Computer Algebra
Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/306-6.3.1

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May 18, 2024 Compiled on May 18, 2024 at 8:24am

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This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**77**]. This is test number [306].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (77)	0.00 (0)
Mathematica	93.51 (72)	6.49 (5)
Maple	89.61 (69)	10.39 (8)
Fricas	83.12 (64)	16.88 (13)
Maxima	81.82 (63)	18.18 (14)
Giac	57.14 (44)	42.86 (33)
Mupad	50.65 (39)	49.35 (38)
Reduce	50.65 (39)	49.35 (38)
Sympy	38.96 (30)	61.04 (47)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

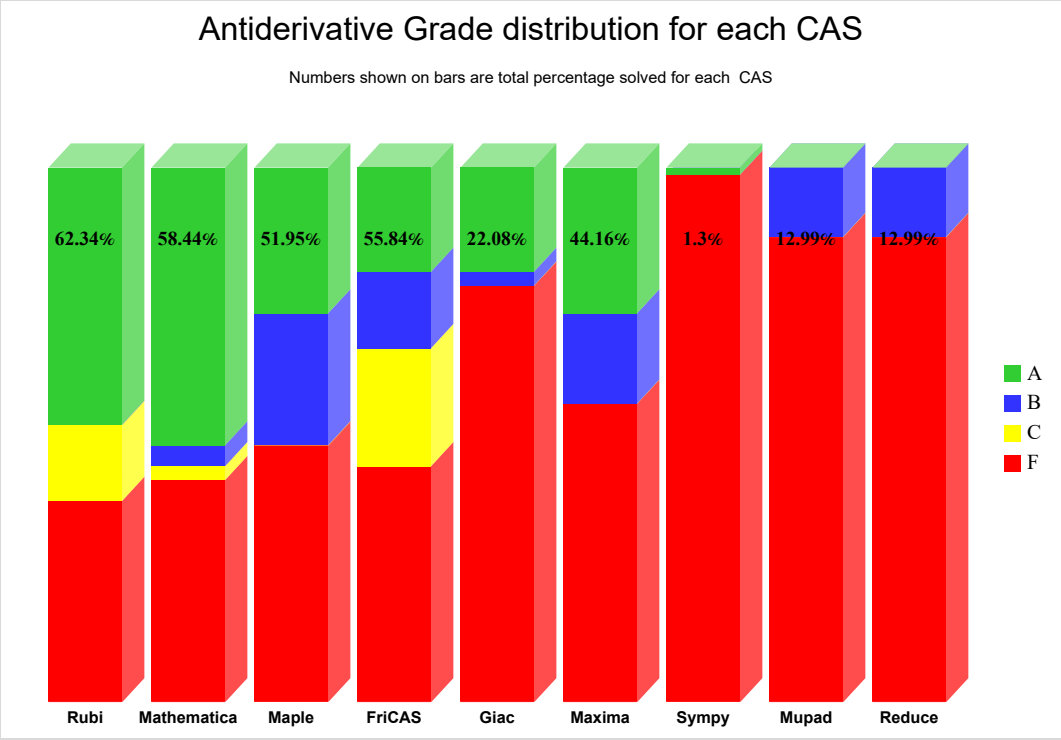
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

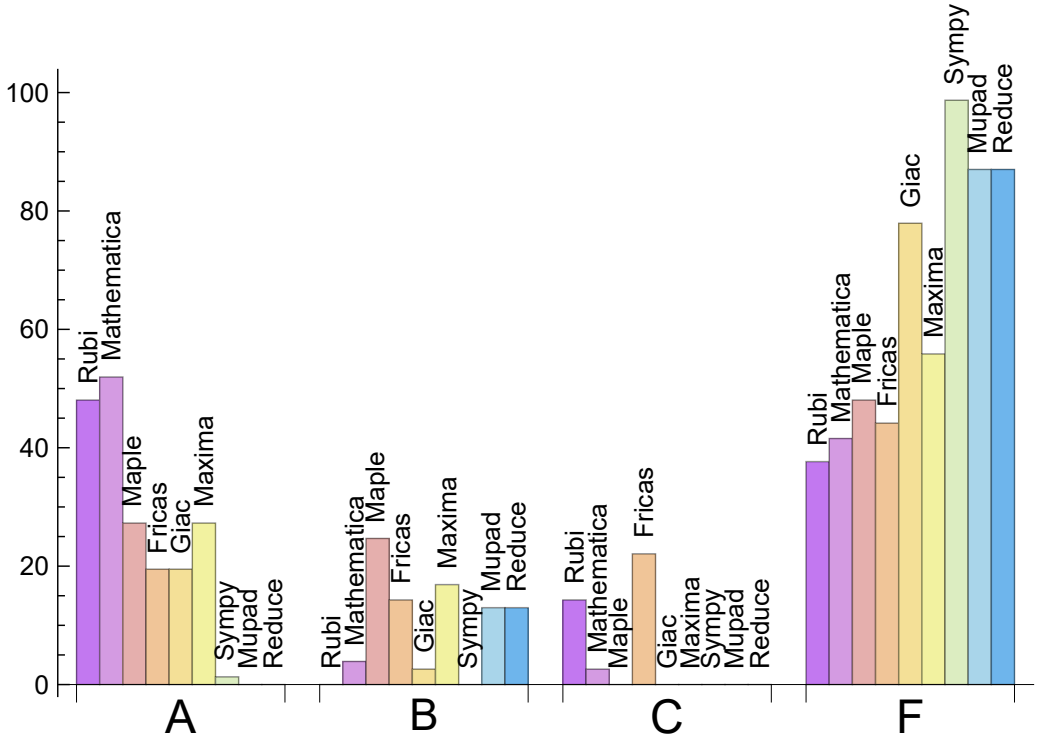
System	% A grade	% B grade	% C grade	% F grade
Mathematica	51.948	3.896	2.597	41.558
Rubi	48.052	0.000	14.286	37.662
Maple	27.273	24.675	0.000	48.052
Maxima	27.273	16.883	0.000	55.844
Fricas	19.481	14.286	22.078	44.156
Giac	19.481	2.597	0.000	77.922
Sympy	1.299	0.000	0.000	98.701
Mupad	0.000	12.987	0.000	87.013
Reduce	0.000	12.987	0.000	87.013

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00	0.00	0.00
Mathematica	5	60.00	40.00	0.00
Fricas	13	0.00	0.00	100.00
Maple	8	100.00	0.00	0.00
Maxima	14	100.00	0.00	0.00
Giac	33	87.88	0.00	12.12
Mupad	38	0.00	100.00	0.00
Reduce	38	100.00	0.00	0.00
Sympy	47	100.00	0.00	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Fricas	0.11
Giac	0.20
Reduce	0.28
Maxima	0.54
Maple	0.54
Rubi	0.76
Sympy	1.29
Mupad	2.31
Mathematica	7.50

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Sympy	19.30	1.00	17.00	0.95
Mupad	61.95	1.09	20.00	1.10
Giac	110.39	1.11	22.00	1.10
Mathematica	212.62	1.25	132.50	1.12
Rubi	213.08	1.02	117.00	1.00
Maxima	265.59	5.70	155.00	2.15
Maple	270.01	1.57	102.00	1.00
Reduce	737.77	36.38	155.00	4.45
Fricas	1029.47	4.65	240.00	2.26

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

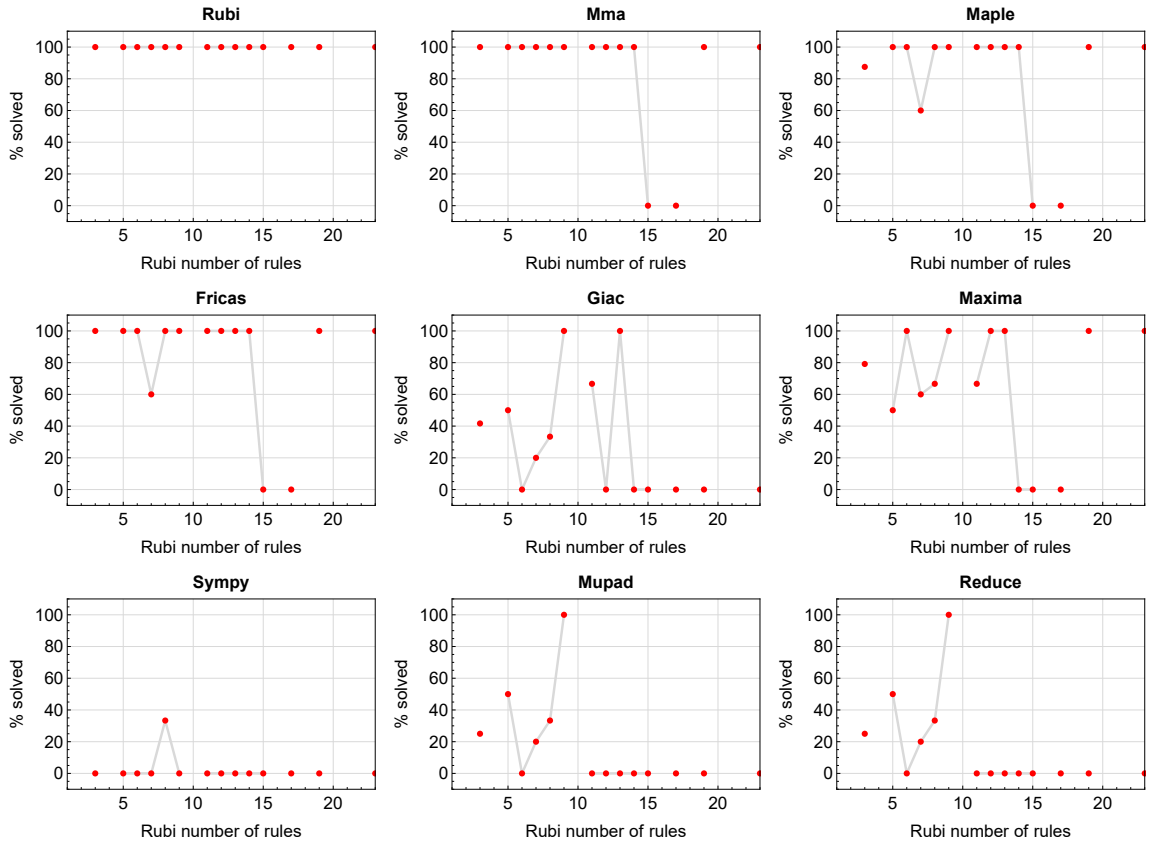


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

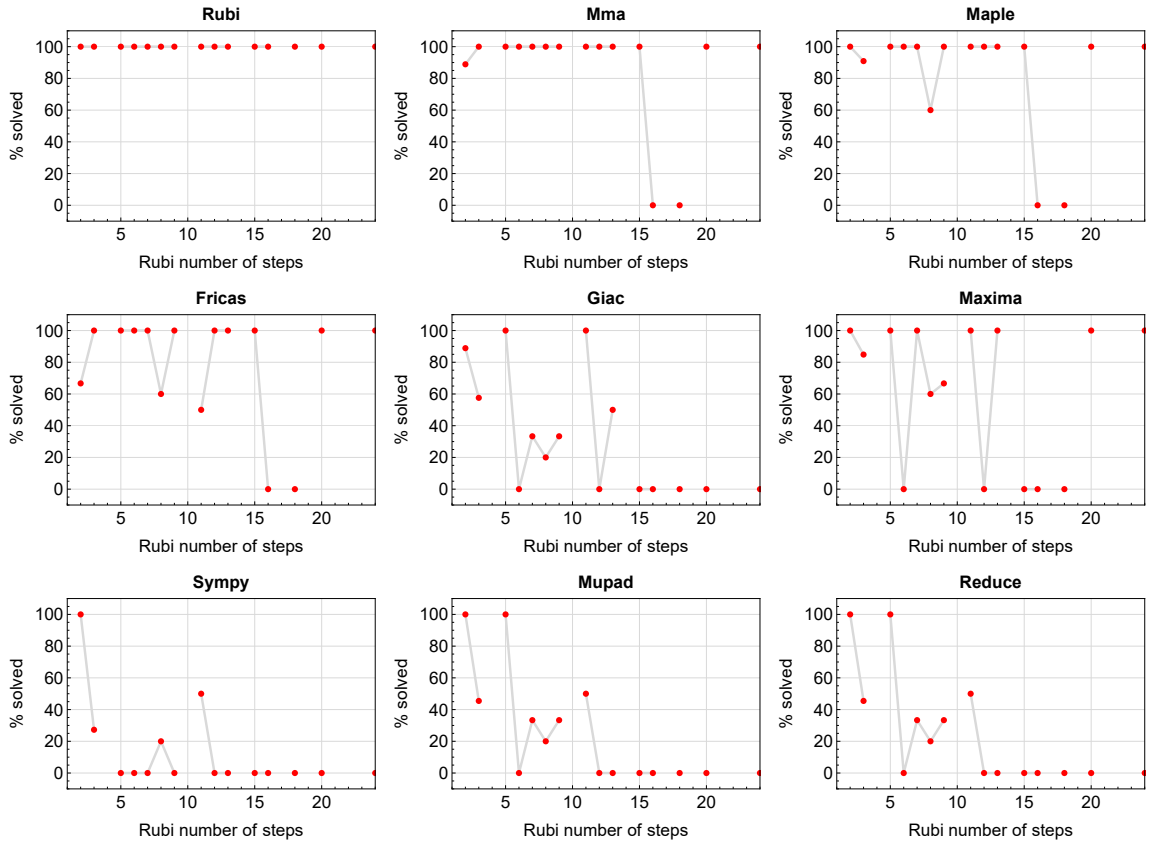


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

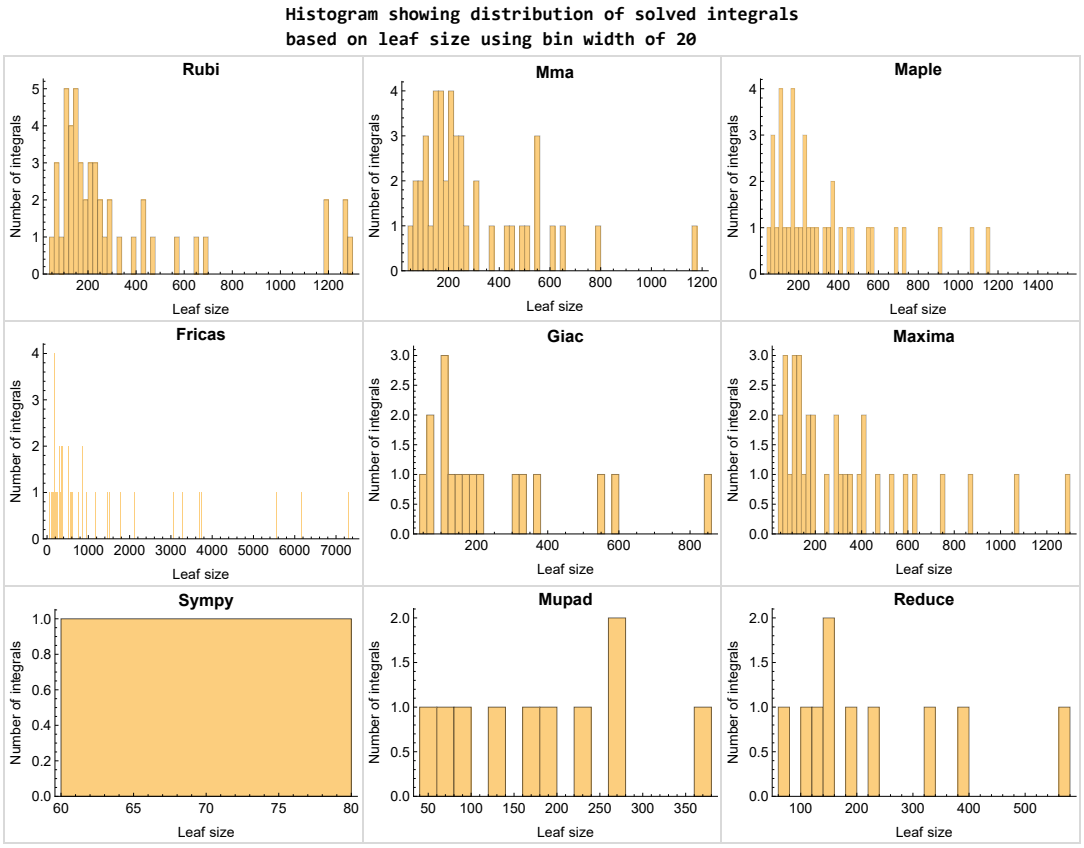


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

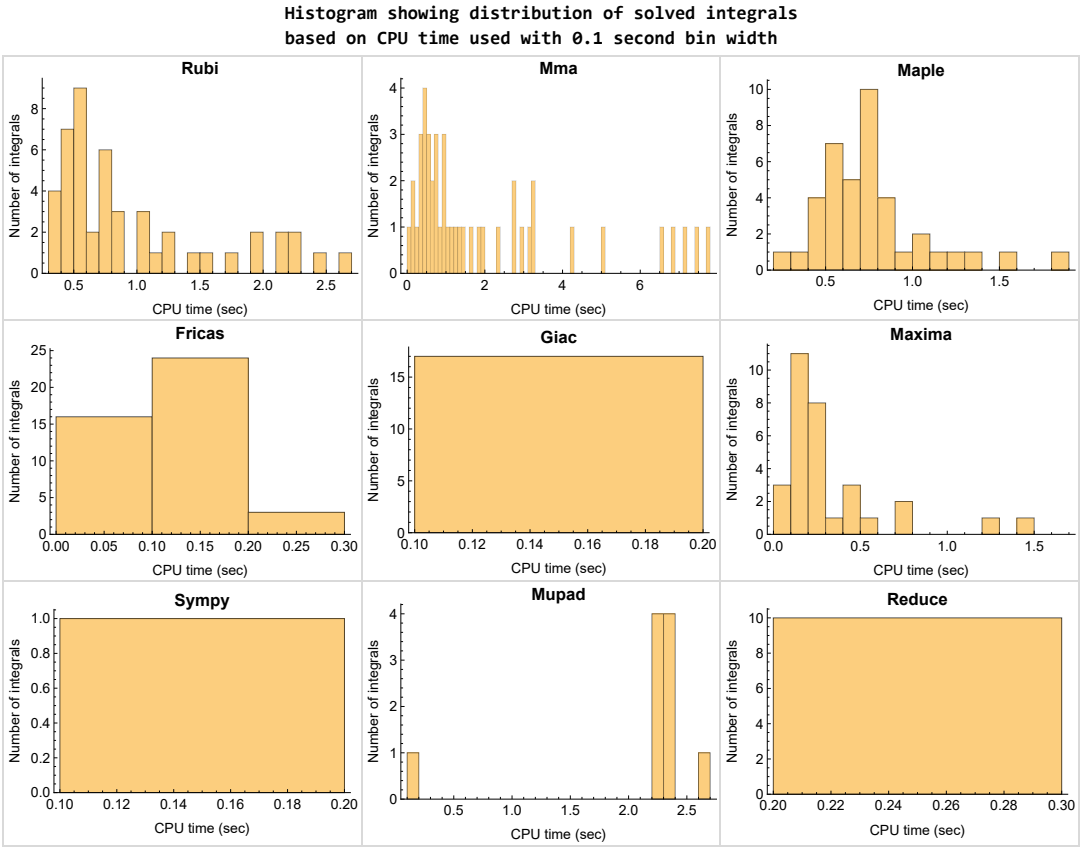


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

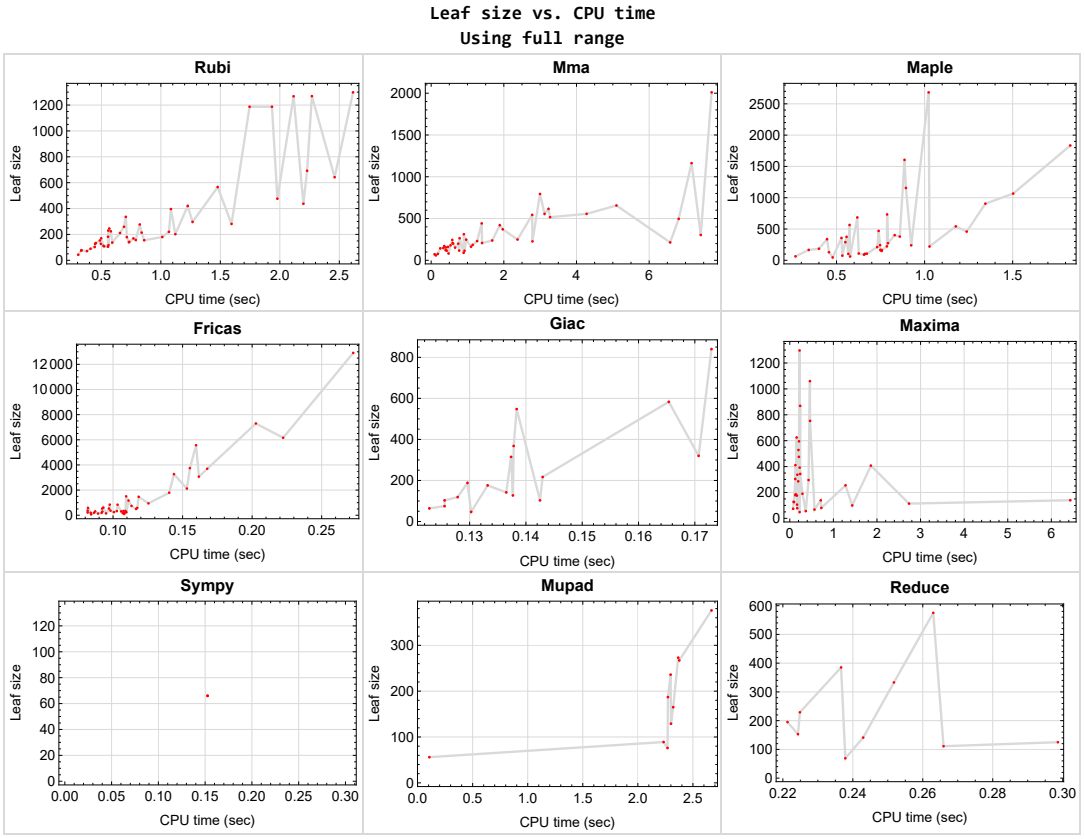


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{4, 5, 9, 10, 14, 15, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 48, 49, 56, 57, 61, 62, 66, 67, 71, 72, 76, 77}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Reduce {}

Mupad {}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {16, 17, 20}

Mathematica {73, 74}

Maple {}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

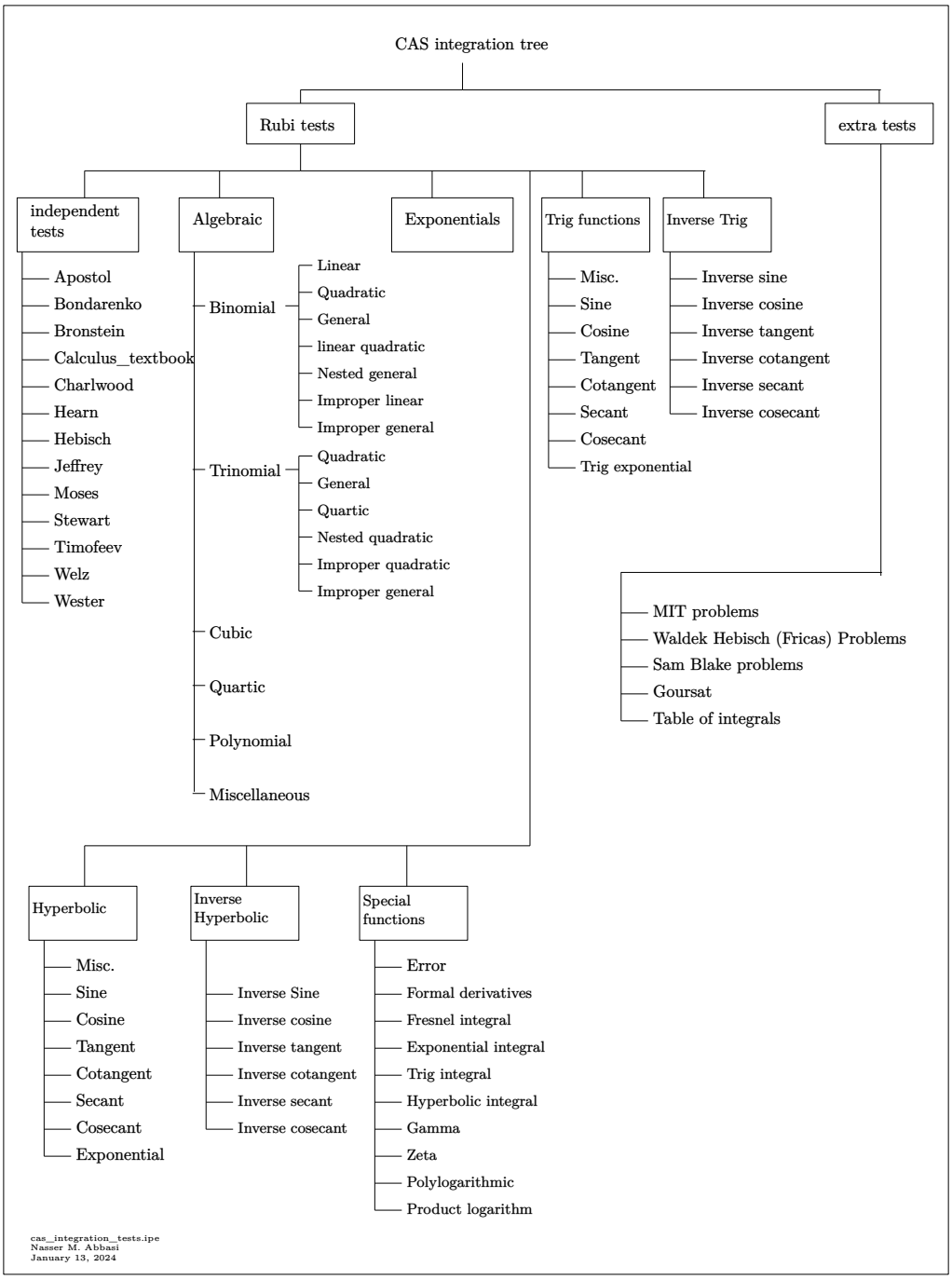
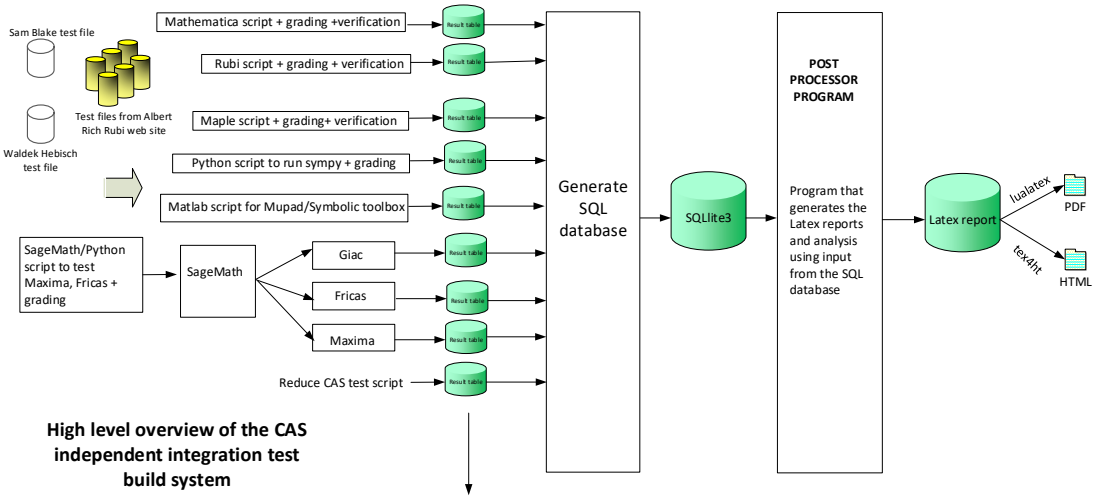


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



High level overview of the CAS independent integration test build system

One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be “A”, “B”, “C”, or “F”
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified. (For mma, rubi and maple only)

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form “{n,n,..}” which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
January 13, 2024
Design.vsdx

CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

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2.1 List of integrals sorted by grade for each CAS

Rubi 26
Mma 26
Maple 27
Fricas 27
Maxima 27
Giac 28
Mupad 28
Sympy 28
Reduce 29

Rubi

A grade { 8, 16, 17, 18, 19, 20, 32, 33, 34, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

B grade { }

C grade { 1, 2, 3, 6, 7, 11, 12, 13, 35, 36, 37 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 6, 7, 8, 12, 13, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 65, 68, 69, 70, 73, 74, 75 }

B grade { 11, 63, 64 }

C grade { 18, 19 }

F normal fail { 16, 17, 20 }

F(-1) timedout fail { 22, 23 }

F(-2) exception fail { }

Maple

A grade { 8, 13, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 55, 60, 65 }

B grade { 1, 2, 3, 6, 7, 11, 12, 53, 54, 58, 59, 63, 64, 68, 69, 70, 73, 74, 75 }

C grade { }

F normal fail { 16, 17, 18, 19, 20, 50, 51, 52 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 32, 33, 34, 35, 36, 37, 40, 41, 42, 45, 46, 47, 50, 51, 52 }

B grade { 8, 38, 39, 43, 44, 68, 69, 70, 73, 74, 75 }

C grade { 1, 2, 3, 6, 7, 11, 12, 13, 53, 54, 55, 58, 59, 60, 63, 64, 65 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28 }

Maxima

A grade { 3, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 54, 65, 73, 74 }

B grade { 1, 2, 6, 8, 11, 12, 53, 58, 59, 63, 64, 68, 69 }

C grade { }

F normal fail { 7, 13, 16, 17, 18, 19, 20, 50, 51, 52, 55, 60, 70, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

Giac

A grade { 32, 33, 34, 35, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47 }

B grade { 8, 36 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 20, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { 18, 19, 22, 23 }

Mupad

A grade { }

B grade { 8, 32, 33, 34, 38, 39, 40, 43, 44, 45 }

C grade { }

F normal fail { }

F(-1) timedout fail { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 35, 36, 37, 41, 42, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

F(-2) exception fail { }

Sympy

A grade { 8 }

B grade { }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 8, 32, 33, 34, 38, 39, 40, 43, 44, 45 }

C grade { }

F normal fail { 1, 2, 3, 6, 7, 11, 12, 13, 16, 17, 18, 19, 20, 35, 36, 37, 41, 42, 46, 47, 50, 51, 52, 53, 54, 55, 58, 59, 60, 63, 64, 65, 68, 69, 70, 73, 74, 75 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	140	154	402	286	531	0	0	171	0
N.S.	1	1.20	1.32	3.44	2.44	4.54	0.00	0.00	1.46	0.00
time (sec)	N/A	0.732	0.405	0.830	0.193	0.097	0.000	0.000	0.245	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	105	143	242	176	329	0	0	119	0
N.S.	1	1.25	1.70	2.88	2.10	3.92	0.00	0.00	1.42	0.00
time (sec)	N/A	0.555	0.250	0.747	0.164	0.103	0.000	0.000	0.221	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	71	59	109	78	171	0	0	67	0
N.S.	1	1.25	1.04	1.91	1.37	3.00	0.00	0.00	1.18	0.00
time (sec)	N/A	0.379	0.128	0.626	0.171	0.107	0.000	0.000	0.240	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	42	16	12	16	48	16
N.S.	1	1.00	1.14	1.00	3.00	1.14	0.86	1.14	3.43	1.14
time (sec)	N/A	0.229	8.703	0.143	0.278	0.079	0.410	0.116	0.266	2.171

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	16	14	71	27	14	16	155	16
N.S.	1	1.00	1.14	1.00	5.07	1.93	1.00	1.14	11.07	1.14
time (sec)	N/A	0.225	20.433	0.138	0.219	0.079	0.594	0.179	0.255	2.177

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	142	179	336	343	1505	0	0	669	0
N.S.	1	1.19	1.50	2.82	2.88	12.65	0.00	0.00	5.62	0.00
time (sec)	N/A	0.734	0.533	0.446	0.239	0.109	0.000	0.000	0.240	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	108	115	185	0	840	0	0	350	0
N.S.	1	1.23	1.31	2.10	0.00	9.55	0.00	0.00	3.98	0.00
time (sec)	N/A	0.523	0.426	0.400	0.000	0.103	0.000	0.000	0.317	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	77	63	127	232	66	127	141	56
N.S.	1	1.00	1.79	1.47	2.95	5.40	1.53	2.95	3.28	1.30
time (sec)	N/A	0.306	0.184	0.267	0.089	0.110	0.153	0.138	0.243	0.107

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	110	18	14	18	87	18
N.S.	1	1.00	1.12	1.00	6.88	1.12	0.88	1.12	5.44	1.12
time (sec)	N/A	0.220	21.496	0.138	0.156	0.076	0.398	0.155	0.247	2.130

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	195	29	15	18	275	18
N.S.	1	1.00	1.12	1.00	12.19	1.81	0.94	1.12	17.19	1.12
time (sec)	N/A	0.230	23.348	0.133	0.199	0.084	0.594	0.233	0.285	2.110

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	B	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	281	496	685	595	5569	0	0	0	0
N.S.	1	1.19	2.09	2.89	2.51	23.50	0.00	0.00	0.00	0.00
time (sec)	N/A	1.594	6.812	0.618	0.212	0.160	0.000	0.000	0.296	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	180	226	375	392	3071	0	0	1076	0
N.S.	1	1.19	1.50	2.48	2.60	20.34	0.00	0.00	7.13	0.00
time (sec)	N/A	1.012	2.787	0.557	0.227	0.162	0.000	0.000	0.277	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	117	89	166	0	1462	0	0	472	0
N.S.	1	1.17	0.89	1.66	0.00	14.62	0.00	0.00	4.72	0.00
time (sec)	N/A	0.560	0.894	0.342	0.000	0.118	0.000	0.000	0.297	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	303	18	14	18	205	18
N.S.	1	1.00	1.12	1.00	18.94	1.12	0.88	1.12	12.81	1.12
time (sec)	N/A	0.221	24.873	0.146	0.258	0.093	0.412	0.198	0.283	2.132

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	18	16	501	29	15	18	705	18
N.S.	1	1.00	1.12	1.00	31.31	1.81	0.94	1.12	44.06	1.12
time (sec)	N/A	0.224	20.723	0.141	0.365	0.091	0.605	0.365	0.270	2.174

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1392	1298	0	0	0	0	0	0	48	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.03	0.00
time (sec)	N/A	2.615	0.000	0.000	0.000	0.000	0.000	0.000	0.256	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1363	1269	0	0	0	0	0	0	59	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.269	0.000	0.000	0.000	0.000	0.000	0.000	0.265	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1280	1187	556	0	0	0	0	0	28	0
N.S.	1	0.93	0.43	0.00	0.00	0.00	0.00	0.00	0.02	0.00
time (sec)	N/A	1.934	4.278	0.000	0.000	0.000	0.000	0.000	0.260	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	F(-2)	F	F(-2)	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1280	1187	556	0	0	0	0	0	48	0
N.S.	1	0.93	0.43	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	1.746	3.121	0.000	0.000	0.000	0.000	0.000	0.287	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	F	F	F	F(-2)	F	F	F	F(-1)
verified	N/A	No	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	1365	1268	0	0	0	0	0	0	48	0
N.S.	1	0.93	0.00	0.00	0.00	0.00	0.00	0.00	0.04	0.00
time (sec)	N/A	2.116	0.000	0.000	0.000	0.000	0.000	0.000	0.266	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	89	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	4.45	1.00
time (sec)	N/A	2.617	27.466	0.156	0.478	0.000	3.023	0.297	0.279	2.535

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	0	18	20	0	19	0	49	20
N.S.	1	1.00	0.00	0.90	1.00	0.00	0.95	0.00	2.45	1.00
time (sec)	N/A	0.263	0.000	0.195	0.477	0.000	1.286	0.000	0.231	2.596

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	F(-1)	N/A	N/A	F(-2)	N/A	F(-2)	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	0	18	20	0	19	0	77	20
N.S.	1	1.00	0.00	0.90	1.00	0.00	0.95	0.00	3.85	1.00
time (sec)	N/A	0.248	0.000	0.184	0.523	0.000	1.260	0.000	0.299	2.494

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	77	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	3.85	1.00
time (sec)	N/A	2.371	28.350	0.159	0.457	0.000	1.430	0.330	0.250	2.539

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	17	20	27	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.85	1.00	1.35	1.00
time (sec)	N/A	0.266	22.891	0.173	0.194	0.000	1.007	0.186	0.248	2.483

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	17	20	20	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.85	1.00	1.00	1.00
time (sec)	N/A	0.262	1.838	0.204	0.194	0.000	0.627	0.135	0.315	2.200

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	36	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	1.80	1.00
time (sec)	N/A	0.248	1.931	0.198	0.336	0.000	0.948	0.142	0.250	2.312

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	F(-2)	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	18	20	0	19	20	40	20
N.S.	1	1.00	1.10	0.90	1.00	0.00	0.95	1.00	2.00	1.00
time (sec)	N/A	0.258	20.062	0.170	0.327	0.000	1.102	0.232	0.225	2.580

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	171	14	12	14	1614	14
N.S.	1	1.00	1.17	1.00	14.25	1.17	1.00	1.17	134.50	1.17
time (sec)	N/A	0.229	0.655	0.105	0.714	0.076	0.627	0.170	0.237	2.188

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	12	12	14	12	144	14	12	14	99	14
N.S.	1	1.00	1.17	1.00	12.00	1.17	1.00	1.17	8.25	1.17
time (sec)	N/A	0.208	0.367	0.106	0.626	0.074	0.485	0.148	0.299	2.268

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	10	10	12	10	100	12	10	12	57	12
N.S.	1	1.00	1.20	1.00	10.00	1.20	1.00	1.20	5.70	1.20
time (sec)	N/A	0.204	0.320	0.109	0.384	0.094	0.396	0.129	0.227	2.070

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	179	244	165	186	304	0	188	195	273
N.S.	1	1.06	1.44	0.98	1.10	1.80	0.00	1.11	1.15	1.62
time (sec)	N/A	0.714	0.580	0.750	0.137	0.087	0.000	0.130	0.221	2.367

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	127	169	103	126	192	0	119	125	187
N.S.	1	1.04	1.39	0.84	1.03	1.57	0.00	0.98	1.02	1.53
time (sec)	N/A	0.496	0.360	0.661	0.098	0.086	0.000	0.128	0.299	2.273

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	78	81	46	74	101	0	64	69	76
N.S.	1	1.05	1.09	0.62	1.00	1.36	0.00	0.86	0.93	1.03
time (sec)	N/A	0.333	0.479	0.478	0.077	0.108	0.000	0.123	0.238	2.269

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	122	61	48	73	0	47	30	0
N.S.	1	1.00	0.78	0.39	0.31	0.46	0.00	0.30	0.19	0.00
time (sec)	N/A	0.789	0.397	0.577	0.225	0.084	0.000	0.130	0.250	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	169	206	90	56	217	0	320	127	0
N.S.	1	1.06	1.30	0.57	0.35	1.36	0.00	2.01	0.80	0.00
time (sec)	N/A	0.769	0.590	0.656	0.367	0.082	0.000	0.171	0.273	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	220	264	211	68	342	0	176	265	0
N.S.	1	1.04	1.25	1.00	0.32	1.62	0.00	0.83	1.26	0.00
time (sec)	N/A	1.068	0.783	0.731	0.567	0.109	0.000	0.133	0.273	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	420	273	296	573	0	368	385	267
N.S.	1	1.00	1.83	1.19	1.29	2.49	0.00	1.60	1.67	1.16
time (sec)	N/A	0.558	1.889	0.792	0.431	0.082	0.000	0.138	0.237	2.376

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	207	163	190	361	0	217	229	165
N.S.	1	1.00	1.22	0.96	1.12	2.12	0.00	1.28	1.35	0.97
time (sec)	N/A	0.498	1.401	0.757	0.290	0.082	0.000	0.143	0.225	2.321

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	114	74	106	192	0	103	111	89
N.S.	1	1.00	0.86	0.56	0.80	1.44	0.00	0.77	0.83	0.67
time (sec)	N/A	0.453	0.918	0.533	0.167	0.084	0.000	0.126	0.266	2.234

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	199	106	81	133	0	75	53	0
N.S.	1	1.00	0.67	0.36	0.27	0.45	0.00	0.25	0.18	0.00
time (sec)	N/A	1.267	0.745	0.672	0.725	0.089	0.000	0.126	0.259	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	442	163	100	609	0	583	249	0
N.S.	1	1.00	1.05	0.39	0.24	1.45	0.00	1.39	0.59	0.00
time (sec)	N/A	1.226	1.391	0.750	1.432	0.093	0.000	0.165	0.239	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	615	379	407	844	0	548	575	376
N.S.	1	1.00	1.83	1.13	1.21	2.51	0.00	1.63	1.71	1.12
time (sec)	N/A	0.706	3.229	0.859	1.860	0.098	0.000	0.138	0.263	2.669

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	371	223	255	532	0	315	333	236
N.S.	1	1.00	1.51	0.91	1.04	2.16	0.00	1.28	1.35	0.96
time (sec)	N/A	0.567	1.971	0.786	1.279	0.092	0.000	0.137	0.252	2.298

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	182	185	102	139	286	0	142	153	129
N.S.	1	0.99	1.01	0.56	0.76	1.56	0.00	0.78	0.84	0.70
time (sec)	N/A	0.556	1.143	0.567	0.710	0.092	0.000	0.137	0.224	2.301

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	437	437	312	151	114	193	0	103	76	0
N.S.	1	1.00	0.71	0.35	0.26	0.44	0.00	0.24	0.17	0.00
time (sec)	N/A	2.198	0.901	0.756	2.735	0.108	0.000	0.143	0.293	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	692	692	794	239	140	1164	0	840	371	0
N.S.	1	1.00	1.15	0.35	0.20	1.68	0.00	1.21	0.54	0.00
time (sec)	N/A	2.229	2.995	0.924	6.434	0.111	0.000	0.173	0.309	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	122	37	44	22	5344	22
N.S.	1	1.00	1.10	1.00	6.10	1.85	2.20	1.10	267.20	1.10
time (sec)	N/A	0.230	26.328	0.234	0.351	0.088	3.409	0.173	0.309	2.414

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	68	20	24	20	86	20
N.S.	1	1.00	1.11	1.00	3.78	1.11	1.33	1.11	4.78	1.11
time (sec)	N/A	0.219	16.051	0.192	0.146	0.091	1.923	0.140	0.236	2.347

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	116	0	0	148	0	0	201	0
N.S.	1	1.00	1.30	0.00	0.00	1.66	0.00	0.00	2.26	0.00
time (sec)	N/A	0.409	0.771	0.000	0.000	0.095	0.000	0.000	0.309	0.000

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	163	0	0	248	0	0	1305	0
N.S.	1	1.00	1.07	0.00	0.00	1.62	0.00	0.00	8.53	0.00
time (sec)	N/A	0.488	1.098	0.000	0.000	0.101	0.000	0.000	0.266	0.000

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	A	F	F	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	230	0	0	345	0	0	0	0
N.S.	1	1.00	1.03	0.00	0.00	1.54	0.00	0.00	0.00	0.00
time (sec)	N/A	0.580	1.265	0.000	0.000	0.107	0.000	0.000	0.251	0.000

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	161	470	304	583	0	0	218	0
N.S.	1	1.00	1.18	3.43	2.22	4.26	0.00	0.00	1.59	0.00
time (sec)	N/A	0.592	0.463	0.739	0.126	0.117	0.000	0.000	0.307	0.000

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	147	290	179	364	0	0	151	0
N.S.	1	1.00	1.43	2.82	1.74	3.53	0.00	0.00	1.47	0.00
time (sec)	N/A	0.444	0.335	0.550	0.127	0.098	0.000	0.000	0.239	0.000

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	72	129	0	191	0	0	84	0
N.S.	1	1.00	0.96	1.72	0.00	2.55	0.00	0.00	1.12	0.00
time (sec)	N/A	0.331	0.095	0.457	0.000	0.092	0.000	0.000	0.253	0.000

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	56	20	15	20	59	20
N.S.	1	1.00	1.11	1.00	3.11	1.11	0.83	1.11	3.28	1.11
time (sec)	N/A	0.225	2.659	0.156	0.103	0.091	0.743	0.131	0.336	2.291

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	20	18	87	31	17	20	162	20
N.S.	1	1.00	1.11	1.00	4.83	1.72	0.94	1.11	9.00	1.11
time (sec)	N/A	0.224	5.138	0.155	0.133	0.088	2.762	0.182	0.266	2.409

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	544	905	625	3744	0	0	0	0
N.S.	1	1.00	1.96	3.27	2.26	13.52	0.00	0.00	0.00	0.00
time (sec)	N/A	0.823	2.781	1.345	0.158	0.155	0.000	0.000	0.253	0.000

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	237	542	411	2123	0	0	1086	0
N.S.	1	1.00	1.12	2.57	1.95	10.06	0.00	0.00	5.15	0.00
time (sec)	N/A	0.657	1.680	1.177	0.129	0.153	0.000	0.000	0.329	0.000

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	214	221	0	944	0	0	484	0
N.S.	1	1.00	1.69	1.74	0.00	7.43	0.00	0.00	3.81	0.00
time (sec)	N/A	0.448	6.578	1.028	0.000	0.125	0.000	0.000	0.260	0.000

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	155	36	17	22	258	22
N.S.	1	1.00	1.10	1.00	7.75	1.80	0.85	1.10	12.90	1.10
time (sec)	N/A	0.243	25.142	0.433	0.240	0.100	1.181	0.172	0.257	2.545

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	280	47	19	22	803	22
N.S.	1	1.00	1.10	1.00	14.00	2.35	0.95	1.10	40.15	1.10
time (sec)	N/A	0.245	18.240	0.434	0.318	0.089	2.048	0.973	0.344	2.808

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	2010	1834	1297	12909	0	0	0	0
N.S.	1	1.00	3.55	3.24	2.29	22.81	0.00	0.00	0.00	0.00
time (sec)	N/A	1.478	7.719	1.827	0.224	0.273	0.000	0.000	9.100	0.000

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	396	396	1163	1066	869	7298	0	0	0	0
N.S.	1	1.00	2.94	2.69	2.19	18.43	0.00	0.00	0.00	0.00
time (sec)	N/A	1.085	7.167	1.503	0.238	0.203	0.000	0.000	0.301	0.000

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	C	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	303	459	475	3262	0	0	1407	0
N.S.	1	1.00	1.17	1.77	1.83	12.59	0.00	0.00	5.43	0.00
time (sec)	N/A	0.690	7.420	1.239	0.208	0.144	0.000	0.000	0.318	0.000

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	455	52	17	22	772	22
N.S.	1	1.00	1.10	1.00	22.75	2.60	0.85	1.10	38.60	1.10
time (sec)	N/A	0.268	31.539	0.842	0.446	0.095	1.400	0.226	0.365	2.390

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	926	63	19	22	2546	22
N.S.	1	1.00	1.10	1.00	46.30	3.15	0.95	1.10	127.30	1.10
time (sec)	N/A	0.261	32.962	0.848	0.578	0.145	2.374	0.585	0.296	2.492

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	202	247	1156	528	740	0	0	542	0
N.S.	1	0.95	1.17	5.45	2.49	3.49	0.00	0.00	2.56	0.00
time (sec)	N/A	1.122	0.964	0.894	0.199	0.113	0.000	0.000	0.276	0.000

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	155	200	733	337	500	0	0	373	0
N.S.	1	0.99	1.27	4.67	2.15	3.18	0.00	0.00	2.38	0.00
time (sec)	N/A	0.860	0.603	0.787	0.174	0.117	0.000	0.000	0.287	0.000

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	112	152	357	0	306	0	0	206	0
N.S.	1	1.04	1.41	3.31	0.00	2.83	0.00	0.00	1.91	0.00
time (sec)	N/A	0.518	0.659	0.528	0.000	0.106	0.000	0.000	0.247	0.000

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	114	27	17	22	271	22
N.S.	1	1.00	1.10	1.00	5.70	1.35	0.85	1.10	13.55	1.10
time (sec)	N/A	0.267	4.685	0.196	0.236	0.092	1.018	0.131	0.299	2.200

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	199	51	19	22	937	22
N.S.	1	1.00	1.10	1.00	9.95	2.55	0.95	1.10	46.85	1.10
time (sec)	N/A	0.247	8.598	0.202	0.442	0.088	1.704	0.214	0.332	2.449

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	642	642	656	2683	1060	6160	0	0	0	0
N.S.	1	1.00	1.02	4.18	1.65	9.60	0.00	0.00	0.00	0.00
time (sec)	N/A	2.461	5.100	1.023	0.462	0.222	0.000	0.000	0.296	0.000

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	F	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	476	476	516	1605	753	3693	0	0	0	0
N.S.	1	1.00	1.08	3.37	1.58	7.76	0.00	0.00	0.00	0.00
time (sec)	N/A	1.979	3.272	0.886	0.465	0.168	0.000	0.000	0.284	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	214	249	565	0	1790	0	0	0	0
N.S.	1	1.09	1.27	2.88	0.00	9.13	0.00	0.00	0.00	0.00
time (sec)	N/A	0.838	2.377	0.574	0.000	0.140	0.000	0.000	0.276	0.000

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	468	55	19	22	2595	22
N.S.	1	1.00	1.10	1.00	23.40	2.75	0.95	1.10	129.75	1.10
time (sec)	N/A	0.273	29.431	0.210	1.202	0.089	1.599	0.176	0.301	2.855

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A	N/A
verified	N/A	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	22	20	784	96	20	22	8964	22
N.S.	1	1.00	1.10	1.00	39.20	4.80	1.00	1.10	448.20	1.10
time (sec)	N/A	0.250	25.809	0.219	2.388	0.096	3.076	0.318	0.456	2.493

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [11] had the largest ratio of [1.43750000000000000]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	C	9	8	1.20	14	0.571
2	C	8	7	1.25	14	0.500
3	C	7	6	1.25	12	0.500
4	N/A	3	0	1.00	14	0.000
5	N/A	3	0	1.00	14	0.000
6	C	13	12	1.19	16	0.750
7	C	12	11	1.23	16	0.688
8	A	8	8	1.00	14	0.571
9	N/A	3	0	1.00	16	0.000
10	N/A	3	0	1.00	16	0.000
11	C	24	23	1.19	16	1.438
12	C	20	19	1.19	16	1.188
13	C	15	14	1.17	14	1.000
14	N/A	3	0	1.00	16	0.000
15	N/A	3	0	1.00	16	0.000
16	A	18	17	0.93	18	0.944
17	A	16	15	0.93	18	0.833
18	A	8	7	0.93	18	0.389
19	A	8	7	0.93	18	0.389
20	A	16	15	0.93	18	0.833

Continued on next page

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	N/A	11	0	1.00	20	0.000
22	N/A	2	0	1.00	20	0.000
23	N/A	2	0	1.00	20	0.000
24	N/A	11	0	1.00	20	0.000
25	N/A	2	0	1.00	20	0.000
26	N/A	2	0	1.00	20	0.000
27	N/A	2	0	1.00	20	0.000
28	N/A	2	0	1.00	20	0.000
29	N/A	3	0	1.00	12	0.000
30	N/A	3	0	1.00	12	0.000
31	N/A	3	0	1.00	10	0.000
32	A	9	9	1.06	20	0.450
33	A	7	7	1.04	20	0.350
34	A	5	5	1.05	18	0.278
35	C	11	11	1.00	20	0.550
36	C	11	11	1.06	20	0.550
37	C	13	13	1.04	20	0.650
38	A	3	3	1.00	20	0.150
39	A	3	3	1.00	20	0.150
40	A	3	3	1.00	18	0.167
41	A	3	3	1.00	20	0.150
42	A	3	3	1.00	20	0.150
43	A	3	3	1.00	20	0.150
44	A	3	3	1.00	20	0.150
45	A	3	3	0.99	18	0.167
46	A	3	3	1.00	20	0.150
47	A	3	3	1.00	20	0.150
48	N/A	2	0	1.00	20	0.000
49	N/A	2	0	1.00	18	0.000
50	A	3	3	1.00	20	0.150
51	A	3	3	1.00	20	0.150

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#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
52	A	3	3	1.00	20	0.150
53	A	3	3	1.00	18	0.167
54	A	3	3	1.00	18	0.167
55	A	3	3	1.00	16	0.188
56	N/A	2	0	1.00	18	0.000
57	N/A	2	0	1.00	18	0.000
58	A	3	3	1.00	20	0.150
59	A	3	3	1.00	20	0.150
60	A	3	3	1.00	18	0.167
61	N/A	2	0	1.00	20	0.000
62	N/A	2	0	1.00	20	0.000
63	A	3	3	1.00	20	0.150
64	A	3	3	1.00	20	0.150
65	A	3	3	1.00	18	0.167
66	N/A	2	0	1.00	20	0.000
67	N/A	2	0	1.00	20	0.000
68	A	8	7	0.95	20	0.350
69	A	7	6	0.99	20	0.300
70	A	6	5	1.04	18	0.278
71	N/A	2	0	1.00	20	0.000
72	N/A	2	0	1.00	20	0.000
73	A	3	3	1.00	20	0.150
74	A	3	3	1.00	20	0.150
75	A	9	8	1.09	18	0.444
76	N/A	2	0	1.00	20	0.000
77	N/A	2	0	1.00	20	0.000

CHAPTER 3

Listing of integrals

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3.1 $\int (c+dx)^3 \tanh(e+fx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 117

$$\begin{aligned}\int (c+dx)^3 \tanh(e+fx)\,dx = &-\frac{(c+dx)^4}{4d} + \frac{(c+dx)^3 \log\left(1+e^{2(e+fx)}\right)}{f} \\ &+ \frac{3d(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} \\ &- \frac{3d^2(c+dx) \operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} \\ &+ \frac{3d^3 \operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}\end{aligned}$$

output

```
-1/4*(d*x+c)^4/d+(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2*d*(d*x+c)^2*polylog(
2,-exp(2*f*x+2*e))/f^2-3/2*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/4*
d^3*polylog(4,-exp(2*f*x+2*e))/f^4
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.32

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \frac{1}{4}\left(\frac{2(c+dx)^4}{d\left(1+e^{2e}\right)} + \frac{4(c+dx)^3 \log\left(1+e^{-2(e+fx)}\right)}{f} - \frac{3d\left(2f^2(c+dx)^2 \operatorname{PolyLog}\left(2,-e^{-2(e+fx)}\right) + d\left(2f(c+dx)\operatorname{PolyLog}\left(3,-e^{-2(e+fx)}\right) + d\operatorname{PolyLog}\left(4,-e\right.\right.\right.}{f^4} + x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\tanh(e)\right)$$

input
```
Integrate[(c + d*x)^3*Tanh[e + f*x],x]
```

output
```
((2*(c + d*x)^4)/(d*(1 + E^(2*e))) + (4*(c + d*x)^3*Log[1 + E^(-2*(e + f*x
))])/f - (3*d*(2*f^2*(c + d*x)^2*PolyLog[2, -E^(-2*(e + f*x))] + d*(2*f*(c
 + d*x)*PolyLog[3, -E^(-2*(e + f*x))] + d*PolyLog[4, -E^(-2*(e + f*x))])))
/f^4 + x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Tanh[e])/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.20, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 26, 4201, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tanh(e+fx)\,dx$$

$$\downarrow \text{3042}$$

$$\int -i(c+dx)^3 \tan(ie+ifx)dx$$

$$\downarrow \text{26}$$

$$-i\int (c+dx)^3\tan(ie+ifx)dx$$

$$\downarrow 4201$$

$$-i\left(2i\int \frac{e^{2(e+fx)}(c+dx)^3}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 2620$$

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\int(c+dx)^2\log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)-\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 3011$$

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)-\frac{i(c+}{4}$$

$$\downarrow 7163$$

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)dx}{2f}\right)}{f}-\frac{(c+dx)^2\operatorname{PolyLog}(2,}{2f}\right.\right.\right.$$

$$\downarrow 2720$$

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)}{f}-\frac{(c}{2f}\right.\right.\right.$$

$$\downarrow 7143$$

$$-i\left(2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\,\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^2}\right)}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2}\right.}{2f}\right.\right.\right.$$

input
```
Int[(c + d*x)^3*Tanh[e + f*x],x]
```

output
```
(-I)*(((-1/4*I)*(c + d*x)^4)/d + (2*I)*(((c + d*x)^3*Log[1 + E^(2*(e + f*x
))])/(2*f) - (3*d*(-1/2*((c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/f + (d*
(((c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f) - (d*PolyLog[4, -E^(2*(e +
 f*x))])/(4*f^2)))/f))/(2*f)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163
```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(109) = 218$.

Time = 0.83 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.44

method	result
risch	$-\frac{6c^2dex}{f} - \frac{3d^3e^4}{2f^4} + \frac{c^3\ln(1+e^{2fx+2e})}{f} - \frac{2c^3\ln(e^{fx+e})}{f} - d^2cx^3 - \frac{3dc^2x^2}{2} + c^3x - \frac{3c^2de^2}{f^2} + \frac{4cd^2e^3}{f^3} + \frac{d^3\ln(1$

input
```
int((d*x+c)^3*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output

```
-6/f*c^2*d*e*x-3/2/f^4*d^3*e^4+1/f*c^3*ln(1+exp(2*f*x+2*e))-2/f*c^3*ln(exp
(f*x+e))-d^2*c*x^3-3/2*d*c^2*x^2+c^3*x-3/f^2*c^2*d*e^2+4/f^3*c*d^2*e^3+1/f
*d^3*ln(1+exp(2*f*x+2*e))*x^3+3/2/f^2*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3
/2/f^3*d^3*polylog(3,-exp(2*f*x+2*e))*x+2/f^4*d^3*e^3*ln(exp(f*x+e))+6/f^2
*c*d^2*e^2*x-2/f^3*d^3*e^3*x+3/2/f^2*c^2*d*polylog(2,-exp(2*f*x+2*e))-3/2/
f^3*c*d^2*polylog(3,-exp(2*f*x+2*e))+3/f*c*d^2*ln(1+exp(2*f*x+2*e))*x^2+3/
f^2*c*d^2*polylog(2,-exp(2*f*x+2*e))*x-6/f^3*c*d^2*e^2*ln(exp(f*x+e))+6/f^
2*c^2*d*e*ln(exp(f*x+e))+3/f*c^2*d*ln(1+exp(2*f*x+2*e))*x-1/4*d^3*x^4+1/4/
d*c^4+3/4*d^3*polylog(4,-exp(2*f*x+2*e))/f^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 531, normalized size of antiderivative = 4.54

$$\int (c+dx)^3 \tanh(e+fx)\,dx = -\frac{d^3 f^4 x^4 + 4\,cd^2 f^4 x^3 + 6\,c^2 df^4 x^2 + 4\,c^3 f^4 x - 24\,d^3 \mathrm{polylog}(4, i\cosh(fx+e) + i\sinh(fx+e)) - 24\,d^3$$

input

```
integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="fricas")
```

output

```
-1/4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x - 24*d
^3*polylog(4, I*cosh(f*x + e) + I*sinh(f*x + e)) - 24*d^3*polylog(4, -I*co
sh(f*x + e) - I*sinh(f*x + e)) - 12*(d^3*f^2*x^2 + 2*c*d^2*f^2*x + c^2*d*f
^2)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 12*(d^3*f^2*x^2 + 2*c*d^2*f
^2*x + c^2*d*f^2)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 4*(d^3*e^3 -
 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e
) + I) + 4*(d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2 - c^3*f^3)*log(cosh(f*
x + e) + sinh(f*x + e) - I) - 4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f
^3*x + d^3*e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*log(I*cosh(f*x + e) + I*si
nh(f*x + e) + 1) - 4*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2 + 3*c^2*d*f^3*x + d^3*
e^3 - 3*c*d^2*e^2*f + 3*c^2*d*e*f^2)*log(-I*cosh(f*x + e) - I*sinh(f*x + e
) + 1) + 24*(d^3*f*x + c*d^2*f)*polylog(3, I*cosh(f*x + e) + I*sinh(f*x +
e)) + 24*(d^3*f*x + c*d^2*f)*polylog(3, -I*cosh(f*x + e) - I*sinh(f*x + e)
))/f^4
```

Sympy [F]

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \int (c+dx)^3 \tanh\left(e+fx\right)\,dx$$

input
```
integrate((d*x+c)**3*tanh(f*x+e),x)
```

output
```
Integral((c + d*x)**3*tanh(e + f*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. 2(108) = 216.

Time = 0.19 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.44

$$\begin{aligned}\int (c+dx)^3 \tanh(e+fx)\,dx &= \frac{1}{4}d^3x^4 + cd^2x^3 + \frac{3}{2}c^2dx^2 + \frac{c^3\log\left(e^{(2fx+2e)}+1\right)}{2f} \\ &+ \frac{c^3\log\left(e^{(-2fx-2e)}+1\right)}{2f} + \frac{3\left(2fx\log\left(e^{(2fx+2e)}+1\right)+\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)c^2d}{2f^2} \\ &+ \frac{3\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right)+2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)cd^2}{2f^3} \\ &+ \frac{\left(4f^3x^3\log\left(e^{(2fx+2e)}+1\right)+6f^2x^2\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-6fx\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)+3\mathrm{Li}_4\left(-e^{(2fx+2e)}\right)\right)d^3}{3f^4} \\ &- \frac{d^3f^4x^4+4cd^2f^4x^3+6c^2df^4x^2}{2f^4}\end{aligned}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="maxima")
```

output
```
1/4*d^3*x^4 + c*d^2*x^3 + 3/2*c^2*d*x^2 + 1/2*c^3*log(e^(2*f*x + 2*e) + 1)
/f + 1/2*c^3*log(e^(-2*f*x - 2*e) + 1)/f + 3/2*(2*f*x*log(e^(2*f*x + 2*e)
+ 1) + dilog(-e^(2*f*x + 2*e)))*c^2*d/f^2 + 3/2*(2*f^2*x^2*log(e^(2*f*x +
2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*
c*d^2/f^3 + 1/3*(4*f^3*x^3*log(e^(2*f*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(
2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*
x + 2*e)))*d^3/f^4 - 1/2*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2)
/f^4
```

Giac [F]

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \int (dx+c)^3 \tanh\left(fx+e\right)\,dx$$

input
```
integrate((d*x+c)^3*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tanh(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tanh(e+fx)\,dx = \int \tanh(e+f\,x)\,\left(c+d\,x\right)^3\,dx$$

input
```
int(tanh(e + f*x)*(c + d*x)^3,x)
```

output
```
int(tanh(e + f*x)*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \tanh(e+fx)\,dx$$
$$= \frac{8e^{2e}\left(\int \frac{e^{2fx}x^3}{e^{2fx+2e}+1}dx\right) d^3 f + 24e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}+1}dx\right) c\,d^2 f + 24e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}+1}dx\right) c^2 df + 4\log\left(e^{2fx+2e}+1\right)}{4f}$$

input
```
int((d*x+c)^3*tanh(f*x+e),x)
```

output

```
(8*e**(2*e)*int((e**(2*f*x)*x**3)/(e**(2*e + 2*f*x) + 1),x)*d**3*f + 24*e*
*(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x) + 1),x)*c*d**2*f + 24*e**(2
*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) + 1),x)*c**2*d*f + 4*log(e**(2*e
+ 2*f*x) + 1)*c**3 - 4*c**3*f*x - 6*c**2*d*f*x**2 - 4*c*d**2*f*x**3 - d**3
*f*x**4)/(4*f)
```

3.2 $\int (c+dx)^2 \tanh(e+fx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 84

$$\int (c+dx)^2 \tanh(e+fx)\,dx = -\frac{(c+dx)^3}{3d} + \frac{(c+dx)^2 \log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} - \frac{d^2\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}$$

output

```
-1/3*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+d*(d*x+c)*polylog(2,-exp
(2*f*x+2*e))/f^2-1/2*d^2*polylog(3,-exp(2*f*x+2*e))/f^3
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.70

$$\int (c+dx)^2 \tanh(e+fx)\,dx = \frac{e^{2e}\left(\frac{4e^{-2e}(c+dx)^3}{d} + \frac{6\left(1+e^{-2e}\right)(c+dx)^2 \log\left(1+e^{-2(e+fx)}\right)}{f} - \frac{3d\left(1+e^{-2e}\right)\left(2f(c+dx)\,\mathrm{PolyLog}\left(2,-e^{-2(e+fx)}\right)+d\,\mathrm{PolyLog}\left(3,-e^{-2(e+fx)}\right)\right)}{f^3}\right.}{6\left(1+e^{2e}\right)} + \frac{1}{3}x\left(3c^2+3cdx+d^2x^2\right)\tanh(e)$$

input
```
Integrate[(c + d*x)^2*Tanh[e + f*x],x]
```

output
```
(E^(2*e)*((4*(c + d*x)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*Log[
1 + E^(-2*(e + f*x))])/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2, -
E^(-2*(e + f*x))] + d*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/(6*(1 + E^(2*e
))) + (x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tanh[e])/3
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tanh(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int -i(c+dx)^2 \tan(ie+ifx)dx$$

$$\downarrow 26$$

$$-i\int (c+dx)^2 \tan(ie+ifx)dx$$

$$\downarrow 4201$$

$$-i\left(2i\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}dx - \frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 2620$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f} - \frac{d\int (c+dx)\log\left(1+e^{2(e+fx)}\right)dx}{f}\right) - \frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 3011$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int \text{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{2f}-\frac{(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 2720$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-$$

$$\downarrow 7143$$

$$-i\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{4f^2}-\frac{(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)$$

input
```
Int[(c + d*x)^2*Tanh[e + f*x],x]
```

output
```
(-I)*(((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((c + d*x)^2*Log[1 + E^(2*(e + f*x
))])/(2*f) - (d*(-1/2*((c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f + (d*Poly
Log[3, -E^(2*(e + f*x))])/(4*f^2)))/f))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(80) = 160$.

Time = 0.75 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.88

method	result
risch	$-\frac{d^2x^3}{3} - dcx^2 + c^2x + \frac{c^3}{3d} - \frac{2dce^2}{f^2} + \frac{2dc\ln(1+e^{2fx+2e})x}{f} + \frac{dc\,\mathrm{polylog}(2,-e^{2fx+2e})}{f^2} - \frac{2e^2d^2\ln(e^{fx+e})}{f^3} + \frac{2d^2e^2}{f^2}$

input
```
int((d*x+c)^2*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^2*x^3-d*c*x^2+c^2*x+1/3/d*c^3-2/f^2*d*c*e^2+2/f*d*c*ln(1+exp(2*f*x+
2*e))*x+1/f^2*d*c*polylog(2,-exp(2*f*x+2*e))-2/f^3*e^2*d^2*ln(exp(f*x+e))+
2/f^2*d^2*e^2*x+1/f^2*d^2*polylog(2,-exp(2*f*x+2*e))*x-1/2*d^2*polylog(3,-
exp(2*f*x+2*e))/f^3+4/f^2*c*d*e*ln(exp(f*x+e))-4/f*d*c*e*x+4/3/f^3*d^2*e^3
-2/f*c^2*ln(exp(f*x+e))+1/f*c^2*ln(1+exp(2*f*x+2*e))+1/f*d^2*ln(1+exp(2*f*
x+2*e))*x^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.92

$$\int (c+dx)^2 \tanh(e+fx)\,dx =$$
$$-\frac{d^2 f^3 x^3 + 3\,cdf^3 x^2 + 3\,c^2 f^3 x + 6\,d^2 \mathrm{polylog}(3, i\,\cosh(fx+e) + i\,\sinh(fx+e)) + 6\,d^2 \mathrm{polylog}(3, -i\,\mathrm{c}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="fricas")
```

output
```
-1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 6*d^2*polylog(3, I*cosh(
f*x + e) + I*sinh(f*x + e)) + 6*d^2*polylog(3, -I*cosh(f*x + e) - I*sinh(f
*x + e)) - 6*(d^2*f*x + c*d*f)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) -
6*(d^2*f*x + c*d*f)*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 3*(d^2*e^2
 - 2*c*d*e*f + c^2*f^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 3*(d^2*e^
2 - 2*c*d*e*f + c^2*f^2)*log(cosh(f*x + e) + sinh(f*x + e) - I) - 3*(d^2*f
^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*log(I*cosh(f*x + e) + I*sinh(f
*x + e) + 1) - 3*(d^2*f^2*x^2 + 2*c*d*f^2*x - d^2*e^2 + 2*c*d*e*f)*log(-I*
cosh(f*x + e) - I*sinh(f*x + e) + 1))/f^3
```

Sympy [F]

$$\int (c+dx)^2 \tanh(e+fx)\,dx = \int (c+dx)^2 \tanh\left(e+fx\right)\,dx$$

input
```
integrate((d*x+c)**2*tanh(f*x+e),x)
```

output
```
Integral((c + d*x)**2*tanh(e + f*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 176 vs. 2(79) = 158.

Time = 0.16 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.10

$$\begin{aligned}
&\int (c+dx)^2 \tanh(e+fx)\,dx \\
&= \frac{1}{3}d^2x^3 + cdx^2 + \frac{c^2 \log\left(e^{(2fx+2e)}+1\right)}{2f} + \frac{c^2 \log\left(e^{(-2fx-2e)}+1\right)}{2f} \\
&\quad + \frac{\left(2fx\log\left(e^{(2fx+2e)}+1\right) + \mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)cd}{f^2} \\
&\quad + \frac{\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right) + 2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right) - \mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)d^2}{2f^3} \\
&\quad - \frac{2\left(d^2f^3x^3 + 3cdf^3x^2\right)}{3f^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="maxima")
```

output
```
1/3*d^2*x^3 + c*d*x^2 + 1/2*c^2*log(e^(2*f*x + 2*e) + 1)/f + 1/2*c^2*log(e
^(-2*f*x - 2*e) + 1)/f + (2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x
 + 2*e)))*c*d/f^2 + 1/2*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(
-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*d^2/f^3 - 2/3*(d^2*f^3*x
^3 + 3*c*d*f^3*x^2)/f^3
```

Giac [F]

$$\int (c+dx)^2 \tanh(e+fx)\,dx = \int (dx+c)^2 \tanh\left(fx+e\right)\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tanh(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tanh(e+fx)\,dx = \int \tanh(e+f\,x)\,\left(c+d\,x\right)^2\,dx$$

input
```
int(tanh(e + f*x)*(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \tanh(e+fx)\,dx$$
$$= \frac{6e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}+1}dx\right) d^2 f + 12e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}+1}dx\right) cdf + 3\log\left(e^{2fx+2e}+1\right) c^2 - 3c^2 fx - 3cdf\,x^2 - d^2 f\,x^3}{3f}$$

input
```
int((d*x+c)^2*tanh(f*x+e),x)
```

output
```
(6*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x) + 1),x)*d**2*f + 12*e*
*(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) + 1),x)*c*d*f + 3*log(e**(2*e
+ 2*f*x) + 1)*c**2 - 3*c**2*f*x - 3*c*d*f*x**2 - d**2*f*x**3)/(3*f)
```

3.3 $\int (c+dx)\tanh(e+fx)\,dx$

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Optimal result

Integrand size = 12, antiderivative size = 57

$$\int (c+dx)\tanh(e+fx)\,dx = -\frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}$$

output

```
-1/2*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*d*polylog(2,-exp(2*f*x
+2*e))/f^2
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.04

$$\int (c+dx)\tanh(e+fx)\,dx = \frac{f\left(dfx^2+2dx\log\left(1+e^{-2(e+fx)}\right)+2c\log(\cosh(e+fx))\right)-d\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{2f^2}$$

input

```
Integrate[(c + d*x)*Tanh[e + f*x],x]
```

output
```
(f*(d*f*x^2 + 2*d*x*Log[1 + E^(-2*(e + f*x))] + 2*c*Log[Cosh[e + f*x]]) -
d*PolyLog[2, -E^(-2*(e + f*x))])/(2*f^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.38 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tanh(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int -i(c+dx)\tan(ie+ifx)dx$$

$$\downarrow 26$$

$$-i\int (c+dx)\tan(ie+ifx)dx$$

$$\downarrow 4201$$

$$-i\left(2i\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}dx - \frac{i(c+dx)^2}{2d}\right)$$

$$\downarrow 2620$$

$$-i\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f} - \frac{d\int \log\left(1+e^{2(e+fx)}\right)dx}{2f}\right) - \frac{i(c+dx)^2}{2d}\right)$$

$$\downarrow 2715$$

$$-i\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f} - \frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right) - \frac{i(c+dx)^2}{2d}\right)$$

$$\downarrow 2838$$

$$-i\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}+\frac{d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)$$

input
```
Int[(c + d*x)*Tanh[e + f*x],x]
```

output
```
(-I)*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c + d*x)*Log[1 + E^(2*(e + f*x))
])/(2*f) + (d*PolyLog[2, -E^(2*(e + f*x))])/(4*f^2)))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 108 vs. $2(53) = 106$.

Time $= 0.63$ (sec) , antiderivative size $= 109$, normalized size of antiderivative $= 1.91$

method	result
risch	$-\frac{dx^2}{2} + cx - \frac{2c\ln(\mathrm{e}^{fx+e})}{f} + \frac{c\ln(1+\mathrm{e}^{2fx+2e})}{f} - \frac{2dex}{f} - \frac{de^2}{f^2} + \frac{d\ln(1+\mathrm{e}^{2fx+2e})x}{f} + \frac{d\,\mathrm{polylog}(2,-\mathrm{e}^{2fx+2e})}{2f^2} + \frac{2de$

input

```
int((d*x+c)*tanh(f*x+e),x,method=_RETURNVERBOSE)
```

output

```
-1/2*d*x^2+c*x-2/f*c*ln(exp(f*x+e))+1/f*c*ln(1+exp(2*f*x+2*e))-2/f*d*e*x-1
/f^2*d*e^2+1/f*d*ln(1+exp(2*f*x+2*e))*x+1/2*d*polylog(2,-exp(2*f*x+2*e))/f
^2+2/f^2*d*e*ln(exp(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time $= 0.11$ (sec) , antiderivative size $= 171$, normalized size of antiderivative $= 3.00$

$$\int (c+dx)\tanh(e+fx)\,dx =$$
$$-\frac{df^2x^2 + 2\,cf^2x - 2\,d\mathrm{Li}_2(i\cosh(fx+e) + i\sinh(fx+e)) - 2\,d\mathrm{Li}_2(-i\cosh(fx+e) - i\sinh(fx+$$

input

```
integrate((d*x+c)*tanh(f*x+e),x, algorithm="fricas")
```

output
```
-1/2*(d*f^2*x^2 + 2*c*f^2*x - 2*d*dilog(I*cosh(f*x + e) + I*sinh(f*x + e))
 - 2*d*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) + 2*(d*e - c*f)*log(cosh(
f*x + e) + sinh(f*x + e) + I) + 2*(d*e - c*f)*log(cosh(f*x + e) + sinh(f*x
 + e) - I) - 2*(d*f*x + d*e)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) -
2*(d*f*x + d*e)*log(-I*cosh(f*x + e) - I*sinh(f*x + e) + 1))/f^2
```

Sympy [F]

$$\int (c+dx)\tanh(e+fx)\,dx = \int (c+dx)\tanh\left(e+fx\right)dx$$

input
```
integrate((d*x+c)*tanh(f*x+e),x)
```

output
```
Integral((c + d*x)*tanh(e + f*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.37

$$\int (c+dx)\tanh(e+fx)\,dx = -\frac{1}{2}\,dx^2 + \frac{c\log\left(e^{(2\,fx+2\,e)}+1\right)}{2\,f} + \frac{c\log\left(e^{(-2\,fx-2\,e)}+1\right)}{2\,f} + \frac{\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right)+\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)d}{2\,f^2}$$

input
```
integrate((d*x+c)*tanh(f*x+e),x, algorithm="maxima")
```

output
```
-1/2*d*x^2 + 1/2*c*log(e^(2*f*x + 2*e) + 1)/f + 1/2*c*log(e^(-2*f*x - 2*e)
 + 1)/f + 1/2*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*d
/f^2
```

Giac [F]

$$\int (c+dx)\tanh(e+fx)\,dx = \int (dx+c)\tanh\left(fx+e\right)\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e),x, algorithm="giac")
```

output
```
integrate((d*x + c)*tanh(f*x + e), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\tanh(e+fx)\,dx = \int \tanh(e+f\,x)\,\left(c+d\,x\right)\,dx$$

input
```
int(tanh(e + f*x)*(c + d*x),x)
```

output
```
int(tanh(e + f*x)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\tanh(e+fx)\,dx$$
$$= \frac{4e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}+1}dx\right)df + 2\log\left(e^{2fx+2e}+1\right)c - 2cfx - df\,x^2}{2f}$$

input
```
int((d*x+c)*tanh(f*x+e),x)
```

output
```
(4*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) + 1),x)*d*f + 2*log(e**(2
*e + 2*f*x) + 1)*c - 2*c*f*x - d*f*x**2)/(2*f)
```

3.4 $\int \frac{\tanh(e+fx)}{c+dx}\,dx$

Optimal result 78
Mathematica [N/A] 78
Rubi [N/A] 79
Maple [N/A] 79
Fricas [N/A] 80
Sympy [N/A] 80
Maxima [N/A] 81
Giac [N/A] 81
Mupad [N/A] 81
Reduce [N/A] 82

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \text{Int}\left(\frac{\tanh(e+fx)}{c+dx}, x\right)$$

output
```
Defer(Int)(tanh(f*x+e)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 8.70 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+fx)}{c+dx}\,dx$$

input
```
Integrate[Tanh[e + f*x]/(c + d*x),x]
```

output
```
Integrate[Tanh[e + f*x]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{i\tan(ie+ifx)}{c+dx}dx$$
$$\downarrow 26$$
$$-i\int \frac{\tan(ie+ifx)}{c+dx}dx$$
$$\downarrow 4222$$
$$\int \frac{\tanh(e+fx)}{c+dx}dx$$

input
```
Int[Tanh[e + f*x]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(fx+e)}{dx+c}dx$$

input
```
int(tanh(f*x+e)/(d*x+c),x)
```

output
```
int(tanh(f*x+e)/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.86

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(e+fx\right)}{c+dx}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x)
```

output
```
Integral(tanh(e + f*x)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 42, normalized size of antiderivative = 3.00

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh(fx+e)}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x, algorithm="maxima")
```

output
```
log(d*x + c)/d - 2*integrate(1/(d*x + (d*x*e^(2*e) + c*e^(2*e))*e^(2*f*x)
+ c), x)
```

Giac [N/A]

Not integrable

Time = 0.12 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh(fx+e)}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+f\,x)}{c+d\,x}\,dx$$

input
```
int(tanh(e + f*x)/(c + d*x),x)
```

output
```
int(tanh(e + f*x)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 48, normalized size of antiderivative = 3.43

$$\int \frac{\tanh(e+fx)}{c+dx}\,dx = \frac{-2\left(\int \frac{1}{e^{2fx+2e}c+e^{2fx+2e}dx+c+dx}dx\right)d+\log(dx+c)}{d}$$

input
```
int(tanh(f*x+e)/(d*x+c),x)
```

output
```
( - 2*int(1/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x + c + d*x),x)*d + l
og(c + d*x))/d
```

3.5 $\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$

Optimal result 83
Mathematica [N/A] 83
Rubi [N/A] 84
Maple [N/A] 84
Fricas [N/A] 85
Sympy [N/A] 85
Maxima [N/A] 86
Giac [N/A] 86
Mupad [N/A] 86
Reduce [N/A] 87

Optimal result

Integrand size = 14, antiderivative size = 14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tanh(e+fx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(tanh(f*x+e)/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.43 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tanh[e + f*x]/(c + d*x)^2,x]
```

output
```
Integrate[Tanh[e + f*x]/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{i\tan(ie+ifx)}{(c+dx)^2}dx$$
$$\downarrow 26$$
$$-i\int \frac{\tan(ie+ifx)}{(c+dx)^2}dx$$
$$\downarrow 4222$$
$$\int \frac{\tanh(e+fx)}{(c+dx)^2}dx$$

input
```
Int[Tanh[e + f*x]/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(fx+e\right)}{\left(dx+c\right)^2}dx$$

input
```
int(tanh(f*x+e)/(d*x+c)^2,x)
```

output
```
int(tanh(f*x+e)/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.93

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(e+fx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)**2,x)
```

output
```
Integral(tanh(e + f*x)/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 71, normalized size of antiderivative = 5.07

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-1/(d^2*x + c*d) - 2*integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^(2*
e) + 2*c*d*x*e^(2*e) + c^2*e^(2*e))*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.14

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+f\,x)}{\left(c+d\,x\right)^2}\,dx$$

input
```
int(tanh(e + f*x)/(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 155, normalized size of antiderivative = 11.07

$$\int \frac{\tanh(e+fx)}{(c+dx)^2}\,dx$$
$$= \frac{-2\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2+c^2+2cdx+d^2x^2}dx\right)c^2 - 2\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2+c^2+2cdx+d^2x^2}dx\right)$$

input
```
int(tanh(f*x+e)/(d*x+c)^2,x)
```

output
```
( - 2*int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e +
2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*c**2 - 2*int(1/(e**(2*e
+ 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2 + c*
*2 + 2*c*d*x + d**2*x**2),x)*c*d*x + x)/(c*(c + d*x))
```

3.6 $\int (c+dx)^3 \tanh^2(e+fx)\,dx$

Optimal result . . . 88
Mathematica [A] (verified) . . . 89
Rubi [C] (verified) . . . 89
Maple [B] (verified) . . . 93
Fricas [C] (verification not implemented) . . . 94
Sympy [F] . . . 95
Maxima [B] (verification not implemented) . . . 95
Giac [F] . . . 96
Mupad [F(-1)] . . . 96
Reduce [F] . . . 97

Optimal result

Integrand size = 16, antiderivative size = 119

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = -\frac{(c+dx)^3}{f} + \frac{(c+dx)^4}{4d} + \frac{3d(c+dx)^2 \log\left(1+e^{2(e+fx)}\right)}{f^2} + \frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3} - \frac{3d^3\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4} - \frac{(c+dx)^3 \tanh(e+fx)}{f}$$

output

```
-(d*x+c)^3/f+1/4*(d*x+c)^4/d+3*d*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f^2+3*d^2*
(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3-3/2*d^3*polylog(3,-exp(2*f*x+2*e))/
f^4-(d*x+c)^3*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.53 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.50

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \frac{1}{4}\left(x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right) + \frac{2de^{2e}\left(\frac{4e^{-2e}(c+dx)^3}{d} + \frac{6\left(1+e^{-2e}\right)(c+dx)^2\log\left(1+e^{-2(e+fx)}\right)}{f} - \frac{3d\left(1+e^{-2e}\right)\left(2f(c+dx)\,\mathrm{PolyLog}\left(2,-e^{-2(e+fx)}\right)+d\,\mathrm{PolyLog}\left(3,-e^{-}\right.\right.\right.}{f^3}}{\left(1+e^{2e}\right)f} - \frac{4(c+dx)^3\mathrm{sech}(e)\mathrm{sech}(e+fx)\sinh(fx)}{f}\right)$$

input

```
Integrate[(c + d*x)^3*Tanh[e + f*x]^2,x]
```

output

```
(x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3) + (2*d*E^(2*e)*((4*(c + d*x
)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*Log[1 + E^(-2*(e + f*x))]
)/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2, -E^(-2*(e + f*x))] + d
*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/((1 + E^(2*e))*f) - (4*(c + d*x)^3*
Sech[e]*Sech[e + f*x]*Sinh[f*x])/f)/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.19, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 25, 4203, 17, 26, 3042, 26, 4201, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^3 \tan(ie+ifx)^2 dx$$

$$\downarrow 25$$

$$-\int (c+dx)^3 \tan(ie+ifx)^2 dx$$

$$\downarrow 4203$$

$$-\frac{3id\int i(c+dx)^2\tanh(e+fx)dx}{f}+\int (c+dx)^3 dx-\frac{(c+dx)^3\tanh(e+fx)}{f}$$

$$\downarrow 17$$

$$-\frac{3id\int i(c+dx)^2\tanh(e+fx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 26$$

$$\frac{3d\int (c+dx)^2\tanh(e+fx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 3042$$

$$\frac{3d\int -i(c+dx)^2\tan(ie+ifx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 26$$

$$-\frac{3id\int (c+dx)^2\tan(ie+ifx)dx}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 4201$$

$$-\frac{3id\left(2i\int \frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 2620$$

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int (c+dx)\log\left(1+e^{2(e+fx)}\right)dx}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 3011$$

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int \mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{2f}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 2720$$

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

$$\downarrow 7143$$

$$-\frac{3id\left(2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{4f^2}-\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)-\frac{i(c+dx)^3}{3d}\right)}{f}-\frac{(c+dx)^3\tanh(e+fx)}{f}+\frac{(c+dx)^4}{4d}$$

input

```
Int[(c + d*x)^3*Tanh[e + f*x]^2,x]
```

output

```
(c + d*x)^4/(4*d) - ((3*I)*d*(((-1/3*I)*(c + d*x)^3)/d + (2*I)*(((c + d*x)
^2*Log[1 + E^(2*(e + f*x))])/(2*f) - (d*(-1/2*((c + d*x)*PolyLog[2, -E^(2*
(e + f*x))])/f + (d*PolyLog[3, -E^(2*(e + f*x))])/(4*f^2)))/f)))/f - ((c +
 d*x)^3*Tanh[e + f*x])/f
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 335 vs. $2(115) = 230$.

Time $= 0.45$ (sec) , antiderivative size $= 336$, normalized size of antiderivative $= 2.82$

method	result
risch	$\frac{d^3x^4}{4} + d^2cx^3 + \frac{3dc^2x^2}{2} + c^3x + \frac{c^4}{4d} + \frac{2d^3x^3+6cd^2x^2+6c^2dx+2c^3}{f\left(1+e^{2fx+2e}\right)} + \frac{6d^2c\ln\left(1+e^{2fx+2e}\right)x}{f^2} + \frac{3d^2c\,\mathrm{polylog}\left(2,-e^{2fx}\right.}{f^3}$

input

```
int((d*x+c)^3*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output

```
1/4*d^3*x^4+d^2*c*x^3+3/2*d*c^2*x^2+c^3*x+1/4/d*c^4+2*(d^3*x^3+3*c*d^2*x^2
+3*c^2*d*x+c^3)/f/(1+exp(2*f*x+2*e))+6/f^2*d^2*c*ln(1+exp(2*f*x+2*e))*x+3/
f^3*d^2*c*polylog(2,-exp(2*f*x+2*e))-6/f*d^2*c*x^2-6/f^3*d^2*c*e^2-2/f*d^3
*x^3+4/f^4*d^3*e^3-6/f^4*d^3*e^2*ln(exp(f*x+e))+6/f^3*d^3*e^2*x+3/f^3*d^3*
polylog(2,-exp(2*f*x+2*e))*x-6/f^2*d*c^2*ln(exp(f*x+e))+3/f^2*d*c^2*ln(1+e
xp(2*f*x+2*e))+12/f^3*d^2*c*e*ln(exp(f*x+e))-12/f^2*d^2*c*e*x+3/f^2*d^3*ln
(1+exp(2*f*x+2*e))*x^2-3/2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.11 (sec) , antiderivative size = 1505, normalized size of antiderivative = 12.65

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="fricas")
```

output

```
1/4*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*c^2*d*f^4*x^2 + 4*c^3*f^4*x - 8*d^3
*e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 8*c^3*f^3 + (d^3*f^4*x^4 - 8*d^3*
e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d^2*f^4 - 2*d^3*f^3)*x^3 + 6*
(c^2*d*f^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6*c^2*d*f^3)*x)*cosh(f*x + e)
^2 + 2*(d^3*f^4*x^4 - 8*d^3*e^3 + 24*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d
^2*f^4 - 2*d^3*f^3)*x^3 + 6*(c^2*d*f^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6
*c^2*d*f^3)*x)*cosh(f*x + e)*sinh(f*x + e) + (d^3*f^4*x^4 - 8*d^3*e^3 + 24
*c*d^2*e^2*f - 24*c^2*d*e*f^2 + 4*(c*d^2*f^4 - 2*d^3*f^3)*x^3 + 6*(c^2*d*f
^4 - 4*c*d^2*f^3)*x^2 + 4*(c^3*f^4 - 6*c^2*d*f^3)*x)*sinh(f*x + e)^2 + 24*
(d^3*f*x + c*d^2*f + (d^3*f*x + c*d^2*f)*cosh(f*x + e)^2 + 2*(d^3*f*x + c*
d^2*f)*cosh(f*x + e)*sinh(f*x + e) + (d^3*f*x + c*d^2*f)*sinh(f*x + e)^2)*
dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) + 24*(d^3*f*x + c*d^2*f + (d^3*f*
x + c*d^2*f)*cosh(f*x + e)^2 + 2*(d^3*f*x + c*d^2*f)*cosh(f*x + e)*sinh(f*
x + e) + (d^3*f*x + c*d^2*f)*sinh(f*x + e)^2)*dilog(-I*cosh(f*x + e) - I*s
inh(f*x + e)) + 12*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2
*e*f + c^2*d*f^2)*cosh(f*x + e)^2 + 2*(d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*
cosh(f*x + e)*sinh(f*x + e) + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*sinh(f*x
 + e)^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) + 12*(d^3*e^2 - 2*c*d^2*e*
f + c^2*d*f^2 + (d^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cosh(f*x + e)^2 + 2*(d
^3*e^2 - 2*c*d^2*e*f + c^2*d*f^2)*cosh(f*x + e)*sinh(f*x + e) + (d^3*e^...
```

Sympy [F]

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \int (c+dx)^3 \tanh^2(e+fx)\,dx$$

input
```
integrate((d*x+c)**3*tanh(f*x+e)**2,x)
```

output
```
Integral((c + d*x)**3*tanh(e + f*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(114) = 228.

