Computer Algebra Independent Integration Tests

Summer 2024

6-Hyperbolic-functions/6.3-Hyperbolic-tangent/308-6.3.7

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Contents

CHAPTER 1

Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [**263**]. This is test number [308].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 14 (January 9, 2024) on windows 10 pro.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 14 on windows 10m pro.
3. Maple 2024 (March 1, 2024) on windows 10 pro.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.4.0 on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
5. FriCAS 1.3.10 built with sbcl 2.3.11 (January 10, 2024) on Linux Manjaro 23.1.2 KDE via sagemath 10.3.
6. Giac/Xcas 1.9.0-99 on Linux via sagemath 10.3.
7. Sympy 1.12 using Python 3.11.6 (Nov 14 2023, 09:36:21) [GCC 13.2.1 20230801] on Linux Manjaro 23.1.2 KDE.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.
9. Reduce CSL rev 6687 (January 9, 2024) on Linux Manjaro 23.1.2 KDE.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

Reduce was called directly.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (263)	0.00 (0)
Mathematica	100.00 (263)	0.00 (0)
Fricas	100.00 (263)	0.00 (0)
Maple	94.68 (249)	5.32 (14)
Giac	86.31 (227)	13.69 (36)
Reduce	73.38 (193)	26.62 (70)
Mupad	70.34 (185)	29.66 (78)
Maxima	67.30 (177)	32.70 (86)
Sympy	14.83 (39)	85.17 (224)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

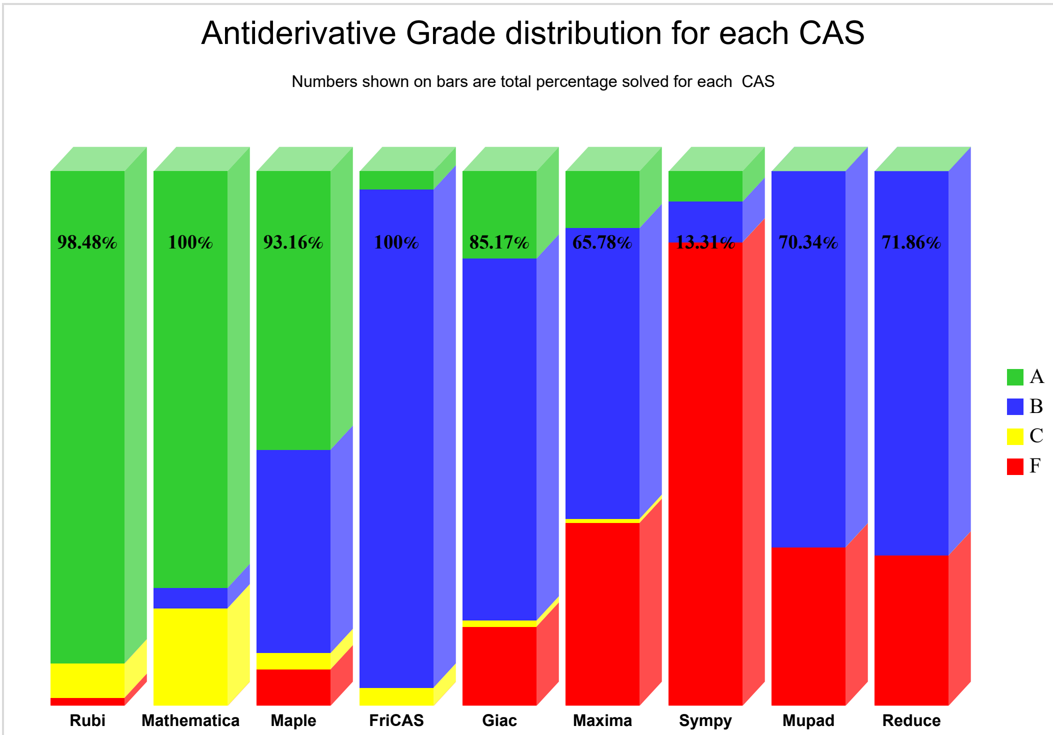
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

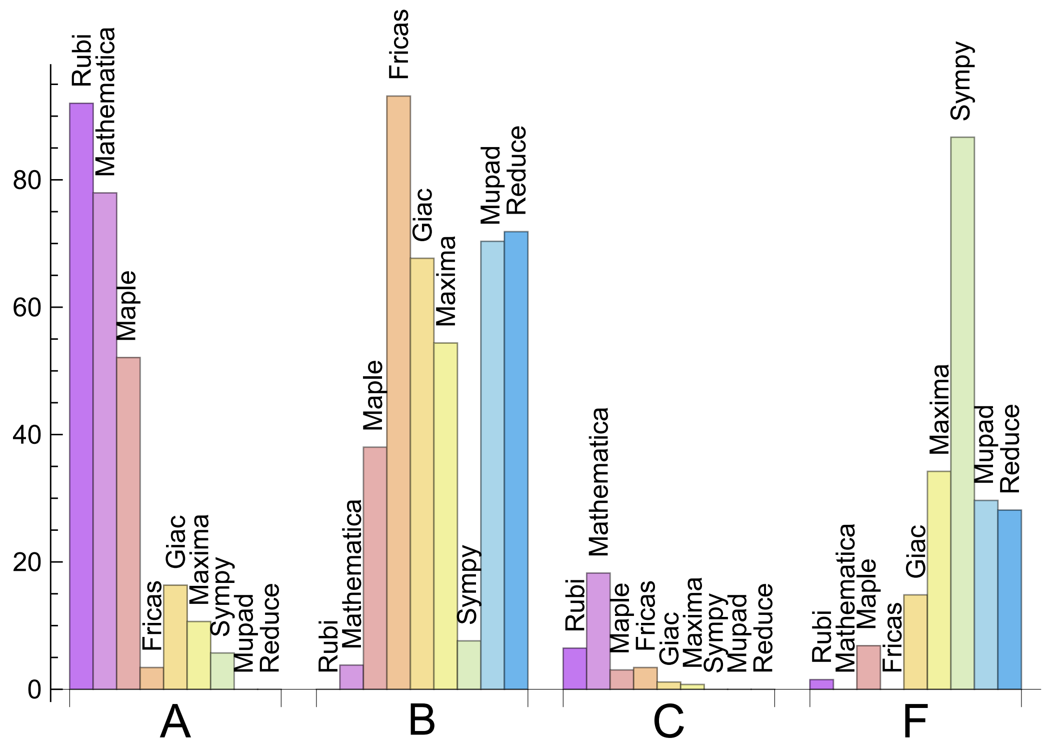
System	% A grade	% B grade	% C grade	% F grade
Rubi	92.015	0.000	6.464	1.521
Mathematica	77.947	3.802	18.251	0.000
Maple	52.091	38.023	3.042	6.844
Giac	16.350	67.681	1.141	14.829
Maxima	10.646	54.373	0.760	34.221
Sympy	5.703	7.605	0.000	86.692
Fricas	3.422	93.156	3.422	0.000
Mupad	0.000	70.342	0.000	29.658
Reduce	0.000	71.863	0.000	28.137

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the grades of the CAS systems.



The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Mathematica	0	0.00	0.00	0.00
Fricas	0	0.00	0.00	0.00
Rubi	0	0.00	0.00	0.00
Maple	14	100.00	0.00	0.00
Giac	36	19.44	0.00	80.56
Reduce	70	100.00	0.00	0.00
Mupad	78	0.00	100.00	0.00
Maxima	86	98.84	0.00	1.16
Sympy	224	83.93	16.07	0.00

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)
Maxima	0.15
Fricas	0.24
Giac	0.32
Rubi	0.42
Reduce	0.65
Mathematica	1.28
Mupad	2.73
Sympy	4.76
Maple	12.18

Table 1.5: Time performance for each CAS

System	Mean size	Normalized mean	Median size	Normalized median
Rubi	96.23	1.07	82.00	1.04
Mathematica	123.02	1.62	86.00	1.00
Sympy	144.59	2.45	117.00	1.93
Maple	178.92	2.14	142.00	1.61
Giac	277.17	3.26	216.00	2.51
Maxima	417.18	4.17	235.00	2.98
Mupad	434.32	6.83	177.00	2.72
Reduce	1341.64	16.81	564.00	6.28
Fricas	3894.45	52.88	1942.00	23.82

Table 1.6: Leaf size performance for each CAS

1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the y axis is the percentage solved which Rubi itself needed the number of rules given the x axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

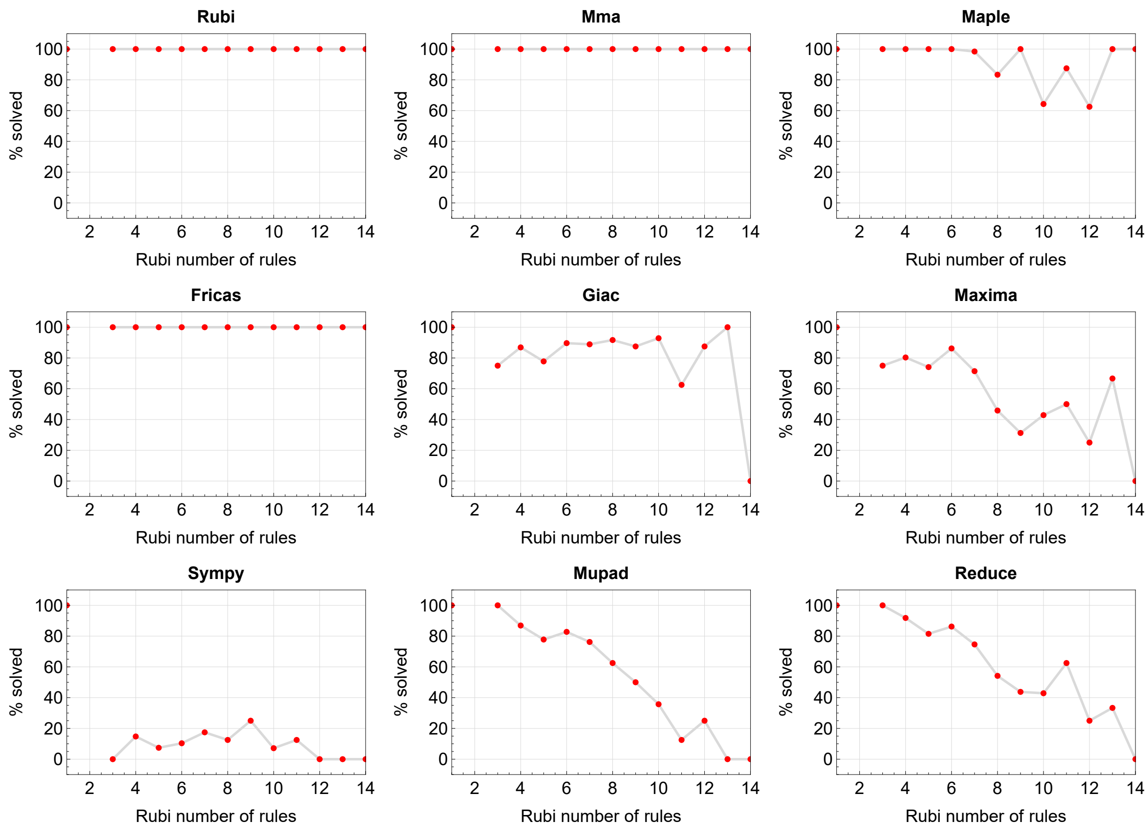


Figure 1.1: Solving statistics per number of Rubi rules used

1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

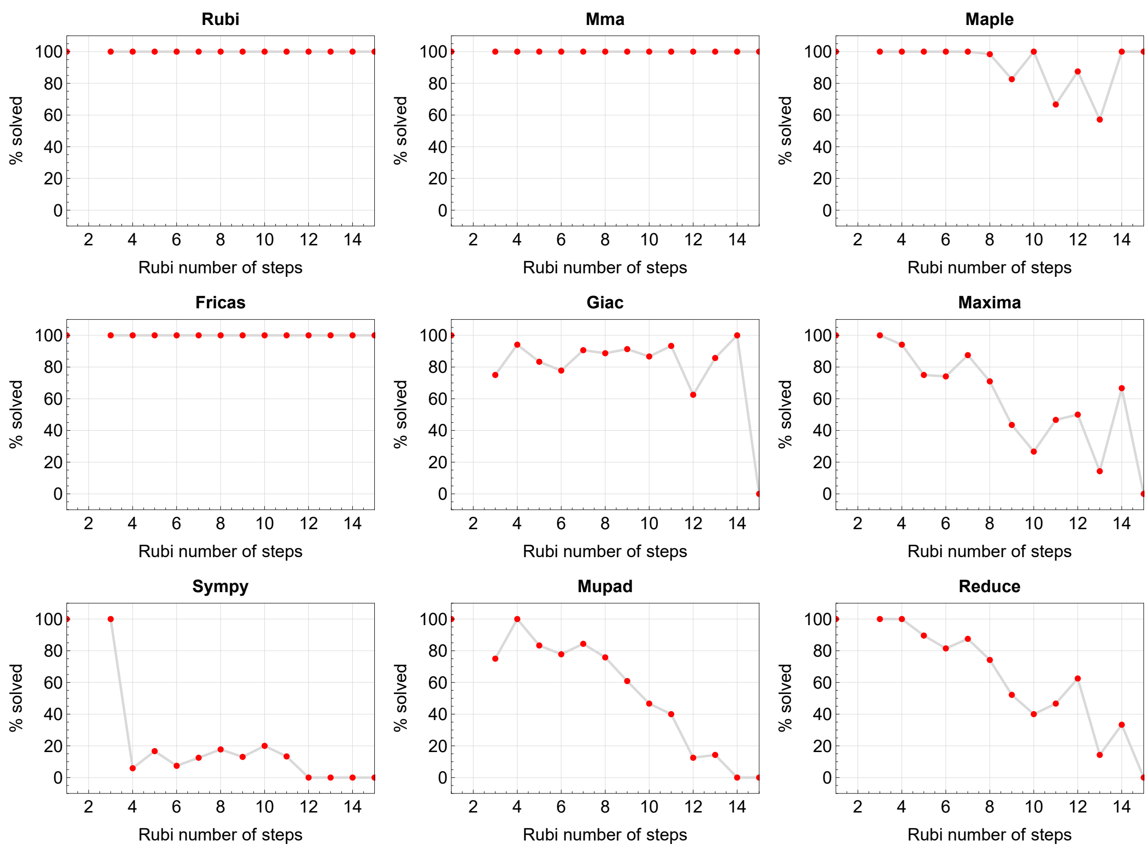


Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

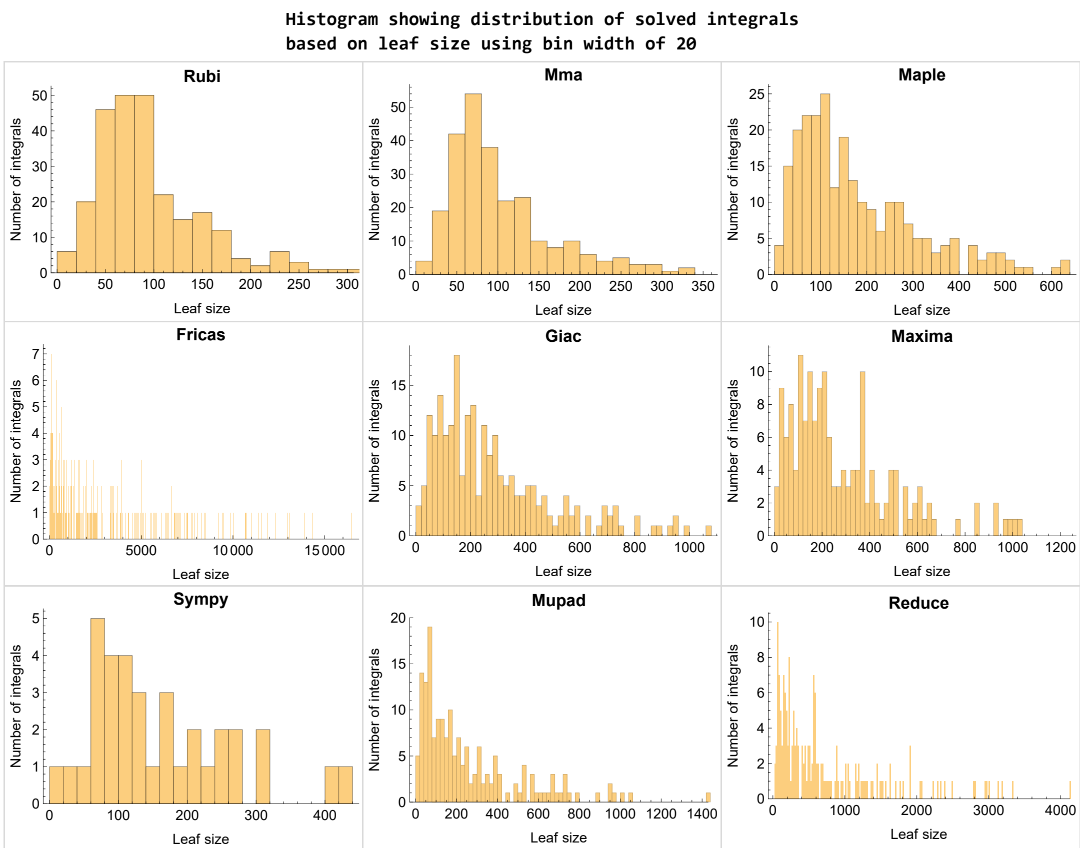


Figure 1.3: Solved integrals based on leaf size distribution

1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

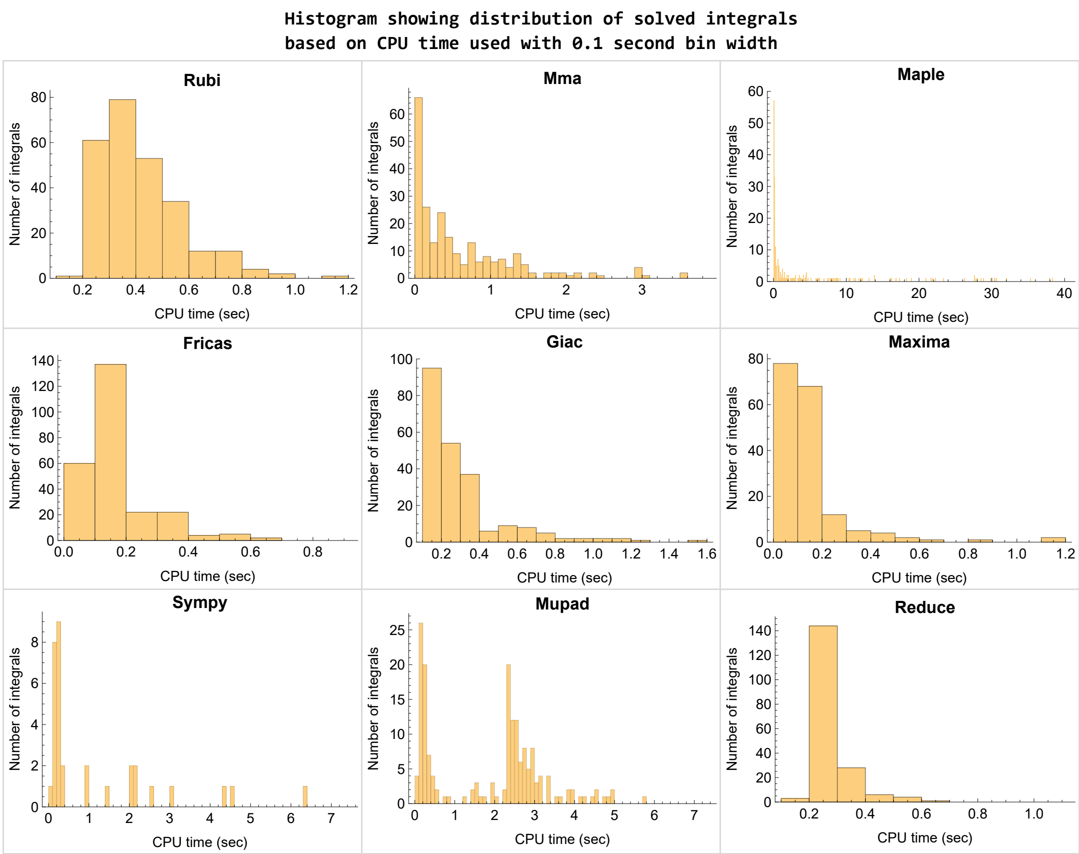


Figure 1.4: Solved integrals histogram based on CPU time used

1.8 Leaf size vs. CPU time used

The following shows the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time. This should also be taken into account when looking at the timing of these three CAS systems. Direct calls not using sagemath would result in faster timings, but current implementation uses sagemath as this makes testing much easier to do.

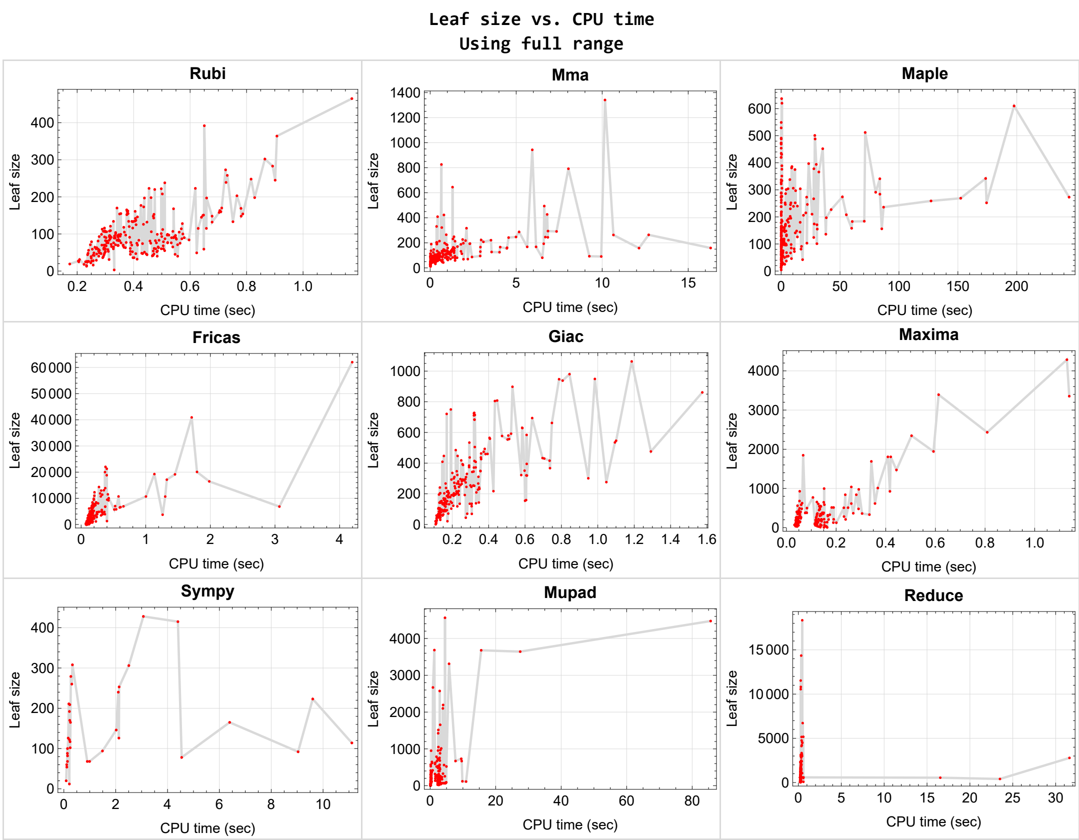


Figure 1.5: Leaf size vs. CPU time. Full range

1.9 list of integrals with no known antiderivative

{74, 76, 77, 79}

1.10 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {74, 76, 77, 79}

Maple {}

Maxima {}

Fricas {74, 76, 77, 79}

Sympy {}

Giac {}

Reduce {}

Mupad {76, 77, 79}

1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

Rubi {151, 153, 155, 163, 165, 167, 178, 188, 199, 216, 218, 237}

Mathematica {93, 95, 98, 101, 217, 236, 243, 245, 250, 252}

Maple {144, 146, 148, 156, 158, 160, 168, 169}

Maxima Verification phase not currently implemented.

Fricas Verification phase not currently implemented.

Sympy Verification phase not currently implemented.

Giac Verification phase not currently implemented.

Reduce Verification phase not currently implemented.

Mupad Verification phase not currently implemented.

1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.14 Important notes about some of the results

Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.15 Current tree layout of integration tests

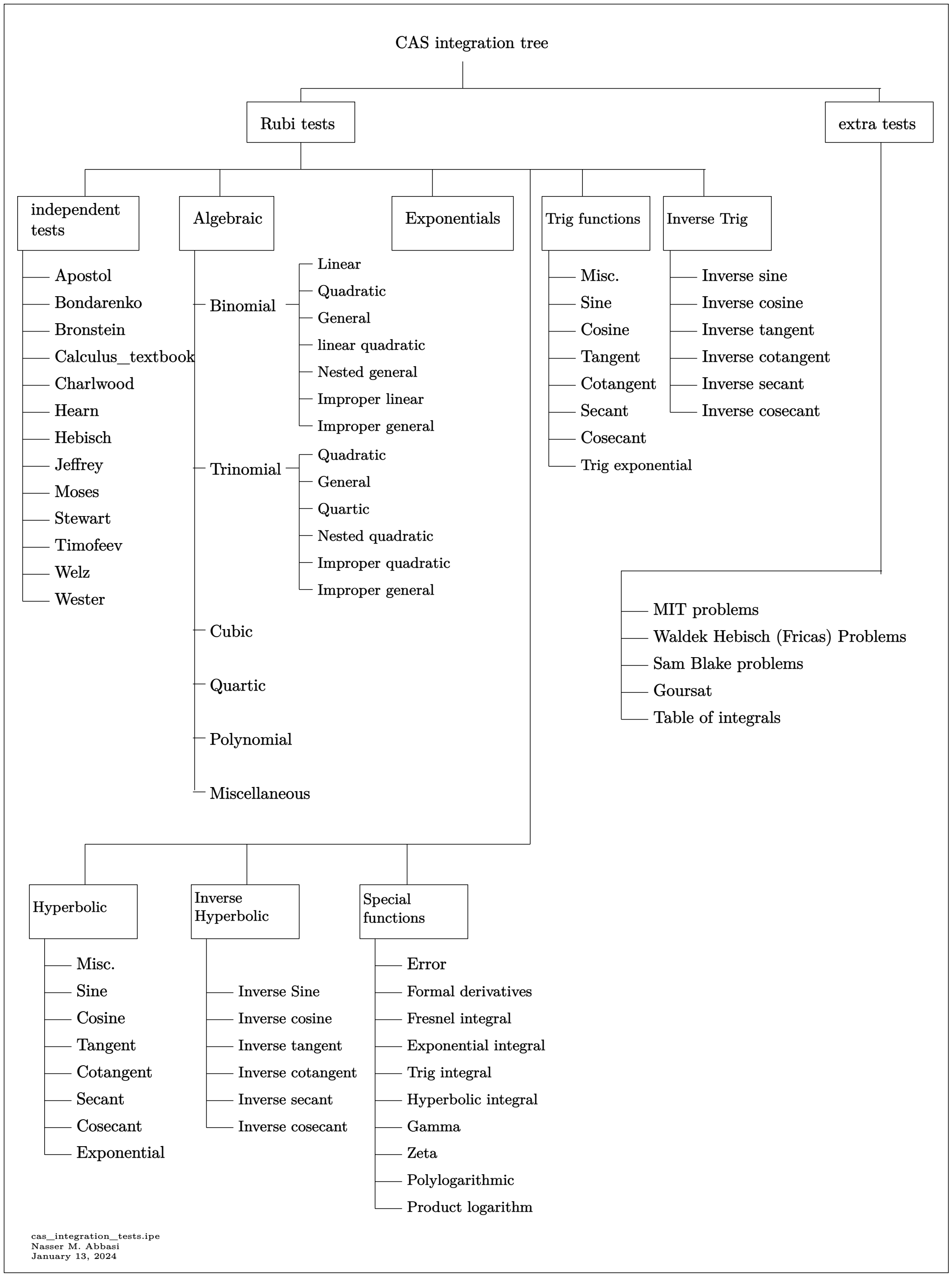
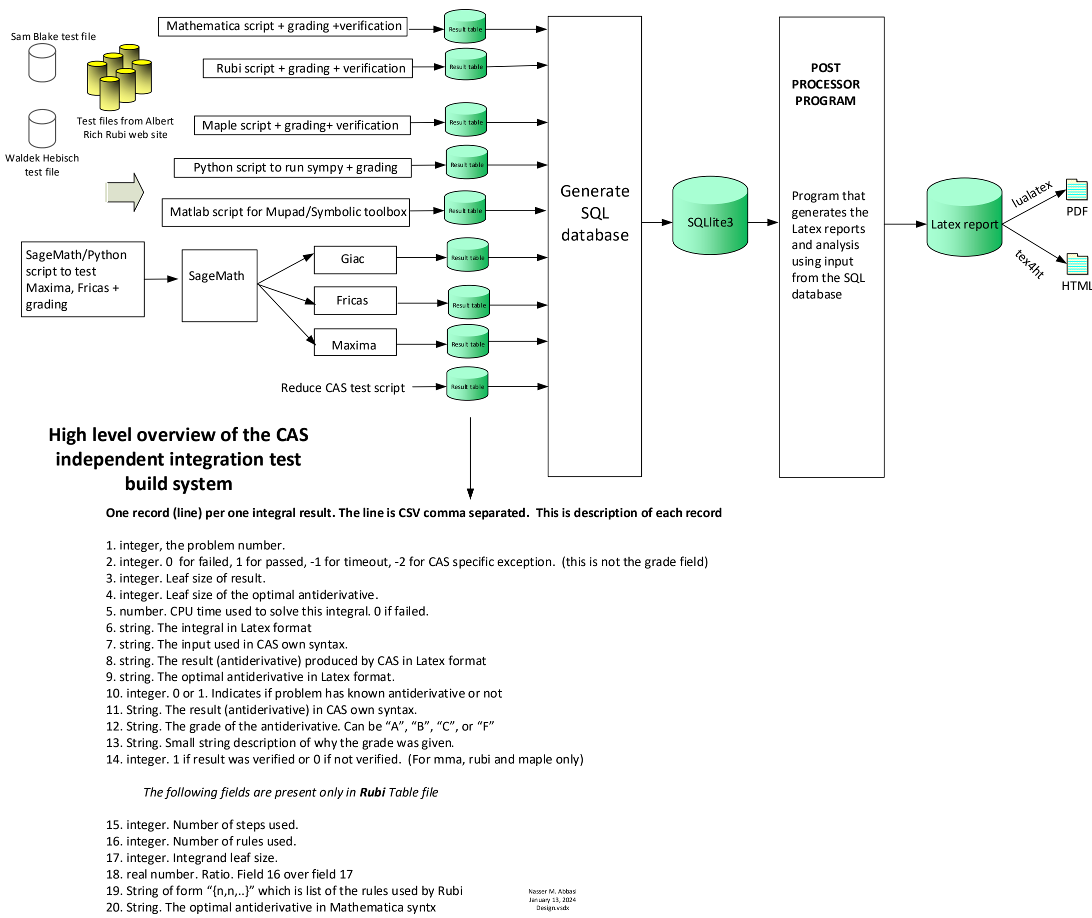


Figure 1.6: CAS integration tests tree

1.16 Design of the test system

The following diagram gives a high level view of the current test build system.



CHAPTER 2

DETAILED SUMMARY TABLES OF RESULTS

2.1 List of integrals sorted by grade for each CAS

Rubi

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 51, 54, 56, 57, 59, 62, 64, 65, 67, 70, 72, 73, 75, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 136, 138, 140, 142, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade { }

C grade { 50, 52, 53, 55, 58, 60, 61, 63, 66, 68, 69, 71, 135, 137, 139, 141, 143 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Mma

A grade { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 30, 32, 33, 35, 38, 40, 41, 43, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 94, 96, 97, 99, 100, 102, 103, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 134, 135, 136, 137, 138, 139, 141, 143, 145, 147, 148, 149, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 203, 204, 205, 206, 207, 208, 210, 212, 213, 214, 216, 218, 219, 221, 222, 223, 225, 226, 227, 228, 229, 231, 233, 234, 235, 237, 240, 248, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

B grade { 7, 74, 76, 77, 79, 104, 144, 146, 202, 241 }

C grade { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 73, 75, 78, 80, 93, 95, 98, 101, 133, 140, 142, 150, 154, 209, 211, 215, 217, 220, 224, 230, 232, 236, 238, 239, 242, 243, 244, 245, 246, 247, 249, 250, 251, 252, 253, 254 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maple

A grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 11, 13, 14, 15, 16, 17, 19, 21, 23, 24, 29, 31, 37, 39, 47, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 63, 64, 65, 66, 67, 68, 69, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 92, 93, 95, 98, 103, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 257, 258, 260, 261 }

B grade { 10, 12, 18, 20, 22, 25, 26, 27, 28, 30, 32, 33, 34, 35, 36, 38, 40, 41, 42, 43, 44, 45, 46, 48, 62, 70, 91, 94, 96, 97, 99, 100, 101, 102, 104, 105, 106, 107, 108, 109, 110, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 156, 157, 208, 209, 210, 211, 212, 213, 219, 220, 221, 222, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 238, 239, 240, 241, 242, 243, 246, 247, 248, 249, 250, 251, 252, 255, 256 }

C grade { 73, 75, 78, 80, 111, 259, 262, 263 }

F normal fail { 214, 215, 216, 217, 218, 223, 224, 235, 236, 237, 244, 245, 253, 254 }

F(-1) timedout fail { }

F(-2) exception fail { }

Fricas

A grade { 1, 3, 4, 81, 82, 83, 89, 138, 206 }

B grade { 2, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 78, 84, 85, 86, 87, 88, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

C grade { 73, 74, 75, 76, 77, 79, 80, 226, 228 }

F normal fail { }

F(-1) timedout fail { }

F(-2) exception fail { }

Maxima

A grade { 5, 6, 30, 49, 52, 53, 54, 66, 68, 81, 86, 94, 102, 110, 112, 121, 138, 139, 140, 150, 172, 174, 175, 176, 178, 202, 204, 206 }

B grade { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 32, 33, 35, 38, 40, 41, 43, 46, 48, 50, 51, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 67, 69, 70, 71, 72, 82, 83, 84, 85, 87, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 101, 103, 104, 105, 107, 114, 116, 119, 123, 125, 128, 130, 132, 134, 135, 136, 137, 141, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 173, 177, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 207, 257, 258 }

C grade { 203, 205 }

F normal fail { 26, 28, 29, 31, 34, 36, 37, 39, 44, 45, 47, 73, 75, 78, 80, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

F(-1) timedout fail { }

F(-2) exception fail { 42 }

Giac

A grade { 6, 8, 14, 30, 32, 50, 52, 53, 54, 58, 60, 61, 63, 66, 68, 78, 80, 81, 84, 110, 112, 137, 138, 139, 140, 141, 150, 153, 171, 173, 174, 175, 176, 177, 178, 189, 202, 203, 205, 206, 207, 257, 258 }

B grade { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 27, 33, 35, 38, 40, 41, 43, 46, 48, 49, 51, 55, 56, 57, 59, 62, 64, 65, 67, 69, 70, 71, 72, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 114, 116, 119, 121, 123, 125, 128, 130, 132, 134, 135, 136, 142, 143, 144, 145, 146, 147, 148, 149, 151, 152, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 172, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 204, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255 }

C grade { 226, 228, 256 }

F normal fail { 73, 75, 259, 260, 261, 262, 263 }

F(-1) timedout fail { }

F(-2) exception fail { 26, 28, 29, 31, 34, 36, 37, 39, 42, 44, 45, 47, 79, 106, 108, 109, 111, 113, 115, 117, 118, 120, 122, 124, 126, 127, 129, 131, 133 }

Mupad

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 201, 202, 203, 204, 205, 206, 207, 208, 210, 212, 219, 221, 225, 226, 227, 229, 231, 233, 234, 238, 240, 242, 247, 249, 251, 255, 256, 257, 258 }

C grade { }

F normal fail { }

F(-1) timedout fail { 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 74, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 186, 187, 188, 189, 197, 198, 199, 200, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 228, 230, 232, 235, 236, 237, 239, 241, 243, 244, 245, 246, 248, 250, 252, 253, 254, 259, 260, 261, 262, 263 }

F(-2) exception fail { }

Sympy

A grade { 134, 136, 138, 148, 160, 207, 208, 210, 212, 219, 221, 233, 242, 251, 257 }

B grade { 135, 137, 140, 144, 145, 146, 147, 156, 157, 158, 159, 168, 169, 170, 171, 172, 173, 174, 175, 258 }

C grade { }

F normal fail { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 34, 36, 37, 38, 39, 40, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 58, 59, 60, 61, 62, 63, 64, 68, 69, 70, 71, 72, 75, 78, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 117, 118, 119, 120, 121, 122, 123, 124, 127, 128, 129, 130, 131, 132, 139, 141, 142, 143, 149, 150, 151, 152, 153, 154, 161, 162, 163, 176, 177, 178, 179, 186, 187, 188, 189, 197, 198, 199, 200, 202, 203, 204, 205, 206, 209, 211, 213, 214, 215, 216, 217, 218, 220, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 234, 235, 236, 237, 238, 239, 240, 241, 243, 244, 245, 246, 247, 248, 249, 250, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

F(-1) timedout fail { 17, 33, 35, 41, 42, 43, 44, 57, 65, 66, 67, 73, 97, 116, 125, 126, 133, 155, 164, 165, 166, 167, 180, 181, 182, 183, 184, 185, 190, 191, 192, 193, 194, 195, 196, 201 }

F(-2) exception fail { }

Reduce

A grade { }

B grade { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 190, 191, 192, 193, 194, 195, 196, 197, 198, 201, 257, 258 }

C grade { }

F normal fail { 73, 75, 78, 80, 165, 166, 167, 189, 199, 200, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 259, 260, 261, 262, 263 }

F(-1) timedout fail { }

F(-2) exception fail { }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	96	56	96	154	120	0	142	200	101
N.S.	1	1.32	0.77	1.32	2.11	1.64	0.00	1.95	2.74	1.38
time (sec)	N/A	0.315	0.438	3.499	0.053	0.095	0.000	0.134	0.238	2.355

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	42	73	75	136	91	0	105	136	99
N.S.	1	0.89	1.55	1.60	2.89	1.94	0.00	2.23	2.89	2.11
time (sec)	N/A	0.260	0.191	1.682	0.044	0.091	0.000	0.132	0.222	2.324

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	60	41	66	101	71	0	107	148	64
N.S.	1	1.36	0.93	1.50	2.30	1.61	0.00	2.43	3.36	1.45
time (sec)	N/A	0.267	0.278	0.973	0.041	0.100	0.000	0.125	0.287	2.302

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	26	45	44	67	42	0	63	81	27
N.S.	1	1.04	1.80	1.76	2.68	1.68	0.00	2.52	3.24	1.08
time (sec)	N/A	0.239	0.140	0.701	0.037	0.087	0.000	0.130	0.251	0.138

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	24	26	27	40	167	0	68	99	64
N.S.	1	0.92	1.00	1.04	1.54	6.42	0.00	2.62	3.81	2.46
time (sec)	N/A	0.236	0.017	0.706	0.037	0.092	0.000	0.123	0.256	0.151

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	24	22	24	25	39	88	0	45	61	43
N.S.	1	0.92	1.00	1.04	1.62	3.67	0.00	1.88	2.54	1.79
time (sec)	N/A	0.248	0.012	1.158	0.040	0.081	0.000	0.128	0.284	0.113

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	59	123	50	152	924	0	142	440	156
N.S.	1	1.16	2.41	0.98	2.98	18.12	0.00	2.78	8.63	3.06
time (sec)	N/A	0.254	0.184	2.939	0.043	0.104	0.000	0.134	0.262	2.383

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	39	61	55	113	244	0	80	100	173
N.S.	1	0.89	1.39	1.25	2.57	5.55	0.00	1.82	2.27	3.93
time (sec)	N/A	0.249	0.222	6.094	0.042	0.087	0.000	0.138	0.230	2.489

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	137	94	166	295	394	0	293	534	293
N.S.	1	1.21	0.83	1.47	2.61	3.49	0.00	2.59	4.73	2.59
time (sec)	N/A	0.408	1.439	6.948	0.053	0.107	0.000	0.196	0.295	0.369

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	69	71	148	265	259	0	205	328	215
N.S.	1	0.90	0.92	1.92	3.44	3.36	0.00	2.66	4.26	2.79
time (sec)	N/A	0.344	1.185	4.135	0.047	0.111	0.000	0.181	0.265	0.343

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	82	70	118	217	291	0	213	446	248
N.S.	1	1.15	0.99	1.66	3.06	4.10	0.00	3.00	6.28	3.49
time (sec)	N/A	0.477	0.983	2.094	0.044	0.088	0.000	0.171	0.245	2.453

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	46	46	98	171	167	0	139	335	154
N.S.	1	0.94	0.94	2.00	3.49	3.41	0.00	2.84	6.84	3.14
time (sec)	N/A	0.445	0.713	1.296	0.042	0.103	0.000	0.157	0.255	2.390

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	46	64	63	196	890	0	123	305	160
N.S.	1	0.90	1.25	1.24	3.84	17.45	0.00	2.41	5.98	3.14
time (sec)	N/A	0.472	0.713	1.076	0.040	0.107	0.000	0.157	0.254	0.188

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	41	43	68	136	264	0	86	176	209
N.S.	1	0.89	0.93	1.48	2.96	5.74	0.00	1.87	3.83	4.54
time (sec)	N/A	0.466	0.764	4.573	0.053	0.085	0.000	0.158	0.224	2.343

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	79	127	67	181	2462	0	153	768	261
N.S.	1	1.07	1.72	0.91	2.45	33.27	0.00	2.07	10.38	3.53
time (sec)	N/A	0.506	3.572	8.029	0.037	0.100	0.000	0.169	0.247	0.189

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	64	59	81	210	393	0	143	176	143
N.S.	1	0.89	0.82	1.12	2.92	5.46	0.00	1.99	2.44	1.99
time (sec)	N/A	0.366	1.448	16.878	0.039	0.074	0.000	0.165	0.285	2.313

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	198	125	246	480	879	0	505	1018	730
N.S.	1	1.46	0.92	1.81	3.53	6.46	0.00	3.71	7.49	5.37
time (sec)	N/A	0.462	4.032	22.275	0.061	0.110	0.000	0.324	0.267	2.761

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	94	91	239	439	540	0	330	598	361
N.S.	1	0.90	0.87	2.28	4.18	5.14	0.00	3.14	5.70	3.44
time (sec)	N/A	0.332	9.947	10.501	0.049	0.090	0.000	0.281	0.256	0.480

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	155	95	180	377	725	0	393	894	668
N.S.	1	1.55	0.95	1.80	3.77	7.25	0.00	3.93	8.94	6.68
time (sec)	N/A	0.381	2.909	6.102	0.054	0.099	0.000	0.257	0.273	2.075

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	63	62	170	321	383	0	236	697	308
N.S.	1	0.90	0.89	2.43	4.59	5.47	0.00	3.37	9.96	4.40
time (sec)	N/A	0.269	0.354	3.612	0.054	0.087	0.000	0.217	0.238	0.173

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	76	93	118	560	2277	0	196	574	317
N.S.	1	0.90	1.11	1.40	6.67	27.11	0.00	2.33	6.83	3.77
time (sec)	N/A	0.305	9.254	2.540	0.045	0.103	0.000	0.219	0.235	1.470

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	56	70	141	348	572	0	202	400	590
N.S.	1	0.88	1.09	2.20	5.44	8.94	0.00	3.16	6.25	9.22
time (sec)	N/A	0.279	2.189	12.014	0.046	0.090	0.000	0.223	0.274	1.484

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	161	159	103	403	5037	0	206	1198	412
N.S.	1	1.59	1.57	1.02	3.99	49.87	0.00	2.04	11.86	4.08
time (sec)	N/A	0.405	16.306	21.891	0.041	0.136	0.000	0.230	0.293	0.167

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	87	87	136	493	925	0	257	484	622
N.S.	1	0.89	0.89	1.39	5.03	9.44	0.00	2.62	4.94	6.35
time (sec)	N/A	0.511	2.431	37.961	0.053	0.091	0.000	0.246	0.262	1.536

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	151	93	215	514	2024	0	276	281	250
N.S.	1	1.28	0.79	1.82	4.36	17.15	0.00	2.34	2.38	2.12
time (sec)	N/A	0.648	0.545	20.981	0.190	0.157	0.000	1.047	0.299	1.724

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	76	135	202	0	1367	0	0	281	955
N.S.	1	1.01	1.80	2.69	0.00	18.23	0.00	0.00	3.75	12.73
time (sec)	N/A	0.520	0.731	7.675	0.000	0.131	0.000	0.000	0.266	2.393

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	97	67	151	316	916	0	159	167	198
N.S.	1	1.24	0.86	1.94	4.05	11.74	0.00	2.04	2.14	2.54
time (sec)	N/A	0.525	0.403	2.616	0.140	0.137	0.000	0.606	0.232	1.584

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	52	107	104	0	666	0	0	217	520
N.S.	1	0.98	2.02	1.96	0.00	12.57	0.00	0.00	4.09	9.81
time (sec)	N/A	0.396	0.399	1.215	0.000	0.117	0.000	0.000	0.250	1.960

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	135	67	0	587	0	0	145	284
N.S.	1	0.96	2.45	1.22	0.00	10.67	0.00	0.00	2.64	5.16
time (sec)	N/A	0.277	0.407	0.981	0.000	0.135	0.000	0.000	0.294	1.973

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	46	48	112	62	618	0	69	165	136
N.S.	1	0.96	1.00	2.33	1.29	12.88	0.00	1.44	3.44	2.83
time (sec)	N/A	0.269	0.337	1.249	0.137	0.120	0.000	0.308	0.267	1.544

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	94	198	111	0	1797	0	0	570	787
N.S.	1	1.11	2.33	1.31	0.00	21.14	0.00	0.00	6.71	9.26
time (sec)	N/A	0.314	0.841	1.884	0.000	0.138	0.000	0.000	0.283	1.927

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	71	246	134	1628	0	119	683	254
N.S.	1	1.00	1.01	3.51	1.91	23.26	0.00	1.70	9.76	3.63
time (sec)	N/A	0.288	0.443	3.596	0.154	0.123	0.000	0.307	0.301	1.646

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	223	132	512	1690	7366	0	476	1361	0
N.S.	1	1.16	0.69	2.67	8.80	38.36	0.00	2.48	7.09	0.00
time (sec)	N/A	0.455	1.031	71.347	0.342	0.222	0.000	1.292	0.312	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	126	160	267	0	5025	0	0	1523	0
N.S.	1	1.02	1.29	2.15	0.00	40.52	0.00	0.00	12.28	0.00
time (sec)	N/A	0.435	1.377	27.618	0.000	0.190	0.000	0.000	0.341	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	152	105	381	840	3918	0	367	1237	0
N.S.	1	1.15	0.80	2.89	6.36	29.68	0.00	2.78	9.37	0.00
time (sec)	N/A	0.376	0.889	8.867	0.237	0.165	0.000	0.737	0.304	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	93	133	167	0	2252	0	0	1296	0
N.S.	1	1.03	1.48	1.86	0.00	25.02	0.00	0.00	14.40	0.00
time (sec)	N/A	0.278	0.862	3.582	0.000	0.131	0.000	0.000	0.284	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	112	188	161	0	2614	0	0	1630	0
N.S.	1	1.09	1.83	1.56	0.00	25.38	0.00	0.00	15.83	0.00
time (sec)	N/A	0.503	0.940	2.085	0.000	0.163	0.000	0.000	0.321	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	86	252	212	2562	0	227	1177	0
N.S.	1	1.00	1.06	3.11	2.62	31.63	0.00	2.80	14.53	0.00
time (sec)	N/A	0.458	0.703	3.358	0.190	0.142	0.000	0.341	0.311	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	150	221	187	0	6335	0	0	2493	0
N.S.	1	1.06	1.57	1.33	0.00	44.93	0.00	0.00	17.68	0.00
time (sec)	N/A	0.645	3.535	5.120	0.000	0.182	0.000	0.000	0.335	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	115	114	338	282	5062	0	209	2398	0
N.S.	1	1.02	1.01	2.99	2.50	44.80	0.00	1.85	21.22	0.00
time (sec)	N/A	0.658	0.994	7.427	0.232	0.160	0.000	0.333	0.396	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	273	184	610	3392	18818	0	861	4498	0
N.S.	1	1.14	0.77	2.54	14.13	78.41	0.00	3.59	18.74	0.00
time (sec)	N/A	0.726	0.986	197.689	0.613	0.396	0.000	1.574	0.470	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F(-2)	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	168	227	341	0	13095	0	0	3335	0
N.S.	1	1.01	1.37	2.05	0.00	78.89	0.00	0.00	20.09	0.00
time (sec)	N/A	0.541	1.888	83.765	0.000	0.352	0.000	0.000	0.383	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	220	158	488	1806	12965	0	535	4124	0
N.S.	1	1.19	0.85	2.64	9.76	70.08	0.00	2.89	22.29	0.00
time (sec)	N/A	0.475	1.388	28.857	0.420	0.310	0.000	1.094	0.365	0.000

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	134	157	252	0	7119	0	0	18347	0
N.S.	1	1.07	1.26	2.02	0.00	56.95	0.00	0.00	146.78	0.00
time (sec)	N/A	0.329	1.415	11.925	0.000	0.203	0.000	0.000	0.453	0.000

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	177	249	304	0	10716	0	0	4756	0
N.S.	1	1.13	1.60	1.95	0.00	68.69	0.00	0.00	30.49	0.00
time (sec)	N/A	0.426	1.384	5.882	0.000	0.304	0.000	0.000	0.380	0.000

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	116	109	306	478	8312	0	351	3001	0
N.S.	1	1.04	0.97	2.73	4.27	74.21	0.00	3.13	26.79	0.00
time (sec)	N/A	0.297	1.180	10.317	0.290	0.236	0.000	0.597	0.437	0.000

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	222	287	304	0	21301	0	0	6723	0
N.S.	1	1.13	1.46	1.55	0.00	108.68	0.00	0.00	34.30	0.00
time (sec)	N/A	0.498	5.170	16.254	0.000	0.397	0.000	0.000	0.503	0.000

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	154	149	397	615	14334	0	395	5176	0
N.S.	1	1.02	0.99	2.63	4.07	94.93	0.00	2.62	34.28	0.00
time (sec)	N/A	0.787	1.282	23.364	0.357	0.276	0.000	0.609	0.579	0.000

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	133	92	100	194	1530	0	203	344	156
N.S.	1	1.21	0.84	0.91	1.76	13.91	0.00	1.85	3.13	1.42
time (sec)	N/A	0.752	0.130	4.108	0.117	0.105	0.000	0.156	0.227	0.298

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	114	112	104	174	1070	0	132	239	171
N.S.	1	1.24	1.22	1.13	1.89	11.63	0.00	1.43	2.60	1.86
time (sec)	N/A	0.569	0.024	2.836	0.122	0.146	0.000	0.147	0.259	0.245

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	90	69	68	141	924	0	140	292	115
N.S.	1	1.25	0.96	0.94	1.96	12.83	0.00	1.94	4.06	1.60
time (sec)	N/A	0.534	0.072	1.583	0.125	0.101	0.000	0.147	0.232	2.220

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	76	77	73	105	528	0	86	127	128
N.S.	1	1.31	1.33	1.26	1.81	9.10	0.00	1.48	2.19	2.21
time (sec)	N/A	0.310	0.019	1.245	0.120	0.121	0.000	0.130	0.242	0.145

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	49	56	83	522	0	74	220	233
N.S.	1	1.20	1.00	1.14	1.69	10.65	0.00	1.51	4.49	4.76
time (sec)	N/A	0.295	0.007	1.190	0.114	0.110	0.000	0.132	0.225	3.599

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	27	29	28	44	141	0	45	85	79
N.S.	1	0.93	1.00	0.97	1.52	4.86	0.00	1.55	2.93	2.72
time (sec)	N/A	0.251	0.014	1.718	0.036	0.101	0.000	0.135	0.234	0.169

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	83	114	54	156	1188	0	143	299	173
N.S.	1	1.17	1.61	0.76	2.20	16.73	0.00	2.01	4.21	2.44
time (sec)	N/A	0.313	0.016	4.774	0.118	0.109	0.000	0.139	0.230	3.619

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	48	74	46	184	1739	0	149	552	162
N.S.	1	0.86	1.32	0.82	3.29	31.05	0.00	2.66	9.86	2.89
time (sec)	N/A	0.297	0.135	8.785	0.133	0.102	0.000	0.135	0.218	2.307

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	197	156	180	379	5034	0	374	894	359
N.S.	1	1.08	0.86	0.99	2.08	27.66	0.00	2.05	4.91	1.97
time (sec)	N/A	0.658	4.459	16.176	0.144	0.156	0.000	0.293	0.240	0.485

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	208	121	195	348	3341	0	289	564	397
N.S.	1	1.17	0.68	1.10	1.96	18.77	0.00	1.62	3.17	2.23
time (sec)	N/A	0.502	1.388	8.213	0.123	0.151	0.000	0.254	0.244	2.544

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	148	137	130	301	3649	0	296	806	306
N.S.	1	1.11	1.03	0.98	2.26	27.44	0.00	2.23	6.06	2.30
time (sec)	N/A	0.781	1.398	4.519	0.135	0.128	0.000	0.245	0.234	2.530

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	145	90	145	253	2230	0	203	611	338
N.S.	1	1.21	0.75	1.21	2.11	18.58	0.00	1.69	5.09	2.82
time (sec)	N/A	0.640	0.962	3.051	0.120	0.121	0.000	0.209	0.216	0.315

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	115	125	111	447	2498	0	178	595	522
N.S.	1	1.17	1.28	1.13	4.56	25.49	0.00	1.82	6.07	5.33
time (sec)	N/A	0.629	0.709	2.407	0.131	0.130	0.000	0.189	0.236	4.968

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	42	94	98	256	518	0	122	274	483
N.S.	1	0.89	2.00	2.09	5.45	11.02	0.00	2.60	5.83	10.28
time (sec)	N/A	0.469	0.576	8.512	0.039	0.095	0.000	0.206	0.259	2.918

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	125	159	91	378	4642	0	192	864	561
N.S.	1	1.17	1.49	0.85	3.53	43.38	0.00	1.79	8.07	5.24
time (sec)	N/A	0.409	1.020	17.316	0.150	0.158	0.000	0.213	0.242	4.240

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	83	147	101	468	4125	0	249	898	344
N.S.	1	0.86	1.52	1.04	4.82	42.53	0.00	2.57	9.26	3.55
time (sec)	N/A	0.341	0.779	30.382	0.134	0.127	0.000	0.214	0.224	0.287

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	283	294	274	647	12323	0	694	1902	682
N.S.	1	0.99	1.03	0.96	2.26	43.09	0.00	2.43	6.65	2.38
time (sec)	N/A	0.892	6.850	51.988	0.150	0.207	0.000	0.639	0.268	2.954

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F(-1)	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	330	392	291	366	604	8462	0	580	1243	757
N.S.	1	1.19	0.88	1.11	1.83	25.64	0.00	1.76	3.77	2.29
time (sec)	N/A	0.650	7.348	32.035	0.128	0.175	0.000	0.508	0.229	2.864

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	223	244	206	544	9862	0	577	1808	617
N.S.	1	0.99	1.08	0.91	2.41	43.64	0.00	2.55	8.00	2.73
time (sec)	N/A	0.619	6.842	18.964	0.132	0.195	0.000	0.474	0.269	0.599

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	260	302	168	297	484	6410	0	463	1184	707
N.S.	1	1.16	0.65	1.14	1.86	24.65	0.00	1.78	4.55	2.72
time (sec)	N/A	0.865	6.157	12.296	0.134	0.157	0.000	0.359	0.240	2.632

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	245	168	245	654	7127	0	414	1304	671
N.S.	1	1.12	0.77	1.12	2.99	32.54	0.00	1.89	5.95	3.06
time (sec)	N/A	0.901	5.642	7.935	0.122	0.165	0.000	0.322	0.252	7.744

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	63	113	171	679	1192	0	311	587	1515
N.S.	1	0.89	1.59	2.41	9.56	16.79	0.00	4.38	8.27	21.34
time (sec)	N/A	0.488	1.843	30.423	0.053	0.084	0.000	0.344	0.263	2.497

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	258	264	208	586	10985	0	426	1702	731
N.S.	1	1.11	1.14	0.90	2.53	47.35	0.00	1.84	7.34	3.15
time (sec)	N/A	0.732	12.705	55.346	0.153	0.204	0.000	0.359	0.243	9.493

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	118	213	156	997	9459	0	437	1626	646
N.S.	1	0.86	1.54	1.13	7.22	68.54	0.00	3.17	11.78	4.68
time (sec)	N/A	0.365	1.446	85.426	0.152	0.197	0.000	0.358	0.308	0.542

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F(-1)	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	491	465	645	452	0	17123	0	0	0	3313
N.S.	1	0.95	1.31	0.92	0.00	34.87	0.00	0.00	0.00	6.75
time (sec)	N/A	1.173	1.301	35.315	0.000	1.326	0.000	0.000	0.370	5.766

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	N/A	N/A	F(-1)
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	826	289	533	62017	20	3	7934	0
N.S.	1	1.00	35.91	12.57	23.17	2696.39	0.87	0.13	344.96	0.00
time (sec)	N/A	0.234	0.655	14.734	0.728	4.195	65.295	2.179	0.344	0.000

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	F	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	384	364	423	301	0	10695	0	0	0	2100
N.S.	1	0.95	1.10	0.78	0.00	27.85	0.00	0.00	0.00	5.47
time (sec)	N/A	0.907	0.800	4.355	0.000	1.003	0.000	0.000	0.332	3.876

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	N/A	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	409	159	250	40923	19	3	2466	4474
N.S.	1	1.00	19.48	7.57	11.90	1948.71	0.90	0.14	117.43	213.05
time (sec)	N/A	0.371	0.432	3.501	0.409	1.711	53.199	1.818	0.308	85.633

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	N/A	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	331	96	160	20085	19	3	784	3679
N.S.	1	1.00	15.76	4.57	7.62	956.43	0.90	0.14	37.33	175.19
time (sec)	N/A	0.399	0.374	1.743	0.278	1.793	0.456	1.385	0.319	15.578

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	158	190	116	0	640	0	21	0	669
N.S.	1	1.01	1.21	0.74	0.00	4.08	0.00	0.13	0.00	4.26
time (sec)	N/A	0.703	0.391	1.863	0.000	0.125	0.000	0.174	0.380	9.626

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	N/A	B	N/A	N/A	C	N/A	F(-2)	N/A	B
verified	N/A	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	23	23	214	136	188	6846	20	0	13742	3643
N.S.	1	1.00	9.30	5.91	8.17	297.65	0.87	0.00	597.48	158.39
time (sec)	N/A	0.409	0.541	2.941	0.316	3.068	0.365	0.000	0.482	27.489

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	C	F	C	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	198	322	173	0	3726	0	180	0	4563
N.S.	1	0.93	1.51	0.81	0.00	17.49	0.00	0.85	0.00	21.42
time (sec)	N/A	0.829	0.676	3.730	0.000	1.261	0.000	0.183	0.524	4.529

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	79	44	82	104	63	0	104	104	74
N.S.	1	1.25	0.70	1.30	1.65	1.00	0.00	1.65	1.65	1.17
time (sec)	N/A	0.251	0.083	11.040	0.037	0.101	0.000	0.132	0.230	0.211

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	28	44	37	83	45	0	84	100	74
N.S.	1	0.93	1.47	1.23	2.77	1.50	0.00	2.80	3.33	2.47
time (sec)	N/A	0.257	0.009	4.427	0.038	0.091	0.000	0.132	0.222	0.203

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	48	32	54	69	30	0	78	78	27
N.S.	1	1.45	0.97	1.64	2.09	0.91	0.00	2.36	2.36	0.82
time (sec)	N/A	0.239	0.030	1.533	0.032	0.097	0.000	0.130	0.195	0.145

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	25	47	32	55	102	0	47	63	66
N.S.	1	0.93	1.74	1.19	2.04	3.78	0.00	1.74	2.33	2.44
time (sec)	N/A	0.230	0.011	0.828	0.117	0.117	0.000	0.127	0.204	2.396

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	44	49	56	80	323	0	84	157	125
N.S.	1	1.10	1.22	1.40	2.00	8.08	0.00	2.10	3.92	3.12
time (sec)	N/A	0.237	0.009	1.524	0.137	0.099	0.000	0.127	0.226	0.124

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	26	28	25	34	159	0	59	97	59
N.S.	1	0.93	1.00	0.89	1.21	5.68	0.00	2.11	3.46	2.11
time (sec)	N/A	0.234	0.006	3.086	0.044	0.087	0.000	0.130	0.218	0.128

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	71	93	85	181	1046	0	153	341	280
N.S.	1	1.08	1.41	1.29	2.74	15.85	0.00	2.32	5.17	4.24
time (sec)	N/A	0.247	0.006	8.609	0.118	0.106	0.000	0.141	0.231	2.439

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	43	86	42	371	345	0	95	152	304
N.S.	1	0.90	1.79	0.88	7.73	7.19	0.00	1.98	3.17	6.33
time (sec)	N/A	0.257	0.013	18.230	0.044	0.081	0.000	0.132	0.238	0.156

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	A	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	108	63	124	171	95	0	186	184	102
N.S.	1	1.42	0.83	1.63	2.25	1.25	0.00	2.45	2.42	1.34
time (sec)	N/A	0.315	1.030	27.691	0.044	0.096	0.000	0.171	0.236	0.255

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	72	72	161	519	0	152	185	130
N.S.	1	0.91	1.33	1.33	2.98	9.61	0.00	2.81	3.43	2.41
time (sec)	N/A	0.278	0.385	11.278	0.141	0.100	0.000	0.173	0.233	0.239

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	63	54	96	140	105	0	167	232	77
N.S.	1	1.24	1.06	1.88	2.75	2.06	0.00	3.27	4.55	1.51
time (sec)	N/A	0.296	0.665	4.476	0.040	0.088	0.000	0.162	0.225	0.239

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	61	54	97	152	774	0	160	313	182
N.S.	1	1.02	0.90	1.62	2.53	12.90	0.00	2.67	5.22	3.03
time (sec)	N/A	0.388	0.136	2.342	0.121	0.094	0.000	0.155	0.274	0.214

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	101	427	134	199	1373	0	170	476	303
N.S.	1	1.25	5.27	1.65	2.46	16.95	0.00	2.10	5.88	3.74
time (sec)	N/A	0.498	6.802	4.555	0.119	0.109	0.000	0.153	0.259	0.155

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	44	49	126	53	391	0	169	224	482
N.S.	1	0.90	1.00	2.57	1.08	7.98	0.00	3.45	4.57	9.84
time (sec)	N/A	0.447	0.143	13.817	0.035	0.075	0.000	0.160	0.244	2.390

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	128	792	174	345	2824	0	267	781	572
N.S.	1	1.13	7.01	1.54	3.05	24.99	0.00	2.36	6.91	5.06
time (sec)	N/A	0.573	8.038	28.082	0.137	0.117	0.000	0.164	0.257	0.173

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	68	83	158	928	677	0	238	326	732
N.S.	1	0.89	1.09	2.08	12.21	8.91	0.00	3.13	4.29	9.63
time (sec)	N/A	0.505	1.072	59.860	0.053	0.107	0.000	0.166	0.227	2.326

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	106	81	184	267	227	0	282	424	133
N.S.	1	1.16	0.89	2.02	2.93	2.49	0.00	3.10	4.66	1.46
time (sec)	N/A	0.463	6.519	70.574	0.041	0.079	0.000	0.253	0.222	2.507

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	85	494	155	284	1840	0	264	515	232
N.S.	1	0.98	5.68	1.78	3.26	21.15	0.00	3.03	5.92	2.67
time (sec)	N/A	0.346	6.634	30.695	0.121	0.122	0.000	0.252	0.227	0.340

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	87	69	148	256	369	0	265	606	243
N.S.	1	1.12	0.88	1.90	3.28	4.73	0.00	3.40	7.77	3.12
time (sec)	N/A	0.317	1.937	13.167	0.042	0.095	0.000	0.235	0.253	0.280

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	101	89	193	295	2411	0	272	743	355
N.S.	1	1.02	0.90	1.95	2.98	24.35	0.00	2.75	7.51	3.59
time (sec)	N/A	0.336	0.223	6.305	0.166	0.105	0.000	0.211	0.238	0.286

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	B	B	F	B	B	B
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	165	1341	240	362	3465	0	310	971	535
N.S.	1	1.32	10.73	1.92	2.90	27.72	0.00	2.48	7.77	4.28
time (sec)	N/A	0.376	10.164	13.826	0.125	0.108	0.000	0.212	0.233	2.463

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	59	67	227	71	786	0	347	435	1050
N.S.	1	0.88	1.00	3.39	1.06	11.73	0.00	5.18	6.49	15.67
time (sec)	N/A	0.275	0.156	42.457	0.038	0.084	0.000	0.225	0.243	2.522

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	197	158	291	553	6114	0	485	1394	951
N.S.	1	1.15	0.92	1.69	3.22	35.55	0.00	2.82	8.10	5.53
time (sec)	N/A	0.438	12.142	80.239	0.137	0.124	0.000	0.219	0.245	0.282

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	91	218	269	1847	1185	0	447	572	1424
N.S.	1	0.89	2.14	2.64	18.11	11.62	0.00	4.38	5.61	13.96
time (sec)	N/A	0.321	2.057	152.406	0.067	0.092	0.000	0.229	0.269	2.444

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	153	115	273	514	2180	0	301	333	967
N.S.	1	1.28	0.96	2.28	4.28	18.17	0.00	2.51	2.78	8.06
time (sec)	N/A	0.472	0.584	21.668	0.182	0.133	0.000	0.948	0.271	3.123

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	75	79	230	0	1857	0	0	284	2194
N.S.	1	0.94	0.99	2.88	0.00	23.21	0.00	0.00	3.55	27.42
time (sec)	N/A	0.565	0.322	7.809	0.000	0.115	0.000	0.000	0.264	3.954

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	97	77	159	316	948	0	155	181	880
N.S.	1	1.26	1.00	2.06	4.10	12.31	0.00	2.01	2.35	11.43
time (sec)	N/A	0.519	0.435	2.750	0.166	0.156	0.000	0.600	0.279	3.020

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	51	53	149	0	773	0	0	195	154
N.S.	1	0.96	1.00	2.81	0.00	14.58	0.00	0.00	3.68	2.91
time (sec)	N/A	0.438	0.081	1.366	0.000	0.119	0.000	0.000	0.251	2.768

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	102	0	519	0	0	63	147
N.S.	1	1.00	1.00	2.83	0.00	14.42	0.00	0.00	1.75	4.08
time (sec)	N/A	0.398	0.026	1.915	0.000	0.122	0.000	0.000	0.248	0.286

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	32	114	36	455	0	44	61	81
N.S.	1	1.00	1.00	3.56	1.12	14.22	0.00	1.38	1.91	2.53
time (sec)	N/A	0.412	0.305	6.200	0.124	0.108	0.000	0.274	0.282	2.574

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	53	55	144	0	540	0	0	80	449
N.S.	1	0.96	1.00	2.62	0.00	9.82	0.00	0.00	1.45	8.16
time (sec)	N/A	0.311	0.183	16.032	0.000	0.138	0.000	0.000	0.274	3.018

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	48	50	197	63	649	0	70	307	176
N.S.	1	0.96	1.00	3.94	1.26	12.98	0.00	1.40	6.14	3.52
time (sec)	N/A	0.283	0.326	38.356	0.153	0.115	0.000	0.282	0.292	3.135

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	94	79	236	0	1584	0	0	620	1012
N.S.	1	1.09	0.92	2.74	0.00	18.42	0.00	0.00	7.21	11.77
time (sec)	N/A	0.317	0.433	86.950	0.000	0.150	0.000	0.000	0.280	3.382

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	70	71	252	140	2032	0	135	1042	252
N.S.	1	0.93	0.95	3.36	1.87	27.09	0.00	1.80	13.89	3.36
time (sec)	N/A	0.299	0.474	174.331	0.164	0.123	0.000	0.307	0.281	2.891

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	120	111	378	0	6941	0	0	1442	0
N.S.	1	0.94	0.87	2.95	0.00	54.23	0.00	0.00	11.27	0.00
time (sec)	N/A	0.384	0.751	27.800	0.000	0.192	0.000	0.000	0.343	0.000

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	162	110	386	840	4324	0	416	1269	0
N.S.	1	1.16	0.79	2.76	6.00	30.89	0.00	2.97	9.06	0.00
time (sec)	N/A	0.380	0.899	9.181	0.281	0.183	0.000	0.735	0.289	0.000

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	96	84	286	0	3510	0	0	1145	0
N.S.	1	0.95	0.83	2.83	0.00	34.75	0.00	0.00	11.34	0.00
time (sec)	N/A	0.345	0.488	3.810	0.000	0.153	0.000	0.000	0.288	0.000

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	78	252	0	2049	0	0	914	0
N.S.	1	0.98	0.94	3.04	0.00	24.69	0.00	0.00	11.01	0.00
time (sec)	N/A	0.274	0.207	8.314	0.000	0.125	0.000	0.000	0.259	0.000

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	64	63	210	125	1515	0	138	508	0
N.S.	1	0.97	0.95	3.18	1.89	22.95	0.00	2.09	7.70	0.00
time (sec)	N/A	0.263	0.586	26.228	0.201	0.123	0.000	0.331	0.242	0.000

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	69	183	0	1563	0	0	590	0
N.S.	1	0.97	0.96	2.54	0.00	21.71	0.00	0.00	8.19	0.00
time (sec)	N/A	0.363	0.117	60.371	0.000	0.117	0.000	0.000	0.257	0.000

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	A	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	75	83	259	127	1443	0	143	637	0
N.S.	1	0.97	1.08	3.36	1.65	18.74	0.00	1.86	8.27	0.00
time (sec)	N/A	0.461	0.543	127.205	0.189	0.129	0.000	0.346	0.323	0.000

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	107	203	273	0	2140	0	0	950	0
N.S.	1	1.05	1.99	2.68	0.00	20.98	0.00	0.00	9.31	0.00
time (sec)	N/A	0.539	0.369	244.370	0.000	0.136	0.000	0.000	0.278	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	92	102	306	209	2869	0	232	1449	0
N.S.	1	0.95	1.05	3.15	2.15	29.58	0.00	2.39	14.94	0.00
time (sec)	N/A	0.544	0.774	0.711	0.238	0.135	0.000	0.350	0.309	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	163	265	351	0	6396	0	0	2051	0
N.S.	1	1.05	1.71	2.26	0.00	41.26	0.00	0.00	13.23	0.00
time (sec)	N/A	0.705	0.935	0.615	0.000	0.178	0.000	0.000	0.363	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F(-1)	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	238	164	501	1806	13887	0	546	4226	0
N.S.	1	1.20	0.83	2.53	9.12	70.14	0.00	2.76	21.34	0.00
time (sec)	N/A	0.510	1.341	28.587	0.409	0.373	0.000	1.101	0.398	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	146	136	375	0	11399	0	0	3187	0
N.S.	1	0.95	0.88	2.44	0.00	74.02	0.00	0.00	20.69	0.00
time (sec)	N/A	0.414	1.296	11.767	0.000	0.327	0.000	0.000	0.415	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	154	134	394	0	7917	0	0	14353	0
N.S.	1	1.10	0.96	2.81	0.00	56.55	0.00	0.00	102.52	0.00
time (sec)	N/A	0.350	0.686	29.847	0.000	0.184	0.000	0.000	0.329	0.000

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	99	77	284	366	5840	0	320	1906	0
N.S.	1	1.03	0.80	2.96	3.81	60.83	0.00	3.33	19.85	0.00
time (sec)	N/A	0.286	1.450	83.826	0.269	0.167	0.000	0.607	0.335	0.000

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	121	123	342	0	6622	0	0	2228	0
N.S.	1	0.94	0.95	2.65	0.00	51.33	0.00	0.00	17.27	0.00
time (sec)	N/A	0.318	0.507	173.548	0.000	0.188	0.000	0.000	0.287	0.000

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	113	115	334	360	5659	0	319	2949	0
N.S.	1	0.98	1.00	2.90	3.13	49.21	0.00	2.77	25.64	0.00
time (sec)	N/A	0.289	1.168	0.690	0.305	0.167	0.000	0.611	0.326	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	107	87	276	0	5084	0	0	1507	0
N.S.	1	1.03	0.84	2.65	0.00	48.88	0.00	0.00	14.49	0.00
time (sec)	N/A	0.293	0.269	0.648	0.000	0.159	0.000	0.000	0.334	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	143	128	376	332	5233	0	322	2976	0
N.S.	1	1.08	0.97	2.85	2.52	39.64	0.00	2.44	22.55	0.00
time (sec)	N/A	0.471	1.161	0.636	0.334	0.197	0.000	0.579	0.277	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F(-1)	F(-2)	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	170	317	389	0	8070	0	0	2818	0
N.S.	1	1.09	2.03	2.49	0.00	51.73	0.00	0.00	18.06	0.00
time (sec)	N/A	0.712	2.113	0.630	0.000	0.263	0.000	0.000	0.334	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	49	97	68	199	339	82	134	67	50
N.S.	1	0.91	1.80	1.26	3.69	6.28	1.52	2.48	1.24	0.93
time (sec)	N/A	0.623	0.036	0.298	0.045	0.088	0.142	0.156	0.308	0.132

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	59	46	71	168	1205	88	92	484	53
N.S.	1	1.20	0.94	1.45	3.43	24.59	1.80	1.88	9.88	1.08
time (sec)	N/A	0.649	0.164	0.265	0.121	0.104	0.137	0.151	0.243	2.395

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	65	45	105	160	54	86	45	34
N.S.	1	1.00	1.81	1.25	2.92	4.44	1.50	2.39	1.25	0.94
time (sec)	N/A	0.402	0.024	0.263	0.046	0.081	0.114	0.135	0.226	2.381

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	40	39	49	76	399	60	57	249	37
N.S.	1	1.29	1.26	1.58	2.45	12.87	1.94	1.84	8.03	1.19
time (sec)	N/A	0.277	0.047	0.361	0.120	0.100	0.109	0.126	0.295	2.373

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	28	22	31	37	20	29	22	18
N.S.	1	1.00	1.47	1.16	1.63	1.95	1.05	1.53	1.16	0.95
time (sec)	N/A	0.174	0.001	0.214	0.043	0.084	0.089	0.115	0.240	0.072

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	39	25	24	35	70	0	46	54	228
N.S.	1	1.56	1.00	0.96	1.40	2.80	0.00	1.84	2.16	9.12
time (sec)	N/A	0.293	0.029	0.781	0.037	0.098	0.000	0.125	0.229	0.147

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	32	21	31	38	78	30	70	25
N.S.	1	1.00	1.78	1.17	1.72	2.11	4.33	1.67	3.89	1.39
time (sec)	N/A	0.224	0.023	0.471	0.041	0.092	4.545	0.135	0.260	2.385

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	44	39	35	106	407	0	58	345	76
N.S.	1	1.42	1.26	1.13	3.42	13.13	0.00	1.87	11.13	2.45
time (sec)	N/A	0.295	0.051	0.672	0.040	0.086	0.000	0.147	0.213	2.466

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	61	39	105	156	0	86	189	162
N.S.	1	1.00	1.69	1.08	2.92	4.33	0.00	2.39	5.25	4.50
time (sec)	N/A	0.291	0.042	0.149	0.042	0.088	0.000	0.152	0.235	2.530

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	C	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	63	45	56	206	1216	0	93	664	177
N.S.	1	1.29	0.92	1.14	4.20	24.82	0.00	1.90	13.55	3.61
time (sec)	N/A	0.371	0.192	0.184	0.041	0.125	0.000	0.171	0.256	2.420

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	83	81	190	135	369	796	165	300	134	91
N.S.	1	0.98	2.29	1.63	4.45	9.59	1.99	3.61	1.61	1.10
time (sec)	N/A	0.305	0.063	0.093	0.050	0.090	0.255	0.209	0.266	0.174

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	130	333	3441	170	191	1141	100
N.S.	1	1.00	1.00	1.71	4.38	45.28	2.24	2.51	15.01	1.32
time (sec)	N/A	0.329	0.125	0.074	0.123	0.126	0.227	0.195	0.238	2.516

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	137	97	231	483	117	218	96	67
N.S.	1	1.03	2.17	1.54	3.67	7.67	1.86	3.46	1.52	1.06
time (sec)	N/A	0.329	0.051	0.070	0.047	0.093	0.238	0.177	0.249	2.517

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	60	49	92	186	1638	122	116	723	76
N.S.	1	1.05	0.86	1.61	3.26	28.74	2.14	2.04	12.68	1.33
time (sec)	N/A	0.478	0.215	0.056	0.119	0.101	0.214	0.163	0.248	2.568

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	65	60	114	201	68	103	60	47
N.S.	1	1.09	1.51	1.40	2.65	4.67	1.58	2.40	1.40	1.09
time (sec)	N/A	0.408	0.515	0.057	0.037	0.110	0.140	0.124	0.260	2.435

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	54	57	59	104	668	0	141	440	210
N.S.	1	1.10	1.16	1.20	2.12	13.63	0.00	2.88	8.98	4.29
time (sec)	N/A	0.484	0.073	0.154	0.114	0.098	0.000	0.159	0.236	2.572

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	40	64	36	64	97	0	71	147	59
N.S.	1	1.11	1.78	1.00	1.78	2.69	0.00	1.97	4.08	1.64
time (sec)	N/A	0.484	0.075	0.164	0.043	0.083	0.000	0.178	0.228	2.460

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	55	57	62	134	677	0	141	493	211
N.S.	1	1.06	1.10	1.19	2.58	13.02	0.00	2.71	9.48	4.06
time (sec)	N/A	0.510	0.083	0.154	0.051	0.105	0.000	0.198	0.261	0.249

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	47	65	43	114	197	0	103	266	175
N.S.	1	1.09	1.51	1.00	2.65	4.58	0.00	2.40	6.19	4.07
time (sec)	N/A	0.285	0.415	0.131	0.043	0.088	0.000	0.195	0.303	0.160

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	48	77	236	1649	0	118	1014	197
N.S.	1	1.00	0.67	1.07	3.28	22.90	0.00	1.64	14.08	2.74
time (sec)	N/A	0.320	0.253	0.175	0.043	0.111	0.000	0.228	0.337	2.497

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	65	98	58	231	473	0	218	513	529
N.S.	1	1.03	1.56	0.92	3.67	7.51	0.00	3.46	8.14	8.40
time (sec)	N/A	0.487	0.070	0.160	0.046	0.090	0.000	0.235	0.570	0.190

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	92	89	76	92	390	3454	0	192	1566	362
N.S.	1	0.97	0.83	1.00	4.24	37.54	0.00	2.09	17.02	3.93
time (sec)	N/A	0.550	0.172	0.231	0.051	0.138	0.000	0.269	0.236	0.261

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	114	109	123	218	583	1563	260	534	217	138
N.S.	1	0.96	1.08	1.91	5.11	13.71	2.28	4.68	1.90	1.21
time (sec)	N/A	0.318	1.000	0.122	0.059	0.106	0.307	0.294	0.254	0.239

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	104	98	205	540	7502	279	309	2074	155
N.S.	1	0.97	0.92	1.92	5.05	70.11	2.61	2.89	19.38	1.45
time (sec)	N/A	0.336	0.198	0.095	0.139	0.179	0.269	0.276	0.280	2.536

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	94	93	108	164	400	1036	192	418	163	106
N.S.	1	0.99	1.15	1.74	4.26	11.02	2.04	4.45	1.73	1.13
time (sec)	N/A	0.500	1.147	0.099	0.051	0.106	0.214	0.249	0.250	2.629

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	76	151	351	4298	211	216	1493	123
N.S.	1	1.00	0.92	1.82	4.23	51.78	2.54	2.60	17.99	1.48
time (sec)	N/A	0.490	0.174	0.057	0.148	0.133	0.191	0.223	0.245	2.748

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	74	75	95	112	239	567	126	241	111	86
N.S.	1	1.01	1.28	1.51	3.23	7.66	1.70	3.26	1.50	1.16
time (sec)	N/A	0.411	0.412	0.053	0.048	0.097	0.167	0.132	0.247	2.827

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	74	67	80	214	2381	0	267	1069	380
N.S.	1	1.03	0.93	1.11	2.97	33.07	0.00	3.71	14.85	5.28
time (sec)	N/A	0.432	0.373	0.173	0.140	0.128	0.000	0.226	0.269	0.386

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	60	81	59	147	341	0	135	407	218
N.S.	1	1.02	1.37	1.00	2.49	5.78	0.00	2.29	6.90	3.69
time (sec)	N/A	0.506	1.564	0.149	0.055	0.099	0.000	0.241	0.263	2.828

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	70	63	77	203	1686	0	274	712	327
N.S.	1	0.97	0.88	1.07	2.82	23.42	0.00	3.81	9.89	4.54
time (sec)	N/A	0.414	0.307	0.184	0.142	0.143	0.000	0.274	0.268	3.916

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	60	82	56	147	341	0	135	407	219
N.S.	1	1.02	1.39	0.95	2.49	5.78	0.00	2.29	6.90	3.71
time (sec)	N/A	0.498	0.868	0.170	0.044	0.102	0.000	0.297	23.504	0.174

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	67	90	264	2393	0	267	25	381
N.S.	1	1.00	0.81	1.08	3.18	28.83	0.00	3.22	0.30	4.59
time (sec)	N/A	0.419	0.390	0.195	0.059	0.110	0.000	0.343	200.020	0.406

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	76	100	72	239	557	0	241	25	568
N.S.	1	1.03	1.35	0.97	3.23	7.53	0.00	3.26	0.34	7.68
time (sec)	N/A	0.533	1.197	0.176	0.048	0.088	0.000	0.348	200.023	2.904

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	F	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	100	76	106	420	4305	0	217	25	380
N.S.	1	0.97	0.74	1.03	4.08	41.80	0.00	2.11	0.24	3.69
time (sec)	N/A	0.574	0.162	0.240	0.050	0.172	0.000	0.427	200.020	0.314

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	110	108	128	179	410	1176	209	447	178	133
N.S.	1	0.98	1.16	1.63	3.73	10.69	1.90	4.06	1.62	1.21
time (sec)	N/A	0.298	1.230	0.079	0.053	0.097	0.229	0.154	0.211	2.904

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	No	TBD	TBD	TBD	TBD	TBD	TBD
size	160	155	170	262	624	2133	308	721	261	188
N.S.	1	0.97	1.06	1.64	3.90	13.33	1.92	4.51	1.63	1.18
time (sec)	N/A	0.355	1.585	0.132	0.063	0.121	0.330	0.169	0.231	2.495

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	60	79	133	742	415	132	153	72
N.S.	1	1.00	0.91	1.20	2.02	11.24	6.29	2.00	2.32	1.09
time (sec)	N/A	0.322	0.128	0.098	0.127	0.175	4.400	0.183	0.230	0.262

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	69	66	87	509	777	428	87	63	56
N.S.	1	1.17	1.12	1.47	8.63	13.17	7.25	1.47	1.07	0.95
time (sec)	N/A	0.346	0.139	0.148	0.248	0.119	3.067	0.164	0.242	2.341

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	52	44	49	82	118	306	96	119	46
N.S.	1	1.13	0.96	1.07	1.78	2.57	6.65	2.09	2.59	1.00
time (sec)	N/A	0.482	0.028	0.062	0.123	0.121	2.507	0.159	0.222	0.115

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	54	47	72	215	486	253	65	40	38
N.S.	1	1.17	1.02	1.57	4.67	10.57	5.50	1.41	0.87	0.83
time (sec)	N/A	0.470	0.022	0.078	0.152	0.119	2.132	0.151	0.221	0.106

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	47	35	44	58	82	146	61	79	43
N.S.	1	1.12	0.83	1.05	1.38	1.95	3.48	1.45	1.88	1.02
time (sec)	N/A	0.432	0.018	0.062	0.049	0.089	2.019	0.139	0.236	2.317

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	65	71	57	484	240	63	39	37
N.S.	1	1.00	1.44	1.58	1.27	10.76	5.33	1.40	0.87	0.82
time (sec)	N/A	0.547	0.064	0.086	0.141	0.119	2.100	0.130	0.241	2.383

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	65	55	65	101	118	0	97	141	194
N.S.	1	1.08	0.92	1.08	1.68	1.97	0.00	1.62	2.35	3.23
time (sec)	N/A	0.539	0.046	0.151	0.044	0.123	0.000	0.142	0.239	0.389

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	71	67	89	329	784	0	89	227	402
N.S.	1	1.18	1.12	1.48	5.48	13.07	0.00	1.48	3.78	6.70
time (sec)	N/A	0.336	0.137	0.160	0.175	0.122	0.000	0.151	0.263	2.643

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	A	B	B
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	82	60	88	159	747	0	133	661	313
N.S.	1	0.96	0.71	1.04	1.87	8.79	0.00	1.56	7.78	3.68
time (sec)	N/A	0.351	0.127	0.193	0.042	0.152	0.000	0.169	0.353	2.639

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	96	91	109	1038	2368	0	147	550	519
N.S.	1	1.17	1.11	1.33	12.66	28.88	0.00	1.79	6.71	6.33
time (sec)	N/A	0.548	0.453	0.185	0.261	0.139	0.000	0.161	16.547	2.728

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	84	69	91	217	1141	0	194	3130	170
N.S.	1	1.01	0.83	1.10	2.61	13.75	0.00	2.34	37.71	2.05
time (sec)	N/A	0.596	0.339	0.115	0.129	0.180	0.000	0.236	0.274	2.928

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	105	90	104	1010	1950	0	195	228	1655
N.S.	1	1.18	1.01	1.17	11.35	21.91	0.00	2.19	2.56	18.60
time (sec)	N/A	0.561	0.354	0.145	0.368	0.143	0.000	0.205	0.221	3.061

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	71	57	84	170	629	0	149	2332	210
N.S.	1	0.99	0.79	1.17	2.36	8.74	0.00	2.07	32.39	2.92
time (sec)	N/A	0.556	0.301	0.089	0.049	0.120	0.000	0.191	0.223	2.914

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	99	86	100	614	2025	0	177	232	106
N.S.	1	1.16	1.01	1.18	7.22	23.82	0.00	2.08	2.73	1.25
time (sec)	N/A	0.484	0.268	0.129	0.261	0.129	0.000	0.177	0.214	2.987

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	55	86	170	623	0	149	2288	129
N.S.	1	1.00	0.81	1.26	2.50	9.16	0.00	2.19	33.65	1.90
time (sec)	N/A	0.363	0.279	0.087	0.049	0.092	0.000	0.162	0.245	2.755

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	106	97	103	206	1942	0	195	226	110
N.S.	1	1.19	1.09	1.16	2.31	21.82	0.00	2.19	2.54	1.24
time (sec)	N/A	0.304	0.377	0.135	0.170	0.137	0.000	0.127	0.216	0.458

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	94	83	110	235	1148	0	195	1760	0
N.S.	1	0.99	0.87	1.16	2.47	12.08	0.00	2.05	18.53	0.00
time (sec)	N/A	0.352	1.308	0.255	0.058	0.229	0.000	0.179	0.276	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	134	111	120	976	3725	0	336	1911	0
N.S.	1	1.13	0.93	1.01	8.20	31.30	0.00	2.82	16.06	0.00
time (sec)	N/A	0.401	1.115	0.219	0.291	0.176	0.000	0.201	0.317	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	116	93	131	402	3468	0	323	2786	0
N.S.	1	0.94	0.75	1.06	3.24	27.97	0.00	2.60	22.47	0.00
time (sec)	N/A	0.397	0.600	0.359	0.057	0.309	0.000	0.228	31.605	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	A	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	169	139	142	2345	8482	0	281	25	0
N.S.	1	1.06	0.87	0.89	14.75	53.35	0.00	1.77	0.16	0.00
time (sec)	N/A	0.780	0.972	0.258	0.504	0.233	0.000	0.233	200.026	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	173	144	157	3354	7528	0	408	575	2669
N.S.	1	1.20	1.00	1.09	23.29	52.28	0.00	2.83	3.99	18.53
time (sec)	N/A	0.436	0.827	0.219	1.139	0.218	0.000	0.329	0.202	0.847

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	105	91	127	376	2584	0	245	11531	416
N.S.	1	0.96	0.83	1.17	3.45	23.71	0.00	2.25	105.79	3.82
time (sec)	N/A	0.367	0.725	0.111	0.075	0.144	0.000	0.305	0.270	0.770

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	157	135	148	2432	7757	0	384	580	2574
N.S.	1	1.15	0.99	1.08	17.75	56.62	0.00	2.80	4.23	18.79
time (sec)	N/A	0.378	0.741	0.211	0.809	0.220	0.000	0.293	0.197	2.903

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	95	80	115	384	2611	0	245	10556	397
N.S.	1	0.97	0.82	1.17	3.92	26.64	0.00	2.50	107.71	4.05
time (sec)	N/A	0.343	0.328	0.118	0.079	0.140	0.000	0.278	0.273	2.844

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	162	137	152	1472	7791	0	388	578	255
N.S.	1	1.18	1.00	1.11	10.74	56.87	0.00	2.83	4.22	1.86
time (sec)	N/A	0.377	0.783	0.187	0.443	0.206	0.000	0.259	0.197	4.487

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	91	77	116	378	2554	0	245	10812	235
N.S.	1	0.97	0.82	1.23	4.02	27.17	0.00	2.61	115.02	2.50
time (sec)	N/A	0.315	0.339	0.115	0.082	0.145	0.000	0.237	0.284	3.302

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	170	147	156	507	7496	0	409	575	260
N.S.	1	1.20	1.04	1.10	3.57	52.79	0.00	2.88	4.05	1.83
time (sec)	N/A	0.341	0.209	0.178	0.231	0.189	0.000	0.146	0.204	3.110

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	134	117	161	498	4800	0	295	5158	0
N.S.	1	0.97	0.85	1.17	3.61	34.78	0.00	2.14	37.38	0.00
time (sec)	N/A	0.394	1.063	0.647	0.083	0.359	0.000	0.272	0.371	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	B	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	203	166	167	1944	11865	0	437	4537	0
N.S.	1	1.14	0.93	0.94	10.92	66.66	0.00	2.46	25.49	0.00
time (sec)	N/A	0.766	4.072	0.317	0.592	0.294	0.000	0.296	0.445	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	No	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	160	138	184	770	10720	0	474	25	0
N.S.	1	0.94	0.81	1.08	4.50	62.69	0.00	2.77	0.15	0.00
time (sec)	N/A	0.709	1.220	0.905	0.107	0.579	0.000	0.373	200.025	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	248	194	189	4285	22038	0	493	25	0
N.S.	1	1.09	0.85	0.83	18.79	96.66	0.00	2.16	0.11	0.00
time (sec)	N/A	0.816	2.313	0.378	1.130	0.377	0.000	0.379	200.038	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	F(-1)	B	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	239	203	219	925	20020	0	750	1058	3685
N.S.	1	1.19	1.01	1.09	4.60	99.60	0.00	3.73	5.26	18.33
time (sec)	N/A	0.727	0.400	0.303	0.417	0.369	0.000	0.192	0.438	1.288

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	A	A	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	3	3	15	4	5	8	0	5	11	3
N.S.	1	1.00	5.00	1.33	1.67	2.67	0.00	1.67	3.67	1.00
time (sec)	N/A	0.331	0.007	0.069	0.151	0.076	0.000	0.111	0.478	0.042

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	17	15	5	96	0	1	9	14
N.S.	1	1.00	1.06	0.94	0.31	6.00	0.00	0.06	0.56	0.88
time (sec)	N/A	0.257	0.008	0.028	0.164	0.094	0.000	0.109	0.453	0.232

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	26	23	21	28	140	0	45	30	20
N.S.	1	1.18	1.05	0.95	1.27	6.36	0.00	2.05	1.36	0.91
time (sec)	N/A	0.241	0.015	0.069	0.161	0.083	0.000	0.118	0.509	0.089

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	C	B	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	23	28	32	385	0	41	26	27
N.S.	1	1.00	0.66	0.80	0.91	11.00	0.00	1.17	0.74	0.77
time (sec)	N/A	0.250	0.013	0.035	0.159	0.089	0.000	0.114	0.568	2.274

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	A	A	F	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	15	11	14	11	2	0	11	22	2
N.S.	1	1.36	1.00	1.27	1.00	0.18	0.00	1.00	2.00	0.18
time (sec)	N/A	0.234	0.021	0.032	0.137	0.076	0.000	0.112	0.496	0.135

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	12	25	36	12	21	18	14
N.S.	1	1.00	1.00	0.92	1.92	2.77	0.92	1.62	1.38	1.08
time (sec)	N/A	0.233	0.020	0.022	0.130	0.082	0.213	0.108	0.498	0.086

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	92	85	288	0	4529	126	980	149	119
N.S.	1	1.06	0.98	3.31	0.00	52.06	1.45	11.26	1.71	1.37
time (sec)	N/A	0.345	0.349	0.179	0.000	0.330	2.123	0.844	0.555	9.844

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	133	242	337	0	9220	0	938	16	0
N.S.	1	1.10	2.00	2.79	0.00	76.20	0.00	7.75	0.13	0.00
time (sec)	N/A	0.397	4.560	0.067	0.000	0.424	0.000	0.807	0.569	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	68	60	253	0	2329	94	630	95	66
N.S.	1	1.08	0.95	4.02	0.00	36.97	1.49	10.00	1.51	1.05
time (sec)	N/A	0.303	0.114	0.066	0.000	0.191	1.489	0.584	0.562	4.198

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	89	193	276	0	4685	0	554	16	0
N.S.	1	1.05	2.27	3.25	0.00	55.12	0.00	6.52	0.19	0.00
time (sec)	N/A	0.331	2.302	0.060	0.000	0.243	0.000	0.504	0.505	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	49	44	238	0	1543	68	349	75	51
N.S.	1	1.11	1.00	5.41	0.00	35.07	1.55	7.93	1.70	1.16
time (sec)	N/A	0.259	0.024	0.053	0.000	0.134	0.903	0.355	0.565	2.756

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	81	238	0	3303	0	253	11	0
N.S.	1	1.00	1.35	3.97	0.00	55.05	0.00	4.22	0.18	0.00
time (sec)	N/A	0.328	0.221	0.053	0.000	0.189	0.000	0.318	0.534	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	56	0	0	3313	0	255	14	0
N.S.	1	1.09	1.00	0.00	0.00	59.16	0.00	4.55	0.25	0.00
time (sec)	N/A	0.473	0.024	0.000	0.000	0.194	0.000	0.300	0.559	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	42	0	0	1469	0	348	16	0
N.S.	1	1.00	0.88	0.00	0.00	30.60	0.00	7.25	0.33	0.00
time (sec)	N/A	0.429	0.104	0.000	0.000	0.143	0.000	0.355	0.605	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	87	83	0	0	4735	0	557	16	0
N.S.	1	1.05	1.00	0.00	0.00	57.05	0.00	6.71	0.19	0.00
time (sec)	N/A	0.571	0.155	0.000	0.000	0.259	0.000	0.513	0.555	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	82	161	0	0	2285	0	629	16	0
N.S.	1	1.05	2.06	0.00	0.00	29.29	0.00	8.06	0.21	0.00
time (sec)	N/A	0.346	4.471	0.000	0.000	0.189	0.000	0.586	73.738	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	121	134	111	0	0	9488	0	947	17	0
N.S.	1	1.11	0.92	0.00	0.00	78.41	0.00	7.83	0.14	0.00
time (sec)	N/A	0.382	0.457	0.000	0.000	0.403	0.000	0.787	200.048	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	89	86	488	0	4941	223	1063	181	112
N.S.	1	1.09	1.05	5.95	0.00	60.26	2.72	12.96	2.21	1.37
time (sec)	N/A	0.335	0.303	0.100	0.000	0.314	9.602	1.186	0.659	10.963

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	132	247	529	0	10046	0	949	37	0
N.S.	1	1.07	2.01	4.30	0.00	81.67	0.00	7.72	0.30	0.00
time (sec)	N/A	0.447	4.991	0.053	0.000	0.427	0.000	0.984	0.667	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	70	59	473	0	2385	165	662	146	64
N.S.	1	1.11	0.94	7.51	0.00	37.86	2.62	10.51	2.32	1.02
time (sec)	N/A	0.295	0.115	0.043	0.000	0.186	6.396	0.748	0.707	4.592

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	92	110	473	0	4841	0	584	32	0
N.S.	1	1.05	1.25	5.38	0.00	55.01	0.00	6.64	0.36	0.00
time (sec)	N/A	0.297	0.432	0.040	0.000	0.277	0.000	0.609	0.507	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	71	76	71	0	0	3883	0	433	37	0
N.S.	1	1.07	1.00	0.00	0.00	54.69	0.00	6.10	0.52	0.00
time (sec)	N/A	0.481	0.053	0.000	0.000	0.249	0.000	0.696	0.668	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	197	0	0	3913	0	430	41	0
N.S.	1	1.00	2.56	0.00	0.00	50.82	0.00	5.58	0.53	0.00
time (sec)	N/A	0.521	1.931	0.000	0.000	0.240	0.000	0.706	0.694	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	51	97	0	679	0	104	9	68
N.S.	1	1.00	1.65	3.13	0.00	21.90	0.00	3.35	0.29	2.19
time (sec)	N/A	0.313	0.053	0.094	0.000	0.099	0.000	0.127	0.638	0.202

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	53	142	0	226	0	104	11	43
N.S.	1	1.00	1.18	3.16	0.00	5.02	0.00	2.31	0.24	0.96
time (sec)	N/A	0.389	0.035	0.081	0.000	0.104	0.000	0.126	0.647	2.373

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	53	74	158	0	1027	0	202	24	78
N.S.	1	1.06	1.48	3.16	0.00	20.54	0.00	4.04	0.48	1.56
time (sec)	N/A	0.389	0.109	0.066	0.000	0.096	0.000	0.132	0.718	0.265

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	C	F	C	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	70	76	211	0	361	0	204	27	0
N.S.	1	1.04	1.13	3.15	0.00	5.39	0.00	3.04	0.40	0.00
time (sec)	N/A	0.393	0.060	0.048	0.000	0.086	0.000	0.135	0.592	0.000

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	76	68	164	0	2827	0	592	65	65
N.S.	1	1.09	0.97	2.34	0.00	40.39	0.00	8.46	0.93	0.93
time (sec)	N/A	0.355	0.341	0.083	0.000	0.242	0.000	0.523	0.625	3.389

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	95	208	178	0	5494	0	559	26	0
N.S.	1	1.08	2.36	2.02	0.00	62.43	0.00	6.35	0.30	0.00
time (sec)	N/A	0.333	3.095	0.077	0.000	0.320	0.000	0.407	0.711	0.000

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	52	47	129	0	1625	0	345	26	39
N.S.	1	1.11	1.00	2.74	0.00	34.57	0.00	7.34	0.55	0.83
time (sec)	N/A	0.300	0.071	0.057	0.000	0.169	0.000	0.317	0.650	2.725

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	101	137	0	3361	0	252	26	0
N.S.	1	1.00	1.68	2.28	0.00	56.02	0.00	4.20	0.43	0.00
time (sec)	N/A	0.300	0.285	0.067	0.000	0.204	0.000	0.321	0.752	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	114	0	1361	68	188	24	23
N.S.	1	1.00	1.00	3.93	0.00	46.93	2.34	6.48	0.83	0.79
time (sec)	N/A	0.268	0.013	0.135	0.000	0.134	0.992	0.207	0.611	2.749

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	31	114	0	1287	0	188	22	25
N.S.	1	1.00	1.00	3.68	0.00	41.52	0.00	6.06	0.71	0.81
time (sec)	N/A	0.209	0.019	0.051	0.000	0.127	0.000	0.208	0.785	2.634

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	56	61	56	0	0	3371	0	254	24	0
N.S.	1	1.09	1.00	0.00	0.00	60.20	0.00	4.54	0.43	0.00
time (sec)	N/A	0.294	0.032	0.000	0.000	0.223	0.000	0.316	0.746	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	130	0	0	1565	0	343	26	0
N.S.	1	1.00	2.55	0.00	0.00	30.69	0.00	6.73	0.51	0.00
time (sec)	N/A	0.299	2.919	0.000	0.000	0.144	0.000	0.310	0.652	0.000

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	F	F	B	F	B	F	F(-1)
verified	N/A	No	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	88	96	107	0	0	5555	0	565	26	0
N.S.	1	1.09	1.22	0.00	0.00	63.12	0.00	6.42	0.30	0.00
time (sec)	N/A	0.351	0.280	0.000	0.000	0.320	0.000	0.405	3.199	0.000

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	77	67	322	0	3991	0	460	139	70
N.S.	1	1.07	0.93	4.47	0.00	55.43	0.00	6.39	1.93	0.97
time (sec)	N/A	0.390	0.086	0.079	0.000	0.334	0.000	0.394	0.675	3.836

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	99	188	328	0	6833	0	371	38	0
N.S.	1	1.18	2.24	3.90	0.00	81.35	0.00	4.42	0.45	0.00
time (sec)	N/A	0.354	1.789	0.050	0.000	0.374	0.000	0.319	0.685	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	58	52	287	0	2525	0	287	107	45
N.S.	1	1.12	1.00	5.52	0.00	48.56	0.00	5.52	2.06	0.87
time (sec)	N/A	0.309	0.091	0.062	0.000	0.182	0.000	0.265	0.737	3.321

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	B	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	112	289	0	2211	0	293	107	0
N.S.	1	1.00	2.11	5.45	0.00	41.72	0.00	5.53	2.02	0.00
time (sec)	N/A	0.300	1.013	0.056	0.000	0.189	0.000	0.256	0.758	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	54	41	273	0	2277	92	292	36	41
N.S.	1	1.10	0.84	5.57	0.00	46.47	1.88	5.96	0.73	0.84
time (sec)	N/A	0.276	0.024	0.049	0.000	0.200	9.037	0.265	0.631	3.166

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	223	272	0	2439	0	288	34	0
N.S.	1	1.00	3.98	4.86	0.00	43.55	0.00	5.14	0.61	0.00
time (sec)	N/A	0.241	2.964	0.042	0.000	0.202	0.000	0.267	0.599	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	78	98	70	0	0	6799	0	372	36	0
N.S.	1	1.26	0.90	0.00	0.00	87.17	0.00	4.77	0.46	0.00
time (sec)	N/A	0.348	0.068	0.000	0.000	0.372	0.000	0.329	0.667	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	85	91	263	0	0	3859	0	459	38	0
N.S.	1	1.07	3.09	0.00	0.00	45.40	0.00	5.40	0.45	0.00
time (sec)	N/A	0.386	10.655	0.000	0.000	0.331	0.000	0.383	1.520	0.000

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	148	231	549	0	19265	0	805	50	0
N.S.	1	1.25	1.96	4.65	0.00	163.26	0.00	6.82	0.42	0.00
time (sec)	N/A	0.678	1.322	0.081	0.000	1.134	0.000	0.435	0.572	0.000

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	90	68	469	0	7033	0	711	238	92
N.S.	1	1.07	0.81	5.58	0.00	83.73	0.00	8.46	2.83	1.10
time (sec)	N/A	0.541	0.086	0.057	0.000	0.526	0.000	0.319	0.559	4.926

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	99	132	491	0	5719	0	684	218	0
N.S.	1	1.10	1.47	5.46	0.00	63.54	0.00	7.60	2.42	0.00
time (sec)	N/A	0.520	1.792	0.062	0.000	0.514	0.000	0.322	0.593	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	86	63	435	0	6621	0	725	206	82
N.S.	1	1.16	0.85	5.88	0.00	89.47	0.00	9.80	2.78	1.11
time (sec)	N/A	0.552	0.063	0.060	0.000	0.610	0.000	0.319	0.622	4.710

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	97	193	454	0	6507	0	728	50	0
N.S.	1	1.10	2.19	5.16	0.00	73.94	0.00	8.27	0.57	0.00
time (sec)	N/A	0.372	6.602	0.059	0.000	0.597	0.000	0.321	0.588	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	A	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	82	43	420	0	5779	114	683	48	76
N.S.	1	1.17	0.61	6.00	0.00	82.56	1.63	9.76	0.69	1.09
time (sec)	N/A	0.300	0.033	0.052	0.000	0.542	11.111	0.323	0.615	4.816

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	B	F	B	F	B	F	F(-1)
verified	N/A	Yes	No	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	106	943	420	0	6933	0	714	46	0
N.S.	1	1.14	10.14	4.52	0.00	74.55	0.00	7.68	0.49	0.00
time (sec)	N/A	0.299	5.938	0.040	0.000	0.655	0.000	0.324	0.545	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	108	142	73	0	0	19151	0	808	48	0
N.S.	1	1.31	0.68	0.00	0.00	177.32	0.00	7.48	0.44	0.00
time (sec)	N/A	0.406	0.059	0.000	0.000	1.452	0.000	0.447	0.523	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	C	F	F	B	F	B	F	F(-1)
verified	N/A	Yes	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	131	148	246	0	0	10671	0	898	50	0
N.S.	1	1.13	1.88	0.00	0.00	81.46	0.00	6.85	0.38	0.00
time (sec)	N/A	0.470	6.695	0.000	0.000	1.301	0.000	0.531	13.535	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	B	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	35	62	0	543	0	58	18	63
N.S.	1	1.00	1.40	2.48	0.00	21.72	0.00	2.32	0.72	2.52
time (sec)	N/A	0.206	0.017	0.096	0.000	0.119	0.000	0.114	0.656	0.197

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	B	F	B	F	C	F	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	37	66	0	170	0	58	21	22
N.S.	1	1.00	1.37	2.44	0.00	6.30	0.00	2.15	0.78	0.81
time (sec)	N/A	0.204	0.017	0.087	0.000	0.103	0.000	0.115	0.488	2.558

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	A	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	90	95	91	194	2074	100	142	586	91
N.S.	1	1.01	1.07	1.02	2.18	23.30	1.12	1.60	6.58	1.02
time (sec)	N/A	0.303	0.494	0.091	0.121	0.119	0.140	0.130	0.601	2.620

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	B	B	B	A	B	B
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	40	40	41	73	95	102	25	82	38
N.S.	1	1.05	1.05	1.08	1.92	2.50	2.68	0.66	2.16	1.00
time (sec)	N/A	0.556	0.052	0.086	0.117	0.090	0.245	0.115	0.588	0.116

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	132	166	620	0	11528	0	0	173	0
N.S.	1	0.97	1.22	4.56	0.00	84.76	0.00	0.00	1.27	0.00
time (sec)	N/A	0.678	2.923	0.701	0.000	0.413	0.000	0.000	0.770	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	90	86	116	0	5136	0	0	14	0
N.S.	1	1.01	0.97	1.30	0.00	57.71	0.00	0.00	0.16	0.00
time (sec)	N/A	0.579	0.050	0.431	0.000	0.319	0.000	0.000	0.640	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	A	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	43	40	37	0	1286	0	0	24	0
N.S.	1	1.08	1.00	0.92	0.00	32.15	0.00	0.00	0.60	0.00
time (sec)	N/A	0.469	0.015	0.405	0.000	0.400	0.000	0.000	0.547	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	77	73	431	0	3914	0	0	36	0
N.S.	1	1.04	0.99	5.82	0.00	52.89	0.00	0.00	0.49	0.00
time (sec)	N/A	0.374	0.361	0.445	0.000	0.368	0.000	0.000	0.691	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Reduce	Mupad
grade	N/A	A	A	C	F	B	F	F	F	F(-1)
verified	N/A	Yes	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	135	113	637	0	16463	0	0	48	0
N.S.	1	1.14	0.96	5.40	0.00	139.52	0.00	0.00	0.41	0.00
time (sec)	N/A	0.406	0.555	0.429	0.000	1.983	0.000	0.000	0.761	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [259] had the largest ratio of [.9333329999999999968]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	7	1.32	21	0.333
2	A	7	6	0.89	21	0.286
3	A	7	6	1.36	21	0.286
4	A	6	5	1.04	19	0.263
5	A	7	6	0.92	19	0.316
6	A	6	5	0.92	21	0.238
7	A	7	6	1.16	21	0.286
8	A	5	4	0.89	21	0.190
9	A	8	7	1.21	23	0.304
10	A	7	6	0.90	23	0.261
11	A	8	7	1.15	23	0.304
12	A	6	5	0.94	21	0.238
13	A	7	6	0.90	21	0.286
14	A	6	5	0.89	23	0.217
15	A	8	7	1.07	23	0.304
16	A	5	4	0.89	23	0.174
17	A	9	8	1.46	23	0.348
18	A	7	6	0.90	23	0.261
19	A	11	10	1.55	23	0.435
20	A	6	5	0.90	21	0.238
21	A	7	6	0.90	21	0.286

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
22	A	6	5	0.88	23	0.217
23	A	11	10	1.59	23	0.435
24	A	5	4	0.89	23	0.174
25	A	9	8	1.28	23	0.348
26	A	8	7	1.01	23	0.304
27	A	8	7	1.24	23	0.304
28	A	6	5	0.98	21	0.238
29	A	8	7	0.96	21	0.333
30	A	6	5	0.96	23	0.217
31	A	8	7	1.11	23	0.304
32	A	6	5	1.00	23	0.217
33	A	11	10	1.16	23	0.435
34	A	8	7	1.02	23	0.304
35	A	10	9	1.15	23	0.391
36	A	7	6	1.03	21	0.286
37	A	10	9	1.09	21	0.429
38	A	7	6	1.00	23	0.261
39	A	10	9	1.06	23	0.391
40	A	7	6	1.02	23	0.261
41	A	13	12	1.14	23	0.522
42	A	9	8	1.01	23	0.348
43	A	12	11	1.19	23	0.478
44	A	8	7	1.07	21	0.333
45	A	12	11	1.13	21	0.524
46	A	8	7	1.04	23	0.304
47	A	12	11	1.13	23	0.478
48	A	8	7	1.02	23	0.304
49	A	9	8	1.21	21	0.381
50	C	4	4	1.24	21	0.190
51	A	8	7	1.25	21	0.333
52	C	4	4	1.31	19	0.211
53	C	4	4	1.20	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
54	A	6	5	0.93	21	0.238
55	C	4	4	1.17	21	0.190
56	A	5	4	0.86	21	0.190
57	A	9	8	1.08	23	0.348
58	C	4	4	1.17	23	0.174
59	A	8	7	1.11	23	0.304
60	C	4	4	1.21	21	0.190
61	C	4	4	1.17	21	0.190
62	A	6	5	0.89	23	0.217
63	C	4	4	1.17	23	0.174
64	A	5	4	0.86	23	0.174
65	A	9	8	0.99	23	0.348
66	C	4	4	1.19	23	0.174
67	A	8	7	0.99	23	0.304
68	C	4	4	1.16	21	0.190
69	C	4	4	1.12	21	0.190
70	A	6	5	0.89	23	0.217
71	C	4	4	1.11	23	0.174
72	A	5	4	0.86	23	0.174
73	A	5	4	0.95	23	0.174
74	N/A	3	0	1.00	23	0.000
75	A	6	5	0.95	23	0.217
76	N/A	3	0	1.00	21	0.000
77	N/A	3	0	1.00	21	0.000
78	A	13	12	1.01	23	0.522
79	N/A	3	0	1.00	23	0.000
80	A	5	4	0.93	23	0.174
81	A	6	5	1.25	21	0.238
82	A	4	3	0.93	21	0.143
83	A	5	4	1.45	21	0.190
84	A	5	4	0.93	19	0.211
85	A	5	4	1.10	19	0.211

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
86	A	4	3	0.93	21	0.143
87	A	6	5	1.08	21	0.238
88	A	5	4	0.90	21	0.190
89	A	7	6	1.42	23	0.261
90	A	5	4	0.91	23	0.174
91	A	5	4	1.24	23	0.174
92	A	5	4	1.02	21	0.190
93	A	6	5	1.25	21	0.238
94	A	5	4	0.90	23	0.174
95	A	7	6	1.13	23	0.261
96	A	5	4	0.89	23	0.174
97	A	5	4	1.16	23	0.174
98	A	5	4	0.98	23	0.174
99	A	5	4	1.12	23	0.174
100	A	5	4	1.02	21	0.190
101	A	8	7	1.32	21	0.333
102	A	5	4	0.88	23	0.174
103	A	9	8	1.15	23	0.348
104	A	5	4	0.89	23	0.174
105	A	8	7	1.28	23	0.304
106	A	5	4	0.94	23	0.174
107	A	7	6	1.26	23	0.261
108	A	5	4	0.96	21	0.190
109	A	4	3	1.00	21	0.143
110	A	4	3	1.00	23	0.130
111	A	6	5	0.96	23	0.217
112	A	5	4	0.96	23	0.174
113	A	7	6	1.09	23	0.261
114	A	5	4	0.93	23	0.174
115	A	5	4	0.94	23	0.174
116	A	9	8	1.16	23	0.348
117	A	5	4	0.95	21	0.190

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
118	A	5	4	0.98	21	0.190
119	A	5	4	0.97	23	0.174
120	A	5	4	0.97	23	0.174
121	A	5	4	0.97	23	0.174
122	A	7	6	1.05	23	0.261
123	A	5	4	0.95	23	0.174
124	A	9	8	1.05	23	0.348
125	A	11	10	1.20	23	0.435
126	A	5	4	0.95	21	0.190
127	A	6	5	1.10	21	0.238
128	A	6	5	1.03	23	0.217
129	A	6	5	0.94	23	0.217
130	A	6	5	0.98	23	0.217
131	A	6	5	1.03	23	0.217
132	A	7	6	1.08	23	0.261
133	A	8	7	1.09	23	0.304
134	A	9	9	0.91	21	0.429
135	C	11	11	1.20	21	0.524
136	A	8	8	1.00	21	0.381
137	C	7	7	1.29	19	0.368
138	A	1	1	1.00	12	0.083
139	C	7	7	1.56	19	0.368
140	A	4	4	1.00	21	0.190
141	C	8	8	1.42	21	0.381
142	A	7	7	1.00	21	0.333
143	C	12	12	1.29	21	0.571
144	A	5	4	0.98	23	0.174
145	A	8	7	1.00	23	0.304
146	A	7	6	1.03	23	0.261
147	A	8	7	1.05	21	0.333
148	A	5	4	1.09	14	0.286
149	A	8	7	1.10	21	0.333

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
150	A	7	6	1.11	23	0.261
151	A	8	7	1.06	23	0.304
152	A	5	4	1.09	23	0.174
153	A	8	7	1.00	23	0.304
154	A	7	6	1.03	23	0.261
155	A	8	7	0.97	23	0.304
156	A	5	4	0.96	23	0.174
157	A	8	7	0.97	23	0.304
158	A	7	6	0.99	23	0.261
159	A	8	7	1.00	21	0.333
160	A	5	4	1.01	14	0.286
161	A	8	7	1.03	21	0.333
162	A	7	6	1.02	23	0.261
163	A	8	7	0.97	23	0.304
164	A	5	4	1.02	23	0.174
165	A	8	7	1.00	23	0.304
166	A	7	6	1.03	23	0.261
167	A	8	7	0.97	23	0.304
168	A	5	4	0.98	14	0.286
169	A	5	4	0.97	14	0.286
170	A	8	7	1.00	23	0.304
171	A	7	6	1.17	23	0.261
172	A	8	7	1.13	23	0.304
173	A	8	7	1.17	23	0.304
174	A	8	7	1.12	21	0.333
175	A	6	5	1.00	14	0.357
176	A	8	7	1.08	21	0.333
177	A	9	8	1.18	23	0.348
178	A	8	7	0.96	23	0.304
179	A	10	9	1.17	23	0.391
180	A	8	7	1.01	23	0.304
181	A	7	6	1.18	23	0.261

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
182	A	8	7	0.99	23	0.304
183	A	9	8	1.16	23	0.348
184	A	8	7	1.00	21	0.333
185	A	7	6	1.19	14	0.429
186	A	8	7	0.99	21	0.333
187	A	12	11	1.13	23	0.478
188	A	8	7	0.94	23	0.304
189	A	12	11	1.06	23	0.478
190	A	10	9	1.20	23	0.391
191	A	8	7	0.96	23	0.304
192	A	10	9	1.15	23	0.391
193	A	8	7	0.97	23	0.304
194	A	11	10	1.18	23	0.435
195	A	8	7	0.97	21	0.333
196	A	8	7	1.20	14	0.500
197	A	8	7	0.97	21	0.333
198	A	14	13	1.14	23	0.565
199	A	8	7	0.94	23	0.304
200	A	14	13	1.09	23	0.565
201	A	11	10	1.19	14	0.714
202	A	6	5	1.00	12	0.417
203	A	7	6	1.00	10	0.600
204	A	7	6	1.18	12	0.500
205	A	8	7	1.00	10	0.700
206	A	6	5	1.36	12	0.417
207	A	6	5	1.00	10	0.500
208	A	8	7	1.06	17	0.412
209	A	10	9	1.10	17	0.529
210	A	10	9	1.08	17	0.529
211	A	11	10	1.05	17	0.588
212	A	9	8	1.11	15	0.533
213	A	8	7	1.00	12	0.583

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
214	A	9	8	1.09	15	0.533
215	A	9	8	1.00	17	0.471
216	A	11	10	1.05	17	0.588
217	A	8	7	1.05	17	0.412
218	A	13	12	1.11	17	0.706
219	A	11	10	1.09	17	0.588
220	A	14	13	1.07	17	0.765
221	A	10	9	1.11	15	0.600
222	A	10	9	1.05	12	0.750
223	A	11	10	1.07	15	0.667
224	A	11	10	1.00	17	0.588
225	A	7	6	1.00	10	0.600
226	A	8	7	1.00	12	0.583
227	A	9	8	1.06	10	0.800
228	A	10	9	1.04	12	0.750
229	A	8	7	1.09	17	0.412
230	A	9	8	1.08	17	0.471
231	A	9	8	1.11	17	0.471
232	A	10	9	1.00	17	0.529
233	A	8	7	1.00	15	0.467
234	A	5	4	1.00	12	0.333
235	A	9	8	1.09	15	0.533
236	A	9	8	1.00	17	0.471
237	A	11	10	1.09	17	0.588
238	A	8	7	1.07	17	0.412
239	A	9	8	1.18	17	0.471
240	A	9	8	1.12	17	0.471
241	A	8	7	1.00	17	0.412
242	A	9	8	1.10	15	0.533
243	A	6	5	1.00	12	0.417
244	A	11	10	1.26	15	0.667
245	A	12	11	1.07	17	0.647

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
246	A	13	12	1.25	17	0.706
247	A	8	7	1.07	17	0.412
248	A	8	7	1.10	17	0.412
249	A	10	9	1.16	17	0.529
250	A	10	9	1.10	17	0.529
251	A	10	9	1.17	15	0.600
252	A	8	7	1.14	12	0.583
253	A	13	12	1.31	15	0.800
254	A	13	12	1.13	17	0.706
255	A	5	4	1.00	10	0.400
256	A	5	4	1.00	12	0.333
257	A	5	4	1.01	14	0.286
258	A	5	4	1.05	8	0.500
259	A	15	14	0.97	15	0.933
260	A	13	12	1.01	15	0.800
261	A	8	7	1.08	15	0.467
262	A	11	10	1.04	15	0.667
263	A	12	11	1.14	15	0.733

CHAPTER 3

LISTING OF INTEGRALS

3.1 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 73

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{3}{8}(a+5b)x-\frac{(5a+9b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b\tanh(c+dx)}{d}$$

output
```
3/8*(a+5*b)*x-1/8*(5*a+9*b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c
)^3*sinh(d*x+c)/d-b*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.77

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{12(a+5b)(c+dx)-8(a+2b)\sinh(2(c+dx))+(a+b)\sinh(4(c+dx))-32b\tanh(c+dx)}{32d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(12*(a + 5*b)*(c + d*x) - 8*(a + 2*b)*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(
c + d*x)] - 32*b*Tanh[c + d*x])/(32*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4146, 360, 25, 1471, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{1}{4}\int -\frac{4b\tanh^4(c+dx)+4(a+b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{4b\tanh^4(c+dx)+4(a+b)\tanh^2(c+dx)+a+b}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{1471}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b\tanh^2(c+dx)+3a+7b}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{(5a+9b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-8b\tanh(c+dx)\right)-\frac{(5a+9b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{(a+b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(3(a+5b)\text{arctanh}(\tanh(c+dx))-8b\tanh(c+dx))-\frac{(5a+9b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{(a+b)\tanh(c+dx)}{4(1-\tanh^2(c+dx))^2}}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((3*(a + 5*b)*ArcTa
nh[Tanh[c + d*x]] - 8*b*Tanh[c + d*x])/2 - ((5*a + 9*b)*Tanh[c + d*x])/(2*
(1 - Tanh[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1471
```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, Simp[(-R)*x*((d + e*x^2)^(q + 1)/(2*d*(q + 1))), x] + Simp[1/(2*d*(q
 + 1))   Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3),
x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^
2 - b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 3.50 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.32

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+c)}{8}\right)}{d}$
risch	$\frac{3ax}{8}+\frac{15bx}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}a}{8d}-\frac{e^{2dx+2c}b}{4d}+\frac{e^{-2dx-2c}a}{8d}+\frac{e^{-2dx-2c}b}{4d}-\frac{e^{-4dx-4c}a}{64d}-$

input
```
int(sinh(d*x+c)^4*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*
c-15/8*tanh(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.64

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\sinh(dx+c)^5+\left(10\,(a+b)\cosh(dx+c)^2-7\,a-15\,b\right)\sinh(dx+c)^3+8\,(3\,(a+5\,b)dx+8\,b)\,c}{64\,d\cosh(dx}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/64*((a + b)*sinh(d*x + c)^5 + (10*(a + b)*cosh(d*x + c)^2 - 7*a - 15*b)*
sinh(d*x + c)^3 + 8*(3*(a + 5*b)*d*x + 8*b)*cosh(d*x + c) + (5*(a + b)*cos
h(d*x + c)^4 - 3*(7*a + 15*b)*cosh(d*x + c)^2 - 8*a - 80*b)*sinh(d*x + c))
/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\sinh^4(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 154 vs. $2(67) = 134$.

Time = 0.05 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.11

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{64}b\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d(e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c)
 - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/
(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(67) = 134$.

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.95

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{24(dx+c)(a+5b)+ae^{(4dx+4c)}+be^{(4dx+4c)}-8ae^{(2dx+2c)}-16be^{(2dx+2c)}-\left(18ae^{(4dx+4c)}+90be^{(4dx-}\right.}{64d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(24*(d*x + c)*(a + 5*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a
*e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) + 90*b*e^(
4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c) - 16*b*e^(2*d*x + 2*c) + a + b)*e^(-4*d
*x - 4*c) + 128*b/(e^(2*d*x + 2*c) + 1))/d
```

Mupad [B] (verification not implemented)

Time = 2.35 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.38

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{3\,a}{8}+\frac{15\,b}{8}\right)+\frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)}{64\,d} + \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a+2\,b)}{8\,d}-\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a+2\,b)}{8\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*((3*a)/8 + (15*b)/8) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 4*c - 4
*d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x)*(a + b))/(64*d) + (exp(- 2*c - 2
*d*x)*(a + 2*b))/(8*d) - (exp(2*c + 2*d*x)*(a + 2*b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 200, normalized size of antiderivative = 2.74

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{10dx+10c}a+e^{10dx+10c}b-7e^{8dx+8c}a-15e^{8dx+8c}b+24e^{6dx+6c}adx-16e^{6dx+6c}a+120e^{6dx+6c}bdx-160e^{6dx}}{64e^{4dx+4c}d\left(e^{2dx+2c}+1\right)}$$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(10*c + 10*d*x)*a + e**(10*c + 10*d*x)*b - 7*e**(8*c + 8*d*x)*a - 15*e
**(8*c + 8*d*x)*b + 24*e**(6*c + 6*d*x)*a*d*x - 16*e**(6*c + 6*d*x)*a + 12
0*e**(6*c + 6*d*x)*b*d*x - 160*e**(6*c + 6*d*x)*b + 24*e**(4*c + 4*d*x)*a*
d*x + 120*e**(4*c + 4*d*x)*b*d*x + 7*e**(2*c + 2*d*x)*a + 15*e**(2*c + 2*d
*x)*b - a - b)/(64*e**(4*c + 4*d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.2 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 47

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+2b)\cosh(c+dx)}{d}+\frac{(a+b)\cosh^3(c+dx)}{3d}-\frac{b\text{sech}(c+dx)}{d}$$

output
```
-(a+2*b)*cosh(d*x+c)/d+1/3*(a+b)*cosh(d*x+c)^3/d-b*sech(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.55

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{3a\cosh(c+dx)}{4d}-\frac{7b\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{b\cosh(3(c+dx))}{12d}-\frac{b\text{sech}(c+dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-3*a*Cosh[c + d*x])/(4*d) - (7*b*Cosh[c + d*x])/(4*d) + (a*Cosh[3*(c + d*
x)])/(12*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (b*Sech[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$-\frac{\int \left((a+b)\cosh^4(c+dx)+(-a-2b)\cosh^2(c+dx)+b\right)d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{3}(a+b)\cosh^3(c+dx)-(a+2b)\cosh(c+dx)-b\text{sech}(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-((a + 2*b)*Cosh[c + d*x]) + ((a + b)*Cosh[c + d*x]^3)/3 - b*Sech[c + d*x
])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.68 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.60

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)}{d}$
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}-\frac{3e^{dx+c}a}{8d}-\frac{7e^{dx+c}b}{8d}-\frac{3e^{-dx-c}a}{8d}-\frac{7e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}a}{24d}+\frac{e^{-3dx-3c}b}{24d}-\frac{}{d(}$

input
```
int(sinh(d*x+c)^3*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(1/3*sinh(d*x+c)^4/cosh(d*x+
c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 91 vs. $2(45) = 90$.

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.94

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(a+b)\cosh(dx+c)^4+(a+b)\sinh(dx+c)^4-4(2a+5b)\cosh(dx+c)^2+2\left(3(a+b)\cosh(dx+c)^2\right.}{24\,d\cosh(dx+c)}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/24*((a + b)*cosh(d*x + c)^4 + (a + b)*sinh(d*x + c)^4 - 4*(2*a + 5*b)*co
sh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 4*a - 10*b)*sinh(d*x + c)^2
 - 9*a - 45*b)/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2(c+dx)\right)\sinh^3(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(45) = 90$.

Time = 0.04 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=-\frac{1}{24}b\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)+\frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/24*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 6
9*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*
a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*
c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.23

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3 + b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3 - 12\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right) - 24\,b\left(e^{(dx+c)}+e^{(-dx-c)}\right) - \frac{}{e^{(dx}}}{24\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/24*(a*(e^(d*x + c) + e^(-d*x - c))^3 + b*(e^(d*x + c) + e^(-d*x - c))^3
- 12*a*(e^(d*x + c) + e^(-d*x - c)) - 24*b*(e^(d*x + c) + e^(-d*x - c)) -
48*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.11

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{-3\,c-3\,d\,x}\,(a+b)}{24\,d} + \frac{e^{3\,c+3\,d\,x}\,(a+b)}{24\,d} - \frac{e^{c+d\,x}\,(3\,a+7\,b)}{8\,d} - \frac{e^{-c-d\,x}\,(3\,a+7\,b)}{8\,d} - \frac{2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(- 3*c - 3*d*x)*(a + b))/(24*d) + (exp(3*c + 3*d*x)*(a + b))/(24*d) -
(exp(c + d*x)*(3*a + 7*b))/(8*d) - (exp(- c - d*x)*(3*a + 7*b))/(8*d) - (2
*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.89

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{e^{8dx+8c}a+e^{8dx+8c}b-8e^{6dx+6c}a-20e^{6dx+6c}b-18e^{4dx+4c}a-90e^{4dx+4c}b-8e^{2dx+2c}a-20e^{2dx+2c}b+a+b}{24e^{3dx+3c}d\left(e^{2dx+2c}+1\right)}$$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(8*c + 8*d*x)*a + e**(8*c + 8*d*x)*b - 8*e**(6*c + 6*d*x)*a - 20*e**(6
*c + 6*d*x)*b - 18*e**(4*c + 4*d*x)*a - 90*e**(4*c + 4*d*x)*b - 8*e**(2*c
+ 2*d*x)*a - 20*e**(2*c + 2*d*x)*b + a + b)/(24*e**(3*c + 3*d*x)*d*(e**(2*
c + 2*d*x) + 1))
```

3.3 $\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = -\frac{1}{2}(a+3b)x + \frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b\tanh(c+dx)}{d}$$

output
```
-1/2*(a+3*b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d+b*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.93

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{-2(a+3b)(c+dx)+(a+b)\sinh(2(c+dx))+4b\tanh(c+dx)}{4d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-2*(a + 3*b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)] + 4*b*Tanh[c + d*x])/(
4*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 25, 4146, 360, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 360}$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{2b\tanh^2(c+dx)+a+b}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 299}$$

$$\frac{\frac{1}{2}\left(2b\tanh(c+dx)-(a+3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}(2b\tanh(c+dx)-(a+3b)\text{arctanh}(\tanh(c+dx)))+\frac{(a+b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-((a + 3*b)*ArcTanh[Tanh[c + d*x]]) + 2*b*Tanh[c + d*x])/2 + ((a + b)*Ta
nh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.50

method	result	size
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$	66
risch	$-\frac{ax}{2}-\frac{3bx}{2}+\frac{e^{2dx+2c}a}{8d}+\frac{e^{2dx+2c}b}{8d}-\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-2dx-2c}b}{8d}-\frac{2b}{d\left(e^{2dx+2c}+1\right)}$	89

input
```
int(sinh(d*x+c)^2*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^3/co
sh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.61

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(a+b)\sinh\left(dx+c\right)^3-4\left((a+3\,b)dx+2\,b\right)\cosh\left(dx+c\right)+\left(3\,(a+b)\cosh\left(dx+c\right)^2+a+9\,b\right)\sinh\left(dx\right.}{8\,d\cosh\left(dx+c\right)}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*((a + b)*sinh(d*x + c)^3 - 4*((a + 3*b)*d*x + 2*b)*cosh(d*x + c) + (3*
(a + b)*cosh(d*x + c)^2 + a + 9*b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\sinh^2(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(40) = 80$.

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 2.30

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= -\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)\\&\quad-\frac{1}{8}\,b\left(\frac{12\,(dx+c)}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}-\frac{17\,e^{(-2\,dx-2\,c)}+1}{d\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}\right)}\right)\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(12*(d*x + c
)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c)
+ e^(-4*d*x - 4*c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 107 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.43

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{4\,(dx+c)(a+3\,b) - ae^{(2\,dx+2\,c)} - be^{(2\,dx+2\,c)} - \frac{ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-14\,be^{(2\,dx+2\,c)}-a-b}{e^{(4\,dx+4\,c)}+e^{(2\,dx+2\,c)}}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*(a + 3*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (a*e
^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 14*b*e^(2*d*x + 2*c) - a - b)/(e^(4
*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 2.30 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.45

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{2\,c+2\,d\,x}\,(a+b)}{8\,d} - \frac{2\,b}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{e^{-2\,c-2\,d\,x}\,(a+b)}{8\,d} - x\,\left(\frac{a}{2}+\frac{3\,b}{2}\right)$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (exp
(- 2*c - 2*d*x)*(a + b))/(8*d) - x*(a/2 + (3*b)/2)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 148, normalized size of antiderivative = 3.36

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{6dx+6c}a+e^{6dx+6c}b-4e^{4dx+4c}adx+2e^{4dx+4c}a-12e^{4dx+4c}bdx+18e^{4dx+4c}b-4e^{2dx+2c}adx-12e^{2dx+2c}bd}{8e^{2dx+2c}d\left(e^{2dx+2c}+1\right)}$$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(6*c + 6*d*x)*a + e**(6*c + 6*d*x)*b - 4*e**(4*c + 4*d*x)*a*d*x + 2*e*
*(4*c + 4*d*x)*a - 12*e**(4*c + 4*d*x)*b*d*x + 18*e**(4*c + 4*d*x)*b - 4*e
**(2*c + 2*d*x)*a*d*x - 12*e**(2*c + 2*d*x)*b*d*x - a - b)/(8*e**(2*c + 2*
d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.4 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
(a+b)*cosh(d*x+c)/d+b*sech(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.80

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\cosh(c)\cosh(dx)}{d}+\frac{b\cosh(c+dx)}{d}+\frac{b\operatorname{sech}(c+dx)}{d}+\frac{a\sinh(c)\sinh(dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Cosh[c]*Cosh[d*x])/d + (b*Cosh[c + d*x])/d + (b*Sech[c + d*x])/d + (a*S
inh[c]*Sinh[d*x])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.04, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.263$, Rules used = {3042, 26, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4147}$$

$$-\frac{\int \cosh^2(c+dx)\left(-b\text{sech}^2(c+dx)+a+b\right)d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left((a+b)\cosh^2(c+dx)-b\right)d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{-((a+b)\cosh(c+dx))-b\text{sech}(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((-((a + b)*Cosh[c + d*x]) - b*Sech[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 0.70 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.76

method	result	size
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
default	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)}{d}$	44
risch	$\frac{e^{dx+c}a}{2d}+\frac{e^{dx+c}b}{2d}+\frac{e^{-dx-c}a}{2d}+\frac{e^{-dx-c}b}{2d}+\frac{2b\,e^{dx+c}}{d\left(e^{2dx+2c}+1\right)}$	81

input
```
int(sinh(d*x+c)*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c)))
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.68

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\cosh\left(dx+c\right)^2+(a+b)\sinh\left(dx+c\right)^2+a+3\,b}{2\,d\cosh\left(dx+c\right)}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a + 3*b)/(d*cosh(
d*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\sinh\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 67 vs. $2(25) = 50$.

Time = 0.04 (sec) , antiderivative size = 67, normalized size of antiderivative = 2.68

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}b\left(\frac{e^{(-dx-c)}}{d} + \frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right) + \frac{a\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e^(-d*x - c) + e^(-3*
d*x - 3*c)))) + a*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 63 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)+b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(a*(e^(d*x + c) + e^(-d*x - c)) + b*(e^(d*x + c) + e^(-d*x - c)) + 4*b
/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.08

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{b}{d\cosh\left(c+d\,x\right)}+\frac{\cosh(c+d\,x)\,\left(a+b\right)}{d}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
b/(d*cosh(c + d*x)) + (cosh(c + d*x)*(a + b))/d
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 3.24

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{4dx+4c}a+e^{4dx+4c}b+2e^{2dx+2c}a+6e^{2dx+2c}b+a+b}{2e^{dx+c}d\left(e^{2dx+2c}+1\right)}$$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(4*c + 4*d*x)*a + e**(4*c + 4*d*x)*b + 2*e**(2*c + 2*d*x)*a + 6*e**(2*
c + 2*d*x)*b + a + b)/(2*e**(c + d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.5 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 26

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

output
```
-a*arctanh(cosh(d*x+c))/d-b*sech(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b\operatorname{sech}(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Sech[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.92, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.316, Rules used = {3042, 26, 4147, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{-b\text{sech}^2(c+dx)+a+b}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{-b\text{sech}^2(c+dx)+a+b}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{-a\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-b\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{-a\text{arctanh}(\text{sech}(c+dx))-b\text{sech}(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(a*ArcTanh[Sech[c + d*x]]) - b*Sech[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 0.71 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
default	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)-\frac{b}{\cosh(dx+c)}}{d}$	27
risch	$-\frac{2b\,\mathrm{e}^{dx+c}}{d\left(\mathrm{e}^{2dx+2c}+1\right)}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	56

input
```
int(csch(d*x+c)*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*arctanh(exp(d*x+c))-b/cosh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 167, normalized size of antiderivative = 6.42

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{2\,b\cosh\left(dx+c\right)+\left(a\cosh\left(dx+c\right)^2+2\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)+a\sinh\left(dx+c\right)^2+a\right)\log\left(\cosh\right.}{d\,\mathrm{co}}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(2*b*cosh(d*x + c) + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c)
 + a*sinh(d*x + c)^2 + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh
(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*log
(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*b*sinh(d*x + c))/(d*cosh(d*x + c)^
2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x + c)^2 + d)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \int \left(a+b\tanh^2{\left(c+dx \right)}\right) \operatorname{csch}{\left(c+dx \right)}\, dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.54

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{a \log \left(\tanh \left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d} - \frac{2\,b}{d\left(e^{(dx+c)}+e^{(-dx-c)}\right)}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*log(tanh(1/2*d*x + 1/2*c))/d - 2*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 68 vs. 2(26) = 52.

Time = 0.12 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.62

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = -\frac{a \log \left(e^{(dx+c)}+e^{(-dx-c)}+2\right) - a \log \left(e^{(dx+c)}+e^{(-dx-c)}-2\right) + \frac{4\,b}{e^{(dx+c)}+e^{(-dx-c)}}}{2\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(a*log(e^(d*x + c) + e^(-d*x - c) + 2) - a*log(e^(d*x + c) + e^(-d*x
- c) - 2) + 4*b/(e^(d*x + c) + e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 64, normalized size of antiderivative = 2.46

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{2\,\mathrm{atan}\left(\frac{a\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^2}}\right)\,\sqrt{a^2}}{\sqrt{-d^2}}-\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x),x)
```

output
```
- (2*atan((a*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^2)^(1/2)))*(a^2)^(1/2))/(
-d^2)^(1/2) - (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 99, normalized size of antiderivative = 3.81

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{e^{2dx+2c}\log\left(e^{dx+c}-1\right)a-e^{2dx+2c}\log\left(e^{dx+c}+1\right)a-2e^{dx+c}b+\log\left(e^{dx+c}-1\right)a-\log\left(e^{dx+c}+1\right)a}{d\left(e^{2dx+2c}+1\right)}$$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a - e**(2*c + 2*d*x)*log(e**(c + d
*x) + 1)*a - 2*e**(c + d*x)*b + log(e**(c + d*x) - 1)*a - log(e**(c + d*x)
 + 1)*a)/(d*(e**(2*c + 2*d*x) + 1))
```

3.6 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 24

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

output
```
-a*coth(d*x+c)/d+b*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 24, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}+\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((a*Coth[c + d*x])/d) + (b*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 25, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \ 4146$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 244$$

$$\frac{\int \left(a\coth^2(c+dx)+b\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{b\tanh(c+dx)-a\coth(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(a*Coth[c + d*x]) + b*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.16 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.04

method	result	size
derivativedivides	$\frac{b\tanh(dx+c)-\frac{a}{\tanh(dx+c)}}{d}$	25
default	$\frac{b\tanh(dx+c)-\frac{a}{\tanh(dx+c)}}{d}$	25
risch	$-\frac{2(e^{2dx+2c}a+e^{2dx+2c}b+a-b)}{d(e^{2dx+2c}-1)(e^{2dx+2c}+1)}$	59

input
```
int(csch(d*x+c)^2*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*tanh(d*x+c)-a/tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(24) = 48$.

Time = 0.08 (sec) , antiderivative size = 88, normalized size of antiderivative = 3.67

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{4\left(a\cosh\left(dx+c\right)+b\sinh\left(dx+c\right)\right)}{d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+d\sinh\left(dx+c\right)^3-d\cosh\left(dx+c\right)+\left(3\,d\cosh\left(dx\right.\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-4*(a*cosh(d*x + c) + b*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3 - d*cosh(d*x + c) + (3*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}^2\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.62

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2\,b}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
2*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.88

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{2\left(ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b\right)}{d(e^{(4\,dx+4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/(d*(e^(4*d*x + 4*c) - 1
))
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{\frac{2\,(a-b)}{d}+\frac{2\,e^{2\,c+2\,d\,x}\,(a+b)}{d}}{e^{4\,c+4\,d\,x}-1}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^2,x)
```

output
```
-((2*(a - b))/d + (2*exp(2*c + 2*d*x)*(a + b))/d)/(exp(4*c + 4*d*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 2.54

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{2e^{2dx+2c}\left(-e^{2dx+2c}a+e^{2dx+2c}b-a-b\right)}{d\left(e^{4dx+4c}-1\right)}$$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*e**(2*c + 2*d*x)*( - e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b - a - b))/
(d*(e**(4*c + 4*d*x) - 1))
```

3.7 $\int \text{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \text{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a-2b)\text{arctanh}(\cosh(c+dx))}{2d} - \frac{a\coth(c+dx)\text{csch}(c+dx)}{2d} + \frac{b\text{sech}(c+dx)}{d}$$

output
```
1/2*(a-2*b)*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)*csch(d*x+c)/d+b*sech(
d*x+c)/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 123 vs. 2(51) = 102.

Time = 0.18 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.41

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{b\text{sech}(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/8*(a*Csch[(c + d*x)/2]^2)/d + (a*Log[Cosh[(c + d*x)/2]])/(2*d) - (b*Log
[Cosh[(c + d*x)/2]])/d - (a*Log[Sinh[(c + d*x)/2]])/(2*d) + (b*Log[Sinh[(c
 + d*x)/2]])/d - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sech[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.16, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4147, 360, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)}{\sin(ic+idx)^3}dx$$

$\Big\downarrow$ 26

$$-i\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^3}dx$$

$\Big\downarrow$ 4147

$$-\frac{\int \frac{\text{sech}^2(c+dx)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}{\left(1-\text{sech}^2(c+dx)\right)^2}d\text{sech}(c+dx)}{d}$$

$\Big\downarrow$ 360

$$-\frac{\frac{a\,\text{sech}(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}-\frac{1}{2}\int \frac{a-2b\,\text{sech}^2(c+dx)}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{d}$$

$\Big\downarrow$ 299

$$-\frac{\frac{1}{2}\left(-(a-2b)\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-2b\,\text{sech}(c+dx)\right)+\frac{a\,\text{sech}(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 219

$$-\frac{\frac{1}{2}(-((a-2b)\text{arctanh}(\text{sech}(c+dx)))-2b\,\text{sech}(c+dx))+\frac{a\,\text{sech}(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((-((a - 2*b)*ArcTanh[Sech[c + d*x]]) - 2*b*Sech[c + d*x])/2 + (a*Sech[c
 + d*x])/(2*(1 - Sech[c + d*x]^2)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.94 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
risch	$-\frac{\mathrm{e}^{dx+c}\left(\mathrm{e}^{4dx+4c}a-2b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+4\,\mathrm{e}^{2dx+2c}b+a-2b\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}-\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{2d}+\frac{\ln\left(\mathrm{e}^{dx+c}-1\right)b}{d}+\frac{a\ln\left(\mathrm{e}^{dx+c}\right.}{2d}$

input
```
int(csch(d*x+c)^3*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/cosh(d*x+c)
-2*arctanh(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(47) = 94.

Time = 0.10 (sec) , antiderivative size = 924, normalized size of antiderivative = 18.12

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*(a - 2*b)*cosh(d*x + c)^5 + 10*(a - 2*b)*cosh(d*x + c)*sinh(d*x +
c)^4 + 2*(a - 2*b)*sinh(d*x + c)^5 + 4*(a + 2*b)*cosh(d*x + c)^3 + 4*(5*(a
 - 2*b)*cosh(d*x + c)^2 + a + 2*b)*sinh(d*x + c)^3 + 4*(5*(a - 2*b)*cosh(d
*x + c)^3 + 3*(a + 2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(a - 2*b)*cosh(
d*x + c) - ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(d*x + c)*sinh(d*x
 + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x + c)^4 + (15*(a -
 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(a - 2*b)*cosh(d*x
 + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a - 2*b)*cosh(d*x +
c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d*x + c)^2 - a + 2
*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*(a - 2*b)*cosh(d*
x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*b)*log(cosh(d*x
+ c) + sinh(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^6 + 6*(a - 2*b)*cosh(
d*x + c)*sinh(d*x + c)^5 + (a - 2*b)*sinh(d*x + c)^6 - (a - 2*b)*cosh(d*x
+ c)^4 + (15*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^4 + 4*(5*(
a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 - (a -
 2*b)*cosh(d*x + c)^2 + (15*(a - 2*b)*cosh(d*x + c)^4 - 6*(a - 2*b)*cosh(d
*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^5 - 2*
(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2
*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(5*(a - 2*b)*cosh(d*x + c)^
4 + 6*(a + 2*b)*cosh(d*x + c)^2 + a - 2*b)*sinh(d*x + c))/(d*cosh(d*x +...
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}^3\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.98

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right) - b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
 + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*
(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/(d*(e^
(-2*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 142 vs. $2(47) = 94$.

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 2.78

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)-\frac{4\left(a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-2\,b\left(e^{(dx}\right.\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-4\,e^{(dx}}}{4\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) - (a - 2*b)*log(e^(d*x
+ c) + e^(-d*x - c) - 2) - 4*(a*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b*(e^(d
*x + c) + e^(-d*x - c))^2 + 8*b)/((e^(d*x + c) + e^(-d*x - c))^3 - 4*e^(d*
x + c) - 4*e^(-d*x - c)))/d
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 156, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(a\,\sqrt{-d^2}-2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^2-4\,a\,b+4\,b^2}}\right)\,\sqrt{a^2-4\,a\,b+4\,b^2}}{\sqrt{-d^2}}-\frac{a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$
$$+\frac{2\,b\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,a\,\mathrm{e}^{c+d\,x}}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a*(-d^2)^(1/2) - 2*b*(-d^2)^(1/2)))/(d*(a^2 - 4*a*
b + 4*b^2)^(1/2)))*(a^2 - 4*a*b + 4*b^2)^(1/2))/(-d^2)^(1/2) - (a*exp(c +
d*x))/(d*(exp(2*c + 2*d*x) - 1)) + (2*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x)
 + 1)) - (2*a*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)
)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 440, normalized size of antiderivative = 8.63

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{-e^{6dx+6c}\log\left(e^{dx+c}-1\right)a+2e^{6dx+6c}\log\left(e^{dx+c}-1\right)b+e^{6dx+6c}\log\left(e^{dx+c}+1\right)a-2e^{6dx+6c}\log\left(e^{dx+c}+1\right)b}{}$$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

output

```
( - e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a + 2*e**(6*c + 6*d*x)*log(e**(
c + d*x) - 1)*b + e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a - 2*e**(6*c + 6
*d*x)*log(e**(c + d*x) + 1)*b - 2*e**(5*c + 5*d*x)*a + 4*e**(5*c + 5*d*x)*
b + e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a - 2*e**(4*c + 4*d*x)*log(e**(
c + d*x) - 1)*b - e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a + 2*e**(4*c + 4
*d*x)*log(e**(c + d*x) + 1)*b - 4*e**(3*c + 3*d*x)*a - 8*e**(3*c + 3*d*x)*
b + e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a - 2*e**(2*c + 2*d*x)*log(e**(
c + d*x) - 1)*b - e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a + 2*e**(2*c + 2
*d*x)*log(e**(c + d*x) + 1)*b - 2*e**(c + d*x)*a + 4*e**(c + d*x)*b - log(
e**(c + d*x) - 1)*a + 2*log(e**(c + d*x) - 1)*b + log(e**(c + d*x) + 1)*a
 - 2*log(e**(c + d*x) + 1)*b)/(2*d*(e**(6*c + 6*d*x) - e**(4*c + 4*d*x) - e
**(2*c + 2*d*x) + 1))
```

3.8 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a-b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

output
```
(a-b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d-b*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.39

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{2a\coth(c+dx)}{3d}-\frac{b\coth(c+dx)}{d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c
+ d*x]^2)/(3*d) - (b*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right) dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \ 4146$$

$$\frac{\int \coth^4(c+dx) \left(1-\tanh^2(c+dx)\right) \left(b\tanh^2(c+dx)+a\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 355$$

$$\frac{\int \left(a\coth^4(c+dx)+(b-a)\coth^2(c+dx)-b\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{(a-b)\coth(c+dx)-\frac{1}{3}a\coth^3(c+dx)-b\tanh(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
((a - b)*Coth[c + d*x] - (a*Coth[c + d*x]^3)/3 - b*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 6.09 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.25

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	55
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)}{d}$	55
risch	$-\frac{4\left(3\,\mathrm{e}^{4dx+4c}a+3b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a-6\,\mathrm{e}^{2dx+2c}b-a+3b\right)}{3d\left(\mathrm{e}^{2dx+2c}-1\right)^3\left(\mathrm{e}^{2dx+2c}+1\right)}$	87

input
```
int(csch(d*x+c)^4*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(-1/sinh(d*x+c)/cosh(d*x+c)-2
*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(42) = 84$.

Time = 0.09 (sec) , antiderivative size = 244, normalized size of antiderivative = 5.55

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{}{3\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6-2\,d\cosh\left(dx+c\right)^4+\left(15\,d\,\mathrm{co}\right.\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-8/3*((a + 3*b)*cosh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c) + (a + 3
*b)*sinh(d*x + c)^2 + a - 3*b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh
(d*x + c)^5 + d*sinh(d*x + c)^6 - 2*d*cosh(d*x + c)^4 + (15*d*cosh(d*x + c
)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*s
inh(d*x + c)^3 - d*cosh(d*x + c)^2 + (15*d*cosh(d*x + c)^4 - 12*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^5 - 4*d*cosh(d*x + c)^
3 + d*cosh(d*x + c))*sinh(d*x + c) + 2*d)
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{csch}^4\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(42) = 84$.

Time = 0.04 (sec) , antiderivative size = 113, normalized size of antiderivative = 2.57

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)+\frac{4\,b}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1))) + 4*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{2\left(\frac{3\,b}{e^{(2\,dx+2\,c)}+1}-\frac{3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b}{\left(e^{(2\,dx+2\,c)}-1\right)^3}\right)}{3\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
2/3*(3*b/(e^(2*d*x + 2*c) + 1) - (3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c
) - 6*b*e^(2*d*x + 2*c) - 2*a + 3*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.49 (sec) , antiderivative size = 173, normalized size of antiderivative = 3.93

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{\frac{2\,\left(2\,a-b\right)}{3\,d}+\frac{2\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{3\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}-\frac{2\,b}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{\frac{2\,b}{3\,d}+\frac{2\,b\,\mathrm{e}^{4\,c+4\,d\,x}}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,a-b\right)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}$$

input
```
int((a + b*tanh(c + d*x)^2)/sinh(c + d*x)^4,x)
```

output
```
(2*b)/(d*(exp(2*c + 2*d*x) + 1)) - ((2*(2*a - b))/(3*d) + (2*b*exp(2*c + 2
*d*x))/(3*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - (2*b)/(3*d*(ex
p(2*c + 2*d*x) - 1)) - ((2*b)/(3*d) + (2*b*exp(4*c + 4*d*x))/(3*d) + (4*ex
p(2*c + 2*d*x)*(2*a - b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.27

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-4e^{4dx+4c}a-4e^{4dx+4c}b-\frac{8e^{2dx+2c}a}{3}+8e^{2dx+2c}b+\frac{4a}{3}-4b}{d\left(e^{8dx+8c}-2e^{6dx+6c}+2e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

output
```
(4*( - 3*e**(4*c + 4*d*x)*a - 3*e**(4*c + 4*d*x)*b - 2*e**(2*c + 2*d*x)*a
+ 6*e**(2*c + 2*d*x)*b + a - 3*b))/(3*d*(e**(8*c + 8*d*x) - 2*e**(6*c + 6*
d*x) + 2*e**(2*c + 2*d*x) - 1))
```

3.9 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{8}\left(3a^2+30ab+35b^2\right)x-\frac{(a+b)(5a+13b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^2\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b(2a+3b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
1/8*(3*a^2+30*a*b+35*b^2)*x-1/8*(a+b)*(5*a+13*b)*cosh(d*x+c)*sinh(d*x+c)/d
+1/4*(a+b)^2*cosh(d*x+c)^3*sinh(d*x+c)/d-b*(2*a+3*b)*tanh(d*x+c)/d-1/3*b^2
*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.44 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.83

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{12\left(3a^2+30ab+35b^2\right)(c+dx)-24\left(a^2+4ab+3b^2\right)\sinh(2(c+dx))+3(a+b)^2\sinh(4(c+dx))+32b(}{96d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(12*(3*a^2 + 30*a*b + 35*b^2)*(c + d*x) - 24*(a^2 + 4*a*b + 3*b^2)*Sinh[2*
(c + d*x)] + 3*(a + b)^2*Sinh[4*(c + d*x)] + 32*b*(-6*a - 10*b + b*Sech[c
+ d*x]^2)*Tanh[c + d*x])/(96*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.21, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4146, 366, 360, 25, 1467, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{(a+b)^2\tanh^5(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{\tanh^4(c+dx)\left(a^2+10ba+5b^2+4b^2\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{360}$$

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{8b^2\tanh^4(c+dx)+2(a+b)(a+9b)\tanh^2(c+dx)+(a+b)(a+9b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{(a+b)(a+9b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)^2\tanh^5(\cdots}{4(1-\tanh^2(c+\cdots}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int \frac{8b^2\tanh^4(c+dx)+2(a+b)(a+9b)\tanh^2(c+dx)+(a+b)(a+9b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{(a+b)(a+9b)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)^2\tanh^5(c+dx\cdots}{4\left(1-\tanh^2(c+dx)\right)\cdots}}{d}$$

$$\Big\downarrow \text{1467}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int\left(-8b^2\tanh^2(c+dx)-2(a^2+10ba+13b^2)+\frac{3a^2+30ba+35b^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)-\frac{(a+b)(a+9b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{(}{4}}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left((3a^2+30ab+35b^2)\operatorname{arctanh}(\tanh(c+dx))-2(a^2+10ab+13b^2)\tanh(c+dx)-\frac{8}{3}b^2\tanh^3(c+dx)\right)-\frac{(a}{}\right.}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a + b)^2*Tanh[c + d*x]^5)/(4*(1 - Tanh[c + d*x]^2)^2) + (-1/2*((a + b)*
(a + 9*b)*Tanh[c + d*x])/(1 - Tanh[c + d*x]^2) + ((3*a^2 + 30*a*b + 35*b^2
)*ArcTanh[Tanh[c + d*x]] - 2*(a^2 + 10*a*b + 13*b^2)*Tanh[c + d*x] - (8*b^
2*Tanh[c + d*x]^3)/3)/2)/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 360
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[(a + b*x^2)^(p + 1)*Expan
dToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2
- 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /;
FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &
& (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 1467
```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 6.95 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.47

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+}{8}\right.}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx+}{8}\right.}{d}$
risch	$\frac{3a^2x}{8}+\frac{15abx}{4}+\frac{35b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}ab}{2d}-\frac{3\,e^{2dx+2c}b^2}{8d}$

input
```
int(sinh(d*x+c)^4*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x
+15/8*c-15/8*tanh(d*x+c))+b^2*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*sinh(d*
x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x+c)^3)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 394 vs. 2(105) = 210.

Time = 0.11 (sec) , antiderivative size = 394, normalized size of antiderivative = 3.49

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^7+3\left(21\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^2-5\,a^2-26\,ab-21\,b^2\right)\sinh\left(dx+\right.}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/192*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^7 + 3*(21*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 - 5*a^2 - 26*a*b - 21*b^2)*sinh(d*x + c)^5 + 8*(3*(3*a^2 +
30*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)^3 + 24*(3*(3*a^2 + 3
0*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (10
5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 - 30*(5*a^2 + 26*a*b + 21*b^2)*cosh(
d*x + c)^2 - 63*a^2 - 654*a*b - 847*b^2)*sinh(d*x + c)^3 + 24*(3*(3*a^2 +
30*a*b + 35*b^2)*d*x + 48*a*b + 80*b^2)*cosh(d*x + c) + 3*(7*(a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^6 - 5*(5*a^2 + 26*a*b + 21*b^2)*cosh(d*x + c)^4 - (63
*a^2 + 654*a*b + 847*b^2)*cosh(d*x + c)^2 - 15*a^2 - 190*a*b - 175*b^2)*si
nh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*
cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)^2\sinh^4{(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. 2(105) = 210.

Time = 0.05 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.61

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)$$
$$+\frac{1}{192}b^2\left(\frac{840(dx+c)}{d}+\frac{3\left(24e^{(-2dx-2c)}-e^{(-4dx-4c)}\right)}{d}-\frac{63e^{(-2dx-2c)}+1487e^{(-4dx-4c)}+2517e^{(-6dx}}{d(e^{(-4dx-4c)}+3e^{(-6dx-6c)}+3e^{(-8dx}}\right.$$
$$+\frac{1}{32}ab\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d(e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right)$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/192*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x
 - 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*
c) + 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*
c) + 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 1/3
2*a*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*
e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(105) = 210$.

Time = 0.20 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.59

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3\,a^2e^{(4\,dx+4\,c)} + 6\,abe^{(4\,dx+4\,c)} + 3\,b^2e^{(4\,dx+4\,c)} - 24\,a^2e^{(2\,dx+2\,c)} - 96\,abe^{(2\,dx+2\,c)} - 72\,b^2e^{(2\,dx+2\,c)} + 24\,(3\,a^2}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/192*(3*a^2*e^(4*d*x + 4*c) + 6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*x + 4*
c) - 24*a^2*e^(2*d*x + 2*c) - 96*a*b*e^(2*d*x + 2*c) - 72*b^2*e^(2*d*x + 2
*c) + 24*(3*a^2 + 30*a*b + 35*b^2)*(d*x + c) - 3*(18*a^2*e^(4*d*x + 4*c) +
 180*a*b*e^(4*d*x + 4*c) + 210*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c)
 - 32*a*b*e^(2*d*x + 2*c) - 24*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^
(-4*d*x - 4*c) + 256*(3*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) + 6*a*
b*e^(2*d*x + 2*c) + 9*b^2*e^(2*d*x + 2*c) + 3*a*b + 5*b^2)/(e^(2*d*x + 2*c
) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 293, normalized size of antiderivative = 2.59

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\frac{4\,(b^2+a\,b)}{3\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(2\,b^2+a\,b)}{3\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}+x\left(\frac{3\,a^2}{8}+\frac{15\,a\,b}{4}+\frac{35\,b^2}{8}\right)$$
$$+\frac{\frac{4\,(2\,b^2+a\,b)}{3\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(b^2+a\,b)}{3\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(2\,b^2+a\,b)}{3\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}$$
$$+\frac{4\,(2\,b^2+a\,b)}{3\,d\,(e^{2\,c+2\,d\,x}+1)}+\frac{e^{-2\,c-2\,d\,x}\,(a^2+4\,a\,b+3\,b^2)}{8\,d}$$
$$-\frac{e^{2\,c+2\,d\,x}\,(a^2+4\,a\,b+3\,b^2)}{8\,d}-\frac{e^{-4\,c-4\,d\,x}\,(a+b)^2}{64\,d}+\frac{e^{4\,c+4\,d\,x}\,(a+b)^2}{64\,d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((4*(a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(a*b + 2*b^2))/(3*d))/(2*exp(
2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + x*((15*a*b)/4 + (3*a^2)/8 + (35*b^2
)/8) + ((4*(a*b + 2*b^2))/(3*d) + (8*exp(2*c + 2*d*x)*(a*b + b^2))/(3*d) +
 (4*exp(4*c + 4*d*x)*(a*b + 2*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c
 + 4*d*x) + exp(6*c + 6*d*x) + 1) + (4*(a*b + 2*b^2))/(3*d*(exp(2*c + 2*d*
x) + 1)) + (exp(- 2*c - 2*d*x)*(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(2*c + 2
*d*x)*(4*a*b + a^2 + 3*b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)/(64*d)
 + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.73

$$\int \sinh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$$
$$= \frac{3e^{14dx+14c}a^2 + 3e^{14dx+14c}b^2 - 15e^{12dx+12c}a^2 - 63e^{12dx+12c}b^2 - 48e^{10dx+10c}a^2 - 672e^{10dx+10c}b^2 + 72e^{10dx+1}}{}$$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(3*e**(14*c + 14*d*x)*a**2 + 6*e**(14*c + 14*d*x)*a*b + 3*e**(14*c + 14*d*
x)*b**2 - 15*e**(12*c + 12*d*x)*a**2 - 78*e**(12*c + 12*d*x)*a*b - 63*e**(
12*c + 12*d*x)*b**2 + 72*e**(10*c + 10*d*x)*a**2*d*x - 48*e**(10*c + 10*d*
x)*a**2 + 720*e**(10*c + 10*d*x)*a*b*d*x - 464*e**(10*c + 10*d*x)*a*b + 84
0*e**(10*c + 10*d*x)*b**2*d*x - 672*e**(10*c + 10*d*x)*b**2 + 216*e**(8*c
+ 8*d*x)*a**2*d*x + 2160*e**(8*c + 8*d*x)*a*b*d*x + 2520*e**(8*c + 8*d*x)*
b**2*d*x + 216*e**(6*c + 6*d*x)*a**2*d*x + 90*e**(6*c + 6*d*x)*a**2 + 2160
*e**(6*c + 6*d*x)*a*b*d*x + 1140*e**(6*c + 6*d*x)*a*b + 2520*e**(6*c + 6*d
*x)*b**2*d*x + 1050*e**(6*c + 6*d*x)*b**2 + 72*e**(4*c + 4*d*x)*a**2*d*x +
 78*e**(4*c + 4*d*x)*a**2 + 720*e**(4*c + 4*d*x)*a*b*d*x + 844*e**(4*c + 4
*d*x)*a*b + 840*e**(4*c + 4*d*x)*b**2*d*x + 1022*e**(4*c + 4*d*x)*b**2 + 1
5*e**(2*c + 2*d*x)*a**2 + 78*e**(2*c + 2*d*x)*a*b + 63*e**(2*c + 2*d*x)*b*
*2 - 3*a**2 - 6*a*b - 3*b**2)/(192*e**(4*c + 4*d*x)*d*(e**(6*c + 6*d*x) +
3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.10 $\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{(a+b)(a+3b)\cosh(c+dx)}{d}+\frac{(a+b)^2\cosh^3(c+dx)}{3d}-\frac{b(2a+3b)\text{sech}(c+dx)}{d}+\frac{b^2\text{sech}^3(c+dx)}{3d}$$

output
```
-(a+b)*(a+3*b)*cosh(d*x+c)/d+1/3*(a+b)^2*cosh(d*x+c)^3/d-b*(2*a+3*b)*sech(
d*x+c)/d+1/3*b^2*sech(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.19 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.92

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{-3(3a^2+14ab+11b^2)\cosh(c+dx)+(a+b)^2\cosh(3(c+dx))+4b\text{sech}(c+dx)\left(-6a-9b+b\text{sech}^2(c\right.}{12d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-3*(3*a^2 + 14*a*b + 11*b^2)*Cosh[c + d*x] + (a + b)^2*Cosh[3*(c + d*x)]
+ 4*b*Sech[c + d*x]*(-6*a - 9*b + b*Sech[c + d*x]^2))/(12*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \; 3042$$

$$\int i\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \; 26$$

$$i\int \sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int -\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^2 d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^2 d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 355$$

$$-\frac{\int \left((a+b)^2\cosh^4(c+dx)+(-a-3b)(a+b)\cosh^2(c+dx)-b^2\text{sech}^2(c+dx)+b(2a+3b)\right) d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{1}{3}(a+b)^2\cosh^3(c+dx)-(a+b)(a+3b)\cosh(c+dx)-b(2a+3b)\text{sech}(c+dx)+\frac{1}{3}b^2\text{sech}^3(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)*(a + 3*b)*Cosh[c + d*x]) + ((a + b)^2*Cosh[c + d*x]^3)/3 - b*(2
*a + 3*b)*Sech[c + d*x] + (b^2*Sech[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. 2(73) = 146.

Time = 4.14 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx+c}{\cosh(dx+c)}\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(dx+c}{\cosh(dx+c)}\right.}{d}$
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}-\frac{3\,e^{dx+c}a^2}{8d}-\frac{7\,e^{dx+c}ab}{4d}-\frac{11\,e^{dx+c}b^2}{8d}-\frac{3\,e^{-dx-c}a^2}{8d}-\frac{7\,e^{-dx-c}ab}{4d}$

input
```
int(sinh(d*x+c)^3*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(1/3*sinh(d*x+c)^4/cos
h(d*x+c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c))+b^2*(1/3*sinh(d*x+
c)^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8*sinh(d*x+c)^2/cosh(d*x+
c)^3-16/3/cosh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 259 vs. 2(73) = 146.

Time = 0.11 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.36

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{(a^2+2\,ab+b^2)\cosh\left(dx+c\right)^6+(a^2+2\,ab+b^2)\sinh\left(dx+c\right)^6-6\,(a^2+6\,ab+5\,b^2)\cosh\left(dx+c\right)^4+3}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + (a^2 + 2*a*b + b^2)*sinh(d*x +
 c)^6 - 6*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)
*cosh(d*x + c)^2 - 2*a^2 - 12*a*b - 10*b^2)*sinh(d*x + c)^4 - 3*(11*a^2 +
86*a*b + 91*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^
4 - 12*(a^2 + 6*a*b + 5*b^2)*cosh(d*x + c)^2 - 11*a^2 - 86*a*b - 91*b^2)*s
inh(d*x + c)^2 - 26*a^2 - 220*a*b - 210*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh
(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh^3\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(73) = 146$.

Time = 0.05 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.44

$$\begin{aligned}&\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\\&-\frac{1}{24}b^2\left(\frac{33\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{30\,e^{(-2\,dx-2\,c)}+240\,e^{(-4\,dx-4\,c)}+322\,e^{(-6\,dx-6\,c)}+177\,e^{(-8\,dx-8\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)}\right)}\right.\\&-\frac{1}{12}ab\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)\\&+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)\end{aligned}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*b^2*((33*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) +
 240*e^(-4*d*x - 4*c) + 322*e^(-6*d*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(
d*(e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x
- 9*c)))) - 1/12*a*b*((21*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d
*x - 2*c) + 69*e^(-4*d*x - 4*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*
c)))) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d +
 e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 205 vs. $2(73) = 146$.

Time = 0.18 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.66

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+2\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)-}{24}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(a^2*(e^(d*x + c) + e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c) + e^(-d*x -
c))^3 + b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*a^2*(e^(d*x + c) + e^(-d*x
 - c)) - 48*a*b*(e^(d*x + c) + e^(-d*x - c)) - 36*b^2*(e^(d*x + c) + e^(-d
*x - c)) - 16*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 9*b^2*(e^(d*x + c) +
 e^(-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.79

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{e^{-3c-3dx}(a+b)^2}{24d} - \frac{e^{c+dx}(3a^2+14ab+11b^2)}{8d} + \frac{e^{3c+3dx}(a+b)^2}{24d} - \frac{e^{-c-dx}(3a^2+14ab+11b^2)}{8d} - \frac{8b^2 e^{c+dx}}{3d(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)} - \frac{2e^{c+dx}(3b^2+2ab)}{d(e^{2c+2dx}+1)} + \frac{8b^2 e^{c+dx}}{3d(2e^{2c+2dx}+e^{4c+4dx}+1)}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(14*a*b + 3*a^2 + 11
*b^2))/(8*d) + (exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- c - d*x)*(14*a
*b + 3*a^2 + 11*b^2))/(8*d) - (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x
) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b +
 3*b^2))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c
 + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 328, normalized size of antiderivative = 4.26

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{e^{12dx+12c}a^2 + 2e^{12dx+12c}ab + e^{12dx+12c}b^2 - 6e^{10dx+10c}a^2 - 36e^{10dx+10c}ab - 30e^{10dx+10c}b^2 - 33e^{8dx+8c}a^2 - 2}{}$$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(12*c + 12*d*x)*a**2 + 2*e**(12*c + 12*d*x)*a*b + e**(12*c + 12*d*x)*b
**2 - 6*e**(10*c + 10*d*x)*a**2 - 36*e**(10*c + 10*d*x)*a*b - 30*e**(10*c
+ 10*d*x)*b**2 - 33*e**(8*c + 8*d*x)*a**2 - 258*e**(8*c + 8*d*x)*a*b - 273
*e**(8*c + 8*d*x)*b**2 - 52*e**(6*c + 6*d*x)*a**2 - 440*e**(6*c + 6*d*x)*a
*b - 420*e**(6*c + 6*d*x)*b**2 - 33*e**(4*c + 4*d*x)*a**2 - 258*e**(4*c +
4*d*x)*a*b - 273*e**(4*c + 4*d*x)*b**2 - 6*e**(2*c + 2*d*x)*a**2 - 36*e**(
2*c + 2*d*x)*a*b - 30*e**(2*c + 2*d*x)*b**2 + a**2 + 2*a*b + b**2)/(24*e**
(3*c + 3*d*x)*d*(e**(6*c + 6*d*x) + 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x
) + 1))
```

3.11 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{1}{2}(a+b)(a+5b)x+\frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{2b(a+b)\tanh(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
-1/2*(a+b)*(a+5*b)*x+1/2*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d+2*b*(a+b)*tanh(
d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.98 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.99

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-6(a^2+6ab+5b^2)(c+dx)+3(a+b)^2\sinh(2(c+dx))+4b\left(6a+7b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-6*(a^2 + 6*a*b + 5*b^2)*(c + d*x) + 3*(a + b)^2*Sinh[2*(c + d*x)] + 4*b*
(6*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.15, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 366, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{366}$$

$$\frac{\frac{(a+b)^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{\tanh^2(c+dx)\left(a^2+6ba+3b^2+2b^2\tanh^2(c+dx)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{363}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx)-(a+b)(a+5b)\int \frac{\tanh^2(c+dx)}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+\frac{(a+b)^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{262}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx)-(a+b)(a+5b)\left(\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\tanh(c+dx)\right)\right)+\frac{(a+b)^2\tanh^3(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{2}\left(\frac{2}{3}b^2\tanh^3(c+dx)-(a+b)(a+5b)(\text{arctanh}(\tanh(c+dx))-\tanh(c+dx))\right)+\frac{(a+b)^2\tanh^3(c+dx)}{2(1-\tanh^2(c+dx))}}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a + b)^2*Tanh[c + d*x]^3)/(2*(1 - Tanh[c + d*x]^2)) + (-((a + b)*(a + 5
*b)*(ArcTanh[Tanh[c + d*x]] - Tanh[c + d*x])) + (2*b^2*Tanh[c + d*x]^3)/3)
/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 2.09 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.66

method	result
derivativedivides	$\frac{a^{2}\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^{3}}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^{2}\left(\frac{\sinh(dx+c)^{5}}{2\cosh(dx+c)^{3}}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh}{}\right.}{d}$
default	$\frac{a^{2}\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+2ab\left(\frac{\sinh(dx+c)^{3}}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+b^{2}\left(\frac{\sinh(dx+c)^{5}}{2\cosh(dx+c)^{3}}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\tanh}{}\right.}{d}$
risch	$-\frac{a^{2}x}{2}-3abx-\frac{5b^{2}x}{2}+\frac{e^{2dx+2c}a^{2}}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^{2}}{8d}-\frac{e^{-2dx-2c}a^{2}}{8d}-\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2}}{8d}$

input
```
int(sinh(d*x+c)^2*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c
)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^5/cosh
(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 291 vs. $2(65) = 130$.

Time = 0.09 (sec) , antiderivative size = 291, normalized size of antiderivative = 4.10

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{3\left(a^2+2ab+b^2\right)\sinh\left(dx+c\right)^5 - 4\left(3\left(a^2+6ab+5b^2\right)dx+12ab+14b^2\right)\cosh\left(dx+c\right)^3 - 12\left(3\left(a^2+\right.\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^5 - 4*(3*(a^2 + 6*a*b + 5*b^2)*d
*x + 12*a*b + 14*b^2)*cosh(d*x + c)^3 - 12*(3*(a^2 + 6*a*b + 5*b^2)*d*x +
12*a*b + 14*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*(a^2 + 2*a*b + b^2)*c
osh(d*x + c)^2 + 9*a^2 + 66*a*b + 65*b^2)*sinh(d*x + c)^3 - 12*(3*(a^2 + 6
*a*b + 5*b^2)*d*x + 12*a*b + 14*b^2)*cosh(d*x + c) + 3*(5*(a^2 + 2*a*b + b
^2)*cosh(d*x + c)^4 + (9*a^2 + 66*a*b + 65*b^2)*cosh(d*x + c)^2 + 2*a^2 +
20*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sin
h(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2 \sinh^2\left(c+dx\right) dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(65) = 130$.

Time = 0.04 (sec) , antiderivative size = 217, normalized size of antiderivative = 3.06

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{24}b^2\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)})}\right) - \frac{1}{4}ab\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^2*(60*(d*
x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x -
4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c)
 + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 1/4*a*b*(12*(d*x + c)/d + e^
(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*
d*x - 4*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 213 vs. $2(65) = 130$.

Time = 0.17 (sec) , antiderivative size = 213, normalized size of antiderivative = 3.00

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3a^2e^{(2dx+2c)}+6abe^{(2dx+2c)}+3b^2e^{(2dx+2c)}-12(a^2+6ab+5b^2)(dx+c)+3\left(2a^2e^{(2dx+2c)}+12abe^{(2c}\right.}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(3*a^2*e^(2*d*x + 2*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c
) - 12*(a^2 + 6*a*b + 5*b^2)*(d*x + c) + 3*(2*a^2*e^(2*d*x + 2*c) + 12*a*b
*e^(2*d*x + 2*c) + 10*b^2*e^(2*d*x + 2*c) - a^2 - 2*a*b - b^2)*e^(-2*d*x -
 2*c) - 16*(6*a*b*e^(4*d*x + 4*c) + 9*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(2*d*
x + 2*c) + 12*b^2*e^(2*d*x + 2*c) + 6*a*b + 7*b^2)/(e^(2*d*x + 2*c) + 1)^3
)/d
```

Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 248, normalized size of antiderivative = 3.49

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{e^{2c+2dx}(a+b)^2}{8d} - x\left(\frac{a^2}{2}+3ab+\frac{5b^2}{2}\right) - \frac{\frac{2(3b^2+2ab)}{3d}+\frac{4e^{2c+2dx}(b^2+2ab)}{3d}+\frac{2e^{4c+4dx}(3b^2+2ab)}{3d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1} - \frac{2(3b^2+2ab)}{3d(e^{2c+2dx}+1)} - \frac{e^{-2c-2dx}(a+b)^2}{8d} - \frac{\frac{2(b^2+2ab)}{3d}+\frac{2e^{2c+2dx}(3b^2+2ab)}{3d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - x*(3*a*b + a^2/2 + (5*b^2)/2) - ((2*(
2*a*b + 3*b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d) + (2*exp(
4*c + 4*d*x)*(2*a*b + 3*b^2))/(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d
*x) + exp(6*c + 6*d*x) + 1) - (2*(2*a*b + 3*b^2))/(3*d*(exp(2*c + 2*d*x) +
 1)) - (exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - ((2*(2*a*b + b^2))/(3*d) + (
2*exp(2*c + 2*d*x)*(2*a*b + 3*b^2))/(3*d))/(2*exp(2*c + 2*d*x) + exp(4*c +
 4*d*x) + 1)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 446, normalized size of antiderivative = 6.28

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{46e^{8dx+8c}ab+3e^{10dx+10c}a^2+3e^{10dx+10c}b^2-12e^{2dx+2c}a^2dx-12e^{8dx+8c}a^2dx-60e^{8dx+8c}b^2dx-36e^{6dx+6c}\cdots}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(3*e**(10*c + 10*d*x)*a**2 + 6*e**(10*c + 10*d*x)*a*b + 3*e**(10*c + 10*d*
x)*b**2 - 12*e**(8*c + 8*d*x)*a**2*d*x + 7*e**(8*c + 8*d*x)*a**2 - 72*e**(
8*c + 8*d*x)*a*b*d*x + 46*e**(8*c + 8*d*x)*a*b - 60*e**(8*c + 8*d*x)*b**2*
d*x + 55*e**(8*c + 8*d*x)*b**2 - 36*e**(6*c + 6*d*x)*a**2*d*x - 216*e**(6*
c + 6*d*x)*a*b*d*x - 180*e**(6*c + 6*d*x)*b**2*d*x - 36*e**(4*c + 4*d*x)*a
**2*d*x - 12*e**(4*c + 4*d*x)*a**2 - 216*e**(4*c + 4*d*x)*a*b*d*x - 120*e*
*(4*c + 4*d*x)*a*b - 180*e**(4*c + 4*d*x)*b**2*d*x - 60*e**(4*c + 4*d*x)*b
**2 - 12*e**(2*c + 2*d*x)*a**2*d*x - 11*e**(2*c + 2*d*x)*a**2 - 72*e**(2*c
 + 2*d*x)*a*b*d*x - 86*e**(2*c + 2*d*x)*a*b - 60*e**(2*c + 2*d*x)*b**2*d*x
 - 75*e**(2*c + 2*d*x)*b**2 - 3*a**2 - 6*a*b - 3*b**2)/(24*e**(2*c + 2*d*x
)*d*(e**(6*c + 6*d*x) + 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.12 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{2b(a+b)\text{sech}(c+dx)}{d}-\frac{b^2\text{sech}^3(c+dx)}{3d}$$

output
```
(a+b)^2*cosh(d*x+c)/d+2*b*(a+b)*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{3(a+b)^2\cosh(c+dx)+b\text{sech}(c+dx)\left(6(a+b)-b\text{sech}^2(c+dx)\right)}{3d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(3*(a + b)^2*Cosh[c + d*x] + b*Sech[c + d*x]*(6*(a + b) - b*Sech[c + d*x]^
2))/(3*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -i\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 4147}$$

$$-\frac{\int \cosh^2(c+dx)\left(-b\text{sech}^2(c+dx)+a+b\right)^2 d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{ 244}$$

$$-\frac{\int \left((a+b)^2\cosh^2(c+dx)+b^2\text{sech}^2(c+dx)-2b(a+b)\right) d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-(a+b)^2\cosh(c+dx)-2b(a+b)\text{sech}(c+dx)+\frac{1}{3}b^2\text{sech}^3(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*Cosh[c + d*x]) - 2*b*(a + b)*Sech[c + d*x] + (b^2*Sech[c +
d*x]^3)/3)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(47) = 94$.

Time = 1.30 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.00

method	result
derivativedivides	$\frac{a^2\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
default	$\frac{a^2\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^2}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+b^2\left(\frac{\sinh(dx+c)^4}{\cosh(dx+c)^3}+\frac{4\sinh(dx+c)^2}{\cosh(dx+c)^3}+\frac{8}{3\cosh(dx+c)^3}\right)}{d}$
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{e^{dx+c}ab}{d}+\frac{e^{dx+c}b^2}{2d}+\frac{e^{-dx-c}a^2}{2d}+\frac{e^{-dx-c}ab}{d}+\frac{e^{-dx-c}b^2}{2d}+\frac{4b\,e^{dx+c}\left(3\,e^{4dx+4c}a+3b\,e^{4dx+4c}+\cdots\right.}{3d\left(e^{2dx+\cdots}\right.}$

input
```
int(sinh(d*x+c)*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*cosh(d*x+c)+2*a*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+b^2*(
sinh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d*x+c)^
3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time = 0.10 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.41

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3\left(a^2+2\,ab+b^2\right)\cosh\left(dx+c\right)^4+3\left(a^2+2\,ab+b^2\right)\sinh\left(dx+c\right)^4+12\left(a^2+4\,ab+3\,b^2\right)\cosh\left(dx+c\right)^2}{6\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx+c\right)\sin\right.}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/6*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*sinh(d*
x + c)^4 + 12*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + 6*(3*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^2 + 2*a^2 + 8*a*b + 6*b^2)*sinh(d*x + c)^2 + 9*a^2 + 42
*a*b + 25*b^2)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*
d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\sinh\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 3.49

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{6}b^2\left(\frac{3\,e^{(-dx-c)}}{d}+\frac{33\,e^{(-2\,dx-2\,c)}+41\,e^{(-4\,dx-4\,c)}+27\,e^{(-6\,dx-6\,c)}+3}{d(e^{(-dx-c)}+3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right)$$
$$+ab\left(\frac{e^{(-dx-c)}}{d}+\frac{5\,e^{(-2\,dx-2\,c)}+1}{d(e^{(-dx-c)}+e^{(-3\,dx-3\,c)})}\right)+\frac{a^2\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/6*b^2*(3*e^(-d*x - c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 2
7*e^(-6*d*x - 6*c) + 3)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*
x - 5*c) + e^(-7*d*x - 7*c)))) + a*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c)
 + 1)/(d*(e^(-d*x - c) + e^(-3*d*x - 3*c)))) + a^2*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 139 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 139, normalized size of antiderivative = 2.84

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)+3\,b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+\frac{8\left(3\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b^2\left(\cdots\right.\right.}{\left(e^{(dx+c)}+e^{(-\cdots}\right.}}{6\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*a^2*(e^(d*x + c) + e^(-d*x - c)) + 6*a*b*(e^(d*x + c) + e^(-d*x - c
)) + 3*b^2*(e^(d*x + c) + e^(-d*x - c)) + 8*(3*a*b*(e^(d*x + c) + e^(-d*x
- c))^2 + 3*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 2*b^2)/(e^(d*x + c) + e^(
-d*x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 154, normalized size of antiderivative = 3.14

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{e^{c+d\,x}\,(a+b)^2}{2\,d}+\frac{e^{-c-d\,x}\,(a+b)^2}{2\,d}+\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}+\frac{4\,e^{c+d\,x}\,(b^2+a\,b)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a + b)^2)/(2*d) + (8*b^2
*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6
*d*x) + 1)) + (4*exp(c + d*x)*(a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (8
*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 335, normalized size of antiderivative = 6.84

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{-2e^{3dx+3c}\cosh(dx+c)\tanh(dx+c)^2b^2+12e^{3dx+3c}\cosh(dx+c)a^2-4e^{3dx+3c}\cosh(dx+c)b^2-2e^{dx-}}{}$$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 2*e**(3*c + 3*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*b**2 + 12*e**(3*c +
3*d*x)*cosh(c + d*x)*a**2 - 4*e**(3*c + 3*d*x)*cosh(c + d*x)*b**2 - 2*e**(
c + d*x)*cosh(c + d*x)*tanh(c + d*x)**2*b**2 + 12*e**(c + d*x)*cosh(c + d*
x)*a**2 - 4*e**(c + d*x)*cosh(c + d*x)*b**2 + 12*e**(4*c + 4*d*x)*a*b + 9*
e**(4*c + 4*d*x)*b**2 - 4*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)**3*
b**2 + 4*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)*b**2 + 72*e**(2*c +
2*d*x)*a*b + 54*e**(2*c + 2*d*x)*b**2 - 4*e**(c + d*x)*sinh(c + d*x)*tanh(
c + d*x)**3*b**2 + 4*e**(c + d*x)*sinh(c + d*x)*tanh(c + d*x)*b**2 + 12*a*
b + 9*b**2)/(12*e**(c + d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.13 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 51

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b(2a+b)\operatorname{sech}(c+dx)}{d}+\frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

output
```
-a^2*arctanh(cosh(d*x+c))/d-b*(2*a+b)*sech(d*x+c)/d+1/3*b^2*sech(d*x+c)^3/
d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.25

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{3a^2\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)-3b(2a+b)\operatorname{sech}(c+dx)+b^2\operatorname{sech}^3(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(3*a^2*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]) - 3*b*(2*a + b)*
Sech[c + d*x] + b^2*Sech[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4147, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(\frac{a^2}{1-\operatorname{sech}^2(c+dx)}-b^2\operatorname{sech}^2(c+dx)+b(2a+b)\right)d\operatorname{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2\operatorname{arctanh}(\operatorname{sech}(c+dx)) - b(2a+b)\operatorname{sech}(c+dx) + \frac{1}{3}b^2\operatorname{sech}^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a^2*ArcTanh[Sech[c + d*x]]) - b*(2*a + b)*Sech[c + d*x] + (b^2*Sech[c +
 d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.08 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24

method	result	si
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	6
default	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{2ab}{\cosh(dx+c)}+b^2\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)}{d}$	6
risch	$-\frac{2b\,e^{dx+c}\left(6\,e^{4dx+4c}a+3b\,e^{4dx+4c}+12\,e^{2dx+2c}a+2\,e^{2dx+2c}b+6a+3b\right)}{3d\left(e^{2dx+2c}+1\right)^3}+\frac{a^2\ln\left(e^{dx+c}-1\right)}{d}-\frac{a^2\ln\left(e^{dx+c}+1\right)}{d}$	1

input
```
int(csch(d*x+c)*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))-2*a*b/cosh(d*x+c)+b^2*(-sinh(d*x+c)^2/cosh
(d*x+c)^3-2/3/cosh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 890 vs. 2(49) = 98.

Time = 0.11 (sec) , antiderivative size = 890, normalized size of antiderivative = 17.45

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(6*(2*a*b + b^2)*cosh(d*x + c)^5 + 30*(2*a*b + b^2)*cosh(d*x + c)*sin
h(d*x + c)^4 + 6*(2*a*b + b^2)*sinh(d*x + c)^5 + 4*(6*a*b + b^2)*cosh(d*x
+ c)^3 + 4*(15*(2*a*b + b^2)*cosh(d*x + c)^2 + 6*a*b + b^2)*sinh(d*x + c)^
3 + 12*(5*(2*a*b + b^2)*cosh(d*x + c)^3 + (6*a*b + b^2)*cosh(d*x + c))*sin
h(d*x + c)^2 + 6*(2*a*b + b^2)*cosh(d*x + c) + 3*(a^2*cosh(d*x + c)^6 + 6*
a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*cosh(d*x +
 c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*cosh(d*x +
 c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^3 +
3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^2 +
a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(a^2*cosh(d*x +
c)^6 + 6*a^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*sinh(d*x + c)^6 + 3*a^2*c
osh(d*x + c)^4 + 3*(5*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^4 + 3*a^2*c
osh(d*x + c)^2 + 4*(5*a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*a^2*cosh(d*x + c)^4 + 6*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x
+ c)^2 + a^2 + 6*(a^2*cosh(d*x + c)^5 + 2*a^2*cosh(d*x + c)^3 + a^2*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(5*(2*a
*b + b^2)*cosh(d*x + c)^4 + 2*(6*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)
*sinh(d*x + c))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d
*sinh(d*x + c)^6 + 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + d)*si...
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{csch}\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(49) = 98$.

Time = 0.04 (sec) , antiderivative size = 196, normalized size of antiderivative = 3.84

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{2}{3}b^2\left(\frac{3\,e^{(-dx-c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{2\,e^{(-3\,dx-3\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$

$$+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}-\frac{4\,ab}{d(e^{(dx+c)}+e^{(-dx-c)})}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-2/3*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*x - 5*c)/(d*(3*e^(-2*d*x -
 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^2*log(tanh(1/2*d*
x + 1/2*c))/d - 4*a*b/(d*(e^(d*x + c) + e^(-d*x - c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 123 vs. $2(49) = 98$.

Time = 0.16 (sec) , antiderivative size = 123, normalized size of antiderivative = 2.41

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{3\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-3\,a^2\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(6\,ab\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2+3\,b^2\left(e^{(dx+c)}+e\right.\right.}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3}}{6\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/6*(3*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2) - 3*a^2*log(e^(d*x + c) +
e^(-d*x - c) - 2) + 4*(6*a*b*(e^(d*x + c) + e^(-d*x - c))^2 + 3*b^2*(e^(d*
x + c) + e^(-d*x - c))^2 - 4*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 160, normalized size of antiderivative = 3.14

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)} - \frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)} - \frac{2\,\mathrm{e}^{c+d\,x}\,\left(b^2+2\,a\,b\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{2\,\mathrm{atan}\left(\frac{a^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^4}}\right)\,\sqrt{a^4}}{\sqrt{-d^2}}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x),x)
```

output
```
(8*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (
8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1)) - (2*exp(c + d*x)*(2*a*b + b^2))/(d*(exp(2*c + 2*d*x) + 1
)) - (2*atan((a^2*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^4)^(1/2)))*(a^4)^(1/
2))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 305, normalized size of antiderivative = 5.98

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3e^{6dx+6c}\log\left(e^{dx+c}-1\right)a^2 - 3e^{6dx+6c}\log\left(e^{dx+c}+1\right)a^2 - 12e^{5dx+5c}ab - 6e^{5dx+5c}b^2 + 9e^{4dx+4c}\log\left(e^{dx+c}-\right.}{}$$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(3*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**2 - 3*e**(6*c + 6*d*x)*log(e*
*(c + d*x) + 1)*a**2 - 12*e**(5*c + 5*d*x)*a*b - 6*e**(5*c + 5*d*x)*b**2 +
 9*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2 - 9*e**(4*c + 4*d*x)*log(e*
*(c + d*x) + 1)*a**2 - 24*e**(3*c + 3*d*x)*a*b - 4*e**(3*c + 3*d*x)*b**2 +
 9*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2 - 9*e**(2*c + 2*d*x)*log(e*
*(c + d*x) + 1)*a**2 - 12*e**(c + d*x)*a*b - 6*e**(c + d*x)*b**2 + 3*log(e
**(c + d*x) - 1)*a**2 - 3*log(e**(c + d*x) + 1)*a**2)/(3*d*(e**(6*c + 6*d*
x) + 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.14 $\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{a^2\coth(c+dx)}{d}+\frac{2ab\tanh(c+dx)}{d} + \frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
-a^2*coth(d*x+c)/d+2*a*b*tanh(d*x+c)/d+1/3*b^2*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.76 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.93

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{-3a^2\coth(c+dx)+b\left(6a+b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-3*a^2*Coth[c + d*x] + b*(6*a + b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*
d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 244}$$

$$\frac{\int \left(a^2\coth^2(c+dx)+b^2\tanh^2(c+dx)+2ab\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-a^2\coth(c+dx)+2ab\tanh(c+dx)+\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a^2*Coth[c + d*x]) + 2*a*b*Tanh[c + d*x] + (b^2*Tanh[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.57 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.48

method	result
derivativedivides	$\frac{-\coth(dx+c)a^2+2\tanh(dx+c)ab+b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{-\coth(dx+c)a^2+2\tanh(dx+c)ab+b^2\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)}{d}$
risch	$-\frac{2\left(3\,e^{6dx+6c}a^2+6\,e^{6dx+6c}ab+3\,e^{6dx+6c}b^2+9\,e^{4dx+4c}a^2+6\,e^{4dx+4c}ab-3\,e^{4dx+4c}b^2+9\,e^{2dx+2c}a^2-6\,e^{2dx+2c}ba+b^2e\right.}{3d\left(e^{2dx+2c}+1\right)^3\left(e^{2dx+2c}-1\right)}$

input
```
int(csch(d*x+c)^2*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^2+2*tanh(d*x+c)*a*b+b^2*(-1/2*sinh(d*x+c)/cosh(d*x+c)^
3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(44) = 88$.

Time = 0.08 (sec) , antiderivative size = 264, normalized size of antiderivative = 5.74

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(\left(3a^2+b^2\right)\cosh\left(dx+c\right)^3+3\left(3a^2+b^2\right)\cosh\left(dx+c\right)\mathrm{si}\right.}{3\left(d\cosh\left(dx+c\right)^5+5d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+d\sinh\left(dx+c\right)^5+d\cosh\left(dx+c\right)^3+\left(10d\cos\right.\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-4/3*((3*a^2 + b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + b^2)*cosh(d*x + c)*sinh(d
*x + c)^2 + 2*(3*a*b + b^2)*sinh(d*x + c)^3 + (9*a^2 - b^2)*cosh(d*x + c)
+ 2*(3*(3*a*b + b^2)*cosh(d*x + c)^2 + 3*a*b - b^2)*sinh(d*x + c))/(d*cosh
(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + d*co
sh(d*x + c)^3 + (10*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^3 + (10*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 - 2*d*cosh(d*x + c) + (5*d
*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{csch}^2\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(44) = 88$.

Time = 0.05 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.96

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{2}{3}b^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+\frac{4\,ab}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2/3*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) + 1)) + 2*a^2/(d*(e^(-
2*d*x - 2*c) - 1))
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.87

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{2\left(\frac{3\,a^2}{e^{(2\,dx+2\,c)}-1}+\frac{6\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+12\,abe^{(2\,dx+2\,c)}+6\,ab+b^2}{\left(e^{(2\,dx+2\,c)}+1\right)^3}\right)}{3\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/3*(3*a^2/(e^(2*d*x + 2*c) - 1) + (6*a*b*e^(4*d*x + 4*c) + 3*b^2*e^(4*d*
x + 4*c) + 12*a*b*e^(2*d*x + 2*c) + 6*a*b + b^2)/(e^(2*d*x + 2*c) + 1)^3)/
d
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 209, normalized size of antiderivative = 4.54

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2\,(2\,a\,b-b^2)}{3\,d}+\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}-\frac{\frac{2\,(b^2+2\,a\,b)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{3\,d}+\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}\,(2\,a\,b-b^2)}{3\,d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}$$
$$-\frac{2\,a^2}{d\,(\mathrm{e}^{2\,c+2\,d\,x}-1)}-\frac{2\,(b^2+2\,a\,b)}{3\,d\,(\mathrm{e}^{2\,c+2\,d\,x}+1)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^2,x)
```

output
```
- ((2*(2*a*b - b^2))/(3*d) + (2*exp(2*c + 2*d*x)*(2*a*b + b^2))/(3*d))/(2*
exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(2*a*b + b^2))/(3*d) + (2*e
xp(4*c + 4*d*x)*(2*a*b + b^2))/(3*d) + (4*exp(2*c + 2*d*x)*(2*a*b - b^2))/
(3*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
(2*a^2)/(d*(exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(3*d*(exp(2*c + 2*d
*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.83

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3e^{8dx+8c}a^2+6e^{8dx+8c}ab+3e^{8dx+8c}b^2-18e^{4dx+4c}a^2-12e^{4dx+4c}ab+6e^{4dx+4c}b^2-24e^{2dx+2c}a^2-8e^{2dx+2c}b}{3d\left(e^{8dx+8c}+2e^{6dx+6c}-2e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(3*e**(8*c + 8*d*x)*a**2 + 6*e**(8*c + 8*d*x)*a*b + 3*e**(8*c + 8*d*x)*b**
2 - 18*e**(4*c + 4*d*x)*a**2 - 12*e**(4*c + 4*d*x)*a*b + 6*e**(4*c + 4*d*x
)*b**2 - 24*e**(2*c + 2*d*x)*a**2 - 8*e**(2*c + 2*d*x)*b**2 - 9*a**2 + 6*a
*b - b**2)/(3*d*(e**(8*c + 8*d*x) + 2*e**(6*c + 6*d*x) - 2*e**(2*c + 2*d*x
) - 1))
```

3.15 $\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{a(a-4b)\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{2ab\operatorname{sech}(c+dx)}{d} - \frac{b^2\operatorname{sech}^3(c+dx)}{3d}$$

output
```
1/2*a*(a-4*b)*arctanh(cosh(d*x+c))/d-1/2*a^2*coth(d*x+c)*csch(d*x+c)/d+2*a
*b*sech(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 3.57 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.72

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - 12a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 48ab\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right) + 12a^2\log\left(\sinh\left(\frac{1}{2}(\right.\right.}{2}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/24*(3*a^2*Csch[(c + d*x)/2]^2 - 12*a^2*Log[Cosh[(c + d*x)/2]] + 48*a*b*
Log[Cosh[(c + d*x)/2]] + 12*a^2*Log[Sinh[(c + d*x)/2]] - 48*a*b*Log[Sinh[(
c + d*x)/2]] + 3*a^2*Sech[(c + d*x)/2]^2 - 48*a*b*Sech[c + d*x] + 8*b^2*Se
ch[c + d*x]^3)/d
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4147, 366, 363, 262, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{4147}$$

$$-\frac{\int \frac{\operatorname{sech}^2(c+dx)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}{\left(1-\operatorname{sech}^2(c+dx)\right)^2}d\operatorname{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{366}$$

$$-\frac{\frac{a^2\operatorname{sech}^3(c+dx)}{2\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{1}{2}\int \frac{\operatorname{sech}^2(c+dx)\left(3a^2-2(a+b)^2+2b^2\operatorname{sech}^2(c+dx)\right)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{363}$$

$$-\frac{\frac{1}{2}\left(\frac{2}{3}b^2\operatorname{sech}^3(c+dx)-a(a-4b)\int \frac{\operatorname{sech}^2(c+dx)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)\right)+\frac{a^2\operatorname{sech}^3(c+dx)}{2\left(1-\operatorname{sech}^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{262}$$

$$-\frac{\frac{1}{2}\left(\frac{2}{3}b^2\text{sech}^3(c+dx)-a(a-4b)\left(\int\frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)-\text{sech}(c+dx)\right)\right)+\frac{a^2\text{sech}^3(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{a^2\text{sech}^3(c+dx)}{2\left(1-\text{sech}^2(c+dx)\right)}+\frac{1}{2}\left(\frac{2}{3}b^2\text{sech}^3(c+dx)-a(a-4b)(\text{arctanh}(\text{sech}(c+dx))-\text{sech}(c+dx))\right)}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-(((a^2*Sech[c + d*x]^3)/(2*(1 - Sech[c + d*x]^2)) + (-(a*(a - 4*b)*(ArcTa
nh[Sech[c + d*x]] - Sech[c + d*x])) + (2*b^2*Sech[c + d*x]^3)/3)/2)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 262
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[c*(c*x)
^(m - 1)*((a + b*x^2)^(p + 1)/(b*(m + 2*p + 1))), x] - Simp[a*c^2*((m - 1)/
(b*(m + 2*p + 1)))   Int[(c*x)^(m - 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b
, c, p}, x] && GtQ[m, 2 - 1] && NeQ[m + 2*p + 1, 0] && IntBinomialQ[a, b, c
, 2, m, p, x]
```

rule 363
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[d*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(b*e*(m + 2*p + 3))),
 x] - Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(b*(m + 2*p + 3))   Int[(e*x)^
m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d
, 0] && NeQ[m + 2*p + 3, 0]
```

rule 366
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^2,
x_Symbol] :> Simp[(-(b*c - a*d)^2)*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(2*a*
b^2*e*(p + 1))), x] + Simp[1/(2*a*b^2*(p + 1))   Int[(e*x)^m*(a + b*x^2)^(p
 + 1)*Simp[(b*c - a*d)^2*(m + 1) + 2*b^2*c^2*(p + 1) + 2*a*b*d^2*(p + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LtQ[p
, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 8.03 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^{2}\left(-\frac{\operatorname{csch}\left(dx+c\right)\coth\left(dx+c\right)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(\frac{1}{\cosh\left(dx+c\right)}-2\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)-\frac{b^{2}}{3\cosh\left(dx+c\right)^{3}}}{d}$
default	$\frac{a^{2}\left(-\frac{\operatorname{csch}\left(dx+c\right)\coth\left(dx+c\right)}{2}+\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)+2ab\left(\frac{1}{\cosh\left(dx+c\right)}-2\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)\right)-\frac{b^{2}}{3\cosh\left(dx+c\right)^{3}}}{d}$
risch	$-\frac{\mathrm{e}^{dx+c}\left(3\,\mathrm{e}^{8dx+8c}a^{2}-12\,\mathrm{e}^{8dx+8c}ab+12\,\mathrm{e}^{6dx+6c}a^{2}+8\,\mathrm{e}^{6dx+6c}b^{2}+18\,\mathrm{e}^{4dx+4c}a^{2}+24\,\mathrm{e}^{4dx+4c}ab-16\,\mathrm{e}^{4dx+4c}b^{2}+12\,\mathrm{e}^{2dx}\right.}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^{3}\left(\mathrm{e}^{2dx+2c}-1\right)^{2}}$

input
```
int(csch(d*x+c)^3*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(1/cosh(
d*x+c)-2*arctanh(exp(d*x+c)))-1/3*b^2/cosh(d*x+c)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2462 vs. $2(68) = 136$.

Time = 0.10 (sec) , antiderivative size = 2462, normalized size of antiderivative = 33.27

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/6*(6*(a^2 - 4*a*b)*cosh(d*x + c)^9 + 54*(a^2 - 4*a*b)*cosh(d*x + c)*sin
h(d*x + c)^8 + 6*(a^2 - 4*a*b)*sinh(d*x + c)^9 + 8*(3*a^2 + 2*b^2)*cosh(d*
x + c)^7 + 8*(27*(a^2 - 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x +
 c)^7 + 56*(9*(a^2 - 4*a*b)*cosh(d*x + c)^3 + (3*a^2 + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c)^6 + 4*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x + c)^5 + 4*(189*(
a^2 - 4*a*b)*cosh(d*x + c)^4 + 42*(3*a^2 + 2*b^2)*cosh(d*x + c)^2 + 9*a^2
+ 12*a*b - 8*b^2)*sinh(d*x + c)^5 + 4*(189*(a^2 - 4*a*b)*cosh(d*x + c)^5 +
 70*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 5*(9*a^2 + 12*a*b - 8*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^4 + 8*(3*a^2 + 2*b^2)*cosh(d*x + c)^3 + 8*(63*(a^2 - 4
*a*b)*cosh(d*x + c)^6 + 35*(3*a^2 + 2*b^2)*cosh(d*x + c)^4 + 5*(9*a^2 + 12
*a*b - 8*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*b^2)*sinh(d*x + c)^3 + 8*(27*(a^
2 - 4*a*b)*cosh(d*x + c)^7 + 21*(3*a^2 + 2*b^2)*cosh(d*x + c)^5 + 5*(9*a^2
 + 12*a*b - 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*b^2)*cosh(d*x + c))*sinh
(d*x + c)^2 + 6*(a^2 - 4*a*b)*cosh(d*x + c) - 3*((a^2 - 4*a*b)*cosh(d*x +
c)^10 + 10*(a^2 - 4*a*b)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^2 - 4*a*b)*sin
h(d*x + c)^10 + (a^2 - 4*a*b)*cosh(d*x + c)^8 + (45*(a^2 - 4*a*b)*cosh(d*x
 + c)^2 + a^2 - 4*a*b)*sinh(d*x + c)^8 + 8*(15*(a^2 - 4*a*b)*cosh(d*x + c)
^3 + (a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(a^2 - 4*a*b)*cosh(d
*x + c)^6 + 2*(105*(a^2 - 4*a*b)*cosh(d*x + c)^4 + 14*(a^2 - 4*a*b)*cosh(d
*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^6 + 4*(63*(a^2 - 4*a*b)*cosh(d*x...
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. 2(68) = 136.

Time = 0.04 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.45

$$\begin{aligned}
&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx \\
&= \frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right) \\
&\quad - 2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right) \\
&\quad - \frac{8\,b^2}{3\,d(e^{(dx+c)}+e^{(-dx-c)})^3}
\end{aligned}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x -
c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) -
2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c)/
(d*(e^(-2*d*x - 2*c) + 1))) - 8/3*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(68) = 136$.

Time = 0.17 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.07

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{\frac{12\,a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4}-3\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)+3\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-\right.}{12\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/12*(12*a^2*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
 - 4) - 3*(a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 3*(a^2 - 4*a
*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 16*(3*a*b*(e^(d*x + c) + e^(-d*x
 - c))^2 - 2*b^2)/(e^(d*x + c) + e^(-d*x - c))^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 261, normalized size of antiderivative = 3.53

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$=\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^c\left(a^2\,\sqrt{-d^2}-4\,a\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}\right)\sqrt{a^4-8\,a^3\,b+16\,a^2\,b^2}}{\sqrt{-d^2}}$$
$$+\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{a^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$
$$-\frac{2\,a^2\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,b^2\,e^{c+d\,x}}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}+\frac{4\,a\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^2*(-d^2)^(1/2) - 4*a*b*(-d^2)^(1/2)))/(d*(a^4 -
8*a^3*b + 16*a^2*b^2)^(1/2)))*(a^4 - 8*a^3*b + 16*a^2*b^2)^(1/2))/(-d^2)^(
1/2) + (8*b^2*exp(c + d*x))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x)
+ exp(6*c + 6*d*x) + 1)) - (a^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) -
 (2*a^2*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8
*b^2*exp(c + d*x))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (4*
a*b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 768, normalized size of antiderivative = 10.38

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{-3e^{10dx+10c}\log\left(e^{dx+c}-1\right)a^2+3e^{10dx+10c}\log\left(e^{dx+c}+1\right)a^2+24e^{9dx+9c}ab-3e^{8dx+8c}\log\left(e^{dx+c}-1\right)a^2+}{}$$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
( - 3*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a**2 + 12*e**(10*c + 10*d*x
)*log(e**(c + d*x) - 1)*a*b + 3*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a
**2 - 12*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a*b - 6*e**(9*c + 9*d*x)
*a**2 + 24*e**(9*c + 9*d*x)*a*b - 3*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)
*a**2 + 12*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a*b + 3*e**(8*c + 8*d*x)
*log(e**(c + d*x) + 1)*a**2 - 12*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a*
b - 24*e**(7*c + 7*d*x)*a**2 - 16*e**(7*c + 7*d*x)*b**2 + 6*e**(6*c + 6*d*
x)*log(e**(c + d*x) - 1)*a**2 - 24*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*
a*b - 6*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**2 + 24*e**(6*c + 6*d*x)*
log(e**(c + d*x) + 1)*a*b - 36*e**(5*c + 5*d*x)*a**2 - 48*e**(5*c + 5*d*x)
*a*b + 32*e**(5*c + 5*d*x)*b**2 + 6*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)
*a**2 - 24*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*b - 6*e**(4*c + 4*d*x)
*log(e**(c + d*x) + 1)*a**2 + 24*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a*
b - 24*e**(3*c + 3*d*x)*a**2 - 16*e**(3*c + 3*d*x)*b**2 - 3*e**(2*c + 2*d*
x)*log(e**(c + d*x) - 1)*a**2 + 12*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*
a*b + 3*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2 - 12*e**(2*c + 2*d*x)*
log(e**(c + d*x) + 1)*a*b - 6*e**(c + d*x)*a**2 + 24*e**(c + d*x)*a*b - 3*
log(e**(c + d*x) - 1)*a**2 + 12*log(e**(c + d*x) - 1)*a*b + 3*log(e**(c +
d*x) + 1)*a**2 - 12*log(e**(c + d*x) + 1)*a*b)/(6*d*(e**(10*c + 10*d*x) +
e**(8*c + 8*d*x) - 2*e**(6*c + 6*d*x) - 2*e**(4*c + 4*d*x) + e**(2*c + ...
```

3.16 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{(2a-b)b\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
a*(a-2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d-(2*a-b)*b*tanh(d*x+c)/d-1/
3*b^2*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-a\coth(c+dx)\left(-2a+6b+a\operatorname{csch}^2(c+dx)\right)+b\left(-6a+2b+b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{3d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a*Coth[c + d*x]*(-2*a + 6*b + a*Csch[c + d*x]^2)) + b*(-6*a + 2*b + b*S
ech[c + d*x]^2)*Tanh[c + d*x])/(3*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left(a^2\coth^4(c+dx)-a(a-2b)\coth^2(c+dx)-b^2\tanh^2(c+dx)+b(b-2a)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)-b(2a-b)\tanh(c+dx)+a(a-2b)\coth(c+dx)-\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a*(a - 2*b)*Coth[c + d*x] - (a^2*Coth[c + d*x]^3)/3 - (2*a - b)*b*Tanh[c
+ d*x] - (b^2*Tanh[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 16.88 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+b^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{d}$
risch	$-\frac{4(3\,e^{8dx+8c}a^2+6\,e^{8dx+8c}ab+3\,e^{8dx+8c}b^2+8\,e^{6dx+6c}a^2-8\,e^{6dx+6c}b^2+6\,e^{4dx+4c}a^2-12\,e^{4dx+4c}ab+6\,e^{4dx+4c}b^2-a^2}{3d\left(e^{2dx+2c}-1\right)^3\left(e^{2dx+2c}+1\right)^3}$

input
```
int(csch(d*x+c)^4*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1/sinh(d*x+c)/cosh(d*
x+c)-2*tanh(d*x+c))+b^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(68) = 136$.

Time = 0.07 (sec) , antiderivative size = 393, normalized size of antiderivative = 5.46

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{8 \left((a^2+6\,ab+b^2)\cosh\left(dx+c\right)^4+8\,(a^2+b^2)\cosh\left(dx\right.\right.}{3 \left(d\cosh\left(dx+c\right)^8+56\,d\cosh\left(dx+c\right)^3\sinh\left(dx+c\right)^5+28\,d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^6+8\,d\cosh\right.}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-8/3*((a^2 + 6*a*b + b^2)*cosh(d*x + c)^4 + 8*(a^2 + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 6*a*b + b^2)*sinh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d
*x + c)^2 + 2*(3*(a^2 + 6*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - 2*b^2)*sinh
(d*x + c)^2 + 3*a^2 - 6*a*b + 3*b^2 + 8*((a^2 + b^2)*cosh(d*x + c)^3 + (a^
2 - b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 56*d*cosh(d*x
+ c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*d*cosh(d
*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^4 + 2*(35*
d*cosh(d*x + c)^4 - 2*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - d*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(7*d*cosh(d*x + c)^6 - 6*d*cosh(d*x + c)^2)
*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - d*cosh(d*x + c)^3)*sinh(d*x + c)
 + 3*d)
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2 \operatorname{csch}^4\left(c+dx\right) dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(68) = 136$.

Time = 0.04 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.92

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{4}{3}b^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+\frac{4}{3}a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1}\right.$$
$$+\frac{8\,ab}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
4/3*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c
) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
 - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 8*a*b/(d*(e^(-4*d*x - 4*
c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(68) = 136$.

Time = 0.16 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{4\left(3\,a^2e^{(8\,dx+8\,c)}+6\,abe^{(8\,dx+8\,c)}+3\,b^2e^{(8\,dx+8\,c)}+8\,a^2e^{(6\,dx+6\,c)}-8\,b^2e^{(6\,dx+6\,c)}+6\,a^2e^{(4\,dx+4\,c)}-12\,abe^{(}\right.}{3\,d(e^{(4\,dx+4\,c)}-1)^3}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-4/3*(3*a^2*e^(8*d*x + 8*c) + 6*a*b*e^(8*d*x + 8*c) + 3*b^2*e^(8*d*x + 8*c
) + 8*a^2*e^(6*d*x + 6*c) - 8*b^2*e^(6*d*x + 6*c) + 6*a^2*e^(4*d*x + 4*c)
- 12*a*b*e^(4*d*x + 4*c) + 6*b^2*e^(4*d*x + 4*c) - a^2 + 6*a*b - b^2)/(d*(
e^(4*d*x + 4*c) - 1)^3)
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.99

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(6\,a\,b-a^2-b^2+6\,a^2\,\mathrm{e}^{4\,c+4\,d\,x}+8\,a^2\,\mathrm{e}^{6\,c+6\,d\,x}+3\,a^2\,\mathrm{e}^{8\,c+8\,d\,x}+6\,b^2\,\mathrm{e}^{4\,c+4\,d\,x}-8\,b^2\,\mathrm{e}^{6\,c+6\,d\,x}+3\,b^2\,\mathrm{e}^{8\,c+}\right.}{3\,d\left(\mathrm{e}^{4\,c+4\,d\,x}-1\right)^3}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/sinh(c + d*x)^4,x)
```

output
```
-(4*(6*a*b - a^2 - b^2 + 6*a^2*exp(4*c + 4*d*x) + 8*a^2*exp(6*c + 6*d*x) +
 3*a^2*exp(8*c + 8*d*x) + 6*b^2*exp(4*c + 4*d*x) - 8*b^2*exp(6*c + 6*d*x)
+ 3*b^2*exp(8*c + 8*d*x) - 12*a*b*exp(4*c + 4*d*x) + 6*a*b*exp(8*c + 8*d*x
)))/(3*d*(exp(4*c + 4*d*x) - 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 176, normalized size of antiderivative = 2.44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{-\frac{4e^{12dx+12c}a^2}{3}-\frac{8e^{12dx+12c}ab}{3}-\frac{4e^{12dx+12c}b^2}{3}-\frac{32e^{6dx+6c}a^2}{3}+\frac{32e^{6dx+6c}b^2}{3}-12e^{4dx+4c}a^2+8e^{4dx+4c}ab-12e^{4dx+4c}}{d\left(e^{12dx+12c}-3e^{8dx+8c}+3e^{4dx+4c}-1\right)}$$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(4*( - e**(12*c + 12*d*x)*a**2 - 2*e**(12*c + 12*d*x)*a*b - e**(12*c + 12*
d*x)*b**2 - 8*e**(6*c + 6*d*x)*a**2 + 8*e**(6*c + 6*d*x)*b**2 - 9*e**(4*c
+ 4*d*x)*a**2 + 6*e**(4*c + 4*d*x)*a*b - 9*e**(4*c + 4*d*x)*b**2 + 2*a**2
- 4*a*b + 2*b**2))/(3*d*(e**(12*c + 12*d*x) - 3*e**(8*c + 8*d*x) + 3*e**(4
*c + 4*d*x) - 1))
```

3.17 $\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 136

$$\begin{aligned}\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{3}{8}(a+b)\left(a^2+14ab+21b^2\right)x-\frac{(a+b)^2(5a+17b)\cosh(c+dx)\sinh(c+dx)}{8d}\\&\quad+\frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{3b(a+b)(a+2b)\tanh(c+dx)}{d}\\&\quad-\frac{b^2(a+b)\tanh^3(c+dx)}{d}-\frac{b^3\tanh^5(c+dx)}{5d}\end{aligned}$$

output
```
3/8*(a+b)*(a^2+14*a*b+21*b^2)*x-1/8*(a+b)^2*(5*a+17*b)*cosh(d*x+c)*sinh(d*
x+c)/d+1/4*(a+b)^3*cosh(d*x+c)^3*sinh(d*x+c)/d-3*b*(a+b)*(a+2*b)*tanh(d*x+
c)/d-b^2*(a+b)*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 4.03 (sec) , antiderivative size = 125, normalized size of antiderivative = 0.92

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{60(a^3+15a^2b+35ab^2+21b^3)(c+dx)-40(a+b)^2(a+4b)\sinh(2(c+dx))+5(a+b)^3\sinh(4(c+dx)}{160d}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(60*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(c + d*x) - 40*(a + b)^2*(a + 4*b
)*Sinh[2*(c + d*x)] + 5*(a + b)^3*Sinh[4*(c + d*x)] - 32*b*(15*a^2 + 50*a*
b + 36*b^2 - b*(5*a + 7*b)*Sech[c + d*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c +
 d*x])/(160*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.46, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4146, 369, 27, 439, 25, 437, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 369}$$

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int\frac{3\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2\left(3b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\int\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2\left(3b\tanh^2(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$\downarrow$ 439

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{1}{2}\int-\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)\left(b(5a+21b)\tanh^2(c+dx)+a(a+9b)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{(a}{}\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{(a+3b)\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)\left(b(5a+21b)\tanh^2\right.}{1-\tanh^2(c+dx)}\right.}{d}$$

$\downarrow$ 437

$$\frac{\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{(a+3b)\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int\left(-b^2(5a+21b)\tanh^4(c+dx)-b(6a^2\right.\right.}{d}$$

$\downarrow$ 2009

$$\frac{\tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{3}{4}\left(\frac{1}{2}\left(-(a+b)\left(a^2+14ab+21b^2\right)\text{arctanh}(\tanh(c+dx))+\frac{1}{3}b(6a^2+35ab+21b^2\right.\right.$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3)/(4*(1 - Tanh[c + d*x]^2)^2) -
 (3*(((a + 3*b)*Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2)/(2*(1 - Tanh[c
+ d*x]^2)) + (-((a + b)*(a^2 + 14*a*b + 21*b^2)*ArcTanh[Tanh[c + d*x]]) +
(a + b)*(a^2 + 14*a*b + 21*b^2)*Tanh[c + d*x] + (b*(6*a^2 + 35*a*b + 21*b^
2)*Tanh[c + d*x]^3)/3 + (b^2*(5*a + 21*b)*Tanh[c + d*x]^5)/5)/2))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 437
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2)^(r_.), x_Symbol] :> Int[ExpandIntegrand[(g*x)^m*(
a + b*x^2)^p*(c + d*x^2)^q*(e + f*x^2)^r, x], x] /; FreeQ[{a, b, c, d, e, f
, g, m}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

rule 439
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
 + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*a*b*g*(p + 1))), x] + Simp[1/(2*a*b*(p
+ 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 1)*Simp[c*(2*b*e*(
p + 1) + (b*e - a*f)*(m + 1)) + d*(2*b*e*(p + 1) + (b*e - a*f)*(m + 2*q + 1
))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && LtQ[p, -1] && G
tQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a*f])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 22.28 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.81

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx-}{8}\right.}{}$
default	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^5}{4\cosh(dx+c)}-\frac{5\sinh(dx+c)^3}{8\cosh(dx+c)}+\frac{15dx}{8}+\frac{15c}{8}-\frac{15\tanh(dx-}{8}\right.}{}$
risch	$\frac{3a^3x}{8}+\frac{45a^2bx}{8}+\frac{105a\,b^2x}{8}+\frac{63b^3x}{8}+\frac{e^{4dx+4c}a^3}{64d}+\frac{3\,e^{4dx+4c}a^2b}{64d}+\frac{3\,e^{4dx+4c}b^2a}{64d}+\frac{e^{4dx+4c}b^3}{64d}-\frac{e^{2dx+2}}{8d}$

input
```
int(sinh(d*x+c)^4*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d
*x+15/8*c-15/8*tanh(d*x+c))+3*b^2*a*(1/4*sinh(d*x+c)^7/cosh(d*x+c)^3-7/8*s
inh(d*x+c)^5/cosh(d*x+c)^3+35/8*d*x+35/8*c-35/8*tanh(d*x+c)-35/24*tanh(d*x
+c)^3)+b^3*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*sinh(d*x+c)^7/cosh(d*x+c)^
5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x+c)^3-63/40*tanh(d*x+c)^5)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 879 vs. 2(128) = 256.

Time = 0.11 (sec) , antiderivative size = 879, normalized size of antiderivative = 6.46

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/320*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^9 - 15*(a^3 + 11*a^
2*b + 19*a*b^2 + 9*b^3 - 12*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^7 + 8*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2
*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 +
 288*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)*sinh
(d*x + c)^4 + (630*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 - 150*a
^3 - 2010*a^2*b - 4850*a*b^2 - 3054*b^3 - 315*(a^3 + 11*a^2*b + 19*a*b^2 +
 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 40*(120*a^2*b + 400*a*b^2 + 288
*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 5*(8
4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 - 105*(a^3 + 11*a^2*b +
19*a*b^2 + 9*b^3)*cosh(d*x + c)^4 - 62*a^3 - 978*a^2*b - 2282*a*b^2 - 1302
*b^3 - 4*(75*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^3 + 40*(2*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*
b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c)^3 + 3*(120*a^2*b + 400*a*b^2 + 2
88*b^3 + 15*(a^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c))*sinh(
d*x + c)^2 + 80*(120*a^2*b + 400*a*b^2 + 288*b^3 + 15*(a^3 + 15*a^2*b + 35
*a*b^2 + 21*b^3)*d*x)*cosh(d*x + c) + 5*(9*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*cosh(d*x + c)^8 - 21*(a^3 + 11*a^2*b + 19*a*b^2 + 9*b^3)*cosh(d*x + c)^6
- 2*(75*a^3 + 1005*a^2*b + 2425*a*b^2 + 1527*b^3)*cosh(d*x + c)^4 - 36*a^3
 - 612*a^2*b - 1372*a*b^2 - 924*b^3 - 6*(31*a^3 + 489*a^2*b + 1141*a*b^...
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 480 vs. $2(128) = 256$.

Time = 0.06 (sec) , antiderivative size = 480, normalized size of antiderivative = 3.53

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{64}a^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$
$$+\frac{1}{320}b^3\left(\frac{2520\,(dx+c)}{d}+\frac{5\left(32\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}\right)}{d}-\frac{135\,e^{(-2\,dx-2\,c)}+5358\,e^{(-4\,dx-4\,c)}+18190\,e^{(}}{d(e^{(-4\,dx-4\,c)}+5\,e^{(-6\,dx-6\,c)}+10}\right.$$
$$+\frac{1}{64}ab^2\left(\frac{840\,(dx+c)}{d}+\frac{3\left(24\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}\right)}{d}-\frac{63\,e^{(-2\,dx-2\,c)}+1487\,e^{(-4\,dx-4\,c)}+2517\,e^{(-6\,d}}{d(e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}+3\,e^{(-8\,dx}}\right.$$
$$+\frac{3}{64}a^2b\left(\frac{120\,(dx+c)}{d}+\frac{16\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}-\frac{15\,e^{(-2\,dx-2\,c)}+144\,e^{(-4\,dx-4\,c)}-1}{d(e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)})}\right)$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^3*(2520*(d*x + c)/d + 5*(32*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x -
4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x
- 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x
- 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c
) + e^(-14*d*x - 14*c)))) + 1/64*a*b^2*(840*(d*x + c)/d + 3*(24*e^(-2*d*x
- 2*c) - e^(-4*d*x - 4*c))/d - (63*e^(-2*d*x - 2*c) + 1487*e^(-4*d*x - 4*c
) + 2517*e^(-6*d*x - 6*c) + 1608*e^(-8*d*x - 8*c) - 3)/(d*(e^(-4*d*x - 4*c
) + 3*e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)))) + 3/64
*a^2*b*(120*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15
*e^(-2*d*x - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 505 vs. $2(128) = 256$.

Time = 0.32 (sec) , antiderivative size = 505, normalized size of antiderivative = 3.71

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{5\,a^3e^{(4\,dx+4\,c)} + 15\,a^2be^{(4\,dx+4\,c)} + 15\,ab^2e^{(4\,dx+4\,c)} + 5\,b^3e^{(4\,dx+4\,c)} - 40\,a^3e^{(2\,dx+2\,c)} - 240\,a^2be^{(2\,dx+2\,c)} - 36}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/320*(5*a^3*e^(4*d*x + 4*c) + 15*a^2*b*e^(4*d*x + 4*c) + 15*a*b^2*e^(4*d*
x + 4*c) + 5*b^3*e^(4*d*x + 4*c) - 40*a^3*e^(2*d*x + 2*c) - 240*a^2*b*e^(2
*d*x + 2*c) - 360*a*b^2*e^(2*d*x + 2*c) - 160*b^3*e^(2*d*x + 2*c) + 120*(a
^3 + 15*a^2*b + 35*a*b^2 + 21*b^3)*(d*x + c) - 5*(18*a^3*e^(4*d*x + 4*c) +
 270*a^2*b*e^(4*d*x + 4*c) + 630*a*b^2*e^(4*d*x + 4*c) + 378*b^3*e^(4*d*x
+ 4*c) - 8*a^3*e^(2*d*x + 2*c) - 48*a^2*b*e^(2*d*x + 2*c) - 72*a*b^2*e^(2*
d*x + 2*c) - 32*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4
*d*x - 4*c) + 128*(15*a^2*b*e^(8*d*x + 8*c) + 60*a*b^2*e^(8*d*x + 8*c) + 5
0*b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 210*a*b^2*e^(6*d*x + 6*
c) + 150*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 290*a*b^2*e^(4*d
*x + 4*c) + 210*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 190*a*b^2
*e^(2*d*x + 2*c) + 130*b^3*e^(2*d*x + 2*c) + 15*a^2*b + 50*a*b^2 + 36*b^3)
/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 730, normalized size of antiderivative = 5.37

$$\int \sinh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
((2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(9*a*b^2 +
3*a^2*b + 5*b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(8*a*b^2 + 3*a^2*b + 6*b^3)
)/(5*d) + (8*exp(6*c + 6*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(
8*c + 8*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 1
0*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c +
 10*d*x) + 1) + ((2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (2*exp(2*c + 2*d*
x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4
*d*x) + 1) + x*((105*a*b^2)/8 + (45*a^2*b)/8 + (3*a^3)/8 + (63*b^3)/8) + (
(2*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5*d) + (6*exp(2*c + 2*d*x)*(8*a*b^2 + 3*a
^2*b + 6*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(9*a*b^2 + 3*a^2*b + 5*b^3))/(5
*d) + (2*exp(6*c + 6*d*x)*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d))/(4*exp(2*c
 + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1
) + ((2*(8*a*b^2 + 3*a^2*b + 6*b^3))/(5*d) + (4*exp(2*c + 2*d*x)*(9*a*b^2
+ 3*a^2*b + 5*b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(12*a*b^2 + 3*a^2*b + 10*b
^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) +
1) + (2*(12*a*b^2 + 3*a^2*b + 10*b^3))/(5*d*(exp(2*c + 2*d*x) + 1)) - (exp
(- 4*c - 4*d*x)*(a + b)^3)/(64*d) + (exp(4*c + 4*d*x)*(a + b)^3)/(64*d) +
(exp(- 2*c - 2*d*x)*(a + b)^2*(a + 4*b))/(8*d) - (exp(2*c + 2*d*x)*(a + b)
^2*(a + 4*b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1018, normalized size of antiderivative = 7.49

$$\int \sinh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{7132e^{4dx+4c}a\,b^2 - 5b^3 + 15e^{18dx+18c}a^2b + 15e^{18dx+18c}a\,b^2 - 165e^{16dx+16c}a^2b - 285e^{16dx+16c}a\,b^2 - 1032e^{14d}}{}$$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
(5*e**(18*c + 18*d*x)*a**3 + 15*e**(18*c + 18*d*x)*a**2*b + 15*e**(18*c +
18*d*x)*a*b**2 + 5*e**(18*c + 18*d*x)*b**3 - 15*e**(16*c + 16*d*x)*a**3 -
165*e**(16*c + 16*d*x)*a**2*b - 285*e**(16*c + 16*d*x)*a*b**2 - 135*e**(16
*c + 16*d*x)*b**3 + 120*e**(14*c + 14*d*x)*a**3*d*x - 88*e**(14*c + 14*d*x
)*a**3 + 1800*e**(14*c + 14*d*x)*a**2*b*d*x - 1032*e**(14*c + 14*d*x)*a**2
*b + 4200*e**(14*c + 14*d*x)*a*b**2*d*x - 2568*e**(14*c + 14*d*x)*a*b**2 +
 2520*e**(14*c + 14*d*x)*b**3*d*x - 1752*e**(14*c + 14*d*x)*b**3 + 600*e**
(12*c + 12*d*x)*a**3*d*x + 9000*e**(12*c + 12*d*x)*a**2*b*d*x + 21000*e**(
12*c + 12*d*x)*a*b**2*d*x + 12600*e**(12*c + 12*d*x)*b**3*d*x + 1200*e**(1
0*c + 10*d*x)*a**3*d*x + 440*e**(10*c + 10*d*x)*a**3 + 18000*e**(10*c + 10
*d*x)*a**2*b*d*x + 6720*e**(10*c + 10*d*x)*a**2*b + 42000*e**(10*c + 10*d*
x)*a*b**2*d*x + 15960*e**(10*c + 10*d*x)*a*b**2 + 25200*e**(10*c + 10*d*x)
*b**3*d*x + 8400*e**(10*c + 10*d*x)*b**3 + 1200*e**(8*c + 8*d*x)*a**3*d*x
+ 800*e**(8*c + 8*d*x)*a**3 + 18000*e**(8*c + 8*d*x)*a**2*b*d*x + 12840*e*
*(8*c + 8*d*x)*a**2*b + 42000*e**(8*c + 8*d*x)*a*b**2*d*x + 29680*e**(8*c
+ 8*d*x)*a*b**2 + 25200*e**(8*c + 8*d*x)*b**3*d*x + 17640*e**(8*c + 8*d*x)
*b**3 + 600*e**(6*c + 6*d*x)*a**3*d*x + 620*e**(6*c + 6*d*x)*a**3 + 9000*e
**(6*c + 6*d*x)*a**2*b*d*x + 9780*e**(6*c + 6*d*x)*a**2*b + 21000*e**(6*c
+ 6*d*x)*a*b**2*d*x + 22820*e**(6*c + 6*d*x)*a*b**2 + 12600*e**(6*c + 6*d*
x)*b**3*d*x + 13020*e**(6*c + 6*d*x)*b**3 + 120*e**(4*c + 4*d*x)*a**3*d...
```

3.18 $\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 105

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{(a+b)^2(a+4b)\cosh(c+dx)}{d} + \frac{(a+b)^3\cosh^3(c+dx)}{3d} - \frac{3b(a+b)(a+2b)\text{sech}(c+dx)}{d} + \frac{b^2(3a+4b)\text{sech}^3(c+dx)}{3d} - \frac{b^3\text{sech}^5(c+dx)}{5d}$$

output
```
-(a+b)^2*(a+4*b)*cosh(d*x+c)/d+1/3*(a+b)^3*cosh(d*x+c)^3/d-3*b*(a+b)*(a+2*
b)*sech(d*x+c)/d+1/3*b^2*(3*a+4*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 9.95 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.87

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-45(a+b)^2(a+5b)\cosh(c+dx)+5(a+b)^3\cosh(3(c+dx))-180b(a+b)(a+2b)\text{sech}(c+dx)+20b}{60d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-45*(a + b)^2*(a + 5*b)*Cosh[c + d*x] + 5*(a + b)^3*Cosh[3*(c + d*x)] - 1
80*b*(a + b)*(a + 2*b)*Sech[c + d*x] + 20*b^2*(3*a + 4*b)*Sech[c + d*x]^3
- 12*b^3*Sech[c + d*x]^5)/(60*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 26, 4147, 25, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{26}$$

$$i\int \sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$-\frac{\int \cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)\left(-b\text{sech}^2(c+dx)+a+b\right)^3d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{ 355}$$

$$-\frac{\int \left((a+b)^3\cosh^4(c+dx)-(a+b)^2(a+4b)\cosh^2(c+dx)+b^3\text{sech}^4(c+dx)-b^2(3a+4b)\text{sech}^2(c+dx)+3b\right.}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{3}b^2(3a+4b)\text{sech}^3(c+dx)+\frac{1}{3}(a+b)^3\cosh^3(c+dx)-(a+b)^2(a+4b)\cosh(c+dx)-3b(a+b)(a+2b)\text{sech}(c}{d}$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-((a + b)^2*(a + 4*b)*Cosh[c + d*x]) + ((a + b)^3*Cosh[c + d*x]^3)/3 - 3*
b*(a + b)*(a + 2*b)*Sech[c + d*x] + (b^2*(3*a + 4*b)*Sech[c + d*x]^3)/3 -
(b^3*Sech[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(99) = 198$.

Time $= 10.50$ (sec) , antiderivative size $= 239$, normalized size of antiderivative $= 2.28$

method	result
derivativedivides	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+3b^2a\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(d}{\cosh(d}\right.}{}$
default	$\frac{a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^4}{3\cosh(dx+c)}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)}-\frac{8}{3\cosh(dx+c)}\right)+3b^2a\left(\frac{\sinh(dx+c)^6}{3\cosh(dx+c)^3}-\frac{2\sinh(d}{\cosh(d}\right.}{}$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{e^{3dx+3c}b^2a}{8d}+\frac{e^{3dx+3c}b^3}{24d}-\frac{3\,e^{dx+c}a^3}{8d}-\frac{21\,e^{dx+c}a^2b}{8d}-\frac{33\,e^{dx+c}b^2a}{8d}-\frac{15\,e^{dx+}}{8d}$

input
```
int(sinh(d*x+c)^3*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(1/3*sinh(d*x+c)^4/c
osh(d*x+c)-4/3*sinh(d*x+c)^2/cosh(d*x+c)-8/3/cosh(d*x+c))+3*b^2*a*(1/3*sin
h(d*x+c)^6/cosh(d*x+c)^3-2*sinh(d*x+c)^4/cosh(d*x+c)^3-8*sinh(d*x+c)^2/cos
h(d*x+c)^3-16/3/cosh(d*x+c)^3)+b^3*(1/3*sinh(d*x+c)^8/cosh(d*x+c)^5-8/3*si
nh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+c)^5-64/3*sinh(d*x+c)^
2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(99) = 198$.

Time = 0.09 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.14

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{5\left(a^3+3a^2b+3ab^2+b^3\right)\cosh\left(dx+c\right)^8+5\left(a^3+3a^2b+3ab^2+b^3\right)\sinh\left(dx+c\right)^8-20\left(a^3+12a^2b+\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/120*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^8 + 5*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*sinh(d*x + c)^8 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^
3)*cosh(d*x + c)^6 - 20*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3 - 7*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 20*(11*a^3 + 123*
a^2*b + 249*a*b^2 + 137*b^3)*cosh(d*x + c)^4 + 10*(35*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^4 - 22*a^3 - 246*a^2*b - 498*a*b^2 - 274*b^3 - 3
0*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 -
425*a^3 - 5235*a^2*b - 10395*a*b^2 - 5649*b^3 - 20*(31*a^3 + 372*a^2*b + 7
47*a*b^2 + 390*b^3)*cosh(d*x + c)^2 + 20*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*cosh(d*x + c)^6 - 15*(a^3 + 12*a^2*b + 21*a*b^2 + 10*b^3)*cosh(d*x + c)^
4 - 31*a^3 - 372*a^2*b - 747*a*b^2 - 390*b^3 - 6*(11*a^3 + 123*a^2*b + 249
*a*b^2 + 137*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x +
 c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh^3\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(99) = 198$.

Time = 0.05 (sec) , antiderivative size = 439, normalized size of antiderivative = 4.18

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{1}{120}b^3\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8}}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-}}\right.$$

$$-\frac{1}{8}ab^2\left(\frac{33\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{30\,e^{(-2\,dx-2\,c)}+240\,e^{(-4\,dx-4\,c)}+322\,e^{(-6\,dx-6\,c)}+177\,e^{(-8\,dx-8\,c)}-1}{d(e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}+3\,e^{(-7\,dx-7\,c)}+e^{(-9\,dx-9\,c)})}\right.$$

$$-\frac{1}{8}a^2b\left(\frac{21\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{20\,e^{(-2\,dx-2\,c)}+69\,e^{(-4\,dx-4\,c)}-1}{d(e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)$$

$$+\frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/120*b^3*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*
c) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c)
 + 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x -
3*c) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*
e^(-11*d*x - 11*c) + e^(-13*d*x - 13*c)))) - 1/8*a*b^2*((33*e^(-d*x - c) -
 e^(-3*d*x - 3*c))/d + (30*e^(-2*d*x - 2*c) + 240*e^(-4*d*x - 4*c) + 322*e
^(-6*d*x - 6*c) + 177*e^(-8*d*x - 8*c) - 1)/(d*(e^(-3*d*x - 3*c) + 3*e^(-5
*d*x - 5*c) + 3*e^(-7*d*x - 7*c) + e^(-9*d*x - 9*c)))) - 1/8*a^2*b*((21*e^
(-d*x - c) - e^(-3*d*x - 3*c))/d + (20*e^(-2*d*x - 2*c) + 69*e^(-4*d*x - 4
*c) - 1)/(d*(e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 1/24*a^3*(e^(3*d*x +
 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 330 vs. $2(99) = 198$.

Time = 0.28 (sec) , antiderivative size = 330, normalized size of antiderivative = 3.14

$$\int \sinh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+15\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3+5\,b^3\left(e^{(dx+c)}+e^{(-dx}\right.}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/120*(5*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 15*a^2*b*(e^(d*x + c) + e^(-
d*x - c))^3 + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 5*b^3*(e^(d*x + c)
 + e^(-d*x - c))^3 - 60*a^3*(e^(d*x + c) + e^(-d*x - c)) - 360*a^2*b*(e^(d
*x + c) + e^(-d*x - c)) - 540*a*b^2*(e^(d*x + c) + e^(-d*x - c)) - 240*b^3
*(e^(d*x + c) + e^(-d*x - c)) - 16*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^
4 + 135*a*b^2*(e^(d*x + c) + e^(-d*x - c))^4 + 90*b^3*(e^(d*x + c) + e^(-d
*x - c))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 80*b^3*(e^(d*x + c)
 + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 361, normalized size of antiderivative = 3.44

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{e^{-3c-3dx}(a+b)^3}{24d} + \frac{e^{3c+3dx}(a+b)^3}{24d} + \frac{8e^{c+dx}(4b^3+3ab^2)}{3d\,(2e^{2c+2dx}+e^{4c+4dx}+1)} + \frac{64b^3e^{c+dx}}{5d\,(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1)} - \frac{8e^{c+dx}(32b^3+15ab^2)}{15d\,(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1)} - \frac{32b^3e^{c+dx}}{5d\,(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1)} - \frac{3e^{c+dx}(a+b)^2(a+5b)}{8d} - \frac{6e^{c+dx}(a^2b+3ab^2+2b^3)}{d\,(e^{2c+2dx}+1)} - \frac{3e^{-c-dx}(a+b)^2(a+5b)}{8d}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*(a + b)^3)/(24*d
) + (8*exp(c + d*x)*(3*a*b^2 + 4*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c
+ 4*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*
c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(c + d*x)
*(15*a*b^2 + 32*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp
(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*
exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 1
0*d*x) + 1)) - (3*exp(c + d*x)*(a + b)^2*(a + 5*b))/(8*d) - (6*exp(c + d*x
)*(3*a*b^2 + a^2*b + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x
)*(a + b)^2*(a + 5*b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 598, normalized size of antiderivative = 5.70

$$\int \sinh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{-4980e^{4dx+4c}a\,b^2+5b^3+15e^{16dx+16c}a^2b+15e^{16dx+16c}a\,b^2-240e^{14dx+14c}a^2b-420e^{14dx+14c}a\,b^2-7440e^{1}}{}$$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(5*e**(16*c + 16*d*x)*a**3 + 15*e**(16*c + 16*d*x)*a**2*b + 15*e**(16*c +
16*d*x)*a*b**2 + 5*e**(16*c + 16*d*x)*b**3 - 20*e**(14*c + 14*d*x)*a**3 -
240*e**(14*c + 14*d*x)*a**2*b - 420*e**(14*c + 14*d*x)*a*b**2 - 200*e**(14
*c + 14*d*x)*b**3 - 220*e**(12*c + 12*d*x)*a**3 - 2460*e**(12*c + 12*d*x)*
a**2*b - 4980*e**(12*c + 12*d*x)*a*b**2 - 2740*e**(12*c + 12*d*x)*b**3 - 6
20*e**(10*c + 10*d*x)*a**3 - 7440*e**(10*c + 10*d*x)*a**2*b - 14940*e**(10
*c + 10*d*x)*a*b**2 - 7800*e**(10*c + 10*d*x)*b**3 - 850*e**(8*c + 8*d*x)*
a**3 - 10470*e**(8*c + 8*d*x)*a**2*b - 20790*e**(8*c + 8*d*x)*a*b**2 - 112
98*e**(8*c + 8*d*x)*b**3 - 620*e**(6*c + 6*d*x)*a**3 - 7440*e**(6*c + 6*d*
x)*a**2*b - 14940*e**(6*c + 6*d*x)*a*b**2 - 7800*e**(6*c + 6*d*x)*b**3 - 2
20*e**(4*c + 4*d*x)*a**3 - 2460*e**(4*c + 4*d*x)*a**2*b - 4980*e**(4*c + 4
*d*x)*a*b**2 - 2740*e**(4*c + 4*d*x)*b**3 - 20*e**(2*c + 2*d*x)*a**3 - 240
*e**(2*c + 2*d*x)*a**2*b - 420*e**(2*c + 2*d*x)*a*b**2 - 200*e**(2*c + 2*d
*x)*b**3 + 5*a**3 + 15*a**2*b + 15*a*b**2 + 5*b**3)/(120*e**(3*c + 3*d*x)*
d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6*c + 6*d*x) + 10*e**(
4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.19 $\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 100

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{1}{2}(a+b)^2(a+7b)x+\frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{3b(a+b)^2\tanh(c+dx)}{d}+\frac{b^2(3a+2b)\tanh^3(c+dx)}{3d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
-1/2*(a+b)^2*(a+7*b)*x+1/2*(a+b)^3*cosh(d*x+c)*sinh(d*x+c)/d+3*b*(a+b)^2*t
anh(d*x+c)/d+1/3*b^2*(3*a+2*b)*tanh(d*x+c)^3/d+1/5*b^3*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 2.91 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.95

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-30(a+b)^2(a+7b)(c+dx)+15(a+b)^3\sinh(2(c+dx))+4b\big(45a^2+105ab+58b^2-b(15a+16b)\mathrm{sech}}{60d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-30*(a + b)^2*(a + 7*b)*(c + d*x) + 15*(a + b)^3*Sinh[2*(c + d*x)] + 4*b*
(45*a^2 + 105*a*b + 58*b^2 - b*(15*a + 16*b)*Sech[c + d*x]^2 + 3*b^2*Sech[
c + d*x]^4)*Tanh[c + d*x])/(60*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.55, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 25, 4146, 369, 403, 25, 403, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{25}$$

$$-\int \sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{369}$$

$$\frac{\frac{\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{\left(b\tanh^2(c+dx)+a\right)^2\left(7b\tanh^2(c+dx)+a\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{403}$$

$$\frac{\frac{1}{2}\left(\frac{1}{5}\int -\frac{\left(b\tanh^2(c+dx)+a\right)\left(b(33a+35b)\tanh^2(c+dx)+a(5a+7b)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{7}{5}b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\right)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{2}\left(\frac{7}{5}b\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2-\frac{1}{5}\int\frac{\left(b\tanh^2(c+dx)+a\right)\left(b(33a+35b)\tanh^2(c+dx)+a(5a+7b)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+}{d}$$

$\downarrow$ 403

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\int-\frac{b(81a^2+190ba+105b^2)\tanh^2(c+dx)+a(15a^2+54ba+35b^2)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{1}{3}b(33a+35b)\tanh(c+dx)\left(a+b\,\mathrm{ta}\right.\right.\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}b(33a+35b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)-\frac{1}{3}\int\frac{b(81a^2+190ba+105b^2)\tanh^2(c+dx)+a(15a^2+54ba+35b^2)}{1-\tanh^2(c+dx)}d\,\mathrm{ta}\right.\right.}{d}$$

$\downarrow$ 299

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(b(81a^2+190ab+105b^2)\tanh(c+dx)-15(a+b)^2(a+7b)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)\right)+\frac{1}{3}b(33a+\right.\right.}{}$$

$\downarrow$ 219

$$\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(b(81a^2+190ab+105b^2)\tanh(c+dx)-15(a+b)^2(a+7b)\mathrm{arctanh}(\tanh(c+dx))\right)+\frac{1}{3}b(33a+35b)\,\mathrm{tan}\right.\right.}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3)/(2*(1 - Tanh[c + d*x]^2)) + ((7
*b*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2)/5 + ((-15*(a + b)^2*(a + 7*b)*
ArcTanh[Tanh[c + d*x]] + b*(81*a^2 + 190*a*b + 105*b^2)*Tanh[c + d*x])/3 +
 (b*(33*a + 35*b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/3)/5)/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 6.10 (sec) , antiderivative size = 180, normalized size of antiderivative = 1.80

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3b^2a\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\,\mathrm{t}}{}\right.}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)+3b^2a\left(\frac{\sinh(dx+c)^5}{2\cosh(dx+c)^3}-\frac{5dx}{2}-\frac{5c}{2}+\frac{5\,\mathrm{t}}{}\right.}{d}$
risch	$-\frac{a^3x}{2}-\frac{9a^2bx}{2}-\frac{15a\,b^2x}{2}-\frac{7b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3\,e^{2dx+2c}a^2b}{8d}+\frac{3\,e^{2dx+2c}b^2a}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2}}{8d}$

input
```
int(sinh(d*x+c)^2*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x
+c)^3/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+3*b^2*a*(1/2*sinh(d*x+c)^
5/cosh(d*x+c)^3-5/2*d*x-5/2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3)+b^3*(1/2*
sinh(d*x+c)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^
3+7/10*tanh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(92) = 184$.

Time = 0.10 (sec) , antiderivative size = 725, normalized size of antiderivative = 7.25

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/120*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^7 - 4*(90*a^2*b +
210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x
+ c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2
 + 7*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 + (75*a^3 + 585*a^2*b + 1065*
a*b^2 + 539*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^5 - 20*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15
*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c)^3 + 5*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b
^3)*cosh(d*x + c)^4 + 27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3 + 2*(75*a^3
 + 585*a^2*b + 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 20
*(2*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3
)*d*x)*cosh(d*x + c)^3 + 3*(90*a^2*b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a
^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 - 40*(90*a^2*
b + 210*a*b^2 + 116*b^3 + 15*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)*d*x)*cosh(
d*x + c) + 5*(21*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + (75*a^3
 + 585*a^2*b + 1065*a*b^2 + 539*b^3)*cosh(d*x + c)^4 + 15*a^3 + 189*a^2*b
+ 285*a*b^2 + 175*b^3 + 3*(27*a^3 + 297*a^2*b + 489*a*b^2 + 203*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x
 + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)
)*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh^2\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(92) = 184$.

Time = 0.05 (sec) , antiderivative size = 377, normalized size of antiderivative = 3.77

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{120}b^3\left(\frac{420(dx+c)}{d}+\frac{15e^{(-2dx-2c)}}{d}-\frac{1003e^{(-2dx-2c)}+3350e^{(-4dx-4c)}+5590e^{(-6dx-6c)}+3915e^{(}}{d(e^{(-2dx-2c)}+5e^{(-4dx-4c)}+10e^{(-6dx-6c)}+10e^{(-8dx-8c)}}\right. - \frac{1}{8}ab^2\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)})}\right) - \frac{3}{8}a^2b\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^3*(420*(
d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d
*x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*
x - 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c))))
 - 1/8*a*b^2*(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c
) + 201*e^(-4*d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) - 3/8*a^2*
b*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(
-2*d*x - 2*c) + e^(-4*d*x - 4*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(92) = 184$.

Time = 0.26 (sec) , antiderivative size = 393, normalized size of antiderivative = 3.93

$$\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{15a^3e^{(2dx+2c)}+45a^2be^{(2dx+2c)}+45ab^2e^{(2dx+2c)}+15b^3e^{(2dx+2c)}-60(a^3+9a^2b+15ab^2+7b^3)(dx+}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/120*(15*a^3*e^(2*d*x + 2*c) + 45*a^2*b*e^(2*d*x + 2*c) + 45*a*b^2*e^(2*d
*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) - 60*(a^3 + 9*a^2*b + 15*a*b^2 + 7*b^3)
*(d*x + c) + 15*(2*a^3*e^(2*d*x + 2*c) + 18*a^2*b*e^(2*d*x + 2*c) + 30*a*b
^2*e^(2*d*x + 2*c) + 14*b^3*e^(2*d*x + 2*c) - a^3 - 3*a^2*b - 3*a*b^2 - b^
3)*e^(-2*d*x - 2*c) - 16*(45*a^2*b*e^(8*d*x + 8*c) + 135*a*b^2*e^(8*d*x +
8*c) + 90*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) + 450*a*b^2*e^(6
*d*x + 6*c) + 240*b^3*e^(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) + 600*a*
b^2*e^(4*d*x + 4*c) + 340*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c)
+ 390*a*b^2*e^(2*d*x + 2*c) + 200*b^3*e^(2*d*x + 2*c) + 45*a^2*b + 105*a*b
^2 + 58*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.07 (sec) , antiderivative size = 668, normalized size of antiderivative = 6.68

$$
\begin{aligned}
&\int \sinh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{e^{2c+2dx}(a+b)^3}{8d} - \frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{2c+2dx}(a^2b+3ab^2+2b^3)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1} \\
&\quad - \frac{\frac{2(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{6c+6dx}(a^2b+3ab^2+2b^3)}{5d} + \frac{6e^{4c+4dx}(3a^2b+6ab^2+2b^3)}{5d} + \frac{2e^{2c+2dx}(9a^2b+15ab^2+10b^3)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1} \\
&\quad - \frac{\frac{2(9a^2b+15ab^2+10b^3)}{15d} + \frac{6e^{4c+4dx}(a^2b+3ab^2+2b^3)}{5d} + \frac{4e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1} \\
&\quad - \frac{6(a^2b+3ab^2+2b^3)}{5d(e^{2c+2dx}+1)} - \frac{e^{-2c-2dx}(a+b)^3}{8d} \\
&\quad - \frac{\frac{6(a^2b+3ab^2+2b^3)}{5d} + \frac{8e^{2c+2dx}(3a^2b+6ab^2+2b^3)}{5d} + \frac{6e^{8c+8dx}(a^2b+3ab^2+2b^3)}{5d} + \frac{8e^{6c+6dx}(3a^2b+6ab^2+2b^3)}{5d} + 4e^{4c+4\cdots}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1} \\
&\quad - \frac{x(a+b)^2(a+7b)}{2}
\end{aligned}
$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d
) + (6*exp(2*c + 2*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d))/(2*exp(2*c + 2*d
*x) + exp(4*c + 4*d*x) + 1) - ((2*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (6*
exp(6*c + 6*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(6
*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(15*a*b^2 + 9*a^2*b
 + 10*b^3))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c +
6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(15*a*b^2 + 9*a^2*b + 10*b^3))/(15*d)
 + (6*exp(4*c + 4*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (4*exp(2*c + 2*d
*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c +
4*d*x) + exp(6*c + 6*d*x) + 1) - (6*(3*a*b^2 + a^2*b + 2*b^3))/(5*d*(exp(2
*c + 2*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a + b)^3)/(8*d) - ((6*(3*a*b^2 +
a^2*b + 2*b^3))/(5*d) + (8*exp(2*c + 2*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(
5*d) + (6*exp(8*c + 8*d*x)*(3*a*b^2 + a^2*b + 2*b^3))/(5*d) + (8*exp(6*c +
 6*d*x)*(6*a*b^2 + 3*a^2*b + 2*b^3))/(5*d) + (4*exp(4*c + 4*d*x)*(15*a*b^2
 + 9*a^2*b + 10*b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 1
0*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (x*(a
+ b)^2*(a + 7*b))/2
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 894, normalized size of antiderivative = 8.94

$$\int \sinh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{-4890e^{4dx+4c}a\,b^2 - 15b^3 + 45e^{14dx+14c}a^2b + 45e^{14dx+14c}a\,b^2 - 2025e^{8dx+8c}a^2b - 3465e^{8dx+8c}a\,b^2 - 3915e^{6}}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(15*e**(14*c + 14*d*x)*a**3 + 45*e**(14*c + 14*d*x)*a**2*b + 45*e**(14*c +
 14*d*x)*a*b**2 + 15*e**(14*c + 14*d*x)*b**3 - 60*e**(12*c + 12*d*x)*a**3*
d*x + 48*e**(12*c + 12*d*x)*a**3 - 540*e**(12*c + 12*d*x)*a**2*b*d*x + 288
*e**(12*c + 12*d*x)*a**2*b - 900*e**(12*c + 12*d*x)*a*b**2*d*x + 576*e**(1
2*c + 12*d*x)*a*b**2 - 420*e**(12*c + 12*d*x)*b**3*d*x + 336*e**(12*c + 12
*d*x)*b**3 - 300*e**(10*c + 10*d*x)*a**3*d*x - 2700*e**(10*c + 10*d*x)*a**
2*b*d*x - 4500*e**(10*c + 10*d*x)*a*b**2*d*x - 2100*e**(10*c + 10*d*x)*b**
3*d*x - 600*e**(8*c + 8*d*x)*a**3*d*x - 195*e**(8*c + 8*d*x)*a**3 - 5400*e
**(8*c + 8*d*x)*a**2*b*d*x - 2025*e**(8*c + 8*d*x)*a**2*b - 9000*e**(8*c +
 8*d*x)*a*b**2*d*x - 3465*e**(8*c + 8*d*x)*a*b**2 - 4200*e**(8*c + 8*d*x)*
b**3*d*x - 1155*e**(8*c + 8*d*x)*b**3 - 600*e**(6*c + 6*d*x)*a**3*d*x - 34
5*e**(6*c + 6*d*x)*a**3 - 5400*e**(6*c + 6*d*x)*a**2*b*d*x - 3915*e**(6*c
+ 6*d*x)*a**2*b - 9000*e**(6*c + 6*d*x)*a*b**2*d*x - 6315*e**(6*c + 6*d*x)
*a*b**2 - 4200*e**(6*c + 6*d*x)*b**3*d*x - 2905*e**(6*c + 6*d*x)*b**3 - 30
0*e**(4*c + 4*d*x)*a**3*d*x - 270*e**(4*c + 4*d*x)*a**3 - 2700*e**(4*c + 4
*d*x)*a**2*b*d*x - 2970*e**(4*c + 4*d*x)*a**2*b - 4500*e**(4*c + 4*d*x)*a*
b**2*d*x - 4890*e**(4*c + 4*d*x)*a*b**2 - 2100*e**(4*c + 4*d*x)*b**3*d*x -
 2030*e**(4*c + 4*d*x)*b**3 - 60*e**(2*c + 2*d*x)*a**3*d*x - 102*e**(2*c +
 2*d*x)*a**3 - 540*e**(2*c + 2*d*x)*a**2*b*d*x - 882*e**(2*c + 2*d*x)*a**2
*b - 900*e**(2*c + 2*d*x)*a*b**2*d*x - 1554*e**(2*c + 2*d*x)*a*b**2 - 4...
```

3.20 $\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 70

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{3b(a+b)^2\text{sech}(c+dx)}{d}-\frac{b^2(a+b)\text{sech}^3(c+dx)}{d}+\frac{b^3\text{sech}^5(c+dx)}{5d}$$

output
```
(a+b)^3*cosh(d*x+c)/d+3*b*(a+b)^2*sech(d*x+c)/d-b^2*(a+b)*sech(d*x+c)^3/d+
1/5*b^3*sech(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 62, normalized size of antiderivative = 0.89

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =\frac{(a+b)^3\cosh(c+dx)+3b(a+b)^2\text{sech}(c+dx)-b^2(a+b)\text{sech}^3(c+dx)+\frac{1}{5}b^3\text{sech}^5(c+dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*Cosh[c + d*x] + 3*b*(a + b)^2*Sech[c + d*x] - b^2*(a + b)*Sech[
c + d*x]^3 + (b^3*Sech[c + d*x]^5)/5)/d
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4147, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{4147}$$

$$-\frac{\int \cosh^2(c+dx)\left(-b\text{sech}^2(c+dx)+a+b\right)^3 d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$-\frac{\int \left(-b^3\text{sech}^4(c+dx)+3b^2(a+b)\text{sech}^2(c+dx)-3b(a+b)^2+(a+b)^3\cosh^2(c+dx)\right) d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^2(a+b)\text{sech}^3(c+dx)-(a+b)^3\cosh(c+dx)-3b(a+b)^2\text{sech}(c+dx)-\frac{1}{5}b^3\text{sech}^5(c+dx)}{d}$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*Cosh[c + d*x]) - 3*b*(a + b)^2*Sech[c + d*x] + b^2*(a + b)*
Sech[c + d*x]^3 - (b^3*Sech[c + d*x]^5)/5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. $2(68) = 136$.

Time = 3.61 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.43

method	result
derivativedivides	$\frac{a^{3}\cosh(dx+c)+3a^{2}b\left(\frac{\sinh(dx+c)^{2}}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3b^{2}a\left(\frac{\sinh(dx+c)^{4}}{\cosh(dx+c)^{3}}+\frac{4\sinh(dx+c)^{2}}{\cosh(dx+c)^{3}}+\frac{8}{3\cosh(dx+c)^{3}}\right)+b^{3}\left(\frac{\sinh(dx+c)}{\cosh(dx+c)}\right.}{d}$
default	$\frac{a^{3}\cosh(dx+c)+3a^{2}b\left(\frac{\sinh(dx+c)^{2}}{\cosh(dx+c)}+\frac{2}{\cosh(dx+c)}\right)+3b^{2}a\left(\frac{\sinh(dx+c)^{4}}{\cosh(dx+c)^{3}}+\frac{4\sinh(dx+c)^{2}}{\cosh(dx+c)^{3}}+\frac{8}{3\cosh(dx+c)^{3}}\right)+b^{3}\left(\frac{\sinh(dx+c)}{\cosh(dx+c)}\right.}{d}$
risch	$\frac{e^{dx+c}a^{3}}{2d}+\frac{3\,e^{dx+c}a^{2}b}{2d}+\frac{3\,e^{dx+c}b^{2}a}{2d}+\frac{e^{dx+c}b^{3}}{2d}+\frac{e^{-dx-c}a^{3}}{2d}+\frac{3\,e^{-dx-c}a^{2}b}{2d}+\frac{3\,e^{-dx-c}b^{2}a}{2d}+\frac{e^{-dx-c}b^{3}}{2d}$

input
```
int(sinh(d*x+c)*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^2/cosh(d*x+c)+2/cosh(d*x+c))+3*b
^2*a*(sinh(d*x+c)^4/cosh(d*x+c)^3+4*sinh(d*x+c)^2/cosh(d*x+c)^3+8/3/cosh(d
*x+c)^3)+b^3*(sinh(d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*
sinh(d*x+c)^2/cosh(d*x+c)^5+16/5/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 383 vs. $2(68) = 136$.

Time = 0.09 (sec) , antiderivative size = 383, normalized size of antiderivative = 5.47

$$\int \sinh(c+dx)\left(a+b\tanh^{2}(c+dx)\right)^{3}\,dx$$
$$=\frac{5\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\cosh\left(dx+c\right)^{6}+5\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sinh\left(dx+c\right)^{6}+30\left(a^{3}+5\,a^{2}b+7\right.}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/10*(5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 5*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*sinh(d*x + c)^6 + 30*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*c
osh(d*x + c)^4 + 15*(2*a^3 + 10*a^2*b + 14*a*b^2 + 6*b^3 + 5*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 50*a^3 + 330*a^2*b +
 430*a*b^2 + 182*b^3 + 5*(15*a^3 + 93*a^2*b + 125*a*b^2 + 47*b^3)*cosh(d*x
 + c)^2 + 5*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 15*a^3 +
 93*a^2*b + 125*a*b^2 + 47*b^3 + 36*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2)/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(
d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x +
 c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)^3\sinh\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(68) = 136.

Time = 0.05 (sec) , antiderivative size = 321, normalized size of antiderivative = 4.59

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{10}b^3\left(\frac{5e^{(-dx-c)}}{d}+\frac{85e^{(-2dx-2c)}+210e^{(-4dx-4c)}+314e^{(-6dx-6c)}+185e^{(-8dx-8c)}+65e^{(-10dx-10c)}+\cdots}{d(e^{(-dx-c)}+5e^{(-3dx-3c)}+10e^{(-5dx-5c)}+10e^{(-7dx-7c)}+5e^{(-9dx-9c)}+e^{(-11dx-11c)}\cdots}\right.$$
$$+\frac{1}{2}ab^2\left(\frac{3e^{(-dx-c)}}{d}+\frac{33e^{(-2dx-2c)}+41e^{(-4dx-4c)}+27e^{(-6dx-6c)}+3}{d(e^{(-dx-c)}+3e^{(-3dx-3c)}+3e^{(-5dx-5c)}+e^{(-7dx-7c)})}\right)$$
$$+\frac{3}{2}a^2b\left(\frac{e^{(-dx-c)}}{d}+\frac{5e^{(-2dx-2c)}+1}{d(e^{(-dx-c)}+e^{(-3dx-3c)})}\right)+\frac{a^3\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/10*b^3*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x - 4*c) +
 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c) + 5)/
(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^(-7*d*x
 - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + 1/2*a*b^2*(3*e^(-d*
x - c)/d + (33*e^(-2*d*x - 2*c) + 41*e^(-4*d*x - 4*c) + 27*e^(-6*d*x - 6*c
) + 3)/(d*(e^(-d*x - c) + 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) + e^(-7*
d*x - 7*c)))) + 3/2*a^2*b*(e^(-d*x - c)/d + (5*e^(-2*d*x - 2*c) + 1)/(d*(e
^(-d*x - c) + e^(-3*d*x - 3*c)))) + a^3*cosh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time = 0.22 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.37

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$= \frac{5\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)+15\,ab^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)+5\,b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right.}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/10*(5*a^3*(e^(d*x + c) + e^(-d*x - c)) + 15*a^2*b*(e^(d*x + c) + e^(-d*x
 - c)) + 15*a*b^2*(e^(d*x + c) + e^(-d*x - c)) + 5*b^3*(e^(d*x + c) + e^(-
d*x - c)) + 4*(15*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 30*a*b^2*(e^(d*x
+ c) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4 - 20*a*b^2*
(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*x + c) + e^(-d*x - c))^2 + 1
6*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 308, normalized size of antiderivative = 4.40

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{e^{c+dx}(a+b)^3}{2d}+\frac{e^{-c-dx}(a+b)^3}{2d}+\frac{6\,e^{c+dx}(a^2b+2ab^2+b^3)}{d\,(e^{2c+2dx}+1)} - \frac{64\,b^3\,e^{c+dx}}{5d\,(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1)} + \frac{8\,e^{c+dx}(9b^3+5ab^2)}{5d\,(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1)} + \frac{32\,b^3\,e^{c+dx}}{5d\,(5\,e^{2c+2dx}+10\,e^{4c+4dx}+10\,e^{6c+6dx}+5\,e^{8c+8dx}+e^{10c+10dx}+1)} - \frac{8\,e^{c+dx}(b^3+ab^2)}{d\,(2\,e^{2c+2dx}+e^{4c+4dx}+1)}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a + b)^3)/(2*d) + (6*exp
(c + d*x)*(2*a*b^2 + a^2*b + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (64*b^3*ex
p(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*
d*x) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(5*a*b^2 + 9*b^3))/(5*d*(3
*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (32*b^3*
exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c
+ 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (8*exp(c + d*x)
*(a*b^2 + b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 697, normalized size of antiderivative = 9.96

$$\int \sinh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{180e^{4dx+4c}ab^2+75b^3+120e^{4dx+4c}a^2b+180ab^2+75e^{4dx+4c}b^3+1080e^{2dx+2c}ab^2+450e^{2dx+2c}b^3-40e^{3dx}}{}$$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 4*e**(3*c + 3*d*x)*cosh(c + d*x)*tanh(c + d*x)**4*b**3 - 40*e**(3*c +
3*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*a*b**2 - 22*e**(3*c + 3*d*x)*cosh(c
+ d*x)*tanh(c + d*x)**2*b**3 + 80*e**(3*c + 3*d*x)*cosh(c + d*x)*a**3 - 80
*e**(3*c + 3*d*x)*cosh(c + d*x)*a*b**2 - 44*e**(3*c + 3*d*x)*cosh(c + d*x)
*b**3 - 4*e**(c + d*x)*cosh(c + d*x)*tanh(c + d*x)**4*b**3 - 40*e**(c + d*
x)*cosh(c + d*x)*tanh(c + d*x)**2*a*b**2 - 22*e**(c + d*x)*cosh(c + d*x)*t
anh(c + d*x)**2*b**3 + 80*e**(c + d*x)*cosh(c + d*x)*a**3 - 80*e**(c + d*x
)*cosh(c + d*x)*a*b**2 - 44*e**(c + d*x)*cosh(c + d*x)*b**3 + 120*e**(4*c
+ 4*d*x)*a**2*b + 180*e**(4*c + 4*d*x)*a*b**2 + 75*e**(4*c + 4*d*x)*b**3 -
 16*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)**5*b**3 - 80*e**(3*c + 3*
d*x)*sinh(c + d*x)*tanh(c + d*x)**3*a*b**2 - 28*e**(3*c + 3*d*x)*sinh(c +
d*x)*tanh(c + d*x)**3*b**3 + 80*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*
x)*a*b**2 + 44*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)*b**3 + 720*e**
(2*c + 2*d*x)*a**2*b + 1080*e**(2*c + 2*d*x)*a*b**2 + 450*e**(2*c + 2*d*x)
*b**3 - 16*e**(c + d*x)*sinh(c + d*x)*tanh(c + d*x)**5*b**3 - 80*e**(c + d
*x)*sinh(c + d*x)*tanh(c + d*x)**3*a*b**2 - 28*e**(c + d*x)*sinh(c + d*x)*
tanh(c + d*x)**3*b**3 + 80*e**(c + d*x)*sinh(c + d*x)*tanh(c + d*x)*a*b**2
 + 44*e**(c + d*x)*sinh(c + d*x)*tanh(c + d*x)*b**3 + 120*a**2*b + 180*a*b
**2 + 75*b**3)/(80*e**(c + d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.21 $\int \operatorname{csch}(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 84

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = -\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)}{d} + \frac{b^2(3a+2b)\operatorname{sech}^3(c+dx)}{3d} - \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

output
```
-a^3*arctanh(cosh(d*x+c))/d-b*(3*a^2+3*a*b+b^2)*sech(d*x+c)/d+1/3*b^2*(3*a
+2*b)*sech(d*x+c)^3/d-1/5*b^3*sech(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 9.25 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.11

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{15a^3\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right)-15b(3a^2+3ab+b^2)\operatorname{sech}(c+dx)+5b^2(3a+}{15d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(15*a^3*(-Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)/2]]) - 15*b*(3*a^2 +
 3*a*b + b^2)*Sech[c + d*x] + 5*b^2*(3*a + 2*b)*Sech[c + d*x]^3 - 3*b^3*Se
ch[c + d*x]^5)/(15*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 26, 4147, 25, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$-\frac{\int \left(b^3\text{sech}^4(c+dx)-b^2(3a+2b)\text{sech}^2(c+dx)+b\left(3a^2+3ba+b^2\right)+\frac{a^3}{1-\text{sech}^2(c+dx)}\right)d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-a^3\text{arctanh}(\text{sech}(c+dx))-b\left(3a^2+3ab+b^2\right)\text{sech}(c+dx)+\frac{1}{3}b^2(3a+2b)\text{sech}^3(c+dx)-\frac{1}{5}b^3\text{sech}^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^3*ArcTanh[Sech[c + d*x]]) - b*(3*a^2 + 3*a*b + b^2)*Sech[c + d*x] + (
b^2*(3*a + 2*b)*Sech[c + d*x]^3)/3 - (b^3*Sech[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.54 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.40

method	result
derivativedivides	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{3a^2b}{\cosh(dx+c)}+3b^2a\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(\cdots}\right.}{d}$
default	$\frac{-2a^3 \operatorname{arctanh}\left(e^{dx+c}\right)-\frac{3a^2b}{\cosh(dx+c)}+3b^2a\left(-\frac{\sinh(dx+c)^2}{\cosh(dx+c)^3}-\frac{2}{3\cosh(dx+c)^3}\right)+b^3\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{8}{15\cosh(\cdots}\right.}{d}$
risch	$-\frac{2b\,e^{dx+c}\left(45\,e^{8dx+8c}a^2+45\,e^{8dx+8c}ab+15\,e^{8dx+8c}b^2+180\,e^{6dx+6c}a^2+120\,e^{6dx+6c}ab+20\,e^{6dx+6c}b^2+270\,e^{4dx+4c}a^2\cdots\right.}{15d\left(e^{2dx+2c}+1\right)^5\cdots}$

input
```
int(csch(d*x+c)*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3*arctanh(exp(d*x+c))-3*a^2*b/cosh(d*x+c)+3*b^2*a*(-sinh(d*x+c)^
2/cosh(d*x+c)^3-2/3/cosh(d*x+c)^3)+b^3*(-sinh(d*x+c)^4/cosh(d*x+c)^5-4/3*s
inh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2277 vs. $2(80) = 160$.

Time = 0.10 (sec) , antiderivative size = 2277, normalized size of antiderivative = 27.11

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/15*(30*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^9 + 270*(3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + 30*(3*a^2*b + 3*a*b^2 + b^3)*sin
h(d*x + c)^9 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^7 + 40*(9*a^2*b
+ 6*a*b^2 + b^3 + 27*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^7 + 280*(9*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (9*a^2*b + 6*a*
b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(135*a^2*b + 75*a*b^2 + 29*b
^3)*cosh(d*x + c)^5 + 4*(945*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 1
35*a^2*b + 75*a*b^2 + 29*b^3 + 210*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)
^2)*sinh(d*x + c)^5 + 20*(189*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^5 +
70*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^3 + (135*a^2*b + 75*a*b^2 + 29*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*
x + c)^3 + 40*(63*(3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(9*a^2*b
+ 6*a*b^2 + b^3)*cosh(d*x + c)^4 + 9*a^2*b + 6*a*b^2 + b^3 + (135*a^2*b +
75*a*b^2 + 29*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 40*(27*(3*a^2*b + 3*
a*b^2 + b^3)*cosh(d*x + c)^7 + 21*(9*a^2*b + 6*a*b^2 + b^3)*cosh(d*x + c)^
5 + (135*a^2*b + 75*a*b^2 + 29*b^3)*cosh(d*x + c)^3 + 3*(9*a^2*b + 6*a*b^2
 + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 30*(3*a^2*b + 3*a*b^2 + b^3)*cosh
(d*x + c) + 15*(a^3*cosh(d*x + c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^
9 + a^3*sinh(d*x + c)^10 + 5*a^3*cosh(d*x + c)^8 + 10*a^3*cosh(d*x + c)^6
+ 5*(9*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^8 + 40*(3*a^3*cosh(d*x ...
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)^3\operatorname{csch}{(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 560 vs. $2(80) = 160$.

Time = 0.04 (sec) , antiderivative size = 560, normalized size of antiderivative = 6.67

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-2/15*b^3*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e
^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x -
5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) +
5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*
e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x
 - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) - 2*a*b^2*(3*e^(-d*x - c)/(d*(3*e^(-2*d*x - 2*c)
+ 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 2*e^(-3*d*x - 3*c)/(d*(3*e
^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 3*e^(-5*d*
x - 5*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) +
1))) + a^3*log(tanh(1/2*d*x + 1/2*c))/d - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*
x - c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(80) = 160$.

