x+d)^3,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + e*x)**3,x)
```

3.324 $\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx=-\frac{e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
-1/4*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(b*ln(f)-2*x
*(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/4*exp(d-b^2*ln(f)^2/(
4*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f)
)^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.20

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\left(e^{\frac{b^2f\log^2(f)}{2f^2-2c^2\log^2(f)}}\mathrm{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(f+c\log(f))(\cosh(d)-\sinh(d))+\right.}{4\left(f^2-c^2\log^2(f)\right)}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
(f^a*Sqrt[Pi]*(E^((b^2*f*Log[f]^2)/(2*f^2 - 2*c^2*Log[f]^2))*Erf[(2*f*x -
(b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log[
f])*(Cosh[d] - Sinh[d]) + Erfi[(2*f*x + (b + 2*c*x)*Log[f])/(2*Sqrt[f + c*
Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[f]]*(Cosh[d] + Sinh[d])))/(4*E^((b
^2*Log[f]^2)/(4*(f + c*Log[f])))*(f^2 - c^2*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh\left(d+fx^2\right) f^{a+bx+cx^2} \, dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2} e^{-d-fx^2} f^{a+bx+cx^2}+\frac{1}{2} e^{d+fx^2} f^{a+bx+cx^2}\right) dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}} \text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d} \text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2],x]
```

output
```
-1/4*(E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f
] - 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/Sqrt[f - c*Log[f]] + (E^(
d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(
f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.54 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+4d\ln(f)c-4df}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f}{4}}}{4\sqrt{-c\ln(f)-f}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(
1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*d*f)/(c*ln(f)-f))-1/
4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^(1/2))/(-c*ln(f)-f)^(
1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4*d*f)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 324 vs. 2(131) = 262.

Time = 0.11 (sec) , antiderivative size = 324, normalized size of antiderivative = 2.10

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx =$$
$$-\frac{\left(\sqrt{\pi}(c\log(f)+f)\cosh\left(-\frac{(b^2-4ac)\log(f)^2-4df+4(cd+af)\log(f)}{4(c\log(f)-f)}\right)+\sqrt{\pi}(c\log(f)+f)\sinh\left(-\frac{(b^2-4ac)\log(f)}{4(}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f +
 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f) + f)*sinh(-1/4
*((b^2 - 4*a*c)*log(f)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*
sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) +
 f)/(c*log(f) - f)) + (sqrt(pi)*(c*log(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*lo
g(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f
) - f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(
c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*s
qrt(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\left(c\log(f)+f\right)}+d\right)}}{4\sqrt{-c\log\left(f\right)-f}}$$
$$+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\left(c\log(f)-f\right)}-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*log(f) -
 f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) + 1/4*sq
rt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + f))*e
^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.18

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)-4\,af\log(f)-4\,df}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)+4\,af\log(f)-4\,df}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4
*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*
log(f) + f)*(2*x + b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c
*log(f)^2 + 4*c*d*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*l
og(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,\cosh\left(f\,x^2+d\right)\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\cosh\left(d+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2+bx}\cosh\left(f\,x^2+d\right)dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d),x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + f*x**2),x)
```

3.325 $\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 225

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}-\frac{e^{-2d+\frac{b^2\log^2(f)}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{b^2\log^2(f)}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(1/
2)/ln(f)^(1/2)-1/8*exp(-2*d+b^2*ln(f)^2/(8*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(
1/2*(b*ln(f)-2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f))^(1/2)+1
/8*exp(2*d-b^2*ln(f)^2/(8*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(b*ln(f)+2*x
*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (verified)

Time = 1.43 (sec) , antiderivative size = 257, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \frac{1}{8}f^a\sqrt{\pi}\left(\frac{2f^{-\frac{b^2}{4c}}\operatorname{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-\frac{b^2\log^2(f)}{8f+4c\log(f)}}\left(e^{\frac{b^2f\log^2(f)}{4f^2-c^2\log^2(f)}}\operatorname{erf}\left(\frac{4fx-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2f+c\log(f))(\cosh(2d)-\sinh(2d))+\right.}{-4f^2+c^2\log^2(f)}\right.$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^a*Sqrt[Pi]*((2*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/(Sqrt[c]*f
^(b^2/(4*c))*Sqrt[Log[f]]) - (E^((b^2*f*Log[f]^2)/(4*f^2 - c^2*Log[f]^2))*
Erf[(4*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[f]])]*Sqrt[2*f - c*Lo
g[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + Erfi[(4*f*x + (b + 2*c*x)
*Log[f])/(2*Sqrt[2*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(
Cosh[2*d] + Sinh[2*d]))/(E^((b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c
^2*Log[f]^2))))/8
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2\left(d+fx^2\right)f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \; 6039$$

$$\int \left(\frac{1}{4}e^{-2d-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2fx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \; 2009$$

$$-\frac{\sqrt{\pi} f^a e^{\frac{b^2 \log^2(f)}{8f-4c\log(f)}-2d} \operatorname{erf}\left(\frac{b\log(f)-2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a e^{2d-\frac{b^2\log^2(f)}{4c\log(f)+8f}} \operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}+\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (E^(-2*d + (b^2*Log[f]^2)/(8*f - 4*c*Log[f]))*f
^a*Sqrt[Pi]*Erf[(b*Log[f] - 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f - c*Log[f]])
])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (b^2*Log[f]^2)/(8*f + 4*c*Log[f]))
*f^a*Sqrt[Pi]*Erfi[(b*Log[f] + 2*x*(2*f + c*Log[f]))/(2*Sqrt[2*f + c*Log[f
]])])/(8*Sqrt[2*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.31 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.96

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{-\frac{b^2\ln(f)^2+8d\ln(f)c-16df}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-2f}}\right)\sqrt{\pi}\, f^a \mathrm{e}^{-b}}{8\sqrt{-c\ln(f)-2f}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(2*f-c*ln(f))^(1/2))/(2*f-c*ln
(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+8*d*ln(f)*c-16*d*f)/(c*ln(f)
-2*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-2*f)^(1/2))/(
-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-8*d*ln(f)*c-16*d*f)
/(2*f+c*ln(f)))-1/4*f^a*Pi^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*
ln(f))^(1/2)*x+1/2*ln(f)*b/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(185) = 370$.

Time = 0.10 (sec) , antiderivative size = 466, normalized size of antiderivative = 2.07

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log
(f)^2 - 16*d*f + 8*(c*d + a*f)*log(f))/(c*log(f) - 2*f)) + sqrt(pi)*(c^2*l
og(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f + 8*(c*
d + a*f)*log(f))/(c*log(f) - 2*f)))*sqrt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x
- (2*c*x + b)*log(f))*sqrt(-c*log(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*
(c^2*log(f)^2 - 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f -
 8*(c*d + a*f)*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*
log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 16*d*f - 8*(c*d + a*f)*log(f))
/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)*log
(f))*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*log(f)^2 -
 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)^2 - 4*f^2
)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*c*x + b)*s
qrt(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.88

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-2f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)-2f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)+2f)}+2d\right)}}{8\sqrt{-c\log(f)-2f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+2f}x-\frac{b\log(f)}{2\sqrt{-c\log(f)+2f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4(c\log(f)-2f)}-2d\right)}}{8\sqrt{-c\log(f)+2f}}+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
 - 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-c*log(f) - 2*f
) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*b*log(f)/sqrt(-c*lo
g(f) + 2*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 2*f) - 2*d)/sqrt(-c*log(f) +
 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x - 1/2*b*log(f)/sqrt(-c*log(
f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.06

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-2f}\left(2x+\frac{b\log(f)}{c\log(f)+2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-8cd\log(f)-8af\log(f)-16df}{4(c\log(f)+2f)}\right)}}{8\sqrt{-c\log(f)-2f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+2f}\left(2x+\frac{b\log(f)}{c\log(f)-2f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+8cd\log(f)+8af\log(f)-16df}{4(c\log(f)-2f)}\right)}}{8\sqrt{-c\log(f)+2f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)}\left(2x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4ac\log(f)}{4c}\right)}}{4\sqrt{-c\log(f)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + b*log(f)/(c*log(f) + 2
*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) - 8*a*f*log(f)
 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f) - 2*f) - 1/8*sqrt(pi)*erf(-1/2
*sqrt(-c*log(f) + 2*f)*(2*x + b*log(f)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log
(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(f) + 8*a*f*log(f) - 16*d*f)/(c*log(f) -
 2*f))/sqrt(-c*log(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x
+ b/c))*e^(-1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx = \int f^{c\,x^2+b\,x+a}\,{\cosh\left(f\,x^2+d\right)}^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\cosh^2\left(d+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2+bx}\cosh\left(f\,x^2+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + f*x**2)**2,x)
```

3.326 $\int f^{a+bx+cx^2} \cosh^3(d+fx^2)\,dx$

Optimal result

Integrand size = 23, antiderivative size = 323

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx = -\frac{3e^{-d+\frac{b^2\log^2(f)}{4f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}} - \frac{e^{-3d+\frac{b^2\log^2(f)}{12f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}} + \frac{3e^{d-\frac{b^2\log^2(f)}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}} + \frac{e^{3d-\frac{b^2\log^2(f)}{4(3f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}$$

output
```
-3/16*exp(-d+b^2*ln(f)^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(b*ln(f)-2*
x*(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)-1/16*exp(-3*d+b^2*ln(f
)^2/(12*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(b*ln(f)-2*x*(3*f-c*ln(f)))/(3*
f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2)+3/16*exp(d-b^2*ln(f)^2/(4*f+4*c*ln(f
)))*f^a*Pi^(1/2)*erfi(1/2*(b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln(f))^(1/2))/(f+
c*ln(f))^(1/2)+1/16*exp(3*d-b^2*ln(f)^2/(12*f+4*c*ln(f)))*f^a*Pi^(1/2)*erf
i(1/2*(b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.52 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.55

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx$$
$$=\frac{e^{-\frac{b^2\log^2(f)(2f+c\log(f))}{2(f+c\log(f))(3f+c\log(f))}}f^a\sqrt{\pi}\left(3e^{\frac{1}{4}b^2\log^2(f)\left(\frac{1}{f-c\log(f)}+\frac{1}{f+c\log(f)}+\frac{1}{3f+c\log(f)}\right)}\operatorname{erf}\left(\frac{2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(9\cdots}{\cdots}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*(3*E^((b^2*Log[f]^2*((f - c*Log[f])^(-1) + (f + c*Log[f])^(-
1) + (3*f + c*Log[f])^(-1)))/4)*Erf[(2*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[f
 - c*Log[f]])]*Sqrt[f - c*Log[f]]*(9*f^3 + 9*c*f^2*Log[f] - c^2*f*Log[f]^2
 - c^3*Log[f]^3)*(Cosh[d] - Sinh[d]) + (f - c*Log[f])*(E^((b^2*Log[f]^2*((
3*f - c*Log[f])^(-1) + (f + c*Log[f])^(-1) + (3*f + c*Log[f])^(-1)))/4)*Er
f[(6*f*x - (b + 2*c*x)*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[
f]]*(3*f^2 + 4*c*f*Log[f] + c^2*Log[f]^2)*(Cosh[3*d] - Sinh[3*d]) + (3*f -
 c*Log[f])*(3*E^((b^2*Log[f]^2)/(12*f + 4*c*Log[f]))*Erfi[(2*f*x + (b + 2*
c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*Sqrt[f + c*Log[f]]*(3*f + c*Log[f])*(
Cosh[d] + Sinh[d]) + E^((b^2*Log[f]^2)/(4*(f + c*Log[f])))*Erfi[(6*f*x + (
b + 2*c*x)*Log[f])/(2*Sqrt[3*f + c*Log[f]])]*(f + c*Log[f])*Sqrt[3*f + c*L
og[f]]*(Cosh[3*d] + Sinh[3*d])))))/(16*E^((b^2*Log[f]^2*(2*f + c*Log[f]))/
(2*(f + c*Log[f])*(3*f + c*Log[f])))*(9*f^4 - 10*c^2*f^2*Log[f]^2 + c^4*Lo
g[f]^4))
```

Rubi [A] (verified)

Time = 0.85 (sec) , antiderivative size = 323, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3\left(d+fx^2\right) f^{a+bx+cx^2}\,dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{8}e^{-3d-3fx^2} f^{a+bx+cx^2}+\frac{3}{8}e^{-d-fx^2} f^{a+bx+cx^2}+\frac{3}{8}e^{d+fx^2} f^{a+bx+cx^2}+\frac{1}{8}e^{3d+3fx^2} f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-\frac{3\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{4f-4c\log(f)}-d}\text{erf}\left(\frac{b\log(f)-2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}-\frac{\sqrt{\pi} f^a e^{\frac{b^2\log^2(f)}{12f-4c\log(f)}-3d}\text{erf}\left(\frac{b\log(f)-2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi} f^a e^{d-\frac{b^2\log^2(f)}{4(c\log(f)+f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}+\frac{\sqrt{\pi} f^a e^{3d-\frac{b^2\log^2(f)}{4(c\log(f)+3f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + f*x^2]^3,x]
```