Time = 0.24 (sec) , antiderivative size = 343, normalized size of antiderivative = 2.88

$$\begin{aligned}
\int (c+dx)^3 \tanh^2(e+fx)\,dx &= c^3\left(x+\frac{e}{f}-\frac{2}{f\left(e^{(-2fx-2e)}+1\right)}\right) \\
&-\frac{3}{2}c^2 d\left(\frac{2xe^{(2fx+2e)}}{fe^{(2fx+2e)}+f}-\frac{fx^2+\left(fx^2e^{(2e)}-2xe^{(2e)}\right)e^{(2fx)}}{fe^{(2fx+2e)}+f}-\frac{2\log\left(\left(e^{(2fx+2e)}+1\right)e^{(-2e)}\right)}{f^2}\right) \\
&+\frac{3\left(2fx\log\left(e^{(2fx+2e)}+1\right)+\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)cd^2}{f^3} \\
&+\frac{d^3fx^4+24cd^2x^2+4\left(cd^2f+2d^3\right)x^3+\left(d^3fx^4e^{(2e)}+4cd^2fx^3e^{(2e)}\right)e^{(2fx)}}{4\left(fe^{(2fx+2e)}+f\right)} \\
&+\frac{3\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right)+2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)d^3}{2f^4} \\
&-\frac{2\left(d^3f^3x^3+3cd^2f^3x^2\right)}{f^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
c^3*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - 3/2*c^2*d*(2*x*e^(2*f*x + 2
*e)/(f*e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*
f*x))/(f*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2)
 + 3*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*c*d^2/f^3
+ 1/4*(d^3*f*x^4 + 24*c*d^2*x^2 + 4*(c*d^2*f + 2*d^3)*x^3 + (d^3*f*x^4*e^(
2*e) + 4*c*d^2*f*x^3*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 3/2*(2*
f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog
(3, -e^(2*f*x + 2*e)))*d^3/f^4 - 2*(d^3*f^3*x^3 + 3*c*d^2*f^3*x^2)/f^4
```

Giac [F]

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \int (dx+c)^3 \tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tanh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx = \int \tanh(e+f\,x)^2\,(c+d\,x)^3\,dx$$

input
```
int(tanh(e + f*x)^2*(c + d*x)^3,x)
```

output
```
int(tanh(e + f*x)^2*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \tanh^2(e+fx)\,dx$$

$$= \frac{-24\left(\int \frac{x}{e^{4fx+4e}+2e^{2fx+2e}+1}dx\right) d^3f^2 - 24\left(\int \frac{x^2}{e^{4fx+4e}+2e^{2fx+2e}+1}dx\right) d^3f^3 + 6e^{2fx+2e}\log\left(e^{2fx+2e}+1\right) d^3 - 8e^{2}$$

input

```
int((d*x+c)^3*tanh(f*x+e)^2,x)
```

output

```
( - 24*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) +
1),x)*d**3*f**3 - 48*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 2*e**(2*e
+ 2*f*x) + 1),x)*c*d**2*f**3 - 24*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x)
 + 2*e**(2*e + 2*f*x) + 1),x)*d**3*f**2 + 12*e**(2*e + 2*f*x)*log(e**(2*e
+ 2*f*x) + 1)*c**2*d*f**2 + 12*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*
c*d**2*f + 6*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*d**3 + 4*e**(2*e +
 2*f*x)*c**3*f**4*x - 8*e**(2*e + 2*f*x)*c**3*f**3 + 6*e**(2*e + 2*f*x)*c*
*2*d*f**4*x**2 - 24*e**(2*e + 2*f*x)*c**2*d*f**3*x + 4*e**(2*e + 2*f*x)*c*
d**2*f**4*x**3 - 24*e**(2*e + 2*f*x)*c*d**2*f**2*x + e**(2*e + 2*f*x)*d**3
*f**4*x**4 - 12*e**(2*e + 2*f*x)*d**3*f*x - 24*int(x**2/(e**(4*e + 4*f*x)
+ 2*e**(2*e + 2*f*x) + 1),x)*d**3*f**3 - 48*int(x/(e**(4*e + 4*f*x) + 2*e*
*(2*e + 2*f*x) + 1),x)*c*d**2*f**3 - 24*int(x/(e**(4*e + 4*f*x) + 2*e**(2*
e + 2*f*x) + 1),x)*d**3*f**2 + 12*log(e**(2*e + 2*f*x) + 1)*c**2*d*f**2 +
12*log(e**(2*e + 2*f*x) + 1)*c*d**2*f + 6*log(e**(2*e + 2*f*x) + 1)*d**3 +
 4*c**3*f**4*x + 6*c**2*d*f**4*x**2 + 4*c*d**2*f**4*x**3 + 24*c*d**2*f**3*
x**2 + d**3*f**4*x**4 + 8*d**3*f**3*x**3 + 12*d**3*f**2*x**2)/(4*f**4*(e**
(2*e + 2*f*x) + 1))
```

3.7 $\int (c+dx)^2 \tanh^2(e+fx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 88

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = -\frac{(c+dx)^2}{f} + \frac{(c+dx)^3}{3d} + \frac{2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2} + \frac{d^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3} - \frac{(c+dx)^2\tanh(e+fx)}{f}$$

output

```
-(d*x+c)^2/f+1/3*(d*x+c)^3/d+2*d*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^2+d^2*poly
log(2,-exp(2*f*x+2*e))/f^3-(d*x+c)^2*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.31

$$\begin{aligned}&\int (c+dx)^2 \tanh^2(e+fx)\,dx \\ &= c^2x + cdx^2 + \frac{d^2x^3}{3} \\ &\quad + \frac{\frac{2f(c+dx)\left(f(c+dx)+d\left(1+e^{2e}\right)\log\left(1+e^{-2(e+fx)}\right)\right)}{1+e^{2e}} - d^2\,\mathrm{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{f^3} \\ &\quad - \frac{(c+dx)^2\mathrm{sech}(e)\mathrm{sech}(e+fx)\sinh(fx)}{f}\end{aligned}$$

input
```
Integrate[(c + d*x)^2*Tanh[e + f*x]^2,x]
```

output
```
c^2*x + c*d*x^2 + (d^2*x^3)/3 + ((2*f*(c + d*x)*(f*(c + d*x) + d*(1 + E^(2
*e))*Log[1 + E^(-2*(e + f*x))]))/(1 + E^(2*e)) - d^2*PolyLog[2, -E^(-2*(e
+ f*x))])/f^3 - ((c + d*x)^2*Sech[e]*Sech[e + f*x]*Sinh[f*x])/f
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.52 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.23, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.688$, Rules used = {3042, 25, 4203, 17, 26, 3042, 26, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int -(c+dx)^2 \tan(ie+ifx)^2 dx$$

$$\downarrow 25$$

$$-\int (c+dx)^2 \tan(ie+ifx)^2 dx$$

$$\downarrow 4203$$

$$-\frac{2id\int i(c+dx)\tanh(e+fx)dx}{f} + \int (c+dx)^2 dx - \frac{(c+dx)^2\tanh(e+fx)}{f}$$

$$\downarrow 17$$

$$-\frac{2id\int i(c+dx)\tanh(e+fx)dx}{f} - \frac{(c+dx)^2\tanh(e+fx)}{f} + \frac{(c+dx)^3}{3d}$$

$$\downarrow 26$$

$$\frac{2d\int (c+dx)\tanh(e+fx)dx}{f} - \frac{(c+dx)^2\tanh(e+fx)}{f} + \frac{(c+dx)^3}{3d}$$

$$\downarrow 3042$$

$$\frac{2d\int -i(c+dx)\tan(ie+ifx)dx}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\downarrow 26$$

$$-\frac{2id\int (c+dx)\tan(ie+ifx)dx}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\downarrow 4201$$

$$-\frac{2id\left(2i\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}dx-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\downarrow 2620$$

$$-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int \log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\downarrow 2715$$

$$-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

$$\downarrow 2838$$

$$-\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}+\frac{d\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}-\frac{(c+dx)^2\tanh(e+fx)}{f}+\frac{(c+dx)^3}{3d}$$

input

```
Int[(c + d*x)^2*Tanh[e + f*x]^2,x]
```

output

```
(c + d*x)^3/(3*d) - ((2*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c + d*x)
*Log[1 + E^(2*(e + f*x))])/(2*f) + (d*PolyLog[2, -E^(2*(e + f*x))])/(4*f^2
))))/f - ((c + d*x)^2*Tanh[e + f*x])/f
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(86) = 172$.

Time = 0.40 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.10

method	result
risch	$\frac{d^2x^3}{3} + dcx^2 + c^2x + \frac{c^3}{3d} + \frac{2x^2d^2+4cdx+2c^2}{f\left(1+\mathrm{e}^{2fx+2e}\right)} - \frac{4dc\ln\left(\mathrm{e}^{fx+e}\right)}{f^2} + \frac{2dc\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f^2} - \frac{2d^2x^2}{f} - \frac{4d^2ex}{f^2} - \frac{2d^2e^2}{f^3}$

input

```
int((d*x+c)^2*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output

```
1/3*d^2*x^3+d*c*x^2+c^2*x+1/3/d*c^3+2*(d^2*x^2+2*c*d*x+c^2)/f/(1+exp(2*f*x
+2*e))-4/f^2*d*c*ln(exp(f*x+e))+2/f^2*d*c*ln(1+exp(2*f*x+2*e))-2/f*d^2*x^2
-4/f^2*d^2*e*x-2/f^3*d^2*e^2+2/f^2*d^2*ln(1+exp(2*f*x+2*e))*x+d^2*polylog(
2,-exp(2*f*x+2*e))/f^3+4/f^3*d^2*e*ln(exp(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 840, normalized size of antiderivative = 9.55

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="fricas")
```

output

```
1/3*(d^2*f^3*x^3 + 3*c*d*f^3*x^2 + 3*c^2*f^3*x + 6*d^2*e^2 - 12*c*d*e*f +
6*c^2*f^2 + (d^2*f^3*x^3 + 6*d^2*e^2 - 12*c*d*e*f + 3*(c*d*f^3 - 2*d^2*f^2
)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)*x)*cosh(f*x + e)^2 + 2*(d^2*f^3*x^3 + 6*d^
2*e^2 - 12*c*d*e*f + 3*(c*d*f^3 - 2*d^2*f^2)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)
*x)*cosh(f*x + e)*sinh(f*x + e) + (d^2*f^3*x^3 + 6*d^2*e^2 - 12*c*d*e*f +
3*(c*d*f^3 - 2*d^2*f^2)*x^2 + 3*(c^2*f^3 - 4*c*d*f^2)*x)*sinh(f*x + e)^2 +
 6*(d^2*cosh(f*x + e)^2 + 2*d^2*cosh(f*x + e)*sinh(f*x + e) + d^2*sinh(f*x
 + e)^2 + d^2)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) + 6*(d^2*cosh(f*x
+ e)^2 + 2*d^2*cosh(f*x + e)*sinh(f*x + e) + d^2*sinh(f*x + e)^2 + d^2)*di
log(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 6*(d^2*e - c*d*f + (d^2*e - c*d*
f)*cosh(f*x + e)^2 + 2*(d^2*e - c*d*f)*cosh(f*x + e)*sinh(f*x + e) + (d^2*
e - c*d*f)*sinh(f*x + e)^2)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 6*(d^
2*e - c*d*f + (d^2*e - c*d*f)*cosh(f*x + e)^2 + 2*(d^2*e - c*d*f)*cosh(f*x
 + e)*sinh(f*x + e) + (d^2*e - c*d*f)*sinh(f*x + e)^2)*log(cosh(f*x + e) +
 sinh(f*x + e) - I) + 6*(d^2*f*x + d^2*e + (d^2*f*x + d^2*e)*cosh(f*x + e)
^2 + 2*(d^2*f*x + d^2*e)*cosh(f*x + e)*sinh(f*x + e) + (d^2*f*x + d^2*e)*s
inh(f*x + e)^2)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 6*(d^2*f*x +
d^2*e + (d^2*f*x + d^2*e)*cosh(f*x + e)^2 + 2*(d^2*f*x + d^2*e)*cosh(f*x +
 e)*sinh(f*x + e) + (d^2*f*x + d^2*e)*sinh(f*x + e)^2)*log(-I*cosh(f*x + e
) - I*sinh(f*x + e) + 1))/(f^3*cosh(f*x + e)^2 + 2*f^3*cosh(f*x + e)*si...
```

Sympy [F]

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int (c+dx)^2 \tanh^2(e+fx)\,dx$$

input

```
integrate((d*x+c)**2*tanh(f*x+e)**2,x)
```

output

```
Integral((c + d*x)**2*tanh(e + f*x)**2, x)
```

Maxima [F]

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int (dx+c)^2 \tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
c^2*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - c*d*(2*x*e^(2*f*x + 2*e)/(f
*e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*f*x))/
(f*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) + 1/3
*d^2*((f*x^3*e^(2*f*x + 2*e) + f*x^3 + 6*x^2)/(f*e^(2*f*x + 2*e) + f) - 12
*integrate(x/(f*e^(2*f*x + 2*e) + f), x))
```

Giac [F]

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int (dx+c)^2 \tanh\left(fx+e\right)^2\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tanh(f*x + e)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx = \int \tanh(e+f\,x)^2\,(c+d\,x)^2\,dx$$

input
```
int(tanh(e + f*x)^2*(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \tanh^2(e+fx)\,dx$$

$$= \frac{-12e^{2fx+2e}\left(\int \frac{x}{e^{4fx+4e}+2e^{2fx+2e}+1}dx\right) d^2f^2 + 6e^{2fx+2e}\log\left(e^{2fx+2e}+1\right) cdf + 3e^{2fx+2e}\log\left(e^{2fx+2e}+1\right) d^2 +$$

input

```
int((d*x+c)^2*tanh(f*x+e)^2,x)
```

output

```
( - 12*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),
x)*d**2*f**2 + 6*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*c*d*f + 3*e**(
2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*d**2 + 3*e**(2*e + 2*f*x)*c**2*f**3
*x - 6*e**(2*e + 2*f*x)*c**2*f**2 + 3*e**(2*e + 2*f*x)*c*d*f**3*x**2 - 12*
e**(2*e + 2*f*x)*c*d*f**2*x + e**(2*e + 2*f*x)*d**2*f**3*x**3 - 6*e**(2*e
+ 2*f*x)*d**2*f*x - 12*int(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x
)*d**2*f**2 + 6*log(e**(2*e + 2*f*x) + 1)*c*d*f + 3*log(e**(2*e + 2*f*x) +
 1)*d**2 + 3*c**2*f**3*x + 3*c*d*f**3*x**2 + d**2*f**3*x**3 + 6*d**2*f**2*
x**2)/(3*f**3*(e**(2*e + 2*f*x) + 1))
```

3.8 $\int (c+dx)\tanh^2(e+fx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 43

$$\int (c+dx)\tanh^2(e+fx)\,dx = \frac{(c+dx)^2}{2d} + \frac{d\log(\cosh(e+fx))}{f^2} - \frac{(c+dx)\tanh(e+fx)}{f}$$

output
```
1/2*(d*x+c)^2/d+d*ln(cosh(f*x+e))/f^2-(d*x+c)*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.79

$$\begin{aligned}\int (c+dx)\tanh^2(e+fx)\,dx = {} & \frac{c\operatorname{arctanh}(\tanh(e+fx))}{f} + \frac{d\log(\cosh(e+fx))}{f^2} \\ & + \frac{dx\operatorname{sech}(e)(fx\cosh(e) - 2\sinh(e))}{2f} \\ & - \frac{dx\operatorname{sech}(e)\operatorname{sech}(e+fx)\sinh(fx)}{f} - \frac{c\tanh(e+fx)}{f}\end{aligned}$$

input
```
Integrate[(c + d*x)*Tanh[e + f*x]^2,x]
```

output

```
(c*ArcTanh[Tanh[e + f*x]])/f + (d*Log[Cosh[e + f*x]])/f^2 + (d*x*Sech[e]*(
f*x*Cosh[e] - 2*Sinh[e]))/(2*f) - (d*x*Sech[e]*Sech[e + f*x]*Sinh[f*x])/f
- (c*Tanh[e + f*x])/f
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 25, 4203, 17, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tanh^2(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int -((c+dx)\tan(ie+ifx)^2)\,dx$$

$$\downarrow 25$$

$$-\int (c+dx)\tan(ie+ifx)^2 dx$$

$$\downarrow 4203$$

$$\int (c+dx)dx - \frac{id\int i\tanh(e+fx)dx}{f} - \frac{(c+dx)\tanh(e+fx)}{f}$$

$$\downarrow 17$$

$$-\frac{id\int i\tanh(e+fx)dx}{f} - \frac{(c+dx)\tanh(e+fx)}{f} + \frac{(c+dx)^2}{2d}$$

$$\downarrow 26$$

$$\frac{d\int \tanh(e+fx)dx}{f} - \frac{(c+dx)\tanh(e+fx)}{f} + \frac{(c+dx)^2}{2d}$$

$$\downarrow 3042$$

$$\frac{d\int -i\tan(ie+ifx)dx}{f} - \frac{(c+dx)\tanh(e+fx)}{f} + \frac{(c+dx)^2}{2d}$$

$$\downarrow 26$$

$$-\frac{id\int \tan(ie+ifx)dx}{f}-\frac{(c+dx)\tanh(e+fx)}{f}+\frac{(c+dx)^2}{2d}$$

$$\downarrow 3956$$

$$-\frac{(c+dx)\tanh(e+fx)}{f}+\frac{(c+dx)^2}{2d}+\frac{d\log(\cosh(e+fx))}{f^2}$$

input
```
Int[(c + d*x)*Tanh[e + f*x]^2,x]
```

output
```
(c + d*x)^2/(2*d) + (d*Log[Cosh[e + f*x]])/f^2 - ((c + d*x)*Tanh[e + f*x])
/f
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.47

method	result	size
parallelrisch	$-\frac{-d x^2 f^2+2\tanh(fx+e)dfx-2cx f^2+2dxf+2\tanh(fx+e)cf+2\ln(1-\tanh(fx+e))d}{2f^2}$	63
risch	$\frac{d x^2}{2}+cx-\frac{2dx}{f}-\frac{2de}{f^2}+\frac{2dx+2c}{f\left(1+\mathrm{e}^{2fx+2e}\right)}+\frac{d\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f^2}$	65

input

```
int((d*x+c)*tanh(f*x+e)^2,x,method=_RETURNVERBOSE)
```

output

```
-1/2*(-d*x^2*f^2+2*tanh(f*x+e)*d*f*x-2*c*x*f^2+2*d*x*f+2*tanh(f*x+e)*c*f+2
*ln(1-tanh(f*x+e))*d)/f^2
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(41) = 82$.

Time = 0.11 (sec) , antiderivative size = 232, normalized size of antiderivative = 5.40

$$\int (c+dx)\tanh^2(e+fx)\,dx$$

$$= \frac{df^2x^2+2\,cf^2x+(df^2x^2+2\,(cf^2-2\,df)x)\cosh(fx+e)^2+2\,(df^2x^2+2\,(cf^2-2\,df)x)\cosh(fx+e)\sin}{2\left(f^2\cosh\right.}$$

input

```
integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="fricas")
```

output
```
1/2*(d*f^2*x^2 + 2*c*f^2*x + (d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*cosh(f*x +
e)^2 + 2*(d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*cosh(f*x + e)*sinh(f*x + e) + (
d*f^2*x^2 + 2*(c*f^2 - 2*d*f)*x)*sinh(f*x + e)^2 + 4*c*f + 2*(d*cosh(f*x +
 e)^2 + 2*d*cosh(f*x + e)*sinh(f*x + e) + d*sinh(f*x + e)^2 + d)*log(2*cos
h(f*x + e)/(cosh(f*x + e) - sinh(f*x + e))))/(f^2*cosh(f*x + e)^2 + 2*f^2*
cosh(f*x + e)*sinh(f*x + e) + f^2*sinh(f*x + e)^2 + f^2)
```

Sympy [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.53

$$\int (c+dx)\tanh^2(e+fx)\,dx = \begin{cases} cx - \frac{c\tanh(e+fx)}{f} + \frac{dx^2}{2} - \frac{dx\tanh(e+fx)}{f} + \frac{dx}{f} - \frac{d\log(\tanh(e+fx)+1)}{f^2} & \text{for } f \neq 0 \\ \left(cx + \frac{dx^2}{2}\right)\tanh^2(e) & \text{otherwise} \end{cases}$$

input
```
integrate((d*x+c)*tanh(f*x+e)**2,x)
```

output
```
Piecewise((c*x - c*tanh(e + f*x)/f + d*x**2/2 - d*x*tanh(e + f*x)/f + d*x/
f - d*log(tanh(e + f*x) + 1)/f**2, Ne(f, 0)), ((c*x + d*x**2/2)*tanh(e)**2
, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. 2(41) = 82.

Time = 0.09 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.95

$$\int (c+dx)\tanh^2(e+fx)\,dx = c\left(x + \frac{e}{f} - \frac{2}{f\left(e^{(-2fx-2e)}+1\right)}\right) - \frac{1}{2}d\left(\frac{2\,xe^{(2fx+2e)}}{fe^{(2fx+2e)}+f} - \frac{fx^2 + \left(fx^2e^{(2e)} - 2\,xe^{(2e)}\right)e^{(2fx)}}{fe^{(2fx+2e)}+f} - \frac{2\log\left(\left(e^{(2fx+2e)}+1\right)e^{(-2e)}\right)}{f^2}\right)$$

input
```
integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="maxima")
```

output
```
c*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) - 1/2*d*(2*x*e^(2*f*x + 2*e)/(f
*e^(2*f*x + 2*e) + f) - (f*x^2 + (f*x^2*e^(2*e) - 2*x*e^(2*e))*e^(2*f*x))/
(f*e^(2*f*x + 2*e) + f) - 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(41) = 82$.

Time = 0.14 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.95

$$\int (c+dx)\tanh^2(e+fx)\,dx$$
$$= \frac{df^2x^2e^{(2fx+2e)} + df^2x^2 + 2cf^2xe^{(2fx+2e)} + 2cf^2x - 4dfxe^{(2fx+2e)} + 2de^{(2fx+2e)}\log\left(e^{(2fx+2e)}+1\right) +}{2\left(f^2e^{(2fx+2e)} + f^2\right)}$$

input
```
integrate((d*x+c)*tanh(f*x+e)^2,x, algorithm="giac")
```

output
```
1/2*(d*f^2*x^2*e^(2*f*x + 2*e) + d*f^2*x^2 + 2*c*f^2*x*e^(2*f*x + 2*e) + 2
*c*f^2*x - 4*d*f*x*e^(2*f*x + 2*e) + 2*d*e^(2*f*x + 2*e)*log(e^(2*f*x + 2*
e) + 1) + 4*c*f + 2*d*log(e^(2*f*x + 2*e) + 1))/(f^2*e^(2*f*x + 2*e) + f^2
)
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.30

$$\int (c+dx)\tanh^2(e+fx)\,dx = x\left(c+\frac{d}{f}\right) + \frac{d\,x^2}{2} - \frac{d\,\ln\left(\tanh(e+f\,x)+1\right)}{f^2} - \frac{c\tanh(e+f\,x)}{f} - \frac{d\,x\tanh(e+f\,x)}{f}$$

input
```
int(tanh(e + f*x)^2*(c + d*x),x)
```

output
```
x*(c + d/f) + (d*x^2)/2 - (d*log(tanh(e + f*x) + 1))/f^2 - (c*tanh(e + f*x
))/f - (d*x*tanh(e + f*x))/f
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.28

$$\int (c+dx)\tanh^2(e+fx)\,dx$$
$$= \frac{2e^{2fx+2e}\log\left(e^{2fx+2e}+1\right)d + 2e^{2fx+2e}cf^2x - 4e^{2fx+2e}cf + e^{2fx+2e}df^2x^2 - 4e^{2fx+2e}dfx + 2\log\left(e^{2fx+2e}+\right.}{2f^2\left(e^{2fx+2e}+1\right)}$$

input
```
int((d*x+c)*tanh(f*x+e)^2,x)
```

output
```
(2*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*d + 2*e**(2*e + 2*f*x)*c*f**
2*x - 4*e**(2*e + 2*f*x)*c*f + e**(2*e + 2*f*x)*d*f**2*x**2 - 4*e**(2*e +
2*f*x)*d*f*x + 2*log(e**(2*e + 2*f*x) + 1)*d + 2*c*f**2*x + d*f**2*x**2)/(
2*f**2*(e**(2*e + 2*f*x) + 1))
```

3.9 $\int \frac{\tanh^2(e+fx)}{c+dx}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \text{Int}\left(\frac{\tanh^2(e+fx)}{c+dx}, x\right)$$

output
```
Defer(Int)(tanh(f*x+e)^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 21.50 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh^2(e+fx)}{c+dx}\,dx$$

input
```
Integrate[Tanh[e + f*x]^2/(c + d*x),x]
```

output
```
Integrate[Tanh[e + f*x]^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\tan(ie+ifx)^2}{c+dx}dx$$
$$\downarrow 25$$
$$-\int \frac{\tan(ie+ifx)^2}{c+dx}dx$$
$$\downarrow 4222$$
$$\int \frac{\tanh^2(e+fx)}{c+dx}dx$$

input
```
Int[Tanh[e + f*x]^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(fx+e\right)^2}{dx+c}dx$$

input
```
int(tanh(f*x+e)^2/(d*x+c),x)
```

output
```
int(tanh(f*x+e)^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)^2}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^2/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh^2\left(e+fx\right)}{c+dx}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(d*x+c),x)
```

output
```
Integral(tanh(e + f*x)**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 110, normalized size of antiderivative = 6.88

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)^2}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="maxima")
```

output
```
2*d*integrate(1/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^(2*e) + 2*c*
d*f*x*e^(2*e) + c^2*f*e^(2*e))*e^(2*f*x)), x) + log(d*x + c)/d + 2/(d*f*x
+ c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(2*f*x))
```

Giac [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)^2}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+f\,x)^2}{c+d\,x}\,dx$$

input
```
int(tanh(e + f*x)^2/(c + d*x),x)
```

output
```
int(tanh(e + f*x)^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 87, normalized size of antiderivative = 5.44

$$\int \frac{\tanh^2(e+fx)}{c+dx}\,dx = \frac{-4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c+e^{4fx+4e}dx+2e^{2fx+2e}c+2e^{2fx+2e}dx+c+dx}dx\right)d+\log(dx+c)}{d}$$

input
```
int(tanh(f*x+e)^2/(d*x+c),x)
```

output
```
( - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c + e**(4*e + 4*f*x)*d*x + 
 2*e**(2*e + 2*f*x)*c + 2*e**(2*e + 2*f*x)*d*x + c + d*x),x)*d + log(c + d
*x))/d
```

3.10 $\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$

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Rubi [N/A] 119
Maple [N/A] 119
Fricas [N/A] 120
Sympy [N/A] 120
Maxima [N/A] 121
Giac [N/A] 121
Mupad [N/A] 122
Reduce [N/A] 122

Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tanh^2(e+fx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(tanh(f*x+e)^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 23.35 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tanh[e + f*x]^2/(c + d*x)^2,x]
```

output
```
Integrate[Tanh[e + f*x]^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int -\frac{\tan(ie+ifx)^2}{(c+dx)^2}dx$$
$$\downarrow 25$$
$$-\int \frac{\tan(ie+ifx)^2}{(c+dx)^2}dx$$
$$\downarrow 4222$$
$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}dx$$

input
```
Int[Tanh[e + f*x]^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(fx+e\right)^2}{\left(dx+c\right)^2}dx$$

input
```
int(tanh(f*x+e)^2/(d*x+c)^2,x)
```

output
```
int(tanh(f*x+e)^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^2/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^2\left(e+fx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)**2/(d*x+c)**2,x)
```

output
```
Integral(tanh(e + f*x)**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 12.19

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
4*d*integrate(1/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f + (d^3*f*
x^3*e^(2*e) + 3*c*d^2*f*x^2*e^(2*e) + 3*c^2*d*f*x*e^(2*e) + c^3*f*e^(2*e))
*e^(2*f*x)), x) - (d*f*x + c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^(2*f*x) -
 2*d)/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2*e^(2*e) + 2*c*d^2*f*
x*e^(2*e) + c^2*d*f*e^(2*e))*e^(2*f*x))
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^2}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.11 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+f\,x)^2}{(c+d\,x)^2}\,dx$$

input
```
int(tanh(e + f*x)^2/(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 275, normalized size of antiderivative = 17.19

$$\int \frac{\tanh^2(e+fx)}{(c+dx)^2}\,dx$$
$$= \frac{-4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c^2+2e^{4fx+4e}cdx+e^{4fx+4e}d^2x^2+2e^{2fx+2e}c^2+4e^{2fx+2e}cdx+2e^{2fx+2e}d^2x^2+c^2+2cdx+d^2x^2}dx\right)c^2 - 4e^{2e}\left(\int \frac{e^{4fx}}{}\right.}{c\,(dx+c)}$$

input
```
int(tanh(f*x+e)^2/(d*x+c)^2,x)
```

output
```
( - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*
c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*e**(2*e + 2*f*x)*c**2 + 4*e**(2*e +
 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2)
,x)*c**2 - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e +
4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*e**(2*e + 2*f*x)*c**2 + 4*e*
*(2*e + 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**
2*x**2),x)*c*d*x + x)/(c*(c + d*x))
```

3.11 $\int (c+dx)^3 \tanh^3(e+fx)\,dx$

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Giac [F] . . . 133
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Reduce [F] . . . 134

Optimal result

Integrand size = 16, antiderivative size = 237

$$\begin{aligned}\int (c+dx)^3 \tanh^3(e+fx)\,dx = &-\frac{3d(c+dx)^2}{2f^2}+\frac{(c+dx)^3}{2f}-\frac{(c+dx)^4}{4d}\\&+\frac{3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3}\\&+\frac{(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f}\\&+\frac{3d^3\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^4}\\&+\frac{3d(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}\\&-\frac{3d^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}\\&+\frac{3d^3\,\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}\\&-\frac{3d(c+dx)^2\tanh(e+fx)}{2f^2}-\frac{(c+dx)^3\tanh^2(e+fx)}{2f}\end{aligned}$$

output
```
-3/2*d*(d*x+c)^2/f^2+1/2*(d*x+c)^3/f-1/4*(d*x+c)^4/d+3*d^2*(d*x+c)*ln(1+ex
p(2*f*x+2*e))/f^3+(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2*d^3*polylog(2,-exp(
2*f*x+2*e))/f^4+3/2*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*d^2*(d*
x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/4*d^3*polylog(4,-exp(2*f*x+2*e))/f^4
-3/2*d*(d*x+c)^2*tanh(f*x+e)/f^2-1/2*(d*x+c)^3*tanh(f*x+e)^2/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 496 vs. $2(237) = 474$.

Time = 6.81 (sec) , antiderivative size = 496, normalized size of antiderivative = 2.09

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx$$

$$= \frac{-8c(1+e^{2e})\left(3d^2+c^2f^2\right)x + \frac{2\left(c^2f^2+2cdf^2x+d^2\left(3+f^2x^2\right)\right)^2}{df^2} + \frac{12d\left(1+e^{2e}\right)\left(d^2+c^2f^2\right)x\log\left(1+e^{-2(e+fx)}\right)}{f} + 12cd^2(1+e}{}$$

$$+ \frac{(c+dx)^3\operatorname{sech}^2(e+fx)}{2f}$$

$$- \frac{3\operatorname{sech}(e)\operatorname{sech}(e+fx)\left(c^2d\sinh(fx)+2cd^2x\sinh(fx)+d^3x^2\sinh(fx)\right)}{2f^2}$$

$$+ \frac{1}{4}x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)\tanh(e)$$

input
```
Integrate[(c + d*x)^3*Tanh[e + f*x]^3,x]
```

output
```
(-8*c*(1 + E^(2*e))*(3*d^2 + c^2*f^2)*x + (2*(c^2*f^2 + 2*c*d*f^2*x + d^2*
(3 + f^2*x^2))^2)/(d*f^2) + (12*d*(1 + E^(2*e))*(d^2 + c^2*f^2)*x*Log[1 +
E^(-2*(e + f*x))])/f + 12*c*d^2*(1 + E^(2*e))*f*x^2*Log[1 + E^(-2*(e + f*x
))] + 4*d^3*(1 + E^(2*e))*f*x^3*Log[1 + E^(-2*(e + f*x))] + (4*c*(1 + E^(2
*e))*(3*d^2 + c^2*f^2)*Log[1 + E^(2*(e + f*x))])/f - (6*d*(1 + E^(2*e))*(d
^2 + c^2*f^2)*PolyLog[2, -E^(-2*(e + f*x))])/f^2 - 12*c*d^2*(1 + E^(2*e))*
x*PolyLog[2, -E^(-2*(e + f*x))] - 6*d^3*(1 + E^(2*e))*x^2*PolyLog[2, -E^(-
2*(e + f*x))] - (6*c*d^2*(1 + E^(2*e))*PolyLog[3, -E^(-2*(e + f*x))])/f -
(6*d^3*(1 + E^(2*e))*x*PolyLog[3, -E^(-2*(e + f*x))])/f - (3*d^3*(1 + E^(2
*e))*PolyLog[4, -E^(-2*(e + f*x))])/f^2)/(4*(1 + E^(2*e))*f^2) + ((c + d*x
)^3*Sech[e + f*x]^2)/(2*f) - (3*Sech[e]*Sech[e + f*x]*(c^2*d*Sinh[f*x] + 2
*c*d^2*x*Sinh[f*x] + d^3*x^2*Sinh[f*x]))/(2*f^2) + (x*(4*c^3 + 6*c^2*d*x +
 4*c*d^2*x^2 + d^3*x^3)*Tanh[e])/4
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.59 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.19, number of steps used = 24, number of rules used = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 1.438$, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 4203, 17, 26, 3042, 26, 4201, 2620, 2715, 2838, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int i(c+dx)^3 \tan(ie+ifx)^3 dx$$

$$\downarrow 26$$

$$i\int (c+dx)^3 \tan(ie+ifx)^3 dx$$

$$\downarrow 4203$$

$$i\left(\frac{3id\int -(c+dx)^2\tanh^2(e+fx)dx}{2f} - \int i(c+dx)^3\tanh(e+fx)dx + \frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 25$$

$$i\left(-\frac{3id\int (c+dx)^2\tanh^2(e+fx)dx}{2f} - \int i(c+dx)^3\tanh(e+fx)dx + \frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 26$$

$$i\left(-\frac{3id\int (c+dx)^2\tanh^2(e+fx)dx}{2f} - i\int (c+dx)^3\tanh(e+fx)dx + \frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 3042$$

$$i\left(-i\int -i(c+dx)^3\tan(ie+ifx)dx - \frac{3id\int -(c+dx)^2\tan(ie+ifx)^2dx}{2f} + \frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 25$$

$$i\left(-i\int -i(c+dx)^3\tan(ie+ifx)dx+\frac{3id\int(c+dx)^2\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 26$$

$$i\left(-\int(c+dx)^3\tan(ie+ifx)dx+\frac{3id\int(c+dx)^2\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 4201$$

$$i\left(-2i\int\frac{e^{2(e+fx)}(c+dx)^3}{1+e^{2(e+fx)}}dx+\frac{3id\int(c+dx)^2\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)^3\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^4}{4d}\right)$$

$$\downarrow 2620$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\int(c+dx)^2\log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)+\frac{3id\int(c+dx)^2\tan(ie+ifx)^2dx}{2f}\right.$$

$$\downarrow 3011$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\int}{}\right.$$

$$\downarrow 4203$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(}{}\right.$$

$$\downarrow 17$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(}{}\right.$$

$$\downarrow 26$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(-\right.}{}$$

$$\downarrow 3042$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(-\right.}{}$$

$$\downarrow 26$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(\right.}{}$$

$$\downarrow 4201$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(\right.}{}$$

$$\downarrow 2620$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(\right.}{}$$

$$\downarrow 2715$$

$$i\left(\frac{3id\left(\frac{2id\left(2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)-\frac{i(c+dx)^2}{2d}\right)}{f}+\frac{(c+dx)^2\tanh(e+fx)}{f}-\frac{(c+dx)^3}{3d}\right)}{2f}-$$

$$\downarrow 2838$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\int(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{2f}\right)+\frac{3id\left(\right.}{}$$

$$\downarrow 7163$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)dx}{2f}\right)}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,\right.}{2f}\right)}{2f}\right)$$

$$\downarrow 2720$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\int e^{-2(e+fx)}\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)}{f}-\frac{(c\right.}{2f}\right)$$

$$\downarrow 7143$$

$$i\left(-2i\left(\frac{(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{3d\left(\frac{d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f}-\frac{d\,\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^2}\right)}{f}-\frac{(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2}\right.}{2f}\right.}{2f}\right.$$

input
```
Int[(c + d*x)^3*Tanh[e + f*x]^3,x]
```

output
```
I*(((I/4)*(c + d*x)^4)/d - (2*I)*(((c + d*x)^3*Log[1 + E^(2*(e + f*x))])/(
2*f) - (3*d*(-1/2*((c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/f + (d*(((c +
 d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f) - (d*PolyLog[4, -E^(2*(e + f*x))
])/(4*f^2)))/f))/(2*f)) + ((I/2)*(c + d*x)^3*Tanh[e + f*x]^2)/f + (((3*I)/
2)*d*(-1/3*(c + d*x)^3/d + ((2*I)*d*(((-1/2*I)*(c + d*x)^2)/d + (2*I)*(((c
 + d*x)*Log[1 + E^(2*(e + f*x))])/(2*f) + (d*PolyLog[2, -E^(2*(e + f*x))])
/(4*f^2))))/f + ((c + d*x)^2*Tanh[e + f*x])/f))/f)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715

```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 2838

```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4201

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))  Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(219) = 438$.

Time = 0.62 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.89

method	result
risch	$\frac{c^3 \ln(1+e^{2fx+2e})}{f} - \frac{6c^2 dex}{f} - \frac{3d^3 e^4}{2f^4} - \frac{2c^3 \ln(e^{fx+e})}{f} - \frac{6d^2 c \ln(e^{fx+e})}{f^3} + \frac{3d^2 c \ln(1+e^{2fx+2e})}{f^3} + \frac{6e\,d^3 \ln(e^{fx+e})}{f^4} + \frac{3}{}$

input

```
int((d*x+c)^3*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output

```
1/f*c^3*ln(1+exp(2*f*x+2*e))-6/f*c^2*d*e*x-3/2/f^4*d^3*e^4-2/f*c^3*ln(exp(
f*x+e))-6/f^3*d^2*c*ln(exp(f*x+e))+3/f^3*d^2*c*ln(1+exp(2*f*x+2*e))+6/f^4*
e*d^3*ln(exp(f*x+e))+3/f^3*d^3*ln(1+exp(2*f*x+2*e))*x+1/f*d^3*ln(1+exp(2*f
*x+2*e))*x^3-d^2*c*x^3-3/2*d*c^2*x^2+c^3*x-6/f^3*d^3*e*x-3/f^2*c^2*d*e^2+4
/f^3*c*d^2*e^3+3/2/f^2*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/f^3*d^3*poly
log(3,-exp(2*f*x+2*e))*x+2/f^4*d^3*e^3*ln(exp(f*x+e))+(2*d^3*f*x^3*exp(2*f
*x+2*e)+6*c*d^2*f*x^2*exp(2*f*x+2*e)+6*c^2*d*f*x*exp(2*f*x+2*e)+3*d^3*x^2*
exp(2*f*x+2*e)+2*c^3*f*exp(2*f*x+2*e)+6*c*d^2*x*exp(2*f*x+2*e)+3*c^2*d*exp
(2*f*x+2*e)+3*d^3*x^2+6*c*d^2*x+3*d*c^2)/f^2/(1+exp(2*f*x+2*e))^2+6/f^2*c*
d^2*e^2*x+3/f*d^2*c*ln(1+exp(2*f*x+2*e))*x^2-2/f^3*d^3*e^3*x+3/2/f^2*c^2*d
*polylog(2,-exp(2*f*x+2*e))-3/2/f^3*c*d^2*polylog(3,-exp(2*f*x+2*e))+3/f^2
*c*d^2*polylog(2,-exp(2*f*x+2*e))*x-6/f^3*c*d^2*e^2*ln(exp(f*x+e))+6/f^2*c
^2*d*e*ln(exp(f*x+e))+3/f*c^2*d*ln(1+exp(2*f*x+2*e))*x-1/4*d^3*x^4+1/4/d*c
^4+3/2*d^3*polylog(2,-exp(2*f*x+2*e))/f^4+3/4*d^3*polylog(4,-exp(2*f*x+2*e
))/f^4-3/f^2*d^3*x^2-3/f^4*e^2*d^3
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 5569, normalized size of antiderivative = 23.50

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \int (c+dx)^3 \tanh^3(e+fx)\,dx$$

input
```
integrate((d*x+c)**3*tanh(f*x+e)**3,x)
```

output
```
Integral((c + d*x)**3*tanh(e + f*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 595 vs. 2(217) = 434.

Time = 0.21 (sec) , antiderivative size = 595, normalized size of antiderivative = 2.51

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
c^3*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-
2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) - 6*c*d^2*x/f^2 + 3/2*(2*f^2*x^2*lo
g(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*
f*x + 2*e)))*c*d^2/f^3 + 3*c*d^2*log(e^(2*f*x + 2*e) + 1)/f^3 + 1/4*(d^3*f
^2*x^4 + 4*c*d^2*f^2*x^3 + 24*c*d^2*x + 12*c^2*d + 6*(c^2*d*f^2 + 2*d^3)*x
^2 + (d^3*f^2*x^4*e^(4*e) + 4*c*d^2*f^2*x^3*e^(4*e) + 6*c^2*d*f^2*x^2*e^(4
*e))*e^(4*f*x) + 2*(d^3*f^2*x^4*e^(2*e) + 4*(c*d^2*f^2*e^(2*e) + d^3*f*e^(
2*e))*x^3 + 6*c^2*d*e^(2*e) + 6*(c^2*d*f^2*e^(2*e) + 2*c*d^2*f*e^(2*e) + d
^3*e^(2*e))*x^2 + 12*(c^2*d*f*e^(2*e) + c*d^2*e^(2*e))*x)*e^(2*f*x))/(f^2*
e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/3*(4*f^3*x^3*log(e^(2*f
*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(
2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*x + 2*e)))*d^3/f^4 + 3/2*(c^2*d*f^2 +
 d^3)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^4 - 1/2
*(d^3*f^4*x^4 + 4*c*d^2*f^4*x^3 + 6*(c^2*d*f^2 + d^3)*f^2*x^2)/f^4
```

Giac [F]

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \int (dx+c)^3 \tanh\left(fx+e\right)^3\,dx$$

input
```
integrate((d*x+c)^3*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*tanh(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \int \tanh(e+f\,x)^3\,(c+d\,x)^3\,dx$$

input
```
int(tanh(e + f*x)^3*(c + d*x)^3,x)
```

output
```
int(tanh(e + f*x)^3*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3 \tanh^3(e+fx)\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*tanh(f*x+e)^3,x)
```

output
```
( - 128*e**(4*e + 4*f*x)*int(x**3/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) +
 3*e**(2*e + 2*f*x) + 1),x)*d**3*f**4 - 384*e**(4*e + 4*f*x)*int(x**2/(e**
(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*c*d**2*f**
4 - 288*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) +
 3*e**(2*e + 2*f*x) + 1),x)*d**3*f**3 - 384*e**(4*e + 4*f*x)*int(x/(e**(6*
e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*c**2*d*f**4 -
 576*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**
(2*e + 2*f*x) + 1),x)*c*d**2*f**3 - 720*e**(4*e + 4*f*x)*int(x/(e**(6*e +
6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*d**3*f**2 + 64*e*
*(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*c**3*f**3 + 144*e**(4*e + 4*f*x)*
log(e**(2*e + 2*f*x) + 1)*c**2*d*f**2 + 360*e**(4*e + 4*f*x)*log(e**(2*e +
 2*f*x) + 1)*c*d**2*f + 234*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*d**
3 - 64*e**(4*e + 4*f*x)*c**3*f**4*x - 64*e**(4*e + 4*f*x)*c**3*f**3 + 96*e
**(4*e + 4*f*x)*c**2*d*f**4*x**2 - 288*e**(4*e + 4*f*x)*c**2*d*f**3*x - 72
*e**(4*e + 4*f*x)*c**2*d*f**2 + 64*e**(4*e + 4*f*x)*c*d**2*f**4*x**3 - 720
*e**(4*e + 4*f*x)*c*d**2*f**2*x + 36*e**(4*e + 4*f*x)*c*d**2*f + 16*e**(4*
e + 4*f*x)*d**3*f**4*x**4 - 468*e**(4*e + 4*f*x)*d**3*f*x + 45*e**(4*e + 4
*f*x)*d**3 - 256*e**(2*e + 2*f*x)*int(x**3/(e**(6*e + 6*f*x) + 3*e**(4*e +
 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*d**3*f**4 - 768*e**(2*e + 2*f*x)*int(
x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x...
```

3.12 $\int (c+dx)^2 \tanh^3(e+fx)\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 151

$$\begin{aligned}\int (c+dx)^2 \tanh^3(e+fx)\,dx = {} & \frac{(c+dx)^2}{2f} - \frac{(c+dx)^3}{3d} + \frac{(c+dx)^2 \log\left(1+e^{2(e+fx)}\right)}{f} \\ & + \frac{d^2 \log(\cosh(e+fx))}{f^3} \\ & + \frac{d(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} \\ & - \frac{d^2\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} \\ & - \frac{d(c+dx)\tanh(e+fx)}{f^2} - \frac{(c+dx)^2\tanh^2(e+fx)}{2f}\end{aligned}$$

output

```
1/2*(d*x+c)^2/f-1/3*(d*x+c)^3/d+(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+d^2*ln(co
sh(f*x+e))/f^3+d*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*d^2*polylog(3,
-exp(2*f*x+2*e))/f^3-d*(d*x+c)*tanh(f*x+e)/f^2-1/2*(d*x+c)^2*tanh(f*x+e)^2
/f
```

Mathematica [A] (verified)

Time = 2.79 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.50

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \frac{1}{6}\left(\frac{4x(-3c^2e^{2e}f^2+3cdf^2x+d^2(-3e^{2e}+f^2x^2))}{(1+e^{2e})f^2} + \frac{6dx(2c+dx)\log\left(1+e^{-2(e+fx)}\right)}{f} + \frac{6(d^2+c^2f^2)\log\left(1+e^{2(e+fx)}\right)}{f^3} - \frac{6d(c+dx)\,\mathrm{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{f^2} - \frac{3d^2\,\mathrm{PolyLog}\left(3,-e^{-2(e+fx)}\right)}{f^3} + \frac{3(c+dx)^2\mathrm{sech}^2(e+fx)}{f} - \frac{6d(c+dx)\mathrm{sech}(e)\mathrm{sech}(e+fx)\sinh(fx)}{f^2} + 2x\left(3c^2+3cdx+d^2x^2\right)\tanh(e)\right)$$

input
```
Integrate[(c + d*x)^2*Tanh[e + f*x]^3,x]
```

output
```
((4*x*(-3*c^2*E^(2*e)*f^2 + 3*c*d*f^2*x + d^2*(-3*E^(2*e) + f^2*x^2)))/((1
 + E^(2*e))*f^2) + (6*d*x*(2*c + d*x)*Log[1 + E^(-2*(e + f*x))])/f + (6*(d
^2 + c^2*f^2)*Log[1 + E^(2*(e + f*x))])/f^3 - (6*d*(c + d*x)*PolyLog[2, -E
^(-2*(e + f*x))])/f^2 - (3*d^2*PolyLog[3, -E^(-2*(e + f*x))])/f^3 + (3*(c
+ d*x)^2*Sech[e + f*x]^2)/f - (6*d*(c + d*x)*Sech[e]*Sech[e + f*x]*Sinh[f*
x])/f^2 + 2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Tanh[e])/6
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.01 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.19, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 1.188$, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 4201, 2620, 3011, 2720, 4203, 17, 26, 3042, 26, 3956, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int i(c+dx)^2 \tan(ie+ifx)^3 dx$$

$$\downarrow 26$$

$$i\int (c+dx)^2 \tan(ie+ifx)^3 dx$$

$$\downarrow 4203$$

$$i\left(\frac{id\int -((c+dx)\tanh^2(e+fx))\,dx}{f} - \int i(c+dx)^2\tanh(e+fx)dx + \frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 25$$

$$i\left(-\frac{id\int (c+dx)\tanh^2(e+fx)dx}{f} - \int i(c+dx)^2\tanh(e+fx)dx + \frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 26$$

$$i\left(-\frac{id\int (c+dx)\tanh^2(e+fx)dx}{f} - i\int (c+dx)^2\tanh(e+fx)dx + \frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 3042$$

$$i\left(-i\int -i(c+dx)^2\tan(ie+ifx)dx - \frac{id\int -((c+dx)\tan(ie+ifx)^2)\,dx}{f} + \frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 25$$

$$i\left(-i\int -i(c+dx)^2\tan(ie+ifx)dx+\frac{id\int(c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 26$$

$$i\left(-\int(c+dx)^2\tan(ie+ifx)dx+\frac{id\int(c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 4201$$

$$i\left(-2i\int\frac{e^{2(e+fx)}(c+dx)^2}{1+e^{2(e+fx)}}dx+\frac{id\int(c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(c+dx)^2\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^3}{3d}\right)$$

$$\downarrow 2620$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int(c+dx)\log\left(1+e^{2(e+fx)}\right)dx}{f}\right)+\frac{id\int(c+dx)\tan(ie+ifx)^2dx}{f}+\frac{i(}{}\right.$$

$$\downarrow 3011$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)dx}{2f}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+\frac{id\int(c+dx)}{}\right.$$

$$\downarrow 2720$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+\right.$$

$$\downarrow 4203$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+\right.$$

$$\downarrow 17$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+$$

$$\downarrow 26$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+$$

$$\downarrow 3042$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+$$

$$\downarrow 26$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+$$

$$\downarrow 3956$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\int e^{-2(e+fx)}\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+$$

$$\downarrow 7143$$

$$i\left(-2i\left(\frac{(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\left(\frac{d\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{4f^2}-\frac{(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f}\right)}{f}\right)+\frac{id\left(\frac{(c+dx)\tanh(e+}{f}\right.}$$

input

```
Int[(c + d*x)^2*Tanh[e + f*x]^3,x]
```

output
```
I*(((I/3)*(c + d*x)^3)/d - (2*I)*(((c + d*x)^2*Log[1 + E^(2*(e + f*x))])/(
2*f) - (d*(-1/2*((c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f + (d*PolyLog[3,
 -E^(2*(e + f*x))])/(4*f^2)))/f) + ((I/2)*(c + d*x)^2*Tanh[e + f*x]^2)/f +
 (I*d*(-1/2*(c + d*x)^2/d - (d*Log[Cosh[e + f*x]])/f^2 + ((c + d*x)*Tanh[e
 + f*x])/f))/f)
```

Defintions of rubi rules used

rule 17
```
Int[(c_.)*((a_.) + (b_.)*(x_))^(m_.), x_Symbol] :> Simp[c*((a + b*x)^(m + 1
)/(b*(m + 1))), x] /; FreeQ[{a, b, c, m}, x] && NeQ[m, -1]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4201
```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(143) = 286$.

Time $= 0.56$ (sec) , antiderivative size $= 375$, normalized size of antiderivative $= 2.48$

method	result
risch	$-\frac{d^2x^3}{3} - dcx^2 + c^2x + \frac{c^3}{3d} + \frac{2d^2fx^2e^{2fx+2e}+4cdfxe^{2fx+2e}+2c^2fe^{2fx+2e}+2d^2xe^{2fx+2e}+2cde^{2fx+2e}+2d^2x+2cd}{f^2\left(1+e^{2fx+2e}\right)^2} -$

input
```
int((d*x+c)^2*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/3*d^2*x^3-d*c*x^2+c^2*x+1/3/d*c^3+2*(d^2*f*x^2*exp(2*f*x+2*e)+2*c*d*f*x
*exp(2*f*x+2*e)+c^2*f*exp(2*f*x+2*e)+d^2*x*exp(2*f*x+2*e)+c*d*exp(2*f*x+2*
e)+d^2*x+c*d)/f^2/(1+exp(2*f*x+2*e))^2-2/f^3*d^2*ln(exp(f*x+e))+1/f^3*d^2*
ln(1+exp(2*f*x+2*e))-2/f*c^2*ln(exp(f*x+e))+1/f*c^2*ln(1+exp(2*f*x+2*e))-2
/f^3*e^2*d^2*ln(exp(f*x+e))+4/3/f^3*d^2*e^3-1/2*d^2*polylog(3,-exp(2*f*x+2
*e))/f^3+2/f^2*d^2*e^2*x+1/f*d^2*ln(1+exp(2*f*x+2*e))*x^2+1/f^2*d^2*polylo
g(2,-exp(2*f*x+2*e))*x-2/f^2*d*c*e^2+1/f^2*d*c*polylog(2,-exp(2*f*x+2*e))+
2/f*d*c*ln(1+exp(2*f*x+2*e))*x-4/f*d*c*e*x+4/f^2*c*d*e*ln(exp(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 3071, normalized size of antiderivative = 20.34

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \int (c+dx)^2 \tanh^3(e+fx)\,dx$$

input
```
integrate((d*x+c)**2*tanh(f*x+e)**3,x)
```

output
```
Integral((c + d*x)**2*tanh(e + f*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 392 vs. 2(142) = 284.

Time = 0.23 (sec) , antiderivative size = 392, normalized size of antiderivative = 2.60

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx$$

$$= c^2\left(x + \frac{e}{f} + \frac{\log\left(e^{(-2fx-2e)}+1\right)}{f} + \frac{2e^{(-2fx-2e)}}{f\left(2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1\right)}\right)$$
$$+ \frac{\left(2fx\log\left(e^{(2fx+2e)}+1\right)+\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)cd}{f^2} - \frac{2d^2x}{f^2}$$
$$+ \frac{d^2f^2x^3+3cdf^2x^2+6d^2x+6cd+\left(d^2f^2x^3e^{(4e)}+3cdf^2x^2e^{(4e)}\right)e^{(4fx)}+2\left(d^2f^2x^3e^{(2e)}+3\left(cdf^2e^{(2e)}\right.\right.}{3\left(f^2e^{(4fx+4e)}+2f^2e^{(2fx+2e)}+f^2\right)}$$
$$+ \frac{\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right)+2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)d^2}{2f^3}$$
$$+ \frac{d^2\log\left(e^{(2fx+2e)}+1\right)}{f^3} - \frac{2\left(d^2f^3x^3+3cdf^3x^2\right)}{3f^3}$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
c^2*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-
2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + (2*f*x*log(e^(2*f*x + 2*e) + 1) +
 dilog(-e^(2*f*x + 2*e)))*c*d/f^2 - 2*d^2*x/f^2 + 1/3*(d^2*f^2*x^3 + 3*c*d
*f^2*x^2 + 6*d^2*x + 6*c*d + (d^2*f^2*x^3*e^(4*e) + 3*c*d*f^2*x^2*e^(4*e))
*e^(4*f*x) + 2*(d^2*f^2*x^3*e^(2*e) + 3*(c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*
x^2 + 3*c*d*e^(2*e) + 3*(2*c*d*f*e^(2*e) + d^2*e^(2*e))*x)*e^(2*f*x))/(f^2
*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/2*(2*f^2*x^2*log(e^(2*
f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2
*e)))*d^2/f^3 + d^2*log(e^(2*f*x + 2*e) + 1)/f^3 - 2/3*(d^2*f^3*x^3 + 3*c*
d*f^3*x^2)/f^3
```

Giac [F]

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \int (dx+c)^2 \tanh\left(fx+e\right)^3\,dx$$

input
```
integrate((d*x+c)^2*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*tanh(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \int \tanh(e+f\,x)^3\,(c+d\,x)^2\,dx$$

input
```
int(tanh(e + f*x)^3*(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^3*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2 \tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*tanh(f*x+e)^3,x)
```

output

```
( - 96*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) +
3*e**(2*e + 2*f*x) + 1),x)*d**2*f**3 - 192*e**(4*e + 4*f*x)*int(x/(e**(6*e
 + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*c*d*f**3 - 144
*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e
 + 2*f*x) + 1),x)*d**2*f**2 + 48*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1
)*c**2*f**2 + 72*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*c*d*f + 90*e**
(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*d**2 - 48*e**(4*e + 4*f*x)*c**2*f*
*3*x - 48*e**(4*e + 4*f*x)*c**2*f**2 + 48*e**(4*e + 4*f*x)*c*d*f**3*x**2 -
 144*e**(4*e + 4*f*x)*c*d*f**2*x - 36*e**(4*e + 4*f*x)*c*d*f + 16*e**(4*e
+ 4*f*x)*d**2*f**3*x**3 - 180*e**(4*e + 4*f*x)*d**2*f*x + 9*e**(4*e + 4*f*
x)*d**2 - 192*e**(2*e + 2*f*x)*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*
f*x) + 3*e**(2*e + 2*f*x) + 1),x)*d**2*f**3 - 384*e**(2*e + 2*f*x)*int(x/(
e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*c*d*f**
3 - 288*e**(2*e + 2*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*
e**(2*e + 2*f*x) + 1),x)*d**2*f**2 + 96*e**(2*e + 2*f*x)*log(e**(2*e + 2*f
*x) + 1)*c**2*f**2 + 144*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*c*d*f
+ 180*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*d**2 - 96*e**(2*e + 2*f*x
)*c**2*f**3*x + 96*e**(2*e + 2*f*x)*c*d*f**3*x**2 + 32*e**(2*e + 2*f*x)*d*
*2*f**3*x**3 + 144*e**(2*e + 2*f*x)*d**2*f**2*x**2 - 216*e**(2*e + 2*f*x)*
d**2*f*x - 96*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2...
```

3.13 $\int (c+dx)\tanh^3(e+fx)\,dx$

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Optimal result

Integrand size = 14, antiderivative size = 100

$$\int (c+dx)\tanh^3(e+fx)\,dx = \frac{dx}{2f} - \frac{(c+dx)^2}{2d} + \frac{(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} - \frac{d\tanh(e+fx)}{2f^2} - \frac{(c+dx)\tanh^2(e+fx)}{2f}$$

output

```
1/2*d*x/f-1/2*(d*x+c)^2/d+(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*d*polylog(2,-
exp(2*f*x+2*e))/f^2-1/2*d*tanh(f*x+e)/f^2-1/2*(d*x+c)*tanh(f*x+e)^2/f
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.89

$$\int (c+dx)\tanh^3(e+fx)\,dx = \frac{f\left(df x^2 + 2dx\log\left(1+e^{-2(e+fx)}\right) + 2c\log(\cosh(e+fx))\right) - d\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right) + f(c+dx)\text{sech}^2}{2f^2}$$

input

```
Integrate[(c + d*x)*Tanh[e + f*x]^3,x]
```

output
```
(f*(d*f*x^2 + 2*d*x*Log[1 + E^(-2*(e + f*x))] + 2*c*Log[Cosh[e + f*x]]) -
d*PolyLog[2, -E^(-2*(e + f*x))] + f*(c + d*x)*Sech[e + f*x]^2 - d*Sech[e]*
Sech[e + f*x]*Sinh[f*x])/(2*f^2)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.56 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.17, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 1.000$, Rules used = {3042, 26, 4203, 25, 26, 3042, 25, 26, 3954, 24, 4201, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\tanh^3(e+fx)\,dx$$

$$\downarrow 3042$$

$$\int i(c+dx)\tan(ie+ifx)^3 dx$$

$$\downarrow 26$$

$$i\int (c+dx)\tan(ie+ifx)^3 dx$$

$$\downarrow 4203$$

$$i\left(-\int i(c+dx)\tanh(e+fx)dx+\frac{id\int -\tanh^2(e+fx)dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 25$$

$$i\left(-\int i(c+dx)\tanh(e+fx)dx-\frac{id\int \tanh^2(e+fx)dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 26$$

$$i\left(-i\int (c+dx)\tanh(e+fx)dx-\frac{id\int \tanh^2(e+fx)dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 3042$$

$$i\left(-i\int -i(c+dx)\tan(ie+ifx)dx-\frac{id\int -\tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 25$$

$$i\left(-i\int -i(c+dx)\tan(ie+ifx)dx+\frac{id\int \tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 26$$

$$i\left(-\int (c+dx)\tan(ie+ifx)dx+\frac{id\int \tan(ie+ifx)^2dx}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 3954$$

$$i\left(-\int (c+dx)\tan(ie+ifx)dx+\frac{id\left(\frac{\tanh(e+fx)}{f}-\int 1dx\right)}{2f}+\frac{i(c+dx)\tanh^2(e+fx)}{2f}\right)$$

$$\downarrow 24$$

$$i\left(-\int (c+dx)\tan(ie+ifx)dx+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{id\left(\frac{\tanh(e+fx)}{f}-x\right)}{2f}\right)$$

$$\downarrow 4201$$

$$i\left(-2i\int \frac{e^{2(e+fx)}(c+dx)}{1+e^{2(e+fx)}}dx+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^2}{2d}+\frac{id\left(\frac{\tanh(e+fx)}{f}-x\right)}{2f}\right)$$

$$\downarrow 2620$$

$$i\left(-2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int \log\left(1+e^{2(e+fx)}\right)dx}{2f}\right)+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^2}{2d}+\frac{id\left(}{}\right.$$

$$\downarrow 2715$$

$$i\left(-2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}-\frac{d\int e^{-2(e+fx)}\log\left(1+e^{2(e+fx)}\right)de^{2(e+fx)}}{4f^2}\right)+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i}{}\right.$$

$$\downarrow 2838$$

$$i\left(-2i\left(\frac{(c+dx)\log\left(e^{2(e+fx)}+1\right)}{2f}+\frac{d\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{4f^2}\right)+\frac{i(c+dx)\tanh^2(e+fx)}{2f}+\frac{i(c+dx)^2}{2d}+\frac{id\left(}{}\right.$$

input
```
Int[(c + d*x)*Tanh[e + f*x]^3,x]
```

output
```
I*(((I/2)*(c + d*x)^2)/d - (2*I)*(((c + d*x)*Log[1 + E^(2*(e + f*x))])/(2*
f) + (d*PolyLog[2, -E^(2*(e + f*x))])/(4*f^2)) + ((I/2)*(c + d*x)*Tanh[e +
 f*x]^2)/f + ((I/2)*d*(-x + Tanh[e + f*x]/f))/f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4201

```
Int[((c_.) + (d_.)*(x_))^(m_.)*tan[(e_.) + (Complex[0, fz_])*(f_.)*(x_)], x
_Symbol] :> Simp[(-I)*((c + d*x)^(m + 1)/(d*(m + 1))), x] + Simp[2*I   Int[
(c + d*x)^m*(E^(2*((-I)*e + f*fz*x))/(1 + E^(2*((-I)*e + f*fz*x)))), x], x]
 /; FreeQ[{c, d, e, f, fz}, x] && IGtQ[m, 0]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.34 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.66

method	result
risch	$-\frac{dx^2}{2}+cx+\frac{2dfx\,\mathrm{e}^{2fx+2e}+2cf\,\mathrm{e}^{2fx+2e}+\mathrm{e}^{2fx+2e}d+d}{f^2\left(1+\mathrm{e}^{2fx+2e}\right)^2}-\frac{2c\ln\left(\mathrm{e}^{fx+e}\right)}{f}+\frac{c\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f}-\frac{2dex}{f}-\frac{de^2}{f^2}+\frac{d\ln(1+e}{}$

input

```
int((d*x+c)*tanh(f*x+e)^3,x,method=_RETURNVERBOSE)
```

output

```
-1/2*d*x^2+c*x+(2*d*f*x*exp(2*f*x+2*e)+2*c*f*exp(2*f*x+2*e)+exp(2*f*x+2*e)
*d+d)/f^2/(1+exp(2*f*x+2*e))^2-2/f*c*ln(exp(f*x+e))+1/f*c*ln(1+exp(2*f*x+2
*e))-2/f*d*e*x-1/f^2*d*e^2+1/f*d*ln(1+exp(2*f*x+2*e))*x+1/2*d*polylog(2,-e
xp(2*f*x+2*e))/f^2+2/f^2*d*e*ln(exp(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 1462, normalized size of antiderivative = 14.62

$$\int (c+dx)\tanh^3(e+fx)\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="fricas")
```

output
```
-1/2*(d*f^2*x^2 + (d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e
)^4 + 4*(d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e)*sinh(f*x
 + e)^3 + (d*f^2*x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*sinh(f*x + e)^4 + 2*
c*f^2*x - 2*d*e^2 + 4*c*e*f + 2*(d*f^2*x^2 - 2*d*e^2 + 2*(2*c*e - c)*f + 2
*(c*f^2 - d*f)*x - d)*cosh(f*x + e)^2 + 2*(d*f^2*x^2 - 2*d*e^2 + 3*(d*f^2*
x^2 + 2*c*f^2*x - 2*d*e^2 + 4*c*e*f)*cosh(f*x + e)^2 + 2*(2*c*e - c)*f + 2
*(c*f^2 - d*f)*x - d)*sinh(f*x + e)^2 - 2*(d*cosh(f*x + e)^4 + 4*d*cosh(f*
x + e)*sinh(f*x + e)^3 + d*sinh(f*x + e)^4 + 2*d*cosh(f*x + e)^2 + 2*(3*d*
cosh(f*x + e)^2 + d)*sinh(f*x + e)^2 + 4*(d*cosh(f*x + e)^3 + d*cosh(f*x +
 e))*sinh(f*x + e) + d)*dilog(I*cosh(f*x + e) + I*sinh(f*x + e)) - 2*(d*co
sh(f*x + e)^4 + 4*d*cosh(f*x + e)*sinh(f*x + e)^3 + d*sinh(f*x + e)^4 + 2*
d*cosh(f*x + e)^2 + 2*(3*d*cosh(f*x + e)^2 + d)*sinh(f*x + e)^2 + 4*(d*cos
h(f*x + e)^3 + d*cosh(f*x + e))*sinh(f*x + e) + d)*dilog(-I*cosh(f*x + e)
- I*sinh(f*x + e)) + 2*((d*e - c*f)*cosh(f*x + e)^4 + 4*(d*e - c*f)*cosh(f
*x + e)*sinh(f*x + e)^3 + (d*e - c*f)*sinh(f*x + e)^4 + 2*(d*e - c*f)*cosh
(f*x + e)^2 + 2*(3*(d*e - c*f)*cosh(f*x + e)^2 + d*e - c*f)*sinh(f*x + e)^
2 + d*e - c*f + 4*((d*e - c*f)*cosh(f*x + e)^3 + (d*e - c*f)*cosh(f*x + e)
)*sinh(f*x + e))*log(cosh(f*x + e) + sinh(f*x + e) + I) + 2*((d*e - c*f)*c
osh(f*x + e)^4 + 4*(d*e - c*f)*cosh(f*x + e)*sinh(f*x + e)^3 + (d*e - c*f)
*sinh(f*x + e)^4 + 2*(d*e - c*f)*cosh(f*x + e)^2 + 2*(3*(d*e - c*f)*cos...
```

Sympy [F]

$$\int (c+dx)\tanh^3(e+fx)\,dx = \int (c+dx)\tanh^3(e+fx)\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e)**3,x)
```

output
```
Integral((c + d*x)*tanh(e + f*x)**3, x)
```

Maxima [F]

$$\int (c+dx)\tanh^3(e+fx)\,dx = \int (dx+c)\tanh(fx+e)^3\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="maxima")
```

output
```
c*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*
f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + 1/2*d*((f^2*x^2*e^(4*f*x + 4*e) + f
^2*x^2 + 2*(f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(2*f*x) + 2)/(f^2
*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) - 4*integrate(x/(e^(2*f*x
+ 2*e) + 1), x))
```

Giac [F]

$$\int (c+dx)\tanh^3(e+fx)\,dx = \int (dx+c)\tanh(fx+e)^3\,dx$$

input
```
integrate((d*x+c)*tanh(f*x+e)^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*tanh(f*x + e)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\tanh^3(e+fx)\,dx = \int \tanh(e+f\,x)^3\,(c+d\,x)\;dx$$

input
```
int(tanh(e + f*x)^3*(c + d*x),x)
```

output
```
int(tanh(e + f*x)^3*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\tanh^3(e+fx)\,dx$$
$$= \frac{-16e^{4fx+4e}\left(\int \frac{x}{e^{6fx+6e}+3e^{4fx+4e}+3e^{2fx+2e}+1}dx\right) d\,f^2 + 8e^{4fx+4e}\log\left(e^{2fx+2e}+1\right) cf + 6e^{4fx+4e}\log\left(e^{2fx+2e}+\right.}{}$$

input
```
int((d*x+c)*tanh(f*x+e)^3,x)
```

output
```
( - 16*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e
**(2*e + 2*f*x) + 1),x)*d*f**2 + 8*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) +
 1)*c*f + 6*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*d - 8*e**(4*e + 4*f
*x)*c*f**2*x - 8*e**(4*e + 4*f*x)*c*f + 4*e**(4*e + 4*f*x)*d*f**2*x**2 - 1
2*e**(4*e + 4*f*x)*d*f*x - 3*e**(4*e + 4*f*x)*d - 32*e**(2*e + 2*f*x)*int(
x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*d*f*
*2 + 16*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*c*f + 12*e**(2*e + 2*f*
x)*log(e**(2*e + 2*f*x) + 1)*d - 16*e**(2*e + 2*f*x)*c*f**2*x + 8*e**(2*e
+ 2*f*x)*d*f**2*x**2 - 16*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3
*e**(2*e + 2*f*x) + 1),x)*d*f**2 + 8*log(e**(2*e + 2*f*x) + 1)*c*f + 6*log
(e**(2*e + 2*f*x) + 1)*d - 8*c*f**2*x - 8*c*f + 4*d*f**2*x**2 + 3*d)/(8*f*
*2*(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1))
```

3.14 $\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \text{Int}\left(\frac{\tanh^3(e+fx)}{c+dx}, x\right)$$

output
```
Defer(Int)(tanh(f*x+e)^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 24.87 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$

input
```
Integrate[Tanh[e + f*x]^3/(c + d*x),x]
```

output
```
Integrate[Tanh[e + f*x]^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{i\tan(ie+ifx)^3}{c+dx}\,dx$$
$$\downarrow 26$$
$$i\int \frac{\tan(ie+ifx)^3}{c+dx}\,dx$$
$$\downarrow 4222$$
$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$

input
```
Int[Tanh[e + f*x]^3/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(fx+e\right)^3}{dx+c}\,dx$$

input
```
int(tanh(f*x+e)^3/(d*x+c),x)
```

output
```
int(tanh(f*x+e)^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)^3}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^3/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh^3\left(e+fx\right)}{c+dx}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(d*x+c),x)
```

output
```
Integral(tanh(e + f*x)**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 303, normalized size of antiderivative = 18.94

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)^3}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="maxima")
```

output
```
((2*d*f*x*e^(2*e) + 2*c*f*e^(2*e) - d*e^(2*e))*e^(2*f*x) - d)/(d^2*f^2*x^2
 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e) + c^
2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2*e) +
c^2*f^2*e^(2*e))*e^(2*f*x)) + log(d*x + c)/d - integrate(2*(d^2*f^2*x^2 +
2*c*d*f^2*x + c^2*f^2 + d^2)/(d^3*f^2*x^3 + 3*c*d^2*f^2*x^2 + 3*c^2*d*f^2*
x + c^3*f^2 + (d^3*f^2*x^3*e^(2*e) + 3*c*d^2*f^2*x^2*e^(2*e) + 3*c^2*d*f^2
*x*e^(2*e) + c^3*f^2*e^(2*e))*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh\left(fx+e\right)^3}{dx+c}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.13 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx = \int \frac{\tanh(e+f\,x)^3}{c+d\,x}\,dx$$

input
```
int(tanh(e + f*x)^3/(c + d*x),x)
```

output
```
int(tanh(e + f*x)^3/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 205, normalized size of antiderivative = 12.81

$$\int \frac{\tanh^3(e+fx)}{c+dx}\,dx$$
$$= \frac{-6e^{4e}\left(\int \frac{e^{4fx}}{e^{6fx+6e}c+e^{6fx+6e}dx+3e^{4fx+4e}c+3e^{4fx+4e}dx+3e^{2fx+2e}c+3e^{2fx+2e}dx+c+dx}dx\right)d-2\left(\int \frac{}{e^{6fx+6e}c+e^{6fx+6e}dx+3e^{4fx+}}\right.}{d}$$

input
```
int(tanh(f*x+e)^3/(d*x+c),x)
```

output
```
( - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x +
 3*e**(4*e + 4*f*x)*c + 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*
e**(2*e + 2*f*x)*d*x + c + d*x),x)*d - 2*int(1/(e**(6*e + 6*f*x)*c + e**(6
*e + 6*f*x)*d*x + 3*e**(4*e + 4*f*x)*c + 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*
e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x + c + d*x),x)*d + log(c + d*x))/d
```

3.15 $\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 16

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{\tanh^3(e+fx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)(tanh(f*x+e)^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 20.72 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[Tanh[e + f*x]^3/(c + d*x)^2,x]
```

output
```
Integrate[Tanh[e + f*x]^3/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{i\tan(ie+ifx)^3}{(c+dx)^2}\,dx$$
$$\downarrow 26$$
$$i\int \frac{\tan(ie+ifx)^3}{(c+dx)^2}\,dx$$
$$\downarrow 4222$$
$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$

input
```
Int[Tanh[e + f*x]^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00

$$\int \frac{\tanh\left(fx+e\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
int(tanh(f*x+e)^3/(d*x+c)^2,x)
```

output
```
int(tanh(f*x+e)^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.81

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral(tanh(f*x + e)^3/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 0.60 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh^3\left(e+fx\right)}{\left(c+dx\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)**3/(d*x+c)**2,x)
```

output
```
Integral(tanh(e + f*x)**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 501, normalized size of antiderivative = 31.31

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 2*d^2 + (d^2*f^2*x^2*e^(4*e) + 2*c
*d*f^2*x*e^(4*e) + c^2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + c
^2*f^2*e^(2*e) - c*d*f*e^(2*e) + d^2*e^(2*e) + (2*c*d*f^2*e^(2*e) - d^2*f*
e^(2*e))*x)*e^(2*f*x))/(d^4*f^2*x^3 + 3*c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x +
c^3*d*f^2 + (d^4*f^2*x^3*e^(4*e) + 3*c*d^3*f^2*x^2*e^(4*e) + 3*c^2*d^2*f^2
*x*e^(4*e) + c^3*d*f^2*e^(4*e))*e^(4*f*x) + 2*(d^4*f^2*x^3*e^(2*e) + 3*c*d
^3*f^2*x^2*e^(2*e) + 3*c^2*d^2*f^2*x*e^(2*e) + c^3*d*f^2*e^(2*e))*e^(2*f*x
)) - integrate(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + 3*d^2)/(d^4*f^2*x^
4 + 4*c*d^3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f
^2*x^4*e^(2*e) + 4*c*d^3*f^2*x^3*e^(2*e) + 6*c^2*d^2*f^2*x^2*e^(2*e) + 4*c
^3*d*f^2*x*e^(2*e) + c^4*f^2*e^(2*e))*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh\left(fx+e\right)^3}{\left(dx+c\right)^2}\,dx$$

input
```
integrate(tanh(f*x+e)^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate(tanh(f*x + e)^3/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx = \int \frac{\tanh(e+f\,x)^3}{(c+d\,x)^2}\,dx$$

input
```
int(tanh(e + f*x)^3/(c + d*x)^2,x)
```

output
```
int(tanh(e + f*x)^3/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 705, normalized size of antiderivative = 44.06

$$\int \frac{\tanh^3(e+fx)}{(c+dx)^2}\,dx$$
$$= \frac{-6e^{4e}\left(\int \frac{e^{4fx}}{e^{6fx+6e}c^2+2e^{6fx+6e}cdx+e^{6fx+6e}d^2x^2+3e^{4fx+4e}c^2+6e^{4fx+4e}cdx+3e^{4fx+4e}d^2x^2+3e^{2fx+2e}c^2+6e^{2fx+2e}cdx+3e^{2fx+2e}d^2x^2+}\right.}{}$$

input
```
int(tanh(f*x+e)^3/(d*x+c)^2,x)
```

output

```
( - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*
c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e + 4*f*x)*c**2 + 6*e**(4*e +
 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e + 2*f*x)*c**2 + 6
*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x +
d**2*x**2),x)*c**2 - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**2 + 2*
e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e + 4*f*x)*c
**2 + 6*e**(4*e + 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e
+ 2*f*x)*c**2 + 6*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 +
c**2 + 2*c*d*x + d**2*x**2),x)*c*d*x - 2*int(1/(e**(6*e + 6*f*x)*c**2 + 2*
e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e + 4*f*x)*c
**2 + 6*e**(4*e + 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e
+ 2*f*x)*c**2 + 6*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 +
c**2 + 2*c*d*x + d**2*x**2),x)*c**2 - 2*int(1/(e**(6*e + 6*f*x)*c**2 + 2*e
**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e + 4*f*x)*c*
*2 + 6*e**(4*e + 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e +
 2*f*x)*c**2 + 6*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 + c
**2 + 2*c*d*x + d**2*x**2),x)*c*d*x + x)/(c*(c + d*x))
```

3.16 $\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$

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Giac [F] . . . 175
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Optimal result

Integrand size = 18, antiderivative size = 1392

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \text{Too large to display}$$

output

```
2/3*b^(5/2)*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2-(-b)^(5/2)*(d*x+c)
*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f-1/2*(-b)^(5/2)*d*arctanh((b*t
anh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2+2/3*b^(5/2)*d*arctanh((b*tanh(f*x+e))^
(1/2)/b^(1/2))/f^2+b^(5/2)*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/
f+1/2*b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/f^2-b^(5/2)*d*arc
tanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^
(1/2)))/f^2+b^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/
(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5/2)*d*arctanh((b*tanh(f*x+e))
^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2
)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(5/2)*d*arctanh((b*t
anh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))
/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+(-b)^(5/2)*d*ar
ctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)
^(1/2)))/f^2-1/2*(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln
(2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))
^(1/2)/(-b)^(1/2)))/f^2-1/2*(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b
)^(1/2))*ln((-2*b^(1/2)-2*(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(
b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-(-b)^(5/2)*d*arctanh((b*tanh(f*x+e))
^(1/2)/(-b)^(1/2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*b^(5
/2)*d*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^...
```

Mathematica [F]

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$$

input

```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(5/2),x]
```

output

```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(5/2), x]
```

Rubi [A] (warning: unable to verify)

Time = 2.61 (sec) , antiderivative size = 1298, normalized size of antiderivative = 0.93, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.944$, Rules used = {3042, 4203, 3042, 3954, 3042, 3957, 25, 266, 756, 216, 219, 4219, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)(-ib\tan(ie+ifx))^{5/2}dx$$

$$\downarrow 4203$$

$$b^2\int (c+dx)\sqrt{b\tanh(e+fx)}dx+\frac{2bd\int (b\tanh(e+fx))^{3/2}dx}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 3042$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\int (-ib\tan(ie+ifx))^{3/2}dx}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 3954$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(b^2\int \frac{1}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 3042$$

$$b^2\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx+\frac{2bd\left(-\frac{2b\sqrt{b\tanh(e+fx)}}{f}+b^2\int \frac{1}{\sqrt{-ib\tan(ie+ifx)}}dx\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 3957$$

$$\frac{b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx+2bd\left(-\frac{b^3\int-\frac{1}{\sqrt{b\tanh(e+fx)}\left(b^2-b^2\tanh^2(e+fx)\right)}d(b\tanh(e+fx))}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 25$$

$$\frac{b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx+2bd\left(\frac{b^3\int\frac{1}{\sqrt{b\tanh(e+fx)}\left(b^2-b^2\tanh^2(e+fx)\right)}d(b\tanh(e+fx))}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 266$$

$$\frac{b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx+2bd\left(\frac{2b^3\int\frac{1}{b^2-b^4\tanh^4(e+fx)}d\sqrt{b\tanh(e+fx)}}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 756$$

$$\frac{b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx+2bd\left(\frac{2b^3\left(\frac{\int\frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\int\frac{1}{b^2\tanh^2(e+fx)+b}d\sqrt{b\tanh(e+fx)}}{2b}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 216$$

$$\frac{b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx+2bd\left(\frac{2b^3\left(\frac{\int\frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}+b^2\int(c+dx)\sqrt{-ib\tan(ie+ifx)}dx-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 219$$

$$b^2 \int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx + \frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f} - \frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f} - \frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 4219$$

$$b^2\left(\frac{\sqrt{-b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f} - \frac{\sqrt{b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f} - \frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+f}}{\sqrt{-b}}\right.\right.$$

$$\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f} - \frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f} - \frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}$$

$$\downarrow 4853$$

$$b^2\left(\frac{\sqrt{-b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2} - \frac{\sqrt{b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2} - \frac{\sqrt{-b}(c+dx}{}\right.$$

$$\left.\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f} - \frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f} - \frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}\right)$$

$$\downarrow 7267$$

$$b^2\left(\frac{2\sqrt{-b}d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-\frac{2d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}}{\sqrt{b}f^2}\right.$$

$$\left.\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}\right)$$

$$\downarrow 27$$

$$b^2\left(\frac{2\sqrt{-b}bd\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b^{3/2}d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+f}}{b^2-b^2\tanh^2(e+fx)}}{f^2}\right.$$

$$\left.\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}\right)$$

$$\downarrow 7276$$

$$b^2\left(-\frac{2b^{3/2}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+\right.$$

$$\left.\frac{2bd\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)}{3f}-\frac{2b(c+dx)(b\tanh(e+fx))^{3/2}}{3f}\right)$$

$$\downarrow 2009$$

$$\left(-\frac{2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{b}}\right)}{2b}\right.}{3f}\right.$$

$$\left.\left.\frac{2(c+dx)(b\tanh(e+fx))^{3/2}b}{3f}+\frac{2d\left(\frac{2b^3\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f}-\frac{2b\sqrt{b\tanh(e+fx)}}{f}\right)b}{3f}\right)$$

input
```
Int[(c + d*x)*(b*Tanh[e + f*x])^(5/2),x]
```

output
```
b^2*(-((Sqrt[-b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]])/f) + (
Sqrt[b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/f - (2*b^(3/2)*d
*(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e
 + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b
) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqr
t[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[
(2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt
[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqr
t[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt
[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b
])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(
Sqrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[
-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh
[e + f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e
 + f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b))
)/f^2 + (2*Sqrt[-b]*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]^2/(4*b) -
 (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 - Sqrt[b*Tanh[e + f*x]]
/Sqrt[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 + S
qrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sq
rt[-b]]*Log[(-2*(Sqrt[b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b]...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2  Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3957

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d  Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))  Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2  Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4219

```
Int[((c_.) + (d_.)*(x_))*Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*Rt[a - I*b, 2]*((c + d*x)/f)*ArcTanh[Sqrt[a + b*Tan[e + f
*x]]/Rt[a - I*b, 2]], x] + (Simp[I*Rt[a + I*b, 2]*((c + d*x)/f)*ArcTanh[Sqr
t[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*d*(Rt[a - I*b, 2]/f)  I
nt[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*d*(Rt[
a + I*b, 2]/f)  Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853

```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]  Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267

```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]  Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int (dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{5}{2}}\,dx$$

input

```
int((d*x+c)*(b*tanh(f*x+e))^(5/2),x)
```

output

```
int((d*x+c)*(b*tanh(f*x+e))^(5/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \text{Exception raised: TypeError}$$

input

```
integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="fricas")
```

output

```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \int \left(b\tanh\left(e+fx\right)\right)^{\frac{5}{2}}(c+dx)\,dx$$

input

```
integrate((d*x+c)*(b*tanh(f*x+e))**(5/2),x)
```

output

```
Integral((b*tanh(e + f*x))**(5/2)*(c + d*x), x)
```

Maxima [F]

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \int (dx+c)(b\tanh(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(5/2), x)
```

Giac [F]

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \int (dx+c)(b\tanh(fx+e))^{\frac{5}{2}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(5/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(b\tanh(e+fx))^{5/2}\,dx = \int (b\tanh(e+f\,x))^{5/2}\,(c+d\,x)\,dx$$

input
```
int((b*tanh(e + f*x))^(5/2)*(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(5/2)*(c + d*x), x)
```

Reduce [F]

$$\int (c + dx)(b\tanh(e+fx))^{5/2}\,dx = \sqrt{b}\,b^2\left(\left(\int \sqrt{\tanh(fx+e)}\,\tanh(fx+e)^2\,x dx\right)d + \left(\int \sqrt{\tanh(fx+e)}\,\tanh(fx+e)^2\,dx\right)c\right)$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(5/2),x)
```

output
```
sqrt(b)*b**2*(int(sqrt(tanh(e + f*x))*tanh(e + f*x)**2*x,x)*d + int(sqrt(t
anh(e + f*x))*tanh(e + f*x)**2,x)*c)
```

3.17 $\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 1363

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \text{Too large to display}$$

output

```
-2*b^(3/2)*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2-(-b)^(3/2)*(d*x+c)*
arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f-1/2*(-b)^(3/2)*d*arctanh((b*ta
nh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2+2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/
2)/b^(1/2))/f^2+b^(3/2)*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f+1
/2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/f^2-b^(3/2)*d*arctan
h((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/
2)))/f^2+b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^
(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)*d*arctanh((b*tanh(f*x+e))^(1
/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b
^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/2)*d*arctanh((b*tanh
(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((
-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2+(-b)^(3/2)*d*arcta
nh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1
/2)))/f^2-1/2*(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*
(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1
/2)/(-b)^(1/2)))/f^2-1/2*(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(
1/2))*ln((-2*b^(1/2)-2*(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*t
anh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-(-b)^(3/2)*d*arctanh((b*tanh(f*x+e))^(1
/2)/(-b)^(1/2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*b^(3/2)
*d*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(3/...
```

Mathematica [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$$

input

```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(3/2),x]
```

output

```
Integrate[(c + d*x)*(b*Tanh[e + f*x])^(3/2), x]
```

Rubi [A] (warning: unable to verify)

Time = 2.27 (sec) , antiderivative size = 1269, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4203, 3042, 3957, 25, 266, 827, 216, 219, 4221, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)(-ib\tan(ie+ifx))^{3/2}dx$$

$$\downarrow 4203$$

$$b^2\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}dx + \frac{2bd\int \sqrt{b\tanh(e+fx)}dx}{f} - \frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 3042$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx + \frac{2bd\int \sqrt{-ib\tan(ie+ifx)}dx}{f} - \frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 3957$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx - \frac{2b^2d\int -\frac{\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d(b\tanh(e+fx))}{f^2} - \frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 25$$

$$b^2\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx + \frac{2b^2d\int \frac{\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d(b\tanh(e+fx))}{f^2} - \frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 266$$

$$b^2\int\frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{4b^2d\int\frac{b^2\tanh^2(e+fx)}{b^2-b^4\tanh^4(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 827$$

$$b^2\int\frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{4b^2d\left(\frac{1}{2}\int\frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}-\frac{1}{2}\int\frac{1}{b^2\tanh^2(e+fx)+b}d\sqrt{b\tanh(e+fx)}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 216$$

$$\frac{4b^2d\left(\frac{1}{2}\int\frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}+b^2\int\frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 219$$

$$b^2\int\frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx+\frac{4b^2d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 4221$$

$$b^2\left(\frac{d\int\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f}-\frac{d\int\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+d}{}\right.$$

$$\left.\frac{4b^2d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}\right)$$

$$\downarrow 4853$$

$$b^2\left(\frac{d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{-b}f^2}-\frac{d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{b}f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{}}{\sqrt{-b}f}\right.\right.$$

$$\left.\left.\frac{4b^2d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}\right.\right.$$

$$\downarrow 7267$$

$$b^2\left(\frac{2d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}bf^2}-\frac{2d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b}}{b^{3/2}f^2}\right.$$

$$\left.\frac{4b^2d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}\right)$$

$$\downarrow 27$$

$$b^2\left(\frac{2bd\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}bd\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{}}{f^2}\right.$$

$$\left.\frac{4b^2d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}\right)$$

$$\downarrow 7276$$

$$b^2\left(\frac{2bd\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}}{}\right.$$

$$\left.\frac{4b^2d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)}{f^2}-\frac{2b(c+dx)\sqrt{b\tanh(e+fx)}}{f}\right)$$

$$\downarrow 2009$$

$$\frac{4d\left(\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}-\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2\sqrt{b}}\right)b^2}{f^2}+$$

$$\left(-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{2\sqrt{b}d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log}{}\right.\right.$$

$$\left.\left.\frac{2(c+dx)\sqrt{b\tanh(e+fx)}b}{f}\right.\right.$$

input
```
Int[(c + d*x)*(b*Tanh[e + f*x])^(3/2),x]
```

output
```
(4*b^2*d*(-1/2*ArcTan[Sqrt[b]*Tanh[e + f*x]]/Sqrt[b] + ArcTanh[Sqrt[b]*Tan
h[e + f*x]]/(2*Sqrt[b])))/f^2 + b^2*(-(((c + d*x)*ArcTanh[Sqrt[b*Tanh[e +
f*x]]/Sqrt[-b]])/(Sqrt[-b]*f)) + ((c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/
Sqrt[b]])/(Sqrt[b]*f) - (2*Sqrt[b]*d*(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/S
qrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqr
t[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqr
t[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh
[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e +
 f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b)
+ (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[
b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])
)])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/
(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b
) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-
b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b) - PolyLog[2, 1 - (
2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[
b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b)))/f^2 + (2*b*d*(ArcTanh[Sqrt[b*Tanh[e
+ f*x]]/Sqrt[-b]]^2/(4*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2
/(1 - Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f
*x]]/Sqrt[-b]]*Log[2/(1 + Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (Ar...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 827
```
Int[(x_)^2/((a_) + (b_.)*(x_)^4), x_Symbol] :> With[{r = Numerator[Rt[-a/b,
 2]], s = Denominator[Rt[-a/b, 2]]}, Simp[s/(2*b)   Int[1/(r + s*x^2), x],
x] - Simp[s/(2*b)   Int[1/(r - s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4203