Time = 0.22 (sec) , antiderivative size = 196, normalized size of antiderivative = 2.33

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx =$$

$$-\frac{15\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)-15\,a^3\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)+\frac{4\left(45\,a^2b\left(e^{(dx+c)}+e^{(-dx-c)}\right)^4+45\,ab^2\left(e^{(\cdots\right.\right.}{\cdots}}{30\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/30*(15*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2) - 15*a^3*log(e^(d*x + c)
 + e^(-d*x - c) - 2) + 4*(45*a^2*b*(e^(d*x + c) + e^(-d*x - c))^4 + 45*a*b
^2*(e^(d*x + c) + e^(-d*x - c))^4 + 15*b^3*(e^(d*x + c) + e^(-d*x - c))^4
- 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 40*b^3*(e^(d*x + c) + e^(-d*x
- c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.47 (sec) , antiderivative size = 317, normalized size of antiderivative = 3.77

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{8\,e^{c+d\,x}\,(2\,b^3+3\,a\,b^2)}{3\,d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)} - \frac{2\,\mathrm{atan}\left(\frac{a^3\,e^{d\,x}\,e^{c}\,\sqrt{-d^2}}{d\,\sqrt{a^6}}\right)\,\sqrt{a^6}}{\sqrt{-d^2}}$$
$$- \frac{2\,e^{c+d\,x}\,(3\,a^2\,b+3\,a\,b^2+b^3)}{d\,(e^{2\,c+2\,d\,x}+1)}$$
$$+ \frac{64\,b^3\,e^{c+d\,x}}{5\,d\,(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1)}$$
$$- \frac{8\,e^{c+d\,x}\,(22\,b^3+15\,a\,b^2)}{15\,d\,(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1)}$$
$$- \frac{32\,b^3\,e^{c+d\,x}}{5\,d\,(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x),x)
```

output
```
(8*exp(c + d*x)*(3*a*b^2 + 2*b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*
d*x) + 1)) - (2*atan((a^3*exp(d*x)*exp(c)*(-d^2)^(1/2))/(d*(a^6)^(1/2)))*(
a^6)^(1/2))/(-d^2)^(1/2) - (2*exp(c + d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(d*(
exp(2*c + 2*d*x) + 1)) + (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) +
6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (8*exp(
c + d*x)*(15*a*b^2 + 22*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*
x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(
10*c + 10*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 574, normalized size of antiderivative = 6.83

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{15e^{10dx+10c}\log\left(e^{dx+c}-1\right)a^3-15e^{10dx+10c}\log\left(e^{dx+c}+1\right)a^3-90e^{9dx+9c}a^2b-30e^{9dx+9c}b^3-40e^{7dx+7c}b^3-}{}$$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(15*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a**3 - 15*e**(10*c + 10*d*x)*
log(e**(c + d*x) + 1)*a**3 - 90*e**(9*c + 9*d*x)*a**2*b - 90*e**(9*c + 9*d
*x)*a*b**2 - 30*e**(9*c + 9*d*x)*b**3 + 75*e**(8*c + 8*d*x)*log(e**(c + d*
x) - 1)*a**3 - 75*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**3 - 360*e**(7*
c + 7*d*x)*a**2*b - 240*e**(7*c + 7*d*x)*a*b**2 - 40*e**(7*c + 7*d*x)*b**3
 + 150*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**3 - 150*e**(6*c + 6*d*x)*
log(e**(c + d*x) + 1)*a**3 - 540*e**(5*c + 5*d*x)*a**2*b - 300*e**(5*c + 5
*d*x)*a*b**2 - 116*e**(5*c + 5*d*x)*b**3 + 150*e**(4*c + 4*d*x)*log(e**(c
+ d*x) - 1)*a**3 - 150*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**3 - 360*e
**(3*c + 3*d*x)*a**2*b - 240*e**(3*c + 3*d*x)*a*b**2 - 40*e**(3*c + 3*d*x)
*b**3 + 75*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**3 - 75*e**(2*c + 2*d*
x)*log(e**(c + d*x) + 1)*a**3 - 90*e**(c + d*x)*a**2*b - 90*e**(c + d*x)*a
*b**2 - 30*e**(c + d*x)*b**3 + 15*log(e**(c + d*x) - 1)*a**3 - 15*log(e**(
c + d*x) + 1)*a**3)/(15*d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e*
*(6*c + 6*d*x) + 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.22 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 64

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh(c+dx)}{d}+\frac{ab^2\tanh^3(c+dx)}{d}+\frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
-a^3*coth(d*x+c)/d+3*a^2*b*tanh(d*x+c)/d+a*b^2*tanh(d*x+c)^3/d+1/5*b^3*tan
h(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 2.19 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.09

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{-5a^3\coth(c+dx)+b\left(15a^2+5ab+b^2-b(5a+2b)\operatorname{sech}^2(c+dx)+b^2\operatorname{sech}^4(c+dx)\right)\tanh(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-5*a^3*Coth[c + d*x] + b*(15*a^2 + 5*a*b + b^2 - b*(5*a + 2*b)*Sech[c + d
*x]^2 + b^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 244, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{244}$$

$$\frac{\int \left(b^3\tanh^4(c+dx)+3ab^2\tanh^2(c+dx)+a^3\coth^2(c+dx)+3a^2b\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^3\coth(c+dx)+3a^2b\tanh(c+dx)+ab^2\tanh^3(c+dx)+\frac{1}{5}b^3\tanh^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^3*Coth[c + d*x]) + 3*a^2*b*Tanh[c + d*x] + a*b^2*Tanh[c + d*x]^3 + (b
^3*Tanh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 244
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[Expand
Integrand[(c*x)^m*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x] && IGtQ[p
, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(62) = 124$.

Time = 12.01 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.20

method	result
derivativedivides	$\frac{-\coth(dx+c)a^3+3\tanh(dx+c)a^2b+3b^2a\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^3\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3}{8}\right.}{d}$
default	$\frac{-\coth(dx+c)a^3+3\tanh(dx+c)a^2b+3b^2a\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^3\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3}{8}\right.}{d}$
risch	$-\frac{2(5\,e^{10dx+10c}a^3+15a^2b\,e^{10dx+10c}+15a\,b^2e^{10dx+10c}+5b^3e^{10dx+10c}+25a^3e^{8dx+8c}+45a^2b\,e^{8dx+8c}+15a\,b^2e^{8dx+8c}}{}$

input
```
int(csch(d*x+c)^2*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^3+3*tanh(d*x+c)*a^2*b+3*b^2*a*(-1/2*sinh(d*x+c)/cosh(d
*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^3/c
osh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/1
5*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 572 vs. $2(62) = 124$.

Time = 0.09 (sec) , antiderivative size = 572, normalized size of antiderivative = 8.94

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-4/5*((5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 5*a*b^2 + 2*b
^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (15*a^2*b + 10*a*b^2 + 3*b^3)*sinh(d*x
 + c)^5 + (25*a^3 + 5*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + (45*a^2*b + 10*a*b^
2 - 3*b^3 + 10*(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^3 + (10*(5*a^3 + 5*a*b^2 + 2*b^3)*cosh(d*x + c)^3 + 3*(25*a^3 + 5*a*b^2
- 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 - a*b^2)*cosh(d*x + c)
 + (5*(15*a^2*b + 10*a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 30*a^2*b + 10*b^3 +
3*(45*a^2*b + 10*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*
x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cos
h(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(
d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*
d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*c
osh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 -
 5*d*cosh(d*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*co
sh(d*x + c)^2 + 5*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^2{(c+dx)}\right)^3 \operatorname{csch}^2{(c+dx)}\, dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(62) = 124$.

Time = 0.05 (sec) , antiderivative size = 348, normalized size of antiderivative = 5.44

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{2}{5}b^3\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}\ldots}\right.$$
$$+2\,ab^2\left(\frac{3\,e^{(-4\,dx-4\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1\ldots}\right.$$
$$+\frac{6\,a^2b}{d(e^{(-2\,dx-2\,c)}+1)}+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
2/5*b^3*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*
e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 2*a*b^2*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*
x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x
 - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 6*a^2*b/(d*(e^(-2
*d*x - 2*c) + 1)) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(62) = 124$.

Time = 0.22 (sec) , antiderivative size = 202, normalized size of antiderivative = 3.16

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{2\left(\frac{5\,a^3}{e^{(2\,dx+2\,c)}-1}+\frac{15\,a^2be^{(8\,dx+8\,c)}+15\,ab^2e^{(8\,dx+8\,c)}+5\,b^3e^{(8\,dx+8\,c)}+60\,a^2be^{(6\,dx+6\,c)}+30\,ab^2e^{(6\,dx+6\,c)}+90\,a^2be^{(4\,dx+4\,c)}+20\,ab^2e^{(4\ldots}}{\left(e^{(2\,dx+2\,c)}+1\right)^5}\ldots}{5\,d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(8*d*x + 8*c) + 15*a*b^2*e
^(8*d*x + 8*c) + 5*b^3*e^(8*d*x + 8*c) + 60*a^2*b*e^(6*d*x + 6*c) + 30*a*b
^2*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 20*a*b^2*e^(4*d*x + 4*c) +
 10*b^3*e^(4*d*x + 4*c) + 60*a^2*b*e^(2*d*x + 2*c) + 10*a*b^2*e^(2*d*x + 2
*c) + 15*a^2*b + 5*a*b^2 + b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.48 (sec) , antiderivative size = 590, normalized size of antiderivative = 9.22

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{\frac{2\left(3a^2b-b^3\right)}{5d}+\frac{2e^{2c+2dx}\left(3a^2b+3ab^2+b^3\right)}{5d}}{2e^{2c+2dx}+e^{4c+4dx}+1}$$
$$-\frac{\frac{2\left(3a^2b-ab^2+b^3\right)}{5d}+\frac{2e^{4c+4dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{4e^{2c+2dx}\left(3a^2b-b^3\right)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1}$$
$$-\frac{\frac{2\left(3a^2b-b^3\right)}{5d}+\frac{6e^{2c+2dx}\left(3a^2b-ab^2+b^3\right)}{5d}+\frac{2e^{6c+6dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{6e^{4c+4dx}\left(3a^2b-b^3\right)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1}$$
$$-\frac{\frac{2\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{12e^{4c+4dx}\left(3a^2b-ab^2+b^3\right)}{5d}+\frac{2e^{8c+8dx}\left(3a^2b+3ab^2+b^3\right)}{5d}+\frac{8e^{2c+2dx}\left(3a^2b-b^3\right)}{5d}+\frac{8e^{6c+6dx}\left(3a^2b-\right.}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1}$$
$$-\frac{2a^3}{d\left(e^{2c+2dx}-1\right)}-\frac{2\left(3a^2b+3ab^2+b^3\right)}{5d\left(e^{2c+2dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^2,x)
```

output
```
- ((2*(3*a^2*b - b^3))/(5*d) + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*b + b^
3))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(3*a^2*b - a*
b^2 + b^3))/(5*d) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) +
 (4*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4
*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a^2*b - b^3))/(5*d) + (6*exp(
2*c + 2*d*x)*(3*a^2*b - a*b^2 + b^3))/(5*d) + (2*exp(6*c + 6*d*x)*(3*a*b^2
 + 3*a^2*b + b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a^2*b - b^3))/(5*d))/(4*
exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d
*x) + 1) - ((2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) + (12*exp(4*c + 4*d*x)*(3*
a^2*b - a*b^2 + b^3))/(5*d) + (2*exp(8*c + 8*d*x)*(3*a*b^2 + 3*a^2*b + b^3
))/(5*d) + (8*exp(2*c + 2*d*x)*(3*a^2*b - b^3))/(5*d) + (8*exp(6*c + 6*d*x
)*(3*a^2*b - b^3))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*e
xp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (2*a^3)/(
d*(exp(2*c + 2*d*x) - 1)) - (2*(3*a*b^2 + 3*a^2*b + b^3))/(5*d*(exp(2*c +
2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 400, normalized size of antiderivative = 6.25

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{5e^{12dx+12c}a^3+15e^{12dx+12c}a^2b+15e^{12dx+12c}a\,b^2+5e^{12dx+12c}b^3-75e^{8dx+8c}a^3-105e^{8dx+8c}a^2b+15e^{8dx+8c}}{}$$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(5*e**(12*c + 12*d*x)*a**3 + 15*e**(12*c + 12*d*x)*a**2*b + 15*e**(12*c +
12*d*x)*a*b**2 + 5*e**(12*c + 12*d*x)*b**3 - 75*e**(8*c + 8*d*x)*a**3 - 10
5*e**(8*c + 8*d*x)*a**2*b + 15*e**(8*c + 8*d*x)*a*b**2 + 45*e**(8*c + 8*d*
x)*b**3 - 200*e**(6*c + 6*d*x)*a**3 - 120*e**(6*c + 6*d*x)*a**2*b + 40*e**
(6*c + 6*d*x)*a*b**2 - 40*e**(6*c + 6*d*x)*b**3 - 225*e**(4*c + 4*d*x)*a**
3 + 45*e**(4*c + 4*d*x)*a**2*b - 35*e**(4*c + 4*d*x)*a*b**2 + 15*e**(4*c +
 4*d*x)*b**3 - 120*e**(2*c + 2*d*x)*a**3 + 120*e**(2*c + 2*d*x)*a**2*b - 4
0*e**(2*c + 2*d*x)*a*b**2 - 24*e**(2*c + 2*d*x)*b**3 - 25*a**3 + 45*a**2*b
 + 5*a*b**2 - b**3)/(10*d*(e**(12*c + 12*d*x) + 4*e**(10*c + 10*d*x) + 5*e
**(8*c + 8*d*x) - 5*e**(4*c + 4*d*x) - 4*e**(2*c + 2*d*x) - 1))
```

3.23 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 101

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{a^2(a-6b)\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{3a^2b\operatorname{sech}(c+dx)}{d} - \frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d} + \frac{b^3\operatorname{sech}^5(c+dx)}{5d}$$

output
```
1/2*a^2*(a-6*b)*arctanh(cosh(d*x+c))/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d+3
*a^2*b*sech(d*x+c)/d-1/3*b^2*(3*a+b)*sech(d*x+c)^3/d+1/5*b^3*sech(d*x+c)^5
/d
```

Mathematica [A] (verified)

Time = 16.31 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.57

$$\begin{aligned}&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=-\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{(a^3-6a^2b)\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}\\&\quad+\frac{(-a^3+6a^2b)\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}\\&\quad+\frac{3a^2b\operatorname{sech}(c+dx)}{d}-\frac{b^2(3a+b)\operatorname{sech}^3(c+dx)}{3d}+\frac{b^3\operatorname{sech}^5(c+dx)}{5d}\end{aligned}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/8*(a^3*Csch[(c + d*x)/2]^2)/d + ((a^3 - 6*a^2*b)*Log[Cosh[(c + d*x)/2]]
)/(2*d) + ((-a^3 + 6*a^2*b)*Log[Sinh[(c + d*x)/2]])/(2*d) - (a^3*Sech[(c +
 d*x)/2]^2)/(8*d) + (3*a^2*b*Sech[c + d*x])/d - (b^2*(3*a + b)*Sech[c + d*
x]^3)/(3*d) + (b^3*Sech[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.59, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 26, 4147, 369, 403, 25, 403, 25, 299, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^3}dx$$

$\downarrow$ 4147

$$-\frac{\int \frac{\operatorname{sech}^2(c+dx)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^3}{\left(1-\operatorname{sech}^2(c+dx)\right)^2}d\operatorname{sech}(c+dx)}{d}$$

$\downarrow$ 369

$$-\frac{\frac{\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+dx)+b\right)^3}{2\left(1-\operatorname{sech}^2(c+dx)\right)}-\frac{1}{2}\int \frac{\left(-7b\operatorname{sech}^2(c+dx)+a+b\right)\left(-b\operatorname{sech}^2(c+dx)+a+b\right)^2}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)}{d}$$

$\downarrow$ 403

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\int -\frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)\left((5a-2b)(a+b)-(33a-2b)b\operatorname{sech}^2(c+dx)\right)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)-\frac{7}{5}b\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+\right.}{d}$$

$\downarrow$ 25

$$-\frac{\frac{1}{2}\left(-\frac{1}{5}\int \frac{\left(-b\operatorname{sech}^2(c+dx)+a+b\right)\left((5a-2b)(a+b)-(33a-2b)b\operatorname{sech}^2(c+dx)\right)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)-\frac{7}{5}b\operatorname{sech}(c+dx)\left(a-b\operatorname{sech}^2(c+\right.}{d}$$

$\downarrow$ 403

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\int -\frac{(a+b)(15a^2-24ba-4b^2)-b(81a^2-28ba-4b^2)\operatorname{sech}^2(c+dx)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)-\frac{1}{3}b(33a-2b)\operatorname{sech}(c+dx)\left(a-b\operatorname{sec}\right.\right.\right.}{d}$$

$\downarrow$ 25

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(-\frac{1}{3}\int \frac{(a+b)(15a^2-24ba-4b^2)-b(81a^2-28ba-4b^2)\operatorname{sech}^2(c+dx)}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)-\frac{1}{3}b(33a-2b)\operatorname{sech}(c+dx)\left(a-b\operatorname{sec}\right.\right.\right.}{d}$$

$\downarrow$ 299

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(-15a^2(a-6b)\int \frac{1}{1-\operatorname{sech}^2(c+dx)}d\operatorname{sech}(c+dx)-b\left(81a^2-28ab-4b^2\right)\operatorname{sech}(c+dx)\right)-\frac{1}{3}b(33a-2b)\operatorname{se}\right.\right.}{}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{1}{2}\left(\frac{1}{5}\left(\frac{1}{3}\left(-15a^2(a-6b)\text{arctanh}(\text{sech}(c+dx))-b\left(81a^2-28ab-4b^2\right)\text{sech}(c+dx)\right)-\frac{1}{3}b(33a-2b)\text{sech}(c+dx\right.\right.}{d}$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-(((Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^3)/(2*(1 - Sech[c + d*x]^2))
 + ((-7*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2)^2)/5 + ((-15*a^2*(a -
6*b)*ArcTanh[Sech[c + d*x]] - b*(81*a^2 - 28*a*b - 4*b^2)*Sech[c + d*x])/3
 - ((33*a - 2*b)*b*Sech[c + d*x]*(a + b - b*Sech[c + d*x]^2))/3)/5)/2)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 369
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(2*
b*(p + 1))), x] - Simp[e^2/(2*b*(p + 1))   Int[(e*x)^(m - 2)*(a + b*x^2)^(p
 + 1)*(c + d*x^2)^(q - 1)*Simp[c*(m - 1) + d*(m + 2*q - 1)*x^2, x], x], x]
/; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[q, 0
] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 403
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(
x_)^2), x_Symbol] :> Simp[f*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*(2*(p +
 q + 1) + 1))), x] + Simp[1/(b*(2*(p + q + 1) + 1))   Int[(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[c*(b*e - a*f + b*e*2*(p + q + 1)) + (d*(b*e - a*f) +
f*2*q*(b*c - a*d) + b*d*e*2*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c,
 d, e, f, p}, x] && GtQ[q, 0] && NeQ[2*(p + q + 1) + 1, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 21.89 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.02

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{b^2a}{\cosh(dx+c)^3}+b^3\left(-\frac{\sinh(dx+c}{3\cosh(dx+}\right.}{d}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{1}{\cosh(dx+c)}-2\operatorname{arctanh}\left(e^{dx+c}\right)\right)-\frac{b^2a}{\cosh(dx+c)^3}+b^3\left(-\frac{\sinh(dx+c}{3\cosh(dx+}\right.}{d}$
risch	$-\frac{e^{dx+c}\left(15a^3e^{12dx+12c}-90a^2b\,e^{12dx+12c}+90\,e^{10dx+10c}a^3-180a^2b\,e^{10dx+10c}+120a\,b^2e^{10dx+10c}+40b^3e^{10dx+10c}+2\right.}{}$

input
```
int(csch(d*x+c)^3*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(1/cos
h(d*x+c)-2*arctanh(exp(d*x+c)))-b^2*a/cosh(d*x+c)^3+b^3*(-1/3*sinh(d*x+c)^
2/cosh(d*x+c)^5-2/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5037 vs. $2(93) = 186$.

Time = 0.14 (sec) , antiderivative size = 5037, normalized size of antiderivative = 49.87

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{csch}^3\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(93) = 186$.

Time = 0.04 (sec) , antiderivative size = 403, normalized size of antiderivative = 3.99

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$- 3\,a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} - \frac{2\,e^{(-dx-c)}}{d(e^{(-2\,dx-2\,c)}+1)}\right)$$
$$- \frac{8}{15}\,b^3\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-}}\right.$$
$$- \frac{8\,ab^2}{d(e^{(dx+c)}+e^{(-dx-c)})^3}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x -
c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) -
3*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d - 2*e^(-d*x - c
)/(d*(e^(-2*d*x - 2*c) + 1))) - 8/15*b^3*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*
x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
1))) - 8*a*b^2/(d*(e^(d*x + c) + e^(-d*x - c))^3)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(93) = 186$.

Time = 0.23 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.04

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{\frac{60\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4} - 15\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right) + 15\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-}\right.}{60\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/60*(60*a^3*(e^(d*x + c) + e^(-d*x - c))/((e^(d*x + c) + e^(-d*x - c))^2
 - 4) - 15*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2) + 15*(a^3 -
 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2) - 8*(45*a^2*b*(e^(d*x + c) +
 e^(-d*x - c))^4 - 60*a*b^2*(e^(d*x + c) + e^(-d*x - c))^2 - 20*b^3*(e^(d*
x + c) + e^(-d*x - c))^2 + 48*b^3)/(e^(d*x + c) + e^(-d*x - c))^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 412, normalized size of antiderivative = 4.08

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^c\,\left(a^3\,\sqrt{-d^2}-6\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}\right)\,\sqrt{a^6-12\,a^5\,b+36\,a^4\,b^2}}{\sqrt{-d^2}} - \frac{64\,b^3\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)} + \frac{8\,e^{c+d\,x}\,\left(17\,b^3+15\,a\,b^2\right)}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)} + \frac{32\,b^3\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)} - \frac{a^3\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}-1\right)} - \frac{8\,e^{c+d\,x}\,\left(b^3+3\,a\,b^2\right)}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)} - \frac{2\,a^3\,e^{c+d\,x}}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} + \frac{6\,a^2\,b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(a^3*(-d^2)^(1/2) - 6*a^2*b*(-d^2)^(1/2)))/(d*(a^6
- 12*a^5*b + 36*a^4*b^2)^(1/2)))*(a^6 - 12*a^5*b + 36*a^4*b^2)^(1/2))/(-d^
2)^(1/2) - (64*b^3*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*
d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (8*exp(c + d*x)*(15*a
*b^2 + 17*b^3))/(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c +
 6*d*x) + 1)) + (32*b^3*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*
c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x)
 + 1)) - (a^3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (8*exp(c + d*x)*(
3*a*b^2 + b^3))/(3*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (2*a^3
*exp(c + d*x))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (6*a^2*b*
exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1198, normalized size of antiderivative = 11.86

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
( - 15*e**(14*c + 14*d*x)*log(e**(c + d*x) - 1)*a**3 + 90*e**(14*c + 14*d*
x)*log(e**(c + d*x) - 1)*a**2*b + 15*e**(14*c + 14*d*x)*log(e**(c + d*x) +
 1)*a**3 - 90*e**(14*c + 14*d*x)*log(e**(c + d*x) + 1)*a**2*b - 30*e**(13*
c + 13*d*x)*a**3 + 180*e**(13*c + 13*d*x)*a**2*b - 45*e**(12*c + 12*d*x)*l
og(e**(c + d*x) - 1)*a**3 + 270*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a
**2*b + 45*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*a**3 - 270*e**(12*c +
12*d*x)*log(e**(c + d*x) + 1)*a**2*b - 180*e**(11*c + 11*d*x)*a**3 + 360*e
**(11*c + 11*d*x)*a**2*b - 240*e**(11*c + 11*d*x)*a*b**2 - 80*e**(11*c + 1
1*d*x)*b**3 - 15*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a**3 + 90*e**(10
*c + 10*d*x)*log(e**(c + d*x) - 1)*a**2*b + 15*e**(10*c + 10*d*x)*log(e**(
c + d*x) + 1)*a**3 - 90*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a**2*b -
450*e**(9*c + 9*d*x)*a**3 - 180*e**(9*c + 9*d*x)*a**2*b + 192*e**(9*c + 9*
d*x)*b**3 + 75*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**3 - 450*e**(8*c +
 8*d*x)*log(e**(c + d*x) - 1)*a**2*b - 75*e**(8*c + 8*d*x)*log(e**(c + d*x
) + 1)*a**3 + 450*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**2*b - 600*e**(
7*c + 7*d*x)*a**3 - 720*e**(7*c + 7*d*x)*a**2*b + 480*e**(7*c + 7*d*x)*a*b
**2 - 224*e**(7*c + 7*d*x)*b**3 + 75*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1
)*a**3 - 450*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**2*b - 75*e**(6*c +
6*d*x)*log(e**(c + d*x) + 1)*a**3 + 450*e**(6*c + 6*d*x)*log(e**(c + d*x)
+ 1)*a**2*b - 450*e**(5*c + 5*d*x)*a**3 - 180*e**(5*c + 5*d*x)*a**2*b +...
```

3.24 $\int \text{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \text{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{a^2(a-3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{3a(a-b)b\tanh(c+dx)}{d} - \frac{(3a-b)b^2\tanh^3(c+dx)}{3d} - \frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
a^2*(a-3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-3*a*(a-b)*b*tanh(d*x+c)/
d-1/3*(3*a-b)*b^2*tanh(d*x+c)^3/d-1/5*b^3*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 2.43 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89

$$\int \text{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{-5a^2\coth(c+dx)\left(-2a+9b+a\text{csch}^2(c+dx)\right)+b\left(-45a^2+30ab+2b^2+b(15a+b)\text{sech}^2(c+dx)-3\right.}{15d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-5*a^2*Coth[c + d*x]*(-2*a + 9*b + a*Csch[c + d*x]^2) + b*(-45*a^2 + 30*a
*b + 2*b^2 + b*(15*a + b)*Sech[c + d*x]^2 - 3*b^2*Sech[c + d*x]^4)*Tanh[c
+ d*x])/(15*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 355, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{355}$$

$$\frac{\int \left(a^3\coth^4(c+dx)-a^2(a-3b)\coth^2(c+dx)-b^3\tanh^4(c+dx)-(3a-b)b^2\tanh^2(c+dx)-3a(a-b)b\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)+a^2(a-3b)\coth(c+dx)-\frac{1}{3}b^2(3a-b)\tanh^3(c+dx)-3ab(a-b)\tanh(c+dx)-\frac{1}{5}b^3\tan}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^2*(a - 3*b)*Coth[c + d*x] - (a^3*Coth[c + d*x]^3)/3 - 3*a*(a - b)*b*Tan
h[c + d*x] - ((3*a - b)*b^2*Tanh[c + d*x]^3)/3 - (b^3*Tanh[c + d*x]^5)/5)/
d
```

Defintions of rubi rules used

rule 355
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.), x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*(a + b*x^2)^p*(c + d*x^2)^q,
x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] &
& IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 37.96 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.39

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3b^2a\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(-\frac{1}{\sinh(dx+c)\cosh(dx+c)}-2\tanh(dx+c)\right)+3b^2a\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c}{d}$
risch	$-\frac{4(65b^3e^{8dx+8c}-44b^3e^{6dx+6c}+17b^3e^{4dx+4c}-10\,e^{2dx+2c}a^3-15b^2a-5a^3+45a^2b+45a^2b\,e^{12dx+12c}+90a^2b\,e^{10dx+10c}}{}$

input
```
int(csch(d*x+c)^4*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-1/sinh(d*x+c)/cosh(
d*x+c)-2*tanh(d*x+c))+3*b^2*a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b^3*(-1/
4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2
)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(92) = 184$.

Time = 0.09 (sec) , antiderivative size = 925, normalized size of antiderivative = 9.44

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-8/15*((5*a^3 + 45*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^6 + 12*(5*a^3 +
 15*a*b^2 + 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (5*a^3 + 45*a^2*b + 15*
a*b^2 + 7*b^3)*sinh(d*x + c)^6 + 2*(15*a^3 + 45*a^2*b - 15*a*b^2 - 13*b^3)
*cosh(d*x + c)^4 + (30*a^3 + 90*a^2*b - 30*a*b^2 - 26*b^3 + 15*(5*a^3 + 45
*a^2*b + 15*a*b^2 + 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(5*a^3
+ 15*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + 4*(5*a^3 - 3*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 + 50*a^3 - 90*a^2*b + 30*a*b^2 - 22*b^3 + (75*a^3 - 45*a^2*b
- 15*a*b^2 + 41*b^3)*cosh(d*x + c)^2 + (15*(5*a^3 + 45*a^2*b + 15*a*b^2 +
7*b^3)*cosh(d*x + c)^4 + 75*a^3 - 45*a^2*b - 15*a*b^2 + 41*b^3 + 12*(15*a^
3 + 45*a^2*b - 15*a*b^2 - 13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*
(5*a^3 + 15*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + 8*(5*a^3 - 3*b^3)*cosh(d*x +
c)^3 + (25*a^3 - 45*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(
d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 2*
d*cosh(d*x + c)^8 + (45*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 8*(15*d
*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^7 - 3*d*cosh(d*x + c)^
6 + (210*d*cosh(d*x + c)^4 + 56*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 +
 2*(126*d*cosh(d*x + c)^5 + 56*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh
(d*x + c)^5 - 8*d*cosh(d*x + c)^4 + (210*d*cosh(d*x + c)^6 + 140*d*cosh(d*
x + c)^4 - 45*d*cosh(d*x + c)^2 - 8*d)*sinh(d*x + c)^4 + 4*(30*d*cosh(d*x
+ c)^7 + 28*d*cosh(d*x + c)^5 - 5*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c)...
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{csch}^4\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(92) = 184$.

Time = 0.05 (sec) , antiderivative size = 493, normalized size of antiderivative = 5.03

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{4}{15}b^3\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c)}}\right.$$
$$+4\,ab^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)} + \frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1}\right.$$
$$+\frac{4}{3}a^3\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)} - \frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1}\right.$$
$$+\frac{12\,a^2b}{d(e^{(-4\,dx-4\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
4/15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x -
 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) +
 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c)
+ 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*
d*x - 10*c) + 1))) + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) +
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/
(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(
d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 12*
a^2*b/(d*(e^(-4*d*x - 4*c) - 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 257 vs. $2(92) = 184$.

Time = 0.25 (sec) , antiderivative size = 257, normalized size of antiderivative = 2.62

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{2\left(\frac{5\left(9\,a^2be^{(4\,dx+4\,c)}+6\,a^3e^{(2\,dx+2\,c)}-18\,a^2be^{(2\,dx+2\,c)}-2\,a^3+9\,a^2b\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}-\frac{45\,a^2be^{(8\,dx+8\,c)}+180\,a^2be^{(6\,dx+6\,c)}-90\,ab^2e^{(6\,dx+6\,c)}-30\,b^3e^{(6}}{}\right.}{1}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/15*(5*(9*a^2*b*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*
d*x + 2*c) - 2*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1)^3 - (45*a^2*b*e^(8*d*x
 + 8*c) + 180*a^2*b*e^(6*d*x + 6*c) - 90*a*b^2*e^(6*d*x + 6*c) - 30*b^3*e^
(6*d*x + 6*c) + 270*a^2*b*e^(4*d*x + 4*c) - 210*a*b^2*e^(4*d*x + 4*c) + 10
*b^3*e^(4*d*x + 4*c) + 180*a^2*b*e^(2*d*x + 2*c) - 150*a*b^2*e^(2*d*x + 2*
c) - 10*b^3*e^(2*d*x + 2*c) + 45*a^2*b - 30*a*b^2 - 2*b^3)/(e^(2*d*x + 2*c
) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 622, normalized size of antiderivative = 6.35

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\frac{2\,(9\,a^2\,b-12\,a\,b^2+4\,b^3)}{15\,d}-\frac{4\,e^{2\,c+2\,d\,x}\,(-3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}+\frac{6\,a^2\,b\,e^{4\,c+4\,d\,x}}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}$$
$$-\frac{\frac{2\,(-3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(-3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(9\,a^2\,b-12\,a\,b^2+4\,b^3)}{5\,d}-\frac{6\,a^2\,b\,e^{6\,c+6\,d\,x}}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$+\frac{\frac{6\,a^2\,b}{5\,d}-\frac{8\,e^{6\,c+6\,d\,x}\,(-3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}-\frac{8\,e^{2\,c+2\,d\,x}\,(-3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(9\,a^2\,b-12\,a\,b^2+4\,b^3)}{5\,d}+\frac{6\,a^2\,b\,e^{8\,c+8\,d\,x}}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}$$
$$-\frac{\frac{2\,(-3\,a^2\,b+3\,a\,b^2+b^3)}{5\,d}-\frac{6\,a^2\,b\,e^{2\,c+2\,d\,x}}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}-\frac{4\,a^3}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$
$$-\frac{8\,a^3}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}-\frac{6\,a^2\,b}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}+\frac{6\,a^2\,b}{5\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/sinh(c + d*x)^4,x)
```

output
```
((2*(9*a^2*b - 12*a*b^2 + 4*b^3))/(15*d) - (4*exp(2*c + 2*d*x)*(3*a*b^2 -
3*a^2*b + b^3))/(5*d) + (6*a^2*b*exp(4*c + 4*d*x))/(5*d))/(3*exp(2*c + 2*d
*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(3*a*b^2 - 3*a^2*b
+ b^3))/(5*d) + (6*exp(4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + b^3))/(5*d) - (2*
exp(2*c + 2*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) - (6*a^2*b*exp(6*c +
6*d*x))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*
x) + exp(8*c + 8*d*x) + 1) + ((6*a^2*b)/(5*d) - (8*exp(6*c + 6*d*x)*(3*a*b
^2 - 3*a^2*b + b^3))/(5*d) - (8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + b^3)
)/(5*d) + (4*exp(4*c + 4*d*x)*(9*a^2*b - 12*a*b^2 + 4*b^3))/(5*d) + (6*a^2
*b*exp(8*c + 8*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10
*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(3*
a*b^2 - 3*a^2*b + b^3))/(5*d) - (6*a^2*b*exp(2*c + 2*d*x))/(5*d))/(2*exp(2
*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp
(2*c + 2*d*x) + 1)) - (8*a^3)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x
) + exp(6*c + 6*d*x) - 1)) - (6*a^2*b)/(d*(exp(2*c + 2*d*x) - 1)) + (6*a^2
*b)/(5*d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 484, normalized size of antiderivative = 4.94

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{-20e^{4dx+4c}a\,b^2+\frac{4b^3}{15}-24e^{10dx+10c}a^2b+8e^{10dx+10c}a\,b^2+12e^{8dx+8c}a^2b+28e^{8dx+8c}a\,b^2+48e^{6dx+6c}a^2b-1}{}$$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(4*( - 15*e**(12*c + 12*d*x)*a**3 - 45*e**(12*c + 12*d*x)*a**2*b - 45*e**(
12*c + 12*d*x)*a*b**2 - 15*e**(12*c + 12*d*x)*b**3 - 70*e**(10*c + 10*d*x)
*a**3 - 90*e**(10*c + 10*d*x)*a**2*b + 30*e**(10*c + 10*d*x)*a*b**2 + 50*e
**(10*c + 10*d*x)*b**3 - 125*e**(8*c + 8*d*x)*a**3 + 45*e**(8*c + 8*d*x)*a
**2*b + 105*e**(8*c + 8*d*x)*a*b**2 - 65*e**(8*c + 8*d*x)*b**3 - 100*e**(6
*c + 6*d*x)*a**3 + 180*e**(6*c + 6*d*x)*a**2*b - 60*e**(6*c + 6*d*x)*a*b**
2 + 44*e**(6*c + 6*d*x)*b**3 - 25*e**(4*c + 4*d*x)*a**3 + 45*e**(4*c + 4*d
*x)*a**2*b - 75*e**(4*c + 4*d*x)*a*b**2 - 17*e**(4*c + 4*d*x)*b**3 + 10*e*
*(2*c + 2*d*x)*a**3 - 90*e**(2*c + 2*d*x)*a**2*b + 30*e**(2*c + 2*d*x)*a*b
**2 + 2*e**(2*c + 2*d*x)*b**3 + 5*a**3 - 45*a**2*b + 15*a*b**2 + b**3))/(1
5*d*(e**(16*c + 16*d*x) + 2*e**(14*c + 14*d*x) - 2*e**(12*c + 12*d*x) - 6*
e**(10*c + 10*d*x) + 6*e**(6*c + 6*d*x) + 2*e**(4*c + 4*d*x) - 2*e**(2*c +
 2*d*x) - 1))
```

3.25 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 118

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2-6ab-b^2)\,x}{8(a+b)^3} + \frac{a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^3d} - \frac{(5a+b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

output
```
1/8*(3*a^2-6*a*b-b^2)*x/(a+b)^3+a^(3/2)*b^(1/2)*arctan(b^(1/2)*tanh(d*x+c)
/a^(1/2))/(a+b)^3/d-1/8*(5*a+b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d+1/4*cosh
(d*x+c)^3*sinh(d*x+c)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.79

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{4(3a^2-6ab-b^2)(c+dx)+32a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-8a(a+b)\sinh(2(c+dx))+(a+b)^2\sinh}{32(a+b)^3d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(4*(3*a^2 - 6*a*b - b^2)*(c + d*x) + 32*a^(3/2)*Sqrt[b]*ArcTan[(Sqrt[b]*Ta
nh[c + d*x])/Sqrt[a]] - 8*a*(a + b)*Sinh[2*(c + d*x)] + (a + b)^2*Sinh[4*(
c + d*x)])/(32*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.28, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 372, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sin(ic+idx)^4}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))^3(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 372}$$

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\int \frac{(4a+b)\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{4(a+b)}}{d}$$

$\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{\int -\frac{a(3a-b)-b(5a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}+\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\int \frac{a(3a-b)-b(5a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}}{4(a+b)}}{d}$$

$\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{8a^2b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2(a+b)}}{4(a+b)}}{d}$$

$\downarrow$ 218

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}}{2(a+b)}}{4(a+b)}}{d}$$

$\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a+b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}-\frac{\frac{8a^{3/2}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}+\frac{\left(3a^2-6ab-b^2\right)\text{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)}}{4(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2) - (-1/2*((8*a^(3/2)*Sqr
t[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a + b) + ((3*a^2 - 6*a*b -
b^2)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + ((5*a + b)*Tanh[c + d*x])/
(2*(a + b)*(1 - Tanh[c + d*x]^2)))/(4*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(104) = 208$.

Time = 20.98 (sec) , antiderivative size = 215, normalized size of antiderivative = 1.82

method	result
risch	$\frac{3x\,a^{2}}{8(a+b)^{3}}-\frac{3xab}{4(a+b)^{3}}-\frac{x\,b^{2}}{8(a+b)^{3}}+\frac{\mathrm{e}^{4dx+4c}}{64(a+b)d}-\frac{a\,\mathrm{e}^{2dx+2c}}{8(a+b)^{2}d}+\frac{a\,\mathrm{e}^{-2dx-2c}}{8(a^{2}+2ab+b^{2})d}-\frac{\mathrm{e}^{-4dx-4c}}{64(a+b)d}+\frac{\sqrt{-ab}\,a\ln\left(\mathrm{e}^{2d\cdots}\right.}{2(\cdots}$
derivativedivides	$\frac{\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a-3b}{8(a+b)^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3a-b}{8(a+b)^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\cdots}{\cdots}$
default	$\frac{\frac{8}{(32a+32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}+\frac{32}{(64a+64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{a-3b}{8(a+b)^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{3a-b}{8(a+b)^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\cdots}{\cdots}$

input
```
int(sinh(d*x+c)^4/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/(a+b)^3*a^2-3/4*x/(a+b)^3*a*b-1/8*x/(a+b)^3*b^2+1/64/(a+b)/d*exp(4*d
*x+4*c)-1/8*a/(a+b)^2/d*exp(2*d*x+2*c)+1/8*a/(a^2+2*a*b+b^2)/d*exp(-2*d*x-
2*c)-1/64/(a+b)/d*exp(-4*d*x-4*c)+1/2*(-a*b)^(1/2)*a/(a+b)^3/d*ln(exp(2*d*
x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))-1/2*(-a*b)^(1/2)*a/(a+b)^3/d*ln(exp(2*d
*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 860 vs. $2(104) = 208$.

Time = 0.16 (sec) , antiderivative size = 2024, normalized size of antiderivative = 17.15

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 -
6*a*b - b^2)*d*x*cosh(d*x + c)^4 - 8*(a^2 + a*b)*cosh(d*x + c)^6 + 4*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 - 2*a^2 - 2*a*b)*sinh(d*x + c)^6 + 8*(7*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 6*(a^2 + a*b)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 - 6*a*b -
b^2)*d*x - 60*(a^2 + a*b)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^5 + 4*(3*a^2 - 6*a*b - b^2)*d*x*cosh(d*x + c) -
20*(a^2 + a*b)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a^2 + a*b)*cosh(d*x +
 c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 - 6*a*b - b^2
)*d*x*cosh(d*x + c)^2 - 30*(a^2 + a*b)*cosh(d*x + c)^4 + 2*a^2 + 2*a*b)*si
nh(d*x + c)^2 + 32*(a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)^3*sinh(d*x + c)
+ 6*a*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3
+ a*sinh(d*x + c)^4)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 +
 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)
*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2
)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x +
c) + 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
(a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 +...
```

Sympy [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(104) = 208$.

Time = 0.19 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.36

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(ab-b^2)(dx+c)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d} + \frac{\left(8\,be^{(-2\,dx-2\,c)}+a+b\right)e^{(4\,dx+4\,c)}}{64\,(a^2+2\,ab+b^2)d} - \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} + \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} + \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} - \frac{(a^2b-6\,ab^2+b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}d} - \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} - \frac{3\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}(a+b)d} - \frac{8\,be^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}}{64\,(a^2+2\,ab+b^2)d} + \frac{3\,(dx+c)}{8\,(a+b)d} - \frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d} + \frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}$$

input

```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b
*e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) - 1/4*b
*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 +
2*a*b + b^2)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x
 - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a
+ b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d)
 - 1/8*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) - 1/4*(a*b - b
^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b
 + b^2)*sqrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)
/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e
^(-4*d*x - 4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) - 1/8
*e^(2*d*x + 2*c)/((a + b)*d) + 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(104) = 208$.

Time = 1.05 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.34

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{64\,a^2b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} + \frac{8\,(3\,a^2-6\,ab-b^2)(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{(18\,a^2e^{(4\,dx+4\,c)}-36\,abe^{(4\,dx+4\,c)}-6\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+}}{a^3+3\,a^2b+3\,ab^2+}}{64\,d}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(64*a^2*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/
sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) + 8*(3*a^2 - 6*a*b
- b^2)*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (18*a^2*e^(4*d*x + 4*c)
 - 36*a*b*e^(4*d*x + 4*c) - 6*b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c)
- 8*a*b*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-4*d*x - 4*c)/(a^3 + 3*a^2
*b + 3*a*b^2 + b^3) + (a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a*e^(2*d*
x + 2*c))/(a^2 + 2*a*b + b^2))/d
```

Mupad [B] (verification not implemented)

Time = 1.72 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.12

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{e^{4c+4dx}}{64d(a+b)} - \frac{e^{-4c-4dx}}{64d(a+b)} - \frac{x(-3a^2+6ab+b^2)}{8(a+b)^3} + \frac{a\,e^{-2c-2dx}}{8d(a+b)^2} - \frac{a\,e^{2c+2dx}}{8d(a+b)^2}$$
$$+ \frac{(-a)^{3/2}\sqrt{b}\ln\left((-a)^{3/2}b^{3/2}\left(e^{2c+2dx}-1\right)-2a^2b\,e^{2c+2dx}+(-a)^{5/2}\sqrt{b}\left(e^{2c+2dx}+1\right)\right)}{2d(a+b)^3}$$
$$- \frac{(-a)^{3/2}\sqrt{b}\ln\left(2a^2b\,e^{2c+2dx}+(-a)^{3/2}b^{3/2}\left(e^{2c+2dx}-1\right)+(-a)^{5/2}\sqrt{b}\left(e^{2c+2dx}+1\right)\right)}{2d(a+b)^3}$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(4*c + 4*d*x)/(64*d*(a + b)) - exp(- 4*c - 4*d*x)/(64*d*(a + b)) - (x*(
6*a*b - 3*a^2 + b^2))/(8*(a + b)^3) + (a*exp(- 2*c - 2*d*x))/(8*d*(a + b)^
2) - (a*exp(2*c + 2*d*x))/(8*d*(a + b)^2) + ((-a)^(3/2)*b^(1/2)*log((-a)^(
3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) - 2*a^2*b*exp(2*c + 2*d*x) + (-a)^(5/2
)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3) - ((-a)^(3/2)*b^(1/2)*l
og(2*a^2*b*exp(2*c + 2*d*x) + (-a)^(3/2)*b^(3/2)*(exp(2*c + 2*d*x) - 1) +
(-a)^(5/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1)))/(2*d*(a + b)^3)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 281, normalized size of antiderivative = 2.38

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{64e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)a - 64e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)a + e^{8dx+8c}a^2 + 2e^{8dx+8c}ab}{64e^{4}}$$

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

output
```
(64*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a - 64*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*s
qrt(a + b) + sqrt(b))/sqrt(a))*a + e**(8*c + 8*d*x)*a**2 + 2*e**(8*c + 8*d
*x)*a*b + e**(8*c + 8*d*x)*b**2 - 8*e**(6*c + 6*d*x)*a**2 - 8*e**(6*c + 6*
d*x)*a*b + 24*e**(4*c + 4*d*x)*a**2*d*x - 48*e**(4*c + 4*d*x)*a*b*d*x - 8*
e**(4*c + 4*d*x)*b**2*d*x + 8*e**(2*c + 2*d*x)*a**2 + 8*e**(2*c + 2*d*x)*a
*b - a**2 - 2*a*b - b**2)/(64*e**(4*c + 4*d*x)*d*(a**3 + 3*a**2*b + 3*a*b*
*2 + b**3))
```

3.26 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{a\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}d} - \frac{a\cosh(c+dx)}{(a+b)^2d} + \frac{\cosh^3(c+dx)}{3(a+b)d}$$

output
```
a*b^(1/2)*arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))/(a+b)^(5/2)/d-a*cosh(d*
x+c)/(a+b)^2/d+1/3*cosh(d*x+c)^3/(a+b)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{12ia\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right) + \arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right) - 3(3a-b)\sqrt{a+b}\cosh(c}{12(a+b)^{5/2}d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((12*I)*a*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/S
qrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])
- 3*(3*a - b)*Sqrt[a + b]*Cosh[c + d*x] + (a + b)^(3/2)*Cosh[3*(c + d*x)])
/(12*(a + b)^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4147, 25, 359, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\sin(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{359}$$

$$\frac{\frac{a\int \frac{\cosh^2(c+dx)}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}+\frac{\cosh^3(c+dx)}{3(a+b)}}{d}$$

$$\downarrow \text{264}$$

$$\frac{\frac{a\left(\frac{b\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}\right)}{a+b}+\frac{\cosh^3(c+dx)}{3(a+b)}}{d}$$

$$\downarrow \text{221}$$

$$\frac{\frac{a\left(\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}\right)}{a+b}+\frac{\cosh^3(c+dx)}{3(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Cosh[c + d*x]^3/(3*(a + b)) + (a*((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x]
)/Sqrt[a + b]])/(a + b)^(3/2) - Cosh[c + d*x]/(a + b)))/(a + b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(65) = 130$.

Time = 7.68 (sec) , antiderivative size = 202, normalized size of antiderivative = 2.69

method	result
derivativedivides	$\frac{-\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(16a+1 \cdots}}{d}$
default	$\frac{-\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(16a+16b)}-\frac{a-b}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(16a+1 \cdots}}{d}$
risch	$\frac{e^{3dx+3c}}{24(a+b)d}-\frac{3\,e^{dx+c}a}{8(a+b)^2d}+\frac{e^{dx+c}b}{8(a+b)^2d}-\frac{3\,e^{-dx-c}a}{8(a+b)^2d}+\frac{e^{-dx-c}b}{8(a+b)^2d}+\frac{e^{-3dx-3c}}{24(a+b)d}+\frac{\sqrt{(a+b)b}\,a\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+ \cdots}}{}\right.}{2(a+b)^3d}$

input
```
int(sinh(d*x+c)^3/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)+1)^2+16/3/(tanh(1/2*d*x+1/2*c)+1)
^3/(16*a+16*b)-1/2*(a-b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)-16/3/(tanh(1/2*d*
x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b
)^2*(-a+b)/(tanh(1/2*d*x+1/2*c)-1)+a*b/(a+b)^2/(a*b+b^2)^(1/2)*arctanh(1/4
*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 1367, normalized size of antiderivative = 18.23

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 +
 (a + b)*sinh(d*x + c)^6 - 3*(3*a - b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh
(d*x + c)^2 - 3*a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 - 3*
(3*a - b)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3*a - b)*cosh(d*x + c)^2 + 3
*(5*(a + b)*cosh(d*x + c)^4 - 6*(3*a - b)*cosh(d*x + c)^2 - 3*a + b)*sinh(
d*x + c)^2 + 12*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3
*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*sqrt(b/(a + b))*log(
((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*
cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh
(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c
) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a
 + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d
*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*
cosh(d*x + c))*sinh(d*x + c) + a + b)) + 6*((a + b)*cosh(d*x + c)^5 - 2*(3
*a - b)*cosh(d*x + c)^3 - (3*a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)/
((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x
+ c)^2*sinh(d*x + c) + 3*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x +...
```

Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh^3{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^3}{b\tanh{(dx+c)}^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) - 3*(3*a*e^(4*c) - b*e^(4*c))*e^(4
*d*x) - 3*(3*a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-3*d*x)/(a^2*d*e
^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) - 1/8*integrate(16*(a*b*e^(3*d*x
 + 3*c) - a*b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*c) +
 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*
c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 955, normalized size of antiderivative = 12.73

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(- 3*c - 3*d*x)/(24*d*(a + b)) + exp(3*c + 3*d*x)/(24*d*(a + b)) - ((a^
2*b)^(1/2)*(2*atan(((exp(d*x)*exp(c)*((4*(2*a^2*b^3*d*(a^2*b)^(1/2) + 4*a^
3*b^2*d*(a^2*b)^(1/2) + 2*a^4*b*d*(a^2*b)^(1/2)))/(a*(a + b)*(-d^2*(a + b)
^5)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(- a^5*d^2 -
 b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1
/2)) + (2*a^3*b)/(d*(a + b)^3*(a^2*b)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 +
 3*a^2*b + a^3 + b^3))) + (2*a^3*b*exp(3*c)*exp(3*d*x))/(d*(a + b)^3*(a^2*
b)^(1/2)*(2*a*b + a^2 + b^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)))*(a^6*(- a^5
*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d
^2)^(1/2) + b^6*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*
b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 15*a^2*b^4*(- a^5*d^2 - b^5*d^2 - 5*a*b^
4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 20*a^3*b^3*
(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3
*b^2*d^2)^(1/2) + 15*a^4*b^2*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*
d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6*a*b^5*(- a^5*d^2 - b^5*d^
2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2) + 6
*a^5*b*(- a^5*d^2 - b^5*d^2 - 5*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 -
 10*a^3*b^2*d^2)^(1/2)))/(4*a^2*b)) - 2*atan((a*exp(d*x)*exp(c)*(-d^2*(a +
 b)^5)^(1/2))/(2*d*(a + b)^2*(a^2*b)^(1/2)))))/(2*(- a^5*d^2 - b^5*d^2 - 5
*a*b^4*d^2 - 5*a^4*b*d^2 - 10*a^2*b^3*d^2 - 10*a^3*b^2*d^2)^(1/2)) - (e...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 281, normalized size of antiderivative = 3.75

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{-12e^{3dx+3c}\sqrt{b}\,\sqrt{a+b}\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}-2e^{dx+c}\sqrt{b}\right)a+12e^{3dx+3c}\sqrt{b}\,\sqrt{a+b}\log\left(e^{2dx+2c}\sqrt{a}\right.}{}$$

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - 12*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a +
b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a + 12*e**(3*c + 3*d*x)*sqrt(b)
*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*
x)*sqrt(b))*a + e**(6*c + 6*d*x)*a**2 + 2*e**(6*c + 6*d*x)*a*b + e**(6*c +
 6*d*x)*b**2 - 9*e**(4*c + 4*d*x)*a**2 - 6*e**(4*c + 4*d*x)*a*b + 3*e**(4*
c + 4*d*x)*b**2 - 9*e**(2*c + 2*d*x)*a**2 - 6*e**(2*c + 2*d*x)*a*b + 3*e**
(2*c + 2*d*x)*b**2 + a**2 + 2*a*b + b**2)/(24*e**(3*c + 3*d*x)*d*(a**3 + 3
*a**2*b + 3*a*b**2 + b**3))
```

3.27 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(a-b)x}{2(a+b)^2} - \frac{\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a+b)^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

output
```
-1/2*(a-b)*x/(a+b)^2-a^(1/2)*b^(1/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/(
a+b)^2/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.86

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-2(a-b)(c+dx) - 4\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + (a+b)\sinh(2(c+dx))}{4(a+b)^2d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-2*(a - b)*(c + d*x) - 4*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/S
qrt[a]] + (a + b)*Sinh[2*(c + d*x)])/(4*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.24, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 373, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))^2(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{373}$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))}-\frac{\int \frac{a-b\tanh^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))}-\frac{\frac{(a-b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{2ab\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2(a+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))}-\frac{\frac{(a-b)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}}{2(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))}-\frac{\frac{2\sqrt{a}\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a+b}+\frac{(a-b)\text{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/2*((2*Sqrt[a]*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a + b)
 + ((a - b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + Tanh[c + d*x]/(2*(a
 + b)*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 150 vs. $2(66) = 132$.

Time = 2.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.94

method	result
risch	$-\frac{ax}{2(a+b)^2}+\frac{xb}{2(a+b)^2}+\frac{e^{2dx+2c}}{8(a+b)d}-\frac{e^{-2dx-2c}}{8(a+b)d}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2(a+b)^2d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{}}{}\right.}{2(a+b)^2d}$
derivativedivides	$\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{(16a}$
default	$\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{(16a}$

input
```
int(sinh(d*x+c)^2/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-1/2*a*x/(a+b)^2+1/2*x/(a+b)^2*b+1/8/(a+b)/d*exp(2*d*x+2*c)-1/8/(a+b)/d*ex
p(-2*d*x-2*c)+1/2*(-a*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)
-a+b)/(a+b))-1/2*(-a*b)^(1/2)/(a+b)^2/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+
a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(66) = 132$.

Time = 0.14 (sec) , antiderivative size = 916, normalized size of antiderivative = 11.74

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh(d*x + c)^4 - 4*(a + b)
*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x + c)^4 + 2*(2*(a - b)*d*
x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 4*sqrt(-a*b)*(cosh(d*x +
c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)*log(((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)
^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 +
 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a
^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*co
sh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d
*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a +
 b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*si
nh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 +
 a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x +
c))*sinh(d*x + c) + a + b)) + 4*(2*(a - b)*d*x*cosh(d*x + c) - (a + b)*cos
h(d*x + c)^3)*sinh(d*x + c) + a + b)/((a^2 + 2*a*b + b^2)*d*cosh(d*x + c)^
2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + 2*a*b + b
^2)*d*sinh(d*x + c)^2), -1/8*(4*(a - b)*d*x*cosh(d*x + c)^2 - (a + b)*cosh
(d*x + c)^4 - 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 - (a + b)*sinh(d*x +
 c)^4 + 2*(2*(a - b)*d*x - 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
sqrt(a*b)*(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x +...
```

Sympy [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. 2(66) = 132.

Time = 0.14 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.05

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx =& \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &- \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &- \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &+ \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &+ \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}(a+b)d} \\ &- \frac{dx+c}{2\,(a+b)d} + \frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d} - \frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-
4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2
*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*
b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/2*(d*x + c)/((a
+ b)*d) + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*
d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(66) = 132$.

Time = 0.61 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.04

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{8\,ab\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}+\frac{4\,(dx+c)(a-b)}{a^2+2\,ab+b^2}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}-a-b\right)e^{(-2\,dx-2\,c)}}{a^2+2\,ab+b^2}-\frac{e^{(2\,dx+2\,c)}}{a+b}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/8*(8*a*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqr
t(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 4*(d*x + c)*(a - b)/(a^2 + 2*a*b
 + b^2) - (2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) - a - b)*e^(-2*d*x -
2*c)/(a^2 + 2*a*b + b^2) - e^(2*d*x + 2*c)/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 1.58 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.54

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{e^{2c+2dx}}{8d(a+b)} - \frac{e^{-2c-2dx}}{8d(a+b)} - \frac{x(a-b)}{2(a+b)^2}$$
$$- \frac{\sqrt{-a}\sqrt{b}\ln\left(\sqrt{-a}\,b^{3/2}\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\sqrt{b}\left(e^{2c+2dx}+1\right)-2ab\,e^{2c+2dx}\right)}{2d(a+b)^2}$$
$$+ \frac{\sqrt{-a}\sqrt{b}\ln\left(\sqrt{-a}\,b^{3/2}\left(e^{2c+2dx}-1\right)+(-a)^{3/2}\sqrt{b}\left(e^{2c+2dx}+1\right)+2ab\,e^{2c+2dx}\right)}{2d(a+b)^2}$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) - (x*(a
- b))/(2*(a + b)^2) - ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(exp(2*c
+ 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) - 2*a*b*exp(2*c
+ 2*d*x)))/(2*d*(a + b)^2) + ((-a)^(1/2)*b^(1/2)*log((-a)^(1/2)*b^(3/2)*(e
xp(2*c + 2*d*x) - 1) + (-a)^(3/2)*b^(1/2)*(exp(2*c + 2*d*x) + 1) + 2*a*b*e
xp(2*c + 2*d*x)))/(2*d*(a + b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 167, normalized size of antiderivative = 2.14

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{-8e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)+8e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)+e^{4dx+4c}a+e^{4dx+4c}b-4e^{2c}}{8e^{2dx+2c}d\left(a^2+2ab+b^2\right)}$$

input
```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - 8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a)) + 8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sq
rt(a + b) + sqrt(b))/sqrt(a)) + e**(4*c + 4*d*x)*a + e**(4*c + 4*d*x)*b -
4*e**(2*c + 2*d*x)*a*d*x + 4*e**(2*c + 2*d*x)*b*d*x - a - b)/(8*e**(2*c +
2*d*x)*d*(a**2 + 2*a*b + b**2))
```

3.28 $\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\,\mathrm{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}d} + \frac{\cosh(c+dx)}{(a+b)d}$$

output
```
-b^(1/2)*arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))/(a+b)^(3/2)/d+cosh(d*x+c
)/(a+b)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.40 (sec) , antiderivative size = 107, normalized size of antiderivative = 2.02

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)+\sqrt{a+b}\cosh(c+dx)}{(a+b)^{3/2}d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[
b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]) + Sq
rt[a + b]*Cosh[c + d*x])/((a + b)^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 52, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 26, 4147, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\sin(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\sin(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4147}$$

$$-\frac{\int \frac{\cosh^2(c+dx)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{ 264}$$

$$-\frac{\frac{b\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \text{ 221}$$

$$-\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2) -
Cosh[c + d*x]/(a + b))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(45) = 90$.

Time = 1.22 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.96

method	result
derivativedivides	$\frac{\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{(a+b)\sqrt{ab+b^{2}}}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
default	$\frac{\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{(a+b)\sqrt{ab+b^{2}}}-\frac{4}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}}{d}$
risch	$\frac{e^{dx+c}}{2(a+b)d}+\frac{e^{-dx-c}}{2(a+b)d}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)^{2}d}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right.}{2(a+b)^{2}d}$

input
```
int(sinh(d*x+c)/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4/(4*a+4*b)/(tanh(1/2*d*x+1/2*c)+1)-b/(a+b)/(a*b+b^2)^(1/2)*arctanh(1
/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))-4/(4*a+4*b)/(tanh(
1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 247 vs. $2(45) = 90$.

Time = 0.12 (sec) , antiderivative size = 666, normalized size of antiderivative = 12.57

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*log(((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)
^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)
*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*s
inh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*c
osh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a + b)) + a + b)/((a + b)*c
osh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*
x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 + 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)),
 -1/2*(2*sqrt(-b/(a + b))*(cosh(d*x + c) + sinh(d*x + c))*arctan(1/2*((a +
 b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*si
nh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a -
 3*b)*sinh(d*x + c))*sqrt(-b/(a + b))/b) - 2*sqrt(-b/(a + b))*(cosh(d*x +
c) + sinh(d*x + c))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x +
 c))*sqrt(-b/(a + b))/b) - cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c)
 - sinh(d*x + c)^2 - 1)/((a + b)*d*cosh(d*x + c) + (a + b)*d*sinh(d*x + c)
)]
```

Sympy [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\sinh\left(dx+c\right)}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(
b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a
*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d
*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 520, normalized size of antiderivative = 9.81

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{c+d\,x}}{2\,d\,\left(a+b\right)} + \frac{e^{-c-d\,x}}{2\,d\,\left(a+b\right)} - \frac{\sqrt{b}\left(2\,\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\sqrt{-d^2\,{\left(a+b\right)}^3}}{2\,\sqrt{b}\,d\,\left(a+b\right)}\right) - 2\,\mathrm{atan}\left(\frac{\left(e^{d\,x}\,e^{c}\left(\frac{2\,a\,\sqrt{b}}{d\,{\left(a+b\right)}^2\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)} + \frac{4\,\left(2\,a^2\,b^{3/2}\,d+\cdots\right.}{\left(a+b\right)\,\sqrt{-d^2\,{\left(a+b\right)}^3}\,\sqrt{-a^3\,d^2-3\,a^2\,b\,d^2-3\cdots}}\right.\right.}{\cdots}\right.\right.}{\cdots}$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(c + d*x)/(2*d*(a + b)) + exp(- c - d*x)/(2*d*(a + b)) - (b^(1/2)*(2*at
an((exp(d*x)*exp(c)*(-d^2*(a + b)^3)^(1/2))/(2*b^(1/2)*d*(a + b))) - 2*ata
n(((exp(d*x)*exp(c)*((2*a*b^(1/2))/(d*(a + b)^2*(3*a*b^2 + 3*a^2*b + a^3 +
 b^3)) + (4*(2*a^2*b^(3/2)*d + 2*a*b^(5/2)*d))/((a + b)*(-d^2*(a + b)^3)^(
1/2)*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)*(3*a*b^2 + 3*
a^2*b + a^3 + b^3))) + (2*a*b^(1/2)*exp(3*c)*exp(3*d*x))/(d*(a + b)^2*(3*a
*b^2 + 3*a^2*b + a^3 + b^3)))*(a^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*
a^2*b*d^2)^(1/2) + b^4*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(
1/2) + 4*a*b^3*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 4
*a^3*b*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2) + 6*a^2*b^2
*(- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2)))/(4*a*b))))/(2*(
- a^3*d^2 - b^3*d^2 - 3*a*b^2*d^2 - 3*a^2*b*d^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 4.09

$$\int \frac{\sinh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2e^{dx+c}\cosh(dx+c)\,a^2+4e^{dx+c}\cosh(dx+c)\,ab+2e^{dx+c}\cosh(dx+c)\,b^2+e^{dx+c}\sqrt{b}\,\sqrt{a+b}\log\Big(e^{2dx+2c}}{}$$

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*e**(c + d*x)*cosh(c + d*x)*a**2 + 4*e**(c + d*x)*cosh(c + d*x)*a*b + 2*
e**(c + d*x)*cosh(c + d*x)*b**2 + e**(c + d*x)*sqrt(b)*sqrt(a + b)*log(e**
(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a - e**(
c + d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b
) + 2*e**(c + d*x)*sqrt(b))*a - e**(2*c + 2*d*x)*a*b - e**(2*c + 2*d*x)*b*
*2 - a*b - b**2)/(2*e**(c + d*x)*a*d*(a**2 + 2*a*b + b**2))
```

3.29 $\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 55

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx=-\frac{\operatorname{arctanh}(\cosh(c+dx))}{ad}+\frac{\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}d}$$

output

```
-arctanh(cosh(d*x+c))/a/d+b^(1/2)*arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))
/a/(a+b)^(1/2)/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.45

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$=\frac{\frac{i\sqrt{b}\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}+\frac{i\sqrt{b}\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}\right.\right.}{ad}$$

input

```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]
)/Sqrt[a + b] + (I*Sqrt[b]*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*
x)/2])/Sqrt[b]])/Sqrt[a + b] - Log[Cosh[(c + d*x)/2]] + Log[Sinh[(c + d*x)
/2]])/(a*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4147, 25, 303, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{303}$$

$$\frac{\frac{b\int \frac{1}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}-\frac{\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{a}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}-\frac{\text{arctanh}(\text{sech}(c+dx))}{a}}{d}$$

$$\Big\downarrow \textbf{221}$$

$$\frac{\frac{\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}-\frac{\text{arctanh}(\text{sech}(c+dx))}{a}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(ArcTanh[Sech[c + d*x]]/a) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.22

method	result
derivativedivides	$\frac{\frac{b\,\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{a\sqrt{ab+b^{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{\frac{b\,\operatorname{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{a\sqrt{ab+b^{2}}}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$-\frac{\ln\left(e^{dx+c}+1\right)}{ad}+\frac{\ln\left(e^{dx+c}-1\right)}{ad}+\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{(a+b)b}\,e^{dx+c}}{a+b}+1\right)}{2(a+b)da}-\frac{\sqrt{(a+b)b}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{(a+\cdots}}{\cdots}\right)}{2(a+b)da}$

input
```
int(csch(d*x+c)/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/a/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(
a*b+b^2)^(1/2))+1/a*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(47) = 94$.

Time = 0.13 (sec) , antiderivative size = 587, normalized size of antiderivative = 10.67

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(b/(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2
 + 2*(3*(a + b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*co
sh(d*x + c)^3 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d
*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c
)^3 + (a + b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x
 + c))*sqrt(b/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x +
c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*
cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 2*log(c
osh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d*x + c) -
1))/(a*d), (sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a +
b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a - 3*b)*cos
h(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c))*sqrt(-b/
(a + b))/b) - sqrt(-b/(a + b))*arctan(1/2*((a + b)*cosh(d*x + c) + (a + b)
*sinh(d*x + c))*sqrt(-b/(a + b))/b) - log(cosh(d*x + c) + sinh(d*x + c) +
1) + log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
 - 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*
c) + a*b*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.97 (sec) , antiderivative size = 284, normalized size of antiderivative = 5.16

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx=-\frac{2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\left(9\,b^4\,\sqrt{-a^2\,d^2}+16\,a^2\,b^2\,\sqrt{-a^2\,d^2}+24\,a\,b^3\,\sqrt{-a^2\,d^2}\right)}{16\,d\,a^3\,b^2+24\,d\,a^2\,b^3+9\,d\,a\,b^4}\right)}{\sqrt{-a^2\,d^2}}-\frac{\sqrt{b}\,\left(2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}\,\sqrt{-a^2\,d^2\,\left(a+b\right)}+{\mathrm{e}}^{3\,c}\,{\mathrm{e}}^{3\,d\,x}\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}\,\sqrt{-a^2\,d^2\,\left(a+b\right)}+4\,a^2\,b\,d^2\,{\mathrm{e}}^{d\,x}\,{\mathrm{e}}^c}{2\,a\,\sqrt{b}\,d\,\sqrt{-a^2\,d^2\,\left(a+b\right)}}\right)-2\,\mathrm{atan}\left(\frac{{\mathrm{e}}^{d\,x}}{}\right.\right.}{2\,\sqrt{-a^3\,d^2-b\,a^2\,d^2}}$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

output
```
- (2*atan((exp(d*x)*exp(c)*(9*b^4*(-a^2*d^2)^(1/2) + 16*a^2*b^2*(-a^2*d^2)
^(1/2) + 24*a*b^3*(-a^2*d^2)^(1/2)))/(24*a^2*b^3*d + 16*a^3*b^2*d + 9*a*b^
4*d)))/(-a^2*d^2)^(1/2) - (b^(1/2)*(2*atan((exp(d*x)*exp(c)*(- a^3*d^2 - a
^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(- a^3*d^2
- a^2*b*d^2)^(1/2)*(-a^2*d^2*(a + b))^(1/2) + 4*a^2*b*d^2*exp(d*x)*exp(c))
/(2*a*b^(1/2)*d*(-a^2*d^2*(a + b))^(1/2))) - 2*atan((exp(d*x)*exp(c)*(-a^2
*d^2*(a + b))^(1/2))/(2*a*b^(1/2)*d))))/(2*(- a^3*d^2 - a^2*b*d^2)^(1/2))
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.64

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a+b}\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}-2e^{dx+c}\sqrt{b}\right)+\sqrt{b}\,\sqrt{a+b}\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}+2e^{d}\right.}{2ad\,(a+b)}$$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2
*e**(c + d*x)*sqrt(b)) + sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a +
 b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b)) + 2*log(e**(c + d*x) - 1)*a +
2*log(e**(c + d*x) - 1)*b - 2*log(e**(c + d*x) + 1)*a - 2*log(e**(c + d*x)
 + 1)*b)/(2*a*d*(a + b))
```

3.30 $\int \frac{\text{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 48

$$\int \frac{\text{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

output
```
-b^(1/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/d-coth(d*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00

$$\int \frac{\text{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d} - \frac{\coth(c+dx)}{ad}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - Coth[c
+ d*x]/(a*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 25, 4146, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{-\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\coth(c+dx)}{a}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) - Coth[c + d
*x]/a)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 111 vs. $2(40) = 80$.

Time = 1.25 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.33

method	result
risch	$-\frac{2}{ad(e^{2dx+2c}-1)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{2\sqrt{-ab}-a+b}{a+b}\right)}{2a^2d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^2d}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\ldots\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b\ldots}\right.}}\right.}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+2b\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\ldots\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b\ldots}\right.}}\right.}{d}$

input
```
int(csch(d*x+c)^2/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-2/a/d/(exp(2*d*x+2*c)-1)+1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*
b)^(1/2)-a+b)/(a+b))-1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1
/2)+a-b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(40) = 80$.

Time = 0.12 (sec) , antiderivative size = 618, normalized size of antiderivative = 12.88

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 -
 1)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4
+ 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)
*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*
b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*
b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(
a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)
*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b
)) - 4)/(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sin
h(d*x + c)^2 - a*d), -((cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
sinh(d*x + c)^2 - 1)*sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a
+ b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(b
/a)/b) + 2)/(a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d
*sinh(d*x + c)^2 - a*d)]
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{csch}^2{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.29

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}ad}+\frac{2}{(ae^{(-2\,dx-2\,c)}-a)d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
 + 2/((a*e^(-2*d*x - 2*c) - a)*d)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.44

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a}+\frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*a) + 2/(a*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 1.54 (sec) , antiderivative size = 136, normalized size of antiderivative = 2.83

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2}{a\,d-a\,d\,\mathrm{e}^{2\,c+2\,d\,x}}-\frac{\sqrt{b}\,\mathrm{atan}\left(\frac{\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d}-\frac{\sqrt{b}\,\sqrt{a^3\,d^2}}{2\,a^2\,d}+\frac{\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a\,\sqrt{b}\,d}+\frac{\sqrt{b}\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{a^3\,d^2}}{2\,a^2\,d}\right)}{\sqrt{a^3\,d^2}}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
```

output
```
2/(a*d - a*d*exp(2*c + 2*d*x)) - (b^(1/2)*atan((a^3*d^2)^(1/2)/(2*a*b^(1/2
)*d) - (b^(1/2)*(a^3*d^2)^(1/2))/(2*a^2*d) + (exp(2*c)*exp(2*d*x)*(a^3*d^2
)^(1/2))/(2*a*b^(1/2)*d) + (b^(1/2)*exp(2*c)*exp(2*d*x)*(a^3*d^2)^(1/2))/(
2*a^2*d)))/(a^3*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 165, normalized size of antiderivative = 3.44

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{-e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)+e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)}{a^2d\left(e^{2dx+2c}-1\right)}$$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a)) + sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/
sqrt(a)) + e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
 + sqrt(b))/sqrt(a)) - sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sq
rt(b))/sqrt(a)) - 2*e**(2*c + 2*d*x)*a)/(a**2*d*(e**(2*c + 2*d*x) - 1))
```

3.31 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+2b)\operatorname{arctanh}(\cosh(c+dx))}{2a^2d} - \frac{\sqrt{b}\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{a^2d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

output
```
1/2*(a+2*b)*arctanh(cosh(d*x+c))/a^2/d-b^(1/2)*(a+b)^(1/2)*arctanh(b^(1/2)
*sech(d*x+c)/(a+b)^(1/2))/a^2/d-1/2*coth(d*x+c)*csch(d*x+c)/a/d
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.84 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{8i\sqrt{b}\sqrt{a+b}\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right) + 8i\sqrt{b}\sqrt{a+b}\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right) + a\operatorname{csch}^2}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/8*((8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c
 + d*x)/2])/Sqrt[b]] + (8*I)*Sqrt[b]*Sqrt[a + b]*ArcTan[((-I)*Sqrt[a + b]
+ Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]] + a*Csch[(c + d*x)/2]^2 - 4*a*Log[Co
sh[(c + d*x)/2]] - 8*b*Log[Cosh[(c + d*x)/2]] + 4*a*Log[Sinh[(c + d*x)/2]]
 + 8*b*Log[Sinh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^2)/(a^2*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 4147, 373, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4147}$$

$$-\frac{\int \frac{\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)^2\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{373}$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)}-\frac{\int \frac{b\,\text{sech}^2(c+dx)+a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a}}{d}$$

$$\Big\downarrow \text{397}$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)}-\frac{\frac{(a+2b)\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{a}-\frac{2b(a+b)\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)}-\frac{\frac{(a+2b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{2b(a+b)\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)}-\frac{\frac{(a+2b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{2\sqrt{b}\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a}}{2a}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((-1/2*(((a + 2*b)*ArcTanh[Sech[c + d*x]])/a - (2*Sqrt[b]*Sqrt[a + b]*Arc
Tanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/a)/a + Sech[c + d*x]/(2*a*(1 -
Sech[c + d*x]^2)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 1.88 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.31

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{2}}-\frac{b(a+b)\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{a^{2}\sqrt{ab+b^{2}}}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8a}-\frac{1}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}+\frac{(-2a-4b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^{2}}-\frac{b(a+b)\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{a^{2}\sqrt{ab+b^{2}}}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{da\left(e^{2dx+2c}-1\right)^{2}}-\frac{\ln\left(e^{dx+c}-1\right)}{2ad}-\frac{b\ln\left(e^{dx+c}-1\right)}{d\,a^{2}}+\frac{\ln\left(e^{dx+c}+1\right)}{2ad}+\frac{b\ln\left(e^{dx+c}+1\right)}{d\,a^{2}}+\frac{\sqrt{ab+b^{2}}\,\ln\left(e^{2dx}\right.}{}$

input
```
int(csch(d*x+c)^3/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/8/a/tanh(1/2*d*x+1/2*c)^2+1/4/a^2*(-2*a
-4*b)*ln(tanh(1/2*d*x+1/2*c))-b*(a+b)/a^2/(a*b+b^2)^(1/2)*arctanh(1/4*(2*t
anh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(73) = 146$.

Time = 0.14 (sec) , antiderivative size = 1797, normalized size of antiderivative = 21.14

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(
d*x + c)^3 - (cosh(d*x + c)^4 + 4*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x
 + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 +
4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*x + c) + 1)*sqrt(a*b + b^2)*log
(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
 b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x
+ c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + 3*b)
*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(
d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cos
h(d*x + c))*sqrt(a*b + b^2) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d
*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a
+ b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 2*
a*cosh(d*x + c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 +
 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*
cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(co
sh(d*x + c) + sinh(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*
cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + ...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2
*c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a
+ 2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*((a*b*e^(3*
c) + b^2*e^(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3 + a^2*b +
(a^3*e^(4*c) + a^2*b*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*
e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 787, normalized size of antiderivative = 9.26

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,\left(18\,b^7\,\sqrt{-a^4\,d^2}+48\,a^2\,b^5\,\sqrt{-a^4\,d^2}+27\,a^3\,b^4\,\sqrt{-a^4\,d^2}+8\,a^4\,b^3\,\sqrt{-a^4\,d^2}+a^5\,b^2\,\sqrt{-a^4\,d^2}+45\,a\,b^6\,\sqrt{-a^4\,d^2}\right)}{9\,a^2\,b^6\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+18\,a^3\,b^5\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+15\,a^4\,b^4\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+6\,a^5\,b^3\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}+a^6\,b^2\,d\,\sqrt{a^2+4\,a\,b+4\,b^2}}\right)}{\sqrt{-a^4\,d^2}}$$
$$-\frac{\left(2\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\,(a+b)\,\sqrt{-a^4\,d^2}}{2\,a^2\,d\,\sqrt{b\,(a+b)}}\right)+2\operatorname{atan}\left(\frac{\left(e^{dx}\,e^{c}\left(\frac{64\left(2\,a^4\,b\,d\,\sqrt{b^2+a\,b}+6\,a^2\,b^3\,d\,\sqrt{b^2+a\,b}+6\,a^3\,b^2\,d\,\sqrt{b^2+a\,b}\right)}{a^9\,d^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}-\frac{32\left(3\,b^4\,\sqrt{-a^4\,d^2}+4\right.}{a^7\,d\,(a+b}\right.\right.}{}\right)\right.}{}$$
$$-\frac{e^{c+dx}}{a\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,e^{c+dx}}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)
```

output
```
(atan((exp(d*x)*exp(c)*(18*b^7*(-a^4*d^2)^(1/2) + 48*a^2*b^5*(-a^4*d^2)^(1
/2) + 27*a^3*b^4*(-a^4*d^2)^(1/2) + 8*a^4*b^3*(-a^4*d^2)^(1/2) + a^5*b^2*(
-a^4*d^2)^(1/2) + 45*a*b^6*(-a^4*d^2)^(1/2)))/(9*a^2*b^6*d*(4*a*b + a^2 +
4*b^2)^(1/2) + 18*a^3*b^5*d*(4*a*b + a^2 + 4*b^2)^(1/2) + 15*a^4*b^4*d*(4*
a*b + a^2 + 4*b^2)^(1/2) + 6*a^5*b^3*d*(4*a*b + a^2 + 4*b^2)^(1/2) + a^6*b
^2*d*(4*a*b + a^2 + 4*b^2)^(1/2)))*(4*a*b + a^2 + 4*b^2)^(1/2))/(-a^4*d^2)
^(1/2) - ((2*atan((exp(d*x)*exp(c)*(a + b)*(-a^4*d^2)^(1/2))/(2*a^2*d*(b*(
a + b))^(1/2))) + 2*atan(((exp(d*x)*exp(c)*((64*(2*a^4*b*d*(a*b + b^2)^(1/
2) + 6*a^2*b^3*d*(a*b + b^2)^(1/2) + 6*a^3*b^2*d*(a*b + b^2)^(1/2)))/(a^9*
d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b
^2*(-a^4*d^2)^(1/2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/
(a^7*d*(a + b)*(-a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2))) -
(32*exp(3*c)*exp(3*d*x)*(3*b^4*(-a^4*d^2)^(1/2) + 4*a^2*b^2*(-a^4*d^2)^(1/
2) + 6*a*b^3*(-a^4*d^2)^(1/2) + a^3*b*(-a^4*d^2)^(1/2)))/(a^7*d*(a + b)*(-
a^4*d^2)^(1/2)*(b*(a + b))^(1/2)*(2*a*b + a^2 + b^2)))*(a^8*(-a^4*d^2)^(1/
2) + a^5*b^3*(-a^4*d^2)^(1/2) + 3*a^6*b^2*(-a^4*d^2)^(1/2) + 3*a^7*b*(-a^4
*d^2)^(1/2)))/(192*a*b^2 + 64*a^2*b + 192*b^3)))*(a*b + b^2)^(1/2))/(2*(-a
^4*d^2)^(1/2)) - exp(c + d*x)/(a*d*(exp(2*c + 2*d*x) - 1)) - (2*exp(c + d*
x))/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 570, normalized size of antiderivative = 6.71

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{e^{4dx+4c}\sqrt{b}\,\sqrt{a+b}\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}-2e^{dx+c}\sqrt{b}\right)-e^{4dx+4c}\sqrt{b}\,\sqrt{a+b}\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{}\right.}{}$$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + s
qrt(a + b) - 2*e**(c + d*x)*sqrt(b)) - e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b
)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))
 - 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b)
 + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b)) + 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt
(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sq
rt(b)) + sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b
) - 2*e**(c + d*x)*sqrt(b)) - sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqr
t(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b)) - e**(4*c + 4*d*x)*log(e*
*(c + d*x) - 1)*a - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b + e**(4*c +
 4*d*x)*log(e**(c + d*x) + 1)*a + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)
*b - 2*e**(3*c + 3*d*x)*a + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a + 4
*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b - 2*e**(2*c + 2*d*x)*log(e**(c +
 d*x) + 1)*a - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b - 2*e**(c + d*x)
*a - log(e**(c + d*x) - 1)*a - 2*log(e**(c + d*x) - 1)*b + log(e**(c + d*x
) + 1)*a + 2*log(e**(c + d*x) + 1)*b)/(2*a**2*d*(e**(4*c + 4*d*x) - 2*e**(
2*c + 2*d*x) + 1))
```

3.32 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 70

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

output
```
b^(1/2)*(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/d+(a+b)*coth(d*x
+c)/a^2/d-1/3*coth(d*x+c)^3/a/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{3\sqrt{b}(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\coth(c+dx)\left(2a+3b-a\operatorname{csch}^2(c+dx)\right)}{3a^{5/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(3*Sqrt[b]*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Coth[
c + d*x]*(2*a + 3*b - a*Csch[c + d*x]^2))/(3*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4146, 359, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{359}$$

$$\frac{-\frac{(a+b)\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{-\frac{(a+b)\left(-\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}\right)}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{-\frac{(a+b)\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\coth(c+dx)}{a}\right)}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/3*Coth[c + d*x]^3/a - ((a + b)*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*
x])/Sqrt[a]])/a^(3/2)) - Coth[c + d*x]/a))/a)/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 359
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2), x
_Symbol] :> Simp[c*(e*x)^(m + 1)*((a + b*x^2)^(p + 1)/(a*e*(m + 1))), x] +
Simp[(a*d*(m + 1) - b*c*(m + 2*p + 3))/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0]
&& LtQ[m, -1] &&  !ILtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(60) = 120$.

Time = 3.60 (sec) , antiderivative size = 246, normalized size of antiderivative = 3.51

method	result
risch	$-\frac{2\left(-3b\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+6\,\mathrm{e}^{2dx+2c}b-2a-3b\right)}{3a^{2}d\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a^{2}d}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{\cdots}}{\cdots}\right.}{2a^{3}d}\cdots$
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{2}}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-3a-4b}{8a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b(a+b)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)a\cdots}{2a\sqrt{(a+b)\cdots}}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{2}}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}-\frac{-3a-4b}{8a^{2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b(a+b)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)a\cdots}{2a\sqrt{(a+b)\cdots}}\cdots\right.}{\cdots}}{d}$

input
```
int(csch(d*x+c)^4/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-2/3*(-3*b*exp(4*d*x+4*c)+6*exp(2*d*x+2*c)*a+6*exp(2*d*x+2*c)*b-2*a-3*b)/a
^2/d/(exp(2*d*x+2*c)-1)^3+1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*
b)^(1/2)+a-b)/(a+b))+1/2/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1
/2)+a-b)/(a+b))*b-1/2/a^2*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)
-a+b)/(a+b))-1/2/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)
/(a+b))*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(60) = 120$.

Time = 0.12 (sec) , antiderivative size = 1628, normalized size of antiderivative = 23.26

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*b*cosh(d*x + c)^4 + 48*b*cosh(d*x + c)*sinh(d*x + c)^3 + 12*b*sin
h(d*x + c)^4 - 24*(a + b)*cosh(d*x + c)^2 + 24*(3*b*cosh(d*x + c)^2 - a -
b)*sinh(d*x + c)^2 + 3*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*
sinh(d*x + c)^5 + (a + b)*sinh(d*x + c)^6 - 3*(a + b)*cosh(d*x + c)^4 + 3*
(5*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*
x + c)^3 - 3*(a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a + b)*cosh(d*x +
 c)^2 + 3*(5*(a + b)*cosh(d*x + c)^4 - 6*(a + b)*cosh(d*x + c)^2 + a + b)*
sinh(d*x + c)^2 + 6*((a + b)*cosh(d*x + c)^5 - 2*(a + b)*cosh(d*x + c)^3 +
 (a + b)*cosh(d*x + c))*sinh(d*x + c) - a - b)*sqrt(-b/a)*log(((a^2 + 2*a*
b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2
 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 +
 a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*
cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a
*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)
*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 48*(b*cosh(d*x + c)^3 - (a
 + b)*cosh(d*x + c))*sinh(d*x + c) + 8*a + 12*b)/(a^2*d*cosh(d*x + c)^6...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(60) = 120$.

Time = 0.15 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.91

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\left(6\,(a+b)e^{(-2\,dx-2\,c)}-3\,be^{(-4\,dx-4\,c)}-2\,a-3\,b\right)}{3\left(3\,a^2e^{(-2\,dx-2\,c)}-3\,a^2e^{(-4\,dx-4\,c)}+a^2e^{(-6\,dx-6\,c)}-a^2\right)d} - \frac{(ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
2/3*(6*(a + b)*e^(-2*d*x - 2*c) - 3*b*e^(-4*d*x - 4*c) - 2*a - 3*b)/((3*a^
2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*
d) - (a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.70

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{3\,(ab+b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2}+\frac{2\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+2\,a+3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
 b)/sqrt(a*b))/(sqrt(a*b)*a^2) + 2*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2
*c) - 6*b*e^(2*d*x + 2*c) + 2*a + 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 1.65 (sec) , antiderivative size = 254, normalized size of antiderivative = 3.63

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\,b}{a^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{8}{3\,a\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{4}{a\,d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}+\frac{\sqrt{-b}\,\ln\left(-\frac{4\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{a^2}-\frac{2\,\sqrt{-b}\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{5/2}\,d}\right)\,\left(a+b\right)}{2\,a^{5/2}\,d}-\frac{\sqrt{-b}\,\ln\left(\frac{2\,\sqrt{-b}\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{a^{5/2}\,d}-\frac{4\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{a^2}\right)\,\left(a+b\right)}{2\,a^{5/2}\,d}$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)
```

output
```
(2*b)/(a^2*d*(exp(2*c + 2*d*x) - 1)) - 8/(3*a*d*(3*exp(2*c + 2*d*x) - 3*ex
p(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - 4/(a*d*(exp(4*c + 4*d*x) - 2*exp
(2*c + 2*d*x) + 1)) + ((-b)^(1/2)*log(- (4*b*exp(2*c + 2*d*x))/a^2 - (2*(-
b)^(1/2)*(a*d + b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/
2)*d))*(a + b))/(2*a^(5/2)*d) - ((-b)^(1/2)*log((2*(-b)^(1/2)*(a*d + b*d +
 a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/(a^(5/2)*d) - (4*b*exp(2*c
+ 2*d*x))/a^2)*(a + b))/(2*a^(5/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 683, normalized size of antiderivative = 9.76

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{3e^{6dx+6c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)a+3e^{6dx+6c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b-9e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx-}}{}\right.}{}$$

input
```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

output
```
(3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a + 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqr
t(a + b) - sqrt(b))/sqrt(a))*b - 9*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((
e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a - 9*e**(4*c + 4*d*x)*sqrt(b
)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b + 9*e**(2*c
 + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*a + 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*b - 3*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a - 3*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*b - 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)
*sqrt(a + b) + sqrt(b))/sqrt(a))*a - 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*at
an((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b + 9*e**(4*c + 4*d*x)*sq
rt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a + 9*e**
(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sq
rt(a))*b - 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
b) + sqrt(b))/sqrt(a))*a - 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c
+ d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b + 3*sqrt(b)*sqrt(a)*atan((e**(c +
 d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a + 3*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b + 2*e**(6*c + 6*d*x)*a*b - 12*e**(2
*c + 2*d*x)*a**2 - 6*e**(2*c + 2*d*x)*a*b + 4*a**2 + 4*a*b)/(3*a**3*d*(...
```

3.33 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 192

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{3(a^2-6ab+b^2)\,x}{8(a+b)^4} + \frac{3\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a+b)^4d} - \frac{(5a-b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)} + \frac{3(3a-b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
3/8*(a^2-6*a*b+b^2)*x/(a+b)^4+3/2*a^(1/2)*(a-b)*b^(1/2)*arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))/(a+b)^4/d-1/8*(5*a-b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d/
(a+b*tanh(d*x+c)^2)+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)
^2)+3/8*(3*a-b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.69

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{12(a^2-6ab+b^2)\,(c+dx)+48\sqrt{a}(a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-8(a-b)(a+b)\sinh(2(c+dx))+}{32(a+b)^4d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(12*(a^2 - 6*a*b + b^2)*(c + d*x) + 48*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqr
t[b]*Tanh[c + d*x])/Sqrt[a]] - 8*(a - b)*(a + b)*Sinh[2*(c + d*x)] + (16*a
*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + (a + b
)^2*Sinh[4*(c + d*x)])/(32*(a + b)^4*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.16, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 4146, 372, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sin(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 372

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\int \frac{(4a-b)\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4(a+b)}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{\int -\frac{3\left(a(a-b)-(5a-b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{2(a+b)}+\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\int \frac{a(a-b)-(5a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{2(a+b)}}{4(a+b)}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\left(\frac{b(3a-b)\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{2a\left(a(a-3b)-(3a-b\ldots\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\ldots\right)}}{2\ldots}\right)}{2(a+b)}}{4(a+b)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\left(\frac{\int \frac{a(a-3b)-(3a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+d\ldots}{a+b}\ldots\right)}{2(a+b)}}{4(a+b)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\left(\frac{\frac{\left(a^2-6ab+b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{4ab\ldots}{\ldots}}{a+b}\ldots\right)}{\ldots}}{4(a+b)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\left(\frac{\frac{\left(a^2-6ab+b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{4\sqrt{\ldots}}{\ldots}}{a+b}\ldots\right)}{\ldots}}{4(a+b)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{(5a-b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{3\left(\frac{\frac{\left(a^2-6ab+b^2\right)\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{4\sqrt{a}\sqrt{b}(a-\ldots}{\ldots}}{a+b}\ldots\right)}{2(\ldots}}{4(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2*(a + b*Tanh[c + d*x]^2))
 - (((5*a - b)*Tanh[c + d*x])/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh
[c + d*x]^2)) - (3*(((4*Sqrt[a]*(a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/(a + b) + ((a^2 - 6*a*b + b^2)*ArcTanh[Tanh[c + d*x]])/(a +
 b))/(a + b) + ((3*a - b)*b*Tanh[c + d*x])/((a + b)*(a + b*Tanh[c + d*x]^2
))))/(2*(a + b)))/(4*(a + b)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 511 vs. $2(174) = 348$.

Time = 71.35 (sec) , antiderivative size = 512, normalized size of antiderivative = 2.67

method	result
derivativedivides	$-\frac{2ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}+\frac{(3a-3b)a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{2}\right)}{(a+b)^{4}}$
default	$-\frac{2ab\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}+\frac{(3a-3b)a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right.}{2}\right)}{(a+b)^{4}}$
risch	$\frac{3x\,a^{2}}{8(a+b)^{2}(a^{2}+2ab+b^{2})}-\frac{9xab}{4(a+b)^{2}(a^{2}+2ab+b^{2})}+\frac{3x\,b^{2}}{8(a+b)^{2}(a^{2}+2ab+b^{2})}+\frac{e^{4dx+4c}}{64(a^{2}+2ab+b^{2})d}-\frac{e^{2dx+2c}a}{8(a+b)(a^{2}+2ab}$

input

```
int(sinh(d*x+c)^4/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*b/(a+b)^4*(((-1/2*a-1/2*b)*tanh(1/2*d*x+1/2*c)^3+(-1/2*a-1/2*b)*
tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*
b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(3*a-3*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((
a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a
+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+1/4/(a+b)^2/(tanh(1/2*d*x+1/2*
c)-1)^4+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^3-1/8*(a-7*b)/(a+b)^3/(tanh(1/
2*d*x+1/2*c)-1)^2-1/8*(3*a-5*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/8/(a+b)^
4*(-3*a^2+18*a*b-3*b^2)*ln(tanh(1/2*d*x+1/2*c)-1)-1/4/(a+b)^2/(tanh(1/2*d*
x+1/2*c)+1)^4+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*(-a+7*b)/(a+b)^3/(
tanh(1/2*d*x+1/2*c)+1)^2-1/8*(3*a-5*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)+1/8
/(a+b)^4*(3*a^2-18*a*b+3*b^2)*ln(tanh(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3531 vs. $2(174) = 348$.

Time = 0.22 (sec) , antiderivative size = 7366, normalized size of antiderivative = 38.36

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1690 vs. 2(174) = 348.

Time = 0.34 (sec) , antiderivative size = 1690, normalized size of antiderivative = 8.80

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(a*b - 2*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
 + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 1/2*b*log((a +
 b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d) + 1/4*(a*b - 2*b^2)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a
 + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^
4)*d) + 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) +
a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/32*(3*a^3*b - 33*a^2*b^2 +
13*a*b^3 + b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((
a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) + 1/8*(3*a^2*b
 - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 33*a^2
*b^2 + 13*a*b^3 + b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(
a*b))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)*d) - 1/8*
(3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sq
rt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 3/16*(3*a*b +
 b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a
^2*b + a*b^2)*sqrt(a*b)*d) - 1/16*(a^3*b - 5*a^2*b^2 - 5*a*b^3 + b^4 + (a^
3*b - 15*a^2*b^2 + 15*a*b^3 - b^4)*e^(2*d*x + 2*c))/((a^6 + 5*a^5*b + 10*a
^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10
*a^3*b^3 + 5*a^2*b^4 + a*b^5)*e^(4*d*x + 4*c) + 2*(a^6 + 3*a^5*b + 2*a^...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 476 vs. $2(174) = 348$.

Time = 1.29 (sec) , antiderivative size = 476, normalized size of antiderivative = 2.48

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

$$= \frac{\frac{24\,(a^2-6\,ab+b^2)(dx+c)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{(18\,a^2e^{(4\,dx+4\,c)}-108\,abe^{(4\,dx+4\,c)}+18\,b^2e^{(4\,dx+4\,c)}-8\,a^2e^{(2\,dx+2\,c)}+8\,b^2e^{(2\,dx+2\,c)}+a^2+2\,ab+b^2)e^{(-4\,dx-4\,c}}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}}{}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/64*(24*(a^2 - 6*a*b + b^2)*(d*x + c)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) - (18*a^2*e^(4*d*x + 4*c) - 108*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4
*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b
+ b^2)*e^(-4*d*x - 4*c)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + 96*(
a^2*b - a*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/
sqrt(a*b))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a*b)) + (a^2*
e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) - 8*a^2*e^(2
*d*x + 2*c) + 8*b^2*e^(2*d*x + 2*c))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4) - 64*(a^2*b*e^(2*d*x + 2*c) - a*b^2*e^(2*d*x + 2*c) + a^2*b + a*b^2
)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e^(4
*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1361, normalized size of antiderivative = 7.09

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input

```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(96*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a**2 - 96*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x
)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 192*e**(6*c + 6*d*x)*sqrt(b)*sqrt
(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 - 384*e**(6*c
+ 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)
)*a*b + 192*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b
) - sqrt(b))/sqrt(a))*b**2 + 96*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**
(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 - 96*e**(4*c + 4*d*x)*sqrt(
b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 - 96*e*
*(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/s
qrt(a))*a**2 + 96*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt
(a + b) + sqrt(b))/sqrt(a))*b**2 - 192*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*at
an((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 + 384*e**(6*c + 6*d*
x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b
- 192*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sq
rt(b))/sqrt(a))*b**2 - 96*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d
*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 + 96*e**(4*c + 4*d*x)*sqrt(b)*sqr
t(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + e**(12*c +
12*d*x)*a**3 + 3*e**(12*c + 12*d*x)*a**2*b + 3*e**(12*c + 12*d*x)*a*b**2 +
 e**(12*c + 12*d*x)*b**3 - 6*e**(10*c + 10*d*x)*a**3 - 6*e**(10*c + 10*...
```

3.34 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(3a-2b)\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}d} - \frac{(a-b)\cosh(c+dx)}{(a+b)^3d} + \frac{\cosh^3(c+dx)}{3(a+b)^2d} + \frac{ab\,\text{sech}(c+dx)}{2(a+b)^3d\left(a+b-b\,\text{sech}^2(c+dx)\right)}$$

output

```
1/2*(3*a-2*b)*b^(1/2)*arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))/(a+b)^(7/2)
/d-(a-b)*cosh(d*x+c)/(a+b)^3/d+1/3*cosh(d*x+c)^3/(a+b)^2/d+1/2*a*b*sech(d*
x+c)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 160, normalized size of antiderivative = 1.29

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$
$$= \frac{\frac{6i(3a-2b)\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}}+\frac{3\cosh(c+dx)\left(5b+a\left(-3+\frac{4b}{a-b+(a+b)\cosh(2(}\right.\right.}{(a+b)^3}}{12d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((6*I)*(3*a - 2*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c +
d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/
Sqrt[b]]))/(a + b)^(7/2) + (3*Cosh[c + d*x]*(5*b + a*(-3 + (4*b)/(a - b +
(a + b)*Cosh[2*(c + d*x)]))))/(a + b)^3 + Cosh[3*(c + d*x)]/(a + b)^2)/(12
*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4147, 25, 361, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{i\sin(ic+idx)^3}{(a-b\tan(ic+idx)^2)^2}dx$$
$$\Big\downarrow \ 26$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$\frac{\int -\frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{\int \frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{ab\text{sech}(c+dx)}{2(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{1}{2}b\int \frac{\cosh^4(c+dx)\left(-\frac{a\text{sech}^4(c+dx)}{(a+b)^3}-\frac{2a\text{sech}^2(c+dx)}{b(a+b)^2}+\frac{2}{b(a+b)}\right)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\frac{ab\text{sech}(c+dx)}{2(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{1}{2}b\int \left(\frac{2\cosh^4(c+dx)}{b(a+b)^2}-\frac{2(a-b)\cosh^2(c+dx)}{b(a+b)^3}+\frac{2b-3a}{(a+b)^3\left(-b\text{sech}^2(c+dx)+a+b\right)}\right)d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{ab\text{sech}(c+dx)}{2(a+b)^3\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{1}{2}b\left(-\frac{(3a-2b)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}(a+b)^{7/2}}-\frac{2\cosh^3(c+dx)}{3b(a+b)^2}+\frac{2(a-b)\cosh(c+dx)}{b(a+b)^3}\right)}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*(b*(-(((3*a - 2*b)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(Sq
rt[b]*(a + b)^(7/2))) + (2*(a - b)*Cosh[c + d*x])/(b*(a + b)^3) - (2*Cosh[
c + d*x]^3)/(3*b*(a + b)^2))) + (a*b*Sech[c + d*x])/(2*(a + b)^3*(a + b -
b*Sech[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 266 vs. 2(110) = 220.

Time = 27.62 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.15

method	result
derivativedivides	$\frac{\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a-3b}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{\cdots}{2(a+\cdots}}{\cdots}$
default	$\frac{\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a-3b}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{3(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{\cdots}{2(a+\cdots}}{\cdots}$
risch	$\frac{e^{3dx+3c}}{24(a^2+2ab+b^2)d}-\frac{3\,e^{dx+c}a}{8(a+b)(a^2+2ab+b^2)d}+\frac{5\,e^{dx+c}b}{8(a+b)(a^2+2ab+b^2)d}-\frac{3\,e^{-dx-c}a}{8(a^3+3a^2b+3b^2a+b^3)d}+\frac{5\,e^{-dx-\cdots}}{8(a^3+3a^2b+3\cdots}$

input
```
int(sinh(d*x+c)^3/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c
)+1)^2-1/2*(a-3*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)-1/3/(a+b)^2/(tanh(1/2*d
*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^3*(-a+3*b)/
(tanh(1/2*d*x+1/2*c)-1)-4*b/(a+b)^3*(((-1/4*a-1/2*b)*tanh(1/2*d*x+1/2*c)^2
-1/4*a)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*
x+1/2*c)^2+a)-1/8*(3*a-2*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/
2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2650 vs. 2(113) = 226.

Time = 0.19 (sec) , antiderivative size = 5025, normalized size of antiderivative = 40.52

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh^3\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/24*(a^2 + 2*a*b + b^2 + (a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*c))*e
^(10*d*x) - (7*a^2*e^(8*c) - 6*a*b*e^(8*c) - 13*b^2*e^(8*c))*e^(8*d*x) - 2
*(13*a^2*e^(6*c) - 40*a*b*e^(6*c) + 7*b^2*e^(6*c))*e^(6*d*x) - 2*(13*a^2*e
^(4*c) - 40*a*b*e^(4*c) + 7*b^2*e^(4*c))*e^(4*d*x) - (7*a^2*e^(2*c) - 6*a*
b*e^(2*c) - 13*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(7*c) + 4*a^3*b*d*e^(7*c)
 + 6*a^2*b^2*d*e^(7*c) + 4*a*b^3*d*e^(7*c) + b^4*d*e^(7*c))*e^(7*d*x) + 2*
(a^4*d*e^(5*c) + 2*a^3*b*d*e^(5*c) - 2*a*b^3*d*e^(5*c) - b^4*d*e^(5*c))*e^
(5*d*x) + (a^4*d*e^(3*c) + 4*a^3*b*d*e^(3*c) + 6*a^2*b^2*d*e^(3*c) + 4*a*b
^3*d*e^(3*c) + b^4*d*e^(3*c))*e^(3*d*x)) - 1/8*integrate(8*((3*a*b*e^(3*c)
 - 2*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^4 + 4*a^
3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b
^2*e^(4*c) + 4*a*b^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2
*a^3*b*e^(2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1523, normalized size of antiderivative = 12.28

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 18*e**(7*c + 7*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a +
b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2 - 6*e**(7*c + 7*d*x)*sqrt(
b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c +
d*x)*sqrt(b))*a*b + 12*e**(7*c + 7*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c +
2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2 + 18*e**(7
*c + 7*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a
+ b) + 2*e**(c + d*x)*sqrt(b))*a**2 + 6*e**(7*c + 7*d*x)*sqrt(b)*sqrt(a +
b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b)
)*a*b - 12*e**(7*c + 7*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(
a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b**2 - 36*e**(5*c + 5*d*x)*
sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**
(c + d*x)*sqrt(b))*a**2 + 60*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a + b)*log(e**(
2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a*b - 24*
e**(5*c + 5*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sq
rt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2 + 36*e**(5*c + 5*d*x)*sqrt(b)*sqr
t(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*s
qrt(b))*a**2 - 60*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x
)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*b + 24*e**(5*c + 5
*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) +
 2*e**(c + d*x)*sqrt(b))*b**2 - 18*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a + b)...
```

3.35 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(a-3b)x}{2(a+b)^3} - \frac{(3a-b)\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a+b)^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{b\tanh(c+dx)}{(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/2*(a-3*b)*x/(a+b)^3-1/2*(3*a-b)*b^(1/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1
/2))/a^(1/2)/(a+b)^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c
)^2)-b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.89 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.80

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-2(a-3b)(c+dx)+\frac{2\sqrt{b}(-3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}+(a+b)\sinh(2(c+dx))-\frac{2b(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{4(a+b)^3d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-2*(a - 3*b)*(c + d*x) + (2*Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d
*x])/Sqrt[a]])/Sqrt[a] + (a + b)*Sinh[2*(c + d*x)] - (2*b*(a + b)*Sinh[2*(
c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(4*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4146, 373, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 373

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int \frac{a-3b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{2(a+b)}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{2a\left(-2b\tanh^2(c+dx)+a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{\int \frac{-2b\tanh^2(c+dx)+a-b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a+b}+\frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{\frac{(a-3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b(3a-b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a+b}+\frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{\frac{(a-3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a+b}+\frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{\frac{\sqrt{b}(3a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{(a-3b)\text{arctanh}(\tanh(c+dx))}{a+b}}{a+b}+\frac{2b\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}}{d}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)) -
 ((((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a
 + b)) + ((a - 3*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + (2*b*Tanh[c
 + d*x])/((a + b)*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 380 vs. $2(118) = 236$.

Time = 8.87 (sec) , antiderivative size = 381, normalized size of antiderivative = 2.89

method	result
risch	$-\frac{xa}{2(a+b)(a^2+2ab+b^2)}+\frac{3xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2dx+2c}}{8(a^2+2ab+b^2)d}-\frac{e^{-2dx-2c}}{8(a^2+2ab+b^2)d}+\frac{b(e^{2dx}}{d(a+b)^3(e^{4dx+4c}a+b\,e}$
derivativedivides	$\frac{-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-a+3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2(a+b)^3}+\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}\right.}\right.}{}}{}$
default	$\frac{-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(-a+3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2(a+b)^3}+\frac{4b\left(\frac{\left(-\frac{a}{4}-\frac{b}{4}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}\right.}\right.}{}}{}$

input
```
int(sinh(d*x+c)^2/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/2*x/(a+b)/(a^2+2*a*b+b^2)*a+3/2*x/(a+b)/(a^2+2*a*b+b^2)*b+1/8/(a^2+2*a*
b+b^2)/d*exp(2*d*x+2*c)-1/8/(a^2+2*a*b+b^2)/d*exp(-2*d*x-2*c)+b*(exp(2*d*x
+2*c)*a-exp(2*d*x+2*c)*b+a+b)/d/(a+b)^3/(exp(4*d*x+4*c)*a+b*exp(4*d*x+4*c)
+2*exp(2*d*x+2*c)*a-2*exp(2*d*x+2*c)*b+a+b)+3/4*(-a*b)^(1/2)/(a+b)^3/d*ln(
exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))-1/4/a*(-a*b)^(1/2)/(a+b)^3/d*ln
(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))*b-3/4*(-a*b)^(1/2)/(a+b)^3/d*l
n(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))+1/4/a*(-a*b)^(1/2)/(a+b)^3/d*
ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))*b
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1798 vs. $2(118) = 236$.

Time = 0.16 (sec) , antiderivative size = 3918, normalized size of antiderivative = 29.68

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. $2(118) = 236$.

Time = 0.24 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.36

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/
8*(3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b
 - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/4*(3*a*b + b^2)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*
b^2)*sqrt(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x +
2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b
^3 - a*b^4)*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^
3)*e^(-2*d*x - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(
a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a
^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) - 1/2*(a*b + b^2 + (a*b -
 b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3
*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*e^(-4*d*x - 4*c))*d) - 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2
*d*x + 2*c)/((a^2 + 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b +
 b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 367 vs. $2(118) = 236$.

Time = 0.74 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.78

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$

$$-\frac{\frac{12\,(dx+c)(a-3\,b)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{12\,(3\,ab-b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}}-\frac{3\,e^{(2\,dx+2\,c)}}{a^2+2\,ab+b^2}-\frac{2\,a^2e^{(6\,dx+6\,c)}-4\,abe^{(6\,dx+6\,c)}-6\,b^2e^{(6\,d}}{(a^3+3\,a^2b+}}{24\,d}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/24*(12*(d*x + c)*(a - 3*b)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(3*a*b
- b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b
))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) - 3*e^(2*d*x + 2*c)/(a^2 +
2*a*b + b^2) - (2*a^2*e^(6*d*x + 6*c) - 4*a*b*e^(6*d*x + 6*c) - 6*b^2*e^(6
*d*x + 6*c) + a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) - 15*b^2*e^(4*d*
x + 4*c) - 4*a^2*e^(2*d*x + 2*c) + 20*a*b*e^(2*d*x + 2*c) + 24*b^2*e^(2*d*
x + 2*c) - 3*a^2 - 6*a*b - 3*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(a*e^(6
*d*x + 6*c) + b*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c
) + a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{{\mathrm{sinh}\left(c+d\,x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(c+d\,x\right)}^2+a\right)}^2} \,d x$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 1237, normalized size of antiderivative = 9.37

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
( - 12*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**2 - 8*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d
*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b + 4*e**(6*c + 6*d*x)*sqrt(b)*sqrt(
a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 - 24*e**(4*c +
4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*
a**2 + 32*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
- sqrt(b))/sqrt(a))*a*b - 8*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
 d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 - 12*e**(2*c + 2*d*x)*sqrt(b)*s
qrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 - 8*e**(2*c
 + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*a*b + 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
 - sqrt(b))/sqrt(a))*b**2 + 12*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(
c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 + 8*e**(6*c + 6*d*x)*sqrt(b)
*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b - 4*e**(6*
c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(
a))*b**2 + 24*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
 b) + sqrt(b))/sqrt(a))*a**2 - 32*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e
**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b + 8*e**(4*c + 4*d*x)*sqrt(
b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + 12*e*
*(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b)...
```

3.36 $\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 90

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{3\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}d}+\frac{\cosh(c+dx)}{(a+b)^2d}-\frac{b\operatorname{sech}(c+dx)}{2(a+b)^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
-3/2*b^(1/2)*arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))/(a+b)^(5/2)/d+cosh(d
*x+c)/(a+b)^2/d-1/2*b*sech(d*x+c)/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.48

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{3i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{5/2}}+\frac{2\cosh(c+dx)\left(1-\frac{b}{a-b+(a+b)\cosh(2(c+dx))}\right)}{(a+b)^2}}{2d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((-3*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/Sq
rt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))/
(a + b)^(5/2) + (2*Cosh[c + d*x]*(1 - b/(a - b + (a + b)*Cosh[2*(c + d*x)]
)))/(a + b)^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 26, 4147, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4147}$$

$$-\frac{\int \frac{\cosh^2(c+dx)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{253}$$

$$-\frac{\frac{3\int \frac{\cosh^2(c+dx)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{264}$$

$$-\frac{\frac{3\left(\frac{b\int \frac{1}{-b\,\text{sech}^2(c+dx)+a+b}d\,\text{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{3\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-(((3*((Sqrt[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2
) - Cosh[c + d*x]/(a + b)))/(2*(a + b)) + Cosh[c + d*x]/(2*(a + b)*(a + b
- b*Sech[c + d*x]^2)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 166 vs. $2(78) = 156$.

Time = 3.58 (sec) , antiderivative size = 167, normalized size of antiderivative = 1.86

method	result
derivativedivides	$\frac{\frac{2b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{1}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{3\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}}\right)}{(a+b)^2}+\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\right.\right.}}{d}$
default	$\frac{\frac{2b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{2a}-\frac{1}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{3\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4b}{4\sqrt{ab+b^2}}\right)}{4\sqrt{ab+b^2}}\right)}{(a+b)^2}+\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\right.\right.}}{d}$
risch	$\frac{e^{dx+c}}{2(a^2+2ab+b^2)d}+\frac{e^{-dx-c}}{2(a^2+2ab+b^2)d}-\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{d(a+b)^2\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}+\frac{3\sqrt{(a+b)}}{}$

input
```
int(sinh(d*x+c)/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a+b)^2*b*((-1/2*(a+2*b)/a*tanh(1/2*d*x+1/2*c)^2-1/2)/(tanh(1/2*d*x
+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)-3/4/(a*
b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/
2)))+1/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1114 vs. $2(81) = 162$.

Time = 0.13 (sec) , antiderivative size = 2252, normalized size of antiderivative = 25.02

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(2*(a + b)*cosh(d*x + c)^6 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5
 + 2*(a + b)*sinh(d*x + c)^6 + 6*(a - b)*cosh(d*x + c)^4 + 6*(5*(a + b)*co
sh(d*x + c)^2 + a - b)*sinh(d*x + c)^4 + 8*(5*(a + b)*cosh(d*x + c)^3 + 3*
(a - b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)^2 + 6*(5*
(a + b)*cosh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)
^2 + 3*((a + b)*cosh(d*x + c)^5 + 5*(a + b)*cosh(d*x + c)*sinh(d*x + c)^4
+ (a + b)*sinh(d*x + c)^5 + 2*(a - b)*cosh(d*x + c)^3 + 2*(5*(a + b)*cosh(
d*x + c)^2 + a - b)*sinh(d*x + c)^3 + 2*(5*(a + b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c))*sinh(d*x + c)^2 + (a + b)*cosh(d*x + c) + (5*(a + b)*c
osh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/
(a + b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a +
b)*cosh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a + 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^3 + 3
*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)
*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b
/(a + b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(
a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)
^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 12*((a + b)*cosh(...
```

Sympy [F]

$$\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\sinh(dx+c)}{(b\tanh(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(a*e^(4*c) - b*e^(4*c))*e^(4*d*
x) + 3*(a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)/((a^3*d*e^(5*c) + 3*a^2*
b*d*e^(5*c) + 3*a*b^2*d*e^(5*c) + b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^3*d*e^(3
*c) + a^2*b*d*e^(3*c) - a*b^2*d*e^(3*c) - b^3*d*e^(3*c))*e^(3*d*x) + (a^3*
d*e^c + 3*a^2*b*d*e^c + 3*a*b^2*d*e^c + b^3*d*e^c)*e^(d*x)) + 1/2*integrat
e(6*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (
a^3*e^(4*c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) +
 2*(a^3*e^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)),
 x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\sinh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1296, normalized size of antiderivative = 14.40

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(4*e**(5*c + 5*d*x)*cosh(c + d*x)*a**4 + 16*e**(5*c + 5*d*x)*cosh(c + d*x)
*a**3*b + 24*e**(5*c + 5*d*x)*cosh(c + d*x)*a**2*b**2 + 16*e**(5*c + 5*d*x
)*cosh(c + d*x)*a*b**3 + 4*e**(5*c + 5*d*x)*cosh(c + d*x)*b**4 + 8*e**(3*c
 + 3*d*x)*cosh(c + d*x)*a**4 + 16*e**(3*c + 3*d*x)*cosh(c + d*x)*a**3*b -
16*e**(3*c + 3*d*x)*cosh(c + d*x)*a*b**3 - 8*e**(3*c + 3*d*x)*cosh(c + d*x
)*b**4 + 4*e**(c + d*x)*cosh(c + d*x)*a**4 + 16*e**(c + d*x)*cosh(c + d*x)
*a**3*b + 24*e**(c + d*x)*cosh(c + d*x)*a**2*b**2 + 16*e**(c + d*x)*cosh(c
 + d*x)*a*b**3 + 4*e**(c + d*x)*cosh(c + d*x)*b**4 + 3*e**(5*c + 5*d*x)*sq
rt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c
 + d*x)*sqrt(b))*a**3 + 3*e**(5*c + 5*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c
 + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b - 3*e
**(5*c + 5*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqr
t(a + b) + 2*e**(c + d*x)*sqrt(b))*a**3 - 3*e**(5*c + 5*d*x)*sqrt(b)*sqrt(
a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqr
t(b))*a**2*b + 6*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)
*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**3 - 6*e**(3*c + 3*
d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) -
2*e**(c + d*x)*sqrt(b))*a**2*b - 6*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a + b)*lo
g(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**
3 + 6*e**(3*c + 3*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a ...
```

3.37 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 103

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{a^2d} + \frac{\sqrt{b}(3a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b\operatorname{sech}(c+dx)}{2a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
-arctanh(cosh(d*x+c))/a^2/d+1/2*b^(1/2)*(3*a+2*b)*arctanh(b^(1/2)*sech(d*x
+c)/(a+b)^(1/2))/a^2/(a+b)^(3/2)/d+1/2*b*sech(d*x+c)/a/(a+b)/d/(a+b-b*sech
(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 0.94 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.83

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{i\sqrt{b}(3a+2b)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{i\sqrt{b}(3a+2b)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}} + \frac{2ab\cosh(c+dx)}{(a+b)(a-b+(a+b)\cosh(2(c+dx)}}{2a^2d}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/
2])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(3*a + 2*b)*ArcTan[((-I)*Sqrt[a +
 b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (2*a*b*Cosh[c +
 d*x])/((a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])) - 2*Log[Cosh[(c + d*x
)/2]] + 2*Log[Sinh[(c + d*x)/2]])/(2*a^2*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.09, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 26, 4147, 25, 316, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$
$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \ 4147$$

$$\frac{\int -\frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \ 25$$

$$-\frac{\int \frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \ 316$$

$$\frac{\frac{\int -\frac{b\,\text{sech}^2(c+dx)+2a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}+\frac{b\,\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{b\,\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{\int \frac{b\,\text{sech}^2(c+dx)+2a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{b\,\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{\frac{2(a+b)\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{a}-\frac{b(3a+2b)\int \frac{1}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a(a+b)}}{d}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{b\,\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{\frac{2(a+b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{b(3a+2b)\int \frac{1}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a(a+b)}}{d}$$

$$\Big\downarrow \ 221$$

$$\frac{\frac{b\,\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{\frac{2(a+b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((2*(a + b)*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(3*a + 2*b)*ArcTanh
[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(a*(a + b)) + (b*S
ech[c + d*x])/(2*a*(a + b)*(a + b - b*Sech[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.56

method	result
derivativedivides	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a+b)}-\frac{a}{4(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{(3a+2b)\,\text{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4}{4\sqrt{ab+b^2}}\right.}{8(a+b)\sqrt{ab+b^2}}}{a^2}}{d}$
default	$\frac{\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}-\frac{4b\left(\frac{-\frac{(a+2b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{4(a+b)}-\frac{a}{4(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{(3a+2b)\,\text{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+2a+4}{4\sqrt{ab+b^2}}\right.}{8(a+b)\sqrt{ab+b^2}}}{a^2}}{d}$
risch	$\frac{b\,e^{dx+c}\left(e^{2dx+2c}+1\right)}{ad(a+b)\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}-\frac{\ln\left(e^{dx+c}+1\right)}{a^2d}+\frac{\ln\left(e^{dx+c}-1\right)}{a^2d}+\frac{3\sqrt{(a+b)b}\,\ln\left(\right.}{}$

input
```
int(csch(d*x+c)/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/a^2*ln(tanh(1/2*d*x+1/2*c))-4*b/a^2*((-1/4*(a+2*b)/(a+b)*tanh(1/2*d
*x+1/2*c)^2-1/4*a/(a+b))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*
a+4*b*tanh(1/2*d*x+1/2*c)^2+a)-1/8*(3*a+2*b)/(a+b)/(a*b+b^2)^(1/2)*arctanh
(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1324 vs. $2(94) = 188$.

Time = 0.16 (sec) , antiderivative size = 2614, normalized size of antiderivative = 25.38

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) + ((3*a^2 + 5*a*b + 2*b^2)*cosh(d*x
 + c)^4 + 4*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2
 + 5*a*b + 2*b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - a*b - 2*b^2)*cosh(d*x + c)^
2 + 2*(3*(3*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 3*a^2 - a*b - 2*b^2)*si
nh(d*x + c)^2 + 3*a^2 + 5*a*b + 2*b^2 + 4*((3*a^2 + 5*a*b + 2*b^2)*cosh(d*
x + c)^3 + (3*a^2 - a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a +
 b))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*sinh(d*x + c)^4 + 2*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(a + b)*c
osh(d*x + c)^2 + a + 3*b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*((a + b)*cosh(d*x + c)^3 + 3*(a
+ b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (a + b)*cos
h(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c))*sqrt(b/(a
+ b)) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a +
b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 +
 (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) - 4*((a^2 + 2*a*b + b^2)*c
osh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^
2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 + ...
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}(dx+c)}{\left(b\tanh(dx+c)^2+a\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*
e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) +
log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) +
2*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 2*b^2*e^c)*e^(d*x))/(a^4 + 2*a^3*b
 + a^2*b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) +
 2*(a^4*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1630, normalized size of antiderivative = 15.83

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 3*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b
) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2 - 5*e**(4*c + 4*d*x)*sqrt(b
)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d
*x)*sqrt(b))*a*b - 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*
d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2 + 3*e**(4*c
+ 4*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b
) + 2*e**(c + d*x)*sqrt(b))*a**2 + 5*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b)*
log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a
*b + 2*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a +
b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b**2 - 6*e**(2*c + 2*d*x)*sqrt(
b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c +
d*x)*sqrt(b))*a**2 + 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c +
2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a*b + 4*e**(2*c
 + 2*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a +
b) - 2*e**(c + d*x)*sqrt(b))*b**2 + 6*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a + b)
*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*
a**2 - 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a
+ b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*b - 4*e**(2*c + 2*d*x)*sqrt
(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c +
 d*x)*sqrt(b))*b**2 - 3*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a...
```

3.38 $\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 81

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d} - \frac{\coth(c+dx)}{a^2d} - \frac{b\tanh(c+dx)}{2a^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-3/2*b^(1/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/d-coth(d*x+c)/a^2
/d-1/2*b*tanh(d*x+c)/a^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.06

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{-3\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) - 2\sqrt{a}\coth(c+dx) - \frac{\sqrt{a}b\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{5/2}d}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-3*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 2*Sqrt[a]*Coth[c + d
*x] - (Sqrt[a]*b*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(
2*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 25, 4146, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{253}$$

$$\frac{\frac{3\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{264}$$

$$\frac{\frac{3\left(-\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a(a+b\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a(a+b\tanh^2(c+dx))}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((3*(-((Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) - Coth[c
 + d*x]/a))/(2*a) + Coth[c + d*x]/(2*a*(a + b*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(69) = 138$.

Time = 3.36 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.11

method	result
risch	$-\frac{2\,\mathrm{e}^{4dx+4c}a^2+3\,\mathrm{e}^{4dx+4c}ab+3\,\mathrm{e}^{4dx+4c}b^2+4\,\mathrm{e}^{2dx+2c}a^2-6b^2\mathrm{e}^{2dx+2c}+2a^2+5ab+3b^2}{d\,a^2\left(\mathrm{e}^{2dx+2c}-1\right)(a+b)\left(\mathrm{e}^{4dx+4c}a+b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a-2\,\mathrm{e}^{2dx+2c}b+a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{2\sqrt{-ab}}{a+\cdots}\right.}{4a^3d}$
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}+\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}\cdots\right.}{\sqrt{\left(2\sqrt{(a+b)b}+\cdots\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)\cdots}}\right.}{\cdots}\right.}{a^2}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^2}+\frac{4b\left(\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}\cdots\right.}{\sqrt{\left(2\sqrt{(a+b)b}+\cdots\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)\cdots}}\right.}{\cdots}\right.}{a^2}}{d}$

input
```
int(csch(d*x+c)^2/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(2*exp(4*d*x+4*c)*a^2+3*exp(4*d*x+4*c)*a*b+3*exp(4*d*x+4*c)*b^2+4*exp(2*d
*x+2*c)*a^2-6*b^2*exp(2*d*x+2*c)+2*a^2+5*a*b+3*b^2)/d/a^2/(exp(2*d*x+2*c)-
1)/(a+b)/(exp(4*d*x+4*c)*a+b*exp(4*d*x+4*c)+2*exp(2*d*x+2*c)*a-2*exp(2*d*x
+2*c)*b+a+b)+3/4/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)
/(a+b))-3/4/a^3*(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1120 vs. $2(69) = 138$.

Time = 0.14 (sec) , antiderivative size = 2562, normalized size of antiderivative = 31.63

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(2*a^2 + 3*a*b + 3*b^2)*cosh(d*x + c)^4 + 16*(2*a^2 + 3*a*b + 3*b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(2*a^2 + 3*a*b + 3*b^2)*sinh(d*x + c
)^4 + 8*(2*a^2 - 3*b^2)*cosh(d*x + c)^2 + 8*(3*(2*a^2 + 3*a*b + 3*b^2)*cos
h(d*x + c)^2 + 2*a^2 - 3*b^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cos
h(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2
+ 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^4 + (
15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b - 3*b^2)*sinh(d*x + c
)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b - 3*b^2)*cos
h(d*x + c))*sinh(d*x + c)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^2 + (15*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c
)^2 - a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 2*(3*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 - 2*a*b - 3*b^2)*cosh(d*x + c)^3
- (a^2 - 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log(((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
- b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2
 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*...
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{csch}^2\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 212 vs. $2(69) = 138$.

Time = 0.19 (sec) , antiderivative size = 212, normalized size of antiderivative = 2.62

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{2\,a^2+5\,ab+3\,b^2+2\,(2\,a^2-3\,b^2)e^{(-2\,dx-2\,c)}+(2\,a^2+3\,ab+3\,b^2)e^{(-4\,dx-4\,c)}}{(a^4+2\,a^3b+a^2b^2+(a^4-2\,a^3b-3\,a^2b^2)e^{(-2\,dx-2\,c)}-(a^4-2\,a^3b-3\,a^2b^2)e^{(-4\,dx-4\,c)}-(a^4+2\,a^3b+} + \frac{3\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}a^2d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x - 2*c) + (2*a^2 + 3*
a*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2 + (a^4 - 2*a^3*b
- 3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-4*d*x - 4*
c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) + 3/2*b*arctan(1/2*((a
 + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(69) = 138$.

Time = 0.34 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.80

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{3\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a^2} + \frac{2\,(2\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}+3\,b^2e^{(4\,dx+4\,c)}+4\,a^2e^{(2\,dx+2\,c)}-6\,b^2e^{(2\,dx+2\,c)}+2\,a^2+5\,ab}{(a^3+a^2b)(ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}}}{2\,d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*(3*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(
a*b))/(sqrt(a*b)*a^2) + 2*(2*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) +
 3*b^2*e^(4*d*x + 4*c) + 4*a^2*e^(2*d*x + 2*c) - 6*b^2*e^(2*d*x + 2*c) + 2
*a^2 + 5*a*b + 3*b^2)/((a^3 + a^2*b)*(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c
) + a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*e^(2*d*x + 2*c) + 3*b*e^(2
*d*x + 2*c) - a - b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1177, normalized size of antiderivative = 14.53

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**2 + 6*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*
x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b + 9*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 - 3*e**(4*c + 4*
d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*
*2 + 18*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a*b - 27*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 3*e**(2*c + 2*d*x)*sqrt(b)*sqr
t(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 - 18*e**(2*c
+ 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)
)*a*b + 27*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
 - sqrt(b))/sqrt(a))*b**2 + 3*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
b) - sqrt(b))/sqrt(a))*a**2 - 6*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) - sqrt(b))/sqrt(a))*a*b - 9*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
 + b) - sqrt(b))/sqrt(a))*b**2 + 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((
e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 - 6*e**(6*c + 6*d*x)*sqr
t(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b - 9*e*
*(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/s
qrt(a))*b**2 + 3*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(
a + b) + sqrt(b))/sqrt(a))*a**2 - 18*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*a...
```

3.39 $\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 141

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{(a+4b)\operatorname{arctanh}(\cosh(c+dx))}{2a^3d} - \frac{\sqrt{b}(3a+4b)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{2a^3\sqrt{a+b}d} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)} - \frac{b\operatorname{sech}(c+dx)}{a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
1/2*(a+4*b)*arctanh(cosh(d*x+c))/a^3/d-1/2*b^(1/2)*(3*a+4*b)*arctanh(b^(1/
2)*sech(d*x+c)/(a+b)^(1/2))/a^3/(a+b)^(1/2)/d-1/2*coth(d*x+c)*csch(d*x+c)/
a/d/(a+b-b*sech(d*x+c)^2)-b*sech(d*x+c)/a^2/d/(a+b-b*sech(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 3.53 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.57

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{4i\sqrt{b}(3a+4b)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}+\frac{4i\sqrt{b}(3a+4b)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{\sqrt{a+b}}+\frac{8ab\cosh(c+dx)}{a-b+(a+b)\cosh(2(c+dx))}}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/8*(((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(
c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + ((4*I)*Sqrt[b]*(3*a + 4*b)*ArcTan[((-
I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/Sqrt[a + b] + (8*a*b
*Cosh[c + d*x])/(a - b + (a + b)*Cosh[2*(c + d*x)]) + a*Csch[(c + d*x)/2]^
2 - 4*(a + 4*b)*Log[Cosh[(c + d*x)/2]] + 4*(a + 4*b)*Log[Sinh[(c + d*x)/2]
] + a*Sech[(c + d*x)/2]^2)/(a^3*d)
```

Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 150, normalized size of antiderivative = 1.06, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 26, 4147, 373, 402, 27, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{1}{\sin(ic+idx)^3\,(a-b\tan(ic+idx)^2)^2}dx$$

$$\Big\downarrow \; 4147$$

$$-\frac{\int \frac{\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)^2\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 373$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{\int \frac{3b\,\text{sech}^2(c+dx)+a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{2a}}{d}$$

$$\Big\downarrow \; 402$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{-\frac{\int -\frac{2(a+b)\left(2b\,\text{sech}^2(c+dx)+a+2b\right)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}-\frac{2b\,\text{sech}(c+dx)}{a\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{\frac{\int \frac{2b\,\text{sech}^2(c+dx)+a+2b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{a}-\frac{2b\,\text{sech}(c+dx)}{a\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)}-\frac{\frac{\frac{(a+4b)\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{a}-\frac{b(3a+4b)\int \frac{1}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{a}-\frac{\cdots}{a(a\cdots}}{2a}}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{\frac{\frac{(a+4b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{b(3a+4b)\int\frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{a}-\frac{2b\text{sech}(c+dx)}{a\left(a-b\text{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

$$\Big\downarrow \ 221$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\text{sech}^2(c+dx)+b\right)}-\frac{\frac{\frac{(a+4b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{\sqrt{b}(3a+4b)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{a}-\frac{2b\text{sech}(c+dx)}{a\left(a-b\text{sech}^2(c+dx)+b\right)}}{2a}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((Sech[c + d*x]/(2*a*(1 - Sech[c + d*x]^2)*(a + b - b*Sech[c + d*x]^2)) -
 ((((a + 4*b)*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(3*a + 4*b)*ArcTanh[(Sq
rt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/a - (2*b*Sech[c + d*x]
)/(a*(a + b - b*Sech[c + d*x]^2)))/(2*a))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 5.12 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8a^{2}}+\frac{2b\left(\frac{\left(-\frac{a}{2}-b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}-\frac{(3a+4b)\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{4\sqrt{ab+b^{2}}}\right)}{a^{3}}}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}}{8a^{2}}+\frac{2b\left(\frac{\left(-\frac{a}{2}-b\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}-\frac{a}{2}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}-\frac{(3a+4b)\,\mathrm{arctanh}\left(\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+2a+4b}{4\sqrt{ab+b^{2}}}\right)}{4\sqrt{ab+b^{2}}}\right)}{a^{3}}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{6dx+6c}a+2\,e^{6dx+6c}b+3\,e^{4dx+4c}a-2b\,e^{4dx+4c}+3\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+2b\right)}{d\,a^{2}\left(e^{2dx+2c}-1\right)^{2}\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}-\frac{\ln\left(e^{dx+c}-1\right)}{2a^{2}d}-\frac{2\ln(e}{}$

input
```
int(csch(d*x+c)^3/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^2+2*b/a^3*(((-1/2*a-b)*tanh(1/2*d*x+1/2*c
)^2-1/2*a)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2
*d*x+1/2*c)^2+a)-1/4*(3*a+4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x
+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/8/a^2/tanh(1/2*d*x+1/2*c)^2+1/4/a
^3*(-2*a-8*b)*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3368 vs. 2(132) = 264.

Time = 0.18 (sec) , antiderivative size = 6335, normalized size of antiderivative = 44.93

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(b\tanh(dx+c)^2+a)^2}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a*e^(7*c) + 2*b*e^(7*c))*e^(7*d*x) + (3*a*e^(5*c) - 2*b*e^(5*c))*e^(5*d*
x) + (3*a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x) + (a*e^c + 2*b*e^c)*e^(d*x))/(4
*a^2*b*d*e^(6*d*x + 6*c) + 4*a^2*b*d*e^(2*d*x + 2*c) - a^3*d - a^2*b*d - (
a^3*d*e^(8*c) + a^2*b*d*e^(8*c))*e^(8*d*x) + 2*(a^3*d*e^(4*c) - 3*a^2*b*d*
e^(4*c))*e^(4*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^3*d)
- 1/2*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integrate(1/8*((
3*a*b*e^(3*c) + 4*b^2*e^(3*c))*e^(3*d*x) - (3*a*b*e^c + 4*b^2*e^c)*e^(d*x)
)/(a^4 + a^3*b + (a^4*e^(4*c) + a^3*b*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c)
- a^3*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{1}{\sinh(c+d\,x)^3\,\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 2493, normalized size of antiderivative = 17.68

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(3*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) +
 sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2 + 7*e**(8*c + 8*d*x)*sqrt(b)*s
qrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)
*sqrt(b))*a*b + 4*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x
)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2 - 3*e**(8*c + 8
*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) +
 2*e**(c + d*x)*sqrt(b))*a**2 - 7*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log
(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*b
- 4*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b)
+ sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b**2 - 12*e**(6*c + 6*d*x)*sqrt(b)
*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*
x)*sqrt(b))*a*b - 16*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*
d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2 + 12*e**(6*c
 + 6*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a +
b) + 2*e**(c + d*x)*sqrt(b))*a*b + 16*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a + b)
*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*
b**2 - 6*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a
+ b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2 + 10*e**(4*c + 4*d*x)*sq
rt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c
 + d*x)*sqrt(b))*a*b + 24*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a + b)*log(e**(...
```

3.40 $\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d} + \frac{(a+2b)\coth(c+dx)}{a^3d} - \frac{\coth^3(c+dx)}{3a^2d} + \frac{b(a+b)\tanh(c+dx)}{2a^3d\,(a+b\tanh^2(c+dx))}$$

output
```
1/2*b^(1/2)*(3*a+5*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/d+(a+2*b
)*coth(d*x+c)/a^3/d-1/3*coth(d*x+c)^3/a^2/d+1/2*b*(a+b)*tanh(d*x+c)/a^3/d/
(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.01

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$=\frac{3\sqrt{b}(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)+2\sqrt{a}\coth(c+dx)\left(2a+6b-a\operatorname{csch}^2(c+dx)\right)+\frac{3\sqrt{a}b(a+b)\sinh(2(c+}{a-b+(a+b)\cosh(2(c+}}{6a^{7/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(3*Sqrt[b]*(3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + 2*Sqrt[a]
*Coth[c + d*x]*(2*a + 6*b - a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(a + b)*Sinh
[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(6*a^(7/2)*d)
```

Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4146, 361, 25, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{b(a+b)\tanh(c+dx)}{2a^3(a+b\tanh^2(c+dx))}-\frac{1}{2}b\int -\frac{\coth^4(c+dx)\left(\frac{(a+b)\tanh^4(c+dx)}{a^3}-\frac{2(a+b)\tanh^2(c+dx)}{a^2b}+\frac{2}{ab}\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{2}b\int \frac{\coth^4(c+dx)\left(\frac{(a+b)\tanh^4(c+dx)}{a^3}-\frac{2(a+b)\tanh^2(c+dx)}{a^2b}+\frac{2}{ab}\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)+\frac{b(a+b)\tanh(c+dx)}{2a^3(a+b\tanh^2(c+dx))}}{d}$$

$\downarrow$ 1584

$$\frac{\frac{1}{2}b\int \left(\frac{2\coth^4(c+dx)}{a^2b}-\frac{2(a+2b)\coth^2(c+dx)}{a^3b}+\frac{3a+5b}{a^3(b\tanh^2(c+dx)+a)}\right)d\tanh(c+dx)+\frac{b(a+b)\tanh(c+dx)}{2a^3(a+b\tanh^2(c+dx))}}{d}$$

$\downarrow$ 2009

$$\frac{\frac{b(a+b)\tanh(c+dx)}{2a^3(a+b\tanh^2(c+dx))}+\frac{1}{2}b\left(\frac{(3a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{b}}+\frac{2(a+2b)\coth(c+dx)}{a^3b}-\frac{2\coth^3(c+dx)}{3a^2b}\right)}{d}$$

input

```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output

```
((b*(((3*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[b
]) + (2*(a + 2*b)*Coth[c + d*x])/(a^3*b) - (2*Coth[c + d*x]^3)/(3*a^2*b)))
/2 + (b*(a + b)*Tanh[c + d*x])/(2*a^3*(a + b*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25

```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
 + 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
 - a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(99) = 198$.

Time = 7.43 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.99

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(3a+5\cdots}{\cdots}\right.}{\cdots}$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3 a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-8b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^3}-\frac{2b\left(\frac{\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3+\left(-\frac{a}{2}-\frac{b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(3a+5\cdots}{\cdots}\right.}{\cdots}$
risch	$-\frac{-9\,e^{8dx+8c}ab-15\,e^{8dx+8c}b^2+12\,e^{6dx+6c}a^2+6\,e^{6dx+6c}ab+60\,e^{6dx+6c}b^2+20\,e^{4dx+4c}a^2-4\,e^{4dx+4c}ab-90\,e^{4dx+4c}b^2\cdots}{3d\,a^3\left(e^{2dx+2c}-1\right)^3\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+\cdots}\right.}$

input
```
int(csch(d*x+c)^4/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/a^3*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-8*b*tan
h(1/2*d*x+1/2*c))-2*b/a^3*(((-1/2*a-1/2*b)*tanh(1/2*d*x+1/2*c)^3+(-1/2*a-1
/2*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^
2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(3*a+5*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b
)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)
/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/24/a^2/tanh(1/2*d*x+1/
2*c)^3-1/8/a^3*(-3*a-8*b)/tanh(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2370 vs. $2(99) = 198$.

Time = 0.16 (sec) , antiderivative size = 5062, normalized size of antiderivative = 44.80

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(99) = 198$.

Time = 0.23 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.50

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{4a^2+19ab+15b^2-2(2a^2+13ab+30b^2)e^{(-2dx-2c)}-2(10a^2-2ab-45b^2)e^{(-4dx-4c)}-6(2a^2+a}{3(a^4+a^3b-(a^4+5a^3b)e^{(-2dx-2c)}-2(a^4-5a^3b)e^{(-4dx-4c)}+2(a^4-5a^3b)e^{(-6dx-6c)}+(a^4+5} - \frac{(3ab+5b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}a^3d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*(4*a^2 + 19*a*b + 15*b^2 - 2*(2*a^2 + 13*a*b + 30*b^2)*e^(-2*d*x - 2*c
) - 2*(10*a^2 - 2*a*b - 45*b^2)*e^(-4*d*x - 4*c) - 6*(2*a^2 + a*b + 10*b^2
)*e^(-6*d*x - 6*c) + 3*(3*a*b + 5*b^2)*e^(-8*d*x - 8*c))/((a^4 + a^3*b - (
a^4 + 5*a^3*b)*e^(-2*d*x - 2*c) - 2*(a^4 - 5*a^3*b)*e^(-4*d*x - 4*c) + 2*(
a^4 - 5*a^3*b)*e^(-6*d*x - 6*c) + (a^4 + 5*a^3*b)*e^(-8*d*x - 8*c) - (a^4
+ a^3*b)*e^(-10*d*x - 10*c))*d) - 1/2*(3*a*b + 5*b^2)*arctan(1/2*((a + b)*
e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(99) = 198$.

Time = 0.33 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.85

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$= \frac{\frac{3\,(3\,ab+5\,b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{6\,\left(abe^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+ab+b^2\right)}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)a^3} + \frac{8\,(3\,be^{(4\,dx+4\,c)}}{}}{6\,d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*(3*a*b + 5*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) +
 a - b)/sqrt(a*b))/(sqrt(a*b)*a^3) - 6*(a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x
 + 2*c) + a*b + b^2)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*a^3) + 8*(3*b*e^(4*d*x + 4*c) - 3*
a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + a + 3*b)/(a^3*(e^(2*d*x + 2*c) -
 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)^4\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 2398, normalized size of antiderivative = 21.22

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(9*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqr
t(b))/sqrt(a))*a**3 + 69*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b + 135*e**(10*c + 10*d*x)*sqrt(
b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 + 75*
e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b
))/sqrt(a))*b**3 - 9*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*s
qrt(a + b) - sqrt(b))/sqrt(a))*a**3 - 105*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)
*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b - 375*e**(8*c +
 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))
*a*b**2 - 375*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
 b) - sqrt(b))/sqrt(a))*b**3 - 18*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e
**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3 - 30*e**(6*c + 6*d*x)*sqr
t(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b + 4
50*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a*b**2 + 750*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d
*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 + 18*e**(4*c + 4*d*x)*sqrt(b)*sqr
t(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3 + 30*e**(4*c
+ 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)
)*a**2*b - 450*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) - sqrt(b))/sqrt(a))*a*b**2 - 750*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*...
```

3.41 $\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 240

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3(a^2-10ab+5b^2)\,x}{8(a+b)^5} + \frac{3\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a+b)^5d} - \frac{(5a-3b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{(7a-5b)b\tanh(c+dx)}{8(a+b)^3d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{3(a-b)b\tanh(c+dx)}{2(a+b)^4d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
3/8*(a^2-10*a*b+5*b^2)*x/(a+b)^5+3/8*b^(1/2)*(5*a^2-10*a*b+b^2)*arctan(b^(
1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a+b)^5/d-1/8*(5*a-3*b)*cosh(d*x+c)*sinh
(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2+1/4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b
)/d/(a+b*tanh(d*x+c)^2)^2+1/8*(7*a-5*b)*b*tanh(d*x+c)/(a+b)^3/d/(a+b*tanh(
d*x+c)^2)^2+3/2*(a-b)*b*tanh(d*x+c)/(a+b)^4/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.99 (sec) , antiderivative size = 184, normalized size of antiderivative = 0.77

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{12(a^2-10ab+5b^2)(c+dx)+\frac{12\sqrt{b}(5a^2-10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}-8(a-2b)(a+b)\sinh(2(c+dx))+}{32(a+b)^5d}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(12*(a^2 - 10*a*b + 5*b^2)*(c + d*x) + (12*Sqrt[b]*(5*a^2 - 10*a*b + b^2)*
ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - 8*(a - 2*b)*(a + b)*Sin
h[2*(c + d*x)] + (16*a*b^2*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cos
h[2*(c + d*x)])^2 + (4*(9*a - 5*b)*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (
a + b)*Cosh[2*(c + d*x)]) + (a + b)^2*Sinh[4*(c + d*x)])/(32*(a + b)^5*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 273, normalized size of antiderivative = 1.14, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.522, Rules used = {3042, 4146, 372, 402, 25, 402, 27, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 372

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int \frac{(4a-3b)\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{4(a+b)}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int -\frac{a(3a-5b)-5(5a-3b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{2(a+b)}+\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx\right.}}{4(a+b)}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int \frac{a(3a-5b)-5(5a-3b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+d}{2(a+b)}}{4(a+b)}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{b(7a-5b)\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int -\frac{12a\left(a(a-3b)-(7a-\right.}{\left(1-\tanh^2(c+dx)\right)\left(\right.}}{}}{2(a+b)}}{4(a+b)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{3\int \frac{a(a-3b)-(7a-5b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+}{a+b}}{2(a+b)}}{4(a+b)}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{4b(a-b)\tanh(c+dx)}{(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{2a\left(a^2-6ba+b^2-\cdots\right.}{\left(1-\tanh^2(c+dx)\cdots\right.}\cdots}{a+b}\cdots\right.}{\cdots}}{4(a+b)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{\int\frac{a^2-6ba+b^2-4(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c\cdots}{a+b}\cdots\right.}{a+b\cdots}}{4(a+b)}}{d}$$

$\downarrow$ 397

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{\left(a^2-10ab+5b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}\cdots\right.}{\cdots}}{\cdots}}{d}$$

$\downarrow$ 218

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{\frac{\left(a^2-10ab+5b^2\right)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}\cdots}{a+\cdots}\right.}{\cdots}}{4\cdots}}{d}$$

$\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(5a-3b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{3\left(\frac{\frac{\sqrt{b}\left(5a^2-10ab+b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{(\cdots}{}}{a+b}\cdots\right.}{\cdots}}{4(a+\cdots}}{d}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2*(a + b*Tanh[c + d*x]^2)^
2) - (((5*a - 3*b)*Tanh[c + d*x])/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*
Tanh[c + d*x]^2)^2) - (((7*a - 5*b)*b*Tanh[c + d*x])/((a + b)*(a + b*Tanh[
c + d*x]^2)^2) + (3*(((Sqrt[b]*(5*a^2 - 10*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((a^2 - 10*a*b + 5*b^2)*ArcTanh[T
anh[c + d*x]])/(a + b))/(a + b) + (4*(a - b)*b*Tanh[c + d*x])/((a + b)*(a
+ b*Tanh[c + d*x]^2))))/(a + b))/(2*(a + b)))/(4*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(220) = 440$.

Time = 197.69 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.54

method	result
derivativedivides	$\frac{\frac{1}{4(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3a-9b}{8(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{-11b+a}{8(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{(-3a}{}}{}$
default	$\frac{\frac{1}{4(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}+\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{3a-9b}{8(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{-11b+a}{8(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{(-3a}{}}{}$
risch	$\frac{3x\,a^2}{8(a^3+3a^2b+3b^2a+b^3)(a+b)^2}-\frac{15xab}{4(a^3+3a^2b+3b^2a+b^3)(a+b)^2}+\frac{15x\,b^2}{8(a^3+3a^2b+3b^2a+b^3)(a+b)^2}+\frac{e^{4dx+4c}}{64(a+b)^3d}-$

input
```
int(sinh(d*x+c)^4/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/4/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c
)-1)^3-1/8*(3*a-9*b)/(a+b)^4/(tanh(1/2*d*x+1/2*c)-1)-1/8*(-11*b+a)/(a+b)^4
/(tanh(1/2*d*x+1/2*c)-1)^2+1/8/(a+b)^5*(-3*a^2+30*a*b-15*b^2)*ln(tanh(1/2*
d*x+1/2*c)-1)-2*b/(a+b)^5*((-3/8*a*(3*a^2+2*a*b-b^2)*tanh(1/2*d*x+1/2*c)^7
+(-27/8*a^3-23/4*a^2*b+1/8*b^2*a+5/2*b^3)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^3
-23/4*a^2*b+1/8*b^2*a+5/2*b^3)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^3-3/4*a^2*b+3
/8*b^2*a)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2
*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8*(15*a^2-30*a*b+3*b^2)*a*(1/2*(a
+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+
((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/4/(a
+b)^3/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3-1/8*
(3*a-9*b)/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)-1/8*(11*b-a)/(a+b)^4/(tanh(1/2*d
*x+1/2*c)+1)^2+1/8/(a+b)^5*(3*a^2-30*a*b+15*b^2)*ln(tanh(1/2*d*x+1/2*c)+1)
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9248 vs. $2(220) = 440$.

Time = 0.40 (sec) , antiderivative size = 18818, normalized size of antiderivative = 78.41

$$\int \frac{\sinh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3392 vs. 2(220) = 440.

Time = 0.61 (sec) , antiderivative size = 3392, normalized size of antiderivative = 14.13

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-3/8*(a*b - 3*b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
 + a + b)/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) -
3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) + 3/8*(a*b - 3*b^2)*log(2*(a
- b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5 + 5*a^4*b
+ 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d) + 3/4*b*log(2*(a - b)*e^(-2*
d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*d) + 3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4
+ b^5)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a
^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 3/32*
(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c)
 + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sq
rt(a*b)*d) - 3/128*(5*a^4*b - 80*a^3*b^2 + 50*a^2*b^3 + 8*a*b^4 + b^5)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 5*a^6*b + 10
*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) - 3/32*(5*a^3*b
- 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b
)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*
d) - 3/64*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*
c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d)
 - 1/64*(9*a^5*b - 65*a^4*b^2 - 134*a^3*b^3 - 34*a^2*b^4 + 29*a*b^5 + 3...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(220) = 440$.

Time = 1.57 (sec) , antiderivative size = 861, normalized size of antiderivative = 3.59

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/64*(24*(a^2 - 10*a*b + 5*b^2)*(d*x + c)/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10
*a^2*b^3 + 5*a*b^4 + b^5) + 24*(5*a^2*b - 10*a*b^2 + b^3)*arctan(1/2*(a*e^
(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*sqrt(a*b)) + (a^3*e^(4*d*x + 4*c) +
 3*a^2*b*e^(4*d*x + 4*c) + 3*a*b^2*e^(4*d*x + 4*c) + b^3*e^(4*d*x + 4*c) -
 8*a^3*e^(2*d*x + 2*c) + 24*a*b^2*e^(2*d*x + 2*c) + 16*b^3*e^(2*d*x + 2*c)
)/(a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6) -
 (6*a^4*e^(12*d*x + 12*c) - 48*a^3*b*e^(12*d*x + 12*c) - 84*a^2*b^2*e^(12*
d*x + 12*c) + 30*b^4*e^(12*d*x + 12*c) + 16*a^4*e^(10*d*x + 10*c) - 104*a^
3*b*e^(10*d*x + 10*c) - 24*a^2*b^2*e^(10*d*x + 10*c) + 72*a*b^3*e^(10*d*x
+ 10*c) - 24*b^4*e^(10*d*x + 10*c) + 5*a^4*e^(8*d*x + 8*c) + 84*a^3*b*e^(8
*d*x + 8*c) + 30*a^2*b^2*e^(8*d*x + 8*c) + 84*a*b^3*e^(8*d*x + 8*c) - 123*
b^4*e^(8*d*x + 8*c) - 20*a^4*e^(6*d*x + 6*c) + 280*a^3*b*e^(6*d*x + 6*c) -
 64*a^2*b^2*e^(6*d*x + 6*c) - 152*a*b^3*e^(6*d*x + 6*c) + 212*b^4*e^(6*d*x
 + 6*c) - 20*a^4*e^(4*d*x + 4*c) + 136*a^3*b*e^(4*d*x + 4*c) + 224*a^2*b^2
*e^(4*d*x + 4*c) - 40*a*b^3*e^(4*d*x + 4*c) - 108*b^4*e^(4*d*x + 4*c) - 4*
a^4*e^(2*d*x + 2*c) + 24*a^2*b^2*e^(2*d*x + 2*c) + 32*a*b^3*e^(2*d*x + 2*c
) + 12*b^4*e^(2*d*x + 2*c) + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)/((
a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(a*e^(6*d*x + 6*c
) + b*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + a*e...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 4498, normalized size of antiderivative = 18.74

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(120*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**5 - 120*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*atan((e**(c
 + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b - 336*e**(12*c + 12*d*x)*sq
rt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2
 + 144*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a**2*b**3 + 216*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*atan
((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**4 - 24*e**(12*c + 12*d
*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**
5 + 480*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
- sqrt(b))/sqrt(a))*a**5 - 1440*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e
**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b + 576*e**(10*c + 10*d*x
)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*
b**2 + 1344*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
 b) - sqrt(b))/sqrt(a))*a**2*b**3 - 1056*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**4 + 96*e**(10*c
+ 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*b**5 + 720*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
 b) - sqrt(b))/sqrt(a))*a**5 - 2640*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan(
(e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b + 3744*e**(8*c + 8*d*
x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a...
```

3.42 $\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 166

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{5(3a-4b)\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}d} - \frac{(a-2b)\cosh(c+dx)}{(a+b)^4d} + \frac{\cosh^3(c+dx)}{3(a+b)^3d} + \frac{ab\operatorname{sech}(c+dx)}{4(a+b)^3d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{(7a-4b)b\operatorname{sech}(c+dx)}{8(a+b)^4d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
5/8*(3*a-4*b)*b^(1/2)*arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))/(a+b)^(9/2)
/d-(a-2*b)*cosh(d*x+c)/(a+b)^4/d+1/3*cosh(d*x+c)^3/(a+b)^3/d+1/4*a*b*sech(
d*x+c)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)^2+1/8*(7*a-4*b)*b*sech(d*x+c)/(a+b)
^4/d/(a+b-b*sech(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.89 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.37

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\frac{15i(3a-4b)\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{9/2}} - \frac{6\cosh(c+dx)(3a^3-24a^2b+30ab^2-13b^3+(\cdots}{(a\cdots}}{24d}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((15*I)*(3*a - 4*b)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c +
 d*x)/2])/Sqrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])
/Sqrt[b]]))/(a + b)^(9/2) - (6*Cosh[c + d*x]*(3*a^3 - 24*a^2*b + 30*a*b^2
- 13*b^3 + (6*a^3 - 27*a^2*b - 11*a*b^2 + 22*b^3)*Cosh[2*(c + d*x)] + 3*(a
 - 3*b)*(a + b)^2*Cosh[2*(c + d*x)]^2))/((a + b)^4*(a - b + (a + b)*Cosh[2
*(c + d*x)])^2) + (2*Cosh[3*(c + d*x)])/(a + b)^3)/(24*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 168, normalized size of antiderivative = 1.01, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 26, 4147, 25, 361, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i\sin(ic+idx)^3}{(a-b\tan(ic+idx)^2)^3}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{\sin(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$\downarrow$ 4147

$$\frac{\int -\frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$\downarrow$ 25

$$-\frac{\int \frac{\cosh^4(c+dx)\left(1-\text{sech}^2(c+dx)\right)}{\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$\downarrow$ 361

$$\frac{\frac{ab\,\text{sech}(c+dx)}{4(a+b)^3\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\int \frac{\cosh^4(c+dx)\left(-\frac{3a\,\text{sech}^4(c+dx)}{(a+b)^3}-\frac{4a\,\text{sech}^2(c+dx)}{b(a+b)^2}+\frac{4}{b(a+b)}\right)}{\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{d}$$

$\downarrow$ 1582

$$\frac{\frac{ab\,\text{sech}(c+dx)}{4(a+b)^3\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\left(\frac{\int \frac{\cosh^4(c+dx)\left(-\frac{(7a-4b)b^2\text{sech}^4(c+dx)}{a+b}-8(a-b)b\,\text{sech}^2(c+dx)+8b(a+b)\right)}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{2b^2(a+b)^3}-\frac{(7a-}{2(a+b)^4(}\right.}{d}$$

$\downarrow$ 1584

$$\frac{\frac{ab\,\text{sech}(c+dx)}{4(a+b)^3\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\left(\frac{\int \left(8b\cosh^4(c+dx)+\frac{8b(2b-a)\cosh^2(c+dx)}{a+b}-\frac{5b^2(4b-3a)}{(a+b)\left(b\,\text{sech}^2(c+dx)-a-b\right)}\right)d\text{sech}(c+dx)}{2b^2(a+b)^3}-\frac{(7a-}{2(a+b)^4(}\right.}{d}$$

$\downarrow$ 2009

$$\frac{\frac{ab\,\text{sech}(c+dx)}{4(a+b)^3\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{1}{4}b\left(\frac{-\frac{5b^{3/2}(3a-4b)\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{8b(a-2b)\cosh(c+dx)}{a+b}-\frac{8}{3}b\cosh^3(c+dx)}{2b^2(a+b)^3}-\frac{(7a-4b)\text{s}}{2(a+b)^4\left(a-b\text{s}\right.}\right.}{d}$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a*b*Sech[c + d*x])/(4*(a + b)^3*(a + b - b*Sech[c + d*x]^2)^2) - (b*(((-
5*(3*a - 4*b)*b^(3/2)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b
)^(3/2) + (8*(a - 2*b)*b*Cosh[c + d*x])/(a + b) - (8*b*Cosh[c + d*x]^3)/3)
/(2*b^2*(a + b)^3) - ((7*a - 4*b)*Sech[c + d*x])/(2*(a + b)^4*(a + b - b*S
ech[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(150) = 300$.

Time = 83.76 (sec) , antiderivative size = 341, normalized size of antiderivative = 2.05

method	result
derivativedivides	$\frac{\frac{1}{3(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a-5b}{2(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b\left(\frac{-\frac{(9a+20b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8}-\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
default	$\frac{\frac{1}{3(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{1}{2(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{a-5b}{2(a+b)^4\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{2b\left(\frac{-\frac{(9a+20b)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8}-\cdots}{\cdots}\right.}{\cdots}}{\cdots}$
risch	$\frac{e^{3dx+3c}}{24(a^3+3a^2b+3b^2a+b^3)d}-\frac{3\,e^{dx+c}a}{8(a+b)(a^3+3a^2b+3b^2a+b^3)d}+\frac{9\,e^{dx+c}b}{8(a+b)(a^3+3a^2b+3b^2a+b^3)d}-\frac{3\,e^{-dx-c\cdots}}{8(a^4+4a^3b+6a^2b^2+\cdots}$

input
```
int(sinh(d*x+c)^3/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c
)+1)^2-1/2*(a-5*b)/(a+b)^4/(tanh(1/2*d*x+1/2*c)+1)-2*b/(a+b)^4*((-1/8*(9*a
+20*b)*a*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+66*a^2*b+56*a*b^2-16*b^3)/a*tan
h(1/2*d*x+1/2*c)^4+(-27/8*a^2-11/2*a*b+2*b^2)*tanh(1/2*d*x+1/2*c)^2-9/8*a^
2+1/4*a*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2
*d*x+1/2*c)^2+a)^2-5/16*(3*a-4*b)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*
d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2)))-1/3/(a+b)^3/(tanh(1/2*d*x+1/2*c)
-1)^3-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/(a+b)^4*(-a+5*b)/(tanh(1/2
*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7129 vs. $2(156) = 312$.

Time = 0.35 (sec) , antiderivative size = 13095, normalized size of antiderivative = 78.89

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: RuntimeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is un
defined.
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 3335, normalized size of antiderivative = 20.09

$$\int \frac{\sinh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 45*e**(11*c + 11*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a
+ b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**3 - 30*e**(11*c + 11*d*x)*
sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**
(c + d*x)*sqrt(b))*a**2*b + 75*e**(11*c + 11*d*x)*sqrt(b)*sqrt(a + b)*log(
e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a*b**
2 + 60*e**(11*c + 11*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a
+ b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**3 + 45*e**(11*c + 11*d*x)*
sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**
(c + d*x)*sqrt(b))*a**3 + 30*e**(11*c + 11*d*x)*sqrt(b)*sqrt(a + b)*log(e*
*(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**2*b
- 75*e**(11*c + 11*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a +
b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*b**2 - 60*e**(11*c + 11*d*x)*
sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**
(c + d*x)*sqrt(b))*b**3 - 180*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a + b)*log(e**
(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**3 + 2
40*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) +
 sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b + 180*e**(9*c + 9*d*x)*sqrt(
b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c +
d*x)*sqrt(b))*a*b**2 - 240*e**(9*c + 9*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*
c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**3 + 1...
```

3.43 $\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 185

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = &-\frac{(a-5b)x}{2(a+b)^4} \\ &-\frac{\sqrt{b}(15a^2-10ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^4d} \\ &+\frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} \\ &-\frac{3b\tanh(c+dx)}{4(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} \\ &-\frac{(11a-b)b\tanh(c+dx)}{8a(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output
```
-1/2*(a-5*b)*x/(a+b)^4-1/8*b^(1/2)*(15*a^2-10*a*b-b^2)*arctan(b^(1/2)*tanh
(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+
b*tanh(d*x+c)^2)^2-3/4*b*tanh(d*x+c)/(a+b)^2/d/(a+b*tanh(d*x+c)^2)^2-1/8*(
11*a-b)*b*tanh(d*x+c)/a/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.85

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{-4(a-5b)(c+dx)+\frac{\sqrt{b}(-15a^2+10ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+2(a+b)\sinh(2(c+dx))-\frac{4b^2(a+b)\sinh(2(c+d}{(a-b+(a+b)\cosh(2(c+}}{8(a+b)^4d}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-4*(a - 5*b)*(c + d*x) + (Sqrt[b]*(-15*a^2 + 10*a*b + b^2)*ArcTan[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + 2*(a + b)*Sinh[2*(c + d*x)] - (4*b^2*
(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2 - ((9*a -
 b)*b*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/
(8*(a + b)^4*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 220, normalized size of antiderivative = 1.19, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 4146, 373, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \; 25$$
$$-\int \frac{\sin(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \ 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 373$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int \frac{a-5b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{2(a+b)}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{3b\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int -\frac{2a\left(-9b\tanh^2(c+dx)+2a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}}{2(a+b)}}{d}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int \frac{-9b\tanh^2(c+dx)+2a-b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{2(a+b)}+\frac{3b\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}}{d}$$

$$\Big\downarrow \ 402$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{b(11a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{4a^2-9ba-b^2-(11a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{2(a+b)}+\frac{3b\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{\int \frac{4a^2-9ba-b^2-(11a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b(11a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{3b\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}}{d}$$

$$\Big\downarrow \ 397$$

$$\frac{\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{\frac{b\left(15a^2-10ab-b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{4a(a-5b)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2a(a+b)}+\frac{b\cdots}{2a(a\cdots}}{2(a+b)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{\frac{4a(a-5b)\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}+\frac{b(11a-\cdots}{2a(a+b)\left(a\cdots\right.}}{2(a+b)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{\frac{\sqrt{b}\left(15a^2-10ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{4a(a-5b)\text{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)}+\frac{b(11a-b)\tanh\cdots}{2a(a+b)\left(a+b\tanh\cdots\right.}}{2(a+b)}}{2(a+b)}}{d}$$

input

```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output

```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)^2)
 - ((3*b*Tanh[c + d*x])/(2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + (((Sqrt[b]
*(15*a^2 - 10*a*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]
*(a + b)) + (4*a*(a - 5*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*a*(a + b))
+ ((11*a - b)*b*Tanh[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*(
a + b)))/(2*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(167) = 334$.

Time = 28.86 (sec) , antiderivative size = 488, normalized size of antiderivative = 2.64

method	result
derivativedivides	$2b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7-\frac{\left(27a^3+58a^2b+27b^2a-4b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a}-\frac{\left(27a^3+58a^2b+27b^2a-4b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.$
default	$2b\left(\frac{\left(-\frac{9}{8}a^2-\frac{5}{4}ab-\frac{1}{8}b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7-\frac{\left(27a^3+58a^2b+27b^2a-4b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a}-\frac{\left(27a^3+58a^2b+27b^2a-4b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.}{8a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.$
risch	$-\frac{xa}{2(a+b)(a^3+3a^2b+3b^2a+b^3)}+\frac{5xb}{2(a+b)(a^3+3a^2b+3b^2a+b^3)}+\frac{e^{2dx+2c}}{8(a^3+3a^2b+3b^2a+b^3)d}-\frac{e^{-2dx-2c}}{8(a^3+3a^2b+3b^2a+b^3}$

input
```
int(sinh(d*x+c)^2/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*b/(a+b)^4*(((-9/8*a^2-5/4*a*b-1/8*b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(2
7*a^3+58*a^2*b+27*a*b^2-4*b^3)/a*tanh(1/2*d*x+1/2*c)^5-1/8*(27*a^3+58*a^2*
b+27*a*b^2-4*b^3)/a*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^2-5/4*a*b-1/8*b^2)*tanh(
1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tan
h(1/2*d*x+1/2*c)^2+a)^2+1/8*(15*a^2-10*a*b-b^2)*(1/2*(a+((a+b)*b)^(1/2)+b)
/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/
a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+1/2/(a+b)^3/(tanh(1/2*d*x
+1/2*c)-1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/2*(a-5*b)/(a+b)^4*ln(ta
nh(1/2*d*x+1/2*c)-1)-1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/(a+b)^3/(ta
nh(1/2*d*x+1/2*c)+1)+1/2/(a+b)^4*(-a+5*b)*ln(tanh(1/2*d*x+1/2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6319 vs. $2(167) = 334$.

Time = 0.31 (sec) , antiderivative size = 12965, normalized size of antiderivative = 70.08

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. $2(167) = 334$.

Time = 0.42 (sec) , antiderivative size = 1806, normalized size of antiderivative = 9.76

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d
*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1
/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b
^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 +
 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 1/16*(15*a^2*b
+ 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b
))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4
*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^
2*b^3 + 6*a*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2
*b^3 - 36*a*b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2
*b^3 + 38*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 2
0*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2
 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8
 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c)
+ 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5
+ 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 -
4*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 535 vs. $2(167) = 334$.

Time = 1.09 (sec) , antiderivative size = 535, normalized size of antiderivative = 2.89

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{4\,(dx+c)(a-5\,b)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{\left(2\,ae^{(2\,dx+2\,c)}-10\,be^{(2\,dx+2\,c)}-a-b\right)e^{(-2\,dx-2\,c)}}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{\left(15\,a^2b-10\,ab^2-b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}}{2\,\sqrt{ab}}\right.}{\left(a^5+4\,a^4b+6\,a^3b^2+4\,a^2b^3+ab^4\right)\sqrt{ab}}}{}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*(a - 5*b)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) -
(2*a*e^(2*d*x + 2*c) - 10*b*e^(2*d*x + 2*c) - a - b)*e^(-2*d*x - 2*c)/(a^4
 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (15*a^2*b - 10*a*b^2 - b^3)*arct
an(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 +
4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*sqrt(a*b)) - e^(2*d*x + 2*c)/(a^3
 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(9*a^3*b*e^(6*d*x + 6*c) - 5*a^2*b^2*e^(6*
d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) + b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(
4*d*x + 4*c) - 21*a^2*b^2*e^(4*d*x + 4*c) + 29*a*b^3*e^(4*d*x + 4*c) - 3*b
^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + a^2*b^2*e^(2*d*x + 2*c) -
23*a*b^3*e^(2*d*x + 2*c) + 3*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 17*a^2*b^2 +
7*a*b^3 - b^4)/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*(a*e^(4*d*
x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
 a + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\sinh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 4124, normalized size of antiderivative = 22.29

$$\int \frac{\sinh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
( - 60*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a**5 - 20*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(
c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b + 104*e**(10*c + 10*d*x)*s
qrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**
2 + 24*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a**2*b**3 - 44*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan(
(e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**4 - 4*e**(10*c + 10*d*x
)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5
- 240*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**5 + 400*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b + 96*e**(8*c + 8*d*x)*sqrt(b)*
sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 - 416
*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*a**2*b**3 + 144*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**4 + 16*e**(8*c + 8*d*x)*sqrt(b)*
sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5 - 360*e**(
6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqr
t(a))*a**5 + 840*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(
a + b) - sqrt(b))/sqrt(a))*a**4*b - 976*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*a
tan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 + 720*e**(6...
```

3.44 $\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \frac{\sinh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{15\sqrt{b}\,\mathrm{arctanh}\left(\frac{\sqrt{b}\,\mathrm{sech}(c+dx)}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}d} + \frac{\cosh(c+dx)}{(a+b)^3d} - \frac{b\,\mathrm{sech}(c+dx)}{4(a+b)^2d\left(a+b-b\,\mathrm{sech}^2(c+dx)\right)^2} - \frac{7b\,\mathrm{sech}(c+dx)}{8(a+b)^3d\left(a+b-b\,\mathrm{sech}^2(c+dx)\right)}$$

output
```
-15/8*b^(1/2)*arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))/(a+b)^(7/2)/d+cosh(
d*x+c)/(a+b)^3/d-1/4*b*sech(d*x+c)/(a+b)^2/d/(a+b-b*sech(d*x+c)^2)^2-7/8*b
*sech(d*x+c)/(a+b)^3/d/(a+b-b*sech(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.41 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.26

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{-\frac{15i\sqrt{b}\left(\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)+\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)\right)}{(a+b)^{7/2}}+\frac{2\cosh(c+dx)\left(4-\frac{4b^2}{(a-b+(a+b)\cosh(2(c+dx)))^2}-\frac{}{a\cdots}\right.}{(a+b)^3}}{8d}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((-15*I)*Sqrt[b]*(ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*Tanh[(c + d*x)/2])/S
qrt[b]] + ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]]))
/(a + b)^(7/2) + (2*Cosh[c + d*x]*(4 - (4*b^2)/(a - b + (a + b)*Cosh[2*(c
+ d*x)])^2 - (9*b)/(a - b + (a + b)*Cosh[2*(c + d*x)])))/(a + b)^3)/(8*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 4147, 253, 253, 264, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$
$$\Big\downarrow \; 3042$$
$$\int -\frac{i\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \; 26$$
$$-i\int \frac{\sin(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4147}$$

$$-\frac{\int \frac{\cosh^2(c+dx)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \text{253}$$

$$-\frac{\frac{5\int \frac{\cosh^2(c+dx)}{\left(-b\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{253}$$

$$-\frac{\frac{5\left(\frac{3\int \frac{\cosh^2(c+dx)}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{264}$$

$$-\frac{\frac{5\left(\frac{3\left(\frac{b\int \frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a+b}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \text{221}$$

$$-\frac{\frac{5\left(\frac{3\left(\frac{\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{\cosh(c+dx)}{a+b}\right)}{2(a+b)}+\frac{\cosh(c+dx)}{2(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)}\right)}{4(a+b)}+\frac{\cosh(c+dx)}{4(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}}{d}$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((Cosh[c + d*x]/(4*(a + b)*(a + b - b*Sech[c + d*x]^2)^2) + (5*((3*((Sqrt
[b]*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a + b)^(3/2) - Cosh[c +
 d*x]/(a + b)))/(2*(a + b)) + Cosh[c + d*x]/(2*(a + b)*(a + b - b*Sech[c +
 d*x]^2))))/(4*(a + b)))/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(111) = 222$.

Time = 11.92 (sec) , antiderivative size = 252, normalized size of antiderivative = 2.02

method	result
derivativedivides	$\frac{\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{\left(9a^2+24ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a}-\frac{\left(27a^3+78a^2b+88b^2a+16b^3\right)}{8a^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}}{d}$
default	$\frac{\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{\left(9a^2+24ab+8b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8a}-\frac{\left(27a^3+78a^2b+88b^2a+16b^3\right)}{8a^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}}{d}$
risch	$\frac{e^{dx+c}}{2(a^3+3a^2b+3b^2a+b^3)d}+\frac{e^{-dx-c}}{2(a^3+3a^2b+3b^2a+b^3)d}-\frac{(9\,e^{6dx+6c}a+9\,e^{6dx+6c}b+27\,e^{4dx+4c}a-b\,e^{4dx+4c}+27\,e^{2dx+2}}{4(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b}$

input
```
int(sinh(d*x+c)/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+2
*b/(a+b)^3*((-1/8*(9*a^2+24*a*b+8*b^2)/a*tanh(1/2*d*x+1/2*c)^6-1/8/a^2*(27
*a^3+78*a^2*b+88*a*b^2+16*b^3)*tanh(1/2*d*x+1/2*c)^4-1/8*(27*a^2+56*a*b+8*
b^2)/a*tanh(1/2*d*x+1/2*c)^2-9/8*a-1/4*b)/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(
1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-15/16/(a*b+b^2)^(1/2)*ar
ctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3792 vs. $2(117) = 234$.

Time = 0.20 (sec) , antiderivative size = 7119, normalized size of antiderivative = 56.95

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\sinh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(2*a^2 + 4*a*b + 2*b^2 + 2*(a^2*e^(10*c) + 2*a*b*e^(10*c) + b^2*e^(10*
c))*e^(10*d*x) + 5*(2*a^2*e^(8*c) - a*b*e^(8*c) - 3*b^2*e^(8*c))*e^(8*d*x)
 + 5*(4*a^2*e^(6*c) - 7*a*b*e^(6*c) + b^2*e^(6*c))*e^(6*d*x) + 5*(4*a^2*e^
(4*c) - 7*a*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 5*(2*a^2*e^(2*c) - a*b*e^
(2*c) - 3*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(9*c) + 5*a^4*b*d*e^(9*c) + 10
*a^3*b^2*d*e^(9*c) + 10*a^2*b^3*d*e^(9*c) + 5*a*b^4*d*e^(9*c) + b^5*d*e^(9
*c))*e^(9*d*x) + 4*(a^5*d*e^(7*c) + 3*a^4*b*d*e^(7*c) + 2*a^3*b^2*d*e^(7*c
) - 2*a^2*b^3*d*e^(7*c) - 3*a*b^4*d*e^(7*c) - b^5*d*e^(7*c))*e^(7*d*x) + 2
*(3*a^5*d*e^(5*c) + 7*a^4*b*d*e^(5*c) + 6*a^3*b^2*d*e^(5*c) + 6*a^2*b^3*d*
e^(5*c) + 7*a*b^4*d*e^(5*c) + 3*b^5*d*e^(5*c))*e^(5*d*x) + 4*(a^5*d*e^(3*c
) + 3*a^4*b*d*e^(3*c) + 2*a^3*b^2*d*e^(3*c) - 2*a^2*b^3*d*e^(3*c) - 3*a*b^
4*d*e^(3*c) - b^5*d*e^(3*c))*e^(3*d*x) + (a^5*d*e^c + 5*a^4*b*d*e^c + 10*a
^3*b^2*d*e^c + 10*a^2*b^3*d*e^c + 5*a*b^4*d*e^c + b^5*d*e^c)*e^(d*x)) + 1/
2*integrate(15/2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^4 + 4*a^3*b + 6*a^
2*b^2 + 4*a*b^3 + b^4 + (a^4*e^(4*c) + 4*a^3*b*e^(4*c) + 6*a^2*b^2*e^(4*c)
 + 4*a*b^3*e^(4*c) + b^4*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + 2*a^3*b*e^(
2*c) - 2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\sinh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 18347, normalized size of antiderivative = 146.78

$$\int \frac{\sinh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(32*e**(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**4*a**6*b**2 + 192*e**(9*
c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**4*a**5*b**3 + 480*e**(9*c + 9*d*x)
*cosh(c + d*x)*tanh(c + d*x)**4*a**4*b**4 + 640*e**(9*c + 9*d*x)*cosh(c +
d*x)*tanh(c + d*x)**4*a**3*b**5 + 480*e**(9*c + 9*d*x)*cosh(c + d*x)*tanh(
c + d*x)**4*a**2*b**6 + 192*e**(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**
4*a*b**7 + 32*e**(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**4*b**8 + 64*e*
*(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*a**7*b + 400*e**(9*c + 9*d*x
)*cosh(c + d*x)*tanh(c + d*x)**2*a**6*b**2 + 1056*e**(9*c + 9*d*x)*cosh(c
+ d*x)*tanh(c + d*x)**2*a**5*b**3 + 1520*e**(9*c + 9*d*x)*cosh(c + d*x)*ta
nh(c + d*x)**2*a**4*b**4 + 1280*e**(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*
x)**2*a**3*b**5 + 624*e**(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*a**2
*b**6 + 160*e**(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*a*b**7 + 16*e*
*(9*c + 9*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*b**8 + 32*e**(9*c + 9*d*x)*c
osh(c + d*x)*a**8 + 208*e**(9*c + 9*d*x)*cosh(c + d*x)*a**7*b + 576*e**(9*
c + 9*d*x)*cosh(c + d*x)*a**6*b**2 + 880*e**(9*c + 9*d*x)*cosh(c + d*x)*a*
*5*b**3 + 800*e**(9*c + 9*d*x)*cosh(c + d*x)*a**4*b**4 + 432*e**(9*c + 9*d
*x)*cosh(c + d*x)*a**3*b**5 + 128*e**(9*c + 9*d*x)*cosh(c + d*x)*a**2*b**6
 + 16*e**(9*c + 9*d*x)*cosh(c + d*x)*a*b**7 + 128*e**(7*c + 7*d*x)*cosh(c
+ d*x)*tanh(c + d*x)**4*a**6*b**2 + 512*e**(7*c + 7*d*x)*cosh(c + d*x)*tan
h(c + d*x)**4*a**5*b**3 + 640*e**(7*c + 7*d*x)*cosh(c + d*x)*tanh(c + d...
```

3.45 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 156

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{arctanh}(\cosh(c+dx))}{a^3d} + \frac{\sqrt{b}(15a^2+20ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^3(a+b)^{5/2}d} + \frac{b\operatorname{sech}(c+dx)}{4a(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} + \frac{b(7a+4b)\operatorname{sech}(c+dx)}{8a^2(a+b)^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}$$

output
```
-arctanh(cosh(d*x+c))/a^3/d+1/8*b^(1/2)*(15*a^2+20*a*b+8*b^2)*arctanh(b^(1
/2)*sech(d*x+c)/(a+b)^(1/2))/a^3/(a+b)^(5/2)/d+1/4*b*sech(d*x+c)/a/(a+b)/d
/(a+b-b*sech(d*x+c)^2)^2+1/8*b*(7*a+4*b)*sech(d*x+c)/a^2/(a+b)^2/d/(a+b-b*
sech(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 1.38 (sec) , antiderivative size = 249, normalized size of antiderivative = 1.60

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{i\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}} + \frac{i\sqrt{b}(15a^2+20ab+8b^2)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{5/2}} + \frac{8a^2\cdots}{(a+b)^2(a-b\cdots}}{8a^3\cdots}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((I*Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqrt[a]*T
anh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(5/2) + (I*Sqrt[b]*(15*a^2 + 20*a*b +
8*b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a
+ b)^(5/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*
(c + d*x)])^2) + (2*a*b*(9*a + 4*b)*Cosh[c + d*x])/((a + b)^2*(a - b + (a
+ b)*Cosh[2*(c + d*x)])) - 8*Log[Cosh[(c + d*x)/2]] + 8*Log[Sinh[(c + d*x)
/2]])/(8*a^3*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 177, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 26, 4147, 25, 316, 25, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{i}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 26$$

$$i\int \frac{1}{\sin(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 4147$$

$$\frac{\int -\frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\int \frac{1}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int -\frac{3b\,\text{sech}^2(c+dx)+4a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{4a(a+b)}+\frac{b\,\text{sech}(c+dx)}{4a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{b\,\text{sech}(c+dx)}{4a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\int \frac{3b\,\text{sech}^2(c+dx)+4a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{4a(a+b)}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{b\,\text{sech}(c+dx)}{4a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{8a^2+9ba+4b^2+b(7a+4b)\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}-\frac{b(7a+4b)\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{b\,\text{sech}(c+dx)}{4a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{8a^2+9ba+4b^2+b(7a+4b)\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}-\frac{b(7a+4b)\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{8(a+b)^2\int\frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a(a+b)}-\frac{b(7a\cdots}{2a(a+b)\left(\cdots\right.}}{4a(a+b)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{8(a+b)^2\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{b\left(15a^2+20ab+8b^2\right)\int\frac{1}{-b\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a(a+b)}-\frac{b(7a+4b)\text{sec}\cdots}{2a(a+b)\left(a-b\text{sec}\cdots\right.}}{4a(a+b)}}{d}$$

$\Big\downarrow$ 221

$$\frac{\frac{b\text{sech}(c+dx)}{4a(a+b)\left(a-b\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{8(a+b)^2\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{\sqrt{b}\left(15a^2+20ab+8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-\frac{b(7a+4b)\text{sech}(c+\cdots}{2a(a+b)\left(a-b\text{sech}^2(c+\cdots\right.}}{4a(a+b)}}{d}$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b*Sech[c + d*x])/(4*a*(a + b)*(a + b - b*Sech[c + d*x]^2)^2) - (((8*(a +
 b)^2*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(15*a^2 + 20*a*b + 8*b^2)*ArcTa
nh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a + b)) -
(b*(7*a + 4*b)*Sech[c + d*x])/(2*a*(a + b)*(a + b - b*Sech[c + d*x]^2)))/(
4*a*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 303 vs. $2(142) = 284$.

Time = 5.88 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.95

method	result
derivativedivides	$-\frac{2b\left(\frac{-\frac{(9a^2+28ab+16b^2)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a^2+2ab+b^2)}-\frac{3(9a^3+30a^2b+40b^2a+16b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a^2+2ab+b^2)}-\frac{a(27a^2+68ab+32b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8(a^2+2ab+b^2)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{a^3 \; d}$
default	$-\frac{2b\left(\frac{-\frac{(9a^2+28ab+16b^2)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a^2+2ab+b^2)}-\frac{3(9a^3+30a^2b+40b^2a+16b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4}{8(a^2+2ab+b^2)}-\frac{a(27a^2+68ab+32b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8(a^2+2ab+b^2)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{a^3 \; d}$
risch	$\frac{(9\,e^{6dx+6c}a^2+13\,e^{6dx+6c}ab+4\,e^{6dx+6c}b^2+27\,e^{4dx+4c}a^2+11\,e^{4dx+4c}ab-4\,e^{4dx+4c}b^2+27\,e^{2dx+2c}a^2+11\,e^{2dx+2c}ba-4}{4(a^2+2ab+b^2)(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b)^2 d\,a^2}$

input
```
int(csch(d*x+c)/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b/a^3*((-1/8*(9*a^2+28*a*b+16*b^2)*a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^6-3/8*(9*a^3+30*a^2*b+40*a*b^2+16*b^3)/(a^2+2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^4-1/8*a*(27*a^2+68*a*b+32*b^2)/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^2-3/8*a^2*(3*a+2*b)/(a^2+2*a*b+b^2))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/16*(15*a^2+20*a*b+8*b^2)/(
a^2+2*a*b+b^2)/(a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+
4*b)/(a*b+b^2)^(1/2)))+1/a^3*ln(tanh(1/2*d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5783 vs. $2(148) = 296$.

Time = 0.30 (sec) , antiderivative size = 10716, normalized size of antiderivative = 68.69

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)}{(b\tanh(dx+c)^2+a)^3}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((9*a^2*b*e^(7*c) + 13*a*b^2*e^(7*c) + 4*b^3*e^(7*c))*e^(7*d*x) + (27*
a^2*b*e^(5*c) + 11*a*b^2*e^(5*c) - 4*b^3*e^(5*c))*e^(5*d*x) + (27*a^2*b*e^
(3*c) + 11*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) + (9*a^2*b*e^c + 13*a*
b^2*e^c + 4*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3
*d + a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c)
+ 4*a^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) +
2*a^5*b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2
*(3*a^6*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*
e^(4*c) + 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^
(2*c) - 2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x
+ c) + 1)*e^(-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*inte
grate(1/8*((15*a^2*b*e^(3*c) + 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x)
 - (15*a^2*b*e^c + 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^6 + 3*a^5*b + 3*a
^4*b^2 + a^3*b^3 + (a^6*e^(4*c) + 3*a^5*b*e^(4*c) + 3*a^4*b^2*e^(4*c) + a^
3*b^3*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + a^5*b*e^(2*c) - a^4*b^2*e^(2*c
) - a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.38 (sec) , antiderivative size = 4756, normalized size of antiderivative = 30.49

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 15*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a +
b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**4 - 50*e**(8*c + 8*d*x)*sqrt
(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c +
 d*x)*sqrt(b))*a**3*b - 63*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*
c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b**2 -
 36*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b)
+ sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a*b**3 - 8*e**(8*c + 8*d*x)*sqrt(b
)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d
*x)*sqrt(b))*b**4 + 15*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c +
2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**4 + 50*e**(8
*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a
+ b) + 2*e**(c + d*x)*sqrt(b))*a**3*b + 63*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a
 + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt
(b))*a**2*b**2 + 36*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d
*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*b**3 + 8*e**(8*c
 + 8*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a +
b) + 2*e**(c + d*x)*sqrt(b))*b**4 - 60*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a + b
)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))
*a**4 - 80*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(
a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**3*b + 28*e**(6*c + 6*...
```

3.46 $\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 112

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d} - \frac{\coth(c+dx)}{a^3d} - \frac{b\tanh(c+dx)}{4a^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{7b\tanh(c+dx)}{8a^3d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-15/8*b^(1/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2)/d-coth(d*x+c)/a^
3/d-1/4*b*tanh(d*x+c)/a^2/d/(a+b*tanh(d*x+c)^2)^2-7/8*b*tanh(d*x+c)/a^3/d/
(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-15\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-8\sqrt{a}\coth(c+dx)-\frac{\sqrt{a}b(9a-7b+(9a+7b)\cosh(2(c+dx)))\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{7/2}d}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-15*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a]*Coth[c +
d*x] - (Sqrt[a]*b*(9*a - 7*b + (9*a + 7*b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(7/2)*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.04, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 253, 253, 264, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\sin(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(b\tanh^2(c+dx)+a)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 253

$$\frac{\frac{5\int \frac{\coth^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a}+\frac{\coth(c+dx)}{4a(a+b\tanh^2(c+dx))^2}}{d}$$

$\Big\downarrow$ 253

$$\frac{\frac{5\left(\frac{3\int \frac{\coth^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\coth(c+dx)}{4a(a+b\tanh^2(c+dx))^2}}{d}$$

$\Big\downarrow$ 264

$$\frac{\frac{5\left(\frac{3\left(-\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\coth(c+dx)}{4a(a+b\tanh^2(c+dx))^2}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{5\left(\frac{3\left(-\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\frac{\coth(c+dx)}{a}\right)}{2a}+\frac{\coth(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\coth(c+dx)}{4a(a+b\tanh^2(c+dx))^2}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Coth[c + d*x]/(4*a*(a + b*Tanh[c + d*x]^2)^2) + (5*((3*(-((Sqrt[b]*ArcTan
[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) - Coth[c + d*x]/a))/(2*a) + Co
th[c + d*x]/(2*a*(a + b*Tanh[c + d*x]^2))))/(4*a))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 253
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-(c*x
)^(m + 1))*((a + b*x^2)^(p + 1)/(2*a*c*(p + 1))), x] + Simp[(m + 2*p + 3)/(
2*a*(p + 1))   Int[(c*x)^m*(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b, c, m
}, x] && LtQ[p, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 264
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(c*x)^(
m + 1)*((a + b*x^2)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + 2*p + 3)/(a*c
^2*(m + 1)))   Int[(c*x)^(m + 2)*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, p
}, x] && LtQ[m, -1] && IntBinomialQ[a, b, c, 2, m, p, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(98) = 196$.

Time = 10.32 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.73

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\frac{9\,\mathrm{t}}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{}}{}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a^3}-\frac{1}{2a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\frac{-\frac{9a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8}+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5+\left(-\frac{27a}{8}-\frac{7b}{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3-\frac{9\,\mathrm{t}}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{}}{}$
risch	$-\frac{8\,e^{8dx+8c}a^4+23\,e^{8dx+8c}a^3b+45\,e^{8dx+8c}a^2b^2+45\,e^{8dx+8c}a\,b^3+15\,e^{8dx+8c}b^4+32\,e^{6dx+6c}a^4+46b\,a^3e^{6dx+6c}-90\,e^{6d}}{}$

input
```
int(csch(d*x+c)^2/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a^3*tanh(1/2*d*x+1/2*c)-1/2/a^3/tanh(1/2*d*x+1/2*c)+2*b/a^3*((-9
/8*a*tanh(1/2*d*x+1/2*c)^7+(-27/8*a-7/2*b)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a-
7/2*b)*tanh(1/2*d*x+1/2*c)^3-9/8*tanh(1/2*d*x+1/2*c)*a)/(tanh(1/2*d*x+1/2*
c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+15/8*a*(1/
2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*
(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/
2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3995 vs. $2(98) = 196$.

Time = 0.24 (sec) , antiderivative size = 8312, normalized size of antiderivative = 74.21

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(98) = 196$.

Time = 0.29 (sec) , antiderivative size = 478, normalized size of antiderivative = 4.27

$$\int \frac{\operatorname{csch}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx =$$

$$-\frac{8a^4 + 41a^3b + 73a^2b^2 + 55ab^3 + 15b^4 + 2(16a^4 + 41a^3b - 55ab^3 - 30b^4)e^{(-\cdots}}{4(a^7 + 4a^6b + 6a^5b^2 + 4a^4b^3 + a^3b^4 + (3a^7 + 4a^6b - 6a^5b^2 - 12a^4b^3 - 5a^3b^4)e^{(-2dx-2c)} + 2(a^7 + \cdots}$$
$$+\frac{15b\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^3d}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(8*a^4 + 41*a^3*b + 73*a^2*b^2 + 55*a*b^3 + 15*b^4 + 2*(16*a^4 + 41*a
^3*b - 55*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) + 2*(24*a^4 + 32*a^3*b + 5*a^2*
b^2 + 50*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 2*(16*a^4 + 23*a^3*b - 45*a*b^
3 - 30*b^4)*e^(-6*d*x - 6*c) + (8*a^4 + 23*a^3*b + 45*a^2*b^2 + 45*a*b^3 +
 15*b^4)*e^(-8*d*x - 8*c))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b
^4 + (3*a^7 + 4*a^6*b - 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-2*d*x - 2*
c) + 2*(a^7 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-4*d*x - 4*c) - 2*(a^7
 + 2*a^5*b^2 + 8*a^4*b^3 + 5*a^3*b^4)*e^(-6*d*x - 6*c) - (3*a^7 + 4*a^6*b
- 6*a^5*b^2 - 12*a^4*b^3 - 5*a^3*b^4)*e^(-8*d*x - 8*c) - (a^7 + 4*a^6*b +
6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*e^(-10*d*x - 10*c))*d) + 15/8*b*arctan(1/
2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(98) = 196$.

Time = 0.60 (sec) , antiderivative size = 351, normalized size of antiderivative = 3.13

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{15\,b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a^3} - \frac{2\,(9\,a^3be^{(6\,dx+6\,c)}+3\,a^2b^2e^{(6\,dx+6\,c)}-13\,ab^3e^{(6\,dx+6\,c)}-7\,b^4e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+}{(a^5+2\,a}}{}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*(15*b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt
(a*b))/(sqrt(a*b)*a^3) - 2*(9*a^3*b*e^(6*d*x + 6*c) + 3*a^2*b^2*e^(6*d*x +
 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 7*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d
*x + 4*c) + 3*a^2*b^2*e^(4*d*x + 4*c) + 13*a*b^3*e^(4*d*x + 4*c) + 21*b^4*
e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 25*a^2*b^2*e^(2*d*x + 2*c) -
23*a*b^3*e^(2*d*x + 2*c) - 21*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 25*a^2*b^2 +
 23*a*b^3 + 7*b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*
d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a
^3*(e^(2*d*x + 2*c) - 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\sinh\left(c+d\,x\right)^2\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 3001, normalized size of antiderivative = 26.79

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
( - 45*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a**4 - 60*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(
c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b + 90*e**(10*c + 10*d*x)*sq
rt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2
 + 180*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a*b**3 + 75*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e*
*(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**4 - 135*e**(8*c + 8*d*x)*sqr
t(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4 + 180
*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*a**3*b + 390*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x
)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 - 300*e**(8*c + 8*d*x)*sqrt(b)
*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 - 375*e
**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/
sqrt(a))*b**4 - 90*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqr
t(a + b) - sqrt(b))/sqrt(a))*a**4 + 240*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*a
tan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b - 420*e**(6*c + 6
*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a
**2*b**2 + 750*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) - sqrt(b))/sqrt(a))*b**4 + 90*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((
e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4 - 240*e**(4*c + 4*d*x...
```

3.47 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 196

$$\begin{aligned}\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{(a+6b)\operatorname{arctanh}(\cosh(c+dx))}{2a^4d} \\ &\quad - \frac{\sqrt{b}(15a^2+40ab+24b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\operatorname{sech}(c+dx)}{\sqrt{a+b}}\right)}{8a^4(a+b)^{3/2}d} \\ &\quad - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} \\ &\quad - \frac{3b\operatorname{sech}(c+dx)}{4a^2d\left(a+b-b\operatorname{sech}^2(c+dx)\right)^2} \\ &\quad - \frac{b(11a+12b)\operatorname{sech}(c+dx)}{8a^3(a+b)d\left(a+b-b\operatorname{sech}^2(c+dx)\right)}\end{aligned}$$

output
```
1/2*(a+6*b)*arctanh(cosh(d*x+c))/a^4/d-1/8*b^(1/2)*(15*a^2+40*a*b+24*b^2)*
arctanh(b^(1/2)*sech(d*x+c)/(a+b)^(1/2))/a^4/(a+b)^(3/2)/d-1/2*coth(d*x+c)
*csch(d*x+c)/a/d/(a+b-b*sech(d*x+c)^2)^2-3/4*b*sech(d*x+c)/a^2/d/(a+b-b*se
ch(d*x+c)^2)^2-1/8*b*(11*a+12*b)*sech(d*x+c)/a^3/(a+b)/d/(a+b-b*sech(d*x+c
)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 5.17 (sec) , antiderivative size = 287, normalized size of antiderivative = 1.46

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$
$$-\frac{\frac{i\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{-i\sqrt{a+b}-\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}}+\frac{i\sqrt{b}(15a^2+40ab+24b^2)\arctan\left(\frac{-i\sqrt{a+b}+\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b}}\right)}{(a+b)^{3/2}}+\frac{}{(a+b)(}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/8*((I*Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2)*ArcTan[((-I)*Sqrt[a + b] - Sqr
t[a]*Tanh[(c + d*x)/2])/Sqrt[b]])/(a + b)^(3/2) + (I*Sqrt[b]*(15*a^2 + 40*
a*b + 24*b^2)*ArcTan[((-I)*Sqrt[a + b] + Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[b
]])/(a + b)^(3/2) + (8*a^2*b^2*Cosh[c + d*x])/((a + b)*(a - b + (a + b)*Co
sh[2*(c + d*x)])^2) + (2*a*b*(9*a + 8*b)*Cosh[c + d*x])/((a + b)*(a - b +
(a + b)*Cosh[2*(c + d*x)])) + a*Csch[(c + d*x)/2]^2 - 4*(a + 6*b)*Log[Cosh
[(c + d*x)/2]] + 4*(a + 6*b)*Log[Sinh[(c + d*x)/2]] + a*Sech[(c + d*x)/2]^
2)/(a^4*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 26, 4147, 373, 402, 27, 402, 25, 397, 219, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 4147$$

$$-\frac{\int \frac{\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)^2\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{d}$$

$$\Big\downarrow \; 373$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\int \frac{5b\,\text{sech}^2(c+dx)+a+b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^3}d\text{sech}(c+dx)}{2a}}{d}$$

$$\Big\downarrow \; 402$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{-\frac{\int -\frac{2(a+b)\left(9b\,\text{sech}^2(c+dx)+2a+3b\right)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{4a(a+b)}-\frac{3b\,\text{sech}(c+dx)}{2a\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}}{2a}}{d}$$

$$\Big\downarrow \; 27$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\int \frac{9b\,\text{sech}^2(c+dx)+2a+3b}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)^2}d\text{sech}(c+dx)}{2a}-\frac{3b\,\text{sech}(c+dx)}{2a\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}}{2a}}{d}$$

$$\Big\downarrow \; 402$$

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{-\frac{\int -\frac{4a^2+17ba+12b^2+b(11a+12b)\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}-\frac{b(11a+12b)\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+\right.}}{2a}}{2a}}{d}$$

$\downarrow$ 25

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\int \frac{4a^2+17ba+12b^2+b(11a+12b)\text{sech}^2(c+dx)}{\left(1-\text{sech}^2(c+dx)\right)\left(-b\,\text{sech}^2(c+dx)+a+b\right)}d\text{sech}(c+dx)}{2a(a+b)}-\frac{b(11a+12b)\text{sech}(c+dx)}{2a(a+b)\left(a-b\,\text{sech}^2(c+dx)+b\right)}}{2a}}{2a}}{d}$$

$\downarrow$ 397

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{4(a+b)(a+6b)\int \frac{1}{1-\text{sech}^2(c+dx)}d\text{sech}(c+dx)}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a(a+b)}}{2a}}{2a}}{d}$$

$\downarrow$ 219

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{4(a+b)(a+6b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{b\left(15a^2+40ab+24b^2\right)\int \frac{1}{-b\,\text{sech}^2(c+dx)+a+b}d\text{sech}(c+dx)}{a}}{2a(a+b)}}{2a}}{2a}}{d}$$

$\downarrow$ 221

$$-\frac{\frac{\text{sech}(c+dx)}{2a\left(1-\text{sech}^2(c+dx)\right)\left(a-b\,\text{sech}^2(c+dx)+b\right)^2}-\frac{\frac{\frac{\frac{4(a+b)(a+6b)\text{arctanh}(\text{sech}(c+dx))}{a}-\frac{\sqrt{b}\left(15a^2+40ab+24b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\,\text{sech}(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}}}{2a(a+b)}-}{2a}}{2a}}{d}$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((Sech[c + d*x]/(2*a*(1 - Sech[c + d*x]^2)*(a + b - b*Sech[c + d*x]^2)^2)
 - ((-3*b*Sech[c + d*x])/(2*a*(a + b - b*Sech[c + d*x]^2)^2) + (((4*(a + b
)*(a + 6*b)*ArcTanh[Sech[c + d*x]])/a - (Sqrt[b]*(15*a^2 + 40*a*b + 24*b^2
)*ArcTanh[(Sqrt[b]*Sech[c + d*x])/Sqrt[a + b]])/(a*Sqrt[a + b]))/(2*a*(a +
 b)) - (b*(11*a + 12*b)*Sech[c + d*x])/(2*a*(a + b)*(a + b - b*Sech[c + d*
x]^2)))/(2*a))/(2*a))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4147
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sec[e + f*x], x]}, Simp[1/(f*ff^
m)   Subst[Int[(-1 + ff^2*x^2)^((m - 1)/2)*((a - b + b*ff^2*x^2)^p/x^(m + 1
)), x], x, Sec[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(
m - 1)/2]
```

Maple [A] (verified)

Time = 16.25 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.55

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}-\frac{1}{8a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^4}+\frac{2b\left(\frac{-\frac{\left(9a^2+32ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a+b)}-\frac{\left(27a^3+102a\right.}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{}}{}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a^3}-\frac{1}{8a^3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{(-2a-12b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4a^4}+\frac{2b\left(\frac{-\frac{\left(9a^2+32ab+24b^2\right)a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^6}{8(a+b)}-\frac{\left(27a^3+102a\right.}{}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}\right.}{}}{}$
risch	$-\frac{\left(4\,e^{10dx+10c}a^3+21a^2b\,e^{10dx+10c}+29a\,b^2e^{10dx+10c}+12b^3e^{10dx+10c}+20a^3e^{8dx+8c}+37a^2b\,e^{8dx+8c}-15a\,b^2e^{8dx+8c}-\right.}{}$

input
```
int(csch(d*x+c)^3/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a^3-1/8/a^3/tanh(1/2*d*x+1/2*c)^2+1/4/a^4*(
-2*a-12*b)*ln(tanh(1/2*d*x+1/2*c))+2*b/a^4*((-1/8*(9*a^2+32*a*b+24*b^2)*a/
(a+b)*tanh(1/2*d*x+1/2*c)^6-1/8*(27*a^3+102*a^2*b+152*a*b^2+80*b^3)/(a+b)*
tanh(1/2*d*x+1/2*c)^4-1/8*a*(27*a^2+80*a*b+56*b^2)/(a+b)*tanh(1/2*d*x+1/2*
c)^2-1/8*a^2*(9*a+10*b)/(a+b))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2
*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/16*(15*a^2+40*a*b+24*b^2)/(a+b)/(
a*b+b^2)^(1/2)*arctanh(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+2*a+4*b)/(a*b+b^2)^(
1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11576 vs. $2(187) = 374$.

Time = 0.40 (sec) , antiderivative size = 21301, normalized size of antiderivative = 108.68

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{csch}(dx+c)^3}{(b\tanh(dx+c)^2+a)^3}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((4*a^3*e^(11*c) + 21*a^2*b*e^(11*c) + 29*a*b^2*e^(11*c) + 12*b^3*e^(
11*c))*e^(11*d*x) + (20*a^3*e^(9*c) + 37*a^2*b*e^(9*c) - 15*a*b^2*e^(9*c)
- 36*b^3*e^(9*c))*e^(9*d*x) + 2*(20*a^3*e^(7*c) + 3*a^2*b*e^(7*c) - 7*a*b^
2*e^(7*c) + 12*b^3*e^(7*c))*e^(7*d*x) + 2*(20*a^3*e^(5*c) + 3*a^2*b*e^(5*c
) - 7*a*b^2*e^(5*c) + 12*b^3*e^(5*c))*e^(5*d*x) + (20*a^3*e^(3*c) + 37*a^2
*b*e^(3*c) - 15*a*b^2*e^(3*c) - 36*b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 2
1*a^2*b*e^c + 29*a*b^2*e^c + 12*b^3*e^c)*e^(d*x))/(a^6*d + 3*a^5*b*d + 3*a
^4*b^2*d + a^3*b^3*d + (a^6*d*e^(12*c) + 3*a^5*b*d*e^(12*c) + 3*a^4*b^2*d*
e^(12*c) + a^3*b^3*d*e^(12*c))*e^(12*d*x) + 2*(a^6*d*e^(10*c) - a^5*b*d*e^
(10*c) - 5*a^4*b^2*d*e^(10*c) - 3*a^3*b^3*d*e^(10*c))*e^(10*d*x) - (a^6*d*
e^(8*c) + 3*a^5*b*d*e^(8*c) - 13*a^4*b^2*d*e^(8*c) - 15*a^3*b^3*d*e^(8*c))
*e^(8*d*x) - 4*(a^6*d*e^(6*c) - a^5*b*d*e^(6*c) + 3*a^4*b^2*d*e^(6*c) + 5*
a^3*b^3*d*e^(6*c))*e^(6*d*x) - (a^6*d*e^(4*c) + 3*a^5*b*d*e^(4*c) - 13*a^4
*b^2*d*e^(4*c) - 15*a^3*b^3*d*e^(4*c))*e^(4*d*x) + 2*(a^6*d*e^(2*c) - a^5*
b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) - 3*a^3*b^3*d*e^(2*c))*e^(2*d*x)) + 1/2*
(a + 6*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a + 6*b)*log((e^(d*
x + c) - 1)*e^(-c))/(a^4*d) + 8*integrate(1/32*((15*a^2*b*e^(3*c) + 40*a*b
^2*e^(3*c) + 24*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^c + 40*a*b^2*e^c + 24
*b^3*e^c)*e^(d*x))/(a^6 + 2*a^5*b + a^4*b^2 + (a^6*e^(4*c) + 2*a^5*b*e^(4*
c) + a^4*b^2*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) - a^4*b^2*e^(2*c))*e^(...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{1}{\sinh{(c+d\,x)}^3\,\left(b\tanh{(c+d\,x)}^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 6723, normalized size of antiderivative = 34.30

$$\int \frac{\operatorname{csch}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(15*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b
) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**4 + 70*e**(12*c + 12*d*x)*sqr
t(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c
+ d*x)*sqrt(b))*a**3*b + 119*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a + b)*log(e*
*(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b*
*2 + 88*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a
 + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a*b**3 + 24*e**(12*c + 12*d*
x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*
e**(c + d*x)*sqrt(b))*b**4 - 15*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a + b)*log
(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**4
 - 70*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a +
 b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**3*b - 119*e**(12*c + 12*d*x
)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e
**(c + d*x)*sqrt(b))*a**2*b**2 - 88*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a + b)
*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*
a*b**3 - 24*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sq
rt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b**4 + 30*e**(10*c + 10*
d*x)*sqrt(b)*sqrt(a + b)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) -
2*e**(c + d*x)*sqrt(b))*a**4 + 20*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a + b)*l
og(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))...
```

3.48 $\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\tanh^2(c+dx))^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 151

$$\int \frac{\operatorname{csch}^4(c+dx)}{(a+b\tanh^2(c+dx))^3} dx = \frac{5\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d} + \frac{(a+3b)\coth(c+dx)}{a^4d} - \frac{\coth^3(c+dx)}{3a^3d} + \frac{b(a+b)\tanh(c+dx)}{4a^3d\,(a+b\tanh^2(c+dx))^2} + \frac{b(7a+11b)\tanh(c+dx)}{8a^4d\,(a+b\tanh^2(c+dx))}$$

output
```
5/8*b^(1/2)*(3*a+7*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(9/2)/d+(a+3*b
)*coth(d*x+c)/a^4/d-1/3*coth(d*x+c)^3/a^3/d+1/4*b*(a+b)*tanh(d*x+c)/a^3/d/
(a+b*tanh(d*x+c)^2)^2+1/8*b*(7*a+11*b)*tanh(d*x+c)/a^4/d/(a+b*tanh(d*x+c)^
2)
```

Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 149, normalized size of antiderivative = 0.99

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{15\sqrt{b}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-8\sqrt{a}\coth(c+dx)\left(-2a-9b+a\operatorname{csch}^2(c+dx)\right)+\frac{3\sqrt{a}b(9a^2+6ab-1}{}}{24a^{9/2}d}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(15*Sqrt[b]*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - 8*Sqrt[a
]*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*x]^2) + (3*Sqrt[a]*b*(9*a^2 + 6
*a*b - 11*b^2 + (9*a^2 + 20*a*b + 11*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c + d
*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(24*a^(9/2)*d)
```

Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.02, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4146, 361, 25, 1582, 1584, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sin(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)}{\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{361}$$

$$\frac{\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}-\frac{1}{4}b\int -\frac{\coth^4(c+dx)\left(\frac{3(a+b)\tanh^4(c+dx)}{a^3}-\frac{4(a+b)\tanh^2(c+dx)}{a^2b}+\frac{4}{ab}\right)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}b\int \frac{\coth^4(c+dx)\left(\frac{3(a+b)\tanh^4(c+dx)}{a^3}-\frac{4(a+b)\tanh^2(c+dx)}{a^2b}+\frac{4}{ab}\right)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)+\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{1582}$$

$$\frac{\frac{1}{4}b\left(\frac{\int \frac{\coth^4(c+dx)\left(\frac{b^2(7a+11b)\tanh^4(c+dx)}{a}-8b(a+2b)\tanh^2(c+dx)+8ab\right)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a^3b^2}+\frac{(7a+11b)\tanh(c+dx)}{2a^4\left(a+b\tanh^2(c+dx)\right)}\right)+\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{1584}$$

$$\frac{\frac{1}{4}b\left(\frac{\int \left(8b\coth^4(c+dx)-\frac{8b(a+3b)\coth^2(c+dx)}{a}+\frac{5b^2(3a+7b)}{a\left(b\tanh^2(c+dx)+a\right)}\right)d\tanh(c+dx)}{2a^3b^2}+\frac{(7a+11b)\tanh(c+dx)}{2a^4\left(a+b\tanh^2(c+dx)\right)}\right)+\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b(a+b)\tanh(c+dx)}{4a^3\left(a+b\tanh^2(c+dx)\right)^2}+\frac{1}{4}b\left(\frac{(7a+11b)\tanh(c+dx)}{2a^4\left(a+b\tanh^2(c+dx)\right)}+\frac{\frac{5b^{3/2}(3a+7b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+\frac{8b(a+3b)\coth(c+dx)}{a}-\frac{8}{3}b\coth^3(c+dx)}{2a^3b^2}\right)}{d}$$

input

```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output

```
((b*(a + b)*Tanh[c + d*x])/(4*a^3*(a + b*Tanh[c + d*x]^2)^2) + (b*(((5*b^(
3/2)*(3*a + 7*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + (8*b*(
a + 3*b)*Coth[c + d*x])/a - (8*b*Coth[c + d*x]^3)/3)/(2*a^3*b^2) + ((7*a +
 11*b)*Tanh[c + d*x])/(2*a^4*(a + b*Tanh[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 361
```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[(-a)^(m/2 - 1)*(b*c - a*d)*x*((a + b*x^2)^(p + 1)/(2*b^(m/2 + 1)*(p
+ 1))), x] + Simp[1/(2*b^(m/2 + 1)*(p + 1))   Int[x^m*(a + b*x^2)^(p + 1)*E
xpandToSum[2*b*(p + 1)*Together[(b^(m/2)*(c + d*x^2) - (-a)^(m/2 - 1)*(b*c
- a*d)*x^(-m + 2))/(a + b*x^2)] - ((-a)^(m/2 - 1)*(b*c - a*d))/x^m, x], x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && ILtQ[m/
2, 0] && (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

rule 1582
```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[(-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*((d
+ e*x^2)^(q + 1)/(2*e^(2*p + m/2)*(q + 1))), x] + Simp[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1))   Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1/(d + e
*x^2))*(2*(-d)^(-m/2 + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 -
 b*d*e + a*e^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2))], x], x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
&& ILtQ[m/2, 0]
```

rule 1584
```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 396 vs. $2(135) = 270$.

Time = 23.36 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.63

method	result
derivativedivides	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3} a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{4}}-\frac{2b\left(\frac{-\frac{a(9a+13b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8}+\left(-\frac{27}{8}a^{2}-\frac{67}{8}ab-\frac{11}{2}b^{2}\right)\tanh\left(\frac{dx}{2}+\cdots\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\cdots\right.}\cdots\right.}{\cdots}$
default	$-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3} a}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)a-12b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a^{4}}-\frac{2b\left(\frac{-\frac{a(9a+13b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8}+\left(-\frac{27}{8}a^{2}-\frac{67}{8}ab-\frac{11}{2}b^{2}\right)\tanh\left(\frac{dx}{2}+\cdots\right.}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\cdots\right.}\cdots\right.}{\cdots}$
risch	$-\frac{-105b^{4}+270a^{2}b^{2}e^{10dx+10c}-325a\,b^{3}-351a^{2}b^{2}-260a^{2}b^{2}e^{6dx+6c}-16a^{4}-147a^{3}b-64\,e^{2dx+2c}a^{3}b+502\,e^{2dx+2c}a^{2}b^{2}\cdots}{\cdots}$

input
```
int(csch(d*x+c)^4/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/a^4*(1/3*tanh(1/2*d*x+1/2*c)^3*a-3*tanh(1/2*d*x+1/2*c)*a-12*b*ta
nh(1/2*d*x+1/2*c))-2*b/a^4*((-1/8*a*(9*a+13*b)*tanh(1/2*d*x+1/2*c)^7+(-27/
8*a^2-67/8*a*b-11/2*b^2)*tanh(1/2*d*x+1/2*c)^5+(-27/8*a^2-67/8*a*b-11/2*b^
2)*tanh(1/2*d*x+1/2*c)^3+(-9/8*a^2-13/8*a*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+
1/8*(15*a+35*b)*a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*
b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+
a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b
)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-
a-2*b)*a)^(1/2))))-1/24/a^3/tanh(1/2*d*x+1/2*c)^3-1/8/a^4*(-3*a-12*b)/tanh
(1/2*d*x+1/2*c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7006 vs. $2(135) = 270$.

Time = 0.28 (sec) , antiderivative size = 14334, normalized size of antiderivative = 94.93

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 615 vs. $2(135) = 270$.

Time = 0.36 (sec) , antiderivative size = 615, normalized size of antiderivative = 4.07

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{16\,a^4+147\,a^3b+351\,a^2b^2+325\,ab^3+105\,b^4+2\,(8\,a^4+32\,a^3b-251\,a^2b^2-590\,ab^3-315\,b^4)e^{(-2\,dx-2\,c)}}{12\,(a^7+3\,a^6b+3\,a^5b^2+a^4b^3+(a^7-5\,a^6b-13\,a^5b^2-7\,a^4b^3)e^{(-2\,dx-2\,c)}-(3\,a^7+}$$
$$-\frac{5\,(3\,ab+7\,b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}a^4d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/12*(16*a^4 + 147*a^3*b + 351*a^2*b^2 + 325*a*b^3 + 105*b^4 + 2*(8*a^4 +
32*a^3*b - 251*a^2*b^2 - 590*a*b^3 - 315*b^4)*e^(-2*d*x - 2*c) - (96*a^4 +
 313*a^3*b + 19*a^2*b^2 - 1725*a*b^3 - 1575*b^4)*e^(-4*d*x - 4*c) - 4*(56*
a^4 + 80*a^3*b - 65*a^2*b^2 + 400*a*b^3 + 525*b^4)*e^(-6*d*x - 6*c) - (176
*a^4 + 135*a^3*b + 15*a^2*b^2 - 1375*a*b^3 - 1575*b^4)*e^(-8*d*x - 8*c) -
6*(8*a^4 + 45*a^2*b^2 + 150*a*b^3 + 105*b^4)*e^(-10*d*x - 10*c) + 15*(3*a^
3*b + 13*a^2*b^2 + 17*a*b^3 + 7*b^4)*e^(-12*d*x - 12*c))/((a^7 + 3*a^6*b +
 3*a^5*b^2 + a^4*b^3 + (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b^3)*e^(-2*d*x
- 2*c) - (3*a^7 + a^6*b - 23*a^5*b^2 - 21*a^4*b^3)*e^(-4*d*x - 4*c) - (3*a
^7 - 7*a^6*b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-6*d*x - 6*c) + (3*a^7 - 7*a^6*
b + 25*a^5*b^2 + 35*a^4*b^3)*e^(-8*d*x - 8*c) + (3*a^7 + a^6*b - 23*a^5*b^
2 - 21*a^4*b^3)*e^(-10*d*x - 10*c) - (a^7 - 5*a^6*b - 13*a^5*b^2 - 7*a^4*b
^3)*e^(-12*d*x - 12*c) - (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*e^(-14*d*x
- 14*c))*d) - 5/8*(3*a*b + 7*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/(sqrt(a*b)*a^4*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(135) = 270$.

Time = 0.61 (sec) , antiderivative size = 395, normalized size of antiderivative = 2.62

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{6\left(9\,a^3be^{(6\,dx+6\,c)}+7\,a^2b^2e^{(6\,dx+6\,c)}-13\,ab^3e^{(6\,dx+6\,c)}-11\,b^4e^{(6\,dx+6\,c)}+27\,a^3be^{(4\,dx+4\,c)}+15\,a^2b^2e^{(4\,dx+4\,c)}+5\,ab^3e^{(4\,dx+4\,c)}+33\,b^4e^{(4\,dx-}\right.}{(a^5+a^4b)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-\right.}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/24*(6*(9*a^3*b*e^(6*d*x + 6*c) + 7*a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e
^(6*d*x + 6*c) - 11*b^4*e^(6*d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) + 15*a^
2*b^2*e^(4*d*x + 4*c) + 5*a*b^3*e^(4*d*x + 4*c) + 33*b^4*e^(4*d*x + 4*c) +
 27*a^3*b*e^(2*d*x + 2*c) + 37*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x
 + 2*c) - 33*b^4*e^(2*d*x + 2*c) + 9*a^3*b + 29*a^2*b^2 + 31*a*b^3 + 11*b^
4)/((a^5 + a^4*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 15*(3*a*b + 7*b^2)*arctan(1/2*(a*
e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^4) -
16*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 18*b*e^(2*d*x + 2*c) + 2*a
 + 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\sinh\left(c+d\,x\right)^4\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 5176, normalized size of antiderivative = 34.28

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(45*e**(14*c + 14*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**5 - 75*e**(14*c + 14*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
 d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b - 1230*e**(14*c + 14*d*x)*sqr
t(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2
- 2790*e**(14*c + 14*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a**2*b**3 - 2415*e**(14*c + 14*d*x)*sqrt(b)*sqrt(a)*ata
n((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**4 - 735*e**(14*c + 14
*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b
**5 + 45*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
 - sqrt(b))/sqrt(a))*a**5 - 435*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*atan((e
**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b - 270*e**(12*c + 12*d*x
)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*
b**2 + 6090*e**(12*c + 12*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
 b) - sqrt(b))/sqrt(a))*a**2*b**3 + 11025*e**(12*c + 12*d*x)*sqrt(b)*sqrt(
a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**4 + 5145*e**(12
*c + 12*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqr
t(a))*b**5 - 135*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqr
t(a + b) - sqrt(b))/sqrt(a))*a**5 + 585*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)
*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b + 3450*e**(10*c
 + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sq...
```

3.49 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 110

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{3ax}{8}-\frac{3b\cosh^2(c+dx)}{2d}+\frac{b\cosh^4(c+dx)}{4d}+\frac{3b\log(\cosh(c+dx))}{d}-\frac{3a\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{a\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{b\tanh^2(c+dx)}{2d}$$

output

```
3/8*a*x-3/2*b*cosh(d*x+c)^2/d+1/4*b*cosh(d*x+c)^4/d+3*b*ln(cosh(d*x+c))/d-
3/8*a*cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*cosh(d*x+c)*sinh(d*x+c)^3/d-1/2*b*ta
nh(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.84

$$
\begin{aligned}
&\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx \\
&= \frac{3a(c+dx)}{8d} \\
&\quad + \frac{b(12\log(\cosh(c+dx))+2\mathrm{sech}^2(c+dx)-4\sinh^2(c+dx)+\sinh^4(c+dx))}{4d} \\
&\quad - \frac{a\sinh(2(c+dx))}{4d} + \frac{a\sinh(4(c+dx))}{32d}
\end{aligned}
$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(3*a*(c + d*x))/(8*d) + (b*(12*Log[Cosh[c + d*x]] + 2*Sech[c + d*x]^2 - 4*
Sinh[c + d*x]^2 + Sinh[c + d*x]^4))/(4*d) - (a*Sinh[2*(c + d*x)])/(4*d) +
(a*Sinh[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 4146, 2335, 25, 2335, 27, 523, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(ic+idx)^4 \left(a+ib\tan(ic+idx)^3\right) dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^3(c+dx)+a\right)}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2335}$$

$$\frac{\frac{1}{4}\int -\frac{\tanh^3(c+dx)(4b\tanh^2(c+dx)+a\tanh(c+dx)+4b)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)+\frac{\tanh^4(c+dx)(a\tanh(c+dx)+b)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\tanh^4(c+dx)(a\tanh(c+dx)+b)}{4(1-\tanh^2(c+dx))^2}-\frac{1}{4}\int \frac{\tanh^3(c+dx)(4b\tanh^2(c+dx)+a\tanh(c+dx)+4b)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2335}$$

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{3\tanh^2(c+dx)(a+8b\tanh(c+dx))}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{\tanh^3(c+dx)(a+8b\tanh(c+dx))}{2(1-\tanh^2(c+dx))}\right)+\frac{\tanh^4(c+dx)(a\tanh(c+dx)+b)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}\int \frac{\tanh^2(c+dx)(a+8b\tanh(c+dx))}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{\tanh^3(c+dx)(a+8b\tanh(c+dx))}{2(1-\tanh^2(c+dx))}\right)+\frac{\tanh^4(c+dx)(a\tanh(c+dx)+b)}{4(1-\tanh^2(c+dx))^2}}{d}$$

$$\Big\downarrow \text{523}$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}\int \left(-a-8b\tanh(c+dx)+\frac{a+8b\tanh(c+dx)}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)-\frac{\tanh^3(c+dx)(a+8b\tanh(c+dx))}{2(1-\tanh^2(c+dx))}\right)+\frac{\tanh^4(c+dx)(a\tanh}{4(1-\tanh^2(c+}}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{4}\left(\frac{3}{2}\left(a\text{arctanh}(\tanh(c+dx))-a\tanh(c+dx)-4b\tanh^2(c+dx)-4b\log\left(1-\tanh^2(c+dx)\right)\right)-\frac{\tanh^3(c+dx)(a}{2(1-\tanh}}{d}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
((Tanh[c + d*x]^4*(b + a*Tanh[c + d*x]))/(4*(1 - Tanh[c + d*x]^2)^2) + (-1
/2*(Tanh[c + d*x]^3*(a + 8*b*Tanh[c + d*x]))/(1 - Tanh[c + d*x]^2) + (3*(a
*ArcTanh[Tanh[c + d*x]] - 4*b*Log[1 - Tanh[c + d*x]^2] - a*Tanh[c + d*x] -
 4*b*Tanh[c + d*x]^2))/2)/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 523
```
Int[((x_)^(m_.)*((c_) + (d_.)*(x_)))/((a_) + (b_.)*(x_)^2), x_Symbol] :> In
t[ExpandIntegrand[x^m*((c + d*x)/(a + b*x^2)), x], x] /; FreeQ[{a, b, c, d}
, x] && IntegerQ[m]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.11 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh(\cdots}{2}\right.}{d}$
default	$\frac{a\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\tanh(\cdots}{2}\right.}{d}$
risch	$\frac{3ax}{8}-3bx+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}-\frac{e^{2dx+2c}a}{8d}-\frac{5\,e^{2dx+2c}b}{16d}+\frac{e^{-2dx-2c}a}{8d}-\frac{5\,e^{-2dx-2c}b}{16d}-\frac{e^{-4dx-4c}a}{64d}$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*ln(cosh(
d*x+c))-3/2*tanh(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1530 vs. 2(98) = 196.

Time = 0.10 (sec) , antiderivative size = 1530, normalized size of antiderivative = 13.91

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/64*((a + b)*cosh(d*x + c)^12 + 12*(a + b)*cosh(d*x + c)*sinh(d*x + c)^11
 + (a + b)*sinh(d*x + c)^12 - 6*(a + 3*b)*cosh(d*x + c)^10 + 6*(11*(a + b)
*cosh(d*x + c)^2 - a - 3*b)*sinh(d*x + c)^10 + 20*(11*(a + b)*cosh(d*x + c
)^3 - 3*(a + 3*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 3*(8*(a - 8*b)*d*x - 5*
a - 13*b)*cosh(d*x + c)^8 + 3*(165*(a + b)*cosh(d*x + c)^4 + 8*(a - 8*b)*d
*x - 90*(a + 3*b)*cosh(d*x + c)^2 - 5*a - 13*b)*sinh(d*x + c)^8 + 24*(33*(
a + b)*cosh(d*x + c)^5 - 30*(a + 3*b)*cosh(d*x + c)^3 + (8*(a - 8*b)*d*x -
 5*a - 13*b)*cosh(d*x + c))*sinh(d*x + c)^7 + 8*(6*(a - 8*b)*d*x + 11*b)*c
osh(d*x + c)^6 + 4*(231*(a + b)*cosh(d*x + c)^6 - 315*(a + 3*b)*cosh(d*x +
 c)^4 + 12*(a - 8*b)*d*x + 21*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)
^2 + 22*b)*sinh(d*x + c)^6 + 24*(33*(a + b)*cosh(d*x + c)^7 - 63*(a + 3*b)
*cosh(d*x + c)^5 + 7*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^3 + 2*(6
*(a - 8*b)*d*x + 11*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(8*(a - 8*b)*d*x
 + 5*a - 13*b)*cosh(d*x + c)^4 + 3*(165*(a + b)*cosh(d*x + c)^8 - 420*(a +
 3*b)*cosh(d*x + c)^6 + 70*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^4
+ 8*(a - 8*b)*d*x + 40*(6*(a - 8*b)*d*x + 11*b)*cosh(d*x + c)^2 + 5*a - 13
*b)*sinh(d*x + c)^4 + 4*(55*(a + b)*cosh(d*x + c)^9 - 180*(a + 3*b)*cosh(d
*x + c)^7 + 42*(8*(a - 8*b)*d*x - 5*a - 13*b)*cosh(d*x + c)^5 + 40*(6*(a -
 8*b)*d*x + 11*b)*cosh(d*x + c)^3 + 3*(8*(a - 8*b)*d*x + 5*a - 13*b)*cosh(
d*x + c))*sinh(d*x + c)^3 + 6*(a - 3*b)*cosh(d*x + c)^2 + 6*(11*(a + b)...
```

Sympy [F]

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int\left(a+b\tanh^3\left(c+dx\right)\right)\sinh^4\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.76

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{1}{64}\,a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$
$$+\frac{1}{64}\,b\left(\frac{192\,(dx+c)}{d}-\frac{20\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}+\frac{192\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{18\,e^{(-2\,dx-2\,c)}+39\,e^{(-4}}{d(e^{(-4\,dx-4\,c)}+2\,e^{(}}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) + 1/64*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*c)
 - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x -
 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*
c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. 2(98) = 196.

Time = 0.16 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.85

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{24\,(dx+c)(a-8\,b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}-8\,ae^{(2\,dx+2\,c)}-20\,be^{(2\,dx+2\,c)}+192\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-}{64\,d}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/64*(24*(d*x + c)*(a - 8*b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) - 8*a
*e^(2*d*x + 2*c) - 20*b*e^(2*d*x + 2*c) + 192*b*log(e^(2*d*x + 2*c) + 1) -
 (9*a*e^(8*d*x + 8*c) + 72*b*e^(8*d*x + 8*c) + 10*a*e^(6*d*x + 6*c) + 36*b
*e^(6*d*x + 6*c) - 6*a*e^(4*d*x + 4*c) + 111*b*e^(4*d*x + 4*c) - 6*a*e^(2*
d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + a - b)/(e^(4*d*x + 4*c) + e^(2*d*x + 2
*c))^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.42

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = x\left(\frac{3a}{8}-3b\right)+\frac{2b}{d\left(e^{2c+2dx}+1\right)} + \frac{e^{4c+4dx}(a+b)}{64d} - \frac{2b}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} - \frac{e^{-4c-4dx}(a-b)}{64d} + \frac{3b\ln\left(e^{2c}e^{2dx}+1\right)}{d} + \frac{e^{-2c-2dx}(2a-5b)}{16d} - \frac{e^{2c+2dx}(2a+5b)}{16d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3),x)
```

output
```
x*((3*a)/8 - 3*b) + (2*b)/(d*(exp(2*c + 2*d*x) + 1)) + (exp(4*c + 4*d*x)*(
a + b))/(64*d) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (
exp(- 4*c - 4*d*x)*(a - b))/(64*d) + (3*b*log(exp(2*c)*exp(2*d*x) + 1))/d
+ (exp(- 2*c - 2*d*x)*(2*a - 5*b))/(16*d) - (exp(2*c + 2*d*x)*(2*a + 5*b))
/(16*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.13

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{e^{12dx+12c}a + e^{12dx+12c}b - 6e^{10dx+10c}a - 18e^{10dx+10c}b + 192e^{8dx+8c}\log\left(e^{2dx+2c}+1\right)b + 24e^{8dx+8c}adx - 15e}{}$$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3),x)
```

output
```
(e**(12*c + 12*d*x)*a + e**(12*c + 12*d*x)*b - 6*e**(10*c + 10*d*x)*a - 18
*e**(10*c + 10*d*x)*b + 192*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*b +
 24*e**(8*c + 8*d*x)*a*d*x - 15*e**(8*c + 8*d*x)*a - 192*e**(8*c + 8*d*x)*
b*d*x - 83*e**(8*c + 8*d*x)*b + 384*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x)
+ 1)*b + 48*e**(6*c + 6*d*x)*a*d*x - 384*e**(6*c + 6*d*x)*b*d*x + 192*e**(
4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*b + 24*e**(4*c + 4*d*x)*a*d*x + 15*
e**(4*c + 4*d*x)*a - 192*e**(4*c + 4*d*x)*b*d*x - 83*e**(4*c + 4*d*x)*b +
6*e**(2*c + 2*d*x)*a - 18*e**(2*c + 2*d*x)*b - a + b)/(64*e**(4*c + 4*d*x)
*d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.50 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 92

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{5b\arctan(\sinh(c+dx))}{2d}-\frac{a\cosh(c+dx)}{d}+\frac{a\cosh^3(c+dx)}{3d}-\frac{2b\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}-\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
5/2*b*arctan(sinh(d*x+c))/d-a*cosh(d*x+c)/d+1/3*a*cosh(d*x+c)^3/d-2*b*sinh
(d*x+c)/d+1/3*b*sinh(d*x+c)^3/d-1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.22

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{5b\arctan(\sinh(c+dx))}{2d} - \frac{3a\cosh(c+dx)}{4d} + \frac{a\cosh(3(c+dx))}{12d} - \frac{5b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} - \frac{5b\sinh(c+dx)\tanh^2(c+dx)}{3d} + \frac{b\sinh^3(c+dx)\tanh^2(c+dx)}{3d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(5*b*ArcTan[Sinh[c + d*x]])/(2*d) - (3*a*Cosh[c + d*x])/(4*d) + (a*Cosh[3*
(c + d*x)])/(12*d) - (5*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) - (5*b*Sinh[c
 + d*x]*Tanh[c + d*x]^2)/(3*d) + (b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/(3*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.57 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.24, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int i\sin(ic+idx)^3\left(a+ib\tan(ic+idx)^3\right)dx$$

$$\Big\downarrow \; 26$$

$$i\int \sin(ic+idx)^3\left(ib\tan(ic+idx)^3+a\right)dx$$

$$\Big\downarrow \; 4149$$

$$i\int \left(-ib\tanh^3(c+dx)\sinh^3(c+dx)-ia\sinh^3(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$i\left(-\frac{ia\cosh^3(c+dx)}{3d}+\frac{ia\cosh(c+dx)}{d}-\frac{5ib\arctan(\sinh(c+dx))}{2d}-\frac{5ib\sinh^3(c+dx)}{6d}+\frac{5ib\sinh(c+dx)}{2d}+\frac{ib}{}\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
I*((((-5*I)/2)*b*ArcTan[Sinh[c + d*x]])/d + (I*a*Cosh[c + d*x])/d - ((I/3)
*a*Cosh[c + d*x]^3)/d + (((5*I)/2)*b*Sinh[c + d*x])/d - (((5*I)/6)*b*Sinh[
c + d*x]^3)/d + ((I/2)*b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 2.84 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\right.}{d}$
default	$\frac{a\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\arctan(\right.}{d}$
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}-\frac{3\,e^{dx+c}a}{8d}-\frac{9\,e^{dx+c}b}{8d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{9\,e^{-dx-c}b}{8d}+\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-3c}b}{24d}-\frac{b\,e}{}$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b*(1/3*sinh(d*x+c)^5/cosh(d*x+
c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2*sech(
d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1070 vs. 2(84) = 168.

Time = 0.15 (sec) , antiderivative size = 1070, normalized size of antiderivative = 11.63

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/24*((a + b)*cosh(d*x + c)^10 + 10*(a + b)*cosh(d*x + c)*sinh(d*x + c)^9
+ (a + b)*sinh(d*x + c)^10 - (7*a + 25*b)*cosh(d*x + c)^8 + (45*(a + b)*co
sh(d*x + c)^2 - 7*a - 25*b)*sinh(d*x + c)^8 + 8*(15*(a + b)*cosh(d*x + c)^
3 - (7*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(13*a + 25*b)*cosh(d*x
 + c)^6 + 2*(105*(a + b)*cosh(d*x + c)^4 - 14*(7*a + 25*b)*cosh(d*x + c)^2
 - 13*a - 25*b)*sinh(d*x + c)^6 + 4*(63*(a + b)*cosh(d*x + c)^5 - 14*(7*a
+ 25*b)*cosh(d*x + c)^3 - 3*(13*a + 25*b)*cosh(d*x + c))*sinh(d*x + c)^5 -
 2*(13*a - 25*b)*cosh(d*x + c)^4 + 2*(105*(a + b)*cosh(d*x + c)^6 - 35*(7*
a + 25*b)*cosh(d*x + c)^4 - 15*(13*a + 25*b)*cosh(d*x + c)^2 - 13*a + 25*b
)*sinh(d*x + c)^4 + 8*(15*(a + b)*cosh(d*x + c)^7 - 7*(7*a + 25*b)*cosh(d*
x + c)^5 - 5*(13*a + 25*b)*cosh(d*x + c)^3 - (13*a - 25*b)*cosh(d*x + c))*
sinh(d*x + c)^3 - (7*a - 25*b)*cosh(d*x + c)^2 + (45*(a + b)*cosh(d*x + c)
^8 - 28*(7*a + 25*b)*cosh(d*x + c)^6 - 30*(13*a + 25*b)*cosh(d*x + c)^4 -
12*(13*a - 25*b)*cosh(d*x + c)^2 - 7*a + 25*b)*sinh(d*x + c)^2 + 120*(b*co
sh(d*x + c)^7 + 7*b*cosh(d*x + c)*sinh(d*x + c)^6 + b*sinh(d*x + c)^7 + 2*
b*cosh(d*x + c)^5 + (21*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x + c)^5 + 5*(7*b*
cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^4 + b*cosh(d*x + c)^3 +
 (35*b*cosh(d*x + c)^4 + 20*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + (21*b
*cosh(d*x + c)^5 + 20*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)
^2 + (7*b*cosh(d*x + c)^6 + 10*b*cosh(d*x + c)^4 + 3*b*cosh(d*x + c)^2)...
```

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)\sinh^3\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 174 vs. $2(84) = 168$.

Time = 0.12 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.89

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{1}{24}b\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)})}\right.$$
$$+\frac{1}{24}a\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/24*b*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/
d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(
d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a*(e
^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d
)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.43

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{120\,b\arctan\left(e^{(dx+c)}\right)+ae^{(3\,dx+3\,c)}+be^{(3\,dx+3\,c)}-9\,ae^{(dx+c)}-27\,be^{(dx+c)}-\left(9\,ae^{(2\,dx+2\,c)}-27\,be^{(2\,dx+2\,c)}\right.}{24\,d}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/24*(120*b*arctan(e^(d*x + c)) + a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) -
9*a*e^(d*x + c) - 27*b*e^(d*x + c) - (9*a*e^(2*d*x + 2*c) - 27*b*e^(2*d*x
+ 2*c) - a + b)*e^(-3*d*x - 3*c) - 24*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/
(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 171, normalized size of antiderivative = 1.86

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{e^{3c+3dx}(a+b)}{24d}+\frac{5\,\mathrm{atan}\left(\frac{b\,e^{dx}\,e^{c}\sqrt{d^2}}{d\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{d^2}}+\frac{e^{-3c-3dx}(a-b)}{24d}-\frac{e^{c+dx}(3a+9b)}{8d}-\frac{e^{-c-dx}(3a-9b)}{8d}-\frac{b\,e^{c+dx}}{d\left(e^{2c+2dx}+1\right)}+\frac{2b\,e^{c+dx}}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3),x)
```

output
```
(exp(3*c + 3*d*x)*(a + b))/(24*d) + (5*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2)
)/(d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(- 3*c - 3*d*x)*(a - b))
/(24*d) - (exp(c + d*x)*(3*a + 9*b))/(8*d) - (exp(- c - d*x)*(3*a - 9*b))/
(8*d) - (b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(
d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 239, normalized size of antiderivative = 2.60

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$=\frac{120e^{7dx+7c}atan\left(e^{dx+c}\right)b+240e^{5dx+5c}atan\left(e^{dx+c}\right)b+120e^{3dx+3c}atan\left(e^{dx+c}\right)b+e^{10dx+10c}a+e^{10dx+10c}b-}{24e^{3dx+3c}d\left(e^{4c}\right.}$$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3),x)
```

output
```
(120*e**(7*c + 7*d*x)*atan(e**(c + d*x))*b + 240*e**(5*c + 5*d*x)*atan(e**
(c + d*x))*b + 120*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b + e**(10*c + 10*d
*x)*a + e**(10*c + 10*d*x)*b - 7*e**(8*c + 8*d*x)*a - 25*e**(8*c + 8*d*x)*
b - 26*e**(6*c + 6*d*x)*a - 50*e**(6*c + 6*d*x)*b - 26*e**(4*c + 4*d*x)*a
+ 50*e**(4*c + 4*d*x)*b - 7*e**(2*c + 2*d*x)*a + 25*e**(2*c + 2*d*x)*b + a
 - b)/(24*e**(3*c + 3*d*x)*d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.51 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{ax}{2}+\frac{b\cosh^2(c+dx)}{2d}-\frac{2b\log(\cosh(c+dx))}{d}+\frac{a\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b\tanh^2(c+dx)}{2d}$$

output
```
-1/2*a*x+1/2*b*cosh(d*x+c)^2/d-2*b*ln(cosh(d*x+c))/d+1/2*a*cosh(d*x+c)*sin
h(d*x+c)/d+1/2*b*tanh(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{a(-c-dx)}{2d}-\frac{b\left(4\log(\cosh(c+dx))+\operatorname{sech}^2(c+dx)-\sinh^2(c+dx)\right)}{2d}+\frac{a\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(a*(-c - d*x))/(2*d) - (b*(4*Log[Cosh[c + d*x]] + Sech[c + d*x]^2 - Sinh[c
 + d*x]^2))/(2*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.25, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 25, 4146, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\sin(ic+idx)^2\left(a+ib\tan(ic+idx)^3\right)dx$$

$$\Big\downarrow \ 25$$

$$-\int \sin(ic+idx)^2\left(ib\tan(ic+idx)^3+a\right)dx$$

$$\Big\downarrow \ 4146$$

$$\frac{\int \frac{\tanh^2(c+dx)(b\tanh^3(c+dx)+a)}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 2335$$

$$\frac{\frac{1}{2}\int -\frac{\tanh(c+dx)(2b\tanh^2(c+dx)+a\tanh(c+dx)+2b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh^2(c+dx)(a\tanh(c+dx)+b)}{2(1-\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\frac{\tanh^2(c+dx)(a\tanh(c+dx)+b)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}\int \frac{\tanh(c+dx)(2b\tanh^2(c+dx)+a\tanh(c+dx)+2b)}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 2333$$

$$\frac{\frac{\tanh^2(c+dx)(a\tanh(c+dx)+b)}{2(1-\tanh^2(c+dx))}-\frac{1}{2}\int\left(-a-2b\tanh(c+dx)+\frac{a+4b\tanh(c+dx)}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$\downarrow$ 2009

$$\frac{\frac{1}{2}\left(-a\text{arctanh}(\tanh(c+dx))+a\tanh(c+dx)+b\tanh^2(c+dx)+2b\log\left(1-\tanh^2(c+dx)\right)\right)+\frac{\tanh^2(c+dx)(a\tan}{2(1-\tanh^2(c}}{d}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
((Tanh[c + d*x]^2*(b + a*Tanh[c + d*x]))/(2*(1 - Tanh[c + d*x]^2)) + (-(a*
ArcTanh[Tanh[c + d*x]]) + 2*b*Log[1 - Tanh[c + d*x]^2] + a*Tanh[c + d*x] +
 b*Tanh[c + d*x]^2)/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.58 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.94

method	result
derivativedivides	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)}{d}$
default	$\frac{a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)}{d}$
risch	$-\frac{ax}{2}+2bx+\frac{\mathrm{e}^{2dx+2c}a}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}+\frac{\mathrm{e}^{-2dx-2c}b}{8d}+\frac{4bc}{d}-\frac{2b\,\mathrm{e}^{2dx+2c}}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}-\frac{2b\ln(\mathrm{e}^{2dx+}}{d}$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b*(1/2*sinh(d*x+c)^4/co
sh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. 2(64) = 128.

Time = 0.10 (sec) , antiderivative size = 924, normalized size of antiderivative = 12.83

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/8*((a + b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (
a + b)*sinh(d*x + c)^8 - 2*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^6 - 2*(
2*(a - 4*b)*d*x - 14*(a + b)*cosh(d*x + c)^2 - a - b)*sinh(d*x + c)^6 + 4*
(14*(a + b)*cosh(d*x + c)^3 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c))*s
inh(d*x + c)^5 - 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^4 + 2*(35*(a + b)
*cosh(d*x + c)^4 - 4*(a - 4*b)*d*x - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x
 + c)^2 - 7*b)*sinh(d*x + c)^4 + 8*(7*(a + b)*cosh(d*x + c)^5 - 5*(2*(a -
4*b)*d*x - a - b)*cosh(d*x + c)^3 - (4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c))
*sinh(d*x + c)^3 - 2*(2*(a - 4*b)*d*x + a - b)*cosh(d*x + c)^2 + 2*(14*(a
+ b)*cosh(d*x + c)^6 - 15*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^4 - 2*(a
 - 4*b)*d*x - 6*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^2 - a + b)*sinh(d*x
+ c)^2 - 16*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sin
h(d*x + c)^6 + 2*b*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 + 2*b)*sinh(d*x
 + c)^4 + 4*(5*b*cosh(d*x + c)^3 + 2*b*cosh(d*x + c))*sinh(d*x + c)^3 + b*
cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 + 12*b*cosh(d*x + c)^2 + b)*sinh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c)
)*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*
(2*(a + b)*cosh(d*x + c)^7 - 3*(2*(a - 4*b)*d*x - a - b)*cosh(d*x + c)^5 -
 2*(4*(a - 4*b)*d*x + 7*b)*cosh(d*x + c)^3 - (2*(a - 4*b)*d*x + a - b)*cos
h(d*x + c))*sinh(d*x + c) - a + b)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + ...
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int\left(a+b\tanh^3(c+dx)\right)\sinh^2(c+dx)\,dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(64) = 128$.

Time = 0.12 (sec) , antiderivative size = 141, normalized size of antiderivative = 1.96

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{1}{8}\,a\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right) -\frac{1}{8}\,b\left(\frac{16\,(dx+c)}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}+\frac{16\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+1}{d(e^{(-2\,dx-2\,c)}+2\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)})}\right)$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b*(16*(d*x + c
)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x -
 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(64) = 128$.

Time = 0.15 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.94

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{4\,(dx+c)(a-4\,b)-ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}-8\,be^{(2\,dx+2\,c)}-a+b\right)e^{(-2\,dx-2\,c)}+16\,b\,\mathrm{l}}{8\,d}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/8*(4*(d*x + c)*(a - 4*b) - a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) - (2*a
*e^(2*d*x + 2*c) - 8*b*e^(2*d*x + 2*c) - a + b)*e^(-2*d*x - 2*c) + 16*b*lo
g(e^(2*d*x + 2*c) + 1) - 8*(3*b*e^(4*d*x + 4*c) + 4*b*e^(2*d*x + 2*c) + 3*
b)/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.22 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.60

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{e^{2c+2dx}(a+b)}{8d} - \frac{2b}{d\left(e^{2c+2dx}+1\right)} - x\left(\frac{a}{2}-2b\right) + \frac{2b}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} - \frac{e^{-2c-2dx}(a-b)}{8d} - \frac{2b\ln\left(e^{2c}e^{2dx}+1\right)}{d}$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3),x)
```

output
```
(exp(2*c + 2*d*x)*(a + b))/(8*d) - (2*b)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a
/2 - 2*b) + (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (exp(-
 2*c - 2*d*x)*(a - b))/(8*d) - (2*b*log(exp(2*c)*exp(2*d*x) + 1))/d
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 292, normalized size of antiderivative = 4.06

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{e^{8dx+8c}a + e^{8dx+8c}b - 16e^{6dx+6c}\log\left(e^{2dx+2c}+1\right)b - 4e^{6dx+6c}adx + 2e^{6dx+6c}a + 16e^{6dx+6c}bdx + 9e^{6dx+6c}b}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3),x)
```

output
```
(e**(8*c + 8*d*x)*a + e**(8*c + 8*d*x)*b - 16*e**(6*c + 6*d*x)*log(e**(2*c
 + 2*d*x) + 1)*b - 4*e**(6*c + 6*d*x)*a*d*x + 2*e**(6*c + 6*d*x)*a + 16*e*
*(6*c + 6*d*x)*b*d*x + 9*e**(6*c + 6*d*x)*b - 32*e**(4*c + 4*d*x)*log(e**(
2*c + 2*d*x) + 1)*b - 8*e**(4*c + 4*d*x)*a*d*x + 32*e**(4*c + 4*d*x)*b*d*x
 - 16*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*b - 4*e**(2*c + 2*d*x)*a*
d*x - 2*e**(2*c + 2*d*x)*a + 16*e**(2*c + 2*d*x)*b*d*x + 9*e**(2*c + 2*d*x
)*b - a + b)/(8*e**(2*c + 2*d*x)*d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x)
+ 1))
```

3.52 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 58

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{3b\arctan(\sinh(c+dx))}{2d} + \frac{a\cosh(c+dx)}{d} + \frac{b\sinh(c+dx)}{d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
-3/2*b*arctan(sinh(d*x+c))/d+a*cosh(d*x+c)/d+b*sinh(d*x+c)/d+1/2*b*sech(d*
x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.33

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{3b\arctan(\sinh(c+dx))}{2d} + \frac{a\cosh(c)\cosh(dx)}{d} + \frac{a\sinh(c)\sinh(dx)}{d} + \frac{3b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} + \frac{b\sinh(c+dx)\tanh^2(c+dx)}{d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-3*b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Cosh[c]*Cosh[d*x])/d + (a*Sinh[c]*
Sinh[d*x])/d + (3*b*Sech[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sinh[c + d*x]*
Tanh[c + d*x]^2)/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.31, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\sin(ic+idx)\left(a+ib\tan(ic+idx)^3\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \sin(ic+idx)\left(ib\tan(ic+idx)^3+a\right)dx$$

$$\Big\downarrow \text{4149}$$

$$-i\int \left(ib\sinh(c+dx)\tanh^3(c+dx)+ia\sinh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia\cosh(c+dx)}{d}-\frac{3ib\arctan(\sinh(c+dx))}{2d}+\frac{3ib\sinh(c+dx)}{2d}-\frac{ib\sinh(c+dx)\tanh^2(c+dx)}{2d}\right)$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-I)*((((-3*I)/2)*b*ArcTan[Sinh[c + d*x]])/d + (I*a*Cosh[c + d*x])/d + (((
3*I)/2)*b*Sinh[c + d*x])/d - ((I/2)*b*Sinh[c + d*x]*Tanh[c + d*x]^2)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.24 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.26

method	result	s
derivativedivides	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	7
default	$\frac{a\cosh(dx+c)+b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	7
risch	$\frac{\mathrm{e}^{dx+c}a}{2d}+\frac{\mathrm{e}^{dx+c}b}{2d}+\frac{\mathrm{e}^{-dx-c}a}{2d}-\frac{\mathrm{e}^{-dx-c}b}{2d}+\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^2}+\frac{3ib\ln\left(\mathrm{e}^{dx+c}-i\right)}{2d}-\frac{3ib\ln\left(\mathrm{e}^{dx+c}+i\right)}{2d}$	1

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*cosh(d*x+c)+b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c
)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(54) = 108$.

Time = 0.12 (sec) , antiderivative size = 528, normalized size of antiderivative = 9.10

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/2*((a + b)*cosh(d*x + c)^6 + 6*(a + b)*cosh(d*x + c)*sinh(d*x + c)^5 + (
a + b)*sinh(d*x + c)^6 + 3*(a + b)*cosh(d*x + c)^4 + 3*(5*(a + b)*cosh(d*x
 + c)^2 + a + b)*sinh(d*x + c)^4 + 4*(5*(a + b)*cosh(d*x + c)^3 + 3*(a + b
)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)^2 + 3*(5*(a + b
)*cosh(d*x + c)^4 + 6*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 - 6
*(b*cosh(d*x + c)^5 + 5*b*cosh(d*x + c)*sinh(d*x + c)^4 + b*sinh(d*x + c)^
5 + 2*b*cosh(d*x + c)^3 + 2*(5*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^3 + 2*
(5*b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c))*sinh(d*x + c)^2 + b*cosh(d*x + c
) + (5*b*cosh(d*x + c)^4 + 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c))*arctan(
cosh(d*x + c) + sinh(d*x + c)) + 6*((a + b)*cosh(d*x + c)^5 + 2*(a + b)*co
sh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a - b)/(d*cosh(d*x
+ c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(
d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x
 + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh
(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)\sinh{(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*sinh(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.81

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{1}{2}\,b\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)+\frac{a\cosh{(dx+c)}}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) + a*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.48

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -\frac{6\,b\arctan\left(e^{(dx+c)}\right)-ae^{(dx+c)}-be^{(dx+c)}-(a-b)e^{(-dx-c)}-\frac{2\,\left(be^{(3\,dx+3\,c)}-be^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/2*(6*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) - (a - b)*e^
(-d*x - c) - 2*(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2
)/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 128, normalized size of antiderivative = 2.21

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{e^{-c-dx}(a-b)}{2d} - \frac{3\,\mathrm{atan}\left(\frac{b\,e^{dx}\,e^{c}\sqrt{d^2}}{d\sqrt{b^2}}\right)\sqrt{b^2}}{\sqrt{d^2}} + \frac{e^{c+dx}(a+b)}{2d} + \frac{b\,e^{c+dx}}{d\left(e^{2c+2dx}+1\right)} - \frac{2\,b\,e^{c+dx}}{d\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3),x)
```

output
```
(exp(- c - d*x)*(a - b))/(2*d) - (3*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(
d*(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) + (exp(c + d*x)*(a + b))/(2*d) +
(b*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b*exp(c + d*x))/(d*(2*exp
(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 127, normalized size of antiderivative = 2.19

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{-12e^{dx+c}atan\left(e^{dx+c}\right)b - 2e^{dx+c}\cosh(dx+c)\tanh(dx+c)\,b + 4e^{dx+c}\cosh(dx+c)\,a + 3e^{2dx+2c}b - 2e^{dx}}{4e^{dx+c}d}$$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3),x)
```

output
```
( - 12*e**(c + d*x)*atan(e**(c + d*x))*b - 2*e**(c + d*x)*cosh(c + d*x)*ta
nh(c + d*x)*b + 4*e**(c + d*x)*cosh(c + d*x)*a + 3*e**(2*c + 2*d*x)*b - 2*
e**(c + d*x)*sinh(c + d*x)*tanh(c + d*x)**2*b + 2*e**(c + d*x)*sinh(c + d*
x)*b - 3*b)/(4*e**(c + d*x)*d)
```

3.53 $\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 49

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx = \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b*arctan(sinh(d*x+c))/d-a*arctanh(cosh(d*x+c))/d-1/2*b*sech(d*x+c)*tan
h(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx = \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\operatorname{arctanh}(\cosh(c+dx))}{d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*ArcTanh[Cosh[c + d*x]])/d - (b*Sech[c
 + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i(a+ib\tan(ic+idx)^3)}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{ib\tan(ic+idx)^3+a}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4149}$$

$$i\int \left(-ib\text{sech}(c+dx)\tanh^2(c+dx)-ia\text{csch}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{ia\text{arctanh}(\cosh(c+dx))}{d}-\frac{ib\arctan(\sinh(c+dx))}{2d}+\frac{ib\tanh(c+dx)\text{sech}(c+dx)}{2d}\right)$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3),x]
```

output
```
I*(((-1/2*I)*b*ArcTan[Sinh[c + d*x]])/d + (I*a*ArcTanh[Cosh[c + d*x]])/d +
 ((I/2)*b*Sech[c + d*x]*Tanh[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 1.19 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result	size
derivativedivides	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	56
default	$\frac{-2a\,\operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$	56
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{d\left(\mathrm{e}^{2dx+2c}+1\right)^{2}}+\frac{ib\ln\left(\mathrm{e}^{dx+c}+i\right)}{2d}-\frac{ib\ln\left(\mathrm{e}^{dx+c}-i\right)}{2d}+\frac{a\ln\left(\mathrm{e}^{dx+c}-1\right)}{d}-\frac{a\ln\left(\mathrm{e}^{dx+c}+1\right)}{d}$	101

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a*arctanh(exp(d*x+c))+b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d*x+c
)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 522 vs. $2(45) = 90$.

Time = 0.11 (sec) , antiderivative size = 522, normalized size of antiderivative = 10.65

$$\int \operatorname{csch}(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^
3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*arctan(cosh(d*
x + c) + sinh(d*x + c)) - b*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(
d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*cosh(d*x + c)^2 + 2*(3*
a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 + a*cosh(d*x
 + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh
(d*x + c)^4 + 4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 + 2*a*
cosh(d*x + c)^2 + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c)^2 + 4*(a*cosh(
d*x + c)^3 + a*cosh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(
d*x + c) - 1) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x + c))/(d*cosh(d*x + c)^
4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x +
 c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3
 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)\operatorname{csch}\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.69

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.51

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=\frac{b\arctan\left(e^{(dx+c)}\right)-a\log\left(e^{(dx+c)}+1\right)+a\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{be^{(3\,dx+3\,c)}-be^{(dx+c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
(b*arctan(e^(d*x + c)) - a*log(e^(d*x + c) + 1) + a*log(abs(e^(d*x + c) -
1)) - (b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 3.60 (sec) , antiderivative size = 233, normalized size of antiderivative = 4.76

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{2\,b\,\mathrm{e}^{c+d\,x}}{d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}} - \frac{a\,\ln\left(-8\,a\,b^2-32\,a^3-32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-8\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d}$$
$$+ \frac{a\,\ln\left(8\,a\,b^2+32\,a^3-32\,a^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-8\,a\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\right)}{d}$$
$$- \frac{b\left(\ln\left(4\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+16\,a^2\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}-a^2\,b\,16\mathrm{i}-b^3\,4\mathrm{i}\right)\,1\mathrm{i}-\ln\left(4\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+16\,a^2\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^{c}+a^2\,b\,16\mathrm{i}+b^3\,4\mathrm{i}\right)}{2\,d}$$
$$- \frac{b\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x),x)
```

output
```
(2*b*exp(c + d*x))/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*lo
g(- 8*a*b^2 - 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*exp(c)))/
d + (a*log(8*a*b^2 + 32*a^3 - 32*a^3*exp(d*x)*exp(c) - 8*a*b^2*exp(d*x)*ex
p(c)))/d - (b*(log(4*b^3*exp(d*x)*exp(c) - b^3*4i - a^2*b*16i + 16*a^2*b*e
xp(d*x)*exp(c))*1i - log(a^2*b*16i + b^3*4i + 4*b^3*exp(d*x)*exp(c) + 16*a
^2*b*exp(d*x)*exp(c))*1i))/(2*d) - (b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x
))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 220, normalized size of antiderivative = 4.49

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{e^{4dx+4c}atan\left(e^{dx+c}\right)b+2e^{2dx+2c}atan\left(e^{dx+c}\right)b+atan\left(e^{dx+c}\right)b+e^{4dx+4c}\log\left(e^{dx+c}-1\right)a-e^{4dx+4c}\log\left(e^{dx+c}\right.}{d\left(e^{4}\right.}$$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3),x)
```

output
```
(e**(4*c + 4*d*x)*atan(e**(c + d*x))*b + 2*e**(2*c + 2*d*x)*atan(e**(c + d
*x))*b + atan(e**(c + d*x))*b + e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a -
 e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a - e**(3*c + 3*d*x)*b + 2*e**(2*c
 + 2*d*x)*log(e**(c + d*x) - 1)*a - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) +
1)*a + e**(c + d*x)*b + log(e**(c + d*x) - 1)*a - log(e**(c + d*x) + 1)*a)
/(d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.54 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 29

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}+\frac{b\tanh^2(c+dx)}{2d}$$

output
```
-a*coth(d*x+c)/d+1/2*b*tanh(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{a\coth(c+dx)}{d}-\frac{b\operatorname{sech}^2(c+dx)}{2d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
-((a*Coth[c + d*x])/d) - (b*Sech[c + d*x]^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3042, 25, 4146, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{a+ib\tan(ic+idx)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{ib\tan(ic+idx)^3+a}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^3(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 802}$$

$$\frac{\int \left(a\coth^2(c+dx)+b\tanh(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}b\tanh^2(c+dx)-a\coth(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-(a*Coth[c + d*x]) + (b*Tanh[c + d*x]^2)/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 1.72 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.97

method	result	size
derivativedivides	$\frac{-\frac{a}{\tanh(dx+c)}+\frac{\tanh(dx+c)^2 b}{2}}{d}$	28
default	$\frac{-\frac{a}{\tanh(dx+c)}+\frac{\tanh(dx+c)^2 b}{2}}{d}$	28
risch	$-\frac{2\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-e^{2dx+2c}b+a\right)}{d\left(e^{2dx+2c}+1\right)^2\left(e^{2dx+2c}-1\right)}$	80

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/tanh(d*x+c)+1/2*tanh(d*x+c)^2*b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(27) = 54$.

Time = 0.10 (sec) , antiderivative size = 141, normalized size of antiderivative = 4.86

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx =$$
$$-\frac{2\left((2a+b)\cosh\left(dx+c\right)^2+2b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+(2a+b\right.}{d\cosh\left(dx+c\right)^4+6d\cosh\left(dx+c\right)^2\sinh\left(dx+c\right)^2+4d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-2*((2*a + b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (2*a + b
)*sinh(d*x + c)^2 + 2*a - b)/(d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2*sinh
(d*x + c)^2 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*(d
*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)\operatorname{csch}^2\left(c+dx\right) dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.52

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{2\,a}{d(e^{(-2\,dx-2\,c)}-1)}-\frac{2\,b}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
2*a/(d*(e^(-2*d*x - 2*c) - 1)) - 2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.55

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=-\frac{2\left(\frac{a}{e^{(2\,dx+2\,c)}-1}+\frac{be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}\right)}{d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-2*(a/(e^(2*d*x + 2*c) - 1) + b*e^(2*d*x + 2*c)/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 79, normalized size of antiderivative = 2.72

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=-\frac{2\left(a+2\,a\,\mathrm{e}^{2\,c+2\,d\,x}+a\,\mathrm{e}^{4\,c+4\,d\,x}-b\,\mathrm{e}^{2\,c+2\,d\,x}+b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^2}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^2,x)
```

output
```
-(2*(a + 2*a*exp(2*c + 2*d*x) + a*exp(4*c + 4*d*x) - b*exp(2*c + 2*d*x) +
b*exp(4*c + 4*d*x)))/(d*(exp(2*c + 2*d*x) - 1)*(exp(2*c + 2*d*x) + 1)^2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 85, normalized size of antiderivative = 2.93

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{2e^{6dx+6c}a+2e^{6dx+6c}b-6e^{2dx+2c}a-4a-2b}{d\left(e^{6dx+6c}+e^{4dx+4c}-e^{2dx+2c}-1\right)}$$

input

```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3),x)
```

output

```
(2*(e**(6*c + 6*d*x)*a + e**(6*c + 6*d*x)*b - 3*e**(2*c + 2*d*x)*a - 2*a -
 b))/(d*(e**(6*c + 6*d*x) + e**(4*c + 4*d*x) - e**(2*c + 2*d*x) - 1))
```

3.55 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 71

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan(\sinh(c+dx))}{2d} + \frac{a\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b*arctan(sinh(d*x+c))/d+1/2*a*arctanh(cosh(d*x+c))/d-1/2*a*coth(d*x+c)
*csch(d*x+c)/d+1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.61

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} - \frac{a\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} + \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(b*ArcTan[Sinh[c + d*x]])/(2*d) - (a*Csch[(c + d*x)/2]^2)/(8*d) + (a*Log[C
osh[(c + d*x)/2]])/(2*d) - (a*Log[Sinh[(c + d*x)/2]])/(2*d) - (a*Sech[(c +
 d*x)/2]^2)/(8*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a+ib\tan(ic+idx)^3\right)}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{ib\tan(ic+idx)^3+a}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \; 4149$$

$$-i\int \left(ia\text{csch}^3(c+dx)+ib\text{sech}^3(c+dx)\right)dx$$

$$\Big\downarrow \; 2009$$

$$-i\left(\frac{ia\text{arctanh}(\cosh(c+dx))}{2d}-\frac{ia\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{ib\arctan(\sinh(c+dx))}{2d}+\frac{ib\tanh(c+dx)\text{sech}(c\right.$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-I)*(((I/2)*b*ArcTan[Sinh[c + d*x]])/d + ((I/2)*a*ArcTanh[Cosh[c + d*x]])
/d - ((I/2)*a*Coth[c + d*x]*Csch[c + d*x])/d + ((I/2)*b*Sech[c + d*x]*Tanh
[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 4.77 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.76

method	result
derivativedivides	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
default	$\frac{a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$
risch	$-\frac{e^{dx+c}\left(e^{6dx+6c}a-e^{6dx+6c}b+3\,e^{4dx+4c}a+3b\,e^{4dx+4c}+3\,e^{2dx+2c}a-3\,e^{2dx+2c}b+a+b\right)}{d\left(e^{2dx+2c}-1\right)^{2}\left(e^{2dx+2c}+1\right)^{2}}+\frac{a\ln\left(e^{dx+c}+1\right)}{2d}-\frac{a\ln(e^{\ldots}}{\ldots}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*(1/2*sech(d*x+
c)*tanh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1188 vs. $2(63) = 126$.

Time = 0.11 (sec) , antiderivative size = 1188, normalized size of antiderivative = 16.73

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/2*(2*(a - b)*cosh(d*x + c)^7 + 14*(a - b)*cosh(d*x + c)*sinh(d*x + c)^6
 + 2*(a - b)*sinh(d*x + c)^7 + 6*(a + b)*cosh(d*x + c)^5 + 6*(7*(a - b)*co
sh(d*x + c)^2 + a + b)*sinh(d*x + c)^5 + 10*(7*(a - b)*cosh(d*x + c)^3 + 3
*(a + b)*cosh(d*x + c))*sinh(d*x + c)^4 + 6*(a - b)*cosh(d*x + c)^3 + 2*(3
5*(a - b)*cosh(d*x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a - 3*b)*sinh(d
*x + c)^3 + 6*(7*(a - b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*
(a - b)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b*cosh(d*x + c)^8 + 56*b*cosh(
d*x + c)^3*sinh(d*x + c)^5 + 28*b*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*b*co
sh(d*x + c)*sinh(d*x + c)^7 + b*sinh(d*x + c)^8 - 2*b*cosh(d*x + c)^4 + 2*
(35*b*cosh(d*x + c)^4 - b)*sinh(d*x + c)^4 + 8*(7*b*cosh(d*x + c)^5 - b*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b*cosh(d*x + c)^6 - 3*b*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^7 - b*cosh(d*x + c)^3)*sinh(d*x +
c) + b)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(a + b)*cosh(d*x + c) -
(a*cosh(d*x + c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*cosh(d*x
+ c)^2*sinh(d*x + c)^6 + 8*a*cosh(d*x + c)*sinh(d*x + c)^7 + a*sinh(d*x +
c)^8 - 2*a*cosh(d*x + c)^4 + 2*(35*a*cosh(d*x + c)^4 - a)*sinh(d*x + c)^4
+ 8*(7*a*cosh(d*x + c)^5 - a*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a*cosh(
d*x + c)^6 - 3*a*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a*cosh(d*x + c)^7 -
 a*cosh(d*x + c)^3)*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) +
 1) + (a*cosh(d*x + c)^8 + 56*a*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*a*...
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3(c+dx)\right)\operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(63) = 126.

Time = 0.12 (sec) , antiderivative size = 156, normalized size of antiderivative = 2.20

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) + \frac{1}{2}\,a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(63) = 126$.

Time = 0.14 (sec) , antiderivative size = 143, normalized size of antiderivative = 2.01

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{2\,b\arctan\left(e^{(dx+c)}\right)+a\log\left(e^{(dx+c)}+1\right)-a\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{2\left(ae^{(7\,dx+7\,c)}-be^{(7\,dx+7\,c)}+3\,ae^{(5\,dx+5\,c)}+3\,be^{(5\,dx+}\right.}{\left(e^{(4\,dx-}\right.}}{2\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/2*(2*b*arctan(e^(d*x + c)) + a*log(e^(d*x + c) + 1) - a*log(abs(e^(d*x +
 c) - 1)) - 2*(a*e^(7*d*x + 7*c) - b*e^(7*d*x + 7*c) + 3*a*e^(5*d*x + 5*c)
 + 3*b*e^(5*d*x + 5*c) + 3*a*e^(3*d*x + 3*c) - 3*b*e^(3*d*x + 3*c) + a*e^(
d*x + c) + b*e^(d*x + c))/(e^(4*d*x + 4*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 3.62 (sec) , antiderivative size = 173, normalized size of antiderivative = 2.44

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{a\,\ln\left(\mathrm{e}^{c+d\,x}+1\right)}{2\,d}-\frac{\frac{4\,\mathrm{e}^{3\,c+3\,d\,x}\,(a-b)}{d}+\frac{4\,\mathrm{e}^{c+d\,x}\,(a+b)}{d}}{\mathrm{e}^{8\,c+8\,d\,x}-2\,\mathrm{e}^{4\,c+4\,d\,x}+1}$$
$$-\frac{a\,\ln\left(\mathrm{e}^{c+d\,x}-1\right)}{2\,d}$$
$$-\frac{\frac{\mathrm{e}^{3\,c+3\,d\,x}\,(a-b)}{d}+\frac{3\,\mathrm{e}^{c+d\,x}\,(a+b)}{d}}{\mathrm{e}^{4\,c+4\,d\,x}-1}$$
$$-\frac{b\,\ln\left(\mathrm{e}^{c+d\,x}-\mathrm{i}\right)\,1\mathrm{i}}{2\,d}+\frac{b\,\ln\left(\mathrm{e}^{c+d\,x}+1\mathrm{i}\right)\,1\mathrm{i}}{2\,d}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^3,x)
```

output
```
(a*log(exp(c + d*x) + 1))/(2*d) - ((4*exp(3*c + 3*d*x)*(a - b))/d + (4*exp
(c + d*x)*(a + b))/d)/(exp(8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (a*log
(exp(c + d*x) - 1))/(2*d) - ((exp(3*c + 3*d*x)*(a - b))/d + (3*exp(c + d*x
)*(a + b))/d)/(exp(4*c + 4*d*x) - 1) - (b*log(exp(c + d*x) - 1i)*1i)/(2*d)
 + (b*log(exp(c + d*x) + 1i)*1i)/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 299, normalized size of antiderivative = 4.21

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{2e^{8dx+8c}atan\left(e^{dx+c}\right)b-4e^{4dx+4c}atan\left(e^{dx+c}\right)b+2atan\left(e^{dx+c}\right)b-e^{8dx+8c}\log\left(e^{dx+c}-1\right)a+e^{8dx+8c}\log\left(e\right.}{}$$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3),x)
```

output
```
(2*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b - 4*e**(4*c + 4*d*x)*atan(e**(c +
 d*x))*b + 2*atan(e**(c + d*x))*b - e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)
*a + e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a - 2*e**(7*c + 7*d*x)*a + 2*e
**(7*c + 7*d*x)*b - 6*e**(5*c + 5*d*x)*a - 6*e**(5*c + 5*d*x)*b + 2*e**(4*
c + 4*d*x)*log(e**(c + d*x) - 1)*a - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) +
 1)*a - 6*e**(3*c + 3*d*x)*a + 6*e**(3*c + 3*d*x)*b - 2*e**(c + d*x)*a - 2
*e**(c + d*x)*b - log(e**(c + d*x) - 1)*a + log(e**(c + d*x) + 1)*a)/(2*d*
(e**(8*c + 8*d*x) - 2*e**(4*c + 4*d*x) + 1))
```

3.56 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 56

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}+\frac{b\log(\tanh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

output
```
a*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d+b*ln(tanh(d*x+c))/d-1/2*b*tanh(d*x+c
)^2/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.32

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{b\left(2\log(\cosh(c+dx))-2\log(\sinh(c+dx))-\operatorname{sech}^2(c+dx)\right)}{2d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*(
2*Log[Cosh[c + d*x]] - 2*Log[Sinh[c + d*x]] - Sech[c + d*x]^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4146, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a+ib\tan(ic+idx)^3}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^3(c+dx)+a\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2333}$$

$$\frac{\int \left(a\coth^4(c+dx)-a\coth^2(c+dx)+b\coth(c+dx)-b\tanh(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{3}a\coth^3(c+dx)+a\coth(c+dx)-\frac{1}{2}b\tanh^2(c+dx)+b\log(\tanh(c+dx))}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3),x]
```

output
```
(a*Coth[c + d*x] - (a*Coth[c + d*x]^3)/3 + b*Log[Tanh[c + d*x]] - (b*Tanh[
c + d*x]^2)/2)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 8.78 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.82

method	result
derivativedivides	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+b\left(\frac{1}{2\cosh(dx+c)^{2}}+\ln(\tanh(dx+c))\right)}{d}$
default	$\frac{a\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^{2}}{3}\right)\coth(dx+c)+b\left(\frac{1}{2\cosh(dx+c)^{2}}+\ln(\tanh(dx+c))\right)}{d}$
risch	$-\frac{2\left(-3\,\mathrm{e}^{8dx+8c}b+6\,\mathrm{e}^{6dx+6c}a+9\,\mathrm{e}^{6dx+6c}b+10\,\mathrm{e}^{4dx+4c}a-9b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a+3\,\mathrm{e}^{2dx+2c}b-2a\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^{2}\left(\mathrm{e}^{2dx+2c}-1\right)^{3}}+\frac{b\ln\left(\mathrm{e}^{2dx+2c}-\right.}{d}$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b*(1/2/cosh(d*x+c)^2+ln(tanh(d*
x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1739 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 1739, normalized size of antiderivative = 31.05

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^3(c+dx)\right) \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
1/3*(6*b*cosh(d*x + c)^8 + 48*b*cosh(d*x + c)*sinh(d*x + c)^7 + 6*b*sinh(d
*x + c)^8 - 6*(2*a + 3*b)*cosh(d*x + c)^6 + 6*(28*b*cosh(d*x + c)^2 - 2*a
- 3*b)*sinh(d*x + c)^6 + 12*(28*b*cosh(d*x + c)^3 - 3*(2*a + 3*b)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 2*(10*a - 9*b)*cosh(d*x + c)^4 + 2*(210*b*cosh(d*
x + c)^4 - 45*(2*a + 3*b)*cosh(d*x + c)^2 - 10*a + 9*b)*sinh(d*x + c)^4 +
8*(42*b*cosh(d*x + c)^5 - 15*(2*a + 3*b)*cosh(d*x + c)^3 - (10*a - 9*b)*co
sh(d*x + c))*sinh(d*x + c)^3 - 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(84*b*cos
h(d*x + c)^6 - 45*(2*a + 3*b)*cosh(d*x + c)^4 - 6*(10*a - 9*b)*cosh(d*x +
c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 - 3*(b*cosh(d*x + c)^10 + 10*b*cosh(d*x
+ c)*sinh(d*x + c)^9 + b*sinh(d*x + c)^10 - b*cosh(d*x + c)^8 + (45*b*cosh
(d*x + c)^2 - b)*sinh(d*x + c)^8 + 8*(15*b*cosh(d*x + c)^3 - b*cosh(d*x +
c))*sinh(d*x + c)^7 - 2*b*cosh(d*x + c)^6 + 2*(105*b*cosh(d*x + c)^4 - 14*
b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^6 + 4*(63*b*cosh(d*x + c)^5 - 14*b*co
sh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^5 + 2*b*cosh(d*x + c)^4 +
 2*(105*b*cosh(d*x + c)^6 - 35*b*cosh(d*x + c)^4 - 15*b*cosh(d*x + c)^2 +
b)*sinh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^7 - 7*b*cosh(d*x + c)^5 - 5*b*c
osh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c)^3 + b*cosh(d*x + c)^2 + (4
5*b*cosh(d*x + c)^8 - 28*b*cosh(d*x + c)^6 - 30*b*cosh(d*x + c)^4 + 12*b*c
osh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^9 - 4*b*cosh(d*
x + c)^7 - 6*b*cosh(d*x + c)^5 + 4*b*cosh(d*x + c)^3 + b*cosh(d*x + c))...
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx=\int \left(a+b\tanh^3{\left(c+dx\right)}\right)\operatorname{csch}^4{\left(c+dx\right)}\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(52) = 104$.

Time = 0.13 (sec) , antiderivative size = 184, normalized size of antiderivative = 3.29

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$=b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)$$
$$+\frac{4}{3}\,a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x - 2*c
) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c
) + e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(52) = 104$.