output
```
(-3*E^(-d + (b^2*Log[f]^2)/(4*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f]
- 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]) - (
E^(-3*d + (b^2*Log[f]^2)/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(b*Log[f] -
 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[3*f - c*Log[f]]
) + (3*E^(d - (b^2*Log[f]^2)/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[
f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16*Sqrt[f + c*Log[f]])
+ (E^(3*d - (b^2*Log[f]^2)/(4*(3*f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(b*Log[
f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*Log[f]])])/(16*Sqrt[3*f + c*Log
[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.01

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{\ln(f)b}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-\frac{b^{2}\ln(f)^{2}+12d\ln(f)c-36df}{4(c\ln(f)-3f)}}}{16\sqrt{3f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{\ln(f)b}{2\sqrt{-c\ln(f)-3f}}\right)\sqrt{\pi}\,f^{a}\mathrm{e}^{-}}{16\sqrt{-c\ln(f)-3f}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*ln(f)*b/(3*f-c*ln(f))^(1/2))/(3*f-c*l
n(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+12*d*ln(f)*c-36*d*f)/(c*ln(
f)-3*f))-1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-3*f)^(1/2)
)/(-c*ln(f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-12*d*ln(f)*c-36*
d*f)/(3*f+c*ln(f)))-3/16*erf(-x*(f-c*ln(f))^(1/2)+1/2*ln(f)*b/(f-c*ln(f))^
(1/2))/(f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+4*d*ln(f)*c-4*
d*f)/(c*ln(f)-f))-3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*ln(f)*b/(-c*ln(f)-f)^
(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*d*ln(f)*c-4
*d*f)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 851 vs. $2(275) = 550$.

Time = 0.12 (sec) , antiderivative size = 851, normalized size of antiderivative = 2.63

$$\int f^{a+bx+cx^2} \cosh^3\left(d+fx^2\right)\,dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/(c*log
(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3
*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f + 12*(c*d + a*f)*log(f))/
(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)*erf(-1/2*(6*f*x - (2*c*x + b)*log
(f))*sqrt(-c*log(f) + 3*f)/(c*log(f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 +
 c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^
2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3
 + c^2*f*log(f)^2 - 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f
)^2 - 4*d*f + 4*(c*d + a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*e
rf(-1/2*(2*f*x - (2*c*x + b)*log(f))*sqrt(-c*log(f) + f)/(c*log(f) - f)) +
 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*cosh
(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f) +
f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*si
nh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 4*d*f - 4*(c*d + a*f)*log(f))/(c*log(f)
+ f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f))*sqrt(-c*lo
g(f) - f)/(c*log(f) + f)) + (sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c
*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f - 12*(c*d
+ a*f)*log(f))/(c*log(f) + 3*f)) + sqrt(pi)*(c^3*log(f)^3 - 3*c^2*f*log(f)
^2 - c*f^2*log(f) + 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 - 36*d*f -...
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+d)**3,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + f*x**2)**3, x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.89

$$\begin{aligned}&\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx\\&=\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-3\,f}x-\frac{b\log(f)}{2\,\sqrt{-c\log(f)-3\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+3\,f)}+3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)-3\,f}}\\&+\frac{3\,\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)}{2\,\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)+f)}+d\right)}}{16\,\sqrt{-c\log\left(f\right)-f}}\\&+\frac{3\,\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)}{2\,\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-f)}-d\right)}}{16\,\sqrt{-c\log\left(f\right)+f}}\\&+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log\left(f\right)+3\,f}x-\frac{b\log(f)}{2\,\sqrt{-c\log(f)+3\,f}}\right)e^{\left(-\frac{b^2\log(f)^2}{4\,(c\log(f)-3\,f)}-3\,d\right)}}{16\,\sqrt{-c\log\left(f\right)+3\,f}}\end{aligned}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f
) - 3*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + 3*f) + 3*d)/sqrt(-c*log(f) - 3*
f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*b*log(f)/sqrt(-c*lo
g(f) - f))*e^(-1/4*b^2*log(f)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f) +
3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*b*log(f)/sqrt(-c*log(f)
+ f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - f) - d)/sqrt(-c*log(f) + f) + 1/16*
sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x - 1/2*b*log(f)/sqrt(-c*log(f) + 3
*f))*e^(-1/4*b^2*log(f)^2/(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 369, normalized size of antiderivative = 1.14

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx = -\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)-12af\log(f)-36df}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)-4af\log(f)-4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}} - \frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)+4af\log(f)-4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}} - \frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+3f}\left(2x+\frac{b\log(f)}{c\log(f)-3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)+12af\log(f)-36df}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + b*log(f)/(c*log(f) +
3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) - 12*a*f*log
(f) - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log(f) - 3*f) - 3/16*sqrt(pi)*erf(
-1/2*sqrt(-c*log(f) - f)*(2*x + b*log(f)/(c*log(f) + f)))*e^(-1/4*(b^2*log
(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) - 4*a*f*log(f) - 4*d*f)/(c*log(f) +
f))/sqrt(-c*log(f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x
+ b*log(f)/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d
*log(f) + 4*a*f*log(f) - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f) - 1/16
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + b*log(f)/(c*log(f) - 3*f))
)*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f) + 12*a*f*log(f) -
 36*d*f)/(c*log(f) - 3*f))/sqrt(-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,\cosh\left(f\,x^2+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\cosh^3\left(d+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2+bx}\cosh\left(f\,x^2+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+d)^3,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + f*x**2)**3,x)
```

3.327 $\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 24, antiderivative size = 161

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=\frac{e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{4\sqrt{f+c\log(f)}}$$

output
```
1/4*exp(-d+(e-b*ln(f))^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e-b*ln(f)+
2*x*(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/4*exp(d-(e+b*ln(f)
)^2/(4*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+
c*ln(f))^(1/2))/(f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.97 (sec) , antiderivative size = 251, normalized size of antiderivative = 1.56

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$

$$=\frac{e^{-\frac{e^2+b^2\log^2(f)}{4(f+c\log(f))}}f^{a+\frac{bef}{-f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(e^2+b^2\log^2(f)\right)}{2\left(f^2-c^2\log^2(f)\right)}}f^{\frac{be}{2(f+c\log(f))}}\mathrm{erf}\left(\frac{e+2fx-(b+2cx)\log(f)}{2\sqrt{f-c\log(f)}}\right)\sqrt{f-c\log(f)}(f+c\log\right.}{4\left(f^2\right.}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(f^(a + (b*e*f)/(-f^2 + c^2*Log[f]^2))*Sqrt[Pi]*(E^((f*(e^2 + b^2*Log[f]^2
))/(2*(f^2 - c^2*Log[f]^2)))*f^((b*e)/(2*(f + c*Log[f])))*Erf[(e + 2*f*x -
 (b + 2*c*x)*Log[f])/(2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]]*(f + c*Log
[f])*(Cosh[d] - Sinh[d]) + f^((b*e)/(2*f - 2*c*Log[f]))*Erfi[(e + 2*f*x +
(b + 2*c*x)*Log[f])/(2*Sqrt[f + c*Log[f]])]*(f - c*Log[f])*Sqrt[f + c*Log[
f]]*(Cosh[d] + Sinh[d])))/(4*E^((e^2 + b^2*Log[f]^2)/(4*(f + c*Log[f])))*(
f^2 - c^2*Log[f]^2))
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{1}{2}e^{-d-ex-fx^2}f^{a+bx+cx^2}+\frac{1}{2}e^{d+ex+fx^2}f^{a+bx+cx^2}\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{4\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{4\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2],x]
```

output
```
(E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Log[
f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(4*Sqrt[f - c*Log[f]]) +
 (E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt[Pi]*Erfi[(e + b*Log
[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(4*Sqrt[f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.64 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.16

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{f-c\ln(f)}+\frac{b\ln(f)-e}{2\sqrt{f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a}\, \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-2\ln(f) b e+4d\ln(f) c-4df+e^{2}}{4(c\ln(f)-f)}}}{4\sqrt{f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-f}\,x+\frac{e+b\ln(f)}{2\sqrt{-c\ln(f)-f}}\right)\sqrt{\pi}}{4\sqrt{-c}}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x,method=_RETURNVERBOSE)
```

output
```
-1/4*erf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))/(f-c*ln(f
))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+
e^2)/(c*ln(f)-f))-1/4*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-
f)^(1/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*
e-4*d*ln(f)*c-4*d*f+e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(139) = 278$.

Time = 0.12 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.25

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx =$$
$$-\frac{\left(\sqrt{\pi}(c\log(f)+f)\cosh\left(-\frac{(b^2-4\,ac)\log(f)^2+e^2-4\,df+2\,(2\,cd-be+2\,af)\log(f)}{4\,(c\log(f)-f)}\right)+\sqrt{\pi}(c\log(f)+f)\sinh\left(-\frac{(b^2-}{}\right.\right.}{}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(pi)*(c*log(f) + f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4
*d*f + 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c*log(f
) + f)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e +
2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf(-1/2*(2*f*x - (2*c
*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f)) + (sqrt(pi)*(c*log
(f) - f)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e
+ 2*a*f)*log(f))/(c*log(f) + f)) + sqrt(pi)*(c*log(f) - f)*sinh(-1/4*((b^2
 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(
f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*x + (2*c*x + b)*log(f) + e)*sqr
t(-c*log(f) - f)/(c*log(f) + f)))/(c^2*log(f)^2 - f^2)
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx = \int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d),x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x + f*x**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.94

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)-f}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4\,(c\log(f)+f)}+d\right)}}{4\sqrt{-c\log\left(f\right)-f}}$$
$$+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log\left(f\right)+f}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4\,(c\log(f)-f)}-d\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="maxima")
```

output
```
1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*log(f) + e)/sqrt(-c*lo
g(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d)/sqrt(-c*log(f) - f
) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*(b*log(f) - e)/sqrt(-
c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f) - d)/sqrt(-c*log(f)
 + f)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.29

$$\int f^{a+bx+cx^2}\cosh\left(d+ex+fx^2\right)\,dx=$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)-f}\left(2\,x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-4\,cd\log(f)+2\,be\log(f)-4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)+f)}\right)}}{4\sqrt{-c\log\left(f\right)-f}}$$
$$-\frac{\sqrt{\pi}\operatorname{erf}\left(-\frac{1}{2}\sqrt{-c\log\left(f\right)+f}\left(2\,x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+4\,cd\log(f)-2\,be\log(f)+4\,af\log(f)+e^2-4\,df}{4\,(c\log(f)-f)}\right)}}{4\sqrt{-c\log\left(f\right)+f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x, algorithm="giac")
```

output
```
-1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) + e)/(c*log(f)
 + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f) + 2*b*e*log(
f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(f) - f) - 1/4
*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)/(c*log(f) - f
)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) - 2*b*e*log(f) +
 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f) + f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \cosh \left(d+ex+fx^2\right) \, dx = \int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+e\,x+d\right) \,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2),x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2), x)
```

Reduce [F]

$$\int f^{a+bx+cx^2} \cosh \left(d+ex+fx^2\right) \, dx = f^a \left(\int f^{c\,x^2+bx} \cosh \left(f\,x^2+ex+d\right)dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d),x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + e*x + f*x**2),x)
```

3.328 $\int f^{a+bx+cx^2} \cosh^2(d+ex+fx^2)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 239

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx = \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\mathrm{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{e^{-2d+\frac{(2e-b\log(f))^2}{8f-4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erf}\left(\frac{2e-b\log(f)+2x(2f-c\log(f))}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{e^{2d-\frac{(2e+b\log(f))^2}{8f+4c\log(f)}}f^a\sqrt{\pi}\,\mathrm{erfi}\left(\frac{2e+b\log(f)+2x(2f+c\log(f))}{2\sqrt{2f+c\log(f)}}\right)}{8\sqrt{2f+c\log(f)}}$$

output
```
1/4*f^(a-1/4*b^2/c)*Pi^(1/2)*erfi(1/2*(2*c*x+b)*ln(f)^(1/2)/c^(1/2))/c^(1/
2)/ln(f)^(1/2)+1/8*exp(-2*d+(2*e-b*ln(f))^2/(8*f-4*c*ln(f)))*f^a*Pi^(1/2)*
erf(1/2*(2*e-b*ln(f)+2*x*(2*f-c*ln(f)))/(2*f-c*ln(f))^(1/2))/(2*f-c*ln(f))
^(1/2)+1/8*exp(2*d-(2*e+b*ln(f))^2/(8*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*
(2*e+b*ln(f)+2*x*(2*f+c*ln(f)))/(2*f+c*ln(f))^(1/2))/(2*f+c*ln(f))^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 4.03 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.42