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[b*(c + d*x)^m*((b*Tan[e + f*x])^(n - 1)/(f*(n - 1))), x] + (-Si
mp[b*d*(m/(f*(n - 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n - 1), x]
, x] - Simp[b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n - 2), x], x]) /; Free
Q[{b, c, d, e, f}, x] && GtQ[n, 1] && GtQ[m, 0]
```

rule 4221

```
Int[((c_.) + (d_.)*(x_))/Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*((c + d*x)/(f*Rt[a - I*b, 2]))*ArcTanh[Sqrt[a + b*Tan[e +
 f*x]]/Rt[a - I*b, 2]], x] + (Simp[I*((c + d*x)/(f*Rt[a + I*b, 2]))*ArcTanh
[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*(d/(f*Rt[a - I*b, 2]
))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*
(d/(f*Rt[a + I*b, 2]))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2
]], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853

```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267

```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int (dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}\,dx$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)*(b*tanh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int \left(b\tanh\left(e+fx\right)\right)^{\frac{3}{2}}(c+dx)\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((b*tanh(e + f*x))**(3/2)*(c + d*x), x)
```

Maxima [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)(b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(3/2), x)
```

Giac [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)(b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \int (b\tanh(e+f\,x))^{3/2}\,(c+d\,x)\,dx$$

input
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)(b\tanh(e+fx))^{3/2}\,dx = \frac{\sqrt{b}\,b\left(-2\sqrt{\tanh(fx+e)}\,c+\left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)}dx\right)cf+\left(\int \sqrt{\tanh(fx+e)}\,\tanh(fx+e)\,xdx\right.\right.}{f}$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*b*( - 2*sqrt(tanh(e + f*x))*c + int(sqrt(tanh(e + f*x))/tanh(e +
f*x),x)*c*f + int(sqrt(tanh(e + f*x))*tanh(e + f*x)*x,x)*d*f))/f
```

3.18 $\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 1280

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \text{Too large to display}$$

output

```
-(-b)^(1/2)*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f-1/2*(-b)^(
1/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^2+b^(1/2)*(d*x+c)*arc
tanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f+1/2*b^(1/2)*d*arctanh((b*tanh(f*x+e)
)^(1/2)/b^(1/2))^2/f^2-b^(1/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln
(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^2+b^(1/2)*d*arctanh((b*tanh(
f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^2-1
/2*b^(1/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/
2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1
/2)))/f^2-1/2*b^(1/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2
)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh
(f*x+e))^(1/2)))/f^2+(-b)^(1/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2)
)*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*(-b)^(1/2)*d*arctanh(
(b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-
b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*(-b)^(1/2)
*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln((-2*b^(1/2)-2*(b*tanh(f*x+
e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^2-
(-b)^(1/2)*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1+(b*tanh(f*x
+e))^(1/2)/(-b)^(1/2)))/f^2-1/2*b^(1/2)*d*polylog(2,1-2*b^(1/2)/(b^(1/2)-(
b*tanh(f*x+e))^(1/2)))/f^2-1/2*b^(1/2)*d*polylog(2,1-2*b^(1/2)/(b^(1/2)+(b
*tanh(f*x+e))^(1/2)))/f^2+1/4*b^(1/2)*d*polylog(2,1-2*b^(1/2)*((-b)^(1/...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 4.28 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.43

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$$

$$= \frac{\left(-4f(c+dx)\left(2\arctan\left(\sqrt{\tanh(e+fx)}\right)+\log\left(1-\sqrt{\tanh(e+fx)}\right)-\log\left(1+\sqrt{\tanh(e+fx)}\right)\right)\right.}{}$$

input

```
Integrate[(c + d*x)*Sqrt[b*Tanh[e + f*x]],x]
```

output

```
((-4*f*(c + d*x)*(2*ArcTan[Sqrt[Tanh[e + f*x]]] + Log[1 - Sqrt[Tanh[e + f*
x]]] - Log[1 + Sqrt[Tanh[e + f*x]]]) + d*((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]
]^2 - 4*ArcTan[Sqrt[Tanh[e + f*x]]]*Log[1 + E^((4*I)*ArcTan[Sqrt[Tanh[e +
f*x]]])] - Log[1 - Sqrt[Tanh[e + f*x]]]^2 + 2*Log[1 - Sqrt[Tanh[e + f*x]]]
*Log[(1/2 + I/2)*(-I + Sqrt[Tanh[e + f*x]])] + 2*Log[1 - Sqrt[Tanh[e + f*x
]]]*Log[(1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])] - 2*Log[1 - Sqrt[Tanh[e + f
*x]]]*Log[(1 + Sqrt[Tanh[e + f*x]])/2] - 2*Log[1 - (1/2 - I/2)*(1 + Sqrt[T
anh[e + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + 2*Log[(1 - Sqrt[Tanh[e + f*
x]])/2]*Log[1 + Sqrt[Tanh[e + f*x]]] - 2*Log[(-1/2 - I/2)*(I + Sqrt[Tanh[e
 + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + Log[1 + Sqrt[Tanh[e + f*x]]]^2 +
 I*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]])] - 2*PolyLog[2, (1 -
Sqrt[Tanh[e + f*x]])/2] + 2*PolyLog[2, (-1/2 - I/2)*(-1 + Sqrt[Tanh[e + f*
x]])] + 2*PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*PolyLog[
2, (1 + Sqrt[Tanh[e + f*x]])/2] - 2*PolyLog[2, (1/2 - I/2)*(1 + Sqrt[Tanh[
e + f*x]])] - 2*PolyLog[2, (1/2 + I/2)*(1 + Sqrt[Tanh[e + f*x]])]))*Sqrt[b
*Tanh[e + f*x]])/(8*f^2*Sqrt[Tanh[e + f*x]])
```

Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 1187, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4219, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx$$

$$\downarrow 4219$$

$$\frac{\sqrt{-b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f} - \frac{\sqrt{b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f} - \frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f} + \frac{\sqrt{b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}$$

$$\downarrow 4853$$

$$\frac{\sqrt{-b}d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{b}d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-$$
$$\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}$$

$$\downarrow 7267$$

$$\frac{2\sqrt{-b}d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-$$
$$\frac{2d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{b}f^2}-$$
$$\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}$$

$$\downarrow 27$$

$$\frac{2\sqrt{-b}bd\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-$$
$$\frac{2b^{3/2}d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}+$$
$$\frac{\sqrt{b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}$$

$$\downarrow 7276$$

$$-\frac{2b^{3/2}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+$$
$$\frac{2\sqrt{-b}bd\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+$$
$$\frac{\sqrt{b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{f}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}$$

$$\downarrow 2009$$

$$\frac{2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tan}\right.}{2b}\right.}{-}$$

$$\frac{2\sqrt{-b}d\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{4b}-\frac{\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{2b}+\frac{\log\left(\frac{2}{\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}+1}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\ta}\right.}{2b}\right.}{+}$$

$$\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b}}{f}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}$$

input

```
Int[(c + d*x)*Sqrt[b*Tanh[e + f*x]],x]
```

output

```
-((Sqrt[-b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]])/f) + (Sqrt[
b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/f - (2*b^(3/2)*d*(-1/
4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e + f*
x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b) - (
ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqrt[b*T
anh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sq
rt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] +
 Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]
*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*
(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(S
qrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(Sqrt[
b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] -
 Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e +
f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x
]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b)))/f^2
 + (2*Sqrt[-b]*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]^2/(4*b) - (Arc
Tanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 - Sqrt[b*Tanh[e + f*x]]/Sqrt
[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 + Sqrt[b
*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b
]]*Log[(-2*(Sqrt[b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(1 ...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4219
```
Int[((c_.) + (d_.)*(x_))*Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*Rt[a - I*b, 2]*((c + d*x)/f)*ArcTanh[Sqrt[a + b*Tan[e + f
*x]]/Rt[a - I*b, 2]], x] + (Simp[I*Rt[a + I*b, 2]*((c + d*x)/f)*ArcTanh[Sqr
t[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*d*(Rt[a - I*b, 2]/f)   I
nt[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*d*(Rt[
a + I*b, 2]/f)   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int (dx+c)\sqrt{b\tanh(fx+e)}dx$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)*(b*tanh(f*x+e))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int \sqrt{b\tanh(e+fx)}(c+dx)\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tanh(e + f*x))*(c + d*x), x)
```

Maxima [F]

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int (dx+c)\sqrt{b\tanh\left(fx+e\right)}\,dx$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)*sqrt(b*tanh(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x+c)*(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \int \sqrt{b\tanh\left(e+f\,x\right)}\,(c+d\,x)\,dx$$

input
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)\sqrt{b\tanh(e+fx)}\,dx = \sqrt{b}\left(\left(\int \sqrt{\tanh(fx+e)}dx\right)c + \left(\int \sqrt{\tanh(fx+e)}\,xdx\right)d\right)$$

input
```
int((d*x+c)*(b*tanh(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*(int(sqrt(tanh(e + f*x)),x)*c + int(sqrt(tanh(e + f*x))*x,x)*d)
```

3.19 $\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 1280

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Too large to display}$$

output

```
-(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/(-b)^(1/2)/f-1/2*d*arct
anh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/(-b)^(1/2)/f^2+(d*x+c)*arctanh((b*
tanh(f*x+e))^(1/2)/b^(1/2))/b^(1/2)/f+1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/
b^(1/2))^2/b^(1/2)/f^2-d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/
2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(1/2)/f^2+d*arctanh((b*tanh(f*x+e))^
(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(1/2)/f^2-1
/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*ta
nh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^
(1/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^
(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))
^(1/2)))/b^(1/2)/f^2+d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(
b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(1/2)/f^2-1/2*d*arctanh((b*tanh(f*x
+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^
(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(1/2)/f^2-1/2*d*arctanh(
(b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln((-2*b^(1/2)-2*(b*tanh(f*x+e))^(1/2))/
((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(1/2)/f^2-
d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/
(-b)^(1/2)))/(-b)^(1/2)/f^2-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f
*x+e))^(1/2)))/b^(1/2)/f^2-1/2*d*polylog(2,1-2*b^(1/2)/(b^(1/2)+(b*tanh(f*
x+e))^(1/2)))/b^(1/2)/f^2+1/4*d*polylog(2,1-2*b^(1/2)*((-b)^(1/2)-(b*ta...
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.12 (sec) , antiderivative size = 556, normalized size of antiderivative = 0.43

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$$

$$= \frac{\left(4f(c+dx)\left(2\arctan\left(\sqrt{\tanh(e+fx)}\right)-\log\left(1-\sqrt{\tanh(e+fx)}\right)+\log\left(1+\sqrt{\tanh(e+fx)}\right)\right)+\right.}{}$$

input

```
Integrate[(c + d*x)/Sqrt[b*Tanh[e + f*x]],x]
```

output

```
((4*f*(c + d*x)*(2*ArcTan[Sqrt[Tanh[e + f*x]]] - Log[1 - Sqrt[Tanh[e + f*x
]]] + Log[1 + Sqrt[Tanh[e + f*x]]]) + d*((-4*I)*ArcTan[Sqrt[Tanh[e + f*x]]
]^2 + 4*ArcTan[Sqrt[Tanh[e + f*x]]]*Log[1 + E^((4*I)*ArcTan[Sqrt[Tanh[e +
f*x]]])] - Log[1 - Sqrt[Tanh[e + f*x]]]^2 + 2*Log[1 - Sqrt[Tanh[e + f*x]]]
*Log[(1/2 + I/2)*(-I + Sqrt[Tanh[e + f*x]])] + 2*Log[1 - Sqrt[Tanh[e + f*x
]]]*Log[(1/2 - I/2)*(I + Sqrt[Tanh[e + f*x]])] - 2*Log[1 - Sqrt[Tanh[e + f
*x]]]*Log[(1 + Sqrt[Tanh[e + f*x]])/2] - 2*Log[1 - (1/2 - I/2)*(1 + Sqrt[T
anh[e + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + 2*Log[(1 - Sqrt[Tanh[e + f*
x]])/2]*Log[1 + Sqrt[Tanh[e + f*x]]] - 2*Log[(-1/2 - I/2)*(I + Sqrt[Tanh[e
 + f*x]])]*Log[1 + Sqrt[Tanh[e + f*x]]] + Log[1 + Sqrt[Tanh[e + f*x]]]^2 -
 I*PolyLog[2, -E^((4*I)*ArcTan[Sqrt[Tanh[e + f*x]]])] - 2*PolyLog[2, (1 -
Sqrt[Tanh[e + f*x]])/2] + 2*PolyLog[2, (-1/2 - I/2)*(-1 + Sqrt[Tanh[e + f*
x]])] + 2*PolyLog[2, (-1/2 + I/2)*(-1 + Sqrt[Tanh[e + f*x]])] + 2*PolyLog[
2, (1 + Sqrt[Tanh[e + f*x]])/2] - 2*PolyLog[2, (1/2 - I/2)*(1 + Sqrt[Tanh[
e + f*x]])] - 2*PolyLog[2, (1/2 + I/2)*(1 + Sqrt[Tanh[e + f*x]])]))*Sqrt[T
anh[e + f*x]])/(8*f^2*Sqrt[b*Tanh[e + f*x]])
```

Rubi [A] (verified)

Time = 1.75 (sec) , antiderivative size = 1187, normalized size of antiderivative = 0.93, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.389$, Rules used = {3042, 4221, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx$$

$$\downarrow 4221$$

$$\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f} - \frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f} - \frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f} + \frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f} -$$

$$\downarrow 4853$$

$$\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{-b}f^2}-\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{b}f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$$\downarrow 7267$$

$$\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}bf^2}-\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{b^{3/2}f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$$\downarrow 27$$

$$\frac{2bd\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$$\downarrow 7276$$

$$\frac{2bd\int \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int \left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}$$

$$\downarrow 2009$$

$$-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-$$

$$2\sqrt{b}d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}\right.$$

$$2bd\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)^2}{4b}-\frac{\log\left(\frac{2}{1-\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{2b}+\frac{\log\left(\frac{2}{\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}+1}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{2b}\right.$$

input

```
Int[(c + d*x)/Sqrt[b*Tanh[e + f*x]],x]
```

output

```
-(((c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]])/(Sqrt[-b]*f)) + ((c
+ d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]])/(Sqrt[b]*f) - (2*Sqrt[b]*d*
(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b + (ArcTanh[Sqrt[b*Tanh[e
+ f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])])/(2*b)
 - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] + Sqrt
[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(
2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[
b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt
[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[
b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + PolyLog[2, 1 - (2*Sqrt[b]
)/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyLog[2, 1 - (2*Sqrt[b])/(S
qrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-
b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[
e + f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e +
 f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b)))
/f^2 + (2*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]^2/(4*b) - (ArcTanh[
Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 - Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])
])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1 + Sqrt[b*Tanh
[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Lo
g[(-2*(Sqrt[b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])*(1 + Sqr...
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4221
```
Int[((c_.) + (d_.)*(x_))/Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*((c + d*x)/(f*Rt[a - I*b, 2]))*ArcTanh[Sqrt[a + b*Tan[e +
 f*x]]/Rt[a - I*b, 2]], x] + (Simp[I*((c + d*x)/(f*Rt[a + I*b, 2]))*ArcTanh
[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*(d/(f*Rt[a - I*b, 2]
))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*
(d/(f*Rt[a + I*b, 2]))   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2
]], x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853
```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267
```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{dx+c}{\sqrt{b\tanh(fx+e)}}dx$$

input
```
int((d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*x)/sqrt(b*tanh(e + f*x)), x)
```

Maxima [F]

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{dx+c}{\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/sqrt(b*tanh(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Degree mismatch inside factorisation over extensionDegr
ee mismatch inside factorisation over extensionindex.cc index_m i_lex_is_g
reater Error
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{c+d\,x}{\sqrt{b\tanh\left(e+f\,x\right)}}\,dx$$

input
```
int((c + d*x)/(b*tanh(e + f*x))^(1/2),x)
```

output
```
int((c + d*x)/(b*tanh(e + f*x))^(1/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)}dx\right)c + \left(\int \frac{\sqrt{\tanh(fx+e)}\,x}{\tanh(fx+e)}dx\right)d\right)}{b}$$

input
```
int((d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(tanh(e + f*x))/tanh(e + f*x),x)*c + int((sqrt(tanh(e +
f*x))*x)/tanh(e + f*x),x)*d))/b
```

3.20 $\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 1365

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

output

```
2*d*arctan((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f^2-(d*x+c)*arctanh((b*t
anh(f*x+e))^(1/2)/(-b)^(1/2))/(-b)^(3/2)/f-1/2*d*arctanh((b*tanh(f*x+e))^(
1/2)/(-b)^(1/2))^2/(-b)^(3/2)/f^2+2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2
))/b^(3/2)/f^2+(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f+1/
2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))^2/b^(3/2)/f^2-d*arctanh((b*tanh
(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3
/2)/f^2+d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*
tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/
2))*ln(2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(
b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^
(1/2)/b^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)
+b^(1/2))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2+d*arctanh((b*tanh(f
*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(
3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*
tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/
2)))/(-b)^(3/2)/f^2-1/2*d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln((-2
*b^(1/2)-2*(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^
(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-d*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/
2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^2-1/2*d*polylo
g(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^2-1/2*d*poly...
```

Mathematica [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$$

input

```
Integrate[(c + d*x)/(b*Tanh[e + f*x])^(3/2),x]
```

output

```
Integrate[(c + d*x)/(b*Tanh[e + f*x])^(3/2), x]
```

Rubi [A] (warning: unable to verify)

Time = 2.12 (sec) , antiderivative size = 1268, normalized size of antiderivative = 0.93, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.833$, Rules used = {3042, 4204, 3042, 3957, 25, 266, 756, 216, 219, 4219, 4853, 7267, 27, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{c+dx}{(-ib\tan(ie+ifx))^{3/2}}dx$$

$$\downarrow 4204$$

$$\frac{\int (c+dx)\sqrt{b\tanh(e+fx)}dx}{b^2}+\frac{2d\int \frac{1}{\sqrt{b\tanh(e+fx)}}dx}{bf}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{2d\int \frac{1}{\sqrt{-ib\tan(ie+ifx)}}dx}{bf}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 3957$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}-\frac{2d\int -\frac{1}{\sqrt{b\tanh(e+fx)}(b^2-b^2\tanh^2(e+fx))}d(b\tanh(e+fx))}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 25$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{2d\int \frac{1}{\sqrt{b\tanh(e+fx)}(b^2-b^2\tanh^2(e+fx))}d(b\tanh(e+fx))}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 266$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\int \frac{1}{b^2-b^4\tanh^4(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 756$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\left(\frac{\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\int \frac{1}{b^2\tanh^2(e+fx)+b}d\sqrt{b\tanh(e+fx)}}{2b}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 216$$

$$\frac{4d\left(\frac{\int \frac{1}{b-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{2b}+\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}+\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 219$$

$$\frac{\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 4219$$

$$\frac{\frac{\sqrt{-b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{-b}}\right)dx}{f}-\frac{\sqrt{b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{b}}\right)dx}{f}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+dx)\operatorname{arct}}{b^2}$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 4853$$

$$\frac{\sqrt{-b}d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{b}d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{-b}}\right.}{f}$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 7267$$

$$\frac{2\sqrt{-b}d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-\frac{2d\int\frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{b}f^2}-$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 27$$

$$\frac{2\sqrt{-b}bd\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b^{3/2}d\int\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}+$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 7276$$

$$-\frac{2b^{3/2}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(e+fx)}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+\frac{2\sqrt{-b}bd\int\left(\operatorname{arctanh}\right.}{}$$

$$\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 2009$$

$$-\frac{2(c+dx)}{bf\sqrt{b\tanh(e+fx)}}+\frac{4d\left(\frac{\arctan\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}+\frac{\operatorname{arctanh}\left(\sqrt{b}\tanh(e+fx)\right)}{2b^{3/2}}\right)}{f^2}+$$

$$2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right.}{2b}\right.$$

$$-$$

input
```
Int[(c + d*x)/(b*Tanh[e + f*x])^(3/2),x]
```

output
```
(4*d*(ArcTan[Sqrt[b]*Tanh[e + f*x]]/(2*b^(3/2)) + ArcTanh[Sqrt[b]*Tanh[e +
 f*x]]/(2*b^(3/2))))/f^2 + (-((Sqrt[-b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e +
f*x]]/Sqrt[-b]])/f) + (Sqrt[b]*(c + d*x)*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqr
t[b]])/f - (2*b^(3/2)*d*(-1/4*ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]^2/b +
 (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b])/(Sqrt[b] - Sqrt[b
*Tanh[e + f*x]])])/(2*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*
Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[
e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sq
rt[-b] - Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + (ArcTanh[Sq
rt[b*Tanh[e + f*x]]/Sqrt[b]]*Log[(2*Sqrt[b]*(Sqrt[-b] + Sqrt[b*Tanh[e + f*
x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))])/(4*b) + P
olyLog[2, 1 - (2*Sqrt[b])/(Sqrt[b] - Sqrt[b*Tanh[e + f*x]])]/(4*b) + PolyL
og[2, 1 - (2*Sqrt[b])/(Sqrt[b] + Sqrt[b*Tanh[e + f*x]])]/(4*b) - PolyLog[2
, 1 - (2*Sqrt[b]*(Sqrt[-b] - Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] - Sqrt[b])
*(Sqrt[b] + Sqrt[b*Tanh[e + f*x]]))]/(8*b) - PolyLog[2, 1 - (2*Sqrt[b]*(Sq
rt[-b] + Sqrt[b*Tanh[e + f*x]]))/((Sqrt[-b] + Sqrt[b])*(Sqrt[b] + Sqrt[b*T
anh[e + f*x]]))]/(8*b)))/f^2 + (2*Sqrt[-b]*b*d*(ArcTanh[Sqrt[b*Tanh[e + f*
x]]/Sqrt[-b]]^2/(4*b) - (ArcTanh[Sqrt[b*Tanh[e + f*x]]/Sqrt[-b]]*Log[2/(1
- Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) + (ArcTanh[Sqrt[b*Tanh[e + f*x]]
/Sqrt[-b]]*Log[2/(1 + Sqrt[b*Tanh[e + f*x]]/Sqrt[-b])])/(2*b) - (ArcTan...
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 266
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{k = De
nominator[m]}, Simp[k/c   Subst[Int[x^(k*(m + 1) - 1)*(a + b*(x^(2*k)/c^2))
^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && FractionQ[m] && I
ntBinomialQ[a, b, c, 2, m, p, x]
```

rule 756
```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-a/b, 2
]], s = Denominator[Rt[-a/b, 2]]}, Simp[r/(2*a)   Int[1/(r - s*x^2), x], x]
 + Simp[r/(2*a)   Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ[a
/b, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3957

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b/d   Subst[Int
[x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&
!IntegerQ[n]
```

rule 4204

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_), x_Symb
ol] :> Simp[(c + d*x)^m*((b*Tan[e + f*x])^(n + 1)/(b*f*(n + 1))), x] + (-Si
mp[d*(m/(b*f*(n + 1)))   Int[(c + d*x)^(m - 1)*(b*Tan[e + f*x])^(n + 1), x]
, x] - Simp[1/b^2   Int[(c + d*x)^m*(b*Tan[e + f*x])^(n + 2), x], x]) /; Fr
eeQ[{b, c, d, e, f}, x] && LtQ[n, -1] && GtQ[m, 0]
```

rule 4219

```
Int[((c_.) + (d_.)*(x_))*Sqrt[(a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)]], x_Sym
bol] :> Simp[(-I)*Rt[a - I*b, 2]*((c + d*x)/f)*ArcTanh[Sqrt[a + b*Tan[e + f
*x]]/Rt[a - I*b, 2]], x] + (Simp[I*Rt[a + I*b, 2]*((c + d*x)/f)*ArcTanh[Sqr
t[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x] + Simp[I*d*(Rt[a - I*b, 2]/f)   I
nt[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a - I*b, 2]], x], x] - Simp[I*d*(Rt[
a + I*b, 2]/f)   Int[ArcTanh[Sqrt[a + b*Tan[e + f*x]]/Rt[a + I*b, 2]], x],
x]) /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2, 0]
```

rule 4853

```
Int[u_, x_Symbol] :> With[{v = FunctionOfTrig[u, x]}, Simp[With[{d = FreeFa
ctors[Tan[v], x]}, d/Coefficient[v, x, 1]   Subst[Int[SubstFor[1/(1 + d^2*x
^2), Tan[v]/d, u, x], x], x, Tan[v]/d]], x] /;  !FalseQ[v] && FunctionOfQ[N
onfreeFactors[Tan[v], x], u, x, True] && TryPureTanSubst[ActivateTrig[u], x
]]
```

rule 7267

```
Int[u_, x_Symbol] :> With[{lst = SubstForFractionalPowerOfLinear[u, x]}, Si
mp[lst[[2]]*lst[[4]]   Subst[Int[lst[[1]], x], x, lst[[3]]^(1/lst[[2]])], x
] /;  !FalseQ[lst] && SubstForFractionalPowerQ[u, lst[[3]], x]]
```

rule 7276

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [F]

$$\int \frac{dx+c}{(b\tanh(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{c+dx}{(b\tanh(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*x)/(b*tanh(e + f*x))**(3/2), x)
```

Maxima [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{dx+c}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e))^(3/2), x)
```

Giac [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{dx+c}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e))^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{c+d\,x}{(b\tanh(e+f\,x))^{3/2}}\,dx$$

input
```
int((c + d*x)/(b*tanh(e + f*x))^(3/2),x)
```

output
```
int((c + d*x)/(b*tanh(e + f*x))^(3/2), x)
```

Reduce [F]

$$\int \frac{c+dx}{(b\tanh(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)^2}dx\right)c+\left(\int \frac{\sqrt{\tanh(fx+e)}\,x}{\tanh(fx+e)^2}dx\right)d\right)}{b^2}$$

input
```
int((d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(tanh(e + f*x))/tanh(e + f*x)**2,x)*c + int((sqrt(tanh(e
 + f*x))*x)/tanh(e + f*x)**2,x)*d))/b**2
```

3.21 $\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \text{Too large to display}$$

output

```
4*(-b)^(3/2)*d*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/f^2+2*(-b
)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/f^3+4*b^(3/2)*d*(d
*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/f^2+2*b^(3/2)*d^2*arctanh((b*
tanh(f*x+e))^(1/2)/b^(1/2))^2/f^3-4*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1
/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^3+4*b^(3/2)*d
^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*
x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(
2*b^(1/2)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)
+(b*tanh(f*x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b
^(1/2))*ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2
))/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^3-4*(-b)^(3/2)*d^2*arctanh((b*tanh(f
*x+e))^(1/2)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3+2*
(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*
tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/
2)))/f^3+2*(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln((-2
*b^(1/2)-2*(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^
(1/2)/(-b)^(1/2)))/f^3+4*(-b)^(3/2)*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)
^(1/2))*ln(2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/f^3-2*b^(3/2)*d^2*polyl
og(2,1-2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/f^3-2*b^(3/2)*d^2*polylo
g(2,1-2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/f^3+b^(3/2)*d^2*polylo...
```

Mathematica [N/A]

Not integrable

Time = 27.47 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx = \int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx$$

input

```
Integrate[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2),x]
```

output

```
Integrate[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 2.62 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 (-ib\tan(ie+ifx))^{3/2} dx$$

$$\downarrow 4203$$

$$b^2 \int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}} dx + \frac{4bd\int (c+dx)\sqrt{b\tanh(e+fx)}dx}{f} - \frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 3042$$

$$b^2 \int \frac{(c+dx)^2}{\sqrt{-ib\tan(ie+ifx)}} dx + \frac{4bd\int (c+dx)\sqrt{-ib\tan(ie+ifx)}dx}{f} - \frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 4219$$

$$\frac{4bd\left(\frac{\sqrt{-b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f} - \frac{\sqrt{b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f} - \frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f} + \frac{\sqrt{b}(c+d}{}\right.}{f}$$

$$b^2 \int \frac{(c+dx)^2}{\sqrt{-ib\tan(ie+ifx)}} dx - \frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 4223$$

$$\frac{4bd\left(\frac{\sqrt{-b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{f}-\frac{\sqrt{b}d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{f}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{f}+\frac{\sqrt{b}(c+d}{}\right.}{f}$$

$$b^2\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 4853$$

$$\frac{4bd\left(\frac{\sqrt{-b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{f^2}-\frac{\sqrt{-b}(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tan}}{}\right.}{f}\right.}{f}$$

$$b^2\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 7267$$

$$\frac{4bd\left(\frac{2\sqrt{-b}d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{bf^2}-\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+}{\sqrt{b}f^2}\right.}{f}$$

$$b^2\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 27$$

$$\frac{4bd\left(\frac{2\sqrt{-b}bd\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\frac{2b^{3/2}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e-}{f^2}\right.}{f}$$

$$b^2\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 7276$$

$$4bd\left(-\frac{2b^{3/2}d\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{f^2}+\frac{2\sqrt{-b}bd\int\left(\text{ar}\right.}{}\right.$$

$$b^2\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx-\frac{2b(c+dx)^2\sqrt{b\tanh(e+fx)}}{f}$$

$$\downarrow 2009$$

$$\int\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dxb^2+$$

$$4d\left(-\frac{2d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{2b}-\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}+\sqrt{b\tanh(e+fx)}}\right)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(}\right.}{}\right.}{}\right.$$

$$\frac{2(c+dx)^2\sqrt{b\tanh(e+fx)}b}{f}$$

input
```
Int[(c + d*x)^2*(b*Tanh[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (dx+c)^2\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}dx$$

input
```
int((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 3.02 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx = \int (b\tanh(e+fx))^{\frac{3}{2}} (c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((b*tanh(e + f*x))**(3/2)*(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)^2(b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e))^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \int (dx+c)^2(b\tanh(fx+e))^{\frac{3}{2}}\,dx$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e))^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2(b\tanh(e+fx))^{3/2}\,dx = \int (b\tanh(e+f\,x))^{3/2}\,(c+d\,x)^2\,dx$$

input
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x)^2,x)
```

output
```
int((b*tanh(e + f*x))^(3/2)*(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 89, normalized size of antiderivative = 4.45

$$\int (c+dx)^2 (b\tanh(e+fx))^{3/2}\,dx = \frac{\sqrt{b}\,b\left(-2\sqrt{\tanh(fx+e)}\,c^2 + \left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)}dx\right)c^2 f + \left(\int \sqrt{\tanh(fx+e)}\,\tanh(fx+e)\,x\right.\right.}{f}$$

input
```
int((d*x+c)^2*(b*tanh(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*b*( - 2*sqrt(tanh(e + f*x))*c**2 + int(sqrt(tanh(e + f*x))/tanh(e
 + f*x),x)*c**2*f + int(sqrt(tanh(e + f*x))*tanh(e + f*x)*x**2,x)*d**2*f +
 2*int(sqrt(tanh(e + f*x))*tanh(e + f*x)*x,x)*c*d*f))/f
```

3.22 $\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx$

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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx = \text{Int}\left((c+dx)^2 \sqrt{b \tanh(e+fx)}, x\right)$$

output
```
Defer(Int)((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

Mathematica [F(-1)]

Timed out.

$$\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx = \$\text{Aborted}$$

input
```
Integrate[(c + d*x)^2*Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2 \sqrt{-ib \tan(ie+ifx)}\, dx$$

$$\downarrow 4223$$

$$\int (c+dx)^2 \sqrt{b \tanh(e+fx)}\, dx$$

input
```
Int[(c + d*x)^2*Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int (dx+c)^2 \sqrt{b \tanh(fx+e)}\, dx$$

input
```
int((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \int \sqrt{b\tanh\left(e+fx\right)}(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral(sqrt(b*tanh(e + f*x))*(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \int (dx+c)^2\sqrt{b\tanh\left(fx+e\right)}\,dx$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2*sqrt(b*tanh(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:index.cc index_m i_lex_is_greater Error: Bad Argument V
alue
```

Mupad [N/A]

Not integrable

Time = 2.60 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \int \sqrt{b\tanh(e+fx)}\,(c+dx)^2\,dx$$

input
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x)^2,x)
```

output
```
int((b*tanh(e + f*x))^(1/2)*(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 49, normalized size of antiderivative = 2.45

$$\int (c+dx)^2\sqrt{b\tanh(e+fx)}\,dx = \sqrt{b}\left(\left(\int \sqrt{\tanh(fx+e)}dx\right)c^2 + \left(\int \sqrt{\tanh(fx+e)}\,x^2dx\right)d^2 + 2\left(\int \sqrt{\tanh(fx+e)}\,xdx\right)cd\right)$$

input
```
int((d*x+c)^2*(b*tanh(f*x+e))^(1/2),x)
```

output
```
sqrt(b)*(int(sqrt(tanh(e + f*x)),x)*c**2 + int(sqrt(tanh(e + f*x))*x**2,x)
*d**2 + 2*int(sqrt(tanh(e + f*x))*x,x)*c*d)
```

3.23 $\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$

Optimal result . . . 230
Mathematica [F(-1)] . . . 230
Rubi [N/A] . . . 231
Maple [N/A] . . . 231
Fricas [F(-2)] . . . 232
Sympy [N/A] . . . 232
Maxima [N/A] . . . 233
Giac [F(-2)] . . . 233
Mupad [N/A] . . . 233
Reduce [N/A] . . . 234

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Int}\left(\frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}, x\right)$$

output
```
Defer(Int)((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

Mathematica [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \$\text{Aborted}$$

input
```
Integrate[(c + d*x)^2/Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c+dx)^2}{\sqrt{-ib\tan(ie+ifx)}}dx$$
$$\downarrow 4223$$
$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}dx$$

input
```
Int[(c + d*x)^2/Sqrt[b*Tanh[e + f*x]],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(dx+c)^2}{\sqrt{b\tanh(fx+e)}}dx$$

input
```
int((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

output
```
int((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.26 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{(c+dx)^2}{\sqrt{b\tanh\left(e+fx\right)}}\,dx$$

input
```
integrate((d*x+c)**2/(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral((c + d*x)**2/sqrt(b*tanh(e + f*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.52 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{(dx+c)^2}{\sqrt{b\tanh(fx+e)}}\,dx$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/sqrt(b*tanh(f*x + e)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Degree mismatch inside factorisation over extensionDegr
ee mismatch inside factorisation over extensionindex.cc index_m i_lex_is_g
reater Error
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{(c+d\,x)^2}{\sqrt{b\tanh(e+f\,x)}}\,dx$$

input
```
int((c + d*x)^2/(b*tanh(e + f*x))^(1/2),x)
```

output
```
int((c + d*x)^2/(b*tanh(e + f*x))^(1/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.85

$$\int \frac{(c+dx)^2}{\sqrt{b\tanh(e+fx)}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)}dx\right)c^2 + \left(\int \frac{\sqrt{\tanh(fx+e)}\,x^2}{\tanh(fx+e)}dx\right)d^2 + 2\left(\int \frac{\sqrt{\tanh(fx+e)}\,x}{\tanh(fx+e)}dx\right)cd\right)}{b}$$

input
```
int((d*x+c)^2/(b*tanh(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*(int(sqrt(tanh(e + f*x))/tanh(e + f*x),x)*c**2 + int((sqrt(tanh(e
 + f*x))*x**2)/tanh(e + f*x),x)*d**2 + 2*int((sqrt(tanh(e + f*x))*x)/tanh(
e + f*x),x)*c*d))/b
```

3.24 $\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$

Optimal result 235
Mathematica [N/A] 236
Rubi [N/A] 237
Maple [N/A] 239
Fricas [F(-2)] 240
Sympy [N/A] 240
Maxima [N/A] 241
Giac [N/A] 241
Mupad [N/A] 241
Reduce [N/A] 242

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Too large to display}$$

output

```
4*d*(d*x+c)*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))/(-b)^(3/2)/f^2+2*d^2
*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))^2/(-b)^(3/2)/f^3+4*d*(d*x+c)*ar
ctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))/b^(3/2)/f^2+2*d^2*arctanh((b*tanh(f*x
+e))^(1/2)/b^(1/2))^2/b^(3/2)/f^3-4*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1
/2))*ln(2*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3+4*d^2*arcta
nh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1
/2)))/b^(3/2)/f^3-2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*ln(2*b^(1/2
)*((-b)^(1/2)-(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(b^(1/2)+(b*tanh
(f*x+e))^(1/2)))/b^(3/2)/f^3-2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/b^(1/2))*
ln(2*b^(1/2)*((-b)^(1/2)+(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)+b^(1/2))/(b^(1
/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3-4*d^2*arctanh((b*tanh(f*x+e))^(1/2
)/(-b)^(1/2))*ln(2/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3+2*
d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(2*(b^(1/2)-(b*tanh(f*x+e)
)^(1/2))/((-b)^(1/2)+b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(
3/2)/f^3+2*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln((-2*b^(1/2)-2*
(b*tanh(f*x+e))^(1/2))/((-b)^(1/2)-b^(1/2))/(1-(b*tanh(f*x+e))^(1/2)/(-b)^
(1/2)))/(-b)^(3/2)/f^3+4*d^2*arctanh((b*tanh(f*x+e))^(1/2)/(-b)^(1/2))*ln(
2/(1+(b*tanh(f*x+e))^(1/2)/(-b)^(1/2)))/(-b)^(3/2)/f^3-2*d^2*polylog(2,1-2
*b^(1/2)/(b^(1/2)-(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3-2*d^2*polylog(2,1-2*
b^(1/2)/(b^(1/2)+(b*tanh(f*x+e))^(1/2)))/b^(3/2)/f^3+d^2*polylog(2,1-2*...
```

Mathematica [N/A]

Not integrable

Time = 28.35 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$$

input

```
Integrate[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2),x]
```

output

```
Integrate[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2), x]
```

Rubi [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c+dx)^2}{(-ib\tan(ie+ifx))^{3/2}}dx$$

$$\downarrow 4204$$

$$\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}+\frac{4d\int \frac{c+dx}{\sqrt{b\tanh(e+fx)}}dx}{bf}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 3042$$

$$\frac{\int (c+dx)^2\sqrt{-ib\tan(ie+ifx)}dx}{b^2}+\frac{4d\int \frac{c+dx}{\sqrt{-ib\tan(ie+ifx)}}dx}{bf}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 4221$$

$$\frac{4d\left(\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f}-\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{}}{}\right.}{\sqrt{b}f}\right.}{bf}$$

$$\frac{\int (c+dx)^2\sqrt{-ib\tan(ie+ifx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 4223$$

$$4d\left(\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)dx}{\sqrt{-b}f}-\frac{d\int \operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)dx}{\sqrt{b}f}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b}}{\sqrt{b}}\right)}{\sqrt{b}f}\right.$$

$$\left.\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}\right)$$

$$bf$$

$$\downarrow 4853$$

$$4d\left(\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{-b}f^2}-\frac{d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{1-\tanh^2(e+fx)}d\tanh(e+fx)}{\sqrt{b}f^2}-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\right.$$

$$\left.\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}\right)$$

$$bf$$

$$\downarrow 7267$$

$$4d\left(\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}bf^2}-\frac{2d\int \frac{b^2\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{b^{3/2}f^2}-\right.$$

$$\left.\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}\right)$$

$$bf$$

$$\downarrow 27$$

$$4d\left(\frac{2bd\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int \frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)\sqrt{b\tanh(e+fx)}}{b^2-b^2\tanh^2(e+fx)}d\sqrt{b\tanh(e+fx)}}{f^2}-\right.$$

$$\left.\frac{\int (c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}\right)$$

$$bf$$

$$\downarrow 7276$$

$$4d\left(\frac{2bd\int\left(\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(\tanh(e+fx)b+b)}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)\sqrt{b\tanh(e+fx)}}{2b(b\tanh(e+fx)-b)}\right)d\sqrt{b\tanh(e+fx)}}{\sqrt{-b}f^2}-\frac{2\sqrt{b}d\int\left(\frac{\operatorname{arctanh}\left(\right.}{}\right.$$

$$\frac{\int(c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}$$

$$\downarrow 2009$$

$$-\frac{2(c+dx)^2}{bf\sqrt{b\tanh(e+fx)}}+$$

$$4d\left(-\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{-b}}\right)}{\sqrt{-b}f}+\frac{(c+dx)\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)}{\sqrt{b}f}-\frac{2\sqrt{b}d\left(-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b\tanh(e+fx)}}{\sqrt{b}}\right)^2}{4b}+\frac{\log\left(\frac{2\sqrt{b}}{\sqrt{b}-\sqrt{b\tanh(}\right.}{}\right.}{}\right.$$

$$\frac{\int(c+dx)^2\sqrt{b\tanh(e+fx)}dx}{b^2}$$

input
```
Int[(c + d*x)^2/(b*Tanh[e + f*x])^(3/2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int\frac{(dx+c)^2}{(b\tanh(fx+e))^{\frac{3}{2}}}dx$$

input
```
int((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x)
```

output
```
int((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.43 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(c+dx)^2}{(b\tanh(e+fx))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)**2/(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral((c + d*x)**2/(b*tanh(e + f*x))**(3/2), x)
```

Maxima [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(dx+c)^2}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e))^(3/2), x)
```

Giac [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(dx+c)^2}{(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e))^(3/2), x)
```

Mupad [N/A]

Not integrable

Time = 2.54 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{(c+d\,x)^2}{(b\tanh(e+f\,x))^{3/2}}\,dx$$

input
```
int((c + d*x)^2/(b*tanh(e + f*x))^(3/2),x)
```

output
```
int((c + d*x)^2/(b*tanh(e + f*x))^(3/2), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 77, normalized size of antiderivative = 3.85

$$\int \frac{(c+dx)^2}{(b\tanh(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)^2}dx\right)c^2 + \left(\int \frac{\sqrt{\tanh(fx+e)}\,x^2}{\tanh(fx+e)^2}dx\right)d^2 + 2\left(\int \frac{\sqrt{\tanh(fx+e)}\,x}{\tanh(fx+e)^2}dx\right)c\right.}{b^2}$$

input
```
int((d*x+c)^2/(b*tanh(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*(int(sqrt(tanh(e + f*x))/tanh(e + f*x)**2,x)*c**2 + int((sqrt(tan
h(e + f*x))*x**2)/tanh(e + f*x)**2,x)*d**2 + 2*int((sqrt(tanh(e + f*x))*x)
/tanh(e + f*x)**2,x)*c*d))/b**2
```

3.25 $\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$

Optimal result . . . 243
Mathematica [N/A] . . . 243
Rubi [N/A] . . . 244
Maple [N/A] . . . 244
Fricas [F(-2)] . . . 245
Sympy [N/A] . . . 245
Maxima [N/A] . . . 245
Giac [N/A] . . . 246
Mupad [N/A] . . . 246
Reduce [N/A] . . . 247

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \text{Int}\left(\frac{(b\tanh(e+fx))^{3/2}}{c+dx}, x\right)$$

output
```
Defer(Int)((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 22.89 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$

input
```
Integrate[(b*Tanh[e + f*x])^(3/2)/(c + d*x),x]
```

output
```
Integrate[(b*Tanh[e + f*x])^(3/2)/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(-ib\tan(ie+ifx))^{3/2}}{c+dx}\,dx$$
$$\downarrow 4223$$
$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx$$

input
```
Int[(b*Tanh[e + f*x])^(3/2)/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{(b\tanh(fx+e))^{\frac{3}{2}}}{dx+c}\,dx$$

input
```
int((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

output
```
int((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(e+fx))^{\frac{3}{2}}}{c+dx}\,dx$$

input
```
integrate((b*tanh(f*x+e))**(3/2)/(d*x+c),x)
```

output
```
Integral((b*tanh(e + f*x))**(3/2)/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(fx+e))^{\frac{3}{2}}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate((b*tanh(f*x + e))^(3/2)/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(fx+e))^{\frac{3}{2}}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(3/2)/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e))^(3/2)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.48 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \int \frac{(b\tanh(e+f\,x))^{3/2}}{c+d\,x}\,dx$$

input
```
int((b*tanh(e + f*x))^(3/2)/(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(3/2)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{(b\tanh(e+fx))^{3/2}}{c+dx}\,dx = \sqrt{b}\left(\int \frac{\sqrt{\tanh(fx+e)}\,\tanh(fx+e)}{dx+c}dx\right)b$$

input
```
int((b*tanh(f*x+e))^(3/2)/(d*x+c),x)
```

output
```
sqrt(b)*int((sqrt(tanh(e + f*x))*tanh(e + f*x))/(c + d*x),x)*b
```

3.26 $\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx$

Optimal result . . . 248
Mathematica [N/A] . . . 248
Rubi [N/A] . . . 249
Maple [N/A] . . . 249
Fricas [F(-2)] . . . 250
Sympy [N/A] . . . 250
Maxima [N/A] . . . 250
Giac [N/A] . . . 251
Mupad [N/A] . . . 251
Reduce [N/A] . . . 252

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \text{Int}\left(\frac{\sqrt{b\tanh(e+fx)}}{c+dx}, x\right)$$

output
```
Defer(Int)((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 1.84 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx$$

input
```
Integrate[Sqrt[b*Tanh[e + f*x]]/(c + d*x),x]
```

output
```
Integrate[Sqrt[b*Tanh[e + f*x]]/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{\sqrt{-ib\tan(ie+ifx)}}{c+dx}\,dx$$
$$\downarrow 4223$$
$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx$$

input
```
Int[Sqrt[b*Tanh[e + f*x]]/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{\sqrt{b\tanh\left(fx+e\right)}}{dx+c}\,dx$$

input
```
int((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

output
```
int((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx$$

input
```
integrate((b*tanh(f*x+e))**(1/2)/(d*x+c),x)
```

output
```
Integral(sqrt(b*tanh(e + f*x))/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b \tanh(fx+e)}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(f*x + e))/(d*x + c), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b \tanh(fx+e)}}{dx+c}\,dx$$

input
```
integrate((b*tanh(f*x+e))^(1/2)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tanh(f*x + e))/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b \tanh(e+fx)}}{c+dx}\,dx = \int \frac{\sqrt{b \tanh(e+f\,x)}}{c+d\,x}\,dx$$

input
```
int((b*tanh(e + f*x))^(1/2)/(c + d*x),x)
```

output
```
int((b*tanh(e + f*x))^(1/2)/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{\sqrt{b\tanh(e+fx)}}{c+dx}\,dx = \sqrt{b}\left(\int \frac{\sqrt{\tanh(fx+e)}}{dx+c}dx\right)$$

input
```
int((b*tanh(f*x+e))^(1/2)/(d*x+c),x)
```

output
```
sqrt(b)*int(sqrt(tanh(e + f*x))/(c + d*x),x)
```

3.27 $\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$

Optimal result . . . 253
Mathematica [N/A] . . . 253
Rubi [N/A] . . . 254
Maple [N/A] . . . 254
Fricas [F(-2)] . . . 255
Sympy [N/A] . . . 255
Maxima [N/A] . . . 255
Giac [N/A] . . . 256
Mupad [N/A] . . . 256
Reduce [N/A] . . . 257

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \text{Int}\left(\frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}, x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

Mathematica [N/A]

Not integrable

Time = 1.93 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$$

input
```
Integrate[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]),x]
```

output
```
Integrate[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)\sqrt{-ib\tan(ie+ifx)}}dx$$
$$\downarrow 4223$$
$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}dx$$

input
```
Int[1/((c + d*x)*Sqrt[b*Tanh[e + f*x]]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{(dx+c)\sqrt{b\tanh(fx+e)}}dx$$

input
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

output
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 0.95 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tanh(e+fx)}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))**(1/2),x)
```

output
```
Integral(1/(sqrt(b*tanh(e + f*x))*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{(dx+c)\sqrt{b\tanh(fx+e)}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*sqrt(b*tanh(f*x + e))), x)
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{(dx+c)\sqrt{b\tanh\left(fx+e\right)}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*sqrt(b*tanh(f*x + e))), x)
```

Mupad [N/A]

Not integrable

Time = 2.31 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \int \frac{1}{\sqrt{b\tanh\left(e+f\,x\right)}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((b*tanh(e + f*x))^(1/2)*(c + d*x)),x)
```

output
```
int(1/((b*tanh(e + f*x))^(1/2)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{1}{(c+dx)\sqrt{b\tanh(e+fx)}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)c+\tanh(fx+e)dx}dx\right)}{b}$$

input
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(1/2),x)
```

output
```
(sqrt(b)*int(sqrt(tanh(e + f*x))/(tanh(e + f*x)*c + tanh(e + f*x)*d*x),x))
/b
```

3.28 $\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$

Optimal result . . . 258
Mathematica [N/A] . . . 258
Rubi [N/A] . . . 259
Maple [N/A] . . . 259
Fricas [F(-2)] . . . 260
Sympy [N/A] . . . 260
Maxima [N/A] . . . 260
Giac [N/A] . . . 261
Mupad [N/A] . . . 261
Reduce [N/A] . . . 262

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \text{Int}\left(\frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}},x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

Mathematica [N/A]

Not integrable

Time = 20.06 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$$

input
```
Integrate[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)),x]
```

output
```
Integrate[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)), x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)(-ib\tan(ie+ifx))^{3/2}}dx$$
$$\downarrow 4223$$
$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}dx$$

input
```
Int[1/((c + d*x)*(b*Tanh[e + f*x])^(3/2)),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.90

$$\int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}dx$$

input
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

output
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

Fricas [F(-2)]

Exception generated.

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="fricas")
```

output
```
Exception raised: TypeError >>  Error detected within library code:   inte
grate: implementation incomplete (constant residues)
```

Sympy [N/A]

Not integrable

Time = 1.10 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(b\tanh(e+fx))^{\frac{3}{2}}\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))**(3/2),x)
```

output
```
Integral(1/((b*tanh(e + f*x))**(3/2)*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(dx+c)\,(b\tanh(fx+e))^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e))^(3/2)), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{(dx+c)\left(b\tanh\left(fx+e\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e))^(3/2)), x)
```

Mupad [N/A]

Not integrable

Time = 2.58 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \int \frac{1}{\left(b\tanh\left(e+f\,x\right)\right)^{3/2}\,\left(c+d\,x\right)}\,dx$$

input
```
int(1/((b*tanh(e + f*x))^(3/2)*(c + d*x)),x)
```

output
```
int(1/((b*tanh(e + f*x))^(3/2)*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 40, normalized size of antiderivative = 2.00

$$\int \frac{1}{(c+dx)(b\tanh(e+fx))^{3/2}}\,dx = \frac{\sqrt{b}\left(\int \frac{\sqrt{\tanh(fx+e)}}{\tanh(fx+e)^2 c+\tanh(fx+e)^2 dx}dx\right)}{b^2}$$

input
```
int(1/(d*x+c)/(b*tanh(f*x+e))^(3/2),x)
```

output
```
(sqrt(b)*int(sqrt(tanh(e + f*x))/(tanh(e + f*x)**2*c + tanh(e + f*x)**2*d*
x),x))/b**2
```

3.29 $\int x^m \tanh^3(a+bx)\,dx$

Optimal result 263
Mathematica [N/A] 263
Rubi [N/A] 264
Maple [N/A] 264
Fricas [N/A] 265
Sympy [N/A] 265
Maxima [N/A] 266
Giac [N/A] 266
Mupad [N/A] 267
Reduce [N/A] 267

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^3(a+bx)\,dx = \text{Int}\left(x^m \tanh^3(a+bx), x\right)$$

output
```
Defer(Int)(x^m*tanh(b*x+a)^3,x)
```

Mathematica [N/A]

Not integrable

Time = 0.66 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x]^3,x]
```

output
```
Integrate[x^m*Tanh[a + b*x]^3, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh^3(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int ix^m \tan(ia+ibx)^3 dx$$

$$\downarrow 26$$

$$i\int x^m \tan(ia+ibx)^3 dx$$

$$\downarrow 4222$$

$$\int x^m \tanh^3(a+bx)dx$$

input
```
Int[x^m*Tanh[a + b*x]^3,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh\left(bx+a\right)^3\,dx$$

input
```
int(x^m*tanh(b*x+a)^3,x)
```

output
```
int(x^m*tanh(b*x+a)^3,x)
```

Fricas [N/A]

Not integrable

Time = 0.08 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^3,x, algorithm="fricas")
```

output
```
integral(x^m*tanh(b*x + a)^3, x)
```

Sympy [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh^3\left(a+bx\right)\,dx$$

input
```
integrate(x**m*tanh(b*x+a)**3,x)
```

output
```
Integral(x**m*tanh(a + b*x)**3, x)
```

Maxima [N/A]

Not integrable

Time = 0.71 (sec) , antiderivative size = 171, normalized size of antiderivative = 14.25

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^3,x, algorithm="maxima")
```

output
```
x*e^(6*b*x + m*log(x) + 6*a)/((m + 1)*e^(6*b*x + 6*a) + 3*(m + 1)*e^(4*b*x
 + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate((3*(2*b*x*e^(6*a)
 + (m + 1)*e^(6*a))*e^(6*b*x) - 2*(m + 1)*e^(2*b*x + 2*a) + m + 1)*x^m/((m
 + 1)*e^(8*b*x + 8*a) + 4*(m + 1)*e^(6*b*x + 6*a) + 6*(m + 1)*e^(4*b*x + 4
*a) + 4*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)^3\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^3,x, algorithm="giac")
```

output
```
integrate(x^m*tanh(b*x + a)^3, x)
```

Mupad [N/A]

Not integrable

Time = 2.19 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^3(a+bx)\,dx = \int x^m \tanh(a+b\,x)^3\,dx$$

input
```
int(x^m*tanh(a + b*x)^3,x)
```

output
```
int(x^m*tanh(a + b*x)^3, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 1614, normalized size of antiderivative = 134.50

$$\int x^m \tanh^3(a+bx)\,dx = \text{Too large to display}$$

input
```
int(x^m*tanh(b*x+a)^3,x)
```

output

```
( - 6*e**(8*a + 4*b*x)*int((x**m*e**(4*b*x))/(e**(6*a + 6*b*x)*x + 3*e**(4
*a + 4*b*x)*x + 3*e**(2*a + 2*b*x)*x + x),x)*m**2 - 6*e**(8*a + 4*b*x)*int
((x**m*e**(4*b*x))/(e**(6*a + 6*b*x)*x + 3*e**(4*a + 4*b*x)*x + 3*e**(2*a
+ 2*b*x)*x + x),x)*m - 9*e**(6*a + 4*b*x)*int((x**m*e**(2*b*x))/(e**(6*a +
 6*b*x)*x + 3*e**(4*a + 4*b*x)*x + 3*e**(2*a + 2*b*x)*x + x),x)*m**2 - 9*e
**(6*a + 4*b*x)*int((x**m*e**(2*b*x))/(e**(6*a + 6*b*x)*x + 3*e**(4*a + 4*
b*x)*x + 3*e**(2*a + 2*b*x)*x + x),x)*m + 2*x**m*e**(4*a + 4*b*x)*b*x - 4*
e**(4*a + 4*b*x)*int(x**m/(e**(6*a + 6*b*x) + 3*e**(4*a + 4*b*x) + 3*e**(2
*a + 2*b*x) + 1),x)*b*m - 4*e**(4*a + 4*b*x)*int(x**m/(e**(6*a + 6*b*x) +
3*e**(4*a + 4*b*x) + 3*e**(2*a + 2*b*x) + 1),x)*b - 3*e**(4*a + 4*b*x)*int
(x**m/(e**(6*a + 6*b*x)*x + 3*e**(4*a + 4*b*x)*x + 3*e**(2*a + 2*b*x)*x +
x),x)*m**2 - 3*e**(4*a + 4*b*x)*int(x**m/(e**(6*a + 6*b*x)*x + 3*e**(4*a +
 4*b*x)*x + 3*e**(2*a + 2*b*x)*x + x),x)*m - 12*e**(6*a + 2*b*x)*int((x**m
*e**(4*b*x))/(e**(6*a + 6*b*x)*x + 3*e**(4*a + 4*b*x)*x + 3*e**(2*a + 2*b*
x)*x + x),x)*m**2 - 12*e**(6*a + 2*b*x)*int((x**m*e**(4*b*x))/(e**(6*a + 6
*b*x)*x + 3*e**(4*a + 4*b*x)*x + 3*e**(2*a + 2*b*x)*x + x),x)*m - 18*e**(4
*a + 2*b*x)*int((x**m*e**(2*b*x))/(e**(6*a + 6*b*x)*x + 3*e**(4*a + 4*b*x)
*x + 3*e**(2*a + 2*b*x)*x + x),x)*m**2 - 18*e**(4*a + 2*b*x)*int((x**m*e**
(2*b*x))/(e**(6*a + 6*b*x)*x + 3*e**(4*a + 4*b*x)*x + 3*e**(2*a + 2*b*x)*x
 + x),x)*m + 4*x**m*e**(2*a + 2*b*x)*b*x + 6*x**m*e**(2*a + 2*b*x)*m + ...
```

3.30 $\int x^m \tanh^2(a+bx)\,dx$

Optimal result . . . 269
Mathematica [N/A] . . . 269
Rubi [N/A] . . . 270
Maple [N/A] . . . 270
Fricas [N/A] . . . 271
Sympy [N/A] . . . 271
Maxima [N/A] . . . 272
Giac [N/A] . . . 272
Mupad [N/A] . . . 273
Reduce [N/A] . . . 273

Optimal result

Integrand size = 12, antiderivative size = 12

$$\int x^m \tanh^2(a+bx)\,dx = \text{Int}\left(x^m \tanh^2(a+bx), x\right)$$

output
```
Defer(Int)(x^m*tanh(b*x+a)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x]^2,x]
```

output
```
Integrate[x^m*Tanh[a + b*x]^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh^2(a+bx)\,dx$$

$$\downarrow 3042$$

$$\int -x^m \tan(ia+ibx)^2 dx$$

$$\downarrow 25$$

$$-\int x^m \tan(ia+ibx)^2 dx$$

$$\downarrow 4222$$

$$\int x^m \tanh^2(a+bx) dx$$

input
```
Int[x^m*Tanh[a + b*x]^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh\left(bx+a\right)^2 dx$$

input
```
int(x^m*tanh(b*x+a)^2,x)
```

output
```
int(x^m*tanh(b*x+a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.07 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^2,x, algorithm="fricas")
```

output
```
integral(x^m*tanh(b*x + a)^2, x)
```

Sympy [N/A]

Not integrable

Time = 0.49 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.00

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh^2\left(a+bx\right)\,dx$$

input
```
integrate(x**m*tanh(b*x+a)**2,x)
```

output
```
Integral(x**m*tanh(a + b*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.63 (sec) , antiderivative size = 144, normalized size of antiderivative = 12.00

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^2,x, algorithm="maxima")
```

output
```
x*e^(4*b*x + m*log(x) + 4*a)/((m + 1)*e^(4*b*x + 4*a) + 2*(m + 1)*e^(2*b*x
 + 2*a) + m + 1) - integrate((2*(2*b*x*e^(4*a) + (m + 1)*e^(4*a))*e^(4*b*x
) + (m + 1)*e^(2*b*x + 2*a) - m - 1)*x^m/((m + 1)*e^(6*b*x + 6*a) + 3*(m +
 1)*e^(4*b*x + 4*a) + 3*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)^2\,dx$$

input
```
integrate(x^m*tanh(b*x+a)^2,x, algorithm="giac")
```

output
```
integrate(x^m*tanh(b*x + a)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.27 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.17

$$\int x^m \tanh^2(a+bx)\,dx = \int x^m \tanh(a+b\,x)^2\,dx$$

input
```
int(x^m*tanh(a + b*x)^2,x)
```

output
```
int(x^m*tanh(a + b*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 99, normalized size of antiderivative = 8.25

$$\int x^m \tanh^2(a+bx)\,dx$$
$$= \frac{-x^m \tanh(bx+a)\,m - x^m \tanh(bx+a) + x^m bx + x^m m + x^m - 2\left(\int \frac{x^m}{e^{2bx+2a}x+x}dx\right)m^2 - 2\left(\int \frac{x^m}{e^{2bx+2a}x+}\right.}{b(m+1)}$$

input
```
int(x^m*tanh(b*x+a)^2,x)
```

output
```
( - x**m*tanh(a + b*x)*m - x**m*tanh(a + b*x) + x**m*b*x + x**m*m + x**m -
 2*int(x**m/(e**(2*a + 2*b*x)*x + x),x)*m**2 - 2*int(x**m/(e**(2*a + 2*b*x
)*x + x),x)*m)/(b*(m + 1))
```

3.31 $\int x^m \tanh(a+bx)\,dx$

Optimal result . . . 274
Mathematica [N/A] . . . 274
Rubi [N/A] . . . 275
Maple [N/A] . . . 275
Fricas [N/A] . . . 276
Sympy [N/A] . . . 276
Maxima [N/A] . . . 277
Giac [N/A] . . . 277
Mupad [N/A] . . . 277
Reduce [N/A] . . . 278

Optimal result

Integrand size = 10, antiderivative size = 10

$$\int x^m \tanh(a+bx)\,dx = \text{Int}(x^m \tanh(a+bx), x)$$

output
```
Defer(Int)(x^m*tanh(b*x+a),x)
```

Mathematica [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+bx)\,dx$$

input
```
Integrate[x^m*Tanh[a + b*x],x]
```

output
```
Integrate[x^m*Tanh[a + b*x], x]
```

Rubi [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int x^m \tanh(a+bx)\,dx$$
$$\downarrow 3042$$
$$\int -ix^m \tan(ia+ibx)dx$$
$$\downarrow 26$$
$$-i\int x^m \tan(ia+ibx)dx$$
$$\downarrow 4222$$
$$\int x^m \tanh(a+bx)dx$$

input
```
Int[x^m*Tanh[a + b*x],x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.11 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \tanh\left(bx+a\right)dx$$

input
```
int(x^m*tanh(b*x+a),x)
```

output
```
int(x^m*tanh(b*x+a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*tanh(b*x+a),x, algorithm="fricas")
```

output
```
integral(x^m*tanh(b*x + a), x)
```

Sympy [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 10, normalized size of antiderivative = 1.00

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh\left(a+bx\right)\,dx$$

input
```
integrate(x**m*tanh(b*x+a),x)
```

output
```
Integral(x**m*tanh(a + b*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 100, normalized size of antiderivative = 10.00

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*tanh(b*x+a),x, algorithm="maxima")
```

output
```
x*e^(2*b*x + m*log(x) + 2*a)/((m + 1)*e^(2*b*x + 2*a) + m + 1) - integrate
(((2*b*x*e^(2*a) + (m + 1)*e^(2*a))*e^(2*b*x) + m + 1)*x^m/((m + 1)*e^(4*b
*x + 4*a) + 2*(m + 1)*e^(2*b*x + 2*a) + m + 1), x)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh\left(bx+a\right)\,dx$$

input
```
integrate(x^m*tanh(b*x+a),x, algorithm="giac")
```

output
```
integrate(x^m*tanh(b*x + a), x)
```

Mupad [N/A]

Not integrable

Time = 2.07 (sec) , antiderivative size = 12, normalized size of antiderivative = 1.20

$$\int x^m \tanh(a+bx)\,dx = \int x^m \tanh(a+b\,x)\,dx$$

input
```
int(x^m*tanh(a + b*x),x)
```

output
```
int(x^m*tanh(a + b*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 57, normalized size of antiderivative = 5.70

$$\int x^m \tanh(a+bx)\,dx = \frac{x^m x - 2\left(\int \frac{x^m}{e^{2bx+2a}+1}dx\right) m - 2\left(\int \frac{x^m}{e^{2bx+2a}+1}dx\right)}{m+1}$$

input
```
int(x^m*tanh(b*x+a),x)
```

output
```
(x**m*x - 2*int(x**m/(e**(2*a + 2*b*x) + 1),x)*m - 2*int(x**m/(e**(2*a + 2
*b*x) + 1),x))/(m + 1)
```

3.32 $\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$

Optimal result . . . 279
Mathematica [A] (verified) . . . 279
Rubi [A] (verified) . . . 280
Maple [A] (verified) . . . 282
Fricas [A] (verification not implemented) . . . 283
Sympy [F] . . . 283
Maxima [A] (verification not implemented) . . . 284
Giac [A] (verification not implemented) . . . 284
Mupad [B] (verification not implemented) . . . 285
Reduce [B] (verification not implemented) . . . 285

Optimal result

Integrand size = 20, antiderivative size = 169

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{3d^3x}{8af^3} + \frac{3d(c+dx)^2}{8af^2} + \frac{(c+dx)^3}{4af} + \frac{(c+dx)^4}{8ad} - \frac{3d^3}{8f^4(a+a\tanh(e+fx))} - \frac{3d^2(c+dx)}{4f^3(a+a\tanh(e+fx))} - \frac{3d(c+dx)^2}{4f^2(a+a\tanh(e+fx))} - \frac{(c+dx)^3}{2f(a+a\tanh(e+fx))}$$

output

```
3/8*d^3*x/a/f^3+3/8*d*(d*x+c)^2/a/f^2+1/4*(d*x+c)^3/a/f+1/8*(d*x+c)^4/a/d-
3/8*d^3/f^4/(a+a*tanh(f*x+e))-3/4*d^2*(d*x+c)/f^3/(a+a*tanh(f*x+e))-3/4*d*
(d*x+c)^2/f^2/(a+a*tanh(f*x+e))-1/2*(d*x+c)^3/f/(a+a*tanh(f*x+e))
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.44

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx = \frac{\operatorname{sech}(e+fx)(\cosh(fx)+\sinh(fx))\left((4c^3f^3+6c^2df^2(1+2fx)+6cd^2f(1+2fx+2f^2x^2)+d^3(3+6fx\right.$$

input
```
Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x]),x]
```

output
```
(Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2*f*
x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4*f^
3*x^3))*Cosh[2*f*x]*(-Cosh[e] + Sinh[e]) + 2*f^4*x*(4*c^3 + 6*c^2*d*x + 4*
c*d^2*x^2 + d^3*x^3)*(Cosh[e] + Sinh[e]) + (4*c^3*f^3 + 6*c^2*d*f^2*(1 + 2
*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2 + 4
*f^3*x^3))*(Cosh[e] - Sinh[e])*Sinh[2*f*x]))/(16*a*f^4*(1 + Tanh[e + f*x])
)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.450$, Rules used = {3042, 4206, 3042, 4206, 3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a\tanh(e+fx)+a}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c+dx)^3}{a-ia\tan(ie+ifx)}dx$$
$$\downarrow 4206$$
$$\frac{3d\int \frac{(c+dx)^2}{\tanh(e+fx)a+a}dx}{2f} - \frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^4}{8ad}$$
$$\downarrow 3042$$
$$\frac{3d\int \frac{(c+dx)^2}{a-ia\tan(ie+ifx)}dx}{2f} - \frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^4}{8ad}$$
$$\downarrow 4206$$
$$\frac{3d\left(\frac{d\int \frac{c+dx}{\tanh(e+fx)a+a}dx}{f} - \frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^3}{6ad}\right)}{2f} - \frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^4}{8ad}$$

$$\downarrow 3042$$

$$\frac{3d\left(\frac{d\int \frac{c+dx}{a-ia\tan(ie+ifx)}dx}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\downarrow 4206$$

$$\frac{3d\left(\frac{d\left(\frac{d\int \frac{1}{\tanh(e+fx)a+a}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\downarrow 3042$$

$$\frac{3d\left(\frac{d\left(\frac{d\int \frac{1}{a-ia\tan(ie+ifx)}dx}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\downarrow 3960$$

$$\frac{3d\left(\frac{d\left(\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}\right)}{2f}-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^4}{8ad}$$

$$\downarrow 24$$

$$-\frac{(c+dx)^3}{2f(a\tanh(e+fx)+a)}+\frac{3d\left(-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{d\left(-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}\right)}{f}+\frac{(c+dx)^3}{6ad}\right)}{2f}+\frac{(c+dx)^4}{8ad}$$

input
```
Int[(c + d*x)^3/(a + a*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^4/(8*a*d) - (c + d*x)^3/(2*f*(a + a*Tanh[e + f*x])) + (3*d*((c +
 d*x)^3/(6*a*d) - (c + d*x)^2/(2*f*(a + a*Tanh[e + f*x])) + (d*((c + d*x)^
2/(4*a*d) - (c + d*x)/(2*f*(a + a*Tanh[e + f*x])) + (d*(x/(2*a) - 1/(2*f*(
a + a*Tanh[e + f*x])))/(2*f)))/f))/(2*f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.98

method	result
risch	$\frac{d^3x^4}{8a}+\frac{d^2cx^3}{2a}+\frac{3dc^2x^2}{4a}+\frac{c^3x}{2a}+\frac{c^4}{8ad}-\frac{(4d^3x^3f^3+12cd^2f^3x^2+12c^2df^3x+6d^3f^2x^2+4c^3f^3+12cd^2f^2x+6c^2df^2-$
parallelrisch	$\frac{-3d^3+d^3x^4f^4+4xc^3f^4-6c^2df^3x-6cd^2f^3x^2-3d^3f^2x^2-6c^2df^2-6cd^2f^2x-3d^3fx+6x^2c^2df^4+4d^2cx^3f^4-2d^3x^3f^3-4c^3$

input
```
int((d*x+c)^3/(a+tanh(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
1/8/a*d^3*x^4+1/2/a*d^2*c*x^3+3/4/a*d*c^2*x^2+1/2/a*c^3*x+1/8/a/d*c^4-1/16
*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2+4*c^3*f^3+12
*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a/f^4*exp(-2*f*x-2*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.80

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{(2\,d^3f^4x^4 - 4\,c^3f^3 - 6\,c^2df^2 - 6\,cd^2f + 4\,(2\,cd^2f^4 - d^3f^3)x^3 - 3\,d^3 + 6\,(2\,c^2df^4 - 2\,cd^2f^3 - d^3f^2)x^2 + 2}{}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/16*((2*d^3*f^4*x^4 - 4*c^3*f^3 - 6*c^2*d*f^2 - 6*c*d^2*f + 4*(2*c*d^2*f^
4 - d^3*f^3)*x^3 - 3*d^3 + 6*(2*c^2*d*f^4 - 2*c*d^2*f^3 - d^3*f^2)*x^2 + 2
*(4*c^3*f^4 - 6*c^2*d*f^3 - 6*c*d^2*f^2 - 3*d^3*f)*x)*cosh(f*x + e) + (2*d
^3*f^4*x^4 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f + 4*(2*c*d^2*f^4 + d^3*f^
3)*x^3 + 3*d^3 + 6*(2*c^2*d*f^4 + 2*c*d^2*f^3 + d^3*f^2)*x^2 + 2*(4*c^3*f^
4 + 6*c^2*d*f^3 + 6*c*d^2*f^2 + 3*d^3*f)*x)*sinh(f*x + e))/(a*f^4*cosh(f*x
 + e) + a*f^4*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{\int \frac{c^3}{\tanh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\tanh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\tanh(e+fx)+1}\,dx + \int \frac{3c^2dx}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**3/(a+a*tanh(f*x+e)),x)
```

output
```
(Integral(c**3/(tanh(e + f*x) + 1), x) + Integral(d**3*x**3/(tanh(e + f*x)
 + 1), x) + Integral(3*c*d**2*x**2/(tanh(e + f*x) + 1), x) + Integral(3*c*
*2*d*x/(tanh(e + f*x) + 1), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.10

$$\begin{aligned}
&\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx \\
&= \frac{1}{4}c^3\left(\frac{2\,(fx+e)}{af} - \frac{e^{(-2\,fx-2\,e)}}{af}\right) + \frac{3\left(2\,f^2x^2e^{(2\,e)} - (2\,fx+1)e^{(-2\,fx)}\right)c^2de^{(-2\,e)}}{8\,af^2} \\
&\quad + \frac{\left(4\,f^3x^3e^{(2\,e)} - 3\,(2\,f^2x^2+2\,fx+1)e^{(-2\,fx)}\right)cd^2e^{(-2\,e)}}{8\,af^3} \\
&\quad + \frac{\left(2\,f^4x^4e^{(2\,e)} - (4\,f^3x^3+6\,f^2x^2+6\,fx+3)e^{(-2\,fx)}\right)d^3e^{(-2\,e)}}{16\,af^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c^3*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 3/8*(2*f^2*x^2*e^(2
*e) - (2*f*x + 1)*e^(-2*f*x))*c^2*d*e^(-2*e)/(a*f^2) + 1/8*(4*f^3*x^3*e^(2
*e) - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*c*d^2*e^(-2*e)/(a*f^3) + 1/16*
(2*f^4*x^4*e^(2*e) - (4*f^3*x^3 + 6*f^2*x^2 + 6*f*x + 3)*e^(-2*f*x))*d^3*e
^(-2*e)/(a*f^4)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.11

$$\begin{aligned}
&\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx \\
&= \frac{\left(2\,d^3f^4x^4e^{(2\,fx+2\,e)} + 8\,cd^2f^4x^3e^{(2\,fx+2\,e)} + 12\,c^2df^4x^2e^{(2\,fx+2\,e)} - 4\,d^3f^3x^3 + 8\,c^3f^4xe^{(2\,fx+2\,e)} - 12\,cd^2f^3\right.}{16\,af^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/16*(2*d^3*f^4*x^4*e^(2*f*x + 2*e) + 8*c*d^2*f^4*x^3*e^(2*f*x + 2*e) + 12
*c^2*d*f^4*x^2*e^(2*f*x + 2*e) - 4*d^3*f^3*x^3 + 8*c^3*f^4*x*e^(2*f*x + 2*
e) - 12*c*d^2*f^3*x^2 - 12*c^2*d*f^3*x - 6*d^3*f^2*x^2 - 4*c^3*f^3 - 12*c*
d^2*f^2*x - 6*c^2*d*f^2 - 6*d^3*f*x - 6*c*d^2*f - 3*d^3)*e^(-2*f*x - 2*e)/
(a*f^4)
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{\mathrm{e}^{-2e-2fx}\left(8c^3x\,\mathrm{e}^{2e+2fx}+2d^3x^4\,\mathrm{e}^{2e+2fx}+12c^2dx^2\,\mathrm{e}^{2e+2fx}+8cd^2x^3\,\mathrm{e}^{2e+2fx}\right)}{16a}$$
$$-\frac{\frac{\mathrm{e}^{-2e-2fx}\left(3d^3-3d^3\,\mathrm{e}^{2e+2fx}\right)}{16}+\frac{f^2\,\mathrm{e}^{-2e-2fx}\left(6c^2d+6d^3x^2-6c^2d\,\mathrm{e}^{2e+2fx}+12cd^2x\right)}{16}+\frac{f\,\mathrm{e}^{-2e-2fx}\left(6cd^2+6d^3x-6cd^2\,\mathrm{e}^{2e+2}\right.}{16}}{af^4}$$

input
```
int((c + d*x)^3/(a + a*tanh(e + f*x)),x)
```

output
```
(exp(- 2*e - 2*f*x)*(8*c^3*x*exp(2*e + 2*f*x) + 2*d^3*x^4*exp(2*e + 2*f*x)
 + 12*c^2*d*x^2*exp(2*e + 2*f*x) + 8*c*d^2*x^3*exp(2*e + 2*f*x)))/(16*a) -
 ((exp(- 2*e - 2*f*x)*(3*d^3 - 3*d^3*exp(2*e + 2*f*x)))/16 + (f^2*exp(- 2*
e - 2*f*x)*(6*c^2*d + 6*d^3*x^2 - 6*c^2*d*exp(2*e + 2*f*x) + 12*c*d^2*x))/
16 + (f*exp(- 2*e - 2*f*x)*(6*c*d^2 + 6*d^3*x - 6*c*d^2*exp(2*e + 2*f*x)))
/16 + (f^3*exp(- 2*e - 2*f*x)*(4*c^3 - 4*c^3*exp(2*e + 2*f*x) + 4*d^3*x^3
+ 12*c*d^2*x^2 + 12*c^2*d*x))/16)/(a*f^4)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.15

$$\int \frac{(c+dx)^3}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{8e^{2fx+2e}c^3f^4x+12e^{2fx+2e}c^2df^4x^2+8e^{2fx+2e}cd^2f^4x^3+2e^{2fx+2e}d^3f^4x^4-4c^3f^3-12c^2df^3x-6c^2df^2}{16e^{2fx+2e}af^4}$$

input
```
int((d*x+c)^3/(a+a*tanh(f*x+e)),x)
```

output
```
(8*e**(2*e + 2*f*x)*c**3*f**4*x + 12*e**(2*e + 2*f*x)*c**2*d*f**4*x**2 + 8
*e**(2*e + 2*f*x)*c*d**2*f**4*x**3 + 2*e**(2*e + 2*f*x)*d**3*f**4*x**4 - 4
*c**3*f**3 - 12*c**2*d*f**3*x - 6*c**2*d*f**2 - 12*c*d**2*f**3*x**2 - 12*c
*d**2*f**2*x - 6*c*d**2*f - 4*d**3*f**3*x**3 - 6*d**3*f**2*x**2 - 6*d**3*f
*x - 3*d**3)/(16*e**(2*e + 2*f*x)*a*f**4)
```

3.33 $\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 122

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{d^2x}{4af^2} + \frac{(c+dx)^2}{4af} + \frac{(c+dx)^3}{6ad} - \frac{d^2}{4f^3(a+a\tanh(e+fx))} - \frac{d(c+dx)}{2f^2(a+a\tanh(e+fx))} - \frac{(c+dx)^2}{2f(a+a\tanh(e+fx))}$$

output

```
1/4*d^2*x/a/f^2+1/4*(d*x+c)^2/a/f+1/6*(d*x+c)^3/a/d-1/4*d^2/f^3/(a+a*tanh(
f*x+e))-1/2*d*(d*x+c)/f^2/(a+a*tanh(f*x+e))-1/2*(d*x+c)^2/f/(a+a*tanh(f*x+
e))
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.39

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{\operatorname{sech}(e+fx)(\cosh(fx)+\sinh(fx))\left((2c^2f^2+2cdf(1+2fx)+d^2(1+2fx+2f^2x^2))\cosh(2fx)(-\cosh\right.$$

input

```
Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x]),x]
```

output

```
(Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*((2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) +
 d^2*(1 + 2*f*x + 2*f^2*x^2))*Cosh[2*f*x]*(-Cosh[e] + Sinh[e]) + (4*f^3*x*
(3*c^2 + 3*c*d*x + d^2*x^2)*(Cosh[e] + Sinh[e]))/3 + (2*c^2*f^2 + 2*c*d*f*
(1 + 2*f*x) + d^2*(1 + 2*f*x + 2*f^2*x^2))*(Cosh[e] - Sinh[e])*Sinh[2*f*x]
))/(8*a*f^3*(1 + Tanh[e + f*x]))
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.04, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4206, 3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a\tanh(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c+dx)^2}{a-ia\tan(ie+ifx)}dx$$

$$\downarrow 4206$$

$$\frac{d\int \frac{c+dx}{\tanh(e+fx)a+a}dx}{f} - \frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^3}{6ad}$$

$$\downarrow 3042$$

$$\frac{d\int \frac{c+dx}{a-ia\tan(ie+ifx)}dx}{f} - \frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^3}{6ad}$$

$$\downarrow 4206$$

$$\frac{d\left(\frac{d\int \frac{1}{\tanh(e+fx)a+a}dx}{2f} - \frac{c+dx}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^2}{4ad}\right)}{f} - \frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^3}{6ad}$$

$$\downarrow 3042$$

$$\frac{d\left(\frac{d\int \frac{1}{a-ia\tan(ie+ifx)}dx}{2f} - \frac{c+dx}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^2}{4ad}\right)}{f} - \frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^3}{6ad}$$

$$\downarrow 3960$$

$$\frac{d\left(\frac{d\left(\frac{\int 1dx}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}\right)}{f}-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^3}{6ad}$$

$$\downarrow 24$$

$$-\frac{(c+dx)^2}{2f(a\tanh(e+fx)+a)}+\frac{d\left(-\frac{c+dx}{2f(a\tanh(e+fx)+a)}+\frac{(c+dx)^2}{4ad}+\frac{d\left(\frac{x}{2a}-\frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}\right)}{f}+\frac{(c+dx)^3}{6ad}$$

input
```
Int[(c + d*x)^2/(a + a*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^3/(6*a*d) - (c + d*x)^2/(2*f*(a + a*Tanh[e + f*x])) + (d*((c + d
*x)^2/(4*a*d) - (c + d*x)/(2*f*(a + a*Tanh[e + f*x])) + (d*(x/(2*a) - 1/(2
*f*(a + a*Tanh[e + f*x])))/(2*f)))/f
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.84

method	result
risch	$\frac{d^2x^3}{6a}+\frac{dcx^2}{2a}+\frac{c^2x}{2a}+\frac{c^3}{6ad}-\frac{(2d^2x^2f^2+4cdf^2x+2c^2f^2+2d^2fx+2cdf+d^2)\mathrm{e}^{-2fx-2e}}{8af^3}$
parallelrisch	$\frac{-6c^2f^2-3d^2x^2f^2-6cdf^2x+2d^2\tanh(fx+e)x^3f^3+6x^2cdf^3+6x\tanh(fx+e)c^2f^3+3x\tanh(fx+e)d^2f+3x^2\tanh(fx+e)d^2}{12f^3a(\tanh(fx+e)+1)}$

input

```
int((d*x+c)^2/(a+tanh(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output

```
1/6/a*d^2*x^3+1/2/a*d*c*x^2+1/2/a*c^2*x+1/6/a/d*c^3-1/8*(2*d^2*f^2*x^2+4*c
*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a/f^3*exp(-2*f*x-2*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.57

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{(4\,d^2f^3x^3-6\,c^2f^2-6\,cdf+6\,(2\,cdf^3-d^2f^2)x^2-3\,d^2+6\,(2\,c^2f^3-2\,cdf^2-d^2f)x)\cosh(fx+e)+(4}{24\,(af^3\cosh(fx+e)+a}$$

input

```
integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/24*((4*d^2*f^3*x^3 - 6*c^2*f^2 - 6*c*d*f + 6*(2*c*d*f^3 - d^2*f^2)*x^2 -
 3*d^2 + 6*(2*c^2*f^3 - 2*c*d*f^2 - d^2*f)*x)*cosh(f*x + e) + (4*d^2*f^3*x
^3 + 6*c^2*f^2 + 6*c*d*f + 6*(2*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 6*(2*c^2*
f^3 + 2*c*d*f^2 + d^2*f)*x)*sinh(f*x + e))/(a*f^3*cosh(f*x + e) + a*f^3*si
nh(f*x + e))
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = \frac{\int \frac{c^2}{\tanh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\tanh(e+fx)+1}\,dx + \int \frac{2cdx}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**2/(a+a*tanh(f*x+e)),x)
```

output
```
(Integral(c**2/(tanh(e + f*x) + 1), x) + Integral(d**2*x**2/(tanh(e + f*x)
 + 1), x) + Integral(2*c*d*x/(tanh(e + f*x) + 1), x))/a
```

Maxima [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.03

$$\begin{aligned}\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx = {} & \frac{1}{4}c^2\left(\frac{2\,(fx+e)}{af} - \frac{e^{(-2\,fx-2\,e)}}{af}\right) \\ & + \frac{\left(2\,f^2x^2e^{(2\,e)} - (2\,fx+1)e^{(-2\,fx)}\right)cde^{(-2\,e)}}{4\,af^2} \\ & + \frac{\left(4\,f^3x^3e^{(2\,e)} - 3\,(2\,f^2x^2+2\,fx+1)e^{(-2\,fx)}\right)d^2e^{(-2\,e)}}{24\,af^3}\end{aligned}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c^2*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 1/4*(2*f^2*x^2*e^(2
*e) - (2*f*x + 1)*e^(-2*f*x))*c*d*e^(-2*e)/(a*f^2) + 1/24*(4*f^3*x^3*e^(2*
e) - 3*(2*f^2*x^2 + 2*f*x + 1)*e^(-2*f*x))*d^2*e^(-2*e)/(a*f^3)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.98

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{\left(4\,d^2f^3x^3e^{(2\,fx+2\,e)} + 12\,cdf^3x^2e^{(2\,fx+2\,e)} + 12\,c^2f^3xe^{(2\,fx+2\,e)} - 6\,d^2f^2x^2 - 12\,cdf^2x - 6\,c^2f^2 - 6\,d^2fx -\right.}{24\,af^3}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/24*(4*d^2*f^3*x^3*e^(2*f*x + 2*e) + 12*c*d*f^3*x^2*e^(2*f*x + 2*e) + 12*
c^2*f^3*x*e^(2*f*x + 2*e) - 6*d^2*f^2*x^2 - 12*c*d*f^2*x - 6*c^2*f^2 - 6*d
^2*f*x - 6*c*d*f - 3*d^2)*e^(-2*f*x - 2*e)/(a*f^3)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.53

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{\mathrm{e}^{-2\,e-2\,f\,x}\left(12\,c^2\,x\,\mathrm{e}^{2\,e+2\,f\,x} + 4\,d^2\,x^3\,\mathrm{e}^{2\,e+2\,f\,x} + 12\,c\,d\,x^2\,\mathrm{e}^{2\,e+2\,f\,x}\right)}{24\,a}$$
$$- \frac{\frac{\mathrm{e}^{-2\,e-2\,f\,x}\left(3\,d^2-3\,d^2\,\mathrm{e}^{2\,e+2\,f\,x}\right)}{24} + \frac{f\,\mathrm{e}^{-2\,e-2\,f\,x}\left(6\,c\,d+6\,d^2\,x-6\,c\,d\,\mathrm{e}^{2\,e+2\,f\,x}\right)}{24} + \frac{f^2\,\mathrm{e}^{-2\,e-2\,f\,x}\left(6\,c^2-6\,c^2\,\mathrm{e}^{2\,e+2\,f\,x}+6\,d^2\,x^2+12\,c\,d\,x\right)}{24}}{a\,f^3}$$

input
```
int((c + d*x)^2/(a + a*tanh(e + f*x)),x)
```

output
```
(exp(- 2*e - 2*f*x)*(12*c^2*x*exp(2*e + 2*f*x) + 4*d^2*x^3*exp(2*e + 2*f*x
) + 12*c*d*x^2*exp(2*e + 2*f*x)))/(24*a) - ((exp(- 2*e - 2*f*x)*(3*d^2 - 3
*d^2*exp(2*e + 2*f*x)))/24 + (f*exp(- 2*e - 2*f*x)*(6*c*d + 6*d^2*x - 6*c*
d*exp(2*e + 2*f*x)))/24 + (f^2*exp(- 2*e - 2*f*x)*(6*c^2 - 6*c^2*exp(2*e +
 2*f*x) + 6*d^2*x^2 + 12*c*d*x))/24)/(a*f^3)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^2}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{12e^{2fx+2e}c^2f^3x + 12e^{2fx+2e}cd\,f^3x^2 + 4e^{2fx+2e}d^2f^3x^3 - 6c^2f^2 - 12cd\,f^2x - 6cdf - 6d^2f^2x^2 - 6d^2fx - 3}{24e^{2fx+2e}a\,f^3}$$

input
```
int((d*x+c)^2/(a+a*tanh(f*x+e)),x)
```

output
```
(12*e**(2*e + 2*f*x)*c**2*f**3*x + 12*e**(2*e + 2*f*x)*c*d*f**3*x**2 + 4*e
**(2*e + 2*f*x)*d**2*f**3*x**3 - 6*c**2*f**2 - 12*c*d*f**2*x - 6*c*d*f - 6
*d**2*f**2*x**2 - 6*d**2*f*x - 3*d**2)/(24*e**(2*e + 2*f*x)*a*f**3)
```

3.34 $\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx$

Optimal result . . . 294
Mathematica [A] (verified) . . . 294
Rubi [A] (verified) . . . 295
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Optimal result

Integrand size = 18, antiderivative size = 74

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{dx}{4af} + \frac{(c+dx)^2}{4ad} - \frac{d}{4f^2(a+a\tanh(e+fx))} - \frac{c+dx}{2f(a+a\tanh(e+fx))}$$

output
```
1/4*d*x/a/f+1/4*(d*x+c)^2/a/d-1/4*d/f^2/(a+a*tanh(f*x+e))-1/2*(d*x+c)/f/(a
+a*tanh(f*x+e))
```

Mathematica [A] (verified)