Time = 0.14 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.66

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx =$$

$$-\frac{6\,b\log\left(e^{(2\,dx+2\,c)}+1\right)-6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)-\frac{3\,\left(3\,be^{(4\,dx+4\,c)}+10\,be^{(2\,dx+2\,c)}+3\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^2}+\frac{11\,be^{(6\,dx+6\,c)}-33\,be^{(4\,dx+4}}{}}{6\,d}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-1/6*(6*b*log(e^(2*d*x + 2*c) + 1) - 6*b*log(abs(e^(2*d*x + 2*c) - 1)) - 3
*(3*b*e^(4*d*x + 4*c) + 10*b*e^(2*d*x + 2*c) + 3*b)/(e^(2*d*x + 2*c) + 1)^
2 + (11*b*e^(6*d*x + 6*c) - 33*b*e^(4*d*x + 4*c) + 24*a*e^(2*d*x + 2*c) +
33*b*e^(2*d*x + 2*c) - 8*a - 11*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 162, normalized size of antiderivative = 2.89

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx = \frac{2\,b}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{4\,a}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}-\frac{8\,a}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}-\frac{2\,\mathrm{atan}\left(\frac{b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{-d^2}}$$

input
```
int((a + b*tanh(c + d*x)^3)/sinh(c + d*x)^4,x)
```

output
```
(2*b)/(d*(exp(2*c + 2*d*x) + 1)) - (4*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c
+ 2*d*x) + 1)) - (2*b)/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (
8*a)/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)
) - (2*atan((b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(b^2)^(1/2)))*(b^2)^(1
/2))/(-d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 552, normalized size of antiderivative = 9.86

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)\,dx$$
$$= \frac{4a-6b+3\log\left(e^{2dx+2c}+1\right)b-3e^{2dx+2c}\log\left(e^{2dx+2c}+1\right)b+6e^{10dx+10c}b+30e^{4dx+4c}b-20e^{4dx+4c}a-30}{}$$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3),x)
```

output
```
( - 3*e**(10*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*b + 3*e**(10*c + 10*d*x
)*log(e**(c + d*x) - 1)*b + 3*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*b +
 6*e**(10*c + 10*d*x)*b + 3*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*b -
 3*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*b - 3*e**(8*c + 8*d*x)*log(e**(c
 + d*x) + 1)*b + 6*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*b - 6*e**(6*
c + 6*d*x)*log(e**(c + d*x) - 1)*b - 6*e**(6*c + 6*d*x)*log(e**(c + d*x) +
 1)*b - 12*e**(6*c + 6*d*x)*a - 30*e**(6*c + 6*d*x)*b - 6*e**(4*c + 4*d*x)
*log(e**(2*c + 2*d*x) + 1)*b + 6*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b
+ 6*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b - 20*e**(4*c + 4*d*x)*a + 30*
e**(4*c + 4*d*x)*b - 3*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*b + 3*e*
*(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b + 3*e**(2*c + 2*d*x)*log(e**(c + d*
x) + 1)*b - 4*e**(2*c + 2*d*x)*a + 3*log(e**(2*c + 2*d*x) + 1)*b - 3*log(e
**(c + d*x) - 1)*b - 3*log(e**(c + d*x) + 1)*b + 4*a - 6*b)/(3*d*(e**(10*c
 + 10*d*x) - e**(8*c + 8*d*x) - 2*e**(6*c + 6*d*x) + 2*e**(4*c + 4*d*x) +
e**(2*c + 2*d*x) - 1))
```

3.57 $\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 182

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{3}{8}\left(a^2+21b^2\right)x - \frac{3ab\cosh^2(c+dx)}{d} + \frac{ab\cosh^4(c+dx)}{2d} + \frac{6ab\log(\cosh(c+dx))}{d} - \frac{(5a^2+17b^2)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a^2+b^2)\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{6b^2\tanh(c+dx)}{d} - \frac{ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
3/8*(a^2+21*b^2)*x-3*a*b*cosh(d*x+c)^2/d+1/2*a*b*cosh(d*x+c)^4/d+6*a*b*ln(
cosh(d*x+c))/d-1/8*(5*a^2+17*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a^2+b^2)*
cosh(d*x+c)^3*sinh(d*x+c)/d-6*b^2*tanh(d*x+c)/d-a*b*tanh(d*x+c)^2/d-b^2*ta
nh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 4.46 (sec), antiderivative size = 156, normalized size of antiderivative = 0.86

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{60(a^2+21b^2)(c+dx)-200ab\cosh(2(c+dx))+10ab\cosh(4(c+dx))+960ab\log(\cosh(c+dx))+16}{}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(60*(a^2 + 21*b^2)*(c + d*x) - 200*a*b*Cosh[2*(c + d*x)] + 10*a*b*Cosh[4*(
c + d*x)] + 960*a*b*Log[Cosh[c + d*x]] + 160*a*b*Sech[c + d*x]^2 - 40*(a^2
 + 4*b^2)*Sinh[2*(c + d*x)] + 5*(a^2 + b^2)*Sinh[4*(c + d*x)] - 1152*b^2*T
anh[c + d*x] + 224*b^2*Sech[c + d*x]^2*Tanh[c + d*x] - 32*b^2*Sech[c + d*x
]^4*Tanh[c + d*x])/(160*d)
```

Rubi [A] (verified)

Time = 0.66 (sec), antiderivative size = 197, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 2335, 25, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sin(ic+idx)^4\left(a+ib\tan(ic+idx)^3\right)^2\,dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^3(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2335$$

$$\frac{\frac{1}{4}\int -\frac{\tanh^3(c+dx)\left(4b^2\tanh^5(c+dx)+4b^2\tanh^3(c+dx)+8ab\tanh^2(c+dx)+(a^2+5b^2)\tanh(c+dx)+8ab\right)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{\tanh^4(c+dx)((}{4(1-}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\tanh^4(c+dx)\left((a^2+b^2)\tanh(c+dx)+2ab\right)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int\frac{\tanh^3(c+dx)\left(4b^2\tanh^5(c+dx)+4b^2\tanh^3(c+dx)+8ab\tanh^2(c+dx)+(a^2+5b^2)\tanh(c+dx)+\right.}{\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 2335

$$\frac{\frac{1}{4}\left(-\frac{1}{2}\int -\frac{\tanh^2(c+dx)\left(8b^2\tanh^4(c+dx)+16b^2\tanh^2(c+dx)+48ab\tanh(c+dx)+3(a^2+13b^2)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{\tanh^3(c+dx)(a^2+16ab}{2(1-\tanh^2}\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int\frac{\tanh^2(c+dx)\left(8b^2\tanh^4(c+dx)+16b^2\tanh^2(c+dx)+48ab\tanh(c+dx)+3(a^2+13b^2)\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)-\frac{\tanh^3(c+dx)(a^2+16ab\tanh}{2(1-\tanh^2(c+d}\right.}{d}$$

$\downarrow$ 2333

$$\frac{\frac{1}{4}\left(\frac{1}{2}\int\left(-8b^2\tanh^4(c+dx)-24b^2\tanh^2(c+dx)-48ab\tanh(c+dx)-3(a^2+21b^2)+\frac{3(a^2+16b\tanh(c+dx)a+21b^2}{1-\tanh^2(c+dx)}\right.\right.}{d}$$

$\downarrow$ 2009

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3(a^2+21b^2)\operatorname{arctanh}(\tanh(c+dx))-3(a^2+21b^2)\tanh(c+dx)-24ab\tanh^2(c+dx)-24ab\log\left(1-\tan\right.\right.\right.}{}$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
((Tanh[c + d*x]^4*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]))/(4*(1 - Tanh[c + d*
x]^2)^2) + (-1/2*(Tanh[c + d*x]^3*(a^2 + 13*b^2 + 16*a*b*Tanh[c + d*x]))/(
1 - Tanh[c + d*x]^2) + (3*(a^2 + 21*b^2)*ArcTanh[Tanh[c + d*x]] - 24*a*b*L
og[1 - Tanh[c + d*x]^2] - 3*(a^2 + 21*b^2)*Tanh[c + d*x] - 24*a*b*Tanh[c +
 d*x]^2 - 8*b^2*Tanh[c + d*x]^3 - (8*b^2*Tanh[c + d*x]^5)/5)/2)/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 16.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.99

method	result
derivativedivides	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\,\mathrm{ta}}{}\right.}{d}$
default	$\frac{a^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\,\mathrm{ta}}{}\right.}{d}$
risch	$\frac{3a^2x}{8}-6abx+\frac{63b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}-\frac{e^{2dx+2c}a^2}{8d}-\frac{5\,e^{2dx+2c}ab}{8d}-\frac{e^{2dx+2c}b^2}{2d}$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*ln
(cosh(d*x+c))-3/2*tanh(d*x+c)^2)+b^2*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^5-9/8*
sinh(d*x+c)^7/cosh(d*x+c)^5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*tanh(d*x
+c)^3-63/40*tanh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5034 vs. $2(172) = 344$.

Time = 0.16 (sec) , antiderivative size = 5034, normalized size of antiderivative = 27.66

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 379 vs. $2(172) = 344$.

Time $= 0.14$ (sec) , antiderivative size $= 379$, normalized size of antiderivative $= 2.08$

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{1}{64}a^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$
$$+\frac{1}{320}b^2\left(\frac{2520\,(dx+c)}{d}+\frac{5\left(32\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}\right)}{d}-\frac{135\,e^{(-2\,dx-2\,c)}+5358\,e^{(-4\,dx-4\,c)}+18190\,e^{(}}{d(e^{(-4\,dx-4\,c)}+5\,e^{(-6\,dx-6\,c)}+10}\right.$$
$$+\frac{1}{32}ab\left(\frac{192\,(dx+c)}{d}-\frac{20\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}+\frac{192\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{18\,e^{(-2\,dx-2\,c)}+39\,e^{(-}}{d(e^{(-4\,dx-4\,c)}+2\,e}\right.$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/320*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x -
4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*x
- 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*x
- 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12*c
) + e^(-14*d*x - 14*c)))) + 1/32*a*b*(192*(d*x + c)/d - (20*e^(-2*d*x - 2*
c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*d*x - 2*c) + 1)/d - (18*e^(-2*d*x
 - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6*d*x - 6*c) - 1)/(d*(e^(-4*d*x -
4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. $2(172) = 344$.

Time = 0.29 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.05

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2 dx = \frac{5\,a^2e^{(4\,dx+4\,c)} + 10\,abe^{(4\,dx+4\,c)} + 5\,b^2e^{(4\,dx+4\,c)} - 40\,a^2e^{(2\,dx+2\,c)} - 200\,abe^{(2\,dx+2\,c)} - 160\,b^2e^{(2\,dx+2\,c)} + 1920}{}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/320*(5*a^2*e^(4*d*x + 4*c) + 10*a*b*e^(4*d*x + 4*c) + 5*b^2*e^(4*d*x + 4
*c) - 40*a^2*e^(2*d*x + 2*c) - 200*a*b*e^(2*d*x + 2*c) - 160*b^2*e^(2*d*x
+ 2*c) + 1920*a*b*log(e^(2*d*x + 2*c) + 1) + 120*(a^2 - 16*a*b + 21*b^2)*(
d*x + c) - 5*(18*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 378*b^2*e
^(4*d*x + 4*c) - 8*a^2*e^(2*d*x + 2*c) + 40*a*b*e^(2*d*x + 2*c) - 32*b^2*e
^(2*d*x + 2*c) + a^2 - 2*a*b + b^2)*e^(-4*d*x - 4*c) - 32*(137*a*b*e^(10*d
*x + 10*c) + 645*a*b*e^(8*d*x + 8*c) - 200*b^2*e^(8*d*x + 8*c) + 1250*a*b*
e^(6*d*x + 6*c) - 600*b^2*e^(6*d*x + 6*c) + 1250*a*b*e^(4*d*x + 4*c) - 840
*b^2*e^(4*d*x + 4*c) + 645*a*b*e^(2*d*x + 2*c) - 520*b^2*e^(2*d*x + 2*c) +
 137*a*b - 144*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.49 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.97

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= x\left(\frac{3a^2}{8}-6ab+\frac{63b^2}{8}\right)+\frac{4(5b^2+ab)}{d\left(e^{2c+2dx}+1\right)}+\frac{e^{-2c-2dx}\left(a^2-5ab+4b^2\right)}{8d}$$
$$-\frac{e^{2c+2dx}\left(a^2+5ab+4b^2\right)}{8d}+\frac{e^{4c+4dx}(a+b)^2}{64d}-\frac{4(5b^2+ab)}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$
$$+\frac{24b^2}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}-\frac{e^{-4c-4dx}(a-b)^2}{64d}$$
$$-\frac{16b^2}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$+\frac{32b^2}{5d\left(5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1\right)}$$
$$+\frac{6ab\ln\left(e^{2c}e^{2dx}+1\right)}{d}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^2,x)
```

output
```
x*((3*a^2)/8 - 6*a*b + (63*b^2)/8) + (4*(a*b + 5*b^2))/(d*(exp(2*c + 2*d*x
) + 1)) + (exp(- 2*c - 2*d*x)*(a^2 - 5*a*b + 4*b^2))/(8*d) - (exp(2*c + 2*
d*x)*(5*a*b + a^2 + 4*b^2))/(8*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d) -
(4*(a*b + 5*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (24*b^
2)/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) -
(exp(- 4*c - 4*d*x)*(a - b)^2)/(64*d) - (16*b^2)/(d*(4*exp(2*c + 2*d*x) +
6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (32*b^2
)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5
*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*a*b*log(exp(2*c)*exp(2*d
*x) + 1))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 894, normalized size of antiderivative = 4.91

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{10e^{18dx+18c}ab-150e^{16dx+16c}ab+2540e^{8dx+8c}ab-88e^{14dx+14c}a^2-1752e^{14dx+14c}b^2+440e^{10dx+10c}a^2+84}{}$$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x)
```

output
```
(5*e**(18*c + 18*d*x)*a**2 + 10*e**(18*c + 18*d*x)*a*b + 5*e**(18*c + 18*d
*x)*b**2 - 15*e**(16*c + 16*d*x)*a**2 - 150*e**(16*c + 16*d*x)*a*b - 135*e
**(16*c + 16*d*x)*b**2 + 1920*e**(14*c + 14*d*x)*log(e**(2*c + 2*d*x) + 1)
*a*b + 120*e**(14*c + 14*d*x)*a**2*d*x - 88*e**(14*c + 14*d*x)*a**2 - 1920
*e**(14*c + 14*d*x)*a*b*d*x - 736*e**(14*c + 14*d*x)*a*b + 2520*e**(14*c +
 14*d*x)*b**2*d*x - 1752*e**(14*c + 14*d*x)*b**2 + 9600*e**(12*c + 12*d*x)
*log(e**(2*c + 2*d*x) + 1)*a*b + 600*e**(12*c + 12*d*x)*a**2*d*x - 9600*e*
*(12*c + 12*d*x)*a*b*d*x + 12600*e**(12*c + 12*d*x)*b**2*d*x + 19200*e**(1
0*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 1200*e**(10*c + 10*d*x)*a**2
*d*x + 440*e**(10*c + 10*d*x)*a**2 - 19200*e**(10*c + 10*d*x)*a*b*d*x + 25
40*e**(10*c + 10*d*x)*a*b + 25200*e**(10*c + 10*d*x)*b**2*d*x + 8400*e**(1
0*c + 10*d*x)*b**2 + 19200*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b
+ 1200*e**(8*c + 8*d*x)*a**2*d*x + 800*e**(8*c + 8*d*x)*a**2 - 19200*e**(8
*c + 8*d*x)*a*b*d*x + 2540*e**(8*c + 8*d*x)*a*b + 25200*e**(8*c + 8*d*x)*b
**2*d*x + 17640*e**(8*c + 8*d*x)*b**2 + 9600*e**(6*c + 6*d*x)*log(e**(2*c
+ 2*d*x) + 1)*a*b + 600*e**(6*c + 6*d*x)*a**2*d*x + 620*e**(6*c + 6*d*x)*a
**2 - 9600*e**(6*c + 6*d*x)*a*b*d*x + 12600*e**(6*c + 6*d*x)*b**2*d*x + 13
020*e**(6*c + 6*d*x)*b**2 + 1920*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1
)*a*b + 120*e**(4*c + 4*d*x)*a**2*d*x + 212*e**(4*c + 4*d*x)*a**2 - 1920*e
**(4*c + 4*d*x)*a*b*d*x - 736*e**(4*c + 4*d*x)*a*b + 2520*e**(4*c + 4*d...
```

3.58 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{5ab\arctan(\sinh(c+dx))}{d}-\frac{a^2\cosh(c+dx)}{d}\\ &\quad-\frac{4b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}\\ &\quad+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{6b^2\text{sech}(c+dx)}{d}\\ &\quad+\frac{4b^2\text{sech}^3(c+dx)}{3d}-\frac{b^2\text{sech}^5(c+dx)}{5d}\\ &\quad-\frac{4ab\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}\\ &\quad-\frac{ab\,\text{sech}(c+dx)\tanh(c+dx)}{d}\end{aligned}$$

output
```
5*a*b*arctan(sinh(d*x+c))/d-a^2*cosh(d*x+c)/d-4*b^2*cosh(d*x+c)/d+1/3*a^2*
cosh(d*x+c)^3/d+1/3*b^2*cosh(d*x+c)^3/d-6*b^2*sech(d*x+c)/d+4/3*b^2*sech(d
*x+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-4*a*b*sinh(d*x+c)/d+2/3*a*b*sinh(d*x+c)^
3/d-a*b*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.39 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.68

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{-45(a^2+5b^2)\cosh(c+dx)+5(a^2+b^2)\cosh(3(c+dx))-2b\left(-40b\operatorname{sech}^3(c+dx)+6b\operatorname{sech}^5(c+dx)-\right.}{}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-45*(a^2 + 5*b^2)*Cosh[c + d*x] + 5*(a^2 + b^2)*Cosh[3*(c + d*x)] - 2*b*(
-40*b*Sech[c + d*x]^3 + 6*b*Sech[c + d*x]^5 - 5*a*(60*ArcTan[Tanh[(c + d*x
)/2]] - 27*Sinh[c + d*x] + Sinh[3*(c + d*x)]) + 30*Sech[c + d*x]*(6*b + a*
Tanh[c + d*x])))/(60*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.50 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int i\sin(ic+idx)^3\left(a+ib\tan(ic+idx)^3\right)^2dx$$
$$\Big\downarrow \text{26}$$
$$i\int \sin(ic+idx)^3\left(ib\tan(ic+idx)^3+a\right)^2dx$$
$$\Big\downarrow \text{4149}$$

$$i\int\left(-ib^2\sinh^3(c+dx)\tanh^6(c+dx)-2iab\sinh^3(c+dx)\tanh^3(c+dx)-ia^2\sinh^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(-\frac{ia^2\cosh^3(c+dx)}{3d}+\frac{ia^2\cosh(c+dx)}{d}-\frac{5iab\arctan(\sinh(c+dx))}{d}-\frac{5iab\sinh^3(c+dx)}{3d}+\frac{5iab\sinh(c+dx)}{d}\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
I*(((-5*I)*a*b*ArcTan[Sinh[c + d*x]])/d + (I*a^2*Cosh[c + d*x])/d + ((4*I)
*b^2*Cosh[c + d*x])/d - ((I/3)*a^2*Cosh[c + d*x]^3)/d - ((I/3)*b^2*Cosh[c
+ d*x]^3)/d + ((6*I)*b^2*Sech[c + d*x])/d - (((4*I)/3)*b^2*Sech[c + d*x]^3
)/d + ((I/5)*b^2*Sech[c + d*x]^5)/d + ((5*I)*a*b*Sinh[c + d*x])/d - (((5*I
)/3)*a*b*Sinh[c + d*x]^3)/d + (I*a*b*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 8.21 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\,\mathrm{arct}\right.}{d}$
default	$\frac{a^2\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}+5\,\mathrm{arct}\right.}{d}$
risch	$\frac{e^{3dx+3c}a^2}{24d}+\frac{e^{3dx+3c}ab}{12d}+\frac{e^{3dx+3c}b^2}{24d}-\frac{3\,e^{dx+c}a^2}{8d}-\frac{9\,e^{dx+c}ab}{4d}-\frac{15\,e^{dx+c}b^2}{8d}-\frac{3\,e^{-dx-c}a^2}{8d}+\frac{9\,e^{-dx-c}ab}{4d}$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(1/3*sinh(d*x+c)^5/cos
h(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5/2
*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c)))+b^2*(1/3*sinh(d*x+c)^8/cosh
(d*x+c)^5-8/3*sinh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+c)^5-6
4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3341 vs. $2(168) = 336$.

Time = 0.15 (sec) , antiderivative size = 3341, normalized size of antiderivative = 18.77

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)^2\sinh^3{(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. 2(168) = 336.

Time = 0.12 (sec) , antiderivative size = 348, normalized size of antiderivative = 1.96

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=$$
$$-\frac{1}{120}b^2\left(\frac{5\left(45\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}\right)}{d}+\frac{200\,e^{(-2\,dx-2\,c)}+2515\,e^{(-4\,dx-4\,c)}+6680\,e^{(-6\,dx-6\,c)}+9073\,e^{(-8}}{d(e^{(-3\,dx-3\,c)}+5\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+10\,e^{(-}}\right.$$
$$+\frac{1}{12}ab\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c}}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c}}\right.$$
$$+\frac{1}{24}a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/120*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x - 2*
c) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x - 8*c)
 + 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d*x -
3*c) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c) + 5*
e^(-11*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/12*a*b*((27*e^(-d*x - c) -
e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) +
77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(
-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e^(d
*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 289, normalized size of antiderivative = 1.62

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{1200\,ab\arctan\left(e^{(dx+c)}\right)+5\,a^2e^{(3\,dx+3\,c)}+10\,abe^{(3\,dx+3\,c)}+5\,b^2e^{(3\,dx+3\,c)}-45\,a^2e^{(dx+c)}-270\,abe^{(dx+c)}-}{}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/120*(1200*a*b*arctan(e^(d*x + c)) + 5*a^2*e^(3*d*x + 3*c) + 10*a*b*e^(3*
d*x + 3*c) + 5*b^2*e^(3*d*x + 3*c) - 45*a^2*e^(d*x + c) - 270*a*b*e^(d*x +
 c) - 225*b^2*e^(d*x + c) - 5*(9*a^2*e^(2*d*x + 2*c) - 54*a*b*e^(2*d*x + 2
*c) + 45*b^2*e^(2*d*x + 2*c) - a^2 + 2*a*b - b^2)*e^(-3*d*x - 3*c) - 16*(1
5*a*b*e^(9*d*x + 9*c) + 90*b^2*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) +
280*b^2*e^(7*d*x + 7*c) + 428*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c)
 + 280*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c) + 90*b^2*e^(d*x + c))/(e^(
2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 397, normalized size of antiderivative = 2.23

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{e^{3\,c+3\,d\,x}\,(a+b)^2}{24\,d}-\frac{e^{c+d\,x}\,(3\,a^2+18\,a\,b+15\,b^2)}{8\,d}$$
$$-\frac{e^{-c-d\,x}\,(3\,a^2-18\,a\,b+15\,b^2)}{8\,d}+\frac{e^{-3\,c-3\,d\,x}\,(a-b)^2}{24\,d}$$
$$+\frac{10\,\mathrm{atan}\left(\frac{a\,b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{d^2}}-\frac{256\,b^2\,e^{c+d\,x}}{15\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$+\frac{64\,b^2\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$-\frac{32\,b^2\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$-\frac{2\,e^{c+d\,x}\,(6\,b^2+a\,b)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{4\,e^{c+d\,x}\,(8\,b^2+3\,a\,b)}{3\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^2,x)
```

output
```
(exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(c + d*x)*(18*a*b + 3*a^2 + 15*b
^2))/(8*d) - (exp(- c - d*x)*(3*a^2 - 18*a*b + 15*b^2))/(8*d) + (exp(- 3*c
 - 3*d*x)*(a - b)^2)/(24*d) + (10*atan((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(
d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/(d^2)^(1/2) - (256*b^2*exp(c + d*x))/
(15*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) +
(64*b^2*exp(c + d*x))/(5*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*ex
p(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2*exp(c + d*x))/(5*d*(5*ex
p(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8
*d*x) + exp(10*c + 10*d*x) + 1)) - (2*exp(c + d*x)*(a*b + 6*b^2))/(d*(exp(
2*c + 2*d*x) + 1)) + (4*exp(c + d*x)*(3*a*b + 8*b^2))/(3*d*(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 564, normalized size of antiderivative = 3.17

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{10e^{16dx+16c}ab - 20e^{14dx+14c}a^2 - 200e^{14dx+14c}b^2 - 220e^{12dx+12c}a^2 - 2740e^{12dx+12c}b^2 - 620e^{10dx+10c}a^2 - 78}{}$$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x)
```

output
```
(1200*e**(13*c + 13*d*x)*atan(e**(c + d*x))*a*b + 6000*e**(11*c + 11*d*x)*
atan(e**(c + d*x))*a*b + 12000*e**(9*c + 9*d*x)*atan(e**(c + d*x))*a*b + 1
2000*e**(7*c + 7*d*x)*atan(e**(c + d*x))*a*b + 6000*e**(5*c + 5*d*x)*atan(
e**(c + d*x))*a*b + 1200*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a*b + 5*e**(1
6*c + 16*d*x)*a**2 + 10*e**(16*c + 16*d*x)*a*b + 5*e**(16*c + 16*d*x)*b**2
 - 20*e**(14*c + 14*d*x)*a**2 - 220*e**(14*c + 14*d*x)*a*b - 200*e**(14*c
+ 14*d*x)*b**2 - 220*e**(12*c + 12*d*x)*a**2 - 1220*e**(12*c + 12*d*x)*a*b
 - 2740*e**(12*c + 12*d*x)*b**2 - 620*e**(10*c + 10*d*x)*a**2 - 1740*e**(1
0*c + 10*d*x)*a*b - 7800*e**(10*c + 10*d*x)*b**2 - 850*e**(8*c + 8*d*x)*a*
*2 - 11298*e**(8*c + 8*d*x)*b**2 - 620*e**(6*c + 6*d*x)*a**2 + 1740*e**(6*
c + 6*d*x)*a*b - 7800*e**(6*c + 6*d*x)*b**2 - 220*e**(4*c + 4*d*x)*a**2 +
1220*e**(4*c + 4*d*x)*a*b - 2740*e**(4*c + 4*d*x)*b**2 - 20*e**(2*c + 2*d*
x)*a**2 + 220*e**(2*c + 2*d*x)*a*b - 200*e**(2*c + 2*d*x)*b**2 + 5*a**2 -
10*a*b + 5*b**2)/(120*e**(3*c + 3*d*x)*d*(e**(10*c + 10*d*x) + 5*e**(8*c +
 8*d*x) + 10*e**(6*c + 6*d*x) + 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) +
 1))
```

3.59 $\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 133

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= -\frac{1}{2}\left(a^2+7b^2\right)x+\frac{ab\cosh^2(c+dx)}{d} \\ &\quad -\frac{4ab\log(\cosh(c+dx))}{d} \\ &\quad +\frac{\left(a^2+b^2\right)\cosh(c+dx)\sinh(c+dx)}{2d} \\ &\quad +\frac{3b^2\tanh(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d} \\ &\quad +\frac{2b^2\tanh^3(c+dx)}{3d}+\frac{b^2\tanh^5(c+dx)}{5d}\end{aligned}$$

output
```
-1/2*(a^2+7*b^2)*x+a*b*cosh(d*x+c)^2/d-4*a*b*ln(cosh(d*x+c))/d+1/2*(a^2+b^
2)*cosh(d*x+c)*sinh(d*x+c)/d+3*b^2*tanh(d*x+c)/d+a*b*tanh(d*x+c)^2/d+2/3*b
^2*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 1.40 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.03

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{-30a^2c-210b^2c-30a^2dx-210b^2dx+30ab\cosh(2(c+dx))-240ab\log(\cosh(c+dx))+15a^2\sinh(2(}{}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-30*a^2*c - 210*b^2*c - 30*a^2*d*x - 210*b^2*d*x + 30*a*b*Cosh[2*(c + d*x
)] - 240*a*b*Log[Cosh[c + d*x]] + 15*a^2*Sinh[2*(c + d*x)] + 15*b^2*Sinh[2
*(c + d*x)] + 232*b^2*Tanh[c + d*x] + 12*b^2*Sech[c + d*x]^4*Tanh[c + d*x]
 - 4*b*Sech[c + d*x]^2*(15*a + 16*b*Tanh[c + d*x]))/(60*d)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.11, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4146, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\sin(ic+idx)^2\left(a+ib\tan(ic+idx)^3\right)^2dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \sin(ic+idx)^2\left(ib\tan(ic+idx)^3+a\right)^2dx$$

$$\Big\downarrow \text{ 4146}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 2335

$$\frac{\frac{1}{2}\int -\frac{\tanh(c+dx)\left(2b^2\tanh^5(c+dx)+2b^2\tanh^3(c+dx)+4ab\tanh^2(c+dx)+\left(a^2+3b^2\right)\tanh(c+dx)+4ab\right)}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh^2(c+dx)\left(\left(\right.\right.}{2\left(1-\right.}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\tanh^2(c+dx)\left(\left(a^2+b^2\right)\tanh(c+dx)+2ab\right)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{\tanh(c+dx)\left(2b^2\tanh^5(c+dx)+2b^2\tanh^3(c+dx)+4ab\tanh^2(c+dx)+\left(a^2+3b^2\right)\tanh(c+dx)+\right.}{1-\tanh^2(c+dx)}}{d}$$

$\Big\downarrow$ 2333

$$\frac{\frac{\tanh^2(c+dx)\left(\left(a^2+b^2\right)\tanh(c+dx)+2ab\right)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \left(-2b^2\tanh^4(c+dx)-4b^2\tanh^2(c+dx)-4ab\tanh(c+dx)-a^2-7b^2\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{\frac{1}{2}\left(-\left(a^2+7b^2\right)\text{arctanh}(\tanh(c+dx))+\left(a^2+7b^2\right)\tanh(c+dx)+2ab\tanh^2(c+dx)+4ab\log\left(1-\tanh^2(c+\right.\right.}{d}$$

input

```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output

```
((Tanh[c + d*x]^2*(2*a*b + (a^2 + b^2)*Tanh[c + d*x]))/(2*(1 - Tanh[c + d*
x]^2)) + (-((a^2 + 7*b^2)*ArcTanh[Tanh[c + d*x]]) + 4*a*b*Log[1 - Tanh[c +
 d*x]^2] + (a^2 + 7*b^2)*Tanh[c + d*x] + 2*a*b*Tanh[c + d*x]^2 + (4*b^2*Ta
nh[c + d*x]^3)/3 + (2*b^2*Tanh[c + d*x]^5)/5)/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 4.52 (sec) , antiderivative size = 130, normalized size of antiderivative = 0.98

method	result
derivativedivides	$\frac{a^2(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2})+2ab(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2)+b^2(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}-}{d}$
default	$\frac{a^2(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2})+2ab(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2)+b^2(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7dx}{2}-}{d}$
risch	$-\frac{a^2x}{2}+4abx-\frac{7b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2}}{8d}$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+2*a*b*(1/2*sinh(d*x+c
)^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+b^2*(1/2*sinh(d*x+c)^7/
cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*tanh(d*
x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3649 vs. 2(125) = 250.

Time = 0.13 (sec) , antiderivative size = 3649, normalized size of antiderivative = 27.44

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)^2\sinh^2\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. 2(125) = 250.

Time = 0.13 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.26

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{1}{8}\,a^2\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$
$$-\frac{1}{120}\,b^2\left(\frac{420\,(dx+c)}{d}+\frac{15\,e^{(-2\,dx-2\,c)}}{d}-\frac{1003\,e^{(-2\,dx-2\,c)}+3350\,e^{(-4\,dx-4\,c)}+5590\,e^{(-6\,dx-6\,c)}+3915\,e^{(}}{d(e^{(-2\,dx-2\,c)}+5\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+10\,e^{(-8\,dx-8\,c)}}\right.$$
$$-\frac{1}{4}\,ab\left(\frac{16\,(dx+c)}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}+\frac{16\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}-\frac{2\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+1}{d(e^{(-2\,dx-2\,c)}+2\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)})}\right.$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/120*b^2*(420*(
d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*d
*x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d*
x - 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))
 - 1/4*a*b*(16*(d*x + c)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c)
+ 1)/d - (2*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*
c) + 2*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(125) = 250$.

Time = 0.24 (sec) , antiderivative size = 296, normalized size of antiderivative = 2.23

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{15\,a^2e^{(2\,dx+2\,c)}+30\,abe^{(2\,dx+2\,c)}+15\,b^2e^{(2\,dx+2\,c)}-480\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)-60\,(a^2-8\,ab+7\,b^2)(dx+}{}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/120*(15*a^2*e^(2*d*x + 2*c) + 30*a*b*e^(2*d*x + 2*c) + 15*b^2*e^(2*d*x +
 2*c) - 480*a*b*log(e^(2*d*x + 2*c) + 1) - 60*(a^2 - 8*a*b + 7*b^2)*(d*x +
 c) + 15*(2*a^2*e^(2*d*x + 2*c) - 16*a*b*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x
 + 2*c) - a^2 + 2*a*b - b^2)*e^(-2*d*x - 2*c) + 8*(137*a*b*e^(10*d*x + 10*
c) + 625*a*b*e^(8*d*x + 8*c) - 180*b^2*e^(8*d*x + 8*c) + 1190*a*b*e^(6*d*x
 + 6*c) - 480*b^2*e^(6*d*x + 6*c) + 1190*a*b*e^(4*d*x + 4*c) - 680*b^2*e^(
4*d*x + 4*c) + 625*a*b*e^(2*d*x + 2*c) - 400*b^2*e^(2*d*x + 2*c) + 137*a*b
 - 116*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.30

$$\begin{aligned}
\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{e^{2\,c+2\,d\,x}\,(a+b)^2}{8\,d}-\frac{4\,(3\,b^2+a\,b)}{d\,(e^{2\,c+2\,d\,x}+1)}-x\left(\frac{a^2}{2}-4\,a\,b+\frac{7\,b^2}{2}\right)\\
&+\frac{4\,(4\,b^2+a\,b)}{d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)}-\frac{64\,b^2}{3\,d\,(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1)}\\
&-\frac{e^{-2\,c-2\,d\,x}\,(a-b)^2}{8\,d}+\frac{16\,b^2}{d\,(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1)}\\
&-\frac{32\,b^2}{5\,d\,(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1)}\\
&-\frac{4\,a\,b\,\ln\left(e^{2\,c}\,e^{2\,d\,x}+1\right)}{d}
\end{aligned}$$

input
```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^2,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (4*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x
) + 1)) - x*(a^2/2 - 4*a*b + (7*b^2)/2) + (4*(a*b + 4*b^2))/(d*(2*exp(2*c
+ 2*d*x) + exp(4*c + 4*d*x) + 1)) - (64*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*
exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a - b)^2)
/(8*d) + (16*b^2)/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c
+ 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10
*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c +
10*d*x) + 1)) - (4*a*b*log(exp(2*c)*exp(2*d*x) + 1))/d
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 806, normalized size of antiderivative = 6.06

$$\int \sinh^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 dx = \frac{-690e^{8dx+8c}ab + 15e^{14dx+14c}a^2 + 15e^{14dx+14c}b^2 + 48e^{12dx+12c}a^2 + 336e^{12dx+12c}b^2 - 60e^{2dx+2c}a^2dx - 300e}{}$$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x)
```

output

```
(15*e**(14*c + 14*d*x)*a**2 + 30*e**(14*c + 14*d*x)*a*b + 15*e**(14*c + 14
*d*x)*b**2 - 480*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b - 60*e**
(12*c + 12*d*x)*a**2*d*x + 48*e**(12*c + 12*d*x)*a**2 + 480*e**(12*c + 12*
d*x)*a*b*d*x + 180*e**(12*c + 12*d*x)*a*b - 420*e**(12*c + 12*d*x)*b**2*d*
x + 336*e**(12*c + 12*d*x)*b**2 - 2400*e**(10*c + 10*d*x)*log(e**(2*c + 2*
d*x) + 1)*a*b - 300*e**(10*c + 10*d*x)*a**2*d*x + 2400*e**(10*c + 10*d*x)*
a*b*d*x - 2100*e**(10*c + 10*d*x)*b**2*d*x - 4800*e**(8*c + 8*d*x)*log(e**
(2*c + 2*d*x) + 1)*a*b - 600*e**(8*c + 8*d*x)*a**2*d*x - 195*e**(8*c + 8*d
*x)*a**2 + 4800*e**(8*c + 8*d*x)*a*b*d*x - 690*e**(8*c + 8*d*x)*a*b - 4200
*e**(8*c + 8*d*x)*b**2*d*x - 1155*e**(8*c + 8*d*x)*b**2 - 4800*e**(6*c + 6
*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b - 600*e**(6*c + 6*d*x)*a**2*d*x - 345*
e**(6*c + 6*d*x)*a**2 + 4800*e**(6*c + 6*d*x)*a*b*d*x - 690*e**(6*c + 6*d*
x)*a*b - 4200*e**(6*c + 6*d*x)*b**2*d*x - 2905*e**(6*c + 6*d*x)*b**2 - 240
0*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b - 300*e**(4*c + 4*d*x)*a*
*2*d*x - 270*e**(4*c + 4*d*x)*a**2 + 2400*e**(4*c + 4*d*x)*a*b*d*x - 2100*
e**(4*c + 4*d*x)*b**2*d*x - 2030*e**(4*c + 4*d*x)*b**2 - 480*e**(2*c + 2*d
*x)*log(e**(2*c + 2*d*x) + 1)*a*b - 60*e**(2*c + 2*d*x)*a**2*d*x - 102*e**
(2*c + 2*d*x)*a**2 + 480*e**(2*c + 2*d*x)*a*b*d*x + 180*e**(2*c + 2*d*x)*a
*b - 420*e**(2*c + 2*d*x)*b**2*d*x - 742*e**(2*c + 2*d*x)*b**2 - 15*a**2 +
 30*a*b - 15*b**2)/(120*e**(2*c + 2*d*x)*d*(e**(10*c + 10*d*x) + 5*e**(...
```

3.60 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 120

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{3ab\arctan(\sinh(c+dx))}{d} + \frac{a^2\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)}{d} + \frac{3b^2\text{sech}(c+dx)}{d} - \frac{b^2\text{sech}^3(c+dx)}{d} + \frac{b^2\text{sech}^5(c+dx)}{5d} + \frac{2ab\sinh(c+dx)}{d} + \frac{ab\text{sech}(c+dx)\tanh(c+dx)}{d}$$

output
```
-3*a*b*arctan(sinh(d*x+c))/d+a^2*cosh(d*x+c)/d+b^2*cosh(d*x+c)/d+3*b^2*sec
h(d*x+c)/d-b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+2*a*b*sinh(d*x+c)/d
+a*b*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.96 (sec) , antiderivative size = 90, normalized size of antiderivative = 0.75

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{5(a^2+b^2)\cosh(c+dx)+b\left(-5b\operatorname{sech}^3(c+dx)+b\operatorname{sech}^5(c+dx)+10a\left(-3\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+\right.\right.}{5d}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(5*(a^2 + b^2)*Cosh[c + d*x] + b*(-5*b*Sech[c + d*x]^3 + b*Sech[c + d*x]^5
 + 10*a*(-3*ArcTan[Tanh[(c + d*x)/2]] + Sinh[c + d*x]) + 5*Sech[c + d*x]*(
3*b + a*Tanh[c + d*x])))/(5*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.64 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -i\sin(ic+idx)\left(a+ib\tan(ic+idx)^3\right)^2dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \sin(ic+idx)\left(ib\tan(ic+idx)^3+a\right)^2dx$$
$$\Big\downarrow \text{ 4149}$$

$$-i\int \left(ib^2\sinh(c+dx)\tanh^6(c+dx)+2iab\sinh(c+dx)\tanh^3(c+dx)+ia^2\sinh(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia^2\cosh(c+dx)}{d}-\frac{3iab\arctan(\sinh(c+dx))}{d}+\frac{3iab\sinh(c+dx)}{d}-\frac{iab\sinh(c+dx)\tanh^2(c+dx)}{d}+\frac{ib^2\mathrm{c}}{}\right.$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-I)*(((-3*I)*a*b*ArcTan[Sinh[c + d*x]])/d + (I*a^2*Cosh[c + d*x])/d + (I*
b^2*Cosh[c + d*x])/d + ((3*I)*b^2*Sech[c + d*x])/d - (I*b^2*Sech[c + d*x]^
3)/d + ((I/5)*b^2*Sech[c + d*x]^5)/d + ((3*I)*a*b*Sinh[c + d*x])/d - (I*a*
b*Sinh[c + d*x]*Tanh[c + d*x]^2)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 3.05 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.21

method	result
derivativedivides	$\frac{a^{2}\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^{3}}{\cosh(dx+c)^{2}}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^{2}}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}\left(\frac{\sinh(dx+c)^{6}}{\cosh(dx+c)^{5}}+\frac{6\sinh}{\cosh(}\right.}{d}$
default	$\frac{a^{2}\cosh(dx+c)+2ab\left(\frac{\sinh(dx+c)^{3}}{\cosh(dx+c)^{2}}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^{2}}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^{2}\left(\frac{\sinh(dx+c)^{6}}{\cosh(dx+c)^{5}}+\frac{6\sinh}{\cosh(}\right.}{d}$
risch	$\frac{\mathrm{e}^{dx+c}a^{2}}{2d}+\frac{\mathrm{e}^{dx+c}ab}{d}+\frac{\mathrm{e}^{dx+c}b^{2}}{2d}+\frac{\mathrm{e}^{-dx-c}a^{2}}{2d}-\frac{\mathrm{e}^{-dx-c}ab}{d}+\frac{\mathrm{e}^{-dx-c}b^{2}}{2d}+\frac{2b\,\mathrm{e}^{dx+c}\left(5\,\mathrm{e}^{8dx+8c}a+15\,\mathrm{e}^{8dx+8c}b+\right.}{}$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*cosh(d*x+c)+2*a*b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh
(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c)))+b^2*(sinh(d*x+
c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sinh(d*x+c)^2/cosh(d*x+
c)^5+16/5/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2230 vs. 2(118) = 236.

Time = 0.12 (sec) , antiderivative size = 2230, normalized size of antiderivative = 18.58

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/10*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^12 + 60*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)*sinh(d*x + c)^11 + 5*(a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 30*(
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^10 + 30*(11*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^2 + a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^10 + 100*(11*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c
)^9 + 5*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^8 + 5*(495*(a^2 + 2*a*b +
 b^2)*cosh(d*x + c)^4 + 270*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 15*a^2
 + 18*a*b + 47*b^2)*sinh(d*x + c)^8 + 40*(99*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^5 + 90*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + (15*a^2 + 18*a*b + 47*
b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 4*(25*a^2 + 91*b^2)*cosh(d*x + c)^6
+ 4*(1155*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 1575*(a^2 + 2*a*b + 3*b^2)
*cosh(d*x + c)^4 + 35*(15*a^2 + 18*a*b + 47*b^2)*cosh(d*x + c)^2 + 25*a^2
+ 91*b^2)*sinh(d*x + c)^6 + 8*(495*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 9
45*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 35*(15*a^2 + 18*a*b + 47*b^2)*c
osh(d*x + c)^3 + 3*(25*a^2 + 91*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(1
5*a^2 - 18*a*b + 47*b^2)*cosh(d*x + c)^4 + 5*(495*(a^2 + 2*a*b + b^2)*cosh
(d*x + c)^8 + 1260*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^6 + 70*(15*a^2 + 18
*a*b + 47*b^2)*cosh(d*x + c)^4 + 12*(25*a^2 + 91*b^2)*cosh(d*x + c)^2 + 15
*a^2 - 18*a*b + 47*b^2)*sinh(d*x + c)^4 + 20*(55*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^9 + 180*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 14*(15*a^2 + 1...
```

Sympy [F]

$$\int \sinh(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 \, dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)^2 \sinh\left(c+dx\right) dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*sinh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(118) = 236$.

Time = 0.12 (sec) , antiderivative size = 253, normalized size of antiderivative = 2.11

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= ab\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)$$
$$+\frac{1}{10}b^2\left(\frac{5\,e^{(-dx-c)}}{d}+\frac{85\,e^{(-2\,dx-2\,c)}+210\,e^{(-4\,dx-4\,c)}+314\,e^{(-6\,dx-6\,c)}+185\,e^{(-8\,dx-8\,c)}+65\,e^{(-10\,dx-10\,c)}}{d(e^{(-dx-c)}+5\,e^{(-3\,dx-3\,c)}+10\,e^{(-5\,dx-5\,c)}+10\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,dx-9\,c)}+e^{(-11\,dx-}}\right.$$
$$+\frac{a^2\cosh(dx+c)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
a*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(
-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c
)))) + 1/10*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x - 2*c) + 210*e^(-4*d*x -
 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*c) + 65*e^(-10*d*x - 10*c
) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10*e^(-5*d*x - 5*c) + 10*e^
(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x - 11*c)))) + a^2*cosh(d*x
 + c)/d
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.69

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx =$$
$$-\frac{60\,ab\arctan\left(e^{(dx+c)}\right)-5\,a^2e^{(dx+c)}-10\,abe^{(dx+c)}-5\,b^2e^{(dx+c)}-5\,(a^2-2\,ab+b^2)e^{(-dx-c)}-\frac{4\,(5\,abe^{(9\,dx}}{}}{10\,}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/10*(60*a*b*arctan(e^(d*x + c)) - 5*a^2*e^(d*x + c) - 10*a*b*e^(d*x + c)
 - 5*b^2*e^(d*x + c) - 5*(a^2 - 2*a*b + b^2)*e^(-d*x - c) - 4*(5*a*b*e^(9*
d*x + 9*c) + 15*b^2*e^(9*d*x + 9*c) + 10*a*b*e^(7*d*x + 7*c) + 40*b^2*e^(7
*d*x + 7*c) + 66*b^2*e^(5*d*x + 5*c) - 10*a*b*e^(3*d*x + 3*c) + 40*b^2*e^(
3*d*x + 3*c) - 5*a*b*e^(d*x + c) + 15*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) +
1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.82

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{e^{c+dx}(a+b)^2}{2d} + \frac{e^{-c-dx}(a-b)^2}{2d} - \frac{6\,\mathrm{atan}\left(\frac{a\,b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{a^2\,b^2}}\right)\sqrt{a^2\,b^2}}{\sqrt{d^2}}$$
$$+ \frac{72\,b^2\,e^{c+d\,x}}{5\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$- \frac{64\,b^2\,e^{c+d\,x}}{5\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+ \frac{32\,b^2\,e^{c+d\,x}}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+ \frac{2\,e^{c+d\,x}\left(3\,b^2+a\,b\right)}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{4\,e^{c+d\,x}\left(2\,b^2+a\,b\right)}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^2,x)
```

output
```
(exp(c + d*x)*(a + b)^2)/(2*d) + (exp(- c - d*x)*(a - b)^2)/(2*d) - (6*ata
n((a*b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2)^(1/2))/
(d^2)^(1/2) + (72*b^2*exp(c + d*x))/(5*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c +
 4*d*x) + exp(6*c + 6*d*x) + 1)) - (64*b^2*exp(c + d*x))/(5*d*(4*exp(2*c +
 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
 + (32*b^2*exp(c + d*x))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) +
10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (2*e
xp(c + d*x)*(a*b + 3*b^2))/(d*(exp(2*c + 2*d*x) + 1)) - (4*exp(c + d*x)*(a
*b + 2*b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 611, normalized size of antiderivative = 5.09

$$\int \sinh(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 \, dx$$
$$= \frac{-480e^{dx+c}atan\left(e^{dx+c}\right)ab-4e^{3dx+3c}\cosh\left(dx+c\right)\tanh\left(dx+c\right)^4 b^2-4e^{dx+c}\cosh\left(dx+c\right)\tanh\left(dx+c\right)}{}$$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x)
```

output
```
( - 480*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a*b - 480*e**(c + d*x)*atan(e*
*(c + d*x))*a*b - 4*e**(3*c + 3*d*x)*cosh(c + d*x)*tanh(c + d*x)**4*b**2 -
 22*e**(3*c + 3*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*b**2 - 80*e**(3*c + 3*
d*x)*cosh(c + d*x)*tanh(c + d*x)*a*b + 80*e**(3*c + 3*d*x)*cosh(c + d*x)*a
**2 - 44*e**(3*c + 3*d*x)*cosh(c + d*x)*b**2 - 4*e**(c + d*x)*cosh(c + d*x
)*tanh(c + d*x)**4*b**2 - 22*e**(c + d*x)*cosh(c + d*x)*tanh(c + d*x)**2*b
**2 - 80*e**(c + d*x)*cosh(c + d*x)*tanh(c + d*x)*a*b + 80*e**(c + d*x)*co
sh(c + d*x)*a**2 - 44*e**(c + d*x)*cosh(c + d*x)*b**2 + 120*e**(4*c + 4*d*
x)*a*b + 75*e**(4*c + 4*d*x)*b**2 - 16*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh
(c + d*x)**5*b**2 - 28*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)**3*b**
2 - 80*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)**2*a*b + 44*e**(3*c +
3*d*x)*sinh(c + d*x)*tanh(c + d*x)*b**2 + 80*e**(3*c + 3*d*x)*sinh(c + d*x
)*a*b + 450*e**(2*c + 2*d*x)*b**2 - 16*e**(c + d*x)*sinh(c + d*x)*tanh(c +
 d*x)**5*b**2 - 28*e**(c + d*x)*sinh(c + d*x)*tanh(c + d*x)**3*b**2 - 80*e
**(c + d*x)*sinh(c + d*x)*tanh(c + d*x)**2*a*b + 44*e**(c + d*x)*sinh(c +
d*x)*tanh(c + d*x)*b**2 + 80*e**(c + d*x)*sinh(c + d*x)*a*b - 120*a*b + 75
*b**2)/(80*e**(c + d*x)*d*(e**(2*c + 2*d*x) + 1))
```

3.61 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 98

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{ab\arctan(\sinh(c+dx))}{d}-\frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{b^2\operatorname{sech}(c+dx)}{d}\\&\quad+\frac{2b^2\operatorname{sech}^3(c+dx)}{3d}-\frac{b^2\operatorname{sech}^5(c+dx)}{5d}-\frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}\end{aligned}$$

output
```
a*b*arctan(sinh(d*x+c))/d-a^2*arctanh(cosh(d*x+c))/d-b^2*sech(d*x+c)/d+2/3
*b^2*sech(d*x+c)^3/d-1/5*b^2*sech(d*x+c)^5/d-a*b*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.71 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.28

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx &= \frac{2ab\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\&\quad+\frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{b^2\operatorname{sech}(c+dx)}{d}+\frac{2b^2\operatorname{sech}^3(c+dx)}{3d}\\&\quad-\frac{b^2\operatorname{sech}^5(c+dx)}{5d}-\frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}\end{aligned}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Log[Cosh[(c + d*x)/2]])/d + (a^
2*Log[Sinh[(c + d*x)/2]])/d - (b^2*Sech[c + d*x])/d + (2*b^2*Sech[c + d*x]
^3)/(3*d) - (b^2*Sech[c + d*x]^5)/(5*d) - (a*b*Sech[c + d*x]*Tanh[c + d*x]
)/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.63 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(ib\tan(ic+idx)^3+a\right)^2}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4149}$$

$$i\int \left(-ib^2\text{sech}(c+dx)\tanh^5(c+dx)-2iab\text{sech}(c+dx)\tanh^2(c+dx)-ia^2\text{csch}(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{ia^2\text{arctanh}(\cosh(c+dx))}{d}-\frac{iab\arctan(\sinh(c+dx))}{d}+\frac{iab\tanh(c+dx)\text{sech}(c+dx)}{d}+\frac{ib^2\text{sech}^5(c+dx)}{5d}-\right.$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
I*(((-I)*a*b*ArcTan[Sinh[c + d*x]])/d + (I*a^2*ArcTanh[Cosh[c + d*x]])/d +
 (I*b^2*Sech[c + d*x])/d - (((2*I)/3)*b^2*Sech[c + d*x]^3)/d + ((I/5)*b^2*
Sech[c + d*x]^5)/d + (I*a*b*Sech[c + d*x]*Tanh[c + d*x])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 2.41 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}\right.}{d}$
default	$\frac{-2a^2 \operatorname{arctanh}\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx+c)^2}{3\cosh(dx+c)^5}\right.}{d}$
risch	$-\frac{2b\,e^{dx+c}\left(15\,e^{8dx+8c}a+15\,e^{8dx+8c}b+30\,e^{6dx+6c}a+20\,e^{6dx+6c}b+58b\,e^{4dx+4c}-30\,e^{2dx+2c}a+20\,e^{2dx+2c}b-15a+15b\right)}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech
(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-sinh(d*x+c)^4/cosh(d*x+c)^5-
4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(94) = 188$.

Time = 0.13 (sec) , antiderivative size = 2498, normalized size of antiderivative = 25.49

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-1/15*(30*(a*b + b^2)*cosh(d*x + c)^9 + 270*(a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^8 + 30*(a*b + b^2)*sinh(d*x + c)^9 + 20*(3*a*b + 2*b^2)*cosh(d*x
 + c)^7 + 20*(54*(a*b + b^2)*cosh(d*x + c)^2 + 3*a*b + 2*b^2)*sinh(d*x + c
)^7 + 116*b^2*cosh(d*x + c)^5 + 140*(18*(a*b + b^2)*cosh(d*x + c)^3 + (3*a
*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 4*(945*(a*b + b^2)*cosh(d*x +
 c)^4 + 105*(3*a*b + 2*b^2)*cosh(d*x + c)^2 + 29*b^2)*sinh(d*x + c)^5 + 20
*(189*(a*b + b^2)*cosh(d*x + c)^5 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^3 + 2
9*b^2*cosh(d*x + c))*sinh(d*x + c)^4 - 20*(3*a*b - 2*b^2)*cosh(d*x + c)^3
+ 20*(126*(a*b + b^2)*cosh(d*x + c)^6 + 35*(3*a*b + 2*b^2)*cosh(d*x + c)^4
 + 58*b^2*cosh(d*x + c)^2 - 3*a*b + 2*b^2)*sinh(d*x + c)^3 + 20*(54*(a*b +
 b^2)*cosh(d*x + c)^7 + 21*(3*a*b + 2*b^2)*cosh(d*x + c)^5 + 58*b^2*cosh(d
*x + c)^3 - 3*(3*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 30*(a*b*cos
h(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^1
0 + 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^
8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c)
)*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 + 14*a*b*cosh(d*x + c)^2 +
a*b)*sinh(d*x + c)^6 + 10*a*b*cosh(d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5
+ 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*
a*b*cosh(d*x + c)^6 + 35*a*b*cosh(d*x + c)^4 + 15*a*b*cosh(d*x + c)^2 + a*
b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b*cosh(d*x + c)^7 ...
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)^2\operatorname{csch}{(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.

Time = 0.13 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.56

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$-\frac{2}{15}\,b^2\left(\frac{15\,e^{(-dx-c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{\quad}{d(5\,e^{(-2\,dx-2}}\right.$$
$$+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e
^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - 2/15*b^2*(15*e^(-d*x - c)/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x
 - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1)) + 20*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
 - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
1)) + 15*e^(-9*d*x - 9*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*
log(tanh(1/2*d*x + 1/2*c))/d
```

Giac [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.82

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$= \frac{30\,ab\arctan\left(e^{(dx+c)}\right) - 15\,a^2\log\left(e^{(dx+c)}+1\right) + 15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right) - \frac{2\left(15\,abe^{(9\,dx+9\,c)}+15\,b^2e^{(9\,dx+9\,c)}+30\right.}{}}{15\,d}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/15*(30*a*b*arctan(e^(d*x + c)) - 15*a^2*log(e^(d*x + c) + 1) + 15*a^2*lo
g(abs(e^(d*x + c) - 1)) - 2*(15*a*b*e^(9*d*x + 9*c) + 15*b^2*e^(9*d*x + 9*
c) + 30*a*b*e^(7*d*x + 7*c) + 20*b^2*e^(7*d*x + 7*c) + 58*b^2*e^(5*d*x + 5
*c) - 30*a*b*e^(3*d*x + 3*c) + 20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c)
 + 15*b^2*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.97 (sec) , antiderivative size = 522, normalized size of antiderivative = 5.33

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{a^2\ln\left(32a^6+32a^4b^2-32a^6e^{dx}e^c-32a^4b^2e^{dx}e^c\right)}{d} - \frac{176b^2e^{c+dx}}{15\left(d+3de^{2c+2dx}+3de^{4c+4dx}+de^{6c+6dx}\right)} - \frac{32b^2e^{c+dx}}{5\left(d+5de^{2c+2dx}+10de^{4c+4dx}+10de^{6c+6dx}+5de^{8c+8dx}+de^{10c+10dx}\right)} - \frac{a^2\ln\left(-32a^6-32a^4b^2-32a^6e^{dx}e^c-32a^4b^2e^{dx}e^c\right)}{d} - \frac{2b^2e^{c+dx}}{d+de^{2c+2dx}} + \frac{16b^2e^{c+dx}}{3\left(d+2de^{2c+2dx}+de^{4c+4dx}\right)} + \frac{64b^2e^{c+dx}}{5\left(d+4de^{2c+2dx}+6de^{4c+4dx}+4de^{6c+6dx}+de^{8c+8dx}\right)} - \frac{2abe^{c+dx}}{d+de^{2c+2dx}} + \frac{4abe^{c+dx}}{d+2de^{2c+2dx}+de^{4c+4dx}} - \frac{ab\left(\ln\left(32a^3b^3e^{dx}e^c+32a^5be^{dx}e^c-a^5b\,32\mathrm{i}-a^3b^3\,32\mathrm{i}\right)1\mathrm{i}-\ln\left(32a^3b^3e^{dx}e^c+32a^5be^{dx}e^c+a^5\right.\right.}{d}$$

input
```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x),x)
```

output
```
(a^2*log(32*a^6 + 32*a^4*b^2 - 32*a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x
)*exp(c)))/d - (176*b^2*exp(c + d*x))/(15*(d + 3*d*exp(2*c + 2*d*x) + 3*d*
exp(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) - (32*b^2*exp(c + d*x))/(5*(d + 5*
d*exp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x) + 10*d*exp(6*c + 6*d*x) + 5*d*e
xp(8*c + 8*d*x) + d*exp(10*c + 10*d*x))) - (a^2*log(- 32*a^6 - 32*a^4*b^2
- 32*a^6*exp(d*x)*exp(c) - 32*a^4*b^2*exp(d*x)*exp(c)))/d - (2*b^2*exp(c +
 d*x))/(d + d*exp(2*c + 2*d*x)) + (16*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*
c + 2*d*x) + d*exp(4*c + 4*d*x))) + (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(
2*c + 2*d*x) + 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8
*d*x))) - (2*a*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) + (4*a*b*exp(c + d
*x))/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^
3*exp(d*x)*exp(c) - a^3*b^3*32i - a^5*b*32i + 32*a^5*b*exp(d*x)*exp(c))*1i
 - log(a^5*b*32i + a^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 32*a^5*b*exp
(d*x)*exp(c))*1i))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 595, normalized size of antiderivative = 6.07

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{30e^{10dx+10c}atan\left(e^{dx+c}\right)ab+150e^{8dx+8c}atan\left(e^{dx+c}\right)ab+300e^{6dx+6c}atan\left(e^{dx+c}\right)ab+300e^{4dx+4c}atan\left(e^{dx+}\right.}{}$$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^2,x)
```

output
```
(30*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a*b + 150*e**(8*c + 8*d*x)*atan(
e**(c + d*x))*a*b + 300*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b + 300*e**(
4*c + 4*d*x)*atan(e**(c + d*x))*a*b + 150*e**(2*c + 2*d*x)*atan(e**(c + d*
x))*a*b + 30*atan(e**(c + d*x))*a*b + 15*e**(10*c + 10*d*x)*log(e**(c + d*
x) - 1)*a**2 - 15*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a**2 - 30*e**(9
*c + 9*d*x)*a*b - 30*e**(9*c + 9*d*x)*b**2 + 75*e**(8*c + 8*d*x)*log(e**(c
 + d*x) - 1)*a**2 - 75*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**2 - 60*e*
*(7*c + 7*d*x)*a*b - 40*e**(7*c + 7*d*x)*b**2 + 150*e**(6*c + 6*d*x)*log(e
**(c + d*x) - 1)*a**2 - 150*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**2 -
116*e**(5*c + 5*d*x)*b**2 + 150*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**
2 - 150*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2 + 60*e**(3*c + 3*d*x)*
a*b - 40*e**(3*c + 3*d*x)*b**2 + 75*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)
*a**2 - 75*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2 + 30*e**(c + d*x)*a
*b - 30*e**(c + d*x)*b**2 + 15*log(e**(c + d*x) - 1)*a**2 - 15*log(e**(c +
 d*x) + 1)*a**2)/(15*d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6
*c + 6*d*x) + 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.62 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 47

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
-a^2*coth(d*x+c)/d+a*b*tanh(d*x+c)^2/d+1/5*b^2*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 94, normalized size of antiderivative = 2.00

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=-\frac{a^2\coth(c+dx)}{d}-\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{b^2\tanh(c+dx)}{5d}-\frac{2b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}+\frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
-((a^2*Coth[c + d*x])/d) - (a*b*Sech[c + d*x]^2)/d + (b^2*Tanh[c + d*x])/(
5*d) - (2*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) + (b^2*Sech[c + d*x]^4*
Tanh[c + d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(ib\tan(ic+idx)^3+a\right)^2}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^2d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{802}$$

$$\frac{\int \left(b^2\tanh^4(c+dx)+2ab\tanh(c+dx)+a^2\coth^2(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^2\coth(c+dx)+ab\tanh^2(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-(a^2*Coth[c + d*x]) + a*b*Tanh[c + d*x]^2 + (b^2*Tanh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. $2(45) = 90$.

Time = 8.51 (sec) , antiderivative size = 98, normalized size of antiderivative = 2.09

method	result
derivativedivides	$\frac{-\coth(dx+c)a^2-\frac{ab}{\cosh(dx+c)^2}+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
default	$\frac{-\coth(dx+c)a^2-\frac{ab}{\cosh(dx+c)^2}+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{8}\right)}{d}$
risch	$-\frac{2(5\,e^{10dx+10c}a^2+10\,e^{10dx+10c}ab+5\,e^{10dx+10c}b^2+25\,e^{8dx+8c}a^2+20\,e^{8dx+8c}ab-5\,e^{8dx+8c}b^2+50\,e^{6dx+6c}a^2+10\,e^{6d}}{5d(e^{2dx+2c}+1)^5(e^{2c}}$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^2-a*b/cosh(d*x+c)^2+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c
)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*
x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 518 vs. $2(45) = 90$.

Time = 0.10 (sec) , antiderivative size = 518, normalized size of antiderivative = 11.02

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx =$$

$$-\frac{4\left(\left(5a^2+5ab+2b^2\right)\cosh(dx+c)^5+5\left(5a^2+5ab+2b^2\right)\cosh(dx+c)\sinh(dx+c)^4+\right.}{5\left(d\cosh(dx+c)^7+7d\cosh(dx+c)\sinh(dx+c)^6+d\sinh(dx+c)^7+3d\cosh(dx+c)^5+\left(21\,d\cos\right.\right.}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
-4/5*((5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 5*a*b + 2*b^2)*
cosh(d*x + c)*sinh(d*x + c)^4 + (5*a*b + 3*b^2)*sinh(d*x + c)^5 + (25*a^2
+ 5*a*b - 2*b^2)*cosh(d*x + c)^3 + (10*(5*a*b + 3*b^2)*cosh(d*x + c)^2 + 1
5*a*b - 3*b^2)*sinh(d*x + c)^3 + (10*(5*a^2 + 5*a*b + 2*b^2)*cosh(d*x + c)
^3 + 3*(25*a^2 + 5*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2
 - a*b)*cosh(d*x + c) + (5*(5*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(5*a*b - b^
2)*cosh(d*x + c)^2 + 10*a*b + 10*b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^7 +
7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 3*d*cosh(d*x + c)^
5 + (21*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
+ 3*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + 3*(7*d*cosh(d*x + c
)^5 + 10*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 - 5*d*cosh(d
*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)
^2 + 5*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int \left(a+b\tanh^3{(c+dx)}\right)^2\operatorname{csch}^2{(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(45) = 90$.

Time = 0.04 (sec) , antiderivative size = 256, normalized size of antiderivative = 5.45

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{2}{5}b^2\left(\frac{10\,e^{(-4\,dx-4\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}+\frac{}{d(5\,e^{(-2\,dx-2\,c)}\ldots}\right.$$
$$+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}-1)}-\frac{4\,ab}{d(e^{(dx+c)}+e^{(-dx-c)})^2}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*
e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 2*a^2/(d*(e^(-2*d*x - 2*c) - 1)) - 4*a*b/(
d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(45) = 90$.

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.60

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{2\left(\frac{5a^2}{e^{(2dx+2c)}-1}+\frac{10abe^{(8dx+8c)}+5b^2e^{(8dx+8c)}+30abe^{(6dx+6c)}+30abe^{(4dx+4c)}+10b^2e^{(4dx+4c)}+10abe^{(2dx+2c)}+b^2}{\left(e^{(2dx+2c)}+1\right)^5}\right)}{5d}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-2/5*(5*a^2/(e^(2*d*x + 2*c) - 1) + (10*a*b*e^(8*d*x + 8*c) + 5*b^2*e^(8*d
*x + 8*c) + 30*a*b*e^(6*d*x + 6*c) + 30*a*b*e^(4*d*x + 4*c) + 10*b^2*e^(4*
d*x + 4*c) + 10*a*b*e^(2*d*x + 2*c) + b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 483, normalized size of antiderivative = 10.28

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2\,e^{8\,c+8\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{8\,e^{2\,c+2\,d\,x}\,(b^2+a\,b)}{5\,d}-\frac{2\,(2\,a\,b-b^2)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(a\,b-b^2)}{5\,d}+\frac{12\,b^2\,e^{4\,c+4\,d\,x}}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1}$$
$$-\frac{\frac{2\,b^2}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(a\,b-b^2)}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}-\frac{\frac{2\,(a\,b-b^2)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}$$
$$-\frac{\frac{2\,e^{6\,c+6\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{2\,(b^2+a\,b)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(a\,b-b^2)}{5\,d}+\frac{6\,b^2\,e^{2\,c+2\,d\,x}}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$-\frac{2\,a^2}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{2\,\left(b^2+2\,a\,b\right)}{5\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input

```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^2,x)
```

output

```
- ((2*exp(8*c + 8*d*x)*(2*a*b + b^2))/(5*d) - (8*exp(2*c + 2*d*x)*(a*b + b
^2))/(5*d) - (2*(2*a*b - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a*b - b^2))/(5
*d) + (12*b^2*exp(4*c + 4*d*x))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c +
4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1
) - ((2*b^2)/(5*d) + (2*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) + (4*exp(2*c
 + 2*d*x)*(a*b - b^2))/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1) - ((2*(a*b - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(2*a*b
 + b^2))/(5*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*exp(6*c
+ 6*d*x)*(2*a*b + b^2))/(5*d) - (2*(a*b + b^2))/(5*d) + (6*exp(4*c + 4*d*x
)*(a*b - b^2))/(5*d) + (6*b^2*exp(2*c + 2*d*x))/(5*d))/(4*exp(2*c + 2*d*x)
 + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - (2*a^
2)/(d*(exp(2*c + 2*d*x) - 1)) - (2*(2*a*b + b^2))/(5*d*(exp(2*c + 2*d*x) +
 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 274, normalized size of antiderivative = 5.83

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{5e^{12dx+12c}a^2+10e^{12dx+12c}ab+5e^{12dx+12c}b^2-75e^{8dx+8c}a^2-30e^{8dx+8c}ab+45e^{8dx+8c}b^2-200e^{6dx+6c}a^2-}{10d\left(e^{12dx+12c}+4e^{10dx+10c}+5e^{\cdots}\right.}$$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^2,x)
```

output
```
(5*e**(12*c + 12*d*x)*a**2 + 10*e**(12*c + 12*d*x)*a*b + 5*e**(12*c + 12*d
*x)*b**2 - 75*e**(8*c + 8*d*x)*a**2 - 30*e**(8*c + 8*d*x)*a*b + 45*e**(8*c
 + 8*d*x)*b**2 - 200*e**(6*c + 6*d*x)*a**2 - 40*e**(6*c + 6*d*x)*b**2 - 22
5*e**(4*c + 4*d*x)*a**2 + 30*e**(4*c + 4*d*x)*a*b + 15*e**(4*c + 4*d*x)*b*
*2 - 120*e**(2*c + 2*d*x)*a**2 - 24*e**(2*c + 2*d*x)*b**2 - 25*a**2 - 10*a
*b - b**2)/(10*d*(e**(12*c + 12*d*x) + 4*e**(10*c + 10*d*x) + 5*e**(8*c +
8*d*x) - 5*e**(4*c + 4*d*x) - 4*e**(2*c + 2*d*x) - 1))
```

3.63 $\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 dx = \frac{ab\arctan(\sinh(c+dx))}{d} + \frac{a^2\operatorname{arctanh}(\cosh(c+dx))}{2d} - \frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{b^2\operatorname{sech}^3(c+dx)}{3d} + \frac{b^2\operatorname{sech}^5(c+dx)}{5d} + \frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

output
```
a*b*arctan(sinh(d*x+c))/d+1/2*a^2*arctanh(cosh(d*x+c))/d-1/2*a^2*coth(d*x+
c)*csch(d*x+c)/d-1/3*b^2*sech(d*x+c)^3/d+1/5*b^2*sech(d*x+c)^5/d+a*b*sech(
d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.02 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.49

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{2ab\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{d}-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^2\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{b^2\operatorname{sech}^3(c+dx)}{3d}+\frac{b^2\operatorname{sech}^5(c+dx)}{5d}+\frac{ab\operatorname{sech}(c+dx)\tanh(c+dx)}{d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(2*a*b*ArcTan[Tanh[(c + d*x)/2]])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) + (a
^2*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^2*Log[Sinh[(c + d*x)/2]])/(2*d) - (a
^2*Sech[(c + d*x)/2]^2)/(8*d) - (b^2*Sech[c + d*x]^3)/(3*d) + (b^2*Sech[c
+ d*x]^5)/(5*d) + (a*b*Sech[c + d*x]*Tanh[c + d*x])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.41 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.17, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -\frac{i\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)^3}dx$$
$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\left(ib\tan(ic+idx)^3+a\right)^2}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \text{4149}$$

$$-i\int \left(ia^2\text{csch}^3(c+dx)+2iab\text{sech}^3(c+dx)+ib^2\text{sech}^3(c+dx)\tanh^3(c+dx)\right)dx$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia^2\text{arctanh}(\cosh(c+dx))}{2d}-\frac{ia^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{iab\arctan(\sinh(c+dx))}{d}+\frac{iab\tanh(c+dx)\text{se}}{d}\right.$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(-I)*((I*a*b*ArcTan[Sinh[c + d*x]])/d + ((I/2)*a^2*ArcTanh[Cosh[c + d*x]])
/d - ((I/2)*a^2*Coth[c + d*x]*Csch[c + d*x])/d - ((I/3)*b^2*Sech[c + d*x]^
3)/d + ((I/5)*b^2*Sech[c + d*x]^5)/d + (I*a*b*Sech[c + d*x]*Tanh[c + d*x])
/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 17.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.85

method	result
derivativedivides	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15}\right.}{d}$
default	$\frac{a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+2ab\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}-\frac{}{15}\right.}{d}$
risch	$-\frac{e^{dx+c}\left(15\,e^{12dx+12c}a^2-30\,e^{12dx+12c}ab+90\,e^{10dx+10c}a^2+40\,e^{10dx+10c}b^2+225\,e^{8dx+8c}a^2+90\,e^{8dx+8c}ab-96\,e^{8dx+8}\right.}{15d\left(e^{2dx+2}\right.}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(1/2*sec
h(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-1/3*sinh(d*x+c)^2/cosh(d*x+
c)^5-2/15/cosh(d*x+c)^5))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4642 vs. 2(99) = 198.

Time = 0.16 (sec) , antiderivative size = 4642, normalized size of antiderivative = 43.38

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int\left(a+b\tanh^3(c+dx)\right)^2\operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. $2(99) = 198$.

Time = 0.15 (sec) , antiderivative size = 378, normalized size of antiderivative = 3.53

$$\begin{aligned}&\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx\\&=-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\&\quad+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\\&\quad-\frac{8}{15}\,b^2\left(\frac{5\,e^{(-3\,dx-3\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{}{d(5\,e^{(-2\,dx-}}\right.\end{aligned}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
-2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e
^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 1/2*a^2*(log(e^(-d*x - c) + 1)
/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e
^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 8/15*b^2*(5*e^(-3*d*x - 3*c)/(
d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d
*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) + 1)) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*
x - 10*c) + 1)))
```

Giac [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 1.79

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{60\,ab\arctan\left(e^{(dx+c)}\right)+15\,a^2\log\left(e^{(dx+c)}+1\right)-15\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)-\frac{30\left(a^2e^{(3\,dx+3\,c)}+a^2e^{(dx+c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^2}+\frac{4\,(15}{}}{30\,d}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/30*(60*a*b*arctan(e^(d*x + c)) + 15*a^2*log(e^(d*x + c) + 1) - 15*a^2*lo
g(abs(e^(d*x + c) - 1)) - 30*(a^2*e^(3*d*x + 3*c) + a^2*e^(d*x + c))/(e^(2
*d*x + 2*c) - 1)^2 + 4*(15*a*b*e^(9*d*x + 9*c) + 30*a*b*e^(7*d*x + 7*c) -
20*b^2*e^(7*d*x + 7*c) + 8*b^2*e^(5*d*x + 5*c) - 30*a*b*e^(3*d*x + 3*c) -
20*b^2*e^(3*d*x + 3*c) - 15*a*b*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 4.24 (sec) , antiderivative size = 561, normalized size of antiderivative = 5.24

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{a^2\,\mathrm{e}^{c+d\,x}}{d-d\,\mathrm{e}^{2\,c+2\,d\,x}}+\frac{136\,b^2\,\mathrm{e}^{c+d\,x}}{15\,\left(d+3\,d\,\mathrm{e}^{2\,c+2\,d\,x}+3\,d\,\mathrm{e}^{4\,c+4\,d\,x}+d\,\mathrm{e}^{6\,c+6\,d\,x}\right)}$$
$$+\frac{32\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+5\,d\,\mathrm{e}^{2\,c+2\,d\,x}+10\,d\,\mathrm{e}^{4\,c+4\,d\,x}+10\,d\,\mathrm{e}^{6\,c+6\,d\,x}+5\,d\,\mathrm{e}^{8\,c+8\,d\,x}+d\,\mathrm{e}^{10\,c+10\,d\,x}\right)}$$
$$-\frac{a^2\,\ln\left(4\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-16\,a^4\,b^2-4\,a^6+16\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{2\,d}$$
$$+\frac{a^2\,\ln\left(4\,a^6+16\,a^4\,b^2+4\,a^6\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+16\,a^4\,b^2\,\mathrm{e}^{d\,x}\,\mathrm{e}^c\right)}{2\,d}$$
$$-\frac{2\,a^2\,\mathrm{e}^{c+d\,x}}{d-2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}}-\frac{8\,b^2\,\mathrm{e}^{c+d\,x}}{3\,\left(d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}\right)}$$
$$-\frac{64\,b^2\,\mathrm{e}^{c+d\,x}}{5\,\left(d+4\,d\,\mathrm{e}^{2\,c+2\,d\,x}+6\,d\,\mathrm{e}^{4\,c+4\,d\,x}+4\,d\,\mathrm{e}^{6\,c+6\,d\,x}+d\,\mathrm{e}^{8\,c+8\,d\,x}\right)}$$
$$+\frac{2\,a\,b\,\mathrm{e}^{c+d\,x}}{d+d\,\mathrm{e}^{2\,c+2\,d\,x}}-\frac{4\,a\,b\,\mathrm{e}^{c+d\,x}}{d+2\,d\,\mathrm{e}^{2\,c+2\,d\,x}+d\,\mathrm{e}^{4\,c+4\,d\,x}}$$
$$-\frac{a\,b\,\left(\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+8\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c-a^5\,b\,8\mathrm{i}-a^3\,b^3\,32\mathrm{i}\right)\,1\mathrm{i}-\ln\left(32\,a^3\,b^3\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+8\,a^5\,b\,\mathrm{e}^{d\,x}\,\mathrm{e}^c+a^5\,b\,8\mathrm{i}\right.}{d}$$

input
```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^3,x)
```

output
```
(a^2*exp(c + d*x))/(d - d*exp(2*c + 2*d*x)) + (136*b^2*exp(c + d*x))/(15*(
d + 3*d*exp(2*c + 2*d*x) + 3*d*exp(4*c + 4*d*x) + d*exp(6*c + 6*d*x))) + (
32*b^2*exp(c + d*x))/(5*(d + 5*d*exp(2*c + 2*d*x) + 10*d*exp(4*c + 4*d*x)
+ 10*d*exp(6*c + 6*d*x) + 5*d*exp(8*c + 8*d*x) + d*exp(10*c + 10*d*x))) -
(a^2*log(4*a^6*exp(d*x)*exp(c) - 16*a^4*b^2 - 4*a^6 + 16*a^4*b^2*exp(d*x)*
exp(c)))/(2*d) + (a^2*log(4*a^6 + 16*a^4*b^2 + 4*a^6*exp(d*x)*exp(c) + 16*
a^4*b^2*exp(d*x)*exp(c)))/(2*d) - (2*a^2*exp(c + d*x))/(d - 2*d*exp(2*c +
2*d*x) + d*exp(4*c + 4*d*x)) - (8*b^2*exp(c + d*x))/(3*(d + 2*d*exp(2*c +
2*d*x) + d*exp(4*c + 4*d*x))) - (64*b^2*exp(c + d*x))/(5*(d + 4*d*exp(2*c
+ 2*d*x) + 6*d*exp(4*c + 4*d*x) + 4*d*exp(6*c + 6*d*x) + d*exp(8*c + 8*d*x
))) + (2*a*b*exp(c + d*x))/(d + d*exp(2*c + 2*d*x)) - (4*a*b*exp(c + d*x))
/(d + 2*d*exp(2*c + 2*d*x) + d*exp(4*c + 4*d*x)) - (a*b*(log(32*a^3*b^3*ex
p(d*x)*exp(c) - a^3*b^3*32i - a^5*b*8i + 8*a^5*b*exp(d*x)*exp(c))*1i - log
(a^5*b*8i + a^3*b^3*32i + 32*a^3*b^3*exp(d*x)*exp(c) + 8*a^5*b*exp(d*x)*ex
p(c))*1i))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 864, normalized size of antiderivative = 8.07

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^3(c+dx)\right)^2 dx$$
$$= \frac{60e^{10dx+10c}atan\left(e^{dx+c}\right)ab-300e^{8dx+8c}atan\left(e^{dx+c}\right)ab-300e^{6dx+6c}atan\left(e^{dx+c}\right)ab+60e^{4dx+4c}atan\left(e^{dx+c}\right)}{}$$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^2,x)
```

output
```
(60*e**(14*c + 14*d*x)*atan(e**(c + d*x))*a*b + 180*e**(12*c + 12*d*x)*ata
n(e**(c + d*x))*a*b + 60*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a*b - 300*e
**(8*c + 8*d*x)*atan(e**(c + d*x))*a*b - 300*e**(6*c + 6*d*x)*atan(e**(c +
 d*x))*a*b + 60*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b + 180*e**(2*c + 2*
d*x)*atan(e**(c + d*x))*a*b + 60*atan(e**(c + d*x))*a*b - 15*e**(14*c + 14
*d*x)*log(e**(c + d*x) - 1)*a**2 + 15*e**(14*c + 14*d*x)*log(e**(c + d*x)
+ 1)*a**2 - 30*e**(13*c + 13*d*x)*a**2 + 60*e**(13*c + 13*d*x)*a*b - 45*e*
*(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a**2 + 45*e**(12*c + 12*d*x)*log(e*
*(c + d*x) + 1)*a**2 - 180*e**(11*c + 11*d*x)*a**2 - 80*e**(11*c + 11*d*x)
*b**2 - 15*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a**2 + 15*e**(10*c + 1
0*d*x)*log(e**(c + d*x) + 1)*a**2 - 450*e**(9*c + 9*d*x)*a**2 - 180*e**(9*
c + 9*d*x)*a*b + 192*e**(9*c + 9*d*x)*b**2 + 75*e**(8*c + 8*d*x)*log(e**(c
 + d*x) - 1)*a**2 - 75*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**2 - 600*e
**(7*c + 7*d*x)*a**2 - 224*e**(7*c + 7*d*x)*b**2 + 75*e**(6*c + 6*d*x)*log
(e**(c + d*x) - 1)*a**2 - 75*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a**2 -
 450*e**(5*c + 5*d*x)*a**2 + 180*e**(5*c + 5*d*x)*a*b + 192*e**(5*c + 5*d*
x)*b**2 - 15*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2 + 15*e**(4*c + 4*
d*x)*log(e**(c + d*x) + 1)*a**2 - 180*e**(3*c + 3*d*x)*a**2 - 80*e**(3*c +
 3*d*x)*b**2 - 45*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2 + 45*e**(2*c
 + 2*d*x)*log(e**(c + d*x) + 1)*a**2 - 30*e**(c + d*x)*a**2 - 60*e**(c ...
```

3.64 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}+\frac{2ab\log(\tanh(c+dx))}{d}-\frac{ab\tanh^2(c+dx)}{d}+\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
a^2*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d+2*a*b*ln(tanh(d*x+c))/d-a*b*tanh
(d*x+c)^2/d+1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.52

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{2a^2\coth(c+dx)}{3d}-\frac{a^2\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{2ab\log(\cosh(c+dx))}{d}+\frac{2ab\log(\sinh(c+dx))}{d}+\frac{ab\operatorname{sech}^2(c+dx)}{d}+\frac{2b^2\tanh(c+dx)}{15d}+\frac{b^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d}-\frac{b^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(2*a^2*Coth[c + d*x])/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) -
(2*a*b*Log[Cosh[c + d*x]])/d + (2*a*b*Log[Sinh[c + d*x]])/d + (a*b*Sech[c
+ d*x]^2)/d + (2*b^2*Tanh[c + d*x])/(15*d) + (b^2*Sech[c + d*x]^2*Tanh[c +
 d*x])/(15*d) - (b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4146, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+ib\tan(ic+idx)^3\right)^2}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^3(c+dx)+a\right)^2d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 2333

$$\frac{\int \left(a^2\coth^4(c+dx)-a^2\coth^2(c+dx)+2ab\coth(c+dx)-b^2\tanh^4(c+dx)+b^2\tanh^2(c+dx)-2ab\tanh(c+\right.}{d}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)+a^2\coth(c+dx)-ab\tanh^2(c+dx)+2ab\log(\tanh(c+dx))-\frac{1}{5}b^2\tanh^5(c+dx)+\frac{1}{3}b^2\text{ta}}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
(a^2*Coth[c + d*x] - (a^2*Coth[c + d*x]^3)/3 + 2*a*b*Log[Tanh[c + d*x]] -
a*b*Tanh[c + d*x]^2 + (b^2*Tanh[c + d*x]^3)/3 - (b^2*Tanh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 30.38 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.04

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{s}}{}\right.}{}\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+2ab\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+b^2\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\,\mathrm{s}}{}\right.}{}\right.}{d}$
risch	$-\frac{4\left(-15ab\,\mathrm{e}^{14dx+14c}+15\,\mathrm{e}^{12dx+12c}a^2+15\,\mathrm{e}^{12dx+12c}b^2+70\,\mathrm{e}^{10dx+10c}a^2+45\,\mathrm{e}^{10dx+10c}ab-50\,\mathrm{e}^{10dx+10c}b^2+125\,\mathrm{e}^{8dx+}\right.}{}$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(1/2/cosh(d*x+c)^2+ln(t
anh(d*x+c)))+b^2*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)
^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4125 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 4125, normalized size of antiderivative = 42.53

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx=\int\left(a+b\tanh^3(c+dx)\right)^2\operatorname{csch}^4(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**2*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(91) = 182$.

Time = 0.13 (sec) , antiderivative size = 468, normalized size of antiderivative = 4.82

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=2\,ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}-\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right.$$
$$+\frac{4}{15}\,b^2\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}-\frac{\cdots}{d(5\,e^{(-2\,dx-\cdots}}\right.$$
$$+\frac{4}{3}\,a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}-\frac{1}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1}\right.$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
2*a*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x -
 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c
) + 1))) + 4/15*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*
e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*
d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c
) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*
x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x
 - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(91) = 182$.

Time = 0.21 (sec) , antiderivative size = 249, normalized size of antiderivative = 2.57

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = -\frac{60\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)-60\,ab\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{10\,\left(11\,abe^{(6\,dx+6\,c)}-33\,abe^{(4\,dx+4\,c)}+12\,a^2e^{(2\,dx+2\,c)}+33\,abe^{(2}}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
-1/30*(60*a*b*log(e^(2*d*x + 2*c) + 1) - 60*a*b*log(abs(e^(2*d*x + 2*c) -
1)) + 10*(11*a*b*e^(6*d*x + 6*c) - 33*a*b*e^(4*d*x + 4*c) + 12*a^2*e^(2*d*
x + 2*c) + 33*a*b*e^(2*d*x + 2*c) - 4*a^2 - 11*a*b)/(e^(2*d*x + 2*c) - 1)^
3 - (137*a*b*e^(10*d*x + 10*c) + 805*a*b*e^(8*d*x + 8*c) + 1730*a*b*e^(6*d
*x + 6*c) - 120*b^2*e^(6*d*x + 6*c) + 1730*a*b*e^(4*d*x + 4*c) + 40*b^2*e^
(4*d*x + 4*c) + 805*a*b*e^(2*d*x + 2*c) - 40*b^2*e^(2*d*x + 2*c) + 137*a*b
 - 8*b^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 344, normalized size of antiderivative = 3.55

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$= \frac{40\,b^2}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}-\frac{4\,a^2}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}$$
$$-\frac{8\,a^2}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}-\frac{4\,(b^2+a\,b)}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$-\frac{16\,b^2}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{32\,b^2}{5\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$-\frac{4\,\mathrm{atan}\left(\frac{a\,b\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2\,b^2}}\right)\,\sqrt{a^2\,b^2}}{\sqrt{-d^2}}+\frac{4\,a\,b}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^3)^2/sinh(c + d*x)^4,x)
```

output
```
(40*b^2)/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x)
+ 1)) - (4*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/
(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (
4*(a*b + b^2))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (16*b^2)/
(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
 + 8*d*x) + 1)) + (32*b^2)/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x)
+ 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (4
*atan((a*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^2*b^2)^(1/2)))*(a^2*b^2
)^(1/2))/(-d^2)^(1/2) + (4*a*b)/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 898, normalized size of antiderivative = 9.26

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^2,x)
```

output

```
(2*( - 15*e**(16*c + 16*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 15*e**(16*c +
 16*d*x)*log(e**(c + d*x) - 1)*a*b + 15*e**(16*c + 16*d*x)*log(e**(c + d*x
) + 1)*a*b - 15*e**(16*c + 16*d*x)*a*b - 30*e**(14*c + 14*d*x)*log(e**(2*c
 + 2*d*x) + 1)*a*b + 30*e**(14*c + 14*d*x)*log(e**(c + d*x) - 1)*a*b + 30*
e**(14*c + 14*d*x)*log(e**(c + d*x) + 1)*a*b + 30*e**(12*c + 12*d*x)*log(e
**(2*c + 2*d*x) + 1)*a*b - 30*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a*b
 - 30*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*a*b - 30*e**(12*c + 12*d*x)
*a**2 + 30*e**(12*c + 12*d*x)*a*b - 30*e**(12*c + 12*d*x)*b**2 + 90*e**(10
*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b - 90*e**(10*c + 10*d*x)*log(e**
(c + d*x) - 1)*a*b - 90*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a*b - 140
*e**(10*c + 10*d*x)*a**2 + 100*e**(10*c + 10*d*x)*b**2 - 250*e**(8*c + 8*d
*x)*a**2 - 130*e**(8*c + 8*d*x)*b**2 - 90*e**(6*c + 6*d*x)*log(e**(2*c + 2
*d*x) + 1)*a*b + 90*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a*b + 90*e**(6*
c + 6*d*x)*log(e**(c + d*x) + 1)*a*b - 200*e**(6*c + 6*d*x)*a**2 + 88*e**(
6*c + 6*d*x)*b**2 - 30*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 30
*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*b + 30*e**(4*c + 4*d*x)*log(e**(
c + d*x) + 1)*a*b - 50*e**(4*c + 4*d*x)*a**2 - 30*e**(4*c + 4*d*x)*a*b - 3
4*e**(4*c + 4*d*x)*b**2 + 30*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a*
b - 30*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a*b - 30*e**(2*c + 2*d*x)*lo
g(e**(c + d*x) + 1)*a*b + 20*e**(2*c + 2*d*x)*a**2 + 4*e**(2*c + 2*d*x)...
```

3.65 $\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 286

$$\begin{aligned}\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx &= \frac{3}{8}a\left(a^2+63b^2\right)x - \frac{3b(3a^2+2b^2)\cosh^2(c+dx)}{2d} + \frac{b(3a^2+b^2)\cosh^4(c+dx)}{4d} \\ &+ \frac{3b(3a^2+5b^2)\log(\cosh(c+dx))}{d} - \frac{a(5a^2+51b^2)\cosh(c+dx)\sinh(c+dx)}{8d} \\ &+ \frac{a(a^2+3b^2)\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{18ab^2\tanh(c+dx)}{d} \\ &- \frac{b(3a^2+10b^2)\tanh^2(c+dx)}{2d} - \frac{3ab^2\tanh^3(c+dx)}{d} - \frac{3b^3\tanh^4(c+dx)}{2d} \\ &- \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^6(c+dx)}{2d} - \frac{b^3\tanh^8(c+dx)}{8d}\end{aligned}$$

output
```
3/8*a*(a^2+63*b^2)*x-3/2*b*(3*a^2+2*b^2)*cosh(d*x+c)^2/d+1/4*b*(3*a^2+b^2)
*cosh(d*x+c)^4/d+3*b*(3*a^2+5*b^2)*ln(cosh(d*x+c))/d-1/8*a*(5*a^2+51*b^2)*
cosh(d*x+c)*sinh(d*x+c)/d+1/4*a*(a^2+3*b^2)*cosh(d*x+c)^3*sinh(d*x+c)/d-18
*a*b^2*tanh(d*x+c)/d-1/2*b*(3*a^2+10*b^2)*tanh(d*x+c)^2/d-3*a*b^2*tanh(d*x
+c)^3/d-3/2*b^3*tanh(d*x+c)^4/d-3/5*a*b^2*tanh(d*x+c)^5/d-1/2*b^3*tanh(d*x
+c)^6/d-1/8*b^3*tanh(d*x+c)^8/d
```

Mathematica [A] (verified)

Time = 6.85 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.03

$$\begin{aligned}&\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx\\&=\frac{3a(a^2+63b^2)\,(c+dx)}{8d}-\frac{b(15a^2+11b^2)\cosh(2(c+dx))}{8d}\\&+\frac{b(3a^2+b^2)\cosh(4(c+dx))}{32d}+\frac{3(3a^2b+5b^3)\log(\cosh(c+dx))}{d}\\&+\frac{b(3a^2+20b^2)\,\mathrm{sech}^2(c+dx)}{2d}-\frac{15b^3\mathrm{sech}^4(c+dx)}{4d}\\&+\frac{b^3\mathrm{sech}^6(c+dx)}{d}-\frac{b^3\mathrm{sech}^8(c+dx)}{8d}-\frac{a(a^2+12b^2)\sinh(2(c+dx))}{4d}\\&+\frac{a(a^2+3b^2)\sinh(4(c+dx))}{32d}-\frac{108ab^2\tanh(c+dx)}{5d}\\&+\frac{21ab^2\mathrm{sech}^2(c+dx)\tanh(c+dx)}{5d}-\frac{3ab^2\mathrm{sech}^4(c+dx)\tanh(c+dx)}{5d}\end{aligned}$$

input
```
Integrate[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(3*a*(a^2 + 63*b^2)*(c + d*x))/(8*d) - (b*(15*a^2 + 11*b^2)*Cosh[2*(c + d*
x)])/(8*d) + (b*(3*a^2 + b^2)*Cosh[4*(c + d*x)])/(32*d) + (3*(3*a^2*b + 5*
b^3)*Log[Cosh[c + d*x]])/d + (b*(3*a^2 + 20*b^2)*Sech[c + d*x]^2)/(2*d) -
(15*b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]^6)/d - (b^3*Sech[c + d
*x]^8)/(8*d) - (a*(a^2 + 12*b^2)*Sinh[2*(c + d*x)])/(4*d) + (a*(a^2 + 3*b^
2)*Sinh[4*(c + d*x)])/(32*d) - (108*a*b^2*Tanh[c + d*x])/(5*d) + (21*a*b^2
*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tanh[c +
d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 283, normalized size of antiderivative = 0.99, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4146, 2335, 25, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$\big\downarrow$ 3042

$$\int \sin(ic+idx)^4\left(a+ib\tan(ic+idx)^3\right)^3\,dx$$

$\big\downarrow$ 4146

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^3(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$\big\downarrow$ 2335

$$\frac{\frac{1}{4}\int -\frac{\tanh^3(c+dx)\left(4b^3\tanh^8(c+dx)+4b^3\tanh^6(c+dx)+12ab^2\tanh^5(c+dx)+4b^3\tanh^4(c+dx)+12ab^2\tanh^3(c+dx)+4b\left(3a^2+b^2\right)\tanh^2(c+dx\right.}{\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{\tanh^4(c+dx)\left(a\left(a^2+3b^2\right)\tanh(c+dx)+b\left(3a^2+b^2\right)\right)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int \frac{\tanh^3(c+dx)\left(4b^3\tanh^8(c+dx)+4b^3\tanh^6(c+dx)+12ab^2\tanh^5(c+dx)+4b^3\tanh^4(\right.}{(1-}}{d}$$

$\big\downarrow$ 2335

$$\frac{1}{4}\left(-\frac{1}{2}\int -\frac{\tanh^2(c+dx)\left(8b^3\tanh^7(c+dx)+16b^3\tanh^5(c+dx)+24ab^2\tanh^4(c+dx)+24b^3\tanh^3(c+dx)+48ab^2\tanh^2(c+dx)+72b\left(a^2+b^2\right)\tan\right.}{1-\tanh^2(c+dx)}\right.$$

$\big\downarrow$ 25

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\tanh^2(c+dx)\left(8b^3\tanh^7(c+dx)+16b^3\tanh^5(c+dx)+24ab^2\tanh^4(c+dx)+24b^3\tanh^3(c+dx)+48ab^2\tanh^2(c+dx)+72b\left(a^2+b^2\right)\tanh(c+\right.}{1-\tanh^2(c+dx)}\right.$$

$\big\downarrow$ 2333

$$\frac{1}{4}\left(\frac{1}{2}\int\left(-8b^3\tanh^7(c+dx)-24b^3\tanh^5(c+dx)-24ab^2\tanh^4(c+dx)-48b^3\tanh^3(c+dx)-72ab^2\tanh^2(c+\right.\right.$$

$\big\downarrow$ 2009

$$\frac{1}{4}\left(\frac{1}{2}\left(3a\left(a^2+63b^2\right)\text{arctanh}(\tanh(c+dx))-12b\left(3a^2+5b^2\right)\tanh^2(c+dx)-3a\left(a^2+63b^2\right)\tanh(c+dx)-12b\right.\right.$$

input
```
Int[Sinh[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
((Tanh[c + d*x]^4*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/(4*(1
 - Tanh[c + d*x]^2)^2) + (-1/2*(Tanh[c + d*x]^3*(a*(a^2 + 39*b^2) + 4*b*(6
*a^2 + 5*b^2)*Tanh[c + d*x]))/(1 - Tanh[c + d*x]^2) + (3*a*(a^2 + 63*b^2)*
ArcTanh[Tanh[c + d*x]] - 12*b*(3*a^2 + 5*b^2)*Log[1 - Tanh[c + d*x]^2] - 3
*a*(a^2 + 63*b^2)*Tanh[c + d*x] - 12*b*(3*a^2 + 5*b^2)*Tanh[c + d*x]^2 - 2
4*a*b^2*Tanh[c + d*x]^3 - 12*b^3*Tanh[c + d*x]^4 - (24*a*b^2*Tanh[c + d*x]
^5)/5 - 4*b^3*Tanh[c + d*x]^6 - b^3*Tanh[c + d*x]^8)/2)/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 51.99 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.96

method	result
derivativedivides	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\,\mathrm{t}}{}\right.}{}$
default	$\frac{a^3\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)^6}{4\cosh(dx+c)^2}-\frac{3\sinh(dx+c)^4}{4\cosh(dx+c)^2}+3\ln(\cosh(dx+c))-\frac{3\,\mathrm{t}}{}\right.}{}$
risch	$\frac{3\,e^{4dx+4c}a^2b}{64d}+\frac{3\,e^{4dx+4c}b^2a}{64d}-\frac{15\,e^{2dx+2c}a^2b}{16d}-\frac{3\,e^{2dx+2c}b^2a}{2d}-\frac{e^{-4dx-4c}a^3}{64d}+\frac{e^{-4dx-4c}b^3}{64d}+\frac{e^{4dx+4c}a^3}{64d}$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/4*sinh(d*x+c)^6/cosh(d*x+c)^2-3/4*sinh(d*x+c)^4/cosh(d*x+c)^2+3*
ln(cosh(d*x+c))-3/2*tanh(d*x+c)^2)+3*b^2*a*(1/4*sinh(d*x+c)^9/cosh(d*x+c)^
5-9/8*sinh(d*x+c)^7/cosh(d*x+c)^5+63/8*d*x+63/8*c-63/8*tanh(d*x+c)-21/8*ta
nh(d*x+c)^3-63/40*tanh(d*x+c)^5)+b^3*(1/4*sinh(d*x+c)^12/cosh(d*x+c)^8-3/2
*sinh(d*x+c)^10/cosh(d*x+c)^8+15*ln(cosh(d*x+c))-15/2*tanh(d*x+c)^2-15/4*t
anh(d*x+c)^4-5/2*tanh(d*x+c)^6-15/8*tanh(d*x+c)^8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 12323 vs. $2(266) = 532$.

Time = 0.21 (sec) , antiderivative size = 12323, normalized size of antiderivative = 43.09

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 647 vs. $2(266) = 532$.

Time = 0.15 (sec) , antiderivative size = 647, normalized size of antiderivative = 2.26

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/320*a*b^2*(2520*(d*x + c)/d + 5*(32*e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c))/d - (135*e^(-2*d*x - 2*c) + 5358*e^(-4*d*x
- 4*c) + 18190*e^(-6*d*x - 6*c) + 28455*e^(-8*d*x - 8*c) + 19995*e^(-10*d*
x - 10*c) + 6560*e^(-12*d*x - 12*c) - 5)/(d*(e^(-4*d*x - 4*c) + 5*e^(-6*d*
x - 6*c) + 10*e^(-8*d*x - 8*c) + 10*e^(-10*d*x - 10*c) + 5*e^(-12*d*x - 12
*c) + e^(-14*d*x - 14*c)))) + 1/64*b^3*(960*(d*x + c)/d - (44*e^(-2*d*x -
2*c) - e^(-4*d*x - 4*c))/d + 960*log(e^(-2*d*x - 2*c) + 1)/d - (36*e^(-2*d
*x - 2*c) + 324*e^(-4*d*x - 4*c) - 1384*e^(-6*d*x - 6*c) - 9126*e^(-8*d*x
- 8*c) - 24112*e^(-10*d*x - 10*c) - 31868*e^(-12*d*x - 12*c) - 25912*e^(-1
4*d*x - 14*c) - 11169*e^(-16*d*x - 16*c) - 2516*e^(-18*d*x - 18*c) - 1)/(d
*(e^(-4*d*x - 4*c) + 8*e^(-6*d*x - 6*c) + 28*e^(-8*d*x - 8*c) + 56*e^(-10*
d*x - 10*c) + 70*e^(-12*d*x - 12*c) + 56*e^(-14*d*x - 14*c) + 28*e^(-16*d*
x - 16*c) + 8*e^(-18*d*x - 18*c) + e^(-20*d*x - 20*c)))) + 3/64*a^2*b*(192
*(d*x + c)/d - (20*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d + 192*log(e^(-2*
d*x - 2*c) + 1)/d - (18*e^(-2*d*x - 2*c) + 39*e^(-4*d*x - 4*c) - 108*e^(-6
*d*x - 6*c) - 1)/(d*(e^(-4*d*x - 4*c) + 2*e^(-6*d*x - 6*c) + e^(-8*d*x - 8
*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(266) = 532$.

Time = 0.64 (sec) , antiderivative size = 694, normalized size of antiderivative = 2.43

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/2240*(35*a^3*e^(4*d*x + 4*c) + 105*a^2*b*e^(4*d*x + 4*c) + 105*a*b^2*e^(
4*d*x + 4*c) + 35*b^3*e^(4*d*x + 4*c) - 280*a^3*e^(2*d*x + 2*c) - 2100*a^2
*b*e^(2*d*x + 2*c) - 3360*a*b^2*e^(2*d*x + 2*c) - 1540*b^3*e^(2*d*x + 2*c)
 + 840*(a^3 - 24*a^2*b + 63*a*b^2 - 40*b^3)*(d*x + c) - 35*(18*a^3*e^(4*d*
x + 4*c) - 432*a^2*b*e^(4*d*x + 4*c) + 1134*a*b^2*e^(4*d*x + 4*c) - 720*b^
3*e^(4*d*x + 4*c) - 8*a^3*e^(2*d*x + 2*c) + 60*a^2*b*e^(2*d*x + 2*c) - 96*
a*b^2*e^(2*d*x + 2*c) + 44*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*e^(-4*d*x - 4*c) + 6720*(3*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) + 1) -
 8*(6849*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 53112*a^2
*b*e^(14*d*x + 14*c) - 16800*a*b^2*e^(14*d*x + 14*c) + 80120*b^3*e^(14*d*x
 + 14*c) + 181692*a^2*b*e^(12*d*x + 12*c) - 100800*a*b^2*e^(12*d*x + 12*c)
 + 269220*b^3*e^(12*d*x + 12*c) + 358344*a^2*b*e^(10*d*x + 10*c) - 272160*
a*b^2*e^(10*d*x + 10*c) + 520520*b^3*e^(10*d*x + 10*c) + 445830*a^2*b*e^(8
*d*x + 8*c) - 423360*a*b^2*e^(8*d*x + 8*c) + 648970*b^3*e^(8*d*x + 8*c) +
358344*a^2*b*e^(6*d*x + 6*c) - 405216*a*b^2*e^(6*d*x + 6*c) + 520520*b^3*e
^(6*d*x + 6*c) + 181692*a^2*b*e^(4*d*x + 4*c) - 237888*a*b^2*e^(4*d*x + 4*
c) + 269220*b^3*e^(4*d*x + 4*c) + 53112*a^2*b*e^(2*d*x + 2*c) - 79968*a*b^
2*e^(2*d*x + 2*c) + 80120*b^3*e^(2*d*x + 2*c) + 6849*a^2*b - 12096*a*b^2 +
 11415*b^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.95 (sec) , antiderivative size = 682, normalized size of antiderivative = 2.38

$$\int \sinh^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)^3,x)
```

output
```
x*((189*a*b^2)/8 - 9*a^2*b + (3*a^3)/8 - 15*b^3) - (4*(12*a*b^2 + 71*b^3))
/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*
c + 8*d*x) + 1)) - (256*b^3)/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x)
+ 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(1
2*c + 12*d*x) + 1)) + (log(exp(2*c)*exp(2*d*x) + 1)*(9*a^2*b + 15*b^3))/d
+ (2*(30*a*b^2 + 3*a^2*b + 20*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (32*(3*a*
b^2 + 50*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c
 + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (128*b^3)/(d*(
7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*
c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*
d*x) + 1)) - (2*(30*a*b^2 + 3*a^2*b + 50*b^3))/(d*(2*exp(2*c + 2*d*x) + ex
p(4*c + 4*d*x) + 1)) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*
x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 2
8*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) + (
exp(4*c + 4*d*x)*(a + b)^3)/(64*d) - (exp(- 4*c - 4*d*x)*(a - b)^3)/(64*d)
 + (8*(9*a*b^2 + 23*b^3))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + ex
p(6*c + 6*d*x) + 1)) - (exp(2*c + 2*d*x)*(a + b)^2*(2*a + 11*b))/(16*d) +
(exp(- 2*c - 2*d*x)*(a - b)^2*(2*a - 11*b))/(16*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1902, normalized size of antiderivative = 6.65

$$\int \sinh^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x)
```

output

```
(10*e**(24*c + 24*d*x)*a**3 + 30*e**(24*c + 24*d*x)*a**2*b + 30*e**(24*c +
 24*d*x)*a*b**2 + 10*e**(24*c + 24*d*x)*b**3 - 360*e**(22*c + 22*d*x)*a**2
*b - 720*e**(22*c + 22*d*x)*a*b**2 - 360*e**(22*c + 22*d*x)*b**3 + 5760*e*
*(20*c + 20*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 9600*e**(20*c + 20*d*x
)*log(e**(2*c + 2*d*x) + 1)*b**3 + 240*e**(20*c + 20*d*x)*a**3*d*x - 160*e
**(20*c + 20*d*x)*a**3 - 5760*e**(20*c + 20*d*x)*a**2*b*d*x - 2475*e**(20*
c + 20*d*x)*a**2*b + 15120*e**(20*c + 20*d*x)*a*b**2*d*x - 8610*e**(20*c +
 20*d*x)*a*b**2 - 9600*e**(20*c + 20*d*x)*b**3*d*x - 4915*e**(20*c + 20*d*
x)*b**3 + 46080*e**(18*c + 18*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 7680
0*e**(18*c + 18*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 + 1920*e**(18*c + 18*d
*x)*a**3*d*x - 46080*e**(18*c + 18*d*x)*a**2*b*d*x + 120960*e**(18*c + 18*
d*x)*a*b**2*d*x - 76800*e**(18*c + 18*d*x)*b**3*d*x + 161280*e**(16*c + 16
*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 268800*e**(16*c + 16*d*x)*log(e**
(2*c + 2*d*x) + 1)*b**3 + 6720*e**(16*c + 16*d*x)*a**3*d*x + 2450*e**(16*c
 + 16*d*x)*a**3 - 161280*e**(16*c + 16*d*x)*a**2*b*d*x + 28350*e**(16*c +
16*d*x)*a**2*b + 423360*e**(16*c + 16*d*x)*a*b**2*d*x + 136830*e**(16*c +
16*d*x)*a*b**2 - 268800*e**(16*c + 16*d*x)*b**3*d*x + 40850*e**(16*c + 16*
d*x)*b**3 + 322560*e**(14*c + 14*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 5
37600*e**(14*c + 14*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 + 13440*e**(14*c +
 14*d*x)*a**3*d*x + 8320*e**(14*c + 14*d*x)*a**3 - 322560*e**(14*c + 14...
```

3.66 $\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 330

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= \frac{15a^2b\arctan(\sinh(c+dx))}{2d}+\frac{1155b^3\arctan(\sinh(c+dx))}{128d}-\frac{a^3\cosh(c+dx)}{d}\\&-\frac{12ab^2\cosh(c+dx)}{d}+\frac{a^3\cosh^3(c+dx)}{3d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{18ab^2\text{sech}(c+dx)}{d}\\&+\frac{4ab^2\text{sech}^3(c+dx)}{d}-\frac{3ab^2\text{sech}^5(c+dx)}{5d}-\frac{6a^2b\sinh(c+dx)}{d}-\frac{5b^3\sinh(c+dx)}{d}\\&+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}-\frac{3a^2b\,\text{sech}(c+dx)\tanh(c+dx)}{2d}\\&-\frac{765b^3\text{sech}(c+dx)\tanh(c+dx)}{128d}+\frac{515b^3\text{sech}^3(c+dx)\tanh(c+dx)}{192d}\\&-\frac{41b^3\text{sech}^5(c+dx)\tanh(c+dx)}{48d}+\frac{b^3\text{sech}^7(c+dx)\tanh(c+dx)}{8d}\end{aligned}$$

output
```
15/2*a^2*b*arctan(sinh(d*x+c))/d+1155/128*b^3*arctan(sinh(d*x+c))/d-a^3*co
sh(d*x+c)/d-12*a*b^2*cosh(d*x+c)/d+1/3*a^3*cosh(d*x+c)^3/d+a*b^2*cosh(d*x+
c)^3/d-18*a*b^2*sech(d*x+c)/d+4*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*sech(d*x+c
)^5/d-6*a^2*b*sinh(d*x+c)/d-5*b^3*sinh(d*x+c)/d+a^2*b*sinh(d*x+c)^3/d+1/3*
b^3*sinh(d*x+c)^3/d-3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d-765/128*b^3*sech(d
*x+c)*tanh(d*x+c)/d+515/192*b^3*sech(d*x+c)^3*tanh(d*x+c)/d-41/48*b^3*sech
(d*x+c)^5*tanh(d*x+c)/d+1/8*b^3*sech(d*x+c)^7*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 7.35 (sec) , antiderivative size = 291, normalized size of antiderivative = 0.88

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{15(64a^2b+77b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d} - \frac{3a(a^2+15b^2)\cosh(c+dx)}{4d} + \frac{a(a^2+3b^2)\cosh(3(c+dx))}{12d} - \frac{18ab^2\text{sech}(c+dx)}{d} + \frac{4ab^2\text{sech}^3(c+dx)}{d} - \frac{3ab^2\text{sech}^5(c+dx)}{5d} - \frac{3b(9a^2+7b^2)\sinh(c+dx)}{4d} - \frac{3\text{sech}^2(c+dx)\left(64a^2b\sinh(c+dx)+255b^3\sinh(c+dx)\right)}{128d} + \frac{b(3a^2+b^2)\sinh(3(c+dx))}{12d} + \frac{515b^3\text{sech}^3(c+dx)\tanh(c+dx)}{192d} - \frac{41b^3\text{sech}^5(c+dx)\tanh(c+dx)}{48d} + \frac{b^3\text{sech}^7(c+dx)\tanh(c+dx)}{8d}$$

input
```
Integrate[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(15*(64*a^2*b + 77*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (3*a*(a^2 + 15
*b^2)*Cosh[c + d*x])/(4*d) + (a*(a^2 + 3*b^2)*Cosh[3*(c + d*x)])/(12*d) -
(18*a*b^2*Sech[c + d*x])/d + (4*a*b^2*Sech[c + d*x]^3)/d - (3*a*b^2*Sech[c
 + d*x]^5)/(5*d) - (3*b*(9*a^2 + 7*b^2)*Sinh[c + d*x])/(4*d) - (3*Sech[c +
 d*x]^2*(64*a^2*b*Sinh[c + d*x] + 255*b^3*Sinh[c + d*x]))/(128*d) + (b*(3*
a^2 + b^2)*Sinh[3*(c + d*x)])/(12*d) + (515*b^3*Sech[c + d*x]^3*Tanh[c + d
*x])/(192*d) - (41*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) + (b^3*Sech[c
 + d*x]^7*Tanh[c + d*x])/(8*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 392, normalized size of antiderivative = 1.19, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\sin(ic+idx)^3\left(a+ib\tan(ic+idx)^3\right)^3dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \sin(ic+idx)^3\left(ib\tan(ic+idx)^3+a\right)^3dx$$

$$\Big\downarrow \text{ 4149}$$

$$i\int \left(-ib^3\sinh^3(c+dx)\tanh^9(c+dx)-3iab^2\sinh^3(c+dx)\tanh^6(c+dx)-3ia^2b\sinh^3(c+dx)\tanh^3(c+dx)\right.$$

$$\Big\downarrow \text{ 2009}$$

$$i\left(-\frac{ia^3\cosh^3(c+dx)}{3d}+\frac{ia^3\cosh(c+dx)}{d}-\frac{15ia^2b\arctan(\sinh(c+dx))}{2d}-\frac{5ia^2b\sinh^3(c+dx)}{2d}+\frac{15ia^2b\sinh(c}{2d}\right.$$

input
```
Int[Sinh[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
I*((((-15*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d - (((1155*I)/128)*b^3*ArcTa
n[Sinh[c + d*x]])/d + (I*a^3*Cosh[c + d*x])/d + ((12*I)*a*b^2*Cosh[c + d*x
])/d - ((I/3)*a^3*Cosh[c + d*x]^3)/d - (I*a*b^2*Cosh[c + d*x]^3)/d + ((18*
I)*a*b^2*Sech[c + d*x])/d - ((4*I)*a*b^2*Sech[c + d*x]^3)/d + (((3*I)/5)*a
*b^2*Sech[c + d*x]^5)/d + (((15*I)/2)*a^2*b*Sinh[c + d*x])/d + (((1155*I)/
128)*b^3*Sinh[c + d*x])/d - (((5*I)/2)*a^2*b*Sinh[c + d*x]^3)/d - (((385*I
)/128)*b^3*Sinh[c + d*x]^3)/d + (((3*I)/2)*a^2*b*Sinh[c + d*x]^3*Tanh[c +
d*x]^2)/d + (((231*I)/128)*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^2)/d + (((33*
I)/64)*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^4)/d + (((11*I)/48)*b^3*Sinh[c +
d*x]^3*Tanh[c + d*x]^6)/d + ((I/8)*b^3*Sinh[c + d*x]^3*Tanh[c + d*x]^8)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 32.04 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.11

method	result
derivativedivides	$a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+5\,\mathrm{arc}$
default	$a^3\left(-\frac{2}{3}+\frac{\sinh(dx+c)^2}{3}\right)\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^5}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)^3}{3\cosh(dx+c)^2}-\frac{5\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{5\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+5\,\mathrm{arc}$
risch	$\frac{e^{3dx+3c}a^3}{24d}+\frac{e^{3dx+3c}a^2b}{8d}+\frac{b^2a\,e^{3dx+3c}}{8d}+\frac{b^3e^{3dx+3c}}{24d}-\frac{3\,e^{dx+c}a^3}{8d}-\frac{27\,e^{dx+c}a^2b}{8d}-\frac{45\,e^{dx+c}b^2a}{8d}-\frac{21\,e^{dx}}{8d}$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(1/3*sinh(d*x+c)^5/c
osh(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2+5
/2*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c)))+3*b^2*a*(1/3*sinh(d*x+c)^
8/cosh(d*x+c)^5-8/3*sinh(d*x+c)^6/cosh(d*x+c)^5-16*sinh(d*x+c)^4/cosh(d*x+
c)^5-64/3*sinh(d*x+c)^2/cosh(d*x+c)^5-128/15/cosh(d*x+c)^5)+b^3*(1/3*sinh(
d*x+c)^11/cosh(d*x+c)^8-11/3*sinh(d*x+c)^9/cosh(d*x+c)^8-33*sinh(d*x+c)^7/
cosh(d*x+c)^8-77*sinh(d*x+c)^5/cosh(d*x+c)^8-77*sinh(d*x+c)^3/cosh(d*x+c)^
8-33*sinh(d*x+c)/cosh(d*x+c)^8+33*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35
/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+1155/64*arctan(exp(d*x+
c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8462 vs. $2(310) = 620$.

Time = 0.18 (sec) , antiderivative size = 8462, normalized size of antiderivative = 25.64

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 604, normalized size of antiderivative = 1.83

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/192*b^3*(8*(63*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 3465*arctan(e^(-d*x
- c))/d - (440*e^(-2*d*x - 2*c) + 6103*e^(-4*d*x - 4*c) + 21019*e^(-6*d*x
- 6*c) + 41207*e^(-8*d*x - 8*c) + 40243*e^(-10*d*x - 10*c) + 22589*e^(-12*
d*x - 12*c) + 505*e^(-14*d*x - 14*c) - 3331*e^(-16*d*x - 16*c) - 1791*e^(-
18*d*x - 18*c) - 8)/(d*(e^(-3*d*x - 3*c) + 8*e^(-5*d*x - 5*c) + 28*e^(-7*d
*x - 7*c) + 56*e^(-9*d*x - 9*c) + 70*e^(-11*d*x - 11*c) + 56*e^(-13*d*x -
13*c) + 28*e^(-15*d*x - 15*c) + 8*e^(-17*d*x - 17*c) + e^(-19*d*x - 19*c))
)) - 1/40*a*b^2*(5*(45*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + (200*e^(-2*d*x
 - 2*c) + 2515*e^(-4*d*x - 4*c) + 6680*e^(-6*d*x - 6*c) + 9073*e^(-8*d*x -
 8*c) + 5600*e^(-10*d*x - 10*c) + 1665*e^(-12*d*x - 12*c) - 5)/(d*(e^(-3*d
*x - 3*c) + 5*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 10*e^(-9*d*x - 9*c)
 + 5*e^(-11*d*x - 11*c) + e^(-13*d*x - 13*c)))) + 1/8*a^2*b*((27*e^(-d*x -
 c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2
*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) +
 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 580, normalized size of antiderivative = 1.76

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/960*(40*a^3*e^(3*d*x + 3*c) + 120*a^2*b*e^(3*d*x + 3*c) + 120*a*b^2*e^(3
*d*x + 3*c) + 40*b^3*e^(3*d*x + 3*c) - 360*a^3*e^(d*x + c) - 3240*a^2*b*e^
(d*x + c) - 5400*a*b^2*e^(d*x + c) - 2520*b^3*e^(d*x + c) + 225*(64*a^2*b
+ 77*b^3)*arctan(e^(d*x + c)) - 40*(9*a^3*e^(2*d*x + 2*c) - 81*a^2*b*e^(2*
d*x + 2*c) + 135*a*b^2*e^(2*d*x + 2*c) - 63*b^3*e^(2*d*x + 2*c) - a^3 + 3*
a^2*b - 3*a*b^2 + b^3)*e^(-3*d*x - 3*c) - (2880*a^2*b*e^(15*d*x + 15*c) +
34560*a*b^2*e^(15*d*x + 15*c) + 11475*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*
e^(13*d*x + 13*c) + 211200*a*b^2*e^(13*d*x + 13*c) + 36775*b^3*e^(13*d*x +
 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) + 590592*a*b^2*e^(11*d*x + 11*c) +
67715*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 957696*a*b^2*e
^(9*d*x + 9*c) + 27055*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) +
 957696*a*b^2*e^(7*d*x + 7*c) - 27055*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^
(5*d*x + 5*c) + 590592*a*b^2*e^(5*d*x + 5*c) - 67715*b^3*e^(5*d*x + 5*c) -
 14400*a^2*b*e^(3*d*x + 3*c) + 211200*a*b^2*e^(3*d*x + 3*c) - 36775*b^3*e^
(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) + 34560*a*b^2*e^(d*x + c) - 11475*b
^3*e^(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 757, normalized size of antiderivative = 2.29

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)^3,x)
```

output
```
(exp(3*c + 3*d*x)*(a + b)^3)/(24*d) + (exp(- 3*c - 3*d*x)*(a - b)^3)/(24*d
) + (15*atan((exp(d*x)*exp(c)*(77*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))
/(d*(5929*b^6 + 9856*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(5929*b^6 + 9856*a^2*
b^4 + 4096*a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) - (3*exp(- c - d*x)*(a - b)^2*
(a - 7*b))/(8*d) - (exp(c + d*x)*(6144*a*b^2 + 11005*b^3))/(120*d*(3*exp(2
*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (exp(c + d*x)*
(768*a*b^2 + 3365*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4
*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (596*b^3*exp(c + d*x))/(3*d*(
6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*
c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (3*exp(c +
d*x)*(a + b)^2*(a + 7*b))/(8*d) - (3*exp(c + d*x)*(768*a*b^2 + 64*a^2*b +
255*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) - (2*exp(c + d*x)*(144*a*b^2 + 162
5*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d
*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (112*b^3*exp(c + d*x
))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35
*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*
c + 14*d*x) + 1)) + (exp(c + d*x)*(3072*a*b^2 + 576*a^2*b + 4355*b^3))/(96
*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (32*b^3*exp(c + d*x))/(d
*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(
8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(14...
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 1243, normalized size of antiderivative = 3.77

$$\int \sinh^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x)
```

output
```
(14400*e**(19*c + 19*d*x)*atan(e**(c + d*x))*a**2*b + 17325*e**(19*c + 19*
d*x)*atan(e**(c + d*x))*b**3 + 115200*e**(17*c + 17*d*x)*atan(e**(c + d*x)
)*a**2*b + 138600*e**(17*c + 17*d*x)*atan(e**(c + d*x))*b**3 + 403200*e**(
15*c + 15*d*x)*atan(e**(c + d*x))*a**2*b + 485100*e**(15*c + 15*d*x)*atan(
e**(c + d*x))*b**3 + 806400*e**(13*c + 13*d*x)*atan(e**(c + d*x))*a**2*b +
 970200*e**(13*c + 13*d*x)*atan(e**(c + d*x))*b**3 + 1008000*e**(11*c + 11
*d*x)*atan(e**(c + d*x))*a**2*b + 1212750*e**(11*c + 11*d*x)*atan(e**(c +
d*x))*b**3 + 806400*e**(9*c + 9*d*x)*atan(e**(c + d*x))*a**2*b + 970200*e*
*(9*c + 9*d*x)*atan(e**(c + d*x))*b**3 + 403200*e**(7*c + 7*d*x)*atan(e**(
c + d*x))*a**2*b + 485100*e**(7*c + 7*d*x)*atan(e**(c + d*x))*b**3 + 11520
0*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a**2*b + 138600*e**(5*c + 5*d*x)*ata
n(e**(c + d*x))*b**3 + 14400*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a**2*b +
17325*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b**3 + 40*e**(22*c + 22*d*x)*a**
3 + 120*e**(22*c + 22*d*x)*a**2*b + 120*e**(22*c + 22*d*x)*a*b**2 + 40*e**
(22*c + 22*d*x)*b**3 - 40*e**(20*c + 20*d*x)*a**3 - 2280*e**(20*c + 20*d*x
)*a**2*b - 4440*e**(20*c + 20*d*x)*a*b**2 - 2200*e**(20*c + 20*d*x)*b**3 -
 2120*e**(18*c + 18*d*x)*a**3 - 22200*e**(18*c + 18*d*x)*a**2*b - 79800*e*
*(18*c + 18*d*x)*a*b**2 - 27995*e**(18*c + 18*d*x)*b**3 - 10680*e**(16*c +
 16*d*x)*a**3 - 72600*e**(16*c + 16*d*x)*a**2*b - 398760*e**(16*c + 16*d*x
)*a*b**2 - 84975*e**(16*c + 16*d*x)*b**3 - 27120*e**(14*c + 14*d*x)*a**...
```

3.67 $\int \sinh^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 226

$$\begin{aligned}
&\int \sinh^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 dx \\
&= -\frac{1}{2}a\left(a^2+21b^2\right)x + \frac{b(3a^2+b^2)\cosh^2(c+dx)}{2d} - \frac{b(6a^2+5b^2)\log(\cosh(c+dx))}{d} \\
&\quad + \frac{a(a^2+3b^2)\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{9ab^2\tanh(c+dx)}{d} \\
&\quad + \frac{b(3a^2+4b^2)\tanh^2(c+dx)}{2d} + \frac{2ab^2\tanh^3(c+dx)}{d} + \frac{3b^3\tanh^4(c+dx)}{4d} \\
&\quad + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^6(c+dx)}{3d} + \frac{b^3\tanh^8(c+dx)}{8d}
\end{aligned}$$

output
```
-1/2*a*(a^2+21*b^2)*x+1/2*b*(3*a^2+b^2)*cosh(d*x+c)^2/d-b*(6*a^2+5*b^2)*ln
(cosh(d*x+c))/d+1/2*a*(a^2+3*b^2)*cosh(d*x+c)*sinh(d*x+c)/d+9*a*b^2*tanh(d
*x+c)/d+1/2*b*(3*a^2+4*b^2)*tanh(d*x+c)^2/d+2*a*b^2*tanh(d*x+c)^3/d+3/4*b^
3*tanh(d*x+c)^4/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/3*b^3*tanh(d*x+c)^6/d+1/8*b^
3*tanh(d*x+c)^8/d
```

Mathematica [A] (verified)

Time = 6.84 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.08

$$\int \sinh^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 dx$$
$$= -\frac{a(a^2+21b^2)(c+dx)}{2d} + \frac{b(3a^2+b^2)\cosh(2(c+dx))}{4d} + \frac{(-6a^2b-5b^3)\log(\cosh(c+dx))}{d} - \frac{b(3a^2+10b^2)\operatorname{sech}^2(c+dx)}{2d} + \frac{5b^3\operatorname{sech}^4(c+dx)}{2d} - \frac{5b^3\operatorname{sech}^6(c+dx)}{6d} + \frac{b^3\operatorname{sech}^8(c+dx)}{8d} + \frac{a(a^2+3b^2)\sinh(2(c+dx))}{4d} + \frac{58ab^2\tanh(c+dx)}{5d} - \frac{16ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d} + \frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

input
```
Integrate[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
-1/2*(a*(a^2 + 21*b^2)*(c + d*x))/d + (b*(3*a^2 + b^2)*Cosh[2*(c + d*x)])/
(4*d) + ((-6*a^2*b - 5*b^3)*Log[Cosh[c + d*x]])/d - (b*(3*a^2 + 10*b^2)*Se
ch[c + d*x]^2)/(2*d) + (5*b^3*Sech[c + d*x]^4)/(2*d) - (5*b^3*Sech[c + d*x
]^6)/(6*d) + (b^3*Sech[c + d*x]^8)/(8*d) + (a*(a^2 + 3*b^2)*Sinh[2*(c + d*
x)])/(4*d) + (58*a*b^2*Tanh[c + d*x])/(5*d) - (16*a*b^2*Sech[c + d*x]^2*Ta
nh[c + d*x])/(5*d) + (3*a*b^2*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 25, 4146, 2335, 25, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\sin(ic+idx)^2\left(a+ib\tan(ic+idx)^3\right)^3dx$$

$\downarrow$ 25

$$-\int \sin(ic+idx)^2\left(ib\tan(ic+idx)^3+a\right)^3dx$$

$\downarrow$ 4146

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$\downarrow$ 2335

$$\frac{\frac{1}{2}\int -\frac{\tanh(c+dx)\left(2b^3\tanh^8(c+dx)+2b^3\tanh^6(c+dx)+6ab^2\tanh^5(c+dx)+2b^3\tanh^4(c+dx)+6ab^2\tanh^3(c+dx)+2b\left(3a^2+b^2\right)\tanh^2(c+dx)+a}{1-\tanh^2(c+dx)}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\tanh^2(c+dx)\left(a\left(a^2+3b^2\right)\tanh(c+dx)+b\left(3a^2+b^2\right)\right)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \frac{\tanh(c+dx)\left(2b^3\tanh^8(c+dx)+2b^3\tanh^6(c+dx)+6ab^2\tanh^5(c+dx)+2b^3\tanh^4(c+\right.}{1-\text{ta}}}{d}$$

$\downarrow$ 2333

$$\frac{\frac{\tanh^2(c+dx)\left(a\left(a^2+3b^2\right)\tanh(c+dx)+b\left(3a^2+b^2\right)\right)}{2\left(1-\tanh^2(c+dx)\right)}-\frac{1}{2}\int \left(-2b^3\tanh^7(c+dx)-4b^3\tanh^5(c+dx)-6ab^2\tanh^4(c+dx)-\right.}{}$$

$\downarrow$ 2009

$$\frac{\frac{1}{2}\left(-a\left(a^2+21b^2\right)\text{arctanh}(\tanh(c+dx))+b\left(3a^2+4b^2\right)\tanh^2(c+dx)+a\left(a^2+21b^2\right)\tanh(c+dx)+b\left(6a^2+5\right.\right.}{}$$

input
```
Int[Sinh[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
((Tanh[c + d*x]^2*(b*(3*a^2 + b^2) + a*(a^2 + 3*b^2)*Tanh[c + d*x]))/(2*(1
 - Tanh[c + d*x]^2)) + (-(a*(a^2 + 21*b^2)*ArcTanh[Tanh[c + d*x]]) + b*(6*
a^2 + 5*b^2)*Log[1 - Tanh[c + d*x]^2] + a*(a^2 + 21*b^2)*Tanh[c + d*x] + b
*(3*a^2 + 4*b^2)*Tanh[c + d*x]^2 + 4*a*b^2*Tanh[c + d*x]^3 + (3*b^3*Tanh[c
 + d*x]^4)/2 + (6*a*b^2*Tanh[c + d*x]^5)/5 + (2*b^3*Tanh[c + d*x]^6)/3 + (
b^3*Tanh[c + d*x]^8)/4)/2)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 2335
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq
, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 1]}, Simp[(c*x)^m*(a + b*x^2)^(p + 1)*((a*g - b*f*x)/(2*a*b*(p + 1))), x]
+ Simp[c/(2*a*b*(p + 1))   Int[(c*x)^(m - 1)*(a + b*x^2)^(p + 1)*ExpandToSu
m[2*a*b*(p + 1)*x*Q - a*g*m + b*f*(m + 2*p + 3)*x, x], x], x]] /; FreeQ[{a,
 b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && GtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 18.96 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.91

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+3b^2a\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7c}{2}\right.}{}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3a^2b\left(\frac{\sinh(dx+c)^4}{2\cosh(dx+c)^2}-2\ln(\cosh(dx+c))+\tanh(dx+c)^2\right)+3b^2a\left(\frac{\sinh(dx+c)^7}{2\cosh(dx+c)^5}-\frac{7c}{2}\right.}{}$
risch	$-\frac{a^3x}{2}+6a^2bx-\frac{21ab^2x}{2}+5b^3x+\frac{e^{2dx+2c}a^3}{8d}+\frac{3e^{2dx+2c}a^2b}{8d}+\frac{3e^{2dx+2c}b^2a}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-}}{8d}$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*a^2*b*(1/2*sinh(d*x
+c)^4/cosh(d*x+c)^2-2*ln(cosh(d*x+c))+tanh(d*x+c)^2)+3*b^2*a*(1/2*sinh(d*x
+c)^7/cosh(d*x+c)^5-7/2*d*x-7/2*c+7/2*tanh(d*x+c)+7/6*tanh(d*x+c)^3+7/10*t
anh(d*x+c)^5)+b^3*(1/2*sinh(d*x+c)^10/cosh(d*x+c)^8-5*ln(cosh(d*x+c))+5/2*
tanh(d*x+c)^2+5/4*tanh(d*x+c)^4+5/6*tanh(d*x+c)^6+5/8*tanh(d*x+c)^8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9862 vs. $2(210) = 420$.

Time = 0.20 (sec) , antiderivative size = 9862, normalized size of antiderivative = 43.64

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 544 vs. $2(210) = 420$.

Time = 0.13 (sec) , antiderivative size = 544, normalized size of antiderivative = 2.41

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/40*a*b^2*(420*
(d*x + c)/d + 15*e^(-2*d*x - 2*c)/d - (1003*e^(-2*d*x - 2*c) + 3350*e^(-4*
d*x - 4*c) + 5590*e^(-6*d*x - 6*c) + 3915*e^(-8*d*x - 8*c) + 1455*e^(-10*d
*x - 10*c) + 15)/(d*(e^(-2*d*x - 2*c) + 5*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 10*e^(-8*d*x - 8*c) + 5*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)))
) - 1/24*b^3*(120*(d*x + c)/d - 3*e^(-2*d*x - 2*c)/d + 120*log(e^(-2*d*x -
 2*c) + 1)/d - (24*e^(-2*d*x - 2*c) - 396*e^(-4*d*x - 4*c) - 1752*e^(-6*d*
x - 6*c) - 4430*e^(-8*d*x - 8*c) - 5464*e^(-10*d*x - 10*c) - 4556*e^(-12*d
*x - 12*c) - 1896*e^(-14*d*x - 14*c) - 477*e^(-16*d*x - 16*c) + 3)/(d*(e^(
-2*d*x - 2*c) + 8*e^(-4*d*x - 4*c) + 28*e^(-6*d*x - 6*c) + 56*e^(-8*d*x -
8*c) + 70*e^(-10*d*x - 10*c) + 56*e^(-12*d*x - 12*c) + 28*e^(-14*d*x - 14*
c) + 8*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c)))) - 3/8*a^2*b*(16*(d*x + c
)/d - e^(-2*d*x - 2*c)/d + 16*log(e^(-2*d*x - 2*c) + 1)/d - (2*e^(-2*d*x -
 2*c) - 15*e^(-4*d*x - 4*c) + 1)/(d*(e^(-2*d*x - 2*c) + 2*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 577 vs. $2(210) = 420$.

Time = 0.47 (sec) , antiderivative size = 577, normalized size of antiderivative = 2.55

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/840*(105*a^3*e^(2*d*x + 2*c) + 315*a^2*b*e^(2*d*x + 2*c) + 315*a*b^2*e^(
2*d*x + 2*c) + 105*b^3*e^(2*d*x + 2*c) - 420*(a^3 - 12*a^2*b + 21*a*b^2 -
10*b^3)*(d*x + c) + 105*(2*a^3*e^(2*d*x + 2*c) - 24*a^2*b*e^(2*d*x + 2*c)
+ 42*a*b^2*e^(2*d*x + 2*c) - 20*b^3*e^(2*d*x + 2*c) - a^3 + 3*a^2*b - 3*a*
b^2 + b^3)*e^(-2*d*x - 2*c) - 840*(6*a^2*b + 5*b^3)*log(e^(2*d*x + 2*c) +
1) + (13698*a^2*b*e^(16*d*x + 16*c) + 11415*b^3*e^(16*d*x + 16*c) + 104544
*a^2*b*e^(14*d*x + 14*c) - 30240*a*b^2*e^(14*d*x + 14*c) + 74520*b^3*e^(14
*d*x + 14*c) + 353304*a^2*b*e^(12*d*x + 12*c) - 171360*a*b^2*e^(12*d*x + 1
2*c) + 252420*b^3*e^(12*d*x + 12*c) + 691488*a^2*b*e^(10*d*x + 10*c) - 446
880*a*b^2*e^(10*d*x + 10*c) + 476840*b^3*e^(10*d*x + 10*c) + 858060*a^2*b*
e^(8*d*x + 8*c) - 682080*a*b^2*e^(8*d*x + 8*c) + 601930*b^3*e^(8*d*x + 8*c
) + 691488*a^2*b*e^(6*d*x + 6*c) - 644448*a*b^2*e^(6*d*x + 6*c) + 476840*b
^3*e^(6*d*x + 6*c) + 353304*a^2*b*e^(4*d*x + 4*c) - 374304*a*b^2*e^(4*d*x
+ 4*c) + 252420*b^3*e^(4*d*x + 4*c) + 104544*a^2*b*e^(2*d*x + 2*c) - 12566
4*a*b^2*e^(2*d*x + 2*c) + 74520*b^3*e^(2*d*x + 2*c) + 13698*a^2*b - 19488*
a*b^2 + 11415*b^3)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 617, normalized size of antiderivative = 2.73

$$\int \sinh^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$= \frac{8\,(29\,b^3+6\,a\,b^2)}{d\,\left(4\,\mathrm{e}^{2\,c+2\,d\,x}+6\,\mathrm{e}^{4\,c+4\,d\,x}+4\,\mathrm{e}^{6\,c+6\,d\,x}+\mathrm{e}^{8\,c+8\,d\,x}+1\right)}$$

$$+\frac{736\,b^3}{3\,d\,\left(6\,\mathrm{e}^{2\,c+2\,d\,x}+15\,\mathrm{e}^{4\,c+4\,d\,x}+20\,\mathrm{e}^{6\,c+6\,d\,x}+15\,\mathrm{e}^{8\,c+8\,d\,x}+6\,\mathrm{e}^{10\,c+10\,d\,x}+\mathrm{e}^{12\,c+12\,d\,x}+1\right)}$$

$$-\frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+1\right)\,(6\,a^2\,b+5\,b^3)}{d}-\frac{2\,(3\,a^2\,b+18\,a\,b^2+10\,b^3)}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

$$-\frac{96\,(15\,b^3+a\,b^2)}{5\,d\,\left(5\,\mathrm{e}^{2\,c+2\,d\,x}+10\,\mathrm{e}^{4\,c+4\,d\,x}+10\,\mathrm{e}^{6\,c+6\,d\,x}+5\,\mathrm{e}^{8\,c+8\,d\,x}+\mathrm{e}^{10\,c+10\,d\,x}+1\right)}$$

$$-\frac{128\,b^3}{d\,\left(7\,\mathrm{e}^{2\,c+2\,d\,x}+21\,\mathrm{e}^{4\,c+4\,d\,x}+35\,\mathrm{e}^{6\,c+6\,d\,x}+35\,\mathrm{e}^{8\,c+8\,d\,x}+21\,\mathrm{e}^{10\,c+10\,d\,x}+7\,\mathrm{e}^{12\,c+12\,d\,x}+\mathrm{e}^{14\,c+14\,d\,x}+1\right)}$$

$$-\frac{x\,(a-b)^2\,(a-10\,b)}{2}+\frac{6\,(a^2\,b+8\,a\,b^2+10\,b^3)}{d\,\left(2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1\right)}$$

$$+\frac{32\,b^3}{d\,\left(8\,\mathrm{e}^{2\,c+2\,d\,x}+28\,\mathrm{e}^{4\,c+4\,d\,x}+56\,\mathrm{e}^{6\,c+6\,d\,x}+70\,\mathrm{e}^{8\,c+8\,d\,x}+56\,\mathrm{e}^{10\,c+10\,d\,x}+28\,\mathrm{e}^{12\,c+12\,d\,x}+8\,\mathrm{e}^{14\,c+14\,d\,x}+\mathrm{e}^{16}\right.}$$

$$+\frac{\mathrm{e}^{2\,c+2\,d\,x}\,(a+b)^3}{8\,d}-\frac{\mathrm{e}^{-2\,c-2\,d\,x}\,(a-b)^3}{8\,d}-\frac{16\,(25\,b^3+12\,a\,b^2)}{3\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1\right)}$$

input

```
int(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)^3,x)
```

output
```
(8*(6*a*b^2 + 29*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (736*b^3)/(3*d*(6*exp(2*c + 2*d*x
) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*ex
p(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (log(exp(2*c)*exp(2*d*x) + 1
)*(6*a^2*b + 5*b^3))/d - (2*(18*a*b^2 + 3*a^2*b + 10*b^3))/(d*(exp(2*c + 2
*d*x) + 1)) - (96*(a*b^2 + 15*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c
+ 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) +
 1)) - (128*b^3)/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c
 + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*
d*x) + exp(14*c + 14*d*x) + 1)) - (x*(a - b)^2*(a - 10*b))/2 + (6*(8*a*b^2
 + a^2*b + 10*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) + (32*
b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) +
70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*ex
p(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) + (exp(2*c + 2*d*x)*(a + b)^3)
/(8*d) - (exp(- 2*c - 2*d*x)*(a - b)^3)/(8*d) - (16*(12*a*b^2 + 25*b^3))/(
3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1808, normalized size of antiderivative = 8.00

$$\int \sinh^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x)
```

output
```
(120*e**(20*c + 20*d*x)*a**3 + 360*e**(20*c + 20*d*x)*a**2*b + 360*e**(20*
c + 20*d*x)*a*b**2 + 120*e**(20*c + 20*d*x)*b**3 - 5760*e**(18*c + 18*d*x)
*log(e**(2*c + 2*d*x) + 1)*a**2*b - 4800*e**(18*c + 18*d*x)*log(e**(2*c +
2*d*x) + 1)*b**3 - 480*e**(18*c + 18*d*x)*a**3*d*x + 555*e**(18*c + 18*d*x
)*a**3 + 5760*e**(18*c + 18*d*x)*a**2*b*d*x + 2295*e**(18*c + 18*d*x)*a**2
*b - 10080*e**(18*c + 18*d*x)*a*b**2*d*x + 5985*e**(18*c + 18*d*x)*a*b**2
+ 4800*e**(18*c + 18*d*x)*b**3*d*x + 2925*e**(18*c + 18*d*x)*b**3 - 46080*
e**(16*c + 16*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b - 38400*e**(16*c + 16*
d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 3840*e**(16*c + 16*d*x)*a**3*d*x + 4
6080*e**(16*c + 16*d*x)*a**2*b*d*x - 80640*e**(16*c + 16*d*x)*a*b**2*d*x +
 38400*e**(16*c + 16*d*x)*b**3*d*x - 161280*e**(14*c + 14*d*x)*log(e**(2*c
 + 2*d*x) + 1)*a**2*b - 134400*e**(14*c + 14*d*x)*log(e**(2*c + 2*d*x) + 1
)*b**3 - 13440*e**(14*c + 14*d*x)*a**3*d*x - 5580*e**(14*c + 14*d*x)*a**3
+ 161280*e**(14*c + 14*d*x)*a**2*b*d*x - 27900*e**(14*c + 14*d*x)*a**2*b -
 282240*e**(14*c + 14*d*x)*a*b**2*d*x - 91620*e**(14*c + 14*d*x)*a*b**2 +
134400*e**(14*c + 14*d*x)*b**3*d*x - 14100*e**(14*c + 14*d*x)*b**3 - 32256
0*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b - 268800*e**(12*c +
12*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 26880*e**(12*c + 12*d*x)*a**3*d*x
 - 17640*e**(12*c + 12*d*x)*a**3 + 322560*e**(12*c + 12*d*x)*a**2*b*d*x -
83880*e**(12*c + 12*d*x)*a**2*b - 564480*e**(12*c + 12*d*x)*a*b**2*d*x ...
```

3.68 $\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 260

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= -\frac{9a^2b\arctan(\sinh(c+dx))}{2d}-\frac{315b^3\arctan(\sinh(c+dx))}{128d}+\frac{a^3\cosh(c+dx)}{d}\\ &+\frac{3ab^2\cosh(c+dx)}{d}+\frac{9ab^2\operatorname{sech}(c+dx)}{d}-\frac{3ab^2\operatorname{sech}^3(c+dx)}{d}+\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}\\ &+\frac{3a^2b\sinh(c+dx)}{d}+\frac{b^3\sinh(c+dx)}{d}+\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\\ &+\frac{325b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}-\frac{105b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{64d}\\ &+\frac{11b^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{16d}-\frac{b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{8d}\end{aligned}$$

output
```
-9/2*a^2*b*arctan(sinh(d*x+c))/d-315/128*b^3*arctan(sinh(d*x+c))/d+a^3*cos
h(d*x+c)/d+3*a*b^2*cosh(d*x+c)/d+9*a*b^2*sech(d*x+c)/d-3*a*b^2*sech(d*x+c)
^3/d+3/5*a*b^2*sech(d*x+c)^5/d+3*a^2*b*sinh(d*x+c)/d+b^3*sinh(d*x+c)/d+3/2
*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+325/128*b^3*sech(d*x+c)*tanh(d*x+c)/d-105
/64*b^3*sech(d*x+c)^3*tanh(d*x+c)/d+11/16*b^3*sech(d*x+c)^5*tanh(d*x+c)/d-
1/8*b^3*sech(d*x+c)^7*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 6.16 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.65

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{640a(a^2+3b^2)\cosh(c+dx)+b\left(-90(64a^2+35b^2)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+640(3a^2+b^2)\sinh(c+\right.}{}$$

input
```
Integrate[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(640*a*(a^2 + 3*b^2)*Cosh[c + d*x] + b*(-90*(64*a^2 + 35*b^2)*ArcTan[Tanh[
(c + d*x)/2]] + 640*(3*a^2 + b^2)*Sinh[c + d*x] - 80*b^2*Sech[c + d*x]^7*T
anh[c + d*x] - 30*b*Sech[c + d*x]^3*(64*a + 35*b*Tanh[c + d*x]) + 8*b*Sech
[c + d*x]^5*(48*a + 55*b*Tanh[c + d*x]) + 5*Sech[c + d*x]*(1152*a*b + (192
*a^2 + 325*b^2)*Tanh[c + d*x])))/(640*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.86 (sec) , antiderivative size = 302, normalized size of antiderivative = 1.16, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -i\sin(ic+idx)\left(a+ib\tan(ic+idx)^3\right)^3dx$$
$$\Big\downarrow \text{26}$$
$$-i\int \sin(ic+idx)\left(ib\tan(ic+idx)^3+a\right)^3dx$$

$$\Big\downarrow \text{4149}$$

$$-i\int\left(ib^3\sinh(c+dx)\tanh^9(c+dx)+3iab^2\sinh(c+dx)\tanh^6(c+dx)+3ia^2b\sinh(c+dx)\tanh^3(c+dx)+ic\right.$$

$$\Big\downarrow \text{2009}$$

$$-i\left(\frac{ia^3\cosh(c+dx)}{d}-\frac{9ia^2b\arctan(\sinh(c+dx))}{2d}+\frac{9ia^2b\sinh(c+dx)}{2d}-\frac{3ia^2b\sinh(c+dx)\tanh^2(c+dx)}{2d}+\right.$$

input
```
Int[Sinh[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-I)*((((-9*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d - (((315*I)/128)*b^3*ArcT
an[Sinh[c + d*x]])/d + (I*a^3*Cosh[c + d*x])/d + ((3*I)*a*b^2*Cosh[c + d*x
])/d + ((9*I)*a*b^2*Sech[c + d*x])/d - ((3*I)*a*b^2*Sech[c + d*x]^3)/d + (
((3*I)/5)*a*b^2*Sech[c + d*x]^5)/d + (((9*I)/2)*a^2*b*Sinh[c + d*x])/d + (
((315*I)/128)*b^3*Sinh[c + d*x])/d - (((3*I)/2)*a^2*b*Sinh[c + d*x]*Tanh[c
 + d*x]^2)/d - (((105*I)/128)*b^3*Sinh[c + d*x]*Tanh[c + d*x]^2)/d - (((21
*I)/64)*b^3*Sinh[c + d*x]*Tanh[c + d*x]^4)/d - (((3*I)/16)*b^3*Sinh[c + d*
x]*Tanh[c + d*x]^6)/d - ((I/8)*b^3*Sinh[c + d*x]*Tanh[c + d*x]^8)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 12.30 (sec) , antiderivative size = 297, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(e^{dx+c}\right)\right)+3b^2a\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+6\right.}{}$
default	$\frac{a^3\cosh(dx+c)+3a^2b\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\left(e^{dx+c}\right)\right)+3b^2a\left(\frac{\sinh(dx+c)^6}{\cosh(dx+c)^5}+6\right.}{}$
risch	$\frac{e^{dx+c}a^3}{2d}+\frac{3\,e^{dx+c}a^2b}{2d}+\frac{3\,e^{dx+c}b^2a}{2d}+\frac{e^{dx+c}b^3}{2d}+\frac{e^{-dx-c}a^3}{2d}-\frac{3\,e^{-dx-c}a^2b}{2d}+\frac{3b^2a\,e^{-dx-c}}{2d}-\frac{b^3e^{-dx-c}}{2d}$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*cosh(d*x+c)+3*a^2*b*(sinh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/co
sh(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+c)-3*arctan(exp(d*x+c)))+3*b^2*a*(sin
h(d*x+c)^6/cosh(d*x+c)^5+6*sinh(d*x+c)^4/cosh(d*x+c)^5+8*sinh(d*x+c)^2/cos
h(d*x+c)^5+16/5/cosh(d*x+c)^5)+b^3*(sinh(d*x+c)^9/cosh(d*x+c)^8+9*sinh(d*x
+c)^7/cosh(d*x+c)^8+21*sinh(d*x+c)^5/cosh(d*x+c)^8+21*sinh(d*x+c)^3/cosh(d
*x+c)^8+9*sinh(d*x+c)/cosh(d*x+c)^8-9*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^
5+35/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)-315/64*arctan(exp(d
*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6410 vs. 2(244) = 488.

Time = 0.16 (sec) , antiderivative size = 6410, normalized size of antiderivative = 24.65

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)^3\sinh\left(c+dx\right)dx$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*sinh(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 484, normalized size of antiderivative = 1.86

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{1}{64}b^3\left(\frac{315\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{32\,e^{(-dx-c)}}{d} + \frac{581\,e^{(-2\,dx-2\,c)} + 1681\,e^{(-4\,dx-4\,c)} + 3605\,e^{(-6\,dx-6\,c)} + 2569}{d\left(e^{(-dx-c)} + 8\,e^{(-3\,dx-3\,c)} + 28\,e^{(-5\,dx-5\,c)} + 56\,e^{(-7\,dx-}\right.}\right.$$
$$+ \frac{3}{2}a^2b\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d\left(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)}\right)}\right)$$
$$+ \frac{3}{10}ab^2\left(\frac{5\,e^{(-dx-c)}}{d} + \frac{85\,e^{(-2\,dx-2\,c)} + 210\,e^{(-4\,dx-4\,c)} + 314\,e^{(-6\,dx-6\,c)} + 185\,e^{(-8\,dx-8\,c)} + 65\,e^{(-10\,dx-10\,c}}{d\left(e^{(-dx-c)} + 5\,e^{(-3\,dx-3\,c)} + 10\,e^{(-5\,dx-5\,c)} + 10\,e^{(-7\,dx-7\,c)} + 5\,e^{(-9\,dx-9\,c)} + e^{(-11\,dx}\right.}\right.$$
$$+ \frac{a^3\cosh\left(dx+c\right)}{d}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
1/64*b^3*(315*arctan(e^(-d*x - c))/d - 32*e^(-d*x - c)/d + (581*e^(-2*d*x
- 2*c) + 1681*e^(-4*d*x - 4*c) + 3605*e^(-6*d*x - 6*c) + 2569*e^(-8*d*x -
8*c) + 1463*e^(-10*d*x - 10*c) - 917*e^(-12*d*x - 12*c) - 529*e^(-14*d*x -
 14*c) - 293*e^(-16*d*x - 16*c) + 32)/(d*(e^(-d*x - c) + 8*e^(-3*d*x - 3*c
) + 28*e^(-5*d*x - 5*c) + 56*e^(-7*d*x - 7*c) + 70*e^(-9*d*x - 9*c) + 56*e
^(-11*d*x - 11*c) + 28*e^(-13*d*x - 13*c) + 8*e^(-15*d*x - 15*c) + e^(-17*
d*x - 17*c)))) + 3/2*a^2*b*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4
*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x -
 3*c) + e^(-5*d*x - 5*c)))) + 3/10*a*b^2*(5*e^(-d*x - c)/d + (85*e^(-2*d*x
 - 2*c) + 210*e^(-4*d*x - 4*c) + 314*e^(-6*d*x - 6*c) + 185*e^(-8*d*x - 8*
c) + 65*e^(-10*d*x - 10*c) + 5)/(d*(e^(-d*x - c) + 5*e^(-3*d*x - 3*c) + 10
*e^(-5*d*x - 5*c) + 10*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) + e^(-11*d*x
- 11*c)))) + a^3*cosh(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.78

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{160\,a^3e^{(dx+c)} + 480\,a^2be^{(dx+c)} + 480\,ab^2e^{(dx+c)} + 160\,b^3e^{(dx+c)} - 45\,(64\,a^2b + 35\,b^3)\arctan\left(e^{(dx+c)}\right) + 160}{}$$

input
```
integrate(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/320*(160*a^3*e^(d*x + c) + 480*a^2*b*e^(d*x + c) + 480*a*b^2*e^(d*x + c)
 + 160*b^3*e^(d*x + c) - 45*(64*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + 160*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*e^(-d*x - c) + (960*a^2*b*e^(15*d*x + 15*c
) + 5760*a*b^2*e^(15*d*x + 15*c) + 1625*b^3*e^(15*d*x + 15*c) + 4800*a^2*b
*e^(13*d*x + 13*c) + 32640*a*b^2*e^(13*d*x + 13*c) + 3925*b^3*e^(13*d*x +
13*c) + 8640*a^2*b*e^(11*d*x + 11*c) + 88704*a*b^2*e^(11*d*x + 11*c) + 906
5*b^3*e^(11*d*x + 11*c) + 4800*a^2*b*e^(9*d*x + 9*c) + 143232*a*b^2*e^(9*d
*x + 9*c) + 1645*b^3*e^(9*d*x + 9*c) - 4800*a^2*b*e^(7*d*x + 7*c) + 143232
*a*b^2*e^(7*d*x + 7*c) - 1645*b^3*e^(7*d*x + 7*c) - 8640*a^2*b*e^(5*d*x +
5*c) + 88704*a*b^2*e^(5*d*x + 5*c) - 9065*b^3*e^(5*d*x + 5*c) - 4800*a^2*b
*e^(3*d*x + 3*c) + 32640*a*b^2*e^(3*d*x + 3*c) - 3925*b^3*e^(3*d*x + 3*c)
- 960*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1625*b^3*e^(d*x + c))/(
e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 707, normalized size of antiderivative = 2.72

$$\int \sinh(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)^3,x)
```

output
```
(exp(c + d*x)*(a + b)^3)/(2*d) + (exp(- c - d*x)*(a - b)^3)/(2*d) - (9*ata
n((exp(d*x)*exp(c)*(35*b^3*(d^2)^(1/2) + 64*a^2*b*(d^2)^(1/2)))/(d*(1225*b
^6 + 4480*a^2*b^4 + 4096*a^4*b^2)^(1/2)))*(1225*b^6 + 4480*a^2*b^4 + 4096*
a^4*b^2)^(1/2))/(64*(d^2)^(1/2)) + (exp(c + d*x)*(1728*a*b^2 + 2455*b^3))/
(40*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) -
(exp(c + d*x)*(768*a*b^2 + 2605*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*
c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (188*b^3*exp(c
+ d*x))/(d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x)
 + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) +
 (exp(c + d*x)*(1152*a*b^2 + 192*a^2*b + 325*b^3))/(64*d*(exp(2*c + 2*d*x)
 + 1)) + (2*exp(c + d*x)*(48*a*b^2 + 475*b^3))/(5*d*(5*exp(2*c + 2*d*x) +
10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c
+ 10*d*x) + 1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4
*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10
*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(7
68*a*b^2 + 192*a^2*b + 745*b^3))/(32*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d
*x) + 1)) - (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*
d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) +
 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1184, normalized size of antiderivative = 4.55

$$\int \sinh(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x)
```

output
```
( - 80640*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a**2*b - 44100*e**(3*c + 3*d
*x)*atan(e**(c + d*x))*b**3 - 80640*e**(c + d*x)*atan(e**(c + d*x))*a**2*b
 - 44100*e**(c + d*x)*atan(e**(c + d*x))*b**3 - 160*e**(3*c + 3*d*x)*cosh(
c + d*x)*tanh(c + d*x)**7*b**3 - 528*e**(3*c + 3*d*x)*cosh(c + d*x)*tanh(c
 + d*x)**5*b**3 - 1344*e**(3*c + 3*d*x)*cosh(c + d*x)*tanh(c + d*x)**4*a*b
**2 - 1684*e**(3*c + 3*d*x)*cosh(c + d*x)*tanh(c + d*x)**3*b**3 - 7392*e**
(3*c + 3*d*x)*cosh(c + d*x)*tanh(c + d*x)**2*a*b**2 - 13440*e**(3*c + 3*d*
x)*cosh(c + d*x)*tanh(c + d*x)*a**2*b - 10718*e**(3*c + 3*d*x)*cosh(c + d*
x)*tanh(c + d*x)*b**3 + 8960*e**(3*c + 3*d*x)*cosh(c + d*x)*a**3 - 14784*e
**(3*c + 3*d*x)*cosh(c + d*x)*a*b**2 - 160*e**(c + d*x)*cosh(c + d*x)*tanh
(c + d*x)**7*b**3 - 528*e**(c + d*x)*cosh(c + d*x)*tanh(c + d*x)**5*b**3 -
 1344*e**(c + d*x)*cosh(c + d*x)*tanh(c + d*x)**4*a*b**2 - 1684*e**(c + d*
x)*cosh(c + d*x)*tanh(c + d*x)**3*b**3 - 7392*e**(c + d*x)*cosh(c + d*x)*t
anh(c + d*x)**2*a*b**2 - 13440*e**(c + d*x)*cosh(c + d*x)*tanh(c + d*x)*a*
*2*b - 10718*e**(c + d*x)*cosh(c + d*x)*tanh(c + d*x)*b**3 + 8960*e**(c +
d*x)*cosh(c + d*x)*a**3 - 14784*e**(c + d*x)*cosh(c + d*x)*a*b**2 + 20160*
e**(4*c + 4*d*x)*a**2*b + 25200*e**(4*c + 4*d*x)*a*b**2 + 11025*e**(4*c +
4*d*x)*b**3 - 1120*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)**8*b**3 -
1520*e**(3*c + 3*d*x)*sinh(c + d*x)*tanh(c + d*x)**6*b**3 - 5376*e**(3*c +
 3*d*x)*sinh(c + d*x)*tanh(c + d*x)**5*a*b**2 - 2412*e**(3*c + 3*d*x)*s...
```

3.69 $\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 219

$$\begin{aligned}
&\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx \\
&= \frac{3a^2b\arctan(\sinh(c+dx))}{2d}+\frac{35b^3\arctan(\sinh(c+dx))}{128d} \\
&\quad-\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{d}-\frac{3ab^2\operatorname{sech}(c+dx)}{d}+\frac{2ab^2\operatorname{sech}^3(c+dx)}{d} \\
&\quad-\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}-\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d} \\
&\quad-\frac{35b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}-\frac{35b^3\operatorname{sech}(c+dx)\tanh^3(c+dx)}{192d} \\
&\quad-\frac{7b^3\operatorname{sech}(c+dx)\tanh^5(c+dx)}{48d}-\frac{b^3\operatorname{sech}(c+dx)\tanh^7(c+dx)}{8d}
\end{aligned}$$

output
```
3/2*a^2*b*arctan(sinh(d*x+c))/d+35/128*b^3*arctan(sinh(d*x+c))/d-a^3*arcta
nh(cosh(d*x+c))/d-3*a*b^2*sech(d*x+c)/d+2*a*b^2*sech(d*x+c)^3/d-3/5*a*b^2*
sech(d*x+c)^5/d-3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d-35/128*b^3*sech(d*x+c)
*tanh(d*x+c)/d-35/192*b^3*sech(d*x+c)*tanh(d*x+c)^3/d-7/48*b^3*sech(d*x+c)
*tanh(d*x+c)^5/d-1/8*b^3*sech(d*x+c)*tanh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 5.64 (sec) , antiderivative size = 168, normalized size of antiderivative = 0.77

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{30\left(b(192a^2+35b^2)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+64a^3\left(-\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.}{}$$

input
```
Integrate[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(30*(b*(192*a^2 + 35*b^2)*ArcTan[Tanh[(c + d*x)/2]] + 64*a^3*(-Log[Cosh[(c
 + d*x)/2]] + Log[Sinh[(c + d*x)/2]])) + 240*b^3*Sech[c + d*x]^7*Tanh[c +
d*x] - 8*b^2*Sech[c + d*x]^5*(144*a + 125*b*Tanh[c + d*x]) + 10*b^2*Sech[c
 + d*x]^3*(384*a + 163*b*Tanh[c + d*x]) - 45*b*Sech[c + d*x]*(128*a*b + (6
4*a^2 + 31*b^2)*Tanh[c + d*x]))/(1920*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.90 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\left(ib\tan(ic+idx)^3+a\right)^3}{\sin(ic+idx)}dx$$

$$\Big\downarrow \text{4149}$$

$$i\int\left(-ib^3\text{sech}(c+dx)\tanh^8(c+dx)-3iab^2\text{sech}(c+dx)\tanh^5(c+dx)-3ia^2b\text{sech}(c+dx)\tanh^2(c+dx)-ia^3\right.$$

$$\Big\downarrow \text{2009}$$

$$i\left(\frac{ia^3\text{arctanh}(\cosh(c+dx))}{d}-\frac{3ia^2b\arctan(\sinh(c+dx))}{2d}+\frac{3ia^2b\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{3iab^2\text{sech}^5(c+\right.$$

input
```
Int[Csch[c + d*x]*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
I*((((-3*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d - (((35*I)/128)*b^3*ArcTan[S
inh[c + d*x]])/d + (I*a^3*ArcTanh[Cosh[c + d*x]])/d + ((3*I)*a*b^2*Sech[c
+ d*x])/d - ((2*I)*a*b^2*Sech[c + d*x]^3)/d + (((3*I)/5)*a*b^2*Sech[c + d*
x]^5)/d + (((3*I)/2)*a^2*b*Sech[c + d*x]*Tanh[c + d*x])/d + (((35*I)/128)*
b^3*Sech[c + d*x]*Tanh[c + d*x])/d + (((35*I)/192)*b^3*Sech[c + d*x]*Tanh[
c + d*x]^3)/d + (((7*I)/48)*b^3*Sech[c + d*x]*Tanh[c + d*x]^5)/d + ((I/8)*
b^3*Sech[c + d*x]*Tanh[c + d*x]^7)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 7.94 (sec) , antiderivative size = 245, normalized size of antiderivative = 1.12

method	result
derivativedivides	$\frac{-2a^3 \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+3a^2 b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+3b^2 a\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx}{3\cosh(dx}\right.}{}$
default	$\frac{-2a^3 \operatorname{arctanh}\left(\mathrm{e}^{dx+c}\right)+3a^2 b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(\mathrm{e}^{dx+c}\right)\right)+3b^2 a\left(-\frac{\sinh(dx+c)^4}{\cosh(dx+c)^5}-\frac{4\sinh(dx}{3\cosh(dx}\right.}{}$
risch	$-\frac{b\,\mathrm{e}^{dx+c}\left(2880a^2\mathrm{e}^{14dx+14c}+5760ab\,\mathrm{e}^{14dx+14c}+1395b^2\mathrm{e}^{14dx+14c}+14400\,\mathrm{e}^{12dx+12c}a^2+24960\,\mathrm{e}^{12dx+12c}ab+455\,\mathrm{e}^{12dx}\right.}{}$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*se
ch(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*b^2*a*(-sinh(d*x+c)^4/cosh(d*x
+c)^5-4/3*sinh(d*x+c)^2/cosh(d*x+c)^5-8/15/cosh(d*x+c)^5)+b^3*(-sinh(d*x+c
)^7/cosh(d*x+c)^8-7/3*sinh(d*x+c)^5/cosh(d*x+c)^8-7/3*sinh(d*x+c)^3/cosh(d
*x+c)^8-sinh(d*x+c)/cosh(d*x+c)^8+(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35
/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x+c)+35/64*arctan(exp(d*x+c)
)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7127 vs. 2(203) = 406.

Time = 0.16 (sec) , antiderivative size = 7127, normalized size of antiderivative = 32.54

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\int \left(a+b\tanh^3\left(c+dx\right)\right)^3\operatorname{csch}\left(c+dx\right)dx$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. 2(203) = 406.

Time = 0.12 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.99

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/192*b^3*(105*arctan(e^(-d*x - c))/d + (279*e^(-d*x - c) + 91*e^(-3*d*x
- 3*c) + 1799*e^(-5*d*x - 5*c) - 1085*e^(-7*d*x - 7*c) + 1085*e^(-9*d*x -
9*c) - 1799*e^(-11*d*x - 11*c) - 91*e^(-13*d*x - 13*c) - 279*e^(-15*d*x -
15*c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c)
+ 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*
e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x -
 c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*
d*x - 4*c) + 1))) - 2/5*a*b^2*(15*e^(-d*x - c)/(d*(5*e^(-2*d*x - 2*c) + 10
*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) + 20*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x
- 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1
)) + 58*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10
*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 20*e^(
-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-9*d*x - 9*
c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*
e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^3*log(tanh(1/2*d*x + 1/2*
c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 414 vs. $2(203) = 406$.

Time = 0.32 (sec) , antiderivative size = 414, normalized size of antiderivative = 1.89

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx =$$

$$-\frac{960\,a^3\log\left(e^{(dx+c)}+1\right)-960\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)-15\left(192\,a^2b+35\,b^3\right)\arctan\left(e^{(dx+c)}\right)+\frac{2880\,a^2be^{(15\cdots}}{\cdots}}{\cdots}$$

input
```
integrate(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/960*(960*a^3*log(e^(d*x + c) + 1) - 960*a^3*log(abs(e^(d*x + c) - 1)) -
 15*(192*a^2*b + 35*b^3)*arctan(e^(d*x + c)) + (2880*a^2*b*e^(15*d*x + 15*
c) + 5760*a*b^2*e^(15*d*x + 15*c) + 1395*b^3*e^(15*d*x + 15*c) + 14400*a^2
*b*e^(13*d*x + 13*c) + 24960*a*b^2*e^(13*d*x + 13*c) + 455*b^3*e^(13*d*x +
 13*c) + 25920*a^2*b*e^(11*d*x + 11*c) + 62592*a*b^2*e^(11*d*x + 11*c) + 8
995*b^3*e^(11*d*x + 11*c) + 14400*a^2*b*e^(9*d*x + 9*c) + 103296*a*b^2*e^(
9*d*x + 9*c) - 5425*b^3*e^(9*d*x + 9*c) - 14400*a^2*b*e^(7*d*x + 7*c) + 10
3296*a*b^2*e^(7*d*x + 7*c) + 5425*b^3*e^(7*d*x + 7*c) - 25920*a^2*b*e^(5*d
*x + 5*c) + 62592*a*b^2*e^(5*d*x + 5*c) - 8995*b^3*e^(5*d*x + 5*c) - 14400
*a^2*b*e^(3*d*x + 3*c) + 24960*a*b^2*e^(3*d*x + 3*c) - 455*b^3*e^(3*d*x +
3*c) - 2880*a^2*b*e^(d*x + c) + 5760*a*b^2*e^(d*x + c) - 1395*b^3*e^(d*x +
 c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 7.74 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.06

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x),x)
```

output
```
(a^3*log(exp(c + d*x) - 1))/d - (a^3*log(exp(c + d*x) + 1))/d - (exp(c + d
*x)*(4224*a*b^2 + 4445*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*
x) + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 35*b^2)
*1i)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 35*b^2)*1i)/(128*d) +
(exp(c + d*x)*(768*a*b^2 + 1925*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*
c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (532*b^3*exp(c
+ d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*
x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
 - (3*exp(c + d*x)*(128*a*b^2 + 64*a^2*b + 31*b^3))/(64*d*(exp(2*c + 2*d*x
) + 1)) - (2*exp(c + d*x)*(144*a*b^2 + 1225*b^3))/(15*d*(5*exp(2*c + 2*d*x
) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(1
0*c + 10*d*x) + 1)) - (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*e
xp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c
+ 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) + (exp(c + d*x
)*(1536*a*b^2 + 576*a^2*b + 931*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c
+ 4*d*x) + 1)) + (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c
 + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d
*x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) +
1))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 1304, normalized size of antiderivative = 5.95

$$\int \operatorname{csch}(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)*(a+b*tanh(d*x+c)^3)^3,x)
```

output
```
(2880*e**(16*c + 16*d*x)*atan(e**(c + d*x))*a**2*b + 525*e**(16*c + 16*d*x
)*atan(e**(c + d*x))*b**3 + 23040*e**(14*c + 14*d*x)*atan(e**(c + d*x))*a*
*2*b + 4200*e**(14*c + 14*d*x)*atan(e**(c + d*x))*b**3 + 80640*e**(12*c +
12*d*x)*atan(e**(c + d*x))*a**2*b + 14700*e**(12*c + 12*d*x)*atan(e**(c +
d*x))*b**3 + 161280*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a**2*b + 29400*e
**(10*c + 10*d*x)*atan(e**(c + d*x))*b**3 + 201600*e**(8*c + 8*d*x)*atan(e
**(c + d*x))*a**2*b + 36750*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**3 + 161
280*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**2*b + 29400*e**(6*c + 6*d*x)*at
an(e**(c + d*x))*b**3 + 80640*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b +
 14700*e**(4*c + 4*d*x)*atan(e**(c + d*x))*b**3 + 23040*e**(2*c + 2*d*x)*a
tan(e**(c + d*x))*a**2*b + 4200*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**3 +
 2880*atan(e**(c + d*x))*a**2*b + 525*atan(e**(c + d*x))*b**3 + 960*e**(16
*c + 16*d*x)*log(e**(c + d*x) - 1)*a**3 - 960*e**(16*c + 16*d*x)*log(e**(c
 + d*x) + 1)*a**3 - 2880*e**(15*c + 15*d*x)*a**2*b - 5760*e**(15*c + 15*d*
x)*a*b**2 - 1395*e**(15*c + 15*d*x)*b**3 + 7680*e**(14*c + 14*d*x)*log(e**
(c + d*x) - 1)*a**3 - 7680*e**(14*c + 14*d*x)*log(e**(c + d*x) + 1)*a**3 -
 14400*e**(13*c + 13*d*x)*a**2*b - 24960*e**(13*c + 13*d*x)*a*b**2 - 455*e
**(13*c + 13*d*x)*b**3 + 26880*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a*
*3 - 26880*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*a**3 - 25920*e**(11*c
+ 11*d*x)*a**2*b - 62592*e**(11*c + 11*d*x)*a*b**2 - 8995*e**(11*c + 11...
```

3.70 $\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 71

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=-\frac{a^3\coth(c+dx)}{d}+\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^8(c+dx)}{8d}$$

output
```
-a^3*coth(d*x+c)/d+3/2*a^2*b*tanh(d*x+c)^2/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/8
*b^3*tanh(d*x+c)^8/d
```

Mathematica [A] (verified)

Time = 1.84 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.59

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{-40a^3\coth(c+dx)+b\left(-20b^2\operatorname{sech}^6(c+dx)+5b^2\operatorname{sech}^8(c+dx)+24ab\tanh(c+dx)+6b\operatorname{sech}^4(c+dx)\right.}{40d}$$

input
```
Integrate[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-40*a^3*Coth[c + d*x] + b*(-20*b^2*Sech[c + d*x]^6 + 5*b^2*Sech[c + d*x]^
8 + 24*a*b*Tanh[c + d*x] + 6*b*Sech[c + d*x]^4*(5*b + 4*a*Tanh[c + d*x]) -
 4*Sech[c + d*x]^2*(15*a^2 + 5*b^2 + 12*a*b*Tanh[c + d*x])))/(40*d)
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 25, 4146, 802, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(ib\tan(ic+idx)^3+a\right)^3}{\sin(ic+idx)^2}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \coth^2(c+dx)\left(b\tanh^3(c+dx)+a\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{802}$$

$$\frac{\int \left(b^3\tanh^7(c+dx)+3ab^2\tanh^4(c+dx)+3a^2b\tanh(c+dx)+a^3\coth^2(c+dx)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-a^3\coth(c+dx)+\frac{3}{2}a^2b\tanh^2(c+dx)+\frac{3}{5}ab^2\tanh^5(c+dx)+\frac{1}{8}b^3\tanh^8(c+dx)}{d}$$

input
```
Int[Csch[c + d*x]^2*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-(a^3*Coth[c + d*x]) + (3*a^2*b*Tanh[c + d*x]^2)/2 + (3*a*b^2*Tanh[c + d*
x]^5)/5 + (b^3*Tanh[c + d*x]^8)/8)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 802
```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(65) = 130$.

Time = 30.42 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.41

method	result
derivativedivides	$\frac{-\coth(dx+c)a^3-\frac{3a^2b}{2\cosh(dx+c)^2}+3b^2a\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c}{8}\right.}{d}$
default	$\frac{-\coth(dx+c)a^3-\frac{3a^2b}{2\cosh(dx+c)^2}+3b^2a\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c}{8}\right.}{d}$
risch	$-\frac{2(35b^3e^{8dx+8c}-35b^3e^{6dx+6c}+5b^3e^{4dx+4c}+40e^{2dx+2c}a^3-3b^2a+5a^3+135a^2b\,e^{12dx+12c}+75a^2b\,e^{10dx+10c}+30a\,b^2e}{}$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-coth(d*x+c)*a^3-3/2*a^2*b/cosh(d*x+c)^2+3*b^2*a*(-1/2*sinh(d*x+c)^3/
cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/
15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^6/cosh(d*x+c)^8-3/4*s
inh(d*x+c)^4/cosh(d*x+c)^8-1/2*sinh(d*x+c)^2/cosh(d*x+c)^8-1/8/cosh(d*x+c)
^8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1192 vs. $2(65) = 130$.

Time = 0.08 (sec) , antiderivative size = 1192, normalized size of antiderivative = 16.79

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
-2/5*((10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 8*(15*a^2*b
 + 18*a*b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (10*a^3 + 15*a^2*b +
12*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^
3)*cosh(d*x + c)^6 + 2*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3 + 14*(10*a^3
+ 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(14*(1
5*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 27*(5*a^2*b + 2*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 20*(14*a^3 + 3*a^2*b + 2*b^3)*cosh(d*x + c)^4
+ 10*(7*(10*a^3 + 15*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 28*a^3 +
6*a^2*b + 4*b^3 + 3*(40*a^3 + 30*a^2*b + 12*a*b^2 - 5*b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 8*(7*(15*a^2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 4
5*(5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 15*(7*a^2*b + 2*a*b^2 + b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + 350*a^3 - 75*a^2*b - 12*a*b^2 + 35*b^3 + 2*(28
0*a^3 - 30*a^2*b - 12*a*b^2 - 35*b^3)*cosh(d*x + c)^2 + 2*(14*(10*a^3 + 15
*a^2*b + 12*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 15*(40*a^3 + 30*a^2*b + 12*a*
b^2 - 5*b^3)*cosh(d*x + c)^4 + 280*a^3 - 30*a^2*b - 12*a*b^2 - 35*b^3 + 60
*(14*a^3 + 3*a^2*b + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(2*(15*a^
2*b + 18*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 27*(5*a^2*b + 2*a*b^2)*cosh(d*x
+ c)^5 + 30*(7*a^2*b + 2*a*b^2 + b^3)*cosh(d*x + c)^3 + 21*(5*a^2*b + 2*a*
b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 + 6*d*cosh(d*x + c)^8 + 3*(15*d*...
```

Sympy [F]

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^3{(c+dx)}\right)^3 \operatorname{csch}^2{(c+dx)} \, dx$$

input
```
integrate(csch(d*x+c)**2*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(65) = 130$.

Time = 0.05 (sec) , antiderivative size = 679, normalized size of antiderivative = 9.56

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-2*b^3*(e^(-2*d*x - 2*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56
*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-1
2*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-6*
d*x - 6*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6
*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c)
+ 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 7*e^(-10*d*x - 10*c)/(
d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(
-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d
*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + e^(-14*d*x - 14*c)/(d*(8*e^(-2*d*x
 - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c)
+ 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) + e
^(-16*d*x - 16*c) + 1))) + 6/5*a*b^2*(10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d
*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^3/(d*(e^(
-2*d*x - 2*c) - 1)) - 6*a^2*b/(d*(e^(d*x + c) + e^(-d*x - c))^2)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 311 vs. $2(65) = 130$.

Time = 0.34 (sec) , antiderivative size = 311, normalized size of antiderivative = 4.38

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx =$$

$$-\frac{2\left(\frac{5\,a^3}{e^{(2\,dx+2\,c)}-1}+\frac{15\,a^2be^{(14\,dx+14\,c)}+15\,ab^2e^{(14\,dx+14\,c)}+5\,b^3e^{(14\,dx+14\,c)}+90\,a^2be^{(12\,dx+12\,c)}+45\,ab^2e^{(12\,dx+12\,c)}+225\,a^2be^{(10\,dx+10}}{}\right.}{}$$

input
```
integrate(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-2/5*(5*a^3/(e^(2*d*x + 2*c) - 1) + (15*a^2*b*e^(14*d*x + 14*c) + 15*a*b^2
*e^(14*d*x + 14*c) + 5*b^3*e^(14*d*x + 14*c) + 90*a^2*b*e^(12*d*x + 12*c)
+ 45*a*b^2*e^(12*d*x + 12*c) + 225*a^2*b*e^(10*d*x + 10*c) + 75*a*b^2*e^(1
0*d*x + 10*c) + 35*b^3*e^(10*d*x + 10*c) + 300*a^2*b*e^(8*d*x + 8*c) + 105
*a*b^2*e^(8*d*x + 8*c) + 225*a^2*b*e^(6*d*x + 6*c) + 93*a*b^2*e^(6*d*x + 6
*c) + 35*b^3*e^(6*d*x + 6*c) + 90*a^2*b*e^(4*d*x + 4*c) + 39*a*b^2*e^(4*d*
x + 4*c) + 15*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x + 2*c) + 5*b^3*e^(2
*d*x + 2*c) + 3*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 1515, normalized size of antiderivative = 21.34

$$\int \operatorname{csch}^2(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^2,x)
```

output
```
((3*a*b^2 - 15*a^2*b + 7*b^3)/(28*d) - (exp(2*c + 2*d*x)*(3*a*b^2 + 3*a^2*
b + b^3))/(4*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((9*a*b^2 +
 15*a^2*b - 35*b^3)/(140*d) + (exp(6*c + 6*d*x)*(3*a*b^2 + 3*a^2*b + b^3))
/(4*d) + (3*exp(2*c + 2*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(28*d) - (3*exp(
4*c + 4*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d))/(4*exp(2*c + 2*d*x) + 6
*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((exp(10*
c + 10*d*x)*(3*a*b^2 + 3*a^2*b + b^3))/(4*d) - (3*a*b^2 + 9*a^2*b + 7*b^3)
/(28*d) + (5*exp(6*c + 6*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*d) - (5*exp
(8*c + 8*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(28*d) + (exp(2*c + 2*d*x)*(9*
a*b^2 - 15*a^2*b + 35*b^3))/(28*d) + (exp(4*c + 4*d*x)*(9*a*b^2 + 15*a^2*b
 - 35*b^3))/(14*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c
 + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x
) + 1) - ((9*a^2*b - 3*a*b^2 + 7*b^3)/(28*d) + (exp(4*c + 4*d*x)*(3*a*b^2
+ 3*a^2*b + b^3))/(4*d) - (exp(2*c + 2*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/
(14*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
 ((9*a*b^2 - 15*a^2*b + 35*b^3)/(140*d) + (exp(8*c + 8*d*x)*(3*a*b^2 + 3*a
^2*b + b^3))/(4*d) + (3*exp(4*c + 4*d*x)*(9*a^2*b - 3*a*b^2 + 7*b^3))/(14*
d) - (exp(6*c + 6*d*x)*(3*a*b^2 - 15*a^2*b + 7*b^3))/(7*d) + (exp(2*c + 2*
d*x)*(9*a*b^2 + 15*a^2*b - 35*b^3))/(35*d))/(5*exp(2*c + 2*d*x) + 10*exp(4
*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 587, normalized size of antiderivative = 8.27

$$\int \operatorname{csch}^2(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{-\frac{36e^{4dx+4c}a\,b^2}{7}-\frac{2b^3}{7}+\frac{6e^{18dx+18c}a^2b}{7}+\frac{6e^{18dx+18c}a\,b^2}{7}-\frac{90e^{14dx+14c}a^2b}{7}+\frac{36e^{14dx+14c}a\,b^2}{7}-18e^{10dx+10c}a^2b+18e^{8dx+\cdots}}{\cdots}$$

input
```
int(csch(d*x+c)^2*(a+b*tanh(d*x+c)^3)^3,x)
```

output
```
(2*(5*e**(18*c + 18*d*x)*a**3 + 15*e**(18*c + 18*d*x)*a**2*b + 15*e**(18*c
 + 18*d*x)*a*b**2 + 5*e**(18*c + 18*d*x)*b**3 - 180*e**(14*c + 14*d*x)*a**
3 - 225*e**(14*c + 14*d*x)*a**2*b + 90*e**(14*c + 14*d*x)*a*b**2 + 135*e**
(14*c + 14*d*x)*b**3 - 840*e**(12*c + 12*d*x)*a**3 - 525*e**(12*c + 12*d*x
)*a**2*b + 210*e**(12*c + 12*d*x)*a*b**2 - 105*e**(12*c + 12*d*x)*b**3 - 1
890*e**(10*c + 10*d*x)*a**3 - 315*e**(10*c + 10*d*x)*a**2*b + 315*e**(10*c
 + 10*d*x)*b**3 - 2520*e**(8*c + 8*d*x)*a**3 + 315*e**(8*c + 8*d*x)*a**2*b
 - 126*e**(8*c + 8*d*x)*a*b**2 - 315*e**(8*c + 8*d*x)*b**3 - 2100*e**(6*c
+ 6*d*x)*a**3 + 525*e**(6*c + 6*d*x)*a**2*b - 42*e**(6*c + 6*d*x)*a*b**2 +
 105*e**(6*c + 6*d*x)*b**3 - 1080*e**(4*c + 4*d*x)*a**3 + 225*e**(4*c + 4*
d*x)*a**2*b - 90*e**(4*c + 4*d*x)*a*b**2 - 135*e**(4*c + 4*d*x)*b**3 - 315
*e**(2*c + 2*d*x)*a**3 - 63*e**(2*c + 2*d*x)*a*b**2 - 40*a**3 - 15*a**2*b
+ 6*a*b**2 - 5*b**3))/(35*d*(e**(18*c + 18*d*x) + 7*e**(16*c + 16*d*x) + 2
0*e**(14*c + 14*d*x) + 28*e**(12*c + 12*d*x) + 14*e**(10*c + 10*d*x) - 14*
e**(8*c + 8*d*x) - 28*e**(6*c + 6*d*x) - 20*e**(4*c + 4*d*x) - 7*e**(2*c +
 2*d*x) - 1))
```

3.71 $\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 232

$$\begin{aligned}\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx &= \frac{3a^2b\arctan(\sinh(c+dx))}{2d}+\frac{5b^3\arctan(\sinh(c+dx))}{128d}\\ &\quad+\frac{a^3\operatorname{arctanh}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}\\ &\quad-\frac{ab^2\operatorname{sech}^3(c+dx)}{d}+\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}+\frac{3a^2b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}\\ &\quad+\frac{5b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{128d}-\frac{5b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{64d}\\ &\quad-\frac{5b^3\operatorname{sech}^3(c+dx)\tanh^3(c+dx)}{48d}-\frac{b^3\operatorname{sech}^3(c+dx)\tanh^5(c+dx)}{8d}\end{aligned}$$

output
```
3/2*a^2*b*arctan(sinh(d*x+c))/d+5/128*b^3*arctan(sinh(d*x+c))/d+1/2*a^3*ar
ctanh(cosh(d*x+c))/d-1/2*a^3*coth(d*x+c)*csch(d*x+c)/d-a*b^2*sech(d*x+c)^3
/d+3/5*a*b^2*sech(d*x+c)^5/d+3/2*a^2*b*sech(d*x+c)*tanh(d*x+c)/d+5/128*b^3
*sech(d*x+c)*tanh(d*x+c)/d-5/64*b^3*sech(d*x+c)^3*tanh(d*x+c)/d-5/48*b^3*s
ech(d*x+c)^3*tanh(d*x+c)^3/d-1/8*b^3*sech(d*x+c)^3*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 12.70 (sec) , antiderivative size = 264, normalized size of antiderivative = 1.14

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \frac{(192a^2b+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{64d}-\frac{a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a^3\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{ab^2\operatorname{sech}^3(c+dx)}{d}+\frac{3ab^2\operatorname{sech}^5(c+dx)}{5d}+\frac{\operatorname{sech}^2(c+dx)\left(192a^2b\sinh(c+dx)+5b^3\sinh(c+dx)\right)}{128d}-\frac{59b^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d}+\frac{17b^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{48d}-\frac{b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{8d}$$

input
```
Integrate[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
((192*a^2*b + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]])/(64*d) - (a^3*Csch[(c + d*
x)/2]^2)/(8*d) + (a^3*Log[Cosh[(c + d*x)/2]])/(2*d) - (a^3*Log[Sinh[(c + d
*x)/2]])/(2*d) - (a^3*Sech[(c + d*x)/2]^2)/(8*d) - (a*b^2*Sech[c + d*x]^3)
/d + (3*a*b^2*Sech[c + d*x]^5)/(5*d) + (Sech[c + d*x]^2*(192*a^2*b*Sinh[c
+ d*x] + 5*b^3*Sinh[c + d*x]))/(128*d) - (59*b^3*Sech[c + d*x]^3*Tanh[c +
d*x])/(192*d) + (17*b^3*Sech[c + d*x]^5*Tanh[c + d*x])/(48*d) - (b^3*Sech[
c + d*x]^7*Tanh[c + d*x])/(8*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.73 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.11, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 26, 4149, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\left(ib\tan(ic+idx)^3+a\right)^3}{\sin(ic+idx)^3}dx$$

$$\Big\downarrow \; 4149$$

$$-i\int \left(ib^3\text{sech}^3(c+dx)\tanh^6(c+dx)+3iab^2\text{sech}^3(c+dx)\tanh^3(c+dx)+ia^3\text{csch}^3(c+dx)+3ia^2b\text{sech}^3(c+\right.$$

$$\Big\downarrow \; 2009$$

$$-i\left(\frac{ia^3\text{arctanh}(\cosh(c+dx))}{2d}-\frac{ia^3\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{3ia^2b\arctan(\sinh(c+dx))}{2d}+\frac{3ia^2b\tanh(c+d}{2a}\right.$$

input
```
Int[Csch[c + d*x]^3*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(-I)*((((3*I)/2)*a^2*b*ArcTan[Sinh[c + d*x]])/d + (((5*I)/128)*b^3*ArcTan[
Sinh[c + d*x]])/d + ((I/2)*a^3*ArcTanh[Cosh[c + d*x]])/d - ((I/2)*a^3*Coth
[c + d*x]*Csch[c + d*x])/d - (I*a*b^2*Sech[c + d*x]^3)/d + (((3*I)/5)*a*b^
2*Sech[c + d*x]^5)/d + (((3*I)/2)*a^2*b*Sech[c + d*x]*Tanh[c + d*x])/d + (
((5*I)/128)*b^3*Sech[c + d*x]*Tanh[c + d*x])/d - (((5*I)/64)*b^3*Sech[c +
d*x]^3*Tanh[c + d*x])/d - (((5*I)/48)*b^3*Sech[c + d*x]^3*Tanh[c + d*x]^3)
/d - ((I/8)*b^3*Sech[c + d*x]^3*Tanh[c + d*x]^5)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4149
```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> Int[ExpandTrig[(d*sin[e + f*x])^m*(a
 + b*(c*tan[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Maple [A] (verified)

Time = 55.35 (sec) , antiderivative size = 208, normalized size of antiderivative = 0.90

method	result
derivativedivides	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3b^2a\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}\right.}{}$
default	$\frac{a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{arctanh}\left(e^{dx+c}\right)\right)+3a^2b\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3b^2a\left(-\frac{\sinh(dx+c)^2}{3\cosh(dx+c)^5}\right.}{}$
risch	$-\frac{e^{dx+c}\left(30950b^3e^{8dx+8c}-19760b^3e^{6dx+6c}+8520b^3e^{4dx+4c}+8640\,e^{2dx+2c}a^3+960a^3+2880a^2b+23040a^2b\,e^{12dx+12c}\right.}{}$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(1/2*s
ech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*b^2*a*(-1/3*sinh(d*x+c)^2/cos
h(d*x+c)^5-2/15/cosh(d*x+c)^5)+b^3*(-1/3*sinh(d*x+c)^5/cosh(d*x+c)^8-1/3*s
inh(d*x+c)^3/cosh(d*x+c)^8-1/7*sinh(d*x+c)/cosh(d*x+c)^8+1/7*(1/8*sech(d*x
+c)^7+7/48*sech(d*x+c)^5+35/192*sech(d*x+c)^3+35/128*sech(d*x+c))*tanh(d*x
+c)+5/64*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10985 vs. $2(212) = 424$.

Time = 0.20 (sec) , antiderivative size = 10985, normalized size of antiderivative = 47.35

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \int \left(a+b\tanh^3(c+dx)\right)^3\operatorname{csch}^3(c+dx)\,dx$$

input
```
integrate(csch(d*x+c)**3*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(212) = 424$.

Time = 0.15 (sec) , antiderivative size = 586, normalized size of antiderivative = 2.53

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
-1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
 3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*
c) - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*
c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 7
0*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(
-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 3*a^2*b*(arctan(e^(-d*x - c)
)/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
 - 4*c) + 1))) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/
d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x
 - 4*c) - 1))) - 8/5*a*b^2*(5*e^(-3*d*x - 3*c)/(d*(5*e^(-2*d*x - 2*c) + 10
*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
- 10*c) + 1)) - 2*e^(-5*d*x - 5*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x -
 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)
) + 5*e^(-7*d*x - 7*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 426 vs. $2(212) = 424$.

Time = 0.36 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.84

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{480\,a^3\log\left(e^{(dx+c)}+1\right)-480\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)+15\left(192\,a^2b+5\,b^3\right)\arctan\left(e^{(dx+c)}\right)-\frac{960\,\left(a^3e^{(3\,dx+3\,c)}\ldots}{\left(e^{(2\,dx+2\,c)}\ldots}}{\ldots}$$

input
```
integrate(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
1/960*(480*a^3*log(e^(d*x + c) + 1) - 480*a^3*log(abs(e^(d*x + c) - 1)) +
15*(192*a^2*b + 5*b^3)*arctan(e^(d*x + c)) - 960*(a^3*e^(3*d*x + 3*c) + a^
3*e^(d*x + c))/(e^(2*d*x + 2*c) - 1)^2 + (2880*a^2*b*e^(15*d*x + 15*c) + 7
5*b^3*e^(15*d*x + 15*c) + 14400*a^2*b*e^(13*d*x + 13*c) - 7680*a*b^2*e^(13
*d*x + 13*c) - 1985*b^3*e^(13*d*x + 13*c) + 25920*a^2*b*e^(11*d*x + 11*c)
- 19968*a*b^2*e^(11*d*x + 11*c) + 4475*b^3*e^(11*d*x + 11*c) + 14400*a^2*b
*e^(9*d*x + 9*c) - 21504*a*b^2*e^(9*d*x + 9*c) - 8825*b^3*e^(9*d*x + 9*c)
- 14400*a^2*b*e^(7*d*x + 7*c) - 21504*a*b^2*e^(7*d*x + 7*c) + 8825*b^3*e^(
7*d*x + 7*c) - 25920*a^2*b*e^(5*d*x + 5*c) - 19968*a*b^2*e^(5*d*x + 5*c) -
 4475*b^3*e^(5*d*x + 5*c) - 14400*a^2*b*e^(3*d*x + 3*c) - 7680*a*b^2*e^(3*
d*x + 3*c) + 1985*b^3*e^(3*d*x + 3*c) - 2880*a^2*b*e^(d*x + c) - 75*b^3*e^
(d*x + c))/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 9.49 (sec) , antiderivative size = 731, normalized size of antiderivative = 3.15

$$\int \operatorname{csch}^3(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^3,x)
```

output
```
(a^3*log(exp(c + d*x) + 1))/(2*d) - (a^3*log(exp(c + d*x) - 1))/(2*d) + (e
xp(c + d*x)*(192*a^2*b + 5*b^3))/(64*d*(exp(2*c + 2*d*x) + 1)) + (exp(c +
d*x)*(3264*a*b^2 + 2245*b^3))/(120*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d
*x) + exp(6*c + 6*d*x) + 1)) - (b*log(exp(c + d*x) - 1i)*(192*a^2 + 5*b^2)
*1i)/(128*d) + (b*log(exp(c + d*x) + 1i)*(192*a^2 + 5*b^2)*1i)/(128*d) - (
exp(c + d*x)*(768*a*b^2 + 1325*b^3))/(20*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c
 + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (500*b^3*exp(c +
 d*x))/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x
) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1))
+ (2*exp(c + d*x)*(144*a*b^2 + 1025*b^3))/(15*d*(5*exp(2*c + 2*d*x) + 10*e
xp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10
*d*x) + 1)) + (112*b^3*exp(c + d*x))/(d*(7*exp(2*c + 2*d*x) + 21*exp(4*c +
 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x
) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (exp(c + d*x)*(768*a
*b^2 + 576*a^2*b + 251*b^3))/(96*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x)
+ 1)) - (32*b^3*exp(c + d*x))/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x)
 + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*
exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1)) - (a^
3*exp(c + d*x))/(d*(exp(2*c + 2*d*x) - 1)) - (2*a^3*exp(c + d*x))/(d*(exp(
4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1702, normalized size of antiderivative = 7.34

$$\int \operatorname{csch}^3(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3*(a+b*tanh(d*x+c)^3)^3,x)
```

output
```
(2880*e**(20*c + 20*d*x)*atan(e**(c + d*x))*a**2*b + 75*e**(20*c + 20*d*x)
*atan(e**(c + d*x))*b**3 + 17280*e**(18*c + 18*d*x)*atan(e**(c + d*x))*a**
2*b + 450*e**(18*c + 18*d*x)*atan(e**(c + d*x))*b**3 + 37440*e**(16*c + 16
*d*x)*atan(e**(c + d*x))*a**2*b + 975*e**(16*c + 16*d*x)*atan(e**(c + d*x)
)*b**3 + 23040*e**(14*c + 14*d*x)*atan(e**(c + d*x))*a**2*b + 600*e**(14*c
 + 14*d*x)*atan(e**(c + d*x))*b**3 - 40320*e**(12*c + 12*d*x)*atan(e**(c +
 d*x))*a**2*b - 1050*e**(12*c + 12*d*x)*atan(e**(c + d*x))*b**3 - 80640*e*
*(10*c + 10*d*x)*atan(e**(c + d*x))*a**2*b - 2100*e**(10*c + 10*d*x)*atan(
e**(c + d*x))*b**3 - 40320*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**2*b - 10
50*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**3 + 23040*e**(6*c + 6*d*x)*atan(
e**(c + d*x))*a**2*b + 600*e**(6*c + 6*d*x)*atan(e**(c + d*x))*b**3 + 3744
0*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b + 975*e**(4*c + 4*d*x)*atan(e
**(c + d*x))*b**3 + 17280*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2*b + 450
*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**3 + 2880*atan(e**(c + d*x))*a**2*b
 + 75*atan(e**(c + d*x))*b**3 - 480*e**(20*c + 20*d*x)*log(e**(c + d*x) -
1)*a**3 + 480*e**(20*c + 20*d*x)*log(e**(c + d*x) + 1)*a**3 - 960*e**(19*c
 + 19*d*x)*a**3 + 2880*e**(19*c + 19*d*x)*a**2*b + 75*e**(19*c + 19*d*x)*b
**3 - 2880*e**(18*c + 18*d*x)*log(e**(c + d*x) - 1)*a**3 + 2880*e**(18*c +
 18*d*x)*log(e**(c + d*x) + 1)*a**3 - 8640*e**(17*c + 17*d*x)*a**3 + 8640*
e**(17*c + 17*d*x)*a**2*b - 7680*e**(17*c + 17*d*x)*a*b**2 - 2135*e**(1...
```

3.72 $\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 138

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\frac{a^3\coth(c+dx)}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{3a^2b\log(\tanh(c+dx))}{d}-\frac{3a^2b\tanh^2(c+dx)}{2d}+\frac{ab^2\tanh^3(c+dx)}{d}-\frac{3ab^2\tanh^5(c+dx)}{5d}+\frac{b^3\tanh^6(c+dx)}{6d}-\frac{b^3\tanh^8(c+dx)}{8d}$$

output
```
a^3*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d+3*a^2*b*ln(tanh(d*x+c))/d-3/2*a^
2*b*tanh(d*x+c)^2/d+a*b^2*tanh(d*x+c)^3/d-3/5*a*b^2*tanh(d*x+c)^5/d+1/6*b^
3*tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/d
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 213, normalized size of antiderivative = 1.54

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$=\frac{2a^3\coth(c+dx)}{3d}-\frac{a^3\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}-\frac{3a^2b\log(\cosh(c+dx))}{d}+\frac{3a^2b\log(\sinh(c+dx))}{d}+\frac{3a^2b\operatorname{sech}^2(c+dx)}{2d}-\frac{b^3\operatorname{sech}^4(c+dx)}{4d}+\frac{b^3\operatorname{sech}^6(c+dx)}{3d}-\frac{b^3\operatorname{sech}^8(c+dx)}{8d}+\frac{2ab^2\tanh(c+dx)}{5d}+\frac{ab^2\operatorname{sech}^2(c+dx)\tanh(c+dx)}{5d}-\frac{3ab^2\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}$$

input
```
Integrate[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(2*a^3*Coth[c + d*x])/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) -
(3*a^2*b*Log[Cosh[c + d*x]])/d + (3*a^2*b*Log[Sinh[c + d*x]])/d + (3*a^2*b
*Sech[c + d*x]^2)/(2*d) - (b^3*Sech[c + d*x]^4)/(4*d) + (b^3*Sech[c + d*x]
^6)/(3*d) - (b^3*Sech[c + d*x]^8)/(8*d) + (2*a*b^2*Tanh[c + d*x])/(5*d) +
(a*b^2*Sech[c + d*x]^2*Tanh[c + d*x])/(5*d) - (3*a*b^2*Sech[c + d*x]^4*Tan
h[c + d*x])/(5*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 118, normalized size of antiderivative = 0.86, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4146, 2333, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a+ib\tan(ic+idx)^3\right)^3}{\sin(ic+idx)^4}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \coth^4(c+dx)\left(1-\tanh^2(c+dx)\right)\left(b\tanh^3(c+dx)+a\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2333}$$

$$\frac{\int \left(-b^3\tanh^7(c+dx)+b^3\tanh^5(c+dx)-3ab^2\tanh^4(c+dx)+3ab^2\tanh^2(c+dx)-3a^2b\tanh(c+dx)+a^3\text{co}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)+a^3\coth(c+dx)-\frac{3}{2}a^2b\tanh^2(c+dx)+3a^2b\log(\tanh(c+dx))-\frac{3}{5}ab^2\tanh^5(c+dx)+a}{d}$$

input
```
Int[Csch[c + d*x]^4*(a + b*Tanh[c + d*x]^3)^3,x]
```

output
```
(a^3*Coth[c + d*x] - (a^3*Coth[c + d*x]^3)/3 + 3*a^2*b*Log[Tanh[c + d*x]]
- (3*a^2*b*Tanh[c + d*x]^2)/2 + a*b^2*Tanh[c + d*x]^3 - (3*a*b^2*Tanh[c +
d*x]^5)/5 + (b^3*Tanh[c + d*x]^6)/6 - (b^3*Tanh[c + d*x]^8)/8)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2333
```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [A] (verified)

Time = 85.43 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.13

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+3b^2a\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}\right.}{}\right.}{d}$
default	$\frac{a^3\left(\frac{2}{3}-\frac{\operatorname{csch}(dx+c)^2}{3}\right)\coth(dx+c)+3a^2b\left(\frac{1}{2\cosh(dx+c)^2}+\ln(\tanh(dx+c))\right)+3b^2a\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}\right.}{}\right.}{d}$
risch	$-\frac{2\left(-310b^3e^{8dx+8c}+130b^3e^{6dx+6c}-30b^3e^{4dx+4c}-50\,e^{2dx+2c}a^3-6b^2a-10a^3+270a^2b\,e^{12dx+12c}-270a^2b\,e^{10dx+10c}\right.}{}$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(1/2/cosh(d*x+c)^2+ln
(tanh(d*x+c)))+3*b^2*a*(-1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(
d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/4*sinh(d*x+c)^4/cosh(d*x
+c)^8-1/6*sinh(d*x+c)^2/cosh(d*x+c)^8-1/24/cosh(d*x+c)^8))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9459 vs. 2(128) = 256.

Time = 0.20 (sec) , antiderivative size = 9459, normalized size of antiderivative = 68.54

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^3\left(c+dx\right)\right)^3 \operatorname{csch}^4\left(c+dx\right) dx$$

input
```
integrate(csch(d*x+c)**4*(a+b*tanh(d*x+c)**3)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**3)**3*csch(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 997 vs. 2(128) = 256.

Time = 0.15 (sec) , antiderivative size = 997, normalized size of antiderivative = 7.22

$$\int \operatorname{csch}^4(c+dx) \left(a+b\tanh^3(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="maxima")
```

output
```
3*a^2*b*(log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d - log(e^(-2*d*x
 - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4
*c) + 1))) + 4/5*a*b^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) +
15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-
6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(
-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x -
8*c) + e^(-10*d*x - 10*c) + 1))) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2
*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) - 4/3*b^3*(3*e^(
-4*d*x - 4*c)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x
- 6*c) + 70*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*
c) + 8*e^(-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) - 4*e^(-6*d*x - 6*c)/
(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^
(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*
d*x - 14*c) + e^(-16*d*x - 16*c) + 1)) + 10*e^(-8*d*x - 8*c)/(d*(8*e^(-2*d
*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*e^(-8*d*x - 8*c
) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(128) = 256$.

Time = 0.36 (sec) , antiderivative size = 437, normalized size of antiderivative = 3.17

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx = -\frac{2520\,a^2b\log\left(e^{(2\,dx+2\,c)}+1\right)-2520\,a^2b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{140\,\left(33\,a^2be^{(6\,dx+6\,c)}-99\,a^2be^{(4\,dx+4\,c)}+24\,a^3e^{(2\,dx+}}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{}$$

input
```
integrate(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x, algorithm="giac")
```

output
```
-1/840*(2520*a^2*b*log(e^(2*d*x + 2*c) + 1) - 2520*a^2*b*log(abs(e^(2*d*x
+ 2*c) - 1)) + 140*(33*a^2*b*e^(6*d*x + 6*c) - 99*a^2*b*e^(4*d*x + 4*c) +
24*a^3*e^(2*d*x + 2*c) + 99*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 33*a^2*b)/(e^(
2*d*x + 2*c) - 1)^3 - (6849*a^2*b*e^(16*d*x + 16*c) + 59832*a^2*b*e^(14*d*
x + 14*c) + 222012*a^2*b*e^(12*d*x + 12*c) - 10080*a*b^2*e^(12*d*x + 12*c)
 - 3360*b^3*e^(12*d*x + 12*c) + 459144*a^2*b*e^(10*d*x + 10*c) - 26880*a*b
^2*e^(10*d*x + 10*c) + 4480*b^3*e^(10*d*x + 10*c) + 580230*a^2*b*e^(8*d*x
+ 8*c) - 23520*a*b^2*e^(8*d*x + 8*c) - 11200*b^3*e^(8*d*x + 8*c) + 459144*
a^2*b*e^(6*d*x + 6*c) - 10752*a*b^2*e^(6*d*x + 6*c) + 4480*b^3*e^(6*d*x +
6*c) + 222012*a^2*b*e^(4*d*x + 4*c) - 8736*a*b^2*e^(4*d*x + 4*c) - 3360*b^
3*e^(4*d*x + 4*c) + 59832*a^2*b*e^(2*d*x + 2*c) - 5376*a*b^2*e^(2*d*x + 2*
c) + 6849*a^2*b - 672*a*b^2)/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 0.54 (sec) , antiderivative size = 646, normalized size of antiderivative = 4.68

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3\,dx$$
$$= \frac{96\,(10\,b^3+a\,b^2)}{5\,d\,(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1)}$$
$$-\frac{640\,b^3}{3\,d\,(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1)}$$
$$-\frac{4\,(25\,b^3+12\,a\,b^2)}{d\,(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1)}$$
$$+\frac{128\,b^3}{d\,(7\,e^{2\,c+2\,d\,x}+21\,e^{4\,c+4\,d\,x}+35\,e^{6\,c+6\,d\,x}+35\,e^{8\,c+8\,d\,x}+21\,e^{10\,c+10\,d\,x}+7\,e^{12\,c+12\,d\,x}+e^{14\,c+14\,d\,x}+1)}$$
$$-\frac{2\,(3\,a^2\,b+6\,a\,b^2+2\,b^3)}{d\,(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1)}-\frac{6\,\operatorname{atan}\left(\frac{a^2\,b\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^4\,b^2}}\right)\,\sqrt{a^4\,b^2}}{\sqrt{-d^2}}$$
$$-\frac{32\,b^3}{d\,(8\,e^{2\,c+2\,d\,x}+28\,e^{4\,c+4\,d\,x}+56\,e^{6\,c+6\,d\,x}+70\,e^{8\,c+8\,d\,x}+56\,e^{10\,c+10\,d\,x}+28\,e^{12\,c+12\,d\,x}+8\,e^{14\,c+14\,d\,x}+e^{16}}$$
$$-\frac{4\,a^3}{d\,(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1)}-\frac{8\,a^3}{3\,d\,(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1)}$$
$$+\frac{8\,(11\,b^3+15\,a\,b^2)}{3\,d\,(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1)}+\frac{6\,a^2\,b}{d\,(e^{2\,c+2\,d\,x}+1)}$$

input
```
int((a + b*tanh(c + d*x)^3)^3/sinh(c + d*x)^4,x)
```

output
```
(96*(a*b^2 + 10*b^3))/(5*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*
exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) - (640*b^
3)/(3*d*(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) +
15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (4
*(12*a*b^2 + 25*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(
6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) + (128*b^3)/(d*(7*exp(2*c + 2*d*x) +
 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(
10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1)) - (2*(6*a
*b^2 + 3*a^2*b + 2*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) -
 (6*atan((a^2*b*exp(2*c)*exp(2*d*x)*(-d^2)^(1/2))/(d*(a^4*b^2)^(1/2)))*(a^
4*b^2)^(1/2))/(-d^2)^(1/2) - (32*b^3)/(d*(8*exp(2*c + 2*d*x) + 28*exp(4*c
+ 4*d*x) + 56*exp(6*c + 6*d*x) + 70*exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*
x) + 28*exp(12*c + 12*d*x) + 8*exp(14*c + 14*d*x) + exp(16*c + 16*d*x) + 1
)) - (4*a^3)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^3)/(3*
d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) + (8*(
15*a*b^2 + 11*b^3))/(3*d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*
c + 6*d*x) + 1)) + (6*a^2*b)/(d*(exp(2*c + 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1626, normalized size of antiderivative = 11.78

$$\int \operatorname{csch}^4(c+dx)\left(a+b\tanh^3(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^4*(a+b*tanh(d*x+c)^3)^3,x)
```

output
```
( - 45*e**(22*c + 22*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 45*e**(22*c +
 22*d*x)*log(e**(c + d*x) - 1)*a**2*b + 45*e**(22*c + 22*d*x)*log(e**(c +
d*x) + 1)*a**2*b - 18*e**(22*c + 22*d*x)*a**2*b - 225*e**(20*c + 20*d*x)*l
og(e**(2*c + 2*d*x) + 1)*a**2*b + 225*e**(20*c + 20*d*x)*log(e**(c + d*x)
- 1)*a**2*b + 225*e**(20*c + 20*d*x)*log(e**(c + d*x) + 1)*a**2*b - 315*e*
*(18*c + 18*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 315*e**(18*c + 18*d*x)
*log(e**(c + d*x) - 1)*a**2*b + 315*e**(18*c + 18*d*x)*log(e**(c + d*x) +
1)*a**2*b - 60*e**(18*c + 18*d*x)*a**3 + 144*e**(18*c + 18*d*x)*a**2*b - 1
80*e**(18*c + 18*d*x)*a*b**2 - 60*e**(18*c + 18*d*x)*b**3 + 225*e**(16*c +
 16*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b - 225*e**(16*c + 16*d*x)*log(e**
(c + d*x) - 1)*a**2*b - 225*e**(16*c + 16*d*x)*log(e**(c + d*x) + 1)*a**2*
b - 460*e**(16*c + 16*d*x)*a**3 + 90*e**(16*c + 16*d*x)*a**2*b + 60*e**(16
*c + 16*d*x)*a*b**2 + 260*e**(16*c + 16*d*x)*b**3 + 990*e**(14*c + 14*d*x)
*log(e**(2*c + 2*d*x) + 1)*a**2*b - 990*e**(14*c + 14*d*x)*log(e**(c + d*x
) - 1)*a**2*b - 990*e**(14*c + 14*d*x)*log(e**(c + d*x) + 1)*a**2*b - 1520
*e**(14*c + 14*d*x)*a**3 - 324*e**(14*c + 14*d*x)*a**2*b + 480*e**(14*c +
14*d*x)*a*b**2 - 620*e**(14*c + 14*d*x)*b**3 + 630*e**(12*c + 12*d*x)*log(
e**(2*c + 2*d*x) + 1)*a**2*b - 630*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1
)*a**2*b - 630*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*a**2*b - 2800*e**(
12*c + 12*d*x)*a**3 - 288*e**(12*c + 12*d*x)*a**2*b - 192*e**(12*c + 12...
```

3.73 $\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 491

$$\begin{aligned}\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = &-\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{4/3}b^{2/3}-b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^{4/3}+a^{2/3}b^{2/3}+b^{4/3}\right)^3 d}\\
&-\frac{3a(a-5b)\log(1-\tanh(c+dx))}{16(a+b)^3d}+\frac{3a(a+5b)\log(1+\tanh(c+dx))}{16(a-b)^3d}\\
&-\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^3 d}\\
&+\frac{a^{2/3}\sqrt[3]{b}\left(a^4+7a^2b^2+b^4+3a^{2/3}b^{4/3}(2a^2+b^2)\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6\left(a^2-b^2\right)^3 d}\\
&-\frac{a^2b(a^2+2b^2)\log\left(a+b\tanh^3(c+dx)\right)}{\left(a^2-b^2\right)^3 d}\\
&+\frac{1}{16(a+b)d(1-\tanh(c+dx))^2}-\frac{5a-b}{16(a+b)^2d(1-\tanh(c+dx))}\\
&-\frac{1}{16(a-b)d(1+\tanh(c+dx))^2}+\frac{5a+b}{16(a-b)^2d(1+\tanh(c+dx))}\end{aligned}$$

output
```
-1/3*a^(2/3)*b^(1/3)*(a^2+3*a^(4/3)*b^(2/3)-b^2)*arctan(1/3*(a^(1/3)-2*b^(
1/3)*tanh(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/(a^(4/3)+a^(2/3)*b^(2/3)+b^(4/3
))^3/d-3/16*a*(a-5*b)*ln(1-tanh(d*x+c))/(a+b)^3/d+3/16*a*(a+5*b)*ln(1+tanh
(d*x+c))/(a-b)^3/d-1/3*a^(2/3)*b^(1/3)*(a^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3
)*(2*a^2+b^2))*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/(a^2-b^2)^3/d+1/6*a^(2/3)*b
^(1/3)*(a^4+7*a^2*b^2+b^4+3*a^(2/3)*b^(4/3)*(2*a^2+b^2))*ln(a^(2/3)-a^(1/3
)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2)^3/d-a^2*b*(a^2+2*b^
2)*ln(a+b*tanh(d*x+c)^3)/(a^2-b^2)^3/d+1/16/(a+b)/d/(1-tanh(d*x+c))^2-1/16
*(5*a-b)/(a+b)^2/d/(1-tanh(d*x+c))-1/16/(a-b)/d/(1+tanh(d*x+c))^2+1/16*(5*
a+b)/(a-b)^2/d/(1+tanh(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 1.30 (sec) , antiderivative size = 645, normalized size of antiderivative = 1.31

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{-32ab\text{RootSum}\Big[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\frac{-6a^3c-12ab^2c-6a^3dx-12ab^2dx+3}{}}{}$$

input
```
Integrate[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-32*a*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 +
b*#1^3 & , (-6*a^3*c - 12*a*b^2*c - 6*a^3*d*x - 12*a*b^2*d*x + 3*a^3*Log[E
^(2*(c + d*x)) - #1] + 6*a*b^2*Log[E^(2*(c + d*x)) - #1] - 8*a^3*c*#1 + 4*
a^2*b*c*#1 + 8*a*b^2*c*#1 - 4*b^3*c*#1 - 8*a^3*d*x*#1 + 4*a^2*b*d*x*#1 + 8
*a*b^2*d*x*#1 - 4*b^3*d*x*#1 + 4*a^3*Log[E^(2*(c + d*x)) - #1]*#1 - 2*a^2*
b*Log[E^(2*(c + d*x)) - #1]*#1 - 4*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*
b^3*Log[E^(2*(c + d*x)) - #1]*#1 - 10*a^3*c*#1^2 + 20*a^2*b*c*#1^2 - 20*a*
b^2*c*#1^2 + 4*b^3*c*#1^2 - 10*a^3*d*x*#1^2 + 20*a^2*b*d*x*#1^2 - 20*a*b^2
*d*x*#1^2 + 4*b^3*d*x*#1^2 + 5*a^3*Log[E^(2*(c + d*x)) - #1]*#1^2 - 10*a^2
*b*Log[E^(2*(c + d*x)) - #1]*#1^2 + 10*a*b^2*Log[E^(2*(c + d*x)) - #1]*#1^
2 - 2*b^3*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^
2 - b*#1^2) & ] + 3*(4*b*(5*a^3 + 5*a^2*b + a*b^2 + b^3)*Cosh[2*(c + d*x)]
 - (a - b)*b*(a + b)^2*Cosh[4*(c + d*x)] - 8*a*(a^3 + a^2*b + 2*a*b^2 + 2*
b^3)*Sinh[2*(c + d*x)] + a*(a - b)*(12*(a^2 - 6*a*b + 5*b^2)*(c + d*x) + (
a + b)^2*Sinh[4*(c + d*x)])))/(96*(a - b)^2*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 1.17 (sec) , antiderivative size = 465, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sin(ic+idx)^4}{a+ib\tan(ic+idx)^3}dx$$

$$\Big\downarrow \; 4146$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)^3\left(b\tanh^3(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 7276$$

$$\int \left(-\frac{3a(a-5b)}{16(a+b)^3(\tanh(c+dx)-1)} + \frac{3a(a+5b)}{16(a-b)^3(\tanh(c+dx)+1)} + \frac{ab(-3ab(a^2+2b^2)\tanh^2(c+dx)+(a^4+7b^2a^2+b^4)\tanh(c+dx)-3ab(2a^2+b^2}{(a^2-b^2)^3(b\tanh^3(c+dx)+a)}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2b(a^2+2b^2)\log(a+b\tanh^3(c+dx))}{(a^2-b^2)^3} - \frac{a^{2/3}\sqrt[3]{b}(3a^{4/3}b^{2/3}+a^2-b^2)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}(a^{2/3}b^{2/3}+a^{4/3}+b^{4/3})^3} + \frac{a^{2/3}\sqrt[3]{b}(a^4+7a^2b^2+3a^{2/3}b^{4/3}(2}{}$$

input
```
Int[Sinh[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-((a^(2/3)*b^(1/3)*(a^2 + 3*a^(4/3)*b^(2/3) - b^2)*ArcTan[(a^(1/3) - 2*b^
(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^(4/3) + a^(2/3)*b^(2/
3) + b^(4/3))^3)) - (3*a*(a - 5*b)*Log[1 - Tanh[c + d*x]])/(16*(a + b)^3)
+ (3*a*(a + 5*b)*Log[1 + Tanh[c + d*x]])/(16*(a - b)^3) - (a^(2/3)*b^(1/3)
*(a^4 + 7*a^2*b^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(1/3) + b
^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^3) + (a^(2/3)*b^(1/3)*(a^4 + 7*a^2*b
^2 + b^4 + 3*a^(2/3)*b^(4/3)*(2*a^2 + b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*
Tanh[c + d*x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^3) - (a^2*b*(a^2
+ 2*b^2)*Log[a + b*Tanh[c + d*x]^3])/(a^2 - b^2)^3 + 1/(16*(a + b)*(1 - Ta
nh[c + d*x])^2) - (5*a - b)/(16*(a + b)^2*(1 - Tanh[c + d*x])) - 1/(16*(a
- b)*(1 + Tanh[c + d*x])^2) + (5*a + b)/(16*(a - b)^2*(1 + Tanh[c + d*x]))
)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 35.32 (sec) , antiderivative size = 452, normalized size of antiderivative = 0.92

method	result
derivativedivides	$-\frac{8}{(32a-32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{32}{(64a-64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-a-5b}{8(a-b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{3a+3b}{8(a-b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-$
default	$-\frac{8}{(32a-32b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^4}+\frac{32}{(64a-64b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3}-\frac{-a-5b}{8(a-b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{3a+3b}{8(a-b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-8/(32*a-32*b)/(tanh(1/2*d*x+1/2*c)+1)^4+32/(64*a-64*b)/(tanh(1/2*d*x
+1/2*c)+1)^3-1/8*(-a-5*b)/(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)^2-1/8*(3*a+3*b)/
(a-b)^2/(tanh(1/2*d*x+1/2*c)+1)+3/8*a*(a+5*b)/(a-b)^3*ln(tanh(1/2*d*x+1/2*
c)+1)+8/(32*a+32*b)/(tanh(1/2*d*x+1/2*c)-1)^4+32/(64*a+64*b)/(tanh(1/2*d*x
+1/2*c)-1)^3-1/8*(a-5*b)/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/8*(3*a-3*b)/(
a+b)^2/(tanh(1/2*d*x+1/2*c)-1)-3/8*a*(a-5*b)/(a+b)^3*ln(tanh(1/2*d*x+1/2*c
)-1)-1/3/(a-b)^3*a/(a+b)^3*b*sum((3*a^2*(a^2+2*b^2)*_R^5+3*a*b*(-2*a^2-b^2
)*_R^4+2*(4*a^4+13*a^2*b^2+b^4)*_R^3+12*a*b*(a^2+2*b^2)*_R^2+(a^4-8*a^2*b^
2-2*b^4)*_R+6*a^3*b+3*a*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d
*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.33 (sec) , antiderivative size = 17123, normalized size of antiderivative = 34.87

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(sinh(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^4}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-6*a^4*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) +
 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x
+ 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b
^4 - b^6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) - 12*a^2*b^
3*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a +
3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c)
+ 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^
6) - (d*x + c)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*d)) + 10*a^4*b*integra
te(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5) - 20*a^3*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x
+ 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x
)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 20*a^2*b^3*integra
te(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) +
3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^
3 + a*b^4 + b^5) - 4*a*b^4*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6
*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(
a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 + a*b^4 + b^5) + 8*a^4*b*integrate(e^(
2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a +
 b)*e^(2*d*x + 2*c) + a - b), x)/(a^5 + a^4*b - 2*a^3*b^2 - 2*a^2*b^3 +...
```

Giac [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)^4}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 5.77 (sec) , antiderivative size = 3313, normalized size of antiderivative = 6.75

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^4/(a + b*tanh(c + d*x)^3),x)
```

output
```
symsum(log(- root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3
 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*
z - a^2*b, z, k)*((96*(a^2*b^10*d + 20*a^3*b^9*d - 89*a^4*b^8*d + 270*a^5*
b^7*d - 417*a^6*b^6*d + 408*a^7*b^5*d - 190*a^8*b^4*d + 58*a^9*b^3*d - 7*a
^10*b^2*d - a^2*b^10*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3
+ 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2
 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 52*a^3*b^9*d*exp(2*root(81*
a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 1
62*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp
(2*d*x) + 59*a^4*b^8*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3
+ 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2
 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) - 218*a^5*b^7*d*exp(2*root(81
*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 -
162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*ex
p(2*d*x) + 241*a^6*b^6*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^
3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3 - 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z
^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*exp(2*d*x) + 220*a^7*b^5*d*exp(2*root(
81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d^3*z^3 + 27*b^6*d^3*z^3 - 27*a^6*d^3*z^3
- 162*a^2*b^3*d^2*z^2 - 81*a^4*b*d^2*z^2 - 27*a^2*b^2*d*z - a^2*b, z, k))*
exp(2*d*x) - 298*a^8*b^4*d*exp(2*root(81*a^4*b^2*d^3*z^3 - 81*a^2*b^4*d...
```

Reduce [F]

$$\int \frac{\sinh^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^4/(a+b*tanh(d*x+c)^3),x)
```

output
```
(e**(8*c + 8*d*x)*a**5 - 33*e**(8*c + 8*d*x)*a**4*b - 34*e**(8*c + 8*d*x)*
a**3*b**2 + 34*e**(8*c + 8*d*x)*a**2*b**3 + 33*e**(8*c + 8*d*x)*a*b**4 - e
**(8*c + 8*d*x)*b**5 - 8*e**(6*c + 6*d*x)*a**5 + 276*e**(6*c + 6*d*x)*a**4
*b - 136*e**(6*c + 6*d*x)*a**3*b**2 - 272*e**(6*c + 6*d*x)*a**2*b**3 + 144
*e**(6*c + 6*d*x)*a*b**4 - 4*e**(6*c + 6*d*x)*b**5 - 1792*e**(6*c + 4*d*x)
*int(1/(e**(10*c + 8*d*x)*a**7 - 31*e**(10*c + 8*d*x)*a**6*b - 99*e**(10*c
 + 8*d*x)*a**5*b**2 - 67*e**(10*c + 8*d*x)*a**4*b**3 + 67*e**(10*c + 8*d*x
)*a**3*b**4 + 99*e**(10*c + 8*d*x)*a**2*b**5 + 31*e**(10*c + 8*d*x)*a*b**6
 - e**(10*c + 8*d*x)*b**7 + 3*e**(8*c + 6*d*x)*a**7 - 99*e**(8*c + 6*d*x)*
a**6*b - 105*e**(8*c + 6*d*x)*a**5*b**2 + 201*e**(8*c + 6*d*x)*a**4*b**3 +
 201*e**(8*c + 6*d*x)*a**3*b**4 - 105*e**(8*c + 6*d*x)*a**2*b**5 - 99*e**(
8*c + 6*d*x)*a*b**6 + 3*e**(8*c + 6*d*x)*b**7 + 3*e**(6*c + 4*d*x)*a**7 -
93*e**(6*c + 4*d*x)*a**6*b - 297*e**(6*c + 4*d*x)*a**5*b**2 - 201*e**(6*c
+ 4*d*x)*a**4*b**3 + 201*e**(6*c + 4*d*x)*a**3*b**4 + 297*e**(6*c + 4*d*x)
*a**2*b**5 + 93*e**(6*c + 4*d*x)*a*b**6 - 3*e**(6*c + 4*d*x)*b**7 + e**(4*
c + 2*d*x)*a**7 - 33*e**(4*c + 2*d*x)*a**6*b - 35*e**(4*c + 2*d*x)*a**5*b*
*2 + 67*e**(4*c + 2*d*x)*a**4*b**3 + 67*e**(4*c + 2*d*x)*a**3*b**4 - 35*e*
*(4*c + 2*d*x)*a**2*b**5 - 33*e**(4*c + 2*d*x)*a*b**6 + e**(4*c + 2*d*x)*b
**7),x)*a**11*b*d + 32256*e**(6*c + 4*d*x)*int(1/(e**(10*c + 8*d*x)*a**7 -
 31*e**(10*c + 8*d*x)*a**6*b - 99*e**(10*c + 8*d*x)*a**5*b**2 - 67*e**(...
```

3.74 $\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\text{Int}\left(-\frac{i\sinh^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

output
```
I*Defer(Int)(-I*sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 826 vs. 2(33) = 66.

Time = 0.66 (sec) , antiderivative size = 826, normalized size of antiderivative = 35.91

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
Integrate[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-9*a*(a^2 + 3*b^2)*Cosh[c + d*x] + a^3*Cosh[3*(c + d*x)] - a*b^2*Cosh[3*(
c + d*x)] - 2*a*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^
4 + a*#1^6 + b*#1^6 & , (3*a^2*c + 3*a*b*c + 3*b^2*c + 3*a^2*d*x + 3*a*b*d
*x + 3*b^2*d*x + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a*b*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*b^2*Log
[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] + 2*a^2*c*#1^2 - 2*b^2*c*#1^2 + 2*a^2*d*x*#1^2 - 2*b^2*d*x*#1
^2 + 4*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*
#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 4*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a^2*c*#1^
4 - 3*a*b*c*#1^4 + 3*b^2*c*#1^4 + 3*a^2*d*x*#1^4 - 3*a*b*d*x*#1^4 + 3*b^2*
d*x*#1^4 + 6*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 6*a*b*Log[-Cosh[(c + d*x)/2] - Sin
h[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 6*b^2
*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[
(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^
5) & ] + 27*a^2*b*Sinh[c + d*x] + 9*b^3*Sinh[c + d*x] - a^2*b*Sinh[3*(c +
d*x)] + b^3*Sinh[3*(c + d*x)])/(12*(a - b)^2*(a + b)^2*d)
```

Rubi [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\sin(ic+idx)^3}{a+ib\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\sin(ic+idx)^3}{ib\tan(ic+idx)^3+a}dx$$
$$\Big\downarrow \text{4151}$$
$$i\int -\frac{i\sinh^3(c+dx)}{b\tanh^3(c+dx)+a}dx$$

input
```
Int[Sinh[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A] (verified)

Time = 14.73 (sec) , antiderivative size = 289, normalized size of antiderivative = 12.57

method	result
derivativedivides	$-\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(16a+16b)}-\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a+2b}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{ab\left(\sum_{\_R=\text{RootOf}(a\_Z^6+3a}\right.}{}$
default	$-\frac{16}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(16a+16b)}-\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{-a+2b}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{ab\left(\sum_{\_R=\text{RootOf}(a\_Z^6+3a}\right.}{}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-16/3/(tanh(1/2*d*x+1/2*c)-1)^3/(16*a+16*b)-8/(16*a+16*b)/(tanh(1/2*d
*x+1/2*c)-1)^2-1/2/(a+b)^2*(-a+2*b)/(tanh(1/2*d*x+1/2*c)-1)-1/3*a*b/(a+b)^
2/(a-b)^2*sum(((2*a^2+b^2)*_R^4-6*a*b*_R^3+2*(4*a^2+5*b^2)*_R^2-6*a*b*_R+2
*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Ro
otOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-8/(16*a-16*b)/(tanh(1/2*d*x+1/2
*c)+1)^2+16/3/(tanh(1/2*d*x+1/2*c)+1)^3/(16*a-16*b)-1/2*(a+2*b)/(a-b)^2/(t
anh(1/2*d*x+1/2*c)+1))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 4.19 (sec) , antiderivative size = 62017, normalized size of antiderivative = 2696.39

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [N/A]

Not integrable

Time = 65.29 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh^3{(c+dx)}}{a+b\tanh^3{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 533, normalized size of antiderivative = 23.17

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^3}{b\tanh{(dx+c)}^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/24*(a^3 + a^2*b - a*b^2 - b^3 + (a^3*e^(6*c) - a^2*b*e^(6*c) - a*b^2*e^(
6*c) + b^3*e^(6*c))*e^(6*d*x) - 9*(a^3*e^(4*c) - 3*a^2*b*e^(4*c) + 3*a*b^2
*e^(4*c) - b^3*e^(4*c))*e^(4*d*x) - 9*(a^3*e^(2*c) + 3*a^2*b*e^(2*c) + 3*a
*b^2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x))*e^(-3*d*x)/(a^4*d*e^(3*c) - 2*a^2*b
^2*d*e^(3*c) + b^4*d*e^(3*c)) - 1/8*integrate(16*(3*(a^3*b*e^(5*c) - a^2*b
^2*e^(5*c) + a*b^3*e^(5*c))*e^(5*d*x) + 2*(a^3*b*e^(3*c) - a*b^3*e^(3*c))*
e^(3*d*x) + 3*(a^3*b*e^c + a^2*b^2*e^c + a*b^3*e^c)*e^(d*x))/(a^5 - a^4*b
- 2*a^3*b^2 + 2*a^2*b^3 + a*b^4 - b^5 + (a^5*e^(6*c) + a^4*b*e^(6*c) - 2*a
^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c) + b^5*e^(6*c))*e^(6*d*x
) + 3*(a^5*e^(4*c) - a^4*b*e^(4*c) - 2*a^3*b^2*e^(4*c) + 2*a^2*b^3*e^(4*c)
 + a*b^4*e^(4*c) - b^5*e^(4*c))*e^(4*d*x) + 3*(a^5*e^(2*c) + a^4*b*e^(2*c)
 - 2*a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c) + b^5*e^(2*c))*e^
(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 2.18 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.13

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh{(dx+c)}^3}{b\tanh{(dx+c)}^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Hanged}$$

input
```
int(sinh(c + d*x)^3/(a + b*tanh(c + d*x)^3),x)
```

output
```
\text{Hanged}
```

Reduce [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 7934, normalized size of antiderivative = 344.96

$$\int \frac{\sinh^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

output
```
(e**(6*c + 6*d*x)*a**3 + 3*e**(6*c + 6*d*x)*a**2*b + 3*e**(6*c + 6*d*x)*a*
b**2 + e**(6*c + 6*d*x)*b**3 - 9*e**(4*c + 4*d*x)*a**3 - 9*e**(4*c + 4*d*x
)*a**2*b + 9*e**(4*c + 4*d*x)*a*b**2 + 9*e**(4*c + 4*d*x)*b**3 - 720*e**(4
*c + 3*d*x)*int(e**(d*x)/(e**(6*c + 6*d*x)*a**5 + 5*e**(6*c + 6*d*x)*a**4*
b + 10*e**(6*c + 6*d*x)*a**3*b**2 + 10*e**(6*c + 6*d*x)*a**2*b**3 + 5*e**(
6*c + 6*d*x)*a*b**4 + e**(6*c + 6*d*x)*b**5 + 3*e**(4*c + 4*d*x)*a**5 + 9*
e**(4*c + 4*d*x)*a**4*b + 6*e**(4*c + 4*d*x)*a**3*b**2 - 6*e**(4*c + 4*d*x
)*a**2*b**3 - 9*e**(4*c + 4*d*x)*a*b**4 - 3*e**(4*c + 4*d*x)*b**5 + 3*e**(
2*c + 2*d*x)*a**5 + 15*e**(2*c + 2*d*x)*a**4*b + 30*e**(2*c + 2*d*x)*a**3*
b**2 + 30*e**(2*c + 2*d*x)*a**2*b**3 + 15*e**(2*c + 2*d*x)*a*b**4 + 3*e**(
2*c + 2*d*x)*b**5 + a**5 + 3*a**4*b + 2*a**3*b**2 - 2*a**2*b**3 - 3*a*b**4
 - b**5),x)*a**7*b*d - 432*e**(4*c + 3*d*x)*int(e**(d*x)/(e**(6*c + 6*d*x)
*a**5 + 5*e**(6*c + 6*d*x)*a**4*b + 10*e**(6*c + 6*d*x)*a**3*b**2 + 10*e**
(6*c + 6*d*x)*a**2*b**3 + 5*e**(6*c + 6*d*x)*a*b**4 + e**(6*c + 6*d*x)*b**
5 + 3*e**(4*c + 4*d*x)*a**5 + 9*e**(4*c + 4*d*x)*a**4*b + 6*e**(4*c + 4*d*
x)*a**3*b**2 - 6*e**(4*c + 4*d*x)*a**2*b**3 - 9*e**(4*c + 4*d*x)*a*b**4 -
3*e**(4*c + 4*d*x)*b**5 + 3*e**(2*c + 2*d*x)*a**5 + 15*e**(2*c + 2*d*x)*a*
*4*b + 30*e**(2*c + 2*d*x)*a**3*b**2 + 30*e**(2*c + 2*d*x)*a**2*b**3 + 15*
e**(2*c + 2*d*x)*a*b**4 + 3*e**(2*c + 2*d*x)*b**5 + a**5 + 3*a**4*b + 2*a*
*3*b**2 - 2*a**2*b**3 - 3*a*b**4 - b**5),x)*a**6*b**2*d + 4752*e**(4*c ...
```

3.75 $\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 384

$$\begin{aligned}
\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx &= \frac{a^{2/3}\sqrt[3]{b}\left(a^2-3a^{2/3}b^{4/3}+2b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^2-b^2\right)^2 d} \\
&+\frac{(a-2b)\log(1-\tanh(c+dx))}{4(a+b)^2d}-\frac{(a+2b)\log(1+\tanh(c+dx))}{4(a-b)^2d} \\
&+\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} \\
&-\frac{a^{2/3}\sqrt[3]{b}\left(a^2+3a^{2/3}b^{4/3}+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6\left(a^2-b^2\right)^2 d} \\
&+\frac{b(2a^2+b^2)\log\left(a+b\tanh^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} \\
&+\frac{1}{4(a+b)d(1-\tanh(c+dx))}-\frac{1}{4(a-b)d(1+\tanh(c+dx))}
\end{aligned}$$

output
```
1/3*a^(2/3)*b^(1/3)*(a^2-3*a^(2/3)*b^(4/3)+2*b^2)*arctan(1/3*(a^(1/3)-2*b^
(1/3)*tanh(d*x+c))*3^(1/2)/a^(1/3))*3^(1/2)/(a^2-b^2)^2/d+1/4*(a-2*b)*ln(1
-tanh(d*x+c))/(a+b)^2/d-1/4*(a+2*b)*ln(1+tanh(d*x+c))/(a-b)^2/d+1/3*a^(2/3
)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/(a
^2-b^2)^2/d-1/6*a^(2/3)*b^(1/3)*(a^2+3*a^(2/3)*b^(4/3)+2*b^2)*ln(a^(2/3)-a
^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tanh(d*x+c)^2)/(a^2-b^2)^2/d+1/3*b*(2*a
^2+b^2)*ln(a+b*tanh(d*x+c)^3)/(a^2-b^2)^2/d+1/4/(a+b)/d/(1-tanh(d*x+c))-1/
4/(a-b)/d/(1+tanh(d*x+c))
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.80 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.10

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{6(a^2-3ab+2b^2)(c+dx)+3b(a+b)\cosh(2(c+dx))+4b\text{RootSum}\Big[a-b+3a\#1+3b\#1+3a\#1}{}$$

input
```
Integrate[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/12*(6*(a^2 - 3*a*b + 2*b^2)*(c + d*x) + 3*b*(a + b)*Cosh[2*(c + d*x)] +
 4*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 + b*#1
^3 & , (4*a^2*c + 2*b^2*c + 4*a^2*d*x + 2*b^2*d*x - 2*a^2*Log[E^(2*(c + d*
x)) - #1] - b^2*Log[E^(2*(c + d*x)) - #1] + 4*a^2*c*#1 - 4*b^2*c*#1 + 4*a^
2*d*x*#1 - 4*b^2*d*x*#1 - 2*a^2*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b^2*Log[E
^(2*(c + d*x)) - #1]*#1 + 8*a^2*c*#1^2 - 8*a*b*c*#1^2 + 2*b^2*c*#1^2 + 8*a
^2*d*x*#1^2 - 8*a*b*d*x*#1^2 + 2*b^2*d*x*#1^2 - 4*a^2*Log[E^(2*(c + d*x))
- #1]*#1^2 + 4*a*b*Log[E^(2*(c + d*x)) - #1]*#1^2 - b^2*Log[E^(2*(c + d*x)
) - #1]*#1^2)/(a - b + 2*a*#1 + 2*b*#1 + a*#1^2 - b*#1^2) & ] - 3*a*(a + b
)*Sinh[2*(c + d*x)])/((a - b)*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 25, 4146, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sin(ic+idx)^2}{a+ib\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sin(ic+idx)^2}{ib\tan(ic+idx)^3+a}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^3(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{7276}$$

$$\frac{\int \left(\frac{-a-2b}{4(a-b)^2(\tanh(c+dx)+1)}+\frac{a-2b}{4(a+b)^2(\tanh(c+dx)-1)}+\frac{b\left(3ba^2-\left(a^2+2b^2\right)\tanh(c+dx)a+b\left(2a^2+b^2\right)\tanh^2(c+dx)\right)}{\left(a^2-b^2\right)^2\left(b\tanh^3(c+dx)+a\right)}+\frac{1}{4(a+b)(\tanh(c+\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b\left(2a^2+b^2\right)\log\left(a+b\tanh^3(c+dx)\right)}{3\left(a^2-b^2\right)^2}+\frac{a^{2/3}\sqrt[3]{b}\left(-3a^{2/3}b^{4/3}+a^2+2b^2\right)\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\left(a^2-b^2\right)^2}-\frac{a^{2/3}\sqrt[3]{b}\left(3a^{2/3}b^{4/3}+a^2+2b^2\right)\log\left(\right.}{}}{}$$

input
```
Int[Sinh[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
((a^(2/3)*b^(1/3)*(a^2 - 3*a^(2/3)*b^(4/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^
(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*(a^2 - b^2)^2) + ((a - 2
*b)*Log[1 - Tanh[c + d*x]])/(4*(a + b)^2) - ((a + 2*b)*Log[1 + Tanh[c + d*
x]])/(4*(a - b)^2) + (a^(2/3)*b^(1/3)*(a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Lo
g[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*(a^2 - b^2)^2) - (a^(2/3)*b^(1/3)*(
a^2 + 3*a^(2/3)*b^(4/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*
x] + b^(2/3)*Tanh[c + d*x]^2])/(6*(a^2 - b^2)^2) + (b*(2*a^2 + b^2)*Log[a
+ b*Tanh[c + d*x]^3])/(3*(a^2 - b^2)^2) + 1/(4*(a + b)*(1 - Tanh[c + d*x])
) - 1/(4*(a - b)*(1 + Tanh[c + d*x])))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 4.36 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.78

method	result
derivativedivides	$\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{(16}$
default	$\frac{4}{(8a+8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{8}{(16a+16b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(a-2b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{4}{(8a-8b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{(16}$
risch	$-\frac{ax}{2(a+b)^2}+\frac{xb}{(a+b)^2}+\frac{e^{2dx+2c}}{8(a+b)d}-\frac{e^{-2dx-2c}}{8(a-b)d}-\frac{4a^2bd^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{2b^3d^3x}{a^4d^3-2a^2b^2d^3+b^4d^3}-\frac{4a^2b}{a^4d^3-2a^2b}$

input
```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4/(8*a+8*b)/(tanh(1/2*d*x+1/2*c)-1)^2+8/(16*a+16*b)/(tanh(1/2*d*x+1/2
*c)-1)+1/2*(a-2*b)/(a+b)^2*ln(tanh(1/2*d*x+1/2*c)-1)-4/(8*a-8*b)/(tanh(1/2
*d*x+1/2*c)+1)^2+8/(16*a-16*b)/(tanh(1/2*d*x+1/2*c)+1)+1/2/(a-b)^2*(-a-2*b
)*ln(tanh(1/2*d*x+1/2*c)+1)+1/3*b/(a-b)^2/(a+b)^2*sum((a*(2*a^2+b^2)*_R^5-
3*a^2*b*_R^4+6*a*(a^2+b^2)*_R^3+4*b*(2*a^2+b^2)*_R^2-3*a*_R*b^2+3*a^2*b)/(
_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a
+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.00 (sec) , antiderivative size = 10695, normalized size of antiderivative = 27.85

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\sinh^2{(c+dx)}}{a+b\tanh^3{(c+dx)}}\,dx$$

input
```
integrate(sinh(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\sinh{(dx+c)}^2}{b\tanh{(dx+c)}^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
4*a^2*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) +
3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x +
 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (
d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)) + 2*b^3*(integrate(((a + b)*e^(4*d*x
 + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*
e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) +
a - b), x)/(a^4 - 2*a^2*b^2 + b^4) - (d*x + c)/((a^4 - 2*a^2*b^2 + b^4)*d)
) - 8*a^2*b*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)
*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 + a^2*b - a
*b^2 - b^3) + 8*a*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) +
 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 +
 a^2*b - a*b^2 - b^3) - 2*b^3*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x
+ 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x
)/(a^3 + a^2*b - a*b^2 - b^3) - 4*a^2*b*integrate(e^(2*d*x + 2*c)/((a + b)
*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) +
 a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) + 4*b^3*integrate(e^(2*d*x + 2*c)/
((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x
+ 2*c) + a - b), x)/(a^3 + a^2*b - a*b^2 - b^3) - 1/8*(4*(a^2*d*e^(2*c) -
3*a*b*d*e^(2*c) + 2*b^2*d*e^(2*c))*x*e^(2*d*x) + a^2 + 2*a*b + b^2 - (a^2*
e^(4*c) - b^2*e^(4*c))*e^(4*d*x))*e^(-2*d*x)/(a^3*d*e^(2*c) + a^2*b*d*e...
```

Giac [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\sinh\left(dx+c\right)^2}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 3.88 (sec) , antiderivative size = 2100, normalized size of antiderivative = 5.47

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(c + d*x)^2/(a + b*tanh(c + d*x)^3),x)
```

output
```
symsum(log(root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*
a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k)*((2304*root(54*a^2*b
^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d
^2*z^2 - 9*b^2*d*z + b, z, k)*(146*a^5*b^5*d^2 - 133*a^4*b^6*d^2 - 24*a^3*
b^7*d^2 - 12*a^6*b^4*d^2 + 22*a^7*b^3*d^2 + a^8*b^2*d^2 + 32*a^3*b^7*d^2*e
xp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a^2*b*
d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 577*a^4*b^6*
d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 + 54*a
^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 548*a^5
*b^5*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z^3 +
 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x) + 70
*a^6*b^4*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d^3*z
^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d*x)
+ 68*a^7*b^3*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d
^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*d
*x) + a^8*b^2*d^2*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*
d^3*z^3 + 54*a^2*b*d^2*z^2 + 27*b^3*d^2*z^2 - 9*b^2*d*z + b, z, k))*exp(2*
d*x)))/((a + b)^8*(a - b)^2*(a^2 - 2*a*b + b^2)) + (1536*(24*a^3*b^8*d + 1
05*a^4*b^7*d - 156*a^5*b^6*d + 51*a^6*b^5*d - 30*a^7*b^4*d + 6*a^8*b^3*d -
 32*a^3*b^8*d*exp(2*root(54*a^2*b^2*d^3*z^3 - 27*b^4*d^3*z^3 - 27*a^4*d...
```

Reduce [F]

$$\int \frac{\sinh^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{too large to display}$$

input
```
int(sinh(d*x+c)^2/(a+b*tanh(d*x+c)^3),x)
```

output

```
(3*e**(4*c + 4*d*x)*a**3 - 27*e**(4*c + 4*d*x)*a**2*b - 27*e**(4*c + 4*d*x
)*a*b**2 + 3*e**(4*c + 4*d*x)*b**3 - 288*e**(2*c + 2*d*x)*int(1/(e**(8*c +
 8*d*x)*a**5 - 7*e**(8*c + 8*d*x)*a**4*b - 26*e**(8*c + 8*d*x)*a**3*b**2 -
 26*e**(8*c + 8*d*x)*a**2*b**3 - 7*e**(8*c + 8*d*x)*a*b**4 + e**(8*c + 8*d
*x)*b**5 + 3*e**(6*c + 6*d*x)*a**5 - 27*e**(6*c + 6*d*x)*a**4*b - 30*e**(6
*c + 6*d*x)*a**3*b**2 + 30*e**(6*c + 6*d*x)*a**2*b**3 + 27*e**(6*c + 6*d*x
)*a*b**4 - 3*e**(6*c + 6*d*x)*b**5 + 3*e**(4*c + 4*d*x)*a**5 - 21*e**(4*c
+ 4*d*x)*a**4*b - 78*e**(4*c + 4*d*x)*a**3*b**2 - 78*e**(4*c + 4*d*x)*a**2
*b**3 - 21*e**(4*c + 4*d*x)*a*b**4 + 3*e**(4*c + 4*d*x)*b**5 + e**(2*c + 2
*d*x)*a**5 - 9*e**(2*c + 2*d*x)*a**4*b - 10*e**(2*c + 2*d*x)*a**3*b**2 + 1
0*e**(2*c + 2*d*x)*a**2*b**3 + 9*e**(2*c + 2*d*x)*a*b**4 - e**(2*c + 2*d*x
)*b**5),x)*a**7*b*d + 2208*e**(2*c + 2*d*x)*int(1/(e**(8*c + 8*d*x)*a**5 -
 7*e**(8*c + 8*d*x)*a**4*b - 26*e**(8*c + 8*d*x)*a**3*b**2 - 26*e**(8*c +
8*d*x)*a**2*b**3 - 7*e**(8*c + 8*d*x)*a*b**4 + e**(8*c + 8*d*x)*b**5 + 3*e
**(6*c + 6*d*x)*a**5 - 27*e**(6*c + 6*d*x)*a**4*b - 30*e**(6*c + 6*d*x)*a*
*3*b**2 + 30*e**(6*c + 6*d*x)*a**2*b**3 + 27*e**(6*c + 6*d*x)*a*b**4 - 3*e
**(6*c + 6*d*x)*b**5 + 3*e**(4*c + 4*d*x)*a**5 - 21*e**(4*c + 4*d*x)*a**4*
b - 78*e**(4*c + 4*d*x)*a**3*b**2 - 78*e**(4*c + 4*d*x)*a**2*b**3 - 21*e**
(4*c + 4*d*x)*a*b**4 + 3*e**(4*c + 4*d*x)*b**5 + e**(2*c + 2*d*x)*a**5 - 9
*e**(2*c + 2*d*x)*a**4*b - 10*e**(2*c + 2*d*x)*a**3*b**2 + 10*e**(2*c +...
```

3.76 $\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -i\text{Int}\left(\frac{i\sinh(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

output
```
-I*Defer(Int)(I*sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 409 vs. 2(31) = 62.

Time = 0.43 (sec) , antiderivative size = 409, normalized size of antiderivative = 19.48

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{6a\cosh(c+dx)+b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\frac{2ac+bc+2adx+\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(6*a*Cosh[c + d*x] + b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*
b*#1^4 + a*#1^6 + b*#1^6 & , (2*a*c + b*c + 2*a*d*x + b*d*x + 4*a*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1] + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*a*c*#1^4 - b*c*#1^4 + 2*a*d*x*#1^4 - b*d
*x*#1^4 + 4*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c
+ d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#
1 + 2*a*#1^3 - 2*b*#1^3 + a*#1^5 + b*#1^5) & ] - 6*b*Sinh[c + d*x])/(6*(a
- b)*(a + b)*d)
```

Rubi [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\sin(ic+idx)}{a+ib\tan(ic+idx)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\sin(ic+idx)}{ib\tan(ic+idx)^3+a}dx$$

$$\Big\downarrow \; 4151$$

$$-i\int \frac{i\sinh(c+dx)}{b\tanh^3(c+dx)+a}dx$$

input
```
Int[Sinh[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A] (verified)

Time = 3.50 (sec) , antiderivative size = 159, normalized size of antiderivative = 7.57

method	result
derivativedivides	$\frac{\frac{4}{(4a-4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4a-2\_R^3b+6\_R^2a-2\_Rb+a\right.}{\_R^5a+2\_R^3a+4\_R}\right.}{3(a-b)(a+b)}}{d}$
default	$\frac{\frac{4}{(4a-4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{b\left(\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4a-2\_R^3b+6\_R^2a-2\_Rb+a\right.}{\_R^5a+2\_R^3a+4\_R}\right.}{3(a-b)(a+b)}}{d}$
risch	Expression too large to display

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(4/(4*a-4*b)/(tanh(1/2*d*x+1/2*c)+1)+1/3*b/(a-b)/(a+b)*sum((_R^4*a-2*_
R^3*b+6*_R^2*a-2*_R*b+a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))-4/(4*a+4*b)/(tan
h(1/2*d*x+1/2*c)-1))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.71 (sec) , antiderivative size = 40923, normalized size of antiderivative = 1948.71

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [N/A]

Not integrable

Time = 53.20 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(sinh(c + d*x)/(a + b*tanh(c + d*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 250, normalized size of antiderivative = 11.90

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\sinh(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
1/2*((a*e^(2*c) - b*e^(2*c))*e^(2*d*x) + a + b)*e^(-d*x)/(a^2*d*e^c - b^2*
d*e^c) + 1/2*integrate(4*((2*a*b*e^(5*c) - b^2*e^(5*c))*e^(5*d*x) + (2*a*b
*e^c + b^2*e^c)*e^(d*x))/(a^3 - a^2*b - a*b^2 + b^3 + (a^3*e^(6*c) + a^2*b
*e^(6*c) - a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) + 3*(a^3*e^(4*c) - a^2*b
*e^(4*c) - a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 3*(a^3*e^(2*c) + a^2*b
*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 1.82 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\sinh(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 85.63 (sec) , antiderivative size = 4474, normalized size of antiderivative = 213.05

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\text{Too large to display}$$

input
```
int(sinh(c + d*x)/(a + b*tanh(c + d*x)^3),x)
```

output
```
exp(- c - d*x)/(2*(a*d - b*d)) + symsum(log((81920*a^2*b^5*exp(d*x)*exp(ro
ot(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729
*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2
*z^2 - b^2, z, k)) + 221184*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z
^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^
2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^8*d^3 - 3538944*ro
ot(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729
*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2
*z^2 - b^2, z, k)^3*a^3*b^7*d^3 + 1990656*root(2187*a^6*b^2*d^6*z^6 - 2187
*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^
4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^4*b^6*d^
3 + 3538944*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6
*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 +
81*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^5*d^3 - 2211840*root(2187*a^6*b^2*
d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1
458*a^4*b^2*d^4*z^4 - 729*a^2*b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k
)^3*a^6*b^4*d^3 + 7962624*root(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6
 + 729*a^2*b^6*d^6*z^6 - 729*a^8*d^6*z^6 - 1458*a^4*b^2*d^4*z^4 - 729*a^2*
b^4*d^4*z^4 + 81*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^3*b^9*d^5 + 15925248*roo
t(2187*a^6*b^2*d^6*z^6 - 2187*a^4*b^4*d^6*z^6 + 729*a^2*b^6*d^6*z^6 - 7...
```

Reduce [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 2466, normalized size of antiderivative = 117.43

$$\int \frac{\sinh(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sinh(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

output
```
(2*e**(c + d*x)*cosh(c + d*x)*a**2 + 4*e**(c + d*x)*cosh(c + d*x)*a*b + 2*
e**(c + d*x)*cosh(c + d*x)*b**2 - e**(2*c + 2*d*x)*a*b - e**(2*c + 2*d*x)*
b**2 - 24*e**(4*c + d*x)*int(e**(3*d*x)/(e**(6*c + 6*d*x)*a**3 + 3*e**(6*c
 + 6*d*x)*a**2*b + 3*e**(6*c + 6*d*x)*a*b**2 + e**(6*c + 6*d*x)*b**3 + 3*e
**(4*c + 4*d*x)*a**3 + 3*e**(4*c + 4*d*x)*a**2*b - 3*e**(4*c + 4*d*x)*a*b*
*2 - 3*e**(4*c + 4*d*x)*b**3 + 3*e**(2*c + 2*d*x)*a**3 + 9*e**(2*c + 2*d*x
)*a**2*b + 9*e**(2*c + 2*d*x)*a*b**2 + 3*e**(2*c + 2*d*x)*b**3 + a**3 + a*
*2*b - a*b**2 - b**3),x)*a**4*b*d - 36*e**(4*c + d*x)*int(e**(3*d*x)/(e**(
6*c + 6*d*x)*a**3 + 3*e**(6*c + 6*d*x)*a**2*b + 3*e**(6*c + 6*d*x)*a*b**2
+ e**(6*c + 6*d*x)*b**3 + 3*e**(4*c + 4*d*x)*a**3 + 3*e**(4*c + 4*d*x)*a**
2*b - 3*e**(4*c + 4*d*x)*a*b**2 - 3*e**(4*c + 4*d*x)*b**3 + 3*e**(2*c + 2*
d*x)*a**3 + 9*e**(2*c + 2*d*x)*a**2*b + 9*e**(2*c + 2*d*x)*a*b**2 + 3*e**(
2*c + 2*d*x)*b**3 + a**3 + a**2*b - a*b**2 - b**3),x)*a**3*b**2*d + 12*e**
(4*c + d*x)*int(e**(3*d*x)/(e**(6*c + 6*d*x)*a**3 + 3*e**(6*c + 6*d*x)*a**
2*b + 3*e**(6*c + 6*d*x)*a*b**2 + e**(6*c + 6*d*x)*b**3 + 3*e**(4*c + 4*d*
x)*a**3 + 3*e**(4*c + 4*d*x)*a**2*b - 3*e**(4*c + 4*d*x)*a*b**2 - 3*e**(4*
c + 4*d*x)*b**3 + 3*e**(2*c + 2*d*x)*a**3 + 9*e**(2*c + 2*d*x)*a**2*b + 9*
e**(2*c + 2*d*x)*a*b**2 + 3*e**(2*c + 2*d*x)*b**3 + a**3 + a**2*b - a*b**2
 - b**3),x)*a*b**4*d - 16*e**(2*c + d*x)*int(e**(d*x)/(e**(6*c + 6*d*x)*a*
*3 + 3*e**(6*c + 6*d*x)*a**2*b + 3*e**(6*c + 6*d*x)*a*b**2 + e**(6*c + ...
```

3.77 $\int \frac{\text{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 21

$$\int \frac{\text{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = i\text{Int}\left(-\frac{i\text{csch}(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

output
```
I*Defer(Int)(-I*csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 331 vs. 2(31) = 62.

Time = 0.37 (sec) , antiderivative size = 331, normalized size of antiderivative = 15.76

$$\int \frac{\text{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$

$$-\frac{6\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)-6\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-\right.}{}$$

input
```
Integrate[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/6*(6*Log[Cosh[(c + d*x)/2]] - 6*Log[Sinh[(c + d*x)/2]] + b*RootSum[a -
b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*#1^6 + b*#1^6 & , (c + d
*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 +
c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
 + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(a*#1 + b*#1 + 2*a*#1^3 - 2*b*
#1^3 + a*#1^5 + b*#1^5) & ])/(a*d)
```

Rubi [N/A]

Not integrable

Time = 0.40 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\sin(ic+idx)\,(a+ib\tan(ic+idx)^3)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\sin(ic+idx)\,(ib\tan(ic+idx)^3+a)}dx$$

$$\Big\downarrow \text{4151}$$

$$i\int -\frac{i\operatorname{csch}(c+dx)}{b\tanh^3(c+dx)+a}dx$$

input
```
Int[Csch[c + d*x]/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A] (verified)

Time = 1.74 (sec) , antiderivative size = 96, normalized size of antiderivative = 4.57

method	result
derivativedivides	$\frac{-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
default	$\frac{-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}+\frac{\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{d}$
risch	$-\frac{\ln(e^{dx+c}+1)}{ad}+2\left(\sum_{\_R=\mathrm{RootOf}\left((46656a^8d^6-46656a^6b^2d^6)\_Z^6+3888a^4b^2d^4\_Z^4-108a^2d^2\_Z^2b^2+b^2\right)}\_R\ln\right.$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-4/3*b/a*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2
*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))+1/a*ln(tanh(1/2*d*
x+1/2*c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 1.79 (sec) , antiderivative size = 20085, normalized size of antiderivative = 956.43

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [N/A]

Not integrable

Time = 0.46 (sec) , antiderivative size = 19, normalized size of antiderivative = 0.90

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(c+dx\right)}{a+b\tanh^3\left(c+dx\right)}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)/(a + b*tanh(c + d*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.28 (sec) , antiderivative size = 160, normalized size of antiderivative = 7.62

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}\left(dx+c\right)}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
 - 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(
a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(4*c) - a*b*e
^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [N/A]

Not integrable

Time = 1.38 (sec) , antiderivative size = 3, normalized size of antiderivative = 0.14

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\operatorname{csch}(dx+c)}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
sage0*x
```

Mupad [B] (verification not implemented)

Time = 15.58 (sec) , antiderivative size = 3679, normalized size of antiderivative = 175.19

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)*(a + b*tanh(c + d*x)^3)),x)
```

output
```
symsum(log(-(1409286144*b^6*exp(d*x)*exp(root(729*a^6*b^2*d^6*z^6 - 729*a^
8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)) + 13421
7728*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27
*a^2*b^2*d^2*z^2 - b^2, z, k)*b^7*d + 1879048192*root(729*a^6*b^2*d^6*z^6
- 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)*
a*b^6*d - 2818572288*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*
b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^2*b^7*d^3 - 40869298176*
root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*
b^2*d^2*z^2 - b^2, z, k)^3*a^3*b^6*d^3 + 28185722880*root(729*a^6*b^2*d^6*
z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z,
 k)^3*a^4*b^5*d^3 + 15502147584*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6
 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^3*a^5*b^4*d^3 + 1
8119393280*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^
4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^4*b^7*d^5 + 235552112640*root(729*
a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z
^2 - b^2, z, k)^5*a^5*b^6*d^5 + 14495514624*root(729*a^6*b^2*d^6*z^6 - 729
*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^6
*b^5*d^5 - 219244658688*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a
^4*b^2*d^4*z^4 + 27*a^2*b^2*d^2*z^2 - b^2, z, k)^5*a^7*b^4*d^5 - 489223618
56*root(729*a^6*b^2*d^6*z^6 - 729*a^8*d^6*z^6 - 243*a^4*b^2*d^4*z^4 + 2...
```

Reduce [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 784, normalized size of antiderivative = 37.33

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{12e^{5c}\left(\int \frac{e^{5dx}}{e^{8dx+8c}a^2+2e^{8dx+8c}ab+e^{8dx+8c}b^2+2e^{6dx+6c}a^2-2e^{6dx+6c}ab-4e^{6dx+6c}b^2+6e^{4dx+4c}ab+6e^{4dx+4c}b^2-2e^{2dx+2c}a^2-6e^{2dx+2c}ab-\cdots}\right.}{\cdots}$$

input
```
int(csch(d*x+c)/(a+b*tanh(d*x+c)^3),x)
```

output
```
(12*e**(5*c)*int(e**(5*d*x)/(e**(8*c + 8*d*x)*a**2 + 2*e**(8*c + 8*d*x)*a*
b + e**(8*c + 8*d*x)*b**2 + 2*e**(6*c + 6*d*x)*a**2 - 2*e**(6*c + 6*d*x)*a
*b - 4*e**(6*c + 6*d*x)*b**2 + 6*e**(4*c + 4*d*x)*a*b + 6*e**(4*c + 4*d*x)
*b**2 - 2*e**(2*c + 2*d*x)*a**2 - 6*e**(2*c + 2*d*x)*a*b - 4*e**(2*c + 2*d
*x)*b**2 - a**2 + b**2),x)*a*b*d + 12*e**(5*c)*int(e**(5*d*x)/(e**(8*c + 8
*d*x)*a**2 + 2*e**(8*c + 8*d*x)*a*b + e**(8*c + 8*d*x)*b**2 + 2*e**(6*c +
6*d*x)*a**2 - 2*e**(6*c + 6*d*x)*a*b - 4*e**(6*c + 6*d*x)*b**2 + 6*e**(4*c
 + 4*d*x)*a*b + 6*e**(4*c + 4*d*x)*b**2 - 2*e**(2*c + 2*d*x)*a**2 - 6*e**(
2*c + 2*d*x)*a*b - 4*e**(2*c + 2*d*x)*b**2 - a**2 + b**2),x)*b**2*d + 4*e*
*c*int(e**(d*x)/(e**(8*c + 8*d*x)*a**2 + 2*e**(8*c + 8*d*x)*a*b + e**(8*c
+ 8*d*x)*b**2 + 2*e**(6*c + 6*d*x)*a**2 - 2*e**(6*c + 6*d*x)*a*b - 4*e**(6
*c + 6*d*x)*b**2 + 6*e**(4*c + 4*d*x)*a*b + 6*e**(4*c + 4*d*x)*b**2 - 2*e*
*(2*c + 2*d*x)*a**2 - 6*e**(2*c + 2*d*x)*a*b - 4*e**(2*c + 2*d*x)*b**2 - a
**2 + b**2),x)*a*b*d + 4*e**c*int(e**(d*x)/(e**(8*c + 8*d*x)*a**2 + 2*e**(
8*c + 8*d*x)*a*b + e**(8*c + 8*d*x)*b**2 + 2*e**(6*c + 6*d*x)*a**2 - 2*e**
(6*c + 6*d*x)*a*b - 4*e**(6*c + 6*d*x)*b**2 + 6*e**(4*c + 4*d*x)*a*b + 6*e
**(4*c + 4*d*x)*b**2 - 2*e**(2*c + 2*d*x)*a**2 - 6*e**(2*c + 2*d*x)*a*b -
4*e**(2*c + 2*d*x)*b**2 - a**2 + b**2),x)*b**2*d + log(e**(c + d*x) - 1) -
 log(e**(c + d*x) + 1))/(d*(a + b))
```

3.78 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 157

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d} - \frac{\coth(c+dx)}{ad} + \frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}d} - \frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}d}$$

output
```
1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))*3^(1/2)/a^(1/3))*3^
(1/2)/a^(4/3)/d-coth(d*x+c)/a/d+1/3*b^(1/3)*ln(a^(1/3)+b^(1/3)*tanh(d*x+c)
)/a^(4/3)/d-1/6*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2/3)*tan
h(d*x+c)^2)/a^(4/3)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.21

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx =$$
$$-\frac{3\coth(c+dx)+2b\text{RootSum}\left[a-b+3a\#1+3b\#1+3a\#1^2-3b\#1^2+a\#1^3+b\#1^3\&,\frac{-c-dx-\log(-\cdots}{\cdots}\right]}{\cdots}$$

input
```
Integrate[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/3*(3*Coth[c + d*x] + 2*b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3
*b*#1^2 + a*#1^3 + b*#1^3 & , (-c - d*x - Log[-Cosh[c + d*x] - Sinh[c + d*
x] + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1] + c*#1 + d*x*#1 + Log[-Cosh[c +
d*x] - Sinh[c + d*x] + Cosh[c + d*x]*#1 - Sinh[c + d*x]*#1]*#1)/(a + b + 2
*a*#1 - 2*b*#1 + a*#1^2 + b*#1^2) & ])/(a*d)
```

Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 158, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.522$, Rules used = {3042, 25, 4146, 847, 821, 16, 1142, 25, 27, 1082, 217, 1103}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{1}{\sin(ic+idx)^2\left(a+ib\tan(ic+idx)^3\right)}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{1}{\sin(ic+idx)^2\left(ib\tan(ic+idx)^3+a\right)}dx$$

$$\Big\downarrow \textbf{4146}$$

$$\frac{\int \frac{\coth^2(c+dx)}{b\tanh^3(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \textbf{847}$$

$$\frac{-\frac{b\int \frac{\tanh(c+dx)}{b\tanh^3(c+dx)+a}d\tanh(c+dx)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\Big\downarrow \textbf{821}$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}\tanh(c+dx)+\sqrt[3]{a}}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\int \frac{1}{\sqrt[3]{b}\tanh(c+dx)+\sqrt[3]{a}}d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}\right)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\Big\downarrow \textbf{16}$$

$$\frac{-\frac{b\left(\frac{\int \frac{\sqrt[3]{b}\tanh(c+dx)+\sqrt[3]{a}}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

$$\Big\downarrow \textbf{1142}$$

$$\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)+\frac{\int -\frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)\right)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+ \right.}{3\sqrt[3]{}}\right.}{a}}{d}$$

$$\Big\downarrow \textbf{25}$$

$$-\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)-\frac{\int \frac{\sqrt[3]{b}\left(\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)\right)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{2\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{\ldots}\right.}{3\sqrt[3]{\ldots}}\ldots\right.}{a}\ldots}{d}$$

$\downarrow$ 27

$$-\frac{-\frac{b\left(\frac{\frac{3}{2}\sqrt[3]{a}\int \frac{1}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\ldots\right.}{3\sqrt[3]{\ldots}}\ldots\right.}{a}\ldots}{d}$$

$\downarrow$ 1082

$$-\frac{-\frac{b\left(\frac{\frac{3\int \frac{1}{-\left(1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)^2-3}d\left(1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}\right)}{\sqrt[3]{b}}-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh\ldots\right.}{3\sqrt[3]{a}b^{2/3}}\ldots\right.}{a}\ldots}{d}$$

$\downarrow$ 217

$$-\frac{-\frac{b\left(\frac{-\frac{1}{2}\int \frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{b^{2/3}\tanh^2(c+dx)-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+a^{2/3}}d\tanh(c+dx)-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{\coth(c\ldots}{a}\ldots}{d}$$

$\downarrow$ 1103

$$\frac{-\frac{b\left(\frac{\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{2\sqrt[3]{b}}-\frac{\sqrt{3}\arctan\left(\frac{1-\frac{2\sqrt[3]{b}\tanh(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt[3]{b}}}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3\sqrt[3]{a}b^{2/3}}\right)}{a}-\frac{\coth(c+dx)}{a}}{d}$$

input
```
Int[Csch[c + d*x]^2/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-(Coth[c + d*x]/a) - (b*(-1/3*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]]/(a^(1/
3)*b^(2/3)) + (-((Sqrt[3]*ArcTan[(1 - (2*b^(1/3)*Tanh[c + d*x])/a^(1/3))/S
qrt[3]])/b^(1/3)) + Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2/3)*
Tanh[c + d*x]^2]/(2*b^(1/3)))/(3*a^(1/3)*b^(1/3))))/a)/d
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 217
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[-b, 2])^(
-1))*ArcTan[Rt[-b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] &
& (LtQ[a, 0] || LtQ[b, 0])
```

rule 821
```
Int[(x_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> Simp[-(3*Rt[a, 3]*Rt[b, 3])^(-
1)   Int[1/(Rt[a, 3] + Rt[b, 3]*x), x], x] + Simp[1/(3*Rt[a, 3]*Rt[b, 3])
 Int[(Rt[a, 3] + Rt[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2
*x^2), x], x] /; FreeQ[{a, b}, x]
```

rule 847
```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c*x
)^(m + 1)*((a + b*x^n)^(p + 1)/(a*c*(m + 1))), x] - Simp[b*((m + n*(p + 1)
+ 1)/(a*c^n*(m + 1)))   Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a
, b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p
, x]
```

rule 1082
```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[a*(c/b^2)]}, Simp[-2/b   Subst[Int[1/(q - x^2), x], x, 1 + 2*c*(x/b
)], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Fre
eQ[{a, b, c}, x]
```

rule 1103
```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[d*(Log[RemoveContent[a + b*x + c*x^2, x]]/b), x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

rule 1142
```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(2*c*d - b*e)/(2*c)   Int[1/(a + b*x + c*x^2), x], x] + Simp[e/(2*c)
Int[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 1.86 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.74

method	result
derivativedivides	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\_R^5a+2\_R^3a+4\_R^2b+}\right.}{3a}}{d}$
default	$\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}-\frac{1}{2a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}+\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3-\_R\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{\_R^5a+2\_R^3a+4\_R^2b+}\right.}{3a}}{d}$
risch	$-\frac{2}{ad\left(e^{2dx+2c}-1\right)}+4\left(\sum_{\_R=\mathrm{RootOf}\left(1728a^4d^3\_Z^3-b\right)}\_R\ln\left(e^{2dx+2c}+\frac{288a^3d^2\_R^2}{(a+b)\left(\frac{b}{a+b}+\frac{a}{a+b}\right)}-\frac{24a^2}{(a+b)\left(\right.}\right.\right.$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/a*tanh(1/2*d*x+1/2*c)-1/2/a/tanh(1/2*d*x+1/2*c)+2/3*b/a*sum((_R^
3-_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf
(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 640 vs. 2(123) = 246.

Time = 0.12 (sec) , antiderivative size = 640, normalized size of antiderivative = 4.08

$$\int\frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\text{Too large to display}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
-1/6*(2*(sqrt(3)*cosh(d*x + c)^2 + 2*sqrt(3)*cosh(d*x + c)*sinh(d*x + c) +
 sqrt(3)*sinh(d*x + c)^2 - sqrt(3))*(b/a)^(1/3)*arctan(-1/3*(sqrt(3)*b*cos
h(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d*
x + c)^2 - (sqrt(3)*a*cosh(d*x + c)^2 + 2*sqrt(3)*a*cosh(d*x + c)*sinh(d*x
 + c) + sqrt(3)*a*sinh(d*x + c)^2 + sqrt(3)*a)*(b/a)^(2/3) - (sqrt(3)*b*co
sh(d*x + c)^2 + 2*sqrt(3)*b*cosh(d*x + c)*sinh(d*x + c) + sqrt(3)*b*sinh(d
*x + c)^2 - sqrt(3)*b)*(b/a)^(1/3))/b) + (cosh(d*x + c)^2 + 2*cosh(d*x + c
)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a + b)*cosh(d*x +
c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 +
 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) - 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x +
 c)^2 - a)*(b/a)^(2/3) + 2*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x
 + c) + a*sinh(d*x + c)^2 + a)*(b/a)^(1/3) + a + b) - 2*(cosh(d*x + c)^2 +
 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 1)*(b/a)^(1/3)*log((a +
 b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh
(d*x + c)^2 + 2*a*(b/a)^(2/3) - 2*a*(b/a)^(1/3) + a - b) + 12)/(a*d*cosh(d
*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)
```

Sympy [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\operatorname{csch}^2{(c+dx)}}{a+b\tanh^3{(c+dx)}}\,dx$$

input
```
integrate(csch(d*x+c)**2/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)**2/(a + b*tanh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^2}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d
*x + 2*c))/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) + 3*(a^2*e^(
4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e^(2*c))*e^(2*d*x)),
x)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.13

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{2}{ad(e^{(2\,dx+2\,c)}-1)}$$

input
```
integrate(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
-2/(a*d*(e^(2*d*x + 2*c) - 1))
```

Mupad [B] (verification not implemented)

Time = 9.63 (sec) , antiderivative size = 669, normalized size of antiderivative = 4.26

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^2*(a + b*tanh(c + d*x)^3)),x)
```

output
```
(b^(1/3)*log(a^(1/3) - b^(1/3) + a^(1/3)*exp(2*c + 2*d*x) + b^(1/3)*exp(2*
c + 2*d*x)))/(3*a^(4/3)*d) - 2/(a*d*(exp(2*c + 2*d*x) - 1)) + (b^(1/3)*log
((256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x) + b^
3*exp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c + 2*d*x
)))/(a^4*(a + b)^6) + (b^(1/3)*((3^(1/2)*1i)/2 - 1/2)*((1536*b^3*d*(8*a^2
- 8*b^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*exp(2
*c + 2*d*x)))/(a^2*(a + b)^6) + (768*b^(7/3)*d*((3^(1/2)*1i)/2 - 1/2)*(24*
a^2*b - 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c + 2*
d*x) + 113*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7/3)*(
a + b)^6)))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 - 1/2))/(3*a^(4/3)*d) - (b^(1/3
)*log((256*b^3*(19*a^2*b - 24*a*b^2 + 6*a^3 - b^3 + 8*a^3*exp(2*c + 2*d*x)
 + b^3*exp(2*c + 2*d*x) + 70*a*b^2*exp(2*c + 2*d*x) + 113*a^2*b*exp(2*c +
2*d*x)))/(a^4*(a + b)^6) - (b^(1/3)*((3^(1/2)*1i)/2 + 1/2)*((1536*b^3*d*(8
*a^2 - 8*b^2 + 15*a^2*exp(2*c + 2*d*x) + 15*b^2*exp(2*c + 2*d*x) + 66*a*b*
exp(2*c + 2*d*x)))/(a^2*(a + b)^6) - (768*b^(7/3)*d*((3^(1/2)*1i)/2 + 1/2)
*(24*a^2*b - 19*a*b^2 + a^3 - 6*b^3 + a^3*exp(2*c + 2*d*x) + 8*b^3*exp(2*c
 + 2*d*x) + 113*a*b^2*exp(2*c + 2*d*x) + 70*a^2*b*exp(2*c + 2*d*x)))/(a^(7
/3)*(a + b)^6)))/(3*a^(4/3)*d))*((3^(1/2)*1i)/2 + 1/2))/(3*a^(4/3)*d)
```

Reduce [F]

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{too large to display}$$

input
```
int(csch(d*x+c)^2/(a+b*tanh(d*x+c)^3),x)
```

output
```
(80*e**(4*c + 2*d*x)*int(e**(2*d*x)/(e**(10*c + 10*d*x)*a**3 - 12*e**(10*c
 + 10*d*x)*a**2*b - 9*e**(10*c + 10*d*x)*a*b**2 + 4*e**(10*c + 10*d*x)*b**
3 + e**(8*c + 8*d*x)*a**3 - 18*e**(8*c + 8*d*x)*a**2*b + 69*e**(8*c + 8*d*
x)*a*b**2 - 20*e**(8*c + 8*d*x)*b**3 - 2*e**(6*c + 6*d*x)*a**3 + 36*e**(6*
c + 6*d*x)*a**2*b - 138*e**(6*c + 6*d*x)*a*b**2 + 40*e**(6*c + 6*d*x)*b**3
 - 2*e**(4*c + 4*d*x)*a**3 + 16*e**(4*c + 4*d*x)*a**2*b + 122*e**(4*c + 4*
d*x)*a*b**2 - 40*e**(4*c + 4*d*x)*b**3 + e**(2*c + 2*d*x)*a**3 - 8*e**(2*c
 + 2*d*x)*a**2*b - 61*e**(2*c + 2*d*x)*a*b**2 + 20*e**(2*c + 2*d*x)*b**3 +
 a**3 - 14*a**2*b + 17*a*b**2 - 4*b**3),x)*a**4*b*d - 864*e**(4*c + 2*d*x)
*int(e**(2*d*x)/(e**(10*c + 10*d*x)*a**3 - 12*e**(10*c + 10*d*x)*a**2*b -
9*e**(10*c + 10*d*x)*a*b**2 + 4*e**(10*c + 10*d*x)*b**3 + e**(8*c + 8*d*x)
*a**3 - 18*e**(8*c + 8*d*x)*a**2*b + 69*e**(8*c + 8*d*x)*a*b**2 - 20*e**(8
*c + 8*d*x)*b**3 - 2*e**(6*c + 6*d*x)*a**3 + 36*e**(6*c + 6*d*x)*a**2*b -
138*e**(6*c + 6*d*x)*a*b**2 + 40*e**(6*c + 6*d*x)*b**3 - 2*e**(4*c + 4*d*x
)*a**3 + 16*e**(4*c + 4*d*x)*a**2*b + 122*e**(4*c + 4*d*x)*a*b**2 - 40*e**
(4*c + 4*d*x)*b**3 + e**(2*c + 2*d*x)*a**3 - 8*e**(2*c + 2*d*x)*a**2*b - 6
1*e**(2*c + 2*d*x)*a*b**2 + 20*e**(2*c + 2*d*x)*b**3 + a**3 - 14*a**2*b +
17*a*b**2 - 4*b**3),x)*a**3*b**2*d - 1968*e**(4*c + 2*d*x)*int(e**(2*d*x)/
(e**(10*c + 10*d*x)*a**3 - 12*e**(10*c + 10*d*x)*a**2*b - 9*e**(10*c + 10*
d*x)*a*b**2 + 4*e**(10*c + 10*d*x)*b**3 + e**(8*c + 8*d*x)*a**3 - 18*e*...
```

3.79 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 23

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -i\text{Int}\left(\frac{i\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)},x\right)$$

output
```
-I*Defer(Int)(I*csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 214 vs. 2(33) = 66.

Time = 0.54 (sec) , antiderivative size = 214, normalized size of antiderivative = 9.30

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = -\frac{16b\text{RootSum}\left[a-b+3a\#1^2+3b\#1^2+3a\#1^4-3b\#1^4+a\#1^6+b\#1^6\&,\frac{c\#1+dx\#1+2\log\left(-\cosh\left(\frac{1}{2}(c+\right.\right.}{a\cdot}\right.}{}$$

input
```
Integrate[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output
```
-1/24*(16*b*RootSum[a - b + 3*a*#1^2 + 3*b*#1^2 + 3*a*#1^4 - 3*b*#1^4 + a*
#1^6 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(a + b + 2*a*#1^2
 - 2*b*#1^2 + a*#1^4 + b*#1^4) & ] + 3*(Csch[(c + d*x)/2]^2 - 4*Log[Cosh[(
c + d*x)/2]] + 4*Log[Sinh[(c + d*x)/2]] + Sech[(c + d*x)/2]^2))/(a*d)
```

Rubi [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i}{\sin(ic+idx)^3\left(a+ib\tan(ic+idx)^3\right)}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{1}{\sin(ic+idx)^3\left(ib\tan(ic+idx)^3+a\right)}dx$$

$$\Big\downarrow \; 4151$$

$$-i\int \frac{i\operatorname{csch}^3(c+dx)}{b\tanh^3(c+dx)+a}dx$$

input
```
Int[Csch[c + d*x]^3/(a + b*Tanh[c + d*x]^3),x]
```

output
```
$Aborted
```

Maple [N/A] (verified)

Time = 2.94 (sec) , antiderivative size = 136, normalized size of antiderivative = 5.91

method	result
derivativedivides	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-}{d}$
default	$\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2}{8a}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3a}-}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}+1\right)}{da\left(e^{2dx+2c}-1\right)^2}-\frac{\ln\left(e^{dx+c}-1\right)}{2ad}+8\left(\sum_{\_R=\mathrm{RootOf}\left(191102976d^6\_Z^6a^{10}+1728a^4b^2d^2\_Z^2+a^2b^2-b^4\right)}\_R\right.$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/8*tanh(1/2*d*x+1/2*c)^2/a-1/3*b/a*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^
3*a+4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*
_Z^3*b+3*_Z^2*a+a))-1/8/a/tanh(1/2*d*x+1/2*c)^2-1/2/a*ln(tanh(1/2*d*x+1/2*
c)))
```

Fricas [C] (verification not implemented)

Result contains higher order function than in optimal. Order 3 vs. order 1.