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx = \frac{f^{a-\frac{b^2}{4c}}\sqrt{\pi}\,\text{erfi}\left(\frac{(b+2cx)\sqrt{\log(f)}}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}} - \frac{e^{-\frac{4e^2+b^2\log^2(f)}{8f+4c\log(f)}}f^{a+\frac{4bef}{-4f^2+c^2\log^2(f)}}\sqrt{\pi}\left(e^{\frac{f\left(4e^2+b^2\log^2(f)\right)}{4f^2-c^2\log^2(f)}}f^{\frac{be}{2f+c\log(f)}}\text{erf}\left(\frac{2(e+2fx)-(b+2cx)\log(f)}{2\sqrt{2f-c\log(f)}}\right)\sqrt{2f-c\log(f)}(2\ldots\right.}{}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) - (f^(a + (4*b*e*f)/(-4*f^2 + c^2*Log[f]^2))*Sqrt
[Pi]*(E^((f*(4*e^2 + b^2*Log[f]^2))/(4*f^2 - c^2*Log[f]^2))*f^((b*e)/(2*f
+ c*Log[f]))*Erf[(2*(e + 2*f*x) - (b + 2*c*x)*Log[f])/(2*Sqrt[2*f - c*Log[
f]])]*Sqrt[2*f - c*Log[f]]*(2*f + c*Log[f])*(Cosh[2*d] - Sinh[2*d]) + f^((
b*e)/(2*f - c*Log[f]))*Erfi[(2*(e + 2*f*x) + (b + 2*c*x)*Log[f])/(2*Sqrt[2
*f + c*Log[f]])]*(2*f - c*Log[f])*Sqrt[2*f + c*Log[f]]*(Cosh[2*d] + Sinh[2
*d])))/(8*E^((4*e^2 + b^2*Log[f]^2)/(8*f + 4*c*Log[f]))*(-4*f^2 + c^2*Log[
f]^2))
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$

$$\Big\downarrow \; 6039$$

$$\int \left(\frac{1}{4}e^{-2d-2ex-2fx^2}f^{a+bx+cx^2}+\frac{1}{4}e^{2d+2ex+2fx^2}f^{a+bx+cx^2}+\frac{1}{2}f^{a+bx+cx^2}\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{\sqrt{\pi} f^{a-\frac{b^2}{4c}} \operatorname{erfi}\left(\frac{\sqrt{\log(f)}(b+2cx)}{2\sqrt{c}}\right)}{4\sqrt{c}\sqrt{\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(\frac{(2e-b\log(f))^2}{8f-4c\log(f)}-2d\right) \operatorname{erf}\left(\frac{-b\log(f)+2x(2f-c\log(f))+2e}{2\sqrt{2f-c\log(f)}}\right)}{8\sqrt{2f-c\log(f)}}+\frac{\sqrt{\pi} f^a \exp\left(2d-\frac{(b\log(f)+2e)^2}{4c\log(f)+8f}\right) \operatorname{erfi}\left(\frac{b\log(f)+2x(c\log(f)+2f)+2e}{2\sqrt{c\log(f)+2f}}\right)}{8\sqrt{c\log(f)+2f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^2,x]
```

output
```
(f^(a - b^2/(4*c))*Sqrt[Pi]*Erfi[((b + 2*c*x)*Sqrt[Log[f]])/(2*Sqrt[c])])/
(4*Sqrt[c]*Sqrt[Log[f]]) + (E^(-2*d + (2*e - b*Log[f])^2/(8*f - 4*c*Log[f]
))*f^a*Sqrt[Pi]*Erf[(2*e - b*Log[f] + 2*x*(2*f - c*Log[f]))/(2*Sqrt[2*f -
c*Log[f]])])/(8*Sqrt[2*f - c*Log[f]]) + (E^(2*d - (2*e + b*Log[f])^2/(8*f
+ 4*c*Log[f]))*f^a*Sqrt[Pi]*Erfi[(2*e + b*Log[f] + 2*x*(2*f + c*Log[f]))/(
2*Sqrt[2*f + c*Log[f]])])/(8*Sqrt[2*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.78 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.04

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{2f-c\ln(f)}+\frac{b\ln(f)-2e}{2\sqrt{2f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a} \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-4\ln(f)be+8d\ln(f)c-16df+4e^{2}}{4(c\ln(f)-2f)}}}{8\sqrt{2f-c\ln(f)}}-\frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-2f}\,x+\frac{2e+b\ln(f)}{2\sqrt{-c\ln(f)-2}}\right.}{8}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/8*erf(-x*(2*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-2*e)/(2*f-c*ln(f))^(1/2))/(2*
f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-4*ln(f)*b*e+8*d*ln(f)*
c-16*d*f+4*e^2)/(c*ln(f)-2*f))-1/8*erf(-(-c*ln(f)-2*f)^(1/2)*x+1/2*(2*e+b*
ln(f))/(-c*ln(f)-2*f)^(1/2))/(-c*ln(f)-2*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b
^2*ln(f)^2+4*ln(f)*b*e-8*d*ln(f)*c-16*d*f+4*e^2)/(2*f+c*ln(f)))-1/4*f^a*Pi
^(1/2)*f^(-1/4*b^2/c)/(-c*ln(f))^(1/2)*erf(-(-c*ln(f))^(1/2)*x+1/2*ln(f)*b
/(-c*ln(f))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 516 vs. $2(197) = 394$.

Time = 0.10 (sec) , antiderivative size = 516, normalized size of antiderivative = 2.16

$$\int f^{a+bx+cx^2} \cosh^2\left(d+ex+fx^2\right) dx = \text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="fricas")
```

output
```
-1/8*((sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*cosh(-1/4*((b^2 - 4*a*c)*log
(f)^2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f))
 + sqrt(pi)*(c^2*log(f)^2 + 2*c*f*log(f))*sinh(-1/4*((b^2 - 4*a*c)*log(f)^
2 + 4*e^2 - 16*d*f + 4*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 2*f)))*sq
rt(-c*log(f) + 2*f)*erf(-1/2*(4*f*x - (2*c*x + b)*log(f) + 2*e)*sqrt(-c*lo
g(f) + 2*f)/(c*log(f) - 2*f)) + (sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)
*log(f))/(c*log(f) + 2*f)) + sqrt(pi)*(c^2*log(f)^2 - 2*c*f*log(f))*sinh(-
1/4*((b^2 - 4*a*c)*log(f)^2 + 4*e^2 - 16*d*f - 4*(2*c*d - b*e + 2*a*f)*log
(f))/(c*log(f) + 2*f)))*sqrt(-c*log(f) - 2*f)*erf(1/2*(4*f*x + (2*c*x + b)
*log(f) + 2*e)*sqrt(-c*log(f) - 2*f)/(c*log(f) + 2*f)) + 2*(sqrt(pi)*(c^2*
log(f)^2 - 4*f^2)*cosh(-1/4*(b^2 - 4*a*c)*log(f)/c) + sqrt(pi)*(c^2*log(f)
^2 - 4*f^2)*sinh(-1/4*(b^2 - 4*a*c)*log(f)/c))*sqrt(-c*log(f))*erf(1/2*(2*
c*x + b)*sqrt(-c*log(f))/c))/(c^3*log(f)^3 - 4*c*f^2*log(f))
```

Sympy [F]

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)dx$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**2,x)
```

output
```
Integral(f**(a + b*x + c*x**2)*cosh(d + e*x + f*x**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.90

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)-2f}x-\frac{b\log(f)+2e}{2\sqrt{-c\log(f)-2f}}\right)e^{\left(-\frac{(b\log(f)+2e)^2}{4(c\log(f)+2f)}+2d\right)}}{8\sqrt{-c\log(f)-2f}}$$
$$+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)+2f}x-\frac{b\log(f)-2e}{2\sqrt{-c\log(f)+2f}}\right)e^{\left(-\frac{(b\log(f)-2e)^2}{4(c\log(f)-2f)}-2d\right)}}{8\sqrt{-c\log(f)+2f}}$$
$$+\frac{\sqrt{\pi}f^a\,\mathrm{erf}\left(\sqrt{-c\log(f)}x-\frac{b\log(f)}{2\sqrt{-c\log(f)}}\right)}{4\sqrt{-c\log(f)}f^{\frac{b^2}{4c}}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="maxima")
```

output
```
1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 2*f)*x - 1/2*(b*log(f) + 2*e)/sqrt(-
c*log(f) - 2*f))*e^(-1/4*(b*log(f) + 2*e)^2/(c*log(f) + 2*f) + 2*d)/sqrt(-
c*log(f) - 2*f) + 1/8*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 2*f)*x - 1/2*(b*lo
g(f) - 2*e)/sqrt(-c*log(f) + 2*f))*e^(-1/4*(b*log(f) - 2*e)^2/(c*log(f) -
2*f) - 2*d)/sqrt(-c*log(f) + 2*f) + 1/4*sqrt(pi)*f^a*erf(sqrt(-c*log(f))*x
 - 1/2*b*log(f)/sqrt(-c*log(f)))/(sqrt(-c*log(f))*f^(1/4*b^2/c))
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.13

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)-2\,f}\left(2\,x+\frac{b\log(f)+2\,e}{c\log(f)+2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2-8\,cd\log(f)+4\,be\log(f)-8\,af\log(f)+4\,e^2-16\,df}{4\,(c\log(f)+2\,f)}\right.}}{8\,\sqrt{-c\log\left(f\right)-2\,f}}$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)+2\,f}\left(2\,x+\frac{b\log(f)-2\,e}{c\log(f)-2\,f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4\,ac\log(f)^2+8\,cd\log(f)-4\,be\log(f)+8\,af\log(f)+4\,e^2-16\,df}{4\,(c\log(f)-2\,f)}\right.}}{8\,\sqrt{-c\log\left(f\right)+2\,f}}$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\,\sqrt{-c\log\left(f\right)}\left(2\,x+\frac{b}{c}\right)\right)e^{\left(-\frac{b^2\log(f)-4\,ac\log(f)}{4\,c}\right)}}{4\,\sqrt{-c\log\left(f\right)}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x, algorithm="giac")
```

output
```
-1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 2*f)*(2*x + (b*log(f) + 2*e)/(c*lo
g(f) + 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 8*c*d*log(f) + 4*b*
e*log(f) - 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) + 2*f))/sqrt(-c*log(f)
 - 2*f) - 1/8*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 2*f)*(2*x + (b*log(f) - 2
*e)/(c*log(f) - 2*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 8*c*d*log(
f) - 4*b*e*log(f) + 8*a*f*log(f) + 4*e^2 - 16*d*f)/(c*log(f) - 2*f))/sqrt(
-c*log(f) + 2*f) - 1/4*sqrt(pi)*erf(-1/2*sqrt(-c*log(f))*(2*x + b/c))*e^(-
1/4*(b^2*log(f) - 4*a*c*log(f))/c)/sqrt(-c*log(f))
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\,{\cosh\left(f\,x^2+e\,x+d\right)}^2\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^2, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\cosh^2\left(d+ex+fx^2\right)\,dx = f^a\left(\int f^{c\,x^2+bx}\cosh\left(f\,x^2+ex+d\right)^2dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^2,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + e*x + f*x**2)**2,x)
```