Time = 0.48 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.09

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{2cf(-1+2fx)+d(-1-2fx+2f^2x^2)+(2cf(1+2fx)+d(1+2fx+2f^2x^2))\tanh(e+fx)}{8af^2(1+\tanh(e+fx))}$$

input
```
Integrate[(c + d*x)/(a + a*Tanh[e + f*x]),x]
```

output
```
(2*c*f*(-1 + 2*f*x) + d*(-1 - 2*f*x + 2*f^2*x^2) + (2*c*f*(1 + 2*f*x) + d*
(1 + 2*f*x + 2*f^2*x^2))*Tanh[e + f*x])/(8*a*f^2*(1 + Tanh[e + f*x]))
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4206, 3042, 3960, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a\tanh(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{c+dx}{a-ia\tan(ie+ifx)}dx$$

$$\downarrow 4206$$

$$\frac{d\int \frac{1}{\tanh(e+fx)a+a}dx}{2f} - \frac{c+dx}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^2}{4ad}$$

$$\downarrow 3042$$

$$\frac{d\int \frac{1}{a-ia\tan(ie+ifx)}dx}{2f} - \frac{c+dx}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^2}{4ad}$$

$$\downarrow 3960$$

$$\frac{d\left(\frac{\int 1dx}{2a} - \frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f} - \frac{c+dx}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^2}{4ad}$$

$$\downarrow 24$$

$$-\frac{c+dx}{2f(a\tanh(e+fx)+a)} + \frac{(c+dx)^2}{4ad} + \frac{d\left(\frac{x}{2a} - \frac{1}{2f(a\tanh(e+fx)+a)}\right)}{2f}$$

input
```
Int[(c + d*x)/(a + a*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^2/(4*a*d) - (c + d*x)/(2*f*(a + a*Tanh[e + f*x])) + (d*(x/(2*a)
- 1/(2*f*(a + a*Tanh[e + f*x]))))/(2*f)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3960
```
Int[((a_) + (b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[a*((a +
b*Tan[c + d*x])^n/(2*b*d*n)), x] + Simp[1/(2*a)   Int[(a + b*Tan[c + d*x])^
(n + 1), x], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 + b^2, 0] && LtQ[n, 0]
```

rule 4206
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + (Simp[a*d*(m/(2*b*f))
   Int[(c + d*x)^(m - 1)/(a + b*Tan[e + f*x]), x], x] - Simp[a*((c + d*x)^m
/(2*b*f*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[
a^2 + b^2, 0] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.48 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.62

method	result	size
risch	$\frac{dx^2}{4a}+\frac{cx}{2a}-\frac{(2dxf+2cf+d)e^{-2fx-2e}}{8af^2}$	46
parallelrisch	$\frac{d\tanh(fx+e)x^2f^2+dx^2f^2+2x\tanh(fx+e)cf^2+\tanh(fx+e)dfx+2cxf^2-dxf-2cf-d}{4f^2a(\tanh(fx+e)+1)}$	84

input
```
int((d*x+c)/(a+tanh(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
1/4/a*d*x^2+1/2/a*c*x-1/8*(2*d*f*x+2*c*f+d)/a/f^2*exp(-2*f*x-2*e)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.36

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{(2\,df^2x^2 - 2\,cf + 2\,(2\,cf^2 - df)x - d)\cosh(fx+e) + (2\,df^2x^2 + 2\,cf + 2\,(2\,cf^2 + df)x + d)\sinh(fx+}{8\,(af^2\cosh(fx+e) + af^2\sinh(fx+e))}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/8*((2*d*f^2*x^2 - 2*c*f + 2*(2*c*f^2 - d*f)*x - d)*cosh(f*x + e) + (2*d*
f^2*x^2 + 2*c*f + 2*(2*c*f^2 + d*f)*x + d)*sinh(f*x + e))/(a*f^2*cosh(f*x
+ e) + a*f^2*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{\int \frac{c}{\tanh(e+fx)+1}\,dx + \int \frac{dx}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x)
```

output
```
(Integral(c/(tanh(e + f*x) + 1), x) + Integral(d*x/(tanh(e + f*x) + 1), x)
)/a
```

Maxima [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{1}{4}\,c\left(\frac{2\,(fx+e)}{af} - \frac{e^{(-2\,fx-2\,e)}}{af}\right) + \frac{\left(2\,f^2x^2e^{(2\,e)} - (2\,fx+1)e^{(-2\,fx)}\right)de^{(-2\,e)}}{8\,af^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*c*(2*(f*x + e)/(a*f) - e^(-2*f*x - 2*e)/(a*f)) + 1/8*(2*f^2*x^2*e^(2*e
) - (2*f*x + 1)*e^(-2*f*x))*d*e^(-2*e)/(a*f^2)
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{\left(2\,df^2x^2e^{(2\,fx+2\,e)}+4\,cf^2xe^{(2\,fx+2\,e)}-2\,dfx-2\,cf-d\right)e^{(-2\,fx-2\,e)}}{8\,af^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/8*(2*d*f^2*x^2*e^(2*f*x + 2*e) + 4*c*f^2*x*e^(2*f*x + 2*e) - 2*d*f*x - 2
*c*f - d)*e^(-2*f*x - 2*e)/(a*f^2)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{\frac{d\,x^2}{4}+\left(\frac{c}{2}+\frac{d}{4\,f}\right)x}{a} - \frac{\frac{\frac{d}{4}+\frac{c\,f}{2}}{f^2}-x\left(\frac{c}{2}-\frac{d}{4\,f}\right)+x\left(\frac{c}{2}+\frac{d}{4\,f}\right)}{a+a\tanh\left(e+f\,x\right)}$$

input
```
int((c + d*x)/(a + a*tanh(e + f*x)),x)
```

output
```
(x*(c/2 + d/(4*f)) + (d*x^2)/4)/a - ((d/4 + (c*f)/2)/f^2 - x*(c/2 - d/(4*f
)) + x*(c/2 + d/(4*f)))/(a + a*tanh(e + f*x))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.93

$$\int \frac{c+dx}{a+a\tanh(e+fx)}\,dx = \frac{4e^{2fx+2e}cf^2x + 2e^{2fx+2e}df^2x^2 - 2cf - 2dfx - d}{8e^{2fx+2e}af^2}$$

input

```
int((d*x+c)/(a+a*tanh(f*x+e)),x)
```

output

```
(4*e**(2*e + 2*f*x)*c*f**2*x + 2*e**(2*e + 2*f*x)*d*f**2*x**2 - 2*c*f - 2*
d*f*x - d)/(8*e**(2*e + 2*f*x)*a*f**2)
```

3.35 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx$

Optimal result . . . 300
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Rubi [C] (verified) . . . 301
Maple [A] (verified) . . . 304
Fricas [A] (verification not implemented) . . . 305
Sympy [F] . . . 305
Maxima [A] (verification not implemented) . . . 305
Giac [A] (verification not implemented) . . . 306
Mupad [F(-1)] . . . 306
Reduce [F] . . . 307

Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{2ad} + \frac{\log(c+dx)}{2ad} - \frac{\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2ad} - \frac{\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{2ad} + \frac{\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{2ad}$$

output

```
1/2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a/d+1/2*ln(d*x+c)/a/d+1/2*Chi(2*
c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d-1/2*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f
*x)/a/d-1/2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.78

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx$$
$$= \frac{\operatorname{sech}(e+fx)(\cosh(fx)+\sinh(fx))\left(\log(f(c+dx))(\cosh(e)+\sinh(e))+\operatorname{Chi}\left(\frac{2f(c+dx)}{d}\right)\left(\cosh\left(e-\frac{2cf}{d}\right)\right.\right.}{2ad(1+\tanh(e+fx))}$$

input
```
Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])),x]
```

output
```
(Sech[e + f*x]*(Cosh[f*x] + Sinh[f*x])*(Log[f*(c + d*x)]*(Cosh[e] + Sinh[e
]) + CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[e - (2*c*f)/d] - Sinh[e - (2*c*
f)/d]) + (-Cosh[e - (2*c*f)/d] + Sinh[e - (2*c*f)/d])*SinhIntegral[(2*f*(c
 + d*x))/d]))/(2*a*d*(1 + Tanh[e + f*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.79 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3042, 4209, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\tanh(e+fx)+a)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)(a-ia\tan(ie+ifx))}\,dx$$
$$\downarrow 4209$$
$$\frac{i\int \frac{i\sinh(2e+2fx)}{c+dx}\,dx}{2a}+\frac{\int \frac{\cosh(2e+2fx)}{c+dx}\,dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 26$$

$$-\frac{\int \frac{\sinh(2e+2fx)}{c+dx}dx}{2a}+\frac{\int \frac{\cosh(2e+2fx)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 3042$$

$$-\frac{\int -\frac{i\sin(2ie+2ifx)}{c+dx}dx}{2a}+\frac{\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 26$$

$$\frac{i\int \frac{\sin(2ie+2ifx)}{c+dx}dx}{2a}+\frac{\int \frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 3784$$

$$\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{2a}+$$
$$\frac{\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 26$$

$$\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{2a}+$$
$$\frac{\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 3042$$

$$\frac{\sinh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+$$
$$\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int \frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int -\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{2a}+$$
$$\frac{\log(c+dx)}{2ad}$$

$$\downarrow 26$$

$$\frac{\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx}{2a}+$$
$$\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 3779$$

$$\frac{\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx}{2a}+$$
$$\frac{i\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{2a}+\frac{\log(c+dx)}{2ad}$$

$$\downarrow 3782$$

$$\frac{i\left(\frac{i\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{2a}+$$
$$\frac{\frac{\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}}{2a}+\frac{\log(c+dx)}{2ad}$$

input
```
Int[1/((c + d*x)*(a + a*Tanh[e + f*x])),x]
```

output
```
Log[c + d*x]/(2*a*d) + ((I/2)*((I*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e
 - (2*c*f)/d])/d + (I*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x
])/d))/a + ((Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (S
inh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d)/(2*a)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4209
```
Int[1/(((c_.) + (d_.)*(x_))*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Symb
ol] :> Simp[Log[c + d*x]/(2*a*d), x] + (Simp[1/(2*a)   Int[Cos[2*e + 2*f*x]
/(c + d*x), x], x] + Simp[1/(2*b)   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x])
 /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.58 (sec) , antiderivative size = 61, normalized size of antiderivative = 0.39

method	result	size
risch	$\frac{\ln(dx+c)}{2ad} - \frac{e^{\frac{2cf-2de}{d}} \operatorname{expIntegral}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{2ad}$	61

input
```
int(1/(d*x+c)/(a+tanh(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
1/2*ln(d*x+c)/a/d-1/2/a/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.46

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right)+\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\sinh\left(-\frac{2\,(de-cf)}{d}\right)+\log\left(dx+c\right)}{2\,ad}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-2*(d*f*x + c*f)/d
)*sinh(-2*(d*e - c*f)/d) + log(d*x + c))/(a*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\int \frac{1}{c\tanh(e+fx)+c+dx\tanh(e+fx)+dx}\,dx}{a}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x)
```

output
```
Integral(1/(c*tanh(e + f*x) + c + d*x*tanh(e + f*x) + d*x), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.31

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = -\frac{e^{\left(-2\,e+\frac{2\,cf}{d}\right)}\,E_1\left(\frac{2\,(dx+c)f}{d}\right)}{2\,ad}+\frac{\log\left(dx+c\right)}{2\,ad}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a*d) + 1/2*log
(d*x + c)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.30

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\left(\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(\frac{2\,cf}{d}\right)} + e^{(2\,e)}\log\left(dx+c\right)\right)e^{(-2\,e)}}{2\,ad}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/2*(Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + e^(2*e)*log(d*x + c))*e^(-2*e)/(
a*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \int \frac{1}{(a+a\tanh(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))}\,dx = \frac{\int \frac{1}{\tanh(fx+e)c+\tanh(fx+e)dx+c+dx}\,dx}{a}$$

input
```
int(1/(d*x+c)/(a+a*tanh(f*x+e)),x)
```

output
```
int(1/(tanh(e + f*x)*c + tanh(e + f*x)*d*x + c + d*x),x)/a
```

3.36 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 159

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = &-\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^2}\\ &+\frac{f\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^2}\\ &+\frac{f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^2}\\ &-\frac{f\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^2}\\ &-\frac{1}{d(c+dx)(a+a\tanh(e+fx))}\end{aligned}$$

output

```
-f*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a/d^2-f*Chi(2*c*f/d+2*f*x)*sinh(-
2*e+2*c*f/d)/a/d^2+f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d^2+f*sinh(-2
*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d^2-1/d/(d*x+c)/(a+a*tanh(f*x+e))
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.30

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx =$$
$$-\frac{\operatorname{sech}(e+fx)\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)\left(d\left(\cosh\left(e+f\left(-\frac{c}{d}+x\right)\right)+\cosh\left(e+f\left(\frac{c}{d}+x\right)\right)+\sinh\left(e+f\right.\right.\right.}{}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])),x]
```

output
```
-1/2*(Sech[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(Cosh[e + f*(-(c/d)
 + x)] + Cosh[e + f*(c/d + x)] + Sinh[e + f*(-(c/d) + x)] - Sinh[e + f*(c/
d + x)]) + 2*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[e - (f*(c +
 d*x))/d] - Sinh[e - (f*(c + d*x))/d]) + 2*f*(c + d*x)*(-Cosh[e - (f*(c +
d*x))/d] + Sinh[e - (f*(c + d*x))/d])*SinhIntegral[(2*f*(c + d*x))/d]))/(a
*d^2*(c + d*x)*(1 + Tanh[e + f*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.77 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.550$, Rules used = {3042, 4207, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\tanh(e+fx)+a)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))}dx$$
$$\downarrow 4207$$

$$-\frac{if\int\frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int\frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$\frac{f\int\frac{\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int\frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 3042$$

$$\frac{f\int-\frac{i\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int\frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$-\frac{if\int\frac{\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int\frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 3784$$

$$-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-$$

$$\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-$$

$$\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-$$

$$\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-$$

$$\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 3042$$

$$-\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int-\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-$$
$$\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int-\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-$$
$$\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$-\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-$$
$$\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-$$
$$\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 3779$$

$$-\frac{f\left(\frac{\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-$$
$$\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-$$
$$\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

$$\downarrow 3782$$

$$-\frac{if\left(\frac{i\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-$$
$$\frac{f\left(\frac{\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])),x]
```

output
```
((-I)*f*((I*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (I*
Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - (f*((Co
sh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (Sinh[2*e - (2*c*
f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - 1/(d*(c + d*x)*(a + a*T
anh[e + f*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784
```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4207
```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Simp[f/(a*d)
   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Simp[f/(b*d)   Int[Cos[2*e + 2*
f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

Maple [A] (verified)

Time = 0.66 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.57

method	result	size
risch	$-\frac{1}{2da(dx+c)}-\frac{f\,\mathrm{e}^{-2fx-2e}}{2ad(dxf+cf)}+\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{expIntegral}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{a\,d^2}$	90

input
```
int(1/(d*x+c)^2/(a+tanh(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output
```
-1/2/d/a/(d*x+c)-1/2*f/a*exp(-2*f*x-2*e)/d/(d*f*x+c*f)+f/a/d^2*exp(2*(c*f-
d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.36

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx =$$
$$-\frac{(dfx+cf)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh(fx+e)\sinh\left(-\frac{2\,(de-cf)}{d}\right)+\left((dfx+cf)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-}{(ad^3x+acd^2}\right.$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
-((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/d
) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + d)*cosh
(f*x + e) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) +
 (d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d))*sinh(f*x + e
))/((a*d^3*x + a*c*d^2)*cosh(f*x + e) + (a*d^3*x + a*c*d^2)*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = \frac{\int \frac{1}{c^2\tanh(e+fx)+c^2+2cdx\tanh(e+fx)+2cdx+d^2x^2\tanh(e+fx)+d^2x^2}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e)),x)
```

output
```
Integral(1/(c**2*tanh(e + f*x) + c**2 + 2*c*d*x*tanh(e + f*x) + 2*c*d*x + d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.35

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = -\frac{1}{2\,(ad^2x+acd)} - \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{2\,(dx+c)ad}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/2/(a*d^2*x + a*c*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2, 2*(d*x + c)*f/d)/((d*x + c)*a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. 2(159) = 318.

Time = 0.17 (sec) , antiderivative size = 320, normalized size of antiderivative = 2.01

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = -\frac{\left(2\,(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf\right)}{d}\right)e^{\left(-\frac{2\,(de-cf)}{d}\right)} - 2\,def^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\right.\right.\right.}{2\,((dx+c)}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
-1/2*(2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d)
- 2*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c
*f)/d)*e^(-2*(d*e - c*f)/d) + 2*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + d*f^2*e^(-2*(d*x +
 c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + d*f^2)*d^2/(((d*x + c)*a*d^4*
(d*e/(d*x + c) - c*f/(d*x + c) + f) - a*d^5*e + a*c*d^4*f)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx = \int \frac{1}{(a+a\tanh(e+fx))\,(c+dx)^2}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))}\,dx$$
$$= \frac{\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2}\,dx\right)c^2 + \left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2}\,dx\right)cdx + x}{2ac\,(dx+c)}$$

input
```
int(1/(d*x+c)^2/(a+a*tanh(f*x+e)),x)
```

output
```
(int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x
)*d**2*x**2),x)*c**2 + int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c
*d*x + e**(2*e + 2*f*x)*d**2*x**2),x)*c*d*x + x)/(2*a*c*(c + d*x))
```

3.37 $\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 211

$$\begin{aligned}\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = &-\frac{f}{2ad^2(c+dx)}\\ &+\frac{f^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{ad^3}\\ &-\frac{f^2\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{ad^3}\\ &-\frac{f^2\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^3}\\ &+\frac{f^2\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{ad^3}\\ &-\frac{1}{2d(c+dx)^2(a+a\tanh(e+fx))}\\ &+\frac{f}{d^2(c+dx)(a+a\tanh(e+fx))}\end{aligned}$$

output

```
-1/2*f/a/d^2/(d*x+c)+f^2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a/d^3+f^2*C
hi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a/d^3-f^2*cosh(-2*e+2*c*f/d)*Shi(2*c*
f/d+2*f*x)/a/d^3-f^2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a/d^3-1/2/d/(d*
x+c)^2/(a+a*tanh(f*x+e))+f/d^2/(d*x+c)/(a+a*tanh(f*x+e))
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.25

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx =$$
$$-\frac{\operatorname{sech}(e+fx)\left(\cosh\left(\frac{cf}{d}\right)+\sinh\left(\frac{cf}{d}\right)\right)\left(d\left(d\cosh\left(e+f\left(-\frac{c}{d}+x\right)\right)+(d-2cf-2dfx)\cosh\left(e+f\left(\frac{c}{d}+\right.\right.\right.\right.}{\text{}}$$

input
```
Integrate[1/((c + d*x)^3*(a + a*Tanh[e + f*x])),x]
```

output
```
-1/4*(Sech[e + f*x]*(Cosh[(c*f)/d] + Sinh[(c*f)/d])*(d*(d*Cosh[e + f*(-(c/
d) + x)] + (d - 2*c*f - 2*d*f*x)*Cosh[e + f*(c/d + x)] + d*Sinh[e + f*(-(c
/d) + x)] - d*Sinh[e + f*(c/d + x)] + 2*c*f*Sinh[e + f*(c/d + x)] + 2*d*f*
x*Sinh[e + f*(c/d + x)]) + 4*f^2*(c + d*x)^2*CoshIntegral[(2*f*(c + d*x))/
d]*(-Cosh[e - (f*(c + d*x))/d] + Sinh[e - (f*(c + d*x))/d]) + 4*f^2*(c + d
*x)^2*(Cosh[e - (f*(c + d*x))/d] - Sinh[e - (f*(c + d*x))/d])*SinhIntegral
[(2*f*(c + d*x))/d]))/(a*d^3*(c + d*x)^2*(1 + Tanh[e + f*x]))
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 1.07 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.04, number of steps used = 13, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.650$, Rules used = {3042, 4208, 3042, 4207, 26, 3042, 26, 3784, 26, 3042, 26, 3779, 3782}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^3(a\tanh(e+fx)+a)}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)^3(a-ia\tan(ie+ifx))}dx$$
$$\downarrow 4208$$

$$-\frac{f\int\frac{1}{(c+dx)^2(\tanh(e+fx)a+a)}dx}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 3042$$

$$-\frac{f\int\frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))}dx}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 4207$$

$$-\frac{f\left(-\frac{if\int\frac{i\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int\frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$-\frac{f\left(\frac{f\int\frac{\sinh(2e+2fx)}{c+dx}dx}{ad}-\frac{f\int\frac{\cosh(2e+2fx)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 3042$$

$$-\frac{f\left(\frac{f\int-\frac{i\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int\frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$-\frac{f\left(-\frac{if\int\frac{\sin(2ie+2ifx)}{c+dx}dx}{ad}-\frac{f\int\frac{\sin\left(2ie+2ifx+\frac{\pi}{2}\right)}{c+dx}dx}{ad}-\frac{1}{d(c+dx)(a\tanh(e+fx)+a)}\right)}{d}-\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 3784$$

$$f\left(-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{i\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx-i\sinh\left(2e\right.\right.}{ad}\right)$$

$$-\frac{}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$f\left(-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\cosh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+i\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sinh\left(2xf+\frac{2cf}{d}\right)}{c+dx}dx+\cosh\left(2e-\right.\right.}{ad}\right)$$

$$-\frac{}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 3042$$

$$f\left(-\frac{f\left(\sinh\left(2e-\frac{2cf}{d}\right)\int-\frac{i\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\right.}{ad}\right)$$

$$-\frac{}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 26$$

$$f\left(-\frac{f\left(\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx-i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+c\right.}{ad}\right)$$

$$-\frac{}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 3779$$

$$f\left(-\frac{f\left(\frac{\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}+\cosh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx\right)}{ad}-\frac{if\left(i\sinh\left(2e-\frac{2cf}{d}\right)\int\frac{\sin\left(2ixf+\frac{2icf}{d}+\frac{\pi}{2}\right)}{c+dx}dx+\frac{i\cosh\left(2e-\right.\right.}{ad}\right)$$

$$-\frac{}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

$$\downarrow 3782$$

$$-\frac{f\left(-\frac{if\left(\frac{i\,\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{i\cosh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2cf}{d}\right)}{d}\right)}{ad}-\frac{f\left(\frac{\mathrm{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{d}+\frac{\sinh\left(2e-\frac{2cf}{d}\right)\mathrm{Shi}\left(2xf+\frac{2c}{d}\right)}{d}\right)}{ad}\right)}{d}$$

$$\frac{f}{2ad^2(c+dx)}-\frac{1}{2d(c+dx)^2(a\tanh(e+fx)+a)}$$

input
```
Int[1/((c + d*x)^3*(a + a*Tanh[e + f*x])),x]
```

output
```
-1/2*f/(a*d^2*(c + d*x)) - 1/(2*d*(c + d*x)^2*(a + a*Tanh[e + f*x])) - (f*
(((-I)*f*((I*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/d + (I
*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - (f*((C
osh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/d + (Sinh[2*e - (2*c
*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/d))/(a*d) - 1/(d*(c + d*x)*(a + a*
Tanh[e + f*x])))/d
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3779
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[I*(SinhIntegral[c*f*(fz/d) + f*fz*x]/d), x] /; FreeQ[{c, d, e, f
, fz}, x] && EqQ[d*e - c*f*fz*I, 0]
```

rule 3782
```
Int[sin[(e_.) + (Complex[0, fz_])*(f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbo
l] :> Simp[CoshIntegral[c*f*(fz/d) + f*fz*x]/d, x] /; FreeQ[{c, d, e, f, fz
}, x] && EqQ[d*(e - Pi/2) - c*f*fz*I, 0]
```

rule 3784

```
Int[sin[(e_.) + (f_.)*(x_)]/((c_.) + (d_.)*(x_)), x_Symbol] :> Simp[Cos[(d*
e - c*f)/d]   Int[Sin[c*(f/d) + f*x]/(c + d*x), x], x] + Simp[Sin[(d*e - c*
f)/d]   Int[Cos[c*(f/d) + f*x]/(c + d*x), x], x] /; FreeQ[{c, d, e, f}, x]
&& NeQ[d*e - c*f, 0]
```

rule 4207

```
Int[1/(((c_.) + (d_.)*(x_))^2*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])), x_Sy
mbol] :> -Simp[(d*(c + d*x)*(a + b*Tan[e + f*x]))^(-1), x] + (-Simp[f/(a*d)
   Int[Sin[2*e + 2*f*x]/(c + d*x), x], x] + Simp[f/(b*d)   Int[Cos[2*e + 2*
f*x]/(c + d*x), x], x]) /; FreeQ[{a, b, c, d, e, f}, x] && EqQ[a^2 + b^2, 0
]
```

rule 4208

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[f*((c + d*x)^(m + 2)/(b*d^2*(m + 1)*(m + 2))), x] + (Simp[2*b*
(f/(a*d*(m + 1)))   Int[(c + d*x)^(m + 1)/(a + b*Tan[e + f*x]), x], x] + Si
mp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + b*Tan[e + f*x])), x]) /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && LtQ[m, -1] && NeQ[m, -2]
```

Maple [A] (verified)

Time = 0.73 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00

method	result
risch	$-\frac{1}{4ad(dx+c)^2}+\frac{f^3 e^{-2fx-2e}x}{2ad(d^2x^2f^2+2cd\,f^2x+c^2f^2)}+\frac{f^3 e^{-2fx-2e}c}{2a\,d^2(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 e^{-2fx-2e}}{4ad(d^2x^2f^2+2cd\,f^2x+c^2f^2)}-\frac{f^2 e^{\frac{2cf-}{d}}}{}$

input

```
int(1/(d*x+c)^3/(a+tanh(f*x+e)*a),x,method=_RETURNVERBOSE)
```

output

```
-1/4/a/d/(d*x+c)^2+1/2*f^3/a*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^
2*f^2)*x+1/2*f^3/a*exp(-2*f*x-2*e)/d^2/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)*c
-1/4*f^2/a*exp(-2*f*x-2*e)/d/(d^2*f^2*x^2+2*c*d*f^2*x+c^2*f^2)-f^2/a/d^3*e
xp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.62

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$$
$$= \frac{2\left(d^2f^2x^2+2\,cdf^2x+c^2f^2\right)\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(fx+e\right)\sinh\left(-\frac{2\,(de-cf)}{d}\right)+\left(d^2fx+cdf+2\left(d^2f^2x^2+\right.\right.}{}$$

input
```
integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*(2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(f
*x + e)*sinh(-2*(d*e - c*f)/d) + (d^2*f*x + c*d*f + 2*(d^2*f^2*x^2 + 2*c*d
*f^2*x + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - d^2)*cos
h(f*x + e) - (d^2*f*x + c*d*f - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x + c^2*f^2)*Ei
(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) - 2*(d^2*f^2*x^2 + 2*c*d*f^2*x
 + c^2*f^2)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d))*sinh(f*x + e))/
((a*d^5*x^2 + 2*a*c*d^4*x + a*c^2*d^3)*cosh(f*x + e) + (a*d^5*x^2 + 2*a*c*
d^4*x + a*c^2*d^3)*sinh(f*x + e))
```

Sympy [F]

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$$
$$= \frac{\int \frac{1}{c^3\tanh(e+fx)+c^3+3c^2dx\tanh(e+fx)+3c^2dx+3cd^2x^2\tanh(e+fx)+3cd^2x^2+d^3x^3\tanh(e+fx)+d^3x^3}\,dx}{a}$$

input
```
integrate(1/(d*x+c)**3/(a+a*tanh(f*x+e)),x)
```

output
```
Integral(1/(c**3*tanh(e + f*x) + c**3 + 3*c**2*d*x*tanh(e + f*x) + 3*c**2*
d*x + 3*c*d**2*x**2*tanh(e + f*x) + 3*c*d**2*x**2 + d**3*x**3*tanh(e + f*x
) + d**3*x**3), x)/a
```

Maxima [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.32

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = -\frac{1}{4\left(ad^3x^2+2\,acd^2x+ac^2d\right)} - \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_3\left(\frac{2(dx+c)f}{d}\right)}{2(dx+c)^2ad}$$

input
```
integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-1/4/(a*d^3*x^2 + 2*a*c*d^2*x + a*c^2*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_inte
gral_e(3, 2*(d*x + c)*f/d)/((d*x + c)^2*a*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 176, normalized size of antiderivative = 0.83

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$$
$$= \frac{4\,d^2f^2x^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(\frac{2cf}{d}\right)} + 8\,cdf^2x\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(\frac{2cf}{d}\right)} + 4\,c^2f^2\mathrm{Ei}\left(-\frac{2(dfx+cf)}{d}\right)e^{\left(\frac{2cf}{d}\right)} + 2\,d^2fx}{4\left(ad^5x^2e^{(2e)} + 2\,acd^4xe^{(2e)} + ac^2d^3e^{(2e)}\right)}$$

input
```
integrate(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
1/4*(4*d^2*f^2*x^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d) + 8*c*d*f^2*x*Ei(-2*
(d*f*x + c*f)/d)*e^(2*c*f/d) + 4*c^2*f^2*Ei(-2*(d*f*x + c*f)/d)*e^(2*c*f/d
) + 2*d^2*f*x*e^(-2*f*x) + 2*c*d*f*e^(-2*f*x) - d^2*e^(-2*f*x) - d^2*e^(2*
e))/(a*d^5*x^2*e^(2*e) + 2*a*c*d^4*x*e^(2*e) + a*c^2*d^3*e^(2*e))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx = \int \frac{1}{(a+a\tanh(e+fx))\,(c+dx)^3}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^3),x)
```

output
```
int(1/((a + a*tanh(e + f*x))*(c + d*x)^3), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^3(a+a\tanh(e+fx))}\,dx$$
$$= \frac{2\left(\int \frac{1}{e^{2fx+2e}c^3+3e^{2fx+2e}c^2dx+3e^{2fx+2e}cd^2x^2+e^{2fx+2e}d^3x^3}dx\right)c^2d+4\left(\int \frac{1}{e^{2fx+2e}c^3+3e^{2fx+2e}c^2dx+3e^{2fx+2e}cd^2x^2+e^{2fx+2e}d^3}\right.}{4ad\left(d^2x^2+2cdx+c^2\right)}$$

input
```
int(1/(d*x+c)^3/(a+a*tanh(f*x+e)),x)
```

output
```
(2*int(1/(e**(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*e
+ 2*f*x)*c*d**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c**2*d + 4*int(1/(e*
*(2*e + 2*f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d
**2*x**2 + e**(2*e + 2*f*x)*d**3*x**3),x)*c*d**2*x + 2*int(1/(e**(2*e + 2*
f*x)*c**3 + 3*e**(2*e + 2*f*x)*c**2*d*x + 3*e**(2*e + 2*f*x)*c*d**2*x**2 +
 e**(2*e + 2*f*x)*d**3*x**3),x)*d**3*x**2 - 1)/(4*a*d*(c**2 + 2*c*d*x + d*
*2*x**2))
```

3.38 $\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 230

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx = &-\frac{3d^3e^{-4e-4fx}}{512a^2f^4} - \frac{3d^3e^{-2e-2fx}}{16a^2f^4} - \frac{3d^2e^{-4e-4fx}(c+dx)}{128a^2f^3}\\ &- \frac{3d^2e^{-2e-2fx}(c+dx)}{8a^2f^3} - \frac{3de^{-4e-4fx}(c+dx)^2}{64a^2f^2}\\ &- \frac{3de^{-2e-2fx}(c+dx)^2}{8a^2f^2} - \frac{e^{-4e-4fx}(c+dx)^3}{16a^2f}\\ &- \frac{e^{-2e-2fx}(c+dx)^3}{4a^2f} + \frac{(c+dx)^4}{16a^2d}\end{aligned}$$

output

```
-3/512*d^3*exp(-4*f*x-4*e)/a^2/f^4-3/16*d^3*exp(-2*f*x-2*e)/a^2/f^4-3/128*
d^2*exp(-4*f*x-4*e)*(d*x+c)/a^2/f^3-3/8*d^2*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^
3-3/64*d*exp(-4*f*x-4*e)*(d*x+c)^2/a^2/f^2-3/8*d*exp(-2*f*x-2*e)*(d*x+c)^2
/a^2/f^2-1/16*exp(-4*f*x-4*e)*(d*x+c)^3/a^2/f-1/4*exp(-2*f*x-2*e)*(d*x+c)^
3/a^2/f+1/16*(d*x+c)^4/a^2/d
```

Mathematica [A] (verified)

Time = 1.89 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\operatorname{sech}^2(e+fx)(\cosh(fx)+\sinh(fx))^2\left(-((4c^3f^3+6c^2df^2(1+2fx)+6cd^2f(1+2fx+2f^2x^2)+d^3(3+$$

input
```
Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(Cosh[f*x] + Sinh[f*x])^2*(-((4*c^3*f^3 + 6*c^2*d*f^2*(1
+ 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x^2
+ 4*f^3*x^3))*Cosh[2*f*x]) + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 12*
c*d^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*x^
3))*Cosh[4*f*x]*(-Cosh[2*e] + Sinh[2*e]))/32 + f^4*x*(4*c^3 + 6*c^2*d*x +
4*c*d^2*x^2 + d^3*x^3)*(Cosh[2*e] + Sinh[2*e]) + (4*c^3*f^3 + 6*c^2*d*f^2*
(1 + 2*f*x) + 6*c*d^2*f*(1 + 2*f*x + 2*f^2*x^2) + d^3*(3 + 6*f*x + 6*f^2*x
^2 + 4*f^3*x^3))*Sinh[2*f*x] + ((32*c^3*f^3 + 24*c^2*d*f^2*(1 + 4*f*x) + 1
2*c*d^2*f*(1 + 4*f*x + 8*f^2*x^2) + d^3*(3 + 12*f*x + 24*f^2*x^2 + 32*f^3*
x^3))*(Cosh[2*e] - Sinh[2*e])*Sinh[4*f*x])/32))/(16*a^2*f^4*(1 + Tanh[e +
f*x])^2)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\tanh(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c+dx)^3}{(a-ia\tan(ie+ifx))^2}dx$$

$$\downarrow 4212$$

$$\int \left(\frac{(c+dx)^3 e^{-4e-4fx}}{4a^2} + \frac{(c+dx)^3 e^{-2e-2fx}}{2a^2} + \frac{(c+dx)^3}{4a^2} \right) dx$$

$$\downarrow 2009$$

$$-\frac{3d^2(c+dx)e^{-4e-4fx}}{128a^2f^3} - \frac{3d^2(c+dx)e^{-2e-2fx}}{8a^2f^3} - \frac{3d(c+dx)^2e^{-4e-4fx}}{64a^2f^2} - \frac{3d(c+dx)^2e^{-2e-2fx}}{8a^2f^2} - \frac{(c+dx)^3e^{-4e-4fx}}{16a^2f} - \frac{(c+dx)^3e^{-2e-2fx}}{4a^2f} + \frac{(c+dx)^4}{16a^2d} - \frac{3d^3e^{-4e-4fx}}{512a^2f^4} - \frac{3d^3e^{-2e-2fx}}{16a^2f^4}$$

input
```
Int[(c + d*x)^3/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(-3*d^3*E^(-4*e - 4*f*x))/(512*a^2*f^4) - (3*d^3*E^(-2*e - 2*f*x))/(16*a^2
*f^4) - (3*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(128*a^2*f^3) - (3*d^2*E^(-2*e
- 2*f*x)*(c + d*x))/(8*a^2*f^3) - (3*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(64*a
^2*f^2) - (3*d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(8*a^2*f^2) - (E^(-4*e - 4*f*
x)*(c + d*x)^3)/(16*a^2*f) - (E^(-2*e - 2*f*x)*(c + d*x)^3)/(4*a^2*f) + (c
 + d*x)^4/(16*a^2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.19

method	result
risch	$\frac{d^3x^4}{16a^2}+\frac{d^2cx^3}{4a^2}+\frac{3dc^2x^2}{8a^2}+\frac{c^3x}{4a^2}+\frac{c^4}{16a^2d}-\frac{(4d^3x^3f^3+12cd^2f^3x^2+12c^2df^3x+6d^3f^2x^2+4c^3f^3+12cd^2f^2x+6c^2df$
parallelrisch	$\frac{-48d^3+8d^3x^4f^4+32xc^3f^4-120c^2df^3x-120cd^2f^3x^2-54d^3f^2x^2-96c^2df^2-108cd^2f^2x-32\tanh(fx+e)c^3f^3-51d^3fx+4$

input
```
int((d*x+c)^3/(a+tanh(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/16/a^2*d^3*x^4+1/4/a^2*d^2*c*x^3+3/8/a^2*d*c^2*x^2+1/4/a^2*c^3*x+1/16/a^
2/d*c^4-1/16*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2+
4*c^3*f^3+12*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a^2/f^4*ex
p(-2*f*x-2*e)-1/512*(32*d^3*f^3*x^3+96*c*d^2*f^3*x^2+96*c^2*d*f^3*x+24*d^3
*f^2*x^2+32*c^3*f^3+48*c*d^2*f^2*x+24*c^2*d*f^2+12*d^3*f*x+12*c*d^2*f+3*d^
3)/a^2/f^4*exp(-4*f*x-4*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(204) = 408.

Time = 0.08 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx =$$
$$-\frac{128\,d^3f^3x^3+128\,c^3f^3+192\,c^2df^2+192\,cd^2f+96\,d^3+192\,(2\,cd^2f^3+d^3f^2)x^2-(32\,d^3f^4x^4-32\,c^3f$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output

```
-1/512*(128*d^3*f^3*x^3 + 128*c^3*f^3 + 192*c^2*d*f^2 + 192*c*d^2*f + 96*d
^3 + 192*(2*c*d^2*f^3 + d^3*f^2)*x^2 - (32*d^3*f^4*x^4 - 32*c^3*f^3 - 24*c
^2*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 - 3*d^3 + 24*(8*c^2
*d*f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c^2*d*f^3 - 12*c*
d^2*f^2 - 3*d^3*f)*x)*cosh(f*x + e)^2 - 2*(32*d^3*f^4*x^4 + 32*c^3*f^3 + 2
4*c^2*d*f^2 + 12*c*d^2*f + 32*(4*c*d^2*f^4 + d^3*f^3)*x^3 + 3*d^3 + 24*(8*
c^2*d*f^4 + 4*c*d^2*f^3 + d^3*f^2)*x^2 + 4*(32*c^3*f^4 + 24*c^2*d*f^3 + 12
*c*d^2*f^2 + 3*d^3*f)*x)*cosh(f*x + e)*sinh(f*x + e) - (32*d^3*f^4*x^4 + 3
2*c^3*f^3 - 24*c^2*d*f^2 - 12*c*d^2*f + 32*(4*c*d^2*f^4 - d^3*f^3)*x^3 + 3
*d^3 + 24*(8*c^2*d*f^4 - 4*c*d^2*f^3 - d^3*f^2)*x^2 + 4*(32*c^3*f^4 - 24*c
^2*d*f^3 - 12*c*d^2*f^2 - 3*d^3*f)*x)*sinh(f*x + e)^2 + 192*(2*c^2*d*f^3 +
 2*c*d^2*f^2 + d^3*f)*x)/(a^2*f^4*cosh(f*x + e)^2 + 2*a^2*f^4*cosh(f*x + e
)*sinh(f*x + e) + a^2*f^4*sinh(f*x + e)^2)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^3}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{3cd^2x^2}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{}{\tanh^2(}}{a^2}$$

input

```
integrate((d*x+c)**3/(a+a*tanh(f*x+e))**2,x)
```

output

```
(Integral(c**3/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d**
3*x**3/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(3*c*d**2*x*
*2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(tan
h(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 296, normalized size of antiderivative = 1.29

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx = \frac{1}{16}c^3\left(\frac{4\,(fx+e)}{a^2 f} - \frac{4\,e^{(-2fx-2e)} + e^{(-4fx-4e)}}{a^2 f}\right)$$
$$+ \frac{3\left(8f^2x^2e^{(4e)} - 8\left(2fxe^{(2e)} + e^{(2e)}\right)e^{(-2fx)} - (4fx+1)e^{(-4fx)}\right)c^2 d e^{(-4e)}}{64\,a^2 f^2}$$
$$+ \frac{\left(32f^3x^3e^{(4e)} - 48\left(2f^2x^2e^{(2e)} + 2fxe^{(2e)} + e^{(2e)}\right)e^{(-2fx)} - 3\left(8f^2x^2 + 4fx + 1\right)e^{(-4fx)}\right)cd^2e^{(-4e)}}{128\,a^2 f^3}$$
$$+ \frac{\left(32f^4x^4e^{(4e)} - 32\left(4f^3x^3e^{(2e)} + 6f^2x^2e^{(2e)} + 6fxe^{(2e)} + 3e^{(2e)}\right)e^{(-2fx)} - (32f^3x^3 + 24f^2x^2 + 12f\right.}{512\,a^2 f^4}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c^3*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a
^2*f)) + 3/64*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x)
- (4*f*x + 1)*e^(-4*f*x))*c^2*d*e^(-4*e)/(a^2*f^2) + 1/128*(32*f^3*x^3*e^(
4*e) - 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*
f^2*x^2 + 4*f*x + 1)*e^(-4*f*x))*c*d^2*e^(-4*e)/(a^2*f^3) + 1/512*(32*f^4*
x^4*e^(4*e) - 32*(4*f^3*x^3*e^(2*e) + 6*f^2*x^2*e^(2*e) + 6*f*x*e^(2*e) +
3*e^(2*e))*e^(-2*f*x) - (32*f^3*x^3 + 24*f^2*x^2 + 12*f*x + 3)*e^(-4*f*x))
*d^3*e^(-4*e)/(a^2*f^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.60

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\left(32d^3f^4x^4e^{(4fx+4e)} + 128cd^2f^4x^3e^{(4fx+4e)} + 192c^2df^4x^2e^{(4fx+4e)} - 128d^3f^3x^3e^{(2fx+2e)} - 32d^3f^3x^3 +\right.}{}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output

```
1/512*(32*d^3*f^4*x^4*e^(4*f*x + 4*e) + 128*c*d^2*f^4*x^3*e^(4*f*x + 4*e)
+ 192*c^2*d*f^4*x^2*e^(4*f*x + 4*e) - 128*d^3*f^3*x^3*e^(2*f*x + 2*e) - 32
*d^3*f^3*x^3 + 128*c^3*f^4*x*e^(4*f*x + 4*e) - 384*c*d^2*f^3*x^2*e^(2*f*x
+ 2*e) - 96*c*d^2*f^3*x^2 - 384*c^2*d*f^3*x*e^(2*f*x + 2*e) - 192*d^3*f^2*
x^2*e^(2*f*x + 2*e) - 96*c^2*d*f^3*x - 24*d^3*f^2*x^2 - 128*c^3*f^3*e^(2*f
*x + 2*e) - 384*c*d^2*f^2*x*e^(2*f*x + 2*e) - 32*c^3*f^3 - 48*c*d^2*f^2*x
- 192*c^2*d*f^2*e^(2*f*x + 2*e) - 192*d^3*f*x*e^(2*f*x + 2*e) - 24*c^2*d*f
^2 - 12*d^3*f*x - 192*c*d^2*f*e^(2*f*x + 2*e) - 12*c*d^2*f - 96*d^3*e^(2*f
*x + 2*e) - 3*d^3)*e^(-4*f*x - 4*e)/(a^2*f^4)
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 267, normalized size of antiderivative = 1.16

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx = \frac{c^3 x}{4a^2} - e^{-4e-4fx}\left(\frac{32c^3f^3+24c^2df^2+12cd^2f+3d^3}{512a^2f^4} + \frac{d^3x^3}{16a^2f} + \frac{3dx(8c^2f^2+4cdf+d^2)}{128a^2f^3} + \frac{3d^2x^2(d+4cf)}{64a^2f^2}\right) - e^{-2e-2fx}\left(\frac{4c^3f^3+6c^2df^2+6cd^2f+3d^3}{16a^2f^4} + \frac{d^3x^3}{4a^2f} + \frac{3dx(2c^2f^2+2cdf+d^2)}{8a^2f^3} + \frac{3d^2x^2(d+2cf)}{8a^2f^2}\right) + \frac{d^3x^4}{16a^2} + \frac{3c^2dx^2}{8a^2} + \frac{cd^2x^3}{4a^2}$$

input

```
int((c + d*x)^3/(a + a*tanh(e + f*x))^2,x)
```

output

```
(c^3*x)/(4*a^2) - exp(- 4*e - 4*f*x)*((3*d^3 + 32*c^3*f^3 + 24*c^2*d*f^2 +
 12*c*d^2*f)/(512*a^2*f^4) + (d^3*x^3)/(16*a^2*f) + (3*d*x*(d^2 + 8*c^2*f^
2 + 4*c*d*f))/(128*a^2*f^3) + (3*d^2*x^2*(d + 4*c*f))/(64*a^2*f^2)) - exp(
- 2*e - 2*f*x)*((3*d^3 + 4*c^3*f^3 + 6*c^2*d*f^2 + 6*c*d^2*f)/(16*a^2*f^4)
 + (d^3*x^3)/(4*a^2*f) + (3*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(8*a^2*f^3) +
 (3*d^2*x^2*(d + 2*c*f))/(8*a^2*f^2)) + (d^3*x^4)/(16*a^2) + (3*c^2*d*x^2)
/(8*a^2) + (c*d^2*x^3)/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 385, normalized size of antiderivative = 1.67

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{128e^{4fx+4e}c^3f^4x + 192e^{4fx+4e}c^2d\,f^4x^2 + 128e^{4fx+4e}c\,d^2f^4x^3 + 32e^{4fx+4e}d^3f^4x^4 - 128e^{2fx+2e}c^3f^3 - 384e$$

input
```
int((d*x+c)^3/(a+a*tanh(f*x+e))^2,x)
```

output
```
(128*e**(4*e + 4*f*x)*c**3*f**4*x + 192*e**(4*e + 4*f*x)*c**2*d*f**4*x**2
+ 128*e**(4*e + 4*f*x)*c*d**2*f**4*x**3 + 32*e**(4*e + 4*f*x)*d**3*f**4*x*
*4 - 128*e**(2*e + 2*f*x)*c**3*f**3 - 384*e**(2*e + 2*f*x)*c**2*d*f**3*x -
 192*e**(2*e + 2*f*x)*c**2*d*f**2 - 384*e**(2*e + 2*f*x)*c*d**2*f**3*x**2
- 384*e**(2*e + 2*f*x)*c*d**2*f**2*x - 192*e**(2*e + 2*f*x)*c*d**2*f - 128
*e**(2*e + 2*f*x)*d**3*f**3*x**3 - 192*e**(2*e + 2*f*x)*d**3*f**2*x**2 - 1
92*e**(2*e + 2*f*x)*d**3*f*x - 96*e**(2*e + 2*f*x)*d**3 - 32*c**3*f**3 - 9
6*c**2*d*f**3*x - 24*c**2*d*f**2 - 96*c*d**2*f**3*x**2 - 48*c*d**2*f**2*x
- 12*c*d**2*f - 32*d**3*f**3*x**3 - 24*d**3*f**2*x**2 - 12*d**3*f*x - 3*d*
*3)/(512*e**(4*e + 4*f*x)*a**2*f**4)
```

3.39 $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 170

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = -\frac{d^2e^{-4e-4fx}}{128a^2f^3} - \frac{d^2e^{-2e-2fx}}{8a^2f^3} - \frac{de^{-4e-4fx}(c+dx)}{32a^2f^2} - \frac{de^{-2e-2fx}(c+dx)}{4a^2f^2} - \frac{e^{-4e-4fx}(c+dx)^2}{16a^2f} - \frac{e^{-2e-2fx}(c+dx)^2}{4a^2f} + \frac{(c+dx)^3}{12a^2d}$$

output

```
-1/128*d^2*exp(-4*f*x-4*e)/a^2/f^3-1/8*d^2*exp(-2*f*x-2*e)/a^2/f^3-1/32*d*
exp(-4*f*x-4*e)*(d*x+c)/a^2/f^2-1/4*d*exp(-2*f*x-2*e)*(d*x+c)/a^2/f^2-1/16
*exp(-4*f*x-4*e)*(d*x+c)^2/a^2/f-1/4*exp(-2*f*x-2*e)*(d*x+c)^2/a^2/f+1/12*
(d*x+c)^3/a^2/d
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.22

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\operatorname{sech}^2(e+fx)\left(-48(2c^2f^2+2cdf(1+2fx)+d^2(1+2fx+2f^2x^2))+(24c^2f^2(-1+4fx)+12cdf(-1-$$

input
```
Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(-48*(2*c^2*f^2 + 2*c*d*f*(1 + 2*f*x) + d^2*(1 + 2*f*x +
2*f^2*x^2)) + (24*c^2*f^2*(-1 + 4*f*x) + 12*c*d*f*(-1 - 4*f*x + 8*f^2*x^2)
 + d^2*(-3 - 12*f*x - 24*f^2*x^2 + 32*f^3*x^3))*Cosh[2*(e + f*x)] + (24*c^
2*f^2*(1 + 4*f*x) + 12*c*d*f*(1 + 4*f*x + 8*f^2*x^2) + d^2*(3 + 12*f*x + 2
4*f^2*x^2 + 32*f^3*x^3))*Sinh[2*(e + f*x)]))/(384*a^2*f^3*(1 + Tanh[e + f*
x])^2)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\tanh(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c+dx)^2}{(a-ia\tan(ie+ifx))^2}dx$$
$$\downarrow 4212$$
$$\int \left(\frac{(c+dx)^2e^{-4e-4fx}}{4a^2}+\frac{(c+dx)^2e^{-2e-2fx}}{2a^2}+\frac{(c+dx)^2}{4a^2}\right)dx$$

$$\downarrow 2009$$

$$-\frac{d(c+dx)e^{-4e-4fx}}{32a^2f^2}-\frac{d(c+dx)e^{-2e-2fx}}{4a^2f^2}-\frac{(c+dx)^2e^{-4e-4fx}}{16a^2f}-\frac{(c+dx)^2e^{-2e-2fx}}{4a^2f}+\frac{(c+dx)^3}{12a^2d}-\frac{d^2e^{-4e-4fx}}{128a^2f^3}-\frac{d^2e^{-2e-2fx}}{8a^2f^3}$$

input
```
Int[(c + d*x)^2/(a + a*Tanh[e + f*x])^2,x]
```

output
```
-1/128*(d^2*E^(-4*e - 4*f*x))/(a^2*f^3) - (d^2*E^(-2*e - 2*f*x))/(8*a^2*f^
3) - (d*E^(-4*e - 4*f*x)*(c + d*x))/(32*a^2*f^2) - (d*E^(-2*e - 2*f*x)*(c
 + d*x))/(4*a^2*f^2) - (E^(-4*e - 4*f*x)*(c + d*x)^2)/(16*a^2*f) - (E^(-2*e
 - 2*f*x)*(c + d*x)^2)/(4*a^2*f) + (c + d*x)^3/(12*a^2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.96

method	result
risch	$\frac{d^2x^3}{12a^2}+\frac{dcx^2}{4a^2}+\frac{c^2x}{4a^2}+\frac{c^3}{12a^2d}-\frac{(2d^2x^2f^2+4cdf^2x+2c^2f^2+2d^2fx+2cdf+d^2)e^{-2fx-2e}}{8a^2f^3}-\frac{(8d^2x^2f^2+16cdf^2x+8c}{}$
parallelrisch	$\frac{-48c^2f^2-30d^2x^2f^2-60cdf^2x+16d^2\tanh(fx+e)x^3f^3+24x^2cdf^3+48x\tanh(fx+e)c^2f^3+6x\tanh(fx+e)d^2f+12x^2\tanh(}{}$

input
```
int((d*x+c)^2/(a+tanh(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12/a^2*d^2*x^3+1/4/a^2*d*c*x^2+1/4/a^2*c^2*x+1/12/a^2/d*c^3-1/8*(2*d^2*f
^2*x^2+4*c*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a^2/f^3*exp(-2*f*x-2*e
)-1/128*(8*d^2*f^2*x^2+16*c*d*f^2*x+8*c^2*f^2+4*d^2*f*x+4*c*d*f+d^2)/a^2/f
^3*exp(-4*f*x-4*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(150) = 300$.

Time $= 0.08$ (sec) , antiderivative size $= 361$, normalized size of antiderivative $= 2.12$

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = -\frac{96\,d^2f^2x^2+96\,c^2f^2+96\,cdf-(32\,d^2f^3x^3-24\,c^2f^2-12\,cdf+24\,(4\,cdf^3-d^2f^2)x^2-3\,d^2+12\,(8\,c^2$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/384*(96*d^2*f^2*x^2 + 96*c^2*f^2 + 96*c*d*f - (32*d^2*f^3*x^3 - 24*c^2*
f^2 - 12*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*
c*d*f^2 - d^2*f)*x)*cosh(f*x + e)^2 - 2*(32*d^2*f^3*x^3 + 24*c^2*f^2 + 12*
c*d*f + 24*(4*c*d*f^3 + d^2*f^2)*x^2 + 3*d^2 + 12*(8*c^2*f^3 + 4*c*d*f^2 +
 d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e) - (32*d^2*f^3*x^3 - 24*c^2*f^2 - 12
*c*d*f + 24*(4*c*d*f^3 - d^2*f^2)*x^2 - 3*d^2 + 12*(8*c^2*f^3 - 4*c*d*f^2
- d^2*f)*x)*sinh(f*x + e)^2 + 48*d^2 + 96*(2*c*d*f^2 + d^2*f)*x)/(a^2*f^3*
cosh(f*x + e)^2 + 2*a^2*f^3*cosh(f*x + e)*sinh(f*x + e) + a^2*f^3*sinh(f*x
 + e)^2)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\int \frac{c^2}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx + \int \frac{2cdx}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**2/(a+a*tanh(f*x+e))**2,x)
```

output
```
(Integral(c**2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d**
2*x**2/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(2*c*d*x/(ta
nh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = \frac{1}{16}c^2\left(\frac{4(fx+e)}{a^2f} - \frac{4e^{(-2fx-2e)}+e^{(-4fx-4e)}}{a^2f}\right)$$
$$+ \frac{\left(8f^2x^2e^{(4e)} - 8\left(2fxe^{(2e)}+e^{(2e)}\right)e^{(-2fx)} - (4fx+1)e^{(-4fx)}\right)cde^{(-4e)}}{32a^2f^2}$$
$$+ \frac{\left(32f^3x^3e^{(4e)} - 48\left(2f^2x^2e^{(2e)}+2fxe^{(2e)}+e^{(2e)}\right)e^{(-2fx)} - 3(8f^2x^2+4fx+1)e^{(-4fx)}\right)d^2e^{(-4e)}}{384a^2f^3}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c^2*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a
^2*f)) + 1/32*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x)
- (4*f*x + 1)*e^(-4*f*x))*c*d*e^(-4*e)/(a^2*f^2) + 1/384*(32*f^3*x^3*e^(4*
e) - 48*(2*f^2*x^2*e^(2*e) + 2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) - 3*(8*f^
2*x^2 + 4*f*x + 1)*e^(-4*f*x))*d^2*e^(-4*e)/(a^2*f^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\left(32\,d^2f^3x^3e^{(4fx+4e)} + 96\,cdf^3x^2e^{(4fx+4e)} + 96\,c^2f^3xe^{(4fx+4e)} - 96\,d^2f^2x^2e^{(2fx+2e)} - 24\,d^2f^2x^2 - 192\,cd\right.}{}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/384*(32*d^2*f^3*x^3*e^(4*f*x + 4*e) + 96*c*d*f^3*x^2*e^(4*f*x + 4*e) + 9
6*c^2*f^3*x*e^(4*f*x + 4*e) - 96*d^2*f^2*x^2*e^(2*f*x + 2*e) - 24*d^2*f^2*
x^2 - 192*c*d*f^2*x*e^(2*f*x + 2*e) - 48*c*d*f^2*x - 96*c^2*f^2*e^(2*f*x +
 2*e) - 96*d^2*f*x*e^(2*f*x + 2*e) - 24*c^2*f^2 - 12*d^2*f*x - 96*c*d*f*e^
(2*f*x + 2*e) - 12*c*d*f - 48*d^2*e^(2*f*x + 2*e) - 3*d^2)*e^(-4*f*x - 4*e
)/(a^2*f^3)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.97

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx = \frac{c^2x}{4a^2} - e^{-4e-4fx}\left(\frac{8c^2f^2+4cdf+d^2}{128a^2f^3} + \frac{d^2x^2}{16a^2f} + \frac{dx(d+4cf)}{32a^2f^2}\right) - e^{-2e-2fx}\left(\frac{2c^2f^2+2cdf+d^2}{8a^2f^3} + \frac{d^2x^2}{4a^2f} + \frac{dx(d+2cf)}{4a^2f^2}\right) + \frac{d^2x^3}{12a^2} + \frac{cdx^2}{4a^2}$$

input
```
int((c + d*x)^2/(a + a*tanh(e + f*x))^2,x)
```

output
```
(c^2*x)/(4*a^2) - exp(- 4*e - 4*f*x)*((d^2 + 8*c^2*f^2 + 4*c*d*f)/(128*a^2
*f^3) + (d^2*x^2)/(16*a^2*f) + (d*x*(d + 4*c*f))/(32*a^2*f^2)) - exp(- 2*e
 - 2*f*x)*((d^2 + 2*c^2*f^2 + 2*c*d*f)/(8*a^2*f^3) + (d^2*x^2)/(4*a^2*f) +
 (d*x*(d + 2*c*f))/(4*a^2*f^2)) + (d^2*x^3)/(12*a^2) + (c*d*x^2)/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 229, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{96e^{4fx+4e}c^2f^3x + 96e^{4fx+4e}cd\,f^3x^2 + 32e^{4fx+4e}d^2f^3x^3 - 96e^{2fx+2e}c^2f^2 - 192e^{2fx+2e}cd\,f^2x - 96e^{2fx+2e}c}{384}$$

input
```
int((d*x+c)^2/(a+a*tanh(f*x+e))^2,x)
```

output
```
(96*e**(4*e + 4*f*x)*c**2*f**3*x + 96*e**(4*e + 4*f*x)*c*d*f**3*x**2 + 32*
e**(4*e + 4*f*x)*d**2*f**3*x**3 - 96*e**(2*e + 2*f*x)*c**2*f**2 - 192*e**(
2*e + 2*f*x)*c*d*f**2*x - 96*e**(2*e + 2*f*x)*c*d*f - 96*e**(2*e + 2*f*x)*
d**2*f**2*x**2 - 96*e**(2*e + 2*f*x)*d**2*f*x - 48*e**(2*e + 2*f*x)*d**2 -
 24*c**2*f**2 - 48*c*d*f**2*x - 12*c*d*f - 24*d**2*f**2*x**2 - 12*d**2*f*x
 - 3*d**2)/(384*e**(4*e + 4*f*x)*a**2*f**3)
```

3.40 $\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx$

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Giac [A] (verification not implemented) . . . 344
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Reduce [B] (verification not implemented) . . . 345

Optimal result

Integrand size = 18, antiderivative size = 133

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{3dx}{16a^2f} - \frac{dx^2}{8a^2} + \frac{x(c+dx)}{4a^2} - \frac{d}{16f^2(a+a\tanh(e+fx))^2} - \frac{c+dx}{4f(a+a\tanh(e+fx))^2} - \frac{3d}{16f^2\left(a^2+a^2\tanh(e+fx)\right)} - \frac{c+dx}{4f\left(a^2+a^2\tanh(e+fx)\right)}$$

output

```
3/16*d*x/a^2/f-1/8*d*x^2/a^2+1/4*x*(d*x+c)/a^2-1/16*d/f^2/(a+a*tanh(f*x+e)
)^2-1/4*(d*x+c)/f/(a+a*tanh(f*x+e))^2-3/16*d/f^2/(a^2+a^2*tanh(f*x+e))-1/4
*(d*x+c)/f/(a^2+a^2*tanh(f*x+e))
```

Mathematica [A] (verified)

Time = 0.92 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.86

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\operatorname{sech}^2(e+fx)\left(-8(d+2cf+2dfx)+(4cf(-1+4fx)+d(-1-4fx+8f^2x^2))\cosh(2(e+fx))+(4cf\right.}{64a^2f^2(1+\tanh(e+fx))^2}$$

input
```
Integrate[(c + d*x)/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(-8*(d + 2*c*f + 2*d*f*x) + (4*c*f*(-1 + 4*f*x) + d*(-1 -
 4*f*x + 8*f^2*x^2))*Cosh[2*(e + f*x)] + (4*c*f*(1 + 4*f*x) + d*(1 + 4*f*x
 + 8*f^2*x^2))*Sinh[2*(e + f*x)]))/(64*a^2*f^2*(1 + Tanh[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\tanh(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{c+dx}{(a-ia\tan(ie+ifx))^2}dx$$
$$\downarrow 4213$$
$$-d\int\left(\frac{x}{4a^2}-\frac{1}{4f\left(\tanh(e+fx)a^2+a^2\right)}-\frac{1}{4f(\tanh(e+fx)a+a)^2}\right)dx-$$
$$\frac{c+dx}{4f\left(a^2\tanh(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-\frac{c+dx}{4f(a\tanh(e+fx)+a)^2}$$
$$\downarrow 2009$$

$$-\frac{c+dx}{4f\left(a^2\tanh(e+fx)+a^2\right)}+\frac{x(c+dx)}{4a^2}-d\left(\frac{3}{16f^2\left(a^2\tanh(e+fx)+a^2\right)}-\frac{3x}{16a^2f}+\frac{x^2}{8a^2}+\frac{1}{16f^2(a\tanh(e+fx)+a)^2}\right)-\frac{c+dx}{4f(a\tanh(e+fx)+a)^2}$$

input
```
Int[(c + d*x)/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(x*(c + d*x))/(4*a^2) - (c + d*x)/(4*f*(a + a*Tanh[e + f*x])^2) - (c + d*x
)/(4*f*(a^2 + a^2*Tanh[e + f*x])) - d*((-3*x)/(16*a^2*f) + x^2/(8*a^2) + 1
/(16*f^2*(a + a*Tanh[e + f*x])^2) + 3/(16*f^2*(a^2 + a^2*Tanh[e + f*x])))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4213
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Simp[(c + d*x)
^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)   u, x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.53 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.56

method	result
risch	$\frac{dx^2}{8a^2}+\frac{cx}{4a^2}-\frac{(2dxf+2cf+d)e^{-2fx-2e}}{8a^2f^2}-\frac{(4dxf+4cf+d)e^{-4fx-4e}}{64a^2f^2}$
parallelrisch	$\frac{-4d+3\tanh(fx+e)^2dfx-8cf-5dxf+4d\tanh(fx+e)x^2f^2+8x\tanh(fx+e)cf^2-3d\tanh(fx+e)+2\tanh(fx+e)dfx+4x\tan}{16f^2a^2(\tanh(fx+e)+1)^2}$

input
```
int((d*x+c)/(a+tanh(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/8*d*x^2/a^2+1/4/a^2*c*x-1/8*(2*d*f*x+2*c*f+d)/a^2/f^2*exp(-2*f*x-2*e)-1/
64*(4*d*f*x+4*c*f+d)/a^2/f^2*exp(-4*f*x-4*e)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.44

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = -\frac{16\,dfx-(8\,df^2x^2-4\,cf+4\,(4\,cf^2-df)x-d)\cosh(fx+e)^2-2\,(8\,df^2x^2+4\,cf+4\,(4\,cf^2+df)x+}{64\,(a^2f^2\cosh(fx+e)^2+2\,a^2f^2\cosh(fx}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/64*(16*d*f*x - (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*cosh(f*x
 + e)^2 - 2*(8*d*f^2*x^2 + 4*c*f + 4*(4*c*f^2 + d*f)*x + d)*cosh(f*x + e)*
sinh(f*x + e) - (8*d*f^2*x^2 - 4*c*f + 4*(4*c*f^2 - d*f)*x - d)*sinh(f*x +
 e)^2 + 16*c*f + 8*d)/(a^2*f^2*cosh(f*x + e)^2 + 2*a^2*f^2*cosh(f*x + e)*s
inh(f*x + e) + a^2*f^2*sinh(f*x + e)^2)
```

Sympy [F]

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{c}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx+\int \frac{dx}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))**2,x)
```

output
```
(Integral(c/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x) + Integral(d*x/(t
anh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x))/a**2
```

Maxima [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.80

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{1}{16}c\left(\frac{4\,(fx+e)}{a^2 f}-\frac{4\,e^{(-2\,fx-2\,e)}+e^{(-4\,fx-4\,e)}}{a^2 f}\right) + \frac{\left(8\,f^2x^2e^{(4\,e)}-8\left(2\,fxe^{(2\,e)}+e^{(2\,e)}\right)e^{(-2\,fx)}-(4\,fx+1)e^{(-4\,fx)}\right)de^{(-4\,e)}}{64\,a^2f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/16*c*(4*(f*x + e)/(a^2*f) - (4*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e))/(a^2
*f)) + 1/64*(8*f^2*x^2*e^(4*e) - 8*(2*f*x*e^(2*e) + e^(2*e))*e^(-2*f*x) -
(4*f*x + 1)*e^(-4*f*x))*d*e^(-4*e)/(a^2*f^2)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.77

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{\left(8\,df^2x^2e^{(4\,fx+4\,e)}+16\,cf^2xe^{(4\,fx+4\,e)}-16\,dfxe^{(2\,fx+2\,e)}-4\,dfx-16\,cfe^{(2\,fx+2\,e)}-4\,cf-8\,de^{(2\,fx+2\,e)}-\right.}{64\,a^2f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/64*(8*d*f^2*x^2*e^(4*f*x + 4*e) + 16*c*f^2*x*e^(4*f*x + 4*e) - 16*d*f*x*
e^(2*f*x + 2*e) - 4*d*f*x - 16*c*f*e^(2*f*x + 2*e) - 4*c*f - 8*d*e^(2*f*x
+ 2*e) - d)*e^(-4*f*x - 4*e)/(a^2*f^2)
```

Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.67

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{d\,x^2}{8\,a^2} - \mathrm{e}^{-4\,e-4\,f\,x}\left(\frac{d+4\,c\,f}{64\,a^2\,f^2}+\frac{d\,x}{16\,a^2\,f}\right) - \mathrm{e}^{-2\,e-2\,f\,x}\left(\frac{d+2\,c\,f}{8\,a^2\,f^2}+\frac{d\,x}{4\,a^2\,f}\right)+\frac{c\,x}{4\,a^2}$$

input
```
int((c + d*x)/(a + a*tanh(e + f*x))^2,x)
```

output
```
(d*x^2)/(8*a^2) - exp(- 4*e - 4*f*x)*((d + 4*c*f)/(64*a^2*f^2) + (d*x)/(16
*a^2*f)) - exp(- 2*e - 2*f*x)*((d + 2*c*f)/(8*a^2*f^2) + (d*x)/(4*a^2*f))
+ (c*x)/(4*a^2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.83

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^2}\,dx = \frac{16e^{4fx+4e}cf^2x+8e^{4fx+4e}df^2x^2-16e^{2fx+2e}cf-16e^{2fx+2e}dfx-8e^{2fx+2e}d-4cf-4dfx-d}{64e^{4fx+4e}a^2f^2}$$

input
```
int((d*x+c)/(a+a*tanh(f*x+e))^2,x)
```

output
```
(16*e**(4*e + 4*f*x)*c*f**2*x + 8*e**(4*e + 4*f*x)*d*f**2*x**2 - 16*e**(2*
e + 2*f*x)*c*f - 16*e**(2*e + 2*f*x)*d*f*x - 8*e**(2*e + 2*f*x)*d - 4*c*f
- 4*d*f*x - d)/(64*e**(4*e + 4*f*x)*a**2*f**2)
```

3.41 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 297

$$\begin{aligned}\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = {}& \frac{\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} \\ &+ \frac{\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d} \\ &+ \frac{\log(c+dx)}{4a^2d} - \frac{\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d} \\ &- \frac{\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d} \\ &- \frac{\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} \\ &+ \frac{\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{2a^2d} \\ &- \frac{\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d} \\ &+ \frac{\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{4a^2d}\end{aligned}$$

output
```
1/2*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^2/d+1/4*cosh(-4*e+4*c*f/d)*Chi
(4*c*f/d+4*f*x)/a^2/d+1/4*ln(d*x+c)/a^2/d+1/4*Chi(4*c*f/d+4*f*x)*sinh(-4*e
+4*c*f/d)/a^2/d+1/2*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d-1/2*cosh(-
2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^2/d-1/2*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2
*f*x)/a^2/d-1/4*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d-1/4*sinh(-4*e+
4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 199, normalized size of antiderivative = 0.67

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\left(\cosh\left(2e-\frac{2cf}{d}\right)-\sinh\left(2e-\frac{2cf}{d}\right)\right)\left(2\mathrm{Chi}\left(\frac{2f(c+dx)}{d}\right)+\cosh\left(2e-\frac{2cf}{d}\right)\log(f(c+dx))+\mathrm{Chi}\left(\frac{4f(c+dx)}{d}\right)\right.}{}$$

input
```
Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])^2),x]
```

output
```
((Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d])*(2*CoshIntegral[(2*f*(c +
 d*x))/d] + Cosh[2*e - (2*c*f)/d]*Log[f*(c + d*x)] + CoshIntegral[(4*f*(c
+ d*x))/d]*(Cosh[2*e - (2*c*f)/d] - Sinh[2*e - (2*c*f)/d]) + Log[f*(c + d*
x)]*Sinh[2*e - (2*c*f)/d] - 2*SinhIntegral[(2*f*(c + d*x))/d] - Cosh[2*e -
 (2*c*f)/d]*SinhIntegral[(4*f*(c + d*x))/d] + Sinh[2*e - (2*c*f)/d]*SinhIn
tegral[(4*f*(c + d*x))/d]))/(4*a^2*d)
```

Rubi [A] (verified)

Time = 1.27 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\tanh(e+fx)+a)^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(c+dx)(a-ia\tan(ie+ifx))^2}\,dx$$

$$\downarrow 4211$$

$$\int \left(\frac{\sinh^2(2e+2fx)}{4a^2(c+dx)} - \frac{\sinh(2e+2fx)}{2a^2(c+dx)} - \frac{\sinh(4e+4fx)}{4a^2(c+dx)} + \frac{\cosh^2(2e+2fx)}{4a^2(c+dx)} + \frac{\cosh(2e+2fx)}{2a^2(c+dx)} + \frac{1}{4a^2(c+dx)}\right.$$

$$\downarrow 2009$$

$$-\frac{\operatorname{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{2a^2d} - \frac{\operatorname{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{4a^2d} + \frac{\operatorname{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{2a^2d} + \frac{\operatorname{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{4a^2d} + \frac{\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(2xf+\frac{2cf}{d}\right)}{2a^2d} + \frac{\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(4xf+\frac{4cf}{d}\right)}{4a^2d} - \frac{\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(2xf+\frac{2cf}{d}\right)}{2a^2d} - \frac{\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(4xf+\frac{4cf}{d}\right)}{4a^2d} + \frac{\log(c+dx)}{4a^2d}$$

input

```
Int[1/((c + d*x)*(a + a*Tanh[e + f*x])^2),x]
```

output

```
(Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Cosh[
4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + Log[c + d*x]
/(4*a^2*d) - (CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(4*a^
2*d) - (CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(2*a^2*d) -
 (Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) + (Sinh
[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(2*a^2*d) - (Cosh[4*e -
 (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d) + (Sinh[4*e - (4*c*
f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(4*a^2*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 106, normalized size of antiderivative = 0.36

method	result	size
risch	$\frac{\ln(dx+c)}{4a^2d} - \frac{e^{\frac{4cf-4de}{d}}\,\mathrm{expIntegral}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{4a^2d} - \frac{e^{\frac{2cf-2de}{d}}\,\mathrm{expIntegral}_1\left(2fx+2e+\frac{2cf-2de}{d}\right)}{2a^2d}$	106

input
```
int(1/(d*x+c)/(a+tanh(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*ln(d*x+c)/a^2/d-1/4/a^2/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e
)/d)-1/2/a^2/d*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.45

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{2\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right) + \mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)\cosh\left(-\frac{4\,(de-cf)}{d}\right) + 2\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\sinh\left(-\frac{2\,(de-cf)}{d}\right)}{4\,a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/4*(2*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + Ei(-4*(d*f*x + c*f)
/d)*cosh(-4*(d*e - c*f)/d) + 2*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/
d) + Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + log(d*x + c))/(a^2*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{1}{c\tanh^2(e+fx)+2c\tanh(e+fx)+c+dx\tanh^2(e+fx)+2dx\tanh(e+fx)+dx}\,dx}{a^2}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))**2,x)
```

output
```
Integral(1/(c*tanh(e + f*x)**2 + 2*c*tanh(e + f*x) + c + d*x*tanh(e + f*x)
**2 + 2*d*x*tanh(e + f*x) + d*x), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 0.72 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.27

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = -\frac{e^{\left(-4e+\frac{4cf}{d}\right)}E_1\left(\frac{4(dx+c)f}{d}\right)}{4a^2d} - \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{2a^2d} + \frac{\log(dx+c)}{4a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^2*d) - 1/2*e
^(-2*e + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^2*d) + 1/4*log(d*x
 + c)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.25

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = \frac{\left(2\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(2e+\frac{2cf}{d}\right)} + \mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)e^{\left(\frac{4cf}{d}\right)} + e^{(4e)}\log(dx+c)\right)e^{(-4e)}}{4a^2d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
1/4*(2*Ei(-2*(d*f*x + c*f)/d)*e^(2*e + 2*c*f/d) + Ei(-4*(d*f*x + c*f)/d)*e
^(4*c*f/d) + e^(4*e)*log(d*x + c))*e^(-4*e)/(a^2*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+a\tanh(e+fx))^2\,(c+dx)}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{1}{\tanh(fx+e)^2c+\tanh(fx+e)^2dx+2\tanh(fx+e)c+2\tanh(fx+e)dx+c+dx}\,dx}{a^2}$$

input
```
int(1/(d*x+c)/(a+a*tanh(f*x+e))^2,x)
```

output

```
int(1/(tanh(e + f*x)**2*c + tanh(e + f*x)**2*d*x + 2*tanh(e + f*x)*c + 2*t
anh(e + f*x)*d*x + c + d*x),x)/a**2
```

3.42 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 420

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = &-\frac{1}{4a^2d(c+dx)} - \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} \\ &- \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} \\ &- \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2} \\ &- \frac{f\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2} \\ &+ \frac{f\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2} \\ &+ \frac{f\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2} \\ &+ \frac{\sinh(2e+2fx)}{2a^2d(c+dx)} \\ &- \frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{\sinh(4e+4fx)}{4a^2d(c+dx)} \\ &+ \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2} \\ &- \frac{f\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{a^2d^2} \\ &+ \frac{f\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2} \\ &- \frac{f\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{a^2d^2}\end{aligned}$$

output

```
-1/4/a^2/d/(d*x+c)-1/2*cosh(2*f*x+2*e)/a^2/d/(d*x+c)-1/4*cosh(2*f*x+2*e)^2
/a^2/d/(d*x+c)-f*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^2/d^2-f*cosh(-4*e
+4*c*f/d)*Chi(4*c*f/d+4*f*x)/a^2/d^2-f*Chi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/
d)/a^2/d^2-f*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c*f/d)/a^2/d^2+1/2*sinh(2*f*x+
2*e)/a^2/d/(d*x+c)-1/4*sinh(2*f*x+2*e)^2/a^2/d/(d*x+c)+1/4*sinh(4*f*x+4*e)
/a^2/d/(d*x+c)+f*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^2/d^2+f*sinh(-2*e
+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^2/d^2+f*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*
x)/a^2/d^2+f*sinh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^2/d^2
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.05

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{(-\cosh(2(e+f(-\frac{c}{d}+x)))+\sinh(2(e+f(-\frac{c}{d}+x))))\left(2d\cosh\left(\frac{2cf}{d}\right)+d\cosh(2(e+f(-\frac{c}{d}+x)))\right.}{}$$

input
```
Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^2),x]
```

output
```
((-Cosh[2*(e + f*(-(c/d) + x))] + Sinh[2*(e + f*(-(c/d) + x))])*(2*d*Cosh[
(2*c*f)/d] + d*Cosh[2*(e + f*(-(c/d) + x))] + d*Cosh[2*(e + f*(c/d + x))]
- 2*d*Sinh[(2*c*f)/d] + 4*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x))/d]*(Cos
h[2*f*x] + Sinh[2*f*x]) + d*Sinh[2*(e + f*(-(c/d) + x))] - d*Sinh[2*(e + f
*(c/d + x))] + 4*f*(c + d*x)*CoshIntegral[(4*f*(c + d*x))/d]*(Cosh[2*e - (
2*f*(c + d*x))/d] - Sinh[2*e - (2*f*(c + d*x))/d]) - 4*c*f*Cosh[2*f*x]*Sin
hIntegral[(2*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*f*x]*SinhIntegral[(2*f*(c +
d*x))/d] - 4*c*f*Sinh[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 4*d*f*x*Sin
h[2*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 4*c*f*Cosh[2*e - (2*f*(c + d*x)
)/d]*SinhIntegral[(4*f*(c + d*x))/d] - 4*d*f*x*Cosh[2*e - (2*f*(c + d*x))/
d]*SinhIntegral[(4*f*(c + d*x))/d] + 4*c*f*Sinh[2*e - (2*f*(c + d*x))/d]*S
inhIntegral[(4*f*(c + d*x))/d] + 4*d*f*x*Sinh[2*e - (2*f*(c + d*x))/d]*Sin
hIntegral[(4*f*(c + d*x))/d]))/(4*a^2*d^2*(c + d*x))
```

Rubi [A] (verified)

Time = 1.23 (sec) , antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\tanh(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$

$$\int \frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))^2}dx$$

$$\downarrow 4211$$

$$\int \left(\frac{\sinh^2(2e+2fx)}{4a^2(c+dx)^2} - \frac{\sinh(2e+2fx)}{2a^2(c+dx)^2} - \frac{\sinh(4e+4fx)}{4a^2(c+dx)^2} + \frac{\cosh^2(2e+2fx)}{4a^2(c+dx)^2} + \frac{\cosh(2e+2fx)}{2a^2(c+dx)^2} + \frac{1}{4a^2(c+dx)^2}\right.$$

$$\downarrow 2009$$

$$\frac{f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{a^2d^2} + \frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{a^2d^2} - \frac{f\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{a^2d^2} - \frac{f\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{a^2d^2} - \frac{f\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2} - \frac{f\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2} + \frac{f\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{a^2d^2} + \frac{f\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{a^2d^2} - \frac{\sinh^2(2e+2fx)}{4a^2d(c+dx)} + \frac{\sinh(2e+2fx)}{2a^2d(c+dx)} + \frac{\sinh(4e+4fx)}{4a^2d(c+dx)} - \frac{\cosh^2(2e+2fx)}{4a^2d(c+dx)} - \frac{\cosh(2e+2fx)}{2a^2d(c+dx)} - \frac{1}{4a^2d(c+dx)}$$

input

```
Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^2),x]
```

output

```
-1/4*1/(a^2*d*(c + d*x)) - Cosh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Cosh[2*
e + 2*f*x]^2/(4*a^2*d*(c + d*x)) - (f*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(
2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Cosh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*
f)/d + 4*f*x])/(a^2*d^2) + (f*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (
4*c*f)/d])/(a^2*d^2) + (f*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*
f)/d])/(a^2*d^2) + Sinh[2*e + 2*f*x]/(2*a^2*d*(c + d*x)) - Sinh[2*e + 2*f*
x]^2/(4*a^2*d*(c + d*x)) + Sinh[4*e + 4*f*x]/(4*a^2*d*(c + d*x)) + (f*Cosh
[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) - (f*Sinh[2*e
 - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(a^2*d^2) + (f*Cosh[4*e - (
4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2) - (f*Sinh[4*e - (4*c*
f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(a^2*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.75 (sec) , antiderivative size = 163, normalized size of antiderivative = 0.39

method	result
risch	$-\frac{1}{4a^2d(dx+c)}-\frac{f\,\mathrm{e}^{-4fx-4e}}{4a^2d(dxf+cf)}+\frac{f\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{expIntegral}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{a^2d^2}-\frac{f\,\mathrm{e}^{-2fx-2e}}{2a^2d(dxf+cf)}+\frac{f\,\mathrm{e}^{\frac{2cf-2de}{d}}\,\mathrm{expIntegr}$

input
```
int(1/(d*x+c)^2/(a+tanh(f*x+e)*a)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/4/a^2/d/(d*x+c)-1/4*f/a^2*exp(-4*f*x-4*e)/d/(d*f*x+c*f)+f/a^2/d^2*exp(4
*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)-1/2*f/a^2*exp(-2*f*x-2*e)/d/(d
*f*x+c*f)+f/a^2/d^2*exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 609, normalized size of antiderivative = 1.45

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output

```
-1/2*(2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*(d*e
- c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-4
*(d*e - c*f)/d) + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c
*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + d
)*cosh(f*x + e)^2 + (2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e -
 c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) +
 2*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 2*(d*f*x
+ c*f)*Ei(-4*(d*f*x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + d)*sinh(f*x + e)^2
+ 4*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f
)/d) + (d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-4*(d*e - c
*f)/d) + ((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + (d
*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d))*cosh(f*x + e))*
sinh(f*x + e) + d)/((a^2*d^3*x + a^2*c*d^2)*cosh(f*x + e)^2 + 2*(a^2*d^3*x
 + a^2*c*d^2)*cosh(f*x + e)*sinh(f*x + e) + (a^2*d^3*x + a^2*c*d^2)*sinh(f
*x + e)^2)
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{\int \frac{1}{c^2\tanh^2(e+fx)+2c^2\tanh(e+fx)+c^2+2cdx\tanh^2(e+fx)+4cdx\tanh(e+fx)+2cdx+d^2x^2\tanh^2(e+fx)+2d^2x^2\tanh(e+fx)+d^2x^2}\,dx}{a^2}$$

input

```
integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e))**2,x)
```

output

```
Integral(1/(c**2*tanh(e + f*x)**2 + 2*c**2*tanh(e + f*x) + c**2 + 2*c*d*x*
tanh(e + f*x)**2 + 4*c*d*x*tanh(e + f*x) + 2*c*d*x + d**2*x**2*tanh(e + f*
x)**2 + 2*d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a**2
```

Maxima [A] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.24

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = -\frac{1}{4\left(a^2d^2x+a^2cd\right)} - \frac{e^{\left(-4e+\frac{4cf}{d}\right)}E_2\left(\frac{4(dx+c)f}{d}\right)}{4(dx+c)a^2d} - \frac{e^{\left(-2e+\frac{2cf}{d}\right)}E_2\left(\frac{2(dx+c)f}{d}\right)}{2(dx+c)a^2d}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-1/4/(a^2*d^2*x + a^2*c*d) - 1/4*e^(-4*e + 4*c*f/d)*exp_integral_e(2, 4*(d
*x + c)*f/d)/((d*x + c)*a^2*d) - 1/2*e^(-2*e + 2*c*f/d)*exp_integral_e(2,
2*(d*x + c)*f/d)/((d*x + c)*a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 583, normalized size of antiderivative = 1.39

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = -\left(4(dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)f^2\mathrm{Ei}\left(-\frac{2\left((dx+c)\left(\frac{de}{dx+c}-\frac{cf}{dx+c}+f\right)-de+cf\right)}{d}\right)e^{\left(-\frac{2(de-cf)}{d}\right)} - 4\,def^2\mathrm{Ei}\left(-\frac{2\left((dx-\right.\right.\right.$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output

```
-1/4*(4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d)
- 4*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c
*f)/d)*e^(-2*(d*e - c*f)/d) + 4*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + 4*(d*x + c)*(d*e/(
d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d
*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)/d) - 4*d*e*f^2*Ei(-4*((d*x
+ c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)
/d) + 4*c*f^3*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
 c*f)/d)*e^(-4*(d*e - c*f)/d) + 2*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c
*f/(d*x + c) + f)/d) + d*f^2*e^(-4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c
) + f)/d) + d*f^2)*d^2/(((d*x + c)*a^2*d^4*(d*e/(d*x + c) - c*f/(d*x + c)
+ f) - a^2*d^5*e + a^2*c*d^4*f)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+a\tanh(e+fx))^2\,(c+dx)^2}\,dx$$

input

```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)^2),x)
```

output

```
int(1/((a + a*tanh(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{2e^{2e}\left(\int \frac{1}{e^{2fx+4e}c^2+2e^{2fx+4e}cdx+e^{2fx+4e}d^2x^2}dx\right)c^2 + 2e^{2e}\left(\int \frac{1}{e^{2fx+4e}c^2+2e^{2fx+4e}cdx+e^{2fx+4e}d^2x^2}dx\right)cdx + \left(\int \frac{}{e^{4fx+4e}}\right.}{4a^2c\,(dx+c)}$$

input

```
int(1/(d*x+c)^2/(a+a*tanh(f*x+e))^2,x)
```

output

```
(2*e**(2*e)*int(1/(e**(4*e + 2*f*x)*c**2 + 2*e**(4*e + 2*f*x)*c*d*x + e**(
4*e + 2*f*x)*d**2*x**2),x)*c**2 + 2*e**(2*e)*int(1/(e**(4*e + 2*f*x)*c**2
+ 2*e**(4*e + 2*f*x)*c*d*x + e**(4*e + 2*f*x)*d**2*x**2),x)*c*d*x + int(1/
(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*
x**2),x)*c**2 + int(1/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d*x +
e**(4*e + 4*f*x)*d**2*x**2),x)*c*d*x + x)/(4*a**2*c*(c + d*x))
```

3.43 $\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 336

$$\begin{aligned}\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx = &-\frac{d^3e^{-6e-6fx}}{1728a^3f^4} - \frac{9d^3e^{-4e-4fx}}{1024a^3f^4} - \frac{9d^3e^{-2e-2fx}}{64a^3f^4}\\ &- \frac{d^2e^{-6e-6fx}(c+dx)}{288a^3f^3} - \frac{9d^2e^{-4e-4fx}(c+dx)}{256a^3f^3}\\ &- \frac{9d^2e^{-2e-2fx}(c+dx)}{32a^3f^3} - \frac{de^{-6e-6fx}(c+dx)^2}{96a^3f^2}\\ &- \frac{9de^{-4e-4fx}(c+dx)^2}{128a^3f^2} - \frac{9de^{-2e-2fx}(c+dx)^2}{32a^3f^2}\\ &- \frac{e^{-6e-6fx}(c+dx)^3}{48a^3f} - \frac{3e^{-4e-4fx}(c+dx)^3}{32a^3f}\\ &- \frac{3e^{-2e-2fx}(c+dx)^3}{16a^3f} + \frac{(c+dx)^4}{32a^3d}\end{aligned}$$

output

```
-1/1728*d^3*exp(-6*f*x-6*e)/a^3/f^4-9/1024*d^3*exp(-4*f*x-4*e)/a^3/f^4-9/6
4*d^3*exp(-2*f*x-2*e)/a^3/f^4-1/288*d^2*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^3-9/
256*d^2*exp(-4*f*x-4*e)*(d*x+c)/a^3/f^3-9/32*d^2*exp(-2*f*x-2*e)*(d*x+c)/a
^3/f^3-1/96*d*exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f^2-9/128*d*exp(-4*f*x-4*e)*(d
*x+c)^2/a^3/f^2-9/32*d*exp(-2*f*x-2*e)*(d*x+c)^2/a^3/f^2-1/48*exp(-6*f*x-6
*e)*(d*x+c)^3/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^3/a^3/f-3/16*exp(-2*f*x-2
*e)*(d*x+c)^3/a^3/f+1/32*(d*x+c)^4/a^3/d
```

Mathematica [A] (verified)

Time = 3.23 (sec) , antiderivative size = 615, normalized size of antiderivative = 1.83

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)\left(-243(32c^3f^3+8c^2df^2(5+12fx)+4cd^2f(9+20fx+24f^2x^2)+d^3(17+36fx+40f^2x\right.}{}$$

input
```
Integrate[(c + d*x)^3/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(Sech[e + f*x]^3*(-243*(32*c^3*f^3 + 8*c^2*d*f^2*(5 + 12*f*x) + 4*c*d^2*f*
(9 + 20*f*x + 24*f^2*x^2) + d^3*(17 + 36*f*x + 40*f^2*x^2 + 32*f^3*x^3))*C
osh[e + f*x] + 16*(36*c^3*f^3*(-1 + 6*f*x) + 18*c^2*d*f^2*(-1 - 6*f*x + 18
*f^2*x^2) + 6*c*d^2*f*(-1 - 6*f*x - 18*f^2*x^2 + 36*f^3*x^3) + d^3*(-1 - 6
*f*x - 18*f^2*x^2 - 36*f^3*x^3 + 54*f^4*x^4))*Cosh[3*(e + f*x)] - 3645*d^3
*Sinh[e + f*x] - 6804*c*d^2*f*Sinh[e + f*x] - 5832*c^2*d*f^2*Sinh[e + f*x]
 - 2592*c^3*f^3*Sinh[e + f*x] - 6804*d^3*f*x*Sinh[e + f*x] - 11664*c*d^2*f
^2*x*Sinh[e + f*x] - 7776*c^2*d*f^3*x*Sinh[e + f*x] - 5832*d^3*f^2*x^2*Sin
h[e + f*x] - 7776*c*d^2*f^3*x^2*Sinh[e + f*x] - 2592*d^3*f^3*x^3*Sinh[e +
f*x] + 16*d^3*Sinh[3*(e + f*x)] + 96*c*d^2*f*Sinh[3*(e + f*x)] + 288*c^2*d
*f^2*Sinh[3*(e + f*x)] + 576*c^3*f^3*Sinh[3*(e + f*x)] + 96*d^3*f*x*Sinh[3
*(e + f*x)] + 576*c*d^2*f^2*x*Sinh[3*(e + f*x)] + 1728*c^2*d*f^3*x*Sinh[3*
(e + f*x)] + 3456*c^3*f^4*x*Sinh[3*(e + f*x)] + 288*d^3*f^2*x^2*Sinh[3*(e
+ f*x)] + 1728*c*d^2*f^3*x^2*Sinh[3*(e + f*x)] + 5184*c^2*d*f^4*x^2*Sinh[3
*(e + f*x)] + 576*d^3*f^3*x^3*Sinh[3*(e + f*x)] + 3456*c*d^2*f^4*x^3*Sinh[
3*(e + f*x)] + 864*d^3*f^4*x^4*Sinh[3*(e + f*x)]))/(27648*a^3*f^4*(1 + Tan
h[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a\tanh(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c+dx)^3}{(a-ia\tan(ie+ifx))^3}\,dx$$

$$\downarrow 4212$$

$$\int \left(\frac{(c+dx)^3 e^{-6e-6fx}}{8a^3}+\frac{3(c+dx)^3 e^{-4e-4fx}}{8a^3}+\frac{3(c+dx)^3 e^{-2e-2fx}}{8a^3}+\frac{(c+dx)^3}{8a^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{d^2(c+dx)e^{-6e-6fx}}{288a^3f^3}-\frac{9d^2(c+dx)e^{-4e-4fx}}{256a^3f^3}-\frac{9d^2(c+dx)e^{-2e-2fx}}{32a^3f^3}-\frac{d(c+dx)^2e^{-6e-6fx}}{96a^3f^2}-\frac{9d(c+dx)^2e^{-4e-4fx}}{128a^3f^2}-\frac{9d(c+dx)^2e^{-2e-2fx}}{32a^3f^2}-\frac{(c+dx)^3e^{-6e-6fx}}{48a^3f}-\frac{3(c+dx)^3e^{-4e-4fx}}{32a^3f}-\frac{3(c+dx)^3e^{-2e-2fx}}{16a^3f}+\frac{(c+dx)^4}{32a^3d}-\frac{d^3e^{-6e-6fx}}{1728a^3f^4}-\frac{9d^3e^{-4e-4fx}}{1024a^3f^4}-\frac{9d^3e^{-2e-2fx}}{64a^3f^4}$$

input

```
Int[(c + d*x)^3/(a + a*Tanh[e + f*x])^3,x]
```

output

```
-1/1728*(d^3*E^(-6*e - 6*f*x))/(a^3*f^4) - (9*d^3*E^(-4*e - 4*f*x))/(1024*
a^3*f^4) - (9*d^3*E^(-2*e - 2*f*x))/(64*a^3*f^4) - (d^2*E^(-6*e - 6*f*x)*(
c + d*x))/(288*a^3*f^3) - (9*d^2*E^(-4*e - 4*f*x)*(c + d*x))/(256*a^3*f^3)
 - (9*d^2*E^(-2*e - 2*f*x)*(c + d*x))/(32*a^3*f^3) - (d*E^(-6*e - 6*f*x)*(
c + d*x)^2)/(96*a^3*f^2) - (9*d*E^(-4*e - 4*f*x)*(c + d*x)^2)/(128*a^3*f^2
) - (9*d*E^(-2*e - 2*f*x)*(c + d*x)^2)/(32*a^3*f^2) - (E^(-6*e - 6*f*x)*(c
 + d*x)^3)/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^3)/(32*a^3*f) - (3*E
^(-2*e - 2*f*x)*(c + d*x)^3)/(16*a^3*f) + (c + d*x)^4/(32*a^3*d)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212