Time = 3.07 (sec) , antiderivative size = 6846, normalized size of antiderivative = 297.65

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [N/A]

Not integrable

Time = 0.37 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.87

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\operatorname{csch}^3{(c+dx)}}{a+b\tanh^3{(c+dx)}}\,dx$$

input
```
integrate(csch(d*x+c)**3/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)**3/(a + b*tanh(c + d*x)**3), x)
```

Maxima [N/A]

Not integrable

Time = 0.32 (sec) , antiderivative size = 188, normalized size of antiderivative = 8.17

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx=\int \frac{\operatorname{csch}\left(dx+c\right)^3}{b\tanh\left(dx+c\right)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
-8*b*integrate(e^(3*d*x + 3*c)/(a^2 - a*b + (a^2*e^(6*c) + a*b*e^(6*c))*e^
(6*d*x) + 3*(a^2*e^(4*c) - a*b*e^(4*c))*e^(4*d*x) + 3*(a^2*e^(2*c) + a*b*e
^(2*c))*e^(2*d*x)), x) - (e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4
*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*
d) - 1/2*log((e^(d*x + c) - 1)*e^(-c))/(a*d)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Exception raised: AttributeError}$$

input
```
integrate(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
Exception raised: AttributeError >> type
```

Mupad [B] (verification not implemented)

Time = 27.49 (sec) , antiderivative size = 3643, normalized size of antiderivative = 158.39

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^3*(a + b*tanh(c + d*x)^3)),x)
```

output
```
exp(c + d*x)/(a*d - a*d*exp(2*c + 2*d*x)) - (2*exp(c + d*x))/(a*d - 2*a*d*
exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + symsum(log((570425344*a^4*b^6*e
xp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z,
 k)) - 33554432*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4
, z, k)*a*b^10*d - 553648128*a^2*b^8*exp(d*x)*exp(root(729*a^10*d^6*z^6 +
27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 167772160*a^3*b^7*exp(d*x)*ex
p(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) - 167
77216*b^10*exp(d*x)*exp(root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b
^2 - b^4, z, k)) + 192937984*a^5*b^5*exp(d*x)*exp(root(729*a^10*d^6*z^6 +
27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)) + 2617245696*root(729*a^10*d^6*
z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^5*b^8*d^3 - 150994944*
root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^6*b^
7*d^3 - 1384120320*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 -
b^4, z, k)^3*a^7*b^6*d^3 + 2415919104*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d
^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^8*b^5*d^3 - 3498049536*root(729*a^10*d^6
*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^3*a^9*b^4*d^3 + 543581798
4*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^8*
b^7*d^5 + 679477248*root(729*a^10*d^6*z^6 + 27*a^4*b^2*d^2*z^2 + a^2*b^2 -
 b^4, z, k)^5*a^9*b^6*d^5 - 70665633792*root(729*a^10*d^6*z^6 + 27*a^4*b^2
*d^2*z^2 + a^2*b^2 - b^4, z, k)^5*a^10*b^5*d^5 + 52319748096*root(729*a...
```

Reduce [N/A]

Not integrable

Time = 0.48 (sec) , antiderivative size = 13742, normalized size of antiderivative = 597.48

$$\int \frac{\operatorname{csch}^3(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(csch(d*x+c)^3/(a+b*tanh(d*x+c)^3),x)
```

output

```
( - 288*e**(9*c + 4*d*x)*int(e**(5*d*x)/(e**(12*c + 12*d*x)*a**3 + 3*e**(1
2*c + 12*d*x)*a**2*b + 3*e**(12*c + 12*d*x)*a*b**2 + e**(12*c + 12*d*x)*b*
*3 - 6*e**(10*c + 10*d*x)*a**2*b - 12*e**(10*c + 10*d*x)*a*b**2 - 6*e**(10
*c + 10*d*x)*b**3 - 3*e**(8*c + 8*d*x)*a**3 + 9*e**(8*c + 8*d*x)*a**2*b +
27*e**(8*c + 8*d*x)*a*b**2 + 15*e**(8*c + 8*d*x)*b**3 - 20*e**(6*c + 6*d*x
)*a**2*b - 40*e**(6*c + 6*d*x)*a*b**2 - 20*e**(6*c + 6*d*x)*b**3 + 3*e**(4
*c + 4*d*x)*a**3 + 21*e**(4*c + 4*d*x)*a**2*b + 33*e**(4*c + 4*d*x)*a*b**2
 + 15*e**(4*c + 4*d*x)*b**3 - 6*e**(2*c + 2*d*x)*a**2*b - 12*e**(2*c + 2*d
*x)*a*b**2 - 6*e**(2*c + 2*d*x)*b**3 - a**3 - a**2*b + a*b**2 + b**3),x)*a
**3*b*d - 288*e**(9*c + 4*d*x)*int(e**(5*d*x)/(e**(12*c + 12*d*x)*a**3 + 3
*e**(12*c + 12*d*x)*a**2*b + 3*e**(12*c + 12*d*x)*a*b**2 + e**(12*c + 12*d
*x)*b**3 - 6*e**(10*c + 10*d*x)*a**2*b - 12*e**(10*c + 10*d*x)*a*b**2 - 6*
e**(10*c + 10*d*x)*b**3 - 3*e**(8*c + 8*d*x)*a**3 + 9*e**(8*c + 8*d*x)*a**
2*b + 27*e**(8*c + 8*d*x)*a*b**2 + 15*e**(8*c + 8*d*x)*b**3 - 20*e**(6*c +
 6*d*x)*a**2*b - 40*e**(6*c + 6*d*x)*a*b**2 - 20*e**(6*c + 6*d*x)*b**3 + 3
*e**(4*c + 4*d*x)*a**3 + 21*e**(4*c + 4*d*x)*a**2*b + 33*e**(4*c + 4*d*x)*
a*b**2 + 15*e**(4*c + 4*d*x)*b**3 - 6*e**(2*c + 2*d*x)*a**2*b - 12*e**(2*c
 + 2*d*x)*a*b**2 - 6*e**(2*c + 2*d*x)*b**3 - a**3 - a**2*b + a*b**2 + b**3
),x)*a**2*b**2*d + 288*e**(9*c + 4*d*x)*int(e**(5*d*x)/(e**(12*c + 12*d*x)
*a**3 + 3*e**(12*c + 12*d*x)*a**2*b + 3*e**(12*c + 12*d*x)*a*b**2 + e**...
```

3.80 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 213

$$\begin{aligned}\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx &= -\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}\\&\quad-\frac{b\log(\tanh(c+dx))}{a^2d}-\frac{\left(a^{2/3}-b^{2/3}\right)\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^2d}\\&\quad+\frac{\left(a^{2/3}+2b^{2/3}\right)\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^2d}\end{aligned}$$

output
```
-1/3*b^(1/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*tanh(d*x+c))*3^(1/2)/a^(1/3))*3
^(1/2)/a^(4/3)/d+coth(d*x+c)/a/d-1/3*coth(d*x+c)^3/a/d-b*ln(tanh(d*x+c))/a
^2/d-1/3*(a^(2/3)-b^(2/3))*b^(1/3)*ln(a^(1/3)+b^(1/3)*tanh(d*x+c))/a^2/d+1
/6*(a^(2/3)+2*b^(2/3))*b^(1/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*tanh(d*x+c)+b^(2
/3)*tanh(d*x+c)^2)/a^2/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 322, normalized size of antiderivative = 1.51

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$= \frac{-a\coth(c+dx)\left(-2+\operatorname{csch}^2(c+dx)\right)+3b(c+dx-\log(\sinh(c+dx)))+b\text{RootSum}\Big[a-b+3a\#1+}{}$$

input
```
Integrate[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-(a*Coth[c + d*x]*(-2 + Csch[c + d*x]^2)) + 3*b*(c + d*x - Log[Sinh[c + d
*x]]) + b*RootSum[a - b + 3*a*#1 + 3*b*#1 + 3*a*#1^2 - 3*b*#1^2 + a*#1^3 +
 b*#1^3 & , (-2*a*c + 2*b*c - 2*a*d*x + 2*b*d*x + a*Log[E^(2*(c + d*x)) -
#1] - b*Log[E^(2*(c + d*x)) - #1] - 8*a*c*#1 - 4*b*c*#1 - 8*a*d*x*#1 - 4*b
*d*x*#1 + 4*a*Log[E^(2*(c + d*x)) - #1]*#1 + 2*b*Log[E^(2*(c + d*x)) - #1]
*#1 + 2*a*c*#1^2 + 2*b*c*#1^2 + 2*a*d*x*#1^2 + 2*b*d*x*#1^2 - a*Log[E^(2*(
c + d*x)) - #1]*#1^2 - b*Log[E^(2*(c + d*x)) - #1]*#1^2)/(a - b + 2*a*#1 +
 2*b*#1 + a*#1^2 - b*#1^2) & ])/(3*a^2*d)
```

Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 198, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4146, 2373, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sin(ic+idx)^4\left(a+ib\tan(ic+idx)^3\right)}dx$$

$$\Big\downarrow \text{4146}$$

$$\frac{\int \frac{\coth^4(c+dx)(1-\tanh^2(c+dx))}{b\tanh^3(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2373}$$

$$\frac{\int \left(\frac{\coth^4(c+dx)}{a}-\frac{\coth^2(c+dx)}{a}-\frac{b\coth(c+dx)}{a^2}+\frac{b\tanh(c+dx)(a+b\tanh(c+dx))}{a^2(b\tanh^3(c+dx)+a)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\sqrt[3]{b}\arctan\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\tanh(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{4/3}}+\frac{\sqrt[3]{b}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\tanh(c+dx)+b^{2/3}\tanh^2(c+dx)\right)}{6a^{4/3}}-\frac{\sqrt[3]{b}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\tanh(c+dx)\right)}{3a^{4/3}}+}{d}$$

input
```
Int[Csch[c + d*x]^4/(a + b*Tanh[c + d*x]^3),x]
```

output
```
(-((b^(1/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Tanh[c + d*x])/(Sqrt[3]*a^(1/3))])
/(Sqrt[3]*a^(4/3))) + Coth[c + d*x]/a - Coth[c + d*x]^3/(3*a) - (b*Log[Tan
h[c + d*x]])/a^2 - (b^(1/3)*Log[a^(1/3) + b^(1/3)*Tanh[c + d*x]])/(3*a^(4/
3)) + (b^(1/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Tanh[c + d*x] + b^(2/3)*Tanh[
c + d*x]^2])/(6*a^(4/3)) + (b*Log[a + b*Tanh[c + d*x]^3])/(3*a^2))/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2373
```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[(c*x)^m*(Pq/(a + b*x^n)), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] && !IGtQ[m, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4146
```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[c*(ff^(m + 1)/f)   Subst[Int[x^m*((a + b*(ff*x)^n)^p/(c^2 + ff^2*x^2)^(m/
2 + 1)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x
] && IntegerQ[m/2]
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 9 vs. order 3.

Time = 3.73 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.81

method	result
derivativedivides	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\right.}\right.}{}}{d}$
default	$\frac{-\frac{\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{3}-3\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}-\frac{1}{24a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}+\frac{3}{8a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\right.}\right.}{}}{d}$
risch	$-\frac{4\left(3\,\mathrm{e}^{2dx+2c}-1\right)}{3ad\left(\mathrm{e}^{2dx+2c}-1\right)^3}-\frac{b\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{d\,a^2}+16\left(\sum_{\_R=\mathrm{RootOf}\left(110592a^6d^3\_Z^3-6912a^4b\,d^2\_Z^2+144a^2b^2d\_Z+a\right.}\right.$

input
```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/8/a*(1/3*tanh(1/2*d*x+1/2*c)^3-3*tanh(1/2*d*x+1/2*c))-1/24/a/tanh(
1/2*d*x+1/2*c)^3+3/8/a/tanh(1/2*d*x+1/2*c)-b/a^2*ln(tanh(1/2*d*x+1/2*c))+1
/3*b/a^2*sum((_R^5*a+4*_R^2*b+3*_R*a)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(t
anh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 1.26 (sec) , antiderivative size = 3726, normalized size of antiderivative = 17.49

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx$$

input
```
integrate(csch(d*x+c)**4/(a+b*tanh(d*x+c)**3),x)
```

output
```
Integral(csch(c + d*x)**4/(a + b*tanh(c + d*x)**3), x)
```

Maxima [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \int \frac{\operatorname{csch}(dx+c)^4}{b\tanh(dx+c)^3+a}\,dx$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="maxima")
```

output
```
2*a*b*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^(2*d*x + 2*c) + 3*
a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4
*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*b) - (d*x + c)/((a
^3 - a^2*b)*d)) - 2*b^2*(integrate(((a + b)*e^(4*d*x + 4*c) + 3*(a - b)*e^
(2*d*x + 2*c) + 3*a + 3*b)*e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a
 - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/(a^3 - a^2*
b) - (d*x + c)/((a^3 - a^2*b)*d)) + 2*b*integrate(e^(4*d*x + 4*c)/((a + b)
*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) +
 a - b), x)/a + 2*b^2*integrate(e^(4*d*x + 4*c)/((a + b)*e^(6*d*x + 6*c) +
 3*(a - b)*e^(4*d*x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a^2 -
8*b*integrate(e^(2*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*
x + 4*c) + 3*(a + b)*e^(2*d*x + 2*c) + a - b), x)/a - 4*b^2*integrate(e^(2
*d*x + 2*c)/((a + b)*e^(6*d*x + 6*c) + 3*(a - b)*e^(4*d*x + 4*c) + 3*(a +
b)*e^(2*d*x + 2*c) + a - b), x)/a^2 + 2/3*(3*b*d*x*e^(6*d*x + 6*c) - 9*b*d
*x*e^(4*d*x + 4*c) - 3*b*d*x + 3*(3*b*d*x*e^(2*c) - 2*a*e^(2*c))*e^(2*d*x)
 + 2*a)/(a^2*d*e^(6*d*x + 6*c) - 3*a^2*d*e^(4*d*x + 4*c) + 3*a^2*d*e^(2*d*
x + 2*c) - a^2*d) - b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b*log((e^(d*
x + c) - 1)*e^(-c))/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.85

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \frac{\frac{2\,b\log\left(\left|ae^{(6\,dx+6\,c)}+be^{(6\,dx+6\,c)}+3\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}+3\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx+2\,c)}+a-b\right|\right)}{a^2}-\frac{6\,b\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2}+\frac{11\,be^{(6\,dx+}}{}}{6\,d}$$

input
```
integrate(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x, algorithm="giac")
```

output
```
1/6*(2*b*log(abs(a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 3*a*e^(4*d*x + 4*
c) - 3*b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + a -
 b))/a^2 - 6*b*log(abs(e^(2*d*x + 2*c) - 1))/a^2 + (11*b*e^(6*d*x + 6*c) -
 33*b*e^(4*d*x + 4*c) - 24*a*e^(2*d*x + 2*c) + 33*b*e^(2*d*x + 2*c) + 8*a
- 11*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 4.53 (sec) , antiderivative size = 4563, normalized size of antiderivative = 21.42

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(sinh(c + d*x)^4*(a + b*tanh(c + d*x)^3)),x)
```

output
```
8/(3*(a*d - 3*a*d*exp(2*c + 2*d*x) + 3*a*d*exp(4*c + 4*d*x) - a*d*exp(6*c
+ 6*d*x))) - 4/(a*d - 2*a*d*exp(2*c + 2*d*x) + a*d*exp(4*c + 4*d*x)) + sym
sum(log((1507328*a*b^9 + 1572864*b^10 - 5242880*a^2*b^8 - 7479296*a^3*b^7
+ 3948544*a^4*b^6 + 5963776*a^5*b^5 - 278528*a^6*b^4 + 8192*a^7*b^3 - 1572
864*b^10*exp(2*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^
2*b - b^3, z, k))*exp(2*d*x) - 1769472*a*b^9*exp(2*root(27*a^6*d^3*z^3 - 2
7*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k))*exp(2*d*x) + 4246732
8*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z,
 k)^2*a^4*b^8*d^2 + 21626880*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^
2*b^2*d*z + a^2*b - b^3, z, k)^2*a^5*b^7*d^2 - 70189056*root(27*a^6*d^3*z^
3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^6*b^6*d^2 +
18038784*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b -
b^3, z, k)^2*a^7*b^5*d^2 - 11993088*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2
 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^8*b^4*d^2 + 147456*root(27*a^6*d
^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^2*a^9*b^3*d
^2 - 98304*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b
- b^3, z, k)^2*a^10*b^2*d^2 - 42467328*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*
z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^6*b^7*d^3 - 12091392*root(27*
a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d*z + a^2*b - b^3, z, k)^3*a^7*
b^6*d^3 + 22708224*root(27*a^6*d^3*z^3 - 27*a^4*b*d^2*z^2 + 9*a^2*b^2*d...
```

Reduce [F]

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\tanh^3(c+dx)}\,dx = \text{too large to display}$$

input
```
int(csch(d*x+c)^4/(a+b*tanh(d*x+c)^3),x)
```

output

```
( - 1344*e**(8*c + 6*d*x)*int(e**(2*d*x)/(e**(14*c + 14*d*x)*a**4 - 44*e**
(14*c + 14*d*x)*a**3*b + 51*e**(14*c + 14*d*x)*a**2*b**2 + 76*e**(14*c + 1
4*d*x)*a*b**3 - 20*e**(14*c + 14*d*x)*b**4 - e**(12*c + 12*d*x)*a**4 + 38*
e**(12*c + 12*d*x)*a**3*b + 219*e**(12*c + 12*d*x)*a**2*b**2 - 652*e**(12*
c + 12*d*x)*a*b**3 + 140*e**(12*c + 12*d*x)*b**4 - 3*e**(10*c + 10*d*x)*a*
*4 + 156*e**(10*c + 10*d*x)*a**3*b - 1233*e**(10*c + 10*d*x)*a**2*b**2 + 2
076*e**(10*c + 10*d*x)*a*b**3 - 420*e**(10*c + 10*d*x)*b**4 + 3*e**(8*c +
8*d*x)*a**4 - 170*e**(8*c + 8*d*x)*a**3*b + 1863*e**(8*c + 8*d*x)*a**2*b**
2 - 3420*e**(8*c + 8*d*x)*a*b**3 + 700*e**(8*c + 8*d*x)*b**4 + 3*e**(6*c +
 6*d*x)*a**4 - 100*e**(6*c + 6*d*x)*a**3*b - 1287*e**(6*c + 6*d*x)*a**2*b*
*2 + 3300*e**(6*c + 6*d*x)*a*b**3 - 700*e**(6*c + 6*d*x)*b**4 - 3*e**(4*c
+ 4*d*x)*a**4 + 114*e**(4*c + 4*d*x)*a**3*b + 657*e**(4*c + 4*d*x)*a**2*b*
*2 - 1956*e**(4*c + 4*d*x)*a*b**3 + 420*e**(4*c + 4*d*x)*b**4 - e**(2*c +
2*d*x)*a**4 + 52*e**(2*c + 2*d*x)*a**3*b - 411*e**(2*c + 2*d*x)*a**2*b**2
+ 692*e**(2*c + 2*d*x)*a*b**3 - 140*e**(2*c + 2*d*x)*b**4 + a**4 - 46*a**3
*b + 141*a**2*b**2 - 116*a*b**3 + 20*b**4),x)*a**7*b*d + 50880*e**(8*c + 6
*d*x)*int(e**(2*d*x)/(e**(14*c + 14*d*x)*a**4 - 44*e**(14*c + 14*d*x)*a**3
*b + 51*e**(14*c + 14*d*x)*a**2*b**2 + 76*e**(14*c + 14*d*x)*a*b**3 - 20*e
**(14*c + 14*d*x)*b**4 - e**(12*c + 12*d*x)*a**4 + 38*e**(12*c + 12*d*x)*a
**3*b + 219*e**(12*c + 12*d*x)*a**2*b**2 - 652*e**(12*c + 12*d*x)*a*b**...
```

3.81 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 63

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{8}(3a-b)x + \frac{(3a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
1/8*(3*a-b)*x+1/8*(3*a-b)*cosh(d*x+c)*sinh(d*x+c)/d+1/4*(a+b)*cosh(d*x+c)^
3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.70

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{-4bdx+12a(c+dx)+8a\sinh(2(c+dx))+(a+b)\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-4*b*d*x + 12*a*(c + d*x) + 8*a*Sinh[2*(c + d*x)] + (a + b)*Sinh[4*(c + d
*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4158, 298, 215, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)^4}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{b\tanh^2(c+dx)+a}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}(3a-b)\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}(3a-b)\left(\frac{1}{2}\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}(3a-b)\left(\frac{1}{2}\text{arctanh}(\tanh(c+dx))+\frac{\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + ((3*a - b)*(ArcTanh
[Tanh[c + d*x]]/2 + Tanh[c + d*x]/(2*(1 - Tanh[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 11.04 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.30

method	result
derivativedivides	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
default	$\frac{a\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right)}{d}$
risch	$\frac{3ax}{8}-\frac{bx}{8}+\frac{e^{4dx+4c}a}{64d}+\frac{e^{4dx+4c}b}{64d}+\frac{e^{2dx+2c}a}{8d}-\frac{e^{-2dx-2c}a}{8d}-\frac{e^{-4dx-4c}a}{64d}-\frac{e^{-4dx-4c}b}{64d}$

input
```
int(cosh(d*x+c)^4*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+b*(
1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.00

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(a+b)\cosh(dx+c)\sinh(dx+c)^3+(3a-b)dx+\left((a+b)\cosh(dx+c)^3+4a\cosh(dx+c)\right)\sinh(dx}{8d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/8*((a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a - b)*d*x + ((a + b)*cosh
(d*x + c)^3 + 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)\cosh^4\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**4, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{64}a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}+\frac{8\,e^{(2\,dx+2\,c)}}{d}-\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right) - \frac{1}{64}b\left(\frac{8\,(dx+c)}{d}-\frac{e^{(4\,dx+4\,c)}}{d}+\frac{e^{(-4\,dx-4\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c
)/d - e^(-4*d*x - 4*c)/d) - 1/64*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^
(-4*d*x - 4*c)/d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{8\,(dx+c)(3\,a-b)+ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}-\left(18\,ae^{(4\,dx+4\,c)}-6\,be^{(4\,dx+4\,c)}+8\,ae^{(2\,dx+2\,c)}\right.}{64\,d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(8*(d*x + c)*(3*a - b) + a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 8*a*
e^(2*d*x + 2*c) - (18*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e^(2*d
*x + 2*c) + a + b)*e^(-4*d*x - 4*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.17

$$\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = x\left(\frac{3\,a}{8}-\frac{b}{8}\right)-\frac{\mathrm{e}^{-4\,c-4\,d\,x}\,(a+b)}{64\,d}+\frac{\mathrm{e}^{4\,c+4\,d\,x}\,(a+b)}{64\,d}-\frac{a\,\mathrm{e}^{-2\,c-2\,d\,x}}{8\,d}+\frac{a\,\mathrm{e}^{2\,c+2\,d\,x}}{8\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*((3*a)/8 - b/8) - (exp(- 4*c - 4*d*x)*(a + b))/(64*d) + (exp(4*c + 4*d*x
)*(a + b))/(64*d) - (a*exp(- 2*c - 2*d*x))/(8*d) + (a*exp(2*c + 2*d*x))/(8
*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.65

$$\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{e^{8dx+8c}a+e^{8dx+8c}b+8e^{6dx+6c}a+24e^{4dx+4c}adx-8e^{4dx+4c}bdx-8e^{2dx+2c}a-a-b}{64e^{4dx+4c}d}$$

input
```
int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(8*c + 8*d*x)*a + e**(8*c + 8*d*x)*b + 8*e**(6*c + 6*d*x)*a + 24*e**(4
*c + 4*d*x)*a*d*x - 8*e**(4*c + 4*d*x)*b*d*x - 8*e**(2*c + 2*d*x)*a - a -
b)/(64*e**(4*c + 4*d*x)*d)
```

3.82 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 30

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}$$

output
```
a*sinh(d*x+c)/d+1/3*(a+b)*sinh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.47

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\sinh(c+dx)}{d}+\frac{a\sinh^3(c+dx)}{3d}+\frac{b\sinh^3(c+dx)}{3d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Sinh[c + d*x])/d + (a*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^3)/(3*d
)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4159, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)^3}dx$$

$$\Big\downarrow \ 4159$$

$$\frac{\int \left((a+b)\sinh^2(c+dx)+a\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{3}(a+b)\sinh^3(c+dx)+a\sinh(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Sinh[c + d*x] + ((a + b)*Sinh[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.43 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.23

method	result	size
derivativedivides	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{b\sinh(dx+c)^3}{3}}{d}$	37
default	$\frac{a\left(\frac{2}{3}+\frac{\cosh(dx+c)^2}{3}\right)\sinh(dx+c)+\frac{b\sinh(dx+c)^3}{3}}{d}$	37
risch	$\frac{e^{3dx+3c}a}{24d}+\frac{e^{3dx+3c}b}{24d}+\frac{3\,e^{dx+c}a}{8d}-\frac{e^{dx+c}b}{8d}-\frac{3\,e^{-dx-c}a}{8d}+\frac{e^{-dx-c}b}{8d}-\frac{e^{-3dx-3c}a}{24d}-\frac{e^{-3dx-3c}b}{24d}$	116

input
```
int(cosh(d*x+c)^3*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+1/3*b*sinh(d*x+c)^3)
```

Fricas [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.50

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(a+b)\sinh\left(dx+c\right)^3+3\left((a+b)\cosh\left(dx+c\right)^2+3\,a-b\right)\sinh\left(dx+c\right)}{12\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/12*((a + b)*sinh(d*x + c)^3 + 3*((a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh
(d*x + c))/d
```

Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)\cosh^3{(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 83 vs. $2(28) = 56$.

Time = 0.04 (sec) , antiderivative size = 83, normalized size of antiderivative = 2.77

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{24\,d}+\frac{1}{24}\,a\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*b*(e^(d*x + c) - e^(-d*x - c))^3/d + 1/24*a*(e^(3*d*x + 3*c)/d + 9*e^
(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(28) = 56$.

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.80

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{ae^{(3\,dx+3\,c)}+be^{(3\,dx+3\,c)}+9\,ae^{(dx+c)}-3\,be^{(dx+c)}-\left(9\,ae^{(2\,dx+2\,c)}-3\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/24*(a*e^(3*d*x + 3*c) + b*e^(3*d*x + 3*c) + 9*a*e^(d*x + c) - 3*b*e^(d*x
 + c) - (9*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + a + b)*e^(-3*d*x - 3*
c))/d
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 74, normalized size of antiderivative = 2.47

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{e^{3\,c+3\,d\,x}\,(a+b)}{24\,d}-\frac{e^{-3\,c-3\,d\,x}\,(a+b)}{24\,d}+\frac{e^{c+d\,x}\,(3\,a-b)}{8\,d}-\frac{e^{-c-d\,x}\,(3\,a-b)}{8\,d}$$

input
```
int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(3*c + 3*d*x)*(a + b))/(24*d) - (exp(- 3*c - 3*d*x)*(a + b))/(24*d) +
(exp(c + d*x)*(3*a - b))/(8*d) - (exp(- c - d*x)*(3*a - b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 100, normalized size of antiderivative = 3.33

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{e^{6dx+6c}a+e^{6dx+6c}b+9e^{4dx+4c}a-3e^{4dx+4c}b-9e^{2dx+2c}a+3e^{2dx+2c}b-a-b}{24e^{3dx+3c}d}$$

input
```
int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(6*c + 6*d*x)*a + e**(6*c + 6*d*x)*b + 9*e**(4*c + 4*d*x)*a - 3*e**(4*
c + 4*d*x)*b - 9*e**(2*c + 2*d*x)*a + 3*e**(2*c + 2*d*x)*b - a - b)/(24*e*
*(3*c + 3*d*x)*d)
```

3.83 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 33

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{1}{2}(a-b)x+\frac{(a+b)\cosh(c+dx)\sinh(c+dx)}{2d}$$

output
```
1/2*(a-b)*x+1/2*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{2(a-b)(c+dx)+(a+b)\sinh(2(c+dx))}{4d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*(a - b)*(c + d*x) + (a + b)*Sinh[2*(c + d*x)])/(4*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.45, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4158, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \frac{b\tanh^2(c+dx)+a}{(1-\tanh^2(c+dx))^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{2}(a-b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{(a+b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{1}{2}(a-b)\text{arctanh}(\tanh(c+dx))+\frac{(a+b)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a - b)*ArcTanh[Tanh[c + d*x]])/2 + ((a + b)*Tanh[c + d*x])/(2*(1 - Tanh
[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 1.53 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.64

method	result	size
derivativedivides	$\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
default	$\frac{b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)}{d}$	54
risch	$\frac{ax}{2}-\frac{bx}{2}+\frac{\mathrm{e}^{2dx+2c}a}{8d}+\frac{\mathrm{e}^{2dx+2c}b}{8d}-\frac{\mathrm{e}^{-2dx-2c}a}{8d}-\frac{\mathrm{e}^{-2dx-2c}b}{8d}$	70

input
```
int(cosh(d*x+c)^2*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a*(1/2*cosh(d*x+c)*sinh
(d*x+c)+1/2*d*x+1/2*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.91

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a-b)dx+(a+b)\cosh(dx+c)\sinh(dx+c)}{2\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a - b)*d*x + (a + b)*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2(c+dx)\right)\cosh^2(c+dx)\,dx$$

input
```
integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 69 vs. 2(29) = 58.

Time = 0.03 (sec) , antiderivative size = 69, normalized size of antiderivative = 2.09

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{1}{8}\,a\left(4\,x+\frac{e^{(2\,dx+2\,c)}}{d}-\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{8}\,b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/8*b*(4*x - e^(2*d
*x + 2*c)/d + e^(-2*d*x - 2*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 78 vs. $2(29) = 58$.

Time = 0.13 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{4\,(dx+c)(a-b)+ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}-\left(2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(4*(d*x + c)*(a - b) + a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) - (2*a*e^
(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.82

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\left(\frac{a}{2}-\frac{b}{2}\right)+\frac{\sinh(2\,c+2\,d\,x)\,\left(a+b\right)}{4\,d}$$

input
```
int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a/2 - b/2) + (sinh(2*c + 2*d*x)*(a + b))/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 78, normalized size of antiderivative = 2.36

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{4dx+4c}a+e^{4dx+4c}b+4e^{2dx+2c}adx-4e^{2dx+2c}bdx-a-b}{8e^{2dx+2c}d}$$

input
```
int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(4*c + 4*d*x)*a + e**(4*c + 4*d*x)*b + 4*e**(2*c + 2*d*x)*a*d*x - 4*e*
*(2*c + 2*d*x)*b*d*x - a - b)/(8*e**(2*c + 2*d*x)*d)
```

3.84 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 27

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{b\arctan(\sinh(c+dx))}{d}+\frac{(a+b)\sinh(c+dx)}{d}$$

output
```
-b*arctan(sinh(d*x+c))/d+(a+b)*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{b\arctan(\sinh(c+dx))}{d}+\frac{a\cosh(dx)\sinh(c)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\sinh(c+dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((b*ArcTan[Sinh[c + d*x]])/d) + (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh
[d*x])/d + (b*Sinh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4159, 299, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{a-b\tan(ic+idx)^2}{\sec(ic+idx)}dx$$

$$\Big\downarrow \ 4159$$

$$\frac{\int \frac{(a+b)\sinh^2(c+dx)+a}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 299$$

$$\frac{(a+b)\sinh(c+dx)-b\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 216$$

$$\frac{(a+b)\sinh(c+dx)-b\arctan(\sinh(c+dx))}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(b*ArcTan[Sinh[c + d*x]]) + (a + b)*Sinh[c + d*x])/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 0.83 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.19

method	result	size
derivativedivides	$\frac{a\sinh(dx+c)+b(\sinh(dx+c)-2\arctan(\mathrm{e}^{dx+c}))}{d}$	32
default	$\frac{a\sinh(dx+c)+b(\sinh(dx+c)-2\arctan(\mathrm{e}^{dx+c}))}{d}$	32
risch	$\frac{\mathrm{e}^{dx+c}a}{2d}+\frac{\mathrm{e}^{dx+c}b}{2d}-\frac{\mathrm{e}^{-dx-c}a}{2d}-\frac{\mathrm{e}^{-dx-c}b}{2d}+\frac{ib\ln(\mathrm{e}^{dx+c}-i)}{d}-\frac{ib\ln(\mathrm{e}^{dx+c}+i)}{d}$	90

input
```
int(cosh(d*x+c)*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*sinh(d*x+c)+b*(sinh(d*x+c)-2*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 102, normalized size of antiderivative = 3.78

$$\int \cosh(c + dx)\left(a + b\tanh^2(c + dx)\right) dx$$
$$= \frac{(a+b)\cosh(dx+c)^2 + 2(a+b)\cosh(dx+c)\sinh(dx+c) + (a+b)\sinh(dx+c)^2 - 4(b\cosh(dx+c}{2(d\cosh(dx+c) + d\sinh(dx+c))}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a
+ b)*sinh(d*x + c)^2 - 4*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(cosh(d
*x + c) + sinh(d*x + c)) - a - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F]

$$\int \cosh(c + dx)\left(a + b\tanh^2(c + dx)\right) dx = \int \left(a + b\tanh^2(c + dx)\right)\cosh(c + dx)\,dx$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*cosh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 55 vs. $2(27) = 54$.

Time = 0.12 (sec) , antiderivative size = 55, normalized size of antiderivative = 2.04

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{2}b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right) + \frac{a\sinh\left(dx+c\right)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*sinh
(d*x + c)/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.74

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{4\,b\arctan\left(e^{(dx+c)}\right)-ae^{(dx+c)}-be^{(dx+c)}+(a+b)e^{(-dx-c)}}{2\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(4*b*arctan(e^(d*x + c)) - a*e^(d*x + c) - b*e^(d*x + c) + (a + b)*e^
(-d*x - c))/d
```

Mupad [B] (verification not implemented)

Time = 2.40 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

$$\int \cosh(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{e^{c+d\,x}\,(a+b)}{2\,d} - \frac{2\,\mathrm{atan}\left(\frac{b\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^2}}\right)\,\sqrt{b^2}}{\sqrt{d^2}} - \frac{e^{-c-d\,x}\,(a+b)}{2\,d}$$

input
```
int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
(exp(c + d*x)*(a + b))/(2*d) - (2*atan((b*exp(d*x)*exp(c)*(d^2)^(1/2))/(d*
(b^2)^(1/2)))*(b^2)^(1/2))/(d^2)^(1/2) - (exp(- c - d*x)*(a + b))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.33

$$\int \cosh(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{-4e^{dx+c} atan\left(e^{dx+c}\right) b + e^{2dx+2c}a + e^{2dx+2c}b - a - b}{2e^{dx+c}d}$$

input
```
int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

output
```
( - 4*e**(c + d*x)*atan(e**(c + d*x))*b + e**(2*c + 2*d*x)*a + e**(2*c + 2
*d*x)*b - a - b)/(2*e**(c + d*x)*d)
```

3.85 $\int \operatorname{sech}(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 19, antiderivative size = 40

$$\int \operatorname{sech}(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{(2a+b)\arctan(\sinh(c+dx))}{2d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*(2*a+b)*arctan(sinh(d*x+c))/d-1/2*b*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.22

$$\int \operatorname{sech}(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = -\frac{a\cot^{-1}(\sinh(c+dx))}{d} + \frac{b\arctan(\sinh(c+dx))}{2d} - \frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((a*ArcCot[Sinh[c + d*x]])/d) + (b*ArcTan[Sinh[c + d*x]])/(2*d) - (b*Sech
[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.10, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {3042, 4159, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{(a+b)\sinh^2(c+dx)+a}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{2}(2a+b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)-\frac{b\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{2}(2a+b)\arctan(\sinh(c+dx))-\frac{b\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((2*a + b)*ArcTan[Sinh[c + d*x]])/2 - (b*Sinh[c + d*x])/(2*(1 + Sinh[c +
d*x]^2)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 1.52 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.40

method	result	size
derivativedivides	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	56
default	$\frac{2a\arctan\left(e^{dx+c}\right)+b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^{2}}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)}{d}$	56
risch	$-\frac{b\,e^{dx+c}\left(e^{2dx+2c}-1\right)}{d\left(e^{2dx+2c}+1\right)^{2}}+\frac{i\ln\left(e^{dx+c}+i\right)a}{d}+\frac{ib\ln\left(e^{dx+c}+i\right)}{2d}-\frac{i\ln\left(e^{dx+c}-i\right)a}{d}-\frac{ib\ln\left(e^{dx+c}-i\right)}{2d}$	106

input
```
int(sech(d*x+c)*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a*arctan(exp(d*x+c))+b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d*x+c)*
tanh(d*x+c)+arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(36) = 72$.

Time = 0.10 (sec) , antiderivative size = 323, normalized size of antiderivative = 8.08

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{b\cosh\left(dx+c\right)^3+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+b\sinh\left(dx+c\right)^3-\left(\left(2\,a+b\right)\cosh\left(dx+c\right)^4+4\,(2\right.}{}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^
3 - ((2*a + b)*cosh(d*x + c)^4 + 4*(2*a + b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (2*a + b)*sinh(d*x + c)^4 + 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*(2*a + b
)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 + 4*((2*a + b)*cosh(d*x + c)^
3 + (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + b)*arctan(cosh(d*x + c)
 + sinh(d*x + c)) - b*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 - b)*sinh(d*x +
 c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*sinh(d*x +
 c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2
+ 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{sech}\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(36) = 72$.

Time = 0.14 (sec) , antiderivative size = 80, normalized size of antiderivative = 2.00

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right) + \frac{a\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2
*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*arctan(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 84, normalized size of antiderivative = 2.10

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)(2\,a+b)-\frac{4\,b\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4}}{4\,d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/4*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(2*a + b) - 4
*b*(e^(d*x + c) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 125, normalized size of antiderivative = 3.12

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(2\,a\,\sqrt{d^2}+b\,\sqrt{d^2}\right)}{d\,\sqrt{4\,a^2+4\,a\,b+b^2}}\right)\,\sqrt{4\,a^2+4\,a\,b+b^2}}{\sqrt{d^2}}-\frac{b\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{2\,b\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x),x)
```

output
```
(atan((exp(d*x)*exp(c)*(2*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(4*a*b + 4*a^
2 + b^2)^(1/2)))*(4*a*b + 4*a^2 + b^2)^(1/2))/(d^2)^(1/2) - (b*exp(c + d*x
))/(d*(exp(2*c + 2*d*x) + 1)) + (2*b*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x)
+ exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 157, normalized size of antiderivative = 3.92

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{2e^{4dx+4c}atan\left(e^{dx+c}\right)a+e^{4dx+4c}atan\left(e^{dx+c}\right)b+4e^{2dx+2c}atan\left(e^{dx+c}\right)a+2e^{2dx+2c}atan\left(e^{dx+c}\right)b+2atan\left(e^{dx+c}\right)\ldots}{d\left(e^{4dx+4c}+2e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a + e**(4*c + 4*d*x)*atan(e**(c + d
*x))*b + 4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a + 2*e**(2*c + 2*d*x)*atan
(e**(c + d*x))*b + 2*atan(e**(c + d*x))*a + atan(e**(c + d*x))*b - e**(3*c
 + 3*d*x)*b + e**(c + d*x)*b)/(d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) +
1))
```

3.86 $\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 28

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

output
```
a*tanh(d*x+c)/d+1/3*b*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.00

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d + (b*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 26, normalized size of antiderivative = 0.93, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4158, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \text{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ic+idx)^2 \left(a-b\tan(ic+idx)^2\right) dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(b\tanh^2(c+dx)+a\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a\tanh(c+dx)+\frac{1}{3}b\tanh^3(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x] + (b*Tanh[c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 3.09 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.89

method	result	size
derivativedivides	$\frac{\frac{b\tanh(dx+c)^3}{3}+a\tanh(dx+c)}{d}$	25
default	$\frac{\frac{b\tanh(dx+c)^3}{3}+a\tanh(dx+c)}{d}$	25
risch	$-\frac{2\left(3\,\mathrm{e}^{4dx+4c}a+3b\,\mathrm{e}^{4dx+4c}+6\,\mathrm{e}^{2dx+2c}a+3a+b\right)}{3d\left(\mathrm{e}^{2dx+2c}+1\right)^3}$	60

input
```
int(sech(d*x+c)^2*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/3*b*tanh(d*x+c)^3+a*tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 159 vs. 2(26) = 52.

Time = 0.09 (sec) , antiderivative size = 159, normalized size of antiderivative = 5.68

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{4\left((3\,a+2\,b)\cosh\left(dx+c\right)^2+2\,b\cosh\left(dx+c\right)\sinh\right.}{3\left(d\cosh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+d\sinh\left(dx+c\right)^4+4\,d\cosh\left(dx+c\right)^2+2\left(3\,d\cosh\right.\right.}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-4/3*((3*a + 2*b)*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + (3*a
 + 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x +
c)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh
(d*x + c) + 3*d)
```

Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)\operatorname{sech}^2{(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.21

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{b\tanh\left(dx+c\right)^3}{3\,d}+\frac{2\,a}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*b*tanh(d*x + c)^3/d + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 59 vs. $2(26) = 52$.

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\left(3\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+3\,a+b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^3}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-2/3*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 3*
a + b)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 59, normalized size of antiderivative = 2.11

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{2\left(3\,a+b+6\,a\,\mathrm{e}^{2\,c+2\,d\,x}+3\,a\,\mathrm{e}^{4\,c+4\,d\,x}+3\,b\,\mathrm{e}^{4\,c+4\,d\,x}\right)}{3\,d\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)^3}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^2,x)
```

output
```
-(2*(3*a + b + 6*a*exp(2*c + 2*d*x) + 3*a*exp(4*c + 4*d*x) + 3*b*exp(4*c +
 4*d*x)))/(3*d*(exp(2*c + 2*d*x) + 1)^3)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.46

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\frac{2e^{6dx+6c}a}{3}+\frac{2e^{6dx+6c}b}{3}-2e^{2dx+2c}a+2e^{2dx+2c}b-\frac{4a}{3}}{d\left(e^{6dx+6c}+3e^{4dx+4c}+3e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*(e**(6*c + 6*d*x)*a + e**(6*c + 6*d*x)*b - 3*e**(2*c + 2*d*x)*a + 3*e**
(2*c + 2*d*x)*b - 2*a))/(3*d*(e**(6*c + 6*d*x) + 3*e**(4*c + 4*d*x) + 3*e*
*(2*c + 2*d*x) + 1))
```

3.87 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 66

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(4a+b)\arctan(\sinh(c+dx))}{8d}+\frac{(4a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}-\frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
1/8*(4*a+b)*arctan(sinh(d*x+c))/d+1/8*(4*a+b)*sech(d*x+c)*tanh(d*x+c)/d-1/
4*b*sech(d*x+c)^3*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.41

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\arctan(\sinh(c+dx))}{2d}+\frac{b\arctan(\sinh(c+dx))}{8d}+\frac{a\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{b\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}-\frac{b\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(8*d) + (a*Sec
h[c + d*x]*Tanh[c + d*x])/(2*d) + (b*Sech[c + d*x]*Tanh[c + d*x])/(8*d) -
(b*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.08, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4159, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{(a+b)\sinh^2(c+dx)+a}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 298}$$

$$\frac{\frac{1}{4}(4a+b)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{b\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{ 215}$$

$$\frac{\frac{1}{4}(4a+b)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{1}{4}(4a+b)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/4*(b*Sinh[c + d*x])/(1 + Sinh[c + d*x]^2)^2 + ((4*a + b)*(ArcTan[Sinh[
c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 8.61 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.29

method	result
derivativedivides	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+}\right.}{4}\right.}{d}$
default	$\frac{a\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan\left(e^{dx+}\right.}{4}\right.}{d}$
risch	$\frac{e^{dx+c}\left(4e^{6dx+6c}a+e^{6dx+6c}b+4e^{4dx+4c}a-7b\,e^{4dx+4c}-4e^{2dx+2c}a+7e^{2dx+2c}b-4a-b\right)}{4d\left(e^{2dx+2c}+1\right)^4}+\frac{i\ln\left(e^{dx+c}+i\right)a}{2d}+\frac{ib\ln(e}{}$

input
```
int(sech(d*x+c)^3*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b*(-1/3/cosh(d*x+c
)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+1/4*ar
ctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1046 vs. $2(60) = 120$.

Time = 0.11 (sec) , antiderivative size = 1046, normalized size of antiderivative = 15.85

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/4*((4*a + b)*cosh(d*x + c)^7 + 7*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
 + (4*a + b)*sinh(d*x + c)^7 + (4*a - 7*b)*cosh(d*x + c)^5 + (21*(4*a + b)
*cosh(d*x + c)^2 + 4*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(4*a + b)*cosh(d*x +
c)^3 + (4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (4*a - 7*b)*cosh(d*x +
 c)^3 + (35*(4*a + b)*cosh(d*x + c)^4 + 10*(4*a - 7*b)*cosh(d*x + c)^2 - 4
*a + 7*b)*sinh(d*x + c)^3 + (21*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a - 7*b)
*cosh(d*x + c)^3 - 3*(4*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((4*a +
b)*cosh(d*x + c)^8 + 8*(4*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a + b)
*sinh(d*x + c)^8 + 4*(4*a + b)*cosh(d*x + c)^6 + 4*(7*(4*a + b)*cosh(d*x +
 c)^2 + 4*a + b)*sinh(d*x + c)^6 + 8*(7*(4*a + b)*cosh(d*x + c)^3 + 3*(4*a
 + b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(4*a + b)*cosh(d*x + c)^4 + 2*(35
*(4*a + b)*cosh(d*x + c)^4 + 30*(4*a + b)*cosh(d*x + c)^2 + 12*a + 3*b)*si
nh(d*x + c)^4 + 8*(7*(4*a + b)*cosh(d*x + c)^5 + 10*(4*a + b)*cosh(d*x + c
)^3 + 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(4*a + b)*cosh(d*x +
c)^2 + 4*(7*(4*a + b)*cosh(d*x + c)^6 + 15*(4*a + b)*cosh(d*x + c)^4 + 9*(
4*a + b)*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c)^2 + 8*((4*a + b)*cosh(d*
x + c)^7 + 3*(4*a + b)*cosh(d*x + c)^5 + 3*(4*a + b)*cosh(d*x + c)^3 + (4*
a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*a + b)*arctan(cosh(d*x + c) + sinh
(d*x + c)) - (4*a + b)*cosh(d*x + c) + (7*(4*a + b)*cosh(d*x + c)^6 + 5*(4
*a - 7*b)*cosh(d*x + c)^4 - 3*(4*a - 7*b)*cosh(d*x + c)^2 - 4*a - b)*si...
```

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2(c+dx)\right)\operatorname{sech}^3(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 181 vs. $2(60) = 120$.

Time = 0.12 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.74

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{1}{4}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right) - a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^
(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x -
4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a*(arctan(e^(-d*x -
c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d
*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(60) = 120$.

Time = 0.14 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.32

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)(4a+b) + \frac{4\left(4a\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+16a\left(e^{(dx+c)}-\right.\right.}{\left(\left(e^{(dx+c)}-e^{(-dx-c)}\right)^2+4\right)^2}}{16d}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(4*a + b) +
4*(4*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3 +
 16*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/((e
^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 280, normalized size of antiderivative = 4.24

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(4a\sqrt{d^2}+b\sqrt{d^2}\right)}{d\sqrt{16a^2+8ab+b^2}}\right)\sqrt{16a^2+8ab+b^2}}{4\sqrt{d^2}} - \frac{\frac{e^{5c+5dx}(a+b)}{d}+\frac{2e^{3c+3dx}(a-b)}{d}+\frac{e^{c+dx}(a+b)}{d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1} - \frac{e^{c+dx}(2a+3b)}{2d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} + \frac{e^{c+dx}(4a+b)}{4d\left(e^{2c+2dx}+1\right)} + \frac{2b\,e^{c+dx}}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(4*a*(d^2)^(1/2) + b*(d^2)^(1/2)))/(d*(8*a*b + 16*a
^2 + b^2)^(1/2)))*(8*a*b + 16*a^2 + b^2)^(1/2))/(4*(d^2)^(1/2)) - ((exp(5*
c + 5*d*x)*(a + b))/d + (2*exp(3*c + 3*d*x)*(a - b))/d + (exp(c + d*x)*(a
+ b))/d)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + e
xp(8*c + 8*d*x) + 1) - (exp(c + d*x)*(2*a + 3*b))/(2*d*(2*exp(2*c + 2*d*x)
 + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(4*a + b))/(4*d*(exp(2*c + 2*d*x
) + 1)) + (2*b*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) +
 exp(6*c + 6*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.17

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{4e^{8dx+8c}atan\left(e^{dx+c}\right)a+e^{8dx+8c}atan\left(e^{dx+c}\right)b+16e^{6dx+6c}atan\left(e^{dx+c}\right)a+4e^{6dx+6c}atan\left(e^{dx+c}\right)b+24e^{4dx}}{}$$

input
```
int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

output
```
(4*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a + e**(8*c + 8*d*x)*atan(e**(c + d
*x))*b + 16*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a + 4*e**(6*c + 6*d*x)*ata
n(e**(c + d*x))*b + 24*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a + 6*e**(4*c +
 4*d*x)*atan(e**(c + d*x))*b + 16*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a +
4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b + 4*atan(e**(c + d*x))*a + atan(e*
*(c + d*x))*b + 4*e**(7*c + 7*d*x)*a + e**(7*c + 7*d*x)*b + 4*e**(5*c + 5*
d*x)*a - 7*e**(5*c + 5*d*x)*b - 4*e**(3*c + 3*d*x)*a + 7*e**(3*c + 3*d*x)*
b - 4*e**(c + d*x)*a - e**(c + d*x)*b)/(4*d*(e**(8*c + 8*d*x) + 4*e**(6*c
+ 6*d*x) + 6*e**(4*c + 4*d*x) + 4*e**(2*c + 2*d*x) + 1))
```

3.88 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 48

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}-\frac{b\tanh^5(c+dx)}{5d}$$

output
```
a*tanh(d*x+c)/d-1/3*(a-b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.79

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\tanh(c+dx)}{d}+\frac{2b\tanh(c+dx)}{15d}+\frac{b\operatorname{sech}^2(c+dx)\tanh(c+dx)}{15d}-\frac{b\operatorname{sech}^4(c+dx)\tanh(c+dx)}{5d}-\frac{a\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh
[c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (a*Tanh[c +
d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(ic+idx)^4 \left(a-b\tan(ic+idx)^2\right) dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right) \left(b\tanh^2(c+dx)+a\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 290}$$

$$\frac{\int \left(-b\tanh^4(c+dx)-(a-b)\tanh^2(c+dx)+a\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{1}{3}(a-b)\tanh^3(c+dx)+a\tanh(c+dx)-\frac{1}{5}b\tanh^5(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Tanh[c + d*x] - ((a - b)*Tanh[c + d*x]^3)/3 - (b*Tanh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 18.23 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

method	result	size
derivativedivides	$-\frac{\frac{\tanh(dx+c)^5 b}{5}+\frac{(a-b)\tanh(dx+c)^3}{3}-a\tanh(dx+c)}{d}$	42
default	$-\frac{\frac{\tanh(dx+c)^5 b}{5}+\frac{(a-b)\tanh(dx+c)^3}{3}-a\tanh(dx+c)}{d}$	42
risch	$-\frac{4\left(15\,{\mathrm e}^{6dx+6c}a+15\,{\mathrm e}^{6dx+6c}b+35\,{\mathrm e}^{4dx+4c}a-5b\,{\mathrm e}^{4dx+4c}+25\,{\mathrm e}^{2dx+2c}a+5\,{\mathrm e}^{2dx+2c}b+5a+b\right)}{15d\left({\mathrm e}^{2dx+2c}+1\right)^5}$	96

input
```
int(sech(d*x+c)^4*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/5*tanh(d*x+c)^5*b+1/3*(a-b)*tanh(d*x+c)^3-a*tanh(d*x+c))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(44) = 88$.

Time = 0.08 (sec) , antiderivative size = 345, normalized size of antiderivative = 7.19

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{}{15\left(d\cosh\left(dx+c\right)^7+7\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6+d\sinh\left(dx+c\right)^7+5\,d\cosh\left(dx+c\right)^5+\left(21\,d\,c\right.\right.}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-8/15*(2*(5*a + 4*b)*cosh(d*x + c)^3 + 6*(5*a + 4*b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (5*a + 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) + (3*(5*a + 7*
b)*cosh(d*x + c)^2 + 5*a - 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*co
sh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (2
1*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*
cosh(d*x + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c
)^4 + 50*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5
+ 50*d*cosh(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d
*x + c) + (7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)
^2 + 5*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)\operatorname{sech}^4\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*sech(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. $2(44) = 88$.

Time = 0.04 (sec) , antiderivative size = 371, normalized size of antiderivative = 7.73

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{4}{15}b\left(\frac{5\,e^{(-2\,dx-2\,c)}}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)} - \frac{}{d(5\,e^{(-2\,dx-2\,c)}}\right.$$
$$+\frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}+\frac{1}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right.$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6
*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5
*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) +
10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*
x - 10*c) + 1))) + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^
(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(44) = 88$.

Time = 0.13 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.98

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{4\left(15\,ae^{(6\,dx+6\,c)}+15\,be^{(6\,dx+6\,c)}+35\,ae^{(4\,dx+4\,c)}-5\,be^{(4\,dx+4\,c)}+25\,ae^{(2\,dx+2\,c)}+5\,be^{(2\,dx+2\,c)}+5\,a+b\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^5}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-4/15*(15*a*e^(6*d*x + 6*c) + 15*b*e^(6*d*x + 6*c) + 35*a*e^(4*d*x + 4*c)
- 5*b*e^(4*d*x + 4*c) + 25*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 5*a +
 b)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 6.33

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{\frac{8(a-b)}{15d}+\frac{4e^{2c+2dx}(a+b)}{5d}}{3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1} - \frac{\frac{8e^{2c+2dx}(a+b)}{5d}+\frac{8e^{6c+6dx}(a+b)}{5d}+\frac{16e^{4c+4dx}(a-b)}{5d}}{5e^{2c+2dx}+10e^{4c+4dx}+10e^{6c+6dx}+5e^{8c+8dx}+e^{10c+10dx}+1} - \frac{\frac{2(a+b)}{5d}+\frac{6e^{4c+4dx}(a+b)}{5d}+\frac{8e^{2c+2dx}(a-b)}{5d}}{4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1} - \frac{2(a+b)}{5d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)/cosh(c + d*x)^4,x)
```

output
```
- ((8*(a - b))/(15*d) + (4*exp(2*c + 2*d*x)*(a + b))/(5*d))/(3*exp(2*c + 2
*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((8*exp(2*c + 2*d*x)*
(a + b))/(5*d) + (8*exp(6*c + 6*d*x)*(a + b))/(5*d) + (16*exp(4*c + 4*d*x)
*(a - b))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c +
6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((2*(a + b))/(5*d)
 + (6*exp(4*c + 4*d*x)*(a + b))/(5*d) + (8*exp(2*c + 2*d*x)*(a - b))/(5*d)
)/(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c
+ 8*d*x) + 1) - (2*(a + b))/(5*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) +
1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 152, normalized size of antiderivative = 3.17

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{-4e^{6dx+6c}a-4e^{6dx+6c}b-\frac{28e^{4dx+4c}a}{3}+\frac{4e^{4dx+4c}b}{3}-\frac{20e^{2dx+2c}a}{3}-\frac{4e^{2dx+2c}b}{3}-\frac{4a}{3}-\frac{4b}{15}}{d\left(e^{10dx+10c}+5e^{8dx+8c}+10e^{6dx+6c}+10e^{4dx+4c}+5e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

output
```
(4*( - 15*e**(6*c + 6*d*x)*a - 15*e**(6*c + 6*d*x)*b - 35*e**(4*c + 4*d*x)
*a + 5*e**(4*c + 4*d*x)*b - 25*e**(2*c + 2*d*x)*a - 5*e**(2*c + 2*d*x)*b -
 5*a - b))/(15*d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6*c + 6
*d*x) + 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.89 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{8}\left(3a^2-2ab+3b^2\right)x+\frac{(3a-5b)(a+b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{(a+b)^2\cosh^3(c+dx)\sinh(c+dx)}{4d}$$

output
```
1/8*(3*a^2-2*a*b+3*b^2)*x+1/8*(3*a-5*b)*(a+b)*cosh(d*x+c)*sinh(d*x+c)/d+1/
4*(a+b)^2*cosh(d*x+c)^3*sinh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.83

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{4(3a^2-2ab+3b^2)(c+dx)+8(a^2-b^2)\sinh(2(c+dx))+(a+b)^2\sinh(4(c+dx))}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(4*(3*a^2 - 2*a*b + 3*b^2)*(c + d*x) + 8*(a^2 - b^2)*Sinh[2*(c + d*x)] + (
a + b)^2*Sinh[4*(c + d*x)])/(32*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.42, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4158, 315, 25, 298, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)^4}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{315}$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4\left(1-\tanh^2(c+dx)\right)^2}-\frac{1}{4}\int -\frac{(a-3b)b\tanh^2(c+dx)+a(3a-b)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{1}{4}\int \frac{(a-3b)b\tanh^2(c+dx)+a(3a-b)}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)+\frac{(a+b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2-2ab+3b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)+\frac{3\left(a^2-b^2\right)\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}\right)+\frac{(a+b)\tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)}{4\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}\left(3a^2-2ab+3b^2\right)\operatorname{arctanh}(\tanh(c+dx))+\frac{3(a^2-b^2)\tanh(c+dx)}{2(1-\tanh^2(c+dx))}\right)+\frac{(a+b)\tanh(c+dx)(a+b\tanh^2(c+dx))}{4(1-\tanh^2(c+dx))^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a + b)*Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2))/(4*(1 - Tanh[c + d*x]^2)^
2) + (((3*a^2 - 2*a*b + 3*b^2)*ArcTanh[Tanh[c + d*x]])/2 + (3*(a^2 - b^2)*
Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 27.69 (sec) , antiderivative size = 124, normalized size of antiderivative = 1.63

method	result
derivativedivides	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right.}{d}$
default	$\frac{b^2\left(\left(\frac{\sinh(dx+c)^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\frac{c}{8}\right.}{d}$
risch	$\frac{3a^2x}{8}-\frac{abx}{4}+\frac{3b^2x}{8}+\frac{e^{4dx+4c}a^2}{64d}+\frac{e^{4dx+4c}ab}{32d}+\frac{e^{4dx+4c}b^2}{64d}+\frac{e^{2dx+2c}a^2}{8d}-\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}+$

input
```
int(cosh(d*x+c)^4*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2
*a*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/
8*c)+a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c))
```

Fricas [A] (verification not implemented)

Time = 0.10 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a^2+2\,ab+b^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(3\,a^2-2\,ab+3\,b^2)dx+\left((a^2+2\,ab+b^2)\cosh\left(dx+c\right)^3+\right.}{8\,d}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a^2 - 2*a*b +
3*b^2)*d*x + ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 4*(a^2 - b^2)*cosh(d*x
 + c))*sinh(d*x + c))/d
```

Sympy [F]

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)^2\cosh^4\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(70) = 140$.

Time = 0.04 (sec) , antiderivative size = 171, normalized size of antiderivative = 2.25

$$\begin{aligned}&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx\\&=\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)\\&\quad+\frac{1}{64}b^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)\\&\quad-\frac{1}{32}ab\left(\frac{8(dx+c)}{d}-\frac{e^{(4dx+4c)}}{d}+\frac{e^{(-4dx-4c)}}{d}\right)\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*
d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) - 1/32*a*b*(8*(d
*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(70) = 140$.

Time = 0.17 (sec) , antiderivative size = 186, normalized size of antiderivative = 2.45

$$\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$$
$$= \frac{a^2e^{(4\,dx+4\,c)} + 2\,abe^{(4\,dx+4\,c)} + b^2e^{(4\,dx+4\,c)} + 8\,a^2e^{(2\,dx+2\,c)} - 8\,b^2e^{(2\,dx+2\,c)} + 8\,(3\,a^2 - 2\,ab + 3\,b^2)(dx+c) -}{}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/64*(a^2*e^(4*d*x + 4*c) + 2*a*b*e^(4*d*x + 4*c) + b^2*e^(4*d*x + 4*c) +
8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + 8*(3*a^2 - 2*a*b + 3*b^2)*
(d*x + c) - (18*a^2*e^(4*d*x + 4*c) - 12*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4
*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b
+ b^2)*e^(-4*d*x - 4*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.34

$$\int \cosh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = x\left(\frac{3\,a^2}{8} - \frac{a\,b}{4} + \frac{3\,b^2}{8}\right) - \frac{e^{-2\,c-2\,d\,x}\,(a^2-b^2)}{8\,d} + \frac{e^{2\,c+2\,d\,x}\,(a^2-b^2)}{8\,d} - \frac{e^{-4\,c-4\,d\,x}\,(a+b)^2}{64\,d} + \frac{e^{4\,c+4\,d\,x}\,(a+b)^2}{64\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*((3*a^2)/8 - (a*b)/4 + (3*b^2)/8) - (exp(- 2*c - 2*d*x)*(a^2 - b^2))/(8*
d) + (exp(2*c + 2*d*x)*(a^2 - b^2))/(8*d) - (exp(- 4*c - 4*d*x)*(a + b)^2)
/(64*d) + (exp(4*c + 4*d*x)*(a + b)^2)/(64*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.42

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{e^{8dx+8c}a^2+2e^{8dx+8c}ab+e^{8dx+8c}b^2+8e^{6dx+6c}a^2-8e^{6dx+6c}b^2+24e^{4dx+4c}a^2dx-16e^{4dx+4c}abdx+24e^{4dx+}}{64e^{4dx+4c}d}$$

input
```
int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(8*c + 8*d*x)*a**2 + 2*e**(8*c + 8*d*x)*a*b + e**(8*c + 8*d*x)*b**2 +
8*e**(6*c + 6*d*x)*a**2 - 8*e**(6*c + 6*d*x)*b**2 + 24*e**(4*c + 4*d*x)*a*
*2*d*x - 16*e**(4*c + 4*d*x)*a*b*d*x + 24*e**(4*c + 4*d*x)*b**2*d*x - 8*e*
*(2*c + 2*d*x)*a**2 + 8*e**(2*c + 2*d*x)*b**2 - a**2 - 2*a*b - b**2)/(64*e
**(4*c + 4*d*x)*d)
```

3.90 $\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 54

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{b^2 \arctan(\sinh(c+dx))}{d} + \frac{(a^2-b^2)\sinh(c+dx)}{d} + \frac{(a+b)^2 \sinh^3(c+dx)}{3d}$$

output
```
b^2*arctan(sinh(d*x+c))/d+(a^2-b^2)*sinh(d*x+c)/d+1/3*(a+b)^2*sinh(d*x+c)^
3/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{\sinh(c+dx) \left(\frac{3b^2 \operatorname{arctanh}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}} + (a+b)\left(3(a-b)+(a+b)\sinh^2(c+dx)\right)\right)}{3d}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Sinh[c + d*x]*((3*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c + d*x
]^2] + (a + b)*(3*(a - b) + (a + b)*Sinh[c + d*x]^2)))/(3*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)^3}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^2}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(a^2-b^2+(a+b)^2\sinh^2(c+dx)+\frac{b^2}{\sinh^2(c+dx)+1}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2-b^2\right)\sinh(c+dx)+\frac{1}{3}(a+b)^2\sinh^3(c+dx)+b^2\arctan(\sinh(c+dx))}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(b^2*ArcTan[Sinh[c + d*x]] + (a^2 - b^2)*Sinh[c + d*x] + ((a + b)^2*Sinh[c
 + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 11.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.33

method	result
derivativedivides	$\frac{a^{2}\left(\frac{2}{3}+\frac{\cosh(dx+c)^{2}}{3}\right)\sinh(dx+c)+\frac{2ab\sinh(dx+c)^{3}}{3}+b^{2}\left(\frac{\sinh(dx+c)^{3}}{3}-\sinh(dx+c)+2\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
default	$\frac{a^{2}\left(\frac{2}{3}+\frac{\cosh(dx+c)^{2}}{3}\right)\sinh(dx+c)+\frac{2ab\sinh(dx+c)^{3}}{3}+b^{2}\left(\frac{\sinh(dx+c)^{3}}{3}-\sinh(dx+c)+2\arctan\left(\mathrm{e}^{dx+c}\right)\right)}{d}$
risch	$\frac{\mathrm{e}^{3dx+3c}a^{2}}{24d}+\frac{\mathrm{e}^{3dx+3c}ab}{12d}+\frac{\mathrm{e}^{3dx+3c}b^{2}}{24d}+\frac{3\,\mathrm{e}^{dx+c}a^{2}}{8d}-\frac{\mathrm{e}^{dx+c}ab}{4d}-\frac{5\,\mathrm{e}^{dx+c}b^{2}}{8d}-\frac{3\,\mathrm{e}^{-dx-c}a^{2}}{8d}+\frac{\mathrm{e}^{-dx-c}ab}{4d}+$

input
```
int(cosh(d*x+c)^3*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+2/3*a*b*sinh(d*x+c)^3+b^2*(1/
3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 519 vs. $2(52) = 104$.

Time = 0.10 (sec) , antiderivative size = 519, normalized size of antiderivative = 9.61

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$$
$$= \frac{(a^2+2ab+b^2)\cosh(dx+c)^6+6(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^5+(a^2+2ab+b^2)\sinh(a}{}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 3*(3*a^2 - 2
*a*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 +
 3*a^2 - 2*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)^3 + 3*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(3
*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*(5*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^4 + 6*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^2 - 3*a^2 + 2*a*b + 5*b^2
)*sinh(d*x + c)^2 - a^2 - 2*a*b - b^2 + 48*(b^2*cosh(d*x + c)^3 + 3*b^2*co
sh(d*x + c)^2*sinh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*si
nh(d*x + c)^3)*arctan(cosh(d*x + c) + sinh(d*x + c)) + 6*((a^2 + 2*a*b + b
^2)*cosh(d*x + c)^5 + 2*(3*a^2 - 2*a*b - 5*b^2)*cosh(d*x + c)^3 - (3*a^2 -
 2*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cos
h(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*
x + c)^3)
```

Sympy [F]

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\cosh^3\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 161 vs. $2(52)=104$.

Time = 0.14 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.98

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx&=\frac{ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{12\,d}\\&\quad-\frac{1}{24}\,b^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d}-\frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}+\frac{48\,\arctan\left(e^{(-dx-c)}\right)}{d}\right)\\&\quad+\frac{1}{24}\,a^2\left(\frac{e^{(3\,dx+3\,c)}}{d}+\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}-\frac{e^{(-3\,dx-3\,c)}}{d}\right)\end{aligned}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/12*a*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/24*b^2*((15*e^(-2*d*x - 2*c)
 - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arct
an(e^(-d*x - c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^
(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(52) = 104$.

Time = 0.17 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.81

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{48\,b^2\arctan\left(e^{(dx+c)}\right)+a^2e^{(3\,dx+3\,c)}+2\,abe^{(3\,dx+3\,c)}+b^2e^{(3\,dx+3\,c)}+9\,a^2e^{(dx+c)}-6\,abe^{(dx+c)}-15\,b^2e^{(dx+c)}}{24\,d}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(48*b^2*arctan(e^(d*x + c)) + a^2*e^(3*d*x + 3*c) + 2*a*b*e^(3*d*x +
3*c) + b^2*e^(3*d*x + 3*c) + 9*a^2*e^(d*x + c) - 6*a*b*e^(d*x + c) - 15*b^
2*e^(d*x + c) - (9*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) - 15*b^2*e^
(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*e^(-3*d*x - 3*c))/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.41

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{e^{3\,c+3\,d\,x}\,(a+b)^2}{24\,d}-\frac{e^{-3\,c-3\,d\,x}\,(a+b)^2}{24\,d} - \frac{e^{c+d\,x}\,(-3\,a^2+2\,a\,b+5\,b^2)}{8\,d} + \frac{2\,\mathrm{atan}\left(\frac{b^2\,e^{d\,x}\,e^{c}\,\sqrt{d^2}}{d\,\sqrt{b^4}}\right)\sqrt{b^4}}{\sqrt{d^2}} + \frac{e^{-c-d\,x}\,(-3\,a^2+2\,a\,b+5\,b^2)}{8\,d}$$

input
```
int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(3*c + 3*d*x)*(a + b)^2)/(24*d) - (exp(- 3*c - 3*d*x)*(a + b)^2)/(24*d
) - (exp(c + d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d) + (2*atan((b^2*exp(d*x)*e
xp(c)*(d^2)^(1/2))/(d*(b^4)^(1/2)))*(b^4)^(1/2))/(d^2)^(1/2) + (exp(- c -
d*x)*(2*a*b - 3*a^2 + 5*b^2))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 185, normalized size of antiderivative = 3.43

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$
$$= \frac{48e^{3dx+3c}atan\left(e^{dx+c}\right)b^2+e^{6dx+6c}a^2+2e^{6dx+6c}ab+e^{6dx+6c}b^2+9e^{4dx+4c}a^2-6e^{4dx+4c}ab-15e^{4dx+4c}b^2-}{24e^{3dx+3c}d}$$

input
```
int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(48*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b**2 + e**(6*c + 6*d*x)*a**2 + 2*e
**(6*c + 6*d*x)*a*b + e**(6*c + 6*d*x)*b**2 + 9*e**(4*c + 4*d*x)*a**2 - 6*
e**(4*c + 4*d*x)*a*b - 15*e**(4*c + 4*d*x)*b**2 - 9*e**(2*c + 2*d*x)*a**2
+ 6*e**(2*c + 2*d*x)*a*b + 15*e**(2*c + 2*d*x)*b**2 - a**2 - 2*a*b - b**2)
/(24*e**(3*c + 3*d*x)*d)
```

3.91 $\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 51

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{1}{2}(a-3b)(a+b)x + \frac{(a+b)^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\tanh(c+dx)}{d}$$

output
```
1/2*(a-3*b)*(a+b)*x+1/2*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/d+b^2*tanh(d*x+c)/
d
```

Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.06

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{(a-3b)(a+b)(c+dx)}{2d} + \frac{(a+b)^2\sinh(2(c+dx))}{4d} + \frac{b^2\tanh(c+dx)}{d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a - 3*b)*(a + b)*(c + d*x))/(2*d) + ((a + b)^2*Sinh[2*(c + d*x)])/(4*d)
+ (b^2*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.24, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(b^2+\frac{a^2-b^2+2b(a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{1}{2}(a-3b)(a+b)\text{arctanh}(\tanh(c+dx))+\frac{(a+b)^2\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}+b^2\tanh(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((a - 3*b)*(a + b)*ArcTanh[Tanh[c + d*x]])/2 + b^2*Tanh[c + d*x] + ((a +
b)^2*Tanh[c + d*x])/(2*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(47) = 94$.

Time = 4.48 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.88

method	result
derivativedivides	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
default	$\frac{a^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2ab\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+b^2\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)}{d}$
risch	$\frac{a^2x}{2}-abx-\frac{3b^2x}{2}+\frac{e^{2dx+2c}a^2}{8d}+\frac{e^{2dx+2c}ab}{4d}+\frac{e^{2dx+2c}b^2}{8d}-\frac{e^{-2dx-2c}a^2}{8d}-\frac{e^{-2dx-2c}ab}{4d}-\frac{e^{-2dx-2c}b^2}{8d}$

input
```
int(cosh(d*x+c)^2*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+2*a*b*(1/2*cosh(d*x+c
)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^2*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*d*x-3/
2*c+3/2*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(47) = 94$.

Time = 0.09 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.06

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(a^2+2\,ab+b^2)\sinh{(dx+c)}^3+4\left((a^2-2\,ab-3\,b^2)dx-2\,b^2\right)\cosh{(dx+c)}+\left(3\,(a^2+2\,ab+b^2)\cosh\right.}{8\,d\cosh{(dx+c)}}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/8*((a^2 + 2*a*b + b^2)*sinh(d*x + c)^3 + 4*((a^2 - 2*a*b - 3*b^2)*d*x -
2*b^2)*cosh(d*x + c) + (3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*
b + 9*b^2)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2\cosh^2{(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(47) = 94$.

Time = 0.04 (sec) , antiderivative size = 140, normalized size of antiderivative = 2.75

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{4}ab\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$
$$-\frac{1}{8}b^2\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}\right)}\right)$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/4*a*b*(4*x - e^
(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/8*b^2*(12*(d*x + c)/d + e^(-2*d*
x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x -
4*c))))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.27

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{a^2e^{(2dx+2c)}+2abe^{(2dx+2c)}+b^2e^{(2dx+2c)}+4(a^2-2ab-3b^2)(dx+c)-\frac{a^2e^{(4dx+4c)}-2abe^{(4dx+4c)}-3b^2e^{(4dx+4}}{e^{(4dx+}}}{8d}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/8*(a^2*e^(2*d*x + 2*c) + 2*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + 4
*(a^2 - 2*a*b - 3*b^2)*(d*x + c) - (a^2*e^(4*d*x + 4*c) - 2*a*b*e^(4*d*x +
 4*c) - 3*b^2*e^(4*d*x + 4*c) + 2*a^2*e^(2*d*x + 2*c) + 14*b^2*e^(2*d*x +
2*c) + a^2 + 2*a*b + b^2)/(e^(4*d*x + 4*c) + e^(2*d*x + 2*c)))/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.51

$$\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{e^{2c+2dx}(a+b)^2}{8d} - \frac{2b^2}{d\left(e^{2c+2dx}+1\right)} - \frac{e^{-2c-2dx}(a+b)^2}{8d} - x\left(-\frac{a^2}{2}+ab+\frac{3b^2}{2}\right)$$

input
```
int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^2)/(8*d) - (2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) -
(exp(- 2*c - 2*d*x)*(a + b)^2)/(8*d) - x*(a*b - a^2/2 + (3*b^2)/2)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 232, normalized size of antiderivative = 4.55

$$\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{e^{6dx+6c}a^2 + 2e^{6dx+6c}ab + e^{6dx+6c}b^2 + 4e^{4dx+4c}a^2dx + 2e^{4dx+4c}a^2 - 8e^{4dx+4c}abdx + 4e^{4dx+4c}ab - 12e^{4dx+4c}}{8e^{2dx+2c}d\left(e^{2dx+2c}+1\right.}$$

input
```
int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(6*c + 6*d*x)*a**2 + 2*e**(6*c + 6*d*x)*a*b + e**(6*c + 6*d*x)*b**2 +
4*e**(4*c + 4*d*x)*a**2*d*x + 2*e**(4*c + 4*d*x)*a**2 - 8*e**(4*c + 4*d*x)
*a*b*d*x + 4*e**(4*c + 4*d*x)*a*b - 12*e**(4*c + 4*d*x)*b**2*d*x + 18*e**(
4*c + 4*d*x)*b**2 + 4*e**(2*c + 2*d*x)*a**2*d*x - 8*e**(2*c + 2*d*x)*a*b*d
*x - 12*e**(2*c + 2*d*x)*b**2*d*x - a**2 - 2*a*b - b**2)/(8*e**(2*c + 2*d*
x)*d*(e**(2*c + 2*d*x) + 1))
```

3.92 $\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{b(4a+3b)\arctan(\sinh(c+dx))}{2d} + \frac{(a+b)^2\sinh(c+dx)}{d} + \frac{b^2\text{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
-1/2*b*(4*a+3*b)*arctan(sinh(d*x+c))/d+(a+b)^2*sinh(d*x+c)/d+1/2*b^2*sech(
d*x+c)*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2(a+b)^2\sinh(c+dx)+b(-((4a+3b)\arctan(\sinh(c+dx)))+b\text{sech}(c+dx)\tanh(c+dx))}{2d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(a + b)^2*Sinh[c + d*x] + b*(-((4*a + 3*b)*ArcTan[Sinh[c + d*x]]) + b*S
ech[c + d*x]*Tanh[c + d*x]))/(2*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\sec(ic+idx)}dx$$

$$\Big\downarrow \; 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 300$$

$$\frac{\int \left((a+b)^2-\frac{2b(a+b)\sinh^2(c+dx)+b(2a+b)}{\left(\sinh^2(c+dx)+1\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{2}b(4a+3b)\arctan(\sinh(c+dx))+(a+b)^2\sinh(c+dx)+\frac{b^2\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*(b*(4*a + 3*b)*ArcTan[Sinh[c + d*x]]) + (a + b)^2*Sinh[c + d*x] + (b
^2*Sinh[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 2.34 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

method	result
derivativedivides	$\frac{\sinh(dx+c)a^2+2ab(\sinh(dx+c)-2\arctan(e^{dx+c}))+b^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\right.}{d}$
default	$\frac{\sinh(dx+c)a^2+2ab(\sinh(dx+c)-2\arctan(e^{dx+c}))+b^2\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\arctan\right.}{d}$
risch	$\frac{e^{dx+c}a^2}{2d}+\frac{e^{dx+c}ab}{d}+\frac{e^{dx+c}b^2}{2d}-\frac{e^{-dx-c}a^2}{2d}-\frac{e^{-dx-c}ab}{d}-\frac{e^{-dx-c}b^2}{2d}+\frac{b^2e^{dx+c}(e^{2dx+2c}-1)}{d(e^{2dx+2c}+1)^2}+\frac{2iba\ln(e}{a}$

input
```
int(cosh(d*x+c)*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(sinh(d*x+c)*a^2+2*a*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+b^2*(sinh(d*
x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tanh(d*x+
c)-3*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(56) = 112$.

Time = 0.09 (sec) , antiderivative size = 774, normalized size of antiderivative = 12.90

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/2*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + (a^2 + 2*a*b
+ 3*b^2)*cosh(d*x + c)^4 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 +
 2*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3
 + (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (a^2 + 2*a*b + 3
*b^2)*cosh(d*x + c)^2 + (15*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 +
 2*a*b + 3*b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - 3*b^2)*sinh(d*x + c)^2 - a
^2 - 2*a*b - b^2 - 2*((4*a*b + 3*b^2)*cosh(d*x + c)^5 + 5*(4*a*b + 3*b^2)*
cosh(d*x + c)*sinh(d*x + c)^4 + (4*a*b + 3*b^2)*sinh(d*x + c)^5 + 2*(4*a*b
 + 3*b^2)*cosh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*a*b +
 3*b^2)*sinh(d*x + c)^3 + 2*(5*(4*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(4*a*b
+ 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (4*a*b + 3*b^2)*cosh(d*x + c) +
(5*(4*a*b + 3*b^2)*cosh(d*x + c)^4 + 6*(4*a*b + 3*b^2)*cosh(d*x + c)^2 + 4
*a*b + 3*b^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) + 2*(3*
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + 3*b^2)*cosh(d*x + c
)^3 - (a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)
^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 + 2*d*cosh(d*x
+ c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c
)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x
 + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\cosh\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*cosh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 152 vs. $2(56) = 112$.

Time = 0.12 (sec) , antiderivative size = 152, normalized size of antiderivative = 2.53

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx &= \frac{1}{2}b^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)})}\right)\\&\quad+ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\sinh\left(dx+c\right)}{d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
 e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x -
 5*c)))) + a*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d)
 + a^2*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. $2(56) = 112$.

Time = 0.15 (sec) , antiderivative size = 160, normalized size of antiderivative = 2.67

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{2\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)+4\,ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)+2\,b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2}\right.\right.\right.}{4\,d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/4*(2*a^2*(e^(d*x + c) - e^(-d*x - c)) + 4*a*b*(e^(d*x + c) - e^(-d*x - c
)) + 2*b^2*(e^(d*x + c) - e^(-d*x - c)) - (pi + 2*arctan(1/2*(e^(2*d*x + 2
*c) - 1)*e^(-d*x - c)))*(4*a*b + 3*b^2) + 4*b^2*(e^(d*x + c) - e^(-d*x - c
))/((e^(d*x + c) - e^(-d*x - c))^2 + 4))/d
```

Mupad [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 182, normalized size of antiderivative = 3.03

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx &= \frac{e^{c+d\,x}\,(a+b)^2}{2\,d}-\frac{e^{-c-d\,x}\,(a+b)^2}{2\,d}\\ &\quad-\frac{\mathrm{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(3\,b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{16\,a^2\,b^2+24\,a\,b^3+9\,b^4}}\right)\sqrt{16\,a^2\,b^2+24\,a\,b^3+9\,b^4}}{\sqrt{d^2}}\\ &\quad+\frac{b^2\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{2\,b^2\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}\end{aligned}$$

input
```
int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(exp(c + d*x)*(a + b)^2)/(2*d) - (exp(- c - d*x)*(a + b)^2)/(2*d) - (atan(
(exp(d*x)*exp(c)*(3*b^2*(d^2)^(1/2) + 4*a*b*(d^2)^(1/2)))/(d*(24*a*b^3 + 9
*b^4 + 16*a^2*b^2)^(1/2)))*(24*a*b^3 + 9*b^4 + 16*a^2*b^2)^(1/2))/(d^2)^(1
/2) + (b^2*exp(c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (2*b^2*exp(c + d*x))
/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 313, normalized size of antiderivative = 5.22

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{-8e^{5dx+5c}atan\left(e^{dx+c}\right)ab-6e^{5dx+5c}atan\left(e^{dx+c}\right)b^2-16e^{3dx+3c}atan\left(e^{dx+c}\right)ab-12e^{3dx+3c}atan\left(e^{dx+c}\right)b^2}{}$$

input
```
int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 8*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a*b - 6*e**(5*c + 5*d*x)*atan(e*
*(c + d*x))*b**2 - 16*e**(3*c + 3*d*x)*atan(e**(c + d*x))*a*b - 12*e**(3*c
 + 3*d*x)*atan(e**(c + d*x))*b**2 - 8*e**(c + d*x)*atan(e**(c + d*x))*a*b
- 6*e**(c + d*x)*atan(e**(c + d*x))*b**2 + e**(6*c + 6*d*x)*a**2 + 2*e**(6
*c + 6*d*x)*a*b + e**(6*c + 6*d*x)*b**2 + e**(4*c + 4*d*x)*a**2 + 2*e**(4*
c + 4*d*x)*a*b + 3*e**(4*c + 4*d*x)*b**2 - e**(2*c + 2*d*x)*a**2 - 2*e**(2
*c + 2*d*x)*a*b - 3*e**(2*c + 2*d*x)*b**2 - a**2 - 2*a*b - b**2)/(2*e**(c
+ d*x)*d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.93 $\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 81

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(8a^2+8ab+3b^2)\arctan(\sinh(c+dx))}{8d} - \frac{b(8a+5b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{b^2\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctan(sinh(d*x+c))/d-1/8*b*(8*a+5*b)*sech(d*x+c)*
tanh(d*x+c)/d+1/4*b^2*sech(d*x+c)^3*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.80 (sec) , antiderivative size = 427, normalized size of antiderivative = 5.27

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{\operatorname{csch}^3(c+dx)\left(128\,{}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{9}{2};-\sinh^2(c+dx)\right)\sinh^6(c+dx)\left(a+a\sinh^2(c+dx)+b\,\mathrm{s}\right.\right.}{}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/6720*(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + a*Sinh[c + d*x]^2 + b*Sinh
[c + d*x]^2)^2 + 128*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(5*b^2*Sinh[c + d*x]^4 + 2*a*b*Sinh[c + d*x]^2
*(6 + 5*Sinh[c + d*x]^2) + a^2*(7 + 12*Sinh[c + d*x]^2 + 5*Sinh[c + d*x]^4
)) + 35*(b^2*Sinh[c + d*x]^4*(1947 + 485*Sinh[c + d*x]^2) + 2*a*b*Sinh[c +
 d*x]^2*(2625 + 2554*Sinh[c + d*x]^2 + 485*Sinh[c + d*x]^4) + a^2*(3375 +
5907*Sinh[c + d*x]^2 + 3161*Sinh[c + d*x]^4 + 485*Sinh[c + d*x]^6)) - (105
*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^2*Sinh[c + d*x]^4*(649 + 378*Sinh[c +
d*x]^2 + 9*Sinh[c + d*x]^4) + 2*a*b*Sinh[c + d*x]^2*(875 + 1143*Sinh[c + d
*x]^2 + 389*Sinh[c + d*x]^4 + 9*Sinh[c + d*x]^6) + a^2*(1125 + 2344*Sinh[c
 + d*x]^2 + 1674*Sinh[c + d*x]^4 + 400*Sinh[c + d*x]^6 + 9*Sinh[c + d*x]^8
)))/Sqrt[-Sinh[c + d*x]^2]))/d
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.25, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4159, 315, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 315}$$

$$\frac{\frac{1}{4}\int \frac{(a+b)(4a+3b)\sinh^2(c+dx)+a(4a+b)}{(\sinh^2(c+dx)+1)^2}d\sinh(c+dx)-\frac{b\sinh(c+dx)((a+b)\sinh^2(c+dx)+a)}{4(\sinh^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8a^2+8ab+3b^2)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)-\frac{3b(2a+b)\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}\right)-\frac{b\sinh(c+dx)((a+b)\sinh^2(c+dx)+a)}{4(\sinh^2(c+dx)+1)^2}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{4}\left(\frac{1}{2}(8a^2+8ab+3b^2)\arctan(\sinh(c+dx))-\frac{3b(2a+b)\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}\right)-\frac{b\sinh(c+dx)((a+b)\sinh^2(c+dx)+a)}{4(\sinh^2(c+dx)+1)^2}}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/4*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))/(1 + Sinh[c + d*x]^2
)^2 + (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/2 - (3*b*(2*a + b)*
Sinh[c + d*x])/(2*(1 + Sinh[c + d*x]^2)))/4)/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 4.56 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.65

method	result
derivativedivides	$\frac{2a^2 \arctan\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\right.\right.}{d}$
default	$\frac{2a^2 \arctan\left(e^{dx+c}\right)+2ab\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+b^2\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}+\left(\right.\right.}{d}$
risch	$-\frac{b\,e^{dx+c}\left(8\,e^{6dx+6c}a+5\,e^{6dx+6c}b+8\,e^{4dx+4c}a-3b\,e^{4dx+4c}-8\,e^{2dx+2c}a+3\,e^{2dx+2c}b-8a-5b\right)}{4d\left(e^{2dx+2c}+1\right)^4}+\frac{i\ln\left(e^{dx+c}+i\right)a^2}{d}+$

input
```
int(sech(d*x+c)*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^2*arctan(exp(d*x+c))+2*a*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech(d
*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+b^2*(-1/cosh(d*x+c)^4*sinh(d*x+c)^3-
1/cosh(d*x+c)^4*sinh(d*x+c)+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c
)+3/4*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1373 vs. $2(75) = 150$.

Time = 0.11 (sec) , antiderivative size = 1373, normalized size of antiderivative = 16.95

$$\int \operatorname{sech}(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/4*((8*a*b + 5*b^2)*cosh(d*x + c)^7 + 7*(8*a*b + 5*b^2)*cosh(d*x + c)*si
nh(d*x + c)^6 + (8*a*b + 5*b^2)*sinh(d*x + c)^7 + (8*a*b - 3*b^2)*cosh(d*x
 + c)^5 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*sinh(d*x +
c)^5 + 5*(7*(8*a*b + 5*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c
))*sinh(d*x + c)^4 - (8*a*b - 3*b^2)*cosh(d*x + c)^3 + (35*(8*a*b + 5*b^2)
*cosh(d*x + c)^4 + 10*(8*a*b - 3*b^2)*cosh(d*x + c)^2 - 8*a*b + 3*b^2)*sin
h(d*x + c)^3 + (21*(8*a*b + 5*b^2)*cosh(d*x + c)^5 + 10*(8*a*b - 3*b^2)*co
sh(d*x + c)^3 - 3*(8*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((8*a^2
 + 8*a*b + 3*b^2)*cosh(d*x + c)^8 + 8*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c
)*sinh(d*x + c)^7 + (8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(8*a^2 + 8
*a*b + 3*b^2)*cosh(d*x + c)^6 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)
^2 + 8*a^2 + 8*a*b + 3*b^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)
*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 6*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(8*a^2 + 8*a*b + 3*b
^2)*cosh(d*x + c)^4 + 30*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 24*a^2
+ 24*a*b + 9*b^2)*sinh(d*x + c)^4 + 8*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x
+ c)^5 + 10*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(8*a^2 + 8*a*b + 3
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x
+ c)^2 + 4*(7*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^6 + 15*(8*a^2 + 8*a*b
+ 3*b^2)*cosh(d*x + c)^4 + 9*(8*a^2 + 8*a*b + 3*b^2)*cosh(d*x + c)^2 + ...
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{sech}\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(75) = 150$.

Time = 0.12 (sec) , antiderivative size = 199, normalized size of antiderivative = 2.46

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=&\\ -\frac{1}{4}b^2\Bigg(&\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\Bigg)\\ -2\,ab\Bigg(&\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\Bigg)\\ +\frac{a^2\arctan\left(\sinh\left(dx+c\right)\right)}{d}&\end{aligned}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c)
+ 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-
4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 2*a*b*(arcta
n(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + 1))) + a^2*arctan(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(75) = 150$.

Time = 0.15 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.10

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(8a^2+8ab+3b^2\right)-\frac{4\left(8ab\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3+5b^2\left(e^{(dx+c)}-e^{(-dx-c)}\cdots\right.\right.}{\left(\left(e^{(dx+c)}-e\cdots\right.\right.}}{16d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/16*((pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 8*a
*b + 3*b^2) - 4*(8*a*b*(e^(d*x + c) - e^(-d*x - c))^3 + 5*b^2*(e^(d*x + c)
 - e^(-d*x - c))^3 + 32*a*b*(e^(d*x + c) - e^(-d*x - c)) + 12*b^2*(e^(d*x
+ c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 303, normalized size of antiderivative = 3.74

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(8a^2\sqrt{d^2}+3b^2\sqrt{d^2}+8ab\sqrt{d^2}\right)}{d\sqrt{64a^4+128a^3b+112a^2b^2+48ab^3+9b^4}}\right)\sqrt{64a^4+128a^3b+112a^2b^2+48ab^3+9b^4}}{4\sqrt{d^2}}$$
$$-\frac{6b^2\,e^{c+dx}}{d\left(3\,e^{2c+2dx}+3\,e^{4c+4dx}+e^{6c+6dx}+1\right)}$$
$$+\frac{4b^2\,e^{c+dx}}{d\left(4\,e^{2c+2dx}+6\,e^{4c+4dx}+4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$-\frac{e^{c+dx}\left(5b^2+8ab\right)}{4d\left(e^{2c+2dx}+1\right)}+\frac{e^{c+dx}\left(9b^2+8ab\right)}{2d\left(2\,e^{2c+2dx}+e^{4c+4dx}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x),x)
```

output
```
(atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + 3*b^2*(d^2)^(1/2) + 8*a*b*(d^2
)^(1/2)))/(d*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2)))
*(48*a*b^3 + 128*a^3*b + 64*a^4 + 9*b^4 + 112*a^2*b^2)^(1/2))/(4*(d^2)^(1/
2)) - (6*b^2*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + e
xp(6*c + 6*d*x) + 1)) + (4*b^2*exp(c + d*x))/(d*(4*exp(2*c + 2*d*x) + 6*ex
p(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (exp(c + d*
x)*(8*a*b + 5*b^2))/(4*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(8*a*b +
9*b^2))/(2*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 476, normalized size of antiderivative = 5.88

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{8e^{8dx+8c}atan\left(e^{dx+c}\right)a^2+8e^{8dx+8c}atan\left(e^{dx+c}\right)ab+3e^{8dx+8c}atan\left(e^{dx+c}\right)b^2+32e^{6dx+6c}atan\left(e^{dx+c}\right)a^2+3}{}$$

input
```
int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(8*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**2 + 8*e**(8*c + 8*d*x)*atan(e**(
c + d*x))*a*b + 3*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**2 + 32*e**(6*c +
6*d*x)*atan(e**(c + d*x))*a**2 + 32*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*
b + 12*e**(6*c + 6*d*x)*atan(e**(c + d*x))*b**2 + 48*e**(4*c + 4*d*x)*atan
(e**(c + d*x))*a**2 + 48*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b + 18*e**(
4*c + 4*d*x)*atan(e**(c + d*x))*b**2 + 32*e**(2*c + 2*d*x)*atan(e**(c + d*
x))*a**2 + 32*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b + 12*e**(2*c + 2*d*x
)*atan(e**(c + d*x))*b**2 + 8*atan(e**(c + d*x))*a**2 + 8*atan(e**(c + d*x
))*a*b + 3*atan(e**(c + d*x))*b**2 - 8*e**(7*c + 7*d*x)*a*b - 5*e**(7*c +
7*d*x)*b**2 - 8*e**(5*c + 5*d*x)*a*b + 3*e**(5*c + 5*d*x)*b**2 + 8*e**(3*c
 + 3*d*x)*a*b - 3*e**(3*c + 3*d*x)*b**2 + 8*e**(c + d*x)*a*b + 5*e**(c + d
*x)*b**2)/(4*d*(e**(8*c + 8*d*x) + 4*e**(6*c + 6*d*x) + 6*e**(4*c + 4*d*x)
 + 4*e**(2*c + 2*d*x) + 1))
```

3.94 $\int \mathrm{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 49

$$\int \mathrm{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{a^2\tanh(c+dx)}{d} + \frac{2ab\tanh^3(c+dx)}{3d} + \frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
a^2*tanh(d*x+c)/d+2/3*a*b*tanh(d*x+c)^3/d+1/5*b^2*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \mathrm{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{a^2\tanh(c+dx)}{d} + \frac{2ab\tanh^3(c+dx)}{3d} + \frac{b^2\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x])/d + (2*a*b*Tanh[c + d*x]^3)/(3*d) + (b^2*Tanh[c + d*x]
^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.90, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4158, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2\,dx$$

$$\Big\downarrow \; 4158$$

$$\frac{\int \left(b\tanh^2(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 210$$

$$\frac{\int \left(b^2\tanh^4(c+dx)+2ab\tanh^2(c+dx)+a^2\right) d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{a^2\tanh(c+dx)+\frac{2}{3}ab\tanh^3(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x] + (2*a*b*Tanh[c + d*x]^3)/3 + (b^2*Tanh[c + d*x]^5)/5)/
d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(45) = 90$.

Time = 13.82 (sec) , antiderivative size = 126, normalized size of antiderivative = 2.57

method	result
derivativedivides	$\frac{a^2\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\cdots\right.}{\cdots}\right.}{d}$
default	$\frac{a^2\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+b^2\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\frac{3\left(\frac{8}{15}+\cdots\right.}{\cdots}\right.}{d}$
risch	$-\frac{2\left(15\,e^{8dx+8c}a^2+30\,e^{8dx+8c}ab+15\,e^{8dx+8c}b^2+60\,e^{6dx+6c}a^2+60\,e^{6dx+6c}ab+90\,e^{4dx+4c}a^2+40\,e^{4dx+4c}ab+30\,e^{4dx+\cdots}\right.}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(sech(d*x+c)^2*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*se
ch(d*x+c)^2)*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d
*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d
*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 391 vs. $2(45) = 90$.

Time = 0.08 (sec) , antiderivative size = 391, normalized size of antiderivative = 7.98

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left((15\,a^2+20\,ab+9\,b^2)\cosh\left(dx+c\right)^4+8\,(5\,ab+3\,b^2)\cosh\left(dx+c\right)\sinh\left(dx+\cdots\right.\right.}{15\left(d\cosh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+d\sinh\left(dx+c\right)^6+6\,d\cosh\left(dx+c\right)^4+3\left(5\,d\cdots\right.\right.}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-4/15*((15*a^2 + 20*a*b + 9*b^2)*cosh(d*x + c)^4 + 8*(5*a*b + 3*b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (15*a^2 + 20*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*
(3*a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(15*a^2 + 20*a*b + 9*b^2)*cosh(d*x
+ c)^2 + 30*a^2 + 20*a*b)*sinh(d*x + c)^2 + 45*a^2 + 20*a*b + 15*b^2 + 8*(
(5*a*b + 3*b^2)*cosh(d*x + c)^3 + 5*a*b*cosh(d*x + c))*sinh(d*x + c))/(d*c
osh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 + 6
*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 + 4*(5*
d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c
)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2
 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(
d*x + c) + 10*d)
```

Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)^2\operatorname{sech}^2{(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{b^2\tanh{(dx+c)}^5}{5\,d}+\frac{2\,ab\tanh{(dx+c)}^3}{3\,d}+\frac{2\,a^2}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/5*b^2*tanh(d*x + c)^5/d + 2/3*a*b*tanh(d*x + c)^3/d + 2*a^2/(d*(e^(-2*d*
x - 2*c) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 169 vs. 2(45) = 90.

Time = 0.16 (sec) , antiderivative size = 169, normalized size of antiderivative = 3.45

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=$$
$$-\frac{2\left(15\,a^2e^{(8\,dx+8\,c)}+30\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}+60\,a^2e^{(6\,dx+6\,c)}+60\,abe^{(6\,dx+6\,c)}+90\,a^2e^{(4\,dx+4\,c)}+\right.}{15\,d(e^{(2\,dx+2\,c)}+}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-2/15*(15*a^2*e^(8*d*x + 8*c) + 30*a*b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x +
 8*c) + 60*a^2*e^(6*d*x + 6*c) + 60*a*b*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x
+ 4*c) + 40*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x
 + 2*c) + 20*a*b*e^(2*d*x + 2*c) + 15*a^2 + 10*a*b + 3*b^2)/(d*(e^(2*d*x +
 2*c) + 1)^5)
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 482, normalized size of antiderivative = 9.84

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= -\frac{\frac{2\,(a^2-b^2)}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(a+b)^2}{5\,d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1} - \frac{\frac{2\,(a^2-b^2)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(a^2-b^2)}{5\,d}+\frac{2\,e^{6\,c+6\,d\,x}\,(a+b)^2}{5\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{5\,d}}{4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1} - \frac{\frac{2\,(a+b)^2}{5\,d}+\frac{8\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{5\,d}+\frac{8\,e^{6\,c+6\,d\,x}\,(a^2-b^2)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,(a+b)^2}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(3\,a^2-2\,a\,b+3\,b^2)}{5\,d}}{5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1} - \frac{\frac{2\,(3\,a^2-2\,a\,b+3\,b^2)}{15\,d}+\frac{4\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(a+b)^2}{5\,d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1} - \frac{2\,(a+b)^2}{5\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^2,x)
```

output
```
- ((2*(a^2 - b^2))/(5*d) + (2*exp(2*c + 2*d*x)*(a + b)^2)/(5*d))/(2*exp(2*
c + 2*d*x) + exp(4*c + 4*d*x) + 1) - ((2*(a^2 - b^2))/(5*d) + (6*exp(4*c +
 4*d*x)*(a^2 - b^2))/(5*d) + (2*exp(6*c + 6*d*x)*(a + b)^2)/(5*d) + (2*exp
(2*c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(5*d))/(4*exp(2*c + 2*d*x) + 6*exp(
4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a + b)^2)
/(5*d) + (8*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d) + (8*exp(6*c + 6*d*x)*(a^2
 - b^2))/(5*d) + (2*exp(8*c + 8*d*x)*(a + b)^2)/(5*d) + (4*exp(4*c + 4*d*x
)*(3*a^2 - 2*a*b + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x
) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - (
(2*(3*a^2 - 2*a*b + 3*b^2))/(15*d) + (4*exp(2*c + 2*d*x)*(a^2 - b^2))/(5*d
) + (2*exp(4*c + 4*d*x)*(a + b)^2)/(5*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c
+ 4*d*x) + exp(6*c + 6*d*x) + 1) - (2*(a + b)^2)/(5*d*(exp(2*c + 2*d*x) +
1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 224, normalized size of antiderivative = 4.57

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{\frac{2e^{10dx+10c}a^2}{5}+\frac{4e^{10dx+10c}ab}{5}+\frac{2e^{10dx+10c}b^2}{5}-4e^{6dx+6c}a^2+4e^{6dx+6c}b^2-8e^{4dx+4c}a^2+\frac{8e^{4dx+4c}ab}{3}-6e^{2dx+2c}a^2+}{d\left(e^{10dx+10c}+5e^{8dx+8c}+10e^{6dx+6c}+10e^{4dx+4c}+5e^{2dx+2c}+1\right)}$$

input
```
int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(2*(3*e**(10*c + 10*d*x)*a**2 + 6*e**(10*c + 10*d*x)*a*b + 3*e**(10*c + 10
*d*x)*b**2 - 30*e**(6*c + 6*d*x)*a**2 + 30*e**(6*c + 6*d*x)*b**2 - 60*e**(
4*c + 4*d*x)*a**2 + 20*e**(4*c + 4*d*x)*a*b - 45*e**(2*c + 2*d*x)*a**2 + 1
0*e**(2*c + 2*d*x)*a*b + 15*e**(2*c + 2*d*x)*b**2 - 12*a**2 - 4*a*b))/(15*
d*(e**(10*c + 10*d*x) + 5*e**(8*c + 8*d*x) + 10*e**(6*c + 6*d*x) + 10*e**(
4*c + 4*d*x) + 5*e**(2*c + 2*d*x) + 1))
```

3.95 $\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 113

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{(8a^2+4ab+b^2)\arctan(\sinh(c+dx))}{16d} + \frac{(8a^2+4ab+b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} - \frac{b(12a+7b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{b^2\operatorname{sech}^5(c+dx)\tanh(c+dx)}{6d}$$

output
```
1/16*(8*a^2+4*a*b+b^2)*arctan(sinh(d*x+c))/d+1/16*(8*a^2+4*a*b+b^2)*sech(d
*x+c)*tanh(d*x+c)/d-1/24*b*(12*a+7*b)*sech(d*x+c)^3*tanh(d*x+c)/d+1/6*b^2*
sech(d*x+c)^5*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 8.04 (sec) , antiderivative size = 792, normalized size of antiderivative = 7.01

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output

```
(a^2*Sinh[c + d*x]*((-23555*(a + b))/a - (32970*(a + b)^2)/a^2 - 14980*Csc
h[c + d*x]^2 - (91875*(a + b)*Csch[c + d*x]^2)/a - 65625*Csch[c + d*x]^4 -
 (8855*(a + b)^2*Sinh[c + d*x]^2)/a^2 - 620*HypergeometricPFQ[{3/2, 2, 2,
2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 160*HypergeometricPFQ
[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 16
*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]
^2]*Sinh[c + d*x]^2 - (968*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1
, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (288*(a + b)*Hypergeometric
PFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/
a - (32*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (380*(a + b)^2*HypergeometricPFQ[{
3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (128*
(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2}, -Sinh[c + d
*x]^2]*Sinh[c + d*x]^6)/a^2 - (16*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2,
2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 + (656
25*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(5/2) + (1680*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4)/(-Sinh[c + d*x]^2)^(5/2) - (36
855*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(-Sinh[c + d*x]^2)^(3/2) - (91875*(a
+ b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*(-Sinh[c + d*x]^2)^(3/2)) + (5418
0*(a + b)*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/(a*Sqrt[-Sinh[c + d*x]^2]) +...
```

Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.13, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4159, 315, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \ 3042$$

$$\int \sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \ 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^2}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{1}{6}\int \frac{3(a+b)(2a+b)\sinh^2(c+dx)+a(6a+b)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+4ab+b^2\right)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{b(8a+3b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \; 215$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+4ab+b^2\right)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b(8a+3b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\frac{b\sinh(c+dx)\left((a+\cdots\right.}{6\left(\sinh^2(c\cdots\right.}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{1}{6}\left(\frac{3}{4}\left(8a^2+4ab+b^2\right)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b(8a+3b)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c\cdots\right.}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/6*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))/(1 + Sinh[c + d*x]^2
)^3 + (-1/4*(b*(8*a + 3*b)*Sinh[c + d*x])/(1 + Sinh[c + d*x]^2)^2 + (3*(8*
a^2 + 4*a*b + b^2)*(ArcTan[Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c
 + d*x]^2))))/4)/6)/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 28.08 (sec) , antiderivative size = 174, normalized size of antiderivative = 1.54

method	result
derivativedivides	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan(\cdots}{4}\right.}{}$
default	$\frac{a^2\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+2ab\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan(\cdots}{4}\right.}{}$
risch	$\frac{e^{dx+c}\left(24\,e^{10dx+10c}a^2+12\,e^{10dx+10c}ab+3\,e^{10dx+10c}b^2+72\,e^{8dx+8c}a^2-60\,e^{8dx+8c}ab-47\,e^{8dx+8c}b^2+48\,e^{6dx+6c}a^2-7\cdots\right.}{2\cdots}$

input
```
int(sech(d*x+c)^3*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+2*a*b*(-1/3/cosh
(d*x+c)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c)+
1/4*arctan(exp(d*x+c)))+b^2*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*sinh(d*x
+c)/cosh(d*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sech(d*x+
c))*tanh(d*x+c)+1/8*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2824 vs. $2(105) = 210$.

Time = 0.12 (sec) , antiderivative size = 2824, normalized size of antiderivative = 24.99

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/24*(3*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^11 + 33*(8*a^2 + 4*a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^10 + 3*(8*a^2 + 4*a*b + b^2)*sinh(d*x + c)^11
+ (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(8*a^2 + 4*a*b + b^2)*
cosh(d*x + c)^2 + 72*a^2 - 60*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(8*a^2
 + 4*a*b + b^2)*cosh(d*x + c)^3 + (72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^8 + 6*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(
8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^4 + 6*(72*a^2 - 60*a*b - 47*b^2)*cosh(d
*x + c)^2 + 8*a^2 - 12*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(8*a^2 + 4*a
*b + b^2)*cosh(d*x + c)^5 + 2*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^3 +
 (8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(8*a^2 - 12*
a*b + 13*b^2)*cosh(d*x + c)^5 + 6*(231*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)
^6 + 21*(72*a^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(8*a^2 - 12*a*b +
13*b^2)*cosh(d*x + c)^2 - 8*a^2 + 12*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(16
5*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^7 + 21*(72*a^2 - 60*a*b - 47*b^2)*co
sh(d*x + c)^5 + 35*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(8*a^2 -
12*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (72*a^2 - 60*a*b - 47*b^
2)*cosh(d*x + c)^3 + (495*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c)^8 + 84*(72*a
^2 - 60*a*b - 47*b^2)*cosh(d*x + c)^6 + 210*(8*a^2 - 12*a*b + 13*b^2)*cosh
(d*x + c)^4 - 60*(8*a^2 - 12*a*b + 13*b^2)*cosh(d*x + c)^2 - 72*a^2 + 60*a
*b + 47*b^2)*sinh(d*x + c)^3 + 3*(55*(8*a^2 + 4*a*b + b^2)*cosh(d*x + c...
```

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2(c+dx)\right)^2\operatorname{sech}^3(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(105) = 210$.

Time = 0.14 (sec) , antiderivative size = 345, normalized size of antiderivative = 3.05

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{1}{24}\,b^2\left(\frac{3\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{3\,e^{(-dx-c)}-47\,e^{(-3\,dx-3\,c)}+78\,e^{(-5\,dx-5\,c)}-78\,e^{(-7\,dx-7\,c)}+47\,e^{(-9\,d}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10\,d}}\right.$$
$$-\frac{1}{2}\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-7\,e^{(-3\,dx-3\,c)}+7\,e^{(-5\,dx-5\,c)}-e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$-a^2\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c
) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^
(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*
d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*
c) + 1))) - 1/2*a*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x
- 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6
*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^2*(ar
ctan(e^(-d*x - c))/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x -
 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(105) = 210$.

Time = 0.16 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.36

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{3\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(8\,a^2+4\,ab+b^2\right)+\frac{4\left(24\,a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+12\,ab\left(e^{(dx+c)}-e^{(-dx-}\right.\right.}{}}{}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a^2 + 4
*a*b + b^2) + 4*(24*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 12*a*b*(e^(d*x +
c) - e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 192*a^2*(e^(
d*x + c) - e^(-d*x - c))^3 - 32*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 384*a
^2*(e^(d*x + c) - e^(-d*x - c)) - 192*a*b*(e^(d*x + c) - e^(-d*x - c)) - 4
8*b^2*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3
)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 572, normalized size of antiderivative = 5.06

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(8\,a^2\,\sqrt{d^2}+b^2\,\sqrt{d^2}+4\,a\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4+64\,a^3\,b+32\,a^2\,b^2+8\,a\,b^3+b^4}}\right)\,\sqrt{64\,a^4+64\,a^3\,b+32\,a^2\,b^2+8\,a\,b^3+b^4}}{8\,\sqrt{d^2}}$$
$$-\frac{\frac{2\,e^{c+d\,x}\,{\left(a+b\right)}^2}{3\,d}+\frac{8\,e^{3\,c+3\,d\,x}\,\left(a^2-b^2\right)}{3\,d}+\frac{8\,e^{7\,c+7\,d\,x}\,\left(a^2-b^2\right)}{3\,d}+\frac{2\,e^{9\,c+9\,d\,x}\,{\left(a+b\right)}^2}{3\,d}+\frac{4\,e^{5\,c+5\,d\,x}\,\left(3\,a^2-2\,a\,b+3\,b^2\right)}{3\,d}}{6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1}$$
$$-\frac{2\,e^{c+d\,x}\,\left(15\,b^2+4\,a\,b\right)}{3\,d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{16\,b^2\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\,\left(8\,a^2+4\,a\,b+b^2\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}-\frac{e^{c+d\,x}\,\left(16\,a^2+44\,a\,b+23\,b^2\right)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$
$$+\frac{e^{c+d\,x}\,\left(21\,b^2+20\,a\,b\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(8*a^2*(d^2)^(1/2) + b^2*(d^2)^(1/2) + 4*a*b*(d^2)^
(1/2)))/(d*(8*a*b^3 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2)))*(8*a*b
^3 + 64*a^3*b + 64*a^4 + b^4 + 32*a^2*b^2)^(1/2))/(8*(d^2)^(1/2)) - ((2*ex
p(c + d*x)*(a + b)^2)/(3*d) + (8*exp(3*c + 3*d*x)*(a^2 - b^2))/(3*d) + (8*
exp(7*c + 7*d*x)*(a^2 - b^2))/(3*d) + (2*exp(9*c + 9*d*x)*(a + b)^2)/(3*d)
 + (4*exp(5*c + 5*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(3*d))/(6*exp(2*c + 2*d*x)
 + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp
(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1) - (2*exp(c + d*x)*(4*a*b + 15*b^
2))/(3*d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + e
xp(8*c + 8*d*x) + 1)) + (16*b^2*exp(c + d*x))/(3*d*(5*exp(2*c + 2*d*x) + 1
0*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c +
 10*d*x) + 1)) + (exp(c + d*x)*(4*a*b + 8*a^2 + b^2))/(8*d*(exp(2*c + 2*d*
x) + 1)) - (exp(c + d*x)*(44*a*b + 16*a^2 + 23*b^2))/(12*d*(2*exp(2*c + 2*
d*x) + exp(4*c + 4*d*x) + 1)) + (exp(c + d*x)*(20*a*b + 21*b^2))/(3*d*(3*e
xp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 781, normalized size of antiderivative = 6.91

$$\int \operatorname{sech}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$$
$$= \frac{72e^{10dx+10c}atan\left(e^{dx+c}\right)ab + 180e^{8dx+8c}atan\left(e^{dx+c}\right)ab + 240e^{6dx+6c}atan\left(e^{dx+c}\right)ab + 180e^{4dx+4c}atan\left(e^{dx-}\right.}{}$$

input
```
int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(24*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a**2 + 12*e**(12*c + 12*d*x)*ata
n(e**(c + d*x))*a*b + 3*e**(12*c + 12*d*x)*atan(e**(c + d*x))*b**2 + 144*e
**(10*c + 10*d*x)*atan(e**(c + d*x))*a**2 + 72*e**(10*c + 10*d*x)*atan(e**
(c + d*x))*a*b + 18*e**(10*c + 10*d*x)*atan(e**(c + d*x))*b**2 + 360*e**(8
*c + 8*d*x)*atan(e**(c + d*x))*a**2 + 180*e**(8*c + 8*d*x)*atan(e**(c + d*
x))*a*b + 45*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**2 + 480*e**(6*c + 6*d*
x)*atan(e**(c + d*x))*a**2 + 240*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b +
 60*e**(6*c + 6*d*x)*atan(e**(c + d*x))*b**2 + 360*e**(4*c + 4*d*x)*atan(e
**(c + d*x))*a**2 + 180*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a*b + 45*e**(4
*c + 4*d*x)*atan(e**(c + d*x))*b**2 + 144*e**(2*c + 2*d*x)*atan(e**(c + d*
x))*a**2 + 72*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a*b + 18*e**(2*c + 2*d*x
)*atan(e**(c + d*x))*b**2 + 24*atan(e**(c + d*x))*a**2 + 12*atan(e**(c + d
*x))*a*b + 3*atan(e**(c + d*x))*b**2 + 24*e**(11*c + 11*d*x)*a**2 + 12*e**
(11*c + 11*d*x)*a*b + 3*e**(11*c + 11*d*x)*b**2 + 72*e**(9*c + 9*d*x)*a**2
 - 60*e**(9*c + 9*d*x)*a*b - 47*e**(9*c + 9*d*x)*b**2 + 48*e**(7*c + 7*d*x
)*a**2 - 72*e**(7*c + 7*d*x)*a*b + 78*e**(7*c + 7*d*x)*b**2 - 48*e**(5*c +
 5*d*x)*a**2 + 72*e**(5*c + 5*d*x)*a*b - 78*e**(5*c + 5*d*x)*b**2 - 72*e**
(3*c + 3*d*x)*a**2 + 60*e**(3*c + 3*d*x)*a*b + 47*e**(3*c + 3*d*x)*b**2 -
24*e**(c + d*x)*a**2 - 12*e**(c + d*x)*a*b - 3*e**(c + d*x)*b**2)/(24*d*(e
**(12*c + 12*d*x) + 6*e**(10*c + 10*d*x) + 15*e**(8*c + 8*d*x) + 20*e**...
```

3.96 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{a^2\tanh(c+dx)}{d}-\frac{a(a-2b)\tanh^3(c+dx)}{3d}-\frac{(2a-b)b\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

output
```
a^2*tanh(d*x+c)/d-1/3*a*(a-2*b)*tanh(d*x+c)^3/d-1/5*(2*a-b)*b*tanh(d*x+c)^
5/d-1/7*b^2*tanh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 1.07 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.09

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{(70a^2+28ab+6b^2+(35a^2+14ab+3b^2)\operatorname{sech}^2(c+dx)-6b(7a+4b)\operatorname{sech}^4(c+dx)+15b^2\operatorname{sech}^6(c+dx}{105d}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((70*a^2 + 28*a*b + 6*b^2 + (35*a^2 + 14*a*b + 3*b^2)*Sech[c + d*x]^2 - 6*
b*(7*a + 4*b)*Sech[c + d*x]^4 + 15*b^2*Sech[c + d*x]^6)*Tanh[c + d*x])/(10
5*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(-b^2\tanh^6(c+dx)-(2a-b)b\tanh^4(c+dx)-a(a-2b)\tanh^2(c+dx)+a^2\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^2\tanh(c+dx)-\frac{1}{5}b(2a-b)\tanh^5(c+dx)-\frac{1}{3}a(a-2b)\tanh^3(c+dx)-\frac{1}{7}b^2\tanh^7(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Tanh[c + d*x] - (a*(a - 2*b)*Tanh[c + d*x]^3)/3 - ((2*a - b)*b*Tanh[c
 + d*x]^5)/5 - (b^2*Tanh[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. $2(70) = 140$.

Time = 59.86 (sec) , antiderivative size = 158, normalized size of antiderivative = 2.08

method	result
derivativedivides	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+b^2\left(-\frac{\sinh}{4\cosh}\cdots\right.}{d}$
default	$\frac{a^2\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+2ab\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+b^2\left(-\frac{\sinh}{4\cosh}\cdots\right.}{d}$
risch	$-\frac{4\left(105\,e^{10dx+10c}a^2+210\,e^{10dx+10c}ab+105\,e^{10dx+10c}b^2+455\,e^{8dx+8c}a^2+350\,e^{8dx+8c}ab-105\,e^{8dx+8c}b^2+770\,e^{6dx+6c}\cdots\right.}{\cdots}$

input
```
int(sech(d*x+c)^4*(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/4*sinh(d*x+c)/cosh(
d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b^2*
(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35
+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. $2(70) = 140$.

Time = 0.11 (sec) , antiderivative size = 677, normalized size of antiderivative = 8.91

$$\int \operatorname{sech}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-8/105*(2*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^5 + 10*(35*a^2 + 56*a*b
 + 27*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 + (35*a^2 + 98*a*b + 51*b^2)*sinh
(d*x + c)^5 + 14*(25*a^2 + 16*a*b - 3*b^2)*cosh(d*x + c)^3 + (10*(35*a^2 +
 98*a*b + 51*b^2)*cosh(d*x + c)^2 + 105*a^2 + 126*a*b - 63*b^2)*sinh(d*x +
 c)^3 + 2*(10*(35*a^2 + 56*a*b + 27*b^2)*cosh(d*x + c)^3 + 21*(25*a^2 + 16
*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 28*(25*a^2 + 4*a*b + 3*b^2)
*cosh(d*x + c) + (5*(35*a^2 + 98*a*b + 51*b^2)*cosh(d*x + c)^4 + 63*(5*a^2
 + 6*a*b - 3*b^2)*cosh(d*x + c)^2 + 70*a^2 + 28*a*b + 126*b^2)*sinh(d*x +
c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x +
c)^9 + 7*d*cosh(d*x + c)^7 + (36*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7
+ 7*(12*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 22*d*cosh
(d*x + c)^5 + (126*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 + 20*d)*sinh(
d*x + c)^5 + (126*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 + 110*d*cosh(d
*x + c))*sinh(d*x + c)^4 + 42*d*cosh(d*x + c)^3 + (84*d*cosh(d*x + c)^6 +
245*d*cosh(d*x + c)^4 + 200*d*cosh(d*x + c)^2 + 28*d)*sinh(d*x + c)^3 + (3
6*d*cosh(d*x + c)^7 + 147*d*cosh(d*x + c)^5 + 220*d*cosh(d*x + c)^3 + 126*
d*cosh(d*x + c))*sinh(d*x + c)^2 + 56*d*cosh(d*x + c) + (9*d*cosh(d*x + c)
^8 + 49*d*cosh(d*x + c)^6 + 100*d*cosh(d*x + c)^4 + 84*d*cosh(d*x + c)^2 +
 14*d)*sinh(d*x + c))
```

Sympy [F]

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^2\operatorname{sech}^4\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*sech(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. 2(70) = 140.

Time = 0.05 (sec) , antiderivative size = 928, normalized size of antiderivative = 12.21

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
4/35*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7*e^
(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
- 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)
 + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^
(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x
- 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c)
 + 1)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c
) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*
e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) +
 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-
10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 8/15*a
*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*
d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6
*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(70) = 140$.

Time = 0.17 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.13

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{4\left(105\,a^2e^{(10\,dx+10\,c)}+210\,abe^{(10\,dx+10\,c)}+105\,b^2e^{(10\,dx+10\,c)}+455\,a^2e^{(8\,dx+8\,c)}+350\,abe^{(8\,dx+8\,c)}-105\,b\right.}{}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-4/105*(105*a^2*e^(10*d*x + 10*c) + 210*a*b*e^(10*d*x + 10*c) + 105*b^2*e^
(10*d*x + 10*c) + 455*a^2*e^(8*d*x + 8*c) + 350*a*b*e^(8*d*x + 8*c) - 105*
b^2*e^(8*d*x + 8*c) + 770*a^2*e^(6*d*x + 6*c) + 140*a*b*e^(6*d*x + 6*c) +
210*b^2*e^(6*d*x + 6*c) + 630*a^2*e^(4*d*x + 4*c) + 84*a*b*e^(4*d*x + 4*c)
 - 42*b^2*e^(4*d*x + 4*c) + 245*a^2*e^(2*d*x + 2*c) + 98*a*b*e^(2*d*x + 2*
c) + 21*b^2*e^(2*d*x + 2*c) + 35*a^2 + 14*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c)
 + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 732, normalized size of antiderivative = 9.63

$$\int \operatorname{sech}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^2)^2/cosh(c + d*x)^4,x)
```

output
```
- ((4*(3*a^2 - 2*a*b + 3*b^2))/(35*d) + (32*exp(2*c + 2*d*x)*(a^2 - b^2))/
(35*d) + (4*exp(4*c + 4*d*x)*(a + b)^2)/(7*d))/(4*exp(2*c + 2*d*x) + 6*exp
(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((32*(a^2 - b
^2))/(105*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(21*d))/(3*exp(2*c + 2*d*x)
+ 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((32*(a^2 - b^2))/(105*d) +
 (64*exp(4*c + 4*d*x)*(a^2 - b^2))/(35*d) + (16*exp(6*c + 6*d*x)*(a + b)^2
)/(21*d) + (16*exp(2*c + 2*d*x)*(3*a^2 - 2*a*b + 3*b^2))/(35*d))/(5*exp(2*
c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x
) + exp(10*c + 10*d*x) + 1) - ((32*exp(4*c + 4*d*x)*(a^2 - b^2))/(7*d) + (
32*exp(8*c + 8*d*x)*(a^2 - b^2))/(7*d) + (8*exp(2*c + 2*d*x)*(a + b)^2)/(7
*d) + (8*exp(10*c + 10*d*x)*(a + b)^2)/(7*d) + (16*exp(6*c + 6*d*x)*(3*a^2
 - 2*a*b + 3*b^2))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*e
xp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c
 + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((4*(a + b)^2)/(21*d) + (32*exp(2*c
 + 2*d*x)*(a^2 - b^2))/(21*d) + (64*exp(6*c + 6*d*x)*(a^2 - b^2))/(21*d) +
 (20*exp(8*c + 8*d*x)*(a + b)^2)/(21*d) + (8*exp(4*c + 4*d*x)*(3*a^2 - 2*a
*b + 3*b^2))/(7*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c
 + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x
) + 1) - (4*(a + b)^2)/(21*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 326, normalized size of antiderivative = 4.29

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$= \frac{-4e^{10dx+10c}a^2-8e^{10dx+10c}ab-4e^{10dx+10c}b^2-\frac{52e^{8dx+8c}a^2}{3}-\frac{40e^{8dx+8c}ab}{3}+4e^{8dx+8c}b^2-\frac{88e^{6dx+6c}a^2}{3}-\frac{16e^{6dx+6}}{3}}{d\left(e^{14dx+14c}+7e^{12dx+12c}+21e^{10dx+10c}+35\right.}$$

input

```
int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(4*( - 105*e**(10*c + 10*d*x)*a**2 - 210*e**(10*c + 10*d*x)*a*b - 105*e**(
10*c + 10*d*x)*b**2 - 455*e**(8*c + 8*d*x)*a**2 - 350*e**(8*c + 8*d*x)*a*b
 + 105*e**(8*c + 8*d*x)*b**2 - 770*e**(6*c + 6*d*x)*a**2 - 140*e**(6*c + 6
*d*x)*a*b - 210*e**(6*c + 6*d*x)*b**2 - 630*e**(4*c + 4*d*x)*a**2 - 84*e**
(4*c + 4*d*x)*a*b + 42*e**(4*c + 4*d*x)*b**2 - 245*e**(2*c + 2*d*x)*a**2 -
 98*e**(2*c + 2*d*x)*a*b - 21*e**(2*c + 2*d*x)*b**2 - 35*a**2 - 14*a*b - 3
*b**2))/(105*d*(e**(14*c + 14*d*x) + 7*e**(12*c + 12*d*x) + 21*e**(10*c +
10*d*x) + 35*e**(8*c + 8*d*x) + 35*e**(6*c + 6*d*x) + 21*e**(4*c + 4*d*x)
+ 7*e**(2*c + 2*d*x) + 1))
```

3.97 $\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 91

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)x+\frac{3(a-3b)(a+b)^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{(a+b)^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{b^3\tanh(c+dx)}{d}$$

output
```
3/8*(a+b)*(a^2-2*a*b+5*b^2)*x+3/8*(a-3*b)*(a+b)^2*cosh(d*x+c)*sinh(d*x+c)/
d+1/4*(a+b)^3*cosh(d*x+c)^3*sinh(d*x+c)/d-b^3*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 6.52 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.89

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{12(a^3-a^2b+3ab^2+5b^3)(c+dx)+8(a-2b)(a+b)^2\sinh(2(c+dx))+(a+b)^3\sinh(4(c+dx))-32b}{32d}$$

input
```
Integrate[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(12*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(c + d*x) + 8*(a - 2*b)*(a + b)^2*Sinh
[2*(c + d*x)] + (a + b)^3*Sinh[4*(c + d*x)] - 32*b^3*Tanh[c + d*x])/(32*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sec(ic+idx)^4}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(\frac{3b^2(a+b)\tanh^4(c+dx)+3b(a^2-b^2)\tanh^2(c+dx)+a^3+b^3}{\left(1-\tanh^2(c+dx)\right)^3}-b^3\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{3}{8}(a+b)\left(a^2-2ab+5b^2\right)\text{arctanh}(\tanh(c+dx))+\frac{3(a-3b)(a+b)^2\tanh(c+dx)}{8\left(1-\tanh^2(c+dx)\right)}+\frac{(a+b)^3\tanh(c+dx)}{4\left(1-\tanh^2(c+dx)\right)^2}+b^3(-\tanh(c+d}{d}$$

input
```
Int[Cosh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*(a + b)*(a^2 - 2*a*b + 5*b^2)*ArcTanh[Tanh[c + d*x]])/8 - b^3*Tanh[c +
 d*x] + ((a + b)^3*Tanh[c + d*x])/(4*(1 - Tanh[c + d*x]^2)^2) + (3*(a - 3*
b)*(a + b)^2*Tanh[c + d*x])/(8*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 183 vs. $2(85) = 170$.

Time = 70.57 (sec) , antiderivative size = 184, normalized size of antiderivative = 2.02

method	result
derivativedivides	$a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\right.$
default	$a^3\left(\left(\frac{\cosh(dx+c)^3}{4}+\frac{3\cosh(dx+c)}{8}\right)\sinh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+3a^2b\left(\frac{\sinh(dx+c)\cosh(dx+c)^3}{4}-\frac{\cosh(dx+c)\sinh(dx+c)}{8}-\frac{dx}{8}-\right.$
risch	$\frac{3a^3x}{8}-\frac{3a^2bx}{8}+\frac{9a\,b^2x}{8}+\frac{15b^3x}{8}+\frac{e^{4dx+4c}a^3}{64d}+\frac{3\,e^{4dx+4c}a^2b}{64d}+\frac{3\,e^{4dx+4c}b^2a}{64d}+\frac{e^{4dx+4c}b^3}{64d}+\frac{e^{2dx+2c}a^3}{8d}$

input
```
int(cosh(d*x+c)^4*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c)+3
*a^2*b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-
1/8*c)+3*b^2*a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/
8*c)+b^3*(1/4*sinh(d*x+c)^5/cosh(d*x+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8
*d*x+15/8*c-15/8*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 227 vs. $2(85) = 170$.

Time = 0.08 (sec) , antiderivative size = 227, normalized size of antiderivative = 2.49

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{(a^3+3a^2b+3ab^2+b^3)\sinh(dx+c)^5+\left(9a^3+3a^2b-21ab^2-15b^3+10(a^3+3a^2b+3ab^2+b^3)\cos\right.}{}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/64*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 + (9*a^3 + 3*a^2*b -
 21*a*b^2 - 15*b^3 + 10*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + 8*(8*b^3 + 3*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*d*x)*cosh(d*
x + c) + (5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 8*a^3 - 24*a
*b^2 - 80*b^3 + 9*(3*a^3 + a^2*b - 7*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c))/(d*cosh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. 2(85) = 170.

Time = 0.04 (sec) , antiderivative size = 267, normalized size of antiderivative = 2.93

$$\begin{aligned}
&\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&= \frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad+\frac{3}{64}ab^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right) \\
&\quad+\frac{1}{64}b^3\left(\frac{120(dx+c)}{d}+\frac{16e^{(-2dx-2c)}-e^{(-4dx-4c)}}{d}-\frac{15e^{(-2dx-2c)}+144e^{(-4dx-4c)}-1}{d(e^{(-4dx-4c)}+e^{(-6dx-6c)})}\right) \\
&\quad-\frac{3}{64}a^2b\left(\frac{8(dx+c)}{d}-\frac{e^{(4dx+4c)}}{d}+\frac{e^{(-4dx-4c)}}{d}\right)
\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2
*c)/d - e^(-4*d*x - 4*c)/d) + 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(
2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)/d - e^(-4*d*x - 4*c)/d) + 1/64*b^3*(12
0*(d*x + c)/d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x
 - 2*c) + 144*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
)))) - 3/64*a^2*b*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 282 vs. $2(85) = 170$.

Time = 0.25 (sec) , antiderivative size = 282, normalized size of antiderivative = 3.10

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{a^3e^{(4\,dx+4\,c)}+3\,a^2be^{(4\,dx+4\,c)}+3\,ab^2e^{(4\,dx+4\,c)}+b^3e^{(4\,dx+4\,c)}+8\,a^3e^{(2\,dx+2\,c)}-24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+}}{}$$

input
```
integrate(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/64*(a^3*e^(4*d*x + 4*c) + 3*a^2*b*e^(4*d*x + 4*c) + 3*a*b^2*e^(4*d*x + 4
*c) + b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c) - 24*a*b^2*e^(2*d*x + 2*
c) - 16*b^3*e^(2*d*x + 2*c) + 24*(a^3 - a^2*b + 3*a*b^2 + 5*b^3)*(d*x + c)
 + 128*b^3/(e^(2*d*x + 2*c) + 1) - (18*a^3*e^(4*d*x + 4*c) - 18*a^2*b*e^(4
*d*x + 4*c) + 54*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 8*a^3*e^
(2*d*x + 2*c) - 24*a*b^2*e^(2*d*x + 2*c) - 16*b^3*e^(2*d*x + 2*c) + a^3 +
3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x - 4*c))/d
```

Mupad [B] (verification not implemented)

Time = 2.51 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.46

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = x\left(\frac{3\,a^3}{8}-\frac{3\,a^2\,b}{8}+\frac{9\,a\,b^2}{8}+\frac{15\,b^3}{8}\right) + \frac{2\,b^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} - \frac{\mathrm{e}^{-4\,c-4\,d\,x}\,{\left(a+b\right)}^3}{64\,d} + \frac{\mathrm{e}^{4\,c+4\,d\,x}\,{\left(a+b\right)}^3}{64\,d} - \frac{\mathrm{e}^{-2\,c-2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-2\,b\right)}{8\,d} + \frac{\mathrm{e}^{2\,c+2\,d\,x}\,{\left(a+b\right)}^2\,\left(a-2\,b\right)}{8\,d}$$

input
```
int(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*((9*a*b^2)/8 - (3*a^2*b)/8 + (3*a^3)/8 + (15*b^3)/8) + (2*b^3)/(d*(exp(2
*c + 2*d*x) + 1)) - (exp(- 4*c - 4*d*x)*(a + b)^3)/(64*d) + (exp(4*c + 4*d
*x)*(a + b)^3)/(64*d) - (exp(- 2*c - 2*d*x)*(a + b)^2*(a - 2*b))/(8*d) + (
exp(2*c + 2*d*x)*(a + b)^2*(a - 2*b))/(8*d)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 4.66

$$\int \cosh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{-b^3+3e^{10dx+10c}a^2b+3e^{10dx+10c}a\,b^2+3e^{8dx+8c}a^2b-21e^{8dx+8c}a\,b^2-48e^{6dx+6c}a\,b^2-3a\,b^2+21e^{2dx+2c}a\,b}{}$$

input
```
int(cosh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(e**(10*c + 10*d*x)*a**3 + 3*e**(10*c + 10*d*x)*a**2*b + 3*e**(10*c + 10*d
*x)*a*b**2 + e**(10*c + 10*d*x)*b**3 + 9*e**(8*c + 8*d*x)*a**3 + 3*e**(8*c
 + 8*d*x)*a**2*b - 21*e**(8*c + 8*d*x)*a*b**2 - 15*e**(8*c + 8*d*x)*b**3 +
 24*e**(6*c + 6*d*x)*a**3*d*x + 16*e**(6*c + 6*d*x)*a**3 - 24*e**(6*c + 6*
d*x)*a**2*b*d*x + 72*e**(6*c + 6*d*x)*a*b**2*d*x - 48*e**(6*c + 6*d*x)*a*b
**2 + 120*e**(6*c + 6*d*x)*b**3*d*x - 160*e**(6*c + 6*d*x)*b**3 + 24*e**(4
*c + 4*d*x)*a**3*d*x - 24*e**(4*c + 4*d*x)*a**2*b*d*x + 72*e**(4*c + 4*d*x
)*a*b**2*d*x + 120*e**(4*c + 4*d*x)*b**3*d*x - 9*e**(2*c + 2*d*x)*a**3 - 3
*e**(2*c + 2*d*x)*a**2*b + 21*e**(2*c + 2*d*x)*a*b**2 + 15*e**(2*c + 2*d*x
)*b**3 - a**3 - 3*a**2*b - 3*a*b**2 - b**3)/(64*e**(4*c + 4*d*x)*d*(e**(2*
c + 2*d*x) + 1))
```

3.98 $\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 87

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{b^2(6a+5b)\arctan(\sinh(c+dx))}{2d}+\frac{(a-2b)(a+b)^2\sinh(c+dx)}{d}+\frac{(a+b)^3\sinh^3(c+dx)}{3d}-\frac{b^3\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}$$

output
```
1/2*b^2*(6*a+5*b)*arctan(sinh(d*x+c))/d+(a-2*b)*(a+b)^2*sinh(d*x+c)/d+1/3*
(a+b)^3*sinh(d*x+c)^3/d-1/2*b^3*sech(d*x+c)*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.63 (sec) , antiderivative size = 494, normalized size of antiderivative = 5.68

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$
$$= \frac{\operatorname{csch}^5(c+dx) \left(-256\, {}_5F_4\left(\frac{3}{2},2,2,2,2;1,1,1,\frac{11}{2};-\sinh^2(c+dx)\right) \sinh^8(c+dx) \left(a+a\sinh^2(c+dx)+b\right.\right.}{}$$

input
```
Integrate[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2
}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + a*Sinh[c + d*x]^2 + b*Sinh[c + d
*x]^2)^3 - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(b^3*Sinh[c + d*x]^6*(2161
 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + a^3*Cos
h[c + d*x]^6*(2401 + 1875*Sinh[c + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c +
 d*x]^6) + 3*a^2*b*(Sinh[c + d*x] + Sinh[c + d*x]^3)^2*(2401 + 1875*Sinh[c
 + d*x]^2 + 243*Sinh[c + d*x]^4 + Sinh[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]
^4*(2401 + 4180*Sinh[c + d*x]^2 + 2118*Sinh[c + d*x]^4 + 244*Sinh[c + d*x]
^6 + Sinh[c + d*x]^8)))/Sqrt[-Sinh[c + d*x]^2] + 21*(b^3*Sinh[c + d*x]^6*(
32415 + 17320*Sinh[c + d*x]^2 + 753*Sinh[c + d*x]^4) + 3*a*b^2*Sinh[c + d*
x]^4*(36015 + 50695*Sinh[c + d*x]^2 + 18073*Sinh[c + d*x]^4 + 753*Sinh[c +
 d*x]^6) + 3*a^2*b*Sinh[c + d*x]^2*(36015 + 88150*Sinh[c + d*x]^2 + 69728*
Sinh[c + d*x]^4 + 18826*Sinh[c + d*x]^6 + 753*Sinh[c + d*x]^8) + a^3*(3601
5 + 124165*Sinh[c + d*x]^2 + 157878*Sinh[c + d*x]^4 + 89514*Sinh[c + d*x]^
6 + 19579*Sinh[c + d*x]^8 + 753*Sinh[c + d*x]^10))))/(30240*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{(a-b\tan(ic+idx)^2)^3}{\sec(ic+idx)^3}dx$$

$$\Big\downarrow \ 4159$$

$$\frac{\int \frac{((a+b)\sinh^2(c+dx)+a)^3}{(\sinh^2(c+dx)+1)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 300$$

$$\frac{\int \left(\sinh^2(c+dx)(a+b)^3+(a-2b)(a+b)^2+\frac{3(a+b)\sinh^2(c+dx)b^2+(3a+2b)b^2}{(\sinh^2(c+dx)+1)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \ 2009$$

$$\frac{\frac{1}{2}b^2(6a+5b)\arctan(\sinh(c+dx))+\frac{1}{3}(a+b)^3\sinh^3(c+dx)+(a-2b)(a+b)^2\sinh(c+dx)-\frac{b^3\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}}{d}$$

input
```
Int[Cosh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b^2*(6*a + 5*b)*ArcTan[Sinh[c + d*x]])/2 + (a - 2*b)*(a + b)^2*Sinh[c +
d*x] + ((a + b)^3*Sinh[c + d*x]^3)/3 - (b^3*Sinh[c + d*x])/(2*(1 + Sinh[c
+ d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 30.70 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.78

method	result
derivativedivides	$\frac{a^{3}\left(\frac{2}{3}+\frac{\cosh(dx+c)^{2}}{3}\right)\sinh(dx+c)+a^{2}b\sinh(dx+c)^{3}+3b^{2}a\left(\frac{\sinh(dx+c)^{3}}{3}-\sinh(dx+c)+2\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^{3}\left(\frac{\sinh(d}{3\cosh(}\right.}{d}$
default	$\frac{a^{3}\left(\frac{2}{3}+\frac{\cosh(dx+c)^{2}}{3}\right)\sinh(dx+c)+a^{2}b\sinh(dx+c)^{3}+3b^{2}a\left(\frac{\sinh(dx+c)^{3}}{3}-\sinh(dx+c)+2\arctan\left(\mathrm{e}^{dx+c}\right)\right)+b^{3}\left(\frac{\sinh(d}{3\cosh(}\right.}{d}$
risch	$\frac{\mathrm{e}^{3dx+3c}a^{3}}{24d}+\frac{\mathrm{e}^{3dx+3c}a^{2}b}{8d}+\frac{\mathrm{e}^{3dx+3c}b^{2}a}{8d}+\frac{\mathrm{e}^{3dx+3c}b^{3}}{24d}+\frac{3\,\mathrm{e}^{dx+c}a^{3}}{8d}-\frac{3\,\mathrm{e}^{dx+c}a^{2}b}{8d}-\frac{15\,\mathrm{e}^{dx+c}b^{2}a}{8d}-\frac{9\,\mathrm{e}^{dx+c}b}{8d}$

input
```
int(cosh(d*x+c)^3*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c)+a^2*b*sinh(d*x+c)^3+3*b^2*a*(
1/3*sinh(d*x+c)^3-sinh(d*x+c)+2*arctan(exp(d*x+c)))+b^3*(1/3*sinh(d*x+c)^5
/cosh(d*x+c)^2-5/3*sinh(d*x+c)^3/cosh(d*x+c)^2-5*sinh(d*x+c)/cosh(d*x+c)^2
+5/2*sech(d*x+c)*tanh(d*x+c)+5*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1840 vs. 2(81) = 162.

Time = 0.12 (sec) , antiderivative size = 1840, normalized size of antiderivative = 21.15

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 10*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*sinh(d*x + c)^10 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x
 + c)^8 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3 + 45*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*cosh(d*x + c)^3 + (11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*
x + c))*sinh(d*x + c)^7 + 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x
 + c)^6 + 2*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 5*a^3 -
 3*a^2*b - 21*a*b^2 - 25*b^3 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
sh(d*x + c)^5 + 14*(11*a^3 - 3*a^2*b - 39*a*b^2 - 25*b^3)*cosh(d*x + c)^3
+ 3*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 -
 2*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^6 + 35*(11*a^3 - 3*a^2*b - 39*a*b^2
- 25*b^3)*cosh(d*x + c)^4 - 5*a^3 + 3*a^2*b + 21*a*b^2 + 25*b^3 + 15*(5*a^
3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 7*(11*a^3 - 3*a^2*b - 39
*a*b^2 - 25*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)
*cosh(d*x + c)^3 - (5*a^3 - 3*a^2*b - 21*a*b^2 - 25*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 - (11*a^3 - 3*a^2*b - 39*...
```

Sympy [F]

$$\int \cosh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \cosh^3\left(c+dx\right) dx$$

input
```
integrate(cosh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 284 vs. $2(81) = 162$.

Time = 0.12 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.26

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{8\,d} - \frac{1}{8}ab^2\left(\frac{\left(15\,e^{(-2\,dx-2\,c)}-1\right)e^{(3\,dx+3\,c)}}{d} - \frac{15\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d} + \frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right) + \frac{1}{24}b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d} - \frac{120\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c}}{d(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c}}\right. + \frac{1}{24}a^3\left(\frac{e^{(3\,dx+3\,c)}}{d} + \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} - \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3/d - 1/8*a*b^2*((15*e^(-2*d*x - 2*
c) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*ar
ctan(e^(-d*x - c))/d) + 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d -
 120*arctan(e^(-d*x - c))/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) +
 3*e^(-6*d*x - 6*c) - 1)/(d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7
*d*x - 7*c)))) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x
 - c)/d - e^(-3*d*x - 3*c)/d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(81) = 162$.

Time = 0.25 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.03

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 + 3\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 + 3\,ab^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3 + b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{}$$

input
```
integrate(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(a^3*(e^(d*x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x
- c))^3 + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + b^3*(e^(d*x + c) - e^(-
d*x - c))^3 + 12*a^3*(e^(d*x + c) - e^(-d*x - c)) - 36*a*b^2*(e^(d*x + c)
- e^(-d*x - c)) - 24*b^3*(e^(d*x + c) - e^(-d*x - c)) - 24*b^3*(e^(d*x + c
) - e^(-d*x - c))/((e^(d*x + c) - e^(-d*x - c))^2 + 4) + 6*(pi + 2*arctan(
1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(6*a*b^2 + 5*b^3))/d
```

Mupad [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.67

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{dx}\,e^{c}\left(5\,b^3\sqrt{d^2}+6\,a\,b^2\sqrt{d^2}\right)}{d\sqrt{36\,a^2\,b^4+60\,a\,b^5+25\,b^6}}\right)\sqrt{36\,a^2\,b^4+60\,a\,b^5+25\,b^6}}{\sqrt{d^2}} - \frac{e^{-3\,c-3\,d\,x}\,(a+b)^3}{24\,d} + \frac{e^{3\,c+3\,d\,x}\,(a+b)^3}{24\,d} + \frac{3\,e^{c+d\,x}\,(a+b)^2\,(a-3\,b)}{8\,d} - \frac{b^3\,e^{c+d\,x}}{d\,\left(e^{2\,c+2\,d\,x}+1\right)} - \frac{3\,e^{-c-d\,x}\,(a+b)^2\,(a-3\,b)}{8\,d} + \frac{2\,b^3\,e^{c+d\,x}}{d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 6*a*b^2*(d^2)^(1/2)))/(d*(60*a
*b^5 + 25*b^6 + 36*a^2*b^4)^(1/2)))*(60*a*b^5 + 25*b^6 + 36*a^2*b^4)^(1/2)
)/(d^2)^(1/2) - (exp(- 3*c - 3*d*x)*(a + b)^3)/(24*d) + (exp(3*c + 3*d*x)*
(a + b)^3)/(24*d) + (3*exp(c + d*x)*(a + b)^2*(a - 3*b))/(8*d) - (b^3*exp(
c + d*x))/(d*(exp(2*c + 2*d*x) + 1)) - (3*exp(- c - d*x)*(a + b)^2*(a - 3*
b))/(8*d) + (2*b^3*exp(c + d*x))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x)
 + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 515, normalized size of antiderivative = 5.92

$$\int \cosh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{120e^{7dx+7c}atan\left(e^{dx+c}\right)b^3+240e^{5dx+5c}atan\left(e^{dx+c}\right)b^3+120e^{3dx+3c}atan\left(e^{dx+c}\right)b^3+42e^{4dx+4c}a\,b^2-b^3+\cdots}{\cdots}$$

input
```
int(cosh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(144*e**(7*c + 7*d*x)*atan(e**(c + d*x))*a*b**2 + 120*e**(7*c + 7*d*x)*ata
n(e**(c + d*x))*b**3 + 288*e**(5*c + 5*d*x)*atan(e**(c + d*x))*a*b**2 + 24
0*e**(5*c + 5*d*x)*atan(e**(c + d*x))*b**3 + 144*e**(3*c + 3*d*x)*atan(e**
(c + d*x))*a*b**2 + 120*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b**3 + e**(10*
c + 10*d*x)*a**3 + 3*e**(10*c + 10*d*x)*a**2*b + 3*e**(10*c + 10*d*x)*a*b*
*2 + e**(10*c + 10*d*x)*b**3 + 11*e**(8*c + 8*d*x)*a**3 - 3*e**(8*c + 8*d*
x)*a**2*b - 39*e**(8*c + 8*d*x)*a*b**2 - 25*e**(8*c + 8*d*x)*b**3 + 10*e**
(6*c + 6*d*x)*a**3 - 6*e**(6*c + 6*d*x)*a**2*b - 42*e**(6*c + 6*d*x)*a*b**
2 - 50*e**(6*c + 6*d*x)*b**3 - 10*e**(4*c + 4*d*x)*a**3 + 6*e**(4*c + 4*d*
x)*a**2*b + 42*e**(4*c + 4*d*x)*a*b**2 + 50*e**(4*c + 4*d*x)*b**3 - 11*e**
(2*c + 2*d*x)*a**3 + 3*e**(2*c + 2*d*x)*a**2*b + 39*e**(2*c + 2*d*x)*a*b**
2 + 25*e**(2*c + 2*d*x)*b**3 - a**3 - 3*a**2*b - 3*a*b**2 - b**3)/(24*e**(
3*c + 3*d*x)*d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.99 $\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 78

$$\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{1}{2}(a-5b)(a+b)^2x + \frac{(a+b)^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2(3a+2b)\tanh(c+dx)}{d} + \frac{b^3\tanh^3(c+dx)}{3d}$$

output
```
1/2*(a-5*b)*(a+b)^2*x+1/2*(a+b)^3*cosh(d*x+c)*sinh(d*x+c)/d+b^2*(3*a+2*b)*
tanh(d*x+c)/d+1/3*b^3*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 1.94 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.88

$$\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{6(a-5b)(a+b)^2(c+dx) + 3(a+b)^3\sinh(2(c+dx)) + 4b^2\left(9a+7b-b\operatorname{sech}^2(c+dx)\right)\tanh(c+dx)}{12d}$$

input
```
Integrate[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(6*(a - 5*b)*(a + b)^2*(c + d*x) + 3*(a + b)^3*Sinh[2*(c + d*x)] + 4*b^2*(
9*a + 7*b - b*Sech[c + d*x]^2)*Tanh[c + d*x])/(12*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.12, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sec(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4158}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{\left(1-\tanh^2(c+dx)\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 300}$$

$$\frac{\int \left(\tanh^2(c+dx)b^3+(3a+2b)b^2+\frac{3b\tanh^2(c+dx)(a+b)^2+(a-2b)(a+b)^2}{\left(1-\tanh^2(c+dx)\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{1}{2}(a-5b)(a+b)^2\text{arctanh}(\tanh(c+dx))+b^2(3a+2b)\tanh(c+dx)+\frac{(a+b)^3\tanh(c+dx)}{2\left(1-\tanh^2(c+dx)\right)}+\frac{1}{3}b^3\tanh^3(c+dx)}{d}$$

input
```
Int[Cosh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a - 5*b)*(a + b)^2*ArcTanh[Tanh[c + d*x]])/2 + b^2*(3*a + 2*b)*Tanh[c +
 d*x] + (b^3*Tanh[c + d*x]^3)/3 + ((a + b)^3*Tanh[c + d*x])/(2*(1 - Tanh[c
 + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(72) = 144$.

Time = 13.17 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.90

method	result
derivativedivides	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3b^2a\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c}{2}\right.}{d}$
default	$\frac{a^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a^2b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)+3b^2a\left(\frac{\sinh(dx+c)^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c}{2}\right.}{d}$
risch	$\frac{a^3x}{2}-\frac{3a^2bx}{2}-\frac{9ab^2x}{2}-\frac{5b^3x}{2}+\frac{e^{2dx+2c}a^3}{8d}+\frac{3e^{2dx+2c}a^2b}{8d}+\frac{3e^{2dx+2c}b^2a}{8d}+\frac{e^{2dx+2c}b^3}{8d}-\frac{e^{-2dx-2c}a^3}{8d}$

input
```
int(cosh(d*x+c)^2*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c)+3*a^2*b*(1/2*cosh(d*x
+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3*b^2*a*(1/2*sinh(d*x+c)^3/cosh(d*x+c)-3/2*
d*x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/
2*c+5/2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(72) = 144$.

Time = 0.10 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.73

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sinh\left(dx+c\right)^5-4\left(18\,ab^2+14\,b^3-3\left(a^3-3\,a^2b-9\,ab^2-5\,b^3\right)dx\right)\cosh\left(dx+\right.}{}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/24*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^5 - 4*(18*a*b^2 + 14
*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(18*a
*b^2 + 14*b^3 - 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sin
h(d*x + c)^2 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3 + 30*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 12*(18*a*b^2 + 14*b^3 -
 3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c) + 3*(5*(a^3 + 3*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*a^3 + 6*a^2*b + 30*a*b^2 + 10*b^3
 + (9*a^3 + 27*a^2*b + 99*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/
(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c)
)
```

Sympy [F]

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh^2\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 256 vs. $2(72) = 144$.

Time = 0.04 (sec) , antiderivative size = 256, normalized size of antiderivative = 3.28

$$\begin{aligned}
&\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx \\
&=\frac{1}{8}a^3\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{3}{8}a^2b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right) \\
&\quad-\frac{1}{24}b^3\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201e^{(-4dx-4c)}+147e^{(-6dx-6c)}+3}{d(e^{(-2dx-2c)}+3e^{(-4dx-4c)}+3e^{(-6dx-6c)}+e^{(-8dx-8c)})}\right) \\
&\quad-\frac{3}{8}ab^2\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d(e^{(-2dx-2c)}+e^{(-4dx-4c)})}\right)
\end{aligned}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 3/8*a^2*b*(4*x -
e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/24*b^3*(60*(d*x + c)/d + 3*e^(
-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*d*x - 4*c) + 147*e^(-6
*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 3*e^(-6*d*x -
 6*c) + e^(-8*d*x - 8*c))) - 3/8*a*b^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)
/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 265 vs. $2(72) = 144$.

Time = 0.23 (sec) , antiderivative size = 265, normalized size of antiderivative = 3.40

$$\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{3\,a^3e^{(2\,dx+2\,c)}+9\,a^2be^{(2\,dx+2\,c)}+9\,ab^2e^{(2\,dx+2\,c)}+3\,b^3e^{(2\,dx+2\,c)}+12\,(a^3-3\,a^2b-9\,ab^2-5\,b^3)(dx+c)-3}{}$$

input
```
integrate(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*a^3*e^(2*d*x + 2*c) + 9*a^2*b*e^(2*d*x + 2*c) + 9*a*b^2*e^(2*d*x +
 2*c) + 3*b^3*e^(2*d*x + 2*c) + 12*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*(d*x
+ c) - 3*(2*a^3*e^(2*d*x + 2*c) - 6*a^2*b*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*
d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-2
*d*x - 2*c) - 16*(9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 18*a*b
^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 7*b^3)/(e^(2*d*x +
 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 243, normalized size of antiderivative = 3.12

$$\begin{aligned}\int \cosh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= \frac{e^{2\,c+2\,d\,x}\,(a+b)^3}{8\,d}-\frac{\frac{2\,(b^3+3\,a\,b^2)}{3\,d}+\frac{2\,e^{2\,c+2\,d\,x}\,(b^3+a\,b^2)}{d}}{2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1}\\&\quad-\frac{2\,(b^3+a\,b^2)}{d\,(e^{2\,c+2\,d\,x}+1)}-\frac{e^{-2\,c-2\,d\,x}\,(a+b)^3}{8\,d}\\&\quad-\frac{\frac{2\,(b^3+a\,b^2)}{d}+\frac{4\,e^{2\,c+2\,d\,x}\,(b^3+3\,a\,b^2)}{3\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(b^3+a\,b^2)}{d}}{3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1}+\frac{x\,(a+b)^2\,(a-5\,b)}{2}\end{aligned}$$

input
```
int(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(2*c + 2*d*x)*(a + b)^3)/(8*d) - ((2*(3*a*b^2 + b^3))/(3*d) + (2*exp(2
*c + 2*d*x)*(a*b^2 + b^3))/d)/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)
- (2*(a*b^2 + b^3))/(d*(exp(2*c + 2*d*x) + 1)) - (exp(- 2*c - 2*d*x)*(a +
b)^3)/(8*d) - ((2*(a*b^2 + b^3))/d + (4*exp(2*c + 2*d*x)*(3*a*b^2 + b^3))/
(3*d) + (2*exp(4*c + 4*d*x)*(a*b^2 + b^3))/d)/(3*exp(2*c + 2*d*x) + 3*exp(
4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) + (x*(a + b)^2*(a - 5*b))/2
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 606, normalized size of antiderivative = 7.77

$$\int \cosh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{-180e^{4dx+4c}a\,b^2-3b^3+9e^{10dx+10c}a^2b+9e^{10dx+10c}a\,b^2+21e^{8dx+8c}a^2b+69e^{8dx+8c}a\,b^2-36e^{4dx+4c}a^2b-9}{}$$

input
```
int(cosh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(3*e**(10*c + 10*d*x)*a**3 + 9*e**(10*c + 10*d*x)*a**2*b + 9*e**(10*c + 10
*d*x)*a*b**2 + 3*e**(10*c + 10*d*x)*b**3 + 12*e**(8*c + 8*d*x)*a**3*d*x +
7*e**(8*c + 8*d*x)*a**3 - 36*e**(8*c + 8*d*x)*a**2*b*d*x + 21*e**(8*c + 8*
d*x)*a**2*b - 108*e**(8*c + 8*d*x)*a*b**2*d*x + 69*e**(8*c + 8*d*x)*a*b**2
 - 60*e**(8*c + 8*d*x)*b**3*d*x + 55*e**(8*c + 8*d*x)*b**3 + 36*e**(6*c +
6*d*x)*a**3*d*x - 108*e**(6*c + 6*d*x)*a**2*b*d*x - 324*e**(6*c + 6*d*x)*a
*b**2*d*x - 180*e**(6*c + 6*d*x)*b**3*d*x + 36*e**(4*c + 4*d*x)*a**3*d*x -
 12*e**(4*c + 4*d*x)*a**3 - 108*e**(4*c + 4*d*x)*a**2*b*d*x - 36*e**(4*c +
 4*d*x)*a**2*b - 324*e**(4*c + 4*d*x)*a*b**2*d*x - 180*e**(4*c + 4*d*x)*a*
b**2 - 180*e**(4*c + 4*d*x)*b**3*d*x - 60*e**(4*c + 4*d*x)*b**3 + 12*e**(2
*c + 2*d*x)*a**3*d*x - 11*e**(2*c + 2*d*x)*a**3 - 36*e**(2*c + 2*d*x)*a**2
*b*d*x - 33*e**(2*c + 2*d*x)*a**2*b - 108*e**(2*c + 2*d*x)*a*b**2*d*x - 12
9*e**(2*c + 2*d*x)*a*b**2 - 60*e**(2*c + 2*d*x)*b**3*d*x - 75*e**(2*c + 2*
d*x)*b**3 - 3*a**3 - 9*a**2*b - 9*a*b**2 - 3*b**3)/(24*e**(2*c + 2*d*x)*d*
(e**(6*c + 6*d*x) + 3*e**(4*c + 4*d*x) + 3*e**(2*c + 2*d*x) + 1))
```

3.100 $\int \cosh(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 99

$$\int \cosh(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = -\frac{3b(4(a+b)^2+(2a+b)^2)\arctan(\sinh(c+dx))}{8d} + \frac{(a+b)^3\sinh(c+dx)}{d} + \frac{3b^2(4a+3b)\text{sech}(c+dx)\tanh(c+dx)}{8d} - \frac{b^3\text{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

output
```
-3/8*b*(4*(a+b)^2+(2*a+b)^2)*arctan(sinh(d*x+c))/d+(a+b)^3*sinh(d*x+c)/d+3
/8*b^2*(4*a+3*b)*sech(d*x+c)*tanh(d*x+c)/d-1/4*b^3*sech(d*x+c)^3*tanh(d*x+
c)/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.90

$$\int \cosh(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{-3b(8a^2+12ab+5b^2)\arctan(\sinh(c+dx))+8(a+b)^3\sinh(c+dx)+3b^2(4a+3b)\text{sech}(c+dx)\tanh(}{8d}$$

input
```
Integrate[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]] + 8*(a + b)^3*Sinh[c
+ d*x] + 3*b^2*(4*a + 3*b)*Sech[c + d*x]*Tanh[c + d*x] - 2*b^3*Sech[c + d*
x]^3*Tanh[c + d*x])/(8*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\sec(ic+idx)}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left((a+b)^3-\frac{3b(a+b)^2\sinh^4(c+dx)+3b(a+b)(2a+b)\sinh^2(c+dx)+b\left(3a^2+3ba+b^2\right)}{\left(\sinh^2(c+dx)+1\right)^3}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{3}{8}b\left(4(a+b)^2+(2a+b)^2\right)\arctan(\sinh(c+dx))+\frac{3b^2(4a+3b)\sinh(c+dx)}{8\left(\sinh^2(c+dx)+1\right)}+(a+b)^3\sinh(c+dx)-\frac{b^3\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)}}{d}$$

input
```
Int[Cosh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((-3*b*(4*(a + b)^2 + (2*a + b)^2)*ArcTan[Sinh[c + d*x]])/8 + (a + b)^3*Si
nh[c + d*x] - (b^3*Sinh[c + d*x])/(4*(1 + Sinh[c + d*x]^2)^2) + (3*b^2*(4*
a + 3*b)*Sinh[c + d*x])/(8*(1 + Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(93) = 186$.

Time = 6.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 1.95

method	result
derivativedivides	$\frac{\sinh(dx+c)a^3+3a^2b\left(\sinh(dx+c)-2\arctan\left(\mathrm{e}^{dx+c}\right)\right)+3b^2a\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\,\mathrm{arc}\right.}{}$
default	$\frac{\sinh(dx+c)a^3+3a^2b\left(\sinh(dx+c)-2\arctan\left(\mathrm{e}^{dx+c}\right)\right)+3b^2a\left(\frac{\sinh(dx+c)^3}{\cosh(dx+c)^2}+\frac{3\sinh(dx+c)}{\cosh(dx+c)^2}-\frac{3\,\mathrm{sech}(dx+c)\tanh(dx+c)}{2}-3\,\mathrm{arc}\right.}{}$
risch	$\frac{\mathrm{e}^{dx+c}a^3}{2d}+\frac{3\,\mathrm{e}^{dx+c}a^2b}{2d}+\frac{3\,\mathrm{e}^{dx+c}b^2a}{2d}+\frac{\mathrm{e}^{dx+c}b^3}{2d}-\frac{\mathrm{e}^{-dx-c}a^3}{2d}-\frac{3\,\mathrm{e}^{-dx-c}a^2b}{2d}-\frac{3\,\mathrm{e}^{-dx-c}b^2a}{2d}-\frac{\mathrm{e}^{-dx-c}b^3}{2d}$

input
```
int(cosh(d*x+c)*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(sinh(d*x+c)*a^3+3*a^2*b*(sinh(d*x+c)-2*arctan(exp(d*x+c)))+3*b^2*a*(s
inh(d*x+c)^3/cosh(d*x+c)^2+3*sinh(d*x+c)/cosh(d*x+c)^2-3/2*sech(d*x+c)*tan
h(d*x+c)-3*arctan(exp(d*x+c)))+b^3*(sinh(d*x+c)^5/cosh(d*x+c)^4+5/cosh(d*x
+c)^4*sinh(d*x+c)^3+5/cosh(d*x+c)^4*sinh(d*x+c)-5*(1/4*sech(d*x+c)^3+3/8*s
ech(d*x+c))*tanh(d*x+c)-15/4*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2411 vs. $2(93) = 186$.

Time = 0.10 (sec) , antiderivative size = 2411, normalized size of antiderivative = 24.35

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/4*(2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^10 + 20*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^9 + 2*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*sinh(d*x + c)^10 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(
d*x + c)^8 + 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3 + 30*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 24*(10*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(
d*x + c))*sinh(d*x + c)^7 + (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x
 + c)^6 + (420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*a^3 + 1
2*a^2*b + 24*a*b^2 + 5*b^3 + 84*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 6*(84*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d
*x + c)^5 + 28*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (4*a
^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (4*a^3
+ 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cosh(d*x + c)^6 + 210*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^3)*cos
h(d*x + c)^4 - 4*a^3 - 12*a^2*b - 24*a*b^2 - 5*b^3 + 15*(4*a^3 + 12*a^2*b
+ 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(60*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*cosh(d*x + c)^7 + 42*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5*b^
3)*cosh(d*x + c)^5 + 5*(4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)
^3 - (4*a^3 + 12*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
- 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 - 3*(2*a^3 + 6*a^2*b + 10*a*b^2 + 5...
```

Sympy [F]

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)^3\cosh\left(c+dx\right)dx$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*cosh(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(93) = 186$.

Time = 0.17 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.98

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{4}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{2\,e^{(-dx-c)}}{d} + \frac{17\,e^{(-2\,dx-2\,c)} + 13\,e^{(-4\,dx-4\,c)} + 7\,e^{(-6\,dx-6\,c)} - 7\,e^{(-8\,dx-8\,c)}}{d\left(e^{(-dx-c)} + 4\,e^{(-3\,dx-3\,c)} + 6\,e^{(-5\,dx-5\,c)} + 4\,e^{(-7\,dx-7\,c)} + e^{(-9\,d}\right.}\right.$$
$$+ \frac{3}{2}ab^2\left(\frac{6\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{e^{(-dx-c)}}{d} + \frac{4\,e^{(-2\,dx-2\,c)} - e^{(-4\,dx-4\,c)} + 1}{d\left(e^{(-dx-c)} + 2\,e^{(-3\,dx-3\,c)} + e^{(-5\,dx-5\,c)}\right)}\right)$$
$$+ \frac{3}{2}a^2b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d} + \frac{e^{(dx+c)}}{d} - \frac{e^{(-dx-c)}}{d}\right) + \frac{a^3\sinh(dx+c)}{d}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*
c) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d
*(e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7
*c) + e^(-9*d*x - 9*c)))) + 3/2*a*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x
- c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*
e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))
/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a^3*sinh(d*x + c)/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(93) = 186$.

Time = 0.21 (sec) , antiderivative size = 272, normalized size of antiderivative = 2.75

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{8\,a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 24\,a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 24\,ab^2\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 8\,b^3\left(e^{(dx+c)} - e^{(-dx-c}\right.}{}$$

input
```
integrate(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/16*(8*a^3*(e^(d*x + c) - e^(-d*x - c)) + 24*a^2*b*(e^(d*x + c) - e^(-d*x
 - c)) + 24*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 8*b^3*(e^(d*x + c) - e^(-
d*x - c)) - 3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(8*a
^2*b + 12*a*b^2 + 5*b^3) + 4*(12*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*
b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 48*a*b^2*(e^(d*x + c) - e^(-d*x - c))
 + 28*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 +
4)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.59

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{e^{c+dx}\,(a+b)^3}{2\,d} - \frac{e^{-c-dx}\,(a+b)^3}{2\,d} - \frac{3\,\mathrm{atan}\left(\frac{e^{dx}\,e^{c}\,\left(5\,b^3\,\sqrt{d^2}+12\,a\,b^2\,\sqrt{d^2}+8\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{64\,a^4\,b^2+192\,a^3\,b^3+224\,a^2\,b^4+120\,a\,b^5+25\,b^6}}\right)\sqrt{64\,a^4\,b^2+192\,a^3\,b^3+224\,a^2\,b^4+120\,a\,b^5+25\,b^6}}{4\,\sqrt{d^2}} + \frac{3\,e^{c+dx}\,(3\,b^3+4\,a\,b^2)}{4\,d\,(e^{2\,c+2\,dx}+1)} + \frac{6\,b^3\,e^{c+dx}}{d\,(3\,e^{2\,c+2\,dx}+3\,e^{4\,c+4\,dx}+e^{6\,c+6\,dx}+1)} - \frac{e^{c+dx}\,(13\,b^3+12\,a\,b^2)}{2\,d\,(2\,e^{2\,c+2\,dx}+e^{4\,c+4\,dx}+1)} - \frac{4\,b^3\,e^{c+dx}}{d\,(4\,e^{2\,c+2\,dx}+6\,e^{4\,c+4\,dx}+4\,e^{6\,c+6\,dx}+e^{8\,c+8\,dx}+1)}$$

input
```
int(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(exp(c + d*x)*(a + b)^3)/(2*d) - (exp(- c - d*x)*(a + b)^3)/(2*d) - (3*ata
n((exp(d*x)*exp(c)*(5*b^3*(d^2)^(1/2) + 12*a*b^2*(d^2)^(1/2) + 8*a^2*b*(d^
2)^(1/2)))/(d*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2
)^(1/2)))*(120*a*b^5 + 25*b^6 + 224*a^2*b^4 + 192*a^3*b^3 + 64*a^4*b^2)^(1
/2))/(4*(d^2)^(1/2)) + (3*exp(c + d*x)*(4*a*b^2 + 3*b^3))/(4*d*(exp(2*c +
2*d*x) + 1)) + (6*b^3*exp(c + d*x))/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4
*d*x) + exp(6*c + 6*d*x) + 1)) - (exp(c + d*x)*(12*a*b^2 + 13*b^3))/(2*d*(
2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3*exp(c + d*x))/(d*(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 743, normalized size of antiderivative = 7.51

$$\int \cosh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{-15e^{9dx+9c}atan\left(e^{dx+c}\right)b^3-60e^{7dx+7c}atan\left(e^{dx+c}\right)b^3-90e^{5dx+5c}atan\left(e^{dx+c}\right)b^3-60e^{3dx+3c}atan\left(e^{dx+c}\right)b}{}$$

input
```
int(cosh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 24*e**(9*c + 9*d*x)*atan(e**(c + d*x))*a**2*b - 36*e**(9*c + 9*d*x)*at
an(e**(c + d*x))*a*b**2 - 15*e**(9*c + 9*d*x)*atan(e**(c + d*x))*b**3 - 96
*e**(7*c + 7*d*x)*atan(e**(c + d*x))*a**2*b - 144*e**(7*c + 7*d*x)*atan(e*
*(c + d*x))*a*b**2 - 60*e**(7*c + 7*d*x)*atan(e**(c + d*x))*b**3 - 144*e**
(5*c + 5*d*x)*atan(e**(c + d*x))*a**2*b - 216*e**(5*c + 5*d*x)*atan(e**(c
+ d*x))*a*b**2 - 90*e**(5*c + 5*d*x)*atan(e**(c + d*x))*b**3 - 96*e**(3*c
+ 3*d*x)*atan(e**(c + d*x))*a**2*b - 144*e**(3*c + 3*d*x)*atan(e**(c + d*x
))*a*b**2 - 60*e**(3*c + 3*d*x)*atan(e**(c + d*x))*b**3 - 24*e**(c + d*x)*
atan(e**(c + d*x))*a**2*b - 36*e**(c + d*x)*atan(e**(c + d*x))*a*b**2 - 15
*e**(c + d*x)*atan(e**(c + d*x))*b**3 + 2*e**(10*c + 10*d*x)*a**3 + 6*e**(
10*c + 10*d*x)*a**2*b + 6*e**(10*c + 10*d*x)*a*b**2 + 2*e**(10*c + 10*d*x)
*b**3 + 6*e**(8*c + 8*d*x)*a**3 + 18*e**(8*c + 8*d*x)*a**2*b + 30*e**(8*c
+ 8*d*x)*a*b**2 + 15*e**(8*c + 8*d*x)*b**3 + 4*e**(6*c + 6*d*x)*a**3 + 12*
e**(6*c + 6*d*x)*a**2*b + 24*e**(6*c + 6*d*x)*a*b**2 + 5*e**(6*c + 6*d*x)*
b**3 - 4*e**(4*c + 4*d*x)*a**3 - 12*e**(4*c + 4*d*x)*a**2*b - 24*e**(4*c +
 4*d*x)*a*b**2 - 5*e**(4*c + 4*d*x)*b**3 - 6*e**(2*c + 2*d*x)*a**3 - 18*e*
*(2*c + 2*d*x)*a**2*b - 30*e**(2*c + 2*d*x)*a*b**2 - 15*e**(2*c + 2*d*x)*b
**3 - 2*a**3 - 6*a**2*b - 6*a*b**2 - 2*b**3)/(4*e**(c + d*x)*d*(e**(8*c +
8*d*x) + 4*e**(6*c + 6*d*x) + 6*e**(4*c + 4*d*x) + 4*e**(2*c + 2*d*x) + 1)
)
```

3.101 $\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 125

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))}{16d} - \frac{b(24a^2+30ab+11b^2)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{b^2(18a+13b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} - \frac{b^3\operatorname{sech}^5(c+dx)\tanh(c+dx)}{6d}$$

output
```
1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan(sinh(d*x+c))/d-1/16*b*(24*a^2+30*a
*b+11*b^2)*sech(d*x+c)*tanh(d*x+c)/d+1/24*b^2*(18*a+13*b)*sech(d*x+c)^3*ta
nh(d*x+c)/d-1/6*b^3*sech(d*x+c)^5*tanh(d*x+c)/d
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.16 (sec) , antiderivative size = 1341, normalized size of antiderivative = 10.73

$$\int \operatorname{sech}(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \text{Too large to display}$$

input
```
Integrate[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Sinh[c + d*x]*((9514449*(a + b))/a + (135323370*(a + b)^2)/a^2 + (580
09455*(a + b)^3)/a^3 + 4093425*Csch[c + d*x]^2 + (168951510*(a + b)*Csch[c
 + d*x]^2)/a + (215549775*(a + b)^2*Csch[c + d*x]^2)/a^2 + 70189350*Csch[c
 + d*x]^4 + (274542345*(a + b)*Csch[c + d*x]^4)/a + 117228825*Csch[c + d*x
]^6 + (7808535*(a + b)^2*Sinh[c + d*x]^2)/a^2 + (36772890*(a + b)^3*Sinh[c
 + d*x]^2)/a^3 - 75520*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2
}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 13824*HypergeometricPFQ[{3/2, 2, 2,
 2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^2 - 1024*Hy
pergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c +
d*x]^2]*Sinh[c + d*x]^2 + (2160711*(a + b)^3*Sinh[c + d*x]^4)/a^3 - (18969
6*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c +
d*x]^2]*Sinh[c + d*x]^4)/a - (38400*(a + b)*HypergeometricPFQ[{3/2, 2, 2,
2, 2, 2}, {1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (3072
*(a + b)*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^4)/a - (158976*(a + b)^2*HypergeometricPF
Q[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a
^2 - (35328*(a + b)^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1,
 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6)/a^2 - (3072*(a + b)^2*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]
*Sinh[c + d*x]^6)/a^2 - (44800*(a + b)^3*HypergeometricPFQ[{3/2, 2, 2, ...
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.32, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4159, 315, 401, 25, 298, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3dx$$

$$\Big\downarrow \; 3042$$

$$\int \sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \; 4159$$

$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 315$$

$$\frac{\frac{1}{6}\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)\left((a+b)(6a+5b)\sinh^2(c+dx)+a(6a+b)\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)^2}{6\left(\sinh^2(c+dx)+1\right)^3}}{d}$$

$$\Big\downarrow \; 401$$

$$\frac{\frac{1}{6}\left(-\frac{1}{4}\int -\frac{(a+b)\left(24a^2+34ba+15b^2\right)\sinh^2(c+dx)+a\left(24a^2+14ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{5b(2a+b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\int \frac{(a+b)\left(24a^2+34ba+15b^2\right)\sinh^2(c+dx)+a\left(24a^2+14ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{5b(2a+b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\ldots}{d}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(2a+b)\left(8a^2+8ab+5b^2\right)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)-\frac{b\left(44a^2+44ab+15b^2\right)\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{5b(2a+b)\sinh(c+dx}{4(\sinh^2\ldots}\ldots}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{1}{6}\left(\frac{1}{4}\left(\frac{3}{2}(2a+b)\left(8a^2+8ab+5b^2\right)\arctan(\sinh(c+dx))-\frac{b(44a^2+44ab+15b^2)\sinh(c+dx)}{2(\sinh^2(c+dx)+1)}\right)-\frac{5b(2a+b)\sinh(c+dx)((a+b)\sinh}{4(\sinh^2(c+dx)+1}\right.}{d}$$

input
```
Int[Sech[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-1/6*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2)^2)/(1 + Sinh[c + d*x]
^2)^3 + ((-5*b*(2*a + b)*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))/(4*(
1 + Sinh[c + d*x]^2)^2) + ((3*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[Sin
h[c + d*x]])/2 - (b*(44*a^2 + 44*a*b + 15*b^2)*Sinh[c + d*x])/(2*(1 + Sinh
[c + d*x]^2)))/4)/6)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(117) = 234$.

Time = 13.83 (sec) , antiderivative size = 240, normalized size of antiderivative = 1.92

method	result
derivativedivides	$\frac{2a^3 \arctan\left(e^{dx+c}\right)+3a^2 b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3b^2 a\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}\right.}{}$
default	$\frac{2a^3 \arctan\left(e^{dx+c}\right)+3a^2 b\left(-\frac{\sinh(dx+c)}{\cosh(dx+c)^2}+\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3b^2 a\left(-\frac{\sinh(dx+c)^3}{\cosh(dx+c)^4}-\frac{\sinh(dx+c)}{\cosh(dx+c)^4}\right.}{}$
risch	$-\frac{b\,e^{dx+c}\left(72\,e^{10dx+10c}a^2+90\,e^{10dx+10c}ab+33\,e^{10dx+10c}b^2+216\,e^{8dx+8c}a^2+126\,e^{8dx+8c}ab-5\,e^{8dx+8c}b^2+144\,e^{6dx+\cdots}\right.}{}$

input
```
int(sech(d*x+c)*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*a^3*arctan(exp(d*x+c))+3*a^2*b*(-sinh(d*x+c)/cosh(d*x+c)^2+1/2*sech
(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*b^2*a*(-1/cosh(d*x+c)^4*sinh(d*x
+c)^3-1/cosh(d*x+c)^4*sinh(d*x+c)+(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh
(d*x+c)+3/4*arctan(exp(d*x+c)))+b^3*(-sinh(d*x+c)^5/cosh(d*x+c)^6-5/3*sinh
(d*x+c)^3/cosh(d*x+c)^6-sinh(d*x+c)/cosh(d*x+c)^6+(1/6*sech(d*x+c)^5+5/24*
sech(d*x+c)^3+5/16*sech(d*x+c))*tanh(d*x+c)+5/8*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3465 vs. $2(117) = 234$.

Time = 0.11 (sec) , antiderivative size = 3465, normalized size of antiderivative = 27.72

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2(c+dx)\right)^3\operatorname{sech}(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 362 vs. $2(117) = 234$.

Time = 0.12 (sec) , antiderivative size = 362, normalized size of antiderivative = 2.90

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{1}{24}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{33\,e^{(-dx-c)}-5\,e^{(-3\,dx-3\,c)}+90\,e^{(-5\,dx-5\,c)}-90\,e^{(-7\,dx-7\,c)}+5\,e^{(-9\,d}}{d(6\,e^{(-2\,dx-2\,c)}+15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6\,dx-6\,c)}+15\,e^{(-8\,dx-8\,c)}+6\,e^{(-10}}\right.$$
$$-\frac{3}{4}ab^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5\,e^{(-dx-c)}-3\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}-5\,e^{(-7\,dx-7\,c)}}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$-3\,a^2b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{a^3\arctan\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*
c) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e
^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(
-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x -
 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1)))
- 3*a^2*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(
2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*arctan(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(117) = 234$.

Time = 0.21 (sec) , antiderivative size = 310, normalized size of antiderivative = 2.48

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)-\frac{4\left(72\,a^2b\left(e^{(dx+c)}-e^{(-dx-c)}\right)^5+90\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/96*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(16*a^3 +
24*a^2*b + 18*a*b^2 + 5*b^3) - 4*(72*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5
+ 90*a*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 33*b^3*(e^(d*x + c) - e^(-d*x
- c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 + 576*a*b^2*(e^(d*x + c
) - e^(-d*x - c))^3 + 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 1152*a^2*b*
(e^(d*x + c) - e^(-d*x - c)) + 864*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 24
0*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)^3
)/d
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 535, normalized size of antiderivative = 4.28

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{\operatorname{atan}\left(\frac{e^{d\,x}\,e^{c}\,\left(16\,a^3\,\sqrt{d^2}+5\,b^3\,\sqrt{d^2}+18\,a\,b^2\,\sqrt{d^2}+24\,a^2\,b\,\sqrt{d^2}\right)}{d\,\sqrt{256\,a^6+768\,a^5\,b+1152\,a^4\,b^2+1024\,a^3\,b^3+564\,a^2\,b^4+180\,a\,b^5+25\,b^6}}\right)\sqrt{256\,a^6+768\,a^5\,b+1152\,a^4\,b^2+1024\,a^3\,b^3\cdots}}{8\,\sqrt{d^2}}$$
$$-\frac{e^{c+d\,x}\,\left(55\,b^3+54\,a\,b^2\right)}{3\,d\,\left(3\,e^{2\,c+2\,d\,x}+3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}+1\right)}$$
$$-\frac{80\,b^3\,e^{c+d\,x}}{3\,d\,\left(5\,e^{2\,c+2\,d\,x}+10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}+5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}+1\right)}$$
$$+\frac{6\,e^{c+d\,x}\,\left(5\,b^3+2\,a\,b^2\right)}{d\,\left(4\,e^{2\,c+2\,d\,x}+6\,e^{4\,c+4\,d\,x}+4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$
$$+\frac{32\,b^3\,e^{c+d\,x}}{3\,d\,\left(6\,e^{2\,c+2\,d\,x}+15\,e^{4\,c+4\,d\,x}+20\,e^{6\,c+6\,d\,x}+15\,e^{8\,c+8\,d\,x}+6\,e^{10\,c+10\,d\,x}+e^{12\,c+12\,d\,x}+1\right)}$$
$$-\frac{e^{c+d\,x}\,\left(24\,a^2\,b+30\,a\,b^2+11\,b^3\right)}{8\,d\,\left(e^{2\,c+2\,d\,x}+1\right)}+\frac{e^{c+d\,x}\,\left(72\,a^2\,b+162\,a\,b^2+85\,b^3\right)}{12\,d\,\left(2\,e^{2\,c+2\,d\,x}+e^{4\,c+4\,d\,x}+1\right)}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x),x)
```

output
```
(atan((exp(d*x)*exp(c)*(16*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 18*a*b^2*
(d^2)^(1/2) + 24*a^2*b*(d^2)^(1/2)))/(d*(180*a*b^5 + 768*a^5*b + 256*a^6 +
 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^(1/2)))*(180*a*b^5 +
768*a^5*b + 256*a^6 + 25*b^6 + 564*a^2*b^4 + 1024*a^3*b^3 + 1152*a^4*b^2)^
(1/2))/(8*(d^2)^(1/2)) - (exp(c + d*x)*(54*a*b^2 + 55*b^3))/(3*d*(3*exp(2*
c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) - (80*b^3*exp(c +
 d*x))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x
) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (6*exp(c + d*x)*(2*a*b
^2 + 5*b^3))/(d*(4*exp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1)) + (32*b^3*exp(c + d*x))/(3*d*(6*exp(2*c + 2*d
*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*
exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)*(30*a*b^2 +
24*a^2*b + 11*b^3))/(8*d*(exp(2*c + 2*d*x) + 1)) + (exp(c + d*x)*(162*a*b^
2 + 72*a^2*b + 85*b^3))/(12*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 971, normalized size of antiderivative = 7.77

$$\int \operatorname{sech}(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(48*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a**3 + 72*e**(12*c + 12*d*x)*ata
n(e**(c + d*x))*a**2*b + 54*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a*b**2 +
 15*e**(12*c + 12*d*x)*atan(e**(c + d*x))*b**3 + 288*e**(10*c + 10*d*x)*at
an(e**(c + d*x))*a**3 + 432*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a**2*b +
 324*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a*b**2 + 90*e**(10*c + 10*d*x)*
atan(e**(c + d*x))*b**3 + 720*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3 + 1
080*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**2*b + 810*e**(8*c + 8*d*x)*atan
(e**(c + d*x))*a*b**2 + 225*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**3 + 960
*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**3 + 1440*e**(6*c + 6*d*x)*atan(e**
(c + d*x))*a**2*b + 1080*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b**2 + 300*
e**(6*c + 6*d*x)*atan(e**(c + d*x))*b**3 + 720*e**(4*c + 4*d*x)*atan(e**(c
 + d*x))*a**3 + 1080*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b + 810*e**(
4*c + 4*d*x)*atan(e**(c + d*x))*a*b**2 + 225*e**(4*c + 4*d*x)*atan(e**(c +
 d*x))*b**3 + 288*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3 + 432*e**(2*c +
 2*d*x)*atan(e**(c + d*x))*a**2*b + 324*e**(2*c + 2*d*x)*atan(e**(c + d*x)
)*a*b**2 + 90*e**(2*c + 2*d*x)*atan(e**(c + d*x))*b**3 + 48*atan(e**(c + d
*x))*a**3 + 72*atan(e**(c + d*x))*a**2*b + 54*atan(e**(c + d*x))*a*b**2 +
15*atan(e**(c + d*x))*b**3 - 72*e**(11*c + 11*d*x)*a**2*b - 90*e**(11*c +
11*d*x)*a*b**2 - 33*e**(11*c + 11*d*x)*b**3 - 216*e**(9*c + 9*d*x)*a**2*b
- 126*e**(9*c + 9*d*x)*a*b**2 + 5*e**(9*c + 9*d*x)*b**3 - 144*e**(7*c +...
```

3.102 $\int \text{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 67

$$\int \text{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{a^3\tanh(c+dx)}{d} + \frac{a^2b\tanh^3(c+dx)}{d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^7(c+dx)}{7d}$$

output
```
a^3*tanh(d*x+c)/d+a^2*b*tanh(d*x+c)^3/d+3/5*a*b^2*tanh(d*x+c)^5/d+1/7*b^3*
tanh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.00

$$\int \text{sech}^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{a^3\tanh(c+dx)}{d} + \frac{a^2b\tanh^3(c+dx)}{d} + \frac{3ab^2\tanh^5(c+dx)}{5d} + \frac{b^3\tanh^7(c+dx)}{7d}$$

input
```
Integrate[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x])/d + (a^2*b*Tanh[c + d*x]^3)/d + (3*a*b^2*Tanh[c + d*x]
^5)/(5*d) + (b^3*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 210, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(b\tanh^2(c+dx)+a\right)^3d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{210}$$

$$\frac{\int \left(b^3\tanh^6(c+dx)+3ab^2\tanh^4(c+dx)+3a^2b\tanh^2(c+dx)+a^3\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\tanh(c+dx)+a^2b\tanh^3(c+dx)+\frac{3}{5}ab^2\tanh^5(c+dx)+\frac{1}{7}b^3\tanh^7(c+dx)}{d}$$

input
```
Int[Sech[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x] + a^2*b*Tanh[c + d*x]^3 + (3*a*b^2*Tanh[c + d*x]^5)/5 +
 (b^3*Tanh[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 210
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*x^2
)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 226 vs. $2(63) = 126$.

Time = 42.46 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.39

method	result
derivativedivides	$\frac{a^3\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3b^2a\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\ldots\right.}{}$
default	$\frac{a^3\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{2\cosh(dx+c)^3}+\frac{\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)}{2}\right)+3b^2a\left(-\frac{\sinh(dx+c)^3}{2\cosh(dx+c)^5}-\frac{3\sinh(dx+c)}{8\cosh(dx+c)^5}+\ldots\right.}{}$
risch	$-\frac{2\left(175b^3e^{8dx+8c}+105b^3e^{4dx+4c}+210\,e^{2dx+2c}a^3+21b^2a+35a^3+35a^2b+105a^2b\,e^{12dx+12c}+420a^2b\,e^{10dx+10c}+210a\ldots\right.}{}$

input
```
int(sech(d*x+c)^2*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^3+1/2*(2/3+1/3*
sech(d*x+c)^2)*tanh(d*x+c))+3*b^2*a*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*
sinh(d*x+c)/cosh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*
tanh(d*x+c))+b^3*(-1/2*sinh(d*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(
d*x+c)^7-5/16*sinh(d*x+c)/cosh(d*x+c)^7+5/16*(16/35+1/7*sech(d*x+c)^6+6/35
*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 786 vs. $2(63) = 126$.

Time = 0.08 (sec) , antiderivative size = 786, normalized size of antiderivative = 11.73

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-4/35*((35*a^3 + 70*a^2*b + 63*a*b^2 + 20*b^3)*cosh(d*x + c)^6 + 6*(35*a^2
*b + 42*a*b^2 + 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (35*a^3 + 70*a^2*b
 + 63*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(15*a^3 + 20*a^2*b + 9*a*b^2)*c
osh(d*x + c)^4 + (210*a^3 + 280*a^2*b + 126*a*b^2 + 15*(35*a^3 + 70*a^2*b
+ 63*a*b^2 + 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(35*a^2*b + 4
2*a*b^2 + 15*b^3)*cosh(d*x + c)^3 + 28*(5*a^2*b + 3*a*b^2)*cosh(d*x + c))*
sinh(d*x + c)^3 + 350*a^3 + 280*a^2*b + 210*a*b^2 + 7*(75*a^3 + 70*a^2*b +
 39*a*b^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(35*a^3 + 70*a^2*b + 63*a*b^2 +
20*b^3)*cosh(d*x + c)^4 + 525*a^3 + 490*a^2*b + 273*a*b^2 + 140*b^3 + 84*(
15*a^3 + 20*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(35*a
^2*b + 42*a*b^2 + 15*b^3)*cosh(d*x + c)^5 + 56*(5*a^2*b + 3*a*b^2)*cosh(d*
x + c)^3 + 7*(25*a^2*b + 6*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d
*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^6 + 4*(
14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cosh(d*x
+ c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x +
 c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))
*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d*co
sh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*cosh
(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x...
```

Sympy [F]

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \left(a+b\tanh^2(c+dx)\right)^3\operatorname{sech}^2(c+dx)\,dx$$

input
```
integrate(sech(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.06

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{b^3\tanh(dx+c)^7}{7\,d}+\frac{3\,ab^2\tanh(dx+c)^5}{5\,d}+\frac{a^2b\tanh(dx+c)^3}{d}+\frac{2\,a^3}{d(e^{(-2\,dx-2\,c)}+1)}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/7*b^3*tanh(d*x + c)^7/d + 3/5*a*b^2*tanh(d*x + c)^5/d + a^2*b*tanh(d*x +
 c)^3/d + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 347 vs. 2(63) = 126.

Time = 0.22 (sec) , antiderivative size = 347, normalized size of antiderivative = 5.18

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=-\frac{2\left(35\,a^3e^{(12\,dx+12\,c)}+105\,a^2be^{(12\,dx+12\,c)}+105\,ab^2e^{(12\,dx+12\,c)}+35\,b^3e^{(12\,dx+12\,c)}+210\,a^3e^{(10\,dx+10\,c)}+42\right.}{}$$

input
```
integrate(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-2/35*(35*a^3*e^(12*d*x + 12*c) + 105*a^2*b*e^(12*d*x + 12*c) + 105*a*b^2*
e^(12*d*x + 12*c) + 35*b^3*e^(12*d*x + 12*c) + 210*a^3*e^(10*d*x + 10*c) +
 420*a^2*b*e^(10*d*x + 10*c) + 210*a*b^2*e^(10*d*x + 10*c) + 525*a^3*e^(8*
d*x + 8*c) + 665*a^2*b*e^(8*d*x + 8*c) + 315*a*b^2*e^(8*d*x + 8*c) + 175*b
^3*e^(8*d*x + 8*c) + 700*a^3*e^(6*d*x + 6*c) + 560*a^2*b*e^(6*d*x + 6*c) +
 420*a*b^2*e^(6*d*x + 6*c) + 525*a^3*e^(4*d*x + 4*c) + 315*a^2*b*e^(4*d*x
+ 4*c) + 231*a*b^2*e^(4*d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c) + 210*a^3*e^(
2*d*x + 2*c) + 140*a^2*b*e^(2*d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 35*a
^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^7)
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 1050, normalized size of antiderivative = 15.67

$$\int \mathrm{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^2,x)
```

output
```
- ((2*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (2*exp(6*c + 6*d*x)*(a
 + b)^3)/(7*d) - (6*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*
d) + (6*exp(4*c + 4*d*x)*(a + b)^2*(a - b))/(7*d))/(4*exp(2*c + 2*d*x) + 6
*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1) - ((2*(a +
b)^3)/(7*d) + (2*exp(12*c + 12*d*x)*(a + b)^3)/(7*d) - (6*exp(4*c + 4*d*x)
*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d) - (6*exp(8*c + 8*d*x)*(a*b^2 + a^2
*b - 5*a^3 - 5*b^3))/(7*d) + (8*exp(6*c + 6*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^
3 - 5*b^3))/(7*d) + (12*exp(2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d) + (12*ex
p(10*c + 10*d*x)*(a + b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*
c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*
d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((2*exp(4*c + 4*d*
x)*(a + b)^3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (4*exp(
2*c + 2*d*x)*(a + b)^2*(a - b))/(7*d))/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4
*d*x) + exp(6*c + 6*d*x) + 1) - ((2*(a + b)^2*(a - b))/(7*d) + (2*exp(2*c
+ 2*d*x)*(a + b)^3)/(7*d))/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) - (
(2*exp(8*c + 8*d*x)*(a + b)^3)/(7*d) - (2*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))
/(35*d) - (12*exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(35*d) + (
8*exp(2*c + 2*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(35*d) + (8*exp(6*
c + 6*d*x)*(a + b)^2*(a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*
d*x) + 10*exp(6*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + ...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 435, normalized size of antiderivative = 6.49

$$\int \operatorname{sech}^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\frac{2e^{14dx+14c}a^3}{7}+\frac{6e^{14dx+14c}a^2b}{7}+\frac{6e^{14dx+14c}a\,b^2}{7}+\frac{2e^{14dx+14c}b^3}{7}-6e^{10dx+10c}a^3-6e^{10dx+10c}a^2b+6e^{10dx+10c}a\,b^2+6e}{}$$

input
```
int(sech(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
(2*(5*e**(14*c + 14*d*x)*a**3 + 15*e**(14*c + 14*d*x)*a**2*b + 15*e**(14*c
 + 14*d*x)*a*b**2 + 5*e**(14*c + 14*d*x)*b**3 - 105*e**(10*c + 10*d*x)*a**
3 - 105*e**(10*c + 10*d*x)*a**2*b + 105*e**(10*c + 10*d*x)*a*b**2 + 105*e*
*(10*c + 10*d*x)*b**3 - 350*e**(8*c + 8*d*x)*a**3 - 140*e**(8*c + 8*d*x)*a
**2*b + 210*e**(8*c + 8*d*x)*a*b**2 - 525*e**(6*c + 6*d*x)*a**3 - 35*e**(6
*c + 6*d*x)*a**2*b + 105*e**(6*c + 6*d*x)*a*b**2 + 175*e**(6*c + 6*d*x)*b*
*3 - 420*e**(4*c + 4*d*x)*a**3 + 84*e**(4*c + 4*d*x)*a*b**2 - 175*e**(2*c
+ 2*d*x)*a**3 - 35*e**(2*c + 2*d*x)*a**2*b + 63*e**(2*c + 2*d*x)*a*b**2 +
35*e**(2*c + 2*d*x)*b**3 - 30*a**3 - 20*a**2*b - 6*a*b**2))/(35*d*(e**(14*
c + 14*d*x) + 7*e**(12*c + 12*d*x) + 21*e**(10*c + 10*d*x) + 35*e**(8*c +
8*d*x) + 35*e**(6*c + 6*d*x) + 21*e**(4*c + 4*d*x) + 7*e**(2*c + 2*d*x) +
1))
```

3.103 $\int \operatorname{sech}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 172

$$\begin{aligned}
&\int \operatorname{sech}^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx \\
&= \frac{(64a^3+48a^2b+24ab^2+5b^3)\arctan(\sinh(c+dx))}{128d} \\
&\quad + \frac{(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)\tanh(c+dx)}{128d} \\
&\quad - \frac{b(144a^2+168ab+59b^2)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{192d} \\
&\quad + \frac{b^2(24a+17b)\operatorname{sech}^5(c+dx)\tanh(c+dx)}{48d} - \frac{b^3\operatorname{sech}^7(c+dx)\tanh(c+dx)}{8d}
\end{aligned}$$

output
```
1/128*(64*a^3+48*a^2*b+24*a*b^2+5*b^3)*arctan(sinh(d*x+c))/d+1/128*(64*a^3
+48*a^2*b+24*a*b^2+5*b^3)*sech(d*x+c)*tanh(d*x+c)/d-1/192*b*(144*a^2+168*a
*b+59*b^2)*sech(d*x+c)^3*tanh(d*x+c)/d+1/48*b^2*(24*a+17*b)*sech(d*x+c)^5*
tanh(d*x+c)/d-1/8*b^3*sech(d*x+c)^7*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 12.14 (sec) , antiderivative size = 158, normalized size of antiderivative = 0.92

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{6(64a^3+48a^2b+24ab^2+5b^3)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+3(64a^3+48a^2b+24ab^2+5b^3)\operatorname{sech}(c+dx)}{}$$

input
```
Integrate[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(6*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*ArcTan[Tanh[(c + d*x)/2]] + 3*(6
4*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*Sech[c + d*x]*Tanh[c + d*x] - 2*b*(14
4*a^2 + 168*a*b + 59*b^2)*Sech[c + d*x]^3*Tanh[c + d*x] + 8*b^2*(24*a + 17
*b)*Sech[c + d*x]^5*Tanh[c + d*x] - 48*b^3*Sech[c + d*x]^7*Tanh[c + d*x])/
(384*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 197, normalized size of antiderivative = 1.15, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 4159, 315, 401, 25, 298, 215, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3dx$$
$$\Big\downarrow \ 4159$$
$$\frac{\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)^3}{\left(\sinh^2(c+dx)+1\right)^5}d\sinh(c+dx)}{d}$$
$$\Big\downarrow \ 315$$

$$\frac{\frac{1}{8}\int \frac{\left((a+b)\sinh^2(c+dx)+a\right)\left((a+b)(8a+5b)\sinh^2(c+dx)+a(8a+b)\right)}{\left(\sinh^2(c+dx)+1\right)^4}d\sinh(c+dx)-\frac{b\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)^2}{8\left(\sinh^2(c+dx)+1\right)^4}}{d}$$

$\downarrow$ 401

$$\frac{\frac{1}{8}\left(-\frac{1}{6}\int -\frac{3(a+b)\left(16a^2+14ba+5b^2\right)\sinh^2(c+dx)+a\left(48a^2+18ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)-\frac{b(12a+5b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6\left(\sinh^2(c+dx)+1\right)^3}\right.}{d}$$

$\downarrow$ 25

$$\frac{\frac{1}{8}\left(\frac{1}{6}\int \frac{3(a+b)\left(16a^2+14ba+5b^2\right)\sinh^2(c+dx)+a\left(48a^2+18ba+5b^2\right)}{\left(\sinh^2(c+dx)+1\right)^3}d\sinh(c+dx)-\frac{b(12a+5b)\sinh(c+dx)\left((a+b)\sinh^2(c+dx)+a\right)}{6\left(\sinh^2(c+dx)+1\right)^3}\right)-}{d}$$

$\downarrow$ 298

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+48a^2b+24ab^2+5b^3\right)\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2}d\sinh(c+dx)-\frac{b\left(72a^2+52ab+15b^2\right)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)-\frac{b(12a+5b)\,\mathrm{s}}{}\right.}{d}$$

$\downarrow$ 215

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+48a^2b+24ab^2+5b^3\right)\left(\frac{1}{2}\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\left(72a^2+52ab+15b^2\right)\sinh}{4\left(\sinh^2(c+dx)+1\right)}\right.\right.}{d}$$

$\downarrow$ 216

$$\frac{\frac{1}{8}\left(\frac{1}{6}\left(\frac{3}{4}\left(64a^3+48a^2b+24ab^2+5b^3\right)\left(\frac{1}{2}\arctan(\sinh(c+dx))+\frac{\sinh(c+dx)}{2\left(\sinh^2(c+dx)+1\right)}\right)-\frac{b\left(72a^2+52ab+15b^2\right)\sinh(c+dx)}{4\left(\sinh^2(c+dx)+1\right)^2}\right)\right.}{d}$$

input
```
Int[Sech[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-1/8*(b*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2)^2)/(1 + Sinh[c + d*x]
^2)^4 + (-1/6*(b*(12*a + 5*b)*Sinh[c + d*x]*(a + (a + b)*Sinh[c + d*x]^2))
/(1 + Sinh[c + d*x]^2)^3 + (-1/4*(b*(72*a^2 + 52*a*b + 15*b^2)*Sinh[c + d*
x])/(1 + Sinh[c + d*x]^2)^2 + (3*(64*a^3 + 48*a^2*b + 24*a*b^2 + 5*b^3)*(A
rcTan[Sinh[c + d*x]]/2 + Sinh[c + d*x]/(2*(1 + Sinh[c + d*x]^2))))/4)/6)/8
)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 401
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
q/(a*b*2*(p + 1))), x] + Simp[1/(a*b*2*(p + 1))   Int[(a + b*x^2)^(p + 1)*(
c + d*x^2)^(q - 1)*Simp[c*(b*e*2*(p + 1) + b*e - a*f) + d*(b*e*2*(p + 1) +
(b*e - a*f)*(2*q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && L
tQ[p, -1] && GtQ[q, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [A] (verified)

Time = 80.24 (sec) , antiderivative size = 291, normalized size of antiderivative = 1.69

method	result
derivativedivides	$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan}{}\right.}{}$
default	$\frac{a^3\left(\frac{\operatorname{sech}(dx+c)\tanh(dx+c)}{2}+\arctan\left(e^{dx+c}\right)\right)+3a^2b\left(-\frac{\sinh(dx+c)}{3\cosh(dx+c)^4}+\frac{\left(\frac{\operatorname{sech}(dx+c)^3}{4}+\frac{3\operatorname{sech}(dx+c)}{8}\right)\tanh(dx+c)}{3}+\frac{\arctan}{}\right.}{}$
risch	$\frac{e^{dx+c}\left(-1765b^3e^{8dx+8c}+1765b^3e^{6dx+6c}-895b^3e^{4dx+4c}-960\,e^{2dx+2c}a^3-72b^2a-192a^3-144a^2b-432a^2b\,e^{12dx+12c}-\right.}{}$

input
```
int(sech(d*x+c)^3*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(1/2*sech(d*x+c)*tanh(d*x+c)+arctan(exp(d*x+c)))+3*a^2*b*(-1/3/co
sh(d*x+c)^4*sinh(d*x+c)+1/3*(1/4*sech(d*x+c)^3+3/8*sech(d*x+c))*tanh(d*x+c
)+1/4*arctan(exp(d*x+c)))+3*b^2*a*(-1/3*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5*si
nh(d*x+c)/cosh(d*x+c)^6+1/5*(1/6*sech(d*x+c)^5+5/24*sech(d*x+c)^3+5/16*sec
h(d*x+c))*tanh(d*x+c)+1/8*arctan(exp(d*x+c)))+b^3*(-1/3*sinh(d*x+c)^5/cosh
(d*x+c)^8-1/3*sinh(d*x+c)^3/cosh(d*x+c)^8-1/7*sinh(d*x+c)/cosh(d*x+c)^8+1/
7*(1/8*sech(d*x+c)^7+7/48*sech(d*x+c)^5+35/192*sech(d*x+c)^3+35/128*sech(d
*x+c))*tanh(d*x+c)+5/64*arctan(exp(d*x+c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6114 vs. $2(162) = 324$.

Time = 0.12 (sec) , antiderivative size = 6114, normalized size of antiderivative = 35.55

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\operatorname{sech}^3\left(c+dx\right)dx$$

input
```
integrate(sech(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(162) = 324$.

Time = 0.14 (sec) , antiderivative size = 553, normalized size of antiderivative = 3.22

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/192*b^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) - 397*e^(-3*d*x -
 3*c) + 895*e^(-5*d*x - 5*c) - 1765*e^(-7*d*x - 7*c) + 1765*e^(-9*d*x - 9*
c) - 895*e^(-11*d*x - 11*c) + 397*e^(-13*d*x - 13*c) - 15*e^(-15*d*x - 15*
c))/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 7
0*e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(
-14*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x
- c))/d - (3*e^(-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78
*e^(-7*d*x - 7*c) + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-
2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x -
8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c) + 1))) - 3/4*a^2*b*(arcta
n(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c
) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-
6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d - (e^
(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) +
 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 485 vs. $2(162) = 324$.

Time = 0.22 (sec) , antiderivative size = 485, normalized size of antiderivative = 2.82

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)}-1\right)e^{(-dx-c)}\right)\right)\left(64a^3+48a^2b+24ab^2+5b^3\right)+\frac{4\left(192a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^7+14\right.}{}}{}$$

input
```
integrate(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/768*(3*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(64*a^3 +
 48*a^2*b + 24*a*b^2 + 5*b^3) + 4*(192*a^3*(e^(d*x + c) - e^(-d*x - c))^7
+ 144*a^2*b*(e^(d*x + c) - e^(-d*x - c))^7 + 72*a*b^2*(e^(d*x + c) - e^(-d
*x - c))^7 + 15*b^3*(e^(d*x + c) - e^(-d*x - c))^7 + 2304*a^3*(e^(d*x + c)
 - e^(-d*x - c))^5 + 576*a^2*b*(e^(d*x + c) - e^(-d*x - c))^5 - 480*a*b^2*
(e^(d*x + c) - e^(-d*x - c))^5 - 292*b^3*(e^(d*x + c) - e^(-d*x - c))^5 +
9216*a^3*(e^(d*x + c) - e^(-d*x - c))^3 - 2304*a^2*b*(e^(d*x + c) - e^(-d*
x - c))^3 - 4224*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 - 880*b^3*(e^(d*x +
c) - e^(-d*x - c))^3 + 12288*a^3*(e^(d*x + c) - e^(-d*x - c)) - 9216*a^2*b
*(e^(d*x + c) - e^(-d*x - c)) - 4608*a*b^2*(e^(d*x + c) - e^(-d*x - c)) -
960*b^3*(e^(d*x + c) - e^(-d*x - c)))/((e^(d*x + c) - e^(-d*x - c))^2 + 4)
^4)/d
```

Mupad [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 951, normalized size of antiderivative = 5.53

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^3,x)
```

output
```
(atan((exp(d*x)*exp(c)*(64*a^3*(d^2)^(1/2) + 5*b^3*(d^2)^(1/2) + 24*a*b^2*
(d^2)^(1/2) + 48*a^2*b*(d^2)^(1/2)))/(d*(240*a*b^5 + 6144*a^5*b + 4096*a^6
 + 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4*b^2)^(1/2)))*(240*a*b^5
 + 6144*a^5*b + 4096*a^6 + 25*b^6 + 1056*a^2*b^4 + 2944*a^3*b^3 + 5376*a^4
*b^2)^(1/2))/(64*(d^2)^(1/2)) - ((exp(c + d*x)*(a + b)^3)/(2*d) + (exp(13*
c + 13*d*x)*(a + b)^3)/(2*d) - (3*exp(5*c + 5*d*x)*(a*b^2 + a^2*b - 5*a^3
- 5*b^3))/(2*d) - (3*exp(9*c + 9*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(2*
d) + (2*exp(7*c + 7*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/d + (3*exp(3
*c + 3*d*x)*(a + b)^2*(a - b))/d + (3*exp(11*c + 11*d*x)*(a + b)^2*(a - b)
)/d)/(8*exp(2*c + 2*d*x) + 28*exp(4*c + 4*d*x) + 56*exp(6*c + 6*d*x) + 70*
exp(8*c + 8*d*x) + 56*exp(10*c + 10*d*x) + 28*exp(12*c + 12*d*x) + 8*exp(1
4*c + 14*d*x) + exp(16*c + 16*d*x) + 1) + (2*exp(c + d*x)*(48*a*b^2 + 85*b
^3))/(3*d*(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x)
+ 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1)) + (16*b^3*exp(c + d*x))/(d
*(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x) + 35*exp(6*c + 6*d*x) + 35*exp(
8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*exp(12*c + 12*d*x) + exp(14*c + 1
4*d*x) + 1)) + (exp(c + d*x)*(24*a*b^2 + 48*a^2*b + 64*a^3 + 5*b^3))/(64*d
*(exp(2*c + 2*d*x) + 1)) - (4*exp(c + d*x)*(6*a*b^2 + 35*b^3))/(3*d*(6*exp
(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8
*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c + 12*d*x) + 1)) - (exp(c + d*x)...
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1394, normalized size of antiderivative = 8.10

$$\int \operatorname{sech}^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(192*e**(16*c + 16*d*x)*atan(e**(c + d*x))*a**3 + 144*e**(16*c + 16*d*x)*a
tan(e**(c + d*x))*a**2*b + 72*e**(16*c + 16*d*x)*atan(e**(c + d*x))*a*b**2
 + 15*e**(16*c + 16*d*x)*atan(e**(c + d*x))*b**3 + 1536*e**(14*c + 14*d*x)
*atan(e**(c + d*x))*a**3 + 1152*e**(14*c + 14*d*x)*atan(e**(c + d*x))*a**2
*b + 576*e**(14*c + 14*d*x)*atan(e**(c + d*x))*a*b**2 + 120*e**(14*c + 14*
d*x)*atan(e**(c + d*x))*b**3 + 5376*e**(12*c + 12*d*x)*atan(e**(c + d*x))*
a**3 + 4032*e**(12*c + 12*d*x)*atan(e**(c + d*x))*a**2*b + 2016*e**(12*c +
 12*d*x)*atan(e**(c + d*x))*a*b**2 + 420*e**(12*c + 12*d*x)*atan(e**(c + d
*x))*b**3 + 10752*e**(10*c + 10*d*x)*atan(e**(c + d*x))*a**3 + 8064*e**(10
*c + 10*d*x)*atan(e**(c + d*x))*a**2*b + 4032*e**(10*c + 10*d*x)*atan(e**(
c + d*x))*a*b**2 + 840*e**(10*c + 10*d*x)*atan(e**(c + d*x))*b**3 + 13440*
e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3 + 10080*e**(8*c + 8*d*x)*atan(e**
(c + d*x))*a**2*b + 5040*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a*b**2 + 1050
*e**(8*c + 8*d*x)*atan(e**(c + d*x))*b**3 + 10752*e**(6*c + 6*d*x)*atan(e*
*(c + d*x))*a**3 + 8064*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**2*b + 4032*
e**(6*c + 6*d*x)*atan(e**(c + d*x))*a*b**2 + 840*e**(6*c + 6*d*x)*atan(e**
(c + d*x))*b**3 + 5376*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3 + 4032*e**
(4*c + 4*d*x)*atan(e**(c + d*x))*a**2*b + 2016*e**(4*c + 4*d*x)*atan(e**(c
 + d*x))*a*b**2 + 420*e**(4*c + 4*d*x)*atan(e**(c + d*x))*b**3 + 1536*e**(
2*c + 2*d*x)*atan(e**(c + d*x))*a**3 + 1152*e**(2*c + 2*d*x)*atan(e**(c...
```

3.104 $\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-3b)\tanh^3(c+dx)}{3d}-\frac{3a(a-b)b\tanh^5(c+dx)}{5d}-\frac{(3a-b)b^2\tanh^7(c+dx)}{7d}-\frac{b^3\tanh^9(c+dx)}{9d}$$

output
```
a^3*tanh(d*x+c)/d-1/3*a^2*(a-3*b)*tanh(d*x+c)^3/d-3/5*a*(a-b)*b*tanh(d*x+c
)^5/d-1/7*(3*a-b)*b^2*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x+c)^9/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 218 vs. 2(102) = 204.

Time = 2.06 (sec) , antiderivative size = 218, normalized size of antiderivative = 2.14

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{(5775a^3-1071a^2b+621ab^2-725b^3+10(903a^3-63a^2b-27ab^2+107b^3)\cosh(2(c+dx))+8(525a^3+}{}$$

input
```
Integrate[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((5775*a^3 - 1071*a^2*b + 621*a*b^2 - 725*b^3 + 10*(903*a^3 - 63*a^2*b - 2
7*a*b^2 + 107*b^3)*Cosh[2*(c + d*x)] + 8*(525*a^3 + 126*a^2*b - 81*a*b^2 -
 50*b^3)*Cosh[4*(c + d*x)] + 1050*a^3*Cosh[6*(c + d*x)] + 630*a^2*b*Cosh[6
*(c + d*x)] + 270*a*b^2*Cosh[6*(c + d*x)] + 50*b^3*Cosh[6*(c + d*x)] + 105
*a^3*Cosh[8*(c + d*x)] + 63*a^2*b*Cosh[8*(c + d*x)] + 27*a*b^2*Cosh[8*(c +
 d*x)] + 5*b^3*Cosh[8*(c + d*x)])*Sech[c + d*x]^8*Tanh[c + d*x])/(20160*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 290, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \operatorname{sech}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sec(ic+idx)^4 \left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \left(1-\tanh^2(c+dx)\right) \left(b\tanh^2(c+dx)+a\right)^3 d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{290}$$

$$\frac{\int \left(-b^3\tanh^8(c+dx)-(3a-b)b^2\tanh^6(c+dx)-3a(a-b)b\tanh^4(c+dx)-a^2(a-3b)\tanh^2(c+dx)+a^3\right) d}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\tanh(c+dx)-\frac{1}{3}a^2(a-3b)\tanh^3(c+dx)-\frac{1}{7}b^2(3a-b)\tanh^7(c+dx)-\frac{3}{5}ab(a-b)\tanh^5(c+dx)-\frac{1}{9}b^3\tan}{d}$$

input
```
Int[Sech[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Tanh[c + d*x] - (a^2*(a - 3*b)*Tanh[c + d*x]^3)/3 - (3*a*(a - b)*b*Ta
nh[c + d*x]^5)/5 - ((3*a - b)*b^2*Tanh[c + d*x]^7)/7 - (b^3*Tanh[c + d*x]^
9)/9)/d
```

Defintions of rubi rules used

rule 290
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol] :> I
nt[ExpandIntegrand[(a + b*x^2)^p*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d
}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 268 vs. $2(94) = 188$.

Time = 152.41 (sec) , antiderivative size = 269, normalized size of antiderivative = 2.64

method	result
derivativedivides	$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+3b^2a\left(-\right.}{}$
default	$\frac{a^3\left(\frac{2}{3}+\frac{\operatorname{sech}(dx+c)^2}{3}\right)\tanh(dx+c)+3a^2b\left(-\frac{\sinh(dx+c)}{4\cosh(dx+c)^5}+\frac{\left(\frac{8}{15}+\frac{\operatorname{sech}(dx+c)^4}{5}+\frac{4\operatorname{sech}(dx+c)^2}{15}\right)\tanh(dx+c)}{4}\right)+3b^2a\left(-\right.}{}$
risch	$-\frac{4\left(-945b^3e^{8dx+8c}+945b^3e^{6dx+6c}-135b^3e^{4dx+4c}+945\,e^{2dx+2c}a^3+27b^2a+105a^3+63a^2b+3465a^2b\,e^{12dx+12c}+4725\right.}{}$

input
```
int(sech(d*x+c)^4*(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/4*sinh(d*x+c)/cos
h(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+3*
b^2*a*(-1/4*sinh(d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*
(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c
))+b^3*(-1/4*sinh(d*x+c)^5/cosh(d*x+c)^9-5/24*sinh(d*x+c)^3/cosh(d*x+c)^9-
5/64*sinh(d*x+c)/cosh(d*x+c)^9+5/64*(128/315+1/9*sech(d*x+c)^8+8/63*sech(d
*x+c)^6+16/105*sech(d*x+c)^4+64/315*sech(d*x+c)^2)*tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1185 vs. $2(94) = 188$.

Time = 0.09 (sec) , antiderivative size = 1185, normalized size of antiderivative = 11.62

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-8/315*(2*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^7 + 14*
(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 +
 (105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*sinh(d*x + c)^7 + 6*(245*a^3
+ 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^5 + 3*(175*a^3 + 483*a^2*b
+ 117*a*b^2 - 95*b^3 + 7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(105*a^3 + 252*a^2*b + 243*a*b^2 + 80*
b^3)*cosh(d*x + c)^3 + 3*(245*a^3 + 336*a^2*b + 99*a*b^2 - 40*b^3)*cosh(d*
x + c))*sinh(d*x + c)^4 + 18*(245*a^3 + 168*a^2*b + 27*a*b^2 + 40*b^3)*cos
h(d*x + c)^3 + (35*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x +
c)^4 + 945*a^3 + 1701*a^2*b + 459*a*b^2 + 855*b^3 + 30*(175*a^3 + 483*a^2*
b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 6*(7*(105*a^3 +
 252*a^2*b + 243*a*b^2 + 80*b^3)*cosh(d*x + c)^5 + 10*(245*a^3 + 336*a^2*b
 + 99*a*b^2 - 40*b^3)*cosh(d*x + c)^3 + 9*(245*a^3 + 168*a^2*b + 27*a*b^2
+ 40*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 210*(35*a^3 + 12*a^2*b + 9*a*b^
2)*cosh(d*x + c) + (7*(105*a^3 + 441*a^2*b + 459*a*b^2 + 155*b^3)*cosh(d*x
 + c)^6 + 15*(175*a^3 + 483*a^2*b + 117*a*b^2 - 95*b^3)*cosh(d*x + c)^4 +
525*a^3 + 693*a^2*b + 567*a*b^2 - 945*b^3 + 27*(105*a^3 + 189*a^2*b + 51*a
*b^2 + 95*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*
cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 + 9*d*cosh(d*x + c)^9
+ (55*d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^...
```

Sympy [F]

$$\int \operatorname{sech}^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \operatorname{sech}^4\left(c+dx\right) dx$$

input
```
integrate(sech(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*sech(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1847 vs. $2(94) = 188$.

Time = 0.07 (sec) , antiderivative size = 1847, normalized size of antiderivative = 18.11

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
4/63*b^3*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c)
+ 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84
*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-1
8*d*x - 18*c) + 1)) - 27*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-
4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x
 - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x -
16*c) + e^(-18*d*x - 18*c) + 1)) + 189*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x -
2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) +
126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9
*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) - 189*e^(-8*d*x - 8*c)/(d*(
9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) + 126*e^(-8
*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*c) + 36*e^(-14*d
*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1)) + 315*e^(-10*
d*x - 10*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x -
6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d*x - 12*
c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) + 1
)) - 105*e^(-12*d*x - 12*c)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) +
 84*e^(-6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*
e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18
*d*x - 18*c) + 1)) + 63*e^(-14*d*x - 14*c)/(d*(9*e^(-2*d*x - 2*c) + 36*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(94) = 188$.

Time = 0.23 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.38

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{4\left(315\,a^3e^{(14\,dx+14\,c)}+945\,a^2be^{(14\,dx+14\,c)}+945\,ab^2e^{(14\,dx+14\,c)}+315\,b^3e^{(14\,dx+14\,c)}+1995\,a^3e^{(12\,dx+12\,c)}\right.}{}$$

input
```
integrate(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-4/315*(315*a^3*e^(14*d*x + 14*c) + 945*a^2*b*e^(14*d*x + 14*c) + 945*a*b^
2*e^(14*d*x + 14*c) + 315*b^3*e^(14*d*x + 14*c) + 1995*a^3*e^(12*d*x + 12*
c) + 3465*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) - 525*b^3*
e^(12*d*x + 12*c) + 5355*a^3*e^(10*d*x + 10*c) + 4725*a^2*b*e^(10*d*x + 10
*c) + 945*a*b^2*e^(10*d*x + 10*c) + 1575*b^3*e^(10*d*x + 10*c) + 7875*a^3*
e^(8*d*x + 8*c) + 3213*a^2*b*e^(8*d*x + 8*c) + 2457*a*b^2*e^(8*d*x + 8*c)
- 945*b^3*e^(8*d*x + 8*c) + 6825*a^3*e^(6*d*x + 6*c) + 1827*a^2*b*e^(6*d*x
 + 6*c) + 1323*a*b^2*e^(6*d*x + 6*c) + 945*b^3*e^(6*d*x + 6*c) + 3465*a^3*
e^(4*d*x + 4*c) + 1323*a^2*b*e^(4*d*x + 4*c) + 27*a*b^2*e^(4*d*x + 4*c) -
135*b^3*e^(4*d*x + 4*c) + 945*a^3*e^(2*d*x + 2*c) + 567*a^2*b*e^(2*d*x + 2
*c) + 243*a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*x + 2*c) + 105*a^3 + 63*a^
2*b + 27*a*b^2 + 5*b^3)/(d*(e^(2*d*x + 2*c) + 1)^9)
```

Mupad [B] (verification not implemented)

Time = 2.44 (sec) , antiderivative size = 1424, normalized size of antiderivative = 13.96

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int((a + b*tanh(c + d*x)^2)^3/cosh(c + d*x)^4,x)
```

output
```
- ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(2*c + 2*d*x)*(a + b)^3)/(9*d))/(3
*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) - ((5*exp(8
*c + 8*d*x)*(a + b)^3)/(9*d) - (a*b^2 + a^2*b - 5*a^3 - 5*b^3)/(21*d) - (1
0*exp(4*c + 4*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(2*c +
 2*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(63*d) + (40*exp(6*c + 6*d*x)
*(a + b)^2*(a - b))/(21*d))/(6*exp(2*c + 2*d*x) + 15*exp(4*c + 4*d*x) + 20
*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) + 6*exp(10*c + 10*d*x) + exp(12*c
+ 12*d*x) + 1) - ((4*(a + b)^2*(a - b))/(21*d) + (2*exp(10*c + 10*d*x)*(a
+ b)^3)/(3*d) - (2*exp(2*c + 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(7*d)
 - (20*exp(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (16*exp(
4*c + 4*d*x)*(3*a*b^2 - 3*a^2*b + 5*a^3 - 5*b^3))/(21*d) + (20*exp(8*c + 8
*d*x)*(a + b)^2*(a - b))/(7*d))/(7*exp(2*c + 2*d*x) + 21*exp(4*c + 4*d*x)
+ 35*exp(6*c + 6*d*x) + 35*exp(8*c + 8*d*x) + 21*exp(10*c + 10*d*x) + 7*ex
p(12*c + 12*d*x) + exp(14*c + 14*d*x) + 1) - ((16*(3*a*b^2 - 3*a^2*b + 5*a
^3 - 5*b^3))/(315*d) + (4*exp(6*c + 6*d*x)*(a + b)^3)/(9*d) - (4*exp(2*c +
 2*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(21*d) + (8*exp(4*c + 4*d*x)*(a +
 b)^2*(a - b))/(7*d))/(5*exp(2*c + 2*d*x) + 10*exp(4*c + 4*d*x) + 10*exp(6
*c + 6*d*x) + 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) + 1) - ((8*exp(2*c +
 2*d*x)*(a + b)^3)/(9*d) + (8*exp(14*c + 14*d*x)*(a + b)^3)/(9*d) - (8*exp
(6*c + 6*d*x)*(a*b^2 + a^2*b - 5*a^3 - 5*b^3))/(3*d) - (8*exp(10*c + 10...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 572, normalized size of antiderivative = 5.61

$$\int \operatorname{sech}^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$= \frac{-\frac{12e^{4dx+4c}a\,b^2}{35} - \frac{4b^3}{63} - 12e^{14dx+14c}a^2b - 12e^{14dx+14c}a\,b^2 - 60e^{10dx+10c}a^2b - 12e^{10dx+10c}a\,b^2 - \frac{204e^{8dx+8c}a^2b}{5} - \cdots}{}$$

input
```
int(sech(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(4*( - 315*e**(14*c + 14*d*x)*a**3 - 945*e**(14*c + 14*d*x)*a**2*b - 945*e
**(14*c + 14*d*x)*a*b**2 - 315*e**(14*c + 14*d*x)*b**3 - 1995*e**(12*c + 1
2*d*x)*a**3 - 3465*e**(12*c + 12*d*x)*a**2*b - 945*e**(12*c + 12*d*x)*a*b*
*2 + 525*e**(12*c + 12*d*x)*b**3 - 5355*e**(10*c + 10*d*x)*a**3 - 4725*e**
(10*c + 10*d*x)*a**2*b - 945*e**(10*c + 10*d*x)*a*b**2 - 1575*e**(10*c + 1
0*d*x)*b**3 - 7875*e**(8*c + 8*d*x)*a**3 - 3213*e**(8*c + 8*d*x)*a**2*b -
2457*e**(8*c + 8*d*x)*a*b**2 + 945*e**(8*c + 8*d*x)*b**3 - 6825*e**(6*c +
6*d*x)*a**3 - 1827*e**(6*c + 6*d*x)*a**2*b - 1323*e**(6*c + 6*d*x)*a*b**2
- 945*e**(6*c + 6*d*x)*b**3 - 3465*e**(4*c + 4*d*x)*a**3 - 1323*e**(4*c +
4*d*x)*a**2*b - 27*e**(4*c + 4*d*x)*a*b**2 + 135*e**(4*c + 4*d*x)*b**3 - 9
45*e**(2*c + 2*d*x)*a**3 - 567*e**(2*c + 2*d*x)*a**2*b - 243*e**(2*c + 2*d
*x)*a*b**2 - 45*e**(2*c + 2*d*x)*b**3 - 105*a**3 - 63*a**2*b - 27*a*b**2 -
 5*b**3))/(315*d*(e**(18*c + 18*d*x) + 9*e**(16*c + 16*d*x) + 36*e**(14*c
+ 14*d*x) + 84*e**(12*c + 12*d*x) + 126*e**(10*c + 10*d*x) + 126*e**(8*c +
 8*d*x) + 84*e**(6*c + 6*d*x) + 36*e**(4*c + 4*d*x) + 9*e**(2*c + 2*d*x) +
 1))
```

3.105 $\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 120

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2+10ab+15b^2)\,x}{8(a+b)^3} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3d} + \frac{(3a+7b)\cosh(c+dx)\sinh(c+dx)}{8(a+b)^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4(a+b)d}$$

output
```
1/8*(3*a^2+10*a*b+15*b^2)*x/(a+b)^3+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(
1/2))/a^(1/2)/(a+b)^3/d+1/8*(3*a+7*b)*cosh(d*x+c)*sinh(d*x+c)/(a+b)^2/d+1/
4*cosh(d*x+c)^3*sinh(d*x+c)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.96

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(3a^2+10ab+15b^2)(c+dx)}{8(a+b)^3d} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^3d} + \frac{(a+2b)\sinh(2(c+dx))}{4(a+b)^2d} + \frac{\sinh(4(c+dx))}{32(a+b)d}$$

input
```
Integrate[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((3*a^2 + 10*a*b + 15*b^2)*(c + d*x))/(8*(a + b)^3*d) + (b^(5/2)*ArcTan[(S
qrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3*d) + ((a + 2*b)*Sinh[2*
(c + d*x)])/(4*(a + b)^2*d) + Sinh[4*(c + d*x)]/(32*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.28, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 4158, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^3\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{3b\tanh^2(c+dx)+3a+4b}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{\int \frac{3a^2+7ba+8b^2+b(3a+7b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}+\frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{8b^3\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2(a+b)}+\frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{\frac{\left(3a^2+10ab+15b^2\right)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 219$$

$$\frac{\frac{\frac{\frac{\left(3a^2+10ab+15b^2\right)\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{8b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{(3a+7b)\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{4(a+b)}+\frac{\tanh(c+dx)}{4(a+b)\left(1-\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Tanh[c + d*x]/(4*(a + b)*(1 - Tanh[c + d*x]^2)^2) + (((8*b^(5/2)*ArcTan[(
Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ((3*a^2 + 10*a*b + 15
*b^2)*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*(a + b)) + ((3*a + 7*b)*Tanh[c +
 d*x])/(2*(a + b)*(1 - Tanh[c + d*x]^2)))/(4*(a + b)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(106) = 212$.

Time = 21.67 (sec) , antiderivative size = 273, normalized size of antiderivative = 2.28

method	result
risch	$\frac{3x\,a^2}{8(a+b)^3}+\frac{5xab}{4(a+b)^3}+\frac{15x\,b^2}{8(a+b)^3}+\frac{\mathrm{e}^{4dx+4c}}{64(a+b)d}+\frac{a\,\mathrm{e}^{2dx+2c}}{8(a+b)^2d}+\frac{\mathrm{e}^{2dx+2c}b}{4(a+b)^2d}-\frac{\mathrm{e}^{-2dx-2c}a}{8(a^2+2ab+b^2)d}-\frac{\mathrm{e}^{-2dx-2c}b}{4(a^2+2ab+b^2)}$
derivativedivides	$-\frac{2b^3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{(a+b)^3}-\frac{}{2(2b+2a}$
default	$-\frac{2b^3a\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(-a+\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{(a+b)^3}-\frac{}{2(2b+2a}$

input
```
int(cosh(d*x+c)^4/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
3/8*x/(a+b)^3*a^2+5/4*x/(a+b)^3*a*b+15/8*x/(a+b)^3*b^2+1/64/(a+b)/d*exp(4*
d*x+4*c)+1/8*a/(a+b)^2/d*exp(2*d*x+2*c)+1/4/(a+b)^2/d*exp(2*d*x+2*c)*b-1/8
/(a^2+2*a*b+b^2)/d*exp(-2*d*x-2*c)*a-1/4/(a^2+2*a*b+b^2)/d*exp(-2*d*x-2*c)
*b-1/64/(a+b)/d*exp(-4*d*x-4*c)+1/2/a*(-a*b)^(1/2)*b^2/(a+b)^3/d*ln(exp(2*
d*x+2*c)+(2*(-a*b)^(1/2)+a-b)/(a+b))-1/2/a*(-a*b)^(1/2)*b^2/(a+b)^3/d*ln(e
xp(2*d*x+2*c)-(2*(-a*b)^(1/2)-a+b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 929 vs. $2(106) = 212$.

Time = 0.13 (sec) , antiderivative size = 2180, normalized size of antiderivative = 18.17

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/64*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 8*(3*a^2 +
10*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 + 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x +
c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + 6*a*b + 4*b^2)*s
inh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 6*(a^2 + 3*a*b
 + 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(3*a^2 + 10*a*b + 15*b^2)*d*x + 60*(a^2 + 3*a*b + 2*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
+ 4*(3*a^2 + 10*a*b + 15*b^2)*d*x*cosh(d*x + c) + 20*(a^2 + 3*a*b + 2*b^2)
*cosh(d*x + c)^3)*sinh(d*x + c)^3 - 8*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^
2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 12*(3*a^2 + 10*a*b + 15*b^2
)*d*x*cosh(d*x + c)^2 + 30*(a^2 + 3*a*b + 2*b^2)*cosh(d*x + c)^4 - 2*a^2 -
 6*a*b - 4*b^2)*sinh(d*x + c)^2 + 32*(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*b^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^2*cosh
(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4)*sqrt(-b/a)*log(((a^2 + 2*
a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)
^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2
 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2
)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^...
```

Sympy [F]

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(106) = 212$.

Time = 0.18 (sec) , antiderivative size = 514, normalized size of antiderivative = 4.28

$$\begin{aligned}\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{(ab-b^2)(dx+c)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d} \\
&+\frac{\left(8\,be^{(-2\,dx-2\,c)}+a+b\right)e^{(4\,dx+4\,c)}}{64\,(a^2+2\,ab+b^2)d} \\
&+\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\
&-\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\
&-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\
&-\frac{(a^2b-6\,ab^2+b^3)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}d} \\
&+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\
&-\frac{3\,b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}(a+b)d} \\
&-\frac{8\,be^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}}{64\,(a^2+2\,ab+b^2)d} \\
&+\frac{3\,(dx+c)}{8\,(a+b)d}+\frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d}-\frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*(a*b - b^2)*(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 1/64*(8*b
*e^(-2*d*x - 2*c) + a + b)*e^(4*d*x + 4*c)/((a^2 + 2*a*b + b^2)*d) + 1/4*b
*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 +
2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x
 - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2*((a
+ b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d)
 - 1/8*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a -
b)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*d) + 1/4*(a*b - b
^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b
 + b^2)*sqrt(a*b)*d) - 3/8*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)
/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) - 1/64*(8*b*e^(-2*d*x - 2*c) + (a + b)*e
^(-4*d*x - 4*c))/((a^2 + 2*a*b + b^2)*d) + 3/8*(d*x + c)/((a + b)*d) + 1/8
*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 301 vs. $2(106) = 212$.

Time = 0.95 (sec) , antiderivative size = 301, normalized size of antiderivative = 2.51

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{64\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+3\,a^2b+3\,ab^2+b^3)\sqrt{ab}} + \frac{8\,(3\,a^2+10\,ab+15\,b^2)(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{(18\,a^2e^{(4\,dx+4\,c)}+60\,abe^{(4\,dx+4\,c)}+90\,b^2e^{(4\,dx+4\,c)}+8\,a^2e^{(2}}{a^3+3}}{64\,d}$$

input
```
integrate(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/64*(64*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sq
rt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)) + 8*(3*a^2 + 10*a*b +
 15*b^2)*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (18*a^2*e^(4*d*x + 4*
c) + 60*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*
c) + 24*a*b*e^(2*d*x + 2*c) + 16*b^2*e^(2*d*x + 2*c) + a^2 + 2*a*b + b^2)*
e^(-4*d*x - 4*c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (a*e^(4*d*x + 4*c) + b*
e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + 16*b*e^(2*d*x + 2*c))/(a^2 + 2*a*b
 + b^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.12 (sec) , antiderivative size = 967, normalized size of antiderivative = 8.06

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cosh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output
```
(x*(10*a*b + 3*a^2 + 15*b^2))/(8*(a + b)^3) - exp(- 4*c - 4*d*x)/(64*d*(a
+ b)) + exp(4*c + 4*d*x)/(64*d*(a + b)) + (atan((exp(2*c)*exp(2*d*x)*((4*b
^3)/(d*(a + b)^5*(b^5)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + ((a - b)*(
a^4*d*(b^5)^(1/2) - b^4*d*(b^5)^(1/2) - 2*a*b^3*d*(b^5)^(1/2) + 2*a^3*b*d*
(b^5)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2 + 3*a^2*b +
a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4
*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1
/2) + b^4*d*(b^5)^(1/2) + 4*a*b^3*d*(b^5)^(1/2) + 4*a^3*b*d*(b^5)^(1/2) +
6*a^2*b^2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a*d^2*(a + b)^6)^(1/2)*(3*a*b^2
+ 3*a^2*b + a^3 + b^3)*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2
+ 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*((a^4*(a^7*d^2
 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d
^2 + 15*a^5*b^2*d^2)^(1/2))/2 + (b^4*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 +
6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2))/2
 + 2*a*b^3*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4
*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2) + 2*a^3*b*(a^7*d^2 + a*b^6*d
^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^
5*b^2*d^2)^(1/2) + 3*a^2*b^2*(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^
5*d^2 + 15*a^3*b^4*d^2 + 20*a^4*b^3*d^2 + 15*a^5*b^2*d^2)^(1/2)))*(b^5)^(1
/2))/(a^7*d^2 + a*b^6*d^2 + 6*a^6*b*d^2 + 6*a^2*b^5*d^2 + 15*a^3*b^4*d^...
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 333, normalized size of antiderivative = 2.78

$$\int \frac{\cosh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{64e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b^2-64e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)b^2+e^{8dx+8c}a^3+2e^{8dx+8c}\cdots}{}$$

input
```
int(cosh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

output
```
(64*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*b**2 - 64*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x
)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + e**(8*c + 8*d*x)*a**3 + 2*e**(8*c
 + 8*d*x)*a**2*b + e**(8*c + 8*d*x)*a*b**2 + 8*e**(6*c + 6*d*x)*a**3 + 24*
e**(6*c + 6*d*x)*a**2*b + 16*e**(6*c + 6*d*x)*a*b**2 + 24*e**(4*c + 4*d*x)
*a**3*d*x + 80*e**(4*c + 4*d*x)*a**2*b*d*x + 120*e**(4*c + 4*d*x)*a*b**2*d
*x - 8*e**(2*c + 2*d*x)*a**3 - 24*e**(2*c + 2*d*x)*a**2*b - 16*e**(2*c + 2
*d*x)*a*b**2 - a**3 - 2*a**2*b - a*b**2)/(64*e**(4*c + 4*d*x)*a*d*(a**3 +
3*a**2*b + 3*a*b**2 + b**3))
```

3.106 $\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 80

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}d} + \frac{(a+2b)\sinh(c+dx)}{(a+b)^2d} + \frac{\sinh^3(c+dx)}{3(a+b)d}$$

output
```
b^2*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/(a+b)^(5/2)/d+(a+2*b)*
sinh(d*x+c)/(a+b)^2/d+1/3*sinh(d*x+c)^3/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.99

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\frac{12b^2\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}(a+b)^{5/2}} + \frac{3(3a+7b)\sinh(c+dx)}{(a+b)^2} + \frac{\sinh(3(c+dx))}{a+b}}{12d}$$

input
```
Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-12*b^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(Sqrt[a]*(a + b)^(5
/2)) + (3*(3*a + 7*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[3*(c + d*x)]/(a + b)
)/(12*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{b^2}{(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh^2(c+dx)}{a+b}+\frac{a+2b}{(a+b)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{5/2}}+\frac{\sinh^3(c+dx)}{3(a+b)}+\frac{(a+2b)\sinh(c+dx)}{(a+b)^2}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((b^2*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2))
 + ((a + 2*b)*Sinh[c + d*x])/(a + b)^2 + Sinh[c + d*x]^3/(3*(a + b)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 229 vs. $2(70) = 140$.

Time = 7.81 (sec) , antiderivative size = 230, normalized size of antiderivative = 2.88

method	result
risch	$\frac{e^{3dx+3c}}{24(a+b)d}+\frac{3\,e^{dx+c}a}{8(a+b)^2 d}+\frac{7\,e^{dx+c}b}{8(a+b)^2 d}-\frac{3\,e^{-dx-c}a}{8(a+b)^2 d}-\frac{7\,e^{-dx-c}b}{8(a+b)^2 d}-\frac{e^{-3dx-3c}}{24(a+b)d}-\frac{b^2\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)^2 d}$
derivativedivides	$\frac{-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(2b+2a)}-\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+2b}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(2b+2a)}+\frac{}{(2}}{}$
default	$\frac{-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3(2b+2a)}-\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a+2b}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^3(2b+2a)}+\frac{}{(2}}{}$

input
```
int(cosh(d*x+c)^3/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/24/(a+b)/d*exp(3*d*x+3*c)+3/8/(a+b)^2/d*exp(d*x+c)*a+7/8/(a+b)^2/d*exp(d
*x+c)*b-3/8/(a+b)^2/d*exp(-d*x-c)*a-7/8/(a+b)^2/d*exp(-d*x-c)*b-1/24/(a+b)
/d*exp(-3*d*x-3*c)-1/2/(-a^2-a*b)^(1/2)*b^2/(a+b)^2/d*ln(exp(2*d*x+2*c)-2*
a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/2/(-a^2-a*b)^(1/2)*b^2/(a+b)^2/d*ln(exp
(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 867 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 1857, normalized size of antiderivative = 23.21

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/24*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^6 + 6*(a^3 + 2*a^2*b + a*b^2)
*cosh(d*x + c)*sinh(d*x + c)^5 + (a^3 + 2*a^2*b + a*b^2)*sinh(d*x + c)^6 +
 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x + c)^4 + 3*(3*a^3 + 10*a^2*b + 7*
a*b^2 + 5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*
(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^3 + 3*(3*a^3 + 10*a^2*b + 7*a*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 - a^3 - 2*a^2*b - a*b^2 - 3*(3*a^3 + 10*a^2*
b + 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*(a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^
4 - 3*a^3 - 10*a^2*b - 7*a*b^2 + 6*(3*a^3 + 10*a^2*b + 7*a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^2 - 12*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)^2*s
inh(d*x + c) + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 + b^2*sinh(d*x + c)^3)*
sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*
(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*
x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3
*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)
*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*
x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)
^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sin
h(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*
x + c) + a + b)) + 6*((a^3 + 2*a^2*b + a*b^2)*cosh(d*x + c)^5 + 2*(3*a^...
```

Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^3{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

input
```
integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)^3}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/24*((a*e^(6*c) + b*e^(6*c))*e^(6*d*x) + 3*(3*a*e^(4*c) + 7*b*e^(4*c))*e^
(4*d*x) - 3*(3*a*e^(2*c) + 7*b*e^(2*c))*e^(2*d*x) - a - b)*e^(-3*d*x)/(a^2
*d*e^(3*c) + 2*a*b*d*e^(3*c) + b^2*d*e^(3*c)) + 1/8*integrate(16*(b^2*e^(3
*d*x + 3*c) + b^2*e^(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3 + (a^3*e^(4*
c) + 3*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(a^3*e
^(2*c) + a^2*b*e^(2*c) - a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.95 (sec) , antiderivative size = 2194, normalized size of antiderivative = 27.42

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(3*c + 3*d*x)/(24*d*(a + b)) - exp(- 3*c - 3*d*x)/(24*d*(a + b)) + ((b^
4)^(1/2)*(2*atan((exp(d*x)*exp(c)*((4*(10*a^2*d*(b^4)^(5/2) + 12*a^6*d*(b^
4)^(3/2) + 2*a*b^9*d*(b^4)^(1/2) + 10*a^3*b^3*d*(b^4)^(3/2) + 2*a^2*b^8*d*
(b^4)^(1/2) + 20*a^3*b^7*d*(b^4)^(1/2) + 40*a^4*b^6*d*(b^4)^(1/2) + 30*a^5
*b^5*d*(b^4)^(1/2) + 2*a^7*b^3*d*(b^4)^(1/2)))/(a*b^5*(a + b)^5*(a*d^2*(a
+ b)^5)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5*a^4
*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d
^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)) - (2*(b^9*(a^
6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*
b^2*d^2)^(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^
2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 6*a^2*b^7*(a^6*d^2 + a*b^5*d^
2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) +
 4*a^3*b^6*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3
*d^2 + 10*a^4*b^2*d^2)^(1/2) + a^4*b^5*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2
+ 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2)))/(a^2*b^3*d*(a +
 b)^7*(b^4)^(1/2)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)*(5*a*b^4 + 5
*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2)*(a^6*d^2 + a*b^5*d^2 + 5*a^5
*b*d^2 + 5*a^2*b^4*d^2 + 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2))) + (2*exp
(3*c)*exp(3*d*x)*(b^9*(a^6*d^2 + a*b^5*d^2 + 5*a^5*b*d^2 + 5*a^2*b^4*d^2 +
 10*a^3*b^3*d^2 + 10*a^4*b^2*d^2)^(1/2) + 4*a*b^8*(a^6*d^2 + a*b^5*d^2 ...
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 284, normalized size of antiderivative = 3.55

$$\int \frac{\cosh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$= \frac{24e^{3dx+3c}\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b^2 + 24e^{3dx+3c}\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)b^2 + e^{6dx+6c}a^3 + \cdots}{24e^{3}\cdots}$$

input
```
int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

output
```
(24*e**(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*b**2 + 24*e**(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((e**
(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + e**(6*c + 6*d*x)*a**3 + 2
*e**(6*c + 6*d*x)*a**2*b + e**(6*c + 6*d*x)*a*b**2 + 9*e**(4*c + 4*d*x)*a*
*3 + 30*e**(4*c + 4*d*x)*a**2*b + 21*e**(4*c + 4*d*x)*a*b**2 - 9*e**(2*c +
 2*d*x)*a**3 - 30*e**(2*c + 2*d*x)*a**2*b - 21*e**(2*c + 2*d*x)*a*b**2 - a
**3 - 2*a**2*b - a*b**2)/(24*e**(3*c + 3*d*x)*a*d*(a**3 + 3*a**2*b + 3*a*b
**2 + b**3))
```

3.107 $\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+3b)x}{2(a+b)^2} + \frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d}$$

output
```
1/2*(a+3*b)*x/(a+b)^2+b^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/
(a+b)^2/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\sqrt{a}(a+3b)(c+dx)+4b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)+\sqrt{a}(a+b)\sinh(2(c+dx))}{4\sqrt{a}(a+b)^2d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*Sqrt[a]*(a + 3*b)*(c + d*x) + 4*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a]] + Sqrt[a]*(a + b)*Sinh[2*(c + d*x)])/(4*Sqrt[a]*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.26, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4158, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2b^2\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{(a+3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{(a+3b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{2b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{(a+3b)\text{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)(1-\tanh^2(c+dx))}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((2*b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) +
((a + 3*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*(a + b)) + Tanh[c + d*x]/(2
*(a + b)*(1 - Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 158 vs. $2(65) = 130$.

Time = 2.75 (sec) , antiderivative size = 159, normalized size of antiderivative = 2.06

method	result
risch	$\frac{ax}{2(a+b)^2}+\frac{3xb}{2(a+b)^2}+\frac{e^{2dx+2c}}{8(a+b)d}-\frac{e^{-2dx-2c}}{8(a+b)d}+\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}+\frac{2\sqrt{-ab}+a-b}{a+b}\right)}{2a(a+b)^2d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}-2\sqrt{\cdots}\right.}{2a(a+b)^2d}$
derivativedivides	$\frac{\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-a-3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\cdots}{(4a\cdots}}{}$
default	$\frac{\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{2}{(4a+4b)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-a-3b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^2}-\frac{1}{(2b+2a)\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{\cdots}{(4a\cdots}}{}$

input
```
int(cosh(d*x+c)^2/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/2*a*x/(a+b)^2+3/2*x/(a+b)^2*b+1/8/(a+b)/d*exp(2*d*x+2*c)-1/8/(a+b)/d*exp
(-2*d*x-2*c)+1/2/a*(-a*b)^(1/2)*b/(a+b)^2/d*ln(exp(2*d*x+2*c)+(2*(-a*b)^(1
/2)+a-b)/(a+b))-1/2/a*(-a*b)^(1/2)*b/(a+b)^2/d*ln(exp(2*d*x+2*c)-(2*(-a*b)
^(1/2)-a+b)/(a+b))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(65) = 130$.

Time = 0.16 (sec) , antiderivative size = 948, normalized size of antiderivative = 12.31

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/8*(4*(a + 3*b)*d*x*cosh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)
*d*x + 3*(a + b)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^2 +
 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2)*sqrt(-b/a)*log(((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x
 + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2
- b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(
a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2
 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*
(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c
)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 4*(2*(a + 3*b)*d*x*
cosh(d*x + c) + (a + b)*cosh(d*x + c)^3)*sinh(d*x + c) - a - b)/((a^2 + 2*
a*b + b^2)*d*cosh(d*x + c)^2 + 2*(a^2 + 2*a*b + b^2)*d*cosh(d*x + c)*sinh(
d*x + c) + (a^2 + 2*a*b + b^2)*d*sinh(d*x + c)^2), 1/8*(4*(a + 3*b)*d*x*co
sh(d*x + c)^2 + (a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(2*(a + 3*b)*d*x + 3*(a + b)*cosh(d*
x + c)^2)*sinh(d*x + c)^2 + 8*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si...
```

Sympy [F]

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 316 vs. $2(65) = 130$.

Time = 0.17 (sec) , antiderivative size = 316, normalized size of antiderivative = 4.10

$$\begin{aligned}\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx &= \frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &- \frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+2\,ab+b^2)d} \\ &- \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &+ \frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+2\,ab+b^2)\sqrt{ab}d} \\ &- \frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}(a+b)d} \\ &+ \frac{dx+c}{2\,(a+b)d} + \frac{e^{(2\,dx+2\,c)}}{8\,(a+b)d} - \frac{e^{(-2\,dx-2\,c)}}{8\,(a+b)d}\end{aligned}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^2 + 2*a*b + b^2)*d) - 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-
4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/4*(a*b - b^2)*arctan(1/2
*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*
b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt
(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)*d) - 1/2*b*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/2*(d*x + c)/((a
+ b)*d) + 1/8*e^(2*d*x + 2*c)/((a + b)*d) - 1/8*e^(-2*d*x - 2*c)/((a + b)*
d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 155 vs. $2(65) = 130$.

Time = 0.60 (sec) , antiderivative size = 155, normalized size of antiderivative = 2.01

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{8\,b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}}+\frac{4\,(dx+c)(a+3\,b)}{a^2+2\,ab+b^2}-\frac{\left(2\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}}{a^2+2\,ab+b^2}+\frac{e^{(2\,dx+2\,c)}}{a+b}}{8\,d}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/8*(8*b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt
(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 4*(d*x + c)*(a + 3*b)/(a^2 + 2*a*
b + b^2) - (2*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x -
 2*c)/(a^2 + 2*a*b + b^2) + e^(2*d*x + 2*c)/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 880, normalized size of antiderivative = 11.43

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{2c+2dx}}{8d\,(a+b)} - \frac{e^{-2c-2dx}}{8d\,(a+b)} + \frac{x\,(a+3b)}{2\,(a+b)^2}$$
$$+ \frac{\operatorname{atan}\left(\left(e^{2c}\,e^{2dx}\left(\frac{2\left(2\,b^3\,\sqrt{a^5\,d^2+4\,a^4\,b\,d^2+6\,a^3\,b^2\,d^2+4\,a^2\,b^3\,d^2+a\,b^4\,d^2}+2\,a\,b^2\,\sqrt{a^5\,d^2+4\,a^4\,b\,d^2+6\,a^3\,b^2\,d^2+4\,a^2\,b^3\,d^2+a\,b^4\,d^2}\right)}{d\,(a+b)^5\,\sqrt{b^3}\,(a^3+3\,a^2\,b+3\,a\,b^2+b^3)\,\sqrt{a^5\,d^2+4\,a^4\,b\,d^2+6\,a^3\,b^2\,d^2+4\,a^2\,b^3\,d^2+a\,b^4\,d^2}} - \cdots\right.\right.\right.}{\cdots}$$

input
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(2*c + 2*d*x)/(8*d*(a + b)) - exp(- 2*c - 2*d*x)/(8*d*(a + b)) + (x*(a
+ 3*b))/(2*(a + b)^2) + (atan((exp(2*c)*exp(2*d*x)*((2*(2*b^3*(a^5*d^2 + a
*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2) + 2*a*b^2*(a
^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))/
(d*(a + b)^5*(b^3)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*
d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)) - ((a - b)*(2*a*
d*(b^3)^(3/2) + b*d*(b^3)^(3/2) - a^4*d*(b^3)^(1/2) - 2*a^3*b*d*(b^3)^(1/2
)))/(b^2*(a + b)^3*(a*d^2*(a + b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)
)) + ((a - b)*(4*a*d*(b^3)^(3/2) + b*d*(b^3)^(3/2) + a^4*d*(b^3)^(1/2) + 4
*a^3*b*d*(b^3)^(1/2) + 6*a^2*b^2*d*(b^3)^(1/2)))/(b^2*(a + b)^3*(a*d^2*(a
+ b)^4)^(1/2)*(3*a*b^2 + 3*a^2*b + a^3 + b^3)*(a^5*d^2 + a*b^4*d^2 + 4*a^4
*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)))*((a^4*(a^5*d^2 + a*b^4*d^2
 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + (b^4*(a^5*d^2 +
 a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2))/2 + 3*a^2
*b^2*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(
1/2) + 2*a*b^3*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^2 + 6*a^3*
b^2*d^2)^(1/2) + 2*a^3*b*(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2 + 4*a^2*b^3*d^
2 + 6*a^3*b^2*d^2)^(1/2)))*(b^3)^(1/2))/(a^5*d^2 + a*b^4*d^2 + 4*a^4*b*d^2
 + 4*a^2*b^3*d^2 + 6*a^3*b^2*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 181, normalized size of antiderivative = 2.35

$$\int \frac{\cosh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{8e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b-8e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)b+e^{4dx+4c}a^2+e^{4dx+4c}ab+}{8e^{2dx+2c}ad\left(a^2+2ab+b^2\right)}$$

input
```
int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

output
```
(8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*b - 8*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqr
t(a + b) + sqrt(b))/sqrt(a))*b + e**(4*c + 4*d*x)*a**2 + e**(4*c + 4*d*x)*
a*b + 4*e**(2*c + 2*d*x)*a**2*d*x + 12*e**(2*c + 2*d*x)*a*b*d*x - a**2 - a
*b)/(8*e**(2*c + 2*d*x)*a*d*(a**2 + 2*a*b + b**2))
```

3.108 $\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 53

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

output
```
b*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/(a+b)^(3/2)/d+sinh(d*x+c
)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}d} + \frac{\sinh(c+dx)}{(a+b)d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)*d)
+ Sinh[c + d*x]/((a + b)*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 4159, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{b\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{a+b}+\frac{\sinh(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{b\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)^{3/2}}+\frac{\sinh(c+dx)}{a+b}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((b*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) +
 Sinh[c + d*x]/(a + b))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 148 vs. $2(45) = 90$.

Time = 1.37 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.81

method	result
risch	$\frac{e^{dx+c}}{2(a+b)d}-\frac{e^{-dx-c}}{2(a+b)d}-\frac{b\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}+\frac{b\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,(a+b)d}$
derivativedivides	$\frac{\frac{2ba\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{a+b}-\frac{2}{(2b+2a)\left(\tanh\left(\frac{dx}{2}\right.\right.}}{d}$
default	$\frac{\frac{2ba\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{a+b}-\frac{2}{(2b+2a)\left(\tanh\left(\frac{dx}{2}\right.\right.}}{d}$

input
```
int(cosh(d*x+c)/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)/d*exp(d*x+c)-1/2/(a+b)/d*exp(-d*x-c)-1/2/(-a^2-a*b)^(1/2)*b/(a+b
)/d*ln(exp(2*d*x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/2/(-a^2-a*b)^(1
/2)*b/(a+b)/d*ln(exp(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(45) = 90$.

Time = 0.12 (sec) , antiderivative size = 773, normalized size of antiderivative = 14.58

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x +
 c) + (a^2 + a*b)*sinh(d*x + c)^2 - sqrt(-a^2 - a*b)*(b*cosh(d*x + c) + b*
sinh(d*x + c))*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3
*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x
+ c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*c
osh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*s
inh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x
+ c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4
 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(
d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
+ c) + a + b)) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) + (a^
3 + 2*a^2*b + a*b^2)*d*sinh(d*x + c)), 1/2*((a^2 + a*b)*cosh(d*x + c)^2 +
2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 +
2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b*sinh(d*x + c))*arctan(1/2*((a + b)*
cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d
*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b
)*sinh(d*x + c))/sqrt(a^2 + a*b)) - 2*sqrt(a^2 + a*b)*(b*cosh(d*x + c) + b
*sinh(d*x + c))*arctan(2*sqrt(a^2 + a*b)/((a + b)*cosh(d*x + c) + (a + b)*
sinh(d*x + c))) - a^2 - a*b)/((a^3 + 2*a^2*b + a*b^2)*d*cosh(d*x + c) +...
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\cosh(dx+c)}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x)/(a*d*e^c + b*d*e^c) + 1/2*integrate(4*(
b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2 + 2*a*b + b^2 + (a^2*e^(4*c) + 2*a
*b*e^(4*c) + b^2*e^(4*c))*e^(4*d*x) + 2*(a^2*e^(2*c) - b^2*e^(2*c))*e^(2*d
*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 2.77 (sec) , antiderivative size = 154, normalized size of antiderivative = 2.91

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{c+dx}}{2\,d\,(a+b)} - \frac{e^{-c-dx}}{2\,d\,(a+b)} - \frac{b\,\ln\left(\sqrt{-a}\,\sqrt{a+b}+2\,a\,e^{c+dx}-\sqrt{-a}\,e^{2\,c+2\,dx}\,\sqrt{a+b}\right)}{2\,\sqrt{-a}\,d\,(a+b)^{3/2}} + \frac{b\,\ln\left(2\,a\,e^{c+dx}-\sqrt{-a}\,\sqrt{a+b}+\sqrt{-a}\,e^{2\,c+2\,dx}\,\sqrt{a+b}\right)}{2\,\sqrt{-a}\,d\,(a+b)^{3/2}}$$

input
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output
```
exp(c + d*x)/(2*d*(a + b)) - exp(- c - d*x)/(2*d*(a + b)) - (b*log((-a)^(1
/2)*(a + b)^(1/2) + 2*a*exp(c + d*x) - (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)
^(1/2)))/(2*(-a)^(1/2)*d*(a + b)^(3/2)) + (b*log(2*a*exp(c + d*x) - (-a)^(
1/2)*(a + b)^(1/2) + (-a)^(1/2)*exp(2*c + 2*d*x)*(a + b)^(1/2)))/(2*(-a)^(
1/2)*d*(a + b)^(3/2))
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 195, normalized size of antiderivative = 3.68

$$\int \frac{\cosh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2e^{dx+c}\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b+2e^{dx+c}\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)b-e^{2dx+2c}ab-e^{2dx+2c}b}{2e^{dx+c}ad\,(a^2+2ab+}$$

input
```
int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*e**(c + d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*b + 2*e**(c + d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqr
t(a + b) + sqrt(b))/sqrt(a))*b - e**(2*c + 2*d*x)*a*b - e**(2*c + 2*d*x)*b
**2 + 2*e**(c + d*x)*sinh(c + d*x)*a**2 + 4*e**(c + d*x)*sinh(c + d*x)*a*b
 + 2*e**(c + d*x)*sinh(c + d*x)*b**2 + a*b + b**2)/(2*e**(c + d*x)*a*d*(a*
*2 + 2*a*b + b**2))
```

3.109 $\int \frac{\text{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \frac{\text{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

output
```
arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(1/2)/(a+b)^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a+b}d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3042, 4159, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a+b}}$$

input
```
Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 101 vs. $2(28) = 56$.

Time = 1.92 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.83

method	result	size
risch	$-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{2\sqrt{-a^2-ab}\,d}$	102
derivativedivides	$\frac{2a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	152
default	$\frac{2a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	152

input
```
int(sech(d*x+c)/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-
1)+1/2/(-a^2-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c
)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. $2(28) = 56$.

Time = 0.12 (sec) , antiderivative size = 519, normalized size of antiderivative = 14.42

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*c
osh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^
2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*c
osh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*
x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*s
inh(d*x + c) + a + b))/((a^2 + a*b)*d), (sqrt(a^2 + a*b)*arctan(1/2*((a +
b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sin
h(d*x + c)^3 + (3*a - b)*cosh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a
- b)*sinh(d*x + c))/sqrt(a^2 + a*b)) - sqrt(a^2 + a*b)*arctan(2*sqrt(a^2 +
 a*b)/((a + b)*cosh(d*x + c) + (a + b)*sinh(d*x + c))))/((a^2 + a*b)*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
integrate(sech(d*x + c)/(b*tanh(d*x + c)^2 + a), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.08

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\operatorname{atan}\left(\frac{4\,a^2\,d^2\,e^{d\,x}\,e^c-e^{d\,x}\,e^c\,\sqrt{a^2\,d^2+b\,a\,d^2}\,\sqrt{a\,d^2\,(a+b)}+e^{3\,c}\,e^{3\,d\,x}\,\sqrt{a^2\,d^2+b\,a\,d^2}\,\sqrt{a\,d^2\,(a+b)}}{2\,a\,d\,\sqrt{a\,d^2\,(a+b)}}\right)+\operatorname{atan}\left(\frac{e^{d\,x}\,e^c\,\sqrt{a\,d^2\,(a+b)}}{2\,a\,d}\right)}{\sqrt{a^2\,d^2+b\,a\,d^2}}$$

input
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)),x)
```

output
```
(atan((4*a^2*d^2*exp(d*x)*exp(c) - exp(d*x)*exp(c)*(a^2*d^2 + a*b*d^2)^(1/
2)*(a*d^2*(a + b))^(1/2) + exp(3*c)*exp(3*d*x)*(a^2*d^2 + a*b*d^2)^(1/2)*(
a*d^2*(a + b))^(1/2))/(2*a*d*(a*d^2*(a + b))^(1/2))) + atan((exp(d*x)*exp(
c)*(a*d^2*(a + b))^(1/2))/(2*a*d)))/(a^2*d^2 + a*b*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.75

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{a}\,\sqrt{a+b}\,\left(atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)+atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)\right)}{ad\,(a+b)}$$

input
```
int(sech(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

output
```
(sqrt(a)*sqrt(a + b)*(atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)) +
 atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))))/(a*d*(a + b))
```

3.110 $\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 32

$$\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

output
```
arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/b^(1/2)/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(ic+idx)^2}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \; 4158$$

$$\frac{\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(24) = 48$.

Time = 6.20 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.56

method	result	size
risch	$-\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}$	114
derivativedivides	$\frac{2a\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\operatorname{arctan}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	160
default	$\frac{2a\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\operatorname{arctan}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{d}$	160

input
```
int(sech(d*x+c)^2/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)-2*a*b
)/(a+b)/(-a*b)^(1/2))+1/2/(-a*b)^(1/2)/d*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)
-b*(-a*b)^(1/2)+2*a*b)/(a+b)/(-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 75 vs. $2(24) = 48$.

Time = 0.11 (sec) , antiderivative size = 455, normalized size of antiderivative = 14.22

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \left[-\frac{\sqrt{-ab}\log\left(\frac{(a^2+2ab+b^2)\cosh(dx+c)^4+4(a^2+2ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2ab+b^2)\sinh(dx+c)^4+2(a^2-b^2)\cosh(dx+\cdots}{(a+b)\cosh(dx+c)^4+4(a+b)\cosh(dx+c)\sinh(d\cdots}\right.}{\cdots}\right.$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^
4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)
^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)
*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*
x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)
^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*co
sh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b))/(a*b*d), sq
rt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh
(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b))/(a*b*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.12

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=-\frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*d)
```

Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.38

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqr
t(a*b)*d)
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 81, normalized size of antiderivative = 2.53

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{\operatorname{atan}\left(\frac{a\,\sqrt{a\,b\,d^2}-b\,\sqrt{a\,b\,d^2}+a\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{a\,b\,d^2}+b\,e^{2\,c}\,e^{2\,d\,x}\,\sqrt{a\,b\,d^2}}{2\,a\,b\,d}\right)}{\sqrt{a\,b\,d^2}}$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)),x)
```

output
```
atan((a*(a*b*d^2)^(1/2) - b*(a*b*d^2)^(1/2) + a*exp(2*c)*exp(2*d*x)*(a*b*d
^2)^(1/2) + b*exp(2*c)*exp(2*d*x)*(a*b*d^2)^(1/2))/(2*a*b*d))/(a*b*d^2)^(1
/2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.91

$$\int \frac{\mathrm{sech}^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}\,\sqrt{a}\left(atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)-atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)\right)}{abd}$$

input
```
int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*(atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)) - ata
n((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))))/(a*b*d)
```

3.111 $\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 55

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=-\frac{\arctan(\sinh(c+dx))}{bd}+\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

output
```
-arctan(sinh(d*x+c))/b/d+(a+b)^(1/2)*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2
))/a^(1/2)/b/d
```

Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=-\frac{\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}}+2\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{bd}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((Sqrt[a + b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a] + 2*A
rcTan[Tanh[(c + d*x)/2]])/(b*d))
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3042, 4159, 303, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\sec(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4159}$$

$$\frac{\int \frac{1}{(\sinh^2(c+dx)+1)((a+b)\sinh^2(c+dx)+a)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{ 303}$$

$$\frac{\frac{(a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 216}$$

$$\frac{\frac{(a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{\arctan(\sinh(c+dx))}{b}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{\arctan(\sinh(c+dx))}{b}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(ArcTan[Sinh[c + d*x]]/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Sinh[c + d*
x])/Sqrt[a]])/(Sqrt[a]*b))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 303
```
Int[1/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Simp[b/(b
*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x
^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [C] (verified)

Result contains complex when optimal does not.

Time = 16.03 (sec) , antiderivative size = 144, normalized size of antiderivative = 2.62

method	result
risch	$\frac{i\ln(\mathrm{e}^{dx+c}-i)}{db}-\frac{i\ln(\mathrm{e}^{dx+c}+i)}{db}+\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{2\sqrt{-a(a+b)}\,\mathrm{e}^{dx+c}}{a+b}-1\right)}{2adb}-\frac{\sqrt{-a(a+b)}\,\ln\left(\mathrm{e}^{2dx+2c}-2\right.}{2adb}$
derivativedivides	$\frac{-\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{2a(a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\right.}{\sqrt{\left(2\sqrt{(a+b)b}-\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right.}}\right.}{b}}{d}$
default	$\frac{-\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b}+\frac{2a(a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\right.}{\sqrt{\left(2\sqrt{(a+b)b}-\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right.}}\right.}{b}}{d}$

input
```
int(sech(d*x+c)^3/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
I/d/b*ln(exp(d*x+c)-I)-I/d/b*ln(exp(d*x+c)+I)+1/2/a*(-a*(a+b))^(1/2)/d/b*l
n(exp(2*d*x+2*c)+2*(-a*(a+b))^(1/2)/(a+b)*exp(d*x+c)-1)-1/2/a*(-a*(a+b))^(
1/2)/d/b*ln(exp(2*d*x+2*c)-2*(-a*(a+b))^(1/2)/(a+b)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 171 vs. $2(47) = 94$.

Time = 0.14 (sec) , antiderivative size = 540, normalized size of antiderivative = 9.82

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(sqrt(-(a + b)/a)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x +
 c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^
2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*c
osh(d*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x +
c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x
+ c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-(a + b)/a) + a + b)/
((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a +
b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c
)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d
*x + c))*sinh(d*x + c) + a + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c))
)/(b*d), (sqrt((a + b)/a)*arctan(1/2*sqrt((a + b)/a)*(cosh(d*x + c) + sinh
(d*x + c))) + sqrt((a + b)/a)*arctan(1/2*((a + b)*cosh(d*x + c)^3 + 3*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a + b)*sinh(d*x + c)^3 + (3*a - b)*co
sh(d*x + c) + (3*(a + b)*cosh(d*x + c)^2 + 3*a - b)*sinh(d*x + c))*sqrt((a
 + b)/a)/(a + b)) - 2*arctan(cosh(d*x + c) + sinh(d*x + c)))/(b*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^3{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^3}{b\tanh\left(dx+c\right)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-2*arctan(e^(d*x + c))/(b*d) + 8*integrate(1/4*((a*e^(3*c) + b*e^(3*c))*e^
(3*d*x) + (a*e^c + b*e^c)*e^(d*x))/(a*b + b^2 + (a*b*e^(4*c) + b^2*e^(4*c)
)*e^(4*d*x) + 2*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.02 (sec) , antiderivative size = 449, normalized size of antiderivative = 8.16

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\sqrt{a+b}\left(2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\sqrt{a+b}\,\sqrt{a\,b^2\,d^2}}{2\,a\,b\,d}\right)-2\,\mathrm{atan}\left(\frac{\left(\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(\frac{64\,\left(2\,a\,b^2\,d\,\sqrt{a+b}-6\,a^2\,b\,d\,\sqrt{a+b}\right)}{a^3\,b^3\,d^2\,(a+b)^2\,\left(a^2+2\,a\,b+b^2\right)}+\frac{32\,\left(3\,a^2\,\sqrt{a\,b^2\,d^2}-b^2\,\sqrt{a\,b^2\,d^2}+2\,a\,b\,\sqrt{\cdots}\right.}{a^3\,b^2\,d\,(a+b)^{3/2}\,\left(a^2+2\,a\,b+b^2\right)\,\sqrt{a\,\cdots}}\cdots}{\cdots}\right.\right.}{2\,\sqrt{a\,b\cdots}}$$
$$-\frac{2\,\mathrm{atan}\left(\frac{\mathrm{e}^{d\,x}\,\mathrm{e}^{c}\,\left(9\,a^2\,\sqrt{b^2\,d^2}+b^2\,\sqrt{b^2\,d^2}-6\,a\,b\,\sqrt{b^2\,d^2}\right)}{9\,d\,a^2\,b-6\,d\,a\,b^2+d\,b^3}\right)}{\sqrt{b^2\,d^2}}$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)),x)
```

output
```
((a + b)^(1/2)*(2*atan((exp(d*x)*exp(c)*(a + b)^(1/2)*(a*b^2*d^2)^(1/2))/(
2*a*b*d)) - 2*atan(((exp(d*x)*exp(c)*((64*(2*a*b^2*d*(a + b)^(1/2) - 6*a^2
*b*d*(a + b)^(1/2)))/(a^3*b^3*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) + (32*(3*
a^2*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*(a*b^2*d^2)^(1/2)))/
(a^3*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^2)^(1/2))) - (32*exp
(3*c)*exp(3*d*x)*(3*a^2*(a*b^2*d^2)^(1/2) - b^2*(a*b^2*d^2)^(1/2) + 2*a*b*
(a*b^2*d^2)^(1/2)))/(a^3*b^2*d*(a + b)^(3/2)*(2*a*b + a^2 + b^2)*(a*b^2*d^
2)^(1/2)))*(a^4*b*(a + b)*(a*b^2*d^2)^(1/2) + a^2*b^3*(a + b)*(a*b^2*d^2)^
(1/2) + 2*a^3*b^2*(a + b)*(a*b^2*d^2)^(1/2)))/(192*a - 64*b))))/(2*(a*b^2*
d^2)^(1/2)) - (2*atan((exp(d*x)*exp(c)*(9*a^2*(b^2*d^2)^(1/2) + b^2*(b^2*d
^2)^(1/2) - 6*a*b*(b^2*d^2)^(1/2)))/(b^3*d - 6*a*b^2*d + 9*a^2*b*d)))/(b^2
*d^2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.45

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-2atan\left(e^{dx+c}\right)a+\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)+\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{b}}{\sqrt{a}}\right)}{abd}$$

input
```
int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - 2*atan(e**(c + d*x))*a + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a
 + b) - sqrt(b))/sqrt(a)) + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a
+ b) + sqrt(b))/sqrt(a)))/(a*b*d)
```

3.112 $\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 50

$$\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

output
```
(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/b^(3/2)/d-tanh(d*x+c)/b/
d
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00

$$\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}-\frac{\tanh(c+dx)}{bd}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d) - Ta
nh[c + d*x]/(b*d)
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 299, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^4}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{299}$$

$$\frac{\frac{(a+b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{b}-\frac{\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}-\frac{\tanh(c+dx)}{b}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Tan
h[c + d*x]/b)/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 299
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[d*x
*((a + b*x^2)^(p + 1)/(b*(2*p + 3))), x] - Simp[(a*d - b*c*(2*p + 3))/(b*(2
*p + 3))   Int[(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
 a*d, 0] && NeQ[2*p + 3, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(42) = 84$.

Time = 38.36 (sec) , antiderivative size = 197, normalized size of antiderivative = 3.94

method	result
derivativedivides	$\frac{-\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}+\frac{2a(a+b)\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh}{\sqrt{(2\sqrt{(a+}}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a\right.}}\right.}{b}}{d}$
default	$\frac{-\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}+\frac{2a(a+b)\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh}{\sqrt{(2\sqrt{(a+}}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a\right.}}\right.}{b}}{d}$
risch	$\frac{2}{bd\left(e^{2dx+2c}+1\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a}{2\sqrt{-ab}\,db}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{2\sqrt{-ab}\,d}+\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-}}{(}\right.}{2\sqrt{-}}$

input
```
int(sech(d*x+c)^4/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)+2/b*a*(a+b)*(1/2*(
-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(a
-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(42) = 84$.

Time = 0.11 (sec) , antiderivative size = 649, normalized size of antiderivative = 12.98

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) +
(a + b)*sinh(d*x + c)^2 + a + b)*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(
d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 +
2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 +
 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b +
 b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))
*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh
(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x
 + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^
4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh
(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x
 + c) + a + b)) - 4*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c
)*sinh(d*x + c) + a*b^2*d*sinh(d*x + c)^2 + a*b^2*d), (((a + b)*cosh(d*x +
 c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a + b)*sqrt(a*b)*arctan(1/2*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x
+ c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(a*b)/(a*b)) + 2
*a*b)/(a*b^2*d*cosh(d*x + c)^2 + 2*a*b^2*d*cosh(d*x + c)*sinh(d*x + c) + a
*b^2*d*sinh(d*x + c)^2 + a*b^2*d)]
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.26

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(a+b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}bd} - \frac{2}{(be^{(-2\,dx-2\,c)}+b)d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(a + b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*
b)*b*d) - 2/((b*e^(-2*d*x - 2*c) + b)*d)
```

Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.40

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{(a+b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}b} + \frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
((a + b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a
*b))/(sqrt(a*b)*b) + 2/(b*(e^(2*d*x + 2*c) + 1)))/d
```

Mupad [B] (verification not implemented)

Time = 3.14 (sec) , antiderivative size = 176, normalized size of antiderivative = 3.52

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2}{b\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}-\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}\right)\,(a+b)}{2\,\sqrt{-a}\,b^{3/2}\,d} - \frac{\ln\left(\frac{2\,\left(a\,d+b\,d+a\,d\,\mathrm{e}^{2\,c+2\,d\,x}-b\,d\,\mathrm{e}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{3/2}\,d}-\frac{4\,\mathrm{e}^{2\,c+2\,d\,x}}{b}\right)\,(a+b)}{2\,\sqrt{-a}\,b^{3/2}\,d}$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)),x)
```

output
```
2/(b*d*(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x))/b - (2*(a*d +
 b*d + a*d*exp(2*c + 2*d*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d
))*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d) - (log((2*(a*d + b*d + a*d*exp(2*c +
2*d*x) - b*d*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(3/2)*d) - (4*exp(2*c + 2*d*
x))/b)*(a + b))/(2*(-a)^(1/2)*b^(3/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 307, normalized size of antiderivative = 6.14

$$\int \frac{\text{sech}^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)a+e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)}{}$$

input
```
int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*a + e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) - sqrt(b))/sqrt(a))*b + sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b
) - sqrt(b))/sqrt(a))*a + sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*b - e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)
*sqrt(a + b) + sqrt(b))/sqrt(a))*a - e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan
((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b - sqrt(b)*sqrt(a)*atan((e
**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a - sqrt(b)*sqrt(a)*atan((e**(
c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b - 2*e**(2*c + 2*d*x)*a*b)/(a*b*
*2*d*(e**(2*c + 2*d*x) + 1))
```

3.113 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 86

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(2a+3b)\arctan(\sinh(c+dx))}{2b^2d} + \frac{(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd}$$

output
```
-1/2*(2*a+3*b)*arctan(sinh(d*x+c))/b^2/d+(a+b)^(3/2)*arctan((a+b)^(1/2)*si
nh(d*x+c)/a^(1/2))/a^(1/2)/b^2/d-1/2*sech(d*x+c)*tanh(d*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 79, normalized size of antiderivative = 0.92

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{2(a+b)^{3/2}\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{a}} + 2(2a+3b)\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + b\operatorname{sech}(c+dx)\tanh(c+dx)}{2b^2d}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/2*((2*(a + b)^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/Sqrt[a
] + 2*(2*a + 3*b)*ArcTan[Tanh[(c + d*x)/2]] + b*Sech[c + d*x]*Tanh[c + d*x
])/(b^2*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.09, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4159, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^5}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{-\left((a+b)\sinh^2(c+dx)\right)+a+2b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2b}-\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{(2a+3b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}}{2b}-\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{\frac{2(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{(2a+3b)\arctan(\sinh(c+dx))}{b}}{2b}-\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)}}{d}$$

$$\Big\downarrow \ 218$$

$$\frac{\frac{\frac{2(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{(2a+3b)\arctan(\sinh(c+dx))}{b}}{2b}-\frac{\sinh(c+dx)}{2b(\sinh^2(c+dx)+1)}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((-(((2*a + 3*b)*ArcTan[Sinh[c + d*x]])/b) + (2*(a + b)^(3/2)*ArcTan[(Sqrt
[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(2*b) - Sinh[c + d*x]/(2*b*(
1 + Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(74) = 148$.

Time = 86.95 (sec) , antiderivative size = 236, normalized size of antiderivative = 2.74

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\,\mathrm{ta}}{\sqrt{\left(2\sqrt{(}\right.}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b}\right.}}\right.}{}}{d}$
default	$\frac{-\frac{2\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+1\right)^2}+\frac{(2a+3b)\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}\right)}{b^2}+\frac{2\left(a^2+2ab+b^2\right)a\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\,\mathrm{ta}}{\sqrt{\left(2\sqrt{(}\right.}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b}\right.}}\right.}{}}{d}$
risch	$-\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{db\left(e^{2dx+2c}+1\right)^2}+\frac{i\ln\left(e^{dx+c}-i\right)a}{d\,b^2}+\frac{3i\ln\left(e^{dx+c}-i\right)}{2db}-\frac{i\ln\left(e^{dx+c}+i\right)a}{d\,b^2}-\frac{3i\ln\left(e^{dx+c}+i\right)}{2db}+\frac{\sqrt{-a(a+b)}}{}$

input
```
int(sech(d*x+c)^5/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2/b^2*((-1/2*b*tanh(1/2*d*x+1/2*c)^3+1/2*b*tanh(1/2*d*x+1/2*c))/(tan
h(1/2*d*x+1/2*c)^2+1)^2+1/2*(2*a+3*b)*arctan(tanh(1/2*d*x+1/2*c)))+2/b^2*(
a^2+2*a*b+b^2)*a*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^
(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2
*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2
)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)
*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(74) = 148$.