3.329 $\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$

Optimal result

Integrand size = 26, antiderivative size = 344

$$\begin{aligned}\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx &= \frac{3e^{-d+\frac{(e-b\log(f))^2}{4(f-c\log(f))}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{e-b\log(f)+2x(f-c\log(f))}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}\\ &+\frac{e^{-3d+\frac{(3e-b\log(f))^2}{12f-4c\log(f)}}f^a\sqrt{\pi}\operatorname{erf}\left(\frac{3e-b\log(f)+2x(3f-c\log(f))}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}\\ &+\frac{3e^{d-\frac{(e+b\log(f))^2}{4(f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{e+b\log(f)+2x(f+c\log(f))}{2\sqrt{f+c\log(f)}}\right)}{16\sqrt{f+c\log(f)}}\\ &+\frac{e^{3d-\frac{(3e+b\log(f))^2}{4(3f+c\log(f))}}f^a\sqrt{\pi}\operatorname{erfi}\left(\frac{3e+b\log(f)+2x(3f+c\log(f))}{2\sqrt{3f+c\log(f)}}\right)}{16\sqrt{3f+c\log(f)}}\end{aligned}$$

output
```
3/16*exp(-d+(e-b*ln(f))^2/(4*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(e-b*ln(f)
+2*x*(f-c*ln(f)))/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/2)+1/16*exp(-3*d+(3*e-
b*ln(f))^2/(12*f-4*c*ln(f)))*f^a*Pi^(1/2)*erf(1/2*(3*e-b*ln(f)+2*x*(3*f-c*
ln(f)))/(3*f-c*ln(f))^(1/2))/(3*f-c*ln(f))^(1/2)+3/16*exp(d-(e+b*ln(f))^2/
(4*f+4*c*ln(f)))*f^a*Pi^(1/2)*erfi(1/2*(e+b*ln(f)+2*x*(f+c*ln(f)))/(f+c*ln
(f))^(1/2))/(f+c*ln(f))^(1/2)+1/16*exp(3*d-(3*e+b*ln(f))^2/(12*f+4*c*ln(f)
))*f^a*Pi^(1/2)*erfi(1/2*(3*e+b*ln(f)+2*x*(3*f+c*ln(f)))/(3*f+c*ln(f))^(1/
2))/(3*f+c*ln(f))^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2991 vs. $2(344) = 688$.

Time = 6.43 (sec) , antiderivative size = 2991, normalized size of antiderivative = 8.69

$$\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right)\,dx = \text{Result too large to show}$$

input
```
Integrate[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(f^a*Sqrt[Pi]*((27*f^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(
2*Sqrt[f - c*Log[f]])]*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*L
og[f]^2)/(4*(f - c*Log[f]))) + (27*c*f^2*Cosh[d]*Erf[(e + 2*f*x - b*Log[f]
 - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]*Sqrt[f - c*Log[f]])/E^((-e
^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[f]))) - (3*c^2*f*Cosh[d]*E
rf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f - c*Log[f]])]*Log[f]^2*
Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*Log[f]^2)/(4*(f - c*Log[
f]))) - (3*c^3*Cosh[d]*Erf[(e + 2*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[f
 - c*Log[f]])]*Log[f]^3*Sqrt[f - c*Log[f]])/E^((-e^2 + 2*b*e*Log[f] - b^2*
Log[f]^2)/(4*(f - c*Log[f]))) + (3*f^3*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[
f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Sqrt[3*f - c*Log[f]])/E^((-9*
e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) + (c*f^2*Cosh[3*d
]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Lo
g[f]*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3
*f - c*Log[f]))) - (3*c^2*f*Cosh[3*d]*Erf[(3*e + 6*f*x - b*Log[f] - 2*c*x*
Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]^2*Sqrt[3*f - c*Log[f]])/E^((-9*e^
2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f - c*Log[f]))) - (c^3*Cosh[3*d]*Er
f[(3*e + 6*f*x - b*Log[f] - 2*c*x*Log[f])/(2*Sqrt[3*f - c*Log[f]])]*Log[f]
^3*Sqrt[3*f - c*Log[f]])/E^((-9*e^2 + 6*b*e*Log[f] - b^2*Log[f]^2)/(4*(3*f
 - c*Log[f]))) + (27*f^3*Cosh[d]*Erfi[(e + 2*f*x + b*Log[f] + 2*c*x*Log...
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.077$, Rules used = {6039, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right) dx$$

$$\Big\downarrow \text{6039}$$

$$\int \left(\frac{3}{8} f^{a+bx+cx^2} \exp\left(-3\left(d+ex+fx^2\right)+2d+2ex+2fx^2\right)+\frac{3}{8} f^{a+bx+cx^2} \exp\left(-3\left(d+ex+fx^2\right)+4d+4ex+\right.\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{\sqrt{\pi}f^a \exp\left(\frac{(3e-b\log(f))^2}{12f-4c\log(f)}-3d\right)\text{erf}\left(\frac{-b\log(f)+2x(3f-c\log(f))+3e}{2\sqrt{3f-c\log(f)}}\right)}{16\sqrt{3f-c\log(f)}}+\frac{3\sqrt{\pi}f^a e^{\frac{(e-b\log(f))^2}{4(f-c\log(f))}-d}\text{erf}\left(\frac{-b\log(f)+2x(f-c\log(f))+e}{2\sqrt{f-c\log(f)}}\right)}{16\sqrt{f-c\log(f)}}+\frac{\sqrt{\pi}f^a \exp\left(3d-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}\right)\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+3f)+3e}{2\sqrt{c\log(f)+3f}}\right)}{16\sqrt{c\log(f)+3f}}+\frac{3\sqrt{\pi}f^a e^{d-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}}\text{erfi}\left(\frac{b\log(f)+2x(c\log(f)+f)+e}{2\sqrt{c\log(f)+f}}\right)}{16\sqrt{c\log(f)+f}}$$

input
```
Int[f^(a + b*x + c*x^2)*Cosh[d + e*x + f*x^2]^3,x]
```

output
```
(3*E^(-d + (e - b*Log[f])^2/(4*(f - c*Log[f])))*f^a*Sqrt[Pi]*Erf[(e - b*Lo
g[f] + 2*x*(f - c*Log[f]))/(2*Sqrt[f - c*Log[f]])])/(16*Sqrt[f - c*Log[f]]
) + (E^(-3*d + (3*e - b*Log[f])^2/(12*f - 4*c*Log[f]))*f^a*Sqrt[Pi]*Erf[(3
*e - b*Log[f] + 2*x*(3*f - c*Log[f]))/(2*Sqrt[3*f - c*Log[f]])])/(16*Sqrt[
3*f - c*Log[f]]) + (3*E^(d - (e + b*Log[f])^2/(4*(f + c*Log[f])))*f^a*Sqrt
[Pi]*Erfi[(e + b*Log[f] + 2*x*(f + c*Log[f]))/(2*Sqrt[f + c*Log[f]])])/(16
*Sqrt[f + c*Log[f]]) + (E^(3*d - (3*e + b*Log[f])^2/(4*(3*f + c*Log[f])))*
f^a*Sqrt[Pi]*Erfi[(3*e + b*Log[f] + 2*x*(3*f + c*Log[f]))/(2*Sqrt[3*f + c*
Log[f]])])/(16*Sqrt[3*f + c*Log[f]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 6039
```
Int[Cosh[v_]^(n_.)*(F_)^(u_), x_Symbol] :> Int[ExpandTrigToExp[F^u, Cosh[v]
^n, x], x] /; FreeQ[F, x] && (LinearQ[u, x] || PolyQ[u, x, 2]) && (LinearQ[
v, x] || PolyQ[v, x, 2]) && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 5.05 (sec) , antiderivative size = 384, normalized size of antiderivative = 1.12

method	result
risch	$-\frac{\operatorname{erf}\left(-x\sqrt{3f-c\ln(f)}+\frac{b\ln(f)-3e}{2\sqrt{3f-c\ln(f)}}\right)\sqrt{\pi}\, f^{a}\, \mathrm{e}^{-\frac{b^{2}\ln(f)^{2}-6\ln(f)be+12d\ln(f)c-36df+9e^{2}}{4(c\ln(f)-3f)}}}{16\sqrt{3f-c\ln(f)}} - \frac{\operatorname{erf}\left(-\sqrt{-c\ln(f)-3f}\,x+\frac{3e+b\ln(f}{2\sqrt{-c\ln(f)-}}\right.}{1}$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/16*erf(-x*(3*f-c*ln(f))^(1/2)+1/2*(b*ln(f)-3*e)/(3*f-c*ln(f))^(1/2))/(3
*f-c*ln(f))^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-6*ln(f)*b*e+12*d*ln(f
)*c-36*d*f+9*e^2)/(c*ln(f)-3*f))-1/16*erf(-(-c*ln(f)-3*f)^(1/2)*x+1/2*(3*e
+b*ln(f))/(-c*ln(f)-3*f)^(1/2))/(-c*ln(f)-3*f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4
*(b^2*ln(f)^2+6*ln(f)*b*e-12*d*ln(f)*c-36*d*f+9*e^2)/(3*f+c*ln(f)))-3/16*e
rf(-x*(f-c*ln(f))^(1/2)+1/2*(b*ln(f)-e)/(f-c*ln(f))^(1/2))/(f-c*ln(f))^(1/
2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2-2*ln(f)*b*e+4*d*ln(f)*c-4*d*f+e^2)/(
c*ln(f)-f))-3/16*erf(-(-c*ln(f)-f)^(1/2)*x+1/2*(e+b*ln(f))/(-c*ln(f)-f)^(1
/2))/(-c*ln(f)-f)^(1/2)*Pi^(1/2)*f^a*exp(-1/4*(b^2*ln(f)^2+2*ln(f)*b*e-4*d
*ln(f)*c-4*d*f+e^2)/(f+c*ln(f)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 939 vs. 2(295) = 590.

Time = 0.11 (sec) , antiderivative size = 939, normalized size of antiderivative = 2.73

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\text{Too large to display}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="fricas")
```

output
```
-1/16*((sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 - c*f^2*log(f) - 3*f^3)*
cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f + 6*(2*c*d - b*e + 2*a*
f)*log(f))/(c*log(f) - 3*f)) + sqrt(pi)*(c^3*log(f)^3 + 3*c^2*f*log(f)^2 -
 c*f^2*log(f) - 3*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f
+ 6*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - 3*f)))*sqrt(-c*log(f) + 3*f)
*erf(-1/2*(6*f*x - (2*c*x + b)*log(f) + 3*e)*sqrt(-c*log(f) + 3*f)/(c*log(
f) - 3*f)) + 3*(sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2 - 9*c*f^2*log(f) -
 9*f^3)*cosh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f + 2*(2*c*d - b*e +
 2*a*f)*log(f))/(c*log(f) - f)) + sqrt(pi)*(c^3*log(f)^3 + c^2*f*log(f)^2
- 9*c*f^2*log(f) - 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f
+ 2*(2*c*d - b*e + 2*a*f)*log(f))/(c*log(f) - f)))*sqrt(-c*log(f) + f)*erf
(-1/2*(2*f*x - (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) + f)/(c*log(f) - f))
 + 3*(sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*log(f) + 9*f^3)*co
sh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d - b*e + 2*a*f)*lo
g(f))/(c*log(f) + f)) + sqrt(pi)*(c^3*log(f)^3 - c^2*f*log(f)^2 - 9*c*f^2*
log(f) + 9*f^3)*sinh(-1/4*((b^2 - 4*a*c)*log(f)^2 + e^2 - 4*d*f - 2*(2*c*d
 - b*e + 2*a*f)*log(f))/(c*log(f) + f)))*sqrt(-c*log(f) - f)*erf(1/2*(2*f*
x + (2*c*x + b)*log(f) + e)*sqrt(-c*log(f) - f)/(c*log(f) + f)) + (sqrt(pi
)*(c^3*log(f)^3 - 3*c^2*f*log(f)^2 - c*f^2*log(f) + 3*f^3)*cosh(-1/4*((b^2
 - 4*a*c)*log(f)^2 + 9*e^2 - 36*d*f - 6*(2*c*d - b*e + 2*a*f)*log(f))/(...
```

Sympy [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2} \cosh^3\left(d+ex+fx^2\right) dx = \text{Timed out}$$

input
```
integrate(f**(c*x**2+b*x+a)*cosh(f*x**2+e*x+d)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 315, normalized size of antiderivative = 0.92

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx$$
$$=\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-3f}x-\frac{b\log(f)+3e}{2\sqrt{-c\log(f)-3f}}\right)e^{\left(-\frac{(b\log(f)+3e)^2}{4(c\log(f)+3f)}+3d\right)}}{16\sqrt{-c\log(f)-3f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)-f}x-\frac{b\log(f)+e}{2\sqrt{-c\log(f)-f}}\right)e^{\left(-\frac{(b\log(f)+e)^2}{4(c\log(f)+f)}+d\right)}}{16\sqrt{-c\log(f)-f}}$$
$$+\frac{3\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+f}x-\frac{b\log(f)-e}{2\sqrt{-c\log(f)+f}}\right)e^{\left(-\frac{(b\log(f)-e)^2}{4(c\log(f)-f)}-d\right)}}{16\sqrt{-c\log(f)+f}}$$
$$+\frac{\sqrt{\pi}f^a\operatorname{erf}\left(\sqrt{-c\log(f)+3f}x-\frac{b\log(f)-3e}{2\sqrt{-c\log(f)+3f}}\right)e^{\left(-\frac{(b\log(f)-3e)^2}{4(c\log(f)-3f)}-3d\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="maxima")
```

output
```
1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - 3*f)*x - 1/2*(b*log(f) + 3*e)/sqrt(
-c*log(f) - 3*f))*e^(-1/4*(b*log(f) + 3*e)^2/(c*log(f) + 3*f) + 3*d)/sqrt(
-c*log(f) - 3*f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) - f)*x - 1/2*(b*lo
g(f) + e)/sqrt(-c*log(f) - f))*e^(-1/4*(b*log(f) + e)^2/(c*log(f) + f) + d
)/sqrt(-c*log(f) - f) + 3/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + f)*x - 1/2*
(b*log(f) - e)/sqrt(-c*log(f) + f))*e^(-1/4*(b*log(f) - e)^2/(c*log(f) - f
) - d)/sqrt(-c*log(f) + f) + 1/16*sqrt(pi)*f^a*erf(sqrt(-c*log(f) + 3*f)*x
 - 1/2*(b*log(f) - 3*e)/sqrt(-c*log(f) + 3*f))*e^(-1/4*(b*log(f) - 3*e)^2/
(c*log(f) - 3*f) - 3*d)/sqrt(-c*log(f) + 3*f)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 427, normalized size of antiderivative = 1.24