```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.86 (sec) , antiderivative size = 379, normalized size of antiderivative = 1.13

method	result
risch	$\frac{d^3x^4}{32a^3}+\frac{d^2cx^3}{8a^3}+\frac{3dc^2x^2}{16a^3}+\frac{c^3x}{8a^3}+\frac{c^4}{32a^3d}-\frac{3(4d^3x^3f^3+12cd^2f^3x^2+12c^2df^3x+6d^3f^2x^2+4c^3f^3+12cd^2f^2x+6c^2d$
parallelrisch	$\frac{-1952d^3+216d^3x^4f^4+864xc^3f^4-6264c^2df^3x-6264cd^2f^3x^2-2502d^3f^2x^2-4032c^2df^2-5004cd^2f^2x-2592\tanh(fx+e)c$

input

```
int((d*x+c)^3/(a+tanh(f*x+e)*a)^3,x,method=_RETURNVERBOSE)
```

output

```
1/32/a^3*d^3*x^4+1/8/a^3*d^2*c*x^3+3/16/a^3*d*c^2*x^2+1/8/a^3*c^3*x+1/32/a
^3/d*c^4-3/64*(4*d^3*f^3*x^3+12*c*d^2*f^3*x^2+12*c^2*d*f^3*x+6*d^3*f^2*x^2
+4*c^3*f^3+12*c*d^2*f^2*x+6*c^2*d*f^2+6*d^3*f*x+6*c*d^2*f+3*d^3)/a^3/f^4*e
xp(-2*f*x-2*e)-3/1024*(32*d^3*f^3*x^3+96*c*d^2*f^3*x^2+96*c^2*d*f^3*x+24*d
^3*f^2*x^2+32*c^3*f^3+48*c*d^2*f^2*x+24*c^2*d*f^2+12*d^3*f*x+12*c*d^2*f+3*
d^3)/a^3/f^4*exp(-4*f*x-4*e)-1/1728*(36*d^3*f^3*x^3+108*c*d^2*f^3*x^2+108*
c^2*d*f^3*x+18*d^3*f^2*x^2+36*c^3*f^3+36*c*d^2*f^2*x+18*c^2*d*f^2+6*d^3*f*
x+6*c*d^2*f+d^3)/a^3/f^4*exp(-6*f*x-6*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 844 vs. $2(298) = 596$.

Time = 0.10 (sec) , antiderivative size = 844, normalized size of antiderivative = 2.51

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output

```
1/27648*(16*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*d^2*f + 36*(
6*c*d^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*f^3 - d^3*f^
2)*x^2 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x)*cosh(f*x +
 e)^3 + 48*(54*d^3*f^4*x^4 - 36*c^3*f^3 - 18*c^2*d*f^2 - 6*c*d^2*f + 36*(6
*c*d^2*f^4 - d^3*f^3)*x^3 - d^3 + 18*(18*c^2*d*f^4 - 6*c*d^2*f^3 - d^3*f^2
)*x^2 + 6*(36*c^3*f^4 - 18*c^2*d*f^3 - 6*c*d^2*f^2 - d^3*f)*x)*cosh(f*x +
e)*sinh(f*x + e)^2 + 16*(54*d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*
d^2*f + 36*(6*c*d^2*f^4 + d^3*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*
f^3 + d^3*f^2)*x^2 + 6*(36*c^3*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x
)*sinh(f*x + e)^3 - 243*(32*d^3*f^3*x^3 + 32*c^3*f^3 + 40*c^2*d*f^2 + 36*c
*d^2*f + 17*d^3 + 8*(12*c*d^2*f^3 + 5*d^3*f^2)*x^2 + 4*(24*c^2*d*f^3 + 20*
c*d^2*f^2 + 9*d^3*f)*x)*cosh(f*x + e) - 3*(864*d^3*f^3*x^3 + 864*c^3*f^3 +
 1944*c^2*d*f^2 + 2268*c*d^2*f + 1215*d^3 + 648*(4*c*d^2*f^3 + 3*d^3*f^2)*
x^2 - 16*(54*d^3*f^4*x^4 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f + 36*(6*c
*d^2*f^4 + d^3*f^3)*x^3 + d^3 + 18*(18*c^2*d*f^4 + 6*c*d^2*f^3 + d^3*f^2)*
x^2 + 6*(36*c^3*f^4 + 18*c^2*d*f^3 + 6*c*d^2*f^2 + d^3*f)*x)*cosh(f*x + e)
^2 + 324*(8*c^2*d*f^3 + 12*c*d^2*f^2 + 7*d^3*f)*x)*sinh(f*x + e))/(a^3*f^4
*cosh(f*x + e)^3 + 3*a^3*f^4*cosh(f*x + e)^2*sinh(f*x + e) + 3*a^3*f^4*cos
h(f*x + e)*sinh(f*x + e)^2 + a^3*f^4*sinh(f*x + e)^3)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^3}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{d^3x^3}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{}{\tanh^3(e+fx)+3}}{a^3}$$

input

```
integrate((d*x+c)**3/(a+a*tanh(f*x+e))**3,x)
```

output

```
(Integral(c**3/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) +
1), x) + Integral(d**3*x**3/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tan
h(e + f*x) + 1), x) + Integral(3*c*d**2*x**2/(tanh(e + f*x)**3 + 3*tanh(e
+ f*x)**2 + 3*tanh(e + f*x) + 1), x) + Integral(3*c**2*d*x/(tanh(e + f*x)*
*3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 1.86 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.21

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{1}{96}c^3\left(\frac{12\,(fx+e)}{a^3 f} - \frac{18\,e^{(-2fx-2e)} + 9\,e^{(-4fx-4e)} + 2\,e^{(-6fx-6e)}}{a^3 f}\right)$$
$$+ \frac{\left(72\,f^2x^2e^{(6e)} - 108\left(2\,fxe^{(4e)} + e^{(4e)}\right)e^{(-2fx)} - 27\left(4\,fxe^{(2e)} + e^{(2e)}\right)e^{(-4fx)} - 4\,(6fx+1)e^{(-6fx)}\right)c^2}{384\,a^3f^2}$$
$$+ \frac{\left(288\,f^3x^3e^{(6e)} - 648\left(2\,f^2x^2e^{(4e)} + 2\,fxe^{(4e)} + e^{(4e)}\right)e^{(-2fx)} - 81\left(8\,f^2x^2e^{(2e)} + 4\,fxe^{(2e)} + e^{(2e)}\right)e^{(-}\right.}{2304\,a^3f^3}$$
$$+ \frac{\left(864\,f^4x^4e^{(6e)} - 1296\left(4\,f^3x^3e^{(4e)} + 6\,f^2x^2e^{(4e)} + 6\,fxe^{(4e)} + 3\,e^{(4e)}\right)e^{(-2fx)} - 81\left(32\,f^3x^3e^{(2e)} + 24\right.\right.}{27648\,a^3f^{4}}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c^3*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e)
 + 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/384*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e
^(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4
*(6*f*x + 1)*e^(-6*f*x))*c^2*d*e^(-6*e)/(a^3*f^2) + 1/2304*(288*f^3*x^3*e^
(6*e) - 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*
(8*f^2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 8*(18*f^2*x^2 +
 6*f*x + 1)*e^(-6*f*x))*c*d^2*e^(-6*e)/(a^3*f^3) + 1/27648*(864*f^4*x^4*e^
(6*e) - 1296*(4*f^3*x^3*e^(4*e) + 6*f^2*x^2*e^(4*e) + 6*f*x*e^(4*e) + 3*e^
(4*e))*e^(-2*f*x) - 81*(32*f^3*x^3*e^(2*e) + 24*f^2*x^2*e^(2*e) + 12*f*x*e
^(2*e) + 3*e^(2*e))*e^(-4*f*x) - 16*(36*f^3*x^3 + 18*f^2*x^2 + 6*f*x + 1)*
e^(-6*f*x))*d^3*e^(-6*e)/(a^3*f^4)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 548, normalized size of antiderivative = 1.63

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\left(864\,d^3f^4x^4e^{(6fx+6e)} + 3456\,cd^2f^4x^3e^{(6fx+6e)} + 5184\,c^2df^4x^2e^{(6fx+6e)} - 5184\,d^3f^3x^3e^{(4fx+4e)} - 2592\,d^{3}\right.}$$

input
```
integrate((d*x+c)^3/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
1/27648*(864*d^3*f^4*x^4*e^(6*f*x + 6*e) + 3456*c*d^2*f^4*x^3*e^(6*f*x + 6
*e) + 5184*c^2*d*f^4*x^2*e^(6*f*x + 6*e) - 5184*d^3*f^3*x^3*e^(4*f*x + 4*e
) - 2592*d^3*f^3*x^3*e^(2*f*x + 2*e) - 576*d^3*f^3*x^3 + 3456*c^3*f^4*x*e^
(6*f*x + 6*e) - 15552*c*d^2*f^3*x^2*e^(4*f*x + 4*e) - 7776*c*d^2*f^3*x^2*e
^(2*f*x + 2*e) - 1728*c*d^2*f^3*x^2 - 15552*c^2*d*f^3*x*e^(4*f*x + 4*e) -
7776*d^3*f^2*x^2*e^(4*f*x + 4*e) - 7776*c^2*d*f^3*x*e^(2*f*x + 2*e) - 1944
*d^3*f^2*x^2*e^(2*f*x + 2*e) - 1728*c^2*d*f^3*x - 288*d^3*f^2*x^2 - 5184*c
^3*f^3*e^(4*f*x + 4*e) - 15552*c*d^2*f^2*x*e^(4*f*x + 4*e) - 2592*c^3*f^3*
e^(2*f*x + 2*e) - 3888*c*d^2*f^2*x*e^(2*f*x + 2*e) - 576*c^3*f^3 - 576*c*d
^2*f^2*x - 7776*c^2*d*f^2*e^(4*f*x + 4*e) - 7776*d^3*f*x*e^(4*f*x + 4*e) -
 1944*c^2*d*f^2*e^(2*f*x + 2*e) - 972*d^3*f*x*e^(2*f*x + 2*e) - 288*c^2*d*
f^2 - 96*d^3*f*x - 7776*c*d^2*f*e^(4*f*x + 4*e) - 972*c*d^2*f*e^(2*f*x + 2
*e) - 96*c*d^2*f - 3888*d^3*e^(4*f*x + 4*e) - 243*d^3*e^(2*f*x + 2*e) - 16
*d^3)*e^(-6*f*x - 6*e)/(a^3*f^4)
```

Mupad [B] (verification not implemented)

Time = 2.67 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.12

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx = \frac{c^3 x}{8a^3} - e^{-4e-4fx}\left(\frac{96c^3f^3+72c^2df^2+36cd^2f+9d^3}{1024a^3f^4} + \frac{3d^3x^3}{32a^3f} + \frac{9dx(8c^2f^2+4cdf+d^2)}{256a^3f^3} + \frac{9d^2x^2(d+4cf)}{128a^3f^2}\right) - e^{-6e-6fx}\left(\frac{36c^3f^3+18c^2df^2+6cd^2f+d^3}{1728a^3f^4} + \frac{d^3x^3}{48a^3f} + \frac{dx(18c^2f^2+6cdf+d^2)}{288a^3f^3} + \frac{d^2x^2(d+6cf)}{96a^3f^2}\right) - e^{-2e-2fx}\left(\frac{12c^3f^3+18c^2df^2+18cd^2f+9d^3}{64a^3f^4} + \frac{3d^3x^3}{16a^3f} + \frac{9dx(2c^2f^2+2cdf+d^2)}{32a^3f^3} + \frac{9d^2x^2(d+2cf)}{32a^3f^2}\right) + \frac{d^3x^4}{32a^3} + \frac{3c^2dx^2}{16a^3} + \frac{cd^2x^3}{8a^3}$$

input

```
int((c + d*x)^3/(a + a*tanh(e + f*x))^3,x)
```

output

```
(c^3*x)/(8*a^3) - exp(- 4*e - 4*f*x)*((9*d^3 + 96*c^3*f^3 + 72*c^2*d*f^2 +
 36*c*d^2*f)/(1024*a^3*f^4) + (3*d^3*x^3)/(32*a^3*f) + (9*d*x*(d^2 + 8*c^2
*f^2 + 4*c*d*f))/(256*a^3*f^3) + (9*d^2*x^2*(d + 4*c*f))/(128*a^3*f^2)) -
exp(- 6*e - 6*f*x)*((d^3 + 36*c^3*f^3 + 18*c^2*d*f^2 + 6*c*d^2*f)/(1728*a^
3*f^4) + (d^3*x^3)/(48*a^3*f) + (d*x*(d^2 + 18*c^2*f^2 + 6*c*d*f))/(288*a^
3*f^3) + (d^2*x^2*(d + 6*c*f))/(96*a^3*f^2)) - exp(- 2*e - 2*f*x)*((9*d^3
+ 12*c^3*f^3 + 18*c^2*d*f^2 + 18*c*d^2*f)/(64*a^3*f^4) + (3*d^3*x^3)/(16*a
^3*f) + (9*d*x*(d^2 + 2*c^2*f^2 + 2*c*d*f))/(32*a^3*f^3) + (9*d^2*x^2*(d +
 2*c*f))/(32*a^3*f^2)) + (d^3*x^4)/(32*a^3) + (3*c^2*d*x^2)/(16*a^3) + (c*
d^2*x^3)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 575, normalized size of antiderivative = 1.71

$$\int \frac{(c+dx)^3}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{-288c^2df^2 - 96cd^2f - 96d^3fx - 2592e^{2fx+2e}c^3f^3 - 576d^3f^3x^3 - 288d^3f^2x^2 + 3456e^{6fx+6e}c^3f^4x + 86}{}$$

input
```
int((d*x+c)^3/(a+a*tanh(f*x+e))^3,x)
```

output
```
(3456*e**(6*e + 6*f*x)*c**3*f**4*x + 5184*e**(6*e + 6*f*x)*c**2*d*f**4*x**
2 + 3456*e**(6*e + 6*f*x)*c*d**2*f**4*x**3 + 864*e**(6*e + 6*f*x)*d**3*f**
4*x**4 - 5184*e**(4*e + 4*f*x)*c**3*f**3 - 15552*e**(4*e + 4*f*x)*c**2*d*f
**3*x - 7776*e**(4*e + 4*f*x)*c**2*d*f**2 - 15552*e**(4*e + 4*f*x)*c*d**2*
f**3*x**2 - 15552*e**(4*e + 4*f*x)*c*d**2*f**2*x - 7776*e**(4*e + 4*f*x)*c
*d**2*f - 5184*e**(4*e + 4*f*x)*d**3*f**3*x**3 - 7776*e**(4*e + 4*f*x)*d**
3*f**2*x**2 - 7776*e**(4*e + 4*f*x)*d**3*f*x - 3888*e**(4*e + 4*f*x)*d**3
- 2592*e**(2*e + 2*f*x)*c**3*f**3 - 7776*e**(2*e + 2*f*x)*c**2*d*f**3*x -
1944*e**(2*e + 2*f*x)*c**2*d*f**2 - 7776*e**(2*e + 2*f*x)*c*d**2*f**3*x**2
 - 3888*e**(2*e + 2*f*x)*c*d**2*f**2*x - 972*e**(2*e + 2*f*x)*c*d**2*f - 2
592*e**(2*e + 2*f*x)*d**3*f**3*x**3 - 1944*e**(2*e + 2*f*x)*d**3*f**2*x**2
 - 972*e**(2*e + 2*f*x)*d**3*f*x - 243*e**(2*e + 2*f*x)*d**3 - 576*c**3*f*
*3 - 1728*c**2*d*f**3*x - 288*c**2*d*f**2 - 1728*c*d**2*f**3*x**2 - 576*c*
d**2*f**2*x - 96*c*d**2*f - 576*d**3*f**3*x**3 - 288*d**3*f**2*x**2 - 96*d
**3*f*x - 16*d**3)/(27648*e**(6*e + 6*f*x)*a**3*f**4)
```

3.44 $\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 246

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx = -\frac{d^2e^{-6e-6fx}}{864a^3f^3} - \frac{3d^2e^{-4e-4fx}}{256a^3f^3} - \frac{3d^2e^{-2e-2fx}}{32a^3f^3} - \frac{de^{-6e-6fx}(c+dx)}{144a^3f^2} - \frac{3de^{-4e-4fx}(c+dx)}{64a^3f^2} - \frac{3de^{-2e-2fx}(c+dx)}{16a^3f^2} - \frac{e^{-6e-6fx}(c+dx)^2}{48a^3f} - \frac{3e^{-4e-4fx}(c+dx)^2}{32a^3f} - \frac{3e^{-2e-2fx}(c+dx)^2}{16a^3f} + \frac{(c+dx)^3}{24a^3d}$$

output

```
-1/864*d^2*exp(-6*f*x-6*e)/a^3/f^3-3/256*d^2*exp(-4*f*x-4*e)/a^3/f^3-3/32*
d^2*exp(-2*f*x-2*e)/a^3/f^3-1/144*d*exp(-6*f*x-6*e)*(d*x+c)/a^3/f^2-3/64*d
*exp(-4*f*x-4*e)*(d*x+c)/a^3/f^2-3/16*d*exp(-2*f*x-2*e)*(d*x+c)/a^3/f^2-1/
48*exp(-6*f*x-6*e)*(d*x+c)^2/a^3/f-3/32*exp(-4*f*x-4*e)*(d*x+c)^2/a^3/f-3/
16*exp(-2*f*x-2*e)*(d*x+c)^2/a^3/f+1/24*(d*x+c)^3/a^3/d
```

Mathematica [A] (verified)

Time = 1.97 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.51

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)\left(-81(24c^2f^2+4cdf(5+12fx)+d^2(9+20fx+24f^2x^2))\cosh(e+fx)+8(18c^2f^2(-1+$$

input
```
Integrate[(c + d*x)^2/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(Sech[e + f*x]^3*(-81*(24*c^2*f^2 + 4*c*d*f*(5 + 12*f*x) + d^2*(9 + 20*f*x
 + 24*f^2*x^2))*Cosh[e + f*x] + 8*(18*c^2*f^2*(-1 + 6*f*x) + 6*c*d*f*(-1 -
 6*f*x + 18*f^2*x^2) + d^2*(-1 - 6*f*x - 18*f^2*x^2 + 36*f^3*x^3))*Cosh[3*
(e + f*x)] - 567*d^2*Sinh[e + f*x] - 972*c*d*f*Sinh[e + f*x] - 648*c^2*f^2
*Sinh[e + f*x] - 972*d^2*f*x*Sinh[e + f*x] - 1296*c*d*f^2*x*Sinh[e + f*x]
- 648*d^2*f^2*x^2*Sinh[e + f*x] + 8*d^2*Sinh[3*(e + f*x)] + 48*c*d*f*Sinh[
3*(e + f*x)] + 144*c^2*f^2*Sinh[3*(e + f*x)] + 48*d^2*f*x*Sinh[3*(e + f*x)
] + 288*c*d*f^2*x*Sinh[3*(e + f*x)] + 864*c^2*f^3*x*Sinh[3*(e + f*x)] + 14
4*d^2*f^2*x^2*Sinh[3*(e + f*x)] + 864*c*d*f^3*x^2*Sinh[3*(e + f*x)] + 288*
d^2*f^3*x^3*Sinh[3*(e + f*x)]))/(6912*a^3*f^3*(1 + Tanh[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a\tanh(e+fx)+a)^3}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{(c+dx)^2}{(a-ia\tan(ie+ifx))^3}dx$$

$$\downarrow 4212$$

$$\int \left(\frac{(c+dx)^2 e^{-6e-6fx}}{8a^3} + \frac{3(c+dx)^2 e^{-4e-4fx}}{8a^3} + \frac{3(c+dx)^2 e^{-2e-2fx}}{8a^3} + \frac{(c+dx)^2}{8a^3} \right) dx$$

$$\downarrow 2009$$

$$-\frac{d(c+dx)e^{-6e-6fx}}{144a^3f^2} - \frac{3d(c+dx)e^{-4e-4fx}}{64a^3f^2} - \frac{3d(c+dx)e^{-2e-2fx}}{16a^3f^2} - \frac{(c+dx)^2 e^{-6e-6fx}}{48a^3f} - \frac{3(c+dx)^2 e^{-4e-4fx}}{32a^3f} - \frac{3(c+dx)^2 e^{-2e-2fx}}{16a^3f} + \frac{(c+dx)^3}{24a^3d} - \frac{d^2 e^{-6e-6fx}}{864a^3f^3} - \frac{3d^2 e^{-4e-4fx}}{256a^3f^3} - \frac{3d^2 e^{-2e-2fx}}{32a^3f^3}$$

input
```
Int[(c + d*x)^2/(a + a*Tanh[e + f*x])^3,x]
```

output
```
-1/864*(d^2*E^(-6*e - 6*f*x))/(a^3*f^3) - (3*d^2*E^(-4*e - 4*f*x))/(256*a^
3*f^3) - (3*d^2*E^(-2*e - 2*f*x))/(32*a^3*f^3) - (d*E^(-6*e - 6*f*x)*(c +
d*x))/(144*a^3*f^2) - (3*d*E^(-4*e - 4*f*x)*(c + d*x))/(64*a^3*f^2) - (3*d
*E^(-2*e - 2*f*x)*(c + d*x))/(16*a^3*f^2) - (E^(-6*e - 6*f*x)*(c + d*x)^2)
/(48*a^3*f) - (3*E^(-4*e - 4*f*x)*(c + d*x)^2)/(32*a^3*f) - (3*E^(-2*e - 2
*f*x)*(c + d*x)^2)/(16*a^3*f) + (c + d*x)^3/(24*a^3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.79 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.91

method	result
risch	$\frac{d^2x^3}{24a^3}+\frac{dcx^2}{8a^3}+\frac{c^2x}{8a^3}+\frac{c^3}{24a^3d}-\frac{3(2d^2x^2f^2+4cdf^2x+2c^2f^2+2d^2fx+2cdf+d^2)e^{-2fx-2e}}{32a^3f^3}-\frac{3(8d^2x^2f^2+16cdf^2x+}{}$
parallelrisch	$\frac{-720c^2f^2-522d^2x^2f^2-1044cdf^2x+216d^2\tanh(fx+e)x^3f^3+216x^2cdf^3+648x\tanh(fx+e)c^2f^3-207x\tanh(fx+e)d^2f-5}{}$

input
```
int((d*x+c)^2/(a+tanh(f*x+e)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/24/a^3*d^2*x^3+1/8/a^3*d*c*x^2+1/8/a^3*c^2*x+1/24/a^3/d*c^3-3/32*(2*d^2*
f^2*x^2+4*c*d*f^2*x+2*c^2*f^2+2*d^2*f*x+2*c*d*f+d^2)/a^3/f^3*exp(-2*f*x-2*
e)-3/256*(8*d^2*f^2*x^2+16*c*d*f^2*x+8*c^2*f^2+4*d^2*f*x+4*c*d*f+d^2)/a^3/
f^3*exp(-4*f*x-4*e)-1/864*(18*d^2*f^2*x^2+36*c*d*f^2*x+18*c^2*f^2+6*d^2*f*
x+6*c*d*f+d^2)/a^3/f^3*exp(-6*f*x-6*e)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 532 vs. 2(217) = 434.

Time = 0.09 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.16

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{8\left(36\,d^2f^3x^3-18\,c^2f^2-6\,cdf+18\left(6\,cdf^3-d^2f^2\right)x^2-d^2+6\left(18\,c^2f^3-6\,cdf^2-d^2f\right)x\right)\cosh\left(fx+e\right)^3}{}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output

```
1/6912*(8*(36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2
)*x^2 - d^2 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)^3 + 24*(
36*d^2*f^3*x^3 - 18*c^2*f^2 - 6*c*d*f + 18*(6*c*d*f^3 - d^2*f^2)*x^2 - d^2
 + 6*(18*c^2*f^3 - 6*c*d*f^2 - d^2*f)*x)*cosh(f*x + e)*sinh(f*x + e)^2 + 8
*(36*d^2*f^3*x^3 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d
^2 + 6*(18*c^2*f^3 + 6*c*d*f^2 + d^2*f)*x)*sinh(f*x + e)^3 - 81*(24*d^2*f^
2*x^2 + 24*c^2*f^2 + 20*c*d*f + 9*d^2 + 4*(12*c*d*f^2 + 5*d^2*f)*x)*cosh(f
*x + e) - 3*(216*d^2*f^2*x^2 + 216*c^2*f^2 + 324*c*d*f - 8*(36*d^2*f^3*x^3
 + 18*c^2*f^2 + 6*c*d*f + 18*(6*c*d*f^3 + d^2*f^2)*x^2 + d^2 + 6*(18*c^2*f
^3 + 6*c*d*f^2 + d^2*f)*x)*cosh(f*x + e)^2 + 189*d^2 + 108*(4*c*d*f^2 + 3*
d^2*f)*x)*sinh(f*x + e))/(a^3*f^3*cosh(f*x + e)^3 + 3*a^3*f^3*cosh(f*x + e
)^2*sinh(f*x + e) + 3*a^3*f^3*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^3*sinh
(f*x + e)^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c^2}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{d^2x^2}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{}{\tanh^3(e+fx)+3}}{a^3}$$

input

```
integrate((d*x+c)**2/(a+a*tanh(f*x+e))**3,x)
```

output

```
(Integral(c**2/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) +
1), x) + Integral(d**2*x**2/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tan
h(e + f*x) + 1), x) + Integral(2*c*d*x/(tanh(e + f*x)**3 + 3*tanh(e + f*x)
**2 + 3*tanh(e + f*x) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.04

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{1}{96}c^2\left(\frac{12\,(fx+e)}{a^3 f} - \frac{18\,e^{(-2\,fx-2\,e)} + 9\,e^{(-4\,fx-4\,e)} + 2\,e^{(-6\,fx-6\,e)}}{a^3 f}\right)$$
$$+ \frac{\left(72\,f^2x^2e^{(6\,e)} - 108\left(2\,fxe^{(4\,e)} + e^{(4\,e)}\right)e^{(-2\,fx)} - 27\left(4\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-4\,fx)} - 4\,(6\,fx+1)e^{(-6\,fx)}\right)cd}{576\,a^3 f^2}$$
$$+ \frac{\left(288\,f^3x^3e^{(6\,e)} - 648\left(2\,f^2x^2e^{(4\,e)} + 2\,fxe^{(4\,e)} + e^{(4\,e)}\right)e^{(-2\,fx)} - 81\left(8\,f^2x^2e^{(2\,e)} + 4\,fxe^{(2\,e)} + e^{(2\,e)}\right)e^{(-4}\right.}{6912\,a^3 f^3}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c^2*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e)
 + 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/576*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e
^(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4
*(6*f*x + 1)*e^(-6*f*x))*c*d*e^(-6*e)/(a^3*f^2) + 1/6912*(288*f^3*x^3*e^(6
*e) - 648*(2*f^2*x^2*e^(4*e) + 2*f*x*e^(4*e) + e^(4*e))*e^(-2*f*x) - 81*(8
*f^2*x^2*e^(2*e) + 4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 8*(18*f^2*x^2 + 6
*f*x + 1)*e^(-6*f*x))*d^2*e^(-6*e)/(a^3*f^3)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.28

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\left(288\,d^2f^3x^3e^{(6\,fx+6\,e)} + 864\,cdf^3x^2e^{(6\,fx+6\,e)} + 864\,c^2f^3xe^{(6\,fx+6\,e)} - 1296\,d^2f^2x^2e^{(4\,fx+4\,e)} - 648\,d^2f^2x^2e^{(}\right.}$$

input
```
integrate((d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output

```
1/6912*(288*d^2*f^3*x^3*e^(6*f*x + 6*e) + 864*c*d*f^3*x^2*e^(6*f*x + 6*e)
+ 864*c^2*f^3*x*e^(6*f*x + 6*e) - 1296*d^2*f^2*x^2*e^(4*f*x + 4*e) - 648*d
^2*f^2*x^2*e^(2*f*x + 2*e) - 144*d^2*f^2*x^2 - 2592*c*d*f^2*x*e^(4*f*x + 4
*e) - 1296*c*d*f^2*x*e^(2*f*x + 2*e) - 288*c*d*f^2*x - 1296*c^2*f^2*e^(4*f
*x + 4*e) - 1296*d^2*f*x*e^(4*f*x + 4*e) - 648*c^2*f^2*e^(2*f*x + 2*e) - 3
24*d^2*f*x*e^(2*f*x + 2*e) - 144*c^2*f^2 - 48*d^2*f*x - 1296*c*d*f*e^(4*f*
x + 4*e) - 324*c*d*f*e^(2*f*x + 2*e) - 48*c*d*f - 648*d^2*e^(4*f*x + 4*e)
- 81*d^2*e^(2*f*x + 2*e) - 8*d^2)*e^(-6*f*x - 6*e)/(a^3*f^3)
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 236, normalized size of antiderivative = 0.96

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx = \frac{c^2 x}{8a^3} - e^{-2e-2fx}\left(\frac{6c^2f^2+6cdf+3d^2}{32a^3f^3} + \frac{3d^2x^2}{16a^3f} + \frac{3dx(d+2cf)}{16a^3f^2}\right) - e^{-4e-4fx}\left(\frac{24c^2f^2+12cdf+3d^2}{256a^3f^3} + \frac{3d^2x^2}{32a^3f} + \frac{3dx(d+4cf)}{64a^3f^2}\right) - e^{-6e-6fx}\left(\frac{18c^2f^2+6cdf+d^2}{864a^3f^3} + \frac{d^2x^2}{48a^3f} + \frac{dx(d+6cf)}{144a^3f^2}\right) + \frac{d^2x^3}{24a^3} + \frac{cdx^2}{8a^3}$$

input

```
int((c + d*x)^2/(a + a*tanh(e + f*x))^3,x)
```

output

```
(c^2*x)/(8*a^3) - exp(- 2*e - 2*f*x)*((3*d^2 + 6*c^2*f^2 + 6*c*d*f)/(32*a^
3*f^3) + (3*d^2*x^2)/(16*a^3*f) + (3*d*x*(d + 2*c*f))/(16*a^3*f^2)) - exp(
- 4*e - 4*f*x)*((3*d^2 + 24*c^2*f^2 + 12*c*d*f)/(256*a^3*f^3) + (3*d^2*x^2
)/(32*a^3*f) + (3*d*x*(d + 4*c*f))/(64*a^3*f^2)) - exp(- 6*e - 6*f*x)*((d^
2 + 18*c^2*f^2 + 6*c*d*f)/(864*a^3*f^3) + (d^2*x^2)/(48*a^3*f) + (d*x*(d +
 6*c*f))/(144*a^3*f^2)) + (d^2*x^3)/(24*a^3) + (c*d*x^2)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 333, normalized size of antiderivative = 1.35

$$\int \frac{(c+dx)^2}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{864e^{6fx+6e}c^2f^3x + 864e^{6fx+6e}cd\,f^3x^2 + 288e^{6fx+6e}d^2f^3x^3 - 1296e^{4fx+4e}c^2f^2 - 2592e^{4fx+4e}cd\,f^2x - 129}{}$$

input
```
int((d*x+c)^2/(a+a*tanh(f*x+e))^3,x)
```

output
```
(864*e**(6*e + 6*f*x)*c**2*f**3*x + 864*e**(6*e + 6*f*x)*c*d*f**3*x**2 + 2
88*e**(6*e + 6*f*x)*d**2*f**3*x**3 - 1296*e**(4*e + 4*f*x)*c**2*f**2 - 259
2*e**(4*e + 4*f*x)*c*d*f**2*x - 1296*e**(4*e + 4*f*x)*c*d*f - 1296*e**(4*e
 + 4*f*x)*d**2*f**2*x**2 - 1296*e**(4*e + 4*f*x)*d**2*f*x - 648*e**(4*e +
4*f*x)*d**2 - 648*e**(2*e + 2*f*x)*c**2*f**2 - 1296*e**(2*e + 2*f*x)*c*d*f
**2*x - 324*e**(2*e + 2*f*x)*c*d*f - 648*e**(2*e + 2*f*x)*d**2*f**2*x**2 -
 324*e**(2*e + 2*f*x)*d**2*f*x - 81*e**(2*e + 2*f*x)*d**2 - 144*c**2*f**2
- 288*c*d*f**2*x - 48*c*d*f - 144*d**2*f**2*x**2 - 48*d**2*f*x - 8*d**2)/(
6912*e**(6*e + 6*f*x)*a**3*f**3)
```

3.45 $\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 183

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx = \frac{11dx}{96a^3f} - \frac{dx^2}{16a^3} + \frac{x(c+dx)}{8a^3} - \frac{d}{36f^2(a+a\tanh(e+fx))^3} - \frac{c+dx}{6f(a+a\tanh(e+fx))^3} - \frac{5d}{96af^2(a+a\tanh(e+fx))^2} - \frac{c+dx}{8af(a+a\tanh(e+fx))^2} - \frac{11d}{96f^2\left(a^3+a^3\tanh(e+fx)\right)} - \frac{c+dx}{8f\left(a^3+a^3\tanh(e+fx)\right)}$$

output

```
11/96*d*x/a^3/f-1/16*d*x^2/a^3+1/8*x*(d*x+c)/a^3-1/36*d/f^2/(a+a*tanh(f*x+
e))^3-1/6*(d*x+c)/f/(a+a*tanh(f*x+e))^3-5/96*d/a/f^2/(a+a*tanh(f*x+e))^2-1
/8*(d*x+c)/a/f/(a+a*tanh(f*x+e))^2-11/96*d/f^2/(a^3+a^3*tanh(f*x+e))-1/8*(
d*x+c)/f/(a^3+a^3*tanh(f*x+e))
```

Mathematica [A] (verified)

Time = 1.14 (sec) , antiderivative size = 185, normalized size of antiderivative = 1.01

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)\left(-27(12cf+d(5+12fx))\cosh(e+fx)+4(6cf(-1+6fx)+d(-1-6fx+18f^2x^2))\cosh(3(e+fx))-81d\sinh(e+fx)-108cf\sinh(e+fx)-108dfx\sinh(e+fx)+4d\sinh(3(e+fx))+24cf\sinh(3(e+fx))+24dfx\sinh(3(e+fx))+144cf^2x\sinh(3(e+fx))+72df^2x^2\sinh(3(e+fx))\right)}{1152a^3f^2(1+\tanh(e+fx))^3}$$

input
```
Integrate[(c + d*x)/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(Sech[e + f*x]^3*(-27*(12*c*f + d*(5 + 12*f*x))*Cosh[e + f*x] + 4*(6*c*f*(
-1 + 6*f*x) + d*(-1 - 6*f*x + 18*f^2*x^2))*Cosh[3*(e + f*x)] - 81*d*Sinh[e
 + f*x] - 108*c*f*Sinh[e + f*x] - 108*d*f*x*Sinh[e + f*x] + 4*d*Sinh[3*(e
+ f*x)] + 24*c*f*Sinh[3*(e + f*x)] + 24*d*f*x*Sinh[3*(e + f*x)] + 144*c*f^
2*x*Sinh[3*(e + f*x)] + 72*d*f^2*x^2*Sinh[3*(e + f*x)]))/(1152*a^3*f^2*(1
+ Tanh[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.99, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4213, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a\tanh(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{c+dx}{(a-ia\tan(ie+ifx))^3}\,dx$$
$$\downarrow 4213$$

$$-d\int\left(\frac{x}{8a^3}-\frac{1}{8f\left(\tanh(e+fx)a^3+a^3\right)}-\frac{1}{8af(\tanh(e+fx)a+a)^2}-\frac{1}{6f(\tanh(e+fx)a+a)^3}\right)dx-\frac{c+dx}{8f\left(a^3\tanh(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-\frac{c+dx}{8af(a\tanh(e+fx)+a)^2}-\frac{c+dx}{6f(a\tanh(e+fx)+a)^3}$$

$$\downarrow \text{2009}$$

$$-\frac{c+dx}{8f\left(a^3\tanh(e+fx)+a^3\right)}+\frac{x(c+dx)}{8a^3}-d\left(\frac{11}{96f^2\left(a^3\tanh(e+fx)+a^3\right)}-\frac{11x}{96a^3f}+\frac{x^2}{16a^3}+\frac{5}{96af^2(a\tanh(e+fx)+a)^2}+\frac{1}{36f^2(a\tanh(e+fx)+a)^3}\right)-\frac{c+dx}{8af(a\tanh(e+fx)+a)^2}-\frac{c+dx}{6f(a\tanh(e+fx)+a)^3}$$

input

```
Int[(c + d*x)/(a + a*Tanh[e + f*x])^3,x]
```

output

```
(x*(c + d*x))/(8*a^3) - (c + d*x)/(6*f*(a + a*Tanh[e + f*x])^3) - (c + d*x
)/(8*a*f*(a + a*Tanh[e + f*x])^2) - (c + d*x)/(8*f*(a^3 + a^3*Tanh[e + f*x
])) - d*((-11*x)/(96*a^3*f) + x^2/(16*a^3) + 1/(36*f^2*(a + a*Tanh[e + f*x
])^3) + 5/(96*a*f^2*(a + a*Tanh[e + f*x])^2) + 11/(96*f^2*(a^3 + a^3*Tanh[
e + f*x])))
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4213

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> With[{u = IntHide[(a + b*Tan[e + f*x])^n, x]}, Simp[(c + d*x)
^m   u, x] - Simp[d*m   Int[(c + d*x)^(m - 1)   u, x], x]] /; FreeQ[{a, b,
c, d, e, f}, x] && EqQ[a^2 + b^2, 0] && ILtQ[n, -1] && GtQ[m, 0]
```

Maple [A] (verified)

Time = 0.57 (sec) , antiderivative size = 102, normalized size of antiderivative = 0.56

method	result
risch	$\frac{dx^2}{16a^3}+\frac{cx}{8a^3}-\frac{3(2dxf+2cf+d)e^{-2fx-2e}}{32a^3f^2}-\frac{3(4dxf+4cf+d)e^{-4fx-4e}}{128a^3f^2}-\frac{(6dxf+6cf+d)e^{-6fx-6e}}{288a^3f^2}$
parallelrisch	$\frac{-36\tanh(fx+e)^2cf-56d+63\tanh(fx+e)^2dfx-120cf+33\tanh(fx+e)^3dfx-87dxf+54d\tanh(fx+e)x^2f^2+108x\tanh(fx-$

input
```
int((d*x+c)/(a+tanh(f*x+e)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/16*d*x^2/a^3+1/8/a^3*c*x-3/32*(2*d*f*x+2*c*f+d)/a^3/f^2*exp(-2*f*x-2*e)-
3/128*(4*d*f*x+4*c*f+d)/a^3/f^2*exp(-4*f*x-4*e)-1/288*(6*d*f*x+6*c*f+d)/a^
3/f^2*exp(-6*f*x-6*e)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 286, normalized size of antiderivative = 1.56

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{4\left(18\,df^2x^2-6\,cf+6\left(6\,cf^2-df\right)x-d\right)\cosh\left(fx+e\right)^3+12\left(18\,df^2x^2-6\,cf+6\left(6\,cf^2-df\right)x-d\right)\cos}{1}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/1152*(4*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)^3
 + 12*(18*d*f^2*x^2 - 6*c*f + 6*(6*c*f^2 - d*f)*x - d)*cosh(f*x + e)*sinh(
f*x + e)^2 + 4*(18*d*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*sinh(f*x +
 e)^3 - 27*(12*d*f*x + 12*c*f + 5*d)*cosh(f*x + e) - 3*(36*d*f*x - 4*(18*d
*f^2*x^2 + 6*c*f + 6*(6*c*f^2 + d*f)*x + d)*cosh(f*x + e)^2 + 36*c*f + 27*
d)*sinh(f*x + e))/(a^3*f^2*cosh(f*x + e)^3 + 3*a^3*f^2*cosh(f*x + e)^2*sin
h(f*x + e) + 3*a^3*f^2*cosh(f*x + e)*sinh(f*x + e)^2 + a^3*f^2*sinh(f*x +
e)^3)
```

Sympy [F]

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{c}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx + \int \frac{dx}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx}{a^3}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))**3,x)
```

output
```
(Integral(c/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x) + 1),
 x) + Integral(d*x/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e + f*x
) + 1), x))/a**3
```

Maxima [A] (verification not implemented)

Time = 0.71 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.76

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{1}{96}c\left(\frac{12(fx+e)}{a^3f} - \frac{18e^{(-2fx-2e)} + 9e^{(-4fx-4e)} + 2e^{(-6fx-6e)}}{a^3f}\right)$$
$$+ \frac{\left(72f^2x^2e^{(6e)} - 108\left(2fxe^{(4e)} + e^{(4e)}\right)e^{(-2fx)} - 27\left(4fxe^{(2e)} + e^{(2e)}\right)e^{(-4fx)} - 4(6fx+1)e^{(-6fx)}\right)de}{1152a^3f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/96*c*(12*(f*x + e)/(a^3*f) - (18*e^(-2*f*x - 2*e) + 9*e^(-4*f*x - 4*e) +
 2*e^(-6*f*x - 6*e))/(a^3*f)) + 1/1152*(72*f^2*x^2*e^(6*e) - 108*(2*f*x*e^
(4*e) + e^(4*e))*e^(-2*f*x) - 27*(4*f*x*e^(2*e) + e^(2*e))*e^(-4*f*x) - 4*
(6*f*x + 1)*e^(-6*f*x))*d*e^(-6*e)/(a^3*f^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.78

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\left(72\,df^2x^2e^{(6fx+6e)} + 144\,cf^2xe^{(6fx+6e)} - 216\,dfxe^{(4fx+4e)} - 108\,dfxe^{(2fx+2e)} - 24\,dfx - 216\,cfe^{(4fx+4}\right.}{1152\,a^3f^2}$$

input
```
integrate((d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
1/1152*(72*d*f^2*x^2*e^(6*f*x + 6*e) + 144*c*f^2*x*e^(6*f*x + 6*e) - 216*d
*f*x*e^(4*f*x + 4*e) - 108*d*f*x*e^(2*f*x + 2*e) - 24*d*f*x - 216*c*f*e^(4
*f*x + 4*e) - 108*c*f*e^(2*f*x + 2*e) - 24*c*f - 108*d*e^(4*f*x + 4*e) - 2
7*d*e^(2*f*x + 2*e) - 4*d)*e^(-6*f*x - 6*e)/(a^3*f^2)
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.70

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx = \frac{d\,x^2}{16\,a^3} - \mathrm{e}^{-2e-2fx}\left(\frac{3\,d+6\,c\,f}{32\,a^3\,f^2} + \frac{3\,d\,x}{16\,a^3\,f}\right) - \mathrm{e}^{-4e-4fx}\left(\frac{3\,d+12\,c\,f}{128\,a^3\,f^2} + \frac{3\,d\,x}{32\,a^3\,f}\right) - \mathrm{e}^{-6e-6fx}\left(\frac{d+6\,c\,f}{288\,a^3\,f^2} + \frac{d\,x}{48\,a^3\,f}\right) + \frac{c\,x}{8\,a^3}$$

input
```
int((c + d*x)/(a + a*tanh(e + f*x))^3,x)
```

output
```
(d*x^2)/(16*a^3) - exp(- 2*e - 2*f*x)*((3*d + 6*c*f)/(32*a^3*f^2) + (3*d*x
)/(16*a^3*f)) - exp(- 4*e - 4*f*x)*((3*d + 12*c*f)/(128*a^3*f^2) + (3*d*x)
/(32*a^3*f)) - exp(- 6*e - 6*f*x)*((d + 6*c*f)/(288*a^3*f^2) + (d*x)/(48*a
^3*f)) + (c*x)/(8*a^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 153, normalized size of antiderivative = 0.84

$$\int \frac{c+dx}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{144e^{6fx+6e}c\,f^2x + 72e^{6fx+6e}d\,f^2x^2 - 216e^{4fx+4e}cf - 216e^{4fx+4e}dfx - 108e^{4fx+4e}d - 108e^{2fx+2e}cf - 108}{1152e^{6fx+6e}a^3f^2}$$

input
```
int((d*x+c)/(a+a*tanh(f*x+e))^3,x)
```

output
```
(144*e**(6*e + 6*f*x)*c*f**2*x + 72*e**(6*e + 6*f*x)*d*f**2*x**2 - 216*e**
(4*e + 4*f*x)*c*f - 216*e**(4*e + 4*f*x)*d*f*x - 108*e**(4*e + 4*f*x)*d -
108*e**(2*e + 2*f*x)*c*f - 108*e**(2*e + 2*f*x)*d*f*x - 27*e**(2*e + 2*f*x
)*d - 24*c*f - 24*d*f*x - 4*d)/(1152*e**(6*e + 6*f*x)*a**3*f**2)
```

3.46 $\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 437

$$\begin{aligned}\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = {} & \frac{3\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} \\ & + \frac{3\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d} \\ & + \frac{\cosh\left(6e-\frac{6cf}{d}\right)\operatorname{Chi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} \\ & + \frac{\log(c+dx)}{8a^3d} - \frac{\operatorname{Chi}\left(\frac{6cf}{d}+6fx\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d} \\ & - \frac{3\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d} \\ & - \frac{3\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d} \\ & - \frac{3\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} \\ & + \frac{3\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{8a^3d} \\ & - \frac{3\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d} \\ & + \frac{3\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{8a^3d} \\ & - \frac{\cosh\left(6e-\frac{6cf}{d}\right)\operatorname{Shi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d} \\ & + \frac{\sinh\left(6e-\frac{6cf}{d}\right)\operatorname{Shi}\left(\frac{6cf}{d}+6fx\right)}{8a^3d}\end{aligned}$$

output

```
3/8*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^3/d+3/8*cosh(-4*e+4*c*f/d)*Chi
(4*c*f/d+4*f*x)/a^3/d+1/8*cosh(-6*e+6*c*f/d)*Chi(6*c*f/d+6*f*x)/a^3/d+1/8*
ln(d*x+c)/a^3/d+1/8*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d+3/8*Chi(4*
c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/d+3/8*Chi(2*c*f/d+2*f*x)*sinh(-2*e+2*c
*f/d)/a^3/d-3/8*cosh(-2*e+2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d-3/8*sinh(-2*e+
2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d-3/8*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x
)/a^3/d-3/8*sinh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d-1/8*cosh(-6*e+6*c*
f/d)*Shi(6*c*f/d+6*f*x)/a^3/d-1/8*sinh(-6*e+6*c*f/d)*Shi(6*c*f/d+6*f*x)/a^
3/d
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.71

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\operatorname{sech}^3(e+fx)(\cosh(fx)+\sinh(fx))^3\left(\cosh(3e)\log(f(c+dx))+\log(f(c+dx))\sinh(3e)+\left(\cosh\left(e-\right.\right.\right.}$$

input
```
Integrate[1/((c + d*x)*(a + a*Tanh[e + f*x])^3),x]
```

output
```
(Sech[e + f*x]^3*(Cosh[f*x] + Sinh[f*x])^3*(Cosh[3*e]*Log[f*(c + d*x)] + L
og[f*(c + d*x)]*Sinh[3*e] + (Cosh[e - (4*c*f)/d] - Sinh[e - (4*c*f)/d])*(3
*CoshIntegral[(4*f*(c + d*x))/d] + Cosh[2*e - (2*c*f)/d]*CoshIntegral[(6*f
*(c + d*x))/d] - CoshIntegral[(6*f*(c + d*x))/d]*Sinh[2*e - (2*c*f)/d] + 3
*CoshIntegral[(2*f*(c + d*x))/d]*(Cosh[2*e - (2*c*f)/d] + Sinh[2*e - (2*c*
f)/d]) - 3*Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*Sinh[
2*e - (2*c*f)/d]*SinhIntegral[(2*f*(c + d*x))/d] - 3*SinhIntegral[(4*f*(c
+ d*x))/d] - Cosh[2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d] + Sinh[
2*e - (2*c*f)/d]*SinhIntegral[(6*f*(c + d*x))/d])))/(8*a^3*d*(1 + Tanh[e +
 f*x])^3)
```

Rubi [A] (verified)

Time = 2.20 (sec) , antiderivative size = 437, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a\tanh(e+fx)+a)^3}\,dx$$
$$\downarrow \text{3042}$$
$$\int \frac{1}{(c+dx)(a-ia\tan(ie+ifx))^3}dx$$

$$\downarrow 4211$$

$$\int \left(-\frac{\sinh^3(2e+2fx)}{8a^3(c+dx)}+\frac{3\sinh^2(2e+2fx)}{8a^3(c+dx)}-\frac{3\sinh(2e+2fx)}{8a^3(c+dx)}+\frac{3\sinh(2e+2fx)\sinh(4e+4fx)}{16a^3(c+dx)}-\frac{3\sinh(4e$$

$$\downarrow 2009$$

$$-\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{8a^3d}-\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{8a^3d}-\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{3\text{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{8a^3d}+\frac{3\text{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{8a^3d}+\frac{\text{Chi}\left(6xf+\frac{6cf}{d}\right)\cosh\left(6e-\frac{6cf}{d}\right)}{8a^3d}+\frac{3\sinh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}+\frac{3\sinh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}+\frac{\sinh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}-\frac{3\cosh\left(2e-\frac{2cf}{d}\right)\text{Shi}\left(2xf+\frac{2cf}{d}\right)}{8a^3d}-\frac{3\cosh\left(4e-\frac{4cf}{d}\right)\text{Shi}\left(4xf+\frac{4cf}{d}\right)}{8a^3d}-\frac{\cosh\left(6e-\frac{6cf}{d}\right)\text{Shi}\left(6xf+\frac{6cf}{d}\right)}{8a^3d}+\frac{\log(c+dx)}{8a^3d}$$

input

```
Int[1/((c + d*x)*(a + a*Tanh[e + f*x])^3),x]
```

output

```
(3*Cosh[2*e - (2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*C
osh[4*e - (4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (Cosh[6*
e - (6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + Log[c + d*x]/(
8*a^3*d) - (CoshIntegral[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(8*a^3*
d) - (3*CoshIntegral[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(8*a^3*d) -
 (3*CoshIntegral[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(8*a^3*d) - (3*
Cosh[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) + (3*Sinh
[2*e - (2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(8*a^3*d) - (3*Cosh[4*e
 - (4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) + (3*Sinh[4*e - (
4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(8*a^3*d) - (Cosh[6*e - (6*c*f)
/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d) + (Sinh[6*e - (6*c*f)/d]*Si
nhIntegral[(6*c*f)/d + 6*f*x])/(8*a^3*d)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.76 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.35

method	result
risch	$\frac{\ln(dx+c)}{8a^3d} - \frac{e^{\frac{6cf-6de}{d}}\,\mathrm{expIntegral}_1\left(6fx+6e+\frac{6cf-6de}{d}\right)}{8a^3d} - \frac{3e^{\frac{4cf-4de}{d}}\,\mathrm{expIntegral}_1\left(4fx+4e+\frac{4cf-4de}{d}\right)}{8a^3d} - \frac{3e^{\frac{2cf-2de}{d}}\,\mathrm{expIn}$

input
```
int(1/(d*x+c)/(a+tanh(f*x+e)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
1/8*ln(d*x+c)/a^3/d-1/8/a^3/d*exp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e
)/d)-3/8/a^3/d*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/d)-3/8/a^3/d*
exp(2*(c*f-d*e)/d)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 193, normalized size of antiderivative = 0.44

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{3\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)\cosh\left(-\frac{2\,(de-cf)}{d}\right) + 3\,\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)\cosh\left(-\frac{4\,(de-cf)}{d}\right) + \mathrm{Ei}\left(-\frac{6\,(dfx+cf)}{d}\right)\cosh\left(-\frac{6\,(de$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
1/8*(3*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + 3*Ei(-4*(d*f*x + c*
f)/d)*cosh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/
d) + 3*Ei(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 3*Ei(-4*(d*f*x + c*
f)/d)*sinh(-4*(d*e - c*f)/d) + Ei(-6*(d*f*x + c*f)/d)*sinh(-6*(d*e - c*f)/
d) + log(d*x + c))/(a^3*d)
```

Sympy [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{1}{c\tanh^3(e+fx)+3c\tanh^2(e+fx)+3c\tanh(e+fx)+c+dx\tanh^3(e+fx)+3dx\tanh^2(e+fx)+3dx\tanh(e+fx)+dx}\,dx}{a^3}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))**3,x)
```

output
```
Integral(1/(c*tanh(e + f*x)**3 + 3*c*tanh(e + f*x)**2 + 3*c*tanh(e + f*x)
+ c + d*x*tanh(e + f*x)**3 + 3*d*x*tanh(e + f*x)**2 + 3*d*x*tanh(e + f*x)
+ d*x), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 2.74 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.26

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = -\frac{e^{\left(-6e+\frac{6cf}{d}\right)}E_1\left(\frac{6(dx+c)f}{d}\right)}{8a^3d} - \frac{3e^{\left(-4e+\frac{4cf}{d}\right)}E_1\left(\frac{4(dx+c)f}{d}\right)}{8a^3d} - \frac{3e^{\left(-2e+\frac{2cf}{d}\right)}E_1\left(\frac{2(dx+c)f}{d}\right)}{8a^3d} + \frac{\log(dx+c)}{8a^3d}$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(1, 6*(d*x + c)*f/d)/(a^3*d) - 3/8*e
^(-4*e + 4*c*f/d)*exp_integral_e(1, 4*(d*x + c)*f/d)/(a^3*d) - 3/8*e^(-2*e
 + 2*c*f/d)*exp_integral_e(1, 2*(d*x + c)*f/d)/(a^3*d) + 1/8*log(d*x + c)/
(a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 0.24

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\left(3\,\mathrm{Ei}\left(-\frac{2\,(dfx+cf)}{d}\right)e^{\left(4\,e+\frac{2\,cf}{d}\right)} + 3\,\mathrm{Ei}\left(-\frac{4\,(dfx+cf)}{d}\right)e^{\left(2\,e+\frac{4\,cf}{d}\right)} + \mathrm{Ei}\left(-\frac{6\,(dfx+cf)}{d}\right)e^{\left(\frac{6\,cf}{d}\right)} + e^{(6\,e)}\log\left(dx+c\right)\right.$$
$$\left. 8\,a^3d\right.$$

input
```
integrate(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
1/8*(3*Ei(-2*(d*f*x + c*f)/d)*e^(4*e + 2*c*f/d) + 3*Ei(-4*(d*f*x + c*f)/d)
*e^(2*e + 4*c*f/d) + Ei(-6*(d*f*x + c*f)/d)*e^(6*c*f/d) + e^(6*e)*log(d*x
 + c))*e^(-6*e)/(a^3*d)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = \int \frac{1}{(a+a\tanh(e+fx))^3\,(c+dx)}\,dx$$

input
```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)),x)
```

output
```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)(a+a\tanh(e+fx))^3}\,dx = \frac{\int \frac{1}{\tanh(fx+e)^3 c+\tanh(fx+e)^3 dx+3\tanh(fx+e)^2 c+3\tanh(fx+e)^2 dx+3\tanh(fx+e)c+3\tanh(fx+e)dx+c+dx}\,dx}{a^3}$$

input
```
int(1/(d*x+c)/(a+a*tanh(f*x+e))^3,x)
```

output
```
int(1/(tanh(e + f*x)**3*c + tanh(e + f*x)**3*d*x + 3*tanh(e + f*x)**2*c + 
3*tanh(e + f*x)**2*d*x + 3*tanh(e + f*x)*c + 3*tanh(e + f*x)*d*x + c + d*x
),x)/a**3
```

3.47 $\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx$

Optimal result 395
Mathematica [A] (verified) 396
Rubi [A] (verified) 397
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Giac [A] (verification not implemented) 402
Mupad [F(-1)] 403
Reduce [F] 404

Optimal result

Integrand size = 20, antiderivative size = 692

$$\begin{aligned}\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = &-\frac{1}{8a^3d(c+dx)} - \frac{9\cosh(2e+2fx)}{32a^3d(c+dx)} \\ &- \frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)} \\ &- \frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)} - \frac{3\cosh(6e+6fx)}{32a^3d(c+dx)} \\ &- \frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} \\ &- \frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} \\ &- \frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\operatorname{Chi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2} \\ &+ \frac{3f\operatorname{Chi}\left(\frac{6cf}{d}+6fx\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2} \\ &+ \frac{3f\operatorname{Chi}\left(\frac{4cf}{d}+4fx\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2} \\ &+ \frac{3f\operatorname{Chi}\left(\frac{2cf}{d}+2fx\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2} \\ &+ \frac{15\sinh(2e+2fx)}{32a^3d(c+dx)} \\ &- \frac{3\sinh^2(2e+2fx)}{8a^3d(c+dx)} + \frac{\sinh^3(2e+2fx)}{8a^3d(c+dx)} \\ &+ \frac{3\sinh(4e+4fx)}{8a^3d(c+dx)} + \frac{3\sinh(6e+6fx)}{32a^3d(c+dx)} \\ &+ \frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} \\ &- \frac{3f\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(\frac{2cf}{d}+2fx\right)}{4a^3d^2} \\ &+ \frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} \\ &- \frac{3f\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(\frac{4cf}{d}+4fx\right)}{2a^3d^2} \\ &+ \frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\operatorname{Shi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2} \\ &- \frac{3f\sinh\left(6e-\frac{6cf}{d}\right)\operatorname{Shi}\left(\frac{6cf}{d}+6fx\right)}{4a^3d^2}\end{aligned}$$

output

```
-1/8/a^3/d/(d*x+c)-9/32*cosh(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*cosh(2*f*x+2*e)^
2/a^3/d/(d*x+c)-1/8*cosh(2*f*x+2*e)^3/a^3/d/(d*x+c)-3/32*cosh(6*f*x+6*e)/a
^3/d/(d*x+c)-3/4*f*cosh(-2*e+2*c*f/d)*Chi(2*c*f/d+2*f*x)/a^3/d^2-3/2*f*cos
h(-4*e+4*c*f/d)*Chi(4*c*f/d+4*f*x)/a^3/d^2-3/4*f*cosh(-6*e+6*c*f/d)*Chi(6*
c*f/d+6*f*x)/a^3/d^2-3/4*f*Chi(6*c*f/d+6*f*x)*sinh(-6*e+6*c*f/d)/a^3/d^2-3
/2*f*Chi(4*c*f/d+4*f*x)*sinh(-4*e+4*c*f/d)/a^3/d^2-3/4*f*Chi(2*c*f/d+2*f*x
)*sinh(-2*e+2*c*f/d)/a^3/d^2+15/32*sinh(2*f*x+2*e)/a^3/d/(d*x+c)-3/8*sinh(
2*f*x+2*e)^2/a^3/d/(d*x+c)+1/8*sinh(2*f*x+2*e)^3/a^3/d/(d*x+c)+3/8*sinh(4*
f*x+4*e)/a^3/d/(d*x+c)+3/32*sinh(6*f*x+6*e)/a^3/d/(d*x+c)+3/4*f*cosh(-2*e+
2*c*f/d)*Shi(2*c*f/d+2*f*x)/a^3/d^2+3/4*f*sinh(-2*e+2*c*f/d)*Shi(2*c*f/d+2
*f*x)/a^3/d^2+3/2*f*cosh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d^2+3/2*f*si
nh(-4*e+4*c*f/d)*Shi(4*c*f/d+4*f*x)/a^3/d^2+3/4*f*cosh(-6*e+6*c*f/d)*Shi(6
*c*f/d+6*f*x)/a^3/d^2+3/4*f*sinh(-6*e+6*c*f/d)*Shi(6*c*f/d+6*f*x)/a^3/d^2
```

Mathematica [A] (verified)

Time = 3.00 (sec) , antiderivative size = 794, normalized size of antiderivative = 1.15

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \text{Too large to display}$$

input

```
Integrate[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^3),x]
```

output

```
-1/8*(Sech[e + f*x]^3*(Cosh[(3*c*f)/d] + Sinh[(3*c*f)/d])*(3*d*Cosh[e + f*
((-3*c)/d + x)] + d*Cosh[3*(e + f*(-(c/d) + x))] + d*Cosh[3*(e + f*(c/d +
x))] + 3*d*Cosh[e + f*((3*c)/d + x)] + 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]
*CoshIntegral[(6*f*(c + d*x))/d] + 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*C
oshIntegral[(6*f*(c + d*x))/d] + 6*f*(c + d*x)*CoshIntegral[(2*f*(c + d*x)
)/d]*(Cosh[e - (c*f)/d + 3*f*x] + Sinh[e - (c*f)/d + 3*f*x]) + 3*d*Sinh[e
+ f*((-3*c)/d + x)] + d*Sinh[3*(e + f*(-(c/d) + x))] - d*Sinh[3*(e + f*(c/
d + x))] - 3*d*Sinh[e + f*((3*c)/d + x)] - 6*c*f*CoshIntegral[(6*f*(c + d*
x))/d]*Sinh[3*e - (3*f*(c + d*x))/d] - 6*d*f*x*CoshIntegral[(6*f*(c + d*x)
)/d]*Sinh[3*e - (3*f*(c + d*x))/d] + 12*f*(c + d*x)*CoshIntegral[(4*f*(c +
 d*x))/d]*(Cosh[e - (f*(c + 3*d*x))/d] - Sinh[e - (f*(c + 3*d*x))/d]) - 6*
c*f*Cosh[e - (c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Co
sh[e - (c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*c*f*Sinh[e - (
c*f)/d + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 6*d*f*x*Sinh[e - (c*f)/d
 + 3*f*x]*SinhIntegral[(2*f*(c + d*x))/d] - 12*c*f*Cosh[e - (f*(c + 3*d*x)
)/d]*SinhIntegral[(4*f*(c + d*x))/d] - 12*d*f*x*Cosh[e - (f*(c + 3*d*x))/d
]*SinhIntegral[(4*f*(c + d*x))/d] + 12*c*f*Sinh[e - (f*(c + 3*d*x))/d]*Sin
hIntegral[(4*f*(c + d*x))/d] + 12*d*f*x*Sinh[e - (f*(c + 3*d*x))/d]*SinhIn
tegral[(4*f*(c + d*x))/d] - 6*c*f*Cosh[3*e - (3*f*(c + d*x))/d]*SinhIntegr
al[(6*f*(c + d*x))/d] - 6*d*f*x*Cosh[3*e - (3*f*(c + d*x))/d]*SinhInteg...
```

Rubi [A] (verified)

Time = 2.23 (sec) , antiderivative size = 692, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4211, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a\tanh(e+fx)+a)^3}\,dx$$

$$\downarrow 3042$$

$$\int \frac{1}{(c+dx)^2(a-ia\tan(ie+ifx))^3}dx$$

$$\downarrow 4211$$

$$\int \left(-\frac{\sinh^3(2e+2fx)}{8a^3(c+dx)^2}+\frac{3\sinh^2(2e+2fx)}{8a^3(c+dx)^2}-\frac{3\sinh(2e+2fx)}{8a^3(c+dx)^2}+\frac{3\sinh(2e+2fx)\sinh(4e+4fx)}{16a^3(c+dx)^2}-\frac{3\sinh(4e\right.$$

$$\downarrow \text{2009}$$

$$\frac{3f\operatorname{Chi}\left(6xf+\frac{6cf}{d}\right)\sinh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\operatorname{Chi}\left(4xf+\frac{4cf}{d}\right)\sinh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}+$$
$$\frac{3f\operatorname{Chi}\left(2xf+\frac{2cf}{d}\right)\sinh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\operatorname{Chi}\left(2xf+\frac{2cf}{d}\right)\cosh\left(2e-\frac{2cf}{d}\right)}{4a^3d^2}-$$
$$\frac{3f\operatorname{Chi}\left(4xf+\frac{4cf}{d}\right)\cosh\left(4e-\frac{4cf}{d}\right)}{2a^3d^2}-\frac{3f\operatorname{Chi}\left(6xf+\frac{6cf}{d}\right)\cosh\left(6e-\frac{6cf}{d}\right)}{4a^3d^2}-$$
$$\frac{3f\sinh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}-\frac{3f\sinh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}-$$
$$\frac{3f\sinh\left(6e-\frac{6cf}{d}\right)\operatorname{Shi}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}+\frac{3f\cosh\left(2e-\frac{2cf}{d}\right)\operatorname{Shi}\left(2xf+\frac{2cf}{d}\right)}{4a^3d^2}+$$
$$\frac{3f\cosh\left(4e-\frac{4cf}{d}\right)\operatorname{Shi}\left(4xf+\frac{4cf}{d}\right)}{2a^3d^2}+\frac{3f\cosh\left(6e-\frac{6cf}{d}\right)\operatorname{Shi}\left(6xf+\frac{6cf}{d}\right)}{4a^3d^2}+\frac{\sinh^3(2e+2fx)}{8a^3d(c+dx)}-$$
$$\frac{3\sinh^2(2e+2fx)}{8a^3d(c+dx)}+\frac{15\sinh(2e+2fx)}{32a^3d(c+dx)}+\frac{3\sinh(4e+4fx)}{8a^3d(c+dx)}+\frac{3\sinh(6e+6fx)}{32a^3d(c+dx)}-$$
$$\frac{\cosh^3(2e+2fx)}{8a^3d(c+dx)}-\frac{3\cosh^2(2e+2fx)}{8a^3d(c+dx)}-\frac{9\cosh(2e+2fx)}{32a^3d(c+dx)}-\frac{3\cosh(6e+6fx)}{32a^3d(c+dx)}-\frac{1}{8a^3d(c+dx)}$$

input
```
Int[1/((c + d*x)^2*(a + a*Tanh[e + f*x])^3),x]
```

output
```
-1/8*1/(a^3*d*(c + d*x)) - (9*Cosh[2*e + 2*f*x])/(32*a^3*d*(c + d*x)) - (3
*Cosh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) - Cosh[2*e + 2*f*x]^3/(8*a^3*d*(
c + d*x)) - (3*Cosh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) - (3*f*Cosh[2*e - (
2*c*f)/d]*CoshIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) - (3*f*Cosh[4*e - (
4*c*f)/d]*CoshIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Cosh[6*e - (
6*c*f)/d]*CoshIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) + (3*f*CoshIntegral
[(6*c*f)/d + 6*f*x]*Sinh[6*e - (6*c*f)/d])/(4*a^3*d^2) + (3*f*CoshIntegral
[(4*c*f)/d + 4*f*x]*Sinh[4*e - (4*c*f)/d])/(2*a^3*d^2) + (3*f*CoshIntegral
[(2*c*f)/d + 2*f*x]*Sinh[2*e - (2*c*f)/d])/(4*a^3*d^2) + (15*Sinh[2*e + 2*
f*x])/(32*a^3*d*(c + d*x)) - (3*Sinh[2*e + 2*f*x]^2)/(8*a^3*d*(c + d*x)) +
 Sinh[2*e + 2*f*x]^3/(8*a^3*d*(c + d*x)) + (3*Sinh[4*e + 4*f*x])/(8*a^3*d*
(c + d*x)) + (3*Sinh[6*e + 6*f*x])/(32*a^3*d*(c + d*x)) + (3*f*Cosh[2*e -
(2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) - (3*f*Sinh[2*e -
(2*c*f)/d]*SinhIntegral[(2*c*f)/d + 2*f*x])/(4*a^3*d^2) + (3*f*Cosh[4*e -
(4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) - (3*f*Sinh[4*e -
(4*c*f)/d]*SinhIntegral[(4*c*f)/d + 4*f*x])/(2*a^3*d^2) + (3*f*Cosh[6*e -
(6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2) - (3*f*Sinh[6*e -
(6*c*f)/d]*SinhIntegral[(6*c*f)/d + 6*f*x])/(4*a^3*d^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4211
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + Cos[2*e + 2*f*x]/(
2*a) + Sin[2*e + 2*f*x]/(2*b))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
 && EqQ[a^2 + b^2, 0] && ILtQ[m, 0] && ILtQ[n, 0]
```

Maple [A] (verified)

Time = 0.92 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.35

method	result
risch	$-\frac{1}{8a^3d(dx+c)}-\frac{f\,\mathrm{e}^{-6fx-6e}}{8a^3d(dxf+cf)}+\frac{3f\,\mathrm{e}^{\frac{6cf-6de}{d}}\,\mathrm{expIntegral}_1\left(6fx+6e+\frac{6cf-6de}{d}\right)}{4a^3d^2}-\frac{3f\,\mathrm{e}^{-4fx-4e}}{8a^3d(dxf+cf)}+\frac{3f\,\mathrm{e}^{\frac{4cf-4de}{d}}\,\mathrm{expInte}$

input
```
int(1/(d*x+c)^2/(a+tanh(f*x+e)*a)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/8/a^3/d/(d*x+c)-1/8*f/a^3*exp(-6*f*x-6*e)/d/(d*f*x+c*f)+3/4*f/a^3/d^2*e
xp(6*(c*f-d*e)/d)*Ei(1,6*f*x+6*e+6*(c*f-d*e)/d)-3/8*f/a^3*exp(-4*f*x-4*e)/
d/(d*f*x+c*f)+3/2*f/a^3/d^2*exp(4*(c*f-d*e)/d)*Ei(1,4*f*x+4*e+4*(c*f-d*e)/
d)-3/8*f/a^3*exp(-2*f*x-2*e)/d/(d*f*x+c*f)+3/4*f/a^3/d^2*exp(2*(c*f-d*e)/d
)*Ei(1,2*f*x+2*e+2*(c*f-d*e)/d)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1164, normalized size of antiderivative = 1.68

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output

```
-1/4*(3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-2*(d*e
- c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^3*sinh(-4
*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(f*x + e)^3*s
inh(-6*(d*e - c*f)/d) + (3*(d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d
*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/
d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + d)*co
sh(f*x + e)^3 + 3*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f
)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + (d*
f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*cosh(-6*(d*e - c*f)/d) + (d*f*x + c*f)*E
i(-2*(d*f*x + c*f)/d)*sinh(-2*(d*e - c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*
x + c*f)/d)*sinh(-4*(d*e - c*f)/d) + (d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/d)*
sinh(-6*(d*e - c*f)/d))*sinh(f*x + e)^3 + 3*(3*(d*f*x + c*f)*Ei(-2*(d*f*x
+ c*f)/d)*cosh(f*x + e)*sinh(-2*(d*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*
f*x + c*f)/d)*cosh(f*x + e)*sinh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6
*(d*f*x + c*f)/d)*cosh(f*x + e)*sinh(-6*(d*e - c*f)/d) + (3*(d*f*x + c*f)*
Ei(-2*(d*f*x + c*f)/d)*cosh(-2*(d*e - c*f)/d) + 6*(d*f*x + c*f)*Ei(-4*(d*f
*x + c*f)/d)*cosh(-4*(d*e - c*f)/d) + 3*(d*f*x + c*f)*Ei(-6*(d*f*x + c*f)/
d)*cosh(-6*(d*e - c*f)/d) + d)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*d*cosh(f
*x + e) + 9*((d*f*x + c*f)*Ei(-2*(d*f*x + c*f)/d)*cosh(f*x + e)^2*sinh(-2*
(d*e - c*f)/d) + 2*(d*f*x + c*f)*Ei(-4*(d*f*x + c*f)/d)*cosh(f*x + e)^2...
```

Sympy [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{\int \frac{1}{c^2\tanh^3(e+fx)+3c^2\tanh^2(e+fx)+3c^2\tanh(e+fx)+c^2+2cdx\tanh^3(e+fx)+6cdx\tanh^2(e+fx)+6cdx\tanh(e+fx)+2cdx+d^2x^2\tanh^3(}}{a^3}$$

input

```
integrate(1/(d*x+c)**2/(a+a*tanh(f*x+e))**3,x)
```

output

```
Integral(1/(c**2*tanh(e + f*x)**3 + 3*c**2*tanh(e + f*x)**2 + 3*c**2*tanh(
e + f*x) + c**2 + 2*c*d*x*tanh(e + f*x)**3 + 6*c*d*x*tanh(e + f*x)**2 + 6*
c*d*x*tanh(e + f*x) + 2*c*d*x + d**2*x**2*tanh(e + f*x)**3 + 3*d**2*x**2*t
anh(e + f*x)**2 + 3*d**2*x**2*tanh(e + f*x) + d**2*x**2), x)/a**3
```

Maxima [A] (verification not implemented)

Time = 6.43 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.20

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = -\frac{1}{8\,(a^3d^2x+a^3cd)} - \frac{e^{\left(-6\,e+\frac{6\,cf}{d}\right)}E_2\left(\frac{6\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d} - \frac{3\,e^{\left(-4\,e+\frac{4\,cf}{d}\right)}E_2\left(\frac{4\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d} - \frac{3\,e^{\left(-2\,e+\frac{2\,cf}{d}\right)}E_2\left(\frac{2\,(dx+c)f}{d}\right)}{8\,(dx+c)a^3d}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
-1/8/(a^3*d^2*x + a^3*c*d) - 1/8*e^(-6*e + 6*c*f/d)*exp_integral_e(2, 6*(d
*x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-4*e + 4*c*f/d)*exp_integral_e(2,
4*(d*x + c)*f/d)/((d*x + c)*a^3*d) - 3/8*e^(-2*e + 2*c*f/d)*exp_integral_e
(2, 2*(d*x + c)*f/d)/((d*x + c)*a^3*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 840, normalized size of antiderivative = 1.21

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output

```
-1/8*(6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-2*((d*x + c)
*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d)
- 6*d*e*f^2*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c
*f)/d)*e^(-2*(d*e - c*f)/d) + 6*c*f^3*Ei(-2*((d*x + c)*(d*e/(d*x + c) - c*
f/(d*x + c) + f) - d*e + c*f)/d)*e^(-2*(d*e - c*f)/d) + 12*(d*x + c)*(d*e/
(d*x + c) - c*f/(d*x + c) + f)*f^2*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(
d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*f)/d) - 12*d*e*f^2*Ei(-4*((d*
x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-4*(d*e - c*
f)/d) + 12*c*f^3*Ei(-4*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*
e + c*f)/d)*e^(-4*(d*e - c*f)/d) + 6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x +
 c) + f)*f^2*Ei(-6*((d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e +
c*f)/d)*e^(-6*(d*e - c*f)/d) - 6*d*e*f^2*Ei(-6*((d*x + c)*(d*e/(d*x + c) -
 c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-6*(d*e - c*f)/d) + 6*c*f^3*Ei(-6*(
(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f) - d*e + c*f)/d)*e^(-6*(d*e -
 c*f)/d) + 3*d*f^2*e^(-2*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d)
+ 3*d*f^2*e^(-4*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + d*f^2*e
^(-6*(d*x + c)*(d*e/(d*x + c) - c*f/(d*x + c) + f)/d) + d*f^2)*d^2/(((d*x
+ c)*a^3*d^4*(d*e/(d*x + c) - c*f/(d*x + c) + f) - a^3*d^5*e + a^3*c*d^4*f
)*f)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx = \int \frac{1}{(a+a\tanh(e+fx))^3\,(c+dx)^2}\,dx$$

input

```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)^2),x)
```

output

```
int(1/((a + a*tanh(e + f*x))^3*(c + d*x)^2), x)
```

Reduce [F]

$$\int \frac{1}{(c+dx)^2(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{3e^{4e}\left(\int \frac{1}{e^{2fx+6e}c^2+2e^{2fx+6e}cdx+e^{2fx+6e}d^2x^2}dx\right)c^2 + 3e^{4e}\left(\int \frac{1}{e^{2fx+6e}c^2+2e^{2fx+6e}cdx+e^{2fx+6e}d^2x^2}dx\right)cdx + 3e^{2e}\left(\int \frac{}{}\right.$$

input
```
int(1/(d*x+c)^2/(a+a*tanh(f*x+e))^3,x)
```

output
```
(3*e**(4*e)*int(1/(e**(6*e + 2*f*x)*c**2 + 2*e**(6*e + 2*f*x)*c*d*x + e**(
6*e + 2*f*x)*d**2*x**2),x)*c**2 + 3*e**(4*e)*int(1/(e**(6*e + 2*f*x)*c**2
+ 2*e**(6*e + 2*f*x)*c*d*x + e**(6*e + 2*f*x)*d**2*x**2),x)*c*d*x + 3*e**(
2*e)*int(1/(e**(6*e + 4*f*x)*c**2 + 2*e**(6*e + 4*f*x)*c*d*x + e**(6*e + 4
*f*x)*d**2*x**2),x)*c**2 + 3*e**(2*e)*int(1/(e**(6*e + 4*f*x)*c**2 + 2*e**
(6*e + 4*f*x)*c*d*x + e**(6*e + 4*f*x)*d**2*x**2),x)*c*d*x + int(1/(e**(6*
e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2),x
)*c**2 + int(1/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e
 + 6*f*x)*d**2*x**2),x)*c*d*x + x)/(8*a**3*c*(c + d*x))
```

3.48 $\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \text{Int}\big((c+dx)^m (a+a\tanh(e+fx))^2, x\big)$$

output
```
Defer(Int)((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 26.33 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \int (c+dx)^m (a+a\tanh(e+fx))^2\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x])^2,x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x])^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a\tanh(e+fx)+a)^2\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m (a-ia\tan(ie+ifx))^2 dx$$

$$\downarrow 4223$$

$$\int (c+dx)^m (a\tanh(e+fx)+a)^2 dx$$

input
```
Int[(c + d*x)^m*(a + a*Tanh[e + f*x])^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (a+\tanh(fx+e)a)^2\,dx$$

input
```
int((d*x+c)^m*(a+tanh(f*x+e)*a)^2,x)
```

output
```
int((d*x+c)^m*(a+tanh(f*x+e)*a)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.85

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \int (a\tanh(fx+e)+a)^2 (dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral((a^2*tanh(f*x + e)^2 + 2*a^2*tanh(f*x + e) + a^2)*(d*x + c)^m, x)
```

Sympy [N/A]

Not integrable

Time = 3.41 (sec) , antiderivative size = 44, normalized size of antiderivative = 2.20

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = a^2\left(\int 2(c+dx)^m \tanh(e+fx)\,dx + \int (c+dx)^m \tanh^2(e+fx)\,dx + \int (c+dx)^m\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*tanh(f*x+e))**2,x)
```

output
```
a**2*(Integral(2*(c + d*x)**m*tanh(e + f*x), x) + Integral((c + d*x)**m*ta
nh(e + f*x)**2, x) + Integral((c + d*x)**m, x))
```

Maxima [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 122, normalized size of antiderivative = 6.10

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \int (a\tanh(fx+e)+a)^2 (dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
(d*x + c)^(m + 1)*a^2/(d*(m + 1)) + integrate(2*(d*x + c)^m*a^2*(e^(f*x +
e) - e^(-f*x - e))/(e^(f*x + e) + e^(-f*x - e)) + (d*x + c)^m*a^2*(e^(f*x
+ e) - e^(-f*x - e))^2/(e^(f*x + e) + e^(-f*x - e))^2, x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \int (a\tanh(fx+e)+a)^2 (dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((a*tanh(f*x + e) + a)^2*(d*x + c)^m, x)
```

Mupad [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \int (a+a\tanh(e+f\,x))^2\,(c+d\,x)^m\,dx$$

input
```
int((a + a*tanh(e + f*x))^2*(c + d*x)^m,x)
```

output
```
int((a + a*tanh(e + f*x))^2*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 5344, normalized size of antiderivative = 267.20

$$\int (c+dx)^m (a+a\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m*(a+a*tanh(f*x+e))^2,x)
```

output
```
(4*a**2*(2*e**(2*e + 2*f*x)*(c + d*x)**m*c**2*f + 2*e**(2*e + 2*f*x)*(c +
d*x)**m*c*d*f*x - e**(2*e + 2*f*x)*(c + d*x)**m*c*d*m - e**(2*e + 2*f*x)*(
c + d*x)**m*d**2*m*x + 2*(c + d*x)**m*c**2*f + 2*(c + d*x)**m*c*d*f*x + (c
 + d*x)**m*c*d*m + 2*(c + d*x)**m*c*d - (c + d*x)**m*d**2*m*x + 4*e**(4*e
+ 2*f*x)*int((e**(2*f*x)*(c + d*x)**m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2*e*
*(4*e + 4*f*x)*c*d*f*x - e**(4*e + 4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2*m*
x + 4*e**(2*e + 2*f*x)*c**2*f + 4*e**(2*e + 2*f*x)*c*d*f*x - 2*e**(2*e + 2
*f*x)*c*d*m - 2*e**(2*e + 2*f*x)*d**2*m*x + 2*c**2*f + 2*c*d*f*x - c*d*m -
 d**2*m*x),x)*c*d**3*f*m**2 + 4*e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)
**m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2*e**(4*e + 4*f*x)*c*d*f*x - e**(4*e +
 4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2*m*x + 4*e**(2*e + 2*f*x)*c**2*f + 4*
e**(2*e + 2*f*x)*c*d*f*x - 2*e**(2*e + 2*f*x)*c*d*m - 2*e**(2*e + 2*f*x)*d
**2*m*x + 2*c**2*f + 2*c*d*f*x - c*d*m - d**2*m*x),x)*c*d**3*f*m - 2*e**(4
*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)**m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2
*e**(4*e + 4*f*x)*c*d*f*x - e**(4*e + 4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2
*m*x + 4*e**(2*e + 2*f*x)*c**2*f + 4*e**(2*e + 2*f*x)*c*d*f*x - 2*e**(2*e
+ 2*f*x)*c*d*m - 2*e**(2*e + 2*f*x)*d**2*m*x + 2*c**2*f + 2*c*d*f*x - c*d*
m - d**2*m*x),x)*d**4*m**3 - 2*e**(4*e + 2*f*x)*int((e**(2*f*x)*(c + d*x)*
*m*x)/(2*e**(4*e + 4*f*x)*c**2*f + 2*e**(4*e + 4*f*x)*c*d*f*x - e**(4*e +
4*f*x)*c*d*m - e**(4*e + 4*f*x)*d**2*m*x + 4*e**(2*e + 2*f*x)*c**2*f + ...
```

3.49 $\int (c+dx)^m(a+a\tanh(e+fx))\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 18

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \text{Int}((c+dx)^m(a+a\tanh(e+fx)),x)$$

output
```
Defer(Int)((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 16.05 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (c+dx)^m(a+a\tanh(e+fx))\,dx$$

input
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x]),x]
```

output
```
Integrate[(c + d*x)^m*(a + a*Tanh[e + f*x]), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^m (a\tanh(e+fx)+a)\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^m (a-ia\tan(ie+ifx))dx$$

$$\downarrow 4223$$

$$\int (c+dx)^m (a\tanh(e+fx)+a)dx$$

input
```
Int[(c + d*x)^m*(a + a*Tanh[e + f*x]),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.19 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int (dx+c)^m (a+\tanh(fx+e)\,a)\,dx$$

input
```
int((d*x+c)^m*(a+tanh(f*x+e)*a),x)
```

output
```
int((d*x+c)^m*(a+tanh(f*x+e)*a),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (a\tanh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
integral((a*tanh(f*x + e) + a)*(d*x + c)^m, x)
```

Sympy [N/A]

Not integrable

Time = 1.92 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.33

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = a\left(\int (c+dx)^m\tanh(e+fx)\,dx + \int (c+dx)^m\,dx\right)$$

input
```
integrate((d*x+c)**m*(a+a*tanh(f*x+e)),x)
```

output
```
a*(Integral((c + d*x)**m*tanh(e + f*x), x) + Integral((c + d*x)**m, x))
```

Maxima [N/A]

Not integrable

Time = 0.15 (sec) , antiderivative size = 68, normalized size of antiderivative = 3.78

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (a\tanh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
a*integrate((d*x + c)^m*(e^(f*x + e) - e^(-f*x - e))/(e^(f*x + e) + e^(-f*
x - e)), x) + (d*x + c)^(m + 1)*a/(d*(m + 1))
```

Giac [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (a\tanh(fx+e)+a)(dx+c)^m\,dx$$

input
```
integrate((d*x+c)^m*(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((a*tanh(f*x + e) + a)*(d*x + c)^m, x)
```

Mupad [N/A]

Not integrable

Time = 2.35 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int (c+dx)^m(a+a\tanh(e+fx))\,dx = \int (a+a\tanh(e+f\,x))\,(c+d\,x)^m\,dx$$

input
```
int((a + a*tanh(e + f*x))*(c + d*x)^m,x)
```

output
```
int((a + a*tanh(e + f*x))*(c + d*x)^m, x)
```

Reduce [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 86, normalized size of antiderivative = 4.78

$$\int (c+dx)^m (a+a\tanh(e+fx))\,dx = \frac{2a\left((dx+c)^m c+(dx+c)^m dx-\left(\int \frac{(dx+c)^m}{e^{2fx+2e}+1}dx\right) dm-\left(\int \frac{(dx+c)^m}{e^{2fx+2e}+1}dx\right) d\right)}{d\,(m+1)}$$

input
```
int((d*x+c)^m*(a+a*tanh(f*x+e)),x)
```

output
```
(2*a*((c + d*x)**m*c + (c + d*x)**m*d*x - int((c + d*x)**m/(e**(2*e + 2*f*
x) + 1),x)*d*m - int((c + d*x)**m/(e**(2*e + 2*f*x) + 1),x)*d))/(d*(m + 1)
)
```

3.50 $\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 89

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \frac{(c+dx)^{1+m}}{2ad(1+m)} - \frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{af}$$

output

```
1/2*(d*x+c)^(1+m)/a/d/(1+m)-2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1+m
,2*f*(d*x+c)/d)/a/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \frac{\left(\frac{e^e f(c+dx)^{1+m}}{d(1+m)} - 2^{-1-m}e^{-e+\frac{2cf}{d}}(c+dx)^m\left(\frac{cf}{d}+fx\right)^{-m}\Gamma\left(1+m,2\left(\frac{cf}{d}+fx\right)\right)\right)\operatorname{sech}(e+fx)(\cosh(fx)+s}{2f(a+a\tanh(e+fx))}$$

input

```
Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x]),x]
```

output
```
(((E^e*f*(c + d*x)^(1 + m))/(d*(1 + m)) - (2^(-1 - m)*E^(-e + (2*c*f)/d)*(
c + d*x)^m*Gamma[1 + m, 2*((c*f)/d + f*x)])/((c*f)/d + f*x)^m)*Sech[e + f*
x]*(Cosh[f*x] + Sinh[f*x]))/(2*f*(a + a*Tanh[e + f*x]))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4210, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{a\tanh(e+fx)+a}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c+dx)^m}{a-ia\tan(ie+ifx)}dx$$

$$\downarrow 4210$$

$$\frac{\int e^{-2(e+fx)}(c+dx)^m dx}{2a} + \frac{(c+dx)^{m+1}}{2ad(m+1)}$$

$$\downarrow 2612$$

$$\frac{(c+dx)^{m+1}}{2ad(m+1)} - \frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{af}$$

input
```
Int[(c + d*x)^m/(a + a*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^(1 + m)/(2*a*d*(1 + m)) - (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c +
d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2612

```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4210

```
Int[((c_.) + (d_.)*(x_))^(m_)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sym
bol] :> Simp[(c + d*x)^(m + 1)/(2*a*d*(m + 1)), x] + Simp[1/(2*a)   Int[(c
+ d*x)^m*E^(2*(a/b)*(e + f*x)), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] &
& EqQ[a^2 + b^2, 0] &&  !IntegerQ[m]
```

Maple [F]

$$\int \frac{(dx+c)^m}{a+\tanh(fx+e)\,a}dx$$

input

```
int((d*x+c)^m/(a+tanh(f*x+e)*a),x)
```

output

```
int((d*x+c)^m/(a+tanh(f*x+e)*a),x)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.66

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx =$$

$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)-(dm+d)\Gamma\left(m+1,\frac{2(dfx+cf)}{d}\right)\sinh\left(\frac{dm\log}{4(adfm+adf}\right.\right.$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
-1/4*((d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gamma(m + 1, 2*(d
*f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2
*f/d) + 2*d*e - 2*c*f)/d) - 2*(d*f*x + c*f)*cosh(m*log(d*x + c)) - 2*(d*f*
x + c*f)*sinh(m*log(d*x + c)))/(a*d*f*m + a*d*f)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \frac{\int \frac{(c+dx)^m}{\tanh(e+fx)+1}\,dx}{a}$$

input
```
integrate((d*x+c)**m/(a+a*tanh(f*x+e)),x)
```

output
```
Integral((c + d*x)**m/(tanh(e + f*x) + 1), x)/a
```

Maxima [F]