Time = 0.15 (sec) , antiderivative size = 1584, normalized size of antiderivative = 18.42

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[-1/2*(2*b*cosh(d*x + c)^3 + 6*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*b*sinh(
d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)
*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (
a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*sqrt(-(a + b)/a)*log(((a + b)
*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(
d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 -
3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x
 + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x +
 c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b
)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh
(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((
a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) +
2*((2*a + 3*b)*cosh(d*x + c)^4 + 4*(2*a + 3*b)*cosh(d*x + c)*sinh(d*x + c)
^3 + (2*a + 3*b)*sinh(d*x + c)^4 + 2*(2*a + 3*b)*cosh(d*x + c)^2 + 2*(3*(2
*a + 3*b)*cosh(d*x + c)^2 + 2*a + 3*b)*sinh(d*x + c)^2 + 4*((2*a + 3*b)*co
sh(d*x + c)^3 + (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 2*a + 3*b)*arct
an(cosh(d*x + c) + sinh(d*x + c)) - 2*b*cosh(d*x + c) + 2*(3*b*cosh(d*x +
c)^2 - b)*sinh(d*x + c))/(b^2*d*cosh(d*x + c)^4 + 4*b^2*d*cosh(d*x + c)...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\int \frac{\operatorname{sech}^5\left(c+dx\right)}{a+b\tanh^2\left(c+dx\right)}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2), x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{b\tanh(dx+c)^2+a}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-(e^(3*d*x + 3*c) - e^(d*x + c))/(b*d*e^(4*d*x + 4*c) + 2*b*d*e^(2*d*x + 2
*c) + b*d) - (2*a*e^c + 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(b^2*d) + 32*i
ntegrate(1/16*((a^2*e^(3*c) + 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^
2*e^c + 2*a*b*e^c + b^2*e^c)*e^(d*x))/(a*b^2 + b^3 + (a*b^2*e^(4*c) + b^3*
e^(4*c))*e^(4*d*x) + 2*(a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [B] (verification not implemented)

Time = 3.38 (sec) , antiderivative size = 1012, normalized size of antiderivative = 11.77

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)),x)
```

output
```
((2*atan(((exp(d*x)*exp(c)*((64*(12*a^2*b^4*d*(3*a*b^2 + 3*a^2*b + a^3 + b
^3)^(1/2) - 2*a*b^5*d*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 18*a^3*b^3*d
*(3*a*b^2 + 3*a^2*b + a^3 + b^3)^(1/2) + 6*a^4*b^2*d*(3*a*b^2 + 3*a^2*b +
a^3 + b^3)^(1/2)))/(a^3*b^9*d^2*(a + b)^2*(2*a*b + a^2 + b^2)) - (32*(3*a^
5*(a*b^4*d^2)^(1/2) - b^5*(a*b^4*d^2)^(1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) +
15*a^4*b*(a*b^4*d^2)^(1/2) + 20*a^2*b^3*(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*
b^4*d^2)^(1/2)))/(a^3*b^7*d*((a + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d
^2)^(1/2))) + (32*exp(3*c)*exp(3*d*x)*(3*a^5*(a*b^4*d^2)^(1/2) - b^5*(a*b^
4*d^2)^(1/2) + 4*a*b^4*(a*b^4*d^2)^(1/2) + 15*a^4*b*(a*b^4*d^2)^(1/2) + 20
*a^2*b^3*(a*b^4*d^2)^(1/2) + 27*a^3*b^2*(a*b^4*d^2)^(1/2)))/(a^3*b^7*d*((a
 + b)^3)^(1/2)*(2*a*b + a^2 + b^2)*(a*b^4*d^2)^(1/2)))*(a^2*b^7*(a*b^4*d^2
)^(1/2) + 2*a^3*b^6*(a*b^4*d^2)^(1/2) + a^4*b^5*(a*b^4*d^2)^(1/2)))/(384*a
*b^2 + 576*a^2*b + 192*a^3 - 64*b^3)) + 2*atan((exp(d*x)*exp(c)*(a + b)^2*
(a*b^4*d^2)^(1/2))/(2*a*b^2*d*((a + b)^3)^(1/2))))*(3*a*b^2 + 3*a^2*b + a^
3 + b^3)^(1/2))/(2*(a*b^4*d^2)^(1/2)) - (atan((exp(d*x)*exp(c)*(18*a^7*(b^
4*d^2)^(1/2) + 3*b^7*(b^4*d^2)^(1/2) + 30*a^2*b^5*(b^4*d^2)^(1/2) + 342*a^
3*b^4*(b^4*d^2)^(1/2) + 555*a^4*b^3*(b^4*d^2)^(1/2) + 396*a^5*b^2*(b^4*d^2
)^(1/2) - 34*a*b^6*(b^4*d^2)^(1/2) + 135*a^6*b*(b^4*d^2)^(1/2)))/(b^8*d*(1
2*a*b + 4*a^2 + 9*b^2)^(1/2) - 12*a*b^7*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) +
 18*a^2*b^6*d*(12*a*b + 4*a^2 + 9*b^2)^(1/2) + 102*a^3*b^5*d*(12*a*b + ...
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 620, normalized size of antiderivative = 7.21

$$\int \frac{\mathrm{sech}^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{-2e^{4dx+4c}atan\left(e^{dx+c}\right)a^2-3e^{4dx+4c}atan\left(e^{dx+c}\right)ab-4e^{2dx+2c}atan\left(e^{dx+c}\right)a^2-6e^{2dx+2c}atan\left(e^{dx+c}\right)ab-}{}$$

input
```
int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - 2*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**2 - 3*e**(4*c + 4*d*x)*atan(e
**(c + d*x))*a*b - 4*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**2 - 6*e**(2*c
+ 2*d*x)*atan(e**(c + d*x))*a*b - 2*atan(e**(c + d*x))*a**2 - 3*atan(e**(c
 + d*x))*a*b + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqr
t(a + b) - sqrt(b))/sqrt(a))*a + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan
((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b + 2*e**(2*c + 2*d*x)*sqrt
(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a + 2*e
**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*b + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqr
t(b))/sqrt(a))*b + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)
*sqrt(a + b) + sqrt(b))/sqrt(a))*a + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*
atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b + 2*e**(2*c + 2*d*x)*
sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a +
 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + s
qrt(b))/sqrt(a))*b + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) +
sqrt(b))/sqrt(a))*a + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) +
 sqrt(b))/sqrt(a))*b - e**(3*c + 3*d*x)*a*b + e**(c + d*x)*a*b)/(a*b**2*d*
(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.114 $\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 75

$$\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{(a+2b)\tanh(c+dx)}{b^2d} + \frac{\tanh^3(c+dx)}{3bd}$$

output
```
(a+b)^2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/b^(5/2)/d-(a+2*b)*tanh
(d*x+c)/b^2/d+1/3*tanh(d*x+c)^3/b/d
```

Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.95

$$\int \frac{\text{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d} - \frac{(3a+5b+b\text{sech}^2(c+dx))\tanh(c+dx)}{3b^2d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) -
((3*a + 5*b + b*Sech[c + d*x]^2)*Tanh[c + d*x])/(3*b^2*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^6}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\tanh^2(c+dx)}{b}-\frac{a+2b}{b^2}+\frac{a^2+2ba+b^2}{b^2\left(b\tanh^2(c+dx)+a\right)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+b)^2\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}}-\frac{(a+2b)\tanh(c+dx)}{b^2}+\frac{\tanh^3(c+dx)}{3b}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a + b)^2*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - (
(a + 2*b)*Tanh[c + d*x])/b^2 + Tanh[c + d*x]^3/(3*b))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(65) = 130$.

Time = 174.33 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.36

method	result
derivativedivides	$\frac{\frac{2a\left(a^2+2ab+b^2\right)\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{b^2}}{d}$
default	$\frac{\frac{2a\left(a^2+2ab+b^2\right)\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(a-\sqrt{(a+b)b}+b\right)\operatorname{arctanh}\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}-a-2b\right)a}}\right)}{b^2}}{d}$
risch	$\frac{2\,e^{4dx+4c}a+2b\,e^{4dx+4c}+4\,e^{2dx+2c}a+8\,e^{2dx+2c}b+2a+\frac{10b}{3}}{b^2d\left(e^{2dx+2c}+1\right)^3}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)a^2}{2\sqrt{-ab}\,d\,b^2}-\frac{\ln\left(e^{2dx+2c}+\right.}{}$

input
```
int(sech(d*x+c)^6/(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*a*(a^2+2*a*b+b^2)*(1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)
/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+
b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/(
(2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b
)*b)^(1/2)-a-2*b)*a)^(1/2)))+2/b^2*((-a-2*b)*tanh(1/2*d*x+1/2*c)^5+(-2*a-8
/3*b)*tanh(1/2*d*x+1/2*c)^3+(-a-2*b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/
2*c)^2+1)^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 864 vs. $2(65) = 130$.

Time = 0.12 (sec) , antiderivative size = 2032, normalized size of antiderivative = 27.09

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/6*(12*(a^2*b + a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b + a*b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + 12*(a^2*b + a*b^2)*sinh(d*x + c)^4 + 12*a^2*b + 20*a*b
^2 + 24*(a^2*b + 2*a*b^2)*cosh(d*x + c)^2 + 24*(a^2*b + 2*a*b^2 + 3*(a^2*b
 + a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((a^2 + 2*a*b + b^2)*cosh(d
*x + c)^6 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^5 + (a^2 + 2
*a*b + b^2)*sinh(d*x + c)^6 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 3*(5
*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^4
+ 4*(5*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*
x + c))*sinh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 3*(5*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 +
 a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 + 2*a*b + b^2 + 6*((a^2 + 2*a*b
+ b^2)*cosh(d*x + c)^5 + 2*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 + 2*
a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 -
 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d
*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
 c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*
cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sin...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. $2(65) = 130$.

Time = 0.16 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.87

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\left(6(a+2b)e^{(-2dx-2c)}+3(a+b)e^{(-4dx-4c)}+3a+5b\right)}{3\left(3b^2e^{(-2dx-2c)}+3b^2e^{(-4dx-4c)}+b^2e^{(-6dx-6c)}+b^2\right)d} - \frac{(a^2+2ab+b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}b^2d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-2/3*(6*(a + 2*b)*e^(-2*d*x - 2*c) + 3*(a + b)*e^(-4*d*x - 4*c) + 3*a + 5*
b)/((3*b^2*e^(-2*d*x - 2*c) + 3*b^2*e^(-4*d*x - 4*c) + b^2*e^(-6*d*x - 6*c
) + b^2)*d) - (a^2 + 2*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/(sqrt(a*b)*b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(65) = 130$.

Time = 0.31 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.80

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{3(a^2+2ab+b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}b^2} + \frac{2\left(3ae^{(4dx+4c)}+3be^{(4dx+4c)}+6ae^{(2dx+2c)}+12be^{(2dx+2c)}+3a+5b\right)}{b^2\left(e^{(2dx+2c)}+1\right)^3}}{3d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(a^2 + 2*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*
c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b^2) + 2*(3*a*e^(4*d*x + 4*c) + 3*b*e^(4
*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 12*b*e^(2*d*x + 2*c) + 3*a + 5*b)/(b^2
*(e^(2*d*x + 2*c) + 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.89 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.36

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{4}{b\,d\,\left(2\,{\mathrm{e}}^{2\,c+2\,d\,x}+{\mathrm{e}}^{4\,c+4\,d\,x}+1\right)}-\frac{8}{3\,b\,d\,\left(3\,{\mathrm{e}}^{2\,c+2\,d\,x}+3\,{\mathrm{e}}^{4\,c+4\,d\,x}+{\mathrm{e}}^{6\,c+6\,d\,x}+1\right)}+\frac{2\,\left(a+b\right)}{b^2\,d\,\left({\mathrm{e}}^{2\,c+2\,d\,x}+1\right)}+\frac{\ln\left(-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{b^2}-\frac{2\,\left(a+b\right)\,\left(a+b+a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}\right)\,{\left(a+b\right)}^2}{2\,\sqrt{-a}\,b^{5/2}\,d}-\frac{\ln\left(\frac{2\,\left(a+b\right)\,\left(a+b+a\,{\mathrm{e}}^{2\,c+2\,d\,x}-b\,{\mathrm{e}}^{2\,c+2\,d\,x}\right)}{\sqrt{-a}\,b^{5/2}}-\frac{4\,{\mathrm{e}}^{2\,c+2\,d\,x}\,\left(a+b\right)}{b^2}\right)\,{\left(a+b\right)}^2}{2\,\sqrt{-a}\,b^{5/2}\,d}$$

input
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)),x)
```

output
```
4/(b*d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - 8/(3*b*d*(3*exp(2*c
+ 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (2*(a + b))/(b^2*
d*(exp(2*c + 2*d*x) + 1)) + (log(- (4*exp(2*c + 2*d*x)*(a + b))/b^2 - (2*(
a + b)*(a + b + a*exp(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5
/2)))*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d) - (log((2*(a + b)*(a + b + a*exp
(2*c + 2*d*x) - b*exp(2*c + 2*d*x)))/((-a)^(1/2)*b^(5/2)) - (4*exp(2*c + 2
*d*x)*(a + b))/b^2)*(a + b)^2)/(2*(-a)^(1/2)*b^(5/2)*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1042, normalized size of antiderivative = 13.89

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2),x)
```

output

```
(3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a**2 + 6*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*
sqrt(a + b) - sqrt(b))/sqrt(a))*a*b + 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*a
tan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 9*e**(4*c + 4*d*x
)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2
+ 18*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqr
t(b))/sqrt(a))*a*b + 9*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)
*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)
*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 + 18*e**(2*c + 2*
d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*
b + 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*b**2 + 3*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a**2 + 6*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
- sqrt(b))/sqrt(a))*a*b + 3*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
 - sqrt(b))/sqrt(a))*b**2 - 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c
 + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 - 6*e**(6*c + 6*d*x)*sqrt(b)*
sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b - 3*e**(6*c
 + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a
))*b**2 - 9*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b
) + sqrt(b))/sqrt(a))*a**2 - 18*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((...
```

3.115 $\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 128

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}d} + \frac{(a+3b)\sinh(c+dx)}{(a+b)^3d} + \frac{\sinh^3(c+dx)}{3(a+b)^2d} + \frac{b^3\sinh(c+dx)}{2a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*b^2*(6*a+b)*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(7/2
)/d+(a+3*b)*sinh(d*x+c)/(a+b)^3/d+1/3*sinh(d*x+c)^3/(a+b)^2/d+1/2*b^3*sinh
(d*x+c)/a/(a+b)^3/d/(a+(a+b)*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.75 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.87

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{-\frac{6b^2(6a+b)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{3/2}(a+b)^{7/2}}+\frac{3\left(3a+11b+\frac{4b^3}{a(a-b+(a+b)\cosh(2(c+dx)))}\right)\sinh(c+dx)}{(a+b)^3}+\frac{\sinh(3(c+dx))}{(a+b)^2}}{12d}$$

input

```
Integrate[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output

```
((-6*b^2*(6*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(3/2)*(
a + b)^(7/2)) + (3*(3*a + 11*b + (4*b^3)/(a*(a - b + (a + b)*Cosh[2*(c + d
*x)])))*Sinh[c + d*x])/(a + b)^3 + Sinh[3*(c + d*x)]/(a + b)^2)/(12*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^2}dx$$
$$\Big\downarrow \text{4159}$$
$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^3}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$
$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{\sinh^2(c+dx)}{(a+b)^2}+\frac{a+3b}{(a+b)^3}+\frac{3(a+b)\sinh^2(c+dx)b^2+(3a+b)b^2}{(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b^2(6a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{2a(a+b)^3\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh^3(c+dx)}{3(a+b)^2}+\frac{(a+3b)\sinh(c+dx)}{(a+b)^3}}{d}$$

input
```
Int[Cosh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a
 + b)^(7/2)) + ((a + 3*b)*Sinh[c + d*x])/(a + b)^3 + Sinh[c + d*x]^3/(3*(a
 + b)^2) + (b^3*Sinh[c + d*x])/(2*a*(a + b)^3*(a + (a + b)*Sinh[c + d*x]^2
)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 377 vs. $2(114) = 228$.

Time = 27.80 (sec) , antiderivative size = 378, normalized size of antiderivative = 2.95

method	result
derivativedivides	$\frac{2b^2\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(6a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\cdots\right)}{2}\cdots\right)}{(a+b)^3}\cdots$
default	$\frac{2b^2\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(6a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\cdots\right)}{2}\cdots\right)}{(a+b)^3}\cdots$
risch	$\frac{e^{3dx+3c}}{24(a^2+2ab+b^2)d}+\frac{3\,e^{dx+c}a}{8(a^2+2ab+b^2)(a+b)d}+\frac{11\,e^{dx+c}b}{8(a^2+2ab+b^2)(a+b)d}-\frac{3\,e^{-dx-c}a}{8(a^3+3a^2b+3b^2a+b^3)d}-\frac{11\,e^{-dx\cdots}}{8(a^3+3a^2b+3\cdots}$

input
```
int(cosh(d*x+c)^3/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/(a+b)^3*b^2*((-1/2*b/a*tanh(1/2*d*x+1/2*c)^3+1/2*b/a*tanh(1/2*d*x+1
/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x
+1/2*c)^2+a)+1/2*(6*a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((
a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(
1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)
*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2
)-a-2*b)*a)^(1/2))))-1/3/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/(a+b)^2/(ta
nh(1/2*d*x+1/2*c)+1)^2-(a+3*b)/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)-1/3/(a+b)^2
/(tanh(1/2*d*x+1/2*c)-1)^3-1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2-(a+3*b)/(
a+b)^3/(tanh(1/2*d*x+1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3649 vs. $2(114) = 228$.

Time = 0.19 (sec) , antiderivative size = 6941, normalized size of antiderivative = 54.23

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(cosh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)^3}{(b\tanh(dx+c)^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/24*(a^3 + 2*a^2*b + a*b^2 - (a^3*e^(10*c) + 2*a^2*b*e^(10*c) + a*b^2*e^
(10*c))*e^(10*d*x) - (11*a^3*e^(8*c) + 42*a^2*b*e^(8*c) + 31*a*b^2*e^(8*c)
)*e^(8*d*x) - 2*(5*a^3*e^(6*c) + 4*a^2*b*e^(6*c) - 49*a*b^2*e^(6*c) + 12*b
^3*e^(6*c))*e^(6*d*x) + 2*(5*a^3*e^(4*c) + 4*a^2*b*e^(4*c) - 49*a*b^2*e^(4
*c) + 12*b^3*e^(4*c))*e^(4*d*x) + (11*a^3*e^(2*c) + 42*a^2*b*e^(2*c) + 31*
a*b^2*e^(2*c))*e^(2*d*x))/((a^5*d*e^(7*c) + 4*a^4*b*d*e^(7*c) + 6*a^3*b^2*
d*e^(7*c) + 4*a^2*b^3*d*e^(7*c) + a*b^4*d*e^(7*c))*e^(7*d*x) + 2*(a^5*d*e^
(5*c) + 2*a^4*b*d*e^(5*c) - 2*a^2*b^3*d*e^(5*c) - a*b^4*d*e^(5*c))*e^(5*d*
x) + (a^5*d*e^(3*c) + 4*a^4*b*d*e^(3*c) + 6*a^3*b^2*d*e^(3*c) + 4*a^2*b^3*
d*e^(3*c) + a*b^4*d*e^(3*c))*e^(3*d*x)) + 1/8*integrate(8*((6*a*b^2*e^(3*c
) + b^3*e^(3*c))*e^(3*d*x) + (6*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^5 + 4*a^4
*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5*e^(4*c) + 4*a^4*b*e^(4*c) + 6*a^
3*b^2*e^(4*c) + 4*a^2*b^3*e^(4*c) + a*b^4*e^(4*c))*e^(4*d*x) + 2*(a^5*e^(2
*c) + 2*a^4*b*e^(2*c) - 2*a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1442, normalized size of antiderivative = 11.27

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(cosh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(72*e**(7*c + 7*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a**2*b**2 + 84*e**(7*c + 7*d*x)*sqrt(a)*sqrt(a + b)*atan
((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 + 12*e**(7*c + 7*d*x
)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b
**4 + 144*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a +
 b) - sqrt(b))/sqrt(a))*a**2*b**2 - 120*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a +
b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 - 24*e**(5*c
+ 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqr
t(a))*b**4 + 72*e**(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sq
rt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 + 84*e**(3*c + 3*d*x)*sqrt(a)*sqrt
(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 + 12*e**
(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*b**4 + 72*e**(7*c + 7*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*
x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2*b**2 + 84*e**(7*c + 7*d*x)*sqrt(a)
*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b**3 + 1
2*e**(7*c + 7*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sq
rt(b))/sqrt(a))*b**4 + 144*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(
c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2*b**2 - 120*e**(5*c + 5*d*x)*
sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b
**3 - 24*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a...
```

3.116 $\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 140

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{(a+5b)x}{2(a+b)^3} + \frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)} - \frac{(a-b)b\tanh(c+dx)}{2a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
1/2*(a+5*b)*x/(a+b)^3+1/2*b^(3/2)*(5*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/
2))/a^(3/2)/(a+b)^3/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)
^2)-1/2*(a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.90 (sec) , antiderivative size = 110, normalized size of antiderivative = 0.79

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$
$$= \frac{2(a+5b)(c+dx)+\frac{2b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}+(a+b)\sinh(2(c+dx))+\frac{2b^2(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{4(a+b)^3d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(a + 5*b)*(c + d*x) + (2*b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x
])/Sqrt[a]])/a^(3/2) + (a + b)*Sinh[2*(c + d*x)] + (2*b^2*(a + b)*Sinh[2*(
c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(4*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4158, 316, 402, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(ic+idx)^2\,(a-b\tan(ic+idx)^2)^2}dx$$
$$\Big\downarrow \text{4158}$$
$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))^2(b\tanh^2(c+dx)+a)^2}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \text{316}$$

$$\frac{\frac{\int \frac{3b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 402

$$\frac{\frac{-\frac{\int -\frac{2\left(a^2+4ba+b^2+(a-b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{a^2+4ba+b^2+(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{b^2(5a+b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{a(a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{a(a+5b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{b^{3/2}(5a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{a(a+5b)\text{arctanh}(\tanh(c+dx))}{a+b}}{a(a+b)}-\frac{b(a-b)\tanh(c+dx)}{a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

input

```
Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)) +
 (((b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a
 + b)) + (a*(a + 5*b)*ArcTanh[Tanh[c + d*x]])/(a + b))/(a*(a + b)) - ((a -
 b)*b*Tanh[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)))/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(124) = 248$.

Time = 9.18 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.76

method	result
derivativedivides	$\frac{\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{2(a}}{}$
default	$\frac{\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-a-5b)\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2(a+b)^3}-\frac{1}{2(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}+\frac{}{2(a}}{}$
risch	$\frac{xa}{2(a+b)(a^2+2ab+b^2)}+\frac{5xb}{2(a+b)(a^2+2ab+b^2)}+\frac{e^{2dx+2c}}{8(a^2+2ab+b^2)d}-\frac{e^{-2dx-2c}}{8(a^2+2ab+b^2)d}-\frac{b^2\left(e^{2dx+}\right.}{d(a+b)^3a\left(e^{4dx+4c}a+b\,e^{4}\right.}$

input

```
int(cosh(d*x+c)^2/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output

```
1/d*(1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c
)-1)+1/2/(a+b)^3*(-a-5*b)*ln(tanh(1/2*d*x+1/2*c)-1)-1/2/(a+b)^2/(tanh(1/2*
d*x+1/2*c)+1)^2+1/2/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)+1/2*(a+5*b)/(a+b)^3*ln
(tanh(1/2*d*x+1/2*c)+1)-2/(a+b)^3*b^2*((-1/2*(a+b)/a*tanh(1/2*d*x+1/2*c)^3
-1/2*(a+b)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+
1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(5*a+b)*(1/2*(a+((a+b)*b)^(1/2
)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)
-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2001 vs. 2(124) = 248.

Time = 0.18 (sec) , antiderivative size = 4324, normalized size of antiderivative = 30.89

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 840 vs. 2(124) = 248.

Time = 0.28 (sec) , antiderivative size = 840, normalized size of antiderivative = 6.00

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) +
(a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 1/
8*(3*a^2*b - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/8*(3*a^2*b
 - 6*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/4*(3*a*b + b^2)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*
b^2)*sqrt(a*b)*d) + 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^3)*e^(2*d*x +
2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + (a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(4*d*x + 4*c) + 2*(a^5 + 2*a^4*b - 2*a^2*b
^3 - a*b^4)*e^(2*d*x + 2*c))*d) - 1/4*(a^2*b - b^3 + (a^2*b - 6*a*b^2 + b^
3)*e^(-2*d*x - 2*c))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4 + 2*(
a^5 + 2*a^4*b - 2*a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^5 + 4*a^4*b + 6*a
^3*b^2 + 4*a^2*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) + 1/2*(a*b + b^2 + (a*b -
 b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3
*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*e^(-4*d*x - 4*c))*d) + 1/2*(d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/8*e^(2
*d*x + 2*c)/((a^2 + 2*a*b + b^2)*d) - 1/8*e^(-2*d*x - 2*c)/((a^2 + 2*a*b +
 b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(124) = 248$.

Time = 0.74 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.97

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$= \frac{\frac{12\,(dx+c)(a+5\,b)}{a^3+3\,a^2b+3\,ab^2+b^3} + \frac{12\,(5\,ab^2+b^3)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^4+3\,a^3b+3\,a^2b^2+ab^3)\sqrt{ab}} + \frac{3\,e^{(2\,dx+2\,c)}}{a^2+2\,ab+b^2} - \frac{2\,a^3e^{(6\,dx+6\,c)}+12\,a^2be^{(6\,dx+6\,c)}+10\,ab^2e^{(}}{}}{}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/24*(12*(d*x + c)*(a + 5*b)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 12*(5*a*b^2
 + b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*
b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) + 3*e^(2*d*x + 2*c)/(a
^2 + 2*a*b + b^2) - (2*a^3*e^(6*d*x + 6*c) + 12*a^2*b*e^(6*d*x + 6*c) + 10
*a*b^2*e^(6*d*x + 6*c) + 7*a^3*e^(4*d*x + 4*c) + 22*a^2*b*e^(4*d*x + 4*c)
+ 7*a*b^2*e^(4*d*x + 4*c) - 24*b^3*e^(4*d*x + 4*c) + 8*a^3*e^(2*d*x + 2*c)
 + 12*a^2*b*e^(2*d*x + 2*c) + 28*a*b^2*e^(2*d*x + 2*c) + 24*b^3*e^(2*d*x +
 2*c) + 3*a^3 + 6*a^2*b + 3*a*b^2)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a
*e^(6*d*x + 6*c) + b*e^(6*d*x + 6*c) + 2*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x
+ 4*c) + a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c))))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\cosh(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1269, normalized size of antiderivative = 9.06

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(20*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a**2*b + 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d
*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 + 4*e**(6*c + 6*d*x)*sqrt(b)*sq
rt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 + 40*e**(4*c
 + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*a**2*b - 32*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) - sqrt(b))/sqrt(a))*a*b**2 - 8*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan(
(e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 + 20*e**(2*c + 2*d*x)*s
qrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b +
 24*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a*b**2 + 4*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*
x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 - 20*e**(6*c + 6*d*x)*sqrt(b)*sqrt
(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2*b - 24*e**(6*c
 + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a
))*a*b**2 - 4*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
 b) + sqrt(b))/sqrt(a))*b**3 - 40*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e
**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2*b + 32*e**(4*c + 4*d*x)*s
qrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b**2 +
 8*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(
b))/sqrt(a))*b**3 - 20*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d...
```

3.117 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 101

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}d}+\frac{\sinh(c+dx)}{(a+b)^2d}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output

```
1/2*b*(4*a+b)*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/
d+sinh(d*x+c)/(a+b)^2/d+1/2*b^2*sinh(d*x+c)/a/(a+b)^2/d/(a+(a+b)*sinh(d*x+
c)^2)
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 84, normalized size of antiderivative = 0.83

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}}+\frac{\sinh(c+dx)\left(2+\frac{b^2}{a\left(a+(a+b)\sinh^2(c+dx)\right)}\right)}{(a+b)^2}}{2d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b
)^(5/2)) + (Sinh[c + d*x]*(2 + b^2/(a*(a + (a + b)*Sinh[c + d*x]^2))))/(a
+ b)^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{2b(a+b)\sinh^2(c+dx)+b(2a+b)}{(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)^2}+\frac{1}{(a+b)^2}\right)d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{b(4a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\sinh(c+dx)}{2a(a+b)^2\left((a+b)\sinh^2(c+dx)+a\right)}+\frac{\sinh(c+dx)}{(a+b)^2}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a +
 b)^(5/2)) + Sinh[c + d*x]/(a + b)^2 + (b^2*Sinh[c + d*x])/(2*a*(a + b)^2*
(a + (a + b)*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 285 vs. $2(89) = 178$.

Time = 3.81 (sec) , antiderivative size = 286, normalized size of antiderivative = 2.83

method	result
derivativedivides	$\frac{-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(4a-\cdots}{\cdots}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{1}{(a+b)^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{2b\left(\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(4a-\cdots}{\cdots}\cdots\right.}{\cdots}}{d}$
risch	$\frac{e^{dx+c}}{2(a^2+2ab+b^2)d}-\frac{e^{-dx-c}}{2(a^2+2ab+b^2)d}+\frac{b^2 e^{dx+c}\left(e^{2dx+2c}-1\right)}{d(a+b)^2 a\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}-\frac{b\ln\left(e^{2d\cdots}\right.}{\sqrt{-\cdots}}$

input
```
int(cosh(d*x+c)/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)-1)-1/(a+b)^2/(tanh(1/2*d*x+1/2*c)+1)+
2/(a+b)^2*b*((-1/2*b/a*tanh(1/2*d*x+1/2*c)^3+1/2*b/a*tanh(1/2*d*x+1/2*c))/
(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)
^2+a)+1/2*(4*a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)
^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+
2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/
2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b
)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1823 vs. $2(89) = 178$.

Time = 0.15 (sec) , antiderivative size = 3510, normalized size of antiderivative = 34.75

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(cosh(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh(dx+c)}{(b\tanh(dx+c)^2+a)^2}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(a^2 + a*b - (a^2*e^(6*c) + a*b*e^(6*c))*e^(6*d*x) - (a^2*e^(4*c) - 3
*a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x) + (a^2*e^(2*c) - 3*a*b*e^(2*c) + 2
*b^2*e^(2*c))*e^(2*d*x))/((a^4*d*e^(5*c) + 3*a^3*b*d*e^(5*c) + 3*a^2*b^2*d
*e^(5*c) + a*b^3*d*e^(5*c))*e^(5*d*x) + 2*(a^4*d*e^(3*c) + a^3*b*d*e^(3*c)
 - a^2*b^2*d*e^(3*c) - a*b^3*d*e^(3*c))*e^(3*d*x) + (a^4*d*e^c + 3*a^3*b*d
*e^c + 3*a^2*b^2*d*e^c + a*b^3*d*e^c)*e^(d*x)) + 1/2*integrate(2*((4*a*b*e
^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (4*a*b*e^c + b^2*e^c)*e^(d*x))/(a^4 + 3*
a^3*b + 3*a^2*b^2 + a*b^3 + (a^4*e^(4*c) + 3*a^3*b*e^(4*c) + 3*a^2*b^2*e^(
4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(a^4*e^(2*c) + a^3*b*e^(2*c) - a^2*b^2
*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\cosh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 1145, normalized size of antiderivative = 11.34

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(4*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**2*b + 5*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**
(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 + e**(5*c + 5*d*x)*sqrt(a
)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 + 8*
e**(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a**2*b - 6*e**(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c
+ d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 - 2*e**(3*c + 3*d*x)*sqrt(a)
*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 + 4*e
**(c + d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/
sqrt(a))*a**2*b + 5*e**(c + d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sq
rt(a + b) - sqrt(b))/sqrt(a))*a*b**2 + e**(c + d*x)*sqrt(a)*sqrt(a + b)*at
an((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 + 4*e**(5*c + 5*d*x)
*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*
*2*b + 5*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a +
b) + sqrt(b))/sqrt(a))*a*b**2 + e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan(
(e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**3 + 8*e**(3*c + 3*d*x)*sq
rt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2*
b - 6*e**(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b)
+ sqrt(b))/sqrt(a))*a*b**2 - 2*e**(3*c + 3*d*x)*sqrt(a)*sqrt(a + b)*atan((
e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**3 + 4*e**(c + d*x)*sqrt...
```

3.118 $\int \frac{\text{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d}+\frac{b\sinh(c+dx)}{2a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*(2*a+b)*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+
1/2*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.94

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}}+\frac{b\sinh(c+dx)}{a\left(a+(a+b)\sinh^2(c+dx)\right)}}{2(a+b)d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a +
 b]) + (b*Sinh[c + d*x])/(a*(a + (a + b)*Sinh[c + d*x]^2)))/(2*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$
$$\Big\downarrow \text{4159}$$
$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$
$$\Big\downarrow \text{298}$$
$$\frac{\frac{(2a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a(a+b)}+\frac{b\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$
$$\Big\downarrow \text{218}$$
$$\frac{\frac{(2a+b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{b\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(((2*a + b)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b
)^(3/2)) + (b*Sinh[c + d*x])/(2*a*(a + b)*(a + (a + b)*Sinh[c + d*x]^2)))/
d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 251 vs. $2(71) = 142$.

Time = 8.31 (sec) , antiderivative size = 252, normalized size of antiderivative = 3.04

method	result
derivativedivides	$\frac{\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}+\frac{(2a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)\ldots}\right.}{2\ldots}\right.}{a+b}}{d}$
default	$\frac{\frac{-\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{a(a+b)}+\frac{b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a(a+b)}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}+\frac{(2a+b)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)\ldots}\right.}{2\ldots}\right.}{a+b}}{d}$
risch	$\frac{b\,\mathrm{e}^{dx+c}\left(\mathrm{e}^{2dx+2c}-1\right)}{(a+b)ad\left(\mathrm{e}^{4dx+4c}a+b\,\mathrm{e}^{4dx+4c}+2\,\mathrm{e}^{2dx+2c}a-2\,\mathrm{e}^{2dx+2c}b+a+b\right)}-\frac{\ln\left(\mathrm{e}^{2dx+2c}-\frac{2a\,\mathrm{e}^{dx+c}}{\sqrt{-a^{2}-ab}}-1\right)}{2\sqrt{-a^{2}-ab}\,(a+b)d}-\frac{\ln\left(\mathrm{e}^{2dx+2c}-\frac{2a\,\mathrm{e}\ldots}{\sqrt{-\ldots}}\right.}{4\sqrt{-a^{2}-ab}\,(\ldots}$

input
```
int(sech(d*x+c)/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/2*b/a/(a+b)*tanh(1/2*d*x+1/2*c)^3+1/2*b/a/(a+b)*tanh(1/2*d*x+1/
2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+
1/2*c)^2+a)+(2*a+b)/(a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*(
(a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^
(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b
)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/
2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1037 vs. $2(71) = 142$.

Time = 0.13 (sec) , antiderivative size = 2049, normalized size of antiderivative = 24.69

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*b + a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b + a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b + a*b^2)*sinh(d*x + c)^3 - ((2*a^2 + 3*a*b + b
^2)*cosh(d*x + c)^4 + 4*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^
3 + (2*a^2 + 3*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - a*b - b^2)*cosh(d*x
 + c)^2 + 2*(3*(2*a^2 + 3*a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 - a*b - b^2)*
sinh(d*x + c)^2 + 2*a^2 + 3*a*b + b^2 + 4*((2*a^2 + 3*a*b + b^2)*cosh(d*x
+ c)^3 + (2*a^2 - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 - a*b
)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a
 + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*
sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) - 4*(a^2*b + a*b^2)*cosh(d*x + c) - 4*(a^2*b + a*b^2 - 3*(a^2*b + a*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*
cosh(d*x + c)^4 + 4*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)...
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(b\tanh(dx+c)^2+a)^2}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^3*d + 2*a^2*b*d + a*b^2*d + (a^3*d*
e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^3*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/2*((2*a*e^(3*c) + b*e^(3*c
))*e^(3*d*x) + (2*a*e^c + b*e^c)*e^(d*x))/(a^3 + 2*a^2*b + a*b^2 + (a^3*e^
(4*c) + 2*a^2*b*e^(4*c) + a*b^2*e^(4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a*b^
2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 914, normalized size of antiderivative = 11.01

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**2 + 3*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c
 + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b + e**(4*c + 4*d*x)*sqrt(a)*sqr
t(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 4*e**(2
*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/
sqrt(a))*a**2 - 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*
sqrt(a + b) - sqrt(b))/sqrt(a))*a*b - 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a +
b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 2*sqrt(a)*sqr
t(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 + 3*sqrt(
a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b + sq
rt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2
+ 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) +
sqrt(b))/sqrt(a))*a**2 + 3*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(
c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b + e**(4*c + 4*d*x)*sqrt(a)*sq
rt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + 4*e**(
2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))
/sqrt(a))*a**2 - 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)
*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b - 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a +
 b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + 2*sqrt(a)*sq
rt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 + 3*s...
```

3.119 $\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx=\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\tanh(c+dx)}{2ad\,(a+b\tanh^2(c+dx))}$$

output
```
1/2*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(1/2)/d+1/2*tanh(d*x+c)/
a/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx=\frac{\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}+\frac{\sqrt{a}\tanh(c+dx)}{a+b\tanh^2(c+dx)}}{2a^{3/2}d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[b] + (Sqrt[a]*Tanh[c + d*x])
/(a + b*Tanh[c + d*x]^2))/(2*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 64, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Tanh[c + d*
x]/(2*a*(a + b*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(54) = 108$.

Time = 26.23 (sec) , antiderivative size = 210, normalized size of antiderivative = 3.18

method	result
risch	$-\frac{e^{2dx+2c}a-e^{2dx+2c}b+a+b}{(a+b)ad\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}-2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,da}+\frac{\ln(e^{2dx+}{}$
derivativedivides	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}+\frac{(-a+\sqrt{(a+}{2a}}{d}$
default	$\frac{-\frac{2\left(-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}+\frac{(-a+\sqrt{(a+}{2a}}{d}$

input
```
int(sech(d*x+c)^2/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
-(exp(2*d*x+2*c)*a-exp(2*d*x+2*c)*b+a+b)/(a+b)/a/d/(exp(4*d*x+4*c)*a+b*exp
(4*d*x+4*c)+2*exp(2*d*x+2*c)*a-2*exp(2*d*x+2*c)*b+a+b)-1/4/(-a*b)^(1/2)/d/
a*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)-2*a*b)/(a+b)/(-a*b)^(1/
2))+1/4/(-a*b)^(1/2)/d/a*ln(exp(2*d*x+2*c)+(a*(-a*b)^(1/2)-b*(-a*b)^(1/2)+
2*a*b)/(a+b)/(-a*b)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 606 vs. $2(54) = 108$.

Time = 0.12 (sec) , antiderivative size = 1515, normalized size of antiderivative = 22.95

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b -
a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 + (
(a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh
(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d
*x + c)^2 + a^2 + 2*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (
a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^
2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(
3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 -
 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d
*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x +
 c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*
cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d
*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a -
 b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*
sinh(d*x + c) + a + b)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4
+ 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b
 + 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^4*b - a^2*b^3)*d*cosh(d*x
 + c)^2 + 2*(3*(a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (a^4*b...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 125 vs. $2(54) = 108$.

Time = 0.20 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.89

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{\left(a^3+2\,a^2b+ab^2+2\,(a^3-ab^2)e^{(-2\,dx-2\,c)}+(a^3+2\,a^2b+ab^2)e^{(-4\,dx-4\,c)}\right)d} - \frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}ad}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^
2)*e^(-2*d*x - 2*c) + (a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) - 1/2*a
rctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 138 vs. $2(54) = 108$.

Time = 0.33 (sec) , antiderivative size = 138, normalized size of antiderivative = 2.09

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a} - \frac{2\,\left(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b\right)}{(a^2+ab)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}}{2\,d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))
/(sqrt(a*b)*a) - 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 +
 a*b)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e
^(2*d*x + 2*c) + a + b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 508, normalized size of antiderivative = 7.70

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$=\frac{e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)a+e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b+2e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{}}{}\right.}{}$$

input
```
int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*a + e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) - sqrt(b))/sqrt(a))*b + 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(
c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a - 2*e**(2*c + 2*d*x)*sqrt(b)*sq
rt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b + sqrt(b)*sqrt(
a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a + sqrt(b)*sqrt(a)*
atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b - e**(4*c + 4*d*x)*sq
rt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a - e**(4
*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt
(a))*b - 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
 + sqrt(b))/sqrt(a))*a + 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b - sqrt(b)*sqrt(a)*atan((e**(c + d*x
)*sqrt(a + b) + sqrt(b))/sqrt(a))*a - sqrt(b)*sqrt(a)*atan((e**(c + d*x)*s
qrt(a + b) + sqrt(b))/sqrt(a))*b + e**(4*c + 4*d*x)*a*b - a*b)/(2*a**2*b*d
*(e**(4*c + 4*d*x)*a + e**(4*c + 4*d*x)*b + 2*e**(2*c + 2*d*x)*a - 2*e**(2
*c + 2*d*x)*b + a + b))
```

3.120 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}d}+\frac{\sinh(c+dx)}{2ad\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(1/2)/d+1/2*sinh
(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.96

$$\int \frac{\text{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+\frac{\sqrt{a}\sinh(c+dx)}{a+(a+b)\sinh^2(c+dx)}}{2a^{3/2}d}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/Sqrt[a + b] + (Sqrt[a]*Sinh[c
 + d*x])/(a + (a + b)*Sinh[c + d*x]^2))/(2*a^(3/2)*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3042, 4159, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$
$$\Big\downarrow \text{4159}$$
$$\frac{\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$
$$\Big\downarrow \text{215}$$
$$\frac{\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$
$$\Big\downarrow \text{218}$$
$$\frac{\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

input
```
Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Sin
h[c + d*x]/(2*a*(a + (a + b)*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 182 vs. $2(60) = 120$.

Time = 60.37 (sec) , antiderivative size = 183, normalized size of antiderivative = 2.54

method	result
risch	$\frac{e^{dx+c}\left(e^{2dx+2c}-1\right)}{ad\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}-\frac{\ln\left(e^{2dx+2c}-\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}-1\right)}{4\sqrt{-a^2-ab}\,da}+\frac{\ln\left(e^{2dx+2c}+\frac{2a\,e^{dx+c}}{\sqrt{-a^2-ab}}\cdots\right.}{4\sqrt{-a^2-ab}\,da}\cdots$
derivativedivides	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\mathrm{arc}\cdots}{2a\sqrt{(a+b)\cdots}}}{d}$
default	$\frac{\frac{-\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{a}+\frac{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\right)\mathrm{arc}\cdots}{2a\sqrt{(a+b)\cdots}}}{d}$

input
```
int(sech(d*x+c)^3/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
exp(d*x+c)*(exp(2*d*x+2*c)-1)/a/d/(exp(4*d*x+4*c)*a+b*exp(4*d*x+4*c)+2*exp
(2*d*x+2*c)*a-2*exp(2*d*x+2*c)*b+a+b)-1/4/(-a^2-a*b)^(1/2)/d/a*ln(exp(2*d*
x+2*c)-2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)+1/4/(-a^2-a*b)^(1/2)/d/a*ln(exp(
2*d*x+2*c)+2*a/(-a^2-a*b)^(1/2)*exp(d*x+c)-1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(60) = 120$.

Time = 0.12 (sec) , antiderivative size = 1563, normalized size of antiderivative = 21.71

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2 + a*b)*cosh(d*x + c)^3 + 12*(a^2 + a*b)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a^2 + a*b)*sinh(d*x + c)^3 - ((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
*sqrt(-a^2 - a*b)*log(((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*s
inh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2
*(3*(a + b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d
*x + c)^3 - (3*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a^2 - a*b) + a + b)/((a + b)*cosh(d
*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c
)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*si
nh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d
*x + c) + a + b)) - 4*(a^2 + a*b)*cosh(d*x + c) + 4*(3*(a^2 + a*b)*cosh(d*
x + c)^2 - a^2 - a*b)*sinh(d*x + c))/((a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x
 + c)^4 + 4*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a
^4 + 2*a^3*b + a^2*b^2)*d*sinh(d*x + c)^4 + 2*(a^4 - a^2*b^2)*d*cosh(d*x +
 c)^2 + 2*(3*(a^4 + 2*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + (a^4 - a^2*b^2)
*d)*sinh(d*x + c)^2 + (a^4 + 2*a^3*b + a^2*b^2)*d + 4*((a^4 + 2*a^3*b +...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^3\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^3*d + 2*a
^2*b*d + a*b^2*d + (a^3*d*e^(4*c) + 2*a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e
^(4*d*x) + 2*(a^3*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/
8*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^2 + a*b + (a^2*e^(4*c) + a*b*e^(4*c))
*e^(4*d*x) + 2*(a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^3\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 590, normalized size of antiderivative = 8.19

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{e^{4dx+4c}\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)a+e^{4dx+4c}\sqrt{a}\,\sqrt{a+b}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b+2e^{2dx+2c}\sqrt{a}\,\sqrt{a+b}}{}$$

input
```
int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqr
t(b))/sqrt(a))*a + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)
*sqrt(a + b) - sqrt(b))/sqrt(a))*b + 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a - 2*e**(2*c + 2*d*x
)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b
 + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*
a + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))
*b + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) +
 sqrt(b))/sqrt(a))*a + e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c +
d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b + 2*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a
 + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a - 2*e**(2*c + 2
*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a
))*b + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(
a))*a + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt
(a))*b + 2*e**(3*c + 3*d*x)*a**2 + 2*e**(3*c + 3*d*x)*a*b - 2*e**(c + d*x)
*a**2 - 2*e**(c + d*x)*a*b)/(2*a**2*d*(e**(4*c + 4*d*x)*a**2 + 2*e**(4*c +
 4*d*x)*a*b + e**(4*c + 4*d*x)*b**2 + 2*e**(2*c + 2*d*x)*a**2 - 2*e**(2*c
+ 2*d*x)*b**2 + a**2 + 2*a*b + b**2))
```

3.121 $\int \frac{\text{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 77

$$\int \frac{\text{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{2abd\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(3/2)/d+1/2*(a+b)
*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.08

$$\int \frac{\text{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(-a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right) + \frac{\sqrt{a}\sqrt{b}(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2a^{3/2}b^{3/2}d}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] + (Sqrt[a]*Sqrt[b]*(a +
b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]))/(2*a^(3/2)*b^(3
/2)*d)
```

Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 75, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{\sec(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$
$$\Big\downarrow \text{ 4158}$$
$$\frac{\int \frac{1-\tanh^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \text{ 298}$$
$$\frac{\frac{(a+b)\tanh(c+dx)}{2ab\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2ab}}{d}$$
$$\Big\downarrow \text{ 218}$$
$$\frac{\frac{(a+b)\tanh(c+dx)}{2ab\left(a+b\tanh^2(c+dx)\right)}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*b^(3/2))
+ ((a + b)*Tanh[c + d*x])/(2*a*b*(a + b*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 258 vs. $2(65) = 130$.

Time = 127.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 3.36

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{(a-b)\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\right.}{b}}{d}$
default	$\frac{-\frac{2\left(-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2ab}-\frac{(a+b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ab}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}-\frac{(a-b)\left(\frac{\left(-a-\sqrt{(a+b)b}-b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\,\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\right.}{b}}{d}$
risch	$-\frac{e^{2dx+2c}a-e^{2dx+2c}b+a+b}{abd\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,db}+$

input
```
int(sech(d*x+c)^4/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-1/2*(a+b)/a/b*tanh(1/2*d*x+1/2*c)^3-1/2*(a+b)/a/b*tanh(1/2*d*x+1
/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x
+1/2*c)^2+a)-(a-b)/b*(1/2*(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a
+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1
/2)+a+2*b)*a)^(1/2))-1/2*(a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b
)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/
2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 569 vs. $2(65) = 130$.

Time = 0.13 (sec) , antiderivative size = 1443, normalized size of antiderivative = 18.74

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*a^2*b + 4*a*b^2 + 4*(a^2*b - a*b^2)*cosh(d*x + c)^2 + 8*(a^2*b -
a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(a^2*b - a*b^2)*sinh(d*x + c)^2 - (
(a^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3
+ (a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*
(3*(a^2 - b^2)*cosh(d*x + c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2
- b^2 + 4*((a^2 - b^2)*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c)
)*sinh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(
a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sin
h(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*co
sh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 +
 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) -
 4*((a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a +
 b)*sinh(d*x + c)^2 + a - b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a +
 b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*co
sh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*
((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)))
/((a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 + a^2*b^3)*d*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(a^3*b^2
 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2
 + (a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 + a^2*b^3)*d + 4*(...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.65

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a-b)e^{(-2\,dx-2\,c)}+a+b}{\left(a^2b+ab^2+2\,(a^2b-ab^2)e^{(-2\,dx-2\,c)}+(a^2b+ab^2)e^{(-4\,dx-4\,c)}\right)d} + \frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}abd}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*b - a*b^2)*e^(
-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/2*(a - b)*arctan(
1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(65) = 130$.

Time = 0.35 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.86

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(a-b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}ab}+\frac{2\left(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b\right)}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)ab}}{2\,d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/(sqrt(a*b)*a*b) + 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a +
 b)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^
(2*d*x + 2*c) + a + b)*a*b))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 637, normalized size of antiderivative = 8.27

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

$$= \frac{-e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)a^2+e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b^2-2e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(e^{d}\right.}{}$$

input

```
int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
( - e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a**2 + e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*s
qrt(a + b) - sqrt(b))/sqrt(a))*b**2 - 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*a
tan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 + 4*e**(2*c + 2*d*x
)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b -
 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*b**2 - sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a**2 + sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*b**2 + e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sq
rt(a + b) + sqrt(b))/sqrt(a))*a**2 - e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan
((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + 2*e**(2*c + 2*d*x)*s
qrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 - 4
*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b)
)/sqrt(a))*a*b + 2*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqr
t(a + b) + sqrt(b))/sqrt(a))*b**2 + sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqr
t(a + b) + sqrt(b))/sqrt(a))*a**2 - sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqr
t(a + b) + sqrt(b))/sqrt(a))*b**2 + e**(4*c + 4*d*x)*a**2*b + e**(4*c + 4*
d*x)*a*b**2 - a**2*b - a*b**2)/(2*a**2*b**2*d*(e**(4*c + 4*d*x)*a + e**(4*
c + 4*d*x)*b + 2*e**(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b + a + b))
```

3.122 $\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 102

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\arctan(\sinh(c+dx))}{b^2d} - \frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} + \frac{(a+b)\sinh(c+dx)}{2abd\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
arctan(sinh(d*x+c))/b^2/d-1/2*(2*a-b)*(a+b)^(1/2)*arctan((a+b)^(1/2)*sinh(
d*x+c)/a^(1/2))/a^(3/2)/b^2/d+1/2*(a+b)*sinh(d*x+c)/a/b/d/(a+(a+b)*sinh(d*
x+c)^2)
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.99

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(a-b)\left((2a^2+ab-b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+4a^{3/2}\sqrt{a+b}\arctan\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a+b)\left((2\right.}{2a^{3/2}b^2\sqrt{a+}}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a - b)*((2*a^2 + a*b - b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]]
+ 4*a^(3/2)*Sqrt[a + b]*ArcTan[Tanh[(c + d*x)/2]]) + (a + b)*((2*a^2 + a*b
 - b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 4*a^(3/2)*Sqrt[a + b
]*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)] + 2*Sqrt[a]*b*(a + b)^(3/2)
*Sinh[c + d*x])/(2*a^(3/2)*b^2*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c +
d*x)]))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 4159, 316, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{(\sinh^2(c+dx)+1)((a+b)\sinh^2(c+dx)+a)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab((a+b)\sinh^2(c+dx)+a)}-\frac{\int \frac{-\left((a+b)\sinh^2(c+dx)\right)+a-b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2ab}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab((a+b)\sinh^2(c+dx)+a)}-\frac{\frac{(2a-b)(a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{2a\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}}{2ab}}{d}$$

$$\Big\downarrow \text{216}$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab((a+b)\sinh^2(c+dx)+a)}-\frac{\frac{(2a-b)(a+b)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{2a\arctan(\sinh(c+dx))}{b}}{2ab}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{2ab((a+b)\sinh^2(c+dx)+a)}-\frac{\frac{(2a-b)\sqrt{a+b}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{2a\arctan(\sinh(c+dx))}{b}}{2ab}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((-2*a*ArcTan[Sinh[c + d*x]])/b + ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqr
t[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b) + ((a + b)*Sinh[c + d
*x])/(2*a*b*(a + (a + b)*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(90) = 180$.

Time $= 244.37$ (sec) , antiderivative size $= 273$, normalized size of antiderivative $= 2.68$

method	result
derivativedivides	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2\left(\frac{\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(2a^2+ab-b^2)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right.}{2a\sqrt{(a+b)b}\sqrt{(\cdots}}\cdots\right.}{\cdots}\cdots\right.}{b^2}\cdots}{d}$
default	$\frac{\frac{2\arctan\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}-\frac{2\left(\frac{\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{(a+b)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{(2a^2+ab-b^2)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right.}{2a\sqrt{(a+b)b}\sqrt{(\cdots}}\cdots\right.}{\cdots}\cdots\right.}{b^2}\cdots}{d}$
risch	$\frac{e^{dx+c}(a+b)\left(e^{2dx+2c}-1\right)}{abd\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)}+\frac{i\ln\left(e^{dx+c}+i\right)}{d\,b^2}-\frac{i\ln\left(e^{dx+c}-i\right)}{d\,b^2}+\frac{\sqrt{-a(a+b)}\,\ln\left(e^{2c\cdots}\right.}{\cdots}$

input
```
int(sech(d*x+c)^5/(a+tanh(d*x+c)^2*b)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2/b^2*arctan(tanh(1/2*d*x+1/2*c))-2/b^2*((1/2*(a+b)*b/a*tanh(1/2*d*x+
1/2*c)^3-1/2*(a+b)*b/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tan
h(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(2*a^2+a*b-b^2)*(1/2
*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((
a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1078 vs. $2(90) = 180$.

Time = 0.14 (sec) , antiderivative size = 2140, normalized size of antiderivative = 20.98

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*b + b^2)*cosh(d*x + c)^3 + 12*(a*b + b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b + b^2)*sinh(d*x + c)^3 - ((2*a^2 + a*b - b^2)*cosh(d*x +
 c)^4 + 4*(2*a^2 + a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a^2 + a*b
 - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(
2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2
+ 2*a^2 + a*b - b^2 + 4*((2*a^2 + a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*
a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a + b)/a)*log(((a + b)*cos
h(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x
+ c)^4 - 2*(3*a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 - 3*a
- b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (3*a + b)*cosh(d*x + c
))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^
2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d
*x + c))*sqrt(-(a + b)/a) + a + b)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*co
sh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x
 + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a +
b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*((
a^2 + a*b)*cosh(d*x + c)^4 + 4*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c)^3 +
 (a^2 + a*b)*sinh(d*x + c)^4 + 2*(a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*(a^2 +
 a*b)*cosh(d*x + c)^2 + a^2 - a*b)*sinh(d*x + c)^2 + a^2 + a*b + 4*((a^2 +
 a*b)*cosh(d*x + c)^3 + (a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arcta...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}^5{(c+dx)}}{\left(a+b\tanh^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^5}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) - (a*e^c + b*e^c)*e^(d*x))/(a^2*b*d + a
*b^2*d + (a^2*b*d*e^(4*c) + a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(a^2*b*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(b^2*d) - 32*integ
rate(1/32*((2*a^2*e^(3*c) + a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (2*a^2*
e^c + a*b*e^c - b^2*e^c)*e^(d*x))/(a^2*b^2 + a*b^3 + (a^2*b^2*e^(4*c) + a*
b^3*e^(4*c))*e^(4*d*x) + 2*(a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x
)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^5\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 950, normalized size of antiderivative = 9.31

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(4*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**3 + 4*e**(4*c + 4*d*x)*atan(e**(
c + d*x))*a**2*b + 8*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**3 - 8*e**(2*c
+ 2*d*x)*atan(e**(c + d*x))*a**2*b + 4*atan(e**(c + d*x))*a**3 + 4*atan(e*
*(c + d*x))*a**2*b - 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 - e**(4*c + 4*d*x)*sqrt(a)*sqrt(
a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b + e**(4*c +
4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(
a))*b**2 - 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(
a + b) - sqrt(b))/sqrt(a))*a**2 + 6*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*a
tan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b - 2*e**(2*c + 2*d*x)
*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b*
*2 - 2*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(
a))*a**2 - sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/s
qrt(a))*a*b + sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*b**2 - 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x
)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 - e**(4*c + 4*d*x)*sqrt(a)*sqrt(a +
 b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b + e**(4*c + 4*d
*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))
*b**2 - 4*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a +
 b) + sqrt(b))/sqrt(a))*a**2 + 6*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*a...
```

3.123 $\int \frac{\mathrm{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 97

$$\int \frac{\mathrm{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\tanh(c+dx)}{b^2d} + \frac{(a+b)^2\tanh(c+dx)}{2ab^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/2*(3*a-b)*(a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/b^(5/2)/d+t
anh(d*x+c)/b^2/d+1/2*(a+b)^2*tanh(d*x+c)/a/b^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.77 (sec) , antiderivative size = 102, normalized size of antiderivative = 1.05

$$\int \frac{\mathrm{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{-\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\sqrt{b}(a+b)^2\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))} + 2\sqrt{b}\tanh(c+dx)}{2b^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2)) +
(Sqrt[b]*(a + b)^2*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)
])) + 2*Sqrt[b]*Tanh[c + d*x])/(2*b^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 92, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4158, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{1}{b^2}-\frac{a^2-b^2+2b(a+b)\tanh^2(c+dx)}{b^2\left(b\tanh^2(c+dx)+a\right)^2}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{(3a-b)(a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\tanh(c+dx)}{2ab^2\left(a+b\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{b^2}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)
*b^(5/2)) + Tanh[c + d*x]/b^2 + ((a + b)^2*Tanh[c + d*x])/(2*a*b^2*(a + b*
Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 305 vs. $2(85) = 170$.

Time = 0.71 (sec) , antiderivative size = 306, normalized size of antiderivative = 3.15

$$\frac{\frac{2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{b^{2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+1\right)}+\frac{\frac{2\left(\frac{\left(a^{2}+2ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{2a}+\frac{\left(a^{2}+2ab+b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}\right)}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a}+\left(3a^{2}+2ab-b^{2}\right)\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\right.}{\sqrt{\left(2\sqrt{(a+b)b}+a\right.}}\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)}}\right.}{b^{2}}}{d}$$

input
```
int(sech(d*x+c)^6/(a+tanh(d*x+c)^2*b)^2,x)
```

output
```
1/d*(2/b^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)+2/b^2*((1/2*(a^2+
2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3+1/2*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*
c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/
2*c)^2+a)+1/2*(3*a^2+2*a*b-b^2)*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/
2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((
a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2
)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((
a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1282 vs. $2(85) = 170$.

Time = 0.14 (sec) , antiderivative size = 2869, normalized size of antiderivative = 29.58

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[-1/4*(4*(3*a^3*b + 2*a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 16*(3*a^3*b + 2*a
^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(3*a^3*b + 2*a^2*b^2 - a
*b^3)*sinh(d*x + c)^4 + 12*a^3*b + 16*a^2*b^2 + 4*a*b^3 + 8*(3*a^3*b - a^2
*b^2)*cosh(d*x + c)^2 + 8*(3*a^3*b - a^2*b^2 + 3*(3*a^3*b + 2*a^2*b^2 - a*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^3 + 5*a^2*b + a*b^2 - b^3)*c
osh(d*x + c)^6 + 6*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x
+ c)^5 + (3*a^3 + 5*a^2*b + a*b^2 - b^3)*sinh(d*x + c)^6 + (9*a^3 + 3*a^2*
b - 5*a*b^2 + b^3)*cosh(d*x + c)^4 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 15
*(3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(
3*a^3 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^
2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*a^3 + 5*a^2*b + a*b^2 - b^3 +
(9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(3*a^3 + 5*a^2*b +
 a*b^2 - b^3)*cosh(d*x + c)^4 + 9*a^3 + 3*a^2*b - 5*a*b^2 + b^3 + 6*(9*a^3
 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(3*a^3
 + 5*a^2*b + a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(9*a^3 + 3*a^2*b - 5*a*b^2 +
 b^3)*cosh(d*x + c)^3 + (9*a^3 + 3*a^2*b - 5*a*b^2 + b^3)*cosh(d*x + c))*s
inh(d*x + c))*sqrt(-a*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d
*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 +...
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(85) = 170$.

Time = 0.24 (sec) , antiderivative size = 209, normalized size of antiderivative = 2.15

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{3a^2+4ab+b^2+2(3a^2-ab)e^{(-2dx-2c)}+(3a^2+2ab-b^2)e^{(-4dx-4c)}}{(a^2b^2+ab^3+(3a^2b^2-ab^3)e^{(-2dx-2c)}+(3a^2b^2-ab^3)e^{(-4dx-4c)}+(a^2b^2+ab^3)e^{(-6dx-6c)})d} + \frac{(3a^2+2ab-b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{2\sqrt{ab}ab^2d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
(3*a^2 + 4*a*b + b^2 + 2*(3*a^2 - a*b)*e^(-2*d*x - 2*c) + (3*a^2 + 2*a*b -
 b^2)*e^(-4*d*x - 4*c))/((a^2*b^2 + a*b^3 + (3*a^2*b^2 - a*b^3)*e^(-2*d*x
- 2*c) + (3*a^2*b^2 - a*b^3)*e^(-4*d*x - 4*c) + (a^2*b^2 + a*b^3)*e^(-6*d*
x - 6*c))*d) + 1/2*(3*a^2 + 2*a*b - b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2
*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 232 vs. $2(85) = 170$.

Time = 0.35 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.39

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{(3a^2+2ab-b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}ab^2} + \frac{2\,(3a^2e^{(4dx+4c)}+2abe^{(4dx+4c)}-b^2e^{(4dx+4c)}+6a^2e^{(2dx+2c)}-2abe^{(2dx+2c}}{(ae^{(6dx+6c)}+be^{(6dx+6c)}+3ae^{(4dx+4c)}-be^{(4dx+4c)}+3ae^{(2dx+2c)}-be^{(2dx}}}{2d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((3*a^2 + 2*a*b - b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2
*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a*b^2) + 2*(3*a^2*e^(4*d*x + 4*c) + 2*a
*b*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 6*a^2*e^(2*d*x + 2*c) - 2*a*b*e
^(2*d*x + 2*c) + 3*a^2 + 4*a*b + b^2)/((a*e^(6*d*x + 6*c) + b*e^(6*d*x + 6
*c) + 3*a*e^(4*d*x + 4*c) - b*e^(4*d*x + 4*c) + 3*a*e^(2*d*x + 2*c) - b*e^
(2*d*x + 2*c) + a + b)*a*b^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^6\,{\left(b\,{\tanh\left(c+d\,x\right)}^2+a\right)}^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1449, normalized size of antiderivative = 14.94

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
( - 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**3 - 5*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*
x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b - e**(6*c + 6*d*x)*sqrt(b)*sqrt(
a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 + e**(6*c + 6
*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b
**3 - 9*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a**3 - 3*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b + 5*e**(4*c + 4*d*x)*sqrt(b)*s
qrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 - e**(4*c
 + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*b**3 - 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b
) - sqrt(b))/sqrt(a))*a**3 - 3*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(
c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b + 5*e**(2*c + 2*d*x)*sqrt(
b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 - e**
(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sq
rt(a))*b**3 - 3*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/
sqrt(a))*a**3 - 5*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*a**2*b - sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a*b**2 + sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqr
t(b))/sqrt(a))*b**3 + 3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + ...
```

3.124 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 155

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(4a+5b)\arctan(\sinh(c+dx))}{2b^3d} - \frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{(a+b)(2a+b)\sinh(c+dx)}{2ab^2d\left(a+(a+b)\sinh^2(c+dx)\right)} - \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2bd\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/2*(4*a+5*b)*arctan(sinh(d*x+c))/b^3/d-1/2*(4*a-b)*(a+b)^(3/2)*arctan((a+
b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(3/2)/b^3/d+1/2*(a+b)*(2*a+b)*sinh(d*x+c)/
a/b^2/d/(a+(a+b)*sinh(d*x+c)^2)-1/2*sech(d*x+c)*tanh(d*x+c)/b/d/(a+(a+b)*s
inh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.94 (sec) , antiderivative size = 265, normalized size of antiderivative = 1.71

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$
$$= \frac{2\sqrt{a}b(a+b)^{5/2}\sinh(c+dx)+(a-b)\left((4a-b)(a+b)^2\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)+2a^{3/2}\sqrt{a+b}(4a+5b)\right.}{}$$

input
```
Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*Sqrt[a]*b*(a + b)^(5/2)*Sinh[c + d*x] + (a - b)*((4*a - b)*(a + b)^2*Ar
cTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5
*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c
 + d*x]) + (a + b)*Cosh[2*(c + d*x)]*((4*a - b)*(a + b)^2*ArcTan[(Sqrt[a]*
Csch[c + d*x])/Sqrt[a + b]] + 2*a^(3/2)*Sqrt[a + b]*(4*a + 5*b)*ArcTan[Tan
h[(c + d*x)/2]] + a^(3/2)*b*Sqrt[a + b]*Sech[c + d*x]*Tanh[c + d*x]))/(2*a
^(3/2)*b^3*Sqrt[a + b]*d*(a - b + (a + b)*Cosh[2*(c + d*x)]))
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.05, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 4159, 316, 402, 27, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^7}{(a-b\tan(ic+idx)^2)^2}\,dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)^2\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int \frac{-3(a+b)\sinh^2(c+dx)+a+2b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{2b}-\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\int \frac{2\left(2a^2+2ba-b^2-(a+b)(2a+b)\sinh^2(c+dx)\right)}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2ab}}{2b}-\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\int \frac{2a^2+2ba-b^2-(a+b)(2a+b)\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{ab}}{2b}-\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{(4a-b)(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{a(4a+5b)\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}}{ab}}{2b}-\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{(4a-b)(a+b)^2\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{a(4a+5b)\arctan(\sinh(c+dx))}{b}}{ab}}{2b}-\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{\frac{(a+b)(2a+b)\sinh(c+dx)}{ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{(4a-b)(a+b)^{3/2}\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{a(4a+5b)\arctan(\sinh(c+dx))}{b}}{ab}}{2b}-\frac{\sinh(c+dx)}{2b\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}}{d}$$

input
```
Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*Sinh[c + d*x]/(b*(1 + Sinh[c + d*x]^2)*(a + (a + b)*Sinh[c + d*x]^2)
) + (-((-((a*(4*a + 5*b)*ArcTan[Sinh[c + d*x]])/b) + ((4*a - b)*(a + b)^(3
/2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b)) + ((a
 + b)*(2*a + b)*Sinh[c + d*x])/(a*b*(a + (a + b)*Sinh[c + d*x]^2)))/(2*b))
/d
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 350 vs. $2(139) = 278$.

Time = 0.62 (sec) , antiderivative size = 351, normalized size of antiderivative = 2.26

$$-\frac{\frac{2\left(\frac{\frac{b\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{2a}-\frac{b\left(a^2+2ab+b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{2a}}{\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4 a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2 a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a}+\frac{\left(4a^3+7a^2b+2b^2a-b^3\right)\left(\frac{\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{2a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{\left(\sqrt{(a+b)b}-b\cdots\right)}{2a\sqrt{(\cdots}}\right)}{2}\cdots\right)}{b^3}\cdots}{d}$$

input
```
int(sech(d*x+c)^7/(a+tanh(d*x+c)^2*b)^2,x)
```

output
```
1/d*(-2/b^3*((1/2*b*(a^2+2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^3-1/2*b*(a^2+2*a
*b+b^2)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2
*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)+1/2*(4*a^3+7*a^2*b+2*a*b^2-b^3)*(1/2*
(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a
+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))+2/b^3*((-
1/2*b*tanh(1/2*d*x+1/2*c)^3+1/2*b*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c
)^2+1)^2+1/2*(5*b+4*a)*arctan(tanh(1/2*d*x+1/2*c))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3490 vs. $2(139) = 278$.

Time = 0.18 (sec) , antiderivative size = 6396, normalized size of antiderivative = 41.26

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(sech(c + d*x)**7/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\operatorname{sech}\left(dx+c\right)^7}{\left(b\tanh\left(dx+c\right)^2+a\right)^2}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
((2*a^2*e^(7*c) + 3*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (2*a^2*e^(5*c)
- a*b*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (2*a^2*e^(3*c) - a*b*e^(3*c) + b^
2*e^(3*c))*e^(3*d*x) - (2*a^2*e^c + 3*a*b*e^c + b^2*e^c)*e^(d*x))/(4*a^2*b
^2*d*e^(6*d*x + 6*c) + 4*a^2*b^2*d*e^(2*d*x + 2*c) + a^2*b^2*d + a*b^3*d +
 (a^2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x) + 2*(3*a^2*b^2*d*e^(4*c)
- a*b^3*d*e^(4*c))*e^(4*d*x)) + (4*a*e^c + 5*b*e^c)*arctan(e^(d*x + c))*e^
(-c)/(b^3*d) - 128*integrate(1/128*((4*a^3*e^(3*c) + 7*a^2*b*e^(3*c) + 2*a
*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (4*a^3*e^c + 7*a^2*b*e^c + 2*a*b^2
*e^c - b^3*e^c)*e^(d*x))/(a^2*b^3 + a*b^4 + (a^2*b^3*e^(4*c) + a*b^4*e^(4*
c))*e^(4*d*x) + 2*(a^2*b^3*e^(2*c) - a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^7\,\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2),x)
```

output
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2), x)
```

Reduce [B] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 2051, normalized size of antiderivative = 13.23

$$\int \frac{\mathrm{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(8*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**4 + 18*e**(8*c + 8*d*x)*atan(e**
(c + d*x))*a**3*b + 10*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**2*b**2 + 32*
e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**4 + 40*e**(6*c + 6*d*x)*atan(e**(c
+ d*x))*a**3*b + 48*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**4 + 44*e**(4*c
+ 4*d*x)*atan(e**(c + d*x))*a**3*b - 20*e**(4*c + 4*d*x)*atan(e**(c + d*x)
)*a**2*b**2 + 32*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**4 + 40*e**(2*c + 2
*d*x)*atan(e**(c + d*x))*a**3*b + 8*atan(e**(c + d*x))*a**4 + 18*atan(e**(
c + d*x))*a**3*b + 10*atan(e**(c + d*x))*a**2*b**2 - 4*e**(8*c + 8*d*x)*sq
rt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3
- 7*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a**2*b - 2*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e*
*(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 + e**(8*c + 8*d*x)*sqrt(
a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 - 1
6*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**3 - 12*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c
 + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b + 4*e**(6*c + 6*d*x)*sqrt(a
)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 -
24*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**3 - 10*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(
c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b + 12*e**(4*c + 4*d*x)*s...
```

3.125 $\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 198

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(a+7b)x}{2(a+b)^4} + \frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^4d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(2a-b)b\tanh(c+dx)}{4a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)b(4a+b)\tanh(c+dx)}{8a^2(a+b)^3d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
1/2*(a+7*b)*x/(a+b)^4+1/8*b^(3/2)*(35*a^2+14*a*b+3*b^2)*arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^4/d+1/2*cosh(d*x+c)*sinh(d*x+c)/(a+b)/d/(a
+b*tanh(d*x+c)^2)^2-1/4*(2*a-b)*b*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)
^2)^2-1/8*(a-3*b)*b*(4*a+b)*tanh(d*x+c)/a^2/(a+b)^3/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.34 (sec) , antiderivative size = 164, normalized size of antiderivative = 0.83

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{4(a+7b)(c+dx)+\frac{b^{3/2}(35a^2+14ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}+2(a+b)\sinh(2(c+dx))+\frac{4b^3(a+b)\sinh(2(c+dx}{a(a-b+(a+b)\cosh(2(c+d}}{8(a+b)^4d}$$

input
```
Integrate[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(4*(a + 7*b)*(c + d*x) + (b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b
]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2) + 2*(a + b)*Sinh[2*(c + d*x)] + (4*b^3*
(a + b)*Sinh[2*(c + d*x)])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (b^
2*(a + b)*(13*a + 3*b)*Sinh[2*(c + d*x)])/(a^2*(a - b + (a + b)*Cosh[2*(c
+ d*x)])))/(8*(a + b)^4*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3042, 4158, 316, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sec(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \ 4158$$
$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)^2\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{\int \frac{5b\tanh^2(c+dx)+a+2b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{-\frac{\int -\frac{2\left(2a^2+8ba+3b^2+3(2a-b)b\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{\frac{\int \frac{2a^2+8ba+3b^2+3(2a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{2a(a+b)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}+\frac{\tanh(c+dx)}{2(a+b)\left(1-\tanh^2(c+dx)\right)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{-\frac{\frac{\int -\frac{4a^3+24ba^2+11b^2a+3b^3+(a-3b)b(4a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2a(a+b)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}+\frac{\cdots}{2(a+b)(1-\tanh^2\cdots}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{\frac{\frac{\int \frac{4a^3+24ba^2+11b^2a+3b^3+(a-3b)b(4a+b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2a(a+b)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}+\frac{\tan\cdots}{2(a+b)(1-\tanh^2(c\cdots}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{\frac{\frac{\frac{b^2\left(35a^2+14ab+3b^2\right)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{4a^2(a+7b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2a(a+b)}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2a(a+b)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx\cdots\right.}}{2(a+b)}\cdots}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{\frac{\frac{\frac{4a^2(a+7b)\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2a(a+b)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}\ \cdots}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{\frac{\frac{b^{3/2}\left(35a^2+14ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{4a^2(a+7b)\text{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)}-\frac{b(a-3b)(4a+b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{2a(a+b)}-\frac{b(2a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{2(a+b)}+\frac{\cdots}{2(a\cdots}}{d}$$

input
```
Int[Cosh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(2*(a + b)*(1 - Tanh[c + d*x]^2)*(a + b*Tanh[c + d*x]^2)^2)
 + (-1/2*((2*a - b)*b*Tanh[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)^2)
 + (((b^(3/2)*(35*a^2 + 14*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqr
t[a]])/(Sqrt[a]*(a + b)) + (4*a^2*(a + 7*b)*ArcTanh[Tanh[c + d*x]])/(a + b
))/(2*a*(a + b)) - ((a - 3*b)*b*(4*a + b)*Tanh[c + d*x])/(2*a*(a + b)*(a +
 b*Tanh[c + d*x]^2)))/(2*a*(a + b)))/(2*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 500 vs. $2(180) = 360$.

Time = 28.59 (sec) , antiderivative size = 501, normalized size of antiderivative = 2.53

method	result
derivativedivides	$-\frac{2b^2\left(\frac{-\frac{(13a^2+18ab+5b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{(39a^3+98a^2b+71b^2a+12b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{(39a^3+98a^2b+71b^2a+12b^3)\tanh\left(\frac{d}{}\right.}{8a^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{}$
default	$-\frac{2b^2\left(\frac{-\frac{(13a^2+18ab+5b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{(39a^3+98a^2b+71b^2a+12b^3)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}-\frac{(39a^3+98a^2b+71b^2a+12b^3)\tanh\left(\frac{d}{}\right.}{8a^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}\right.}{}$
risch	$\frac{xa}{2(a^3+3a^2b+3b^2a+b^3)(a+b)}+\frac{7xb}{2(a^3+3a^2b+3b^2a+b^3)(a+b)}+\frac{e^{2dx+2c}}{8(a^3+3a^2b+3b^2a+b^3)d}-\frac{e^{-2dx-2c}}{8(a^3+3a^2b+3b^2a+b^3)d}$

input
```
int(cosh(d*x+c)^2/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*b^2/(a+b)^4*((-1/8*(13*a^2+18*a*b+5*b^2)/a*tanh(1/2*d*x+1/2*c)^7-1
/8*(39*a^3+98*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^5-1/8*(39*a^3
+98*a^2*b+71*a*b^2+12*b^3)/a^2*tanh(1/2*d*x+1/2*c)^3-1/8*(13*a^2+18*a*b+5*
b^2)/a*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)
^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/8/a*(35*a^2+14*a*b+3*b^2)*(1/2*(a+((
a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arc
tan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a
+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/2/(a+b)
^3/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)+1)+1/2*(a+7*
b)/(a+b)^4*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)^2
+1/2/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+1/2/(a+b)^4*(-a-7*b)*ln(tanh(1/2*d*x+
1/2*c)-1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6780 vs. $2(180) = 360$.

Time = 0.37 (sec) , antiderivative size = 13887, normalized size of antiderivative = 70.14

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1806 vs. 2(180) = 360.

Time = 0.41 (sec) , antiderivative size = 1806, normalized size of antiderivative = 9.12

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
3/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d) - 3/4*b*log(2*(a - b)*e^(-2*d
*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 4*a^3*b + 6*a^2*b^2
+ 4*a*b^3 + b^4)*d) - 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b^3 - b^4)*arctan(1
/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 4*a^5*b + 6*a^4*b^
2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) + 3/32*(5*a^3*b - 15*a^2*b^2 - 5*a*b
^3 - b^4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^6 +
 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/16*(15*a^2*b
+ 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b
))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/16*(9*a^4*b + 4
*a^3*b^2 - 22*a^2*b^3 - 20*a*b^4 - 3*b^5 + 3*(3*a^4*b - 22*a^3*b^2 - 20*a^
2*b^3 + 6*a*b^4 + b^5)*e^(6*d*x + 6*c) + (27*a^4*b - 156*a^3*b^2 + 110*a^2
*b^3 - 36*a*b^4 - 9*b^5)*e^(4*d*x + 4*c) + (27*a^4*b - 86*a^3*b^2 - 84*a^2
*b^3 + 38*a*b^4 + 9*b^5)*e^(2*d*x + 2*c))/((a^8 + 6*a^7*b + 15*a^6*b^2 + 2
0*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6 + (a^8 + 6*a^7*b + 15*a^6*b^2
 + 20*a^5*b^3 + 15*a^4*b^4 + 6*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*(a^8
 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 - 4*a^3*b^5 - a^2*b^6)*e^(6*d*x + 6*c)
+ 2*(3*a^8 + 10*a^7*b + 13*a^6*b^2 + 12*a^5*b^3 + 13*a^4*b^4 + 10*a^3*b^5
+ 3*a^2*b^6)*e^(4*d*x + 4*c) + 4*(a^8 + 4*a^7*b + 5*a^6*b^2 - 5*a^4*b^4 -
4*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) - 1/16*(9*a^4*b + 4*a^3*b^2 - ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 546 vs. 2(180) = 360.

Time = 1.10 (sec) , antiderivative size = 546, normalized size of antiderivative = 2.76

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{4\,(dx+c)(a+7\,b)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} - \frac{\left(2\,ae^{(2\,dx+2\,c)}+14\,be^{(2\,dx+2\,c)}+a+b\right)e^{(-2\,dx-2\,c)}}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4} + \frac{\left(35\,a^2b^2+14\,ab^3+3\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}}{2\,\sqrt{ab}}\right.}{\left(a^6+4\,a^5b+6\,a^4b^2+4\,a^3b^3+a^2b^4\right)\sqrt{ab}}}{}$$

input
```
integrate(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*(4*(d*x + c)*(a + 7*b)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - (
2*a*e^(2*d*x + 2*c) + 14*b*e^(2*d*x + 2*c) + a + b)*e^(-2*d*x - 2*c)/(a^4
+ 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) + (35*a^2*b^2 + 14*a*b^3 + 3*b^4)*a
rctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6
 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*sqrt(a*b)) + e^(2*d*x + 2*c)
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(13*a^3*b^2*e^(6*d*x + 6*c) - a^2*b^3
*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 3*b^5*e^(6*d*x + 6*c) + 39*a
^3*b^2*e^(4*d*x + 4*c) - 17*a^2*b^3*e^(4*d*x + 4*c) + 33*a*b^4*e^(4*d*x +
4*c) + 9*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d*x + 2*c) + 13*a^2*b^3*e^(
2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 9*b^5*e^(2*d*x + 2*c) + 13*a^3*b
^2 + 29*a^2*b^3 + 19*a*b^4 + 3*b^5)/((a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^
3 + a^2*b^4)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c)
- 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh(c+d\,x)^2}{\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 4226, normalized size of antiderivative = 21.34

$$\int \frac{\cosh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(cosh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(140*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**5*b + 196*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**
(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b**2 - 72*e**(10*c + 10*d*x
)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*
b**3 - 184*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
b) - sqrt(b))/sqrt(a))*a**2*b**4 - 68*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*a
tan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**5 - 12*e**(10*c + 1
0*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*
b**6 + 560*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
 - sqrt(b))/sqrt(a))*a**5*b - 336*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e
**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b**2 - 736*e**(8*c + 8*d*
x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3
*b**3 + 288*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b
) - sqrt(b))/sqrt(a))*a**2*b**4 + 176*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*ata
n((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**5 + 48*e**(8*c + 8*d*
x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**6
 + 840*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**5*b - 1064*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(
c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b**2 + 912*e**(6*c + 6*d*x)*
sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3...
```

3.126 $\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 154

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}d} + \frac{\sinh(c+dx)}{(a+b)^3d} + \frac{b^3\sinh(c+dx)}{4a(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3b^2(4a+b)\sinh(c+dx)}{8a^2(a+b)^3d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output

```
3/8*b*(8*a^2+4*a*b+b^2)*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/(a
+b)^(7/2)/d+sinh(d*x+c)/(a+b)^3/d+1/4*b^3*sinh(d*x+c)/a/(a+b)^3/d/(a+(a+b)
*sinh(d*x+c)^2)^2+3/8*b^2*(4*a+b)*sinh(d*x+c)/a^2/(a+b)^3/d/(a+(a+b)*sinh(
d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.30 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.88

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$=\frac{\frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^{7/2}}+\frac{\sinh(c+dx)\left(8+\frac{3b^3}{a^2\left(a+(a+b)\sinh^2(c+dx)\right)}+\frac{2b^2\left(6a+b+6(a+b)\sinh^2(c+dx)\right)}{a\left(a+(a+b)\sinh^2(c+dx)\right)^2}\right)}{(a+b)^3}}{8d}$$

input
```
Integrate[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(
a^(5/2)*(a + b)^(7/2)) + (Sinh[c + d*x]*(8 + (3*b^3)/(a^2*(a + (a + b)*Sin
h[c + d*x]^2)) + (2*b^2*(6*a + b + 6*(a + b)*Sinh[c + d*x]^2))/(a*(a + (a
+ b)*Sinh[c + d*x]^2)^2)))/(a + b)^3)/(8*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.95, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3042, 4159, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{1}{\sec(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{4159}$$
$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^3}{\left((a+b)\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\downarrow \text{300}$$

$$\frac{\int \left(\frac{3b(a+b)^2\sinh^4(c+dx)+3b(a+b)(2a+b)\sinh^2(c+dx)+b(3a^2+3ba+b^2)}{(a+b)^3((a+b)\sinh^2(c+dx)+a)^3}+\frac{1}{(a+b)^3}\right)d\sinh(c+dx)}{d}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{3b^2(4a+b)\sinh(c+dx)}{8a^2(a+b)^3((a+b)\sinh^2(c+dx)+a)}+\frac{3b(8a^2+4ab+b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{7/2}}+\frac{b^3\sinh(c+dx)}{4a(a+b)^3((a+b)\sinh^2(c+dx)+a)^2}+\frac{\sinh(c+dx)}{(a+b)^3}}{d}$$

input
```
Int[Cosh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*b*(8*a^2 + 4*a*b + b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(
8*a^(5/2)*(a + b)^(7/2)) + Sinh[c + d*x]/(a + b)^3 + (b^3*Sinh[c + d*x])/(
4*a*(a + b)^3*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*b^2*(4*a + b)*Sinh[c +
 d*x])/(8*a^2*(a + b)^3*(a + (a + b)*Sinh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 374 vs. 2(140) = 280.

Time = 11.77 (sec) , antiderivative size = 375, normalized size of antiderivative = 2.44

method	result
derivativedivides	$-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}+\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\right.}{8a^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right.}\right.}{}$
default	$-\frac{1}{(a+b)^3\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{2b\left(\frac{-\frac{b(12a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8a}-\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2}+\frac{3\left(4a^2+15ab+4b^2\right)b\tanh\left(\right.}{8a^2}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right.}\right.}{}$
risch	$\frac{e^{dx+c}}{2(a^3+3a^2b+3b^2a+b^3)d}-\frac{e^{-dx-c}}{2(a^3+3a^2b+3b^2a+b^3)d}+\frac{(12\,e^{6dx+6c}a^2+15\,e^{6dx+6c}ab+3\,e^{6dx+6c}b^2+12\,e^{4dx+4c}a^2-2}{4(e^{4dx+4c}a+b\,e^{4dx+4c}}$

input
```
int(cosh(d*x+c)/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)^3/(tanh(1/2*d*x+1/2*c)-1)+2/(a+b)^3*b*((-1/8*b*(12*a+5*b)/a*
tanh(1/2*d*x+1/2*c)^7-3/8*(4*a^2+15*a*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^5
+3/8*(4*a^2+15*a*b+4*b^2)/a^2*b*tanh(1/2*d*x+1/2*c)^3+1/8*b*(12*a+5*b)/a*t
anh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b
*tanh(1/2*d*x+1/2*c)^2+a)^2+3/8/a*(8*a^2+4*a*b+b^2)*(1/2*(((a+b)*b)^(1/2)+
b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a
/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-1/(a+b)^3/(tanh(1/2*d*x+1/
2*c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6267 vs. $2(140) = 280$.

Time = 0.33 (sec) , antiderivative size = 11399, normalized size of antiderivative = 74.02

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\cosh\left(dx+c\right)}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - 2*(a^4*e^(10*c) + 2*a^3*b*e^(10*c) + a
^2*b^2*e^(10*c))*e^(10*d*x) - (6*a^4*e^(8*c) - 4*a^3*b*e^(8*c) + 2*a^2*b^2
*e^(8*c) + 15*a*b^3*e^(8*c) + 3*b^4*e^(8*c))*e^(8*d*x) - (4*a^4*e^(6*c) -
8*a^3*b*e^(6*c) + 32*a^2*b^2*e^(6*c) - 25*a*b^3*e^(6*c) - 9*b^4*e^(6*c))*e
^(6*d*x) + (4*a^4*e^(4*c) - 8*a^3*b*e^(4*c) + 32*a^2*b^2*e^(4*c) - 25*a*b^
3*e^(4*c) - 9*b^4*e^(4*c))*e^(4*d*x) + (6*a^4*e^(2*c) - 4*a^3*b*e^(2*c) +
2*a^2*b^2*e^(2*c) + 15*a*b^3*e^(2*c) + 3*b^4*e^(2*c))*e^(2*d*x))/((a^7*d*e
^(9*c) + 5*a^6*b*d*e^(9*c) + 10*a^5*b^2*d*e^(9*c) + 10*a^4*b^3*d*e^(9*c) +
 5*a^3*b^4*d*e^(9*c) + a^2*b^5*d*e^(9*c))*e^(9*d*x) + 4*(a^7*d*e^(7*c) + 3
*a^6*b*d*e^(7*c) + 2*a^5*b^2*d*e^(7*c) - 2*a^4*b^3*d*e^(7*c) - 3*a^3*b^4*d
*e^(7*c) - a^2*b^5*d*e^(7*c))*e^(7*d*x) + 2*(3*a^7*d*e^(5*c) + 7*a^6*b*d*e
^(5*c) + 6*a^5*b^2*d*e^(5*c) + 6*a^4*b^3*d*e^(5*c) + 7*a^3*b^4*d*e^(5*c) +
 3*a^2*b^5*d*e^(5*c))*e^(5*d*x) + 4*(a^7*d*e^(3*c) + 3*a^6*b*d*e^(3*c) + 2
*a^5*b^2*d*e^(3*c) - 2*a^4*b^3*d*e^(3*c) - 3*a^3*b^4*d*e^(3*c) - a^2*b^5*d
*e^(3*c))*e^(3*d*x) + (a^7*d*e^c + 5*a^6*b*d*e^c + 10*a^5*b^2*d*e^c + 10*a
^4*b^3*d*e^c + 5*a^3*b^4*d*e^c + a^2*b^5*d*e^c)*e^(d*x)) + 1/2*integrate(3
/2*((8*a^2*b*e^(3*c) + 4*a*b^2*e^(3*c) + b^3*e^(3*c))*e^(3*d*x) + (8*a^2*b
*e^c + 4*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*
b^3 + a^2*b^4 + (a^6*e^(4*c) + 4*a^5*b*e^(4*c) + 6*a^4*b^2*e^(4*c) + 4*a^3
*b^3*e^(4*c) + a^2*b^4*e^(4*c))*e^(4*d*x) + 2*(a^6*e^(2*c) + 2*a^5*b*e^...
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\cosh(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(cosh(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 3187, normalized size of antiderivative = 20.69

$$\int \frac{\cosh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(cosh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(24*e**(9*c + 9*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a**4*b + 60*e**(9*c + 9*d*x)*sqrt(a)*sqrt(a + b)*atan((e
**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 + 51*e**(9*c + 9*d*x
)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a
**2*b**3 + 18*e**(9*c + 9*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt
(a + b) - sqrt(b))/sqrt(a))*a*b**4 + 3*e**(9*c + 9*d*x)*sqrt(a)*sqrt(a + b
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5 + 96*e**(7*c + 7
*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*a**4*b + 48*e**(7*c + 7*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqr
t(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 - 84*e**(7*c + 7*d*x)*sqrt(a)*sqrt(
a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**3 - 48*e
**(7*c + 7*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a*b**4 - 12*e**(7*c + 7*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c
+ d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5 + 144*e**(5*c + 5*d*x)*sqrt(a)
*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b - 2
4*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**3*b**2 + 114*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan(
(e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**3 + 60*e**(5*c + 5*d
*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))
*a*b**4 + 18*e**(5*c + 5*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sq...
```

3.127 $\int \frac{\text{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 140

$$\int \frac{\text{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d} + \frac{b^2\sinh(c+dx)}{4a(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{b(8a+3b)\sinh(c+dx)}{8a^2(a+b)^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/8*(8*a^2+8*a*b+3*b^2)*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/(a
+b)^(5/2)/d+1/4*b^2*sinh(d*x+c)/a/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)^2)^2+1/8*
b*(8*a+3*b)*sinh(d*x+c)/a^2/(a+b)^2/d/(a+(a+b)*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.69 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.96

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-\frac{(8a^2+8ab+3b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{2\sqrt{a}b(8a^2-ab-3b^2+(8a^2+11ab+3b^2)\cosh(2(c+dx)))\sinh(c+dx)}{(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[a + b]])/(
a + b)^(5/2)) + (2*Sqrt[a]*b*(8*a^2 - a*b - 3*b^2 + (8*a^2 + 11*a*b + 3*b^
2)*Cosh[2*(c + d*x)])*Sinh[c + d*x])/((a + b)^2*(a - b + (a + b)*Cosh[2*(c
 + d*x)])^2))/(8*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.238$, Rules used = {3042, 4159, 315, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\left(\sinh^2(c+dx)+1\right)^2}{\left((a+b)\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \color{blue}{315}$$

$$\frac{\frac{\int \frac{(4a+b)\sinh^2(c+dx)+4a+3b}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4a(a+b)}+\frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \color{blue}{298}$$

$$\frac{\frac{\frac{\left(8a^2+8ab+3b^2\right)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a(a+b)}+\frac{3b(2a+b)\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{4a(a+b)}+\frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \color{blue}{218}$$

$$\frac{\frac{\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}}+\frac{3b(2a+b)\sinh(c+dx)}{2a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)}}{4a(a+b)}+\frac{b\sinh(c+dx)\left(\sinh^2(c+dx)+1\right)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b*Sinh[c + d*x]*(1 + Sinh[c + d*x]^2))/(4*a*(a + b)*(a + (a + b)*Sinh[c
+ d*x]^2)^2) + (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Sinh[c + d*x]
)/Sqrt[a]])/(2*a^(3/2)*(a + b)^(3/2)) + (3*b*(2*a + b)*Sinh[c + d*x])/(2*a
*(a + b)*(a + (a + b)*Sinh[c + d*x]^2)))/(4*a*(a + b)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(126) = 252$.

Time = 29.85 (sec) , antiderivative size = 394, normalized size of antiderivative = 2.81

method	result
derivativedivides	$\frac{\frac{-\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a\left(a^2+2ab+b^2\right)}-\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2\left(a^2+2ab+b^2\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a\left(a^2+2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}-}{d}$
default	$\frac{\frac{-\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a\left(a^2+2ab+b^2\right)}-\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2\left(a^2+2ab+b^2\right)}+\frac{\left(8a^2+29ab+12b^2\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2\left(a^2+2ab+b^2\right)}+\frac{b(8a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a\left(a^2+2ab+b^2\right)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}-}{d}$
risch	$\frac{(8\,e^{6dx+6c}a^2+11\,e^{6dx+6c}ab+3\,e^{6dx+6c}b^2+8\,e^{4dx+4c}a^2-13\,e^{4dx+4c}ab-9\,e^{4dx+4c}b^2-8\,e^{2dx+2c}a^2+13\,e^{2dx+2c}ba+9b^2}{4(a^2+2ab+b^2)\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)^2d\,a^2}$

input
```
int(sech(d*x+c)/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/8*b*(8*a+5*b)/a/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/8*(8*a^
2+29*a*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/8*(8*a^2+29
*a*b+12*b^2)/a^2*b/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+1/8*b*(8*a+5*b)/a
/(a^2+2*a*b+b^2)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*
d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/4/a*(8*a^2+8*a*b+3*b^2)/(a
^2+2*a*b+b^2)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/
2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)
*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4385 vs. $2(126) = 252$.

Time = 0.18 (sec) , antiderivative size = 7917, normalized size of antiderivative = 56.55

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)}{(b\tanh(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((8*a^2*b*e^(7*c) + 11*a*b^2*e^(7*c) + 3*b^3*e^(7*c))*e^(7*d*x) + (8*a
^2*b*e^(5*c) - 13*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) - (8*a^2*b*e^(3
*c) - 13*a*b^2*e^(3*c) - 9*b^3*e^(3*c))*e^(3*d*x) - (8*a^2*b*e^c + 11*a*b^
2*e^c + 3*b^3*e^c)*e^(d*x))/(a^6*d + 4*a^5*b*d + 6*a^4*b^2*d + 4*a^3*b^3*d
 + a^2*b^4*d + (a^6*d*e^(8*c) + 4*a^5*b*d*e^(8*c) + 6*a^4*b^2*d*e^(8*c) +
4*a^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^6*d*e^(6*c) + 2*
a^5*b*d*e^(6*c) - 2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(
3*a^6*d*e^(4*c) + 4*a^5*b*d*e^(4*c) + 2*a^4*b^2*d*e^(4*c) + 4*a^3*b^3*d*e^
(4*c) + 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^6*d*e^(2*c) + 2*a^5*b*d*e^(2
*c) - 2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*integrate(1/
8*((8*a^2*e^(3*c) + 8*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (8*a^2*e^c
+ 8*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (
a^5*e^(4*c) + 3*a^4*b*e^(4*c) + 3*a^3*b^2*e^(4*c) + a^2*b^3*e^(4*c))*e^(4*
d*x) + 2*(a^5*e^(2*c) + a^4*b*e^(2*c) - a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))
*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 14353, normalized size of antiderivative = 102.52

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(32*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*tanh(c + d*x)**4*a**5*b**2 + 104*e**(8*c + 8*d*x)*sqrt(a
)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*tanh(c +
d*x)**4*a**4*b**3 + 132*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c +
 d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*tanh(c + d*x)**4*a**3*b**4 + 83*e**(
8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))
/sqrt(a))*tanh(c + d*x)**4*a**2*b**5 + 26*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a
+ b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*tanh(c + d*x)**4*a
*b**6 + 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a +
 b) - sqrt(b))/sqrt(a))*tanh(c + d*x)**4*b**7 + 64*e**(8*c + 8*d*x)*sqrt(a
)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*tanh(c +
d*x)**2*a**6*b + 208*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*
x)*sqrt(a + b) - sqrt(b))/sqrt(a))*tanh(c + d*x)**2*a**5*b**2 + 264*e**(8*
c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/s
qrt(a))*tanh(c + d*x)**2*a**4*b**3 + 166*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a +
 b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*tanh(c + d*x)**2*a*
*3*b**4 + 52*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(
a + b) - sqrt(b))/sqrt(a))*tanh(c + d*x)**2*a**2*b**5 + 6*e**(8*c + 8*d*x)
*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*ta
nh(c + d*x)**2*a*b**6 + 32*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e...
```

3.128 $\int \frac{\text{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 96

$$\int \frac{\text{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\tanh(c+dx)}{4ad\,(a+b\tanh^2(c+dx))^2} + \frac{3\tanh(c+dx)}{8a^2d\,(a+b\tanh^2(c+dx))}$$

output
```
3/8*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(1/2)/d+1/4*tanh(d*x+c)/
a/d/(a+b*tanh(d*x+c)^2)^2+3/8*tanh(d*x+c)/a^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.45 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.80

$$\int \frac{\text{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\frac{3\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{b}} + \frac{\tanh(c+dx)(5a+3b\tanh^2(c+dx))}{a^2(a+b\tanh^2(c+dx))^2}}{8d}$$

input
```
Integrate[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[b]) + (Tanh[c +
d*x]*(5*a + 3*b*Tanh[c + d*x]^2))/(a^2*(a + b*Tanh[c + d*x]^2)^2))/(8*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4158, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\text{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1}{\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a}+\frac{\tanh(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\left(\frac{\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\tanh(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4a}+\frac{\tanh(c+dx)}{4a\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]/(4*a*(a + b*Tanh[c + d*x]^2)^2) + (3*(ArcTan[(Sqrt[b]*Tanh[
c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Tanh[c + d*x]/(2*a*(a + b*Tanh[c
+ d*x]^2))))/(4*a))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(82) = 164$.

Time = 83.83 (sec) , antiderivative size = 284, normalized size of antiderivative = 2.96

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{3\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\cdots\right.}}\cdots\right.}{\cdots}}{d}$
default	$\frac{-\frac{2\left(-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}}-\frac{3(5a+4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}}-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{3\left(\frac{\left(a+\sqrt{(a+b)b}+b\right)\arctan\left(\cdots\right.}{2a\sqrt{(a+b)b}\sqrt{\left(2\cdots\right.}}\cdots\right.}{\cdots}}{d}$
risch	$-\frac{5a^{3}e^{6dx+6c}-a^{2}b\,e^{6dx+6c}-9a\,b^{2}e^{6dx+6c}-3b^{3}e^{6dx+6c}+15\,e^{4dx+4c}a^{3}-a^{2}b\,e^{4dx+4c}+9a\,b^{2}e^{4dx+4c}+9b^{3}e^{4dx+4c}+15\,e\cdots}{4(a^{2}+2ab+b^{2})\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e\cdots\right.}$

input
```
int(sech(d*x+c)^2/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c
)^5-3/8*(5*a+4*b)/a^2*tanh(1/2*d*x+1/2*c)^3-5/8/a*tanh(1/2*d*x+1/2*c))/(ta
nh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+
a)^2-3/4/a*(1/2*(a+((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2
)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*
a)^(1/2))-1/2*(-a+((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)
-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*
a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2768 vs. $2(82) = 164$.

Time = 0.17 (sec) , antiderivative size = 5840, normalized size of antiderivative = 60.83

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(82) = 164$.

Time = 0.27 (sec) , antiderivative size = 366, normalized size of antiderivative = 3.81

$$\int \frac{\operatorname{sech}^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$
$$= \frac{5a^3 + 13a^2b + 11ab^2 + 3b^3 + (15a^3 + 13a^2b - 11ab^2 - 9b^3)e^{(-2}}{4(a^6 + 4a^5b + 6a^4b^2 + 4a^3b^3 + a^2b^4 + 4(a^6 + 2a^5b - 2a^3b^3 - a^2b^4)e^{(-2dx-2c)} + 2(3a^6 + 4a^5b + 2a^4b}$$
$$- \frac{3\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(5*a^3 + 13*a^2*b + 11*a*b^2 + 3*b^3 + (15*a^3 + 13*a^2*b - 11*a*b^2 -
 9*b^3)*e^(-2*d*x - 2*c) + (15*a^3 - a^2*b + 9*a*b^2 + 9*b^3)*e^(-4*d*x -
4*c) + (5*a^3 - a^2*b - 9*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^6 + 4*a^5*b
 + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4 + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^
4)*e^(-2*d*x - 2*c) + 2*(3*a^6 + 4*a^5*b + 2*a^4*b^2 + 4*a^3*b^3 + 3*a^2*b
^4)*e^(-4*d*x - 4*c) + 4*(a^6 + 2*a^5*b - 2*a^3*b^3 - a^2*b^4)*e^(-6*d*x -
 6*c) + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c)
)*d) - 3/8*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(
a*b)*a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(82) = 164$.

Time = 0.61 (sec) , antiderivative size = 320, normalized size of antiderivative = 3.33

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}a^2} - \frac{2\left(5\,a^3e^{(6\,dx+6\,c)}-a^2be^{(6\,dx+6\,c)}-9\,ab^2e^{(6\,dx+6\,c)}-3\,b^3e^{(6\,dx+6\,c)}+15\,a^3e^{(4\,dx+4\,c)}-a^2be^{(4\,dx+4}\right.}{(a^4+2\,a^3b+a^2b^2)\left(ae^{(}\right.}}{8\,d}$$

input
```
integrate(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*(3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b
))/(sqrt(a*b)*a^2) - 2*(5*a^3*e^(6*d*x + 6*c) - a^2*b*e^(6*d*x + 6*c) - 9*
a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 15*a^3*e^(4*d*x + 4*c) - a
^2*b*e^(4*d*x + 4*c) + 9*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) + 1
5*a^3*e^(2*d*x + 2*c) + 13*a^2*b*e^(2*d*x + 2*c) - 11*a*b^2*e^(2*d*x + 2*c
) - 9*b^3*e^(2*d*x + 2*c) + 5*a^3 + 13*a^2*b + 11*a*b^2 + 3*b^3)/((a^4 + 2
*a^3*b + a^2*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^2\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1906, normalized size of antiderivative = 19.85

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a**4 + 12*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)
*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b - 12*e**(8*c + 8*d*x)*sqrt(b)*sqrt
(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 - 6*e**(8*c
+ 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)
)*b**4 + 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b
) - sqrt(b))/sqrt(a))*a**4 - 48*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**
(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 + 24*e**(6*c + 6*d*x)*
sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**4 +
36*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a**4 - 24*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)
*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b + 24*e**(4*c + 4*d*x)*sqrt(b)*sqrt
(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 - 36*e**(4*c
 + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*b**4 + 24*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
b) - sqrt(b))/sqrt(a))*a**4 - 48*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e*
*(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 + 24*e**(2*c + 2*d*x)
*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**4 +
 6*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4
 + 12*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)...
```

3.129 $\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 129

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(4a+3b)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{3/2}d} + \frac{b\sinh(c+dx)}{4a(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{(4a+3b)\sinh(c+dx)}{8a^2(a+b)d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
1/8*(4*a+3*b)*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^(3/2)/
d+1/4*b*sinh(d*x+c)/a/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)^2+1/8*(4*a+3*b)*sinh
(d*x+c)/a^2/(a+b)/d/(a+(a+b)*sinh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.95

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{-\frac{8\sinh(c+dx)}{\left(a+(a+b)\sinh^2(c+dx)\right)^2}+(4a+3b)\left(\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}}+\frac{5a\sinh(c+dx)+3(a+b)\sinh^3(c+dx)}{a^2\left(a+(a+b)\sinh^2(c+dx)\right)^2}\right)}{24(a+b)d}$$

input
```
Integrate[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((-8*Sinh[c + d*x])/(a + (a + b)*Sinh[c + d*x]^2)^2 + (4*a + 3*b)*((3*ArcT
an[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]) + (5*a*Sinh
[c + d*x] + 3*(a + b)*Sinh[c + d*x]^3)/(a^2*(a + (a + b)*Sinh[c + d*x]^2)^
2)))/(24*(a + b)*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 121, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4159, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{\sinh^2(c+dx)+1}{\left((a+b)\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{1}{4}\left(\frac{1}{a+b}+\frac{3}{a}\right)\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)+\frac{b\sinh(c+dx)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{1}{4}\left(\frac{1}{a+b}+\frac{3}{a}\right)\left(\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)+\frac{b\sinh(c+dx)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{1}{4}\left(\frac{1}{a+b}+\frac{3}{a}\right)\left(\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)+\frac{b\sinh(c+dx)}{4a(a+b)\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

input

```
Int[Sech[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output

```
((b*Sinh[c + d*x])/(4*a*(a + b)*(a + (a + b)*Sinh[c + d*x]^2)^2) + ((3/a +
 (a + b)^(-1))*(ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqr
t[a + b]) + Sinh[c + d*x]/(2*a*(a + (a + b)*Sinh[c + d*x]^2))))/4)/d
```

Defintions of rubi rules used

rule 215

```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(115) = 230$.

Time = 173.55 (sec) , antiderivative size = 342, normalized size of antiderivative = 2.65

method	result
derivativedivides	$\frac{\frac{-\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a(a+b)}-\frac{(4a^2+13ab+12b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2(a+b)}+\frac{(4a^2+13ab+12b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}+\cdots}{d}$
default	$\frac{\frac{-\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{4a(a+b)}-\frac{(4a^2+13ab+12b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{4a^2(a+b)}+\frac{(4a^2+13ab+12b^2)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{4a^2(a+b)}+\frac{(5b+4a)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a(a+b)}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}+\cdots}{d}$
risch	$\frac{e^{dx+c}\left(4\,e^{6dx+6c}a^2+7\,e^{6dx+6c}ab+3\,e^{6dx+6c}b^2+4\,e^{4dx+4c}a^2-e^{4dx+4c}ab-9\,e^{4dx+4c}b^2-4\,e^{2dx+2c}a^2+e^{2dx+2c}ba+9b^2e\cdots\right.}{4\left(e^{4dx+4c}a+b\,e^{4dx+4c}+2\,e^{2dx+2c}a-2\,e^{2dx+2c}b+a+b\right)^2d\,a^2(a+b)}$

input
```
int(sech(d*x+c)^3/(a+tanh(d*x+c)^2*b)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(2*(-1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c)^7-1/8*(4*a^2+13*a*b+12*
b^2)/a^2/(a+b)*tanh(1/2*d*x+1/2*c)^5+1/8*(4*a^2+13*a*b+12*b^2)/a^2/(a+b)*t
anh(1/2*d*x+1/2*c)^3+1/8*(5*b+4*a)/a/(a+b)*tanh(1/2*d*x+1/2*c))/(tanh(1/2*
d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+1/
4/a*(4*a+3*b)/(a+b)*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*
b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+
a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(
1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2
*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3635 vs. $2(115) = 230$.

Time = 0.19 (sec) , antiderivative size = 6622, normalized size of antiderivative = 51.33

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{\operatorname{sech}\left(dx+c\right)^3}{\left(b\tanh\left(dx+c\right)^2+a\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*((4*a^2*e^(7*c) + 7*a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^
(5*c) - a*b*e^(5*c) - 9*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - a*b*e^(3
*c) - 9*b^2*e^(3*c))*e^(3*d*x) - (4*a^2*e^c + 7*a*b*e^c + 3*b^2*e^c)*e^(d*
x))/(a^5*d + 3*a^4*b*d + 3*a^3*b^2*d + a^2*b^3*d + (a^5*d*e^(8*c) + 3*a^4*
b*d*e^(8*c) + 3*a^3*b^2*d*e^(8*c) + a^2*b^3*d*e^(8*c))*e^(8*d*x) + 4*(a^5*
d*e^(6*c) + a^4*b*d*e^(6*c) - a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*
d*x) + 2*(3*a^5*d*e^(4*c) + a^4*b*d*e^(4*c) + a^3*b^2*d*e^(4*c) + 3*a^2*b^
3*d*e^(4*c))*e^(4*d*x) + 4*(a^5*d*e^(2*c) + a^4*b*d*e^(2*c) - a^3*b^2*d*e^
(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/32*((4*a*e^(3*c) + 3
*b*e^(3*c))*e^(3*d*x) + (4*a*e^c + 3*b*e^c)*e^(d*x))/(a^4 + 2*a^3*b + a^2*
b^2 + (a^4*e^(4*c) + 2*a^3*b*e^(4*c) + a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(a^4
*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{1}{\cosh(c+d\,x)^3\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 2228, normalized size of antiderivative = 17.27

$$\int \frac{\operatorname{sech}^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(4*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**3 + 11*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(
c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b + 10*e**(8*c + 8*d*x)*sqrt
(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2
+ 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*b**3 + 16*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**
(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3 + 12*e**(6*c + 6*d*x)*sqrt(
a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b -
 16*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a*b**2 - 12*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e
**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3 + 24*e**(4*c + 4*d*x)*sqr
t(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3 +
 2*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**2*b + 12*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e*
*(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**2 + 18*e**(4*c + 4*d*x)*sq
rt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**3
+ 16*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a**3 + 12*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e*
*(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b - 16*e**(2*c + 2*d*x)*sq
rt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*...
```

3.130 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 115

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx = -\frac{(a-3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} + \frac{(a+b)\tanh(c+dx)}{4abd\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(a-3b)\tanh(c+dx)}{8a^2bd\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/8*(a-3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(3/2)/d+1/4*(a+
b)*tanh(d*x+c)/a/b/d/(a+b*tanh(d*x+c)^2)^2-1/8*(a-3*b)*tanh(d*x+c)/a^2/b/d
/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.17 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.00

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{(-a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}}+\frac{\sqrt{a}(a^2+6ab-3b^2+(a^2+4ab+3b^2)\cosh(2(c+dx)))\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((-a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/b^(3/2) + (Sqrt[a]*(
a^2 + 6*a*b - 3*b^2 + (a^2 + 4*a*b + 3*b^2)*Cosh[2*(c + d*x)])*Sinh[2*(c +
 d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3042, 4158, 298, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^4}{(a-b\tan(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{4158}$$

$$\frac{\int \frac{1-\tanh^2(c+dx)}{\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{298}$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(a-3b)\int\frac{1}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4ab}}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(a-3b)\left(\frac{\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{2a}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4ab}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{(a+b)\tanh(c+dx)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{(a-3b)\left(\frac{\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\tanh(c+dx)}{2a\left(a+b\tanh^2(c+dx)\right)}\right)}{4ab}}{d}$$

input
```
Int[Sech[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a + b)*Tanh[c + d*x])/(4*a*b*(a + b*Tanh[c + d*x]^2)^2) - ((a - 3*b)*(A
rcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Tanh[c + d*x]
/(2*a*(a + b*Tanh[c + d*x]^2))))/(4*a*b))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(101) = 202$.

Time = 0.69 (sec) , antiderivative size = 334, normalized size of antiderivative = 2.90

$$\frac{-\frac{2\left(-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8ab}-\frac{\left(3a^{2}+11ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}b}-\frac{\left(3a^{2}+11ab+12b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}b}-\frac{(a+5b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ab}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}-\frac{(a-3b)\left(\frac{\left(-a-\sqrt{(a+b)b}\right.}{2a\sqrt{(}}\right.}{}}{d}$$

input
```
int(sech(d*x+c)^4/(a+tanh(d*x+c)^2*b)^3,x)
```

output
```
1/d*(-2*(-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c)^7-1/8*(3*a^2+11*a*b+12*b^2)/
a^2/b*tanh(1/2*d*x+1/2*c)^5-1/8*(3*a^2+11*a*b+12*b^2)/a^2/b*tanh(1/2*d*x+1
/2*c)^3-1/8*(a+5*b)/a/b*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*ta
nh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/4/a*(a-3*b)/b*(1/2*
(-a-((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(
a-((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2677 vs. $2(101) = 202$.

Time = 0.17 (sec) , antiderivative size = 5659, normalized size of antiderivative = 49.21

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 360 vs. $2(101) = 202$.

Time = 0.30 (sec) , antiderivative size = 360, normalized size of antiderivative = 3.13

$$\int \frac{\operatorname{sech}^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$
$$= \frac{a^3+5a^2b+7ab^2+3b^3+(3a^3+13a^2b+ab^2-9b^3)e^{(-2dx-2c)}+(3a^3+}{4(a^5b+3a^4b^2+3a^3b^3+a^2b^4+4(a^5b+a^4b^2-a^3b^3-a^2b^4)e^{(-2dx-2c)}+2(3a^5b+a^4b^2+a^3b^3+3a^2b^4}$$
$$+ \frac{(a-3b)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2bd}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3 + (3*a^3 + 13*a^2*b + a*b^2 - 9*b^3)*
e^(-2*d*x - 2*c) + (3*a^3 + 7*a^2*b - 3*a*b^2 + 9*b^3)*e^(-4*d*x - 4*c) +
(a^3 - a^2*b - 5*a*b^2 - 3*b^3)*e^(-6*d*x - 6*c))/((a^5*b + 3*a^4*b^2 + 3*
a^3*b^3 + a^2*b^4 + 4*(a^5*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-2*d*x - 2*
c) + 2*(3*a^5*b + a^4*b^2 + a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^5
*b + a^4*b^2 - a^3*b^3 - a^2*b^4)*e^(-6*d*x - 6*c) + (a^5*b + 3*a^4*b^2 +
3*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) + 1/8*(a - 3*b)*arctan(1/2*((a +
 b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(101) = 202$.

Time = 0.61 (sec) , antiderivative size = 319, normalized size of antiderivative = 2.77

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{(a-3b)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2b} + \frac{2\left(a^3e^{(6dx+6c)}-a^2be^{(6dx+6c)}-5ab^2e^{(6dx+6c)}-3b^3e^{(6dx+6c)}+3a^3e^{(4dx+4c)}+7a^2b\right.}{(a^3b+a^2b^2)(a}}{8d}$$

input
```
integrate(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((a - 3*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)
/sqrt(a*b))/(sqrt(a*b)*a^2*b) + 2*(a^3*e^(6*d*x + 6*c) - a^2*b*e^(6*d*x +
6*c) - 5*a*b^2*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 3*a^3*e^(4*d*x +
4*c) + 7*a^2*b*e^(4*d*x + 4*c) - 3*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x
+ 4*c) + 3*a^3*e^(2*d*x + 2*c) + 13*a^2*b*e^(2*d*x + 2*c) + a*b^2*e^(2*d*x
 + 2*c) - 9*b^3*e^(2*d*x + 2*c) + a^3 + 5*a^2*b + 7*a*b^2 + 3*b^3)/((a^3*b
 + a^2*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\mathrm{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^4\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 2949, normalized size of antiderivative = 25.64

$$\int \frac{\mathrm{sech}^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 2*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**5 + 2*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*
x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b + 12*e**(8*c + 8*d*x)*sqrt(b)*sq
rt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 + 4*e**
(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sq
rt(a))*a**2*b**3 - 10*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*
sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**4 - 6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5 - 8*e**(6*c + 6*
d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*
*5 + 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) -
sqrt(b))/sqrt(a))*a**4*b + 16*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c
 + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 - 48*e**(6*c + 6*d*x)*sq
rt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**3
 - 8*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqr
t(b))/sqrt(a))*a*b**4 + 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5 - 12*e**(4*c + 4*d*x)*sqrt(b)*sq
rt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**5 + 44*e**(4*c
 + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*a**4*b - 24*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) - sqrt(b))/sqrt(a))*a**3*b**2 - 8*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)...
```

3.131 $\int \frac{\text{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 104

$$\int \frac{\text{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a+b}d} + \frac{\sinh(c+dx)}{4ad\left(a+(a+b)\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+(a+b)\sinh^2(c+dx)\right)}$$

output
```
3/8*arctan((a+b)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^(1/2)/d+1/4*sinh
(d*x+c)/a/d/(a+(a+b)*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+(a+b)*sinh(
d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 87, normalized size of antiderivative = 0.84

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\frac{\frac{3\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+\frac{\sqrt{a}\sinh(c+dx)\left(5a+3(a+b)\sinh^2(c+dx)\right)}{\left(a+(a+b)\sinh^2(c+dx)\right)^2}}{8a^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*ArcTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/Sqrt[a + b] + (Sqrt[a]*Si
nh[c + d*x]*(5*a + 3*(a + b)*Sinh[c + d*x]^2))/(a + (a + b)*Sinh[c + d*x]^
2)^2)/(8*a^(5/2)*d)
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.03, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}=0.217$, Rules used = {3042, 4159, 215, 215, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\sec(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4159}$$

$$\frac{\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \text{215}$$

$$\frac{\frac{3\int \frac{1}{\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4a}+\frac{\sinh(c+dx)}{4a\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$\Big\downarrow$ 215

$$\frac{\frac{3\left(\frac{\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{2a}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)}{4a}+\frac{\sinh(c+dx)}{4a\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{3\left(\frac{\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\sinh(c+dx)}{2a\left((a+b)\sinh^2(c+dx)+a\right)}\right)}{4a}+\frac{\sinh(c+dx)}{4a\left((a+b)\sinh^2(c+dx)+a\right)^2}}{d}$$

input
```
Int[Sech[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Sinh[c + d*x]/(4*a*(a + (a + b)*Sinh[c + d*x]^2)^2) + (3*(ArcTan[(Sqrt[a
+ b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Sinh[c + d*x]/(2*a*
(a + (a + b)*Sinh[c + d*x]^2))))/(4*a))/d
```

Defintions of rubi rules used

rule 215
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[(-x)*((a + b*x^2)^(p + 1)
/(2*a*(p + 1))), x] + Simp[(2*p + 3)/(2*a*(p + 1))   Int[(a + b*x^2)^(p + 1
), x], x] /; FreeQ[{a, b}, x] && LtQ[p, -1] && (IntegerQ[4*p] || IntegerQ[6
*p])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(90) = 180$.

Time = 0.65 (sec) , antiderivative size = 276, normalized size of antiderivative = 2.65

$$\frac{\frac{-\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{4a}+\frac{3(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{4a^{2}}-\frac{3(a-4b)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{4a^{2}}+\frac{5\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{4a}}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}+\frac{\frac{3\left(\sqrt{(a+b)b}+b\right)\arctan\left(\frac{a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}\right)}{8a\sqrt{(a+b)b}\sqrt{\left(2\sqrt{(a+b)b}+a+2b\right)a}}-\frac{3\left(\sqrt{(a+b)b}-b\right.}{8a\sqrt{(}}}{a}}{d}$$

input
```
int(sech(d*x+c)^5/(a+tanh(d*x+c)^2*b)^3,x)
```

output
```
1/d*(2*(-5/8/a*tanh(1/2*d*x+1/2*c)^7+3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^5
-3/8*(a-4*b)/a^2*tanh(1/2*d*x+1/2*c)^3+5/8/a*tanh(1/2*d*x+1/2*c))/(tanh(1/
2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(1/2*d*x+1/2*c)^2+a)^2+
3/4/a*(1/2*(((a+b)*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2
))-1/2*(((a+b)*b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2719 vs. $2(90) = 180$.

Time = 0.16 (sec) , antiderivative size = 5084, normalized size of antiderivative = 48.88

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(sech(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**5/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [F]

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^5}{(b\tanh(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/4*(3*(a*e^(7*c) + b*e^(7*c))*e^(7*d*x) + (11*a*e^(5*c) - 9*b*e^(5*c))*e^
(5*d*x) - (11*a*e^(3*c) - 9*b*e^(3*c))*e^(3*d*x) - 3*(a*e^c + b*e^c)*e^(d*
x))/(a^4*d + 2*a^3*b*d + a^2*b^2*d + (a^4*d*e^(8*c) + 2*a^3*b*d*e^(8*c) +
a^2*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*
d*x) + 2*(3*a^4*d*e^(4*c) - 2*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*
d*x) + 4*(a^4*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 32*integrate(3/1
28*(e^(3*d*x + 3*c) + e^(d*x + c))/(a^3 + a^2*b + (a^3*e^(4*c) + a^2*b*e^(
4*c))*e^(4*d*x) + 2*(a^3*e^(2*c) - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{1}{\cosh(c+d\,x)^5\,\left(b\tanh(c+d\,x)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 1507, normalized size of antiderivative = 14.49

$$\int \frac{\operatorname{sech}^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input

```
int(sech(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
(3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - s
qrt(b))/sqrt(a))*a**2 + 6*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c
 + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b + 3*e**(8*c + 8*d*x)*sqrt(a)*s
qrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 12*e*
*(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b
))/sqrt(a))*a**2 - 12*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d
*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 18*e**(4*c + 4*d*x)*sqrt(a)*sqr
t(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 - 12*e**(
4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))
/sqrt(a))*a*b + 18*e**(4*c + 4*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)
*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 12*e**(2*c + 2*d*x)*sqrt(a)*sqrt(a
 + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2 - 12*e**(2*c
 + 2*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sq
rt(a))*b**2 + 3*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a**2 + 6*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) -
 sqrt(b))/sqrt(a))*a*b + 3*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a +
 b) - sqrt(b))/sqrt(a))*b**2 + 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan
((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a**2 + 6*e**(8*c + 8*d*x)*s
qrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*a*b
+ 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b)...
```

3.132 $\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 132

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} + \frac{(a+b)^2\tanh(c+dx)}{4ab^2d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(5a-3b)(a+b)\tanh(c+dx)}{8a^2b^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
1/8*(3*a^2-2*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/b^(5/2
)/d+1/4*(a+b)^2*tanh(d*x+c)/a/b^2/d/(a+b*tanh(d*x+c)^2)^2-1/8*(5*a-3*b)*(a
+b)*tanh(d*x+c)/a^2/b^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.16 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.97

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$=\frac{(3a^2-2ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)-\frac{\sqrt{a}\sqrt{b}(a+b)\left(3a^2-10ab+3b^2+3\left(a^2-b^2\right)\cosh(2(c+dx))\right)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}}{8a^{5/2}b^{5/2}d}$$

input
```
Integrate[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((3*a^2 - 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]] - (Sqrt[a
]*Sqrt[b]*(a + b)*(3*a^2 - 10*a*b + 3*b^2 + 3*(a^2 - b^2)*Cosh[2*(c + d*x)
])*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)])^2)/(8*a^(5/2)*b^
(5/2)*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 143, normalized size of antiderivative = 1.08, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4158, 315, 25, 298, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{\sec(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{4158}$$
$$\frac{\int \frac{\left(1-\tanh^2(c+dx)\right)^2}{\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \text{315}$$

$$\frac{\frac{\int -\frac{-\left((3a-b)\tanh^2(c+dx)\right)+a-3b}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4ab}+\frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int \frac{-\left((3a-b)\tanh^2(c+dx)\right)+a-3b}{\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4ab}}{d}$$

$$\Big\downarrow \; 298$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{1}{2}\left(-\frac{3a}{b}-\frac{3b}{a}+2\right)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)+\frac{3\left(\frac{a}{b}-\frac{b}{a}\right)\tanh(c+dx)}{2\left(a+b\tanh^2(c+dx)\right)}}{4ab}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{(a+b)\tanh(c+dx)\left(1-\tanh^2(c+dx)\right)}{4ab\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\left(-\frac{3a}{b}-\frac{3b}{a}+2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}}+\frac{3\left(\frac{a}{b}-\frac{b}{a}\right)\tanh(c+dx)}{2\left(a+b\tanh^2(c+dx)\right)}}{4ab}}{d}$$

input
```
Int[Sech[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a + b)*Tanh[c + d*x]*(1 - Tanh[c + d*x]^2))/(4*a*b*(a + b*Tanh[c + d*x]
^2)^2) - (((2 - (3*a)/b - (3*b)/a)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]]
)/(2*Sqrt[a]*Sqrt[b]) + (3*(a/b - b/a)*Tanh[c + d*x])/(2*(a + b*Tanh[c + d
*x]^2)))/(4*a*b))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 298
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[(-(
b*c - a*d))*x*((a + b*x^2)^(p + 1)/(2*a*b*(p + 1))), x] - Simp[(a*d - b*c*(
2*p + 3))/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1), x], x] /; FreeQ[{a, b,
 c, d, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/2 + p, 0])
```

rule 315
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(a*d - c*b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(2*a*b*(p + 1))),
x] - Simp[1/(2*a*b*(p + 1))   Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^(q - 2)*S
imp[c*(a*d - c*b*(2*p + 3)) + d*(a*d*(2*(q - 1) + 1) - b*c*(2*(p + q) + 1))
*x^2, x], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -
1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(118) = 236$.

Time = 0.64 (sec) , antiderivative size = 376, normalized size of antiderivative = 2.85

$$-\frac{\frac{2\left(\frac{\left(3a^2-2ab-5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^7}{8b^2a}+\frac{\left(9a^3+14a^2b-7b^2a-12b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^5}{8a^2b^2}+\frac{\left(9a^3+14a^2b-7b^2a-12b^3\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^3}{8a^2b^2}+\frac{\left(3a^2-2ab-5b^2\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8b^2a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^4a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^2+a\right)^2}}{d}$$

input
```
int(sech(d*x+c)^6/(a+tanh(d*x+c)^2*b)^3,x)
```

output
```
1/d*(-2*(1/8*(3*a^2-2*a*b-5*b^2)/b^2/a*tanh(1/2*d*x+1/2*c)^7+1/8*(9*a^3+14
*a^2*b-7*a*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^5+1/8*(9*a^3+14*a^2*b-7
*a*b^2-12*b^3)/a^2/b^2*tanh(1/2*d*x+1/2*c)^3+1/8*(3*a^2-2*a*b-5*b^2)/b^2/a
*tanh(1/2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4
*b*tanh(1/2*d*x+1/2*c)^2+a)^2-1/4/a*(3*a^2-2*a*b+3*b^2)/b^2*(1/2*(a+((a+b)
*b)^(1/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(-a+((a+b)*
b)^(1/2)-b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2464 vs. $2(118) = 236$.

Time = 0.20 (sec) , antiderivative size = 5233, normalized size of antiderivative = 39.64

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\operatorname{sech}^6\left(c+dx\right)}{\left(a+b\tanh^2\left(c+dx\right)\right)^3}\,dx$$

input
```
integrate(sech(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(sech(c + d*x)**6/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 332 vs. $2(118) = 236$.

Time = 0.33 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.52

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{3a^3+3a^2b-3ab^2-3b^3+(9a^3-13a^2b-13ab^2+9b^3)e^{(-2dx-2c)}+3(3a^3-5a^2b+5ab^2-3b}{4(a^4b^2+2a^3b^3+a^2b^4+4(a^4b^2-a^2b^4)e^{(-2dx-2c)}+2(3a^4b^2-2a^3b^3+3a^2b^4)e^{(-4dx-4c)}+4(a^4b^2-} - \frac{(3a^2-2ab+3b^2)\arctan\left(\frac{(a+b)e^{(-2dx-2c)}+a-b}{2\sqrt{ab}}\right)}{8\sqrt{ab}a^2b^2d}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(3*a^3 + 3*a^2*b - 3*a*b^2 - 3*b^3 + (9*a^3 - 13*a^2*b - 13*a*b^2 + 9
*b^3)*e^(-2*d*x - 2*c) + 3*(3*a^3 - 5*a^2*b + 5*a*b^2 - 3*b^3)*e^(-4*d*x -
 4*c) + (3*a^3 + a^2*b + a*b^2 + 3*b^3)*e^(-6*d*x - 6*c))/((a^4*b^2 + 2*a^
3*b^3 + a^2*b^4 + 4*(a^4*b^2 - a^2*b^4)*e^(-2*d*x - 2*c) + 2*(3*a^4*b^2 -
2*a^3*b^3 + 3*a^2*b^4)*e^(-4*d*x - 4*c) + 4*(a^4*b^2 - a^2*b^4)*e^(-6*d*x
- 6*c) + (a^4*b^2 + 2*a^3*b^3 + a^2*b^4)*e^(-8*d*x - 8*c))*d) - 1/8*(3*a^2
 - 2*a*b + 3*b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/(sqrt(a*b)*a^2*b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(118) = 236$.

Time = 0.58 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.44

$$\int \frac{\operatorname{sech}^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\frac{(3a^2-2ab+3b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}a^2b^2} + \frac{2(3a^3e^{(6dx+6c)}+a^2be^{(6dx+6c)}+ab^2e^{(6dx+6c)}+3b^3e^{(6dx+6c)}+9a^3e^{(4dx+4c)}}{(}}{}$$

input
```
integrate(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*((3*a^2 - 2*a*b + 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x +
2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*a^2*b^2) + 2*(3*a^3*e^(6*d*x + 6*c) +
a^2*b*e^(6*d*x + 6*c) + a*b^2*e^(6*d*x + 6*c) + 3*b^3*e^(6*d*x + 6*c) + 9*
a^3*e^(4*d*x + 4*c) - 15*a^2*b*e^(4*d*x + 4*c) + 15*a*b^2*e^(4*d*x + 4*c)
- 9*b^3*e^(4*d*x + 4*c) + 9*a^3*e^(2*d*x + 2*c) - 13*a^2*b*e^(2*d*x + 2*c)
 - 13*a*b^2*e^(2*d*x + 2*c) + 9*b^3*e^(2*d*x + 2*c) + 3*a^3 + 3*a^2*b - 3*
a*b^2 - 3*b^3)/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*
c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*a^2*b^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\int \frac{1}{\cosh\left(c+d\,x\right)^6\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2976, normalized size of antiderivative = 22.55

$$\int \frac{\operatorname{sech}^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
int(sech(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
(6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*a**5 + 2*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*
sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b - 4*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 + 4*e**(8*c
 + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a
))*a**2*b**3 - 2*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(
a + b) - sqrt(b))/sqrt(a))*a*b**4 - 6*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*ata
n((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5 + 24*e**(6*c + 6*d*x)
*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**5 -
 40*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*a**4*b + 16*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d
*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b**2 + 16*e**(6*c + 6*d*x)*sqrt(b
)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**3 - 4
0*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b
))/sqrt(a))*a*b**4 + 24*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x
)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**5 + 36*e**(4*c + 4*d*x)*sqrt(b)*sqrt(
a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**5 - 84*e**(4*c +
4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*
a**4*b + 136*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
b) - sqrt(b))/sqrt(a))*a**3*b**2 - 136*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)...
```

3.133 $\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 156

$$\begin{aligned}\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = &-\frac{\arctan(\sinh(c+dx))}{b^3d}\\ &+\frac{\sqrt{a+b}(8a^2-4ab+3b^2)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}\\ &+\frac{(a+b)\sinh(c+dx)}{4abd\left(a+(a+b)\sinh^2(c+dx)\right)^2}\\ &-\frac{(4a-3b)(a+b)\sinh(c+dx)}{8a^2b^2d\left(a+(a+b)\sinh^2(c+dx)\right)}\end{aligned}$$

output
```
-arctan(sinh(d*x+c))/b^3/d+1/8*(a+b)^(1/2)*(8*a^2-4*a*b+3*b^2)*arctan((a+b
)^(1/2)*sinh(d*x+c)/a^(1/2))/a^(5/2)/b^3/d+1/4*(a+b)*sinh(d*x+c)/a/b/d/(a+
(a+b)*sinh(d*x+c)^2)^2-1/8*(4*a-3*b)*(a+b)*sinh(d*x+c)/a^2/b^2/d/(a+(a+b)*
sinh(d*x+c)^2)
```

Mathematica [C] (verified)

Result contains complex when optimal does not.

Time = 2.11 (sec) , antiderivative size = 317, normalized size of antiderivative = 2.03

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx =$$

$$-\frac{\frac{2\sqrt{a+b}(8a^2-4ab+3b^2)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}}+\frac{2(8a^3+4a^2b-ab^2+3b^3)\arctan\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{a+b}}\right)}{a^{5/2}\sqrt{a+b}}+64\arctan\left(\tanh\left(\frac{1}{2}(c+\right.\right.}{}$$

input
```
Integrate[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/32*((2*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x
])/Sqrt[a + b]])/a^(5/2) + (2*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*ArcTan[(Sq
rt[a]*Csch[c + d*x])/Sqrt[a + b]])/(a^(5/2)*Sqrt[a + b]) + 64*ArcTan[Tanh[
(c + d*x)/2]] + (I*Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Log[a - b + (a + b)
*Cosh[2*(c + d*x)]])/a^(5/2) - (I*(8*a^3 + 4*a^2*b - a*b^2 + 3*b^3)*Log[a
- b + (a + b)*Cosh[2*(c + d*x)]])/(a^(5/2)*Sqrt[a + b]) - (32*b^2*(a + b)*
Sinh[c + d*x])/(a*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (8*b*(4*a^2 + a
*b - 3*b^2)*Sinh[c + d*x])/(a^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(b^3
*d)
```

Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 4159, 316, 402, 397, 216, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sec(ic+idx)^7}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 4159$$

$$\frac{\int \frac{1}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)^3}d\sinh(c+dx)}{d}$$

$$\Big\downarrow \; 316$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\int \frac{-3(a+b)\sinh^2(c+dx)+a-3b}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)^2}d\sinh(c+dx)}{4ab}}{d}$$

$$\Big\downarrow \; 402$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\int \frac{4a^2-ba+3b^2-(4a-3b)(a+b)\sinh^2(c+dx)}{\left(\sinh^2(c+dx)+1\right)\left((a+b)\sinh^2(c+dx)+a\right)}d\sinh(c+dx)}{2ab}}{4ab}}{d}$$

$$\Big\downarrow \; 397$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{(a+b)\left(8a^2-4ab+3b^2\right)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{8a^2\int \frac{1}{\sinh^2(c+dx)+1}d\sinh(c+dx)}{b}}{2ab}}{4ab}}{d}$$

$$\Big\downarrow \; 216$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{(a+b)\left(8a^2-4ab+3b^2\right)\int \frac{1}{(a+b)\sinh^2(c+dx)+a}d\sinh(c+dx)}{b}-\frac{8a^2\arctan(\sinh(c+dx))}{b}}{2ab}}{4ab}}{d}$$

$$\Big\downarrow \; 218$$

$$\frac{\frac{(a+b)\sinh(c+dx)}{4ab\left((a+b)\sinh^2(c+dx)+a\right)^2}-\frac{\frac{(4a-3b)(a+b)\sinh(c+dx)}{2ab\left((a+b)\sinh^2(c+dx)+a\right)}-\frac{\frac{\sqrt{a+b}\left(8a^2-4ab+3b^2\right)\arctan\left(\frac{\sqrt{a+b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b}-\frac{8a^2\arctan(\sinh(c+dx))}{b}}{2ab}}{4ab}}{d}$$

input

```
Int[Sech[c + d*x]^7/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(((a + b)*Sinh[c + d*x])/(4*a*b*(a + (a + b)*Sinh[c + d*x]^2)^2) - (-1/2*(
(-8*a^2*ArcTan[Sinh[c + d*x]])/b + (Sqrt[a + b]*(8*a^2 - 4*a*b + 3*b^2)*Ar
cTan[(Sqrt[a + b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b))/(a*b) + ((4*a - 3*
b)*(a + b)*Sinh[c + d*x])/(2*a*b*(a + (a + b)*Sinh[c + d*x]^2)))/(4*a*b))/
d
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4159
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Simp[ff/f
  Subst[Int[ExpandToSum[b*(ff*x)^n + a*(1 - ff^2*x^2)^(n/2), x]^p/(1 - ff^2
*x^2)^((m + n*p + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}
, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2] && IntegerQ[p]
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(142) = 284$.

Time = 0.63 (sec) , antiderivative size = 389, normalized size of antiderivative = 2.49

$$\frac{\frac{2\left(\frac{b\left(4a^{2}-ab-5b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{7}}{8a}+\frac{\left(4a^{3}+23a^{2}b+7b^{2}a-12b^{3}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{5}}{8a^{2}}-\frac{\left(4a^{3}+23a^{2}b+7b^{2}a-12b^{3}\right)b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{3}}{8a^{2}}-\frac{b\left(4a^{2}-ab-5b^{2}\right)\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)}{8a}\right)}{\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{4}a+2\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}a+4b\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)^{2}+a\right)^{2}}}{b^{3}}$$

input
```
int(sech(d*x+c)^7/(a+tanh(d*x+c)^2*b)^3,x)
```

output
```
1/d*(2/b^3*((1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/2*d*x+1/2*c)^7+1/8*(4*a^3+23
*a^2*b+7*a*b^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^5-1/8*(4*a^3+23*a^2*b+7*a
*b^2-12*b^3)/a^2*b*tanh(1/2*d*x+1/2*c)^3-1/8*b*(4*a^2-a*b-5*b^2)/a*tanh(1/
2*d*x+1/2*c))/(tanh(1/2*d*x+1/2*c)^4*a+2*tanh(1/2*d*x+1/2*c)^2*a+4*b*tanh(
1/2*d*x+1/2*c)^2+a)^2+1/8/a*(8*a^3+4*a^2*b-a*b^2+3*b^3)*(1/2*(((a+b)*b)^(1
/2)+b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)+a+2*b)*a)^(1/2))-1/2*(((a+b)*b)^(1/2)-
b)/a/((a+b)*b)^(1/2)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*((a+b)*b)^(1/2)-a-2*b)*a)^(1/2))))-2/b^3*arctan(tanh(1/2*
d*x+1/2*c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4457 vs. 2(142) = 284.

Time = 0.26 (sec) , antiderivative size = 8070, normalized size of antiderivative = 51.73

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(sech(d*x+c)**7/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [F]

$$\int \frac{\operatorname{sech}^7(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\operatorname{sech}(dx+c)^7}{(b\tanh(dx+c)^2+a)^3}\,dx$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*((4*a^3*e^(7*c) + 5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*
e^(7*d*x) + (4*a^3*e^(5*c) - 19*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e
^(5*c))*e^(5*d*x) - (4*a^3*e^(3*c) - 19*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) +
 9*b^3*e^(3*c))*e^(3*d*x) - (4*a^3*e^c + 5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3
*e^c)*e^(d*x))/(a^4*b^2*d + 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) +
 2*a^3*b^3*d*e^(8*c) + a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(a^4*b^2*d*e^(6*c)
 - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(3*a^4*b^2*d*e^(4*c) - 2*a^3*b^3*d*e^(
4*c) + 3*a^2*b^4*d*e^(4*c))*e^(4*d*x) + 4*(a^4*b^2*d*e^(2*c) - a^2*b^4*d*e
^(2*c))*e^(2*d*x)) - 2*arctan(e^(d*x + c))/(b^3*d) + 128*integrate(1/512*(
(8*a^3*e^(3*c) + 4*a^2*b*e^(3*c) - a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x
) + (8*a^3*e^c + 4*a^2*b*e^c - a*b^2*e^c + 3*b^3*e^c)*e^(d*x))/(a^3*b^3 +
a^2*b^4 + (a^3*b^3*e^(4*c) + a^2*b^4*e^(4*c))*e^(4*d*x) + 2*(a^3*b^3*e^(2*
c) - a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Giac [F(-2)]

Exception generated.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Exception raised: TypeError}$$

input
```
integrate(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
Exception raised: TypeError >> an error occurred running a Giac command:IN
PUT:sage2:=int(sage0,sageVARx):;OUTPUT:Limit: Max order reached or unable
to make series expansion Error: Bad Argument Value
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{1}{\cosh\left(c+d\,x\right)^7\,\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3),x)
```

output
```
int(1/(cosh(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3), x)
```

Reduce [B] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 2818, normalized size of antiderivative = 18.06

$$\int \frac{\operatorname{sech}^7(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(sech(d*x+c)^7/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 16*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**5 - 32*e**(8*c + 8*d*x)*atan
(e**(c + d*x))*a**4*b - 16*e**(8*c + 8*d*x)*atan(e**(c + d*x))*a**3*b**2 -
 64*e**(6*c + 6*d*x)*atan(e**(c + d*x))*a**5 + 64*e**(6*c + 6*d*x)*atan(e*
*(c + d*x))*a**3*b**2 - 96*e**(4*c + 4*d*x)*atan(e**(c + d*x))*a**5 + 64*e
**(4*c + 4*d*x)*atan(e**(c + d*x))*a**4*b - 96*e**(4*c + 4*d*x)*atan(e**(c
 + d*x))*a**3*b**2 - 64*e**(2*c + 2*d*x)*atan(e**(c + d*x))*a**5 + 64*e**(
2*c + 2*d*x)*atan(e**(c + d*x))*a**3*b**2 - 16*atan(e**(c + d*x))*a**5 - 3
2*atan(e**(c + d*x))*a**4*b - 16*atan(e**(c + d*x))*a**3*b**2 + 8*e**(8*c
+ 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqr
t(a))*a**4 + 12*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sq
rt(a + b) - sqrt(b))/sqrt(a))*a**3*b + 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a +
 b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 + 2*e**(8
*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/
sqrt(a))*a*b**3 + 3*e**(8*c + 8*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x
)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**4 + 32*e**(6*c + 6*d*x)*sqrt(a)*sqrt(
a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4 - 16*e**(6*
c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/s
qrt(a))*a**3*b - 20*e**(6*c + 6*d*x)*sqrt(a)*sqrt(a + b)*atan((e**(c + d*x
)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 + 16*e**(6*c + 6*d*x)*sqrt(a)*
sqrt(a + b)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 -...
```

3.134 $\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$

Optimal result

Integrand size = 21, antiderivative size = 54

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = (a+b)x - \frac{(a+b)\tanh(c+dx)}{d} - \frac{(a+b)\tanh^3(c+dx)}{3d} - \frac{b\tanh^5(c+dx)}{5d}$$

output
```
(a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*(a+b)*tanh(d*x+c)^3/d-1/5*b*tanh(d*x+c)^5/
d
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.80

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx = \frac{a\text{arctanh}(\tanh(c+dx))}{d} + \frac{b\text{arctanh}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} - \frac{b\tanh(c+dx)}{d} - \frac{a\tanh^3(c+dx)}{3d} - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c +
d*x])/d - (b*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) - (b*Tanh[c + d*
x]^3)/(3*d) - (b*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.91, number of steps used = 9, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3042, 4114, 3042, 3954, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) dx$$

$$\Big\downarrow \text{3042}$$

$$\int \tan(ic+idx)^4 \left(a-b\tan(ic+idx)^2\right) dx$$

$$\Big\downarrow \text{4114}$$

$$(a+b)\int \tanh^4(c+dx)dx-\frac{b\tanh^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\int \tan(ic+idx)^4dx$$

$$\Big\downarrow \text{3954}$$

$$(a+b)\left(-\int -\tanh^2(c+dx)dx-\frac{\tanh^3(c+dx)}{3d}\right)-\frac{b\tanh^5(c+dx)}{5d}$$

$$\Big\downarrow \text{25}$$

$$(a+b)\left(\int \tanh^2(c+dx)dx-\frac{\tanh^3(c+dx)}{3d}\right)-\frac{b\tanh^5(c+dx)}{5d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\left(-\frac{\tanh^3(c+dx)}{3d}+\int -\tan(ic+idx)^2dx\right)$$

$\downarrow$ 25

$$-\frac{b\tanh^5(c+dx)}{5d}+(a+b)\left(-\frac{\tanh^3(c+dx)}{3d}-\int \tan(ic+idx)^2dx\right)$$

$\downarrow$ 3954

$$(a+b)\left(\int 1dx-\frac{\tanh^3(c+dx)}{3d}-\frac{\tanh(c+dx)}{d}\right)-\frac{b\tanh^5(c+dx)}{5d}$$

$\downarrow$ 24

$$(a+b)\left(-\frac{\tanh^3(c+dx)}{3d}-\frac{\tanh(c+dx)}{d}+x\right)-\frac{b\tanh^5(c+dx)}{5d}$$

input
```
Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/5*(b*Tanh[c + d*x]^5)/d + (a + b)*(x - Tanh[c + d*x]/d - Tanh[c + d*x]^
3/(3*d))
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.26

method	result
parallelrisch	$-\frac{3\tanh(dx+c)^5 b+5\tanh(dx+c)^3 a+5b\tanh(dx+c)^3-15dxa-15dxb+15a\tanh(dx+c)+15b\tanh(dx+c)}{15d}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^5 b}{5}-\frac{\tanh(dx+c)^3 a}{3}-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^5 b}{5}-\frac{\tanh(dx+c)^3 a}{3}-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx}{2}}{d}$
parts	$\frac{a\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+\right.}{d}$
risch	$ax+bx+\frac{4\,e^{8dx+8c}a+6\,e^{8dx+8c}b+12\,e^{6dx+6c}a+12\,e^{6dx+6c}b+\frac{44\,e^{4dx+4c}a}{3}+\frac{56b\,e^{4dx+4c}}{3}+\frac{28\,e^{2dx+2c}a}{3}+\frac{28\,e^{2dx+}}{3}}{d\left(e^{2dx+2c}+1\right)^5}$

input
```
int(tanh(d*x+c)^4*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-1/15*(3*tanh(d*x+c)^5*b+5*tanh(d*x+c)^3*a+5*b*tanh(d*x+c)^3-15*d*x*a-15*d
*x*b+15*a*tanh(d*x+c)+15*b*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 339 vs. 2(50) = 100.

Time = 0.09 (sec) , antiderivative size = 339, normalized size of antiderivative = 6.28

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(15\,(a+b)dx+20\,a+23\,b)\cosh\left(dx+c\right)^5+5\,(15\,(a+b)dx+20\,a+23\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4}{}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/15*((15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^5 + 5*(15*(a + b)*d*x +
 20*a + 23*b)*cosh(d*x + c)*sinh(d*x + c)^4 - (20*a + 23*b)*sinh(d*x + c)^
5 + 5*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c)^3 - 5*(2*(20*a + 23*b)*
cosh(d*x + c)^2 + 8*a + 5*b)*sinh(d*x + c)^3 + 5*(2*(15*(a + b)*d*x + 20*a
 + 23*b)*cosh(d*x + c)^3 + 3*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c))
*sinh(d*x + c)^2 + 10*(15*(a + b)*d*x + 20*a + 23*b)*cosh(d*x + c) - 5*((2
0*a + 23*b)*cosh(d*x + c)^4 + 3*(8*a + 5*b)*cosh(d*x + c)^2 + 4*a + 10*b)*
sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*
d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
 c)^2 + 10*d*cosh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.52

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \begin{cases} ax - \frac{a\tanh^3(c+dx)}{3d} - \frac{a\tanh(c+dx)}{d} + bx - \frac{b\tanh^5(c+dx)}{5d} - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^4(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*tanh(c + d*x)**3/(3*d) - a*tanh(c + d*x)/d + b*x - b*ta
nh(c + d*x)**5/(5*d) - b*tanh(c + d*x)**3/(3*d) - b*tanh(c + d*x)/d, Ne(d,
 0)), (x*(a + b*tanh(c)**2)*tanh(c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 199 vs. $2(50) = 100$.

Time = 0.04 (sec) , antiderivative size = 199, normalized size of antiderivative = 3.69

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$$
$$= \frac{1}{15} b \left(15 x + \frac{15 c}{d} - \frac{2 \left(70 e^{(-2 dx - 2 c)} + 140 e^{(-4 dx - 4 c)} + 90 e^{(-6 dx - 6 c)} + 45 e^{(-8 dx - 8 c)} + 23\right)}{d(5 e^{(-2 dx - 2 c)} + 10 e^{(-4 dx - 4 c)} + 10 e^{(-6 dx - 6 c)} + 5 e^{(-8 dx - 8 c)} + e^{(-10 dx - 10 c)} + 1)}\right)$$
$$+ \frac{1}{3} a \left(3 x + \frac{3 c}{d} - \frac{4 \left(3 e^{(-2 dx - 2 c)} + 3 e^{(-4 dx - 4 c)} + 2\right)}{d(3 e^{(-2 dx - 2 c)} + 3 e^{(-4 dx - 4 c)} + e^{(-6 dx - 6 c)} + 1)}\right)$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/15*b*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90
*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
 10*c) + 1))) + 1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
+ 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

Time = 0.16 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.48

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right) \, dx$$
$$= \frac{15 \, (dx + c)(a + b) + \frac{2 \left(30 \, a e^{(8 \, dx + 8 \, c)} + 45 \, b e^{(8 \, dx + 8 \, c)} + 90 \, a e^{(6 \, dx + 6 \, c)} + 90 \, b e^{(6 \, dx + 6 \, c)} + 110 \, a e^{(4 \, dx + 4 \, c)} + 140 \, b e^{(4 \, dx + 4 \, c)} + 70 \, a e^{(2 \, dx + 2 \,}}{\left(e^{(2 \, dx + 2 \, c)} + 1\right)^5}}{15 \, d}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/15*(15*(d*x + c)*(a + b) + 2*(30*a*e^(8*d*x + 8*c) + 45*b*e^(8*d*x + 8*c
) + 90*a*e^(6*d*x + 6*c) + 90*b*e^(6*d*x + 6*c) + 110*a*e^(4*d*x + 4*c) +
140*b*e^(4*d*x + 4*c) + 70*a*e^(2*d*x + 2*c) + 70*b*e^(2*d*x + 2*c) + 20*a
 + 23*b)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.93

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{\tanh(c+d\,x)^3\,(a+b)}{3\,d}-\frac{b\tanh(c+d\,x)^5}{5\,d}-\frac{\tanh(c+d\,x)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (tanh(c + d*x)^3*(a + b))/(3*d) - (b*tanh(c + d*x)^5)/(5*d) -
(tanh(c + d*x)*(a + b))/d
```

Reduce [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.24

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{-3\tanh(dx+c)^5 b-5\tanh(dx+c)^3 a-5\tanh(dx+c)^3 b-15\tanh(dx+c)\,a-15\tanh(dx+c)\,b+}{15d}$$

input
```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

output
```
( - 3*tanh(c + d*x)**5*b - 5*tanh(c + d*x)**3*a - 5*tanh(c + d*x)**3*b - 1
5*tanh(c + d*x)*a - 15*tanh(c + d*x)*b + 15*a*d*x + 15*b*d*x)/(15*d)
```

3.135 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{(a+b)\tanh^2(c+dx)}{2d}-\frac{b\tanh^4(c+dx)}{4d}$$

output
```
(a+b)*ln(cosh(d*x+c))/d-1/2*(a+b)*tanh(d*x+c)^2/d-1/4*b*tanh(d*x+c)^4/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.94

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{4(a+b)\log(\cosh(c+dx))+2(a+2b)\text{sech}^2(c+dx)-b\text{sech}^4(c+dx)}{4d}$$

input
```
Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(4*(a + b)*Log[Cosh[c + d*x]] + 2*(a + 2*b)*Sech[c + d*x]^2 - b*Sech[c + d
*x]^4)/(4*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.65 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20, number of steps used = 11, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {3042, 26, 4114, 26, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$i\int \tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4114}$$

$$i\left((a+b)\int -i\tanh^3(c+dx)dx+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ib\tanh^4(c+dx)}{4d}-i(a+b)\int \tanh^3(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ib\tanh^4(c+dx)}{4d}-i(a+b)\int i\tan(ic+idx)^3dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left((a+b)\int \tan(ic+idx)^3dx+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3954}$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-\int i\tanh(c+dx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-i\int\tanh(c+dx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-i\int -i\tan(ic+idx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-\int \tan(ic+idx)dx\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left((a+b)\left(\frac{i\tanh^2(c+dx)}{2d}-\frac{i\log(\cosh(c+dx))}{d}\right)+\frac{ib\tanh^4(c+dx)}{4d}\right)$$

input
```
Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
I*(((I/4)*b*Tanh[c + d*x]^4)/d + (a + b)*(((-I)*Log[Cosh[c + d*x]])/d + ((
I/2)*Tanh[c + d*x]^2)/d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.45

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4 b}{4}-\frac{a\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^2 b}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4 b}{4}-\frac{a\tanh(dx+c)^2}{2}-\frac{\tanh(dx+c)^2 b}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$
parallelrisch	$-\frac{\tanh(dx+c)^4 b+4dxa+4dxb+2a\tanh(dx+c)^2+2\tanh(dx+c)^2 b+4\ln(1-\tanh(dx+c))a+4\ln(1-\tanh(dx+c))b}{4d}$
parts	$\frac{a\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh \cdots\right.}{d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2\,e^{2dx+2c}\left(e^{4dx+4c}a+2b\,e^{4dx+4c}+2\,e^{2dx+2c}a+2\,e^{2dx+2c}b+a+2b\right)}{d\left(e^{2dx+2c}+1\right)^4}+\frac{\ln\left(e^{2dx+2c}+1 \cdots\right.}{d}$

input
```
int(tanh(d*x+c)^3*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*tanh(d*x+c)^4*b-1/2*a*tanh(d*x+c)^2-1/2*tanh(d*x+c)^2*b-1/2*(a+b
)*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(tanh(d*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1205 vs. $2(45) = 90$.

Time = 0.10 (sec) , antiderivative size = 1205, normalized size of antiderivative = 24.59

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
7 + (a + b)*d*x*sinh(d*x + c)^8 + 2*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c
)^6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 + 2*(a + b)*d*x - a - 2*b)*sinh(d*
x + c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 + 3*(2*(a + b)*d*x - a - 2*b)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x +
 c)^4 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x + 15*(2*(a + b)*
d*x - a - 2*b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)
*d*x*cosh(d*x + c)^5 + 5*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^3 + (3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x + 2*(2
*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^
6 + 15*(2*(a + b)*d*x - a - 2*b)*cosh(d*x + c)^4 + 2*(a + b)*d*x + 6*(3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 - ((a +
b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sin
h(d*x + c)^8 + 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 +
a + b)*sinh(d*x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*
x + c)^4 + 30*(a + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*
(a + b)*cosh(d*x + c)^5 + 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x
+ c)^6 + 15*(a + b)*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 + a + b...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(42) = 84$.

Time = 0.14 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.80

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} - \frac{a\tanh^2(c+dx)}{2d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^4(c+dx)}{4d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^3(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d - a*tanh(c + d*x)**2/(2*d) + b
*x - b*log(tanh(c + d*x) + 1)/d - b*tanh(c + d*x)**4/(4*d) - b*tanh(c + d*
x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(45) = 90$.

Time = 0.12 (sec) , antiderivative size = 168, normalized size of antiderivative = 3.43

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x
 - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) +
4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a*(x + c/d + log(e^(-2*d*x
- 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*
c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 92 vs. $2(45) = 90$.

Time = 0.15 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$
$$-\frac{(dx+c)(a+b)-(a+b)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\left((a+2\,b)e^{(6\,dx+6\,c)}+2\,(a+b)e^{(4\,dx+4\,c)}+(a+2\,b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*((a + 2*b)*e^(6
*d*x + 6*c) + 2*(a + b)*e^(4*d*x + 4*c) + (a + 2*b)*e^(2*d*x + 2*c))/(e^(2
*d*x + 2*c) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.08

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{\tanh(c+d\,x)^2\,(a+b)}{2\,d}-\frac{b\tanh(c+d\,x)^4}{4\,d}-\frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (tanh(c + d*x)^2*(a + b))/(2*d) - (b*tanh(c + d*x)^4)/(4*d) -
(log(tanh(c + d*x) + 1)*(a + b))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 484, normalized size of antiderivative = 9.88

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{-a-2b+2\log\left(e^{2dx+2c}+1\right)b+8e^{2dx+2c}\log\left(e^{2dx+2c}+1\right)b-2bdx+12e^{4dx+4c}\log\left(e^{2dx+2c}+1\right)a+8e^{2dx}}{}$$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a + 2*e**(8*c + 8*d*x)*log(e
**(2*c + 2*d*x) + 1)*b - 2*e**(8*c + 8*d*x)*a*d*x - e**(8*c + 8*d*x)*a - 2
*e**(8*c + 8*d*x)*b*d*x - 2*e**(8*c + 8*d*x)*b + 8*e**(6*c + 6*d*x)*log(e*
*(2*c + 2*d*x) + 1)*a + 8*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*b - 8
*e**(6*c + 6*d*x)*a*d*x - 8*e**(6*c + 6*d*x)*b*d*x + 12*e**(4*c + 4*d*x)*l
og(e**(2*c + 2*d*x) + 1)*a + 12*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)
*b - 12*e**(4*c + 4*d*x)*a*d*x + 2*e**(4*c + 4*d*x)*a - 12*e**(4*c + 4*d*x
)*b*d*x - 4*e**(4*c + 4*d*x)*b + 8*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) +
 1)*a + 8*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*b - 8*e**(2*c + 2*d*x
)*a*d*x - 8*e**(2*c + 2*d*x)*b*d*x + 2*log(e**(2*c + 2*d*x) + 1)*a + 2*log
(e**(2*c + 2*d*x) + 1)*b - 2*a*d*x - a - 2*b*d*x - 2*b)/(2*d*(e**(8*c + 8*
d*x) + 4*e**(6*c + 6*d*x) + 6*e**(4*c + 4*d*x) + 4*e**(2*c + 2*d*x) + 1))
```

3.136 $\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) dx = (a+b)x - \frac{(a+b)\tanh(c+dx)}{d} - \frac{b\tanh^3(c+dx)}{3d}$$

output
```
(a+b)*x-(a+b)*tanh(d*x+c)/d-1/3*b*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.81

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right) dx = \frac{a\text{arctanh}(\tanh(c+dx))}{d} + \frac{b\text{arctanh}(\tanh(c+dx))}{d} - \frac{a\tanh(c+dx)}{d} - \frac{b\tanh(c+dx)}{d} - \frac{b\tanh^3(c+dx)}{3d}$$

input
```
Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*ArcTanh[Tanh[c + d*x]])/d + (b*ArcTanh[Tanh[c + d*x]])/d - (a*Tanh[c +
d*x])/d - (b*Tanh[c + d*x])/d - (b*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {3042, 25, 4114, 25, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4114}$$

$$-(a+b)\int -\tanh^2(c+dx)dx-\frac{b\tanh^3(c+dx)}{3d}$$

$$\Big\downarrow \text{25}$$

$$(a+b)\int \tanh^2(c+dx)dx-\frac{b\tanh^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{b\tanh^3(c+dx)}{3d}+(a+b)\int -\tan(ic+idx)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{b\tanh^3(c+dx)}{3d}-(a+b)\int \tan(ic+idx)^2dx$$

$$\Big\downarrow \text{3954}$$

$$-(a+b)\left(\frac{\tanh(c+dx)}{d}-\int 1dx\right)-\frac{b\tanh^3(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$-(a+b)\left(\frac{\tanh(c+dx)}{d}-x\right)-\frac{b\tanh^3(c+dx)}{3d}$$

input
```
Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/3*(b*Tanh[c + d*x]^3)/d - (a + b)*(-x + Tanh[c + d*x]/d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

method	result
parallelrisch	$-\frac{b\tanh(dx+c)^3-3dxa-3dxb+3a\tanh(dx+c)+3b\tanh(dx+c)}{3d}$
derivativedivides	$\frac{-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)+1)}{2}}{d}$
default	$\frac{-\frac{b\tanh(dx+c)^3}{3}-a\tanh(dx+c)-b\tanh(dx+c)-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(a+b)\ln(\tanh(dx+c)+1)}{2}}{d}$
risch	$ax+bx+\frac{2\,e^{4dx+4c}a+4b\,e^{4dx+4c}+4\,e^{2dx+2c}a+4\,e^{2dx+2c}b+2a+\frac{8b}{3}}{d\left(e^{2dx+2c}+1\right)^3}$
parts	$\frac{a\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}+\frac{b\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tan}{}\right.}{d}$

input
```
int(tanh(d*x+c)^2*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b*tanh(d*x+c)^3-3*d*x*a-3*d*x*b+3*a*tanh(d*x+c)+3*b*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 160 vs. 2(34) = 68.

Time = 0.08 (sec) , antiderivative size = 160, normalized size of antiderivative = 4.44

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{(3\,(a+b)dx+3\,a+4\,b)\cosh\left(dx+c\right)^3+3\,(3\,(a+b)dx+3\,a+4\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-(3\,a+}{3\left(d\cosh\left(dx+c\right)^3+3\,d\cosh\left(dx\right.\right.}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
1/3*((3*(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c)^3 + 3*(3*(a + b)*d*x + 3*a
+ 4*b)*cosh(d*x + c)*sinh(d*x + c)^2 - (3*a + 4*b)*sinh(d*x + c)^3 + 3*(3*
(a + b)*d*x + 3*a + 4*b)*cosh(d*x + c) - 3*((3*a + 4*b)*cosh(d*x + c)^2 +
a)*sinh(d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 +
 3*d*cosh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.50

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \begin{cases} ax - \frac{a\tanh(c+dx)}{d} + bx - \frac{b\tanh^3(c+dx)}{3d} - \frac{b\tanh(c+dx)}{d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh^2(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*tanh(c + d*x)/d + b*x - b*tanh(c + d*x)**3/(3*d) - b*ta
nh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. 2(34) = 68.

Time = 0.05 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{1}{3}b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right) + a\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3
*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a*(x +
c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.13 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{3\,(dx+c)(a+b)+\frac{2\left(3\,ae^{(4\,dx+4\,c)}+6\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+3\,a+4\,b\right)}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*(a + b) + 2*(3*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) +
6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) + 3*a + 4*b)/(e^(2*d*x + 2*c) +
1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.94

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{b\tanh(c+d\,x)^3}{3\,d}-\frac{\tanh(c+d\,x)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (b*tanh(c + d*x)^3)/(3*d) - (tanh(c + d*x)*(a + b))/d
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.25

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{-\tanh(dx+c)^3 b-3\tanh(dx+c)a-3\tanh(dx+c)b+3adx+3bdx}{3d}$$

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

output
```
( - tanh(c + d*x)**3*b - 3*tanh(c + d*x)*a - 3*tanh(c + d*x)*b + 3*a*d*x +
 3*b*d*x)/(3*d)
```

3.137 $\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 31

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{(a+b)\log(\cosh(c+dx))}{d}-\frac{b\tanh^2(c+dx)}{2d}$$

output
```
(a+b)*ln(cosh(d*x+c))/d-1/2*b*tanh(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{a\log(\cosh(c+dx))}{d}+\frac{b(2\log(\cosh(c+dx))+\operatorname{sech}^2(c+dx))}{2d}$$

input
```
Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a*Log[Cosh[c + d*x]])/d + (b*(2*Log[Cosh[c + d*x]] + Sech[c + d*x]^2))/(2
*d)
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.29, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 26, 4114, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)dx$$

$$\Big\downarrow \text{4114}$$

$$-i\left((a+b)\int i\tanh(c+dx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left(i(a+b)\int \tanh(c+dx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3042}$$

$$-i\left(i(a+b)\int -i\tan(ic+idx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{26}$$

$$-i\left((a+b)\int \tan(ic+idx)dx-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3956}$$

$$-i\left(\frac{i(a+b)\log(\cosh(c+dx))}{d}-\frac{ib\tanh^2(c+dx)}{2d}\right)$$

input
```
Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-I)*((I*(a + b)*Log[Cosh[c + d*x]])/d - ((I/2)*b*Tanh[c + d*x]^2)/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4114
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (C_.)*tan[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> Simp[C*((a + b*Tan[e + f*x])^(m + 1)/(b*f*(m
 + 1))), x] + Simp[(A - C)   Int[(a + b*Tan[e + f*x])^m, x], x] /; FreeQ[{a
, b, e, f, A, C, m}, x] && NeQ[A*b^2 + a^2*C, 0] &&  !LeQ[m, -1]
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$\frac{-\frac{\tanh(dx+c)^2 b}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	49
default	$\frac{-\frac{\tanh(dx+c)^2 b}{2}-\frac{(a+b)\ln(\tanh(dx+c)-1)}{2}+\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	49
parts	$\frac{a\ln(\cosh(dx+c))}{d}+\frac{b\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(\tanh(dx+c)-1)}{2}-\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	52
parallelrisch	$-\frac{2dxa+2dxb+\tanh(dx+c)^2 b+2\ln(1-\tanh(dx+c))a+2\ln(1-\tanh(dx+c))b}{2d}$	55
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}+\frac{2b\,e^{2dx+2c}}{d\left(e^{2dx+2c}+1\right)^2}+\frac{\ln\left(e^{2dx+2c}+1\right)a}{d}+\frac{\ln\left(e^{2dx+2c}+1\right)b}{d}$	86

input
```
int(tanh(d*x+c)*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tanh(d*x+c)^2*b-1/2*(a+b)*ln(tanh(d*x+c)-1)+1/2*(-a-b)*ln(tanh(d
*x+c)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(29) = 58$.

Time = 0.10 (sec) , antiderivative size = 399, normalized size of antiderivative = 12.87

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(a+b)dx\cosh\left(dx+c\right)^4+4\left(a+b\right)dx\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+(a+b)dx\sinh\left(dx+c\right)^4+(a+}{}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x + 2*((a + b)*d*x - b)*cosh(d
*x + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a + b)*d*x - b)*sinh(d*x +
 c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a + b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 + ((a + b)*d*x - b)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*
x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
+ c) + d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 60 vs. $2(26) = 52$.

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.94

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \begin{cases} ax - \frac{a\log(\tanh(c+dx)+1)}{d} + bx - \frac{b\log(\tanh(c+dx)+1)}{d} - \frac{b\tanh^2(c+dx)}{2d} & \text{for } d \neq 0 \\ x\left(a+b\tanh^2(c)\right)\tanh(c) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((a*x - a*log(tanh(c + d*x) + 1)/d + b*x - b*log(tanh(c + d*x) +
1)/d - b*tanh(c + d*x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)*tanh(c),
 True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 76 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = b\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d} + \frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)} + e^{(-4\,dx-4\,c)} + 1)}\right) + \frac{a\log(\cosh(dx+c))}{d}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a*log(cosh(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.84

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-(a+b)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{2\,be^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}+1\right)^2}}{d}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(e^(2*d*x + 2*c) + 1) - 2*b*e^(2*d*x + 2*
c)/(e^(2*d*x + 2*c) + 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.19

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = x\,(a+b)-\frac{b\tanh(c+d\,x)^2}{2\,d}-\frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a+b)}{d}$$

input
```
int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (b*tanh(c + d*x)^2)/(2*d) - (log(tanh(c + d*x) + 1)*(a + b))/d
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 249, normalized size of antiderivative = 8.03

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{e^{4dx+4c}\log\left(e^{2dx+2c}+1\right)a+e^{4dx+4c}\log\left(e^{2dx+2c}+1\right)b-e^{4dx+4c}adx-e^{4dx+4c}bdx-e^{4dx+4c}b+2e^{2dx+2c}\log}{}$$

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a + e**(4*c + 4*d*x)*log(e**(2
*c + 2*d*x) + 1)*b - e**(4*c + 4*d*x)*a*d*x - e**(4*c + 4*d*x)*b*d*x - e**
(4*c + 4*d*x)*b + 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a + 2*e**(2
*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*b - 2*e**(2*c + 2*d*x)*a*d*x - 2*e**
(2*c + 2*d*x)*b*d*x + log(e**(2*c + 2*d*x) + 1)*a + log(e**(2*c + 2*d*x) +
 1)*b - a*d*x - b*d*x - b)/(d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.138 $\int \left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 12, antiderivative size = 19

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=ax+bx-\frac{b\tanh(c+dx)}{d}$$

output
```
a*x+b*x-b*tanh(d*x+c)/d
```

Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.47

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=ax+\frac{b\text{arctanh}(\tanh(c+dx))}{d}-\frac{b\tanh(c+dx)}{d}$$

input
```
Integrate[a + b*Tanh[c + d*x]^2,x]
```

output
```
a*x + (b*ArcTanh[Tanh[c + d*x]])/d - (b*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.17 (sec) , antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, $\frac{\text{number of rules}}{\text{integrand size}} = 0.083$, Rules used = {2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 2009$$

$$ax-\frac{b\tanh(c+dx)}{d}+bx$$

input
```
Int[a + b*Tanh[c + d*x]^2,x]
```

output
```
a*x + b*x - (b*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

method	result	size
parallelrisch	$-\frac{b(-dx+\tanh(dx+c))}{d}+ax$	22
risch	$ax+bx+\frac{2b}{d(e^{2dx+2c}+1)}$	27
default	$ax+\frac{b\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	41
parts	$ax+\frac{b\left(-\tanh(dx+c)-\frac{\ln(\tanh(dx+c)-1)}{2}+\frac{\ln(\tanh(dx+c)+1)}{2}\right)}{d}$	41
derivativedivides	$\frac{-b\tanh(dx+c)+\frac{(-a-b)\ln(\tanh(dx+c)-1)}{2}-\frac{(-a-b)\ln(\tanh(dx+c)+1)}{2}}{d}$	51

input
```
int(a+tanh(d*x+c)^2*b,x,method=_RETURNVERBOSE)
```

output
```
-b*(-d*x+tanh(d*x+c))/d+a*x
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.95

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=\frac{((a+b)dx+b)\cosh(dx+c)-b\sinh(dx+c)}{d\cosh(dx+c)}$$

input
```
integrate(a+b*tanh(d*x+c)^2,x, algorithm="fricas")
```

output
```
(((a + b)*d*x + b)*cosh(d*x + c) - b*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 1.05

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx = ax + b\left(\begin{cases} x - \frac{\tanh{(c+dx)}}{d} & \text{for}\ d \neq 0 \\ x\tanh^2{(c)} & \text{otherwise} \end{cases}\right)$$

input
```
integrate(a+b*tanh(d*x+c)**2,x)
```

output
```
a*x + b*Piecewise((x - tanh(c + d*x)/d, Ne(d, 0)), (x*tanh(c)**2, True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.63

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx = b\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right) + ax$$

input
```
integrate(a+b*tanh(d*x+c)^2,x, algorithm="maxima")
```

output
```
b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a*x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.53

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx = ax + \frac{\left(dx + c + \frac{2}{e^{(2\,dx+2\,c)}+1}\right)b}{d}$$

input
```
integrate(a+b*tanh(d*x+c)^2,x, algorithm="giac")
```

output
```
a*x + (d*x + c + 2/(e^(2*d*x + 2*c) + 1))*b/d
```

Mupad [B] (verification not implemented)

Time = 0.07 (sec) , antiderivative size = 18, normalized size of antiderivative = 0.95

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=x\,(a+b)-\frac{b\tanh(c+d\,x)}{d}$$

input
```
int(a + b*tanh(c + d*x)^2,x)
```

output
```
x*(a + b) - (b*tanh(c + d*x))/d
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 22, normalized size of antiderivative = 1.16

$$\int \left(a+b\tanh^2(c+dx)\right)\,dx=\frac{-\tanh\left(dx+c\right)b+adx+bdx}{d}$$

input
```
int(a+b*tanh(d*x+c)^2,x)
```

output
```
( - tanh(c + d*x)*b + a*d*x + b*d*x)/d
```

3.139 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 19, antiderivative size = 25

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{b\log(\cosh(c+dx))}{d}+\frac{a\log(\sinh(c+dx))}{d}$$

output
```
b*ln(cosh(d*x+c))/d+a*ln(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\frac{b\log(\cosh(c+dx))}{d}+\frac{a\log(\sinh(c+dx))}{d}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(b*Log[Cosh[c + d*x]])/d + (a*Log[Sinh[c + d*x]])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.29 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.56, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.368$, Rules used = {3042, 26, 4108, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{4108}$$

$$i(a\int -i\coth(c+dx)dx-b\int i\tanh(c+dx)dx)$$

$$\Big\downarrow \text{26}$$

$$i(-ia\int \coth(c+dx)dx-ib\int \tanh(c+dx)dx)$$

$$\Big\downarrow \text{3042}$$

$$i\left(-ia\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx-ib\int -i\tan(ic+idx)dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(-a\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx-b\int \tan(ic+idx)dx\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left(-\frac{ia\log(-i\sinh(c+dx))}{d}-\frac{ib\log(\cosh(c+dx))}{d}\right)$$

input
```
Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2),x]
```

output
```
I*(((-I)*b*Log[Cosh[c + d*x]])/d - (I*a*Log[(-I)*Sinh[c + d*x]])/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4108
```
Int[((A_) + (C_.)*tan[(e_.) + (f_.)*(x_)]^2)/tan[(e_.) + (f_.)*(x_)], x_Sym
bol] :> Simp[A   Int[1/Tan[e + f*x], x], x] + Simp[C   Int[Tan[e + f*x], x]
, x] /; FreeQ[{e, f, A, C}, x] && NeQ[A, C]
```

Maple [A] (verified)

Time = 0.78 (sec) , antiderivative size = 24, normalized size of antiderivative = 0.96

method	result	size
derivativedivides	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
default	$\frac{a\ln(\sinh(dx+c))+b\ln(\cosh(dx+c))}{d}$	24
parallelrisch	$\frac{(-a-b)\ln(1-\tanh(dx+c))+a\ln(\tanh(dx+c))-(a+b)xd}{d}$	41
risch	$-ax-bx-\frac{2bc}{d}-\frac{2ac}{d}+\frac{\ln\left(e^{2dx+2c}+1\right)b}{d}+\frac{a\ln\left(e^{2dx+2c}-1\right)}{d}$	58

input
```
int(coth(d*x+c)*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*ln(sinh(d*x+c))+b*ln(cosh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 70 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 70, normalized size of antiderivative = 2.80

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(a+b)dx - b\log\left(\frac{2\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right) - a\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x - b*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) - a
*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/d
```

Sympy [F]

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \int \left(a+b\tanh^2(c+dx)\right)\coth(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right) dx = \frac{b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d} + \frac{a\log\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
b*log(e^(d*x + c) + e^(-d*x - c))/d + a*log(sinh(d*x + c))/d
```

Giac [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.84

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right) dx = -\frac{(dx+c)(a+b) - b\log\left(e^{(2\,dx+2\,c)}+1\right) - a\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - b*log(e^(2*d*x + 2*c) + 1) - a*log(abs(e^(2*d*x + 2*
c) - 1)))/d
```

Mupad [B] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 228, normalized size of antiderivative = 9.12

$$\begin{aligned}\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right) dx &= \frac{a\,\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+4\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-8\,a\,b\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}\right)}{2\,d} \\ &\quad - b\,x - \frac{\mathrm{atan}\left(\frac{a\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}} - \frac{b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\,\sqrt{-d^2}}{d\,\sqrt{a^2-2\,a\,b+b^2}}\right)\,\sqrt{a^2-2\,a\,b+b^2}}{\sqrt{-d^2}} - a\,x \\ &\quad + \frac{b\,\ln\left(8\,a\,b-4\,a^2-4\,b^2+4\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+4\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-8\,a\,b\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}\right)}{2\,d}\end{aligned}$$

input
```
int(coth(c + d*x)*(a + b*tanh(c + d*x)^2),x)
```

output
```
(a*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(4*d*x) + 4*b^2*exp(4*c)*
exp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d) - b*x - (atan((a*exp(2*c)*e
xp(2*d*x)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)) - (b*exp(2*c)*exp(2*
d*x)*(-d^2)^(1/2))/(d*(a^2 - 2*a*b + b^2)^(1/2)))*(a^2 - 2*a*b + b^2)^(1/2
))/(-d^2)^(1/2) - a*x + (b*log(8*a*b - 4*a^2 - 4*b^2 + 4*a^2*exp(4*c)*exp(
4*d*x) + 4*b^2*exp(4*c)*exp(4*d*x) - 8*a*b*exp(4*c)*exp(4*d*x)))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 54, normalized size of antiderivative = 2.16

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\log\left(e^{2dx+2c}+1\right)b+\log\left(e^{dx+c}-1\right)a+\log\left(e^{dx+c}+1\right)a-adx-bdx}{d}$$

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2),x)
```

output
```
(log(e**(2*c + 2*d*x) + 1)*b + log(e**(c + d*x) - 1)*a + log(e**(c + d*x)
+ 1)*a - a*d*x - b*d*x)/d
```

3.140 $\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 18

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=(a+b)x-\frac{a\coth(c+dx)}{d}$$

output
```
(a+b)*x-a*coth(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 32, normalized size of antiderivative = 1.78

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=bx-\frac{a\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
b*x - (a*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3042, 25, 4112, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \ 4112$$

$$-\int (-a-b)dx-\frac{a\coth(c+dx)}{d}$$

$$\Big\downarrow \ 24$$

$$x(a+b)-\frac{a\coth(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(a + b)*x - (a*Coth[c + d*x])/d
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.47 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.17

method	result	size
parallelrisch	$\frac{-\coth(dx+c)a+(a+b)xd}{d}$	21
risch	$ax+bx-\frac{2a}{d\left(e^{2dx+2c}-1\right)}$	27
derivativedivides	$\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\ln(\tanh(dx+c)+1)+\left(-\frac{a}{2}-\frac{b}{2}\right)\ln(\tanh(dx+c)-1)-\frac{a}{\tanh(dx+c)}}{d}$	51
default	$\frac{\left(\frac{a}{2}+\frac{b}{2}\right)\ln(\tanh(dx+c)+1)+\left(-\frac{a}{2}-\frac{b}{2}\right)\ln(\tanh(dx+c)-1)-\frac{a}{\tanh(dx+c)}}{d}$	51

input
```
int(coth(d*x+c)^2*(a+tanh(d*x+c)^2*b),x,method=_RETURNVERBOSE)
```

output
```
(-coth(d*x+c)*a+(a+b)*x*d)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 38 vs. $2(18) = 36$.

Time = 0.09 (sec) , antiderivative size = 38, normalized size of antiderivative = 2.11

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{a\cosh(dx+c)-((a+b)dx+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-(a*cosh(d*x + c) - ((a + b)*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 66 vs. $2(14) = 28$.

Time = 4.55 (sec) , antiderivative size = 78, normalized size of antiderivative = 4.33

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = a\left(\begin{cases} x\coth^2(c) & \text{for } d=0 \\ x\coth^2\left(dx+\log\left(-e^{-dx}\right)\right) & \text{for } c=\log\left(-e^{-dx}\right) \\ x\coth^2\left(dx+\log\left(e^{-dx}\right)\right) & \text{for } c=\log\left(e^{-dx}\right) \\ x-\frac{1}{d\tanh(c+dx)} & \text{otherwise} \end{cases}\right) + b\left(\begin{cases} x & \text{for } |x|<1 \\ G_{2,2}^{1,1}\left(\begin{matrix} 1 & 2 \\ 1 & 0 \end{matrix}\middle|\,x\right)+G_{2,2}^{0,2}\left(\begin{matrix} 2,1 & \\ & 1,0 \end{matrix}\middle|\,x\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2),x)
```

output
```
a*Piecewise((x*coth(c)**2, Eq(d, 0)), (x*coth(d*x + log(-exp(-d*x)))**2, E
q(c, log(-exp(-d*x)))), (x*coth(d*x + log(exp(-d*x)))**2, Eq(c, log(exp(-d
*x)))), (x - 1/(d*tanh(c + d*x)), True)) + b*Piecewise((x, Abs(x) < 1), (m
eijerg(((1,), (2,)), ((1,), (0,)), x) + meijerg(((2, 1), ()), ((), (1, 0))
, x), True))
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.72

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = a\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+bx$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 30, normalized size of antiderivative = 1.67

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{(dx+c)(a+b)-\frac{2\,a}{e^{(2\,dx+2\,c)}-1}}{d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
((d*x + c)*(a + b) - 2*a/(e^(2*d*x + 2*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 2.39 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.39

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=x\,(a+b)-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2),x)
```

output
```
x*(a + b) - (2*a)/(d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 70, normalized size of antiderivative = 3.89

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{e^{2dx+2c}adx-2e^{2dx+2c}a+e^{2dx+2c}bdx-adx-bdx}{d\left(e^{2dx+2c}-1\right)}$$

input
```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(2*c + 2*d*x)*a*d*x - 2*e**(2*c + 2*d*x)*a + e**(2*c + 2*d*x)*b*d*x -
a*d*x - b*d*x)/(d*(e**(2*c + 2*d*x) - 1))
```

3.141 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 31

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\coth^2(c+dx)}{2d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

output
```
-1/2*a*coth(d*x+c)^2/d+(a+b)*ln(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.26

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\left(\operatorname{csch}^2(c+dx)-2\log(\sinh(c+dx))\right)}{2d}+\frac{b\log(\sinh(c+dx))}{d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/2*(a*(Csch[c + d*x]^2 - 2*Log[Sinh[c + d*x]]))/d + (b*Log[Sinh[c + d*x]
])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.30 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.42, number of steps used = 8, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3042, 26, 4112, 26, 27, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \; 4112$$

$$-i\left(\int i(a+b)\coth(c+dx)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left(i\int (a+b)\coth(c+dx)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \; 27$$

$$-i\left(i(a+b)\int \coth(c+dx)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \; 3042$$

$$-i\left(i(a+b)\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \; 26$$

$$-i\left((a+b)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx-\frac{ia\coth^2(c+dx)}{2d}\right)$$

$$\Big\downarrow \text{3956}$$

$$-i\left(\frac{i(a+b)\log(-i\sinh(c+dx))}{d}-\frac{ia\coth^2(c+dx)}{2d}\right)$$

input
```
Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-I)*(((-1/2*I)*a*Coth[c + d*x]^2)/d + (I*(a + b)*Log[(-I)*Sinh[c + d*x]])
/d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.13

method	result	size
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)+b\ln(\sinh(dx+c))}{d}$	35
parallelrisch	$\frac{(-2a-2b)\ln(1-\tanh(dx+c))+(2a+2b)\ln(\tanh(dx+c))-a\coth(dx+c)^2-2dx(a+b)}{2d}$	59
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2a\,\mathrm{e}^{2dx+2c}}{d\left(\mathrm{e}^{2dx+2c}-1\right)^2}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)a}{d}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)b}{d}$	86

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2)+b*ln(sinh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. 2(29) = 58.

Time = 0.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 13.13

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx =$$

$$-\frac{(a+b)dx\cosh(dx+c)^4+4(a+b)dx\cosh(dx+c)\sinh(dx+c)^3+(a+b)dx\sinh(dx+c)^4+(a+}{}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^4 + 4*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
3 + (a + b)*d*x*sinh(d*x + c)^4 + (a + b)*d*x - 2*((a + b)*d*x - a)*cosh(d
*x + c)^2 + 2*(3*(a + b)*d*x*cosh(d*x + c)^2 - (a + b)*d*x + a)*sinh(d*x +
 c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + b)*sinh(d*x + c)^4 - 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh
(d*x + c)^2 - a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 - (a + b
)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c)
 - sinh(d*x + c))) + 4*((a + b)*d*x*cosh(d*x + c)^3 - ((a + b)*d*x - a)*co
sh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*
x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^
2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x
+ c) + d)
```

Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int\left(a+b\tanh^2{(c+dx)}\right)\coth^3{(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 106 vs. $2(29) = 58$.

Time = 0.04 (sec) , antiderivative size = 106, normalized size of antiderivative = 3.42

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$+\frac{b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d
*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b*log(e^(d*x
+ c) - e^(-d*x - c))/d
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.87

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\,ae^{(2\,dx+2\,c)}}{\left(e^{(2\,dx+2\,c)}-1\right)^2}}{d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*a*e^(2*d*x
 + 2*c)/(e^(2*d*x + 2*c) - 1)^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.47 (sec) , antiderivative size = 76, normalized size of antiderivative = 2.45

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\ln\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-1\right)\,(a+b)}{d}-\frac{2\,a}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}-\frac{2\,a}{d\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-x\,(a+b)$$

input
```
int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2),x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*a)/(d*(exp(2*c + 2*d*x) - 1)
) - (2*a)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - x*(a + b)
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 345, normalized size of antiderivative = 11.13

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{e^{4dx+4c}\log\left(e^{dx+c}-1\right)a+e^{4dx+4c}\log\left(e^{dx+c}-1\right)b+e^{4dx+4c}\log\left(e^{dx+c}+1\right)a+e^{4dx+4c}\log\left(e^{dx+c}+1\right)b-e}{}$$

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2),x)
```

output
```
(e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a + e**(4*c + 4*d*x)*log(e**(c + d
*x) - 1)*b + e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a + e**(4*c + 4*d*x)*l
og(e**(c + d*x) + 1)*b - e**(4*c + 4*d*x)*a*d*x - e**(4*c + 4*d*x)*a - e**
(4*c + 4*d*x)*b*d*x - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a - 2*e**(2
*c + 2*d*x)*log(e**(c + d*x) - 1)*b - 2*e**(2*c + 2*d*x)*log(e**(c + d*x)
+ 1)*a - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b + 2*e**(2*c + 2*d*x)*a
*d*x + 2*e**(2*c + 2*d*x)*b*d*x + log(e**(c + d*x) - 1)*a + log(e**(c + d*
x) - 1)*b + log(e**(c + d*x) + 1)*a + log(e**(c + d*x) + 1)*b - a*d*x - a
- b*d*x)/(d*(e**(4*c + 4*d*x) - 2*e**(2*c + 2*d*x) + 1))
```

3.142 $\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 36

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=(a+b)x-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}$$

output
```
(a+b)*x-(a+b)*coth(d*x+c)/d-1/3*a*coth(d*x+c)^3/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.04 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.69

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{a\coth^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)}{3d}-\frac{b\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/3*(a*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*x]^2])
/d - (b*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 4112, 27, 3042, 25, 3954, 24}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^4}dx$$

$$\Big\downarrow \text{4112}$$

$$\int (a+b)\coth^2(c+dx)dx-\frac{a\coth^3(c+dx)}{3d}$$

$$\Big\downarrow \text{27}$$

$$(a+b)\int \coth^2(c+dx)dx-\frac{a\coth^3(c+dx)}{3d}$$

$$\Big\downarrow \text{3042}$$

$$-\frac{a\coth^3(c+dx)}{3d}+(a+b)\int -\tan\left(ic+idx+\frac{\pi}{2}\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\frac{a\coth^3(c+dx)}{3d}-(a+b)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^2dx$$

$$\Big\downarrow \text{3954}$$

$$-(a+b)\left(\frac{\coth(c+dx)}{d}-\int 1dx\right)-\frac{a\coth^3(c+dx)}{3d}$$

$$\Big\downarrow \text{24}$$

$$-(a+b)\left(\frac{\coth(c+dx)}{d}-x\right)-\frac{a\coth^3(c+dx)}{3d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/3*(a*Coth[c + d*x]^3)/d - (a + b)*(-x + Coth[c + d*x]/d)
```

Defintions of rubi rules used

rule 24
```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 39, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-\coth(dx+c)^3 a+(-3a-3b)\coth(dx+c)+3dx(a+b)}{3d}$	39
derivativedivides	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
default	$\frac{a\left(dx+c-\coth(dx+c)-\frac{\coth(dx+c)^3}{3}\right)+b(dx+c-\coth(dx+c))}{d}$	46
risch	$ax+bx-\frac{2(6a\,e^{4dx+4c}+3b\,e^{4dx+4c}-6a\,e^{2dx+2c}-6b\,e^{2dx+2c}+4a+3b)}{3d(e^{2dx+2c}-1)^3}$	81

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/3*(-coth(d*x+c)^3*a+(-3*a-3*b)*coth(d*x+c)+3*d*x*(a+b))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 156 vs. 2(34) = 68.

Time = 0.09 (sec) , antiderivative size = 156, normalized size of antiderivative = 4.33

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(4\,a+3\,b)\cosh\left(dx+c\right)^3+3\,(4\,a+3\,b)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-(3\,(a+b)dx+4\,a+3\,b)\sinh\left(c\right.}{3\left(d\sinh\left(dx+c\right)^3+3\left(d\right.\right.}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/3*((4*a + 3*b)*cosh(d*x + c)^3 + 3*(4*a + 3*b)*cosh(d*x + c)*sinh(d*x +
 c)^2 - (3*(a + b)*d*x + 4*a + 3*b)*sinh(d*x + c)^3 - 3*b*cosh(d*x + c) +
3*(3*(a + b)*d*x - (3*(a + b)*d*x + 4*a + 3*b)*cosh(d*x + c)^2 + 4*a + 3*b
)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x +
 c))
```

Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)\coth^4{(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(34) = 68$.

Time = 0.04 (sec) , antiderivative size = 105, normalized size of antiderivative = 2.92

$$\begin{aligned}&\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx\\&=\frac{1}{3}\,a\left(3\,x+\frac{3\,c}{d}-\frac{4\,\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)\\&\quad+b\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)\end{aligned}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/3*a*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*(3
*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b*(x +
c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 86 vs. $2(34) = 68$.

Time = 0.15 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.39

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$= \frac{3\,(dx+c)(a+b) - \frac{2\left(6\,ae^{(4\,dx+4\,c)}+3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+4\,a+3\,b\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*(d*x + c)*(a + b) - 2*(6*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) -
6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + 4*a + 3*b)/(e^(2*d*x + 2*c) -
1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.53 (sec) , antiderivative size = 162, normalized size of antiderivative = 4.50

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \frac{\frac{2\,b}{3\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(2\,a+b)}{3\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1} - \frac{\frac{2\,(2\,a+b)}{3\,d}-\frac{4\,b\,e^{2\,c+2\,d\,x}}{3\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(2\,a+b)}{3\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1} + x\,(a+b) - \frac{2\,(2\,a+b)}{3\,d\,\left(e^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2),x)
```

output
```
((2*b)/(3*d) - (2*exp(2*c + 2*d*x)*(2*a + b))/(3*d))/(exp(4*c + 4*d*x) - 2
*exp(2*c + 2*d*x) + 1) - ((2*(2*a + b))/(3*d) - (4*b*exp(2*c + 2*d*x))/(3*
d) + (2*exp(4*c + 4*d*x)*(2*a + b))/(3*d))/(3*exp(2*c + 2*d*x) - 3*exp(4*c
 + 4*d*x) + exp(6*c + 6*d*x) - 1) + x*(a + b) - (2*(2*a + b))/(3*d*(exp(2*
c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 189, normalized size of antiderivative = 5.25

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{3e^{6dx+6c}adx-4e^{6dx+6c}a+3e^{6dx+6c}bdx-2e^{6dx+6c}b-9e^{4dx+4c}adx-9e^{4dx+4c}bdx+9e^{2dx+2c}adx+9e^{2dx+}}{3d\left(e^{6dx+6c}-3e^{4dx+4c}+3e^{2dx+2c}-1\right)}$$

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2),x)
```

output
```
(3*e**(6*c + 6*d*x)*a*d*x - 4*e**(6*c + 6*d*x)*a + 3*e**(6*c + 6*d*x)*b*d*
x - 2*e**(6*c + 6*d*x)*b - 9*e**(4*c + 4*d*x)*a*d*x - 9*e**(4*c + 4*d*x)*b
*d*x + 9*e**(2*c + 2*d*x)*a*d*x + 9*e**(2*c + 2*d*x)*b*d*x + 6*e**(2*c + 2
*d*x)*b - 3*a*d*x - 4*a - 3*b*d*x - 4*b)/(3*d*(e**(6*c + 6*d*x) - 3*e**(4*
c + 4*d*x) + 3*e**(2*c + 2*d*x) - 1))
```

3.143 $\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{(a+b)\coth^2(c+dx)}{2d}-\frac{a\coth^4(c+dx)}{4d}+\frac{(a+b)\log(\sinh(c+dx))}{d}$$

output
```
-1/2*(a+b)*coth(d*x+c)^2/d-1/4*a*coth(d*x+c)^4/d+(a+b)*ln(sinh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.92

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=-\frac{2(2a+b)\text{csch}^2(c+dx)+a\text{csch}^4(c+dx)-4(a+b)\log(\sinh(c+dx))}{4d}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/4*(2*(2*a + b)*Csch[c + d*x]^2 + a*Csch[c + d*x]^4 - 4*(a + b)*Log[Sinh
[c + d*x]])/d
```

Rubi [C] (verified)

Result contains complex when optimal does not.

Time = 0.37 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.29, number of steps used = 12, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.571$, Rules used = {3042, 26, 4112, 26, 27, 3042, 26, 3954, 26, 3042, 26, 3956}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{a-b\tan(ic+idx)^2}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{4112}$$

$$i\left(\int -i(a+b)\coth^3(c+dx)dx+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left(\frac{ia\coth^4(c+dx)}{4d}-i\int (a+b)\coth^3(c+dx)dx\right)$$

$$\Big\downarrow \text{27}$$

$$i\left(\frac{ia\coth^4(c+dx)}{4d}-i(a+b)\int \coth^3(c+dx)dx\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left(\frac{ia\coth^4(c+dx)}{4d}-i(a+b)\int i\tan\left(ic+idx+\frac{\pi}{2}\right)^3dx\right)$$

$$\Big\downarrow \text{26}$$

$$i\left((a+b)\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)^3dx+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3954}$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-\int i\coth(c+dx)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-i\int \coth(c+dx)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3042}$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-i\int -i\tan\left(ic+idx+\frac{\pi}{2}\right)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{26}$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-\int \tan\left(\frac{1}{2}(2ic+\pi)+idx\right)dx\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

$$\Big\downarrow \text{3956}$$

$$i\left((a+b)\left(\frac{i\coth^2(c+dx)}{2d}-\frac{i\log(-i\sinh(c+dx))}{d}\right)+\frac{ia\coth^4(c+dx)}{4d}\right)$$

input
```
Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2),x]
```

output
```
I*(((I/4)*a*Coth[c + d*x]^4)/d + (a + b)*(((I/2)*Coth[c + d*x]^2)/d - (I*L
og[(-I)*Sinh[c + d*x]])/d))
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 3954
```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[b*((b*Tan[c + d
*x])^(n - 1)/(d*(n - 1))), x] - Simp[b^2   Int[(b*Tan[c + d*x])^(n - 2), x]
, x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

rule 3956
```
Int[tan[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-Log[RemoveContent[Cos[c + d
*x], x]]/d, x] /; FreeQ[{c, d}, x]
```

rule 4112
```
Int[((a_.) + (b_.)*tan[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (C_.)*tan[(e_.) +
 (f_.)*(x_)]^2), x_Symbol] :> Simp[(A*b^2 + a^2*C)*((a + b*Tan[e + f*x])^(m
 + 1)/(b*f*(m + 1)*(a^2 + b^2))), x] + Simp[1/(a^2 + b^2)   Int[(a + b*Tan[
e + f*x])^(m + 1)*Simp[a*(A - C) - (A*b - b*C)*Tan[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, C}, x] && NeQ[A*b^2 + a^2*C, 0] && LtQ[m, -1] && NeQ[
a^2 + b^2, 0]
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.14

method	result
derivativedivides	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)}{d}$
default	$\frac{a\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}-\frac{\coth(dx+c)^4}{4}\right)+b\left(\ln(\sinh(dx+c))-\frac{\coth(dx+c)^2}{2}\right)}{d}$
parallelrisch	$\frac{(-4a-4b)\ln(1-\tanh(dx+c))+(4a+4b)\ln(\tanh(dx+c))-\coth(dx+c)^4a+(-2a-2b)\coth(dx+c)^2-4dx(a+b)}{4d}$
risch	$-ax-bx-\frac{2ac}{d}-\frac{2bc}{d}-\frac{2\,e^{2dx+2c}\left(2a\,e^{4dx+4c}+b\,e^{4dx+4c}-2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+2a+b\right)}{d\left(e^{2dx+2c}-1\right)^4}+\frac{\ln\left(e^{2dx+2c}-\right.}{d}$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(a*(ln(sinh(d*x+c))-1/2*coth(d*x+c)^2-1/4*coth(d*x+c)^4)+b*(ln(sinh(d*
x+c))-1/2*coth(d*x+c)^2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1216 vs. $2(45) = 90$.

Time = 0.13 (sec) , antiderivative size = 1216, normalized size of antiderivative = 24.82

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-((a + b)*d*x*cosh(d*x + c)^8 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x + c)^
7 + (a + b)*d*x*sinh(d*x + c)^8 - 2*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c
)^6 + 2*(14*(a + b)*d*x*cosh(d*x + c)^2 - 2*(a + b)*d*x + 2*a + b)*sinh(d*
x + c)^6 + 4*(14*(a + b)*d*x*cosh(d*x + c)^3 - 3*(2*(a + b)*d*x - 2*a - b)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a + b)*d*x - 2*a - 2*b)*cosh(d*x +
 c)^4 + 2*(35*(a + b)*d*x*cosh(d*x + c)^4 + 3*(a + b)*d*x - 15*(2*(a + b)*
d*x - 2*a - b)*cosh(d*x + c)^2 - 2*a - 2*b)*sinh(d*x + c)^4 + 8*(7*(a + b)
*d*x*cosh(d*x + c)^5 - 5*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^3 + (3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c))*sinh(d*x + c)^3 + (a + b)*d*x - 2*(2
*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^2 + 2*(14*(a + b)*d*x*cosh(d*x + c)^
6 - 15*(2*(a + b)*d*x - 2*a - b)*cosh(d*x + c)^4 - 2*(a + b)*d*x + 6*(3*(a
 + b)*d*x - 2*a - 2*b)*cosh(d*x + c)^2 + 2*a + b)*sinh(d*x + c)^2 - ((a +
b)*cosh(d*x + c)^8 + 8*(a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (a + b)*sin
h(d*x + c)^8 - 4*(a + b)*cosh(d*x + c)^6 + 4*(7*(a + b)*cosh(d*x + c)^2 -
a - b)*sinh(d*x + c)^6 + 8*(7*(a + b)*cosh(d*x + c)^3 - 3*(a + b)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 6*(a + b)*cosh(d*x + c)^4 + 2*(35*(a + b)*cosh(d*
x + c)^4 - 30*(a + b)*cosh(d*x + c)^2 + 3*a + 3*b)*sinh(d*x + c)^4 + 8*(7*
(a + b)*cosh(d*x + c)^5 - 10*(a + b)*cosh(d*x + c)^3 + 3*(a + b)*cosh(d*x
+ c))*sinh(d*x + c)^3 - 4*(a + b)*cosh(d*x + c)^2 + 4*(7*(a + b)*cosh(d*x
+ c)^6 - 15*(a + b)*cosh(d*x + c)^4 + 9*(a + b)*cosh(d*x + c)^2 - a - b...
```

Sympy [F]

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx=\int \left(a+b\tanh^2{(c+dx)}\right)\coth^5{(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)*coth(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(45) = 90$.

Time = 0.04 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.20

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=a\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\right.$$
$$+b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
a*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*
d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) -
 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + b*(x
+ c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x -
2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(45) = 90$.

Time = 0.17 (sec) , antiderivative size = 93, normalized size of antiderivative = 1.90

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx = -\frac{(dx+c)(a+b)-(a+b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{2\left((2\,a+b)e^{(6\,dx+6\,c)}-2\,(a+b)e^{(4\,dx+4\,c)}+(2\,a+b)e^{(2\,dx+2\,c)}\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-((d*x + c)*(a + b) - (a + b)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*((2*a + b)
*e^(6*d*x + 6*c) - 2*(a + b)*e^(4*d*x + 4*c) + (2*a + b)*e^(2*d*x + 2*c))/
(e^(2*d*x + 2*c) - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 2.42 (sec) , antiderivative size = 177, normalized size of antiderivative = 3.61

$$\begin{aligned}\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx &= \frac{\ln\left(e^{2\,c}\,e^{2\,d\,x}-1\right)\,(a+b)}{d}-\frac{2\,(2\,a+b)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-x\,(a+b)\\&\quad-\frac{2\,(4\,a+b)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{8\,a}{d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}\\&\quad-\frac{4\,a}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}\end{aligned}$$

input
```
int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2),x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(a + b))/d - (2*(2*a + b))/(d*(exp(2*c + 2*d
*x) - 1)) - x*(a + b) - (2*(4*a + b))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2
*d*x) + 1)) - (8*a)/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c
+ 6*d*x) - 1)) - (4*a)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 664, normalized size of antiderivative = 13.55

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)\,dx$$
$$=\frac{-2a-b-2bdx-2e^{8dx+8c}adx-2e^{8dx+8c}bdx-8e^{6dx+6c}\log\left(e^{dx+c}-1\right)a-8e^{6dx+6c}\log\left(e^{dx+c}+1\right)a+1}{}$$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a + 2*e**(8*c + 8*d*x)*log(e**(c
 + d*x) - 1)*b + 2*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a + 2*e**(8*c +
8*d*x)*log(e**(c + d*x) + 1)*b - 2*e**(8*c + 8*d*x)*a*d*x - 2*e**(8*c + 8*
d*x)*a - 2*e**(8*c + 8*d*x)*b*d*x - e**(8*c + 8*d*x)*b - 8*e**(6*c + 6*d*x
)*log(e**(c + d*x) - 1)*a - 8*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*b - 8
*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a - 8*e**(6*c + 6*d*x)*log(e**(c +
 d*x) + 1)*b + 8*e**(6*c + 6*d*x)*a*d*x + 8*e**(6*c + 6*d*x)*b*d*x + 12*e*
*(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a + 12*e**(4*c + 4*d*x)*log(e**(c + d
*x) - 1)*b + 12*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a + 12*e**(4*c + 4*
d*x)*log(e**(c + d*x) + 1)*b - 12*e**(4*c + 4*d*x)*a*d*x - 4*e**(4*c + 4*d
*x)*a - 12*e**(4*c + 4*d*x)*b*d*x + 2*e**(4*c + 4*d*x)*b - 8*e**(2*c + 2*d
*x)*log(e**(c + d*x) - 1)*a - 8*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b -
 8*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a - 8*e**(2*c + 2*d*x)*log(e**(c
 + d*x) + 1)*b + 8*e**(2*c + 2*d*x)*a*d*x + 8*e**(2*c + 2*d*x)*b*d*x + 2*l
og(e**(c + d*x) - 1)*a + 2*log(e**(c + d*x) - 1)*b + 2*log(e**(c + d*x) +
1)*a + 2*log(e**(c + d*x) + 1)*b - 2*a*d*x - 2*a - 2*b*d*x - b)/(2*d*(e**(
8*c + 8*d*x) - 4*e**(6*c + 6*d*x) + 6*e**(4*c + 4*d*x) - 4*e**(2*c + 2*d*x
) + 1))
```

3.144 $\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = (a+b)^2x-\frac{(a+b)^2\tanh(c+dx)}{d}-\frac{(a+b)^2\tanh^3(c+dx)}{3d}-\frac{b(2a+b)\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

output
```
(a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*(a+b)^2*tanh(d*x+c)^3/d-1/5*b*(2*a+b)*
tanh(d*x+c)^5/d-1/7*b^2*tanh(d*x+c)^7/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 190 vs. 2(83) = 166.

Time = 0.06 (sec) , antiderivative size = 190, normalized size of antiderivative = 2.29

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{a^2\text{arctanh}(\tanh(c+dx))}{d}+\frac{2ab\text{arctanh}(\tanh(c+dx))}{d}+\frac{b^2\text{arctanh}(\tanh(c+dx))}{d}-\frac{a^2\tanh(c+dx)}{d}-\frac{2ab\tanh(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}-\frac{a^2\tanh^3(c+dx)}{3d}-\frac{2ab\tanh^3(c+dx)}{3d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{2ab\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^7(c+dx)}{7d}$$

input
```
Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*A
rcTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d
 - (b^2*Tanh[c + d*x])/d - (a^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d
*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^3)/(3*d) - (2*a*b*Tanh[c + d*x]^5)/(5*d)
 - (b^2*Tanh[c + d*x]^5)/(5*d) - (b^2*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 364

$$\frac{\int \left(-b^2\tanh^6(c+dx)-b(2a+b)\tanh^4(c+dx)-(a+b)^2\tanh^2(c+dx)-(a+b)^2+\frac{a^2+2ba+b^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+}{d}$$

$\Big\downarrow$ 2009

$$\frac{(a+b)^2\text{arctanh}(\tanh(c+dx))-\frac{1}{5}b(2a+b)\tanh^5(c+dx)-\frac{1}{3}(a+b)^2\tanh^3(c+dx)-(a+b)^2\tanh(c+dx)-}{d}$$

input
```
Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*ArcTanh[Tanh[c + d*x]] - (a + b)^2*Tanh[c + d*x] - ((a + b)^2*T
anh[c + d*x]^3)/3 - (b*(2*a + b)*Tanh[c + d*x]^5)/5 - (b^2*Tanh[c + d*x]^7
)/7)/d
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (warning: unable to verify)

Time = 0.09 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.63

method	result
parallelrisch	$-\frac{15b^2\tanh(dx+c)^7+42ab\tanh(dx+c)^5+21b^2\tanh(dx+c)^5+35a^2\tanh(dx+c)^3+70ab\tanh(dx+c)^3+35b^2\tanh(dx+}{105d}$
derivativedivides	$\frac{-2ab\tanh(dx+c)-\frac{2ab\tanh(dx+c)^5}{5}-\frac{2ab\tanh(dx+c)^3}{3}+\frac{(a^2+2ab+b^2)\ln(1+\tanh(dx+c))}{2}-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c)}{2}}{d}$
default	$\frac{-2ab\tanh(dx+c)-\frac{2ab\tanh(dx+c)^5}{5}-\frac{2ab\tanh(dx+c)^3}{3}+\frac{(a^2+2ab+b^2)\ln(1+\tanh(dx+c))}{2}-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c)}{2}}{d}$
parts	$\frac{a^2\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\tanh(\right.}{}$
risch	$a^2x+2abx+b^2x+\frac{4a^2e^{12dx+12c}+12ab\,e^{12dx+12c}+8b^2e^{12dx+12c}+20a^2e^{10dx+10c}+48ab\,e^{10dx+10c}+24b^2e^{10c}}{}$

input
```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(15*b^2*tanh(d*x+c)^7+42*a*b*tanh(d*x+c)^5+21*b^2*tanh(d*x+c)^5+35*
a^2*tanh(d*x+c)^3+70*a*b*tanh(d*x+c)^3+35*b^2*tanh(d*x+c)^3-105*a^2*d*x-21
0*a*b*d*x-105*b^2*d*x+105*a^2*tanh(d*x+c)+210*a*b*tanh(d*x+c)+105*b^2*tanh
(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 796 vs. $2(77) = 154$.

Time = 0.09 (sec) , antiderivative size = 796, normalized size of antiderivative = 9.59

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/105*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*
x + c)^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*c
osh(d*x + c)*sinh(d*x + c)^6 - 2*(70*a^2 + 161*a*b + 88*b^2)*sinh(d*x + c)
^7 + 7*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*
x + c)^5 - 14*(3*(70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^2 + 40*a^2 + 71
*a*b + 28*b^2)*sinh(d*x + c)^5 + 35*((105*(a^2 + 2*a*b + b^2)*d*x + 140*a^
2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 + (105*(a^2 + 2*a*b + b^2)*d*x + 14
0*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 21*(105*(a^2 +
 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^3 - 14*(5*(
70*a^2 + 161*a*b + 88*b^2)*cosh(d*x + c)^4 + 10*(40*a^2 + 71*a*b + 28*b^2)
*cosh(d*x + c)^2 + 60*a^2 + 123*a*b + 84*b^2)*sinh(d*x + c)^3 + 7*(3*(105*
(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c)^5 + 1
0*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d*x + c
)^3 + 9*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322*a*b + 176*b^2)*cosh(d
*x + c))*sinh(d*x + c)^2 + 35*(105*(a^2 + 2*a*b + b^2)*d*x + 140*a^2 + 322
*a*b + 176*b^2)*cosh(d*x + c) - 14*((70*a^2 + 161*a*b + 88*b^2)*cosh(d*x +
 c)^6 + 5*(40*a^2 + 71*a*b + 28*b^2)*cosh(d*x + c)^4 + 9*(20*a^2 + 41*a*b
+ 28*b^2)*cosh(d*x + c)^2 + 30*a^2 + 75*a*b)*sinh(d*x + c))/(d*cosh(d*x +
c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + 7*d*cosh(d*x + c)^5 + 35*(d*cos
h(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^4 + 21*d*cosh(d*x + c)^3 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 165 vs. $2(70) = 140$.

Time = 0.25 (sec) , antiderivative size = 165, normalized size of antiderivative = 1.99

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$$
$$= \begin{cases} a^2x - \frac{a^2\tanh^3(c+dx)}{3d} - \frac{a^2\tanh(c+dx)}{d} + 2abx - \frac{2ab\tanh^5(c+dx)}{5d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{2ab\tanh(c+dx)}{d} + b^2x - \frac{b^2\tan}{} \\ x\left(a+b\tanh^2(c)\right)^2\tanh^4(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*tanh(c + d*x)**3/(3*d) - a**2*tanh(c + d*x)/d + 2
*a*b*x - 2*a*b*tanh(c + d*x)**5/(5*d) - 2*a*b*tanh(c + d*x)**3/(3*d) - 2*a
*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**7/(7*d) - b**2*tanh(c +
d*x)**5/(5*d) - b**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d,
0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(77) = 154$.

Time = 0.05 (sec) , antiderivative size = 369, normalized size of antiderivative = 4.45

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$$
$$= \frac{1}{105}b^2\left(105\,x + \frac{105\,c}{d} - \frac{8\left(203\,e^{(-2\,dx-2\,c)} + 609\,e^{(-4\,dx-4\,c)} + 770\,e^{(-6\,dx-6\,c)} + 770\,e^{(-8\,dx-8\,c)} + 315\,e^{(-}\right.}{d(7\,e^{(-2\,dx-2\,c)} + 21\,e^{(-4\,dx-4\,c)} + 35\,e^{(-6\,dx-6\,c)} + 35\,e^{(-8\,dx-8\,c)} + 21\,e^{(-10\,dx-10\,c)}}\right.$$
$$+ \frac{2}{15}ab\left(15\,x + \frac{15\,c}{d} - \frac{2\left(70\,e^{(-2\,dx-2\,c)} + 140\,e^{(-4\,dx-4\,c)} + 90\,e^{(-6\,dx-6\,c)} + 45\,e^{(-8\,dx-8\,c)} + 23\right)}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)}\right.$$
$$+ \frac{1}{3}a^2\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/105*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c
) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) +
 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/15*a*b*(15*x + 15*c/d -
2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*e
^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^
(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 1/3*a^2*
(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2
*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 300 vs. $2(77) = 154$.

Time = 0.21 (sec) , antiderivative size = 300, normalized size of antiderivative = 3.61

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{105\left(a^2+2ab+b^2\right)(dx+c)+\frac{4\left(105a^2e^{(12dx+12c)}+315abe^{(12dx+12c)}+210b^2e^{(12dx+12c)}+525a^2e^{(10dx+10c)}+1260abe^{(10dx+\cdots}{\cdots}}{\cdots}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/105*(105*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(105*a^2*e^(12*d*x + 12*c) +
315*a*b*e^(12*d*x + 12*c) + 210*b^2*e^(12*d*x + 12*c) + 525*a^2*e^(10*d*x
+ 10*c) + 1260*a*b*e^(10*d*x + 10*c) + 630*b^2*e^(10*d*x + 10*c) + 1120*a^
2*e^(8*d*x + 8*c) + 2555*a*b*e^(8*d*x + 8*c) + 1540*b^2*e^(8*d*x + 8*c) +
1330*a^2*e^(6*d*x + 6*c) + 3080*a*b*e^(6*d*x + 6*c) + 1540*b^2*e^(6*d*x +
6*c) + 945*a^2*e^(4*d*x + 4*c) + 2121*a*b*e^(4*d*x + 4*c) + 1218*b^2*e^(4*
d*x + 4*c) + 385*a^2*e^(2*d*x + 2*c) + 812*a*b*e^(2*d*x + 2*c) + 406*b^2*e
^(2*d*x + 2*c) + 70*a^2 + 161*a*b + 88*b^2)/(e^(2*d*x + 2*c) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = x\,(a^2+2\,a\,b+b^2)-\frac{\tanh(c+d\,x)\,\left(a+b\right)^2}{d}-\frac{\tanh(c+d\,x)^5\,(b^2+2\,a\,b)}{5\,d}-\frac{b^2\tanh(c+d\,x)^7}{7\,d}-\frac{\tanh(c+d\,x)^3\,(a^2+2\,a\,b+b^2)}{3\,d}$$

input
```
int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^5*(2*
a*b + b^2))/(5*d) - (b^2*tanh(c + d*x)^7)/(7*d) - (tanh(c + d*x)^3*(2*a*b
+ a^2 + b^2))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.61

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{-15\tanh\left(dx+c\right)^7b^2-42\tanh\left(dx+c\right)^5ab-21\tanh\left(dx+c\right)^5b^2-35\tanh\left(dx+c\right)^3a^2-70\tanh\left(dx\right.}{}$$

input
```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 15*tanh(c + d*x)**7*b**2 - 42*tanh(c + d*x)**5*a*b - 21*tanh(c + d*x)*
*5*b**2 - 35*tanh(c + d*x)**3*a**2 - 70*tanh(c + d*x)**3*a*b - 35*tanh(c +
 d*x)**3*b**2 - 105*tanh(c + d*x)*a**2 - 210*tanh(c + d*x)*a*b - 105*tanh(
c + d*x)*b**2 + 105*a**2*d*x + 210*a*b*d*x + 105*b**2*d*x)/(105*d)
```

3.145 $\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 76

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\log(\cosh(c+dx))}{d}-\frac{(a+b)^2\tanh^2(c+dx)}{2d}-\frac{b(2a+b)\tanh^4(c+dx)}{4d}-\frac{b^2\tanh^6(c+dx)}{6d}$$

output
```
(a+b)^2*ln(cosh(d*x+c))/d-1/2*(a+b)^2*tanh(d*x+c)^2/d-1/4*b*(2*a+b)*tanh(d
*x+c)^4/d-1/6*b^2*tanh(d*x+c)^6/d
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{12(a+b)^2\log(\cosh(c+dx))+6(a^2+4ab+3b^2)\operatorname{sech}^2(c+dx)-3b(2a+3b)\operatorname{sech}^4(c+dx)+2b^2\operatorname{sech}^6(c}{12d}$$

input
```
Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(12*(a + b)^2*Log[Cosh[c + d*x]] + 6*(a^2 + 4*a*b + 3*b^2)*Sech[c + d*x]^2
 - 3*b*(2*a + 3*b)*Sech[c + d*x]^4 + 2*b^2*Sech[c + d*x]^6)/(12*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int i\tan(ic+idx)^3 \left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \tan(ic+idx)^3 \left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\int \frac{\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 86}$$

$$\frac{\int \left(-b^2\tanh^4(c+dx)-b(2a+b)\tanh^2(c+dx)-(a+b)^2-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}b(2a+b)\tanh^4(c+dx)-(a+b)^2\tanh^2(c+dx)-(a+b)^2\log\left(1-\tanh^2(c+dx)\right)-\frac{1}{3}b^2\tanh^6(c+dx)}{2d}$$

input
```
Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)^2*Log[1 - Tanh[c + d*x]^2]) - (a + b)^2*Tanh[c + d*x]^2 - (b*(2
*a + b)*Tanh[c + d*x]^4)/2 - (b^2*Tanh[c + d*x]^6)/3)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 130, normalized size of antiderivative = 1.71

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4ab}{2}-\tanh(dx+c)^2ab+\frac{(-a^2-2ab-b^2)\ln(1+\tanh(dx+c))}{2}-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c))}{2}-\frac{\tanh(dx+c)^4b^2}{4}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4ab}{2}-\tanh(dx+c)^2ab+\frac{(-a^2-2ab-b^2)\ln(1+\tanh(dx+c))}{2}-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c))}{2}-\frac{\tanh(dx+c)^4b^2}{4}}{d}$
parallelrisch	$-\frac{2\tanh(dx+c)^6b^2+6\tanh(dx+c)^4ab+3\tanh(dx+c)^4b^2+12a^2dx+24abdx+12b^2dx+6\tanh(dx+c)^2a^2+12\tanh(dx+}{12d}$
parts	$\frac{a^2\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(-1+\tanh(dx+c))}{2}-\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\ln(-1\right.}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}+\frac{2\,e^{2dx+2c}(3a^2e^{8dx+8c}+12ab\,e^{8dx+8c}+9b^2e^{8dx+8c}+12a^2e^{6}}{}$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tanh(d*x+c)^4*a*b-tanh(d*x+c)^2*a*b+1/2*(-a^2-2*a*b-b^2)*ln(1+ta
nh(d*x+c))-1/2*(a^2+2*a*b+b^2)*ln(-1+tanh(d*x+c))-1/4*tanh(d*x+c)^4*b^2-1/
2*tanh(d*x+c)^2*a^2-1/2*b^2*tanh(d*x+c)^2-1/6*tanh(d*x+c)^6*b^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3441 vs. 2(70) = 140.

Time = 0.13 (sec) , antiderivative size = 3441, normalized size of antiderivative = 45.28

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(65) = 130$.

Time = 0.23 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.24

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} - \frac{a^2\tanh^2(c+dx)}{2d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^4(c+dx)}{2d} - \frac{ab\tanh^2(c+dx)}{d} + b \\ x\left(a+b\tanh^2(c)\right)^2\tanh^3(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d - a**2*tanh(c + d*x)**2/
(2*d) + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**4/(2
*d) - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d*x) + 1)/d - b*
*2*tanh(c + d*x)**6/(6*d) - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*
x)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 333 vs. $2(70) = 140$.

Time = 0.12 (sec) , antiderivative size = 333, normalized size of antiderivative = 4.38

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{3}b^2\left(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}+18e^{(-4dx-4c)}+34e^{(-6dx-6c)}+18e^{(-8dx-8c)}\right.}{d(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}}\right.$$
$$+2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}+e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1)}\right)$$
$$+a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*
c) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e
^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x -
 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^
(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + a^2*(x + c
/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*
c) + e^(-4*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(70) = 140$.

Time = 0.20 (sec) , antiderivative size = 191, normalized size of antiderivative = 2.51

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=$$
$$-\frac{3\left(a^2+2ab+b^2\right)(dx+c)-3\left(a^2+2ab+b^2\right)\log\left(e^{(2dx+2c)}+1\right)-\frac{2\left(3\left(a^2+4ab+3b^2\right)e^{(10dx+10c)}+6\left(2a^2+6ab\right.\right.}{}}{3d}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(e^(2*d*x
 + 2*c) + 1) - 2*(3*(a^2 + 4*a*b + 3*b^2)*e^(10*d*x + 10*c) + 6*(2*a^2 + 6
*a*b + 3*b^2)*e^(8*d*x + 8*c) + 2*(9*a^2 + 24*a*b + 17*b^2)*e^(6*d*x + 6*c
) + 6*(2*a^2 + 6*a*b + 3*b^2)*e^(4*d*x + 4*c) + 3*(a^2 + 4*a*b + 3*b^2)*e^
(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.32

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = x\,\left(a^2+2\,a\,b+b^2\right)-\frac{\tanh(c+d\,x)^4\,\left(b^2+2\,a\,b\right)}{4\,d}-\frac{\ln\left(\tanh(c+d\,x)+1\right)\,\left(a^2+2\,a\,b+b^2\right)}{d}-\frac{b^2\,\tanh(c+d\,x)^6}{6\,d}-\frac{\tanh(c+d\,x)^2\,\left(a^2+2\,a\,b+b^2\right)}{2\,d}$$

input
```
int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^4*(2*a*b + b^2))/(4*d) - (log(tanh(
c + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^6)/(6*d) - (tanh
(c + d*x)^2*(2*a*b + a^2 + b^2))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1141, normalized size of antiderivative = 15.01

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(3*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2 + 6*e**(12*c + 12*d*x
)*log(e**(2*c + 2*d*x) + 1)*a*b + 3*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x
) + 1)*b**2 - 3*e**(12*c + 12*d*x)*a**2*d*x - e**(12*c + 12*d*x)*a**2 - 6*
e**(12*c + 12*d*x)*a*b*d*x - 4*e**(12*c + 12*d*x)*a*b - 3*e**(12*c + 12*d*
x)*b**2*d*x - 3*e**(12*c + 12*d*x)*b**2 + 18*e**(10*c + 10*d*x)*log(e**(2*
c + 2*d*x) + 1)*a**2 + 36*e**(10*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b
 + 18*e**(10*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*b**2 - 18*e**(10*c + 10
*d*x)*a**2*d*x - 36*e**(10*c + 10*d*x)*a*b*d*x - 18*e**(10*c + 10*d*x)*b**
2*d*x + 45*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2 + 90*e**(8*c +
8*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 45*e**(8*c + 8*d*x)*log(e**(2*c + 2
*d*x) + 1)*b**2 - 45*e**(8*c + 8*d*x)*a**2*d*x + 9*e**(8*c + 8*d*x)*a**2 -
 90*e**(8*c + 8*d*x)*a*b*d*x + 12*e**(8*c + 8*d*x)*a*b - 45*e**(8*c + 8*d*
x)*b**2*d*x - 9*e**(8*c + 8*d*x)*b**2 + 60*e**(6*c + 6*d*x)*log(e**(2*c +
2*d*x) + 1)*a**2 + 120*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 60
*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*b**2 - 60*e**(6*c + 6*d*x)*a**
2*d*x + 16*e**(6*c + 6*d*x)*a**2 - 120*e**(6*c + 6*d*x)*a*b*d*x + 16*e**(6
*c + 6*d*x)*a*b - 60*e**(6*c + 6*d*x)*b**2*d*x + 8*e**(6*c + 6*d*x)*b**2 +
 45*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2 + 90*e**(4*c + 4*d*x)*
log(e**(2*c + 2*d*x) + 1)*a*b + 45*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) +
 1)*b**2 - 45*e**(4*c + 4*d*x)*a**2*d*x + 9*e**(4*c + 4*d*x)*a**2 - 90*...
```

3.146 $\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = (a+b)^2x - \frac{(a+b)^2\tanh(c+dx)}{d} - \frac{b(2a+b)\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
(a+b)^2*x-(a+b)^2*tanh(d*x+c)/d-1/3*b*(2*a+b)*tanh(d*x+c)^3/d-1/5*b^2*tanh
(d*x+c)^5/d
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 137 vs. 2(63) = 126.

Time = 0.05 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.17

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{a^2\text{arctanh}(\tanh(c+dx))}{d} + \frac{2ab\text{arctanh}(\tanh(c+dx))}{d} + \frac{b^2\text{arctanh}(\tanh(c+dx))}{d} - \frac{a^2\tanh(c+dx)}{d} - \frac{2ab\tanh(c+dx)}{d} - \frac{b^2\tanh(c+dx)}{d} - \frac{2ab\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

input
```
Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*ArcTanh[Tanh[c + d*x]])/d + (2*a*b*ArcTanh[Tanh[c + d*x]])/d + (b^2*A
rcTanh[Tanh[c + d*x]])/d - (a^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x])/d
 - (b^2*Tanh[c + d*x])/d - (2*a*b*Tanh[c + d*x]^3)/(3*d) - (b^2*Tanh[c + d
*x]^3)/(3*d) - (b^2*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2dx$$

$$\Big\downarrow \text{ 4153}$$

$$-\frac{\int -\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 364}$$

$$\frac{\int \left(-b^2\tanh^4(c+dx)-b(2a+b)\tanh^2(c+dx)-(a+b)^2+\frac{a^2+2ba+b^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{-(a+b)^2\text{arctanh}(\tanh(c+dx))+\frac{1}{3}b(2a+b)\tanh^3(c+dx)+(a+b)^2\tanh(c+dx)+\frac{1}{5}b^2\tanh^5(c+dx)}{d}$$

input
```
Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*ArcTanh[Tanh[c + d*x]]) + (a + b)^2*Tanh[c + d*x] + (b*(2*a
 + b)*Tanh[c + d*x]^3)/3 + (b^2*Tanh[c + d*x]^5)/5)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (warning: unable to verify)

Time = 0.07 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.54

method	result
parallelrisch	$-\frac{3b^2\tanh(dx+c)^5+10ab\tanh(dx+c)^3+5b^2\tanh(dx+c)^3-15a^2dx-30abdx-15b^2dx+15a^2\tanh(dx+c)+30ab\tanh(}{15d}$
derivativedivides	$\frac{-2ab\tanh(dx+c)-\frac{2ab\tanh(dx+c)^3}{3}+\frac{(a^2+2ab+b^2)\ln(1+\tanh(dx+c))}{2}-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c))}{2}-\frac{b^2\tanh(dx+c)^5}{5}}{d}$
default	$\frac{-2ab\tanh(dx+c)-\frac{2ab\tanh(dx+c)^3}{3}+\frac{(a^2+2ab+b^2)\ln(1+\tanh(dx+c))}{2}-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c))}{2}-\frac{b^2\tanh(dx+c)^5}{5}}{d}$
parts	$\frac{a^2\left(-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(-1+}{}\right.}{d}$
risch	$a^2x+2abx+b^2x+\frac{2a^2e^{8dx+8c}+8ab\,e^{8dx+8c}+6b^2e^{8dx+8c}+8a^2e^{6dx+6c}+24ab\,e^{6dx+6c}+12b^2e^{6dx+6c}+12a^2e^{4}}{d(e^{2}}$

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(3*b^2*tanh(d*x+c)^5+10*a*b*tanh(d*x+c)^3+5*b^2*tanh(d*x+c)^3-15*a^2
*d*x-30*a*b*d*x-15*b^2*d*x+15*a^2*tanh(d*x+c)+30*a*b*tanh(d*x+c)+15*b^2*ta
nh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 483 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 483, normalized size of antiderivative = 7.67

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{(15\,(a^2+2\,ab+b^2)dx+15\,a^2+40\,ab+23\,b^2)\cosh{(dx+c)}^5+5\,(15\,(a^2+2\,ab+b^2)dx+15\,a^2+40\,ab}{}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/15*((15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c
)^5 + 5*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x +
 c)*sinh(d*x + c)^4 - (15*a^2 + 40*a*b + 23*b^2)*sinh(d*x + c)^5 + 5*(15*(
a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^3 - 5*(2*
(15*a^2 + 40*a*b + 23*b^2)*cosh(d*x + c)^2 + 9*a^2 + 16*a*b + 5*b^2)*sinh(
d*x + c)^3 + 5*(2*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2)*
cosh(d*x + c)^3 + 3*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2 + 40*a*b + 23*b^2
)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(15*(a^2 + 2*a*b + b^2)*d*x + 15*a^2
 + 40*a*b + 23*b^2)*cosh(d*x + c) - 5*((15*a^2 + 40*a*b + 23*b^2)*cosh(d*x
 + c)^4 + 3*(9*a^2 + 16*a*b + 5*b^2)*cosh(d*x + c)^2 + 6*a^2 + 8*a*b + 10*
b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4
 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(
d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(53) = 106$.

Time = 0.24 (sec) , antiderivative size = 117, normalized size of antiderivative = 1.86

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \begin{cases} a^2x - \frac{a^2\tanh{(c+dx)}}{d} + 2abx - \frac{2ab\tanh^3{(c+dx)}}{3d} - \frac{2ab\tanh{(c+dx)}}{d} + b^2x - \frac{b^2\tanh^5{(c+dx)}}{5d} - \frac{b^2\tanh^3{(c+dx)}}{3d} - \frac{b^2\tanh}{} \\ x\left(a+b\tanh^2{(c)}\right)^2\tanh^2{(c)} \end{cases}$$

input
```
integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*tanh(c + d*x)/d + 2*a*b*x - 2*a*b*tanh(c + d*x)**
3/(3*d) - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b
**2*tanh(c + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*ta
nh(c)**2)**2*tanh(c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.

Time = 0.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.67

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{15}b^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1)}\right)$$
$$+\frac{2}{3}ab\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)$$
$$+a^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d
*x - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6
*c) + 1))) + a^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(59) = 118$.

Time = 0.18 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.46

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$$
$$= \frac{15\left(a^2+2ab+b^2\right)(dx+c)+\frac{2\left(15a^2e^{(8dx+8c)}+60abe^{(8dx+8c)}+45b^2e^{(8dx+8c)}+60a^2e^{(6dx+6c)}+180abe^{(6dx+6c)}+90b^2e^{(6dx+}\right.}{}}{15d}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) + 2*(15*a^2*e^(8*d*x + 8*c) + 60*a*
b*e^(8*d*x + 8*c) + 45*b^2*e^(8*d*x + 8*c) + 60*a^2*e^(6*d*x + 6*c) + 180*
a*b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 90*a^2*e^(4*d*x + 4*c) + 22
0*a*b*e^(4*d*x + 4*c) + 140*b^2*e^(4*d*x + 4*c) + 60*a^2*e^(2*d*x + 2*c) +
 140*a*b*e^(2*d*x + 2*c) + 70*b^2*e^(2*d*x + 2*c) + 15*a^2 + 40*a*b + 23*b
^2)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.06

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = x\left(a^2+2ab+b^2\right) - \frac{\tanh(c+dx)\,(a+b)^2}{d} - \frac{\tanh(c+dx)^3\,(b^2+2ab)}{3d} - \frac{b^2\tanh(c+dx)^5}{5d}$$

input
```
int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)*(a + b)^2)/d - (tanh(c + d*x)^3*(2*
a*b + b^2))/(3*d) - (b^2*tanh(c + d*x)^5)/(5*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.52

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{-3\tanh(dx+c)^5 b^2 - 10\tanh(dx+c)^3 ab - 5\tanh(dx+c)^3 b^2 - 15\tanh(dx+c)a^2 - 30\tanh(dx+c}{15d}$$

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 3*tanh(c + d*x)**5*b**2 - 10*tanh(c + d*x)**3*a*b - 5*tanh(c + d*x)**3
*b**2 - 15*tanh(c + d*x)*a**2 - 30*tanh(c + d*x)*a*b - 15*tanh(c + d*x)*b*
*2 + 15*a**2*d*x + 30*a*b*d*x + 15*b**2*d*x)/(15*d)
```

3.147 $\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 57

$$\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{(a+b)^2\log(\cosh(c+dx))}{d} - \frac{b(a+b)\tanh^2(c+dx)}{2d} - \frac{\left(a+b\tanh^2(c+dx)\right)^2}{4d}$$

output
```
(a+b)^2*ln(cosh(d*x+c))/d-1/2*b*(a+b)*tanh(d*x+c)^2/d-1/4*(a+b*tanh(d*x+c)
^2)^2/d
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 49, normalized size of antiderivative = 0.86

$$\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \frac{4(a+b)^2\log(\cosh(c+dx)) + 4b(a+b)\text{sech}^2(c+dx) - b^2\text{sech}^4(c+dx)}{4d}$$

input
```
Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(4*(a + b)^2*Log[Cosh[c + d*x]] + 4*b*(a + b)*Sech[c + d*x]^2 - b^2*Sech[c
 + d*x]^4)/(4*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(a+b)^2}{1-\tanh^2(c+dx)}-b(a+b)-b\left(b\tanh^2(c+dx)+a\right)\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-b(a+b)\tanh^2(c+dx)-\frac{1}{2}\left(a+b\tanh^2(c+dx)\right)^2+(a+b)^2\left(-\log\left(1-\tanh^2(c+dx)\right)\right)}{2d}$$

input
```
Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)^2*Log[1 - Tanh[c + d*x]^2]) - b*(a + b)*Tanh[c + d*x]^2 - (a +
b*Tanh[c + d*x]^2)^2/2)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.61

method	result
derivativedivides	$\frac{-\frac{\tanh(dx+c)^4 b^2}{4}-\tanh(dx+c)^2 ab-\frac{b^2\tanh(dx+c)^2}{2}-\frac{\left(a^2+2ab+b^2\right)\ln(-1+\tanh(dx+c))}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(1+\tanh(dx+c))}{2}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^4 b^2}{4}-\tanh(dx+c)^2 ab-\frac{b^2\tanh(dx+c)^2}{2}-\frac{\left(a^2+2ab+b^2\right)\ln(-1+\tanh(dx+c))}{2}+\frac{\left(-a^2-2ab-b^2\right)\ln(1+\tanh(dx+c))}{2}}{d}$
parts	$\frac{a^2\ln(\cosh(dx+c))}{d}+\frac{b^2\left(-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(-1+\tanh(dx+c))}{2}-\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{2ab\left(-\frac{\tanh(dx+c}{2}\right.}{}$
parallelrisch	$-\frac{\tanh(dx+c)^4 b^2+4a^2dx+8abdx+4b^2dx+4\tanh(dx+c)^2ab+2b^2\tanh(dx+c)^2+4\ln(1-\tanh(dx+c))a^2+8\ln(1-\tanh}{4d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}+\frac{4b\,e^{2dx+2c}\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}+b\,e^{2dx+2c}+a\right.}{d\left(e^{2dx+2c}+1\right)^4}$

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/4*tanh(d*x+c)^4*b^2-tanh(d*x+c)^2*a*b-1/2*b^2*tanh(d*x+c)^2-1/2*(a
^2+2*a*b+b^2)*ln(-1+tanh(d*x+c))+1/2*(-a^2-2*a*b-b^2)*ln(1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1638 vs. 2(53) = 106.

Time = 0.10 (sec) , antiderivative size = 1638, normalized size of antiderivative = 28.74

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 + 4*((
a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b +
b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 + 2*a*b + b
^2)*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b +
b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*
cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x + 30*((a^2 + 2*a*b + b^2)*d*x
- a*b - b^2)*cosh(d*x + c)^2 - 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 8*(7*(a^2
+ 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 10*((a^2 + 2*a*b + b^2)*d*x - a*b - b
^2)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x + 4*((a^2 + 2*a*b + b^2)*
d*x - a*b - b^2)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x +
 c)^6 + 15*((a^2 + 2*a*b + b^2)*d*x - a*b - b^2)*cosh(d*x + c)^4 + (a^2 +
2*a*b + b^2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 4*a*b - 2*b^2)*cosh(d*x
+ c)^2 - a*b - b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8
 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 122 vs. $2(48) = 96$.

Time = 0.21 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.14

$$\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$$
$$= \begin{cases} a^2x - \frac{a^2\log(\tanh(c+dx)+1)}{d} + 2abx - \frac{2ab\log(\tanh(c+dx)+1)}{d} - \frac{ab\tanh^2(c+dx)}{d} + b^2x - \frac{b^2\log(\tanh(c+dx)+1)}{d} - \frac{b^2\tan}{} \\ x\left(a+b\tanh^2(c)\right)^2\tanh(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x - a**2*log(tanh(c + d*x) + 1)/d + 2*a*b*x - 2*a*b*log(ta
nh(c + d*x) + 1)/d - a*b*tanh(c + d*x)**2/d + b**2*x - b**2*log(tanh(c + d
*x) + 1)/d - b**2*tanh(c + d*x)**4/(4*d) - b**2*tanh(c + d*x)**2/(2*d), Ne
(d, 0)), (x*(a + b*tanh(c)**2)**2*tanh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 186 vs. $2(53) = 106$.

Time = 0.12 (sec) , antiderivative size = 186, normalized size of antiderivative = 3.26

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{a^2\log\left(\cosh\left(dx+c\right)\right)}{d}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d
*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c)
+ 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 2*a*b*(x + c/d + log(e^(-
2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*
x - 4*c) + 1))) + a^2*log(cosh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 116 vs. $2(53) = 106$.

Time = 0.16 (sec) , antiderivative size = 116, normalized size of antiderivative = 2.04

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$

$$-\frac{(a^2+2\,ab+b^2)(dx+c)-(a^2+2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+1\right)-\frac{4\left((ab+b^2)e^{(6\,dx+6\,c)}+(2\,ab+b^2)e^{(4\,dx+4\,c)}+(ab+\right.}{\left(e^{(2\,dx+2\,c)}+1\right)^4}}{d}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(e^(2*d*x + 2*c)
+ 1) - 4*((a*b + b^2)*e^(6*d*x + 6*c) + (2*a*b + b^2)*e^(4*d*x + 4*c) + (a
*b + b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.33

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\left(a^2+2\,a\,b+b^2\right)-\frac{\tanh(c+d\,x)^2\,(b^2+2\,a\,b)}{2\,d} - \frac{\ln\left(\tanh(c+d\,x)+1\right)\,(a^2+2\,a\,b+b^2)}{d} - \frac{b^2\tanh(c+d\,x)^4}{4\,d}$$

input
```
int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (tanh(c + d*x)^2*(2*a*b + b^2))/(2*d) - (log(tanh(
c + d*x) + 1)*(2*a*b + a^2 + b^2))/d - (b^2*tanh(c + d*x)^4)/(4*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 723, normalized size of antiderivative = 12.68

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{-e^{8dx+8c}ab-4e^{2dx+2c}a^2dx-e^{8dx+8c}a^2dx-e^{8dx+8c}b^2dx-4e^{6dx+6c}a^2dx-4e^{6dx+6c}b^2dx-6e^{4dx+4c}a^2dx-}{}$$

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2 + 2*e**(8*c + 8*d*x)*log(
e**(2*c + 2*d*x) + 1)*a*b + e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*b**
2 - e**(8*c + 8*d*x)*a**2*d*x - 2*e**(8*c + 8*d*x)*a*b*d*x - e**(8*c + 8*d
*x)*a*b - e**(8*c + 8*d*x)*b**2*d*x - e**(8*c + 8*d*x)*b**2 + 4*e**(6*c +
6*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2 + 8*e**(6*c + 6*d*x)*log(e**(2*c + 2
*d*x) + 1)*a*b + 4*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*b**2 - 4*e**
(6*c + 6*d*x)*a**2*d*x - 8*e**(6*c + 6*d*x)*a*b*d*x - 4*e**(6*c + 6*d*x)*b
**2*d*x + 6*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2 + 12*e**(4*c +
 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 6*e**(4*c + 4*d*x)*log(e**(2*c + 2
*d*x) + 1)*b**2 - 6*e**(4*c + 4*d*x)*a**2*d*x - 12*e**(4*c + 4*d*x)*a*b*d*
x + 2*e**(4*c + 4*d*x)*a*b - 6*e**(4*c + 4*d*x)*b**2*d*x - 2*e**(4*c + 4*d
*x)*b**2 + 4*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2 + 8*e**(2*c +
 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 4*e**(2*c + 2*d*x)*log(e**(2*c + 2
*d*x) + 1)*b**2 - 4*e**(2*c + 2*d*x)*a**2*d*x - 8*e**(2*c + 2*d*x)*a*b*d*x
 - 4*e**(2*c + 2*d*x)*b**2*d*x + log(e**(2*c + 2*d*x) + 1)*a**2 + 2*log(e*
*(2*c + 2*d*x) + 1)*a*b + log(e**(2*c + 2*d*x) + 1)*b**2 - a**2*d*x - 2*a*
b*d*x - a*b - b**2*d*x - b**2)/(d*(e**(8*c + 8*d*x) + 4*e**(6*c + 6*d*x) +
 6*e**(4*c + 4*d*x) + 4*e**(2*c + 2*d*x) + 1))
```

3.148 $\int \left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 14, antiderivative size = 43

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx=(a+b)^2x-\frac{b(2a+b)\tanh(c+dx)}{d}-\frac{b^2\tanh^3(c+dx)}{3d}$$

output
```
(a+b)^2*x-b*(2*a+b)*tanh(d*x+c)/d-1/3*b^2*tanh(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.51 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{\tanh(c+dx)\left(\frac{3(a+b)^2\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(6a+b\left(3+\tanh^2(c+dx)\right)\right)\right)}{3d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(Tanh[c + d*x]*((3*(a + b)^2*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c +
 d*x]^2] - b*(6*a + b*(3 + Tanh[c + d*x]^2))))/(3*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\tan(ic+idx)^2\right)^2 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(\frac{(a+b)^2}{1-\tanh^2(c+dx)}-b^2\tanh^2(c+dx)-b(2a+b)\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{(a+b)^2\text{arctanh}(\tanh(c+dx))-b(2a+b)\tanh(c+dx)-\frac{1}{3}b^2\tanh^3(c+dx)}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*ArcTanh[Tanh[c + d*x]] - b*(2*a + b)*Tanh[c + d*x] - (b^2*Tanh[
c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (warning: unable to verify)

Time = 0.06 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

method	result
parallelrisch	$-\frac{b^{2} \tanh \left(d x+c\right)^{3}-3 a^{2} d x-6 a b d x-3 b^{2} d x+6 a b \tanh \left(d x+c\right)+3 b^{2} \tanh \left(d x+c\right)}{3 d}$
derivativedivides	$\frac{-\frac{b^{2} \tanh \left(d x+c\right)^{3}}{3}-2 a b \tanh \left(d x+c\right)-b^{2} \tanh \left(d x+c\right)-\frac{\left(a^{2}+2 a b+b^{2}\right) \ln \left(-1+\tanh \left(d x+c\right)\right)}{2}+\frac{\left(a^{2}+2 a b+b^{2}\right) \ln \left(1+\tanh \left(d x+c\right)\right)}{2}}{d}$
default	$\frac{-\frac{b^{2} \tanh \left(d x+c\right)^{3}}{3}-2 a b \tanh \left(d x+c\right)-b^{2} \tanh \left(d x+c\right)-\frac{\left(a^{2}+2 a b+b^{2}\right) \ln \left(-1+\tanh \left(d x+c\right)\right)}{2}+\frac{\left(a^{2}+2 a b+b^{2}\right) \ln \left(1+\tanh \left(d x+c\right)\right)}{2}}{d}$
risch	$a^{2} x+2 a b x+b^{2} x+\frac{4 b \left(3 a \,{\mathrm e}^{4 d x+4 c}+3 b \,{\mathrm e}^{4 d x+4 c}+6 a \,{\mathrm e}^{2 d x+2 c}+3 b \,{\mathrm e}^{2 d x+2 c}+3 a+2 b \right)}{3 d \left({\mathrm e}^{2 d x+2 c}+1\right)^{3}}$
parts	$a^{2} x+\frac{b^{2} \left(-\frac{\tanh \left(d x+c\right)^{3}}{3}-\tanh \left(d x+c\right)-\frac{\ln \left(-1+\tanh \left(d x+c\right)\right)}{2}+\frac{\ln \left(1+\tanh \left(d x+c\right)\right)}{2}\right)}{d}+\frac{2 a b \left(-\tanh \left(d x+c\right)-\frac{\ln \left(-1+\tan\right.}{2}\right.}{d}$

input
```
int((a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/3*(b^2*tanh(d*x+c)^3-3*a^2*d*x-6*a*b*d*x-3*b^2*d*x+6*a*b*tanh(d*x+c)+3*
b^2*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 201 vs. $2(41) = 82$.

Time = 0.11 (sec) , antiderivative size = 201, normalized size of antiderivative = 4.67

$$\int \left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{(3\,(a^2+2\,ab+b^2)dx+6\,ab+4\,b^2)\cosh\,(dx+c)^3+3\,(3\,(a^2+2\,ab+b^2)dx+6\,ab+4\,b^2)\cosh\,(dx+c)\,\mathrm{s}}{3\,\big(d\cosh\,(dx}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
1/3*((3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*(a
^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 - 2*(
3*a*b + 2*b^2)*sinh(d*x + c)^3 + 3*(3*(a^2 + 2*a*b + b^2)*d*x + 6*a*b + 4*
b^2)*cosh(d*x + c) - 6*((3*a*b + 2*b^2)*cosh(d*x + c)^2 + a*b)*sinh(d*x +
c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x
+ c))
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.58

$$\int \left(a+b\tanh^2(c+dx)\right)^2 dx = \begin{cases} a^2x+2abx-\frac{2ab\tanh{(c+dx)}}{d}+b^2x-\frac{b^2\tanh^3{(c+dx)}}{3d}-\frac{b^2\tanh{(c+dx)}}{d} & \text{for } d\neq 0 \\ x\left(a+b\tanh^2{(c)}\right)^2 & \text{otherwise} \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x - 2*a*b*tanh(c + d*x)/d + b**2*x - b**2*tanh(c
 + d*x)**3/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)*
*2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. 2(41) = 82.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.65

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right) + 2ab\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+a^2x$$

input
```
integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2*a*b
*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. 2(41) = 82.

Time = 0.12 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{3\,(a^2+2ab+b^2)(dx+c)+\frac{4\left(3abe^{(4dx+4c)}+3b^2e^{(4dx+4c)}+6abe^{(2dx+2c)}+3b^2e^{(2dx+2c)}+3ab+2b^2\right)}{\left(e^{(2dx+2c)}+1\right)^3}}{3d}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) + 4*(3*a*b*e^(4*d*x + 4*c) + 3*b^2*e^
(4*d*x + 4*c) + 6*a*b*e^(2*d*x + 2*c) + 3*b^2*e^(2*d*x + 2*c) + 3*a*b + 2*
b^2)/(e^(2*d*x + 2*c) + 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx=x\,\left(a^2+2\,a\,b+b^2\right)-\frac{b^2\tanh(c+d\,x)^3}{3\,d}-\frac{b\tanh(c+d\,x)\,\left(2\,a+b\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x)^3)/(3*d) - (b*tanh(c + d*x)*(2*
a + b))/d
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.40

$$\int \left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{-\tanh\left(dx+c\right)^3b^2-6\tanh\left(dx+c\right)ab-3\tanh\left(dx+c\right)b^2+3a^2dx+6abdx+3b^2dx}{3d}$$

input
```
int((a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - tanh(c + d*x)**3*b**2 - 6*tanh(c + d*x)*a*b - 3*tanh(c + d*x)*b**2 + 3
*a**2*d*x + 6*a*b*d*x + 3*b**2*d*x)/(3*d)
```

3.149 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 21, antiderivative size = 49

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a^2\log(\tanh(c+dx))}{d}-\frac{b^2\tanh^2(c+dx)}{2d}$$

output
```
(a+b)^2*ln(cosh(d*x+c))/d+a^2*ln(tanh(d*x+c))/d-1/2*b^2*tanh(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.16

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{2ab\log(\cosh(c+dx))}{d}+\frac{a^2\log(\sinh(c+dx))}{d}+\frac{b^2\left(2\log(\cosh(c+dx))+\operatorname{sech}^2(c+dx)\right)}{2d}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*a*b*Log[Cosh[c + d*x]])/d + (a^2*Log[Sinh[c + d*x]])/d + (b^2*(2*Log[Co
sh[c + d*x]] + Sech[c + d*x]^2))/(2*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\coth(c+dx)a^2-b^2-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2 \log \left(\tanh^2(c+dx)\right) - (a+b)^2 \log \left(1-\tanh^2(c+dx)\right) - b^2 \tanh^2(c+dx)}{2d}$$

input
```
Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(a^2*Log[Tanh[c + d*x]^2] - (a + b)^2*Log[1 - Tanh[c + d*x]^2] - b^2*Tanh[
c + d*x]^2)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.20

method	result
parallelrisch	$\frac{-2(a+b)^2\ln(1-\tanh(dx+c))+2a^2\ln(\tanh(dx+c))-b^2\tanh(dx+c)^2-2dx(a+b)^2}{2d}$
derivativedivides	$-\frac{\frac{b^2\tanh(dx+c)^2}{2}-a^2\ln(\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh(dx+c))}{d}$
default	$-\frac{\frac{b^2\tanh(dx+c)^2}{2}-a^2\ln(\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh(dx+c))}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}+\frac{2b^2e^{2dx+2c}}{d\left(e^{2dx+2c}+1\right)^2}+\frac{a^2\ln\left(e^{2dx+2c}-1\right)}{d}+\frac{2\ln\left(e^{2dx+2c}+1\right)}{d}$

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*(a+b)^2*ln(1-tanh(d*x+c))+2*a^2*ln(tanh(d*x+c))-b^2*tanh(d*x+c)^2-
2*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 668 vs. 2(47) = 94.

Time = 0.10 (sec) , antiderivative size = 668, normalized size of antiderivative = 13.63

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2
 + 2*a*b + b^2)*d*x + 2*((a^2 + 2*a*b + b^2)*d*x - b^2)*cosh(d*x + c)^2 +
2*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + (a^2 + 2*a*b + b^2)*d*x - b
^2)*sinh(d*x + c)^2 - ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2)*cos
h(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a*b + b^
2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 2*a*b + b^2)*sin
h(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (2*a*b + b
^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sin
h(d*x + c))) - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3
+ a^2*sinh(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 +
 a^2)*sinh(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + ((a^2 + 2*a*b + b^2)*d*x - b^2)*c
osh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d
*x + c)^3 + d*sinh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^2 + d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

Sympy [F]

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = \int \left(a+b\tanh^2(c+dx)\right)^2 \coth(c+dx)\, dx$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(47) = 94$.

Time = 0.11 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.12

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$$
$$= b^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{2\,ab\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a^2\log\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-
2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + 2*a*b*log(e^(d*x + c) + e^(-d*x -
 c))/d + a^2*log(sinh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(47) = 94$.

Time = 0.16 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.88

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$$
$$= \frac{a^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+(2\,ab+b^2)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{2\,ab\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right.}{e^{(2\,dx+}}}{2\,d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(a^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + (2*a*b + b^2)*log(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (2*a*b*(e^(2*d*x + 2*c) + e^(-2*d
*x - 2*c)) + b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*a*b - 2*b^2)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2))/d
```

Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 210, normalized size of antiderivative = 4.29

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{2b^2}{d\left(e^{2c+2dx}+1\right)} - x(a+b)^2 - \frac{2b^2}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} + \frac{\ln\left(e^{4c+4dx}-1\right)\left(d\left(b^2+2ab\right)+a^2d\right)}{2d^2} + \frac{\operatorname{atan}\left(\frac{e^{2c}e^{2dx}\left(b^2\sqrt{-d^2}-a^2\sqrt{-d^2}+2ab\sqrt{-d^2}\right)}{d\sqrt{a^4-4a^3b+2a^2b^2+4ab^3+b^4}}\right)\sqrt{a^4-4a^3b+2a^2b^2+4ab^3+b^4}}{\sqrt{-d^2}}$$

input
```
int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(2*b^2)/(d*(exp(2*c + 2*d*x) + 1)) - x*(a + b)^2 - (2*b^2)/(d*(2*exp(2*c +
 2*d*x) + exp(4*c + 4*d*x) + 1)) + (log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b +
b^2) + a^2*d))/(2*d^2) + (atan((exp(2*c)*exp(2*d*x)*(b^2*(-d^2)^(1/2) - a^
2*(-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a*b^3 - 4*a^3*b + a^4 + b^4 +
2*a^2*b^2)^(1/2)))*(4*a*b^3 - 4*a^3*b + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^
2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 440, normalized size of antiderivative = 8.98

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = \frac{2e^{4dx+4c}\log\left(e^{2dx+2c}+1\right)ab+e^{4dx+4c}\log\left(e^{2dx+2c}+1\right)b^2+e^{4dx+4c}\log\left(e^{dx+c}-1\right)a^2+e^{4dx+4c}\log\left(e^{dx+c}+\right.}{}$$

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(2*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + e**(4*c + 4*d*x)*log(e
**(2*c + 2*d*x) + 1)*b**2 + e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2 +
e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2 - e**(4*c + 4*d*x)*a**2*d*x -
2*e**(4*c + 4*d*x)*a*b*d*x - e**(4*c + 4*d*x)*b**2*d*x - e**(4*c + 4*d*x)*
b**2 + 4*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 2*e**(2*c + 2*d*
x)*log(e**(2*c + 2*d*x) + 1)*b**2 + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) -
1)*a**2 + 2*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2 - 2*e**(2*c + 2*d*
x)*a**2*d*x - 4*e**(2*c + 2*d*x)*a*b*d*x - 2*e**(2*c + 2*d*x)*b**2*d*x + 2
*log(e**(2*c + 2*d*x) + 1)*a*b + log(e**(2*c + 2*d*x) + 1)*b**2 + log(e**(
c + d*x) - 1)*a**2 + log(e**(c + d*x) + 1)*a**2 - a**2*d*x - 2*a*b*d*x - b
**2*d*x - b**2)/(d*(e**(4*c + 4*d*x) + 2*e**(2*c + 2*d*x) + 1))
```

3.150 $\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 36

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = (a+b)^2x-\frac{a^2\coth(c+dx)}{d}-\frac{b^2\tanh(c+dx)}{d}$$

output
```
(a+b)^2*x-a^2*coth(d*x+c)/d-b^2*tanh(d*x+c)/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\begin{aligned}&\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx\\&=2abx+\frac{b^2\text{arctanh}(\tanh(c+dx))}{d}\\&\quad-\frac{a^2\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}-\frac{b^2\tanh(c+dx)}{d}\end{aligned}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
2*a*b*x + (b^2*ArcTanh[Tanh[c + d*x]])/d - (a^2*Coth[c + d*x]*Hypergeometr
ic2F1[-1/2, 1, 1/2, Tanh[c + d*x]^2])/d - (b^2*Tanh[c + d*x])/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.11, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(-b^2+a^2\coth^2(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^2\coth(c+dx)-(a+b)^2\text{arctanh}(\tanh(c+dx))+b^2\tanh(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*ArcTanh[Tanh[c + d*x]]) + a^2*Coth[c + d*x] + b^2*Tanh[c +
d*x])/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 36, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-\coth(dx+c)a^2-b^2\tanh(dx+c)+dx(a+b)^2}{d}$	36
derivativedivides	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
default	$\frac{a^2(dx+c-\coth(dx+c))+2ab(dx+c)+b^2(dx+c-\tanh(dx+c))}{d}$	49
risch	$a^2x+2abx+b^2x-\frac{2(a^2e^{2dx+2c}-b^2e^{2dx+2c}+a^2+b^2)}{d(e^{2dx+2c}+1)(e^{2dx+2c}-1)}$	82

input
```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
(-coth(d*x+c)*a^2-b^2*tanh(d*x+c)+d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 97 vs. 2(36) = 72.

Time = 0.08 (sec) , antiderivative size = 97, normalized size of antiderivative = 2.69

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{(a^2+b^2)\cosh(dx+c)^2-2\left((a^2+2\,ab+b^2)dx+a^2+b^2\right)\cosh(dx+c)\sinh(dx+c)+(a^2+b^2)\sinh}{2\,d\cosh(dx+c)\sinh(dx+c)}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*((a^2 + b^2)*cosh(d*x + c)^2 - 2*((a^2 + 2*a*b + b^2)*d*x + a^2 + b^2
)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2)/(
d*cosh(d*x + c)*sinh(d*x + c))
```

Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\int \left(a+b\tanh^2(c+dx)\right)^2\coth^2(c+dx)\,dx$$

input
```
integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**2, x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.78

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=b^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)+2\,abx$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
b^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^2*(x + c/d + 2/(d*(e^(-2*
d*x - 2*c) - 1))) + 2*a*b*x
```

Giac [A] (verification not implemented)

Time = 0.18 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.97

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{(a^2+2\,ab+b^2)(dx+c)-\frac{2\,\left(a^2e^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+a^2+b^2\right)}{e^{(4\,dx+4\,c)}-1}}{d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
((a^2 + 2*a*b + b^2)*(d*x + c) - 2*(a^2*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2
*c) + a^2 + b^2)/(e^(4*d*x + 4*c) - 1))/d
```

Mupad [B] (verification not implemented)

Time = 2.46 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.64

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\,(a+b)^2-\frac{\frac{2\,(a^2+b^2)}{d}+\frac{2\,e^{2\,c+2\,d\,x}\,(a^2-b^2)}{d}}{e^{4\,c+4\,d\,x}-1}$$

input
```
int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(a + b)^2 - ((2*(a^2 + b^2))/d + (2*exp(2*c + 2*d*x)*(a^2 - b^2))/d)/(ex
p(4*c + 4*d*x) - 1)
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 147, normalized size of antiderivative = 4.08

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{e^{4dx+4c}a^2dx-2e^{4dx+4c}a^2+2e^{4dx+4c}abdx+e^{4dx+4c}b^2dx-2e^{4dx+4c}b^2-2e^{2dx+2c}a^2+2e^{2dx+2c}b^2-a^2dx-}{d\left(e^{4dx+4c}-1\right)}$$

input
```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(4*c + 4*d*x)*a**2*d*x - 2*e**(4*c + 4*d*x)*a**2 + 2*e**(4*c + 4*d*x)*
a*b*d*x + e**(4*c + 4*d*x)*b**2*d*x - 2*e**(4*c + 4*d*x)*b**2 - 2*e**(2*c
+ 2*d*x)*a**2 + 2*e**(2*c + 2*d*x)*b**2 - a**2*d*x - 2*a*b*d*x - b**2*d*x)
/(d*(e**(4*c + 4*d*x) - 1))
```

3.151 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 52

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=-\frac{a^2\coth^2(c+dx)}{2d}+\frac{(a+b)^2\log(\cosh(c+dx))}{d}+\frac{a(a+2b)\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a^2*coth(d*x+c)^2/d+(a+b)^2*ln(cosh(d*x+c))/d+a*(a+2*b)*ln(tanh(d*x+c
))/d
```

Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.10

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\frac{b^2\log(\cosh(c+dx))}{d}-\frac{a^2\left(\operatorname{csch}^2(c+dx)-2\log(\sinh(c+dx))\right)}{2d}+\frac{2ab\log(\sinh(c+dx))}{d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(b^2*Log[Cosh[c + d*x]])/d - (a^2*(Csch[c + d*x]^2 - 2*Log[Sinh[c + d*x]])
)/(2*d) + (2*a*b*Log[Sinh[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 55, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \; 26$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \; 4153$$

$$-\frac{i\int \frac{i\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 354$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(-\frac{(a+b)^2}{\tanh^2(c+dx)-1}+a^2\coth^2(c+dx)+a(a+2b)\coth(c+dx)\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{a^2(-\coth(c+dx))+a(a+2b)\log\left(\tanh^2(c+dx)\right)-(a+b)^2\log\left(1-\tanh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a^2*Coth[c + d*x]) + a*(a + 2*b)*Log[Tanh[c + d*x]^2] - (a + b)^2*Log[1
 - Tanh[c + d*x]^2])/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.19

method	result
parallelrisch	$\frac{-2(a+b)^2 \ln(1-\tanh(dx+c))+2a(a+2b)\ln(\tanh(dx+c))-\coth(dx+c)^2 a^2-2dx(a+b)^2}{2d}$
derivativedivides	$-\frac{\frac{a^2}{2\tanh(dx+c)^2}-a(a+2b)\ln(\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh(dx+c)}{d}$
default	$-\frac{\frac{a^2}{2\tanh(dx+c)^2}-a(a+2b)\ln(\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh(dx+c)}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}-\frac{2a^2 e^{2dx+2c}}{d\left(e^{2dx+2c}-1\right)^2}+\frac{a^2\ln\left(e^{2dx+2c}-1\right)}{d}+\frac{2a\ln\left(e^{2dx+2c}-\right.}{d}$

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*(a+b)^2*ln(1-tanh(d*x+c))+2*a*(a+2*b)*ln(tanh(d*x+c))-coth(d*x+c)^
2*a^2-2*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 677 vs. 2(50) = 100.

Time = 0.11 (sec) , antiderivative size = 677, normalized size of antiderivative = 13.02

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^4 + (a^2
 + 2*a*b + b^2)*d*x - 2*((a^2 + 2*a*b + b^2)*d*x - a^2)*cosh(d*x + c)^2 +
2*(3*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a
^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^2*sinh(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x +
 c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x
 + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c)))
 - ((a^2 + 2*a*b)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b)*cosh(d*x + c)*sinh(d*x
 + c)^3 + (a^2 + 2*a*b)*sinh(d*x + c)^4 - 2*(a^2 + 2*a*b)*cosh(d*x + c)^2
+ 2*(3*(a^2 + 2*a*b)*cosh(d*x + c)^2 - a^2 - 2*a*b)*sinh(d*x + c)^2 + a^2
+ 2*a*b + 4*((a^2 + 2*a*b)*cosh(d*x + c)^3 - (a^2 + 2*a*b)*cosh(d*x + c))*
sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 4*((
a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - ((a^2 + 2*a*b + b^2)*d*x - a^2)*c
osh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d
*x + c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^2 - d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x
 + c) + d)
```

Sympy [F]

$$\int \coth^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2 \coth^3{(c+dx)} \, dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 134 vs. $2(50) = 100$.

Time = 0.05 (sec) , antiderivative size = 134, normalized size of antiderivative = 2.58

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$+\frac{b^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{2\,ab\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2
*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(
d*x + c) + e^(-d*x - c))/d + 2*a*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(50) = 100$.

Time = 0.20 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.71

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{b^2\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+(a^2+2\,ab)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{a^2\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)\cdots}{e^{(2\,dx+\cdots}}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(b^2*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + (a^2 + 2*a*b)*log(e
^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^2*(e^(2*d*x + 2*c) + e^(-2*d*x
 - 2*c)) + 2*a*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a^2 - 4*a*b)/(e^
(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2))/d
```

Mupad [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 211, normalized size of antiderivative = 4.06

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{\ln\left(e^{4c+4dx}-1\right)\left(d\left(a^2+2ba\right)+b^2d\right)}{2d^2}-\frac{2a^2}{d\left(e^{2c+2dx}-1\right)}-\frac{2a^2}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}-x\left(a+b\right)^2-\frac{\operatorname{atan}\left(\frac{e^{2c}e^{2dx}\left(a^2\sqrt{-d^2}-b^2\sqrt{-d^2}+2ab\sqrt{-d^2}\right)}{d\sqrt{a^4+4a^3b+2a^2b^2-4ab^3+b^4}}\right)\sqrt{a^4+4a^3b+2a^2b^2-4ab^3+b^4}}{\sqrt{-d^2}}$$

input
```
int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(d*(2*a*b + a^2) + b^2*d))/(2*d^2) - (2*a^2)/(d
*(exp(2*c + 2*d*x) - 1)) - (2*a^2)/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*
x) + 1)) - x*(a + b)^2 - (atan((exp(2*c)*exp(2*d*x)*(a^2*(-d^2)^(1/2) - b^
2*(-d^2)^(1/2) + 2*a*b*(-d^2)^(1/2)))/(d*(4*a^3*b - 4*a*b^3 + a^4 + b^4 +
2*a^2*b^2)^(1/2)))*(4*a^3*b - 4*a*b^3 + a^4 + b^4 + 2*a^2*b^2)^(1/2))/(-d^
2)^(1/2)
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 493, normalized size of antiderivative = 9.48

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{2e^{2dx+2c}a^2dx+2\log\left(e^{dx+c}-1\right)ab+2\log\left(e^{dx+c}+1\right)ab-e^{4dx+4c}a^2dx-e^{4dx+4c}b^2dx+e^{4dx+4c}\log\left(e^{dx+c}\right.}{}$$

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*b**2 + e**(4*c + 4*d*x)*log(e*
*(c + d*x) - 1)*a**2 + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*b + e**(
4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2 + 2*e**(4*c + 4*d*x)*log(e**(c + d
*x) + 1)*a*b - e**(4*c + 4*d*x)*a**2*d*x - e**(4*c + 4*d*x)*a**2 - 2*e**(4
*c + 4*d*x)*a*b*d*x - e**(4*c + 4*d*x)*b**2*d*x - 2*e**(2*c + 2*d*x)*log(e
**(2*c + 2*d*x) + 1)*b**2 - 2*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2
 - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a*b - 2*e**(2*c + 2*d*x)*log(e*
*(c + d*x) + 1)*a**2 - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a*b + 2*e*
*(2*c + 2*d*x)*a**2*d*x + 4*e**(2*c + 2*d*x)*a*b*d*x + 2*e**(2*c + 2*d*x)*
b**2*d*x + log(e**(2*c + 2*d*x) + 1)*b**2 + log(e**(c + d*x) - 1)*a**2 + 2
*log(e**(c + d*x) - 1)*a*b + log(e**(c + d*x) + 1)*a**2 + 2*log(e**(c + d*
x) + 1)*a*b - a**2*d*x - a**2 - 2*a*b*d*x - b**2*d*x)/(d*(e**(4*c + 4*d*x)
 - 2*e**(2*c + 2*d*x) + 1))
```

3.152 $\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 43

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = (a+b)^2x - \frac{a(a+2b)\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d}$$

output
```
(a+b)^2*x-a*(a+2*b)*coth(d*x+c)/d-1/3*a^2*coth(d*x+c)^3/d
```

Mathematica [A] (verified)

Time = 0.42 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.51

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = -\frac{\coth(c+dx)\left(a\left(3a+6b+a\coth^2(c+dx)\right) - 3(a+b)^2\text{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)\sqrt{\tanh^2(c+dx)}\right)}{3d}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/3*(Coth[c + d*x]*(a*(3*a + 6*b + a*Coth[c + d*x]^2) - 3*(a + b)^2*ArcTa
nh[Sqrt[Tanh[c + d*x]^2]]*Sqrt[Tanh[c + d*x]^2]))/d
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.09, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^4}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(a^2\coth^4(c+dx)+a(a+2b)\coth^2(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)+(a+b)^2\text{arctanh}(\tanh(c+dx))-a(a+2b)\coth(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((a + b)^2*ArcTanh[Tanh[c + d*x]] - a*(a + 2*b)*Coth[c + d*x] - (a^2*Coth[
c + d*x]^3)/3)/d
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

method	result	size
parallelrisch	$\frac{-\coth(dx+c)^3 a^2-3a(a+2b)\coth(dx+c)+3dx(a+b)^2}{3d}$	43
derivativedivides	$-\frac{\frac{a^2}{3\tanh(dx+c)^3}+\frac{a(a+2b)}{\tanh(dx+c)}+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh(dx+c))}{d}$	84
default	$-\frac{\frac{a^2}{3\tanh(dx+c)^3}+\frac{a(a+2b)}{\tanh(dx+c)}+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh(dx+c))}{d}$	84
risch	$a^2x+2abx+b^2x-\frac{4a\left(3a\,e^{4dx+4c}+3b\,e^{4dx+4c}-3a\,e^{2dx+2c}-6b\,e^{2dx+2c}+2a+3b\right)}{3d\left(e^{2dx+2c}-1\right)^3}$	91

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-coth(d*x+c)^3*a^2-3*a*(a+2*b)*coth(d*x+c)+3*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 197 vs. 2(41) = 82.

Time = 0.09 (sec) , antiderivative size = 197, normalized size of antiderivative = 4.58

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{2\left(2a^2+3ab\right)\cosh\left(dx+c\right)^3+6\left(2a^2+3ab\right)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2-\left(3\left(a^2+2ab+b^2\right)dx+4\right.}{3\left(d\,s\right.}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/3*(2*(2*a^2 + 3*a*b)*cosh(d*x + c)^3 + 6*(2*a^2 + 3*a*b)*cosh(d*x + c)*
sinh(d*x + c)^2 - (3*(a^2 + 2*a*b + b^2)*d*x + 4*a^2 + 6*a*b)*sinh(d*x + c
)^3 - 6*a*b*cosh(d*x + c) + 3*(3*(a^2 + 2*a*b + b^2)*d*x - (3*(a^2 + 2*a*b
 + b^2)*d*x + 4*a^2 + 6*a*b)*cosh(d*x + c)^2 + 4*a^2 + 6*a*b)*sinh(d*x + c
))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F]

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\coth^4\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**4, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 114 vs. $2(41) = 82$.

Time = 0.04 (sec) , antiderivative size = 114, normalized size of antiderivative = 2.65

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{1}{3}a^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)+2ab\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+b^2x$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + 2*a*b
*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 103 vs. $2(41) = 82$.

Time = 0.19 (sec) , antiderivative size = 103, normalized size of antiderivative = 2.40

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=\frac{3\,(a^2+2\,ab+b^2)(dx+c)-\frac{4\left(3\,a^2e^{(4\,dx+4\,c)}+3\,abe^{(4\,dx+4\,c)}-3\,a^2e^{(2\,dx+2\,c)}-6\,abe^{(2\,dx+2\,c)}+2\,a^2+3\,ab\right)}{\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 4*(3*a^2*e^(4*d*x + 4*c) + 3*a*b*e^
(4*d*x + 4*c) - 3*a^2*e^(2*d*x + 2*c) - 6*a*b*e^(2*d*x + 2*c) + 2*a^2 + 3*
a*b)/(e^(2*d*x + 2*c) - 1)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 175, normalized size of antiderivative = 4.07

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = x\,(a+b)^2 - \frac{\frac{4\,e^{2\,c+2\,d\,x}\,(a^2+b\,a)}{3\,d}-\frac{4\,a\,b}{3\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1} - \frac{\frac{4\,(a^2+b\,a)}{3\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(a^2+b\,a)}{3\,d}-\frac{8\,a\,b\,e^{2\,c+2\,d\,x}}{3\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1} - \frac{4\,(a^2+b\,a)}{3\,d\,(e^{2\,c+2\,d\,x}-1)}$$

input
```
int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
x*(a + b)^2 - ((4*exp(2*c + 2*d*x)*(a*b + a^2))/(3*d) - (4*a*b)/(3*d))/(ex
p(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((4*(a*b + a^2))/(3*d) + (4*exp
(4*c + 4*d*x)*(a*b + a^2))/(3*d) - (8*a*b*exp(2*c + 2*d*x))/(3*d))/(3*exp(
2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (4*(a*b + a^2)
)/(3*d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 266, normalized size of antiderivative = 6.19

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{3e^{6dx+6c}a^2dx - 4e^{6dx+6c}a^2 + 6e^{6dx+6c}abdx - 4e^{6dx+6c}ab + 3e^{6dx+6c}b^2dx - 9e^{4dx+4c}a^2dx - 18e^{4dx+4c}abdx}{3d\,(e^{6dx+6c} -}$$

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(3*e**(6*c + 6*d*x)*a**2*d*x - 4*e**(6*c + 6*d*x)*a**2 + 6*e**(6*c + 6*d*x
)*a*b*d*x - 4*e**(6*c + 6*d*x)*a*b + 3*e**(6*c + 6*d*x)*b**2*d*x - 9*e**(4
*c + 4*d*x)*a**2*d*x - 18*e**(4*c + 4*d*x)*a*b*d*x - 9*e**(4*c + 4*d*x)*b*
*2*d*x + 9*e**(2*c + 2*d*x)*a**2*d*x + 18*e**(2*c + 2*d*x)*a*b*d*x + 12*e*
*(2*c + 2*d*x)*a*b + 9*e**(2*c + 2*d*x)*b**2*d*x - 3*a**2*d*x - 4*a**2 - 6
*a*b*d*x - 8*a*b - 3*b**2*d*x)/(3*d*(e**(6*c + 6*d*x) - 3*e**(4*c + 4*d*x)
 + 3*e**(2*c + 2*d*x) - 1))
```

3.153 $\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = -\frac{a(a+2b)\coth^2(c+dx)}{2d} - \frac{a^2\coth^4(c+dx)}{4d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a*(a+2*b)*coth(d*x+c)^2/d-1/4*a^2*coth(d*x+c)^4/d+(a+b)^2*ln(cosh(d*x
+c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.67

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx = -\frac{4a(a+b)\operatorname{csch}^2(c+dx) + a^2\operatorname{csch}^4(c+dx) - 4(a+b)^2\log(\sinh(c+dx))}{4d}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/4*(4*a*(a + b)*Csch[c + d*x]^2 + a^2*Csch[c + d*x]^4 - 4*(a + b)^2*Log[
Sinh[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.32 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(a^2\coth^3(c+dx)+a(a+2b)\coth^2(c+dx)+(a+b)^2\coth(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^2\coth^2(c+dx)-a(a+2b)\coth(c+dx)+(a+b)^2\log\left(\tanh^2(c+dx)\right)-(a+b)^2\log\left(1-\tanh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(a*(a + 2*b)*Coth[c + d*x]) - (a^2*Coth[c + d*x]^2)/2 + (a + b)^2*Log[Ta
nh[c + d*x]^2] - (a + b)^2*Log[1 - Tanh[c + d*x]^2])/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{-4(a+b)^2\ln(1-\tanh(dx+c))+4(a+b)^2\ln(\tanh(dx+c))-\coth(dx+c)^4a^2-2a\coth(dx+c)^2(a+2b)-4dx(a+b)^2}{4d}$
derivativedivides	$-\frac{(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{4\tanh(dx+c)^4}+\frac{a(a+2b)}{2\tanh(dx+c)^2}+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}\right.}{d}$
default	$-\frac{(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{4\tanh(dx+c)^4}+\frac{a(a+2b)}{2\tanh(dx+c)^2}+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}\right.}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}-\frac{4a\,e^{2dx+2c}\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}-a\,e^{2dx+2c}-2b\,e^{2dx+2c}+\right.}{d\left(e^{2dx+2c}-1\right)^4}$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*(a+b)^2*ln(1-tanh(d*x+c))+4*(a+b)^2*ln(tanh(d*x+c))-coth(d*x+c)^4*
a^2-2*a*coth(d*x+c)^2*(a+2*b)-4*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1649 vs. 2(68) = 136.

Time = 0.11 (sec) , antiderivative size = 1649, normalized size of antiderivative = 22.90

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-((a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*d*x*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)^8 - 4*((
a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a*b +
b^2)*d*x*cosh(d*x + c)^2 - (a^2 + 2*a*b + b^2)*d*x + a^2 + a*b)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 - 3*((a^2 + 2*a*b + b
^2)*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(3*(a^2 + 2*a*b +
b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x + c)^4 + 2*(35*(a^2 + 2*a*b + b^2)*d*x*
cosh(d*x + c)^4 + 3*(a^2 + 2*a*b + b^2)*d*x - 30*((a^2 + 2*a*b + b^2)*d*x
- a^2 - a*b)*cosh(d*x + c)^2 - 2*a^2 - 4*a*b)*sinh(d*x + c)^4 + 8*(7*(a^2
+ 2*a*b + b^2)*d*x*cosh(d*x + c)^5 - 10*((a^2 + 2*a*b + b^2)*d*x - a^2 - a
*b)*cosh(d*x + c)^3 + (3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x
 + c))*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*d*x - 4*((a^2 + 2*a*b + b^2)*
d*x - a^2 - a*b)*cosh(d*x + c)^2 + 4*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x +
 c)^6 - 15*((a^2 + 2*a*b + b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^4 - (a^2 +
2*a*b + b^2)*d*x + 3*(3*(a^2 + 2*a*b + b^2)*d*x - 2*a^2 - 4*a*b)*cosh(d*x
+ c)^2 + a^2 + a*b)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x + c)^8
 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*b + b^
2)*sinh(d*x + c)^8 - 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 - a^2 - 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*(a
^2 + 2*a*b + b^2)*cosh(d*x + c)^3 - 3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)...
```

Sympy [F]

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^2\coth^5\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**5, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(68) = 136$.