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx =$$

$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-3f}\left(2x+\frac{b\log(f)+3e}{c\log(f)+3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-12cd\log(f)+6be\log(f)-12af\log(f)+9e^2-36}{4(c\log(f)+3f)}\right)}}{16\sqrt{-c\log(f)-3f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)-f}\left(2x+\frac{b\log(f)+e}{c\log(f)+f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2-4cd\log(f)+2be\log(f)-4af\log(f)+e^2-4df}{4(c\log(f)+f)}\right)}}{16\sqrt{-c\log(f)-f}}$$
$$-\frac{3\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+f}\left(2x+\frac{b\log(f)-e}{c\log(f)-f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+4cd\log(f)-2be\log(f)+4af\log(f)+e^2-4df}{4(c\log(f)-f)}\right)}}{16\sqrt{-c\log(f)+f}}$$
$$-\frac{\sqrt{\pi}\,\mathrm{erf}\left(-\frac{1}{2}\sqrt{-c\log(f)+3f}\left(2x+\frac{b\log(f)-3e}{c\log(f)-3f}\right)\right)e^{\left(-\frac{b^2\log(f)^2-4ac\log(f)^2+12cd\log(f)-6be\log(f)+12af\log(f)+9e^2-36}{4(c\log(f)-3f)}\right)}}{16\sqrt{-c\log(f)+3f}}$$

input
```
integrate(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x, algorithm="giac")
```

output
```
-1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - 3*f)*(2*x + (b*log(f) + 3*e)/(c*l
og(f) + 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 12*c*d*log(f) + 6*
b*e*log(f) - 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) + 3*f))/sqrt(-c*log
(f) - 3*f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) - f)*(2*x + (b*log(f) +
 e)/(c*log(f) + f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 - 4*c*d*log(f)
 + 2*b*e*log(f) - 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) + f))/sqrt(-c*log(
f) - f) - 3/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + f)*(2*x + (b*log(f) - e)
/(c*log(f) - f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 4*c*d*log(f) -
2*b*e*log(f) + 4*a*f*log(f) + e^2 - 4*d*f)/(c*log(f) - f))/sqrt(-c*log(f)
+ f) - 1/16*sqrt(pi)*erf(-1/2*sqrt(-c*log(f) + 3*f)*(2*x + (b*log(f) - 3*e
)/(c*log(f) - 3*f)))*e^(-1/4*(b^2*log(f)^2 - 4*a*c*log(f)^2 + 12*c*d*log(f
) - 6*b*e*log(f) + 12*a*f*log(f) + 9*e^2 - 36*d*f)/(c*log(f) - 3*f))/sqrt(
-c*log(f) + 3*f)
```

Mupad [F(-1)]

Timed out.

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=\int f^{c\,x^2+b\,x+a}\cosh\left(f\,x^2+e\,x+d\right)^3\,dx$$

input
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3,x)
```

output
```
int(f^(a + b*x + c*x^2)*cosh(d + e*x + f*x^2)^3, x)
```

Reduce [F]

$$\int f^{a+bx+cx^2}\cosh^3\left(d+ex+fx^2\right)\,dx=f^a\left(\int f^{c\,x^2+bx}\cosh\left(f\,x^2+ex+d\right)^3dx\right)$$

input
```
int(f^(c*x^2+b*x+a)*cosh(f*x^2+e*x+d)^3,x)
```

output
```
f**a*int(f**(b*x + c*x**2)*cosh(d + e*x + f*x**2)**3,x)
```

3.330 $\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx$

Optimal result

Integrand size = 17, antiderivative size = 20

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=-4\sqrt{\cosh(x)}+\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}$$

output
```
-4*cosh(x)^(1/2)+2*x*sinh(x)/cosh(x)^(1/2)
```

Mathematica [B] (warning: unable to verify)

Leaf count is larger than twice the leaf count of optimal. 46 vs. 2(20) = 40.

Time = 0.22 (sec) , antiderivative size = 46, normalized size of antiderivative = 2.30

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)\,dx=\frac{2\sinh(x)\left(x-\frac{2\cosh(x)\sinh(x)\sqrt{\tanh^2\left(\frac{x}{2}\right)}}{(-1+\cosh(x))^{3/2}\sqrt{1+\cosh(x)}}\right)}{\sqrt{\cosh(x)}}$$

input
```
Integrate[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

output
```
(2*Sinh[x]*(x - (2*Cosh[x]*Sinh[x]*Sqrt[Tanh[x/2]^2])/((-1 + Cosh[x])^(3/2
)*Sqrt[1 + Cosh[x]])))/Sqrt[Cosh[x]]
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.059$, Rules used = $\{2009\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right) dx$$
$$\Big\downarrow \text{2009}$$
$$\frac{2x\sinh(x)}{\sqrt{\cosh(x)}}-4\sqrt{\cosh(x)}$$

input
```
Int[x/Cosh[x]^(3/2) + x*Sqrt[Cosh[x]],x]
```

output
```
-4*Sqrt[Cosh[x]] + (2*x*Sinh[x])/Sqrt[Cosh[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\cosh(x)^{\frac{3}{2}}}+x\sqrt{\cosh(x)}\right)dx$$

input
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right)dx = \int \frac{x(\cosh^2(x)+1)}{\cosh^{\frac{3}{2}}(x)}\,dx$$

input
```
integrate(x/cosh(x)**(3/2)+x*cosh(x)**(1/2),x)
```

output
```
Integral(x*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = \int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = \int x\sqrt{\cosh(x)} + \frac{x}{\cosh(x)^{\frac{3}{2}}} \, dx$$

input
```
integrate(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x*sqrt(cosh(x)) + x/cosh(x)^(3/2), x)
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.95

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)} + x\sqrt{\cosh(x)}\right) dx = -\frac{2\sqrt{\frac{e^{-x}}{2} + \frac{e^{x}}{2}}\,(x + 2\,e^{2\,x} - x\,e^{2\,x} + 2)}{e^{2\,x} + 1}$$

input
```
int(x*cosh(x)^(1/2) + x/cosh(x)^(3/2),x)
```

output
```
-(2*(exp(-x)/2 + exp(x)/2)^(1/2)*(x + 2*exp(2*x) - x*exp(2*x) + 2))/(exp(2
*x) + 1)
```

Reduce [F]

$$\int \left(\frac{x}{\cosh^{\frac{3}{2}}(x)}+x\sqrt{\cosh(x)}\right) dx = \int \frac{\sqrt{\cosh (x)}\, x}{\cosh (x)^{2}}dx + \int \sqrt{\cosh (x)}\, x dx$$

input
```
int(x/cosh(x)^(3/2)+x*cosh(x)^(1/2),x)
```

output
```
int((sqrt(cosh(x))*x)/cosh(x)**2,x) + int(sqrt(cosh(x))*x,x)
```

3.331 $\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right) dx$

Optimal result

Integrand size = 20, antiderivative size = 24

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right) dx=\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

output
```
4/3/cosh(x)^(1/2)+2/3*x*sinh(x)/cosh(x)^(3/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 16, normalized size of antiderivative = 0.67

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right) dx=\frac{2(2+x\tanh(x))}{3\sqrt{\cosh(x)}}$$

input
```
Integrate[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

output
```
(2*(2 + x*Tanh[x]))/(3*Sqrt[Cosh[x]])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)}-\frac{x}{3\sqrt{\cosh(x)}}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{4}{3\sqrt{\cosh(x)}}+\frac{2x\sinh(x)}{3\cosh^{\frac{3}{2}}(x)}$$

input
```
Int[x/Cosh[x]^(5/2) - x/(3*Sqrt[Cosh[x]]),x]
```

output
```
4/(3*Sqrt[Cosh[x]]) + (2*x*Sinh[x])/(3*Cosh[x]^(3/2))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\cosh\left(x\right)^{\frac{5}{2}}}-\frac{x}{3\sqrt{\cosh\left(x\right)}}\right)dx$$

input
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 109 vs. $2(16) = 32$.

Time = 0.09 (sec) , antiderivative size = 109, normalized size of antiderivative = 4.54

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right)\,dx$$
$$= \frac{4\left((x+2)\cosh{(x)}^3 + 3\,(x+2)\cosh{(x)}\sinh{(x)}^2 + (x+2)\sinh{(x)}^3 - (x-2)\cosh{(x)} + \left(3\,(x+2)\cos\right.\right.}{3\left(\cosh{(x)}^4 + 4\,\cosh{(x)}\sinh{(x)}^3 + \sinh{(x)}^4 + 2\left(3\,\cosh{(x)}^2+1\right)\sinh{(x)}^2 + 2\,\cosh{(x)}^2 + 4\,(\cos\right.}$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
4/3*((x + 2)*cosh(x)^3 + 3*(x + 2)*cosh(x)*sinh(x)^2 + (x + 2)*sinh(x)^3 -
 (x - 2)*cosh(x) + (3*(x + 2)*cosh(x)^2 - x + 2)*sinh(x))*sqrt(cosh(x))/(c
osh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2
 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1)
```

Sympy [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right)\,dx = -\frac{\int \left(-\frac{3x}{\cosh^{\frac{5}{2}}(x)}\right)\,dx + \int \frac{x}{\sqrt{\cosh(x)}}\,dx}{3}$$

input
```
integrate(x/cosh(x)**(5/2)-1/3*x/cosh(x)**(1/2),x)
```

output
```
-(Integral(-3*x/cosh(x)**(5/2), x) + Integral(x/sqrt(cosh(x)), x))/3
```

Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \int -\frac{x}{3\sqrt{\cosh(x)}} + \frac{x}{\cosh(x)^{\frac{5}{2}}}\, dx$$

input
```
integrate(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(-1/3*x/sqrt(cosh(x)) + x/cosh(x)^(5/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.02 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.75

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = \frac{4\,e^x\,\sqrt{\frac{e^{-x}}{2}+\frac{e^x}{2}}\,\left(2\,e^{2\,x}-x+x\,e^{2\,x}+2\right)}{3\,{\left(e^{2\,x}+1\right)}^2}$$

input
```
int(x/cosh(x)^(5/2) - x/(3*cosh(x)^(1/2)),x)
```

output
```
(4*exp(x)*(exp(-x)/2 + exp(x)/2)^(1/2)*(2*exp(2*x) - x + x*exp(2*x) + 2))/
(3*(exp(2*x) + 1)^2)
```

Reduce [F]

$$\int \left(\frac{x}{\cosh^{\frac{5}{2}}(x)} - \frac{x}{3\sqrt{\cosh(x)}}\right) dx = -\frac{\left(\int \frac{\sqrt{\cosh(x)}\,x}{\cosh(x)} dx\right)}{3} + \int \frac{\sqrt{\cosh(x)}\,x}{\cosh(x)^3} dx$$

input
```
int(x/cosh(x)^(5/2)-1/3*x/cosh(x)^(1/2),x)
```

output
```
( - int((sqrt(cosh(x))*x)/cosh(x),x) + 3*int((sqrt(cosh(x))*x)/cosh(x)**3,
x))/3
```

3.332 $\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx$

Optimal result

Integrand size = 20, antiderivative size = 47

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

output

```
4/15/cosh(x)^(3/2)-12/5*cosh(x)^(1/2)+2/5*x*sinh(x)/cosh(x)^(5/2)+6/5*x*si
nh(x)/cosh(x)^(1/2)
```

Mathematica [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.36

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx =\frac{1}{5}\sqrt{\cosh(x)}\left(-\frac{12\sinh^2(x)}{\sqrt{-1+\cosh(x)}(1+\cosh(x))^{3/2}\sqrt{\tanh^2\left(\frac{x}{2}\right)}} +6x\tanh(x)+\operatorname{sech}^2(x)\left(\frac{4}{3}+2x\tanh(x)\right)\right)$$

input
```
Integrate[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

output
```
(Sqrt[Cosh[x]]*((-12*Sinh[x]^2)/(Sqrt[-1 + Cosh[x]]*(1 + Cosh[x])^(3/2)*Sq
rt[Tanh[x/2]^2]) + 6*x*Tanh[x] + Sech[x]^2*(4/3 + 2*x*Tanh[x])))/5
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{4}{15\cosh^{\frac{3}{2}}(x)}-\frac{12\sqrt{\cosh(x)}}{5}+\frac{2x\sinh(x)}{5\cosh^{\frac{5}{2}}(x)}+\frac{6x\sinh(x)}{5\sqrt{\cosh(x)}}$$

input
```
Int[x/Cosh[x]^(7/2) + (3*x*Sqrt[Cosh[x]])/5,x]
```

output
```
4/(15*Cosh[x]^(3/2)) - (12*Sqrt[Cosh[x]])/5 + (2*x*Sinh[x])/(5*Cosh[x]^(5/
2)) + (6*x*Sinh[x])/(5*Sqrt[Cosh[x]])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}+\frac{3x\sqrt{\cosh\left(x\right)}}{5}\right)dx$$

input
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

output
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)dx = \text{Exception raised: TypeError}$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F(-1)]

Timed out.

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\text{Timed out}$$

input
```
integrate(x/cosh(x)**(7/2)+3/5*x*cosh(x)**(1/2),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Giac [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\int \frac{3}{5}\,x\sqrt{\cosh\left(x\right)}+\frac{x}{\cosh\left(x\right)^{\frac{7}{2}}}\,dx$$

input
```
integrate(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(3/5*x*sqrt(cosh(x)) + x/cosh(x)^(7/2), x)
```