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a), x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \int \frac{(dx+c)^m}{a\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx = \int \frac{(c+d\,x)^m}{a+a\tanh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^m/(a + a*tanh(e + f*x)),x)
```

output
```
int((c + d*x)^m/(a + a*tanh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{a+a\tanh(e+fx)}\,dx$$
$$= \frac{2e^{2fx+2e}(dx+c)^m\,cf + 2e^{2fx+2e}(dx+c)^m\,dfx - (dx+c)^m\,dm - (dx+c)^m\,d + e^{2fx+2e}\left(\int \frac{(dx+c)^m}{e^{2fx+2e}c+e^{2fx+2}}\right.}{4e^{2fx+2e}adf\,(m+1)}$$

input
```
int((d*x+c)^m/(a+a*tanh(f*x+e)),x)
```

output

```
(2*e**(2*e + 2*f*x)*(c + d*x)**m*c*f + 2*e**(2*e + 2*f*x)*(c + d*x)**m*d*f
*x - (c + d*x)**m*d*m - (c + d*x)**m*d + e**(2*e + 2*f*x)*int((c + d*x)**m
/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m**2 + e**(2*e + 2*f*
x)*int((c + d*x)**m/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x),x)*d**2*m)
/(4*e**(2*e + 2*f*x)*a*d*f*(m + 1))
```

3.51 $\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx$

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Rubi [A] (verified) . . . 422
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Optimal result

Integrand size = 20, antiderivative size = 153

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \frac{(c+dx)^{1+m}}{4a^2d(1+m)} - \frac{2^{-2-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^2f} - \frac{4^{-2-m}e^{-4e+\frac{4cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4f(c+dx)}{d}\right)}{a^2f}$$

output

```
1/4*(d*x+c)^(1+m)/a^2/d/(1+m)-2^(-2-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA(1
+m,2*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)-4^(-2-m)*exp(-4*e+4*c*f/d)*(d*x+
c)^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^2/f/((f*(d*x+c)/d)^m)
```

Mathematica [A] (verified)

Time = 1.10 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.07

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx$$
$$= \frac{(c+dx)^m\left(\frac{4e^{2e}f(c+dx)}{d(1+m)} - 2^{2-m}e^{\frac{2cf}{d}}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right) - 4^{-m}e^{-2e+\frac{4cf}{d}}\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m\right.\right.}{16a^2f(1+\tanh(e+fx))^2}$$

input
```
Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x])^2,x]
```

output
```
((c + d*x)^m*((4*E^(2*e)*f*(c + d*x))/(d*(1 + m)) - (2^(2 - m)*E^((2*c*f)/
d)*Gamma[1 + m, (2*f*(c + d*x))/d])/(f*(c/d + x))^m - (E^(-2*e + (4*c*f)/d
)*Gamma[1 + m, (4*f*(c + d*x))/d])/(4^m*((f*(c + d*x))/d)^m))*Sech[e + f*x
]^2*(Cosh[f*x] + Sinh[f*x])^2)/(16*a^2*f*(1 + Tanh[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\tanh(e+fx)+a)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c+dx)^m}{(a-ia\tan(ie+ifx))^2}dx$$
$$\downarrow 4212$$
$$\int \left(\frac{e^{-4e-4fx}(c+dx)^m}{4a^2} + \frac{e^{-2e-2fx}(c+dx)^m}{2a^2} + \frac{(c+dx)^m}{4a^2}\right)dx$$
$$\downarrow 2009$$

$$-\frac{2^{-m-2}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^2f}-\frac{4^{-m-2}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^2f}+\frac{(c+dx)^{m+1}}{4a^2d(m+1)}$$

input
```
Int[(c + d*x)^m/(a + a*Tanh[e + f*x])^2,x]
```

output
```
(c + d*x)^(1 + m)/(4*a^2*d*(1 + m)) - (2^(-2 - m)*E^(-2*e + (2*c*f)/d)*(c
+ d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^2*f*((f*(c + d*x))/d)^m) - (4
^(-2 - m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*x))/d]
)/(a^2*f*((f*(c + d*x))/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{(a+\tanh(fx+e)a)^2}dx$$

input
```
int((d*x+c)^m/(a+tanh(f*x+e)*a)^2,x)
```

output
```
int((d*x+c)^m/(a+tanh(f*x+e)*a)^2,x)
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.62

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx =$$
$$-\frac{(dm+d)\cosh\left(\frac{dm\log\left(\frac{4f}{d}\right)+4de-4cf}{d}\right)\Gamma\left(m+1,\frac{4(dfx+cf)}{d}\right)+4(dm+d)\cosh\left(\frac{dm\log\left(\frac{2f}{d}\right)+2de-2cf}{d}\right)\Gamma\left(1\right.$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
-1/16*((d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*gamma(m + 1, 4*(
d*f*x + c*f)/d) + 4*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d)*gam
ma(m + 1, 2*(d*f*x + c*f)/d) - (d*m + d)*gamma(m + 1, 4*(d*f*x + c*f)/d)*s
inh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 4*(d*m + d)*gamma(m + 1, 2*(d*f*
x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e - 2*c*f)/d) - 4*(d*f*x + c*f)*cos
h(m*log(d*x + c)) - 4*(d*f*x + c*f)*sinh(m*log(d*x + c)))/(a^2*d*f*m + a^2
*d*f)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \frac{\int \frac{(c+dx)^m}{\tanh^2(e+fx)+2\tanh(e+fx)+1}\,dx}{a^2}$$

input
```
integrate((d*x+c)**m/(a+a*tanh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**m/(tanh(e + f*x)**2 + 2*tanh(e + f*x) + 1), x)/a**2
```

Maxima [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^2, x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^m}{(a+a\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^m/(a+a*tanh(f*x+e))^2,x)
```

output
```
(4*e**(4*e + 4*f*x)*(c + d*x)**m*tanh(e + f*x)*c*f + 4*e**(4*e + 4*f*x)*(c
 + d*x)**m*tanh(e + f*x)*d*f*x + 4*e**(4*e + 4*f*x)*(c + d*x)**m*tanh(e +
f*x)*d*m + 4*e**(4*e + 4*f*x)*(c + d*x)**m*tanh(e + f*x)*d + 4*e**(4*e + 4
*f*x)*(c + d*x)**m*c*f + 4*e**(4*e + 4*f*x)*(c + d*x)**m*d*f*x - 4*e**(4*e
 + 4*f*x)*(c + d*x)**m*d*m - 4*e**(4*e + 4*f*x)*(c + d*x)**m*d - (c + d*x)
**m*tanh(e + f*x)*d*m - (c + d*x)**m*tanh(e + f*x)*d - (c + d*x)**m*d*m -
(c + d*x)**m*d + 16*e**(6*e + 4*f*x)*int((c + d*x)**m/(4*e**(4*e + 2*f*x)*
c**2*f + 4*e**(4*e + 2*f*x)*c*d*f*x - e**(4*e + 2*f*x)*c*d*m - e**(4*e + 2
*f*x)*d**2*m*x),x)*tanh(e + f*x)*c*d**2*f*m**2 + 16*e**(6*e + 4*f*x)*int((
c + d*x)**m/(4*e**(4*e + 2*f*x)*c**2*f + 4*e**(4*e + 2*f*x)*c*d*f*x - e**(
4*e + 2*f*x)*c*d*m - e**(4*e + 2*f*x)*d**2*m*x),x)*tanh(e + f*x)*c*d**2*f*
m - 4*e**(6*e + 4*f*x)*int((c + d*x)**m/(4*e**(4*e + 2*f*x)*c**2*f + 4*e**
(4*e + 2*f*x)*c*d*f*x - e**(4*e + 2*f*x)*c*d*m - e**(4*e + 2*f*x)*d**2*m*x
),x)*tanh(e + f*x)*d**3*m**3 - 4*e**(6*e + 4*f*x)*int((c + d*x)**m/(4*e**(
4*e + 2*f*x)*c**2*f + 4*e**(4*e + 2*f*x)*c*d*f*x - e**(4*e + 2*f*x)*c*d*m
- e**(4*e + 2*f*x)*d**2*m*x),x)*tanh(e + f*x)*d**3*m**2 + 16*e**(6*e + 4*f
*x)*int((c + d*x)**m/(4*e**(4*e + 2*f*x)*c**2*f + 4*e**(4*e + 2*f*x)*c*d*f
*x - e**(4*e + 2*f*x)*c*d*m - e**(4*e + 2*f*x)*d**2*m*x),x)*c*d**2*f*m**2
+ 16*e**(6*e + 4*f*x)*int((c + d*x)**m/(4*e**(4*e + 2*f*x)*c**2*f + 4*e**(
4*e + 2*f*x)*c*d*f*x - e**(4*e + 2*f*x)*c*d*m - e**(4*e + 2*f*x)*d**2*m...
```

3.52 $\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx$

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Mathematica [A] (verified) . . . 428
Rubi [A] (verified) . . . 428
Maple [F] . . . 430
Fricas [A] (verification not implemented) . . . 430
Sympy [F] . . . 431
Maxima [F] . . . 431
Giac [F] . . . 431
Mupad [F(-1)] . . . 432
Reduce [F] . . . 432

Optimal result

Integrand size = 20, antiderivative size = 224

$$\begin{aligned}
&\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx \\
&= \frac{(c+dx)^{1+m}}{8a^3d(1+m)} - \frac{3\,2^{-4-m}e^{-2e+\frac{2cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{2f(c+dx)}{d}\right)}{a^3f} \\
&\quad - \frac{3\,2^{-5-2m}e^{-4e+\frac{4cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{4f(c+dx)}{d}\right)}{a^3f} \\
&\quad - \frac{2^{-4-m}3^{-1-m}e^{-6e+\frac{6cf}{d}}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(1+m,\frac{6f(c+dx)}{d}\right)}{a^3f}
\end{aligned}$$

output

```
1/8*(d*x+c)^(1+m)/a^3/d/(1+m)-3*2^(-4-m)*exp(-2*e+2*c*f/d)*(d*x+c)^m*GAMMA
(1+m,2*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)-3*2^(-5-2*m)*exp(-4*e+4*c*f/d)
*(d*x+c)^m*GAMMA(1+m,4*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)/d)^m)-2^(-4-m)*3^(-1
-m)*exp(-6*e+6*c*f/d)*(d*x+c)^m*GAMMA(1+m,6*f*(d*x+c)/d)/a^3/f/((f*(d*x+c)
/d)^m)
```

Mathematica [A] (verified)

Time = 1.27 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.03

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx$$
$$= \frac{2^{-5-2m}3^{-1-m}e^{-3e}\left(f\left(\frac{c}{d}+x\right)\right)^{-m}(c+dx)^m\left(12^{1+m}e^{6e}f\left(f\left(\frac{c}{d}+x\right)\right)^m(c+dx)-2^{1+m}3^{2+m}de^{4e+\frac{2cf}{d}}(1+n\right.}{}$$

input
```
Integrate[(c + d*x)^m/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(2^(-5 - 2*m)*3^(-1 - m)*(c + d*x)^m*(12^(1 + m)*E^(6*e)*f*(f*(c/d + x))^m
*(c + d*x) - 2^(1 + m)*3^(2 + m)*d*E^(4*e + (2*c*f)/d)*(1 + m)*Gamma[1 + m
, (2*f*(c + d*x))/d] - 3^(2 + m)*d*E^(2*e + (4*c*f)/d)*(1 + m)*Gamma[1 + m
, (4*f*(c + d*x))/d] - 2^(1 + m)*d*E^((6*c*f)/d)*(1 + m)*Gamma[1 + m, (6*f
*(c + d*x))/d])*Sech[e + f*x]^3*(Cosh[f*x] + Sinh[f*x])^3)/(a^3*d*E^(3*e)*
f*(1 + m)*(f*(c/d + x))^m*(1 + Tanh[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4212, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^m}{(a\tanh(e+fx)+a)^3}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(c+dx)^m}{(a-ia\tan(ie+ifx))^3}\,dx$$
$$\downarrow 4212$$
$$\int \left(\frac{e^{-6e-6fx}(c+dx)^m}{8a^3}+\frac{3e^{-4e-4fx}(c+dx)^m}{8a^3}+\frac{3e^{-2e-2fx}(c+dx)^m}{8a^3}+\frac{(c+dx)^m}{8a^3}\right)dx$$

$$\downarrow 2009$$

$$-\frac{3\,2^{-m-4}e^{\frac{2cf}{d}-2e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{2f(c+dx)}{d}\right)}{a^3f}-\frac{3\,2^{-2m-5}e^{\frac{4cf}{d}-4e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{4f(c+dx)}{d}\right)}{a^3f}-\frac{2^{-m-4}3^{-m-1}e^{\frac{6cf}{d}-6e}(c+dx)^m\left(\frac{f(c+dx)}{d}\right)^{-m}\Gamma\left(m+1,\frac{6f(c+dx)}{d}\right)}{a^3f}+\frac{(c+dx)^{m+1}}{8a^3d(m+1)}$$

input
```
Int[(c + d*x)^m/(a + a*Tanh[e + f*x])^3,x]
```

output
```
(c + d*x)^(1 + m)/(8*a^3*d*(1 + m)) - (3*2^(-4 - m)*E^(-2*e + (2*c*f)/d)*(
c + d*x)^m*Gamma[1 + m, (2*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))/d)^m) -
(3*2^(-5 - 2*m)*E^(-4*e + (4*c*f)/d)*(c + d*x)^m*Gamma[1 + m, (4*f*(c + d*
x))/d])/(a^3*f*((f*(c + d*x))/d)^m) - (2^(-4 - m)*3^(-1 - m)*E^(-6*e + (6*
c*f)/d)*(c + d*x)^m*Gamma[1 + m, (6*f*(c + d*x))/d])/(a^3*f*((f*(c + d*x))
/d)^m)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4212
```
Int[((c_.) + (d_.)*(x_))^(m_)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(2*a) + E^(2*(a/b)*(e + f*
x))/(2*a))^(-n), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && EqQ[a^2 + b^2
, 0] && ILtQ[n, 0]
```

Maple [F]

$$\int \frac{(dx+c)^m}{(a+\tanh(fx+e)\,a)^3}\,dx$$

input
```
int((d*x+c)^m/(a+tanh(f*x+e)*a)^3,x)
```

output
```
int((d*x+c)^m/(a+tanh(f*x+e)*a)^3,x)
```

Fricas [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 345, normalized size of antiderivative = 1.54

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = -\frac{2\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{6f}{d}\right)+6de-6cf}{d}\right)\Gamma\left(m+1,\frac{6\,(dfx+cf)}{d}\right)+9\,(dm+d)\cosh\left(\frac{dm\log\left(\frac{4f}{d}\right)+4de-4cf}{d}\right)\Gamma$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
-1/96*(2*(d*m + d)*cosh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d)*gamma(m + 1, 6
*(d*f*x + c*f)/d) + 9*(d*m + d)*cosh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d)*g
amma(m + 1, 4*(d*f*x + c*f)/d) + 18*(d*m + d)*cosh((d*m*log(2*f/d) + 2*d*e
 - 2*c*f)/d)*gamma(m + 1, 2*(d*f*x + c*f)/d) - 2*(d*m + d)*gamma(m + 1, 6*
(d*f*x + c*f)/d)*sinh((d*m*log(6*f/d) + 6*d*e - 6*c*f)/d) - 9*(d*m + d)*ga
mma(m + 1, 4*(d*f*x + c*f)/d)*sinh((d*m*log(4*f/d) + 4*d*e - 4*c*f)/d) - 1
8*(d*m + d)*gamma(m + 1, 2*(d*f*x + c*f)/d)*sinh((d*m*log(2*f/d) + 2*d*e -
 2*c*f)/d) - 12*(d*f*x + c*f)*cosh(m*log(d*x + c)) - 12*(d*f*x + c*f)*sinh
(m*log(d*x + c)))/(a^3*d*f*m + a^3*d*f)
```

Sympy [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \frac{\int \frac{(c+dx)^m}{\tanh^3(e+fx)+3\tanh^2(e+fx)+3\tanh(e+fx)+1}\,dx}{a^3}$$

input
```
integrate((d*x+c)**m/(a+a*tanh(f*x+e))**3,x)
```

output
```
Integral((c + d*x)**m/(tanh(e + f*x)**3 + 3*tanh(e + f*x)**2 + 3*tanh(e +
f*x) + 1), x)/a**3
```

Maxima [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^3, x)
```

Giac [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \int \frac{(dx+c)^m}{(a\tanh(fx+e)+a)^3}\,dx$$

input
```
integrate((d*x+c)^m/(a+a*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)^m/(a*tanh(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \int \frac{(c+d\,x)^m}{(a+a\tanh(e+f\,x))^3}\,dx$$

input
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^3,x)
```

output
```
int((c + d*x)^m/(a + a*tanh(e + f*x))^3, x)
```

Reduce [F]

$$\int \frac{(c+dx)^m}{(a+a\tanh(e+fx))^3}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^m/(a+a*tanh(f*x+e))^3,x)
```

output

```
(12*e**(6*e + 6*f*x)*(c + d*x)**m*tanh(e + f*x)**2*c*f + 12*e**(6*e + 6*f*
x)*(c + d*x)**m*tanh(e + f*x)**2*d*f*x + 18*e**(6*e + 6*f*x)*(c + d*x)**m*
tanh(e + f*x)**2*d*m + 18*e**(6*e + 6*f*x)*(c + d*x)**m*tanh(e + f*x)**2*d
 + 24*e**(6*e + 6*f*x)*(c + d*x)**m*tanh(e + f*x)*c*f + 24*e**(6*e + 6*f*x
)*(c + d*x)**m*tanh(e + f*x)*d*f*x + 12*e**(6*e + 6*f*x)*(c + d*x)**m*tanh
(e + f*x)*d*m + 12*e**(6*e + 6*f*x)*(c + d*x)**m*tanh(e + f*x)*d + 12*e**(
6*e + 6*f*x)*(c + d*x)**m*c*f + 12*e**(6*e + 6*f*x)*(c + d*x)**m*d*f*x - 3
0*e**(6*e + 6*f*x)*(c + d*x)**m*d*m - 30*e**(6*e + 6*f*x)*(c + d*x)**m*d +
 6*e**(4*e + 4*f*x)*(c + d*x)**m*tanh(e + f*x)**2*d*m + 6*e**(4*e + 4*f*x)
*(c + d*x)**m*tanh(e + f*x)**2*d + 12*e**(4*e + 4*f*x)*(c + d*x)**m*tanh(e
 + f*x)*d*m + 12*e**(4*e + 4*f*x)*(c + d*x)**m*tanh(e + f*x)*d + 6*e**(4*e
 + 4*f*x)*(c + d*x)**m*d*m + 6*e**(4*e + 4*f*x)*(c + d*x)**m*d - 3*e**(2*e
 + 2*f*x)*(c + d*x)**m*tanh(e + f*x)**2*d*m - 3*e**(2*e + 2*f*x)*(c + d*x)
**m*tanh(e + f*x)**2*d - 6*e**(2*e + 2*f*x)*(c + d*x)**m*tanh(e + f*x)*d*m
 - 6*e**(2*e + 2*f*x)*(c + d*x)**m*tanh(e + f*x)*d - 3*e**(2*e + 2*f*x)*(c
 + d*x)**m*d*m - 3*e**(2*e + 2*f*x)*(c + d*x)**m*d - 2*(c + d*x)**m*tanh(e
 + f*x)**2*d*m - 2*(c + d*x)**m*tanh(e + f*x)**2*d - 4*(c + d*x)**m*tanh(e
 + f*x)*d*m - 4*(c + d*x)**m*tanh(e + f*x)*d - 2*(c + d*x)**m*d*m - 2*(c +
 d*x)**m*d + 18*e**(10*e + 6*f*x)*int((c + d*x)**m/(e**(6*e + 2*f*x)*c + e
**(6*e + 2*f*x)*d*x),x)*tanh(e + f*x)**2*d**2*m**2 + 18*e**(10*e + 6*f*...
```

3.53 $\int (c+dx)^3(a+b\tanh(e+fx))\,dx$

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Rubi [A] (verified) . . . 435
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Optimal result

Integrand size = 18, antiderivative size = 137

$$\begin{aligned}\int (c+dx)^3(a+b\tanh(e+fx))\,dx = {} & \frac{a(c+dx)^4}{4d} - \frac{b(c+dx)^4}{4d} \\ & + \frac{b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f} \\ & + \frac{3bd(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} \\ & - \frac{3bd^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} \\ & + \frac{3bd^3\,\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}\end{aligned}$$

output

```
1/4*a*(d*x+c)^4/d-1/4*b*(d*x+c)^4/d+b*(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+3/2
*b*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*b*d^2*(d*x+c)*polylog(3,
-exp(2*f*x+2*e))/f^3+3/4*b*d^3*polylog(4,-exp(2*f*x+2*e))/f^4
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.18

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \frac{1}{4}\left(\frac{2b(c+dx)^4}{d\left(1+e^{2e}\right)} + \frac{4b(c+dx)^3\log\left(1+e^{-2(e+fx)}\right)}{f} - \frac{3bd\left(2f^2(c+dx)^2\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right)+d\left(2f(c+dx)\,\text{PolyLog}\left(3,-e^{-2(e+fx)}\right)+d\,\text{PolyLog}\left(4,-\right.\right.\right.}{f^4} + x\left(4c^3+6c^2dx+4cd^2x^2+d^3x^3\right)(a+b\tanh(e))\right)$$

input

```
Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x]),x]
```

output

```
((2*b*(c + d*x)^4)/(d*(1 + E^(2*e))) + (4*b*(c + d*x)^3*Log[1 + E^(-2*(e +
 f*x))])/f - (3*b*d*(2*f^2*(c + d*x)^2*PolyLog[2, -E^(-2*(e + f*x))] + d*(
2*f*(c + d*x)*PolyLog[3, -E^(-2*(e + f*x))] + d*PolyLog[4, -E^(-2*(e + f*x
))])))/f^4 + x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*(a + b*Tanh[e])
)/4
```

Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx$$

$$\downarrow \text{3042}$$

$$\int (c+dx)^3(a-ib\tan(ie+ifx))dx$$

$$\downarrow \text{4205}$$

$$\int \left(a(c+dx)^3 + b(c+dx)^3 \tanh(e+fx)\right) dx$$

$$\downarrow 2009$$

$$\frac{a(c+dx)^4}{4d} - \frac{3bd^2(c+dx)\,\text{PolyLog}\left(3, -e^{2(e+fx)}\right)}{2f^3} + \frac{3bd(c+dx)^2\,\text{PolyLog}\left(2, -e^{2(e+fx)}\right)}{2f^2} + \frac{b(c+dx)^3 \log\left(e^{2(e+fx)}+1\right)}{f} - \frac{b(c+dx)^4}{4d} + \frac{3bd^3\,\text{PolyLog}\left(4, -e^{2(e+fx)}\right)}{4f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Tanh[e + f*x]),x]
```

output
```
(a*(c + d*x)^4)/(4*d) - (b*(c + d*x)^4)/(4*d) + (b*(c + d*x)^3*Log[1 + E^(
2*(e + f*x))])/f + (3*b*d*(c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2
) - (3*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3) + (3*b*d^3*Po
lyLog[4, -E^(2*(e + f*x))])/(4*f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 469 vs. 2(127) = 254.

Time = 0.74 (sec) , antiderivative size = 470, normalized size of antiderivative = 3.43

method	result
risch	$-\frac{2b\,d^3e^3x}{f^3}-\frac{3bc\,d^2\,\mathrm{polylog}\left(3,-\mathrm{e}^{2fx+2e}\right)}{2f^3}+\frac{3b\,c^2d\,\mathrm{polylog}\left(2,-\mathrm{e}^{2fx+2e}\right)}{2f^2}+\frac{b\,d^3\ln\left(1+\mathrm{e}^{2fx+2e}\right)x^3}{f}+\frac{3b\,d^3\,\mathrm{polylog}\left(2,-\mathrm{e}^{2fx}\right.}{2f^2}$

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
-2/f^3*b*d^3*e^3*x-3/2/f^3*b*c*d^2*polylog(3,-exp(2*f*x+2*e))+3/2/f^2*b*c^
2*d*polylog(2,-exp(2*f*x+2*e))+1/f*b*d^3*ln(1+exp(2*f*x+2*e))*x^3+3/2/f^2*
b*d^3*polylog(2,-exp(2*f*x+2*e))*x^2-3/2/f^3*b*d^3*polylog(3,-exp(2*f*x+2*
e))*x+2/f^4*b*d^3*e^3*ln(exp(f*x+e))+4/f^3*b*c*d^2*e^3-3/f^2*b*c^2*d*e^2+d
^2*a*c*x^3+1/4*d^3*a*x^4-1/4*d^3*b*x^4+1/4/d*a*c^4+1/4/d*c^4*b+3/4*b*d^3*p
olylog(4,-exp(2*f*x+2*e))/f^4-6/f*b*c^2*d*e*x+6/f^2*b*c*d^2*e^2*x+6/f^2*b*
c^2*d*e*ln(exp(f*x+e))+3/f*b*c*d^2*ln(1+exp(2*f*x+2*e))*x^2+3/f*b*c^2*d*ln
(1+exp(2*f*x+2*e))*x-6/f^3*b*c*d^2*e^2*ln(exp(f*x+e))-3/2/f^4*b*d^3*e^4-2/
f*b*c^3*ln(exp(f*x+e))+1/f*b*c^3*ln(1+exp(2*f*x+2*e))+3/f^2*b*c*d^2*polylo
g(2,-exp(2*f*x+2*e))*x-d^2*b*c*x^3+3/2*d*a*c^2*x^2-3/2*d*b*c^2*x^2+a*c^3*x
+b*c^3*x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 583, normalized size of antiderivative = 4.26

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx$$
$$=\frac{(a-b)d^3f^4x^4+4\,(a-b)cd^2f^4x^3+6\,(a-b)c^2df^4x^2+4\,(a-b)c^3f^4x+24\,bd^3\mathrm{polylog}(4,i\,\cosh\,(fx+e)}{}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*((a - b)*d^3*f^4*x^4 + 4*(a - b)*c*d^2*f^4*x^3 + 6*(a - b)*c^2*d*f^4*x
^2 + 4*(a - b)*c^3*f^4*x + 24*b*d^3*polylog(4, I*cosh(f*x + e) + I*sinh(f*
x + e)) + 24*b*d^3*polylog(4, -I*cosh(f*x + e) - I*sinh(f*x + e)) + 12*(b*
d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(I*cosh(f*x + e) + I*sin
h(f*x + e)) + 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-I*
cosh(f*x + e) - I*sinh(f*x + e)) - 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^
2*d*e*f^2 - b*c^3*f^3)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 4*(b*d^3*e
^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(cosh(f*x + e) + si
nh(f*x + e) - I) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x
+ b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(I*cosh(f*x + e) + I*s
inh(f*x + e) + 1) + 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x
 + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-I*cosh(f*x + e) - I
*sinh(f*x + e) + 1) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, I*cosh(f*x + e
) + I*sinh(f*x + e)) - 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, -I*cosh(f*x +
 e) - I*sinh(f*x + e)))/f^4
```

Sympy [F]

$$\int (c+dx)^3 (a+b\tanh(e+fx))\,dx = \int (a+b\tanh(e+fx))\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*tanh(f*x+e)),x)
```

output
```
Integral((a + b*tanh(e + f*x))*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(126) = 252.

Time = 0.13 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.22

$$\begin{aligned}
&\int (c+dx)^3(a+b\tanh(e+fx))\,dx \\
&= \frac{1}{4}ad^3x^4 + \frac{1}{4}bd^3x^4 + acd^2x^3 + bcd^2x^3 + \frac{3}{2}ac^2dx^2 + \frac{3}{2}bc^2dx^2 + ac^3x \\
&\quad + \frac{bc^3\log\left(\cosh\left(fx+e\right)\right)}{f} + \frac{3\left(2fx\log\left(e^{(2fx+2e)}+1\right)+\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)bc^2d}{2f^2} \\
&\quad + \frac{3\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right)+2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)\right)bcd^2}{2f^3} \\
&\quad + \frac{\left(4f^3x^3\log\left(e^{(2fx+2e)}+1\right)+6f^2x^2\mathrm{Li}_2\left(-e^{(2fx+2e)}\right)-6fx\mathrm{Li}_3\left(-e^{(2fx+2e)}\right)+3\mathrm{Li}_4\left(-e^{(2fx+2e)}\right)\right)bd^3}{3f^4} \\
&\quad - \frac{bd^3f^4x^4+4bcd^2f^4x^3+6bc^2df^4x^2}{2f^4}
\end{aligned}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/4*a*d^3*x^4 + 1/4*b*d^3*x^4 + a*c*d^2*x^3 + b*c*d^2*x^3 + 3/2*a*c^2*d*x^
2 + 3/2*b*c^2*d*x^2 + a*c^3*x + b*c^3*log(cosh(f*x + e))/f + 3/2*(2*f*x*lo
g(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))*b*c^2*d/f^2 + 3/2*(2*f^2
*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3,
 -e^(2*f*x + 2*e)))*b*c*d^2/f^3 + 1/3*(4*f^3*x^3*log(e^(2*f*x + 2*e) + 1)
+ 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*polylog(3, -e^(2*f*x + 2*e)) +
 3*polylog(4, -e^(2*f*x + 2*e)))*b*d^3/f^4 - 1/2*(b*d^3*f^4*x^4 + 4*b*c*d^
2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/f^4
```

Giac [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \int (dx+c)^3(b\tanh\left(fx+e\right)+a)\,dx$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*tanh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx = \int (a+b\tanh(e+f\,x))\,(c+d\,x)^3\,dx$$

input
```
int((a + b*tanh(e + f*x))*(c + d*x)^3,x)
```

output
```
int((a + b*tanh(e + f*x))*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))\,dx$$
$$= \frac{8e^{2e}\left(\int \frac{e^{2fx}x^3}{e^{2fx+2e}+1}dx\right) b\,d^3f + 24e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}+1}dx\right) bc\,d^2f + 24e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}+1}dx\right) b\,c^2df + 4\log\left(e^{2fx+2e}+\right.}{4f}$$

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e)),x)
```

output
```
(8*e**(2*e)*int((e**(2*f*x)*x**3)/(e**(2*e + 2*f*x) + 1),x)*b*d**3*f + 24*
e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x) + 1),x)*b*c*d**2*f + 24*e
**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) + 1),x)*b*c**2*d*f + 4*log(e*
*(2*e + 2*f*x) + 1)*b*c**3 + 4*a*c**3*f*x + 6*a*c**2*d*f*x**2 + 4*a*c*d**2
*f*x**3 + a*d**3*f*x**4 - 4*b*c**3*f*x - 6*b*c**2*d*f*x**2 - 4*b*c*d**2*f*
x**3 - b*d**3*f*x**4)/(4*f)
```

3.54 $\int (c+dx)^2(a+b\tanh(e+fx))\,dx$

Optimal result . . . 441
Mathematica [A] (verified) . . . 442
Rubi [A] (verified) . . . 442
Maple [B] (verified) . . . 443
Fricas [C] (verification not implemented) . . . 444
Sympy [F] . . . 445
Maxima [A] (verification not implemented) . . . 445
Giac [F] . . . 446
Mupad [F(-1)] . . . 446
Reduce [F] . . . 446

Optimal result

Integrand size = 18, antiderivative size = 103

$$\begin{aligned}\int (c+dx)^2(a+b\tanh(e+fx))\,dx = {} & \frac{a(c+dx)^3}{3d} - \frac{b(c+dx)^3}{3d} \\ & + \frac{b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f} \\ & + \frac{bd(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} \\ & - \frac{bd^2\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}\end{aligned}$$

output

```
1/3*a*(d*x+c)^3/d-1/3*b*(d*x+c)^3/d+b*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+b*d
*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*b*d^2*polylog(3,-exp(2*f*x+2*e
))/f^3
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.43

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx$$

$$= \frac{1}{6}\left(\frac{be^{2e}\left(\frac{4e^{-2e}(c+dx)^3}{d}+\frac{6\left(1+e^{-2e}\right)(c+dx)^2\log\left(1+e^{-2(e+fx)}\right)}{f}-\frac{3d\left(1+e^{-2e}\right)\left(2f(c+dx)\,\mathrm{PolyLog}\left(2,-e^{-2(e+fx)}\right)+d\,\mathrm{PolyLog}\left(3,-e^{-}\right.\right.\right.}{1+e^{2e}}\right.$$

$$\left.+2x\left(3c^2+3cdx+d^2x^2\right)(a+b\tanh(e))\right)$$

input
```
Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x]),x]
```

output
```
((b*E^(2*e)*((4*(c + d*x)^3)/(d*E^(2*e)) + (6*(1 + E^(-2*e))*(c + d*x)^2*L
og[1 + E^(-2*(e + f*x))])/f - (3*d*(1 + E^(-2*e))*(2*f*(c + d*x)*PolyLog[2
, -E^(-2*(e + f*x))] + d*PolyLog[3, -E^(-2*(e + f*x))]))/f^3))/(1 + E^(2*e
)) + 2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*(a + b*Tanh[e]))/6
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx$$

$$\downarrow \text{3042}$$

$$\int (c+dx)^2(a-ib\tan(ie+ifx))dx$$

$$\downarrow \text{4205}$$

$$\int \left(a(c+dx)^2 + b(c+dx)^2 \tanh(e+fx)\right) dx$$

$$\downarrow 2009$$

$$\frac{a(c+dx)^3}{3d} + \frac{bd(c+dx)\,\text{PolyLog}\left(2, -e^{2(e+fx)}\right)}{f^2} + \frac{b(c+dx)^2 \log\left(e^{2(e+fx)}+1\right)}{f} - \frac{b(c+dx)^3}{3d} - \frac{bd^2\,\text{PolyLog}\left(3, -e^{2(e+fx)}\right)}{2f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Tanh[e + f*x]),x]
```

output
```
(a*(c + d*x)^3)/(3*d) - (b*(c + d*x)^3)/(3*d) + (b*(c + d*x)^2*Log[1 + E^(
2*(e + f*x))])/f + (b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (b*d
^2*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. $2(97) = 194$.

Time = 0.55 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.82

method	result
risch	$\frac{d^2 a x^3}{3} - \frac{d^2 b x^3}{3} + dac x^2 - dbc x^2 + a c^2 x + b c^2 x + \frac{a c^3}{3d} + \frac{b c^3}{3d} - \frac{2b c^2 \ln(e^{fx+e})}{f} + \frac{b c^2 \ln(1+e^{2fx+2e})}{f} + 2$

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/3*d^2*a*x^3-1/3*d^2*b*x^3+d*a*c*x^2-d*b*c*x^2+a*c^2*x+b*c^2*x+1/3/d*a*c^
3+1/3/d*b*c^3-2/f*b*c^2*ln(exp(f*x+e))+1/f*b*c^2*ln(1+exp(2*f*x+2*e))+2/f^
2*b*d^2*e^2*x+4/3/f^3*b*d^2*e^3+1/f^2*b*d^2*polylog(2,-exp(2*f*x+2*e))*x-2
/f^2*b*d*c*e^2+2/f*b*d*c*ln(1+exp(2*f*x+2*e))*x+1/f^2*b*d*c*polylog(2,-exp
(2*f*x+2*e))+4/f^2*b*d*c*e*ln(exp(f*x+e))-2/f^3*b*d^2*e^2*ln(exp(f*x+e))+1
/f*b*d^2*ln(1+exp(2*f*x+2*e))*x^2-1/2*b*d^2*polylog(3,-exp(2*f*x+2*e))/f^3
-4/f*b*d*c*e*x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 364, normalized size of antiderivative = 3.53

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx$$
$$= \frac{(a-b)d^2f^3x^3+3\,(a-b)cdf^3x^2+3\,(a-b)c^2f^3x-6\,bd^2\mathrm{polylog}(3,i\,\cosh\,(fx+e)+i\,\sinh\,(fx+e))-}{}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/3*((a - b)*d^2*f^3*x^3 + 3*(a - b)*c*d*f^3*x^2 + 3*(a - b)*c^2*f^3*x - 6
*b*d^2*polylog(3, I*cosh(f*x + e) + I*sinh(f*x + e)) - 6*b*d^2*polylog(3,
-I*cosh(f*x + e) - I*sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(I*cosh
(f*x + e) + I*sinh(f*x + e)) + 6*(b*d^2*f*x + b*c*d*f)*dilog(-I*cosh(f*x +
 e) - I*sinh(f*x + e)) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(cosh(
f*x + e) + sinh(f*x + e) + I) + 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*lo
g(cosh(f*x + e) + sinh(f*x + e) - I) + 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x -
b*d^2*e^2 + 2*b*c*d*e*f)*log(I*cosh(f*x + e) + I*sinh(f*x + e) + 1) + 3*(b
*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-I*cosh(f*x +
e) - I*sinh(f*x + e) + 1))/f^3
```

Sympy [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \int (a+b\tanh(e+fx))\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*tanh(f*x+e)),x)
```

output
```
Integral((a + b*tanh(e + f*x))*(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.74

$$\begin{aligned}
&\int (c+dx)^2(a+b\tanh(e+fx))\,dx \\
&= \frac{1}{3}ad^2x^3 + \frac{1}{3}bd^2x^3 + acdx^2 + bcdx^2 + ac^2x + \frac{bc^2\log\left(\cosh\left(fx+e\right)\right)}{f} \\
&\quad + \frac{\left(2\,fx\log\left(e^{(2\,fx+2\,e)}+1\right) + \mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right)\right)bcd}{f^2} \\
&\quad + \frac{\left(2\,f^2x^2\log\left(e^{(2\,fx+2\,e)}+1\right) + 2\,fx\mathrm{Li}_2\left(-e^{(2\,fx+2\,e)}\right) - \mathrm{Li}_3(-e^{(2\,fx+2\,e)})\right)bd^2}{2\,f^3} \\
&\quad - \frac{2\left(bd^2f^3x^3 + 3\,bcdf^3x^2\right)}{3\,f^3}
\end{aligned}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/3*a*d^2*x^3 + 1/3*b*d^2*x^3 + a*c*d*x^2 + b*c*d*x^2 + a*c^2*x + b*c^2*lo
g(cosh(f*x + e))/f + (2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2
*e)))*b*c*d/f^2 + 1/2*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2*f*x*dilog(-e
^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*b*d^2/f^3 - 2/3*(b*d^2*f^3
*x^3 + 3*b*c*d*f^3*x^2)/f^3
```

Giac [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \int (dx+c)^2(b\tanh(fx+e)+a)\,dx$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx = \int (a+b\tanh(e+f\,x))\,(c+d\,x)^2\,dx$$

input
```
int((a + b*tanh(e + f*x))*(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))\,dx$$
$$= \frac{6e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}+1}dx\right)b\,d^2f + 12e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}+1}dx\right)bcdf + 3\log\left(e^{2fx+2e}+1\right)b\,c^2 + 3a\,c^2fx + 3acdf\,x^2 +}{3f}$$

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e)),x)
```

output
```
(6*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x) + 1),x)*b*d**2*f + 12*
e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) + 1),x)*b*c*d*f + 3*log(e**(
2*e + 2*f*x) + 1)*b*c**2 + 3*a*c**2*f*x + 3*a*c*d*f*x**2 + a*d**2*f*x**3 -
 3*b*c**2*f*x - 3*b*c*d*f*x**2 - b*d**2*f*x**3)/(3*f)
```

3.55 $\int (c+dx)(a+b\tanh(e+fx))\,dx$

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Optimal result

Integrand size = 16, antiderivative size = 75

$$\int (c+dx)(a+b\tanh(e+fx))\,dx = \frac{a(c+dx)^2}{2d} - \frac{b(c+dx)^2}{2d} + \frac{b(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} + \frac{bd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}$$

output

```
1/2*a*(d*x+c)^2/d-1/2*b*(d*x+c)^2/d+b*(d*x+c)*ln(1+exp(2*f*x+2*e))/f+1/2*b
*d*polylog(2,-exp(2*f*x+2*e))/f^2
```

Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.96

$$\int (c+dx)(a+b\tanh(e+fx))\,dx = \frac{f\left(fx(2ac+adx+bdx)+2bdx\log\left(1+e^{-2(e+fx)}\right)+2bc\log(\cosh(e+fx))\right)-bd\,\text{PolyLog}\left(2,-e^{-2(e+fx}\right.}{2f^2}$$

input

```
Integrate[(c + d*x)*(a + b*Tanh[e + f*x]),x]
```

output
```
(f*(f*x*(2*a*c + a*d*x + b*d*x) + 2*b*d*x*Log[1 + E^(-2*(e + f*x))] + 2*b*
c*Log[Cosh[e + f*x]]) - b*d*PolyLog[2, -E^(-2*(e + f*x))])/(2*f^2)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\tanh(e+fx))\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)(a-ib\tan(ie+ifx))dx$$

$$\downarrow 4205$$

$$\int (a(c+dx)+b(c+dx)\tanh(e+fx))dx$$

$$\downarrow 2009$$

$$\frac{a(c+dx)^2}{2d}+\frac{b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b(c+dx)^2}{2d}+\frac{bd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}$$

input
```
Int[(c + d*x)*(a + b*Tanh[e + f*x]),x]
```

output
```
(a*(c + d*x)^2)/(2*d) - (b*(c + d*x)^2)/(2*d) + (b*(c + d*x)*Log[1 + E^(2*
(e + f*x))])/f + (b*d*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 0.46 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.72

method	result
risch	$\frac{ad\,x^2}{2}+acx-\frac{bd\,x^2}{2}+bcx-\frac{2bc\ln\left(\mathrm{e}^{fx+e}\right)}{f}+\frac{bc\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f}-\frac{2bdex}{f}-\frac{bd\,e^2}{f^2}+\frac{bd\ln\left(1+\mathrm{e}^{2fx+2e}\right)x}{f}+\frac{bd\,\mathrm{polyl}$

input
```
int((d*x+c)*(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*d*x^2+a*c*x-1/2*b*d*x^2+b*c*x-2/f*b*c*ln(exp(f*x+e))+1/f*b*c*ln(1+ex
p(2*f*x+2*e))-2/f*b*d*e*x-1/f^2*b*d*e^2+1/f*b*d*ln(1+exp(2*f*x+2*e))*x+1/2
*b*d*polylog(2,-exp(2*f*x+2*e))/f^2+2/f^2*b*d*e*ln(exp(f*x+e))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.55

$$\int (c+dx)(a+b\tanh(e+fx))\,dx$$
$$=\frac{(a-b)df^2x^2+2\,(a-b)cf^2x+2\,bd\mathrm{Li}_2(i\,\cosh\left(fx+e\right)+i\,\sinh\left(fx+e\right))+2\,bd\mathrm{Li}_2(-i\,\cosh\left(fx+e\right)-$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((a - b)*d*f^2*x^2 + 2*(a - b)*c*f^2*x + 2*b*d*dilog(I*cosh(f*x + e) +
 I*sinh(f*x + e)) + 2*b*d*dilog(-I*cosh(f*x + e) - I*sinh(f*x + e)) - 2*(b
*d*e - b*c*f)*log(cosh(f*x + e) + sinh(f*x + e) + I) - 2*(b*d*e - b*c*f)*l
og(cosh(f*x + e) + sinh(f*x + e) - I) + 2*(b*d*f*x + b*d*e)*log(I*cosh(f*x
 + e) + I*sinh(f*x + e) + 1) + 2*(b*d*f*x + b*d*e)*log(-I*cosh(f*x + e) -
I*sinh(f*x + e) + 1))/f^2
```

Sympy [F]

$$\int (c+dx)(a+b\tanh(e+fx))\,dx = \int (a+b\tanh{(e+fx)})\,(c+dx)\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x)
```

output
```
Integral((a + b*tanh(e + f*x))*(c + d*x), x)
```

Maxima [F]

$$\int (c+dx)(a+b\tanh(e+fx))\,dx = \int (dx+c)(b\tanh{(fx+e)}+a)\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*a*d*x^2 + 1/2*(x^2 - 4*integrate(x/(e^(2*f*x + 2*e) + 1), x))*b*d + a*
c*x + b*c*log(cosh(f*x + e))/f
```

Giac [F]

$$\int (c+dx)(a+b\tanh(e+fx))\,dx = \int (dx+c)(b\tanh(fx+e)+a)\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\tanh(e+fx))\,dx = \int (a+b\tanh(e+f\,x))\,(c+d\,x)\,dx$$

input
```
int((a + b*tanh(e + f*x))*(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)(a+b\tanh(e+fx))\,dx$$
$$= \frac{4e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}+1}dx\right)bdf + 2\log\left(e^{2fx+2e}+1\right)bc + 2acfx + adf\,x^2 - 2bcfx - bdf\,x^2}{2f}$$

input
```
int((d*x+c)*(a+b*tanh(f*x+e)),x)
```

output
```
(4*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x) + 1),x)*b*d*f + 2*log(e**
(2*e + 2*f*x) + 1)*b*c + 2*a*c*f*x + a*d*f*x**2 - 2*b*c*f*x - b*d*f*x**2)/
(2*f)
```

3.56 $\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \text{Int}\left(\frac{a+b\tanh(e+fx)}{c+dx}, x\right)$$

output
```
Defer(Int)((a+b*tanh(f*x+e))/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 2.66 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x),x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\tanh(e + fx)}{c + dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{a - ib\tan(ie + ifx)}{c + dx}\,dx$$
$$\downarrow 4223$$
$$\int \frac{a + b\tanh(e + fx)}{c + dx}\,dx$$

input
```
Int[(a + b*Tanh[e + f*x])/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\tanh(fx + e)}{dx + c}\,dx$$

input
```
int((a+b*tanh(f*x+e))/(d*x+c),x)
```

output
```
int((a+b*tanh(f*x+e))/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \int \frac{b\tanh(fx+e)+a}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b*tanh(f*x + e) + a)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 0.74 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.83

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \int \frac{a+b\tanh(e+fx)}{c+dx}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x)
```

output
```
Integral((a + b*tanh(e + f*x))/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 56, normalized size of antiderivative = 3.11

$$\int \frac{a+b\tanh(e+fx)}{c+dx}\,dx = \int \frac{b\tanh(fx+e)+a}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="maxima")
```

output
```
b*(log(d*x + c)/d - 2*integrate(1/(d*x + (d*x*e^(2*e) + c*e^(2*e))*e^(2*f*
x) + c), x)) + a*log(d*x + c)/d
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\tanh(e + fx)}{c + dx}\,dx = \int \frac{b\tanh(fx + e) + a}{dx + c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\tanh(e + fx)}{c + dx}\,dx = \int \frac{a + b\tanh(e + f\,x)}{c + d\,x}\,dx$$

input
```
int((a + b*tanh(e + f*x))/(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 59, normalized size of antiderivative = 3.28

$$\int \frac{a + b\tanh(e + fx)}{c + dx}\,dx = \frac{-2\left(\int \frac{1}{e^{2fx+2e}c + e^{2fx+2e}dx + c + dx}dx\right)bd + \log(dx + c)\,a + \log(dx + c)\,b}{d}$$

input
```
int((a+b*tanh(f*x+e))/(d*x+c),x)
```

output
```
( - 2*int(1/(e**(2*e + 2*f*x)*c + e**(2*e + 2*f*x)*d*x + c + d*x),x)*b*d +
 log(c + d*x)*a + log(c + d*x)*b)/d
```

3.57 $\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$

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Rubi [N/A] . . . 458
Maple [N/A] . . . 458
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Sympy [N/A] . . . 459
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Giac [N/A] . . . 460
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Reduce [N/A] . . . 461

Optimal result

Integrand size = 18, antiderivative size = 18

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \text{Int}\left(\frac{a+b\tanh(e+fx)}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 5.14 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a + b\tanh(e+fx)}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{a - ib\tan(ie+ifx)}{(c+dx)^2}\,dx$$
$$\downarrow 4223$$
$$\int \frac{a + b\tanh(e+fx)}{(c+dx)^2}\,dx$$

input
```
Int[(a + b*Tanh[e + f*x])/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.16 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00

$$\int \frac{a + b\tanh\left(fx+e\right)}{\left(dx+c\right)^2}\,dx$$

input
```
int((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

output
```
int((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{b\tanh(fx+e)+a}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b*tanh(f*x + e) + a)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.76 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.94

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 87, normalized size of antiderivative = 4.83

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx = \int \frac{b\tanh(fx+e)+a}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-b*(1/(d^2*x + c*d) + 2*integrate(1/(d^2*x^2 + 2*c*d*x + c^2 + (d^2*x^2*e^
(2*e) + 2*c*d*x*e^(2*e) + c^2*e^(2*e))*e^(2*f*x)), x)) - a/(d^2*x + c*d)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\tanh(e + fx)}{(c + dx)^2}\,dx = \int \frac{b\tanh(fx + e) + a}{(dx + c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.41 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.11

$$\int \frac{a + b\tanh(e + fx)}{(c + dx)^2}\,dx = \int \frac{a + b\tanh(e + f\,x)}{(c + d\,x)^2}\,dx$$

input
```
int((a + b*tanh(e + f*x))/(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 162, normalized size of antiderivative = 9.00

$$\int \frac{a+b\tanh(e+fx)}{(c+dx)^2}\,dx$$
$$= \frac{-2\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2+c^2+2cdx+d^2x^2}\,dx\right)bc^2-2\left(\int \frac{1}{e^{2fx+2e}c^2+2e^{2fx+2e}cdx+e^{2fx+2e}d^2x^2+c^2+2cdx+d^2}\right.}{c\,(dx+c)}$$

input
```
int((a+b*tanh(f*x+e))/(d*x+c)^2,x)
```

output
```
( - 2*int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*b*c**2 - 2*int(1/(e**(2*e + 2*f*x)*c**2 + 2*e**(2*e + 2*f*x)*c*d*x + e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*b*c*d*x + a*x + b*x)/(c*(c + d*x))
```

3.58 $\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx$

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Giac [F] . . . 468
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Reduce [F] . . . 468

Optimal result

Integrand size = 20, antiderivative size = 277

$$\begin{aligned}\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = &-\frac{b^2(c+dx)^3}{f}+\frac{a^2(c+dx)^4}{4d}-\frac{ab(c+dx)^4}{2d}\\&+\frac{b^2(c+dx)^4}{4d}+\frac{3b^2d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2}\\&+\frac{2ab(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f}\\&+\frac{3b^2d^2(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}\\&+\frac{3abd(c+dx)^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}\\&-\frac{3b^2d^3\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4}\\&-\frac{3abd^2(c+dx)\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}\\&+\frac{3abd^3\,\mathrm{PolyLog}\left(4,-e^{2(e+fx)}\right)}{2f^4}\\&-\frac{b^2(c+dx)^3\tanh(e+fx)}{f}\end{aligned}$$

output
```
-b^2*(d*x+c)^3/f+1/4*a^2*(d*x+c)^4/d-1/2*a*b*(d*x+c)^4/d+1/4*b^2*(d*x+c)^4
/d+3*b^2*d*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f^2+2*a*b*(d*x+c)^3*ln(1+exp(2*f
*x+2*e))/f+3*b^2*d^2*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3+3*a*b*d*(d*x+c
)^2*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/
f^4-3*a*b*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+3/2*a*b*d^3*polylog(4
,-exp(2*f*x+2*e))/f^4-b^2*(d*x+c)^3*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 2.78 (sec) , antiderivative size = 544, normalized size of antiderivative = 1.96

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx$$
$$= \frac{-16bc^2(3bd+2acf)x + \frac{16b^2(c+dx)^3}{1+e^{2e}} + \frac{8abf(c+dx)^4}{d(1+e^{2e})} + \frac{48bcd(bd+acf)x\log\left(1+e^{-2(e+fx)}\right)}{f} + \frac{24bd^2(bd+2acf)x^2\log(1+e^{-2(e+}}{f}$$

input
```
Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x])^2,x]
```

output
```
(-16*b*c^2*(3*b*d + 2*a*c*f)*x + (16*b^2*(c + d*x)^3)/(1 + E^(2*e)) + (8*a
*b*f*(c + d*x)^4)/(d*(1 + E^(2*e))) + (48*b*c*d*(b*d + a*c*f)*x*Log[1 + E^
(-2*(e + f*x))])/f + (24*b*d^2*(b*d + 2*a*c*f)*x^2*Log[1 + E^(-2*(e + f*x)
)])/f + 16*a*b*d^3*x^3*Log[1 + E^(-2*(e + f*x))] + (8*b*c^2*(3*b*d + 2*a*c
*f)*Log[1 + E^(2*(e + f*x))])/f - (24*b*c*d*(b*d + a*c*f)*PolyLog[2, -E^(-
2*(e + f*x))])/f^2 - (24*b*d^2*(b*d + 2*a*c*f)*x*PolyLog[2, -E^(-2*(e + f*
x))])/f^2 - (24*a*b*d^3*x^2*PolyLog[2, -E^(-2*(e + f*x))])/f - (12*b*d^2*(
b*d + 2*a*c*f)*PolyLog[3, -E^(-2*(e + f*x))])/f^3 - (24*a*b*d^3*x*PolyLog[
3, -E^(-2*(e + f*x))])/f^2 - (12*a*b*d^3*PolyLog[4, -E^(-2*(e + f*x))])/f^
3 + Sech[e]*Sech[e + f*x]*((a^2 + b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^
2 + d^3*x^3)*Cosh[f*x] + (a^2 + b^2)*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2
+ d^3*x^3)*Cosh[2*e + f*x] - 2*b*(4*b*(c + d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d
*x + 4*c*d^2*x^2 + d^3*x^3))*Sinh[f*x] + 2*a*b*f*x*(4*c^3 + 6*c^2*d*x + 4*
c*d^2*x^2 + d^3*x^3)*Sinh[2*e + f*x]))/(8*f)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3(a-ib\tan(ie+ifx))^2 dx$$

$$\downarrow 4205$$

$$\int \left(a^2(c+dx)^3+2ab(c+dx)^3\tanh(e+fx)+b^2(c+dx)^3\tanh^2(e+fx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2(c+dx)^4}{4d}-\frac{3abd^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}+\frac{3abd(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{ab(c+dx)^4}{2d}+\frac{3abd^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{2f^4}+\frac{3b^2d^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{3b^2d(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{b^2(c+dx)^3\tanh(e+fx)}{f}-\frac{b^2(c+dx)^3}{f}+\frac{b^2(c+dx)^4}{4d}-\frac{3b^2d^3\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4}$$

input

```
Int[(c + d*x)^3*(a + b*Tanh[e + f*x])^2,x]
```

output

```
-((b^2*(c + d*x)^3)/f) + (a^2*(c + d*x)^4)/(4*d) - (a*b*(c + d*x)^4)/(2*d)
 + (b^2*(c + d*x)^4)/(4*d) + (3*b^2*d*(c + d*x)^2*Log[1 + E^(2*(e + f*x))]
)/f^2 + (2*a*b*(c + d*x)^3*Log[1 + E^(2*(e + f*x))])/f + (3*b^2*d^2*(c + d
*x)*PolyLog[2, -E^(2*(e + f*x))])/f^3 + (3*a*b*d*(c + d*x)^2*PolyLog[2, -E
^(2*(e + f*x))])/f^2 - (3*b^2*d^3*PolyLog[3, -E^(2*(e + f*x))])/(2*f^4) -
(3*a*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/f^3 + (3*a*b*d^3*PolyLo
g[4, -E^(2*(e + f*x))])/(2*f^4) - (b^2*(c + d*x)^3*Tanh[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 904 vs. $2(267) = 534$.

Time = 1.34 (sec) , antiderivative size = 905, normalized size of antiderivative = 3.27

method	result
risch	$\frac{12b\,d^2ca\,e^2x}{f^2} - \frac{12ba\,c^2dex}{f} + \frac{12bea\,c^2d\ln\left(e^{fx+e}\right)}{f^2} - \frac{12b\,e^2d^2ca\ln\left(e^{fx+e}\right)}{f^3} + \frac{2b^2\left(d^3x^3+3c\,d^2x^2+3c^2dx+c^3\right)}{f\left(1+e^{2fx+2e}\right)} + \frac{6b\,d^2ca\ln($

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output

```
12/f^2*b*d^2*c*a*e^2*x-12/f*b*a*c^2*d*e*x+12/f^2*b*e*a*c^2*d*ln(exp(f*x+e)
)-12/f^3*b*e^2*d^2*c*a*ln(exp(f*x+e))+2/f*b^2*(d^3*x^3+3*c*d^2*x^2+3*c^2*d
*x+c^3)/(1+exp(2*f*x+2*e))+6/f*b*d^2*c*a*ln(1+exp(2*f*x+2*e))*x^2+6/f^2*b*
d^2*c*a*polylog(2,-exp(2*f*x+2*e))*x+6/f*b*a*c^2*d*ln(1+exp(2*f*x+2*e))*x-
3/2*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4+3/2*a*b*d^3*polylog(4,-exp(2*f*
x+2*e))/f^4-2*d^2*a*b*c*x^3-3*d*a*b*c^2*x^2+2*a*b*c^3*x+1/4*d^3*a^2*x^4+1/
4*d^3*b^2*x^4+1/4/d*a^2*c^4+1/4/d*b^2*c^4-1/2*d^3*a*b*x^4+d^2*a^2*c*x^3+d^
2*b^2*c*x^3+3/2*d*a^2*c^2*x^2+3/2*d*b^2*c^2*x^2+a^2*c^3*x+b^2*c^3*x+1/2/d*
a*b*c^4+6/f^3*b^2*d^3*e^2*x+3/f^3*b^2*c*d^2*polylog(2,-exp(2*f*x+2*e))-6/f
^4*b^2*e^2*d^3*ln(exp(f*x+e))+3/f^2*b^2*d^3*ln(1+exp(2*f*x+2*e))*x^2+3/f^3
*b^2*d^3*polylog(2,-exp(2*f*x+2*e))*x-4/f*b*a*c^3*ln(exp(f*x+e))+2/f*b*a*c
^3*ln(1+exp(2*f*x+2*e))-6/f^2*b^2*c^2*d*ln(exp(f*x+e))+3/f^2*b^2*c^2*d*ln(
1+exp(2*f*x+2*e))-2/f*b^2*d^3*x^3+4/f^4*b^2*d^3*e^3-3/f^4*b*d^3*a*e^4-6/f*
b^2*c*d^2*x^2-6/f^3*b^2*c*d^2*e^2-12/f^2*b^2*c*d^2*e*x+12/f^3*b^2*e*c*d^2*
ln(exp(f*x+e))+6/f^2*b^2*c*d^2*ln(1+exp(2*f*x+2*e))*x+3/f^2*b*d^3*a*polylo
g(2,-exp(2*f*x+2*e))*x^2-3/f^3*b*d^3*a*polylog(3,-exp(2*f*x+2*e))*x+3/f^2*
b*a*c^2*d*polylog(2,-exp(2*f*x+2*e))-3/f^3*b*d^2*c*a*polylog(3,-exp(2*f*x+
2*e))+4/f^4*b*e^3*d^3*a*ln(exp(f*x+e))+2/f*b*d^3*a*ln(1+exp(2*f*x+2*e))*x^
3-6/f^2*b*a*c^2*d*e^2+8/f^3*b*d^2*c*a*e^3-4/f^3*b*d^3*a*e^3*x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 3744, normalized size of antiderivative = 13.52

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh(e+fx))^2\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**2*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 625 vs. $2(265) = 530$.

Time $= 0.16$ (sec) , antiderivative size $= 625$, normalized size of antiderivative $= 2.26$

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/4*a^2*d^3*x^4 + a^2*c*d^2*x^3 + 3/2*a^2*c^2*d*x^2 + b^2*c^3*(x + e/f - 2
/(f*(e^(-2*f*x - 2*e) + 1))) + a^2*c^3*x + 3/2*b^2*c^2*d*((f*x^2 + (f*x^2*
e^(2*e) - 4*x*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 2*log((e^(2*f*
x + 2*e) + 1)*e^(-2*e))/f^2) + 2*a*b*c^3*log(cosh(f*x + e))/f + 2/3*(4*f^3
*x^3*log(e^(2*f*x + 2*e) + 1) + 6*f^2*x^2*dilog(-e^(2*f*x + 2*e)) - 6*f*x*
polylog(3, -e^(2*f*x + 2*e)) + 3*polylog(4, -e^(2*f*x + 2*e)))*a*b*d^3/f^4
 + 1/4*((2*a*b*d^3*f + b^2*d^3*f)*x^4 + 4*(2*a*b*c*d^2*f + (c*d^2*f + 2*d^
3)*b^2)*x^3 + 12*(a*b*c^2*d*f + 2*b^2*c*d^2)*x^2 + (12*a*b*c^2*d*f*x^2*e^(
2*e) + (2*a*b*d^3*f*e^(2*e) + b^2*d^3*f*e^(2*e))*x^4 + 4*(2*a*b*c*d^2*f*e^
(2*e) + b^2*c*d^2*f*e^(2*e))*x^3)*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 3*(
a*b*c^2*d*f + b^2*c*d^2)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x
 + 2*e)))/f^3 + 3/2*(2*a*b*c*d^2*f + b^2*d^3)*(2*f^2*x^2*log(e^(2*f*x + 2*
e) + 1) + 2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))/f^
4 - (a*b*d^3*f^4*x^4 + 2*(2*a*b*c*d^2*f + b^2*d^3)*f^3*x^3 + 6*(a*b*c^2*d*
f^2 + b^2*c*d^2*f)*f^2*x^2)/f^4
```

Giac [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \int (dx+c)^3(b\tanh(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^3*(b*tanh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh(e+f\,x))^2\,(c+d\,x)^3\,dx$$

input
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^3,x)
```

output
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^2\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e))^2,x)
```

output

```
( - 16*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) +
1),x)*a*b*d**3*f**4 - 48*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x) + 2*e
**(2*e + 2*f*x) + 1),x)*a*b*c*d**2*f**4 - 24*e**(2*e + 2*f*x)*int(x**2/(e*
*(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**3*f**3 - 24*e**(2*e + 2
*f*x)*int(x**2/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*b**2*d**3*f*
*3 - 48*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1)
,x)*a*b*c**2*d*f**4 - 48*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 2*e**(
2*e + 2*f*x) + 1),x)*a*b*c*d**2*f**3 - 24*e**(2*e + 2*f*x)*int(x/(e**(4*e
+ 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**3*f**2 - 48*e**(2*e + 2*f*x)*
int(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*b**2*c*d**2*f**3 - 24
*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*b**
2*d**3*f**2 + 8*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*a*b*c**3*f**3 +
 12*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*a*b*c**2*d*f**2 + 12*e**(2*
e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*a*b*c*d**2*f + 6*e**(2*e + 2*f*x)*log
(e**(2*e + 2*f*x) + 1)*a*b*d**3 + 12*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x)
 + 1)*b**2*c**2*d*f**2 + 12*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*b**
2*c*d**2*f + 6*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*b**2*d**3 + 4*e*
*(2*e + 2*f*x)*a**2*c**3*f**4*x + 6*e**(2*e + 2*f*x)*a**2*c**2*d*f**4*x**2
 + 4*e**(2*e + 2*f*x)*a**2*c*d**2*f**4*x**3 + e**(2*e + 2*f*x)*a**2*d**3*f
**4*x**4 - 8*e**(2*e + 2*f*x)*a*b*c**3*f**4*x + 12*e**(2*e + 2*f*x)*a*b...
```

3.59 $\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 211

$$\begin{aligned}\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = &-\frac{b^2(c+dx)^2}{f}+\frac{a^2(c+dx)^3}{3d}-\frac{2ab(c+dx)^3}{3d}\\&+\frac{b^2(c+dx)^3}{3d}+\frac{2b^2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2}\\&+\frac{2ab(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f}\\&+\frac{b^2d^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}\\&+\frac{2abd(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}\\&-\frac{abd^2\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}\\&-\frac{b^2(c+dx)^2\tanh(e+fx)}{f}\end{aligned}$$

output

```
-b^2*(d*x+c)^2/f+1/3*a^2*(d*x+c)^3/d-2/3*a*b*(d*x+c)^3/d+1/3*b^2*(d*x+c)^3
/d+2*b^2*d*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^2+2*a*b*(d*x+c)^2*ln(1+exp(2*f*x
+2*e))/f+b^2*d^2*polylog(2,-exp(2*f*x+2*e))/f^3+2*a*b*d*(d*x+c)*polylog(2,
-exp(2*f*x+2*e))/f^2-a*b*d^2*polylog(3,-exp(2*f*x+2*e))/f^3-b^2*(d*x+c)^2*
tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 1.68 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.12

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx$$

$$=\frac{1}{3}\left(\frac{2b\left(\frac{fx(3bd(-2ce^{2e}+dx)+2af(-3c^2e^{2e}+3cdx+d^2x^2))}{1+e^{2e}}+3dx(bd+af(2c+dx))\log\left(1+e^{-2(e+fx)}\right)+3c(bd+ac\right.}{f^2}\right.$$

$$-\frac{3bd(bd+2af(c+dx))\,\text{PolyLog}\left(2,-e^{-2(e+fx)}\right)}{f^3}-\frac{3abd^2\,\text{PolyLog}\left(3,-e^{-2(e+fx)}\right)}{f^3}$$

$$-\frac{3b^2(c+dx)^2\text{sech}(e)\text{sech}(e+fx)\sinh(fx)}{f}$$

$$\left.+x\left(3c^2+3cdx+d^2x^2\right)\left(a^2+b^2+2ab\tanh(e)\right)\right)$$

input
```
Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x])^2,x]
```

output
```
((2*b*((f*x*(3*b*d*(-2*c*E^(2*e) + d*x) + 2*a*f*(-3*c^2*E^(2*e) + 3*c*d*x
+ d^2*x^2)))/(1 + E^(2*e)) + 3*d*x*(b*d + a*f*(2*c + d*x))*Log[1 + E^(-2*(
e + f*x))] + 3*c*(b*d + a*c*f)*Log[1 + E^(2*(e + f*x))]))/f^2 - (3*b*d*(b*
d + 2*a*f*(c + d*x))*PolyLog[2, -E^(-2*(e + f*x))])/f^3 - (3*a*b*d^2*PolyL
og[3, -E^(-2*(e + f*x))])/f^3 - (3*b^2*(c + d*x)^2*Sech[e]*Sech[e + f*x]*S
inh[f*x])/f + x*(3*c^2 + 3*c*d*x + d^2*x^2)*(a^2 + b^2 + 2*a*b*Tanh[e]))/3
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2(a-ib\tan(ie+ifx))^2dx$$

$$\downarrow 4205$$

$$\int \left(a^2(c+dx)^2+2ab(c+dx)^2\tanh(e+fx)+b^2(c+dx)^2\tanh^2(e+fx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2(c+dx)^3}{3d}+\frac{2abd(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{2ab(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{2ab(c+dx)^3}{3d}-\frac{abd^2\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{f^3}+\frac{2b^2d(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{b^2(c+dx)^2\tanh(e+fx)}{f}-\frac{b^2(c+dx)^2}{f}+\frac{b^2(c+dx)^3}{3d}+\frac{b^2d^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}$$

input
```
Int[(c + d*x)^2*(a + b*Tanh[e + f*x])^2,x]
```

output
```
-((b^2*(c + d*x)^2)/f) + (a^2*(c + d*x)^3)/(3*d) - (2*a*b*(c + d*x)^3)/(3*
d) + (b^2*(c + d*x)^3)/(3*d) + (2*b^2*d*(c + d*x)*Log[1 + E^(2*(e + f*x))]
)/f^2 + (2*a*b*(c + d*x)^2*Log[1 + E^(2*(e + f*x))])/f + (b^2*d^2*PolyLog[
2, -E^(2*(e + f*x))])/f^3 + (2*a*b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))
])/f^2 - (a*b*d^2*PolyLog[3, -E^(2*(e + f*x))])/f^3 - (b^2*(c + d*x)^2*Tan
h[e + f*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(205) = 410$.

Time = 1.18 (sec) , antiderivative size = 542, normalized size of antiderivative = 2.57

method	result
risch	$-\frac{8bcdaex}{f} + \frac{8becda\ln(e^{fx+e})}{f^2} + \frac{4bcda\ln(1+e^{2fx+2e})x}{f} - \frac{abd^2\,\mathrm{polylog}(3,-e^{2fx+2e})}{f^3} + \frac{b^2d^2\,\mathrm{polylog}(2,-e^{2fx+2e})}{f^3} + \frac{d^2}{}$

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
-8/f*b*c*d*a*e*x+8/f^2*b*e*c*d*a*ln(exp(f*x+e))+4/f*b*c*d*a*ln(1+exp(2*f*x
+2*e))*x-a*b*d^2*polylog(3,-exp(2*f*x+2*e))/f^3+b^2*d^2*polylog(2,-exp(2*f
*x+2*e))/f^3+1/3*d^2*a^2*x^3+1/3*d^2*b^2*x^3+1/3/d*c^3*a^2+1/3/d*b^2*c^3-2
/3*d^2*a*b*x^3+d*a^2*c*x^2+d*x^2*c*b^2+a^2*c^2*x+x*b^2*c^2+2/f*b^2*(d^2*x^
2+2*c*d*x+c^2)/(1+exp(2*f*x+2*e))+2/3/d*c^3*a*b-4/f^2*b^2*d^2*e*x+8/3/f^3*
b*a*d^2*e^3+4/f^3*b^2*e*d^2*ln(exp(f*x+e))-4/f*b*a*c^2*ln(exp(f*x+e))+2/f*
b*a*c^2*ln(1+exp(2*f*x+2*e))-4/f^2*b^2*c*d*ln(exp(f*x+e))+2/f^2*b^2*c*d*ln
(1+exp(2*f*x+2*e))+2/f^2*b^2*d^2*ln(1+exp(2*f*x+2*e))*x-2/f*b^2*d^2*x^2-2/
f^3*b^2*d^2*e^2-2*d*a*b*c*x^2+2*a*b*c^2*x+2/f*b*a*d^2*ln(1+exp(2*f*x+2*e))
*x^2+2/f^2*b*a*d^2*polylog(2,-exp(2*f*x+2*e))*x+2/f^2*b*c*d*a*polylog(2,-e
xp(2*f*x+2*e))-4/f^3*b*e^2*a*d^2*ln(exp(f*x+e))+4/f^2*b*a*d^2*e^2*x-4/f^2*
b*c*d*a*e^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.15 (sec) , antiderivative size = 2123, normalized size of antiderivative = 10.06

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output

```
1/3*((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 + 3*(a^2 - 2*a*b + b^2)*c*d*f^3*x^2 -
 4*a*b*d^2*e^3 + 3*(a^2 - 2*a*b + b^2)*c^2*f^3*x + 6*b^2*d^2*e^2 - 6*(2*a*
b*c^2*e - b^2*c^2)*f^2 + ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3
- 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2
)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2
- 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)^2 + 2*((a^2 - 2*a*b + b^2)*d^2*f^
3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 + 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^
2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*
(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f^3)*x)*cosh(f*x + e)*sinh(f*x +
e) + ((a^2 - 2*a*b + b^2)*d^2*f^3*x^3 - 4*a*b*d^2*e^3 - 12*a*b*c^2*e*f^2 +
 6*b^2*d^2*e^2 - 3*(2*b^2*d^2*f^2 - (a^2 - 2*a*b + b^2)*c*d*f^3)*x^2 + 12*
(a*b*c*d*e^2 - b^2*c*d*e)*f - 3*(4*b^2*c*d*f^2 - (a^2 - 2*a*b + b^2)*c^2*f
^3)*x)*sinh(f*x + e)^2 + 12*(a*b*c*d*e^2 - b^2*c*d*e)*f + 6*(2*a*b*d^2*f*x
 + 2*a*b*c*d*f + b^2*d^2 + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*
x + e)^2 + 2*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f*
x + e) + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(I*
cosh(f*x + e) + I*sinh(f*x + e)) + 6*(2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^
2 + (2*a*b*d^2*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)^2 + 2*(2*a*b*d^2
*f*x + 2*a*b*c*d*f + b^2*d^2)*cosh(f*x + e)*sinh(f*x + e) + (2*a*b*d^2*f*x
 + 2*a*b*c*d*f + b^2*d^2)*sinh(f*x + e)^2)*dilog(-I*cosh(f*x + e) - I*s...
```

Sympy [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh{(e+fx)})^2\,(c+dx)^2\,dx$$

input

```
integrate((d*x+c)**2*(a+b*tanh(f*x+e))**2,x)
```

output

```
Integral((a + b*tanh(e + f*x))**2*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 411 vs. 2(203) = 406.

Time = 0.13 (sec) , antiderivative size = 411, normalized size of antiderivative = 1.95

$$
\begin{aligned}
&\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx \\
&= \frac{1}{3}a^2d^2x^3 + a^2cdx^2 + b^2c^2\left(x + \frac{e}{f} - \frac{2}{f(e^{(-2fx-2e)}+1)}\right) + a^2c^2x \\
&\quad + b^2cd\left(\frac{fx^2 + \left(fx^2e^{(2e)} - 4xe^{(2e)}\right)e^{(2fx)}}{fe^{(2fx+2e)}+f} + \frac{2\log\left(\left(e^{(2fx+2e)}+1\right)e^{(-2e)}\right)}{f^2}\right) \\
&\quad + \frac{2abc^2\log\left(\cosh\left(fx+e\right)\right)}{f} \\
&\quad + \frac{\left(2f^2x^2\log\left(e^{(2fx+2e)}+1\right) + 2fx\mathrm{Li}_2\left(-e^{(2fx+2e)}\right) - \mathrm{Li}_3(-e^{(2fx+2e)})\right)abd^2}{f^3} \\
&\quad + \frac{(2abd^2f + b^2d^2f)x^3 + 6\left(abcdf + b^2d^2\right)x^2 + \left(6abcdfx^2e^{(2e)} + \left(2abd^2fe^{(2e)} + b^2d^2fe^{(2e)}\right)x^3\right)e^{(2fx)}}{3\left(fe^{(2fx+2e)}+f\right)} \\
&\quad + \frac{\left(2abcdf + b^2d^2\right)\left(2fx\log\left(e^{(2fx+2e)}+1\right) + \mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)}{f^3} \\
&\quad - \frac{2\left(2abd^2f^3x^3 + 3\left(2abcdf + b^2d^2\right)f^2x^2\right)}{3f^3}
\end{aligned}
$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/3*a^2*d^2*x^3 + a^2*c*d*x^2 + b^2*c^2*(x + e/f - 2/(f*(e^(-2*f*x - 2*e)
+ 1))) + a^2*c^2*x + b^2*c*d*((f*x^2 + (f*x^2*e^(2*e) - 4*x*e^(2*e))*e^(2*
f*x))/(f*e^(2*f*x + 2*e) + f) + 2*log((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2)
 + 2*a*b*c^2*log(cosh(f*x + e))/f + (2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) +
2*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))*a*b*d^2/f^3
+ 1/3*((2*a*b*d^2*f + b^2*d^2*f)*x^3 + 6*(a*b*c*d*f + b^2*d^2)*x^2 + (6*a*
b*c*d*f*x^2*e^(2*e) + (2*a*b*d^2*f*e^(2*e) + b^2*d^2*f*e^(2*e))*x^3)*e^(2*
f*x))/(f*e^(2*f*x + 2*e) + f) + (2*a*b*c*d*f + b^2*d^2)*(2*f*x*log(e^(2*f*
x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^3 - 2/3*(2*a*b*d^2*f^3*x^3 + 3*
(2*a*b*c*d*f + b^2*d^2)*f^2*x^2)/f^3
```

Giac [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \int (dx+c)^2(b\tanh(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2*(b*tanh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh(e+f\,x))^2\,(c+d\,x)^2\,dx$$

input
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^2*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e))^2,x)
```

output

```
( - 12*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) +
1),x)*a*b*d**2*f**3 - 24*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 2*e**(
2*e + 2*f*x) + 1),x)*a*b*c*d*f**3 - 12*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4
*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**2*f**2 - 12*e**(2*e + 2*f*x)*int
(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*b**2*d**2*f**2 + 6*e**(2
*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*a*b*c**2*f**2 + 6*e**(2*e + 2*f*x)*l
og(e**(2*e + 2*f*x) + 1)*a*b*c*d*f + 3*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*
x) + 1)*a*b*d**2 + 6*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*b**2*c*d*f
 + 3*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*b**2*d**2 + 3*e**(2*e + 2*
f*x)*a**2*c**2*f**3*x + 3*e**(2*e + 2*f*x)*a**2*c*d*f**3*x**2 + e**(2*e +
2*f*x)*a**2*d**2*f**3*x**3 - 6*e**(2*e + 2*f*x)*a*b*c**2*f**3*x + 6*e**(2*
e + 2*f*x)*a*b*c*d*f**3*x**2 - 12*e**(2*e + 2*f*x)*a*b*c*d*f**2*x + 2*e**(
2*e + 2*f*x)*a*b*d**2*f**3*x**3 - 6*e**(2*e + 2*f*x)*a*b*d**2*f*x + 3*e**(
2*e + 2*f*x)*b**2*c**2*f**3*x - 6*e**(2*e + 2*f*x)*b**2*c**2*f**2 + 3*e**(
2*e + 2*f*x)*b**2*c*d*f**3*x**2 - 12*e**(2*e + 2*f*x)*b**2*c*d*f**2*x + e*
*(2*e + 2*f*x)*b**2*d**2*f**3*x**3 - 6*e**(2*e + 2*f*x)*b**2*d**2*f*x - 12
*int(x**2/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**2*f**3 - 2
4*int(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*a*b*c*d*f**3 - 12*i
nt(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*a*b*d**2*f**2 - 12*int
(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x)*b**2*d**2*f**2 + 6*lo...
```

3.60 $\int (c+dx)(a+b\tanh(e+fx))^2\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 127

$$\begin{aligned}\int (c+dx)(a+b\tanh(e+fx))^2\,dx = {} & \frac{a^2(c+dx)^2}{2d} - \frac{ab(c+dx)^2}{d} + \frac{b^2(c+dx)^2}{2d} \\ & + \frac{2ab(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f} \\ & + \frac{b^2 d\log(\cosh(e+fx))}{f^2} \\ & + \frac{abd\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2} \\ & - \frac{b^2(c+dx)\tanh(e+fx)}{f}\end{aligned}$$

output

```
1/2*a^2*(d*x+c)^2/d-a*b*(d*x+c)^2/d+1/2*b^2*(d*x+c)^2/d+2*a*b*(d*x+c)*ln(1
+exp(2*f*x+2*e))/f+b^2*d*ln(cosh(f*x+e))/f^2+a*b*d*polylog(2,-exp(2*f*x+2*
e))/f^2-b^2*(d*x+c)*tanh(f*x+e)/f
```

Mathematica [A] (verified)

Time = 6.58 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.69

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx$$
$$= \frac{\cosh(e+fx)\left(-((a^2+b^2)(e+fx)(-2cf+d(e-fx))\cosh(e+fx))+2b\cosh(e+fx)\left(\frac{af^2(c+dx)^2}{d}+b\right.\right.}{}$$

input
```
Integrate[(c + d*x)*(a + b*Tanh[e + f*x])^2,x]
```

output
```
(Cosh[e + f*x]*(-((a^2 + b^2)*(e + f*x)*(-2*c*f + d*(e - f*x))*Cosh[e + f*
x]) + 2*b*Cosh[e + f*x]*((a*f^2*(c + d*x)^2)/d + b*d*(e + f*x) - 2*(b*d -
2*a*d*e + 2*a*c*f)*(e + f*x) + 2*a*d*(e + f*x)*Log[1 + E^(-2*(e + f*x))] +
 (b*d - 2*a*d*e + 2*a*c*f)*Log[1 + E^(2*(e + f*x))] - a*d*PolyLog[2, -E^(-
2*(e + f*x))]) - 2*b^2*f*(c + d*x)*Sinh[e + f*x])*(a + b*Tanh[e + f*x])^2)
/(2*f^2*(a*Cosh[e + f*x] + b*Sinh[e + f*x])^2)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx$$
$$\downarrow 3042$$
$$\int (c+dx)(a-ib\tan(ie+ifx))^2 dx$$
$$\downarrow 4205$$
$$\int \left(a^2(c+dx)+2ab(c+dx)\tanh(e+fx)+b^2(c+dx)\tanh^2(e+fx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^2(c+dx)^2}{2d}+\frac{2ab(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{ab(c+dx)^2}{d}+\frac{abd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{b^2(c+dx)\tanh(e+fx)}{f}+\frac{b^2(c+dx)^2}{2d}+\frac{b^2d\log(\cosh(e+fx))}{f^2}$$

input
```
Int[(c + d*x)*(a + b*Tanh[e + f*x])^2,x]
```

output
```
(a^2*(c + d*x)^2)/(2*d) - (a*b*(c + d*x)^2)/d + (b^2*(c + d*x)^2)/(2*d) +
(2*a*b*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f + (b^2*d*Log[Cosh[e + f*x]])/
f^2 + (a*b*d*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (b^2*(c + d*x)*Tanh[e + f
*x])/f
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.03 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.74

method	result
risch	$\frac{a^2dx^2}{2}-abdx^2+\frac{b^2dx^2}{2}+a^2cx+2abcx+b^2cx+\frac{2(dx+c)b^2}{f\left(1+e^{2fx+2e}\right)}-\frac{2b^2d\ln\left(e^{fx+e}\right)}{f^2}+\frac{b^2d\ln\left(1+e^{2fx+2e}\right)}{f^2}-$

input
```
int((d*x+c)*(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*a^2*d*x^2-a*b*d*x^2+1/2*b^2*d*x^2+a^2*c*x+2*a*b*c*x+b^2*c*x+2/f*(d*x+c
)*b^2/(1+exp(2*f*x+2*e))-2/f^2*b^2*d*ln(exp(f*x+e))+1/f^2*b^2*d*ln(1+exp(2
*f*x+2*e))-4/f*b*a*c*ln(exp(f*x+e))+2/f*b*a*c*ln(1+exp(2*f*x+2*e))+4/f^2*b
*e*a*d*ln(exp(f*x+e))-4/f*b*a*d*e*x-2/f^2*b*a*d*e^2+2/f*b*a*d*ln(1+exp(2*f
*x+2*e))*x+a*b*d*polylog(2,-exp(2*f*x+2*e))/f^2
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 944, normalized size of antiderivative = 7.43

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 + 2*(a^2 - 2*a*b + b^2)*c
*f^2*x - 4*b^2*d*e + ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*
c*e*f - 4*b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x
+ e)^2 + 2*((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4*
b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*cosh(f*x + e)*sinh(
f*x + e) + ((a^2 - 2*a*b + b^2)*d*f^2*x^2 + 4*a*b*d*e^2 - 8*a*b*c*e*f - 4*
b^2*d*e - 2*(2*b^2*d*f - (a^2 - 2*a*b + b^2)*c*f^2)*x)*sinh(f*x + e)^2 - 4
*(2*a*b*c*e - b^2*c)*f + 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*
sinh(f*x + e) + a*b*d*sinh(f*x + e)^2 + a*b*d)*dilog(I*cosh(f*x + e) + I*s
inh(f*x + e)) + 4*(a*b*d*cosh(f*x + e)^2 + 2*a*b*d*cosh(f*x + e)*sinh(f*x
+ e) + a*b*d*sinh(f*x + e)^2 + a*b*d)*dilog(-I*cosh(f*x + e) - I*sinh(f*x
+ e)) - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d + (2*a*b*d*e - 2*a*b*c*f - b^2*d)
*cosh(f*x + e)^2 + 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh(f*x + e)*sinh(f*
x + e) + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2)*log(cosh(f*x + e
) + sinh(f*x + e) + I) - 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d + (2*a*b*d*e - 2
*a*b*c*f - b^2*d)*cosh(f*x + e)^2 + 2*(2*a*b*d*e - 2*a*b*c*f - b^2*d)*cosh
(f*x + e)*sinh(f*x + e) + (2*a*b*d*e - 2*a*b*c*f - b^2*d)*sinh(f*x + e)^2)
*log(cosh(f*x + e) + sinh(f*x + e) - I) + 4*(a*b*d*f*x + a*b*d*e + (a*b*d*
f*x + a*b*d*e)*cosh(f*x + e)^2 + 2*(a*b*d*f*x + a*b*d*e)*cosh(f*x + e)*sin
h(f*x + e) + (a*b*d*f*x + a*b*d*e)*sinh(f*x + e)^2)*log(I*cosh(f*x + e)...
```

Sympy [F]

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh(e+fx))^2\,(c+dx)\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**2*(c + d*x), x)
```

Maxima [F]

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = \int (dx+c)(b\tanh(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*a^2*d*x^2 + (x^2 - 4*integrate(x/(e^(2*f*x + 2*e) + 1), x))*a*b*d + b^
2*c*(x + e/f - 2/(f*(e^(-2*f*x - 2*e) + 1))) + a^2*c*x + 1/2*b^2*d*((f*x^2
 + (f*x^2*e^(2*e) - 4*x*e^(2*e))*e^(2*f*x))/(f*e^(2*f*x + 2*e) + f) + 2*lo
g((e^(2*f*x + 2*e) + 1)*e^(-2*e))/f^2) + 2*a*b*c*log(cosh(f*x + e))/f
```

Giac [F]

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = \int (dx+c)(b\tanh(fx+e)+a)^2\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx = \int (a+b\tanh(e+f\,x))^2\,(c+d\,x)\,dx$$

input
```
int((a + b*tanh(e + f*x))^2*(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^2*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)(a+b\tanh(e+fx))^2\,dx$$
$$= \frac{-8e^{2fx+2e}\left(\int \frac{x}{e^{4fx+4e}+2e^{2fx+2e}+1}dx\right) abd\,f^2 + 4e^{2fx+2e}\log\left(e^{2fx+2e}+1\right) abcf + 2e^{2fx+2e}\log\left(e^{2fx+2e}+1\right) ab}{}$$

input
```
int((d*x+c)*(a+b*tanh(f*x+e))^2,x)
```

output
```
( - 8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x) + 2*e**(2*e + 2*f*x) + 1),x
)*a*b*d*f**2 + 4*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*a*b*c*f + 2*e*
*(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*a*b*d + 2*e**(2*e + 2*f*x)*log(e*
*(2*e + 2*f*x) + 1)*b**2*d + 2*e**(2*e + 2*f*x)*a**2*c*f**2*x + e**(2*e +
2*f*x)*a**2*d*f**2*x**2 - 4*e**(2*e + 2*f*x)*a*b*c*f**2*x + 2*e**(2*e + 2*
f*x)*a*b*d*f**2*x**2 - 4*e**(2*e + 2*f*x)*a*b*d*f*x + 2*e**(2*e + 2*f*x)*b
**2*c*f**2*x - 4*e**(2*e + 2*f*x)*b**2*c*f + e**(2*e + 2*f*x)*b**2*d*f**2*
x**2 - 4*e**(2*e + 2*f*x)*b**2*d*f*x - 8*int(x/(e**(4*e + 4*f*x) + 2*e**(2
*e + 2*f*x) + 1),x)*a*b*d*f**2 + 4*log(e**(2*e + 2*f*x) + 1)*a*b*c*f + 2*l
og(e**(2*e + 2*f*x) + 1)*a*b*d + 2*log(e**(2*e + 2*f*x) + 1)*b**2*d + 2*a*
*2*c*f**2*x + a**2*d*f**2*x**2 - 4*a*b*c*f**2*x + 2*a*b*d*f**2*x**2 + 2*b*
*2*c*f**2*x + b**2*d*f**2*x**2)/(2*f**2*(e**(2*e + 2*f*x) + 1))
```

3.61 $\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$

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Maple [N/A] 485
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Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^2}{c+dx}, x\right)$$

output
```
Defer(Int)((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 25.14 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x),x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\tanh(e + fx))^2}{c + dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a - ib\tan(ie + ifx))^2}{c + dx}\,dx$$
$$\downarrow 4223$$
$$\int \frac{(a + b\tanh(e + fx))^2}{c + dx}\,dx$$

input
```
Int[(a + b*Tanh[e + f*x])^2/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\tanh(fx + e))^2}{dx + c}\,dx$$

input
```
int((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

output
```
int((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.80

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b^2*tanh(f*x + e)^2 + 2*a*b*tanh(f*x + e) + a^2)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.18 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**2/(d*x+c),x)
```

output
```
Integral((a + b*tanh(e + f*x))**2/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 155, normalized size of antiderivative = 7.75

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="maxima")
```

output
```
a^2*log(d*x + c)/d + 2*b^2/(d*f*x + c*f + (d*f*x*e^(2*e) + c*f*e^(2*e))*e^
(2*f*x)) + (2*a*b + b^2)*log(d*x + c)/d - integrate(2*(2*a*b*d*f*x + 2*a*b
*c*f - b^2*d)/(d^2*f*x^2 + 2*c*d*f*x + c^2*f + (d^2*f*x^2*e^(2*e) + 2*c*d*
f*x*e^(2*e) + c^2*f*e^(2*e))*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^2/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.55 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx = \int \frac{(a+b\tanh(e+f\,x))^2}{c+d\,x}\,dx$$

input
```
int((a + b*tanh(e + f*x))^2/(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^2/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 258, normalized size of antiderivative = 12.90

$$\int \frac{(a+b\tanh(e+fx))^2}{c+dx}\,dx$$
$$= \frac{-4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c+e^{4fx+4e}dx+2e^{2fx+2e}c+2e^{2fx+2e}dx+c+dx}dx\right)abd-4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c+e^{4fx+4e}dx+2e^{2fx+2e}c+2e^{2fx+2e}dx+}\right.}{}$$

input
```
int((a+b*tanh(f*x+e))^2/(d*x+c),x)
```

output
```
( - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c + e**(4*e + 4*f*x)*d*x +
 2*e**(2*e + 2*f*x)*c + 2*e**(2*e + 2*f*x)*d*x + c + d*x),x)*a*b*d - 4*e**
(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c + e**(4*e + 4*f*x)*d*x + 2*e**(2*
e + 2*f*x)*c + 2*e**(2*e + 2*f*x)*d*x + c + d*x),x)*b**2*d - 4*int(1/(e**(
4*e + 4*f*x)*c + e**(4*e + 4*f*x)*d*x + 2*e**(2*e + 2*f*x)*c + 2*e**(2*e +
 2*f*x)*d*x + c + d*x),x)*a*b*d + log(c + d*x)*a**2 + 2*log(c + d*x)*a*b +
 log(c + d*x)*b**2)/d
```