Time = 0.04 (sec) , antiderivative size = 236, normalized size of antiderivative = 3.28

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= a^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c}}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-}}\right.$$
$$+2\,ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)$$
$$+\frac{b^2\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
a^2*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c)
 - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 2*a
*b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*
d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^2*log(e^(d
*x + c) - e^(-d*x - c))/d
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.64

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{(a^2+2\,ab+b^2)(dx+c)-(a^2+2\,ab+b^2)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)+\frac{4\left((a^2+ab)e^{(6\,dx+6\,c)}-(a^2+2\,ab)e^{(4\,dx+4\,c)}+(a}{\left(e^{(2\,dx+2\,c)}-1\right)^4}}{d}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-((a^2 + 2*a*b + b^2)*(d*x + c) - (a^2 + 2*a*b + b^2)*log(abs(e^(2*d*x + 2
*c) - 1)) + 4*((a^2 + a*b)*e^(6*d*x + 6*c) - (a^2 + 2*a*b)*e^(4*d*x + 4*c)
 + (a^2 + a*b)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^4)/d
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.74

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\ln\left(e^{2c}\,e^{2dx}-1\right)\left(a^2+2ab+b^2\right)}{d}-x\,(a+b)^2 - \frac{4\,(2\,a^2+b\,a)}{d\,\left(e^{4c+4dx}-2\,e^{2c+2dx}+1\right)}-\frac{8\,a^2}{d\,\left(3\,e^{2c+2dx}-3\,e^{4c+4dx}+e^{6c+6dx}-1\right)} - \frac{4\,a^2}{d\,\left(6\,e^{4c+4dx}-4\,e^{2c+2dx}-4\,e^{6c+6dx}+e^{8c+8dx}+1\right)}-\frac{4\,(a^2+b\,a)}{d\,\left(e^{2c+2dx}-1\right)}$$

input
```
int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - x*(a + b)^2 - (4*(a
*b + 2*a^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*a^2)/(d*
(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*a^2
)/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8
*c + 8*d*x) + 1)) - (4*(a*b + a^2))/(d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1014, normalized size of antiderivative = 14.08

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**2 + 2*e**(8*c + 8*d*x)*log(e**(
c + d*x) - 1)*a*b + e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*b**2 + e**(8*c
+ 8*d*x)*log(e**(c + d*x) + 1)*a**2 + 2*e**(8*c + 8*d*x)*log(e**(c + d*x)
+ 1)*a*b + e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*b**2 - e**(8*c + 8*d*x)*
a**2*d*x - e**(8*c + 8*d*x)*a**2 - 2*e**(8*c + 8*d*x)*a*b*d*x - e**(8*c +
8*d*x)*a*b - e**(8*c + 8*d*x)*b**2*d*x - 4*e**(6*c + 6*d*x)*log(e**(c + d*
x) - 1)*a**2 - 8*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a*b - 4*e**(6*c +
6*d*x)*log(e**(c + d*x) - 1)*b**2 - 4*e**(6*c + 6*d*x)*log(e**(c + d*x) +
1)*a**2 - 8*e**(6*c + 6*d*x)*log(e**(c + d*x) + 1)*a*b - 4*e**(6*c + 6*d*x
)*log(e**(c + d*x) + 1)*b**2 + 4*e**(6*c + 6*d*x)*a**2*d*x + 8*e**(6*c + 6
*d*x)*a*b*d*x + 4*e**(6*c + 6*d*x)*b**2*d*x + 6*e**(4*c + 4*d*x)*log(e**(c
 + d*x) - 1)*a**2 + 12*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*b + 6*e**(
4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**2 + 6*e**(4*c + 4*d*x)*log(e**(c + d
*x) + 1)*a**2 + 12*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a*b + 6*e**(4*c
+ 4*d*x)*log(e**(c + d*x) + 1)*b**2 - 6*e**(4*c + 4*d*x)*a**2*d*x - 2*e**(
4*c + 4*d*x)*a**2 - 12*e**(4*c + 4*d*x)*a*b*d*x + 2*e**(4*c + 4*d*x)*a*b -
 6*e**(4*c + 4*d*x)*b**2*d*x - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a*
*2 - 8*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a*b - 4*e**(2*c + 2*d*x)*log
(e**(c + d*x) - 1)*b**2 - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a**2 -
8*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*a*b - 4*e**(2*c + 2*d*x)*log(e...
```

3.154 $\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 63

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx=(a+b)^2x-\frac{(a+b)^2\coth(c+dx)}{d}-\frac{a(a+2b)\coth^3(c+dx)}{3d}-\frac{a^2\coth^5(c+dx)}{5d}$$

output
```
(a+b)^2*x-(a+b)^2*coth(d*x+c)/d-1/3*a*(a+2*b)*coth(d*x+c)^3/d-1/5*a^2*coth
(d*x+c)^5/d
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.56

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$=-\frac{a^2\coth^5(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{5}{2},1,-\frac{3}{2},\tanh^2(c+dx)\right)}{5d}-\frac{2ab\coth^3(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\tanh^2(c+dx)\right)}{3d}-\frac{b^2\coth(c+dx)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\tanh^2(c+dx)\right)}{d}$$

input
```
Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/5*(a^2*Coth[c + d*x]^5*Hypergeometric2F1[-5/2, 1, -3/2, Tanh[c + d*x]^2
])/d - (2*a*b*Coth[c + d*x]^3*Hypergeometric2F1[-3/2, 1, -1/2, Tanh[c + d*
x]^2])/(3*d) - (b^2*Coth[c + d*x]*Hypergeometric2F1[-1/2, 1, 1/2, Tanh[c +
 d*x]^2])/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^6}dx$$
$$\Big\downarrow \text{25}$$
$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^6}dx$$

$$\Big\downarrow \; 4153$$

$$-\frac{\int -\frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 364$$

$$\frac{\int \left(a^2\coth^6(c+dx)+a(a+2b)\coth^4(c+dx)+(a+b)^2\coth^2(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$-\frac{\frac{1}{5}a^2\coth^5(c+dx)-(a+b)^2\text{arctanh}(\tanh(c+dx))+\frac{1}{3}a(a+2b)\coth^3(c+dx)+(a+b)^2\coth(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-((a + b)^2*ArcTanh[Tanh[c + d*x]]) + (a + b)^2*Coth[c + d*x] + (a*(a +
 2*b)*Coth[c + d*x]^3)/3 + (a^2*Coth[c + d*x]^5)/5)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 58, normalized size of antiderivative = 0.92

method	result
parallelrisch	$\frac{-3\coth(dx+c)^5a^2-5a\coth(dx+c)^3(a+2b)-15(a+b)^2\coth(dx+c)+15dx(a+b)^2}{15d}$
derivativedivides	$-\frac{-\frac{-a^2-2ab-b^2}{\tanh(dx+c)}+\frac{a^2}{5\tanh(dx+c)^5}+\frac{a(a+2b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh}{d}$
default	$-\frac{-\frac{-a^2-2ab-b^2}{\tanh(dx+c)}+\frac{a^2}{5\tanh(dx+c)^5}+\frac{a(a+2b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^2-ab-\frac{1}{2}b^2\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^2+ab+\frac{1}{2}b^2\right)\ln(-1+\tanh}{d}$
risch	$a^2x+2abx+b^2x-\frac{2(45a^2e^{8dx+8c}+60ab\,e^{8dx+8c}+15b^2e^{8dx+8c}-90a^2e^{6dx+6c}-180ab\,e^{6dx+6c}-60b^2e^{6dx+6c}}{}$

input
```
int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*coth(d*x+c)^5*a^2-5*a*coth(d*x+c)^3*(a+2*b)-15*(a+b)^2*coth(d*x+c
)+15*d*x*(a+b)^2)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(59) = 118$.

Time = 0.09 (sec) , antiderivative size = 473, normalized size of antiderivative = 7.51

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{(23\,a^2+40\,ab+15\,b^2)\cosh{(dx+c)}^5+5\,(23\,a^2+40\,ab+15\,b^2)\cosh{(dx+c)}\sinh{(dx+c)}^4-(15\,(a^2$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/15*((23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^5 + 5*(23*a^2 + 40*a*b + 1
5*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (15*(a^2 + 2*a*b + b^2)*d*x + 23*a^
2 + 40*a*b + 15*b^2)*sinh(d*x + c)^5 - 5*(5*a^2 + 16*a*b + 9*b^2)*cosh(d*x
 + c)^3 + 5*(15*(a^2 + 2*a*b + b^2)*d*x - 2*(15*(a^2 + 2*a*b + b^2)*d*x +
23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 23*a^2 + 40*a*b + 15*b^2)*sinh
(d*x + c)^3 + 5*(2*(23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 +
 16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^2 + 4*a*b + 3*b^
2)*cosh(d*x + c) - 5*((15*(a^2 + 2*a*b + b^2)*d*x + 23*a^2 + 40*a*b + 15*b
^2)*cosh(d*x + c)^4 + 30*(a^2 + 2*a*b + b^2)*d*x - 3*(15*(a^2 + 2*a*b + b^
2)*d*x + 23*a^2 + 40*a*b + 15*b^2)*cosh(d*x + c)^2 + 46*a^2 + 80*a*b + 30*
b^2)*sinh(d*x + c))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(
d*x + c)^3 + 5*(d*cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x +
c))
```

Sympy [F]

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \int \left(a+b\tanh^2{(c+dx)}\right)^2\coth^6{(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**2*coth(c + d*x)**6, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(59) = 118$.

Time = 0.05 (sec) , antiderivative size = 231, normalized size of antiderivative = 3.67

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{15}a^2\left(15\,x+\frac{15\,c}{d}-\frac{2\left(70\,e^{(-2\,dx-2\,c)}-140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}-45\,e^{(-8\,dx-8\,c)}-23\right)}{d(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}-5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}-1)}\right)$$
$$+\frac{2}{3}ab\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}-2\right)}{d(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1)}\right)$$
$$+b^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2\,dx-2\,c)}-1)}\right)$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/15*a^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1))) + 2/3*a*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d
*x - 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6
*c) - 1))) + b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 218 vs. $2(59) = 118$.

Time = 0.24 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.46

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{15\left(a^2+2\,ab+b^2\right)(dx+c)-\frac{2\left(45\,a^2e^{(8\,dx+8\,c)}+60\,abe^{(8\,dx+8\,c)}+15\,b^2e^{(8\,dx+8\,c)}-90\,a^2e^{(6\,dx+6\,c)}-180\,abe^{(6\,dx+6\,c)}-60\,b^2e^{(6\,dx+}}{}}{15\,d}$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/15*(15*(a^2 + 2*a*b + b^2)*(d*x + c) - 2*(45*a^2*e^(8*d*x + 8*c) + 60*a*
b*e^(8*d*x + 8*c) + 15*b^2*e^(8*d*x + 8*c) - 90*a^2*e^(6*d*x + 6*c) - 180*
a*b*e^(6*d*x + 6*c) - 60*b^2*e^(6*d*x + 6*c) + 140*a^2*e^(4*d*x + 4*c) + 2
20*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 70*a^2*e^(2*d*x + 2*c) -
 140*a*b*e^(2*d*x + 2*c) - 60*b^2*e^(2*d*x + 2*c) + 23*a^2 + 40*a*b + 15*b
^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 529, normalized size of antiderivative = 8.40

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1} + \frac{\frac{2\,(b^2+2\,a\,b)}{5\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{2\,e^{6\,c+6\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(5\,a^2+4\,a\,b+3\,b^2)}{5\,d}}{6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1} + x\,(a+b)^2 - \frac{\frac{2\,(3\,a^2+4\,a\,b+b^2)}{5\,d}-\frac{8\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}-\frac{8\,e^{6\,c+6\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,e^{8\,c+8\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(5\,a^2+4\,a\,b+3\,b^2)}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1} - \frac{\frac{2\,(5\,a^2+4\,a\,b+3\,b^2)}{15\,d}-\frac{4\,e^{2\,c+2\,d\,x}\,(b^2+2\,a\,b)}{5\,d}+\frac{2\,e^{4\,c+4\,d\,x}\,(3\,a^2+4\,a\,b+b^2)}{5\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1} - \frac{2\,(3\,a^2+4\,a\,b+b^2)}{5\,d\,(e^{2\,c+2\,d\,x}-1)}$$

input
```
int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((2*(2*a*b + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d
))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) + ((2*(2*a*b + b^2))/(5*d)
+ (6*exp(4*c + 4*d*x)*(2*a*b + b^2))/(5*d) - (2*exp(6*c + 6*d*x)*(4*a*b +
3*a^2 + b^2))/(5*d) - (2*exp(2*c + 2*d*x)*(4*a*b + 5*a^2 + 3*b^2))/(5*d))/
(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c +
8*d*x) + 1) + x*(a + b)^2 - ((2*(4*a*b + 3*a^2 + b^2))/(5*d) - (8*exp(2*c
+ 2*d*x)*(2*a*b + b^2))/(5*d) - (8*exp(6*c + 6*d*x)*(2*a*b + b^2))/(5*d) +
 (2*exp(8*c + 8*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d) + (4*exp(4*c + 4*d*x)*(4
*a*b + 5*a^2 + 3*b^2))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) +
10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(
4*a*b + 5*a^2 + 3*b^2))/(15*d) - (4*exp(2*c + 2*d*x)*(2*a*b + b^2))/(5*d)
+ (2*exp(4*c + 4*d*x)*(4*a*b + 3*a^2 + b^2))/(5*d))/(3*exp(2*c + 2*d*x) -
3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) - (2*(4*a*b + 3*a^2 + b^2))/(5*
d*(exp(2*c + 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 513, normalized size of antiderivative = 8.14

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{-18e^{10dx+10c}a^2 - 6e^{10dx+10c}b^2 + 75e^{2dx+2c}a^2dx + 15e^{10dx+10c}a^2dx + 15e^{10dx+10c}b^2dx - 75e^{8dx+8c}a^2dx - 7}{}$$

input
```
int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(15*e**(10*c + 10*d*x)*a**2*d*x - 18*e**(10*c + 10*d*x)*a**2 + 30*e**(10*c
 + 10*d*x)*a*b*d*x - 24*e**(10*c + 10*d*x)*a*b + 15*e**(10*c + 10*d*x)*b**
2*d*x - 6*e**(10*c + 10*d*x)*b**2 - 75*e**(8*c + 8*d*x)*a**2*d*x - 150*e**
(8*c + 8*d*x)*a*b*d*x - 75*e**(8*c + 8*d*x)*b**2*d*x + 150*e**(6*c + 6*d*x
)*a**2*d*x + 300*e**(6*c + 6*d*x)*a*b*d*x + 120*e**(6*c + 6*d*x)*a*b + 150
*e**(6*c + 6*d*x)*b**2*d*x + 60*e**(6*c + 6*d*x)*b**2 - 150*e**(4*c + 4*d*
x)*a**2*d*x - 100*e**(4*c + 4*d*x)*a**2 - 300*e**(4*c + 4*d*x)*a*b*d*x - 2
00*e**(4*c + 4*d*x)*a*b - 150*e**(4*c + 4*d*x)*b**2*d*x - 120*e**(4*c + 4*
d*x)*b**2 + 75*e**(2*c + 2*d*x)*a**2*d*x + 50*e**(2*c + 2*d*x)*a**2 + 150*
e**(2*c + 2*d*x)*a*b*d*x + 160*e**(2*c + 2*d*x)*a*b + 75*e**(2*c + 2*d*x)*
b**2*d*x + 90*e**(2*c + 2*d*x)*b**2 - 15*a**2*d*x - 28*a**2 - 30*a*b*d*x -
 56*a*b - 15*b**2*d*x - 24*b**2)/(15*d*(e**(10*c + 10*d*x) - 5*e**(8*c + 8
*d*x) + 10*e**(6*c + 6*d*x) - 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x) - 1
))
```

3.155 $\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$

Optimal result

Integrand size = 23, antiderivative size = 92

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{(a+b)^2\coth^2(c+dx)}{2d} - \frac{a(a+2b)\coth^4(c+dx)}{4d} - \frac{a^2\coth^6(c+dx)}{6d} + \frac{(a+b)^2\log(\cosh(c+dx))}{d} + \frac{(a+b)^2\log(\tanh(c+dx))}{d}$$

output
```
-1/2*(a+b)^2*coth(d*x+c)^2/d-1/4*a*(a+2*b)*coth(d*x+c)^4/d-1/6*a^2*coth(d*
x+c)^6/d+(a+b)^2*ln(cosh(d*x+c))/d+(a+b)^2*ln(tanh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.83

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = -\frac{6(3a^2+4ab+b^2)\operatorname{csch}^2(c+dx)+3a(3a+2b)\operatorname{csch}^4(c+dx)+2a^2\operatorname{csch}^6(c+dx)-12(a+b)^2\log(\sinh}{12d}$$

input
```
Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/12*(6*(3*a^2 + 4*a*b + b^2)*Csch[c + d*x]^2 + 3*a*(3*a + 2*b)*Csch[c +
d*x]^4 + 2*a^2*Csch[c + d*x]^6 - 12*(a + b)^2*Log[Sinh[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 89, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^2}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 26$$

$$\frac{\int \frac{\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 354$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \; 99$$

$$\frac{\int \left(a^2\coth^4(c+dx)+a(a+2b)\coth^3(c+dx)+(a+b)^2\coth^2(c+dx)+(a+b)^2\coth(c+dx)-\frac{(a+b)^2}{\tanh^2(c+dx)-1}\right.}{2d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{3}a^2\coth^3(c+dx)-\frac{1}{2}a(a+2b)\coth^2(c+dx)-(a+b)^2\coth(c+dx)+(a+b)^2\log\left(\tanh^2(c+dx)\right)-(a+b)}{2d}$$

input
```
Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-((a + b)^2*Coth[c + d*x]) - (a*(a + 2*b)*Coth[c + d*x]^2)/2 - (a^2*Coth[
c + d*x]^3)/3 + (a + b)^2*Log[Tanh[c + d*x]^2] - (a + b)^2*Log[1 - Tanh[c
+ d*x]^2])/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.23 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{-12(a+b)^2\ln(1-\tanh(dx+c))+12(a+b)^2\ln(\tanh(dx+c))-2\coth(dx+c)^6a^2-3a\coth(dx+c)^4(a+2b)-6\coth(dx+c)^2}{12d}$
derivativedivides	$-\frac{-\frac{-a^2-2ab-b^2}{2\tanh(dx+c)^2}+(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{6\tanh(dx+c)^6}+\frac{a(a+2b)}{4\tanh(dx+c)^4}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(1+\tanh(dx+c}{d}$
default	$-\frac{-\frac{-a^2-2ab-b^2}{2\tanh(dx+c)^2}+(-a^2-2ab-b^2)\ln(\tanh(dx+c))+\frac{a^2}{6\tanh(dx+c)^6}+\frac{a(a+2b)}{4\tanh(dx+c)^4}+(\frac{1}{2}a^2+ab+\frac{1}{2}b^2)\ln(1+\tanh(dx+c}{d}$
risch	$-a^2x-2abx-b^2x-\frac{2a^2c}{d}-\frac{4abc}{d}-\frac{2b^2c}{d}-\frac{2\,e^{2dx+2c}(9a^2e^{8dx+8c}+12ab\,e^{8dx+8c}+3b^2e^{8dx+8c}-18a^2e^6}{}$

input
```
int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/12*(-12*(a+b)^2*ln(1-tanh(d*x+c))+12*(a+b)^2*ln(tanh(d*x+c))-2*coth(d*x+
c)^6*a^2-3*a*coth(d*x+c)^4*(a+2*b)-6*coth(d*x+c)^2*(a+b)^2-12*d*x*(a+b)^2)
/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3454 vs. 2(86) = 172.

Time = 0.14 (sec) , antiderivative size = 3454, normalized size of antiderivative = 37.54

$$\int \coth^7(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 \, dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 390 vs. $2(86) = 172$.

Time = 0.05 (sec) , antiderivative size = 390, normalized size of antiderivative = 4.24

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx$$
$$= \frac{1}{3}a^2\left(3x + \frac{3c}{d} + \frac{3\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(9e^{(-2dx-2c)} - 18e^{(-4dx-4c)}\right.}{d(6e^{(-2dx-2c)} - 15e^{(-4dx-4c)} + 20e^{(-6c}}\right.$$
$$+2ab\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} - e^{(-4dx-4c)} + e^{(-6dx}\right.}{d(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} -}\right.$$
$$+b^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} - e^{(-4dx-4c)} - 1)}\right)$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/3*a^2*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)
/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 1
8*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(
-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x -
 10*c) - e^(-12*d*x - 12*c) - 1))) + 2*a*b*(x + c/d + log(e^(-d*x - c) + 1
)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) + e
^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1))) + b^2*(x + c/d + log(e^(-d*x - c) + 1)/d
 + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e
^(-4*d*x - 4*c) - 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(86) = 172$.

Time = 0.27 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.09

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx =$$
$$-\frac{3(a^2+2ab+b^2)(dx+c) - 3(a^2+2ab+b^2)\log\left(\left|e^{(2dx+2c)}-1\right|\right) + \frac{2\left(3\left(3a^2+4ab+b^2\right)e^{(10dx+10c)} - 6\left(3a^2+6\right.\right.}{}}{3d}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/3*(3*(a^2 + 2*a*b + b^2)*(d*x + c) - 3*(a^2 + 2*a*b + b^2)*log(abs(e^(2
*d*x + 2*c) - 1)) + 2*(3*(3*a^2 + 4*a*b + b^2)*e^(10*d*x + 10*c) - 6*(3*a^
2 + 6*a*b + 2*b^2)*e^(8*d*x + 8*c) + 2*(17*a^2 + 24*a*b + 9*b^2)*e^(6*d*x
+ 6*c) - 6*(3*a^2 + 6*a*b + 2*b^2)*e^(4*d*x + 4*c) + 3*(3*a^2 + 4*a*b + b^
2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 362, normalized size of antiderivative = 3.93

$$\int \coth^7(c+dx) \left(a+b\tanh^2(c+dx)\right)^2 dx$$
$$= \frac{\ln\left(e^{2c}\,e^{2dx}-1\right)\left(a^2+2ab+b^2\right)}{d} - \frac{2\left(3a^2+4ab+b^2\right)}{d\left(e^{2c+2dx}-1\right)}$$
$$- \frac{32a^2}{3d\left(15e^{4c+4dx}-6e^{2c+2dx}-20e^{6c+6dx}+15e^{8c+8dx}-6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$
$$- x(a+b)^2 - \frac{2\left(9a^2+8ab+b^2\right)}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}$$
$$- \frac{8\left(13a^2+6ba\right)}{3d\left(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1\right)}$$
$$- \frac{4\left(11a^2+2ba\right)}{d\left(6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$- \frac{32a^2}{d\left(5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1\right)}$$

input
```
int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(2*a*b + a^2 + b^2))/d - (2*(4*a*b + 3*a^2 +
 b^2))/(d*(exp(2*c + 2*d*x) - 1)) - (32*a^2)/(3*d*(15*exp(4*c + 4*d*x) - 6
*exp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c
 + 10*d*x) + exp(12*c + 12*d*x) + 1)) - x*(a + b)^2 - (2*(8*a*b + 9*a^2 +
b^2))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (8*(6*a*b + 13*a^2
))/(3*d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1))
- (4*(2*a*b + 11*a^2))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp
(6*c + 6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^2)/(d*(5*exp(2*c + 2*d*x) -
 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c
 + 10*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1566, normalized size of antiderivative = 17.02

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^2\,dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(3*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*a**2 + 6*e**(12*c + 12*d*x)*lo
g(e**(c + d*x) - 1)*a*b + 3*e**(12*c + 12*d*x)*log(e**(c + d*x) - 1)*b**2
+ 3*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*a**2 + 6*e**(12*c + 12*d*x)*l
og(e**(c + d*x) + 1)*a*b + 3*e**(12*c + 12*d*x)*log(e**(c + d*x) + 1)*b**2
 - 3*e**(12*c + 12*d*x)*a**2*d*x - 3*e**(12*c + 12*d*x)*a**2 - 6*e**(12*c
+ 12*d*x)*a*b*d*x - 4*e**(12*c + 12*d*x)*a*b - 3*e**(12*c + 12*d*x)*b**2*d
*x - e**(12*c + 12*d*x)*b**2 - 18*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)
*a**2 - 36*e**(10*c + 10*d*x)*log(e**(c + d*x) - 1)*a*b - 18*e**(10*c + 10
*d*x)*log(e**(c + d*x) - 1)*b**2 - 18*e**(10*c + 10*d*x)*log(e**(c + d*x)
+ 1)*a**2 - 36*e**(10*c + 10*d*x)*log(e**(c + d*x) + 1)*a*b - 18*e**(10*c
+ 10*d*x)*log(e**(c + d*x) + 1)*b**2 + 18*e**(10*c + 10*d*x)*a**2*d*x + 36
*e**(10*c + 10*d*x)*a*b*d*x + 18*e**(10*c + 10*d*x)*b**2*d*x + 45*e**(8*c
+ 8*d*x)*log(e**(c + d*x) - 1)*a**2 + 90*e**(8*c + 8*d*x)*log(e**(c + d*x)
 - 1)*a*b + 45*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*b**2 + 45*e**(8*c +
8*d*x)*log(e**(c + d*x) + 1)*a**2 + 90*e**(8*c + 8*d*x)*log(e**(c + d*x) +
 1)*a*b + 45*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*b**2 - 45*e**(8*c + 8*
d*x)*a**2*d*x - 9*e**(8*c + 8*d*x)*a**2 - 90*e**(8*c + 8*d*x)*a*b*d*x + 12
*e**(8*c + 8*d*x)*a*b - 45*e**(8*c + 8*d*x)*b**2*d*x + 9*e**(8*c + 8*d*x)*
b**2 - 60*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**2 - 120*e**(6*c + 6*d*
x)*log(e**(c + d*x) - 1)*a*b - 60*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1...
```

3.156 $\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 114

$$\int \tanh^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{(a+b)^3\tanh^3(c+dx)}{3d} - \frac{b(3a^2+3ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^2(3a+b)\tanh^7(c+dx)}{7d} - \frac{b^3\tanh^9(c+dx)}{9d}$$

output
```
(a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*(a+b)^3*tanh(d*x+c)^3/d-1/5*b*(3*a^2+3
*a*b+b^2)*tanh(d*x+c)^5/d-1/7*b^2*(3*a+b)*tanh(d*x+c)^7/d-1/9*b^3*tanh(d*x
+c)^9/d
```

Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.08

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{\tanh(c+dx)\left(-315(a+b)^3-105(a+b)^3\tanh^2(c+dx)-63b(3a^2+3ab+b^2)\tanh^4(c+dx)-45b^2(3\right.}{315d}$$

input
```
Integrate[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*(-315*(a + b)^3 - 105*(a + b)^3*Tanh[c + d*x]^2 - 63*b*(3*a
^2 + 3*a*b + b^2)*Tanh[c + d*x]^4 - 45*b^2*(3*a + b)*Tanh[c + d*x]^6 - 35*
b^3*Tanh[c + d*x]^8 + (315*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[
Tanh[c + d*x]^2]))/(315*d)
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.96, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3dx$$

$$\Big\downarrow \ 4153$$

$$\frac{\int \frac{\tanh^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 364$$

$$\frac{\int \left(-b^3 \tanh^8(c+dx) - b^2(3a+b)\tanh^6(c+dx) - b(3a^2+3ba+b^2)\tanh^4(c+dx) - (a+b)^3\tanh^2(c+dx) -}{d}$$

$\big\downarrow$ 2009

$$\frac{-\frac{1}{5}b(3a^2+3ab+b^2)\tanh^5(c+dx) + (a+b)^3\text{arctanh}(\tanh(c+dx)) - \frac{1}{7}b^2(3a+b)\tanh^7(c+dx) - \frac{1}{3}(a+b)^3 \text{ta}}{d}$$

input
```
Int[Tanh[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*ArcTanh[Tanh[c + d*x]] - (a + b)^3*Tanh[c + d*x] - ((a + b)^3*T
anh[c + d*x]^3)/3 - (b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^5)/5 - (b^2*(3*
a + b)*Tanh[c + d*x]^7)/7 - (b^3*Tanh[c + d*x]^9)/9)/d
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (warning: unable to verify)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(106) = 212$.

Time = 0.12 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.91

method	result
parallelrisch	$-\frac{35b^3\tanh(dx+c)^9+135a\,b^2\tanh(dx+c)^7+45b^3\tanh(dx+c)^7+189a^2b\tanh(dx+c)^5+189a\,b^2\tanh(dx+c)^5+63b^3\tanh\ldots}{}$
derivativedivides	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-\frac{3a\,b^2\tanh(dx+c)^7}{7}-\frac{3a^2b\tanh(dx+c)^5}{5}-\frac{3a\,b^2\tanh(dx+c)^5}{5}-a^2b\tanh(dx+c)^3-a\ldots}{}$
default	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-\frac{3a\,b^2\tanh(dx+c)^7}{7}-\frac{3a^2b\tanh(dx+c)^5}{5}-\frac{3a\,b^2\tanh(dx+c)^5}{5}-a^2b\tanh(dx+c)^3-a\ldots}{}$
parts	$\frac{a^3\left(-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^9}{9}-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(\ldots}{}\right.}{}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x+\frac{\frac{8a^3}{3}+\frac{46a^2b}{5}+\frac{352a\,b^2}{35}+\frac{1126b^3}{315}+\frac{3104b^3e^{4dx+4c}}{35}+\frac{1252b^3e^{8dx+8c}}{5}+\frac{412a^3e^{6dx+\ldots}}{3}}{}$

input
```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(35*b^3*tanh(d*x+c)^9+135*a*b^2*tanh(d*x+c)^7+45*b^3*tanh(d*x+c)^7+
189*a^2*b*tanh(d*x+c)^5+189*a*b^2*tanh(d*x+c)^5+63*b^3*tanh(d*x+c)^5+105*a
^3*tanh(d*x+c)^3+315*a^2*b*tanh(d*x+c)^3+315*a*b^2*tanh(d*x+c)^3+105*b^3*t
anh(d*x+c)^3-315*a^3*d*x-945*a^2*b*d*x-945*a*b^2*d*x-315*b^3*d*x+315*a^3*t
anh(d*x+c)+945*a^2*b*tanh(d*x+c)+945*a*b^2*tanh(d*x+c)+315*b^3*tanh(d*x+c)
)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1563 vs. $2(106) = 212$.

Time = 0.11 (sec) , antiderivative size = 1563, normalized size of antiderivative = 13.71

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/315*((420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^9 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^
2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(
d*x + c)^8 - (420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*sinh(d*x + c)^9
 + 9*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 - 9*(280*a^3 + 819*a^2*b + 744*a*b^2 +
213*b^3 + 4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^7 + 21*(4*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 31
5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 3*(420*a^3 + 1449
*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*c
osh(d*x + c))*sinh(d*x + c)^6 + 36*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 56
3*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 - 9*(14*(
420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3)*cosh(d*x + c)^4 + 700*a^3 + 2
016*a^2*b + 2136*a*b^2 + 852*b^3 + 21*(280*a^3 + 819*a^2*b + 744*a*b^2 + 2
13*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 9*(14*(420*a^3 + 1449*a^2*b + 1
584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c)^5 + 35*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*
b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + 20*(420*a^3 + 1449*a^2*b + 1584*
a*b^2 + 563*b^3 + 315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*
sinh(d*x + c)^4 + 84*(420*a^3 + 1449*a^2*b + 1584*a*b^2 + 563*b^3 + 315...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 260 vs. $2(99) = 198$.

Time = 0.31 (sec) , antiderivative size = 260, normalized size of antiderivative = 2.28

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} a^3x - \frac{a^3\tanh^3(c+dx)}{3d} - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{3a^2b\tanh^5(c+dx)}{5d} - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \\ x\left(a+b\tanh^2(c)\right)^3\tanh^4(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*tanh(c + d*x)**3/(3*d) - a**3*tanh(c + d*x)/d + 3
*a**2*b*x - 3*a**2*b*tanh(c + d*x)**5/(5*d) - a**2*b*tanh(c + d*x)**3/d -
3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**7/(7*d) -
3*a*b**2*tanh(c + d*x)**5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tan
h(c + d*x)/d + b**3*x - b**3*tanh(c + d*x)**9/(9*d) - b**3*tanh(c + d*x)**
7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3
*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 583 vs. $2(106) = 212$.

Time = 0.06 (sec) , antiderivative size = 583, normalized size of antiderivative = 5.11

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/315*b^3*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x -
4*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x
- 10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16
*d*x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-
6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d
*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 1
8*c) + 1))) + 1/35*a*b^2*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*
e^(-4*d*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-1
0*d*x - 10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e
^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*
x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a^2*b*(
15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d
*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*
x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1))) + 1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c)
+ 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 534 vs. $2(106) = 212$.

Time = 0.29 (sec) , antiderivative size = 534, normalized size of antiderivative = 4.68

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{315\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)+\frac{2\,(630\,a^3e^{(16\,dx+16\,c)}+2835\,a^2be^{(16\,dx+16\,c)}+3780\,ab^2e^{(16\,dx+16\,c)}+1575\,b^3e^{(16\,dx+16\,c)}}{}}{}$$

input
```
integrate(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/315*(315*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(630*a^3*e^(16*d*
x + 16*c) + 2835*a^2*b*e^(16*d*x + 16*c) + 3780*a*b^2*e^(16*d*x + 16*c) +
1575*b^3*e^(16*d*x + 16*c) + 4410*a^3*e^(14*d*x + 14*c) + 17010*a^2*b*e^(1
4*d*x + 14*c) + 18900*a*b^2*e^(14*d*x + 14*c) + 6300*b^3*e^(14*d*x + 14*c)
 + 13650*a^3*e^(12*d*x + 12*c) + 48510*a^2*b*e^(12*d*x + 12*c) + 54180*a*b
^2*e^(12*d*x + 12*c) + 21000*b^3*e^(12*d*x + 12*c) + 24570*a^3*e^(10*d*x +
 10*c) + 85050*a^2*b*e^(10*d*x + 10*c) + 94500*a*b^2*e^(10*d*x + 10*c) + 3
1500*b^3*e^(10*d*x + 10*c) + 28350*a^3*e^(8*d*x + 8*c) + 97524*a^2*b*e^(8*
d*x + 8*c) + 105084*a*b^2*e^(8*d*x + 8*c) + 39438*b^3*e^(8*d*x + 8*c) + 21
630*a^3*e^(6*d*x + 6*c) + 73206*a^2*b*e^(6*d*x + 6*c) + 78876*a*b^2*e^(6*d
*x + 6*c) + 26292*b^3*e^(6*d*x + 6*c) + 10710*a^3*e^(4*d*x + 4*c) + 35154*
a^2*b*e^(4*d*x + 4*c) + 38124*a*b^2*e^(4*d*x + 4*c) + 13968*b^3*e^(4*d*x +
 4*c) + 3150*a^3*e^(2*d*x + 2*c) + 10206*a^2*b*e^(2*d*x + 2*c) + 10476*a*b
^2*e^(2*d*x + 2*c) + 3492*b^3*e^(2*d*x + 2*c) + 420*a^3 + 1449*a^2*b + 158
4*a*b^2 + 563*b^3)/(e^(2*d*x + 2*c) + 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 138, normalized size of antiderivative = 1.21

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\left(a^3+3a^2b+3ab^2+b^3\right)-\frac{\tanh(c+dx)\,(a+b)^3}{d}-\frac{\tanh(c+dx)^5\,(3a^2b+3ab^2+b^3)}{5d}-\frac{\tanh(c+dx)^7\,(b^3+3ab^2)}{7d}-\frac{b^3\tanh(c+dx)^9}{9d}-\frac{\tanh(c+dx)^3\,(a^3+3a^2b+3ab^2+b^3)}{3d}$$

input
```
int(tanh(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c
+ d*x)^5*(3*a*b^2 + 3*a^2*b + b^3))/(5*d) - (tanh(c + d*x)^7*(3*a*b^2 + b^
3))/(7*d) - (b^3*tanh(c + d*x)^9)/(9*d) - (tanh(c + d*x)^3*(3*a*b^2 + 3*a^
2*b + a^3 + b^3))/(3*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.90

$$\int \tanh^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{-35\tanh(dx+c)^9b^3-135\tanh(dx+c)^7ab^2-45\tanh(dx+c)^7b^3-189\tanh(dx+c)^5a^2b-189\tan}{}$$

input
```
int(tanh(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 35*tanh(c + d*x)**9*b**3 - 135*tanh(c + d*x)**7*a*b**2 - 45*tanh(c + d
*x)**7*b**3 - 189*tanh(c + d*x)**5*a**2*b - 189*tanh(c + d*x)**5*a*b**2 -
63*tanh(c + d*x)**5*b**3 - 105*tanh(c + d*x)**3*a**3 - 315*tanh(c + d*x)**
3*a**2*b - 315*tanh(c + d*x)**3*a*b**2 - 105*tanh(c + d*x)**3*b**3 - 315*t
anh(c + d*x)*a**3 - 945*tanh(c + d*x)*a**2*b - 945*tanh(c + d*x)*a*b**2 -
315*tanh(c + d*x)*b**3 + 315*a**3*d*x + 945*a**2*b*d*x + 945*a*b**2*d*x +
315*b**3*d*x)/(315*d)
```

3.157 $\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 107

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{(a+b)^3\log(\cosh(c+dx))}{d} - \frac{(a+b)^3\tanh^2(c+dx)}{2d} - \frac{b(3a^2+3ab+b^2)\tanh^4(c+dx)}{4d} - \frac{b^2(3a+b)\tanh^6(c+dx)}{6d} - \frac{b^3\tanh^8(c+dx)}{8d}$$

output
```
(a+b)^3*ln(cosh(d*x+c))/d-1/2*(a+b)^3*tanh(d*x+c)^2/d-1/4*b*(3*a^2+3*a*b+b
^2)*tanh(d*x+c)^4/d-1/6*b^2*(3*a+b)*tanh(d*x+c)^6/d-1/8*b^3*tanh(d*x+c)^8/
d
```

Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.92

$$\int \tanh^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{2(a+b)^3\log(\cosh(c+dx)) - (a+b)^3\tanh^2(c+dx) - \frac{1}{2}b(3a^2+3ab+b^2)\tanh^4(c+dx) - \frac{1}{3}b^2(3a+b)}{2d}$$

input
```
Integrate[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(2*(a + b)^3*Log[Cosh[c + d*x]] - (a + b)^3*Tanh[c + d*x]^2 - (b*(3*a^2 +
3*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3 - (b^3
*Tanh[c + d*x]^8)/4)/(2*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\Big\downarrow \text{26}$$

$$i\int \tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3\,dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$\downarrow$ 86

$$\frac{\int \left(-b^3 \tanh^6(c+dx) - b^2(3a+b)\tanh^4(c+dx) - b\left(3a^2+3ba+b^2\right)\tanh^2(c+dx) - (a+b)^3 - \frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)}{2d}$$

$\downarrow$ 2009

$$\frac{-\frac{1}{2}b\left(3a^2+3ab+b^2\right)\tanh^4(c+dx) - \frac{1}{3}b^2(3a+b)\tanh^6(c+dx) - (a+b)^3\tanh^2(c+dx) - (a+b)^3\log\left(1-\tan\right.}{2d}$$

input
```
Int[Tanh[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-((a + b)^3*Log[1 - Tanh[c + d*x]^2]) - (a + b)^3*Tanh[c + d*x]^2 - (b*(3
*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^4)/2 - (b^2*(3*a + b)*Tanh[c + d*x]^6)/3
 - (b^3*Tanh[c + d*x]^8)/4)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 204 vs. $2(99) = 198$.

Time = 0.10 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.92

method	result
derivativedivides	$-\frac{\tanh(dx+c)^6 a\,b^2}{2}-\frac{3\tanh(dx+c)^4 a^2 b}{4}-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}+\frac{(-a^3-3a^2b-3a\,b^2-b^3)}{2}$
default	$-\frac{\tanh(dx+c)^6 a\,b^2}{2}-\frac{3\tanh(dx+c)^4 a^2 b}{4}-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}+\frac{(-a^3-3a^2b-3a\,b^2-b^3)}{2}$
parts	$\frac{a^3\left(-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(-1+\tanh(dx+c))}{2}-\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^8}{8}-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\tanh(\right.}{d}$
parallelrisch	$-\frac{3\tanh(dx+c)^8b^3+12\tanh(dx+c)^6a\,b^2+4\tanh(dx+c)^6b^3+18\tanh(dx+c)^4a^2b+18\tanh(dx+c)^4a\,b^2+6b^3\tanh(dx+}{}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6a^2bc}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}+\frac{2\,e^{2dx+2c}(3a^3+18a^2b+27a\,b^2+1}{}$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*tanh(d*x+c)^6*a*b^2-3/4*tanh(d*x+c)^4*a^2*b-3/4*tanh(d*x+c)^4*a*
b^2-3/2*tanh(d*x+c)^2*a^2*b-3/2*a*b^2*tanh(d*x+c)^2+1/2*(-a^3-3*a^2*b-3*a*
b^2-b^3)*ln(1+tanh(d*x+c))-1/2*(a^3+3*a^2*b+3*a*b^2+b^3)*ln(-1+tanh(d*x+c)
)-1/6*tanh(d*x+c)^6*b^3-1/4*b^3*tanh(d*x+c)^4-1/2*tanh(d*x+c)^2*a^3-1/2*b^
3*tanh(d*x+c)^2-1/8*tanh(d*x+c)^8*b^3)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7502 vs. $2(99) = 198$.

Time = 0.18 (sec) , antiderivative size = 7502, normalized size of antiderivative = 70.11

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 279 vs. $2(94) = 188$.

Time = 0.27 (sec) , antiderivative size = 279, normalized size of antiderivative = 2.61

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} - \frac{a^3\tanh^2(c+dx)}{2d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^4(c+dx)}{4d} - \frac{3a^2b\tanh^2(c+d}{2d} \\ x\left(a+b\tanh^2(c)\right)^3\tanh^3(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d - a**3*tanh(c + d*x)**2/
(2*d) + 3*a**2*b*x - 3*a**2*b*log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c +
 d*x)**4/(4*d) - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x - 3*a*b**2*l
og(tanh(c + d*x) + 1)/d - a*b**2*tanh(c + d*x)**6/(2*d) - 3*a*b**2*tanh(c
+ d*x)**4/(4*d) - 3*a*b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh
(c + d*x) + 1)/d - b**3*tanh(c + d*x)**8/(8*d) - b**3*tanh(c + d*x)**6/(6*
d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x)**2/(2*d), Ne(d, 0)),
 (x*(a + b*tanh(c)**2)**3*tanh(c)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 540 vs. $2(99) = 198$.

Time = 0.14 (sec) , antiderivative size = 540, normalized size of antiderivative = 5.05

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a*b^2*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*c)
 + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e^(
-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d
*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c
) + 1))) + 1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 8*(3*e^(
-2*d*x - 2*c) + 9*e^(-4*d*x - 4*c) + 25*e^(-6*d*x - 6*c) + 26*e^(-8*d*x -
8*c) + 25*e^(-10*d*x - 10*c) + 9*e^(-12*d*x - 12*c) + 3*e^(-14*d*x - 14*c)
)/(d*(8*e^(-2*d*x - 2*c) + 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) + 70*
e^(-8*d*x - 8*c) + 56*e^(-10*d*x - 10*c) + 28*e^(-12*d*x - 12*c) + 8*e^(-1
4*d*x - 14*c) + e^(-16*d*x - 16*c) + 1))) + 3*a^2*b*(x + c/d + log(e^(-2*d
*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*
c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(
-8*d*x - 8*c) + 1))) + a^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-
2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(99) = 198$.

Time = 0.28 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.89

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+1\right)-\frac{2\left(3\left(a^3+6a^2b+9ab^2-\right.\right.}{}}{}$$

input
```
integrate(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b
^3)*e^(14*d*x + 14*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2*b^3)*e^(12*d*x + 1
2*c) + (45*a^3 + 198*a^2*b + 237*a*b^2 + 100*b^3)*e^(10*d*x + 10*c) + 4*(1
5*a^3 + 63*a^2*b + 78*a*b^2 + 26*b^3)*e^(8*d*x + 8*c) + (45*a^3 + 198*a^2*
b + 237*a*b^2 + 100*b^3)*e^(6*d*x + 6*c) + 18*(a^3 + 5*a^2*b + 6*a*b^2 + 2
*b^3)*e^(4*d*x + 4*c) + 3*(a^3 + 6*a^2*b + 9*a*b^2 + 4*b^3)*e^(2*d*x + 2*c
))/(e^(2*d*x + 2*c) + 1)^8)/d
```

Mupad [B] (verification not implemented)

Time = 2.54 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.45

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\left(a^3+3a^2b+3ab^2+b^3\right)-\frac{\tanh(c+dx)^4\left(3a^2b+3ab^2+b^3\right)}{4d}$$
$$-\frac{\ln\left(\tanh(c+dx)+1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d}-\frac{\tanh(c+dx)^6\left(b^3+3ab^2\right)}{6d}$$
$$-\frac{b^3\tanh(c+dx)^8}{8d}-\frac{\tanh(c+dx)^2\left(a^3+3a^2b+3ab^2+b^3\right)}{2d}$$

input
```
int(tanh(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^4*(3*a*b^2 + 3*a^2*b +
b^3))/(4*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d -
 (tanh(c + d*x)^6*(3*a*b^2 + b^3))/(6*d) - (b^3*tanh(c + d*x)^8)/(8*d) - (
tanh(c + d*x)^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*d)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 2074, normalized size of antiderivative = 19.38

$$\int \tanh^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
int(tanh(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(12*e**(16*c + 16*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3 + 36*e**(16*c + 16*d
*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 36*e**(16*c + 16*d*x)*log(e**(2*c +
 2*d*x) + 1)*a*b**2 + 12*e**(16*c + 16*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3
 - 12*e**(16*c + 16*d*x)*a**3*d*x - 3*e**(16*c + 16*d*x)*a**3 - 36*e**(16*
c + 16*d*x)*a**2*b*d*x - 18*e**(16*c + 16*d*x)*a**2*b - 36*e**(16*c + 16*d
*x)*a*b**2*d*x - 27*e**(16*c + 16*d*x)*a*b**2 - 12*e**(16*c + 16*d*x)*b**3
*d*x - 12*e**(16*c + 16*d*x)*b**3 + 96*e**(14*c + 14*d*x)*log(e**(2*c + 2*
d*x) + 1)*a**3 + 288*e**(14*c + 14*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b +
 288*e**(14*c + 14*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 + 96*e**(14*c + 1
4*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 96*e**(14*c + 14*d*x)*a**3*d*x - 2
88*e**(14*c + 14*d*x)*a**2*b*d*x - 288*e**(14*c + 14*d*x)*a*b**2*d*x - 96*
e**(14*c + 14*d*x)*b**3*d*x + 336*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x)
+ 1)*a**3 + 1008*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 100
8*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 + 336*e**(12*c + 12*
d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 336*e**(12*c + 12*d*x)*a**3*d*x + 60
*e**(12*c + 12*d*x)*a**3 - 1008*e**(12*c + 12*d*x)*a**2*b*d*x + 216*e**(12
*c + 12*d*x)*a**2*b - 1008*e**(12*c + 12*d*x)*a*b**2*d*x + 108*e**(12*c +
12*d*x)*a*b**2 - 336*e**(12*c + 12*d*x)*b**3*d*x - 48*e**(12*c + 12*d*x)*b
**3 + 672*e**(10*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3 + 2016*e**(10*
c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 2016*e**(10*c + 10*d*x)*...
```

3.158 $\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 94

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = (a+b)^3x - \frac{(a+b)^3\tanh(c+dx)}{d} - \frac{b(3a^2+3ab+b^2)\tanh^3(c+dx)}{3d} - \frac{b^2(3a+b)\tanh^5(c+dx)}{5d} - \frac{b^3\tanh^7(c+dx)}{7d}$$

output

```
(a+b)^3*x-(a+b)^3*tanh(d*x+c)/d-1/3*b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)^3/d-1/
5*b^2*(3*a+b)*tanh(d*x+c)^5/d-1/7*b^3*tanh(d*x+c)^7/d
```

Mathematica [A] (verified)

Time = 1.15 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.15

$$\int \tanh^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$= \frac{\tanh(c+dx)\Big(-105(a+b)^3 - 35b(3a^2+3ab+b^2)\tanh^2(c+dx) - 21b^2(3a+b)\tanh^4(c+dx) - 15b^3}{105d}$$

input
```
Integrate[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*(-105*(a + b)^3 - 35*b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]^
2 - 21*b^2*(3*a + b)*Tanh[c + d*x]^4 - 15*b^3*Tanh[c + d*x]^6 + (105*(a +
b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c + d*x]^2]))/(105*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.99, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \; 3042$$

$$\int -\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \; 25$$

$$-\int \tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \; 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\tanh^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 364$$

$$\frac{\int \left(-b^3\tanh^6(c+dx)-b^2(3a+b)\tanh^4(c+dx)-b\left(3a^2+3ba+b^2\right)\tanh^2(c+dx)-(a+b)^3+\frac{a^3+3ba^2+3b^2a+b}{1-\tanh^2(c+dx)}\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{3}b(3a^2+3ab+b^2)\tanh^3(c+dx)-(a+b)^3\text{arctanh}(\tanh(c+dx))+\frac{1}{5}b^2(3a+b)\tanh^5(c+dx)+(a+b)^3\tan}{d}$$

input
```
Int[Tanh[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*ArcTanh[Tanh[c + d*x]]) + (a + b)^3*Tanh[c + d*x] + (b*(3*a
^2 + 3*a*b + b^2)*Tanh[c + d*x]^3)/3 + (b^2*(3*a + b)*Tanh[c + d*x]^5)/5 +
 (b^3*Tanh[c + d*x]^7)/7)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (warning: unable to verify)

Time = 0.10 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.74

method	result
parallelrisch	$-\frac{15b^3\tanh(dx+c)^7+63a\,b^2\tanh(dx+c)^5+21b^3\tanh(dx+c)^5+105a^2b\tanh(dx+c)^3+105a\,b^2\tanh(dx+c)^3+35b^3\tan}{}$
derivativedivides	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-\frac{3a\,b^2\tanh(dx+c)^5}{5}-a^2b\tanh(dx+c)^3-a\,b^2\tanh(dx+c)^3+\frac{(a^3+3a^2b+3a\,b^2+b^3)}{2}}{}$
default	$\frac{-3a^2b\tanh(dx+c)-3a\,b^2\tanh(dx+c)-\frac{3a\,b^2\tanh(dx+c)^5}{5}-a^2b\tanh(dx+c)^3-a\,b^2\tanh(dx+c)^3+\frac{(a^3+3a^2b+3a\,b^2+b^3)}{2}}{}$
parts	$\frac{a^3\left(-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x+\frac{2a^3+8a^2b+\frac{46a\,b^2}{5}+\frac{352b^3}{105}+\frac{232b^3e^{4dx+4c}}{5}+\frac{176b^3e^{8dx+8c}}{3}+40a^3e^{6dx+6c}+17}{}$

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(15*b^3*tanh(d*x+c)^7+63*a*b^2*tanh(d*x+c)^5+21*b^3*tanh(d*x+c)^5+1
05*a^2*b*tanh(d*x+c)^3+105*a*b^2*tanh(d*x+c)^3+35*b^3*tanh(d*x+c)^3-105*a^
3*d*x-315*a^2*b*d*x-315*a*b^2*d*x-105*b^3*d*x+105*a^3*tanh(d*x+c)+315*a^2*
b*tanh(d*x+c)+315*a*b^2*tanh(d*x+c)+105*b^3*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1036 vs. 2(88) = 176.

Time = 0.11 (sec) , antiderivative size = 1036, normalized size of antiderivative = 11.02

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/105*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^7 + 7*(105*a^3 + 420*a^2*b + 483*a*b^2 +
176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x
+ c)^6 - (105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*sinh(d*x + c)^7 + 7*(
105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*d*x)*cosh(d*x + c)^5 - 7*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3 +
3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^5 + 35*((105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b
 + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (105*a^3 + 420*a^2*b + 483*a*b^2
+ 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d
*x + c)^4 + 21*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 7*(5*(105*a^3 + 420*a^2*b + 4
83*a*b^2 + 176*b^3)*cosh(d*x + c)^4 + 135*a^3 + 360*a^2*b + 369*a*b^2 + 16
8*b^3 + 10*(75*a^3 + 240*a^2*b + 213*a*b^2 + 56*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^3 + 7*(3*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^5 + 10*(105*a^3 + 420*a^2*b +
 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
 c)^3 + 9*(105*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(105*a^3 + 420*a
^2*b + 483*a*b^2 + 176*b^3 + 105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*c...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 192 vs. $2(82) = 164$.

Time = 0.21 (sec) , antiderivative size = 192, normalized size of antiderivative = 2.04

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} a^3x - \frac{a^3\tanh(c+dx)}{d} + 3a^2bx - \frac{a^2b\tanh^3(c+dx)}{d} - \frac{3a^2b\tanh(c+dx)}{d} + 3ab^2x - \frac{3ab^2\tanh^5(c+dx)}{5d} - \frac{ab^2\tanh^3(c+dx)}{d} - \\ x\left(a+b\tanh^2(c)\right)^3\tanh^2(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*tanh(c + d*x)/d + 3*a**2*b*x - a**2*b*tanh(c + d*
x)**3/d - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - 3*a*b**2*tanh(c + d*x)**
5/(5*d) - a*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x -
b**3*tanh(c + d*x)**7/(7*d) - b**3*tanh(c + d*x)**5/(5*d) - b**3*tanh(c +
d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*
tanh(c)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(88) = 176$.

Time = 0.05 (sec) , antiderivative size = 400, normalized size of antiderivative = 4.26

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{105}b^3\left(105\,x + \frac{105\,c}{d} - \frac{8\left(203\,e^{(-2\,dx-2\,c)} + 609\,e^{(-4\,dx-4\,c)} + 770\,e^{(-6\,dx-6\,c)} + 770\,e^{(-8\,dx-8\,c)} + 315\,e^{(-\cdots}\right.}{d(7\,e^{(-2\,dx-2\,c)} + 21\,e^{(-4\,dx-4\,c)} + 35\,e^{(-6\,dx-6\,c)} + 35\,e^{(-8\,dx-8\,c)} + 21\,e^{(-10\,dx-10\,c)}\cdots}\right.$$
$$+ \frac{1}{5}ab^2\left(15\,x + \frac{15\,c}{d} - \frac{2\left(70\,e^{(-2\,dx-2\,c)} + 140\,e^{(-4\,dx-4\,c)} + 90\,e^{(-6\,dx-6\,c)} + 45\,e^{(-8\,dx-8\,c)} + 23\right)}{d(5\,e^{(-2\,dx-2\,c)} + 10\,e^{(-4\,dx-4\,c)} + 10\,e^{(-6\,dx-6\,c)} + 5\,e^{(-8\,dx-8\,c)} + e^{(-10\,dx-10\,c)} + 1)}\right)$$
$$+ a^2b\left(3\,x + \frac{3\,c}{d} - \frac{4\left(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + 2\right)}{d(3\,e^{(-2\,dx-2\,c)} + 3\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)} + 1)}\right)$$
$$+ a^3\left(x + \frac{c}{d} - \frac{2}{d(e^{(-2\,dx-2\,c)} + 1)}\right)$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/105*b^3*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c
) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) +
 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 1/5*a*b^2*(15*x + 15*c/d -
 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45*
e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e
^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + a^2*b*(
3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e^(-2*
d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + a^3*(x + c/d -
 2/(d*(e^(-2*d*x - 2*c) + 1)))
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(88) = 176$.

Time = 0.25 (sec) , antiderivative size = 418, normalized size of antiderivative = 4.45

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{105\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c)+\frac{2\,(105\,a^3e^{(12\,dx+12\,c)}+630\,a^2be^{(12\,dx+12\,c)}+945\,ab^2e^{(12\,dx+12\,c)}+420\,b^3e^{(12\,dx+12\,c)}+63}{}}{}$$

input
```
integrate(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/105*(105*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(105*a^3*e^(12*d*
x + 12*c) + 630*a^2*b*e^(12*d*x + 12*c) + 945*a*b^2*e^(12*d*x + 12*c) + 42
0*b^3*e^(12*d*x + 12*c) + 630*a^3*e^(10*d*x + 10*c) + 3150*a^2*b*e^(10*d*x
 + 10*c) + 3780*a*b^2*e^(10*d*x + 10*c) + 1260*b^3*e^(10*d*x + 10*c) + 157
5*a^3*e^(8*d*x + 8*c) + 6720*a^2*b*e^(8*d*x + 8*c) + 7665*a*b^2*e^(8*d*x +
 8*c) + 3080*b^3*e^(8*d*x + 8*c) + 2100*a^3*e^(6*d*x + 6*c) + 7980*a^2*b*e
^(6*d*x + 6*c) + 9240*a*b^2*e^(6*d*x + 6*c) + 3080*b^3*e^(6*d*x + 6*c) + 1
575*a^3*e^(4*d*x + 4*c) + 5670*a^2*b*e^(4*d*x + 4*c) + 6363*a*b^2*e^(4*d*x
 + 4*c) + 2436*b^3*e^(4*d*x + 4*c) + 630*a^3*e^(2*d*x + 2*c) + 2310*a^2*b*
e^(2*d*x + 2*c) + 2436*a*b^2*e^(2*d*x + 2*c) + 812*b^3*e^(2*d*x + 2*c) + 1
05*a^3 + 420*a^2*b + 483*a*b^2 + 176*b^3)/(e^(2*d*x + 2*c) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.13

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)-\frac{\tanh(c+d\,x)\,\left(a+b\right)^3}{d} - \frac{\tanh(c+d\,x)^3\,\left(3\,a^2\,b+3\,a\,b^2+b^3\right)}{3\,d} - \frac{\tanh(c+d\,x)^5\,\left(b^3+3\,a\,b^2\right)}{5\,d}-\frac{b^3\tanh(c+d\,x)^7}{7\,d}$$

input
```
int(tanh(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)*(a + b)^3)/d - (tanh(c
+ d*x)^3*(3*a*b^2 + 3*a^2*b + b^3))/(3*d) - (tanh(c + d*x)^5*(3*a*b^2 + b^
3))/(5*d) - (b^3*tanh(c + d*x)^7)/(7*d)
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 163, normalized size of antiderivative = 1.73

$$\int \tanh^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{-15\tanh(dx+c)^7 b^3 - 63\tanh(dx+c)^5 a b^2 - 21\tanh(dx+c)^5 b^3 - 105\tanh(dx+c)^3 a^2 b - 105\tanh}{}$$

input
```
int(tanh(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 15*tanh(c + d*x)**7*b**3 - 63*tanh(c + d*x)**5*a*b**2 - 21*tanh(c + d*
x)**5*b**3 - 105*tanh(c + d*x)**3*a**2*b - 105*tanh(c + d*x)**3*a*b**2 - 3
5*tanh(c + d*x)**3*b**3 - 105*tanh(c + d*x)*a**3 - 315*tanh(c + d*x)*a**2*
b - 315*tanh(c + d*x)*a*b**2 - 105*tanh(c + d*x)*b**3 + 105*a**3*d*x + 315
*a**2*b*d*x + 315*a*b**2*d*x + 105*b**3*d*x)/(105*d)
```

3.159 $\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 21, antiderivative size = 83

$$\int \tanh(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{(a+b)^3 \log(\cosh(c+dx))}{d} - \frac{b(a+b)^2 \tanh^2(c+dx)}{2d} - \frac{(a+b)\left(a+b\tanh^2(c+dx)\right)^2}{4d} - \frac{\left(a+b\tanh^2(c+dx)\right)^3}{6d}$$

output
```
(a+b)^3*ln(cosh(d*x+c))/d-1/2*b*(a+b)^2*tanh(d*x+c)^2/d-1/4*(a+b)*(a+b*tan
h(d*x+c)^2)^2/d-1/6*(a+b*tanh(d*x+c)^2)^3/d
```

Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.92

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{-2(a+b)^3\log(\cosh(c+dx))+b(a+b)^2\tanh^2(c+dx)+\frac{1}{2}(a+b)\left(a+b\tanh^2(c+dx)\right)^2+\frac{1}{3}(a+b\,\mathrm{t}}{2d}$$

input
```
Integrate[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(-2*(a + b)^3*Log[Cosh[c + d*x]] + b*(a + b)^2*Tanh[c + d*x]^2 + ((a
+ b)*(a + b*Tanh[c + d*x]^2)^2)/2 + (a + b*Tanh[c + d*x]^2)^3/3)/d
```

Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 4153, 26, 353, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3dx$$
$$\Big\downarrow \; 3042$$
$$\int -i\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3dx$$
$$\Big\downarrow \; 26$$
$$-i\int \tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3dx$$
$$\Big\downarrow \; 4153$$
$$-\frac{i\int \frac{i\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{\int \left(\frac{(a+b)^3}{1-\tanh^2(c+dx)}-b(a+b)^2-b\left(b\tanh^2(c+dx)+a\right)(a+b)-b\left(b\tanh^2(c+dx)+a\right)^2\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-b(a+b)^2\tanh^2(c+dx)-\frac{1}{2}(a+b)\left(a+b\tanh^2(c+dx)\right)^2-\frac{1}{3}\left(a+b\tanh^2(c+dx)\right)^3+(a+b)^3\left(-\log\left(1-\text{ta}\right.\right.}{2d}$$

input
```
Int[Tanh[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-((a + b)^3*Log[1 - Tanh[c + d*x]^2]) - b*(a + b)^2*Tanh[c + d*x]^2 - ((a
 + b)*(a + b*Tanh[c + d*x]^2)^2)/2 - (a + b*Tanh[c + d*x]^2)^3/3)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.82

method	result
derivativedivides	$\frac{-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}+\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(1+\tanh(dx+c))}{2}-\frac{\left(a^3+3a^2b+3a\,b^2+b\right.}{}}{d}$
default	$\frac{-\frac{3\tanh(dx+c)^4 a\,b^2}{4}-\frac{3\tanh(dx+c)^2 a^2 b}{2}-\frac{3a\,b^2\tanh(dx+c)^2}{2}+\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(1+\tanh(dx+c))}{2}-\frac{\left(a^3+3a^2b+3a\,b^2+b\right.}{}}{d}$
parts	$\frac{a^3\ln(\cosh(dx+c))}{d}+\frac{b^3\left(-\frac{\tanh(dx+c)^6}{6}-\frac{\tanh(dx+c)^4}{4}-\frac{\tanh(dx+c)^2}{2}-\frac{\ln(-1+\tanh(dx+c))}{2}-\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{3a}{}$
parallelrisch	$-\frac{2\tanh(dx+c)^6b^3+9\tanh(dx+c)^4a\,b^2+3b^3\tanh(dx+c)^4+12a^3dx+36a^2bdx+36a\,b^2dx+12b^3dx+18\tanh(dx+c)^2a^2}{}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6a^2bc}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}+\frac{2b\,e^{2dx+2c}\left(9a^2e^{8dx+8c}+18ab\,e^{8d}\right.}{}$

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-3/4*tanh(d*x+c)^4*a*b^2-3/2*tanh(d*x+c)^2*a^2*b-3/2*a*b^2*tanh(d*x+c
)^2+1/2*(-a^3-3*a^2*b-3*a*b^2-b^3)*ln(1+tanh(d*x+c))-1/2*(a^3+3*a^2*b+3*a*
b^2+b^3)*ln(-1+tanh(d*x+c))-1/6*tanh(d*x+c)^6*b^3-1/4*b^3*tanh(d*x+c)^4-1/
2*b^3*tanh(d*x+c)^2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4298 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 4298, normalized size of antiderivative = 51.78

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(71) = 142$.

Time = 0.19 (sec) , antiderivative size = 211, normalized size of antiderivative = 2.54

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \begin{cases} a^3x - \frac{a^3\log(\tanh(c+dx)+1)}{d} + 3a^2bx - \frac{3a^2b\log(\tanh(c+dx)+1)}{d} - \frac{3a^2b\tanh^2(c+dx)}{2d} + 3ab^2x - \frac{3ab^2\log(\tanh(c+dx)+1)}{d} \\ x\left(a+b\tanh^2(c)\right)^3\tanh(c) \end{cases}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x - a**3*log(tanh(c + d*x) + 1)/d + 3*a**2*b*x - 3*a**2*b*
log(tanh(c + d*x) + 1)/d - 3*a**2*b*tanh(c + d*x)**2/(2*d) + 3*a*b**2*x -
3*a*b**2*log(tanh(c + d*x) + 1)/d - 3*a*b**2*tanh(c + d*x)**4/(4*d) - 3*a*
b**2*tanh(c + d*x)**2/(2*d) + b**3*x - b**3*log(tanh(c + d*x) + 1)/d - b**
3*tanh(c + d*x)**6/(6*d) - b**3*tanh(c + d*x)**4/(4*d) - b**3*tanh(c + d*x
)**2/(2*d), Ne(d, 0)), (x*(a + b*tanh(c)**2)**3*tanh(c), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 351 vs. $2(77) = 154$.

Time = 0.15 (sec) , antiderivative size = 351, normalized size of antiderivative = 4.23

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$= \frac{1}{3}b^3\left(3x + \frac{3c}{d} + \frac{3\log\left(e^{(-2dx-2c)}+1\right)}{d} + \frac{2\left(9e^{(-2dx-2c)} + 18e^{(-4dx-4c)} + 34e^{(-6dx-6c)} + 18\right.}{d(6e^{(-2dx-2c)} + 15e^{(-4dx-4c)} + 20e^{(-6dx-6c)} + 15e^{(-8dx-8c)}}\right.$$
$$+3ab^2\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{d} + \frac{4\left(e^{(-2dx-2c)} + e^{(-4dx-4c)} + e^{(-6dx-6c)}\right)}{d(4e^{(-2dx-2c)} + 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} + e^{(-8dx-8c)} + 1)}\right)$$
$$+3a^2b\left(x + \frac{c}{d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{d} + \frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)} + e^{(-4dx-4c)} + 1)}\right)$$
$$+\frac{a^3\log\left(\cosh\left(dx+c\right)\right)}{d}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*b^3*(3*x + 3*c/d + 3*log(e^(-2*d*x - 2*c) + 1)/d + 2*(9*e^(-2*d*x - 2*
c) + 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) + 18*e^(-8*d*x - 8*c) + 9*e
^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) + 3*a*b^2*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x
 - 2*c) + e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*
e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a^2*b*
(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*
x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^3*log(cosh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 216 vs. $2(77) = 154$.

Time = 0.22 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.60

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+1\right)-\frac{2\left(9\left(a^2b+2ab^2+b^3\right)e^{(\cdots}\right.}{\cdots}}{\cdots}$$

input
```
integrate(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*log(e^(2*d*x + 2*c) + 1) - 2*(9*(a^2*b + 2*a*b^2 + b^3)*e^(10*
d*x + 10*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(8*d*x + 8*c) + 2*(27*a^2*b +
 36*a*b^2 + 17*b^3)*e^(6*d*x + 6*c) + 18*(2*a^2*b + 3*a*b^2 + b^3)*e^(4*d*
x + 4*c) + 9*(a^2*b + 2*a*b^2 + b^3)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c) + 1
)^6)/d
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 123, normalized size of antiderivative = 1.48

$$\begin{aligned}\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx &= x\left(a^3+3a^2b+3ab^2+b^3\right)-\frac{\tanh(c+dx)^2\left(3a^2b+3ab^2+b^3\right)}{2d}\\&\quad-\frac{\ln\left(\tanh(c+dx)+1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d}\\&\quad-\frac{\tanh(c+dx)^4\left(b^3+3ab^2\right)}{4d}-\frac{b^3\tanh(c+dx)^6}{6d}\end{aligned}$$

input
```
int(tanh(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^2*(3*a*b^2 + 3*a^2*b +
b^3))/(2*d) - (log(tanh(c + d*x) + 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d -
 (tanh(c + d*x)^4*(3*a*b^2 + b^3))/(4*d) - (b^3*tanh(c + d*x)^6)/(6*d)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 1493, normalized size of antiderivative = 17.99

$$\int \tanh(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(3*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3 + 9*e**(12*c + 12*d*x
)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 9*e**(12*c + 12*d*x)*log(e**(2*c + 2*
d*x) + 1)*a*b**2 + 3*e**(12*c + 12*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 3
*e**(12*c + 12*d*x)*a**3*d*x - 9*e**(12*c + 12*d*x)*a**2*b*d*x - 3*e**(12*
c + 12*d*x)*a**2*b - 9*e**(12*c + 12*d*x)*a*b**2*d*x - 6*e**(12*c + 12*d*x
)*a*b**2 - 3*e**(12*c + 12*d*x)*b**3*d*x - 3*e**(12*c + 12*d*x)*b**3 + 18*
e**(10*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3 + 54*e**(10*c + 10*d*x)*
log(e**(2*c + 2*d*x) + 1)*a**2*b + 54*e**(10*c + 10*d*x)*log(e**(2*c + 2*d
*x) + 1)*a*b**2 + 18*e**(10*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 1
8*e**(10*c + 10*d*x)*a**3*d*x - 54*e**(10*c + 10*d*x)*a**2*b*d*x - 54*e**(
10*c + 10*d*x)*a*b**2*d*x - 18*e**(10*c + 10*d*x)*b**3*d*x + 45*e**(8*c +
8*d*x)*log(e**(2*c + 2*d*x) + 1)*a**3 + 135*e**(8*c + 8*d*x)*log(e**(2*c +
 2*d*x) + 1)*a**2*b + 135*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**
2 + 45*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 45*e**(8*c + 8*d*
x)*a**3*d*x - 135*e**(8*c + 8*d*x)*a**2*b*d*x + 27*e**(8*c + 8*d*x)*a**2*b
 - 135*e**(8*c + 8*d*x)*a*b**2*d*x + 18*e**(8*c + 8*d*x)*a*b**2 - 45*e**(8
*c + 8*d*x)*b**3*d*x - 9*e**(8*c + 8*d*x)*b**3 + 60*e**(6*c + 6*d*x)*log(e
**(2*c + 2*d*x) + 1)*a**3 + 180*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)
*a**2*b + 180*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 + 60*e**(6
*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*b**3 - 60*e**(6*c + 6*d*x)*a**3*d...
```

3.160 $\int \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 74

$$\int \left(a+b\tanh^2(c+dx)\right)^3 \, dx = (a+b)^3x-\frac{b(3a^2+3ab+b^2)\tanh(c+dx)}{d} -\frac{b^2(3a+b)\tanh^3(c+dx)}{3d}-\frac{b^3\tanh^5(c+dx)}{5d}$$

output
```
(a+b)^3*x-b*(3*a^2+3*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(3*a+b)*tanh(d*x+c)^3/
d-1/5*b^3*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.28

$$\int \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{\tanh(c+dx)\left(\frac{15(a+b)^3\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}}-b\left(45a^2+15ab\left(3+\tanh^2(c+dx)\right)+b^2\left(15+5\tanh^2(c-\right.\right.\right.}{15d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*((15*(a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
+ d*x]^2] - b*(45*a^2 + 15*a*b*(3 + Tanh[c + d*x]^2) + b^2*(15 + 5*Tanh[c
+ d*x]^2 + 3*Tanh[c + d*x]^4))))/(15*d)
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\tan(ic+idx)^2\right)^3 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-b^3\tanh^4(c+dx)-b^2(3a+b)\tanh^2(c+dx)-b\left(3a^2+3ba+b^2\right)+\frac{(a+b)^3}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-b\left(3a^2+3ab+b^2\right)\tanh(c+dx)+(a+b)^3\text{arctanh}(\tanh(c+dx))-\frac{1}{3}b^2(3a+b)\tanh^3(c+dx)-\frac{1}{5}b^3\tanh^5(c+}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^3,x]
```

output

```
((a + b)^3*ArcTanh[Tanh[c + d*x]] - b*(3*a^2 + 3*a*b + b^2)*Tanh[c + d*x]
- (b^2*(3*a + b)*Tanh[c + d*x]^3)/3 - (b^3*Tanh[c + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 300

```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042

```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144

```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (warning: unable to verify)

Time = 0.05 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.51

method	result
parallelrisch	$-\frac{3b^{3}\tanh(dx+c)^{5}+15a\,b^{2}\tanh(dx+c)^{3}+5b^{3}\tanh(dx+c)^{3}-15a^{3}dx-45a^{2}bdx-45a\,b^{2}dx-15b^{3}dx+45a^{2}b\tanh(dx+c}{15d}$
derivativedivides	$\frac{-3a^{2}b\tanh(dx+c)-3a\,b^{2}\tanh(dx+c)-a\,b^{2}\tanh(dx+c)^{3}+\frac{\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\ln(1+\tanh(dx+c))}{2}-\frac{\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right.}{}}{d}$
default	$\frac{-3a^{2}b\tanh(dx+c)-3a\,b^{2}\tanh(dx+c)-a\,b^{2}\tanh(dx+c)^{3}+\frac{\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right)\ln(1+\tanh(dx+c))}{2}-\frac{\left(a^{3}+3a^{2}b+3a\,b^{2}+b^{3}\right.}{}}{d}$
parts	$a^{3}x+\frac{b^{3}\left(-\frac{\tanh(dx+c)^{5}}{5}-\frac{\tanh(dx+c)^{3}}{3}-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{3a^{2}b\left(-\tanh(dx+\right.}{}$
risch	$a^{3}x+3a^{2}bx+3a\,b^{2}x+b^{3}x+\frac{2b\left(45a^{2}e^{8dx+8c}+90ab\,e^{8dx+8c}+45b^{2}e^{8dx+8c}+180a^{2}e^{6dx+6c}+270ab\,e^{6dx+6}\right.}{}$

input
```
int((a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(3*b^3*tanh(d*x+c)^5+15*a*b^2*tanh(d*x+c)^3+5*b^3*tanh(d*x+c)^3-15*a
^3*d*x-45*a^2*b*d*x-45*a*b^2*d*x-15*b^3*d*x+45*a^2*b*tanh(d*x+c)+45*a*b^2*
tanh(d*x+c)+15*b^3*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(70) = 140$.

Time = 0.10 (sec) , antiderivative size = 567, normalized size of antiderivative = 7.66

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{(45\,a^2b+60\,ab^2+23\,b^3+15\,(a^3+3\,a^2b+3\,ab^2+b^3)dx)\cosh\left(dx+c\right)^5+5\,(45\,a^2b+60\,ab^2+23\,b^3+1}{}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
1/15*((45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d
*x)*cosh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^4 - (45*a^2*b + 60*a*b^2
 + 23*b^3)*sinh(d*x + c)^5 + 5*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3
*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - 5*(27*a^2*b + 24*a*b^2 + 5*
b^3 + 2*(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
5*(2*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x)*cosh(d*x + c)^3 + 3*(45*a^2*b + 60*a*b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(45*a^2*b + 60*a*
b^2 + 23*b^3 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c) - 5*(
(45*a^2*b + 60*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 18*a^2*b + 12*a*b^2 + 10*
b^3 + 3*(27*a^2*b + 24*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*c
osh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 +
 5*(2*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d
*x + c))
```

Sympy [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.70

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\begin{cases} a^3x+3a^2bx-\frac{3a^2b\tanh(c+dx)}{d}+3ab^2x-\frac{ab^2\tanh^3(c+dx)}{d}-\frac{3ab^2\tanh(c+dx)}{d}+b^3x-\frac{b^3\tanh^5(c+dx)}{5d}-\frac{b^3\tanh^3(c+dx)}{3d} \\ x\left(a+b\tanh^2(c)\right)^3 \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Piecewise((a**3*x + 3*a**2*b*x - 3*a**2*b*tanh(c + d*x)/d + 3*a*b**2*x - a
*b**2*tanh(c + d*x)**3/d - 3*a*b**2*tanh(c + d*x)/d + b**3*x - b**3*tanh(c
 + d*x)**5/(5*d) - b**3*tanh(c + d*x)**3/(3*d) - b**3*tanh(c + d*x)/d, Ne(
d, 0)), (x*(a + b*tanh(c)**2)**3, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. 2(70) = 140.

Time = 0.05 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.23

$$\int \left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{15}b^3\left(15\,x+\frac{15\,c}{d}-\frac{2\left(70\,e^{(-2\,dx-2\,c)}+140\,e^{(-4\,dx-4\,c)}+90\,e^{(-6\,dx-6\,c)}+45\,e^{(-8\,dx-8\,c)}+23\right)}{d(5\,e^{(-2\,dx-2\,c)}+10\,e^{(-4\,dx-4\,c)}+10\,e^{(-6\,dx-6\,c)}+5\,e^{(-8\,dx-8\,c)}+e^{(-10\,dx-10\,c)}+1)}\right)$$
$$+ab^2\left(3\,x+\frac{3\,c}{d}-\frac{4\left(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+2\right)}{d(3\,e^{(-2\,dx-2\,c)}+3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}+1)}\right)$$
$$+3\,a^2b\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2\,dx-2\,c)}+1)}\right)+a^3x$$

input
```
integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) + a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
 - 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) + 1))) + 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. $2(70) = 140$.

Time = 0.13 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.26

$$\int \left(a + b\tanh^2(c + dx)\right)^3 \, dx$$
$$= \frac{15\,(a^3 + 3\,a^2b + 3\,ab^2 + b^3)(dx + c) + \frac{2\,(45\,a^2be^{(8\,dx+8\,c)}+90\,ab^2e^{(8\,dx+8\,c)}+45\,b^3e^{(8\,dx+8\,c)}+180\,a^2be^{(6\,dx+6\,c)}+270\,ab^2e^{(6\,dx+}}{}}{}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 2*(45*a^2*b*e^(8*d*x
+ 8*c) + 90*a*b^2*e^(8*d*x + 8*c) + 45*b^3*e^(8*d*x + 8*c) + 180*a^2*b*e^(
6*d*x + 6*c) + 270*a*b^2*e^(6*d*x + 6*c) + 90*b^3*e^(6*d*x + 6*c) + 270*a^
2*b*e^(4*d*x + 4*c) + 330*a*b^2*e^(4*d*x + 4*c) + 140*b^3*e^(4*d*x + 4*c)
+ 180*a^2*b*e^(2*d*x + 2*c) + 210*a*b^2*e^(2*d*x + 2*c) + 70*b^3*e^(2*d*x
+ 2*c) + 45*a^2*b + 60*a*b^2 + 23*b^3)/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16

$$\int \left(a + b\tanh^2(c + dx)\right)^3 \, dx = x\left(a^3 + 3\,a^2\,b + 3\,a\,b^2 + b^3\right) - \frac{\tanh(c + d\,x)^3\,(b^3 + 3\,a\,b^2)}{3\,d} - \frac{b^3\tanh(c + d\,x)^5}{5\,d} - \frac{b\tanh(c + d\,x)\,\left(3\,a^2 + 3\,a\,b + b^2\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c + d*x)^3*(3*a*b^2 + b^3))/(3*d
) - (b^3*tanh(c + d*x)^5)/(5*d) - (b*tanh(c + d*x)*(3*a*b + 3*a^2 + b^2))/
d
```

Reduce [B] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 111, normalized size of antiderivative = 1.50

$$\int \left(a + b\tanh^2(c + dx)\right)^3 \, dx$$
$$= \frac{-3\tanh\left(dx+c\right)^5 b^3 - 15\tanh\left(dx+c\right)^3 a\, b^2 - 5\tanh\left(dx+c\right)^3 b^3 - 45\tanh\left(dx+c\right) a^2 b - 45\tanh\left(dx\right.}{15d}$$

input
```
int((a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 3*tanh(c + d*x)**5*b**3 - 15*tanh(c + d*x)**3*a*b**2 - 5*tanh(c + d*x)
**3*b**3 - 45*tanh(c + d*x)*a**2*b - 45*tanh(c + d*x)*a*b**2 - 15*tanh(c +
 d*x)*b**3 + 15*a**3*d*x + 45*a**2*b*d*x + 45*a*b**2*d*x + 15*b**3*d*x)/(1
5*d)
```

3.161 $\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 21, antiderivative size = 72

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\frac{(a+b)^3\log(\cosh(c+dx))}{d}+\frac{a^3\log(\tanh(c+dx))}{d}-\frac{b^2(3a+b)\tanh^2(c+dx)}{2d}-\frac{b^3\tanh^4(c+dx)}{4d}$$

output

```
(a+b)^3*ln(cosh(d*x+c))/d+a^3*ln(tanh(d*x+c))/d-1/2*b^2*(3*a+b)*tanh(d*x+c
)^2/d-1/4*b^3*tanh(d*x+c)^4/d
```

Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =\frac{2(a+b)^3\log(\cosh(c+dx))+2a^3\log(\tanh(c+dx))-b^2(3a+b)\tanh^2(c+dx)-\frac{1}{2}b^3\tanh^4(c+dx)}{2d}$$

input
```
Integrate[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(2*(a + b)^3*Log[Cosh[c + d*x]] + 2*a^3*Log[Tanh[c + d*x]] - b^2*(3*a + b)
*Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.03, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\coth(c+dx)a^3-b^3\tanh^2(c+dx)-b^2(3a+b)-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{a^3\log\left(\tanh^2(c+dx)\right)-b^2(3a+b)\tanh^2(c+dx)-(a+b)^3\log\left(1-\tanh^2(c+dx)\right)-\frac{1}{2}b^3\tanh^4(c+dx)}{2d}$$

input
```
Int[Coth[c + d*x]*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(a^3*Log[Tanh[c + d*x]^2] - (a + b)^3*Log[1 - Tanh[c + d*x]^2] - b^2*(3*a
+ b)*Tanh[c + d*x]^2 - (b^3*Tanh[c + d*x]^4)/2)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 80, normalized size of antiderivative = 1.11

method	result
parallelrisch	$-\frac{-4(a+b)^3\ln(1-\tanh(dx+c))+4a^3\ln(\tanh(dx+c))-b^3\tanh(dx+c)^4+(-6a\,b^2-2b^3)\tanh(dx+c)^2-4dx(a+b)^3}{4d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^4}{4}+\frac{3a\,b^2\tanh(dx+c)^2}{2}+\frac{b^3\tanh(dx+c)^2}{2}-a^3\ln(\tanh(dx+c))+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(1+\tanh(dx+}{d}$
default	$-\frac{\frac{b^3\tanh(dx+c)^4}{4}+\frac{3a\,b^2\tanh(dx+c)^2}{2}+\frac{b^3\tanh(dx+c)^2}{2}-a^3\ln(\tanh(dx+c))+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3\right)\ln(1+\tanh(dx+}{d}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{6a^2bc}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a^3c}{d}+\frac{2b^2e^{2dx+2c}\left(3a\,e^{4dx+4c}+2b\,e^{4dx+}{d\left(e^{2d}}$

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*(a+b)^3*ln(1-tanh(d*x+c))+4*a^3*ln(tanh(d*x+c))-b^3*tanh(d*x+c)^4+
(-6*a*b^2-2*b^3)*tanh(d*x+c)^2-4*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2381 vs. 2(68) = 136.

Time = 0.13 (sec) , antiderivative size = 2381, normalized size of antiderivative = 33.07

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x*sinh(d*x + c)^8 - 2*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x*cosh(d*x + c)^2 - 3*a*b^2 - 2*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x)*sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^3 - 3*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^5 - 2*(6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^4 - 6*a*b^2 - 2*b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x
- 15*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^5 - 5*(3*a*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^3 - (6*a*b^2 + 2*b^3 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(3*a
*b^2 + 2*b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*
(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 - 15*(3*a*b^2 + 2*
b^3 - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 3*a*b^2 - 2
*b^3 + 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 6*(6*a*b^2 + 2*b^3 - 3*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3...
```

Sympy [F]

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3 \coth\left(c+dx\right) dx$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 214 vs. $2(68) = 136$.

Time = 0.14 (sec) , antiderivative size = 214, normalized size of antiderivative = 2.97

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}+6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}+e^{(-8\,dx-8\,c)}+1)}\right)$$
$$+3\,ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)$$
$$+\frac{3\,a^2b\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{a^3\log\left(\sinh\left(dx+c\right)\right)}{d}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
b^3*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 4*(e^(-2*d*x - 2*c) + e^(-4*d
*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c)
+ 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) + 3*a*b^2*(x + c/d + log(e^
(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*
d*x - 4*c) + 1))) + 3*a^2*b*log(e^(d*x + c) + e^(-d*x - c))/d + a^3*log(si
nh(d*x + c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(68) = 136$.

Time = 0.23 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.71

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{2\,a^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)+2\left(3\,a^2b+3\,ab^2+b^3\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)-\frac{9\,a^2b(e^{(2\,dx+}}{}}{}$$

input
```
integrate(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*a^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) + 2*(3*a^2*b + 3*a*
b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) - (9*a^2*b*(e^(2*d*
x + 2*c) + e^(-2*d*x - 2*c))^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*
c))^2 + 3*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 36*a^2*b*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c)) + 12*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))
 - 4*b^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 36*a^2*b - 12*a*b^2 - 4*b^
3)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 380, normalized size of antiderivative = 5.28

$$\int \coth(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{\ln\left(e^{4c+4dx}-1\right)\left(a^3 d + d\left(3a^2b+3ab^2+b^3\right)\right)}{2d^2} - x\left(a+b\right)^3$$
$$+ \frac{2\left(2b^3+3ab^2\right)}{d\left(e^{2c+2dx}+1\right)} + \frac{8b^3}{d\left(3e^{2c+2dx}+3e^{4c+4dx}+e^{6c+6dx}+1\right)}$$
$$+ \frac{\operatorname{atan}\left(\frac{e^{2c}e^{2dx}\left(b^3\sqrt{-d^2}-a^3\sqrt{-d^2}+3ab^2\sqrt{-d^2}+3a^2b\sqrt{-d^2}\right)}{d\sqrt{a^6-6a^5b+3a^4b^2+16a^3b^3+15a^2b^4+6ab^5+b^6}}\right)\sqrt{a^6-6a^5b+3a^4b^2+16a^3b^3+15a^2b^4+6ab}}{\sqrt{-d^2}}$$
$$- \frac{2\left(4b^3+3ab^2\right)}{d\left(2e^{2c+2dx}+e^{4c+4dx}+1\right)} - \frac{4b^3}{d\left(4e^{2c+2dx}+6e^{4c+4dx}+4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$

input
```
int(coth(c + d*x)*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(a^3*d + d*(3*a*b^2 + 3*a^2*b + b^3)))/(2*d^2)
- x*(a + b)^3 + (2*(3*a*b^2 + 2*b^3))/(d*(exp(2*c + 2*d*x) + 1)) + (8*b^3)
/(d*(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1)) + (a
tan((exp(2*c)*exp(2*d*x)*(b^3*(-d^2)^(1/2) - a^3*(-d^2)^(1/2) + 3*a*b^2*(-
d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a*b^5 - 6*a^5*b + a^6 + b^6 + 15
*a^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2)))*(6*a*b^5 - 6*a^5*b + a^6 + b^6
+ 15*a^2*b^4 + 16*a^3*b^3 + 3*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a*b^2 +
 4*b^3))/(d*(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1)) - (4*b^3)/(d*(4*e
xp(2*c + 2*d*x) + 6*exp(4*c + 4*d*x) + 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 1069, normalized size of antiderivative = 14.85

$$\int \coth(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(6*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 6*e**(8*c + 8*d*x)*
log(e**(2*c + 2*d*x) + 1)*a*b**2 + 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)
 + 1)*b**3 + 2*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**3 + 2*e**(8*c + 8
*d*x)*log(e**(c + d*x) + 1)*a**3 - 2*e**(8*c + 8*d*x)*a**3*d*x - 6*e**(8*c
 + 8*d*x)*a**2*b*d*x - 6*e**(8*c + 8*d*x)*a*b**2*d*x - 3*e**(8*c + 8*d*x)*
a*b**2 - 2*e**(8*c + 8*d*x)*b**3*d*x - 2*e**(8*c + 8*d*x)*b**3 + 24*e**(6*
c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 24*e**(6*c + 6*d*x)*log(e**(
2*c + 2*d*x) + 1)*a*b**2 + 8*e**(6*c + 6*d*x)*log(e**(2*c + 2*d*x) + 1)*b*
*3 + 8*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**3 + 8*e**(6*c + 6*d*x)*lo
g(e**(c + d*x) + 1)*a**3 - 8*e**(6*c + 6*d*x)*a**3*d*x - 24*e**(6*c + 6*d*
x)*a**2*b*d*x - 24*e**(6*c + 6*d*x)*a*b**2*d*x - 8*e**(6*c + 6*d*x)*b**3*d
*x + 36*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 36*e**(4*c + 4
*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 + 12*e**(4*c + 4*d*x)*log(e**(2*c +
 2*d*x) + 1)*b**3 + 12*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**3 + 12*e*
*(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**3 - 12*e**(4*c + 4*d*x)*a**3*d*x -
 36*e**(4*c + 4*d*x)*a**2*b*d*x - 36*e**(4*c + 4*d*x)*a*b**2*d*x + 6*e**(4
*c + 4*d*x)*a*b**2 - 12*e**(4*c + 4*d*x)*b**3*d*x - 4*e**(4*c + 4*d*x)*b**
3 + 24*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a**2*b + 24*e**(2*c + 2*
d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 + 8*e**(2*c + 2*d*x)*log(e**(2*c + 2
*d*x) + 1)*b**3 + 8*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**3 + 8*e**...
```

3.162 $\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = (a+b)^3x - \frac{a^3\coth(c+dx)}{d} - \frac{b^2(3a+b)\tanh(c+dx)}{d} - \frac{b^3\tanh^3(c+dx)}{3d}$$

output
```
(a+b)^3*x-a^3*coth(d*x+c)/d-b^2*(3*a+b)*tanh(d*x+c)/d-1/3*b^3*tanh(d*x+c)^
3/d
```

Mathematica [A] (verified)

Time = 1.56 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.37

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{\tanh(c+dx)\left(-3a^3\coth^2(c+dx) + 3(a+b)^3\text{arctanh}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(c+dx)} - b^2(9a+3\right.}{3d}$$

input
```
Integrate[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Tanh[c + d*x]*(-3*a^3*Coth[c + d*x]^2 + 3*(a + b)^3*ArcTanh[Sqrt[Coth[c +
 d*x]^2]]*Sqrt[Coth[c + d*x]^2] - b^2*(9*a + 3*b + b*Tanh[c + d*x]^2)))/(3
*d)
```

Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{\int -\frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{364}$$

$$\frac{\int \left(\coth^2(c+dx)a^3-b^3\tanh^2(c+dx)-b^2(3a+b)-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{a^3\coth(c+dx)-(a+b)^3\text{arctanh}(\tanh(c+dx))+b^2(3a+b)\tanh(c+dx)+\frac{1}{3}b^3\tanh^3(c+dx)}{d}$$

input
```
Int[Coth[c + d*x]^2*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*ArcTanh[Tanh[c + d*x]]) + a^3*Coth[c + d*x] + b^2*(3*a + b)
*Tanh[c + d*x] + (b^3*Tanh[c + d*x]^3)/3)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 59, normalized size of antiderivative = 1.00

method	result
parallelrisch	$\frac{-b^3\tanh(dx+c)^3+(-9a\,b^2-3b^3)\tanh(dx+c)-3\coth(dx+c)a^3+3dx(a+b)^3}{3d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^3}{3}+3a\,b^2\tanh(dx+c)+b^3\tanh(dx+c)+(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3)\ln(1+\tanh(dx+c))+\frac{a^3}{\tanh(dx+c)}+(}{d}$
default	$-\frac{\frac{b^3\tanh(dx+c)^3}{3}+3a\,b^2\tanh(dx+c)+b^3\tanh(dx+c)+(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3)\ln(1+\tanh(dx+c))+\frac{a^3}{\tanh(dx+c)}+(}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x-\frac{2(3a^3\mathrm{e}^{6dx+6c}-9a\,b^2\mathrm{e}^{6dx+6c}-6b^3\mathrm{e}^{6dx+6c}+9a^3\mathrm{e}^{4dx+4c}-9a\,b^2\mathrm{e}^{4dx+4c}+9a^3}{3d(\mathrm{e}^{2dx+2c}+1)^3(\mathrm{e}^{2dx+2c}}$

input
```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-b^3*tanh(d*x+c)^3+(-9*a*b^2-3*b^3)*tanh(d*x+c)-3*coth(d*x+c)*a^3+3*d
*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(57) = 114$.

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.78

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$
$$-\frac{(3\,a^3+9\,ab^2+4\,b^3)\cosh{(dx+c)}^4-4\,(3\,a^3+9\,ab^2+4\,b^3+3\,(a^3+3\,a^2b+3\,ab^2+b^3)dx)\cosh{(dx+}}{}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/12*((3*a^3 + 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 - 4*(3*a^3 + 9*a*b^2 + 4*
b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3
 + (3*a^3 + 9*a*b^2 + 4*b^3)*sinh(d*x + c)^4 + 9*a^3 - 9*a*b^2 + 4*(3*a^3
- b^3)*cosh(d*x + c)^2 + 2*(6*a^3 - 2*b^3 + 3*(3*a^3 + 9*a*b^2 + 4*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((3*a^3 + 9*a*b^2 + 4*b^3 + 3*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 + (3*a^3 + 9*a*b^2 + 4*b^3 + 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*
x + c))
```

Sympy [F]

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \int \left(a+b\tanh^2\left(c+dx\right)\right)^3\coth^2\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**2*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.

Time = 0.05 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.49

$$\begin{aligned}&\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=\frac{1}{3}b^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1)}\right)\\&\quad+3ab^2\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+a^3\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+3a^2bx\end{aligned}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*b^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*
(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 3*a*b
^2*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^3*(x + c/d + 2/(d*(e^(-2*d
*x - 2*c) - 1))) + 3*a^2*b*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 0.24 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.29

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{3\,(a^3+3\,a^2b+3\,ab^2+b^3)(dx+c) - \frac{6\,a^3}{e^{(2\,dx+2\,c)}-1} + \frac{2\,(9\,ab^2e^{(4\,dx+4\,c)}+6\,b^3e^{(4\,dx+4\,c)}+18\,ab^2e^{(2\,dx+2\,c)}+6\,b^3e^{(2\,dx+2\,c)}+9\,ab}{\left(e^{(2\,dx+2\,c)}+1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 6*a^3/(e^(2*d*x + 2*c)
- 1) + 2*(9*a*b^2*e^(4*d*x + 4*c) + 6*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*
d*x + 2*c) + 6*b^3*e^(2*d*x + 2*c) + 9*a*b^2 + 4*b^3)/(e^(2*d*x + 2*c) + 1
)^3)/d
```

Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 218, normalized size of antiderivative = 3.69

$$\int \coth^2(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\,(a+b)^3 + \frac{\frac{2\,a\,b^2}{d} + \frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d}}{2\,\mathrm{e}^{2\,c+2\,d\,x}+\mathrm{e}^{4\,c+4\,d\,x}+1}$$
$$+ \frac{\frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d} + \frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d} + \frac{4\,a\,b^2\,\mathrm{e}^{2\,c+2\,d\,x}}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}+3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}+1}$$
$$- \frac{2\,a^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)} + \frac{2\,\left(2\,b^3+3\,a\,b^2\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}$$

input
```
int(coth(c + d*x)^2*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(a + b)^3 + ((2*a*b^2)/d + (2*exp(2*c + 2*d*x)*(3*a*b^2 + 2*b^3))/(3*d))
/(2*exp(2*c + 2*d*x) + exp(4*c + 4*d*x) + 1) + ((2*(3*a*b^2 + 2*b^3))/(3*d
) + (2*exp(4*c + 4*d*x)*(3*a*b^2 + 2*b^3))/(3*d) + (4*a*b^2*exp(2*c + 2*d*
x))/d)/(3*exp(2*c + 2*d*x) + 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) + 1) -
(2*a^3)/(d*(exp(2*c + 2*d*x) - 1)) + (2*(3*a*b^2 + 2*b^3))/(3*d*(exp(2*c +
 2*d*x) + 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \coth^2(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{18e^{4dx+4c}a\,b^2 - 2b^3 - 9e^{8dx+8c}a\,b^2 - 9a\,b^2 - 9a^3 + 8e^{2dx+2c}b^3 - 18e^{2dx+2c}a^2bdx - 18e^{2dx+2c}a\,b^2dx + 3e^{8dx}}{}$$

input
```
int(coth(d*x+c)^2*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(3*e**(8*c + 8*d*x)*a**3*d*x + 3*e**(8*c + 8*d*x)*a**3 + 9*e**(8*c + 8*d*x
)*a**2*b*d*x + 9*e**(8*c + 8*d*x)*a*b**2*d*x - 9*e**(8*c + 8*d*x)*a*b**2 +
 3*e**(8*c + 8*d*x)*b**3*d*x - 6*e**(8*c + 8*d*x)*b**3 + 6*e**(6*c + 6*d*x
)*a**3*d*x + 18*e**(6*c + 6*d*x)*a**2*b*d*x + 18*e**(6*c + 6*d*x)*a*b**2*d
*x + 6*e**(6*c + 6*d*x)*b**3*d*x - 18*e**(4*c + 4*d*x)*a**3 + 18*e**(4*c +
 4*d*x)*a*b**2 - 6*e**(2*c + 2*d*x)*a**3*d*x - 24*e**(2*c + 2*d*x)*a**3 -
18*e**(2*c + 2*d*x)*a**2*b*d*x - 18*e**(2*c + 2*d*x)*a*b**2*d*x - 6*e**(2*
c + 2*d*x)*b**3*d*x + 8*e**(2*c + 2*d*x)*b**3 - 3*a**3*d*x - 9*a**3 - 9*a*
*2*b*d*x - 9*a*b**2*d*x - 9*a*b**2 - 3*b**3*d*x - 2*b**3)/(3*d*(e**(8*c +
8*d*x) + 2*e**(6*c + 6*d*x) - 2*e**(2*c + 2*d*x) - 1))
```

3.163 $\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^2(c+dx)}{2d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a^2(a+3b)\log(\tanh(c+dx))}{d} - \frac{b^3\tanh^2(c+dx)}{2d}$$

output
```
-1/2*a^3*coth(d*x+c)^2/d+(a+b)^3*ln(cosh(d*x+c))/d+a^2*(a+3*b)*ln(tanh(d*x
+c))/d-1/2*b^3*tanh(d*x+c)^2/d
```

Mathematica [A] (verified)

Time = 0.31 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.88

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a^3\coth^2(c+dx)-2(a+b)^3\log(\cosh(c+dx))-2a^2(a+3b)\log(\tanh(c+dx))+b^3\tanh^2(c+dx)}{2d}$$

input
```
Integrate[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(a^3*Coth[c + d*x]^2 - 2*(a + b)^3*Log[Cosh[c + d*x]] - 2*a^2*(a + 3*
b)*Log[Tanh[c + d*x]] + b^3*Tanh[c + d*x]^2)/d
```

Rubi [A] (warning: unable to verify)

Time = 0.41 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\downarrow 99$$

$$\frac{\int \left(\coth^2(c+dx)a^3+(a+3b)\coth(c+dx)a^2-b^3-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{a^3(-\coth(c+dx))+a^2(a+3b)\log\left(\tanh^2(c+dx)\right)-(a+b)^3\log\left(1-\tanh^2(c+dx)\right)-b^3\tanh^2(c+dx)}{2d}$$

input
```
Int[Coth[c + d*x]^3*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^3*Coth[c + d*x]) + a^2*(a + 3*b)*Log[Tanh[c + d*x]^2] - (a + b)^3*Log
[1 - Tanh[c + d*x]^2] - b^3*Tanh[c + d*x]^2)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07

method	result
parallelrisch	$\frac{-2(a+b)^3\ln(1-\tanh(dx+c))+2a^2(a+3b)\ln(\tanh(dx+c))-\coth(dx+c)^2a^3-b^3\tanh(dx+c)^2-2dx(a+b)^3}{2d}$
derivativedivides	$-\frac{\frac{b^3\tanh(dx+c)^2}{2}+\frac{a^3}{2\tanh(dx+c)^2}-a^2(a+3b)\ln(\tanh(dx+c))+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(1+\tanh(dx+c))+(\frac{1}{2}a^3+\frac{3}{2}}{d}$
default	$-\frac{\frac{b^3\tanh(dx+c)^2}{2}+\frac{a^3}{2\tanh(dx+c)^2}-a^2(a+3b)\ln(\tanh(dx+c))+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(1+\tanh(dx+c))+(\frac{1}{2}a^3+\frac{3}{2}}{d}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a^3c}{d}-\frac{6a^2bc}{d}-\frac{2\,e^{2dx+2c}(a^3e^{4dx+4c}-b^3e^{4dx+4c}}{d(e^{2dx+2c}-1}$

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/2*(-2*(a+b)^3*ln(1-tanh(d*x+c))+2*a^2*(a+3*b)*ln(tanh(d*x+c))-coth(d*x+c
)^2*a^3-b^3*tanh(d*x+c)^2-2*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1686 vs. $2(68) = 136$.

Time = 0.14 (sec) , antiderivative size = 1686, normalized size of antiderivative = 23.42

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x*sinh(d*x + c)^8 + 2*(a^3 - b^3)*cosh(d*x + c)^6 + 2*(14*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^2 + a^3 - b^3)*sinh(d*x + c)
^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + 3*(a^3 -
b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*x*cosh(d*x + c)^4 + 2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*
x + 15*(a^3 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^5 + 5*(a^3 - b^3)*cosh(d*x + c)^3 + (2*
a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x
 + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(a^3 - b^3)*cosh(d*x + c
)^2 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(a^3
- b^3)*cosh(d*x + c)^4 + a^3 - b^3 + 6*(2*a^3 + 2*b^3 - (a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a*b^2 + b^3)*cos
h(d*x + c)^8 + 56*(3*a*b^2 + b^3)*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*(3*
a*b^2 + b^3)*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*(3*a*b^2 + b^3)*cosh(d*x
+ c)*sinh(d*x + c)^7 + (3*a*b^2 + b^3)*sinh(d*x + c)^8 - 2*(3*a*b^2 + b^3)
*cosh(d*x + c)^4 + 2*(35*(3*a*b^2 + b^3)*cosh(d*x + c)^4 - 3*a*b^2 - b^3)*
sinh(d*x + c)^4 + 8*(7*(3*a*b^2 + b^3)*cosh(d*x + c)^5 - (3*a*b^2 + b^3...
```

Sympy [F]

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int\left(a+b\tanh^2\left(c+dx\right)\right)^3\coth^3\left(c+dx\right)dx$$

input
```
integrate(coth(d*x+c)**3*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral((a + b*tanh(c + d*x)**2)**3*coth(c + d*x)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 203 vs. $2(68) = 136$.

Time = 0.14 (sec) , antiderivative size = 203, normalized size of antiderivative = 2.82

$$\begin{aligned}&\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\\&\quad+b^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1)}\right)\\&\quad+\frac{3\,ab^2\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,a^2b\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2
*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*(x + c/
d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c
) + e^(-4*d*x - 4*c) + 1))) + 3*a*b^2*log(e^(d*x + c) + e^(-d*x - c))/d +
3*a^2*b*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(68) = 136$.

Time = 0.27 (sec) , antiderivative size = 274, normalized size of antiderivative = 3.81

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{2\left(3ab^2+b^3\right)\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}+2\right)+2\left(a^3+3a^2b\right)\log\left(e^{(2dx+2c)}+e^{(-2dx-2c)}-2\right)-\frac{a^3\left(e^{(2dx+}\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*(3*a*b^2 + b^3)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^
3 + 3*a^2*b)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (a^3*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c))^2 + 3*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)
)^2 + 3*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + b^3*(e^(2*d*x + 2*c
) + e^(-2*d*x - 2*c))^2 + 8*a^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 8*b
^3*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 12*a^3 - 12*a^2*b - 12*a*b^2 + 1
2*b^3)/((e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 - 4))/d
```

Mupad [B] (verification not implemented)

Time = 3.92 (sec) , antiderivative size = 327, normalized size of antiderivative = 4.54

$$\int \coth^3(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{\ln\left(e^{4c+4dx}-1\right)\left(d\,a^3+3\,d\,a^2\,b+3\,d\,a\,b^2+d\,b^3\right)}{2\,d^2}$$
$$-\frac{\frac{4\left(a^3+b^3\right)}{d}+\frac{2\,e^{2c+2dx}\left(a^3-b^3\right)}{d}}{e^{4c+4dx}-1}-\frac{\frac{4\left(a^3+b^3\right)}{d}+\frac{4\,e^{2c+2dx}\left(a^3-b^3\right)}{d}}{e^{8c+8dx}-2\,e^{4c+4dx}+1}$$
$$-\frac{\mathrm{atan}\left(\frac{e^{2c}\,e^{2dx}\left(a^3\sqrt{-d^2}-b^3\sqrt{-d^2}-3\,a\,b^2\sqrt{-d^2}+3\,a^2\,b\sqrt{-d^2}\right)}{d\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5+b^6}}\right)\sqrt{a^6+6\,a^5\,b+3\,a^4\,b^2-20\,a^3\,b^3+3\,a^2\,b^4+6\,a\,b^5}}{\sqrt{-d^2}}$$
$$-x\left(a+b\right)^3$$

input
```
int(coth(c + d*x)^3*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(a^3*d + b^3*d + 3*a*b^2*d + 3*a^2*b*d))/(2*d^2
) - ((4*(a^3 + b^3))/d + (2*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(4*c + 4*
d*x) - 1) - ((4*(a^3 + b^3))/d + (4*exp(2*c + 2*d*x)*(a^3 - b^3))/d)/(exp(
8*c + 8*d*x) - 2*exp(4*c + 4*d*x) + 1) - (atan((exp(2*c)*exp(2*d*x)*(a^3*(
-d^2)^(1/2) - b^3*(-d^2)^(1/2) - 3*a*b^2*(-d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/
2)))/(d*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4*b^
2)^(1/2)))*(6*a*b^5 + 6*a^5*b + a^6 + b^6 + 3*a^2*b^4 - 20*a^3*b^3 + 3*a^4
*b^2)^(1/2))/(-d^2)^(1/2) - x*(a + b)^3
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 712, normalized size of antiderivative = 9.89

$$\int \coth^3(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$$
$$= \frac{3\log\left(e^{dx+c}-1\right)a^2b+3\log\left(e^{dx+c}+1\right)a^2b-2b^3+e^{8dx+8c}\log\left(e^{2dx+2c}+1\right)b^3+\log\left(e^{2dx+2c}+1\right)b^3+\log}{}$$

input
```
int(coth(d*x+c)^3*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(3*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 + e**(8*c + 8*d*x)*lo
g(e**(2*c + 2*d*x) + 1)*b**3 + e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**3
 + 3*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**2*b + e**(8*c + 8*d*x)*log(
e**(c + d*x) + 1)*a**3 + 3*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**2*b -
 e**(8*c + 8*d*x)*a**3*d*x - 2*e**(8*c + 8*d*x)*a**3 - 3*e**(8*c + 8*d*x)*
a**2*b*d*x - 3*e**(8*c + 8*d*x)*a*b**2*d*x - e**(8*c + 8*d*x)*b**3*d*x - 2
*e**(8*c + 8*d*x)*b**3 - 2*e**(6*c + 6*d*x)*a**3 + 2*e**(6*c + 6*d*x)*b**3
 - 6*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b**2 - 2*e**(4*c + 4*d*x
)*log(e**(2*c + 2*d*x) + 1)*b**3 - 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1
)*a**3 - 6*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2*b - 2*e**(4*c + 4*d
*x)*log(e**(c + d*x) + 1)*a**3 - 6*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*
a**2*b + 2*e**(4*c + 4*d*x)*a**3*d*x + 6*e**(4*c + 4*d*x)*a**2*b*d*x + 6*e
**(4*c + 4*d*x)*a*b**2*d*x + 2*e**(4*c + 4*d*x)*b**3*d*x - 2*e**(2*c + 2*d
*x)*a**3 + 2*e**(2*c + 2*d*x)*b**3 + 3*log(e**(2*c + 2*d*x) + 1)*a*b**2 +
log(e**(2*c + 2*d*x) + 1)*b**3 + log(e**(c + d*x) - 1)*a**3 + 3*log(e**(c
+ d*x) - 1)*a**2*b + log(e**(c + d*x) + 1)*a**3 + 3*log(e**(c + d*x) + 1)*
a**2*b - a**3*d*x - 2*a**3 - 3*a**2*b*d*x - 3*a*b**2*d*x - b**3*d*x - 2*b*
*3)/(d*(e**(8*c + 8*d*x) - 2*e**(4*c + 4*d*x) + 1))
```

3.164 $\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = (a+b)^3x - \frac{a^2(a+3b)\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{b^3\tanh(c+dx)}{d}$$

output
```
(a+b)^3*x-a^2*(a+3*b)*coth(d*x+c)/d-1/3*a^3*coth(d*x+c)^3/d-b^3*tanh(d*x+c
)/d
```

Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.39

$$\int \coth^4(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = \frac{\left(-3b^3 - 3a^2(a+3b)\coth^2(c+dx) - a^3\coth^4(c+dx) + 3(a+b)^3\text{arctanh}\left(\sqrt{\coth^2(c+dx)}\right)\sqrt{\coth^2(\right.}{3d}$$

input
```
Integrate[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((-3*b^3 - 3*a^2*(a + 3*b)*Coth[c + d*x]^2 - a^3*Coth[c + d*x]^4 + 3*(a +
b)^3*ArcTanh[Sqrt[Coth[c + d*x]^2]]*Sqrt[Coth[c + d*x]^2])*Tanh[c + d*x])/
(3*d)
```

Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.02, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3042, 4153, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^4}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 364$$

$$\frac{\int \left(a^3\coth^4(c+dx)+a^2(a+3b)\coth^2(c+dx)-b^3-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)-a^2(a+3b)\coth(c+dx)+(a+b)^3\text{arctanh}(\tanh(c+dx))+b^3(-\tanh(c+dx))}{d}$$

input
```
Int[Coth[c + d*x]^4*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a + b)^3*ArcTanh[Tanh[c + d*x]] - a^2*(a + 3*b)*Coth[c + d*x] - (a^3*Cot
h[c + d*x]^3)/3 - b^3*Tanh[c + d*x])/d
```

Defintions of rubi rules used

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.17 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

method	result
parallelrisch	$\frac{-\coth(dx+c)^3 a^3-3a^2(a+3b)\coth(dx+c)-3b^3\tanh(dx+c)+3dx(a+b)^3}{3d}$
derivativedivides	$-\frac{\frac{\coth(dx+c)^3a^3}{3}+\coth(dx+c)a^3+3a^2b\coth(dx+c)+\frac{b^3}{\coth(dx+c)}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3\right)\ln(\coth(dx+c)+1)+\left(\frac{1}{2}\right.}{d}$
default	$-\frac{\frac{\coth(dx+c)^3a^3}{3}+\coth(dx+c)a^3+3a^2b\coth(dx+c)+\frac{b^3}{\coth(dx+c)}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3\right)\ln(\coth(dx+c)+1)+\left(\frac{1}{2}\right.}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x-\frac{2\left(6a^3e^{6dx+6c}+9a^2b\,e^{6dx+6c}-3b^3e^{6dx+6c}-9a^2b\,e^{4dx+4c}+9b^3e^{4dx+4c}-2a^3\right.}{3d\left(e^{2dx+2c}-1\right)^3\left(e^{2dx+2c}\right.}$

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/3*(-coth(d*x+c)^3*a^3-3*a^2*(a+3*b)*coth(d*x+c)-3*b^3*tanh(d*x+c)+3*d*x*
(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 341 vs. $2(57) = 114$.

Time = 0.10 (sec) , antiderivative size = 341, normalized size of antiderivative = 5.78

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{(4a^3+9a^2b+3b^3)\cosh(dx+c)^4-4(4a^3+9a^2b+3b^3+3(a^3+3a^2b+3ab^2+b^3)dx)\cosh(dx+c)}{}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/12*((4*a^3 + 9*a^2*b + 3*b^3)*cosh(d*x + c)^4 - 4*(4*a^3 + 9*a^2*b + 3*
b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^3
 + (4*a^3 + 9*a^2*b + 3*b^3)*sinh(d*x + c)^4 - 9*a^2*b + 9*b^3 + 4*(a^3 -
3*b^3)*cosh(d*x + c)^2 + 2*(2*a^3 - 6*b^3 + 3*(4*a^3 + 9*a^2*b + 3*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^2 - 4*((4*a^3 + 9*a^2*b + 3*b^3 + 3*(a^3 + 3*
a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b + 3*b^3 + 3
*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)*sinh(d*x + c)^3 + (d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**4*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(57) = 114$.

Time = 0.04 (sec) , antiderivative size = 147, normalized size of antiderivative = 2.49

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{3}a^3\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$
$$+b^3\left(x+\frac{c}{d}-\frac{2}{d(e^{(-2dx-2c)}+1)}\right)+3a^2b\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+3ab^2x$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*a^3*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - 2)/(d*
(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))) + b^3*(
x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 3*a^2*b*(x + c/d + 2/(d*(e^(-2*d
*x - 2*c) - 1))) + 3*a*b^2*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 135 vs. $2(57) = 114$.

Time = 0.30 (sec) , antiderivative size = 135, normalized size of antiderivative = 2.29

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)+\frac{6b^3}{e^{(2dx+2c)}+1}-\frac{2\left(6a^3e^{(4dx+4c)}+9a^2be^{(4dx+4c)}-6a^3e^{(2dx+2c)}-18a^2be^{(2dx+2c)}+4a\right.}{\left(e^{(2dx+2c)}-1\right)^3}}{3d}$$

input
```
integrate(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) + 6*b^3/(e^(2*d*x + 2*c)
+ 1) - 2*(6*a^3*e^(4*d*x + 4*c) + 9*a^2*b*e^(4*d*x + 4*c) - 6*a^3*e^(2*d*x
 + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) + 4*a^3 + 9*a^2*b)/(e^(2*d*x + 2*c) - 1
)^3)/d
```

Mupad [B] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 219, normalized size of antiderivative = 3.71

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= x\,(a+b)^3+\frac{\frac{2\,a^2\,b}{d}-\frac{2\,\mathrm{e}^{2\,c+2\,d\,x}\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d}}{\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1}$$
$$-\frac{\frac{2\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d}+\frac{2\,\mathrm{e}^{4\,c+4\,d\,x}\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d}-\frac{4\,a^2\,b\,\mathrm{e}^{2\,c+2\,d\,x}}{d}}{3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1}$$
$$+\frac{2\,b^3}{d\,\left(\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,\left(2\,a^3+3\,b\,a^2\right)}{3\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^4*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(a + b)^3 + ((2*a^2*b)/d - (2*exp(2*c + 2*d*x)*(3*a^2*b + 2*a^3))/(3*d))
/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1) - ((2*(3*a^2*b + 2*a^3))/(3*d
) + (2*exp(4*c + 4*d*x)*(3*a^2*b + 2*a^3))/(3*d) - (4*a^2*b*exp(2*c + 2*d*
x))/d)/(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) +
(2*b^3)/(d*(exp(2*c + 2*d*x) + 1)) - (2*(3*a^2*b + 2*a^3))/(3*d*(exp(2*c +
 2*d*x) - 1))
```

Reduce [B] (verification not implemented)

Time = 23.50 (sec) , antiderivative size = 407, normalized size of antiderivative = 6.90

$$\int \coth^4(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{-9b^3-9e^{8dx+8c}a^2b+18e^{4dx+4c}a^2b-18e^{4dx+4c}b^3-2a^3+24e^{2dx+2c}b^3+18e^{2dx+2c}a^2bdx+18e^{2dx+2c}a\,b^2d}{}$$

input
```
int(coth(d*x+c)^4*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(3*e**(8*c + 8*d*x)*a**3*d*x - 6*e**(8*c + 8*d*x)*a**3 + 9*e**(8*c + 8*d*x
)*a**2*b*d*x - 9*e**(8*c + 8*d*x)*a**2*b + 9*e**(8*c + 8*d*x)*a*b**2*d*x +
 3*e**(8*c + 8*d*x)*b**3*d*x + 3*e**(8*c + 8*d*x)*b**3 - 6*e**(6*c + 6*d*x
)*a**3*d*x - 18*e**(6*c + 6*d*x)*a**2*b*d*x - 18*e**(6*c + 6*d*x)*a*b**2*d
*x - 6*e**(6*c + 6*d*x)*b**3*d*x + 18*e**(4*c + 4*d*x)*a**2*b - 18*e**(4*c
 + 4*d*x)*b**3 + 6*e**(2*c + 2*d*x)*a**3*d*x - 8*e**(2*c + 2*d*x)*a**3 + 1
8*e**(2*c + 2*d*x)*a**2*b*d*x + 18*e**(2*c + 2*d*x)*a*b**2*d*x + 6*e**(2*c
 + 2*d*x)*b**3*d*x + 24*e**(2*c + 2*d*x)*b**3 - 3*a**3*d*x - 2*a**3 - 9*a*
*2*b*d*x - 9*a**2*b - 9*a*b**2*d*x - 3*b**3*d*x - 9*b**3)/(3*d*(e**(8*c +
8*d*x) - 2*e**(6*c + 6*d*x) + 2*e**(2*c + 2*d*x) - 1))
```

3.165 $\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = -\frac{a^2(a+3b)\coth^2(c+dx)}{2d} - \frac{a^3\coth^4(c+dx)}{4d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{a(a^2+3ab+3b^2)\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a^2*(a+3*b)*coth(d*x+c)^2/d-1/4*a^3*coth(d*x+c)^4/d+(a+b)^3*ln(cosh(d
*x+c))/d+a*(a^2+3*a*b+3*b^2)*ln(tanh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.39 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.81

$$\int \coth^5(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \frac{-a^2(a+3b)\coth^2(c+dx) - \frac{1}{2}a^3\coth^4(c+dx) + 2(a+b)^3\log(\sinh(c+dx)) - 2b^3\log(\tanh(c+dx))}{2d}$$

input
```
Integrate[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^2*(a + 3*b)*Coth[c + d*x]^2) - (a^3*Coth[c + d*x]^4)/2 + 2*(a + b)^3*
Log[Sinh[c + d*x]] - 2*b^3*Log[Tanh[c + d*x]])/(2*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.42 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^5}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth^5(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth^3(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(-\frac{(a+b)^3}{\tanh^2(c+dx)-1}+a^3\coth^3(c+dx)+a^2(a+3b)\coth^2(c+dx)+a\left(a^2+3ba+3b^2\right)\coth(c+dx)\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{2}a^3\coth^2(c+dx)+a\left(a^2+3ab+3b^2\right)\log\left(\tanh^2(c+dx)\right)-a^2(a+3b)\coth(c+dx)-(a+b)^3\log\left(1-\tanh^2(c+dx)\right)}{2d}$$

input
```
Int[Coth[c + d*x]^5*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a^2*(a + 3*b)*Coth[c + d*x]) - (a^3*Coth[c + d*x]^2)/2 + a*(a^2 + 3*a*b
 + 3*b^2)*Log[Tanh[c + d*x]^2] - (a + b)^3*Log[1 - Tanh[c + d*x]^2])/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.20 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.08

method	result
parallelrisch	$\frac{-4(a+b)^3\ln(1-\tanh(dx+c))+4(a^3+3a^2b+3ab^2)\ln(\tanh(dx+c))-\coth(dx+c)^4a^3-2a^2\coth(dx+c)^2(a+3b)-4dx(a}{4d}$
derivativedivides	$-\frac{\frac{a^3}{4\tanh(dx+c)^4}-a(a^2+3ab+3b^2)\ln(\tanh(dx+c))+\frac{a^2(a+3b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(1+\tanh(dx+c))+(\frac{1}{2}}{d}$
default	$-\frac{\frac{a^3}{4\tanh(dx+c)^4}-a(a^2+3ab+3b^2)\ln(\tanh(dx+c))+\frac{a^2(a+3b)}{2\tanh(dx+c)^2}+(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}ab^2+\frac{1}{2}b^3)\ln(1+\tanh(dx+c))+(\frac{1}{2}}{d}$
risch	$-a^3x-3a^2bx-3ab^2x-b^3x-\frac{2a^3c}{d}-\frac{6a^2bc}{d}-\frac{6ab^2c}{d}-\frac{2b^3c}{d}-\frac{2a^2e^{2dx+2c}(2a\,e^{4dx+4c}+3b\,e^{4dx+}}{d(e^{2d}}$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/4*(-4*(a+b)^3*ln(1-tanh(d*x+c))+4*(a^3+3*a^2*b+3*a*b^2)*ln(tanh(d*x+c))-
coth(d*x+c)^4*a^3-2*a^2*coth(d*x+c)^2*(a+3*b)-4*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2393 vs. $2(79) = 158$.

Time = 0.11 (sec) , antiderivative size = 2393, normalized size of antiderivative = 28.83

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^8 + 8*(a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*d*x*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*x*sinh(d*x + c)^8 + 2*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*
a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 + 2*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x*cosh(d*x + c)^2 + 2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
d*x)*sinh(d*x + c)^6 + 4*(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^3 + 3*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^5 - 2*(2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*(35*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*
cosh(d*x + c)^4 - 2*a^3 - 6*a^2*b + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x
+ 15*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 8*(7*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x
+ c)^5 + 5*(2*a^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(
d*x + c)^3 - (2*a^3 + 6*a^2*b - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x + 2*(2*a
^3 + 3*a^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2 + 2*
(14*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x*cosh(d*x + c)^6 + 15*(2*a^3 + 3*a^
2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 2*a^3 + 3*a
^2*b - 2*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 6*(2*a^3 + 6*a^2*b - 3*(a^3
 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - (b^...
```

Sympy [F(-1)]

Timed out.

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\text{Timed out}$$

input
```
integrate(coth(d*x+c)**5*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 264 vs. $2(79) = 158$.

Time = 0.06 (sec) , antiderivative size = 264, normalized size of antiderivative = 3.18

$$\begin{aligned}&\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx\\&=a^3\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}{d(4\,e^{(-2\,dx-2\,c)}-6\,e^{(-4\,dx-4\,c)}+4\,e^{(-6\,dx-6\,c)}-e^{(-8\,dx-8\,c)}-1)}\right)\\&\quad+3\,a^2b\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\,e^{(-2\,dx-2\,c)}}{d(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1)}\right)\\&\quad+\frac{b^3\log\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{d}+\frac{3\,ab^2\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
a^3*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c)
 - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 3*a
^2*b*(x + c/d + log(e^(-d*x - c) + 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-
2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) + b^3*log(e^
(d*x + c) + e^(-d*x - c))/d + 3*a*b^2*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 267 vs. $2(79) = 158$.

Time = 0.34 (sec) , antiderivative size = 267, normalized size of antiderivative = 3.22

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{2\,b^3\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2\right)+2\left(a^3+3\,a^2b+3\,ab^2\right)\log\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2\right)-\frac{3\,a^3\left(e^{(2\,dx+2}\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*b^3*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) + 2) + 2*(a^3 + 3*a^2*b
+ 3*a*b^2)*log(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2) - (3*a^3*(e^(2*d*x
+ 2*c) + e^(-2*d*x - 2*c))^2 + 9*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)
)^2 + 9*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 4*a^3*(e^(2*d*x + 2
*c) + e^(-2*d*x - 2*c)) - 12*a^2*b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) -
36*a*b^2*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*a^3 - 12*a^2*b + 36*a*b^
2)/(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c) - 2)^2)/d
```

Mupad [B] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 381, normalized size of antiderivative = 4.59

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$= \frac{\ln\left(e^{4\,c+4\,d\,x}-1\right)\,\left(b^3\,d+d\,\left(a^3+3\,a^2\,b+3\,a\,b^2\right)\right)}{2\,d^2}-x\,{\left(a+b\right)}^3$$
$$-\frac{2\,\left(2\,a^3+3\,b\,a^2\right)}{d\,\left(e^{2\,c+2\,d\,x}-1\right)}-\frac{8\,a^3}{d\,\left(3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{\mathrm{atan}\left(\frac{e^{2\,c}\,e^{2\,d\,x}\,\left(a^3\,\sqrt{-d^2}-b^3\,\sqrt{-d^2}+3\,a\,b^2\,\sqrt{-d^2}+3\,a^2\,b\,\sqrt{-d^2}\right)}{d\,\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+16\,a^3\,b^3+3\,a^2\,b^4-6\,a\,b^5+b^6}}\right)\,\sqrt{a^6+6\,a^5\,b+15\,a^4\,b^2+16\,a^3\,b^3+3\,a^2\,b^4-6\,a\,b}{\sqrt{-d^2}}$$
$$-\frac{2\,\left(4\,a^3+3\,b\,a^2\right)}{d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)}-\frac{4\,a^3}{d\,\left(6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1\right)}$$

input
```
int(coth(c + d*x)^5*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(log(exp(4*c + 4*d*x) - 1)*(b^3*d + d*(3*a*b^2 + 3*a^2*b + a^3)))/(2*d^2)
- x*(a + b)^3 - (2*(3*a^2*b + 2*a^3))/(d*(exp(2*c + 2*d*x) - 1)) - (8*a^3)
/(d*(3*exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (a
tan((exp(2*c)*exp(2*d*x)*(a^3*(-d^2)^(1/2) - b^3*(-d^2)^(1/2) + 3*a*b^2*(-
d^2)^(1/2) + 3*a^2*b*(-d^2)^(1/2)))/(d*(6*a^5*b - 6*a*b^5 + a^6 + b^6 + 3*
a^2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2)))*(6*a^5*b - 6*a*b^5 + a^6 + b^6
+ 3*a^2*b^4 + 16*a^3*b^3 + 15*a^4*b^2)^(1/2))/(-d^2)^(1/2) - (2*(3*a^2*b +
 4*a^3))/(d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (4*a^3)/(d*(6*e
xp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d*x) + exp(8*c + 8*d*
x) + 1))
```

Reduce [F]

$$\int \coth^5(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \coth\left(dx+c\right)^5\left(\tanh\left(dx+c\right)^2b+a\right)^3dx$$

input
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
int(coth(d*x+c)^5*(a+b*tanh(d*x+c)^2)^3,x)
```

3.166 $\int \coth^6(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 74

$$\int \coth^6(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = (a+b)^3x - \frac{a(a^2+3ab+3b^2)\coth(c+dx)}{d} - \frac{a^2(a+3b)\coth^3(c+dx)}{3d} - \frac{a^3\coth^5(c+dx)}{5d}$$

output
```
(a+b)^3*x-a*(a^2+3*a*b+3*b^2)*coth(d*x+c)/d-1/3*a^2*(a+3*b)*coth(d*x+c)^3/
d-1/5*a^3*coth(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 1.20 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.35

$$\int \coth^6(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{a\coth(c+dx)\left(15(a^2+3ab+3b^2)+5a(a+3b)\coth^2(c+dx)+3a^2\coth^4(c+dx)\right)}{15d} + \frac{(a+b)^3\operatorname{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)\tanh(c+dx)}{d\sqrt{\tanh^2(c+dx)}}$$

input
```
Integrate[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/15*(a*Coth[c + d*x]*(15*(a^2 + 3*a*b + 3*b^2) + 5*a*(a + 3*b)*Coth[c +
d*x]^2 + 3*a^2*Coth[c + d*x]^4))/d + ((a + b)^3*ArcTanh[Sqrt[Tanh[c + d*x]
^2]]*Tanh[c + d*x])/(d*Sqrt[Tanh[c + d*x]^2])
```

Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.03, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3042, 25, 4153, 25, 364, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^6}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^6}dx$$

$$\Big\downarrow \text{ 4153}$$

$$-\frac{\int -\frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{\int \frac{\coth^6(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 364}$$

$$\frac{\int \left(a^3\coth^6(c+dx)+a^2(a+3b)\coth^4(c+dx)+a(a^2+3ba+3b^2)\coth^2(c+dx)-\frac{(a+b)^3}{\tanh^2(c+dx)-1}\right)d\tanh(c+d}{d}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{\frac{1}{5}a^3\coth^5(c+dx)+a(a^2+3ab+3b^2)\coth(c+dx)+\frac{1}{3}a^2(a+3b)\coth^3(c+dx)-(a+b)^3\text{arctanh}(\tanh(c+}{d}$$

input
```
Int[Coth[c + d*x]^6*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-((a + b)^3*ArcTanh[Tanh[c + d*x]]) + a*(a^2 + 3*a*b + 3*b^2)*Coth[c +
d*x] + (a^2*(a + 3*b)*Coth[c + d*x]^3)/3 + (a^3*Coth[c + d*x]^5)/5)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 364
```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_))/((c_) + (d_.)*(x_)^2),
x_Symbol] :> Int[ExpandIntegrand[(e*x)^m*((a + b*x^2)^p/(c + d*x^2)), x], x
] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && (In
tegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.97

method	result
parallelrisch	$\frac{-3\coth(dx+c)^5a^3-5a^2\coth(dx+c)^3(a+3b)+\left(-15a^3-45a^2b-45a\,b^2\right)\coth(dx+c)+15dx(a+b)^3}{15d}$
derivativedivides	$-\frac{\frac{a^3}{5\tanh(dx+c)^5}+\frac{a\left(a^2+3ab+3b^2\right)}{\tanh(dx+c)}+\frac{a^2(a+3b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\right.}{d}$
default	$-\frac{\frac{a^3}{5\tanh(dx+c)^5}+\frac{a\left(a^2+3ab+3b^2\right)}{\tanh(dx+c)}+\frac{a^2(a+3b)}{3\tanh(dx+c)^3}+\left(-\frac{1}{2}a^3-\frac{3}{2}a^2b-\frac{3}{2}a\,b^2-\frac{1}{2}b^3\right)\ln(1+\tanh(dx+c))+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\right.}{d}$
risch	$a^3x+3a^2bx+3a\,b^2x+b^3x-\frac{2a\left(45a^2\mathrm{e}^{8dx+8c}+90ab\,\mathrm{e}^{8dx+8c}+45b^2\mathrm{e}^{8dx+8c}-90a^2\mathrm{e}^{6dx+6c}-270ab\,\mathrm{e}^{6dx+6c}\right.}{}$

input
```
int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/15*(-3*coth(d*x+c)^5*a^3-5*a^2*coth(d*x+c)^3*(a+3*b)+(-15*a^3-45*a^2*b-4
5*a*b^2)*coth(d*x+c)+15*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. 2(70) = 140.

Time = 0.09 (sec) , antiderivative size = 557, normalized size of antiderivative = 7.53

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{(23\,a^3+60\,a^2b+45\,ab^2)\cosh\left(dx+c\right)^5+5\,(23\,a^3+60\,a^2b+45\,ab^2)\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-(2}{}$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/15*((23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^5 + 5*(23*a^3 + 60*a^2
*b + 45*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^4 - (23*a^3 + 60*a^2*b + 45*a*b
^2 + 15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*sinh(d*x + c)^5 - 5*(5*a^3 +
24*a^2*b + 27*a*b^2)*cosh(d*x + c)^3 + 5*(23*a^3 + 60*a^2*b + 45*a*b^2 + 1
5*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 2*(23*a^3 + 60*a^2*b + 45*a*b^2 +
15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 +
 5*(2*(23*a^3 + 60*a^2*b + 45*a*b^2)*cosh(d*x + c)^3 - 3*(5*a^3 + 24*a^2*b
 + 27*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 10*(5*a^3 + 6*a^2*b + 9*a*b^
2)*cosh(d*x + c) - 5*((23*a^3 + 60*a^2*b + 45*a*b^2 + 15*(a^3 + 3*a^2*b +
3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 + 46*a^3 + 120*a^2*b + 90*a*b^2 + 30*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x - 3*(23*a^3 + 60*a^2*b + 45*a*b^2 + 15*
(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*si
nh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*
x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \coth^6(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 \, dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**6*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(70) = 140$.

Time = 0.05 (sec) , antiderivative size = 239, normalized size of antiderivative = 3.23

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{15}a^3\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}-140e^{(-4dx-4c)}+90e^{(-6dx-6c)}-45e^{(-8dx-8c)}-23\right)}{d(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1)}\right)$$
$$+a^2b\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}-2\right)}{d(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1)}\right)$$
$$+3ab^2\left(x+\frac{c}{d}+\frac{2}{d(e^{(-2dx-2c)}-1)}\right)+b^3x$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/15*a^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) - 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) - 45*e^(-8*d*x - 8*c) - 23)/(d*(5*e^(-2*d*x - 2*c) - 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) - 1))) + a^2*b*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x
 - 4*c) - 2)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
) - 1))) + 3*a*b^2*(x + c/d + 2/(d*(e^(-2*d*x - 2*c) - 1))) + b^3*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(70) = 140.

Time = 0.35 (sec) , antiderivative size = 241, normalized size of antiderivative = 3.26

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{15\,(a^3+3a^2b+3ab^2+b^3)(dx+c)-\frac{2\left(45a^3e^{(8dx+8c)}+90a^2be^{(8dx+8c)}+45ab^2e^{(8dx+8c)}-90a^3e^{(6dx+6c)}-270a^2be^{(6dx+6}\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 2*(45*a^3*e^(8*d*x +
8*c) + 90*a^2*b*e^(8*d*x + 8*c) + 45*a*b^2*e^(8*d*x + 8*c) - 90*a^3*e^(6*d
*x + 6*c) - 270*a^2*b*e^(6*d*x + 6*c) - 180*a*b^2*e^(6*d*x + 6*c) + 140*a^
3*e^(4*d*x + 4*c) + 330*a^2*b*e^(4*d*x + 4*c) + 270*a*b^2*e^(4*d*x + 4*c)
- 70*a^3*e^(2*d*x + 2*c) - 210*a^2*b*e^(2*d*x + 2*c) - 180*a*b^2*e^(2*d*x
+ 2*c) + 23*a^3 + 60*a^2*b + 45*a*b^2)/(e^(2*d*x + 2*c) - 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 568, normalized size of antiderivative = 7.68

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = x\,(a+b)^3$$
$$-\frac{\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{6\,e^{8\,c+8\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{24\,e^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{24\,e^{6\,c+6\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}+\frac{4\,e^{4\,c+4\,d\,x}\,(5\,a^3+6\,a^2\,b+9}{5\,d}}{5\,e^{2\,c+2\,d\,x}-10\,e^{4\,c+4\,d\,x}+10\,e^{6\,c+6\,d\,x}-5\,e^{8\,c+8\,d\,x}+e^{10\,c+10\,d\,x}-1}$$
$$-\frac{\frac{2\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{15\,d}+\frac{6\,e^{4\,c+4\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}-\frac{12\,e^{2\,c+2\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}}{3\,e^{2\,c+2\,d\,x}-3\,e^{4\,c+4\,d\,x}+e^{6\,c+6\,d\,x}-1}$$
$$+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,e^{6\,c+6\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}+\frac{18\,e^{4\,c+4\,d\,x}\,(a^2\,b+a\,b^2)}{5\,d}-\frac{2\,e^{2\,c+2\,d\,x}\,(5\,a^3+6\,a^2\,b+9\,a\,b^2)}{5\,d}}{6\,e^{4\,c+4\,d\,x}-4\,e^{2\,c+2\,d\,x}-4\,e^{6\,c+6\,d\,x}+e^{8\,c+8\,d\,x}+1}$$
$$+\frac{\frac{6\,(a^2\,b+a\,b^2)}{5\,d}-\frac{6\,e^{2\,c+2\,d\,x}\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d}}{e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1}-\frac{6\,(a^3+2\,a^2\,b+a\,b^2)}{5\,d\,(e^{2\,c+2\,d\,x}-1)}$$

input
```
int(coth(c + d*x)^6*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
x*(a + b)^3 - ((6*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (6*exp(8*c + 8*d*x)*(a*
b^2 + 2*a^2*b + a^3))/(5*d) - (24*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d)
- (24*exp(6*c + 6*d*x)*(a*b^2 + a^2*b))/(5*d) + (4*exp(4*c + 4*d*x)*(9*a*b
^2 + 6*a^2*b + 5*a^3))/(5*d))/(5*exp(2*c + 2*d*x) - 10*exp(4*c + 4*d*x) +
10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) + exp(10*c + 10*d*x) - 1) - ((2*(
9*a*b^2 + 6*a^2*b + 5*a^3))/(15*d) + (6*exp(4*c + 4*d*x)*(a*b^2 + 2*a^2*b
+ a^3))/(5*d) - (12*exp(2*c + 2*d*x)*(a*b^2 + a^2*b))/(5*d))/(3*exp(2*c +
2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1) + ((6*(a*b^2 + a^2*b))
/(5*d) - (6*exp(6*c + 6*d*x)*(a*b^2 + 2*a^2*b + a^3))/(5*d) + (18*exp(4*c
+ 4*d*x)*(a*b^2 + a^2*b))/(5*d) - (2*exp(2*c + 2*d*x)*(9*a*b^2 + 6*a^2*b +
 5*a^3))/(5*d))/(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c + 6*d
*x) + exp(8*c + 8*d*x) + 1) + ((6*(a*b^2 + a^2*b))/(5*d) - (6*exp(2*c + 2*
d*x)*(a*b^2 + 2*a^2*b + a^3))/(5*d))/(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x
) + 1) - (6*(a*b^2 + 2*a^2*b + a^3))/(5*d*(exp(2*c + 2*d*x) - 1))
```

Reduce [F]

$$\int \coth^6(c+dx)\left(a+b\tanh^2(c+dx)\right)^3 dx = \int \coth(dx+c)^6\left(\tanh(dx+c)^2b+a\right)^3 dx$$

input
```
int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
int(coth(d*x+c)^6*(a+b*tanh(d*x+c)^2)^3,x)
```

3.167 $\int \coth^7(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx$

Optimal result

Integrand size = 23, antiderivative size = 103

$$\int \coth^7(c+dx) \left(a+b\tanh^2(c+dx)\right)^3 dx = -\frac{a(a^2+3ab+3b^2)\coth^2(c+dx)}{2d} - \frac{a^2(a+3b)\coth^4(c+dx)}{4d} - \frac{a^3\coth^6(c+dx)}{6d} + \frac{(a+b)^3\log(\cosh(c+dx))}{d} + \frac{(a+b)^3\log(\tanh(c+dx))}{d}$$

output
```
-1/2*a*(a^2+3*a*b+3*b^2)*coth(d*x+c)^2/d-1/4*a^2*(a+3*b)*coth(d*x+c)^4/d-1
/6*a^3*coth(d*x+c)^6/d+(a+b)^3*ln(cosh(d*x+c))/d+(a+b)^3*ln(tanh(d*x+c))/d
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.74

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = -\frac{a(a+b)^2\coth^2(c+dx)+\frac{1}{2}(a+b)\left(b+a\coth^2(c+dx)\right)^2+\frac{1}{3}\left(b+a\coth^2(c+dx)\right)^3-2(a+b)^3\log(s$$

$$2d$$

input
```
Integrate[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(a*(a + b)^2*Coth[c + d*x]^2 + ((a + b)*(b + a*Coth[c + d*x]^2)^2)/2
+ (b + a*Coth[c + d*x]^2)^3/3 - 2*(a + b)^3*Log[Sinh[c + d*x]])/d
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 100, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\left(a-b\tan(ic+idx)^2\right)^3}{\tan(ic+idx)^7}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth^7(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth^4(c+dx)\left(b\tanh^2(c+dx)+a\right)^3}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(a^3\coth^4(c+dx)+a^2(a+3b)\coth^3(c+dx)+a\left(a^2+3ba+3b^2\right)\coth^2(c+dx)+(a+b)^3\coth(c+dx)-\frac{}{\tan}\right.}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}a^3\coth^3(c+dx)-a\left(a^2+3ab+3b^2\right)\coth(c+dx)-\frac{1}{2}a^2(a+3b)\coth^2(c+dx)+(a+b)^3\log\left(\tanh^2(c+dx\right.}{2d}$$

input
```
Int[Coth[c + d*x]^7*(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x]) - (a^2*(a + 3*b)*Coth[c + d*x]^2
)/2 - (a^3*Coth[c + d*x]^3)/3 + (a + b)^3*Log[Tanh[c + d*x]^2] - (a + b)^3
*Log[1 - Tanh[c + d*x]^2])/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.24 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.03

method	result
parallelrisch	$\frac{-12(a+b)^3 \ln(1-\tanh(dx+c))+12(a+b)^3 \ln(\tanh(dx+c))-2\coth(dx+c)^6 a^3-3a^2 \coth(dx+c)^4 (a+3b)+\left(-6a^3-18a^2 \cdots\right.}{12d}$
derivativedivides	$-\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(\tanh(dx+c))+\frac{a^3}{6\tanh(dx+c)^6}+\frac{a\left(a^2+3ab+3b^2\right)}{2\tanh(dx+c)^2}+\frac{a^2(a+3b)}{4\tanh(dx+c)^4}+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3 \cdots\right.}{d}$
default	$-\frac{\left(-a^3-3a^2b-3a\,b^2-b^3\right)\ln(\tanh(dx+c))+\frac{a^3}{6\tanh(dx+c)^6}+\frac{a\left(a^2+3ab+3b^2\right)}{2\tanh(dx+c)^2}+\frac{a^2(a+3b)}{4\tanh(dx+c)^4}+\left(\frac{1}{2}a^3+\frac{3}{2}a^2b+\frac{3}{2}a\,b^2+\frac{1}{2}b^3 \cdots\right.}{d}$
risch	$-a^3x-3a^2bx-3a\,b^2x-b^3x-\frac{2a^3c}{d}-\frac{6a^2bc}{d}-\frac{6a\,b^2c}{d}-\frac{2b^3c}{d}-\frac{2a\,e^{2dx+2c}\left(9a^2 e^{8dx+8c}+18ab\,e^{8c \cdots}\right.}{\cdots}$

input
```
int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/12*(-12*(a+b)^3*ln(1-tanh(d*x+c))+12*(a+b)^3*ln(tanh(d*x+c))-2*coth(d*x+
c)^6*a^3-3*a^2*coth(d*x+c)^4*(a+3*b)+(-6*a^3-18*a^2*b-18*a*b^2)*coth(d*x+c
)^2-12*d*x*(a+b)^3)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4305 vs. $2(97) = 194$.

Time = 0.17 (sec) , antiderivative size = 4305, normalized size of antiderivative = 41.80

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \text{Timed out}$$

input
```
integrate(coth(d*x+c)**7*(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(97) = 194$.

Time = 0.05 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.08

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx$$
$$=\frac{1}{3}a^3\Bigg(3x+\frac{3c}{d}+\frac{3\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{3\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(9e^{(-2dx-2c)}-18e^{(-4dx-4c)}\right.}{d(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6c}}$$
$$+3a^2b\Bigg(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{4\left(e^{(-2dx-2c)}-e^{(-4dx-4c)}+e^{(-6c}\right.}{d(4e^{(-2dx-2c)}-6e^{(-4dx-4c)}+4e^{(-6dx-6c)}}$$
$$+3ab^2\left(x+\frac{c}{d}+\frac{\log\left(e^{(-dx-c)}+1\right)}{d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d(2e^{(-2dx-2c)}-e^{(-4dx-4c)}-1)}\right)$$
$$+\frac{b^3\log\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{d}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/3*a^3*(3*x + 3*c/d + 3*log(e^(-d*x - c) + 1)/d + 3*log(e^(-d*x - c) - 1)
/d + 2*(9*e^(-2*d*x - 2*c) - 18*e^(-4*d*x - 4*c) + 34*e^(-6*d*x - 6*c) - 1
8*e^(-8*d*x - 8*c) + 9*e^(-10*d*x - 10*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(
-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x -
 10*c) - e^(-12*d*x - 12*c) - 1))) + 3*a^2*b*(x + c/d + log(e^(-d*x - c) +
 1)/d + log(e^(-d*x - c) - 1)/d + 4*(e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d
*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 3*a*b^2*(x + c/d + log(e^(-d*x - c)
+ 1)/d + log(e^(-d*x - c) - 1)/d + 2*e^(-2*d*x - 2*c)/(d*(2*e^(-2*d*x - 2*
c) - e^(-4*d*x - 4*c) - 1))) + b^3*log(e^(d*x + c) - e^(-d*x - c))/d
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(97) = 194$.

Time = 0.43 (sec) , antiderivative size = 217, normalized size of antiderivative = 2.11

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx =$$

$$-\frac{3\left(a^3+3a^2b+3ab^2+b^3\right)(dx+c)-3\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(\left|e^{(2dx+2c)}-1\right|\right)+\frac{2\left(9\left(a^3+2a^2b+ab^2\right)\right.}{}}{}$$

input
```
integrate(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/3*(3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(d*x + c) - 3*(a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*log(abs(e^(2*d*x + 2*c) - 1)) + 2*(9*(a^3 + 2*a^2*b + a*b^2)*e
^(10*d*x + 10*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^(8*d*x + 8*c) + 2*(17*a^
3 + 36*a^2*b + 27*a*b^2)*e^(6*d*x + 6*c) - 18*(a^3 + 3*a^2*b + 2*a*b^2)*e^
(4*d*x + 4*c) + 9*(a^3 + 2*a^2*b + a*b^2)*e^(2*d*x + 2*c))/(e^(2*d*x + 2*c
) - 1)^6)/d
```

Mupad [B] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 380, normalized size of antiderivative = 3.69

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx = \frac{\ln\left(e^{2c}\,e^{2dx}-1\right)\left(a^3+3a^2b+3ab^2+b^3\right)}{d}$$
$$-\frac{4\left(11a^3+3ba^2\right)}{d\left(6e^{4c+4dx}-4e^{2c+2dx}-4e^{6c+6dx}+e^{8c+8dx}+1\right)}$$
$$-\frac{32a^3}{3d\left(15e^{4c+4dx}-6e^{2c+2dx}-20e^{6c+6dx}+15e^{8c+8dx}-6e^{10c+10dx}+e^{12c+12dx}+1\right)}$$
$$-\frac{6\left(3a^3+4a^2b+ab^2\right)}{d\left(e^{4c+4dx}-2e^{2c+2dx}+1\right)}-\frac{6\left(a^3+2a^2b+ab^2\right)}{d\left(e^{2c+2dx}-1\right)}$$
$$-\frac{8\left(13a^3+9ba^2\right)}{3d\left(3e^{2c+2dx}-3e^{4c+4dx}+e^{6c+6dx}-1\right)}$$
$$-\frac{32a^3}{d\left(5e^{2c+2dx}-10e^{4c+4dx}+10e^{6c+6dx}-5e^{8c+8dx}+e^{10c+10dx}-1\right)}-x(a+b)^3$$

input
```
int(coth(c + d*x)^7*(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(log(exp(2*c)*exp(2*d*x) - 1)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/d - (4*(3*a
^2*b + 11*a^3))/(d*(6*exp(4*c + 4*d*x) - 4*exp(2*c + 2*d*x) - 4*exp(6*c +
6*d*x) + exp(8*c + 8*d*x) + 1)) - (32*a^3)/(3*d*(15*exp(4*c + 4*d*x) - 6*e
xp(2*c + 2*d*x) - 20*exp(6*c + 6*d*x) + 15*exp(8*c + 8*d*x) - 6*exp(10*c +
 10*d*x) + exp(12*c + 12*d*x) + 1)) - (6*(a*b^2 + 4*a^2*b + 3*a^3))/(d*(ex
p(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (6*(a*b^2 + 2*a^2*b + a^3))/(d
*(exp(2*c + 2*d*x) - 1)) - (8*(9*a^2*b + 13*a^3))/(3*d*(3*exp(2*c + 2*d*x)
 - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (32*a^3)/(d*(5*exp(2*c +
2*d*x) - 10*exp(4*c + 4*d*x) + 10*exp(6*c + 6*d*x) - 5*exp(8*c + 8*d*x) +
exp(10*c + 10*d*x) - 1)) - x*(a + b)^3
```

Reduce [F]

$$\int \coth^7(c+dx)\left(a+b\tanh^2(c+dx)\right)^3\,dx=\int \coth(dx+c)^7\left(\tanh(dx+c)^2b+a\right)^3dx$$

input
```
int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
int(coth(d*x+c)^7*(a+b*tanh(d*x+c)^2)^3,x)
```

3.168 $\int \left(a+b\tanh^2(c+dx)\right)^4 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 110

$$\int \left(a+b\tanh^2(c+dx)\right)^4 \, dx = (a+b)^4x - \frac{b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)}{d} - \frac{b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)}{3d} - \frac{b^3(4a+b)\tanh^5(c+dx)}{5d} - \frac{b^4\tanh^7(c+dx)}{7d}$$

output
```
(a+b)^4*x-b*(2*a+b)*(2*a^2+2*a*b+b^2)*tanh(d*x+c)/d-1/3*b^2*(6*a^2+4*a*b+b
^2)*tanh(d*x+c)^3/d-1/5*b^3*(4*a+b)*tanh(d*x+c)^5/d-1/7*b^4*tanh(d*x+c)^7/
d
```

Mathematica [A] (verified)

Time = 1.23 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.16

$$\int \left(a+b\tanh^2(c+dx)\right)^4 \, dx$$

$$= \frac{\tanh(c+dx)\left(\frac{105(a+b)^4\text{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(105\left(4a^3+6a^2b+4ab^2+b^3\right)+35b\left(6a^2+4ab+b^2\right)\text{ta}\right.\right.}{105d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^4,x]
```

output
```
(Tanh[c + d*x]*((105*(a + b)^4*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
 + d*x]^2] - b*(105*(4*a^3 + 6*a^2*b + 4*a*b^2 + b^3) + 35*b*(6*a^2 + 4*a*
b + b^2)*Tanh[c + d*x]^2 + 21*b^2*(4*a + b)*Tanh[c + d*x]^4 + 15*b^3*Tanh[
c + d*x]^6)))/(105*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 108, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tanh^2(c+dx)\right)^4 dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a-b\tan(ic+idx)^2\right)^4 dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^4}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-b^4\tanh^6(c+dx)-b^3(4a+b)\tanh^4(c+dx)-b^2\left(6a^2+4ba+b^2\right)\tanh^2(c+dx)-b(2a+b)\left(2a^2+2ba+b\right.\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{3}b^2\left(6a^2+4ab+b^2\right)\tanh^3(c+dx)-b(2a+b)\left(2a^2+2ab+b^2\right)\tanh(c+dx)+(a+b)^4\text{arctanh}(\tanh(c+dx)}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^4,x]
```

output
```
((a + b)^4*ArcTanh[Tanh[c + d*x]] - b*(2*a + b)*(2*a^2 + 2*a*b + b^2)*Tanh
[c + d*x] - (b^2*(6*a^2 + 4*a*b + b^2)*Tanh[c + d*x]^3)/3 - (b^3*(4*a + b)
*Tanh[c + d*x]^5)/5 - (b^4*Tanh[c + d*x]^7)/7)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (warning: unable to verify)

Time = 0.08 (sec) , antiderivative size = 179, normalized size of antiderivative = 1.63

method	result
parallelrisch	$-\frac{15b^4\tanh(dx+c)^7+84a\,b^3\tanh(dx+c)^5+21b^4\tanh(dx+c)^5+210a^2b^2\tanh(dx+c)^3+140a\,b^3\tanh(dx+c)^3+35b^4\tanh$
derivativedivides	$\frac{-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-4a\,b^3\tanh(dx+c)-\frac{4a\,b^3\tanh(dx+c)^5}{5}-2a^2b^2\tanh(dx+c)^3-\frac{4a\,b^3\tanh(dx+c)^3}{3}}{}$
default	$\frac{-4a^3b\tanh(dx+c)-6a^2b^2\tanh(dx+c)-4a\,b^3\tanh(dx+c)-\frac{4a\,b^3\tanh(dx+c)^5}{5}-2a^2b^2\tanh(dx+c)^3-\frac{4a\,b^3\tanh(dx+c)^3}{3}}{}$
parts	$a^4x+\frac{b^4\left(-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{4a}{}$
risch	$a^4x+4a^3bx+6a^2b^2x+4a\,b^3x+b^4x+\frac{8b\left(105a^3+210a^2b+161a\,b^2+44b^3+609b^3e^{4dx+4c}+770b^3e^{8dx+}\right.}{}$

input
```
int((a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
-1/105*(15*b^4*tanh(d*x+c)^7+84*a*b^3*tanh(d*x+c)^5+21*b^4*tanh(d*x+c)^5+2
10*a^2*b^2*tanh(d*x+c)^3+140*a*b^3*tanh(d*x+c)^3+35*b^4*tanh(d*x+c)^3-105*
a^4*d*x-420*a^3*b*d*x-630*a^2*b^2*d*x-420*a*b^3*d*x-105*b^4*d*x+420*a^3*b*
tanh(d*x+c)+630*a^2*b^2*tanh(d*x+c)+420*a*b^3*tanh(d*x+c)+105*b^4*tanh(d*x
+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1176 vs. $2(104) = 208$.

Time = 0.10 (sec) , antiderivative size = 1176, normalized size of antiderivative = 10.69

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
1/105*((420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^7 + 7*(420*a^3*b + 840*a^
2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*d*x)*cosh(d*x + c)*sinh(d*x + c)^6 - 4*(105*a^3*b + 210*a^2*b^2 + 161*
a*b^3 + 44*b^4)*sinh(d*x + c)^7 + 7*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 +
 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x +
 c)^5 - 28*(75*a^3*b + 120*a^2*b^2 + 71*a*b^3 + 14*b^4 + 3*(105*a^3*b + 21
0*a^2*b^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 35*((42
0*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)^3 + (420*a^3*b + 840*a^2*b^2 + 644*
a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cos
h(d*x + c))*sinh(d*x + c)^4 + 21*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 17
6*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x + c)
^3 - 28*(5*(105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4)*cosh(d*x + c)^4
+ 135*a^3*b + 180*a^2*b^2 + 123*a*b^3 + 42*b^4 + 10*(75*a^3*b + 120*a^2*b^
2 + 71*a*b^3 + 14*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(3*(420*a^3*b
+ 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a
*b^3 + b^4)*d*x)*cosh(d*x + c)^5 + 10*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3
 + 176*b^4 + 105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*x)*cosh(d*x
 + c)^3 + 9*(420*a^3*b + 840*a^2*b^2 + 644*a*b^3 + 176*b^4 + 105*(a^4 +...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 209 vs. $2(99) = 198$.

Time = 0.23 (sec) , antiderivative size = 209, normalized size of antiderivative = 1.90

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx$$
$$= \begin{cases} a^4x+4a^3bx-\frac{4a^3b\tanh(c+dx)}{d}+6a^2b^2x-\frac{2a^2b^2\tanh^3(c+dx)}{d}-\frac{6a^2b^2\tanh(c+dx)}{d}+4ab^3x-\frac{4ab^3\tanh^5(c+dx)}{5d}-4 \\ x\left(a+b\tanh^2(c)\right)^4 \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**2)**4,x)
```

output
```
Piecewise((a**4*x + 4*a**3*b*x - 4*a**3*b*tanh(c + d*x)/d + 6*a**2*b**2*x
- 2*a**2*b**2*tanh(c + d*x)**3/d - 6*a**2*b**2*tanh(c + d*x)/d + 4*a*b**3*
x - 4*a*b**3*tanh(c + d*x)**5/(5*d) - 4*a*b**3*tanh(c + d*x)**3/(3*d) - 4*
a*b**3*tanh(c + d*x)/d + b**4*x - b**4*tanh(c + d*x)**7/(7*d) - b**4*tanh(
c + d*x)**5/(5*d) - b**4*tanh(c + d*x)**3/(3*d) - b**4*tanh(c + d*x)/d, Ne
(d, 0)), (x*(a + b*tanh(c)**2)**4, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 410 vs. $2(104) = 208$.

Time = 0.05 (sec) , antiderivative size = 410, normalized size of antiderivative = 3.73

$$\int \left(a + b\tanh^2(c + dx)\right)^4 \, dx$$

$$= \frac{1}{105} b^4 \left(105 x + \frac{105 c}{d} - \frac{8 \left(203 e^{(-2 dx - 2 c)} + 609 e^{(-4 dx - 4 c)} + 770 e^{(-6 dx - 6 c)} + 770 e^{(-8 dx - 8 c)} + 315 e^{(-}\right.}{d(7 e^{(-2 dx - 2 c)} + 21 e^{(-4 dx - 4 c)} + 35 e^{(-6 dx - 6 c)} + 35 e^{(-8 dx - 8 c)} + 21 e^{(-10 dx - 10 c)}}\right.$$

$$+ \frac{4}{15} ab^3 \left(15 x + \frac{15 c}{d} - \frac{2 \left(70 e^{(-2 dx - 2 c)} + 140 e^{(-4 dx - 4 c)} + 90 e^{(-6 dx - 6 c)} + 45 e^{(-8 dx - 8 c)} + 23\right)}{d(5 e^{(-2 dx - 2 c)} + 10 e^{(-4 dx - 4 c)} + 10 e^{(-6 dx - 6 c)} + 5 e^{(-8 dx - 8 c)} + e^{(-10 dx - 10 c)} + 1}\right.$$

$$+ 2 a^2 b^2 \left(3 x + \frac{3 c}{d} - \frac{4 \left(3 e^{(-2 dx - 2 c)} + 3 e^{(-4 dx - 4 c)} + 2\right)}{d(3 e^{(-2 dx - 2 c)} + 3 e^{(-4 dx - 4 c)} + e^{(-6 dx - 6 c)} + 1)}\right)$$

$$+ 4 a^3 b \left(x + \frac{c}{d} - \frac{2}{d(e^{(-2 dx - 2 c)} + 1)}\right) + a^4 x$$

input
```
integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
1/105*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*e^(-4*d*x - 4*c
) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-10*d*x - 10*c) +
 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e
^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 4/15*a*b^3*(15*x + 15*c/d
- 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6*d*x - 6*c) + 45
*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*
e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2*a^2*
b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*e
^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4*a^3*b*(
x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^4*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 447 vs. $2(104) = 208$.

Time = 0.15 (sec) , antiderivative size = 447, normalized size of antiderivative = 4.06

$$\int \left(a+b\tanh^2(c+dx)\right)^4 \, dx = \frac{105\,(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)(dx+c)+\frac{8\,(105\,a^3be^{(12\,dx+12\,c)}+315\,a^2b^2e^{(12\,dx+12\,c)}+315\,ab^3e^{(12\,dx+12\,c)}+105\,b^4e^{(}}{}}{}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(d*x + c) + 8*(105*
a^3*b*e^(12*d*x + 12*c) + 315*a^2*b^2*e^(12*d*x + 12*c) + 315*a*b^3*e^(12*
d*x + 12*c) + 105*b^4*e^(12*d*x + 12*c) + 630*a^3*b*e^(10*d*x + 10*c) + 15
75*a^2*b^2*e^(10*d*x + 10*c) + 1260*a*b^3*e^(10*d*x + 10*c) + 315*b^4*e^(1
0*d*x + 10*c) + 1575*a^3*b*e^(8*d*x + 8*c) + 3360*a^2*b^2*e^(8*d*x + 8*c)
+ 2555*a*b^3*e^(8*d*x + 8*c) + 770*b^4*e^(8*d*x + 8*c) + 2100*a^3*b*e^(6*d
*x + 6*c) + 3990*a^2*b^2*e^(6*d*x + 6*c) + 3080*a*b^3*e^(6*d*x + 6*c) + 77
0*b^4*e^(6*d*x + 6*c) + 1575*a^3*b*e^(4*d*x + 4*c) + 2835*a^2*b^2*e^(4*d*x
 + 4*c) + 2121*a*b^3*e^(4*d*x + 4*c) + 609*b^4*e^(4*d*x + 4*c) + 630*a^3*b
*e^(2*d*x + 2*c) + 1155*a^2*b^2*e^(2*d*x + 2*c) + 812*a*b^3*e^(2*d*x + 2*c
) + 203*b^4*e^(2*d*x + 2*c) + 105*a^3*b + 210*a^2*b^2 + 161*a*b^3 + 44*b^4
)/(e^(2*d*x + 2*c) + 1)^7)/d
```

Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.21

$$\int \left(a+b\tanh^2(c+dx)\right)^4 \, dx = x\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right) - \frac{\tanh(c+d\,x)^3\,(6\,a^2\,b^2+4\,a\,b^3+b^4)}{3\,d} - \frac{\tanh(c+d\,x)^5\,(b^4+4\,a\,b^3)}{5\,d} - \frac{b^4\tanh(c+d\,x)^7}{7\,d} - \frac{b\tanh(c+d\,x)\,\left(4\,a^3+6\,a^2\,b+4\,a\,b^2+b^3\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^4,x)
```

output
```
x*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) - (tanh(c + d*x)^3*(4*a*b^3
+ b^4 + 6*a^2*b^2))/(3*d) - (tanh(c + d*x)^5*(4*a*b^3 + b^4))/(5*d) - (b^4
*tanh(c + d*x)^7)/(7*d) - (b*tanh(c + d*x)*(4*a*b^2 + 6*a^2*b + 4*a^3 + b^
3))/d
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 178, normalized size of antiderivative = 1.62

$$\int \left(a+b\tanh^2(c+dx)\right)^4\,dx$$
$$= \frac{-15\tanh\left(dx+c\right)^7b^4-84\tanh\left(dx+c\right)^5a\,b^3-21\tanh\left(dx+c\right)^5b^4-210\tanh\left(dx+c\right)^3a^2b^2-140\tan}{}$$

input
```
int((a+b*tanh(d*x+c)^2)^4,x)
```

output
```
( - 15*tanh(c + d*x)**7*b**4 - 84*tanh(c + d*x)**5*a*b**3 - 21*tanh(c + d*
x)**5*b**4 - 210*tanh(c + d*x)**3*a**2*b**2 - 140*tanh(c + d*x)**3*a*b**3
- 35*tanh(c + d*x)**3*b**4 - 420*tanh(c + d*x)*a**3*b - 630*tanh(c + d*x)*
a**2*b**2 - 420*tanh(c + d*x)*a*b**3 - 105*tanh(c + d*x)*b**4 + 105*a**4*d
*x + 420*a**3*b*d*x + 630*a**2*b**2*d*x + 420*a*b**3*d*x + 105*b**4*d*x)/(
105*d)
```

3.169 $\int (a+b\tanh^2(c+dx))^5\,dx$

Optimal result

Integrand size = 14, antiderivative size = 160

$$\int (a+b\tanh^2(c+dx))^5\,dx = (a+b)^5x - \frac{b(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)\tanh(c+dx)}{d} - \frac{b^2(10a^3+10a^2b+5ab^2+b^3)\tanh^3(c+dx)}{3d} - \frac{b^3(10a^2+5ab+b^2)\tanh^5(c+dx)}{5d} - \frac{b^4(5a+b)\tanh^7(c+dx)}{7d} - \frac{b^5\tanh^9(c+dx)}{9d}$$

output
```
(a+b)^5*x-b*(5*a^4+10*a^3*b+10*a^2*b^2+5*a*b^3+b^4)*tanh(d*x+c)/d-1/3*b^2*
(10*a^3+10*a^2*b+5*a*b^2+b^3)*tanh(d*x+c)^3/d-1/5*b^3*(10*a^2+5*a*b+b^2)*t
anh(d*x+c)^5/d-1/7*b^4*(5*a+b)*tanh(d*x+c)^7/d-1/9*b^5*tanh(d*x+c)^9/d
```

Mathematica [A] (verified)

Time = 1.58 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.06

$$\int \left(a+b\tanh^2(c+dx)\right)^5 \, dx$$

$$= \frac{\tanh(c+dx)\left(\frac{315(a+b)^5\text{arctanh}\left(\sqrt{\tanh^2(c+dx)}\right)}{\sqrt{\tanh^2(c+dx)}} - b\left(315(5a^4+10a^3b+10a^2b^2+5ab^3+b^4)+105b(10a^3+\right.\right.}{}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^5,x]
```

output
```
(Tanh[c + d*x]*((315*(a + b)^5*ArcTanh[Sqrt[Tanh[c + d*x]^2]])/Sqrt[Tanh[c
 + d*x]^2] - b*(315*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a*b^3 + b^4) + 105*
b*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[c + d*x]^2 + 63*b^2*(10*a^2 + 5
*a*b + b^2)*Tanh[c + d*x]^4 + 45*b^3*(5*a + b)*Tanh[c + d*x]^6 + 35*b^4*Ta
nh[c + d*x]^8)))/(315*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 155, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 300, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tanh^2(c+dx)\right)^5 \, dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a-b\tan(ic+idx)^2\right)^5 dx$$

$$\Big\downarrow \text{ 4144}$$

$$\frac{\int \frac{\left(b\tanh^2(c+dx)+a\right)^5}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{300}$$

$$\frac{\int \left(-b^5 \tanh^8(c+dx) - b^4(5a+b)\tanh^6(c+dx) - b^3\left(10a^2+5ba+b^2\right)\tanh^4(c+dx) - b^2\left(10a^3+10ba^2+5b^2\right.}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{5}b^3\left(10a^2+5ab+b^2\right)\tanh^5(c+dx) - \frac{1}{3}b^2\left(10a^3+10a^2b+5ab^2+b^3\right)\tanh^3(c+dx) - b\left(5a^4+10a^3b+10a^2b^2\right.}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^5,x]
```

output
```
((a + b)^5*ArcTanh[Tanh[c + d*x]] - b*(5*a^4 + 10*a^3*b + 10*a^2*b^2 + 5*a
*b^3 + b^4)*Tanh[c + d*x] - (b^2*(10*a^3 + 10*a^2*b + 5*a*b^2 + b^3)*Tanh[
c + d*x]^3)/3 - (b^3*(10*a^2 + 5*a*b + b^2)*Tanh[c + d*x]^5)/5 - (b^4*(5*a
 + b)*Tanh[c + d*x]^7)/7 - (b^5*Tanh[c + d*x]^9)/9)/d
```

Defintions of rubi rules used

rule 300
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Int
[PolynomialDivide[(a + b*x^2)^p, (c + d*x^2)^(-q), x], x] /; FreeQ[{a, b, c
, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && ILtQ[q, 0] && GeQ[p, -q]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (warning: unable to verify)

Time = 0.13 (sec) , antiderivative size = 262, normalized size of antiderivative = 1.64

method	result
parallelrisch	$-\frac{1575a^4b\tanh(dx+c)+3150a^3b^2\tanh(dx+c)+3150a^2b^3\tanh(dx+c)+1575a\,b^4\tanh(dx+c)+225a\,b^4\tanh(dx+c)^7+}{}$
derivativedivides	$\frac{-5a^4b\tanh(dx+c)-10a^3b^2\tanh(dx+c)-10a^2b^3\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{5a\,b^4\tanh(dx+c)^7}{7}-2a^2b^3\tanh(dx+}{}$
default	$\frac{-5a^4b\tanh(dx+c)-10a^3b^2\tanh(dx+c)-10a^2b^3\tanh(dx+c)-5a\,b^4\tanh(dx+c)-\frac{5a\,b^4\tanh(dx+c)^7}{7}-2a^2b^3\tanh(dx+}{}$
parts	$a^5x+\frac{b^5\left(-\frac{\tanh(dx+c)^9}{9}-\frac{\tanh(dx+c)^7}{7}-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(da}{2}\right.}{d}$
risch	$a^5x+5a^4bx+10a^3b^2x+10a^2b^3x+5a\,b^4x+b^5x+\frac{2b(4200a^3b+2640a\,b^3+4830a^2b^2+26292b^4e^{6c}}{}$

input
```
int((a+b*tanh(d*x+c)^2)^5,x,method=_RETURNVERBOSE)
```

output
```
-1/315*(1575*a^4*b*tanh(d*x+c)+3150*a^3*b^2*tanh(d*x+c)+3150*a^2*b^3*tanh(
d*x+c)+1575*a*b^4*tanh(d*x+c)+225*a*b^4*tanh(d*x+c)^7+630*a^2*b^3*tanh(d*x
+c)^5+315*a*b^4*tanh(d*x+c)^5+1050*a^3*b^2*tanh(d*x+c)^3+1050*a^2*b^3*tanh
(d*x+c)^3+525*a*b^4*tanh(d*x+c)^3+45*b^5*tanh(d*x+c)^7+63*b^5*tanh(d*x+c)^
5+105*b^5*tanh(d*x+c)^3+315*b^5*tanh(d*x+c)+35*b^5*tanh(d*x+c)^9-1575*a^4*
b*d*x-3150*a^3*b^2*d*x-3150*a^2*b^3*d*x-1575*a*b^4*d*x-315*a^5*d*x-315*b^5
*d*x)/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2133 vs. 2(152) = 304.

Time = 0.12 (sec) , antiderivative size = 2133, normalized size of antiderivative = 13.33

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="fricas")
```

output
```
1/315*((1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 +
315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*
x + c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*
b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*c
osh(d*x + c)*sinh(d*x + c)^8 - (1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 +
 2640*a*b^4 + 563*b^5)*sinh(d*x + c)^9 + 9*(1575*a^4*b + 4200*a^3*b^2 + 48
30*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a
^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x + c)^7 - 9*(1225*a^4*b + 2800*a^3*b^
2 + 2730*a^2*b^3 + 1240*a*b^4 + 213*b^5 + 4*(1575*a^4*b + 4200*a^3*b^2 + 4
830*a^2*b^3 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 21*
(4*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5 + 315*
(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(d*x +
c)^3 + 3*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563*b^5
+ 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*d*x)*cosh(
d*x + c))*sinh(d*x + c)^6 + 36*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 +
 2640*a*b^4 + 563*b^5 + 315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a
*b^4 + b^5)*d*x)*cosh(d*x + c)^5 - 9*(3500*a^4*b + 7000*a^3*b^2 + 6720*a^2
*b^3 + 3560*a*b^4 + 852*b^5 + 14*(1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3
 + 2640*a*b^4 + 563*b^5)*cosh(d*x + c)^4 + 21*(1225*a^4*b + 2800*a^3*b^2 +
 2730*a^2*b^3 + 1240*a*b^4 + 213*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^5 ...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 308 vs. $2(148) = 296$.

Time = 0.33 (sec) , antiderivative size = 308, normalized size of antiderivative = 1.92

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx$$
$$= \begin{cases} a^5x+5a^4bx-\frac{5a^4b\tanh(c+dx)}{d}+10a^3b^2x-\frac{10a^3b^2\tanh^3(c+dx)}{3d}-\frac{10a^3b^2\tanh(c+dx)}{d}+10a^2b^3x-\frac{2a^2b^3\tanh^5(c+d}{d} \\ x\left(a+b\tanh^2(c)\right)^5 \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**2)**5,x)
```

output
```
Piecewise((a**5*x + 5*a**4*b*x - 5*a**4*b*tanh(c + d*x)/d + 10*a**3*b**2*x
 - 10*a**3*b**2*tanh(c + d*x)**3/(3*d) - 10*a**3*b**2*tanh(c + d*x)/d + 10
*a**2*b**3*x - 2*a**2*b**3*tanh(c + d*x)**5/d - 10*a**2*b**3*tanh(c + d*x)
**3/(3*d) - 10*a**2*b**3*tanh(c + d*x)/d + 5*a*b**4*x - 5*a*b**4*tanh(c +
d*x)**7/(7*d) - a*b**4*tanh(c + d*x)**5/d - 5*a*b**4*tanh(c + d*x)**3/(3*d
) - 5*a*b**4*tanh(c + d*x)/d + b**5*x - b**5*tanh(c + d*x)**9/(9*d) - b**5
*tanh(c + d*x)**7/(7*d) - b**5*tanh(c + d*x)**5/(5*d) - b**5*tanh(c + d*x)
**3/(3*d) - b**5*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**2)**5, Tru
e))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(152) = 304$.

Time = 0.06 (sec) , antiderivative size = 624, normalized size of antiderivative = 3.90

$$\int \left(a + b\tanh^2(c + dx)\right)^5 dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="maxima")
```

output
```
1/315*b^5*(315*x + 315*c/d - 2*(3492*e^(-2*d*x - 2*c) + 13968*e^(-4*d*x -
4*c) + 26292*e^(-6*d*x - 6*c) + 39438*e^(-8*d*x - 8*c) + 31500*e^(-10*d*x
- 10*c) + 21000*e^(-12*d*x - 12*c) + 6300*e^(-14*d*x - 14*c) + 1575*e^(-16
*d*x - 16*c) + 563)/(d*(9*e^(-2*d*x - 2*c) + 36*e^(-4*d*x - 4*c) + 84*e^(-
6*d*x - 6*c) + 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) + 84*e^(-12*d
*x - 12*c) + 36*e^(-14*d*x - 14*c) + 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 1
8*c) + 1))) + 1/21*a*b^4*(105*x + 105*c/d - 8*(203*e^(-2*d*x - 2*c) + 609*
e^(-4*d*x - 4*c) + 770*e^(-6*d*x - 6*c) + 770*e^(-8*d*x - 8*c) + 315*e^(-1
0*d*x - 10*c) + 105*e^(-12*d*x - 12*c) + 44)/(d*(7*e^(-2*d*x - 2*c) + 21*e
^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*
x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/3*a^2*b^3
*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 90*e^(-6
*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*
d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c)
 + 1))) + 10/3*a^3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x
- 4*c) + 2)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)
 + 1))) + 5*a^4*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + a^5*x
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(152) = 304$.

Time = 0.17 (sec) , antiderivative size = 721, normalized size of antiderivative = 4.51

$$\int \left(a + b\tanh^2(c + dx)\right)^5 dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^2)^5,x, algorithm="giac")
```

output
```
1/315*(315*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(d*x
+ c) + 2*(1575*a^4*b*e^(16*d*x + 16*c) + 6300*a^3*b^2*e^(16*d*x + 16*c) +
9450*a^2*b^3*e^(16*d*x + 16*c) + 6300*a*b^4*e^(16*d*x + 16*c) + 1575*b^5*e
^(16*d*x + 16*c) + 12600*a^4*b*e^(14*d*x + 14*c) + 44100*a^3*b^2*e^(14*d*x
 + 14*c) + 56700*a^2*b^3*e^(14*d*x + 14*c) + 31500*a*b^4*e^(14*d*x + 14*c)
 + 6300*b^5*e^(14*d*x + 14*c) + 44100*a^4*b*e^(12*d*x + 12*c) + 136500*a^3
*b^2*e^(12*d*x + 12*c) + 161700*a^2*b^3*e^(12*d*x + 12*c) + 90300*a*b^4*e^
(12*d*x + 12*c) + 21000*b^5*e^(12*d*x + 12*c) + 88200*a^4*b*e^(10*d*x + 10
*c) + 245700*a^3*b^2*e^(10*d*x + 10*c) + 283500*a^2*b^3*e^(10*d*x + 10*c)
+ 157500*a*b^4*e^(10*d*x + 10*c) + 31500*b^5*e^(10*d*x + 10*c) + 110250*a^
4*b*e^(8*d*x + 8*c) + 283500*a^3*b^2*e^(8*d*x + 8*c) + 325080*a^2*b^3*e^(8
*d*x + 8*c) + 175140*a*b^4*e^(8*d*x + 8*c) + 39438*b^5*e^(8*d*x + 8*c) + 8
8200*a^4*b*e^(6*d*x + 6*c) + 216300*a^3*b^2*e^(6*d*x + 6*c) + 244020*a^2*b
^3*e^(6*d*x + 6*c) + 131460*a*b^4*e^(6*d*x + 6*c) + 26292*b^5*e^(6*d*x + 6
*c) + 44100*a^4*b*e^(4*d*x + 4*c) + 107100*a^3*b^2*e^(4*d*x + 4*c) + 11718
0*a^2*b^3*e^(4*d*x + 4*c) + 63540*a*b^4*e^(4*d*x + 4*c) + 13968*b^5*e^(4*d
*x + 4*c) + 12600*a^4*b*e^(2*d*x + 2*c) + 31500*a^3*b^2*e^(2*d*x + 2*c) +
34020*a^2*b^3*e^(2*d*x + 2*c) + 17460*a*b^4*e^(2*d*x + 2*c) + 3492*b^5*e^(
2*d*x + 2*c) + 1575*a^4*b + 4200*a^3*b^2 + 4830*a^2*b^3 + 2640*a*b^4 + 563
*b^5)/(e^(2*d*x + 2*c) + 1)^9)/d
```

Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 188, normalized size of antiderivative = 1.18

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx = x\left(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5\right) - \frac{\tanh(c+dx)^3\left(10a^3b^2+10a^2b^3+5ab^4+b^5\right)}{3d} - \frac{\tanh(c+dx)^5\left(10a^2b^3+5ab^4+b^5\right)}{5d} - \frac{\tanh(c+dx)^7\left(b^5+5ab^4\right)}{7d} - \frac{b^5\tanh(c+dx)^9}{9d} - \frac{b\tanh(c+dx)\left(5a^4+10a^3b+10a^2b^2+5ab^3+b^4\right)}{d}$$

input
```
int((a + b*tanh(c + d*x)^2)^5,x)
```

output
```
x*(5*a*b^4 + 5*a^4*b + a^5 + b^5 + 10*a^2*b^3 + 10*a^3*b^2) - (tanh(c + d*
x)^3*(5*a*b^4 + b^5 + 10*a^2*b^3 + 10*a^3*b^2))/(3*d) - (tanh(c + d*x)^5*(
5*a*b^4 + b^5 + 10*a^2*b^3))/(5*d) - (tanh(c + d*x)^7*(5*a*b^4 + b^5))/(7*
d) - (b^5*tanh(c + d*x)^9)/(9*d) - (b*tanh(c + d*x)*(5*a*b^3 + 10*a^3*b +
5*a^4 + b^4 + 10*a^2*b^2))/d
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.63

$$\int \left(a+b\tanh^2(c+dx)\right)^5\,dx = \frac{-35\tanh(dx+c)^9 b^5 - 225\tanh(dx+c)^7 a b^4 - 45\tanh(dx+c)^7 b^5 - 630\tanh(dx+c)^5 a^2b^3 - 315\,\mathrm{ta}}{}$$

input
```
int((a+b*tanh(d*x+c)^2)^5,x)
```

output
```
( - 35*tanh(c + d*x)**9*b**5 - 225*tanh(c + d*x)**7*a*b**4 - 45*tanh(c + d
*x)**7*b**5 - 630*tanh(c + d*x)**5*a**2*b**3 - 315*tanh(c + d*x)**5*a*b**4
 - 63*tanh(c + d*x)**5*b**5 - 1050*tanh(c + d*x)**3*a**3*b**2 - 1050*tanh(
c + d*x)**3*a**2*b**3 - 525*tanh(c + d*x)**3*a*b**4 - 105*tanh(c + d*x)**3
*b**5 - 1575*tanh(c + d*x)*a**4*b - 3150*tanh(c + d*x)*a**3*b**2 - 3150*ta
nh(c + d*x)*a**2*b**3 - 1575*tanh(c + d*x)*a*b**4 - 315*tanh(c + d*x)*b**5
 + 315*a**5*d*x + 1575*a**4*b*d*x + 3150*a**3*b**2*d*x + 3150*a**2*b**3*d*
x + 1575*a*b**4*d*x + 315*b**5*d*x)/(315*d)
```

3.170 $\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 66

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)d} - \frac{\tanh^2(c+dx)}{2bd}$$

output
```
ln(cosh(d*x+c))/(a+b)/d+1/2*a^2*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)/d-1/2*tanh
(d*x+c)^2/b/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.91

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{-\frac{2\log(\cosh(c+dx))}{a+b} - \frac{a^2\log\left(a+b\tanh^2(c+dx)\right)}{b^2(a+b)} + \frac{\tanh^2(c+dx)}{b}}{2d}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/2*((-2*Log[Cosh[c + d*x]])/(a + b) - (a^2*Log[a + b*Tanh[c + d*x]^2])/(
b^2*(a + b)) + Tanh[c + d*x]^2/b)/d
```

Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)^5}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)^5}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\tanh^5(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh^5(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{a^2}{b(a+b)(b\tanh^2(c+dx)+a)}-\frac{1}{b}-\frac{1}{(a+b)(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{a^2\log(a+b\tanh^2(c+dx))}{b^2(a+b)}-\frac{\log(1-\tanh^2(c+dx))}{a+b}-\frac{\tanh^2(c+dx)}{b}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)) + (a^2*Log[a + b*Tanh[c + d*x]^2])/(b
^2*(a + b)) - Tanh[c + d*x]^2/b)/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.10 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.20

method	result
parallelrisch	$-\frac{2b^2dx+\tanh(dx+c)^2ab+b^2\tanh(dx+c)^2+2\ln(1-\tanh(dx+c))b^2-a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2b^2d(a+b)}$
derivativedivides	$\frac{-\frac{\tanh(dx+c)^2}{2b}+\frac{a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b^2}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$
default	$\frac{-\frac{\tanh(dx+c)^2}{2b}+\frac{a^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)b^2}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2a^2x}{b^2(a+b)}-\frac{2a^2c}{b^2d(a+b)}+\frac{2ax}{b^2}+\frac{2ac}{b^2d}-\frac{2x}{b}-\frac{2c}{bd}+\frac{2\,e^{2dx+2c}}{bd\left(e^{2dx+2c}+1\right)^2}+\frac{a^2\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2}}{a+b}\right.}{2b^2d(a+b)}$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b^2*d*x+tanh(d*x+c)^2*a*b+b^2*tanh(d*x+c)^2+2*ln(1-tanh(d*x+c))*b^
2-a^2*ln(a+b*tanh(d*x+c)^2))/b^2/d/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 742 vs. 2(62) = 124.

Time = 0.17 (sec) , antiderivative size = 742, normalized size of antiderivative = 11.24

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b^2*d*x*cosh(d*x + c)^4 + 8*b^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 2*b^2*d*x*sinh(d*x + c)^4 + 2*b^2*d*x + 4*(b^2*d*x - a*b - b^2)*cosh(d*x
 + c)^2 + 4*(3*b^2*d*x*cosh(d*x + c)^2 + b^2*d*x - a*b - b^2)*sinh(d*x + c
)^2 - (a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*sin
h(d*x + c)^4 + 2*a^2*cosh(d*x + c)^2 + 2*(3*a^2*cosh(d*x + c)^2 + a^2)*sin
h(d*x + c)^2 + a^2 + 4*(a^2*cosh(d*x + c)^3 + a^2*cosh(d*x + c))*sinh(d*x
+ c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(c
osh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 2*((a
^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 - b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 -
b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 -
b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*cos
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(b^2*d*x*cosh(d*x + c)^3 +
 (b^2*d*x - a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^2 + b^3)*d*cosh
(d*x + c)^4 + 4*(a*b^2 + b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^2 + b
^3)*d*sinh(d*x + c)^4 + 2*(a*b^2 + b^3)*d*cosh(d*x + c)^2 + 2*(3*(a*b^2 +
b^3)*d*cosh(d*x + c)^2 + (a*b^2 + b^3)*d)*sinh(d*x + c)^2 + (a*b^2 + b^3)*
d + 4*((a*b^2 + b^3)*d*cosh(d*x + c)^3 + (a*b^2 + b^3)*d*cosh(d*x + c))*si
nh(d*x + c))
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 415 vs. $2(53) = 106$.

Time = 4.40 (sec) , antiderivative size = 415, normalized size of antiderivative = 6.29

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \begin{cases} \tilde{\infty}x\tanh^3(c) \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}-\frac{\tanh^4(c+dx)}{4d}-\frac{\tanh^2(c+dx)}{2d}}{a} \\ \frac{4dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}-\frac{4dx}{2bd\tanh^2(c+dx)-2bd}-\frac{4\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{4\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd}-\frac{\tanh^4(c+dx)}{2bd\tanh^2(c+dx)} \\ \frac{x\tanh^5(c)}{a+b\tanh^2(c)} \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d}+\frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d+2b^3d}-\frac{ab\tanh^2(c+dx)}{2ab^2d+2b^3d}+\frac{2b^2dx}{2ab^2d+2b^3d}-\frac{2b^2\log(\tanh(c+dx)+1)}{2ab^2d+2b^3d}-\frac{b^2\mathrm{t}}{2a} \end{cases}$$

input
```
integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*tanh(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(ta
nh(c + d*x) + 1)/d - tanh(c + d*x)**4/(4*d) - tanh(c + d*x)**2/(2*d))/a, E
q(b, 0)), (4*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*d*x
/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 4*log(tanh(c + d*x) + 1)*tanh(c + d*x)
**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 4*log(tanh(c + d*x) + 1)/(2*b*d*tan
h(c + d*x)**2 - 2*b*d) - tanh(c + d*x)**4/(2*b*d*tanh(c + d*x)**2 - 2*b*d)
 + 2/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**5/(a + b*ta
nh(c)**2), Eq(d, 0)), (a**2*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d +
 2*b**3*d) + a**2*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d + 2*b**3*d)
- a*b*tanh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d) + 2*b**2*d*x/(2*a*b**2*d +
2*b**3*d) - 2*b**2*log(tanh(c + d*x) + 1)/(2*a*b**2*d + 2*b**3*d) - b**2*t
anh(c + d*x)**2/(2*a*b**2*d + 2*b**3*d), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 133 vs. $2(62) = 124$.

Time = 0.13 (sec) , antiderivative size = 133, normalized size of antiderivative = 2.02

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{a^2\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(ab^2+b^3)d} + \frac{dx+c}{(a+b)d} + \frac{2\,e^{(-2\,dx-2\,c)}}{\left(2\,be^{(-2\,dx-2\,c)}+be^{(-4\,dx-4\,c)}+b\right)d} - \frac{(a-b)\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*a^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)
/((a*b^2 + b^3)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*b*e^(-
2*d*x - 2*c) + b*e^(-4*d*x - 4*c) + b)*d) - (a - b)*log(e^(-2*d*x - 2*c) +
 1)/(b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. $2(62) = 124$.

Time = 0.18 (sec) , antiderivative size = 132, normalized size of antiderivative = 2.00

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{a^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab^2+b^3}-\frac{2\,(dx+c)}{a+b}-\frac{2\,(a-b)\log\left(e^{(2\,dx+2\,c)}+1\right)}{b^2}+\frac{4\,e^{(2\,dx+2\,c)}}{b\left(e^{(2\,dx+2\,c)}+1\right)^2}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(a^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)/(a*b^2 + b^3) - 2*(d*x + c)/(a + b) - 2*(a -
 b)*log(e^(2*d*x + 2*c) + 1)/b^2 + 4*e^(2*d*x + 2*c)/(b*(e^(2*d*x + 2*c) +
 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.09

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{b^2\left(\ln\left(\tanh(c+d\,x)+1\right)-d\,x+\frac{\tanh(c+d\,x)^2}{2}\right)-\frac{a^2\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}+\frac{a\,b\tanh(c+d\,x)^2}{2}}{b^2\,d\,\left(a+b\right)}$$

input
```
int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2),x)
```

output
```
-(b^2*(log(tanh(c + d*x) + 1) - d*x + tanh(c + d*x)^2/2) - (a^2*log(a + b*
tanh(c + d*x)^2))/2 + (a*b*tanh(c + d*x)^2)/2)/(b^2*d*(a + b))
```

Reduce [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 153, normalized size of antiderivative = 2.32

$$\int \frac{\tanh^5(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\log\left(\tanh(dx+c)^2 b+a\right)a^2-\log\left(\tanh(dx+c)^2 b+a\right)b^2+\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}-2e^{dx+c}\sqrt{b}\right)}{2b^2d\,(a+}$$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2),x)
```

output
```
(log(tanh(c + d*x)**2*b + a)*a**2 - log(tanh(c + d*x)**2*b + a)*b**2 + log
(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2
 + log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b)
)*b**2 - tanh(c + d*x)**2*a*b - tanh(c + d*x)**2*b**2 - 2*b**2*d*x)/(2*b**
2*d*(a + b))
```

3.171 $\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 59

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{x}{a+b}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d}-\frac{\tanh(c+dx)}{bd}$$

output
```
x/(a+b)+a^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/(a+b)/d-tanh(d
*x+c)/b/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.12

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{c+dx}{(a+b)d}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}(a+b)d}-\frac{\tanh(c+dx)}{bd}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(c + d*x)/((a + b)*d) + (a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(b^(3/2)*(a + b)*d) - Tanh[c + d*x]/(b*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4153, 381, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{\tan(ic+idx)^4}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 381}$$

$$\frac{\frac{\int \frac{a-(a-b)\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{b}-\frac{\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 397}$$

$$\frac{\frac{\frac{a^2\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{b}-\frac{\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 218}$$

$$\frac{\frac{\frac{b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}}{b}-\frac{\tanh(c+dx)}{b}}{d}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{\frac{\frac{a^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}+\frac{b\text{arctanh}(\tanh(c+dx))}{a+b}}{b}-\frac{\tanh(c+dx)}{b}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(((a^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a + b)) + (b
*ArcTanh[Tanh[c + d*x]])/(a + b))/b - Tanh[c + d*x]/b)/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

method	result	size
derivativedivides	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{b(a+b)\sqrt{ab}}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	87
default	$\frac{-\frac{\tanh(dx+c)}{b}+\frac{a^2\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{b(a+b)\sqrt{ab}}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	87
risch	$\frac{x}{a+b}+\frac{2}{bd(e^{2dx+2c}+1)}+\frac{\sqrt{-ab}\,a\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2b^2(a+b)d}-\frac{\sqrt{-ab}\,a\ln\left(e^{2dx+2c}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2b^2(a+b)d}$	131

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/b*tanh(d*x+c)+1/b/(a+b)*a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)
^(1/2))+1/(2*a+2*b)*ln(1+tanh(d*x+c))-1/(2*a+2*b)*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 225 vs. 2(51) = 102.

Time = 0.12 (sec) , antiderivative size = 777, normalized size of antiderivative = 13.17

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*b*d*x*cosh(d*x + c)^2 + 4*b*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*b*
d*x*sinh(d*x + c)^2 + 2*b*d*x + (a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sin
h(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
c))*sinh(d*x + c) + 4*((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c) + (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)) + 4*a + 4*b)/((a*b + b^2)*d*cosh(d*x +
c)^2 + 2*(a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*sinh(d*
x + c)^2 + (a*b + b^2)*d), (b*d*x*cosh(d*x + c)^2 + 2*b*d*x*cosh(d*x + c)*
sinh(d*x + c) + b*d*x*sinh(d*x + c)^2 + b*d*x + (a*cosh(d*x + c)^2 + 2*a*c
osh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2 + a)*sqrt(a/b)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*
sinh(d*x + c)^2 + a - b)*sqrt(a/b)/a) + 2*a + 2*b)/((a*b + b^2)*d*cosh(d*x
 + c)^2 + 2*(a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2)*d*s...
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(48) = 96$.

Time = 3.07 (sec) , antiderivative size = 428, normalized size of antiderivative = 7.25

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \begin{cases} \tilde{\infty} x \tanh^2(c) \\ \frac{x - \frac{\tanh^3(c+dx)}{3d} - \frac{\tanh(c+dx)}{d}}{a} \\ \frac{x - \frac{\tanh(c+dx)}{d}}{b} \\ \frac{3dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{3dx}{2bd\tanh^2(c+dx)-2bd} - \frac{2\tanh^3(c+dx)}{2bd\tanh^2(c+dx)-2bd} + \frac{3\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} \\ \frac{x\tanh^4(c)}{a+b\tanh^2(c)} \\ \frac{a^2\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{a^2\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{2ab\sqrt{-\frac{a}{b}}\tanh(c+dx)}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} + \frac{2b^2dx\sqrt{-\frac{a}{b}}}{2ab^2d\sqrt{-\frac{a}{b}}+2b^3d\sqrt{-\frac{a}{b}}} - \frac{2b^2\sqrt{-\frac{a}{b}}\,\mathrm{ta}}{2ab^2d\sqrt{-\frac{a}{b}}+} \end{cases}$$

input
```
integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c
 + d*x)**3/(3*d) - tanh(c + d*x)/d)/a, Eq(b, 0)), ((x - tanh(c + d*x)/d)/b
, Eq(a, 0)), (3*d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 3*
d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*tanh(c + d*x)**3/(2*b*d*tanh(c +
d*x)**2 - 2*b*d) + 3*tanh(c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a,
 -b)), (x*tanh(c)**4/(a + b*tanh(c)**2), Eq(d, 0)), (a**2*log(-sqrt(-a/b)
+ tanh(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b)) - a**2*log(
sqrt(-a/b) + tanh(c + d*x))/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b))
- 2*a*b*sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a
/b)) + 2*b**2*d*x*sqrt(-a/b)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(-a/b))
 - 2*b**2*sqrt(-a/b)*tanh(c + d*x)/(2*a*b**2*d*sqrt(-a/b) + 2*b**3*d*sqrt(
-a/b)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 509 vs. $2(51) = 102$.

Time = 0.25 (sec) , antiderivative size = 509, normalized size of antiderivative = 8.63

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= -\frac{(a-b)\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{8\,(ab+b^2)d}$$
$$+\frac{(a-b)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{8\,(ab+b^2)d}$$
$$+\frac{(a^2-6\,ab+b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{16\,(ab+b^2)\sqrt{ab}d}+\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}bd}$$
$$-\frac{(a^2-6\,ab+b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{16\,(ab+b^2)\sqrt{ab}d}$$
$$-\frac{3\,(a+b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\sqrt{ab}bd}-\frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}bd}$$
$$-\frac{\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,bd}$$
$$+\frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,bd}+\frac{3\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{4\,bd}$$
$$-\frac{3\,\log\left(e^{(-2\,dx-2\,c)}+1\right)}{4\,bd}+\frac{5}{8\,(be^{(2\,dx+2\,c)}+b)d}-\frac{11}{8\,(be^{(-2\,dx-2\,c)}+b)d}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(a - b)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a +
 b)/((a*b + b^2)*d) + 1/8*(a - b)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)
*e^(-4*d*x - 4*c) + a + b)/((a*b + b^2)*d) + 1/16*(a^2 - 6*a*b + b^2)*arct
an(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)
*d) + 1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*b*d) - 1/16*(a^2 - 6*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x -
 2*c) + a - b)/sqrt(a*b))/((a*b + b^2)*sqrt(a*b)*d) - 3/8*(a + b)*arctan(1
/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b*d) - 1/4*(a
- b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*b
*d) - 1/4*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)
/(b*d) + 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a
 + b)/(b*d) + 3/4*log(e^(2*d*x + 2*c) + 1)/(b*d) - 3/4*log(e^(-2*d*x - 2*c
) + 1)/(b*d) + 5/8/((b*e^(2*d*x + 2*c) + b)*d) - 11/8/((b*e^(-2*d*x - 2*c)
 + b)*d)
```

Giac [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{a^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\sqrt{ab}}\right)}{(ab+b^2)\sqrt{ab}}+\frac{dx+c}{a+b}+\frac{2}{b\left(e^{(2\,dx+2\,c)}+1\right)}}{d}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
(a^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))
/((a*b + b^2)*sqrt(a*b)) + (d*x + c)/(a + b) + 2/(b*(e^(2*d*x + 2*c) + 1))
)/d
```

Mupad [B] (verification not implemented)

Time = 2.34 (sec) , antiderivative size = 56, normalized size of antiderivative = 0.95

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\tanh(c+d\,x)}{b\,d} + \frac{a^2\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{b\,d\,\sqrt{a\,b}\,(a+b)}$$

input
```
int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - tanh(c + d*x)/(b*d) + (a^2*atan((b*tanh(c + d*x))/(a*b)^(1/2))
)/(b*d*(a*b)^(1/2)*(a + b))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.07

$$\int \frac{\tanh^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)a - \tanh(dx+c)\,ab - \tanh(dx+c)\,b^2 + b^2dx}{b^2d\,(a+b)}$$

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a - tanh(c + d*
x)*a*b - tanh(c + d*x)*b**2 + b**2*d*x)/(b**2*d*(a + b))
```

3.172 $\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)d}$$

output
```
ln(cosh(d*x+c))/(a+b)/d-1/2*a*ln(a+b*tanh(d*x+c)^2)/b/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.96

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{\log(\cosh(c+dx))}{a+b} - \frac{a\log\left(a+b\tanh^2(c+dx)\right)}{2b(a+b)}}{d}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Log[Cosh[c + d*x]]/(a + b) - (a*Log[a + b*Tanh[c + d*x]^2])/(2*b*(a + b))
)/d
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.13, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ic+idx)^3}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh^3(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{a}{(a+b)(b\tanh^2(c+dx)+a)}-\frac{1}{(a+b)(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{\log(1-\tanh^2(c+dx))}{a+b}-\frac{a\log(a+b\tanh^2(c+dx))}{b(a+b)}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)) - (a*Log[a + b*Tanh[c + d*x]^2])/(b*(
a + b)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.07

method	result	size
parallelrisch	$-\frac{2bdx+2\ln(1-\tanh(dx+c))b+a\ln\left(a+b\tanh(dx+c)^{2}\right)}{2db(a+b)}$	49
derivativedivides	$\frac{-\frac{a\ln\left(a+b\tanh(dx+c)^{2}\right)}{2(a+b)b}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	70
default	$\frac{-\frac{a\ln\left(a+b\tanh(dx+c)^{2}\right)}{2(a+b)b}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	70
risch	$\frac{x}{a+b}-\frac{2x}{b}-\frac{2c}{bd}+\frac{2ax}{b(a+b)}+\frac{2ac}{db(a+b)}+\frac{\ln\left(e^{2dx+2c}+1\right)}{bd}-\frac{a\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2db(a+b)}$	117

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*b*d*x+2*ln(1-tanh(d*x+c))*b+a*ln(a+b*tanh(d*x+c)^2))/d/b/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(44) = 88.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.57

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,bdx+a\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2\,(a+b)\log\left(\frac{2\,\cosh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(ab+b^2)d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*b*d*x + a*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2)) - 2*(a + b)*log(2*cosh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a
*b + b^2)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(36) = 72$.

Time = 2.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 6.65

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \tilde{\infty}x\tanh(c) \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}-\frac{\tanh^2(c+dx)}{2d}}{a} \\ \frac{2dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}-\frac{2dx}{2bd\tanh^2(c+dx)-2bd}-\frac{2\log(\tanh(c+dx)+1)\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{2\log(\tanh(c+dx)+1)}{2bd\tanh^2(c+dx)-2bd}+\frac{1}{2bd\tanh^2(c+dx)} \\ \frac{x\tanh^3(c)}{a+b\tanh^2(c)} \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}-\frac{a\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd+2b^2d}+\frac{2bdx}{2abd+2b^2d}-\frac{2b\log(\tanh(c+dx)+1)}{2abd+2b^2d} \end{cases}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x*tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(
c + d*x) + 1)/d - tanh(c + d*x)**2/(2*d))/a, Eq(b, 0)), (2*d*x*tanh(c + d*
x)**2/(2*b*d*tanh(c + d*x)**2 - 2*b*d) - 2*d*x/(2*b*d*tanh(c + d*x)**2 - 2
*b*d) - 2*log(tanh(c + d*x) + 1)*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)**2
- 2*b*d) + 2*log(tanh(c + d*x) + 1)/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + 1/(
2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**3/(a + b*tanh(c)*
*2), Eq(d, 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d)
- a*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d + 2*b**2*d) + 2*b*d*x/(2*a*b*
d + 2*b**2*d) - 2*b*log(tanh(c + d*x) + 1)/(2*a*b*d + 2*b**2*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.78

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{a\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(ab+b^2)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{(-2dx-2c)}+1\right)}{bd}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*a*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a*b + b^2)*d) + (d*x + c)/((a + b)*d) + log(e^(-2*d*x - 2*c) + 1)/(b*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(44) = 88$.

Time = 0.16 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.09

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{ab+b^2}+\frac{2\,(dx+c)}{a+b}-\frac{2\,\log\left(e^{(2\,dx+2\,c)}+1\right)}{b}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(a*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a*b + b^2) + 2*(d*x + c)/(a + b) - 2*log(e^(
2*d*x + 2*c) + 1)/b)/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}+b\left(\ln\left(\tanh(c+d\,x)+1\right)-d\,x\right)}{b\,d\,(a+b)}$$

input
```
int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
-((a*log(a + b*tanh(c + d*x)^2))/2 + b*(log(tanh(c + d*x) + 1) - d*x))/(b*
d*(a + b))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 119, normalized size of antiderivative = 2.59

$$\int \frac{\tanh^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{-\log\left(\tanh\left(dx+c\right)^2 b+a\right) a-\log\left(\tanh\left(dx+c\right)^2 b+a\right) b+\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}-2e^{dx+c}\sqrt{b}\right)}{2bd\left(a+b\right)}$$

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - log(tanh(c + d*x)**2*b + a)*a - log(tanh(c + d*x)**2*b + a)*b + log(e*
*(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b + log
(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b -
2*b*d*x)/(2*b*d*(a + b))
```

3.173 $\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 46

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)d}$$

output
```
x/(a+b)-a^(1/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(1/2)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.02

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}} + \operatorname{arctanh}(\tanh(c+dx))}{(a+b)d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[b]) + ArcTanh[Ta
nh[c + d*x]])/((a + b)*d)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.17, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 25, 4153, 25, 383, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{\tan(ic+idx)^2}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{\tan(ic+idx)^2}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \ 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 383$$

$$-\frac{\frac{a\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}-\frac{\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{d}$$

$$\Big\downarrow \ 219$$

$$-\frac{\frac{\sqrt{a}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{\text{arctanh}(\tanh(c+dx))}{a+b}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-(((Sqrt[a]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b]*(a + b)) - A
rcTanh[Tanh[c + d*x]]/(a + b))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 383
```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_Sym
bol] :> Simp[(-a)*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(a + b*x^2), x], x]
 + Simp[c*(e^2/(b*c - a*d))   Int[(e*x)^(m - 2)/(c + d*x^2), x], x] /; Free
Q[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.08 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.57

method	result	size
derivativedivides	$\frac{-\frac{a \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	72
default	$\frac{-\frac{a \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	72
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2b(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2b(a+b)d}$	108

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))+1/(2*a+2*b)*ln
(1+tanh(d*x+c))-1/(2*a+2*b)*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 81 vs. 2(38) = 76.

Time = 0.12 (sec) , antiderivative size = 486, normalized size of antiderivative = 10.57

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[\frac{2\,dx+\sqrt{-\frac{a}{b}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cosh}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(dx}\right.}{}\right.$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d
*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*
a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*
((a*b + b^2)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) +
 (a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x +
c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 +
 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) + a + b)))/((a + b)*d), (d*x - sqrt(a/b)*arctan(1/2*((a + b)*cosh(d*x +
 c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(a/b)/a))/((a + b)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(37) = 74$.

Time = 2.13 (sec) , antiderivative size = 253, normalized size of antiderivative = 5.50

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \begin{cases} \tilde{\infty}x & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x-\frac{\tanh(c+dx)}{d}}{a} & \text{for } b=0 \\ \frac{x}{b} & \text{for } a=0 \\ \frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd} - \frac{dx}{2bd\tanh^2(c+dx)-2bd} + \frac{\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for } a=-b \\ \frac{x\tanh^2(c)}{a+b\tanh^2(c)} & \text{for } d=0 \\ -\frac{a\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}} + \frac{a\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}} + \frac{2bdx\sqrt{-\frac{a}{b}}}{2abd\sqrt{-\frac{a}{b}}+2b^2d\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - tanh(c + d*x)/d)/
a, Eq(b, 0)), (x/b, Eq(a, 0)), (d*x*tanh(c + d*x)**2/(2*b*d*tanh(c + d*x)*
*2 - 2*b*d) - d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(c + d*x)/(2*b*d*
tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x*tanh(c)**2/(a + b*tanh(c)**2), E
q(d, 0)), (-a*log(-sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) + 2*b**
2*d*sqrt(-a/b)) + a*log(sqrt(-a/b) + tanh(c + d*x))/(2*a*b*d*sqrt(-a/b) +
2*b**2*d*sqrt(-a/b)) + 2*b*d*x*sqrt(-a/b)/(2*a*b*d*sqrt(-a/b) + 2*b**2*d*s
qrt(-a/b)), True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 215 vs. $2(38) = 76$.

Time = 0.15 (sec) , antiderivative size = 215, normalized size of antiderivative = 4.67

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{(a-b)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}(a+b)d} + \frac{\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}d} + \frac{(a-b)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,\sqrt{ab}(a+b)d} + \frac{\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a+b)d} - \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a+b)d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*(a - b)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(sqrt
(a*b)*(a + b)*d) + 1/2*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(
a*b))/(sqrt(a*b)*d) + 1/4*(a - b)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a
 - b)/sqrt(a*b))/(sqrt(a*b)*(a + b)*d) + 1/4*log((a + b)*e^(4*d*x + 4*c) +
 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a + b)*d) - 1/4*log(2*(a - b)*e^(-2*
d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a + b)*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.41

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{a\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}(a+b)} - \frac{dx+c}{a+b}}{d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(a*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/
(sqrt(a*b)*(a + b)) - (d*x + c)/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 38, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{a\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,\left(a+b\right)}$$

input
```
int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - (a*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b)
)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.87

$$\int \frac{\tanh^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)+bdx}{bd\,(a+b)}$$

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a))) + b*d*x)/(b*
d*(a + b))
```

3.174 $\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 42

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)d}$$

output
```
ln(cosh(d*x+c))/(a+b)/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 0.83

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{2\log(\cosh(c+dx)) + \log\left(a+b\tanh^2(c+dx)\right)}{2ad+2bd}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2])/(2*a*d + 2*b*d)
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 4153, 26, 353, 47, 16}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\tanh(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{1}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{47}$$

$$\frac{\frac{\int \frac{1}{1-\tanh^2(c+dx)}d\tanh^2(c+dx)}{a+b}+\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh^2(c+dx)}{a+b}}{2d}$$

$$\Big\downarrow \text{16}$$

$$\frac{\frac{\log(a+b\tanh^2(c+dx))}{a+b}-\frac{\log(1-\tanh^2(c+dx))}{a+b}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)) + Log[a + b*Tanh[c + d*x]^2]/(a + b))
/(2*d)
```

Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 47
```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Simp[b/(b*c
 - a*d)   Int[1/(a + b*x), x], x] - Simp[d/(b*c - a*d)   Int[1/(c + d*x), x
], x] /; FreeQ[{a, b, c, d}, x]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.06 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.05

method	result	size
parallelrisch	$-\frac{2dx+2\ln(1-\tanh(dx+c))-\ln\left(a+b\tanh(dx+c)^{2}\right)}{2d(a+b)}$	44
risch	$-\frac{x}{a+b}-\frac{2c}{d(a+b)}+\frac{\ln\left(e^{4dx+4c}+\frac{2(a-b)e^{2dx+2c}}{a+b}+1\right)}{2d(a+b)}$	64
derivativedivides	$\frac{\frac{\ln\left(a+b\tanh(dx+c)^{2}\right)}{2a+2b}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	66
default	$\frac{\frac{\ln\left(a+b\tanh(dx+c)^{2}\right)}{2a+2b}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	66

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(2*d*x+2*ln(1-tanh(d*x+c))-ln(a+b*tanh(d*x+c)^2))/d/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 82 vs. 2(40) = 80.

Time = 0.09 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.95

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{2\,dx-\log\left(\frac{2\left((a+b)\cosh(dx+c)^{2}+(a+b)\sinh(dx+c)^{2}+a-b\right)}{\cosh(dx+c)^{2}-2\,\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^{2}}\right)}{2\,(a+b)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*d*x - log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a
 - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2))
)/((a + b)*d)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. $2(32) = 64$.

Time = 2.02 (sec) , antiderivative size = 146, normalized size of antiderivative = 3.48

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\tanh(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x-\frac{\log(\tanh(c+dx)+1)}{d}}{a} & \text{for } b=0 \\ \frac{1}{2bd\tanh^2(c+dx)-2bd} & \text{for } a=-b \\ \frac{x\tanh(c)}{a+b\tanh^2(c)} & \text{for } d=0 \\ \frac{2dx}{2ad+2bd}+\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad+2bd}+\frac{\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad+2bd}-\frac{2\log(\tanh(c+dx)+1)}{2ad+2bd} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/tanh(c), Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((x - log(tanh(
c + d*x) + 1)/d)/a, Eq(b, 0)), (1/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a,
-b)), (x*tanh(c)/(a + b*tanh(c)**2), Eq(d, 0)), (2*d*x/(2*a*d + 2*b*d) + l
og(-sqrt(-a/b) + tanh(c + d*x))/(2*a*d + 2*b*d) + log(sqrt(-a/b) + tanh(c
+ d*x))/(2*a*d + 2*b*d) - 2*log(tanh(c + d*x) + 1)/(2*a*d + 2*b*d), True))
```

Maxima [A] (verification not implemented)

Time = 0.05 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.38

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{dx+c}{(a+b)d} + \frac{\log\left(2(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2(a+b)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
(d*x + c)/((a + b)*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4
*d*x - 4*c) + a + b)/((a + b)*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.45

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{2\,(a+b)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c)) + 2*a - 2*b))/((a + b)*d)
```

Mupad [B] (verification not implemented)

Time = 2.32 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.02

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\ln\left(\tanh(c+d\,x)+1\right) - \frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2}}{d\,\left(a+b\right)}$$

input
```
int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - (log(tanh(c + d*x) + 1) - log(a + b*tanh(c + d*x)^2)/2)/(d*(a
+ b))
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.88

$$\int \frac{\tanh(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}-2e^{dx+c}\sqrt{b}\right)+\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}+2e^{dx+c}\sqrt{b}\right)-2dx}{2d\,(a+b)}$$

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

output
```
(log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))
+ log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))
 - 2*d*x)/(2*d*(a + b))
```

3.175 $\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 45

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx=\frac{x}{a+b}+\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)d}$$

output
```
x/(a+b)+b^(1/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.44

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx =\frac{\frac{2\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}-\log(1-\tanh(c+dx))+\log(1+\tanh(c+dx))}{2ad+2bd}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

output
```
((2*Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a] - Log[1 - Tan
h[c + d*x]] + Log[1 + Tanh[c + d*x]])/(2*a*d + 2*b*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3042, 4143, 3042, 4158, 218}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{a-b\tan(ic+idx)^2}dx$$

$$\Big\downarrow \text{4143}$$

$$\frac{b\int \frac{\text{sech}^2(c+dx)}{b\tanh^2(c+dx)+a}dx}{a+b}+\frac{x}{a+b}$$

$$\Big\downarrow \text{3042}$$

$$\frac{x}{a+b}+\frac{b\int \frac{\sec(ic+idx)^2}{a-b\tan(ic+idx)^2}dx}{a+b}$$

$$\Big\downarrow \text{4158}$$

$$\frac{b\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{d(a+b)}+\frac{x}{a+b}$$

$$\Big\downarrow \text{218}$$

$$\frac{\sqrt{b}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a+b)}+\frac{x}{a+b}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-1),x]
```

output
```
x/(a + b) + (Sqrt[b]*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a
+ b)*d)
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4143
```
Int[((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> Simp[x/(a -
 b), x] - Simp[b/(a - b)   Int[Sec[e + f*x]^2/(a + b*Tan[e + f*x]^2), x], x
] /; FreeQ[{a, b, e, f}, x] && NeQ[a, b]
```

rule 4158
```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_
)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Sim
p[ff/(c^(m - 1)*f)   Subst[Int[(c^2 + ff^2*x^2)^(m/2 - 1)*(a + b*(ff*x)^n)^
p, x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
ntegerQ[m/2] && (IntegersQ[n, p] || IGtQ[m, 0] || IGtQ[p, 0] || EqQ[n^2, 4]
 || EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

method	result	size
derivativedivides	$\frac{\frac{b \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	71
default	$\frac{\frac{b \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}-\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	71
risch	$\frac{x}{a+b}+\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a(a+b)d}-\frac{\sqrt{-ab}\,\ln\left(\mathrm{e}^{2dx+2c}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2a(a+b)d}$	108

input
```
int(1/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/d*(b/(a+b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))+1/(2*a+2*b)*ln(
1+tanh(d*x+c))-1/(2*a+2*b)*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 80 vs. $2(37) = 74$.

Time = 0.12 (sec) , antiderivative size = 484, normalized size of antiderivative = 10.76

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$$
$$= \left[\frac{2\,dx + \sqrt{-\frac{b}{a}}\log\left(\frac{(a^2+2\,ab+b^2)\cosh(dx+c)^4+4\,(a^2+2\,ab+b^2)\cosh(dx+c)\sinh(dx+c)^3+(a^2+2\,ab+b^2)\sinh(dx+c)^4+2\,(a^2-b^2)\cos}{(a+b)\cosh(dx+c)^4+4\,(a+b)\cosh(d}\right.}{} \right.$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*d*x + sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2
 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d
*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*
a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*
((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) +
 (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x +
c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 +
 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*
x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x +
c) + a + b)))/((a + b)*d), (d*x + sqrt(b/a)*arctan(1/2*((a + b)*cosh(d*x +
 c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 +
a - b)*sqrt(b/a)/b))/((a + b)*d)]
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 240 vs. $2(37) = 74$.

Time = 2.10 (sec) , antiderivative size = 240, normalized size of antiderivative = 5.33

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx$$
$$= \begin{cases} \frac{\tilde{\infty}x}{\tanh^2(c)} & \text{for } a=0 \wedge b=0 \wedge d=0 \\ \frac{x}{a} & \text{for } b=0 \\ \frac{x-\frac{1}{d\tanh(c+dx)}}{b} & \text{for } a=0 \\ -\frac{dx\tanh^2(c+dx)}{2bd\tanh^2(c+dx)-2bd}+\frac{dx}{2bd\tanh^2(c+dx)-2bd}+\frac{\tanh(c+dx)}{2bd\tanh^2(c+dx)-2bd} & \text{for } a=-b \\ \frac{x}{a+b\tanh^2(c)} & \text{for } d=0 \\ \frac{2dx\sqrt{-\frac{a}{b}}}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}}+\frac{\log\left(-\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}}-\frac{\log\left(\sqrt{-\frac{a}{b}}+\tanh(c+dx)\right)}{2ad\sqrt{-\frac{a}{b}}+2bd\sqrt{-\frac{a}{b}}} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2),x)
```

output
```
Piecewise((zoo*x/tanh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (x/a, Eq(b,
0)), ((x - 1/(d*tanh(c + d*x)))/b, Eq(a, 0)), (-d*x*tanh(c + d*x)**2/(2*b*
d*tanh(c + d*x)**2 - 2*b*d) + d*x/(2*b*d*tanh(c + d*x)**2 - 2*b*d) + tanh(
c + d*x)/(2*b*d*tanh(c + d*x)**2 - 2*b*d), Eq(a, -b)), (x/(a + b*tanh(c)**
2), Eq(d, 0)), (2*d*x*sqrt(-a/b)/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) + l
og(-sqrt(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)) - lo
g(sqrt(-a/b) + tanh(c + d*x))/(2*a*d*sqrt(-a/b) + 2*b*d*sqrt(-a/b)), True)
)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.27

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = -\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\sqrt{ab}}\right)}{\sqrt{ab}(a+b)d}+\frac{dx+c}{(a+b)d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/(sqrt(a*b)*(a
+ b)*d) + (d*x + c)/((a + b)*d)
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.40

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx=\frac{\frac{b\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\sqrt{ab}(a+b)}+\frac{dx+c}{a+b}}{d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
(b*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/(
sqrt(a*b)*(a + b)) + (d*x + c)/(a + b))/d
```

Mupad [B] (verification not implemented)

Time = 2.38 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.82

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx=\frac{x}{a+b}+\frac{b\,\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)}{d\,\sqrt{a\,b}\,(a+b)}$$

input
```
int(1/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) + (b*atan((b*tanh(c + d*x))/(a*b)^(1/2)))/(d*(a*b)^(1/2)*(a + b)
)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.87

$$\int \frac{1}{a+b\tanh^2(c+dx)}\,dx = \frac{\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)+adx}{ad\,(a+b)}$$

input
```
int(1/(a+b*tanh(d*x+c)^2),x)
```

output
```
(sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a))) + a*d*x)/(a*d*(
a + b))
```

3.176 $\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 60

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)d} + \frac{\log(\tanh(c+dx))}{ad} - \frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)d}$$

output
```
ln(cosh(d*x+c))/(a+b)/d+ln(tanh(d*x+c))/a/d-1/2*b*ln(a+b*tanh(d*x+c)^2)/a/
(a+b)/d
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.92

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{\log(\cosh(c+dx))}{a+b} + \frac{\log(\tanh(c+dx))}{a} - \frac{b\log\left(a+b\tanh^2(c+dx)\right)}{2a(a+b)}}{d}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Log[Cosh[c + d*x]]/(a + b) + Log[Tanh[c + d*x]]/a - (b*Log[a + b*Tanh[c +
 d*x]^2])/(2*a*(a + b)))/d
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\coth(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(-\frac{b^2}{a(a+b)\left(b\tanh^2(c+dx)+a\right)}+\frac{\coth(c+dx)}{a}-\frac{1}{(a+b)\left(\tanh^2(c+dx)-1\right)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow 2009$$

$$\frac{-\frac{\log(1-\tanh^2(c+dx))}{a+b}-\frac{b\log(a+b\tanh^2(c+dx))}{a(a+b)}+\frac{\log(\tanh^2(c+dx))}{a}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(Log[Tanh[c + d*x]^2]/a - Log[1 - Tanh[c + d*x]^2]/(a + b) - (b*Log[a + b*
Tanh[c + d*x]^2])/(a*(a + b)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.15 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.08

method	result	size
parallelrisch	$\frac{-b\ln\left(a+b\tanh\left(dx+c\right)^{2}\right)-2\ln\left(1-\tanh\left(dx+c\right)\right)a+\left(2a+2b\right)\ln\left(\tanh\left(dx+c\right)\right)-2adx}{2ad\left(a+b\right)}$	65
derivativedivides	$-\frac{\frac{b\ln\left(a+b\tanh\left(dx+c\right)^{2}\right)}{2\left(a+b\right)a}-\frac{\ln\left(\tanh\left(dx+c\right)\right)}{a}+\frac{\ln\left(1+\tanh\left(dx+c\right)\right)}{2a+2b}+\frac{\ln\left(-1+\tanh\left(dx+c\right)\right)}{2a+2b}}{d}$	81
default	$-\frac{\frac{b\ln\left(a+b\tanh\left(dx+c\right)^{2}\right)}{2\left(a+b\right)a}-\frac{\ln\left(\tanh\left(dx+c\right)\right)}{a}+\frac{\ln\left(1+\tanh\left(dx+c\right)\right)}{2a+2b}+\frac{\ln\left(-1+\tanh\left(dx+c\right)\right)}{2a+2b}}{d}$	81
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2c}{ad}+\frac{2bx}{a\left(a+b\right)}+\frac{2bc}{ad\left(a+b\right)}+\frac{\ln\left(\mathrm{e}^{2dx+2c}-1\right)}{ad}-\frac{b\ln\left(\mathrm{e}^{4dx+4c}+\frac{2\left(a-b\right)\mathrm{e}^{2dx+2c}}{a+b}+1\right)}{2ad\left(a+b\right)}$	117

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(-b*ln(a+b*tanh(d*x+c)^2)-2*ln(1-tanh(d*x+c))*a+(2*a+2*b)*ln(tanh(d*x+
c))-2*a*d*x)/a/d/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 118 vs. 2(58) = 116.

Time = 0.12 (sec) , antiderivative size = 118, normalized size of antiderivative = 1.97

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx =$$
$$-\frac{2\,adx+b\log\left(\frac{2\left((a+b)\cosh(dx+c)^2+(a+b)\sinh(dx+c)^2+a-b\right)}{\cosh(dx+c)^2-2\cosh(dx+c)\sinh(dx+c)+\sinh(dx+c)^2}\right)-2\,(a+b)\log\left(\frac{2\sinh(dx+c)}{\cosh(dx+c)-\sinh(dx+c)}\right)}{2\,(a^2+ab)d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a*d*x + b*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2
 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2)) - 2*(a + b)*log(2*sinh(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))))/((a
^2 + a*b)*d)
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+ab)d} + \frac{dx+c}{(a+b)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{ad} + \frac{\log\left(e^{(-dx-c)}-1\right)}{ad}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/2*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^2 + a*b)*d) + (d*x + c)/((a + b)*d) + log(e^(-d*x - c) + 1)/(a*d) + lo
g(e^(-d*x - c) - 1)/(a*d)
```

Giac [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.62

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{b\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^2+ab}+\frac{2\,(dx+c)}{a+b}-\frac{2\,\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a}}{2\,d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-1/2*(b*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)/(a^2 + a*b) + 2*(d*x + c)/(a + b) - 2*log(abs
(e^(2*d*x + 2*c) - 1))/a)/d
```

Mupad [B] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 194, normalized size of antiderivative = 3.23

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\ln\left(12\,a\,b^2+4\,a^2\,b+9\,b^3-9\,b^3\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-12\,a\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}-4\,a^2\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\right)}{a\,d} - \frac{b\,\ln\left(5\,a\,b+2\,a^2+3\,b^2+4\,a^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+2\,a^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}-6\,b^2\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+3\,b^2\,\mathrm{e}^{4\,c}\,\mathrm{e}^{4\,d\,x}+2\,a\,b\,\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}+5\right.}{2\,d\,a^2+2\,b\,d\,a} - \frac{x}{a+b}$$

input
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2),x)
```

output
```
log(12*a*b^2 + 4*a^2*b + 9*b^3 - 9*b^3*exp(2*c)*exp(2*d*x) - 12*a*b^2*exp(
2*c)*exp(2*d*x) - 4*a^2*b*exp(2*c)*exp(2*d*x))/(a*d) - (b*log(5*a*b + 2*a^
2 + 3*b^2 + 4*a^2*exp(2*c)*exp(2*d*x) + 2*a^2*exp(4*c)*exp(4*d*x) - 6*b^2*
exp(2*c)*exp(2*d*x) + 3*b^2*exp(4*c)*exp(4*d*x) + 2*a*b*exp(2*c)*exp(2*d*x
) + 5*a*b*exp(4*c)*exp(4*d*x)))/(2*a^2*d + 2*a*b*d) - x/(a + b)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 141, normalized size of antiderivative = 2.35

$$\int \frac{\coth(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{2\log\left(e^{dx+c}-1\right)a+2\log\left(e^{dx+c}-1\right)b+2\log\left(e^{dx+c}+1\right)a+2\log\left(e^{dx+c}+1\right)b-\log\left(e^{2dx+2c}\sqrt{a+b}+\right.}{2ad\,(a+b)}$$

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*log(e**(c + d*x) - 1)*a + 2*log(e**(c + d*x) - 1)*b + 2*log(e**(c + d*x
) + 1)*a + 2*log(e**(c + d*x) + 1)*b - log(e**(2*c + 2*d*x)*sqrt(a + b) +
sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b - log(e**(2*c + 2*d*x)*sqrt(a + b)
 + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b - 2*a*d*x)/(2*a*d*(a + b))
```

3.177 $\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)} dx$

Optimal result

Integrand size = 23, antiderivative size = 60

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{x}{a+b}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d}-\frac{\coth(c+dx)}{ad}$$

output
```
x/(a+b)-b^(3/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/(a+b)/d-coth(d
*x+c)/a/d
```

Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 67, normalized size of antiderivative = 1.12

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx=\frac{c+dx}{(a+b)d}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)d}-\frac{\coth(c+dx)}{ad}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(c + d*x)/((a + b)*d) - (b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(a^(3/2)*(a + b)*d) - Coth[c + d*x]/(a*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3042, 25, 4153, 25, 382, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \ 25$$

$$-\int \frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \ 4153$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 25$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 382$$

$$-\frac{\frac{\coth(c+dx)}{a}-\frac{\int \frac{b\tanh^2(c+dx)+a-b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}}{d}$$

$$\Big\downarrow \ 397$$

$$-\frac{\frac{\coth(c+dx)}{a}-\frac{\frac{a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^2\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}}{d}$$

$$\Big\downarrow \ 218$$

$$-\frac{\frac{\coth(c+dx)}{a}-\frac{\frac{a\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\frac{\coth(c+dx)}{a}-\frac{\frac{a\text{arctanh}(\tanh(c+dx))}{a+b}-\frac{b^{3/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{d}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-((-((-((b^(3/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)
)) + (a*ArcTanh[Tanh[c + d*x]])/(a + b))/a) + Coth[c + d*x]/a)/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.16 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

method	result	size
derivativedivides	$-\frac{\frac{b^2 \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a\sqrt{ab}}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}+\frac{1}{a\tanh(dx+c)}+\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	89
default	$-\frac{\frac{b^2 \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a\sqrt{ab}}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}+\frac{1}{a\tanh(dx+c)}+\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$	89
risch	$\frac{x}{a+b}-\frac{2}{ad\left(e^{2dx+2c}-1\right)}+\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right)}{2a^2(a+b)d}-\frac{\sqrt{-ab}\,b\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a^2(a+b)d}$	131

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(b^2/(a+b)/a/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))-1/(2*a+2*b
)*ln(1+tanh(d*x+c))+1/a/tanh(d*x+c)+1/(2*a+2*b)*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 230 vs. $2(52) = 104$.

Time = 0.12 (sec) , antiderivative size = 784, normalized size of antiderivative = 13.07

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
[1/2*(2*a*d*x*cosh(d*x + c)^2 + 4*a*d*x*cosh(d*x + c)*sinh(d*x + c) + 2*a*
d*x*sinh(d*x + c)^2 - 2*a*d*x + (b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*co
sh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2
 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^
2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*
b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x +
c))*sinh(d*x + c) - 4*((a^2 + a*b)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*
x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a)
)/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a
+ b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x +
 c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh
(d*x + c))*sinh(d*x + c) + a + b)) - 4*a - 4*b)/((a^2 + a*b)*d*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*sinh(d*
x + c)^2 - (a^2 + a*b)*d), (a*d*x*cosh(d*x + c)^2 + 2*a*d*x*cosh(d*x + c)*
sinh(d*x + c) + a*d*x*sinh(d*x + c)^2 - a*d*x - (b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - b)*sqrt(b/a)*arctan(1/2*(
(a + b)*cosh(d*x + c)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*
sinh(d*x + c)^2 + a - b)*sqrt(b/a)/b) - 2*a - 2*b)/((a^2 + a*b)*d*cosh(d*x
 + c)^2 + 2*(a^2 + a*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*d*s...
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(52) = 104.

Time = 0.17 (sec) , antiderivative size = 329, normalized size of antiderivative = 5.48

$$\begin{aligned}\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = &-\frac{b\log\left((a+b)e^{(4\,dx+4\,c)}+2\,(a-b)e^{(2\,dx+2\,c)}+a+b\right)}{4\,(a^2+ab)d} \\ &+\frac{b\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{4\,(a^2+ab)d} \\ &+\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+ab)\sqrt{ab}d} \\ &-\frac{(ab-b^2)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{4\,(a^2+ab)\sqrt{ab}d} \\ &+\frac{b\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\sqrt{ab}ad} \\ &+\frac{\log\left(e^{(2\,dx+2\,c)}-1\right)}{2\,ad}-\frac{\log\left(e^{(-2\,dx-2\,c)}-1\right)}{2\,ad} \\ &-\frac{1}{2\,(ae^{(2\,dx+2\,c)}-a)d}+\frac{3}{2\,(ae^{(-2\,dx-2\,c)}-a)d}\end{aligned}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/4*b*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((
a^2 + a*b)*d) + 1/4*b*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
 4*c) + a + b)/((a^2 + a*b)*d) + 1/4*(a*b - b^2)*arctan(1/2*((a + b)*e^(2*
d*x + 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqrt(a*b)*d) - 1/4*(a*b - b^2)
*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^2 + a*b)*sqr
t(a*b)*d) + 1/2*b*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))
/(sqrt(a*b)*a*d) + 1/2*log(e^(2*d*x + 2*c) - 1)/(a*d) - 1/2*log(e^(-2*d*x
- 2*c) - 1)/(a*d) - 1/2/((a*e^(2*d*x + 2*c) - a)*d) + 3/2/((a*e^(-2*d*x -
2*c) - a)*d)
```

Giac [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.48

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{b^2\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+ab)\sqrt{ab}} - \frac{dx+c}{a+b} + \frac{2}{a\left(e^{(2\,dx+2\,c)}-1\right)}}{d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
-(b^2*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b)
)/((a^2 + a*b)*sqrt(a*b)) - (d*x + c)/(a + b) + 2/(a*(e^(2*d*x + 2*c) - 1)
))/d
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 402, normalized size of antiderivative = 6.70

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} - \frac{\mathrm{atan}\left(\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(\frac{4\,b^2}{a\,d\,(a+b)^3\,(a^2+b\,a)\,\sqrt{b^3}} + \frac{\left(a^3\,d\,\sqrt{b^3}-a\,b^2\,d\,\sqrt{b^3}\right)(a-b)}{b^2\,(a+b)^2\,(a^2+b\,a)\,\sqrt{a^5\,d^2+2\,a^4\,b\,d^2+a^3\,b^2\,d^2}\,\sqrt{a^3\,d^2\,(a+b)^2}}\right) + \frac{(a-b)\,\big(\cdots}{b^2\,(a+b)^2\,(a^2+b\,a\cdots}\right.\right.}{\sqrt{\cdots}} - \frac{2}{a\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)}$$

input
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) - (atan((exp(2*c)*exp(2*d*x)*((4*b^2)/(a*d*(a + b)^3*(a*b + a^2)
*(b^3)^(1/2)) + ((a^3*d*(b^3)^(1/2) - a*b^2*d*(b^3)^(1/2))*(a - b))/(b^2*(
a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)*(a^3*d^2*
(a + b)^2)^(1/2))) + ((a - b)*(a^3*d*(b^3)^(1/2) + a*b^2*d*(b^3)^(1/2) + 2
*a^2*b*d*(b^3)^(1/2)))/(b^2*(a + b)^2*(a*b + a^2)*(a^5*d^2 + 2*a^4*b*d^2 +
 a^3*b^2*d^2)^(1/2)*(a^3*d^2*(a + b)^2)^(1/2)))*((a^3*(a^5*d^2 + 2*a^4*b*d
^2 + a^3*b^2*d^2)^(1/2))/2 + (a*b^2*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^
(1/2))/2 + a^2*b*(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2)))*(b^3)^(1/2)
)/(a^5*d^2 + 2*a^4*b*d^2 + a^3*b^2*d^2)^(1/2) - 2/(a*d*(exp(2*c + 2*d*x) -
 1))
```

Reduce [B] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 227, normalized size of antiderivative = 3.78

$$\int \frac{\coth^2(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{-e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b+e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}+\sqrt{}}{\sqrt{a}}\right.}{a^2d\left(e^{2dx+2c}a+e^{2dx+2c}b-\right.}$$

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2),x)
```

output
```
( - e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt
(b))/sqrt(a))*b + sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b)
)/sqrt(a))*b + e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
+ b) + sqrt(b))/sqrt(a))*b - sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b
) + sqrt(b))/sqrt(a))*b + e**(2*c + 2*d*x)*a**2*d*x - 2*e**(2*c + 2*d*x)*a
**2 - 2*e**(2*c + 2*d*x)*a*b - a**2*d*x)/(a**2*d*(e**(2*c + 2*d*x)*a + e**
(2*c + 2*d*x)*b - a - b))
```

3.178 $\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\coth^2(c+dx)}{2ad}+\frac{\log(\cosh(c+dx))}{(a+b)d}+\frac{(a-b)\log(\tanh(c+dx))}{a^2d}+\frac{b^2\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)d}$$

output
```
-1/2*coth(d*x+c)^2/a/d+ln(cosh(d*x+c))/(a+b)/d+(a-b)*ln(tanh(d*x+c))/a^2/d
+1/2*b^2*ln(a+b*tanh(d*x+c)^2)/a^2/(a+b)/d
```

Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.71

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = -\frac{\frac{\coth^2(c+dx)}{a}-\frac{b^2\log\left(b+a\coth^2(c+dx)\right)}{a^2(a+b)}-\frac{2\log(\sinh(c+dx))}{a+b}}{2d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
-1/2*(Coth[c + d*x]^2/a - (b^2*Log[b + a*Coth[c + d*x]^2])/(a^2*(a + b)) -
 (2*Log[Sinh[c + d*x]])/(a + b))/d
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\coth^3(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth^3(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(\frac{b^3}{a^2(a+b)(b\tanh^2(c+dx)+a)}+\frac{\coth^2(c+dx)}{a}+\frac{(a-b)\coth(c+dx)}{a^2}-\frac{1}{(a+b)(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+dx)}{2d}$$

$$\downarrow \text{2009}$$

$$\frac{\frac{b^2\log(a+b\tanh^2(c+dx))}{a^2(a+b)}+\frac{(a-b)\log(\tanh^2(c+dx))}{a^2}-\frac{\log(1-\tanh^2(c+dx))}{a+b}-\frac{\coth(c+dx)}{a}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-(Coth[c + d*x]/a) + ((a - b)*Log[Tanh[c + d*x]^2])/a^2 - Log[1 - Tanh[c
+ d*x]^2]/(a + b) + (b^2*Log[a + b*Tanh[c + d*x]^2])/(a^2*(a + b)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.04

method	result
parallelrisch	$\frac{b^2 \ln\left(a+b\tanh(dx+c)^2\right)-2\ln(1-\tanh(dx+c))a^2+(2a^2-2b^2)\ln(\tanh(dx+c))-\coth(dx+c)^2(a+b)a-2a^2dx}{2a^2d(a+b)}$
derivativedivides	$-\frac{-\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)a^2}+\frac{(-a+b)\ln(\tanh(dx+c))}{a^2}+\frac{1}{2a\tanh(dx+c)^2}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}+\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$
default	$-\frac{-\frac{b^2\ln\left(a+b\tanh(dx+c)^2\right)}{2(a+b)a^2}+\frac{(-a+b)\ln(\tanh(dx+c))}{a^2}+\frac{1}{2a\tanh(dx+c)^2}+\frac{\ln(1+\tanh(dx+c))}{2a+2b}+\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2x}{a}-\frac{2c}{ad}+\frac{2bx}{a^2}+\frac{2bc}{a^2d}-\frac{2b^2x}{a^2(a+b)}-\frac{2b^2c}{a^2d(a+b)}-\frac{2\,e^{2dx+2c}}{ad\left(e^{2dx+2c}-1\right)^2}+\frac{\ln\left(e^{2dx+2c}-1\right)}{ad}-\frac{\ln(e^{2dx+}}{a^2}$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
1/2*(b^2*ln(a+b*tanh(d*x+c)^2)-2*ln(1-tanh(d*x+c))*a^2+(2*a^2-2*b^2)*ln(ta
nh(d*x+c))-coth(d*x+c)^2*(a+b)*a-2*a^2*d*x)/a^2/d/(a+b)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 747 vs. $2(81) = 162$.

Time = 0.15 (sec) , antiderivative size = 747, normalized size of antiderivative = 8.79

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output
```
-1/2*(2*a^2*d*x*cosh(d*x + c)^4 + 8*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 2*a^2*d*x*sinh(d*x + c)^4 + 2*a^2*d*x - 4*(a^2*d*x - a^2 - a*b)*cosh(d*x
 + c)^2 + 4*(3*a^2*d*x*cosh(d*x + c)^2 - a^2*d*x + a^2 + a*b)*sinh(d*x + c
)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sin
h(d*x + c)^4 - 2*b^2*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 - b^2)*sin
h(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x
+ c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(c
osh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2)) - 2*((a
^2 - b^2)*cosh(d*x + c)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 +
(a^2 - b^2)*sinh(d*x + c)^4 - 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 -
b^2)*cosh(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 -
b^2)*cosh(d*x + c)^3 - (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*sin
h(d*x + c)/(cosh(d*x + c) - sinh(d*x + c))) + 8*(a^2*d*x*cosh(d*x + c)^3 -
 (a^2*d*x - a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + a^2*b)*d*cosh
(d*x + c)^4 + 4*(a^3 + a^2*b)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 + a^2
*b)*d*sinh(d*x + c)^4 - 2*(a^3 + a^2*b)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + a^
2*b)*d*cosh(d*x + c)^2 - (a^3 + a^2*b)*d)*sinh(d*x + c)^2 + (a^3 + a^2*b)*
d + 4*((a^3 + a^2*b)*d*cosh(d*x + c)^3 - (a^3 + a^2*b)*d*cosh(d*x + c))*si
nh(d*x + c))
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^3{(c+dx)}}{a+b\tanh^2{(c+dx)}}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 159, normalized size of antiderivative = 1.87

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+a^2b)d} + \frac{dx+c}{(a+b)d} + \frac{2\,e^{(-2\,dx-2\,c)}}{(2\,ae^{(-2\,dx-2\,c)}-ae^{(-4\,dx-4\,c)}-a)d} + \frac{(a-b)\log\left(e^{(-dx-c)}+1\right)}{a^2d} + \frac{(a-b)\log\left(e^{(-dx-c)}-1\right)}{a^2d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
1/2*b^2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)
/((a^3 + a^2*b)*d) + (d*x + c)/((a + b)*d) + 2*e^(-2*d*x - 2*c)/((2*a*e^(-
2*d*x - 2*c) - a*e^(-4*d*x - 4*c) - a)*d) + (a - b)*log(e^(-d*x - c) + 1)/
(a^2*d) + (a - b)*log(e^(-d*x - c) - 1)/(a^2*d)
```

Giac [A] (verification not implemented)

Time = 0.17 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.56

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{b^2\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^3+a^2b} - \frac{2\,(dx+c)}{a+b} + \frac{2\,(a-b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^2} - \frac{4\,e^{(2\,dx+2\,c)}}{a\left(e^{(2\,dx+2\,c)}-1\right)^2}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/2*(b^2*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
 2*b*e^(2*d*x + 2*c) + a + b)/(a^3 + a^2*b) - 2*(d*x + c)/(a + b) + 2*(a -
 b)*log(abs(e^(2*d*x + 2*c) - 1))/a^2 - 4*e^(2*d*x + 2*c)/(a*(e^(2*d*x + 2
*c) - 1)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.64 (sec) , antiderivative size = 313, normalized size of antiderivative = 3.68

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{b^2\ \ln\left(3\,a\,b^2-2\,a^2\,b-2\,a^3+3\,b^3-4\,a^3\,e^{2\,c}\,e^{2\,d\,x}-2\,a^3\,e^{4\,c}\,e^{4\,d\,x}-6\,b^3\,e^{2\,c}\,e^{2\,d\,x}+3\,b^3\,e^{4\,c}\,e^{4\,d\,x}+6\,a\,b^2\,e^{2\,c}\right.}{2\,d\,a^3+2\,b\,d\,a^2} - \frac{x}{a+b} - \frac{2}{a\,d\,\left(e^{4\,c+4\,d\,x}-2\,e^{2\,c+2\,d\,x}+1\right)} + \frac{\ln\left(4\,a^4\,b+9\,b^5-12\,a^2\,b^3-9\,b^5\,e^{2\,c}\,e^{2\,d\,x}-4\,a^4\,b\,e^{2\,c}\,e^{2\,d\,x}+12\,a^2\,b^3\,e^{2\,c}\,e^{2\,d\,x}\right)\,(a-b)}{a^2\,d} - \frac{2\,(a^2+b\,a)}{a^2\,d\,\left(e^{2\,c+2\,d\,x}-1\right)\,(a+b)}$$

input
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2),x)
```

output
```
(b^2*log(3*a*b^2 - 2*a^2*b - 2*a^3 + 3*b^3 - 4*a^3*exp(2*c)*exp(2*d*x) - 2
*a^3*exp(4*c)*exp(4*d*x) - 6*b^3*exp(2*c)*exp(2*d*x) + 3*b^3*exp(4*c)*exp(
4*d*x) + 6*a*b^2*exp(2*c)*exp(2*d*x) + 4*a^2*b*exp(2*c)*exp(2*d*x) + 3*a*b
^2*exp(4*c)*exp(4*d*x) - 2*a^2*b*exp(4*c)*exp(4*d*x)))/(2*a^3*d + 2*a^2*b*
d) - x/(a + b) - 2/(a*d*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) + (lo
g(4*a^4*b + 9*b^5 - 12*a^2*b^3 - 9*b^5*exp(2*c)*exp(2*d*x) - 4*a^4*b*exp(2
*c)*exp(2*d*x) + 12*a^2*b^3*exp(2*c)*exp(2*d*x))*(a - b))/(a^2*d) - (2*(a*
b + a^2))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

Reduce [B] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 661, normalized size of antiderivative = 7.78

$$\int \frac{\coth^3(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{-2e^{2dx+2c}\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}-2e^{dx+c}\sqrt{b}\right)b^2-2e^{2dx+2c}\log\left(e^{2dx+2c}\sqrt{a+b}+\sqrt{a+b}+2e^{dx+c}\right.}{}$$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2),x)
```

output
```
(2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**2 - 2*e**(4*c + 4*d*x)*log(e*
*(c + d*x) - 1)*b**2 + 2*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**2 - 2*e
**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*b**2 + e**(4*c + 4*d*x)*log(e**(2*c
+ 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2 + e**(4*
c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)
*sqrt(b))*b**2 - 2*e**(4*c + 4*d*x)*a**2*d*x - 2*e**(4*c + 4*d*x)*a**2 - 2
*e**(4*c + 4*d*x)*a*b - 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*a**2 + 4*
e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**2 - 4*e**(2*c + 2*d*x)*log(e**(c
 + d*x) + 1)*a**2 + 4*e**(2*c + 2*d*x)*log(e**(c + d*x) + 1)*b**2 - 2*e**(
2*c + 2*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*
x)*sqrt(b))*b**2 - 2*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + s
qrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b**2 + 4*e**(2*c + 2*d*x)*a**2*d*x +
2*log(e**(c + d*x) - 1)*a**2 - 2*log(e**(c + d*x) - 1)*b**2 + 2*log(e**(c
+ d*x) + 1)*a**2 - 2*log(e**(c + d*x) + 1)*b**2 + log(e**(2*c + 2*d*x)*sqr
t(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**2 + log(e**(2*c + 2*d*
x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*b**2 - 2*a**2*d*x -
 2*a**2 - 2*a*b)/(2*a**2*d*(e**(4*c + 4*d*x)*a + e**(4*c + 4*d*x)*b - 2*e*
*(2*c + 2*d*x)*a - 2*e**(2*c + 2*d*x)*b + a + b))
```

3.179 $\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 82

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b} + \frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)d} - \frac{(a-b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

output
```
x/(a+b)+b^(5/2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2)/(a+b)/d-(a-b)*
coth(d*x+c)/a^2/d-1/3*coth(d*x+c)^3/a/d
```

Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.11

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{\frac{6\left(c+dx+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}}\right)}{a+b} - \frac{(-2a+3b+(4a-3b)\cosh(2(c+dx)))\coth(c+dx)\operatorname{csch}^2(c+dx)}{a^2}}{6d}$$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
((6*(c + d*x + (b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(5/2)))
/(a + b) - ((-2*a + 3*b + (4*a - 3*b)*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csc
h[c + d*x]^2)/a^2)/(6*d)
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4153, 382, 27, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\tan(ic+idx)^4\,(a-b\tan(ic+idx)^2)}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\coth^4(c+dx)}{(1-\tanh^2(c+dx))\,(b\tanh^2(c+dx)+a)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{382}$$

$$\frac{\frac{\int \frac{3\coth^2(c+dx)\left(b\tanh^2(c+dx)+a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{3a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{\int \frac{\coth^2(c+dx)\left(b\tanh^2(c+dx)+a-b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\frac{\int -\frac{a^2-ba+b^2+(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{a^2-ba+b^2+(a-b)b\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^3\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

$\Big\downarrow$ 219

$$\frac{\frac{\frac{\frac{a^2\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{b^{5/2}\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(a-b)\coth(c+dx)}{a}}{a}-\frac{\coth^3(c+dx)}{3a}}{d}$$

input
```
Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2),x]
```

output
```
(-1/3*Coth[c + d*x]^3/a + (((b^(5/2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a
]])/(Sqrt[a]*(a + b)) + (a^2*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((a - b)
*Coth[c + d*x])/a)/a)/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 109, normalized size of antiderivative = 1.33

method	result
derivativedivides	$-\frac{-\frac{b^3 \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a^2\sqrt{ab}}-\frac{-a+b}{a^2\tanh(dx+c)}+\frac{1}{3a\tanh(dx+c)^3}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}+\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$
default	$-\frac{-\frac{b^3 \arctan\left(\frac{b \tanh(dx+c)}{\sqrt{ab}}\right)}{(a+b)a^2\sqrt{ab}}-\frac{-a+b}{a^2\tanh(dx+c)}+\frac{1}{3a\tanh(dx+c)^3}-\frac{\ln(1+\tanh(dx+c))}{2a+2b}+\frac{\ln(-1+\tanh(dx+c))}{2a+2b}}{d}$
risch	$\frac{x}{a+b}-\frac{2\left(6a\,\mathrm{e}^{4dx+4c}-3b\,\mathrm{e}^{4dx+4c}-6a\,\mathrm{e}^{2dx+2c}+6b\,\mathrm{e}^{2dx+2c}+4a-3b\right)}{3d\,a^2\left(\mathrm{e}^{2dx+2c}-1\right)^3}+\frac{\sqrt{-ab}\,b^2\ln\left(\mathrm{e}^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{2a^3(a+b)d}-\sqrt{-}$

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-b^3/(a+b)/a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2))-1/a^2*(
-a+b)/tanh(d*x+c)+1/3/a/tanh(d*x+c)^3-1/(2*a+2*b)*ln(1+tanh(d*x+c))+1/(2*a
+2*b)*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1023 vs. $2(72) = 144$.

Time = 0.14 (sec) , antiderivative size = 2368, normalized size of antiderivative = 28.88

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input

```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="fricas")
```

output

```
[1/6*(6*a^2*d*x*cosh(d*x + c)^6 + 36*a^2*d*x*cosh(d*x + c)*sinh(d*x + c)^5
 + 6*a^2*d*x*sinh(d*x + c)^6 - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(
d*x + c)^4 + 6*(15*a^2*d*x*cosh(d*x + c)^2 - 3*a^2*d*x - 4*a^2 - 2*a*b + 2
*b^2)*sinh(d*x + c)^4 - 6*a^2*d*x + 24*(5*a^2*d*x*cosh(d*x + c)^3 - (3*a^2
*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(3*a^2*d*
x + 4*a^2 - 4*b^2)*cosh(d*x + c)^2 + 6*(15*a^2*d*x*cosh(d*x + c)^4 + 3*a^2
*d*x - 6*(3*a^2*d*x + 4*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*b
^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*
x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x
 + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x
 + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4
- 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c
)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)
*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(d*x
 + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 - b^2
)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*
sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x + c)
^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b)*cosh(d*x +
c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b)*sinh(d*x +
c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh...
```

Sympy [F]

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2),x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2), x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1038 vs. $2(72) = 144$.

Time = 0.26 (sec) , antiderivative size = 1038, normalized size of antiderivative = 12.66

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="maxima")
```

output
```
-1/8*(a*b - b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) +
 a + b)/((a^3 + a^2*b)*d) + 1/8*(a*b - b^2)*log(2*(a - b)*e^(-2*d*x - 2*c)
 + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^3 + a^2*b)*d) + 1/16*(a^2*b - 6*a
*b^2 + b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3
+ a^2*b)*sqrt(a*b)*d) - 1/16*(a^2*b - 6*a*b^2 + b^3)*arctan(1/2*((a + b)*e
^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + a^2*b)*sqrt(a*b)*d) - 1/24*(3*
(12*a - b)*e^(4*d*x + 4*c) - 6*(9*a - b)*e^(2*d*x + 2*c) + 22*a - 3*b)/((a
^2*e^(6*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - a^2)*
d) - 1/6*(3*(4*a - b)*e^(4*d*x + 4*c) - 6*(2*a - b)*e^(2*d*x + 2*c) + 4*a
- 3*b)/((a^2*e^(6*d*x + 6*c) - 3*a^2*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*
c) - a^2)*d) - 1/24*(6*(9*a - b)*e^(-2*d*x - 2*c) - 3*(12*a - b)*e^(-4*d*x
 - 4*c) - 22*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-4*d*x - 4*c) +
a^2*e^(-6*d*x - 6*c) - a^2)*d) - 1/6*(6*(2*a - b)*e^(-2*d*x - 2*c) - 3*(4*
a - b)*e^(-4*d*x - 4*c) - 4*a + 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2*e^(-
4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*(6*(a + b)*e^(-2*d*x -
 2*c) - 3*b*e^(-4*d*x - 4*c) - 2*a - 3*b)/((3*a^2*e^(-2*d*x - 2*c) - 3*a^2
*e^(-4*d*x - 4*c) + a^2*e^(-6*d*x - 6*c) - a^2)*d) + 1/4*b*log((a + b)*e^(
4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/(a^2*d) - 1/4*b*log(2*(a
 - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/(a^2*d) + 1/4*(
2*a - b)*log(e^(2*d*x + 2*c) - 1)/(a^2*d) - 1/2*b*log(e^(2*d*x + 2*c) -...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(72) = 144$.

Time = 0.16 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.79

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx$$
$$= \frac{\frac{3\,b^3\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+a^2b)\sqrt{ab}}+\frac{3\,(dx+c)}{a+b}-\frac{2\,\left(6\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+6\,be^{(2\,dx+2\,c)}+4\,a-3\,b\right)}{a^2\left(e^{(2\,dx+2\,c)}-1\right)^3}}{3\,d}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x, algorithm="giac")
```

output
```
1/3*(3*b^3*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt
(a*b))/((a^3 + a^2*b)*sqrt(a*b)) + 3*(d*x + c)/(a + b) - 2*(6*a*e^(4*d*x +
 4*c) - 3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) +
4*a - 3*b)/(a^2*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 519, normalized size of antiderivative = 6.33

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{x}{a+b}$$
$$+\frac{\mathrm{atan}\left(\left(\mathrm{e}^{2\,c}\,\mathrm{e}^{2\,d\,x}\left(\frac{4\,b^3}{a^2\,d\,(a+b)^3\,(a^3+b\,a^2)\,\sqrt{b^5}}+\frac{\left(a^4\,d\,\sqrt{b^5}-a^2\,b^2\,d\,\sqrt{b^5}\right)(a-b)}{b^3\,(a+b)^2\,(a^3+b\,a^2)\,\sqrt{a^7\,d^2+2\,a^6\,b\,d^2+a^5\,b^2\,d^2}\,\sqrt{a^5\,d^2\,(a+b)^2}}\right)+\frac{(a-b}{b^3\,(a+b)^2\,(a^3-}\right.\right.}{}$$
$$-\frac{8}{3\,a\,d\,\left(3\,\mathrm{e}^{2\,c+2\,d\,x}-3\,\mathrm{e}^{4\,c+4\,d\,x}+\mathrm{e}^{6\,c+6\,d\,x}-1\right)}$$
$$-\frac{4\,(a^2+b\,a)}{a^2\,d\,(a+b)\,\left(\mathrm{e}^{4\,c+4\,d\,x}-2\,\mathrm{e}^{2\,c+2\,d\,x}+1\right)}-\frac{2\,(2\,a^2+a\,b-b^2)}{a^2\,d\,\left(\mathrm{e}^{2\,c+2\,d\,x}-1\right)\,(a+b)}$$

input
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2),x)
```

output
```
x/(a + b) + (atan((exp(2*c)*exp(2*d*x)*((4*b^3)/(a^2*d*(a + b)^3*(a^2*b +
a^3)*(b^5)^(1/2)) + ((a^4*d*(b^5)^(1/2) - a^2*b^2*d*(b^5)^(1/2))*(a - b))/
(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2)*(
a^5*d^2*(a + b)^2)^(1/2))) + ((a - b)*(a^4*d*(b^5)^(1/2) + 2*a^3*b*d*(b^5)
^(1/2) + a^2*b^2*d*(b^5)^(1/2)))/(b^3*(a + b)^2*(a^2*b + a^3)*(a^7*d^2 + 2
*a^6*b*d^2 + a^5*b^2*d^2)^(1/2)*(a^5*d^2*(a + b)^2)^(1/2)))*((a^4*(a^7*d^2
 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2 + a^3*b*(a^7*d^2 + 2*a^6*b*d^2 + a^
5*b^2*d^2)^(1/2) + (a^2*b^2*(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2))/2
))*(b^5)^(1/2))/(a^7*d^2 + 2*a^6*b*d^2 + a^5*b^2*d^2)^(1/2) - 8/(3*a*d*(3*
exp(2*c + 2*d*x) - 3*exp(4*c + 4*d*x) + exp(6*c + 6*d*x) - 1)) - (4*(a*b +
 a^2))/(a^2*d*(a + b)*(exp(4*c + 4*d*x) - 2*exp(2*c + 2*d*x) + 1)) - (2*(a
*b + 2*a^2 - b^2))/(a^2*d*(exp(2*c + 2*d*x) - 1)*(a + b))
```

Reduce [B] (verification not implemented)

Time = 16.55 (sec) , antiderivative size = 550, normalized size of antiderivative = 6.71

$$\int \frac{\coth^4(c+dx)}{a+b\tanh^2(c+dx)}\,dx = \frac{3e^{6dx+6c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b^2-9e^{4dx+4c}\sqrt{b}\,\sqrt{a}\,atan\left(\frac{e^{dx+c}\sqrt{a+b}-\sqrt{b}}{\sqrt{a}}\right)b^2+9e^{2dx+2c}\sqrt{b}\,\sqrt{a}\,atan\left(e\right.}{}$$

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2),x)
```

output
```
(3*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(
b))/sqrt(a))*b**2 - 9*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*
sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 + 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*
atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 - 3*sqrt(b)*sqrt(a
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**2 - 3*e**(6*c + 6*
d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b*
*2 + 9*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) + s
qrt(b))/sqrt(a))*b**2 - 9*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d
*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + 3*sqrt(b)*sqrt(a)*atan((e**(c +
 d*x)*sqrt(a + b) + sqrt(b))/sqrt(a))*b**2 + 3*e**(6*c + 6*d*x)*a**3*d*x -
 4*e**(6*c + 6*d*x)*a**3 - 2*e**(6*c + 6*d*x)*a**2*b + 2*e**(6*c + 6*d*x)*
a*b**2 - 9*e**(4*c + 4*d*x)*a**3*d*x + 9*e**(2*c + 2*d*x)*a**3*d*x - 6*e**
(2*c + 2*d*x)*a**2*b - 6*e**(2*c + 2*d*x)*a*b**2 - 3*a**3*d*x - 4*a**3 + 4
*a*b**2)/(3*a**3*d*(e**(6*c + 6*d*x)*a + e**(6*c + 6*d*x)*b - 3*e**(4*c +
4*d*x)*a - 3*e**(4*c + 4*d*x)*b + 3*e**(2*c + 2*d*x)*a + 3*e**(2*c + 2*d*x
)*b - a - b))
```

3.180 $\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 83

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2d} - \frac{a(a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2b^2(a+b)^2d} - \frac{a^2}{2b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d-1/2*a*(a+2*b)*ln(a+b*tanh(d*x+c)^2)/b^2/(a+b)^2/
d-1/2*a^2/b^2/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 69, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{-2\log(\cosh(c+dx)) + \frac{a(a+2b)\log(a+b\tanh^2(c+dx))}{b^2} + \frac{a^2(a+b)}{b^2(a+b\tanh^2(c+dx))}}{2(a+b)^2d}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/2*(-2*Log[Cosh[c + d*x]] + (a*(a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/b^2
 + (a^2*(a + b))/(b^2*(a + b*Tanh[c + d*x]^2)))/((a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.01, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \ 3042$$

$$\int -\frac{i\tan(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \ 26$$

$$-i\int \frac{\tan(ic+idx)^5}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \ 4153$$

$$-\frac{i\int \frac{i\tanh^5(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 26$$

$$\frac{\int \frac{\tanh^5(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \ 354$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \ 99$$

$$\frac{\int \left(\frac{a^2}{b(a+b)(b\tanh^2(c+dx)+a)^2}-\frac{(a+2b)a}{b(a+b)^2(b\tanh^2(c+dx)+a)}-\frac{1}{(a+b)^2(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{a^2}{b^2(a+b)(a+b\tanh^2(c+dx))}-\frac{a(a+2b)\log(a+b\tanh^2(c+dx))}{b^2(a+b)^2}-\frac{\log(1-\tanh^2(c+dx))}{(a+b)^2}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^2) - (a*(a + 2*b)*Log[a + b*Tanh[c + d
*x]^2])/(b^2*(a + b)^2) - a^2/(b^2*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{(a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b^2}+\frac{a(a+b)}{b^2\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{-\frac{a\left(\frac{(a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b^2}+\frac{a(a+b)}{b^2\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{a^3+a^2b+\ln\left(a+b\tanh(dx+c)^2\right)a^3+2a\,b^2dx+2\ln(1-\tanh(dx+c))a\,b^2+2\ln\left(a+b\tanh(dx+c)^2\right)a^2b+2\ln(1-\tanh(\cdots}{2\left(a+b\tan\cdots\right.}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{b^2}-\frac{2c}{b^2d}+\frac{2a^2x}{b^2(a^2+2ab+b^2)}+\frac{2a^2c}{b^2d(a^2+2ab+b^2)}+\frac{4ax}{b(a^2+2ab+b^2)}+\frac{4ac}{bd(a^2+2ab+b^2)}-\frac{\cdots}{bd(\cdots}$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2*a/(a+b)^2*((a+2*b)/b^2*ln(a+b*tanh(d*x+c)^2)+a*(a+b)/b^2/(a+b*ta
nh(d*x+c)^2))-1/2/(a+b)^2*ln(1+tanh(d*x+c))-1/2/(a+b)^2*ln(-1+tanh(d*x+c))
)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1141 vs. $2(79) = 158$.

Time = 0.18 (sec) , antiderivative size = 1141, normalized size of antiderivative = 13.75

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a*b^2 + b^3)*d*x*cosh(d*x + c)^4 + 8*(a*b^2 + b^3)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^3 + 2*(a*b^2 + b^3)*d*x*sinh(d*x + c)^4 + 2*(a*b^2 + b^3
)*d*x + 4*(a^2*b + (a*b^2 - b^3)*d*x)*cosh(d*x + c)^2 + 4*(3*(a*b^2 + b^3)
*d*x*cosh(d*x + c)^2 + a^2*b + (a*b^2 - b^3)*d*x)*sinh(d*x + c)^2 + ((a^3
+ 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*
x + c)*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 2*a*b^2)*sinh(d*x + c)^4 + a^3 +
 3*a^2*b + 2*a*b^2 + 2*(a^3 + a^2*b - 2*a*b^2)*cosh(d*x + c)^2 + 2*(a^3 +
a^2*b - 2*a*b^2 + 3*(a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*((a^3 + 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + (a^3 + a^2*b - 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)
*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4
+ 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 -
 b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^
2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*cosh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) + 8*((a*b^2 + b^3)*d*x*cosh(d*x + c)^3 + (a^2*b + (a*b^2
 - b^3)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b^2 + 3*a^2*b^3 + 3*a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 217 vs. $2(79) = 158$.

Time $= 0.13$ (sec) , antiderivative size $= 217$, normalized size of antiderivative $= 2.61$

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{2\,a^2e^{(-2\,dx-2\,c)}}{(a^3b+3\,a^2b^2+3\,ab^3+b^4+2\,(a^3b+a^2b^2-ab^3-b^4)e^{(-2\,dx-2\,c)}+(a^3b+3\,a^2b^2+3\,ab^3+b^4)e^{(-4\,dx-4\,c)}} - \frac{(a^2+2\,ab)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2b^2+2\,ab^3+b^4)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-2\,dx-2\,c)}+1\right)}{b^2d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-2*a^2*e^(-2*d*x - 2*c)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4 + 2*(a^3*b + a
^2*b^2 - a*b^3 - b^4)*e^(-2*d*x - 2*c) + (a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^
4)*e^(-4*d*x - 4*c))*d) - 1/2*(a^2 + 2*a*b)*log(2*(a - b)*e^(-2*d*x - 2*c)
 + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*d) + (d*x
+ c)/((a^2 + 2*a*b + b^2)*d) + log(e^(-2*d*x - 2*c) + 1)/(b^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. $2(79) = 158$.

Time = 0.24 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.34

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{\frac{(a^2+2\,ab)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^2b^2+2\,ab^3+b^4}+\frac{2\,(dx+c)}{a^2+2\,ab+b^2}+\frac{4\,a^2e^{(2\,dx+2\,c)}}{(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-\cdots}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((a^2 + 2*a*b)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^2*b^2 + 2*a*b^3 + b^4) + 2*(d*
x + c)/(a^2 + 2*a*b + b^2) + 4*a^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b
*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a +
 b)^2*b) - 2*log(e^(2*d*x + 2*c) + 1)/b^2)/d
```

Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.05

$$\begin{aligned}\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx &= -\frac{a^2}{2\,\left(d\,a^2\,b^2+d\,a\,b^3\,\tanh\left(c+d\,x\right)^2+d\,a\,b^3+d\,b^4\,\tanh\left(c+d\,x\right)^2\right)} \\ &\quad -\frac{\ln\left(\tanh(c+d\,x)^2-1\right)}{2\,\left(d\,a^2+2\,d\,a\,b+d\,b^2\right)}-\frac{a^2\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\,\left(d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4\right)} \\ &\quad -\frac{a\,b\,\ln\left(b\tanh(c+d\,x)^2+a\right)}{d\,a^2\,b^2+2\,d\,a\,b^3+d\,b^4}\end{aligned}$$

input
```
int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
- a^2/(2*(a^2*b^2*d + b^4*d*tanh(c + d*x)^2 + a*b^3*d + a*b^3*d*tanh(c + d
*x)^2)) - log(tanh(c + d*x)^2 - 1)/(2*(a^2*d + b^2*d + 2*a*b*d)) - (a^2*lo
g(a + b*tanh(c + d*x)^2))/(2*(b^4*d + a^2*b^2*d + 2*a*b^3*d)) - (a*b*log(a
 + b*tanh(c + d*x)^2))/(b^4*d + a^2*b^2*d + 2*a*b^3*d)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 3130, normalized size of antiderivative = 37.71

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - e**(4*c + 4*d*x)*log(tanh(c + d*x)**2*b + a)*tanh(c + d*x)**2*a**5*b -
 2*e**(4*c + 4*d*x)*log(tanh(c + d*x)**2*b + a)*tanh(c + d*x)**2*a**4*b**2
 + 2*e**(4*c + 4*d*x)*log(tanh(c + d*x)**2*b + a)*tanh(c + d*x)**2*a**2*b*
*4 + e**(4*c + 4*d*x)*log(tanh(c + d*x)**2*b + a)*tanh(c + d*x)**2*a*b**5
- e**(4*c + 4*d*x)*log(tanh(c + d*x)**2*b + a)*a**6 - 2*e**(4*c + 4*d*x)*l
og(tanh(c + d*x)**2*b + a)*a**5*b + 2*e**(4*c + 4*d*x)*log(tanh(c + d*x)**
2*b + a)*a**3*b**3 + e**(4*c + 4*d*x)*log(tanh(c + d*x)**2*b + a)*a**2*b**
4 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e*
*(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**3*b**3 - e**(4*c + 4*d*x)*log(e**(
2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c +
d*x)**2*a*b**5 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(
a + b) - 2*e**(c + d*x)*sqrt(b))*a**4*b**2 - e**(4*c + 4*d*x)*log(e**(2*c
+ 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b**4 + e
**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c +
 d*x)*sqrt(b))*tanh(c + d*x)**2*a**3*b**3 - e**(4*c + 4*d*x)*log(e**(2*c +
 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)*
*2*a*b**5 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b
) + 2*e**(c + d*x)*sqrt(b))*a**4*b**2 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d
*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**2*b**4 + e**(4*
c + 4*d*x)*tanh(c + d*x)**2*a**5*b + e**(4*c + 4*d*x)*tanh(c + d*x)**2*...
```

3.181 $\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 89

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{x}{(a+b)^2} - \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^{3/2}(a+b)^2d} + \frac{a\tanh(c+dx)}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2-1/2*a^(1/2)*(a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/b^(3/2)/
(a+b)^2/d+1/2*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{2(c+dx) - \frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{3/2}} + \frac{a(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{2(a+b)^2d}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(c + d*x) - (Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])
/b^(3/2) + (a*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (a + b)*Cosh[2*(c + d
*x)])))/(2*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3042, 4153, 372, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ic+idx)^4}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int \frac{a-(a+2b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2b(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{a(a+3b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}-\frac{2b\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2b(a+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)(a+b\tanh^2(c+dx))}-\frac{\frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{2b\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2b(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\tanh(c+dx)}{2b(a+b)(a+b\tanh^2(c+dx))}-\frac{\frac{\sqrt{a}(a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{2b\,\text{arctanh}(\tanh(c+dx))}{a+b}}{2b(a+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-1/2*((Sqrt[a]*(a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[b
]*(a + b)) - (2*b*ArcTanh[Tanh[c + d*x]])/(a + b))/(b*(a + b)) + (a*Tanh[c
 + d*x])/(2*b*(a + b)*(a + b*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\tanh(dx+c)^2\right)}+\frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{-\frac{a\left(-\frac{(a+b)\tanh(dx+c)}{2b\left(a+b\tanh(dx+c)^2\right)}+\frac{(a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2b\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{a\left(a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b\right)}{d(a+b)^2b\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{\sqrt{-ab}\,\ln\left(e^{2dx+2c}-\frac{-a+2\sqrt{-ab}+b}{a+b}\right.}{4b^2(a+b)^2d}$

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/(a+b)^2*(-1/2*(a+b)/b*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(a+3*b)/
b/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(1+tanh(d*x
+c))-1/2/(a+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 814 vs. $2(77) = 154$.

Time = 0.14 (sec) , antiderivative size = 1950, normalized size of antiderivative = 21.91

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a*b + b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b + b^2)*d*x*cosh(d*x + c)
*sinh(d*x + c)^3 + 4*(a*b + b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b + b^2)*d*x +
 4*(2*(a*b - b^2)*d*x - a^2 + a*b)*cosh(d*x + c)^2 + 4*(6*(a*b + b^2)*d*x*
cosh(d*x + c)^2 + 2*(a*b - b^2)*d*x - a^2 + a*b)*sinh(d*x + c)^2 + ((a^2 +
 4*a*b + 3*b^2)*cosh(d*x + c)^4 + 4*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (a^2 + 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 2*(a^2 + 2*a*b - 3
*b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 4*a*b + 3*b^2)*cosh(d*x + c)^2 + a^2 +
 2*a*b - 3*b^2)*sinh(d*x + c)^2 + a^2 + 4*a*b + 3*b^2 + 4*((a^2 + 4*a*b +
3*b^2)*cosh(d*x + c)^3 + (a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-a/b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 +
 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*
cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a*b + b^2
)*cosh(d*x + c)^2 + 2*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b + b^2
)*sinh(d*x + c)^2 + a*b - b^2)*sqrt(-a/b))/((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) - 4*a^2 - 4*a*b + 8*(2*(a*b + b^2)*d*x*cosh(d*x + c)^3 + (2*(a*b - b^...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1010 vs. $2(77) = 154$.

Time = 0.37 (sec) , antiderivative size = 1010, normalized size of antiderivative = 11.35

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/32*(a^3 + 9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(2*d*x + 2*c)
+ a - b)/sqrt(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) + 1/32*(a^3
+ 9*a^2*b - 9*a*b^2 - b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/s
qrt(a*b))/((a^3*b + 2*a^2*b^2 + a*b^3)*sqrt(a*b)*d) - 1/16*(a^3 - 5*a^2*b
- 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 - b^3)*e^(2*d*x + 2*c))/((a^4
*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + (a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^
4)*e^(4*d*x + 4*c) + 2*(a^4*b + a^3*b^2 - a^2*b^3 - a*b^4)*e^(2*d*x + 2*c)
)*d) + 1/16*(a^3 - 5*a^2*b - 5*a*b^2 + b^3 + (a^3 - 15*a^2*b + 15*a*b^2 -
b^3)*e^(-2*d*x - 2*c))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4 + 2*(a^4*b
+ a^3*b^2 - a^2*b^3 - a*b^4)*e^(-2*d*x - 2*c) + (a^4*b + 3*a^3*b^2 + 3*a^2
*b^3 + a*b^4)*e^(-4*d*x - 4*c))*d) - 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*
e^(2*d*x + 2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + (a^3*b + 2*a^2*b^2 + a*b^3)
*e^(4*d*x + 4*c) + 2*(a^3*b - a*b^3)*e^(2*d*x + 2*c))*d) + 1/4*(a^2 - b^2
+ (a^2 - 6*a*b + b^2)*e^(-2*d*x - 2*c))/((a^3*b + 2*a^2*b^2 + a*b^3 + 2*(a
^3*b - a*b^3)*e^(-2*d*x - 2*c) + (a^3*b + 2*a^2*b^2 + a*b^3)*e^(-4*d*x - 4
*c))*d) + 3/8*((a - b)*e^(-2*d*x - 2*c) + a + b)/((a^2*b + a*b^2 + 2*(a^2*
b - a*b^2)*e^(-2*d*x - 2*c) + (a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*l
og((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*
a*b + b^2)*d) - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4
*c) + a + b)/((a^2 + 2*a*b + b^2)*d) - 1/8*(a + b)*arctan(1/2*((a + b)*...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(77) = 154$.