Mupad [B] (verification not implemented)

Time = 2.17 (sec) , antiderivative size = 110, normalized size of antiderivative = 2.34

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\frac{e^{2\,x}\,\left(\frac{8\,x}{5}+\frac{16}{15}\right)\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{{\left(e^{2\,x}+1\right)}^2}-\left(\frac{6\,x}{5}+\frac{12}{5}\right)\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}+\frac{12\,x\,e^{2\,x}\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\,\left(e^{2\,x}+1\right)}-\frac{16\,x\,e^{2\,x}\,\sqrt{\frac{e^{-x}}{2}+\frac{e^{x}}{2}}}{5\,{\left(e^{2\,x}+1\right)}^3}$$

input
```
int((3*x*cosh(x)^(1/2))/5 + x/cosh(x)^(7/2),x)
```

output
```
(exp(2*x)*((8*x)/5 + 16/15)*(exp(-x)/2 + exp(x)/2)^(1/2))/(exp(2*x) + 1)^2
 - ((6*x)/5 + 12/5)*(exp(-x)/2 + exp(x)/2)^(1/2) + (12*x*exp(2*x)*(exp(-x)
/2 + exp(x)/2)^(1/2))/(5*(exp(2*x) + 1)) - (16*x*exp(2*x)*(exp(-x)/2 + exp
(x)/2)^(1/2))/(5*(exp(2*x) + 1)^3)
```

Reduce [F]

$$\int \left(\frac{x}{\cosh^{\frac{7}{2}}(x)}+\frac{3}{5}x\sqrt{\cosh(x)}\right)\,dx=\int \frac{\sqrt{\cosh(x)}\,x}{\cosh(x)^4}dx+\frac{3\left(\int \sqrt{\cosh(x)}\,x dx\right)}{5}$$

input
```
int(x/cosh(x)^(7/2)+3/5*x*cosh(x)^(1/2),x)
```

output
```
(5*int((sqrt(cosh(x))*x)/cosh(x)**4,x) + 3*int(sqrt(cosh(x))*x,x))/5
```

3.333 $\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)+\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}$$

output
```
-8*x*cosh(x)^(1/2)-16*I*EllipticE(I*sinh(1/2*x),2^(1/2))+2*x^2*sinh(x)/cos
h(x)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

Time = 0.10 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.11

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx$$
$$=\frac{4\sqrt{\cosh(x)}(\cosh(x)+\sinh(x))\left(-4(-2+x)\cosh(x)+x^2\sinh(x)+8\,\text{Hypergeometric2F1}\left(-\frac{1}{4},\frac{1}{2},\frac{3}{4},-\right.\right.}{1+e^{2x}}$$

input
```
Integrate[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

output
```
(4*Sqrt[Cosh[x]]*(Cosh[x] + Sinh[x])*(-4*(-2 + x)*Cosh[x] + x^2*Sinh[x] +
8*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^(2*x)]*(-Cosh[x] + Sinh[x])*Sqrt[1
+ Cosh[2*x] + Sinh[2*x]]))/(1 + E^(2*x))
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.048$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)dx$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{2x^2\sinh(x)}{\sqrt{\cosh(x)}}-8x\sqrt{\cosh(x)}-16iE\left(\frac{ix}{2}\middle|2\right)$$

input
```
Int[x^2/Cosh[x]^(3/2) + x^2*Sqrt[Cosh[x]],x]
```

output
```
-8*x*Sqrt[Cosh[x]] - (16*I)*EllipticE[(I/2)*x, 2] + (2*x^2*Sinh[x])/Sqrt[C
osh[x]]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [F]

$$\int \left(\frac{x^2}{\cosh(x)^{\frac{3}{2}}} + x^2\sqrt{\cosh(x)}\right) dx$$

input
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

output
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)}\right) dx = \text{Exception raised: TypeError}$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (has polynomial part)
```

Sympy [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)}\right) dx = \int \frac{x^2(\cosh^2(x) + 1)}{\cosh^{\frac{3}{2}}(x)}\, dx$$

input
```
integrate(x**2/cosh(x)**(3/2)+x**2*cosh(x)**(1/2),x)
```

output
```
Integral(x**2*(cosh(x)**2 + 1)/cosh(x)**(3/2), x)
```

Maxima [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=\int x^2\sqrt{\cosh{(x)}}+\frac{x^2}{\cosh{(x)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="maxima")
```

output
```
integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Giac [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=\int x^2\sqrt{\cosh{(x)}}+\frac{x^2}{\cosh{(x)}^{\frac{3}{2}}}\,dx$$

input
```
integrate(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x, algorithm="giac")
```

output
```
integrate(x^2*sqrt(cosh(x)) + x^2/cosh(x)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)}+x^2\sqrt{\cosh(x)}\right)\,dx=\int x^2\,\sqrt{\cosh{(x)}}+\frac{x^2}{{\cosh{(x)}}^{3/2}}\,dx$$

input
```
int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2),x)
```

output
```
int(x^2*cosh(x)^(1/2) + x^2/cosh(x)^(3/2), x)
```

Reduce [F]

$$\int \left(\frac{x^2}{\cosh^{\frac{3}{2}}(x)} + x^2\sqrt{\cosh(x)}\right) dx = \int \frac{\sqrt{\cosh(x)}\, x^2}{\cosh(x)^2} dx + \int \sqrt{\cosh(x)}\, x^2 dx$$

input
```
int(x^2/cosh(x)^(3/2)+x^2*cosh(x)^(1/2),x)
```

output
```
int((sqrt(cosh(x))*x**2)/cosh(x)**2,x) + int(sqrt(cosh(x))*x**2,x)
```

3.334 $\int (x+\cosh(x))^2 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 30

$$\int (x+\cosh(x))^2 \, dx = \frac{x}{2}+\frac{x^3}{3}-2\cosh(x)+2x\sinh(x)+\frac{1}{2}\cosh(x)\sinh(x)$$

output
```
1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*cosh(x)*sinh(x)
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.87

$$\int (x+\cosh(x))^2 \, dx = \frac{1}{6}\left(3\cosh(x)(-4+\sinh(x))+x\left(3+2x^2+12\sinh(x)\right)\right)$$

input
```
Integrate[(x + Cosh[x])^2,x]
```

output
```
(3*Cosh[x]*(-4 + Sinh[x]) + x*(3 + 2*x^2 + 12*Sinh[x]))/6
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\cosh(x))^2\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^2+\cosh^2(x)+2x\cosh(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^3}{3}+\frac{x}{2}+2x\sinh(x)-2\cosh(x)+\frac{1}{2}\sinh(x)\cosh(x)$$

input
```
Int[(x + Cosh[x])^2,x]
```

output
```
x/2 + x^3/3 - 2*Cosh[x] + 2*x*Sinh[x] + (Cosh[x]*Sinh[x])/2
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.83

method	result
default	$\frac{x}{2}+\frac{x^{3}}{3}-2\cosh\left(x\right)+2x\sinh\left(x\right)+\frac{\sinh(x)\cosh(x)}{2}$
parts	$\frac{x}{2}+\frac{x^{3}}{3}-2\cosh\left(x\right)+2x\sinh\left(x\right)+\frac{\sinh(x)\cosh(x)}{2}$
parallelrisch	$\frac{\sinh(2x)}{4}+2x\sinh\left(x\right)+\frac{x^{3}}{3}-2\cosh\left(x\right)-2+\frac{x}{2}$
risch	$\frac{x^{3}}{3}+\frac{x}{2}+\frac{\mathrm{e}^{2x}}{8}+\left(-1+x\right)\mathrm{e}^{x}+\left(-1-x\right)\mathrm{e}^{-x}-\frac{\mathrm{e}^{-2x}}{8}$
orering	$\frac{x(9x^{4}-58x^{2}-12)(x+\cosh(x))^{2}}{27x^{4}+42x^{2}-12}+\frac{(69x^{4}+22x^{2}+76)(x+\cosh(x))(1+\sinh(x))}{18x^{4}+28x^{2}-8}-\frac{x(45x^{4}-50x^{2}+228)\left(2(1+\sinh(x))^{2}+2(\right.}{12(9x^{4}+14x^{2}-4)}$

input
```
int((x+cosh(x))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*x+1/3*x^3-2*cosh(x)+2*x*sinh(x)+1/2*sinh(x)*cosh(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.77

$$\int (x+\cosh(x))^2\,dx=\frac{1}{3}\,x^{3}+\frac{1}{2}\,\left(4\,x+\cosh\left(x\right)\right)\sinh\left(x\right)+\frac{1}{2}\,x-2\,\cosh\left(x\right)$$

input
```
integrate((x+cosh(x))^2,x, algorithm="fricas")
```

output
```
1/3*x^3 + 1/2*(4*x + cosh(x))*sinh(x) + 1/2*x - 2*cosh(x)
```

Sympy [A] (verification not implemented)

Time = 0.06 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.37

$$\int (x+\cosh(x))^2\,dx = \frac{x^3}{3} - \frac{x\sinh^2(x)}{2} + 2x\sinh(x) + \frac{x\cosh^2(x)}{2} + \frac{\sinh(x)\cosh(x)}{2} - 2\cosh(x)$$

input
```
integrate((x+cosh(x))**2,x)
```

output
```
x**3/3 - x*sinh(x)**2/2 + 2*x*sinh(x) + x*cosh(x)**2/2 + sinh(x)*cosh(x)/2
 - 2*cosh(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int (x+\cosh(x))^2\,dx = \frac{1}{3}x^3 - (x+1)e^{(-x)} + (x-1)e^x + \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

input
```
integrate((x+cosh(x))^2,x, algorithm="maxima")
```

output
```
1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x
)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.20

$$\int (x+\cosh(x))^2\,dx = \frac{1}{3}x^3 - (x+1)e^{(-x)} + (x-1)e^x + \frac{1}{2}x + \frac{1}{8}e^{(2x)} - \frac{1}{8}e^{(-2x)}$$

input
```
integrate((x+cosh(x))^2,x, algorithm="giac")
```

output
```
1/3*x^3 - (x + 1)*e^(-x) + (x - 1)*e^x + 1/2*x + 1/8*e^(2*x) - 1/8*e^(-2*x
)
```

Mupad [B] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.80

$$\int (x+\cosh(x))^2\,dx=\frac{x}{2}-2\cosh(x)+\frac{\cosh(x)\,\sinh(x)}{2}+2\,x\sinh(x)+\frac{x^3}{3}$$

input
```
int((x + cosh(x))^2,x)
```

output
```
x/2 - 2*cosh(x) + (cosh(x)*sinh(x))/2 + 2*x*sinh(x) + x^3/3
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.07

$$\int (x+\cosh(x))^2\,dx=\frac{3e^{4x}+24e^{3x}x-24e^{3x}+8e^{2x}x^3+12e^{2x}x-24e^{x}x-24e^{x}-3}{24e^{2x}}$$

input
```
int((x+cosh(x))^2,x)
```

output
```
(3*e**(4*x) + 24*e**(3*x)*x - 24*e**(3*x) + 8*e**(2*x)*x**3 + 12*e**(2*x)*
x - 24*e**x*x - 24*e**x - 3)/(24*e**(2*x))
```

3.335 $\int (x+\cosh(x))^3 \, dx$

Optimal result

Integrand size = 6, antiderivative size = 56

$$\int (x+\cosh(x))^3 \, dx = \frac{3x^2}{4} + \frac{x^4}{4} - 6x\cosh(x) - \frac{3\cosh^2(x)}{4} + 7\sinh(x) + 3x^2\sinh(x) + \frac{3}{2}x\cosh(x)\sinh(x) + \frac{\sinh^3(x)}{3}$$

output
```
3/4*x^2+1/4*x^4-6*x*cosh(x)-3/4*cosh(x)^2+7*sinh(x)+3*x^2*sinh(x)+3/2*x*co
sh(x)*sinh(x)+1/3*sinh(x)^3
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.91

$$\int (x+\cosh(x))^3 \, dx = -6x\cosh(x) - \frac{3}{8}\cosh(2x) + \frac{1}{12}\left(9x^2 + 3x^4 + 9(9+4x^2)\sinh(x) + 9x\sinh(2x) + \sinh(3x)\right)$$

input
```
Integrate[(x + Cosh[x])^3,x]
```

output
```
-6*x*Cosh[x] - (3*Cosh[2*x])/8 + (9*x^2 + 3*x^4 + 9*(9 + 4*x^2)*Sinh[x] +
9*x*Sinh[2*x] + Sinh[3*x])/12
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {7293, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (x+\cosh(x))^3\,dx$$

$$\Big\downarrow \text{7293}$$

$$\int \left(x^3+3x^2\cosh(x)+\cosh^3(x)+3x\cosh^2(x)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$\frac{x^4}{4}+\frac{3x^2}{4}+3x^2\sinh(x)+\frac{\sinh^3(x)}{3}+7\sinh(x)-\frac{3\cosh^2(x)}{4}-6x\cosh(x)+\frac{3}{2}x\sinh(x)\cosh(x)$$

input
```
Int[(x + Cosh[x])^3,x]
```

output
```
(3*x^2)/4 + x^4/4 - 6*x*Cosh[x] - (3*Cosh[x]^2)/4 + 7*Sinh[x] + 3*x^2*Sinh
[x] + (3*x*Cosh[x]*Sinh[x])/2 + Sinh[x]^3/3
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7293
```
Int[u_, x_Symbol] :> With[{v = ExpandIntegrand[u, x]}, Int[v, x] /; SumQ[v]
]
```