3.62 $\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$

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Mathematica [N/A] . . . 489
Rubi [N/A] . . . 490
Maple [N/A] . . . 490
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Giac [N/A] . . . 492
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Reduce [N/A] . . . 493

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 18.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^2/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a-ib\tan(ie+ifx))^2}{(c+dx)^2}\,dx$$
$$\downarrow 4223$$
$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
Int[(a + b*Tanh[e + f*x])^2/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tanh(fx+e))^2}{(dx+c)^2}\,dx$$

input
```
int((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

output
```
int((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 47, normalized size of antiderivative = 2.35

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b^2*tanh(f*x + e)^2 + 2*a*b*tanh(f*x + e) + a^2)/(d^2*x^2 + 2*c*
d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.05 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**2/(d*x+c)**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**2/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 280, normalized size of antiderivative = 14.00

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-a^2/(d^2*x + c*d) - (2*a*b*c*f + (c*f - 2*d)*b^2 + (2*a*b*d*f + b^2*d*f)*
x + (2*a*b*c*f*e^(2*e) + b^2*c*f*e^(2*e) + (2*a*b*d*f*e^(2*e) + b^2*d*f*e^
(2*e))*x)*e^(2*f*x))/(d^3*f*x^2 + 2*c*d^2*f*x + c^2*d*f + (d^3*f*x^2*e^(2*
e) + 2*c*d^2*f*x*e^(2*e) + c^2*d*f*e^(2*e))*e^(2*f*x)) - integrate(4*(a*b*
d*f*x + a*b*c*f - b^2*d)/(d^3*f*x^3 + 3*c*d^2*f*x^2 + 3*c^2*d*f*x + c^3*f
+ (d^3*f*x^3*e^(2*e) + 3*c*d^2*f*x^2*e^(2*e) + 3*c^2*d*f*x*e^(2*e) + c^3*f
*e^(2*e))*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.97 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^2}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^2/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^2/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+f\,x))^2}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*tanh(e + f*x))^2/(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^2/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 803, normalized size of antiderivative = 40.15

$$\int \frac{(a+b\tanh(e+fx))^2}{(c+dx)^2}\,dx$$
$$= \frac{-4e^{2e}\left(\int \frac{e^{2fx}}{e^{4fx+4e}c^2+2e^{4fx+4e}cdx+e^{4fx+4e}d^2x^2+2e^{2fx+2e}c^2+4e^{2fx+2e}cdx+2e^{2fx+2e}d^2x^2+c^2+2cdx+d^2x^2}dx\right)abc^2 - 4e^{2e}\left(\int \frac{}{e}\right.}$$

input
```
int((a+b*tanh(f*x+e))^2/(d*x+c)^2,x)
```

output
```
( - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*
c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*e**(2*e + 2*f*x)*c**2 + 4*e**(2*e +
 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2)
,x)*a*b*c**2 - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**2 + 2*e**(4*
e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*e**(2*e + 2*f*x)*c**2 +
4*e**(2*e + 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x +
 d**2*x**2),x)*a*b*c*d*x - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*c**
2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*e**(2*e + 2*
f*x)*c**2 + 4*e**(2*e + 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2
 + 2*c*d*x + d**2*x**2),x)*b**2*c**2 - 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e
+ 4*f*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*
e**(2*e + 2*f*x)*c**2 + 4*e**(2*e + 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2
*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*b**2*c*d*x - 4*int(1/(e**(4*e + 4*f
*x)*c**2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*e**(2
*e + 2*f*x)*c**2 + 4*e**(2*e + 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2*x**2
 + c**2 + 2*c*d*x + d**2*x**2),x)*a*b*c**2 - 4*int(1/(e**(4*e + 4*f*x)*c**
2 + 2*e**(4*e + 4*f*x)*c*d*x + e**(4*e + 4*f*x)*d**2*x**2 + 2*e**(2*e + 2*
f*x)*c**2 + 4*e**(2*e + 2*f*x)*c*d*x + 2*e**(2*e + 2*f*x)*d**2*x**2 + c**2
 + 2*c*d*x + d**2*x**2),x)*a*b*c*d*x + a**2*x + 2*a*b*x + b**2*x)/(c*(c +
d*x))
```

3.63 $\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx$

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Rubi [A] (verified) 497
Maple [B] (verified) 499
Fricas [C] (verification not implemented) 500
Sympy [F] 501
Maxima [B] (verification not implemented) 501
Giac [F] 502
Mupad [F(-1)] 503
Reduce [F] 503

Optimal result

Integrand size = 20, antiderivative size = 566

$$\begin{aligned}\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = &-\frac{3b^3d(c+dx)^2}{2f^2}-\frac{3ab^2(c+dx)^3}{f}+\frac{b^3(c+dx)^3}{2f}\\&+\frac{a^3(c+dx)^4}{4d}-\frac{3a^2b(c+dx)^4}{4d}+\frac{3ab^2(c+dx)^4}{4d}\\&-\frac{b^3(c+dx)^4}{4d}+\frac{3b^3d^2(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^3}\\&+\frac{9ab^2d(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f^2}\\&+\frac{3a^2b(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f}\\&+\frac{b^3(c+dx)^3\log\left(1+e^{2(e+fx)}\right)}{f}\\&+\frac{3b^3d^3\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^4}\\&+\frac{9ab^2d^2(c+dx)\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}\\&+\frac{9a^2bd(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}\\&+\frac{3b^3d(c+dx)^2\operatorname{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}\\&-\frac{9ab^2d^3\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4}\\&-\frac{9a^2bd^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}\\&-\frac{3b^3d^2(c+dx)\operatorname{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}\\&+\frac{9a^2bd^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}\\&+\frac{3b^3d^3\operatorname{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}\\&-\frac{3b^3d(c+dx)^2\tanh(e+fx)}{2f^2}\\&-\frac{3ab^2(c+dx)^3\tanh(e+fx)}{f}\\&-\frac{b^3(c+dx)^3\tanh^2(e+fx)}{2f}\end{aligned}$$

output

```
-3/2*b^3*d*(d*x+c)^2/f^2-3*a*b^2*(d*x+c)^3/f+1/2*b^3*(d*x+c)^3/f+1/4*a^3*(
d*x+c)^4/d-3/4*a^2*b*(d*x+c)^4/d+3/4*a*b^2*(d*x+c)^4/d-1/4*b^3*(d*x+c)^4/d
+3*b^3*d^2*(d*x+c)*ln(1+exp(2*f*x+2*e))/f^3+9*a*b^2*d*(d*x+c)^2*ln(1+exp(2
*f*x+2*e))/f^2+3*a^2*b*(d*x+c)^3*ln(1+exp(2*f*x+2*e))/f+b^3*(d*x+c)^3*ln(1
+exp(2*f*x+2*e))/f+3/2*b^3*d^3*polylog(2,-exp(2*f*x+2*e))/f^4+9*a*b^2*d^2*
(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^3+9/2*a^2*b*d*(d*x+c)^2*polylog(2,-ex
p(2*f*x+2*e))/f^2+3/2*b^3*d*(d*x+c)^2*polylog(2,-exp(2*f*x+2*e))/f^2-9/2*a
*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4-9/2*a^2*b*d^2*(d*x+c)*polylog(3,-e
xp(2*f*x+2*e))/f^3-3/2*b^3*d^2*(d*x+c)*polylog(3,-exp(2*f*x+2*e))/f^3+9/4*
a^2*b*d^3*polylog(4,-exp(2*f*x+2*e))/f^4+3/4*b^3*d^3*polylog(4,-exp(2*f*x+
2*e))/f^4-3/2*b^3*d*(d*x+c)^2*tanh(f*x+e)/f^2-3*a*b^2*(d*x+c)^3*tanh(f*x+e
)/f-1/2*b^3*(d*x+c)^3*tanh(f*x+e)^2/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2010 vs. $2(566) = 1132$.

Time $= 7.72$ (sec) , antiderivative size $= 2010$, normalized size of antiderivative $= 3.55$

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \text{Result too large to show}$$

input

```
Integrate[(c + d*x)^3*(a + b*Tanh[e + f*x])^3,x]
```

output

```
(b*E^(2*e)*(-24*b^2*c*d^2*x - 72*a*b*c^2*d*f*x - 24*a^2*c^3*f^2*x - 8*b^2*
c^3*f^2*x - 12*b^2*d^3*x^2 - 72*a*b*c*d^2*f*x^2 - 36*a^2*c^2*d*f^2*x^2 - 1
2*b^2*c^2*d*f^2*x^2 - 24*a*b*d^3*f*x^3 - 24*a^2*c*d^2*f^2*x^3 - 8*b^2*c*d^
2*f^2*x^3 - 6*a^2*d^3*f^2*x^4 - 2*b^2*d^3*f^2*x^4 + 36*a*b*c^2*d*Log[1 + E
^(2*(e + f*x))] + (36*a*b*c^2*d*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (12*b^
2*c*d^2*Log[1 + E^(2*(e + f*x))])/f + (12*b^2*c*d^2*Log[1 + E^(2*(e + f*x)
)])/(E^(2*e)*f) + 12*a^2*c^3*f*Log[1 + E^(2*(e + f*x))] + 4*b^2*c^3*f*Log[
1 + E^(2*(e + f*x))] + (12*a^2*c^3*f*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (
4*b^2*c^3*f*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 72*a*b*c*d^2*x*Log[1 + E^(
2*(e + f*x))] + (72*a*b*c*d^2*x*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (12*b^
2*d^3*x*Log[1 + E^(2*(e + f*x))])/f + (12*b^2*d^3*x*Log[1 + E^(2*(e + f*x)
)])/(E^(2*e)*f) + 36*a^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))] + 12*b^2*c^2*d
*f*x*Log[1 + E^(2*(e + f*x))] + (36*a^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))]
)/E^(2*e) + (12*b^2*c^2*d*f*x*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 36*a*b*d
^3*x^2*Log[1 + E^(2*(e + f*x))] + (36*a*b*d^3*x^2*Log[1 + E^(2*(e + f*x))]
)/E^(2*e) + 36*a^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))] + 12*b^2*c*d^2*f*x
^2*Log[1 + E^(2*(e + f*x))] + (36*a^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))]
)/E^(2*e) + (12*b^2*c*d^2*f*x^2*Log[1 + E^(2*(e + f*x))])/E^(2*e) + 12*a^2
*d^3*f*x^3*Log[1 + E^(2*(e + f*x))] + 4*b^2*d^3*f*x^3*Log[1 + E^(2*(e + f*
x))] + (12*a^2*d^3*f*x^3*Log[1 + E^(2*(e + f*x))])/E^(2*e) + (4*b^2*d^3...
```

Rubi [A] (verified)

Time = 1.48 (sec) , antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^3(a-ib\tan(ie+ifx))^3 dx$$

$$\downarrow 4205$$

$$\int \left(a^3(c+dx)^3 + 3a^2b(c+dx)^3\tanh(e+fx) + 3ab^2(c+dx)^3\tanh^2(e+fx) + b^3(c+dx)^3\tanh^3(e+fx)\right)dx$$

$$\downarrow \text{2009}$$

$$\frac{a^3(c+dx)^4}{4d} - \frac{9a^2bd^2(c+dx)\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} + \frac{9a^2bd(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} + \frac{3a^2b(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f} - \frac{3a^2b(c+dx)^4}{4d} + \frac{9a^2bd^3\,\text{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4} + \frac{9ab^2d^2(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3} + \frac{9ab^2d(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f^2} - \frac{3ab^2(c+dx)^3\tanh(e+fx)}{f} - \frac{3ab^2(c+dx)^3}{f} + \frac{3ab^2(c+dx)^4}{4d} - \frac{9ab^2d^3\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^4} - \frac{3b^3d^2(c+dx)\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3} + \frac{3b^3d^2(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^3} + \frac{3b^3d(c+dx)^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2} - \frac{3b^3d(c+dx)^2\tanh(e+fx)}{2f^2} + \frac{b^3(c+dx)^3\log\left(e^{2(e+fx)}+1\right)}{f} - \frac{b^3(c+dx)^3\tanh^2(e+fx)}{2f} - \frac{3b^3d(c+dx)^2}{2f^2} + \frac{b^3(c+dx)^3}{2f} - \frac{b^3(c+dx)^4}{4d} + \frac{3b^3d^3\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^4} + \frac{3b^3d^3\,\text{PolyLog}\left(4,-e^{2(e+fx)}\right)}{4f^4}$$

input
```
Int[(c + d*x)^3*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(-3*b^3*d*(c + d*x)^2)/(2*f^2) - (3*a*b^2*(c + d*x)^3)/f + (b^3*(c + d*x)^
3)/(2*f) + (a^3*(c + d*x)^4)/(4*d) - (3*a^2*b*(c + d*x)^4)/(4*d) + (3*a*b^
2*(c + d*x)^4)/(4*d) - (b^3*(c + d*x)^4)/(4*d) + (3*b^3*d^2*(c + d*x)*Log[
1 + E^(2*(e + f*x))])/f^3 + (9*a*b^2*d*(c + d*x)^2*Log[1 + E^(2*(e + f*x))
])/f^2 + (3*a^2*b*(c + d*x)^3*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)
^3*Log[1 + E^(2*(e + f*x))])/f + (3*b^3*d^3*PolyLog[2, -E^(2*(e + f*x))])/
(2*f^4) + (9*a*b^2*d^2*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^3 + (9*a^
2*b*d*(c + d*x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) + (3*b^3*d*(c + d*
x)^2*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (9*a*b^2*d^3*PolyLog[3, -E^(2
*(e + f*x))])/(2*f^4) - (9*a^2*b*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))
])/(2*f^3) - (3*b^3*d^2*(c + d*x)*PolyLog[3, -E^(2*(e + f*x))])/(2*f^3) +
(9*a^2*b*d^3*PolyLog[4, -E^(2*(e + f*x))])/(4*f^4) + (3*b^3*d^3*PolyLog[4,
 -E^(2*(e + f*x))])/(4*f^4) - (3*b^3*d*(c + d*x)^2*Tanh[e + f*x])/(2*f^2)
- (3*a*b^2*(c + d*x)^3*Tanh[e + f*x])/f - (b^3*(c + d*x)^3*Tanh[e + f*x]^2
)/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1833 vs. $2(534) = 1068$.

Time = 1.83 (sec) , antiderivative size = 1834, normalized size of antiderivative = 3.24

method	result	size
risch	Expression too large to display	1834

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
-9/2*a*b^2*d^3*polylog(3,-exp(2*f*x+2*e))/f^4+9/4*a^2*b*d^3*polylog(4,-exp
(2*f*x+2*e))/f^4+3/2*b^3*d^3*polylog(2,-exp(2*f*x+2*e))/f^4+3/4*b^3*d^3*po
lylog(4,-exp(2*f*x+2*e))/f^4+9/f^2*b^2*a*d^3*ln(1+exp(2*f*x+2*e))*x^2+9/f^
3*b^2*a*d^3*polylog(2,-exp(2*f*x+2*e))*x+6/f^4*b*e^3*a^2*d^3*ln(exp(f*x+e)
)-6/f^3*b^3*e^2*c*d^2*ln(exp(f*x+e))-9/2/f^3*b*d^2*c*a^2*polylog(3,-exp(2*
f*x+2*e))+3/f*b*a^2*d^3*ln(1+exp(2*f*x+2*e))*x^3+9/2/f^2*b*a^2*d^3*polylog
(2,-exp(2*f*x+2*e))*x^2-9/2/f^3*b*a^2*d^3*polylog(3,-exp(2*f*x+2*e))*x+9/f
^3*b^2*a*c*d^2*polylog(2,-exp(2*f*x+2*e))+9/2/f^2*b*d*c^2*a^2*polylog(2,-e
xp(2*f*x+2*e))-3/f^4*b^3*e^2*d^3-3/f^2*b^3*d^3*x^2-3/2/f^4*b^3*d^3*e^4-2/f
*b^3*c^3*ln(exp(f*x+e))+1/f*b^3*c^3*ln(1+exp(2*f*x+2*e))+1/4*d^3*a^3*x^4-1
/4*d^3*b^3*x^4+1/4/d*c^4*a^3+1/4/d*c^4*b^3+b^2*(6*a*d^3*f*x^3*exp(2*f*x+2*
e)+2*b*d^3*f*x^3*exp(2*f*x+2*e)+18*a*c*d^2*f*x^2*exp(2*f*x+2*e)+6*b*c*d^2*
f*x^2*exp(2*f*x+2*e)+18*a*c^2*d*f*x*exp(2*f*x+2*e)+6*a*d^3*f*x^3+6*b*c^2*d
*f*x*exp(2*f*x+2*e)+3*b*d^3*x^2*exp(2*f*x+2*e)+6*a*c^3*f*exp(2*f*x+2*e)+18
*a*c*d^2*f*x^2+2*b*c^3*f*exp(2*f*x+2*e)+6*b*c*d^2*x*exp(2*f*x+2*e)+18*a*c^
2*d*f*x+3*b*c^2*d*exp(2*f*x+2*e)+3*b*d^3*x^2+6*a*c^3*f+6*b*c*d^2*x+3*b*c^2
*d)/f^2/(1+exp(2*f*x+2*e))^2+6/f^2*b^3*c*d^2*e^2*x-6/f*b^3*d*c^2*e*x+18/f^
3*b^2*e^2*a*d^3*x-18/f*b^2*a*c*d^2*x^2+12/f^3*b*d^2*c*a^2*e^3-6/f^3*b*a^2*
d^3*e^3*x-18/f^3*b^2*a*c*d^2*e^2-9/f^2*b*d*c^2*a^2*e^2+3/f*b^3*c*d^2*ln(1+
exp(2*f*x+2*e))*x^2+3/f^2*b^3*c*d^2*polylog(2,-exp(2*f*x+2*e))*x-18/f^2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.27 (sec) , antiderivative size = 12909, normalized size of antiderivative = 22.81

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \int (a+b\tanh(e+fx))^3\,(c+dx)^3\,dx$$

input
```
integrate((d*x+c)**3*(a+b*tanh(f*x+e))**3,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3*(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1297 vs. $2(530) = 1060$.

Time $= 0.22$ (sec) , antiderivative size $= 1297$, normalized size of antiderivative $= 2.29$

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

output

```
1/4*a^3*d^3*x^4 + a^3*c*d^2*x^3 + 3/2*a^3*c^2*d*x^2 + b^3*c^3*(x + e/f + l
og(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e
^(-4*f*x - 4*e) + 1))) + a^3*c^3*x + 3*a^2*b*c^3*log(cosh(f*x + e))/f + 1/
4*(24*a*b^2*c^3*f + 12*b^3*c^2*d + (3*a^2*b*d^3*f^2 + 3*a*b^2*d^3*f^2 + b^
3*d^3*f^2)*x^4 + 4*(3*a^2*b*c*d^2*f^2 + b^3*c*d^2*f^2 + 3*(c*d^2*f^2 + 2*d
^3*f)*a*b^2)*x^3 + 6*(3*a^2*b*c^2*d*f^2 + 3*(c^2*d*f^2 + 4*c*d^2*f)*a*b^2
+ (c^2*d*f^2 + 2*d^3)*b^3)*x^2 + 12*(2*b^3*c*d^2 + (c^3*f^2 + 6*c^2*d*f)*a
*b^2)*x + (12*a*b^2*c^3*f^2*x*e^(4*e) + (3*a^2*b*d^3*f^2*e^(4*e) + 3*a*b^2
*d^3*f^2*e^(4*e) + b^3*d^3*f^2*e^(4*e))*x^4 + 4*(3*a^2*b*c*d^2*f^2*e^(4*e)
 + 3*a*b^2*c*d^2*f^2*e^(4*e) + b^3*c*d^2*f^2*e^(4*e))*x^3 + 6*(3*a^2*b*c^2
*d*f^2*e^(4*e) + 3*a*b^2*c^2*d*f^2*e^(4*e) + b^3*c^2*d*f^2*e^(4*e))*x^2)*e
^(4*f*x) + 2*(12*a*b^2*c^3*f*e^(2*e) + 6*b^3*c^2*d*e^(2*e) + (3*a^2*b*d^3*
f^2*e^(2*e) + 3*a*b^2*d^3*f^2*e^(2*e) + b^3*d^3*f^2*e^(2*e))*x^4 + 4*(3*a^
2*b*c*d^2*f^2*e^(2*e) + 3*(c*d^2*f^2*e^(2*e) + d^3*f*e^(2*e))*a*b^2 + (c*d
^2*f^2*e^(2*e) + d^3*f*e^(2*e))*b^3)*x^3 + 6*(3*a^2*b*c^2*d*f^2*e^(2*e) +
3*(c^2*d*f^2*e^(2*e) + 2*c*d^2*f*e^(2*e))*a*b^2 + (c^2*d*f^2*e^(2*e) + 2*c
*d^2*f*e^(2*e) + d^3*e^(2*e))*b^3)*x^2 + 12*((c^3*f^2*e^(2*e) + 3*c^2*d*f*
e^(2*e))*a*b^2 + (c^2*d*f*e^(2*e) + c*d^2*e^(2*e))*b^3)*x)*e^(2*f*x))/(f^2
*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) - 6*(3*a*b^2*c^2*d*f + b^3
*c*d^2)*x/f^2 + 3*(3*a*b^2*c^2*d*f + b^3*c*d^2)*log(e^(2*f*x + 2*e) + 1...
```

Giac [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \int (dx+c)^3(b\tanh(fx+e)+a)^3\,dx$$

input

```
integrate((d*x+c)^3*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3*(b*tanh(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \int (a+b\tanh(e+f\,x))^3\,(c+d\,x)^3\,dx$$

input
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^3,x)
```

output
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^3, x)
```

Reduce [F]

$$\int (c+dx)^3(a+b\tanh(e+fx))^3\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3*(a+b*tanh(f*x+e))^3,x)
```

output

```
( - 384*e**(4*e + 4*f*x)*int(x**3/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) +
 3*e**(2*e + 2*f*x) + 1),x)*a**2*b*d**3*f**4 - 128*e**(4*e + 4*f*x)*int(x*
*3/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*b**
3*d**3*f**4 - 1152*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e
 + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*a**2*b*c*d**2*f**4 - 864*e**(4*e +
4*f*x)*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x
) + 1),x)*a**2*b*d**3*f**3 - 1152*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f
*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*a*b**2*d**3*f**3 - 3
84*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e*
*(2*e + 2*f*x) + 1),x)*b**3*c*d**2*f**4 - 288*e**(4*e + 4*f*x)*int(x**2/(e
**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*b**3*d**
3*f**3 - 1152*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x
) + 3*e**(2*e + 2*f*x) + 1),x)*a**2*b*c**2*d*f**4 - 1728*e**(4*e + 4*f*x)*
int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*
a**2*b*c*d**2*f**3 - 1008*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**
(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*a**2*b*d**3*f**2 - 2304*e**(4*e
 + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x
) + 1),x)*a*b**2*c*d**2*f**3 - 1728*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*
x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*a*b**2*d**3*f**2 - 38
4*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**...
```

3.64 $\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx$

Optimal result 506
Mathematica [B] (verified) 507
Rubi [A] (verified) 508
Maple [B] (verified) 510
Fricas [C] (verification not implemented) 511
Sympy [F] 511
Maxima [B] (verification not implemented) 511
Giac [F] 512
Mupad [F(-1)] 513
Reduce [F] 513

Optimal result

Integrand size = 20, antiderivative size = 396

$$\begin{aligned}\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = &-\frac{3ab^2(c+dx)^2}{f}+\frac{b^3(c+dx)^2}{2f}+\frac{a^3(c+dx)^3}{3d}\\&-\frac{a^2b(c+dx)^3}{d}+\frac{ab^2(c+dx)^3}{d}-\frac{b^3(c+dx)^3}{3d}\\&+\frac{6ab^2d(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f^2}\\&+\frac{3a^2b(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f}\\&+\frac{b^3(c+dx)^2\log\left(1+e^{2(e+fx)}\right)}{f}\\&+\frac{b^3d^2\log(\cosh(e+fx))}{f^3}\\&+\frac{3ab^2d^2\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}\\&+\frac{3a^2bd(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}\\&+\frac{b^3d(c+dx)\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}\\&-\frac{3a^2bd^2\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}\\&-\frac{b^3d^2\,\mathrm{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}\\&-\frac{b^3d(c+dx)\tanh(e+fx)}{f^2}\\&-\frac{3ab^2(c+dx)^2\tanh(e+fx)}{f}\\&-\frac{b^3(c+dx)^2\tanh^2(e+fx)}{2f}\end{aligned}$$

output

```
-3*a*b^2*(d*x+c)^2/f+1/2*b^3*(d*x+c)^2/f+1/3*a^3*(d*x+c)^3/d-a^2*b*(d*x+c)
^3/d+a*b^2*(d*x+c)^3/d-1/3*b^3*(d*x+c)^3/d+6*a*b^2*d*(d*x+c)*ln(1+exp(2*f*
x+2*e))/f^2+3*a^2*b*(d*x+c)^2*ln(1+exp(2*f*x+2*e))/f+b^3*(d*x+c)^2*ln(1+ex
p(2*f*x+2*e))/f+b^3*d^2*ln(cosh(f*x+e))/f^3+3*a*b^2*d^2*polylog(2,-exp(2*f
*x+2*e))/f^3+3*a^2*b*d*(d*x+c)*polylog(2,-exp(2*f*x+2*e))/f^2+b^3*d*(d*x+c
)*polylog(2,-exp(2*f*x+2*e))/f^2-3/2*a^2*b*d^2*polylog(3,-exp(2*f*x+2*e))/
f^3-1/2*b^3*d^2*polylog(3,-exp(2*f*x+2*e))/f^3-b^3*d*(d*x+c)*tanh(f*x+e)/f
^2-3*a*b^2*(d*x+c)^2*tanh(f*x+e)/f-1/2*b^3*(d*x+c)^2*tanh(f*x+e)^2/f
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1163 vs. $2(396) = 792$.

Time $= 7.17$ (sec) , antiderivative size $= 1163$, normalized size of antiderivative $= 2.94$

$$\int (c + dx)^2 (a + b \tanh(e + fx))^3 \, dx = \text{Too large to display}$$

input

```
Integrate[(c + d*x)^2*(a + b*Tanh[e + f*x])^3,x]
```

output

```
(b*(-4*E^(2*e)*f*x*(9*a*b*d*f*(2*c + d*x) + 3*a^2*f^2*(3*c^2 + 3*c*d*x + d
^2*x^2) + b^2*(3*c^2*f^2 + 3*c*d*f^2*x + d^2*(3 + f^2*x^2))) + 6*(1 + E^(2
*e))*(6*a*b*d*f*(c + d*x) + 3*a^2*f^2*(c + d*x)^2 + b^2*(c^2*f^2 + 2*c*d*f
^2*x + d^2*(1 + f^2*x^2)))*Log[1 + E^(2*(e + f*x))] + 6*d*(1 + E^(2*e))*(3
*a*b*d + 3*a^2*f*(c + d*x) + b^2*f*(c + d*x))*PolyLog[2, -E^(2*(e + f*x))]
 - 3*(3*a^2 + b^2)*d^2*(1 + E^(2*e))*PolyLog[3, -E^(2*(e + f*x))]))/(6*(1
+ E^(2*e))*f^3) + (Sech[e]*Sech[e + f*x]^2*(6*b^3*c^2*f*Cosh[e] + 12*b^3*c
*d*f*x*Cosh[e] + 6*a^3*c^2*f^2*x*Cosh[e] + 18*a*b^2*c^2*f^2*x*Cosh[e] + 6*
b^3*d^2*f*x^2*Cosh[e] + 6*a^3*c*d*f^2*x^2*Cosh[e] + 18*a*b^2*c*d*f^2*x^2*C
osh[e] + 2*a^3*d^2*f^2*x^3*Cosh[e] + 6*a*b^2*d^2*f^2*x^3*Cosh[e] + 3*a^3*c
^2*f^2*x*Cosh[e + 2*f*x] + 9*a*b^2*c^2*f^2*x*Cosh[e + 2*f*x] + 3*a^3*c*d*f
^2*x^2*Cosh[e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*Cosh[e + 2*f*x] + a^3*d^2*f^2
*x^3*Cosh[e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*Cosh[e + 2*f*x] + 3*a^3*c^2*f^2
*x*Cosh[3*e + 2*f*x] + 9*a*b^2*c^2*f^2*x*Cosh[3*e + 2*f*x] + 3*a^3*c*d*f^2
*x^2*Cosh[3*e + 2*f*x] + 9*a*b^2*c*d*f^2*x^2*Cosh[3*e + 2*f*x] + a^3*d^2*f
^2*x^3*Cosh[3*e + 2*f*x] + 3*a*b^2*d^2*f^2*x^3*Cosh[3*e + 2*f*x] + 6*b^3*c
*d*Sinh[e] + 18*a*b^2*c^2*f*Sinh[e] + 6*b^3*d^2*x*Sinh[e] + 36*a*b^2*c*d*f
*x*Sinh[e] + 18*a^2*b*c^2*f^2*x*Sinh[e] + 6*b^3*c^2*f^2*x*Sinh[e] + 18*a*b
^2*d^2*f*x^2*Sinh[e] + 18*a^2*b*c*d*f^2*x^2*Sinh[e] + 6*b^3*c*d*f^2*x^2*Si
nh[e] + 6*a^2*b*d^2*f^2*x^3*Sinh[e] + 2*b^3*d^2*f^2*x^3*Sinh[e] - 6*b^3...
```

Rubi [A] (verified)

Time = 1.08 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)^2(a-ib\tan(ie+ifx))^3 dx$$

$$\downarrow 4205$$

$$\int \left(a^3(c+dx)^2+3a^2b(c+dx)^2\tanh(e+fx)+3ab^2(c+dx)^2\tanh^2(e+fx)+b^3(c+dx)^2\tanh^3(e+fx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^3(c+dx)^3}{3d}+\frac{3a^2bd(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}+\frac{3a^2b(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{a^2b(c+dx)^3}{d}-\frac{3a^2bd^2\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{6ab^2d(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f^2}-\frac{3ab^2(c+dx)^2\tanh(e+fx)}{f}-\frac{3ab^2(c+dx)^2}{f}+\frac{ab^2(c+dx)^3}{d}+\frac{3ab^2d^2\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^3}+\frac{b^3d(c+dx)\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{f^2}-\frac{b^3d(c+dx)\tanh(e+fx)}{f^2}+\frac{b^3(c+dx)^2\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b^3(c+dx)^2\tanh^2(e+fx)}{2f}+\frac{b^3(c+dx)^2}{2f}-\frac{b^3(c+dx)^3}{3d}-\frac{b^3d^2\,\text{PolyLog}\left(3,-e^{2(e+fx)}\right)}{2f^3}+\frac{b^3d^2\log(\cosh(e+fx))}{f^3}$$

input

```
Int[(c + d*x)^2*(a + b*Tanh[e + f*x])^3,x]
```

output

```
(-3*a*b^2*(c + d*x)^2)/f + (b^3*(c + d*x)^2)/(2*f) + (a^3*(c + d*x)^3)/(3*
d) - (a^2*b*(c + d*x)^3)/d + (a*b^2*(c + d*x)^3)/d - (b^3*(c + d*x)^3)/(3*
d) + (6*a*b^2*d*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f^2 + (3*a^2*b*(c + d*
x)^2*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)^2*Log[1 + E^(2*(e + f*x)
)])/f + (b^3*d^2*Log[Cosh[e + f*x]])/f^3 + (3*a*b^2*d^2*PolyLog[2, -E^(2*(
e + f*x))])/f^3 + (3*a^2*b*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 +
 (b^3*d*(c + d*x)*PolyLog[2, -E^(2*(e + f*x))])/f^2 - (3*a^2*b*d^2*PolyLog
[3, -E^(2*(e + f*x))])/(2*f^3) - (b^3*d^2*PolyLog[3, -E^(2*(e + f*x))])/(2
*f^3) - (b^3*d*(c + d*x)*Tanh[e + f*x])/f^2 - (3*a*b^2*(c + d*x)^2*Tanh[e
+ f*x])/f - (b^3*(c + d*x)^2*Tanh[e + f*x]^2)/(2*f)
```

Defintions of rubi rules used

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205

```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1065 vs. 2(384) = 768.

Time = 1.50 (sec) , antiderivative size = 1066, normalized size of antiderivative = 2.69

method	result	size
risch	Expression too large to display	1066

input

```
int((d*x+c)^2*(a+b*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output

```
3*a*b^2*d^2*polylog(2,-exp(2*f*x+2*e))/f^3-3/2*a^2*b*d^2*polylog(3,-exp(2*
f*x+2*e))/f^3-1/2*b^3*d^2*polylog(3,-exp(2*f*x+2*e))/f^3-3*d*a^2*b*c*x^2+3
*d*a*b^2*c*x^2+3*a^2*b*c^2*x+3*a*b^2*c^2*x-4/f*b^3*c*d*e*x-d^2*a^2*b*x^3+d
^2*a*b^2*x^3+d*a^3*c*x^2-d*b^3*c*x^2+a^3*c^2*x+b^3*c^2*x+1/d*a^2*b*c^3+1/d
*a*b^2*c^3-12/f^2*b^2*a*d^2*e*x+6/f^2*b*d^2*a^2*e^2*x-6/f^2*b*d*c*a^2*e^2-
6/f^3*b*e^2*d^2*a^2*ln(exp(f*x+e))-12/f^2*b^2*a*c*d*ln(exp(f*x+e))+6/f^2*b
^2*a*c*d*ln(1+exp(2*f*x+2*e))+4/f^2*b^3*e*c*d*ln(exp(f*x+e))+2/f*b^3*c*d*l
n(1+exp(2*f*x+2*e))*x+6/f^2*b^2*a*d^2*ln(1+exp(2*f*x+2*e))*x+3/f*b*d^2*a^2
*ln(1+exp(2*f*x+2*e))*x^2+3/f^2*b*d^2*a^2*polylog(2,-exp(2*f*x+2*e))*x+12/
f^3*b^2*e*a*d^2*ln(exp(f*x+e))+3/f^2*b*d*c*a^2*polylog(2,-exp(2*f*x+2*e))+
4/3/f^3*b^3*d^2*e^3-2/f*b^3*c^2*ln(exp(f*x+e))+1/f*b^3*c^2*ln(1+exp(2*f*x+
2*e))-2/f^3*b^3*d^2*ln(exp(f*x+e))+1/f^3*b^3*d^2*ln(1+exp(2*f*x+2*e))+2*b^
2*(3*a*d^2*f*x^2*exp(2*f*x+2*e)+b*d^2*f*x^2*exp(2*f*x+2*e)+6*a*c*d*f*x*exp
(2*f*x+2*e)+2*b*c*d*f*x*exp(2*f*x+2*e)+3*a*c^2*f*exp(2*f*x+2*e)+3*a*d^2*f*
x^2+b*c^2*f*exp(2*f*x+2*e)+b*d^2*x*exp(2*f*x+2*e)+6*a*c*d*f*x+b*c*d*exp(2*
f*x+2*e)+3*a*c^2*f+b*d^2*x+b*c*d)/f^2/(1+exp(2*f*x+2*e))^2-12/f*b*d*c*a^2*
e*x+12/f^2*b*e*d*c*a^2*ln(exp(f*x+e))+6/f*b*d*c*a^2*ln(1+exp(2*f*x+2*e))*x
+1/3*d^2*a^3*x^3-1/3*d^2*b^3*x^3+1/3/d*a^3*c^3+1/3/d*b^3*c^3-6/f*b^2*a*d^2
*x^2-2/f^2*b^3*c*d*e^2+1/f^2*b^3*d^2*polylog(2,-exp(2*f*x+2*e))*x+1/f^2*b^
3*c*d*polylog(2,-exp(2*f*x+2*e))-6/f*b*a^2*c^2*ln(exp(f*x+e))+3/f*b*a^2...
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.20 (sec) , antiderivative size = 7298, normalized size of antiderivative = 18.43

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \int (a+b\tanh(e+fx))^3\,(c+dx)^2\,dx$$

input
```
integrate((d*x+c)**2*(a+b*tanh(f*x+e))**3,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3*(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 869 vs. 2(381) = 762.

Time = 0.24 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.19

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

output

```
1/3*a^3*d^2*x^3 + a^3*c*d*x^2 + b^3*c^2*(x + e/f + log(e^(-2*f*x - 2*e) +
1)/f + 2*e^(-2*f*x - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1)))
 + a^3*c^2*x + 3*a^2*b*c^2*log(cosh(f*x + e))/f + 1/3*(18*a*b^2*c^2*f + 6*
b^3*c*d + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2 + b^3*d^2*f^2)*x^3 + 3*(3*a^2
*b*c*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 + 2*d^2*f)*a*b^2)*x^2 + 3*(2*b^3*d^2
 + 3*(c^2*f^2 + 4*c*d*f)*a*b^2)*x + (9*a*b^2*c^2*f^2*x*e^(4*e) + (3*a^2*b*
d^2*f^2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*e))*x^3 + 3*(
3*a^2*b*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f^2*e^(4*e))*x
^2)*e^(4*f*x) + 2*(9*a*b^2*c^2*f*e^(2*e) + 3*b^3*c*d*e^(2*e) + (3*a^2*b*d^
2*f^2*e^(2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*x^3 + 3*(3*
a^2*b*c*d*f^2*e^(2*e) + 3*(c*d*f^2*e^(2*e) + d^2*f*e^(2*e))*a*b^2 + (c*d*f
^2*e^(2*e) + d^2*f*e^(2*e))*b^3)*x^2 + 3*(3*(c^2*f^2*e^(2*e) + 2*c*d*f*e^(
2*e))*a*b^2 + (2*c*d*f*e^(2*e) + d^2*e^(2*e))*b^3)*x)*e^(2*f*x))/(f^2*e^(4
*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) - 2*(6*a*b^2*c*d*f + b^3*d^2)*x
/f^2 + 1/2*(3*a^2*b*d^2 + b^3*d^2)*(2*f^2*x^2*log(e^(2*f*x + 2*e) + 1) + 2
*f*x*dilog(-e^(2*f*x + 2*e)) - polylog(3, -e^(2*f*x + 2*e)))/f^3 + (3*a^2*
b*c*d*f + b^3*c*d*f + 3*a*b^2*d^2)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog
(-e^(2*f*x + 2*e)))/f^3 + (6*a*b^2*c*d*f + b^3*d^2)*log(e^(2*f*x + 2*e) +
1)/f^3 - 2/3*((3*a^2*b*d^2 + b^3*d^2)*f^3*x^3 + 3*(3*a^2*b*c*d*f + b^3*c*d
*f + 3*a*b^2*d^2)*f^2*x^2)/f^3
```

Giac [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \int (dx+c)^2(b\tanh(fx+e)+a)^3\,dx$$

input

```
integrate((d*x+c)^2*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

output

```
integrate((d*x + c)^2*(b*tanh(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \int (a+b\tanh(e+f\,x))^3\,(c+d\,x)^2\,dx$$

input
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^3*(c + d*x)^2, x)
```

Reduce [F]

$$\int (c+dx)^2(a+b\tanh(e+fx))^3\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2*(a+b*tanh(f*x+e))^3,x)
```

output

```
( - 288*e**(4*e + 4*f*x)*int(x**2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) +
 3*e**(2*e + 2*f*x) + 1),x)*a**2*b*d**2*f**3 - 96*e**(4*e + 4*f*x)*int(x**
2/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*b**3
*d**2*f**3 - 576*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*
f*x) + 3*e**(2*e + 2*f*x) + 1),x)*a**2*b*c*d*f**3 - 432*e**(4*e + 4*f*x)*i
nt(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*a
**2*b*d**2*f**2 - 576*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e
 + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*a*b**2*d**2*f**2 - 192*e**(4*e + 4*
f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1
),x)*b**3*c*d*f**3 - 144*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(
4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*b**3*d**2*f**2 + 144*e**(4*e + 4
*f*x)*log(e**(2*e + 2*f*x) + 1)*a**2*b*c**2*f**2 + 216*e**(4*e + 4*f*x)*lo
g(e**(2*e + 2*f*x) + 1)*a**2*b*c*d*f + 126*e**(4*e + 4*f*x)*log(e**(2*e +
2*f*x) + 1)*a**2*b*d**2 + 288*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*a
*b**2*c*d*f + 216*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*a*b**2*d**2 +
 48*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*b**3*c**2*f**2 + 72*e**(4*e
 + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*b**3*c*d*f + 90*e**(4*e + 4*f*x)*log(e
**(2*e + 2*f*x) + 1)*b**3*d**2 + 48*e**(4*e + 4*f*x)*a**3*c**2*f**3*x + 48
*e**(4*e + 4*f*x)*a**3*c*d*f**3*x**2 + 16*e**(4*e + 4*f*x)*a**3*d**2*f**3*
x**3 - 144*e**(4*e + 4*f*x)*a**2*b*c**2*f**3*x + 144*e**(4*e + 4*f*x)*a...
```

3.65 $\int (c+dx)(a+b\tanh(e+fx))^3\,dx$

Optimal result . . . 515
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Rubi [A] (verified) . . . 516
Maple [A] (verified) . . . 518
Fricas [C] (verification not implemented) . . . 519
Sympy [F] . . . 519
Maxima [A] (verification not implemented) . . . 519
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Reduce [F] . . . 521

Optimal result

Integrand size = 18, antiderivative size = 259

$$\begin{aligned}\int (c+dx)(a+b\tanh(e+fx))^3\,dx = {} & \frac{b^3dx}{2f}+\frac{a^3(c+dx)^2}{2d}-\frac{3a^2b(c+dx)^2}{2d}\\ & +\frac{3ab^2(c+dx)^2}{2d}-\frac{b^3(c+dx)^2}{2d}\\ & +\frac{3a^2b(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}\\ & +\frac{b^3(c+dx)\log\left(1+e^{2(e+fx)}\right)}{f}\\ & +\frac{3ab^2d\log(\cosh(e+fx))}{f^2}\\ & +\frac{3a^2bd\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}\\ & +\frac{b^3d\,\mathrm{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{b^3d\tanh(e+fx)}{2f^2}\\ & -\frac{3ab^2(c+dx)\tanh(e+fx)}{f}\\ & -\frac{b^3(c+dx)\tanh^2(e+fx)}{2f}\end{aligned}$$

output
```
1/2*b^3*d*x/f+1/2*a^3*(d*x+c)^2/d-3/2*a^2*b*(d*x+c)^2/d+3/2*a*b^2*(d*x+c)^
2/d-1/2*b^3*(d*x+c)^2/d+3*a^2*b*(d*x+c)*ln(1+exp(2*f*x+2*e))/f+b^3*(d*x+c)
*ln(1+exp(2*f*x+2*e))/f+3*a*b^2*d*ln(cosh(f*x+e))/f^2+3/2*a^2*b*d*polylog(
2,-exp(2*f*x+2*e))/f^2+1/2*b^3*d*polylog(2,-exp(2*f*x+2*e))/f^2-1/2*b^3*d*
tanh(f*x+e)/f^2-3*a*b^2*(d*x+c)*tanh(f*x+e)/f-1/2*b^3*(d*x+c)*tanh(f*x+e)^
2/f
```

Mathematica [A] (verified)

Time = 7.42 (sec) , antiderivative size = 303, normalized size of antiderivative = 1.17

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx$$
$$= \frac{\cosh(e+fx)\left(b^3 f(c+dx) - a(a^2+3b^2)\,(e+fx)(-2cf+d(e-fx))\cosh^2(e+fx) + b\cosh^2(e+fx)\right.}{}$$

input
```
Integrate[(c + d*x)*(a + b*Tanh[e + f*x])^3,x]
```

output
```
(Cosh[e + f*x]*(b^3*f*(c + d*x) - a*(a^2 + 3*b^2)*(e + f*x)*(-2*c*f + d*(e
 - f*x))*Cosh[e + f*x]^2 + b*Cosh[e + f*x]^2*((3*a^2*f^2*(c + d*x)^2)/d +
(b^2*f^2*(c + d*x)^2)/d - 6*a*b*d*(e + f*x) + 4*(3*a^2 + b^2)*(d*e - c*f)*
(e + f*x) + 2*(3*a^2 + b^2)*d*(e + f*x)*Log[1 + E^(-2*(e + f*x))] + 6*a*b*
d*Log[1 + E^(2*(e + f*x))] - 2*(3*a^2 + b^2)*(d*e - c*f)*Log[1 + E^(2*(e +
 f*x))] - (3*a^2 + b^2)*d*PolyLog[2, -E^(-2*(e + f*x))]) - (b^2*(b*d + 6*a
*f*(c + d*x))*Sinh[2*(e + f*x)])/2)*(a + b*Tanh[e + f*x])^3)/(2*f^2*(a*Cos
h[e + f*x] + b*Sinh[e + f*x])^3)
```

Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3042, 4205, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx$$

$$\downarrow 3042$$

$$\int (c+dx)(a-ib\tan(ie+ifx))^3 dx$$

$$\downarrow 4205$$

$$\int \left(a^3(c+dx)+3a^2b(c+dx)\tanh(e+fx)+3ab^2(c+dx)\tanh^2(e+fx)+b^3(c+dx)\tanh^3(e+fx)\right)dx$$

$$\downarrow 2009$$

$$\frac{a^3(c+dx)^2}{2d}+\frac{3a^2b(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{3a^2b(c+dx)^2}{2d}+\frac{3a^2bd\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{3ab^2(c+dx)\tanh(e+fx)}{f}+\frac{3ab^2(c+dx)^2}{2d}+\frac{3ab^2d\log(\cosh(e+fx))}{f^2}+\frac{b^3(c+dx)\log\left(e^{2(e+fx)}+1\right)}{f}-\frac{b^3(c+dx)\tanh^2(e+fx)}{2f}-\frac{b^3(c+dx)^2}{2d}+\frac{b^3d\,\text{PolyLog}\left(2,-e^{2(e+fx)}\right)}{2f^2}-\frac{b^3d\tanh(e+fx)}{2f^2}+\frac{b^3dx}{2f}$$

input

```
Int[(c + d*x)*(a + b*Tanh[e + f*x])^3,x]
```

output

```
(b^3*d*x)/(2*f) + (a^3*(c + d*x)^2)/(2*d) - (3*a^2*b*(c + d*x)^2)/(2*d) +
(3*a*b^2*(c + d*x)^2)/(2*d) - (b^3*(c + d*x)^2)/(2*d) + (3*a^2*b*(c + d*x)
*Log[1 + E^(2*(e + f*x))])/f + (b^3*(c + d*x)*Log[1 + E^(2*(e + f*x))])/f
+ (3*a*b^2*d*Log[Cosh[e + f*x]])/f^2 + (3*a^2*b*d*PolyLog[2, -E^(2*(e + f*
x))])/(2*f^2) + (b^3*d*PolyLog[2, -E^(2*(e + f*x))])/(2*f^2) - (b^3*d*Tanh
[e + f*x])/(2*f^2) - (3*a*b^2*(c + d*x)*Tanh[e + f*x])/f - (b^3*(c + d*x)*
Tanh[e + f*x]^2)/(2*f)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4205
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.)
, x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (a + b*Tan[e + f*x])^n, x],
 x] /; FreeQ[{a, b, c, d, e, f, m}, x] && IGtQ[m, 0] && IGtQ[n, 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 459, normalized size of antiderivative = 1.77

method	result
risch	$\frac{3a^2bd\,\mathrm{polylog}\left(2,-\mathrm{e}^{2fx+2e}\right)}{2f^2}+\frac{b^3d\,\mathrm{polylog}\left(2,-\mathrm{e}^{2fx+2e}\right)}{2f^2}-\frac{b^3de^2}{f^2}-\frac{2b^3c\ln\left(\mathrm{e}^{fx+e}\right)}{f}+\frac{b^3c\ln\left(1+\mathrm{e}^{2fx+2e}\right)}{f}-\frac{3a^2bdx^2}{2}+$

input
```
int((d*x+c)*(a+b*tanh(f*x+e))^3,x,method=_RETURNVERBOSE)
```

output
```
3/2*a^2*b*d*polylog(2,-exp(2*f*x+2*e))/f^2+1/2*b^3*d*polylog(2,-exp(2*f*x+
2*e))/f^2-1/f^2*b^3*d*e^2-2/f*b^3*c*ln(exp(f*x+e))+1/f*b^3*c*ln(1+exp(2*f*
x+2*e))-3/2*a^2*b*d*x^2+3/2*a*b^2*d*x^2+3*a^2*b*c*x+3*a*b^2*c*x+b^2*(6*a*d
*f*x*exp(2*f*x+2*e)+2*b*d*f*x*exp(2*f*x+2*e)+6*a*c*f*exp(2*f*x+2*e)+2*b*c*
f*exp(2*f*x+2*e)+6*a*d*f*x+b*exp(2*f*x+2*e)*d+6*a*c*f+b*d)/f^2/(1+exp(2*f*
x+2*e))^2-3/f^2*b*d*a^2*e^2+1/f*b^3*d*ln(1+exp(2*f*x+2*e))*x-6/f^2*b^2*a*d
*ln(exp(f*x+e))+3/f^2*b^2*a*d*ln(1+exp(2*f*x+2*e))-6/f*b*a^2*c*ln(exp(f*x+
e))+3/f*b*a^2*c*ln(1+exp(2*f*x+2*e))+2/f^2*b^3*e*d*ln(exp(f*x+e))-2/f*b^3*
d*e*x+6/f^2*b*e*d*a^2*ln(exp(f*x+e))+3/f*b*d*a^2*ln(1+exp(2*f*x+2*e))*x-6/
f*b*d*a^2*e*x+1/2*a^3*d*x^2-1/2*b^3*d*x^2+a^3*c*x+b^3*c*x
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.14 (sec) , antiderivative size = 3262, normalized size of antiderivative = 12.59

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx = \int (a+b\tanh(e+fx))^3\,(c+dx)\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))**3,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3*(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 475, normalized size of antiderivative = 1.83

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx = \frac{1}{2}\,a^3dx^2$$
$$+ b^3c\left(x + \frac{e}{f} + \frac{\log\left(e^{(-2fx-2e)}+1\right)}{f} + \frac{2\,e^{(-2fx-2e)}}{f\left(2\,e^{(-2fx-2e)}+e^{(-4fx-4e)}+1\right)}\right) + a^3cx$$
$$- \frac{6\,ab^2dx}{f} + \frac{3\,a^2bc\log\left(\cosh\left(fx+e\right)\right)}{f} - \left(3\,a^2bd+b^3d\right)x^2 + \frac{3\,ab^2d\log\left(e^{(2fx+2e)}+1\right)}{f^2}$$
$$+ \frac{12\,ab^2cf + 6\left(cf^2+2\,df\right)ab^2x + 2\,b^3d + \left(3\,a^2bdf^2+3\,ab^2df^2+b^3df^2\right)x^2 + \left(6\,ab^2cf^2xe^{(4e)} + \left(3\,a^2bdf^2\right.\right.}{}$$
$$+ \frac{\left(3\,a^2bd+b^3d\right)\left(2\,fx\log\left(e^{(2fx+2e)}+1\right) + \mathrm{Li}_2\left(-e^{(2fx+2e)}\right)\right)}{2\,f^2}$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="maxima")
```

output
```
1/2*a^3*d*x^2 + b^3*c*(x + e/f + log(e^(-2*f*x - 2*e) + 1)/f + 2*e^(-2*f*x
 - 2*e)/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))) + a^3*c*x - 6*a*b
^2*d*x/f + 3*a^2*b*c*log(cosh(f*x + e))/f - (3*a^2*b*d + b^3*d)*x^2 + 3*a*
b^2*d*log(e^(2*f*x + 2*e) + 1)/f^2 + 1/2*(12*a*b^2*c*f + 6*(c*f^2 + 2*d*f)
*a*b^2*x + 2*b^3*d + (3*a^2*b*d*f^2 + 3*a*b^2*d*f^2 + b^3*d*f^2)*x^2 + (6*
a*b^2*c*f^2*x*e^(4*e) + (3*a^2*b*d*f^2*e^(4*e) + 3*a*b^2*d*f^2*e^(4*e) + b
^3*d*f^2*e^(4*e))*x^2)*e^(4*f*x) + 2*(6*a*b^2*c*f*e^(2*e) + b^3*d*e^(2*e)
+ (3*a^2*b*d*f^2*e^(2*e) + 3*a*b^2*d*f^2*e^(2*e) + b^3*d*f^2*e^(2*e))*x^2
+ 2*(b^3*d*f*e^(2*e) + 3*(c*f^2*e^(2*e) + d*f*e^(2*e))*a*b^2)*x)*e^(2*f*x)
)/(f^2*e^(4*f*x + 4*e) + 2*f^2*e^(2*f*x + 2*e) + f^2) + 1/2*(3*a^2*b*d + b
^3*d)*(2*f*x*log(e^(2*f*x + 2*e) + 1) + dilog(-e^(2*f*x + 2*e)))/f^2
```

Giac [F]

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx = \int (dx+c)(b\tanh(fx+e)+a)^3\,dx$$

input
```
integrate((d*x+c)*(a+b*tanh(f*x+e))^3,x, algorithm="giac")
```

output
```
integrate((d*x + c)*(b*tanh(f*x + e) + a)^3, x)
```

Mupad [F(-1)]

Timed out.

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx = \int (a+b\tanh(e+f\,x))^3\,(c+d\,x)\,dx$$

input
```
int((a + b*tanh(e + f*x))^3*(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^3*(c + d*x), x)
```

Reduce [F]

$$\int (c+dx)(a+b\tanh(e+fx))^3\,dx = \text{Too large to display}$$

input
```
int((d*x+c)*(a+b*tanh(f*x+e))^3,x)
```

output
```
( - 48*e**(4*e + 4*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e
**(2*e + 2*f*x) + 1),x)*a**2*b*d*f**2 - 16*e**(4*e + 4*f*x)*int(x/(e**(6*e
 + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*b**3*d*f**2 +
24*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*a**2*b*c*f + 18*e**(4*e + 4*
f*x)*log(e**(2*e + 2*f*x) + 1)*a**2*b*d + 24*e**(4*e + 4*f*x)*log(e**(2*e
+ 2*f*x) + 1)*a*b**2*d + 8*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*b**3
*c*f + 6*e**(4*e + 4*f*x)*log(e**(2*e + 2*f*x) + 1)*b**3*d + 8*e**(4*e + 4
*f*x)*a**3*c*f**2*x + 4*e**(4*e + 4*f*x)*a**3*d*f**2*x**2 - 24*e**(4*e + 4
*f*x)*a**2*b*c*f**2*x + 12*e**(4*e + 4*f*x)*a**2*b*d*f**2*x**2 - 36*e**(4*
e + 4*f*x)*a**2*b*d*f*x + 3*e**(4*e + 4*f*x)*a**2*b*d + 24*e**(4*e + 4*f*x
)*a*b**2*c*f**2*x - 24*e**(4*e + 4*f*x)*a*b**2*c*f + 12*e**(4*e + 4*f*x)*a
*b**2*d*f**2*x**2 - 48*e**(4*e + 4*f*x)*a*b**2*d*f*x - 8*e**(4*e + 4*f*x)*
b**3*c*f**2*x - 8*e**(4*e + 4*f*x)*b**3*c*f + 4*e**(4*e + 4*f*x)*b**3*d*f*
*2*x**2 - 12*e**(4*e + 4*f*x)*b**3*d*f*x - 3*e**(4*e + 4*f*x)*b**3*d - 96*
e**(2*e + 2*f*x)*int(x/(e**(6*e + 6*f*x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e
+ 2*f*x) + 1),x)*a**2*b*d*f**2 - 32*e**(2*e + 2*f*x)*int(x/(e**(6*e + 6*f*
x) + 3*e**(4*e + 4*f*x) + 3*e**(2*e + 2*f*x) + 1),x)*b**3*d*f**2 + 48*e**(
2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*a**2*b*c*f + 36*e**(2*e + 2*f*x)*lo
g(e**(2*e + 2*f*x) + 1)*a**2*b*d + 48*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x
) + 1)*a*b**2*d + 16*e**(2*e + 2*f*x)*log(e**(2*e + 2*f*x) + 1)*b**3*c*...
```

3.66 $\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$

Optimal result 522
Mathematica [N/A] 522
Rubi [N/A] 523
Maple [N/A] 523
Fricas [N/A] 524
Sympy [N/A] 524
Maxima [N/A] 524
Giac [N/A] 525
Mupad [N/A] 525
Reduce [N/A] 526

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^3}{c+dx}, x\right)$$

output
```
Defer(Int)((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

Mathematica [N/A]

Not integrable

Time = 31.54 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x),x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a + b\tanh(e + fx))^3}{c + dx}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a - ib\tan(ie + ifx))^3}{c + dx}\,dx$$
$$\downarrow 4223$$
$$\int \frac{(a + b\tanh(e + fx))^3}{c + dx}\,dx$$

input
```
Int[(a + b*Tanh[e + f*x])^3/(c + d*x),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.84 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a + b\tanh(fx + e))^3}{dx + c}\,dx$$

input
```
int((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

output
```
int((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 52, normalized size of antiderivative = 2.60

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="fricas")
```

output
```
integral((b^3*tanh(f*x + e)^3 + 3*a*b^2*tanh(f*x + e)^2 + 3*a^2*b*tanh(f*x
 + e) + a^3)/(d*x + c), x)
```

Sympy [N/A]

Not integrable

Time = 1.40 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**3/(d*x+c),x)
```

output
```
Integral((a + b*tanh(e + f*x))**3/(c + d*x), x)
```

Maxima [N/A]

Not integrable

Time = 0.45 (sec) , antiderivative size = 455, normalized size of antiderivative = 22.75

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="maxima")
```

output
```
a^3*log(d*x + c)/d + (3*a^2*b + 3*a*b^2 + b^3)*log(d*x + c)/d + (6*a*b^2*d
*f*x + 6*a*b^2*c*f - b^3*d + (6*a*b^2*c*f*e^(2*e) + (2*c*f*e^(2*e) - d*e^(
2*e))*b^3 + 2*(3*a*b^2*d*f*e^(2*e) + b^3*d*f*e^(2*e))*x)*e^(2*f*x))/(d^2*f
^2*x^2 + 2*c*d*f^2*x + c^2*f^2 + (d^2*f^2*x^2*e^(4*e) + 2*c*d*f^2*x*e^(4*e
) + c^2*f^2*e^(4*e))*e^(4*f*x) + 2*(d^2*f^2*x^2*e^(2*e) + 2*c*d*f^2*x*e^(2
*e) + c^2*f^2*e^(2*e))*e^(2*f*x)) - integrate(2*(3*a^2*b*c^2*f^2 - 3*a*b^2
*c*d*f + (c^2*f^2 + d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 + (6*a^
2*b*c*d*f^2 + 2*b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^3*f^2*x^3 + 3*c*d^2*f^2
*x^2 + 3*c^2*d*f^2*x + c^3*f^2 + (d^3*f^2*x^3*e^(2*e) + 3*c*d^2*f^2*x^2*e^
(2*e) + 3*c^2*d*f^2*x*e^(2*e) + c^3*f^2*e^(2*e))*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{dx+c}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c),x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^3/(d*x + c), x)
```

Mupad [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx = \int \frac{(a+b\tanh(e+f\,x))^3}{c+d\,x}\,dx$$

input
```
int((a + b*tanh(e + f*x))^3/(c + d*x),x)
```

output
```
int((a + b*tanh(e + f*x))^3/(c + d*x), x)
```

Reduce [N/A]

Not integrable

Time = 0.36 (sec) , antiderivative size = 772, normalized size of antiderivative = 38.60

$$\int \frac{(a+b\tanh(e+fx))^3}{c+dx}\,dx$$
$$= \frac{-6e^{4e}\left(\int \frac{e^{4fx}}{e^{6fx+6e}c+e^{6fx+6e}dx+3e^{4fx+4e}c+3e^{4fx+4e}dx+3e^{2fx+2e}c+3e^{2fx+2e}dx+c+dx}dx\right)a^2bd-12e^{4e}\left(\int \frac{}{e^{6fx+6e}c+e^{6fx+6e}d}\right.}{}$$

input
```
int((a+b*tanh(f*x+e))^3/(d*x+c),x)
```

output
```
( - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x +
 3*e**(4*e + 4*f*x)*c + 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*
e**(2*e + 2*f*x)*d*x + c + d*x),x)*a**2*b*d - 12*e**(4*e)*int(e**(4*f*x)/(
e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x + 3*e**(4*e + 4*f*x)*c + 3*e**(4
*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x + c + d*x)
,x)*a*b**2*d - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c + e**(6*e + 6
*f*x)*d*x + 3*e**(4*e + 4*f*x)*c + 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*
f*x)*c + 3*e**(2*e + 2*f*x)*d*x + c + d*x),x)*b**3*d - 12*e**(2*e)*int(e**
(2*f*x)/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x + 3*e**(4*e + 4*f*x)*c
+ 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x +
 c + d*x),x)*a**2*b*d - 12*e**(2*e)*int(e**(2*f*x)/(e**(6*e + 6*f*x)*c + e
**(6*e + 6*f*x)*d*x + 3*e**(4*e + 4*f*x)*c + 3*e**(4*e + 4*f*x)*d*x + 3*e*
*(2*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x + c + d*x),x)*a*b**2*d - 6*int(1
/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x + 3*e**(4*e + 4*f*x)*c + 3*e**
(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2*e + 2*f*x)*d*x + c + d*
x),x)*a**2*b*d - 2*int(1/(e**(6*e + 6*f*x)*c + e**(6*e + 6*f*x)*d*x + 3*e*
*(4*e + 4*f*x)*c + 3*e**(4*e + 4*f*x)*d*x + 3*e**(2*e + 2*f*x)*c + 3*e**(2
*e + 2*f*x)*d*x + c + d*x),x)*b**3*d + log(c + d*x)*a**3 + 3*log(c + d*x)*
a**2*b + 3*log(c + d*x)*a*b**2 + log(c + d*x)*b**3)/d
```

3.67 $\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$

Optimal result 527
Mathematica [N/A] 527
Rubi [N/A] 528
Maple [N/A] 528
Fricas [N/A] 529
Sympy [N/A] 529
Maxima [N/A] 529
Giac [N/A] 530
Mupad [N/A] 531
Reduce [N/A] 531

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \text{Int}\left(\frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}, x\right)$$

output
```
Defer(Int)((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

Mathematica [N/A]

Not integrable

Time = 32.96 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

input
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x)^2,x]
```

output
```
Integrate[(a + b*Tanh[e + f*x])^3/(c + d*x)^2, x]
```

Rubi [N/A]

Not integrable

Time = 0.26 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{(a-ib\tan(ie+ifx))^3}{(c+dx)^2}\,dx$$
$$\downarrow 4223$$
$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

input
```
Int[(a + b*Tanh[e + f*x])^3/(c + d*x)^2,x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.85 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{(a+b\tanh(fx+e))^3}{(dx+c)^2}\,dx$$

input
```
int((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

output
```
int((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 63, normalized size of antiderivative = 3.15

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="fricas")
```

output
```
integral((b^3*tanh(f*x + e)^3 + 3*a*b^2*tanh(f*x + e)^2 + 3*a^2*b*tanh(f*x
 + e) + a^3)/(d^2*x^2 + 2*c*d*x + c^2), x)
```

Sympy [N/A]

Not integrable

Time = 2.37 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))**3/(d*x+c)**2,x)
```

output
```
Integral((a + b*tanh(e + f*x))**3/(c + d*x)**2, x)
```

Maxima [N/A]

Not integrable

Time = 0.58 (sec) , antiderivative size = 926, normalized size of antiderivative = 46.30

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="maxima")
```

output
```
-a^3/(d^2*x + c*d) - (3*a^2*b*c^2*f^2 + 3*(c^2*f^2 - 2*c*d*f)*a*b^2 + (c^2
*f^2 + 2*d^2)*b^3 + (3*a^2*b*d^2*f^2 + 3*a*b^2*d^2*f^2 + b^3*d^2*f^2)*x^2
+ 2*(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 + 3*(c*d*f^2 - d^2*f)*a*b^2)*x + (3*a^2
*b*c^2*f^2*e^(4*e) + 3*a*b^2*c^2*f^2*e^(4*e) + b^3*c^2*f^2*e^(4*e) + (3*a^
2*b*d^2*f^2*e^(4*e) + 3*a*b^2*d^2*f^2*e^(4*e) + b^3*d^2*f^2*e^(4*e))*x^2 +
 2*(3*a^2*b*c*d*f^2*e^(4*e) + 3*a*b^2*c*d*f^2*e^(4*e) + b^3*c*d*f^2*e^(4*e
))*x)*e^(4*f*x) + 2*(3*a^2*b*c^2*f^2*e^(2*e) + 3*(c^2*f^2*e^(2*e) - c*d*f*
e^(2*e))*a*b^2 + (c^2*f^2*e^(2*e) - c*d*f*e^(2*e) + d^2*e^(2*e))*b^3 + (3*
a^2*b*d^2*f^2*e^(2*e) + 3*a*b^2*d^2*f^2*e^(2*e) + b^3*d^2*f^2*e^(2*e))*x^2
 + (6*a^2*b*c*d*f^2*e^(2*e) + 3*(2*c*d*f^2*e^(2*e) - d^2*f*e^(2*e))*a*b^2
+ (2*c*d*f^2*e^(2*e) - d^2*f*e^(2*e))*b^3)*x)*e^(2*f*x))/(d^4*f^2*x^3 + 3*
c*d^3*f^2*x^2 + 3*c^2*d^2*f^2*x + c^3*d*f^2 + (d^4*f^2*x^3*e^(4*e) + 3*c*d
^3*f^2*x^2*e^(4*e) + 3*c^2*d^2*f^2*x*e^(4*e) + c^3*d*f^2*e^(4*e))*e^(4*f*x
) + 2*(d^4*f^2*x^3*e^(2*e) + 3*c*d^3*f^2*x^2*e^(2*e) + 3*c^2*d^2*f^2*x*e^(
2*e) + c^3*d*f^2*e^(2*e))*e^(2*f*x)) - integrate(2*(3*a^2*b*c^2*f^2 - 6*a*
b^2*c*d*f + (c^2*f^2 + 3*d^2)*b^3 + (3*a^2*b*d^2*f^2 + b^3*d^2*f^2)*x^2 +
2*(3*a^2*b*c*d*f^2 + b^3*c*d*f^2 - 3*a*b^2*d^2*f)*x)/(d^4*f^2*x^4 + 4*c*d^
3*f^2*x^3 + 6*c^2*d^2*f^2*x^2 + 4*c^3*d*f^2*x + c^4*f^2 + (d^4*f^2*x^4*e^(
2*e) + 4*c*d^3*f^2*x^3*e^(2*e) + 6*c^2*d^2*f^2*x^2*e^(2*e) + 4*c^3*d*f^2*x
*e^(2*e) + c^4*f^2*e^(2*e))*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.59 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(b\tanh(fx+e)+a)^3}{(dx+c)^2}\,dx$$

input
```
integrate((a+b*tanh(f*x+e))^3/(d*x+c)^2,x, algorithm="giac")
```

output
```
integrate((b*tanh(f*x + e) + a)^3/(d*x + c)^2, x)
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \int \frac{(a+b\tanh(e+f\,x))^3}{(c+d\,x)^2}\,dx$$

input
```
int((a + b*tanh(e + f*x))^3/(c + d*x)^2,x)
```

output
```
int((a + b*tanh(e + f*x))^3/(c + d*x)^2, x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 2546, normalized size of antiderivative = 127.30

$$\int \frac{(a+b\tanh(e+fx))^3}{(c+dx)^2}\,dx = \text{Too large to display}$$

input
```
int((a+b*tanh(f*x+e))^3/(d*x+c)^2,x)
```

output

```
( - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*
c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e + 4*f*x)*c**2 + 6*e**(4*e +
 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e + 2*f*x)*c**2 + 6
*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x +
d**2*x**2),x)*a**2*b*c**2 - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c*
*2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e + 4
*f*x)*c**2 + 6*e**(4*e + 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e
**(2*e + 2*f*x)*c**2 + 6*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*
x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*a**2*b*c*d*x - 12*e**(4*e)*int(e**(4
*f*x)/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)
*d**2*x**2 + 3*e**(4*e + 4*f*x)*c**2 + 6*e**(4*e + 4*f*x)*c*d*x + 3*e**(4*
e + 4*f*x)*d**2*x**2 + 3*e**(2*e + 2*f*x)*c**2 + 6*e**(2*e + 2*f*x)*c*d*x
+ 3*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x + d**2*x**2),x)*a*b**2*c**
2 - 12*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*c**2 + 2*e**(6*e + 6*f*x)
*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e + 4*f*x)*c**2 + 6*e**(4*e
+ 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 + 3*e**(2*e + 2*f*x)*c**2 +
6*e**(2*e + 2*f*x)*c*d*x + 3*e**(2*e + 2*f*x)*d**2*x**2 + c**2 + 2*c*d*x +
 d**2*x**2),x)*a*b**2*c*d*x - 6*e**(4*e)*int(e**(4*f*x)/(e**(6*e + 6*f*x)*
c**2 + 2*e**(6*e + 6*f*x)*c*d*x + e**(6*e + 6*f*x)*d**2*x**2 + 3*e**(4*e +
 4*f*x)*c**2 + 6*e**(4*e + 4*f*x)*c*d*x + 3*e**(4*e + 4*f*x)*d**2*x**2 ...
```

3.68 $\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$

Optimal result . . . 533
Mathematica [A] (verified) . . . 534
Rubi [A] (verified) . . . 534
Maple [B] (verified) . . . 538
Fricas [B] (verification not implemented) . . . 539
Sympy [F] . . . 539
Maxima [B] (verification not implemented) . . . 540
Giac [F] . . . 541
Mupad [F(-1)] . . . 541
Reduce [F] . . . 541

Optimal result

Integrand size = 20, antiderivative size = 212

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^4}{4(a+b)d} - \frac{b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{3bd(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^2} + \frac{3bd^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^3} + \frac{3bd^3\,\mathrm{PolyLog}\left(4,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4(a^2-b^2)f^4}$$

output

```
1/4*(d*x+c)^4/(a+b)/d-b*(d*x+c)^3*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^
2)/f+3/2*b*d*(d*x+c)^2*polylog(2,-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^
2+3/2*b*d^2*(d*x+c)*polylog(3,-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3+3
/4*b*d^3*polylog(4,-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^4
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{1}{4}\left(-\frac{2b(c+dx)^4}{(a+b)d\left(b\left(-1+e^{2e}\right)+a\left(1+e^{2e}\right)\right)} - \frac{4b(c+dx)^3\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f}\right.$$
$$+\frac{3bd\left(2f^2(c+dx)^2\,\text{PolyLog}\left(2,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)+d\left(2f(c+dx)\,\text{PolyLog}\left(3,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)+d\,\text{PolyL}\right.\right.}{(a-b)(a+b)f^4}$$
$$\left.\left.+\frac{x(4c^3+6c^2dx+4cd^2x^2+d^3x^3)\cosh(e)}{a\cosh(e)+b\sinh(e)}\right)\right)$$

input
```
Integrate[(c + d*x)^3/(a + b*Tanh[e + f*x]),x]
```

output
```
((-2*b*(c + d*x)^4)/((a + b)*d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (4*
b*(c + d*x)^3*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)
*f) + (3*b*d*(2*f^2*(c + d*x)^2*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x
)))] + d*(2*f*(c + d*x)*PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))] + d
*PolyLog[4, (-a + b)/((a + b)*E^(2*(e + f*x)))])))/((a - b)*(a + b)*f^4) +
 (x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[e])/(a*Cosh[e] + b*Si
nh[e]))/4
```

Rubi [A] (verified)

Time = 1.12 (sec) , antiderivative size = 202, normalized size of antiderivative = 0.95, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {3042, 4215, 2620, 3011, 7163, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c+dx)^3}{a-ib\tan(ie+ifx)}dx$$

$$\downarrow 4215$$

$$2b\int \frac{e^{-2(e+fx)}(c+dx)^3}{(a+b)^2+(a^2-b^2)\,e^{-2(e+fx)}}dx+\frac{(c+dx)^4}{4d(a+b)}$$

$$\downarrow 2620$$

$$2b\left(\frac{3d\int(c+dx)^2\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{2f\,(a^2-b^2)}-\frac{(c+dx)^3\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\,(a^2-b^2)}\right)+$$

$$\frac{(c+dx)^4}{4d(a+b)}$$

$$\downarrow 3011$$

$$2b\left(\frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\int(c+dx)\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{f}\right)}{2f\,(a^2-b^2)}-\frac{(c+dx)^3\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}\right.}{2f\,(a^2-b^2)}\right.$$

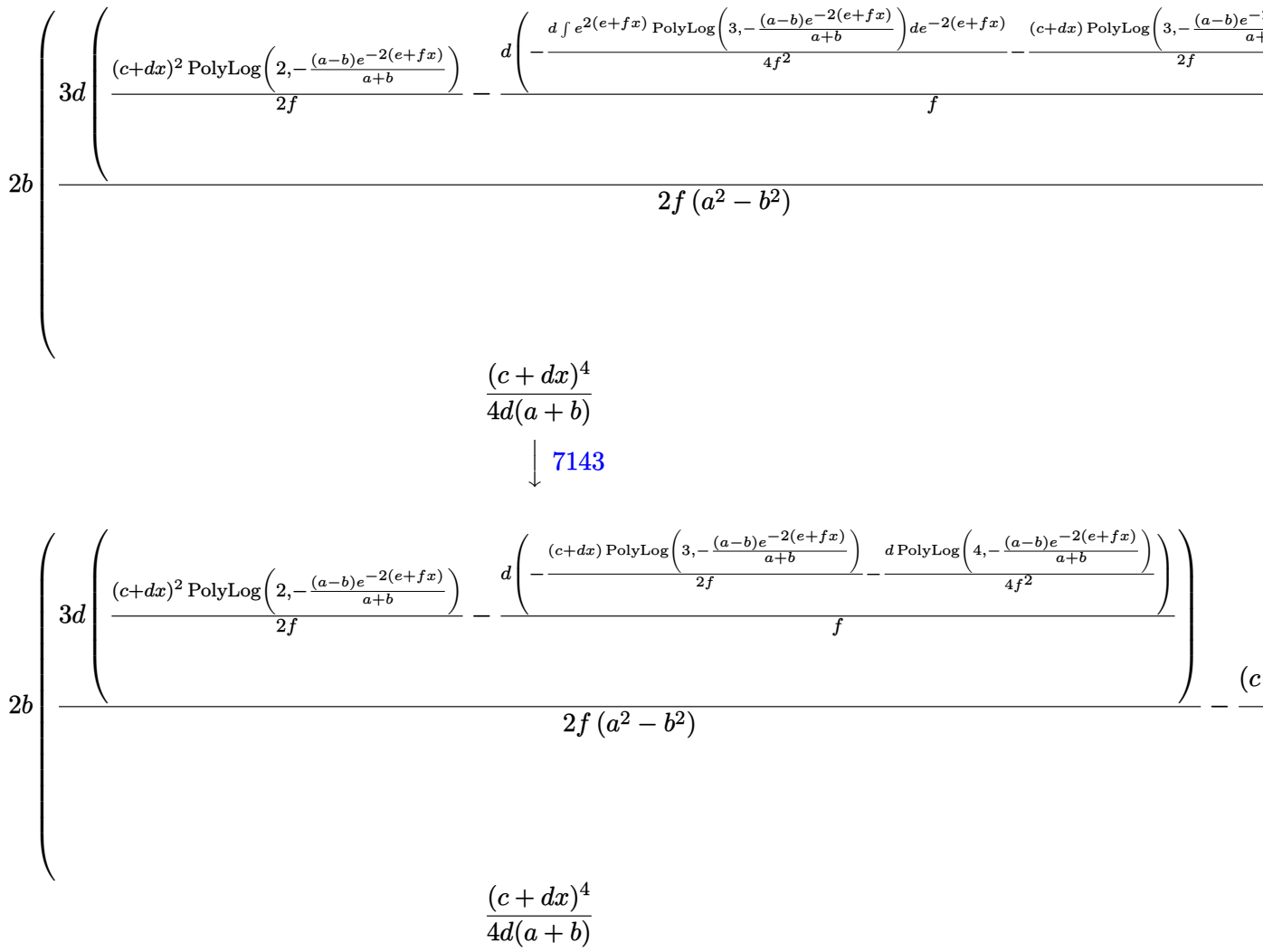
$$\frac{(c+dx)^4}{4d(a+b)}$$

$$\downarrow 7163$$

$$2b\left(\frac{3d\left(\frac{(c+dx)^2\operatorname{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\left(\frac{d\int\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f}-\frac{(c+dx)\operatorname{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}\right)}{f}\right)}{2f\,(a^2-b^2)}-\frac{(}{}\right.$$

$$\frac{(c+dx)^4}{4d(a+b)}$$

$$\downarrow 2720$$



$$\frac{(c+dx)^4}{4d(a+b)}$$

$$\downarrow 7143$$

$$\frac{(c+dx)^4}{4d(a+b)}$$

input
```
Int[(c + d*x)^3/(a + b*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^4/(4*(a + b)*d) + 2*b*(-1/2*((c + d*x)^3*Log[1 + (a - b)/((a + b
)*E^(2*(e + f*x)))])/((a^2 - b^2)*f) + (3*d*(((c + d*x)^2*PolyLog[2, -((a
 - b)/((a + b)*E^(2*(e + f*x))))])/(2*f) - (d*(-1/2*((c + d*x)*PolyLog[3, -
((a - b)/((a + b)*E^(2*(e + f*x))))])/f - (d*PolyLog[4, -((a - b)/((a + b)
*E^(2*(e + f*x))))])/(4*f^2)))/f))/(2*(a^2 - b^2)*f))
```

Defintions of rubi rules used

rule 2620

```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720

```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011

```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

rule 7163

```
Int[((e_.) + (f_.)*(x_))^(m_.)*PolyLog[n_, (d_.)*((F_)^((c_.)*((a_.) + (b_.
)*(x_))))^(p_.)], x_Symbol] :> Simp[(e + f*x)^m*(PolyLog[n + 1, d*(F^(c*(a
+ b*x)))^p]/(b*c*p*Log[F])), x] - Simp[f*(m/(b*c*p*Log[F]))   Int[(e + f*x)
^(m - 1)*PolyLog[n + 1, d*(F^(c*(a + b*x)))^p], x], x] /; FreeQ[{F, a, b, c
, d, e, f, n, p}, x] && GtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1155 vs. 2(212) = 424.

Time = 0.89 (sec) , antiderivative size = 1156, normalized size of antiderivative = 5.45

method	result	size
risch	Expression too large to display	1156

input

```
int((d*x+c)^3/(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
-3/2/f^3*b/(a+b)/(-a+b)*d^2*c*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))+1/f*b
/(a+b)/(-a+b)*d^3*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^3+3/2/f^2*b/(a+b)/(-
a+b)*d*c^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))+3/2/f^2*b/(a+b)/(-a+b)*d
^3*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))*x^2+1/f^4*b/(a+b)/(-a+b)*d^3*ln(
1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e^3-3/2/f^3*b/(a+b)/(-a+b)*d^3*polylog(3,(a
+b)*exp(2*f*x+2*e)/(-a+b))*x+1/f^4*b/(a+b)*d^3*e^3/(a-b)*ln(a*exp(2*f*x+2*
e)+exp(2*f*x+2*e)*b+a-b)-2/f^4*b/(a+b)*d^3*e^3/(a-b)*ln(exp(f*x+e))-3*b/(a
+b)/(-a+b)*d*c^2*x^2-3/f^2*b/(a+b)/(-a+b)*d*c^2*e^2-2/f^3*b/(a+b)/(-a+b)*d
^3*e^3*x-2*b/(a+b)/(-a+b)*d^2*c*x^3+4/f^3*b/(a+b)/(-a+b)*d^2*c*e^3+1/(a+b)
*d^2*c*x^3+3/2/(a+b)*d*c^2*x^2+1/(a+b)*c^3*x+3/f^2*b/(a+b)/(-a+b)*d^2*c*po
lylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))*x-6/f^2*b/(a+b)*c^2*d*e/(a-b)*ln(exp(
f*x+e))+3/f*b/(a+b)/(-a+b)*d*c^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x+3/f^2
*b/(a+b)/(-a+b)*d*c^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e+6/f^3*b/(a+b)*c*
d^2*e^2/(a-b)*ln(exp(f*x+e))+3/f^2*b/(a+b)*c^2*d*e/(a-b)*ln(a*exp(2*f*x+2*
e)+exp(2*f*x+2*e)*b+a-b)-3/f^3*b/(a+b)*c*d^2*e^2/(a-b)*ln(a*exp(2*f*x+2*e)
+exp(2*f*x+2*e)*b+a-b)+3/f*b/(a+b)/(-a+b)*d^2*c*ln(1-(a+b)*exp(2*f*x+2*e)/
(-a+b))*x^2-3/f^3*b/(a+b)/(-a+b)*d^2*c*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e
^2+6/f^2*b/(a+b)/(-a+b)*d^2*c*e^2*x-6/f*b/(a+b)/(-a+b)*d*c^2*e*x-1/2*b/(a+
b)/(-a+b)*d^3*x^4-3/2/f^4*b/(a+b)/(-a+b)*d^3*e^4+3/4/f^4*b/(a+b)/(-a+b)*d^
3*polylog(4,(a+b)*exp(2*f*x+2*e)/(-a+b))-1/f*b/(a+b)*c^3/(a-b)*ln(a*exp...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(203) = 406$.

Time $= 0.11$ (sec) , antiderivative size $= 740$, normalized size of antiderivative $= 3.49$

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/4*((a + b)*d^3*f^4*x^4 + 4*(a + b)*c*d^2*f^4*x^3 + 6*(a + b)*c^2*d*f^4*x
^2 + 4*(a + b)*c^3*f^4*x - 24*b*d^3*polylog(4, sqrt(-(a + b)/(a - b))*(cos
h(f*x + e) + sinh(f*x + e))) - 24*b*d^3*polylog(4, -sqrt(-(a + b)/(a - b))
*(cosh(f*x + e) + sinh(f*x + e))) - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x +
b*c^2*d*f^2)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)))
 - 12*(b*d^3*f^2*x^2 + 2*b*c*d^2*f^2*x + b*c^2*d*f^2)*dilog(-sqrt(-(a + b)
/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 4*(b*d^3*e^3 - 3*b*c*d^2*e^2*
f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*s
inh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) + 4*(b*d^3*e^3 - 3*b*c*d^
2*e^2*f + 3*b*c^2*d*e*f^2 - b*c^3*f^3)*log(2*(a + b)*cosh(f*x + e) + 2*(a
+ b)*sinh(f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b))) - 4*(b*d^3*f^3*x^3
+ 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 - 3*b*c*d^2*e^2*f + 3*b*
c^2*d*e*f^2)*log(sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) +
1) - 4*(b*d^3*f^3*x^3 + 3*b*c*d^2*f^3*x^2 + 3*b*c^2*d*f^3*x + b*d^3*e^3 -
3*b*c*d^2*e^2*f + 3*b*c^2*d*e*f^2)*log(-sqrt(-(a + b)/(a - b))*(cosh(f*x +
 e) + sinh(f*x + e)) + 1) + 24*(b*d^3*f*x + b*c*d^2*f)*polylog(3, sqrt(-(a
 + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) + 24*(b*d^3*f*x + b*c*d^2*
f)*polylog(3, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))))/((
a^2 - b^2)*f^4)
```

Sympy [F]

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+dx)^3}{a+b\tanh\left(e+fx\right)}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*tanh(f*x+e)),x)
```

output
```
Integral((c + d*x)**3/(a + b*tanh(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(203) = 406.

Time = 0.20 (sec) , antiderivative size = 528, normalized size of antiderivative = 2.49

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$
$$= -\frac{3\left(2fx\log\left(\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}\right)\right)bc^2d}{2\left(a^2f^2-b^2f^2\right)}$$
$$-\frac{3\left(2f^2x^2\log\left(\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}+1\right)+2fx\mathrm{Li}_2\left(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}\right)-\mathrm{Li}_3(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b})\right)bcd}{2\left(a^2f^3-b^2f^3\right)}$$
$$-\frac{\left(4f^3x^3\log\left(\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}+1\right)+6f^2x^2\mathrm{Li}_2\left(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}\right)-6fx\mathrm{Li}_3(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b})\right.}{3\left(a^2f^4-b^2f^4\right)}$$
$$-c^3\left(\frac{b\log\left(-(a-b)e^{(-2fx-2e)}-a-b\right)}{(a^2-b^2)f}-\frac{fx+e}{(a+b)f}\right)$$
$$+\frac{bd^3f^4x^4+4bcd^2f^4x^3+6bc^2df^4x^2}{2\left(a^2f^4-b^2f^4\right)}+\frac{d^3x^4+4cd^2x^3+6c^2dx^2}{4(a+b)}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-3/2*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(-(a
*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c^2*d/(a^2*f^2 - b^2*f^2) - 3/
2*(2*f^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*di
log(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) +
 b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c*d^2/(a^2*f^3 - b^2*f^3) - 1/3*(4*f^3*x
^3*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog(-(
a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, -(a*e^(2*e) +
 b*e^(2*e))*e^(2*f*x)/(a - b)) + 3*polylog(4, -(a*e^(2*e) + b*e^(2*e))*e^(
2*f*x)/(a - b)))*b*d^3/(a^2*f^4 - b^2*f^4) - c^3*(b*log(-(a - b)*e^(-2*f*x
 - 2*e) - a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 1/2*(b*d^3*f^4
*x^4 + 4*b*c*d^2*f^4*x^3 + 6*b*c^2*d*f^4*x^2)/(a^2*f^4 - b^2*f^4) + 1/4*(d
^3*x^4 + 4*c*d^2*x^3 + 6*c^2*d*x^2)/(a + b)
```

Giac [F]

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \int \frac{(dx+c)^3}{b\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^3/(b*tanh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+d\,x)^3}{a+b\tanh(e+f\,x)}\,dx$$

input
```
int((c + d*x)^3/(a + b*tanh(e + f*x)),x)
```

output
```
int((c + d*x)^3/(a + b*tanh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{-8e^{2e}\left(\int \frac{e^{2fx}x^3}{e^{2fx+2e}a^2-e^{2fx+2e}b^2+a^2-2ab+b^2}dx\right)a^2b\,d^3f+8e^{2e}\left(\int \frac{e^{2fx}x^3}{e^{2fx+2e}a^2-e^{2fx+2e}b^2+a^2-2ab+b^2}dx\right)b^3d^3f-24e^{2e}(}{}$$

input
```
int((d*x+c)^3/(a+b*tanh(f*x+e)),x)
```

output

```
( - 8*e**(2*e)*int((e**(2*f*x)*x**3)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*
f*x)*b**2 + a**2 - 2*a*b + b**2),x)*a**2*b*d**3*f + 8*e**(2*e)*int((e**(2*
f*x)*x**3)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 + a**2 - 2*a*b +
 b**2),x)*b**3*d**3*f - 24*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x
)*a**2 - e**(2*e + 2*f*x)*b**2 + a**2 - 2*a*b + b**2),x)*a**2*b*c*d**2*f +
 24*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*
x)*b**2 + a**2 - 2*a*b + b**2),x)*b**3*c*d**2*f - 24*e**(2*e)*int((e**(2*f
*x)*x)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 + a**2 - 2*a*b + b**
2),x)*a**2*b*c**2*d*f + 24*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a
**2 - e**(2*e + 2*f*x)*b**2 + a**2 - 2*a*b + b**2),x)*b**3*c**2*d*f - 4*lo
g(e**(2*e + 2*f*x)*a + e**(2*e + 2*f*x)*b + a - b)*b*c**3 + 4*a*c**3*f*x +
 6*a*c**2*d*f*x**2 + 4*a*c*d**2*f*x**3 + a*d**3*f*x**4 + 4*b*c**3*f*x + 6*
b*c**2*d*f*x**2 + 4*b*c*d**2*f*x**3 + b*d**3*f*x**4)/(4*f*(a**2 - b**2))
```

3.69 $\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 157

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^3}{3(a+b)d} - \frac{b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{bd(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f^2} + \frac{bd^2\,\mathrm{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^3}$$

output

```
1/3*(d*x+c)^3/(a+b)/d-b*(d*x+c)^2*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^
2)/f+b*d*(d*x+c)*polylog(2,-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2+1/2*
b*d^2*polylog(3,-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^3
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 200, normalized size of antiderivative = 1.27

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{1}{6}\left(-\frac{4b(c+dx)^3}{(a+b)d\left(b\left(-1+e^{2e}\right)+a\left(1+e^{2e}\right)\right)} - \frac{6b(c+dx)^2\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f}\right.$$
$$+ \frac{3bd\left(2f(c+dx)\,\text{PolyLog}\left(2,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)+d\,\text{PolyLog}\left(3,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)\right)}{(a-b)(a+b)f^3}$$
$$\left. + \frac{2x(3c^2+3cdx+d^2x^2)\cosh(e)}{a\cosh(e)+b\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)^2/(a + b*Tanh[e + f*x]),x]
```

output
```
((-4*b*(c + d*x)^3)/((a + b)*d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (6*
b*(c + d*x)^2*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)
*f) + (3*b*d*(2*f*(c + d*x)*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))]
 + d*PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))]))/((a - b)*(a + b)*f^3
) + (2*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[e])/(a*Cosh[e] + b*Sinh[e]))/6
```

Rubi [A] (verified)

Time = 0.86 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3042, 4215, 2620, 3011, 2720, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$
$$\downarrow \text{3042}$$

$$\int \frac{(c+dx)^2}{a-ib\tan(ie+ifx)}dx$$

$$\downarrow 4215$$

$$2b\int \frac{e^{-2(e+fx)}(c+dx)^2}{(a+b)^2+(a^2-b^2)\,e^{-2(e+fx)}}dx+\frac{(c+dx)^3}{3d(a+b)}$$

$$\downarrow 2620$$

$$2b\left(\frac{d\int(c+dx)\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)+\frac{(c+dx)^3}{3d(a+b)}$$

$$\downarrow 3011$$

$$2b\left(\frac{d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}-\frac{d\int\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)dx}{2f}\right)}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)$$

$$\frac{(c+dx)^3}{3d(a+b)}$$

$$\downarrow 2720$$

$$2b\left(\frac{d\left(\frac{d\int e^{2(e+fx)}\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)de^{-2(e+fx)}}{4f^2}+\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}\right)}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-}}{a-}\right.}{2f\left(a^2-b^{2}\right.}$$

$$\frac{(c+dx)^3}{3d(a+b)}$$

$$\downarrow 7143$$

$$2b\left(\frac{d\left(\frac{(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f}+\frac{d\,\mathrm{PolyLog}\left(3,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^2}\right)}{f\left(a^2-b^2\right)}-\frac{(c+dx)^2\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)+$$

$$\frac{(c+dx)^3}{3d(a+b)}$$

input
```
Int[(c + d*x)^2/(a + b*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^3/(3*(a + b)*d) + 2*b*(-1/2*((c + d*x)^2*Log[1 + (a - b)/((a + b
)*E^(2*(e + f*x)))])/((a^2 - b^2)*f) + (d*((c + d*x)*PolyLog[2, -((a - b)
/((a + b)*E^(2*(e + f*x))))])/(2*f) + (d*PolyLog[3, -((a - b)/((a + b)*E^(
2*(e + f*x))))])/(4*f^2)))/((a^2 - b^2)*f))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2720
```
Int[u_, x_Symbol] :> With[{v = FunctionOfExponential[u, x]}, Simp[v/D[v, x]
   Subst[Int[FunctionOfExponentialFunction[u, x]/x, x], x, v], x]] /; Funct
ionOfExponentialQ[u, x] &&  !MatchQ[u, (w_)*((a_.)*(v_)^(n_))^(m_) /; FreeQ
[{a, m, n}, x] && IntegerQ[m*n]] &&  !MatchQ[u, E^((c_.)*((a_.) + (b_.)*x))
*(F_)[v_] /; FreeQ[{a, b, c}, x] && InverseFunctionQ[F[x]]]
```

rule 3011
```
Int[Log[1 + (e_.)*((F_)^((c_.)*((a_.) + (b_.)*(x_))))^(n_.)]*((f_.) + (g_.)
*(x_))^(m_.), x_Symbol] :> Simp[(-(f + g*x)^m)*(PolyLog[2, (-e)*(F^(c*(a +
b*x)))^n]/(b*c*n*Log[F])), x] + Simp[g*(m/(b*c*n*Log[F]))   Int[(f + g*x)^(
m - 1)*PolyLog[2, (-e)*(F^(c*(a + b*x)))^n], x], x] /; FreeQ[{F, a, b, c, e
, f, g, n}, x] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

rule 7143

```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 732 vs. $2(159) = 318$.

Time = 0.79 (sec) , antiderivative size = 733, normalized size of antiderivative = 4.67

method	result
risch	$\frac{d^2x^3}{3a+3b}+\frac{dcx^2}{a+b}+\frac{c^2x}{a+b}+\frac{c^3}{3(a+b)d}+\frac{2bcde\ln\left(a\,e^{2fx+2e}+e^{2fx+2e}b+a-b\right)}{f^2(a+b)(a-b)}-\frac{4bcde\ln\left(e^{fx+e}\right)}{f^2(a+b)(a-b)}-\frac{4bcdex}{f(a+b)(-a+b)}+\frac{2bcdl}{}$

input

```
int((d*x+c)^2/(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
1/3/(a+b)*d^2*x^3+1/(a+b)*d*c*x^2+1/(a+b)*c^2*x+1/3/(a+b)/d*c^3+2/f^2*b/(a
+b)*c*d*e/(a-b)*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)-4/f^2*b/(a+b)*c*
d*e/(a-b)*ln(exp(f*x+e))-4/f*b/(a+b)/(-a+b)*c*d*e*x+2/f^2*b/(a+b)/(-a+b)*c
*d*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e+2/f*b/(a+b)/(-a+b)*c*d*ln(1-(a+b)*e
xp(2*f*x+2*e)/(-a+b))*x+1/f^2*b/(a+b)/(-a+b)*c*d*polylog(2,(a+b)*exp(2*f*x
+2*e)/(-a+b))-2/3*b/(a+b)/(-a+b)*d^2*x^3+4/3/f^3*b/(a+b)/(-a+b)*d^2*e^3+1/
f*b/(a+b)/(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^2-1/2/f^3*b/(a+b)
/(-a+b)*d^2*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))-1/f^3*b/(a+b)*e^2*d^2/(
a-b)*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)+1/f^2*b/(a+b)/(-a+b)*d^2*po
lylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))*x-2*b/(a+b)/(-a+b)*c*d*x^2-2/f^2*b/(a
+b)/(-a+b)*c*d*e^2+2/f^3*b/(a+b)*e^2*d^2/(a-b)*ln(exp(f*x+e))+2/f^2*b/(a+b
)/(-a+b)*d^2*e^2*x-1/f^3*b/(a+b)/(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+
b))*e^2+2/f*b/(a+b)*c^2/(a-b)*ln(exp(f*x+e))-1/f*b/(a+b)*c^2/(a-b)*ln(a*ex
p(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(152) = 304$.