Time = 0.20 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{\frac{(a^2+3ab)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^2b+2ab^2+b^3)\sqrt{ab}} - \frac{2(dx+c)}{a^2+2ab+b^2} + \frac{2\left(a^2e^{(2dx+2c)}-abe^{(2dx+2c)}+a^2+ab\right)}{(a^2b+2ab^2+b^3)\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx\cdots}}}{2d}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((a^2 + 3*a*b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a
- b)/sqrt(a*b))/((a^2*b + 2*a*b^2 + b^3)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2
*a*b + b^2) + 2*(a^2*e^(2*d*x + 2*c) - a*b*e^(2*d*x + 2*c) + a^2 + a*b)/((
a^2*b + 2*a*b^2 + b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 1655, normalized size of antiderivative = 18.60

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^2*d + 2*b^2*d + 4*a*b*d) - log(tanh(c + d*x) -
 1)/(2*d*(a + b)^2) - (atan((((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*
a^3*b + a^4 + 4*b^4 + 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))
+ ((a + 3*b)*(-a*b^3)^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2
 + 8*a^4*b^3*d^2 + 2*a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a
^2*b^2*d^3) - (tanh(c + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b
^7*d^2 + 32*a^2*b^6*d^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2
))/(8*(b^5*d + a^2*b^3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))
))/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^
4*d)) + ((a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 +
 9*a^2*b^2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) - ((a + 3*b)*(-a*b^3)
^(1/2)*((2*a*b^6*d^2 + 8*a^2*b^5*d^2 + 12*a^3*b^4*d^2 + 8*a^4*b^3*d^2 + 2*
a^5*b^2*d^2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) + (tanh(c
 + d*x)*(a + 3*b)*(-a*b^3)^(1/2)*(16*b^8*d^2 + 48*a*b^7*d^2 + 32*a^2*b^6*d
^2 - 32*a^3*b^5*d^2 - 48*a^4*b^4*d^2 - 16*a^5*b^3*d^2))/(8*(b^5*d + a^2*b^
3*d + 2*a*b^4*d)*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2))))/(4*(b^5*d + a^2*b^
3*d + 2*a*b^4*d)))*1i)/(4*(b^5*d + a^2*b^3*d + 2*a*b^4*d)))/((3*a*b^2 + (5
*a^2*b)/2 + a^3/2)/(b^4*d^3 + 3*a*b^3*d^3 + a^3*b*d^3 + 3*a^2*b^2*d^3) - (
(a + 3*b)*(-a*b^3)^(1/2)*((tanh(c + d*x)*(6*a^3*b + a^4 + 4*b^4 + 9*a^2*b^
2))/(2*(b^3*d^2 + 2*a*b^2*d^2 + a^2*b*d^2)) + ((a + 3*b)*(-a*b^3)^(1/2)...
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 228, normalized size of antiderivative = 2.56

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tanh\left(dx+c\right)^2 ab - 3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tanh\left(dx+c\right)^2 b^2 - \sqrt{b}\,\sqrt{a}\,atan}{2b^2d\left(\tanh\left(dx+c\right)^2 a^2b + 2\tanh\left(\right.\right.}$$

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x
)**2*a*b - 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tan
h(c + d*x)**2*b**2 - sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(
a)))*a**2 - 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a*
b + 2*tanh(c + d*x)**2*b**3*d*x + tanh(c + d*x)*a**2*b + tanh(c + d*x)*a*b
**2 + 2*a*b**2*d*x)/(2*b**2*d*(tanh(c + d*x)**2*a**2*b + 2*tanh(c + d*x)**
2*a*b**2 + tanh(c + d*x)**2*b**3 + a**3 + 2*a**2*b + a*b**2))
```

3.182 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 72

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}+\frac{a}{2b(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d+1/2*a/b/(a+b
)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 57, normalized size of antiderivative = 0.79

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{2\log(\cosh(c+dx))+\log\left(a+b\tanh^2(c+dx)\right)+\frac{a(a+b)}{b(a+b\tanh^2(c+dx))}}{2(a+b)^2d}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*Log[Cosh[c + d*x]] + Log[a + b*Tanh[c + d*x]^2] + (a*(a + b))/(b*(a + b
*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 71, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{86}$$

$$\frac{\int \left(-\frac{a}{(a+b)(b\tanh^2(c+dx)+a)^2}-\frac{1}{(a+b)^2(\tanh^2(c+dx)-1)}+\frac{b}{(a+b)^2(b\tanh^2(c+dx)+a)}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{a}{b(a+b)(a+b\tanh^2(c+dx))}-\frac{\log(1-\tanh^2(c+dx))}{(a+b)^2}+\frac{\log(a+b\tanh^2(c+dx))}{(a+b)^2}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^2) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^2 + a/(b*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{-\frac{-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{-ab-a^2+2abdx-\ln\left(a+b\tanh(dx+c)^2\right)ab+2\ln(1-\tanh(dx+c))\tanh(dx+c)^2b^2+2\ln(1-\tanh(dx+c))ab-b^2\ln\left(a\right.}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2b}$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}+\frac{2a\,e^{2dx+2c}}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{4dx+4c}\right.}{2d(}$

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)^2*(-ln(a+b*tanh(d*x+c)^2)-a*(a+b)/b/(a+b*tanh(d*x+c)^2))-1
/2/(a+b)^2*ln(1+tanh(d*x+c))-1/2/(a+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. 2(68) = 136.

Time = 0.12 (sec) , antiderivative size = 629, normalized size of antiderivative = 8.74

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
 + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x -
 a)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x - a)*
sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(
d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x
+ c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((
a - b)*d*x - a)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(
a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)
^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*
x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(68) = 136$.

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{2\,ae^{(-2\,dx-2\,c)}}{\left(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)}\right)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*a*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a*
b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x -
4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*x
 - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(68) = 136$.

Time = 0.19 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.07

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2} - \frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}-2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c) - 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d
*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 2.91 (sec) , antiderivative size = 210, normalized size of antiderivative = 2.92

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx =$$
$$-\frac{-a^2+a\,b\left(-1+\operatorname{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c+d\,x)^2\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^2+b\tanh(c+d\,x)^2}\right)\,2\mathrm{i}\right)+b^2\tanh(c+d\,x)^2\operatorname{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(}{2\,a-a\tanh(c+d\,x)^2+b\tan}\right.}{2\,d\,a^3\,b+2\,d\,a^2\,b^2\tanh\left(c+d\,x\right)^2+4\,d\,a^2\,b^2+4\,d\,a\,b^3\tanh\left(c+d\,x\right)^2+2\,d\,a\,b^3+2\,d\,b^4\tanh\left(c+\right.}$$

input
```
int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
-(a*b*(atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c
+ d*x)^2 + b*tanh(c + d*x)^2))*2i - 1) - a^2 + b^2*tanh(c + d*x)^2*atan((a
*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*t
anh(c + d*x)^2))*2i)/(4*a^2*b^2*d + 2*b^4*d*tanh(c + d*x)^2 + 2*a*b^3*d +
2*a^3*b*d + 2*a^2*b^2*d*tanh(c + d*x)^2 + 4*a*b^3*d*tanh(c + d*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 2332, normalized size of antiderivative = 32.39

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

output

```
(e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c
 + d*x)*sqrt(b))*tanh(c + d*x)**2*a**3*b - e**(4*c + 4*d*x)*log(e**(2*c +
2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**
2*a*b**3 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b)
 - 2*e**(c + d*x)*sqrt(b))*a**4 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sq
rt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b**2 + e**(4*c + 4*
d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(
b))*tanh(c + d*x)**2*a**3*b - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a
 + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a*b**3 + e*
*(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c +
d*x)*sqrt(b))*a**4 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + s
qrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**2*b**2 - e**(4*c + 4*d*x)*tanh(c +
 d*x)**2*a**4 - 2*e**(4*c + 4*d*x)*tanh(c + d*x)**2*a**3*b*d*x - 2*e**(4*c
 + 4*d*x)*tanh(c + d*x)**2*a**3*b + 2*e**(4*c + 4*d*x)*tanh(c + d*x)**2*a*
*2*b**2 + 2*e**(4*c + 4*d*x)*tanh(c + d*x)**2*a*b**3*d*x + 4*e**(4*c + 4*d
*x)*tanh(c + d*x)**2*a*b**3 + e**(4*c + 4*d*x)*tanh(c + d*x)**2*b**4 - 2*e
**(4*c + 4*d*x)*a**4*d*x + 2*e**(4*c + 4*d*x)*a**3*b + 2*e**(4*c + 4*d*x)*
a**2*b**2*d*x + 2*e**(4*c + 4*d*x)*a**2*b**2 + 2*e**(2*c + 2*d*x)*log(e**(
2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c +
d*x)**2*a**3*b - 4*e**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + ...
```

3.183 $\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 85

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{x}{(a+b)^2} - \frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\sqrt{b}(a+b)^2 d} - \frac{\tanh(c+dx)}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2-1/2*(a-b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2)/b^(1/2)/(a
+b)^2/d-1/2*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.27 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.01

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{2(c+dx) + \frac{(-a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}} - \frac{(a+b)\sinh(2(c+dx))}{a-b+(a+b)\cosh(2(c+dx))}}{2(a+b)^2 d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(2*(c + d*x) + ((-a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]
*Sqrt[b]) - ((a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*Cosh[2*(c + d*x)]
))/(2*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.16, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3042, 25, 4153, 25, 373, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{373}$$

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int \frac{\tanh^2(c+dx)+1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}}{d}$$

$\Big\downarrow$ 397

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{(a-b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2(a+b)}}{d}$$

$\Big\downarrow$ 218

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2(a+b)}}{d}$$

$\Big\downarrow$ 219

$$-\frac{\frac{\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\frac{2\text{arctanh}(\tanh(c+dx))}{a+b}-\frac{(a-b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2(a+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-1/2*(-(((a - b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*Sqrt
[b]*(a + b))) + (2*ArcTanh[Tanh[c + d*x]])/(a + b))/(a + b) + Tanh[c + d*x
]/(2*(a + b)*(a + b*Tanh[c + d*x]^2)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.13 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.18

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{-\frac{\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(a-b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}+\frac{a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}-\frac{\ln\left(e^{2dx+2c}+\frac{a\sqrt{-ab}-b\sqrt{-ab}+2ab}{(a+b)\sqrt{-ab}}\right)}{4\sqrt{-ab}\,(a+b)^2d}$

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)^2*((1/2*b+1/2*a)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(a-b)/(
a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(1+tanh(d*x+c)
)-1/2/(a+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(73) = 146$.

Time = 0.13 (sec) , antiderivative size = 2025, normalized size of antiderivative = 23.82

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^4 + 16*(a^2*b + a*b^2)*d*x*cosh(
d*x + c)*sinh(d*x + c)^3 + 4*(a^2*b + a*b^2)*d*x*sinh(d*x + c)^4 + 4*a^2*b
 + 4*a*b^2 + 4*(a^2*b + a*b^2)*d*x + 4*(a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*
d*x)*cosh(d*x + c)^2 + 4*(6*(a^2*b + a*b^2)*d*x*cosh(d*x + c)^2 + a^2*b -
a*b^2 + 2*(a^2*b - a*b^2)*d*x)*sinh(d*x + c)^2 + ((a^2 - b^2)*cosh(d*x + c
)^4 + 4*(a^2 - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - b^2)*sinh(d*x +
 c)^4 + 2*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 - b^2)*cosh(d*x
+ c)^2 + a^2 - 2*a*b + b^2)*sinh(d*x + c)^2 + a^2 - b^2 + 4*((a^2 - b^2)*c
osh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a
*b)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b + b^2)*cosh(
d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 2*(a^2 -
b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^
2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*cosh(d*x +
 c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*((a + b)*cosh(d*x + c
)^2 + 2*(a + b)*cosh(d*x + c)*sinh(d*x + c) + (a + b)*sinh(d*x + c)^2 + a
- b)*sqrt(-a*b))/((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(d
*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
+ b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)) + 8*(2*(a^2*b + a*b^2)*d
*x*cosh(d*x + c)^3 + (a^2*b - a*b^2 + 2*(a^2*b - a*b^2)*d*x)*cosh(d*x +...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. 2(73) = 146.

Time = 0.26 (sec) , antiderivative size = 614, normalized size of antiderivative = 7.22

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/8*(a^2 - 4*a*b - b^2)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt
(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + 1/8*(a^2 - 4*a*b - b^2)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*
b^2)*sqrt(a*b)*d) + 1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(2*d*x + 2*c))/
((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*
e^(4*d*x + 4*c) + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(2*d*x + 2*c))*d) -
1/4*(a^2 - b^2 + (a^2 - 6*a*b + b^2)*e^(-2*d*x - 2*c))/((a^4 + 3*a^3*b + 3
*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x - 4*c))*d) - 1/2*((a - b)*e^(-
2*d*x - 2*c) + a + b)/((a^3 + 2*a^2*b + a*b^2 + 2*(a^3 - a*b^2)*e^(-2*d*x
- 2*c) + (a^3 + 2*a^2*b + a*b^2)*e^(-4*d*x - 4*c))*d) + 1/4*log((a + b)*e^
(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
 - 1/4*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^2 + 2*a*b + b^2)*d) + 1/4*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b
)/sqrt(a*b))/(sqrt(a*b)*a*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(73) = 146$.

Time = 0.18 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = -\frac{\frac{(a-b)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^2+2\,ab+b^2)\sqrt{ab}} - \frac{2\,(dx+c)}{a^2+2\,ab+b^2} - \frac{2\,(ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+a+b)}{(a^2+2\,ab+b^2)(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b)}}{2\,d}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((a - b)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a - b)/s
qrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) - 2*(d*x + c)/(a^2 + 2*a*b + b^2
) - 2*(a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + a + b)/((a^2 + 2*a*b + b^2)
*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 2.99 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.25

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{\frac{a\,x}{(a+b)^2} - \frac{\tanh(c+d\,x)}{2\,a\,d+2\,b\,d} + \frac{b\,x\tanh(c+d\,x)^2}{(a+b)^2}}{b\tanh\left(c+d\,x\right)^2+a} - \frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,(a-b)}{\sqrt{a\,b}\,\left(2\,d\,a^2+4\,d\,a\,b+2\,d\,b^2\right)}$$

input
```
int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((a*x)/(a + b)^2 - tanh(c + d*x)/(2*a*d + 2*b*d) + (b*x*tanh(c + d*x)^2)/(
a + b)^2)/(a + b*tanh(c + d*x)^2) - (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(
a - b))/((a*b)^(1/2)*(2*a^2*d + 2*b^2*d + 4*a*b*d))
```

Reduce [B] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 232, normalized size of antiderivative = 2.73

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$
$$= \frac{-\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tanh\left(dx+c\right)^2ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tanh\left(dx+c\right)^2b^2-\sqrt{b}\,\sqrt{a}\,atan\left(\right.}{2abd\left(\tanh\left(dx+c\right)^2a^2b+2\tanh\left(\right.\right.}$$

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x
)**2*a*b + sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(
c + d*x)**2*b**2 - sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)
))*a**2 + sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a*b +
2*tanh(c + d*x)**2*a*b**2*d*x - tanh(c + d*x)*a**2*b - tanh(c + d*x)*a*b**
2 + 2*a**2*b*d*x)/(2*a*b*d*(tanh(c + d*x)**2*a**2*b + 2*tanh(c + d*x)**2*a
*b**2 + tanh(c + d*x)**2*b**3 + a**3 + 2*a**2*b + a*b**2))
```

3.184 $\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 68

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^2d}-\frac{1}{2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^2/d-1/2/(a+b)/d/
(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 55, normalized size of antiderivative = 0.81

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=-\frac{-2\log(\cosh(c+dx))-\log\left(a+b\tanh^2(c+dx)\right)+\frac{a+b}{a+b\tanh^2(c+dx)}}{2(a+b)^2d}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/2*(-2*Log[Cosh[c + d*x]] - Log[a + b*Tanh[c + d*x]^2] + (a + b)/(a + b*
Tanh[c + d*x]^2))/((a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{b}{(a+b)^2\left(b\tanh^2(c+dx)+a\right)}+\frac{b}{(a+b)\left(b\tanh^2(c+dx)+a\right)^2}-\frac{1}{(a+b)^2\left(\tanh^2(c+dx)-1\right)}\right)d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{(a+b)(a+b\tanh^2(c+dx))}-\frac{\log(1-\tanh^2(c+dx))}{(a+b)^2}+\frac{\log(a+b\tanh^2(c+dx))}{(a+b)^2}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^2) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^2 - 1/((a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.26

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
parallelrisch	$-\frac{-b^2\tanh(dx+c)^2+2\ln(1-\tanh(dx+c))a^2+2a^2dx+2\ln(1-\tanh(dx+c))\tanh(dx+c)^2ab-\ln\left(a+b\tanh(dx+c)^2\right)\text{ta}}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2a}$
risch	$-\frac{x}{a^2+2ab+b^2}-\frac{2c}{d(a^2+2ab+b^2)}-\frac{2b\,e^{2dx+2c}}{d(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{\ln\left(e^{4dx+4c}\right.}{2d(}$

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2/(a+b)^2*b*(1/b*ln(a+b*tanh(d*x+c)^2)-(a+b)/b/(a+b*tanh(d*x+c)^2))
-1/2/(a+b)^2*ln(1+tanh(d*x+c))-1/2/(a+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 623 vs. $2(64) = 128$.

Time = 0.09 (sec) , antiderivative size = 623, normalized size of antiderivative = 9.16

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a + b)*d*x*cosh(d*x + c)^4 + 8*(a + b)*d*x*cosh(d*x + c)*sinh(d*x
 + c)^3 + 2*(a + b)*d*x*sinh(d*x + c)^4 + 2*(a + b)*d*x + 4*((a - b)*d*x +
 b)*cosh(d*x + c)^2 + 4*(3*(a + b)*d*x*cosh(d*x + c)^2 + (a - b)*d*x + b)*
sinh(d*x + c)^2 - ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh(
d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^
3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*log(2*((a + b)*cosh(d*x
+ c)^2 + (a + b)*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2)) + 8*((a + b)*d*x*cosh(d*x + c)^3 + ((
a - b)*d*x + b)*cosh(d*x + c))*sinh(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*d*cosh(d*x + c)^4 + 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*sinh(d*x + c)^4 + 2*(
a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*d*cosh(d*x + c)^2 + (a^3 + a^2*b - a*b^2 - b^3)*d)*sinh(d*x + c)
^2 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d + 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3
)*d*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*
x + c))
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(64) = 128$.

Time = 0.05 (sec) , antiderivative size = 170, normalized size of antiderivative = 2.50

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{2\,be^{(-2\,dx-2\,c)}}{\left(a^3+3\,a^2b+3\,ab^2+b^3+2\,(a^3+a^2b-ab^2-b^3)e^{(-2\,dx-2\,c)}+(a^3+3\,a^2b+3\,ab^2+b^3)e^{(-4\,dx-4\,c)}\right)d} + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^2+2\,ab+b^2)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-2*b*e^(-2*d*x - 2*c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 + a^2*b - a
*b^2 - b^3)*e^(-2*d*x - 2*c) + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*e^(-4*d*x -
 4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d) + 1/2*log(2*(a - b)*e^(-2*d*
x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^2 + 2*a*b + b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(64) = 128$.

Time = 0.16 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.19

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^2+2\,ab+b^2} - \frac{e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}+2}{\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2}}{2\,d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*(log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) +
 e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^2 + 2*a*b + b^2) - (e^(2*d*x + 2*c) +
e^(-2*d*x - 2*c) + 2)/((a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d
*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b)*(a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 129, normalized size of antiderivative = 1.90

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{a\,x}{a^2+2\,a\,b+b^2}+\frac{b\,x\,\tanh(c+d\,x)^2}{a^2+2\,a\,b+b^2}+\frac{b\,\tanh(c+d\,x)^2}{2\,a\,d\,(a+b)}}{b\,\tanh\left(c+d\,x\right)^2+a} + \frac{\ln\left(b\,\tanh(c+d\,x)^2+a\right)}{2\,d\,\left(a^2+2\,a\,b+b^2\right)} - \frac{\ln\left(\tanh(c+d\,x)+1\right)}{d\,(a+b)^2}$$

input
```
int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((a*x)/(2*a*b + a^2 + b^2) + (b*x*tanh(c + d*x)^2)/(2*a*b + a^2 + b^2) + (
b*tanh(c + d*x)^2)/(2*a*d*(a + b)))/(a + b*tanh(c + d*x)^2) + log(a + b*ta
nh(c + d*x)^2)/(2*d*(2*a*b + a^2 + b^2)) - log(tanh(c + d*x) + 1)/(d*(a +
b)^2)
```

Reduce [B] (verification not implemented)

Time = 0.24 (sec) , antiderivative size = 2288, normalized size of antiderivative = 33.65

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c
 + d*x)*sqrt(b))*tanh(c + d*x)**2*a**3*b - e**(4*c + 4*d*x)*log(e**(2*c +
2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**
2*a*b**3 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b)
 - 2*e**(c + d*x)*sqrt(b))*a**4 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sq
rt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b**2 + e**(4*c + 4*
d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(
b))*tanh(c + d*x)**2*a**3*b - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a
 + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a*b**3 + e*
*(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c +
d*x)*sqrt(b))*a**4 - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + s
qrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**2*b**2 + e**(4*c + 4*d*x)*tanh(c +
 d*x)**2*a**4 - 2*e**(4*c + 4*d*x)*tanh(c + d*x)**2*a**3*b*d*x - 2*e**(4*c
 + 4*d*x)*tanh(c + d*x)**2*a**2*b**2 + 2*e**(4*c + 4*d*x)*tanh(c + d*x)**2
*a*b**3*d*x - 2*e**(4*c + 4*d*x)*tanh(c + d*x)**2*a*b**3 - e**(4*c + 4*d*x
)*tanh(c + d*x)**2*b**4 - 2*e**(4*c + 4*d*x)*a**4*d*x - 2*e**(4*c + 4*d*x)
*a**4 - 2*e**(4*c + 4*d*x)*a**3*b + 2*e**(4*c + 4*d*x)*a**2*b**2*d*x + 2*e
**(2*c + 2*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c +
 d*x)*sqrt(b))*tanh(c + d*x)**2*a**3*b - 4*e**(2*c + 2*d*x)*log(e**(2*c +
2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x...
```

3.185 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{x}{(a+b)^2}+\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2d}+\frac{b\tanh(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2+1/2*b^(1/2)*(3*a+b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2)/
(a+b)^2/d+1/2*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.38 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.09

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\frac{\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}}-\log(1-\tanh(c+dx))+\log(1+\tanh(c+dx))+\frac{b(a+b)\tanh(c+dx)}{a\left(a+b\tanh^2(c+dx)\right)}}{2(a+b)^2d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

output
```
((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) - Log
[1 - Tanh[c + d*x]] + Log[1 + Tanh[c + d*x]] + (b*(a + b)*Tanh[c + d*x])/(
a*(a + b*Tanh[c + d*x]^2)))/(2*(a + b)^2*d)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.19, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.429, Rules used = {3042, 4144, 316, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int \frac{b\tanh^2(c+dx)+b-2(a+b)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{-\frac{2a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b(3a+b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{-\frac{2a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b\tanh(c+dx)}{2a(a+b)(a+b\tanh^2(c+dx))} - \frac{-\frac{\sqrt{b}(3a+b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)} - \frac{2a\text{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)}}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-2),x]
```

output
```
(-1/2*(-((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt
[a]*(a + b))) - (2*a*ArcTanh[Tanh[c + d*x]])/(a + b))/(a*(a + b)) + (b*Tan
h[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.14 (sec) , antiderivative size = 103, normalized size of antiderivative = 1.16

method	result
derivativedivides	$\frac{\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$\frac{\frac{b\left(\frac{(a+b)\tanh(dx+c)}{2a\left(a+b\tanh(dx+c)^2\right)}+\frac{(3a+b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2a\sqrt{ab}}\right)}{(a+b)^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{b\left(a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b\right)}{d(a+b)^2a\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)}+\frac{3\sqrt{-ab}\,\ln\left(e^{2dx+2c}+\frac{a+2\sqrt{-ab}-b}{a+b}\right)}{4a(a+b)^2d}$

input
```
int(1/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^2*b*(1/2*(a+b)/a*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(3*a+b)/
a/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^2*ln(1+tanh(d*x
+c))-1/2/(a+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 810 vs. $2(77) = 154$.

Time = 0.14 (sec) , antiderivative size = 1942, normalized size of antiderivative = 21.82

$$\int \frac{1}{\left(a + b\tanh^2(c + dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
[1/4*(4*(a^2 + a*b)*d*x*cosh(d*x + c)^4 + 16*(a^2 + a*b)*d*x*cosh(d*x + c)
*sinh(d*x + c)^3 + 4*(a^2 + a*b)*d*x*sinh(d*x + c)^4 + 4*(a^2 + a*b)*d*x +
 4*(2*(a^2 - a*b)*d*x - a*b + b^2)*cosh(d*x + c)^2 + 4*(6*(a^2 + a*b)*d*x*
cosh(d*x + c)^2 + 2*(a^2 - a*b)*d*x - a*b + b^2)*sinh(d*x + c)^2 + ((3*a^2
 + 4*a*b + b^2)*cosh(d*x + c)^4 + 4*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)*si
nh(d*x + c)^3 + (3*a^2 + 4*a*b + b^2)*sinh(d*x + c)^4 + 2*(3*a^2 - 2*a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*(3*a^2 + 4*a*b + b^2)*cosh(d*x + c)^2 + 3*a^2
 - 2*a*b - b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + b^2 + 4*((3*a^2 + 4*a*b
+ b^2)*cosh(d*x + c)^3 + (3*a^2 - 2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-b/a)*log(((a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 4*(a^2 + 2*a*b +
b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 +
 2*(a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2
+ a^2 - b^2)*sinh(d*x + c)^2 + a^2 - 6*a*b + b^2 + 4*((a^2 + 2*a*b + b^2)*
cosh(d*x + c)^3 + (a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a^2 + a*b
)*cosh(d*x + c)^2 + 2*(a^2 + a*b)*cosh(d*x + c)*sinh(d*x + c) + (a^2 + a*b
)*sinh(d*x + c)^2 + a^2 - a*b)*sqrt(-b/a))/((a + b)*cosh(d*x + c)^4 + 4*(a
 + b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a - b)*
cosh(d*x + c)^2 + 2*(3*(a + b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c)^2 +
4*((a + b)*cosh(d*x + c)^3 + (a - b)*cosh(d*x + c))*sinh(d*x + c) + a + b)
) - 4*a*b - 4*b^2 + 8*(2*(a^2 + a*b)*d*x*cosh(d*x + c)^3 + (2*(a^2 - a*...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^{2}(c+dx)\right)^{2}}\,dx=\text{Timed out}$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. 2(77) = 154.

Time = 0.17 (sec) , antiderivative size = 206, normalized size of antiderivative = 2.31

$$\int \frac{1}{\left(a+b\tanh^{2}(c+dx)\right)^{2}}\,dx=-\frac{\left(3\,ab+b^{2}\right)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{2\,\left(a^{3}+2\,a^{2}b+ab^{2}\right)\sqrt{ab}d}+\frac{ab+b^{2}+\left(ab-b^{2}\right)e^{(-2\,dx-2\,c)}}{\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}+2\,\left(a^{4}+a^{3}b-a^{2}b^{2}-ab^{3}\right)e^{(-2\,dx-2\,c)}+\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)e^{(-4\,dx-4}}+\frac{dx+c}{\left(a^{2}+2\,ab+b^{2}\right)d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/2*(3*a*b + b^2)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b)
)/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)*d) + (a*b + b^2 + (a*b - b^2)*e^(-2*d
*x - 2*c))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*b - a^2*b^2
- a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*e^(-4*d*x
- 4*c))*d) + (d*x + c)/((a^2 + 2*a*b + b^2)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(77) = 154$.

Time = 0.13 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.19

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{(3\,ab+b^2)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^3+2\,a^2b+ab^2)\sqrt{ab}}+\frac{2\,(dx+c)}{a^2+2\,ab+b^2}-\frac{2\,\left(abe^{(2\,dx+2\,c)}-b^2e^{(2\,dx+2\,c)}+ab+b^2\right)}{(a^3+2\,a^2b+ab^2)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c}\right)}}{2\,d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((3*a*b + b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c) + a -
 b)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 2*(d*x + c)/(a^2 + 2*
a*b + b^2) - 2*(a*b*e^(2*d*x + 2*c) - b^2*e^(2*d*x + 2*c) + a*b + b^2)/((a
^3 + 2*a^2*b + a*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)))/d
```

Mupad [B] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.24

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{a\,x}{(a+b)^2}+\frac{b\,x\tanh(c+d\,x)^2}{(a+b)^2}+\frac{b\tanh(c+d\,x)}{2\,a\,d\,(a+b)}}{b\tanh\left(c+d\,x\right)^2+a} + \frac{\mathrm{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,\left(b^2+3\,a\,b\right)}{\sqrt{a\,b}\,\left(2\,a^3\,d+a\,b\,\left(4\,a\,d+2\,b\,d\right)\right)}$$

input
```
int(1/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
((a*x)/(a + b)^2 + (b*x*tanh(c + d*x)^2)/(a + b)^2 + (b*tanh(c + d*x))/(2*
a*d*(a + b)))/(a + b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2
))*(3*a*b + b^2))/((a*b)^(1/2)*(2*a^3*d + a*b*(4*a*d + 2*b*d)))
```

Reduce [B] (verification not implemented)

Time = 0.22 (sec) , antiderivative size = 226, normalized size of antiderivative = 2.54

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{3\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tanh\left(dx+c\right)^2 ab+\sqrt{b}\,\sqrt{a}\,atan\left(\frac{\tanh(dx+c)b}{\sqrt{b}\,\sqrt{a}}\right)\tanh\left(dx+c\right)^2 b^2+3\sqrt{b}\,\sqrt{a}\,atan}{2a^2d\left(\tanh\left(dx+c\right)^2 a^2b+2\tanh\left(c\right.\right.}$$

input
```
int(1/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)
**2*a*b + sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c
 + d*x)**2*b**2 + 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a
)))*a**2 + sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a*b +
 2*tanh(c + d*x)**2*a**2*b*d*x + tanh(c + d*x)*a**2*b + tanh(c + d*x)*a*b*
*2 + 2*a**3*d*x)/(2*a**2*d*(tanh(c + d*x)**2*a**2*b + 2*tanh(c + d*x)**2*a
*b**2 + tanh(c + d*x)**2*b**3 + a**3 + 2*a**2*b + a*b**2))
```

3.186 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 95

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{\log(\tanh(c+dx))}{a^2d}-\frac{b(2a+b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^2(a+b)^2d}+\frac{b}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^2/d+ln(tanh(d*x+c))/a^2/d-1/2*b*(2*a+b)*ln(a+b*tanh(
d*x+c)^2)/a^2/(a+b)^2/d+1/2*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 0.87

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{\frac{2\log(\cosh(c+dx))}{(a+b)^2}+\frac{2\log(\tanh(c+dx))+\frac{b\left(-\left((2a+b)\log\left(a+b\tanh^2(c+dx)\right)\right)+\frac{a(a+b)}{a+b\tanh^2(c+dx)}\right)}{(a+b)^2}}{a^2}}{2d}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((2*Log[Cosh[c + d*x]])/(a + b)^2 + (2*Log[Tanh[c + d*x]] + (b*(-((2*a + b
)*Log[a + b*Tanh[c + d*x]^2]) + (a*(a + b))/(a + b*Tanh[c + d*x]^2)))/(a +
 b)^2)/a^2)/(2*d)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 94, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{1}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$\frac{i\int -\frac{i\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{93}$$

$$\frac{\int \left(-\frac{(2a+b)b^2}{a^2(a+b)^2(b\tanh^2(c+dx)+a)} - \frac{b^2}{a(a+b)(b\tanh^2(c+dx)+a)^2} + \frac{\coth(c+dx)}{a^2} - \frac{1}{(a+b)^2(\tanh^2(c+dx)-1)}\right) d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(2a+b)\log(a+b\tanh^2(c+dx))}{a^2(a+b)^2} + \frac{\log(\tanh^2(c+dx))}{a^2} + \frac{b}{a(a+b)(a+b\tanh^2(c+dx))} - \frac{\log(1-\tanh^2(c+dx))}{(a+b)^2}}{2d}$$

input

```
Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^2,x]
```

output

```
(Log[Tanh[c + d*x]^2]/a^2 - Log[1 - Tanh[c + d*x]^2]/(a + b)^2 - (b*(2*a +
 b)*Log[a + b*Tanh[c + d*x]^2])/(a^2*(a + b)^2) + b/(a*(a + b)*(a + b*Tanh
[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26

```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93

```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009

```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.16

method	result
derivativedivides	$-\frac{\frac{b^2\left(\frac{(2a+b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^2}-\frac{\ln(\tanh(dx+c))}{a^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}+\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$-\frac{\frac{b^2\left(\frac{(2a+b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^2}-\frac{\ln(\tanh(dx+c))}{a^2}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}+\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
parallelrisch	$\frac{-2\left(a+\frac{b}{2}\right)\left(a+b\tanh(dx+c)^2\right)b\ln\left(a+b\tanh(dx+c)^2\right)+\left(-2\tanh(dx+c)^2a^2b-2a^3\right)\ln(1-\tanh(dx+c))+2(a+b)^2\left(a\right.}{2\left(a+b\tanh(dx+c)^2\right)d(a+b)^2a^2}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{a^2}-\frac{2c}{a^2d}+\frac{4bx}{a(a^2+2ab+b^2)}+\frac{4bc}{ad(a^2+2ab+b^2)}+\frac{2b^2x}{a^2(a^2+2ab+b^2)}+\frac{2b^2c}{a^2d(a^2+2ab+b^2)}+\frac{}{ad}$

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/2*b^2/(a+b)^2/a^2*((2*a+b)/b*ln(a+b*tanh(d*x+c)^2)-a*(a+b)/b/(a+b*
tanh(d*x+c)^2))-1/a^2*ln(tanh(d*x+c))+1/2/(a+b)^2*ln(1+tanh(d*x+c))+1/2/(a
+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1148 vs. $2(91) = 182$.

Time = 0.23 (sec) , antiderivative size = 1148, normalized size of antiderivative = 12.08

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^3 + a^2*b)*d*x*cosh(d*x + c)^4 + 8*(a^3 + a^2*b)*d*x*cosh(d*x +
 c)*sinh(d*x + c)^3 + 2*(a^3 + a^2*b)*d*x*sinh(d*x + c)^4 + 2*(a^3 + a^2*b
)*d*x - 4*(a*b^2 - (a^3 - a^2*b)*d*x)*cosh(d*x + c)^2 + 4*(3*(a^3 + a^2*b)
*d*x*cosh(d*x + c)^2 - a*b^2 + (a^3 - a^2*b)*d*x)*sinh(d*x + c)^2 + ((2*a^
2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4 + 4*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*
x + c)*sinh(d*x + c)^3 + (2*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + 2*a^2
*b + 3*a*b^2 + b^3 + 2*(2*a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(2*a^2*
b - a*b^2 - b^3 + 3*(2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 4*((2*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (2*a^2*b - a*b^2 - b
^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*((a + b)*cosh(d*x + c)^2 + (a + b)
*sinh(d*x + c)^2 + a - b)/(cosh(d*x + c)^2 - 2*cosh(d*x + c)*sinh(d*x + c)
 + sinh(d*x + c)^2)) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^4
+ 4*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*sinh(d*x + c)^4 + a^3 + 3*a^2*b + 3*a*b^2 + b^3
+ 2*(a^3 + a^2*b - a*b^2 - b^3)*cosh(d*x + c)^2 + 2*(a^3 + a^2*b - a*b^2 -
 b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 4*((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b - a*b^
2 - b^3)*cosh(d*x + c))*sinh(d*x + c))*log(2*sinh(d*x + c)/(cosh(d*x + c)
- sinh(d*x + c))) + 8*((a^3 + a^2*b)*d*x*cosh(d*x + c)^3 - (a*b^2 - (a^3 -
 a^2*b)*d*x)*cosh(d*x + c))*sinh(d*x + c))/((a^5 + 3*a^4*b + 3*a^3*b^2 ...
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 235 vs. $2(91) = 182$.

Time = 0.06 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.47

$$\begin{aligned}
&\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx \\
&= \frac{2\,b^2e^{(-2\,dx-2\,c)}}{(a^4+3\,a^3b+3\,a^2b^2+ab^3+2\,(a^4+a^3b-a^2b^2-ab^3)e^{(-2\,dx-2\,c)}+(a^4+3\,a^3b+3\,a^2b^2+ab^3)e^{(-4\,dx-4\,c)})} \\
&\quad - \frac{(2\,ab+b^2)\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^4+2\,a^3b+a^2b^2)d} \\
&\quad + \frac{dx+c}{(a^2+2\,ab+b^2)d} + \frac{\log\left(e^{(-dx-c)}+1\right)}{a^2d} + \frac{\log\left(e^{(-dx-c)}-1\right)}{a^2d}
\end{aligned}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
2*b^2*e^(-2*d*x - 2*c)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 + 2*(a^4 + a^3*
b - a^2*b^2 - a*b^3)*e^(-2*d*x - 2*c) + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3
)*e^(-4*d*x - 4*c))*d) - 1/2*(2*a*b + b^2)*log(2*(a - b)*e^(-2*d*x - 2*c)
+ (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + (d*x +
 c)/((a^2 + 2*a*b + b^2)*d) + log(e^(-d*x - c) + 1)/(a^2*d) + log(e^(-d*x
- c) - 1)/(a^2*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(91) = 182$.

Time = 0.18 (sec) , antiderivative size = 195, normalized size of antiderivative = 2.05

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx =$$
$$-\frac{\frac{(2\,ab+b^2)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^4+2\,a^3b+a^2b^2}+\frac{2\,(dx+c)}{a^2+2\,ab+b^2}-\frac{4\,b^2e^{(2\,dx+2\,c)}}{\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-\right.}}{2\,d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((2*a*b + b^2)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d
*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^4 + 2*a^3*b + a^2*b^2) + 2*(d*
x + c)/(a^2 + 2*a*b + b^2) - 4*b^2*e^(2*d*x + 2*c)/((a*e^(4*d*x + 4*c) + b
*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a +
 b)^2*a) - 2*log(abs(e^(2*d*x + 2*c) - 1))/a^2)/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 1760, normalized size of antiderivative = 18.53

$$\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a**4 + 4*e**(4*c + 4*d*x)*log(e*
*(c + d*x) - 1)*a**3*b - 4*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*a*b**3 -
 2*e**(4*c + 4*d*x)*log(e**(c + d*x) - 1)*b**4 + 2*e**(4*c + 4*d*x)*log(e*
*(c + d*x) + 1)*a**4 + 4*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a**3*b - 4
*e**(4*c + 4*d*x)*log(e**(c + d*x) + 1)*a*b**3 - 2*e**(4*c + 4*d*x)*log(e*
*(c + d*x) + 1)*b**4 - 2*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b)
 + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**3*b - e**(4*c + 4*d*x)*log(e**
(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b**
2 + 2*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*
e**(c + d*x)*sqrt(b))*a*b**3 + e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(
a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**4 - 2*e**(4*c + 4*d*x)*l
og(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*
*3*b - e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2
*e**(c + d*x)*sqrt(b))*a**2*b**2 + 2*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x)
*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*b**3 + e**(4*c + 4*
d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(
b))*b**4 - 2*e**(4*c + 4*d*x)*a**4*d*x + 2*e**(4*c + 4*d*x)*a**2*b**2*d*x
- 2*e**(4*c + 4*d*x)*a**2*b**2 - 2*e**(4*c + 4*d*x)*a*b**3 + 4*e**(2*c + 2
*d*x)*log(e**(c + d*x) - 1)*a**4 - 8*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1
)*a**2*b**2 + 4*e**(2*c + 2*d*x)*log(e**(c + d*x) - 1)*b**4 + 4*e**(2*c...
```

3.187 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 119

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{x}{(a+b)^2} - \frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^2 d} - \frac{(2a+3b)\coth(c+dx)}{2a^2(a+b)d} + \frac{b\coth(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2-1/2*b^(3/2)*(5*a+3*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(5/2
)/(a+b)^2/d-1/2*(2*a+3*b)*coth(d*x+c)/a^2/(a+b)/d+1/2*b*coth(d*x+c)/a/(a+b
)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.12 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.93

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= -\frac{-\frac{2(c+dx)}{(a+b)^2}+\frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a+b)^2}+\frac{2\coth(c+dx)}{a^2}+\frac{b^2\sinh(2(c+dx))}{a^2(a+b)(a-b+(a+b)\cosh(2(c+dx)))}}{2d}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-1/2*((-2*(c + d*x))/(a + b)^2 + (b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh
[c + d*x])/Sqrt[a]])/(a^(5/2)*(a + b)^2) + (2*Coth[c + d*x])/a^2 + (b^2*Si
nh[2*(c + d*x)])/(a^2*(a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/d
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.13, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 25, 4153, 25, 374, 25, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{\int -\frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{374}$$

$$-\frac{\frac{\int -\frac{\coth^2(c+dx)\left(-3b\tanh^2(c+dx)+2a+3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\int \frac{\coth^2(c+dx)\left(-3b\tanh^2(c+dx)+2a+3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{445}$$

$$-\frac{-\frac{-\frac{\int -\frac{2a^2-2ba-3b^2+b(2a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{25}$$

$$-\frac{-\frac{\frac{\int \frac{2a^2-2ba-3b^2+b(2a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{397}$$

$$-\frac{-\frac{\frac{\frac{2a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^2(5a+3b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{218}$$

$$-\frac{-\frac{\frac{\frac{2a^2\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$$\Big\downarrow \text{219}$$

$$-\frac{-\frac{\frac{\frac{2a^2\text{arctanh}(\tanh(c+dx))}{a+b}-\frac{b^{3/2}(5a+3b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{(2a+3b)\coth(c+dx)}{a}}{2a(a+b)}-\frac{b\coth(c+dx)}{2a(a+b)(a+b\tanh^2(c+dx))}}{d}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
-((-1/2*((-((b^(3/2)*(5*a + 3*b)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(Sqrt[a]*(a + b))) + (2*a^2*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((2*a + 3
*b)*Coth[c + d*x])/a)/(a*(a + b)) - (b*Coth[c + d*x])/(2*a*(a + b)*(a + b*
Tanh[c + d*x]^2)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 120, normalized size of antiderivative = 1.01

method	result
derivativedivides	$-\frac{\frac{b^2\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(5a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^2}+\frac{1}{a^2\tanh(dx+c)}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}+\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
default	$-\frac{\frac{b^2\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(5a+3b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^2}+\frac{1}{a^2\tanh(dx+c)}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}+\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^2}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2a^3e^{4dx+4c}+6a^2b\,e^{4dx+4c}+5a\,b^2e^{4dx+4c}+3b^3e^{4dx+4c}+4a^3e^{2dx+2c}+4a^2b\,e^{2dx+2c}-4a\,b^2e^{2dx+2c}-6b}{a^2d\left(e^{2dx+2c}-1\right)(a+b)^2\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+\right.}$

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(b^2/(a+b)^2/a^2*((1/2*b+1/2*a)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*(
5*a+3*b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/a^2/tanh(d*x+c)-
1/2/(a+b)^2*ln(1+tanh(d*x+c))+1/2/(a+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1702 vs. $2(105) = 210$.

Time = 0.18 (sec) , antiderivative size = 3725, normalized size of antiderivative = 31.30

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth^2{(c+dx)}}{\left(a+b\tanh^2{(c+dx)}\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(105) = 210$.

Time = 0.29 (sec) , antiderivative size = 976, normalized size of antiderivative = 8.20

$$\int \frac{\coth^2(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(2*a*b + b^2)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*x + 2*c)
 + a + b)/((a^4 + 2*a^3*b + a^2*b^2)*d) + 1/4*(2*a*b + b^2)*log(2*(a - b)*
e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^4 + 2*a^3*b + a^2
*b^2)*d) + 1/8*(3*a^2*b - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x +
2*c) + a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) - 1/8*(3*
a^2*b - 4*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqr
t(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)*d) + 1/4*(2*a^3 + 5*a^2*b + 6
*a*b^2 + 3*b^3 + (2*a^3 + 7*a^2*b + 3*b^3)*e^(4*d*x + 4*c) + 2*(2*a^3 + 2*
a^2*b + a*b^2 - 3*b^3)*e^(2*d*x + 2*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*
b^3 - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(6*d*x + 6*c) - (a^5 - a^4*b
 - 5*a^3*b^2 - 3*a^2*b^3)*e^(4*d*x + 4*c) + (a^5 - a^4*b - 5*a^3*b^2 - 3*a
^2*b^3)*e^(2*d*x + 2*c))*d) - 1/4*(2*a^3 + 5*a^2*b + 6*a*b^2 + 3*b^3 + 2*(
2*a^3 + 2*a^2*b + a*b^2 - 3*b^3)*e^(-2*d*x - 2*c) + (2*a^3 + 7*a^2*b + 3*b
^3)*e^(-4*d*x - 4*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 - a^4*b
 - 5*a^3*b^2 - 3*a^2*b^3)*e^(-2*d*x - 2*c) - (a^5 - a^4*b - 5*a^3*b^2 - 3*
a^2*b^3)*e^(-4*d*x - 4*c) - (a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-6*d*
x - 6*c))*d) - 1/2*(2*a^2 + 5*a*b + 3*b^2 + 2*(2*a^2 - 3*b^2)*e^(-2*d*x -
2*c) + (2*a^2 + 3*a*b + 3*b^2)*e^(-4*d*x - 4*c))/((a^4 + 2*a^3*b + a^2*b^2
 + (a^4 - 2*a^3*b - 3*a^2*b^2)*e^(-2*d*x - 2*c) - (a^4 - 2*a^3*b - 3*a^2*b
^2)*e^(-4*d*x - 4*c) - (a^4 + 2*a^3*b + a^2*b^2)*e^(-6*d*x - 6*c))*d) +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(105) = 210$.

Time = 0.20 (sec) , antiderivative size = 336, normalized size of antiderivative = 2.82

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\frac{\left(5\,ab^2+3\,b^3\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^4+2\,a^3b+a^2b^2\right)\sqrt{ab}} - \frac{2\,(dx+c)}{a^2+2\,ab+b^2} + \frac{2\,\left(2\,a^3e^{(4\,dx+4\,c)}+6\,a^2be^{(4\,dx+4\,c)}+5\,ab^2e^{(4\,dx+4\,c)}+3\,b^3e^{(4\,dx+4}\right.}{\left(a^4+2\,a^3b+a^2b^2\right)\left(ae^{(6\,dx+6\,c)}+be^{(6\,dx}\right.}}{2\,d}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
-1/2*((5*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c)
+ a - b)/sqrt(a*b))/((a^4 + 2*a^3*b + a^2*b^2)*sqrt(a*b)) - 2*(d*x + c)/(a
^2 + 2*a*b + b^2) + 2*(2*a^3*e^(4*d*x + 4*c) + 6*a^2*b*e^(4*d*x + 4*c) + 5
*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 4*a^3*e^(2*d*x + 2*c) + 4
*a^2*b*e^(2*d*x + 2*c) - 4*a*b^2*e^(2*d*x + 2*c) - 6*b^3*e^(2*d*x + 2*c) +
 2*a^3 + 6*a^2*b + 7*a*b^2 + 3*b^3)/((a^4 + 2*a^3*b + a^2*b^2)*(a*e^(6*d*x
 + 6*c) + b*e^(6*d*x + 6*c) + a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - a*
e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) - a - b)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth(c+d\,x)^2}{\left(b\,\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 1911, normalized size of antiderivative = 16.06

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
( - 5*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sq
rt(b))/sqrt(a))*a**3*b + 7*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 + 21*e**(6*c + 6*d*x)*sqrt(
b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 + 9*e
**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/
sqrt(a))*b**4 - 5*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt
(a + b) - sqrt(b))/sqrt(a))*a**3*b + 27*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*a
tan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 - 27*e**(4*c +
 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))
*a*b**3 - 27*e**(4*c + 4*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
b) - sqrt(b))/sqrt(a))*b**4 + 5*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**
(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b - 27*e**(2*c + 2*d*x)*sqr
t(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2
+ 27*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqr
t(b))/sqrt(a))*a*b**3 + 27*e**(2*c + 2*d*x)*sqrt(b)*sqrt(a)*atan((e**(c +
d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**4 + 5*sqrt(b)*sqrt(a)*atan((e**(c
+ d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**3*b - 7*sqrt(b)*sqrt(a)*atan((e*
*(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**2*b**2 - 21*sqrt(b)*sqrt(a)*
atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**3 - 9*sqrt(b)*sqrt
(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*b**4 + 5*e**(6*c...
```

3.188 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 124

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = -\frac{\coth^2(c+dx)}{2a^2d}+\frac{\log(\cosh(c+dx))}{(a+b)^2d}+\frac{(a-2b)\log(\tanh(c+dx))}{a^3d}+\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^2d}-\frac{b^2}{2a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/2*coth(d*x+c)^2/a^2/d+ln(cosh(d*x+c))/(a+b)^2/d+(a-2*b)*ln(tanh(d*x+c))
/a^3/d+1/2*b^2*(3*a+2*b)*ln(a+b*tanh(d*x+c)^2)/a^3/(a+b)^2/d-1/2*b^2/a^2/(
a+b)/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.60 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.75

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \frac{-\frac{\coth^2(c+dx)}{a^2}+\frac{b^3}{a^3(a+b)(b+a\coth^2(c+dx))}+\frac{b^2(3a+2b)\log(b+a\coth^2(c+dx))}{a^3(a+b)^2}+\frac{2\log(\sinh(c+dx))}{(a+b)^2}}{2d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Coth[c + d*x]^2/a^2) + b^3/(a^3*(a + b)*(b + a*Coth[c + d*x]^2)) + (b^2
*(3*a + 2*b)*Log[b + a*Coth[c + d*x]^2])/(a^3*(a + b)^2) + (2*Log[Sinh[c +
 d*x]])/(a + b)^2)/(2*d)
```

Rubi [A] (warning: unable to verify)

Time = 0.40 (sec) , antiderivative size = 116, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ic+idx)^3\,(a-b\tan(ic+idx)^2)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\tan(ic+idx)^3\,(a-b\tan(ic+idx)^2)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\coth^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 26}$$

$$\frac{\int \frac{\coth^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{ 354}$$

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{ 99}$$

$$\frac{\int \left(\frac{(3a+2b)b^3}{a^3(a+b)^2\left(b\tanh^2(c+dx)+a\right)}+\frac{b^3}{a^2(a+b)\left(b\tanh^2(c+dx)+a\right)^2}+\frac{\coth^2(c+dx)}{a^2}+\frac{(a-2b)\coth(c+dx)}{a^3}-\frac{1}{(a+b)^2\left(\tanh^2(c+dx)-1\right)}\right)d\,t}{2d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{\frac{b^2(3a+2b)\log\left(a+b\tanh^2(c+dx)\right)}{a^3(a+b)^2}+\frac{(a-2b)\log\left(\tanh^2(c+dx)\right)}{a^3}-\frac{b^2}{a^2(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\coth(c+dx)}{a^2}-\frac{\log\left(1-\tanh^2(c+dx)\right)}{(a+b)^2}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
(-(Coth[c + d*x]/a^2) + ((a - 2*b)*Log[Tanh[c + d*x]^2])/a^3 - Log[1 - Tan
h[c + d*x]^2]/(a + b)^2 + (b^2*(3*a + 2*b)*Log[a + b*Tanh[c + d*x]^2])/(a^
3*(a + b)^2) - b^2/(a^2*(a + b)*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.36 (sec) , antiderivative size = 131, normalized size of antiderivative = 1.06

method	result
derivativedivides	$-\frac{-\frac{b^3\left(\frac{(3a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^3}+\frac{(2b-a)\ln(\tanh(dx+c))}{a^3}+\frac{1}{2a^2\tanh(dx+c)^2}+\frac{\ln(1+\tanh(dx+c)}{2(a+b)^2}}{d}$
default	$-\frac{-\frac{b^3\left(\frac{(3a+2b)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a(a+b)}{b\left(a+b\tanh(dx+c)^2\right)}\right)}{2(a+b)^2a^3}+\frac{(2b-a)\ln(\tanh(dx+c))}{a^3}+\frac{1}{2a^2\tanh(dx+c)^2}+\frac{\ln(1+\tanh(dx+c)}{2(a+b)^2}}{d}$
parallelrisch	$\frac{3\left(a+\frac{2b}{3}\right)\left(a+b\tanh(dx+c)^2\right)b^2\ln\left(a+b\tanh(dx+c)^2\right)+\left(-2a^3b\tanh(dx+c)^2-2a^4\right)\ln(1-\tanh(dx+c))+2(a-2b)(a-}{2d(a+b)^2a^3\left(a-\right.}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{2x}{a^2}-\frac{2c}{a^2d}+\frac{4bx}{a^3}+\frac{4bc}{a^3d}-\frac{6b^2x}{a^2(a^2+2ab+b^2)}-\frac{6b^2c}{a^2d(a^2+2ab+b^2)}-\frac{4b^3x}{a^3(a^2+2ab+b^2)}-\frac{}{a^3d(a}$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/2*b^3/(a+b)^2/a^3*((3*a+2*b)/b*ln(a+b*tanh(d*x+c)^2)-a*(a+b)/b/(a
+b*tanh(d*x+c)^2))+(2*b-a)/a^3*ln(tanh(d*x+c))+1/2/a^2/tanh(d*x+c)^2+1/2/(
a+b)^2*ln(1+tanh(d*x+c))+1/2/(a+b)^2*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3468 vs. $2(118) = 236$.

Time = 0.31 (sec) , antiderivative size = 3468, normalized size of antiderivative = 27.97

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 402 vs. 2(118) = 236.

Time = 0.06 (sec) , antiderivative size = 402, normalized size of antiderivative = 3.24

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \frac{(3ab^2+2b^3)\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^5+2a^4b+a^3b^2)d}+\frac{dx+c}{(a^2+2ab+b^2)d} - \frac{2\left((a^3+3a^2b+3ab^2+2b^3)e^{(-2dx-2c)}+2(a^3+a^2b-ab^2-2b^3)e^{(-\cdots}\right.}{(a^5+3a^4b+3a^3b^2+a^2b^3-4(a^4b+2a^3b^2+a^2b^3)e^{(-2dx-2c)}-2(a^5-a^4b-5a^3b^2-3a^2b^3)e^{(-4dx-\cdots}} + \frac{(a-2b)\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{(a-2b)\log\left(e^{(-dx-c)}-1\right)}{a^3d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
1/2*(3*a*b^2 + 2*b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
 4*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + (d*x + c)/((a^2 + 2*a*b + b
^2)*d) - 2*((a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3)*e^(-2*d*x - 2*c) + 2*(a^3 +
a^2*b - a*b^2 - 2*b^3)*e^(-4*d*x - 4*c) + (a^3 + 3*a^2*b + 3*a*b^2 + 2*b^3
)*e^(-6*d*x - 6*c))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 - 4*(a^4*b + 2*a
^3*b^2 + a^2*b^3)*e^(-2*d*x - 2*c) - 2*(a^5 - a^4*b - 5*a^3*b^2 - 3*a^2*b^
3)*e^(-4*d*x - 4*c) - 4*(a^4*b + 2*a^3*b^2 + a^2*b^3)*e^(-6*d*x - 6*c) + (
a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*e^(-8*d*x - 8*c))*d) + (a - 2*b)*log(
e^(-d*x - c) + 1)/(a^3*d) + (a - 2*b)*log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 323 vs. $2(118) = 236$.

Time = 0.23 (sec) , antiderivative size = 323, normalized size of antiderivative = 2.60

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{(3\,ab^2+2\,b^3)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^5+2\,a^4b+a^3b^2} - \frac{2\,(dx+c)}{a^2+2\,ab+b^2} + \frac{2\,(a-2\,b)\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^3} - \frac{4\left(\frac{(a^4+}{}\right.}{}}{2\,d}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/2*((3*a*b^2 + 2*b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(
2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^5 + 2*a^4*b + a^3*b^2) - 2*
(d*x + c)/(a^2 + 2*a*b + b^2) + 2*(a - 2*b)*log(abs(e^(2*d*x + 2*c) - 1))/
a^3 - 4*((a^4 + 3*a^3*b + 3*a^2*b^2 + 2*a*b^3)*e^(6*d*x + 6*c)/(a + b) + 2
*(a^4 + a^3*b - a^2*b^2 - 2*a*b^3)*e^(4*d*x + 4*c)/(a + b) + (a^4 + 3*a^3*
b + 3*a^2*b^2 + 2*a*b^3)*e^(2*d*x + 2*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*
e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)*(a +
b)*a^3*(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx = \int \frac{\coth(c+d\,x)^3}{\left(b\tanh(c+d\,x)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [B] (verification not implemented)

Time = 31.61 (sec) , antiderivative size = 2786, normalized size of antiderivative = 22.47

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
(2*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**4*b + 2*e**(8*c + 8*d*x)*log(
e**(c + d*x) - 1)*a**3*b**2 - 6*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**
2*b**3 - 10*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a*b**4 - 4*e**(8*c + 8*
d*x)*log(e**(c + d*x) - 1)*b**5 + 2*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)
*a**4*b + 2*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**3*b**2 - 6*e**(8*c +
 8*d*x)*log(e**(c + d*x) + 1)*a**2*b**3 - 10*e**(8*c + 8*d*x)*log(e**(c +
d*x) + 1)*a*b**4 - 4*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*b**5 + 3*e**(8
*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x
)*sqrt(b))*a**2*b**3 + 5*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b)
 + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a*b**4 + 2*e**(8*c + 8*d*x)*log(e
**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*b**5 +
 3*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**
(c + d*x)*sqrt(b))*a**2*b**3 + 5*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqr
t(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a*b**4 + 2*e**(8*c + 8*d*
x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b)
)*b**5 - e**(8*c + 8*d*x)*a**5 - 2*e**(8*c + 8*d*x)*a**4*b*d*x - 4*e**(8*c
 + 8*d*x)*a**4*b - 2*e**(8*c + 8*d*x)*a**3*b**2*d*x - 6*e**(8*c + 8*d*x)*a
**3*b**2 - 5*e**(8*c + 8*d*x)*a**2*b**3 - 2*e**(8*c + 8*d*x)*a*b**4 - 8*e*
*(6*c + 6*d*x)*log(e**(c + d*x) - 1)*a**3*b**2 + 24*e**(6*c + 6*d*x)*log(e
**(c + d*x) - 1)*a*b**4 + 16*e**(6*c + 6*d*x)*log(e**(c + d*x) - 1)*b**...
```

3.189 $\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx$

Optimal result

Integrand size = 23, antiderivative size = 159

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2} dx = \frac{x}{(a+b)^2}+\frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^2d} - \frac{(2a^2-2ab-5b^2)\coth(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\coth^3(c+dx)}{6a^2(a+b)d} + \frac{b\coth^3(c+dx)}{2a(a+b)d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^2+1/2*b^(5/2)*(7*a+5*b)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(7/2
)/(a+b)^2/d-1/2*(2*a^2-2*a*b-5*b^2)*coth(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*
coth(d*x+c)^3/a^2/(a+b)/d+1/2*b*coth(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2
)
```

Mathematica [A] (verified)

Time = 0.97 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.87

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{6(c+dx)}{(a+b)^2}+\frac{3b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^2}+\frac{4(-2a+3b)\coth(c+dx)}{a^3}-\frac{2\coth(c+dx)\text{csch}^2(c+dx)}{a^2}+\frac{3b^3\sinh(2(c+dx)}{a^3(a+b)(a-b+(a+b)\cosh(}}{6d}$$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((6*(c + d*x))/(a + b)^2 + (3*b^(5/2)*(7*a + 5*b)*ArcTan[(Sqrt[b]*Tanh[c +
 d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^2) + (4*(-2*a + 3*b)*Coth[c + d*x])/a^3
- (2*Coth[c + d*x]*Csch[c + d*x]^2)/a^2 + (3*b^3*Sinh[2*(c + d*x)])/(a^3*(
a + b)*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(6*d)
```

Rubi [A] (verified)

Time = 0.78 (sec) , antiderivative size = 169, normalized size of antiderivative = 1.06, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.478$, Rules used = {3042, 4153, 374, 25, 445, 27, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^2}dx$$
$$\Big\downarrow \ 4153$$
$$\frac{\int \frac{\coth^4(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{d}$$
$$\Big\downarrow \ 374$$

$$\frac{\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{\coth^4(c+dx)\left(-5b\tanh^2(c+dx)+2a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\int \frac{\coth^4(c+dx)\left(-5b\tanh^2(c+dx)+2a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 445

$$\frac{\frac{-\frac{\int -\frac{3\coth^2(c+dx)\left(2a^2-2ba-5b^2+b(2a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{3a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 27

$$\frac{\frac{\frac{\int \frac{\coth^2(c+dx)\left(2a^2-2ba-5b^2+b(2a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 445

$$\frac{\frac{\frac{-\frac{\int -\frac{2a^3-2ba^2+2b^2a+5b^3+b\left(2a^2-2ba-5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 25

$$\frac{\frac{\frac{\frac{\int \frac{2a^3-2ba^2+2b^2a+5b^3+b\left(2a^2-2ba-5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 397

$$\frac{\frac{\frac{\frac{\frac{2a^3\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^3(7a+5b)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{d}$$

$\Big\downarrow$ 218

$$\frac{\frac{\frac{\frac{2a^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx}{2a(a+b)\left(a+b\tanh^2(\right.}}{d}$$

$\downarrow$ 219

$$\frac{\frac{\frac{\frac{2a^3\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{b^{5/2}(7a+5b)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{a}-\frac{\left(2a^2-2ab-5b^2\right)\coth(c+dx)}{a}-\frac{(2a+5b)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right.}}{d}$$

input
```
Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^2,x]
```

output
```
((-1/3*((2*a + 5*b)*Coth[c + d*x]^3)/a + (((b^(5/2)*(7*a + 5*b)*ArcTan[(Sq
rt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + (2*a^3*ArcTanh[Tanh[c +
 d*x]])/(a + b))/a - ((2*a^2 - 2*a*b - 5*b^2)*Coth[c + d*x])/a)/a)/(2*a*(a
 + b)) + (b*Coth[c + d*x]^3)/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.26 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.89

method	result
derivativedivides	$-\frac{-\frac{b^3\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(7a+5b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^3}-\frac{2b-a}{a^3\tanh(dx+c)}+\frac{1}{3a^2\tanh(dx+c)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}+\frac{\ln(}{}}{d}$
default	$-\frac{-\frac{b^3\left(\frac{\left(\frac{b}{2}+\frac{a}{2}\right)\tanh(dx+c)}{a+b\tanh(dx+c)^2}+\frac{(7a+5b)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{2\sqrt{ab}}\right)}{(a+b)^2a^3}-\frac{2b-a}{a^3\tanh(dx+c)}+\frac{1}{3a^2\tanh(dx+c)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^2}+\frac{\ln(}{}}{d}$
risch	$\frac{x}{a^2+2ab+b^2}-\frac{12a^4e^{8dx+8c}+24a^3b\,e^{8dx+8c}-21a\,b^3e^{8dx+8c}-15b^4e^{8dx+8c}+12a^4e^{6dx+6c}-12a^3b\,e^{6dx+6c}-12a^2b^2e^{6c}}{}$

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-b^3/(a+b)^2/a^3*((1/2*b+1/2*a)*tanh(d*x+c)/(a+b*tanh(d*x+c)^2)+1/2*
(7*a+5*b)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))-(2*b-a)/a^3/tanh(
d*x+c)+1/3/a^2/tanh(d*x+c)^3-1/2/(a+b)^2*ln(1+tanh(d*x+c))+1/2/(a+b)^2*ln(
-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4080 vs. 2(143) = 286.

Time = 0.23 (sec) , antiderivative size = 8482, normalized size of antiderivative = 53.35

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2} \, dx = \int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2} \, dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**2,x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**2, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2345 vs. $2(143) = 286$.

Time = 0.50 (sec) , antiderivative size = 2345, normalized size of antiderivative = 14.75

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^2} \, dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="maxima")
```

output
```
-1/4*(a^2*b - a*b^2 - b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(2*d*
x + 2*c) + a + b)/((a^5 + 2*a^4*b + a^3*b^2)*d) + 1/4*(a^2*b - a*b^2 - b^3
)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^5
 + 2*a^4*b + a^3*b^2)*d) + 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*
arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b +
a^3*b^2)*sqrt(a*b)*d) - 1/32*(3*a^3*b - 29*a^2*b^2 - 11*a*b^3 + 5*b^4)*arc
tan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^
3*b^2)*sqrt(a*b)*d) + 1/48*(44*a^4 + 117*a^3*b + 111*a^2*b^2 + 23*a*b^3 -
15*b^4 + 3*(24*a^4 + 69*a^3*b + 45*a^2*b^2 + 27*a*b^3 - 5*b^4)*e^(8*d*x +
8*c) + 6*(6*a^4 - 31*a^3*b - 50*a^2*b^2 - 51*a*b^3 + 10*b^4)*e^(6*d*x + 6*
c) - 2*(50*a^4 - 78*a^3*b - 225*a^2*b^2 - 196*a*b^3 + 45*b^4)*e^(4*d*x + 4
*c) - 2*(10*a^4 + 115*a^3*b + 182*a^2*b^2 + 95*a*b^3 - 30*b^4)*e^(2*d*x +
2*c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3 - (a^6 + 3*a^5*b + 3*a^4*b^2 +
 a^3*b^3)*e^(10*d*x + 10*c) + (a^6 + 7*a^5*b + 11*a^4*b^2 + 5*a^3*b^3)*e^(
8*d*x + 8*c) + 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(6*d*x + 6*c) -
 2*(a^6 - 3*a^5*b - 9*a^4*b^2 - 5*a^3*b^3)*e^(4*d*x + 4*c) - (a^6 + 7*a^5*
b + 11*a^4*b^2 + 5*a^3*b^3)*e^(2*d*x + 2*c))*d) - 1/48*(44*a^4 + 117*a^3*b
 + 111*a^2*b^2 + 23*a*b^3 - 15*b^4 - 2*(10*a^4 + 115*a^3*b + 182*a^2*b^2 +
 95*a*b^3 - 30*b^4)*e^(-2*d*x - 2*c) - 2*(50*a^4 - 78*a^3*b - 225*a^2*b^2
- 196*a*b^3 + 45*b^4)*e^(-4*d*x - 4*c) + 6*(6*a^4 - 31*a^3*b - 50*a^2*b...
```

Giac [A] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 281, normalized size of antiderivative = 1.77

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx$$
$$= \frac{\frac{3\left(7\,ab^3+5\,b^4\right)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\sqrt{ab}}+\frac{6\,(dx+c)}{a^2+2\,ab+b^2}-\frac{6\left(ab^3e^{(2\,dx+2\,c)}-b^4e^{(2\,dx+2\,c)}+ab^3+b^4\right)}{\left(a^5+2\,a^4b+a^3b^2\right)\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,a}}}{6\,d}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x, algorithm="giac")
```

output
```
1/6*(3*(7*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2*c)
 + a - b)/sqrt(a*b))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt(a*b)) + 6*(d*x + c)/(
a^2 + 2*a*b + b^2) - 6*(a*b^3*e^(2*d*x + 2*c) - b^4*e^(2*d*x + 2*c) + a*b^
3 + b^4)/((a^5 + 2*a^4*b + a^3*b^2)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c)
 + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)) - 8*(3*a*e^(4*d*x +
 4*c) - 3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 6*b*e^(2*d*x + 2*c) +
2*a - 3*b)/(a^3*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\coth(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^2}\,dx$$

input
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2,x)
```

output
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^2, x)
```

Reduce [F]

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^2}\,dx=\int \frac{\coth\left(dx+c\right)^4}{\left(\tanh\left(dx+c\right)^2b+a\right)^2}dx$$

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

output
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^2,x)
```

3.190 $\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 144

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{x}{(a+b)^3} - \frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8b^{5/2}(a+b)^3d} + \frac{a\tanh^3(c+dx)}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(3a+7b)\tanh(c+dx)}{8b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^3-1/8*a^(1/2)*(3*a^2+10*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(
1/2))/b^(5/2)/(a+b)^3/d+1/4*a*tanh(d*x+c)^3/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^
2+1/8*a*(3*a+7*b)*tanh(d*x+c)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.83 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{8(c+dx)-\frac{\sqrt{a}(3a^2+10ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^{5/2}}-\frac{4a^2(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))^2}+\frac{3a(a+b)(a+3b)\sinh(2(c+dx))}{b^2(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

input
```
Integrate[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) - (Sqrt[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c +
 d*x])/Sqrt[a]])/b^(5/2) - (4*a^2*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (
a + b)*Cosh[2*(c + d*x)])^2) + (3*a*(a + b)*(a + 3*b)*Sinh[2*(c + d*x)])/(
b^2*(a - b + (a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.20, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.391$, Rules used = {3042, 25, 4153, 25, 372, 440, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ic+idx)^6}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{\int -\frac{\tanh^6(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\tanh^6(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 372$$

$$-\frac{\frac{\int \frac{\tanh^2(c+dx)\left(3a-(3a+4b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 440$$

$$-\frac{\frac{\frac{\int \frac{a(3a+7b)-\left(3a^2+7ba+8b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\frac{\frac{a\left(3a^2+10ab+15b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}-\frac{8b^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)(a+b\tanh^2(c+dx)}}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\frac{\frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{8b^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\frac{\frac{\sqrt{a}\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}(a+b)}-\frac{8b^2\text{arctanh}(\tanh(c+dx))}{a+b}}{2b(a+b)}-\frac{a(3a+7b)\tanh(c+dx)}{2b(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}-\frac{a\tanh^3(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

input
```
Int[Tanh[c + d*x]^6/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-1/4*(a*Tanh[c + d*x]^3)/(b*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + (((Sqr
t[a]*(3*a^2 + 10*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(S
qrt[b]*(a + b)) - (8*b^2*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*b*(a + b)) -
(a*(3*a + 7*b)*Tanh[c + d*x])/(2*b*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*b*
(a + b)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.22 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{-\frac{a\left(\frac{-\frac{\left(5a^2+14ab+9b^2\right)\tanh(dx+c)^3}{8b}-\frac{a\left(3a^2+10ab+7b^2\right)\tanh(dx+c)}{8b^2}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8b^2\sqrt{ab}}\right)}{(a+b)^3}+\frac{\ln(1+\mathrm{ta}}{2(a}}{d}$
default	$\frac{-\frac{a\left(\frac{-\frac{\left(5a^2+14ab+9b^2\right)\tanh(dx+c)^3}{8b}-\frac{a\left(3a^2+10ab+7b^2\right)\tanh(dx+c)}{8b^2}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(3a^2+10ab+15b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8b^2\sqrt{ab}}\right)}{(a+b)^3}+\frac{\ln(1+\mathrm{ta}}{2(a}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{a\left(3a^3\mathrm{e}^{6dx+6c}+13a^2b\,\mathrm{e}^{6dx+6c}+a\,b^2\mathrm{e}^{6dx+6c}-9b^3\mathrm{e}^{6dx+6c}+9a^3\mathrm{e}^{4dx+4c}+21a^2b\,\mathrm{e}^{4dx+4c}-9a\,b^2}{4\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx}\right.}$

input
```
int(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-a/(a+b)^3*((-1/8*(5*a^2+14*a*b+9*b^2)/b*tanh(d*x+c)^3-1/8*a*(3*a^2+1
0*a*b+7*b^2)/b^2*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(3*a^2+10*a*b+15*b
^2)/b^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^3*ln(1+ta
nh(d*x+c))-1/2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3603 vs. $2(130) = 260$.

Time = 0.22 (sec) , antiderivative size = 7528, normalized size of antiderivative = 52.28

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^6(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**6/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3354 vs. $2(130) = 260$.

Time = 1.14 (sec) , antiderivative size = 3354, normalized size of antiderivative = 23.29

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/512*(3*a^5 + 25*a^4*b + 150*a^3*b^2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*a
rctan(1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b
^3 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*b)*d) + 1/512*(3*a^5 + 25*a^4*b + 150*a^3
*b^2 - 150*a^2*b^3 - 25*a*b^4 - 3*b^5)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c
) + a - b)/sqrt(a*b))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*sqrt(a*
b)*d) - 1/256*(3*a^6 + 30*a^5*b - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 +
30*a*b^5 + 3*b^6 + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 - 28*a*b^
5 - 3*b^6)*e^(6*d*x + 6*c) + (9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^
3 - 905*a^2*b^4 + 66*a*b^5 + 9*b^6)*e^(4*d*x + 4*c) + (9*a^6 + 68*a^5*b -
659*a^4*b^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(2*d*x + 2*c))/((a^7*b^2 +
 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7 + (a^7*b^2 + 5*
a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b^7)*e^(8*d*x + 8*c) +
 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(
6*d*x + 6*c) + 2*(3*a^7*b^2 + 7*a^6*b^3 + 6*a^5*b^4 + 6*a^4*b^5 + 7*a^3*b^
6 + 3*a^2*b^7)*e^(4*d*x + 4*c) + 4*(a^7*b^2 + 3*a^6*b^3 + 2*a^5*b^4 - 2*a^
4*b^5 - 3*a^3*b^6 - a^2*b^7)*e^(2*d*x + 2*c))*d) + 1/256*(3*a^6 + 30*a^5*b
 - 99*a^4*b^2 - 252*a^3*b^3 - 99*a^2*b^4 + 30*a*b^5 + 3*b^6 + (9*a^6 + 68*
a^5*b - 659*a^4*b^2 + 659*a^2*b^4 - 68*a*b^5 - 9*b^6)*e^(-2*d*x - 2*c) + (
9*a^6 + 66*a^5*b - 905*a^4*b^2 + 1148*a^3*b^3 - 905*a^2*b^4 + 66*a*b^5 + 9
*b^6)*e^(-4*d*x - 4*c) + (3*a^6 + 28*a^5*b - 465*a^4*b^2 + 465*a^2*b^4 ...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 408 vs. $2(130) = 260$.

Time = 0.33 (sec) , antiderivative size = 408, normalized size of antiderivative = 2.83

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{(3a^3+10a^2b+15ab^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^3b^2+3a^2b^3+3ab^4+b^5)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(3a^4e^{(6dx+6c)}+13a^3be^{(6dx+6c)}+a^2b^2e^{(6dx\cdots}\right.}{\cdots}}{\cdots}$$

input
```
integrate(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((3*a^3 + 10*a^2*b + 15*a*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2
*d*x + 2*c) + a - b)/sqrt(a*b))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*sqr
t(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(3*a^4*e^(6*d*x
+ 6*c) + 13*a^3*b*e^(6*d*x + 6*c) + a^2*b^2*e^(6*d*x + 6*c) - 9*a*b^3*e^(6
*d*x + 6*c) + 9*a^4*e^(4*d*x + 4*c) + 21*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2
*e^(4*d*x + 4*c) + 27*a*b^3*e^(4*d*x + 4*c) + 9*a^4*e^(2*d*x + 2*c) + 23*a
^3*b*e^(2*d*x + 2*c) - 13*a^2*b^2*e^(2*d*x + 2*c) - 27*a*b^3*e^(2*d*x + 2*
c) + 3*a^4 + 15*a^3*b + 21*a^2*b^2 + 9*a*b^3)/((a^3*b^2 + 3*a^2*b^3 + 3*a*
b^4 + b^5)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.85 (sec) , antiderivative size = 2669, normalized size of antiderivative = 18.53

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^6/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tan
h(c + d*x)^3*(9*a*b + 5*a^2))/(8*b*(2*a*b + a^2 + b^2)) + (a*tanh(c + d*x)
*(7*a*b + 3*a^2))/(8*b^2*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x
)^4 + 2*a*b*d*tanh(c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) -
(atan((((-a*b^5)^(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^
2*b^4 + 300*a^3*b^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5
*d^2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)) + (((224*a*b^10*d^2 + 1440*a^2*b^9*d^
2 + 3936*a^3*b^8*d^2 + 5920*a^4*b^7*d^2 + 5280*a^5*b^6*d^2 + 2784*a^6*b^5*
d^2 + 800*a^7*b^4*d^2 + 96*a^8*b^3*d^2)/(64*(b^9*d^3 + 6*a*b^8*d^3 + 15*a^
2*b^7*d^3 + 20*a^3*b^6*d^3 + 15*a^4*b^5*d^3 + 6*a^5*b^4*d^3 + a^6*b^3*d^3)
) - (tanh(c + d*x)*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2)*(256*b^12*d^2
+ 1280*a*b^11*d^2 + 2304*a^2*b^10*d^2 + 1280*a^3*b^9*d^2 - 1280*a^4*b^8*d^
2 - 2304*a^5*b^7*d^2 - 1280*a^6*b^6*d^2 - 256*a^7*b^5*d^2))/(512*(b^8*d +
3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^
2 + 4*a^3*b^4*d^2 + a^4*b^3*d^2)))*(-a*b^5)^(1/2)*(10*a*b + 3*a^2 + 15*b^2
))/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)))*(10*a*b + 3*a^2 + 1
5*b^2)*1i)/(16*(b^8*d + 3*a^2*b^6*d + a^3*b^5*d + 3*a*b^7*d)) + ((-a*b^5)^
(1/2)*((tanh(c + d*x)*(60*a^5*b + 9*a^6 + 64*b^6 + 225*a^2*b^4 + 300*a^3*b
^3 + 190*a^4*b^2))/(32*(b^7*d^2 + 4*a*b^6*d^2 + 6*a^2*b^5*d^2 + 4*a^3*b^4*
d^2 + a^4*b^3*d^2)) - (((224*a*b^10*d^2 + 1440*a^2*b^9*d^2 + 3936*a^3*b...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 575, normalized size of antiderivative = 3.99

$$\int \frac{\tanh^6(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^6/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d
*x)**4*a**2*b**2 - 10*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt
(a)))*tanh(c + d*x)**4*a*b**3 - 15*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/
(sqrt(b)*sqrt(a)))*tanh(c + d*x)**4*b**4 - 6*sqrt(b)*sqrt(a)*atan((tanh(c
+ d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**3*b - 20*sqrt(b)*sqrt(a)*
atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**2*b**2 - 30*
sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2
*a*b**3 - 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**4
 - 10*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b - 1
5*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*
tanh(c + d*x)**4*b**5*d*x + 5*tanh(c + d*x)**3*a**3*b**2 + 14*tanh(c + d*x
)**3*a**2*b**3 + 9*tanh(c + d*x)**3*a*b**4 + 16*tanh(c + d*x)**2*a*b**4*d*
x + 3*tanh(c + d*x)*a**4*b + 10*tanh(c + d*x)*a**3*b**2 + 7*tanh(c + d*x)*
a**2*b**3 + 8*a**2*b**3*d*x)/(8*b**3*d*(tanh(c + d*x)**4*a**3*b**2 + 3*tan
h(c + d*x)**4*a**2*b**3 + 3*tanh(c + d*x)**4*a*b**4 + tanh(c + d*x)**4*b**
5 + 2*tanh(c + d*x)**2*a**4*b + 6*tanh(c + d*x)**2*a**3*b**2 + 6*tanh(c +
d*x)**2*a**2*b**3 + 2*tanh(c + d*x)**2*a*b**4 + a**5 + 3*a**4*b + 3*a**3*b
**2 + a**2*b**3))
```

3.191 $\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 109

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d} - \frac{a^2}{4b^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{a(a+2b)}{2b^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4*a^2/b^2/
(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/2*a*(a+2*b)/b^2/(a+b)^2/d/(a+b*tanh(d*x+c)
^2)
```

Mathematica [A] (verified)

Time = 0.72 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{-4\log(\cosh(c+dx)) - 2\log\left(a+b\tanh^2(c+dx)\right) + \frac{a^2(a+b)^2}{b^2(a+b\tanh^2(c+dx))^2} - \frac{2a(a+b)(a+2b)}{b^2(a+b\tanh^2(c+dx))}}{4(a+b)^3d}$$

input
```
Integrate[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/4*(-4*Log[Cosh[c + d*x]] - 2*Log[a + b*Tanh[c + d*x]^2] + (a^2*(a + b)^
2)/(b^2*(a + b*Tanh[c + d*x]^2)^2) - (2*a*(a + b)*(a + 2*b))/(b^2*(a + b*T
anh[c + d*x]^2)))/((a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.96, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)^5}{(a-b\tan(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)^5}{(a-b\tan(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\tanh^5(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$\downarrow$ 26

$$\frac{\int \frac{\tanh^5(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$\downarrow$ 354

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh^2(c+dx)}{2d}$$

$\downarrow$ 99

$$\frac{\int \left(\frac{a^2}{b(a+b)\left(b\tanh^2(c+dx)+a\right)^3}-\frac{(a+2b)a}{b(a+b)^2\left(b\tanh^2(c+dx)+a\right)^2}-\frac{1}{(a+b)^3(\tanh^2(c+dx)-1)}+\frac{b}{(a+b)^3(b\tanh^2(c+dx)+a)}\right)d\tanh^2(c+}{2d}$$

$\downarrow$ 2009

$$\frac{-\frac{a^2}{2b^2(a+b)\left(a+b\tanh^2(c+dx)\right)^2}+\frac{a(a+2b)}{b^2(a+b)^2(a+b\tanh^2(c+dx))}-\frac{\log(1-\tanh^2(c+dx))}{(a+b)^3}+\frac{\log(a+b\tanh^2(c+dx))}{(a+b)^3}}{2d}$$

input
```
Int[Tanh[c + d*x]^5/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^3) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^3 - a^2/(2*b^2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + (a*(a + 2*b))/(b^2*(
a + b)^2*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.11 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\tanh(dx+c)^2\right)}-\ln\left(a+b\tanh(dx+c)^2\right)+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\tanh(dx+c)^2\right)^2}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^3}}{d}$
default	$\frac{-\frac{-\frac{a\left(a^2+3ab+2b^2\right)}{b^2\left(a+b\tanh(dx+c)^2\right)}-\ln\left(a+b\tanh(dx+c)^2\right)+\frac{a^2\left(a^2+2ab+b^2\right)}{2b^2\left(a+b\tanh(dx+c)^2\right)^2}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{4\,e^{2dx+2c}a(a\,e^{4dx+4c}+b\,e^{4dx+4c}+a\,e^{2dx+2c}-2b\,e^{2dx+2c}\cdots}{(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b)^2(a+b)(a\cdots}$
parallelrisch	$-\frac{-2a^3b\tanh(dx+c)^2-4a^3b+8\ln(1-\tanh(dx+c))\tanh(dx+c)^2a\,b^3+4b^4\tanh(dx+c)^4xd+8x\tanh(dx+c)^2a\,b^3d-2\ln\cdots}{\cdots}$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)^3*(-a*(a^2+3*a*b+2*b^2)/b^2/(a+b*tanh(d*x+c)^2)-ln(a+b*tan
h(d*x+c)^2)+1/2*a^2*(a^2+2*a*b+b^2)/b^2/(a+b*tanh(d*x+c)^2)^2)-1/2/(a+b)^3
*ln(1+tanh(d*x+c))-1/2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(103) = 206$.

Time = 0.14 (sec) , antiderivative size = 2584, normalized size of antiderivative = 23.71

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^8 + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x - a^2 - a*b)*sinh(d*x + c)^6 +
 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x - a^2
- a*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2
*a^2 + 4*a*b)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c
)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(
d*x + c)^2 - 2*a^2 + 4*a*b)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)^5 + 10*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c)^3 + ((3
*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x + c)^3
+ 2*(a^2 + 2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x - a^2 - a*b)*cosh(d*x + c
)^2 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x -
 a^2 - a*b)*cosh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)
*d*x - 2*a^2 + 4*a*b)*cosh(d*x + c)^2 - a^2 - a*b)*sinh(d*x + c)^2 - ((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x
 + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**5/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 376 vs. $2(103) = 206$.

Time = 0.08 (sec) , antiderivative size = 376, normalized size of antiderivative = 3.45

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{dx+c}{(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

$$+ \frac{}{(a^5+5\,a^4b+10\,a^3b^2+10\,a^2b^3+5\,ab^4+b^5+4\,(a^5+3\,a^4b+2\,a^3b^2-2\,a^2b^3-3\,ab^4-b^5)e^{(-2\,dx-2\,c)}+}$$

$$+ \frac{\log\left(2\,(a-b)e^{(-2\,dx-2\,c)}+(a+b)e^{(-4\,dx-4\,c)}+a+b\right)}{2\,(a^3+3\,a^2b+3\,ab^2+b^3)d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 4*((a^2 + a*b)*e^(-2*d*x -
 2*c) + (a^2 - 2*a*b)*e^(-4*d*x - 4*c) + (a^2 + a*b)*e^(-6*d*x - 6*c))/((a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b
+ 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a
^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d)
 + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(103) = 206$.

Time = 0.30 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.25

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$= \frac{\frac{2\,\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2-}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-}\right.\right.}}{4\,d}$$

input
```
integrate(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
 + e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x
- 2*c))^2 - 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 12*b*(e^(2*d*x + 2*
c) + e^(-2*d*x - 2*c)) - 4*a + 12*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2
*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2
*b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 416, normalized size of antiderivative = 3.82

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{a^4+a^3\,b\left(2\tanh(c+d\,x)^2+4\right)-a\,b^3\left(-4\tanh(c+d\,x)^2+\tanh(c+d\,x)^2\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh\cdots}{2\,a-a\tanh(c+d\,x)^2+b\tanh\cdots}\right.\cdots\right.}{4\,d\,a^5\,b^2+8\,d\,a^4\,b^3\tanh\left(c+d\,x\right)^2+12\,d\,a^4\,b^3+4\,d\,a^3\,b^4\tanh\left(c+d\,x\right)^4+24\,d\,a^3\,b^4\tanh\left(c+d\,x\right)^2+12\cdots}$$

input
```
int(tanh(c + d*x)^5/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
(a^4 + a^3*b*(2*tanh(c + d*x)^2 + 4) - a*b^3*(tanh(c + d*x)^2*atan((a*tanh
(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c
 + d*x)^2))*8i - 4*tanh(c + d*x)^2) + a^2*b^2*(6*tanh(c + d*x)^2 - atan((a
*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*t
anh(c + d*x)^2))*4i + 3) - b^4*tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i
+ b*tanh(c + d*x)^2*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i)
/(4*a^2*b^5*d + 12*a^3*b^4*d + 12*a^4*b^3*d + 4*a^5*b^2*d + 4*b^7*d*tanh(c
 + d*x)^4 + 24*a^2*b^5*d*tanh(c + d*x)^2 + 24*a^3*b^4*d*tanh(c + d*x)^2 +
8*a^4*b^3*d*tanh(c + d*x)^2 + 12*a^2*b^5*d*tanh(c + d*x)^4 + 4*a^3*b^4*d*t
anh(c + d*x)^4 + 8*a*b^6*d*tanh(c + d*x)^2 + 12*a*b^6*d*tanh(c + d*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 11531, normalized size of antiderivative = 105.79

$$\int \frac{\tanh^5(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^5/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**
(c + d*x)*sqrt(b))*tanh(c + d*x)**4*a**5*b**2 + 2*e**(8*c + 8*d*x)*log(e**
(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c +
 d*x)**4*a**4*b**3 - 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) +
 sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**4*a**3*b**4 - 2*e**(
8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*
x)*sqrt(b))*tanh(c + d*x)**4*a**2*b**5 + 4*e**(8*c + 8*d*x)*log(e**(2*c +
2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**
2*a**6*b + 4*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a +
b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**5*b**2 - 4*e**(8*c + 8*d*
x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b)
)*tanh(c + d*x)**2*a**4*b**3 - 4*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqr
t(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**3*b**
4 + 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*
e**(c + d*x)*sqrt(b))*a**7 + 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(
a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**6*b - 2*e**(8*c + 8*d*x)
*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*
a**5*b**2 - 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a +
 b) - 2*e**(c + d*x)*sqrt(b))*a**4*b**3 + 2*e**(8*c + 8*d*x)*log(e**(2*c +
 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*tanh(c + d*...
```

3.192 $\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{x}{(a+b)^3} - \frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}b^{3/2}(a+b)^3d} + \frac{a\tanh(c+dx)}{4b(a+b)d\,(a+b\tanh^2(c+dx))^2} - \frac{(a+5b)\tanh(c+dx)}{8b(a+b)^2d\,(a+b\tanh^2(c+dx))}$$

output
```
x/(a+b)^3-1/8*(a^2+6*a*b-3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(1/2
)/b^(3/2)/(a+b)^3/d+1/4*a*tanh(d*x+c)/b/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*
(a+5*b)*tanh(d*x+c)/b/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.74 (sec) , antiderivative size = 135, normalized size of antiderivative = 0.99

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{8(c+dx)-\frac{(a^2+6ab-3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}+\frac{4a(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2}+\frac{(a-5b)(a+b)\sinh(2(c+dx))}{b(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

input
```
Integrate[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) - ((a^2 + 6*a*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a]*b^(3/2)) + (4*a*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)*
Cosh[2*(c + d*x)])^2 + ((a - 5*b)*(a + b)*Sinh[2*(c + d*x)])/(b*(a - b + (
a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 157, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3042, 4153, 372, 402, 25, 27, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{\tan(ic+idx)^4}{(a-b\tan(ic+idx)^2)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{\int \frac{\tanh^4(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{372}$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int\frac{a-(a+4b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int-\frac{a\left(-\left((a+5b)\tanh^2(c+dx)\right)+a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int\frac{a\left(-\left((a+5b)\tanh^2(c+dx)\right)+a-3b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int\frac{-\left((a+5b)\tanh^2(c+dx)\right)+a-3b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{\left(a^2+6ab-3b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}-\frac{8b\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}-\frac{8b\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{a\tanh(c+dx)}{4b(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}-\frac{8b\text{arctanh}(\tanh(c+dx))}{a+b}}{2(a+b)}+\frac{(a+5b)\tanh(c+dx)}{2(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4b(a+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((a*Tanh[c + d*x])/(4*b*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - ((((a^2 + 6*a
*b - 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a +
 b)) - (8*b*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*(a + b)) + ((a + 5*b)*Tanh
[c + d*x])/(2*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*b*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.21 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.08

method	result
derivativedivides	$\frac{-\frac{\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\tanh(dx+c)^3-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a+b)^3}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-}{d}$
default	$\frac{-\frac{\frac{\left(\frac{1}{8}a^2+\frac{3}{4}ab+\frac{5}{8}b^2\right)\tanh(dx+c)^3-\frac{a\left(a^2-2ab-3b^2\right)\tanh(dx+c)}{8b}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(a^2+6ab-3b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8b\sqrt{ab}}}{(a+b)^3}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{a^3\mathrm{e}^{6dx+6c}-9a^2b\,\mathrm{e}^{6dx+6c}-5a\,b^2\mathrm{e}^{6dx+6c}+5b^3\mathrm{e}^{6dx+6c}+3a^3\mathrm{e}^{4dx+4c}-17a^2b\,\mathrm{e}^{4dx+4c}+13a\,b^2\mathrm{e}^{4c}}{4b\left(a\,\mathrm{e}^{4dx+4c}+b\,\mathrm{e}^{4dx+4c}+2a\,\mathrm{e}^{2dx+2}\right.}$

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)^3*(((1/8*a^2+3/4*a*b+5/8*b^2)*tanh(d*x+c)^3-1/8*a*(a^2-2*a*b
-3*b^2)/b*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(a^2+6*a*b-3*b^2)/b/(a*b)
^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^3*ln(1+tanh(d*x+c))-1/
2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3727 vs. 2(123) = 246.

Time = 0.22 (sec) , antiderivative size = 7757, normalized size of antiderivative = 56.62

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2432 vs. $2(123) = 246$.

Time = 0.81 (sec) , antiderivative size = 2432, normalized size of antiderivative = 17.75

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^4)*arctan(1/2*((a + b
)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^
2*b^4)*sqrt(a*b)*d) + 1/128*(a^4 + 24*a^3*b - 54*a^2*b^2 - 16*a*b^3 - 3*b^
4)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5*b + 3*a^
4*b^2 + 3*a^3*b^3 + a^2*b^4)*sqrt(a*b)*d) - 1/64*(a^5 - 33*a^4*b - 54*a^3*
b^2 - 2*a^2*b^3 + 21*a*b^4 + 3*b^5 + (a^5 - 71*a^4*b + 98*a^3*b^2 + 154*a^
2*b^3 - 19*a*b^4 - 3*b^5)*e^(6*d*x + 6*c) + (3*a^5 - 171*a^4*b + 310*a^3*b
^2 - 254*a^2*b^3 + 39*a*b^4 + 9*b^5)*e^(4*d*x + 4*c) + (3*a^5 - 133*a^4*b
+ 86*a^3*b^2 + 190*a^2*b^3 - 41*a*b^4 - 9*b^5)*e^(2*d*x + 2*c))/((a^7*b +
5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + (a^7*b + 5*a^6
*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6)*e^(8*d*x + 8*c) + 4*
(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3*a^3*b^5 - a^2*b^6)*e^(6*d*x
 + 6*c) + 2*(3*a^7*b + 7*a^6*b^2 + 6*a^5*b^3 + 6*a^4*b^4 + 7*a^3*b^5 + 3*a
^2*b^6)*e^(4*d*x + 4*c) + 4*(a^7*b + 3*a^6*b^2 + 2*a^5*b^3 - 2*a^4*b^4 - 3
*a^3*b^5 - a^2*b^6)*e^(2*d*x + 2*c))*d) + 1/64*(a^5 - 33*a^4*b - 54*a^3*b^
2 - 2*a^2*b^3 + 21*a*b^4 + 3*b^5 + (3*a^5 - 133*a^4*b + 86*a^3*b^2 + 190*a
^2*b^3 - 41*a*b^4 - 9*b^5)*e^(-2*d*x - 2*c) + (3*a^5 - 171*a^4*b + 310*a^3
*b^2 - 254*a^2*b^3 + 39*a*b^4 + 9*b^5)*e^(-4*d*x - 4*c) + (a^5 - 71*a^4*b
+ 98*a^3*b^2 + 154*a^2*b^3 - 19*a*b^4 - 3*b^5)*e^(-6*d*x - 6*c))/((a^7*b +
 5*a^6*b^2 + 10*a^5*b^3 + 10*a^4*b^4 + 5*a^3*b^5 + a^2*b^6 + 4*(a^7*b +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(123) = 246$.

Time = 0.29 (sec) , antiderivative size = 384, normalized size of antiderivative = 2.80

$$\int \frac{\tanh^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = -\frac{\frac{(a^2+6ab-3b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^3b+3a^2b^2+3ab^3+b^4)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2\left(a^3e^{(6dx+6c)}-9a^2be^{(6dx+6c)}-5ab^2e^{(6dx+6c)}+5b\cdots\right.}{\cdots}}{\cdots}$$

input
```
integrate(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((a^2 + 6*a*b - 3*b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2
*c) + a - b)/sqrt(a*b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*sqrt(a*b)) -
8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a^3*e^(6*d*x + 6*c) - 9*a
^2*b*e^(6*d*x + 6*c) - 5*a*b^2*e^(6*d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 3
*a^3*e^(4*d*x + 4*c) - 17*a^2*b*e^(4*d*x + 4*c) + 13*a*b^2*e^(4*d*x + 4*c)
 - 15*b^3*e^(4*d*x + 4*c) + 3*a^3*e^(2*d*x + 2*c) - 11*a^2*b*e^(2*d*x + 2*
c) + a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + a^3 - 3*a^2*b - 9*a*
b^2 - 5*b^3)/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*(a*e^(4*d*x + 4*c) + b*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.90 (sec) , antiderivative size = 2574, normalized size of antiderivative = 18.79

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - ((tan
h(c + d*x)^3*(a + 5*b))/(8*(2*a*b + a^2 + b^2)) - (a*tanh(c + d*x)*(a - 3*
b))/(8*b*(2*a*b + a^2 + b^2)))/(a^2*d + b^2*d*tanh(c + d*x)^4 + 2*a*b*d*ta
nh(c + d*x)^2) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^3) - (atan(((((tanh(c
 + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2))/(32*(b^5*d^2 +
4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)) + (((96*b^9*d^2
+ 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 + 800*a^4*b^5*d^2 +
96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64*(b^7*d^3 + 6*a*b^6*d
^3 + a^6*b*d^3 + 15*a^2*b^5*d^3 + 20*a^3*b^4*d^3 + 15*a^4*b^3*d^3 + 6*a^5*
b^2*d^3)) - (tanh(c + d*x)*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2)*(256*b^10*
d^2 + 1280*a*b^9*d^2 + 2304*a^2*b^8*d^2 + 1280*a^3*b^7*d^2 - 1280*a^4*b^6*
d^2 - 2304*a^5*b^5*d^2 - 1280*a^6*b^4*d^2 - 256*a^7*b^3*d^2))/(512*(3*a^2*
b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*
d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)))*(-a*b^3)^(1/2)*(6*a*b + a^2 - 3*b^2
))/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b^6*d)))*(-a*b^3)^(1/2)*
(6*a*b + a^2 - 3*b^2)*1i)/(16*(3*a^2*b^5*d + 3*a^3*b^4*d + a^4*b^3*d + a*b
^6*d)) + (((tanh(c + d*x)*(12*a^3*b - 36*a*b^3 + a^4 + 73*b^4 + 30*a^2*b^2
))/(32*(b^5*d^2 + 4*a*b^4*d^2 + a^4*b*d^2 + 6*a^2*b^3*d^2 + 4*a^3*b^2*d^2)
) - (((96*b^9*d^2 + 544*a*b^8*d^2 + 1248*a^2*b^7*d^2 + 1440*a^3*b^6*d^2 +
800*a^4*b^5*d^2 + 96*a^5*b^4*d^2 - 96*a^6*b^3*d^2 - 32*a^7*b^2*d^2)/(64...
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 580, normalized size of antiderivative = 4.23

$$\int \frac{\tanh^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

output

```
( - sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x
)**4*a**2*b**2 - 6*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)
))*tanh(c + d*x)**4*a*b**3 + 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqr
t(b)*sqrt(a)))*tanh(c + d*x)**4*b**4 - 2*sqrt(b)*sqrt(a)*atan((tanh(c + d*
x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**3*b - 12*sqrt(b)*sqrt(a)*atan
((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**2*b**2 + 6*sqrt(
b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a*b*
*3 - sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**4 - 6*sq
rt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b + 3*sqrt(b)
*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*tanh(c +
d*x)**4*a*b**4*d*x - tanh(c + d*x)**3*a**3*b**2 - 6*tanh(c + d*x)**3*a**2*
b**3 - 5*tanh(c + d*x)**3*a*b**4 + 16*tanh(c + d*x)**2*a**2*b**3*d*x + tan
h(c + d*x)*a**4*b - 2*tanh(c + d*x)*a**3*b**2 - 3*tanh(c + d*x)*a**2*b**3
+ 8*a**3*b**2*d*x)/(8*a*b**2*d*(tanh(c + d*x)**4*a**3*b**2 + 3*tanh(c + d*
x)**4*a**2*b**3 + 3*tanh(c + d*x)**4*a*b**4 + tanh(c + d*x)**4*b**5 + 2*ta
nh(c + d*x)**2*a**4*b + 6*tanh(c + d*x)**2*a**3*b**2 + 6*tanh(c + d*x)**2*
a**2*b**3 + 2*tanh(c + d*x)**2*a*b**4 + a**5 + 3*a**4*b + 3*a**3*b**2 + a*
*2*b**3))
```

3.193 $\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$

Optimal result

Integrand size = 23, antiderivative size = 98

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx = \frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3d} + \frac{a}{4b(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d+1/4*a/b/(a+b
)/d/(a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.33 (sec) , antiderivative size = 80, normalized size of antiderivative = 0.82

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx = \frac{4\log(\cosh(c+dx)) + 2\log\left(a+b\tanh^2(c+dx)\right) + \frac{a(a+b)^2}{b\left(a+b\tanh^2(c+dx)\right)^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3d}$$

input
```
Integrate[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)^2)/(b*(a
 + b*Tanh[c + d*x]^2)^2) - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)
^3*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 95, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3042, 26, 4153, 26, 354, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\tan(ic+idx)^3}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$\frac{i\int -\frac{i\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh^3(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow 86$$

$$\frac{\int \left(-\frac{a}{(a+b)(b\tanh^2(c+dx)+a)^3}-\frac{1}{(a+b)^3(\tanh^2(c+dx)-1)}+\frac{b}{(a+b)^3(b\tanh^2(c+dx)+a)}+\frac{b}{(a+b)^2(b\tanh^2(c+dx)+a)^2}\right)d\tanh^2(c-}{2d}$$

$$\Big\downarrow 2009$$

$$\frac{\frac{a}{2b(a+b)(a+b\tanh^2(c+dx))^2}-\frac{1}{(a+b)^2(a+b\tanh^2(c+dx))}-\frac{\log(1-\tanh^2(c+dx))}{(a+b)^3}+\frac{\log(a+b\tanh^2(c+dx))}{(a+b)^3}}{2d}$$

input
```
Int[Tanh[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^3) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^3 + a/(2*b*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - 1/((a + b)^2*(a + b*Tanh
[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

method	result
derivativedivides	$\frac{-\frac{-\frac{-a-b}{a+b\tanh(dx+c)^2}-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^3}}{d}$
default	$\frac{-\frac{-\frac{-a-b}{a+b\tanh(dx+c)^2}-\ln\left(a+b\tanh(dx+c)^2\right)-\frac{a\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^3}}{d}$
parallelrisch	$\frac{a^3b-a\,b^3-8\ln(1-\tanh(dx+c))\tanh(dx+c)^2a\,b^3-4b^4\tanh(dx+c)^4xd-8x\tanh(dx+c)^2a\,b^3d+2\ln\left(a+b\tanh(dx+c)^2\right.}{}$
risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}+\frac{2\,e^{2dx+2c}\left(a^2e^{4dx+4c}-b^2e^{4dx+4c}+2a^2e^{2dx+2c}-2ab\,e^{2dx+2c}+2\right.}{\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)^2d(a^2}$

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/2/(a+b)^3*(-(-a-b)/(a+b*tanh(d*x+c)^2)-ln(a+b*tanh(d*x+c)^2)-1/2*a
*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2)-1/2/(a+b)^3*ln(1+tanh(d*x+c))-1/
2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2611 vs. $2(92) = 184$.

Time = 0.14 (sec) , antiderivative size = 2611, normalized size of antiderivative = 26.64

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^8 + 4*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^6 + 4*(14*(a^2 + 2*a*
b + b^2)*d*x*cosh(d*x + c)^2 + 2*(a^2 - b^2)*d*x - a^2 + b^2)*sinh(d*x + c
)^6 + 8*(14*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*(2*(a^2 - b^2)*d*x
 - a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*
d*x - 2*a^2 + 2*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 15*(2*(a^2 - b^2)*d*x -
 a^2 + b^2)*cosh(d*x + c)^2 - 2*a^2 + 2*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*
(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^5 + 5*(2*(a^2 - b^2)*d*x - a^2 +
b^2)*cosh(d*x + c)^3 + ((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^
2)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(a^2 + 2*a*b + b^2)*d*x + 4*(2*(a^2
- b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^2 + 4*(14*(a^2 + 2*a*b + b^2)*d*x*co
sh(d*x + c)^6 + 15*(2*(a^2 - b^2)*d*x - a^2 + b^2)*cosh(d*x + c)^4 + 2*(a^
2 - b^2)*d*x + 6*((3*a^2 - 2*a*b + 3*b^2)*d*x - 2*a^2 + 2*a*b - 2*b^2)*cos
h(d*x + c)^2 - a^2 + b^2)*sinh(d*x + c)^2 - ((a^2 + 2*a*b + b^2)*cosh(d*x
+ c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2 + 2*a*
b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x + c)^6 + 4*(7*(a^2 + 2*a
*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b
 + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 ...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. 2(92) = 184.

Time = 0.08 (sec) , antiderivative size = 384, normalized size of antiderivative = 3.92

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d} + \frac{2}{(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4-b^5)e^{(-2dx-2c)}+} + \frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^3+3a^2b+3ab^2+b^3)d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((a^2 - b^2)*e^(-2*d*x -
 2*c) + 2*(a^2 - a*b + b^2)*e^(-4*d*x - 4*c) + (a^2 - b^2)*e^(-6*d*x - 6*c
))/((a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*
a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5
 + 7*a^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4
*(a^5 + 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c)
+ (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*
c))*d) + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a
 + b)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(92) = 184$.

Time = 0.28 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.50

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$
$$= \frac{\frac{2\log\left(\left|a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)+2\,a-2\,b\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{3\,a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+3\,b\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\,c)}\right)^2+}{(a^2+2\,ab+b^2)\left(a\left(e^{(2\,dx+2\,c)}+e^{(-2\,dx-2\right.\right.}}}{4\,d}$$

input
```
integrate(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
 + e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x
- 2*c))^2 + 4*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) - 4*b*(e^(2*d*x + 2*c
) + e^(-2*d*x - 2*c)) - 4*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2*c
) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2*b
)^2))/d
```

Mupad [B] (verification not implemented)

Time = 2.84 (sec) , antiderivative size = 397, normalized size of antiderivative = 4.05

$$\int \frac{\tanh^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx =$$
$$-\frac{-a^3+b^3\left(2\tanh(c+d\,x)^2+\tanh(c+d\,x)^4\,\mathrm{atan}\left(\frac{a\tanh(c+d\,x)^2\,1\mathrm{i}+b\tanh(c+d\,x)^2\,1\mathrm{i}}{2\,a-a\tanh(c+d\,x)^2+b\tanh(c+d\,x)^2}\right)\,4\mathrm{i}\right)}{4\,d\,a^5\,b+8\,d\,a^4\,b^2\tanh\left(c+d\,x\right)^2+12\,d\,a^4\,b^2+4\,d\,a^3\,b^3\tanh\left(c+d\,x\right)^4+24\,d\,a^3\,b^3\tanh\left(c+d\,x\right)^2+1}$$

input
```
int(tanh(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
-(b^3*(tanh(c + d*x)^4*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/
(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i + 2*tanh(c + d*x)^2) - a
^3 + a*b^2*(tanh(c + d*x)^2*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2
*1i)/(2*a - a*tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*8i + 2*tanh(c + d*x)^2
 + 1) + a^2*b*atan((a*tanh(c + d*x)^2*1i + b*tanh(c + d*x)^2*1i)/(2*a - a*
tanh(c + d*x)^2 + b*tanh(c + d*x)^2))*4i)/(4*a^2*b^4*d + 12*a^3*b^3*d + 12
*a^4*b^2*d + 4*b^6*d*tanh(c + d*x)^4 + 4*a^5*b*d + 24*a^2*b^4*d*tanh(c + d
*x)^2 + 24*a^3*b^3*d*tanh(c + d*x)^2 + 8*a^4*b^2*d*tanh(c + d*x)^2 + 12*a^
2*b^4*d*tanh(c + d*x)^4 + 4*a^3*b^3*d*tanh(c + d*x)^4 + 8*a*b^5*d*tanh(c +
 d*x)^2 + 12*a*b^5*d*tanh(c + d*x)^4)
```

Reduce [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 10556, normalized size of antiderivative = 107.71

$$\int \frac{\tanh^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**
(c + d*x)*sqrt(b))*tanh(c + d*x)**4*a**3*b**3 + 2*e**(8*c + 8*d*x)*log(e**
(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c +
 d*x)**4*a**2*b**4 - 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) +
 sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**4*a*b**5 - 2*e**(8*c
 + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*
sqrt(b))*tanh(c + d*x)**4*b**6 + 4*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*s
qrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**4*b
**2 + 4*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) -
2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**3*b**3 - 4*e**(8*c + 8*d*x)*lo
g(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tan
h(c + d*x)**2*a**2*b**4 - 4*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a +
 b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a*b**5 + 2*e*
*(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c +
d*x)*sqrt(b))*a**5*b + 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b)
 + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**4*b**2 - 2*e**(8*c + 8*d*x)*lo
g(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**
3*b**3 - 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b)
 - 2*e**(c + d*x)*sqrt(b))*a**2*b**4 + 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*
d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)*...
```

3.194 $\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 137

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{x}{(a+b)^3} - \frac{(3a^2-6ab-b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}\sqrt{b}(a+b)^3d} - \frac{\tanh(c+dx)}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{(3a-b)\tanh(c+dx)}{8a(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^3-1/8*(3*a^2-6*a*b-b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(1/2))/a^(3/2
)/b^(1/2)/(a+b)^3/d-1/4*tanh(d*x+c)/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/8*(3*a
-b)*tanh(d*x+c)/a/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.78 (sec) , antiderivative size = 137, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$$
$$= \frac{8(c+dx) + \frac{(-3a^2+6ab+b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{b}} - \frac{4b(a+b)\sinh(2(c+dx))}{(a-b+(a+b)\cosh(2(c+dx)))^2} - \frac{(5a-b)(a+b)\sinh(2(c+dx))}{a(a-b+(a+b)\cosh(2(c+dx)))}}{8(a+b)^3d}$$

input
```
Integrate[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(8*(c + d*x) + ((-3*a^2 + 6*a*b + b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt
[a]])/(a^(3/2)*Sqrt[b]) - (4*b*(a + b)*Sinh[2*(c + d*x)])/(a - b + (a + b)
*Cosh[2*(c + d*x)])^2 - ((5*a - b)*(a + b)*Sinh[2*(c + d*x)])/(a*(a - b +
(a + b)*Cosh[2*(c + d*x)])))/(8*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.18, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.435$, Rules used = {3042, 25, 4153, 25, 373, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3} dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan(ic+idx)^2}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 4153$$

$$-\frac{\int -\frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 25$$

$$\frac{\int \frac{\tanh^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \; 373$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int \frac{3\tanh^2(c+dx)+1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4(a+b)}}{d}$$

$$\Big\downarrow \; 402$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{\int -\frac{(3a-b)\tanh^2(c+dx)+5a+b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\Big\downarrow \; 25$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\int \frac{(3a-b)\tanh^2(c+dx)+5a+b}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\Big\downarrow \; 397$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{8a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\Big\downarrow \; 218$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{8a\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

$$\Big\downarrow \; 219$$

$$-\frac{\frac{\tanh(c+dx)}{4(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\frac{\frac{8a\operatorname{arctanh}(\tanh(c+dx))}{a+b}-\frac{\left(3a^2-6ab-b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}(a+b)}}{2a(a+b)}-\frac{(3a-b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4(a+b)}}{d}$$

input
```
Int[Tanh[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((Tanh[c + d*x]/(4*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - ((-(((3*a^2 - 6*a
*b - b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*Sqrt[b]*(a + b
))) + (8*a*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*a*(a + b)) - ((3*a - b)*Tan
h[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*(a + b)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.19 (sec) , antiderivative size = 152, normalized size of antiderivative = 1.11

method	result
derivativedivides	$\frac{-\frac{\frac{\frac{b(3a^2+2ab-b^2)\tanh(dx+c)^3}{8a}+\left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(3a^2-6ab-b^2)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a+b)^3}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-}{d}$
default	$\frac{-\frac{\frac{\frac{b(3a^2+2ab-b^2)\tanh(dx+c)^3}{8a}+\left(\frac{5}{8}a^2+\frac{3}{4}ab+\frac{1}{8}b^2\right)\tanh(dx+c)}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(3a^2-6ab-b^2)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8a\sqrt{ab}}}{(a+b)^3}+\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}+\frac{5a^3e^{6dx+6c}-5a^2be^{6dx+6c}-9ab^2e^{6dx+6c}+b^3e^{6dx+6c}+15a^3e^{4dx+4c}-13a^2be^{4dx+4c}+17ab^2e}{4a\left(ae^{4dx+4c}+be^{4dx+4c}+2ae^{2dx+2}\right.}$

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(-1/(a+b)^3*((1/8*b*(3*a^2+2*a*b-b^2)/a*tanh(d*x+c)^3+(5/8*a^2+3/4*a*b
+1/8*b^2)*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(3*a^2-6*a*b-b^2)/a/(a*b)
^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^3*ln(1+tanh(d*x+c))-1/
2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3744 vs. $2(123) = 246$.

Time = 0.21 (sec) , antiderivative size = 7791, normalized size of antiderivative = 56.87

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. $2(123) = 246$.

Time = 0.44 (sec) , antiderivative size = 1472, normalized size of antiderivative = 10.74

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(2*d*x +
 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)
*d) + 1/32*(3*a^3 - 21*a^2*b - 11*a*b^2 - 3*b^3)*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqr
t(a*b)*d) + 1/16*(5*a^4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (5*a^4 - 46*a^3*
b - 40*a^2*b^2 + 14*a*b^3 + 3*b^4)*e^(6*d*x + 6*c) + (15*a^4 - 104*a^3*b +
 58*a^2*b^2 - 24*a*b^3 - 9*b^4)*e^(4*d*x + 4*c) + (15*a^4 - 58*a^3*b - 56*
a^2*b^2 + 26*a*b^3 + 9*b^4)*e^(2*d*x + 2*c))/((a^7 + 5*a^6*b + 10*a^5*b^2
+ 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*
b^3 + 5*a^3*b^4 + a^2*b^5)*e^(8*d*x + 8*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2
- 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(6*d*x + 6*c) + 2*(3*a^7 + 7*a^6*b +
6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(4*d*x + 4*c) + 4*(a^7 +
3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(2*d*x + 2*c))*d)
 - 1/16*(5*a^4 - 18*a^2*b^2 - 16*a*b^3 - 3*b^4 + (15*a^4 - 58*a^3*b - 56*a
^2*b^2 + 26*a*b^3 + 9*b^4)*e^(-2*d*x - 2*c) + (15*a^4 - 104*a^3*b + 58*a^2
*b^2 - 24*a*b^3 - 9*b^4)*e^(-4*d*x - 4*c) + (5*a^4 - 46*a^3*b - 40*a^2*b^2
 + 14*a*b^3 + 3*b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a
^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 -
3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6
*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b +...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 388 vs. $2(123) = 246$.

Time = 0.26 (sec) , antiderivative size = 388, normalized size of antiderivative = 2.83

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx =$$

$$-\frac{\frac{(3a^2-6ab-b^2)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^4+3a^3b+3a^2b^2+ab^3)\sqrt{ab}} - \frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3} - \frac{2\left(5a^3e^{(6dx+6c)}-5a^2be^{(6dx+6c)}-9ab^2e^{(6dx+6c)}+b\right.}{}}{}$$

input
```
integrate(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((3*a^2 - 6*a*b - b^2)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*d*x + 2
*c) + a - b)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) -
8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(5*a^3*e^(6*d*x + 6*c) - 5
*a^2*b*e^(6*d*x + 6*c) - 9*a*b^2*e^(6*d*x + 6*c) + b^3*e^(6*d*x + 6*c) + 1
5*a^3*e^(4*d*x + 4*c) - 13*a^2*b*e^(4*d*x + 4*c) + 17*a*b^2*e^(4*d*x + 4*c
) - 3*b^3*e^(4*d*x + 4*c) + 15*a^3*e^(2*d*x + 2*c) + a^2*b*e^(2*d*x + 2*c)
 - 11*a*b^2*e^(2*d*x + 2*c) + 3*b^3*e^(2*d*x + 2*c) + 5*a^3 + 9*a^2*b + 3*
a*b^2 - b^3)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*e^(4*d*x + 4*c) + b*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 4.49 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.86

$$\int \frac{\tanh^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

$$= \frac{\frac{a^2x}{(a+b)(a^2+2ab+b^2)} - \frac{\tanh(c+dx)(5a+b)}{8d(a^2+2ab+b^2)} + \frac{b^2x\tanh(c+dx)^4}{a^3+3a^2b+3ab^2+b^3} + \frac{2abx\tanh(c+dx)^2}{a^3+3a^2b+3ab^2+b^3} - \frac{\tanh(c+dx)^3(3ab-b^2)}{8ad(a^2+2ab+b^2)}}{a^2+2ab\tanh(c+dx)^2+b^2\tanh(c+dx)^4}$$

$$+ \frac{\mathrm{atan}\left(\frac{b\tanh(c+dx)}{\sqrt{ab}}\right)(-3a^2+6ab+b^2)}{\sqrt{ab}\,(8a^4d+ab\,(24da^2+24dab+8db^2))}$$

input
```
int(tanh(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
((a^2*x)/((a + b)*(2*a*b + a^2 + b^2)) - (tanh(c + d*x)*(5*a + b))/(8*d*(2
*a*b + a^2 + b^2)) + (b^2*x*tanh(c + d*x)^4)/(3*a*b^2 + 3*a^2*b + a^3 + b^
3) + (2*a*b*x*tanh(c + d*x)^2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - (tanh(c +
 d*x)^3*(3*a*b - b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*
x)^4 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(6*a*
b - 3*a^2 + b^2))/((a*b)^(1/2)*(8*a^4*d + a*b*(24*a^2*d + 8*b^2*d + 24*a*b
*d)))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 578, normalized size of antiderivative = 4.22

$$\int \frac{\tanh^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d
*x)**4*a**2*b**2 + 6*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(
a)))*tanh(c + d*x)**4*a*b**3 + sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqr
t(b)*sqrt(a)))*tanh(c + d*x)**4*b**4 - 6*sqrt(b)*sqrt(a)*atan((tanh(c + d*
x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**3*b + 12*sqrt(b)*sqrt(a)*atan
((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**2*b**2 + 2*sqrt(
b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a*b*
*3 - 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**4 + 6*
sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b + sqrt(b)
*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*tanh(c +
d*x)**4*a**2*b**3*d*x - 3*tanh(c + d*x)**3*a**3*b**2 - 2*tanh(c + d*x)**3*
a**2*b**3 + tanh(c + d*x)**3*a*b**4 + 16*tanh(c + d*x)**2*a**3*b**2*d*x -
5*tanh(c + d*x)*a**4*b - 6*tanh(c + d*x)*a**3*b**2 - tanh(c + d*x)*a**2*b*
*3 + 8*a**4*b*d*x)/(8*a**2*b*d*(tanh(c + d*x)**4*a**3*b**2 + 3*tanh(c + d*
x)**4*a**2*b**3 + 3*tanh(c + d*x)**4*a*b**4 + tanh(c + d*x)**4*b**5 + 2*ta
nh(c + d*x)**2*a**4*b + 6*tanh(c + d*x)**2*a**3*b**2 + 6*tanh(c + d*x)**2*
a**2*b**3 + 2*tanh(c + d*x)**2*a*b**4 + a**5 + 3*a**4*b + 3*a**3*b**2 + a*
*2*b**3))
```

3.195 $\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3} \, dx$

Optimal result

Integrand size = 21, antiderivative size = 94

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3} \, dx = \frac{\log(\cosh(c+dx))}{(a+b)^3 d} + \frac{\log\left(a+b\tanh^2(c+dx)\right)}{2(a+b)^3 d} - \frac{1}{4(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{1}{2(a+b)^2 d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
ln(cosh(d*x+c))/(a+b)^3/d+1/2*ln(a+b*tanh(d*x+c)^2)/(a+b)^3/d-1/4/(a+b)/d/
(a+b*tanh(d*x+c)^2)^2-1/2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.82

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3} \, dx = \frac{4\log(\cosh(c+dx)) + 2\log\left(a+b\tanh^2(c+dx)\right) - \frac{(a+b)^2}{(a+b\tanh^2(c+dx))^2} - \frac{2(a+b)}{a+b\tanh^2(c+dx)}}{4(a+b)^3 d}$$

input
```
Integrate[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(4*Log[Cosh[c + d*x]] + 2*Log[a + b*Tanh[c + d*x]^2] - (a + b)^2/(a + b*Ta
nh[c + d*x]^2)^2 - (2*(a + b))/(a + b*Tanh[c + d*x]^2))/(4*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 4153, 26, 353, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ic+idx)}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\tanh(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{353}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{54}$$

$$\frac{\int \left(\frac{b}{(a+b)^3(b\tanh^2(c+dx)+a)}+\frac{b}{(a+b)^2(b\tanh^2(c+dx)+a)^2}+\frac{b}{(a+b)(b\tanh^2(c+dx)+a)^3}-\frac{1}{(a+b)^3(\tanh^2(c+dx)-1)}\right)d\tanh^2(c+}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{1}{(a+b)^2(a+b\tanh^2(c+dx))}-\frac{1}{2(a+b)(a+b\tanh^2(c+dx))^2}-\frac{\log(1-\tanh^2(c+dx))}{(a+b)^3}+\frac{\log(a+b\tanh^2(c+dx))}{(a+b)^3}}{2d}$$

input
```
Int[Tanh[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Log[1 - Tanh[c + d*x]^2]/(a + b)^3) + Log[a + b*Tanh[c + d*x]^2]/(a + b
)^3 - 1/(2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - 1/((a + b)^2*(a + b*Tanh[c
 + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.12 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.23

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}-\frac{a^2+2ab+b^2}{2b\left(a+b\tanh(dx+c)^2\right)^2}\right)}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^3}}{d}$
default	$\frac{\frac{b\left(\frac{\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a+b}{b\left(a+b\tanh(dx+c)^2\right)}-\frac{a^2+2ab+b^2}{2b\left(a+b\tanh(dx+c)^2\right)^2}\right)}{2(a+b)^3}-\frac{\ln(1+\tanh(dx+c))}{2(a+b)^3}-\frac{\ln(-1+\tanh(dx+c))}{2(a+b)^3}}{d}$
risch	$-\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2c}{d(a^3+3a^2b+3a\,b^2+b^3)}-\frac{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-b\,e^{2dx+2c}+a+b\right)b\,e^{2dx+2c}}{\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx+2c}-2b\,e^{2dx+2c}+a+b\right)^2d(a+b)^3}$
parallelrisch	$-\frac{4a\,b^3+8\ln(1-\tanh(dx+c))\tanh(dx+c)^2a\,b^3+4b^4\tanh(dx+c)^4xd+8x\tanh(dx+c)^2a\,b^3d-2\ln\left(a+b\tanh(dx+c)^2\right)}{}$

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/2*b/(a+b)^3*(1/b*ln(a+b*tanh(d*x+c)^2)-(a+b)/b/(a+b*tanh(d*x+c)^2)-
1/2*(a^2+2*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)^2)-1/2/(a+b)^3*ln(1+tanh(d*x+c))
-1/2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2554 vs. $2(88) = 176$.

Time = 0.14 (sec) , antiderivative size = 2554, normalized size of antiderivative = 27.17

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
-1/2*(2*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^8 + 16*(a^2 + 2*a*b + b^2)*d
*x*cosh(d*x + c)*sinh(d*x + c)^7 + 2*(a^2 + 2*a*b + b^2)*d*x*sinh(d*x + c)
^8 + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^6 + 8*(7*(a^2 + 2*a*b +
 b^2)*d*x*cosh(d*x + c)^2 + (a^2 - b^2)*d*x + a*b + b^2)*sinh(d*x + c)^6 +
 16*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^3 + 3*((a^2 - b^2)*d*x + a*b
+ b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*((3*a^2 - 2*a*b + 3*b^2)*d*x + 4
*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(35*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c
)^4 + (3*a^2 - 2*a*b + 3*b^2)*d*x + 30*((a^2 - b^2)*d*x + a*b + b^2)*cosh(
d*x + c)^2 + 4*a*b - 2*b^2)*sinh(d*x + c)^4 + 16*(7*(a^2 + 2*a*b + b^2)*d*
x*cosh(d*x + c)^5 + 10*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c)^3 + ((3
*a^2 - 2*a*b + 3*b^2)*d*x + 4*a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3
+ 2*(a^2 + 2*a*b + b^2)*d*x + 8*((a^2 - b^2)*d*x + a*b + b^2)*cosh(d*x + c
)^2 + 8*(7*(a^2 + 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 15*((a^2 - b^2)*d*x +
 a*b + b^2)*cosh(d*x + c)^4 + (a^2 - b^2)*d*x + 3*((3*a^2 - 2*a*b + 3*b^2)
*d*x + 4*a*b - 2*b^2)*cosh(d*x + c)^2 + a*b + b^2)*sinh(d*x + c)^2 - ((a^2
 + 2*a*b + b^2)*cosh(d*x + c)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh
(d*x + c)^7 + (a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 4*(a^2 - b^2)*cosh(d*x
 + c)^6 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + a^2 - b^2)*sinh(d*x +
 c)^6 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - b^2)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(3*a^2 - 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(...
```

Sympy [F(-1)]

Timed out.

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Timed out}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 378 vs. 2(88) = 176.

Time = 0.08 (sec) , antiderivative size = 378, normalized size of antiderivative = 4.02

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d}$$

$$- \frac{\quad}{(a^5+5a^4b+10a^3b^2+10a^2b^3+5ab^4+b^5+4(a^5+3a^4b+2a^3b^2-2a^2b^3-3ab^4-b^5)e^{(-2dx-2c)}+\cdots}$$

$$+ \frac{\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^3+3a^2b+3ab^2+b^3)d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
(d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 4*((a*b + b^2)*e^(-2*d*x -
 2*c) + (2*a*b - b^2)*e^(-4*d*x - 4*c) + (a*b + b^2)*e^(-6*d*x - 6*c))/((a
^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5 + 4*(a^5 + 3*a^4*b
+ 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-2*d*x - 2*c) + 2*(3*a^5 + 7*a
^4*b + 6*a^3*b^2 + 6*a^2*b^3 + 7*a*b^4 + 3*b^5)*e^(-4*d*x - 4*c) + 4*(a^5
+ 3*a^4*b + 2*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 - b^5)*e^(-6*d*x - 6*c) + (a^5
 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*e^(-8*d*x - 8*c))*d)
 + 1/2*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 245 vs. $2(88) = 176$.

Time = 0.24 (sec) , antiderivative size = 245, normalized size of antiderivative = 2.61

$$\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{2\log\left(\left|a\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+b\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)+2a-2b\right|\right)}{a^3+3a^2b+3ab^2+b^3} - \frac{3a\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+3b\left(e^{(2dx+2c)}+e^{(-2dx-2c)}\right)^2+}{(a^2+2ab+b^2)\left(a\left(e^{(2dx+2c)}+e^{(-2dx-}\right.\right.}}{4d}$$

input
```
integrate(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/4*(2*log(abs(a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c)
 + e^(-2*d*x - 2*c)) + 2*a - 2*b))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - (3*a*
(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c))^2 + 3*b*(e^(2*d*x + 2*c) + e^(-2*d*x
- 2*c))^2 + 12*a*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 4*b*(e^(2*d*x + 2*
c) + e^(-2*d*x - 2*c)) + 12*a - 4*b)/((a^2 + 2*a*b + b^2)*(a*(e^(2*d*x + 2
*c) + e^(-2*d*x - 2*c)) + b*(e^(2*d*x + 2*c) + e^(-2*d*x - 2*c)) + 2*a - 2
*b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.30 (sec) , antiderivative size = 235, normalized size of antiderivative = 2.50

$$\begin{aligned}\int \frac{\tanh(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{\ln\left(b\tanh(c+d\,x)^2+a\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} \\ &\quad - \frac{\ln\left(1-\tanh(c+d\,x)\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} \\ &\quad - \frac{\ln\left(\tanh(c+d\,x)+1\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} \\ &\quad + \frac{\frac{\tanh(c+d\,x)^4\,\left(\frac{b^3}{4}+\frac{3\,a\,b^2}{4}\right)}{a^2\,d\,(a^2+2\,a\,b+b^2)}+\frac{\tanh(c+d\,x)^2\,\left(\frac{b^2}{2}+a\,b\right)}{a\,d\,(a^2+2\,a\,b+b^2)}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)^2+b^2\tanh\left(c+d\,x\right)^4}\end{aligned}$$

input
```
int(tanh(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(a + b*tanh(c + d*x)^2)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - l
og(1 - tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(ta
nh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c +
d*x)^4*((3*a*b^2)/4 + b^3/4))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c + d*x)
^2*(a*b + b^2/2))/(a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*x)^4 +
2*a*b*tanh(c + d*x)^2)
```

Reduce [B] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 10812, normalized size of antiderivative = 115.02

$$\int \frac{\tanh(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
int(tanh(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c
 + d*x)*sqrt(b))*tanh(c + d*x)**4*a**4*b**2 + e**(8*c + 8*d*x)*log(e**(2*c
 + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x
)**4*a**3*b**3 - e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(
a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**4*a**2*b**4 - e**(8*c + 8*
d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(
b))*tanh(c + d*x)**4*a*b**5 + 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt
(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**5*b +
2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(
c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**4*b**2 - 2*e**(8*c + 8*d*x)*log(e**(
2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c +
d*x)**2*a**3*b**3 - 2*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) +
sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**2*a**2*b**4 + e**(8*c
 + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*
sqrt(b))*a**6 + e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a
 + b) - 2*e**(c + d*x)*sqrt(b))*a**5*b - e**(8*c + 8*d*x)*log(e**(2*c + 2*
d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**4*b**2 - e**(8
*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x
)*sqrt(b))*a**3*b**3 + e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) +
 sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*tanh(c + d*x)**4*a**4*b**2 + e**...
```

3.196 $\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 142

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3}+\frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^3d} + \frac{b\tanh(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(7a+3b)\tanh(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^3+1/8*b^(1/2)*(15*a^2+10*a*b+3*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^(
1/2))/a^(5/2)/(a+b)^3/d+1/4*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+
1/8*b*(7*a+3*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 0.21 (sec) , antiderivative size = 147, normalized size of antiderivative = 1.04

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{\sqrt{b}(15a^2+10ab+3b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}} - 4\log(1-\tanh(c+dx)) + 4\log(1+\tanh(c+dx)) + \frac{2b(a+b)^2\tanh(c+d}{a(a+b\tanh^2(c+dx)}}{8(a+b)^3d}$$

input

```
Integrate[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

output

```
((Sqrt[b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]
])/a^(5/2) - 4*Log[1 - Tanh[c + d*x]] + 4*Log[1 + Tanh[c + d*x]] + (2*b*(a
 + b)^2*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^2) + (b*(a + b)*(7*a + 3
*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)))/(8*(a + b)^3*d)
```

Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3042, 4144, 316, 402, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{316}$$

$$\frac{\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int\frac{3b\tanh^2(c+dx)+b-4(a+b)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{402}$$

$$\frac{\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{\int\frac{8a^2+7ba+3b^2-b(7a+3b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}-\frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{397}$$

$$\frac{\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{\frac{b\left(15a^2+10ab+3b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}+\frac{8a^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{218}$$

$$\frac{\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{\frac{8a^2\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}-\frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}}{d}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{b\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{-\frac{\frac{\sqrt{b}\left(15a^2+10ab+3b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}+\frac{8a^2\text{arctanh}(\tanh(c+dx))}{a+b}}{2a(a+b)}-\frac{b(7a+3b)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-3),x]
```

output
```
((b*Tanh[c + d*x])/(4*a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - (-1/2*((Sqrt[
b]*(15*a^2 + 10*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqr
t[a]*(a + b)) + (8*a^2*ArcTanh[Tanh[c + d*x]])/(a + b))/(a*(a + b)) - (b*(
7*a + 3*b)*Tanh[c + d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*a*(a +
 b)))/d
```

Defintions of rubi rules used

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.18 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.10

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\frac{b(7a^2+10ab+3b^2)\tanh(dx+c)^3}{8a^2}+\frac{(9a^2+14ab+5b^2)\tanh(dx+c)}{8a}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(15a^2+10ab+3b^2)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a+b)^3}+\frac{\ln(1+\tanh(d}{2(a+b)}}{d}$
default	$\frac{\frac{b\left(\frac{\frac{b(7a^2+10ab+3b^2)\tanh(dx+c)^3}{8a^2}+\frac{(9a^2+14ab+5b^2)\tanh(dx+c)}{8a}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{(15a^2+10ab+3b^2)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8a^2\sqrt{ab}}\right)}{(a+b)^3}+\frac{\ln(1+\tanh(d}{2(a+b)}}{d}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{b\left(9a^3e^{6dx+6c}-a^2b\,e^{6dx+6c}-13a\,b^2e^{6dx+6c}-3b^3e^{6dx+6c}+27a^3e^{4dx+4c}-9a^2b\,e^{4dx+4c}+21a\,b\right.}{4\left(a\,e^{4dx+4c}+b\,e^{4dx+4c}+2a\,e^{2dx}\right.}$

input
```
int(1/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^3*b*((1/8*b*(7*a^2+10*a*b+3*b^2)/a^2*tanh(d*x+c)^3+1/8*(9*a^2
+14*a*b+5*b^2)/a*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(15*a^2+10*a*b+3*b
^2)/a^2/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/2/(a+b)^3*ln(1+ta
nh(d*x+c))-1/2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3587 vs. 2(128) = 256.

Time = 0.19 (sec) , antiderivative size = 7496, normalized size of antiderivative = 52.79

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 507 vs. 2(128) = 256.

Time = 0.23 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.57

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\left(15\,a^2b+10\,ab^2+3\,b^3\right)\arctan\left(\frac{(a+b)e^{(-2\,dx-2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{8\,\left(a^5+3\,a^4b+3\,a^3b^2+a^2b^3\right)\sqrt{ab}d}$$
$$+\frac{9\,a^3b+21\,a^2b^2+15\,ab^3+3\,b^4+(27}{4\,\left(a^7+5\,a^6b+10\,a^5b^2+10\,a^4b^3+5\,a^3b^4+a^2b^5+4\,\left(a^7+3\,a^6b+2\,a^5b^2-2\,a^4b^3-3\,a^3b^4-a^2b^5\right)e^{(-2}}$$
$$+\frac{dx+c}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) +
a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt(a*b)*d) + 1/
4*(9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4 + (27*a^3*b + 13*a^2*b^2 - 23*a
*b^3 - 9*b^4)*e^(-2*d*x - 2*c) + 3*(9*a^3*b - 3*a^2*b^2 + 7*a*b^3 + 3*b^4)
*e^(-4*d*x - 4*c) + (9*a^3*b - a^2*b^2 - 13*a*b^3 - 3*b^4)*e^(-6*d*x - 6*c
))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^
7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c
) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^
(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^
2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3
*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + (d*x + c)/((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(128) = 256$.

Time = 0.15 (sec) , antiderivative size = 409, normalized size of antiderivative = 2.88

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{(15a^2b+10ab^2+3b^3)\arctan\left(\frac{ae^{(2dx+2c)}+be^{(2dx+2c)}+a-b}{2\sqrt{ab}}\right)}{(a^5+3a^4b+3a^3b^2+a^2b^3)\sqrt{ab}}+\frac{8(dx+c)}{a^3+3a^2b+3ab^2+b^3}-\frac{2\left(9a^3be^{(6dx+6c)}-a^2b^2e^{(6dx+6c)}-13ab^3e^{(6dx+6}\right.}{}}{}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/8*((15*a^2*b + 10*a*b^2 + 3*b^3)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e^(2*
d*x + 2*c) + a - b)/sqrt(a*b))/((a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3)*sqrt
(a*b)) + 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(9*a^3*b*e^(6*d*x
 + 6*c) - a^2*b^2*e^(6*d*x + 6*c) - 13*a*b^3*e^(6*d*x + 6*c) - 3*b^4*e^(6*
d*x + 6*c) + 27*a^3*b*e^(4*d*x + 4*c) - 9*a^2*b^2*e^(4*d*x + 4*c) + 21*a*b
^3*e^(4*d*x + 4*c) + 9*b^4*e^(4*d*x + 4*c) + 27*a^3*b*e^(2*d*x + 2*c) + 13
*a^2*b^2*e^(2*d*x + 2*c) - 23*a*b^3*e^(2*d*x + 2*c) - 9*b^4*e^(2*d*x + 2*c
) + 9*a^3*b + 21*a^2*b^2 + 15*a*b^3 + 3*b^4)/((a^5 + 3*a^4*b + 3*a^3*b^2 +
 a^2*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + a + b)^2))/d
```

Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 260, normalized size of antiderivative = 1.83

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\ln\left(\tanh(c+d\,x)+1\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} - \frac{\ln\left(1-\tanh(c+d\,x)\right)}{2\,d\,a^3+6\,d\,a^2\,b+6\,d\,a\,b^2+2\,d\,b^3} + \frac{\frac{\tanh(c+d\,x)^3\,\left(\frac{3\,b^3}{8}+\frac{7\,a\,b^2}{8}\right)}{a^2\,d\,(a^2+2\,a\,b+b^2)}+\frac{\tanh(c+d\,x)\,(5\,b^2+9\,a\,b)}{8\,a\,d\,(a^2+2\,a\,b+b^2)}}{a^2+2\,a\,b\tanh\left(c+d\,x\right)^2+b^2\tanh\left(c+d\,x\right)^4} + \frac{\operatorname{atan}\left(\frac{b\tanh(c+d\,x)}{\sqrt{a\,b}}\right)\,(15\,a^2\,b+10\,a\,b^2+3\,b^3)}{\sqrt{a\,b}\,\left(8\,a^5\,d+a\,b\,\left(24\,a^3\,d+a\,b\,\left(24\,a\,d+8\,b\,d\right)\right)\right)}$$

input
```
int(1/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) - log(1
 - tanh(c + d*x))/(2*a^3*d + 2*b^3*d + 6*a*b^2*d + 6*a^2*b*d) + ((tanh(c +
 d*x)^3*((7*a*b^2)/8 + (3*b^3)/8))/(a^2*d*(2*a*b + a^2 + b^2)) + (tanh(c +
 d*x)*(9*a*b + 5*b^2))/(8*a*d*(2*a*b + a^2 + b^2)))/(a^2 + b^2*tanh(c + d*
x)^4 + 2*a*b*tanh(c + d*x)^2) + (atan((b*tanh(c + d*x))/(a*b)^(1/2))*(10*a
*b^2 + 15*a^2*b + 3*b^3))/((a*b)^(1/2)*(8*a^5*d + a*b*(24*a^3*d + a*b*(24*
a*d + 8*b*d))))
```

Reduce [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 575, normalized size of antiderivative = 4.05

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(15*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x
)**4*a**2*b**2 + 10*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a
)))*tanh(c + d*x)**4*a*b**3 + 3*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sq
rt(b)*sqrt(a)))*tanh(c + d*x)**4*b**4 + 30*sqrt(b)*sqrt(a)*atan((tanh(c +
d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**3*b + 20*sqrt(b)*sqrt(a)*at
an((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**2*b**2 + 6*sqr
t(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a*
b**3 + 15*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**4 +
 10*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**3*b + 3*s
qrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*a**2*b**2 + 8*tan
h(c + d*x)**4*a**3*b**2*d*x + 7*tanh(c + d*x)**3*a**3*b**2 + 10*tanh(c + d
*x)**3*a**2*b**3 + 3*tanh(c + d*x)**3*a*b**4 + 16*tanh(c + d*x)**2*a**4*b*
d*x + 9*tanh(c + d*x)*a**4*b + 14*tanh(c + d*x)*a**3*b**2 + 5*tanh(c + d*x
)*a**2*b**3 + 8*a**5*d*x)/(8*a**3*d*(tanh(c + d*x)**4*a**3*b**2 + 3*tanh(c
 + d*x)**4*a**2*b**3 + 3*tanh(c + d*x)**4*a*b**4 + tanh(c + d*x)**4*b**5 +
 2*tanh(c + d*x)**2*a**4*b + 6*tanh(c + d*x)**2*a**3*b**2 + 6*tanh(c + d*x
)**2*a**2*b**3 + 2*tanh(c + d*x)**2*a*b**4 + a**5 + 3*a**4*b + 3*a**3*b**2
 + a**2*b**3))
```

3.197 $\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 21, antiderivative size = 138

$$\begin{aligned}\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx &= \frac{\log(\cosh(c+dx))}{(a+b)^3d}+\frac{\log(\tanh(c+dx))}{a^3d} \\ &\quad -\frac{b(3a^2+3ab+b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^3(a+b)^3d} \\ &\quad +\frac{b}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} \\ &\quad +\frac{b(2a+b)}{2a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}\end{aligned}$$

output
```
ln(cosh(d*x+c))/(a+b)^3/d+ln(tanh(d*x+c))/a^3/d-1/2*b*(3*a^2+3*a*b+b^2)*ln
(a+b*tanh(d*x+c)^2)/a^3/(a+b)^3/d+1/4*b/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/
2*b*(2*a+b)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.06 (sec) , antiderivative size = 117, normalized size of antiderivative = 0.85

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$=\frac{\frac{4\log(\cosh(c+dx))}{(a+b)^3}+\frac{4\log(\tanh(c+dx))+\frac{b\left(-2\left(3a^2+3ab+b^2\right)\log\left(a+b\tanh^2(c+dx)\right)+\frac{a(a+b)\left(a(5a+3b)+2b(2a+b)\tanh^2(c+dx)\right)}{\left(a+b\tanh^2(c+dx)\right)^2}\right)}{(a+b)^3}}{a^3}}{4d}$$

input
```
Integrate[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((4*Log[Cosh[c + d*x]])/(a + b)^3 + (4*Log[Tanh[c + d*x]] + (b*(-2*(3*a^2
+ 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2] + (a*(a + b)*(a*(5*a + 3*b) + 2*
b*(2*a + b)*Tanh[c + d*x]^2))/(a + b*Tanh[c + d*x]^2)^2))/(a + b)^3)/a^3)/
(4*d)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 26, 4153, 26, 354, 93, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{1}{\tan(ic+idx)\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \textbf{4153}$$

$$\frac{i\int -\frac{i\coth(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \textbf{26}$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \textbf{354}$$

$$\frac{\int \frac{\coth(c+dx)}{(1-\tanh^2(c+dx))(b\tanh^2(c+dx)+a)^3}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \textbf{93}$$

$$\frac{\int \left(-\frac{(3a^2+3ba+b^2)b^2}{a^3(a+b)^3(b\tanh^2(c+dx)+a)}-\frac{(2a+b)b^2}{a^2(a+b)^2(b\tanh^2(c+dx)+a)^2}-\frac{b^2}{a(a+b)(b\tanh^2(c+dx)+a)^3}+\frac{\coth(c+dx)}{a^3}-\frac{1}{(a+b)^3(\tanh^2(c+dx}\right.}{2d}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{\frac{\log(\tanh^2(c+dx))}{a^3}+\frac{b(2a+b)}{a^2(a+b)^2(a+b\tanh^2(c+dx))}-\frac{b(3a^2+3ab+b^2)\log(a+b\tanh^2(c+dx))}{a^3(a+b)^3}+\frac{b}{2a(a+b)(a+b\tanh^2(c+dx))^2}-\frac{\log(1-\tan}{(a+}}{2d}$$

input
```
Int[Coth[c + d*x]/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(Log[Tanh[c + d*x]^2]/a^3 - Log[1 - Tanh[c + d*x]^2]/(a + b)^3 - (b*(3*a^2
 + 3*a*b + b^2)*Log[a + b*Tanh[c + d*x]^2])/(a^3*(a + b)^3) + b/(2*a*(a +
b)*(a + b*Tanh[c + d*x]^2)^2) + (b*(2*a + b))/(a^2*(a + b)^2*(a + b*Tanh[c
 + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 93
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Int[ExpandIntegrand[(e + f*x)^p/((a + b*x)*(c + d*x)), x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && IntegerQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.65 (sec) , antiderivative size = 161, normalized size of antiderivative = 1.17

method	result
derivativedivides	$-\frac{\frac{b^2\left(-\frac{a\left(2a^2+3ab+b^2\right)}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}\right)}{2(a+b)^3a^3}-\frac{\ln(\tanh(dx+c))}{a^3}+\frac{\ln(1+\tanh(}{2(a+b}}{d}$
default	$-\frac{\frac{b^2\left(-\frac{a\left(2a^2+3ab+b^2\right)}{b\left(a+b\tanh(dx+c)^2\right)}+\frac{\left(3a^2+3ab+b^2\right)\ln\left(a+b\tanh(dx+c)^2\right)}{b}-\frac{a^2\left(a^2+2ab+b^2\right)}{2b\left(a+b\tanh(dx+c)^2\right)^2}\right)}{2(a+b)^3a^3}-\frac{\ln(\tanh(dx+c))}{a^3}+\frac{\ln(1+\tanh(}{2(a+b}}{d}$
parallelrisch	$\frac{-24\left(\frac{(a+b)^2\cosh(4dx+4c)}{4}+(a^2-b^2)\cosh(2dx+2c)+\frac{3a^2}{4}-\frac{ab}{2}+\frac{3b^2}{4}\right)(a^2+ab+\frac{1}{3}b^2)b\ln\left(a+b\tanh(dx+c)^2\right)-16\left(\frac{(a+b)}{}\right.}{}$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2x}{a^3}-\frac{2c}{a^3d}+\frac{6bx}{a(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6bc}{ad(a^3+3a^2b+3a\,b^2+b^3)}+\frac{6b^2x}{a^2(a^3+3a^2b+3a\,b^2+}$

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(1/2*b^2/(a+b)^3/a^3*(-a*(2*a^2+3*a*b+b^2)/b/(a+b*tanh(d*x+c)^2)+(3*a
^2+3*a*b+b^2)/b*ln(a+b*tanh(d*x+c)^2)-1/2*a^2*(a^2+2*a*b+b^2)/b/(a+b*tanh(
d*x+c)^2)^2)-1/a^3*ln(tanh(d*x+c))+1/2/(a+b)^3*ln(1+tanh(d*x+c))+1/2/(a+b)
^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4800 vs. $2(132) = 264$.

Time = 0.36 (sec) , antiderivative size = 4800, normalized size of antiderivative = 34.78

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to display
```

Sympy [F]

$$\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. 2(132) = 264.

Time = 0.08 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.61

$$\begin{aligned}&\int \frac{\coth(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx\\&= -\frac{(3a^2b+3ab^2+b^3)\log\left(2(a-b)e^{(-2dx-2c)}+(a+b)e^{(-4dx-4c)}+a+b\right)}{2(a^6+3a^5b+3a^4b^2+a^3b^3)d}\\&\quad+\frac{dx+c}{(a^3+3a^2b+3ab^2+b^3)d}\\&\quad+\frac{2((3a}{(a^7+5a^6b+10a^5b^2+10a^4b^3+5a^3b^4+a^2b^5+4(a^7+3a^6b+2a^5b^2-2a^4b^3-3a^3b^4-a^2b^5)e^{(-2dx}}\\&\quad+\frac{\log\left(e^{(-dx-c)}+1\right)}{a^3d}+\frac{\log\left(e^{(-dx-c)}-1\right)}{a^3d}\end{aligned}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/2*(3*a^2*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^
(-4*d*x - 4*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + (d*x +
 c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) + 2*((3*a^2*b^2 + 4*a*b^3 + b^4)*e
^(-2*d*x - 2*c) + 2*(3*a^2*b^2 - a*b^3 - b^4)*e^(-4*d*x - 4*c) + (3*a^2*b^
2 + 4*a*b^3 + b^4)*e^(-6*d*x - 6*c))/((a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4
*b^3 + 5*a^3*b^4 + a^2*b^5 + 4*(a^7 + 3*a^6*b + 2*a^5*b^2 - 2*a^4*b^3 - 3*
a^3*b^4 - a^2*b^5)*e^(-2*d*x - 2*c) + 2*(3*a^7 + 7*a^6*b + 6*a^5*b^2 + 6*a
^4*b^3 + 7*a^3*b^4 + 3*a^2*b^5)*e^(-4*d*x - 4*c) + 4*(a^7 + 3*a^6*b + 2*a^
5*b^2 - 2*a^4*b^3 - 3*a^3*b^4 - a^2*b^5)*e^(-6*d*x - 6*c) + (a^7 + 5*a^6*b
 + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*e^(-8*d*x - 8*c))*d) + l
og(e^(-d*x - c) + 1)/(a^3*d) + log(e^(-d*x - c) - 1)/(a^3*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(132) = 264$.

Time = 0.27 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.14

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{\frac{(3\,a^2b+3\,ab^2+b^3)\log\left(ae^{(4\,dx+4\,c)}+be^{(4\,dx+4\,c)}+2\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+a+b\right)}{a^6+3\,a^5b+3\,a^4b^2+a^3b^3}+\frac{2\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,\log\left(\left|e^{(2\,dx+2\,c)}-1\right|\right)}{a^3}-}{2\,d}$$

input
```
integrate(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/2*((3*a^2*b + 3*a*b^2 + b^3)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c)
+ 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^6 + 3*a^5*b + 3*a^
4*b^2 + a^3*b^3) + 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*log(abs
(e^(2*d*x + 2*c) - 1))/a^3 - 4*((3*a^2*b^2 + a*b^3)*e^(6*d*x + 6*c) + (3*a
^2*b^2 + a*b^3)*e^(2*d*x + 2*c) + 2*(3*a^3*b^2 - a^2*b^3 - a*b^4)*e^(4*d*x
 + 4*c)/(a + b))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x +
2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2*(a + b)^2*a^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth(c+d\,x)}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 5158, normalized size of antiderivative = 37.38

$$\int \frac{\coth(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
(2*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**6 + 8*e**(8*c + 8*d*x)*log(e*
*(c + d*x) - 1)*a**5*b + 10*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**4*b*
*2 - 10*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1)*a**2*b**4 - 8*e**(8*c + 8*d
*x)*log(e**(c + d*x) - 1)*a*b**5 - 2*e**(8*c + 8*d*x)*log(e**(c + d*x) - 1
)*b**6 + 2*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**6 + 8*e**(8*c + 8*d*x
)*log(e**(c + d*x) + 1)*a**5*b + 10*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)
*a**4*b**2 - 10*e**(8*c + 8*d*x)*log(e**(c + d*x) + 1)*a**2*b**4 - 8*e**(8
*c + 8*d*x)*log(e**(c + d*x) + 1)*a*b**5 - 2*e**(8*c + 8*d*x)*log(e**(c +
d*x) + 1)*b**6 - 3*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqr
t(a + b) - 2*e**(c + d*x)*sqrt(b))*a**5*b - 6*e**(8*c + 8*d*x)*log(e**(2*c
 + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**4*b**2 -
e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c
+ d*x)*sqrt(b))*a**3*b**3 + 5*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a
 + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b))*a**2*b**4 + 4*e**(8*c + 8*d*
x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) - 2*e**(c + d*x)*sqrt(b)
)*a*b**5 + e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b)
 - 2*e**(c + d*x)*sqrt(b))*b**6 - 3*e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*
sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt(b))*a**5*b - 6*e**(8*c + 8
*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sqrt(a + b) + 2*e**(c + d*x)*sqrt
(b))*a**4*b**2 - e**(8*c + 8*d*x)*log(e**(2*c + 2*d*x)*sqrt(a + b) + sq...
```

3.198 $\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 178

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{x}{(a+b)^3} - \frac{b^{3/2}(35a^2+42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^3d} - \frac{(8a^2+27ab+15b^2)\coth(c+dx)}{8a^3(a+b)^2d} + \frac{b\coth(c+dx)}{4a(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} + \frac{b(9a+5b)\coth(c+dx)}{8a^2(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
x/(a+b)^3-1/8*b^(3/2)*(35*a^2+42*a*b+15*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^
(1/2))/a^(7/2)/(a+b)^3/d-1/8*(8*a^2+27*a*b+15*b^2)*coth(d*x+c)/a^3/(a+b)^2
/d+1/4*b*coth(d*x+c)/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(9*a+5*b)*coth(
d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 4.07 (sec) , antiderivative size = 166, normalized size of antiderivative = 0.93

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx =$$

$$-\frac{-\frac{8(c+dx)}{(a+b)^3}+\frac{b^{3/2}(35a^2+42ab+15b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}(a+b)^3}+\frac{8\coth(c+dx)}{a^3}+\frac{4b^3\sinh(2(c+dx))}{a^2(a+b)^2(a-b+(a+b)\cosh(2(c+dx)))^2}+\frac{b^2(}{a^3(a+b)^2}}{8d}$$

input
```
Integrate[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/8*((-8*(c + d*x))/(a + b)^3 + (b^(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTa
n[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*(a + b)^3) + (8*Coth[c + d*x]
)/a^3 + (4*b^3*Sinh[2*(c + d*x)])/(a^2*(a + b)^2*(a - b + (a + b)*Cosh[2*(
c + d*x)])^2) + (b^2*(13*a + 7*b)*Sinh[2*(c + d*x)])/(a^3*(a + b)^2*(a - b
 + (a + b)*Cosh[2*(c + d*x)])))/d
```

Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.14, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 25, 4153, 25, 374, 25, 441, 25, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan(ic+idx)^2\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$\Big\downarrow$ 4153

$$-\frac{\int -\frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 25

$$\frac{\int \frac{\coth^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$\Big\downarrow$ 374

$$-\frac{\frac{\int -\frac{\coth^2(c+dx)\left(-5b\tanh^2(c+dx)+4a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\int \frac{\coth^2(c+dx)\left(-5b\tanh^2(c+dx)+4a+5b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 441

$$-\frac{-\frac{\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{\coth^2(c+dx)\left(8a^2+27ba+15b^2-3b(9a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\int \frac{\coth^2(c+dx)\left(8a^2+27ba+15b^2-3b(9a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}-\frac{b\coth(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$\Big\downarrow$ 445

$$-\frac{-\frac{\frac{-\frac{\int -\frac{8a^3-8ba^2-27b^2a-15b^3+b\left(8a^2+27ba+15b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{a}}{2a(a+b)}+\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}-\frac{\cdots}{4a(a+\cdots}}{d}$$

$\Big\downarrow$ 25

$$-\frac{-\frac{\frac{\frac{\int \frac{8a^3-8ba^2-27b^2a-15b^3+b\left(8a^2+27ba+15b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{a}}{2a(a+b)}+\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}-\frac{b}{4a(a+b)(}}{d}$$

$\downarrow$ 397

$$-\frac{-\frac{\frac{\frac{\frac{8a^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^2\left(35a^2+42ab+15b^2\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}-\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{a}}{2a(a+b)}+\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+\right.}}{4a(a+b)}}{d}$$

$\downarrow$ 218

$$-\frac{-\frac{\frac{\frac{\frac{8a^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}-\frac{b^{3/2}\left(35a^2+42ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{a}}{2a(a+b)}+\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}}{d}$$

$\downarrow$ 219

$$-\frac{-\frac{\frac{\frac{\frac{8a^3\text{arctanh}(\tanh(c+dx))}{a+b}-\frac{b^{3/2}\left(35a^2+42ab+15b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}-\frac{\left(8a^2+27ab+15b^2\right)\coth(c+dx)}{a}}{2a(a+b)}+\frac{b(9a+5b)\coth(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}-\frac{}{4a(}}{d}$$

input
```
Int[Coth[c + d*x]^2/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-((-1/4*(b*Coth[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - (((-((b^
(3/2)*(35*a^2 + 42*a*b + 15*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/
(Sqrt[a]*(a + b))) + (8*a^3*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((8*a^2 +
 27*a*b + 15*b^2)*Coth[c + d*x])/a)/(2*a*(a + b)) + (b*(9*a + 5*b)*Coth[c
+ d*x])/(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*a*(a + b)))/d)
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.32 (sec) , antiderivative size = 167, normalized size of antiderivative = 0.94

method	result
derivativedivides	$-\frac{\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{9}{4}ab^2+\frac{7}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(13a^2+22ab+9b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(35a^2+42ab+15b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a+b)^3a^3}+\frac{}{a}}{d}$
default	$-\frac{\frac{b^2\left(\frac{\left(\frac{11}{8}a^2b+\frac{9}{4}ab^2+\frac{7}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(13a^2+22ab+9b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(35a^2+42ab+15b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a+b)^3a^3}+\frac{}{a}}{d}$
risch	$\frac{x}{a^3+3a^2b+3ab^2+b^3}-\frac{8a^5+40a^4b+93a^3b^2+113a^2b^3+67ab^4+8a^5e^{8dx+8c}+15b^5+90b^5e^{4dx+4c}+32a^5e^{2dx+2c}-60b^5}{}$

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(b^2/(a+b)^3/a^3*(((11/8*a^2*b+9/4*a*b^2+7/8*b^3)*tanh(d*x+c)^3+1/8*a
*(13*a^2+22*a*b+9*b^2)*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(35*a^2+42*a
*b+15*b^2)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))+1/a^3/tanh(d*x+c
)-1/2/(a+b)^3*ln(1+tanh(d*x+c))+1/2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5772 vs. $2(162) = 324$.

Time = 0.29 (sec) , antiderivative size = 11865, normalized size of antiderivative = 66.66

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

input
```
integrate(coth(d*x+c)**2/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**2/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1944 vs. $2(162) = 324$.

Time = 0.59 (sec) , antiderivative size = 1944, normalized size of antiderivative = 10.92

$$\int \frac{\coth^2(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/4*(3*a^2*b + 3*a*b^2 + b^3)*log((a + b)*e^(4*d*x + 4*c) + 2*(a - b)*e^(
2*d*x + 2*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/4*(3*a
^2*b + 3*a*b^2 + b^3)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)*e^(-4*d*x -
 4*c) + a + b)/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d) + 1/32*(15*a^3*b
- 25*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(2*d*x + 2*c) + a
- b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*b)*d) - 1/32
*(15*a^3*b - 25*a^2*b^2 - 39*a*b^3 - 15*b^4)*arctan(1/2*((a + b)*e^(-2*d*x
 - 2*c) + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a*
b)*d) + 1/16*(8*a^5 + 31*a^4*b + 72*a^3*b^2 + 98*a^2*b^3 + 64*a*b^4 + 15*b
^5 + (8*a^5 + 49*a^4*b + 18*a^3*b^2 + 38*a*b^4 + 15*b^5)*e^(8*d*x + 8*c) +
 2*(16*a^5 + 57*a^4*b - 9*a^3*b^2 + 37*a^2*b^3 - 39*a*b^4 - 30*b^5)*e^(6*d
*x + 6*c) + 2*(24*a^5 + 56*a^4*b + 83*a^3*b^2 - 37*a^2*b^3 + 53*a*b^4 + 45
*b^5)*e^(4*d*x + 4*c) + 2*(16*a^5 + 39*a^4*b + 73*a^3*b^2 + 15*a^2*b^3 - 6
5*a*b^4 - 30*b^5)*e^(2*d*x + 2*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b
^3 + 5*a^4*b^4 + a^3*b^5 - (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^
4*b^4 + a^3*b^5)*e^(10*d*x + 10*c) - (3*a^8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5
*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(8*d*x + 8*c) - 2*(a^8 + a^7*b + 2*a^6*b^
2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*e^(6*d*x + 6*c) + 2*(a^8 + a^7*b
+ 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4*b^4 + 5*a^3*b^5)*e^(4*d*x + 4*c) + (3*a^
8 + 7*a^7*b - 2*a^6*b^2 - 18*a^5*b^3 - 17*a^4*b^4 - 5*a^3*b^5)*e^(2*d*x...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 437 vs. $2(162) = 324$.

Time = 0.30 (sec) , antiderivative size = 437, normalized size of antiderivative = 2.46

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{(35\,a^2b^2+42\,ab^3+15\,b^4)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^6+3\,a^5b+3\,a^4b^2+a^3b^3)\sqrt{ab}} - \frac{8\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3} - \frac{2\,(13\,a^3b^2e^{(6\,dx+6\,c)}+3\,a^2b^3e^{(6\,dx+6\,c)}-17\,ab}{}}{}$$

input
```
integrate(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
-1/8*((35*a^2*b^2 + 42*a*b^3 + 15*b^4)*arctan(1/2*(a*e^(2*d*x + 2*c) + b*e
^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*
sqrt(a*b)) - 8*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*(13*a^3*b^2*e
^(6*d*x + 6*c) + 3*a^2*b^3*e^(6*d*x + 6*c) - 17*a*b^4*e^(6*d*x + 6*c) - 7*
b^5*e^(6*d*x + 6*c) + 39*a^3*b^2*e^(4*d*x + 4*c) - 5*a^2*b^3*e^(4*d*x + 4*
c) + 25*a*b^4*e^(4*d*x + 4*c) + 21*b^5*e^(4*d*x + 4*c) + 39*a^3*b^2*e^(2*d
*x + 2*c) + 25*a^2*b^3*e^(2*d*x + 2*c) - 35*a*b^4*e^(2*d*x + 2*c) - 21*b^5
*e^(2*d*x + 2*c) + 13*a^3*b^2 + 33*a^2*b^3 + 27*a*b^4 + 7*b^5)/((a^6 + 3*a
^5*b + 3*a^4*b^2 + a^3*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e
^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) + 16/(a^3*(e^(2*d*x + 2*c
) - 1)))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \int \frac{\coth(c+d\,x)^2}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)^2/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [B] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 4537, normalized size of antiderivative = 25.49

$$\int \frac{\coth^2(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
int(coth(d*x+c)^2/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
( - 105*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b)
- sqrt(b))/sqrt(a))*a**5*b - 161*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((
e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b**2 + 158*e**(10*c + 10
*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a
**3*b**3 + 454*e**(10*c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(
a + b) - sqrt(b))/sqrt(a))*a**2*b**4 + 315*e**(10*c + 10*d*x)*sqrt(b)*sqrt
(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**5 + 75*e**(10*
c + 10*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt
(a))*b**6 - 315*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a
 + b) - sqrt(b))/sqrt(a))*a**5*b + 357*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*at
an((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b**2 + 922*e**(8*c +
 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))
*a**3*b**3 - 350*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(
a + b) - sqrt(b))/sqrt(a))*a**2*b**4 - 975*e**(8*c + 8*d*x)*sqrt(b)*sqrt(a
)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a*b**5 - 375*e**(8*c
+ 8*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a)
)*b**6 - 210*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a +
b) - sqrt(b))/sqrt(a))*a**5*b + 518*e**(6*c + 6*d*x)*sqrt(b)*sqrt(a)*atan(
(e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))*a**4*b**2 - 916*e**(6*c + 6*
d*x)*sqrt(b)*sqrt(a)*atan((e**(c + d*x)*sqrt(a + b) - sqrt(b))/sqrt(a))...
```

3.199 $\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 171

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\coth^2(c+dx)}{2a^3d}+\frac{\log(\cosh(c+dx))}{(a+b)^3d} + \frac{(a-3b)\log(\tanh(c+dx))}{a^4d} + \frac{b^2(6a^2+8ab+3b^2)\log\left(a+b\tanh^2(c+dx)\right)}{2a^4(a+b)^3d} - \frac{b^2}{4a^2(a+b)d\left(a+b\tanh^2(c+dx)\right)^2} - \frac{b^2(3a+2b)}{2a^3(a+b)^2d\left(a+b\tanh^2(c+dx)\right)}$$

output
```
-1/2*coth(d*x+c)^2/a^3/d+ln(cosh(d*x+c))/(a+b)^3/d+(a-3*b)*ln(tanh(d*x+c))
/a^4/d+1/2*b^2*(6*a^2+8*a*b+3*b^2)*ln(a+b*tanh(d*x+c)^2)/a^4/(a+b)^3/d-1/4
*b^2/a^2/(a+b)/d/(a+b*tanh(d*x+c)^2)^2-1/2*b^2*(3*a+2*b)/a^3/(a+b)^2/d/(a+
b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 1.22 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.81

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = -\frac{\frac{\coth^2(c+dx)}{a^3}+\frac{b^4}{2a^4(a+b)\left(b+a\coth^2(c+dx)\right)^2}-\frac{b^3(4a+3b)}{a^4(a+b)^2\left(b+a\coth^2(c+dx)\right)}-\frac{b^2(6a^2+8ab+3b^2)\log\left(b+a\coth^2(c+dx)\right)}{a^4(a+b)^3}-\frac{2\log(\sinh(c+dx))}{(a+b)^3}}{2d}$$

input
```
Integrate[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
-1/2*(Coth[c + d*x]^2/a^3 + b^4/(2*a^4*(a + b)*(b + a*Coth[c + d*x]^2)^2)
- (b^3*(4*a + 3*b))/(a^4*(a + b)^2*(b + a*Coth[c + d*x]^2)) - (b^2*(6*a^2
+ 8*a*b + 3*b^2)*Log[b + a*Coth[c + d*x]^2])/(a^4*(a + b)^3) - (2*Log[Sinh
[c + d*x]])/(a + b)^3)/d
```

Rubi [A] (warning: unable to verify)

Time = 0.71 (sec) , antiderivative size = 160, normalized size of antiderivative = 0.94, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\tan(ic+idx)^3\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$-\frac{i\int \frac{i\coth^3(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{26}$$

$$\frac{\int \frac{\coth^3(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{354}$$

$$\frac{\int \frac{\coth^2(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh^2(c+dx)}{2d}$$

$$\Big\downarrow \text{99}$$

$$\frac{\int \left(\frac{(6a^2+8ba+3b^2)b^3}{a^4(a+b)^3(b\tanh^2(c+dx)+a)}+\frac{(3a+2b)b^3}{a^3(a+b)^2(b\tanh^2(c+dx)+a)^2}+\frac{b^3}{a^2(a+b)(b\tanh^2(c+dx)+a)^3}+\frac{\coth^2(c+dx)}{a^3}+\frac{(a-3b)\coth(c+dx)}{a^4}\right.}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a-3b)\log(\tanh^2(c+dx))}{a^4}-\frac{b^2(3a+2b)}{a^3(a+b)^2(a+b\tanh^2(c+dx))}-\frac{\coth(c+dx)}{a^3}-\frac{b^2}{2a^2(a+b)(a+b\tanh^2(c+dx))^2}+\frac{b^2(6a^2+8ab+3b^2)\log(a+b\,\text{ta}}{a^4(a+b)^3}}{2d}$$

input
```
Int[Coth[c + d*x]^3/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
(-(Coth[c + d*x]/a^3) + ((a - 3*b)*Log[Tanh[c + d*x]^2])/a^4 - Log[1 - Tan
h[c + d*x]^2]/(a + b)^3 + (b^2*(6*a^2 + 8*a*b + 3*b^2)*Log[a + b*Tanh[c +
d*x]^2])/(a^4*(a + b)^3) - b^2/(2*a^2*(a + b)*(a + b*Tanh[c + d*x]^2)^2) -
 (b^2*(3*a + 2*b))/(a^3*(a + b)^2*(a + b*Tanh[c + d*x]^2)))/(2*d)
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 184, normalized size of antiderivative = 1.08

method	result
derivativedivides	$-\frac{-\frac{b^3\left(-\frac{a(3a^2+5ab+2b^2)}{b(a+b\tanh(dx+c)^2)}+\frac{(6a^2+8ab+3b^2)\ln(a+b\tanh(dx+c)^2)}{b}-\frac{a^2(a^2+2ab+b^2)}{2b(a+b\tanh(dx+c)^2)^2}\right)}{2(a+b)^3a^4}+\frac{(3b-a)\ln(\tanh(dx+c))}{a^4}+\cdots}{d}$
default	$-\frac{-\frac{b^3\left(-\frac{a(3a^2+5ab+2b^2)}{b(a+b\tanh(dx+c)^2)}+\frac{(6a^2+8ab+3b^2)\ln(a+b\tanh(dx+c)^2)}{b}-\frac{a^2(a^2+2ab+b^2)}{2b(a+b\tanh(dx+c)^2)^2}\right)}{2(a+b)^3a^4}+\frac{(3b-a)\ln(\tanh(dx+c))}{a^4}+\cdots}{d}$
parallelrisch	$48(a^2+\frac{4}{3}ab+\frac{1}{2}b^2)\left(\frac{(a+b)^2\cosh(4dx+4c)}{4}+(a^2-b^2)\cosh(2dx+2c)+\frac{3a^2}{4}-\frac{ab}{2}+\frac{3b^2}{4}\right)b^2\ln\left(a+b\tanh(dx+c)^2\right)-16\left(\frac{(a+\cdots}{}\right)\cdots$
risch	$\frac{x}{a^3+3a^2b+3a\,b^2+b^3}-\frac{2x}{a^3}-\frac{2c}{a^3d}+\frac{6bx}{a^4}+\frac{6bc}{d\,a^4}-\frac{12b^2x}{a^2(a^3+3a^2b+3a\,b^2+b^3)}-\frac{12b^2c}{a^2d(a^3+3a^2b+3a\,b^2+b^3)}-\cdots$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-1/2*b^3/(a+b)^3/a^4*(-a*(3*a^2+5*a*b+2*b^2)/b/(a+b*tanh(d*x+c)^2)+(
6*a^2+8*a*b+3*b^2)/b*ln(a+b*tanh(d*x+c)^2)-1/2*a^2*(a^2+2*a*b+b^2)/b/(a+b*
tanh(d*x+c)^2)^2)+(3*b-a)/a^4*ln(tanh(d*x+c))+1/2/a^3/tanh(d*x+c)^2+1/2/(a
+b)^3*ln(1+tanh(d*x+c))+1/2/(a+b)^3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10720 vs. $2(163) = 326$.

Time = 0.58 (sec) , antiderivative size = 10720, normalized size of antiderivative = 62.69

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(coth(d*x+c)**3/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**3/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 770 vs. 2(163) = 326.

Time = 0.11 (sec) , antiderivative size = 770, normalized size of antiderivative = 4.50

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
1/2*(6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(2*(a - b)*e^(-2*d*x - 2*c) + (a + b)
*e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d) + (d*
x + c)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d) - 2*((a^5 + 5*a^4*b + 10*a^3*b^
2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-2*d*x - 2*c) + 2*(2*a^5 + 6*a^4*b +
 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-4*d*x - 4*c) + 2*(3*a^5 + 7
*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(-6*d*x - 6*c) + 2*(2
*a^5 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(-8*d*x - 8*c
) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(-10*d*
x - 10*c))/((a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5
 + 2*(a^8 + a^7*b - 6*a^6*b^2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-2
*d*x - 2*c) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b^3 - 43*a^4*b^4 - 15*a^
3*b^5)*e^(-4*d*x - 4*c) - 4*(a^8 + a^7*b + 2*a^6*b^2 + 10*a^5*b^3 + 13*a^4
*b^4 + 5*a^3*b^5)*e^(-6*d*x - 6*c) - (a^8 + 5*a^7*b - 6*a^6*b^2 - 38*a^5*b
^3 - 43*a^4*b^4 - 15*a^3*b^5)*e^(-8*d*x - 8*c) + 2*(a^8 + a^7*b - 6*a^6*b^
2 - 14*a^5*b^3 - 11*a^4*b^4 - 3*a^3*b^5)*e^(-10*d*x - 10*c) + (a^8 + 5*a^7
*b + 10*a^6*b^2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*e^(-12*d*x - 12*c))*d)
 + (a - 3*b)*log(e^(-d*x - c) + 1)/(a^4*d) + (a - 3*b)*log(e^(-d*x - c) -
1)/(a^4*d)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 474 vs. $2(163) = 326$.

Time = 0.37 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.77

$$\int \frac{\coth^3(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \frac{\frac{(6a^2b^2+8ab^3+3b^4)\log\left(ae^{(4dx+4c)}+be^{(4dx+4c)}+2ae^{(2dx+2c)}-2be^{(2dx+2c)}+a+b\right)}{a^7+3a^6b+3a^5b^2+a^4b^3} - \frac{2(dx+c)}{a^3+3a^2b+3ab^2+b^3} + \frac{2(a-3b)\log\left(\left|e^{(2dx+2c)}-\right.\right.}{a^4}}{}$$

input
```
integrate(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/2*((6*a^2*b^2 + 8*a*b^3 + 3*b^4)*log(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*
c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)/(a^7 + 3*a^6*b + 3
*a^5*b^2 + a^4*b^3) - 2*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 2*(a -
 3*b)*log(abs(e^(2*d*x + 2*c) - 1))/a^4 - 4*((a^5 + 5*a^4*b + 10*a^3*b^2 +
 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(10*d*x + 10*c) + 2*(2*a^5 + 6*a^4*b + 4
*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(8*d*x + 8*c) + 2*(3*a^5 + 7*a^
4*b + 6*a^3*b^2 + 2*a^2*b^3 + 15*a*b^4 + 9*b^5)*e^(6*d*x + 6*c) + 2*(2*a^5
 + 6*a^4*b + 4*a^3*b^2 - 4*a^2*b^3 - 13*a*b^4 - 6*b^5)*e^(4*d*x + 4*c) + (
a^5 + 5*a^4*b + 10*a^3*b^2 + 14*a^2*b^3 + 11*a*b^4 + 3*b^5)*e^(2*d*x + 2*c
))/((a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(
2*d*x + 2*c) + a + b)^2*(a + b)^3*a^3*(e^(2*d*x + 2*c) - 1)^2))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\coth(c+d\,x)^3}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)^3/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [F]

$$\int \frac{\coth^3(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\coth\left(dx+c\right)^3}{\left(\tanh\left(dx+c\right)^2b+a\right)^3}dx$$

input
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
int(coth(d*x+c)^3/(a+b*tanh(d*x+c)^2)^3,x)
```

3.200 $\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$

Optimal result

Integrand size = 23, antiderivative size = 228

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \frac{x}{(a+b)^3} + \frac{b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a+b)^3d} - \frac{(8a^3-8a^2b-55ab^2-35b^3)\coth(c+dx)}{8a^4(a+b)^2d} - \frac{(8a^2+55ab+35b^2)\coth^3(c+dx)}{24a^3(a+b)^2d} + \frac{b\coth^3(c+dx)}{4a(a+b)d\,(a+b\tanh^2(c+dx))^2} + \frac{b(11a+7b)\coth^3(c+dx)}{8a^2(a+b)^2d\,(a+b\tanh^2(c+dx))}$$

output

```
x/(a+b)^3+1/8*b^(5/2)*(63*a^2+90*a*b+35*b^2)*arctan(b^(1/2)*tanh(d*x+c)/a^
(1/2))/a^(9/2)/(a+b)^3/d-1/8*(8*a^3-8*a^2*b-55*a*b^2-35*b^3)*coth(d*x+c)/a
^4/(a+b)^2/d-1/24*(8*a^2+55*a*b+35*b^2)*coth(d*x+c)^3/a^3/(a+b)^2/d+1/4*b*
coth(d*x+c)^3/a/(a+b)/d/(a+b*tanh(d*x+c)^2)^2+1/8*b*(11*a+7*b)*coth(d*x+c)
^3/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2)
```

Mathematica [A] (verified)

Time = 2.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.85

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{24(c+dx)}{(a+b)^3}+\frac{3b^{5/2}(63a^2+90ab+35b^2)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{9/2}(a+b)^3}+\frac{8(-4a+9b)\coth(c+dx)}{a^4}-\frac{8\coth(c+dx)\text{csch}^2(c+dx)}{a^3}+\frac{12b^4}{a^3(a+b)^2(a-b}}{24d}$$

input
```
Integrate[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((24*(c + d*x))/(a + b)^3 + (3*b^(5/2)*(63*a^2 + 90*a*b + 35*b^2)*ArcTan[(
Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(a^(9/2)*(a + b)^3) + (8*(-4*a + 9*b)*Cot
h[c + d*x])/a^4 - (8*Coth[c + d*x]*Csch[c + d*x]^2)/a^3 + (12*b^4*Sinh[2*(
c + d*x)])/(a^3*(a + b)^2*(a - b + (a + b)*Cosh[2*(c + d*x)])^2) + (3*b^3*
(17*a + 11*b)*Sinh[2*(c + d*x)])/(a^4*(a + b)^2*(a - b + (a + b)*Cosh[2*(c
 + d*x)])))/(24*d)
```

Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 248, normalized size of antiderivative = 1.09, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3042, 4153, 374, 25, 441, 25, 445, 27, 445, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\tan(ic+idx)^4\left(a-b\tan(ic+idx)^2\right)^3}dx$$

$$\Big\downarrow \; 4153$$

$$\frac{\int \frac{\coth^4(c+dx)}{(1-\tanh^2(c+dx))\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{d}$$

$\downarrow$ 374

$$\frac{\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}-\frac{\int -\frac{\coth^4(c+dx)\left(-7b\tanh^2(c+dx)+4a+7b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\int \frac{\coth^4(c+dx)\left(-7b\tanh^2(c+dx)+4a+7b\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}+\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 441

$$\frac{\frac{\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{\coth^4(c+dx)\left(8a^2+55ba+35b^2-5b(11a+7b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}}{4a(a+b)}+\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 25

$$\frac{\frac{\frac{\int \frac{\coth^4(c+dx)\left(8a^2+55ba+35b^2-5b(11a+7b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}+\frac{b\coth^3(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{d}$$

$\downarrow$ 445

$$\frac{\frac{\frac{-\frac{\int -\frac{3\coth^2(c+dx)\left(8a^3-8ba^2-55b^2a-35b^3+b\left(8a^2+55ba+35b^2\right)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{3a}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{\frac{\frac{\int \frac{\coth^2(c+dx)\left(8a^3-8ba^2-55b^2a-35b^3+b\left(8a^2+55ba+35b^2\right)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{3a}}{2a(a+b)}+\frac{b(11a+7b)\coth^3(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}}{4a(a+b)}+}{d}$$

$\big\downarrow$ 445

$$\frac{\frac{-\frac{\int -\frac{8a^4-8ba^3+8b^2a^2+55b^3a+35b^4+b\left(8a^3-8ba^2-55b^2a-35b^3\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\text{cot}}{3a}}{4a(a+b)}}{d}$$

$\big\downarrow$ 25

$$\frac{\frac{\frac{\frac{\int \frac{8a^4-8ba^3+8b^2a^2+55b^3a+35b^4+b\left(8a^3-8ba^2-55b^2a-35b^3\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{a}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(}{3a}}{4a(a+b)}}{d}$$

$\big\downarrow$ 397

$$\frac{\frac{\frac{\frac{\frac{8a^4\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^3\left(63a^2+90ab+35b^2\right)\int \frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{a}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\text{co}}{3a}}{4a(a+b)}}{d}$$

$\big\downarrow$ 218

$$\frac{\frac{\frac{\frac{\frac{8a^4\int \frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b^{5/2}\left(63a^2+90ab+35b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c}{3a}}{4a(a+b)}}{d}$$

$\big\downarrow$ 219

$$\frac{\frac{\frac{\frac{\frac{8a^4\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{b^{5/2}\left(63a^2+90ab+35b^2\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{a}}{a}-\frac{\left(8a^3-8a^2b-55ab^2-35b^3\right)\coth(c+dx)}{a}}{2a(a+b)}-\frac{\left(8a^2+55ab+35b^2\right)\coth^3(c+dx)}{3a}+}{4a(a+b)}}{d}$$

input
```
Int[Coth[c + d*x]^4/(a + b*Tanh[c + d*x]^2)^3,x]
```

output
```
((b*Coth[c + d*x]^3)/(4*a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) + ((-1/3*((8*
a^2 + 55*a*b + 35*b^2)*Coth[c + d*x]^3)/a + (((b^(5/2)*(63*a^2 + 90*a*b +
35*b^2)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + (8*a^
4*ArcTanh[Tanh[c + d*x]])/(a + b))/a - ((8*a^3 - 8*a^2*b - 55*a*b^2 - 35*b
^3)*Coth[c + d*x])/a)/a)/(2*a*(a + b)) + (b*(11*a + 7*b)*Coth[c + d*x]^3)/
(2*a*(a + b)*(a + b*Tanh[c + d*x]^2)))/(4*a*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.38 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.83

method	result
derivativedivides	$-\frac{-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{13}{4}a\,b^2+\frac{11}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(17a^2+30ab+13b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(63a^2+90ab+35b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a+b)^3a^4}}{d}$
default	$-\frac{-\frac{b^3\left(\frac{\left(\frac{15}{8}a^2b+\frac{13}{4}a\,b^2+\frac{11}{8}b^3\right)\tanh(dx+c)^3+\frac{a\left(17a^2+30ab+13b^2\right)\tanh(dx+c)}{8}}{\left(a+b\tanh(dx+c)^2\right)^2}+\frac{\left(63a^2+90ab+35b^2\right)\arctan\left(\frac{b\tanh(dx+c)}{\sqrt{ab}}\right)}{8\sqrt{ab}}\right)}{(a+b)^3a^4}}{d}$
risch	Expression too large to display

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/d*(-b^3/(a+b)^3/a^4*(((15/8*a^2*b+13/4*a*b^2+11/8*b^3)*tanh(d*x+c)^3+1/
8*a*(17*a^2+30*a*b+13*b^2)*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^2+1/8*(63*a^2+
90*a*b+35*b^2)/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/2)))-(3*b-a)/a^4/
tanh(d*x+c)+1/3/a^3/tanh(d*x+c)^3-1/2/(a+b)^3*ln(1+tanh(d*x+c))+1/2/(a+b)^
3*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10858 vs. $2(210) = 420$.

Time = 0.38 (sec) , antiderivative size = 22038, normalized size of antiderivative = 96.66

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx$$

input
```
integrate(coth(d*x+c)**4/(a+b*tanh(d*x+c)**2)**3,x)
```

output
```
Integral(coth(c + d*x)**4/(a + b*tanh(c + d*x)**2)**3, x)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4285 vs. 2(210) = 420.

Time = 1.13 (sec) , antiderivative size = 4285, normalized size of antiderivative = 18.79

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx = \text{Too large to display}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="maxima")
```

output
```
-1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log((a + b)*e^(4*d*x + 4*c) +
 2*(a - b)*e^(2*d*x + 2*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)
*d) + 1/8*(3*a^3*b - 3*a^2*b^2 - 7*a*b^3 - 3*b^4)*log(2*(a - b)*e^(-2*d*x
 - 2*c) + (a + b)*e^(-4*d*x - 4*c) + a + b)/((a^7 + 3*a^6*b + 3*a^5*b^2 + a
^4*b^3)*d) + 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 + 35*b^5)*arctan(
1/2*((a + b)*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b
^2 + a^4*b^3)*sqrt(a*b)*d) - 1/128*(15*a^4*b - 200*a^3*b^2 - 186*a^2*b^3 +
 35*b^5)*arctan(1/2*((a + b)*e^(-2*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*sqrt(a*b)*d) + 1/192*(176*a^6 + 781*a^5*b +
 1571*a^4*b^2 + 1538*a^3*b^3 + 502*a^2*b^4 - 175*a*b^5 - 105*b^6 + 3*(96*a
^6 + 465*a^5*b + 665*a^4*b^2 + 706*a^3*b^3 + 506*a^2*b^4 + 61*a*b^5 - 35*b
^6)*e^(12*d*x + 12*c) + 6*(120*a^6 + 192*a^5*b - 315*a^4*b^2 - 728*a^3*b^3
 - 1070*a^2*b^4 - 240*a*b^5 + 105*b^6)*e^(10*d*x + 10*c) + (176*a^6 - 281*
a^5*b + 3509*a^4*b^2 + 3950*a^3*b^3 + 12226*a^2*b^4 + 3755*a*b^5 - 1575*b^
6)*e^(8*d*x + 8*c) - 4*(184*a^6 + 48*a^5*b + 473*a^4*b^2 + 970*a^3*b^3 + 3
684*a^2*b^4 + 1070*a*b^5 - 525*b^6)*e^(6*d*x + 6*c) - (384*a^6 + 1127*a^5*
b - 861*a^4*b^2 - 7146*a^3*b^3 - 11386*a^2*b^4 - 1965*a*b^5 + 1575*b^6)*e^
(4*d*x + 4*c) + 2*(136*a^6 - 96*a^5*b - 1309*a^4*b^2 - 2996*a^3*b^3 - 2238
*a^2*b^4 - 4*a*b^5 + 315*b^6)*e^(2*d*x + 2*c))/((a^9 + 5*a^8*b + 10*a^7*b^
2 + 10*a^6*b^3 + 5*a^5*b^4 + a^4*b^5 - (a^9 + 5*a^8*b + 10*a^7*b^2 + 10...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 493 vs. $2(210) = 420$.

Time = 0.38 (sec) , antiderivative size = 493, normalized size of antiderivative = 2.16

$$\int \frac{\coth^4(c+dx)}{\left(a+b\tanh^2(c+dx)\right)^3}\,dx$$
$$= \frac{\frac{3\,(63\,a^2b^3+90\,ab^4+35\,b^5)\arctan\left(\frac{ae^{(2\,dx+2\,c)}+be^{(2\,dx+2\,c)}+a-b}{2\,\sqrt{ab}}\right)}{(a^7+3\,a^6b+3\,a^5b^2+a^4b^3)\sqrt{ab}}+\frac{24\,(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{6\,(17\,a^3b^3e^{(6\,dx+6\,c)}+7\,a^2b^4e^{(6\,dx+6\,c)}-21\,ab^5e}{}}{}$$

input
```
integrate(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x, algorithm="giac")
```

output
```
1/24*(3*(63*a^2*b^3 + 90*a*b^4 + 35*b^5)*arctan(1/2*(a*e^(2*d*x + 2*c) + b
*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3
)*sqrt(a*b)) + 24*(d*x + c)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(17*a^3*b^
3*e^(6*d*x + 6*c) + 7*a^2*b^4*e^(6*d*x + 6*c) - 21*a*b^5*e^(6*d*x + 6*c) -
 11*b^6*e^(6*d*x + 6*c) + 51*a^3*b^3*e^(4*d*x + 4*c) - a^2*b^4*e^(4*d*x +
4*c) + 29*a*b^5*e^(4*d*x + 4*c) + 33*b^6*e^(4*d*x + 4*c) + 51*a^3*b^3*e^(2
*d*x + 2*c) + 37*a^2*b^4*e^(2*d*x + 2*c) - 47*a*b^5*e^(2*d*x + 2*c) - 33*b
^6*e^(2*d*x + 2*c) + 17*a^3*b^3 + 45*a^2*b^4 + 39*a*b^5 + 11*b^6)/((a^7 +
3*a^6*b + 3*a^5*b^2 + a^4*b^3)*(a*e^(4*d*x + 4*c) + b*e^(4*d*x + 4*c) + 2*
a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^2) - 16*(6*a*e^(4*d*x + 4
*c) - 9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 18*b*e^(2*d*x + 2*c) + 4
*a - 9*b)/(a^4*(e^(2*d*x + 2*c) - 1)^3))/d
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx=\int \frac{\coth(c+d\,x)^4}{\left(b\tanh\left(c+d\,x\right)^2+a\right)^3}\,dx$$

input
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3,x)
```

output
```
int(coth(c + d*x)^4/(a + b*tanh(c + d*x)^2)^3, x)
```

Reduce [F]

$$\int \frac{\coth^4(c+dx)}{(a+b\tanh^2(c+dx))^3}\,dx=\int \frac{\coth\left(dx+c\right)^4}{\left(\tanh\left(dx+c\right)^2b+a\right)^3}dx$$

input
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

output
```
int(coth(d*x+c)^4/(a+b*tanh(d*x+c)^2)^3,x)
```

3.201 $\int \frac{1}{(a+b\tanh^2(c+dx))^4}\,dx$

Optimal result

Integrand size = 14, antiderivative size = 201

$$\int \frac{1}{(a+b\tanh^2(c+dx))^4}\,dx = \frac{x}{(a+b)^4} + \frac{\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^4d} + \frac{b\tanh(c+dx)}{6a(a+b)d\,(a+b\tanh^2(c+dx))^3} + \frac{b(11a+5b)\tanh(c+dx)}{24a^2(a+b)^2d\,(a+b\tanh^2(c+dx))^2} + \frac{b(19a^2+16ab+5b^2)\tanh(c+dx)}{16a^3(a+b)^3d\,(a+b\tanh^2(c+dx))}$$

output
```
x/(a+b)^4+1/16*b^(1/2)*(35*a^3+35*a^2*b+21*a*b^2+5*b^3)*arctan(b^(1/2)*tan
h(d*x+c)/a^(1/2))/a^(7/2)/(a+b)^4/d+1/6*b*tanh(d*x+c)/a/(a+b)/d/(a+b*tanh(
d*x+c)^2)^3+1/24*b*(11*a+5*b)*tanh(d*x+c)/a^2/(a+b)^2/d/(a+b*tanh(d*x+c)^2
)^2+1/16*b*(19*a^2+16*a*b+5*b^2)*tanh(d*x+c)/a^3/(a+b)^3/d/(a+b*tanh(d*x+c
)^2)
```

Mathematica [A] (verified)

Time = 0.40 (sec) , antiderivative size = 203, normalized size of antiderivative = 1.01

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$$
$$=\frac{\frac{3\sqrt{b}(35a^3+35a^2b+21ab^2+5b^3)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}}-24\log(1-\tanh(c+dx))+24\log(1+\tanh(c+dx))+\frac{8b(a+}{a(a+b}}{48(a+b)^4d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

output
```
((3*Sqrt[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c
+ d*x])/Sqrt[a]])/a^(7/2) - 24*Log[1 - Tanh[c + d*x]] + 24*Log[1 + Tanh[c
+ d*x]] + (8*b*(a + b)^3*Tanh[c + d*x])/(a*(a + b*Tanh[c + d*x]^2)^3) + (2
*b*(a + b)^2*(11*a + 5*b)*Tanh[c + d*x])/(a^2*(a + b*Tanh[c + d*x]^2)^2) +
 (3*b*(a + b)*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(a^3*(a + b*Tanh[c
+ d*x]^2)))/(48*(a + b)^4*d)
```

Rubi [A] (verified)

Time = 0.73 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.19, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.714$, Rules used = {3042, 4144, 316, 402, 27, 402, 25, 397, 218, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{1}{\left(a-b\tan(ic+idx)^2\right)^4}dx$$

$$\Big\downarrow \; 4144$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^4}d\tanh(c+dx)}{d}$$

$\downarrow$ 316

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{\int \frac{5b\tanh^2(c+dx)+b-6(a+b)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^3}d\tanh(c+dx)}{6a(a+b)}}{d}$$

$\downarrow$ 402

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{\int \frac{3\left(8a^2+11ba+5b^2-b(11a+5b)\tanh^2(c+dx)\right)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b(11a+5b)\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{6a(a+b)}}{d}$$

$\downarrow$ 27

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\int \frac{8a^2+11ba+5b^2-b(11a+5b)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)^2}d\tanh(c+dx)}{4a(a+b)}-\frac{b(11a+5b)\tanh(c+dx)}{4a(a+b)\left(a+b\tanh^2(c+dx)\right)^2}}{6a(a+b)}}{d}$$

$\downarrow$ 402

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}-\frac{\int -\frac{16a^3+19ba^2+16b^2a+5b^3-b\left(19a^2+16ba+5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}\right)}{4a(a+b)}-\cdots}{6a(a+b)}}{d}$$

$\downarrow$ 25

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{\int \frac{16a^3+19ba^2+16b^2a+5b^3-b\left(19a^2+16ba+5b^2\right)\tanh^2(c+dx)}{\left(1-\tanh^2(c+dx)\right)\left(b\tanh^2(c+dx)+a\right)}d\tanh(c+dx)}{2a(a+b)}+\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}\right)}{4a(a+b)}-\cdots}{6a(a+b)}}{d}$$

$\downarrow$ 397

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{\frac{16a^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{b\left(35a^3+35a^2b+21ab^2+5b^3\right)\int\frac{1}{b\tanh^2(c+dx)+a}d\tanh(c+dx)}{a+b}}{2a(a+b)}+\frac{b\left(19a^2+16ab\right.}{2a(a+b)\left(a\right.}}{4a(a+b)}}{6a(a+b)}}{d}$$

$\downarrow$ 218

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{\frac{16a^3\int\frac{1}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{a+b}+\frac{\sqrt{b}\left(35a^3+35a^2b+21ab^2+5b^3\right)\arctan\left(\frac{\sqrt{b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a+b)}}{2a(a+b)}+\frac{b\left(19a^2+16ab+5b^2\right.}{2a(a+b)\left(a+b\tan\right.}}{4a(a+b)}}{6a(a+b)}}{d}$$

$\downarrow$ 219

$$\frac{\frac{b\tanh(c+dx)}{6a(a+b)\left(a+b\tanh^2(c+dx)\right)^3}-\frac{-\frac{3\left(\frac{b\left(19a^2+16ab+5b^2\right)\tanh(c+dx)}{2a(a+b)\left(a+b\tanh^2(c+dx)\right)}+\frac{\frac{16a^3\text{arctanh}(\tanh(c+dx))}{a+b}+\frac{\sqrt{b}\left(35a^3+35a^2b+21ab^2+5b^3\right)\arctan\left(\frac{\sqrt{b}\tanh(c+\right.}{\sqrt{a}}}{\sqrt{a}(a+b)}}{2a(a+b)}\right.}{4a(a+b)}}{6a(a+b)}}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^2)^(-4),x]
```

output
```
((b*Tanh[c + d*x])/(6*a*(a + b)*(a + b*Tanh[c + d*x]^2)^3) - (-1/4*(b*(11*
a + 5*b)*Tanh[c + d*x])/(a*(a + b)*(a + b*Tanh[c + d*x]^2)^2) - (3*(((Sqrt
[b]*(35*a^3 + 35*a^2*b + 21*a*b^2 + 5*b^3)*ArcTan[(Sqrt[b]*Tanh[c + d*x])/
Sqrt[a]])/(Sqrt[a]*(a + b)) + (16*a^3*ArcTanh[Tanh[c + d*x]])/(a + b))/(2*
a*(a + b)) + (b*(19*a^2 + 16*a*b + 5*b^2)*Tanh[c + d*x])/(2*a*(a + b)*(a +
 b*Tanh[c + d*x]^2))))/(4*a*(a + b)))/(6*a*(a + b)))/d
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 218
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 397
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*((c_) + (d_.)*(x_)^2)), x_
Symbol] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[1/(a + b*x^2), x], x] - Simp[
(d*e - c*f)/(b*c - a*d)   Int[1/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d, e
, f}, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.30 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.09

method	result
derivativedivides	$\frac{\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^5}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^3}{6a^2}+\frac{\left(29a^3+61a^2b+43a\,b^2+11b^3\right)\tanh(\ldots}{16a}}{\left(a+b\tanh(dx+c)^2\right)^3}\right.}{(a+b)^4}}{d}$
default	$\frac{\frac{b\left(\frac{\frac{b^2\left(19a^3+35a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^5}{16a^3}+\frac{b\left(17a^3+33a^2b+21a\,b^2+5b^3\right)\tanh(dx+c)^3}{6a^2}+\frac{\left(29a^3+61a^2b+43a\,b^2+11b^3\right)\tanh(\ldots}{16a}}{\left(a+b\tanh(dx+c)^2\right)^3}\right.}{(a+b)^4}}{d}$
risch	Expression too large to display

input
```
int(1/(a+b*tanh(d*x+c)^2)^4,x,method=_RETURNVERBOSE)
```

output
```
1/d*(1/(a+b)^4*b*((1/16*b^2*(19*a^3+35*a^2*b+21*a*b^2+5*b^3)/a^3*tanh(d*x+
c)^5+1/6*b*(17*a^3+33*a^2*b+21*a*b^2+5*b^3)/a^2*tanh(d*x+c)^3+1/16*(29*a^3
+61*a^2*b+43*a*b^2+11*b^3)/a*tanh(d*x+c))/(a+b*tanh(d*x+c)^2)^3+1/16*(35*a
^3+35*a^2*b+21*a*b^2+5*b^3)/a^3/(a*b)^(1/2)*arctan(b*tanh(d*x+c)/(a*b)^(1/
2)))+1/2/(a+b)^4*ln(1+tanh(d*x+c))-1/2/(a+b)^4*ln(-1+tanh(d*x+c)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9849 vs. 2(185) = 370.

Time = 0.37 (sec) , antiderivative size = 20020, normalized size of antiderivative = 99.60

$$\int \frac{1}{(a+b\tanh^2(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+b\tanh^2(c+dx))^4}\,dx = \text{Timed out}$$

input
```
integrate(1/(a+b*tanh(d*x+c)**2)**4,x)
```

output
```
Timed out
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. 2(185) = 370.

Time = 0.42 (sec) , antiderivative size = 925, normalized size of antiderivative = 4.60

$$\int \frac{1}{(a+b\tanh^2(c+dx))^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="maxima")
```

output
```
-1/16*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*((a + b)*e^(-2
*d*x - 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
a^3*b^4)*sqrt(a*b)*d) + 1/24*(87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a
^2*b^4 + 103*a*b^5 + 15*b^6 + 3*(145*a^5*b + 267*a^4*b^2 + 34*a^3*b^3 - 17
8*a^2*b^4 - 115*a*b^5 - 25*b^6)*e^(-2*d*x - 2*c) + 6*(145*a^5*b + 93*a^4*b
^2 - 6*a^3*b^3 + 106*a^2*b^4 + 85*a*b^5 + 25*b^6)*e^(-4*d*x - 4*c) + 2*(43
5*a^5*b + 29*a^4*b^2 + 162*a^3*b^3 - 306*a^2*b^4 - 245*a*b^5 - 75*b^6)*e^(
-6*d*x - 6*c) + 3*(145*a^5*b + 17*a^4*b^2 - 58*a^3*b^3 + 150*a^2*b^4 + 105
*a*b^5 + 25*b^6)*e^(-8*d*x - 8*c) + 3*(29*a^5*b + 23*a^4*b^2 - 62*a^3*b^3
- 82*a^2*b^4 - 31*a*b^5 - 5*b^6)*e^(-10*d*x - 10*c))/((a^10 + 7*a^9*b + 21
*a^8*b^2 + 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7 + 6*
(a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^5*b^5 - 5*a^4*b^
6 - a^3*b^7)*e^(-2*d*x - 2*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*b^2 + 15*a^7
*b^3 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-4*d*x - 4*c)
+ 4*(5*a^10 + 17*a^9*b + 21*a^8*b^2 + 9*a^7*b^3 - 9*a^6*b^4 - 21*a^5*b^5 -
 17*a^4*b^6 - 5*a^3*b^7)*e^(-6*d*x - 6*c) + 3*(5*a^10 + 19*a^9*b + 25*a^8*
b^2 + 15*a^7*b^3 + 15*a^6*b^4 + 25*a^5*b^5 + 19*a^4*b^6 + 5*a^3*b^7)*e^(-8
*d*x - 8*c) + 6*(a^10 + 5*a^9*b + 9*a^8*b^2 + 5*a^7*b^3 - 5*a^6*b^4 - 9*a^
5*b^5 - 5*a^4*b^6 - a^3*b^7)*e^(-10*d*x - 10*c) + (a^10 + 7*a^9*b + 21*a^8
*b^2 + 35*a^7*b^3 + 35*a^6*b^4 + 21*a^5*b^5 + 7*a^4*b^6 + a^3*b^7)*e^(-...
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 750 vs. $2(185) = 370$.

Time = 0.19 (sec) , antiderivative size = 750, normalized size of antiderivative = 3.73

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(d*x+c)^2)^4,x, algorithm="giac")
```

output
```
1/48*(3*(35*a^3*b + 35*a^2*b^2 + 21*a*b^3 + 5*b^4)*arctan(1/2*(a*e^(2*d*x
+ 2*c) + b*e^(2*d*x + 2*c) + a - b)/sqrt(a*b))/((a^7 + 4*a^6*b + 6*a^5*b^2
 + 4*a^4*b^3 + a^3*b^4)*sqrt(a*b)) + 48*(d*x + c)/(a^4 + 4*a^3*b + 6*a^2*b
^2 + 4*a*b^3 + b^4) - 2*(87*a^5*b*e^(10*d*x + 10*c) + 69*a^4*b^2*e^(10*d*x
 + 10*c) - 186*a^3*b^3*e^(10*d*x + 10*c) - 246*a^2*b^4*e^(10*d*x + 10*c) -
 93*a*b^5*e^(10*d*x + 10*c) - 15*b^6*e^(10*d*x + 10*c) + 435*a^5*b*e^(8*d*
x + 8*c) + 51*a^4*b^2*e^(8*d*x + 8*c) - 174*a^3*b^3*e^(8*d*x + 8*c) + 450*
a^2*b^4*e^(8*d*x + 8*c) + 315*a*b^5*e^(8*d*x + 8*c) + 75*b^6*e^(8*d*x + 8*
c) + 870*a^5*b*e^(6*d*x + 6*c) + 58*a^4*b^2*e^(6*d*x + 6*c) + 324*a^3*b^3*
e^(6*d*x + 6*c) - 612*a^2*b^4*e^(6*d*x + 6*c) - 490*a*b^5*e^(6*d*x + 6*c)
- 150*b^6*e^(6*d*x + 6*c) + 870*a^5*b*e^(4*d*x + 4*c) + 558*a^4*b^2*e^(4*d
*x + 4*c) - 36*a^3*b^3*e^(4*d*x + 4*c) + 636*a^2*b^4*e^(4*d*x + 4*c) + 510
*a*b^5*e^(4*d*x + 4*c) + 150*b^6*e^(4*d*x + 4*c) + 435*a^5*b*e^(2*d*x + 2*
c) + 801*a^4*b^2*e^(2*d*x + 2*c) + 102*a^3*b^3*e^(2*d*x + 2*c) - 534*a^2*b
^4*e^(2*d*x + 2*c) - 345*a*b^5*e^(2*d*x + 2*c) - 75*b^6*e^(2*d*x + 2*c) +
87*a^5*b + 319*a^4*b^2 + 450*a^3*b^3 + 306*a^2*b^4 + 103*a*b^5 + 15*b^6)/(
(a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*(a*e^(4*d*x + 4*c) + b*e
^(4*d*x + 4*c) + 2*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + a + b)^3))/d
```

Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 3685, normalized size of antiderivative = 18.33

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a + b*tanh(c + d*x)^2)^4,x)
```

output
```
log(tanh(c + d*x) + 1)/(2*a^4*d + 2*b^4*d + 12*a^2*b^2*d + 8*a*b^3*d + 8*a
^3*b*d) + ((tanh(c + d*x)^3*(16*a*b^3 + 5*b^4 + 17*a^2*b^2))/(6*a^2*(3*a*b
^2 + 3*a^2*b + a^3 + b^3)) + (tanh(c + d*x)*(32*a*b^2 + 29*a^2*b + 11*b^3)
)/(16*a*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (b^2*tanh(c + d*x)^5*(16*a*b^2
+ 19*a^2*b + 5*b^3))/(16*a^2*(a*b^3 + 3*a^3*b + a^4 + 3*a^2*b^2)))/(a^3*d
+ b^3*d*tanh(c + d*x)^6 + 3*a^2*b*d*tanh(c + d*x)^2 + 3*a*b^2*d*tanh(c + d
*x)^4) - log(tanh(c + d*x) - 1)/(2*d*(a + b)^4) - (atan((((-a^7*b)^(1/2)*(
(tanh(c + d*x)*(210*a*b^8 + 25*b^9 + 791*a^2*b^7 + 1820*a^3*b^6 + 2695*a^4
*b^5 + 2450*a^5*b^4 + 1481*a^6*b^3))/(128*(a^12*d^2 + 6*a^11*b*d^2 + a^6*b
^6*d^2 + 6*a^7*b^5*d^2 + 15*a^8*b^4*d^2 + 20*a^9*b^3*d^2 + 15*a^10*b^2*d^2
)) + ((((5*a^3*b^13*d^2)/4 + 14*a^4*b^12*d^2 + (287*a^5*b^11*d^2)/4 + 224*
a^6*b^10*d^2 + (953*a^7*b^9*d^2)/2 + 728*a^8*b^8*d^2 + (1631*a^9*b^7*d^2)/
2 + 668*a^10*b^6*d^2 + (1561*a^11*b^5*d^2)/4 + 154*a^12*b^4*d^2 + (147*a^1
3*b^3*d^2)/4 + 4*a^14*b^2*d^2)/(a^15*d^3 + 9*a^14*b*d^3 + a^6*b^9*d^3 + 9*
a^7*b^8*d^3 + 36*a^8*b^7*d^3 + 84*a^9*b^6*d^3 + 126*a^10*b^5*d^3 + 126*a^1
1*b^4*d^3 + 84*a^12*b^3*d^3 + 36*a^13*b^2*d^3) - (tanh(c + d*x)*(-a^7*b)^(
1/2)*(21*a*b^2 + 35*a^2*b + 35*a^3 + 5*b^3)*(1024*a^6*b^11*d^2 + 7168*a^7*
b^10*d^2 + 20480*a^8*b^9*d^2 + 28672*a^9*b^8*d^2 + 14336*a^10*b^7*d^2 - 14
336*a^11*b^6*d^2 - 28672*a^12*b^5*d^2 - 20480*a^13*b^4*d^2 - 7168*a^14*b^3
*d^2 - 1024*a^15*b^2*d^2))/(4096*(a^11*d + a^7*b^4*d + 4*a^8*b^3*d + 6*...
```

Reduce [B] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 1058, normalized size of antiderivative = 5.26

$$\int \frac{1}{\left(a+b\tanh^2(c+dx)\right)^4}\,dx = \text{Too large to display}$$

input
```
int(1/(a+b*tanh(d*x+c)^2)^4,x)
```

output

```
(105*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*
x)**6*a**3*b**3 + 105*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt
(a)))*tanh(c + d*x)**6*a**2*b**4 + 63*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*
b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**6*a*b**5 + 15*sqrt(b)*sqrt(a)*atan((t
anh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**6*b**6 + 315*sqrt(b)*sqr
t(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**4*a**4*b**2
 + 315*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d
*x)**4*a**3*b**3 + 189*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqr
t(a)))*tanh(c + d*x)**4*a**2*b**4 + 45*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)
*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**4*a*b**5 + 315*sqrt(b)*sqrt(a)*atan(
(tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**5*b + 315*sqrt(b)
*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c + d*x)**2*a**4*b
**2 + 189*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*sqrt(a)))*tanh(c
 + d*x)**2*a**3*b**3 + 45*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(sqrt(b)*
sqrt(a)))*tanh(c + d*x)**2*a**2*b**4 + 105*sqrt(b)*sqrt(a)*atan((tanh(c +
d*x)*b)/(sqrt(b)*sqrt(a)))*a**6 + 105*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*
b)/(sqrt(b)*sqrt(a)))*a**5*b + 63*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(
sqrt(b)*sqrt(a)))*a**4*b**2 + 15*sqrt(b)*sqrt(a)*atan((tanh(c + d*x)*b)/(s
qrt(b)*sqrt(a)))*a**3*b**3 + 48*tanh(c + d*x)**6*a**4*b**3*d*x + 57*tanh(c
 + d*x)**5*a**4*b**3 + 105*tanh(c + d*x)**5*a**3*b**4 + 63*tanh(c + d*x...
```

3.202 $\int \sqrt{1-\tanh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 3

$$\int \sqrt{1-\tanh^2(x)}\,dx = \arcsin(\tanh(x))$$

output
```
arcsin(tanh(x))
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 15 vs. 2(3) = 6.

Time = 0.01 (sec) , antiderivative size = 15, normalized size of antiderivative = 5.00

$$\int \sqrt{1-\tanh^2(x)}\,dx = -\cot^{-1}(\sinh(x))\cosh(x)\sqrt{\operatorname{sech}^2(x)}$$

input
```
Integrate[Sqrt[1 - Tanh[x]^2],x]
```

output
```
-(ArcCot[Sinh[x]]*Cosh[x]*Sqrt[Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4140, 3042, 4610, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{1-\tanh^2(x)}\,dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{1+\tan(ix)^2}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \sqrt{\text{sech}^2(x)}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \sqrt{\sec(ix)^2}dx$$

$$\Big\downarrow \text{ 4610}$$

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{ 223}$$

$$\arcsin(\tanh(x))$$

input
```
Int[Sqrt[1 - Tanh[x]^2],x]
```

output
```
ArcSin[Tanh[x]]
```

Defintions of rubi rules used

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 4, normalized size of antiderivative = 1.33

method	result	size
derivativedivides	$\arcsin\left(\tanh\left(x\right)\right)$	4
default	$\arcsin\left(\tanh\left(x\right)\right)$	4
risch	$i\sqrt{\frac{{\mathrm e}^{2 x}}{\left({\mathrm e}^{2 x}+1\right)^{2}}}\, {\mathrm e}^{-x} \left({\mathrm e}^{2 x}+1\right) \ln\left({\mathrm e}^{x}+i\right)-i\sqrt{\frac{{\mathrm e}^{2 x}}{\left({\mathrm e}^{2 x}+1\right)^{2}}}\, {\mathrm e}^{-x} \left({\mathrm e}^{2 x}+1\right) \ln\left({\mathrm e}^{x}-i\right)$	70

input
```
int((1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
arcsin(tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8 vs. 2(3) = 6.

Time = 0.08 (sec) , antiderivative size = 8, normalized size of antiderivative = 2.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = 2\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)$$

input
```
integrate((1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
2*arctan(cosh(x) + sinh(x))
```

Sympy [F]

$$\int \sqrt{1-\tanh^2(x)}\,dx = \int \sqrt{1-\tanh^2\left(x\right)}\,dx$$

input
```
integrate((1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(1 - tanh(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.15 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = 2\arctan\left(e^x\right)$$

input
```
integrate((1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*arctan(e^x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 5, normalized size of antiderivative = 1.67

$$\int \sqrt{1-\tanh^2(x)}\,dx = 2 \arctan(e^x)$$

input
```
integrate((1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(e^x)
```

Mupad [B] (verification not implemented)

Time = 0.04 (sec) , antiderivative size = 3, normalized size of antiderivative = 1.00

$$\int \sqrt{1-\tanh^2(x)}\,dx = \operatorname{asin}(\tanh(x))$$

input
```
int((1 - tanh(x)^2)^(1/2),x)
```

output
```
asin(tanh(x))
```

Reduce [F]

$$\int \sqrt{1-\tanh^2(x)}\,dx = \int \sqrt{-\tanh(x)^2+1}dx$$

input
```
int((1-tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt( - tanh(x)**2 + 1),x)
```

3.203 $\int \sqrt{-1+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 16

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right)$$

output
```
-arctanh(tanh(x)/(-sech(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 17, normalized size of antiderivative = 1.06

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\cot^{-1}(\sinh(x))\cosh(x)\sqrt{-\text{sech}^2(x)}$$

input
```
Integrate[Sqrt[-1 + Tanh[x]^2],x]
```

output
```
-(ArcCot[Sinh[x]]*Cosh[x]*Sqrt[-Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4140, 3042, 4610, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tanh^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-1-\tan(ix)^2}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \sqrt{-\text{sech}^2(x)}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-\sec(ix)^2}dx$$

$$\Big\downarrow \text{4610}$$

$$-\int \frac{1}{\sqrt{\tanh^2(x)-1}}d\tanh(x)$$

$$\Big\downarrow \text{224}$$

$$-\int \frac{1}{1-\frac{\tanh^2(x)}{\tanh^2(x)-1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}$$

$$\Big\downarrow \text{219}$$

$$-\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}\right)$$

input
```
Int[Sqrt[-1 + Tanh[x]^2],x]
```

output
```
-ArcTanh[Tanh[x]/Sqrt[-1 + Tanh[x]^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 15, normalized size of antiderivative = 0.94

method	result	size
derivativedivides	$-\ln\left(\tanh\left(x\right)+\sqrt{-1+\tanh\left(x\right)^{2}}\right)$	15
default	$-\ln\left(\tanh\left(x\right)+\sqrt{-1+\tanh\left(x\right)^{2}}\right)$	15
risch	$i\sqrt{-\frac{e^{2x}}{\left(e^{2x}+1\right)^{2}}}\, e^{-x}\left(e^{2x}+1\right)\ln\left(e^{x}+i\right)-i\sqrt{-\frac{e^{2x}}{\left(e^{2x}+1\right)^{2}}}\, e^{-x}\left(e^{2x}+1\right)\ln\left(e^{x}-i\right)$	72

input
```
int((-1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(tanh(x)+(-1+tanh(x)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(14) = 28$.

Time = 0.09 (sec) , antiderivative size = 96, normalized size of antiderivative = 6.00

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\log\left(\left(\cosh(x)e^x + e^x\sinh(x) + \sqrt{-\frac{e^{(2x)}}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)\right)e^{(-x)}\right) + \log\left(\left(\cosh(x)e^x + e^x\sinh(x) - \sqrt{-\frac{e^{(2x)}}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)\right)e^{(-x)}\right)$$

input
```
integrate((-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-log((cosh(x)*e^x + e^x*sinh(x) + sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))
*(e^(2*x) + 1))*e^(-x)) + log((cosh(x)*e^x + e^x*sinh(x) - sqrt(-e^(2*x)/(
e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1))*e^(-x))
```

Sympy [F]

$$\int \sqrt{-1+\tanh^2(x)}\,dx = \int \sqrt{\tanh^2(x)-1}\,dx$$

input
```
integrate((-1+tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(tanh(x)**2 - 1), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 5, normalized size of antiderivative = 0.31

$$\int \sqrt{-1+\tanh^2(x)}\,dx = 2i\arctan\left(e^x\right)$$

input
```
integrate((-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
2*I*arctan(e^x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 1, normalized size of antiderivative = 0.06

$$\int \sqrt{-1+\tanh^2(x)}\,dx = 0$$

input
```
integrate((-1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
0
```

Mupad [B] (verification not implemented)

Time = 0.23 (sec) , antiderivative size = 14, normalized size of antiderivative = 0.88

$$\int \sqrt{-1+\tanh^2(x)}\,dx = -\ln\left(\tanh(x)+\sqrt{\tanh{(x)}^2-1}\right)$$

input
```
int((tanh(x)^2 - 1)^(1/2),x)
```

output
```
-log(tanh(x) + (tanh(x)^2 - 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{-1+\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2-1}dx$$

input
```
int((-1+tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2 - 1),x)
```

3.204 $\int \left(1-\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 12, antiderivative size = 22

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{1}{2}\arcsin(\tanh(x)) + \frac{1}{2}\sqrt{\operatorname{sech}^2(x)}\tanh(x)$$

output
```
1/2*arcsin(tanh(x))+1/2*(sech(x)^2)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 1.05

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{\operatorname{sech}(x)(\arctan(\sinh(x)) + \operatorname{sech}(x)\tanh(x))}{2\sqrt{\operatorname{sech}^2(x)}}$$

input
```
Integrate[(1 - Tanh[x]^2)^(3/2),x]
```

output
```
(Sech[x]*(ArcTan[Sinh[x]] + Sech[x]*Tanh[x]))/(2*Sqrt[Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 26, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4140, 3042, 4610, 211, 223}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(1+\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \ 4140$$

$$\int \operatorname{sech}^2(x)^{3/2} dx$$

$$\Big\downarrow \ 3042$$

$$\int \left(\sec(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \ 4610$$

$$\int \sqrt{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 211$$

$$\frac{1}{2}\int \frac{1}{\sqrt{1-\tanh^2(x)}}d\tanh(x)+\frac{1}{2}\sqrt{1-\tanh^2(x)}\tanh(x)$$

$$\Big\downarrow \ 223$$

$$\frac{1}{2}\arcsin(\tanh(x))+\frac{1}{2}\tanh(x)\sqrt{1-\tanh^2(x)}$$

input
```
Int[(1 - Tanh[x]^2)^(3/2),x]
```

output
```
ArcSin[Tanh[x]]/2 + (Tanh[x]*Sqrt[1 - Tanh[x]^2])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 223
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[Rt[-b, 2]*(x/Sqrt
[a])]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.07 (sec) , antiderivative size = 21, normalized size of antiderivative = 0.95

method	result	size
derivativedivides	$\frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
default	$\frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}+\frac{\arcsin(\tanh(x))}{2}$	21
risch	$\frac{\sqrt{\frac{e^{2x}}{(e^{2x}+1)^2}}\,(e^{2x}-1)}{e^{2x}+1}+\frac{i\sqrt{\frac{e^{2x}}{(e^{2x}+1)^2}}\,e^{-x}(e^{2x}+1)\ln(e^x+i)}{2}-\frac{i\sqrt{\frac{e^{2x}}{(e^{2x}+1)^2}}\,e^{-x}(e^{2x}+1)\ln(e^x-i)}{2}$	100

input
```
int((1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/2*tanh(x)*(1-tanh(x)^2)^(1/2)+1/2*arcsin(tanh(x))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 140 vs. 2(16) = 32.

Time = 0.08 (sec) , antiderivative size = 140, normalized size of antiderivative = 6.36

$$\int (1 - \tanh^2(x))^{3/2} \, dx = \frac{\cosh(x)^3 + 3\cosh(x)\sinh(x)^2 + \sinh(x)^3 + (\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh}{\cosh(x)^4 + 4\cosh(x)\sinh(}$$

input
```
integrate((1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
(cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh
(x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(
x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)
*sinh(x) - cosh(x))/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 2*(3*co
sh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)^3 + cosh(x))*sinh(x) + 1
)
```

Sympy [F]

$$\int (1 - \tanh^2(x))^{3/2} \, dx = \int (1 - \tanh^2(x))^{\frac{3}{2}} \, dx$$

input
```
integrate((1-tanh(x)**2)**(3/2),x)
```

output
```
Integral((1 - tanh(x)**2)**(3/2), x)
```

Maxima [A] (verification not implemented)

Time = 0.16 (sec) , antiderivative size = 28, normalized size of antiderivative = 1.27

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{e^{(3x)} - e^x}{e^{(4x)} + 2e^{(2x)} + 1} + \arctan\left(e^x\right)$$

input
```
integrate((1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
(e^(3*x) - e^x)/(e^(4*x) + 2*e^(2*x) + 1) + arctan(e^x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 45 vs. 2(16) = 32.

Time = 0.12 (sec) , antiderivative size = 45, normalized size of antiderivative = 2.05

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{1}{4}\pi - \frac{e^{(-x)} - e^x}{\left(e^{(-x)} - e^x\right)^2 + 4} + \frac{1}{2}\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)$$

input
```
integrate((1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/4*pi - (e^(-x) - e^x)/((e^(-x) - e^x)^2 + 4) + 1/2*arctan(1/2*(e^(2*x) -
 1)*e^(-x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 20, normalized size of antiderivative = 0.91

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \frac{\operatorname{asin}(\tanh(x))}{2} + \frac{\tanh(x)\sqrt{1-\tanh(x)^2}}{2}$$

input
```
int((1 - tanh(x)^2)^(3/2),x)
```

output
```
asin(tanh(x))/2 + (tanh(x)*(1 - tanh(x)^2)^(1/2))/2
```

Reduce [F]

$$\int \left(1-\tanh^2(x)\right)^{3/2} dx = \int \sqrt{-\tanh(x)^2+1}dx - \left(\int \sqrt{-\tanh(x)^2+1}\,\tanh(x)^2\,dx\right)$$

input
```
int((1-tanh(x)^2)^(3/2),x)
```

output
```
int(sqrt( - tanh(x)**2 + 1),x) - int(sqrt( - tanh(x)**2 + 1)*tanh(x)**2,x)
```

3.205 $\int \left(-1+\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 10, antiderivative size = 35

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = \frac{1}{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{-\text{sech}^2(x)}}\right)-\frac{1}{2}\sqrt{-\text{sech}^2(x)}\tanh(x)$$

output
```
1/2*arctanh(tanh(x)/(-sech(x)^2)^(1/2))-1/2*(-sech(x)^2)^(1/2)*tanh(x)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.66

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = -\frac{1}{2}\sqrt{-\text{sech}^2(x)}(\arctan(\sinh(x))\cosh(x)+\tanh(x))$$

input
```
Integrate[(-1 + Tanh[x]^2)^(3/2),x]
```

output
```
-1/2*(Sqrt[-Sech[x]^2]*(ArcTan[Sinh[x]]*Cosh[x] + Tanh[x]))
```

Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.700, Rules used = {3042, 4140, 3042, 4610, 211, 224, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tanh^2(x)-1\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(-1-\tan(ix)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \left(-\text{sech}^2(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(-\sec(ix)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{ 4610}$$

$$-\int \sqrt{\tanh^2(x)-1}d\tanh(x)$$

$$\Big\downarrow \text{ 211}$$

$$\frac{1}{2}\int \frac{1}{\sqrt{\tanh^2(x)-1}}d\tanh(x)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)-1}$$

$$\Big\downarrow \text{ 224}$$

$$\frac{1}{2}\int \frac{1}{1-\frac{\tanh^2(x)}{\tanh^2(x)-1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)-1}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\text{arctanh}\left(\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)-1}$$

input
```
Int[(-1 + Tanh[x]^2)^(3/2),x]
```

output
```
ArcTanh[Tanh[x]/Sqrt[-1 + Tanh[x]^2]]/2 - (Tanh[x]*Sqrt[-1 + Tanh[x]^2])/2
```

Defintions of rubi rules used

rule 211
```
Int[((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[x*((a + b*x^2)^p/(2*p + 1
)), x] + Simp[2*a*(p/(2*p + 1))   Int[(a + b*x^2)^(p - 1), x], x] /; FreeQ[
{a, b}, x] && GtQ[p, 0] && (IntegerQ[4*p] || IntegerQ[6*p])
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.04 (sec) , antiderivative size = 28, normalized size of antiderivative = 0.80

method	result	size
derivativedivides	$-\frac{\tanh(x)\sqrt{-1+\tanh(x)^2}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{-1+\tanh(x)^2}\right)}{2}$	28
default	$-\frac{\tanh(x)\sqrt{-1+\tanh(x)^2}}{2}+\frac{\ln\left(\tanh(x)+\sqrt{-1+\tanh(x)^2}\right)}{2}$	28
risch	$-\frac{\sqrt{-\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\left(e^{2x}-1\right)}{e^{2x}+1}-\frac{i\sqrt{-\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\,e^{-x}\left(e^{2x}+1\right)\ln\left(e^{x}+i\right)}{2}+\frac{i\sqrt{-\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\,e^{-x}\left(e^{2x}+1\right)\ln\left(e^{x}-i\right)}{2}$	104

input
```
int((-1+tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(x)*(-1+tanh(x)^2)^(1/2)+1/2*ln(tanh(x)+(-1+tanh(x)^2)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 385 vs. 2(27) = 54.

Time = 0.09 (sec) , antiderivative size = 385, normalized size of antiderivative = 11.00

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((-1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/2*((4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*si
nh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 +
 1)*e^x)*log((cosh(x)*e^x + e^x*sinh(x) + sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*
x) + 1))*(e^(2*x) + 1))*e^(-x)) - (4*cosh(x)*e^x*sinh(x)^3 + e^x*sinh(x)^4
 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cosh(x)^3 + cosh(x))*e^x*sinh(x)
 + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x)*log((cosh(x)*e^x + e^x*sinh(x) - sqr
t(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1))*e^(-x)) - 2*((e^(2*x)
 + 1)*sinh(x)^3 + cosh(x)^3 + 3*(cosh(x)*e^(2*x) + cosh(x))*sinh(x)^2 + (c
osh(x)^3 - cosh(x))*e^(2*x) + (3*cosh(x)^2 + (3*cosh(x)^2 - 1)*e^(2*x) - 1
)*sinh(x) - cosh(x))*sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1)))/(4*cosh(x)*
e^x*sinh(x)^3 + e^x*sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*e^x*sinh(x)^2 + 4*(cos
h(x)^3 + cosh(x))*e^x*sinh(x) + (cosh(x)^4 + 2*cosh(x)^2 + 1)*e^x)
```

Sympy [F]

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx=\int \left(\tanh^2{(x)}-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1+tanh(x)**2)**(3/2),x)
```

output
```
Integral((tanh(x)**2 - 1)**(3/2), x)
```

Maxima [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.16 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.91

$$\int \left(-1+\tanh^2(x)\right)^{3/2}\,dx=\frac{-i\,e^{(3\,x)}+i\,e^{x}}{e^{(4\,x)}+2\,e^{(2\,x)}+1}-i\,\arctan\left(e^{x}\right)$$

input
```
integrate((-1+tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
(-I*e^(3*x) + I*e^x)/(e^(4*x) + 2*e^(2*x) + 1) - I*arctan(e^x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.17

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = \frac{\sqrt{-e^{(2x)}} + \frac{1}{\sqrt{-e^{(2x)}}}}{\left(\sqrt{-e^{(2x)}} + \frac{1}{\sqrt{-e^{(2x)}}}\right)^2 - 4}$$

input
```
integrate((-1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
(sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))/((sqrt(-e^(2*x)) + 1/sqrt(-e^(2*x)))^2
 - 4)
```

Mupad [B] (verification not implemented)

Time = 2.27 (sec) , antiderivative size = 27, normalized size of antiderivative = 0.77

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = \frac{\ln\left(\tanh(x) + \sqrt{\tanh(x)^2 - 1}\right)}{2} - \frac{\tanh(x)\sqrt{\tanh(x)^2 - 1}}{2}$$

input
```
int((tanh(x)^2 - 1)^(3/2),x)
```

output
```
log(tanh(x) + (tanh(x)^2 - 1)^(1/2))/2 - (tanh(x)*(tanh(x)^2 - 1)^(1/2))/2
```

Reduce [F]

$$\int \left(-1+\tanh^2(x)\right)^{3/2} dx = -\left(\int \sqrt{\tanh(x)^2 - 1}dx\right) + \int \sqrt{\tanh(x)^2 - 1}\tanh(x)^2 dx$$

input
```
int((-1+tanh(x)^2)^(3/2),x)
```

output
```
 - int(sqrt(tanh(x)**2 - 1),x) + int(sqrt(tanh(x)**2 - 1)*tanh(x)**2,x)
```

3.206 $\int \frac{1}{\sqrt{1-\tanh^2(x)}} dx$

Optimal result

Integrand size = 12, antiderivative size = 11

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}} dx = \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

output
```
tanh(x)/(sech(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}} dx = \frac{\tanh(x)}{\sqrt{\operatorname{sech}^2(x)}}$$

input
```
Integrate[1/Sqrt[1 - Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[Sech[x]^2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 15, normalized size of antiderivative = 1.36, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.417, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{1+\tan(ix)^2}}dx$$

$$\Big\downarrow \text{ 4140}$$

$$\int \frac{1}{\sqrt{\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{1}{\sqrt{\sec(ix)^2}}dx$$

$$\Big\downarrow \text{ 4610}$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 208}$$

$$\frac{\tanh(x)}{\sqrt{1-\tanh^2(x)}}$$

input
```
Int[1/Sqrt[1 - Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[1 - Tanh[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.03 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.27

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
default	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
parallelrisch	$\frac{\tanh(x)}{\sqrt{1-\tanh(x)^2}}$	14
risch	$\frac{e^{2x}}{2\sqrt{\frac{e^{2x}}{(e^{2x}+1)^2}}\,(e^{2x}+1)}-\frac{1}{2(e^{2x}+1)\sqrt{\frac{e^{2x}}{(e^{2x}+1)^2}}}$	56

input
```
int(1/(1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/(1-tanh(x)^2)^(1/2)*tanh(x)
```

Fricas [A] (verification not implemented)

Time = 0.08 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx=\sinh(x)$$

input
```
integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
sinh(x)
```

Sympy [F]

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx=\int \frac{1}{\sqrt{1-\tanh^2{(x)}}}\,dx$$

input
```
integrate(1/(1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(1 - tanh(x)**2), x)
```

Maxima [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx=-\frac{1}{2}\,e^{(-x)}+\frac{1}{2}\,e^x$$

input
```
integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*e^(-x) + 1/2*e^x
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 11, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = -\frac{1}{2}\,e^{(-x)} + \frac{1}{2}\,e^x$$

input
```
integrate(1/(1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*e^(-x) + 1/2*e^x
```

Mupad [B] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 2, normalized size of antiderivative = 0.18

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = \sinh(x)$$

input
```
int(1/(1 - tanh(x)^2)^(1/2),x)
```

output
```
sinh(x)
```

Reduce [F]

$$\int \frac{1}{\sqrt{1-\tanh^2(x)}}\,dx = -\left(\int \frac{\sqrt{-\tanh(x)^2+1}}{\tanh(x)^2-1}dx\right)$$

input
```
int(1/(1-tanh(x)^2)^(1/2),x)
```

output
```
- int(sqrt( - tanh(x)**2 + 1)/(tanh(x)**2 - 1),x)
```

3.207 $\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 13

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

output
```
tanh(x)/(-sech(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{-\operatorname{sech}^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 + Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[-Sech[x]^2]
```

Rubi [A] (verified)

Time = 0.23 (sec) , antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4140, 3042, 4610, 208}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\tanh^2(x)-1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-1-\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4140}$$

$$\int \frac{1}{\sqrt{-\text{sech}^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-\sec(ix)^2}}dx$$

$$\Big\downarrow \text{4610}$$

$$-\int \frac{1}{\left(\tanh^2(x)-1\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{208}$$

$$\frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}$$

input
```
Int[1/Sqrt[-1 + Tanh[x]^2],x]
```

output
```
Tanh[x]/Sqrt[-1 + Tanh[x]^2]
```

Defintions of rubi rules used

rule 208
```
Int[((a_) + (b_.)*(x_)^2)^(-3/2), x_Symbol] :> Simp[x/(a*Sqrt[a + b*x^2]),
x] /; FreeQ[{a, b}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4140
```
Int[(u_.)*((a_) + (b_.)*tan[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*sec[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a, b]
```

rule 4610
```
Int[((b_.)*sec[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> With[{ff = FreeFac
tors[Tan[e + f*x], x]}, Simp[b*(ff/f)   Subst[Int[(b + b*ff^2*x^2)^(p - 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{b, e, f, p}, x] &&  !IntegerQ[p]
```

Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\tanh(x)}{\sqrt{-1+\tanh(x)^2}}$	12
default	$\frac{\tanh(x)}{\sqrt{-1+\tanh(x)^2}}$	12
risch	$\frac{e^{2x}}{2\sqrt{-\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}\,(e^{2x}+1)}-\frac{1}{2(e^{2x}+1)\sqrt{-\frac{e^{2x}}{\left(e^{2x}+1\right)^2}}}$	58

input
```
int(1/(-1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
tanh(x)/(-1+tanh(x)^2)^(1/2)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 36 vs. $2(11) = 22$.

Time = 0.08 (sec) , antiderivative size = 36, normalized size of antiderivative = 2.77

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\sqrt{-\frac{e^{(2x)}}{e^{(4x)}+2e^{(2x)}+1}}\left(e^{(2x)}+1\right)e^{(-x)}\sinh(x)$$

input
```
integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-sqrt(-e^(2*x)/(e^(4*x) + 2*e^(2*x) + 1))*(e^(2*x) + 1)*e^(-x)*sinh(x)
```

Sympy [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 12, normalized size of antiderivative = 0.92

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \frac{\tanh(x)}{\sqrt{\tanh^2(x)-1}}$$

input
```
integrate(1/(-1+tanh(x)**2)**(1/2),x)
```

output
```
tanh(x)/sqrt(tanh(x)**2 - 1)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 25 vs. $2(11) = 22$.

Time = 0.13 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.92

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{e^{(-2x)}}{2\sqrt{-e^{(-2x)}}}+\frac{1}{2\sqrt{-e^{(-2x)}}}$$

input
```
integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
-1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 21, normalized size of antiderivative = 1.62

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{1}{2}\sqrt{-e^{(2x)}} - \frac{1}{2\sqrt{-e^{(2x)}}}$$

input
```
integrate(1/(-1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(-e^(2*x)) - 1/2/sqrt(-e^(2*x))
```

Mupad [B] (verification not implemented)

Time = 0.09 (sec) , antiderivative size = 14, normalized size of antiderivative = 1.08

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = -\frac{\sinh(2x)\sqrt{-\frac{1}{\cosh(x)^2}}}{2}$$

input
```
int(1/(tanh(x)^2 - 1)^(1/2),x)
```

output
```
-(sinh(2*x)*(-1/cosh(x)^2)^(1/2))/2
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1+\tanh^2(x)}}\,dx = \int \frac{\sqrt{\tanh(x)^2-1}}{\tanh(x)^2-1}dx$$

input
```
int(1/(-1+tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2 - 1)/(tanh(x)**2 - 1),x)
```

3.208 $\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 87

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \sqrt{a+b\tanh^2(x)} + \frac{(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}$$

output
```
(a+b)^(1/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b*tanh(x)^2)^(1/
2)+1/3*(a-b)*(a+b*tanh(x)^2)^(3/2)/b^2-1/5*(a+b*tanh(x)^2)^(5/2)/b^2
```

Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 85, normalized size of antiderivative = 0.98

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$=\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)+\frac{\sqrt{a+b\tanh^2(x)}\,(2a^2-5ab-15b^2-b(a+5b)\tanh^2(x)-3b^2\tanh^4(x))}{15b^2}$$

input
```
Integrate[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] + (Sqrt[a + b*Tanh[
x]^2]*(2*a^2 - 5*a*b - 15*b^2 - b*(a + 5*b)*Tanh[x]^2 - 3*b^2*Tanh[x]^4))/
(15*b^2)
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int -i\tan(ix)^5\sqrt{a-b\tan(ix)^2}dx$$
$$\Big\downarrow \text{ 26}$$
$$-i\int \tan(ix)^5\sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh^5(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^5(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\tanh^4(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(-\frac{\left(b\tanh^2(x)+a\right)^{3/2}}{b}+\frac{(a-b)\sqrt{b\tanh^2(x)+a}}{b}+\frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}\right)d\tanh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b^2}+\frac{2(a-b)\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}-2\sqrt{a+b\tan}\right.$$

input

```
Int[Tanh[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output

```
(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*T
anh[x]^2] + (2*(a - b)*(a + b*Tanh[x]^2)^(3/2))/(3*b^2) - (2*(a + b*Tanh[x
]^2)^(5/2))/(5*b^2))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. $2(71) = 142$.

Time = 0.18 (sec) , antiderivative size = 288, normalized size of antiderivative = 3.31

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\tanh(x)^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{5b}+\frac{2a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{15b^2}-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)}}{2}$
default	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\tanh(x)^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{5b}+\frac{2a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{15b^2}-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)}}{2}$

input
```
int(tanh(x)^5*(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a+b*tanh(x)^2)^(3/2)/b-1/5*tanh(x)^2*(a+b*tanh(x)^2)^(3/2)/b+2/15*a/
b^2*(a+b*tanh(x)^2)^(3/2)-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-
1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+
a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(
tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2*(b*(tanh(x)+1)^2
-2*b*(tanh(x)+1)+a+b)^(1/2)+1/2*b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(t
anh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(t
anh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh
(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1982 vs. $2(71) = 142$.

Time = 0.33 (sec) , antiderivative size = 4529, normalized size of antiderivative = 52.06

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.45

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\begin{cases} \frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10}+\frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^4(x)}{4}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)**5*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
-Piecewise((2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(sqrt(a +
b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(5/2)
/10 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3, Ne(b, 0)), (sqr
t(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**4/4 + tanh(x)**2/2), True))
```

Maxima [F]

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^5\,dx$$

input
```
integrate(tanh(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(71) = 142$.

Time = 0.84 (sec) , antiderivative size = 980, normalized size of antiderivative = 11.26

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) - 4/15*(15*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^9*(2*a + 3*b) +
 15*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^8*(10*a + 9*b)*sqrt(a + b) + 20*(18*a^2 + 23*a*b + b^2)*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b))^7 + 20*(30*a^2 - 7*a*b - 65*b^2)*(sqrt(a + b)*e^(2*x) - sqrt
(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b)
 + 2*(330*a^3 - 705*a^2*b - 1480*a*b^2 + 19*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 10*(18*
a^3 - 279*a^2*b + 68*a*b^2 + 349*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) - 20*(30
*a^4 + 81*a^3*b - 149*a^2*b^2 - 245*a*b^3 + 19*b^4)*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 20*(
42*a^4 - 33*a^3*b - 139*a^2*b^2 + 69*a*b^3 + 325*b^4)*(sqrt(a + b)*e^(2*x)
 - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*s...
```

Mupad [B] (verification not implemented)

Time = 9.84 (sec) , antiderivative size = 119, normalized size of antiderivative = 1.37

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b^2} - 2\,\mathrm{atan}\left(\frac{2\,\sqrt{b\tanh\left(x\right)^2+a}\,\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}} - \sqrt{b\tanh\left(x\right)^2+a}\left(\left(a+b\right)\left(\frac{a+b}{b^2}-\frac{2\,a}{b^2}\right)+\frac{a^2}{b^2}\right) - \left(\frac{a+b}{3\,b^2}-\frac{2\,a}{3\,b^2}\right)\left(b\tanh(x)^2+a\right)^{3/2}$$

input
```
int(tanh(x)^5*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
- (a + b*tanh(x)^2)^(5/2)/(5*b^2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a
/4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2) - (a + b*tanh(x)^2)^(1/2)*((
a + b)*((a + b)/b^2 - (2*a)/b^2) + a^2/b^2) - ((a + b)/(3*b^2) - (2*a)/(3*
b^2))*(a + b*tanh(x)^2)^(3/2)
```

Reduce [F]

$$\int \tanh^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{-3\sqrt{\tanh\left(x\right)^2b+a}\,\tanh\left(x\right)^4b^2-\sqrt{\tanh\left(x\right)^2b+a}\,\tanh\left(x\right)^2ab-5\sqrt{\tanh\left(x\right)^2b+a}\,\tanh\left(x\right)^2b^2+2}{}$$

input
```
int(tanh(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
( - 3*sqrt(tanh(x)**2*b + a)*tanh(x)**4*b**2 - sqrt(tanh(x)**2*b + a)*tanh
(x)**2*a*b - 5*sqrt(tanh(x)**2*b + a)*tanh(x)**2*b**2 + 2*sqrt(tanh(x)**2*
b + a)*a**2 + 10*sqrt(tanh(x)**2*b + a)*a*b + 15*int((sqrt(tanh(x)**2*b +
a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*a*b**2 + 15*int((sqrt(tanh(x)**2*b +
a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*b**3)/(15*b**2)
```

3.209 $\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{(a^2-4ab-8b^2)\,\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8b^{3/2}} + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{8b} - \frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

output
```
1/8*(a^2-4*a*b-8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/
2)+(a+b)^(1/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/8*(a+4
*b)*tanh(x)*(a+b*tanh(x)^2)^(1/2)/b-1/4*tanh(x)^3*(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 4.56 (sec) , antiderivative size = 242, normalized size of antiderivative = 2.00

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= \frac{\left(4\sqrt{2}a(a+4b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right)-32\sqrt{2}a\right.}{}$$

input
```
Integrate[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((4*Sqrt[2]*a*(a + 4*b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*El
lipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1
] - 32*Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ellipti
cPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt
[2]], 1] - (3*a^2 + 9*a*b + 2*b^2 + 4*(a^2 + 4*a*b - 2*b^2)*Cosh[2*x] + (a
^2 + 7*a*b + 6*b^2)*Cosh[4*x])*Sech[x]^4)*Tanh[x])/(32*Sqrt[2]*b*Sqrt[(a -
 b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 133, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 4153, 380, 444, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \tan(ix)^4\sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tanh^4(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{380}$$

$$\frac{1}{4}\int \frac{\tanh^2(x)\left((a+4b)\tanh^2(x)+3a\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{4}\left(\frac{\int \frac{a(a+4b)-\left(a^2-4ba-8b^2\right)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}\right)-\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{4}\left(\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)+8b(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{(a+4b)\tanh(x)}{}\right.$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{4}\left(\frac{\left(a^2-4ab-8b^2\right)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}+8b(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{(a+4b)\tan}{}\right.$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{4}\left(\frac{8b(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)+\frac{\left(a^2-4ab-8b^2\right)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b}-\frac{(a+4b)\tanh(x)\sqrt{a+}}{2b}\right.$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \; 291$$

$$\frac{1}{4}\left(\frac{8b(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}+\frac{(a^2-4ab-8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}\right)-$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{4}\left(\frac{\frac{(a^2-4ab-8b^2)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}+8b\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b}-\frac{(a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}\right)-$$

$$\frac{1}{4}\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Tanh[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-1/4*(Tanh[x]^3*Sqrt[a + b*Tanh[x]^2]) + ((((a^2 - 4*a*b - 8*b^2)*ArcTanh[
(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] + 8*b*Sqrt[a + b]*ArcTan
h[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(2*b) - ((a + 4*b)*Tanh[x]
*Sqrt[a + b*Tanh[x]^2])/(2*b))/4
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(99) = 198.

Time = 0.07 (sec) , antiderivative size = 337, normalized size of antiderivative = 2.79

method	result
derivativedivides	$-\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4b}+\frac{a\tanh(x)\sqrt{a+}}{8b}$
default	$-\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4b}+\frac{a\tanh(x)\sqrt{a+}}{8b}$

input
```
int(tanh(x)^4*(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(x)*(a+b*tanh(x)^2)^(1/2)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*t
anh(x)^2)^(1/2))-1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)/b+1/8*a/b*tanh(x)*(a+b*
tanh(x)^2)^(1/2)+1/8*a^2/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))
-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x
)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/
2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)
-1)+a+b)^(1/2))/(tanh(x)-1))+1/2*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/
2)-1/2*b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1)^2-2*b*(tanh(x)+
1)+a+b)^(1/2))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(
b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1960 vs. 2(99) = 198.

Time = 0.42 (sec) , antiderivative size = 9220, normalized size of antiderivative = 76.20

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to display
```

Sympy [F]

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\tanh^4{(x)}\,dx$$

input
```
integrate(tanh(x)**4*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**4, x)
```

Maxima [F]

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\tanh{(x)}^4\,dx$$

input
```
integrate(tanh(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 938 vs. $2(99) = 198$.

Time = 0.81 (sec) , antiderivative size = 938, normalized size of antiderivative = 7.75

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) -
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 1/4*(a^2 - 4*a*b - 8*b^2)*arctan(-1/2*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) - 1/2*((a^2 + 12*a*b +
16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))^7 + (7*a^2 + 52*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a
 + b) + (21*a^3 + 109*a^2*b + 28*a*b^2 - 48*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (35*a^3
 + 115*a^2*b - 156*a*b^2 - 176*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (35*a^4
+ 130*a^3*b - 317*a^2*b^2 - 156*a*b^3 + 304*b^4)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 + (21*a^4
 + 94*a^3*b - 379*a^2*b^2 + 476*a*b^3 + 48*b^4)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b
) + (7*a^5 + 53*a^4*b - 135*a^3*b^2 + 271*a^2*b^3 - 140*a*b^4 - 272*b^5...
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx=\int \tanh(x)^4\,\sqrt{b\tanh\left(x\right)^2+a}\,dx$$

input
```
int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \tanh^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2 b+a}\,\tanh(x)^4\,dx$$

input
```
int(tanh(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*tanh(x)**4,x)
```

3.210 $\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 63

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx=\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b}$$

output
```
(a+b)^(1/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b*tanh(x)^2)^(1/
2)-1/3*(a+b*tanh(x)^2)^(3/2)/b
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 60, normalized size of antiderivative = 0.95

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx=\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{\sqrt{a+b\tanh^2(x)}\left(a+3b+b\tanh^2(x)\right)}{3b}$$

input
```
Integrate[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tanh[
x]^2]*(a + 3*b + b*Tanh[x]^2))/(3*b)
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.08, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 26, 4153, 26, 354, 90, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ix)^3\sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \tan(ix)^3\sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$i\int -\frac{i\tanh^3(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^3(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\tanh^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\int \frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}\right)$$

$\Big\downarrow$ 60

$$\frac{1}{2}\left((a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\tanh^2(x)}\right)$$

$\Big\downarrow$ 73

$$\frac{1}{2}\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\tanh^2(x)}\right)$$

$\Big\downarrow$ 221

$$\frac{1}{2}\left(2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b}-2\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Tanh[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*T
anh[x]^2] - (2*(a + b*Tanh[x]^2)^(3/2))/(3*b))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(51) = 102$.

Time = 0.07 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.02

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\text{t}}\right.}{2}$
default	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{3b}-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\text{t}}\right.}{2}$

input
```
int(tanh(x)^3*(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*(a+b*tanh(x)^2)^(3/2)/b-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/
2)-1/2*b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-
1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(
b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2*(b*(tanh(x)+1
)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+1/2*b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b
*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b
*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(t
anh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(51) = 102$.

Time = 0.19 (sec) , antiderivative size = 2329, normalized size of antiderivative = 36.97

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 + 3*b*cosh(x)^
4 + 3*(5*b*cosh(x)^2 + b)*sinh(x)^4 + 4*(5*b*cosh(x)^3 + 3*b*cosh(x))*sinh
(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 + 6*b*cosh(x)^2 + b)*sinh(x)^2 +
6*(b*cosh(x)^5 + 2*b*cosh(x)^3 + b*cosh(x))*sinh(x) + b)*sqrt(a + b)*log((
(a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b
)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a
^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^
2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*
(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*
b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*
b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 +
3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3
 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3
 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*c
osh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(
5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sin
h(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh
(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh
(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a +
 b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2...
```

Sympy [A] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 94, normalized size of antiderivative = 1.49

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\begin{cases} \frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b\neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)**3*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
-Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a +
b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/2)
/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**2/2), True
))
```

Maxima [F]

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)^3\,dx$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 630 vs. $2(51) = 102$.

Time = 0.58 (sec) , antiderivative size = 630, normalized size of antiderivative = 10.00

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) - 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(a + 2*b) + 3*(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^4*(3*a + 2*b)*sqrt(a + b) + 2*(3*a^2 - 3*a*b - 10*b^2)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))^3 - 6*(a^2 + 3*a*b + 6*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) - 3*(3*a^3
 + 4*a^2*b - 9*a*b^2 - 26*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (3*a^3 - 17*a*b^2 + 34*b^3)
*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B] (verification not implemented)

Time = 4.20 (sec) , antiderivative size = 66, normalized size of antiderivative = 1.05

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b\tanh(x)^2+a} - \frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3\,b} - 2\,\mathrm{atan}\left(\frac{2\,\sqrt{b\tanh(x)^2+a}\,\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

input
```
int(tanh(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
- (a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/(3*b) - 2*atan((2*(a +
 b*tanh(x)^2)^(1/2)*(- a/4 - b/4)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

Reduce [F]

$$\int \tanh^3(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= \frac{-\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2 b+2\sqrt{\tanh(x)^2 b+a}\,a+3\left(\int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^3}{\tanh(x)^2 b+a}dx\right)ab+3\left(\int \frac{\sqrt{\tanh(x)^2}}{\tanh(}\right.}{3b}$$

input
```
int(tanh(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
( - sqrt(tanh(x)**2*b + a)*tanh(x)**2*b + 2*sqrt(tanh(x)**2*b + a)*a + 3*i
nt((sqrt(tanh(x)**2*b + a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*a*b + 3*int((
sqrt(tanh(x)**2*b + a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*b**2)/(3*b)
```

3.211 $\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(a+2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2\sqrt{b}} + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-1/2*(a+2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+(a+b)^
(1/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/2*tanh(x)*(a+b*
tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.30 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.27

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$=\frac{\left(\sqrt{2}a\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right)-2\sqrt{2}a\sqrt{\frac{(a-b+(a+}{}}\right.}{2\sqrt{2}\sqrt{(a-}}$$

input
```
Integrate[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((Sqrt[2]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcS
in[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - 2*Sqrt[2
]*a*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b),
ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] - (a -
 b + (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*Sqrt[(a - b + (a +
b)*Cosh[2*x])*Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 25, 4153, 25, 380, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int -\tan(ix)^2\sqrt{a-b\tan(ix)^2}dx$$
$$\Big\downarrow \text{25}$$

$$-\int \tan(ix)^2\sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int -\frac{\tanh^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{380}$$

$$\frac{1}{2}\int \frac{(a+2b)\tanh^2(x)+a}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-(a+2b)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-(a+2b)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(2(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-$$
$$\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 219}$$

$$\frac{1}{2}\left(2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{(a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-$$
$$\frac{1}{2}\tanh(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Tanh[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(-(((a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b]) +
 2*Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 - (
Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 380
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(b*
(m + 2*(p + q) + 1))), x] - Simp[e^2/(b*(m + 2*(p + q) + 1))   Int[(e*x)^(m
 - 2)*(a + b*x^2)^p*(c + d*x^2)^(q - 1)*Simp[a*c*(m - 1) + (a*d*(m - 1) - 2
*q*(b*c - a*d))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
- a*d, 0] && GtQ[q, 0] && GtQ[m, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p,
 q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 275 vs. $2(67) = 134$.

Time = 0.06 (sec) , antiderivative size = 276, normalized size of antiderivative = 3.25

method	result
derivativedivides	$-\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-$
default	$-\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2}-\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-$

input
```
int(tanh(x)^2*(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*tanh(x)*(a+b*tanh(x)^2)^(1/2)-1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*t
anh(x)^2)^(1/2))-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/
2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2
))+1/2*(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2*(b*(tanh(x)+1)^2-2*b*(tan
h(x)+1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1)
^2-2*b*(tanh(x)+1)+a+b)^(1/2))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)
+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 826 vs. $2(67) = 134$.

Time = 0.24 (sec) , antiderivative size = 4685, normalized size of antiderivative = 55.12

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\tanh^2{(x)}\,dx$$

input
```
integrate(tanh(x)**2*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*tanh(x)**2, x)
```

Maxima [F]

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\tanh{(x)}^2\,dx$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 554 vs. $2(67) = 134$.

Time = 0.50 (sec) , antiderivative size = 554, normalized size of antiderivative = 6.52

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-(a + 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/
2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2
*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(ab
s(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b) - sqrt(a + b))) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a +
b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b))^2*(3*a - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a
^2 - a*b + 2*b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a +
 b) + a - 3*b)^2
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \tanh(x)^2\,\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \tanh^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2\,dx$$

input
```
int(tanh(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*tanh(x)**2,x)
```

3.212 $\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 44

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx=\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}$$

output
```
(a+b)^(1/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b*tanh(x)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx=\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\sqrt{a+b\tanh^2(x)}$$

input
```
Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Sqrt[a + b*Tanh[x
]^2]
```

Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.533, Rules used = {3042, 26, 4153, 26, 353, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)\sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ix)\sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-2\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Tanh[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*T
anh[x]^2])/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(36) = 72$.

Time = 0.05 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.41

method	result
derivativedivides	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2}+$
default	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2}+$

input
```
int(tanh(x)*(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x
)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/
2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)
-1)+a+b)^(1/2))/(tanh(x)-1))-1/2*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/
2)+1/2*b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1)^2-2*b*(tanh(x)+
1)+a+b)^(1/2))+1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(
b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 489 vs. $2(36) = 72$.

Time = 0.13 (sec) , antiderivative size = 1543, normalized size of antiderivative = 35.07

$$\int \tanh(x)\sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)*sqrt(a + b)*log(((a^
3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*s
inh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*
b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b
)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^
3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*
cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*
cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a
^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 +
a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 +
3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh
(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a
^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x
)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)
^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)
^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)
*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(
x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*b
)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3*a^2*b
 - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)...
```

Sympy [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 68, normalized size of antiderivative = 1.55

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = -\begin{cases} \frac{2\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b} & \text{for } b\neq 0 \\ \frac{\sqrt{a}\log\left(2\tanh^2(x)-2\right)}{2} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
-Piecewise((2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*tanh
(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)))/b, Ne(b, 0)), (sqrt(a)*log(2*tanh(
x)**2 - 2)/2, True))
```

Maxima [F]

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\tanh(x)\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*tanh(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(36) = 72$.

Time = 0.35 (sec) , antiderivative size = 349, normalized size of antiderivative = 7.93

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-\right.\right.$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)b\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}\right.}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) - 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*b - sqrt(a + b)*b)/((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

Mupad [B] (verification not implemented)

Time = 2.76 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b\tanh(x)^2+a} - 2\,\mathrm{atan}\left(\frac{2\sqrt{b\tanh(x)^2+a}\sqrt{-\frac{a}{4}-\frac{b}{4}}}{a+b}\right)\sqrt{-\frac{a}{4}-\frac{b}{4}}$$

input
```
int(tanh(x)*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
- (a + b*tanh(x)^2)^(1/2) - 2*atan((2*(a + b*tanh(x)^2)^(1/2)*(- a/4 - b/4
)^(1/2))/(a + b))*(- a/4 - b/4)^(1/2)
```

Reduce [F]

$$\int \tanh(x)\sqrt{a+b\tanh^2(x)}\,dx = \frac{\sqrt{\tanh(x)^2 b+a}\,a+\left(\int \frac{\sqrt{\tanh(x)^2b+a}\,\tanh(x)^3}{\tanh(x)^2b+a}dx\right)ab+\left(\int \frac{\sqrt{\tanh(x)^2b+a}\,\tanh(x)^3}{\tanh(x)^2b+a}dx\right)b^2}{b}$$

input
```
int(tanh(x)*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
(sqrt(tanh(x)**2*b + a)*a + int((sqrt(tanh(x)**2*b + a)*tanh(x)**3)/(tanh(
x)**2*b + a),x)*a*b + int((sqrt(tanh(x)**2*b + a)*tanh(x)**3)/(tanh(x)**2*
b + a),x)*b**2)/b
```

3.213 $\int \sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 60

$$\int \sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

output
```
-b^(1/2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))+(a+b)^(1/2)*arctan
h((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.35

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \sqrt{-a-b}\arctan\left(\frac{\sqrt{b}\operatorname{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right) + \sqrt{b}\log\left(-\sqrt{b}\tanh(x)+\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Integrate[Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[-a - b]*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^2])/Sq
rt[-a - b]] + Sqrt[b]*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a + b*Tanh[x]^2]]
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4144, 301, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{a+b\tanh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{a-b\tan(ix)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{301}$$

$$(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \text{224}$$

$$(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}$$

$$\Big\downarrow \text{219}$$

$$(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \text{291}$$

$$(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \text{219}$$

$$\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + Sqrt[a + b]*
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 237 vs. $2(48) = 96$.

Time = 0.05 (sec) , antiderivative size = 238, normalized size of antiderivative = 3.97

method	result
derivativedivides	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2}+$
default	$-\frac{\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{2}-\frac{\sqrt{b}\,\ln\left(\frac{b(\tanh(x)-1)+b}{\sqrt{b}}+\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}\right)}{2}+$

input
```
int((a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)-1/2*b^(1/2)*ln((b*(tanh(x
)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))+1/2*(a+b)^(1/
2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)
-1)+a+b)^(1/2))/(tanh(x)-1))+1/2*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/
2)-1/2*b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1)^2-2*b*(tanh(x)+
1)+a+b)^(1/2))-1/2*(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(
b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 481 vs. $2(48) = 96$.

Time = 0.19 (sec) , antiderivative size = 3303, normalized size of antiderivative = 55.05

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\,dx$$

input
```
integrate((a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. $2(48) = 96$.

Time = 0.32 (sec) , antiderivative size = 253, normalized size of antiderivative = 4.22

$$\int \sqrt{a+b\tanh^2(x)}\,dx = -\frac{2\,b\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}}{2\,\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-\right.\right. - \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right) + \frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)$$

input
```
integrate((a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*b*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*sqrt
(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e
^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) - 1/2*sqrt(
a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log(abs(-sqr
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) - sqrt(a + b)))
```

Mupad [F(-1)]

Timed out.

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\,{\tanh\left(x\right)}^2+a}\,dx$$

input
```
int((a + b*tanh(x)^2)^(1/2),x)
```

output
```
int((a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{\tanh{(x)}^2 b + a}dx$$

input
```
int((a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a),x)
```

3.214 $\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

output
```
-a^(1/2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))+(a+b)^(1/2)*arctanh((a+b*t
anh(x)^2)^(1/2)/(a+b)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = -\sqrt{a}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right) + \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)$$

input
```
Integrate[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a]]) + Sqrt[a + b]*ArcTanh[Sq
rt[a + b*Tanh[x]^2/Sqrt[a + b]]
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4153, 26, 354, 94, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\Big\downarrow \ 3042$$

$$\int \frac{i\sqrt{a-b\tan(ix)^2}}{\tan(ix)}dx$$

$$\Big\downarrow \ 26$$

$$i\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)}dx$$

$$\Big\downarrow \ 4153$$

$$i\int -\frac{i\coth(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 26$$

$$\int \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 354$$

$$\frac{1}{2}\int \frac{\coth(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{ 94}$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+a\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2a\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)\right)$$

input
```
Int[Coth[x]*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + 2*Sqrt[a + b]*ArcTanh
[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 94
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[(b*e - a*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(a + b*x), x],
x] - Simp[(d*e - c*f)/(b*c - a*d)   Int[(e + f*x)^(p - 1)/(c + d*x), x], x]
 /; FreeQ[{a, b, c, d, e, f}, x] && LtQ[0, p, 1]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \coth(x)\sqrt{a+b\tanh(x)^2}dx$$

input
```
int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 473 vs. $2(44) = 88$.

Time = 0.19 (sec) , antiderivative size = 3313, normalized size of antiderivative = 59.16

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\coth{(x)}\,dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x), x)
```

Maxima [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\coth{(x)}\,dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(44) = 88$.

Time = 0.30 (sec) , antiderivative size = 255, normalized size of antiderivative = 4.55

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= \frac{2\,a\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}}{2\,\sqrt{-a}}\right)}{\sqrt{-a}}$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-\right.\right.$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*a*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*sqrt(
a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*sqrt(a
 + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs(-sqrt
(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) - sqrt(a + b)))
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)\,\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(coth(x)*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)*(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2 b + a}\,\coth(x)\,dx$$

input
```
int(coth(x)*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*coth(x),x)
```

3.215 $\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 48

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \coth(x)\sqrt{a+b\tanh^2(x)}$$

output
```
(a+b)^(1/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-coth(x)*(a+
b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.10 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.88

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\coth(x)\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{(a+b)\tanh^2(x)}{a+b\tanh^2(x)}\right)\sqrt{a+b\tanh^2(x)}$$

input
```
Integrate[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(Coth[x]*Hypergeometric2F1[-1/2, 1, 1/2, ((a + b)*Tanh[x]^2)/(a + b*Tanh[
x]^2)]*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 4153, 25, 377, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int -\frac{\coth^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{377}$$

$$\int \frac{a+b}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{27}$$

$$(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{291}$$

$$(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{219}$$

$$\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Coth[x]^2*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - Coth[x]
*Sqrt[a + b*Tanh[x]^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \coth(x)^2 \sqrt{a + b\tanh(x)^2}dx$$

input
```
int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(40) = 80$.

Time = 0.14 (sec) , antiderivative size = 1469, normalized size of antiderivative = 30.60

$$\int \coth^2(x)\sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 - 1)*sqrt(a + b)*log(-((a
*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*
sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b
^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^
3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a
*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)
*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)
*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*
a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2
 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 +
 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cos
h(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*
b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(
x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x
)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x
)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b
)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh
(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b^
3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*a*b^2
- 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x...
```

Sympy [F]

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2(x)}\coth^2(x)\,dx$$

input
```
integrate(coth(x)**2*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\coth{(x)}^2\,dx$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 348 vs. $2(40) = 80$.

Time = 0.36 (sec) , antiderivative size = 348, normalized size of antiderivative = 7.25

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{1}{2}\sqrt{a+b}\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-\right.\right.$$
$$-\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\sqrt{a+b}\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$+\frac{4\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)a\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}\right.}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) -
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 4*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a + sqrt(a + b)*a)/((
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx=\int \coth(x)^2\,\sqrt{b\tanh{(x)}^2+a}\,dx$$

input
```
int(coth(x)^2*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2*(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^2(x)\sqrt{a+b\tanh^2(x)}\,dx=\int \sqrt{\tanh{(x)}^2\,b+a}\,\coth{(x)}^2\,dx$$

input
```
int(coth(x)^2*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*coth(x)**2,x)
```

3.216 $\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 83

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-1/2*(2*a+b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+(a+b)^(1/2)*ar
ctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/
2)
```

Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.00

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2\sqrt{a}} + \sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Integrate[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-1/2*((2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + Sqrt[a +
 b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Coth[x]^2*Sqrt[a + b*Tan
h[x]^2])/2
```

Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.05, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.588, Rules used = {3042, 26, 4153, 26, 354, 110, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\sqrt{a-b\tan(ix)^2}}{\tan(ix)^3}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^3}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\coth^3(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth^3(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\coth^2(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{110}$$

$$\frac{1}{2}\left(\int \frac{\coth(x)\left(b\tanh^2(x)+2a+b\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\coth(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{\coth(x)\left(b\tanh^2(x)+2a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\coth(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(2a+b)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)\right)-\text{cot}\right.$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(\frac{4(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(2a+b)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}\right)-\coth(x)\sqrt{a+b}\right.$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(4\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2(2a+b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)-\coth(x)\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Coth[x]^3*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(((-2*(2*a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a]])/Sqrt[a] + 4*Sqrt[a
 + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a + b]])/2 - Coth[x]*Sqrt[a + b*T
anh[x]^2])/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \coth(x)^3 \sqrt{a + b\tanh(x)^2}\,dx$$

input
```
int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(65) = 130$.

Time = 0.26 (sec) , antiderivative size = 4735, normalized size of antiderivative = 57.05

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\coth^3{(x)}\,dx$$

input
```
integrate(coth(x)**3*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**3, x)
```

Maxima [F]

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\coth{(x)}^3\,dx$$

input
```
integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 557 vs. $2(65) = 130$.

Time = 0.51 (sec) , antiderivative size = 557, normalized size of antiderivative = 6.71

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(2*a + b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2
*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) + 1/2*
sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(
2*x) + a + b) - sqrt(a + b))) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^2*(2*a - 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (2*
a^2 - a*b + b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) -
sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a +
b) - 3*a + b)^2
```

Mupad [F(-1)]

Timed out.

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^3\,\sqrt{b\tanh(x)^2+a}\,dx$$

input
```
int(coth(x)^3*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3*(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^3(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2 b+a}\,\coth(x)^3\,dx$$

input
```
int(coth(x)^3*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*coth(x)**3,x)
```

3.217 $\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 78

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a} - \frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

output

```
(a+b)^(1/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/3*(3*a+b)
*coth(x)*(a+b*tanh(x)^2)^(1/2)/a-1/3*coth(x)^3*(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 4.47 (sec) , antiderivative size = 161, normalized size of antiderivative = 2.06

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= \frac{\cosh^4(x)\coth^3(x)\left(1+\frac{b\tanh^2(x)}{a}\right)\left(-\frac{\operatorname{sech}^4(x)\left(\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}+\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}\right)(a-2b\,t}{\sqrt{\cosh^2(x)+\frac{b\sinh^2(x)}{a}}}\right.}{3\sqrt{a+b\tanh^2(x)}}$$

input

```
Integrate[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

output

```
(Cosh[x]^4*Coth[x]^3*(1 + (b*Tanh[x]^2)/a)*(-((Sech[x]^4*(ArcSin[Sqrt[-(((
a + b)*Sinh[x]^2)/a)]]*Sqrt[-(((a + b)*Sinh[x]^2)/a)] + Sqrt[Cosh[x]^2 + (
b*Sinh[x]^2)/a])*(a - 2*b*Tanh[x]^2))/Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]) -
 (4*(a + b)*Hypergeometric2F1[2, 2, 3/2, -(((a + b)*Sinh[x]^2)/a)]*(a*Tanh
[x] + b*Tanh[x]^3)^2)/a^2))/(3*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.05, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4153, 377, 445, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^4}dx$$

$$\Big\downarrow \; 4153$$

$$\int \frac{\coth^4(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \; 377$$

$$\frac{1}{3}\int \frac{\coth^2(x)\left(2b\tanh^2(x)+3a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \; 445$$

$$\frac{1}{3}\left(-\frac{\int -\frac{3a(a+b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a}-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \; 27$$

$$\frac{1}{3}\left(3(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \; 291$$

$$\frac{1}{3}\left(3(a+b)\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \; 219$$

$$\frac{1}{3}\left(3\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{(3a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{3}\coth^3(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Coth[x]^4*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-1/3*(Coth[x]^3*Sqrt[a + b*Tanh[x]^2]) + (3*Sqrt[a + b]*ArcTanh[(Sqrt[a +
b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - ((3*a + b)*Coth[x]*Sqrt[a + b*Tanh[x]
^2])/a)/3
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 377
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^q/(a*e*(
m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^2)^p*(c
+ d*x^2)^(q - 1)*Simp[b*c*(m + 1) + 2*(b*c*(p + 1) + a*d*q) + d*(b*(m + 1)
+ 2*b*(p + q + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b
*c - a*d, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m
, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \coth(x)^4 \sqrt{a + b\tanh(x)^2}dx$$

input
```
int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(64) = 128$.

Time = 0.19 (sec) , antiderivative size = 2285, normalized size of antiderivative = 29.29

$$\int \coth^4(x)\sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(a*cosh(x)^6 + 6*a*cosh(x)*sinh(x)^5 + a*sinh(x)^6 - 3*a*cosh(x)^
4 + 3*(5*a*cosh(x)^2 - a)*sinh(x)^4 + 4*(5*a*cosh(x)^3 - 3*a*cosh(x))*sinh
(x)^3 + 3*a*cosh(x)^2 + 3*(5*a*cosh(x)^4 - 6*a*cosh(x)^2 + a)*sinh(x)^2 +
6*(a*cosh(x)^5 - 2*a*cosh(x)^3 + a*cosh(x))*sinh(x) - a)*sqrt(a + b)*log(-
((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^
3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2
+ b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2
*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70
*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b
^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b
^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 +
 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*
b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^
3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*
cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*
(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*si
nh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cos
h(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cos
h(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a
+ b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - ...
```

Sympy [F]

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\coth^4{(x)}\,dx$$

input
```
integrate(coth(x)**4*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**4, x)
```

Maxima [F]

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh(x)^2+a}\coth(x)^4\,dx$$

input
```
integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^4, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(64) = 128$.

Time = 0.59 (sec) , antiderivative size = 629, normalized size of antiderivative = 8.06

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) -
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 4/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5*(2*a + b) - 3*(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))^4*(2*a + 3*b)*sqrt(a + b) - 2*(10*a^2 + 3*a*b - 3*b^2)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b))^3 + 6*(6*a^2 + 3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + 3*(26*a^
3 + 9*a^2*b - 4*a*b^2 - 3*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (34*a^3 - 17*a^2*b + 3*b^3)
*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) +
 b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)^3
```

Mupad [F(-1)]

Timed out.

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^4\,\sqrt{b\tanh\left(x\right)^2+a}\,dx$$

input
```
int(coth(x)^4*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^4*(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^4(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{\tanh\left(x\right)^2 b+a}\,\coth\left(x\right)^4 dx$$

input
```
int(coth(x)^4*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*coth(x)**4,x)
```

3.218 $\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 121

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = -\frac{(8a^2+4ab-b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{8a^{3/2}} + \sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{(4a+b)\coth^2(x)\sqrt{a+b\tanh^2(x)}}{8a} - \frac{1}{4}\coth^4(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-1/8*(8*a^2+4*a*b-b^2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+(a+b
)^(1/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-1/8*(4*a+b)*coth(x)^2*(
a+b*tanh(x)^2)^(1/2)/a-1/4*coth(x)^4*(a+b*tanh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.46 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.92

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$= \frac{(-8a^2-4ab+b^2)\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a}\left(8a\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\coth^2(x)\left(4a+b-\right.\right.}{8a^{3/2}}$$

input
```
Integrate[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((-8*a^2 - 4*a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(
8*a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - Coth[x]^2*(4*
a + b + 2*a*Coth[x]^2)*Sqrt[a + b*Tanh[x]^2]))/(8*a^(3/2))
```

Rubi [A] (warning: unable to verify)

Time = 0.38 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.11, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 26, 4153, 26, 354, 110, 27, 168, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx$$
$$\Big\downarrow \text{ 3042}$$
$$\int \frac{i\sqrt{a-b\tan(ix)^2}}{\tan(ix)^5}dx$$
$$\Big\downarrow \text{ 26}$$
$$i\int \frac{\sqrt{a-b\tan(ix)^2}}{\tan(ix)^5}dx$$
$$\Big\downarrow \text{ 4153}$$

$$i\int -\frac{i\coth^5(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \; 26$$

$$\int \frac{\coth^5(x)\sqrt{a+b\tanh^2(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{\coth^3(x)\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \; 110$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{\coth^2(x)\left(3b\tanh^2(x)+4a+b\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{1}{4}\int \frac{\coth^2(x)\left(3b\tanh^2(x)+4a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\frac{1}{2}\coth^2(x)\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \; 168$$

$$\frac{1}{2}\left(\frac{1}{4}\left(-\frac{\int -\frac{\coth(x)\left(8a^2+4ba-b^2+b(4a+b)\tanh^2(x)\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{2}\coth^2(x)\sqrt{a}\right.$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\int \frac{\coth(x)\left(8a^2+4ba-b^2+b(4a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)-\frac{1}{2}\coth^2(x)\sqrt{a+}\right.$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\left(8a^2+4ab-b^2\right)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+8a(a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{(4a+b)\,c}{}\right.\right.$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{\frac{2(8a^2+4ab-b^2)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{16a(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{2a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right.\right.$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{1}{4}\left(\frac{16a\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{2(8a^2+4ab-b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{(4a+b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right.\right.$$

input
```
Int[Coth[x]^5*Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(-1/2*(Coth[x]^2*Sqrt[a + b*Tanh[x]^2]) + (((-2*(8*a^2 + 4*a*b - b^2)*ArcT
anh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + 16*a*Sqrt[a + b]*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a + b]])/(2*a) - ((4*a + b)*Coth[x]*Sqrt[a + b*Tan
h[x]^2])/a)/4)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 110

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[(a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)/((m +
1)*(b*e - a*f))), x] - Simp[1/((m + 1)*(b*e - a*f))   Int[(a + b*x)^(m + 1)
*(c + d*x)^(n - 1)*(e + f*x)^p*Simp[d*e*n + c*f*(m + p + 2) + d*f*(m + n +
p + 2)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && LtQ[m, -1] && Gt
Q[n, 0] && (IntegersQ[2*m, 2*n, 2*p] || IntegersQ[m, n + p] || IntegersQ[p,
 m + n])
```

rule 168

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && ILtQ[m, -1]
```

rule 174

```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \coth(x)^5 \sqrt{a + b\tanh(x)^2}dx$$

input
```
int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. $2(99) = 198$.

Time = 0.40 (sec) , antiderivative size = 9488, normalized size of antiderivative = 78.41

$$\int \coth^5(x)\sqrt{a + b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{a+b\tanh^2{(x)}}\coth^5{(x)}\,dx$$

input
```
integrate(coth(x)**5*(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**2)*coth(x)**5, x)
```

Maxima [F]

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \sqrt{b\tanh{(x)}^2+a}\coth{(x)}^5\,dx$$

input
```
integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^2 + a)*coth(x)^5, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 947 vs. $2(99) = 198$.

Time = 0.79 (sec) , antiderivative size = 947, normalized size of antiderivative = 7.83

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(a + b)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b))) +
1/2*sqrt(a + b)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*sqrt(a + b)*log
(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b
*e^(2*x) + a + b) - sqrt(a + b))) + 1/4*(8*a^2 + 4*a*b - b^2)*arctan(-1/2*
(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) + 1/2*((16*a^2 + 12*a*b
 + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b))^7 - (16*a^2 + 52*a*b + 7*b^2)*(sqrt(a + b)*e^(2*x) -
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a
 + b) - (48*a^3 - 28*a^2*b - 109*a*b^2 - 21*b^3)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + (176*a^
3 + 156*a^2*b - 115*a*b^2 - 35*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + (304*a^4
 - 156*a^3*b - 317*a^2*b^2 + 130*a*b^3 + 35*b^4)*(sqrt(a + b)*e^(2*x) - sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - (48*a^4
 + 476*a^3*b - 379*a^2*b^2 + 94*a*b^3 + 21*b^4)*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b
) - (272*a^5 + 140*a^4*b - 271*a^3*b^2 + 135*a^2*b^3 - 53*a*b^4 - 7*b^5...
```

Mupad [F(-1)]

Timed out.