Maple [A] (verified)

Time = 0.29 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.93

$$\left(\frac{2}{3}+\frac{\cosh\left(x\right)^{2}}{3}\right)\sinh\left(x\right)+\frac{3x\cosh\left(x\right)\sinh\left(x\right)}{2}+\frac{3x^{2}}{4}-\frac{3\cosh\left(x\right)^{2}}{4}+3x^{2}\sinh\left(x\right)-6x\cosh\left(x\right)+6\sinh\left(x\right)+$$

input
```
int((x+cosh(x))^3,x)
```

output
```
(2/3+1/3*cosh(x)^2)*sinh(x)+3/2*x*cosh(x)*sinh(x)+3/4*x^2-3/4*cosh(x)^2+3*
x^2*sinh(x)-6*x*cosh(x)+6*sinh(x)+1/4*x^4
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.96

$$\int (x+\cosh(x))^3\,dx = \frac{1}{4}\,x^{4}+\frac{1}{12}\,\sinh\left(x\right)^{3}+\frac{3}{4}\,x^{2}-6\,x\cosh\left(x\right)-\frac{3}{8}\,\cosh\left(x\right)^{2} + \frac{1}{4}\,\left(12\,x^{2}+6\,x\cosh\left(x\right)+\cosh\left(x\right)^{2}+27\right)\sinh\left(x\right)-\frac{3}{8}\,\sinh\left(x\right)^{2}$$

input
```
integrate((x+cosh(x))^3,x, algorithm="fricas")
```

output
```
1/4*x^4 + 1/12*sinh(x)^3 + 3/4*x^2 - 6*x*cosh(x) - 3/8*cosh(x)^2 + 1/4*(12
*x^2 + 6*x*cosh(x) + cosh(x)^2 + 27)*sinh(x) - 3/8*sinh(x)^2
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.52

$$\int (x+\cosh(x))^3\,dx = \frac{x^{4}}{4}-\frac{3x^{2}\sinh^{2}\left(x\right)}{4}+3x^{2}\sinh\left(x\right)+\frac{3x^{2}\cosh^{2}\left(x\right)}{4} + \frac{3x\sinh\left(x\right)\cosh\left(x\right)}{2}-6x\cosh\left(x\right)-\frac{2\sinh^{3}\left(x\right)}{3} - \frac{3\sinh^{2}\left(x\right)}{4}+\sinh\left(x\right)\cosh^{2}\left(x\right)+6\sinh\left(x\right)$$

input
```
integrate((x+cosh(x))**3,x)
```

output
```
x**4/4 - 3*x**2*sinh(x)**2/4 + 3*x**2*sinh(x) + 3*x**2*cosh(x)**2/4 + 3*x*
sinh(x)*cosh(x)/2 - 6*x*cosh(x) - 2*sinh(x)**3/3 - 3*sinh(x)**2/4 + sinh(x
)*cosh(x)**2 + 6*sinh(x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.45

$$\int (x+\cosh(x))^3\,dx = \frac{1}{4}\,x^4 + \frac{3}{4}\,x^2 + \frac{3}{16}\,(2\,x-1)e^{(2\,x)} - \frac{3}{2}\,\left(x^2+2\,x+2\right)e^{(-x)} - \frac{3}{16}\,(2\,x+1)e^{(-2\,x)} + \frac{3}{2}\,\left(x^2-2\,x+2\right)e^{x} + \frac{1}{24}\,e^{(3\,x)} - \frac{3}{8}\,e^{(-x)} - \frac{1}{24}\,e^{(-3\,x)} + \frac{3}{8}\,e^{x}$$

input
```
integrate((x+cosh(x))^3,x, algorithm="maxima")
```

output
```
1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/2*(x^2 + 2*x + 2)*e^(-x) -
3/16*(2*x + 1)*e^(-2*x) + 3/2*(x^2 - 2*x + 2)*e^x + 1/24*e^(3*x) - 3/8*e^(
-x) - 1/24*e^(-3*x) + 3/8*e^x
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.34

$$\int (x+\cosh(x))^3\,dx = \frac{1}{4}\,x^4 + \frac{3}{4}\,x^2 + \frac{3}{16}\,(2\,x-1)e^{(2\,x)} - \frac{3}{8}\,\left(4\,x^2+8\,x+9\right)e^{(-x)} - \frac{3}{16}\,(2\,x+1)e^{(-2\,x)} + \frac{3}{8}\,\left(4\,x^2-8\,x+9\right)e^{x} + \frac{1}{24}\,e^{(3\,x)} - \frac{1}{24}\,e^{(-3\,x)}$$

input
```
integrate((x+cosh(x))^3,x, algorithm="giac")
```

output
```
1/4*x^4 + 3/4*x^2 + 3/16*(2*x - 1)*e^(2*x) - 3/8*(4*x^2 + 8*x + 9)*e^(-x)
- 3/16*(2*x + 1)*e^(-2*x) + 3/8*(4*x^2 - 8*x + 9)*e^x + 1/24*e^(3*x) - 1/2
4*e^(-3*x)
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86

$$\int (x+\cosh(x))^3\,dx = \frac{20\sinh(x)}{3} + 3\,x^2\sinh(x) - \frac{3\cosh(x)^2}{4} + \frac{\cosh(x)^2\sinh(x)}{3} - 6\,x\cosh(x) + \frac{3\,x^2}{4} + \frac{x^4}{4} + \frac{3\,x\cosh(x)\,\sinh(x)}{2}$$

input
```
int((x + cosh(x))^3,x)
```

output
```
(20*sinh(x))/3 + 3*x^2*sinh(x) - (3*cosh(x)^2)/4 + (cosh(x)^2*sinh(x))/3 -
 6*x*cosh(x) + (3*x^2)/4 + x^4/4 + (3*x*cosh(x)*sinh(x))/2
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.04

$$\int (x+\cosh(x))^3\,dx = \frac{2e^{6x} + 18e^{5x}x - 9e^{5x} + 72e^{4x}x^2 - 144e^{4x}x + 162e^{4x} + 12e^{3x}x^4 + 36e^{3x}x^2 - 72e^{2x}x^2 - 144e^{2x}x - 162e^{2x}}{48e^{3x}}$$

input
```
int((x+cosh(x))^3,x)
```

output
```
(2*e**(6*x) + 18*e**(5*x)*x - 9*e**(5*x) + 72*e**(4*x)*x**2 - 144*e**(4*x)
*x + 162*e**(4*x) + 12*e**(3*x)*x**4 + 36*e**(3*x)*x**2 - 72*e**(2*x)*x**2
 - 144*e**(2*x)*x - 162*e**(2*x) - 18*e**x*x - 9*e**x - 2)/(48*e**(3*x))
```

3.336 $\int \frac{\cosh(a+bx)}{c+dx^2} dx$

Optimal result

Integrand size = 16, antiderivative size = 213

$$\int \frac{\cosh(a+bx)}{c+dx^2} dx = \frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}} - \frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}+bx\right)}{2\sqrt{-c}\sqrt{d}}$$

output

```
1/2*cosh(a+b*(-c)^(1/2)/d^(1/2))*Chi(b*(-c)^(1/2)/d^(1/2)-b*x)/(-c)^(1/2)/
d^(1/2)-1/2*cosh(a-b*(-c)^(1/2)/d^(1/2))*Chi(b*(-c)^(1/2)/d^(1/2)+b*x)/(-c
)^(1/2)/d^(1/2)+1/2*sinh(a+b*(-c)^(1/2)/d^(1/2))*Shi(-b*(-c)^(1/2)/d^(1/2)
+b*x)/(-c)^(1/2)/d^(1/2)-1/2*sinh(a-b*(-c)^(1/2)/d^(1/2))*Shi(b*(-c)^(1/2)
/d^(1/2)+b*x)/(-c)^(1/2)/d^(1/2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.35 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.78

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = -\frac{ie^{-a-\frac{ib\sqrt{c}}{\sqrt{d}}}\left(e^{2a+\frac{2ib\sqrt{c}}{\sqrt{d}}}\,\text{ExpIntegralEi}\left(b\left(-\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)-e^{2a}\,\text{ExpIntegralEi}\left(b\left(\frac{i\sqrt{c}}{\sqrt{d}}+x\right)\right)-e^{\frac{2ib\sqrt{c}}{\sqrt{d}}}\,\text{ExpIn}\right.}{4\sqrt{c}\sqrt{d}}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x^2),x]
```

output
```
((-1/4*I)*E^(-a - (I*b*Sqrt[c])/Sqrt[d])*(E^(2*a + ((2*I)*b*Sqrt[c])/Sqrt[
d])*ExpIntegralEi[b*(((-I)*Sqrt[c])/Sqrt[d] + x)] - E^(2*a)*ExpIntegralEi[
b*((I*Sqrt[c])/Sqrt[d] + x)] - E^(((2*I)*b*Sqrt[c])/Sqrt[d])*ExpIntegralEi
[((-I)*b*Sqrt[c])/Sqrt[d] - b*x] + ExpIntegralEi[(I*b*Sqrt[c])/Sqrt[d] - b
*x]))/(Sqrt[c]*Sqrt[d])
```

Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {5804, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx$$

$$\Big\downarrow \; 5804$$

$$\int \left(\frac{\sqrt{-c}\cosh(a+bx)}{2c\left(\sqrt{-c}-\sqrt{d}x\right)}+\frac{\sqrt{-c}\cosh(a+bx)}{2c\left(\sqrt{-c}+\sqrt{d}x\right)}\right)dx$$

$$\Big\downarrow \; 2009$$

$$\frac{\cosh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\cosh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Chi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a+\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(\frac{b\sqrt{-c}}{\sqrt{d}}-bx\right)}{2\sqrt{-c}\sqrt{d}}-\frac{\sinh\left(a-\frac{b\sqrt{-c}}{\sqrt{d}}\right)\text{Shi}\left(xb+\frac{\sqrt{-c}b}{\sqrt{d}}\right)}{2\sqrt{-c}\sqrt{d}}$$

input
```
Int[Cosh[a + b*x]/(c + d*x^2),x]
```

output
```
(Cosh[a + (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/
(2*Sqrt[-c]*Sqrt[d]) - (Cosh[a - (b*Sqrt[-c])/Sqrt[d]]*CoshIntegral[(b*Sqr
t[-c])/Sqrt[d] + b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh[a + (b*Sqrt[-c])/Sqrt[
d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] - b*x])/(2*Sqrt[-c]*Sqrt[d]) - (Sinh
[a - (b*Sqrt[-c])/Sqrt[d]]*SinhIntegral[(b*Sqrt[-c])/Sqrt[d] + b*x])/(2*Sq
rt[-c]*Sqrt[d])
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 5804
```
Int[Cosh[(c_.) + (d_.)*(x_)]*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> In
t[ExpandIntegrand[Cosh[c + d*x], (a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d
}, x] && ILtQ[p, 0] && IGtQ[n, 0] && (EqQ[n, 2] || EqQ[p, -1])
```

Maple [A] (verified)

Time = 0.52 (sec) , antiderivative size = 212, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{e^{\frac{b\sqrt{-cd}+ad}{d}}\,\text{expIntegral}_1\left(\frac{b\sqrt{-cd}-(bx+a)d+ad}{d}\right)}{4\sqrt{-cd}}+\frac{e^{\frac{-b\sqrt{-cd}+ad}{d}}\,\text{expIntegral}_1\left(-\frac{b\sqrt{-cd}+(bx+a)d-ad}{d}\right)}{4\sqrt{-cd}}-\frac{e^{-\frac{b\sqrt{-cd}+ad}{d}}\,\text{expInt}}{}$

input
```
int(cosh(b*x+a)/(d*x^2+c),x,method=_RETURNVERBOSE)
```

output
```
-1/4/(-c*d)^(1/2)*exp((b*(-c*d)^(1/2)+a*d)/d)*Ei(1,(b*(-c*d)^(1/2)-(b*x+a)
*d+a*d)/d)+1/4/(-c*d)^(1/2)*exp((-b*(-c*d)^(1/2)+a*d)/d)*Ei(1,-(b*(-c*d)^(
1/2)+(b*x+a)*d-a*d)/d)-1/4/(-c*d)^(1/2)*exp(-(b*(-c*d)^(1/2)+a*d)/d)*Ei(1,
-(b*(-c*d)^(1/2)-(b*x+a)*d+a*d)/d)+1/4/(-c*d)^(1/2)*exp(-(-b*(-c*d)^(1/2)+
a*d)/d)*Ei(1,(b*(-c*d)^(1/2)+(b*x+a)*d-a*d)/d)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(157) = 314$.