Time $= 0.12$ (sec) , antiderivative size $= 500$, normalized size of antiderivative $= 3.18$

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$

$$= \frac{(a+b)d^2f^3x^3 + 3\,(a+b)cdf^3x^2 + 3\,(a+b)c^2f^3x + 6\,bd^2\mathrm{polylog}\left(3, \sqrt{-\frac{a+b}{a-b}}(\cosh\,(fx+e) + \sinh\,(fx+\right.}{}$$

input

```
integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output

```
1/3*((a + b)*d^2*f^3*x^3 + 3*(a + b)*c*d*f^3*x^2 + 3*(a + b)*c^2*f^3*x + 6
*b*d^2*polylog(3, sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)))
+ 6*b*d^2*polylog(3, -sqrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e
))) - 6*(b*d^2*f*x + b*c*d*f)*dilog(sqrt(-(a + b)/(a - b))*(cosh(f*x + e)
+ sinh(f*x + e))) - 6*(b*d^2*f*x + b*c*d*f)*dilog(-sqrt(-(a + b)/(a - b))*
(cosh(f*x + e) + sinh(f*x + e))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)
*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(
a + b)/(a - b))) - 3*(b*d^2*e^2 - 2*b*c*d*e*f + b*c^2*f^2)*log(2*(a + b)*c
osh(f*x + e) + 2*(a + b)*sinh(f*x + e) - 2*(a - b)*sqrt(-(a + b)/(a - b)))
 - 3*(b*d^2*f^2*x^2 + 2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(sqrt(-(
a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 3*(b*d^2*f^2*x^2 +
2*b*c*d*f^2*x - b*d^2*e^2 + 2*b*c*d*e*f)*log(-sqrt(-(a + b)/(a - b))*(cosh
(f*x + e) + sinh(f*x + e)) + 1))/((a^2 - b^2)*f^3)
```

Sympy [F]

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+dx)^2}{a+b\tanh\,(e+fx)}\,dx$$

input

```
integrate((d*x+c)**2/(a+b*tanh(f*x+e)),x)
```

output

```
Integral((c + d*x)**2/(a + b*tanh(e + f*x)), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(152) = 304.

Time = 0.17 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.15

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$
$$= -\frac{\left(2fx\log\left(\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}+1\right)+\mathrm{Li}_2\left(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}\right)\right)bcd}{a^2f^2-b^2f^2}$$
$$-\frac{\left(2f^2x^2\log\left(\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}+1\right)+2fx\mathrm{Li}_2\left(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b}\right)-\mathrm{Li}_3(-\frac{(ae^{(2e)}+be^{(2e)})e^{(2fx)}}{a-b})\right)bd^2}{2\left(a^2f^3-b^2f^3\right)}$$
$$-c^2\left(\frac{b\log\left(-(a-b)e^{(-2fx-2e)}-a-b\right)}{(a^2-b^2)f}-\frac{fx+e}{(a+b)f}\right)$$
$$+\frac{2\left(bd^2f^3x^3+3bcdf^3x^2\right)}{3\left(a^2f^3-b^2f^3\right)}+\frac{d^2x^3+3cdx^2}{3(a+b)}$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
-(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(-(a*e^(
2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))*b*c*d/(a^2*f^2 - b^2*f^2) - 1/2*(2*f
^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog(-(
a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) + b*e^(
2*e))*e^(2*f*x)/(a - b)))*b*d^2/(a^2*f^3 - b^2*f^3) - c^2*(b*log(-(a - b)*
e^(-2*f*x - 2*e) - a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f)) + 2/3*(
b*d^2*f^3*x^3 + 3*b*c*d*f^3*x^2)/(a^2*f^3 - b^2*f^3) + 1/3*(d^2*x^3 + 3*c*
d*x^2)/(a + b)
```

Giac [F]

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \int \frac{(dx+c)^2}{b\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx = \int \frac{(c+d\,x)^2}{a+b\tanh\left(e+f\,x\right)}\,dx$$

input
```
int((c + d*x)^2/(a + b*tanh(e + f*x)),x)
```

output
```
int((c + d*x)^2/(a + b*tanh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{-6e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}a^2-e^{2fx+2e}b^2+a^2-2ab+b^2}\,dx\right)a^2b\,d^2f+6e^{2e}\left(\int \frac{e^{2fx}x^2}{e^{2fx+2e}a^2-e^{2fx+2e}b^2+a^2-2ab+b^2}\,dx\right)b^3d^2f-12e^{2e}($$

input
```
int((d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

output
```
( - 6*e**(2*e)*int((e**(2*f*x)*x**2)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*
f*x)*b**2 + a**2 - 2*a*b + b**2),x)*a**2*b*d**2*f + 6*e**(2*e)*int((e**(2*
f*x)*x**2)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 + a**2 - 2*a*b +
 b**2),x)*b**3*d**2*f - 12*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a
**2 - e**(2*e + 2*f*x)*b**2 + a**2 - 2*a*b + b**2),x)*a**2*b*c*d*f + 12*e*
*(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 +
 a**2 - 2*a*b + b**2),x)*b**3*c*d*f - 3*log(e**(2*e + 2*f*x)*a + e**(2*e +
 2*f*x)*b + a - b)*b*c**2 + 3*a*c**2*f*x + 3*a*c*d*f*x**2 + a*d**2*f*x**3
+ 3*b*c**2*f*x + 3*b*c*d*f*x**2 + b*d**2*f*x**3)/(3*f*(a**2 - b**2))
```

3.70 $\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx$

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Optimal result

Integrand size = 18, antiderivative size = 108

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \frac{(c+dx)^2}{2(a+b)d} - \frac{b(c+dx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a^2-b^2)f} + \frac{bd\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2(a^2-b^2)f^2}$$

output

```
1/2*(d*x+c)^2/(a+b)/d-b*(d*x+c)*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)
/f+1/2*b*d*polylog(2,-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)/f^2
```

Mathematica [A] (verified)

Time = 0.66 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.41

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \frac{1}{2}\left(-\frac{2b(c+dx)^2}{(a+b)d\left(b\left(-1+e^{2e}\right)+a\left(1+e^{2e}\right)\right)} - \frac{2b(c+dx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f} + \frac{bd\,\mathrm{PolyLog}\left(2,\frac{(-a+b)e^{-2(e+fx)}}{a+b}\right)}{(a-b)(a+b)f^2} + \frac{x(2c+dx)\cosh(e)}{a\cosh(e)+b\sinh(e)}\right)$$

input
```
Integrate[(c + d*x)/(a + b*Tanh[e + f*x]),x]
```

output
```
((-2*b*(c + d*x)^2)/((a + b)*d*(b*(-1 + E^(2*e)) + a*(1 + E^(2*e)))) - (2*
b*(c + d*x)*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)*f
) + (b*d*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))])/((a - b)*(a + b)*
f^2) + (x*(2*c + d*x)*Cosh[e])/(a*Cosh[e] + b*Sinh[e]))/2
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.278$, Rules used = {3042, 4215, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx$$

$$\downarrow 3042$$

$$\int \frac{c+dx}{a-ib\tan(ie+ifx)}dx$$

$$\downarrow 4215$$

$$2b\int \frac{e^{-2(e+fx)}(c+dx)}{(a+b)^2+(a^2-b^2)\,e^{-2(e+fx)}}dx+\frac{(c+dx)^2}{2d(a+b)}$$

$$\downarrow 2620$$

$$2b\left(\frac{d\int \log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{2f\,(a^2-b^2)}-\frac{(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\,(a^2-b^2)}\right)+\frac{(c+dx)^2}{2d(a+b)}$$

$$\downarrow 2715$$

$$2b\left(-\frac{d\int e^{2(e+fx)}\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)de^{-2(e+fx)}}{4f^2\,(a^2-b^2)}-\frac{(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\,(a^2-b^2)}\right)+$$

$$\frac{(c+dx)^2}{2d(a+b)}$$

$$\downarrow 2838$$

$$2b\left(\frac{d\,\text{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{4f^2\left(a^2-b^2\right)}-\frac{(c+dx)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f\left(a^2-b^2\right)}\right)+\frac{(c+dx)^2}{2d(a+b)}$$

input
```
Int[(c + d*x)/(a + b*Tanh[e + f*x]),x]
```

output
```
(c + d*x)^2/(2*(a + b)*d) + 2*b*(-1/2*((c + d*x)*Log[1 + (a - b)/((a + b)*
E^(2*(e + f*x)))])/((a^2 - b^2)*f) + (d*PolyLog[2, -((a - b)/((a + b)*E^(2
*(e + f*x))))])/(4*(a^2 - b^2)*f^2))
```

Defintions of rubi rules used

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215

```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(108) = 216$.

Time = 0.53 (sec) , antiderivative size = 357, normalized size of antiderivative = 3.31

method	result
risch	$\frac{d\,x^2}{2b+2a}+\frac{cx}{a+b}-\frac{bc\ln\left(a\,\mathrm{e}^{2fx+2e}+\mathrm{e}^{2fx+2e}b+a-b\right)}{f(a+b)(a-b)}+\frac{2bc\ln\left(\mathrm{e}^{fx+e}\right)}{f(a+b)(a-b)}-\frac{bd\,x^2}{(a+b)(-a+b)}+\frac{bd\ln\left(1-\frac{(a+b)\mathrm{e}^{2fx+2e}}{-a+b}\right)x}{f(a+b)(-a+b)}-\frac{}{f(a-}$

input

```
int((d*x+c)/(a+b*tanh(f*x+e)),x,method=_RETURNVERBOSE)
```

output

```
1/2/(a+b)*d*x^2+1/(a+b)*c*x-1/f*b/(a+b)*c/(a-b)*ln(a*exp(2*f*x+2*e)+exp(2*
f*x+2*e)*b+a-b)+2/f*b/(a+b)*c/(a-b)*ln(exp(f*x+e))-b/(a+b)/(-a+b)*d*x^2+1/
f*b/(a+b)/(-a+b)*d*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x-2/f*b/(a+b)/(-a+b)*
d*e*x+1/f^2*b/(a+b)/(-a+b)*d*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e-1/f^2*b/(
a+b)/(-a+b)*d*e^2+1/2/f^2*b/(a+b)/(-a+b)*d*polylog(2,(a+b)*exp(2*f*x+2*e)/
(-a+b))+1/f^2*b/(a+b)*d*e/(a-b)*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)-
2/f^2*b/(a+b)*d*e/(a-b)*ln(exp(f*x+e))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(103) = 206$.

Time = 0.11 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.83

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx$$

$$=\frac{(a+b)df^2x^2+2\,(a+b)cf^2x-2\,bd\mathrm{Li}_2\left(\sqrt{-\frac{a+b}{a-b}}(\cosh(fx+e)+\sinh(fx+e))\right)-2\,bd\mathrm{Li}_2\left(-\sqrt{-\frac{a+b}{a-b}}(c\right.}{}$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
1/2*((a + b)*d*f^2*x^2 + 2*(a + b)*c*f^2*x - 2*b*d*dilog(sqrt(-(a + b)/(a
- b))*(cosh(f*x + e) + sinh(f*x + e))) - 2*b*d*dilog(-sqrt(-(a + b)/(a - b
))*(cosh(f*x + e) + sinh(f*x + e))) + 2*(b*d*e - b*c*f)*log(2*(a + b)*cosh
(f*x + e) + 2*(a + b)*sinh(f*x + e) + 2*(a - b)*sqrt(-(a + b)/(a - b))) +
2*(b*d*e - b*c*f)*log(2*(a + b)*cosh(f*x + e) + 2*(a + b)*sinh(f*x + e) -
2*(a - b)*sqrt(-(a + b)/(a - b))) - 2*(b*d*f*x + b*d*e)*log(sqrt(-(a + b)/
(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1) - 2*(b*d*f*x + b*d*e)*log(-s
qrt(-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e)) + 1))/((a^2 - b^2)*f
^2)
```

Sympy [F]

$$\int \frac{c + dx}{a + b \tanh(e + fx)}\, dx = \int \frac{c + dx}{a + b \tanh\left(e + fx\right)}\, dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x)
```

output
```
Integral((c + d*x)/(a + b*tanh(e + f*x)), x)
```

Maxima [F]

$$\int \frac{c + dx}{a + b \tanh(e + fx)}\, dx = \int \frac{dx + c}{b \tanh\left(fx + e\right) + a}\, dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
1/2*(4*b*integrate(x/(a^2 - b^2 + (a^2*e^(2*e) + 2*a*b*e^(2*e) + b^2*e^(2*
e))*e^(2*f*x)), x) + x^2/(a + b))*d - c*(b*log(-(a - b)*e^(-2*f*x - 2*e) -
 a - b)/((a^2 - b^2)*f) - (f*x + e)/((a + b)*f))
```

Giac [F]

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \int \frac{dx+c}{b\tanh(fx+e)+a}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e) + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx = \int \frac{c+d\,x}{a+b\tanh(e+f\,x)}\,dx$$

input
```
int((c + d*x)/(a + b*tanh(e + f*x)),x)
```

output
```
int((c + d*x)/(a + b*tanh(e + f*x)), x)
```

Reduce [F]

$$\int \frac{c+dx}{a+b\tanh(e+fx)}\,dx$$
$$= \frac{-4e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}a^2-e^{2fx+2e}b^2+a^2-2ab+b^2}\,dx\right)a^2bdf + 4e^{2e}\left(\int \frac{e^{2fx}x}{e^{2fx+2e}a^2-e^{2fx+2e}b^2+a^2-2ab+b^2}\,dx\right)b^3df - 2\log\left(e^{2f}\right.}{2f\left(a^2-b^2\right)}$$

input
```
int((d*x+c)/(a+b*tanh(f*x+e)),x)
```

output

```
( - 4*e**(2*e)*int((e**(2*f*x)*x)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x
)*b**2 + a**2 - 2*a*b + b**2),x)*a**2*b*d*f + 4*e**(2*e)*int((e**(2*f*x)*x
)/(e**(2*e + 2*f*x)*a**2 - e**(2*e + 2*f*x)*b**2 + a**2 - 2*a*b + b**2),x)
*b**3*d*f - 2*log(e**(2*e + 2*f*x)*a + e**(2*e + 2*f*x)*b + a - b)*b*c + 2
*a*c*f*x + a*d*f*x**2 + 2*b*c*f*x + b*d*f*x**2)/(2*f*(a**2 - b**2))
```

3.71 $\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$

Optimal result . . . 558
Mathematica [N/A] . . . 558
Rubi [N/A] . . . 559
Maple [N/A] . . . 559
Fricas [N/A] . . . 560
Sympy [N/A] . . . 560
Maxima [N/A] . . . 560
Giac [N/A] . . . 561
Mupad [N/A] . . . 561
Reduce [N/A] . . . 562

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+b\tanh(e+fx))}, x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 4.68 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)(a-ib\tan(ie+ifx))}dx$$
$$\downarrow 4223$$
$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}dx$$

input
```
Int[1/((c + d*x)*(a + b*Tanh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\,(a+b\tanh\,(fx+e))}dx$$

input
```
int(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

output
```
int(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.35

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d*x + a*c + (b*d*x + b*c)*tanh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.02 (sec) , antiderivative size = 17, normalized size of antiderivative = 0.85

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(a+b\tanh(e+fx))\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 0.24 (sec) , antiderivative size = 114, normalized size of antiderivative = 5.70

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
2*b*integrate(1/(a^2*c - b^2*c + (a^2*d - b^2*d)*x + (a^2*c*e^(2*e) + 2*a*
b*c*e^(2*e) + b^2*c*e^(2*e) + (a^2*d*e^(2*e) + 2*a*b*d*e^(2*e) + b^2*d*e^(
2*e))*x)*e^(2*f*x)), x) + log(d*x + c)/(a*d + b*d)
```

Giac [N/A]

Not integrable

Time = 0.13 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.20 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(a+b\tanh(e+f\,x))\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)),x)
```

output
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 271, normalized size of antiderivative = 13.55

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))}\,dx$$
$$= \frac{2\left(\int \frac{1}{e^{2fx+2e}a^2c+e^{2fx+2e}a^2dx+2e^{2fx+2e}abc+2e^{2fx+2e}abdx+e^{2fx+2e}b^2c+e^{2fx+2e}b^2dx+a^2c+a^2dx-b^2c-b^2dx}dx\right)abd+2\left(\int \frac{e^{2fx+}}{d\,(a+b}\right.}$$

input
```
int(1/(d*x+c)/(a+b*tanh(f*x+e)),x)
```

output
```
(2*int(1/(e**(2*e + 2*f*x)*a**2*c + e**(2*e + 2*f*x)*a**2*d*x + 2*e**(2*e
+ 2*f*x)*a*b*c + 2*e**(2*e + 2*f*x)*a*b*d*x + e**(2*e + 2*f*x)*b**2*c + e*
*(2*e + 2*f*x)*b**2*d*x + a**2*c + a**2*d*x - b**2*c - b**2*d*x),x)*a*b*d
+ 2*int(1/(e**(2*e + 2*f*x)*a**2*c + e**(2*e + 2*f*x)*a**2*d*x + 2*e**(2*e
 + 2*f*x)*a*b*c + 2*e**(2*e + 2*f*x)*a*b*d*x + e**(2*e + 2*f*x)*b**2*c + e
**(2*e + 2*f*x)*b**2*d*x + a**2*c + a**2*d*x - b**2*c - b**2*d*x),x)*b**2*
d + log(c + d*x))/(d*(a + b))
```

3.72 $\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$

Optimal result . . . 563
Mathematica [N/A] . . . 563
Rubi [N/A] . . . 564
Maple [N/A] . . . 564
Fricas [N/A] . . . 565
Sympy [N/A] . . . 565
Maxima [N/A] . . . 565
Giac [N/A] . . . 566
Mupad [N/A] . . . 566
Reduce [N/A] . . . 567

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\tanh(e+fx))}, x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

Mathematica [N/A]

Not integrable

Time = 8.60 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)^2(a-ib\tan(ie+ifx))}\,dx$$
$$\downarrow 4223$$
$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Tanh[e + f*x])),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.20 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\,(a+b\tanh(fx+e))}\,dx$$

input
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

output
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 51, normalized size of antiderivative = 2.55

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="fricas")
```

output
```
integral(1/(a*d^2*x^2 + 2*a*c*d*x + a*c^2 + (b*d^2*x^2 + 2*b*c*d*x + b*c^2
)*tanh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.70 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(a+b\tanh(e+fx))\,(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*tanh(f*x+e)),x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 0.44 (sec) , antiderivative size = 199, normalized size of antiderivative = 9.95

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="maxima")
```

output
```
2*b*integrate(1/(a^2*c^2 - b^2*c^2 + (a^2*d^2 - b^2*d^2)*x^2 + 2*(a^2*c*d
- b^2*c*d)*x + (a^2*c^2*e^(2*e) + 2*a*b*c^2*e^(2*e) + b^2*c^2*e^(2*e) + (a
^2*d^2*e^(2*e) + 2*a*b*d^2*e^(2*e) + b^2*d^2*e^(2*e))*x^2 + 2*(a^2*c*d*e^(
2*e) + 2*a*b*c*d*e^(2*e) + b^2*c*d*e^(2*e))*x)*e^(2*f*x)), x) - 1/(a*c*d +
 b*c*d + (a*d^2 + b*d^2)*x)
```

Giac [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*tanh(f*x + e) + a)), x)
```

Mupad [N/A]

Not integrable

Time = 2.45 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx = \int \frac{1}{(a+b\tanh(e+fx))\,(c+dx)^2}\,dx$$

input
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)^2),x)
```

output
```
int(1/((a + b*tanh(e + f*x))*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 937, normalized size of antiderivative = 46.85

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))}\,dx$$
$$= \frac{2\left(\int \frac{1}{e^{2fx+2e}a^2c^2+2e^{2fx+2e}a^2cdx+e^{2fx+2e}a^2d^2x^2+2e^{2fx+2e}ab\,c^2+4e^{2fx+2e}abcdx+2e^{2fx+2e}ab\,d^2x^2+e^{2fx+2e}b^2c^2+2e^{2fx+2e}b^2cdx+e^{2fx}}\right.}{}$$

input
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e)),x)
```

output
```
(2*int(1/(e**(2*e + 2*f*x)*a**2*c**2 + 2*e**(2*e + 2*f*x)*a**2*c*d*x + e**
(2*e + 2*f*x)*a**2*d**2*x**2 + 2*e**(2*e + 2*f*x)*a*b*c**2 + 4*e**(2*e + 2
*f*x)*a*b*c*d*x + 2*e**(2*e + 2*f*x)*a*b*d**2*x**2 + e**(2*e + 2*f*x)*b**2
*c**2 + 2*e**(2*e + 2*f*x)*b**2*c*d*x + e**(2*e + 2*f*x)*b**2*d**2*x**2 +
a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - b**2*c**2 - 2*b**2*c*d*x - b**
2*d**2*x**2),x)*a*b*c**2 + 2*int(1/(e**(2*e + 2*f*x)*a**2*c**2 + 2*e**(2*e
 + 2*f*x)*a**2*c*d*x + e**(2*e + 2*f*x)*a**2*d**2*x**2 + 2*e**(2*e + 2*f*x
)*a*b*c**2 + 4*e**(2*e + 2*f*x)*a*b*c*d*x + 2*e**(2*e + 2*f*x)*a*b*d**2*x*
*2 + e**(2*e + 2*f*x)*b**2*c**2 + 2*e**(2*e + 2*f*x)*b**2*c*d*x + e**(2*e
 + 2*f*x)*b**2*d**2*x**2 + a**2*c**2 + 2*a**2*c*d*x + a**2*d**2*x**2 - b**2
*c**2 - 2*b**2*c*d*x - b**2*d**2*x**2),x)*a*b*c*d*x + 2*int(1/(e**(2*e + 2
*f*x)*a**2*c**2 + 2*e**(2*e + 2*f*x)*a**2*c*d*x + e**(2*e + 2*f*x)*a**2*d*
*2*x**2 + 2*e**(2*e + 2*f*x)*a*b*c**2 + 4*e**(2*e + 2*f*x)*a*b*c*d*x + 2*e
**(2*e + 2*f*x)*a*b*d**2*x**2 + e**(2*e + 2*f*x)*b**2*c**2 + 2*e**(2*e + 2
*f*x)*b**2*c*d*x + e**(2*e + 2*f*x)*b**2*d**2*x**2 + a**2*c**2 + 2*a**2*c*
d*x + a**2*d**2*x**2 - b**2*c**2 - 2*b**2*c*d*x - b**2*d**2*x**2),x)*b**2*
c**2 + 2*int(1/(e**(2*e + 2*f*x)*a**2*c**2 + 2*e**(2*e + 2*f*x)*a**2*c*d*x
 + e**(2*e + 2*f*x)*a**2*d**2*x**2 + 2*e**(2*e + 2*f*x)*a*b*c**2 + 4*e**(2
*e + 2*f*x)*a*b*c*d*x + 2*e**(2*e + 2*f*x)*a*b*d**2*x**2 + e**(2*e + 2*f*x
)*b**2*c**2 + 2*e**(2*e + 2*f*x)*b**2*c*d*x + e**(2*e + 2*f*x)*b**2*d**...
```

3.73 $\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$

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Optimal result

Integrand size = 20, antiderivative size = 642

$$\begin{aligned}
\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = &-\frac{2b^2(c+dx)^3}{\left(a^2-b^2\right)^2 f} \\
&+\frac{2b^2(c+dx)^3}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right)f} \\
&+\frac{(c+dx)^4}{4(a-b)^2 d}+\frac{3b^2 d(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^2} \\
&-\frac{2b(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} \\
&+\frac{2b^2(c+dx)^3\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f} \\
&+\frac{3b^2d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^3} \\
&-\frac{3bd(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} \\
&+\frac{3b^2d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^2} \\
&-\frac{3b^2d^3\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2\left(a^2-b^2\right)^2 f^4} \\
&+\frac{3bd^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^3} \\
&-\frac{3b^2d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^3} \\
&-\frac{3bd^3\,\mathrm{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2(a-b)^2(a+b)f^4} \\
&+\frac{3b^2d^3\,\mathrm{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2\left(a^2-b^2\right)^2 f^4}
\end{aligned}$$

output

```
-2*b^2*(d*x+c)^3/(a^2-b^2)^2/f+2*b^2*(d*x+c)^3/(a-b)/(a+b)^2/(a-b+(a+b)*ex
p(2*f*x+2*e))/f+1/4*(d*x+c)^4/(a-b)^2/d+3*b^2*d*(d*x+c)^2*ln(1+(a+b)*exp(2
*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^3*ln(1+(a+b)*exp(2*f*x+2*e)/(
a-b))/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^3*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b))/(a^
2-b^2)^2/f+3*b^2*d^2*(d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b
^2)^2/f^3-3*b*d*(d*x+c)^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(
a+b)/f^2+3*b^2*d*(d*x+c)^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2
)^2/f^2-3/2*b^2*d^3*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4
+3*b*d^2*(d*x+c)*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-
3*b^2*d^2*(d*x+c)*polylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-3
/2*b*d^3*polylog(4,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^4+3/2*b^2*
d^3*polylog(4,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^4
```

Mathematica [A] (warning: unable to verify)

Time = 5.10 (sec) , antiderivative size = 656, normalized size of antiderivative = 1.02

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$$

$$= \frac{16bc^2f^3(-3bd+2acf)x + \frac{16(a-b)b^2f^3(c+dx)^3}{b(-1+e^{2e})+a(1+e^{2e})} - \frac{8a(a-b)bf^4(c+dx)^4}{d(b(-1+e^{2e})+a(1+e^{2e}))} + 48bcdf^2(bd-acf)x\log\left(1+\frac{(a-b)e^{-}}{a+}\right.}$$

input

```
Integrate[(c + d*x)^3/(a + b*Tanh[e + f*x])^2,x]
```

output

```
(16*b*c^2*f^3*(-3*b*d + 2*a*c*f)*x + (16*(a - b)*b^2*f^3*(c + d*x)^3)/(b*(
-1 + E^(2*e)) + a*(1 + E^(2*e))) - (8*a*(a - b)*b*f^4*(c + d*x)^4)/(d*(b*(
-1 + E^(2*e)) + a*(1 + E^(2*e)))) + 48*b*c*d*f^2*(b*d - a*c*f)*x*Log[1 + (
a - b)/((a + b)*E^(2*(e + f*x)))] + 24*b*d^2*f^2*(b*d - 2*a*c*f)*x^2*Log[1
 + (a - b)/((a + b)*E^(2*(e + f*x)))] - 16*a*b*d^3*f^3*x^3*Log[1 + (a - b)
/((a + b)*E^(2*(e + f*x)))] + 8*b*c^2*f^2*(3*b*d - 2*a*c*f)*Log[a - b + (a
 + b)*E^(2*(e + f*x))] + 24*b*c*d*f*(-(b*d) + a*c*f)*PolyLog[2, (-a + b)/(
(a + b)*E^(2*(e + f*x)))] - 12*b*d^2*(b*d - 2*a*c*f)*(2*f*x*PolyLog[2, (-a
 + b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[3, (-a + b)/((a + b)*E^(2*(e +
f*x)))]) + 12*a*b*d^3*(2*f^2*x^2*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*
x)))] + 2*f*x*PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))] + PolyLog[4,
(-a + b)/((a + b)*E^(2*(e + f*x)))]) + ((a - b)*(a + b)*f^3*((a^2 + b^2)*f
*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[f*x] + (a^2 - b^2)*f*x
*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3)*Cosh[2*e + f*x] + 2*b*(-4*b*(
c + d*x)^3 + a*f*x*(4*c^3 + 6*c^2*d*x + 4*c*d^2*x^2 + d^3*x^3))*Sinh[f*x])
)/((a*Cosh[e] + b*Sinh[e])*(a*Cosh[e + f*x] + b*Sinh[e + f*x])))/(8*(a - b
)^2*(a + b)^2*f^4)
```

Rubi [A] (verified)

Time = 2.46 (sec) , antiderivative size = 642, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4217, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c+dx)^3}{(a-ib\tan(ie+ifx))^2}dx$$

$$\downarrow 4217$$

$$\int \left(\frac{4b^2(c+dx)^3e^{4e+4fx}}{(a-b)^2\left(a\left(\frac{b}{a}+1\right)e^{2e+2fx}+a\left(1-\frac{b}{a}\right)\right)^2}+\frac{4b(c+dx)^3e^{2e+2fx}}{(a-b)^2\left(-a\left(\frac{b}{a}+1\right)e^{2e+2fx}-a\left(1-\frac{b}{a}\right)\right)}+\frac{(c+dx)^3}{(a-b)^2}\right)dx$$

↓ 2009

$$\frac{3b^2d^2(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-\frac{3b^2d^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}+\frac{3b^2d(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{3b^2d(c+dx)^2\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^3\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f\left(a^2-b^2\right)^2}-\frac{2b^2(c+dx)^3}{f\left(a^2-b^2\right)^2}-\frac{3b^2d^3\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4\left(a^2-b^2\right)^2}+\frac{3b^2d^3\,\mathrm{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^3}{f(a-b)(a+b)^2\left((a+b)e^{2e+2fx}+a-b\right)}+\frac{3bd^2(c+dx)\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a-b)^2(a+b)}-\frac{3bd(c+dx)^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)^2(a+b)}-\frac{2b(c+dx)^3\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f(a-b)^2(a+b)}+\frac{(c+dx)^4}{4d(a-b)^2}-\frac{3bd^3\,\mathrm{PolyLog}\left(4,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{2f^4(a-b)^2(a+b)}$$

input

```
Int[(c + d*x)^3/(a + b*Tanh[e + f*x])^2,x]
```

output

```
(-2*b^2*(c + d*x)^3)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^3)/((a - b)*(a +
 b)^2*(a - b + (a + b)*E^(2*e + 2*f*x))*f) + (c + d*x)^4/(4*(a - b)^2*d) +
 (3*b^2*d*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 -
b^2)^2*f^2) - (2*b*(c + d*x)^3*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])
/((a - b)^2*(a + b)*f) + (2*b^2*(c + d*x)^3*Log[1 + ((a + b)*E^(2*e + 2*f*
x))/(a - b)])/((a^2 - b^2)^2*f) + (3*b^2*d^2*(c + d*x)*PolyLog[2, -(((a +
b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (3*b*d*(c + d*x)^2*Po
lyLog[2, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^2) +
(3*b^2*d*(c + d*x)^2*PolyLog[2, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a
^2 - b^2)^2*f^2) - (3*b^2*d^3*PolyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a -
b))])/(2*(a^2 - b^2)^2*f^4) + (3*b*d^2*(c + d*x)*PolyLog[3, -(((a + b)*E^(
2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^3) - (3*b^2*d^2*(c + d*x)*Po
lyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (3*b
*d^3*PolyLog[4, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/(2*(a - b)^2*(a + b
)*f^4) + (3*b^2*d^3*PolyLog[4, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/(2*(
a^2 - b^2)^2*f^4)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4217
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(622) = 1244$.

Time = 1.02 (sec) , antiderivative size = 2683, normalized size of antiderivative = 4.18

method	result	size
risch	Expression too large to display	2683

input
```
int((d*x+c)^3/(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output
```
12/(a^2+2*a*b+b^2)/f^3*b/(a-b)^2*e^2*d^2*c*a*ln(exp(f*x+e))+3/(a^2+2*a*b+b
^2)/f^2*b/(a-b)*a*c^2*d/(-a+b)*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))-3/(a
^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*d^2*c*a*polylog(3,(a+b)*exp(2*f*x+2*e)/(-
a+b))+2/(a^2+2*a*b+b^2)/f*b/(a-b)/(-a+b)*d^3*a*ln(1-(a+b)*exp(2*f*x+2*e)/(
-a+b))*x^3+3/(a^2+2*a*b+b^2)/f^2*b/(a-b)/(-a+b)*d^3*a*polylog(2,(a+b)*exp(
2*f*x+2*e)/(-a+b))*x^2+2/(a^2+2*a*b+b^2)/f^4*b/(a-b)/(-a+b)*d^3*a*ln(1-(a+
b)*exp(2*f*x+2*e)/(-a+b))*e^3-3/(a^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*d^3*a*p
olylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b))*x+6/(a^2+2*a*b+b^2)/f^2*b/(a-b)^2*e*
a*c^2*d*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)-12/(a^2+2*a*b+b^2)/f^2*b
/(a-b)^2*e*a*c^2*d*ln(exp(f*x+e))-6/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)/(-a+b)*c
*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x-6/(a^2+2*a*b+b^2)/f^3*b^2/(a-b)/(
-a+b)*c*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e-6/(a^2+2*a*b+b^2)/f^2*b/(a
-b)*a*c^2*d/(-a+b)*e^2-4/(a^2+2*a*b+b^2)*b/(a-b)/(-a+b)*d^2*c*a*x^3+8/(a^2
+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*d^2*c*a*e^3-6/(a^2+2*a*b+b^2)*b/(a-b)*a*c^2
*d/(-a+b)*x^2-4/(a^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*d^3*a*e^3*x-6/(a^2+2*a*
b+b^2)/f^3*b/(a-b)^2*e^2*d^2*c*a*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)
-6/(a^2+2*a*b+b^2)/f^3*b^2/(a-b)/(-a+b)*d^3*e^2*x-3/(a^2+2*a*b+b^2)/f^3*b^
2/(a-b)/(-a+b)*c*d^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))-3/(a^2+2*a*b+b
^2)/f^2*b^2/(a-b)/(-a+b)*d^3*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x^2+3/(a^2+
2*a*b+b^2)/f^4*b^2/(a-b)/(-a+b)*d^3*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6160 vs. $2(619) = 1238$.

Time $= 0.22$ (sec) , antiderivative size $= 6160$, normalized size of antiderivative $= 9.60$

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx$$

input
```
integrate((d*x+c)**3/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**3/(a + b*tanh(e + f*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 1060, normalized size of antiderivative = 1.65

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output

```
-6*b^2*c^2*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - 2/3*(4*f^3*x^3*log(
(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 6*f^2*x^2*dilog(-(a*e^(2*
e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - 6*f*x*polylog(3, -(a*e^(2*e) + b*e^(2
*e))*e^(2*f*x)/(a - b)) + 3*polylog(4, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/
(a - b)))*a*b*d^3/(a^4*f^4 - 2*a^2*b^2*f^4 + b^4*f^4) + 3*b^2*c^2*d*log((a
*e^(2*e) + b*e^(2*e))*e^(2*f*x) + a - b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^
2) - c^3*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 +
 b^4)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)
*e^(-2*f*x - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - 3/2*(2*a*b*c*
d^2*f - b^2*d^3)*(2*f^2*x^2*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)
+ 1) + 2*f*x*dilog(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)) - polylog(3
, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^4 - 2*a^2*b^2*f^4 +
b^4*f^4) - 3*(a*b*c^2*d*f - b^2*c*d^2)*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*
e^(2*f*x)/(a - b) + 1) + dilog(-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b))
)/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + (a*b*d^3*f^4*x^4 + 2*(2*a*b*c*d^2*
f - b^2*d^3)*f^3*x^3 + 6*(a*b*c^2*d*f^2 - b^2*c*d^2*f)*f^2*x^2)/(a^4*f^4 -
 2*a^2*b^2*f^4 + b^4*f^4) + 1/4*(24*b^2*c^2*d*x + (a^2*d^3*f - 2*a*b*d^3*f
 + b^2*d^3*f)*x^4 + 4*(a^2*c*d^2*f - 2*a*b*c*d^2*f + (c*d^2*f + 2*d^3)*b^2
)*x^3 + 6*(a^2*c^2*d*f - 2*a*b*c^2*d*f + (c^2*d*f + 4*c*d^2)*b^2)*x^2 + ((
a^2*d^3*f*e^(2*e) - b^2*d^3*f*e^(2*e))*x^4 + 4*(a^2*c*d^2*f*e^(2*e) - b...
```

Giac [F]

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^3}{(b\tanh(fx+e)+a)^2}\,dx$$

input

```
integrate((d*x+c)^3/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output

```
integrate((d*x + c)^3/(b*tanh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^3}{(a+b\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^3/(a + b*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)^3/(a + b*tanh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^3}{(a+b\tanh(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^3/(a+b*tanh(f*x+e))^2,x)
```

output

```
(16*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*
a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*
e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a**3*b - 4*
e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b
**4),x)*a**6*b*d**3*f**4 + 16*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x)*
a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*
e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e*
*(2*e + 2*f*x)*a**3*b - 4*e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**
4 + a**4 - 2*a**2*b**2 + b**4),x)*a**5*b**2*d**3*f**4 - 32*e**(2*e + 2*f*x
)*int(x**3/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e
+ 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 + 2
*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a**3*b - 4*e**(2*e + 2*f*x)*a*
b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**4*b**3*d
**3*f**4 - 32*e**(2*e + 2*f*x)*int(x**3/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e
 + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**
3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a
**3*b - 4*e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**
2*b**2 + b**4),x)*a**3*b**4*d**3*f**4 + 16*e**(2*e + 2*f*x)*int(x**3/(e**(
4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b*
*2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*...
```

3.74 $\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx$

Optimal result 580
Mathematica [A] (warning: unable to verify) 581
Rubi [A] (verified) 581
Maple [B] (verified) 583
Fricas [B] (verification not implemented) 584
Sympy [F] 585
Maxima [A] (verification not implemented) 585
Giac [F] 586
Mupad [F(-1)] 586
Reduce [F] 586

Optimal result

Integrand size = 20, antiderivative size = 476

$$\begin{aligned}\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx &= -\frac{2b^2(c+dx)^2}{\left(a^2-b^2\right)^2 f} \\ &+ \frac{2b^2(c+dx)^2}{(a-b)(a+b)^2\left(a-b+(a+b)e^{2e+2fx}\right) f} \\ &+ \frac{(c+dx)^3}{3(a-b)^2 d} + \frac{2b^2 d(c+dx)\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^2} \\ &- \frac{2b(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f} \\ &+ \frac{2b^2(c+dx)^2\log\left(1+\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f} \\ &+ \frac{b^2 d^2\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^3} \\ &- \frac{2bd(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^2} \\ &+ \frac{2b^2 d(c+dx)\,\mathrm{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^2} \\ &+ \frac{bd^2\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{(a-b)^2(a+b)f^3} \\ &- \frac{b^2 d^2\,\mathrm{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{\left(a^2-b^2\right)^2 f^3}\end{aligned}$$

output

```
-2*b^2*(d*x+c)^2/(a^2-b^2)^2/f+2*b^2*(d*x+c)^2/(a-b)/(a+b)^2/(a-b+(a+b)*ex
p(2*f*x+2*e))/f+1/3*(d*x+c)^3/(a-b)^2/d+2*b^2*d*(d*x+c)*ln(1+(a+b)*exp(2*f
*x+2*e)/(a-b))/(a^2-b^2)^2/f^2-2*b*(d*x+c)^2*ln(1+(a+b)*exp(2*f*x+2*e)/(a-
b))/(a-b)^2/(a+b)/f+2*b^2*(d*x+c)^2*ln(1+(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-
b^2)^2/f+b^2*d^2*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3-2*
b*d*(d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^2+2*b^2
*d*(d*x+c)*polylog(2,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^2+b*d^2*po
lylog(3,-(a+b)*exp(2*f*x+2*e)/(a-b))/(a-b)^2/(a+b)/f^3-b^2*d^2*polylog(3,-
(a+b)*exp(2*f*x+2*e)/(a-b))/(a^2-b^2)^2/f^3
```

Mathematica [A] (warning: unable to verify)

Time = 3.27 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.08

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx$$
$$= \frac{24bcf^2(-bd+acf)x - \frac{24(a-b)bcf^2(-bd+acf)x}{b(-1+e^{2e})+a(1+e^{2e})} - \frac{12(a-b)bdf^2(-bd+2acf)x^2}{b(-1+e^{2e})+a(1+e^{2e})} - \frac{8a(a-b)bd^2f^3x^3}{b(-1+e^{2e})+a(1+e^{2e})} + 12bdf(bd-2ac$$

input
```
Integrate[(c + d*x)^2/(a + b*Tanh[e + f*x])^2,x]
```

output
```
(24*b*c*f^2*(-(b*d) + a*c*f)*x - (24*(a - b)*b*c*f^2*(-(b*d) + a*c*f)*x)/(
b*(-1 + E^(2*e)) + a*(1 + E^(2*e))) - (12*(a - b)*b*d*f^2*(-(b*d) + 2*a*c*
f)*x^2)/(b*(-1 + E^(2*e)) + a*(1 + E^(2*e))) - (8*a*(a - b)*b*d^2*f^3*x^3)
/(b*(-1 + E^(2*e)) + a*(1 + E^(2*e))) + 12*b*d*f*(b*d - 2*a*c*f)*x*Log[1 +
 (a - b)/((a + b)*E^(2*(e + f*x)))] - 12*a*b*d^2*f^2*x^2*Log[1 + (a - b)/(
(a + b)*E^(2*(e + f*x)))] + 12*b*c*f*(b*d - a*c*f)*Log[a - b + (a + b)*E^(
2*(e + f*x))] - 6*b*d*(b*d - 2*a*c*f)*PolyLog[2, (-a + b)/((a + b)*E^(2*(e
 + f*x)))] + 6*a*b*d^2*(2*f*x*PolyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x))
)] + PolyLog[3, (-a + b)/((a + b)*E^(2*(e + f*x)))]) + ((a - b)*(a + b)*f^
2*((a^2 + b^2)*f*x*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[f*x] + (a^2 - b^2)*f*x
*(3*c^2 + 3*c*d*x + d^2*x^2)*Cosh[2*e + f*x] + 2*b*(-3*b*(c + d*x)^2 + a*f
*x*(3*c^2 + 3*c*d*x + d^2*x^2))*Sinh[f*x]))/((a*Cosh[e] + b*Sinh[e])*(a*Co
sh[e + f*x] + b*Sinh[e + f*x])))/(6*(a - b)^2*(a + b)^2*f^3)
```

Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 476, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {3042, 4217, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx$$

$$\downarrow 3042$$

$$\int \frac{(c+dx)^2}{(a-ib\tan(ie+ifx))^2}dx$$

$$\downarrow 4217$$

$$\int \left(\frac{4b^2(c+dx)^2e^{4e+4fx}}{(a-b)^2\left(a\left(\frac{b}{a}+1\right)e^{2e+2fx}+a\left(1-\frac{b}{a}\right)\right)^2}+\frac{4b(c+dx)^2e^{2e+2fx}}{(a-b)^2\left(-a\left(\frac{b}{a}+1\right)e^{2e+2fx}-a\left(1-\frac{b}{a}\right)\right)}+\frac{(c+dx)^2}{(a-b)^2}\right)dx$$

$$\downarrow 2009$$

$$\frac{2b^2d(c+dx)\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2\left(a^2-b^2\right)^2}+\frac{2b^2d(c+dx)\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f^2\left(a^2-b^2\right)^2}+$$
$$\frac{2b^2(c+dx)^2\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f\left(a^2-b^2\right)^2}-\frac{2b^2(c+dx)^2}{f\left(a^2-b^2\right)^2}+\frac{b^2d^2\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}-$$
$$\frac{b^2d^2\operatorname{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3\left(a^2-b^2\right)^2}+\frac{2b^2(c+dx)^2}{f(a-b)(a+b)^2\left((a+b)e^{2e+2fx}+a-b\right)}-$$
$$\frac{2bd(c+dx)\operatorname{PolyLog}\left(2,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^2(a-b)^2(a+b)}-\frac{2b(c+dx)^2\log\left(\frac{(a+b)e^{2e+2fx}}{a-b}+1\right)}{f(a-b)^2(a+b)}+\frac{(c+dx)^3}{3d(a-b)^2}+$$
$$\frac{bd^2\operatorname{PolyLog}\left(3,-\frac{(a+b)e^{2e+2fx}}{a-b}\right)}{f^3(a-b)^2(a+b)}$$

input

```
Int[(c + d*x)^2/(a + b*Tanh[e + f*x])^2,x]
```

output

```
(-2*b^2*(c + d*x)^2)/((a^2 - b^2)^2*f) + (2*b^2*(c + d*x)^2)/((a - b)*(a +
 b)^2*(a - b + (a + b)*E^(2*e + 2*f*x))*f) + (c + d*x)^3/(3*(a - b)^2*d) +
 (2*b^2*d*(c + d*x)*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/((a^2 - b^
2)^2*f^2) - (2*b*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x))/(a - b)])/(
(a - b)^2*(a + b)*f) + (2*b^2*(c + d*x)^2*Log[1 + ((a + b)*E^(2*e + 2*f*x)
)/(a - b)])/((a^2 - b^2)^2*f) + (b^2*d^2*PolyLog[2, -(((a + b)*E^(2*e + 2*
f*x))/(a - b))])/((a^2 - b^2)^2*f^3) - (2*b*d*(c + d*x)*PolyLog[2, -(((a +
 b)*E^(2*e + 2*f*x))/(a - b))])/((a - b)^2*(a + b)*f^2) + (2*b^2*d*(c + d*
x)*PolyLog[2, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2 - b^2)^2*f^2) +
 (b*d^2*PolyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a - b)^2*(a +
b)*f^3) - (b^2*d^2*PolyLog[3, -(((a + b)*E^(2*e + 2*f*x))/(a - b))])/((a^2
 - b^2)^2*f^3)
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4217
```
Int[((c_.) + (d_.)*(x_))^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(n_),
 x_Symbol] :> Int[ExpandIntegrand[(c + d*x)^m, (1/(a - I*b) - 2*I*(b/(a^2 +
 b^2 + (a - I*b)^2*E^(2*I*(e + f*x)))))^(-n), x], x] /; FreeQ[{a, b, c, d,
e, f}, x] && NeQ[a^2 + b^2, 0] && ILtQ[n, 0] && IGtQ[m, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1604 vs. $2(465) = 930$.

Time = 0.89 (sec) , antiderivative size = 1605, normalized size of antiderivative = 3.37

method	result	size
risch	Expression too large to display	1605

input
```
int((d*x+c)^2/(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output

```
d/(a^2+2*a*b+b^2)*c*x^2+1/(a^2+2*a*b+b^2)*c^2*x+2/(a-b)/f/(a^2+2*a*b+b^2)*
(d^2*x^2+2*c*d*x+c^2)*b^2/(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)-4/3/(a^2
+2*a*b+b^2)*b/(a-b)/(-a+b)*a*d^2*x^3+8/3/(a^2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b
)*a*d^2*e^3+4/(a^2+2*a*b+b^2)/f^3*b/(a-b)^2*e^2*a*d^2*ln(exp(f*x+e))-1/(a^
2+2*a*b+b^2)/f^3*b/(a-b)/(-a+b)*a*d^2*polylog(3,(a+b)*exp(2*f*x+2*e)/(-a+b
))-2/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)/(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*e)/(-
a+b))*x-2/(a^2+2*a*b+b^2)/f^3*b^2/(a-b)/(-a+b)*d^2*ln(1-(a+b)*exp(2*f*x+2*
e)/(-a+b))*e-2/(a^2+2*a*b+b^2)/f^3*b/(a-b)^2*e^2*a*d^2*ln(a*exp(2*f*x+2*e)
+exp(2*f*x+2*e)*b+a-b)-8/(a^2+2*a*b+b^2)/f*b/(a-b)*c*d*a/(-a+b)*e*x+4/(a^2
+2*a*b+b^2)/f^2*b/(a-b)*c*d*a/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*e+4
/(a^2+2*a*b+b^2)/f*b/(a-b)*c*d*a/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*
x+4/(a^2+2*a*b+b^2)/f^2*b^2/(a-b)/(-a+b)*d^2*e*x+2/(a^2+2*a*b+b^2)/f*b^2/(
a-b)/(-a+b)*d^2*x^2+2/(a^2+2*a*b+b^2)/f^3*b^2/(a-b)/(-a+b)*d^2*e^2-2/(a^2+
2*a*b+b^2)/f^3*b^2/(a-b)^2*e*d^2*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)
+4/(a^2+2*a*b+b^2)/f^3*b^2/(a-b)^2*e*d^2*ln(exp(f*x+e))-1/(a^2+2*a*b+b^2)/
f^3*b^2/(a-b)/(-a+b)*d^2*polylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))-2/(a^2+2*a
*b+b^2)/f*b/(a-b)^2*a*c^2*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)+4/(a^2
+2*a*b+b^2)/f*b/(a-b)^2*a*c^2*ln(exp(f*x+e))+2/(a^2+2*a*b+b^2)/f^2*b^2/(a-
b)^2*c*d*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)-4/(a^2+2*a*b+b^2)/f^2*b
^2/(a-b)^2*c*d*ln(exp(f*x+e))-8/(a^2+2*a*b+b^2)/f^2*b/(a-b)^2*e*c*d*a*l...
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3693 vs. $2(462) = 924$.

Time $= 0.17$ (sec) , antiderivative size $= 3693$, normalized size of antiderivative $= 7.76$

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output

```
Too large to include
```

Sympy [F]

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx$$

input
```
integrate((d*x+c)**2/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral((c + d*x)**2/(a + b*tanh(e + f*x))**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 753, normalized size of antiderivative = 1.58

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-4*b^2*c*d*f*x/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2) - (2*f^2*x^2*log((a*e^(
2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + 2*f*x*dilog(-(a*e^(2*e) + b*e^(
2*e))*e^(2*f*x)/(a - b)) - polylog(3, -(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(
a - b)))*a*b*d^2/(a^4*f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 2*b^2*c*d*log((a*e^
(2*e) + b*e^(2*e))*e^(2*f*x) + a - b)/(a^4*f^2 - 2*a^2*b^2*f^2 + b^4*f^2)
 - c^2*(2*a*b*log(-(a - b)*e^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 + b^
4)*f) + 2*b^2/((a^4 - 2*a^2*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^
(-2*f*x - 2*e))*f) - (f*x + e)/((a^2 + 2*a*b + b^2)*f)) - (2*a*b*c*d*f - b
^2*d^2)*(2*f*x*log((a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b) + 1) + dilog(
-(a*e^(2*e) + b*e^(2*e))*e^(2*f*x)/(a - b)))/(a^4*f^3 - 2*a^2*b^2*f^3 + b^
4*f^3) + 2/3*(2*a*b*d^2*f^3*x^3 + 3*(2*a*b*c*d*f - b^2*d^2)*f^2*x^2)/(a^4*
f^3 - 2*a^2*b^2*f^3 + b^4*f^3) + 1/3*(12*b^2*c*d*x + (a^2*d^2*f - 2*a*b*d^
2*f + b^2*d^2*f)*x^3 + 3*(a^2*c*d*f - 2*a*b*c*d*f + (c*d*f + 2*d^2)*b^2)*x
^2 + ((a^2*d^2*f*e^(2*e) - b^2*d^2*f*e^(2*e))*x^3 + 3*(a^2*c*d*f*e^(2*e) -
 b^2*c*d*f*e^(2*e))*x^2)*e^(2*f*x))/(a^4*f - 2*a^2*b^2*f + b^4*f + (a^4*f*
e^(2*e) + 2*a^3*b*f*e^(2*e) - 2*a*b^3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x)
)
```

Giac [F]

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(dx+c)^2}{(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)^2/(b*tanh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{(c+d\,x)^2}{(a+b\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)^2/(a + b*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)^2/(a + b*tanh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{(c+dx)^2}{(a+b\tanh(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

output

```
(12*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*
a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*
e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a**3*b - 4*
e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b
**4),x)*a**6*b*d**2*f**3 + 12*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x)*
a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*
e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e*
*(2*e + 2*f*x)*a**3*b - 4*e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**
4 + a**4 - 2*a**2*b**2 + b**4),x)*a**5*b**2*d**2*f**3 - 24*e**(2*e + 2*f*x
)*int(x**2/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e
+ 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 + 2
*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a**3*b - 4*e**(2*e + 2*f*x)*a*
b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**4*b**3*d
**2*f**3 - 24*e**(2*e + 2*f*x)*int(x**2/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e
 + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**
3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a
**3*b - 4*e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**
2*b**2 + b**4),x)*a**3*b**4*d**2*f**3 + 12*e**(2*e + 2*f*x)*int(x**2/(e**(
4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b*
*2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*...
```

3.75 $\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$

Optimal result . . . 588
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Optimal result

Integrand size = 18, antiderivative size = 196

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = -\frac{(c+dx)^2}{2\left(a^2-b^2\right)d} + \frac{(bd-2acf-2adfx)^2}{4a(a-b)(a+b)^2df^2} + \frac{b(bd-2acf-2adfx)\log\left(1+\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)^2f^2} + \frac{abd\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{\left(a^2-b^2\right)^2f^2} + \frac{b(c+dx)}{\left(a^2-b^2\right)f(a+b\tanh(e+fx))}$$

output

```
-1/2*(d*x+c)^2/(a^2-b^2)/d+1/4*(-2*a*d*f*x-2*a*c*f+b*d)^2/a/(a-b)/(a+b)^2/
d/f^2+b*(-2*a*d*f*x-2*a*c*f+b*d)*ln(1+(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2
)^2/f^2+a*b*d*polylog(2,-(a-b)/(a+b)/exp(2*f*x+2*e))/(a^2-b^2)^2/f^2+b*(d*
x+c)/(a^2-b^2)/f/(a+b*tanh(f*x+e))
```

Mathematica [A] (verified)

Time = 2.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.27

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$$

$$= \frac{\operatorname{sech}^2(e+fx)(a\cosh(e+fx)+b\sinh(e+fx))\left(-\frac{2b^2 f(c+dx)\sinh(e+fx)}{a}-(e+fx)(-2cf+d(e-fx))(a\right.}{}$$

input
```
Integrate[(c + d*x)/(a + b*Tanh[e + f*x])^2,x]
```

output
```
(Sech[e + f*x]^2*(a*Cosh[e + f*x] + b*Sinh[e + f*x])*((-2*b^2*f*(c + d*x)*
Sinh[e + f*x])/a - (e + f*x)*(-2*c*f + d*(e - f*x))*(a*Cosh[e + f*x] + b*S
inh[e + f*x]) + (b*((b*d - 2*a*f*(c + d*x))*((a - b)*(-(b*d) + 2*a*f*(c +
d*x)) + 4*a^2*d*Log[1 + (a - b)/((a + b)*E^(2*(e + f*x)))]) + 4*a^3*d^2*Po
lyLog[2, (-a + b)/((a + b)*E^(2*(e + f*x)))])*(a*Cosh[e + f*x] + b*Sinh[e
+ f*x]))/(2*a^2*(a - b)*(a + b)*d)))/(2*(a - b)*(a + b)*f^2*(a + b*Tanh[e
+ f*x])^2)
```

Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {3042, 4216, 26, 3042, 4215, 2620, 2715, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{c+dx}{(a-ib\tan(ie+ifx))^2}dx$$
$$\downarrow 4216$$

$$-\frac{i\int -\frac{i(bd-2afxd-2acf)}{a+b\tanh(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\downarrow 26$$

$$-\frac{\int \frac{bd-2afxd-2acf}{a+b\tanh(e+fx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\downarrow 3042$$

$$-\frac{\int \frac{bd-2afxd-2acf}{a-ib\tan(ie+ifx)}dx}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\downarrow 4215$$

$$-\frac{2b\int \frac{e^{-2(e+fx)}(bd-2afxd-2acf)}{(a+b)^2+(a^2-b^2)e^{-2(e+fx)}}dx-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\downarrow 2620$$

$$-\frac{2b\left(-\frac{ad\int \log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)dx}{a^2-b^2}-\frac{(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\downarrow 2715$$

$$-\frac{2b\left(\frac{ad\int e^{2(e+fx)}\log\left(\frac{e^{-2(e+fx)}(a-b)}{a+b}+1\right)de^{-2(e+fx)}}{2f(a^2-b^2)}-\frac{(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

$$\downarrow 2838$$

$$-\frac{2b\left(-\frac{(-2acf-2adfx+bd)\log\left(\frac{(a-b)e^{-2(e+fx)}}{a+b}+1\right)}{2f(a^2-b^2)}-\frac{ad\,\mathrm{PolyLog}\left(2,-\frac{(a-b)e^{-2(e+fx)}}{a+b}\right)}{2f(a^2-b^2)}\right)-\frac{(-2acf-2adfx+bd)^2}{4adf(a+b)}}{f\left(a^2-b^2\right)}+\frac{b(c+dx)}{f\left(a^2-b^2\right)(a+b\tanh(e+fx))}-\frac{(c+dx)^2}{2d\left(a^2-b^2\right)}$$

input
```
Int[(c + d*x)/(a + b*Tanh[e + f*x])^2,x]
```

output
```
-1/2*(c + d*x)^2/((a^2 - b^2)*d) - (-1/4*(b*d - 2*a*c*f - 2*a*d*f*x)^2/(a*
(a + b)*d*f) + 2*b*(-1/2*((b*d - 2*a*c*f - 2*a*d*f*x)*Log[1 + (a - b)/((a
+ b)*E^(2*(e + f*x)))])/((a^2 - b^2)*f) - (a*d*PolyLog[2, -((a - b)/((a +
b)*E^(2*(e + f*x))))])/(2*(a^2 - b^2)*f)))/((a^2 - b^2)*f) + (b*(c + d*x))
/((a^2 - b^2)*f*(a + b*Tanh[e + f*x]))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2620
```
Int[(((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)*((c_.) + (d_.)*(x_))^(m_.))/
((a_) + (b_.)*((F_)^((g_.)*((e_.) + (f_.)*(x_))))^(n_.)), x_Symbol] :> Simp
[((c + d*x)^m/(b*f*g*n*Log[F]))*Log[1 + b*((F^(g*(e + f*x)))^n/a)], x] - Si
mp[d*(m/(b*f*g*n*Log[F]))   Int[(c + d*x)^(m - 1)*Log[1 + b*((F^(g*(e + f*x
)))^n/a)], x], x] /; FreeQ[{F, a, b, c, d, e, f, g, n}, x] && IGtQ[m, 0]
```

rule 2715
```
Int[Log[(a_) + (b_.)*((F_)^((e_.)*((c_.) + (d_.)*(x_))))^(n_.)], x_Symbol]
:> Simp[1/(d*e*n*Log[F])   Subst[Int[Log[a + b*x]/x, x], x, (F^(e*(c + d*x)
))^n], x] /; FreeQ[{F, a, b, c, d, e, n}, x] && GtQ[a, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4215
```
Int[((c_.) + (d_.)*(x_))^(m_.)/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]), x_Sy
mbol] :> Simp[(c + d*x)^(m + 1)/(d*(m + 1)*(a + I*b)), x] + Simp[2*I*b   In
t[(c + d*x)^m*(E^Simp[2*I*(e + f*x), x]/((a + I*b)^2 + (a^2 + b^2)*E^Simp[2
*I*(e + f*x), x])), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[a^2 + b^2
, 0] && IGtQ[m, 0]
```

rule 4216
```
Int[((c_.) + (d_.)*(x_))/((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)])^2, x_Symbol
] :> Simp[-(c + d*x)^2/(2*d*(a^2 + b^2)), x] + (Simp[1/(f*(a^2 + b^2))   In
t[(b*d + 2*a*c*f + 2*a*d*f*x)/(a + b*Tan[e + f*x]), x], x] - Simp[b*((c + d
*x)/(f*(a^2 + b^2)*(a + b*Tan[e + f*x]))), x]) /; FreeQ[{a, b, c, d, e, f},
 x] && NeQ[a^2 + b^2, 0]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 564 vs. $2(196) = 392$.

Time $= 0.57$ (sec) , antiderivative size $= 565$, normalized size of antiderivative $= 2.88$

method	result
risch	$\frac{d x^2}{2a^2+4ab+2b^2} + \frac{cx}{a^2+2ab+b^2} + \frac{2(dx+c)b^2}{(a-b)f(a^2+2ab+b^2)(a\,e^{2fx+2e}+e^{2fx+2e}b+a-b)} + \frac{b^2 d \ln(a\,e^{2fx+2e}+e^{2fx+2e}b+a-b)}{f^2(a-b)^2(a+b)^2} - \frac{2}{f}$

input
```
int((d*x+c)/(a+b*tanh(f*x+e))^2,x,method=_RETURNVERBOSE)
```

output

```
1/2/(a^2+2*a*b+b^2)*d*x^2+1/(a^2+2*a*b+b^2)*c*x+2/(a-b)/f/(a^2+2*a*b+b^2)*
(d*x+c)*b^2/(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)+1/f^2/(a-b)^2/(a+b)^2*
b^2*d*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)-2/f^2/(a-b)^2/(a+b)^2*b^2*
d*ln(exp(f*x+e))-2/f/(a-b)^2/(a+b)^2*b*a*c*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2
*e)*b+a-b)+4/f/(a-b)^2/(a+b)^2*b*a*c*ln(exp(f*x+e))+2/f^2/(a-b)^2/(a+b)^2*
b*e*a*d*ln(a*exp(2*f*x+2*e)+exp(2*f*x+2*e)*b+a-b)-4/f^2/(a-b)^2/(a+b)^2*b*
e*a*d*ln(exp(f*x+e))-2/(a-b)/(a+b)^2*b*a*d/(-a+b)*x^2+2/f/(a-b)/(a+b)^2*b*
a*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))*x-4/f/(a-b)/(a+b)^2*b*a*d/(-a
+b)*e*x+2/f^2/(a-b)/(a+b)^2*b*a*d/(-a+b)*ln(1-(a+b)*exp(2*f*x+2*e)/(-a+b))
*e-2/f^2/(a-b)/(a+b)^2*b*a*d/(-a+b)*e^2+1/f^2/(a-b)/(a+b)^2*b*a*d/(-a+b)*p
olylog(2,(a+b)*exp(2*f*x+2*e)/(-a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1790 vs. $2(194) = 388$.

Time $= 0.14$ (sec) , antiderivative size $= 1790$, normalized size of antiderivative $= 9.13$

$$\int \frac{c + dx}{(a + b\tanh(e + fx))^2}\, dx = \text{Too large to display}$$

input

```
integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output

```
1/2*((a^3 + a^2*b - a*b^2 - b^3)*d*f^2*x^2 + 2*(a^3 + a^2*b - a*b^2 - b^3)
*c*f^2*x - 4*(a^2*b - a*b^2)*d*e^2 - 4*(a*b^2 - b^3)*d*e + ((a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*f^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*
c*e*f - 4*(a*b^2 + b^3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2
*(a*b^2 + b^3)*d*f)*x)*cosh(f*x + e)^2 + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*f^2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^2
 + b^3)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*d
*f)*x)*cosh(f*x + e)*sinh(f*x + e) + ((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*f^
2*x^2 - 4*(a^2*b + a*b^2)*d*e^2 + 8*(a^2*b + a*b^2)*c*e*f - 4*(a*b^2 + b^3
)*d*e + 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*c*f^2 - 2*(a*b^2 + b^3)*d*f)*x)
*sinh(f*x + e)^2 + 4*(2*(a^2*b - a*b^2)*c*e + (a*b^2 - b^3)*c)*f - 4*((a^2
*b + a*b^2)*d*cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*x
 + e) + (a^2*b + a*b^2)*d*sinh(f*x + e)^2 + (a^2*b - a*b^2)*d)*dilog(sqrt(
-(a + b)/(a - b))*(cosh(f*x + e) + sinh(f*x + e))) - 4*((a^2*b + a*b^2)*d*
cosh(f*x + e)^2 + 2*(a^2*b + a*b^2)*d*cosh(f*x + e)*sinh(f*x + e) + (a^2*b
 + a*b^2)*d*sinh(f*x + e)^2 + (a^2*b - a*b^2)*d)*dilog(-sqrt(-(a + b)/(a -
 b))*(cosh(f*x + e) + sinh(f*x + e))) + 2*(2*(a^2*b - a*b^2)*d*e - 2*(a^2*
b - a*b^2)*c*f + (2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*f + (a*b^2 +
 b^3)*d)*cosh(f*x + e)^2 + 2*(2*(a^2*b + a*b^2)*d*e - 2*(a^2*b + a*b^2)*c*
f + (a*b^2 + b^3)*d)*cosh(f*x + e)*sinh(f*x + e) + (2*(a^2*b + a*b^2)*d...
```

Sympy [F]

$$\int \frac{c + dx}{(a + b\tanh(e + fx))^2}\,dx = \int \frac{c + dx}{(a + b\tanh{(e + fx)})^2}\,dx$$

input

```
integrate((d*x+c)/(a+b*tanh(f*x+e))**2,x)
```

output

```
Integral((c + d*x)/(a + b*tanh(e + f*x))**2, x)
```

Maxima [F]

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{dx+c}{(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
1/2*(8*a*b*f*integrate(x/(a^4*f*e^(2*f*x + 2*e) + 2*a^3*b*f*e^(2*f*x + 2*e
) - 2*a*b^3*f*e^(2*f*x + 2*e) - b^4*f*e^(2*f*x + 2*e) + a^4*f - 2*a^2*b^2*
f + b^4*f), x) - 2*b^2*(2*(f*x + e)/((a^4 - 2*a^2*b^2 + b^4)*f^2) - log((a
 + b)*e^(2*f*x + 2*e) + a - b)/((a^4 - 2*a^2*b^2 + b^4)*f^2)) + ((a^2*f*e^
(2*e) - b^2*f*e^(2*e))*x^2*e^(2*f*x) + 4*b^2*x + (a^2*f - 2*a*b*f + b^2*f)
*x^2)/(a^4*f - 2*a^2*b^2*f + b^4*f + (a^4*f*e^(2*e) + 2*a^3*b*f*e^(2*e) -
2*a*b^3*f*e^(2*e) - b^4*f*e^(2*e))*e^(2*f*x)))*d - c*(2*a*b*log(-(a - b)*e
^(-2*f*x - 2*e) - a - b)/((a^4 - 2*a^2*b^2 + b^4)*f) + 2*b^2/((a^4 - 2*a^2
*b^2 + b^4 + (a^4 - 2*a^3*b + 2*a*b^3 - b^4)*e^(-2*f*x - 2*e))*f) - (f*x +
 e)/((a^2 + 2*a*b + b^2)*f))
```

Giac [F]

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{dx+c}{(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate((d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate((d*x + c)/(b*tanh(f*x + e) + a)^2, x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = \int \frac{c+d\,x}{(a+b\tanh(e+f\,x))^2}\,dx$$

input
```
int((c + d*x)/(a + b*tanh(e + f*x))^2,x)
```

output
```
int((c + d*x)/(a + b*tanh(e + f*x))^2, x)
```

Reduce [F]

$$\int \frac{c+dx}{(a+b\tanh(e+fx))^2}\,dx = \text{too large to display}$$

input
```
int((d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

output

```
(8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3
*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e +
4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a**3*b - 4*e**(
2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4)
,x)*a**6*b*d*f**2 + 8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e*
*(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*
a*b**3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f
*x)*a**3*b - 4*e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 -
2*a**2*b**2 + b**4),x)*a**5*b**2*d*f**2 - 16*e**(2*e + 2*f*x)*int(x/(e**(4
*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**
2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)
*a**4 + 4*e**(2*e + 2*f*x)*a**3*b - 4*e**(2*e + 2*f*x)*a*b**3 - 2*e**(2*e
+ 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**4*b**3*d*f**2 - 16*e**(2*
e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e + 4*f*x)*a**3*b + 6*e*
*(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3 + e**(4*e + 4*f*x)*b*
*4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)*a**3*b - 4*e**(2*e + 2*f
*x)*a*b**3 - 2*e**(2*e + 2*f*x)*b**4 + a**4 - 2*a**2*b**2 + b**4),x)*a**3*
b**4*d*f**2 + 8*e**(2*e + 2*f*x)*int(x/(e**(4*e + 4*f*x)*a**4 + 4*e**(4*e
+ 4*f*x)*a**3*b + 6*e**(4*e + 4*f*x)*a**2*b**2 + 4*e**(4*e + 4*f*x)*a*b**3
 + e**(4*e + 4*f*x)*b**4 + 2*e**(2*e + 2*f*x)*a**4 + 4*e**(2*e + 2*f*x)...
```

3.76 $\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$

Optimal result . . . 598
Mathematica [N/A] . . . 598
Rubi [N/A] . . . 599
Maple [N/A] . . . 599
Fricas [N/A] . . . 600
Sympy [N/A] . . . 600
Maxima [N/A] . . . 600
Giac [N/A] . . . 601
Mupad [N/A] . . . 601
Reduce [N/A] . . . 602

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)(a+b\tanh(e+fx))^2}, x\right)$$

output
```
Defer(Int)(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 29.43 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)*(a + b*Tanh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.27 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)(a-ib\tan(ie+ifx))^2}\,dx$$
$$\downarrow 4223$$
$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx$$

input
```
Int[1/((c + d*x)*(a + b*Tanh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.21 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)\left(a+b\tanh\left(fx+e\right)\right)^2}\,dx$$

input
```
int(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.75

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d*x + a^2*c + (b^2*d*x + b^2*c)*tanh(f*x + e)^2 + 2*(a*b*d
*x + a*b*c)*tanh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 1.60 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.95

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+b\tanh(e+fx))^2\,(c+dx)}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))**2*(c + d*x)), x)
```

Maxima [N/A]

Not integrable

Time = 1.20 (sec) , antiderivative size = 468, normalized size of antiderivative = 23.40

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
2*b^2/(a^4*c*f - 2*a^2*b^2*c*f + b^4*c*f + (a^4*d*f - 2*a^2*b^2*d*f + b^4*
d*f)*x + (a^4*c*f*e^(2*e) + 2*a^3*b*c*f*e^(2*e) - 2*a*b^3*c*f*e^(2*e) - b^
4*c*f*e^(2*e) + (a^4*d*f*e^(2*e) + 2*a^3*b*d*f*e^(2*e) - 2*a*b^3*d*f*e^(2*
e) - b^4*d*f*e^(2*e))*x)*e^(2*f*x)) + log(d*x + c)/(a^2*d + 2*a*b*d + b^2*
d) + integrate(2*(2*a*b*d*f*x + 2*a*b*c*f + b^2*d)/(a^4*c^2*f - 2*a^2*b^2*
c^2*f + b^4*c^2*f + (a^4*d^2*f - 2*a^2*b^2*d^2*f + b^4*d^2*f)*x^2 + 2*(a^4
*c*d*f - 2*a^2*b^2*c*d*f + b^4*c*d*f)*x + (a^4*c^2*f*e^(2*e) + 2*a^3*b*c^2
*f*e^(2*e) - 2*a*b^3*c^2*f*e^(2*e) - b^4*c^2*f*e^(2*e) + (a^4*d^2*f*e^(2*e
) + 2*a^3*b*d^2*f*e^(2*e) - 2*a*b^3*d^2*f*e^(2*e) - b^4*d^2*f*e^(2*e))*x^2
 + 2*(a^4*c*d*f*e^(2*e) + 2*a^3*b*c*d*f*e^(2*e) - 2*a*b^3*c*d*f*e^(2*e) -
b^4*c*d*f*e^(2*e))*x)*e^(2*f*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.18 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)*(b*tanh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.85 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+b\tanh(e+f\,x))^2\,(c+d\,x)}\,dx$$

input
```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)),x)
```

output
```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)), x)
```

Reduce [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 2595, normalized size of antiderivative = 129.75

$$\int \frac{1}{(c+dx)(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input

```
int(1/(d*x+c)/(a+b*tanh(f*x+e))^2,x)
```

output

```
(4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c + e**(4*e + 4*f*x)*a**
4*d*x + 4*e**(4*e + 4*f*x)*a**3*b*c + 4*e**(4*e + 4*f*x)*a**3*b*d*x + 6*e*
*(4*e + 4*f*x)*a**2*b**2*c + 6*e**(4*e + 4*f*x)*a**2*b**2*d*x + 4*e**(4*e
+ 4*f*x)*a*b**3*c + 4*e**(4*e + 4*f*x)*a*b**3*d*x + e**(4*e + 4*f*x)*b**4*
c + e**(4*e + 4*f*x)*b**4*d*x + 2*e**(2*e + 2*f*x)*a**4*c + 2*e**(2*e + 2*
f*x)*a**4*d*x + 4*e**(2*e + 2*f*x)*a**3*b*c + 4*e**(2*e + 2*f*x)*a**3*b*d*
x - 4*e**(2*e + 2*f*x)*a*b**3*c - 4*e**(2*e + 2*f*x)*a*b**3*d*x - 2*e**(2*
e + 2*f*x)*b**4*c - 2*e**(2*e + 2*f*x)*b**4*d*x + a**4*c + a**4*d*x - 2*a*
*2*b**2*c - 2*a**2*b**2*d*x + b**4*c + b**4*d*x),x)*a**3*b*d + 12*e**(2*e)
*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c + e**(4*e + 4*f*x)*a**4*d*x + 4*e
**(4*e + 4*f*x)*a**3*b*c + 4*e**(4*e + 4*f*x)*a**3*b*d*x + 6*e**(4*e + 4*f
*x)*a**2*b**2*c + 6*e**(4*e + 4*f*x)*a**2*b**2*d*x + 4*e**(4*e + 4*f*x)*a*
b**3*c + 4*e**(4*e + 4*f*x)*a*b**3*d*x + e**(4*e + 4*f*x)*b**4*c + e**(4*e
 + 4*f*x)*b**4*d*x + 2*e**(2*e + 2*f*x)*a**4*c + 2*e**(2*e + 2*f*x)*a**4*d
*x + 4*e**(2*e + 2*f*x)*a**3*b*c + 4*e**(2*e + 2*f*x)*a**3*b*d*x - 4*e**(2
*e + 2*f*x)*a*b**3*c - 4*e**(2*e + 2*f*x)*a*b**3*d*x - 2*e**(2*e + 2*f*x)*
b**4*c - 2*e**(2*e + 2*f*x)*b**4*d*x + a**4*c + a**4*d*x - 2*a**2*b**2*c -
 2*a**2*b**2*d*x + b**4*c + b**4*d*x),x)*a**2*b**2*d + 12*e**(2*e)*int(e**
(2*f*x)/(e**(4*e + 4*f*x)*a**4*c + e**(4*e + 4*f*x)*a**4*d*x + 4*e**(4*e +
 4*f*x)*a**3*b*c + 4*e**(4*e + 4*f*x)*a**3*b*d*x + 6*e**(4*e + 4*f*x)*a...
```

3.77 $\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$

Optimal result . . . 603
Mathematica [N/A] . . . 603
Rubi [N/A] . . . 604
Maple [N/A] . . . 604
Fricas [N/A] . . . 605
Sympy [N/A] . . . 605
Maxima [N/A] . . . 605
Giac [N/A] . . . 606
Mupad [N/A] . . . 607
Reduce [N/A] . . . 607

Optimal result

Integrand size = 20, antiderivative size = 20

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \text{Int}\left(\frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}, x\right)$$

output
```
Defer(Int)(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

Mathematica [N/A]

Not integrable

Time = 25.81 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$$

input
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2),x]
```

output
```
Integrate[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2), x]
```

Rubi [N/A]

Not integrable

Time = 0.25 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$$
$$\downarrow 3042$$
$$\int \frac{1}{(c+dx)^2(a-ib\tan(ie+ifx))^2}\,dx$$
$$\downarrow 4223$$
$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx$$

input
```
Int[1/((c + d*x)^2*(a + b*Tanh[e + f*x])^2),x]
```

output
```
$Aborted
```

Maple [N/A]

Not integrable

Time = 0.22 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(dx+c)^2\,(a+b\tanh(fx+e))^2}\,dx$$

input
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

output
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

Fricas [N/A]

Not integrable

Time = 0.10 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.80

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="fricas")
```

output
```
integral(1/(a^2*d^2*x^2 + 2*a^2*c*d*x + a^2*c^2 + (b^2*d^2*x^2 + 2*b^2*c*d
*x + b^2*c^2)*tanh(f*x + e)^2 + 2*(a*b*d^2*x^2 + 2*a*b*c*d*x + a*b*c^2)*ta
nh(f*x + e)), x)
```

Sympy [N/A]

Not integrable

Time = 3.08 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+b\tanh(e+fx))^2(c+dx)^2}\,dx$$

input
```
integrate(1/(d*x+c)**2/(a+b*tanh(f*x+e))**2,x)
```

output
```
Integral(1/((a + b*tanh(e + f*x))**2*(c + d*x)**2), x)
```

Maxima [N/A]

Not integrable

Time = 2.39 (sec) , antiderivative size = 784, normalized size of antiderivative = 39.20

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2(b\tanh(fx+e)+a)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="maxima")
```

output
```
-(a^2*c*f - 2*a*b*c*f + (c*f - 2*d)*b^2 + (a^2*d*f - 2*a*b*d*f + b^2*d*f)*
x + (a^2*c*f*e^(2*e) - b^2*c*f*e^(2*e) + (a^2*d*f*e^(2*e) - b^2*d*f*e^(2*e
))*x)*e^(2*f*x))/(a^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f + (a^4*d^3
*f - 2*a^2*b^2*d^3*f + b^4*d^3*f)*x^2 + 2*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f
 + b^4*c*d^2*f)*x + (a^4*c^2*d*f*e^(2*e) + 2*a^3*b*c^2*d*f*e^(2*e) - 2*a*b
^3*c^2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e) + (a^4*d^3*f*e^(2*e) + 2*a^3*b*d^
3*f*e^(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(2*e))*x^2 + 2*(a^4*c*d^
2*f*e^(2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d^2*f*e^(2*e) - b^4*c*d^
2*f*e^(2*e))*x)*e^(2*f*x)) + integrate(4*(a*b*d*f*x + a*b*c*f + b^2*d)/(a^
4*c^3*f - 2*a^2*b^2*c^3*f + b^4*c^3*f + (a^4*d^3*f - 2*a^2*b^2*d^3*f + b^4
*d^3*f)*x^3 + 3*(a^4*c*d^2*f - 2*a^2*b^2*c*d^2*f + b^4*c*d^2*f)*x^2 + 3*(a
^4*c^2*d*f - 2*a^2*b^2*c^2*d*f + b^4*c^2*d*f)*x + (a^4*c^3*f*e^(2*e) + 2*a
^3*b*c^3*f*e^(2*e) - 2*a*b^3*c^3*f*e^(2*e) - b^4*c^3*f*e^(2*e) + (a^4*d^3*
f*e^(2*e) + 2*a^3*b*d^3*f*e^(2*e) - 2*a*b^3*d^3*f*e^(2*e) - b^4*d^3*f*e^(2
*e))*x^3 + 3*(a^4*c*d^2*f*e^(2*e) + 2*a^3*b*c*d^2*f*e^(2*e) - 2*a*b^3*c*d^
2*f*e^(2*e) - b^4*c*d^2*f*e^(2*e))*x^2 + 3*(a^4*c^2*d*f*e^(2*e) + 2*a^3*b*
c^2*d*f*e^(2*e) - 2*a*b^3*c^2*d*f*e^(2*e) - b^4*c^2*d*f*e^(2*e))*x)*e^(2*f
*x)), x)
```

Giac [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(dx+c)^2\left(b\tanh\left(fx+e\right)+a\right)^2}\,dx$$

input
```
integrate(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*(b*tanh(f*x + e) + a)^2), x)
```

Mupad [N/A]

Not integrable

Time = 2.49 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.10

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \int \frac{1}{(a+b\tanh(e+fx))^2\,(c+dx)^2}\,dx$$

input
```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)^2),x)
```

output
```
int(1/((a + b*tanh(e + f*x))^2*(c + d*x)^2), x)
```

Reduce [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 8964, normalized size of antiderivative = 448.20

$$\int \frac{1}{(c+dx)^2(a+b\tanh(e+fx))^2}\,dx = \text{Too large to display}$$

input
```
int(1/(d*x+c)^2/(a+b*tanh(f*x+e))^2,x)
```

output

```
(4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c**2 + 2*e**(4*e + 4*f*x
)*a**4*c*d*x + e**(4*e + 4*f*x)*a**4*d**2*x**2 + 4*e**(4*e + 4*f*x)*a**3*b
*c**2 + 8*e**(4*e + 4*f*x)*a**3*b*c*d*x + 4*e**(4*e + 4*f*x)*a**3*b*d**2*x
**2 + 6*e**(4*e + 4*f*x)*a**2*b**2*c**2 + 12*e**(4*e + 4*f*x)*a**2*b**2*c*
d*x + 6*e**(4*e + 4*f*x)*a**2*b**2*d**2*x**2 + 4*e**(4*e + 4*f*x)*a*b**3*c
**2 + 8*e**(4*e + 4*f*x)*a*b**3*c*d*x + 4*e**(4*e + 4*f*x)*a*b**3*d**2*x**
2 + e**(4*e + 4*f*x)*b**4*c**2 + 2*e**(4*e + 4*f*x)*b**4*c*d*x + e**(4*e +
 4*f*x)*b**4*d**2*x**2 + 2*e**(2*e + 2*f*x)*a**4*c**2 + 4*e**(2*e + 2*f*x)
*a**4*c*d*x + 2*e**(2*e + 2*f*x)*a**4*d**2*x**2 + 4*e**(2*e + 2*f*x)*a**3*
b*c**2 + 8*e**(2*e + 2*f*x)*a**3*b*c*d*x + 4*e**(2*e + 2*f*x)*a**3*b*d**2*
x**2 - 4*e**(2*e + 2*f*x)*a*b**3*c**2 - 8*e**(2*e + 2*f*x)*a*b**3*c*d*x -
4*e**(2*e + 2*f*x)*a*b**3*d**2*x**2 - 2*e**(2*e + 2*f*x)*b**4*c**2 - 4*e**
(2*e + 2*f*x)*b**4*c*d*x - 2*e**(2*e + 2*f*x)*b**4*d**2*x**2 + a**4*c**2 +
 2*a**4*c*d*x + a**4*d**2*x**2 - 2*a**2*b**2*c**2 - 4*a**2*b**2*c*d*x - 2*
a**2*b**2*d**2*x**2 + b**4*c**2 + 2*b**4*c*d*x + b**4*d**2*x**2),x)*a**3*b
*c**2 + 4*e**(2*e)*int(e**(2*f*x)/(e**(4*e + 4*f*x)*a**4*c**2 + 2*e**(4*e
+ 4*f*x)*a**4*c*d*x + e**(4*e + 4*f*x)*a**4*d**2*x**2 + 4*e**(4*e + 4*f*x)
*a**3*b*c**2 + 8*e**(4*e + 4*f*x)*a**3*b*c*d*x + 4*e**(4*e + 4*f*x)*a**3*b
*d**2*x**2 + 6*e**(4*e + 4*f*x)*a**2*b**2*c**2 + 12*e**(4*e + 4*f*x)*a**2*
b**2*c*d*x + 6*e**(4*e + 4*f*x)*a**2*b**2*d**2*x**2 + 4*e**(4*e + 4*f*x...
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions 609
4.2 Links to plain text integration problems used in this report for each CAS . 627

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*          GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                convert(leaf_count_result,string),"$ vs. $2(",
                                convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                    convert(ExpnType_result,string)," vs. order ",
                    convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False

   except AttributeError as error:
      return False

def expnType(expn):
   debug=False
   if debug:
      print("expn=",expn,"type(expn)=",type(expn))

   if is_atom(expn):
     return 1
   elif isinstance(expn,list):
      return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
   elif  is_sqrt(expn):
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
         return 1
      else:
         return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
   elif isinstance(expn,Pow):   #type(expn,'`^`')
      if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
         return expnType(expn.args[0])   #ExpnType(op(1,expn))
      elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
         if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
         else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
      else:
         return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
   elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#             Albert Rich to use with Sagemath. This is used to
#             grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#             'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#             issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:    #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
      if m:
         print ("func ", func , " is elementary_function")
      else:
         print ("func ", func , " is NOT elementary_function")


   return m

def is_special_function(func):
   #debug=False
   if debug:
      print ("type(func)=", type(func))

   m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
         'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
         'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
         'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
         'elliptic_pi','exp_integral_e','log_integral',
         'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
         'weierstrassPPrime','weierstrassSigma']

   if debug:
      print ("m=",m)
      if m:
         print ("func ", func ," is special_function")
      else:
         print ("func ", func ," is NOT special_function")


   return m


def is_hypergeometric_function(func):
   return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
   return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

   #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
```

```
        return expnType(expn.operands()[0])   #expnType(expn.args[0])
      elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
        if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
          return 1
        else:
          return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
        return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
          #is checked before calling the grading function that is passed.
          #but kept it here.
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
      return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
   if result.has(I):
      if optimal.has(I): #both result and optimal complex
         if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
         else:
            grade = "B"
            grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
      else: #result contains complex but optimal is not
         grade = "C"
         grade_annotation ="Result contains complex when optimal does not."
   else: # result do not contain complex, this assumes optimal do not as well
      if leaf_count_result <= 2*leaf_count_optimal:
         grade = "A"
         grade_annotation ="none"
      else:
         grade = "B"
         grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
   grade = "C"
   grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file