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth(x)^5\,\sqrt{b\tanh{(x)}^2+a}\,dx$$

input
```
int(coth(x)^5*(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^5*(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \coth^5(x)\sqrt{a+b\tanh^2(x)}\,dx = \int \coth\left(x\right)^5 \sqrt{\tanh\left(x\right)^2 b + a}dx$$

input
```
int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^5*(a+b*tanh(x)^2)^(1/2),x)
```

3.219 $\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 82

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - (a+b)\sqrt{a+b\tanh^2(x)} - \frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2} - \frac{\left(a+b\tanh^2(x)\right)^{5/2}}{5b}$$

output
```
(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^
2)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)-1/5*(a+b*tanh(x)^2)^(5/2)/b
```

Mathematica [A] (verified)

Time = 0.30 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.05

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right) - \frac{\sqrt{a+b\tanh^2(x)}\left(3a^2+20ab+15b^2+b(6a+5b)\tanh^2(x)+3b^2\tanh^4(x)\right)}{15b}$$

input
```
Integrate[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (Sqrt[a + b*Tan
h[x]^2]*(3*a^2 + 20*a*b + 15*b^2 + b*(6*a + 5*b)*Tanh[x]^2 + 3*b^2*Tanh[x]
^4))/(15*b)
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 89, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 26, 4153, 26, 354, 90, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int i\tan(ix)^3 \left(a-b\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{26}$$

$$i\int \tan(ix)^3 \left(a-b\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4153}$$

$$i\int -\frac{i\tanh^3(x) \left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\tanh^2(x) \left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow 90$$

$$\frac{1}{2}\left(\int \frac{\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{2}\left((a+b)\int \frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{2}{3}\left(a+b\tanh^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow 60$$

$$\frac{1}{2}\left((a+b)\left((a+b)\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/}}{5b}\right.$$

$$\Big\downarrow 73$$

$$\frac{1}{2}\left((a+b)\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{2}{3}(a+b\right.$$

$$\Big\downarrow 221$$

$$\frac{1}{2}\left((a+b)\left(2\sqrt{a+b}\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2\left(a+b\tanh^2(x)\right)^{5/2}}{5b}-\frac{2}{3}(a+b\tan\right.$$

input
```
Int[Tanh[x]^3*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((-2*(a + b*Tanh[x]^2)^(3/2))/3 - (2*(a + b*Tanh[x]^2)^(5/2))/(5*b) + (a +
 b)*(2*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a +
 b*Tanh[x]^2]))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 487 vs. $2(66) = 132$.

Time = 0.10 (sec) , antiderivative size = 488, normalized size of antiderivative = 5.95

method	result
derivativedivides	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{5}{2}}}{5b}-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh}}{4b}\right.}{}$
default	$-\frac{\left(a+b\tanh(x)^2\right)^{\frac{5}{2}}}{5b}-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh}}{4b}\right.}{}$

input
```
int(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/5*(a+b*tanh(x)^2)^(5/2)/b-1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/
2)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tan
h(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(ta
nh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b
)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))-1/6*(b*
(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)+1/2*b*(1/4*(2*b*(tanh(x)+1)-2*b)/
b*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2
)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)
))-1/2*(a+b)*((b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)-b^(1/2)*ln((b*(t
anh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))-(a+b)^(1
/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x
)+1)+a+b)^(1/2))/(tanh(x)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2188 vs. $2(66) = 132$.

Time = 0.31 (sec) , antiderivative size = 4941, normalized size of antiderivative = 60.26

$$\int \tanh^3(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 9.60 (sec) , antiderivative size = 223, normalized size of antiderivative = 2.72

$$\int \tanh^3(x) \left(a + b\tanh^2(x)\right)^{3/2} dx =$$

$$-a\left(\begin{cases} \frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2} + \frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}} + \frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2} + \frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}\right) - b\left(\begin{cases} \frac{2\left(\frac{b^3\sqrt{a+b\tanh^2(x)}}{2} + \frac{b^3(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}} + \frac{b\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}{10} + \frac{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}\left(-\frac{ab}{2}+\frac{b^2}{2}\right)}{3}\right)}{b^3} & \text{for } b \neq 0 \\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2} + \frac{\tanh^4(x)}{4} + \frac{\tanh^2(x)}{2}\right) & \text{otherwise} \end{cases}\right)$$

input
```
integrate(tanh(x)**3*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
-a*Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b**2*(a + b)*atan(sqrt(a
+ b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2)**(3/
2)/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**2/2), Tr
ue)) - b*Piecewise((2*(b**3*sqrt(a + b*tanh(x)**2)/2 + b**3*(a + b)*atan(s
qrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b*(a + b*tanh(x)**2
)**(5/2)/10 + (a + b*tanh(x)**2)**(3/2)*(-a*b/2 + b**2/2)/3)/b**3, Ne(b, 0
)), (sqrt(a)*(log(tanh(x)**2 - 1)/2 + tanh(x)**4/4 + tanh(x)**2/2), True))
```

Maxima [F]

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} \, dx = \int \left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}} \tanh\left(x\right)^3 \, dx$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^3, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1063 vs. 2(66) = 132.

Time = 1.19 (sec) , antiderivative size = 1063, normalized size of antiderivative = 12.96

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)
*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 4/15*(15*(a^2 +
 4*a*b + 3*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b))^9 + 15*(7*a^2 + 20*a*b + 9*b^2)*(sqrt(a + b)
*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
)^8*sqrt(a + b) + 20*(15*a^3 + 39*a^2*b + 21*a*b^2 + b^3)*(sqrt(a + b)*e^(
2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7
+ 20*(21*a^3 + 21*a^2*b - 57*a*b^2 - 65*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a
*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^6*sqrt(a + b) +
 2*(105*a^4 - 210*a^3*b - 1860*a^2*b^2 - 1590*a*b^3 + 19*b^4)*(sqrt(a + b)
*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
)^5 - 10*(21*a^4 + 126*a^3*b + 288*a^2*b^2 - 390*a*b^3 - 349*b^4)*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b))^4*sqrt(a + b) - 20*(21*a^5 + 63*a^4*b - 18*a^3*b^2 - 378*a^2*b^3 - 2
35*a*b^4 + 19*b^5)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 20*(15*a^5 + 21*a^4*b - 126*a^3*b^...
```

Mupad [B] (verification not implemented)

Time = 10.96 (sec) , antiderivative size = 112, normalized size of antiderivative = 1.37

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = -\frac{\left(b\tanh(x)^2+a\right)^{5/2}}{5\,b} - \left(\frac{a+b}{3\,b}-\frac{a}{3\,b}\right)\left(b\tanh(x)^2+a\right)^{3/2} - (a+b)\left(\frac{a+b}{b}-\frac{a}{b}\right)\sqrt{b\tanh\left(x\right)^2+a} - \mathrm{atan}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh\left(x\right)^2+a}\,1\mathrm{i}}{a^2+2\,a\,b+b^2}\right)(a+b)^{3/2}\,1\mathrm{i}$$

input
```
int(tanh(x)^3*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
- (a + b*tanh(x)^2)^(5/2)/(5*b) - ((a + b)/(3*b) - a/(3*b))*(a + b*tanh(x)
^2)^(3/2) - atan(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2)*1i)/(2*a*b + a^2 +
 b^2))*(a + b)^(3/2)*1i - (a + b)*((a + b)/b - a/b)*(a + b*tanh(x)^2)^(1/2
)
```

Reduce [F]

$$\int \tanh^3(x) \left(a+b\tanh^2(x)\right)^{3/2}\,dx = \frac{-3\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^4 b^2-6\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2 ab-5\sqrt{\tanh(x)^2 b}$$

input
```
int(tanh(x)^3*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
( - 3*sqrt(tanh(x)**2*b + a)*tanh(x)**4*b**2 - 6*sqrt(tanh(x)**2*b + a)*ta
nh(x)**2*a*b - 5*sqrt(tanh(x)**2*b + a)*tanh(x)**2*b**2 + 12*sqrt(tanh(x)*
*2*b + a)*a**2 + 10*sqrt(tanh(x)**2*b + a)*a*b + 15*int((sqrt(tanh(x)**2*b
 + a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*a**2*b + 30*int((sqrt(tanh(x)**2*b
 + a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*a*b**2 + 15*int((sqrt(tanh(x)**2*b
 + a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*b**3)/(15*b)
```

3.220 $\int \tanh^2(x) \left(a + b\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 17, antiderivative size = 123

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = -\frac{(3a^2+12ab+8b^2)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{8\sqrt{b}} + (a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - \frac{1}{8}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)} - \frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-1/8*(3*a^2+12*a*b+8*b^2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b
^(1/2)+(a+b)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/8*
(5*a+4*b)*tanh(x)*(a+b*tanh(x)^2)^(1/2)-1/4*b*tanh(x)^3*(a+b*tanh(x)^2)^(1
/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 4.99 (sec) , antiderivative size = 247, normalized size of antiderivative = 2.01

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \frac{\left(4\sqrt{2}a(5a+4b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csc}}{b}}}{\sqrt{2}}\right.\right.\right.}{}$$

input
```
Integrate[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((4*Sqrt[2]*a*(5*a + 4*b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*
EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]],
 1] - 32*Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]
*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)
/b]/Sqrt[2]], 1] - (15*a^2 + 5*a*b + 2*b^2 + 4*(5*a^2 + 4*a*b - 2*b^2)*Cos
h[2*x] + (5*a^2 + 11*a*b + 6*b^2)*Cosh[4*x])*Sech[x]^4)*Tanh[x])/(32*Sqrt[
2]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.07, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.765$, Rules used = {3042, 25, 4153, 25, 379, 25, 444, 27, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \ 3042$$

$$\int -\tan(ix)^2 \left(a-b\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{25}$$

$$-\int \tan(ix)^2 \left(a-b\tan(ix)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int -\frac{\tanh^2(x)\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{379}$$

$$-\frac{1}{4}\int -\frac{\tanh^2(x)\left(b(5a+4b)\tanh^2(x)+a(4a+3b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{4}\int \frac{\tanh^2(x)\left(b(5a+4b)\tanh^2(x)+a(4a+3b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{444}$$

$$\frac{1}{4}\left(\frac{\int \frac{b\left(\left(3a^2+12ba+8b^2\right)\tanh^2(x)+a(5a+4b)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{1}{2}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}\right)-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{4}\left(\frac{1}{2}\int \frac{\left(3a^2+12ba+8b^2\right)\tanh^2(x)+a(5a+4b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{2}(5a+4b)\tanh(x)\sqrt{a+b\tanh^2(x)}\right)-\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\left(3a^2+12ab+8b^2\right)\int\frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(\right.\right.$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 224

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\left(3a^2+12ab+8b^2\right)\int\frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tan}{\sqrt{b\tanh}}\right.\right.$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 219

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)\right.$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 291

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^2\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a\right.$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 219

$$\frac{1}{4}\left(\frac{1}{2}\left(8(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{\left(3a^2+12ab+8b^2\right)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}\right)-\frac{1}{2}(5a+4b)\,\mathrm{t}\right.$$

$$\frac{1}{4}b\tanh^3(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Tanh[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-1/4*(b*Tanh[x]^3*Sqrt[a + b*Tanh[x]^2]) + ((-(((3*a^2 + 12*a*b + 8*b^2)*A
rcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b]) + 8*(a + b)^(3/2
)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 - ((5*a + 4*b)*T
anh[x]*Sqrt[a + b*Tanh[x]^2])/2)/4
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 379
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[d*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q -
1)/(b*e*(m + 2*(p + q) + 1))), x] + Simp[1/(b*(m + 2*(p + q) + 1))   Int[(e
*x)^m*(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*((b*c - a*d)*(m + 1) + b*c*2
*(p + q)) + (d*(b*c - a*d)*(m + 1) + d*2*(q - 1)*(b*c - a*d) + b*c*d*2*(p +
 q))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a*d, 0
] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 444
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q
_.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[f*g*(g*x)^(m - 1)*(a + b*x^2)^
(p + 1)*((c + d*x^2)^(q + 1)/(b*d*(m + 2*(p + q + 1) + 1))), x] - Simp[g^2/
(b*d*(m + 2*(p + q + 1) + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^p*(c + d*x^2)
^q*Simp[a*f*c*(m - 1) + (a*f*d*(m + 2*q + 1) + b*(f*c*(m + 2*p + 1) - e*d*(
m + 2*(p + q + 1) + 1)))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p,
q}, x] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 528 vs. $2(101) = 202$.

Time = 0.05 (sec) , antiderivative size = 529, normalized size of antiderivative = 4.30

method	result
derivativedivides	$-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4}-\frac{3a\left(\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}\right)}{4}-\frac{\left(b(\tanh(x)-1)^2\right.}{}$
default	$-\frac{\tanh(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}{4}-\frac{3a\left(\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2\sqrt{b}}\right)}{4}-\frac{\left(b(\tanh(x)-1)^2\right.}{}$

input
```
int(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*tanh(x)*(a+b*tanh(x)^2)^(3/2)-3/4*a*(1/2*tanh(x)*(a+b*tanh(x)^2)^(1/2
)+1/2*a/b^(1/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2)))-1/6*(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-1)+2*b)/b*(b*(ta
nh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*
(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)))-1/2*(
a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*ln((b*(tanh(x)-1
)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-(a+b)^(1/2)*ln((
2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(1/2))/(tanh(x)-1)))+1/6*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)-1/2
*b*(1/4*(2*b*(tanh(x)+1)-2*b)/b*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2
)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1
)^2-2*b*(tanh(x)+1)+a+b)^(1/2)))+1/2*(a+b)*((b*(tanh(x)+1)^2-2*b*(tanh(x)+
1)+a+b)^(1/2)-b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tanh(x)+1)^2-2*b*(t
anh(x)+1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2
)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2167 vs. $2(101) = 202$.

Time = 0.43 (sec) , antiderivative size = 10046, normalized size of antiderivative = 81.67

$$\int \tanh^2(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \left(a+b\tanh^2{(x)}\right)^{\frac{3}{2}} \tanh^2{(x)}\, dx$$

input
```
integrate(tanh(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(3/2)*tanh(x)**2, x)
```

Maxima [F]

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}} \tanh\left(x\right)^2 \, dx$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. 2(101) = 202.

Time = 0.98 (sec) , antiderivative size = 949, normalized size of antiderivative = 7.72

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2
)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/4*(3*a^2 + 12*a*b + 8*b^2)*arct
an(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b
+ b^2)*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b)
- 1/2*((5*a^2 + 20*a*b + 16*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b
*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^7 + (35*a^2 + 76*a*b + 16*b
^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^6*sqrt(a + b) + (105*a^3 + 153*a^2*b - 28*a*b^2 - 48*b^3
)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))^5 + (175*a^3 - 25*a^2*b - 260*a*b^2 - 176*b^3)*(sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^4*sqrt(a + b) + (175*a^4 - 110*a^3*b - 417*a^2*b^2 + 60*a*b^3 + 304*b^4
)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))^3 + (105*a^4 - 210*a^3*b - 55*a^2*b^2 + 484*a*b^3 + 48*b^4
)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))^2*sqrt(a + b) + (35*a^5 - 79*a^4*b + 53*a^3*b^2 + 195*a...
```

Mupad [F(-1)]

Timed out.

$$\int \tanh^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=\int \tanh(x)^2\left(b\tanh(x)^2+a\right)^{3/2}\,dx$$

input
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \tanh^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \left(\int \sqrt{\tanh(x)^2 b+a}\,\tanh(x)^4 dx\right) b + \left(\int \sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2 dx\right) a$$

input
```
int(tanh(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*tanh(x)**4,x)*b + int(sqrt(tanh(x)**2*b + a)*ta
nh(x)**2,x)*a
```

3.221 $\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 63

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\sqrt{a+b\tanh^2(x)}-\frac{1}{3}\left(a+b\tanh^2(x)\right)^{3/2}$$

output
```
(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))-(a+b)*(a+b*tanh(x)^
2)^(1/2)-1/3*(a+b*tanh(x)^2)^(3/2)
```

Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 59, normalized size of antiderivative = 0.94

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=(a+b)^{3/2}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-\frac{1}{3}\sqrt{a+b\tanh^2(x)}\left(4a+3b+b\tanh^2(x)\right)$$

input
```
Integrate[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a + b]] - (Sqrt[a + b*Tan
h[x]^2]*(4*a + 3*b + b*Tanh[x]^2))/3
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.11, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3042, 26, 4153, 26, 353, 60, 60, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix) \left(a-b\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ix) \left(a-b\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x) \left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x) \left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left((a+b)\int \frac{\sqrt{b\tanh^2(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{2}{3}\left(a+b\tanh^2(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{60}$$

$$\frac{1}{2}\left((a+b)\left((a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2}{3}(a+b\tanh^2(x))^{3/2}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left((a+b)\left(\frac{2(a+b)\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2}{3}(a+b\tanh^2(x))^{3/2}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left((a+b)\left(2\sqrt{a+b}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2\sqrt{a+b\tanh^2(x)}\right)-\frac{2}{3}(a+b\tanh^2(x))^{3/2}\right)$$

input
```
Int[Tanh[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((-2*(a + b*Tanh[x]^2)^(3/2))/3 + (a + b)*(2*Sqrt[a + b]*ArcTanh[Sqrt[a +
b*Tanh[x]^2]/Sqrt[a + b]] - 2*Sqrt[a + b*Tanh[x]^2]))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 60
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Simp[n*((b*c - a*d)/(
b*(m + n + 1)))   Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
 c, d}, x] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ[m, 0] && ( !Integer
Q[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n + 2, 0] && IntLinear
Q[a, b, c, d, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(51) = 102$.

Time = 0.04 (sec) , antiderivative size = 473, normalized size of antiderivative = 7.51

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{(4b(a+b)-}{}\right.}{2}$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{(4b(a+b)-}{}\right.}{2}$

input
```
int(tanh(x)*(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-
1)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2
)/b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+
b)^(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*
ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-
(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))-1/6*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1
)+a+b)^(3/2)+1/2*b*(1/4*(2*b*(tanh(x)+1)-2*b)/b*(b*(tanh(x)+1)^2-2*b*(tanh
(x)+1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(tanh(x)+1)-b)/b^(1/
2)+(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)))-1/2*(a+b)*((b*(tanh(x)+1)
^2-2*b*(tanh(x)+1)+a+b)^(1/2)-b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tan
h(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)
+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 910 vs. $2(51) = 102$.

Time = 0.19 (sec) , antiderivative size = 2385, normalized size of antiderivative = 37.86

$$\int \tanh(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/12*(3*((a + b)*cosh(x)^6 + 6*(a + b)*cosh(x)*sinh(x)^5 + (a + b)*sinh(x
)^6 + 3*(a + b)*cosh(x)^4 + 3*(5*(a + b)*cosh(x)^2 + a + b)*sinh(x)^4 + 4*
(5*(a + b)*cosh(x)^3 + 3*(a + b)*cosh(x))*sinh(x)^3 + 3*(a + b)*cosh(x)^2
+ 3*(5*(a + b)*cosh(x)^4 + 6*(a + b)*cosh(x)^2 + a + b)*sinh(x)^2 + 6*((a
+ b)*cosh(x)^5 + 2*(a + b)*cosh(x)^3 + (a + b)*cosh(x))*sinh(x) + a + b)*s
qrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^
7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2
*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3
 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*
cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 +
30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 +
10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*si
nh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x
)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3
 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2
+ sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2
*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*
a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)
^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2
 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)...
```

Sympy [A] (verification not implemented)

Time = 6.40 (sec) , antiderivative size = 165, normalized size of antiderivative = 2.62

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx =$$

$$-a\left(\begin{cases}\frac{2\left(\frac{b\sqrt{a+b\tanh^2(x)}}{2}+\frac{b(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}\right)}{b} & \text{for } b\neq 0\\ \frac{\sqrt{a}\log\left(2\tanh^2(x)-2\right)}{2} & \text{otherwise}\end{cases}\right)$$

$$-b\left(\begin{cases}\frac{2\left(\frac{b^2\sqrt{a+b\tanh^2(x)}}{2}+\frac{b^2(a+b)\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}}+\frac{b\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}{6}\right)}{b^2} & \text{for } b\neq 0\\ \sqrt{a}\left(\frac{\log\left(\tanh^2(x)-1\right)}{2}+\frac{\tanh^2(x)}{2}\right) & \text{otherwise}\end{cases}\right)$$

input
```
integrate(tanh(x)*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
-a*Piecewise((2*(b*sqrt(a + b*tanh(x)**2)/2 + b*(a + b)*atan(sqrt(a + b*ta
nh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)))/b, Ne(b, 0)), (sqrt(a)*log(2*tan
h(x)**2 - 2)/2, True)) - b*Piecewise((2*(b**2*sqrt(a + b*tanh(x)**2)/2 + b
**2*(a + b)*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)) + b
*(a + b*tanh(x)**2)**(3/2)/6)/b**2, Ne(b, 0)), (sqrt(a)*(log(tanh(x)**2 -
1)/2 + tanh(x)**2/2), True))
```

Maxima [F]

$$\int \tanh(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \int \left(b\tanh\left(x\right)^2 + a\right)^{\frac{3}{2}} \tanh\left(x\right) dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*tanh(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 662 vs. 2(51) = 102.

Time = 0.75 (sec) , antiderivative size = 662, normalized size of antiderivative = 10.51

$$\int \tanh(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)
*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) -
 2*b*e^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(
sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 8/3*(3*(a*b + b
^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^5 + 3*(3*a*b + b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^4*sqrt(a + b) + 2*(3*a
^2*b - 6*a*b^2 - 5*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3 - 6*(a^2*b + 4*a*b^2 + 3*b^3)*(sqr
t(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b))^2*sqrt(a + b) - 3*(3*a^3*b + a^2*b^2 - 15*a*b^3 - 13*b^4)*(sqrt(
a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
a + b)) - (3*a^3*b - 9*a^2*b^2 + 5*a*b^3 + 17*b^4)*sqrt(a + b))/((sqrt(a +
 b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
 b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
 - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^3
```

Mupad [B] (verification not implemented)

Time = 4.59 (sec) , antiderivative size = 64, normalized size of antiderivative = 1.02

$$\int \tanh(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \operatorname{atanh}\left(\frac{(a+b)^{3/2}\sqrt{b\tanh(x)^2+a}}{a^2+2ab+b^2}\right)(a+b)^{3/2} - (a+b)\sqrt{b\tanh(x)^2+a} - \frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3}$$

input
```
int(tanh(x)*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh(((a + b)^(3/2)*(a + b*tanh(x)^2)^(1/2))/(2*a*b + a^2 + b^2))*(a + b)
^(3/2) - (a + b)*(a + b*tanh(x)^2)^(1/2) - (a + b*tanh(x)^2)^(3/2)/3
```

Reduce [F]

$$\int \tanh(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx = \frac{-\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2 b^2+3\sqrt{\tanh(x)^2 b+a}\,a^2+2\sqrt{\tanh(x)^2 b+a}\,ab+3\left(\right.}{}$$

input
```
int(tanh(x)*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
( - sqrt(tanh(x)**2*b + a)*tanh(x)**2*b**2 + 3*sqrt(tanh(x)**2*b + a)*a**2
 + 2*sqrt(tanh(x)**2*b + a)*a*b + 3*int((sqrt(tanh(x)**2*b + a)*tanh(x)**3
)/(tanh(x)**2*b + a),x)*a**2*b + 6*int((sqrt(tanh(x)**2*b + a)*tanh(x)**3)
/(tanh(x)**2*b + a),x)*a*b**2 + 3*int((sqrt(tanh(x)**2*b + a)*tanh(x)**3)/
(tanh(x)**2*b + a),x)*b**3)/(3*b)
```

3.222 $\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 88

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=-\frac{1}{2}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)+(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-1/2*b^(1/2)*(3*a+2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))+(a+b
)^(3/2)*arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-1/2*b*tanh(x)*(
a+b*tanh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.25

$$\int (a+b\tanh^2(x))^{3/2}\,dx=\frac{1}{2}\left(-2(-a-b)^{3/2}\arctan\left(\frac{\sqrt{b}\text{sech}^2(x)+\tanh(x)\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)+\sqrt{b}(3a+2b)\,\text{lo}\right.$$

input
```
Integrate[(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-2*(-a - b)^(3/2)*ArcTan[(Sqrt[b]*Sech[x]^2 + Tanh[x]*Sqrt[a + b*Tanh[x]^
2])/Sqrt[-a - b]] + Sqrt[b]*(3*a + 2*b)*Log[-(Sqrt[b]*Tanh[x]) + Sqrt[a +
b*Tanh[x]^2]] - b*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.05, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.750$, Rules used = {3042, 4144, 318, 25, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \left(a-b\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{ 4144}$$

$$\int \frac{\left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{ 318}$$

$$-\frac{1}{2}\int -\frac{b(3a+2b)\tanh^2(x)+a(2a+b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{1}{2}\int \frac{b(3a+2b)\tanh^2(x)+a(2a+b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 398}$$

$$\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b(3a+2b)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 224

$$\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b(3a+2b)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 291

$$\frac{1}{2}\left(2(a+b)^2\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

$\Big\downarrow$ 219

$$\frac{1}{2}\left(2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)-\frac{1}{2}b\tanh(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) +
 2*(a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/2 -
 (b*Tanh[x]*Sqrt[a + b*Tanh[x]^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 472 vs. $2(70) = 140$.

Time = 0.04 (sec) , antiderivative size = 473, normalized size of antiderivative = 5.38

method	result
derivativedivides	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{(4b(a+b)-}{}\right.}{2}$
default	$-\frac{\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}{6}-\frac{b\left(\frac{(2b(\tanh(x)-1)+2b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{4b}+\frac{(4b(a+b)-}{}\right.}{2}$

input
```
int((a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)-
1)+2*b)/b*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2
)/b^(3/2)*ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+
b)^(1/2)))-1/2*(a+b)*((b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b^(1/2)*
ln((b*(tanh(x)-1)+b)/b^(1/2)+(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))-
(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1)))+1/6*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1
)+a+b)^(3/2)-1/2*b*(1/4*(2*b*(tanh(x)+1)-2*b)/b*(b*(tanh(x)+1)^2-2*b*(tanh
(x)+1)+a+b)^(1/2)+1/8*(4*b*(a+b)-4*b^2)/b^(3/2)*ln((b*(tanh(x)+1)-b)/b^(1/
2)+(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)))+1/2*(a+b)*((b*(tanh(x)+1)
^2-2*b*(tanh(x)+1)+a+b)^(1/2)-b^(1/2)*ln((b*(tanh(x)+1)-b)/b^(1/2)+(b*(tan
h(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))-(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)
+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1)
))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 865 vs. $2(70) = 140$.

Time = 0.28 (sec) , antiderivative size = 4841, normalized size of antiderivative = 55.01

$$\int \left(a+b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \left(a+b\tanh^2(x)\right)^{\frac{3}{2}} dx$$

input
```
integrate((a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\int \left(b\tanh\left(x\right)^2+a\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 584 vs. $2(70) = 140$.

Time = 0.61 (sec) , antiderivative size = 584, normalized size of antiderivative = 6.64

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/2*(a + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2
)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b))) - (3*a*b + 2*b^2)*arctan(-1/2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a^2 + 2*a*b + b^2)*log(ab
s(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 2*((a*b + 2
*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*
b*e^(2*x) + a + b))^3 + (3*a*b - 2*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(a + b) + (3*a
^2*b - 3*a*b^2 - 2*b^3)*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) + (a^2*b - a*b^2 + 2*b^3)*sqrt(a + b
))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)
+ 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)^2
```

Mupad [F(-1)]

Timed out.

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\int {\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}^{3/2}\,dx$$

input
```
int((a + b*tanh(x)^2)^(3/2),x)
```

output
```
int((a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\left(\int \sqrt{\tanh \left(x\right)^{2} b +a}d x \right) a +\left(\int \sqrt{\tanh \left(x\right)^{2} b +a}\, \tanh \left(x\right)^{2}d x \right) b$$

input
```
int((a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a),x)*a + int(sqrt(tanh(x)**2*b + a)*tanh(x)**2,x)
*b
```

3.223 $\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 71

$$\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=-a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\tanh^2(x)}$$

output
```
-a^(3/2)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))+(a+b)^(3/2)*arctanh((a+b*t
anh(x)^2)^(1/2)/(a+b)^(1/2))-b*(a+b*tanh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx=-a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-b\sqrt{a+b\tanh^2(x)}$$

input
```
Integrate[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-(a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]) + (a + b)^(3/2)*ArcTanh[
Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - b*Sqrt[a + b*Tanh[x]^2]
```

Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.07, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.667$, Rules used = {3042, 26, 4153, 26, 354, 95, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i\left(a-b\tan(ix)^2\right)^{3/2}}{\tan(ix)}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{\left(a-b\tan(ix)^2\right)^{3/2}}{\tan(ix)}dx$$

$$\Big\downarrow \text{4153}$$

$$i\int -\frac{i\coth(x)\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth(x)\left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\coth(x)\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{95}$$

$$\frac{1}{2}\left(-\int -\frac{\coth(x)\left(a^2+b(2a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2b\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\int \frac{\coth(x)\left(a^2+b(2a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2b\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(a^2\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-2b\sqrt{a+b\tanh$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{2a^2\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(a+b)^2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-2b\sqrt{a+b\tanh^2(x)}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(-2a^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+2(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-2b\sqrt{a+b\tanh^2(x)}\right)$$

input
```
Int[Coth[x]*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-2*a^(3/2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + 2*(a + b)^(3/2)*ArcTa
nh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - 2*b*Sqrt[a + b*Tanh[x]^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 95
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p - 1)/(b*d*(p - 1))), x] + Simp[1/(b*d)   Int[(b
*d*e^2 - a*c*f^2 + f*(2*b*d*e - b*c*f - a*d*f)*x)*((e + f*x)^(p - 2)/((a +
b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && GtQ[p, 1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \coth(x)\left(a+b\tanh(x)^2\right)^{\frac{3}{2}} dx$$

input
```
int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 616 vs. $2(57) = 114$.

Time = 0.25 (sec) , antiderivative size = 3883, normalized size of antiderivative = 54.69

$$\int \coth(x)\left(a+b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2} \, dx = \int \left(a+b\tanh^{2}\left(x\right)\right)^{\frac{3}{2}} \coth\left(x\right) dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(3/2)*coth(x), x)
```

Maxima [F]

$$\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2} \, dx = \int \left(b\tanh\left(x\right)^{2}+a\right)^{\frac{3}{2}} \coth\left(x\right) \, dx$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*coth(x), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 433 vs. $2(57) = 114$.

Time = 0.70 (sec) , antiderivative size = 433, normalized size of antiderivative = 6.10

$$\int \coth(x) \left(a + b\tanh^2(x)\right)^{3/2} dx = \frac{2 a^2 \arctan\left(-\frac{\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} + \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right) - \frac{1}{2}(a+b)^{\frac{3}{2}} \log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right) - \frac{(a^2 + 2ab + b^2) \log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(\right.\right.}{2\sqrt{a+b}} - \frac{4\left(\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)b^2 - \right.}{\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^2 + 2\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + \right.}$$

input
```
integrate(coth(x)*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
2*a^2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*
e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) + 1/2*(a
+ b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) - 1/2*(a + b)^(3/2)*log(ab
s(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 4*((sqrt(a + b)*e^(2*x
) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*b^2 -
 sqrt(a + b)*b^2)/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b
)
```

Mupad [F(-1)]

Timed out.

$$\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\int \coth(x)\,\left(b\tanh(x)^2+a\right)^{3/2}\,dx$$

input
```
int(coth(x)*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)*(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \coth(x) \left(a+b\tanh^2(x)\right)^{3/2}\,dx=\left(\int \sqrt{\tanh(x)^2 b+a}\,\coth(x)\tanh(x)^2\,dx\right)b+\left(\int \sqrt{\tanh(x)^2 b+a}\,\coth(x)\,dx\right)a$$

input
```
int(coth(x)*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*coth(x)*tanh(x)**2,x)*b + int(sqrt(tanh(x)**2*b
 + a)*coth(x),x)*a
```

3.224 $\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 77

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2}\,dx = -b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) + (a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right) - a\coth(x)\sqrt{a+b\tanh^2(x)}$$

output
```
-b^(3/2)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))+(a+b)^(3/2)*arctan
h((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))-a*coth(x)*(a+b*tanh(x)^2)^(1/
2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.93 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.56

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx =$$

$$-\frac{a\left((a-b+(a+b)\cosh(2x))\text{csch}^2(x)-\sqrt{2}(a+2b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{(a-}}{}\right.\right.\right.}{\sqrt{2}\sqrt{}}$$

input
```
Integrate[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((a*((a - b + (a + b)*Cosh[2*x])*Csch[x]^2 - Sqrt[2]*(a + 2*b)*Sqrt[((a -
 b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a +
 b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] + Sqrt[2]*(a + b)*Sqrt[((a - b +
 (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a -
b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*Sqrt[
(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]))
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 25, 4153, 25, 376, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{\left(a-b\tan(ix)^2\right)^{3/2}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{ 25}$$

$$-\int \frac{\left(a-b\tan(ix)^2\right)^{3/2}}{\tan(ix)^2}dx$$

$$\Big\downarrow \text{ 4153}$$

$$-\int -\frac{\coth^2(x)\left(b\tanh^2(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{ 25}$$

$$\int \frac{\coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{ 376}$$

$$\int \frac{b^2\tanh^2(x)+a(a+2b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 398}$$

$$b^2\left(-\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)\right)+(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 224}$$

$$b^2\left(-\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}\right)+(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 219}$$

$$(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 291}$$

$$(a+b)^2\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-b^{3/2}\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

$$\Big\downarrow \text{ 219}$$

$$b^{3/2}\left(-\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)\right)+(a+b)^{3/2}\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)-a\coth(x)\sqrt{a+b\tanh^2(x)}$$

input
```
Int[Coth[x]^2*(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-(b^(3/2)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]) + (a + b)^(3/2
)*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]] - a*Coth[x]*Sqrt[a
+ b*Tanh[x]^2]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 376
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[c*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1
)/(a*e*(m + 1))), x] - Simp[1/(a*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*x^
2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b*c - a*d)*(m + 1) + 2*c*(b*c*(p + 1) + a*
d*(q - 1)) + d*((b*c - a*d)*(m + 1) + 2*b*c*(p + q))*x^2, x], x], x] /; Fre
eQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && GtQ[q, 1] && LtQ[m, -1] &
& IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \coth\left(x\right)^{2} \left(a+b\tanh\left(x\right)^{2}\right)^{\frac{3}{2}}dx$$

input
```
int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(63) = 126$.

Time = 0.24 (sec) , antiderivative size = 3913, normalized size of antiderivative = 50.82

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \left(a+b\tanh^2{(x)}\right)^{\frac{3}{2}} \coth^2{(x)}\, dx$$

input
```
integrate(coth(x)**2*(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(3/2)*coth(x)**2, x)
```

Maxima [F]

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \left(b\tanh{(x)}^2+a\right)^{\frac{3}{2}} \coth{(x)}^2\, dx$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(3/2)*coth(x)^2, x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 430 vs. $2(63) = 126$.

Time = 0.71 (sec) , antiderivative size = 430, normalized size of antiderivative = 5.58

$$\int \coth^2(x)\left(a+b\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{2\,b^2\arctan\left(-\frac{\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}}{2\,\sqrt{-b}}\right)}{\sqrt{-b}}$$
$$-\frac{1}{2}\,(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}+\sqrt{a+b}\right|\right)$$
$$+\frac{1}{2}\,(a+b)^{\frac{3}{2}}\log\left(\left|-\sqrt{a+b}e^{(2\,x)}+\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}-\sqrt{a+b}\right|\right)$$
$$-\frac{(a^2+2\,ab+b^2)\log\left(\left|-\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)(a+b)-\sqrt{a+b}(\right.\right.}{2\,\sqrt{a+b}}$$
$$+\frac{4\left(\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)a^2+\right.}{\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\,ae^{(2\,x)}-2\,be^{(2\,x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2\,x)}-\sqrt{ae^{(4\,x)}+be^{(4\,x)}+2\right.}$$

input
```
integrate(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-2*b^2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*(a
 + b)^(3/2)*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))) + 1/2*(a + b)^(3/2)*log(a
bs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b) - sqrt(a + b))) - 1/2*(a^2 + 2*a*b + b^2)*log(abs(-(sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 4*((sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*a^2
+ sqrt(a + b)*a^2)/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*
a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a +
b)
```

Mupad [F(-1)]

Timed out.

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \int \coth(x)^2 \left(b\tanh(x)^2+a\right)^{3/2} dx$$

input
```
int(coth(x)^2*(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2*(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \coth^2(x) \left(a+b\tanh^2(x)\right)^{3/2} dx = \left(\int \sqrt{\tanh(x)^2 b+a}\,\coth(x)^2\tanh(x)^2 dx\right) b + \left(\int \sqrt{\tanh(x)^2 b+a}\,\coth(x)^2 dx\right) a$$

input
```
int(coth(x)^2*(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)*coth(x)**2*tanh(x)**2,x)*b + int(sqrt(tanh(x)**
2*b + a)*coth(x)**2,x)*a
```

3.225 $\int \sqrt{1+\tanh^2(x)}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 31

$$\int \sqrt{1+\tanh^2(x)}\,dx = -\operatorname{arcsinh}(\tanh(x))+\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)$$

output
```
-arcsinh(tanh(x))+2^(1/2)*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.65

$$\int \sqrt{1+\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)-\operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{1+\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

input
```
Integrate[Sqrt[1 + Tanh[x]^2],x]
```

output
```
((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cos
h[x]*Sqrt[1 + Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 4144, 301, 222, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{\tanh^2(x)+1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{1-\tan(ix)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{\sqrt{\tanh^2(x)+1}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{301}$$

$$2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-\int \frac{1}{\sqrt{\tanh^2(x)+1}}d\tanh(x)$$

$$\Big\downarrow \text{222}$$

$$2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-\text{arcsinh}(\tanh(x))$$

$$\Big\downarrow \text{291}$$

$$2\int \frac{1}{1-\frac{2\tanh^2(x)}{\tanh^2(x)+1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)+1}}-\text{arcsinh}(\tanh(x))$$

$$\Big\downarrow \text{219}$$

$$\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-\text{arcsinh}(\tanh(x))$$

input
```
Int[Sqrt[1 + Tanh[x]^2],x]
```

output
```
-ArcSinh[Tanh[x]] + Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 96 vs. $2(25) = 50$.

Time = 0.09 (sec) , antiderivative size = 97, normalized size of antiderivative = 3.13

method	result
derivativedivides	$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(\tanh}{}$
default	$-\frac{\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{2}-\operatorname{arcsinh}(\tanh(x))+\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{2}+\frac{\sqrt{(\tanh}{}$

input
```
int((1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*((tanh(x)-1)^2+2*tanh(x))^(1/2)-arcsinh(tanh(x))+1/2*2^(1/2)*arctanh(
1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/2*((tanh(x)+1
)^2-2*tanh(x))^(1/2)-1/2*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((tanh(
x)+1)^2-2*tanh(x))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 679 vs. $2(25) = 50$.

Time = 0.10 (sec) , antiderivative size = 679, normalized size of antiderivative = 21.90

$$\int \sqrt{1+\tanh^2(x)}\,dx = \text{Too large to display}$$

input
```
integrate((1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh
(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^
5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cos
h(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^
4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)
^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*co
sh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x)
)*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*si
nh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)
^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*
sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/4*sqrt(2)*log(2*(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(
x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*
cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2)) - 1/2*log((cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^
2 + 2*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin...
```

Sympy [F]

$$\int \sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh^2(x)+1}\,dx$$

input
```
integrate((1+tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(tanh(x)**2 + 1), x)
```

Maxima [F]

$$\int \sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh(x)^2+1}\,dx$$

input
```
integrate((1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(tanh(x)^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. 2(25) = 50.

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 3.35

$$\int \sqrt{1+\tanh^2(x)}\,dx = -\frac{1}{2}\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e\right.$$

input
```
integrate((1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(sqr
t(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*x)
 + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x
) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.19

$$\int \sqrt{1+\tanh^2(x)}\,dx = \frac{\sqrt{2}\left(\ln\left(\tanh(x)+1\right)-\ln\left(\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}-\tanh(x)+1\right)\right)}{2}-\mathrm{asinh}(\tanh(x)) + \frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}+1\right)-\ln\left(\tanh(x)-1\right)\right)}{2}$$

input
```
int((tanh(x)^2 + 1)^(1/2),x)
```

output
```
(2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
 1)))/2 - asinh(tanh(x)) + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)
^(1/2) + 1) - log(tanh(x) - 1)))/2
```

Reduce [F]

$$\int \sqrt{1+\tanh^2(x)}\,dx = \int \sqrt{\tanh\left(x\right)^2+1}dx$$

input
```
int((1+tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2 + 1),x)
```

3.226 $\int \sqrt{-1-\tanh^2(x)}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 45

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)$$

output
```
arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))-2^(1/2)*arctan(2^(1/2)*tanh(x)/(-1-ta
nh(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.18

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \frac{\left(\sqrt{2}\text{arcsinh}\left(\sqrt{2}\sinh(x)\right)-\text{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\right)\cosh(x)\sqrt{-1-\tanh^2(x)}}{\sqrt{\cosh(2x)}}$$

input
```
Integrate[Sqrt[-1 - Tanh[x]^2],x]
```

output
```
((Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]] - ArcTanh[Sinh[x]/Sqrt[Cosh[2*x]]])*Cos
h[x]*Sqrt[-1 - Tanh[x]^2])/Sqrt[Cosh[2*x]]
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4144, 301, 224, 216, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \sqrt{-\tanh^2(x)-1}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \sqrt{-1+\tan(ix)^2}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{\sqrt{-\tanh^2(x)-1}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{301}$$

$$\int \frac{1}{\sqrt{-\tanh^2(x)-1}}d\tanh(x)-2\int \frac{1}{\sqrt{-\tanh^2(x)-1}\,(1-\tanh^2(x))}d\tanh(x)$$

$$\Big\downarrow \text{224}$$

$$\int \frac{1}{\frac{\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}-2\int \frac{1}{\sqrt{-\tanh^2(x)-1}\,(1-\tanh^2(x))}d\tanh(x)$$

$$\Big\downarrow \text{216}$$

$$\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-2\int \frac{1}{\sqrt{-\tanh^2(x)-1}\,(1-\tanh^2(x))}d\tanh(x)$$

$$\Big\downarrow \text{291}$$

$$\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-2\int\frac{1}{\frac{2\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}$$

$$\Big\downarrow \ 216$$

$$\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)$$

input
```
Int[Sqrt[-1 - Tanh[x]^2],x]
```

output
```
ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] - Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[x])/Sq
rt[-1 - Tanh[x]^2]]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 301
```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Simp[b/
d   Int[(a + b*x^2)^(p - 1), x], x] - Simp[(b*c - a*d)/d   Int[(a + b*x^2)^
(p - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
&& GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4] || (EqQ[p, 2/3] && E
qQ[b*c + 3*a*d, 0]))
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(37) = 74$.

Time = 0.08 (sec) , antiderivative size = 142, normalized size of antiderivative = 3.16

method	result
derivativedivides	$-\frac{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{2}+\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{2}+\frac{\sqrt{2}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right.}{2}$
default	$-\frac{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{2}+\frac{\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{2}+\frac{\sqrt{2}\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right.}{2}$

input
```
int((-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-(tanh(x)-1)^2-2*tanh(x))^(1/2)+1/2*arctan(tanh(x)/(-(tanh(x)-1)^2-2
*tanh(x))^(1/2))+1/2*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-
1)^2-2*tanh(x))^(1/2))+1/2*(-(tanh(x)+1)^2+2*tanh(x))^(1/2)+1/2*arctan(tan
h(x)/(-(tanh(x)+1)^2+2*tanh(x))^(1/2))-1/2*2^(1/2)*arctan(1/4*(-2+2*tanh(x
))*2^(1/2)/(-(tanh(x)+1)^2+2*tanh(x))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.10 (sec) , antiderivative size = 226, normalized size of antiderivative = 5.02

$$\int \sqrt{-1-\tanh^2(x)}\,dx = -\frac{1}{4}\sqrt{-2}\log\left(-\left(\sqrt{-2}\sqrt{-2\,e^{(4\,x)}-2}+2\,e^{(2\,x)}+2\right)e^{(-2\,x)}\right) + \frac{1}{4}\sqrt{-2}\log\left(\left(\sqrt{-2}\sqrt{-2\,e^{(4\,x)}-2}-2\,e^{(2\,x)}-2\right)e^{(-2\,x)}\right) + \frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2\,e^{(4\,x)}-2}\left(e^{(2\,x)}-2\right)+\sqrt{-2}e^{(4\,x)}-\sqrt{-2}e^{(2\,x)}+2\sqrt{-2}\right)e^{(-4\,x)}\right) - \frac{1}{4}\sqrt{-2}\log\left(-2\left(\sqrt{-2\,e^{(4\,x)}-2}\left(e^{(2\,x)}-2\right)-\sqrt{-2}e^{(4\,x)}+\sqrt{-2}e^{(2\,x)}-2\sqrt{-2}\right)e^{(-4\,x)}\right) + \frac{1}{2}i\,\log\left(-4\left(i\sqrt{-2\,e^{(4\,x)}-2}+e^{(2\,x)}-1\right)e^{(-2\,x)}\right) - \frac{1}{2}i\,\log\left(-4\left(-i\sqrt{-2\,e^{(4\,x)}-2}+e^{(2\,x)}-1\right)e^{(-2\,x)}\right)$$

input
```
integrate((-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
-1/4*sqrt(-2)*log(-(sqrt(-2)*sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x
)) + 1/4*sqrt(-2)*log((sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(2*x) - 2)*e^(-
2*x)) + 1/4*sqrt(-2)*log(-2*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + sqrt(-2)
*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) - 1/4*sqrt(-2)*log(-2*
(sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x)
- 2*sqrt(-2))*e^(-4*x)) + 1/2*I*log(-4*(I*sqrt(-2*e^(4*x) - 2) + e^(2*x) -
 1)*e^(-2*x)) - 1/2*I*log(-4*(-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2
*x))
```

Sympy [F]

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \int \sqrt{-\tanh^2(x)-1}\,dx$$

input
```
integrate((-1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(sqrt(-tanh(x)**2 - 1), x)
```

Maxima [F]

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \int \sqrt{-\tanh(x)^2-1}\,dx$$

input
```
integrate((-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(-tanh(x)^2 - 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.31

$$\int \sqrt{-1-\tanh^2(x)}\,dx = -\frac{1}{2}i\sqrt{2}\left(\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-\right.\right.$$

input
```
integrate((-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*I*sqrt(2)*(sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(s
qrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) + log(sqrt(e^(4*x) + 1) - e^(2*
x) + 1) + log(sqrt(e^(4*x) + 1) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2
*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \sqrt{-1-\tanh^2(x)}\,dx = -\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right) - \ln\left(\tanh(x)-\sqrt{-\tanh(x)^2-1}\,1\mathrm{i}\right)\,1\mathrm{i}$$

input
```
int((- tanh(x)^2 - 1)^(1/2),x)
```

output
```
- log(tanh(x) - (- tanh(x)^2 - 1)^(1/2)*1i)*1i - 2^(1/2)*atan((2^(1/2)*tan
h(x))/(- tanh(x)^2 - 1)^(1/2))
```

Reduce [F]

$$\int \sqrt{-1-\tanh^2(x)}\,dx = \left(\int \sqrt{\tanh(x)^2+1}dx\right)i$$

input
```
int((-1-tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2 + 1),x)*i
```

3.227 $\int \left(1+\tanh^2(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 50

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = -\frac{5}{2}\operatorname{arcsinh}(\tanh(x))+2\sqrt{2}\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)-\frac{1}{2}\tanh(x)\sqrt{1+\tanh^2(x)}$$

output
```
-5/2*arcsinh(tanh(x))+2*2^(1/2)*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2
))-1/2*tanh(x)*(1+tanh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = -\frac{\left(-4\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x)+5\operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x)+\cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)(1}{2\cosh^{\frac{3}{2}}(2x)}$$

input
```
Integrate[(1 + Tanh[x]^2)^(3/2),x]
```

output
```
-1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/S
qrt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(1 + Tanh[x]^
2)^(3/2))/Cosh[2*x]^(3/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.06, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.800$, Rules used = {3042, 4144, 318, 25, 398, 222, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(\tanh^2(x)+1\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(1-\tan(ix)^2\right)^{3/2}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{\left(\tanh^2(x)+1\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{318}$$

$$-\frac{1}{2}\int -\frac{5\tanh^2(x)+3}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-\frac{1}{2}\sqrt{\tanh^2(x)+1}\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{5\tanh^2(x)+3}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(8\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-5\int \frac{1}{\sqrt{\tanh^2(x)+1}}d\tanh(x)\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\Big\downarrow \ 222$$

$$\frac{1}{2}\left(8\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{\tanh^2(x)+1}}d\tanh(x)-5\text{arcsinh}(\tanh(x))\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\Big\downarrow \ 291$$

$$\frac{1}{2}\left(8\int \frac{1}{1-\frac{2\tanh^2(x)}{\tanh^2(x)+1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)+1}}-5\text{arcsinh}(\tanh(x))\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left(4\sqrt{2}\text{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)-5\text{arcsinh}(\tanh(x))\right)-\frac{1}{2}\tanh(x)\sqrt{\tanh^2(x)+1}$$

input
```
Int[(1 + Tanh[x]^2)^(3/2),x]
```

output
```
(-5*ArcSinh[Tanh[x]] + 4*Sqrt[2]*ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x
]^2]])/2 - (Tanh[x]*Sqrt[1 + Tanh[x]^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 222
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(38) = 76.

Time = 0.07 (sec) , antiderivative size = 158, normalized size of antiderivative = 3.16

method	result
derivativedivides	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\,\mathrm{arcsinh}(\tanh(x))}{2}-\sqrt{(\tanh\left(x\right)}$
default	$-\frac{\left((\tanh(x)-1)^2+2\tanh(x)\right)^{\frac{3}{2}}}{6}-\frac{\tanh(x)\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}{4}-\frac{5\,\mathrm{arcsinh}(\tanh(x))}{2}-\sqrt{(\tanh\left(x\right)}$

input
```
int((1+tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*((tanh(x)-1)^2+2*tanh(x))^(3/2)-1/4*tanh(x)*((tanh(x)-1)^2+2*tanh(x))
^(1/2)-5/2*arcsinh(tanh(x))-((tanh(x)-1)^2+2*tanh(x))^(1/2)+2^(1/2)*arctan
h(1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1/2))+1/6*((tanh(x)
+1)^2-2*tanh(x))^(3/2)-1/4*tanh(x)*((tanh(x)+1)^2-2*tanh(x))^(1/2)+((tanh(
x)+1)^2-2*tanh(x))^(1/2)-2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((tanh(
x)+1)^2-2*tanh(x))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. 2(38) = 76.

Time = 0.10 (sec) , antiderivative size = 1027, normalized size of antiderivative = 20.54

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx = \text{Too large to display}$$

input
```
integrate((1+tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sqrt(2)*sinh(x)^
4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*cosh(x)^2 + 4*
(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log(-2*(cosh(x)^8
 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 - 3)*sinh(x)^6 - 3*cosh
(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 - 9*cosh(
x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cosh(x)^5 - 15*cosh(x)^3 + 5*cos
h(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^4 + 30*cosh(x)^2 - 4)*sinh(x)
^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)^5 + 10*cosh(x)^3 - 4*cosh(x)
)*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*cosh(x)*sinh(x)^5 + sqrt(2)*sin
h(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^4 - 3*sqrt(2)*cosh(x)^4
 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x)^3 + (15*sqrt(2)*cos
h(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*sinh(x)^2 + 4*sqrt(2)*cosh(x)^2
 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)^3 + 4*sqrt(2)*cosh(x))*sinh(
x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/(cosh(x)^2 - 2*cosh(x)*sinh(x
) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*cosh(x)^4*sinh(
x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*sinh(x)^4 + 6*cosh(x)*sinh(x)
^5 + sinh(x)^6)) + 2*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 + sq
rt(2)*sinh(x)^4 + 2*(3*sqrt(2)*cosh(x)^2 + sqrt(2))*sinh(x)^2 + 2*sqrt(2)*
cosh(x)^2 + 4*(sqrt(2)*cosh(x)^3 + sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log
(2*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sin...
```

Sympy [F]

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx=\int \left(\tanh^2{(x)}+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+tanh(x)**2)**(3/2),x)
```

output
```
Integral((tanh(x)**2 + 1)**(3/2), x)
```

Maxima [F]

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx=\int \left(\tanh\left(x\right)^2+1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((1+tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((tanh(x)^2 + 1)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 202 vs. $2(38) = 76$.

Time = 0.13 (sec) , antiderivative size = 202, normalized size of antiderivative = 4.04

$$\int \left(1+\tanh^2(x)\right)^{3/2}\,dx=$$

$$-\frac{1}{4}\sqrt{2}\left(5\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(3\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3-\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2\cdots\right.}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}+1}\cdots\right.}\cdots\right.$$

input
```
integrate((1+tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(5*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)/(s
qrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(3*(sqrt(e^(4*x) + 1) - e^(
2*x))^3 - (sqrt(e^(4*x) + 1) - e^(2*x))^2 - sqrt(e^(4*x) + 1) + e^(2*x) -
1)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*e^(2*x) - 1)
^2 + 4*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + 4*log(sqrt(e^(4*x) + 1) - e^
(2*x)) - 4*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 78, normalized size of antiderivative = 1.56

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = \sqrt{2}\left(\ln(\tanh(x)+1) - \ln\left(\sqrt{2}\sqrt{\tanh(x)^2+1} - \tanh(x)+1\right)\right) - \frac{5\,\mathrm{asinh}(\tanh(x))}{2} - \frac{\tanh(x)\,\sqrt{\tanh(x)^2+1}}{2} + \sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh(x)^2+1}+1\right) - \ln(\tanh(x)-1)\right)$$

input
```
int((tanh(x)^2 + 1)^(3/2),x)
```

output
```
2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
1)) - (5*asinh(tanh(x)))/2 - (tanh(x)*(tanh(x)^2 + 1)^(1/2))/2 + 2^(1/2)*(
log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log(tanh(x) - 1))
```

Reduce [F]

$$\int \left(1+\tanh^2(x)\right)^{3/2} dx = \int \sqrt{\tanh(x)^2+1}dx + \int \sqrt{\tanh(x)^2+1}\,\tanh(x)^2\,dx$$

input
```
int((1+tanh(x)^2)^(3/2),x)
```

output
```
int(sqrt(tanh(x)**2 + 1),x) + int(sqrt(tanh(x)**2 + 1)*tanh(x)**2,x)
```

3.228 $\int \left(-1-\tanh^2(x)\right)^{3/2} dx$

Optimal result

Integrand size = 12, antiderivative size = 67

$$\int \left(-1-\tanh^2(x)\right)^{3/2} dx = -\frac{5}{2}\arctan\left(\frac{\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + 2\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right) + \frac{1}{2}\tanh(x)\sqrt{-1-\tanh^2(x)}$$

output
```
-5/2*arctan(tanh(x)/(-1-tanh(x)^2)^(1/2))+2*2^(1/2)*arctan(2^(1/2)*tanh(x)
/(-1-tanh(x)^2)^(1/2))+1/2*tanh(x)*(-1-tanh(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.13

$$\int \left(-1-\tanh^2(x)\right)^{3/2} dx = -\frac{\left(-4\sqrt{2}\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\cosh^3(x) + 5\operatorname{arctanh}\left(\frac{\sinh(x)}{\sqrt{\cosh(2x)}}\right)\cosh^3(x) + \cosh(x)\sqrt{\cosh(2x)}\sinh(x)\right)(-}{2\cosh^{\frac{3}{2}}(2x)}$$

input
```
Integrate[(-1 - Tanh[x]^2)^(3/2),x]
```

output
```
-1/2*((-4*Sqrt[2]*ArcSinh[Sqrt[2]*Sinh[x]]*Cosh[x]^3 + 5*ArcTanh[Sinh[x]/S
qrt[Cosh[2*x]]]*Cosh[x]^3 + Cosh[x]*Sqrt[Cosh[2*x]]*Sinh[x])*(-1 - Tanh[x]
^2)^(3/2))/Cosh[2*x]^(3/2)
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 70, normalized size of antiderivative = 1.04, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3042, 4144, 318, 25, 398, 224, 216, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(-\tanh^2(x)-1\right)^{3/2} dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(-1+\tan(ix)^2\right)^{3/2} dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{\left(-\tanh^2(x)-1\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{318}$$

$$\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}-\frac{1}{2}\int -\frac{5\tanh^2(x)+3}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\int \frac{5\tanh^2(x)+3}{\sqrt{-\tanh^2(x)-1}\left(1-\tanh^2(x)\right)}d\tanh(x)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\Big\downarrow \text{398}$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt{-\tanh^2(x)-1}\,(1-\tanh^2(x))}d\tanh(x)-5\int \frac{1}{\sqrt{-\tanh^2(x)-1}}d\tanh(x)\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\Big\downarrow \text{224}$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt{-\tanh^2(x)-1}\,(1-\tanh^2(x))}d\tanh(x)-5\int \frac{1}{\frac{\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(8\int \frac{1}{\sqrt{-\tanh^2(x)-1}\,(1-\tanh^2(x))}d\tanh(x)-5\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\Big\downarrow \text{291}$$

$$\frac{1}{2}\left(8\int \frac{1}{\frac{2\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}-5\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)\right)+\frac{1}{2}\sqrt{-\tanh^2(x)-1}\tanh(x)$$

$$\Big\downarrow \text{216}$$

$$\frac{1}{2}\left(4\sqrt{2}\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)-5\arctan\left(\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)\right)+\frac{1}{2}\tanh(x)\sqrt{-\tanh^2(x)-1}$$

input
```
Int[(-1 - Tanh[x]^2)^(3/2),x]
```

output
```
(-5*ArcTan[Tanh[x]/Sqrt[-1 - Tanh[x]^2]] + 4*Sqrt[2]*ArcTan[(Sqrt[2]*Tanh[
x])/Sqrt[-1 - Tanh[x]^2]])/2 + (Tanh[x]*Sqrt[-1 - Tanh[x]^2])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 318
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[d*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q - 1)/(b*(2*(p + q) + 1))), x] + S
imp[1/(b*(2*(p + q) + 1))   Int[(a + b*x^2)^p*(c + d*x^2)^(q - 2)*Simp[c*(b
*c*(2*(p + q) + 1) - a*d) + d*(b*c*(2*(p + 2*q - 1) + 1) - a*d*(2*(q - 1) +
 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b*c - a*d, 0] && G
tQ[q, 1] && NeQ[2*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a, b, c,
d, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(53) = 106.

Time = 0.05 (sec) , antiderivative size = 211, normalized size of antiderivative = 3.15

method	result
derivativedivides	$-\frac{\left(-(\tanh(x)-1)^2-2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}$
default	$-\frac{\left(-(\tanh(x)-1)^2-2\tanh(x)\right)^{\frac{3}{2}}}{6}+\frac{\tanh(x)\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}{4}-\frac{5\arctan\left(\frac{\tanh(x)}{\sqrt{-(\tanh(x)-1)^2-2\tanh(x)}}\right)}{4}$

input
```
int((-1-tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6*(-(tanh(x)-1)^2-2*tanh(x))^(3/2)+1/4*tanh(x)*(-(tanh(x)-1)^2-2*tanh(x
))^(1/2)-5/4*arctan(tanh(x)/(-(tanh(x)-1)^2-2*tanh(x))^(1/2))+(-(tanh(x)-1
)^2-2*tanh(x))^(1/2)-2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-
1)^2-2*tanh(x))^(1/2))+1/6*(-(tanh(x)+1)^2+2*tanh(x))^(3/2)+1/4*tanh(x)*(-
(tanh(x)+1)^2+2*tanh(x))^(1/2)-5/4*arctan(tanh(x)/(-(tanh(x)+1)^2+2*tanh(x
))^(1/2))-(-(tanh(x)+1)^2+2*tanh(x))^(1/2)+2^(1/2)*arctan(1/4*(-2+2*tanh(x
))*2^(1/2)/(-(tanh(x)+1)^2+2*tanh(x))^(1/2))
```

Fricas [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.09 (sec) , antiderivative size = 361, normalized size of antiderivative = 5.39

$$\int (-1 -\tanh^2(x))^{3/2} dx = \frac{2\left(\sqrt{-2}e^{(4x)} + 2\sqrt{-2}e^{(2x)} + \sqrt{-2}\right)\log\left(2\left(\sqrt{-2}\sqrt{-2e^{(4x)} - 2} + 2e^{(2x)} + 2\right)e^{(-2x)}\right)}{}$$

input
```
integrate((-1-tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
1/4*(2*(sqrt(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(2*(sqrt(-2)*
sqrt(-2*e^(4*x) - 2) + 2*e^(2*x) + 2)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x) + 2*
sqrt(-2)*e^(2*x) + sqrt(-2))*log(-2*(sqrt(-2)*sqrt(-2*e^(4*x) - 2) - 2*e^(
2*x) - 2)*e^(-2*x)) - 5*(I*e^(4*x) + 2*I*e^(2*x) + I)*log(-4*(I*sqrt(-2*e^
(4*x) - 2) + e^(2*x) - 1)*e^(-2*x)) - 5*(-I*e^(4*x) - 2*I*e^(2*x) - I)*log
(-4*(-I*sqrt(-2*e^(4*x) - 2) + e^(2*x) - 1)*e^(-2*x)) - 2*(sqrt(-2)*e^(4*x
) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*(e^(2*x) -
2) + sqrt(-2)*e^(4*x) - sqrt(-2)*e^(2*x) + 2*sqrt(-2))*e^(-4*x)) + 2*(sqrt
(-2)*e^(4*x) + 2*sqrt(-2)*e^(2*x) + sqrt(-2))*log(4*(sqrt(-2*e^(4*x) - 2)*
(e^(2*x) - 2) - sqrt(-2)*e^(4*x) + sqrt(-2)*e^(2*x) - 2*sqrt(-2))*e^(-4*x)
) + 2*sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 1))/(e^(4*x) + 2*e^(2*x) + 1)
```

Sympy [F]

$$\int (-1 - \tanh^2(x))^{3/2} dx = \int \left(-\tanh^2(x) - 1\right)^{\frac{3}{2}} dx$$

input
```
integrate((-1-tanh(x)**2)**(3/2),x)
```

output
```
Integral((-tanh(x)**2 - 1)**(3/2), x)
```

Maxima [F]

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx = \int \left(-\tanh(x)^2-1\right)^{\frac{3}{2}}\,dx$$

input
```
integrate((-1-tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((-tanh(x)^2 - 1)^(3/2), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.13 (sec) , antiderivative size = 204, normalized size of antiderivative = 3.04

$$\int \left(-1-\tanh^2(x)\right)^{3/2}\,dx =$$

$$-\frac{1}{4}\sqrt{2}\left(-5i\sqrt{2}\log\left(\frac{\sqrt{2}-\sqrt{e^{(4x)}+1}+e^{(2x)}+1}{\sqrt{2}+\sqrt{e^{(4x)}+1}-e^{(2x)}-1}\right)-\frac{4\left(-3i\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^3+i\left(\sqrt{e^{(4x)}+1}-e\cdots\right.\right.}{\left(\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)^2-2\sqrt{e^{(4x)}\cdots}\right.}\right.$$

input
```
integrate((-1-tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(-5*I*sqrt(2)*log((sqrt(2) - sqrt(e^(4*x) + 1) + e^(2*x) + 1)
/(sqrt(2) + sqrt(e^(4*x) + 1) - e^(2*x) - 1)) - 4*(-3*I*(sqrt(e^(4*x) + 1)
 - e^(2*x))^3 + I*(sqrt(e^(4*x) + 1) - e^(2*x))^2 + I*sqrt(e^(4*x) + 1) -
I*e^(2*x) + I)/((sqrt(e^(4*x) + 1) - e^(2*x))^2 - 2*sqrt(e^(4*x) + 1) + 2*
e^(2*x) - 1)^2 - 4*I*log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) - 4*I*log(sqrt(e
^(4*x) + 1) - e^(2*x)) + 4*I*log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [F(-1)]

Timed out.

$$\int \left(-1-\tanh^2(x)\right)^{3/2} dx = \int \left(-\tanh(x)^2-1\right)^{3/2} dx$$

input
```
int((- tanh(x)^2 - 1)^(3/2),x)
```

output
```
int((- tanh(x)^2 - 1)^(3/2), x)
```

Reduce [F]

$$\int \left(-1-\tanh^2(x)\right)^{3/2} dx = -i\left(\int \sqrt{\tanh(x)^2+1}dx+\int \sqrt{\tanh(x)^2+1}\,\tanh(x)^2\,dx\right)$$

input
```
int((-1-tanh(x)^2)^(3/2),x)
```

output
```
- i*(int(sqrt(tanh(x)**2 + 1),x) + int(sqrt(tanh(x)**2 + 1)*tanh(x)**2,x)
)
```

3.229 $\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 70

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}-\frac{\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+(a-b)*(a+b*tanh(x)^
2)^(1/2)/b^2-1/3*(a+b*tanh(x)^2)^(3/2)/b^2
```

Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.97

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}+\frac{(a-b+(a-2b)\cosh(2x))\operatorname{sech}^2(x)\sqrt{a+b\tanh^2(x)}}{3b^2}$$

input
```
Integrate[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2/Sqrt[a + b]]/Sqrt[a + b] + ((a - b + (a - 2*
b)*Cosh[2*x])*Sech[x]^2*Sqrt[a + b*Tanh[x]^2])/(3*b^2)
```

Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 26, 4153, 26, 354, 99, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)^5}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)^5}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh^5(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^5(x)}{(1-\tanh^2(x))\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\tanh^4(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\Big\downarrow \text{99}$$

$$\frac{1}{2}\int \left(\frac{a-b}{b\sqrt{b\tanh^2(x)+a}}-\frac{\sqrt{b\tanh^2(x)+a}}{b}+\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}\right)d\tanh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\left(a+b\tanh^2(x)\right)^{3/2}}{3b^2}+\frac{2(a-b)\sqrt{a+b\tanh^2(x)}}{b^2}\right)$$

input
```
Int[Tanh[x]^5/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b] + (2*(a - b)*S
qrt[a + b*Tanh[x]^2])/b^2 - (2*(a + b*Tanh[x]^2)^(3/2))/(3*b^2))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 99
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (IntegerQ[p] |
| (GtQ[m, 0] && GeQ[n, -1]))
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 163 vs. $2(58) = 116$.

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 2.34

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}-\frac{\tanh(x)^2\sqrt{a+b\tanh(x)^2}}{3b}+\frac{2a\sqrt{a+b\tanh(x)^2}}{3b^2}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(\cdots}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}-\frac{\tanh(x)^2\sqrt{a+b\tanh(x)^2}}{3b}+\frac{2a\sqrt{a+b\tanh(x)^2}}{3b^2}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(\cdots}{\tanh(x)-1}\right.}{2\sqrt{a+b}}$

input
```
int(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*tanh(x)^2)^(1/2)/b-1/3*tanh(x)^2/b*(a+b*tanh(x)^2)^(1/2)+2/3*a/b^2*(
a+b*tanh(x)^2)^(1/2)+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(
1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)^(
1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(
x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1131 vs. $2(58) = 116$.

Time = 0.24 (sec) , antiderivative size = 2827, normalized size of antiderivative = 40.39

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/12*(3*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 + 3*b^2*
cosh(x)^4 + 3*(5*b^2*cosh(x)^2 + b^2)*sinh(x)^4 + 3*b^2*cosh(x)^2 + 4*(5*b
^2*cosh(x)^3 + 3*b^2*cosh(x))*sinh(x)^3 + 3*(5*b^2*cosh(x)^4 + 6*b^2*cosh(
x)^2 + b^2)*sinh(x)^2 + b^2 + 6*(b^2*cosh(x)^5 + 2*b^2*cosh(x)^3 + b^2*cos
h(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*
cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6
+ 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 +
a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b
 - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b
- a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2
*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 +
b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2
*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*c
osh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(
x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*s
inh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^
2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 +
 (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a
^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b -
 b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*s...
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^5{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**5/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh{(x)}^5}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 592 vs. 2(58) = 116.

Time = 0.52 (sec) , antiderivative size = 592, normalized size of antiderivative = 8.46

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) - 8/3*(3*(sqrt(a + b)*e^(2*x) - sqrt(a*e
^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^5 + 3*(sqrt(a + b
)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b
))^4*sqrt(a + b) + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2
*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(3*a - 5*b) + 6*(sqrt(a + b)*e^(2*x)
- sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2*sqrt(
a + b)*(a - 3*b) - 3*(3*a^2 + 6*a*b - 13*b^2)*(sqrt(a + b)*e^(2*x) - sqrt(
a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) - (9*a^2 - 22*
a*b + 17*b^2)*sqrt(a + b))/((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4
*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqr
t(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b)
+ a - 3*b)^3
```

Mupad [B] (verification not implemented)

Time = 3.39 (sec) , antiderivative size = 65, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\left(b\tanh(x)^2+a\right)^{3/2}}{3\,b^2} - \left(\frac{a+b}{b^2} - \frac{2\,a}{b^2}\right)\sqrt{b\tanh\left(x\right)^2+a}$$

input
```
int(tanh(x)^5/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x
)^2)^(3/2)/(3*b^2) - ((a + b)/b^2 - (2*a)/b^2)*(a + b*tanh(x)^2)^(1/2)
```

Reduce [F]

$$\int \frac{\tanh^5(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$
$$= \frac{-\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2 b+2\sqrt{\tanh(x)^2 b+a}\,a+3\left(\int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^3}{\tanh(x)^2 b+a}dx\right)b^2}{3b^2}$$

input
```
int(tanh(x)^5/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
( - sqrt(tanh(x)**2*b + a)*tanh(x)**2*b + 2*sqrt(tanh(x)**2*b + a)*a + 3*i
nt((sqrt(tanh(x)**2*b + a)*tanh(x)**3)/(tanh(x)**2*b + a),x)*b**2)/(3*b**2
)
```

3.230 $\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(a-2b)\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{2b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

output
```
1/2*(a-2*b)*arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh
((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-1/2*tanh(x)*(a+b*t
anh(x)^2)^(1/2)/b
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 3.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.36

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$
$$= \frac{\left(\sqrt{2}a(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}\operatorname{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\operatorname{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right)-2\sqrt{2}ab\sqrt{\cdots}\right.}{2\sqrt{2}b(a\cdots}$$

input
```
Integrate[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((Sqrt[2]*a*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*Ellipt
icF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1] -
2*Sqrt[2]*a*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*EllipticPi[b
/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]],
 1] - (a + b)*(a - b + (a + b)*Cosh[2*x])*Sech[x]^2)*Tanh[x])/(2*Sqrt[2]*b
*(a + b)*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])
```

Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.08, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4153, 381, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$
$$\Big\downarrow \; 3042$$
$$\int \frac{\tan(ix)^4}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{381}$$

$$\frac{\int \frac{a-(a-2b)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\Big\downarrow \text{398}$$

$$\frac{(a-2b)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)+2b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{2b}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\Big\downarrow \text{224}$$

$$\frac{2b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)+(a-2b)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{2b}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{2b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)+\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\Big\downarrow \text{291}$$

$$\frac{2b\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}+\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}}{2b}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{(a-2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{2b\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}}{2b}-\frac{\tanh(x)\sqrt{a+b\tanh^2(x)}}{2b}$$

input
```
Int[Tanh[x]^4/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(((a - 2*b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] + (2
*b*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b])/(2*b
) - (Tanh[x]*Sqrt[a + b*Tanh[x]^2])/(2*b)
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 381
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
+ 1)/(b*d*(m + 2*(p + q) + 1))), x] - Simp[e^4/(b*d*(m + 2*(p + q) + 1))
Int[(e*x)^(m - 4)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*c*(m - 3) + (a*d*(m +
2*q - 1) + b*c*(m + 2*p - 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 3] && IntBinomialQ[a, b, c, d, e, m, 2
, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(70) = 140$.

Time = 0.08 (sec) , antiderivative size = 178, normalized size of antiderivative = 2.02

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}}-\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2b^{\frac{3}{2}}}+\frac{\ln\left(2\cdots\right.}{}$
default	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{\sqrt{b}}-\frac{\tanh(x)\sqrt{a+b\tanh(x)^2}}{2b}+\frac{a\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{2b^{\frac{3}{2}}}+\frac{\ln\left(2\cdots\right.}{}$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)-1/2*tanh(x)*(a+b*tanh(x
)^2)^(1/2)/b+1/2*a/b^(3/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))+1/2/(
a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh
(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)
+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1027 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 5494, normalized size of antiderivative = 62.43

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^4{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**4/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh{(x)}^4}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(70) = 140$.

Time = 0.41 (sec) , antiderivative size = 559, normalized size of antiderivative = 6.35

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(a - 2*b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b) -
 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) -
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) - 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(a + 2*b) + (sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2*(3*a - 2*b)*sqrt(a + b) + (3*a^2 - 3*a*b - 2*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) +
 (a^2 - a*b + 2*b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt
(a + b) + a - 3*b)^2*b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^4}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^4}{\tanh(x)^2 b+a}dx$$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x)**4)/(tanh(x)**2*b + a),x)
```

3.231 $\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 47

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-(a+b*tanh(x)^2)^(1/
2)/b
```

Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 47, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{\sqrt{a+b\tanh^2(x)}}{b}$$

input
```
Integrate[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b] - Sqrt[a + b*Tanh[x
]^2]/b
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.11, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 4153, 26, 354, 90, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\tan(ix)^3}{\sqrt{a-b\tan(ix)^2}}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\tan(ix)^3}{\sqrt{a-b\tan(ix)^2}}dx$$
$$\Big\downarrow \text{4153}$$
$$i\int -\frac{i\tanh^3(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$
$$\Big\downarrow \text{26}$$
$$\int \frac{\tanh^3(x)}{(1-\tanh^2(x))\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2}\int \frac{\tanh^2(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$
$$\Big\downarrow \text{90}$$

$$\frac{1}{2}\left(\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)-\frac{2\sqrt{a+b\tanh^2(x)}}{b}\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}-\frac{2\sqrt{a+b\tanh^2(x)}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\sqrt{a+b\tanh^2(x)}}{b}\right)$$

input
```
Int[Tanh[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b] - (2*Sqrt[a +
b*Tanh[x]^2])/b)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 90
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[b*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(d*f*(n + p + 2))),
 x] + Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p + 1)))/(d*f*(n + p
+ 2))   Int[(c + d*x)^n*(e + f*x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, n,
p}, x] && NeQ[n + p + 2, 0]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 128 vs. $2(39) = 78$.

Time = 0.06 (sec) , antiderivative size = 129, normalized size of antiderivative = 2.74

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(\tanh(x}{}\right.}{}$
default	$-\frac{\sqrt{a+b\tanh(x)^2}}{b}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(\tanh(x}{}\right.}{}$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-(a+b*tanh(x)^2)^(1/2)/b+1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+
b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+
b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(t
anh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(39) = 78$.

Time = 0.17 (sec) , antiderivative size = 1625, normalized size of antiderivative = 34.57

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((b*cosh(x)^2 + 2*b*cosh(x)*sinh(x) + b*sinh(x)^2 + b)*sqrt(a + b)*lo
g(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^
2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3
+ a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 +
 a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (
70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a
^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a
^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3
 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(
a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b -
b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^
2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 +
3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*
sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*c
osh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*c
osh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(
a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2
*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 +
 a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^3 + (2*a^3 + 3
*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*c...
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^3{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 345 vs. 2(39) = 78.

Time = 0.32 (sec) , antiderivative size = 345, normalized size of antiderivative = 7.34

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx =$$

$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\right.}{\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}\right.}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) - 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/((sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) + a - 3*b)
```

Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.83

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\sqrt{b\tanh(x)^2+a}}{b}$$

input
```
int(tanh(x)^3/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2) - (a + b*tanh(x
)^2)^(1/2)/b
```

Reduce [F]

$$\int \frac{\tanh^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^3}{\tanh(x)^2 b+a}\,dx$$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x)**3)/(tanh(x)**2*b + a),x)
```

3.232 $\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 60

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

output
```
-arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(1/2)+arctanh((a+b)^(1/2
)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.28 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.68

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=$$

$$-\frac{a\coth(x)\,\text{EllipticPi}\left(\frac{b}{a+b},\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right)\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(}{b(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}$$

input
```
Integrate[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-((a*Coth[x]*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x]
)*Csch[x]^2)/b]/Sqrt[2]], 1]*Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2])/
(b*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 25, 4153, 25, 385, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^2}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ix)^2}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int -\frac{\tanh^2(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(x)}{(1-\tanh^2(x))\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{385}$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \textbf{224}$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}$$

$$\Big\downarrow \textbf{219}$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \textbf{291}$$

$$\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}$$

input
```
Int[Tanh[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[b]) + ArcTanh[(Sqr
t[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 385
```
Int[(((e_.)*(x_))^(m_)*((c_) + (d_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2),
x_Symbol] :> Simp[e^2/b   Int[(e*x)^(m - 2)*(c + d*x^2)^q, x], x] - Simp[a*
(e^2/b)   Int[(e*x)^(m - 2)*((c + d*x^2)^q/(a + b*x^2)), x], x] /; FreeQ[{a
, b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LeQ[2, m, 3] && IntBinomial
Q[a, b, c, d, e, m, 2, -1, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(48) = 96$.

Time = 0.07 (sec) , antiderivative size = 137, normalized size of antiderivative = 2.28

method	result
derivativedivides	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{\sqrt{b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^{2}+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}-$
default	$-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^{2}}\right)}{\sqrt{b}}+\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^{2}+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}-$

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))/b^(1/2)+1/2/(a+b)^(1/2)*ln((2*a
+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(
1/2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/
2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 496 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 3361, normalized size of antiderivative = 56.02

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh^2{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(tanh(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh{(x)}^2}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 252 vs. $2(48) = 96$.

Time = 0.32 (sec) , antiderivative size = 252, normalized size of antiderivative = 4.20

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sqrt(-b))/sqrt(-b) - 1/2*log(ab
s(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e
^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) - 1/2*log(abs
(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(
2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a + b)*e^(2*
x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqr
t(a + b)))/sqrt(a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh(x)^2}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2}{\tanh(x)^2 b+a}dx$$

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x)**2)/(tanh(x)**2*b + a),x)
```

3.233 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 29

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.467$, Rules used = {3042, 26, 4153, 26, 353, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x)}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)}{(1-\tanh^2(x))\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\Big\downarrow \text{73}$$

$$\frac{\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}$$

$$\Big\downarrow \text{221}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(23) = 46$.

Time = 0.14 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.93

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(\tanh(x)+1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)}}{\tanh(x)+1}\right.}{2\sqrt{a+b}}$
default	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}}+\frac{\ln\left(\frac{2a+2b-2b(\tanh(x)+1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)}}{\tanh(x)+1}\right.}{2\sqrt{a+b}}$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2
+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*
(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(ta
nh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(23) = 46$.

Time = 0.13 (sec) , antiderivative size = 1361, normalized size of antiderivative = 46.93

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*s
inh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^
3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*co
sh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2
+ b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 +
 b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(
x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh
(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)
*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4
+ 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sin
h(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6
+ 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)
^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*
cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*
b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cos
h(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a
 - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*co
sh(x)^7 + 3*(2*a^3 + a^2*b)*cosh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*co
sh(x)^3 + (2*a^3 + 3*a^2*b - b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)
^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(...
```

Sympy [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 68, normalized size of antiderivative = 2.34

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\begin{cases} \frac{\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} & \text{for}\ b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for}\ \sqrt{a}=0 \\ \frac{\log\left(2\sqrt{a}\tanh^2(x)-2\sqrt{a}\right)}{2\sqrt{a}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
-Piecewise((atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/sqrt(-a - b), Ne(b,
0)), (Piecewise((zoo*tanh(x)**2, Eq(sqrt(a), 0)), (log(2*sqrt(a)*tanh(x)**
2 - 2*sqrt(a))/(2*sqrt(a)), True)), True))
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\tanh\left(x\right)}{\sqrt{b\tanh\left(x\right)^2+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. 2(23) = 46.

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 6.48

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2\,ae^{(2x)}-2\,be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.75 (sec) , antiderivative size = 23, normalized size of antiderivative = 0.79

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
int(tanh(x)/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(1/2)
```

Reduce [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)}{\tanh(x)^2 b+a}dx$$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**2*b + a),x)
```

3.234 $\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 31

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

input
```
Integrate[1/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4144, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}dx$$
$$\Big\downarrow \ 3042$$
$$\int \frac{1}{\sqrt{a-b\tan(ix)^2}}dx$$
$$\Big\downarrow \ 4144$$
$$\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$
$$\Big\downarrow \ 291$$
$$\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{a+b\tanh^2(x)}}d\frac{\tanh(x)}{\sqrt{a+b\tanh^2(x)}}$$
$$\Big\downarrow \ 219$$
$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}$$

input
```
Int[1/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 113 vs. $2(25) = 50$.

Time = 0.05 (sec) , antiderivative size = 114, normalized size of antiderivative = 3.68

method	result
derivativedivides	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2b(\tanh(x)+1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)}{\tanh(x)+1}\right.}{2\sqrt{a+b}}$
default	$\frac{\ln\left(\frac{2a+2b+2b(\tanh(x)-1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}{\tanh(x)-1}\right)}{2\sqrt{a+b}} - \frac{\ln\left(\frac{2a+2b-2b(\tanh(x)+1)+2\sqrt{a+b}\,\sqrt{b(\tanh(x)}{\tanh(x)+1}\right.}{2\sqrt{a+b}}$

input
```
int(1/(a+b*tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2
+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)^(1/2)*ln((2*a+2*b-2*b*
(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(ta
nh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. $2(25) = 50$.

Time = 0.13 (sec) , antiderivative size = 1287, normalized size of antiderivative = 41.52

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*(sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*
sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b
^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*c
osh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 +
 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 +
6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh
(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cos
h(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3
)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4
 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*si
nh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6
 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x
)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2
*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a
*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*co
sh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 +
a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*c
osh(x)^7 - 3*(a*b^2 + 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*c
osh(x)^3 + (a^3 - 3*a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x
)^5*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh...
```

Sympy [F]

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(25) = 50$.

Time = 0.21 (sec) , antiderivative size = 188, normalized size of antiderivative = 6.06

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) -
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b)
```

Mupad [B] (verification not implemented)

Time = 2.63 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx=\frac{\operatorname{atanh}\left(\frac{\tanh(x)\sqrt{a+b}}{\sqrt{b\tanh(x)^2+a}}\right)}{\sqrt{a+b}}$$

input
```
int(1/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
atanh((tanh(x)*(a + b)^(1/2))/(a + b*tanh(x)^2)^(1/2))/(a + b)^(1/2)
```

Reduce [F]

$$\int \frac{1}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\sqrt{\tanh(x)^2 b+a}}{\tanh(x)^2 b+a}dx$$

input
```
int(1/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)/(tanh(x)**2*b + a),x)
```

3.235 $\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 56

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

output
```
-arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(1/2)+arctanh((a+b*tanh(x)^2)^(1
/2)/(a+b)^(1/2))/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=-\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}+\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}$$

input
```
Integrate[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-(ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]]/Sqrt[a]) + ArcTanh[Sqrt[a + b*Tan
h[x]^2]/Sqrt[a + b]]/Sqrt[a + b]
```

Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 61, normalized size of antiderivative = 1.09, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4153, 26, 354, 97, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\tan(ix)\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$i\int -\frac{i\coth(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\Big\downarrow \text{97}$$

$$\frac{1}{2}\left(\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)\right)$$

$$\Big\downarrow \text{ 73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}\right)$$

$$\Big\downarrow \text{ 221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}\right)$$

input
```
Int[Coth[x]/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((-2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (2*ArcTanh[Sqrt[a +
 b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 97
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[b/(b*c - a*d)   Int[(e + f*x)^p/(a + b*x), x], x] - Simp[d/(b*c
 - a*d)   Int[(e + f*x)^p/(c + d*x), x], x] /; FreeQ[{a, b, c, d, e, f, p},
 x] &&  !IntegerQ[p]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh(x)^2}}dx$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 488 vs. $2(44) = 88$.

Time = 0.22 (sec) , antiderivative size = 3371, normalized size of antiderivative = 60.20

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(44) = 88.

Time = 0.32 (sec) , antiderivative size = 254, normalized size of antiderivative = 4.54

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2\sqrt{a+b}}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/sqrt(-a) - 1/2*log(abs
(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^
(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) + 1/2*log(abs(
-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2
*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a + b)*e^(2*x
) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt
(a + b)))/sqrt(a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
int(coth(x)/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)/(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^2 b+a}\,\coth\left(x\right)}{\tanh\left(x\right)^2 b+a}dx$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*coth(x))/(tanh(x)**2*b + a),x)
```

3.236 $\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}} \, dx$

Optimal result

Integrand size = 17, antiderivative size = 51

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}} \, dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(1/2)-coth(x)*(a+
b*tanh(x)^2)^(1/2)/a
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.92 (sec) , antiderivative size = 130, normalized size of antiderivative = 2.55

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}} \, dx = -\frac{\cosh^2(x)\coth(x)\left(1+\frac{b\tanh^2(x)}{a}\right)\left(-\frac{4(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{5}{2},-\frac{(a+b)\sinh^2(x)}{a}\right)\sinh^2(x)\left(a+b\tanh^2(x)\right)}{3a^2} + \frac{\arcsin\left(\right.}{a\sqrt{-}}\right.}{\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
-((Cosh[x]^2*Coth[x]*(1 + (b*Tanh[x]^2)/a)*((-4*(a + b)*Hypergeometric2F1[
2, 2, 5/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh[x]^2*(a + b*Tanh[x]^2))/(3*a^2)
+ (ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + 2*b*Tanh[x]^2))/(a*Sqrt[-((
(a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)])))/Sqrt[a + b*Tanh[x]
^2])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 25, 4153, 25, 382, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan(ix)^2\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \text{382}$$

$$\frac{\int \frac{a}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

$$\Big\downarrow \text{27}$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}$$

input
```
Int[Coth[x]^2/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/Sqrt[a + b] - (Coth[x
]*Sqrt[a + b*Tanh[x]^2])/a
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 382
```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_)
, x_Symbol] :> Simp[(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/
(a*c*e*(m + 1))), x] - Simp[1/(a*c*e^2*(m + 1))   Int[(e*x)^(m + 2)*(a + b*
x^2)^p*(c + d*x^2)^q*Simp[(b*c + a*d)*(m + 3) + 2*(b*c*p + a*d*q) + b*d*(m
+ 2*p + 2*q + 5)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[
b*c - a*d, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\coth(x)^2}{\sqrt{a+b\tanh(x)^2}}dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(43) = 86$.

Time = 0.14 (sec) , antiderivative size = 1565, normalized size of antiderivative = 30.69

$$\int \frac{\coth^2(x)}{\sqrt{a + b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*((a*cosh(x)^2 + 2*a*cosh(x)*sinh(x) + a*sinh(x)^2 - a)*sqrt(a + b)*lo
g(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 +
 b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b
^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2
+ 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 +
(70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 +
2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 +
2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^
3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*
(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2
*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b
^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 +
 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))
*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*
cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*
cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt
(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 -
2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2
+ 2*b^3)*cosh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^3 + (a^3 - 3*
a*b^2 - 2*b^3)*cosh(x))*sinh(x))/(cosh(x)^6 + 6*cosh(x)^5*sinh(x) + 15*...
```

Sympy [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth^2{(x)}}{\sqrt{a+b\tanh^2{(x)}}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)**2/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth{(x)}^2}{\sqrt{b\tanh{(x)}^2+a}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(43) = 86$.

Time = 0.31 (sec) , antiderivative size = 343, normalized size of antiderivative = 6.73

$$\int \frac{\coth^2(x)}{\sqrt{a + b\tanh^2(x)}}\,dx = -\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)(a+b) - \sqrt{a+b}(a-b)\right|\right)}{2\sqrt{a+b}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)} + \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} - \sqrt{a+b}\right|\right)}{2\sqrt{a+b}} + \frac{4\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b} + \cdots\right.}{\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)} + 2ae^{(2x)} - 2be^{(2x)} + a + b}\right)^2 - 2\left(\sqrt{a+b}e^{(2x)} - \sqrt{ae^{(4x)} + be^{(4x)}\cdots\right.}$$

input

```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output

```
-1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) -
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) + 4*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x
) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/((sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*
x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) - 3*a + b)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\coth(x)^2}{\sqrt{b\tanh\left(x\right)^2+a}}\,dx$$

input
```
int(coth(x)^2/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^2/(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^2 b+a}\,\coth\left(x\right)^2}{\tanh\left(x\right)^2 b+a}dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*coth(x)**2)/(tanh(x)**2*b + a),x)
```

3.237 $\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = -\frac{(2a-b)\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{2a^{3/2}} + \frac{\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}} - \frac{\coth^2(x)\sqrt{a+b\tanh^2(x)}}{2a}$$

output
```
-1/2*(2*a-b)*arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*t
anh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-1/2*coth(x)^2*(a+b*tanh(x)^2)^(1/
2)/a
```

Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 107, normalized size of antiderivative = 1.22

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \frac{(-2a^2-ab+b^2)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)+\sqrt{a}\left(2a\sqrt{a+b}\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)-(a+b)\coth^2(x)\sqrt{a}\right.}{2a^{3/2}(a+b)}$$

input
```
Integrate[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
((-2*a^2 - a*b + b^2)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]] + Sqrt[a]*(2*
a*Sqrt[a + b]*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]] - (a + b)*Coth[x]
^2*Sqrt[a + b*Tanh[x]^2]))/(2*a^(3/2)*(a + b))
```

Rubi [A] (warning: unable to verify)

Time = 0.35 (sec) , antiderivative size = 96, normalized size of antiderivative = 1.09, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.588$, Rules used = {3042, 26, 4153, 26, 354, 114, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i}{\tan(ix)^3\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{1}{\tan(ix)^3\sqrt{a-b\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\coth^3(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)$$

$$\Big\downarrow \; 26$$

$$\int \frac{\coth^3(x)}{\left(1-\tanh^2(x)\right)\sqrt{a+b\tanh^2(x)}}d\tanh(x)$$

$$\Big\downarrow \; 354$$

$$\frac{1}{2}\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)$$

$$\Big\downarrow \; 114$$

$$\frac{1}{2}\left(-\frac{\int -\frac{\coth(x)\left(b\tanh^2(x)+2a-b\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\Big\downarrow \; 27$$

$$\frac{1}{2}\left(\frac{\int \frac{\coth(x)\left(b\tanh^2(x)+2a-b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(\frac{2a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(2a-b)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{4a\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(2a-b)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{4a\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(2a-b)\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{2a}-\frac{\coth(x)\sqrt{a+b\tanh^2(x)}}{a}\right)$$

input
```
Int[Coth[x]^3/Sqrt[a + b*Tanh[x]^2],x]
```

output
```
(((-2*(2*a - b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (4*a*Arc
Tanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(2*a) - (Coth[x]*Sqr
t[a + b*Tanh[x]^2])/a)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 114
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_] :> Simp[b*(a + b*x)^(m + 1)*(c + d*x)^(n + 1)*((e + f*x)^(p + 1
)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + Simp[1/((m + 1)*(b*c - a*d)*(b*e
 - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n*(e + f*x)^p*Simp[a*d*f*(m + 1)
 - b*(d*e*(m + n + 2) + c*f*(m + p + 2)) - b*d*f*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && ILtQ[m, -1] && (IntegerQ[n] ||
 IntegersQ[2*n, 2*p] || ILtQ[m + n + p + 3, 0])
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\coth(x)^3}{\sqrt{a+b\tanh(x)^2}}dx$$

input
```
int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1033 vs. $2(70) = 140$.

Time = 0.32 (sec) , antiderivative size = 5555, normalized size of antiderivative = 63.12

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx$$

input
```
integrate(coth(x)**3/(a+b*tanh(x)**2)**(1/2),x)
```

output
```
Integral(coth(x)**3/sqrt(a + b*tanh(x)**2), x)
```

Maxima [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{\coth(x)^3}{\sqrt{b\tanh(x)^2+a}}\,dx$$

input
```
integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^3/sqrt(b*tanh(x)^2 + a), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(70) = 140$.

Time = 0.40 (sec) , antiderivative size = 565, normalized size of antiderivative = 6.42

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
(2*a - b)*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) +
2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a) -
 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2
*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/sqrt(a + b) +
1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/sqrt(a + b) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/sqrt(a + b) + 2*((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*
x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^3*(2*a + b) + (sqrt(a
 + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b))^2*(2*a - 3*b)*sqrt(a + b) - (2*a^2 + 3*a*b - 3*b^2)*(sqrt(a + b)*e^
(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)) -
 (2*a^2 - a*b + b^2)*sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*
x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt
(a + b) - 3*a + b)^2*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx = \int \frac{{\mathrm{coth}\left(x\right)}^3}{\sqrt{b\,{\mathrm{tanh}\left(x\right)}^2+a}}\,dx$$

input
```
int(coth(x)^3/(a + b*tanh(x)^2)^(1/2),x)
```

output
```
int(coth(x)^3/(a + b*tanh(x)^2)^(1/2), x)
```

Reduce [F]

$$\int \frac{\coth^3(x)}{\sqrt{a+b\tanh^2(x)}}\,dx=\int \frac{\sqrt{\tanh(x)^2 b+a}\,\coth(x)^3}{\tanh(x)^2 b+a}dx$$

input
```
int(coth(x)^3/(a+b*tanh(x)^2)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*coth(x)**3)/(tanh(x)**2*b + a),x)
```

3.238 $\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 72

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}} - \frac{\sqrt{a+b\tanh^2(x)}}{b^2}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-a^2/b^2/(a+b)/(a+b*
tanh(x)^2)^(1/2)-(a+b*tanh(x)^2)^(1/2)/b^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 67, normalized size of antiderivative = 0.93

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{-b^2\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right) - (a+b)\left(2a-b+b\tanh^2(x)\right)}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-(b^2*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) - (a +
b)*(2*a - b + b*Tanh[x]^2))/(b^2*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.412, Rules used = {3042, 26, 4153, 26, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b(a+b)\left(b\tanh^2(x)+a\right)^{3/2}}-\frac{1}{b\sqrt{b\tanh^2(x)+a}}-\frac{1}{(a+b)\left(\tanh^2(x)-1\right)\sqrt{b\tanh^2(x)+a}}\right)d\tanh^2(x)$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^2}{b^2(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2\sqrt{a+b\tanh^2(x)}}{b^2}\right)$$

input
```
Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - (2*a^2)/(b
^2*(a + b)*Sqrt[a + b*Tanh[x]^2]) - (2*Sqrt[a + b*Tanh[x]^2])/b^2)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. $2(62) = 124$.

Time = 0.08 (sec) , antiderivative size = 322, normalized size of antiderivative = 4.47

method	result
derivativedivides	$\frac{1}{b\sqrt{a+b\tanh(x)^2}}-\frac{\tanh(x)^2}{b\sqrt{a+b\tanh(x)^2}}-\frac{2a}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$
default	$\frac{1}{b\sqrt{a+b\tanh(x)^2}}-\frac{\tanh(x)^2}{b\sqrt{a+b\tanh(x)^2}}-\frac{2a}{b^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+$

input
```
int(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/(a+b*tanh(x)^2)^(1/2)-tanh(x)^2/b/(a+b*tanh(x)^2)^(1/2)-2*a/b^2/(a+b*t
anh(x)^2)^(1/2)-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a
+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-
1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)/(b*(tanh
(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)-b/(a+b)*(2*b*(tanh(x)+1)-2*b)/(4*b*(a+
b)-4*b^2)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((
2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b
)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1713 vs. $2(62) = 124$.

Time = 0.33 (sec) , antiderivative size = 3991, normalized size of antiderivative = 55.43

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh^5{(x)}}{\left(a+b\tanh^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**5/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh{(x)}^5}{\left(b\tanh{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 460 vs. $2(62) = 124$.

Time = 0.39 (sec) , antiderivative size = 460, normalized size of antiderivative = 6.39

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{(a^4b+a^3b^2)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5} + \frac{a^4b+a^3b^2}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2+2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}-\right.\right.}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^4*b + a^3*b^2)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) + (a^4*b + a^3*b
^2)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b
)*(a - b)))/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a +
b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2) - 4*(sqr
t(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) - sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 + 2*(sqrt(a + b)*e^(2*x) - sqrt(
a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) +
a - 3*b)*b)
```

Mupad [B] (verification not implemented)

Time = 3.84 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.97

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2\,a+2\,b)}{2\,(a+b)^{3/2}}\right)}{(a+b)^{3/2}}-\frac{\sqrt{b\tanh\left(x\right)^2+a}}{b^2}-\frac{a^2}{b^2\,\left(a+b\right)\,\sqrt{b\tanh\left(x\right)^2+a}}$$

input
```
int(tanh(x)^5/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(2*a + 2*b))/(2*(a + b)^(3/2)))/(a + b)^(3/
2) - (a + b*tanh(x)^2)^(1/2)/b^2 - a^2/(b^2*(a + b)*(a + b*tanh(x)^2)^(1/2
))
```

Reduce [F]

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{-\sqrt{\tanh\left(x\right)^2b+a}\,\tanh\left(x\right)^2b-2\sqrt{\tanh\left(x\right)^2b+a}\,a+\sqrt{\tanh\left(x\right)^2b+a}\,b+}{b^2\left(\tanh\right.}$$

input
```
int(tanh(x)^5/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
( - sqrt(tanh(x)**2*b + a)*tanh(x)**2*b - 2*sqrt(tanh(x)**2*b + a)*a + sqr
t(tanh(x)**2*b + a)*b + int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**4*b
**2 + 2*tanh(x)**2*a*b + a**2),x)*tanh(x)**2*b**3 + int((sqrt(tanh(x)**2*b
 + a)*tanh(x))/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*b + a**2),x)*a*b**2)/(b**
2*(tanh(x)**2*b + a))
```

3.239 $\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} + \frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

output

```
-arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(3/2)+arctanh((a+b)^(1/2
)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+a*tanh(x)/b/(a+b)/(a+b*tanh(x
)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.79 (sec) , antiderivative size = 188, normalized size of antiderivative = 2.24

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{a\left(-2a-2b+\sqrt{2}(a+b)\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a+b)\cosh(2x))\text{csch}^2(x)}{b}}}{\sqrt{2}}\right),1\right)\right.}{\sqrt{2}b(a+b)^2\sqrt{(a-b+(a}$$

input
```
Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((a*(-2*a - 2*b + Sqrt[2]*(a + b)*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[
x]^2)/b]*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/
Sqrt[2]], 1] + Sqrt[2]*b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]*E
llipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b
]/Sqrt[2]], 1])*Tanh[x])/(Sqrt[2]*b*(a + b)^2*Sqrt[(a - b + (a + b)*Cosh[2
*x])*Sech[x]^2]))
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.18, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 4153, 372, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(ix)^4}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{372}$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\int \frac{a-(a+b)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}$$

$$\Big\downarrow \text{398}$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+b)\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}$$

$$\Big\downarrow \text{224}$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+b)\int \frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-b\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}$$

$$\Big\downarrow \text{291}$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-b\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{b(a+b)}$$

$$\Big\downarrow \text{219}$$

$$\frac{a\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\frac{(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-\frac{b\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}}{b(a+b)}$$

input
```
Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((((a + b)*ArcTanh[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] - (b
*ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b])/(b*(a
+ b))) + (a*Tanh[x])/(b*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 327 vs. $2(70) = 140$.

Time = 0.05 (sec) , antiderivative size = 328, normalized size of antiderivative = 3.90

method	result
derivativedivides	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{b\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{b^{\frac{3}{2}}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\text{ta}}$
default	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{b\sqrt{a+b\tanh(x)^2}}-\frac{\ln\left(\sqrt{b}\,\tanh(x)+\sqrt{a+b\tanh(x)^2}\right)}{b^{\frac{3}{2}}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\text{ta}}$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-tanh(x)/a/(a+b*tanh(x)^2)^(1/2)+tanh(x)/b/(a+b*tanh(x)^2)^(1/2)-1/b^(3/2)
*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*
(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(
tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tan
h(x)-1))+1/2/(a+b)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+b/(a+b)*(2*
b*(tanh(x)+1)-2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)
^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(
x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1364 vs. $2(70) = 140$.

Time = 0.37 (sec) , antiderivative size = 6833, normalized size of antiderivative = 81.35

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh^4{(x)}}{(a+b\tanh^2{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**4/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^4}{(b\tanh(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 371 vs. 2(70) = 140.

Time = 0.32 (sec) , antiderivative size = 371, normalized size of antiderivative = 4.42

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^3b^3+2a^2b^4+ab^5} - \frac{a^3b^2+a^2b^3}{a^3b^3+2a^2b^4+ab^5}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} - \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}}{2\sqrt{-b}}\right)}{\sqrt{-b}b} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^3*b^3 + 2*a^2*b^4 + a*b^5) - (a^3*b^2 + a^
2*b^3)/(a^3*b^3 + 2*a^2*b^4 + a*b^5))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) - 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b))/sq
rt(-b))/(sqrt(-b)*b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a
 - b)))/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(
3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)^4}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\sqrt{\tanh\left(x\right)^2 b+a}\,\tanh\left(x\right)^4}{\tanh\left(x\right)^4 b^2+2\tanh\left(x\right)^2 ab+a^2}dx$$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x)**4)/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*
b + a**2),x)
```

3.240 $\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 52

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)+a/b/(a+b)/(a+b*tanh
(x)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 0.09 (sec) , antiderivative size = 52, normalized size of antiderivative = 1.00

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{b(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]]/(a + b)^(3/2) + a/(b*(a + b)*Sq
rt[a + b*Tanh[x]^2])
```

Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 58, normalized size of antiderivative = 1.12, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.471$, Rules used = {3042, 26, 4153, 26, 354, 87, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int \frac{i\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$i\int \frac{\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$i\int -\frac{i\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{ 354}$$

$$\frac{1}{2}\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{ 87}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}+\frac{2a}{b(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \textbf{73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}+\frac{2a}{b(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \textbf{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{2a}{b(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) + (2*a)/(b*(
a + b)*Sqrt[a + b*Tanh[x]^2]))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(44) = 88$.

Time = 0.06 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.52

method	result
derivativedivides	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}} + \frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\mathrm{t}}}$
default	$\frac{1}{b\sqrt{a+b\tanh(x)^2}} - \frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}} + \frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\mathrm{t}}}$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
1/b/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^
(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b
*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b
)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b
)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)-b/(a+b)*(2*b*(tanh(x)+1)-2*b
)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+1/2/(a+b)^
(3/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh
(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 980 vs. $2(44) = 88$.

Time = 0.18 (sec) , antiderivative size = 2525, normalized size of antiderivative = 48.56

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a*b + b^2)*cosh(x)^4 + 4*(a*b + b^2)*cosh(x)*sinh(x)^3 + (a*b + b^
2)*sinh(x)^4 + 2*(a*b - b^2)*cosh(x)^2 + 2*(3*(a*b + b^2)*cosh(x)^2 + a*b
- b^2)*sinh(x)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(x)^3 + (a*b - b^2)*cosh
(x))*sinh(x))*sqrt(a + b)*log(((a^3 + a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*c
osh(x)*sinh(x)^7 + (a^3 + a^2*b)*sinh(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 +
 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a
^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b
- a*b^2 + b^3)*cosh(x)^4 + (70*(a^3 + a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b -
 a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*
b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*cosh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b
^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*
b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*co
sh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x
)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*si
nh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2
*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 +
(15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^
2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b -
b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(
x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 ...
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\tanh^3\left(x\right)}{\left(a+b\tanh^2\left(x\right)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**3/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\tanh(x)^3}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 287 vs. 2(44) = 88.

Time = 0.26 (sec) , antiderivative size = 287, normalized size of antiderivative = 5.52

$$\begin{aligned}\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx&=\frac{\frac{(a^3+a^2b)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^3+a^2b}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}\\&-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}\\&+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}\\&-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}\end{aligned}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
((a^3 + a^2*b)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^3 + a^2*b)/(a^3*b
+ 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(
a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) - 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 3.32 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.87

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}+\frac{a}{\left(b^2+a\,b\right)\,\sqrt{b\tanh\left(x\right)^2+a}}$$

input
```
int(tanh(x)^3/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) + a/((a*b + b^2
)*(a + b*tanh(x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\sqrt{\tanh\left(x\right)^2 b+a}+\left(\int\frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)}{\tanh(x)^4 b^2+2\tanh(x)^2 ab+a^2}dx\right)\tanh\left(x\right)^2 b^2+\left(\int\frac{\sqrt{\tanh(\ldots}}{\tanh(x)^4 b^2\ldots}\right.}{b\left(\tanh\left(x\right)^2 b+a\right)}$$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
(sqrt(tanh(x)**2*b + a) + int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**4
*b**2 + 2*tanh(x)**2*a*b + a**2),x)*tanh(x)**2*b**2 + int((sqrt(tanh(x)**2
*b + a)*tanh(x))/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*b + a**2),x)*a*b)/(b*(t
anh(x)**2*b + a))
```

3.241 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}} dx$

Optimal result

Integrand size = 17, antiderivative size = 53

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}} dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}} - \frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)-tanh(x)/(a+
b)/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 112 vs. 2(53) = 106.

Time = 1.01 (sec) , antiderivative size = 112, normalized size of antiderivative = 2.11

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}} dx = \frac{\tanh(x)\left(\operatorname{arctanh}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)\left(b+a\coth^2(x)\right)\sqrt{\frac{(a+b)\tanh^2(x)}{a}}-(a+b)\sqrt{}\right.}{(a+b)^2\sqrt{a+b\tanh^2(x)}\sqrt{1+\frac{b\tanh^2(x)}{a}}}$$

input
```
Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(Tanh[x]*(ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a]]*(
b + a*Coth[x]^2)*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*Sqrt[1 + (b*Tanh[x]
^2)/a]))/((a + b)^2*Sqrt[a + b*Tanh[x]^2]*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 25, 4153, 25, 373, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int -\frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{373}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow 219$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) - Tanh[
x]/((a + b)*Sqrt[a + b*Tanh[x]^2])
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(45) = 90$.

Time = 0.06 (sec) , antiderivative size = 289, normalized size of antiderivative = 5.45

method	result
derivativedivides	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2}}$
default	$-\frac{\tanh(x)}{a\sqrt{a+b\tanh(x)^2}}-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2}}$

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-tanh(x)/a/(a+b*tanh(x)^2)^(1/2)-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1
)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1
)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)
+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1
/2/(a+b)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)
+1)-2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)-1/2
/(a+b)^(3/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*
b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 791 vs. $2(45) = 90$.

Time = 0.19 (sec) , antiderivative size = 2211, normalized size of antiderivative = 41.72

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a
 + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(-((a*b
^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*si
nh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6 - 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3
)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2 + b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)
*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b
^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*c
osh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*c
osh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^
2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 +
 b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*cosh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3
*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(
x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*sinh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^
2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)
^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 + (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^
2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^
5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*
sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x
)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b^2 + b^3)*cosh(x)^7 - 3*(a*b^2 + 2*b...
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(tanh(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)^2}{(b\tanh(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(45) = 90$.

Time = 0.26 (sec) , antiderivative size = 293, normalized size of antiderivative = 5.53

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = -\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3} - \frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a^2*b + a*b^2)/(a
^3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b
)))/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2)
 + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\tanh(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{-\sqrt{\tanh\left(x\right)^2b+a}\,\tanh\left(x\right)+\left(\int \frac{\sqrt{\tanh(x)^2b+a}}{\tanh(x)^4b^2+2\tanh(x)^2ab+a^2}dx\right)\tanh\left(x\right)^2ab+\left(\right.}{a\left(\tanh\left(x\right)^2b+a\right)}$$

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
( - sqrt(tanh(x)**2*b + a)*tanh(x) + int(sqrt(tanh(x)**2*b + a)/(tanh(x)**
4*b**2 + 2*tanh(x)**2*a*b + a**2),x)*tanh(x)**2*a*b + int(sqrt(tanh(x)**2*
b + a)/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*b + a**2),x)*a**2)/(a*(tanh(x)**2
*b + a))
```

3.242 $\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 49

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)-1/(a+b)/(a+b*tanh(x
)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=-\frac{\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)}{(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-(Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*Sqrt
[a + b*Tanh[x]^2]))
```

Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 54, normalized size of antiderivative = 1.10, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.533$, Rules used = {3042, 26, 4153, 26, 353, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \text{221}$$

$$\frac{1}{2}\left(\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)
*Sqrt[a + b*Tanh[x]^2]))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 272 vs. $2(41) = 82$.

Time = 0.05 (sec) , antiderivative size = 273, normalized size of antiderivative = 5.57

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}$
default	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(
x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1
/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/2/(a+b)/(b*(tanh(x)+1)^2-2*b*(t
anh(x)+1)+a+b)^(1/2)-b/(a+b)*(2*b*(tanh(x)+1)-2*b)/(4*b*(a+b)-4*b^2)/(b*(t
anh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(ta
nh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(
x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 856 vs. $2(41) = 82$.

Time = 0.20 (sec) , antiderivative size = 2277, normalized size of antiderivative = 46.47

$$\int \frac{\tanh(x)}{\left(a + b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4
 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2 + 4*((a
 + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*log(((a^3
+ a^2*b)*cosh(x)^8 + 8*(a^3 + a^2*b)*cosh(x)*sinh(x)^7 + (a^3 + a^2*b)*sin
h(x)^8 + 2*(2*a^3 + a^2*b)*cosh(x)^6 + 2*(2*a^3 + a^2*b + 14*(a^3 + a^2*b)
*cosh(x)^2)*sinh(x)^6 + 4*(14*(a^3 + a^2*b)*cosh(x)^3 + 3*(2*a^3 + a^2*b)*
cosh(x))*sinh(x)^5 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^4 + (70*(a^3
+ a^2*b)*cosh(x)^4 + 6*a^3 + 4*a^2*b - a*b^2 + b^3 + 30*(2*a^3 + a^2*b)*co
sh(x)^2)*sinh(x)^4 + 4*(14*(a^3 + a^2*b)*cosh(x)^5 + 10*(2*a^3 + a^2*b)*co
sh(x)^3 + (6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2
*b + 3*a*b^2 + b^3 + 2*(2*a^3 + 3*a^2*b - b^3)*cosh(x)^2 + 2*(14*(a^3 + a^
2*b)*cosh(x)^6 + 15*(2*a^3 + a^2*b)*cosh(x)^4 + 2*a^3 + 3*a^2*b - b^3 + 3*
(6*a^3 + 4*a^2*b - a*b^2 + b^3)*cosh(x)^2)*sinh(x)^2 + sqrt(2)*(a^2*cosh(x
)^6 + 6*a^2*cosh(x)*sinh(x)^5 + a^2*sinh(x)^6 + 3*a^2*cosh(x)^4 + 3*(5*a^2
*cosh(x)^2 + a^2)*sinh(x)^4 + 4*(5*a^2*cosh(x)^3 + 3*a^2*cosh(x))*sinh(x)^
3 + (3*a^2 + 2*a*b - b^2)*cosh(x)^2 + (15*a^2*cosh(x)^4 + 18*a^2*cosh(x)^2
 + 3*a^2 + 2*a*b - b^2)*sinh(x)^2 + a^2 + 2*a*b + b^2 + 2*(3*a^2*cosh(x)^5
 + 6*a^2*cosh(x)^3 + (3*a^2 + 2*a*b - b^2)*cosh(x))*sinh(x))*sqrt(a + b)*s
qrt(((a + b)*cosh(x)^2 + (a + b)*sinh(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)
*sinh(x) + sinh(x)^2)) + 4*(2*(a^3 + a^2*b)*cosh(x)^7 + 3*(2*a^3 + a^2*...
```

Sympy [A] (verification not implemented)

Time = 9.04 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.88

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{2(a+b)\sqrt{a+b\tanh^2(x)}}+\frac{b\,\operatorname{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}(a+b)}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } a^{\frac{3}{2}}=0 \\ \frac{\log\left(2a^{\frac{3}{2}}\tanh^2(x)-2a^{\frac{3}{2}}\right)}{2a^{\frac{3}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
-Piecewise((2*(b/(2*(a + b)*sqrt(a + b*tanh(x)**2)) + b*atan(sqrt(a + b*ta
nh(x)**2)/sqrt(-a - b))/(2*sqrt(-a - b)*(a + b)))/b, Ne(b, 0)), (Piecewise
((zoo*tanh(x)**2, Eq(a**(3/2), 0)), (log(2*a**(3/2)*tanh(x)**2 - 2*a**(3/2
))/(2*a**(3/2)), True)), True))
```

Maxima [F]

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\tanh(x)}{(b\tanh(x)^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(41) = 82$.

Time = 0.27 (sec) , antiderivative size = 292, normalized size of antiderivative = 5.96

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = -\frac{\frac{(a^2b+ab^2)e^{(2x)}}{a^3b+2a^2b^2+ab^3}+\frac{a^2b+ab^2}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b + a*b^2)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) + (a^2*b + a*b^2)/(a
^3*b + 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*
e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b
)))/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2)
 - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.84

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{1}{(a+b)\,\sqrt{b\tanh\left(x\right)^2+a}}$$

input
```
int(tanh(x)/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
atanh((a + b*tanh(x)^2)^(1/2)/(a + b)^(1/2))/(a + b)^(3/2) - 1/((a + b)*(a
 + b*tanh(x)^2)^(1/2))
```

Reduce [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^2 b+a}\,\tanh\left(x\right)}{\tanh\left(x\right)^4 b^2+2\tanh\left(x\right)^2 ab+a^2}dx$$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*b +
 a**2),x)
```

3.243 $\int \frac{1}{(a+b\tanh^2(x))^{3/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 56

$$\int \frac{1}{(a+b\tanh^2(x))^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*tanh(x)/a
/(a+b)/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 2.96 (sec) , antiderivative size = 223, normalized size of antiderivative = 3.98

$$\int \frac{1}{(a+b\tanh^2(x))^{3/2}}\,dx=$$

$$-\frac{\sinh^2(x)\left(\frac{15}{4}a(3a-2b+(3a+2b)\cosh(2x))\operatorname{csch}(x)\operatorname{sech}(x)\left((a-b)\arcsin\left(\sqrt{-\frac{(a+b)\sinh^2(x)}{a}}\right)+(a+b\right.\right.}{}$$

input
```
Integrate[(a + b*Tanh[x]^2)^(-3/2),x]
```

output
```
-1/15*(Sinh[x]^2*((15*a*(3*a - 2*b + (3*a + 2*b)*Cosh[2*x])*Csch[x]*Sech[x
]*((a - b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (a + b)*ArcSin[Sqrt[-(
((a + b)*Sinh[x]^2)/a)]]*Cosh[2*x] - 2*a*Sqrt[-(((a + b)*(b + a*Coth[x]^2)
*Sinh[x]^4)/a^2)]))/4 + Sqrt[2]*a^2*(a + b)*Hypergeometric2F1[2, 2, 7/2, -
(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*(a - b + (a + b)*Cosh[2*x])*Sinh[x]^2
)/a^2))^(3/2)*Tanh[x]))/(a^4*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]
^2 + (b*Sinh[x]^2)/a]*Sqrt[a + b*Tanh[x]^2])
```

Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.417$, Rules used = {3042, 4144, 296, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{1}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{296}$$

$$\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Int[(a + b*Tanh[x]^2)^(-3/2),x]
```

output
```
ArcTanh[(Sqrt[a + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]]/(a + b)^(3/2) + (b*Ta
nh[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2])
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 296
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[(b*c + 2*(p + 1)*(b*c - a*d))/(2*a*(p + 1)*(b*c - a*d))   Int[
(a + b*x^2)^(p + 1)*(c + d*x^2)^q, x], x] /; FreeQ[{a, b, c, d, q}, x] && N
eQ[b*c - a*d, 0] && EqQ[2*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ[q, -1
]) && NeQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. 2(48) = 96.

Time = 0.04 (sec) , antiderivative size = 272, normalized size of antiderivative = 4.86

method	result
derivativedivides	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}$
default	$-\frac{1}{2(a+b)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}+\frac{b(2b(\tanh(x)-1)+2b)}{(a+b)(4b(a+b)-4b^2)\sqrt{b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b}}$

input
```
int(1/(a+b*tanh(x)^2)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(
x)-1)+2*b)/(4*b*(a+b)-4*b^2)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1
/2/(a+b)^(3/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+
2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/2/(a+b)/(b*(tanh(x)+1)^2-2*b*(t
anh(x)+1)+a+b)^(1/2)+b/(a+b)*(2*b*(tanh(x)+1)-2*b)/(4*b*(a+b)-4*b^2)/(b*(t
anh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)-1/2/(a+b)^(3/2)*ln((2*a+2*b-2*b*(ta
nh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(
x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(48) = 96$.

Time = 0.20 (sec) , antiderivative size = 2439, normalized size of antiderivative = 43.55

$$\int \frac{1}{\left(a + b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
[1/4*(((a^2 + a*b)*cosh(x)^4 + 4*(a^2 + a*b)*cosh(x)*sinh(x)^3 + (a^2 + a*
b)*sinh(x)^4 + 2*(a^2 - a*b)*cosh(x)^2 + 2*(3*(a^2 + a*b)*cosh(x)^2 + a^2
- a*b)*sinh(x)^2 + a^2 + a*b + 4*((a^2 + a*b)*cosh(x)^3 + (a^2 - a*b)*cosh
(x))*sinh(x))*sqrt(a + b)*log(-((a*b^2 + b^3)*cosh(x)^8 + 8*(a*b^2 + b^3)*
cosh(x)*sinh(x)^7 + (a*b^2 + b^3)*sinh(x)^8 - 2*(a*b^2 + 2*b^3)*cosh(x)^6
- 2*(a*b^2 + 2*b^3 - 14*(a*b^2 + b^3)*cosh(x)^2)*sinh(x)^6 + 4*(14*(a*b^2
+ b^3)*cosh(x)^3 - 3*(a*b^2 + 2*b^3)*cosh(x))*sinh(x)^5 + (a^3 - a^2*b + 4
*a*b^2 + 6*b^3)*cosh(x)^4 + (70*(a*b^2 + b^3)*cosh(x)^4 + a^3 - a^2*b + 4*
a*b^2 + 6*b^3 - 30*(a*b^2 + 2*b^3)*cosh(x)^2)*sinh(x)^4 + 4*(14*(a*b^2 + b
^3)*cosh(x)^5 - 10*(a*b^2 + 2*b^3)*cosh(x)^3 + (a^3 - a^2*b + 4*a*b^2 + 6*
b^3)*cosh(x))*sinh(x)^3 + a^3 + 3*a^2*b + 3*a*b^2 + b^3 + 2*(a^3 - 3*a*b^2
 - 2*b^3)*cosh(x)^2 + 2*(14*(a*b^2 + b^3)*cosh(x)^6 - 15*(a*b^2 + 2*b^3)*c
osh(x)^4 + a^3 - 3*a*b^2 - 2*b^3 + 3*(a^3 - a^2*b + 4*a*b^2 + 6*b^3)*cosh(
x)^2)*sinh(x)^2 + sqrt(2)*(b^2*cosh(x)^6 + 6*b^2*cosh(x)*sinh(x)^5 + b^2*s
inh(x)^6 - 3*b^2*cosh(x)^4 + 3*(5*b^2*cosh(x)^2 - b^2)*sinh(x)^4 + 4*(5*b^
2*cosh(x)^3 - 3*b^2*cosh(x))*sinh(x)^3 - (a^2 - 2*a*b - 3*b^2)*cosh(x)^2 +
 (15*b^2*cosh(x)^4 - 18*b^2*cosh(x)^2 - a^2 + 2*a*b + 3*b^2)*sinh(x)^2 - a
^2 - 2*a*b - b^2 + 2*(3*b^2*cosh(x)^5 - 6*b^2*cosh(x)^3 - (a^2 - 2*a*b - 3
*b^2)*cosh(x))*sinh(x))*sqrt(a + b)*sqrt(((a + b)*cosh(x)^2 + (a + b)*sinh
(x)^2 + a - b)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4*(2*(a*b...
```

Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(a+b\tanh^2{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(-3/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{1}{\left(b\tanh{(x)}^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(-3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 288 vs. $2(48) = 96$.

Time = 0.27 (sec) , antiderivative size = 288, normalized size of antiderivative = 5.14

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \frac{\frac{(ab^2+b^3)e^{(2x)}}{a^3b+2a^2b^2+ab^3} - \frac{ab^2+b^3}{a^3b+2a^2b^2+ab^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
((a*b^2 + b^3)*e^(2*x)/(a^3*b + 2*a^2*b^2 + a*b^3) - (a*b^2 + b^3)/(a^3*b
+ 2*a^2*b^2 + a*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))/(
a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*
x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(3/2) + 1/
2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{1}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}\,dx$$

input
```
int(1/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(1/(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^2 b+a}}{\tanh\left(x\right)^4 b^2+2\tanh\left(x\right)^2 ab+a^2}dx$$

input
```
int(1/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*b + a**2),x)
```

3.244 $\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}} dx$

Optimal result

Integrand size = 15, antiderivative size = 78

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}} dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} + \frac{b}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

output
```
-arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(3/2)+arctanh((a+b*tanh(x)^2)^(1
/2)/(a+b)^(1/2))/(a+b)^(3/2)+b/a/(a+b)/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.90

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}} dx = \frac{-a\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\,\text{Hypergeometric2F1}}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Integrate[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(-(a*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b)
*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Tanh[x]^2)/a])/(a*(a + b)*Sqrt[a +
 b*Tanh[x]^2])
```

Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 98, normalized size of antiderivative = 1.26, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 26, 4153, 26, 354, 96, 25, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\tan(ix)\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$i\int -\frac{i\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\Big\downarrow \; 96$$

$$\frac{1}{2}\left(\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\int -\frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}\right)$$

$$\Big\downarrow \; 25$$

$$\frac{1}{2}\left(\frac{\int \frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \; 174$$

$$\frac{1}{2}\left(\frac{a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(a+b)\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{2a\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(a+b)\int \frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{2a\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)\,\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b}{a(a+b)\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Coth[x]/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(((-2*(a + b)*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (2*a*ArcTa
nh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) + (2*b)/(a
*(a + b)*Sqrt[a + b*Tanh[x]^2]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\coth \left(x \right)}{\left(a +b \tanh \left(x \right)^{2}\right)^{\frac{3}{2}}}d x$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1345 vs. $2(64) = 128$.

Time = 0.37 (sec) , antiderivative size = 6799, normalized size of antiderivative = 87.17

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\coth{(x)}}{(a+b\tanh^2{(x)})^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(coth(x)/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \int \frac{\coth{(x)}}{(b\tanh{(x)}^2+a)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(64) = 128$.

Time = 0.33 (sec) , antiderivative size = 372, normalized size of antiderivative = 4.77

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx = \frac{\frac{(a^3b^2+a^2b^3)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3} + \frac{a^3b^2+a^2b^3}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}} + \frac{2\arctan\left(-\frac{\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}}{2\sqrt{-a}}\right)}{\sqrt{-a}a} - \frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}} + \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}} - \frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$

input

```
integrate(coth(x)/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output

```
((a^3*b^2 + a^2*b^3)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) + (a^3*b^2 + a^
2*b^3)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(
2*x) - 2*b*e^(2*x) + a + b) + 2*arctan(-1/2*(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b))/sq
rt(-a))/(sqrt(-a)*a) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a
 - b)))/(a + b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x)
+ b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a + b)^(
3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a
*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx=\int \frac{\coth(x)}{\left(b\tanh(x)^2+a\right)^{3/2}}\,dx$$

input
```
int(coth(x)/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)/(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{3/2}}\,dx=\int \frac{\sqrt{\tanh(x)^2 b+a}\,\coth(x)}{\tanh(x)^4 b^2+2\tanh(x)^2 ab+a^2}dx$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*coth(x))/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*b +
 a**2),x)
```

3.245 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 85

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a^2(a+b)}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(3/2)+b*coth(x)/a
/(a+b)/(a+b*tanh(x)^2)^(1/2)-(a+2*b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a^2/(a+
b)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 10.65 (sec) , antiderivative size = 263, normalized size of antiderivative = 3.09

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx =$$

$$-\frac{\cosh^2(x)\coth(x)\left(\frac{8(a+b)\cosh^2(x)\,{}_3F_2\left(2,2,2;1,\frac{7}{2};-\frac{(a+b)\sinh^2(x)}{a}\right)\left(ia\tanh(x)+ib\tanh^3(x)\right)^2}{15a^3}-\frac{8(a+b)\,\text{Hypergeometric2F1}\left(2,2,\frac{7}{2},\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
-((Cosh[x]^2*Coth[x]*((8*(a + b)*Cosh[x]^2*HypergeometricPFQ[{2, 2, 2}, {1
, 7/2}, -(((a + b)*Sinh[x]^2)/a)]*(I*a*Tanh[x] + I*b*Tanh[x]^3)^2)/(15*a^3
) - (8*(a + b)*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Sinh[x]^2)/a)]*Sinh
[x]^2*(2*a^2 + 5*a*b*Tanh[x]^2 + 3*b^2*Tanh[x]^4))/(15*a^3) - (Coth[x]^2*(
3*a^2 + 12*a*b*Tanh[x]^2 + 8*b^2*Tanh[x]^4)*(ArcSin[Sqrt[-(((a + b)*Sinh[x
]^2)/a)]]*(-a - b*Tanh[x]^2) + a*Sech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[
x]^2*(a + b*Tanh[x]^2))/a^2)]))/(a^2*(a + b)*Sqrt[-(((a + b)*Cosh[x]^2*Sin
h[x]^2*(a + b*Tanh[x]^2))/a^2)])))/(a*Sqrt[a + b*Tanh[x]^2]))
```

Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.07, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.647$, Rules used = {3042, 25, 4153, 25, 374, 25, 445, 25, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{1}{\tan(ix)^2\left(a-b\tan(ix)^2\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{25}$$

$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{374}$$

$$\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\int -\frac{\coth^2(x)\left(-2b\tanh^2(x)+a+2b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)}$$

$$\Big\downarrow \text{25}$$

$$\frac{\int \frac{\coth^2(x)\left(-2b\tanh^2(x)+a+2b\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \text{445}$$

$$\frac{-\frac{\int -\frac{a^2}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \text{25}$$

$$\frac{\frac{\int \frac{a^2}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a}-\frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \text{27}$$

$$\frac{a\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x) - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \ 291$$

$$\frac{a\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

$$\Big\downarrow \ 219$$

$$\frac{\frac{a\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}} - \frac{(a+2b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)} + \frac{b\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}$$

input
```
Int[Coth[x]^2/(a + b*Tanh[x]^2)^(3/2),x]
```

output
```
(b*Coth[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) + ((a*ArcTanh[(Sqrt[a + b]*T
anh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b] - ((a + 2*b)*Coth[x]*Sqrt[a +
b*Tanh[x]^2])/a)/(a*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\coth(x)^2}{\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1615 vs. $2(75) = 150$.

Time = 0.33 (sec) , antiderivative size = 3859, normalized size of antiderivative = 45.40

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*tanh(x)**2)**(3/2),x)
```

output
```
Integral(coth(x)**2/(a + b*tanh(x)**2)**(3/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = \int \frac{\coth(x)^2}{\left(b\tanh(x)^2+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/(b*tanh(x)^2 + a)^(3/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 459 vs. $2(75) = 150$.

Time = 0.38 (sec) , antiderivative size = 459, normalized size of antiderivative = 5.40

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx = -\frac{\frac{(a^2b^3+ab^4)e^{(2x)}}{a^5b+2a^4b^2+a^3b^3} - \frac{a^2b^3+ab^4}{a^5b+2a^4b^2+a^3b^3}}{\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}}$$
$$-\frac{\log\left(\left|-\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)(a+b)-\sqrt{a+b}(a-b)\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$-\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{\log\left(\left|-\sqrt{a+b}e^{(2x)}+\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}-\sqrt{a+b}\right|\right)}{2(a+b)^{\frac{3}{2}}}$$
$$+\frac{4\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}+\right.}{\left(\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+2ae^{(2x)}-2be^{(2x)}+a+b}\right)^2-2\left(\sqrt{a+b}e^{(2x)}-\sqrt{ae^{(4x)}+be^{(4x)}+\right.\right.}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x, algorithm="giac")
```

output
```
-((a^2*b^3 + a*b^4)*e^(2*x)/(a^5*b + 2*a^4*b^2 + a^3*b^3) - (a^2*b^3 + a*b
^4)/(a^5*b + 2*a^4*b^2 + a^3*b^3))/sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x
) - 2*b*e^(2*x) + a + b) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b
)*(a - b)))/(a + b)^(3/2) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4
*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b)))/(a +
b)^(3/2) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) +
 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a + b)))/(a + b)^(3/2) + 4*(sqr
t(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) + sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x
) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))^2 - 2*(sqrt(a + b)*e^(2*x) - sqrt(
a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*sqrt(a + b) -
3*a + b)*a)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{{\mathrm{coth}\left(x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}^{3/2}} \,d x$$

input
```
int(coth(x)^2/(a + b*tanh(x)^2)^(3/2),x)
```

output
```
int(coth(x)^2/(a + b*tanh(x)^2)^(3/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{3/2}}\,dx=\int \frac{\sqrt{\tanh \left(x \right)^{2} b +a}\, \coth \left(x \right)^{2}}{\tanh \left(x \right)^{4} b^{2}+2 \tanh \left(x \right)^{2} a b +a^{2}}d x$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(3/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*coth(x)**2)/(tanh(x)**4*b**2 + 2*tanh(x)**2*a*
b + a**2),x)
```

3.246 $\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 118

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{b^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{a(a+2b)\tanh(x)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
-arctanh(b^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/b^(5/2)+arctanh((a+b)^(1/2
)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*a*tanh(x)^3/b/(a+b)/(a+b*
tanh(x)^2)^(3/2)+a*(a+2*b)*tanh(x)/b^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.32 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.96

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\sqrt{(a-b+(a+b)\cosh(2x))\operatorname{sech}^2(x)}\,\Big(-\frac{3\sqrt{2}a\coth(x)\big((a^2+3ab+2b^2)\operatorname{EllipticF}\big(\arcsin\big(\cdots}{\cdots}\cdots}{\cdots}$$

input
```
Integrate[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((-3*Sqrt[2]*a*Coth[x]*((a^2
+ 3*a*b + 2*b^2)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x
]^2)/b]/Sqrt[2]], 1] + b^2*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a
+ b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cos
h[2*x])*Csch[x]^2)/b]) + (a*(a + b)*(3*a^2 + 2*a*b - 7*b^2 + (3*a^2 + 10*a
*b + 7*b^2)*Cosh[2*x])*Sinh[2*x])/(a - b + (a + b)*Cosh[2*x])^2))/(3*Sqrt[
2]*b^2*(a + b)^3)
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.25, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.706$, Rules used = {3042, 25, 4153, 25, 372, 27, 440, 398, 224, 219, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^6(x)}{(a+b\tanh^2(x))^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int -\frac{\tan(ix)^6}{(a-b\tan(ix)^2)^{5/2}}dx$$

$$\Big\downarrow \; 25$$

$$-\int \frac{\tan(ix)^6}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$\Big\downarrow$ 4153

$$-\int -\frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$\Big\downarrow$ 25

$$\int \frac{\tanh^6(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$\Big\downarrow$ 372

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int \frac{3\tanh^2(x)\left(a-(a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{3b(a+b)}$$

$\Big\downarrow$ 27

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int \frac{\tanh^2(x)\left(a-(a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{b(a+b)}$$

$\Big\downarrow$ 440

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{\int \frac{a(a+2b)-(a+b)^2\tanh^2(x)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$\Big\downarrow$ 398

$$\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\int \frac{1}{\sqrt{b\tanh^2(x)+a}}d\tanh(x)-b^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$\Big\downarrow$ 224

$$\frac{\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{(a+b)^2\int\frac{1}{1-\frac{b\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-b^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$\Big\downarrow$ 219

$$\frac{\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-b^2\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$\Big\downarrow$ 291

$$\frac{\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-b^2\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

$\Big\downarrow$ 219

$$\frac{\frac{a\tanh^3(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+b)^2\text{arctanh}\left(\frac{\sqrt{b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{b}}-\frac{b^2\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}}{b(a+b)}-\frac{a(a+2b)\tanh(x)}{b(a+b)\sqrt{a+b\tanh^2(x)}}}{b(a+b)}$$

input
```
Int[Tanh[x]^6/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(a*Tanh[x]^3)/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((((a + b)^2*ArcTanh
[(Sqrt[b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[b] - (b^2*ArcTanh[(Sqrt[a
+ b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b])/(b*(a + b)) - (a*(a + 2
*b)*Tanh[x])/(b*(a + b)*Sqrt[a + b*Tanh[x]^2]))/(b*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 398
```
Int[((e_) + (f_.)*(x_)^2)/(((a_) + (b_.)*(x_)^2)*Sqrt[(c_) + (d_.)*(x_)^2])
, x_Symbol] :> Simp[f/b   Int[1/Sqrt[c + d*x^2], x], x] + Simp[(b*e - a*f)/
b   Int[1/((a + b*x^2)*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d, e, f}
, x]
```

rule 440
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[g*(b*e - a*f)*(g*x)^(m - 1)*(a +
 b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] - Simp[
g^2/(2*b*(b*c - a*d)*(p + 1))   Int[(g*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c +
d*x^2)^q*Simp[c*(b*e - a*f)*(m - 1) + (d*(b*e - a*f)*(m + 2*q + 1) - b*2*(c
*f - d*e)*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, q}, x] &&
 LtQ[p, -1] && GtQ[m, 1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 548 vs. $2(100) = 200$.

Time = 0.08 (sec) , antiderivative size = 549, normalized size of antiderivative = 4.65

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)^3}{3b\left(a+b\tanh(x)\right.}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)^3}{3b\left(a+b\tanh(x)\right.}$

input
```
int(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
+1/3*tanh(x)/b/(a+b*tanh(x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)
+1/3*tanh(x)^3/b/(a+b*tanh(x)^2)^(3/2)+1/b^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
-1/b^(5/2)*ln(b^(1/2)*tanh(x)+(a+b*tanh(x)^2)^(1/2))-1/6/(a+b)/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(
x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+
a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)
+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(
a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(tanh(x)+1)^2-2*b*(tanh(
x)+1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)
*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*tanh(
x)+1/2/(a+b)^2/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)+1/2/(a+b)^2/a/(
b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*b*tanh(x)-1/2/(a+b)^(5/2)*ln((2
*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)
^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4472 vs. $2(100) = 200$.

Time = 1.13 (sec) , antiderivative size = 19265, normalized size of antiderivative = 163.26

$$\int \frac{\tanh^6(x)}{\left(a + b\tanh^2(x)\right)^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**6/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**6/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)^6}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^6/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(100) = 200.

Time = 0.43 (sec) , antiderivative size = 805, normalized size of antiderivative = 6.82

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((3*a^9*b^8 + 22*a^8*b^9 + 65*a^7*b^10 + 100*a^6*b^11 + 85*a^5*b^12
+ 38*a^4*b^13 + 7*a^3*b^14)*e^(2*x)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 +
 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16) + 3*(a^9*b^8 + 2*a^8*b
^9 - 9*a^7*b^10 - 36*a^6*b^11 - 49*a^5*b^12 - 30*a^4*b^13 - 7*a^3*b^14)/(a
^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^1
5 + a^2*b^16))*e^(2*x) - 3*(a^9*b^8 + 2*a^8*b^9 - 9*a^7*b^10 - 36*a^6*b^11
 - 49*a^5*b^12 - 30*a^4*b^13 - 7*a^3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6
*b^12 + 20*a^5*b^13 + 15*a^4*b^14 + 6*a^3*b^15 + a^2*b^16))*e^(2*x) - (3*a
^9*b^8 + 22*a^8*b^9 + 65*a^7*b^10 + 100*a^6*b^11 + 85*a^5*b^12 + 38*a^4*b^
13 + 7*a^3*b^14)/(a^8*b^10 + 6*a^7*b^11 + 15*a^6*b^12 + 20*a^5*b^13 + 15*a
^4*b^14 + 6*a^3*b^15 + a^2*b^16))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(
4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a +
b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
+ sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 2*arctan(-1/2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) + sqrt(a + b))/sqrt(-b))/(sqrt(-b)*b^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)^6}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^6/(a + b*tanh(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh^6(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^6}{\tanh(x)^6 b^3+3\tanh(x)^4 a\,b^2+3\tanh(x)^2 a^2 b+a^3}dx$$

input

```
int(tanh(x)^6/(a+b*tanh(x)^2)^(5/2),x)
```

output

```
int((sqrt(tanh(x)**2*b + a)*tanh(x)**6)/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*
b**2 + 3*tanh(x)**2*a**2*b + a**3),x)
```

3.247 $\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 84

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}-\frac{a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/3*a^2/b^2/(a+b)/(
a+b*tanh(x)^2)^(3/2)+a*(a+2*b)/b^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.09 (sec) , antiderivative size = 68, normalized size of antiderivative = 0.81

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx=\frac{-b^2\,\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)+(a+b)\left(2a+b+3b\tanh^2\right.}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Integrate[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(-(b^2*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a +
 b)*(2*a + b + 3*b*Tanh[x]^2))/(3*b^2*(a + b)*(a + b*Tanh[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 26, 4153, 26, 354, 98, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)^5}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh^5(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{98}$$

$$\frac{1}{2}\int \left(\frac{a^2}{b(a+b)\left(b\tanh^2(x)+a\right)^{5/2}}-\frac{(a+2b)a}{b(a+b)^2\left(b\tanh^2(x)+a\right)^{3/2}}-\frac{1}{(a+b)^2\left(\tanh^2(x)-1\right)\sqrt{b\tanh^2(x)+a}}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{1}{2}\left(-\frac{2a^2}{3b^2(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}+\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}}+\frac{2a(a+2b)}{b^2(a+b)^2\sqrt{a+b\tanh^2(x)}}\right)$$

input
```
Int[Tanh[x]^5/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
((2*ArcTanh[Sqrt[a + b*Tanh[x]^2]/Sqrt[a + b]])/(a + b)^(5/2) - (2*a^2)/(3
*b^2*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (2*a*(a + 2*b))/(b^2*(a + b)^2*Sqr
t[a + b*Tanh[x]^2]))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 98
```
Int[(((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))^(p_))/((a_.) + (b_.)*(x
_)), x_] :> Int[ExpandIntegrand[(e + f*x)^FractionalPart[p], (c + d*x)^n*((
e + f*x)^IntegerPart[p]/(a + b*x)), x], x] /; FreeQ[{a, b, c, d, e, f}, x]
&& IGtQ[n, 0] && LtQ[p, -1] && FractionQ[p]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(72) = 144$.

Time = 0.06 (sec) , antiderivative size = 469, normalized size of antiderivative = 5.58

method	result
derivativedivides	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)^2}{b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{2a}{3b^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+\right.}$
default	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{\tanh(x)^2}{b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}+\frac{2a}{3b^2\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+\right.}$

input
```
int(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/(a+b*tanh(x)^2)^(3/2)+tanh(x)^2/b/(a+b*tanh(x)^2)^(3/2)+2/3*a/b^2/(a
+b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+
1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a
+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b
*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2
*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(
x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-
1))-1/6/(a+b)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)-1/6*b/(a+b)/a/(b
*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/(b*(tanh
(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)+1)^2-2
*b*(tanh(x)+1)+a+b)^(1/2)-1/2/(a+b)^2/a/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a
+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1
/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3234 vs. $2(72) = 144$.

Time = 0.53 (sec) , antiderivative size = 7033, normalized size of antiderivative = 83.73

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**5/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**5/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^5}{(b\tanh(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^5/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 711 vs. 2(72) = 144.

Time = 0.32 (sec) , antiderivative size = 711, normalized size of antiderivative = 8.46

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((a^9 + 8*a^8*b + 25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5
+ 3*a^3*b^6)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a
^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^9 + 6*a^8*b + 13*a^7*b^2 + 12*a^6*b^3
 + 3*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20
*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^9 + 6*a^8*b +
 13*a^7*b^2 + 12*a^6*b^3 + 3*a^5*b^4 - 2*a^4*b^5 - a^3*b^6)/(a^8*b^2 + 6*a
^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2
*x) + (a^9 + 8*a^8*b + 25*a^7*b^2 + 40*a^6*b^3 + 35*a^5*b^4 + 16*a^4*b^5 +
 3*a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))
/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b
)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [B] (verification not implemented)

Time = 4.93 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.10

$$\int \frac{\tanh^5(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,\left(2a^2+4ab+2b^2\right)}{2\,(a+b)^{5/2}}\right)}{(a+b)^{5/2}} - \frac{\frac{a^2}{3\,(a+b)}-\frac{\left(a^2+2\,b\,a\right)\,\left(b\tanh(x)^2+a\right)}{(a+b)^2}}{b^2\,\left(b\tanh\left(x\right)^2+a\right)^{3/2}}$$

input
```
int(tanh(x)^5/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))
/(a + b)^(5/2) - (a^2/(3*(a + b)) - ((2*a*b + a^2)*(a + b*tanh(x)^2))/(a +
 b)^2)/(b^2*(a + b*tanh(x)^2)^(3/2))
```

Reduce [F]

$$\int \frac{\tanh^5(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{3\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2 b+2\sqrt{\tanh(x)^2 b+a}\,a+\sqrt{\tanh(x)^2 b+a}\,b+3}{}$$

input
```
int(tanh(x)^5/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
(3*sqrt(tanh(x)**2*b + a)*tanh(x)**2*b + 2*sqrt(tanh(x)**2*b + a)*a + sqrt
(tanh(x)**2*b + a)*b + 3*int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**6*
b**3 + 3*tanh(x)**4*a*b**2 + 3*tanh(x)**2*a**2*b + a**3),x)*tanh(x)**4*b**
4 + 6*int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**6*b**3 + 3*tanh(x)**4
*a*b**2 + 3*tanh(x)**2*a**2*b + a**3),x)*tanh(x)**2*a*b**3 + 3*int((sqrt(t
anh(x)**2*b + a)*tanh(x))/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*b**2 + 3*tanh(
x)**2*a**2*b + a**3),x)*a**2*b**2)/(3*b**2*(tanh(x)**4*b**2 + 2*tanh(x)**2
*a*b + a**2))
```

3.248 $\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 90

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} - \frac{(a+4b)\tanh(x)}{3b(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*a*tanh(
x)/b/(a+b)/(a+b*tanh(x)^2)^(3/2)-1/3*(a+4*b)*tanh(x)/b/(a+b)^2/(a+b*tanh(x
)^2)^(1/2)
```

Mathematica [A] (verified)

Time = 1.79 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.47

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\tanh^3(x)\left(3\operatorname{arctanh}\left(\frac{\sqrt{\frac{(a+b)\tanh^2(x)}{a}}}{\sqrt{1+\frac{b\tanh^2(x)}{a}}}\right)\left(b+a\coth^2(x)\right)^2\sqrt{\frac{(a+b)\tanh^2(x)}{a}} - (a+b}{3(a+b)^3\left(a+b\tanh^2(x)\right)^{3/2}\sqrt{1+\frac{b\tanh}{a}}}$$

input
```
Integrate[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(Tanh[x]^3*(3*ArcTanh[Sqrt[((a + b)*Tanh[x]^2)/a]/Sqrt[1 + (b*Tanh[x]^2)/a
]]*(b + a*Coth[x]^2)^2*Sqrt[((a + b)*Tanh[x]^2)/a] - (a + b)*(a + 4*b + 3*
a*Coth[x]^2)*Sqrt[1 + (b*Tanh[x]^2)/a]))/(3*(a + b)^3*(a + b*Tanh[x]^2)^(3
/2)*Sqrt[1 + (b*Tanh[x]^2)/a])
```

Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 99, normalized size of antiderivative = 1.10, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.412$, Rules used = {3042, 4153, 372, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \; 3042$$

$$\int \frac{\tan(ix)^4}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \; 4153$$

$$\int \frac{\tanh^4(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \; 372$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int \frac{a-(a+3b)\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{3b(a+b)}$$

$$\Big\downarrow \; 402$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\int \frac{3ab}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)}}{3b(a+b)}$$

$$\Big\downarrow \; 27$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{3b\int\frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b}}{3b(a+b)}$$

$$\Big\downarrow \; 291$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{3b\int\frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b}}{3b(a+b)}$$

$$\Big\downarrow \; 219$$

$$\frac{a\tanh(x)}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\frac{(a+4b)\tanh(x)}{(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{3b\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}}{3b(a+b)}$$

input
```
Int[Tanh[x]^4/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(a*Tanh[x])/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((-3*b*ArcTanh[(Sqrt[a
 + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(a + b)^(3/2) + ((a + 4*b)*Tanh[x])
/((a + b)*Sqrt[a + b*Tanh[x]^2]))/(3*b*(a + b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 372
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-a)*e^3*(e*x)^(m - 3)*(a + b*x^2)^(p + 1)*((c + d*x^2
)^(q + 1)/(2*b*(b*c - a*d)*(p + 1))), x] + Simp[e^4/(2*b*(b*c - a*d)*(p + 1
))   Int[(e*x)^(m - 4)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[a*c*(m - 3) +
 (a*d*(m + 2*q - 1) + 2*b*c*(p + 1))*x^2, x], x], x] /; FreeQ[{a, b, c, d,
e, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && GtQ[m, 3] && IntBinomialQ[a
, b, c, d, e, m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 490 vs. $2(76) = 152$.

Time = 0.06 (sec) , antiderivative size = 491, normalized size of antiderivative = 5.46

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}}-\frac{}{6(a+b)\left(b(\tanh(\right.}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}+\frac{\tanh(x)}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{\tanh(x)}{3ab\sqrt{a+b\tanh(x)^2}}-\frac{}{6(a+b)\left(b(\tanh(\right.}$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
+1/3*tanh(x)/b/(a+b*tanh(x)^2)^(3/2)-1/3/a/b*tanh(x)/(a+b*tanh(x)^2)^(1/2)
-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)
*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(t
anh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)+1)^2-2*b*
(tanh(x)+1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)+1)^2-2*b*(tanh(
x)+1)+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)
^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*b*tanh(x)
-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^
2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2545 vs. $2(76) = 152$.

Time = 0.51 (sec) , antiderivative size = 5719, normalized size of antiderivative = 63.54

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh^4{(x)}}{(a+b\tanh^2{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**4/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**4/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^4(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh{(x)}^4}{(b\tanh{(x)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^4/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(76) = 152$.

Time = 0.32 (sec) , antiderivative size = 684, normalized size of antiderivative = 7.60

$$\int \frac{\tanh^4(x)}{(a + b\tanh^2(x))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*((((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b
^7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8) - 3*(a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6 + a^
2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3
*b^7 + a^2*b^8))*e^(2*x) + 3*(a^6*b^3 + 4*a^5*b^4 + 6*a^4*b^5 + 4*a^3*b^6
+ a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6
*a^3*b^7 + a^2*b^8))*e^(2*x) - (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*
b^5 + 5*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5
+ 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a
 + b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^4(x)}{(a + b\tanh^2(x))^{5/2}}\, dx = \int \frac{\tanh(x)^4}{(b\tanh(x)^2 + a)^{5/2}}\, dx$$

input
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^4/(a + b*tanh(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh^4(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{-\sqrt{\tanh\left(x\right)^{2} b+a}\,\tanh\left(x\right)^{3}+3\left(\int \frac{\sqrt{\tanh(x)^{2} b+a}\,\tanh(x)^{2}}{\tanh(x)^{6} b^{3}+3\tanh(x)^{4} a\,b^{2}+3\tanh(x)^{2} a^{2} b+a^{3}}dx\right) \mathrm{t}}{}$$

input
```
int(tanh(x)^4/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
( - sqrt(tanh(x)**2*b + a)*tanh(x)**3 + 3*int((sqrt(tanh(x)**2*b + a)*tanh
(x)**2)/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*b**2 + 3*tanh(x)**2*a**2*b + a**
3),x)*tanh(x)**4*a*b**2 + 6*int((sqrt(tanh(x)**2*b + a)*tanh(x)**2)/(tanh(
x)**6*b**3 + 3*tanh(x)**4*a*b**2 + 3*tanh(x)**2*a**2*b + a**3),x)*tanh(x)*
*2*a**2*b + 3*int((sqrt(tanh(x)**2*b + a)*tanh(x)**2)/(tanh(x)**6*b**3 + 3
*tanh(x)**4*a*b**2 + 3*tanh(x)**2*a**2*b + a**3),x)*a**3)/(3*a*(tanh(x)**4
*b**2 + 2*tanh(x)**2*a*b + a**2))
```

3.249 $\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 74

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{a}{3b(a+b)\,(a+b\tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)+1/3*a/b/(a+b)/(a+b*
tanh(x)^2)^(3/2)-1/(a+b)^2/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 63, normalized size of antiderivative = 0.85

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{a(a+b) - 3b\,\text{Hypergeometric2F1}\left(-\frac{1}{2},1,\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)(a+b\tanh^2(x))}{3b(a+b)^2\,(a+b\tanh^2(x))^{3/2}}$$

input
```
Integrate[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(a*(a + b) - 3*b*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Tanh[x]^2)/(a + b)
]*(a + b*Tanh[x]^2))/(3*b*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 86, normalized size of antiderivative = 1.16, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 26, 4153, 26, 354, 87, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$
$$\Big\downarrow \text{3042}$$
$$\int \frac{i\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$
$$\Big\downarrow \text{26}$$
$$i\int \frac{\tan(ix)^3}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$
$$\Big\downarrow \text{4153}$$
$$i\int -\frac{i\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$
$$\Big\downarrow \text{26}$$
$$\int \frac{\tanh^3(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$
$$\Big\downarrow \text{354}$$
$$\frac{1}{2}\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\Big\downarrow \ 87$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{(1-\tanh^2(x))(b\tanh^2(x)+a)^{3/2}}d\tanh^2(x)}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \ 61$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \ 73$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \ 221$$

$$\frac{1}{2}\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}+\frac{2a}{3b(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]^3/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
((2*a)/(3*b*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((2*ArcTanh[Sqrt[a + b*Tanh
[x]^2]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)*Sqrt[a + b*Tanh[x]^2]))/(a
 + b))/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(62) = 124$.

Time = 0.06 (sec) , antiderivative size = 435, normalized size of antiderivative = 5.88

method	result
derivativedivides	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-\right.}$
default	$\frac{1}{3b\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-\right.}$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
1/3/b/(a+b*tanh(x)^2)^(3/2)-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b
)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+
1/3*b/(a+b)/a^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a
+b)^2/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x
)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*
b*(tanh(x)-1)+2*(a+b)^(1/2)*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(
tanh(x)-1))-1/6/(a+b)/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)-1/6*b/(a
+b)/a/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/
(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x
)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)-1/2/(a+b)^2/a/(b*(tanh(x)+1)^2-2*b*(tanh
(x)+1)+a+b)^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*
(a+b)^(1/2)*(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3028 vs. $2(62) = 124$.

Time = 0.61 (sec) , antiderivative size = 6621, normalized size of antiderivative = 89.47

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**3/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**3/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh(x)^3}{(b\tanh(x)^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^3/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 725 vs. $2(62) = 124$.

Time = 0.32 (sec) , antiderivative size = 725, normalized size of antiderivative = 9.80

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((a^8*b + 2*a^7*b^2 - 5*a^6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b
^6 - 3*a^2*b^7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 1
5*a^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^8*b + 2*a^7*b^2 - a^6*b^3 - 4*a^5*
b^4 - a^4*b^5 + 2*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 2
0*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) + 3*(a^8*b + 2*a^7*
b^2 - a^6*b^3 - 4*a^5*b^4 - a^4*b^5 + 2*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^
7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*
x) + (a^8*b + 2*a^7*b^2 - 5*a^6*b^3 - 20*a^5*b^4 - 25*a^4*b^5 - 14*a^3*b^6
 - 3*a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6
+ 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
 + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e
^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)
))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a +
 b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x)
 + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(
a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [B] (verification not implemented)

Time = 4.71 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.11

$$\int \frac{\tanh^3(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2\,a^2+4\,a\,b+2\,b^2)}{2\,(a+b)^{5/2}}\right)}{(a+b)^{5/2}}+\frac{\frac{a}{3\,(a+b)}-\frac{b\,\left(b\tanh(x)^2+a\right)}{(a+b)^2}}{b\,\left(b\tanh\left(x\right)^2+a\right)^{3/2}}$$

input
```
int(tanh(x)^3/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))
/(a + b)^(5/2) + (a/(3*(a + b)) - (b*(a + b*tanh(x)^2))/(a + b)^2)/(b*(a +
 b*tanh(x)^2)^(3/2))
```

Reduce [F]

$$\int \frac{\tanh^3(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\sqrt{\tanh(x)^2 b+a}+3\left(\int \frac{\sqrt{\tanh(x)^2b+a}\,\tanh(x)}{\tanh(x)^6b^3+3\tanh(x)^4a\,b^2+3\tanh(x)^2a^2b+a^3}dx\right)\tanh(x)^4 b^3 -}{3b}$$

input
```
int(tanh(x)^3/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
(sqrt(tanh(x)**2*b + a) + 3*int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)*
*6*b**3 + 3*tanh(x)**4*a*b**2 + 3*tanh(x)**2*a**2*b + a**3),x)*tanh(x)**4*
b**3 + 6*int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**6*b**3 + 3*tanh(x)
**4*a*b**2 + 3*tanh(x)**2*a**2*b + a**3),x)*tanh(x)**2*a*b**2 + 3*int((sqr
t(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*b**2 + 3*ta
nh(x)**2*a**2*b + a**3),x)*a**2*b)/(3*b*(tanh(x)**4*b**2 + 2*tanh(x)**2*a*
b + a**2))
```

3.250 $\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 88

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{(2a-b)\tanh(x)}{3a(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)-1/3*tanh(x)
/(a+b)/(a+b*tanh(x)^2)^(3/2)-1/3*(2*a-b)*tanh(x)/a/(a+b)^2/(a+b*tanh(x)^2)
^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 6.60 (sec) , antiderivative size = 193, normalized size of antiderivative = 2.19

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\coth(x)\left(-12(a+b)^3\,\text{Hypergeometric2F1}\left(2,2,\frac{9}{2},-\frac{(a+b)\sinh^2(x)}{a}\right)\sinh^4(x)\,\text{ta}\right.}{}$$

input
```
Integrate[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(Coth[x]*(-12*(a + b)^3*Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)
/a)]*Sinh[x]^4*Tanh[x]^2*(a + b*Tanh[x]^2) - (35*a*Cosh[x]^2*(-5*a - 2*b*T
anh[x]^2)*(3*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*(a + b*Tanh[x]^2)^2 -
a*Sech[x]^2*Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Tanh[x]^2))/a^2)]*(
3*a + (a + 4*b)*Tanh[x]^2)))/Sqrt[-(((a + b)*Cosh[x]^2*Sinh[x]^2*(a + b*Ta
nh[x]^2))/a^2)]))/(315*a^3*(a + b)^2*(a + b*Tanh[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 97, normalized size of antiderivative = 1.10, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.529$, Rules used = {3042, 25, 4153, 25, 373, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\tan(ix)^2}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \textbf{4153}$$

$$-\int -\frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \textbf{25}$$

$$\int \frac{\tanh^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \textbf{373}$$

$$\frac{\int \frac{2\tanh^2(x)+1}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{3(a+b)}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \textbf{402}$$

$$\frac{-\frac{\int -\frac{3a}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)}-\frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \textbf{27}$$

$$\frac{\frac{3\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b}-\frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \textbf{291}$$

$$\frac{\frac{3\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b}-\frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \textbf{219}$$

$$\frac{\frac{3\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{(2a-b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3(a+b)}-\frac{\tanh(x)}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Int[Tanh[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
-1/3*Tanh[x]/((a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((3*ArcTanh[(Sqrt[a + b]*
Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(a + b)^(3/2) - ((2*a - b)*Tanh[x])/(a*(a
 + b)*Sqrt[a + b*Tanh[x]^2]))/(3*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 373
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[e*(e*x)^(m - 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q +
1)/(2*(b*c - a*d)*(p + 1))), x] - Simp[e^2/(2*(b*c - a*d)*(p + 1))   Int[(e
*x)^(m - 2)*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(m - 1) + d*(m + 2*p +
 2*q + 3)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d,
 0] && LtQ[p, -1] && GtQ[m, 1] && LeQ[m, 3] && IntBinomialQ[a, b, c, d, e,
m, 2, p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 453 vs. $2(74) = 148$.

Time = 0.06 (sec) , antiderivative size = 454, normalized size of antiderivative = 5.16

method	result
derivativedivides	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{6(a+b)a\left(b\right.}$
default	$-\frac{\tanh(x)}{3a\left(a+b\tanh(x)^2\right)^{\frac{3}{2}}}-\frac{2\tanh(x)}{3a^2\sqrt{a+b\tanh(x)^2}}-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{6(a+b)a\left(b\right.}$

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/3*tanh(x)/a/(a+b*tanh(x)^2)^(3/2)-2/3/a^2*tanh(x)/(a+b*tanh(x)^2)^(1/2)
-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)
*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(t
anh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)+1)^2-2*b*
(tanh(x)+1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)+1)^2-2*b*(tanh(
x)+1)+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)
^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*b*tanh(x)
-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^
2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2939 vs. $2(74) = 148$.

Time = 0.60 (sec) , antiderivative size = 6507, normalized size of antiderivative = 73.94

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\tanh^2(x)}{(a+b\tanh^2(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)**2/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(tanh(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\tanh\left(x\right)^2}{\left(b\tanh\left(x\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 728 vs. 2(74) = 148.

Time = 0.32 (sec) , antiderivative size = 728, normalized size of antiderivative = 8.27

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\text{Too large to display}$$

input
```
integrate(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((((3*a^7*b^2 + 14*a^6*b^3 + 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*
a^2*b^7 - a*b^8)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 +
15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^7*b^2 + 2*a^6*b^3 - a^5*b^4 - 4*a
^4*b^5 - a^3*b^6 + 2*a^2*b^7 + a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 +
20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^7*b^2 + 2*a
^6*b^3 - a^5*b^4 - 4*a^4*b^5 - a^3*b^6 + 2*a^2*b^7 + a*b^8)/(a^8*b^2 + 6*a
^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2
*x) - (3*a^7*b^2 + 14*a^6*b^3 + 25*a^5*b^4 + 20*a^4*b^5 + 5*a^3*b^6 - 2*a^
2*b^7 - a*b^8)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6
 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x
) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*
e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b
)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x
) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt
(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{{\mathrm{tanh}\left(x\right)}^2}{{\left(b\,{\mathrm{tanh}\left(x\right)}^2+a\right)}^{5/2}} \,d x$$

input
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(tanh(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\sqrt{\tanh(x)^2 b+a}\,\tanh(x)^2}{\tanh(x)^6 b^3+3\tanh(x)^4 a\,b^2+3\tanh(x)^2 a^2b+a^3}dx$$

input
```
int(tanh(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x)**2)/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*
b**2 + 3*tanh(x)**2*a**2*b + a**3),x)
```

3.251 $\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 70

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} - \frac{1}{3(a+b)\,(a+b\tanh^2(x))^{3/2}} - \frac{1}{(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b*tanh(x)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)-1/3/(a+b)/(a+b*tanh
(x)^2)^(3/2)-1/(a+b)^2/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.61

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = -\frac{\text{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right)}{3(a+b)\,(a+b\tanh^2(x))^{3/2}}$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
-1/3*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]/((a + b)*
(a + b*Tanh[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 82, normalized size of antiderivative = 1.17, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.600$, Rules used = {3042, 26, 4153, 26, 353, 61, 61, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{353}$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{61}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{(1-\tanh^2(x))\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 61$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a+b}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 73$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b(a+b)}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}}-\frac{2}{(a+b)\sqrt{a+b\tanh^2(x)}}}{a+b}-\frac{2}{3(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(-2/(3*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((2*ArcTanh[Sqrt[a + b*Tanh[x]^2
]/Sqrt[a + b]])/(a + b)^(3/2) - 2/((a + b)*Sqrt[a + b*Tanh[x]^2]))/(a + b)
)/2
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 61
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Simp[d*((
m + n + 2)/((b*c - a*d)*(m + 1)))   Int[(a + b*x)^(m + 1)*(c + d*x)^n, x],
x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[m, -1] &&  !(LtQ[n, -1] && (EqQ[a, 0
] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && IntLinearQ[a, b, c, d
, m, n, x]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 353
```
Int[(x_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^2], x] /; FreeQ[
{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(58) = 116$.

Time = 0.05 (sec) , antiderivative size = 420, normalized size of antiderivative = 6.00

method	result
derivativedivides	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+}$
default	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+}$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)
*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))-1/6/(a+b)/(b*(t
anh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)-1/6*b/(a+b)/a/(b*(tanh(x)+1)^2-2*b*
(tanh(x)+1)+a+b)^(3/2)*tanh(x)-1/3*b/(a+b)/a^2/(b*(tanh(x)+1)^2-2*b*(tanh(
x)+1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)
^(1/2)-1/2/(a+b)^2/a/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*b*tanh(x)
+1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^
2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2607 vs. $2(58) = 116$.

Time = 0.54 (sec) , antiderivative size = 5779, normalized size of antiderivative = 82.56

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [A] (verification not implemented)

Time = 11.11 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.63

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = -\begin{cases} \frac{2\left(\frac{b}{6(a+b)\left(a+b\tanh^2(x)\right)^{\frac{3}{2}}}+\frac{b}{2(a+b)^2\sqrt{a+b\tanh^2(x)}}+\frac{b\,\mathrm{atan}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{-a-b}}\right)}{2\sqrt{-a-b}(a+b)^2}\right)}{b} & \text{for } b\neq 0 \\ \begin{cases} \tilde{\infty}\tanh^2(x) & \text{for } a^{\frac{5}{2}}=0 \\ \frac{\log\left(2a^{\frac{5}{2}}\tanh^2(x)-2a^{\frac{5}{2}}\right)}{2a^{\frac{5}{2}}} & \text{otherwise} \end{cases} & \text{otherwise} \end{cases}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
-Piecewise((2*(b/(6*(a + b)*(a + b*tanh(x)**2)**(3/2)) + b/(2*(a + b)**2*s
qrt(a + b*tanh(x)**2)) + b*atan(sqrt(a + b*tanh(x)**2)/sqrt(-a - b))/(2*sq
rt(-a - b)*(a + b)**2))/b, Ne(b, 0)), (Piecewise((zoo*tanh(x)**2, Eq(a**(5
/2), 0)), (log(2*a**(5/2)*tanh(x)**2 - 2*a**(5/2))/(2*a**(5/2)), True)), T
rue))
```

Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh(x)^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 683 vs. 2(58) = 116.

Time = 0.32 (sec) , antiderivative size = 683, normalized size of antiderivative = 9.76

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-4/3*((((a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*b^5 + 5*a^3*b^6 + a^2*b
^7)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8) + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5 + a^
3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3
*b^7 + a^2*b^8))*e^(2*x) + 3*(a^7*b^2 + 4*a^6*b^3 + 6*a^5*b^4 + 4*a^4*b^5
+ a^3*b^6)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6
*a^3*b^7 + a^2*b^8))*e^(2*x) + (a^7*b^2 + 5*a^6*b^3 + 10*a^5*b^4 + 10*a^4*
b^5 + 5*a^3*b^6 + a^2*b^7)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5
+ 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x)
- 2*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*
e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a
 + b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a
 + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
 + b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.09

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \frac{\operatorname{atanh}\left(\frac{\sqrt{b\tanh(x)^2+a}\,(2\,a^2+4\,a\,b+2\,b^2)}{2\,(a+b)^{5/2}}\right)}{(a+b)^{5/2}} - \frac{\frac{1}{3\,(a+b)}+\frac{b\tanh(x)^2+a}{(a+b)^2}}{\left(b\tanh\left(x\right)^2+a\right)^{3/2}}$$

input
```
int(tanh(x)/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
atanh(((a + b*tanh(x)^2)^(1/2)*(4*a*b + 2*a^2 + 2*b^2))/(2*(a + b)^(5/2)))
/(a + b)^(5/2) - (1/(3*(a + b)) + (a + b*tanh(x)^2)/(a + b)^2)/(a + b*tanh
(x)^2)^(3/2)
```

Reduce [F]

$$\int \frac{\tanh(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\sqrt{\tanh\left(x\right)^2 b+a}\,\tanh\left(x\right)}{\tanh\left(x\right)^6 b^3+3\tanh\left(x\right)^4 a\,b^2+3\tanh\left(x\right)^2 a^2 b+a^3}dx$$

input
```
int(tanh(x)/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*tanh(x))/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*b**
2 + 3*tanh(x)**2*a**2*b + a**3),x)
```

3.252 $\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 93

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\tanh(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(5a+2b)\tanh(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*tanh(
x)/a/(a+b)/(a+b*tanh(x)^2)^(3/2)+1/3*b*(5*a+2*b)*tanh(x)/a^2/(a+b)^2/(a+b*
tanh(x)^2)^(1/2)
```

Mathematica [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 5.94 (sec) , antiderivative size = 943, normalized size of antiderivative = 10.14

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
Integrate[(a + b*Tanh[x]^2)^(-5/2),x]
```

output
```
(Cosh[x]*Sinh[x]*(1575*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]] + (3150*(a +
 b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^2)/a + (1575*(a + b)^2*
ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4)/a^2 + 2100*(-(((a + b)*S
inh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a] + 96*Hypergeometric2
F1[2, 2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*S
qrt[Cosh[x]^2 + (b*Sinh[x]^2)/a] + 24*HypergeometricPFQ[{2, 2, 2}, {1, 9/2
}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Sqrt[Cosh[x]
^2 + (b*Sinh[x]^2)/a] + (2100*b*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Tan
h[x]^2)/a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]
^2*Tanh[x]^2)/a^2 + (2100*b*(a + b)^2*ArcSin[Sqrt[-(((a + b)*Sinh[x]^2)/a)
]]*Sinh[x]^4*Tanh[x]^2)/a^3 + (2800*b*(-(((a + b)*Sinh[x]^2)/a))^(3/2)*Sqr
t[Cosh[x]^2 + (b*Sinh[x]^2)/a]*Tanh[x]^2)/a + (168*b*Hypergeometric2F1[2,
2, 9/2, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Sqrt[Co
sh[x]^2 + (b*Sinh[x]^2)/a]*Tanh[x]^2)/a + (48*b*HypergeometricPFQ[{2, 2, 2
}, {1, 9/2}, -(((a + b)*Sinh[x]^2)/a)]*(-(((a + b)*Sinh[x]^2)/a))^(7/2)*Sq
rt[Cosh[x]^2 + (b*Sinh[x]^2)/a]*Tanh[x]^2)/a + (840*b^2*ArcSin[Sqrt[-(((a
+ b)*Sinh[x]^2)/a)]]*Tanh[x]^4)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqrt[-(((a
+ b)*Sinh[x]^2)/a)]]*Sinh[x]^2*Tanh[x]^4)/a^3 + (840*b^2*(a + b)^2*ArcSin[
Sqrt[-(((a + b)*Sinh[x]^2)/a)]]*Sinh[x]^4*Tanh[x]^4)/a^4 + (1120*b^2*(-(((
a + b)*Sinh[x]^2)/a))^(3/2)*Sqrt[Cosh[x]^2 + (b*Sinh[x]^2)/a]*Tanh[x]^4...
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.14, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.583$, Rules used = {3042, 4144, 316, 402, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{1}{(1-\tanh^2(x))\,(a+b\tanh^2(x))^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{316}$$

$$\frac{b\tanh(x)}{3a(a+b)\,(a+b\tanh^2(x))^{3/2}}-\frac{\int \frac{2b\tanh^2(x)+b-3(a+b)}{(1-\tanh^2(x))\,(b\tanh^2(x)+a)^{3/2}}d\tanh(x)}{3a(a+b)}$$

$$\Big\downarrow \text{402}$$

$$\frac{b\tanh(x)}{3a(a+b)\,(a+b\tanh^2(x))^{3/2}}-\frac{-\frac{\int \frac{3a^2}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)}-\frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

$$\Big\downarrow \text{27}$$

$$\frac{b\tanh(x)}{3a(a+b)\,(a+b\tanh^2(x))^{3/2}}-\frac{-\frac{3a\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a+b}-\frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

$$\Big\downarrow \text{291}$$

$$\frac{b\tanh(x)}{3a(a+b)\,(a+b\tanh^2(x))^{3/2}}-\frac{-\frac{3a\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}}{a+b}-\frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

$$\Big\downarrow \text{219}$$

$$\frac{b\tanh(x)}{3a(a+b)\,(a+b\tanh^2(x))^{3/2}}-\frac{-\frac{3a\,\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{3/2}}-\frac{b(5a+2b)\tanh(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}$$

input
```
Int[(a + b*Tanh[x]^2)^(-5/2),x]
```

output
```
(b*Tanh[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) - ((-3*a*ArcTanh[(Sqrt[a
 + b]*Tanh[x])/Sqrt[a + b*Tanh[x]^2]])/(a + b)^(3/2) - (b*(5*a + 2*b)*Tanh
[x])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]))/(3*a*(a + b))
```

Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 316
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_), x_Symbol] :> Sim
p[(-b)*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(2*a*(p + 1)*(b*c - a*d))
), x] + Simp[1/(2*a*(p + 1)*(b*c - a*d))   Int[(a + b*x^2)^(p + 1)*(c + d*x
^2)^q*Simp[b*c + 2*(p + 1)*(b*c - a*d) + d*b*(2*(p + q + 2) + 1)*x^2, x], x
], x] /; FreeQ[{a, b, c, d, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] &&  !
( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c, d, 2,
 p, q, x]
```

rule 402
```
Int[((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_.)*((e_) + (f_.)*(x
_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^2)^(p + 1)*((c + d*x^2)^
(q + 1)/(a*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[c*(b*e - a*f) + e*2*(b*c - a*d)
*(p + 1) + d*(b*e - a*f)*(2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b
, c, d, e, f, q}, x] && LtQ[p, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 419 vs. $2(79) = 158$.

Time = 0.04 (sec) , antiderivative size = 420, normalized size of antiderivative = 4.52

method	result
derivativedivides	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+}$
default	$-\frac{1}{6(a+b)\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{b\tanh(x)}{6(a+b)a\left(b(\tanh(x)-1)^2+2b(\tanh(x)-1)+a+b\right)^{\frac{3}{2}}}+\frac{}{3(a+}$

input
```
int(1/(a+b*tanh(x)^2)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/6/(a+b)/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(t
anh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)
-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2)*tanh(x)-1/2/(a+b)^2/(b*(tanh(x)-1)^2+2*b*
(tanh(x)-1)+a+b)^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)
^(1/2)*b*tanh(x)+1/2/(a+b)^(5/2)*ln((2*a+2*b+2*b*(tanh(x)-1)+2*(a+b)^(1/2)
*(b*(tanh(x)-1)^2+2*b*(tanh(x)-1)+a+b)^(1/2))/(tanh(x)-1))+1/6/(a+b)/(b*(t
anh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(3/2)+1/6*b/(a+b)/a/(b*(tanh(x)+1)^2-2*b*
(tanh(x)+1)+a+b)^(3/2)*tanh(x)+1/3*b/(a+b)/a^2/(b*(tanh(x)+1)^2-2*b*(tanh(
x)+1)+a+b)^(1/2)*tanh(x)+1/2/(a+b)^2/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)
^(1/2)+1/2/(a+b)^2/a/(b*(tanh(x)+1)^2-2*b*(tanh(x)+1)+a+b)^(1/2)*b*tanh(x)
-1/2/(a+b)^(5/2)*ln((2*a+2*b-2*b*(tanh(x)+1)+2*(a+b)^(1/2)*(b*(tanh(x)+1)^
2-2*b*(tanh(x)+1)+a+b)^(1/2))/(tanh(x)+1))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3152 vs. $2(79) = 158$.

Time = 0.66 (sec) , antiderivative size = 6933, normalized size of antiderivative = 74.55

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a+b\tanh^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral((a + b*tanh(x)**2)**(-5/2), x)
```

Maxima [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tanh{(x)}^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^2 + a)^(-5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(79) = 158.

Time = 0.32 (sec) , antiderivative size = 714, normalized size of antiderivative = 7.68

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
2/3*((((3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*
a*b^8 + b^9)*e^(2*x)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a
^4*b^6 + 6*a^3*b^7 + a^2*b^8) + 3*(a^6*b^3 + 2*a^5*b^4 - 3*a^4*b^5 - 12*a^
3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20
*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2*x) - 3*(a^6*b^3 + 2*a^5
*b^4 - 3*a^4*b^5 - 12*a^3*b^6 - 13*a^2*b^7 - 6*a*b^8 - b^9)/(a^8*b^2 + 6*a
^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 + 6*a^3*b^7 + a^2*b^8))*e^(2
*x) - (3*a^6*b^3 + 16*a^5*b^4 + 35*a^4*b^5 + 40*a^3*b^6 + 25*a^2*b^7 + 8*a
*b^8 + b^9)/(a^8*b^2 + 6*a^7*b^3 + 15*a^6*b^4 + 20*a^5*b^5 + 15*a^4*b^6 +
6*a^3*b^7 + a^2*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(
4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a + b)*(a - b)))
/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a + b)*e^(2*x) + sq
rt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) + sqrt(a + b
)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*e^(2*x) +
 sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b) - sqrt(a
+ b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b))
```

Mupad [F(-1)]

Timed out.

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(1/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(1/(a + b*tanh(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{1}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^2 b+a}}{\tanh\left(x\right)^6 b^3+3\tanh\left(x\right)^4 a\,b^2+3\tanh\left(x\right)^2 a^2b+a^3}dx$$

input
```
int(1/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int(sqrt(tanh(x)**2*b + a)/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*b**2 + 3*tanh
(x)**2*a**2*b + a**3),x)
```

3.253 $\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}} \, dx$

Optimal result

Integrand size = 15, antiderivative size = 108

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}} \, dx = -\frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{a^{5/2}} + \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}} + \frac{b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(2a+b)}{a^2(a+b)^2\sqrt{a+b\tanh^2(x)}}$$

output
```
-arctanh((a+b*tanh(x)^2)^(1/2)/a^(1/2))/a^(5/2)+arctanh((a+b*tanh(x)^2)^(1
/2)/(a+b)^(1/2))/(a+b)^(5/2)+1/3*b/a/(a+b)/(a+b*tanh(x)^2)^(3/2)+b*(2*a+b)
/a^2/(a+b)^2/(a+b*tanh(x)^2)^(1/2)
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.

Time = 0.06 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.68

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}} \, dx = \frac{-a \operatorname{Hypergeometric2F1}\left(-\frac{3}{2},1,-\frac{1}{2},\frac{a+b\tanh^2(x)}{a+b}\right) + (a+b) \operatorname{Hypergeometric2F}}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Integrate[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(-(a*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Tanh[x]^2)/(a + b)]) + (a + b
)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Tanh[x]^2)/a])/(3*a*(a + b)*(a +
 b*Tanh[x]^2)^(3/2))
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.31, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 26, 4153, 26, 354, 96, 25, 169, 27, 174, 73, 221}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{i}{\tan(ix)\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{26}$$

$$i\int \frac{1}{\tan(ix)\left(a-b\tan(ix)^2\right)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$i\int -\frac{i\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{354}$$

$$\frac{1}{2}\int \frac{\coth(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{96}$$

$$\frac{1}{2}\left(\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int -\frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)}{a(a+b)}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{\coth(x)\left(-b\tanh^2(x)+a+b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh^2(x)}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{169}$$

$$\frac{1}{2}\left(\frac{\frac{2\int \frac{\coth(x)\left((a+b)^2-b(2a+b)\tanh^2(x)\right)}{2\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{\int \frac{\coth(x)\left((a+b)^2-b(2a+b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{174}$$

$$\frac{1}{2}\left(\frac{\frac{a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)+(a+b)^2\int \frac{\coth(x)}{\sqrt{b\tanh^2(x)+a}}d\tanh^2(x)}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2\right.}\right.$$

$$\Big\downarrow \text{73}$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2a^2\int\frac{1}{\frac{a+b}{b}-\frac{\tanh^4(x)}{b}}d\sqrt{b\tanh^2(x)+a}}{b}+\frac{2(a+b)^2\int\frac{1}{\frac{\tanh^4(x)}{b}-\frac{a}{b}}d\sqrt{b\tanh^2(x)+a}}{b}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x\right.}\right.$$

$$\Big\downarrow \; 221$$

$$\frac{1}{2}\left(\frac{\frac{\frac{2a^2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{2(a+b)^2\text{arctanh}\left(\frac{\sqrt{a+b\tanh^2(x)}}{\sqrt{a}}\right)}{\sqrt{a}}}{a(a+b)}+\frac{2b(2a+b)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{a(a+b)}+\frac{2b}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}\right.$$

input
```
Int[Coth[x]/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
((2*b)/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + (((-2*(a + b)^2*ArcTanh[Sqr
t[a + b*Tanh[x]^2]/Sqrt[a]])/Sqrt[a] + (2*a^2*ArcTanh[Sqrt[a + b*Tanh[x]^2
]/Sqrt[a + b]])/Sqrt[a + b])/(a*(a + b)) + (2*b*(2*a + b))/(a*(a + b)*Sqrt
[a + b*Tanh[x]^2]))/(a*(a + b)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 73
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Simp[p/b   Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
 d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && Lt
Q[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Denominator[m]] && IntL
inearQ[a, b, c, d, m, n, x]
```

rule 96
```
Int[((e_.) + (f_.)*(x_))^(p_)/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))),
x_] :> Simp[f*((e + f*x)^(p + 1)/((p + 1)*(b*e - a*f)*(d*e - c*f))), x] + S
imp[1/((b*e - a*f)*(d*e - c*f))   Int[(b*d*e - b*c*f - a*d*f - b*d*f*x)*((e
 + f*x)^(p + 1)/((a + b*x)*(c + d*x))), x], x] /; FreeQ[{a, b, c, d, e, f},
 x] && LtQ[p, -1]
```

rule 169
```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_)*((g_.) + (h_.)*(x_)), x_] :> Simp[(b*g - a*h)*(a + b*x)^(m + 1)*(c +
 d*x)^(n + 1)*((e + f*x)^(p + 1)/((m + 1)*(b*c - a*d)*(b*e - a*f))), x] + S
imp[1/((m + 1)*(b*c - a*d)*(b*e - a*f))   Int[(a + b*x)^(m + 1)*(c + d*x)^n
*(e + f*x)^p*Simp[(a*d*f*g - b*(d*e + c*f)*g + b*c*e*h)*(m + 1) - (b*g - a*
h)*(d*e*(n + 1) + c*f*(p + 1)) - d*f*(b*g - a*h)*(m + n + p + 3)*x, x], x],
 x] /; FreeQ[{a, b, c, d, e, f, g, h, n, p}, x] && LtQ[m, -1] && IntegersQ[
2*m, 2*n, 2*p]
```

rule 174
```
Int[(((e_.) + (f_.)*(x_))^(p_)*((g_.) + (h_.)*(x_)))/(((a_.) + (b_.)*(x_))*
((c_.) + (d_.)*(x_))), x_] :> Simp[(b*g - a*h)/(b*c - a*d)   Int[(e + f*x)^
p/(a + b*x), x], x] - Simp[(d*g - c*h)/(b*c - a*d)   Int[(e + f*x)^p/(c + d
*x), x], x] /; FreeQ[{a, b, c, d, e, f, g, h}, x]
```

rule 221
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

rule 354
```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_.), x_S
ymbol] :> Simp[1/2   Subst[Int[x^((m - 1)/2)*(a + b*x)^p*(c + d*x)^q, x], x
, x^2], x] /; FreeQ[{a, b, c, d, p, q}, x] && NeQ[b*c - a*d, 0] && IntegerQ
[(m - 1)/2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\coth (x)}{\left(a+b \tanh (x)^{2}\right)^{\frac{5}{2}}}dx$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4433 vs. $2(90) = 180$.

Time = 1.45 (sec) , antiderivative size = 19151, normalized size of antiderivative = 177.32

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\coth{(x)}}{(a+b\tanh^2{(x)})^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(coth(x)/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \int \frac{\coth{(x)}}{(b\tanh{(x)}^2+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 808 vs. $2(90) = 180$.

Time $= 0.45$ (sec) , antiderivative size $= 808$, normalized size of antiderivative $= 7.48$

$$\int \frac{\coth(x)}{(a+b\tanh^2(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
1/3*((((7*a^14*b^3 + 38*a^13*b^4 + 85*a^12*b^5 + 100*a^11*b^6 + 65*a^10*b^
7 + 22*a^9*b^8 + 3*a^8*b^9)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 +
 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8) + 3*(7*a^14*b^3 + 30*a
^13*b^4 + 49*a^12*b^5 + 36*a^11*b^6 + 9*a^10*b^7 - 2*a^9*b^8 - a^8*b^9)/(a
^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^
7 + a^10*b^8))*e^(2*x) + 3*(7*a^14*b^3 + 30*a^13*b^4 + 49*a^12*b^5 + 36*a^
11*b^6 + 9*a^10*b^7 - 2*a^9*b^8 - a^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a^1
4*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x) + (7*a
^14*b^3 + 38*a^13*b^4 + 85*a^12*b^5 + 100*a^11*b^6 + 65*a^10*b^7 + 22*a^9*
b^8 + 3*a^8*b^9)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a
^12*b^6 + 6*a^11*b^7 + a^10*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2
*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - sqrt(a*e^(
4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) - sqrt(a +
b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-sqrt(a + b)*
e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b)
+ sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqrt(a +
b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a +
b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 2*arctan(-1/2*(sqrt
(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) +
 a + b) - sqrt(a + b))/sqrt(-a))/(sqrt(-a)*a^2)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth(x)}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(coth(x)/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(coth(x)/(a + b*tanh(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\coth(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \int \frac{\sqrt{\tanh(x)^2 b + a}\,\coth(x)}{\tanh(x)^6 b^3 + 3\tanh(x)^4 a\,b^2 + 3\tanh(x)^2 a^2 b + a^3}dx$$

input
```
int(coth(x)/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*coth(x))/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*b**
2 + 3*tanh(x)**2*a**2*b + a**3),x)
```

3.254 $\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$

Optimal result

Integrand size = 17, antiderivative size = 131

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{(a+b)^{5/2}} + \frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}} + \frac{b(7a+4b)\coth(x)}{3a^2(a+b)^2\sqrt{a+b\tanh^2(x)}} - \frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{3a^3(a+b)^2}$$

output
```
arctanh((a+b)^(1/2)*tanh(x)/(a+b*tanh(x)^2)^(1/2))/(a+b)^(5/2)+1/3*b*coth(
x)/a/(a+b)/(a+b*tanh(x)^2)^(3/2)+1/3*b*(7*a+4*b)*coth(x)/a^2/(a+b)^2/(a+b*
tanh(x)^2)^(1/2)-1/3*(3*a+2*b)*(a+4*b)*coth(x)*(a+b*tanh(x)^2)^(1/2)/a^3/(
a+b)^2
```

Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 6.69 (sec) , antiderivative size = 246, normalized size of antiderivative = 1.88

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \frac{\sqrt{(a-b+(a+b)\cosh(2x))\text{sech}^2(x)}\left(\frac{3\sqrt{2}a^3\coth(x)\left((a+b)\,\text{EllipticF}\left(\arcsin\left(\frac{\sqrt{\frac{(a-b+(a}{}}}{}\right.\right.\right.}{}}{}\right.}{}$$

input
```
Integrate[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(Sqrt[(a - b + (a + b)*Cosh[2*x])*Sech[x]^2]*((3*Sqrt[2]*a^3*Coth[x]*((a +
 b)*EllipticF[ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]^2)/b]/Sqrt[
2]], 1] - a*EllipticPi[b/(a + b), ArcSin[Sqrt[((a - b + (a + b)*Cosh[2*x])
*Csch[x]^2)/b]/Sqrt[2]], 1]))/(b*Sqrt[((a - b + (a + b)*Cosh[2*x])*Csch[x]
^2)/b]) - ((a + b)*(3*(a + b)^2*(a - b + (a + b)*Cosh[2*x])^2*Coth[x] + 2*
a*b^3*Sinh[2*x] + b^2*(9*a + 5*b)*(a - b + (a + b)*Cosh[2*x])*Sinh[2*x]))/
(a - b + (a + b)*Cosh[2*x])^2))/(3*Sqrt[2]*a^3*(a + b)^3)
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 148, normalized size of antiderivative = 1.13, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.706, Rules used = {3042, 25, 4153, 25, 374, 25, 441, 25, 445, 27, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{1}{\tan(ix)^2\left(a-b\tan(ix)^2\right)^{5/2}}dx$$
$$\Big\downarrow \ 25$$
$$-\int \frac{1}{\tan(ix)^2\left(a-b\tan(ix)^2\right)^{5/2}}dx$$
$$\Big\downarrow \ 4153$$
$$-\int -\frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{5/2}}d\tanh(x)$$
$$\Big\downarrow \ 25$$
$$\int \frac{\coth^2(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^2(x)\right)^{5/2}}d\tanh(x)$$
$$\Big\downarrow \ 374$$
$$\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}-\frac{\int -\frac{\coth^2(x)\left(-4b\tanh^2(x)+3a+4b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{3a(a+b)}$$
$$\Big\downarrow \ 25$$
$$\frac{\int \frac{\coth^2(x)\left(-4b\tanh^2(x)+3a+4b\right)}{\left(1-\tanh^2(x)\right)\left(b\tanh^2(x)+a\right)^{3/2}}d\tanh(x)}{3a(a+b)}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$
$$\Big\downarrow \ 441$$
$$\frac{\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}-\frac{\int -\frac{\coth^2(x)\left((3a+2b)(a+4b)-2b(7a+4b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)}}{3a(a+b)}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$
$$\Big\downarrow \ 25$$
$$\frac{\frac{\int \frac{\coth^2(x)\left((3a+2b)(a+4b)-2b(7a+4b)\tanh^2(x)\right)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{445}$$

$$\frac{\frac{-\frac{\int -\frac{3a^3}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)}{a}-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{3a^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^2(x)+a}}d\tanh(x)-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{291}$$

$$\frac{\frac{3a^2\int \frac{1}{1-\frac{(a+b)\tanh^2(x)}{b\tanh^2(x)+a}}d\frac{\tanh(x)}{\sqrt{b\tanh^2(x)+a}}-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\frac{\frac{3a^2\text{arctanh}\left(\frac{\sqrt{a+b}\tanh(x)}{\sqrt{a+b\tanh^2(x)}}\right)}{\sqrt{a+b}}-\frac{(3a+2b)(a+4b)\coth(x)\sqrt{a+b\tanh^2(x)}}{a}}{a(a+b)}+\frac{b(7a+4b)\coth(x)}{a(a+b)\sqrt{a+b\tanh^2(x)}}}{3a(a+b)}+\frac{b\coth(x)}{3a(a+b)\left(a+b\tanh^2(x)\right)^{3/2}}$$

input
```
Int[Coth[x]^2/(a + b*Tanh[x]^2)^(5/2),x]
```

output
```
(b*Coth[x])/(3*a*(a + b)*(a + b*Tanh[x]^2)^(3/2)) + ((b*(7*a + 4*b)*Coth[x
])/(a*(a + b)*Sqrt[a + b*Tanh[x]^2]) + ((3*a^2*ArcTanh[(Sqrt[a + b]*Tanh[x
])/Sqrt[a + b*Tanh[x]^2]])/Sqrt[a + b] - ((3*a + 2*b)*(a + 4*b)*Coth[x]*Sq
rt[a + b*Tanh[x]^2])/a)/(a*(a + b)))/(3*a*(a + b))
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 374
```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
), x_Symbol] :> Simp[(-b)*(e*x)^(m + 1)*(a + b*x^2)^(p + 1)*((c + d*x^2)^(q
 + 1)/(a*e*2*(b*c - a*d)*(p + 1))), x] + Simp[1/(a*2*(b*c - a*d)*(p + 1))
 Int[(e*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2)^q*Simp[b*c*(m + 1) + 2*(b*c -
a*d)*(p + 1) + d*b*(m + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b,
c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IntBinomialQ[a, b,
 c, d, e, m, 2, p, q, x]
```

rule 441
```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2)^(q_
)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[(-(b*e - a*f))*(g*x)^(m + 1)*(a
+ b*x^2)^(p + 1)*((c + d*x^2)^(q + 1)/(a*g*2*(b*c - a*d)*(p + 1))), x] + Si
mp[1/(a*2*(b*c - a*d)*(p + 1))   Int[(g*x)^m*(a + b*x^2)^(p + 1)*(c + d*x^2
)^q*Simp[c*(b*e - a*f)*(m + 1) + e*2*(b*c - a*d)*(p + 1) + d*(b*e - a*f)*(m
 + 2*(p + q + 2) + 1)*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m, q},
 x] && LtQ[p, -1]
```

rule 445
```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_.)*((c_) + (d_.)*(x_)^2)^(q_
.)*((e_) + (f_.)*(x_)^2), x_Symbol] :> Simp[e*(g*x)^(m + 1)*(a + b*x^2)^(p
+ 1)*((c + d*x^2)^(q + 1)/(a*c*g*(m + 1))), x] + Simp[1/(a*c*g^2*(m + 1))
 Int[(g*x)^(m + 2)*(a + b*x^2)^p*(c + d*x^2)^q*Simp[a*f*c*(m + 1) - e*(b*c
+ a*d)*(m + 2 + 1) - e*2*(b*c*p + a*d*q) - b*e*d*(m + 2*(p + q + 2) + 1)*x^
2, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && LtQ[m, -1]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [F]

$$\int \frac{\coth\left(x\right)^{2}}{\left(a+b\tanh\left(x\right)^{2}\right)^{\frac{5}{2}}}dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5021 vs. $2(113) = 226$.

Time = 1.30 (sec) , antiderivative size = 10671, normalized size of antiderivative = 81.46

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth^2{(x)}}{\left(a+b\tanh^2{(x)}\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)**2/(a+b*tanh(x)**2)**(5/2),x)
```

output
```
Integral(coth(x)**2/(a + b*tanh(x)**2)**(5/2), x)
```

Maxima [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth{(x)}^2}{\left(b\tanh{(x)}^2+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="maxima")
```

output
```
integrate(coth(x)^2/(b*tanh(x)^2 + a)^(5/2), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 898 vs. $2(113) = 226$.

Time = 0.53 (sec) , antiderivative size = 898, normalized size of antiderivative = 6.85

$$\int \frac{\coth^2(x)}{(a + b\tanh^2(x))^{5/2}}\, dx = \text{Too large to display}$$

input
```
integrate(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x, algorithm="giac")
```

output
```
-1/3*((((9*a^13*b^4 + 50*a^12*b^5 + 115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b
^8 + 34*a^8*b^9 + 5*a^7*b^10)*e^(2*x)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4
 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8) + 3*(3*a^13*b^4 + 6*
a^12*b^5 - 11*a^11*b^6 - 44*a^10*b^7 - 51*a^9*b^8 - 26*a^8*b^9 - 5*a^7*b^1
0)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^
11*b^7 + a^10*b^8))*e^(2*x) - 3*(3*a^13*b^4 + 6*a^12*b^5 - 11*a^11*b^6 - 4
4*a^10*b^7 - 51*a^9*b^8 - 26*a^8*b^9 - 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3
+ 15*a^14*b^4 + 20*a^13*b^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))*e^(2*x
) - (9*a^13*b^4 + 50*a^12*b^5 + 115*a^11*b^6 + 140*a^10*b^7 + 95*a^9*b^8 +
 34*a^8*b^9 + 5*a^7*b^10)/(a^16*b^2 + 6*a^15*b^3 + 15*a^14*b^4 + 20*a^13*b
^5 + 15*a^12*b^6 + 6*a^11*b^7 + a^10*b^8))/(a*e^(4*x) + b*e^(4*x) + 2*a*e^
(2*x) - 2*b*e^(2*x) + a + b)^(3/2) - 1/2*log(abs(-(sqrt(a + b)*e^(2*x) - s
qrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a + b))*(a + b) -
sqrt(a + b)*(a - b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 1/2*log(abs(-sqr
t(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x)
+ a + b) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 1/2*log(abs(-
sqrt(a + b)*e^(2*x) + sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*
x) + a + b) - sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) + 4*(sqrt(a
+ b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x) + 2*a*e^(2*x) - 2*b*e^(2*x) + a
+ b) + sqrt(a + b))/(((sqrt(a + b)*e^(2*x) - sqrt(a*e^(4*x) + b*e^(4*x)...
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\coth(x)^2}{\left(b\tanh\left(x\right)^2+a\right)^{5/2}}\,dx$$

input
```
int(coth(x)^2/(a + b*tanh(x)^2)^(5/2),x)
```

output
```
int(coth(x)^2/(a + b*tanh(x)^2)^(5/2), x)
```

Reduce [F]

$$\int \frac{\coth^2(x)}{\left(a+b\tanh^2(x)\right)^{5/2}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^2b+a}\,\coth\left(x\right)^2}{\tanh\left(x\right)^6b^3+3\tanh\left(x\right)^4a\,b^2+3\tanh\left(x\right)^2a^2b+a^3}dx$$

input
```
int(coth(x)^2/(a+b*tanh(x)^2)^(5/2),x)
```

output
```
int((sqrt(tanh(x)**2*b + a)*coth(x)**2)/(tanh(x)**6*b**3 + 3*tanh(x)**4*a*
b**2 + 3*tanh(x)**2*a**2*b + a**3),x)
```

3.255 $\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 10, antiderivative size = 25

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\operatorname{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{1+\tanh^2(x)}}\right)}{\sqrt{2}}$$

output
```
1/2*2^(1/2)*arctanh(2^(1/2)*tanh(x)/(1+tanh(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.40

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{1+\tanh^2(x)}}$$

input
```
Integrate[1/Sqrt[1 + Tanh[x]^2],x]
```

output
```
(ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[1 + Tanh[
x]^2])
```

Rubi [A] (verified)

Time = 0.21 (sec) , antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3042, 4144, 291, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{\tanh^2(x)+1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{1-\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{1}{(1-\tanh^2(x))\sqrt{\tanh^2(x)+1}}d\tanh(x)$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{1-\frac{2\tanh^2(x)}{\tanh^2(x)+1}}d\frac{\tanh(x)}{\sqrt{\tanh^2(x)+1}}$$

$$\Big\downarrow \text{219}$$

$$\frac{\text{arctanh}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{\tanh^2(x)+1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[1 + Tanh[x]^2],x]
```

output
```
ArcTanh[(Sqrt[2]*Tanh[x])/Sqrt[1 + Tanh[x]^2]]/Sqrt[2]
```

Defintions of rubi rules used

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 61 vs. $2(20) = 40$.

Time = 0.10 (sec) , antiderivative size = 62, normalized size of antiderivative = 2.48

method	result	size
derivativedivides	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{4}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)+1)^2-2\tanh(x)}}\right)}{4}$	62
default	$\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2+2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)-1)^2+2\tanh(x)}}\right)}{4}-\frac{\sqrt{2}\,\operatorname{arctanh}\left(\frac{(2-2\tanh(x))\sqrt{2}}{4\sqrt{(\tanh(x)+1)^2-2\tanh(x)}}\right)}{4}$	62

input
```
int(1/(1+tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/4*2^(1/2)*arctanh(1/4*(2+2*tanh(x))*2^(1/2)/((tanh(x)-1)^2+2*tanh(x))^(1
/2))-1/4*2^(1/2)*arctanh(1/4*(2-2*tanh(x))*2^(1/2)/((tanh(x)+1)^2-2*tanh(x
))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 543 vs. $2(20) = 40$.

Time = 0.12 (sec) , antiderivative size = 543, normalized size of antiderivative = 21.72

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/8*sqrt(2)*log(-2*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh
(x)^2 - 3)*sinh(x)^6 - 3*cosh(x)^6 + 2*(28*cosh(x)^3 - 9*cosh(x))*sinh(x)^
5 + 5*(14*cosh(x)^4 - 9*cosh(x)^2 + 1)*sinh(x)^4 + 5*cosh(x)^4 + 4*(14*cos
h(x)^5 - 15*cosh(x)^3 + 5*cosh(x))*sinh(x)^3 + (28*cosh(x)^6 - 45*cosh(x)^
4 + 30*cosh(x)^2 - 4)*sinh(x)^2 - 4*cosh(x)^2 + 2*(4*cosh(x)^7 - 9*cosh(x)
^5 + 10*cosh(x)^3 - 4*cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^6 + 6*sqrt(2)*co
sh(x)*sinh(x)^5 + sqrt(2)*sinh(x)^6 + 3*(5*sqrt(2)*cosh(x)^2 - sqrt(2))*si
nh(x)^4 - 3*sqrt(2)*cosh(x)^4 + 4*(5*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x)
)*sinh(x)^3 + (15*sqrt(2)*cosh(x)^4 - 18*sqrt(2)*cosh(x)^2 + 4*sqrt(2))*si
nh(x)^2 + 4*sqrt(2)*cosh(x)^2 + 2*(3*sqrt(2)*cosh(x)^5 - 6*sqrt(2)*cosh(x)
^3 + 4*sqrt(2)*cosh(x))*sinh(x) - 4*sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2)/
(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 4)/(cosh(x)^6 + 6*cosh(x)^5
*sinh(x) + 15*cosh(x)^4*sinh(x)^2 + 20*cosh(x)^3*sinh(x)^3 + 15*cosh(x)^2*
sinh(x)^4 + 6*cosh(x)*sinh(x)^5 + sinh(x)^6)) + 1/8*sqrt(2)*log(2*(cosh(x)
^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + (6*cosh(x)^2 + 1)*sinh(x)^2 + cosh(
x)^2 + 2*(2*cosh(x)^3 + cosh(x))*sinh(x) + (sqrt(2)*cosh(x)^2 + 2*sqrt(2)*
cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + sqrt(2))*sqrt((cosh(x)^2 + sinh(x)^2
)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)) + 1)/(cosh(x)^2 + 2*cosh(x)
*sinh(x) + sinh(x)^2))
```

Sympy [F]

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh^2(x)+1}}\,dx$$

input
```
integrate(1/(1+tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(tanh(x)**2 + 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{\tanh(x)^2+1}}\,dx$$

input
```
integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(tanh(x)^2 + 1), x)
```

Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 58 vs. $2(20) = 40$.

Time = 0.11 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.32

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = -\frac{1}{4}\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

input
```
integrate(1/(1+tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
-1/4*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1)
 - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 63, normalized size of antiderivative = 2.52

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \frac{\sqrt{2}\left(\ln\left(\tanh(x)+1\right)-\ln\left(\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}-\tanh(x)+1\right)\right)}{4} + \frac{\sqrt{2}\left(\ln\left(\tanh(x)+\sqrt{2}\sqrt{\tanh\left(x\right)^2+1}+1\right)-\ln\left(\tanh(x)-1\right)\right)}{4}$$

input
```
int(1/(tanh(x)^2 + 1)^(1/2),x)
```

output
```
(2^(1/2)*(log(tanh(x) + 1) - log(2^(1/2)*(tanh(x)^2 + 1)^(1/2) - tanh(x) +
 1)))/4 + (2^(1/2)*(log(tanh(x) + 2^(1/2)*(tanh(x)^2 + 1)^(1/2) + 1) - log
(tanh(x) - 1)))/4
```

Reduce [F]

$$\int \frac{1}{\sqrt{1+\tanh^2(x)}}\,dx = \int \frac{\sqrt{\tanh\left(x\right)^2+1}}{\tanh\left(x\right)^2+1}dx$$

input
```
int(1/(1+tanh(x)^2)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**2 + 1)/(tanh(x)**2 + 1),x)
```

3.256 $\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$

Optimal result

Integrand size = 12, antiderivative size = 27

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx=\frac{\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-1-\tanh^2(x)}}\right)}{\sqrt{2}}$$

output
```
1/2*2^(1/2)*arctan(2^(1/2)*tanh(x)/(-1-tanh(x)^2)^(1/2))
```

Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.37

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx=\frac{\operatorname{arcsinh}\left(\sqrt{2}\sinh(x)\right)\sqrt{\cosh(2x)}\operatorname{sech}(x)}{\sqrt{2}\sqrt{-1-\tanh^2(x)}}$$

input
```
Integrate[1/Sqrt[-1 - Tanh[x]^2],x]
```

output
```
(ArcSinh[Sqrt[2]*Sinh[x]]*Sqrt[Cosh[2*x]]*Sech[x])/(Sqrt[2]*Sqrt[-1 - Tanh
[x]^2])
```

Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3042, 4144, 291, 216}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\sqrt{-\tanh^2(x)-1}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{\sqrt{-1+\tan(ix)^2}}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{1}{\sqrt{-\tanh^2(x)-1}\,(1-\tanh^2(x))}d\tanh(x)$$

$$\Big\downarrow \text{291}$$

$$\int \frac{1}{\frac{2\tanh^2(x)}{-\tanh^2(x)-1}+1}d\frac{\tanh(x)}{\sqrt{-\tanh^2(x)-1}}$$

$$\Big\downarrow \text{216}$$

$$\frac{\arctan\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh^2(x)-1}}\right)}{\sqrt{2}}$$

input
```
Int[1/Sqrt[-1 - Tanh[x]^2],x]
```

output
```
ArcTan[(Sqrt[2]*Tanh[x])/Sqrt[-1 - Tanh[x]^2]]/Sqrt[2]
```

Defintions of rubi rules used

rule 216
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

rule 291
```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)), x_Symbol] :> Subst
[Int[1/(c - (b*c - a*d)*x^2), x], x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b, c,
d}, x] && NeQ[b*c - a*d, 0]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 65 vs. $2(22) = 44$.

Time = 0.09 (sec) , antiderivative size = 66, normalized size of antiderivative = 2.44

method	result	size
derivativedivides	$-\frac{\sqrt{2}\,\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^{2}-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\,\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)+1)^{2}+2\tanh(x)}}\right)}{4}$	66
default	$-\frac{\sqrt{2}\,\arctan\left(\frac{(-2-2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)-1)^{2}-2\tanh(x)}}\right)}{4}+\frac{\sqrt{2}\,\arctan\left(\frac{(-2+2\tanh(x))\sqrt{2}}{4\sqrt{-(\tanh(x)+1)^{2}+2\tanh(x)}}\right)}{4}$	66

input
```
int(1/(-1-tanh(x)^2)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/4*2^(1/2)*arctan(1/4*(-2-2*tanh(x))*2^(1/2)/(-(tanh(x)-1)^2-2*tanh(x))^
(1/2))+1/4*2^(1/2)*arctan(1/4*(-2+2*tanh(x))*2^(1/2)/(-(tanh(x)+1)^2+2*tan
h(x))^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. $2(22) = 44$.

Time = 0.10 (sec) , antiderivative size = 170, normalized size of antiderivative = 6.30

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \frac{1}{4}\sqrt{-\frac{1}{2}}\log\left(\left(\sqrt{-\frac{1}{2}}\sqrt{-2\,e^{(4\,x)}-2}+e^{(2\,x)}+1\right)e^{(-2\,x)}\right) - \frac{1}{4}\sqrt{-\frac{1}{2}}\log\left(-\left(\sqrt{-\frac{1}{2}}\sqrt{-2\,e^{(4\,x)}-2}-e^{(2\,x)}-1\right)e^{(-2\,x)}\right) - \frac{1}{4}\sqrt{-\frac{1}{2}}\log\left(\left(\sqrt{-2\,e^{(4\,x)}-2}\left(e^{(2\,x)}-2\right)+2\sqrt{-\frac{1}{2}}e^{(4\,x)}-2\sqrt{-\frac{1}{2}}e^{(2\,x)}+4\sqrt{-\frac{1}{2}}\right)e^{(-4\,x)}\right) + \frac{1}{4}\sqrt{-\frac{1}{2}}\log\left(\left(\sqrt{-2\,e^{(4\,x)}-2}\left(e^{(2\,x)}-2\right)-2\sqrt{-\frac{1}{2}}e^{(4\,x)}+2\sqrt{-\frac{1}{2}}e^{(2\,x)}-4\sqrt{-\frac{1}{2}}\right)e^{(-4\,x)}\right)$$

input
```
integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="fricas")
```

output
```
1/4*sqrt(-1/2)*log((sqrt(-1/2)*sqrt(-2*e^(4*x) - 2) + e^(2*x) + 1)*e^(-2*x
)) - 1/4*sqrt(-1/2)*log(-(sqrt(-1/2)*sqrt(-2*e^(4*x) - 2) - e^(2*x) - 1)*e
^(-2*x)) - 1/4*sqrt(-1/2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) + 2*sqrt
(-1/2)*e^(4*x) - 2*sqrt(-1/2)*e^(2*x) + 4*sqrt(-1/2))*e^(-4*x)) + 1/4*sqrt
(-1/2)*log((sqrt(-2*e^(4*x) - 2)*(e^(2*x) - 2) - 2*sqrt(-1/2)*e^(4*x) + 2*
sqrt(-1/2)*e^(2*x) - 4*sqrt(-1/2))*e^(-4*x))
```

Sympy [F]

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh^2(x)-1}}\,dx$$

input
```
integrate(1/(-1-tanh(x)**2)**(1/2),x)
```

output
```
Integral(1/sqrt(-tanh(x)**2 - 1), x)
```

Maxima [F]

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx = \int \frac{1}{\sqrt{-\tanh(x)^2-1}}\,dx$$

input
```
integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="maxima")
```

output
```
integrate(1/sqrt(-tanh(x)^2 - 1), x)
```

Giac [C] (verification not implemented)

Result contains complex when optimal does not.

Time = 0.12 (sec) , antiderivative size = 58, normalized size of antiderivative = 2.15

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx$$
$$= \frac{1}{4} i\sqrt{2}\left(\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}+1\right)+\log\left(\sqrt{e^{(4x)}+1}-e^{(2x)}\right)-\log\left(-\sqrt{e^{(4x)}+1}+e^{(2x)}+1\right)\right)$$

input
```
integrate(1/(-1-tanh(x)^2)^(1/2),x, algorithm="giac")
```

output
```
1/4*I*sqrt(2)*(log(sqrt(e^(4*x) + 1) - e^(2*x) + 1) + log(sqrt(e^(4*x) + 1
) - e^(2*x)) - log(-sqrt(e^(4*x) + 1) + e^(2*x) + 1))
```

Mupad [B] (verification not implemented)

Time = 2.56 (sec) , antiderivative size = 22, normalized size of antiderivative = 0.81

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx=\frac{\sqrt{2}\,\mathrm{atan}\left(\frac{\sqrt{2}\tanh(x)}{\sqrt{-\tanh(x)^2-1}}\right)}{2}$$

input
```
int(1/(- tanh(x)^2 - 1)^(1/2),x)
```

output
```
(2^(1/2)*atan((2^(1/2)*tanh(x))/(- tanh(x)^2 - 1)^(1/2)))/2
```

Reduce [F]

$$\int \frac{1}{\sqrt{-1-\tanh^2(x)}}\,dx=-\left(\int \frac{\sqrt{\tanh(x)^2+1}}{\tanh(x)^2+1}dx\right)i$$

input
```
int(1/(-1-tanh(x)^2)^(1/2),x)
```

output
```
 - int(sqrt(tanh(x)**2 + 1)/(tanh(x)**2 + 1),x)*i
```

3.257 $\int \left(a+b\tanh^3(c+dx)\right)^2 \, dx$

Optimal result

Integrand size = 14, antiderivative size = 89

$$\int \left(a+b\tanh^3(c+dx)\right)^2 \, dx = \left(a^2+b^2\right)x + \frac{2ab\log(\cosh(c+dx))}{d} - \frac{b^2\tanh(c+dx)}{d} - \frac{ab\tanh^2(c+dx)}{d} - \frac{b^2\tanh^3(c+dx)}{3d} - \frac{b^2\tanh^5(c+dx)}{5d}$$

output
```
(a^2+b^2)*x+2*a*b*ln(cosh(d*x+c))/d-b^2*tanh(d*x+c)/d-a*b*tanh(d*x+c)^2/d-
1/3*b^2*tanh(d*x+c)^3/d-1/5*b^2*tanh(d*x+c)^5/d
```

Mathematica [A] (verified)

Time = 0.49 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.07

$$\int \left(a+b\tanh^3(c+dx)\right)^2 \, dx = -\frac{15\left((a+b)^2\log(1-\tanh(c+dx))-(a-b)^2\log(1+\tanh(c+dx))\right)+30b^2\tanh(c+dx)+30ab\tanh}{30d}$$

input
```
Integrate[(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
-1/30*(15*((a + b)^2*Log[1 - Tanh[c + d*x]] - (a - b)^2*Log[1 + Tanh[c + d
*x]]) + 30*b^2*Tanh[c + d*x] + 30*a*b*Tanh[c + d*x]^2 + 10*b^2*Tanh[c + d*
x]^3 + 6*b^2*Tanh[c + d*x]^5)/d
```

Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3042, 4144, 2341, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int \left(a+ib\tan(ic+idx)^3\right)^2dx$$

$$\Big\downarrow \text{4144}$$

$$\frac{\int \frac{\left(b\tanh^3(c+dx)+a\right)^2}{1-\tanh^2(c+dx)}d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2341}$$

$$\frac{\int \left(-b^2\tanh^4(c+dx)-b^2\tanh^2(c+dx)-2ab\tanh(c+dx)-b^2+\frac{a^2+2b\tanh(c+dx)a+b^2}{1-\tanh^2(c+dx)}\right)d\tanh(c+dx)}{d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\left(a^2+b^2\right)\text{arctanh}(\tanh(c+dx))-ab\tanh^2(c+dx)-ab\log\left(1-\tanh^2(c+dx)\right)-\frac{1}{5}b^2\tanh^5(c+dx)-\frac{1}{3}b^2\tan}{d}$$

input
```
Int[(a + b*Tanh[c + d*x]^3)^2,x]
```

output
```
((a^2 + b^2)*ArcTanh[Tanh[c + d*x]] - a*b*Log[1 - Tanh[c + d*x]^2] - b^2*T
anh[c + d*x] - a*b*Tanh[c + d*x]^2 - (b^2*Tanh[c + d*x]^3)/3 - (b^2*Tanh[c
 + d*x]^5)/5)/d
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2341
```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

method	result
parallelrisch	$-\frac{3b^2\tanh(dx+c)^5+5b^2\tanh(dx+c)^3-15a^2dx+30abdx-15b^2dx+15\tanh(dx+c)^2ab+30\ln(1-\tanh(dx+c))ab+15b^2}{15d}$
derivativedivides	$\frac{-\frac{b^2\tanh(dx+c)^5}{5}-\frac{b^2\tanh(dx+c)^3}{3}-\tanh(dx+c)^2ab-b^2\tanh(dx+c)-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c))}{2}+\frac{(a^2-2ab+b^2)\ln}{2}}{d}$
default	$\frac{-\frac{b^2\tanh(dx+c)^5}{5}-\frac{b^2\tanh(dx+c)^3}{3}-\tanh(dx+c)^2ab-b^2\tanh(dx+c)-\frac{(a^2+2ab+b^2)\ln(-1+\tanh(dx+c))}{2}+\frac{(a^2-2ab+b^2)\ln}{2}}{d}$
parts	$a^2x+\frac{b^2\left(-\frac{\tanh(dx+c)^5}{5}-\frac{\tanh(dx+c)^3}{3}-\tanh(dx+c)-\frac{\ln(-1+\tanh(dx+c))}{2}+\frac{\ln(1+\tanh(dx+c))}{2}\right)}{d}+\frac{2ab\left(-\frac{\tanh(dx+c}{2}\right.}{}$
risch	$a^2x-2abx+b^2x-\frac{4abc}{d}+\frac{2b(30a\,e^{8dx+8c}+45b\,e^{8dx+8c}+90a\,e^{6dx+6c}+90b\,e^{6dx+6c}+90a\,e^{4dx+4c}+140b\,e^{4d}}{15d\left(e^{2dx+2c}+1\right)^5}$

input
```
int((a+b*tanh(d*x+c)^3)^2,x,method=_RETURNVERBOSE)
```

output
```
-1/15*(3*b^2*tanh(d*x+c)^5+5*b^2*tanh(d*x+c)^3-15*a^2*d*x+30*a*b*d*x-15*b^
2*d*x+15*tanh(d*x+c)^2*a*b+30*ln(1-tanh(d*x+c))*a*b+15*b^2*tanh(d*x+c))/d
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2074 vs. 2(85) = 170.

Time = 0.12 (sec) , antiderivative size = 2074, normalized size of antiderivative = 23.30

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="fricas")
```

output
```
1/15*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^10 + 150*(a^2 - 2*a*b + b^2
)*d*x*cosh(d*x + c)*sinh(d*x + c)^9 + 15*(a^2 - 2*a*b + b^2)*d*x*sinh(d*x
+ c)^10 + 15*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^8 +
 15*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^2 + 5*(a^2 - 2*a*b + b^2)*d*
x + 4*a*b + 6*b^2)*sinh(d*x + c)^8 + 120*(15*(a^2 - 2*a*b + b^2)*d*x*cosh(
d*x + c)^3 + (5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c))*si
nh(d*x + c)^7 + 30*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x +
c)^6 + 30*(105*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^4 + 5*(a^2 - 2*a*b +
b^2)*d*x + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^2
+ 6*a*b + 6*b^2)*sinh(d*x + c)^6 + 60*(63*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x
 + c)^5 + 14*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^3 +
 3*(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c))*sinh(d*x + c
)^5 + 10*(15*(a^2 - 2*a*b + b^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c)^4 +
10*(315*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^6 + 105*(5*(a^2 - 2*a*b + b^
2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^4 + 15*(a^2 - 2*a*b + b^2)*d*x + 45*
(5*(a^2 - 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^2 + 18*a*b + 28*
b^2)*sinh(d*x + c)^4 + 40*(45*(a^2 - 2*a*b + b^2)*d*x*cosh(d*x + c)^7 + 21
*(5*(a^2 - 2*a*b + b^2)*d*x + 4*a*b + 6*b^2)*cosh(d*x + c)^5 + 15*(5*(a^2
- 2*a*b + b^2)*d*x + 6*a*b + 6*b^2)*cosh(d*x + c)^3 + (15*(a^2 - 2*a*b + b
^2)*d*x + 18*a*b + 28*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 15*(a^2 - 2...
```

Sympy [A] (verification not implemented)

Time = 0.14 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx = \begin{cases} a^2x+2abx-\frac{2ab\log(\tanh(c+dx)+1)}{d}-\frac{ab\tanh^2(c+dx)}{d}+b^2x-\frac{b^2\tanh^5(c+dx)}{5d}-\frac{b^2\tanh^3(c+dx)}{3d}-\frac{b^2\tanh(c+dx)}{d} & \text{fo} \\ x\left(a+b\tanh^3(c)\right)^2 & \text{o} \end{cases}$$

input
```
integrate((a+b*tanh(d*x+c)**3)**2,x)
```

output
```
Piecewise((a**2*x + 2*a*b*x - 2*a*b*log(tanh(c + d*x) + 1)/d - a*b*tanh(c
+ d*x)**2/d + b**2*x - b**2*tanh(c + d*x)**5/(5*d) - b**2*tanh(c + d*x)**3
/(3*d) - b**2*tanh(c + d*x)/d, Ne(d, 0)), (x*(a + b*tanh(c)**3)**2, True))
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 194 vs. 2(85) = 170.

Time = 0.12 (sec) , antiderivative size = 194, normalized size of antiderivative = 2.18

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx = \frac{1}{15}b^2\left(15x+\frac{15c}{d}-\frac{2\left(70e^{(-2dx-2c)}+140e^{(-4dx-4c)}+90e^{(-6dx-6c)}+45e^{(-8dx-8c)}+23\right)}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}\right) + 2ab\left(x+\frac{c}{d}+\frac{\log\left(e^{(-2dx-2c)}+1\right)}{d}+\frac{2e^{(-2dx-2c)}}{d\left(2e^{(-2dx-2c)}+e^{(-4dx-4c)}+1\right)}\right)+a^2x$$

input
```
integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="maxima")
```

output
```
1/15*b^2*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) +
90*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 1
0*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x
 - 10*c) + 1))) + 2*a*b*(x + c/d + log(e^(-2*d*x - 2*c) + 1)/d + 2*e^(-2*d
*x - 2*c)/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) + a^2*x
```

Giac [A] (verification not implemented)

Time = 0.13 (sec) , antiderivative size = 142, normalized size of antiderivative = 1.60

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx$$
$$=\frac{30\,ab\log\left(e^{(2\,dx+2\,c)}+1\right)+15\,(a^2-2\,ab+b^2)(dx+c)+\frac{2\,(23\,b^2+15\,(2\,ab+3\,b^2)e^{(8\,dx+8\,c)}+90\,(ab+b^2)e^{(6\,dx+6\,c)}+10\,(9}{\left(e^{(2\,dx+2\,c)}+1\right)^5}}{15\,d}$$

input
```
integrate((a+b*tanh(d*x+c)^3)^2,x, algorithm="giac")
```

output
```
1/15*(30*a*b*log(e^(2*d*x + 2*c) + 1) + 15*(a^2 - 2*a*b + b^2)*(d*x + c) +
 2*(23*b^2 + 15*(2*a*b + 3*b^2)*e^(8*d*x + 8*c) + 90*(a*b + b^2)*e^(6*d*x
+ 6*c) + 10*(9*a*b + 14*b^2)*e^(4*d*x + 4*c) + 10*(3*a*b + 7*b^2)*e^(2*d*x
 + 2*c))/(e^(2*d*x + 2*c) + 1)^5)/d
```

Mupad [B] (verification not implemented)

Time = 2.62 (sec) , antiderivative size = 91, normalized size of antiderivative = 1.02

$$\int \left(a+b\tanh^3(c+dx)\right)^2\,dx = x\left(a^2+2\,a\,b+b^2\right)-\frac{b^2\tanh(c+d\,x)}{d}-\frac{b^2\tanh(c+d\,x)^3}{3\,d}-\frac{b^2\tanh(c+d\,x)^5}{5\,d}-\frac{2\,a\,b\,\ln\left(\tanh(c+d\,x)+1\right)}{d}-\frac{a\,b\tanh(c+d\,x)^2}{d}$$

input
```
int((a + b*tanh(c + d*x)^3)^2,x)
```

output
```
x*(2*a*b + a^2 + b^2) - (b^2*tanh(c + d*x))/d - (b^2*tanh(c + d*x)^3)/(3*d
) - (b^2*tanh(c + d*x)^5)/(5*d) - (2*a*b*log(tanh(c + d*x) + 1))/d - (a*b*
tanh(c + d*x)^2)/d
```

Reduce [B] (verification not implemented)

Time = 0.60 (sec) , antiderivative size = 586, normalized size of antiderivative = 6.58

$$\int \left(a + b\tanh^3(c + dx)\right)^2 \, dx$$
$$= \frac{-18e^{10dx+10c}b^2 + 75e^{2dx+2c}a^2dx + 15e^{10dx+10c}a^2dx + 15e^{10dx+10c}b^2dx + 75e^{8dx+8c}a^2dx + 75e^{8dx+8c}b^2dx +}{}$$

input
```
int((a+b*tanh(d*x+c)^3)^2,x)
```

output
```
(30*e**(10*c + 10*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 15*e**(10*c + 10*d*
x)*a**2*d*x - 30*e**(10*c + 10*d*x)*a*b*d*x - 12*e**(10*c + 10*d*x)*a*b +
15*e**(10*c + 10*d*x)*b**2*d*x - 18*e**(10*c + 10*d*x)*b**2 + 150*e**(8*c
+ 8*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 75*e**(8*c + 8*d*x)*a**2*d*x - 15
0*e**(8*c + 8*d*x)*a*b*d*x + 75*e**(8*c + 8*d*x)*b**2*d*x + 300*e**(6*c +
6*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 150*e**(6*c + 6*d*x)*a**2*d*x - 300
*e**(6*c + 6*d*x)*a*b*d*x + 60*e**(6*c + 6*d*x)*a*b + 150*e**(6*c + 6*d*x)
*b**2*d*x + 300*e**(4*c + 4*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 150*e**(4
*c + 4*d*x)*a**2*d*x - 300*e**(4*c + 4*d*x)*a*b*d*x + 60*e**(4*c + 4*d*x)*
a*b + 150*e**(4*c + 4*d*x)*b**2*d*x + 100*e**(4*c + 4*d*x)*b**2 + 150*e**(
2*c + 2*d*x)*log(e**(2*c + 2*d*x) + 1)*a*b + 75*e**(2*c + 2*d*x)*a**2*d*x
- 150*e**(2*c + 2*d*x)*a*b*d*x + 75*e**(2*c + 2*d*x)*b**2*d*x + 50*e**(2*c
 + 2*d*x)*b**2 + 30*log(e**(2*c + 2*d*x) + 1)*a*b + 15*a**2*d*x - 30*a*b*d
*x - 12*a*b + 15*b**2*d*x + 28*b**2)/(15*d*(e**(10*c + 10*d*x) + 5*e**(8*c
 + 8*d*x) + 10*e**(6*c + 6*d*x) + 10*e**(4*c + 4*d*x) + 5*e**(2*c + 2*d*x)
 + 1))
```

3.258 $\int \frac{1}{1+\tanh^3(x)} dx$

Optimal result

Integrand size = 8, antiderivative size = 38

$$\int \frac{1}{1+\tanh^3(x)} dx = \frac{x}{2} - \frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} - \frac{1}{6(1+\tanh(x))}$$

output
```
1/2*x-2/9*arctan(1/3*(1-2*tanh(x))*3^(1/2))*3^(1/2)-1/(6+6*tanh(x))
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05

$$\int \frac{1}{1+\tanh^3(x)} dx = -\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}} + \frac{1}{2}\text{arctanh}(\tanh(x)) - \frac{1}{6(1+\tanh(x))}$$

input
```
Integrate[(1 + Tanh[x]^3)^(-1),x]
```

output
```
(-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/
(6*(1 + Tanh[x]))
```

Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.05, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.500, Rules used = {3042, 4144, 7276, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\tanh^3(x)+1}dx$$

$$\Big\downarrow \text{3042}$$

$$\int \frac{1}{1+i\tan(ix)^3}dx$$

$$\Big\downarrow \text{4144}$$

$$\int \frac{1}{(1-\tanh^2(x))\left(\tanh^3(x)+1\right)}d\tanh(x)$$

$$\Big\downarrow \text{7276}$$

$$\int \left(\frac{1}{3\left(\tanh^2(x)-\tanh(x)+1\right)}-\frac{1}{2\left(\tanh^2(x)-1\right)}+\frac{1}{6(\tanh(x)+1)^2}\right)d\tanh(x)$$

$$\Big\downarrow \text{2009}$$

$$-\frac{2\arctan\left(\frac{1-2\tanh(x)}{\sqrt{3}}\right)}{3\sqrt{3}}+\frac{1}{2}\text{arctanh}(\tanh(x))-\frac{1}{6(\tanh(x)+1)}$$

input
```
Int[(1 + Tanh[x]^3)^(-1),x]
```

output
```
(-2*ArcTan[(1 - 2*Tanh[x])/Sqrt[3]])/(3*Sqrt[3]) + ArcTanh[Tanh[x]]/2 - 1/
(6*(1 + Tanh[x]))
```

Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4144
```
Int[((a_) + (b_.)*((c_.)*tan[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
With[{ff = FreeFactors[Tan[e + f*x], x]}, Simp[c*(ff/f)   Subst[Int[(a + b*
(ff*x)^n)^p/(c^2 + ff^2*x^2), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a,
 b, c, e, f, n, p}, x] && (IntegersQ[n, p] || IGtQ[p, 0] || EqQ[n^2, 4] ||
EqQ[n^2, 16])
```

rule 7276
```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Maple [A] (verified)

Time = 0.09 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.08

method	result	size
derivativedivides	$-\frac{\ln(\tanh(x)-1)}{4}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9}-\frac{1}{6(\tanh(x)+1)}+\frac{\ln(\tanh(x)+1)}{4}$	41
default	$-\frac{\ln(\tanh(x)-1)}{4}+\frac{2\sqrt{3}\,\arctan\left(\frac{(2\tanh(x)-1)\sqrt{3}}{3}\right)}{9}-\frac{1}{6(\tanh(x)+1)}+\frac{\ln(\tanh(x)+1)}{4}$	41
risch	$\frac{x}{2}-\frac{e^{-2x}}{12}+\frac{i\sqrt{3}\,\ln\left(e^{2x}+i\sqrt{3}\right)}{9}-\frac{i\sqrt{3}\,\ln\left(e^{2x}-i\sqrt{3}\right)}{9}$	47

input
```
int(1/(1+tanh(x)^3),x,method=_RETURNVERBOSE)
```

output
```
-1/4*ln(tanh(x)-1)+2/9*3^(1/2)*arctan(1/3*(2*tanh(x)-1)*3^(1/2))-1/6/(tanh
(x)+1)+1/4*ln(tanh(x)+1)
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 95 vs. $2(29) = 58$.

Time = 0.09 (sec) , antiderivative size = 95, normalized size of antiderivative = 2.50

$$\int \frac{1}{1+\tanh^3(x)}\,dx$$
$$= \frac{18\,x\cosh\left(x\right)^2 + 36\,x\cosh\left(x\right)\sinh\left(x\right) + 18\,x\sinh\left(x\right)^2 - 8\left(\sqrt{3}\cosh\left(x\right)^2 + 2\,\sqrt{3}\cosh\left(x\right)\sinh\left(x\right) + \sqrt{3}\,s\right.}{36\left(\cosh\left(x\right)^2 + 2\,\cosh\left(x\right)\sinh\left(x\right) + \sinh\left(x\right)^2\right)}$$

input
```
integrate(1/(1+tanh(x)^3),x, algorithm="fricas")
```

output
```
1/36*(18*x*cosh(x)^2 + 36*x*cosh(x)*sinh(x) + 18*x*sinh(x)^2 - 8*(sqrt(3)*
cosh(x)^2 + 2*sqrt(3)*cosh(x)*sinh(x) + sqrt(3)*sinh(x)^2)*arctan(-1/3*(sq
rt(3)*cosh(x) + sqrt(3)*sinh(x))/(cosh(x) - sinh(x))) - 3)/(cosh(x)^2 + 2*
cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 102 vs. $2(36) = 72$.

Time = 0.24 (sec) , antiderivative size = 102, normalized size of antiderivative = 2.68

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{9x\tanh\left(x\right)}{18\tanh\left(x\right)+18} + \frac{9x}{18\tanh\left(x\right)+18} + \frac{4\sqrt{3}\tanh\left(x\right)\operatorname{atan}\left(\frac{2\sqrt{3}\tanh\left(x\right)}{3} - \frac{\sqrt{3}}{3}\right)}{18\tanh\left(x\right)+18} + \frac{4\sqrt{3}\operatorname{atan}\left(\frac{2\sqrt{3}\tanh\left(x\right)}{3} - \frac{\sqrt{3}}{3}\right)}{18\tanh\left(x\right)+18} - \frac{3}{18\tanh\left(x\right)+18}$$

input
```
integrate(1/(1+tanh(x)**3),x)
```

output
```
9*x*tanh(x)/(18*tanh(x) + 18) + 9*x/(18*tanh(x) + 18) + 4*sqrt(3)*tanh(x)*
atan(2*sqrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) + 4*sqrt(3)*atan(2
*sqrt(3)*tanh(x)/3 - sqrt(3)/3)/(18*tanh(x) + 18) - 3/(18*tanh(x) + 18)
```

Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(29) = 58$.

Time = 0.12 (sec) , antiderivative size = 73, normalized size of antiderivative = 1.92

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}+3^{\frac{1}{4}}\sqrt{2}\right)\right) - \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{6}\cdot 3^{\frac{3}{4}}\sqrt{2}\left(2\sqrt{3}e^{(-x)}-3^{\frac{1}{4}}\sqrt{2}\right)\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

input
```
integrate(1/(1+tanh(x)^3),x, algorithm="maxima")
```

output
```
2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) + 3^(1/4)*sqrt(2)
)) - 2/9*sqrt(3)*arctan(1/6*3^(3/4)*sqrt(2)*(2*sqrt(3)*e^(-x) - 3^(1/4)*sq
rt(2))) + 1/2*x - 1/12*e^(-2*x)
```

Giac [A] (verification not implemented)

Time = 0.11 (sec) , antiderivative size = 25, normalized size of antiderivative = 0.66

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{2}{9}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}e^{(2x)}\right)+\frac{1}{2}x-\frac{1}{12}e^{(-2x)}$$

input
```
integrate(1/(1+tanh(x)^3),x, algorithm="giac")
```

output
```
2/9*sqrt(3)*arctan(1/3*sqrt(3)*e^(2*x)) + 1/2*x - 1/12*e^(-2*x)
```

Mupad [B] (verification not implemented)

Time = 0.12 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.00

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{\frac{x}{2}+\frac{\tanh(x)}{6}+\frac{x\tanh(x)}{2}}{\tanh(x)+1}+\frac{2\sqrt{3}\,\mathrm{atan}\left(\frac{\sqrt{3}\,(2\tanh(x)-1)}{3}\right)}{9}$$

input
```
int(1/(tanh(x)^3 + 1),x)
```

output
```
(x/2 + tanh(x)/6 + (x*tanh(x))/2)/(tanh(x) + 1) + (2*3^(1/2)*atan((3^(1/2)
*(2*tanh(x) - 1))/3))/9
```

Reduce [B] (verification not implemented)

Time = 0.59 (sec) , antiderivative size = 82, normalized size of antiderivative = 2.16

$$\int \frac{1}{1+\tanh^3(x)}\,dx = \frac{8e^{2x}\sqrt{3}\,atan\left(\frac{\left(2e^{x}-\sqrt{2}\,3^{\frac{1}{4}}\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)-8e^{2x}\sqrt{3}\,atan\left(\frac{\left(2e^{x}+\sqrt{2}\,3^{\frac{1}{4}}\right)3^{\frac{3}{4}}}{3\sqrt{2}}\right)+18e^{2x}x-3}{36e^{2x}}$$

input
```
int(1/(1+tanh(x)^3),x)
```

output
```
(8*e**(2*x)*sqrt(3)*atan((2*e**x - sqrt(2)*3**(1/4))/(sqrt(2)*3**(1/4))) -
 8*e**(2*x)*sqrt(3)*atan((2*e**x + sqrt(2)*3**(1/4))/(sqrt(2)*3**(1/4))) +
 18*e**(2*x)*x - 3)/(36*e**(2*x))
```

3.259 $\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 136

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx = -\frac{1}{4}\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}(a+b)^{3/2}\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}(a+b)\sqrt{a+b\tanh^4(x)} - \frac{1}{4}b\tanh^2(x)\sqrt{a+b\tanh^4(x)} - \frac{1}{6}\left(a+b\tanh^4(x)\right)^{3/2}$$

output
```
-1/4*b^(1/2)*(3*a+2*b)*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))+1/
2*(a+b)^(3/2)*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1
/2*(a+b)*(a+b*tanh(x)^4)^(1/2)-1/4*b*tanh(x)^2*(a+b*tanh(x)^4)^(1/2)-1/6*(
a+b*tanh(x)^4)^(3/2)
```

Mathematica [A] (verified)

Time = 2.92 (sec) , antiderivative size = 166, normalized size of antiderivative = 1.22

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}\,dx=\frac{1}{12}\left(-6\sqrt{b}(a+b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right.$$

$$+6(a+b)^{3/2}\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\left(8a+6b+3b\tanh^2(x)+2b\tanh^4(x)\right)--$$

input
```
Integrate[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
(-6*Sqrt[b]*(a + b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]] + 6
*(a + b)^(3/2)*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4
])] - Sqrt[a + b*Tanh[x]^4]*(8*a + 6*b + 3*b*Tanh[x]^2 + 2*b*Tanh[x]^4) -
(3*Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a]]*Sqrt[a + b*Tanh[x]
^4])/Sqrt[1 + (b*Tanh[x]^4)/a])/12
```

Rubi [A] (verified)

Time = 0.68 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.97, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.933, Rules used = {3042, 26, 4153, 26, 1577, 493, 25, 682, 27, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)\left(a+b\tan(ix)^4\right)^{3/2}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ix)\left(b\tan(ix)^4+a\right)^{3/2}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x)\left(b\tanh^4(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)\left(a+b\tanh^4(x)\right)^{3/2}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{\left(b\tanh^4(x)+a\right)^{3/2}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \text{493}$$

$$\frac{1}{2}\left(-\int -\frac{\left(b\tanh^2(x)+a\right)\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\int \frac{\left(b\tanh^2(x)+a\right)\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}\right)$$

$$\Big\downarrow \text{682}$$

$$\frac{1}{2}\left(\frac{\int \frac{b(b(3a+2b)\tanh^2(x)+a(2a+b))}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{2b}-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}-\frac{1}{2}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b\tanh^4(x)}\right.$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{1}{2}\int \frac{b(3a+2b)\tanh^2(x)+a(2a+b)}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\frac{1}{3}\left(a+b\tanh^4(x)\right)^{3/2}-\frac{1}{2}\left(2(a+b)+b\tanh^2(x)\right)\sqrt{a+b}\right.$$

$$\Big\downarrow \text{719}$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-b(3a+2b)\int \frac{1}{\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)\right)-\right.$$

$$\Big\downarrow 224$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-b(3a+2b)\int \frac{1}{1-b\tanh^4(x)}d\frac{\tanh^2(x)}{\sqrt{b\tanh^4(x)+a}}\right.\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(2(a+b)^2\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)-\right.$$

$$\Big\downarrow 488$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2(a+b)^2\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)-\frac{1}{3}\right.$$

$$\Big\downarrow 219$$

$$\frac{1}{2}\left(\frac{1}{2}\left(-2(a+b)^{3/2}\text{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{b}(3a+2b)\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)\right)-\frac{1}{3}(a+\right.$$

input
```
Int[Tanh[x]*(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
((-(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]
) - 2*(a + b)^(3/2)*ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tan
h[x]^4])])/2 - ((2*(a + b) + b*Tanh[x]^2)*Sqrt[a + b*Tanh[x]^4])/2 - (a +
b*Tanh[x]^4)^(3/2)/3)/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 682
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[(d + e*x)^(m + 1)*(c*e*f*(m + 2*p + 2) - g*c*d*(2*p
+ 1) + g*c*e*(m + 2*p + 1)*x)*((a + c*x^2)^p/(c*e^2*(m + 2*p + 1)*(m + 2*p
+ 2))), x] + Simp[2*(p/(c*e^2*(m + 2*p + 1)*(m + 2*p + 2)))   Int[(d + e*x)
^m*(a + c*x^2)^(p - 1)*Simp[f*a*c*e^2*(m + 2*p + 2) + a*c*d*e*g*m - (c^2*f*
d*e*(m + 2*p + 2) - g*(c^2*d^2*(2*p + 1) + a*c*e^2*(m + 2*p + 1)))*x, x], x
], x] /; FreeQ[{a, c, d, e, f, g, m}, x] && GtQ[p, 0] && (IntegerQ[p] ||  !
RationalQ[m] || (GeQ[m, -1] && LtQ[m, 0])) &&  !ILtQ[m + 2*p, 0] && (Intege
rQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.70 (sec) , antiderivative size = 620, normalized size of antiderivative = 4.56

method	result
derivativedivides	$-\frac{b\tanh(x)^4\sqrt{a+b\tanh(x)^4}}{6}-\frac{b\tanh(x)^2\sqrt{a+b\tanh(x)^4}}{4}-\frac{2\sqrt{a+b\tanh(x)^4}\,a}{3}-\frac{b\sqrt{a+b\tanh(x)^4}}{2}-\frac{(\frac{5}{3}ab+b}{}$
default	$-\frac{b\tanh(x)^4\sqrt{a+b\tanh(x)^4}}{6}-\frac{b\tanh(x)^2\sqrt{a+b\tanh(x)^4}}{4}-\frac{2\sqrt{a+b\tanh(x)^4}\,a}{3}-\frac{b\sqrt{a+b\tanh(x)^4}}{2}-\frac{(\frac{5}{3}ab+b}{}$

input

```
int(tanh(x)*(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output

```
-1/6*b*tanh(x)^4*(a+b*tanh(x)^4)^(1/2)-1/4*b*tanh(x)^2*(a+b*tanh(x)^4)^(1/
2)-2/3*(a+b*tanh(x)^4)^(1/2)*a-1/2*b*(a+b*tanh(x)^4)^(1/2)-1/2*(5/3*a*b+b^
2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^
(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF(tanh(x)*(I/
a^(1/2)*b^(1/2))^(1/2),I)-3/4*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/
2))*b^(1/2)*a-1/2*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(x)^4)^(1/2))*b^(3/2)-
1/2*I*(-7/5*a*b-b^2)*a^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2
)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(
1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-EllipticE(tan
h(x)*(I/a^(1/2)*b^(1/2))^(1/2),I))+1/2*a^2/(a+b)^(1/2)*arctanh(1/2*(2*b*ta
nh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+a*b/(a+b)^(1/2)*arctanh(1/
2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/2*b^2/(a+b)^(1/
2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/2*
(-5/3*a*b-b^2)/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(
1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticF
(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I)-1/2*I*(7/5*a*b+b^2)*a^(1/2)/(I/a^(1/
2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/
2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)/b^(1/2)*(EllipticF(tanh(x)*(I/a^
(1/2)*b^(1/2))^(1/2),I)-EllipticE(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),I))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2646 vs. $2(108) = 216$.

Time = 0.41 (sec) , antiderivative size = 11528, normalized size of antiderivative = 84.76

$$\int \tanh(x) \left(a + b\tanh^4(x)\right)^{3/2} \, dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tanh(x) \left(a + b\tanh^4(x)\right)^{3/2} \, dx = \int \left(a + b\tanh^4(x)\right)^{\frac{3}{2}} \tanh(x) \, dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)**4)**(3/2),x)
```

output
```
Integral((a + b*tanh(x)**4)**(3/2)*tanh(x), x)
```

Maxima [F]

$$\int \tanh(x) \left(a + b\tanh^4(x)\right)^{3/2} \, dx = \int \left(b \tanh(x)^4 + a\right)^{\frac{3}{2}} \tanh(x) \, dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

Giac [F]

$$\int \tanh(x) \left(a+b\tanh^4(x)\right)^{3/2} dx = \int \left(b\tanh\left(x\right)^4+a\right)^{\frac{3}{2}} \tanh\left(x\right) dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
integrate((b*tanh(x)^4 + a)^(3/2)*tanh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh(x) \left(a+b\tanh^4(x)\right)^{3/2} dx = \int \tanh(x) \left(b\tanh(x)^4+a\right)^{3/2} dx$$

input
```
int(tanh(x)*(a + b*tanh(x)^4)^(3/2),x)
```

output
```
int(tanh(x)*(a + b*tanh(x)^4)^(3/2), x)
```

Reduce [F]

$$\begin{aligned}\int \tanh(x) \left(a+b\tanh^4(x)\right)^{3/2} dx &= -\frac{\sqrt{\tanh\left(x\right)^4 b+a}\, \tanh\left(x\right)^4 b}{6} \\ &- \frac{\sqrt{\tanh\left(x\right)^4 b+a}\, \tanh\left(x\right)^2 b}{4} - \frac{2\sqrt{\tanh\left(x\right)^4 b+a}\, a}{3} - \frac{\sqrt{\tanh\left(x\right)^4 b+a}\, b}{2} \\ &+ \frac{3\left(\int \frac{\sqrt{\tanh(x)^4 b+a}\, \tanh(x)^3}{\tanh(x)^4 b+a} dx\right) ab}{2} + \left(\int \frac{\sqrt{\tanh\left(x\right)^4 b+a}\, \tanh\left(x\right)^3}{\tanh\left(x\right)^4 b+a} dx\right) b^2 \\ &+ \left(\int \frac{\sqrt{\tanh\left(x\right)^4 b+a}\, \tanh\left(x\right)}{\tanh\left(x\right)^4 b+a} dx\right) a^2 + \frac{\left(\int \frac{\sqrt{\tanh(x)^4 b+a}\, \tanh(x)}{\tanh(x)^4 b+a} dx\right) ab}{2}\end{aligned}$$

input
```
int(tanh(x)*(a+b*tanh(x)^4)^(3/2),x)
```

output
```
( - 2*sqrt(tanh(x)**4*b + a)*tanh(x)**4*b - 3*sqrt(tanh(x)**4*b + a)*tanh(
x)**2*b - 8*sqrt(tanh(x)**4*b + a)*a - 6*sqrt(tanh(x)**4*b + a)*b + 18*int
((sqrt(tanh(x)**4*b + a)*tanh(x)**3)/(tanh(x)**4*b + a),x)*a*b + 12*int((s
qrt(tanh(x)**4*b + a)*tanh(x)**3)/(tanh(x)**4*b + a),x)*b**2 + 12*int((sqr
t(tanh(x)**4*b + a)*tanh(x))/(tanh(x)**4*b + a),x)*a**2 + 6*int((sqrt(tanh
(x)**4*b + a)*tanh(x))/(tanh(x)**4*b + a),x)*a*b)/12
```

3.260 $\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 89

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = -\frac{1}{2}\sqrt{b}\,\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right) + \frac{1}{2}\sqrt{a+b}\,\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right) - \frac{1}{2}\sqrt{a+b\tanh^4(x)}$$

output
```
-1/2*b^(1/2)*arctanh(b^(1/2)*tanh(x)^2/(a+b*tanh(x)^4)^(1/2))+1/2*(a+b)^(1
/2)*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/2*(a+b*ta
nh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.97

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \frac{1}{2}\left(-\sqrt{b}\operatorname{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)+\sqrt{a+b}\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\right)$$

input
```
Integrate[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

output
```
(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) + Sqrt[a +
b]*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[a
 + b*Tanh[x]^4])/2
```

Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.01, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.800, Rules used = {3042, 26, 4153, 26, 1577, 493, 25, 719, 224, 219, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx$$

$$\Big\downarrow \text{3042}$$

$$\int -i\tan(ix)\sqrt{a+b\tan(ix)^4}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \tan(ix)\sqrt{b\tan(ix)^4+a}dx$$

$$\Big\downarrow \ 4153$$

$$-i\int \frac{i\tanh(x)\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 26$$

$$\int \frac{\tanh(x)\sqrt{a+b\tanh^4(x)}}{1-\tanh^2(x)}d\tanh(x)$$

$$\Big\downarrow \ 1577$$

$$\frac{1}{2}\int \frac{\sqrt{b\tanh^4(x)+a}}{1-\tanh^2(x)}d\tanh^2(x)$$

$$\Big\downarrow \ 493$$

$$\frac{1}{2}\left(-\int -\frac{b\tanh^2(x)+a}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tanh^4(x)}\right)$$

$$\Big\downarrow \ 25$$

$$\frac{1}{2}\left(\int \frac{b\tanh^2(x)+a}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tanh^4(x)}\right)$$

$$\Big\downarrow \ 719$$

$$\frac{1}{2}\left(-b\int \frac{1}{\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)+(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tanh^4(x}\right.$$

$$\Big\downarrow \ 224$$

$$\frac{1}{2}\left(-b\int \frac{1}{1-b\tanh^4(x)}d\frac{\tanh^2(x)}{\sqrt{b\tanh^4(x)+a}}+(a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{a+b\tanh\right.$$

$$\Big\downarrow \ 219$$

$$\frac{1}{2}\left((a+b)\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\right.$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(-(a+b)\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\sqrt{b}\text{arctanh}\left(\frac{\sqrt{b}\tanh^2(x)}{\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b}\text{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)-\sqrt{a+b\tanh^4(x)}\right)$$

input
```
Int[Tanh[x]*Sqrt[a + b*Tanh[x]^4],x]
```

output
```
(-(Sqrt[b]*ArcTanh[(Sqrt[b]*Tanh[x]^2)/Sqrt[a + b*Tanh[x]^4]]) - Sqrt[a +
b]*ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])] - Sqrt[
a + b*Tanh[x]^4])/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 224
```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 493
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(c + d*x)^(n + 1)*((a + b*x^2)^p/(d*(n + 2*p + 1))), x] + Simp[2*(p/(d*(n +
 2*p + 1)))   Int[(c + d*x)^n*(a + b*x^2)^(p - 1)*(a*d - b*c*x), x], x] /;
FreeQ[{a, b, c, d, n}, x] && GtQ[p, 0] && NeQ[n + 2*p + 1, 0] && ( !Rationa
lQ[n] || LtQ[n, 1]) &&  !ILtQ[n + 2*p, 0] && IntQuadraticQ[a, 0, b, c, d, n
, p, x]
```

rule 719
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[g/e   Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] +
Simp[(e*f - d*g)/e   Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c,
d, e, f, g, m, p}, x] &&  !IGtQ[m, 0]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.43 (sec) , antiderivative size = 116, normalized size of antiderivative = 1.30

method	result
derivativedivides	$-\frac{\sqrt{a+b\tanh(x)^4}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,\tanh(x)^2+2\sqrt{a+b\tanh(x)^4}\right)}{2}+\frac{a\,\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}+\frac{b\,\operatorname{arctan}}{}$
default	$-\frac{\sqrt{a+b\tanh(x)^4}}{2}-\frac{\sqrt{b}\,\ln\left(2\sqrt{b}\,\tanh(x)^2+2\sqrt{a+b\tanh(x)^4}\right)}{2}+\frac{a\,\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}+\frac{b\,\operatorname{arctan}}{}$

input
```
int(tanh(x)*(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(a+b*tanh(x)^4)^(1/2)-1/2*b^(1/2)*ln(2*b^(1/2)*tanh(x)^2+2*(a+b*tanh(
x)^4)^(1/2))+1/2*a/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)
/(a+b*tanh(x)^4)^(1/2))+1/2*b/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/
(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. 2(69) = 138.

Time = 0.32 (sec) , antiderivative size = 5136, normalized size of antiderivative = 57.71

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{a+b\tanh^4{(x)}}\tanh{(x)}\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)**4)**(1/2),x)
```

output
```
Integral(sqrt(a + b*tanh(x)**4)*tanh(x), x)
```

Maxima [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{b\tanh{(x)}^4+a}\tanh{(x)}\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

Giac [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{b\tanh{(x)}^4+a}\tanh{(x)}\,dx$$

input
```
integrate(tanh(x)*(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(sqrt(b*tanh(x)^4 + a)*tanh(x), x)
```

Mupad [F(-1)]

Timed out.

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \tanh(x)\ \sqrt{b\tanh\left(x\right)^4+a}\,dx$$

input
```
int(tanh(x)*(a + b*tanh(x)^4)^(1/2),x)
```

output
```
int(tanh(x)*(a + b*tanh(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \tanh(x)\sqrt{a+b\tanh^4(x)}\,dx = \int \sqrt{\tanh\left(x\right)^4 b+a}\ \tanh\left(x\right)dx$$

input
```
int(tanh(x)*(a+b*tanh(x)^4)^(1/2),x)
```

output
```
int(sqrt(tanh(x)**4*b + a)*tanh(x),x)
```

3.261 $\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 40

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

output
```
1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(1/2)
```

Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 40, normalized size of antiderivative = 1.00

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx=\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Integrate[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

output
```
ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(2*Sqrt[a +
 b])
```

Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.08, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.467, Rules used = {3042, 26, 4153, 26, 1577, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(ix)}{\sqrt{a+b\tan(ix)^4}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan(ix)}{\sqrt{b\tan(ix)^4+a}}dx$$

$$\Big\downarrow \text{ 4153}$$

$$-i\int \frac{i\tanh(x)}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh(x)$$

$$\Big\downarrow \text{ 26}$$

$$\int \frac{\tanh(x)}{(1-\tanh^2(x))\sqrt{a+b\tanh^4(x)}}d\tanh(x)$$

$$\Big\downarrow \text{ 1577}$$

$$\frac{1}{2}\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)$$

$$\Big\downarrow \text{ 488}$$

$$-\frac{1}{2}\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}$$

$$\Big\downarrow \text{219}$$

$$-\frac{\text{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2\sqrt{a+b}}$$

input
```
Int[Tanh[x]/Sqrt[a + b*Tanh[x]^4],x]
```

output
```
-1/2*ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/Sqrt[
a + b]
```

Defintions of rubi rules used

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [A] (verified)

Time = 0.40 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.92

method	result	size
derivativedivides	$\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}$	37
default	$\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\,\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}$	37

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(1/2),x,method=_RETURNVERBOSE)
```

output
```
1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4
)^(1/2))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 624 vs. 2(32) = 64.

Time = 0.40 (sec) , antiderivative size = 1286, normalized size of antiderivative = 32.15

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="fricas")
```

output
```
[1/4*log(((a^2 + 2*a*b + b^2)*cosh(x)^8 + 8*(a^2 + 2*a*b + b^2)*cosh(x)*si
nh(x)^7 + (a^2 + 2*a*b + b^2)*sinh(x)^8 + 4*(a^2 - b^2)*cosh(x)^6 + 4*(7*(
a^2 + 2*a*b + b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x)^6 + 8*(7*(a^2 + 2*a*b +
b^2)*cosh(x)^3 + 3*(a^2 - b^2)*cosh(x))*sinh(x)^5 + 2*(3*a^2 + 2*a*b + 3*b
^2)*cosh(x)^4 + 2*(35*(a^2 + 2*a*b + b^2)*cosh(x)^4 + 30*(a^2 - b^2)*cosh(
x)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(x)^4 + 8*(7*(a^2 + 2*a*b + b^2)*cosh(x)
^5 + 10*(a^2 - b^2)*cosh(x)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x))*sinh(x)^3
 + 4*(a^2 - b^2)*cosh(x)^2 + 4*(7*(a^2 + 2*a*b + b^2)*cosh(x)^6 + 15*(a^2
- b^2)*cosh(x)^4 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(x)^2 + a^2 - b^2)*sinh(x
)^2 + sqrt(2)*((a + b)*cosh(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*s
inh(x)^4 + 2*(a - b)*cosh(x)^2 + 2*(3*(a + b)*cosh(x)^2 + a - b)*sinh(x)^2
 + 4*((a + b)*cosh(x)^3 + (a - b)*cosh(x))*sinh(x) + a + b)*sqrt(a + b)*sq
rt(((a + b)*cosh(x)^4 + (a + b)*sinh(x)^4 + 4*(a - b)*cosh(x)^2 + 2*(3*(a
+ b)*cosh(x)^2 + 2*a - 2*b)*sinh(x)^2 + 3*a + 3*b)/(cosh(x)^4 - 4*cosh(x)^
3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 - 4*cosh(x)*sinh(x)^3 + sinh(x)^4)) + a^
2 + 2*a*b + b^2 + 8*((a^2 + 2*a*b + b^2)*cosh(x)^7 + 3*(a^2 - b^2)*cosh(x)
^5 + (3*a^2 + 2*a*b + 3*b^2)*cosh(x)^3 + (a^2 - b^2)*cosh(x))*sinh(x))/(co
sh(x)^4 + 4*cosh(x)^3*sinh(x) + 6*cosh(x)^2*sinh(x)^2 + 4*cosh(x)*sinh(x)^
3 + sinh(x)^4))/sqrt(a + b), -1/2*sqrt(-a - b)*arctan(sqrt(2)*((a + b)*cos
h(x)^4 + 4*(a + b)*cosh(x)*sinh(x)^3 + (a + b)*sinh(x)^4 + 2*(a - b)*co...
```

Sympy [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx=\int \frac{\tanh{(x)}}{\sqrt{a+b\tanh^4{(x)}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**4)**(1/2),x)
```

output
```
Integral(tanh(x)/sqrt(a + b*tanh(x)**4), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

Giac [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(1/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/sqrt(b*tanh(x)^4 + a), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx = \int \frac{\tanh(x)}{\sqrt{b\tanh(x)^4+a}}\,dx$$

input
```
int(tanh(x)/(a + b*tanh(x)^4)^(1/2),x)
```

output
```
int(tanh(x)/(a + b*tanh(x)^4)^(1/2), x)
```

Reduce [F]

$$\int \frac{\tanh(x)}{\sqrt{a+b\tanh^4(x)}}\,dx=\int \frac{\sqrt{\tanh(x)^4 b+a}\,\tanh(x)}{\tanh(x)^4 b+a}dx$$

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(1/2),x)
```

output
```
int((sqrt(tanh(x)**4*b + a)*tanh(x))/(tanh(x)**4*b + a),x)
```

3.262 $\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{3/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 74

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{3/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{2a(a+b)\sqrt{a+b\tanh^4(x)}}$$

output
```
1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(3/2)
-1/2*(a-b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.99

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{3/2}}\,dx = \frac{1}{2}\left(\frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}} - \frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
(ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)^(3
/2) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2
```

Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 77, normalized size of antiderivative = 1.04, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3042, 26, 4153, 26, 1577, 496, 25, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}dx$$

$$\Big\downarrow \text{3042}$$

$$\int -\frac{i\tan(ix)}{\left(a+b\tan(ix)^4\right)^{3/2}}dx$$

$$\Big\downarrow \text{26}$$

$$-i\int \frac{\tan(ix)}{\left(b\tan(ix)^4+a\right)^{3/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x)}{\left(1-\tanh^2(x)\right)\left(b\tanh^4(x)+a\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)}{\left(1-\tanh^2(x)\right)\left(a+b\tanh^4(x)\right)^{3/2}}d\tanh(x)$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{1}{\left(1-\tanh^2(x)\right)\left(b\tanh^4(x)+a\right)^{3/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{496}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{a}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a(a+b)}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{a}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a(a+b)}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\int \frac{1}{(1-\tanh^2(x))\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a+b}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(-\frac{\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}}{a+b}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(-\frac{\text{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}}-\frac{a-b\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^4)^(3/2),x]
```

output
```
(-(ArcTanh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])]/(a + b)
^(3/2)) - (a - b*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.44 (sec) , antiderivative size = 431, normalized size of antiderivative = 5.82

method	result
derivativedivides	$\frac{b\left(\frac{\tanh(x)^3}{4a(a+b)}+\frac{\tanh(x)^2}{4a(a+b)}+\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}-\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}-\frac{\sqrt{1-\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\tanh(x)}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}}{2(a+b)}$
default	$\frac{b\left(\frac{\tanh(x)^3}{4a(a+b)}+\frac{\tanh(x)^2}{4a(a+b)}+\frac{\tanh(x)}{4a(a+b)}-\frac{1}{4(a+b)b}\right)}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}-\frac{-\frac{\operatorname{arctanh}\left(\frac{2b\tanh(x)^2+2a}{2\sqrt{a+b}\sqrt{a+b\tanh(x)^4}}\right)}{2\sqrt{a+b}}-\frac{\sqrt{1-\frac{i\sqrt{b}\tanh(x)^2}{\sqrt{a}}}\sqrt{1+\frac{i\sqrt{b}\tanh(x)}{\sqrt{a}}}}{\sqrt{\frac{i\sqrt{b}}{\sqrt{a}}}}}{2(a+b)}$

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(3/2),x,method=_RETURNVERBOSE)
```

output
```
b*(1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2+1/4/a/(a+b)*tanh(x)-1/4/(a+
b)/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2
*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))-1/(I/a^(1/2)*b^(1/2))
^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^
2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2
),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2))
)+b*(-1/4/a/(a+b)*tanh(x)^3+1/4/a/(a+b)*tanh(x)^2-1/4/a/(a+b)*tanh(x)-1/4/
(a+b)/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)*(-1/2/(a+b)^(1/2)*arctanh(1/2
*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/
2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(
x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(
1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/
2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1935 vs. $2(63) = 126$.

Time = 0.37 (sec) , antiderivative size = 3914, normalized size of antiderivative = 52.89

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh{(x)}}{\left(a+b\tanh^4{(x)}\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**4)**(3/2),x)
```

output
```
Integral(tanh(x)/(a + b*tanh(x)**4)**(3/2), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh\left(x\right)}{\left(b\tanh\left(x\right)^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

Giac [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh\left(x\right)}{\left(b\tanh\left(x\right)^4+a\right)^{\frac{3}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(3/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(3/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx = \int \frac{\tanh(x)}{\left(b\tanh\left(x\right)^4+a\right)^{3/2}}\,dx$$

input
```
int(tanh(x)/(a + b*tanh(x)^4)^(3/2),x)
```

output
```
int(tanh(x)/(a + b*tanh(x)^4)^(3/2), x)
```

Reduce [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{3/2}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^{4}b+a}\,\tanh\left(x\right)}{\tanh\left(x\right)^{8}b^{2}+2\tanh\left(x\right)^{4}ab+a^{2}}dx$$

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(3/2),x)
```

output
```
int((sqrt(tanh(x)**4*b + a)*tanh(x))/(tanh(x)**8*b**2 + 2*tanh(x)**4*a*b +
 a**2),x)
```

3.263 $\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{5/2}}\,dx$

Optimal result

Integrand size = 15, antiderivative size = 118

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{5/2}}\,dx = \frac{\operatorname{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{2(a+b)^{5/2}} - \frac{a-b\tanh^2(x)}{6a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}} - \frac{3a^2-b(5a+2b)\tanh^2(x)}{6a^2(a+b)^2\sqrt{a+b\tanh^4(x)}}$$

output
```
1/2*arctanh((a+b*tanh(x)^2)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))/(a+b)^(5/2)
-1/6*(a-b*tanh(x)^2)/a/(a+b)/(a+b*tanh(x)^4)^(3/2)-1/6*(3*a^2-b*(5*a+2*b)*
tanh(x)^2)/a^2/(a+b)^2/(a+b*tanh(x)^4)^(1/2)
```

Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 113, normalized size of antiderivative = 0.96

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx = \frac{1}{6}\left(\frac{3\text{arctanh}\left(\frac{a+b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{5/2}} + \frac{-a^2(4a+b)+3ab(2a+b)\tanh^2(x)-3a^2b\tanh^4(x)+b^2(5a+2b)\tanh^6(x)}{a^2(a+b)^2\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

input
```
Integrate[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

output
```
((3*ArcTanh[(a + b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/(a + b
)^(5/2) + (-(a^2*(4*a + b)) + 3*a*b*(2*a + b)*Tanh[x]^2 - 3*a^2*b*Tanh[x]^
4 + b^2*(5*a + 2*b)*Tanh[x]^6)/(a^2*(a + b)^2*(a + b*Tanh[x]^4)^(3/2)))/6
```

Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 135, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.733$, Rules used = {3042, 26, 4153, 26, 1577, 496, 25, 686, 27, 488, 219}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 3042}$$

$$\int -\frac{i\tan(ix)}{\left(a+b\tan(ix)^4\right)^{5/2}}dx$$

$$\Big\downarrow \text{ 26}$$

$$-i\int \frac{\tan(ix)}{(b\tan(ix)^4+a)^{5/2}}dx$$

$$\Big\downarrow \text{4153}$$

$$-i\int \frac{i\tanh(x)}{(1-\tanh^2(x))\left(b\tanh^4(x)+a\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{26}$$

$$\int \frac{\tanh(x)}{(1-\tanh^2(x))\left(a+b\tanh^4(x)\right)^{5/2}}d\tanh(x)$$

$$\Big\downarrow \text{1577}$$

$$\frac{1}{2}\int \frac{1}{(1-\tanh^2(x))\left(b\tanh^4(x)+a\right)^{5/2}}d\tanh^2(x)$$

$$\Big\downarrow \text{496}$$

$$\frac{1}{2}\left(-\frac{\int -\frac{-2b\tanh^2(x)+3a+2b}{(1-\tanh^2(x))\left(b\tanh^4(x)+a\right)^{3/2}}d\tanh^2(x)}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{25}$$

$$\frac{1}{2}\left(\frac{\int \frac{-2b\tanh^2(x)+3a+2b}{(1-\tanh^2(x))\left(b\tanh^4(x)+a\right)^{3/2}}d\tanh^2(x)}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{686}$$

$$\frac{1}{2}\left(\frac{-\frac{\int -\frac{3a^2b}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{ab(a+b)}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{2}\left(\frac{\frac{3a\int \frac{1}{\left(1-\tanh^2(x)\right)\sqrt{b\tanh^4(x)+a}}d\tanh^2(x)}{a+b}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{488}$$

$$\frac{1}{2}\left(\frac{-\frac{3a\int \frac{1}{-\tanh^4(x)+a+b}d\frac{-b\tanh^2(x)-a}{\sqrt{b\tanh^4(x)+a}}}{a+b}-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

$$\Big\downarrow \text{219}$$

$$\frac{1}{2}\left(\frac{-\frac{3a^2-b(5a+2b)\tanh^2(x)}{a(a+b)\sqrt{a+b\tanh^4(x)}}-\frac{3a\text{arctanh}\left(\frac{-a-b\tanh^2(x)}{\sqrt{a+b}\sqrt{a+b\tanh^4(x)}}\right)}{(a+b)^{3/2}}}{3a(a+b)}-\frac{a-b\tanh^2(x)}{3a(a+b)\left(a+b\tanh^4(x)\right)^{3/2}}\right)$$

input
```
Int[Tanh[x]/(a + b*Tanh[x]^4)^(5/2),x]
```

output
```
(-1/3*(a - b*Tanh[x]^2)/(a*(a + b)*(a + b*Tanh[x]^4)^(3/2)) + ((-3*a*ArcTa
nh[(-a - b*Tanh[x]^2)/(Sqrt[a + b]*Sqrt[a + b*Tanh[x]^4])])/(a + b)^(3/2)
- (3*a^2 - b*(5*a + 2*b)*Tanh[x]^2)/(a*(a + b)*Sqrt[a + b*Tanh[x]^4]))/(3*
a*(a + b)))/2
```

Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 26
```
Int[(Complex[0, a_])*(Fx_), x_Symbol] :> Simp[(Complex[Identity[0], a])   I
nt[Fx, x], x] /; FreeQ[a, x] && EqQ[a^2, 1]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 219
```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

rule 488
```
Int[1/(((c_) + (d_.)*(x_))*Sqrt[(a_) + (b_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(b*c^2 + a*d^2 - x^2), x], x, (a*d - b*c*x)/Sqrt[a + b*x^2]] /; FreeQ
[{a, b, c, d}, x]
```

rule 496
```
Int[((c_) + (d_.)*(x_))^(n_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> Simp[
(-(a*d + b*c*x))*(c + d*x)^(n + 1)*((a + b*x^2)^(p + 1)/(2*a*(p + 1)*(b*c^2
 + a*d^2))), x] + Simp[1/(2*a*(p + 1)*(b*c^2 + a*d^2))   Int[(c + d*x)^n*(a
 + b*x^2)^(p + 1)*Simp[b*c^2*(2*p + 3) + a*d^2*(n + 2*p + 3) + b*c*d*(n + 2
*p + 4)*x, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && LtQ[p, -1] && IntQuad
raticQ[a, 0, b, c, d, n, p, x]
```

rule 686
```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_), x_Symbol] :> Simp[(-(d + e*x)^(m + 1))*(f*a*c*e - a*g*c*d + c*(c*d*f +
a*e*g)*x)*((a + c*x^2)^(p + 1)/(2*a*c*(p + 1)*(c*d^2 + a*e^2))), x] + Simp[
1/(2*a*c*(p + 1)*(c*d^2 + a*e^2))   Int[(d + e*x)^m*(a + c*x^2)^(p + 1)*Sim
p[f*(c^2*d^2*(2*p + 3) + a*c*e^2*(m + 2*p + 3)) - a*c*d*e*g*m + c*e*(c*d*f
+ a*e*g)*(m + 2*p + 4)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g}, x] && LtQ
[p, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*p])
```

rule 1577
```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Simp[1/2   Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; Free
Q[{a, c, d, e, p, q}, x]
```

rule 3042
```
Int[u_, x_Symbol] :> Int[DeactivateTrig[u, x], x] /; FunctionOfTrigOfLinear
Q[u, x]
```

rule 4153
```
Int[((d_.)*tan[(e_.) + (f_.)*(x_)])^(m_.)*((a_) + (b_.)*((c_.)*tan[(e_.) +
(f_.)*(x_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x],
 x]}, Simp[c*(ff/f)   Subst[Int[(d*ff*(x/c))^m*((a + b*(ff*x)^n)^p/(c^2 + f
f^2*x^2)), x], x, c*(Tan[e + f*x]/ff)], x]] /; FreeQ[{a, b, c, d, e, f, m,
n, p}, x] && (IGtQ[p, 0] || EqQ[n, 2] || EqQ[n, 4] || (IntegerQ[p] && Ratio
nalQ[n]))
```

Maple [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3.

Time = 0.43 (sec) , antiderivative size = 637, normalized size of antiderivative = 5.40

method	result
derivativedivides	$-\frac{\left(-\frac{\tanh(x)^3}{6a(a+b)b}-\frac{\tanh(x)^2}{6a(a+b)b}-\frac{\tanh(x)}{6a(a+b)b}+\frac{1}{6(a+b)b^2}\right)\sqrt{a+b\tanh(x)^4}}{2\left(\tanh(x)^4+\frac{a}{b}\right)^2}+\frac{b\left(\frac{(3a+b)\tanh(x)^3}{8a^2(a+b)^2}+\frac{(5a+2b)\tanh(x)^2}{12a^2(a+b)^2}+\frac{(11a+5b)\,\mathrm{t}}{24a^2(a-}\right.}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}$
default	$-\frac{\left(-\frac{\tanh(x)^3}{6a(a+b)b}-\frac{\tanh(x)^2}{6a(a+b)b}-\frac{\tanh(x)}{6a(a+b)b}+\frac{1}{6(a+b)b^2}\right)\sqrt{a+b\tanh(x)^4}}{2\left(\tanh(x)^4+\frac{a}{b}\right)^2}+\frac{b\left(\frac{(3a+b)\tanh(x)^3}{8a^2(a+b)^2}+\frac{(5a+2b)\tanh(x)^2}{12a^2(a+b)^2}+\frac{(11a+5b)\,\mathrm{t}}{24a^2(a-}\right.}{\sqrt{\left(\tanh(x)^4+\frac{a}{b}\right)b}}$

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(5/2),x,method=_RETURNVERBOSE)
```

output
```
-1/2*(-1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/b*tanh(x)^2-1/6/a/(a+b)/b*tanh(
x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(tanh(x)^4+a/b)^2+b*(1/8*(3*a+b)/a
^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a+b)^2*tanh(x)^2+1/24/a^2*(11*a+5
*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(
-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^
4)^(1/2))-1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2
)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a+b*tanh(x)^4)^(1/2)*EllipticPi(t
anh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(
1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))-1/2*(1/6/a/(a+b)/b*tanh(x)^3-1/6/a/(a+b)/
b*tanh(x)^2+1/6/a/(a+b)/b*tanh(x)+1/6/(a+b)/b^2)*(a+b*tanh(x)^4)^(1/2)/(ta
nh(x)^4+a/b)^2+b*(-1/8*(3*a+b)/a^2/(a+b)^2*tanh(x)^3+1/12*(5*a+2*b)/a^2/(a
+b)^2*tanh(x)^2-1/24/a^2*(11*a+5*b)/(a+b)^2*tanh(x)-1/4/(a+b)^2/b)/((tanh(
x)^4+a/b)*b)^(1/2)-1/2/(a+b)^2*(-1/2/(a+b)^(1/2)*arctanh(1/2*(2*b*tanh(x)^
2+2*a)/(a+b)^(1/2)/(a+b*tanh(x)^4)^(1/2))+1/(I/a^(1/2)*b^(1/2))^(1/2)*(1-I
/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)*(1+I/a^(1/2)*b^(1/2)*tanh(x)^2)^(1/2)/(a
+b*tanh(x)^4)^(1/2)*EllipticPi(tanh(x)*(I/a^(1/2)*b^(1/2))^(1/2),-I*a^(1/2
)/b^(1/2),(-I/a^(1/2)*b^(1/2))^(1/2)/(I/a^(1/2)*b^(1/2))^(1/2)))
```

Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8210 vs. $2(102) = 204$.

Time = 1.98 (sec) , antiderivative size = 16463, normalized size of antiderivative = 139.52

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{5/2}}\,dx = \text{Too large to display}$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="fricas")
```

output
```
Too large to include
```

Sympy [F]

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{5/2}}\,dx = \int \frac{\tanh(x)}{(a+b\tanh^4(x))^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)**4)**(5/2),x)
```

output
```
Integral(tanh(x)/(a + b*tanh(x)**4)**(5/2), x)
```

Maxima [F]

$$\int \frac{\tanh(x)}{(a+b\tanh^4(x))^{5/2}}\,dx = \int \frac{\tanh(x)}{(b\tanh(x)^4+a)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="maxima")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Giac [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx=\int \frac{\tanh\left(x\right)}{\left(b\tanh\left(x\right)^4+a\right)^{\frac{5}{2}}}\,dx$$

input
```
integrate(tanh(x)/(a+b*tanh(x)^4)^(5/2),x, algorithm="giac")
```

output
```
integrate(tanh(x)/(b*tanh(x)^4 + a)^(5/2), x)
```

Mupad [F(-1)]

Timed out.

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx=\int \frac{\tanh(x)}{\left(b\tanh\left(x\right)^4+a\right)^{5/2}}\,dx$$

input
```
int(tanh(x)/(a + b*tanh(x)^4)^(5/2),x)
```

output
```
int(tanh(x)/(a + b*tanh(x)^4)^(5/2), x)
```

Reduce [F]

$$\int \frac{\tanh(x)}{\left(a+b\tanh^4(x)\right)^{5/2}}\,dx=\int \frac{\sqrt{\tanh\left(x\right)^4 b+a}\,\tanh\left(x\right)}{\tanh\left(x\right)^{12} b^3+3\tanh\left(x\right)^8 a\,b^2+3\tanh\left(x\right)^4 a^2 b+a^3}dx$$

input
```
int(tanh(x)/(a+b*tanh(x)^4)^(5/2),x)
```

output
```
int((sqrt(tanh(x)**4*b + a)*tanh(x))/(tanh(x)**12*b**3 + 3*tanh(x)**8*a*b*
*2 + 3*tanh(x)**4*a**2*b + a**3),x)
```

CHAPTER 4

Appendix

4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)
```

```
(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leaf
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A",""}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A",""}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal.
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "
            ,
```

```
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
```

```
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issue
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
```

```
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                     return "A"," ";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of
                                   convert(leaf_count_result,string)," vs. $2 (",
                                   convert(leaf_count_optimal,string)," ) = ",convert(2*leaf
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well
            fi;
```

```
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A"," ";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of
                                  convert(leaf_count_result,string),"$ vs. $2(",
                                  convert(leaf_count_optimal,string),")=",convert(2*leaf_co
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
```

```
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
```

```
  end if
end proc:



ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
```

```
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]
```

```
def is_hypergeometric_function(func):
  return func in [hyper]

def is_appell_function(func):
  return func in [appellf1]

def is_atom(expn):
  try:
    if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
      return True
    else:
      return False

  except AttributeError as error:
    return False

def expnType(expn):
  debug=False
  if debug:
    print("expn=",expn,"type(expn)=",type(expn))

  if is_atom(expn):
    return 1
  elif isinstance(expn,list):
    return max(map(expnType, expn))  #apply(max,map(ExpnType,expn))
  elif  is_sqrt(expn):
    if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
      return 1
    else:
      return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
  elif isinstance(expn,Pow):   #type(expn,'`^`')
    if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
      return expnType(expn.args[0])   #ExpnType(op(1,expn))
    elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
      if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
        return 1
      else:
        return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    else:
      return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),Expr
  elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
```

```
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)  #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)  #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
```

```
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is lar
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.
```

```
from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
```

```
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi','zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
           'elliptic_pi','exp_integral_e','log_integral',
           'weierstrassPInverse','weierstrass','weierstrassP','weierstrassZeta',
           'weierstrassPPrime','weierstrassSigma']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
```

```
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:    #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:    #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
```

```
            return expnType(expn.operands()[0])    #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expr
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isins
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
              #is checked before calling the grading function that is passed.
              #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):


    if debug:
        print ("Enter grade_antiderivative for sagemath")
        print("Enter grade_antiderivative, result=",result)
        print("Enter grade_antiderivative, optimal=",optimal)
        print("type(anti)=",type(result))
        print("type(optimal)=",type(optimal))
```

```
leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_re


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```

4.2 Links to plain text integration problems used in this report for each CAS

1. Mathematica integration problems as .m file
2. Maple integration problems as .txt file
3. Sagemath integration problems as .sage file
4. Reduce integration problems as .txt file
5. Mupad integration problems as .txt file
6. Sympy integration problems as .py file