Time = 0.09 (sec) , antiderivative size = 316, normalized size of antiderivative = 1.48

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = -\frac{\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx-\sqrt{-\frac{b^2c}{d}}\right)+\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(-bx+\sqrt{-\frac{b^2c}{d}}\right)\right)\cosh\left(a+\sqrt{-\frac{b^2c}{d}}\right)-\left(\sqrt{-\frac{b^2c}{d}}\operatorname{Ei}\left(bx+\sqrt{\right.\right.}{}$$

input
```
integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="fricas")
```

output
```
-1/4*((sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x +
sqrt(-b^2*c/d)))*cosh(a + sqrt(-b^2*c/d)) - (sqrt(-b^2*c/d)*Ei(b*x + sqrt(
-b^2*c/d)) + sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*cosh(-a + sqrt(-b^2
*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x - sqrt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x
 + sqrt(-b^2*c/d)))*sinh(a + sqrt(-b^2*c/d)) + (sqrt(-b^2*c/d)*Ei(b*x + sq
rt(-b^2*c/d)) - sqrt(-b^2*c/d)*Ei(-b*x - sqrt(-b^2*c/d)))*sinh(-a + sqrt(-
b^2*c/d)))/(b*c)
```

Sympy [F]

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int \frac{\cosh(a+bx)}{c+dx^2}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x**2+c),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x**2), x)
```

Maxima [F]

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int \frac{\cosh\left(bx+a\right)}{dx^2+c}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="maxima")
```

output
```
integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

Giac [F]

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int \frac{\cosh\left(bx+a\right)}{dx^2+c}\,dx$$

input
```
integrate(cosh(b*x+a)/(d*x^2+c),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)/(d*x^2 + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int \frac{\cosh(a+b\,x)}{d\,x^2+c}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x^2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x^2), x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{c+dx^2}\,dx = \int \frac{\cosh\left(bx+a\right)}{d\,x^2+c}dx$$

input
```
int(cosh(b*x+a)/(d*x^2+c),x)
```

output
```
int(cosh(a + b*x)/(c + d*x**2),x)
```

3.337 $\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$

Optimal result

Integrand size = 19, antiderivative size = 271

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx = \frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\cosh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} + \frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}} - \frac{\sinh\left(a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}\right)\mathrm{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

output
```
cosh(a-1/2*b*(d-(-4*c*e+d^2)^(1/2))/e)*Chi(1/2*b*(d-(-4*c*e+d^2)^(1/2))/e+
b*x)/(-4*c*e+d^2)^(1/2)-cosh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Chi(1/2*b*(
d+(-4*c*e+d^2)^(1/2))/e+b*x)/(-4*c*e+d^2)^(1/2)+sinh(a-1/2*b*(d-(-4*c*e+d^
2)^(1/2))/e)*Shi(1/2*b*(d-(-4*c*e+d^2)^(1/2))/e+b*x)/(-4*c*e+d^2)^(1/2)-si
nh(a-1/2*b*(d+(-4*c*e+d^2)^(1/2))/e)*Shi(1/2*b*(d+(-4*c*e+d^2)^(1/2))/e+b*
x)/(-4*c*e+d^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.81

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$$
$$= \frac{e^{-a-\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}}\left(e^{\frac{bd}{e}}\,\mathrm{ExpIntegralEi}\left(-\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right)+e^{2a+\frac{b\sqrt{d^2-4ce}}{e}}\,\mathrm{ExpIntegralEi}\left(\frac{b\left(d-\sqrt{d^2-4ce}+2ex\right)}{2e}\right.\right.}{2\sqrt{d^2-}}$$

input
```
Integrate[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(E^(-a - (b*(d + Sqrt[d^2 - 4*c*e]))/(2*e))*(E^((b*d)/e)*ExpIntegralEi[-1/
2*(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/e] + E^(2*a + (b*Sqrt[d^2 - 4*c*e])/
e)*ExpIntegralEi[(b*(d - Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)] - E^((b*(d + S
qrt[d^2 - 4*c*e]))/e)*ExpIntegralEi[-1/2*(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x
))/e] - E^(2*a)*ExpIntegralEi[(b*(d + Sqrt[d^2 - 4*c*e] + 2*e*x))/(2*e)]))
/(2*Sqrt[d^2 - 4*c*e])
```

Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 271, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx$$
$$\Big\downarrow \text{7279}$$
$$\int \left(\frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(-\sqrt{d^2-4ce}+d+2ex\right)}-\frac{2e\cosh(a+bx)}{\sqrt{d^2-4ce}\left(\sqrt{d^2-4ce}+d+2ex\right)}\right)dx$$
$$\Big\downarrow \text{2009}$$

$$\frac{\cosh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\cosh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Chi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}+\frac{\sinh\left(a-\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d-\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}-\frac{\sinh\left(a-\frac{b\left(\sqrt{d^2-4ce}+d\right)}{2e}\right)\text{Shi}\left(\frac{b\left(d+\sqrt{d^2-4ce}\right)}{2e}+bx\right)}{\sqrt{d^2-4ce}}$$

input
```
Int[Cosh[a + b*x]/(c + d*x + e*x^2),x]
```

output
```
(Cosh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*CoshIntegral[(b*(d - Sqrt[d^2
 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Cosh[a - (b*(d + Sqrt[d^2 -
 4*c*e]))/(2*e)]*CoshIntegral[(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sq
rt[d^2 - 4*c*e] + (Sinh[a - (b*(d - Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegra
l[(b*(d - Sqrt[d^2 - 4*c*e]))/(2*e) + b*x])/Sqrt[d^2 - 4*c*e] - (Sinh[a -
(b*(d + Sqrt[d^2 - 4*c*e]))/(2*e)]*SinhIntegral[(b*(d + Sqrt[d^2 - 4*c*e])
)/(2*e) + b*x])/Sqrt[d^2 - 4*c*e]
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.39

method	result
risch	$-\frac{b\,e^{\frac{2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expIntegral}_1\left(\frac{-2e(bx+a)+2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}\right)}{2\sqrt{-4b^2ce+b^2d^2}}-\frac{b\,e^{-\frac{2ea-bd+\sqrt{-4b^2ce+b^2d^2}}{2e}}\,\mathrm{expIntegral}_1\left(-\right.}{2\sqrt{-4b^2ce+b}}$

input
```
int(cosh(b*x+a)/(e*x^2+d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2
)^(1/2)))*Ei(1,1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)-
1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d+(-4*b^2*c*e+b^2*d^2
)^(1/2)))*Ei(1,-1/2*(-2*e*(b*x+a)+2*e*a-b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
+1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(-1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^
2)^(1/2)))*Ei(1,1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)+
1/2*b/(-4*b^2*c*e+b^2*d^2)^(1/2)*exp(1/2/e*(2*e*a-b*d-(-4*b^2*c*e+b^2*d^2)
^(1/2)))*Ei(1,-1/2*(2*e*(b*x+a)-2*e*a+b*d+(-4*b^2*c*e+b^2*d^2)^(1/2))/e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. 2(231) = 462.

Time = 0.11 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.48

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx =$$
$$-\frac{\left(e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}\,\mathrm{Ei}\left(-\frac{2\,bex+bd+e\sqrt{\frac{b^2d^2-4\,b^2ce}{e^2}}}{2\,e}\right)\right)\cosh\left(\frac{bd-2\,ae+e\sqrt{}}{2}\right.}{}$$

input
```
integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="fricas")
```

output
```
-1/2*((e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sqrt((b
^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(
2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*cosh(1/2*(b*d - 2*a
*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)
/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) + e*sq
rt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4
*b^2*c*e)/e^2))/e))*cosh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*c*e)/
e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d + e*sq
rt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-
1/2*(2*b*e*x + b*d + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e))*sinh(1/2*(b*d
- 2*a*e + e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) - (e*sqrt((b^2*d^2 - 4*b^2
*c*e)/e^2)*Ei(1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2))/e) -
 e*sqrt((b^2*d^2 - 4*b^2*c*e)/e^2)*Ei(-1/2*(2*b*e*x + b*d - e*sqrt((b^2*d^
2 - 4*b^2*c*e)/e^2))/e))*sinh(-1/2*(b*d - 2*a*e - e*sqrt((b^2*d^2 - 4*b^2*
c*e)/e^2))/e))/(b*d^2 - 4*b*c*e)
```

Sympy [F]

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\cosh{(a+bx)}}{c+dx+ex^2}\,dx$$

input
```
integrate(cosh(b*x+a)/(e*x**2+d*x+c),x)
```

output
```
Integral(cosh(a + b*x)/(c + d*x + e*x**2), x)
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx=\text{Exception raised: ValueError}$$

input
```
integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="maxima")
```

output
```
Exception raised: ValueError >> Computation failed since Maxima requested
additional constraints; using the 'assume' command before evaluation *may*
 help (example of legal syntax is 'assume(4*c*e-d^2>0)', see `assume?` for
 more deta
```

Giac [F]

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\cosh{(bx+a)}}{ex^2+dx+c}\,dx$$

input
```
integrate(cosh(b*x+a)/(e*x^2+d*x+c),x, algorithm="giac")
```

output
```
integrate(cosh(b*x + a)/(e*x^2 + d*x + c), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\cosh(a+b\,x)}{e\,x^2+d\,x+c}\,dx$$

input
```
int(cosh(a + b*x)/(c + d*x + e*x^2),x)
```

output
```
int(cosh(a + b*x)/(c + d*x + e*x^2), x)
```

Reduce [F]

$$\int \frac{\cosh(a+bx)}{c+dx+ex^2}\,dx=\int \frac{\cosh{(bx+a)}}{e\,x^2+dx+c}dx$$

input
```
int(cosh(b*x+a)/(e*x^2+d*x+c),x)
```

output
```
int(cosh(a + b*x)/(c + d*x + e*x**2),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*           GradeAntiderivative[result,optimal] returns*)
(*   "F" if the result fails to integrate an expression that*)
(*       is integrable*)
(*   "C" if result involves higher level functions than necessary*)
(*   "B" if result is more than twice the size of the optimal*)
(*       antiderivative*)
(*   "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                  convert(leaf_count_result,string)," vs. $2 (",
                                  convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                  print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                   convert(leaf_count_result,string),"$ vs. $2(",
                                   convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
else:
    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
            else: #result contains complex but optimal is not
                grade = "C"
                grade_annotation ="Result contains complex when optimal does not."
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =""
            else:
                grade = "B"
                grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
    else:
        grade = "C"
        grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType(


#print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#               Albert Rich to use with Sagemath. This is used to
#               grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#               'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#               issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
   r"""
   Return the tree size of this expression.
   """
   #print("Enter tree_size, expr is ",expr)

   if expr not in SR:
      # deal with lists, tuples, vectors
      return 1 + sum(tree_size(a) for a in expr)
   expr = SR(expr)
   x, aa = expr.operator(), expr.operands()
   if x is None:
      return 1
   else:
      return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
   if expr.operator() == operator.pow:   #isinstance(expr,Pow):
      if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
         if debug: print ("expr is sqrt")
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   #debug=False
   m = func.name() in ['exp','log','ln',
         'sin','cos','tan','cot','sec','csc',
         'arcsin','arccos','arctan','arccot','arcsec','arccsc',
         'sinh','cosh','tanh','coth','sech','csch',
         'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
      'arctan2','floor','abs'
      ]
   if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
            'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
            'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
            'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
            'elliptic_pi','exp_integral_e','log_integral',
            'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
            'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file